Quality provision in duopoly

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à Emilie,

TABLE OF CONTENTS

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That said, I comply with the established custom by affirming that the persons or institutions mentioned above are not responsible for the statements contained in this booklet, and that only I am to blame for its all-too-real shortcomings and the errors it possibly contains.

Stockholm, May 2006.

Chapter 1

Introduction and summary

I believe that economists did not immediately recognize the importance of quality for market performance. I am reluctant to place that affirmation at the beginning of an academic work, as I fear it is too general not to be readily contested by anyone versed in the writings of the field's founding fathers.[1]

Indeed, it is possible to find in their works indications that they thought it imperative to consider the characteristics of the products brought to the market in assessing economic outcomes. Consider for instance the following passage from Adam Smith's *Wealth of Nations*:

> Grain, the food of the common people, is dearer in Scotland than in England, whence Scotland receives almost every year very large supplies.

[1] Isaiah Berlin, in his famous conference about romanticism, given at the National Gallery in Washington, best captured this feature of scientific life. The first paragraph in Berlin (1999) reads:

> I might be expected to begin, or to attempt to begin, with some kind of definition of romanticism, or at least some generalization, in order to make clear what it is that I mean by it. I do not propose to walk into that particular trap. The eminent and wise Professor Northrop Frye points out that whenever anyone embarks on a generalization on the subject of romanticism, even something so innocuous, for example, as to say that a new attitude sprang up among English poets towards nature - in Wordsworth and Coleridge, let us say, as against Racine and Pope - somebody will always be found who will produce countervailing evidence from the writings of Homer, Kalidasa, pre-Muslim Arabian epics, medieval Spanish verse - and finally, Racine and Pope themselves.

> But English corn must be sold dearer in Scotland, the country to which it is brought, than in England, the country from which it comes; and in proportion to its quality it cannot be sold dearer in Scotland than the Scotch corn that comes to the same market in competition with it. The quality of grain depends chiefly upon the quantity of flour or meal which it yields at the mill, and in this respect English grain is so much superior to the Scotch, that, though often dearer in appearance, or in proportion to the measure of its bulk, it is generally cheaper in reality, or in proportion to its quality, or even to the measure of its weight. (Book 1, chapter 8, "Of the wages of labour")

One could, with some exaggeration, argue that Smith had already in mind the idea of a price adjusted for quality, a version of the so-called hedonic price. Similarly, although the word "quality" does not appear too often in his *Principles*, Alfred Marshall had us believe at times that he thought quality differentiation ubiquitous.

> When watching the action of demand and supply with regard to a material commodity, we are constantly met by the difficulty that two things which are being sold under the same name in the same market, are really not of the same quality and not of the same value to the purchasers. (Book VI, chapter 3)

Yet, it is fair to say that such statements never led these distinguished authors to modify their theories in any fundamental way. As a matter of fact, even when product differentiation within an industry started seriously attracting economists' attention, quality was not at the foreground of that revolution. In Edward Chamberlin's various writings about monopolistic competition, quality competition is often opposed to price competition but the expression has a general meaning and refers to any differentiation in attributes that leads consumers not to regard two goods as perfect substitutes. The following statement by one of Chamberlin's followers is typical in that respect.

> [Quality competition] will be conceived in a broad sense, which includes spatial competition, brand competition, competition in the specifications of the product, and competition in terms of sales. (Copeland [1940])

Kelvin Lancaster is traditionally credited with the essential clarification of the issue. Since the parution of his sum (Lancaster [1979]), it has been common to distinguish between two (polar) cases of product differentiation.[2] Two products are said to be *horizontally differentiated* if they both have positive demand when offered at the same price. There is no sense in which one product dominates the other and heterogeneity in consumers' tastes regarding varieties explains why both products are present in the market. Two products are said to be *vertically differentiated* if one product captures the whole demand when both are supplied at the same price. Thus, one product clearly dominates the other and a difference in prices, along with heterogeneity in consumers' willingness to pay for "quality", is necessary for both products to be simultaneously present in the market.

Of course, all questions of definition and modelling surrounding quality have not been resolved by that distinction. In practice, it is hard to find clear-cut examples of attributes about which all consumers agree. Reliability, in the case of a durable product, is a good example (as well as sweetness in the case of melons, as we later argue). Yet, very often, quality is not unidimensional and differences in the weights that each consumer attaches to each dimension may well result in buyers disagreeing over the ranking of products. Besides, there is disagreement about the exact nature

[2]The distinction between horizontal and vertical product differentiation is anterior, although it was often marred by associations with supply-side considerations. The earliest mention of which we are aware is Abbott (1953).

of the index referred to as "quality": does it allude to any feature of the product (or the producer) that increases willingness to pay, or is it associated to some kind of "objective" measure of the product's worth? Economists are enclined to favor the first option and it has proved a fruitful approach. (If only one piece of work should be presented as evidence, it should be John Sutton's 1991 book, testing the implication of the theory which treats advertising as an endogenous, quality-enhancing, sunk cost.) On the other hand, for obvious reasons, quality regulations are set out with reference to some observable features of the products (or production processes). The discrepancy between those features and the attributes that matter in the eyes of consumers is well-known, although there should be no presumption that non-expert buyers care for something else than the observable or certified attributes.[3]

As a matter of fact, models of vertical product differentiation have been used only for a quarter of a century. Early contributions include Mussa and Rosen (1978), Gabszewicz and Thisse (1979), and Shaked and Sutton (1982).[4] Their approaches were unified by Tirole (1988), who popularized what has become, in effect, the canonical model of price competition under vertical differentiation. Over that short time span, a lot was learnt about the effect that the presence of a quality element has on market outcomes. Although many questions have been adressed in the literature, they all revolve around this one: Will the "right amount" of quality tend to be provided? If no, what mechanisms could help solve that market failure?

In those situations in which quality is perfectly observable by consumers before their purchase, the answer is positive in the case of perfect competition, as there is

[3]For instance, the regression study undertaken by Combris, Lecoq and Visser (1997) shows that the market price of Bordeaux wines tends to be essentially determined by the legal mentions displayed on the label, while quality, as judged by a panel of experts, is mainly associated to the sensory characteristics of the beverage.

[4]Gabszewicz and Thisse (1986) is also known for perfectly capturing the spirit of Lancaster's distinction: they cast vertical differentiation within the Hotelling-like spatial framework by assuming that two firms are located outside the market, although on the same side.

a market for each possible quality variant. As soon as firms are endowed with some market power, the result breaks down: price-setters tend to distort their quality choice or product selection. Spence (1975) is the classical reference, dealing with monopoly pricing.

In those situations where quality cannot be learnt upon inspection of the product (in the case of "experience goods" in Nelson [1970]'s terminology), markets always fail, whether competitive or not. Akerlof (1970) is of course credited with putting into light the famous "lemons problem", whereby the presence of low-quality goods is sufficient to decrease the consumers' willingness to pay to such an extent that quality goods are driven out of the market. Considerable energy has since been devoted to the study of the potential remedies, on the part of producers (signalling though price or advertisement, reputation built over long-term interactions, warranties, leasing, and so on) or on the part of the government (minimum quality standard, occupational licensing, and so on).

In this dissertation, we revisit or complement some of these topics. We now set ourselves to outlining the contribution of each chapter.

CHAPTER 2: PRODUCERS BARGAINING OVER A QUALITY STANDARD

The imposition of a minimal quality standard (MQS) has very often looked like a minimally invasive yet promising form of government intervention aimed at solving the lemons problem. An early contribution by Leland (1979) offered an elegant formalization of that idea. Using a continuum of price-taking producers, he showed that quality deterioration indeed takes place on unregulated markets with asymmetric information and that there are a number of instances where a MQS, although generally not first-best, increases social welfare. Leland also showed that, if the standard is set by the producers so as to maximize their joint profits, then it is in general likely, and certain if a typical Spencian (1975) condition is satisfied, that the standard will be "too high" (that is, higher than the welfare-maximizing standard), as producers have the additional monopoly incentive to decrease output so as to drive prices up.

Leland's model assumes that producers are able to maximize joint profits and raise quality by litterally eliminating those producers whose (fixed) quality is too low. These two features are not particularly convincing. Very often, firms cannot merge or transfer profit in a way that leads them to act as a single profit maximizer.

We thus study a simple situation in which two firms are unable to make monetary payments but have the option of agreeing on a minimum quality standard. Like in Akerlof (1970), in our model buyers only observe the average quality supplied. Quantities and cost structures are exogenously given and firms compete in quality. Before choosing their qualities, they bargain over a perfectly enforcable minimum quality standard. (The bargaining outcome is given by the Kalai-Smorodinsky solution.)

Absent any agreement, the firms face a classical public-good problem. Each is tempted to free-ride on the quality investment made by the other, as only part of its investment is reflected in the market price.

We show that an agreement on a binding standard is possible only if the firms are sufficiently similar with respect to the ease with which they can upgrade their product. The agreed-upon standard always falls short of the (second-most) efficient level. It is decreasing in the high-cost producer's cost of quality. Yet, it first increases then decreases with the low-cost producer's cost of quality, showing that the latter's bargaining position can be enhanced by seemingly adverse cost changes.

CHAPTER 3: THE MARKET FOR MELONS: COURNOT COMPETITION WITH UNOBSERVABLE QUALITIES

This paper investigates the effect of letting firms set their quantity (rather than their price, as in the signalling literature) in a "lemons" context. We keep the same demand side as in the previous chapter but now the two firms, whose qualities are fixed and produced at possibly different, constant marginal costs, compete in quantities. The model is a generalization of the standard Cournot duopoly game, which corresponds to the special case where the two qualities are equal.

When the quality differential is small, the outcome of the game is similar to the Cournot case: unless the difference in marginal costs is very large, there is a unique equilibrium in which both firms are active.

When the quality differential is large, however, the fact that the price is determined by the average quality changes the firms' incentives. Indeed, by bringing more goods to the market, a firm realizes that it not only decreases the market price along the current demand curve but also impacts the average quality, which shifts the demand curve altogether. As a result, a high-quality producer may have an incentive for flooding the market in situations where it could not make a profit on the first unit sold at the current price. Similarly, a low-quality producer may have an incentive for increasing its output along with the other firm's, as a way to free-ride on the high price that the increased quality average commands.

As a consequence, the firms' output levels are not always strategic substitutes and their best responses may not even be monotonic. There can be no, or up to three pure-strategy equilibria. The important observation, though, is that as long as the cost differential is not extreme, there always exists a stable duopolistic equilibrium, that is, a situation in which even the high-quality producer brings some, sometimes even most, of the goods to the market. This is in sharp contrast with the quality crowding-out that would occur if firms were price-takers.

In that sense, strategic quantity-setting helps mitigate the adverse-selection problem created by the presence of asymmetric information. This is another instance in which two market distorsions end up being preferable to one only.

CHAPTER 4: EXCLUSIVE QUALITY

So far, in the literature, the issues of vertical foreclosure and entry deterrence have been mostly if not exclusively adressed in the context of homogenous products. A school of thought associated to the University of Chicago has long argued that the existence of long-term exclusivity contracts cannot be explained by the will of an incumbent producer enodwed with some market power to exclude an efficient rival. The reason is that consumers have much more to loose from not buying from an efficient entrant than an incumbent can win from exerting its monopoly power.

This argument is undermined if the potential entrant has to incur a fixed, unrecoverable cost in order to enter the market. In that case, as Rasmusen, Ramseyer and Wiley (1991) and Segal and Whinston (2000) showed, the incumbent can exploit a possible coordination failure among consumers so as to induce enough of them to sign on an exclusivity contract, which prevents the entrant from covering its fixed cost and leads to monopolization.[5]

Fumagalli and Motta (forthcoming) show that this line of reasoning breaks down if one assumes that the buyers of the product are retailers rather than final consumers. In that case, even a moderate degree of competition at the retail level is sufficient to eliminate all the exclusion equilibria, because a deviant retailer who would refuse to enter into the incumbent's deterrence scheme would be at such a competitive advantage vis-à-vis the other retailers that its input demand would be sufficient to induce entry.

[5]Rasmusen, Ramseyer and Wiley (1991) stated the result but their proof contained some mistakes. Corrections and refinements were provided by Segal and Whinston (2000).

We consider the case where the two upstream firms sell vertically differentiated products. We show that the effect of downstream competition on the possibility of using exclusivity contracts to deter entry is reversed: under intense (i.e. Bertrand) competition at the retail level, the incumbent is able to deter the entry of a superior product at no cost. This is always true if firms are restricted to linear pricing, and valid under two-part tariffs as long as the quality difference is not too pronounced.

The rationale for our result is that because of product differentiation, the incumbent's inferior product cannot be priced out of the market. As a result of the ensuing price rivalry, the entrant is not assured to capture all the efficiency gains from the introduction of its superior product, and decidea not to enter unless it has access to all retailers. Since, by assumption, it cannot commit to rebate some profit to the retailers in that event, exclusion emerges as the only equilibrium outcome.

All references are collected at the end of this booklet in a separate bibliography section. The first number after the name of the periodical publication stands for the volume; the second string of numbers, for the pages.

Chapter 2

Producers bargaining over a quality standard

ABSTRACT[1] We study an asymmetric information model in which two firms are active on a market where buyers only observe the average quality supplied. Quantities and cost structures are exogenously given and firms compete in quality. Before choosing their qualities, they bargain over a perfectly enforcable minimum quality standard. The bargaining outcome is given by the Kalai-Smorodinsky (KS) solution. Agreement on a binding standard is possible only if the firms are sufficiently similar with respect to their production costs. The agreed-upon standard always falls short of the joint-profit-maximizing (or, for that matter, the efficient) level. It is decreasing in the high-cost producer's cost of production. Yet, it first increases then decreases with the low-cost producer's cost of production, showing that the latter's bargaining position can be enhanced by seemingly adverse cost changes.

KEYWORDS: asymmetric information, minimum quality standard, duopoly, bargaining, free riding.

JEL CODES: D43, D82, L13, L15.

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2.1 Introduction

Since Akerlof (1970) put into light the negative consequences of asymmetric information on market outcomes in his famous article on "lemons", a great deal of attention has been devoted to the study of potential remedies. Among those, the imposition of a minimal quality standard (MQS) has very often looked like a minimally invasive yet promising form of government intervention.

Leland (1979) offered an elegant formalization of these ideas. Using a continuum of price-taking producers, he showed that quality deterioration indeed takes place on unregulated markets with asymmetric information and that there are a number of instances where a MQS, although generally not first-best, increases social welfare. Leland also showed that, if the standard is set by the producers so as to maximize their joint profits, then it is in general likely, and certain if a typical Spencian (1975) condition is satisfied, that the standard will be "too high" (that is, higher than the welfare-maximizing standard), as producers have the additional monopoly incentive to decrease output so as to drive prices up.

Later on, the issue of the desirability of a MQS was preferrably addressed within the frame of a full-information, vertically-differentiated duopoly à la Mussa-Rosen (1978) where firms first choose qualities and then compete for customers. Ronnen (1991) proved that when firms compete in prices in the second stage and quality affects fixed costs, the introduction of a MQS leads to a narrowing of the quality gap that, because it increases price competition, increases welfare through higher average quality, consumer participation and consumer surplus. This contribution spurred a lot of effort aimed at checking whether this favorable effect of introducing a MQS was robust to the assumed cost structure (Crampes and Hollander [1995]), the possibility of collusion (Ecchia and Lambertini [1997]), the duopoly setting (Scarpa [1998]) or the nature of competition (Valletti [2000]). At a general level, it is possible to say that there exist some instances in which the rise in consumer surplus following the introduction of a MQS is large enough to compensate for the decrease in firm's profits so that total welfare goes up. Nonetheless, in many of these instances, one has to trade the welfare of some consumers (typically, the high-quality buyers) for the welfare of some others (typically, the low-quality buyers).

In our opinion, what has been lost in this last strand of literature was Leland's original consideration of a biased standard-setting process. Nothing indeed guarantees that a benevolent decision-maker will strive to achieve efficiency. Very often, the government, or the standard-setting organization, is not independent of producers, or insensitive to their interests. As a matter of fact, as in the case of technological standards, producers can sometimes choose which authority will certify their compliance with a norm and may therefore engage into "strategic forum shopping" (see Lerner and Tirole [2004]). In addition, there exist many professions whose regulation in general, and quality regulation in particular, is left to "representative bodies". Medicine is a good example of such an auto-regulated industry and there is a long history of suspicion toward the way licensing and other quality requirements are used by medical organizations (for an early and strong statement in the US context, see Kessel [1958]).

The key to Leland's result about the "overprovision of quality" when a MQS is set by the industry is that side-payments can be made among producers. Indeed, in his model, producers succeed in rising quality by eliminating the lowest qualities' suppliers. If profits are transferable, it is possible to compensate the producers evicted from the market. Thus, in the event these producers have a say in the standard-setting process, it is always possible to buy their approval. There are a number of instances, though, where it does not seem appropriate to assume that the required side-payments are possible. Often, especially in oligopolistic markets, antitrust considerations lead to the prohibition of direct payments between firms and to restrictions to the use of hidden-payment vehicles such as joint R&D or marketing efforts. Even the clearest cases of collusion rarely involve direct profit sharing but rather agreements on a scheme to fix prices, allot market shares or coordinate auction bids. Hence, we believe that there is some justification for studying the outcome of a process in which the incumbent suppliers bargain over the choice of a MQS as a result of their inability perfectly to redistribute the cartel profits. This question is not usefully addressed within the framework of the full-information models mentioned above. Indeed, as Crampes and Hollander (1995) have proved, in these models the high-quality firm always loses, and the low-quality firm always benefits, from the imposition of a mildly restrictive MQS. (Exit occurs if the standard is severely restrictive.) Hence, the interests of the firms radically diverge when it comes to adopting a common norm and no common ground can be found. This degeneracy of the bargaining problem does not arise when firms have a common interest in sustaining quality. This element is present under imperfect information whenever consumers care about some measure of the average level of quality in the market.

As a first attempt at tackling this research program, we construct a simple Akerlovian model of MQS bargaining between two firms. Consumers cannot observe or infer the quality of the goods or services produced by a given firm. Instead, demand depends upon the average quality of goods available in the market. Firms are free to choose the quality of their product as long as they do not violate the MQS they might have agreed upon in the first place. They differ in their marginal cost of production given quality. Once qualities have been chosen, the goods are brought to the market, the price set so as to equate supply with demand, and the profits realized. Given these profit opportunities, the Kalai-Smorodinsky (KS) bargaining solution is assumed to capture the outcome of the standard-setting negotiation between the two firms taking place ahead of production.[2]

The unregulated market is characterized by underprovision of quality and free riding on the part of the high-cost producer. We show that the bargaining problem is non-degenerate only if the firms are not too dissimilar with respect to the cost of quality. Under large cost heterogeneity, the high-cost firm does not find it profitable to agree to any quality norm. This is because it cannot profit from a standard that does not force the low-cost firm into raising quality as well. If there is a big difference in costs, it simply does not pay for the high-cost producer to undertake the "jump" in quality needed for this to happen. By contrast, if the firms' costs are not too dissimilar, then there exists a range of mutually profitable standards. Through bargaining, the firms settle for a standard that is too low, when compared to the profit-maximizing, or for that matter the (second-most) efficient, level.

At a conceptual level, these results can be attributed to the non-transferable nature of profit in our model. Joint-profit maximization would require firms to set the MQS at a level that considerably enhances the low-cost producer's profitability relatively to the high-cost producer's. This extremely unequal allocation of profits cannot arise through bargaining because there exist other MQS levels that are more favorable to the high-cost producer and these are the source of its bargaining power.

The agreed-upon MQS often exhibits the intuitive property that an increase in one firm's cost of quality leads to a decrease in the adopted quality standard. Perhaps surprisingly, the converse is true when firms' costs are relatively dissimilar (yet similar enough for the bargaining problem not to be degenerate). In that case, an increase in the efficient producer's cost of quality leads to an *increase* in the

[2]For a characterization of the solution, see Kalai and Smorodinsky (1975). These authors attributed the first mention of this solution to Raiffa. Hence, it is sometimes described as the "Raiffa solution".

adopted standard. This is because, although the high-cost producer's best profit opportunity from adopting a common standard is little affected by the increase in cost, the low-cost producer's maximal gain increases enormously, as the inefficient producer is suddenly in the position to agree to a much larger range of standards. Thus, the reduced dissimilarity between the firms opens up the range of mutually beneficial bargains in a way that is biased toward the low-cost producer. As the KS bargaining solution is monotone in the bargainers' maximal utility gains, that translates into a shift of the solution towards the low-cost producer's interests, which can be achieved only through an increase in the adopted standard.

One can question our choice of the Kalai-Smorodinsky bargaining solution. Numerous bargaining solutions have been proposed in the literature over the years and they all come with different characteristic properties or non-cooperative foundations. The KS, along with the Nash and the egalitarian solutions, are the ones that stand most of the tests that one could possibly devise (see the presentation by Thomson [1994]). One reason for our choice of the KS solution is that in this model, it leads to a tractable quadratic equation, whereas the Nash bargaining solution leads to an unappealing quartic equation. In addition, in the context of standard setting, we find the monotonicity property that differentiates the KS from the Nash solution appealing. This property requires that an expansion of the feasible set in a direction favorable to a particular agent always benefits him. This is clearly desirable as in most cases, firms must devote some energy and resources to convincing the standard-setter, be it a government agency or an assembly of producers, that the norm ought to be set at the level they favor. Like in rent-seeking models, the amount that firms are willing to spend on successful lobbying effort is equal to their potential gain. If one believes that the eventually-adopted standard is a reflection of these lobbying efforts, then monotonicity in maximal profit changes makes for a very defensible assumption.

Our model also assumes that firms produce fixed quantities. This is analytically convenient and facilitates the comparison with Leland, for in his model firms take the market price as given and decide whether or not they want to supply a predetermined quantity. Sophisticated firms could of course realize that their quantity choice affects the market price. These strategic effects are quite complicated and we prefer overlooking them in the present study.[3] We think of the situation as one in which bargaining and production specification takes place well before the choice of output level, or one in which heavy investment in capacities is required previous to any choice concerning product characteristics.

The remainder of this article is organized as follows. In Section 2, we describe the basic model. Some preliminaries concerning alternative market structures and welfare comparisons are developed in Section 3. Section 4 presents the main results. Section 5 presents some extensions of the basic model. Section 6 concludes and suggests the directions in which the model could be taken. All the formal proofs are collected in a final section.

2.2 Model

Two firms, indexed by $i \in \{L, H\}$, each produce a fixed quantity α_i (set at half a unit for most of this paper) of a variant of a given good, whose quality $x_i \geq 0$ they choose. Firms differ with respect to the cost of production at a given quality level. Consumers cannot observe the quality of the individual products and cannot distinguish their origin (i.e. know the identity of their producer). Instead, consumers' demand depends upon their expectation or observation of the prevailing average quality. By assumption, the market price is set so as to equate demand with the fixed aggregate supply (which we always take to equal one unit).

[3]We have given a try at analyzing a quantity-setting game in the same demand environment as this model's. See Chapter 3 of this dissertation.

We model the situation as a two-player two-stage game.[4] In Stage 1, the two firms bargain over a common minimal level of quality, denoted $m \geq 0$. In the absence of agreement $m = 0$, that is, no standard is enforced.[5] The outcome of the negotiation process is captured by the Kalai-Smorodinsky bargaining solution. The KS solution chooses the point on the Pareto frontier of the bargaining set at which the ratio of firms' actual gains over the disagreement outcome equals the ratio of their "ideal" gains, i.e. the gains they can expect under the most favorable negotiation outcome.

In Stage 2, firms take the minimum level of quality agreed upon in Stage 1 as given, and simultaneously choose the quality levels, x_L and x_H. The minimal level of quality, m, is assumed to be strictly and costlessly enforced: firms cannot choose a quality level falling short of that standard.

There is a cost associated with increasing quality. We assume that the cost per unit produced is a quadratic function of quality[6] given by $\theta_i \cdot (x_i)^2/2$, where $\theta_i \geq 0$ is a parameter standing for firm i's *cost of quality*. There are no fixed costs. Hence, each firm's total production cost is equal to $\alpha_i \cdot \theta_i \cdot (x_i)^2/2$. We assume that $\theta_H \geq \theta_L$ and it is understood that in the case when this inequality strictly holds and firms

[4] Because our first "stage" involves a cooperative bargaining solution, our model does not fit the technical definition of a "game" in canonical non-cooperative game theory. It would be possible to cast it into the usual framework by artificially introducing a third player called upon choosing the common standard in Stage 1, whose objective function would involve costly departures from the Kalai-Smorodinsky ratio of firms' profits. Alternatively, one can see the analysis of firms' behavior in Stage 2 as background work for the determination of the bargaining set and take the situation to be a pure bargaining problem. We find the reference to stages useful and will hence continue using it.

[5] Any strictly positive but ineffective standard (one that would not affect firms' behavior in Stage 2) could be equivalently chosen as the disagreement point.

[6] The quadratic specification for marginal cost seems to be the most economical specification to address the problem at hand. In the case where marginal cost is a linear function of quality, there is no equilibrium in pure and interior strategies to the quality "subgame" in general, and this is so independently of the choice of the inverse demand function. There do exist "corner-solution" equilibria but they are uninteresting as they give rise to trivial bargaining problems (degeneracy or immediate agreement on a unique Pareto-efficient outcome).

produce the same quality level, firm L enjoys a lower marginal cost than firm H, justifying our choice of subscripts.

After both firms have determined their quality levels, consumers observe (or infer) the average level of quality $\bar{x}$. Their aggregate demand is an affine function of this average. More precisely, the quantity demanded, at any given price p, is taken to be

$$D(p) = 1 + a + \bar{x} - p, \tag{2.1}$$

where $a \geq 0$ is a demand-shifting parameter introduced to guarantee full market coverage. This demand could arise from a continuum of consumers with valuations of the good in question uniformly distributed between 0 and $1 + a + \bar{x}$.[7]

The equilibrium market price p^* is defined by equating demand with the fixed supply, giving $p^* = a + \bar{x}$. In most of this article, we will consider the situation where both firms produce half a unit. Thus, the market price, as a function of the firms' quality levels, is

$$p^*\left(x_L, x_H\right) = a + \frac{1}{2}\left(x_L + x_H\right). \tag{2.2}$$

Firms strive to maximize their profits, $\Pi_i = \left[p^*\left(x_L, x_H\right) - (\theta_i \cdot (x_i)^2)/2\right]/2$. Hence, viewed as players in stage 2, their payoff functions are

$$\pi_i\left(x_H, x_L\right) = \frac{1}{2}\left[\frac{x_L + x_H}{2} - \frac{1}{2}\theta_i \cdot (x_i)^2 + a\right]. \tag{2.3}$$

[7]For instance, consumers could all be willing to buy one unit of the good only. Their preferences could be defined over that good and all the other goods they possibly care about. Absent any strong income effect, their indirect utility function could then be taken to be linearly separable in the gross utility derived from that unit and its price. The utility from consuming the good could comprise a "baseline" utility level, differing across consumers, and a valuation-of-quality term, identical across consumers. For example, a unit mass of consumers indexed by $t \in [0, 1]$ could have preferences represented by the utility functions $U_t = b_t + x - p$ where each consumer t derives a baseline utility from consuming one unit of the good equal to $a + t$ and a quality-related utility equal to x.

2.3 Preliminaries

Before analyzing the model, it is informative to analyze some related market structures and make some welfare comparisons.

2.3.1 Monopoly

Consider first the monopoly case. Suppose then that there is a single decision-maker who sets x_L and x_H so as to maximize $\pi_L(x_H, x_L) + \pi_H(x_H, x_L)$, the total profit to a corporation owning both production plants or "profit centers." Implicit in the specification of this objective function is the assumption that the difference in costs across plants is not due to a difference in technologies—potentially eliminated by a merger—but to some other cause(s), for instance, heterogeneity in the quality or price of local inputs (such as land in agriculture, metals in the industry, or labor in general).

It is easy to establish that the first-order conditions for profit maximization are

$$x_L^M = \frac{1}{\theta_L} \qquad x_H^M = \frac{1}{\theta_H}, \tag{2.4}$$

resulting in monopoly profit

$$\Pi^M = \frac{1}{4\theta_L} + \frac{1}{4\theta_H} + a. \tag{2.5}$$

The higher the cost parameters, the lower are the qualities, as well as the monopolist's profit.

Using the familiar criterion of total surplus, welfare in our model can be expressed as

$$W(x_H, x_L) = \frac{1}{2} + a + \frac{x_L + x_H}{2} - \frac{1}{4}\theta_L \cdot (x_H)^2 - \frac{1}{4}\theta_H \cdot (x_L)^2. \tag{2.6}$$

The unrestricted maximizers of this function happen to be x_L^M and x_H^M, so that a monopoly achieves economic efficiency. The same is true if the decision-maker is restricted to choose only one quality to be produced by both plants. In that case, we have

$$\Pi^M(x,x) = x - \frac{1}{4}(\theta_L + \theta_H)x^2 + a, \tag{2.7}$$

which is maximized at

$$x^M = \frac{2}{\theta_H + \theta_L}, \tag{2.8}$$

the harmonic mean of x_L^M and x_H^M, just as $W(x,x)$ is. There is nothing surprising in these results. We know from Spence (1975) that the source of the divergence between the socially optimal quality and the one chosen by a profit-maximizer lies in the difference between the marginal consumer's valuation of quality improvements and the average consumer's valuation of quality improvements (where the average is taken over all infra-marginal consumers). This discrepancy does not materialize when demand is linear in quality, i.e. when consumers' marginal valuations of quality are identical, as here.

2.3.2 Perfect information: monopolistic competition

Another relevant case for comparison is when there are two firms but consumers *can* distinguish the two products. Then, these are sold on two separate yet interdependent markets in a monopolistically competitive manner.

If a unit mass of consumers indexed by t is uniformly distributed over $[0,1]$ and endowed with preferences of the form $U_t = a+t+x-p$, and if they randomly patron one firm or the other when they are indifferent, then the residual demand addressed to firm L is given by

$$D_L(x_L, x_H, p_L, p_H) = \begin{cases} 1 & \text{if} \quad x_L - p_L > x_H - p_H \\ 1/2 & \text{if} \quad x_L - p_L = x_H - p_H \\ 0 & \text{if} \quad x_L - p_L < x_H - p_H \end{cases}, \tag{2.9}$$

and conversely for firm H.

If firms simultaneously choose qualities and prices, then under our quantity constraint, firm i's program is

$$\max_{x_i, p_i} \quad \left[\max\left\{\frac{1}{2}, D_i\right\}\right] \left[p_i - \frac{1}{2}\theta_i \cdot (x_i)^2\right].$$

As it is clearly not optimal to price so high as to generate zero demand or to price so low as to create excess residual demand, firm i's best response always involves setting p_i equal to $x_i - x_j + p_j$. Therefore, firm i's program reduces to

$$\max_{x_i} \quad \frac{1}{2}\left[x_i - x_j + p_j - \frac{1}{2}\theta_i (x_i)^2\right],$$

which leads to

$$x_i^S = \frac{1}{\theta_i}. \tag{2.10}$$

Thus, there are infinitely many equilibria but, as long as the market is fully covered, they all share the features that the price differential equals the quality differential, and qualities are socially optimal. The resulting profit to each firm is of the form

$$\pi_i^S = \frac{1}{2}\left[\frac{1}{2\theta_i} + a + c\right], \tag{2.11}$$

where c is a function of the prices selected in a particular equilibrium. As expected, the equilibrium profit to each firm is decreasing in its own cost parameter.

2.3.3 Asymmetric information in the absence of standard

Now suppose that there are two firms, that consumers cannot distinguish the two products, and that there is no quality standard. Formally, this is equivalent to letting m be exogenously fixed at zero in the model outlined in Section 2.

First, observe that each firm's profit function $\pi_i(x_H, x_L)$, defined in equation (2.3), is continuously differentiable and strictly concave in x_i. Hence, a necessary and sufficient first-order condition for unconstrained maximization is

$$x_i^U = \frac{1}{2\theta_i}. \tag{2.12}$$

When firms make their decision, they do not take into account the positive externality that their effort entails for the other firm. This explains why quality is underprovided as compared to the monopoly case or the duopoly case with perfectly informed consumers.[8]

The resulting profit levels are accordingly lower:

$$\pi_i^U = \frac{1}{2}\left[\frac{1}{8\theta_i} + \frac{1}{4\theta_j} + a\right] \tag{2.13}$$

for $i = L, H$ and $j \neq i$. Note that firm H makes a higher profit than firm L! This reversal of the profit ranking, as compared to the separate markets case, is a consequence of the free riding occuring in this market. To see this, let us call the portion of profit that a firm can affect through its own choice of effort its *internal profit*, ι_i. One then has

$$\iota_i = \frac{1}{2}\left[\frac{x_i}{2} - \frac{1}{2}\theta_i\left(x_i\right)^2\right]. \tag{2.14}$$

Similarly, let us call the portion of a firm's profit that results from the impact of the other firm's effort choice on the market price its *external profit*, ε_i:

$$\varepsilon_i = \frac{1}{2}\frac{x_j}{2}, \tag{2.15}$$

where x_j stands for the other firm's effort choice. Then $\pi_i = \iota_i + \varepsilon_i + a/2$.

It is easily verified that in equilibrium firm L's internal profit is higher than firm H's internal profit, that is: $\iota_2 > \iota_1$. On the other hand, firm L does not benefit much from firm H's low quality choice, which contributes little to the market price, while firm H quite gains from the high quality produced by firm L, which raises the price it receives. Hence, $\varepsilon_1 > \varepsilon_2$. For the particular specification of our payoff function,

[8] If the industry were composed of n firms, each selling a fraction $1/n$ of total output, the unconstrained quality choice would be $x_i^U = \frac{1}{n\theta_i}$.

with equal quantities, this latter external effect is so big as to dominate. In other terms: $\varepsilon_{1-}\varepsilon_2 > \iota_2 - \iota_1$.

2.4 Analysis

We now proceed to analyse the model presented in Section 2. We first consider Stage 2 (firms' production decisions after the standard has been set), which determines the boundaries of the profit possibility set defining Stage 1's bargaining problem.

2.4.1 Quality choice

Recall that each firm's payoff π_i is a strictly concave function of its own effort, x_i. Suppose that a standard $m \geq 0$ has been agreed upon in Stage 1, constraining firms' quality choices in Stage 2. By strict concavity, the optimal quality levels are

$$x_i^*(m) = \max\left\{\frac{1}{2\theta_i}, m\right\}. \tag{2.16}$$

By assumption, firm H faces a higher cost of quality. Thus, firm H always chooses a smaller quality level than firm L. From equations (2.3) and (2.16), firms' profits, $\pi_i\left(x_H^*, x_L^*\right)$ can be shown to depend on m, the minimal quality level agreed upon in Stage 1, in the following manner:

$$\begin{array}{lll}
\text{for } m < \frac{1}{2\theta_H}: & \pi_H = \frac{1}{2}\left[\frac{1}{8\theta_H} + \frac{1}{4\theta_L} + a\right] & \pi_L = \frac{1}{2}\left[\frac{1}{4\theta_H} + \frac{1}{8\theta_L} + a\right] \\
\text{for } m \in [\frac{1}{2\theta_H}, \frac{1}{2\theta L}]: & \pi_H = \frac{1}{2}\left[\frac{m}{2} + \frac{1}{4\theta_L} - \frac{1}{2}\theta_H m^2 + a\right] & \pi_L = \frac{1}{2}\left[\frac{m}{2} + \frac{1}{8\theta_L} + a\right] \\
\text{for } m > \frac{1}{2\theta_L}: & \pi_H = \frac{1}{2}\left[m - \frac{1}{2}\theta_H m^2 + a\right] & \pi_L = \frac{1}{2}\left[m - \frac{1}{2}\theta_L m^2 + a\right]
\end{array}$$

Several observations are in order. Firstly, when the standard does not constrain firms' choices, then, as explained in the previous section and as a result of free riding, the high-cost firm makes more profit than the low-cost firm.

Secondly, firm H loses from a binding quality standard, when set at an intermediate level: $m \in (\frac{1}{2\theta_H}, \frac{1}{2\theta_L}]$. This is because in this range, firm L's behavior is unaffected by the standard, whose sole effect is to force firm H to depart from its optimal choice of quality. As a consequence, π_H is decreasing in m in this interval.

Thirdly, once $m > \frac{1}{2\theta_L}$, firm L is forced to raise its quality level as well. That leads to an increase in firm H's revenues through the increase in the market price but profitability also depends on costs. Observe that, when the standard is doubly binding, firm H's profit function is single-peaked at $m = \frac{1}{\theta_H}$. Thus determining the behavior of π_H in $(\frac{1}{2\theta_L}, +\infty)$ requires us to know whether $\frac{1}{\theta_H}$ falls into that interval or not. We have $\frac{1}{\theta_H} > \frac{1}{2\theta_L}$ if and only if firm H is at a cost disadvantage relatively to firm L but this disadvantage is less than twofold. That is:

$$\theta_L \leq \theta_H < 2\theta_L. \tag{2.17}$$

In that case, π_H increases with m over $[\frac{1}{2\theta_L}, \frac{1}{\theta_H}]$ and decreases thereafter. Nonetheless, it is possible for π_H never to reach again its initial level. Indeed, if firm H is at a big cost disadvantage relatively to firm L, then its costs will have become very high when the standard finally reaches the zone where it affects firm L's behavior. So, even if an increase in the standard increases firm L's profit in that zone, it never brings it back to the level associated to low effort and no standard. A simple computation shows that there are gains to be reaped by firm H from the imposition of *some* effort standard only if[9]

$$\frac{2}{3} < \frac{\theta_L}{\theta_H}. \tag{2.18}$$

Under our assumptions, the ratio θ_L/θ_H lies between zero and one and can be interpreted as a measure of cost homogeneity, a magnitude which will play an important role in all that follows. Thus, condition (2.18) requires firms not to be too dissimilar in costs in order for firm H to find it profitable to put an end to free riding.

[9]This condition guarantees that the maximum achieved by π_H on $[\frac{1}{2\theta_L}, \infty)$ is strictly greater than the profit achieved when $m = 0$.

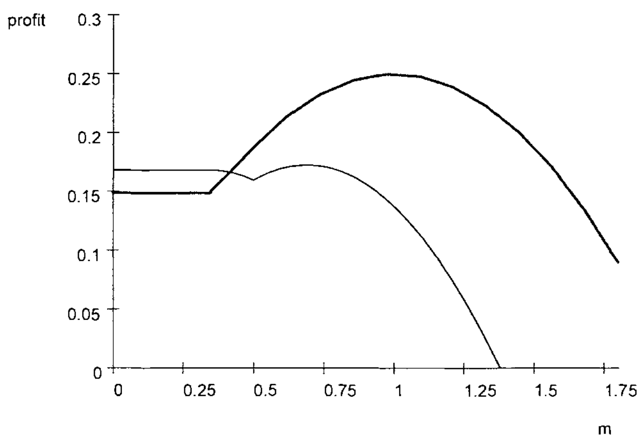

Figure 2.1: The two firms' equilibrium profits, as functions of the common minimum standard, m. Intermediate cost homogeneity.

Fourthly, we observe that firm L's profit starts increasing as soon as the standard impacts firm H's behavior, that is, for all $m > \frac{1}{2\theta_H}$. There is a kink at $m = \frac{1}{2\theta_L}$, when the standard starts constraining firm L as well. Yet, π_L continues increasing with m until $m = \frac{1}{\theta_L}$, which is firm L's preferred standard.

The two diagrams below depict the firms' profits as functions of m. The thin line stands for firm H's profit while the thick line is for firm L. Figure 1 corresponds to the case where $a = 0$, $\theta_H = 1.45$ and $\theta_L = 1$. (These parameters meet condition [2.18] on cost homogeneity.) One can see that firm H is hurt by an intermediate-level standard but recovers once m becomes high enough. There is a range of values for which π_H is higher than the vertical intercept.

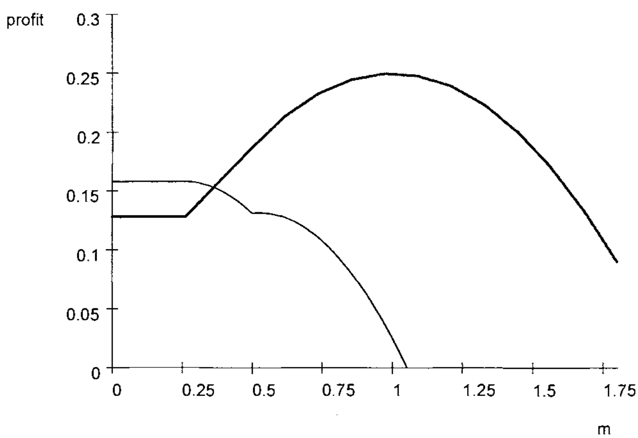

Figure 2.2: The two firms' equilibrium profits, as functions of the common minimum standard, m. Small cost homogeneity.

Figure 2 corresponds to the case where $a = 0$, $\theta_H = 1.9$ and $\theta_L = 1$. (These parameters do *not* meet condition [2.18] on cost homogeneity.) One can see that firm H is hurt by an intermediate-level standard, recovers somewhat once m becomes high enough but not so as to bring π_H back to its initial level.

2.4.2 Bargaining problem

A bargaining problem is usually defined with respect to a set of possible utility allocations, the *feasible set*, and a fallback or *disagreement point* in this set. Here, the disagreement point corresponds to $m = 0$ and is given by $\left(\pi_H^U, \pi_L^U\right)$, the profits to the firms when they do not adopt any standard. The table displaying firm's profits as functions of m in the previous subsection is a parametric characterization of the set of possible utility allocations in $\mathbb{R}^2$. It is well-known that in the case of the KS solution as in many others', the consideration of the set of utility changes from the disagreement allocation utilities leads to the same solution as the consideration of the original feasible set. So we can take the feasible set to be the set F of *gains* over the no-standard-case profits, and the disagreement point to be $(0, 0)$. Because the KS solution satisfies individual rationality, one is in fact only interested in the set of outcomes in which firms' gains are positive. A bargaining problem is said to be *degenerate* if the feasible set does not contain any point corresponding to strict gains over the disagreement utilities for all parties (that is, here, if $F \cap \mathbb{R}^2_{++} = \varnothing$).

Because the KS solution is defined by reference to each party's most favorable outcome, we would like to know which standards lead to the highest gains to firm H and firm L. At the same time, by individual rationality, only the outcomes in which both firms make at least as much profit than in the no-standard case really matter. We saw in the previous subsection that firm H is susceptible to make more profit than in the absence of a standard only if the MQS affects both firms and if cost homogeneity is large enough. By comparing firm H's profit under such a doubly binding standard to firm H's profit when its quality choice is unrestricted, one can find the interval $[z_1, z_2]$ of relevant standards.[10] There is no issue about firm L's gains being non-positive in that interval since its revenues are the same as firm H's but its costs of production are smaller by assumption. So the best feasible outcome for firm H is always $m = 1/\theta_H$. On the other hand, firm L's profit is maximized at $m = 1/\theta_L$ but that standard may or may not lie within $[z_1, z_2]$. Indeed, if cost homogeneity is not so small as to prevent any agreement but still consequential, then

[10]Formally, this is done by computing the roots of the equation

$$\pi_H^U = \pi_H \left[x_H^*(m), x_L^*(m)\right].$$

firm H makes a loss when $m = 1/\theta_L$, as this level of quality is simply too costly to produce. In that case (to which we will refer as the intermediate cost homogeneity case), the best outcome for firm L under the constraint that firm H does not earn less than its disagreement profit corresponds to $m = z_2$, which is smaller than $1/\theta_L$.[11] This is the situation depicted in Figure 1.

Thus, we have to distinguish three cases. Under small cost homogeneity, firm H never benefits from agreeing to a binding standard. Under intermediate cost homogeneity, firm's H favorite standard is $1/\theta_H$, while firm L's maximal feasible gain corresponds to $z_2 < 1/\theta_L$. Under large cost homogeneity, firms' maximal gains correspond to $1/\theta_H$ and $1/\theta_L$, respectively. We characterize these three cases more precisely in the following lemma.

Lemma 1 *Firms' most favorable outcomes under the condition that all parties make at least as much as under disagreement are given by the following table:*

Cost homogeneity	*Parameter range*	*firm L's best outcome*	*firm H's best outcome*
small	$0 \leq \frac{\theta_L}{\theta_H} \leq \frac{2}{3}$	$m = 0$	$m = 0$
intermediate	$\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$	$m = z_2 = \frac{1+\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}$	$m = \frac{1}{\theta_H}$
large	$\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1$	$m = \frac{1}{\theta_L}$	$m = \frac{1}{\theta_H}$

A proof of the lemma can be found in the appendix.

Small cost homogeneity

If condition (2.18) on costs is not satisfied, i.e. if the firms display a small level of cost homogeneity, then firm H never benefits from agreeing to a quality standard.

[11] A variant of the KS solution, suggested by Kalai and Rosenthal (1978), chooses the allocation that sets players' utilities proportional to their most optimistic expectation of gain, even when this gain entails losses for one or more players, i.e. when the corresponding allocation is not individually rational. For the problem at hand, it is hard to argue that such allocations should play a role in the determination of the final bargain.

Hence, the bargaining problem in Stage 1 is degenerate, as there are no "gains from trade" to be shared among the bargainers. Strictly speaking, the Kalai-Smorodinsky solution is not defined in that instance.[12] Intuition nevertheless suggests that firm H will refuse any standard above its unconstrained optimum $x_H^U = \frac{1}{2\theta_H}$ and accept any standard less than or equal to the latter, out of indifference. The limit case when $\theta_L/\theta_H = 2/3$ opens the possibility that firms agree on setting m equal to $\frac{1}{\theta_H}$, as this is profitable to firm L and makes no difference to firm H. We summarize these considerations in the following paragraph.

Remark 2 *In the case of small cost homogeneity ($\theta_L/\theta_H \leq 2/3$), the bargaining problem is degenerate: Any ineffective standard $m \leq 1/(2\theta_H)$ might be chosen. Consequently, firms' qualities and profits are the same as in the absence of a standard. If $\theta_L/\theta_H = 2/3$, then there might also be an outcome in which $m = 1/\theta_H$, in which case quality, price, and firm L's profit are higher than in the absence of a quality standard.*

LARGE COST HOMOGENEITY

Consider now the case when the firms have very similar cost parameters, that is

$$\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1. \tag{2.19}$$

Then, the downward-sloping section of the bargaining set frontier, which corresponds to quality standards in $\left[\frac{1}{\theta_H}, \frac{1}{\theta_L}\right]$, entirely lies to the northeast of the disagreement point. It is understood that in case negotiations break down, no binding standard will be imposed. In that case firm i's payoff will be π_i^U, its *guaranteed,* or *disagreement, profit.* Firm i's *actual gain* following the imposition of a binding standard m

[12]Neither are other bargaining solutions. Thus, there is a sense in which too big a difference in costs prevents agreement on a MQS, independently of the details, or features, of the bargaining process.

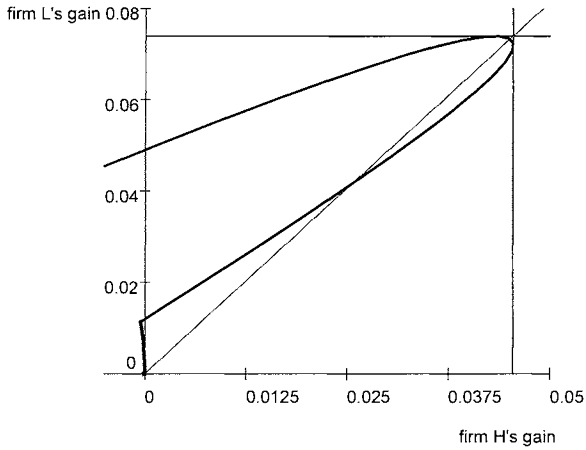

Figure 2.3: The feasible set and the KS solution when costs are very similar

is defined as the difference between firm i's actual payoff and its guaranteed profit. Firm i's *maximal gain* is defined as the highest achievable profit increment under the condition that players get at least as much as their guaranteed profit.

One can picture the situation in the profit space $\pi_H \times \pi_L$. Figure 3 corresponds to $\theta_H = 1.1$, $\theta_L = 1$, $a = 0$

The point $(0,0)$ corresponds to the cases when $m \in [0, \frac{1}{2 \cdot 1.1}]$, i.e. the circumstances in which the quality standard is not binding. The beginning of the line (before the kink) is for standards $m \in [\frac{1}{2 \cdot 1.1}, \frac{1}{2}]$. Recall that in this range, because quality is costly but the norm does not affect firm L's behavior, firm H's profit decreases quadratically while firm L's profit increases linearly. Thus, firm H's gain is negative, while firm L's is positive. The part of the line that follows the kink is for higher standards. Its intersections with the vertical axis correspond to $m = z_1$ and $m = z_2$. The maximal gain for firm H is achieved at $m = \frac{1}{1.1}$ (vertical dashed line). The maximal gain for firm L is achieved at $m = \frac{1}{1}$ (horizontal dashed line). The KS solution for this bargaining problem is found at the intersection of the profit frontier with the upward-sloping dashed line. At the solution one must have that the ratio of firms' actual gains equals the ratio of firms' maximal gains.

It happens that a reasonably simple closed-form solution for the agreed-upon standard $\hat{m}$ is available in that case.

Proposition 3 *If $\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1$, then the standard upon which firms agree is given by*

$$\hat{m} = \frac{5 + \sqrt{25 - 6\frac{(\theta_L+\theta_H)^2}{\theta_L \theta_H}}}{3(\theta_L + \theta_H)}. \tag{2.20}$$

A proof of this claim can be found in the appendix.

Note that when $\theta_H = \theta_L = \theta$, the agreed-upon standard $\hat{m}$ equals $1/\theta$. Indeed, when the two firms are exactly identical, the downward-sloping section of the bargaining set collapses to a single point, corresponding to $1/\theta$, and the feasible set reduces to a segment along the 45-degree line. The set $[z_1, z_2]$ still stands for the range of mutually profitable standards but the interests of the firms are aligned in the sense that, to the left of $1/\theta$, both prefer to increase the standard and, conversely, to the right of $1/\theta$, both prefer to decrease it. Since there exists a unique Pareto-optimal outcome, it is attained by any bargaining solution satisfying Pareto-optimality, in particular by the KS solution.

The comparative statics of the equilibrium standard are as follows.

Proposition 4 *If $\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1$, then (i) for any fixed θ_L, $\hat{m}$ is strictly decreasing in θ_H; (ii) for any fixed θ_L, $\hat{m}$ is strictly decreasing in θ_H.*

A proof of this claim can be found in the appendix.

These small variation effects might look intuitive but they do not necessarily follow from casual observation because in the space of profits, a change in one cost parameter displaces the frontier of the bargaining set as well as the disagreement point.[13]

In equilibrium, firm L makes a higher profit than firm H, which is a reversal of the ranking under disagreement. This outcome is to be expected once both firms produce the same quality, since they have the same revenues but by assumption firm L has smaller costs.

A straightforward computation allows us to make the following claim.

Remark 5 *If* $\frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} < 1$, *then* $\hat{m} < x^M$.

That it, the standard that firms agree upon is lower than the (second-most) profit-maximizing level, which we know from Section 2 is also the (second-most) efficient level. This is a consequence of the non-transferable nature of profit in our model. Joint-profit maximization would require firm H to produce at a higher quality level but this would lead to an even greater enhancement of firm L's relative profitability that is precluded by the bargaining solution. If firm L could somehow share its profit with firm H, it would be possible to achieve efficiency.

In other terms, the initial free riding that plagues the industry in the absence of a standard, by allowing firm H to be (relatively) very profitable, empowers it too

[13]For instance, if θ_H goes up, then firm H's maximal gain goes down. Indeed, at its preferred effort standard, $1/\theta_H$, its payoff decreases to a big extent, as both firms reduce the quality provided to the market. In addition, its guaranteed profit is also affected by the increase in θ_H but to a lesser extent as firm L's quality remains unchanged. At the same time, firm L's maximal gain goes up because there is no change to its payoff under its preferred standard, $1/\theta_L$, and its guaranteed profit goes down as a result of the decrease in external profit. The KS solution thus dictates an increase in the ratio of firm L's actual gain to firm H's actual gain. Precisely, at any given binding standard, firm L's actual gain is mechanically increased through the decrease in its guaranteed profit. So, a priori, it is not clear that the required decrease in the ratio will necessitate a decrease in the equilibrium standard. Our result shows that it does.

much. Indeed, since the geometric mean of two real numbers is smaller than their arithmetic means, it is clear that $\hat{m}$ lies to the left of the midpoint between $1/\theta_H$ and $1/\theta_L$. So, to the extent that the agreed-upon standard is closer to firm H 's favorite standard than to firm L's, the former can be said to have more bargaining power than the latter. To the extent that the agreed-upon standard falls short of the social optimum, it can be said to have too much of it.

This remark obviously does not apply to the case where $\theta_H = \theta_L = \theta$, in which case firms, equal in all respects, agree to implement the first-best standard $1/\theta$.

Intermediate cost homogeneity

Suppose now that $\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$. Then firm L's maximum feasible gain is no longer achieved at $m = \frac{1}{\theta_L}$, but at $m = z_2 = \frac{1+\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}$ as firm H would refuse any standard delivering less than its disagreement profit. One can picture the situation in the profit space $\pi_H \times \pi_L$. Figure 4 corresponds to $\theta_H = 1.4$, $\theta_L = 1$, $a = 0$.

It is easily seen that firm L's maximal gain corresponds to so high a standard that firm H would make less than its guaranteed profit. Thus, the best that firm L can reasonably expect is the standard corresponding to the intersection of F with the vertical axis, which is z_2. Again, at the solution one must have that the ratio of firms' actual gains equals the ratio of firms' maximal gains, which translates into a quadratic equation whose bigger root, $\tilde{m}$, is the solution to our bargaining problem.

It is possible to get a closed-form expression for this root but it is not appealing, its derivatives being too complicated to be easily signed. Comparative statics can nevertheless be studied by recalling that the defining equation of the solution (displaied in the appendix) has the following form:

$$\frac{A}{B} = \frac{C}{D},$$

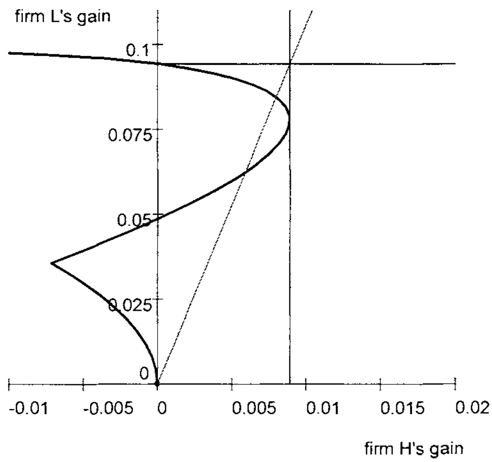

Figure 2.4: The feasible set and the KS solution when costs are somewhat similar

where A is firm L's actual gain, B Firm H's actual gain, C Firm L's maximal gain, and D firm H's maximal gain. Because of the single-peakedness of the polynomial $m - \frac{1}{2}\theta_i m^2$, A/B is strictly increasing in m on $\left[\frac{1}{\theta_H}, \frac{1}{\theta_L}\right]$. So one can start from a situation where the equality prevails, introduce a "small" change to either θ_H or θ_L, and look at the resulting change in C/D. If A/B, evaluated at the initial solution (but at the new θ_H or θ_L), has not changed to the same extent, then it must be that $\tilde{m}$ has changed in order to bring the two ratios back into equality. This way we are able to make the following claim.

Proposition 6 *If* $\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$, *then for a given* θ_L, $\tilde{m}$ *is strictly decreasing in* θ_H.

A proof of this claim can be found in the appendix.

By contrast, the changes in $\tilde{m}$ brought about by small changes in θ_L do not always go in the same direction. It is possible to show that if the homogeneity in costs between firms is quite low, then an increase in θ_L leads to an increase in the agreed-upon standard.

Proposition 7 *There exists* $c \leq \frac{4}{3+\sqrt{5}}$ *such that if* $\frac{\theta_L}{\theta_H} \in \left(\frac{2}{3}, d\right)$, *then for any given* θ_H, $\tilde{m}$ *is strictly increasing in* θ_L.

A proof of this claim can be found in the appendix. The argument is only sketched here.

When $\frac{\theta_L}{\theta_H}$ is very close to $\frac{2}{3}$, there is a very small interval of standards $(\frac{1}{\theta_H}, z_2)$ to the right of $\frac{1}{\theta_H}$ that firm H could agree upon as generating more than the disagreement level of profit. Because we are so close to $\frac{1}{\theta_H}$, we are very much at the top of firm H's profit hill under a doubly-binding standard (when one pictures π_H as a function of m). Any increase in θ_L, by decreasing firm H's guaranteed profit, has the effect of pushing z_2 to the right along a nearly horizontal trajectory. Thus, there is an enormous change in z_2, that increases firm L's maximal gain at an extraordinary rate. As the KS bargaining solution is monotonic in maximal utility gains over the disagreement utility, that translates into a big increase in firm L's actual gain, which is achievable only through a rise in the effort standard. The key point is that firm L's profit under its favorite standard of all, $\frac{1}{\theta_L}$, does not change much following the rise in θ_L but that does not matter as this outcome is so unfavorable to firm H that it cannot be agreed-upon anyway. By contrast, among the outcomes that firm H can rationally accept, firm L's maximal gain changes tremendously because firm H

is suddenly open to a much larger range of standards. Thus, the enhanced similarity between the firms opens up the range of mutually beneficial bargains in a way that is biased towards firm L.

In fact, because the derivative of z_2 with respect to θ_L is infinite at $\frac{\theta_L}{\theta_H} = \frac{2}{3}$, so is the derivative of $\tilde{m}$. That implies that the derivative of firm L's profit function is also infinitely positive at that point. In other terms, when firm L's cost increases in a way that makes firm H suddenly willing to agree to a much broader range of standards, then firm L's profit goes up: *It is profitable to become less efficient if it is the price to pay to become more similar.* Again, this is so because the change in θ_L has a first-order effect on firm L's maximum gain.

This behavior of the agreed-upon standard when cost homogeneity is verging to smallness might raise doubt about the appropriateness of our bargaining solution: Might firm L not be tempted by masquerading as a slightly more inefficient firm, when information about costs is not perfect, in order to get a MQS closer to its favorite one? For instance, if the standard-setter is a public authority, might firm L not lie about its cost in order to fool the arbitrator? This questions are out of the scope of this paper. We note that Moulin (1984) proved that the KS solution was implementable as the unique subgame-perfect Nash equilibrium of a mechanism in which players are asked to bid "fractions of dictatorship."[14]

It is readily observed that, were we to compute the level of the standard that equated firms' actual gains ratio to firms' ideal gains ratio, disregarding individual rationality considerations, the algebraic expression would be given by $\hat{m}$ in Proposi-

[14]Because of its scale invariance and symmetry properties, the KS solution in effect maximizes a Rawlsian social welfare function once the problem is suitably normalized. Thus, implementability is not surprising. For two-person problems, Kalai and Rosenthal (1978) had already offered a mechanism Nash-implementing the KS solution (among other equilibria). Myiagawa (2002) proposes a general game form that implements the KS solution (along with the Nash solution and some weighted utilitarian solutions) in subgame-perfect equilibrium.

tion 3, and we would have $\hat{m} > \tilde{m}$. (The dotted ray would rotate counterclockwise in Figure 4.) From Remark 5 we know that $\hat{m}$ is smaller than the (second-most) efficient standard. As a result, we can draw a similar conclusion for $\tilde{m}$.

Remark 8 *For any θ_H and θ_L such that $\frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}$, we have $\tilde{m} < \hat{m} < x^M$.*

This implies that the adopted standard lies even closer to firm H's favorite choice than firm L's.

2.4.3 Numerical example

Fix θ_H at 1.5. The treshold $\frac{\theta_L}{\theta_H} = \frac{4}{3+\sqrt{5}}$ corresponds to a value of $\frac{6}{3+\sqrt{5}} \simeq 1.14$ for θ_L. Figure 5 depicts how the adopted standard behaves when firm L's cost parameter varies from 1 to 1.5 , the range for which the bargaining problem is not degenerate. Values below the treshold (to the left of the dashed vertical line on the graph) correspond the case of intermediate cost heterogeneity; values above the treshold (to the right of the dashed line) correspond to the case of small cost heterogeneity.

When firm L is very efficient (θ_L close to 1), the adopted standard is very close to the one favored by firm H, $\frac{2}{3}$, which in the limit is the only one that the latter can agree to. As firm L's cost increases, the agreed-upon standard goes up first, peaks a bit before the treshold before decreasing toward $\frac{2}{3}$, the norm that both firms happen to favor as θ_L converges to 1.5.

The graph makes clear that the derivative of the equilibrium standard right of 1 is infinitely positive, a fact used in the proof of Proposition 8, and implying that firm L's profit, as a function of θ_L, is increasing in that region.

A possible measure of firms' bargaining power is the distance between the adopted standard and their favorite standard. One could in principle construct an index of Firm L's bargaining power by expressing the distance from the adopted standard to Firm H's favorite standard as a fraction of $\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)$. The higher this index, the

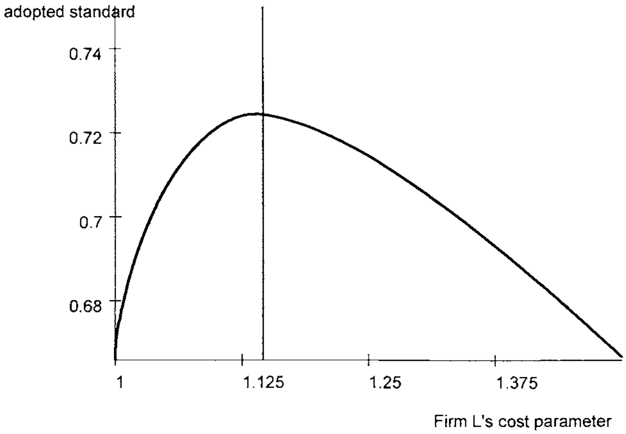

Figure 2.5: The relationship between the adopted standard and Firm 2's cost parameter in the case when $c_1 = 1.5$.

closer to Firm L's ideal point the adopted standard is. The remarks made above imply that this index is bounded above by 0.5. Indeed, in the numerical example it increases monotonically toward this value as θ_L goes up. This is again an indication that similarity is desirable in this context.

2.5 Extensions

2.5.1 Uncertainty in production

The basic model can be straightforwardly extended to deal with uncertainty in the production process making qualities random variables. Under risk-neutrality on the part of producers and consumers, it is sufficient to reinterpret the variable x_i as "effort" determining the mean of the product quality distribution and all the results carry over without modification.

2.5.2 Risk-aversion

The model described in Section 2 assumed that demand linearly depended upon the average quality, implying that the valuation of the good was independent of the other features of the lottery over qualities that consumers faced when purchasing the good. One might argue that consumers' willingness to pay should decrease with the uncertainty associated to the sampling procedure, as a consequence of risk-aversion. If consumers' willingness to pay is taken to correspond to the lottery's certainty equivalent, then all the qualitative features of our model are preserved.

For instance, one can assume that a unit mass of consumers indexed by t is uniformly distributed on $[0,1]$. Consumer t's preferences can be represented by the following utility function:

$$U_t = b_t + \sqrt{\bar{x}} - p, \tag{2.21}$$

where baseline utility, b_t, equals $a+t$. With this structure, the demand associated to a fair lottery between qualities x_L and x_H, whose arithmetic average is $\bar{x}$, can be given by[15]

$$D(p) = 1 + a + \frac{1}{2}\bar{x} + \sqrt{x_L x_H} - p. \tag{2.22}$$

Thus, firm i's profit in the quality subgame is

$$\pi_i(x_H, x_L) = \frac{1}{2}\left[a + \frac{1}{2}\frac{x_L + x_H}{2} + \frac{1}{2}\sqrt{x_L x_H} - \frac{1}{2}\theta_i\,(x_i)^2\right]. \tag{2.23}$$

[15]Under monotonic preferences represented by the Bernoulli utility function U, the certainty equivalent (CE_L) associated to a fair lottery L between x_L and x_H is defined by $U(CE_L) = \frac{1}{2}U(x_L) + \frac{1}{2}U(x_H) \equiv EU(L)$. If $U(x) = \sqrt{x}$, then $CE_L = \frac{1}{2}\bar{x} + \frac{1}{2}\sqrt{x_L x_H}$. The same preferences can be represented by the utility function $V(L) = CE_L$ since for two lotteries L_1 and L_2, $L_1 \succsim L_2$ iff $EU(L_1) \geq EU(L_2)$, which by definition is equivalent to $U(CE_1) \geq U(CE_2)$, which by monotonicity is equivalent to $CE_1 \geq CE_2$, thus justifying our assumption that demand is linear in the certainty equivalent.

This expression is strictly concave in x_i. Because consumers are willing to pay less when they face a large quality differential, there is now an additional incentive for firms to choose qualities that are close to each other. As a result, the optimal choice of quality is no longer independent of the other firm's decision. Nevertheless, there is a unique pure-strategy Nash equilibrium of the quality subgame in which

$$\begin{aligned} x_H^U &= \left(\tfrac{\theta_L}{\theta_H}\right)^{\frac{2}{3}} \cdot x_L^U \\ x_L^U &= \tfrac{1+\left(\frac{\theta_L}{\theta_H}\right)^{\frac{1}{3}}}{4\theta_L}. \end{aligned} \tag{2.24}$$

It is easily verified that firm H's (firm L's) quality is higher (lower) than under risk neutrality. Firm H produces only a fraction of the quality chosen by firm L. A monopolist owning both plants would keep the ratio of x_H to x_L unchanged but set

$$x_L^M(0) = \frac{1+\left(\frac{\theta_L}{\theta_H}\right)^{\frac{1}{3}}}{2\theta_L} = 2x_L^U, \tag{2.25}$$

showing that free riding is as momentous under risk-aversion as under risk-neutrality. Because the marginal valuation of quality improvements is still the same across consumers, the monopolist's choices would be mimicked by a social planner seeking to maximize total surplus. Hence, so far all the qualitative features of the model are preserved.

Observe now that once the MQS is doubly binding, i.e. when $x_L = x_H = m$, the profit function reduces to the one studied in Section 4! So there is no change to the Pareto frontier of the bargaining set and no change either to the second-most efficient MQS. The disagreement point only is affected by the introduction of risk-aversion, moving northwestwardly as a smaller quality differential makes for less initial free riding. Consequently, the agreed-upon standard is higher than under risk-neutrality as firm H's bargaining position is weakened. In turn, that implies that the adopted standard is closer to the second-best standard than under risk-neutrality.

One should be careful with the interpretation of this model. The information structure is such that consumers are in all circumstances aware of the characteristics of the lottery they face. That is, they know the individual product qualities but are unable physically to distinguish the two products at the time of purchase. The absence of labelling or branding is key here.[16]

2.5.3 Unequal market shares

Recall that α_H was taken to be firm H's market share. Since we maintain the assumption that aggregate supply equals one, it is also firm H's fixed quantity. For simplicity we will denote it by α and let $1-\alpha$ stand for firm L's quantity. When considering this situation, it is again essential to be very clear about the information structure characterizing the demand side of the model: here, as in the original "lemons" example, the probability to draw a variant of a given quality equals the market share of that variant.

In Stage 2, since consumers care about the weighted average of qualities, firm i's profits are given by

$$\pi_i\left(x_L, x_H\right) = \alpha_i\left[\alpha_i x_i + (1-\alpha_i)x_{-i} - \frac{1}{2}\theta_i \cdot (x_i)^2 + a\right]. \tag{2.26}$$

A monopolist owning both plants would again select

$$x_H^M = \frac{1}{\theta_H} \qquad x_L^M = \frac{1}{\theta_L}, \tag{2.27}$$

as would a benevolent social planner. If they had to specify quality uniformly, they would both choose

$$x^M = \frac{1}{\alpha\theta_H + (1-\alpha)\theta_L}, \tag{2.28}$$

[16]The entire analysis could be conducted by assuming that $U_t = a + t + x^{\frac{1}{k}} - p$ for $k > 1$. The parameter k, because it governs the curvature of the function U, is a measure of quality risk aversion. Demand could be taken to be linear in the certainty equivalent $CE = \left[\frac{1}{2}\left(x_L\right)^{1/k} + \frac{1}{2}\left(x_H\right)^{1/k}\right]^k$, a familiar CES form.

the weighted harmonic mean of x_H^M and x_L^M.

In the absence of a MQS, the duopolists would choose

$$x_H^U = \frac{\alpha}{\theta_H} \qquad x_L^U = \frac{1-\alpha}{\theta_L}, \tag{2.29}$$

which is a direct generalization of the results in Section 4. Intuitively, the downward quality distortion is caused by the positive externality associated with a quality improvement. Since this externality is proportional to market share, so is the quality distortion: the smaller firm H's market share, the easier it takes it. The associated profits are

$$\begin{aligned} \pi_H^U &= \alpha\left[\tfrac{1}{2}\tfrac{\alpha^2}{\theta_H} + \tfrac{(1-\alpha)^2}{\theta_L} + a\right] \\ \pi_L^U &= (1-\alpha)\left[\tfrac{\alpha^2}{\theta_H} + \tfrac{1}{2}\tfrac{(1-\alpha)^2}{\theta_L} + a\right] \end{aligned} \tag{2.30}$$

It is easy to show that for any $\theta_H > \theta_L$, there exists $d \in \left(\frac{1}{3}, \frac{1}{2}\right)$ such that $\pi_H^U > \pi_L^U$ for any $\alpha \in [d, 1]$. That is, provided it is not minuscule, firm H makes more profit than firm L in the absence of an effective standard.

Since bargaining really takes place over margins (by scale invariance of the KS solution), it is more interesting to look at the conditions under which firm H's margin, denoted by ξ_H, is greater than firm L's, denoted by ξ_L. Observe that

$$\xi_L - \xi_H = \frac{1}{2}\left[\frac{\alpha^2}{\theta_H} - \frac{(1-\alpha)^2}{\theta_L}\right], \tag{2.31}$$

which is strictly increasing in α. It is a matter of computation to show that

$$\xi_H \geq \xi_L \Longleftrightarrow \alpha \leq \frac{1}{1+\sqrt{\frac{\theta_L}{\theta_H}}}, \tag{2.32}$$

the last quantity always being greater than 1/2.

We have to distinguish two cases here. If α is small, then $x_H^U \leq x_L^U$; that is, firm H free-rides on firm L. If α is large enough, then we are in the somewhat perverse case where the low-cost firm enjoys so low a market share that it chooses to free-ride on the high-cost firm. Our results can be straighforwardly generalized in the first case.

From (2.29) above, it is straightforward to derive that

$$x_H^U \leq x_L^U \Longleftrightarrow \alpha \leq \frac{1}{1+\frac{\theta_L}{\theta_H}}. \tag{2.33}$$

Observe that this treshold, which is always equal to, or greater than, $1/2$, is always larger than $1/\left(1+\sqrt{\theta_L/\theta_H}\right)$.

We get a table of profits as functions of the MQS very similar to the one in the equal market shares' case (omitting the a-terms):

for $m < \frac{\alpha}{\theta_H}$:	$\pi_H = \alpha\left[\frac{1}{2}\frac{\alpha^2}{\theta_H} + \frac{(1-\alpha)^2}{\theta_L}\right]$	$\pi_L = (1-\alpha)\left[\frac{\alpha^2}{\theta_H} + \frac{1}{2}\frac{(1-\alpha)^2}{\theta_L}\right]$
for $m \in [\frac{\alpha}{\theta_H}, \frac{1-\alpha}{\theta_L}]$:	$\pi_H = \alpha\left[\alpha m + \frac{(1-\alpha)^2}{\theta_L} - \frac{1}{2}\theta_H m^2\right]$	$\pi_L = (1-\alpha)\left[\alpha m + \frac{1}{2}\frac{(1-\alpha)^2}{\theta_L}\right]$
for $m > \frac{1-\alpha}{\theta_L}$:	$\pi_H = \alpha\left[m - \frac{1}{2}\theta_H m^2\right]$	$\pi_L = (1-\alpha)\left[m - \frac{1}{2}\theta_L m^2\right]$

Observe again that once the standard is binding for both firms (for $m > (1-\alpha)/\theta_L$), the unit margins are exactly the same as in the equal market shares' case. Thus, there is no change to the Pareto frontier of the bargaining set; only the disagreement point is affected by the quantity asymmetry. In particular, for $\alpha \leq 1/2$, there is initially more free riding and that enhances firm H's bargaining position, leading to the adoption of a standard that is lower than in the case of equal market shares. Any decrease in α causes the disagreement point to move southeastwardly, leading to a decrease in the adopted standard (if anything).

It is possible to generalize the results concerning the role of cost homogeneity in the determination of the bargaining outcome. Firm H never benefits from a standard that affects its behavior only. It can benefit from a doubly binding standard only if θ_L/θ_H is greater than $[2(1-\alpha)]/[1+\alpha]$, which provides an upper bound for the "small homogeneity" region. In that case $1/\theta_H$ remains its favorite outcome. This condition cannot be satisfied if $\alpha < 1/3$.[17] Thus, if the market share enjoyed by the high-cost firm is too small, free riding is intense and there is no hope of getting the problem solved through a standard-setting procedure that puts any weight on

[17]Indeed, in that case $[2(1-\alpha)]/[1+\alpha] > 1$.

firm H's profit, whatever the level of cost homogeneity. Moreoever, the bound is decreasing in α. Taken together, these observations justify the following remark.

Remark 9 *In the region where the high-cost firm initially free-rides on the low-cost firm's effort, the smaller firm H's market share, the greater the extent of free riding, and the larger the level of cost homogeneity required for the bargaining problem to be non-degenerate.*

That is, for firms to agree on some effective standard, it is imperative that they not be too dissimilar in all respects. If their size difference is big, then the cost differential must be small in order for them to find a common ground.

Provided cost heterogeneity is not too large, firm H is willing to agree to some standards. The highest standard which it is willing to accept is given by

$$z_2 = \frac{1 + \sqrt{1 - \alpha^2 - 2\left(1-\alpha\right)^2 \frac{\theta_H}{\theta_L}}}{\theta_H}. \tag{2.34}$$

Firm L's most optimistic expectation of profit corresponds to its favorite standard of all, $1/\theta_L$, or z_2, whichever is smaller. Under large cost homogeneity, defined as

$$\theta_L/\theta_H \geq 1/\left[\alpha(2-\alpha) + \alpha\sqrt{\alpha^2 - 4\alpha + 3}\right], \tag{2.35}$$

the former is larger.

Therefore, we still have the three regions identified in Section 4 and the results carry over.

2.6 Conclusion

We have studied a simple model in which consumers cannot observe the quality (or quality effort) choices made by two producers but demand depends instead upon the average quality of the goods available in the market. Before specifying the quality aspect of their products, the two firms engage into bargaining over the adoption of a strictly and costlessly enforced minimum quality (effort) standard. The Kalai-Smorodinsky solution is assumed to capture the outcome of this negotiation.

In the absence of a standard, firms underprovide quality as a result of a classical public good problem. We have shown that if firms have very different costs for quality, then they cannot agree on any common standard, except perhaps a completely ineffective one. When firms are not too dissimilar, then the KS solution selects a standard that is always lower than the joint-profit maximizing, or for that matter the (second-most) efficient, level. The adopted standard always lies closer to the high-cost producer's favorite choice than to the low-cost producer's. Thus, there is a tendency for a duopoly deciding about the minimal level of quality to be provided in a particular industry to set it too low. This somewhat contrasts with Leland (1979)'s finding but it needs to be noted that by prohibiting side transfers and fixing quantities, we have in effect remove any possibility for the industry to use the standard so as to restrict output.

In our model, the adopted standard often decreases in the cost of providing quality of any firm. Nonetheless, when firms' costs are such that the high-cost producer can agree only to a small range of possible norms, the equilibrium standard increases with the low-cost producer's cost as the sudden expansion of the interval of norms to which the inefficient producer is amenable enhances the low-cost producer's bargaining position.

The question of the robustness and generality of these results immediately arises. One may want to get rid of the fixed-quantity assumption but the introduction of a quantity choice considerably complicates the analysis. The extension to larger oligopolies seems more promising. The study of such a "grand bargaining" could be the first step in the study of a more general partition-game, or club-formation, model in which firms "decide" which producers to join to create a "label" or "brand".

Of course, such a model will necessitate a specification of the demand-side less rudimentary than the one in this article and awaits future research.

2.7 Appendix

2.7.1 Intermediate homogeneity case

The equation defining the agreed-upon standard is as follows:

$$\frac{\frac{1}{2}\left[m-\frac{1}{2}\theta_L m^2+a\right]-\frac{1}{2}\left[\frac{1}{4\theta_H}+\frac{1}{2}\frac{1}{4\theta_L}+a\right]}{\frac{1}{2}\left[m-\frac{1}{2}\theta_H m^2+a\right]-\frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H}+\frac{1}{4\theta_L}+a\right]}=\frac{\frac{1}{2}\left[z_2-\frac{1}{2}\theta_L (z_2)^2+a\right]-\frac{1}{2}\left[\frac{1}{4\theta_H}+\frac{1}{2}\frac{1}{4\theta_L}+a\right]}{\frac{1}{2}\left[\frac{1}{\theta_H}-\frac{1}{2}\theta_H\left(\frac{1}{\theta_H}\right)^2+a\right]-\frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H}+\frac{1}{4\theta_L}+a\right]} \tag{2.36}$$

2.7.2 Proofs

Proof of Lemma 1 Let $\gamma : \mathbb{R}_+ \to \mathbb{R}^2$ be the function that to each $m \geq 0$ associates the profit gain vector:

$$\gamma(m) \equiv (\gamma_H(m), \gamma_L(m)) \equiv \left(\pi_H\left[x_H^*(m), x_L^*(m)\right] - \pi_H^U, \quad \pi_L\left[x_H^*(m), x_L^*(m)\right] - \pi_L^U\right). \tag{2.37}$$

Then by definition the feasible set, F, of gains is the range of γ.[18] Since the intersection of F with $\mathbb{R}_+^2$ is what really matters, one looks for the best outcome for firms in the positive orthant only.

It was argued in Section 4.1 that if condition (2.18) was not satisfied, then firm H's profit could never be greater than when the standard was not binding. That is: if $\frac{\theta_L}{\theta_H} < \frac{2}{3}$, then $\gamma_H(m) < \gamma_H(0)$ for all $m > \frac{1}{2\theta_H}$. That takes care of the small homogeneity case.

[18] As a matter of fact, the KS solution is usually defined on convex feasible sets. Technically speaking, we take F to be the convex and comprehensive hull of the range of γ. See Thomson (1994) for an exact definition. Convexity is obtained by assuming that the negotiators can always randomize between outcomes. Comprehensiveness corresponds to a "free-disposal-of-profit assumption" that is innocuous in our context.

Suppose now that condition (2.18) is satisfied. We have to prove that

$$\text{if } \frac{2}{3} < \frac{\theta_L}{\theta_H} < \frac{4}{3+\sqrt{5}}, \text{ then } \begin{cases} \arg \max\limits_{m \in \gamma^{-1}(\mathbb{R}^2_+)} \gamma_H(m) = \frac{1}{\theta_H} \\ \arg \max\limits_{m \in \gamma^{-1}(\mathbb{R}^2_+)} \gamma_L(m) = z_2 = \frac{1+\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H} \end{cases};$$

$$\text{if } \frac{4}{3+\sqrt{5}} \leq \frac{\theta_L}{\theta_H} \leq 1, \text{ then } \begin{cases} \arg \max\limits_{m \in \gamma^{-1}(\mathbb{R}^2_+)} \gamma_H(m) = \frac{1}{\theta_H} \\ \arg \max\limits_{m \in \gamma^{-1}(\mathbb{R}^2_+)} \gamma_L(m) = \frac{1}{\theta_L} \end{cases}.$$

Since condition (2.18) is satisfied, $\gamma_H(m)$ reaches its maximum at $m = \frac{1}{\theta_H}$. Since $\theta_L \leq \theta_H$ and $\frac{1}{\theta_H} > \frac{1}{2\theta_L}$, we necessarily have $\gamma_L(\frac{1}{\theta_H}) \geq \gamma_H(\frac{1}{\theta_H}) > 0$ and so $\gamma(\frac{1}{\theta_H}) \in \mathbb{R}^2_+$. Hence, we always have

$$\arg \max_{m \in \gamma^{-1}(\mathbb{R}^2_+)} \gamma_H(m) = \frac{1}{\theta_H}. \tag{2.38}$$

From the analysis in Section 4.1, we know that the unrestricted maximizer of γ_L is $\frac{1}{\theta_L}$. The range of standards $\gamma^{-1}(\mathbb{R}^2_+)$ is $\{m \mid \gamma_H(m) \geq 0\}$, as $\gamma_L(m) \geq \gamma_H(m)$ for $m \geq \frac{1}{2\theta_H}$ and π_L^U as well as π_H^U are positive. It is found by computing the roots z_1 and z_2 of the following equation:

$$\frac{1}{2}\left[z - \frac{1}{2}\theta_H z^2 + a\right] - \frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H} + \frac{1}{4\theta_L} + a\right] = 0, \tag{2.39}$$

which equates firm H's profit from the imposition of a binding standard[19] to firm H's profit in the absence of a standard. The discriminant of that equation is $\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}$, which is strictly positive if condition (2.18) above is met. So we have:

$$z_1 = \frac{1 - \sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H} \tag{2.40}$$

$$z_2 = \frac{1 + \sqrt{\frac{3}{4} - \frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}. \tag{2.41}$$

[19]That is: binding for *both* firms. We saw that Firm 1's profit is less than its guaranteed profit under no standard if the standard does not affect Firm 2's behavior.

and[20] $\gamma^{-1}(\mathbb{R}^2_+) = (z_1, z_2)$. It is obvious that $z_1 < \frac{1}{\theta_H}$. The question of the comparison between z_2 and $\frac{1}{\theta_L}$ can be settled by studying the equation

$$\frac{1+\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H} = \frac{1}{\theta_L} \tag{2.42}$$

or, equivalently,

$$1+\sqrt{\frac{3}{4}-\frac{1}{2}z} = z, \tag{2.43}$$

where $z = \frac{\theta_H}{\theta_L}$. The roots are given by $\frac{3}{4} \pm \frac{\sqrt{5}}{4}$. Thus, if $1 \leq \frac{\theta_H}{\theta_L} \leq \frac{3}{4} + \frac{\sqrt{5}}{4}$, then

$$\arg \max_{m \in \gamma^{-1}(\mathbb{R}^2_+)} \gamma_L(m) = \frac{1}{\theta_L}. \tag{2.44}$$

If $\frac{3}{4} + \frac{\sqrt{5}}{4} < \frac{\theta_H}{\theta_L} < \frac{3}{2}$, then

$$\arg \max_{m \in \gamma^{-1}(\mathbb{R}^2_+)} \gamma_L(m) = z_2 = \frac{1+\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}}{\theta_H}. \tag{2.45}$$

END OF PROOF.

Proof of Proposition 3 At the KS solution, the following equation is verified:

$$\frac{\frac{1}{2}\left[m-\frac{1}{2}\theta_L m^2+a\right]-\frac{1}{2}\left[\frac{1}{4\theta_H}+\frac{1}{2}\frac{1}{4\theta_L}+a\right]}{\frac{1}{2}\left[m-\frac{1}{2}\theta_H m^2+a\right]-\frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H}+\frac{1}{4\theta_L}+a\right]} = \frac{\frac{1}{2}\left[\frac{1}{\theta_L}-\frac{1}{2}\theta_L\left(\frac{1}{\theta_L}\right)^2+a\right]-\frac{1}{2}\left[\frac{1}{4\theta_H}+\frac{1}{2}\frac{1}{4\theta_L}+a\right]}{\frac{1}{2}\left[\frac{1}{\theta_H}-\frac{1}{2}\theta_H\left(\frac{1}{\theta_H}\right)^2+a\right]-\frac{1}{2}\left[\frac{1}{2}\frac{1}{4\theta_H}+\frac{1}{4\theta_L}+a\right]} \tag{2.46}$$

It can be rewritten

$$-\frac{3(\theta_L-\theta_H)}{16\theta_H\theta_L}m^2 + \frac{5(\theta_L-\theta_H)}{8\theta_H\theta_L}m + \frac{1}{8}\left(\frac{1}{\theta_L}-\frac{1}{\theta_H}\right) = 0 \tag{2.47}$$

or

[20] The range of standards for which there are mutual gains and the interests of the two bargainers diverge (that is, corresponding to the downward-sloping section of the feasible set frontier) is $\left[\frac{1}{\theta_H}, \min(z_2, \frac{1}{\theta_L})\right]$. The interval $\left[\frac{1}{2\theta_L}, \frac{1}{\theta_H}\right]$ corresponds to an upward-sloping section of the feasible set frontier as both firms' profits are going up with m. Similarly, the interval $[\frac{1}{\theta_L}, +\infty)$ also corresponds to an upward-sloping section of the profit possibility set as both firms' profits are going down with m. Over these two latter intervals, the interests of the two bargainers can be said to be aligned.

$$-\frac{3\left(\theta_L+\theta_H\right)}{2}m^2+5m-\frac{\theta_L+\theta_H}{\theta_H\theta_L}=0 \tag{2.48}$$

The discriminant of this equation is

$$\Delta=25-6\frac{\left(\theta_L+\theta_H\right)^2}{\theta_H\theta_L}, \tag{2.49}$$

which is positive for $\theta_H \leq \frac{3}{2}\theta_L$.

Solving for m:

$$m=\frac{5\pm\sqrt{25-6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}}{3(\theta_H+\theta_L)} \tag{2.50}$$

Both roots are positive but it can be shown that only the bigger one lies in $\left[\frac{1}{\theta_H},\frac{1}{\theta_L}\right]$, as the dashed line standing for the ratio of maximal profit increments on Figure 3 always crosses the upward-sloping section of the feasible set before crossing its downward-sloping section. Thus we have that in equilibrium:

$$\hat{m}=\frac{5+\sqrt{25-6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}}{3(\theta_H+\theta_L)} \tag{2.51}$$

END OF PROOF.

Proof of Proposition 4 (i) Observe that

$$\frac{\partial}{\partial\theta_H}\left[\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}\right]=\frac{\theta_H(\theta_H+\theta_L)}{(\theta_H\theta_L)^2}\left(\theta_H-\theta_L\right), \tag{2.52}$$

which is positive for any values of θ_H and θ_L since by assumption $\theta_H > \theta_L$. Thus, when θ_H goes up, the numerator in $\hat{m}$ goes down. Since the denominator goes up, $\hat{m}$ is bound to decrease.

(ii) Observe that

$$\frac{\partial\hat{m}}{\partial\theta_L}=\frac{\frac{3\theta_H(\theta_H-\theta_L)(\theta_H+\theta_L)^2}{(\theta_H\theta_L)^2\sqrt{25-6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}}-\left(5+\sqrt{25-6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}\right)}{3(\theta_H+\theta_L)^2}. \tag{2.53}$$

To know the sign of this derivative, it is sufficient to study the numerator. If we can show that

$$\frac{3\theta_H\left(\theta_H-\theta_L\right)\left(\theta_H+\theta_L\right)^2}{(\theta_H\theta_L)^2\sqrt{25-6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}}<\sqrt{25-6\frac{(\theta_H+\theta_L)^2}{\theta_H\theta_L}}, \tag{2.54}$$

then we are done. Manipulating this inequality, one gets

$$\frac{\theta_H(\theta_H+\theta_L)^3}{(\theta_H\theta_L)^2}<\frac{25}{3}. \tag{2.55}$$

Observe that the left-hand side is strictly increasing in θ_H. By assumption $\theta_H > \theta_L$, so

$$\frac{\theta_H(\theta_H+\theta_L)^3}{(\theta_H\theta_L)^2}<\frac{\theta_H(2\theta_H)^3}{(\theta_H)^2}=\frac{8(\theta_H)^4}{(\theta_H)^4}. \tag{2.56}$$

The right-hand side of this inequality is of course smaller than $\frac{25}{3}$. Hence, $\frac{\partial\tilde{m}}{\partial\theta_L}<0$ for any θ_H.

END OF PROOF.

Proof of Proposition 7 Suppose that, for a given pair (θ_H,θ_L) such that $\frac{3}{4}+\frac{\sqrt{5}}{4}<\frac{\theta_H}{\theta_L}<\frac{3}{2}$, Equation (2.36) holds for a certain $\tilde{m}$, taken to be fixed in what follows.

Suppose that θ_H increases infinitesimally. D will go down as it is a positive linear function of $\frac{1}{\theta_H}$. B will decrease by still a bigger percentage for initially, Firm 1's actual profit increment is less than its maximum profit increment, and the change in Firm 1's cost brought about by the increase in θ_H is proportional to the prevailing effort level, $\tilde{m}$, which is bigger than $\frac{1}{\theta_H}$.

A will go up following the decrease in Firm 2's guaranteed profit. (With higher cost, Firm 1 free-rides even more in the absence of a standard.) Without careful calculations, it is not possible to tell whether C will increase or decrease. On one hand, Firm 2's guaranteed profit goes up but on the other hand, its maximal profit $z_2-\frac{1}{2}\theta_L(z_2)^2$ goes down, as $\partial z_2/\partial\theta_H$ is negative. Yet, from this extra-impact on Firm 2's maximal profit (and from the fact that initially Firm 2's actual profit increment is less than its maximum profit increment), it is clear that in any case C grows by a strictly smaller percentage than A.

Thus, for a fixed $\tilde{m}$, we have that A/B grows by a strictly greater percentage than C/D. Therefore, $\tilde{m}$ must have decreased in order to preserve Equation (2.36).

END OF PROOF.

Proof of Proposition 8 Suppose that, for a given pair (θ_H, θ_L) such that $\frac{3}{4}+\frac{\sqrt{5}}{4} < \frac{\theta_H}{\theta_L} < \frac{3}{2}$, Equation (2.36) holds for a certain $\tilde{m}$, taken to be fixed in what follows.

Suppose that θ_L increases infinitesimally. Firm 1's maximal profit increment, D, goes up as its guaranteed profit goes down and its maximal profit remains unchanged. Firm 1's actual profit increment, B, goes up by a higher percentage as the change in guaranteed profit is the same but applies to a smaller initial value. Firm 2's actual gain, A, goes down. Indeed, one has

$$\frac{dA}{d\theta_L} = \frac{1}{2}\left[\left(\frac{1}{2\theta_L}\right)^2 - (\tilde{m})^2\right] \tag{2.57}$$

and $\tilde{m}$ is of course greater than $\frac{1}{2\theta_L}$. So the left-hand side of Equation (2.36), A/B, goes down.

If it can be shown that the right-hand side goes up, then there will be no doubt that the standard has to increase in order for Equality (2.36) to remain satisfied. Observe that

$$\frac{dC}{d\theta_L} = \frac{1}{8(\theta_L)^2} - \frac{1}{2}(z_2)^2 + (1-\theta_L z_2)\frac{dz_2}{d\theta_L} \tag{2.58}$$

and

$$\frac{dz_2}{d\theta_L} = \frac{1}{4(\theta_L)^2\sqrt{\frac{3}{4}-\frac{1}{2}\frac{\theta_H}{\theta_L}}} \tag{2.59}$$

It is readily seen that

$$\lim_{\theta_L \to \frac{2}{3}\theta_H} \frac{dC}{d\theta_L} = +\infty \tag{2.60}$$

By continuity, $dC/d\theta_L$ is very large in the neighborhood of $\frac{2}{3}\theta_H$. Thus, C/D goes up in this neighborhood, provided it is sufficiently small. As a result, $\tilde{m}$ needs to increase in order for Equation (2.36) to be verified again.

END OF PROOF.

CHAPTER 3

THE MARKET FOR MELONS: COURNOT COMPETITION WITH UNOBSERVABLE QUALITIES

ABSTRACT[1] Two firms produce different qualities at possibly different, constant marginal costs. They compete in quantities on a market where buyers only observe the average quality supplied. The model is a generalization of the standard Cournot duopoly, which corresponds to the special case where the two qualities are equal. When the quality differential is large, the firms' output levels are not always strategic substitutes. There can be no, or up to three pure-strategy equilibria. Yet, as long as the cost differential is not extreme, there always exists a stable duopolistic equilibrium. In that sense, strategic quantity-setting helps prevent market unraveling.

KEYWORDS: Cournot competition, quality, duopoly, asymmetric information, Nash equilibrium.

JEL CODES: D43, D82, L13, L15.

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3.1 Introduction

With summer comes the time of relishing those flavorful and refreshing melons which you find at your local marketplace. Well, they are not always flavorful, are they? As a matter of fact, it is not uncommon to get disappointed with a gourd which looked particularly enticing at the time of purchase. If only you remembered the brand name on the sticker (possibly) affixed to the last one you ate, which you particularly savored, but you don't! That would not be a problem if your local seller consistently supplied melons from the same source but sellers typically don't. They very often adjust their inventory in accordance with the availaibility, characteristics and prices of the different products sold on wholesale markets. Often, retailers sell different batches of a given fruit or vegetable at the same price, indepently of their origin; in supermarkets, cases are just stacked and only the curious and unhurried shoppers pay attention to the slight differences in the boxes' appearance, wondering whether they should act upon this information, and dig the pile or not.

This description makes clear that melons are experience goods, whose quality is not observable at the time of purchase. This quality is also subject to some variation at the retail level. Although we all have a friend who claims to know how to choose a good melon, it is clear that most of us face a lottery in doing so. In these circumstances, our willingness to pay might well depend upon the features of this lottery. We here make the assumption that the quantity demanded is linear in the average quality, a magnitude that can often be observed or infered by consumers. (Indeed, by word-of-mouth, one generally gets an idea of the worth of the season's harvest.)

Abstracting from production and retailing details, we study a game in which two producers of some variants of a given good have to decide about the quantity they will bring to the market on which their undistinguishable products are sold. Given these quantities and the corresponding average quality, which consumers observe or infer, the market is cleared by setting the price so as to equate demand with supply. In effect, the two producers compete *à la Cournot* with given but unobservable qualities when consumers have correct beliefs regarding the average quality in all circumstances. We attempt at characterizing the pure-strategy Nash equilibria of the game, which is a generalization of the standard Cournot game.

The existence of an unobserved difference in quality introduces an additional effect into the Cournot model. When a producer considers an increase in the quantity he or she brings to the market, he or she must anticipate not only that the market price will decrease along the current demand curve but also that the average quality will change, shifting the demand curve altogether. The high-quality producer thus has an incentive to produce more than the typical Cournot quantity, while the low-quality producer is led to produce less. If the marginal cost of production does not increase too quickly with quality, in equilibrium the high-quality producer produces more than the low-quality producer, even if she faces higher costs. This is the case, for instance, whenever quality is determined by an initial investment affecting the fixed cost but not the variable cost of production. In a sense, in this situation, there is favorable, or advantageous, selection. More generally, the strategic behavior of the producers mitigates adverse-selection phenomena of the type described by Akerlof (1970). In particular, it is easy to come up with examples where the only competitive equilibrium involves unraveling of the market for "lemons", whereas on our market for melons, for moderate cost differentials, high-quality products continue being supplied, sometimes on a high scale.

Because of this feature of quantity choice under asymmetric information, there are instances in which consumers would prefer to face two unequally able producers rather than two identical producers displaying the (unweighted) average level of ability. That is, assuming that melon producers can be ranked on a linear quality scale, consumers could prefer their local market being supplied by a first-class producer along with a third-class producer to having the certainty of buying a second-class melon, for in equilibrium the average quality will increase more than the price.

At the same time, the unobserved difference in quality have the potential to give rise to surprising outcomes. Large quality differentials can produce a non-monotonic best-response curve for the low-quality firm, and a discontinuous curve for the high-quality firm. That can lead to the non-existence of an equilibrium in pure strategies. Even in cases where the quality differential is small and firms' programs are well-behaved, up to three pure-strategy equilibria may co-exist, thus raising an equilibrium selection problem.

Importantly, our setting requires that goods be undistinguishable to the eyes of the potential buyers. Thus, either the legal system does not support proprietary brands (as in the important case of counterfeiting), or the costs of establishing or maintaining such brands are prohibitive.

We do not want to claim that a model where goods are undistinguishable and producers do not set their price is general, although we believe that some concentrated agricultural or mineral product markets correspond to that description. We note that, even in environments where producers are straightforwardly identifiable, such as the markets for wines or spirits, where labelling or branding are common, high-quality producers very often express their fear that the market be flooded by low-quality variants taking advantage of the good "reputation" of the product and depressing its price. For example, it is arguably hard to confuse a bottle of Champagne from a *grande maison* with a bottle of sparkling wine produced in any other region by an unknown wine-grower. Yet, Champagne producers have always protested against the use of this name outside the historical region of production. This might well be an anti-competitive strategy but it is also likely that, because the purchase of sparkling wine is not repeated enough (or information acquisition, or processing, costs are high, or consumers have cognitive limitations), purchasers tend to bunch these distinguishable products into the same category. Indeed, concerns of this kind have led members of the World Trade Organization to grant a so-called higher level of protection to the place names used to identify the origin and quality, reputation or other characteristics of wines and spirits. Thus, we believe that there are many markets on which products are not absolutely undistinguishable but the asymmetric information problem we tackle here is present, to some extent, with the same qualitative consequences. Similarly, outside the realm of centralized markets, firms very likely have some pricing power. We take the quantity competition assumption to stand for a form of moderate competition where the law of one price does hold but firms cannot commit to serve any level of demand addressed to them, as opposed to the case of Bertrand competition.

There is of course a voluminous literature on Cournot competition, thoroughly surveyed, most recently, by Vives (1999). Our model is not the first not to guarantee the existence of a pure-strategy Nash equilibrium under quantity competition. Early examples were provided (in a more general context) by Roberts and Sonnenschein (1976). Even in the homogenous product case, some well-accepted demand or cost structures can lead to the non-quasiconcavity of firms' payoffs. In our model, the linear presence of the quality average term in the inverse demand function is sufficient to generate some non-convexity of profits. (We argue that this problem is not due to our functional form but is instead a general feature of the economic situation of interest.) The study of Cournot competition with differentiated products was marked by the seminal contributions of Singh and Vives (1984) and Vives (1985) but to our knowledge, our analysis is the first to incorporate an element of asymmetric information between firms and consumers, in the spirit of Akerlov (1970)'s market for lemons.

We introduce the formal model in Section 2. Section 3 briefly recalls the standard Cournot model, which corresponds to the special case when qualities are identical. Section 4 deals with the general features of the case when qualities are dissimilar. Section 5 attemps at classifying the equilibrium outcomes on the basis of cost and quality heterogeneity. Section 6 develops some welfare considerations. Section 7 concludes.

3.2 Model

We first describe the model in its general form. After showing the unavoidable difficulties it leads to, we describe the special case on which we will focus.

Two firms indexed by $i \in \{L, H\}$ produce two variants of the same good, whose qualities are denoted x_L and x_H, respectively. We assume that $0 < x_L \leq x_H$. It is thus understood that our choice of subscript corresponds to the quality ranking of the variants. The cost of production depends upon quantity and quality and is given by a function $\mu : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$. Because the firms' qualities are exogenously fixed, we will slightly abuse notation by indexing μ with i. That is, $\mu_i(q_i) \equiv \mu(q_i, x_i)$.

Firms face a market (inverse) demand that is given by

$$P = P(Q, \bar{x}), \tag{3.1}$$

a function that strictly increases with $\bar{x}$, the average quality of the units brought to the market, and decreases with Q, the total quantity produced by the duopolists. We make the assumption that for any quadruple (x_L, x_H, q_L, q_H), consumers infer or observe the "true" average quality $\bar{x}$. This is so even when firms consider "deviations" from the prescribed equilibrium behavior.[2]

[2]One could be precise about the time and information structure of the interaction, seen as an extensive-form game. Because consumers cannot observe the variants' quality, their demand obviously depends upon their beliefs regarding this quality. Our assumption here is that in all circumstances, on and off the equilibrium path, consumers' (uniform) belief

To avoid trivialities, we assume that

$$P(0, x_L) > c_L \quad \text{and} \quad P(0, x_H) > c_H.$$

These inequalities reflect the idea that, should one only of these goods be sold on the market, it could be profitably supplied.[3] We will refer to this assumption as the *profitable supply assumption.*

Firms compete *à la Cournot*, simultaneously deciding about the quantity $q_i \geq 0$ they will bring to the market and then letting a fictitious auctioneer set the price that equates market demand with market supply. Yet they are not price-takers: at the time they decide about their volume of production, they recognize that a change in q_i will affect the market price. We attempt at characterizing the (pure-strategy) Nash equilibria of this two-player, simultaneous-move game.

Each firm's profit is thus given by:

$$\pi_i(q_L, q_H) = q_i P\left(q_L + q_H, \frac{q_L x_L + q_H x_H}{q_L + q_H}\right) - \mu_i(q_i). \tag{3.2}$$

Using standard calculus notation, we have that

$$\frac{\partial \pi_i}{\partial q_i} = P(Q, \bar{x}) + q_i \frac{\partial P}{\partial Q} + \frac{\partial P}{\partial \bar{x}} \cdot \frac{q_L q_H (x_i - x_{-i})}{(q_L + q_H)^2} - \frac{\partial \mu_i}{\partial q_i}; \tag{3.3}$$

$$\begin{aligned} \frac{\partial^2 \pi_i}{(\partial q_i)^2} &= 2\frac{\partial P}{\partial Q} + q_i \frac{\partial^2 P}{\partial q_i \partial Q} + \frac{\partial^2 P}{\partial q_i \partial \bar{x}} \frac{q_L q_H (x_i - x_{-i})}{(q_L + q_H)^2} \\ &\quad + \frac{\partial P}{\partial \bar{x}} \cdot \frac{q_{-i}(x_i - x_{-i})}{(q_L + q_H)^2} \cdot \left(1 - \frac{2q_i}{q_L + q_H}\right) - \frac{\partial^2 \mu_i}{(\partial q_i)^2}; \end{aligned} \tag{3.4}$$

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corresponds to the correct average. In effect, we prevent firms from taking advantage of some inertia in the prevailing beliefs: although not directly observable, no "deviation" can take place at unchanged beliefs. This is likely if production takes time and output decisions have to be made well ahead of sales, which prevents "instantaneous deviations".

[3]These inequalities guarantee that the most enthusiastic consumer's willingness to pay will cover the cost of the "first" unit.

$$\begin{aligned}\frac{\partial^2\pi_i}{\partial q_{-i}\partial q_i} &= \frac{\partial P}{\partial Q}+q_i\frac{\partial^2 P}{\partial q_{-i}\partial Q}+q_i\frac{\partial P}{\partial \bar{x}}\frac{(q_H-q_L)(x_H-x_L)}{(q_L+q_H)^3}\\ &\quad+\frac{\partial P}{\partial \bar{x}}\frac{q_i(x_{-i}-x_i)}{(q_L+q_H)^2}+\frac{\partial^2 P}{\partial q_{-i}\partial \bar{x}}\cdot\frac{q_Lq_H(x_i-x_{-i})}{(q_L+q_H)^2}. \end{aligned} \tag{3.5}$$

These expressions make clear that the presence of the average-quality term considerably complicates the duopoly problem.

Assume for instance that the inverse demand curve intersects both axes and all its derivatives are finite. In that case,

$$\lim_{q_H\to 0}\frac{\partial^2\pi_H}{(\partial q_H)^2}=2\frac{\partial P}{\partial Q}-\frac{\partial^2\mu_H}{(\partial q_H)^2}+\frac{\partial P}{\partial \bar{x}}\cdot\frac{q_L(x_H-x_L)}{(q_L+q_H)^2}, \tag{3.6}$$

which is positive for sufficiently low q_L. That means that firm H's program is not always quasi-concave. Hence, the best-response correspondence may fail to be a continuous function and we cannot appeal to the usual fixed-point theorems to prove the existence of (at least) one pure-strategy equilibrium. The reason for the convexity of the profit function in this quantity range will soon become very clear.

Besides, observe that, since the very meaning of quality justifies $\partial P/\partial\bar{x} > 0$, it follows that whatever the assumption one is willing to make about the relationship between the marginal willingness to pay for quality and quantity (i.e. about the sign of $\partial^2 P/\partial q_{-i}\partial\bar{x}$), terms in $(x_i - x_{-i})$ and $(x_{-i} - x_i)$ both appear in one of the two cross-derivatives. That implies that the best-response of one of the two firms might well be non-monotonic, in which case we cannot appeal to the standard existence theorems based on super-, or sub-modularity. Again, the reason for this non-monotonicity will become clear in the special case to which we will restrict our attention.

These issues arise independently on the assumptions made about the curvature of the inverse demand curve or the cost function, as is usually the case in oligopoly theory. Instead, they are rooted in the economics of the problem at hand.

We therefore choose to concentrate on a special case. We take the simplest one, in which the inverse demand curve is linear in both arguments. Thus, the inverse demand curve is given by:

$$P = a + \bar{x} - bQ \tag{3.7}$$

where a is a positive demand-shifting parameter and b, a strictly positive demand-rotating parameter.

We also assume that the marginal cost of production only depends upon quality. That is, technology exhibits constant returns to scale and firms produce at unit costs c_L and c_H, respectively.

3.3 Standard Cournot duopoly

Setting $x_L = x_H = x$ gives the usual Cournot duopoly model as a special case. Indeed, both firms face a demand of the form

$$P = a + x - bQ. \tag{3.8}$$

Still, they can have different unit costs. In order to avoid any confusion, we choose the firm indices so that $c_L < c_H$.

Firm i's profit is given by

$$\pi_i = q_i \left[a + x - b\left(q_i + q_{-i}\right) - c_i\right], \tag{3.9}$$

where q_{-i} stands for the quantity produced by firm i's rival. For any q_{-i}, the profit function, being quadratic and strictly concave in q_i, admits a unique maximizer on $\mathbb{R}_+$, which, if interior, is characterized by the first-order condition:

$$a + x - 2bq_i - bq_{-i} - c_i = 0 \tag{3.10}$$

Disallowing negative quantities, one gets the best-response functions:

$$q_i = \max\left\{0, \frac{a + x - bq_{-i} - c_i}{2b}\right\}; i \in \{L, H\} \tag{3.11}$$

Because $dq_i/dq_{-i} < 0$ at the interior solution, the two choice variables are seen to be strategic substitutes.

Solving for the intersection of the two interior best-response curves, one gets:

$$\begin{aligned} q_L^* &= \frac{a + x + (c_H - 2c_L)}{3b} \\ q_H^* &= \frac{a + x + (c_L - 2c_H)}{3b} \\ Q^* &= \frac{2(a + x) - (c_L + c_H)}{3b} \\ P^* &= \frac{a + x + c_L + c_H}{3} \end{aligned}$$

The producer facing the highest production cost ends up producing less than its more efficient competitor in equilibrium.

One has to check that the values of q_L^* and q_H^* indeed lead both firms to remain active on the market, a situation to which we will refer as a *duopolistic equilibrium*. It is easily verified that if $c_H - c_L \geq a + x - c_H$, then firm H wants to withdraw from the market, for its margin turns negative.

There could then be an equilibrium in which the low-cost firm serves the market and the high-cost producer decides to withdraw. We call such a situation, in which only one firm remains active in equilibrium, a *monopolistic equilibrium*. At this equilibrium, firm L produces the monopoly quantity, q_L^M, and firm H's margin is negative at all quantity levels. From the best-response characterization, one gets the condition under which such an equilibrium exists:

$$bq_L^M \geq a + x - c_H, \tag{3.12}$$

which is verified if and only if

$$c_H - c_L \geq a + x - c_H. \tag{3.13}$$

So if the cost differential is large enough in comparison to firm H's margin on the first unit sold, then a monopolistic equilibrium exists and is unique. Conversely, if the cost differential is low enough, then the duopolistic equilibrium is the only equilibrium. If firms have the same cost function, then the duopolistic equilibrium is unique and symmetric.

In view of the subsequent analysis, we find it convenient to express these results by reference to a (symmetric) cost heterogeneity parameter, δ. So for any fixed c_L and c_H, let c be their arithmetic average and set $\delta = (c_H - c_L)/2$. Then, $c_L = c - \delta$ and $c_H = c + \delta$. We thus summarize this section with the following claim.

Claim 10 *In the case when the two firms produce a homogenous product ($x_L = x_H = x$), the game admits a unique Nash equilibrium. Given a linear inverse demand curve and a cost average, c, the nature of the equilibrium depends upon the level of cost heterogeneity. (i) If $\delta \geq (a + x - c)/3$, then firm L produces $q_L^M = (a + x - c + \delta)/(2b)$ and firm H withdraws from the market. (ii) If $\delta < (a + x - c)/3$, then both firms are active in equilibrium and*

$$\begin{aligned}
q_L^* &= \frac{a + x - c + 3\delta}{3b} \\
q_H^* &= \frac{a + x - c - 3\delta}{3b} \\
Q^* &= \frac{2(a + x - c)}{3b} \\
P^* &= \frac{a + x + 2c}{3}.
\end{aligned}$$

Observe that, at unchanged average, a small increase in the cost differential does not impact the aggregate variables. It only distorts the market shares in favor of the efficient firm.

3.4 Unobservable differences in quality

Suppose now that the two firms do no longer produce a homogenous product, and $x_L < x_H$.

Each firm's profit is given by[4]

$$\pi_i(q_L, q_H) = q_i \left[a + \frac{q_L x_L + q_H x_H}{q_L + q_H} - b(q_L + q_H) - c_i \right]. \tag{3.14}$$

It is a matter of computation to derive the following expressions, which we display here for reference:

$$\frac{\partial \pi_i}{\partial q_i} = a + \bar{x} - bQ - c_i - bq_i + \frac{q_L q_H (x_i - x_{-i})}{(q_L + q_H)^2} \tag{3.15}$$

$$\frac{\partial^2 \pi_i}{(\partial q_i)^2} = 2\frac{(q_{-i})^2 (x_i - x_{-i})}{(q_L + q_H)^3} - 2b \tag{3.16}$$

$$\frac{\partial^2 \pi_i}{\partial q_{-i} \partial q_i} = 2\frac{q_L q_H}{(q_L + q_H)^3}(x_{-i} - x_i) - b \tag{3.17}$$

Firms will now recognize that they can affect the market price in two distinct manners: by producing more, they depress the willingness to pay of the marginal consumer along the current demand curve, but by doing so they also change the average quality, which shifts the demand curve altogether. This additional effect is captured by the last term in equation (3.15), which did not appear in the standard Cournot model.

We first attempt at giving a general description of the nature of the problem that firms face and its consequence on the existence and features of equilibria. In the next section, we try to make these results precise for various combinations of cost and quality heterogeneity levels.

[4] In all rigor, given the usual meaning attached to a demand curve, we should write

$$\pi_i = q_i \left[\max \left\{ a + \frac{q_L x_L + q_H x_H}{q_L + q_H} - b(q_L + q_H), 0 \right\} - c_i \right].$$

3.4.1 Existence of interior solutions to the firms' problems

Observe that $\pi_L(0, q_H) = \pi_H(q_L, 0) = 0$ and $\lim_{q_i \to +\infty} \pi_i = -\infty$. With $x_L < x_H$, π_L is strictly concave in q_L for any $q_H \geq 0$. Since π_L is then single-peaked on $\mathbb{R}_+$, firm L's output decision is characterized by the first-order condition if and only if, given firm H's quality and quantity, firm L's margin on the "first" unit sold is positive (which amounts to saying that $\partial\pi_L(0, q_H)/\partial q_L \geq 0$). In turn, this condition is met when firm H does not depress the price too much by "flooding the market".

Remark 11 *The unique solution to firm L's problem is characterized by the first-order condition if and only if*

$$bq_H \leq a + x_H - c_L. \tag{3.18}$$

Denote ρ_H the supremum of the quantities q_H that elicit a strictly positive response from firm L.

By contrast, π_H may not be strictly concave in q_H everywhere (although it eventually turns so). In particular, there can be a convex section for low values of q_H if the quality differential is big relatively to the demand curve slope. So, either π_H, being continuous and eventually negative, achieves an interior global maximum for a finite $q_H > 0$, or there is a corner solution at $q_H = 0$, or both. The usual first-order condition is necessary in the first case but in general not sufficient. It takes a little work to establish the precise conditions under which firm H chooses to remain active on the market.

Proposition 12 *If it is the case that*

$$bq_L < \max\left\{a + x_L - c_H, \min\left\{x_H - x_L, \frac{(a + x_H - c_H)^2}{4(x_H - x_L)}\right\}\right\}, \tag{3.19}$$

then firm H's program admits a unique solution, and this solution satisfies the first-order condition. If $x_H - x_L > a + x_L - c_H$, then for $q_L = \frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$, firm H's program *admits two solutions, one interior solution satisfying the first-order condition and one corner solution. In all other cases, the unique solution to firm H's problem is* $q_H = 0$.

Proof. The question of whether $\pi_H(q_L, q_H)$ assumes some positive values on $(0, +\infty)$ for a fixed q_L can be settled by studying the "margin function"

$$\mu_H(q_L, q_H) = a + \frac{q_L x_L + q_H x_H}{q_L + q_H} - b\,(q_L + q_H) - c_H, \tag{3.20}$$

a well-defined rational function of q_H on $\mathbb{R}_+$, for any $q_L > 0$.

If μ_H takes positive values on $(0, +\infty)$, then there is an interior solution to the firm's problem as firm H can make a positive profit in that range.

Observe that on $[0, +\infty)$:

$$\frac{\partial \mu_H}{\partial q_H} = \frac{q_L(x_H - x_L)}{(q_L + q_H)^2} - b, \tag{3.21}$$

$$\frac{\partial^2 \mu_H}{(\partial q_H)^2} < 0, \tag{3.22}$$

$$\frac{\partial \mu_H(q_L, 0)}{\partial q_H} = \frac{x_H - x_L}{q_L} - b, \tag{3.23}$$

and

$$\mu_H(q_L, 0) = a + x_L - bq_L - c_H \tag{3.24}$$

(i) If $\mu_H(q_L, 0)$ is strictly greater then zero (i.e. if $bq_L < a + x_L - c_H$), then by continuity π_H must achieve a positive interior maximum. There is a unique interior solution to firm H's problem.

(ii) If $bq_L \geq a + x_L - c_H$, then one must distinguish two cases.

(a) If $\frac{\partial \mu_H(q_L,0)}{\partial q_H} \leq 0$ (i.e. if $bq_L \geq x_H - x_L$), then μ_H assumes strictly negative values on $(0, +\infty)$ by strict concavity. There is a unique corner solution.

(b) If $\frac{\partial \mu_H(q_L,0)}{\partial q_H} > 0$ (i.e. if $bq_L < x_H - x_L$), then it is possible that μ_H increases sufficiently to reach the positive range before starting decreasing inexorably. In any case, from the first-order condition, the maximum is reached at $q_H = \sqrt{\frac{q_L(x_H - x_L)}{b}} - q_L$, leading to an average quality equal to $x_H - \sqrt{bq_L(x_H - x_L)}$, and a margin equal to $a + x_H - c_H - 2\sqrt{bq_L(x_H - x_L)}$. Thus, there is an interior solution if and only if this latter expression is positive, that is, if and only if $bq_L \leq \frac{(a+x_H-c_H)^2}{4(x_H-x_L)}$.

Note that when $x_H - x_L \leq (a + x_H - c_H)/2 \leq a + x_L - c_H$, the previous inequality is mechanically satisfied once $bq_L < x_H - x_L$. So the only instance in which the condition can strictly bind is when $x_H - x_L > (a + x_H - c_H)/2 > a + x_L - c_H$.

In that case, when bq_L exactly equals $\frac{(a+x_H-c_H)^2}{4(x_H-x_L)}$, the best interior margin equals zero. The quantity q_H associated with this margin is given by

$$\frac{a + x_H - c_H}{2b}\left(1 - \frac{a + x_H - c_H}{2(x_H - x_L)}\right) \tag{3.25}$$

and must correspond to a local maximum of the profit function, reaching zero at that point, negative everywhere else on $(0, +\infty)$. Observe that this quantity is strictly positive since $x_H - x_L > a + x_L - c_H$. Thus, firm H's best-response correspondence at this point is pair-valued: the optimal response comprises the interior solution as well as the corner solution. So bq_L must be strictly smaller than $(a + x_H - c_H)^2/[4(x_H - x_L)]$ in order for the first-order condition necessarily to be verified at a solution to the firm's problem. ∎

Denote ρ_L the supremum of the set of quantities q_L that elicit a strictly positive response from firm H.

Numerical examples

We illustrate the conditions above by looking at some numerical examples.

Let $x_L = 1$, $x_H = 2$, $c_L = 1$, $c_H = 3$, $a = 5$, and $b = 1$. We can compute the value of the tresholds:

$$\begin{aligned} a + x_L - c_H &= 3 \\ x_H - x_L &= 1 \\ \frac{(a + x_H - c_H)^2}{4(x_H - x_L)} &= 4. \end{aligned}$$

On figure 3-1, the thin line is for $q_L = 2$. The thick line is for $q_L = 4$. The general shapes of these two curves are the only possible here as the quality differential is too small to generate some convex sections.

Contrast with the following example, depicted on figure 3-2: $x_L = 1$, $x_H = 10$, $c_L = 0$, $c_H = 1$, $a = 5$, and $b = 1$, giving

$$\begin{aligned} a + x_L - c_H &= 5 \\ x_H - x_L &= 9 \\ \frac{(a + x_H - c_H)^2}{4(x_H - x_L)} &\simeq 5.44. \end{aligned}$$

The thin line is for $q_L = 4.8$, the thicker line for $q_L = 5.35$, and the thickest line for $q_L = 5.5$. One can observe the convex sections of the curve that the relatively large quality differential creates near zero. A curve for $q_L > 9$, not shown on the figure, would be downward-sloping and concave everywhere on $[0, +\infty)$.

We now attempt at describing the incentives that firms face and the general shape of the the best-response curves to which these incentives give rise.

3.4.2 Firms' best responses

Consider the first-order conditions necessarily satisfied at the interior solutions to the firms' programs:

$$a + \frac{q_L x_L + q_H x_H}{q_L + q_H} - b\left(q_L + q_H\right) - c_L + \frac{q_L q_H \left(x_L - x_H\right)}{\left(q_L + q_H\right)^2} - b q_L = 0 \qquad \text{(A)}$$

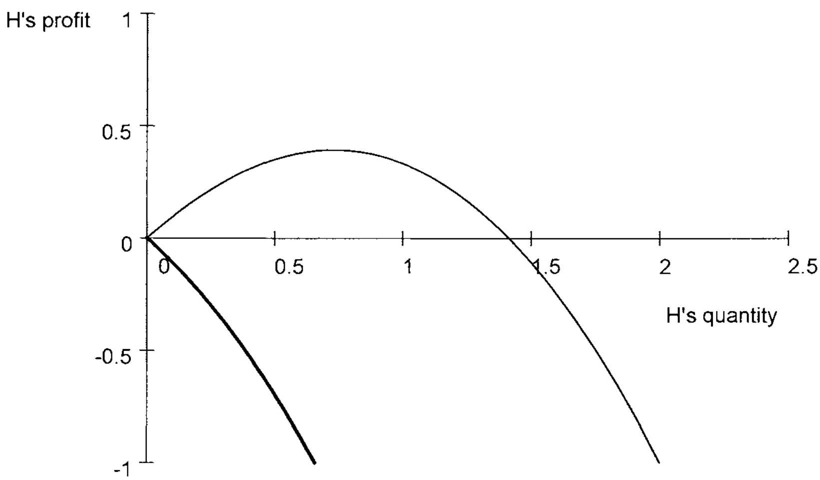

Figure 3.1: Strict concavity of firm H's problem

$$a + \frac{q_L x_L + q_H x_H}{q_L + q_H} - b\left(q_L + q_H\right) - c_H + \frac{q_L q_H \left(x_H - x_L\right)}{\left(q_L + q_H\right)^2} - bq_H = 0. \qquad \text{(B)}$$

INCENTIVES TO PRODUCE

It is possible to rewrite conditions (A) and (B) in a more illustrative manner. After some manipulation (A) gives

$$a - 2bq_L - bq_H + \left[1 - \left(\frac{q_H}{q_L + q_H}\right)^2\right] x_L + \left[\left(\frac{q_H}{q_L + q_H}\right)^2\right] x_H = c_L. \qquad (3.26)$$

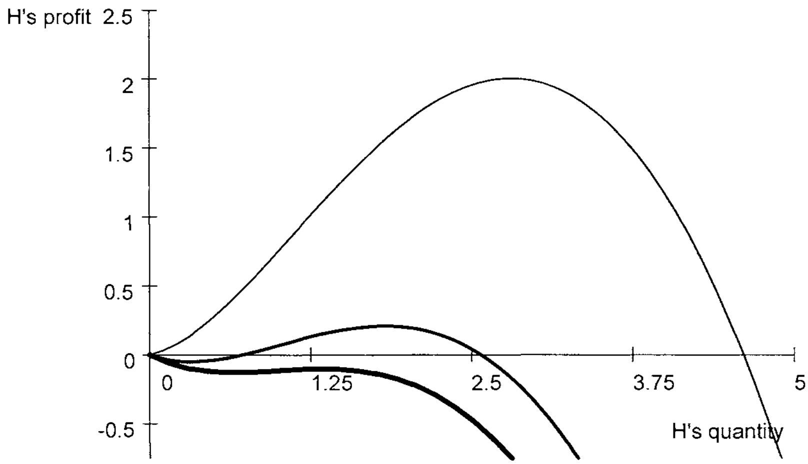

Figure 3.2: An example of the possible non-concavity of firm H's problem

This expression shows that the first-order condition for firm L is the same as the one in the standard Cournot model, except that the demand-intercept term not only depends upon the quantities chosen by the players but, for those quantities, is also a downward-distorted linear combination of x_L and x_H. Therefore, the fact that consumers immediately observe the average quality available on the market decreases firm L's marginal revenue, as any additional output brought to the market not only depresses the market price along the current demand curve but also impacts this average, thus shifting the demand curve down.

Similarly, one can rewrite (B) as

$$a - bq_L - 2bq_H + \left[\left(\frac{q_L}{q_L + q_H}\right)^2\right] x_L + \left[1 - \left(\frac{q_L}{q_L + q_H}\right)^2\right] x_H = c_H. \tag{3.27}$$

This expression shows that for the high-quality firm, the weights on the qualities distort the average upwards. Therefore, for given quantity levels, firm H's marginal revenue from increasing output is higher than in the standard Cournot model, as this increase has the additional effect of increasing the average quality.

STRATEGIC RELATIONSHIP BETWEEN THE FIRMS' ACTIONS

One can also rewrite (A) as:

$$a + x_L - c_L - bq_H - 2bq_L = \left(\frac{q_H}{q_L + q_H}\right)^2 (x_L - x_H). \tag{3.28}$$

For a given value of q_H, the left-hand side (LHS) is a linear function of q_L with negative slope. The right-hand side (RHS) assumes only negative values and monotonically increases towards 0. These geometrical considerations confirm what was infered from the strict concavity of the high-cost firm's problem, i.e. that its best response is unique, and interior as long as $bq_H \leq a + x_H - c_L$. It is clearly continuous in q_H. A change in q_H shifts the straight line down but also pushes the hyperbola down (in the bottom-right quadrant) so that the total effect is a priori undeterminate.

An application of the implicit-function theorem in the neighborhood of the best interior response to q_H gives

$$\frac{dq_L}{dq_H} = \frac{\frac{q_L q_H}{(q_L+q_H)^3}(x_H - x_L) - \frac{1}{2}b}{\frac{(q_H)^2}{(q_L+q_H)^3}(x_H - x_L) + b} \tag{3.29}$$

So, if b is small, and the quality differential is large, an increase in q_H can drive the average quality sufficiently high for firm L to find it profitable to increase its own quantity. Figure 3-3 illustrates this possibility. It depicts firm L's best response

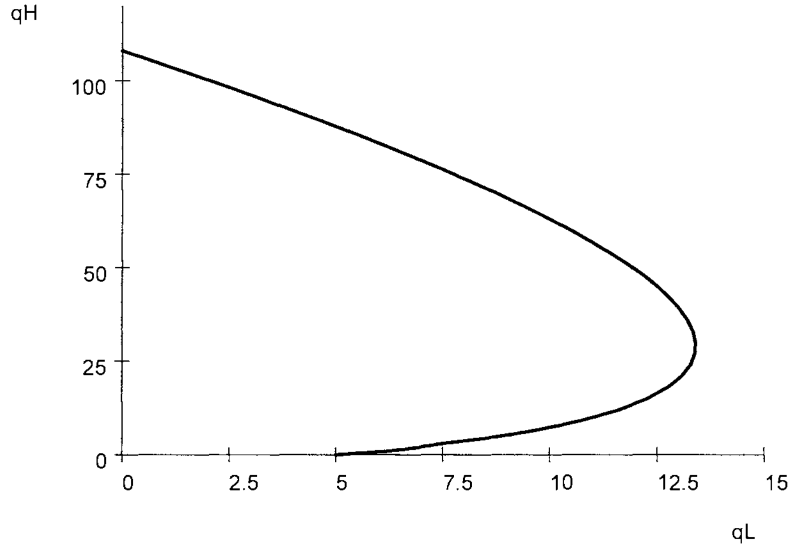

Figure 3.3: Firm L's best response when the quality differential is big

(measured along the horizontal axis) as a function of q_H for the case when $a = 9$, $b = 1$, $x_L = 2$, $x_H = 100$, and $c_L = 1$.

This graph hides the fact that firm L's best response is initially decreasing in q_H (at the rate of $1/2$, like in the standard Cournot model). Figure 3-4 is a zoom around the point $\left(q_L^M, 0\right)$.

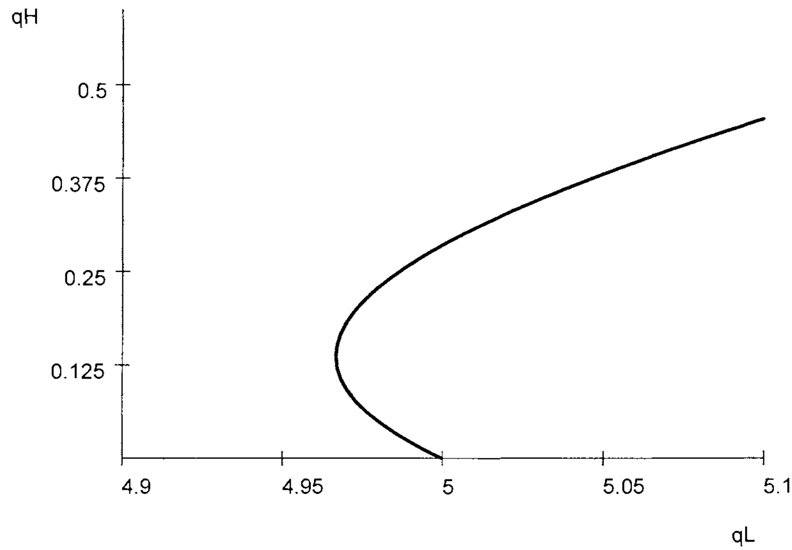

Figure 3.4: Firm L's best response when the quality differential is big (zoom)

Thus, it is not always the case that the two producers' quantities are strategic substitutes, as in the standard Cournot model with linear demand. Observe that

$$\frac{dq_L}{dq_H} \geq 0 \Longleftrightarrow \frac{q_L q_H}{(q_L + q_H)^3}(x_H - x_L) - \frac{1}{2}b \geq 0. \tag{3.30}$$

The first term in the right-most inequality is the product of firm H's market share, denoted z, with the effect of a change in q_H on the average quality $\partial\bar{x}/\partial q_H$, itself equal to $(1-z)\frac{1}{Q}(x_H - x_L)$. From this observation stream several features of firm L's best response. First, BR_L cannot be upward-sloping at points that are too close to the axes or too distant from the origin. Second, being continuous, BR_L has to cross the 45-degree line. When it does, we have

$$\frac{dq_L}{dq_H} = \frac{\frac{(x_H - x_L)}{4Q} - \frac{1}{2}b}{\frac{(x_H - x_L)}{4Q} + b}, \tag{3.31}$$

which is greater than $-1/2$ in all events. So because BR_L is moving away from the main diagonal *and* from the origin at this point, dq_L/dq_H must be decreasing in this neighborhood, implying that BR_L is concave. Once the main diagonal is crossed, because the curve moves away and q_H increases, the slope in (3.29) must converge to $-1/2$ from above, ruling out any inflexion and upward-sloping portion. So, if an upward-sloping portion exists, the inflexion must take place before the curve crosses the 45-degree line.

Similarly, one can rewrite (B) as

$$a + x_H - c_H - bq_L - 2bq_H = \left(\frac{q_L}{q_L + q_H}\right)^2 (x_H - x_L) \tag{3.32}$$

This time, for a given value of q_L, the LHS is a linear function of q_H with negative slope while the RHS is a downward-sloping hyperbola assuming only positive values. Right of $-q_L$, it is thus possible for the curves to intersect once, twice, or not at all. The circumstances in which they intersect only once *in the positive orthant* are of course the same as the ones ensuring the global concavity of π_H on $\mathbb{R}_+$. The circumstances in which they don't intersect at all in the positive orthant are the reverse of the ones ensuring that there is an interior best-response. Observe that in the case where the curves intersect twice, there is no need for formally checking the second-order condition as the higher quantity always corresponds to the local maximum of the profit function on $(0, +\infty)$. Observe also that when q_L goes up, the line shifts down while the hyperbola shifts up. As a result, the q_H-coordinate of the left-most intersection goes up while the q_H-coordinate of the right-most intersection goes down.

Thus, firm H's unique interior best-response is locally strictly decreasing in q_L and concave. Unfortunately, firm H's best-response correspondence is not a continuous function on $\mathbb{R}_+$. Indeed, recall that, when the quality differential is large ($x_H - x_L > a + x_L - c_H$), firm H is indifferent between $q_H = 0$ and $q_H = \frac{a+x_H-c_H}{2b}\left(1 - \frac{a+x_H-c_H}{2(x_H-x_L)}\right) > 0$ for $q_L = \frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$. Thus, at this point, the best-response correspondence is not singleton-valued. It is upper-hemicontinuous, though, and each "branch" of the correspondence graph is nonincreasing in q_L.

We summarize these results in the following claim.[5]

Claim 13 *Firm L's best response $BR_L(q_H)$ is a continuous, possibly non-monotonic function of q_H, although it is strictly decreasing in the neighborhood of 0 and ρ_H. Firm H's best-response correspondence, $BR_H(q_L)$, is singleton-valued at all points, with the possible exception of the point $q_L = \frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$ in the event where $x_H - x_L > a + x_L - c_H$ (in which case the firm's problem admits both an interior and a corner solution). BR_H is nevertheless upper-hemicontinuous and non-increasing in q_L. On $[0, \rho_L)$, it is a strictly decreasing and concave function.*

Again, because of these features of firms' best-responses, one cannot use the tools commonly applied in oligopoly theory to show the general existence, unicity, or stability of Nash equilibria. In particular, owing to the "discontinuity" in BR_H, one can not resort to traditional fixed-point theorems to prove existence. Similarly, owing to the non-monotonicity of BR_L one cannot redefine strategic variables in a way that transforms the model into a supermodular game, as in the standard Cournot duopoly.

[5] In all rigor, by "non-increasing" we mean that if $g \in BR_H(q_L)$ and $h \in BR_H(q_L')$ for any pair (q_L, q_L') such that $0 \leq q_L < q_L'$, then $g \geq h$. Because this last inequality is true for any two *selections* from the correspondence at the two points, this property is sometimes referred to as "strong monotonicity".

3.4.3 Monopolistic equilibria

The conditions for the existence of interior best-responses derived in the previous section allow us to study the existence of monopolistic equilibria in which one firm produces a strictly positive quantity (in fact, the monopoly output) and the other one withdraws from the market. There are two possible classes of monopolistic equilibria: one in which firm L is inactive and one in which firm H is inactive.

Inactive low-quality firm

Take for our candidate equilibrium the situation in which the high-quality firm produces $q_H > 0$ and the low-quality firm shuts down: $q_L = 0$. For this latter behavior to be optimal one needs:

$$q_H \geq \frac{a + x_H - c_L}{b}. \tag{3.33}$$

The question is: Does firm H find it optimal to provide so large a quantity under the assumption that firm L leaves the market? In equilibrium, firm H should take firm L's withdrawal for granted and best-respond by producing the monopoly-profit-maximizing quantity, that is

$$q_H^M = \frac{a + x_H - c_H}{2b}. \tag{3.34}$$

This volume is greater than the treshold iff

$$\frac{a + x_H - c_H}{2b} \geq \frac{a + x_H - c_L}{b}, \tag{3.35}$$

or, equivalently

$$c_L - c_H \geq a + x_H - c_L. \tag{3.36}$$

Under our profitable supply assumption, this inequality is impossible to satisfy for $c_H \geq c_L$ but if marginal cost decreases sufficiently with quality, then there exits a monopolistic equilibrium in which firm L is inactive. The condition resembles the one in the standard Cournot model, in that the cost differential must be sufficiently unfavorable to firm L as to turn negative its margin on the first unit sold.

INACTIVE HIGH-QUALITY FIRM

Our next candidate equilibrium corresponds to the situation in which firm L produces $q_L^M = \frac{a+x_L-c_L}{2b}$ and the firm H shuts down: $q_H = 0$. For this latter behavior to be optimal one needs:

$$q_L^M \geq \frac{\max\left\{a + x_L - c_H, \min\left\{x_H - x_L, \frac{(a+x_H-c_H)^2}{4(x_H-x_L)}\right\}\right\}}{b}. \tag{3.37}$$

It is immediately observed that $\frac{a+x_H-c_H}{2}$ is the arithmetic average of $a + x_L - c_H$ and $x_H - x_L$. So we need to distinguish only two cases, according to the ranking of these two magnitudes.

Large quality differential Suppose that the quality differential is relatively big, so that

$$x_H - x_L \geq \frac{a + x_H - c_H}{2} \geq a + x_L - c_H. \tag{3.38}$$

Then

$$\min\left\{x_H - x_L, \frac{(a + x_H - c_H)^2}{4(x_H - x_L)}\right\} = \frac{(a + x_H - c_H)^2}{4(x_H - x_L)} \tag{3.39}$$

The relevant quantity treshold for q_L is then either $(a + x_L - c_H)/b$ or $\frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$. Note that for $x_H > x_L$:

$$a + x_L - c_H \leq \frac{(a + x_H - c_H)^2}{4(x_H - x_L)} \iff (a + x_L - c_H)(x_H - x_L) \leq \left(\frac{a + x_H - c_H}{2}\right)^2. \tag{3.40}$$

If $a+x_L-c_H<0$, then this proposition is always true, since $x_H>x_L$ by assumption. Else,

$$a+x_L-c_H \leq \frac{(a+x_H-c_H)^2}{4(x_H-x_L)} \Longleftrightarrow \sqrt{(a+x_L-c_H)(x_H-x_L)} \leq \frac{a+x_H-c_H}{2}. \tag{3.41}$$

As the geometric mean of two positive numbers is no greater than their arithmetic mean, we have

$$\sqrt{(a+x_L-c_H)(x_H-x_L)} \leq \frac{a+x_H-c_H}{2}. \tag{3.42}$$

Hence it is always true that

$$a+x_L-c_H \leq \frac{(a+x_H-c_H)^2}{4(x_H-x_L)} \tag{3.43}$$

As a result, the relevant treshold when the quality differential is high is always $\frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$. In a sense, all that matters for firm H is whether, in the event that the margin on the first unit sold is negative (owing to firm L's large production), it can restaure its profitability by driving the average quality up. We thus have

$$bq_L^M \geq \frac{(a+x_H-c_H)^2}{4(x_H-x_L)} \Longleftrightarrow \left(\frac{a+x_L-c_L}{2}\right)(x_H-x_L) \geq \left(\frac{a+x_H-c_H}{2}\right)^2, \tag{3.44}$$

which is true whenever $a+x_L-c_L$ is not much smaller than $a+x_H-c_H$. The exact treshold depends on the difference between (x_H-x_L) and $\frac{a+x_H-c_H}{2}$. A sufficient condition for the inequality to hold is $c_H-c_L \geq x_H-x_L$ (but if a big difference between $a+x_L-c_H$ and x_H-x_L exists, then firm L needs less of a cost advantage).

Small quality differential Suppose that

$$x_H-x_L < \frac{a+x_H-c_H}{2} < a+x_L-c_H. \tag{3.45}$$

Then the relevant treshold is $(a+x_L-c_H)/b$. This occurs whenever the demand for firm L's product is relatively high in comparison to the quality differential. Then, all that matters for firm H is whether the margin on the first unit sold is positive or not because whenever it is possible to enjoy a positive margin by driving quality up through mass production, it is also the case that there is profit to be made on the very first unit. In a monopolistic equilibrium, one has

$$q_L^M \geq \frac{a + x_L - c_H}{b} \Longleftrightarrow \frac{a + x_L - c_L}{2} \geq a + x_L - c_H \Longleftrightarrow c_H - c_L \geq a + x_L - c_H. \quad (3.46)$$

That is, once again, the cost differential must be large enough for a monopolistic equilibrium to exist. A necessary (but not sufficient) condition for it to hold is $c_H - c_L \geq x_H - x_L$.

We summarize the observations made in that section by providing a sufficient condition for the existence of a monopolistic equilibrium. In the standard Cournot model, monopolistic equilibria were sustained by a cost differential larger than the margin on the first unit sold by the evicted firm. This condition carries over but one also has to account for the fact that even if this margin is negative, firm H can (sometimes) drive prices up by increasing its quantity and thus average quality. In that instance, eviction is always possible if the cost differential is larger than the quality differential.

Claim 14 *If the cost differential is sufficiently unfavorable to firm i, that is if*

$$c_i - c_{-i} \geq \max\left\{a + x_{-i} - c_i \ , \ x_i - x_{-i}\right\}, \quad (3.47)$$

then there always exists an equilibrium in which firm $-i$ produces the monopoly output $q_{-i}^M = (a + x_{-i} - c_{-i}) \,/\, (2b)$ and firm i withdraws from the market.

3.4.4 Duopolistic equilibria

Claim 13 above established that firm L's best-response correspondence is in fact a continuous function, although possibly non-monotone, while firm H's best-response correspondence is upper semi-continuous, and singleton-valued at all points except, sometimes, at $q_L = \frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$. It remains to be shown that there are circumstances in which the best-response curves intersect away from the axes, thus proving the existence of a pure-strategy duopolistic equilibrium. As it happens, there are instances in which such an equilibrium fails to exist. We do not immediately tackle this question but instead look at the multiplicity issue. In a fashion that parallels our treatment of cost heterogeneity, we introduce x, taken to be the arithmetic average of qualities x_L and x_H, and let $\varepsilon = (x_H - x_L)/2$. This way, $x_L = x - \varepsilon$ and $x_H = x + \varepsilon$. We are now in the position to state our result on the number of duopolistic equilibria.

Proposition 15 *There exist at most two pure-strategy duopolistic equilibria.*

Proof. Suppose that there exists a duopolistic equilibrium in pure-strategies. The system of necessary conditions (A) and (B) can then be equivalently rewritten as

$$\begin{cases} q_L = \frac{a+x_L+z(x_H-x_L)+c_H-2c_L+3z(z-1)(x_H-x_L)}{3b} \\ q_H = \frac{a+x_L+z(x_H-x_L)+c_L-2c_H+3z(1-z)(x_H-x_L)}{3b} \\ z = \frac{q_H}{q_L+q_H} \end{cases}. \tag{3.48}$$

Observe that q_L and q_H are uniquely determined by z and the parameters of the model.

By substitution of q_L and q_H, as given by the first two equations, into the third one, and by definition of x, ε, c, and δ, one obtains the following equation in z:

$$z = \frac{a + x + (2z-1)\varepsilon - c + 3\left[2\varepsilon z(1-z) - \delta\right]}{2\left[a + x + (2z-1)\varepsilon - c\right]}. \tag{3.49}$$

In a duopolistic equilibrium, $Q = q_L + q_H > 0$. So one can multiply both sides of equation (3.49) by its denominator to obtain a quadratic equation in z. Its discriminant equals $15\varepsilon^2 + M^2 - 30\delta\varepsilon$. If it is negative, then there cannot exist a solution to the system of equations (A) and (B), which contradicts our initial assumption.

Suppose it is positive, then. The roots are given by

$$z^* = \frac{1}{2} + \frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2} - M}{10\varepsilon}, \tag{3.50}$$

$$z^{**} = \frac{1}{2} - \frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2} + M}{10\varepsilon}. \tag{3.51}$$

Each root need not correspond to an equilibrium but the associated quantities are the only candidate equilibria. Therefore, there can be at most two duopolistic equilibria.

■

As a by-product, the proposition singles out the candidate equilibria.

Substituting z^* back into the two first equations, we have:

$$\begin{array}{l} q_H^* = \frac{1}{3b}\left\{\frac{4M+\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}}{5} + \frac{3}{2}(\varepsilon - 2\delta) - \frac{3\left(\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}-M\right)^2}{50\varepsilon}\right\} \\ q_L^* = \frac{1}{3b}\left\{\frac{4M+\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}}{5} - \frac{3}{2}(\varepsilon - 2\delta) + \frac{3\left(\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}-M\right)^2}{50\varepsilon}\right\} \end{array}. \tag{3.52}$$

Thus

$$Q^* = \frac{2}{3b}\frac{4M + \sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2}}{5}. \tag{3.53}$$

Plugging this quantity into the inverse demand curve gives

$$P^* = c + \frac{1}{3}\frac{4M + \sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2}}{5}. \tag{3.54}$$

Substituting z^{**} back into the two first equations, we have:

$$\begin{array}{l} q_H^{**} = \frac{1}{3b}\left\{\frac{4M-\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}}{5} + \frac{3}{2}(\varepsilon - 2\delta) - \frac{3\left(\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}+M\right)^2}{50\varepsilon}\right\} \\ q_L^{**} = \frac{1}{3b}\left\{\frac{4M-\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}}{5} - \frac{3}{2}(\varepsilon - 2\delta) + \frac{3\left(\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}+M\right)^2}{50\varepsilon}\right\} \end{array}. \tag{3.55}$$

Thus

$$Q^{**} = \frac{2}{3b}\frac{4M - \sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2}}{5}. \tag{3.56}$$

Plugging this quantity into the inverse demand curve gives

$$P^{**} = c + \frac{1}{3}\frac{4M - \sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2}}{5}. \tag{3.57}$$

Several remarks are in order. First, in constrast with the standard Cournot duopoly, the market outcomes are not independent on the level of cost heterogeneity: the (candidate) equilibrium prices and quantities are functions of δ, and a change in firms' cost parameters (keeping the average constant) does more than shifting production and profit from one firm to the other.

Second, from the proof of Proposition 15, it is readily observed that $z^{**} < 1/2$ for any choice of parameters (as long as $M > 0$), and that $z^* > 1/2$ if and only if $\varepsilon > 2\delta$. So we have the following corollary.

Corollary 16 *There can exist a pure-strategy duopolistic equilibrium in which firm H produces a higher quantity than firm L ($q_H^* > q_L^*$) only if $2(c_H - c_L) < x_H - x_L$, or, equivalently, $2\delta < \varepsilon$.*

A sufficient condition for the last inequality to hold is that

$$\mu'(x) < \frac{1}{2} \tag{3.58}$$

The condition requires the marginal cost of production not to increase too quickly with quality.

Third, in those cases where firm H's problem is not concave, the pair (q_L^{**}, q_H^{**}) (assuming it involve positive quantities) may not be an equilibrium, as q_H^{**} can then correspond to a local minimum or a non-global maximum of firm H's profit function. In the first case, (q_L^{**}, q_H^{**}) lies on the lower branch of the locus of points satisfying firm H's first-order condition. In the second case, it lies on its upper branch but not on firm H's best-global-response curve, BR_H, because the corner solution is preferred.

Now, we have already argued that BR_H, in the range of quantities that elicit a strictly positive response from firm H (i.e. for $0 \leq q_L < \rho_L$) is a continuous and strictly decreasing function. From the expressions for the candidate equilibrium quantities, it is easy to see that $q_H^{**} < q_H^*$. Thus, if (q_L^{**}, q_H^{**}) lies on BR_H, then (q_L^*, q_H^*) lies on it as well.

Corollary 17 *If $(q_L^{**}, q_H^{**}) \in \mathbb{R}^2_{++}$ is an equilibrium, then (q_L^*, q_H^*) is an equilibrium as well, provided $(q_L^*, q_H^*) \in \mathbb{R}^2_+$.*

In particular, whenever $\varepsilon > 2\delta$, if there exists a duopolistic equilibrium in which $q_L^{**} > q_H^{**}$, then there exists another duopolistic equilibrium (q_L^*, q_H^*) in which $q_L^* < q_H^*$, provided $q_L^* > 0$.

The previous corollary bears on the issue of equilibrium selection. Indeed, recall that firm H's best-interior-response is a strictly increasing and concave function of q_L on $[0, \rho_L)$. So, if (q_L^{**}, q_H^{**}) lies on the upper "branch" of BR_H and therefore is an equilibrium, then it must be the case that BR_L is flatter than BR_H there (in the $q_L \times q_H$ space). That implies that (q_L^{**}, q_H^{**}) is always an unstable equilibrium under the usual (i.e. alternate) best-reply dynamics. For the reverse reason, (q_L^*, q_H^*) is always stable.

Corollary 18 *If an equilibrium, (q_L^*, q_H^*) is always stable under the usual best-reply dynamics. If an equilibrium, (q_L^{**}, q_H^{**}) is always unstable.*

In our detailed treatment below, we will thus focus on the monopolistic equilibria on one hand, and on (q_L^*, q_H^*) on the other hand.

3.5 A TAXONOMY OF EQUILIBRIA

We now want to describe the possible equilibrium outcomes in relation to the levels of cost heterogeneity and quality heterogeneity. The case where $\varepsilon = 0$ corresponds to the standard Cournot model and was covered in Section 3. We take in turn the cases where the cost function μ does not depend on quality, decreases with quality, and increases with it.

3.5.1 Identical marginal costs

Suppose that $c_H = c_L = c$, or $\delta = 0$.

There does not exist any monopolistic equilibrium in this situation as a firm cannot deter its rival from entering the market, which always requires a cost advantage. This is just another way of saying that $\rho_L > q_L^M$ and $\rho_H > q_H^M$ for all values of ε. So we focus on duopolistic equilibria. We claim that, independently on the level of quality heterogeneity, a unique, pure-strategy duopolistic equilibrium always exists when costs are identical.

Proposition 19 *For $\delta = 0$ and for any ε, there exists a unique stable, pure-strategy equilibrium, which is duopolistic and characterized by the following market share for firm H:*

$$z^* = \frac{1}{2} + \frac{\sqrt{15\varepsilon^2 + (a+x-c)^2} - (a+x-c)}{10\varepsilon}, \tag{3.59}$$

the following equilibrium quantity:

$$Q^* = \frac{2}{3b} \frac{\sqrt{15\varepsilon^2 + (a+x-c)^2} + 4(a+x-c)}{5}, \tag{3.60}$$

and equilibrium price:

$$P^* = c + \frac{1}{3} \frac{\sqrt{15\varepsilon^2 + (a+x-c)^2} + 4(a+x-c)}{5}. \tag{3.61}$$

Proof. From Section 3.4.3, we know there cannot exist any monopolistic equilibrium with $\delta = 0$.

From Section 3.4.4, there is only one stable, duopolistic candidate equilibria, characterised by:

$$z^* = \frac{1}{2} + \frac{\sqrt{15\varepsilon^2 + M^2} - M}{10\varepsilon}, \tag{3.62}$$

The equilibrium quantities must be:

$$\begin{aligned} q_H^* &= \frac{1}{3b}\left\{\frac{\sqrt{15\varepsilon^2 + M^2} + 4M}{5} + \left[\frac{3}{2}\varepsilon - \frac{3\left(\sqrt{15\varepsilon^2 + M^2} - M\right)^2}{50\varepsilon}\right]\right\}, \\ q_L^* &= \frac{1}{3b}\left\{\frac{\sqrt{15\varepsilon^2 + M^2} + 4M}{5} - \left[\frac{3}{2}\varepsilon - \frac{3\left(\sqrt{15\varepsilon^2 + M^2} - M\right)^2}{50\varepsilon}\right]\right\}, \end{aligned}$$

giving

$$Q^* = \frac{2}{3b}\frac{\sqrt{15\varepsilon^2 + M^2} + 4M}{5}, \tag{3.63}$$

and

$$P^* = c + \frac{1}{3}\frac{\sqrt{15\varepsilon^2 + M^2} + 4M}{5}. \tag{3.64}$$

The point (q_L^*, q_H^*) will not correspond to an equilibrium in the circumstances where either (i) it lies outside the positive orthant, or (ii) the first-order conditions do not characterize the firm's best *global* responses.

As for (i), observe that it is not possible that both q_L^* and q_H^* be simultaneously negative since $\frac{\sqrt{15\varepsilon^2+M^2}+4M}{5} > 0$. Therefore, if (q_L^*, q_H^*) lies outside the positive orthant, then $\frac{q_H^*}{q_L^*} = \frac{z^*}{1-z^*} \leq 0$, which occurs only if $z^* \leq \frac{1}{2}$. This cannot happen since $z^* > \frac{1}{2}$ for any $\varepsilon > 0$.

As for (ii), there is no issue if the quality differential is small, for then BR_H is a continuous function which coincides with the locus of points satisfying the first-order condition for all $q_L \in [0, \rho_L]$. If the quality differential is large (i.e. if $a + x_L - c_H \leq x_H - x_L$, or, equivalently, $\varepsilon > M/3$), then BR_H exhibits a non-convexity at $\tilde{q}_L = \frac{(a+x_H-c_H)^2}{4b(x_H-x_L)} = \frac{M+\varepsilon}{2b}\frac{M+\varepsilon}{4\varepsilon}$, at which point both $q_H = 0$ and $\tilde{q}_H = \frac{M+\varepsilon}{2b}\left(1 - \frac{M+\varepsilon}{4\varepsilon}\right)$ are optimal responses. A simple computation shows that at the best interior response,

$$\tilde{z} = 1 - \frac{M+\varepsilon}{4\varepsilon} = \frac{3\varepsilon - M}{4\varepsilon}. \tag{3.65}$$

If we can show that $z^* \geq \tilde{z}$ for any choice of parameters, then we will be done as we will be reassured that the intersection of firm L's best *interior* response curve with firm H's best *interior* response curve lies to the left of the possible "jump" in firm H's best *global* response. Now,

$$z^* - \tilde{z} = \frac{3M - 5\varepsilon + 2\sqrt{15\varepsilon^2 + M^2}}{20\varepsilon}, \tag{3.66}$$

which a simple computation shows is positive for any M and ε. ∎

Several remarks are in order before we proceed with illustrations. First, by L'Hospital rule, q_L^* and q_H^* tend to $(a+x-c)/3b$ as ε tends to 0. In that sense, the standard Cournot result with identical marginal costs is robust to the homogeneity assumption, as the introduction of a small quality differential leads to an equilibrium that is "close" to the usual Cournot outcome.

Second, z^* monotonically increases toward $1/2+\sqrt{15}/10 \simeq 0.89$ as ε increases. So, firm H's market share always goes up as this firm's quality advantage increases. Nonetheless, even in the case of an extreme advantage, firm L's always secures at least 10% of the sales. It is never possible for the quality leader to reduce its competitor to insignificance on this market.

Third, this pattern can be explained by the rates of change in the firms' equilibrium quantities. q_H^* is an increasing function of ε for any M and b. Asymptotically, it increases linearly, which implies that it grows with ε at a smaller and smaller rate. q_L^* first decreases then increases with ε. In the limit, it also tends to increase linearly. The convergence of z^* implies that the growth rates of both equilibrium quantities must themselves converge.

Figure 3-5 displays the equilibrium quantities as functions of ε in the case when $M=10$ and $b=1$. The solid line is firm L's quantity. The dotted line is firm H's quantity.

The same data, plotted against a logarithmic scale on figure 3-6, makes clear that after an initial divergence phase, the two quantities grow at the same rate.

We now illustrate the equilibrium itself with two examples, typical of the two possible configurations that arise when $\delta=0$.

Figure 3-7 is for the case when $a=10$, $b=2$, $x_L=1$, $x_H=2$, $c_L=c_H=1$. That corresponds to $x=1.5$, $c=1$, $\delta=0$, $\varepsilon=.5$. The solid line is firm L's best response. The dotted line is firm H's best response. The dashed line is the main diagonal. The quality differential being small, both best responses are continuous,

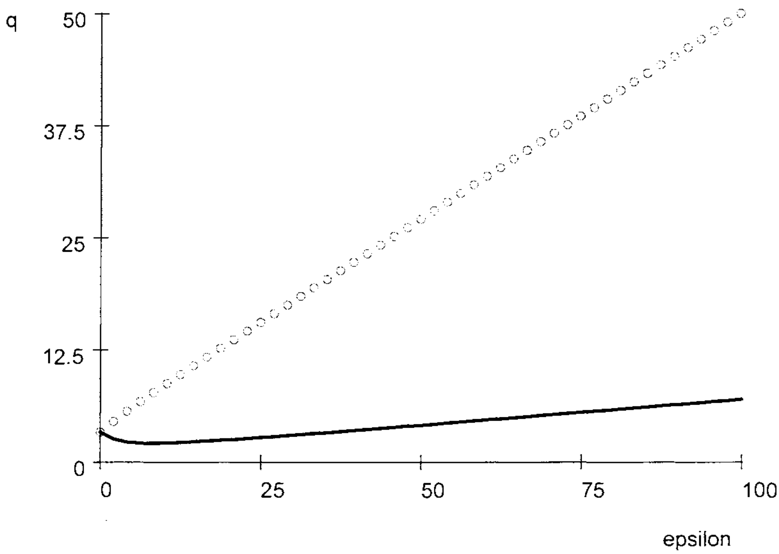

Figure 3.5: Equilibrium quantities for $M = 10$ and $b = 1$

decreasing functions of the other firm's quantity, as in the standard Cournot model. The intersection lies to the left of the main diagonal, implying that firm H dominates the market.

Figure 3-8 is for the case when $a = 10$, $b = 2$, $x_L = 1$, $x_H = 101$, $c_L = c_H = 1$. That corresponds to $x = 51$, $c = 1$, $\delta = 0$, $\varepsilon = 50$. Because of the large quality differential, BR_L becomes non-monotone and BR_H exhibits a nonconvexity at 15.125. Nonetheless, the curves intersect only once, to the left of the main diagonal and the "jump" in BR_H.

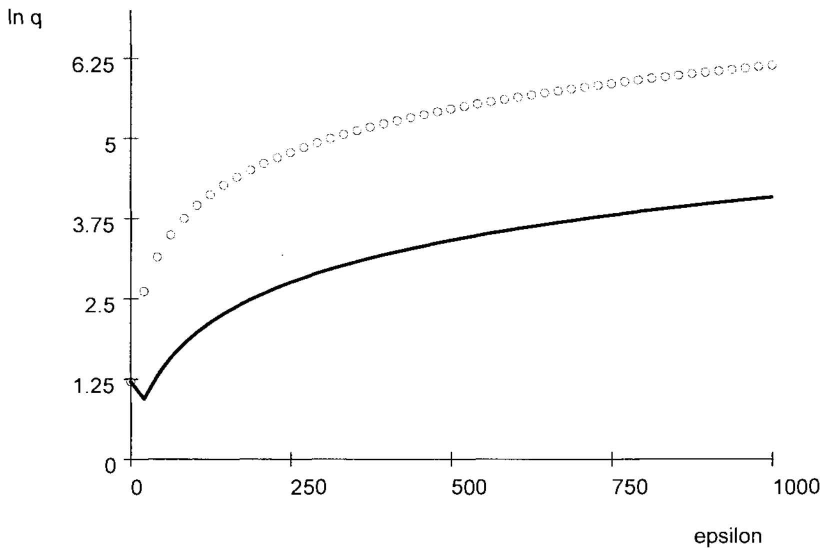

Figure 3.6: Equilibrium quantities for $M = 10$ and $b = 1$ - logarithmic scale

3.5.2 Decreasing marginal cost

Consider the case now where c decreases with quality. This is not as implausible a situation as it might seem. Quality could be associated with the use of technologies requiring big set-up or fixed costs but commanding low marginal costs. In fact, any quality-improving mechanization of the production process would constitute an example of this phenomenon. The market for collected blood is also often mentioned in that context, as donors tend to self-select in such a way that the quality of

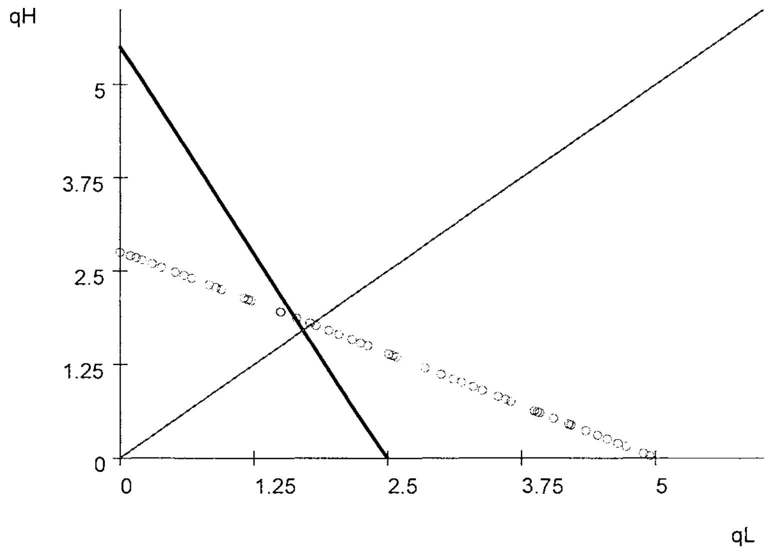

Figure 3.7: The unique equilibrium in the case of a small quality differential ($x_L = 1$; $x_H = 2$) and no cost differential ($c_L = c_H = 1$)

the blood collected from volunteers is on average higher than the quality of the blood collected from profit-motivated donors, which generates an inverse relationship between quality and variable cost.

In any case, assume in that section that $\delta < 0$.

In this configuration, there cannot exist a monopolistic equilibrium in which only firm L is active as it is at a cost disadvantage. By contrast, from Section 4.3 we know that firm H can remain the only active firm if and only if $c_L - c_H \geq a + x_H - c_L$, or

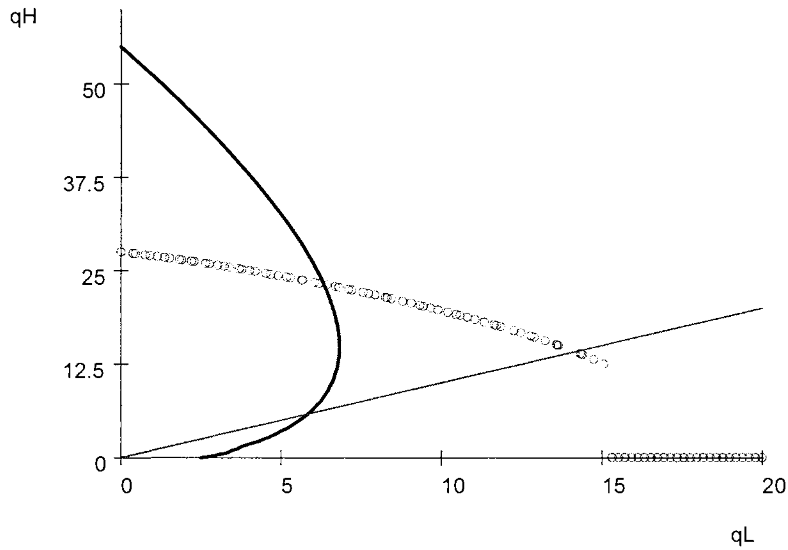

Figure 3.8: The unique equilibrium in the case of a large quality differential ($x_L = 1$; $x_H = 101$) and no cost differential ($c_L = c_H = 1$)

equivalently:

$$-\delta \geq \frac{M + \varepsilon}{3}. \tag{3.67}$$

For a given M, the right-hand side is a linear function of ε. The higher the quality differential, the bigger firm H's cost advantage must be in order for it to monopolize the market. That is, a big quality advantage makes it *harder* for the leading firm to evict its competitor. This is of course because the latter can free-ride on consumers' high valuation of the product and is therefore led to increase its quantity. Therefore, firm H must be in the position to dump a very big quantity on the market in order to preclude firm L's entry.

Two questions remain. When a monopolistic equilibrium exists, can it co-exist with a duopolistic equilibrium? When a monopolistic equilibrium does not exist, does a duopolistic equilibrium always exist? The answers are "no" and "yes", respectively.

Proposition 20 *Suppose that $\delta < 0$. (i) If $-\delta \geq \frac{M+\varepsilon}{3}$, then there is a unique stable, pure-strategy equilibrium, which is H-monopolistic. (ii) If $-\delta < \frac{M+\varepsilon}{3}$, then there is a unique stable pure-strategy equilibrium, which is duopolistic, and characterized by the following market share for firm H:*

$$z^* = \frac{1}{2} + \frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + (a + x - c)^2} - (a + x - c)}{10\varepsilon}, \tag{3.68}$$

the following equilibrium quantity:

$$Q^* = \frac{2}{3b}\frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + (a + x - c)^2} + 4(a + x - c)}{5}, \tag{3.69}$$

and equilibrium price:

$$P^* = c + \frac{1}{3}\frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + (a + x - c)^2} + 4(a + x - c)}{5}. \tag{3.70}$$

Proof. From Section 3.4.4, there is only one stable, pure-strategy duopolistic equilibrium candidate, characterized by

$$z^* = \frac{1}{2} + \frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2} - M}{10\varepsilon} > \frac{1}{2}. \tag{3.71}$$

Plugging z^* back in the system of necessary conditions:

$$\begin{array}{l} q_H^* = \frac{1}{3b}\left\{\frac{\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}+4M}{5} + \left[\frac{3}{2}(\varepsilon - 2\delta) - \frac{3\left(\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}-M\right)^2}{50\varepsilon}\right]\right\} \\ q_L^* = \frac{1}{3b}\left\{\frac{\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}+4M}{5} - \left[\frac{3}{2}(\varepsilon - 2\delta) - \frac{3\left(\sqrt{15\varepsilon(\varepsilon-2\delta)+M^2}-M\right)^2}{50\varepsilon}\right]\right\} \end{array}. \tag{3.72}$$

Because q_L^* and q_H^* cannot be simultaneously negative, one only needs to guarantee that $z^* \in \left(\frac{1}{2}, 1\right)$, which requires

$$0 \leq \frac{\sqrt{15\varepsilon^2 + M^2 - 30\delta\varepsilon} - M}{10\varepsilon} < \frac{1}{2}. \tag{3.73}$$

The left-most inequality is trivially satisfied for $\varepsilon > 0$ and $\delta < 0$. The right-most inequality is satisfied if and only if $-\delta < \frac{M+\varepsilon}{3}$, which is the same condition as the one leading to the non-existence of a monopolistic equilibrium. Thus, a duopolistic equilibrium and a monopolistic equilibrium cannot coexist.

It remains to show that a duopolistic equilibrium never fails to exist when $-\delta < \frac{M+\varepsilon}{3}$. So suppose this inequality holds and distinguish the cases of a small and large quality differential.

Take the case of a small quality differential first. Suppose that

$$x_H - x_L < a + x_L - c_H, \tag{3.74}$$

or, equivalently,

$$-\delta > 3\varepsilon - M. \tag{3.75}$$

Then, BR_H is a non-increasing, continuous *function* of q_L, and we have $q_L^M < \rho_L$ and $q_H^M < \rho_H$. As the two continuous best- response curves must then intersect away from the axes, the pure-strategy duopolistic equilibrium always exists.

Assume now that

$$-\delta \leq 3\varepsilon - M. \tag{3.76}$$

Then BR_H is no longer a continuous curve. Nevertheless, in the parameter region where there is no monopolistic equilibrium, the interior best-response curves intersect in the positive orthant and it only remains to be checked that this intersection lies left of the "jump" in BR_H.

Observe that the non-convexity in BR_H occurs at $\tilde{q}_L = \frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$, at which point the best interior reply is $\tilde{q}_H = \tilde{q}_L\left(1 - \tilde{q}_L\right)$. Firm H's market share at this point is given by

$$\tilde{z} = 1 - \frac{M + \varepsilon - \delta}{4\varepsilon}. \tag{3.77}$$

(q_L^*, q_H^*) lies to the left of the non-convexity only if $z^* \geq \tilde{z}$. We have

$$z^* - \tilde{z} = \frac{3M - 5\varepsilon - 5\delta + 2\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2}}{20\varepsilon}. \tag{3.78}$$

A simple computation shows that this quantity is always positive, so that this constraint never binds. Therefore, (q_L^*, q_H^*) never lies right of the "jump" in BR_H. ■

3.5.3 Increasing marginal cost

We now turn to the case when $\delta > 0$, i.e. when marginal cost increases with quality.

Monopolistic equilibrium region

In this configuration, firm H has a quality advantage over firm L but suffers from a cost disadvantage. Hence, there cannot exist any H-monopolistic equilibrium. An L-monopolistic equilibrium may exist, under conditions that vary with the size of the quality differential. We accordingly distinguish cases.

Small quality differential   Suppose that

$$x_H - x_L \leq a + x_L - c_H, \tag{3.79}$$

or, equivalently,

$$\delta \leq M - 3\varepsilon. \tag{3.80}$$

In that case, there can exist a L-monopolistic equilibrium only if

$$c_H - c_L \geq a + x_L - c_H, \tag{3.81}$$

or, equivalently,

$$\delta \geq \frac{M - \varepsilon}{3}. \tag{3.82}$$

Observe that in that parameter range, a higher quality differential corresponds to a *lower* cost differential treshold. This is caused by our symmetric measure of quality heterogeneity. When ε increases, firm L's quality diminishes, which decreases the willingness to pay on a market dominated by frm L. That makes it "easier" for firm L to evict firm H (in equilibrium), in the sense that it requires a smaller cost advantage.

Large quality differential Suppose now that

$$x_H - x_L > a + x_L - c_H, \tag{3.83}$$

or, equivalently,

$$\delta > M - 3\varepsilon. \tag{3.84}$$

In that case, there can exist a L-monopolistic equilibrium if and only if

$$\left(\frac{a + x_L - c_L}{2}\right)(x_H - x_L) \geq \left(\frac{a + x_H - c_H}{2}\right)^2, \tag{3.85}$$

or, equivalently,

$$\varepsilon\left(M + \delta - \varepsilon\right) \geq \frac{\left(M - \delta + \varepsilon\right)^2}{4}. \tag{3.86}$$

This inequation is quadratic in δ, and is always verified within a closed interval lying in $\mathbb{R}_+$, whose bounds depend on M and ε. More precisely, it is true whenever

$$M + 3\varepsilon - 2\sqrt{\varepsilon^2 + 2M\varepsilon} \leq \delta \leq M + 3\varepsilon + 2\sqrt{\varepsilon^2 + 2M\varepsilon}. \tag{3.87}$$

The right-most inequality is an algebraic artefact. It corresponds to a situation where the original inequality would be true only because firm H would make enormous losses, which is ruled out. So, in effect, the left-most inequality sets the lower bound on δ for firm L to enjoy monopoly in equilibrium. Asymptotically, this bound approaches $M + \varepsilon$ from below. One implication is that, even when the quality differential is extremely large, there always exist circumstances (i.e. combinations

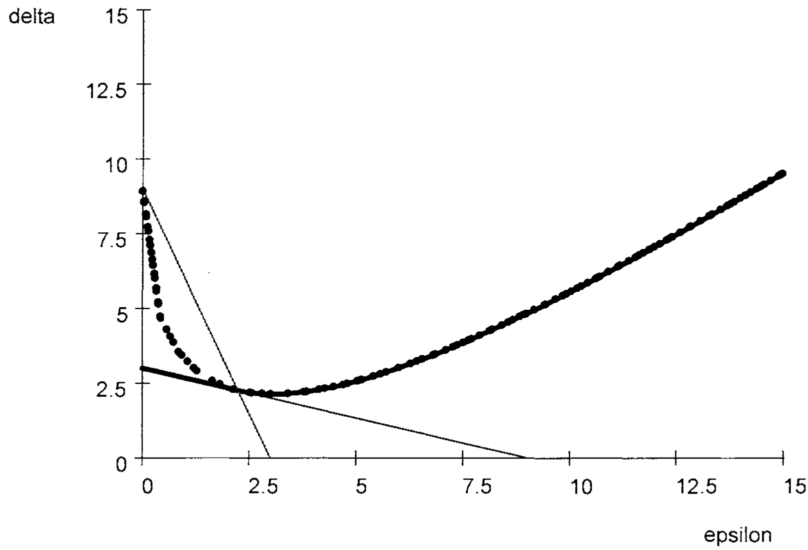

Figure 3.9: The border of the L-monopolistic equilibrium region ($M = 9$)

of δ and ε) in which firm H would have profitably sold its product on a separate market (although barely so) but ends up being inactive in equilibrium when buyers do not distinguish its product from firm L's product.

Figure 3-9 illustrates the determination of the monopolistic equilibrium region's border.

The steeper, downward-sloping, thin line delineates the region corresponding to what we have called a "small quality differential." The less steep, downward-sloping, thin line is the lower contour of the region where the margin "on the first unit sold" by firm H on a market monopolized by firm L is negative. The dashed curve stands for the lower contour of the region in which the best interior margin for firm H is negative. In case of a small quality differential area, the relevant treshold is the margin on the "first unit". By contrast, outside the small quality differential area, the best interior margin constitutes the relevant treshold. Therefore, the thick curve delineates the bottom of the L-monopolistic equilibrium region.

Duopolistic equilibrium region

As the argument for that case is a bit long, we present it verbally rather than in a formal proof.

Recall that there are only two candidate duopolistic equilibria. In order to prove that they indeed correspond to equilibria, one has to check for each of them that several conditions are satisfied:

(i) that the quantities are real numbers; that is, that the best-interior-response curves do intersect;

(ii) that the quantities are strictly positive; that is, that the intersection lies in the positive orthant;

(iii) that q_L is less than $\tilde{q}_L$, the value for which BR_H might exhibit a "jump"; that is, that the intersection of the best-interior-response curves corresponds to an intersection of the best-(global)-response curves.

By means of an example, we first show that it is possible for both candidates to be equilibria, along with a third, monopolistic equilibrium, even in the case of a small quality differential. Figure 3-10 is for the case when $a = 9.4$, $b = 1$, $x_L = 2$, $x_H = 6.4$, $c_L = 1$; and $c_H = 6.21$. That corresponds to $x = 4.2$, $c = 3.605$, $\delta = 2.605$, $\varepsilon = 2.2$.

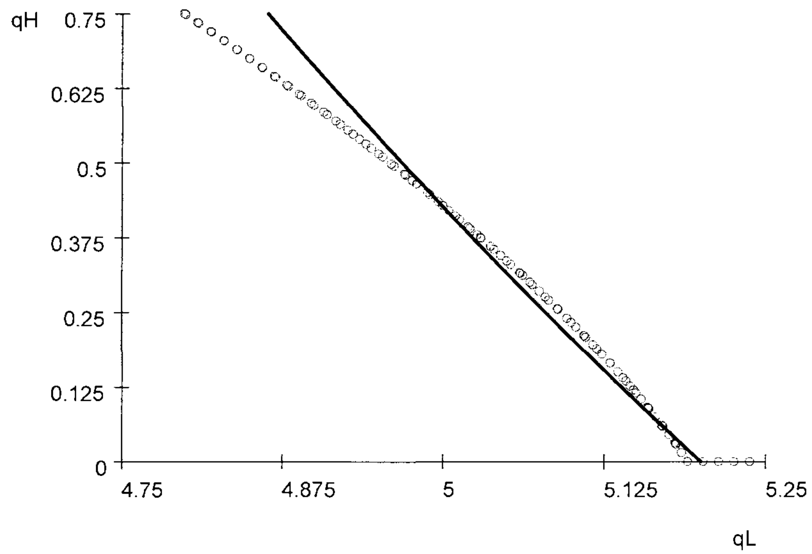

Figure 3.10: Three pure-strategy equilibria ($M = 9.995$, $\delta = 2.605$, $\varepsilon = 2.2$)

Next, as far as condition (i) is concerned, observe from the proof of Proposition 15 that the system of equations (A) and (B) does not admit a solution if

$$15\varepsilon(\varepsilon - 2\delta) + M^2 < 0. \tag{3.88}$$

So, a pure-strategy duopolistic equilibrium fails to exist if

$$\delta > \frac{\varepsilon}{2} + \frac{M^2}{30\varepsilon}. \tag{3.89}$$

As the condition above is at times less stringent than the one corresponding to the existence of a monopolistic equilibrium, there is a whole range of parameters for which a pure-strategy equilibrium simply fails to exist. Such combinations of δ and ε correspond to situations where firm H's problem is not concave. The best-interior-response then decreases with q_L until it corresponds to a simple inflexion point of the profit function, after which point it is no longer well-defined.

We illustrate this possibility with the following example, displayed on figure 3-11: $a = 0$, $b = 1$, $x_L = 4$, $x_H = 34$, $c_L = 1$, $c_H = 19$. That corresponds to $x = 19$, $c = 10$, $M = 9$, $\delta = 9$, and $\varepsilon = 15$. The thin, rounded curve is the locus of all points satisfying the first-order condition. For low q_L, there are two of them, corresponding to a local minimum and a local maximum of π_H. For large q_L, such points do no exist. The circles stand for BR_H, which exhibits a "jump" at 1.875. Clearly, the two best-response curves do not intersect; there is no pure-strategy equilibrium.

We now check conditions (ii) and (iii) for the only stable, duopolistic candidate equilibrium, characterized by

$$z^* = \frac{1}{2} + \frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2} - M}{10\varepsilon}. \tag{3.90}$$

Intersection in the positive orthant Even if the best-interior-response curves intersect, the intersection may lie outside the positive orthant. As it is not possible for q_L^* and q_H^* to be both negative, one only needs to check that $z^* \in (0, 1)$. We have to distinguish several cases.

If $\varepsilon = 2\delta$, then $z^* = \frac{1}{2}$, and the intersection always lies in the positive orthant.

If $\varepsilon > 2\delta$, then $z^* > \frac{1}{2}$ and we must ensure that

$$0 < \frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2} - M}{10\varepsilon} < \frac{1}{2}, \tag{3.91}$$

which is true if and only if

$$\delta > \frac{-M - \varepsilon}{3}. \tag{3.92}$$

This condition is always satisfied when $\delta > 0$.

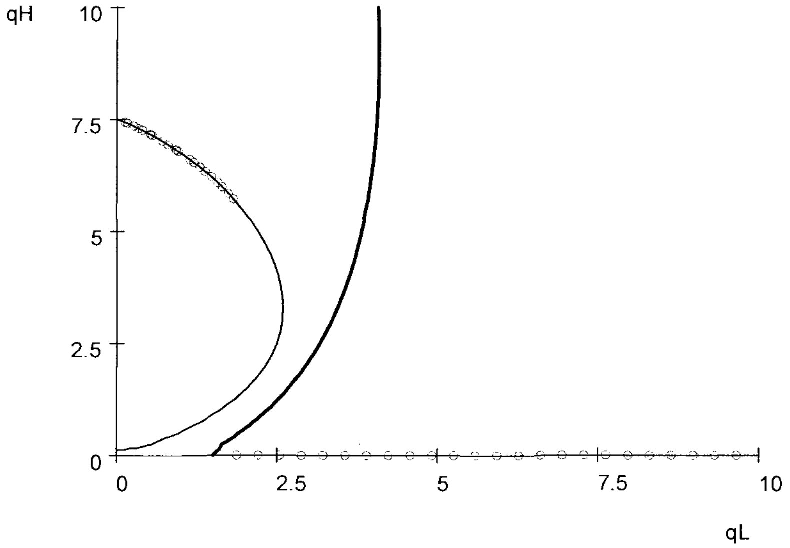

Figure 3.11: No equilibrium in pure strategies ($M = 9$, $\delta = 9$, $\varepsilon = 15$)

If $\varepsilon < 2\delta$, then $z^* < \frac{1}{2}$ and we must ensure that

$$0 > \frac{\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2} - M}{10\varepsilon} > -\frac{1}{2}. \tag{3.93}$$

If $M - 5\varepsilon \geq 0$, then the right-most inequality holds only if

$$\delta < \frac{M - \varepsilon}{3}. \tag{3.94}$$

This condition is the reverse of the one delineating the border of the monopolistic equilibrium region. Therefore, for $\varepsilon \leq \frac{M}{5}$, a monopolistic and a duopolistic equilibrium characterized by z^* cannot co-exist. If $M - 5\varepsilon < 0$, then the condition is always satisfied.

Intersection left of the non-convexity in BR_H Even if the best-interior-response curves do intersect in the positive orthant, their intersection might not lie on the best-global-response curves. We thus have to check that it takes place left of the possible "jump" in BR_H. In the case of a small quality differential, BR_H is a non-increasing, continuous *function* of q_L. As the two continuous best-response curves must then intersect, a pure-strategy equilibrium always exists. If $\delta < \frac{M-\varepsilon}{3}$, we also have $q_L^M < \rho_L$ and $q_H^M < \rho_H$, and this equilibrium must be duopolistic. In that case, (q_L^*, q_H^*) lies in the positive orthant. It must be an equilibrium then, since by Corollary 8 (q_L^{**}, q_H^{**}) cannot be an equilibrium without (q_L^*, q_H^*) being an equilibrium as well.

The non-convexity issue arises only in the case of a large differential. Observe that the non-convexity in BR_H then occurs at $\tilde{q}_L = \frac{(a+x_H-c_H)^2}{4b(x_H-x_L)}$, at which point the best interior reply is $\tilde{q}_H = \frac{a+x_H-c_H}{2b}\left(1 - \frac{a+x_H-c_H}{2(x_H-x_L)}\right)$. Firm H's market share at this point is given by

$$\tilde{z} = 1 - \frac{M+\varepsilon-\delta}{4\varepsilon}. \tag{3.95}$$

(q_L^*, q_H^*) lies to the left of the non-convexity only if $z^* \geq \tilde{z}$.

We have

$$z^* - \tilde{z} = \frac{3M - 5\varepsilon - 5\delta + 2\sqrt{15\varepsilon(\varepsilon - 2\delta) + M^2}}{20\varepsilon}. \tag{3.96}$$

It is possible to show that this quantity is always positive, so that this constraint never binds: (q_L^*, q_H^*) never lies right of the "jump" in BR_H.

We thus conclude with the following statement.

Proposition 21 *When $\delta > 0$, (q_L^*, q_H^*) is a pure-strategy duopolistic equilibrium if and only if the following conditions are satisfied:*

(i) $\delta \leq \frac{\varepsilon}{2} + \frac{M^2}{30\varepsilon}$;

(ii) if $\varepsilon \leq \frac{M}{5}$ *and* $\varepsilon < 2\delta$, *then* $\delta < \frac{M-\varepsilon}{3}$.

3.5.4 Summary

We can now summarize our findings regarding the existence and nature of stable pure-strategy Nash equilibria with figure 3-12 (drawn to scale for the case where $M = 9$).

The quality differential, ε, is on the horizontal axis. The cost differential, δ, is on the vertical axis.

The downward-sloping line marked with circles delineates the small quality differential area. The thick, downward-sloping line that is the closest to the bottom of the figure is the border between the H-monopolistic equilibrium region and the duopolistic equilibrium region. Left of the vertical, dashed line, this region extends upward until the thin, parallel, downward-sloping line is reached. Right of the vertical line, it extends until the dotted curve is reached. The thick, non-monotone curve is the border between the L-monopolistic equilibrium region and the duopolistic equilibrium region. The two latter curves are tangent to each other exactly on the vertical line. Letting the quality differential increase from that point, they delineate a region where the duopolistic and the L-monopolistic region co-exist.

The three most noticeable features of the model are the following.

First, the border of the L-monopolistic area is not monotone. When the quality differential is large, the high-quality firm can restablish its margin by flooding the market and driving quality and price up. Thus, firm L needs a big cost advantage in order to force firm H's shut-down.

Second, when the quality differential and the cost differential are of the same large order of magnitude, two stable equilibria in pure strategies coexist: one duopolistic

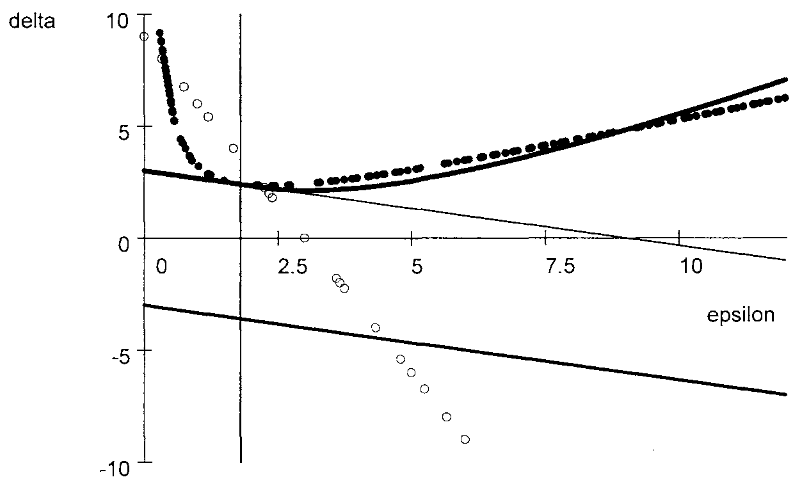

Figure 3.12: A typical equilibrium map ($M = 9$)

equilibrium and one L-monopolistic equilibrium. In effect, the nature of the interection between the two firms is akin to a battle-of-the-sexes game: either the ratio of q_H over q_L is low and so are price and quality; or the ratio of q_H over q_L is high and so are price and quality.

Third, when the quality differential and the cost differential are very large, a pure-strategy equilibrium may fail to exit. This outcome is the combination of the discontinuity in firm H's optimal-response behavior and the non-monotonicity of firm L's optimal-response behavior. In terms of the usual best-reply dymanics, this is a classical instance of cycling. If H exits, then L wants to exert its monopoly power and restrict output. Given this low output, H wants to re-enter and being a large quantity to the market, driving quality and price up. L then wants to take advantage of the high margins and increase its output, forcing H's exit.

3.6 Welfare considerations

Rather than carrying out a detailed welfare analysis, our aim in this section is to underline the important aspects in which our strategic-quantity-setting model may depart from the conventional wisdom about the provision of quality in an environment with asymmetric information. We firstly show by means of an example that strategic behavior can mitigate the adverse selection problem. Secondly, we compare consumer surplus in two different situations to show that buyers can potentially benefit from the variation in quality at the producer level.

3.6.1 Market unraveling

Recall that when the cost disparity does not assume extreme values, there always exists a duopolistic equilibrium. So, in our setting where producers recognize that they have an impact on the market price, in contrast with the market for lemons, the market for melons does not completely unravel and high-quality products continue being supplied.

Consider the following lemons example, which involves only two types of products. To facilitate the comparison with our model, we assume that the good is perfectly divisible. Producers choose the quantity of cars they offer for sale. Producer H has cars whose use she values at c_H. Producer L has cars whose use she values at $c_L < c_H$. Under this constant-returns-to-scale production technology, the individual supply curves are easily derived:

$$S_L(p) = \begin{cases} 0 & \text{if } p < c_L \\ \text{any positive number} & \text{if } p = c_L \\ \infty & \text{if } p > c_L \end{cases}; \tag{3.97}$$

$$S_H(p) = \begin{cases} 0 & \text{if } p < c_H \\ \text{any positive number} & \text{if } p = c_H \\ \infty & \text{if } p > c_H \end{cases}. \tag{3.98}$$

A mass $\Theta + a$ of consumers, indexed by θ, is uniformly distributed over a closed interval $[-\Theta, a]$. We assume that Θ is sufficiently high for the demand curve beloew not to exhibit a kink. θ stands for the "baseline" (dis)utility derived from owning a car of quality zero (a loss). Consumers all have the same willingness to pay for quality improvements. That is, consumer θ's utility from buying car i at price p is given by

$$U(x_i, p; \theta) = \theta + x_i - p. \tag{3.99}$$

There is no utility derived from consuming more than one car and the utility from not owning any car is equal to zero. Consumers maximize expected utility so that when quality is not observable the inverse demand curve is given by

$$P = a + \bar{x} - Q. \tag{3.100}$$

Let us make the assumption that consumers derive more utility from owning the cars than the sellers, and the more so for high-quality cars. Let us have

$$x_L = \alpha c_L, \tag{3.101}$$

$$x_H = \beta c_H, \tag{3.102}$$

with $1 < \alpha < \beta$. Observe that

$$x_H - c_H = (\beta - 1)c_H > (\alpha - 1)\, c_L = x_L - c_L \tag{3.103}$$

It is thus clear that maximizing total surplus requires H cars to be sold to those consumers for which $\theta \geq -(\beta - 1)c_H$.

If the producers are price-takers, then the unique competitive equilibrium is given by:

$$p^c = c_L. \tag{3.104}$$

$$Q^c = a + (\alpha - 1) c_L. \tag{3.105}$$

That is, all consumers for which $\theta \geq -(\alpha - 1) c_L$ buy a "lemon". This of course an instance of adverse selection. Less consumers buy and they get the wrong car, so to speak. The example makes clear that the phenomenon does not arise from the fact that producers make a binary decision to offer a given capacity or not. On the contrary, it is driven by the price-taking behavior of the sellers.

By contrast, if the gap between c_L and c_H is not too pronounced, the previous analysis showed that firm H will not withdraw from the market if it and firm L strategically set their quantity. Moreover, for α or $\frac{\beta}{\alpha}$ high enough, we have $\varepsilon > 2\delta$, and firm H will actually dominate the market.

This said, on one hand, there is no value of β and α for which firm H can enjoy a monopoly position in equilibrium (as this would require $c_H - c_L < 0$, contrary to our assumption). On the other hand, if $c_H - c_L$ is high enough, there might be a L-monopolistic equilibrium that is worse (in terms of welfare) than the competitive equilibrium because firm L will exert its monopoly power. So, there is a sense in which strategic quantity-setting mitigates the adverse selection issue without solving it completely for relatively small cost differentials (but exacerbates it for large ones).

This result is an example of the well-known result in second-best theory according to which two market distorsions may be preferable to only one. Here, endowing the two competitors with some market power is a way to alleviate the problems generated by the information asymmetry between buyers and sellers.

3.6.2 Consumer surplus

An interesting way of looking at the stable duopolistic equilibrium outcome consists in adding (A) to (B), the necessary first-order conditions, which gives:

$$Q^* = \frac{2(a + \bar{x}^*) - (2c)}{3b} \tag{3.106}$$

Plugging this into the demand curve:

$$P^* = \frac{a + \bar{x}^* + (2c)}{3} \tag{3.107}$$

These expressions are identical to the ones in standard Cournot model, with the double caveat that the equilibrium average quality, $\bar{x}$, is endogenously determined in our model, and that $c = (c_H + c_L)/2 = [\mu(x_H) + \mu(x_L)]/2$ depends upon the specification of qualities, for a given marginal cost function μ.

Call the situation where consumers face two producers of different qualities x_L and x_H, Situation 1. Suppose that (q_L^*, q_H^*) is the unique equilibrium. (That is, suppose that the cost differential is not extreme.) Consumer surplus is straightforwardly computed:

$$CS_1 = \frac{4\left(a + \bar{x}^* - c_1\right)^2}{9b}, \tag{3.108}$$

where $c_1 = [\mu(x_H) + \mu(x_L)]/2$.

Imagine that, instead of facing the two firms producing x_L and x_H, consumers were in Situation 2, facing two identical firms producing the average quality x. Then, consumer surplus would be

$$CS_2 = \frac{4\left(a + x - \mu(x)\right)^2}{9b} \tag{3.109}$$

We have

$$CS_1 > CS_2 \Leftrightarrow \bar{x} - \frac{\mu(x_H) + \mu(x_L)}{2} > x - \mu(x). \tag{3.110}$$

Clearly, there are numerous instances in which CS_1 will be greater than CS_2. For a given quality differential, all that is required is that the function μ be not "too convex" (or concave enough). In fact, if μ is a linear function of quality with $\mu' < 1/2$, then the inequality holds for any quality differential. Indeed, in that case, firm H produces more than firm L (for $2\delta < \varepsilon$), so that $\bar{x} > x$, and $[\mu(x_H) + \mu(x_L)]/2 = \mu(x)$ by linearity. This is true, in particular, if marginal cost is constant, as in the case when quality impacts only fixed costs.

3.7 Conclusion

We have analyzed a generalization of the Cournot duopoly game where firms produce different qualities but products cannot be distinguished by consumers, whose willingness to pay for the good depends upon the average quality.

We have shown that when the quality differential is small, the game is a continuous deformation of the standard Cournot game, in the sense that there always exists a unique pure-strategy equilibrium. If the low-quality firm is at a large cost disadvantage, then the high-quality firm is a monopoly in equilibrium. If the low-quality firm is at a large cost advantage, then it enjoys monopoly in equilibrium. If it is on the par or so with its competitor, then both firms remain active in equilibrium.

When the quality differential is large, however, the two quantities are not always strategic substitutes and the high-quality firm's best-response curve may exhibit a "jump". As a result, a pure-strategy equilibrium may fail to exist, or two or three equilibria may co-exist, which raises an equilibrium selection issue. To this end, we argued that when the equilibrium is not unique, there are only two stable equilibria: one in which the low-quality firm produces its monopoly output and the high-quality firm withdraws from the market; and another one in which the high-quality firm dumps a high quantity on the market and thus sustain high levels of quality and price. In these circumstances, we would expect the high-quality firm, operating in a more complicated, multi-stage game, to take steps in order to shape the industry expectations, or commit in advance to produce high quantities, so as to end up in the duopolistic equilibrium.

Because the high-quality firm has the possibility of impacting the average quality (and therefore consumers' willingness to pay for the good) through a rise in its quantity, the range of parameters for which an L-monopolistic equilibrium exists first enlarges, then shrinks with the quality differential. However large the quality differential, there always exists a (vanishing) range of cost parameters for which the high-quality firm would have entered the market if guaranteed a monopoly position but is evicted by the low-quality firm in equilibrium. In the special case where marginal costs are equal, the high-quality firm's market share is capped by a constant that is independent of the quality differential. In other words, when quality upgrades necessitate fixed investments, there is no way for a top-quality firm to prevent a lower-quality competitor from benefiting from these investments.

In all cases where the cost differential is not extreme, there exists a stable equilibrium in which both firms remain active. This is in sharp contrast with the well-known unraveling of markets under asymmetric information and price-taking behavior. It is the sense in which strategic quantity-setting can be said to help mitigate adverse selection problems. This result has direct implications for policy-making as it suggests that a planner (or an anti-trust authority) might come to regard the horizontal concentration of same-quality producers as a way to prevent the disappearance of high-quality products.

As a matter of fact, in our model the high-cost firm ends up producing more than the low-cost firm if it enjoys a sufficiently large quality advantage. On the basis of consumer surplus, consumers may thus prefer facing two producers of unequal qualities to facing two identical producers, since in equilibrium the average quality can increase more than the price. We speculate that this result coud be reproduced under mild risk aversion.

CHAPTER 4

EXCLUSIVE QUALITY

ABSTRACT[1] Fumagalli and Motta (forthcoming) show that in the case of a homogenous product, competition at the retail level prevents an incumbent upstream firm from using exclusivity contracts in order to deter the entry of a more efficient rival. We show here that in the case where the upstream firms sell vertically differentiated products, the result is reversed: Bertrand competition among retailers makes it easy for an incumbent to exclude a higher-quality producer. Indeed, as long as the quality differential is not too large, the incumbent's inferior product cannot be priced out of the market. As a result of the ensuing price competition, the potential entrant is not assured to capture all the efficiency gains from the introduction of its superior product and decides not to enter unless it has access to both retailers. Since, by assumption, it cannot commit to transfer some of its profit to retailers in that event, exclusion emerges as the unique equilibrium outcome.

KEYWORDS: vertical differentiation, contracts, exclusion, monopolization.

JEL CODES: L12, L42.

[1] I sincerely thank the Stockholm-based Knut and Alice Wallenberg Foundation (Knut och Alice Wallenbergs Stifelse) for financial support. The paper has benefited from helpful comments from Jörgen Weibull, Tore Ellingsen, Tim Howington, Stefan Napel and Marcus Salomonsson. The remaining errors and shortcomings are mine only.

4.1 Introduction

US courts and antitrust practitioners gave early credence to the possibility of foreclosing rivals through vertical integration or other vertical arrangements. In the 1970s, some leading scholars mounted a critique of what constituted, in their eyes, the undiscriminating use of a logically-flawed theory. Posner (1976) and Bork (1978) gave particularly eloquent expositions of the argument which came to be associated with the general "Chicago critique" of antitrust government activity. According to these authors, in order to induce a buyer to sign an exclusivity contract, the incumbent should fully compensate it for the loss it suffers from not buying from a more efficient entrant.[2] If the buyers are final consumers, this loss amounts to the difference between the consumer surplus under entry and under the incumbent's monopoly. This loss comprises the monopolist's rents and the deadweight loss. Hence, even by offering to give up the whole of its rents, an incumbent is never able fully to compensate consumers. Therefore, deterring entry is not a feasible policy and exclusivity contracts, if they are observed, must be explained by some other motivations, for instance by some efficiency considerations. As a consequence, competition authorities should not worry about their use.

The Chicago critique forced industrial economists into reconsidering their theories and the past twenty years have witnessed a flurry of contributions addressing the possibility of vertical foreclosure or entry deterrence through the use of vertical arrangements. It is not easy to summarize the vast literature that is now available. One can only hope to make some broad distinctions. One can argue that early on, researchers explored two different routes, one concerned with the possible anti-competitive effect of vertical *mergers* on *existing* rivals, the other concerned with the deterrence of a *potential* rival's entry by an incumbent through the use of

[2] There is no loss if the potential rival produces at a higher cost and would be wiped out by competition upon entry.

vertical arrangements, especially exclusivity *contracts*. The theory of vertical foreclosure now comprises numerous variants but two prominent versions are associated with the pioneering contributions of Ordover, Saloner and Salop (1990) and Hart and Tirole (1990).

The work on exclusivity contracts is associated with Rasmusen, Ramseyer and Wiley (1991) and Segal and Whinston (2000). In their model, the entrant needs to generate enough revenues in order to cover its fixed cost, which translates into supplying a minimum number of consumers. The existence of this minimum viable scale creates scope for entry deterrence, as one retailer's signing on an exclusivity contract exerts an externality on all the other retailers by making the entry of the more efficient rival more difficult. By exploiting this externality, the incumbent is able to prevent entry. Indeed, if all consumers sign up, a unilateral deviation by one consumer will not prove sufficient to induce entry, so that this consumer has no incentive not to sign up as well.

Recently, Fumagalli and Motta (forthcoming) have cast doubt on the validity of this line of reasoning by assuming that the purchasers of the incumbent's product are retailers rather than final consumers, as previsouly assumed. They show that, in this case, even a small degree of competition between retailers is sufficient to disturb the incumbent's deterrence scheme. In effect, competition between retailers introduces another externality across them. A retailer's signing on an exclusivity contract indeed gives the other retailers an additional incentive to market the entrant's product, as they will face less competition in selling it. As a result, first, every retailer becomes pivotal because the demand from only one retailer proves sufficient to induce entry. Second, a retailer anticipates big profits when deviating from the incumbent's exclusivity scheme, which obliges the latter to pay high compensations. The combination of both facts renders the deterrence scheme prohibitively costly to the incumbent.

So far, the discussion has been conducted under the maintained hypothesis that the upstream firms produce a homogeneous product.[3] We show in the present study that, when the two upstream firms sell vertically differentiated products, the effect of downstream competition on the possibility of using exclusivity contracts to deter entry identified by Fumagalli and Motta (2002) is reversed: intense downstream competition helps the incumbent pre-empt the entry of its rival, as long as the quality differential is not too large. Indeed, by signing an exclusivity contract with one retailer, the incumbent wholesaler forces the potential entrant exclusively to deal with the other retailer. Yet, upon entry, because of differentiation, the incumbent's inferior product is *not* eliminated from the market. As a consequence, the potential entrant cannot always capture the efficiency gains associated with the introduction of its superior product. Since, bu assumption, it is also unable to commit to transfer some of the industry profit to the retailers, any exclusivity offer by the incumbent looks attractive to them. As a consequence, in equilibrium, the incumbent is able to exclude *at no cost.*

In the linear-pricing case, the phenomenon we describe is comparable with the vertical foreclosure argument in Ordover, Saloner and Salop (1990). However, the structure of the industry is reversed: the retailing services constitute the essential input and an exclusivity contract is akin to a refusal to deal on the part of one of the bottleneck owners. Another difference is that in our model, this retailer does not need to continue marketing the entrant's product in order to weaken the market

[3]To our knowledge, the only prominent article discussing vertical foreclosure in the case of differentiated upstream goods is Ma (1997). Yet, the author assumes a particular downstream structure since the retailers sell so-called option contracts to the final consumers, who, for a fixed fee, purchase the possibility to buy one of the two goods (of their choice) at pre-specified prices at some time in the future. Stenneck (2006) looks at the role of quality in exclusive distribution agreements but his focus is not on foreclosure. Indeed, there is a single upstream firm in his bargaining model. Fumagalli and Motta (2002) use a (horizontal) differentiation parameter as a proxy for the intensity of competition at the *downstream* level.

power exerted on the latter by the non-integrated retailer. Indeed, the entrant does not take part in the initial bidding for the right to foreclose and does not have the possibility to respond either. As a consequence, there is no need for preventing the foreclosed firm from vertically integrating by keeping its profit up.

The rest of this chapter is organized as follows. Section 2 describes the game and its timing. Section 3 is devoted to the linear-pricing case. In turn, we study the situation when the two goods are sold by independent retail monopolists and when there is Bertrand competition at the retail level. Section 4 is concerned with the case where firms are allowed to offer two-part tariffs. Section 5 contains some conclusive remarks.

4.2 MODEL

We consider an industry in which firms produce vertically differentiated products. The industry is characterized by the presence of two downstream firms (retailers), labelled 1 and 2, and two upstream firms (referred to as producers or wholesalers), I and E. The incumbent upstream firm, I, produces a good of quality q_I at constant marginal cost c_I and sells it to retailer j at unit price w_I^j (along with a fixed fee ϕ_I^j in the two-part tariff variation), for $j = 1, 2$. The potential rival in the upstream segment, E, has the option of entering the market and selling its product of quality $q_E \geq q_I$ to retailers for a price w_E (along with a fixed fee ϕ_E in the two-part tariff variation). Doing so would entail the expense of a fixed and unrecoverable amount $F > 0$, as well as per-unit production costs equal to c_E. Choosing not to enter the market would bring E zero profit. The two upstream firms cannot sell their products directly to consumers. For simplicity, the retailers are assumed to buy and resell the goods at no cost. They compete for final consumers *à la Bertrand.*

There is a unit mass of consumers indexed by θ and uniformly distributed over $[0, 1]$. Consumers value the first unit consumed only. A consumer θ who buys one unit of quality q at price p derives utility

$$U(q, p; \theta) = \theta q - p. \tag{4.1}$$

The utility from not consuming the good is set to zero. Observe that the type θ of a consumer stands for his willingness to pay for quality at the margin.

We follow Fumagalli and Motta (forthcoming) in specifying an extensive-form game that allows the incumbent to offer the retailers exclusivity contracts before the entrant can make its decision. More precisely, the timing of the game is as follows. At time t_0, I offers buyers identical exclusivity contracts in exchange for compensation $y \geq 0$, and buyers decide to accept or not.[4] An exclusivity contract commits the retailer to purchase only from the incumbent but does not constrain the latter's behavior.[5]

At time t_1, the actions previously taken are observed by all players. A distinctive feature of the history of play at this stage is the number S of retailers who have signed on an exclusivity contract, where S can take the values 0, 1, or 2. E must decide on entry. If it enters, it has to pay the unrecoverable amount F. If it does not, its payoff is assumed to be zero.

At time t_2, the action taken by E is observed by all players. The upstream firms that are present on the market simultaneously name their price (or, in the two-part-tariff variation, their price schedule). I is allowed to discriminate between the two retailers only in the case when one has signed an exclusivity contract and the other

[4]Rasmusen, Ramseyer and Wiley (1991), Segal and Whinston (2000), and Fumagalli and Motta (forthcoming) study variations of the game where the incumbent is allowed to make retailers simultaneous yet different offers, or sequential offers. In these papers, the case of simultaneous, identical offers is the less favorable to exclusion. Focusing on it only reinforces our results.

[5]That is, exclusivity goes one-way only and the incumbent upstream firm is free to continue selling its product to any other retailer.

has not. Thus, only in subgames where $S = 1$ can we have $w_I^1 \neq w_I^2$ (or $\phi_1 \neq \phi_2$ in the two-part-tariff variation). E can only sell its product to the retailers who have not signed an exclusivity contract with I (that is, exclusive contracts are perfectly enforced) and is constrained to charge the same price w_E to those.

At time t_3, the retailers compete in prices for final consumers. Each is committed to serve the demand adressed to it at its posted price, and order the corresponding inputs. In case both retailers charge the same price for the same product, we assume that all consumers patronize the firm with the strictly lower marginal cost. If the retailers happen to have the same marginal cost, then a fair coin toss determines which one all consumers buy from.[6]

We look for the subgame-perfect equilibria of this game. Observe that our model is as close as possible to the one studied by Fumagalli and Motta (forthcoming). Indeed, with our notation, their model obtains when setting $q_I = q_E = 1$ and $c_I > c_E = 0$.[7] Because products are homogeneous in their specification, efficiency dictates that firm I, which is cost-inefficient, be shut down (provided E's fixed cost is not too large). In order to reproduce that outcome in the simplest differentiated-product setting, we further restrict attention to the case where $c_I = c_E = 0$ and $q_E > q_I > 0$. In that configuration, E's product is unambiguously superior to I's

[6]This assumption is made to avoid the non-existence of a pure-strategy Bertrand equilibrium. It is well-known that, if prices were chosen on a finite grid, as is realistic, the outcome in the case when the two firms have different marginal costs, would be close to the one assumed here. In the case when the firms have the same cost function, the prescribed rule (i) leads to the same payoff as the one in which every *single* consumer randomizes, under linear pricing; and (ii) with two-part tariffs, leads to an equilibrium (in which firms charge the lowest possible price guaranteeing zero profit to a monopoly) that adequately stands for the industry long-term structure. (By contrast, other rules typically give rise to probabilistic entry.)

[7]With one modification: in their model, retailers incur a fixed cost for being active and have to make a separate decision about entry. This has the advantage of eliminating equilibrium multiplicity in some instances but is not innocuous, as they discuss in Section III of their article. In particular, it gives a retailer buying from the efficient producer the possibility to charge the monopoly price. Our results do not hinge on the relaxation of that assumption.

product: if both were offered at their true resource cost, all consumers would choose to buy q_E and firm I would shut down.[8] Notice that, as θ runs from 0 to 1, there is enough heterogeneity in preferences for both qualities to be profitably supplied in the unique pure-strategy equilibrium of the game where the two producers compete in prices for the direct patronage of final consumers, a game we will refer to as the "canonical (or standard) model of product differentiation."[9]

4.3 Linear pricing

We first look at the benchmark case when the market is divided in two separate submarkets, each served by one retailer, so that there is no strategic interaction between 1 and 2. We then consider the game described in the previous section, when both retailers are present on a unique market.

4.3.1 Local downstream monopolists

Suppose first that 1 and 2 are local retail monopolists in two separate markets, each serving (a randomly drawn) half of the population of consumers.

Retailer's choice

The situation is identical for both monopolists. Given the input prices, w_I and w_E, retailer j can choose to sell either product or both of them. One might want to specify a choice rule for the cases when the retailer is indifferent. Such a rule would

[8] When price is set at marginal cost (here, zero), the rise in consumer surplus from the introduction of E's product is given by $(q_E - q_I)/2$. As long as F is below that level, E's product should be introduced by a social planner seeking to maximize total surplus. This will always be true under the different assumptions (A1 and A2) we later make about the size of the sunk cost.

[9] This model was introduced by Gabzsewicz and Thisse (1979) and Shaked and Sutton (1982). It was popularized by Tirole (1988) through the specification of simple preferences à la Mussa and Rosen (1978). It was used in the specific form under which it appears in the present study by Ronnen (1991).

not affect our results, as will become clear below. For the sake of definiteness, we make the assumption that is the most favorable to the entrant: in case the retailer is indifferent between selling only q_E and only q_I (or indifferent between marketing both products or selling only q_I), it chooses the first option.

Suppose first that the local monopolist markets only one of the two goods. We denote the last consumer willing to buy product q_i by $\tilde{\theta}_i$. Given the consumers' preferences, $\tilde{\theta}_i = p_i/q_i$. So,

$$\pi_j = \frac{1}{2}(p_i - w_i)(1 - \frac{p_i}{q_i}). \tag{4.2}$$

This is a concave problem and the sufficient first-order condition gives

$$p_i = \frac{q_i + w_i}{2}. \tag{4.3}$$

At the optimum, the unit margin is $(q_i - w_i)/2$ and the profit is

$$\begin{aligned} \pi_j &= \frac{1}{2}\left(\frac{q_i - w_i}{2}\right)\left(\frac{q_i - w_i}{2q_i}\right) \\ &= \frac{1}{8}\frac{(q_i - w_i)^2}{q_i}. \end{aligned} \tag{4.4}$$

Of course, if there is no cost difference or there is one in favor of the high-quality product, then it is unambiguously preferable to market only that product. If the high-quality product is also more expensive, then the comparison is less immediate. In particular, it is not enough for the high-quality product to yield a higher margin than the low-quality variant; it must do so by a factor of $\sqrt{q_E/q_I}$ at least.

Suppose now that j sells positive quantities of both variants. There is a consumer $\hat{\theta}$ who is indifferent between buying either product. The retailer seeks to maximize

$$\begin{aligned} \pi_j &= \frac{1}{2}\left[(p_E - w_E)\left(1 - \hat{\theta}\right) + (p_I - w_I)\left(\hat{\theta} - \tilde{\theta}_I\right)\right] \\ &= \frac{1}{2}\left[(p_E - w_E)\left(1 - \frac{p_E - p_I}{q_E - q_I}\right) + (p_I - w_I)\left(\frac{p_E - \frac{q_E}{q_I}p_I}{q_E - q_I}\right)\right]. \end{aligned} \tag{4.5}$$

The program is concave in both prices. Solving the system of first-order conditions gives:

$$p_E = \frac{q_E + w_E}{2}, \tag{4.6}$$

$$p_I = \frac{q_I + w_I}{2}, \tag{4.7}$$

the consumer indifferent between buying from either firm being located at

$$\hat{\theta} = \frac{1}{2} + \frac{1}{2}\frac{w_E - w_I}{q_E - q_I}. \tag{4.8}$$

This set of prices maximizes profit (under the constraint that both products are marketed) only if the so-called hedonic price for the high-quality product (i.e. the price per unit of quality) is higher than the hedonic price for the low-quality product. Otherwise, there is no demand for the latter. This condition is satisfied if:

$$\frac{q_E + w_E}{2q_E} > \frac{q_I + w_I}{2q_I}, \tag{4.9}$$

which reduces to

$$\frac{w_E}{q_E} > \frac{w_I}{q_I}. \tag{4.10}$$

Clearly, if this inequality is reversed, selling E's product only is the optimal policy.

Notice that, when the monopolist markets both products, the prices are the same as when it sold only one of them (a consequence of the linearity of the demand curves). The mass of consumers served by the firm is the same as when it sells only I's product. Yet, it charges higher prices to the top consumers. "Cannibalization" does occur: a fraction $\hat{\theta} - \tilde{\theta}_E$ of consumers who would have bought the high-quality product at a high price if it were the only good available switch when presented with the low-quality alternative. If $q_E - w_E < q_I - w_I$, this is good news for the monopolist as these customers will generate higher profit margins but then, it would be preferable to have all consumers switch, and to drop E's product altogether. If $q_E - w_E \geq q_I - w_I$, then selling both products is at least as profitable as selling good

q_I only. In order to know whether good q_I should be marketed at all, one has to compare the losses due to cannibalization to the gains arising from the additional mass of consumers $\tilde{\theta}_E - \tilde{\theta}_I$ purchasing the low-quality good. The gains generated by these extra consumers are given by

$$\begin{aligned}\left(\tilde{\theta}_E - \tilde{\theta}_I\right)(p_I - w_I) &= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\frac{w_E}{q_E} - \frac{1}{2} - \frac{1}{2}\frac{w_I}{q_I}\right)\left(\frac{q_I - w_I}{2}\right) \\ &= \frac{1}{8}\left(\frac{w_E}{q_E} - \frac{w_I}{q_I}\right)(q_I - w_I). \qquad (4.11)\end{aligned}$$

The loss on those consumers who switch to the low-quality variant are given by

$$\begin{aligned}\left(\hat{\theta} - \tilde{\theta}_E\right)[(p_E - w_E) - (p_I - w_I)] &= \frac{1}{2}\left[\frac{1}{2} + \frac{1}{2}\frac{w_E - w_I}{q_E - q_I} - \left(\frac{1}{2} + \frac{1}{2}\frac{w_E}{q_E}\right)\right] \\ &\quad \cdot \left[\frac{(q_E - w_E) - (q_I - w_I)}{2}\right] \\ &= \frac{1}{8}\left(\frac{w_E - w_I}{q_E - q_I} - \frac{w_E}{q_E}\right) \\ &\quad \cdot [(q_E - w_E) - (q_I - w_I)]. \qquad (4.12)\end{aligned}$$

The gains are greater than the losses if and only if

$$\left(\frac{w_E}{q_E} - \frac{w_I}{q_I}\right)(q_I - w_I) \geq \left(\frac{w_E - w_I}{q_E - q_I} - \frac{w_E}{q_E}\right)[(q_E - w_E) - (q_I - w_I)], \qquad (4.13)$$

or, equivalently,

$$-\frac{w_I}{q_I}(q_I - w_I) \geq \left(\frac{w_E - w_I}{q_E - q_I}\right)[(q_E - w_E) - (q_I - w_I)] - \frac{w_E}{q_E}(q_E - w_E), \qquad (4.14)$$

or, equivalently,

$$\left(\frac{w_E}{q_E} - \frac{w_E - w_I}{q_E - q_I}\right)(q_E - w_E) \geq \left(\frac{w_I}{q_I} - \frac{w_E - w_I}{q_E - q_I}\right)(q_I - w_I), \qquad (4.15)$$

or, equivalently,

$$\left[\frac{w_E(q_E - q_I) - q_E(w_E - w_I)}{q_E(q_E - q_I)}\right](q_E - w_E) \geq \left[\frac{w_I(q_E - q_I) - q_I(w_E - w_I)}{q_I(q_E - q_I)}\right](q_I - w_I), \qquad (4.16)$$

or, equivalently,

$$\left[\frac{w_I q_E - w_E q_I}{q_E(q_E - q_I)}\right](q_E - w_E) \geq \left[\frac{w_I q_E - w_E q_I}{q_I\,(q_E - q_I)}\right](q_I - w_I)\,, \tag{4.17}$$

or, equivalently,

$$\frac{q_E - w_E}{q_E} \leq \frac{q_I - w_I}{q_I} \tag{4.18}$$

(since $w_I q_E - w_E q_I < 0$ by assumption), or, finally,

$$\frac{w_I}{q_I} \leq \frac{w_E}{q_E}, \tag{4.19}$$

which is true by assumption. So the gains from selling product q_I to some additional consumers always compensate for the losses incurred on those consumers who give up on product q_E.

Hence, the conclusion is simple.

Summary 22 *If $\frac{w_E}{q_E} \leq \frac{w_I}{q_I}$, then the monopolist sells only E's product. If $\frac{w_E}{q_E} > \frac{w_I}{q_I}$, as long as $q_E - w_E \geq q_I - w_I$, the monopolist markets both products. When the inequality reverses, it sells only I's product.*

Upstream competition

Given the retail monopolists' optimal policy, what prices will the upstream firms charge? Clearly, under linear pricing, since both firms have the same marginal cost, it is not possible for the high-quality producer to exclude the low-quality variant by means of limit-pricing. That would require $\frac{w_E}{q_E} \leq \frac{w_I}{q_I}$, and since I's marginal cost is zero, either I could cut its price sufficiently to reverse the inequality, or $w_E = w_I = 0$ and E could assuredly avail itself a profitable deviation. Therefore, in equilibrium, both products are sold by the retailers. Firm E seeks to maximize:

$$\begin{aligned} \pi_E &= w_E\left(1 - \frac{1}{2} - \frac{1}{2}\frac{w_E - w_I}{q_E - q_I}\right) \\ &= \frac{1}{2}w_E\left(1 - \frac{w_E - w_I}{q_E - q_I}\right). \end{aligned} \tag{4.20}$$

Meanwhile, firm I seeks to maximize:

$$\begin{aligned}\pi_I &= w_I\left[\frac{1}{2}+\frac{1}{2}\frac{w_E-w_I}{q_E-q_I}-\left(\frac{1}{2}+\frac{1}{2}\frac{w_I}{q_I}\right)\right]\\ &= \frac{1}{2}w_E\left(\frac{w_E-w_I}{q_E-q_I}-\frac{w_I}{q_I}\right). \end{aligned}\tag{4.21}$$

That is, the game is analogous to the standard model of vertical differentiation, with the difference that, because of double marginalization, the upstream firms anticipate that they will end up selling only half of the quantities that would have been demanded by final consumers, had they been directly charged the input prices. We now recall the structure of that standard model.

E's best response Consider firm E's problem, given w_I. We introduce the price a for which $\hat{\theta}=\tilde{\theta}_E$:

$$a = w_I\frac{q_E}{q_I}. \tag{4.22}$$

If firm E prices below a, it becomes a monopolist and so wishes to increase its price as long as market demand is inelastic, that is, as long as

$$w_E < \frac{q_E}{2}. \tag{4.23}$$

So the best response on $[0,a]$, denoted $BR_E^- = \min\left\{\frac{q_E}{2},a\right\}$. Observe that

$$\frac{q_E}{2} \geq a \Longleftrightarrow w_I \leq \frac{q_I}{2} \tag{4.24}$$

If firm E prices above a, then it faces competition from firm I and demand is therefore more elastic. Taking w_I as given, it wishes to price at

$$w_E = \frac{w_I+q_E-q_I}{2}. \tag{4.25}$$

So the best response on $[a,q_E]$, denoted $BR_E^+ = \max\left\{\frac{w_I+q_E-q_I}{2},a\right\}$. Observe that

$$\frac{w_I+q_E-q_I}{2} \leq a \Longleftrightarrow w_I \geq \frac{q_I\left(q_E-q_I\right)}{2q_E-q_I}. \tag{4.26}$$

This last treshold is always smaller than $\frac{q_I}{2}$ as

$$\frac{q_E - q_I}{2q_E - q_I} < \frac{q_E - q_I}{2q_E - 2q_I} = \frac{1}{2}. \tag{4.27}$$

Thus, firm E's best-response function is given by

$$BR_E(w_I) = \begin{cases} \frac{w_I + q_E - q_I}{2} & \text{if } 0 \leq w_I \leq q_I \frac{q_E - q_I}{2q_E - q_I} \\ a & \text{if } q_I \frac{q_E - q_I}{2q_E - q_I} < w_I < \frac{q_I}{2} \\ \frac{q_E}{2} & \text{if } w_I \geq \frac{q_I}{2} \end{cases}. \tag{4.28}$$

I's best response Consider firm I's choice problem, given w_E. We introduce the price b for which $\hat{\theta} = 1$:

$$b = w_E - (q_E - q_I). \tag{4.29}$$

(If w_E is low enough, then b can turn negative. So, in all rigor, we should write

$$b = \max\{0, w_E - (q_E - q_I)\}. \tag{4.30}$$

The derivations below correspond to the case where $b \geq 0$.)

If firm I prices below b, it becomes a monopolist and so wishes to increase its price as long as market demand is inelastic, that is, as long as

$$w_I < \frac{q_I}{2}. \tag{4.31}$$

So the best response on $[0, b]$, denoted BR_I^-, is $\min\left\{\frac{q_I}{2}, b\right\}$. Observe that

$$b \leq \frac{q_I}{2} \Longleftrightarrow w_E \leq \frac{2q_E - q_I}{2}. \tag{4.32}$$

If firm I prices above b, it faces competition from firm E and demand is therefore more elastic. Taking w_E as given, it wishes to serve half of its residual market, i.e. produce a quantity

$$\frac{w_E}{2(q_E - q_I)}, \tag{4.33}$$

which involves pricing at

$$w_I = \frac{w_E}{2}\frac{q_I}{q_E}. \tag{4.34}$$

So the best response on $[b, q_I]$, denoted $BR_I^+ = \max\left\{\frac{w_E}{2}\frac{q_I}{q_E}, b\right\}$. Observe that

$$b \geq \frac{w_E}{2}\frac{q_I}{q_E} \Longleftrightarrow w_E \geq \frac{2q_E\left(q_E - q_I\right)}{2q_E - q_I}. \tag{4.35}$$

Observe also that with $q_E > q_I > 0$, we have

$$\frac{2q_E\left(q_E - q_I\right)}{2q_E - q_I} < \frac{2q_E - q_I}{2}. \tag{4.36}$$

Thus, firm I's best-response function is given by

$$BR_I(w_E) = \begin{cases} \frac{w_E}{2}\frac{q_I}{q_E} & \text{if } 0 \leq w_E \leq \frac{2q_E(q_E - q_I)}{2q_E - q_I} \\ b & \text{if } \frac{2q_E(q_E - q_I)}{2q_E - q_I} < w_E < q_E - \frac{q_I}{2} \\ \frac{q_I}{2} & \text{if } w_E \geq q_E - \frac{q_I}{2} \end{cases}. \tag{4.37}$$

Equilibrium

It is easily checked that the two best-response curves intersect only once, at:

$$w_E^* = \frac{2q_E\left(q_E - q_I\right)}{4q_E - q_I} \tag{4.38}$$

$$w_I^* = \frac{q_I\left(q_E - q_I\right)}{4q_E - q_I}. \tag{4.39}$$

At the equilibrium, in the canonical version without retailers, we would have

$$\hat{\Theta}^* = \frac{2q_E - q_I}{4q_E - q_I}, \tag{4.40}$$

and

$$\tilde{\Theta}_I^* = \frac{q_E - q_I}{4q_E - q_I}, \tag{4.41}$$

giving

$$D_E^* = \frac{2q_E}{4q_E - q_I}, \tag{4.42}$$

$$D_I^* = \frac{q_E}{4q_E - q_I}, \tag{4.43}$$

where D_i stands for the demand adressed to producer $i \in \{E, I\}$. Thus, E serves two thirds of demand in equilibrium.

In our two-local-monopolists benchmark, because of double marginalization, these quantities are divided by two. Evidently, each retailer buys and resells half of these. This can be verified by locating the indifferent consumers:

$$\begin{aligned} \hat{\theta}^* &= \frac{1}{2} + \frac{1}{2}\hat{\Theta}^* \\ &= \frac{1}{2} + \frac{1}{2}\frac{2q_E - q_I}{4q_E - q_I} \\ &= \frac{3q_E - q_I}{4q_E - q_I}, \end{aligned} \tag{4.44}$$

and

$$\begin{aligned} \tilde{\theta}_I^* &= \frac{1}{2} + \frac{1}{2}\tilde{\Theta}_I^* \\ &= \frac{1}{2} + \frac{1}{2}\frac{q_E - q_I}{4q_E - q_I} \\ &= \frac{1}{2}\frac{5q_E - 2q_I}{4q_E - q_I}. \end{aligned} \tag{4.45}$$

Downstream equilibrium prices are given by $p_i = (w_i + q_i)/2$, which gives

$$p_E^* = \frac{1}{2}\frac{q_E(6q_E - 3q_I)}{4q_E - q_I} \tag{4.46}$$

$$p_I^* = \frac{1}{2}\frac{q_I(5q_E - 2q_I)}{4q_E - q_I}. \tag{4.47}$$

The upstream firms' equilibrium profits are

$$\begin{aligned} \pi_E^* &= \tfrac{1}{2}\left(\tfrac{2q_E}{4q_E - q_I}\right)^2 (q_E - q_I) \\ \pi_I^* &= \tfrac{1}{2}\tfrac{q_E q_I}{(4q_E - q_I)^2}(q_E - q_I) \end{aligned} . \tag{4.48}$$

Notice that $\pi_E^*/\pi_I^* = 4q_E/q_I$, which means that E appropriates a very big fraction of the upstream segment's profits.

It is of separate interest to compute the retail-level unit margins on each product:

$$\frac{q_E - w_E^*}{2} = \frac{1}{2}\frac{q_E(2q_E + q_I)}{4q_E - q_I} \tag{4.49}$$

$$\frac{q_I - w_I^*}{2} = \frac{1}{2}\frac{3q_E q_I}{4q_E - q_I}. \tag{4.50}$$

It is therefore verified that the retailers make more profit on the high-quality units than on the low-quality units, a necessary condition for E's product to be sold. The downstream firms' equilibrium profits are given by

$$\pi_j^* = \frac{1}{2}\left[\left(\frac{q_E - w_E^*}{2}\right)D_E^* + \left(\frac{q_I - w_I^*}{2}\right)D_I^*\right], \tag{4.51}$$

or,

$$\begin{aligned} \pi_j^* &= \frac{1}{8}\left[\frac{q_E\left(2q_E + q_I\right)}{4q_E - q_I} \cdot \frac{2q_E}{4q_E - q_I} + \frac{3q_E q_I}{4q_E - q_I}\frac{q_E}{4q_E - q_I}\right] \\ &= \frac{1}{8}\frac{\left(q_E\right)^2\left(4q_E + 5q_I\right)}{\left(4q_E - q_I\right)^2}. \end{aligned} \tag{4.52}$$

Observe that

$$\begin{aligned} \frac{\pi_I^* + \pi_E^*}{2\pi_j^*} &= 2\frac{\left(q_E - q_I\right)\left(4q_E + q_I\right)}{q_E\left(4q_E + 5q_I\right)} \\ &= 2\left(1 - \frac{q_I}{q_E}\frac{8q_E + q_I}{4q_E + 5q_I}\right) \\ &= 2\left(1 - \frac{q_I}{q_E}\frac{8 + \frac{q_I}{q_E}}{4 + 5\frac{q_I}{q_E}}\right), \end{aligned} \tag{4.53}$$

a number that increases monotonically with $\frac{q_E}{q_I}$ from 0 (when $q_I = q_E$) to 2. Hence, when there is no product differentiation, fierce competition at the upstream level benefits the retailers, which capture the entire industry surplus. By contrast, when there is very large differentiation, the entrant is (almost) a monopolist, which manages to capture two thirds of the industry surplus, as expected in this linear environment.

Contracting

Suppose that firms play in accordance with the game form described in Section 2. The question is: is it possible for I to induce one or both retailers to sign on exclusivity contracts that foreclose E?

At time t_2, there are three possible configurations.

- $S = 0$; neither retailers has signed an exclusivity contract; both can purchase good q_E at price w_E.
- $S = 2$; both retailers have signed an exclusivity contract; they can purchase good q_I at price w_I.
- $S = 1$; one retailer has signed an exclusivity contract but the other has not. Since the retailers are ex-ante identical, when looking at such a subgame, there is no loss of generality in assuming that 1 is the retailer who signed on while 2 is free from any commitment. The former can only buy I's product and pay w_I^1 while the latter faces prices w_I^2 (and w_E if E happens to be active).

If neither retailer signs up, then the profits are the equilibrium profits derived above.

If both sign up, then I is a monopolist. As we know, I will optimally set its price at $q_I/2$, so that $1/8$ units will be sold in each market. Hence,

$$\pi_I = \frac{q_I}{8}, \tag{4.54}$$

and

$$\pi_j = \frac{q_I}{32}; j = 1, 2. \tag{4.55}$$

The question is: can the increase in I's profit, $\Delta\pi_I$, compensate for the decrease in the retailers' profits, $2\Delta\pi_j$, *under the maintained hypothesis that E enters the market*? The answer is no, as maintained by the proponents of the Chicago critique. Indeed, there is only one market and one monopoly profit to be reaped, really. On each local market:

$$\begin{aligned} \frac{\Delta\pi_I}{2} &= \frac{q_I}{16} - \frac{4}{16} \frac{q_E q_I}{(4q_E - q_I)^2} (q_E - q_I) && (4.56) \\ &= \frac{q_I}{16} \frac{q_I^2 - 4q_I q_E + 12q_E^2}{(4q_E - q_I)^2}, && (4.57) \end{aligned}$$

and

$$\Delta\pi_j = \frac{1}{8}\frac{(q_E)^2(4q_E+5q_I)}{(4q_E-q_I)^2} - \frac{q_I}{32} \tag{4.58}$$
$$= \frac{1}{32}\frac{4(q_E)^2(4q_E+5q_I) - q_I(4q_E-q_I)^2}{(4q_E-q_I)^2}. \tag{4.59}$$

A direct computation gives

$$\frac{\Delta\pi_I}{2} - \Delta\pi_j = \frac{4q_Iq_E - 4(q_E)^2 - 3(q_I)^2}{32(4q_E-q_I)^2} < 0. \tag{4.60}$$

Hence, given the maximal amount that I can offer in order to induce a retailer to sign up, it is never in the latter's interest to do so if he believes that E will subsequently enter the market. Thus, in the absence of high entry cost, I will never never be able to prevent E' s entry.

We now make the assumption that serving only one local market is not sufficient for E to cover its fixed cost, F, unlike serving both. That is,

$$\frac{1}{2}\left(\frac{2q_E}{4q_E-q_I}\right)^2(q_E-q_I) > F > \frac{1}{4}\left(\frac{2q_E}{4q_E-q_I}\right)^2(q_E-q_I). \tag{A1}$$

Then, I will be able to exclude E only by taking advantage of a potential coordination failure between the two retailers. In our case where the incumbent makes simultaneous, non-discriminatory offers, two types of equilibria will co-exist: equilibria in which neither retailer signs up, and equilibria in which both retailers sign up. The reason is that locking-in one retailer is sufficient to prevent E's entry, so that counting on it, the other retailer has no incentive not to sign on the contract as well. The argument is rigorously identical to Proposition 1 in Segal and Whinston (2000) and is therefore omitted.

Claim 23 *In the case where I makes simultaneous, non-discriminatory exclusivity offers to two independent retail monopolists, and upstream firms are constrained to use linear price schedules, there exist both "exclusion equilibria" and "non-exclusion equilibria".*

4.3.2 Bertrand competition in the downstream market

Suppose now that 1 and 2 compete in prices *à la Bertrand* on the same market, as in the model described in Section 2. Absent any exclusivity arrangement ($S = 0$), it is clear that at the retail level sellers will charge the price they pay for their inputs and each will serve half of the demand for every quality. They will make zero profit. At the upstream level, the game played by the producers is identical to game in the benchmark two-local-monopolists case, except that quantities and thus profits are multiplied by two as double marginalization does no longer occur. In effect, the two upstream firms are playing the canonical model of vertical differentiation.

In order to assess the possibility of exclusion, we have to determine the outcome of the pricing game when 1 (alone) has signed the exclusivity contract and 2 is the only seller of E's product. 2 then seeks to maximize

$$\pi_2 = (p_E - w_E)\left(1 - \frac{p_E - p_I}{q_E - q_I}\right) + (p_I - w_I^2)\left(\frac{p_E - \frac{q_E}{q_I}p_I}{q_E - q_I}\right). \tag{4.61}$$

w_I^2 is the price charged by I to the retailers that have not signed an exclusivity contract. p_I is the outcome of Bertrand competition between the retailers, that is:

$$p_I = \max\left\{w_I^1, w_I^2\right\}. \tag{4.62}$$

It is clear that I will continue charging the same price to both retailers. Doing otherwise would amount to leave a fraction of the revenues generated by its product to the retailer charged the lowest price. Observe that it is not possible that 2 does not carry I's product. If it were so, then 1 would sell it at a profit. Whatever the price it charges for E' s product, 2 could keep it at that level and marginally undercut 1's price, thereby increasing profit.

Hence, retailers will not make any profit on I's product and the first-order condition for 2's program gives:

$$p_E = \frac{p_I + q_E - q_I + w_E}{2}, \tag{4.63}$$

and

$$\hat{\theta} = \frac{1}{2} + \frac{1}{2}\frac{w_E - p_I}{q_E - q_I}. \tag{4.64}$$

Thus, firm E seeks to maximize

$$\pi_E = \frac{1}{2}w_E\left(1 - \frac{w_E - p_I}{q_E - q_I}\right), \tag{4.65}$$

giving

$$w_E = \frac{p_I + q_E - q_I}{2}. \tag{4.66}$$

Observe that this is exactly the same response as in the two-local-monopolists case. By linearity of the demand curves, although the quantities are halved, the price decisions remain unchanged.

Hence, the equilibrium upstream prices are unchanged:

$$w_E^* = \frac{2q_E(q_E - q_I)}{4q_E - q_I}, \tag{4.67}$$

$$w_I^* = \frac{q_I(q_E - q_I)}{4q_E - q_I}. \tag{4.68}$$

At the downstream level, though, the outcome is different, for I, who sells to both retailers, can in practice determine the final price while E suffers from double marginalization. Hence:

$$p_E^* = \frac{3q_E(q_E - q_I)}{4q_E - q_I}, \tag{4.69}$$

$$p_I^* = w_I^*. \tag{4.70}$$

As a result, firm I is now dominant as it serves two thirds of the market:

$$D_E^* = \frac{1}{2}\frac{q_E}{4q_E - q_I} \tag{4.71}$$

$$D_I^* = \frac{q_E}{4q_E - q_I}. \tag{4.72}$$

So, E's equlibrium quantity is halved, as compared to the two-independent-monopolists case and so are its profits. Therefore, we have that

$$\pi_E^* < F. \tag{4.73}$$

That is to say, the demand (and profit) generated by selling through only one retailer is not sufficient to help E cover its development cost. Hence, it will not enter unless neither retailer signs up.

We are now in the position to show that I is able to exclude *at no cost.*

Proposition 24 *In the case when retailers compete à la Bertrand, I makes simultaneous, non-discriminatory exclusivity offers, and firms are constrained to use linear price schedules, there are two subgame-perfect equilibria, in which I excludes at no cost.*

Proof. Let us first show that the following profile of strategies is a subgame-perfect equilibrium:

a) I offers both firms exclusivity contracts in exchange of compensation $y = 0$; both firms accept;

b) E enters if and only if $S = 0$;

c) the following table summarizes the pricing behavior of upstream firms as a function of the number of active firms, $N \in \{1, 2\}$ (where $N = 1$ stands for the case when E does not enter) and S:

$$\begin{array}{cccc} N & S & w_I & w_E \\ 1 & 0 & \frac{q_I}{2} & \text{n/a} \\ 1 & 1 & \frac{q_I}{2} & \text{n/a} \\ 1 & 2 & \frac{q_I}{2} & \text{n/a} \\ 2 & 0 & \frac{q_I(q_E-q_I)}{4q_E-q_I} & \frac{2q_E(q_E-q_I)}{4q_E-q_I} \\ 2 & 1 & \frac{q_I(q_E-q_I)}{4q_E-q_I} & \frac{2q_E(q_E-q_I)}{4q_E-q_I} \\ 2 & 2 & \frac{q_I}{2} & \text{n/a} \end{array} \tag{4.74}$$

d) in all cases but $S = 1$, and $N = 2$, both retailers charge their input prices; if $S = 1$ and $N = 2$, then $p_I^1 = p_I^2 = w_I^*$ and $p_E^2 = \frac{3q_E(q_E-q_I)}{4q_E-q_I}$.

The computations above showed that the strategies in c) and d) are the equilibria in each of the corresponding subgames. We have shown that $\pi_E > F$ only when $S = 0$, so it is optimal for E to enter if and only if $S = 0$.

In a), neither retailer has an incentive to refuse to sign on for any $y \geq 0$, as a unilateral deviation would not trigger E's entry. For his part, I cannot get the retailers to sign on for cheaper.

Second, consider the strategy profile where everything is as above, except that one retailer does not sign on. The free retailer is indifferent between signing up or not; so it does not have a profitable deviation. The retailer that has signed on is also indifferent as its deviation would trigger entry but would not affect its profit (zero in all cases).

It remains to show that no other strategy profile is a subgame-perfect equilibrium.

Suppose any strategy profile in which at least one of the retailers signs on and $y > 0$. Then I can decrease this amount and still induce acceptance.

Suppose that there is an equilibrium in which both retailers decline to sign on for a compensation y. In the continuation equilibrium, E must enter since in the ensuing game it collects enough revenue to cover its sunk cost but retailers will make zero profit. If $y = 0$, then at time t_0, I can deviate and raise y, triggering acceptance of the exclusivity contracts. If $y_0 > 0$, then the original strategy profile was not an equilibrum, as retailers should have taken on the offer, given that they make zero profit following entry. ■

We conclude that the set of equilibria can be described as follows: I offers both retailers to sign up for zero compensation; at least one of them does (out of indifference); E decides against entry; I then charge the monopoly price that is passed onto final consumers. This result is particularly striking because exclusion is the only outcome in the setting that is the less favorable to the incumbent's deterrence scheme according to the previous literature: the case of simultaneous and non-discriminatory offers. Moreover, exclusion is achieved at no cost to the incumbent.

This outcome is a consequence of the differentiated nature of the industry. In Fumagalli and Motta (forthcoming), a unilateral deviator chooses to market only the entrant's product while the other retailer, having signed on, is forced to market only the incumbent's. The situation is symmetric in the sense that each upstream firm competes in the final market through one of the two retailers. The free retailer then captures the entire market, which is sufficient for the entrant to cover its fixed cost. In our model, it is not in a unilateral deviator's interests to refrain from selling product q_I. Thus, the incumbent continues reaching final consumers through both retailers. This introduces an asymmetry in the game that has negative consequences on E's sales.

4.4 Two-part tariffs

Firms are now allowed to use tariffs specifying a proportional part (with coefficient w_i) along with a fixed fee[10], $\phi_i \geq 0$. Absent any strategic considerations, firms would set the marginal cost at the level that is needed to avoid the double-marginalization problem and use the fixed fee to extract all the producer surplus. Yet, strategic considerations *are* present and somewhat complicate the analysis. Throughout this section we continue making the assumption that in the independent-local-monopolists case, E would have to sell to both monopolists in order to recoup its development costs. In the present context, where double marginalization is no longer an issue, that amounts to assuming that

$$\frac{q_E - q_I}{8} < F < \frac{q_E - q_I}{4}. \tag{A2}$$

[10]The fee is incurred in case a strictly positive quantity is ordered.

4.4.1 Local monopolists

We first consider the benchmark case where the market is served by two local retail monopolists.

Proposition 25 *In the case where I makes simultaneous, non-discriminatory exclusivity offers to two independent retail monopolists, and upstream firms offer two-part tariffs, there is a unique equilibrium, in which E enters the market.*

Proof. Firms would like to set w_i at marginal cost in order to avoid the double mark-up. The sole comparison of sales on this basis would lead a local monopolist to market E's product only (because w_I/q_I would then be equal to w_E/q_E) but that cannot be an equilibrium. Indeed, to avoid being priced out of the market, I is willing to bribe retailer j into carrying its product instead of E's. The best offer that I can make is to transfer the entire monopoly profit to the retailer. Yet, this offer can be matched by firm E, whose product commands higher revenues. Thus, in the unique equilibrium, $w_I = 0$, $\phi_I = 0$, $w_E = 0$ and $\phi_E = (q_E - q_I)/8$. (Should I make another, less favorable offer, then E would best-respond by increasing ϕ_E. In turn, the resulting strategy profile would not be an equilibrium as I could profitably deviate by raising its offer and exclude E.) That is, the entrant appropriates the entire efficiency gains derived from the introduction of its superior product, while the retailer is left with the former incumbent's monopoly rents. In equilibrium only q_E is sold. I cannot induce the retailers to sign on its offer at time t_0 as it cannot promise to hand out more profit than the entrant. ∎

This outcome makes clear that as soon as the entrant is allowed to take part in the "bidding for the right to foreclose" (something made possible here by the existence of the fixed fee), it can only win, for by assumption its product generates more revenue than the incumbent's. Observe also that the active (but ultimately unsuccessful) presence of the incumbent after entry is needed in order for E not to engage into some opportunistic behavior that would leave the retailers with zero profit. As has been noted by previous authors, if the incumbent could instead commit to withdraw upon entry (or more generally could commit to policies that are contingent on E's presence or absence), exclusion would be possible even in that setting.

4.4.2 Bertrand competition

In the case of Bertrand competition at the retail level, the outcome of the subgame in which neither retailer signs on the incumbent's offer ($S = 0$) ends up being the same as in the standard model of vertical differentiation. If an upstream firm's tariff does not comprise a fixed fee, retailers cannot refrain from carrying its product (as long as the price is not prohibitive). Both charge the input price and make zero profit on that product. If the upstream firm charges a fixed fee, then the retailers face the same decreasing average-cost curve. If the fixed fee is smaller than the maximum level of revenues, then given our tie-breaking rule, there is a unique equilibrium in pure strategies in which both firms charge the lowest price compatible with (weakly) positive profits and are randomly selected to serve the market. Of course, conditional upon the other wholesaler's active presence, this situation is dominated, from the upstream firm's point of view, by the policy consisting in setting the fixed fee to zero and charging its desired downstream price to both retailers. If the fixed fee is equal to, or greater than, the monopoly surplus, then there is also an equilibria in which one firm refuses to supply the good. An upstream firm still faces the temptation of buying retailers into stopping carrying their rival's product but in this symmetric Bertrand retail configuration, there is no way retailers can make positive profit in equilibrium. As a result, it is not possible for E to eliminate I. (That would require setting $w_E = 0$.) So, the outcome of the canonical vertical product differentiation model makes for the only candidate for an equilibrium.

There is an equilibrium with

$$w_E^* = \frac{2q_E(q_E - q_I)}{4q_E - q_I}, \tag{4.75}$$
$$w_I^* = \frac{q_I(q_E - q_I)}{4q_E - q_I}, \tag{4.76}$$

and no fixed fees, leading to

$$D_E^* = \frac{2q_E}{4q_E - q_I}, \tag{4.77}$$
$$D_I^* = \frac{q_E}{4q_E - q_I}. \tag{4.78}$$

Upstream firms make the corresponding profits while downtream firms make zero profit.

There is another, payoff-equivalent equilibrium in which I charges zero at the margin but extracts all profit from 1 (by charging $\phi_I = \frac{q_E q_I (q_E - q_I)}{(4q_E - q_I)^2}$), while E does the same with 2. (The situation is then equivalent to double vertical integration.) In any case, in this subgame with $S = 0$ and $N = 2$, E always finds its entry decision vindicated. Indeed, one has

$$\pi_E^* = \left(\frac{2q_E}{4q_E - q_I}\right)^2 (q_E - q_I) > \frac{q_E - q_I}{4}. \tag{4.79}$$

Consider now the subgame when $S = 1$ and $N = 2$, that is, only 1 has signed on and E has entered. On the one hand, it is not possible that firm I be priced out of the market. Indeed, for any $w_E \geq 0$, either $p_E = 0$ and the turnover on E's product is zero, in which case 2 has a profitable deviation, or $p_E > 0$ and firm I can always induce 1 to price below $q_I p_E / q_E$ and capture some market share. Thus, I must make a strictly positive profit in equilibrium. On the other hand, if 2 does not make enough profit, I will be tempted to charge a prohibitive tariff to 1 and to offer 2 to share the monopoly profits. In order to prevent that deviation, 2 must be left with at least the difference between I's monopoly profit and its profit in the standard model of vertical differentiation, which is the latter's "willingness to bribe 2":

$$\frac{q_I}{4}-\frac{q_E q_I}{(4q_E-q_I)^2}(q_E-q_I). \tag{4.80}$$

Observe, though, that provided I is outbid by E and 2 makes the decision to carry only the latter's product, firm I is indifferent to the tariff it charges that retailer, as this does not affect its payoff. (Its product can always be efficiently distributed by 1.) In particular, it is possible for firm I to offer to hand out *the whole* of its monopoly profit to 2. So, there are multiple equilibria, all involving 1 and 2 pricing the goods as in the canonical model but with different $\tilde{\phi}_I^2$,$\tilde{w}_I^2$, $\tilde{\phi}_E$, and consequently, $\tilde{\pi}_E$ and $\tilde{\pi}_2$.

Any equilibrium upstream tariff profile must thus have that

$$\begin{aligned}\tilde{w}_E &= 0 && (4.81)\\ \tilde{\phi}_E &\in \left[\frac{4(q_E)^2}{(4q_E-q_I)^2}(q_E-q_I)-\frac{q_I}{4},\ \frac{4(q_E)^2+q_E q_I}{(4q_E-q_I)^2}(q_E-q_I)-\frac{q_I}{4}\right]. && (4.82)\end{aligned}$$

I either charges $\tilde{w}_I^1=\tilde{w}_I^2=w_I^*=\frac{q_I(q_E-q_I)}{4q_E-q_I}$ and no fixed fee, or it charges zero at the margin and extracts all profit from 1, the only retailer to take on its offer, by setting $\tilde{\phi}_I^1=\frac{q_E q_I(q_E-q_I)}{(4q_E-q_I)^2}$. In any case, 1 makes zero profit.

What is of importance here is that, in contrast with the independent-monopolies case, firm E does not reap the entire efficiency gains from introducing a superior product. This is because firm I cannot be eliminated from the market. Its presence forces E not only to hand out part of its profit to 2 but it also diminishes the revenues generated by its product. In these circumstances, it might well be the case that E is not left with enough profit to cover its sunk cost. Indeed, consider the difference between E's highest equilibrium profit and the minimum sunk cost under assumption (A2):

$$\frac{4(q_E)^2+q_Eq_I}{(4q_E-q_I)^2}(q_E-q_I)-\frac{q_I}{4}-\frac{q_E-q_I}{8}=\frac{16(q_E)^3-(q_I)^3-32q_I(q_E)^2-(q_I)^2q_E}{8(4q_E-q_I)^2}. \tag{4.83}$$

Dividing the numerator by $(q_I)^3$ makes clear that its sign depends on q_E/q_I. If this ratio is close to one (little product differentiation), then it is negative. If it is bigger than about 2.045 (more product differentiation), then it is positive.

Besides, consider the highest equilibrium profit that E can collect and the maximum sunk cost under assumption (A2):

$$\frac{4(q_E)^2+q_Eq_I}{(4q_E-q_I)^2}(q_E-q_I)-\frac{q_I}{4}-\frac{q_E-q_I}{4}=-\frac{(5q_I+4q_E)q_Iq_E}{4(4q_E-q_I)^2}, \tag{4.84}$$

which is always negative, so that in the case when the introduction of the superior product is barely efficient, E never finds it profitable to enter when $S=1$.

More generally for any $F\in\left(\frac{q_E-q_I}{8},\frac{q_E-q_I}{4}\right)$ and any admissible ϕ_I^2, it is possible to associate a treshold $d(F,\phi_I^2)$ such that

$$\tilde{\phi}_E\leq F\Longleftrightarrow\frac{q_E}{q_I}\leq d(F,\phi_I^2). \tag{4.85}$$

The general lesson is that the higher F, the higher the quality differential required for E to find it profitable to enter when it cannot sell to both retailers.

We are now in the position to assert our second exclusion result.

Proposition 26 *In the case when retailers compete à la Bertrand, I makes simultaneous, non-discriminatory exclusivity offers, and firms offer two-part tariffs, if either (i)*

$$q_E/q_I<\frac{67}{144\sqrt[3]{\frac{\sqrt{761}}{192}+\frac{301}{864}}}+\sqrt[3]{\frac{\sqrt{761}}{192}+\frac{301}{864}}+\frac{2}{3}\simeq 2.0455, \tag{4.86}$$

and (A2) holds, or (ii)

$$F>\frac{4(q_E)^2+q_Eq_I}{(4q_E-q_I)^2}(q_E-q_I)-\frac{q_I}{4}, \tag{4.87}$$

for any given $q_E/q_I > 1$, then in every subgame-perfect equilibrium, I excludes E at no cost.

Proof. Consider the following strategy profile:

a) I offers exclusivity contracts against compensation $y = 0$; both retailers accept;

b) E enters if and only if $S = 0$;

c) in all cases where I ends up being the only active firm ($N = 1$), it charges the monopoly price and no fixed fee; in all cases where both firms are active ($N = 2$), firms charge invariant prices w_E^*, w_I^* when $S = 2$, and as above when $S = 1$;

d) in all cases where $N = 1$, retailers charge the price they pay for I's product; similarly when $N = 2$ and $S = 0$ or 2; when $N = 2$ and $S = 1$, both firms charge the price they pay for I's product and 2 sets $p_E = \frac{2q_E(q_E-q_I)}{4q_E-q_I}$.

Retailers have no incentive unilaterally to deviate as they make zero profit in all circumstances. E enters only when it can sell to both retailers, in which case it makes the same profit as in the standard model, which is sufficient to cover its fixed cost, for

$$\left(\frac{2q_E}{4q_E - q_I}\right)^2 (q_E - q_I) > \frac{q_E - q_I}{4} > F, \tag{4.88}$$

the last inequality being true by Assumption (A2). In the other cases, either it generates no revenue upon entry ($S = 0$), or, in any continuation equilibrium with $S = 1$, the amount it collects given the competition from I is not sufficient to cover the rebate $\tilde{\phi}_E$ and the sunk cost F. I excludes E at no cost and cannot do better than collecting the entire monopoly profit.

The argument is identical for the cases where either or both of the following modifications are made to the strategy profile: (i) in c) firms charge zero marginal cost and fixed fees $\tilde{\phi}_E = \frac{4(q_E)^2(q_E-q_I)}{(4q_E-q_I)^2}$ and $\tilde{\phi}_I = \frac{q_Iq_E(q_E-q_I)}{(4q_E-q_I)^2}$, which are incurred in the continuation equilibrium in which $S = 2$ by 2 and 1, respectively; (ii) in a) only one retailer signs on.

Consider now any other strategy profile in which both retailers sign on and $y > 0$. Then I can profitably decrease y and continue inducing acceptance.

Consider now any other strategy profile in which one retailer does not sign up and $y > 0$. Then, either the retailer who hasn't signed up should do so (as E does not enter and it makes zero profit selling q_I anyway), or I should decrease the compensation.

Consider finally any other strategy profile in which neither retailer signs up. In that case E enters and in the continuation equilibrium in which $S = 2$, retailers make zero profit. If $y > 0$, retailers gain by signing up. If $y = 0$, it is profitable for I to deviate and offer some $y > 0$, which will be sufficient to elicit acceptance. ■

So, exclusion is not driven by the double marginalization problem that arises when a product is sold through two successive price-setters. Indeed, under two-part tariffs, exclusion is also sustainable, provided the entrant does not enjoy too large a quality advantage or the development cost is sufficiently high. This is due to the fact that E cannot eliminate competition from I's product in presence of two retailers. As a result, a retailer anticipates that, in case it deviates from the exclusion equilibrium candidate by not signing on the exclusivity contract, entry will not happen because firm E cannot cover both its development cost and the rebate it must grant 2 in order to prevent it from succumbing to I's profit-sharing offers. This is very different from the coordination failure story. Indeed, there are no equilibria where neither retailer signs on because, once the entry cost is sunk and E has access to both retail channels, it has no incentive to refrain from bullying its distributors. So, any small compensation offered by I proves sufficient to induce both retailers to sign.

Observe once again that if the potential entrant could compete with the incumbent at time t_0, or promise not to extract all surplus upon entry, it would gain a monopoly position. In truth, inefficient exclusion is generated by the entrant's inability to commit to share part of its profit, not by the nature of pricing. The incumbent's advantage is that, through the use of the exclusivity contracts, it can commit to rebate some profit to the downstream firms.

What happens with those configurations involving some continuation equilibria in which E finds it profitable to enter when $S = 1$ ($\tilde{\phi}_E \geq F$)? It is easy to see that there cannot be equilibria in which both retailers sign on. Indeed, upon deviating unilaterally, a retailer, instead of zero profit, makes at least the amount that firm I would be willing to pay to avoid that deviation. So we are back to a sort of traditional "triangle argument". There are equilibria in which neither retailer signs on. Retailers make zero profit but firm I cannot lure them into signing on. Indeed, if one retailer signs, the other refuses to sign unless it is paid the entire monopoly profit. I cannot offer that compensation to both retailers. There are also equilibria in which I offers zero compensation, one retailer signs on and the other doesn't. I knows that entry occurs whatever it offers at time t_0. Given that one retailer only signs on, offering $y = 0$ is a best response. Retailers cannot profitably change their exclusivity choices: the signer by not signing would still make zero profit; the non-signer would lose the rebate that E grants it upon entry. Given the situation, E is right to enter as soon as it has access to one retailer.

4.5 Conclusion

We have shown in this paper that, when products are vertically differentiated, intense (i.e. Bertrand) downstream competition makes it *easier* for an incumbent firm to use long-term exclusivity contracts so as to prevent a clearly (yet not overwhelmingly) superior product from being introduced. Indeed, under linear pricing, if one retailer signs on the contract but the other doesn't, upon entry, the demand addressed to the incumbent is drastically reduced but continues being positive as products are differentiated. Yet, the situation is not symmetric as the incumbent, whose product continues being marketed by both retailers, does not suffer from double marginalization. As a result, the incumbent captures a higher share of the market than it would otherwise. The entrant's diminished sales then prove insufficient to allow it to recover its development costs. Under non-linear pricing, the same outcome emerges as long as the quality differential is not too large.

Our argument relies on the fact that the incumbent's product continues being sold after entry. This feature might not extend to those cases where differentiation is very large and marginal costs are different from zero, as the seller of the high-quality product might find it profitable to engage in limit-pricing so as to keep the incumbent out of the market. As a consequence, some exclusion equilibria (in particular, in those cases where the sunk cost is large) will disappear. Yet, limit-pricing is not profitable when the products are sufficiently similar the spirit of our results should be preserved, exclusion remaining possible as long as the quality differential is moderate.

Observe that, in our setting, once an innovative product that might be improved upon has been brought to the market, the first order of business for the innovator is to engage into long-term exclusivity relationships with its retailers. Indeed, this might be sufficient to deter any potential competitor from investing into the development of a superior product (over the length of the contracts). Observe also that, in practice, exclusivity can be achieved not only by means of signing a contract explicitely written for that purpose but also by designing incentive schemes (e.g. quantity rebates or loyalty rebates) that give rise to the same outcome.

Thus, anti-trust practitioners should not take pretext of the fact that downstream markets are highly competitive to dismiss claims that vertical restraints threaten the competitive structure of an industry. In the end, our work can be viewed as an analysis dual to Fumagalli and Motta (forthcoming)'s study. These authors conclude that "controlling for other factors [...] it would be more likely to observe exclusive contracts in industries with highly differentiated products than in highly competitive downstream markets." We show that even in these instances where the downstream markets are highly competitive (because retailers are homogenous Bertand competitors), differentiation at the upstream level may restore the possibility to use long-term contracts to deter entry.

Yet, it is worth emphasizing that our model does not provide any reason for the existence of such contracts but the exclusion motive. Hence, our study should not be interpreted as indicating that long-term vertical arrangements are likely to arise from anti-competitive behavior in a vertically-differentiated environment. They might or they might not. Even if they do, the detrimental consequences might well be counter-balanced by some other (efficiency) considerations. In the end, a careful examination of the exact nature of competition in the industry is the only path to a proper assessment of these practices.

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