PRICE DISCRIMINATION, ADVERTISING AND COMPETITION

Witness Simbanegavi

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Price Discrimination, Advertising and Competition

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To My Boys - Tendai and Tanaka

Contents

Acknowledgements

This journey has been characterized by ups and downs, highs and lows and sometimes even odious. But looking back, without a doubt, this milestone is worth every drop of sweat. The hope now is to build on what I learnt in the last five years as I move forward.

Prior to my coming to the Stockholm School of Economics, I spent two rewarding years at the University of Oslo. When I enrolled for my Ph.D. at the SSE, I was full of enthusiasm. My original plan was to major in either Development Economics or Macroeconomics. However, my plan did not last long. When I took a course in Industrial Organization (instructed by Jonas Häckner and Richard Friberg), I fell for it. From then on, my interest in Industrial Organization only grew. I talked to Richard about him being my advisor and he readily accepted. Teamed with Richard, it was time to enter the next phase of this journey – Thesis writing. This proved to be a difficult "battle" and I take this opportunity to thank all those who supported me in one way or another in this effort.

Although the first two years were rather smooth, I laboured to find my feet when it came to Thesis writing. This escapade would not have been successful without the guidance and encouragement of my Supervisor, Richard Friberg. Sometimes it was really frustrating, trying to work hard, but making no progress – for days on end. Then I would turn to Richard. Discussions with Richard were very rewarding, opening my thought process to new ways of approaching and solving problems. I therefore take this opportunity to express my sincere gratitude for all that you have done in helping me realize this goal. I especially applaud your "open door" policy.

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Stockholm, September 2005

Witness Simbanegavi

Summary of Thesis

Introduction

Research on advertising has taken a huge leap in the last few decades (Bagwell, 2003) and deservedly so. From an Industrial Economics perspective, advertising is one of the most important strategic variables in the firm's toolbox. This is evidenced by the huge advertising expenditures that firms are willing to incur. For example, in the first nine months of 2004, Time Warner Inc. spent a whopping $1.345 billion on advertising, Daimler Chrysler AG and Ford Motor Company spent, respectively, $1.219 billion and $1.083 billion advertising cars and trucks while Johnson & Johnson spent $0.916 billion on advertising (Marketing Today, July 2005).

The need for firms to advertise may arise for several reasons. First, advertising may be driven by a desire to create brand loyalty among consumers. This type of advertising is called *persuasive advertising* and is important, especially for experience goods – goods for which pertinent characteristics are only discerned through actual consumption. Second, advertising may be necessitated by the need to furnish consumers with relevant and verifiable information. If consumer search costs are high and the firms' demands positively depend on the amount of information that consumers have, then firms may find it optimal to compliment consumer search by advertising certain information about their products (e.g., prices). This type of advertising is called *informative advertising.* The difference between persuasive and informative advertising lies mainly on whether the attributes being advertised are immediately verifiable or not.

In this thesis, we study aspects of the informative view. One aspect of interest is whether firms can benefit from collusion on advertising even though advertising is only informative. If so, will this enhance or lower welfare? There are several reasons why firms may want to collude on advertising. First, the legal field is tilted in favour of nonprice collusion. In many countries, price collusion is deemed per se illegal – which makes price collusion too risky a venture. In contrast, restrictions on price advertising by competing firms are not condemned as per se illegal, but rather, are normally treated under the "rule of reason" approach – which makes collusion on advertising a less risky venture. Second, it is not at all obvious that collusion on price is more profitable than collusion on advertising and third, the analyses Grossman and Shapiro (1984) and others show that profits can be increased by restricting advertising. In the first paper,

Paper 1, we examine firms' incentives to collude on advertising and the implications for welfare.

Another important issue concerns the effect, on prices and profits, of advertising only a subset of the product range. Many firms, in particular those in the retail sector, sell a wide variety of products but only advertise a few. Recent empirical evidence suggests that prices of unadvertised products are higher (Milyo and Waldfogel, 1999). Theoretically, little is known. Contributions on this issue include Lal and Matutes (1994) and Ellison (2005). These contributions, however, do not explicitly model the firm's advertising and hence cannot study the interaction between advertising and prices. In Paper 2, we study the effects of advertising only a subset of products. We allow for both low and high differentiation and, at the same time, we explicitly model the advertising decision.

We next consider a different subject – price discrimination. Although it is well understood that movements in the exchange rate have a bearing on firm profitability and hence affect firm behaviour, the role of exchange rate variability in the firm's choice of the number of varieties to produce has (to my knowledge) never been explored. This, despite the fact that the product mix is an important aspect of firm strategy. By tinkering with the number of varieties, a firm can bolster its ability to extract consumer surplus. In Paper 3, we explore this issue.

Below we summarize the papers.

Summary of Papers

This thesis is composed of three papers. The first two papers focus on informative (price) advertising while the third paper, Paper 3, studies the relation between exchange rate variability and a monopoly firm's product mix.

Paper 1: Informative Advertising: Competition or Cooperation?

In this paper we examine firms' incentives to collude on advertising in a Grossman and Shapiro (1984) framework. The role of advertising is purely informative – informing consumers about the prices. This paper is motivated by the fact that the analysis of price advertising has been framed exclusively in terms of fully noncooperative interaction, yet, the analyses of Grossman and Shapiro (1984), Christou and Vettas (2003) among others show that higher advertising is detrimental to industry profits – a finding which suggests that firms may be better off colluding on advertising. Another reason why firms may want to collude on advertising is the fact that the legal field is tilted against price collusion – price collusion is deemed per se illegal while collusion on advertising is not. Also, anecdotal as well as empirical evidence show support for the hypothesis that firms collude on advertising (Gasmi, Laffont and Vuong, 1992; Wang, Dhar and Stiegert, 2004).

We consider a duopoly selling differentiated products. Consumers have unit demands and are uniformly distributed on the unit interval. Firms advertise to inform consumers of the prices they charge. Firm advertising is the only source of information for consumers. We compare the profits when firms collude on advertising and compete on prices (semicollusion) to the noncooperative profits (profits when firms compete on both prices and advertising). We find that collusion on advertising and competition on price is more profitable than competition on both price and advertising. Moreover, semicollusion on advertising is detrimental to welfare. Fewer consumers are informed when firms collude on advertising and this lowers welfare. This result is important for policy. Even though advertising is only informative, there is need for monitoring, especially since it is in the interest of firms to restrict price advertising. Left unchecked, firms will be tempted to connive against consumers.

We also compare semicollusion on price to semicollusion on advertising. This is motivated by the fact that the analysis of semicollusion to date has largely been framed

as collusion on price and competition on a nonprice variable. We find that, in general, semicollusion on price does not lead to higher profits compared to semicollusion on advertising. Semicollusion on price is more profitable when the advertising cost is low but is less profitable when the advertising cost is high. This is so because firms advertise "excessively" when they collude on price and, as a result, they incur larger advertising outlays. In contrast, when firms collude on advertising, they advertise less. Therefore, when the advertising cost is high, semicollusion on advertising yields higher profits. In this sense, there is no justification for the theoretical literature's exclusive focus on semicollusion on price. Hence we lend theoretical support to the empirical literature that consistently find evidence of semicollusion on advertising rather than on price.

Paper 2: Informative Advertising When Only Some Goods are Advertised

It is well established that many firms advertise only a subset of the products they sell – a supermarket, for instance, advertises a handful of products but sells hundreds of products. Recent empirical findings suggest that the effect of price advertising on the prices of advertised and unadvertised products differs. In particular, prices of unadvertised products are higher (Milyo and Waldfogel, 1999). Theoretically, little is known.

This paper studies how, in a Grossman and Shapiro (1984) framework, advertising of a subset of the firm's product range affects the equilibrium outcome (prices and advertising). We consider a duopoly, where each firm offers two products, but only advertises one and the same product. Consumers with unit demands (over product lines) are uniformly distributed on the unit interval with each consumer's location corresponding to her most preferred bundle. In this market, the only way to get information about prices is by receiving an advertisement, otherwise consumers are uninformed. In particular, consumers do not actively search.

We find that the extend of differentiation between competing firms plays an important role in the analysis of loss leader pricing. When firms sell products with the same reservation price, loss leader pricing obtains only when differentiation is low. When products are less similar however, price competition is less intense and, as a result, there is less rationale for pricing below cost. Hence, firms advertise prices above marginal cost. In contrast, when reservation prices differ, equilibrium may entail loss-leader pricing even when differentiation is high. This happens when firms advertise the low reservation price good. Here, the incentive to undercut derives from the unadvertised price. The higher the unadvertised price, the greater the incentive to undercut and therefore the lower the advertised price. Our loss leader pricing results enable us to

shed some light on the seemingly paradoxical empirical findings in the marketing literature that loss leader pricing fails to increase store traffic, loss leader sales and hence to increase profits. We also examine the welfare implications of informative advertising.

Paper 3: Exchange Rates and Product Variety

That the exchange rate is one of the most important "exogenous" factors affecting the performance of exporting firms is, in general, well understood. This explains why, for firms facing exchange rate exposure, hedging against exchange rate risk is a popular strategy (see, for example, Friberg, 1999). There is a huge literature on firm behaviour under variable exchange rates, for example, Baum et al (2001), Friberg (1999; 2001), Baldwin and Krugman (1989). However, the relation between exchange rate movements and the firm's product mix has not been studied. This is a bit surprising since one of the principal aspects of firm strategy concerns the number of varieties the firm offers. This paper fills this void. We study the role of exchange rate variability in the firm's choice of whether to offer one or two varieties. The goal of this paper is to deepen our understanding of the effects on product variety (and hence the firm's ability to second degree price discriminate) of movements in the exchange rate.

We consider a monopoly firm selling in the Home market and the Foreign market. We show that variability in the exchange rate induces the firm to vertically segment markets (i.e., offer two varieties in each market). This happens because exchange rate variability affects income dispersion and hence the firm's incentives to extract consumer surplus. To better extract surplus, the firm offers two price-quality menus, high quality variant (priced high) for top-end surplus extraction and a low quality variety (priced low) to address market coverage concerns. We extend the model to allow for horizontal segmentation. We find that the profitability of second degree price discrimination increases as the Home and Foreign markets become horizontally segmented. Hence, when the costs of segmenting markets are not too high, variability in the exchange rate will lead to both greater variety and international market segmentation.

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Papers

PAPER 1

Informative Advertising: Competition or Cooperation?

Witness Simbanegavi

ABSTRACT. We compare the equilibrium outcome when firms collude on advertising (and compete on prices) to the Nash equilibrium outcome in the Grossman and Shapiro (1984) model of informative advertising. It is shown that advertising is lower but prices and profits are higher under semicollusion on advertising. We also show that semicollusion on advertising is detrimental to welfare. Although firms earn higher profits when they semicollude on advertising, fewer consumers are informed, and as a result, welfare is lower. We also contrast semicollusion on price to semicollusion on advertising. We find that, in general, semicollusion on price is not associated with higher profits than under semicollusion on advertising. When the advertising cost is *low*, semicollusion on price is more profitable while semicollusion on advertising yields higher profits when the advertising cost is *high*.

1. Introduction

The importance of advertising as a competitive weapon in sellers' interactions has long been recognized. Typically, a firm that advertises more can expect higher demand and hence higher revenues, other things being equal. In multifirm industries, this possibility to steal customers from competitors often results in costly "advertising wars" as firms try to regain lost market share. If, in addition, advertising conveys price information, such advertising wars inadvertently lead to lower prices – a double blow![1]

Indeed, Grossman and Shapiro (1984), Christou and Vettas (2003), among others, show that increased price advertising raises demand elasticity and thus lowers prices. Hence, excessive advertising may actually hurt firms. Therefore, if firms are sophisticated, they ought to realize the folly of unbridled price advertising. Nevertheless, the analysis of price advertising has been framed exclusively in terms of fully noncooperative interaction. While in many countries price collusion is per se illegal (which may

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[1] Advertising can also lead to lower prices even if it does not convey price information (Benham, 1972; Bagwell and Ramey, 1994).

explain nonprice collusion), collusion on advertising is not[2]. If anything, the existence of advertising agencies – that often handle advertising from several competing firms – provides scope for collusion on advertising[3].

What is more, empirical evidence (on price and advertising strategies in different industries) support the hypothesis of collusion on advertising. Gasmi et al (1992) investigate possible market configurations in the Cola market (Nash behaviour, Stackelberg leadership and several possible configurations of collusion). They use data for the period 1968-1986 to test their hypotheses and thus to select a model of strategic behavior that best fits the data. Noncooperative behaviour in both advertising and prices is rejected by the data. They find support for collusion on advertising (but not price). In a similar study, but for the US butter and margarine industry, Wang et al (2004) reach a similar conclusion. A related study is that of the US cigarette market by Roberts and Samuelson (1988). They find that, particularly for low tar cigarettes, the data does not seem to support the hypothesis of combative advertising. Moreover, they cannot reject the hypothesis of joint profit maximizing choice of advertising.

The gist of this paper is to examine firms' incentives to collude on advertising when advertising is only informative. More precisely, we compare the equilibrium under collusion on advertising to the fully noncooperative equilibrium as well as to the equilibrium under price collusion. We also investigate the welfare implications of collusion on advertising.

We adopt the framework of Grossman and Shapiro (1984) and postulate a linear city in which firms sell a differentiated product. Consumers are uniformly distributed along the unit interval and do not search. Firms advertise to inform consumers. We analyze three cases: no collusion, collusion on advertising only and collusion on price only.

[2] In the US, the pertinent case is that of the California Dental Association (CDA) versus the Federal Trade Commission (FTC). In that case the US Supreme Court ruled that it was not "intuitively obvious" that advertising restrictions by the CDA were anticompetitive. Instead, the Court instructed that the restrictions should be examined (by the Ninth Circuit Court of Appeals) under the rule of reason approach – where the potential benefits are contrasted to the costs (Lande and Marvel, 2000; pages 956-957). When a particular conduct is deemed per se illegal, it is not necessary to evaluate the benefits and costs of such a conduct. Rather, the FTC /Court will move directly to the punishment phase – little wonder why the threshold for per se illegal conducts is high. To paraphrase Commissioner Starek (1997), the majority of the Commissioners in the CDA case contended that the restrictions on price advertising by the CDA were per se illegal. Yet, they went on to examine the competitive consequences of the said restrictions – an examination that the per se rule makes redundant! This, Starek (p. 7) concludes, "made clear why restraints on price advertising should not be labelled per se illegal".

[3] In the US for example, promotion of milk products is cooperatively managed. There are two national programs for dairy advertising – the Producer Dairy Promotion Program and the Fluid Milk Processor Promotion Program. These programs are financed through mandatory contributions by the beneficiaries (Blisard, *Undated*; Lande and Marvel, 2000; footnote 43).

We find that, compared to the noncooperative equilibrium outcome, collusion on advertising leads to reduced advertising but higher prices and profits. By lowering the advertising intensity, collusion on advertising relaxes price competition by raising informational product differentiation and this raises the equilibrium price relative to the noncooperative outcome. Also, lower advertising has a positive direct effect on profit – lower advertising outlay. The lower advertising outlay, coupled with the induced higher prices, enable firms to earn higher profits.

Although firms earn higher profits, semicollusion on advertising is bad for welfare. Consumers lose more than what the firms gain. In particular, consumers not only pay higher prices, rather, in addition to higher prices, fewer consumers get informed when firms collude on advertising – and this exacerbates the loss of consumer surplus. In comparing price collusion to collusion on advertising, we find that the former dominates the latter in terms of revenues. Firms advertise more and charge higher prices when colluding on price. However, price collusion is not, in general, more profitable.

Our work adds to the growing literature on semicollusion. Semicollusion obtains whenever economic agents choose to cooperate along some dimension(s) while at the same time competing along another dimension. The only previous work on semicollusion in advertising that we are aware of is Aluf and Shy (2001)[4]. They study comparison advertising in a duopoly market where products are, in the absence of advertising, homogeneous. In their model, advertising serves to differentiate products in the eyes of the consumers (spurious product differentiation). They show that semicollusion leads to higher advertising, prices and profits relative to the noncooperative outcome.

In an interesting contribution, Fershtman and Gandal (1994) challenge the widely accepted view that price collusion is always beneficial to firms. They argue that semicollusion can be disadvantageous. In particular, they show that when firms noncooperatively choose capacity in the first stage of the game and then collude on price in the second stage, they earn lower profits compared to a fully noncooperative outcome. Steen and Sørgard (1999) adapt the Fershtman and Gandal (1994) model to suit the Norwegian cement market. In their model, firms can also export excess output at the prevailing world price. They show that if each firm's domestic market share is determined by the firm's share of total industry capacity and firms collude on price, a higher domestic demand may induce overinvestment in capacity and this in turn will lead to an increase in exports. They label this effect the "semicollusion effect". They empirically test for and find support for this effect in the Norwegian cement cartel.

[4] The literature has focused on situations where firms cooperate on price while at the same time competing on some other variable – for example capacity (Steen and Sørgard, 1999), (Fershtman and Gandal, 1994); R&D (Fershtman and Gandal, 1994); location (Friedman and Thisse, 1993). See also Steen and Sørgard, (1999; footnote 1).

The paper closest to ours in scope is Aluf and Shy (2001). However, in our framework, unlike Aluf and Shy (2001), advertising does not change consumers' tastes. That is, advertising is purely informative. We also differ with them in that we allow for semicollusion on price. This enables us to make comparisons between semicollusion on advertising and semicollusion on price.

The paper is organized as follows: Section 2 sets out the model. In section 3 we derive the noncooperative and the semicollusive equilibria and in section 4 we study semicollusion on price. In section 5 we contrast the equilibrium when firms semicollude on price to the equilibrium when they semicollude on advertising. Section 6 discusses the pros and cons of semicollusion and section 7 concludes the paper.

2. Model and Preliminaries

We adopt Tirole (1988)'s model – a simplification of the Grossman and Shapiro (1984) model of informative advertising with differentiated products. Two firms, firm 1 and firm 2, sell a horizontally differentiated good. The firms are located at the end points of a linear city of unit length with firm 1 located at point 0 and firm 2 at point 1. Firms randomly send out advertisements (ads) to inform consumers of the prices they charge. That is, every consumer has an equal chance of receiving any ad that is sent by any firm. Let ϕ_i denote the advertising intensity of firm $i; i = 1, 2$ (fraction of the consumer population that is exposed, at least once, to the advertising message of firm i). The cost of reaching fraction ϕ_i of consumers is denoted $A(\phi_i)$, where $A(\phi) = a\phi^2/2; a > t/2$.[5] Each good is produced at a constant marginal cost which we normalize to zero. There is no entry or exit.

Consumers are uniformly distributed on [0,1], have unit demands and attach a dollar value of v to the consumption of a unit of the good. Consumers are uninformed about prices and firm locations unless they are reached by advertising. Thus, uninformed consumers do not participate in the market. Informed consumers incur a shopping cost of t per unit of distance travelled.

Given the firms' advertising intensities, ϕ_1 and ϕ_2, and the consumers' (passive) behavior, the market is delineated as follows; fraction $\phi_1\phi_2$ of consumers receive advertising messages from both firms (fully informed); fraction $\phi_i(1-\phi_j); i, j = 1, 2; j \neq i$ receive ads from firm i but not firm j (partially informed); and fraction $(1-\phi_1)(1-\phi_2)$

[5] We assume $a > t/2$ to allow for some consumers to be uninformed in equilibrium, so that it is possible to study the effects of varying the advertising level. For $a \leq t/2$, the advertising cost is too low and, as a result, we have full information in equilibrium. That is, $\phi_1 = \phi_2 = 1$. See also Tirole (1988; p. 292).

receive no ads from either firm (uninformed). We assume that $\phi_1\phi_2$ is large enough so that firms find it worthwhile to compete for the fully informed consumers[6].

Fully informed consumers purchase from whichever firm guarantees them the greatest surplus. A consumer located at $x \in (0,1)$ gets surplus $v - p_1 - tx$ buying from firm 1 and surplus $v - p_2 - t(1-x)$ buying from firm 2. Let $\widehat{x}$ denote the location of the consumer who is indifferent between buying from firm 1 and buying from firm 2; then, $\widehat{x} = (p_2 - p_1 + t)/2t$. Consumers with locations $x \in [0, \widehat{x})$ buy from firm 1 while those with locations $x \in (\widehat{x}, 1]$ buy from firm 2. Thus, firm i faces the demand $D^i_{full} = (p_j - p_i + t)/2t$ from the fully informed consumers.

For partially informed consumers, demand is determined by individual rationality. Let x_i denote the location of the consumer who receives advertising only from firm i. Buying from firm i yields the surplus $v - p_i - tx_i$ while the consumer gets surplus zero when not purchasing. Hence the demand from partially informed consumers is given by $x_i = \frac{v-p_i}{t}$. All partially informed consumers with locations less than x_i find it worthwhile to purchase while those with locations greater than x_i will not purchase[7].

Thus, each firm's demand is a sum of the demands by the partially informed and the fully informed consumers. That is;

$$D_i(\phi_1, \phi_2; p_1, p_2) = \phi_i \left((1-\phi_j) \frac{v - p_i}{t} + \phi_j \frac{p_j - p_i + t}{2t} \right); i \neq j. \tag{1}$$

In the sequel, we will assume that the market is fully covered. The market is said to be fully covered (differentiation is low) if, given the prices, the transportation cost, t, is such that all partially informed consumers make a purchase. Thus, for the market to be covered, it is necessary and sufficient that the partially informed consumer who travels the entire unit distance gets nonnegative surplus. That is, $p + t \leq v$. This assumption implies that the demand facing firm i reduces to:

$$D_i(\phi_1, \phi_2; p_1, p_2) = \phi_i \left(1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} \right). \tag{1'}$$

3. Competition or Collusion?

In subsections 3.1 and 3.2 we derive, respectively, the noncooperative and the semi-collusive equilibria and in subsection 3.3 we contrast the two.

[6] In Paper 2 (Appendix D) we provide the necessary condition for firms to compete for the fully informed consumers. This can be applied to the present model with minor modifications.

[7] However, if $v - p_i \geq t$, all consumers who receive at least one ad from firm i will make a purchase, that is, $x_i = 1$.

3.1. Noncooperative Equilibrium. We consider first the case of noncooperative interaction between firms[8]. Firms simultaneously and noncooperatively choose both advertising levels and prices (Nash equilibrium). Firm i has the following maximization problem:

$$\pi_i = \max_{p_i,\phi_i} \left\{ p_i\phi_i \left(1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} \right) - \frac{a\phi_i^2}{2} \right\}. \tag{2}$$

The first order necessary conditions are

$$\frac{\partial \pi_i}{\partial p_i} = 1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} - \frac{p_i\phi_j}{2t} = 0 \tag{3}$$

and

$$\frac{\partial \pi_i}{\partial \phi_i} = p_i \left(1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} \right) - a\phi_i = 0. \tag{4}$$

Equation (4) equates the marginal revenue and the marginal cost of raising the advertising reach marginally.

Solving (3) for p at the symmetric equilibrium gives

$$p = \frac{2t}{\phi} - t. \tag{5}$$

It is immediate from (5) that higher advertising is associated with lower prices. This is explained by the fact that when the market is covered, fully informed consumers are price sensitive while partially informed consumers are not. A higher advertising intensity implies a higher proportion of fully informed consumers in the market and this puts pressure on prices[9].

Substituting (5) back into the objective function yields,

$$\pi = 2t - 2t\phi + \frac{1}{2}t\phi^2 - \frac{a\phi^2}{2} \tag{6}$$

as the firm's profit for any given level of advertising. One can easily show that;

LEMMA 1. *Profits are strictly decreasing in the advertising intensity.*

PROOF. $\partial\pi/\partial\phi = (\phi - 2)\,t - a\phi < 0.$ □

To understand why profits decrease with advertising at all levels, we write the profit function as: $\pi = R(\phi) - C(\phi)$, where the revenue, $R(\phi) = 2t - 2t\phi + t\phi^2/2$ and the cost, $C(\phi) = a\phi^2/2$. Differentiating the revenue and cost functions with respect to ϕ

[8] This case has been studied rather extensively (Grossman and Shapiro, 1984; Tirole, 1988; Soberman, 2004; Hamilton, 2004; Simbanegavi, 2005; among others).

[9] Demand elasticity at the symmetric equilibrium is given by: $\varepsilon_p = \phi p/(2-\phi)\,t$ and $\partial\varepsilon_p(\phi)/\partial\phi = 2p/t\,(2-\phi)^2 > 0$. Thus, choosing a high ϕ is tantamount to choosing a more elastic demand and hence leads to lower prices.

gives; $R'(\phi) = (\phi - 2)\,t < 0$ and respectively, $C'(\phi) = a\phi > 0$. That is, a small increase in advertising lowers the firm's revenues but raises the firm's costs. Although demand increases with advertising, the negative effect on price of an increase in ϕ dominates the total effect on revenues. Since revenues fall while costs rise with advertising, it follows that profit decreases with increases in advertising.

Solving (3) and (4) simultaneously gives;

$$\phi^{nc} = 2/\left(1 + \sqrt{2a/t}\right);\ p^{nc} = \sqrt{2at} \tag{7}$$

and substituting (7) back into the objective function gives

$$\pi^{nc} = 2a/\left(1 + \sqrt{2a/t}\right)^2 \tag{8}$$

where nc is a mnemonic for noncooperative.

3.2. Semicollusion. In this section, we study a two period game where firms collude on advertising but compete on prices. The timing of the game is as follows: In the first stage, firms noncooperatively set their prices, reveal the prices to one another and in the second stage, knowing the equilibrium prices chosen in the first stage, they collusively decide on advertising[10].

To bring more realism to this game, we can recast the game as follows: firms noncooperatively choose their prices while delegating the decision on advertising to a third party – the advertising agency[11]. In the first stage of the game, firms noncooperatively set their prices, which they make public in the intervening period. In the second period, knowing the prices chosen by the firms in the first period, the advertising agency chooses the advertising level to maximize firms' joint profits.

As is typical in two stage games, we solve the problem backwards, starting with the collusive phase.

[10] The standard approach in the literature is to let firms set the less flexible variable in the first stage and then set the more flexible choice variable (prices) in the second stage (see for example, Aluf and Shy, 2001; Salvanes et al, 2003). This, however, is not possible in the present model. If we adopt this approach, there does not exist an equilibrium. One way to rationalize our approach is to suppose that the choice of advertising is not separable from the actual sending of advertisements (fliers). In this case, firms set prices first since they advertise their prices.

[11] The role of advertising agencies include market analysis, media buying services, consultation on promotion strategies and techniques (design and packaging) among others (Printadvertising.com; Utah Firms Staff, 2003). Many firms nowadays employ advertising agencies to do the advertising on their behalf. For example, EURO RSCG Worldwide has, among its clients, Volvo, Citroen and Peugeot – firms competing in the same market! Catalpha Advertising and Design has among its clients; Black & Decker, DeWalt, Craftsman – firms selling similar products.

3.2.1. *Collusion phase.* In the collusive phase, the advertising agency sets $\phi_1 = \phi_2 = \phi$, knowing the equilibrium prices, p_1 and p_2, chosen by the firms in the prior (noncooperative) phase[12]. The agency maximizes the following objective function:

$$\text{(9)} \quad \prod \equiv \pi_1 + \pi_2 = \max_{\phi} \left\{ (p_1 + p_2)\,\phi\,(1-\phi) + \frac{\phi^2}{2t}\left(t\,(p_1+p_2) - (p_1-p_2)^2\right) - a\phi^2 \right\}.$$

The first order condition yields

$$\text{(10)} \qquad \phi = \frac{t\,(p_1+p_2)}{2at + t\,(p_1+p_2) + (p_1-p_2)^2}.$$

3.2.2. *Competition phase.* In the competition phase, firms noncooperatively set their prices, knowing that they will collude on advertising afterwards. Given the collusive advertising level in (10), firm i's problem is described by:

$$\text{(11)} \qquad \pi_i = \max_{p_i} \left\{ p_i\phi \left(1 - \phi + \phi\frac{p_j - p_i + t}{2t}\right) - \frac{a\phi^2}{2} \right\} \text{ subject to (10).}$$

Differentiating with respect to p_i, and solving for a symmetric equilibrium gives:

$$\text{(12)} \qquad p^{ac} = t/2 + \sqrt{2at + t^2/4}.$$

Substituting (12) into (10) gives the semicollusive advertising level as:

$$\text{(13)} \qquad \phi^{ac} = \frac{t/2 + \sqrt{2at + t^2/4}}{a + t/2 + \sqrt{2at + t^2/4}}.$$

Finally, substituting (12) and (13) into (9) gives the semicollusive profit as[13]:

$$\text{(14)} \qquad \pi^{ac} = \frac{\left(t/2 + \sqrt{2at + t^2/4}\right)^2}{2\left(a + t/2 + \sqrt{2at + t^2/4}\right)}$$

where ac is a mnemonic for collusion on advertising.

The full coverage assumption implies that $p^{ac} = t/2 + \sqrt{2at + t^2/4} \leq v - t$. For given t and v, simplifying gives $a \leq \overline{a} \equiv \frac{(v-2t)(v-t)}{2t}$. $\overline{a}$ is the highest advertising cost compatible with full market coverage.

[12] We assume that side payments are not feasible. Hence, $\phi_1 = \phi_2$ in the collusive equilibrium.

[13] For simplicity, we assume that the advertising agency gets **no** share of the profits.

3.3. Comparison. The question we seek to address here is the following: Does collusion on advertising and competition on price entail higher or lower prices; higher or lower advertising intensities; higher or lower profits compared to the noncooperative outcome? Comparing equations (7) and (12), we see immediately that $p^{ac} > p^{nc}$. That is, equilibrium prices are higher under collusion on advertising. Also, from (7) and (13), we get (after a bit of algebraic manipulation) that $\phi^{ac} < \phi^{nc}$.[14] Since higher advertising has a negative direct effect on profit, collusion on advertising unambiguously raises profits relative to the noncooperative outcome[15].

The discussion following Lemma 1 gives a concise statement of why firms may want to constrain informative advertising. The mechanism works as follows; Consumers who receive advertising from both firms (fully informed) can make across firm price comparisons and, as a result, these consumers buy from the firm quoting the lowest "delivered" price. Competition to sell to these consumers drives the price down. In contrast, consumers who receive advertising from a single firm only (partially informed) are totally price insensitive (for all prices $p \leq v - t$). Hence, the optimal price applicable to this group is higher compared to that applicable to the fully informed group. Intuitively, because an increase in advertising raises the proportion of fully informed consumers in the market, it elevates the importance of the fully informed (and hence price sensitive) consumers and this puts pressure on prices and by Lemma 1, lowers profits. The idea of collusion on advertising is precisely to try to minimize such competition by constraining the proportion of fully informed consumers. We summarize our findings thus far in the following proposition:

PROPOSITION 1. *Collusion on advertising (and competition on price) gives lower equilibrium advertising but higher equilibrium prices and profits relative to the fully noncooperative equilibrium. That is,* $\phi^{ac} < \phi^{nc}$, $p^{ac} > p^{nc}$ *and* $\pi^{ac} > \pi^{nc}$.

PROOF. (See Appendix A). □

Given that price fixing is per se illegal, the fact that it is possible to sustain higher prices and profits without resorting to price fixing should be comforting for firms. Collusion on advertising is difficult to detect and /or prosecute (unlike price collusion)[16]. The reason being that collusion on advertising is not deemed per se illegal which makes

[14] See Proof of Proposition 1 in Appendix A.

[15] More precisely, since $\phi^{ac} < \phi^{nc}$ and $\frac{\partial \pi}{\partial \phi} < 0$ (Lemma 1), it follows that $\pi^{ac} > \pi^{nc}$.

[16] When the Court of Appeals for the 9th Circuit concurred that the CDA code of conduct was a "naked" restraint on price competition, the Commission thought they had nailed the CDA. However, the Supreme Court was not convinced that the said restrictions could peremptorily be deemed anticompetitive. As a result, the court instructed that the rules be examined under the rule of reason approach. Upon reconsideration, the decision of the 9th Circuit – which had earlier concluded that the rules were anticompetitive – was least expected. The Court concluded that there

it difficult to draw the line between permissible and not permissible conducts (Lande and Marvel, 2000; pages 956-957; Starek, 1997). As a matter of fact, advertising agencies openly handle business on behalf of competing firms (see footnote 11).

From a welfare perspective, an important question is whether semicollusion on advertising improves welfare. Grossman and Shapiro (1984), Hamilton (2004) and Simbanegavi (2005) show that the market may overprovide informative advertising relative to the socially optimal level[17]. Hence collusion on advertising, by restricting advertising, is potentially welfare improving, especially for low advertising costs. On the one hand, when firms collude on advertising, fewer consumers get informed and this lowers aggregate consumer surplus since uninformed consumers do not purchase. On the other hand, firms earn higher profits when they collude on advertising[18]. So, which direction will the welfare effect go? The following Proposition answers this question:

PROPOSITION 2. *Semicollusion on advertising is detrimental to welfare.*

PROOF. Let W^{ac} (CS^{ac}) be the welfare (consumer surplus) when firms collude on advertising but compete on prices and W^{nc} (CS^{nc}) be the welfare (consumer surplus) when firms compete on both prices and advertising. Because the market is covered, $CS = v - p$. Defining welfare as profits plus consumer surplus ($W = \pi + CS$), we get that

$$\begin{aligned} W^{ac} &= \pi^{ac} + v - p^{ac} \\ &= \frac{\left(\frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)^2 + \left(v - \left(\frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)\right) 2\left(a + \frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)}{2\left(a + \frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)} \end{aligned}$$

and

$$\begin{aligned} W^{nc} &= \pi^{nc} + v - p^{nc} \\ &= \frac{2at^2 + \left(v - \sqrt{2at}\right)\left(t + \sqrt{2at}\right)^2}{\left(t + \sqrt{2at}\right)^2}. \end{aligned}$$

was not enough evidence to show that the said restrictions were indeed anticompetitive (Lande and Marvel, 2000; pages 956-957).

[17] The market welfare is lower than the social optimum for the following reasons; For low (and very high) advertising costs, the market advertises "excessively" relative to the socially optimal level. Because the advertising cost function is convex, this raises the total advertising outlay and hence lowers welfare. For intermediate values of the advertising cost, advertising is "too low" and as a result, fewer consumers are informed. This also lowers welfare.

[18] Profits are higher for two reasons; first, prices are higher when firms collude on advertising and second and more importantly, the total advertising outlay is lower.

Subtracting the latter from the former gives

$$W^{ac} - W^{nc} = \frac{-2at^3 - 2a^2t^2 - \frac{1}{2}t^4 - 2at^2\sqrt{2at} - t^3\sqrt{2at + \frac{1}{4}t^2} - 4a^2t\left(\sqrt{2at + \frac{1}{4}t^2} - \sqrt{2at}\right)}{2\left(a + \frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)\left(t + \sqrt{2at}\right)^2} < 0.$$

We conclude therefore that welfare is lower when firms collude on advertising rather than compete. □

Intuitively, when firms collude on advertising, they restrict advertising "too much". In fact, it can be shown that for all advertising costs (in the relevant range), the collusive level is lower than the socially optimal level[19]. That welfare in the semicollusive equilibrium is lower than in the Nash equilibrium is an important result, particularly for competition policy. Although firms may overprovide informative advertising in the noncooperative equilibrium (particularly for low advertising costs), uncontrolled collusion is not a remedy. It is even more inefficient. Under collusion on advertising, the collusive advertising level is "too low" and as a result, too few consumers are informed. This exacerbates the loss of consumer surplus. Since firms have incentives to collude on advertising, there is clearly need for monitoring.

Although in our model, just as in Aluf and Shy (2001), firms charge higher prices and earn higher profits when colluding on advertising, there are significant differences between the two models[20]. First, in our framework, firms advertise less when colluding. Second, the mechanism through which advertising affects prices and profits is different. In our model, advertising does not change consumers' tastes. Instead, it affects informational product differentiation and hence the toughness of price competition. That is, it alters the proportion of fully informed consumers in the market and hence the price elasticity of demand.

In what follows, we contrast price collusion to collusion on advertising. This is motivated by the fact that the analysis of semicollusion to date has largely been framed as collusion on price and competition on a nonprice variable. The question we address is the following: Does price collusion lead to higher profits compared to nonprice collusion – in particular, to collusion on advertising?

[19] The proof is a bit messy and, as a result, we skip it. The proof is available from the author on request.

[20] In a model in which TV channels sell advertising time to firms, Kind et al (2005) find that when TV channels collude on advertising, equilibrium advertising levels are higher and the TV channels earn higher profits than when they compete on advertising. Their model, however, is not directly comparable to either ours or to Aluf and Shy (2001) since the only choice variable in their model is the advertising level.

4. Price Collusion

Many firms have multiple decision variables and for such firms, price collusion does not constitute "full" collusion. If anything, collusion on price may actually trigger more competitive behaviour in other choice variables (Fershtman and Gandal, 1994). We consider here a setting where firms cooperatively set the price at which their merchandise will be sold. However, each firm independently decides on the "measure" of fliers to send out to consumers. We derive the price collusion equilibrium and compare it to the advertising collusion equilibrium derived earlier. Apart from the change of the collusion instrument, everything is as before.

As before, we model the firms' behaviour as a two stage game. In the first stage, firms collude on price and in the second stage, firms compete on advertising.

When firms collude on price, firm i's demand is given by:

$$D_i(\phi_1, \phi_2; p) = \phi_i(1 - \phi_j) + \frac{\phi_i \phi_j}{2}; j \neq i. \tag{15}$$

A peculiar feature of our model is that when firms collude on price, demand is independent of price. This independence is a direct consequence of the full market coverage assumption. Because the market is fully covered, the demand by partially informed (and hence captive) consumers is independent of price. Prices only matter for the partitioning of the fully informed segment of the market (see equation 1'). Therefore, when firms collude on price (charge the same price), they divide the fully informed consumer population equally between them. That is, each firm gets proportion half of the fully informed consumers.

We solve the problem backwards, starting with the second stage. Given the collusive price, p, chosen in the first stage, firm i's second stage maximization program is given by:

$$\pi_i^{pc} = \max_{\phi_i} \left\{ p\left(\phi_i(1 - \phi_j) + \frac{\phi_i \phi_j}{2}\right) - \frac{a\phi_i^2}{2} \right\}. \tag{16}$$

Differentiating (16) with respect to ϕ_i and evaluating the first order condition gives

$$\phi = \frac{2p}{2a + p}. \tag{17}$$

In the first period, anticipating competition on advertising in the second period, firms collude on price. Given full market coverage, there is a unique focal price. Let p^{pc} be the collusive price, where pc is a mnemonic for price collusion. Then:

LEMMA 2. $p^{pc} = v - t$.

PROOF. We prove by contradiction. Let p^{pc} be the profit maximizing collusive price and suppose $p^{pc} \neq v - t$. Then, either $p^{pc} < v - t$ or $p^{pc} > v - t$. First, suppose

$p^{pc} < v - t$. Then (by continuity of price)[21], $\exists \varepsilon > 0 : p^{pc} + \varepsilon < v - t$ and $\pi(p^{pc} + \varepsilon, \phi) > \pi(p^{pc}, \phi)$. Hence, any collusive price, $p^{pc} : p^{pc} < v - t$ cannot be profit maximizing – a contradiction[22]. Therefore, we must have $p^{pc} > v - t$. However, observe that $p^{pc} > v - t$ violates the full market coverage assumption – a contradiction. Hence, it must be the case that $p^{pc} = v - t$. □

Since the collusive price is a corner solution, a question that naturally arises regards the extent to which the full coverage assumption constrains the collusive price. As we show in Appendix B, this assumption is not very restrictive. For a wide range of the advertising cost parameter, a, $p^{pc} = v - t$ is the unconstrained collusive price.

Given Lemma 2, evaluating (16) and (17) gives:

$$\phi^{pc} = \begin{cases} 1 & \text{if } a \leq \frac{v-t}{2} \\ \frac{2(v-t)}{2a+v-t} & \text{if } a > \frac{v-t}{2} \end{cases} \quad \text{and } \pi^{pc} = \begin{cases} \frac{v-t-a}{2} & \text{if } a \leq \frac{v-t}{2} \\ \frac{2a(v-t)^2}{(2a+v-t)^2} & \text{if } a > \frac{v-t}{2} \end{cases}. \tag{18}$$

We see from (18) that price collusion gives rise to a full information equilibrium for lower levels of the advertising cost while it gives rise to a partial information equilibrium for higher levels of the advertising cost. That is:

LEMMA 3. *$\phi^{pc} = 1$ for $a \leq \frac{v-t}{2}$ and $\phi^{pc} < 1$ for $a > \frac{v-t}{2}$.*

PROOF. (See Appendix A) □

Because the price is given (price is unaffected by advertising), each firm wants to inform as many consumers as possible (demand effect). When a is small relative to price, it pays to inform all consumers. However, when a increases beyond $\frac{v-t}{2}$, the advertising outlay becomes large relative to the revenues and the firm responds by reducing the advertising intensity.

5. Collusion on Price or Advertising?

Below we relate the price collusion equilibrium to the advertising collusion equilibrium. Our first result in this section comes from comparing the equilibrium prices and advertising intensities under the two collusive regimes.

PROPOSITION 3. *When $a \in \left(\frac{t}{2}, \frac{(v-2t)(v-t)}{2t}\right)$, collusion on price (and competition on advertising) yields both a higher equilibrium price and a higher equilibrium advertising intensity compared to collusion on advertising (and competition on price). That is, $p^{ac} < p^{pc}$ and $\phi^{ac} < \phi^{pc}$.*

[21] Full market coverage obtains for $t < v/2$ (see the Appendix to Soberman, 2004). Hence, without loss of generality, we can let $t = \alpha v, \alpha \in (0, 1/2)$. Then, $p^{pc} = v - t = (1 - \alpha) v$ which is obviously continuous.

[22] Raising the price to $p = p^{pc} + \varepsilon < v - t$, does not violate any consumer's individual rationality constraint – hence demand is unchanged.

PROOF. By full market coverage, $p + t \leq v$. Therefore from (12), we must have that $p^{ac} + t = t/2 + \sqrt{2at + t^2/4} + t \leq v$. For given t and v, we can solve for a to get; $a \leq \frac{(v-2t)(v-t)}{2t} = \overline{a}$. Furthermore, by assumption, our model is valid for $a > t/2$. Hence, $a \in \left(\frac{t}{2}, \frac{(v-2t)(v-t)}{2t}\right]$. It follows then that for $a \in \left(\frac{t}{2}, \frac{(v-2t)(v-t)}{2t}\right)$, $p^{ac} = t/2 + \sqrt{2at + t^2/4} < v - t = p^{pc}$ as required. The proof of the second claim (that $\phi^{ac} < \phi^{pc}$) is given in Appendix A. □

First, note that $p^{pc} = v - t$ is the highest possible price consistent with full market coverage. Secondly, as we saw in Proposition 1, collusion on advertising is a "proxy" for collusion on price. Being an indirect way of colluding on price, it is sensible that $p^{ac} < p^{pc}$.

That $\phi^{ac} < \phi^{pc}$ is intuitive. First, advertising is important in this model in that it raises demand. Hence, other things being equal, firms always want to increase advertising. Second, when firms collude on price, the negative relationship between price and advertising is broken. Clearly therefore, when firms collude on price, they have greater incentives to advertise than when they collude on advertising[23]. It follows therefore that price collusion induces more advertising.

Since both the price and the advertising level are higher under price collusion, it follows immediately from Proposition 3 that:

COROLLARY 1. *Revenues and advertising outlays are higher when firms collude on price.*

PROOF. Let R denote revenues and D denote the demand. At equilibrium, $\frac{\partial D}{\partial \phi} = 1 - \phi > 0$. That is, demand is increasing in the advertising intensity. Since $p^{pc} > p^{ac}$ and $\phi^{pc} > \phi^{ac}$, it follows that $R^{pc} \equiv p^{pc}D^{pc} > p^{pc}D^{ac} > p^{ac}D^{ac} \equiv R^{ac}$ – where the first inequality follows from the fact that $\phi^{pc} > \phi^{ac}$ and $\frac{\partial D}{\partial \phi} > 0$ and the second inequality follows from the fact that $p^{pc} > p^{ac}$. That the advertising outlay is higher under price collusion follows from convexity of the advertising cost function (and the fact that $\phi^{pc} > \phi^{ac}$). □

A closer look at Propositions 1 and 3 brings to the fore an important difference between price and nonprice collusion. Price collusion exacerbates competition on the variable that is chosen noncooperatively (see also Fershtman and Gandal, 1994 and Steen and Sørgard, 1999). In contrast, nonprice collusion (collusion on advertising or

[23] Whereas firms internalize costs due to business stealing when they collude on advertising (prices fall with advertising), there are no such costs when firms collude on price (price fixed). There is some business stealing going on though. If an ad, by firm 1 for instance, reaches a consumer who hitherto had only received an ad from firm 2, the consumer (who now becomes fully informed) will switch to firm 1 if she is located in $\left(0, \frac{1}{2}\right)$. This possibility to steal business from the competitor creates incentives to increase advertising.

capacity) does not intensify price competition. If anything, it relaxes price competition. In other words, the "semicollusion effect" (the competition intensifying effect of semicollusion) only kicks in under price collusion. To help explain this observation, we invoke Fudenberg and Tirole (1984)'s "taxonomy of business strategies".

As we have shown, when firms collude on advertising, they advertise less. By voluntarily restricting its advertising, each firm signals that it will not be aggressive in the price competition game. This is so because, with low advertising, fewer consumers are informed and with fewer informed consumers, demand is low. Hence profits can only be enhanced by charging a higher (and not a lower) price. Because prices are strategic compliments, collusion on advertising softens the rival firm's pricing behaviour[24]. In this sense, collusion on advertising is a "puppy dog" strategy.

In contrast, collusion on price induces more aggressive behaviour in the advertising competition game. Because prices are fixed, the larger the demand that a firm can generate, the higher the revenues it expects to get. However, since the price is fixed, demand can only be increased by informing more consumers – since uninformed consumers do not purchase. In this sense, price collusion makes each firm tough in the advertising game[25]. In the animal jargon of Fudenberg and Tirole (1984), price collusion is a "top dog" strategy.

The use of the animal terminology here needs to be qualified. Fudenberg and Tirole (1984) use the animal jargon in a setting in which firms move sequentially, with the first mover committing to a particular action, an action which is observed by the follower firm prior to making its own move. In our setting however, firms move simultaneously (rather than sequentially) at each stage, but still the commitment issue comes into play since firms' second period choices will only be made after both firms observe the first period choices.

From our analysis, together with the analyses of Fershtman and Gandal (1994) and Steen and Sørgard (1999), it appears that the semicollusion effect can be explained by whether the collusion and competition instruments are strategic complements or substitutes. When firms collude on a strategic substitute (advertising or capacity/quantity) and compete on a strategic compliment (price), competition is relaxed. However, when

[24] From the first order conditions (see equation (3)), $p_i = \left(2t - t\phi_j + \phi_j p_j\right)/2\phi_j$ and $\partial p_i/\partial p_j > 0$.

[25] Though not apparent, the firms' advertising intensities are strategic substitutes. From (4), $\phi_i = p_i\left(2t + (p_j - p_i - t)\,\phi_j\right)/2at$. In any equilibrium in which firms compete for the fully informed consumers, it must be the case that for $i, j = 1, 2; j \neq i$, $p_j < p_i + t$. For if $p_j > p_i + t$, then all fully informed consumers will buy from firm i – violating the assumption that firms compete for the fully informed consumers. Hence, $\partial\phi_i/\partial\phi_j = p_i\left(p_j - p_i - t\right)/2at < 0$.

firms collude on a strategic compliment (price) and compete on a strategic substitute (advertising or capacity/quantity), competition is exacerbated. We therefore conjecture that a necessary condition for the semicollusion effect to kick in is that the competition variable is a strategic substitute.

The observation that price collusion intensifies competition on the nonprice variable but not the other way round has important implications for firm conduct. If firms are "sophisticated" and have multiple choice variables, they ought to realize that price collusion is more likely to hurt them compared to nonprice collusion. Moreover, price collusion is per se illegal and is heavily punished for when discovered. This suggests then that firms ought to shift focus from price to nonprice collusion. They seem to. There is an increasing number of nonprice collusion cases that the US Federal Trade Commission has had to deal with in recent years. Examples include; the California Dental Association case in which the association instituted rules and regulations that restrict price and quality advertising (FTC Docket No. 9259); the Arizona Automobile Dealers Association case in which the association agreed with some of its members to "restrain truthful and nondeceptive advertising (FTC File No. 931 0056); collusion on advertising by PolyGram (predecessor to Vivendi Universal) and Warner in order to reduce intrabrand competition – competition between the Three Tenors' third album and video and the first and second albums and video (FTC File No. 001 0231; Goldberg, 2005).

To recapitulate, the main question we address in this section is the following: If they had a choice, which strategic variable (price or advertising) would firms use as the collusion instrument? To answer this question, we compare π^{ac} and π^{pc}. As a prelude, it is instructive to analyze the relationship between profits and the advertising cost, a, under semicollusion on price and respectively, semicollusion on advertising. Differentiating equations (14) and (18) with respect to a, we find that:

LEMMA 4. *Semicollusive profits are decreasing (increasing) in the advertising cost under price (advertising) collusion.*

PROOF. See Appendix A. □

Under both collusion on price and collusion on advertising, the effect of an increase in the advertising cost on profit can be decomposed into a direct effect and an indirect effect. Notice that the advertising cost, a, enters directly into the advertising cost function but only enters into the revenue function indirectly – via price and /or advertising level (see equations (9) and (16)). The direct effect of an increase in a is to

raise the advertising outlay, other things being equal[26]. However, other things will not remain equal. An increase in a induces firms to reduce advertising and this increases informational product differentiation – a strategic effect.

Under collusion on advertising, this strategic effect allows firms to raise prices and consequently revenues. The effect on revenues outweighs the direct effect on the advertising outlay and hence profits increase with the advertising cost.

Under semicollusion on price, there are two cases to consider. First, when $a \leq (v-t)/2$, we have full information (that is, $\phi^{pc} = 1$). Moreover, since $p^{pc} = v - t$, it follows that the revenue function is independent of a. Therefore, when a increases, the only component of the profit function that changes is the advertising outlay (which increases with a). Hence, for $a \leq (v-t)/2$, profit necessarily decreases with a. For $a > (v-t)/2$, $\phi^{pc} = \frac{2(v-t)}{2a+v-t} < 1$ and, when the advertising cost increases, firms respond by advertising less. Although informational product differentiation increases, prices cannot be increased and hence revenues must of necessity decrease. Since the direct effect of an increase in a is to raise the advertising outlay, profits fall when the advertising cost, a, increases.

We just showed above (Lemma 4) that profits are decreasing in the advertising cost, a, under semicollusion on price but increasing in the advertising cost under semicollusion on advertising. One may therefore conjecture that there exists an a, (call it $\widehat{a}$) for which the two profit functions intersect. If indeed such an a exits, then, for $a < \widehat{a}$, semicollusion on price should yield higher profits while for $a > \widehat{a}$, semicollusion on advertising should yield higher profits.

Let α denote the ratio of transportation costs to the gross surplus, that is, $\alpha \equiv t/v$. Below we plot π^{ac} and π^{pc} as functions of the advertising cost, a, for $\alpha = 0.25$.

From Figure 1, we see that for "low" values of a, $\pi^{ac}(a) < \pi^{pc}(a)$ while the opposite is true for "high" values of a. In fact, it can be shown, in a general setting, that for a wide range of the parameter α, π^{ac} and π^{pc} intersect. Below we state the main result of this section;

PROPOSITION 4. *Semicollusion on price does not always lead to higher profits compared to semicollusion on advertising. More precisely, let* $a \in \left(\frac{\alpha}{2}, \frac{(1-2\alpha)(1-\alpha)}{2\alpha}\right) v$ *and let* $\widehat{a} \equiv a(\alpha)$ *such that* $\pi^{pc}(a(\alpha)) = \pi^{ac}(a(\alpha))$. *Then,* $\forall \alpha \in \left(\underline{\alpha}, \frac{1}{3}\right)$, $\underline{\alpha} > 0$; $\pi^{pc}(a) > \pi^{ac}(a)$ *for* $a < \widehat{a}$ *and* $\pi^{pc}(a) < \pi^{ac}(a)$ *for* $a > \widehat{a}$.

PROOF. See Appendix A. □

[26] An increase in a has two opposing effects on the advertising outlay. An increase in a directly increases the advertising outlay, other things being equal. However, when a increases, other things will not remain equal. Firms will respond by reducing advertising intensities. This indirect effect works to reduce the advertising outlay as now firms advertise less. However, the direct effect dominates.

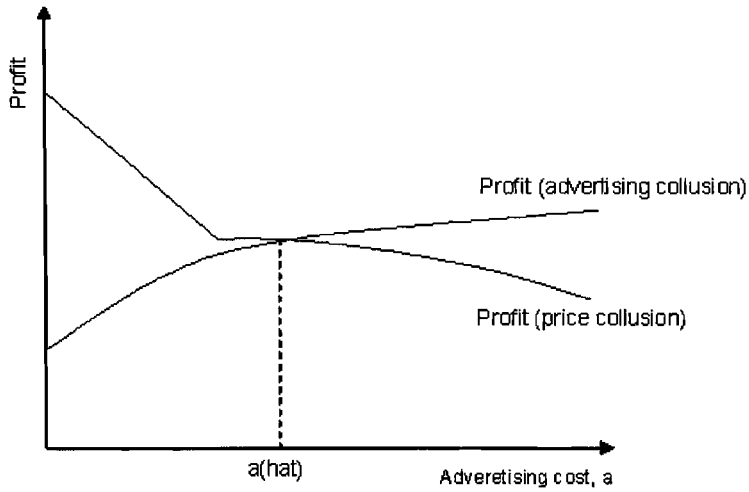

Figure1. Profits from price and advertising collusion

Although price collusion dominates collusion on advertising in terms of revenues (Corollary 1), it is, in general, not superior to the latter. As was shown in Proposition 3, firms advertise rather "excessively" when they collude on price (which increases demand and hence revenues). However, because the advertising cost function is convex, the firms incur higher advertising costs under price collusion (bad for profits). When the advertising cost is *low*, the revenue effect dominates in the profit function and this makes price collusion more profitable. However, for *higher* advertising costs, the revenue effect is weakened by the ballooning advertising outlays. Because firms advertise less when they collude on advertising, they incur lower advertising outlays. As a result, collusion on advertising yields higher profits compared to price collusion when the advertising cost is higher. In summary, collusion on price is not always more profitable compared to collusion on advertising. Depending on parameter values, sometimes price collusion dominates and sometimes it is dominated.

A principal assumption of this paper is that *equilibrium prices are such that the market is fully covered.* One might wonder what the effect of assuming full coverage is on profits, particularly under semicollusion on price. As we have presented it, Proposition 4 is predicated on the assumption that the market is covered. However, it is quite reasonable to conjecture that when firms collude on price, the optimal price may be such that some consumers find it profitable not to purchase. If this is the case, then, by assuming full coverage, we restrict the collusive profits under semicollusion on price, π^{pc}. In fact, as we showed in Lemma 2, the collusive price is a corner solution,

which suggests that if an interior collusive price exists, it has to lie in the interval $(v-t, v)$. Hence, there is a possibility that, by allowing the firms to collude on a price that induces some consumers not to purchase, our Proposition 4 may be fundamentally weakened (or cease to hold altogether). Thus, it is imperative that we undertake a robustness check to see to what extent Proposition 4 depends on the full coverage assumption. We solve this exercise in Appendix B.

So, what have we missed by restricting the collusive price to the interval $(0, v-t]$? It turns out not much has been missed. Although indeed there exist some profitable collusive prices for which the market will not be covered, qualitatively, Proposition 4 is unaltered. The unrestricted collusive price (and hence the associated unrestricted profit) exceeds the restricted collusive price (and hence the associated restricted profit) only for a "narrow" range of the advertising cost, a. For the most part, the restricted profits are higher! More importantly, in terms of comparisons with profits under semicollusion on advertising, π^{ac}, allowing for some prices that lead to less than full coverage is of no consequence. For reasonable parameter values, π^{pc} and π^{ac} intersect, with π^{ac} intersecting π^{pc} from below – which establishes our result.

Our main findings thus far are that (i) collusion on advertising and competition on price is more profitable than competition on both price and advertising and (ii) collusion on price does not always lead to higher profits compared to collusion on advertising. Empirical evidence seem to support both our findings. As stated in the introduction, studies of price and advertising strategies find support for collusion on advertising but not price, which is supportive of our findings. Or, put differently, we lend theoretical support to the empirical findings on firms' price and advertising strategies.

6. Is Semicollusion Disadvantageous?

Fershtman and Gandal (1994) argue that semicollusion typically induces intense competition on the choice variable(s) chosen noncooperatively. If the competitive pressure is sufficiently intense, semicollusion results in lower profits compared to a fully noncooperative outcome. Does this thesis hold in our framework?

To answer this question, we compare the noncooperative profits to the collusive profits. Specifically, we compare π^{nc} and π^{ac} on the one hand and π^{nc} and π^{pc} on the other. We find that;

PROPOSITION 5. *Semicollusion (price/advertising) yields higher equilibrium profits than when firms compete in both price and advertising.*

PROOF. See Appendix A. □

By Proposition 1, $\pi^{nc} < \pi^{ac}$. That is, collusion on advertising is advantageous. As we argued in the discussion following Corollary 1, collusion on advertising relaxes price competition. Collusion on advertising enhances profitability for two reasons: it lowers the advertising outlay and it raises the equilibrium price.

Proposition 5 supports the 'conventional wisdom' that, overall, firms are better off colluding rather than competing. Irrespective of the collusion instrument (price or advertising), firms earn higher profits compared to the profits when the firms compete on both price and advertising. A question that arises is: Why is semicollusion disadvantageous in the models of Fershtman and Gandal (1994), but not in the present model? In Fershtman and Gandal (1994), when firms collude on price, they overinvest in capacity hoping to use the excess capacity as a bargaining chip when it comes to the division of the collusive profits. However, in equilibrium, excess capacity is *totally* redundant in so far as its intended objective is concerned[27,28]. Since capacity is costly to install, price collusion may hurt firms compared to fully noncooperative interaction.

Unlike capacity, advertising has a positive direct effect for the advertising firm – it raises demand. In fact, the reason for "excessive" advertising (under price collusion) is to increase demand. Thus, even though advertising (just like capacity) is costly, it is not totally redundant. This demand expansion effect mitigates the negative effect of higher advertising intensities. Hence, notwithstanding the fact that firms advertise excessively under price collusion, they still earn higher profits compared to competition on both price and advertising.

7. Conclusion

We analyze, in this paper, firms' incentives to collude on advertising when advertising is purely informative. We find that collusion on advertising and competition on price is more profitable than competition on both price and advertising. From a welfare perspective, collusion on advertising is bad. When firms collude on advertising, "too few" consumers are informed and, as a result, welfare is lower than when firms compete on both prices and advertising. This result is important for policy. Although advertising is only informative, there is need for monitoring – more so with the advent of advertising agencies. Left unchecked, firms will be tempted to connive against consumers.

[27] In a symmetric equilibrium, both firms overinvest to the same extend – hence the excess capacity will not in any way enhance a particular firm's bargaining position in the division of the collusive profits.

[28] The same mechanism also operates in Steen and Sørgard (1999) to induce overinvestment in capacity. However, in Steen and Sørgard, the export market provides a leeway that reduces the redundancy of excess capacity.

We also compare price collusion to collusion on advertising. In general, price collusion does not dominate collusion on advertising. In this sense, there is no justification for the theoretical literature's exclusive focus on price collusion. Hence we lend theoretical support to the empirical literature that consistently find evidence of collusion on advertising rather than on price.

In this paper, we use a static model to study firms' incentives to collude on advertising. But, will the firms actually collude on advertising? To answer this question, we need a dynamic setting which permits firms to respond to the actions of competitors. Collusion on advertising is sustainable only if the incentives to deviate are outweighed by the benefits from conforming. This, however, is left for future research.

Appendix

Appendix A: Proofs of Lemmas and Propositions

Proof of Proposition 1

PROOF. Proposition 1 states that $\phi^{nc} > \phi^{ac}; p^{nc} < p^{ac}$ and $\pi^{nc} < \pi^{ac}$. From (7) and (12), $p^{nc} = \sqrt{2at} < \frac{t}{2} + \sqrt{2at + t^2/4} = p^{ac}$. By Lemma 1, $\partial\pi/\partial\phi < 0$. Therefore, if $\phi^{ac} < \phi^{nc}$, it follows that $\pi^{ac} > \pi^{nc}$. Hence, we only need to show that $\phi^{nc} > \phi^{ac}$, for all $a \in \left(t/2, \frac{(v-2t)(v-t)}{2t}\right)$; where $\phi^{nc} = \frac{2t}{t+\sqrt{2at}} = \frac{1}{1+\frac{\sqrt{2at}}{2t}-\frac{1}{2}}$ and $\phi^{ac} = \frac{\frac{t}{2}+\sqrt{2at+t^2/4}}{a+\frac{t}{2}+\sqrt{2at+t^2/4}} = \frac{1}{\frac{2a}{t+2\sqrt{2at+t^2/4}}+1}$. To show that $\phi^{ac} < \phi^{nc}$, it suffices to show that $\frac{\sqrt{2at}}{2t} - \frac{1}{2} < \frac{2a}{t+2\sqrt{2at+t^2/4}}$. Let $\Psi(a) \equiv \frac{\sqrt{2at}}{2t} - \frac{1}{2} - \frac{2a}{t+2\sqrt{2at+t^2/4}} = \frac{\left(\sqrt{2at}-t\right)\left(t+2\sqrt{2at+t^2/4}\right)-4at}{2t\left(t+2\sqrt{2at+t^2/4}\right)}$. $\phi^{ac} < \phi^{nc} \iff \Psi(a) < 0 \iff \left(\sqrt{2at} - t\right)\left(t + 2\sqrt{2at + t^2/4}\right) - 4at < 0$. Expanding, we get that; $\left(\sqrt{2at} - t\right)\left(t + 2\sqrt{2at + t^2/4}\right) - 4at = t\sqrt{2at} - t^2 - 2t\sqrt{2at + t^2/4} + 2\sqrt{2at}\sqrt{2at + t^2/4} - 4at$. Since $\sqrt{2at}\sqrt{2at + t^2/4} < 2at + t^2/4$, it follows that; $\Psi(a) = \frac{t\sqrt{2at}-t^2-2t\sqrt{2at+t^2/4}+2\sqrt{2at}\sqrt{2at+t^2/4}-4at}{2t\left(t+2\sqrt{2at+t^2/4}\right)} < \frac{-\frac{1}{2}t-\left(2\sqrt{2at+t^2/4}-\sqrt{2at}\right)}{2\left(t+2\sqrt{2at+t^2/4}\right)} < 0 \forall a$. We conclude that $\phi^{nc} > \phi^{ac}$, for all $a \in \left(t/2, \frac{(v-2t)(v-t)}{2t}\right)$. □

Proof of Lemma 3

PROOF. The idea of the proof is to show that $\frac{\partial\pi}{\partial\phi} > 0 \forall \phi$ whenever $a \leq \frac{v-t}{2}$ and $\frac{\partial\pi}{\partial\phi} < 0$ for $\phi \to 1$ and $\frac{\partial\pi}{\partial\phi} > 0$ for $\phi \to 0$ whenever $a > \frac{v-t}{2}$. To start with, at the symmetric equilibrium, $\frac{\partial\pi}{\partial\phi} = \frac{v-t}{2}(2-\phi) - a\phi$. The first term on the R.H.S is the marginal revenue resulting from a marginal increase in the advertising intensity, ϕ, while the second term is the marginal cost. Suppose $a \leq \frac{v-t}{2}$. Substituting $a = \frac{v-t}{2}$ into the expression for $\frac{\partial\pi}{\partial\phi}$ and simplifying gives $\frac{\partial\pi}{\partial\phi} = (v-t)(1-\phi) > 0 \forall \phi \in (0,1)$. In other words, a marginal increase in advertising raises profits. Hence firms will expand advertising whenever $\phi < 1$. Thus the equilibrium advertising level is $\phi^{pc} = 1$ whenever $a \leq \frac{v-t}{2}$. Now suppose $a > \frac{v-t}{2}$. Without loss of generality, let $a = \frac{v-t}{2} + \varepsilon$, where $\varepsilon > 0$.

Substituting a into the equilibrium advertising condition gives $\frac{\partial \pi}{\partial \phi} = (v-t)(1-\phi) - \varepsilon\phi$. Clearly, if $\phi = 1$, $\frac{\partial \pi}{\partial \phi} < 0$. Hence the firm wants to lower the advertising level. More generally, by continuity of ϕ and an appropriate choice of ε, this holds too for $\phi \to 1$. Hence the firm also wants to reduce the advertising level for ϕ close enough to 1. On the other hand, if ϕ is small, $\frac{\partial \pi}{\partial \phi} > 0$ and the firm wants to increase the advertising level. More precisely, $\lim_{\phi \to 0} \frac{\partial \pi}{\partial \phi} = v - t > 0$. It follows therefore that the equilibrium advertising level is given by the condition: $\frac{\partial \pi}{\partial \phi} = 0 \iff \phi^{pc} = \frac{v-t}{v-t+\varepsilon} < 1$. Thus for $\phi : \phi > \phi^{pc}$, the firm will reduce advertising while for $\phi : \phi < \phi^{pc}$, the firm will increase advertising whenever $a > \frac{v-t}{2}$. □

Proof of Proposition 3

PROOF. There are two cases to prove. First, we show that $\phi^{ac} < \phi^{pc}$ whenever $a \leq \frac{v-t}{2}$. Second, we show that $\phi^{ac} < \phi^{pc}$ for $a > \frac{v-t}{2}$. The first case is rather trivial. From Lemma 3, $\phi^{pc} = 1$ for $a \leq \frac{v-t}{2}$ and from (13), $\phi^{ac} = \left(\frac{t}{2} + \sqrt{2at + t^2/4}\right) / \left(a + \frac{t}{2} + \sqrt{2at + t^2/4}\right) < 1 \ \forall a$ – which proves the first claim. To prove the second claim, write the advertising intensities as; $\phi^{ac} = \frac{1}{\frac{2a}{t+2\sqrt{2at+t^2/4}}+1}$ and (from (18)) $\phi^{pc} = \frac{2(v-t)}{2a+v-t} = \frac{1}{1+\frac{2a}{2(v-t)}-\frac{1}{2}}$. First note that as a converges to $\left(\frac{v-t}{2}\right)^{+}$, $\phi^{pc} = 1 > \phi^{ac}$. Notice also that $\phi^{pc}(\overline{a}) > \phi^{ac}(\overline{a})$ if and only if $\frac{2\overline{a}}{2(v-t)} - \frac{1}{2} < \frac{2\overline{a}}{t+2\sqrt{2\overline{a}t+t^2/4}}$. Substituting $\overline{a} = \frac{(v-2t)(v-t)}{2t}$, we get; $\frac{2\overline{a}}{2(v-t)} - \frac{1}{2} = \frac{v-2t}{2t} - \frac{1}{2} < \frac{v-2t}{2t} = \frac{2\overline{a}}{t+2\sqrt{2\overline{a}t+t^2/4}}$. It follows therefore that $\phi^{pc}(\overline{a}) > \phi^{ac}(\overline{a})$. Second, because both ϕ^{pc} and ϕ^{ac} are everywhere continuous, if ϕ^{pc} and ϕ^{ac} never cross (intersect) for $a \in (t/2, \overline{a})$, then it must be the case that ϕ^{pc} lies everywhere above ϕ^{ac}. From the above expressions for ϕ^{pc} and ϕ^{ac}, it is obvious that at the point where they intersect, $\frac{2a}{2(v-t)} - \frac{1}{2} = \frac{2a}{t+2\sqrt{2at+t^2/4}}$. The only solution to the equation $\frac{2a}{2(v-t)} - \frac{1}{2} - \frac{2a}{t+2\sqrt{2at+t^2/4}} = 0$ is $a^* = \frac{v(v-t)}{2t}$. However, $a^* > \overline{a}$. Hence, in the interval $(t/2, \overline{a})$, ϕ^{pc} and ϕ^{ac} do not intersect. It follows therefore that, in the interval $(t/2, \overline{a})$, $\phi^{pc} > \phi^{ac}$. □

Proof of Lemma 4

PROOF. Lemma 4 claims that $\frac{\partial \pi^{ac}}{\partial a} > 0$ and $\frac{\partial \pi^{pc}}{\partial a} < 0\ \forall a$. Under collusion on advertising, profits are given by; $\pi^{ac} = \frac{1}{2}\frac{\left(\frac{t}{2}+\sqrt{2at+t^2/4}\right)^2}{a+\frac{t}{2}+\sqrt{2at+t^2/4}}$ and $\frac{\partial \pi^{ac}}{\partial a} = \frac{t\left(t+2\sqrt{2at+t^2/4}\right)}{\sqrt{2at+t^2/4}\left(a+\frac{1}{2}t+\sqrt{2at+t^2/4}\right)} - \frac{t+\sqrt{2at+t^2/4}}{\sqrt{2at+t^2/4}}\frac{\left(\frac{1}{2}t+\sqrt{2at+t^2/4}\right)^2}{\left(a+\frac{1}{2}t+\sqrt{2at+t^2/4}\right)^2} = \frac{\frac{1}{4}t^3+at^2+\frac{1}{2}t^2\sqrt{2at+t^2/4}}{\sqrt{2at+t^2/4}\left(a+\frac{1}{2}t+\sqrt{2at+t^2/4}\right)^2} > 0$ as required. As for price collusion, $\pi^{pc} = \frac{v-t-a}{2}$ for $a \leq \frac{v-t}{2}$ and $\frac{\partial \pi^{pc}}{\partial a} = -\frac{1}{2} < 0$ as required. Secondly, for $a > \frac{v-t}{2}$, $\pi^{pc} = \frac{2a(v-t)^2}{(2a+v-t)^2}$ and $\frac{\partial \pi^{pc}}{\partial a} = \frac{2(v-t)^2}{(2a+v-t)^2} + \frac{2a(v-t)^2(-2)2}{(2a+v-t)^3} = \frac{2(v-t)^2}{(2a+v-t)^3}(v-t-2a) < 0$ since $a > \frac{v-t}{2}$. Hence the lemma. □

Proof of Proposition 4

PROOF. Whenever $\widehat{a}$ exists, the result follows directly from Lemma 4. Hence we only need to show that $\widehat{a}$ indeed exists $\forall \alpha \in \left(\underline{\alpha}, \frac{1}{3}\right)$, for some $\underline{\alpha} > 0$.[29] Notice that for any given α, a is constrained to the interval $(\underline{a}(\alpha), \overline{a}(\alpha))$, where $\underline{a}(\alpha) = \frac{\alpha}{2}v$ and $\overline{a}(\alpha) = \frac{(1-2\alpha)(1-\alpha)}{2\alpha}v$. Since (by Lemma 4) $\pi^{pc}(a)$ is continuous and decreasing in a and $\pi^{ac}(a)$ is continuous and increasing in a, to show that $\widehat{a}$ exists, it suffices to show that $\lim_{a\to\underline{a}(\alpha)^+}\pi^{pc}(a) \geq \lim_{a\to\underline{a}(\alpha)^+}\pi^{ac}(a)$ and $\lim_{a\to\overline{a}(\alpha)}\pi^{pc}(a) \leq \lim_{a\to\overline{a}(\alpha)}\pi^{ac}(a)\,\forall\alpha \in \left(\underline{\alpha}, \frac{1}{3}\right)$. Substituting αv for t, the profit functions (14) and (18) reduce to $\pi^{ac} = \frac{\left(\alpha v/2+\sqrt{2a\alpha v+(\alpha v)^2/4}\right)^2}{2\left(a+\alpha v/2+\sqrt{2a\alpha v+(\alpha v)^2/4}\right)}$ and $\pi^{pc} = \frac{(1-\alpha)v-a}{2}$ for $a \leq \frac{(1-\alpha)v}{2}$ and $\pi^{pc} = \frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2}$ for $a > \frac{(1-\alpha)v}{2}$. To begin with, notice that $\lim_{a\to\left(\frac{\alpha v}{2}\right)^+}\left(\frac{(1-\alpha)v-a}{2}\right) = \left(\frac{1}{2}-\frac{3}{4}\alpha\right)v > \alpha(\alpha-1)^2 v = \lim_{a\to\left(\frac{\alpha v}{2}\right)^+}\left(\frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2}\right)\forall\alpha \in \left(0, \frac{1}{2}\right)$. Hence, the relevant part of π^{pc} as

[29] The restriction that $\alpha < 1/3$ is a direct consequence of the full market coverage assumption. Under low differentiation, $t < v/2$, which implies $\alpha < 1/2$. However, internal consistencies (within the model) impute further restrictions on α. Because $\overline{a} = \frac{(v-2t)(v-t)}{2t}$ and a also takes values greater than $\frac{v-t}{2}$ (see equation (18)), consistency requires that $\frac{v-t}{2} < \overline{a}$. Simplifying this inequality gives the upper bound to the ratio t/v. More precisely, $\alpha \equiv \frac{t}{v} < 1/3$.

$a \to \underline{a}(\alpha)$ is $\pi^{pc} = \frac{(1-\alpha)v-a}{2}$. Notice also that $\lim_{a\to\frac{(1-2\alpha)(1-\alpha)v}{2\alpha}} \left(\frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2}\right) = \frac{2\alpha^2-\alpha}{\alpha-1}v > \frac{(1-\alpha)(4\alpha-1)v}{4\alpha} = \lim_{a\to\frac{(1-2\alpha)(1-\alpha)v}{2\alpha}} \left(\frac{(1-\alpha)v-a}{2}\right) \forall\alpha \in \left(0, \frac{1}{3}\right)$. Hence, the relevant part of π^{pc} as $a \to \overline{a}(\alpha)$ is $\pi^{pc} = \frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2}$. First, $\lim_{a\to\left(\frac{\alpha v}{2}\right)^+} \pi^{pc}(a) - \lim_{a\to\left(\frac{\alpha v}{2}\right)^+} \pi^{ac}(a) = \frac{2-\left(\sqrt{20}+1\right)\alpha}{4}v > 0$ for $\alpha \in (0, 0.365)$. Since for our purposes $\alpha < 1/3$, we conclude that $\lim_{a\to\underline{a}(\alpha)^+} \pi^{pc}(a) > \lim_{a\to\underline{a}(\alpha)^+} \pi^{ac}(a) \forall\alpha \in \left(\underline{\alpha}, \frac{1}{3}\right)$ as required. Second, $\lim_{a\to\overline{a}(\alpha)} \pi^{ac}(a) - \lim_{a\to\overline{a}(\alpha)} \pi^{pc}(a) = \left(\frac{\alpha\left(\alpha+\sqrt{(3\alpha-2)^2}\right)^2}{4(3\alpha^2+1-3\alpha)+\alpha\sqrt{2(3\alpha-2)^2}} - \frac{2\alpha^2-\alpha}{\alpha-1}\right)v > 0 \forall\alpha \in (0,1)$. We conclude that $\lim_{a\to\overline{a}(\alpha)} \pi^{pc}(a) < \lim_{a\to\overline{a}(\alpha)} \pi^{ac}(a) \forall\alpha \in \left(\underline{\alpha}, \frac{1}{3}\right)$. In summary, for $\alpha \in \left(\underline{\alpha}, \frac{1}{3}\right)$, $\lim_{a\to\underline{a}(\alpha)^+} \pi^{pc}(a) > \lim_{a\to\underline{a}(\alpha)^+} \pi^{ac}(a)$ and $\lim_{a\to\overline{a}(\alpha)} \pi^{pc}(a) < \lim_{a\to\overline{a}(\alpha)} \pi^{ac}(a)$. Hence, $\pi^{pc}(a)$ and $\pi^{ac}(a)$ intersect. □

Proof of Proposition 5

PROOF. Proposition 5 claims that $\pi^{ac} > \pi^{nc}$ and $\pi^{pc} > \pi^{nc}$, where $\pi^{pc} = \frac{v-t-a}{2}$ for $a \leq \frac{v-t}{2}$ and $\pi^{pc} = \frac{2a(v-t)^2}{(2a+v-t)^2}$ for $a > \frac{v-t}{2}$ while $\pi^{nc} = \frac{2at^2}{\left(t+\sqrt{2at}\right)^2}$ $\forall a \in (t/2, \overline{a})$. The first claim is covered under Proposition 1. Hence, here we only prove the second claim. The idea of the proof is to show that π^{pc} lies everywhere above π^{nc} in the interval $(t/2, \overline{a})$. To proceed, observe that $\frac{\partial\pi^{nc}}{\partial a} = \frac{2t^3}{\left(t+\sqrt{2at}\right)^3} > 0$ and that $\pi^{pc}(a)$ is continuous and decreasing (Lemma 4). Therefore, to show that $\pi^{pc} > \pi^{nc}$ for all $a \in (t/2, \overline{a})$, it suffices to show that $\pi^{pc}(\overline{a}) > \pi^{nc}(\overline{a})$. Noting that $\pi^{pc} = \frac{2a(v-t)^2}{(2a+v-t)^2}$ for $a > \frac{v-t}{2}$, evaluating at $\overline{a} = \frac{(v-2t)(v-t)}{2t}$ gives $\pi^{pc}(\overline{a}) = \frac{t(v-2t)}{v-t}$ and $\pi^{nc}(\overline{a}) = \frac{t(v-2t)(v-t)}{\left(t+\sqrt{(v-2t)(v-t)}\right)^2}$. Subtracting the latter from the former gives, $\pi^{pc}(\overline{a}) - \pi^{nc}(\overline{a}) = \frac{t^2(v-2t)\left(2t-v+2\sqrt{(v-2t)(v-t)}\right)}{(v-t)\left(t+\sqrt{(v-2t)(v-t)}\right)^2} > 0$ if and only if $2t - v + 2\sqrt{(v-2t)(v-t)} > 0$. Since $\sqrt{(v-2t)(v-t)} > v - 2t$, it follows that $2t^2 - tv + 2t\sqrt{(v-2t)(v-t)} > 0$. Hence, $\pi^{pc}(\overline{a}) > \pi^{nc}(\overline{a})$. We conclude therefore that $\pi^{pc} > \pi^{nc}$ for all $a \in (t/2, \overline{a})$. □

Appendix B: Unconstrained Collusive Price and Profits

As we mentioned before, it is possible (and plausible) that the firms may collude on a "high" price – a price which may induce some consumers not to purchase. The objective of this appendix is to show that assuming full coverage is not very restrictive.

Suppose then that firms collude on a price that induces some consumers not to purchase. Then, for such a price, firm i's demand is given by[30]:

$$\text{(B1)} \qquad D_i(\phi_1, \phi_2; p) = \phi_i (1 - \phi_j) \frac{v - p}{t} + \frac{\phi_i \phi_j}{2}; i, j = 1, 2; j \neq i,$$

where $(v - p)/t < 1$ denotes the purchase probability by a consumer who receives the advertising message from only one of the firms (partially informed). Because firms charge the same price, the purchase decision of the fully informed consumers is not governed by prices (as long as the price does not exceed the reservation price). That is, independent of the price, firms equally split (between them) the population of fully informed consumers.

As before, firms set prices in the first stage and advertising in the second stage. We start by solving for the optimal advertising level in the second stage. Given the collusive price, p, chosen in the first stage, firm i's second stage maximization program is given by:

$$\text{(B2)} \qquad \pi_i^{pc} = \max_{\phi_i} \left\{ p \left(\phi_i (1 - \phi_j) \frac{v - p}{t} + \frac{\phi_i \phi_j}{2} \right) - \frac{a \phi_i^2}{2} \right\}.$$

Differentiating with respect to ϕ_i and solving for a symmetric equilibrium gives,

$$\text{(B3)} \qquad \phi = \frac{2pv - 2p^2}{2at - pt + 2pv - 2p^2}$$

In the first stage, firms choose the collusive price to maximize joint profits. Their problem is described by,

$$\prod^{pc} = \pi_1^{pc} + \pi_2^{pc} = \max_p \left\{ 2p \left(\phi (1 - \phi) \frac{v - p}{t} + \frac{\phi^2}{2} \right) - a \phi^2 \right\} \text{ s.t. (B3).}$$

Substituting for ϕ from (B3), this simplifies to

$$\text{(B4)} \qquad \prod^{pc} = \frac{4p^2 a (v - p)^2}{(2at - pt + 2pv - 2p^2)^2}$$

Differentiating with respect to p and solving the first order condition yields,

$$p \in \left\{ v, 2a + \sqrt{2a(2a - v)}, 2a - \sqrt{2a(2a - v)} \right\}.$$

[30] The demand function in (B1) is valid **only** when the collusive price exceeds $v - t$. If the collusive price is less than or equal to $v - t$, then $(v - p)/t = 1$ and the demand is independent of prices.

Since consumers' reservation price is v, prices higher than v cannot be optimal. Hence, $p \leq v$. But, can firms collude on the price $p = v$? The answer is no! At the price $p = v$, each firm has demand zero and profits can be increased by lowering the price[31]. Therefore we rule out $p = v$. Hence

$$p \in \left\{2a + \sqrt{2a\,(2a - v)}, 2a - \sqrt{2a\,(2a - v)}\right\}$$

Clearly, for p to be an equilibrium, we must have $a \geq v/2$. Suppose first that $a = v/2$. Then, $p = 2a = v$. But, $p = v$ is impossible. Hence, $a > v/2$. But, if $a > v/2$, then, $2a + \sqrt{2a\,(2a - v)} > v$, and hence cannot be an equilibrium collusive price. This leaves

$$p_{ur}^{pc} = 2a - \sqrt{2a\,(2a - v)} \tag{B5}$$

as the collusive price, where the subscript ur stands for unrestricted. Observe that $p_{ur}^{pc}(a)$ is decreasing in a.[32]

Substituting (B5) back into (B3) gives

$$\phi_{ur}^{pc} = \frac{8av - 16a^2 + (8a - 2v)\sqrt{a\,(4a - 2v)}}{8av - 16a^2 + (8a + t - 2v)\sqrt{a\,(4a - 2v)}} < 1. \tag{B6}$$

Note that $\lim_{a\to\infty} p_{ur}^{pc} = v/2$. In fact, p_{ur}^{pc} quickly converges to $v/2$. For example, for $a = 8v$, $p_{ur}^{pc} = 0.508\,07v$. Since $\lim_{a\to v/2} p_{ur}^{pc}(a) = v$ and since p_{ur}^{pc} is decreasing in a and intersects p_r^{pc} (subscript r stands for restricted (we are abusing notation here) – this is the case studied in the main text), there exists a^* such that $p_{ur}^{pc}(a^*)$ ensures that the market is just fully covered[33]. That is, the equation $2a - \sqrt{2a\,(2a - v)} - (v - t) = 0$ has a solution and this solution is given by,

$$a^* = \frac{1}{2v - 4t}\left(t^2 - 2tv + v^2\right) \tag{B7}$$

REMARK 1. *$p_{ur}^{pc} > p_r^{pc}$ for $a \in (v/2, a^*)$ and $p_{ur}^{pc} < p_r^{pc}$ for $a > a^*$. Because $\lim_{t\to 0} a^* = v/2$ it follows that $\lim_{t\to 0}(p_{ur}^{pc} - p_r^{pc}) = 0$. That is, when t is small, the restricted collusive price is close to the unrestricted price. This shows that the full coverage assumption does not constrain the collusive price "too much".*

[31] Note that because consumers are distributed according to a continuous density, the probability that an ad sent by firm 1 (firm 2) will reach a consumer located at point 0 (1) is zero. Because it is costly to visit a store, consumers other than the ones located at 0 and 1 will not purchase at price $p = v$ even if they are informed.

[32] $\partial p_{ur}^{pc}/\partial a = \left(2\sqrt{2a\,(2a - v)} - (4a - v)\right)/\sqrt{a\,(2a - v)} < 0$ if and only if $2\sqrt{2a\,(2a - v)} < (4a - v)$. Squaring both sides and simplifying we get; $16a^2 - 8av = \left(2\sqrt{a\,(4a - 2v)}\right)^2 < (4a - v)^2 = 16a^2 - 8av + v^2$, as required. Hence, $\partial p/\partial a < 0$.

[33] That p_{ur}^{pc} and p_r^{pc} intersect follows from the fact that p_{ur}^{pc} converges to $v/2$, together with the condition for low differentiation ($t < v/2$). The condition $t < v/2$ implies that $p_r^{pc} = v - t > v/2 = \lim_{a\to\infty} p_{ur}^{pc}$. Hence, p_{ur}^{pc} and p_r^{pc} intersect.

We next derive the firms' optimal profits. The objective is to show that the unrestricted profits do not differ much from the "restricted" profits. As we saw above, for $a \leq a^*$, the collusive price is given by $p_{ur}^{pc} = 2a - \sqrt{2a(2a-v)}$ and the collusive profits are given by

$$\pi_{ur}^{pc} = p_{ur}^{pc}\left(\phi(1-\phi)\frac{v-p_{ur}^{pc}}{t} + \frac{\phi^2}{2}\right) - \frac{a\phi^2}{2} \tag{B8}$$

where ϕ is given by (B6). Observe that for $a \in (v/2, a^*)$, the market is not covered. That is, $\frac{v-p_{ur}^{pc}}{t} < 1$.

We claim that; For $a > a^*$, $\pi^{pc} = p_r^{pc} D(\phi_r^{pc}) - a\frac{(\phi_r^{pc})^2}{2}$, where $D(\phi_r^{pc}) = \phi_r^{pc} - (\phi_r^{pc})^2/2$. This is the case studied in section 4.

Observe that for $a > a^*$, $p_{ur}^{pc} < p_r^{pc}$ and the market is covered. If the firm charges the collusive price p_{ur}^{pc}, profits are given by $\pi_{ur}^{fullcov} = p_{ur}^{pc} D(\phi_{ur}^{pc}) - a\frac{(\phi_{ur}^{pc})^2}{2}$, where $D(\phi_{ur}^{pc}) = \phi_{ur}^{pc} - (\phi_{ur}^{pc})^2/2$. Since the market is covered and $p_{ur}^{pc} < p_r^{pc}$, it follows that $\pi_{ur}^{fullcov}$ is strictly dominated by $\pi^{naive} \equiv p_r^{pc} D(\phi_{ur}^{pc}) - a\frac{(\phi_{ur}^{pc})^2}{2}$. Hence, for $a > a^*$, firms cannot collude on price p_{ur}^{pc}. We next show that π^{naive} can be improved upon. Notice that at price p_r^{pc}, ϕ_{ur}^{pc} is not optimal. At this price, the optimal advertising level is given by ϕ_r^{pc}. It follows therefore that π^{naive} is dominated by $\pi_r^{pc} \equiv p_r^{pc} D(\phi_r^{pc}) - a\frac{(\phi_r^{pc})^2}{2}$. Hence the claim.

To summarize, let (with some abuse of notation), π^{pc} denote the optimal collusive profits when there are no a priori restrictions on the collusive price. Then, $\pi^{pc} = \pi_{ur}^{pc}$ for $a \in (v/2, a^*)$ and $\pi^{pc} = \pi_r^{pc}$ for $a > a^*$.

REMARK 2. $a^* - v/2 = \frac{t^2}{2(v-2t)}$. *We see that when t is small, the range of a over which π_{ur}^{pc} is relevant is very "narrow" and moreover, this range diminishes as t converges to zero. Hence, when t is small, π_{ur}^{pc} is largely irrelevant and $\pi^{pc} \approx \pi_r^{pc}$. That is, when t is small, assuming full coverage is not very restrictive.*

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PAPER 2

Informative Advertising when Only Some Goods are Advertised

Witness Simbanegavi

ABSTRACT. We study how price advertising of a subset of products affects equilibrium pricing and advertising under two product differentiation regimes. We find that, when firms sell products with the same reservation price, loss-leader pricing obtains only when differentiation is low. However, when reservation prices differ, equilibrium may entail loss-leader pricing even when differentiation is high. This enables us to shed some light on the seemingly *paradoxical* empirical findings in the marketing literature that loss-leader pricing fails to increase store traffic, loss-leader sales and hence to increase profits. We also examine welfare implications.

1. Introduction

Firms spend a fortune on advertising and this makes advertising such an important strategic variable in the firm's "toolbox". For example, in the first nine months of 2004, Time Warner Inc. spent a whopping $1.345 billion on advertising, Daimler Chrysler AG spent $1.219 billion while Ford Motor Company spent $1.083 billion advertising cars and trucks and Johnson & Johnson spent $0.916 billion on advertising (Marketing Today, July 2005)[1]. Given such huge expenditures on advertising, several questions come to mind, for instance; Why do firms advertise so much? Does advertising lead to higher or lower prices? What are the implications for welfare? The principal focus of this paper is the relation between advertising and prices.

This question – of the relation between advertising and prices – is of course not new. A large number of studies find that, in retail markets, prices are lower when firms advertise (Benham, 1972; Kwoka, 1984). It is, however, well established that many firms advertise only a subset of the products they sell – a supermarket, for instance,

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[1] See also Bagwell (2003; p. 1).

advertises a handful of prices but sells hundreds of products[2]. What is the effect on prices of advertising only a subset of products? Milyo and Waldfogel (1999)'s empirical study of the effect of price advertising on prices suggests that prices of unadvertised products are higher when advertising is allowed. Theoretically, little is known. Indeed, in his extensive survey, Bagwell (2003; p. 51) writes, "Recent work, however, suggests that the distinction between the effect of advertising on the prices of advertised and unadvertised products warrants greater attention".

The goal of this paper is to deepen our understanding of the effects of informative advertising on prices, profits and welfare when firms advertise only a subset of their products. Toward this end, we study price advertising under two mutually exclusive differentiation regimes, namely, "low" and "high" differentiation. We consider two firms, each selling two products but only advertising a single product. Advertising messages are randomly distributed over consumers, who are assumed to be uninformed about prices and firm locations unless they are reached by advertising.

We show that the existence of loss-leader pricing crucially depends on how strong competition between the firms is[3]. In particular, for sufficiently low distance (differentiation) between firms, the advertised good is priced below cost, otherwise the advertised good is priced above cost. By emphasizing the role of the degree of product differentiation, we shed some light on the seemingly paradoxical findings of Walters and MacKenzie (1988) of a "weak" link between loss leader pricing and store traffic and profits. With regard to welfare, we find that the market level of advertising can be excessive or too low depending on parameter values. In particular, for low advertising costs, the market determined advertising level is excessive. We find also that an increase in the advertising cost lowers welfare and the welfare decline is larger the larger the number of products offered.

The paper is organized as follows. Section 2 points out how our work relates to the existing literature and section 3 sets out the model. We analyze the model in section 4, and in section 5 we examine welfare implications. Section 6 extends the model to the case where reservation prices differ and section 7 concludes the paper.

[2] We will not discuss the rationale for advertising only a subset of the product range. We take it as a given that firms advertise only a subset of their products. Ellison (2005; pages 607-611) discusses some of the reasons why firms may want to advertise only a subset of their products. See also Lal and Matutes (1994).

[3] A good is termed a loss leader if it is priced below cost as a deliberate ploy to attract consumers to the advertising store.

2. Related Literature

Economic researchers have devoted a significant amount of effort on trying to understand the price effects of informative advertising when firms sell differentiated products. A seminal paper is Grossman and Shapiro (1984). They consider a model where single product firms sell horizontally differentiated products, consumers are uniformly distributed and do not actively search and product differentiation is low (market fully covered). They show that higher advertising is associated with lower prices. However, since in some cases the degree of product differentiation may be so high that some informed consumers find it optimal not to purchase, the question arose as to the nature of this relationship when markets are not fully covered. Soberman (2004) addresses this question. He extends the Grossman and Shapiro model to include the case of high differentiation[4]. He finds that when differentiation is high, higher advertising is associated with higher (not lower) prices.

The Grossman and Shapiro – Soberman model has limitations however. Many firms sell several products but only advertise a subset. This raises the question of how the advertising of only a subset of their assortments affects the prices of advertised and unadvertised products.

Lal and Matutes (1994) make some strides in addressing this question. They study a model where multiproduct firms advertise in order to inform consumers of their offerings. A firm that advertises reaches all consumers in the market. They show that equilibrium pricing *may* be characterized by loss-leader pricing. Closely related is Ellison, (2005) who studies a vertically differentiated goods model in which firms only advertise the low quality good. Since firms sell a vertically differentiated good, consumers either buy the low quality good or the high quality good but not both. Whether the low quality good is sold below cost or not depends on parameter values and he shows that advertising only a subset of products results in higher equilibrium profits.

However, neither Lal and Matutes (1994) nor Ellison (2005) explicitly model the advertising decision. In reality, advertising is an important strategic tool. Not only do firms determine the prices they charge, they also determine how much to advertise. Modelling the advertising decision allows us to study the interaction between advertising and prices. Moreover, both Lal and Matutes (1994) and Ellison (2005) only consider the case where the market is fully covered. However, as we saw above, for some constellations of the differentiation parameter, some informed consumers may find it profitable not to purchase.

[4] In a similar model, Hamilton (2004) examines the welfare implications of informative advertising.

We extend the Lal and Matutes model in two dimensions. First, we explicitly model firms' advertising decisions and in so doing allow for situations where the market is not fully informed about the advertised prices. Second, we study firms' pricing and advertising strategies when differentiation is high.

Another paper that is closely related to ours is Bagwell and Ramey (1994). In a model in which the marginal costs (and hence prices) are decreasing in store traffic, they show that "ostensibly uninformative" advertising may bring about (and hence enable firms and consumers to benefit from) coordination economies. Such advertising helps direct consumers toward the firms that offer "best deals", that is, firms that offer lowest prices. As a result, there is a negative association between advertising and prices. We differ with Bagwell and Ramey in that in our model, advertising is informative whereas in theirs, it is not. Because advertising does not contain price information in their framework, there is no possibility for undercutting a competitor. In essence, there is no price competition in their model. Another difference is that in the present paper, firms advertise only a subset of the products they sell while in Bagwell and Ramey firms neither advertise the prices nor the products they sell. Rather, firms advertise for instance, their size. Also, in Bagwell and Ramey, product differentiation is unimportant.

Our contribution is to show that the level of differentiation between competing firms is important for understanding pricing and advertising strategies of multiproduct firms. In particular, we have loss-leader pricing only when differentiation is low. We also provide the exact conditions (on the level of differentiation) where the pricing strategy changes from prices above cost to loss-leader pricing.

3. Model and Preliminaries

3.1. Model. Our model is an extension of the Grossman and Shapiro (1984) model to multiproduct firms. We consider a linear city of unit length served by two firms, 1 and 2, located at points 0 and 1 respectively. Each firm sells two independent products, product 1 and product 2. However, firm 1 and firm 2's products are differentiated[5]. Firms advertise only a subset of their products. Both firms advertise the same good (good 1, for instance) and advertising is truthful. Following Butters (1977), advertising is modelled as a series of messages that are randomly sent to consumers. Let ϕ_i denote the advertising intensity of firm $i; i = 1, 2$ (fraction of the consumer population that is exposed, at least once, to the advertising message (ad) of firm i). The cost of reaching fraction ϕ_i of consumers is denoted $A(\phi_i)$, where $A' > 0$ and $A'' > 0$.[6] For what

[5] Firm 1's good k is an imperfect substitute for firm 2's good $k; k = 1, 2$.

[6] In a Butters type model, decreasing returns to scale in advertising are natural. When the advertising intensity is already high, an increasing number of advertising messages must be sent in

follows, let $A(\phi_i) = a\phi_i^2/2; a > t/2$. Each good is produced at a constant marginal cost, c, and firms simultaneously and non-cooperatively choose prices and advertising intensities to maximize profits. There is no entry or exit and there are no fixed costs.

Consumers are uniformly distributed on $[0, 1]$. That is, each consumer is identified by a point on the unit interval that corresponds to her most preferred brand. Consumers are uninformed about prices and firm locations unless they are reached by advertising. Each informed consumer buys at most one unit of each product and uninformed consumers stay out of the market. A unit of each good generates gross surplus of v and consumers incur a shopping cost (independent of the number of products purchased in one store) of t per unit distance. Whenever they find it profitable to purchase, partially informed consumers buy both goods from the same store. This, however, is not obvious with fully informed consumers[7].

3.2. Preliminaries. Given the firms' advertising intensities, ϕ_1 and ϕ_2, the market is delineated as follows: Fraction $\phi_1\phi_2$ of consumers receive advertising messages from both firms 1 and 2 and are thus fully informed. Fraction $\phi_i(1-\phi_j); i, j = 1, 2; j \neq i$ receive ads from firm i but not firm j and hence are partially informed. Fraction $(1-\phi_1)(1-\phi_2)$ receive no ads from either firm and are thus uninformed. We assume that $\phi_1\phi_2$ is large enough so that firms find it worthwhile to compete for the fully informed consumers[8].

Let p_{11} and p_{21} denote firm 1 and respectively, firm 2's advertised prices and let p_{12}^E and p_{22}^E denote the expected prices of the unadvertised products (The first subscript denotes the firm while the second denotes the product). There are three possible configurations of the expected prices. Either $p_{12}^E = p_{22}^E$, $p_{12}^E < p_{22}^E$ or $p_{12}^E > p_{22}^E$. In the second and third configurations, the analysis gets complicated (see Appendix C.). For what follows, we suppose consumer expectations are such that $p_{12}^E = p_{22}^E$. Lemmas 1 and 2 below greatly simplify the construction and respectively, structure of the demand functions.

LEMMA 1. *If $p_{12}^E = p_{22}^E$, fully informed consumers will buy both goods from a single firm.*

order to reach a consumer that hitherto has not been exposed to advertising. Thus the marginal cost of reaching one more *uninformed* consumer increases with the advertising intensity. Indeed, empirical studies have found evidence consistent with decreasing returns to scale in advertising (Bagwell, 2003; p. 34).

[7] We get around this problem by assuming that consumers expect firms to charge the same price for the unadvertised product. This effectively rules out shopping at both stores.

[8] This assumption ensures that charging a high price and only selling to captive consumers is not profitable. Christou and Vettas (2003) argue that when this assumption is not satisfied, a symmetric equilibrium may not exist. In Appendix D, we provide a necessary condition for firms to compete for the fully informed consumers.

PROOF. Let $p_{12}^E = p_{22}^E$. Suppose, without loss of generality, that $p_{11} \leq p_{21}$. Then, either $p_{11} = p_{21}$ or $p_{11} < p_{21}$. To start with, let $p_{11} = p_{21}$. Clearly, a consumer who buys at both stores incurs unnecessary transportation costs. Hence consumers buy both goods at the same store. Now suppose $p_{11} < p_{21}$. Suppose a consumer at $x \in (0,1)$ plans to buy good 1 from firm 1 and good 2 from firm 2. Then, she incurs total transportation cost of $tx + t(1-x) = t$ travelling to firms 1 and 2 respectively. However, if she buys both goods at firm 1, her total transportation cost is $tx < t$. It follows therefore that the consumer at x will buy both goods from firm 1 rather than buying from both firms. This holds for all $x \in (0,1)$. Hence, no consumer will buy from both firms □

Lemma 1 is intuitive. If it is costly to visit a store, a necessary condition for consumers to shop around is that each firm quotes the lowest price in only one of the products. That is, either firm 1 is the cheapest supplier of good 1 and firm 2 is the cheapest supplier of good 2 or vice versa.

Fully informed consumers purchase from whichever firm gives them the greatest surplus. A consumer located at $x \in (0,1)$ gets surplus $2v - p_{11} - p_{12}^E - tx$ buying from firm 1 and surplus $2v - p_{21} - p_{22}^E - t(1-x)$ buying from firm 2. Let $\widehat{x}$ denote the location of the consumer who is equally well-off buying from either firm. Then $\widehat{x} = (p_{21} - p_{11} + t)/2t$.[9] It follows that all fully informed consumers with locations $x \in [0, \widehat{x})$ buy from firm 1 while those with locations $x \in (\widehat{x}, 1]$ buy from firm 2. Thus, firm i faces demand $\widehat{x}$ from the fully informed consumers.

For partially informed consumers, demand is determined (only) by individual rationality. Let y_i denote the location of a consumer who receives only firm i's ad(s). Buying yields surplus $2v - p_{i1} - p_{i2}^E - ty_i$ while not buying yields surplus zero. Denote by $\widetilde{y}_i$ the consumer for whom the individual rationality constraint just binds. Then $\widetilde{y}_i = \left(2v - p_{i1} - p_{i2}^E\right)/t$ and all partially informed consumers with locations $y_i < \widetilde{y}_i$ will make a purchase while those with locations $y_i > \widetilde{y}_i$ will not[10]. Thus firm i's demand from partially informed consumers is $\widetilde{y}_i$. Hence, the firm faces the demand:

$$D_i = \phi_i \left((1-\phi_j) \frac{2v - p_{i1} - p_{i2}^E}{t} + \phi_j \frac{p_{j1} - p_{i1} + t}{2t} \right); i \neq j. \tag{1}$$

To simplify the demand functions even further, we make the following "intuitively obvious" assumption. Let $\overline{p} = p_{12} = p_{22}$ be the equilibrium price of the unadvertised good and p_{11} and p_{21}, as we saw earlier, are respectively, firm 1 and firm 2's advertised prices.

Assumption A1: $\overline{p} \geq \max\{p_{11}, p_{21}\}$.

[9] Since $p_{12}^E = p_{22}^E$.

[10] However, if $2v - p_{i1} - p_{i2}^E > t$, all consumers who receive at least one ad from firm i will make a purchase, that is, $\widetilde{y}_i = 1$.

Assumption A1 simply says that the unadvertised price cannot be lower than the advertised price. If it were, the firm would be better off advertising that price instead, since consumers' visitation decisions are predicated on the advertised price. An immediate consequence of Assumption A1 is that;

LEMMA 2. *If consumers expect firms to charge the same price for the unadvertised good, then, in equilibrium, firms will charge the reservation price. That is, if* $p_{12}^E = p_{22}^E$, *then* $\overline{p} = v$.

PROOF. Let $\overline{p} = p_{12} = p_{22}$ be the equilibrium profit maximizing price of the unadvertised good. Suppose, contrary to the lemma, that $\overline{p} \neq v$. Then either $\overline{p} < v$ or $\overline{p} > v$. To start with, suppose $\overline{p} < v$. Suppose also that price is a continuous variable. Then, $\exists \varepsilon > 0 : p = \overline{p} + \varepsilon < v$. Since consumers have unit demands, the demand that each firm faces is independent of the price of good 2 (the unadvertised price). Therefore, for any firm i with advertised price p_{i1} and for any $\varepsilon > 0$, $\pi_i(p_{i1}, \overline{p} + \varepsilon) > \pi_i(p_{i1}, \overline{p})$. In other words, $\overline{p}$ is not profit maximizing – a contradiction. Hence, we must have $\overline{p} > v$. However, for $\overline{p} > v$, visiting consumers only buy the advertised product. In fact, for any $\varepsilon > 0$, $\pi_i(p_{i1}, v + \varepsilon) < \pi_i(p_{i1}, p)$, for any $p \in (\max\{p_{11}, p_{21}\}, v]$. Hence, $\overline{p} > v$ is not profit maximizing – a contradiction. Hence, it must be the case that $\overline{p} = v$. □

What Lemma 2 says is the following; If consumers expect firms to charge the same price for the unadvertised good, then, the only equilibrium price that satisfies those expectations is $\overline{p} = v$. Given Lemma 2, firm i's demand reduces to

$$D_i = \phi_i \left((1 - \phi_j) \frac{v - p_{i1}}{t} + \phi_j \frac{p_{j1} - p_{i1} + t}{2t} \right); i \neq j. \tag{1'}$$

The literature on informative advertising has focused on single product firms and /or low differentiation. In light of this, we label the above model the "Standard" model (S) when firms each sell a single product and differentiation is low. When instead, firms each sell two products, we label the model "Regime L" (L) when differentiation is low and "Regime H" (H) when differentiation is high.

4. Analysis

This section is divided into three subsections. In subsection 4.1, we study the case of low differentiation – where we jointly analyze the Standard model and regime L. To this effect, we let r denote the regime (S or L) and let I be an indicator variable such that $I = 0$ if $r = S$ and $I = 1$ if $r = L$. In subsection 4.2, we study the case of high differentiation and in subsection 4.3, we give a summary of our main findings.

4.1. Low Differentiation. When $v - p_{i1} \geq t$, all consumers who receive at least one ad make a purchase. In particular, the partially informed consumer who travels the longest distance (the whole unit interval) gets non-negative surplus[11]. Thus, $\widetilde{y}_i \equiv \frac{v-p_{i1}}{t} = 1$. We make the following (technical) feasibility assumption.

Assumption A2: $c + \sqrt{2at} < v$.

Assumption A2 is needed to ensure that, in equilibrium, consumers visit the stores in the single product case. If the equilibrium price is greater than or equal to v, visiting consumers get negative surplus since they incur some positive transportation costs and hence no consumer would visit a store[12].

Let p_{21} and ϕ_2 be the advertised price and respectively, advertising level chosen by firm 2. Since $p_{12} = v$, firm 1's behaviour is described by

$$\pi_1^r = \max_{p_{11},\phi_1} \left(p_{11} - c + I\left(v - c\right)\right) \phi_1 \left(1 - \phi_2 + \phi_2 \frac{p_{21} - p_{11} + t}{2t}\right) - \frac{a\phi_1^2}{2}. \tag{2}$$

The first order conditions, evaluated at the symmetric equilibrium: p^r, ϕ^r are;

$$(\pi_p = 0) \qquad p^r - c + I\left(v - c\right) = \frac{2t}{\phi^r} - t \tag{3}$$

$$(\pi_\phi = 0) \qquad p^r - c + I\left(v - c\right) = \frac{2a\phi^r}{2 - \phi^r}. \tag{4}$$

It is immediate from (3) that

$$p^r = \frac{2t}{\phi^r} + c - t - I\left(v - c\right). \tag{5}$$

From (5), we see that a higher advertising intensity is associated with lower advertised prices. We will revisit this point later. Solving (3) and (4) yields;

$$p^r = c + \sqrt{2at} - I\left(v - c\right) \tag{6}$$

$$\phi^r = 2/\left(1 + \sqrt{2a/t}\right) \tag{7}$$

and substituting (6) and (7) into the objective function gives

$$\pi^r = 2a/\left(1 + \sqrt{2a/t}\right)^2. \tag{8}$$

The advertised price is given by $p^S = c + \sqrt{2at}$ in regime S $(I = 0)$ and by $p_1^L = 2c + \sqrt{2at} - v$ in regime L $(I = 1)$. We see that $p_1^L = 2c + \sqrt{2at} - v = p^S - (v - c) < p^S$. That is, firms advertise lower prices when they sell multiple products but only advertise

[11] See Appendix B for the derivation of limits to the region of low differentiation.

[12] This assumption is implicit in the analyses of, for example, Bagwell (2003; Section. 5), Tirole (1988; Chap. 7) and Soberman (2004).

a subset. Although consumers pay the reservation price for the unadvertised good in regime L, the advertised price is lower as competition is more intense in regime L.

Since ϕ^r and π^r are independent of I in (7) and (8), it follows that $\phi^S = \phi^L$ and $\pi^S = \pi^L$. In particular, firms get the same profit selling a single product as selling two products. This leads us to the following invariance result,

PROPOSITION 1. *When firms advertise only a subset of their products, equilibrium profits are invariant with respect to increases in the number of products.*

The intuition is as follows: When each firm carries multiple products, but only advertises a subset, the incentives to attract consumers change. The ability to extract the entire consumer surplus on the unadvertised good raises the incentives to increase store traffic. This leads firms to offer price reductions on the advertised good. Price cutting continues until the loss from price reductions equals the gain from the sale of the unadvertised good – *with no change in demand.* That is, in equilibrium, no firm can increase its market share at the expense of the rival. Put differently, in equilibrium, price cutting only serves to defend existing market shares.

Our invariance result resembles a standard finding in the switching costs literature. In fact, our model can be reinterpreted as a two period model in which consumers have switching costs and each firm sells a single product in each period. In the first period, firms ferociously compete for market share in order to lock-in more consumers who can then be exploited in the second period (Klemperer, 1987). The second period in a switching costs model is analogous to the stage, in our model, where a consumer has already sunk in the transportation cost (that is, consumer is right in the store). At this point, the firm has monopoly power over the consumer. As in Klemperer (1987), firms offer bargains to entice consumers followed by rip-offs when consumers are locked in and the fierce competition for market share may totally dissipate potential profits from exploiting locked in consumers[13].

4.2. High differentiation. We say that differentiation is high if, given the prices, at least one partially informed consumer does not make a purchase. When differentiation is high, the degree of product differentiation is such that $t \in (v - c, 4(v - c)/3)$ and firm i's demand is given by equation (1') exactly[14].

[13] However, in equilibrium, consumers are not fooled. They know they are going to be ripped. Because firms cannot commit to not fleece consumers once they are locked in, they have to offer price discounts as a way to commit to leave consumers sufficient surplus to make the relationship (shopping trip) worthwhile.

[14] In regime H, we have that $t/2 < v - p < t$. Intuitively, the condition says that for all possible prices, the consumer located at $x = 1/2$ finds it profitable to purchase while the partially informed consumer who travels the entire unit distance finds it not profitable to purchase. The first inequality, evaluated at the full information (highest) price, gives the upper bound to regime H while

Firm 1's behaviour in the high differentiation case is described by

$$\pi_1^H = \max_{p_{11},\phi_1} (p_{11} + v - 2c)\,\phi_1 \left((1-\phi_2)\frac{v - p_{11}}{t} + \phi_2 \frac{p_{21} - p_{11} + t}{2t} \right) - \frac{a\phi_1^2}{2}.$$

The first order necessary conditions for an equilibrium are;

$$\pi_p = 4c - 4p_1^H + \left(3p_1^H + t - 2c - v\right)\phi^H = 0 \tag{9}$$

$$\pi_\phi = \left(p_1^H + v - 2c\right)\left(\frac{\phi^H}{2} + \left(1-\phi^H\right)\frac{v - p_1^H}{t}\right) - a\phi^H = 0. \tag{10}$$

The equilibrium advertising condition above (10) is intuitive. When the firm increases advertising marginally so that it informs one more consumer, the consumer is fully informed with probability ϕ^H, in which case she makes a purchase from the advertising firm half the time. With probability $1-\phi^H$, the additional ad reaches a hitherto uninformed consumer (thus making her partially informed), in which case she buys the advertising firm's brand with probability $\left(v - p_1^H\right)/t$. Hence, expected profit increases by $\left(p_1^H + v - 2c\right)\left(\frac{\phi^H}{2} + \left(1-\phi^H\right)\frac{v-p_1^H}{t}\right)$. The firm equates this incremental profit to the incremental cost of reaching one more consumer, $a\phi^H$.

Solving (9) for p_1^H gives,

$$p_1^H = \left(4c + (t - 2c - v)\,\phi^H\right) / \left(4 - 3\phi^H\right). \tag{11}$$

Equation (11) gives the relationship between the advertising intensity and the advertised price under high differentiation (see Corollary 2 below).

Substituting (11) into (10), the optimal advertising condition becomes,

$$MR\left(p_1^H\left(\phi^H\right), \phi^H\right) - a\phi^H = 0, \tag{12}$$

where $MR\left(\phi^H\right) = \frac{4(v-c)\left(1-\phi^H\right)+t\phi^H}{4-3\phi^H}\left(\frac{\phi^H}{2} + \left(1-\phi^H\right)\frac{2(v-c)\left(2-\phi^H\right)-t\phi^H}{t}\right)$. We show below that, although we cannot solve (12) explicitly for ϕ^H, an equilibrium does exist (though we cannot guarantee uniqueness)[15].

LEMMA 3. *A symmetric equilibrium* $\left(p_1^H, \phi^H\right)$; $\phi^H < 1$, *satisfying the first order necessary conditions given in (9) and (10) exists.*

PROOF. (see Appendix A) □

the second inequality gives the lower bound. Evaluating this condition at $p_{1F}^H = t + 2c - v$ gives $t \in (v - c, 4(v-c)/3)$.

[15] We plotted $MR(\phi)$ for different parameter values and we consistently found that $MR(\phi)$ is strictly decreasing. That is, we have not found any case where uniqueness fails.

The profit function is given by

$$\pi^H = R\left(\phi^H\right) - a\frac{\left(\phi^H\right)^2}{2} \tag{13}$$

where $R\left(\phi^H\right)$ is the firm's total revenue when its reach is ϕ^H.

4.3. Main Results. To recapitulate, p_1^L, ϕ^L and π^L (p_1^H, ϕ^H and π^H) are respectively the equilibrium advertised price, advertising and profit under low (high) differentiation. We summarize equations (6) and (11), in the following result:

PROPOSITION 2. *When firms advertise only a subset of their products, the advertised good is priced below cost when differentiation is low but is priced above marginal cost when differentiation is high; i.e.* $p_1^L < c$ *and* $p_1^H > c \forall \phi$. *The unadvertised good is priced at its reservation value.*

PROOF. First, from (6), $p_1^L = 2c + \sqrt{2at} - v = c + \sqrt{2at} - (v - c)$. Clearly, $p_1^L < c$ if and only if $\sqrt{2at} - (v - c) < 0$ and $\sqrt{2at} - (v - c) < 0$ if and only if $c + \sqrt{2at} < v$. But this is nothing other than Assumption A2. Hence, we conclude that indeed $p_1^L < c$. Second, differentiation is high if the differentiation parameter, t, is such that $v - c < t < \frac{4}{3}(v - c)$. Let $\underline{t} = v - c$. Notice that (from (11)) $\partial p_1^H / \partial t > 0$. Therefore, let $\underline{p_1^H} \equiv p_1^H |_{t=\underline{t}}$. Then, $\underline{p_1^H} = c$. Since $t > \underline{t} = v - c$, it follows that $p_1^H = \left(4c + (t - 2c - v)\phi^H\right) / \left(4 - 3\phi^H\right) > \underline{p_1^H} = c$. The proof of the second part of the result is given in Lemma 2. □

This result is intuitive. Since consumers do not search, in equilibrium, market shares are determined solely by the advertised prices. Holding the firm's advertising reach constant, the lower the advertised price the greater the likelihood that each ad received results in a sale. When differentiation is low, a firm that successfully undercuts a rival can substantially increase its market share. Moreover, since firms can rip-off visiting consumers on the unadvertised good, they compete more aggressively for market share and thus end up pricing below cost. When differentiation is high however, price advertising is primarily informative. Products are less similar and therefore price differences have to be large to induce consumers to switch to the distant supplier. Hence there is less rationale for pricing below cost.

An example of a market in which differentiation is generally low is the grocery retail market. Supermarkets, for instance, sell products that are almost (if not exactly) physically similar. In such cases, differentiation mainly take the form of (quality of) services offered. Competition therefore mainly take the form of prices and to a lesser extent, services. When firms advertise only a subset of their products, competition for market share will be intense since consumers do not have a strong inclination to

buy from a particular store (consumers are "footloose") and firms can rip off visiting consumers on the unadvertised goods. This leads to lower advertised prices.

Observe that our loss-leader pricing result differs from that of Lal and Matutes (1994). In Lal and Matutes, equilibrium advertised prices may well exceed marginal cost. Whether advertised prices are below marginal cost or not depends on parameter values. In contrast, in our model, for all parameter values, the equilibrium necessarily entail advertised prices below marginal cost when differentiation is low. Proposition 2 thus allows us to pin down the sufficient conditions for loss-leader pricing. As a corollary to Proposition 2, we therefore have;

COROLLARY 1. *When firms each sell two products with the same reservation price, the following conditions are sufficient for loss-leader pricing; (i) Low differentiation and (ii) Firms advertise only a subset of their products.*

Low differentiation is indispensable for loss leader pricing since it induces tougher competition for market share and hence the rationale for price undercutting. That firms only advertise a subset of their products is important for at least two reasons. First, by advertising, firms inform consumers of the prices they charge thereby causing the fully informed consumers to choose the store that offers the best deal. Competition to attract these fully informed consumers causes firms to advertise lower prices. Second, by advertising only a subset, firms relax price competition on the unadvertised good. This is important for loss-leader pricing since the price reductions on the advertised good need to be financed. Firms rip-off consumers on the unadvertised good and use the proceeds to offset the losses from the loss-leader[16]. These two conditions are together sufficient for loss-leader pricing[17].

Propositions 1 and 2 together with Corollary 1 allow us to rationalize the "surprising" finding of Walters and MacKenzie (1988) that loss-leader pricing fails to stimulate store traffic and hence is unprofitable. Walters and MacKenzie interpret their finding as "pointing to the fact that locational convenience and overall price perceptions are more important determinants of patronage than weekly specials" (p. 60). We turn their explanation on its head. Because differentiation is typically low in the retail sector, weekly specials are very important to consumers when it comes to visitation. As a result, a firm that offers such specials would substantially increase its market share if rivals would not follow suit. Realizing this, firms always try to match price cutting

[16] Loss leader pricing does not by itself imply that the firm is making losses overall. In fact, the rationale for loss leader pricing is to try to stimulate visitation (store traffic) and hence increase profits by selling add-ons at higher prices.

[17] Sufficiency follows from the fact that when both these conditions are satisfied, the only equilibrium outcome is one where the advertised price is below cost.

by rivals and this enables them to maintain their market shares. This is a typical prisoner's dilemma. A firm that succeeds in undercutting its rival can greatly increase its profits since it then faces a large demand and sells the unadvertised good at its reservation price. On the other hand, if both firms undercut (symmetrically), then each firm maintains its market share. In this sense, undercutting is a dominant strategy for each individual firm. However, this strategy does not maximize joint profits and firms settle for an equilibrium with low advertised prices but with the same level of demand[18]. This makes loss-leader pricing appear as if it were less important[19]. We find our argument more convincing, for if price specials were unimportant, why are retailers placing greater emphasis on *hotter price specials*? (Lal and Matutes, 1994; p. 345)[20].

We next highlight the interaction between advertising and prices. One of the features that distinguish our model from the models of Lal and Matutes (1994) and Ellison (2005) is that with regard to information, consumers are ex-post heterogeneous. In Lal and Matutes and respectively, Ellison, all consumers are fully informed as regards to the advertised prices. However, in our model, in equilibrium some consumers receive ads and some don't ($\phi^* < 1$). Below we compare the "equilibrium advertised" prices to the "full information" prices[21].

PROPOSITION 3. *Let p_{1F}^L (p_{1F}^H) be the full information price in regime L (H). Then, $p_{1F}^L < p_1^L$ and $p_{1F}^H > p_1^H$.*

The full information price is $t+2c-v$ in both regimes[22]. Under full information and low differentiation, the equilibrium of our model collapses to that of Lal and Matutes (1994). However, because our model is an extension of the single product models of Grossman and Shapiro (1984) and Soberman (2004), we can (conclusively) show that the advertised good is priced below marginal cost whereas Lal and Matutes cannot[23].

[18] That loss leader pricing does not increase demand is a consequence of the unit demands assumption. With downward sloping demands, total demand can increase in equilibrium but firms' market shares will not.

[19] Since competitors always match price cutting by rivals, the full effect of loss leader pricing on store traffic and profits is never realized in equilibrium. This gives a biased reading of the importance of loss leader pricing. This suggests a different empirical method to test for the effect of loss leader pricing – counterfactual analysis. What would be the effect on profits of firm i were competitors to not reciprocate when firm i lowers its price?

[20] Emphasis added.

[21] Full information here does not mean that consumers have full information about all prices prior to visitation. By full information we mean that every consumer in the market is aware of the *advertised prices* ($\phi \to 1$) – as in Lal and Matutes (1994). We reserve the term "perfect information" to the case where all consumers are fully informed of all prices.

[22] Prices under low and high product differentiation are not directly comparable since the ratio of t to v is different in the two regimes.

[23] The equilibrium in the single product case provides sufficient restrictions on the parameters to enable us to fully characterize the equilibrium in the multiproduct case. More precisely, in the single

When differentiation is low, the equilibrium advertised price exceeds the full information price. The reason is that when the market is covered, demand is less elastic in the presence of informational product differentiation (Bagwell, 2003; p. 75). This informational differentiation comes in the form of some consumers being informed of the prices at only one of the stores. That is, these consumers (by definition) cannot make across store price comparisons. They are captive to the firm from which they receive an ad and as a result, the price applicable to this group of consumers is "higher". On the other hand, consumers who receive ads from both firms can make price comparisons across stores and, as a result, the price applicable to the fully informed consumer group is "lower". When some consumers are not fully informed, the equilibrium price is a weighted average of the price applicable to the fully informed and that applicable to the partially informed consumers, which obviously exceeds the price applicable to the fully informed consumers. Technically, $\forall \phi \in (0,1)$, $\varepsilon_p(\phi) = \phi p/(2-\phi)t < p/t = \varepsilon_p(\phi = 1)$. Because demand is less elastic, firms can afford to charge higher prices.

In contrast, the full information price exceeds the equilibrium advertised price when the market is only partially covered. In this case, demand is more elastic in the presence of informational product differentiation. When differentiation is high, the elasticity of demand is given by: $\varepsilon_p(\phi) = (2-\phi)p/(t\phi + 2(1-\phi)(v-p))$ and $\partial\varepsilon_p(\phi)/\partial\phi = 2p(v-p-t)/(t\phi + 2(1-\phi)(v-p))^2 < 0$ since $v - p < t$. Thus imperfect information increases firms' market power when differentiation is low but decreases firms' market power when differentiation is high. In other words, the effect of improvements in information is regime dependent – a point that appears to have eluded the literature[24]. Below we characterize the relationship between the advertised prices and the advertising intensity. As a corollary to Proposition 3, we have that:

COROLLARY 2. *An increase in advertising is associated with lower (higher) advertised prices when differentiation is low (high)*[25].

product case, the equilibrium price is given by: $p^S = c + \sqrt{2at}$. Since the reservation price for this good is v and firms advertise to inform consumers, $p^S = c + \sqrt{2at}$ is an equilibrium price only if $p^S < v$. For, if $p^S \geq v$, no consumer will visit a store when faced with positive transport costs. Letting marginal cost equal zero (as in Lal and Matutes), the equilibrium advertised price in our two good model is $p_1^L = t - v$ (same as in Lal and Matutes). But since $\sqrt{2at} < v$ (from the single product model) and $a > t/2$, it follows that $t < v$. Hence, $p_1^L = t - v < 0 =$ marginal cost.

[24] [For example, *Stigler (1961) and Ozga (1960) suggest that consumer ignorance leads to higher prices while advertising leads to lower prices;* Bagwell 2003; p. 75]. Also, Grossman and Shapiro (1984; p. 69), Christou and Vettas (2003), make the same point. Soberman (2004) is an exception.

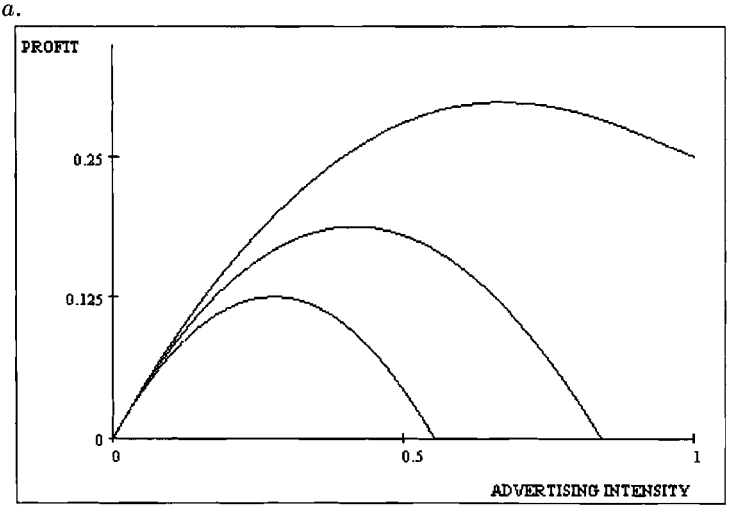
[25] These relations between advertising intensity and price presuppose a change in some exogenous parameter (e.g., the advertising cost) that directly affects the advertising intensity but does not directly enter the price equation. These are not causal relationships.

PROOF. $\partial p_1^L/\partial\phi = -2t/\phi^2 < 0$ and $\partial p_1^H/\partial\phi = 4\,(c+t-v)\,/\,(3\phi-4)^2 > 0$ □

This result has equivalences in Soberman (2004; Propositions 1 and 2). When differentiation is low, demand by partially informed consumers is price inelastic while demand by fully informed consumers is price sensitive. An increase in the advertising intensity raises the share of fully informed (and thus price sensitive) consumers in the market and this puts pressure on prices. Put differently, an increase in the advertising intensity raises demand elasticity and hence lead to lower advertised prices. When differentiation is high however, partially informed consumers are more price sensitive compared to fully informed consumers ($\varepsilon_{part} = p/(v-p) < p/t = \varepsilon_{full}$)[26]. Thus higher advertising, by increasing the share of fully informed consumers, reduces overall demand elasticity and thus enables firms to charge higher prices.

Although we cannot solve explicitly for the optimal advertising level under high differentiation, a numerical exercise shows that both the advertising intensity and profit decrease as the advertising cost, a, increases. There are at least three channels by which an increase in the advertising cost propagates itself into lower profit. Firstly, an increase in the advertising cost directly increases the advertising outlay and hence lowers profit. Secondly, an increase in the advertising cost reduces the advertising level and this directly lowers demand and, other things equal, lowers profit. Thirdly, an increase in the advertising cost, by reducing the advertising intensity, leads to lower prices (Corollary 2) and hence lower profit.

Figure 1 below plots profit as a function of the advertising intensity, for different values of a.



[26] By the definition of high differentiation, $v - p < t$ and hence the result.

Figure 1. Profit as a function of the advertising intensity, for different values of a. ($v = 1, c = 0, t = 1.1$; $a = .6$ (highest curve), $a = 1.4$ (intermediate curve), $a = 2.5$ (lowest curve)).

We summarize the above observations in the following statement:

REMARK 1. *First, for a given advertising intensity, profit increases as the advertising cost decreases. Second, as the advertising cost decreases, the optimal advertising level increases and moreover, profits become less sensitive to small perturbations in the advertising intensity. That is, the profit function becomes flatter in the neighborhood of the optimal advertising level.*

We next consider the effect of changes in the reservation value, v. From (6) and (11), the advertised prices decrease as v increases. As v increases, the gain from selling one more unit of the unadvertised good, $v - c$, increases and as a result, each firm wants to sell more units of the unadvertised good. This induces tough price competition leading to lower prices. In the low differentiation regime, the incentive to undercut is greatest and this results in total dissipation of all potential gains. In the high differentiation regime, however, price cutting is only modest and as a result, prices decrease but less than proportionately[27].

5. Welfare

We consider, in this section, the welfare implications of informative advertising when firms advertise only a subset of their products. In the sequel, we restrict ourselves to the case of low differentiation[28].

5.1. Market Equilibrium. To start with, we consider welfare in the market equilibrium. We define welfare as the sum of total profits and consumer surplus. As we saw earlier (Proposition 1), profits do not change when firms move from the single to the multiproduct configuration. Thus the welfare effect of such a move equals the effect on consumer surplus. Hence,

PROPOSITION 4. *When firms increase the number of products they sell, but only advertise a subset, consumer surplus (and hence welfare) increases.*

PROOF. $\phi^L = \phi^S$ and $\pi^L = \pi^S$ but $p^L < p^S$ □

[27] $\partial p_1^L/\partial v = -1 < \partial p_1^H/\partial v = -\phi/(4 - 3\phi) < 0 \forall \phi \in (0,1)$. For the comparative statics results of t, c and a see Tirole (1988; p. 293).

[28] We cannot compare the optimal outcome to the equilibrium outcome under high differentiation because the regions of t consistent with partial market coverage do not intersect.

Even though consumers pay the reservation price on the unadvertised good (i.e. get zero surplus), the advertised price is lower in the multiproduct case – that is, visiting consumers get higher surplus. Since the advertising reach is the same and profits are equal in the two configurations, the result follows. Intuitively, introducing the second good intensifies competition for market share and this generates spillovers for consumers (in the form of lower prices).

5.2. Social optimum. Suppose the firms studied above are instead run by a benevolent planner. The objective of the planner is to maximize welfare. We derive the socially optimal advertising levels for the single and multiproduct cases.

Let each firm offer n products; $n \in \{1, 2\}$. For consumers who receive ads from both firms (measure ϕ^2), the average transportation cost is $t/4$. Because each good is priced at marginal cost, c, the average net benefit per consumer is $n(v-c) - t/4$.[29] For consumers who only receive a single ad (measure $2\phi(1-\phi)$), the average net benefit per consumer is $n(v-c) - t/2$. The planner chooses the advertising level, ϕ, to maximize,

$$W_n = \phi^2 (n(v-c) - t/4) + 2\phi(1-\phi)(n(v-c) - t/2) - a\phi^2$$

where W_n is the welfare when each firm offers n products. The first order condition for a welfare maximum is,

$$\phi t/4 + (1-\phi)(n(v-c) - t/2) = a\phi. \tag{14}$$

A marginal increase in advertising reduces total transportation costs by on average $t/4$ when the ad reaches a consumer who is partially informed (and thus making him fully informed) and this occurs with probability ϕ. When the ad reaches a hitherto uninformed consumer however, net social surplus increases by $n(v-c) - t/2$, and this occurs with probability $1-\phi$. In equilibrium, the planner equates the increase in social surplus to the cost of informing one more consumer, $a\phi$. This gives,

$$\phi_n^{Social} = (4n(v-c) - 2t) / (4n(v-c) + 4a - 3t) \tag{15}$$

and substituting ϕ_n^{Social} into the objective function gives

$$W_n = (2n(v-c) - t)^2 / (4n(v-c) + 4a - 3t). \tag{16}$$

An important policy question concerns the welfare effects of changes in the cost of advertising and how that relates to the number of products offered. For easy of exposition, we suppose here that n is continuous[30].

[29] Marginal cost pricing ensures that there is no consumption distortion and hence maximizes welfare.

[30] Assuming continuity here is for convenience purposes only. This assumption is innocuous. It can be shown that $\left|\frac{\partial W_2}{\partial a}\right| > \left|\frac{\partial W_1}{\partial a}\right|$ (see the Proof of Proposition 5 in Appendix A).

PROPOSITION 5. *An increase in the cost of advertising leads to a decrease in welfare. Moreover, this negative welfare effect is exacerbated when firms offer multiple products. That is,* $\partial W_n/\partial a < 0$ *and* $\partial^2 W_n/\partial a \partial n < 0$.

PROOF. The first part of the proposition follows immediately from (16). As for the second part, $\frac{\partial^2 W_n}{\partial a \partial n} = \frac{-16(v-c)(2n(v-c)-t)(4a-t)}{(4n(v-c)+4a-3t)^3} < 0$ since $a > t/2$ □

The first part of the proposition is due to Grossman and Shapiro (1984)[31]. Since prices are fixed at the marginal cost, apart from raising the advertising outlay, the only effect of an increase in the cost of advertising is to lower the socially optimal level of advertising. These two (higher advertising outlay and reduced advertising intensity) both work to reduce welfare. The intuition for the second part of the proposition is straight forward. Notice that since transport costs are independent of the number of products purchased from any one supplier, surplus per visiting consumer increases with the number of products. Since an increase in a lowers the advertising level – which means fewer consumers make a purchase, the result follows immediately.

That $\partial^2 W_n/\partial a \partial n < 0$ applies equally to the market welfare. Since welfare increases with the number of products in the market equilibrium (Proposition 4), it follows that reducing the share of informed consumers in the market lowers welfare more the larger the number of products.

5.3. Market versus Planner. Below we compare the market determined to the socially optimal advertising level. The market determined advertising level is given by $\phi^L = 2/\left(1+\sqrt{2a/t}\right)$ while the socially optimal level is given by $\phi_n^{Social} = \frac{8s-2t}{8s+4a-3t}$; $s = v - c$. The question we address in this subsection is whether the market determined advertising level is excessive or too low compared to the socially optimal level. A divergence may occur since the objectives of the planner and private firms differ.

The equation $\phi_2^{Social} - \phi^L = \frac{8s-2t}{8s+4a-3t} - \frac{2t}{t+\sqrt{2at}} = 0$ has the following solutions[32].

$$a_1 = \frac{(4s-3t)^2 + (t-4s)\sqrt{16s^2 - 40st + 17t^2}}{16t}$$

$$a_2 = \frac{(4s-3t)^2 - (t-4s)\sqrt{16s^2 - 40st + 17t^2}}{16t}$$

Observe that since $t - 4s < 0$, $a_1 < a_2$. Since both ϕ_2^{Social} and ϕ^L are monotonically decreasing in a and $\lim_{a\to\frac{t}{2}} \phi^L = 1 > \lim_{a\to\frac{t}{2}} \phi_2^{Social}$, clearly, in the interval $(t/2, a_1)$,

[31] Soberman (2002) finds a similar result, but for market welfare. He shows (numerically) that although the price increases when the advertising cost decreases, the advertising level increases much faster and hence welfare increases (provided the number of firms is not too large).

[32] These solutions are valid for $t \in \left(0, \frac{4s(5-2\sqrt{2})}{17}\right)$.

it must be the case that $\phi^L > \phi_2^{Social}$. In the interval (a_1, a_2), $\phi^L < \phi_2^{Social}$ and for $a > a_2$, $\phi^L > \phi_2^{Social}$. Thus, from a social welfare perspective, the market determined advertising level is excessive for a close to $t/2$, too low for intermediate values of a, and excessive for large values of a.

However, if we fix a and $v - c$, so that both ϕ^L and ϕ_2^{Social} are only functions of the differentiation parameter, t, we get the result (due to Hamilton (2004; Proposition 2)) that: *Compared to the socially optimal level of informative advertising, the market undersupplies informative advertising for sufficiently homogeneous brands but oversupplies advertising for more differentiated brands.*

Below we attempt an explanation for this apparent divergence between the socially optimal and the market determined advertising levels. First, by increasing the number of products they sell, firms create benefits for all visiting consumers (Proposition 4). However, firms cannot appropriate the benefits so created (Proposition 1). This suggests that firms have lower incentives to inform consumers. Thus, compared to the social planner, nonappropriability of consumer surplus leads firms to undersupply informative advertising. Second, when a firm reaches a consumer who already has received advertising from the competitor, there is, on average, realignment of consumers among firms (the matching effect). Fully informed consumers buy from the nearest store, thereby saving on transportation costs. Firms do not care about this benefit and hence tend to underprovide informative advertising. However, firms care about business stealing. When a firm reaches a consumer who already has received advertising from a rival, the resulting realignment of consumers may create business for the advertising firm. If it does, the competitor loses $v + p^L - 2c > t$ while the consumer saves on average $t/2$ on transportation costs[33,34]. This ad thus generates a welfare loss from the social stand point. This suggests that the market determined advertising level may be excessive. We however, cannot make general statements as to whether ϕ^L exceeds ϕ_2^{Social} or not. Depending on parameter values, the market may overprovide, provide the optimal or underprovide informative advertising relative to the socially optimal level.

We end this section with a *caveat*. As Ellison (2005; p. 619) notes: "Models ... with unit demands are poorly suited to welfare analysis". This is because there isn't much consumption distortion in these models. Consumption distortion only takes the form of changes in the number of consumers purchasing and changes in prices have no

[33] The competitor loses $v - c$ on the unadvertised good and $p^L - c$ on the advertised good. $v + p^L - 2c = \sqrt{2at} > t$.

[34] A consumer on firm i's turf, if they only receive ads from firm $j, j \neq i$; travels on average $3t/4$ (the average consumer on firm i's turf is located at $t/4$). Now if firm i sends an ad that reaches a consumer on her turf, then the consumer switches to firm i and thus saves on average $3t/4 - t/4 = t/2$.

effect on the quantity demanded by any individual consumer as long as the price is less than the reservation price[35]. With downward sloping demand curves, one expects that consumers would buy more of the loss-leader (stocking) and less of the unadvertised good. For example, Pesendorfer (2002) finds that demand for ketchup increases sharply during the sale period but also falls sharply after the sale. If we distinguish between "shoppers" – low search cost consumers who only come in to buy the good on offer and "regulars" – high search cost consumers who visit the store with the intention of purchasing multiple products, then loss leader pricing will not be profitable if it disproportionately attracts shoppers.

In reality though, firms tend to offer limited quantities of the loss leader good, or to restrict the quantity purchased, of the loss leader, by any individual consumer. Another strategy is to offer nonstorable products as loss leaders. This is meant to discourage stocking by consumers or even arbitrage. By offering loss leaders and then restricting quantities of the loss leader good purchased, firms indirectly induce consumers to purchase other unadvertised products for two reasons – the income effect (a lower price on the loss leader leaves consumers with more income to spend) and the shopping cost effect (because it is costly to shop around, consumers, once in the store, tend to make all their purchases in that store).

6. Extension: Equilibrium Pricing when Reservation Prices Differ

In this section we allow reservation prices to differ and derive the associated equilibrium under both low and high differentiation. Our model is unchanged except that now the reservation values are v_1 for good 1 and v_2 for good 2. That is, Lemmas 1 and 2 apply.

Let firms advertise good 1 as before. Firm i's demand is given by

$$D_i = \phi_i \left(\left(1 - \phi_j\right) \frac{v_1 - p_{i1}}{t} + \phi_j \frac{p_{j1} - p_{i1} + t}{2t} \right); i \neq j. \tag{17}$$

6.1. Low differentiation. When $t < (v_1 + v_2 - 2c)/2$, all consumers who receive at least one ad make a purchase. That is, $(v_1 - p_{i1})/t = 1$.

We modify Assumption A2 as follows;

Assumption A2': $c + \sqrt{2at} < \min\{v_1, v_2\}$.

Firm 1's behaviour is described by

$$\pi_1^L = \max_{p_{11}, \phi_1} (p_{11} + v_2 - 2c)\, \phi_1 \left(1 - \phi_2 + \phi_2 \frac{p_{21} - p_{11} + t}{2t}\right) - \frac{a\phi_1^2}{2}.$$

[35] I thank Jonas Häckner for this observation.

Differentiating with respect to p_{11}, ϕ_1 and solving the first order conditions yield;

$$p_1^L = 2c + \sqrt{2at} - v_2 \tag{18}$$

$$\phi^L = 2/\left(1 + \sqrt{2a/t}\right) \tag{19}$$

and substituting (18) and (19) into the objective function gives

$$\pi^L = 2a/\left(1 + \sqrt{2a/t}\right)^2. \tag{20}$$

6.2. High differentiation. Under high differentiation, the degree of product differentiation is such that $t \in \left(\frac{v_1+v_2}{2} - c, \frac{2(v_1+v_2-2c)}{3}\right)$ and the demand that firm i faces is given by equation (17).[36]

Firm 1's behaviour is described by

$$\pi_1^H = \max_{p_{11},\phi_1} (p_{11} + v_2 - 2c)\,\phi_1 \left((1-\phi_2)\frac{v_1 - p_{11}}{t} + \phi_2 \frac{p_{21} - p_{11} + t}{2t}\right) - \frac{a\phi_1^2}{2}.$$

The first order necessary conditions for an equilibrium are;

$$\pi_p = 4c + 2(v_1 - v_2) + (t - 2c - (2v_1 - v_2))\,\phi^H - \left(4 - 3\phi^H\right) p_1^H = 0 \tag{21}$$

$$\pi_\phi = \left(p_1^H + v_2 - 2c\right)\left(\frac{\phi^H}{2} + \left(1 - \phi^H\right)\frac{v_1 - p_1^H}{t}\right) - a\phi^H = 0. \tag{22}$$

Solving (21) for p_1^H gives,

$$p_1^H = \frac{4c + 2(v_1 - v_2) + (t - 2c - (2v_1 - v_2))\,\phi^H}{4 - 3\phi^H}. \tag{23}$$

Equation (23) gives the equilibrium price as a function of the advertising intensity. Due to complexity of the first order conditions, we cannot explicitly solve for p_1^H and ϕ^H.

We summarize equations (18)-(23) below.

PROPOSITION 6. *Let the reservation prices be v_1 and v_2 for goods 1 and 2 respectively. If $t < \frac{v_1+v_2-2c}{2}$ and firms advertise only a subset of their products, the advertised good is priced below cost irrespective of whether firms advertise the low or the high reservation price good. The unadvertised good is priced at its reservation price.*

PROOF. Let p_i^L be the advertised price when firms advertise good $i, i = 1, 2$.[37] The idea of the proof is to show that the highest advertised price is below marginal cost. Without loss of generality, let $\min\{v_1, v_2\} = v_2$. Then, $p_1^L = 2c + \sqrt{2at} - v_2$ is the

[36] In Regime H, we have that $t/2 < v_1 - p < t$. Simply substitute $p_{1F}^H = t + 2c - v_2$ for p. See also footnote 14.

[37] We abuse notation here by letting i (and j) denote the products.

highest advertised price. Now, $p_1^L = 2c + \sqrt{2at} - v_2 = c + \sqrt{2at} - (v_2 - c) < c \iff \sqrt{2at} - (v_2 - c) < 0 \iff c + \sqrt{2at} - v_2 < 0$. Since $c + \sqrt{2at} < v_2$ (by Assumption **A2'**), it follows that $p_1^L < c$. That the unadvertised good is priced at its reservation price follows from Lemma 2 □

When firms advertise the low reservation value good, competition for market share is tougher as the firm that succeeds in attracting more consumers will sell more units at the higher unadvertised (reservation) price. The larger the difference between the reservation prices the greater the incentive to undercut. Coupled with the fact that differentiation is low, this leads to a much lower equilibrium advertised price. When firms advertise the high reservation price good instead, there are two opposing effects. On the one hand, low differentiation induces firms to compete more aggressively for market share. On the other hand, firms realize that visiting consumers will pay a lower (reservation) price on the unadvertised good and this restrains the aggression. Firms will try to undercut their rival but not as much as when they advertise the low reservation price good. However, the low differentiation effect dominates and firms advertise prices below marginal cost in either case.

This result contrasts with Lal and Matutes (1994; p. 363) who show under similar circumstances (in a setting of full market coverage) that when reservation values differ, loss-leader pricing obtains only when firms advertise the low reservation price good.

As a corollary to Proposition 6, we have the following;

Corollary 3: *When firms sell products with different reservation prices and differentiation is low, they are indifferent as to which good (the low reservation price good or the high reservation price good) to offer as a loss-leader.*

PROOF. The profit in either case is $\pi = 2a/\left(1 + \sqrt{2a/t}\right)^2$. □

We showed earlier (Proposition 2) that when firms sell products with the same reservation value, loss-leader pricing obtains *only* when differentiation is low. Does this hold in general? It turns out that when reservation prices differ, loss-leader pricing is possible under high differentiation. More precisely,

PROPOSITION 7. *Let* $t \in \left(\frac{v_1+v_2}{2} - c, \frac{2(v_1+v_2-2c)}{3}\right)$. *When firms advertise only a subset of their products and the reservation values are sufficiently different, equilibrium* ***may*** *entail loss-leader pricing when firms advertise the low reservation price good. However, when firms advertise the high reservation value good, the advertised price exceeds marginal cost for all parameter constellations.*

PROOF. Suppose that $|v_i - v_j| > 2c$. Let firms advertise good i so that p_i^H is the advertised price. Notice that (from (23)) $\partial p_i^H / \partial t > 0$. Since $t > \underline{t} = \frac{v_1+v_2}{2} - c$, it follows

therefore that $p_i^H = \frac{4c+t\phi^H-2c\phi^H+2(v_1-v_2)-\phi^H(2v_1-v_2)}{4-3\phi^H} > p_i^H\,|_{t=\underline{t}} \equiv \underline{p_i^H} = c + \frac{(v_i-v_j)}{2}; i,j = 1,2; i \neq j$. If $v_i < v_j$, advertising good i (the low reservation price good) gives $\underline{p_i^H} < c$. Thus, for $t \to \left(\frac{v_1+v_2}{2} - c\right)^+$ and for $|v_i - v_j|$ large, $p_i^H < c$. However, if $v_i > v_j$, then advertising good i (the high reservation price good) gives $\underline{p_i^H} > c$ and hence $p_i^H > c$. □

The intuition is as follows: For $t \to \left(\frac{v_1+v_2}{2} - c\right)^+$, most partially informed consumers make a purchase (the case of almost full coverage) and therefore competition for market share can be intense. Moreover, a higher unadvertised price adds to the incentives to compete vigorously. As a result, when firms advertise the low reservation price good, undercutting may result in prices below cost. However, when the differentiation parameter is sufficiently large, the incentive to undercut is reduced. Hence, for large t, firms advertise prices above marginal cost when they advertise the low valuation good. When firms advertise the high reservation price good however, it is never optimal to advertise prices below marginal cost. As we saw above, the incentive to undercut is positively correlated with the unadvertised price. The higher the unadvertised price, the greater the benefit of attracting more consumers and hence the greater the incentive to offer price discounts. The fact that differentiation is high and the fact that the unadvertised (reservation) price is lower when firms advertise the high reservation price good both induce firms to advertise higher prices since the benefits to cutthroat competition are negligible.

7. Conclusion

We study, in this paper, the effect of price advertising on prices of both advertised and unadvertised products under two product differentiation regimes. We find some support for the empirical findings that price advertising affects advertised and unadvertised prices differently. Price advertising is associated with lower advertised prices when differentiation is low and with higher advertised prices when differentiation is high but has no effect on the prices of the unadvertised goods.

We find that product differentiation plays an important role in the analysis of loss-leader pricing. We have loss-leader pricing only when differentiation is low. When products are less similar, price competition is less intense and firms advertise higher prices. Because firms offer loss-leaders only in highly competitive markets and because firms mimic the behaviour of their rivals, loss-leader pricing only serves to defend existing market shares.

We extend our model to allow for different reservation prices. Irrespective of whether firms advertise the low or the high reservation price good, equilibrium entails loss-leader pricing when differentiation is low. We find also that when firms advertise

the low reservation price good, loss-leader pricing is possible under high differentiation provided the difference in the reservation prices is large enough.

With regard to welfare, we find that the market determined advertising level exceeds the socially optimal level for some parameter constellations while the reverse is true for other constellations. We also find that welfare decreases as the advertising cost increases and, more importantly, the welfare decrease is higher the larger the number of products.

In our analysis, the advertised good is exogenously determined. A worthy extension would be to endogenize the choice of which products to advertise. This would allow firms to advertise different products. Other extensions could include generalizing the model to more than two products.

Appendix

Appendix A: Proofs of Lemmas and Propositions

Proof of Lemma 3

The objective here is to show that $MR(\phi)$ and $A'(\phi)$ intersect at least once. We proceed by showing that (i) $MR(\phi)$ is continuous and (ii) $\lim_{\phi\to 0} MR(\phi) > 0$ and (iii) $\lim_{\phi\to 1} A'(\phi) > \lim_{\phi\to 1} MR(\phi)$, where $MR(\phi) = (p_1 + v - 2c)\left(\frac{\phi}{2} + (1-\phi)\frac{v-p_1}{t}\right)$, $p_1(\phi) = \frac{(4c+(t-2c-v)\phi)}{(4-3\phi)}$ and $A'(\phi) = a\phi$. We will make use of the following results[38]:

Result 1: A product of two continuous functions is continuous.

Result 2: A sum of two continuous functions is continuous.

PROOF. (i). Let $L(\phi) = (p_1(\phi) + v - 2c)$ and $Z(\phi) = \frac{\phi}{2} + (1-\phi)\frac{v-p_1(\phi)}{t}$. It is evident that $p_1(\phi)$ is continuous in ϕ. Clearly therefore, $L(\phi)$ is continuous. We prove that $Z(\phi)$ is continuous, by showing that its constituent parts are all continuous in ϕ; First, we show that $\frac{v-p_1(\phi)}{t}$ is continuous. Secondly we show that $(1-\phi)\frac{v-p_1(\phi)}{t}$ is continuous and lastly, we show that $Z(\phi)$ must be continuous. First, since v and t are constants and $p_1(\phi)$ is continuous, it follows that $\frac{v-p_1(\phi)}{t}$ is continuous. Second, since $1-\phi$ is continuous in $\phi\forall\phi \in [0,1]$ and $\frac{v-p_1(\phi)}{t}$ is continuous in $\phi\forall\phi \in [0,1]$, it follows that $(1-\phi)\frac{v-p_1(\phi)}{t}$ is continuous in ϕ (*Result 1*). Obviously, $\frac{\phi}{2}$ is continuous. Hence $Z(\phi)$ is continuous (*Result 2*). Since $MR(\phi) \equiv L(\phi) * Z(\phi)$, it follows (*Result 1*) that it is continuous. Continuity of MR is needed to rule out jumps in MR which may lead to nonexistence. (ii). $\lim_{\phi\to 0} MR(\phi) = \frac{1}{t}(v-c)^2 > 0$. That $\lim_{\phi\to 0} MR(\phi) > 0$ is necessary to rule out $MR(\phi)$ trajectories that lie everywhere below $A'(\phi)$ for $\phi \in (0,1]$; for example, the trajectory $\omega\phi$, $\omega < a$. Such trajectories lead to nonexistence of equilibrium. Also, $\lim_{\phi\to 1} A'(\phi) = a > t/2 = \lim_{\phi\to 1} MR(\phi)$. The inequality follows from the assumptions of our model. Since $MR(\phi)$ is continuous, and $\lim_{\phi\to 0} MR(\phi) > 0$, it follows that $MR(\phi)$ and $A'(\phi)$ intersect at least once. Hence, a ϕ that solves (12) is guaranteed to exist[39]. □

Proof of Proposition 5

The objective here is to show that a given change in the advertising cost has a larger effect on welfare when firms offer two products as opposed to a single product. From (16), $W_n = \frac{(2n(v-c)-t)^2}{(4n(v-c)+4a-3t)}$. Thus, $W_1 = \frac{(2s-t)^2}{4s+4a-3t}$ and $W_2 = \frac{(4s-t)^2}{8s+4a-3t}$, where $s = v - c$.

PROOF. $\frac{\partial W_2}{\partial a} = -4\left(\frac{4s-t}{8s+4a-3t}\right)^2$ and $\frac{\partial W_1}{\partial a} = -4\left(\frac{2s-t}{4s+4a-3t}\right)^2$. Clearly, $\left|\frac{\partial W_2}{\partial a}\right| > \left|\frac{\partial W_1}{\partial a}\right| \iff \left(\frac{4s-t}{8s+4a-3t}\right)^2 > \left(\frac{2s-t}{4s+4a-3t}\right)^2$ and $\left(\frac{4s-t}{8s+4a-3t}\right)^2 > \left(\frac{2s-t}{4s+4a-3t}\right)^2$ if and only if $\frac{4s-t}{8s+4a-3t} >$

[38] See Sydsaeter and Hammond (1995; p. 188).

[39] Substituting the equilibrium ϕ into (11) gives the equilibrium price.

$\frac{2s-t}{4s+4a-3t}$. Simplifying this inequality, we get that $\frac{4s-t}{8s+4a-3t} > \frac{2s-t}{4s+4a-3t}$ if and only if $2s(4a-t) > 0$. Therefore, $\left|\frac{\partial W_2}{\partial a}\right| > \left|\frac{\partial W_1}{\partial a}\right|$ if and only if $2s(4a-t) > 0$. Since $a > t/2$ (by assumption), we conclude that indeed, $\left|\frac{\partial W_2}{\partial a}\right| > \left|\frac{\partial W_1}{\partial a}\right|$. That is, the welfare effect of a change in the advertising cost gets larger as the number of products increases. □

Appendix B: Derivations of Limits to the region of Low Differentiation

We want to derive restrictions on the differentiation parameter, t, that guarantee that the market will be fully covered[40]. That is, the consumer who travels the entire unit distance finds it profitable to purchase. We proceed by showing that if the advertised price, p_{i1}, exceeds $v_1 - t$, firm i's profits can be increased by reducing the price to below or about $v_1 - t$. In other words, the maximum price observed cannot exceed $v_1 - t$. Technically, if $v_1 - t$ is the maximum price, then, for $p_{i1} > v_1 - t$, we must have $\frac{\partial \pi_1}{\partial p_{i1}} < 0$. That is, any price higher than $v_1 - t$ yields lower profit. Consider firm 1.

Suppose then that $p_{11} > v_1 - t$. Then, $D_1 = \phi_1\left((1-\phi_2)\frac{v_1-p_{11}}{t} + \phi_2\frac{p_{21}-p_{11}+t}{2t}\right)$ and $\pi_1 = (p_{11} + v_2 - 2c)\,\phi_1\left((1-\phi_2)\frac{v_1-p_{11}}{t} + \phi_2\frac{p_{21}-p_{11}+t}{2t}\right) - \frac{a\phi_1^2}{2}$. Differentiating with respect to p_{11} gives, $\frac{\partial \pi_1}{\partial p_{11}} = \phi_1\left((1-\phi_2)\frac{v_1-p_{11}}{t} + \phi_2\frac{p_{21}-p_{11}+t}{2t} + (p_{11}+v_2-2c)\frac{\phi_2-2}{2t}\right)$ and $\frac{\partial^2 \pi_1}{\partial p_{11}^2} = \frac{\phi_1}{t}(\phi_2 - 2) < 0$. That is, firm i's profit is concave in own price. $v_1 - t$ being the maximum price consistent with full market coverage, it follows that $\frac{\partial \pi_1}{\partial p_{11}}\big|_{p=v_1-t} \leq 0$. $\frac{\partial \pi_1}{\partial p_{11}}\big|_{p=v_1-t} = (\phi - 2)(v_1 + v_2 - 2c - 2t) \leq 0$ only if $v_1 + v_2 - 2c - 2t \geq 0 \iff t \leq \frac{v_1+v_2}{2} - c$. Thus, for $t < \frac{v_1+v_2}{2} - c$, prices higher than $v_1 - t$ cannot be observed. That is, $p + t \leq v_1$ – which is the condition for full market coverage when firms advertise good 1.

Appendix C: Consumer Expectations and Purchase Decisions

We take up the issue of the purchase decisions of consumers given their expectations of the prices of the unadvertised good. To recapitulate, there are three cases to consider; In case i, $p_{12}^E = p_{22}^E$ and this is the case studied in the paper; In case ii, $p_{12}^E < p_{22}^E$ and in case iii, $p_{12}^E > p_{22}^E$. We consider only case ii since case iii is similar.

Let $p_{12}^E < p_{22}^E$. Depending on the advertised prices, each fully informed consumer has three options; (i) plan to buy both goods from firm 1, (ii) plan to buy both goods from firm 2 and (iii) plan to buy from both firms.

Option i yields higher surplus than option ii if and only if $2v - p_{11} - p_{12}^E - tx \geq 2v - p_{21} - p_{22}^E - t(1-x) \iff x \leq \widehat{x} = \left(p_{21} + p_{22}^E - p_{11} - p_{12}^E + t\right)/2t$. Consumers with locations $x \in [0, \widehat{x})$ plan to buy both goods at firm 1 while those with locations $x \in (\widehat{x}, 1]$ plan to buy both goods at firm 2. Similarly, option i yields higher surplus

[40] We closely follow Soberman (2004).

than option iii if and only if $2v - p_{11} - p_{12}^E - tx \geq 2v - p_{21} - p_{12}^E - tx - t(1-x) \iff x \leq \widetilde{x}_1 = (p_{21} - p_{11} + t)/t$. Consumers with locations $x \in [0, \widetilde{x}_1)$ plan to buy both goods from firm 1 while those with locations $x \in (\widetilde{x}_1, 1]$ plan to buy good 1 from firm 2 and good 2 from firm 1. This alternative is viable only if $p_{21} < p_{11}$. For, if $p_{21} \geq p_{11}$, you incur unnecessary travel expenses by shopping from both stores. Similarly, option ii yields higher surplus than option iii if and only if $2v - p_{21} - p_{22}^E - t(1-x) \geq 2v - p_{21} - p_{12}^E - tx - t(1-x) \iff x \geq \widetilde{x}_2 = (p_{22}^E - p_{12}^E)/t$. Consumers with locations $x \in [0, \widetilde{x}_2)$ plan to buy from both firms while those with locations $x \in (\widetilde{x}_2, 1]$ plan to buy both goods from firm 2.

Suppose $p_{21} < p_{11}$. If $\widetilde{x}_1 < \widehat{x} < \widetilde{x}_2$, then all consumers with locations $x \in (\widetilde{x}_1, \widetilde{x}_2)$ plan to buy good 1 from firm 2 and good 2 from firm 1. Therefore, firm 1 faces demand $\widetilde{x}_1 = (p_{21} - p_{11} + t)/t$ for the advertised good and demand $\widetilde{x}_2 = (p_{22}^E - p_{12}^E)/t$ for the unadvertised good from the fully informed consumers. Hence,

$$\pi_1 = \left(p_{11} + p_{12}^E - 2c\right)\phi_1(1-\phi_2)\tfrac{2v - p_{11} - p_{12}^E}{t} + \phi_1\phi_2\left\{(p_{11} - c)\widetilde{x}_1 + \left(p_{12}^E - c\right)\widetilde{x}_2\right\}.$$

The profit function is a bit complicated. The first part constitute profit from the captive consumers while the second part constitute profit from the fully informed consumers.

However, if $\widetilde{x}_2 \leq \widetilde{x}_1$, then option iii is dominated and a generic consumer either buys both goods from firm 1 or from firm 2. Firm 1 then faces demand $\widehat{x} = \left(p_{21} + p_{22}^E - p_{11} - p_{12}^E + t\right)/2t$ from the fully informed consumers and profits are given by[41]

$$\pi_1 = \left(p_{11} + p_{12}^E - 2c\right)\left(\phi_1(1-\phi_2)\frac{2v - p_{11} - p_{12}^E}{t} + \phi_1\phi_2\widehat{x}\right).$$

Appendix D: When Does a Symmetric Equilibrium Exist?

In the main text, we assumed that firms find it profitable to compete for the fully informed consumers. In this section, we provide a necessary condition for the existence of a symmetric equilibrium in pure strategies. In this model, each firm has two choices. Each firm can choose to compete for the fully informed consumers or to sell only to its captive consumers.

When firms compete for the fully informed consumers, in equilibrium, each firm gets profit $\pi^c = 2at^2/\left(t + \sqrt{2at}\right)^2$, where superscript c stands for competition.

What if firm 1, for instance, deviates from the symmetric equilibrium and opts to serve only its partially informed consumers? In that case, since differentiation

[41] Also notice that if $p_{11} < p_{12}$, then both goods are cheaper at firm 1 and obviously, no consumer finds it worthwhile to buy from both firms. That is, option iii is dominated.

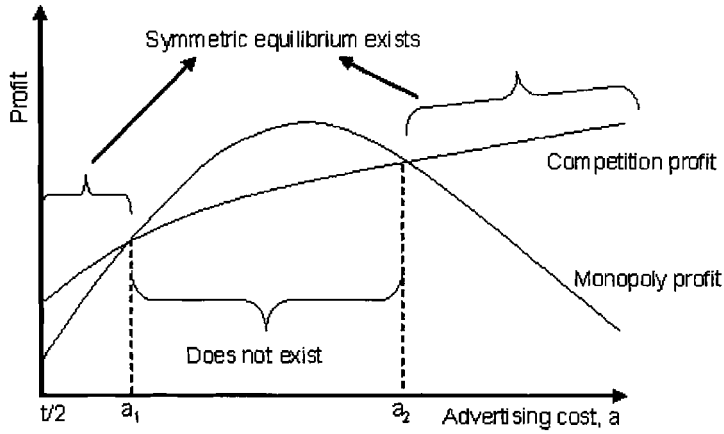


Figure 2. Existence of a symmetric equilibrium

is low, firm 1 will advertise the price $p^m = v - t$ so that it extracts all the consumer surplus of the (partially informed) consumer who travels the farthest distance. In equilibrium, firm 1 faces the demand $D^m = \phi(1-\phi)$ and its profit is given by $\pi^m = (2v - 2c - t)\,\phi\,(1-\phi) - a\phi^2/2$, where superscript m stands for monopoly[42]. Substituting for the equilibrium advertising level (equation (7)), we get:

$\pi^m = \frac{2t}{\left(t+\sqrt{2at}\right)^2}\left((2s-t)\left(\sqrt{2at}-t\right) - at\right)$; $s = v - c$.

For given s and t, firm 1 will choose to compete for the fully informed consumers, rather than serve only its captive consumers if and only if $\pi^c \geq \pi^m$. However, $\pi^c \geq \pi^m$ if and only if $2at - (2s-t)\left(\sqrt{2at}-t\right) \geq 0$. Solving for a gives;

$$a_1^* = \frac{1}{4t}\left(2s - 3t - \sqrt{4s^2 - 12st + 5t^2}\right)(2s-t)$$

$$a_2^* = \frac{1}{4t}\left(2s - 3t + \sqrt{4s^2 - 12st + 5t^2}\right)(2s-t)$$

For $a < a_1^*$, firms earn higher profits when they both serve all informed consumers. Intuitively, when the advertising cost is "low" ($a \in (t/2, a_1^*)$), the advertising intensity will be high. A higher advertising intensity implies that the majority of consumers are fully informed about the advertised prices, i.e., they have seen advertising from both firms. Because the majority of consumers have seen advertising from both firms, it follows that the market composed only of the partially informed consumers is "too thin". Consequently, firms find it profitable to compete for the fully informed consumers.

[42] The firm charges price $v - t$ on the advertised good and price v on the unadvertised good.

When the advertising cost is high however ($a \in (a_1^*, a_2^*)$), the advertising levels are not high, which implies that not many consumers receive advertising messages from both firms. Because fewer consumers are fully informed, the offsetting benefit from competing for the fully informed consumers (market size) is small[43]. On the other hand, selling only to the captive consumers allows the firm to charge the monopoly price. Since the share of fully informed consumers is relatively small, the demand loss from ignoring the fully informed consumer segment is small. Hence, for $a \in (a_1^*, a_2^*)$, at least one of the firms find it profitable to "defect" and only serve its partially informed consumers[44].

For $a > a_2^*$, the advertising levels are low, but the equilibrium price when firms compete for the fully informed consumers is high (converges to $v - t$). Thus, although fewer consumers receive advertising from both firms, the price applicable to this group is not very different from the monopoly price (price applicable to the captive consumers). As a result, it pays to compete for the fully informed consumers. Hence, for $a > a_2^*$, firms find it profitable to compete for the fully informed consumers.

To summarize (See Figure 2. above), a symmetric equilibrium in pure strategies exists for $a \in (t/2, a_1^*)$ and for $a > a_2^*$. For $a \in (a_1^*, a_2^*)$, a symmetric equilibrium in pure strategies does not exist. Firms have incentives to defect from the symmetric equilibrium when $a \in (a_1^*, a_2^*)$. This exercise also shows that "low" advertising cost is not a necessary condition for loss leader pricing. Even for "high" advertising costs ($a > a_2^*$), loss leader pricing obtains.

[43] Remember that competition for the fully informed consumers leads to lower prices.

[44] In the asymmetric equilibrium, when firm i, for instance, defects and only serve its captive consumers, it charges price $p = v - t$ for the advertised good while the other firm, firm j, charges price $v - 2t$. This price, $v - 2t$, ensures that the fully informed consumer who travels the farthest distance (unit interval) is just indifferent between buying from firm i at price $v - t$ and buying from firm j.

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PAPER 3

Exchange Rates and Product Variety

Witness Simbanegavi

ABSTRACT. We study the role of exchange rate variability in the firm's choice of whether to offer one or two varieties. The firm sells in both the Home market and the Foreign market. It is shown that variability in the exchange rate induces the firm to vertically segment markets (i.e., offer two varieties). This happens because variability in the exchange rate affects income dispersion and hence the firm's incentives to extract consumer surplus. To better extract surplus, the firm offers two price-quality menus, high quality variant (priced high) for top-end surplus extraction and a low quality variety (priced low) to address market coverage concerns. We extend the model to allow for horizontal segmentation. We find that the profitability of second degree price discrimination increases as markets become horizontally segmented. Hence, when the costs of segmenting markets are not too high, variability in the exchange rate will lead to both greater variety and international market segmentation

1. Introduction

For firms selling across national borders, the exchange rate is an important factor in strategic planning and behaviour. Fluctuations in the exchange rate have a bearing on exporting firms' competitiveness and hence profitability. Indeed, Baldwin and Krugman (1989) show that the level of the exchange rate matters for the firm's incentives to enter/ exit a foreign market.

Although there exists a large literature on firm behaviour under variable exchange rates (Dornbusch, 1987; Baum et al, 2001; Bodnar et al, 2002; Friberg, 1999; 2001) to name but a few, no study (to my knowledge) has considered the effect of exchange rate variability on firms' product variety. Yet, one of the important aspects of firm strategy concerns the number of varieties the firm offers (the product mix). According to the Economist (November 2001), the launch of the Euro has seen some firms in Europe cutting on the number of varieties they produce. The launch of the Euro meant a permanent reduction in exchange rate variability to zero within the EMU.

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The question we address in this paper is the following: Does exchange rate variability matter for a firm's choice of the number of varieties to produce?

In many cases, firms offer multiple varieties of the same product. These varieties may be differentiated by quality, size or other subtle characteristics. A simple example is that of a bookseller, who can sell the book as a paperback, hardcover or both. A more elaborate example is that of a car manufacturer, for example, BMW. There is the 1 series, 3 series the 5 and 7 series respectively. Moreover, within each series, there are several different varieties – differentiated by such things as engine capacity, leather upholstery, entertainment etc. In such circumstances, firms may not necessarily exit a foreign market when the exchange rate dips too low as Baldwin and Krugman (1989) purports. Rather, the firm may just alter its product range.

The goal of this paper is to increase our understanding of the effects on product variety of exchange rate variability (or lack of). In the context of the EMU, will the adoption of the Euro, by eliminating exchange rate variability within the Euro zone, lead to shrinking product variety across Euroland? To study this effect, we consider a monopoly firm selling in the Home market and the Foreign market. Consumers in both markets are uniformly distributed according to willingness to pay for quality. The firm can not observe each individual consumer's type, but knows the distribution from which types are drawn in each country. We assume that the Law of One Price (LOP) holds. The monopolist's problem is to choose whether to produce a single variety or two varieties.

We find that exchange rate variability engenders product variety. Changes in the exchange rate not only affect the purchasing power of consumers, more importantly, they affect income dispersion and hence the firm's incentives to extract consumer surplus. Higher income dispersion makes it harder for the firm to extract surplus with a single variety. A single variety forces the firm to price out too many consumers but at the same time fails to extract much surplus from the top end of the market. Clearly, these two concerns (market coverage and surplus extraction) are irreconcilable when the firm offers a single variety. Hence exchange rate variability induces the firm to choose the two varieties strategy. This is a classic case of second degree price discrimination – but here driven by exchange rate volatility.

Close in scope to the present study is Friberg (2001), who studies how variability in the exchange rate affects a monopoly firm's incentives to "horizontally" segment international markets. We instead investigate how variability in the exchange rate affects a monopoly firm's incentives to vertically segment markets. Other related literature include Gabszewicz et al, (1986) who study a monopoly firm's optimal product mix when the feasible range of qualities is bounded and Bonnisseau and Lahmandi-Ayed,

(2001) who study a monopoly firm's incentives to vertically segment markets when consumers are distributed according to intensity of preference rather than willingness to pay.

The paper is organized as follows: In section 2 we set out the model and derive the equilibrium profits under the alternative strategies. We state and discuss our results in section 3 and section 4 concludes the paper.

2. The Model

2.1. Model and Preliminaries. Our model is closely related to Lambertini (2000). Consider a monopoly firm selling in Home (H) and Foreign (F) markets. The firm is confronted with two mutually exclusive technologies – a "monovariety" technology that allows it to produce a good of quality q only and a "multivariety" technology that allows it to produce two qualities; a high quality, q_2 and a low quality, q_1. If the firm chooses the multivariety technology, it has to sell both varieties in each market and moreover, it has to pay an additional fixed cost $K > 0$ over and above that associated with the monovariety technology. Thus, K can be interpreted as the cost of vertically segmenting markets. For simplicity, we normalize the fixed cost associated with the monovariety technology to zero. Quality is chosen from the interval $(0, \infty)$. Total production cost for any variety q_ℓ is $\Gamma_\ell = tq_\ell^2 x_\ell$; $t > 0$ is a constant and x_ℓ is the output of variety ℓ. Thus marginal cost of producing a unit of variety q_ℓ is $c_\ell(q_\ell) = tq_\ell^2$. Observe that, for a given quality level, the marginal cost of output is constant whereas the marginal cost of quality is increasing.

Consumers in each market are uniformly distributed on $[\eta\underline{\theta}, \eta\overline{\theta}]$; $\underline{\theta} = \overline{\theta} - 1 > 0$; where θ denotes the marginal willingness to pay for quality and η is a constant measuring the level of affluence. We assume (throughout the paper) that $\eta \equiv 1$ for Home consumers and $\eta \geq 1$ for Foreign consumers. That is, the Foreign consumers are at least as rich as the Home consumers. Consumers have unit demands. A generic consumer's net utility takes the form;

$$U(\theta) = \begin{cases} \eta\theta q_i - P_{ij}; i \in \{1,2\}; j \in \{H, F\} \\ 0 \text{ otherwise} \end{cases} . \tag{1}$$

where P_{ij} is the price of quality i in market j.

Let S be the the exchange rate – the units of Home currency needed to buy one unit of Foreign currency. We make the following assumptions

Assumptions A1: (i). $\overline{\theta} \leq \frac{8}{5}$ and (ii). $S \in [0.718, 2]$.

The first assumption ensures that the market is only partially covered and the second assumption ensures that demands are nonnegative when the firm sells both

varieties in each market[1]. We further assume that the exchange rate is symmetrically distributed with a mean of unit and a finite variance, σ^2. We also assume that LOP holds, that is, $P_{iH} = SP_{iF}$.

It is straight forward to solve for the demand functions. In the single quality case, a consumer with willingness to pay θ gets surplus $\eta\theta q - P_j$ if she buys a unit of the good with quality q and 0 otherwise. Let $\widehat{\theta}$ denote the consumer for whom the individual rationality constraint just binds. Then, $\widehat{\theta} = P_j/\eta q$. Individuals for whom $\theta \in [\widehat{\theta}, \overline{\theta}]$ buy a unit of the good while those with $\theta < \widehat{\theta}$ do not buy. Thus, in each market, j, the firm faces the demand:

$$x_j = \overline{\theta} - P_j/\eta q; j = H, F. \tag{2}$$

In the multivariety case, a consumer with willingness to pay θ gets surplus $\eta\theta q_2 - P_{2j}$ when buying quality q_2 and surplus $\eta\theta q_1 - P_{1j}$ when buying quality q_1 and surplus zero when not buying. The consumer buys quality q_2 rather than quality q_1 only if $\eta\theta q_2 - P_{2j} \geq \eta\theta q_1 - P_{1j}$. Let $\widehat{\theta}_2$ denote the consumer indifferent between buying qualities q_2 and q_1. Then, $\widehat{\theta}_2 = (P_{2j} - P_{1j})/\eta(q_2 - q_1)$ and all consumers with $\theta > \widehat{\theta}_2$ buy quality q_2. Consumers with $\theta < \widehat{\theta}_2$ either buy quality q_1 or do not buy at all. They buy q_1 if and only if $\eta\theta q_1 - P_{1j} \geq 0$. Let $\widehat{\theta}_1$ denote the consumer for whom the individual rationality constraint just binds. Then, $\widehat{\theta}_1 = P_{1j}/\eta q_1$ and consumers with $\theta \in [\widehat{\theta}_1, \widehat{\theta}_2)$ buy q_1 while those with $\theta < \widehat{\theta}_1$ stay out of the market. Thus, in each market, j, the firm faces the following demands for the low and high quality varieties respectively;

$$\begin{pmatrix} x_{1j} \\ x_{2j} \end{pmatrix} = \begin{pmatrix} \frac{P_{2j}-P_{1j}}{\eta(q_2-q_1)} - \frac{P_{1j}}{\eta q_1} \\ \overline{\theta} - \frac{P_{2j}-P_{1j}}{\eta(q_2-q_1)} \end{pmatrix}. \tag{3}$$

As we stated before, our objective is to examine the role of exchange rate variability in the firm's choice of whether to offer one or two varieties. We proceed as follows; first, we derive the firm's profit (as a function of the exchange rate) under the "monovariety" and the "multivariety" strategies. Second, we calculate the difference in profits between the two strategies and study the behaviour of the resulting expression as the exchange rate changes.

2.2. Profits. The objective of the firm is to maximize expected profits. The uncertainty here arises from the fact that the level of the exchange rate will only be known after some irrevocable decisions have already been taken. We model the firm's decision as a two stage game. In the first stage and before the realization of the exchange rate, the firm decides on the quality level(s) q (q_1 and q_2). Since the exchange rate distribution is symmetric, with a mean of unit, we can simplify our analysis by supposing that

[1] For the derivation of the upper bound to $\overline{\theta}$, see Appendix A.

the choice of quality is made assuming that the exchange rate equal its expected value of unit[2]. Thereafter, the exchange rate is revealed and the firm makes a second move, the choice of price(s). We solve the problem backwards, starting with the second stage decision.

Let $n \in \{1, 2\}$ be the number of varieties offered by the firm and π_k, $k = I, II$ be the firm's profit when it offers k varieties. The firm's profits are denominated in Home currency. That is, Foreign earned profits have to be converted into the Home currency equivalent. Given the qualities chosen in the first stage, the firm's second stage behaviour is described by,

$$\prod = \max_{P_{iH}, P_{iF}} \sum_{i=1}^{n} \left\{ \left(P_{iH} - tq_i^2\right) x_{iH} + \left(SP_{iF} - tq_i^2\right) x_{iF} \right\} \text{ s.t. } P_{iH} = SP_{iF}. \tag{4}$$

Substituting the demand functions in (2) and (3) into the objective function, (4), and differentiating with respect to prices gives the equilibrium prices as a function of the quality levels. Substituting the equilibrium prices back into (4) and letting $S = 1$, the firm's first stage behaviour (choice of quality) is described by,

$$\mathbf{q}^* = \arg\max_{\mathbf{q}} \sum_{i=1}^{n} \left(P_i^* - tq_i^2\right) \left(x_{iH}\left(P_i^*\right) + x_{iF}\left(P_i^*\right)\right). \tag{5}$$

Differentiating (5) with respect to quality, q, and simplifying the resulting expressions we get[3]

$$q^* = \frac{2\eta\overline{\theta}}{3t\left(1+\eta\right)} \tag{6}$$

$$P_H^* = \frac{2\left(1+3S+4S\eta\right)\eta^2\overline{\theta}^2}{9t\left(1+S\eta\right)\left(1+\eta\right)^2} = SP_F^* \tag{7}$$

$$\pi_I = \frac{2\left(\left(2\eta+3\right)S-1\right)^2\eta^2\overline{\theta}^3}{27St\left(1+\eta S\right)\left(1+\eta\right)^3} \tag{8}$$

when the firm offers a single variety ($n = 1$) and

$$q_i^* = \frac{2i\eta\overline{\theta}}{5t\left(1+\eta\right)}; i = 1, 2 \tag{9}$$

$$P_{iH}^* = \frac{2i\left(\left(1+S\eta\right)i+5S\left(1+\eta\right)\right)\eta^2\overline{\theta}^2}{25t\left(1+S\eta\right)\left(1+\eta\right)^2} = SP_{iF}^* \tag{10}$$

[2] In an appendix available from the author on request, we discard this assumption and instead let the firm choose the optimal quality, given the distribution of the exchange rate. The conclusions of the paper are qualitatively unaffected but the expressions quickly get messy.

[3] Since the calculations are not of any particular interest here, we relegate all calculations to Appendix B.

$$\pi_{II} = \frac{\left(1-(4+2\eta)S+(5+6\eta+2\eta^2)S^2\right)4\eta^2\overline{\theta}^3}{25St(1+\eta)^3(1+S\eta)} - K \tag{11}$$

when the firm offers two distinct varieties ($n = 2$).

Notice that qualities, prices and profits are increasing in market affluence $(\overline{\theta}, \eta)$ but decreasing in the cost of quality, t. That is, as consumers become wealthier, the firm responds by raising the quality level and this enables the firm to charge higher prices (see equation B2 in Appendix B) and consequently, the firm earns higher profit. Notice also that a depreciation of the Home currency (increase in S) raises the Home price, P_H, but lowers the Foreign price, P_F and an appreciation (decrease in S) has the opposite effect. Clearly, movements in the exchange rate have wealth (income) effects. A depreciation is tantamount to an increase in the wealth of Foreign consumers and the opposite is true for an appreciation.

The literature on vertical product differentiation assert that a single product monopolist pools all consumers on the top quality (Gabszewicz et al (1986), Bonnisseau and Lahmandi-Ayed (2001)). It is therefore interesting to see how the quality levels compare in the mono and the multivariety cases. From (6), $q^* = 2\eta\overline{\theta}/3t(1+\eta)$ and from (9), $q_1^* = 2\eta\overline{\theta}/5t(1+\eta)$ and $q_2^* = 4\eta\overline{\theta}/5t(1+\eta)$. Comparing q^* and q_1^* on the one hand and q^* and q_2^* on the other hand gives $q^* - q_1^* = 4\eta\overline{\theta}/15t(1+\eta) > 0$ and respectively, $q_2^* - q^* = 2\eta\overline{\theta}/15t(1+\eta) > 0$. Hence,

LEMMA 1. *When consumers are uniformly distributed according willingness to pay for quality, and the marginal cost of quality is increasing, a single variety firm produces a good of intermediate quality compared to a multivariety firm. That is,* $q_1^* < q^* < q_2^*$.

This observation is new[4]. The objective of the firm is to extract as much consumer surplus as possible. Observe that under our assumptions, price is convex in the quality level (see equation B2). With prices strictly increasing in quality and consumers uniformly distributed according to willingness to pay for quality, top end surplus extraction calls for a high quality whereas greater market coverage calls for a low quality (and hence low price). When the firm offers a single variety, this poses a dilemma. Top end surplus extraction and greater market coverage are incompatible. Greater market coverage can only be achieved at the expense of top-end surplus extraction and vice-versa. To minimize this incongruence, the firm settles for an intermediate quality – a compromise that permits modest surplus extraction without pricing out

[4] Although Lambertini (2000) employs a similar model to the one of this paper, he does not compare the qualities across the single and the multivariety strategies. Instead, he contrasts the monopoly outcome to the social planner outcome.

too many consumers[5]. When the firm offers two qualities, the tension alluded to above falls away. The firm tailors the high quality variety for surplus extraction and the low quality variety for market coverage. Thus, q_1 is much lower and q_2 is much higher.

The tension that we emphasized above is absent in the models of Gabszewicz et al (1986) and Bonnisseau and Lahmandi-Ayed (2001). In their models, quality is costless. That is, the cost of providing the higher quality is the same as the cost of providing the lower quality. Moreover, in their models, markets are always fully covered. For these reasons, bunching occurs on the top quality. In the present model, however, the marginal cost of quality depends on the quality level. As the quality level increases, so does the price and as a result, lower willingness to pay consumers are driven out of the market – a diminution of the effective market. The need to balance surplus extraction and market coverage implies that bunching occurs on an intermediate quality.

The next issue we address is the relationship between profits and the exchange rate. Figure 1 below plots π_I and π_{II} for $\eta = 1$ (and independent of $\overline{\theta}$ and t).

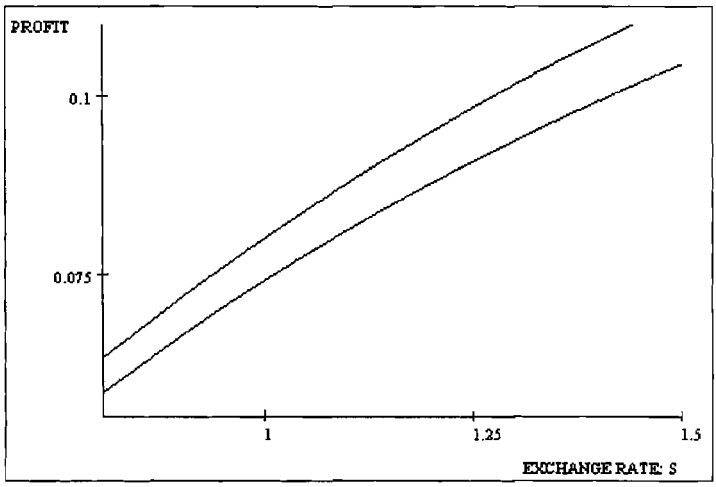

Figure 1. Plot of π_I (bottom curve) and π_{II} (top curve) as functions of the exchange rate, S.

There are two interesting things to note about Figure 1. First, π_I and π_{II} are concave in the exchange rate and second, π_I appears more concave than π_{II} and they diverge as the Home currency depreciates.

[5] A question that naturally comes to mind is the following: Why doesn't the firm offer a higher quality in the single variety case so that it only sells to the richest few? The answer follows from our assumptions. $\underline{\theta}$ and $\overline{\theta}$ are related as follows: $\underline{\theta} + 1 = \overline{\theta}$. That is, the difference in willingness to pay between the poorest and the richest individuals is not huge. Moreover, we restricted θ so that $\overline{\theta} \leq \frac{8}{5}$. That is, the market is relatively poor. Consequently, the degree of market coverage matters.

Concavity of the profit function in the exchange rate, means that the firm loses more in bad times (appreciations) than it gains in good times (depreciations). That is, expected profits are decreasing in exchange rate volatility. Hence, greater variability in the exchange rate leads to lower expected profits. A depreciation of the Home currency lowers Home market profits but raises profits from Foreign market (denominated in foreign currency) at a decreasing rate[6]. Because now Foreign profits are converted at a more favourable exchange rate, the Home currency equivalent of the Foreign profits increases with the exchange rate. Although profits from Home sales fall as the Home currency depreciates, the Home currency equivalent of Foreign profits increases with a depreciation faster than the Home market profits are falling and hence total profits increase with a depreciation. However, since a depreciation raises the Home currency equivalent of Foreign profits at a decreasing rate, total profits also increase at a decreasing rate.

What is most striking about Figure 1 is the observation that π_I is more concave than π_{II}. This means that exchange rate variability hurts the firm more if it offers a single variety. Offering two varieties allows the firm to reduce the sensitivity of profits to exchange rate surprises (i.e., it makes the profit function less concave). Hence, one may conjecture that greater variability in the exchange rate may induce the firm to choose a multivariety strategy. In the next section, we show that this is indeed the case.

3. Variability and the Number of Varieties

The question we seek to answer is the following: How does variability in the exchange rate affect a monopoly firm's incentives to offer multiple varieties? Let $\triangle\pi \equiv \pi_{II} - \pi_I$ denote the profit difference – the difference between the profit with two varieties and the profit with a single variety. That is, $\triangle\pi$ denotes the benefit (to the firm) from vertically segmenting markets when the Home and Foreign markets are integrated. We analyze the relationship between the profit difference and the exchange rate. In subsection 3.1, we consider the case where the Home and Foreign markets are equally affluent. In subsection 3.2 we relax the symmetry assumption and subsection 3.3 relaxes the LOP assumption.

Subtracting π_I from π_{II}, the profit difference, as a function of the exchange rate, S, is given by;

$$\triangle\pi\left(S;\eta\right) = \frac{\overline{\theta}^3}{t}\frac{\left(29-\left(66+8\eta\right)S+\left(45+24\eta+8\eta^2\right)S^2\right)2\eta^2}{675S\left(1+\eta\right)^3\left(1+S\eta\right)} - K. \tag{12}$$

[6] Profit in the Foreign market increases at a decreasing rate because the LOP imposes a much tighter restriction as the Home currency depreciates.

3.1. Symmetric markets ($\eta = 1$). In the symmetric markets case, (12) reduces to: $\triangle\pi(S) = \kappa(77S^2 - 74S + 29)/S(1+S)$, $\kappa = \overline{\theta}^3/2700t$. Let $\overline{S} = 1.19364$. Independent of $\overline{\theta}$ and t; $\triangle\pi(S)$ is convex for $S \leq \overline{S}$.[7] Since S is symmetrically distributed with mean of unit, we restrict ourselves (in this section) to the symmetric interval $[\underline{S}, \overline{S}] = [0.80636, 1.19364]$. Figure 2 below plots the profit difference, $\triangle\pi$, as a function of the exchange rate, S.

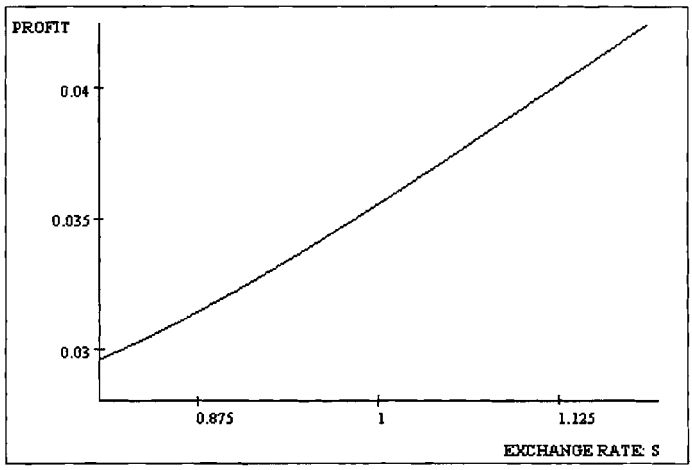

Figure 2. Profit difference as a function of the exchange rate.

We see from Figure 2 that when variability is restricted to the interval $[\underline{S}, \overline{S}]$, the profit difference is convex in S. That is, the expected profit difference is increasing in exchange rate volatility. To be more precise, suppose K – the cost of vertically segmenting markets – is such that $\triangle\pi(1) = 0$. That is, K is such that the firm is indifferent between the single and the two variety strategies when the exchange rate is equal to unit. Then we have the following result;

PROPOSITION 1. *A firm that is indifferent between offering a single variety and two varieties when the exchange rate is fixed and equal to its mean (of unit) strictly prefers to offer two varieties when the exchange rate is mildly stochastic.*

PROOF. We have seen above that $\triangle\pi(S)$ is convex for $S \in [\underline{S}, \overline{S}]$. Because the exchange rate is symmetrically distributed, it follows that $E_S[\triangle\pi(S)] > \triangle\pi(1) = 0$. Hence the firm offers two varieties. □

[7] Formally, $\frac{\partial^2(\triangle\pi(S))}{\partial S^2} = \kappa\frac{174S+174S^2-302S^3+58}{S^3(1+S)^3} > 0 \forall S < 1.19364$.

The intuition rests on the interaction between the following two observations; namely, that exchange rate variability has wealth /income effects and that a single variety makes surplus extraction more difficult (Lemma 1). Exchange rate variability affects the purchasing power of consumers, but more importantly here, it affects income dispersion[8]. It is this effect (effect on income dispersion) that matters for the firm's choice of product range (see, for example, Gabszewicz et al, 1986). For example, a depreciation makes the Foreign market richer but also generates greater dispersion in willingness to pay. This confronts the firm with a dilemma: On the one hand, as the Foreign market gets richer, the firm finds it more costly to price out many consumers (by charging a higher price). On the other hand, the higher spread in willingness to pay creates incentives for the firm to want to charge a higher price so as to extract more surplus from the top end of the market. These two cannot be reconciled with the firm offering a single variety. The firm thus needs to offer both high and low quality varieties. Offering two varieties is more profitable because it enhances surplus extraction at the top and at the same time permits greater market coverage[9].

Since the marginal cost of quality is increasing in the quality level, the higher gross surplus (higher price) associated with the high quality variety does not necessarily imply higher unit margins. If unit costs rise fast enough, it is possible that the unit margin is lower for the high quality variety compared to the unit margin associated with the intermediate variety. However, this is not the case in the present model. In fact, it can be shown that for all S in $[.712, 2]$, *the unit margin is higher for quality* q_2 *compared to quality* q *while it is higher for quality* q *compared to quality* q_1. Thus, the superiority of the two variety strategy over the single variety strategy derives from the former's ability to generate higher unit margins at the top. This discrepancy in the ability to extract *net* surplus ensures that, when the exchange rate is stochastic, second degree price discrimination is more profitable than selling a single variety to all consumers[10].

The ability to extract surplus is however curtailed by arbitrage concerns. A "strong" depreciation of the Home currency implies a significant reduction in Foreign prices and this weakens surplus extraction in the Foreign market. However, the *conversion effect* – the effect of the level of the exchange rate on Foreign earned profits expressed in terms of the Home currency – mitigates this diminished ability to extract surplus by

[8] In our model, consumers have unit demands, so an increase in income will not translate into a corresponding increase in demand by an individual consumer.

[9] Gabszewicz (1983) show that there are instances where a larger heterogeneity in willingness to pay may lead to fewer (and not more) varieties being offered by a monopolist.

[10] In general, it is possible that the unit margin is lower for the high quality variety and higher for the low quality variety (Srinagesh and Bradburd, 1989; p. 103).

converting Foreign profits at a more favourable rate. It is the interplay between the conversion effect and the LOP that determines the curvature of the profit difference. The stronger the depreciation, the more LOP binds and the less convex the profit difference becomes.

The above Proposition parallels Friberg (2001) – exchange rate volatility affects consumers' purchasing power and hence the firm's incentives to price discriminate. In Friberg (2001), the firm responds to exchange rate variability by horizontally segmenting markets in order to third degree price discriminate. Here, we allow for spatial arbitrage and the firm responds to exchange rate variability by vertically segmenting markets in order to second degree price discriminate.

We have shown that exchange rate variability matters for firms' decisions with respect to the number of varieties to produce. Specifically, variability in the exchange rate engenders provision of multiple varieties. We find some support, albeit anecdotal, for our finding. The Economist (November, 2001) discusses how multiproduct firms selling in the EU are responding to the launch of the Euro. The introduction of the Euro meant a permanent reduction of exchange rate variability to zero within the EMU. According to the Economist, some firms, for example, Unilever and Procter & Gamble have started trimming the number of brands they offer so that they can concentrate on a few brands while others, for example, Nestle, Henkel and Danone are increasingly using the same brand name on their products across Euroland. These observed responses are in line with the predictions of our model. Elimination of exchange rate variability diminishes product variety because it reduces (expected) future dispersion in income and hence the incentive to crowd the product space[11].

3.2. Asymmetric markets. In this subsection, we assume that the Foreign market is richer[12], that is, $\eta > 1$. Figure 3 plots $\triangle\pi\,(S;\eta)$ for different values of η.

[11] Prior to the formation of the EMU, the now EMU countries were characterized by semi-fixed exchange rates (mild volatility). That is, exchange rate variability between the currencies of the now EMU member countries was rather low compared to variability with non EMU countries.

[12] It makes no (qualitative) difference whether the Home market or the Foreign market is richer.

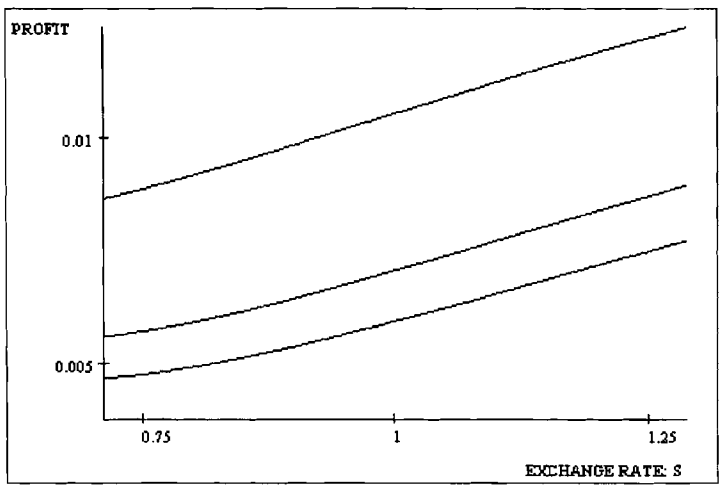

Figure 3. Plot of $\Delta\pi(S;\eta)$ for $\eta = 1$ (lowest curve), $\eta = 1.2$ and $\eta = 2$ (highest curve).

We summarize figure 3 as follows,

Result 1: *Let $I(\eta)$ denote the interval $\left[\underline{S}(\eta), \overline{S}(\eta)\right]$ for which $\Delta\pi(S;\eta)$ is convex in S, given η. Then, (i) the profit difference, $\Delta\pi(S;\eta)$, is increasing in η and (ii) $I(\eta) \subset \left[\underline{S}, \overline{S}\right]$ shrinks as η increases and (iii) The profit difference becomes less convex as the Foreign market gets richer.*

Result 1 says that an increase in affluence raises the profitability of the multivariety strategy. Notice that an increase in η increases the dispersion in consumers' willingness to pay in the Foreign market. In both the single and the two variety strategies, the firm responds by raising the quality level when the Foreign market gets richer. Raising the quality level allows the firm to charge higher prices (see equation B2 in Appendix B). When the firm sells two varieties, both the quality gap and the price gap increase with η and this augurs well for surplus extraction. Moreover, for all S in the relevant range, the price of the high quality variety increases faster with affluence compared to the price of the intermediate variety, which in turn increases faster compared to the price of the low quality variety. That is, gross surplus extraction at the top is higher and market coverage is higher with the two variety strategy.

What is of interest to the firm however, is the net surplus extracted per unit sold – the unit margin. Although the marginal cost of quality increases with η, the increase in price is more than enough to compensate for the increase in the cost of quality. More precisely, the unit margin increases with η and the increase is larger the higher

the quality level[13]. Clearly, the two variety strategy (vertical segmentation) permits greater surplus extraction at the top as well as greater market coverage[14]. Consequently, profits increase more with affluence under vertically segmented markets – hence the profit difference increases with η. As before however, LOP constrains top end surplus extraction more as the Foreign market gets richer and this makes the profit difference less convex.

3.3. Segmented markets. Although it serves as a useful benchmark, the assumption of "fully" integrated goods markets is not completely realistic. As Goldberg and Knetter (1997; p. 1246) point out, LOP holding requires costless transportation, distribution and resale. These three requirements are impossible to satisfy in practice, hence markets in different countries are at best only partially integrated. In this subsection, we allow the firm to "buy itself free" of spatial arbitrage at some (fixed) cost and study the role of exchange rate variability in the firm's choice of whether to offer one or two varieties. Conceivably, one would expect that as markets become (horizontally) segmented, it becomes even more profitable to offer two varieties since the pricing constraint is relaxed.

Let $\triangle\pi^{seg}(S)$ be the profit difference when the Home and Foreign markets are horizontally segmented (see Appendix C). $\triangle\pi^{seg}$ is convex in the exchange rate and it lies everywhere above $\triangle\pi$ (except at $S = 1$, where $\triangle\pi^{seg}(1) = \triangle\pi(1)$). Therefore, $E_S[\triangle\pi^{seg}(S)] > E_S[\triangle\pi(S)]$ whenever the exchange rate is stochastic. Thus, our result is strengthened. Since expected profits are higher when markets are horizontally segmented, it follows that whenever the firm finds it optimal to offer two varieties when LOP holds, it will also offer two varieties when markets are (horizontally) segmented but not the other way round. That is, exchange rate variability raises the profitability of vertically segmenting markets when Home and Foreign markets are horizontally segmented.

We defined $\triangle\pi^{seg}(S)$ as the net benefit of adopting the two variety strategy when Home and Foreign markets are horizontally segmented and $\triangle\pi(S)$ as the net benefit of adopting the two variety strategy when the markets are integrated. It follows therefore that $\triangle\pi^{seg}(S) - \triangle\pi(S)$ measures the net benefit of horizontally segmenting the Home and Foreign markets. Independent of $\overline{\theta}$ and t, the difference, $\triangle\pi^{seg}(S) - \triangle\pi(S)$, is

[13] That is, $\partial\left(P - tq_i^2\right)/\partial\eta = \mu q_i, \mu > 0$; which is increasing in the quality level (P is given by equation B2 in Appendix B).

[14] Although demand for the high quality variety falls when η increases (because prices increase with η), the consumers who can no longer afford to buy the high quality variety will turn to the low quality variety – rather than exit the market. This virtue of the two variety strategy mitigates the negative effect on demand of increases in quality (and hence price). With a single variety however, an increase in η significantly lowers market coverage as all consumers who can no longer afford to buy the good have no alternative but to leave the market.

convex in the exchange rate[15]. Hence, expected profits are increasing in exchange rate volatility. Figure 4 below plots $(\triangle\pi^{seg}(S) - \triangle\pi(S))$, as a function of the exchange rate, S.

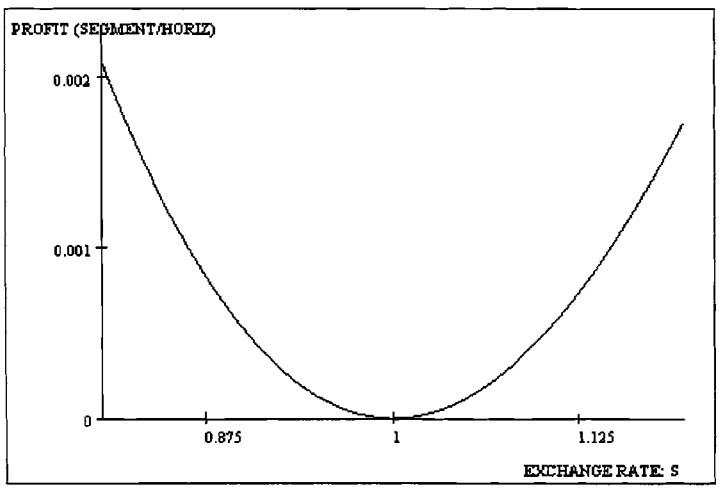

Figure 4. Plot of $\triangle\pi^{seg}(S) - \triangle\pi(S)$ as a function of the exchange rate.

We summarize the above discussion, together with Figure 4 in the following proposition:

PROPOSITION 2. *The expected benefit from horizontally segmenting the Home and Foreign markets increases as exchange rate volatility increases (Friberg, 2001).*

LOP imposes a constraint on the firm's pricing and this makes the Home and Foreign prices perfectly negatively correlated. This negative relationship impacts negatively on the firm's ability to extract consumer surplus[16]. When the markets are segmented however, the Home and Foreign prices are no longer perfectly negatively correlated. That is, increases (decreases) in the Foreign prices need not be accompanied by decreases (increases) in Home prices and this enhances the firm's ability to extract surplus and thus raises profits. Put simply, horizontal segmentation allows the firm to charge the optimal price in each market. Since in general, the optimal prices for

[15] $\triangle\pi^{seg}(S) - \triangle\pi(S) = \lambda(S^2 - 2S + 1)/(60S + 60)$, for some positive constant, λ and $\partial^2(\triangle\pi^{seg} - \triangle\pi)/\partial S^2 = 2\lambda/(45S + 45S^2 + 15S^3 + 15) > 0 \forall S$.

[16] When LOP holds, the price of the high quality variety is more responsive to changes in the exchange rate than the price of the low quality variant. That is, $\left|\frac{\partial P_{2j}}{\partial S}\right| > \left|\frac{\partial P_{1j}}{\partial S}\right|; j \in \{H, F\}$. For example, when the home currency depreciates, P_{2F} falls more than P_{1F} and this weakens the firm's ability to extract surplus from the top end of the market. Therefore, the greater is exchange rate variability, the more LOP constraints surplus extraction.

the Home and Foreign markets differ, this possibility to charge different prices cannot lower the firm's profits[17].

$\triangle\pi^{seg}(S) - \triangle\pi(S)$ can also be written as $(\pi_{II}^{seg} - \pi_{II}) - (\pi_{I}^{seg} - \pi_{I})$ which simplifies to $\frac{(S^2-2S+1)}{60S+60} \geq 0$ for all S in the relevant range. That is, the expected profit from horizontally segmenting is higher when the firm offers two varieties than when the firm sells a single variety. The difference between $(\pi_{II}^{seg} - \pi_{II})$ and $(\pi_{I}^{seg} - \pi_{I})$ therefore measures the net benefit from vertically segmenting (offering two varieties). We can therefore generalize Proposition 1 as follows;

COROLLARY 1. *When the exchange rate is stochastic, the firm is better off with the multivariety strategy as compared to the monovariety strategy.*

Propositions 1 and 2 together with Corollary 1 show that exchange rate variability may induce a monopoly firm to undertake (invest in) both horizontal and vertical market segmentation. Horizontal segmentation allows the firm to sell the same variety (either q_1 or q_2) at different prices in the Home and Foreign markets. This is third degree price discrimination. At the same time, vertical segmentation allows the firm to extract more surplus from the top end of the market (by selling q_2 at a higher price in each market) still maintaining a sizable demand at the low end of the market (by selling q_1 at a lower price). The firm offers two price-quality menus, (P_{2j}, q_2), (P_{1j}, q_1); $j \in H, F$; and let consumers self select. This is second degree price discrimination. In a nutshell, when the costs of segmenting markets are not too high and consumers are uniformly distributed according to willingness to pay for quality, exchange rate variability simultaneously leads to third degree price discrimination across national markets and second degree price discrimination within markets.

3.4. Importance of the result, a quantitative assessment. How much does a variable exchange rate raise expected profits relative to profits when the exchange rate is permanently fixed at unit? Is this increase quantitatively important? As we can see (Figure 2), the *profit difference* function is rather flat. We calculate the percentage increase in the profit difference (gross of the fixed cost, K) for chosen realizations of the exchange rate.

We find a progressive effect of exchange rate variability on percentage increase in profits. For variability within 0.05, 0.15 and 0.18 units from the mean, expected profits increase by as much as 0.1%, 1% and 1.5% respectively.

These numbers are not huge, but they are also non-trivial. For example, if the profit difference under a "fixed" exchange rate were SEK 10 000 000, then a one percent

[17] The firm can still charge the same price in both markets if it so wishes.

increase in expected profits (due to exchange rate variability) would mean a SEK 100 000 increase in expected profits. This is not insignificant.

4. Conclusion

We extend the literature on monopoly and product mix by considering how variability in the exchange rate affects the variety range offered by a monopoly firm selling at home and abroad. Starting in a situation where there is no variability in the exchange rate, we show that introducing variability induces the firm to expand the number of varieties produced. The mechanism works through the effect of exchange rate volatility on the dispersion of income. A higher dispersion of income makes it harder for the firm to significantly extract surplus from the top end of the market under a single variety strategy. Hence, variability in the exchange rate leads to more varieties being offered. In other words, two countries that are served by a single monopoly firm can adopt a single currency (currency union) only at a cost of diminished product variety. This is an interesting implication in light of the adoption of the Euro. The result seems to matter quantitatively, that is, the effect on profits is non-negligible. We also show that the result holds under more realistic assumptions (partially integrated or fully segmented markets).

As an extension, it would be interesting to introduce competition into the model (a duopoly for instance) and study how variability in the exchange rate affects the firms' incentives to vertically segment markets. Other possible extensions include an investigation of potential welfare effects. If consumers value variety per se, then the example from the Economist (Nov. 2001) suggests a welfare loss as the firms mentioned therein all respond to the introduction of the Euro by reducing variety of the products they sell. However, in general, welfare is more than just product variety. Hence there is need for a systematic analysis.

Appendix

Appendix A: Partial Market Coverage

Partial market coverage obtains whenever $\underline{\theta} < \frac{P}{q}$, where $\frac{P}{q}$ is the marginal willingness to pay (MWTP) for quality of the individual indifferent between purchasing a unit of quality q at price P and purchasing nothing. Let $\overline{\theta}_{jk}$ be the highest WTP consistent with partial market coverage in market j when the firm offers k varieties.

Consider the single variety case. For the Home market, the condition $\underline{\theta} < \frac{P_H}{q}$ implies $\overline{\theta}_{H1} \equiv \frac{6(1+S)}{5-S}$ and in the Foreign market, the condition $\underline{\theta} < \frac{P_F}{q}$ implies $\overline{\theta}_{F1} \equiv \frac{6S(1+S)}{(3S+1)(2S-1)}$, where we make use of the fact that $\underline{\theta} = \overline{\theta} - 1$.

In the two varieties case, market coverage is determined by the low quality variety. In the Home market, the condition $\underline{\theta} < \frac{P_{1H}}{q_1}$ implies $\overline{\theta}_{H2} \equiv \frac{10(1+S)}{9-S}$ and in the Foreign market, the partial market coverage condition $\underline{\theta} < \frac{P_{1F}}{q_1}$ implies $\overline{\theta}_{F2} \equiv \frac{10S(1+S)}{10S^2-S-1}$, where P_{1j} is the price of quality 1 in market $j; j = H, F$.

In the Home market, $\overline{\theta}_{H1} - \overline{\theta}_{H2} = \frac{8S+4S^2+4}{S^2-14S+45} > 0$ for all S in the relevant range. That is, the binding constraint is $\overline{\theta}_H \leq \overline{\theta}_{H2}$. In the Foreign market, $\overline{\theta}_{F1} - \overline{\theta}_{F2} = \frac{4S+8S^2+4S^3}{2S-15S^2-16S^3+60S^4+1} > 0$ for all S in the relevant range. Hence, the binding constraint is $\overline{\theta}_F \leq \overline{\theta}_{F2}$. Thus partial market coverage in the two varieties case implies partial market coverage in the single variety case **but not the other way round**. We have partial market coverage in both markets with two varieties if $\overline{\theta} < \min\left\{\overline{\theta}_{H2}, \overline{\theta}_{F2}\right\}$. Evaluating $\overline{\theta}_{H2}$ and $\overline{\theta}_{F2}$ at $S = \{0.7, 2\}$ gives $\min\left\{\overline{\theta}_{H2}, \overline{\theta}_{F2}\right\} = \{1.6216\}$. In a sense, partial market coverage requires that markets be rather poor.

Appendix B: Derivation of Profits

Below we derive the firm's profits under the single, and respectively, the two variety strategies.

Single variety strategy

Let π_I be the profit when the firm sells a single variety in both markets. In the second stage, the firm chooses prices P_H and P_F, given quality q chosen in the first stage, and her behaviour is described by

$$\text{(B1)} \qquad \pi_I = \max_{P_H, P_F} \left\{ \left(P_H - tq^2\right) x_H + \left(SP_F - tq^2\right) x_F \right\} \text{ s.t. } P_H = SP_F.$$

where the demands (x_H and x_F) are given by equation (2).

Differentiating (B1) with respect to P_H and P_F and simplifying the first order conditions yields

$$\text{(B2)} \qquad P_H^* = \frac{1}{2S\eta + 2}\left(2Sq\overline{\theta}\eta + q^2 t + Sq^2 t\eta\right) = SP_F^*.$$

Substituting (B2) into (B1) (assuming $S = 1$) gives π_I as a function of q only. In the first stage, the firm chooses q to maximize $\pi_I(q)$ and her optimal behaviour is described by[18]

$$q^* = \arg\max_q \left(P^* - tq^2\right)\left(x_H(P^*) + x_F(P^*)\right). \tag{B3}$$

Differentiating (B3) with respect to q and simplifying yields

$$q^* = \frac{2\eta\overline{\theta}}{3t(1+\eta)}. \tag{B4}$$

Substituting (B4) into (B2) gives

$$P_H^* = \frac{2(1+3S+4S\eta)\eta^2\overline{\theta}^2}{9t(1+\eta S)(1+\eta)^2} = SP_F^* \tag{B5}$$

and substituting (B5) and (B4) into (B1) gives

$$\pi_I^* = \frac{(4S\eta+6S-2)(2S\eta+3S-1)\eta^2\overline{\theta}^3}{27St(1+\eta S)(1+\eta)^3}. \tag{B6}$$

Two variety strategy

Let π_{II} be the profit when the firm sells two varieties in each market. As before, the firm chooses quality basing on the expected exchange rate and then after the revelation of the exchange rate, choose prices. We solve the problem backwards. In the second stage, given the qualities q_1 and q_2 chosen in stage 1, the firm chooses prices, $P_{iH}, P_{iF}; i = 1, 2$ to solve,

$$\pi_{II} = \max_{P_{iH},P_{iF}} \sum_{i=1}^{2}\left(P_{iH} - tq_i^2\right)x_{iH} + \sum_{i=1}^{2}\left(SP_{iF} - tq_i^2\right)x_{iF} - K \quad \text{s.t. } P_{iH} - SP_{iF} = 0. \tag{B7}$$

where the demands (x_{iH} and x_{iF}) are given by equation (3). Differentiating (B7) with respect to P_{iH} and P_{iF} and simplifying the first order conditions yield

$$P_{iH}^* = \frac{1}{2S\eta+2}\left(2S\overline{\theta}\eta q_i + tq_i^2 + St\eta q_i^2\right) = SP_{iF}^*. \tag{B8}$$

Substituting (B8) into (B7) (assuming $S = 1$) gives $\pi_{II}(q_1, q_2)$. In the first stage, the firm chooses q_1 and q_2 to maximize $\pi_{II}(q_1, q_2)$. That is, her optimal behaviour is described by

$$\mathbf{q}^* = \arg\max_{q_1,q_2} \sum_{i=1}^{2}\left(P_i^* - tq_i^2\right)\left(x_{iH}(P_i^*) + x_{iF}(P_i^*)\right). \tag{B9}$$

[18] Notice that $\pi_I(q) \neq \pi_I$ since now we assume $S = 1$. Observe also that $P_H^* = P_F^* = P^*$.

Differentiating (B9) with respect to q_i and simplifying yields

$$q_i^* = \frac{2i\eta\overline{\theta}}{5t\,(1+\eta)}; i = 1, 2. \tag{B10}$$

Substituting (B10) into (B8) gives

$$P_{iH}^* = \frac{2i\left(\left(1+S\eta\right)i + 5S\left(1+\eta\right)\right)\eta^2\overline{\theta}^2}{25t\left(1+S\eta\right)\left(1+\eta\right)^2} = SP_{iF}^* \tag{B11}$$

and substituting (B11) and (B10) into (B7) gives the profit function under the two variety strategy as

$$\pi_{II}^* = \frac{\left(1 - 4S - 2S\eta + 5S^2 + 6S^2\eta + 2S^2\eta^2\right)4\eta^2\overline{\theta}^3}{25St\left(1+\eta\right)^3\left(1+S\eta\right)} - K\ . \tag{B12}$$

Appendix C: Segmented Markets

The firm chooses prices to solve the problem;

$$\pi_{II}^{seg} = \max_{P_{iH}, P_{iF}} \sum_{i=1}^{n} \left[\left(P_{iH} - tq_i^2\right)x_{iH} + \left(SP_{iF} - tq_i^2\right)x_{iF}\right]. \tag{C1}$$

This is analogous to equation (B7) except that now we do not require Home and Foreign prices to be equal when expressed in terms of a common currency. Solving as we did before gives

$$\left(\pi_I^{seg}, \pi_{II}^{seg}\right) = \left(\frac{\left(1 - 2S + 9S^2\right)}{108St}, \frac{\left(1 - 2S + 5S^2\right)}{50St}\right)\overline{\theta}^3.$$

Hence

$$\triangle\pi^{seg} = \frac{\left(29 - 58S + 45S^2\right)\overline{\theta}^3}{2700St} - K. \tag{C2}$$

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