Macroeconomic Aspects of Capital Flows to Small Open Economies in Transition

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To Anna

Contents

Acknowledgements

When I finished my undergraduate studies, I did not feel that I knew enough about economics. I wanted more, and I am indebted to *Paul Klein* for helping me find my way to the Stockholm Doctoral Course Program in Economics.

I entered the program with a general interest in macroeconomics and a curiosity about the forces at play in a globalized economy where capital can move around the world within seconds. Particularly, I had been fascinated by the effects of the South East Asian crisis on the Swedish economy, which to me seemed very remote from any of the events that hit the Asian markets.

During the first two years of coursework, I was fortunate to take two courses taught by Professor *Lars Ljungqvist*, who impressed me with his deep understanding of macroeconomics, and whose enthusiastic way of teaching conveyed that economics was exciting and important. I am grateful to Lars for accepting a rather crazy term paper on bailouts (it contained something as rare as a time warp in the production process!), and for accepting to become my thesis advisor. Lars has given guidance by indicating the direction and letting me discover things on my own. When after a lot of pondering, Rudolfs and I finally managed to figure out what Lars had been talking about two months ago, Lars always raised new questions and showed us the direction to the next level. Thank you Lars for making the writing of my thesis such a fruitful learning experience! I also want to express my thanks for your support during periods of doubt and for making me apply to the Riksbank, although I had not gotten my act together in time and the deadline had been passed.

One distinguished researcher who in my third year seemed to have a lot of answers to the questions I wanted to learn more about was Professor *Timothy J. Kehoe*. Rudolfs and I were lucky to get accepted to the Workshop on Dynamic Macroeconomics, which Tim helps organizing each year in Vigo, Spain. We presented a slightly less crazy version of the paper on bailouts, and got a first view of the international research community in our field. Tim's visit to Stockholm in November 2002 really gave our research a boost and inspired us to delve deeper into the world of calibrations and simulations. Tim, your take on economics as something very serious but at the same time fun is contagious, and I am grateful to you for your encouragement and for your generous help in different aspects of my academic life!

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The person with whom I spent more time than anyone else during parts of my studies is *Rudolfs Bems* (please forgive me, Anna). I started writing that crazy term paper on bailouts together with Rudolfs, since we had similar interests and since I felt that it would be more fun to work together on a project. I have never regretted that decision. To me, our co-operation has been very fruitful and based on equal contributions. It would be wrong to say that we complement each other in terms of academic knowledge; if one of us does not know how to solve a problem, it is a safe bet that the other does not know either. But the fact that we have been two has enabled us to be bolder and more enduring in our attempts at finding solutions and coming up with ideas. With the roles changing from day to day, innumerable discussions (some of them rather heated) have been played out along the lines: -"Wait, I know. It's like this ...", -"No, no! You are wrong, because...". The right answer usually lay several debates down the road, but our dialogues propelled us forward on a common journey that I enjoyed immensely. Outside of our models, we have developed a real friendship and I am grateful to you, Rudolfs, for putting things into perspective and bringing me down to earth at times when I got overly worked up about things like deadlines , 'final versions' and the future.

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the golf course, where he became my mentor (I once crashed my car while listening to his authoritative advice on the way home from one of our games!). In the last years, *Fredrik Wilander* has become a good friend both on and off the course.

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Introduction and Summary

With the internationalization of financial markets, short-term capital flows to emerging market economies have become an important phenomenon in the world. Over the last two decades, international portfolio investment has rapidly increased, which has made capital flows from rich to developing countries more volatile and prone to sudden reversals. The papers in this dissertation are concerned with investigating the effects of such flows in the receiving countries. The analysis is cast in a dynamic general equilibrium framework for small open economies.

Two of the papers are quantitative investigations of the forces at work in small and relatively poor economies that liberalize trade and capital flows. The common approach of these papers is that of a computational experiment: calibrated simulations constitute a test of whether the models can explain certain dynamics which we observe in the data.

The first paper investigates whether a calibrated two-sector neoclassical growth model can explain the magnitudes and the timing of capital flows in the Baltic countries after the fall of the Soviet Union. The results indicate that it can, and that the large and persistent trade deficits which we observe in the data need not be a reason to worry. However, the model also tells us that a reversal of capital flows and large sectoral adjustments lie ahead of the Baltic countries.

In the second paper, the focus is on modelling the observed co-movement between consumption and the real exchange rate in Spain, which experienced large capital inflows following the entry into the European Community in 1986. In accordance with episodes of trade liberalization elsewhere, consumption in Spain boomed and the real exchange rate appreciated for several years after 1986. Standard two-sector models with traded and non-traded goods have problems accounting for these facts. The paper explores some mechanisms that can improve the standard modelling framework, and evaluates their quantitative importance in calibrated simulations for Spain.

The third paper studies the government's optimal bailout policy in an environment where sudden stops of capital flows cause financial crises in a small open economy. Real world events, such as the financial crises in the South East Asian countries in 1997, motivate the analysis. Compared to the previous essays, the paper is different in its nature in that it develops a highly stylized environment to analytically study the government's optimal bailout policy. The paper shows that the government should optimally commit to a policy that only partially protects private debtors against inefficient liquidation.

Chronologically, the thesis developed from the paper on bailouts during financial crises in emerging markets. Thinking about how the government should deal with financial panics and sudden reversals of capital flows, we became interested in the modelling of capital flows on its own. The result was the paper on trade deficits in the Baltic countries, which took shape after several months of trying to figure out how to solve dynamic models on a computer. The model used in that paper turned out to explain capital flows rather well, but we could not get the dynamics right for the real exchange rate. Curious about the potential of the two-sector framework, I spent my last year in the Ph.D. program looking into possible remedies for this failure of the standard model.

Essay I: "Trade Deficits in the Baltic States: How Long Will the Party Last?" (with Rudolfs Bems)

Since their opening up to international capital markets, the economies of Estonia, Latvia and Lithuania have experienced large and persistent capital inflows and trade deficits. This paper investigates whether a calibrated two-sector neoclassical growth model can explain the magnitudes and the timing of the trade flows in the Baltic countries. The model is calibrated for each of the three countries, which we simulate as small closed economies that suddenly open up to international trade and capital flows. The results show that the model can account for the observed magnitudes of the trade deficits in the 1995-2001 period. Introducing a real interest rate risk premium in the model increases its explanatory power. The model indicates that trade balances will turn positive in the Baltic states around 2010.

Essay II: "Real Exchange Rate and Consumption Fluctuations following Trade Liberalization"

Two-sector models with traded and non-traded goods have problems accounting for the stylized facts that the real exchange rate appreciates and consumption booms for several years following trade liberalization, or exchange-rate-based stabilization programs, in small open economies. The paper investigates some possible solutions to this 'price-consumption puzzle' and evaluates their quantitative importance in calibrated simulations of Spain's accession to the European Community in 1986. Extending the standard two-sector framework, the paper investigates the effects of relative productivity growth in the traded sector along the lines of Balassa-Samuelson, of time-to-build, and of habit formation in preferences. The analysis shows that a calibrated version of the augmented model can account for more of the price-consumption dynamics after trade liberalization than a benchmark two-sector model, without losing explanatory power for other real variables in the Spanish economy after 1986.

Essay III: "Financial Crisis in Emerging Markets and the Optimal Bailout Policy" (with Rudolfs Bems)

This paper develops a framework for analyzing optimal government bailout policy in a dynamic stochastic general equilibrium model where financial crises are exogenous. Important elements of the model are that private borrowers only internalize part of the social cost of foreign borrowing in the emerging market, and that the private sector is illiquid in the event of a crisis. The distinguishing feature of our paper is that it addresses the optimal bailout policy in an environment where there are both costs and benefits of bailouts, and where bailout guarantees potentially distort investment decisions in the private sector. We show that it is always optimal to commit to a bailout policy that only partially protects investment against inefficient liquidation, both in a centralized economy and a market economy. Due to overinvestment in the market economy, the government's optimal level of bailout guarantees is lower than in the social optimum. Further, we show that, in contrast to a social planner, the government in the market economy should optimally bail out a smaller fraction of private investments when the probability of a crisis increases.

Essay I

Trade Deficits in the Baltic States: How Long Will the Party Last?*

Rudolfs Bems and Kristian Jönsson

Stockholm School of Economics

Abstract

Since their opening up to international capital markets, the economies of Estonia, Latvia and Lithuania have experienced large and persistent capital inflows and trade deficits. This paper investigates whether a calibrated two-sector neoclassical growth model can explain the magnitudes and the timing of the trade flows in the Baltic countries. The model is calibrated for each of the three countries, which we simulate as small closed economies that suddenly open up to international trade and capital flows. The results show that the model can account for the observed magnitudes of the trade deficits in the 1995-2001 period. Introducing a real interest rate risk premium in the model increases its explanatory power. The model indicates that trade balances will turn positive in the Baltic states around 2010.

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1 Introduction

As the transition in Estonia, Latvia and Lithuania enters its second decade with trade deficits and capital inflows that show no signs of reversal, the general public and the Baltic politicians in particular are increasingly concerned about the consequences of the large deficits. Most economists approach the issue using the neoclassical framework and make the point that external deficits in poor countries are not a problem, but rather a sign of healthy development, as long as the foreign capital is wisely invested in the local economy. However, an elementary ingredient in the neoclassical message is that the developing country will, sooner or later, have to start repaying its foreign creditors.

Is there really no reason to worry about the size of the trade deficits in the Baltic states, as long as ten years after liberalization? The aim of this paper is to give a quantitative answer to that question. Calibrating and simulating a two-sector neoclassical growth model for each of the Baltic states, we investigate whether the trade deficits implied by the theory are in line with the magnitudes observed in the data. In the simulations of the Baltic countries as initially closed economies that suddenly open up to trade, we pinpoint the predicted timing of capital flow reversals in the model.

The type of model we employ is sometimes referred to as 'the dependent economy model' (Turnovsky, 1997). It is a standard two-sector model of a small open economy with a traded good, a non-traded good, labor, capital and an investment good that augments the capital stock. Traded and non-traded goods are either consumed or used as inputs into the investment sector, in which case we can consider them as equipment and structures. Previous literature includes many applications of the model: Fernandez de Cordoba and Kehoe (2000) apply the model to study the Spanish economy after its entry into the European Community in 1986. Slightly different versions of the same model have been used to study the consequences of exchange-rate based stabilization programs in countries such as Portugal (Rebelo, 1993) and Argentina (Burstein, Neves and Rebelo, 2003).

In this paper, we use the same basic model as in Fernandez de Cordoba and Kehoe (2000), where the authors point to the importance of incorporating frictions in factor mobility for the two-sector growth model to explain data on capital flows and real exchange rates. Our paper builds on this finding and contributes to the development of the dependent economy model by introducing investment transformation costs that have been modeled by Abel and Eberly (1994) and empirically estimated by Eberly (1997) and labor adjustment costs of a form that can be calibrated to each of the Baltic states.

The sudden change of the economic system in the Baltic states after their independence in 1991 and an almost immediate liberalization make the countries well-suited as test cases for the model. Estonia, Latvia and Lithuania were completely closed off from the west before 1991 and upon opening, they were much poorer than their western neighbors. The three countries are small and have become very open; the population ranges from 1.4 million in Estonia to 3.7 million in Lithuania, while trade amounts to more than 110 percent of GDP in all three countries.

In Section 2, we identify the years when trade and capital flows were liberalized in each of the Baltic countries. These years will be used as the first open periods in our simulations. Section 3 looks at data for the Baltic countries and identifies important macroeconomic developments that have been associated with trade deficits in the decade after liberalization. Data is presented for trade deficits, real GDP growth rates, the sectoral composition of GDP and real exchange rates.

Sections 4, 5 and 6 present the model, its calibration, and the results of our basic simulations. The model is found to capture the main dynamics of trade balances and output, but the initial responses to the shock of liberalization are larger in the model than what we observe in the Baltics. Furthermore, the real exchange rate dynamics of the model do not show any of the persistence found in the data. Both of these problems are in the next section addressed with an extension of the model.

In Section 7, we augment the model to account for financial frictions by introducing and calibrating an interest rate risk premium on foreign loans to each of the Baltic states. The model dynamics for trade deficits, real exchange rates and the sectoral composition of GDP now more closely capture the variation in the data. The trade deficits in the model are in line with what we observe in the data, and the predicted year of capital flow reversal is around 2010 for all three countries. The date of reversal is robust to varying the initial capital stocks used in the simulations. If the neoclassical model is an appropriate framework for analyzing the Baltic countries, our results indicate that the current sustained trade deficits should not be a reason to worry and that the reversal of capital flows will come in about seven years.

Section 8 concludes and gives suggestions for future research.

2 Dating the Liberalization in the Baltic Countries

In this paper, the Baltic states will be modeled as initially closed economies that suddenly open up to trade with the rest of the world. To appropriately apply the model, we therefore first need to establish the earliest years for which it can be said that capital and trade flows were liberalized in the Baltic countries.

In their quest to achieve social stability and economic growth, the Baltic states rather closely followed the recommendations of the 'Washington consensus', which advocated a rapid shift from a planned economy to an open market system. With some minor exceptions, Estonia opted for a complete liberalization of import and export flows by 1993, thereby becoming one of the most open countries in the world. Latvia and Lithuania also liberalized their trade flows in 1993, but retained import duties of 15 percent or less on most products, as well as export duties on some products. Capital flows were liberalized by 1993 in Latvia and Lithuania and by 1994 in Estonia.

When dating the liberalization, the degree to which economic outcomes were market determined in the years following the independence from the Soviet Union should also be considered. A relatively large private sector with competing firms is important for a successful application of the model, since our model assumes perfect competition. Similarly, the existence of a credible national currency is of importance because a liberalization of capital flows is incomplete without a reasonably functioning foreign-exchange market. Most prices were liberalized and a process of rapid privatization was begun in the Baltics already in 1992.[1] All three countries substituted away from the Russian rouble as the domestic currency in 1992, with Estonia implementing a currency board in mid-1992. In the same year, Latvia and Lithuania introduced transitional currencies that were allowed to float. In 1993, Latvia adopted a permanent currency, which was pegged to the SDR at the beginning of 1994. The Lithuanian permanent national currency was not issued until the end of 1994, when a currency board was established.

A weighted assessment of these elements of liberalization leads us to treat 1994 as the first open year for Estonia, since most of the liberalization had been completed by that year. For Latvia and Lithuania, we choose 1995 as the year of opening, however, which is motivated by the slower pace of reforms in those countries. Privatization was less rapid in Latvia, which is seen in Table 1 where the share of the private sector in GDP is taken as a proxy for the progress of privatization. Lithuania was slower in implementing an exchange-rate based stabilization program and introducing a credible national currency.

3 Effects of the Liberalization

All three Baltic countries have been running substantial trade deficits since their opening up.[2] Net trade in goods and services as a percentage of nominal GDP is presented in Figure 1, which shows steady trade deficits of around 10 percent of GDP since the liberalization. The contraction of the deficits during the 1999-2000 period was a result of the Russian crisis in 1998-1999, and its negative effect on the Baltic economies. Data for 2002 shows that in Estonia and Latvia, trade deficits are now back at around 10 percent of GDP.

The completion of liberalization coincided with the time when economic growth returned to the Baltic states, following a period of contraction immediately after the independence. During the 1995-2001 period, real economic activity expanded with yearly average growth rates of 4.9 percent in Estonia, 4.8 percent in Latvia and 3.8 percent in

[1] A gradual price liberalization was only applied to the necessary minimum (e.g. public utilities, rents). Such prices were administered to increase in line with the purchasing power of the population.

[2] Before the collapse of the USSR, the Baltic republics were net exporters.

Lithuania. Figure 2 shows annual real GDP growth rates in the three countries after the opening.

In the model which we subsequently develop, the GDP share of traded output and the real exchange rate will be closely associated with trade flows. Next, we therefore examine the development of these variables in data for the Baltic states.

A shift in economic activity from the traded to the non-traded sectors is associated with the large trade deficits in the Baltic countries. Defining manufacturing (except electricity, gas and water), agriculture, mining and transportation as the traded sector and other industries as non-traded, Figure 3 shows the reduction of the traded sector as a share of GDP to vary from 7 percentage points in Estonia to 14 percentage points in Latvia. The shift mainly consists of a contraction in manufacturing and agriculture and an expansion of the wholesale/retail trade, the real estate and the construction industries. In Estonia and Lithuania, after contracting to around 40 percent of GDP, economic activity shifted back towards the traded sector in the period 2000-2001.

Along with sustained trade deficits, the Baltic states have also experienced marked real exchange rate appreciations. Figure 4 presents the development of the logged bilateral real exchange rate with Germany for each of the Baltic countries after the liberalization. We choose to present the real exchange rate with Germany, since it is the largest trading partner for Latvia and Lithuania and the third largest for Estonia. Germany is also the largest economy in the EU, which accounts for more than half of the total trade in all three Baltic states. Nominal exchange rates and Consumer Price Indices from the IFS are used to construct the series labeled *rer* in Figure 4.

In the model that we develop in section 4, there is only one traded good and no nominal variables. Therefore, we can only hope to account for the part of the real exchange rate fluctuations which is due to changes in the relative price of non-tradable to tradable goods in each country. Assuming that Purchasing Power Parity holds for goods in the traded sector, we use the same decomposition as in Betts and Kehoe (2001) to express the real exchange rate in terms of its traded and non-traded components:

$$\begin{aligned} RER_t &= S_t \frac{P_{Tt}^G}{P_{Tt}^C} * \frac{P_t^G}{P_t^C} \frac{P_{Tt}^C}{P_{Tt}^G}, \\ RER_t &= RER_{Tt} * RER_{Nt}, \end{aligned} \tag{1}$$

where S_t stands for the nominal exchange rate expressed in units of Baltic currencies per DM, P_t^G is a price index for Germany, P_t^C, $C \in \{Est, Lat, Lit\}$ is a price index for each of the Baltic states and P_{Tt} is a price index for tradable goods.

In (1), RER_{Tt} captures price changes of traded goods whereas RER_{Nt} captures relative price changes of non-traded goods. Expressing (1) in log form we have:

$$rer_t = rer_{Tt} + rer_{Nt}. \tag{2}$$

The assumption of Purchasing Power Parity in our model implies that $rer_{Tt} = 0, \forall t$ and that fluctuations in the real exchange rate can only be caused by movements in the relative price of non-tradables across countries. Since we cannot account for more than rer_N with our model in section 4, we plot rer_N together with the real exchange rate in Figure 4. When constructing price indices for traded goods (P_{Tt}^C and P_{Tt}^G), we use Producer Price Indices for the manufacturing sector in each country.[3]

In line with the existing empirical evidence for other countries, we find most of the real exchange rate movements in the Baltic states to be explained by changes in the relative prices of traded goods.[4] Relative movements of non-traded prices are, however, also important and account for 43 percent of the movement in the real exchange rate between 1994 and 2001 in Latvia. The same figures for Estonia and Lithuania are 30 percent and 17 percent, respectively. Figure 4 reveals a co-movement between rer and rer_N, with a clear trend of sustained appreciation present in both the real exchange rate and its non-traded component for all three countries. Therefore, we can hope for the model to account for a significant part of the observed real exchange rate fluctuations, although the Purchasing Power Parity assumption considerably limits its explanatory power.

[3]For a more detailed discussion of the suitability of PPI as a measure of price changes for traded goods; see Engel (1999) and Betts and Kehoe (2001).

[4]For evidence on other countries, see Engel (1999), Chari, Kehoe and McGrattan (2001), Betts and Kehoe (2001), and Burstein, Neves and Rebelo (2003).

4 The Model

Each Baltic country is modeled as a small open economy with a representative consumer. There are five goods in any period: a traded good, a non-traded good, capital, labor and an investment good augmenting the capital stock in the subsequent period.

The representative consumer maximizes the sum of discounted utility from the consumption of traded and non-traded goods. Taking prices as given, the consumer solves the following problem:

$$\max_{\{c_{Tt}, c_{Nt}, k_{t+1}, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{1}{\sigma} (\varepsilon c_{Tt}^{\sigma} + (1-\varepsilon)\, c_{Nt}^{\sigma} - 1) \tag{3}$$

$$\begin{aligned} \text{subject to: } c_{Tt} + p_{Nt} c_{Nt} + q_{t+1} k_{t+1} + b_{t+1} &\leq w_t L + (1+r_t) b_t + v_t k_t, \forall t \\ c_{Tt} &\geq 0, \forall t \\ c_{Nt} &\geq 0, \forall t \\ b_{t+1} + q_{t+1} k_{t+1} &\geq -A, \forall t \\ k_0, b_0 &\quad \text{given.} \end{aligned}$$

Here, c_{Tt} is consumption of the traded good, which is numeraire in the model; c_{Nt} is consumption of the non-traded good; p_{Nt} is the relative price of the non-traded good; k_{t+1} is investment in the domestic capital stock, purchased at the relative price of capital q_{t+1}; b_{t+1} is investment in a bond denominated in units of traded goods and earning the interest r_{t+1}; L is the endowment of labor, inelastically supplied at wage w_t; and $v_t k_t$ is income from selling capital at the relative price v_t to firms producing traded or non-traded goods.

Note that q_t is the price at which the consumer acquires capital for period t (the transaction takes place at the end of period $t-1$), whereas v_t is the price at which the consumer sells capital in period t to firms producing traded or non-traded goods.

The utility function exhibits the same constant elasticity of substitution, $1/(1-\sigma)$, both between goods in a period and across time. ε is a preference parameter and β is a subjective discount rate.

If b_{t+1} is negative, the economy is borrowing from the rest of the world. Ponzi schemes are ruled out by assuming that consumer's assets, $b_{t+1} + q_{t+1} k_{t+1}$, in any period cannot be smaller than $-A$, for A sufficiently large.

The first-order conditions for the consumer maximization problem are:

$$\begin{aligned} \varepsilon c_{Tt}^{\sigma-1} - \theta_t &= 0, \\ (1-\varepsilon) c_{Nt}^{\sigma-1} - \theta_t p_{Nt} &= 0, \\ \theta_{t+1}\beta(1+r_{t+1}) - \theta_t &= 0, \\ \theta_{t+1}\beta v_{t+1} - \theta_t q_{t+1} &= 0, \end{aligned} \tag{4}$$

where θ_t is a Lagrange multiplier for the consumer budget constraint.

The model allows for different specifications of interest rate determination. If the economy is closed in period t, there can be no foreign borrowing or lending, $b_{t+1} = 0$, and the return on investment is endogenously determined in the model. If the economy is open, the interest rate is equal to an exogenously given international rate, $r_{t+1} = r^*_{t+1}$ and b_{t+1} is endogenously determined.

A condition of no arbitrage between investments in domestic capital and foreign assets requires the relationship between the prices of capital before and after production to be

$$v_{t+1} = q_{t+1}(1 + r_{t+1}). \tag{5}$$

In addition to being consumed, the traded and non-traded goods can be used as inputs into the investment sector. The economy's resource constraint for non-traded goods is:

$$c_{Nt} + x_{Nt} \leq F_N(k_{Nt}, k_{Nt-1}, l_{Nt}, l_{Nt-1}), \ \forall t \tag{6}$$

where x_{Nt} is the input of non-traded goods into the investment sector. Note that the production process for non-tradables, $F_N(.)$, is a function of inputs of capital and labor into the non-traded sector in both the current and the previous period. Output depends on lagged production factors due to costs associated with frictions in capital and labor mobility.

The resource constraint for traded goods is more complicated, due to the possibility of trading with the rest of the world:

$$c_{Tt} + x_{Tt} + b_{t+1} - b_t(1 + r_t) \leq F_T(k_{Tt}, k_{Tt-1}, l_{Tt}, l_{Tt-1}), \ \forall t \tag{7}$$

where $b_{t+1} - b_t(1 + r_t)$ is the trade balance.

Investment goods are produced using a Cobb-Douglas technology taking traded and non-traded goods as inputs. The investment good augments the capital stock in the subsequent period, which gives the following law of motion for capital:

$$k_{t+1} - (1-\delta)\, k_t \leq G x_{Tt}^{\gamma} x_{Nt}^{1-\gamma}, \ \forall t. \tag{8}$$

Firms in the investment sector are assumed to operate under perfect competition. They choose how much of the traded and non-traded good to buy as inputs, taking prices p_{Nt} and q_{t+1} as given. In every period, firms in the investment sector maximize:

$$\max_{\{x_{Tt},x_{Nt}\}} q_{t+1} G x_{Tt}^{\gamma} x_{Nt}^{1-\gamma} - x_{Tt} - p_{Nt} x_{Nt}. \tag{9}$$

The problem in (9) has the following first-order conditions:

$$\begin{aligned} q_{t+1}\gamma G x_{Tt}^{\gamma-1} x_{Nt}^{1-\gamma} - 1 &\leq 0, \\ q_{t+1}(1-\gamma) G x_{Tt}^{\gamma} x_{Nt}^{-\gamma} - p_{Nt} &\leq 0. \end{aligned} \tag{10}$$

Firms producing traded or non-traded goods maximize infinite horizon profits under perfect competition. Taking prices as given (with p_{Tt} normalized to 1, since the traded good is numeraire), the firms choose how much capital and labor to buy in each period. If $r_t = r\ \forall t$, the firms in the traded and non-traded sectors solve the following problem:

$$\max_{\{k_{jt},l_{jt}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(p_{jt}F_j(k_{jt},k_{jt-1},l_{jt},l_{jt-1}) + q_{t+1}(1-\delta)k_{jt} - w_t l_{jt} - v_t k_{jt}\right), \tag{11}$$

where $j \in \{T, N\}$ and δ is a constant per-period capital depreciation rate[5]. The production functions are assumed to have the following form:

$$\begin{aligned} F_j(k_{jt},k_{jt-1},l_{jt},l_{jt-1}) &= A_j k_{jt}^{\alpha_j} l_{jt}^{1-\alpha_j} - \Phi(k_{jt},k_{jt-1}) - \Psi(l_{jt},l_{jt-1}), \\ \text{where} \quad \Phi(k_{jt},k_{jt-1}) &= \frac{\zeta}{1+\zeta}\left(\frac{|k_{jt}-(1-\delta)k_{jt-1}|}{k_{jt-1}}\right)^{\frac{1+\zeta}{\zeta}} k_{jt-1}, \quad \zeta > 0, \\ \Psi(l_{jt},l_{jt-1}) &= \phi\left(\frac{l_{jt}-l_{jt-1}}{l_{jt-1}}\right)^2 l_{jt-1}, \quad \phi > 0. \end{aligned} \tag{12}$$

Here, $\Phi(.)$ is a convex cost associated with investment, which we model in line with Abel and Eberly (1994) and Eberly (1997). Note that the specification in (12) implies capital frictions to be present in steady state, because the cost is associated with the transformation of investment goods rather than the adjustment of the capital stock. $\Psi(.)$ is a quadratic cost associated with the adjustment of the labor force in a sector. The specification implies that there are costs of both hiring and firing, whenever labor movements between sectors take place.

[5]For expositional purposes, we do not present the profit functions for the general case, when the real interest rate can vary.

The first-order conditions for the profit maximization problem in (11) are:

$$p_{jt}\frac{\partial F_j(k_{jt},k_{jt-1},l_{jt},l_{jt-1})}{\partial k_{jt}}+q_{t+1}(1-\delta)-v_t+\frac{p_{jt+1}}{1+r}\frac{\partial F_j(k_{jt+1},k_{jt},l_{jt+1},l_{jt})}{\partial k_{jt}}\leq 0, \quad (13)$$
$$p_{jt}\frac{\partial F_j(k_{jt},k_{jt-1},l_{jt},l_{jt-1})}{\partial l_{jt}}+\frac{p_{jt+1}}{1+r}\frac{\partial F_j(k_{jt+1},k_{jt},l_{jt+1},l_{jt})}{\partial l_{jt}}-w_t\leq 0.$$

Definition of equilibrium An equilibrium in this model is characterized by sequences of prices $\{\widehat{p}_{Nt},\widehat{w}_t,\widehat{q}_{t+1},\widehat{v}_t,\widehat{r}_{t+1}\}_{t=0}^{\infty}$, consumption and assets $\left\{\widehat{c}_{Tt},\widehat{c}_{Nt},\widehat{k}_{t+1},\widehat{b}_{t+1}\right\}_{t=0}^{\infty}$, sectoral production plans $\left\{\widehat{k}_{Tt},\widehat{l}_{Tt}\right\}_{t=0}^{\infty}$ and $\left\{\widehat{k}_{Nt},\widehat{l}_{Nt}\right\}_{t=0}^{\infty}$, and inputs into the investment sector $\{\widehat{x}_{Tt},\widehat{x}_{Nt}\}_{t=0}^{\infty}$, such that:

(i) given prices $\widehat{p}_{Nt},\widehat{r}_{t+1},\widehat{q}_{t+1}$ and $\widehat{v}_{t+1}$, the representative consumer's first-order conditions in (4) are satisfied in every period.

(ii) given prices $\{\widehat{p}_{Nt},\widehat{w}_t,\widehat{q}_{t+1},\widehat{v}_t\}_{t=0}^{\infty}$, producers in sector $j\in\{T,N\}$ choose factor inputs $\left\{\widehat{k}_{jt},\widehat{l}_{jt}\right\}_{t=0}^{\infty}$ so that the first-order conditions in (13) are satisfied in every period.

(iii) given prices $\widehat{p}_{Nt}$ and $\widehat{q}_{t+1}$, the investment sector's first-order conditions in (10) are satisfied in every period.

(iv) The market clearing conditions in (6), (7) and (8) are satisfied in every period. If the economy is closed in period t, $\widehat{b}_{t+1}=0$. If the economy is open in period t, $\widehat{r}_{t+1}=r^*_{t+1}$.

(v) Factor markets clear in every period:

$$\begin{aligned}\widehat{k}_{Tt}+\widehat{k}_{Nt}&=\widehat{k}_t,\quad\forall t\\ \widehat{l}_{Tt}+\widehat{l}_{Nt}&=L,\quad\forall t.\end{aligned}\quad (14)$$

5 Calibration

In this section, we first discuss the data used in the calibration and then present the way we calibrate the model to the Baltic countries.

5.1 Data issues

Most of the macroeconomic data we need for this paper was readily available at the statistical offices in the Baltic states. The only exceptions, which we now discuss in more detail, were data on capital stocks and input-output tables.

Capital stock estimates Unfortunately, there are no official estimates on capital stocks available for the Baltic countries. Furthermore, since the time series of National Accounts data for the Baltic states are very short, we cannot estimate the capital stock using the perpetual inventory method. Instead, we estimate the capital stock by assuming the total capital stock to be the sum of fixed tangible assets of all enterprises in the economy, and the total stock of residential housing. A detailed explanation of this method is presented in Appendix A. The resulting values of capital stocks in current prices are 30782.2 million Kroons in Estonia in 1993, 2859.6 million Lats in Latvia in 1994 and 21343.4 million Litas in Lithuania in 1994, of which correspondingly 44, 53 and 49 percent come from the value of residential housing. Dividing these estimates with the output for the same year, we obtain capital-output ratios of 1.41 for Estonia, 1.40 for Latvia and 1.33 for Lithuania.

We have confidence in our estimates for two reasons. First, using a different methodology, Hazans (1999) arrives at a similar estimate for the part of the capital stock excluding residential housing for Latvia. Second, the size of our estimates for residential housing stocks relative to real GDP per capita is broadly in line with the averages for countries with similar income levels in PWT 5.6.[6]

Input-output tables Our calibration of the model to the Baltic economies relies on input-output tables. Given the amount of detailed data required, it should not be surprising that input-output matrices were not constructed for the early years of transition in the Baltic states. The first experimental input-output tables in each of the Baltic states were compiled in 1996. Afterwards, statistical offices in Latvia and Estonia published the first official input-output tables in 1997. In Lithuania, the second, still experimental, input-output table was compiled in 1998 and the first official input-output table for the year 2001 is still to be published. We base our calibration on the first official input-output tables for 1997 for Latvia and Estonia and the 1998 experimental input-output table for Lithuania. These tables were preferred to the earlier ones for quality considerations.

Following Fernandez de Cordoba and Kehoe (2000), we aggregate the input-output tables into two-sector (traded and non-traded) input-output tables, which requires us to classify each sector of the economy as traded or non-traded. In deciding whether an industry is traded, we follow a common classification in the literature (see De Gregorio, Giovannini and Wolf, 1994 and Stockman and Tesar, 1991), summarized in Table 2.

In the context of the model, the classification in Table 2 has an intuitive appeal. We can consider the traded sector as producing *equipment* and the non-traded sector as producing *structures.* The traded sector should include services and goods that can be immediately traded and used when the borders open up. Furthermore, the sector should consist of products that are fairly homogenous across countries. Defined this way, the

[6] See Summers et al. (1995).

traded sector can be expected to satisfy the assumption of Purchasing Power Parity.

Based on Table 2, we aggregate the 57 sector input-output matrices compiled by the central statistical offices in the Baltic countries to obtain the two-sector matrices presented in Tables 3 and 4.[7]

5.2 Initial factors and parameter values

When calibrating the model, we follow Fernandez de Cordoba and Kehoe (2000) and use the equilibrium conditions of the model while normalizing all prices to 1 for the initial year, except the rental price of capital.

As explained above, we cannot use input-output tables for the last closed year (1993 for Estonia and 1994 for Latvia and Lithuania) in our calibration. In general, parameters derived from input-output tables for 1997 or 1998 might be more representative for the simulated time span, but caution is called for since relative price changes occur between the beginning of the simulation and the year for which parameter values are calibrated. We deal with this by constructing appropriate price indices for traded and non-traded goods. Price changes for traded goods are captured with the PPI for manufacturing, and for non-traded goods we use GDP deflators for the non-traded sector. These indices connect the normalized prices of the last closed year with prices of the year for which input-output tables are calculated.

The equations presented in the discussion of initial factor and parameter calibration that follows are restricted to one of the Baltic countries - Latvia. This allows us to simplify the equations by suppressing the country superscript and using one set of time subscripts only. An identical procedure is applied to calibrate the model to Estonia and Lithuania. The results of the calibration are summarized in Table 5.

We start by calibrating the production functions for traded and non-traded goods. Output figures for each sector in the last year before liberalization, y_{T0} and y_{N0}, were obtained from National Accounts data. Normalizing the total output in each country to 100, the values of sectoral outputs are presented in the second and third rows of Table 5.

To obtain $k_{T0}, k_{N0}, l_{T0}, l_{N0}, \alpha_T, \alpha_N, A_T$ and A_N, we solve a system of eight equations provided by the equilibrium conditions of the model for the autarky steady state (15)-(19).[8] First, note that output in sector j, where $j = \{T, N\}$, in the last year before liberalization is:

$$y_{j0} = A_j k_{j0}^{\alpha_j} l_{j0}^{1-\alpha_j} - \frac{\zeta}{1+\zeta} \delta^{\frac{1+\zeta}{\zeta}} k_{j0}, \tag{15}$$

where the last term is the cost associated with the transformation of investment goods into capital. Next, we use the fact that capital in each sector in equilibrium must earn

[7] We have not received permission to publish the 1998 unofficial input-output table for Lithuania.

[8] We here use steady state equations since we cannot obtain information on capital stocks *before* the initial year.

its marginal product, which implies that

$$y_{j0}X_j = \left(\alpha_j A_j k_{j0}^{\alpha_j - 1} l_{j0}^{1-\alpha_j} + Z\right) k_{j0}, \tag{16}$$

where $Z = -\delta^{\frac{1}{\zeta}} + \beta\delta^{\frac{1}{\zeta}} - \beta\frac{\zeta}{1+\zeta}\delta^{\frac{1+\zeta}{\zeta}}$ is once more a term stemming from investment transformation costs and where X_j denotes the income share of capital and is obtained from the aggregated input-output tables using the following formula:[9]

$$X_j = \frac{os_{j,97}}{os_{j,97} + wl_{j,97} + mix_{j,97}}. \tag{17}$$

Here, os is the operating surplus (i.e. the operating surplus of incorporated enterprises), wl is remuneration of employees and mix stands for mixed income (i.e. the operating surplus of private unincorporated enterprises). We include all mixed income in labor income, which has been proposed by Gollin (2002) as one way of getting the income shares right. This is a sensible adjustment for self-employment, since household enterprises in the Baltic countries are labor intensive. Unfortunately, we only have data on the division between mixed income and operating surplus for Latvia. Since the three Baltic economies are very similar in structure, we assume the relative size of these subcategories of business profit to be the same for all three countries.

Equilibrium conditions also require that the returns to capital and labor are the same in both sectors, which implies that:

$$\begin{aligned} \alpha_T A_T k_{T0}^{\alpha_T - 1} l_{T0}^{1-\alpha_T} &= \alpha_N A_N k_{N0}^{\alpha_N - 1} l_{N0}^{1-\alpha_N}, \\ (1-\alpha_j) A_j k_{j0}^{\alpha_j} l_{j0}^{-\alpha_j} &= 1, \quad j = \{T, N\}, \end{aligned} \tag{18}$$

where in the last equality, we used the fact that initial wages have been normalized to 1. Finally, we can use the market clearing condition for capital:

$$k_0 = k_{T0} + k_{N0}. \tag{19}$$

When this method of calibrating the production functions is applied to each of the Baltic countries individually, we observe considerable variation in the technology parameter associated with capital, which ranges from $\alpha_T^{Est} = 0.29$ to $\alpha_N^{Lit} = 0.42$. In all three countries, the traded sector is the more labor intensive sector, however. We believe the observed variation to be due to measurement errors and therefore use the averages of α_T and α_N for the Baltic states as the actual parameters for each country. Taking averages puts our calibrated parameter values in line with empirical evidence on income shares (see Gollin (2002)), while preserving the observed relative factor intensities. The remaining

[9]Note that since capital frictions are present in the steady state in the model, the capital income share in a sector differs from α_j.

parameters and the initial production factors are recalibrated using the equations in (15) and in (18)-(19), the results of which are presented in Table 5.

In order to calibrate the parameters of the production function for the investment sector, γ and G, we need to take relative price changes between the base year (1994) and the year of the input-output table into account. The investment equation we use for calibration is:

$$\frac{I_{97}}{q_{98}} = G\left(\frac{x_{T97}}{p_{T97}}\right)^{\gamma}\left(\frac{x_{N97}}{p_{N97}}\right)^{1-\gamma}, \tag{20}$$

where I_{97} is total investments in current prices that can be directly obtained from the input-output table; x_{T97} and x_{N97} are the current price inputs of traded and non-traded goods into the investment sector; and p_{T97} and p_{N97} are the price indices discussed above.[10] Since all prices in the initial period, except the returns to capital, have been normalized to 1 in the initial period, the price index for investment goods is calculated as:

$$q_{98} = p_{T97}^{\gamma} p_{N97}^{1-\gamma}. \tag{21}$$

Using the first-order conditions in (10), we can now calibrate γ as a standard Cobb-Douglas income share.

The resulting values of γ display considerable differences across countries and range from 0.31 in Lithuania to 0.54 in Latvia. To gain further insight into the appropriate values for γ, we turn to the National Accounts data, which is more widely available than input-output tables. PWT 6.1 contains data on gross capital formation in current prices, which is divided into construction, producer durables and changes in inventory. Since construction is the dominant non-tradable input in the investment sector by far, this division should closely correspond to our classification of investment inputs as consisting of either non-tradable or tradable goods, once changes in inventories have been excluded. The values of γ implied by the PWT data for the Baltic countries in 1996 are very similar to the estimates from the input-output tables, with an average value of $\gamma = 0.446$ in the Baltics. When calculating values of γ for all 115 countries included in the PWT data, we observe that, for the OECD countries, these values are in the range of $0.35 - 0.55$, with an average value of 0.44. For the transition economies included in the PWT 6.1, the average value of γ is 0.43, thereby suggesting that the value of γ is the same for different income levels. The data contains more variation for transition economies than for the OECD countries, but we believe the higher variance to be caused by measurement problems, since no systematic deviation from the mean can be observed for poorer countries. Therefore, we use the average value of $\gamma = 0.446$ for the Baltic states that was found in PWT 6.1 as the actual parameter value for each country. Given γ, we calibrate G using (20).

In our simulations, we use a value of $\sigma = -1$, which is a standard value in the

[10] We omit changes in stock and valuables from the investment data, since we have not included inventories in our estimates of the initial capital stocks.

literature (see, for example, Mankiw, Rotemberg and Summers (1985)). To calibrate ε, the consumption entries by sector in the input-output tables need to be adjusted with price indices. Using the first-order conditions for the consumer's problem in (4), we obtain:

$$\varepsilon = \frac{\left(\frac{c_{N97}}{c_{T97}}\right)^{\sigma-1}}{\left(\frac{p_{N97}}{p_{T97}}\right)^{\sigma} + \left(\frac{c_{N97}}{c_{T97}}\right)^{\sigma-1}}, \tag{22}$$

where c_{T97} and c_{T97} are the current price consumption of traded and non-traded goods, which can be directly obtained from the input-output table.

In our simulations of the model, we will take Germany as representing the world outside the Baltic states. The rate of depreciation, δ, is calibrated for Germany, using the Summers et al. (1995) data on capital stocks and expenditures on gross fixed capital formation ($GFCF_t$). The law of motion for capital gives:

$$\delta_t = \frac{(GFCF_t - k_{t+1})}{k_t} + 1. \tag{23}$$

To obtain our parameter value, we use a fifteen-year average for the years 1975-1989, which yields $\delta = 0.081$.

An assumption on which we will base our simulations is that the German economy has reached its steady state. The discount factor, β, is therefore calibrated for Germany, using the average real return on German government bonds over the 1981-2001 period. The measured annual real interest rate is $r^* = 0.0419$, which gives us $\beta = 1/(1+r^*) = 0.9597$.

Eberly (1997) has estimated the convex component of capital frictions of the same form that we specified in (12). Looking at annual data for the OECD countries between 1981-1994, she finds that ζ ranges from 0.65 in Sweden to 1.95 in France. For the Baltic states, we have adopted the convexity parameter that she estimated for the US, $\zeta = 1.22$. Our choice is motivated by the observation that the Baltic states in the 1990's were closer to the US with very liberal capital regulations, more bankruptcies and higher volatility of the capital stock at the firm level than in western Europe.

Finally, we calibrate ϕ so that job creation in the model never exceeds the highest rates of sectoral net job creation observed in the data. For the period from the first open year until 2001, data on employment by economic activity is available at the national statistical offices. After aggregating this data into sectors, we find that the largest observed net job creation rates were 6.4 percent in Estonia, 5.1 percent in Latvia and 2.9 percent in Lithuania. In all three countries, the maximum increase took place in the non-traded sector. The high sectoral adjustment capacities we find in the data are in line with Haltiwanger and Vodopivec (2002) who study job flows in Estonia and reveal a very high degree of flexibility in the labor market of that economy.

6 Simulation of the Model

In this section, we simulate the model introduced and calibrated in the previous sections. Each Baltic country is modeled as an initially closed economy that opens up in the first year after liberalization (1994 for Estonia and 1995 for Latvia as well as Lithuania). Figure 5 presents the simulated time paths of the trade balance, the annual growth rate of real GDP, the GDP share of traded output and the real exchange rate for the three model economies. To contrast the model results with the data, we have included the corresponding Baltic data in the graphs. Note that the relevant data series for the real exchange rate is rer_N and not rer.

After the liberalization, the capital-poor model economies borrow heavily from abroad, as revealed by the large trade deficits in Figures 5a-5c. Naturally, this implies that in steady state, the economies will have to run permanent trade surpluses. In essence, the large net inflows of traded goods are a result of consumption smoothing by consumers. After the shock of liberalization, borrowing allows consumers to transfer the higher levels of future consumption into the early stage of transition. Trade deficits in the model exceed those observed in the data for all three countries, although the model does capture the main dynamics.

The model growth rates of real GDP capture the trends observed in the data, as shown in Figures 5d-5f. The initial contraction of real GDP in the model is a result of costly factor reallocation, as a response to liberalization. For Latvia and Lithuania, we do not observe such effects in the data. After the initial adjustment, however, GDP in the model expands at a rate similar to the data for all three countries.

In line with the data, traded output as a fraction of GDP decreases after liberalization. However, this process exhibits more persistence in the data than in the model, which is seen in Figures 5g-5i. Since consumers in the model can only borrow traded goods, the shock of liberalization makes the non-traded goods relatively scarce. As specified in (8), the investment sector requires both traded and non-traded goods as inputs for augmenting the capital stock. While traded goods can be borrowed, non-traded goods must be produced at home. Thus, non-traded goods become a bottleneck for development in the model. The optimal response of the Baltic model economies is to initially import traded goods and specialize in the production of non-traded goods at home.[11]

In terms of the real exchange rate, Figures 5j-5l show that the model dynamics imply a sharp appreciation in the first open year. The intuition for the marked appreciation in the model follows from our discussion of traded output: the scarcity of the non-traded good results in a sudden substantial increase in its relative price. Similar to the sectoral dynamics of output, the model predicts too strong an appreciation of the real exchange

[11] If the model had no frictions in factor markets, the optimal solution would be for all three countries to completely specialize in producing non-traded goods in the first open period.

rate in the first year after liberalization and fails to capture the persistence in the data.

7 Interest Rate Risk Premium

After the opening of their economies, the Baltic states faced considerably higher real interest rates than Germany, which is something that has so far not been accounted for in our model. Interest rate differentials can slow down capital flows in the model and cause persistence in the movements of the real exchange rate and the sectoral composition of output. We pursue a simple way of investigating such dynamic effects by introducing real interest rate risk premia on foreign loans in the model, which we calibrate to the risk premia that can be observed in data for the Baltic countries.

The real interest rate risk premium ρ_t^C for country $C \in \{Est, Lat, Lit\}$ in period t is defined as:

$$1 + r_t^C = (1 + \rho_t^C)(1 + r_t^*), \tag{24}$$

where $1 + r_t^C$ is the gross interest rate faced by consumers in country C on foreign loans to be repaid in period t, and where r_t^* is the international real interest rate.

Taking Germany and the U.S. as representing the rest of the world, the risk premium is calibrated as:

$$\rho_t^C = \frac{r_t^C - r_t^W}{1 + r_t^W}. \tag{25}$$

Here, r_t^C is the annualized average real interest rate charged on 6-12 month DM denominated loans to the private sector in Estonia or on 6-12 month US dollar denominated loans to the private sector in Latvia and Lithuania; and r_t^W is the real interest rate on loans to enterprises for up to twelve months in Germany (for Estonia) or the real prime loan rate in the U.S. (for Latvia and Lithuania). Yearly CPI inflation rates in Germany (for Estonia) or the U.S. (for Latvia and Lithuania) were used to calculate real interest rates.

There are several reasons for considering DM denominated loans in Estonia and US dollar denominated loans in Latvia and Lithuania. We want to avoid any local currency risk considerations to be reflected in the measured risk premium, which is why foreign currency loans must be considered. It is also the case that most of the lending to the private sector in the Baltic states during the 1990's was denominated in foreign currencies. In Estonia, the majority of the foreign currency loans were DM denominated, while in Latvia and Lithuania almost all such loans were denominated in US dollars.[12] Although theoretically, it would be better to calculate the risk premium in DM (or Euro) denominated loans for all three countries, the market for such loans was very thin in Latvia and

[12] These differences in denomination is a result of the exchange-rate arrangements. The Estonian Krooni was fixed to DM, the Lithuanian Lits was fixed to the US dollar and the Latvian Lats was pegged to the SDR, which is dominated by the US dollar.

Lithuania in the 1990's. By calculating the risk premium using US dollar interest rates, we implicitly assume that, for the sake of our simulations, differences in the U.S. and German real interest rates can be ignored.

The interest rate data for the Baltic countries was taken from the web sites of the national central banks. Interest rate data for the U.S. is from IMF International Financial Statistics, whereas the interest rate data for Germany is from the Eurostat database. The risk premia we arrived at through these calculations are shown in Table 6 and the resulting interest rate differentials in the model are depicted in Figure 6.

Since a non-negligible real interest rate risk premium is still present in 2001, we must take a stand in our simulations about the future size of the premium. For the simulations of the model, we assume that after 2001, the risk premium in the three Baltic states will decrease linearly, reaching zero in 2005. We motivate such a gradual decrease in the future premium by the EU accession in May, 2004.

With a country-specific interest rate risk premium, the problem of the representative consumer in country C now reads:

$$\max_{\{c_{Tt},c_{Nt},k_{t+1},b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{1}{\rho}\left(\varepsilon c_{Tt}^{\rho} + (1-\varepsilon)\, c_{Nt}^{\rho} - 1\right) \tag{26}$$

$$\text{subject to: } c_{Tt} + p_{Nt}c_{Nt} + b_{t+1} + q_{t+1}k_{t+1} \leq w_t L + (1+r_t^C)b_t + v_t k_t, \forall t \tag{27}$$

$$\begin{aligned} c_{Tt} &\geq 0, \forall t \\ c_{Nt} &\geq 0, \forall t \\ b_{t+1} + q_{t+1}k_{t+1} &\geq -A, \forall t \end{aligned}$$

$$b_{t+1}\begin{cases} \leq 0 & \text{if } \rho_{t+1}^C > 0 \\ \in \mathbb{R} & \text{else} \end{cases} \tag{28}$$

$$k_0, b_0 \quad \text{given.}$$

Here, $r_t^C = (1+r^*)\left(1+\rho_t^C\right) - 1$ is the real interest rate at which consumers can borrow in period $t-1$, and ρ_t^C is the risk premium for country C in period $t-1$. Once more, in our simulations, we will assume the rest of the world to be in steady state, so that $r^* = \beta^{-1} - 1$.

Note that the constraint (28) implies that the consumer cannot lend abroad, if there is a positive risk premium. Naturally, a more accurate specification of the investment opportunities would be to allow consumers to lend at the world interest rate in any period. From our simulations of the model in section 6, we know, however, that the representative consumer does not optimally lend until after 2006 at the world interest rate in any of the three countries. The constraint on foreign lending reflects the exogenous nature of the introduced risk premium. Although it would be more satisfactory to obtain the risk premium as an endogenous outcome of the model, the current setup provides a simple way of evaluating the effects of such a premium. The rest of the model is the same as in Section 4.

The model with a risk premium is solved by first finding a solution without imposing the constraint in (28). If the optimal solution in any period t is to lend at an interest rate above the international rate, we set $b_{t+1} = 0$ and resolve the model. It turns out that for Estonia and Lithuania, the optimal solution with the calibrated risk premia never violates (28). In the Latvian model economy, the constraint on b_{t+1} is violated in the first open period. The opening is therefore postponed for a period, since the calibrated interest rate in the period after liberalization is so high that consumers do not find it optimal to borrow.

The dynamics of the model with the interest rate risk premium are presented in Figure 7.[13] The higher interest rates slow down the capital inflows and thus, the initial adjustments in all of the graphs are more gradual. Since the cost of borrowing from abroad now varies between periods in a realistic manner, the model accounts for a larger part of the variations in the data.

In Latvia, the dynamics of the trade balance now closely resemble the magnitudes in the data. For Estonia and Lithuania, the model captures the overall dynamics, although the size of the deficits in the model is still larger than in the data. Part of the discrepancy between the model and the data in Figures 7a-7c is likely to be a consequence of the Russian crisis of 1998-99, which we have not modelled. The Baltic economies were seriously shaken by this event that lead to a slowdown or even a contraction of economic activity. As seen in Figure 2, Latvia was the country least affected by the crisis while Lithuania was most severely hit. The economic slowdown temporarily improved trade balances, but the data for Latvia in 2001 and Estonia in 2002 demonstrates that the pre-crisis deficits returned, once the effects of the Russian crisis had been overcome.

Figures 7d-7f show the initial contraction of GDP in the model to be smaller and more in line with the data when an interest rate risk premium is introduced. The simulated GDP growth rates still capture the trends of the data, but the rates of expansion are slightly lower than in the model without the risk premium.

An interesting result is that the model with a risk premium can generate a more gradual decrease in the relative size of the traded sector, and some sustained appreciation in the real exchange rate. In terms of sectoral adjustment, the model dynamics in Figures 7g-7i are now more similar to the gradual shift towards the non-traded sector that we observe in the Baltics. The overall magnitude of the shift is, however, still larger in the model economies than in the data.

The real exchange rate appreciation in the model now lasts for three periods in Lithuania and for two periods in Estonia and Latvia (see Figures 7j-7l). Furthermore, the overall

[13] In the simulations shown in Figure 7, ϕ has been recalibrated to match the observed maximum net job creation rates: $\phi^{Est} = 2.02$, $\phi^{Lat} = 16.35$, $\phi^{Lit} = 18.22$.

size of the appreciation for Latvia and Lithuania is very close to what we observe in the data. For Estonia, the introduction of the risk premium results in the model real exchange rate not moving enough, however.

Sensitivity analysis The estimates of the initial capital-output ratios in the Baltic countries are crucial for the outcome of our simulations. In the model, trade deficits are a consequence of the Baltic economies being capital poor when liberalization occurs. Next, we therefore investigate the sensitivity of the model dynamics for the trade balances with respect to our estimates of the initial capital stocks.

Table 7 reports a summary of the trade balance dynamics in simulations using three different levels of initial capital stocks: our estimates presented in section 5 and capital stocks that are twenty percent larger or twenty percent smaller. In producing the table, the parameters of the model were recalibrated for each level of the capital stock.

In Table 7, we see that changes in the initial capital stock have a considerable effect on the size of the trade deficits that the model economies experience during the first decade after liberalization. For instance, a twenty-percent decrease in the initial capital stock decreases the Estonian minimum trade balance by three percentage points of GDP in the peak year, while a twenty-percent increase in the capital stock increases the minimum trade balance by three percentage points. At the same time, the year in which capital flows are reversed is rather insensitive to varying the initial capital stocks. The first year of trade surplus is the same for all three initial capital stocks in the simulations for Estonia and only changes by a year up or down for the two other countries.

8 Concluding Remarks

The results of the simulations in this paper show that a calibrated two-sector growth model can actually account for the size of the trade deficits in the Baltic states during the last decade. The model predicts that the current deficits will be reversed into trade surpluses in 2009 for Estonia, in 2011 for Lithuania and in 2012 for Latvia. The predicted years of trade flow reversals have in the simulations proven robust to considerable changes in the initial capital inherited from the Soviet Union. There is reason to have confidence in the results of the neoclassical model, since the implications of the model for real GDP growth, the sectoral composition of GDP and the real exchange rate are in line with the data for the Baltic countries.

The analysis has shown that the reversal of trade flows will have far reaching effects on the Baltic economies. The current boom in the service and real estate sectors will, according to the model, come to an end as capital and labor will have to be shifted into the traded sector to produce export goods. A high degree of flexibility in the labor market therefore seems imperative for the economies to successfully accommodate the pressures for sectoral readjustment in the future. Politicians and the general public in the Baltics should bear in mind that a serious transfer problem, with drastically reduced imports as a result, might arise if the factors of production cannot easily be moved into the export sector.

One important aspect which we have not modelled in this paper is the possibility of a financial crisis in the Baltic countries after the liberalization. As demonstrated by the contagious effects of the Russian crisis in 1998-1999, crises in emerging markets tend to be associated with sudden interruptions of capital flows and sharp increases in the trade balance. It then goes without saying that our predictions for future trade deficits and the timing of their reversals are subject to the condition that a financial crisis does not occur in the Baltic states.

The prospects of productivity growth in the Baltic states have also been omitted in our quantitative exercise. Productivity is likely to increase as the Baltic states catch up with the EU economies. In the model, this would increase the steady-state levels of capital and consumption, thereby inducing larger trade deficits. The Balassa-Samuelson effect of higher relative productivity growth in the traded sector, would in our model increase the relative price of non-traded goods and lead to a more sustained appreciation of the real exchange rate.

The introduction of an interest rate risk premium is found to improve the model's explanatory power. An interesting extension of the present work would therefore be to endogenize financial market frictions, which could be done by incorporating explicit financial contracts in the model. Another improvement would be to explore ways of relaxing the assumption of purchasing power parity for traded goods, since that would allow the model to account for a larger part of the observed real exchange rate movements.

A Estimates of the capital stocks

In this appendix, we describe the methodology used for estimating the capital stocks in the Baltic states. The final estimates are obtained as a sum of the value of fixed tangible assets and the value of residential housing in each of the countries.

Data on fixed tangible assets is provided by the national statistical offices in each of the countries. It is obtained from the balance sheets of all larger companies (more than 100 employees) and a representative sample from small enterprises. For Lithuania, the earliest measurement of fixed tangible assets that we have obtained was for the end of 1995. To obtain the same figure for 1994, we subtract gross fixed capital formation for 1995 ($GFCF_{95}$) and add consumption of fixed capital for 1995 (CFC_{95}) to the fixed tangible assets in 1995 and then deflate the resulting figure with the GDP deflator for 1995. The nominal value of the stocks of fixed tangible assets that we obtain are 1772.6 million lats for Latvia in 1994, 21115.4 million krooni for Estonia in 1993 and 13506.9 million litas for Lithuania in 1994.

To our knowledge, there is no reliable data available for the stock of residential housing in the Baltic states. To obtain an estimate of the residential capital, we therefore look at comparable data for other countries. The country which is closest to the Baltic states in its economic conditions and for which residential housing stock data is available in the PWT 5.6 data set is Poland. To estimate the residential housing stock for the Baltic states using Polish data, we assume that in each of the Baltic countries, the real value of residential capital per capita in the last closed year was the same as the real value of residential capital per capita in Poland in 1990. More formally,

$$\frac{real\ rescap_i^C}{n_i^C} = \frac{real\ rescap_{90}^{Pol}}{n_{90}^{Pol}}, \tag{A.1}$$

where $C \in \{Est, Lat, Lit\}$, i is the last closed year (1993 for Estonia and 1994 for Latvia and Lithuania), $real\ rescap$ is the real value of the residential capital stock and n is population. Such an assumption is quite reasonable, since before the collapse of socialism, the level of economic development in Poland and the Baltic states was similar and, by 1994, the progress of transition in the Baltics was at a level comparable to Poland's progress by 1990.

According to the data in PWT 5.6, the Polish stock of residential housing in 1990 was at a level of 47 per cent of real GDP. Next, we use PWT 6.1 (see Heston, Summers and Aten (2002)), which is the first version of the Penn World Tables to include real GDP data for the Baltic states, but it does not yet include data on residential capital stocks for any country. Using PWT 6.1, we translate the value of Polish residential housing into comparable figures for the Baltic states. This is done by first noting that in PWT 6.1, the Polish ratio of residential housing stock to real GDP in 1990 should be the same as in

PWT 5.6, since the different 'international dollars' in which values are measured in PWT 5.6 and PWT 6.1 cancel out in a ratio. Thus, we get that the real residential capital per capita for Poland in 1990 was 3121.8 '1996 international dollars'. Using our assumption in equation (A.1), the Polish per capita value of the residential housing is applied to each of the Baltic states through the following formula:

$$nominal\ rescap_i^C = \frac{nominal\ GDP_i^C}{real\ GDP_i^C}\frac{real\ rescap_{90}^{Pol}}{n_{90}^{Pol}}n_i^C, \tag{A.2}$$

where the notation is the same as in equation (A.1). $real\ GDP_i^C$ is measured in 1996 international dollars and $nominal\ GDP_i^C$ is measured in year i local currency of country C. The resulting value for the $nominal\ rescap_i^C$ is measured in the same (nominal) units as tangible fixed assets. Using this procedure, we get that the residential capital stock was 1087.0 million lats for Latvia in 1994, 9666.8 million krooni for Estonia in 1993 and 7836.5 million litas for Lithuania in 1994.

Adding up fixed tangible assets and residential housing stock and dividing the sum by nominal GDP, we obtain the following capital to output ratios:

$$\frac{k_{93}^{Est}}{y_{93}^{Est}} = 1.410, \quad \frac{k_{94}^{Lat}}{y_{94}^{Lat}} = 1.399, \quad \frac{k_{94}^{Lit}}{y_{94}^{Lit}} = 1.334. \tag{A.3}$$

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Table 1: The size of the private sector in the Baltic states, percent of GDP

	1991	1992	1993	1994	1995	1996
Estonia	10	25	40	55	65	70
Latvia	10	25	30	40	55	60
Lithuania	10	20	35	60	65	65

Source: EBRD Transition report 1995-2000

Table 2: The division of industries into traded and non-traded sectors

Traded industries	**Non-traded industries**
Agriculture	Energy
Fish	Construction
Mining	Wholesale/retail
Manufacturing	Hotel services
Transport/	Finance
Communications	Real estate/renting
	Public administration
	Education
	Health
	Other community services

Table 3

AGGREGATED INPUT-OUTPUT MATRIX FOR LATVIA 1997

(Million Lats)

EXPENDITURES

		1	2	3	1+2+3	*C+G*	*I*	*X*	*C+G+I +X*	*TOTAL*
	1	1630.2	664.0	69.6	2363.8	1179.7	459.7	1438.9	3078.3	5442.1
R	2	404.5	570.9	76.4	1051.8	1212.8	286.6	207.6	1707.0	2758.9
E	3	14.3	22.1	37.8	74.1	419.8	0.0	17.5	437.4	511.5
C	1+2+3	2049.0	1257.0	183.7	3489.7	2812.3	746.3	1664.1	5222.7	8712.5
E	*wl*	660.5	745.0	197.2	1602.7					1602.7
I	*rk*	614.6	554.3	75.3	1244.1					1244.1
P	*os*	448.9	479.2	55.6	983.7					983.7
T	*mix*	165.7	75.1	19.7	260.5					260.5
S	*T*	324.0	78.9	25.8	428.6					428.6
	wl+rk+T	1599.2	1378.1	298.2	**3275.5**					**3275.5**
	M	1794.0	123.7	29.6	1947.3					1947.3
	TOTAL	5442.1	2758.9	511.5	8712.5	2812.3	746.3	1664.1	5222.7	

Source: derived from Central Statistical Bureau of Latvia (2001)

1. Traded sectors (agriculture, fishing, mining, manufacturing, transport).
2. Non-traded sectors (except services not for sale).
3. Services not for sale (public adm. and defense, other community services).

wl. Remuneration of employees.

rk. Net business income, of which

os. Operating surplus (incorporated enterprises).

mix. Mixed income (unincorporated enterprises).

T. Net indirect taxes and transfers including value added tax.

M. Imports.

C+G. Private consumption plus government consumption.

I. Investment

X. Exports

Table 4

AGGREGATED INPUT-OUTPUT MATRIX FOR ESTONIA 1997

(Billion Kroons)

EXPENDITURES

		1	2	3	1+2+3	*C+G*	*I*	*X*	*C+G+I +X*	*TOTAL*
	1	43.1	18.1	1.5	62.7	21.5	10.6	41.9	74.0	136.7
R	2	10.9	15.2	2.2	28.2	21.5	8.9	7.6	38.0	66.2
E	3	0.3	0.5	0.5	1.3	9.5	0.0	0.7	10.2	11.5
C	1+2+3	54.2	33.7	4.2	92.2	52.4	19.6	50.2	122.2	214.4
E	*wl*	13.7	15.7	4.2	33.6					33.6
I	*rk*	10.2	9.9	1.0	21.1					21.1
P	*os*	7.0	8.7	0.8	16.5					16.5
T	*mix*	3.2	1.2	0.2	4.6					4.6
S	*T*	5.5	3.6	0.9	9.9					9.9
	wl+rk+T	29.4	29.2	6.1	**64.6**					**64.6**
	M	53.1	3.3	1.2	57.7					57.7
	TOTAL	136.7	66.2	11.5	214.4	52.4	19.6	50.2	122.2	

Source: derived from Statistical Office of Estonia (1997)

1. Traded sectors (agriculture, fishing, mining, manufacturing, transport).
2. Non-traded sectors (except services not for sale).
3. Services not for sale (public adm. and defense, other community services).

wl. Remuneration of employees.

rk. Net business income, of which

os. Operating surplus (incorporated enterprises).

mix. Mixed income (unincorporated enterprises).

U. Net indirect taxes and transfers including value added tax.

M. Imports.

C+G. Private consumption plus government consumption.

I. Investment

X. Exports

Table 5: Calibration of the model for the Baltic countries

	Latvia	**Estonia**	**Lithuania**
	Initial factors		
y_0	100.00	100.00	100.00
y_{T0}	54.72	47.21	49.09
y_{N0}	45.28	52.79	50.91
k_0	140.00	141.03	133.45
k_{T0}	74.20	64.16	63.21
k_{N0}	65.80	76.87	70.24
L	62.99	62.80	62.82
l_{T0}	35.11	30.29	31.48
l_{N0}	27.88	32.51	31.34
	Parameters		
A_T	1.1967	1.1958	1.2193
A_N	1.1721	1.1712	1.1959
α_T	0.3634	0.3634	0.3634
α_N	0.3892	0.3892	0.3892
γ	0.4467	0.4467	0.4467
G	2.1013	2.0096	2.0709
σ	-1	-1	-1
ε	0.3789	0.4043	0.4159
δ	0.0809	0.0809	0.0809
β	0.9597	0.9597	0.9597
ζ	1.22	1.22	1.22
ϕ	21.88	7.05	28.05

Table 6: Real interest rate risk premia in the Baltic states

	1994	1995	1996	1997	1998	1999	2000	2001
ρ^{Est}	0.170	0.069	0.037	0.022	0.060	0.022	0.016	0.028
ρ^{Lat}		0.189	0.136	0.051	0.032	0.039	0.018	0.021
ρ^{Lit}		0.168	0.140	0.046	0.023	0.037	0.013	0.012

Table 7: Sensitivity analysis with respect to the initial capital stock

Country	Initial capital stock: % of our estimate	Initial capital stock: k/y	Min. trade balance: % of GDP	Min. trade balance: year	Year of reversal
	120	1.69	-11.6	1999	2009
Estonia	100	1.41	-14.6	1999	2009
	80	1.13	-17.6	1997	2009
	120	1.68	-9.6	2002	2013
Latvia	100	1.40	-11.8	2000	2012
	80	1.12	-14.3	2000	2011
	120	1.60	-12.4	2000	2011
Lithuania	100	1.33	-14.4	1998	2011
	80	1.07	-17.4	1998	2010

Figure 1: Trade balance

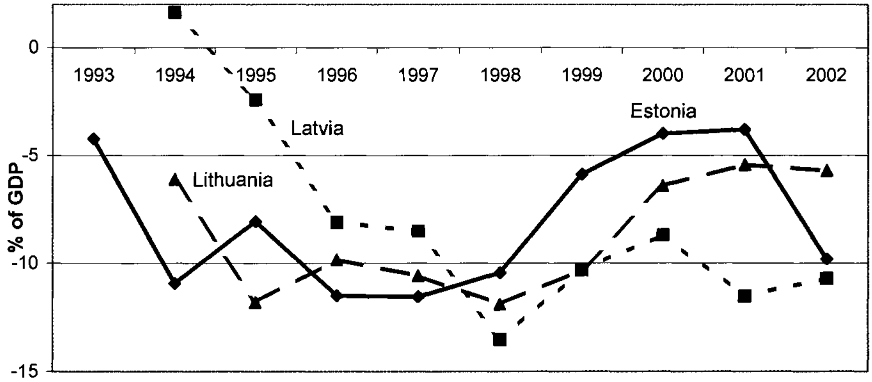


Figure 2: Real GDP growth

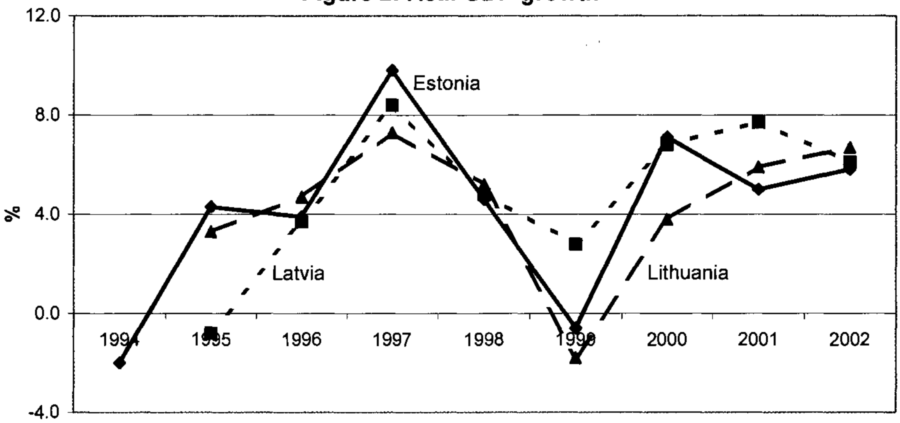


Figure 3: Traded sector

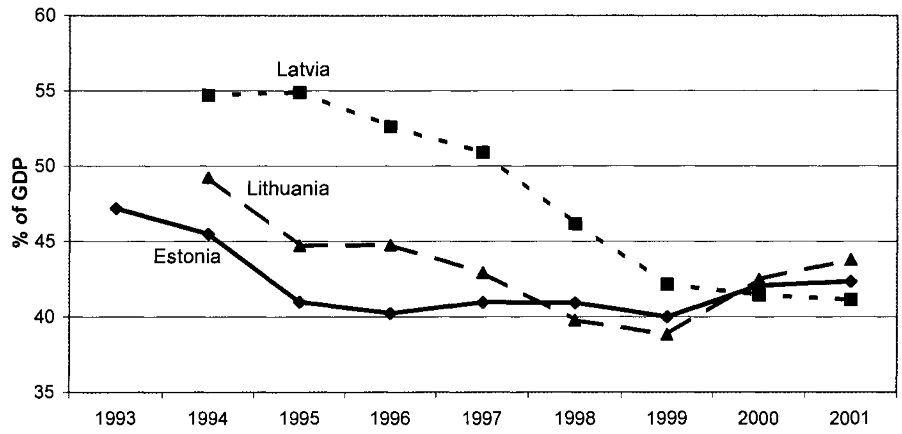


Figure 4: Real exchange rates in the Baltic states

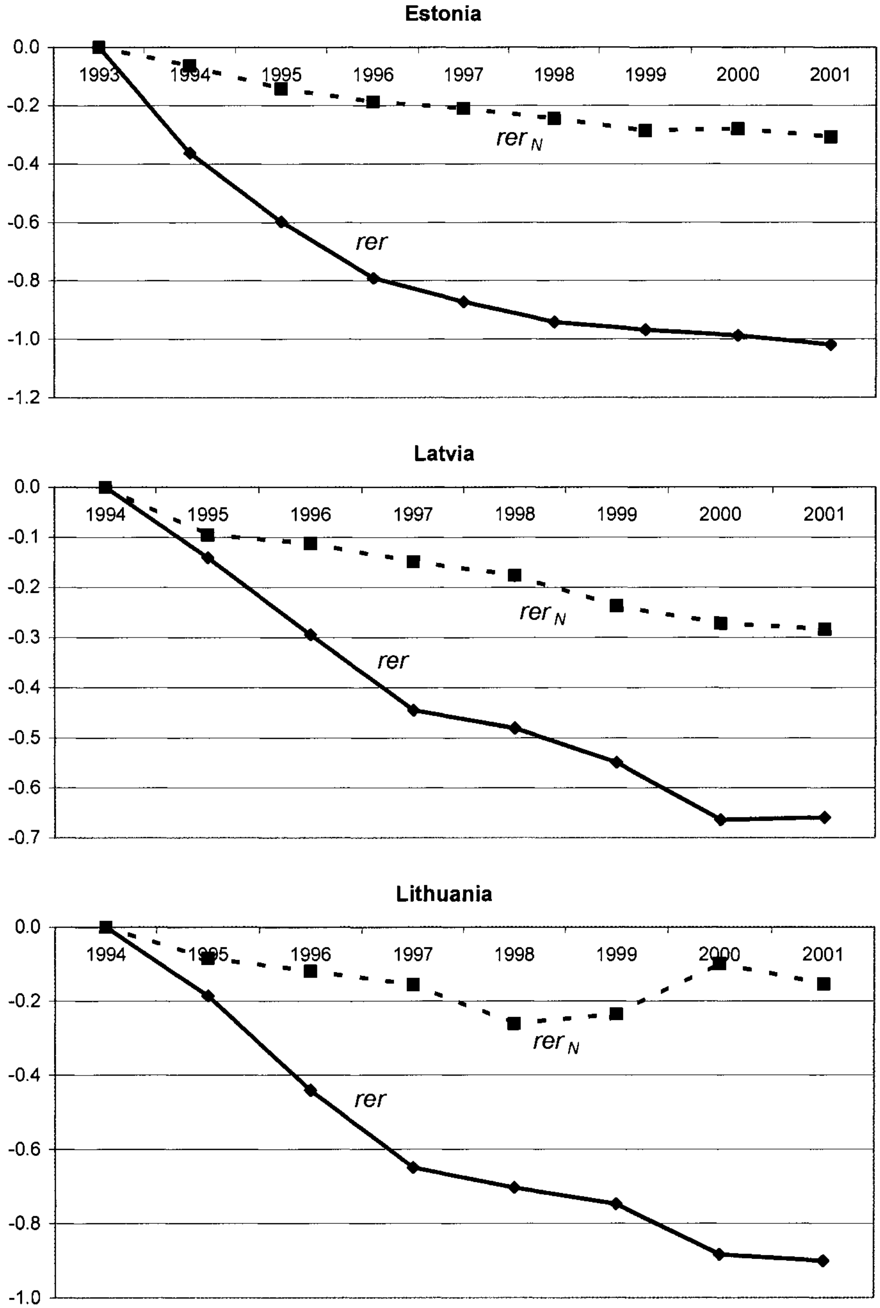


Figure 5: The basic model

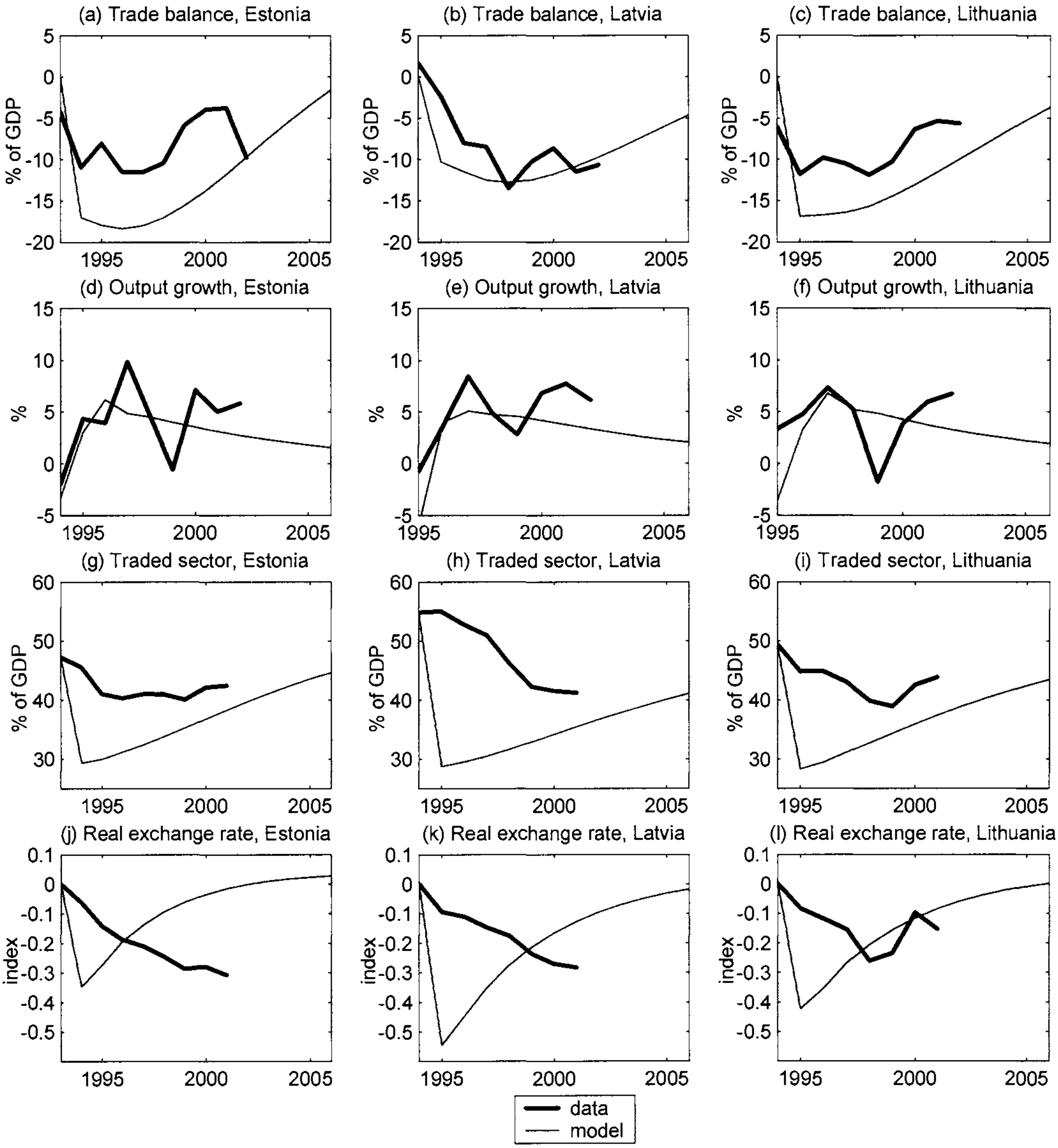


Figure 6: Real gross interest rates in the model

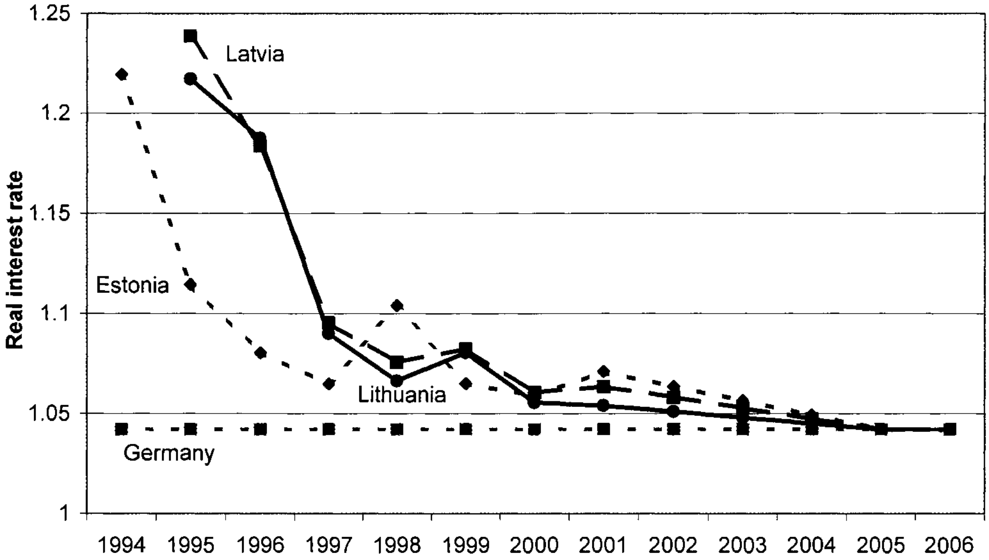
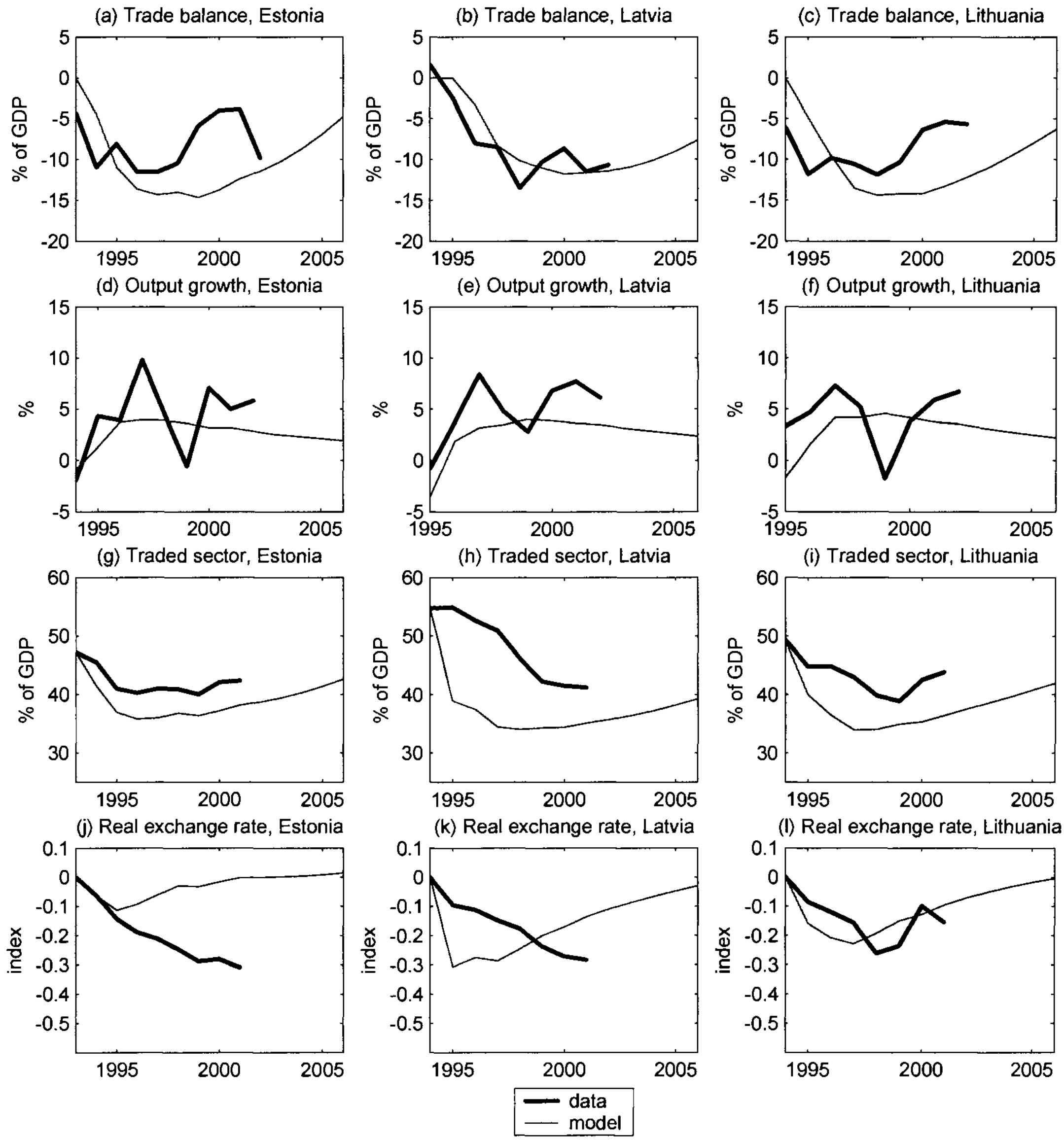


Figure 7: The model with interest rate risk premia



Essay II

Real Exchange Rate and Consumption Fluctuations following Trade Liberalization*

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Abstract

Two-sector models with traded and non-traded goods have problems accounting for the stylized fact that the real exchange rate appreciates and consumption booms for several years following trade liberalization, or exchange-rate-based stabilization programs, in small open economies. The paper investigates some possible solutions to this 'price-consumption puzzle' and evaluates their quantitative importance in calibrated simulations of Spain's accession to the European Community in 1986. Extending the standard two-sector framework, the paper investigates the effects of relative productivity growth in the traded sector along the lines of Balassa-Samuelson, of time-to-build, and of habit formation in preferences. The analysis shows that a calibrated version of the augmented model can account for more of the price-consumption dynamics after trade liberalization than a benchmark two-sector model, without losing explanatory power for other real variables in the Spanish economy after 1986.

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1 Introduction

It is a well documented empirical regularity that the real exchange rate appreciates and consumption booms for several years following trade liberalization or exchange-rate-based stabilization programs in small open economies (Végh (1992) and Uribe (2002)). Two-sector models with traded and non-traded goods that have often been used to analyze such episodes have problems accounting for these stylized facts, as pointed out by Uribe (2002). Typically in these models, the real exchange rate appreciates in the first period of the liberalization or stabilization program and then counterfactually depreciates, while the consumption of non-traded goods increases over time. A class of widely used models thus fails to replicate the observed co-movement between the real exchange rate and consumption.

The paper investigates some possible solutions to this 'price-consumption puzzle', and evaluates their quantitative importance in calibrated simulations of Spain's accession to the European Community in 1986. Extending the two-sector framework of Fernandez de Cordoba and Kehoe (2000), the paper investigates the effects of relative productivity growth in the traded sector along the lines of Balassa-Samuelson, of gestation lags in investment, and of habit formation in preferences.

In the two-sector growth model which we subsequently develop, higher productivity growth in the traded sector makes non-traded goods relatively more expensive to produce over time, as suggested by Balassa (1964) and Samuelson (1964), which implies that the real exchange rate appreciates in the long run. Uribe (1997) shows that gestation lags in investment, combined with convex capital adjustment costs, lead to a gradually increasing investment demand in the initial phase after a positive and permanent shock in a small open economy. Therefore, time-to-build has the potential of making the real exchange rate gradually increase after trade liberalization. Following Uribe (2002), we introduce habit formation in preferences, since it has the potential of causing the consumption of both traded and non-traded goods to increase over time, although the real exchange rate is appreciating in the model.

Ultimately, it is a quantitative question whether the investigated mechanisms are really of importance for improving the model's capacity to replicate the co-movement of the real exchange rate and consumption. Therefore, the paper makes a serious attempt at calibrating habit formation and gestation lags in the model and measuring the relative productivity developments for Spain and Germany through sectoral growth accounting.

The analysis shows that a calibrated version of the augmented model can account for more of the price-consumption dynamics after trade liberalization than the benchmark two-sector model. The magnitudes of the fluctuations in the real exchange rate and consumption improve considerably when relative productivity growth, time-to-build and habit formation are incorporated in the model. The augmented model cannot fully account for the observed co-movement of the real exchange rate and consumption in Spain, but in several periods, the model real exchange rate appreciates while the consumption of both traded and non-traded goods increases. We also show that, except for the trade balance, the augmented model does not lose explanatory power for other real variables in the Spanish economy after 1986.

We next take a look at the data for the real exchange rate and consumption in Spain after 1986. Section 3 develops, calibrates and simulates the basic model, which is similar to the model developed by Fernandez de Cordoba and Kehoe (2000) and will serve as a benchmark for the analysis in the paper. The simulations of the basic model demonstrate the weakness of the standard two-sector framework in the price-consumption dimension. In section 4, we analyze the qualitative effects of introducing higher productivity growth in the traded sector, time-to-build, and habit formation in preferences. Section 5 explains how we calibrate the augmented model and in section 6, we investigate the quantitative relevance of the three mechanisms when simulating the model. The concluding remarks are presented in section 7.

2 The real exchange rate and consumption in Spain after 1986

After joining the European Community, Spain experienced large capital inflows associated with a sustained appreciation of its real exchange rate and a consumption boom. These initial effects of trade liberalization are similar to the effects of the well-documented exchange-rate-based stabilization plans undertaken by several Latin American countries. Uribe (2002) documents the effects of the Argentine convertibility plan of 1991 and identifies a 'price-consumption regularity' in that in the initial phase, the real exchange rate gradually appreciates while the consumption of both traded and non-traded goods increases over time. Végh (1992) and Kiguel and Liviatan (1992) provide extensive evidence on this stylized fact concerning the real exchange rate and consumption for other Latin American countries.

In figure 1, *rer* is the log of the bilateral real exchange rate between Spain and Germany for the years 1986-2002. In the model that we develop in section 3, there is only one traded good and no nominal variables. Therefore, the model can only account for the part of real exchange rate fluctuations that is due to changes in the the relative price of non-traded to tradable goods. Following Fernandez de Cordoba and Kehoe (2000) and Betts and Kehoe (2001), we decompose the real exchange rate into its traded and non-traded components,

$$RER_t = S_t \frac{P_{Tt}^G}{P_{Tt}^S} * \frac{P_t^G}{P_t^S} \frac{P_{Tt}^S}{P_{Tt}^G} \quad (1)$$

$$RER_t = RER_{Tt} * RER_{Nt}, \quad (2)$$

where S_t stands for the nominal exchange rate expressed in units of pesetas per DM, P_t^G is a price index for Germany, P_t^S is price index for Spain and P_{Tt} is a price index for tradable goods.

In equation (2), RER_{Tt} captures relative price changes of traded goods, whereas RER_{Nt} captures relative price changes of non-traded goods. Expressing equation (2) in logarithms, we obtain

$$rer_t = rer_{Tt} + rer_{Nt}. \quad (3)$$

The model assumes Purchasing Power Parity to hold for traded goods, which implies that $rer_{Tt} = 0, \forall t$ and that fluctuations in the real exchange rate can only be caused by movements in the relative price of non-traded goods across countries. When constructing price indices for traded goods, we use Producer Price Indices for the manufacturing sector in each country.[1] The details and the sources of the data are given in Appendix A.

The log of the non-traded component of the real exchange rate, rer_N, is presented together with the log of the real exchange rate in Figure 1. The fact that rer_N appreciates over time means that the relative price of non-traded goods increased faster in Spain than in Germany between 1986 and 2002. The figure reveals a strong co-movement between the two series, with the non-traded component explaining almost two thirds of the gradual appreciation of the real exchange rate up to the currency crisis in 1992.

Figure 2 shows the development of real consumption of traded and non-traded goods in Spain for the years 1986-1998, which grew over the period, with a stronger initial boom in traded consumption. Traded and non-traded goods are defined as in Fernandez de Cordoba and Kehoe (2000). Agriculture and the manufacturing industry constitute the traded sector whereas construction and services for sale constitute the non-traded sector. For details on how we obtain the sectoral consumption data in figure 2, see Appendix A.

Figures 1 and 2 together reveal that the stylized facts concerning the 'price-consumption regularity' apply to Spain after 1986. Following trade liberalization, the real exchange rate and the relative price of non-traded goods gradually appreciated for several years, while aggregate consumption and its non-traded component boomed. The remainder of the paper will therefore try to develop a model that can account for the dynamics of the non-traded component of the real exchange rate, and its co-movement with consumption of both traded and non-traded goods.

[1]For a more detailed discussion of the suitablility of PPI as a measure of price changes for traded goods, see Betts and Kehoe (2001) and Engel (1999).

3 The basic model

The starting point of the analysis in this paper is a model which is representative of the class of two-sector models often used to analyze the effects of trade liberalization or exchange-rate-based stabilization (see Rebelo and Végh (1995) for a survey). In this section, we present a model to which we will refer as the basic model, which is similar to that used in Fernandez de Cordoba and Kehoe (2000). The basic model will provide a benchmark against which to evaluate the quantitative relevance of the mechanisms investigated in the paper for improving the model dynamics for the real exchange rate and consumption.

Spain is modeled as a small open economy with a representative consumer. In the basic model, there are five goods in any period: a traded good, a non-traded good, capital, labor and an investment good augmenting the capital stock in the subsequent period. The traded good is the numeraire in the economy.

The centralized problem consists of maximizing the sum of discounted utility from consumption of traded and non-traded goods, subject to the resource constraints of the economy:

$$\max_{\{c_{Tt},c_{Nt},k_{Tt},k_{Nt},b_{t+1},l_{Tt},x_{Tt},x_{Nt}\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{\sigma}-1}{\sigma}\right], \tag{4}$$

where the consumption of traded and non-traded goods, c_{Tt} and c_{Nt}, constitutes aggregate consumption, c_t, according to

$$\begin{aligned} c_t &= C(c_{Tt}, c_{Nt}) \\ &= \left[\varepsilon c_{Tt}^{\rho} + (1-\varepsilon)\, c_{Nt}^{\rho}\right]^{\frac{1}{\rho}}. \end{aligned} \tag{5}$$

The maximization is subject to the following constraints

$$c_{Nt} + x_{Nt} \leq F_N(k_{Nt}, k_{Nt-1}, l_{Nt}, l_{Nt-1}) \tag{6}$$

$$c_{Tt} + x_{Tt} + b_{t+1} - b_t(1+r) \leq F_T(k_{Tt}, k_{Tt-1}, l_{Tt}, l_{Tt-1}) \tag{7}$$

$$(k_{Tt+1} + k_{Nt+1}) - (1-\delta)(k_{Tt} + k_{Nt}) \leq G x_{Tt}^{\gamma} x_{Nt}^{1-\gamma} \tag{8}$$

$$L = l_{Tt} + l_{Nt}, \tag{9}$$

$$c_{Tt}, c_{Nt}, k_{Tt}, k_{Nt}, l_{Tt}, x_{Tt}, x_{Nt} \geq 0 \quad \forall t$$

$$k_{T0}, k_{N0}, l_{T0}, l_{N0}, b_0 \text{ given},$$

where (6) is the economy's resource constraint for non-traded goods and x_{Nt} is the input of non-traded goods into the investment technology specified on the right-hand side of (8). Note that the production process for non-traded goods, $F_N(.)$, is a function of inputs of capital and labor into the non-traded sector, k_N and l_N, in both the current and the previous period. Output depends on lagged factors of production, due to costs associated with frictions in capital and labor mobility.

The resource constraint for traded goods, (7), includes additional terms that reflect the possibility of trading with the rest of the world; b_{t+1} denotes a foreign bond purchased in period t and redeemed in period $t+1$ at the world interest rate r, which we assume to equal $1/\beta - 1$.

The law of motion for capital is specified in (8). Investment goods are produced using a Cobb-Douglas technology taking traded and non-traded goods as inputs. The model assumes that labor is supplied inelastically, as specified in equation (9), where L is the size of the total labor force.

The utility function exhibits a constant intertemporal elasticity of substitution, which equals $1/(1-\sigma)$, and a constant intratemporal elasticity of substitution, $1/(1-\rho)$. The parameter ε determines relative preferences for traded and non-traded goods, and β is a subjective discount rate.

The production processes are modeled in the following way

$$F_T(k_{Tt}, k_{Tt-1}, l_{Tt}, l_{Tt-1}) = A_T k_{Tt}^{\alpha_T} l_{Tt}^{1-\alpha_T} - \Phi(k_{Tt}, k_{Tt-1}) - \Psi(l_{Tt}, l_{Tt-1}) \tag{10}$$

$$F_N(k_{Nt}, k_{Nt-1}, l_{Nt}, l_{Nt-1}) = A_N k_{Nt}^{\alpha_N} l_{Nt}^{1-\alpha_N} - \Phi(k_{Nt}, k_{Nt-1}) - \Psi(l_{Nt}, l_{Nt-1}), \tag{11}$$

where for $j \in \{T, N\}$

$$\Phi(k_{jt}, k_{jt-1}) = \frac{\zeta}{1+\zeta}\left(\frac{|k_{jt} - (1-\delta)k_{jt-1}|}{k_{jt-1}}\right)^{\frac{1+\zeta}{\zeta}} k_{jt-1}, \quad \zeta > 0 \tag{12}$$

$$\Psi(l_{jt}, l_{jt-1}) = \psi\left(\frac{l_{jt} - l_{jt-1}}{l_{jt-1}}\right)^2 l_{jt-1}, \quad \psi > 0. \tag{13}$$

Here, $\Phi(.)$ is a convex adjustment cost associated with investment, as in Abel and Eberly (1994) and Eberly (1997). Note that the specification in (12) implies that capital frictions are present in steady state, because the cost is associated with the transformation of investment goods rather than the adjustment of the capital stock. $\Psi(.)$ is a quadratic cost associated with the adjustment of the labor force in a sector. The specification implies that there are costs of both hiring and firing when labor moves between sectors. It is in the specification of the factor frictions that the basic model differs from the model in Fernandez de Cordoba and Kehoe (2000). The reason is that the functional forms in equations (12) and (13) are easier to calibrate than the frictions used by Fernandez de Cordoba and Kehoe (2000).

The solution to the centralized problem in (4) corresponds to a decentralized equilibrium where a representative consumer maximizes utility and where firms in the traded, non-traded and investment sectors maximize their profits under perfect competition.

3.1 Calibration of the basic model

To facilitate a comparison between the models used in this paper and the existing literature, the basic model is calibrated as closely as possible to Fernandez de Cordoba and Kehoe (2000). We normalize all prices except the rental price of capital to be 1 in 1986, and use the equilibrium conditions of the model to find the values of the parameters and the initial conditions. Germany, the largest economy in the EC in 1986, will in the paper be used as a proxy for the 'rest of the world' in the small open economy setting.

Aggregating the input-output table for Spain in 1986 under the assumption that Spain was closed to capital flows in 1986, Fernandez de Cordoba and Kehoe calculate values for the parameters ε, γ and G, as well as the sectoral division of output in 1986, y_{T0} and y_{N0}.[2] The initial capital-output ratio is taken from the Penn World Table, where the capital stock estimate includes nonresidential capital, residential construction and transportation equipment. A period is assumed to be a year, and the values of σ, ρ, β and δ are chosen to be identical to the values used in Fernandez de Cordoba and Kehoe (2000).

Eberly (1997) has estimated the convex component of capital frictions of the form specified in (12), using annual data for the OECD countries between 1981-1994. For Spain, we use a value of $\zeta = 1.6133$, which is an average of the values Eberly estimates for France, Germany and the UK. These three countries are the only ones in Europe for which Eberly obtains statistically significant estimates, with values of ζ ranging from 1.29 for Germany to 1.95 for France.

In order to obtain $k_{T0}, k_{N0}, l_{T0}, l_{N0}, \alpha_T, \alpha_N, A_T$ and A_N, we solve a system of eight equations provided by the equilibrium conditions of the model for the autarky steady state in equations (14)-(17). First, note that the output in sector j, where $j = \{T, N\}$, in the last year before liberalization is

$$y_{j0} = A_j k_{j0}^{\alpha_j} l_{j0}^{1-\alpha_j} - \frac{\zeta}{1+\zeta} \delta^{\frac{1+\zeta}{\zeta}} k_{j0}, \tag{14}$$

where the last term is the cost associated with the transformation of investment goods into capital. Next, we use the fact that in equilibrium, capital in each sector earns its marginal product, which implies that

$$y_{j0} X_j = \left(\alpha_j A_j k_{j0}^{\alpha_j - 1} l_{j0}^{1-\alpha_j} + Z \right) k_{j0}, \tag{15}$$

where $Z = -\delta^{\frac{1}{\zeta}} + \beta \delta^{\frac{1}{\zeta}} - \beta \frac{\zeta}{1+\zeta} \delta^{\frac{1+\zeta}{\zeta}}$ is once more a term stemming from investment transformation costs and X_j denotes the income share of capital, which can be obtained from the

[2] Since 1991, the 1986 input-output matrix has been slightly revised (Instituto Nacional de Estadistica, 1986). To facilitate a comparison with the existing literature, we use the same values as in Fernandez de Cordoba and Kehoe (2000).

aggregated input-output matrix for Spain in 1986. Equilibrium conditions also require that the returns to capital and labor are the same in both sectors, which implies that:

$$\begin{aligned} \alpha_T A_T k_{T0}^{\alpha_T - 1} l_{T0}^{1-\alpha_T} &= \alpha_N A_N k_{N0}^{\alpha_N - 1} l_{N0}^{1-\alpha_N}, \\ (1-\alpha_j) A_j k_{j0}^{\alpha_j} l_{j0}^{-\alpha_j} &= 1, \quad j = \{T, N\}, \end{aligned} \tag{16}$$

where in the last equality, we used the fact that the initial wages have been normalized to 1. Finally, we use the market clearing condition for capital:

$$k_0 = k_{T0} + k_{N0}. \tag{17}$$

The labor friction parameter, ψ, is calibrated so that job creation in the model never exceeds the highest rate of sectoral net job creation observed in Spanish data. Aggregating sectoral national accounts data on full-time equivalent jobs, we find that the largest observed net job creation rate between 1986 and 2002 was 2.5 percent in the non-traded sector (OECD, 2004b).

The left-hand column of Table 1 summarizes the initial conditions and the parameter values used when simulating the basic model.

3.2 Simulation of the basic model

When simulating the basic model, we assume that the economy is closed in 1986 and opens up to trade and capital flows in 1987.

Figure 3 presents the simulated time path for the non-traded component of the real exchange rate together with the actual data series for Spain. The model displays an initial appreciation which is too large and it cannot explain the sustained appreciation of the real exchange rate that we observe in the data. The appreciation of the real exchange rate is connected to capital inflows after the opening up to trade in the model. Since the economy can only borrow traded goods, the shock of trade liberalization makes non-traded goods relatively scarce. The investment technology specified in (8) requires the use of both traded and non-traded goods as inputs for augmenting the capital stock. In contrast to traded goods, non-traded goods must be produced at home and thus, become a bottleneck for development. The relative scarcity of the non-traded good is most acute just after liberalization, which causes its relative price to spike in the initial period.

Figures 4 and 5 present the model outcomes for the consumption of non-traded and traded goods, together with the actual data series. We see that the basic model cannot account for the observed dynamics in traded consumption. In the basic model, the consumption of traded goods immediately jumps to its new steady state level in 1987, whereas the data reveals a gradual boom in the consumption of traded goods. The basic model also under-predicts the magnitude of the consumption boom following trade liberalization.

More fundamentally, the basic model cannot account for the observed co-movement between the real exchange rate and consumption. The co-movement between the non-traded component of the real exchange rate and the consumption of non-traded goods is of the incorrect sign from 1987 and onwards in the basic model. In the basic model, the real exchange rate and non-traded consumption move in the same direction, $\Delta c_{Nt} * \Delta rer_{Nt} > 0$, for all periods after liberalization, which is in stark contrast to the data where we observe a negative co-movement, $\Delta c_{Nt} * \Delta rer_{Nt} < 0$, for Spain in many periods.

In the basic model, the consumption of non-traded goods is directly linked to the relative price of non-traded goods. As the relative price of non-traded goods, and therefore the model real exchange rate, depreciates after an initial spike in 1987, the representative consumer chooses to consume relatively more of the non-traded good over time. To successfully model the observed co-movement between the real exchange rate and consumption, mechanisms that can create a sustained appreciation of the real exchange rate and break the direct link between the relative price of non-traded goods and the consumption of non-traded goods must be introduced.

4 Augmenting the model

In this section, we investigate the qualitative effects of introducing relative productivity growth in the traded sector along the lines of Balassa-Samuelson, of gestation lags in investment, and of habit formation in preferences.

4.1 Higher productivity growth in the traded sector

As pointed out in the canonical papers by Balassa (1964) and Samuelson (1964), productivity differentials between the traded and non-traded sectors are of importance for the relative price of non-traded goods in an economy (Asea and Corden (1992) provide a good overview of the theory). Substantial empirical evidence shows countries with higher sectoral difference in total factor productivity growth to have experienced higher levels of relative inflation in the non-traded sector (De Gregorio et al. (1994)).

Differentiated productivity growth is introduced into the model by allowing for time varying total factor productivity in the production of the traded good. Since productivity in Spain has grown in both sectors since 1986, the model thus augmented can only account for effects due to differences in productivity growth between the sectors. The production functions in equations (10) and (11) are changed to

$$F_{Tt}(k_{Tt}, k_{Tt-1}, l_{Tt}, l_{Tt-1}) = A_{Tt} k_{Tt}^{\alpha_T} l_{Tt}^{1-\alpha_T} - \Phi(k_{Tt}, k_{Tt-1}) - \Psi(l_{Tt}, l_{Tt-1}) \tag{18}$$

$$F_N(k_{Nt}, k_{Nt-1}, l_{Nt}, l_{Nt-1}) = A_N k_{Nt}^{\alpha_N} l_{Nt}^{1-\alpha_N} - \Phi(k_{Nt}, k_{Nt-1}) - \Psi(l_{Nt}, l_{Nt-1}), \tag{19}$$

where we have introduced a time subscript in the traded productivity parameter, A_{Tt}. Forming a Lagrangian of the utility maximization problem in (4), we define the Lagrange multiplier on the non-traded resource constraint as p_{Nt} and the Lagrange multiplier on the traded resource constraint as p_{Tt}.

If we ignore adjustment costs, we can analytically study the effect of higher technological growth in the traded sector on the relative price of non-traded goods. Using the price of traded goods as numeraire, we follow Rebelo (1993) and study the optimality conditions for capital and labor. Equating the marginal products of capital and labor in the two sectors, we obtain

$$\alpha_T A_{Tt} k_{Tt}^{\alpha_T - 1} l_{Tt}^{1-\alpha_T} = p_{Nt} \alpha_N A_N k_{Nt}^{\alpha_N - 1} l_{Nt}^{1-\alpha_N} \tag{20}$$

$$(1-\alpha_T) A_{Tt} k_{Tt}^{\alpha_T} l_{Tt}^{-\alpha_T} = p_{Nt}(1-\alpha_N) A_N k_{Nt}^{\alpha_N} l_{Nt}^{-\alpha_N}. \tag{21}$$

Dividing equation (20) with equation (21) and rearranging, we can show the capital-labor ratio in the non-traded sector to be proportional to the capital-labor ratio in the traded sector along the equilibrium path:

$$\frac{k_{Nt}}{l_{Nt}} = \frac{\alpha_N(1-\alpha_T)}{\alpha_T(1-\alpha_N)} * \frac{k_{Tt}}{l_{Tt}}. \tag{22}$$

Plugging equation (22) into equation (21), we obtain the following expression for the relative price of non-traded goods:

$$p_{Nt} = \frac{(1-\alpha_T)}{(1-\alpha_N)} \left[\frac{\alpha_N(1-\alpha_T)}{\alpha_T(1-\alpha_N)}\right]^{-\alpha_N} * \frac{A_{Tt}}{A_N} * \left(\frac{k_{Tt}}{l_{Tt}}\right)^{\alpha_T-\alpha_N}. \tag{23}$$

In equation (23), we see that if there is no technological change in the traded sector, the relative price of non-tradables will depreciate in the long run when $\alpha_T < \alpha_N$, which is the case in our calibration for Spain. This is because the capital-labor ratio will be higher in the new steady state than in the initial period of trade liberalization, thereby making the capital intensive good relatively cheaper in the new steady state.

Faster relative technological growth in the traded sector works in the opposite direction, and tends to appreciate the relative price of non-traded goods over time, since p_{Nt} is positively dependent on the relative productivity factor, A_{Tt}/A_N. The productivity growth in the traded sector causes the real wage to rise, making the production of non-traded goods relatively more expensive. Which of the two effects that dominates is a quantitative question.

What is of importance for the non-traded component of the real exchange rate in equation (2) is the relative price of non-traded goods in Spain, relative to the same price in Germany. However, the model assumes Germany to be in steady state and the German relative price of non-traded goods to remain constant over time. The model real exchange rate is thus entirely driven by changes in the relative price of non-traded goods in Spain.

Figure 6.a compares the dynamics for the real exchange rate in the basic model and in a model where A_{Tt} grows by one percent per year during 15 years from 1986. The relative productivity growth rate was arbitrarily chosen for illustrative purposes. The parameters and initial conditions are the same, except for ψ, which is readjusted to match the maximum sectoral net job creation in the Spanish data. In line with the above discussion, we note that the real exchange rate in the basic model depreciates in the long run, whereas the model with differentiated productivity growth produces a long-run appreciation of the real exchange rate. We also see that relative productivity growth creates an even larger initial appreciation of the real exchange rate, because the economy with productivity growth starts out further away from its new steady state.

With perfect foresight, the larger wealth effect due to productivity growth after liberalization translates into increased consumption and investment demand, which leads to a higher relative price of non-traded goods than in the basic model. Figures 6.b and 6.c show the effects on consumption of non-traded and traded goods. As can be seen in Figure 6.d, the economy with productivity growth finances its higher levels of consumption by running a larger trade deficit, i.e. by borrowing more from abroad against future production. In figures 6.e and 6.f, we see that investment increases and that the economy accumulates a larger capital stock, since the return on capital increases when productivity grows.

On its own, the Balassa-Samuelson effect can only take us part of the way towards improving the dynamics for the real exchange rate in the model. Although the real exchange rate, in accordance with the data, appreciates in the long run, the initial dynamics deteriorate with higher productivity growth in the traded sector. The model with sectoral productivity differences furthermore remains unable to replicate the observed co-movement of the real exchange rate and consumption.

4.2 Time-to-build

The basic model assumes that investment goods can be transformed into capital within a year, which is a stark assumption when considering that augmenting the capital stock in the real world requires investment in infrastructure, as well as the construction of houses and factories.

Kydland and Prescott (1982) emphasized the importance of time-to-build in creating a persistent investment response to shocks in a general equilibrium model. Empirical studies have found evidence on completion times longer than two years for investment projects in many industries (Peeters (1996) and Koeva (2000)). Christiano and Todd (1996) stress that lags between investment decision and project completion also stem from the need to plan before engaging in the physical investment process.

In a two-sector economy, hit by a positive and permanent shock, Uribe (1997) points out that gestation lags combined with convex factor adjustment costs lead to a gradually increasing investment demand. A lower initial investment demand for non-traded goods could potentially improve the dynamics for the relative price of non-traded goods in our model.

With gestation lags in the model, the investment technology and the laws of motion for the capital stocks change to

$$i_{Tt} + i_{Nt} \leq G x_{Tt}^{\gamma} x_{Nt}^{1-\gamma} \tag{24}$$

$$i_{Tt} = \frac{1}{J}\sum_{i=0}^{J-1} s_{Tt-i}, \quad i_{Nt} = \frac{1}{J}\sum_{i=0}^{J-1} s_{Nt-i} \tag{25}$$

$$k_{Tt+J} = (1-\delta)\, k_{Tt+J-1} + s_{Tt} \tag{26}$$

$$k_{Nt+J} = (1-\delta)\, k_{Nt+J-1} + s_{Nt}, \tag{27}$$

where J is the number of gestation lags and s_{Tt} and s_{Nt} are the number of investment projects initiated in period t in the traded and non-traded sectors, respectively. To build s units of capital available in period $t+J$, the economy must invest s/J units of investment goods for J consecutive periods, starting in period t.

Note that the law of motion for the aggregate capital stock can be written

$$\frac{1}{J}\left[k_{Tt+J} + k_{Nt+J} + \delta\sum_{i=1}^{J-1}(k_{Tt+i} + k_{Nt+i}) - (1-\delta)\left(k_{Tt} + k_{Nt}\right)\right] \leq G x_{Tt}^{\gamma} x_{Nt}^{1-\gamma}, \tag{28}$$

which for $J = 1$ is identical to the law of motion for capital in the basic model in equation (8).

Figure 7.a compares the dynamics for the real exchange rate in the basic model and in a model where it takes three years to build capital ($J = 3$). None of the models presented in the figure incorporate productivity growth, whereas convex investment costs are present in both models. The initial conditions and parameters are the same in the models, except for ψ, which is adjusted so that both models match the maximum sectoral net job creation in the data. We observe that the real exchange rate in the model with time-to-build appreciates much less in the initial period and then gradually appreciates for another two years, since the investment demand for non-tradable goods is reduced in the initial period and gradually increases for J periods (see figure 7.e). This leads to a gradual increase in the relative price of the non-traded good, since the supply of non-traded goods is rather inelastic up to period J (the only way of increasing non-traded output up to period J is by moving capital and labor into the non-traded sector, which is costly due to frictions in factor mobility). For higher values of J, time-to-build leads to a longer gradual appreciation of the real exchange rate.

The wealth effect associated with trade liberalization is smaller in the economy with gestation lags than in the basic model economy, since time-to-build constitutes an additional friction in capital accumulation. In figures 7.b and 7.c, this is reflected in lower steady state consumption levels. Figure 7.d shows that the reduced investment demand improves the initial trade deficit, while figure 7.f shows that the long-run capital stock is lower in the model with time-to-build.

Although the introduction of time-to-build enables us to model a gradual initial appreciation of the real exchange rate, figures 7.a-7.c reveal that gestation lags *per se* do not help explain the price-consumption regularity. Counter to what we observe in the data, the model with gestation lags still predict a positive co-movement between the real exchange rate and consumption from 1987 onwards.

To investigate the extent to which time-to-build on its own can slow down capital accumulation in the model, it is of interest to look at a model where the convex investment costs have been switched off. In Appendix B, we compare the basic model to a model where the convex costs in investment have been replaced by a time-to-build technology. The solutions show that for $J = 3$, the model economy with only time-to-build borrows more from abroad, accumulates a higher level of capital and both invests and consumes more than the basic model economy. Consequently, the initial appreciation of the real exchange rate is larger than in the basic model. To dampen the initial demand for non-traded goods in the economy with only time-to-build and labor frictions, a very high number of gestation lags would be required.

4.3 Habit formation in preferences

We now proceed to analyze the effects of introducing habit formation into the basic model.

Habit formation in preferences has the potential of making consumption in the model increase over time, although the real exchange rate is appreciating. Following Uribe (2002), we first analyze why the basic model cannot account for the observed co-movement between the real exchange rate and consumption in Spain after 1986, and then investigate the qualitative effects of introducing habit formation.

Let $u(c_t) = (c_t^\sigma - 1)/\sigma$. In the basic model, the optimality conditions for consumption of traded and non-traded goods are

$$u'(c_t)C_1(c_{Tt}, c_{Nt}) = p_{Tt} \tag{29}$$

$$u'(c_t)C_2(c_{Tt}, c_{Nt}) = p_{Nt}, \tag{30}$$

where C is the consumption aggregator defined in equation (5), and where p_{Tt} and p_{Nt} are the Lagrange multipliers on the resource constraints for the traded and non-traded sectors, respectively. Dividing equation (30) with equation (29), and using the price of traded goods as numeraire, we obtain an expression for the relative price of non-traded goods,

$$p_{Nt} = \frac{(1-\varepsilon)}{\varepsilon} \left(\frac{c_{Tt}}{c_{Nt}} \right)^{1-\rho}, \tag{31}$$

which for negative values of ρ tells us that the relative price of non-traded goods is increasing in the ratio c_{Tt}/c_{Nt}. The optimality condition for foreign bonds, b_{t+1}, is given by

$$p_{Tt} = \beta(1+r)p_{Tt+1}, \tag{32}$$

which, since the world gross interest rate is equal to the inverse of the discount rate in our small open economy, implies that the marginal utility of consuming traded goods should be constant over time. Equation (32) thus explains why consumption of traded goods immediately jumps to its steady state level in the basic model. With consumption of traded goods being constant, equation (31) forces the consumption of non-traded goods to vary negatively with the relative price of non-traded goods, and hence to vary positively with the non-traded component of the real exchange rate. As pointed out in the previous sections, this is at odds with the price-consumption regularity in the data.

When habit formation is introduced in the model, the discounted sum of utility in (4) changes to

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - \theta c_{t-1})^{\sigma} - 1)}{\sigma} \right], \tag{33}$$

where $\theta \in [0,1)$ is a habit stock parameter. The optimality condition for the consumption of traded goods in equation (29) becomes

$$[u'(c_t - \theta c_{t-1}) - \theta\beta u'(c_{t+1} - \theta c_t)]\, C_1(c_{Tt}, c_{Nt}) = p_{Tt}, \tag{34}$$

which together with equation (32) implies that the consumption of traded goods can vary over time in the model. Equation (31) still holds in the model with habit formation, but with the consumption of traded goods varying, the model no longer forces the consumption of non-traded goods and the relative price of non-traded goods to move in different directions. In the model, an increasing relative price of non-traded goods is compatible with an increase in the consumption of non-traded goods, as long as the consumption of traded goods increases more. Interestingly, a look at the data in figures 1 and 2 reveals that this is exactly what happened in Spain after 1986.

Habit formation introduces an addictive element in preferences. The more one eats, the hungrier one wakes up in the next period. The wealth effect associated with trade liberalization therefore results in a gradual increase in the consumption of both goods. This is illustrated in figures 8.b and 8.c, where we compare the consumption dynamics in the basic model and in a model with habit formation. Neither of the models incorporate productivity growth or gestation lags in investment. For the model with habit formation, we have chosen a value of $\theta = 0.8$.

In figure 8.a, we see that the initial reduction and the gradual increase in consumption demand due to habit formation on its own is not sufficient to create a sustained appreciation of the real exchange rate in our model. Non-traded goods are still most scarce in the initial period, and the reduced consumption demand can only dampen the initial spike in the relative price of non-traded goods.

Figures 8.d to 8.f show that the trade deficit improves in the model with habits, whereas initial investment increases to build a larger capital stock that can sustain higher long-run levels of consumption.

The effects of introducing habit formation in the basic model stand in contrast to the model developed by Uribe (2002), where habit formation leads to a gradual appreciation in the real exchange rate. In that model, there is no capital accumulation in the non-traded sector, which makes the supply of non-traded goods more inelastic than in the model presented here. Furthermore, non-traded goods are not used as inputs into investment, which eliminates the effects of investment demand that play an important role in determining the relative price of non-traded goods in our model. A gradually increasing consumption demand can therefore create a gradual appreciation of the real exchange rate in the model presented by Uribe (2002).

5 Calibration of the augmented model

To evaluate the quantitative significance of the three mechanisms discussed in the previous section, we calibrate a model incorporating higher technological growth in the traded sector, gestation lags in investment, and habit formation in preferences. For the parameters governing time-to-build and habit formation, we use standard values in the literature, while productivity differentials are measured in sectoral growth accounting for Spain and Germany.

5.1 Calibration of time-to-build and habit formation

Kydland and Prescott (1982) note the average construction time for plants to be around two years. Peeters (1996) finds that the completion of investment projects often takes more than two years and Koeva (2000) provides evidence of the average time from decision to completion of a plant being more than two years in industries such as food, textile communications, wholesale, transportation and utilities. According to the input-output matrices published by Instituto Nacional de Estadistica (1986-1994, 1995-1998), construction was the most important investment input in Spain between 1986 and 1998. In calibrating the model, we therefore choose a value of $J = 3$, implying that it takes three years to put new capital in place after trade liberalization.

Following Uribe (1997) and Koeva (2001), the augmented model incorporates both convex adjustment costs in investment and time-to-build. As discussed in Appendix B, a model with only time-to-build displays oscillatory solutions and cannot improve the initial dynamics for the relative price of non-traded goods.[3]

Although microeconometric evidence on habit formation is scarce, empirical work using aggregate data has found support for the inclusion of previous consumption levels in the utility function (Constantinides and Ferson (1991) and Fuhrer (2000)). In the macroeconomic literature, Constantinides (1990) used a value of $\theta = 0.8$ to explain the equity premium puzzle, while Jermann (1998) and Boldrin et al. (2001) find that values of θ between 0.73 and 0.82 best enable real business cycle models to replicate the properties of the business cycle and asset prices in the US economy. Therefore, we will use a value of $\theta = 0.8$ in the simulation of our augmented model.

5.2 Measurement of sectoral productivity growth

Our modelling framework assumes Germany to be in steady state and there to be no differences in productivity growth between the traded and non-traded sectors in Germany. To augment the model in a realistic fashion, the observed growth in relative sectoral productivity for Spain should therefore be adjusted for the growth in relative sectoral productivity observed in Germany.

To measure the growth of sectoral productivity in both countries, we first estimate capital stocks consistent with a three period time-to-build technology. Then, we use data on aggregate capital, sectoral output and sectoral labor inputs to compute the sectoral productivities and capital stocks implied by the equilibrium conditions of the model. The data sources and the details of the method used in our sectoral growth accounting exercise are given in Appendix A.

By relative sectoral productivity in period t, we mean A_{Tt}/A_{Nt}. Since the model assumes productivity in the non-traded sector to be constant over time, the growth of relative sectoral productivity in the model is given by the productivity growth in the traded sector. The annual growth rate for traded sector productivity in the model should be set to capture the observed difference between the growth rates in relative sectoral productivity for Spain and Germany,

$$\frac{A_{Tt} - A_{Tt-1}}{A_{Tt-1}} = \frac{\left(A^S_{Tt}/A^S_{Nt}\right)}{\left(A^S_{Tt-1}/A^S_{Nt-1}\right)} - \frac{\left(A^G_{Tt}/A^G_{Nt}\right)}{\left(A^G_{Tt-1}/A^G_{Nt-1}\right)}, \tag{35}$$

where superscripts S and G denote the observed productivities for Spain and Germany, respectively.

[3]One problem with including both types of capital frictions is that the estimation of ζ in Eberly (1997) is based on a model without gestation lags. However, as we will discuss later, the outcomes of the augmented model are not sensitive to the precise value of the investment cost parameter.

Due to the quality of the data and the particular form of the equilibrium conditions used in our exercise, we should not put too much emphasis on any one observation of the measure in equation (35). According to our calculations, the average annual growth rate of relative sectoral productivity between 1986 and 2001 was 4.72 percent in Spain and 3.28 percent in Germany. Therefore, we feed a yearly growth rate of 1.44 percent for productivity in the traded sector between 1986 and 2001 into the augmented model. From 2001, we assume that the difference between Spain and Germany decreases linearly, to completely have vanished by 2010.

Our results on relative productivity are in line with the Balassa-Samuelson framework. As noted in figure 1, Spain has had a higher inflation in the non-traded sector than Germany over the 1986-2001 period and, according to our calculations, it also has had a higher growth in relative sectoral productivity. Furthermore, the measured productivity differential is in line with empirical work by Sinn and Reutter (2001) who find that the relative sectoral labor productivity grew faster in Spain than in Germany after 1986. They estimate the average difference in sectoral relative labor productivity between Spain and Germany to be 1.94 percentage points between 1987 and 1995.[4]

The sectoral growth accounting is robust to the capital stock estimates that we employ. Using the capital stock estimates of Conesa and Kehoe (2003) for Spain and the sectoral capital stocks in the OECD STAN database (OECD, 2004) for Germany, we arrive at an average relative productivity growth differential of 1.51 percentage points over the 1992-2000 period.

The initial conditions and parameter values used in the simulation of the augmented model are presented in the right-hand column of Table 1.

6 Simulation of the augmented model

Figure 9 compares the Spanish data with the dynamics for the real exchange rate and consumption in the basic model and the augmented model. In figures 9.a and 9.b, we see that the augmented model can explain a much larger part of the observed fluctuations in the real exchange rate than the basic model. In the augmented model, time-to-build and habit formation dampen the initial appreciation of the real exchange rate. The three-period time-to-build technology leads the real exchange rate to gradually appreciate during the three initial periods, while relative sectoral productivity growth causes the model real exchange rate to appreciate in the long run.

Figures 9.c to 9.f further reveal that the augmented model can also account for a larger part of the observed co-movement between the real exchange rate and consumption than

[4] Sinn and Reutter (2001) define labor productivity as value added, divided by employment. Replicating their study with our data on real value added and employment, we arrive at an average relative labor productivity differential of 1.61 percentage points per year.

the basic model. The augmented model does not fully account for the observed co-movement, but in several periods the model real exchange rate is appreciating while the consumption of both traded and non-traded goods increases. Habit formation causes the consumption of both traded and non-traded goods to increase over time, although the relative price of non-traded goods is appreciating. We also see that habit formation and the wealth effect of productivity growth lead to consumption booms closer to what we observe in the data than in the basic model.

To evaluate the augmented model, it is important to investigate how it performs for other real variables as compared to the basic model. In figure 10, we compare the two models with Spanish data for the trade balance, the relative size of the traded sector, real GDP, the relative size of the labor force in the traded sector and aggregate investment. The figure reveals that the augmented model matches the data better or equally well in all dimensions, except the trade balance. Higher productivity growth in the traded sector causes the augmented model economy to borrow more against future income in the early stages of transition. The first and second columns of table 2 present the mean square errors for the basic model and the augmented model for all variables examined in figures 9 and 10. The data sources and the methods used in producing the figures are given in Appendix A.

6.1 Sensitivity analysis

The dynamics of the augmented model are robust to varying the parameter governing the elasticity of substitution across goods, ρ, and the parameter governing the intertemporal elasticity of substitution, σ. For values of ρ and σ between -0.5 and -4, the model outcomes for the real exchange rate and its co-movement with consumption only change slightly.

The exact value of the habit stock parameter, θ, does not matter for the ability of the augmented model to better replicate the observed price-consumption dynamics. For a value of $\theta = 0.5$, the model predicts a slightly larger initial appreciation of the real exchange rate and a slightly larger trade deficit, while the consumption of both traded and non-traded goods increases over time. For values of θ above 0.8, the model displays a smaller initial appreciation of the real exchange rate, a smaller trade deficit and a larger consumption boom.

The outcomes of the augmented model are not sensitive to the precise value of the investment cost parameter, ζ, partly because the labor friction parameter, ψ, is calibrated so that each version of the model matches the maximum sectoral job creation rate in the data. A higher value of ζ thus implies a lower value of ψ, and vice versa.

The quantitative results of the augmented model are sensitive to the growth rate of productivity in the traded sector which we use in the model. For a higher annual growth rate than 1.44 percent between 1986 and 2001, the model displays a larger initial appreciation of the real exchange rate and a larger trade deficit. The reason, as discussed in section 4.1, is that the model economy starts out further from its new steady state which, in our environment of perfect foresight, leads to more borrowing against future income to optimally smooth consumption. Similarly, changing our assumption that the relative sectoral productivity differential between Spain and Germany will vanish by 2010 would also affect the dynamics of the model. If the productivity difference were to vanish earlier, the fluctuations in the trade deficit and the initial appreciation of the real exchange rate would be dampened in the model.

The number of gestation lags in the time-to-build technology affects the ability of the augmented model to account for the price-consumption regularity in the initial periods after trade liberalization. In figures 11 and 12, we compare the Spanish data with the dynamics in the basic model and an augmented model with $J = 2$, whereas figures 13 and 14 present the corresponding comparison for a model where $J = 5$. The figures reveal that for any number of gestation lags between $J = 2$ and $J = 5$, the augmented model performs better than the basic model in the price-consumption dimension, while performing worse only for the trade deficit. Although a construction period of five years may be unrealistically long, it is intriguing to see how well the model with $J = 5$ can account for the observed fluctuations in the real exchange rate, consumption and investment. The mean square errors for the two models are reported in the third and fourth columns of table 2.

7 Concluding remarks

The analysis has shown that introducing higher productivity growth in the traded sector, time-to-build and habit formation enhances the quantitative performance of the standard two-sector model we use to simulate the Spanish economy after the accession to the European Community. A calibrated version of the augmented model can better account for the price-consumption dynamics after trade liberalization, than the model developed by Fernandez de Cordoba and Kehoe (2000). The magnitudes of the fluctuations in the real exchange rate and consumption improve considerably in the augmented model. Although it cannot fully account for the observed co-movement of the real exchange rate and consumption in Spain, the augmented model displays dynamics where the real exchange rate appreciates in several periods, while the consumption of both traded and non-traded goods increases.

In enhancing the model's capacity to replicate the observed price-consumption dynamics in Spain, the investigated mechanisms have different effects. By decreasing the initial demand for non-traded goods, time-to-build and habit formation dampen the initial appreciation of the real exchange rate in the augmented model. The time-to-build technology also causes investment demand to gradually increase in the model, which leads to a gradual appreciation of the real exchange rate in the initial periods after trade liberalization, while higher productivity growth in the traded sector causes the model real exchange rate to appreciate in the long run. Habit formation in preferences makes the consumption of both traded and non-traded goods increase over time, although the relative price of non-traded goods is appreciating.

The analysis further showed that the augmented model does not lose explanatory power for other real variables compared to the benchmark model, with the exception of the trade balance, which deteriorates due to productivity growth. We therefore conclude that the mechanisms investigated in the paper constitute extensions of the two-sector framework that are important for the quantitative analysis of small open economies in transition.

A Data

To construct the real exchange rate series labeled rer in figure 1, we use the peseta/DM nominal exchange rate and the Consumer Price Index for each country. When constructing price indices for traded goods (P^S_{Tt} and P^G_{Tt}) to calculate rer_N in figure 1, we use Producer Price Indices for the manufacturing sector in Germany and Industrial Prices for Spain. The real exchange rate data is taken from the IFS database (IMF, 2004).

The sectoral consumption series in figure 2 are obtained by aggregating input-output tables for Spain 1986-1994, and tables of total use for the years 1995-1998. The input-output tables were purchased from the Instituto Nacional de Estadistica (1986-1994), while the tables for 1995-1998 were obtained from the web site of the Instituto Nacional de Estadistica. The sectoral consumption figures are deflated using industrial prices for traded goods and the sectoral value added deflator for non-traded goods. The sectoral value added deflator is obtained from the SourceOECD database (OECD, 2004a).

In line with Peeters (1996), the aggregate capital stock estimates in section 5.2 are calculated by adjusting the Perpetual Inventory Method to accommodate three gestation lags according to equation (28). We use a value of $\delta = 0.056$ for both Germany and Spain and data on Gross Fixed Capital Formation from the IFS (IMF, 2004). The capital-output ratios for 1965-1967 in the Penn World Table were used to obtain the starting values for the capital stocks (Summers et al. (1995)). Using data on nominal GDP from the IFS database (IMF, 2004), we calculate aggregate capital-output ratios for both Germany and Spain, which together with data on real GDP in the sourceOECD database (OECD, 2004a) enable us to calculate the aggregate capital stocks used in equation (38).

For each country, the sectoral productivity parameters and capital stocks are found by solving the following system of equations for $t = (1986, 1987, ..., 2001)$,

$$\begin{aligned} y_{Tt} &= A_{Tt} k_{Tt}^{\alpha_T} l_{Tt}^{1-\alpha_T} - \Phi(k_{Tt}, k_{Tt-1}) && (36) \\ y_{Nt} &= A_{Nt} k_{Nt}^{\alpha_N} l_{Nt}^{1-\alpha_N} - \Phi(k_{Nt}, k_{Nt-1}) && (37) \\ k_t &= k_{Tt} + k_{Nt} && (38) \\ (1-\alpha_T) A_{Tt} k_{Tt}^{\alpha_T} l_{Tt}^{-\alpha_T} &= p_{Nt}(1-\alpha_N) A_{Nt} k_{Nt}^{\alpha_N} l_{Nt}^{-\alpha_N}, && (39) \end{aligned}$$

where the unknowns are A_{Tt}, A_{Nt}, k_{Tt} and k_{Nt} and where we have ignored labor frictions. For 1986, the system is assumed to be in steady state, so that the sectoral capital stocks in 1985 were the same as in 1986. For the convex capital adjustment cost, we use a value of $\zeta = 1.29$ for Germany as estimated by Eberly (1997). The capital intensity parameters α_T and α_N are both assumed to equal 0.3 in Germany, which is a fair approximation according to Gollin (2002).[5] The data on sectoral GDP and labor shares used in equations (36), (37)

[5]We could improve on this approximation using a German input-output table to obtain values for α_T and α_N. The calculated relative sectoral productivities are not very sensitive to the sectoral capital

and (39) are also obtained from the sourceOECD database (OECD, 2004a, 2004b). For Germany, the labor data was complemented for the period 1986-1990 with data from the micro census of Statistisches Bundesamt (2003). The German unification in 1991 causes a jump in the data series which we use for output and investment, but the resulting jump in our productivity measures for Germany are symmetric across the two sectors.

One possible estimate of the relative price of non-traded goods, p_{Nt}, in equation (39) would be the ratio of the non-traded value added deflator and the producer price index. To numerically solve the system of equations in (36)-(39), we use a model-based estimate of p_{N_t} that ignores factor frictions, however.[6] Both the estimate directly available in the data and our model-based estimate appreciate by about 31 percent more in Spain than in Germany during the period 1986 to 2001, which indicates that the effects of ignoring factor frictions are close to symmetric across the two countries.

When solving the system in equations (36) to (39), we ignore labor frictions, since the value of ψ used in the model will depend on the calculated productivity growth (remember that ψ is calibrated so that the model replicates the maximum sectoral job creation rate in the data). The inconsistency resulting from ignoring labor frictions in the sectoral growth accounting for both countries is only likely to have a small effect, however.[7] According to equation (13), labor movement between sectors would have a similar effect on the measured productivity parameters in both sectors for $\psi > 0$, which means that the relative productivity measure would be close to that obtained when ignoring labor frictions. The share of the labor force working in the traded sector has furthermore decreased in both Spain and Germany over the 1986-2001 period, so that part of the errors resulting from ignoring labor frictions can be expected to cancel out in equation (35).

It would be desirable to investigate whether it is possible to develop an algorithm that makes use of the relative price estimate available in the data, that accounts for both labor and capital frictions, and that iteratively calculates sectoral productivities to find a value of ψ consistent with both the model and the data.

In figure 10, the trade balance as a percentage of GDP is calculated using data from IFS (IMF, 2004) on the trade balance and GDP. To construct the series for traded output as a percentage of GDP, for real GDP and for the labor share in the traded sector, we use data from the SourceOECD database (OECD, 2004a, 2004b). The data on investment was

intensities, however.

[6]Ignoring factor frictions, the relative price of non-traded goods can be expressed as

$$p_{Nt} = \frac{(1-\alpha_T) y_{Tt} l_{Nt}}{(1-\alpha_N) y_{Nt} l_{Tt}}.$$

[7]A value of $\psi = 2.29$, as in the basic model, implies that, for both countries, the average cost of labor frictions according to the data would be smaller than 1 percent of output in both the traded and the non-traded sectors.

obtained by deflating and adding the sectoral investment series in the aggregated input-output tables and the tables of total use provided by Instituto Nacional de Estadistica (1986-1994, 1995-1998).

B The effects of substituting convex investment costs with time-to-build

Time-to-build constitutes a friction in investment that slows down capital accumulation in the model. To investigate what effects time-to-build has on its own, it is of interest to look at a model with gestation lags where the convex investment costs have been switched off.

Figure 15 compares the dynamics in the basic model and a model where the convex adjustment costs in investment have been replaced by a three period time-to-build technology ($J = 3$). In both models, the labor friction parameter, ψ, has been adjusted so that the models match the maximum sectoral job creation rate in the data. The first thing to note is the saw-tooth pattern of the dynamics in the model with time-to-build. As explained by Rouwenhorst (1991) and Christiano and Todd (1996), the oscillatory solutions are due to the'Leontief type' technology for producing capital and stem from an effort to concentrate investment activities and consumption in periods of relative abundance.

The solutions show that for $J = 3$, the model economy with only time-to-build borrows more from abroad, accumulates a higher level of capital and both invests and consumes more than the basic model economy. The reason is that a three-period time-to-build technology constitutes a smaller capital friction than the convex investment costs as estimated by Eberly (1997), which implies that the wealth effect from trade liberalization is larger in the model with time-to-build. In figure 15.a, we consequently see that the real exchange rate in the model with only time-to-build initially appreciates more than in the basic model, since non-traded goods are more scarce in the first period after liberalization.

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Table 1: Calibration of the basic and the augmented models

	Basic model	**Augmented Model**
	Initial conditions	
y_0	100.00	100.00
y_{T0}	37.1458	37.1458
y_{N0}	62.8542	62.8542
k_0	207.1086	207.1086
k_{T0}	73.0841	73.0841
k_{N0}	134.0245	134.0245
L	67.9729	67.9729
l_{T0}	25.8416	25.8416
l_{N0}	42.1313	42.1313
	Parameters	
A_{T0}	1.0511	1.0511
A_N	1.0217	1.0217
α_T	0.3125	0.3125
α_N	0.3383	0.3383
γ	0.3802	0.3802
G	1.9434	1.9434
σ	-1	-1
ρ	-1	-1
ε	0.2537	0.2537
δ	0.0576	0.0576
β	0.9463	0.9463
ζ	1.6133	1.6133
ψ	2.2974	6.4014
J		3
θ		0.8

Table 2: Mean square errors for the models presented in figures 9 to 14

Variable	**Basic**	**Augmented, J=3**	**J=2**	**J=5**
rer_N	0.0089	0.0016	0.0024	0.0005
c_N	0.0087	0.0025	0.0024	0.0026
c_T	0.0336	0.0054	0.0050	0.0058
trade balance	6.3197	28.5834	27.2015	31.2914
traded output	66.9278	31.5161	32.9827	29.1583
real GDP	0.0934	0.0598	0.0547	0.0695
traded sector labor share	0.0072	0.0033	0.0035	0.0029
investment	0.0852	0.1099	0.1885	0.0222

Figure 1. Bilateral Spanish-German real exchange rates

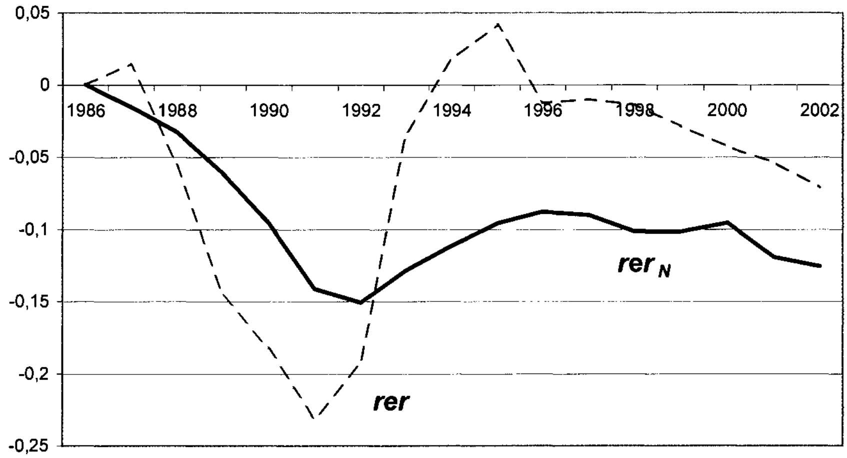
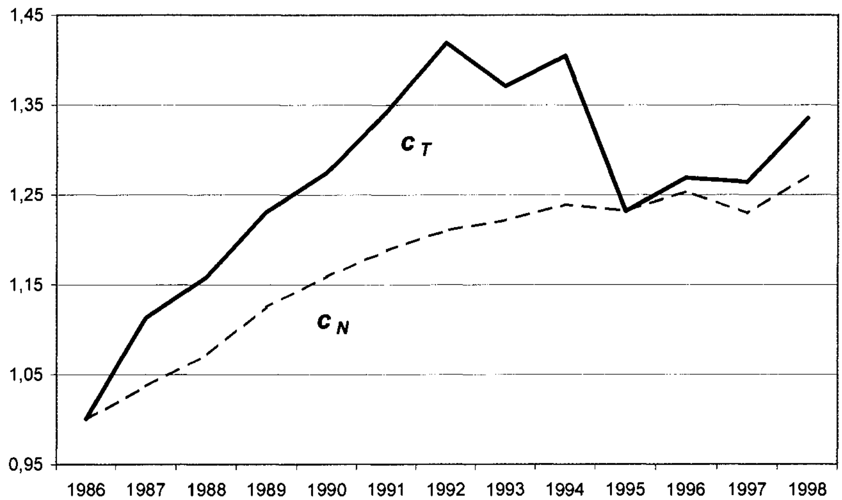


Figure 2. Consumption of traded and non-traded goods in Spain



The Price-Consumption Dynamics in the Basic Model

Figure 3. rer_N

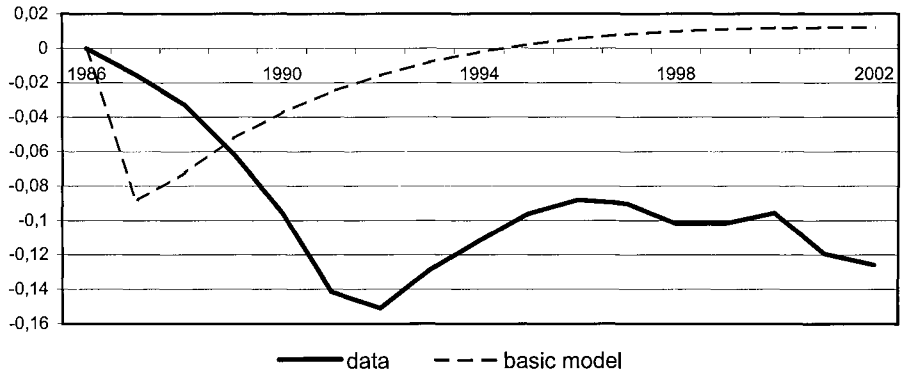


Figure 4. c_N

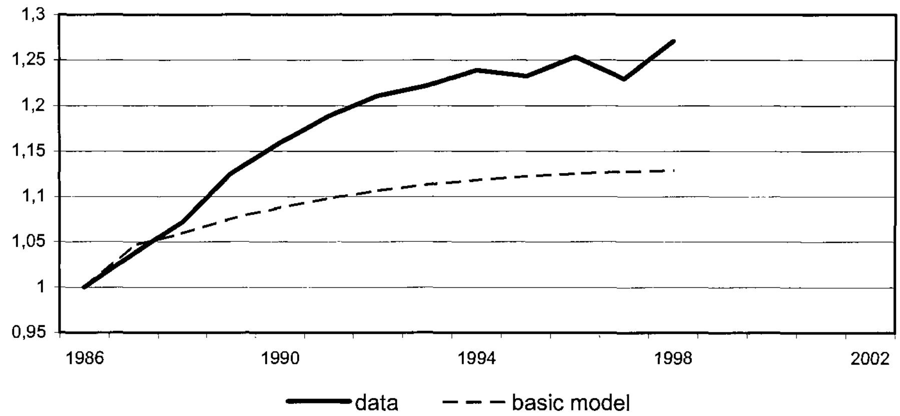


Figure 5. c_T

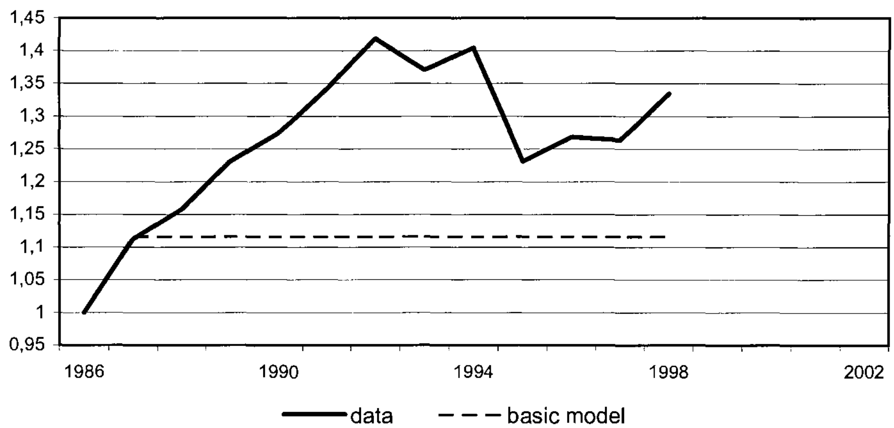


Figure 6. The effects of higher productivity growth in the traded sector

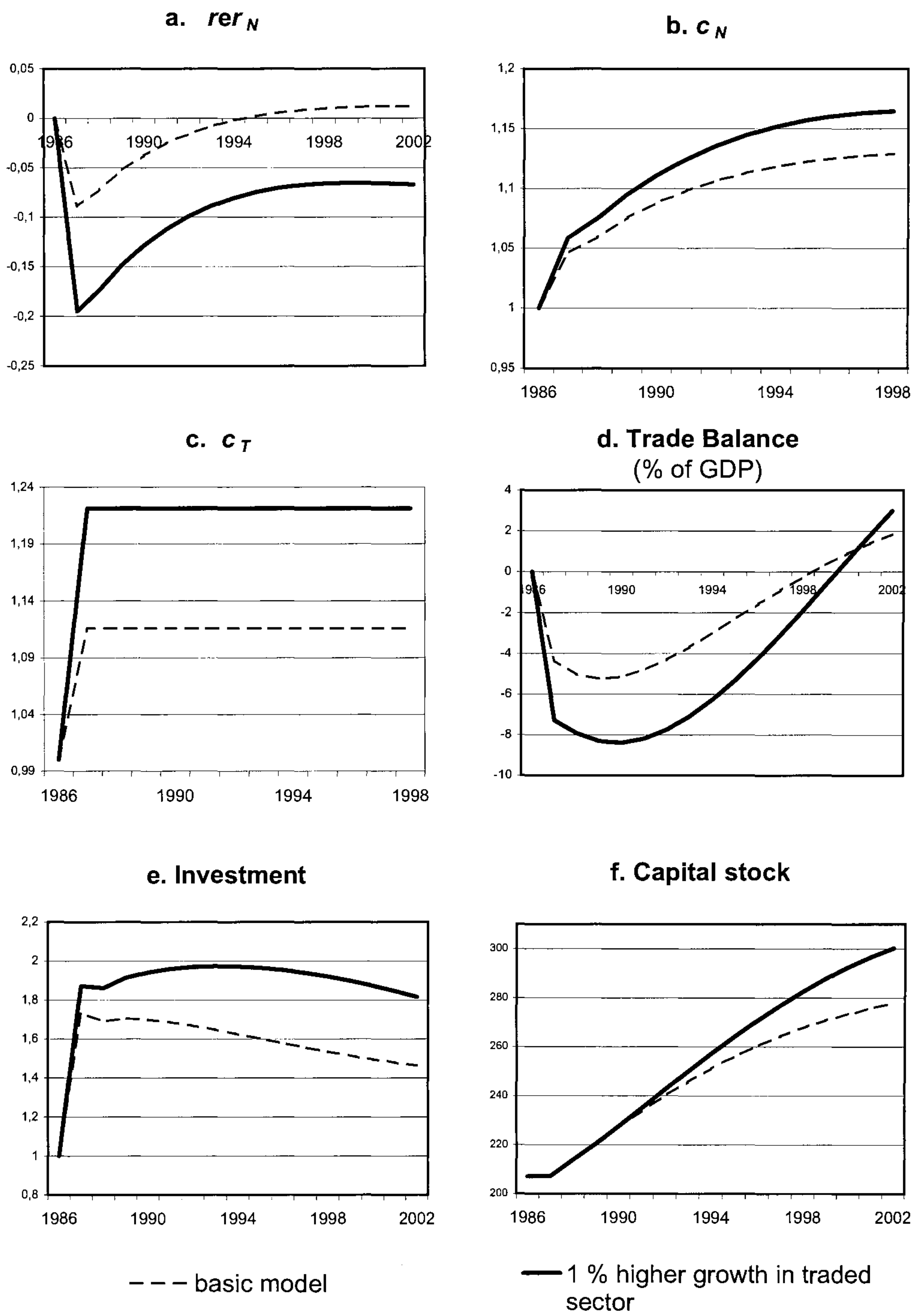


Figure 7. The effects of time-to-build

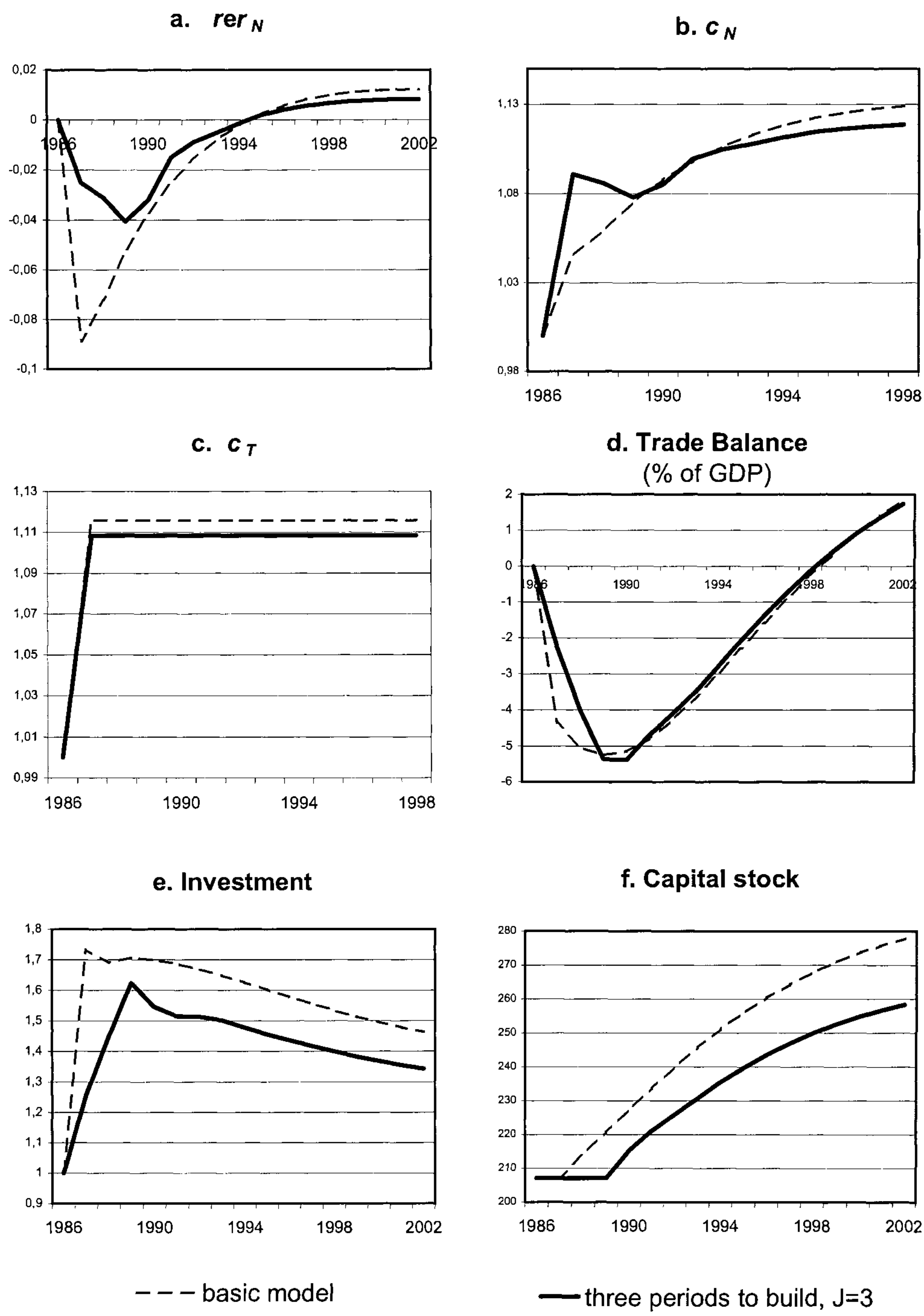


Figure 8. The effects of habit formation in preferences

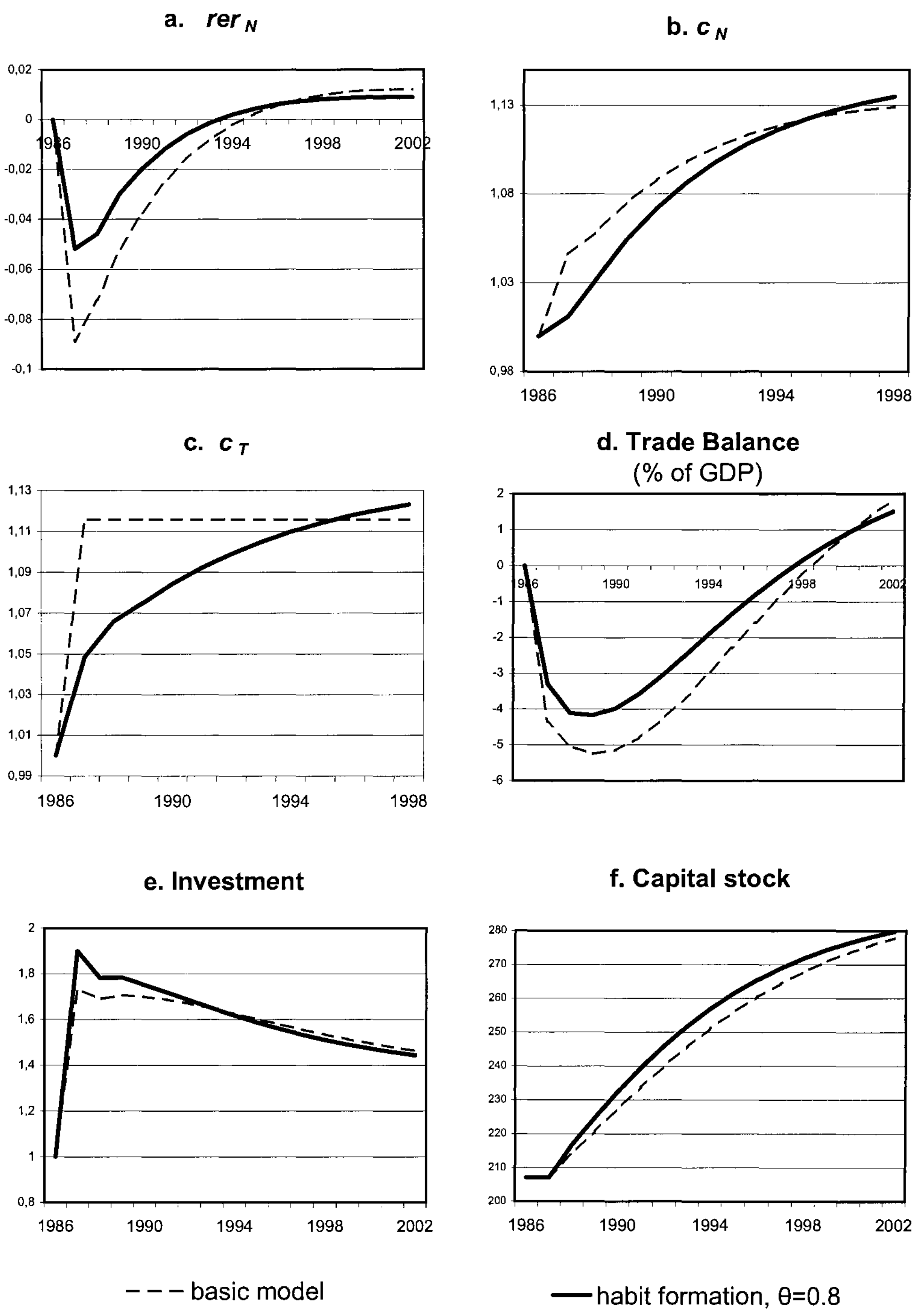
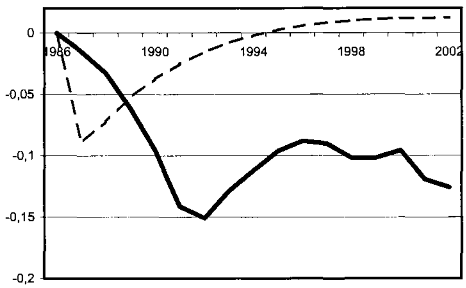
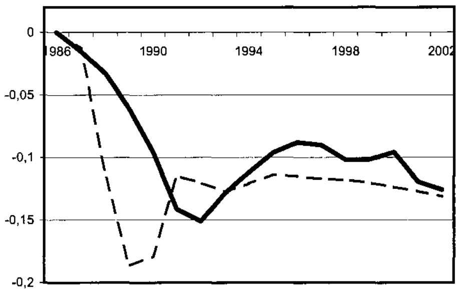


Figure 9. Comparison in the Price-Consumption dimension

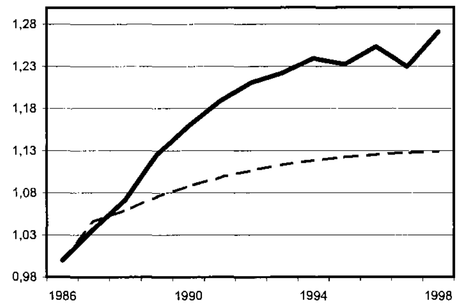
a. Basic model, rer_N



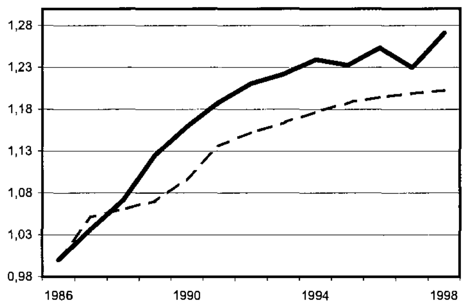
b. Augmented model, rer_N



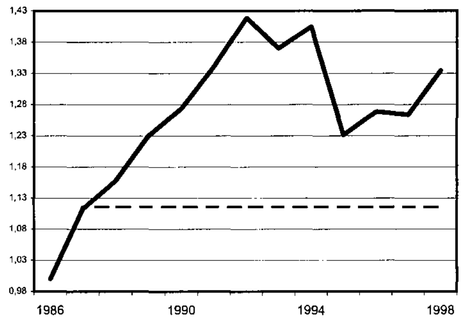
c. Basic model, c_N



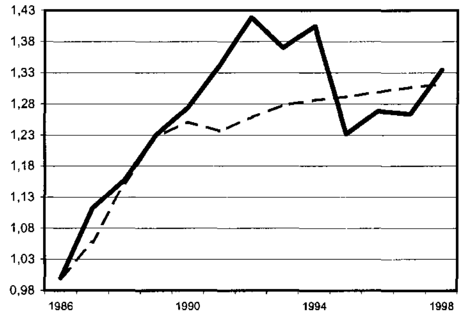
d. Augmented model, c_N



e. Basic model, c_T



f. Augmented model, c_T



data – – – basic model

data – – – augmented model

Figure 10. Comparison for other real variables

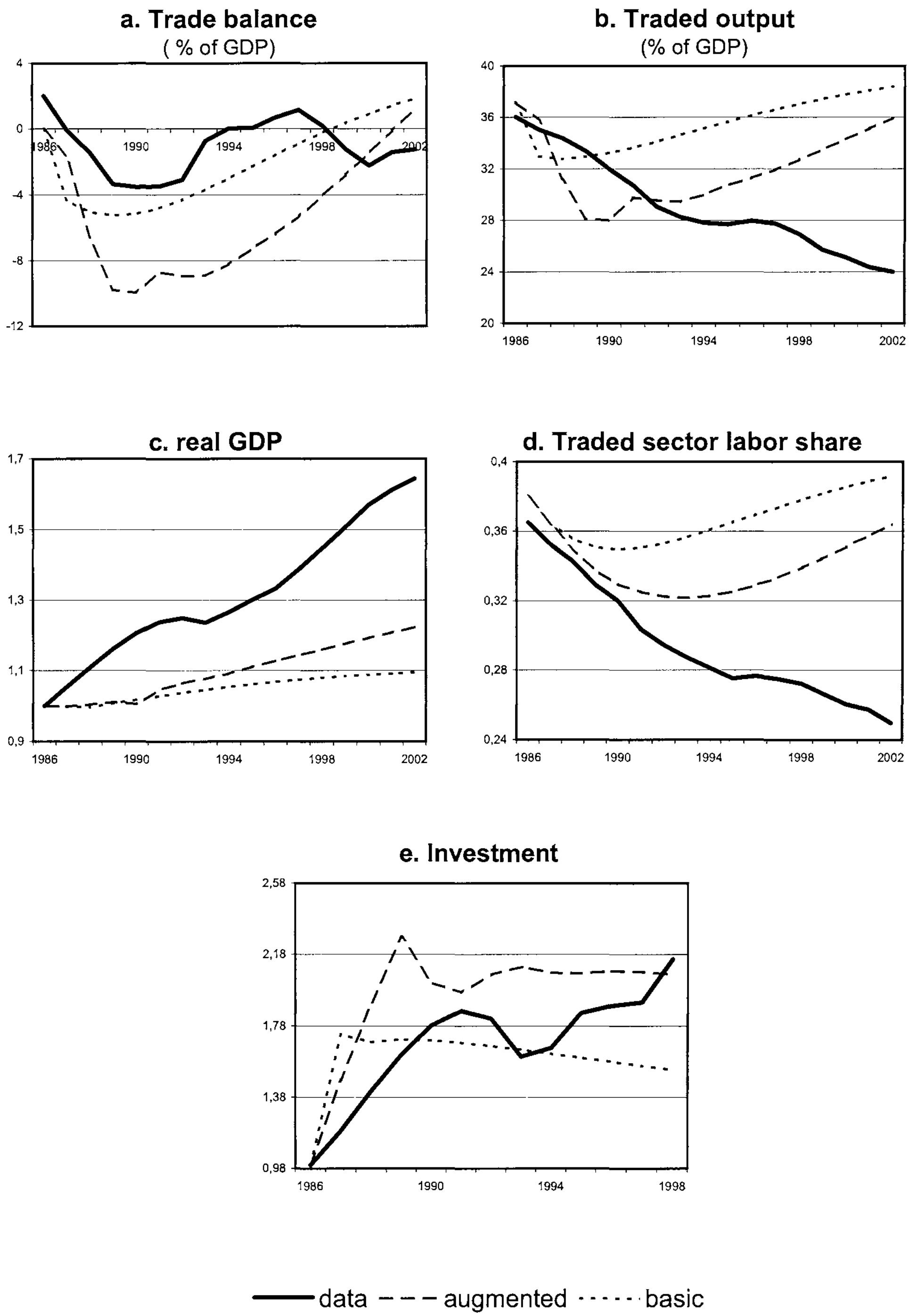


Figure 11. Comparison in the Price-Consumption dimension
(J=2 in augmented model)

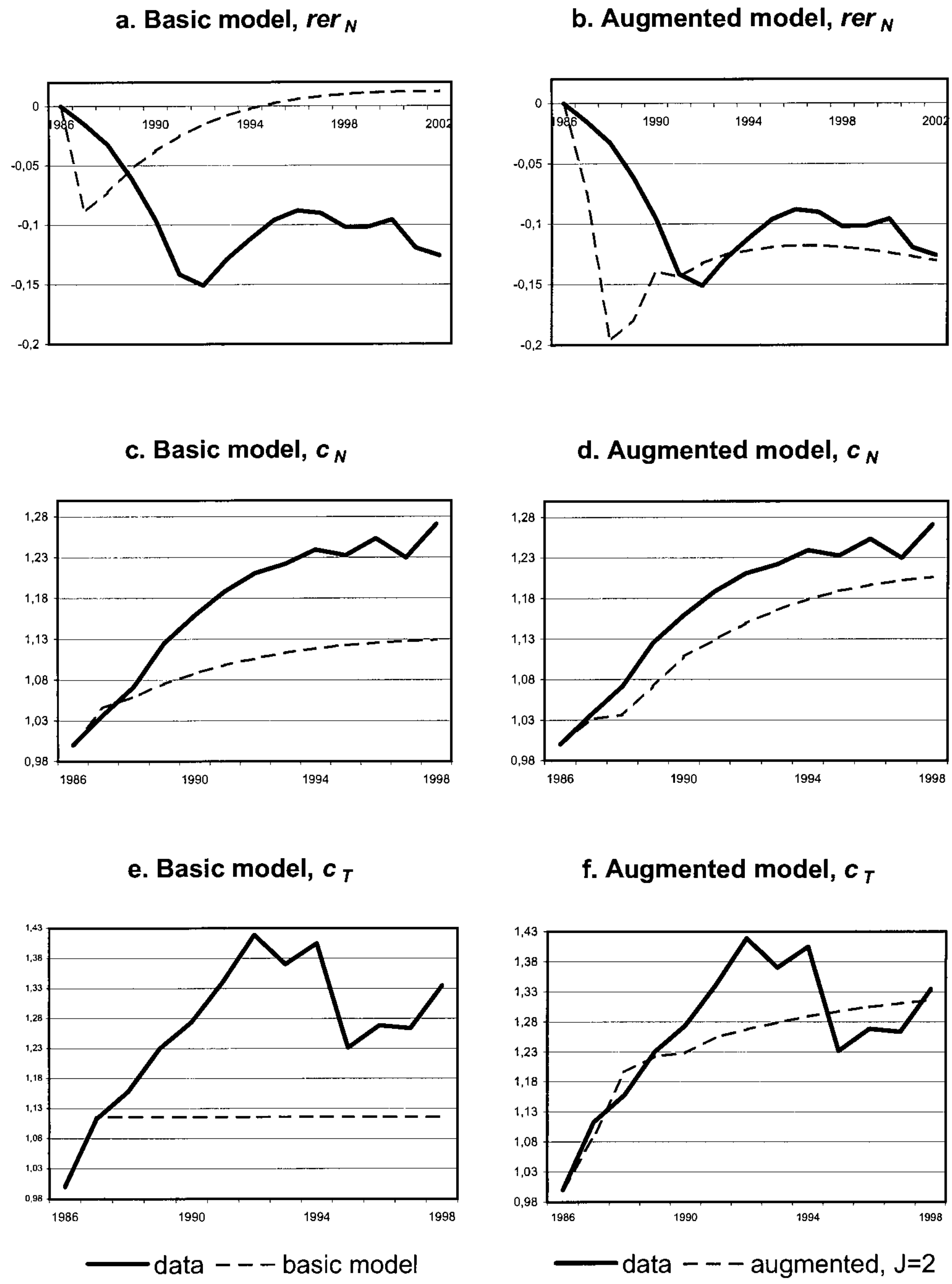


Figure 12. Comparison for other real variables
(J=2 in augmented model)

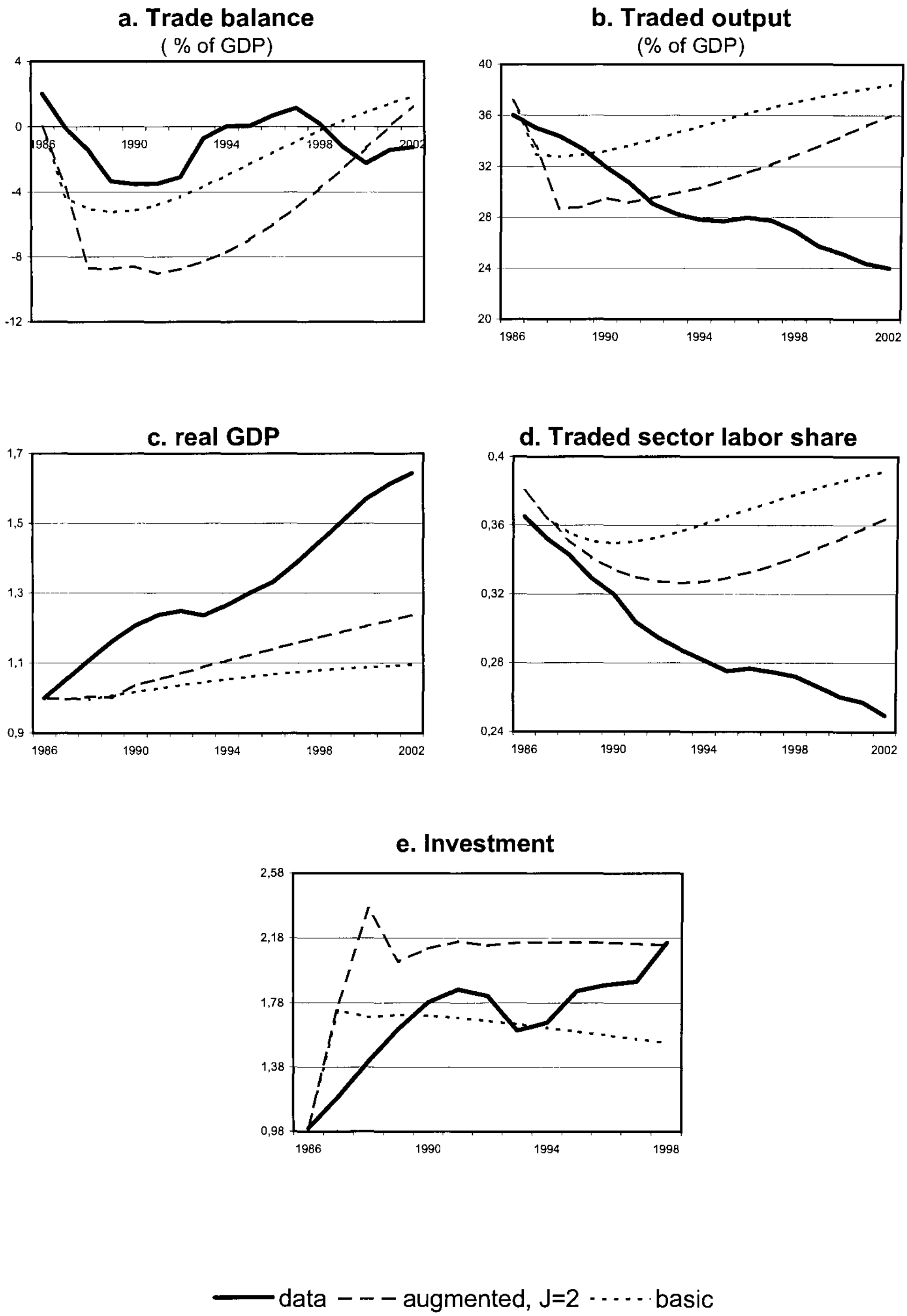


Figure 13. Comparison in the Price-Consumption dimension
(J=5 in augmented model)

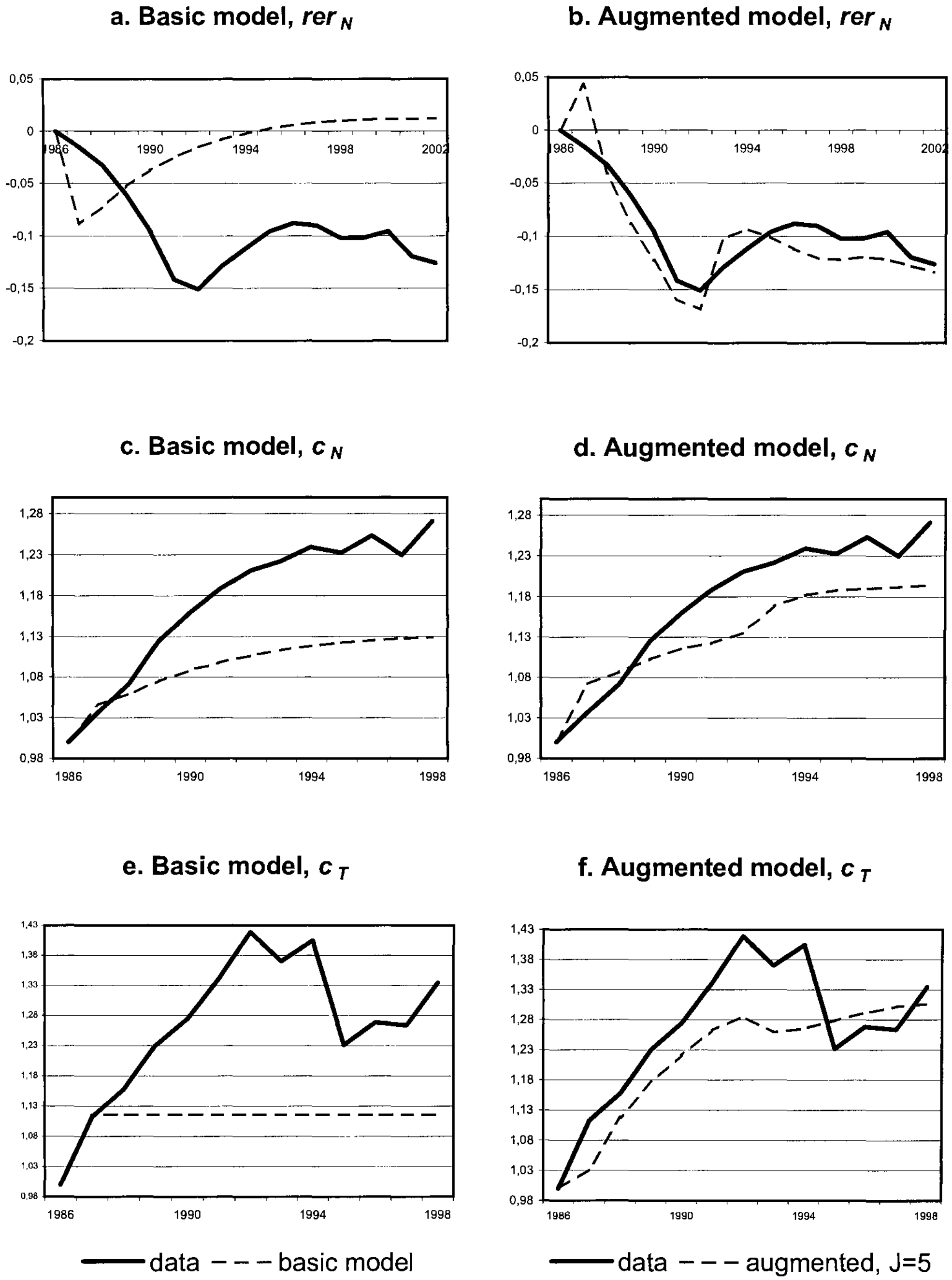


Figure 14. Comparison for other real variables
(J=5 in augmented model)

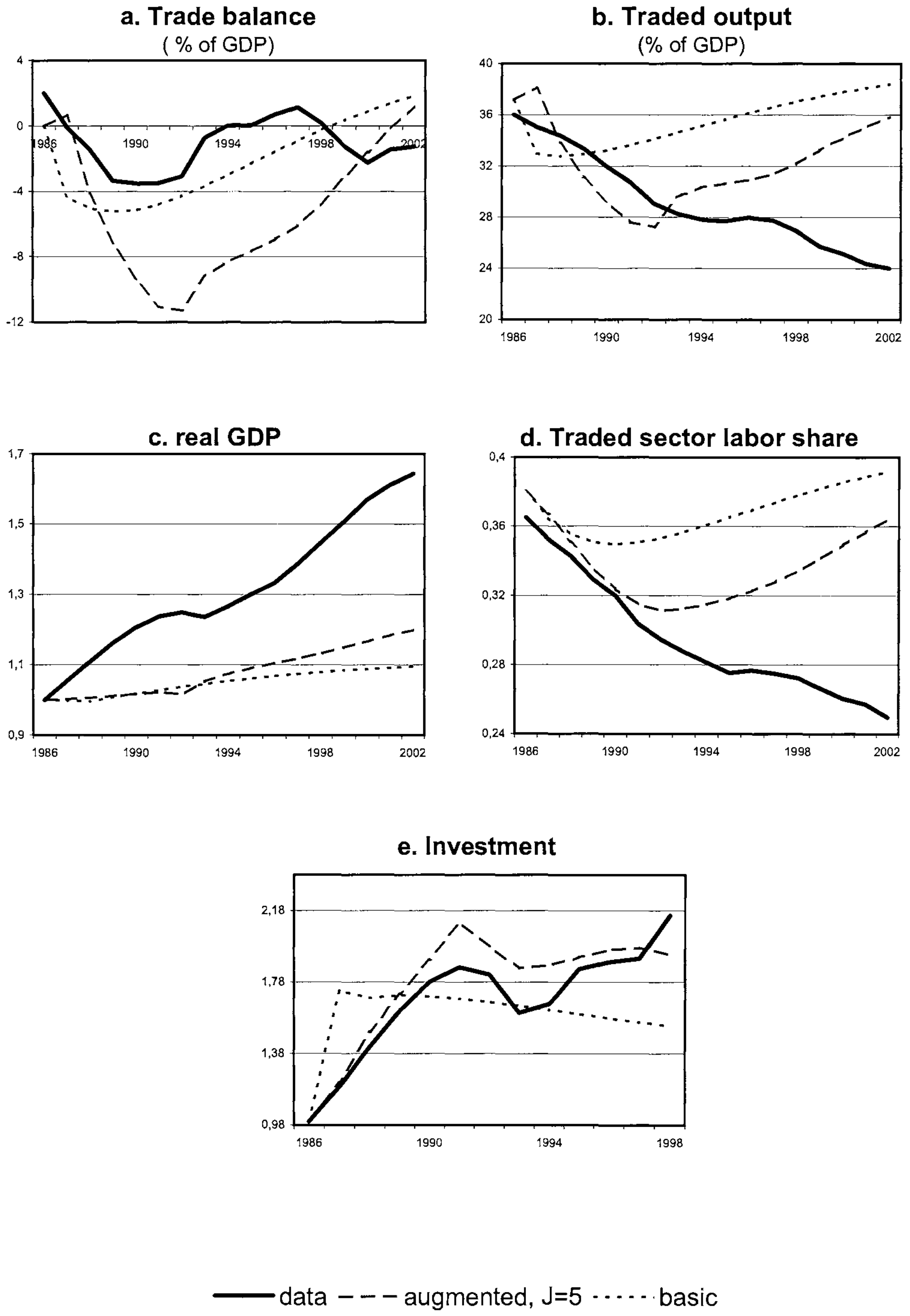
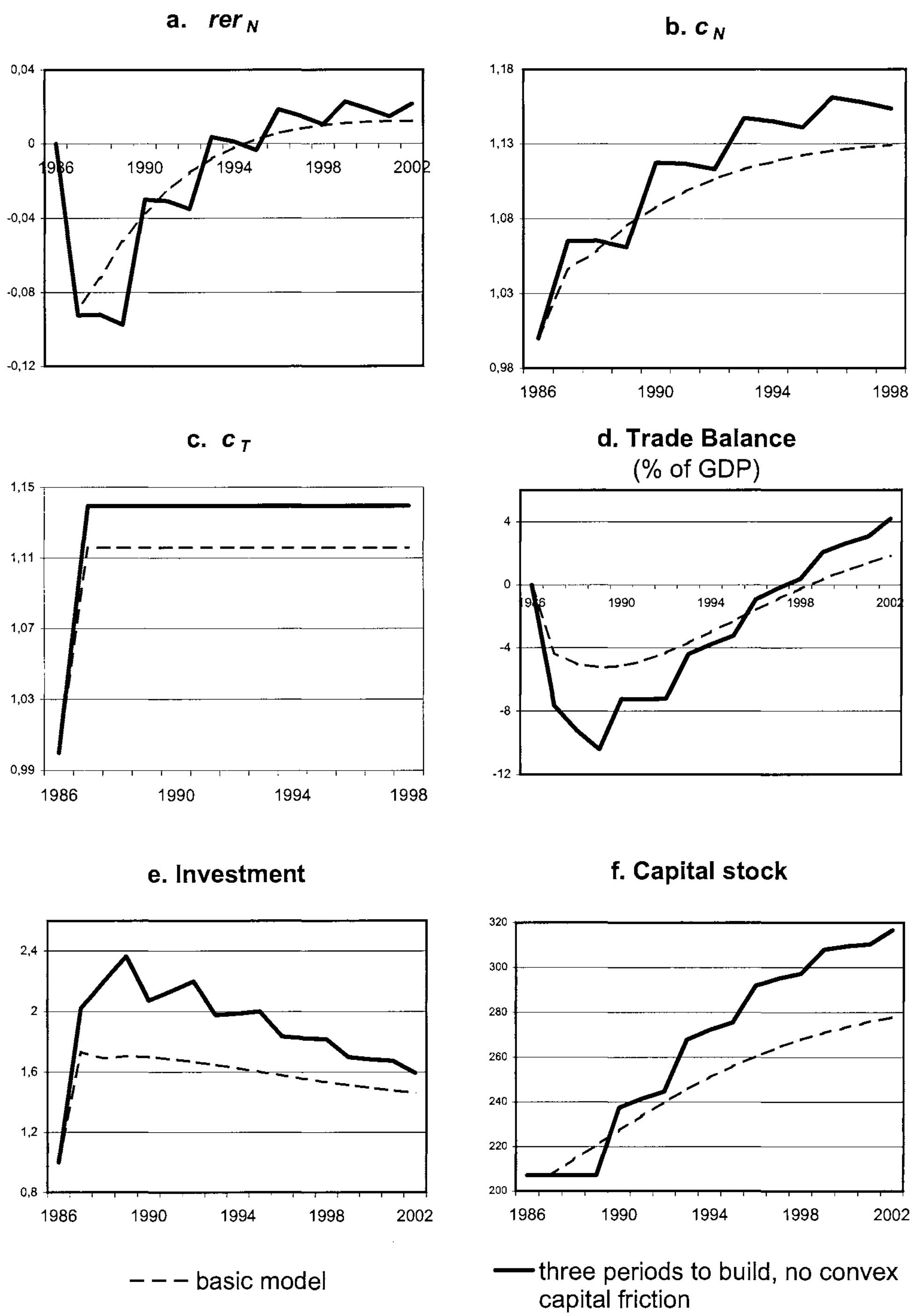


Figure 15. The effects of substituting convex capital frictions with time-to-build



Essay III

Financial Crisis in Emerging Markets and the Optimal Bailout Policy*

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Abstract

This paper develops a framework for analyzing optimal government bailout policy in a dynamic stochastic general equilibrium model where financial crises are exogenous. Important elements of the model are that private borrowers only internalize part of the social cost of foreign borrowing in the emerging market and that the private sector is illiquid in the event of a crisis. The distinguishing feature of our paper is that it addresses the optimal bailout policy in an environment where there are both costs and benefits of bailouts, and where bailout guarantees potentially distort investment decisions in the private sector. We show that it is always optimal to commit to a bailout policy that only partially protects investment against inefficient liquidation, both in a centralized economy and a market economy. Due to overinvestment in the market economy, the government's optimal level of bailout guarantees is lower than in the social optimum. Further, we show that, in contrast to a social planner, the government in the market economy should optimally bail out a smaller fraction of private investments when the probability of a crisis increases.

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1 Introduction

A wide range of emerging market economies have experienced financial crises in recent decades. In the wake of these events, governments of such countries as Chile, Argentina, Mexico, Korea and Indonesia have spent large shares of their GDP, sometimes more than 30 percent, on saving financial systems in distress.[1] In light of the large costs involved, investigating the macroeconomic role and efficiency of such rescue packages must be of prime concern. The economic profession is divided on the role of bailouts in financial crises in emerging markets. On the one hand, governments should limit the provision of guarantees to avoid distorting investment incentives in the private sector, on the other hand, they should stand ready to provide liquidity in times of crisis to minimize the negative consequences of financial panics and maturity mismatch. What advice should we give to policy makers in emerging markets? What is the optimal bailout policy in response to a financial crisis?

In this paper, we analyze optimal bailout policy under commitment in a dynamic stochastic general equilibrium model where financial crises are exogenous. Important elements of the model are that private borrowers only internalize part of the social cost of foreign borrowing in the emerging market and that the private sector is illiquid in the event of a crisis. We model the strategic interaction between the government and the private sector, assuming the government to be benevolent in the sense of maximizing consumer utility.

The distinguishing feature of our paper is that it addresses the optimal bailout policy in an environment where there are both costs and benefits of bailouts, and where bailout guarantees potentially distort investment decisions in the private sector. In the model, the cost of bailouts arises because bailouts lead to more volatile government consumption, while the benefit of bailouts is that they help avoid inefficient liquidation of investments in the private sector. The cost is aggravated by the distortion of private investment incentives which arises from bailout guarantees in the model.

To examine the optimal bailout policy, we consider a range of alternative bailout policies and construct equilibrium with commitment and a Markov structure. We show that in both a centralized economy and a market economy where the government is restricted to providing bailouts for free, it is always optimal to commit to a bailout policy that only partially protects investment against inefficient liquidation. Due to overinvestment in the decentralized economy, the government's optimal level of bailout guarantees is lower than in the social optimum. Further, we show that, in contrast to a social planner, the government in the decentralized economy should optimally bail out a smaller fraction of private investments when the probability of a crisis increases.

[1] See, for example, Dziobek and Pazarbasioglu (1997), Kaminsky and Reinhart (1999) and Eichengreen and Rose (1997).

Previous work on government bailouts has included some of the different elements that we merge into a unified framework in our model. Gale and Vives (2002) model a moral hazard cost of bailouts by making the private managerial effort depend on the size of bailout guarantees. Freixas (1999) pursues a cost-benefit analysis to characterize the optimal bailout policy of the Lender of Last Resort. Mundaca (2001) develops a game theoretic setting for the interaction between the government and the market to address the optimal bailout policy, whereas Schneider and Tornell (2000) investigate the effects of government bailouts on market behavior in an infinite horizon setting. The general structure of our model has been inspired by Cole and Kehoe (2000), who address the issue of optimal government debt policies and self-fulfilling debt crises.

Our paper is related to the strand of research arguing that bailout guarantees are a 'bad policy', in that the provision of these leads to distortions in investment incentives. According to the theoretical work of Corsetti et al. (1998a) and Burnside et al. (2003), bailout guarantees induce moral hazard by providing insurance against future crises to the private sector. Market participants willingly take on excessive risk, which leads to over-investment, excessive external borrowing or unhedged foreign loans. Empirical support for the 'bad policy' argument has been provided by Corsetti et al. (1998b) and Dooley and Shin (2000).

Our model also captures the arguments of the literature on maturity mismatch and financial panics, which holds a more positive view on government bailouts. The provision of emergency liquidity can help avoid inefficient liquidation of investments in times of crisis, as argued by Chang and Velasco (2001) and Allen and Gale (2000). We believe potential illiquidity to be a characteristic feature of producers in emerging markets, who are often forced to borrow abroad at short maturities. Empirical work by Radelet and Sachs (1998) and Rodrik and Velasco (1999) has found evidence supporting this view.

The next section of the paper lays out the model. In Section 3, we define an equilibrium which takes into account the strategic incentives of a government that must commit to a level of bailout guarantees in the first period of the model and adhere to this in all subsequent periods. In section 4, we analyze the market response to bailout guarantees. Section 5 presents the social planner's solution of the model and Section 6 analyzes the government's optimal bailout policy. In Section 7, we show that the formal analysis and the policy conclusions of the paper remain unchanged if the commitment assumptions in the model are relaxed. Section 8 provides a numerical example to illustrate the model, while concluding remarks and suggestions for future research are presented in Section 9.

2 The Model

We model an emerging market as a small economy opening up to the international capital market in period $t = 0$. The economy is inhabited by a continuum of consumers and a government. The consumers receive an endowment of a consumption good in each period. Additional consumption goods can be produced with borrowed foreign capital as the only input. In the first period, the government must commit to a bailout policy, to which it must adhere in all subsequent periods.[2]

2.1 The International Capital Market

A continuum of risk neutral agents act in an environment of perfect competition on the international capital market. This implies that the expected return on any one-period loan must equal $1/\beta$, where β is the universal and subjective discount rate.

In every period, the international capital market offers one-period loans to the consumers in the emerging economy. We assume all international loans in some periods to be recalled before having reached full maturity. Define ζ_t to be an exogenous random variable realized at the beginning of each period and following the process

$$\zeta_t = \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } \pi \end{cases}, \quad t > 0 \tag{1}$$

$$\zeta_0 = 0, \tag{2}$$

where $\pi \in [0, 1)$ is an exogenous parameter.

If $\zeta_t = 1$, all international loans are recalled before they have reached full maturity in period t, which is what we define as a financial crisis in the emerging market. Repayments of international loans are requested after full maturity if $\zeta_t = 0$. Note that a financial crisis in the model economy occurs with the exogenous probability π in every period.

2.2 The Consumers

There is a continuum with measure one of identical and infinitely lived consumers, who consume, invest, borrow from abroad, and pay lump-sum taxes. The individual consumer's utility function is

$$E \sum_{t=0}^{\infty} \beta^t (c_t + v(g_t)),$$

where c_t is private consumption and g_t is government consumption. We assume v to be twice continuously differentiable, strictly concave, monotonically increasing and that $v(0) = -\infty$.

If the probability of crisis is zero (i.e. if $\pi = 0$), the individual consumer is subject to

[2]The formal analysis and the policy conclusions of the paper would remain unchanged if the assumption that the government must commit to its bailout policy in the first period were relaxed to allow for one-period commitment. In Section 7, we discuss the commitment assumption further.

the following budget and investment constraints

$$c_t + k_{t+1} + R_t b_t + Q_t \leq f(k_t) + b_{t+1} + \omega \tag{3}$$

$$k_{t+1} = b_{t+1}. \tag{4}$$

Here, b_{t+1} is a foreign loan to be repaid in period $t+1$. The investment constraint in (4) specifies that international loans can only be used to augment the capital stock, k_{t+1}, and that the capital used in production must be borrowed from abroad. R_t is the gross interest rate on foreign loans, Q_t is a lump-sum tax, and $\omega > 0$ is an endowment received in each period. We assume the production function to be of the functional form

$$f(k) = Ak^{a},\ \alpha < 1. \tag{5}$$

The consumer is endowed with $k_0 = 0$ and $b_0 = 0$ in period $t = 0$. We assume that capital depreciates fully after one period.

When the probability of crisis is positive and a financial crisis occurs in period t, repayment of international loans must be made before production takes place. We assume that in this case, international lenders can liquidate the capital stock, k_t, with a linear return of $1/\beta$. This simplifying assumption implies that the interest rate on international loans in the model economy is constant and equal to the world interest rate, $R_t = 1/\beta$ $\forall t$.[3]

When the probability of a crisis is positive, the individual consumer's budget constraint will depend on government policy, which is the reason why we now turn to describing the government.

2.3 The Government

The government is benevolent in the sense of its objective being to maximize the consumers' utility. The government is the only strategic agent in the model, and when making its decisions, it takes into account the effects of these decisions on the level of the aggregate capital stock K_t, the aggregate level of international private debt B_t, government revenue, and the level of private consumption.

We define a bailout policy x as the fraction of international liabilities the government provides to international lenders in the event of a crisis.

In period $t = 0$, the government can commit to any bailout policy $x \in [0, 1]$, to which it must subsequently adhere forever. Choosing $x = 0$ corresponds to a policy of *No Bailout*, which implies that the government never provides any resources for repaying

[3]Previous versions of this paper included costs of liquidation so that the return to liquidating capital was smaller than unity. The assumption that capital can be liquidated with a return of $1/\beta$ greatly facilitates our analytical investigation, without affecting the policy conclusions of the paper.

international lenders in the event of a crisis. Committing to $x = 1$ corresponds to a policy of *Full Bailout*, by which the government provides RB_t to repay international loans in a crisis.

The consumers must pay an exogenous price, $P \geq 0$, for the bailouts provided by the government. For a bailout policy x, the government will spend xRB_t on bailouts in a financial crisis and receive xPB_t from consumers later in the same period. A price of $P = 1/\beta$ thus implies that the consumers repay exactly what the government spends on bailouts. In the model, the price of bailouts will affect the investment decision of the individual consumer. When analyzing the government's optimal bailout policy in Section 6, the price P will therefore play an important role for the outcomes of the model.

In the first period, the government commits to a bailout policy x, and in every period, it chooses the size of the lump-sum tax, $Q_t \geq 0$, after observing the realization of the crisis variable, ζ_t. The timing of events in the model is such that in every period, the government must spend resources before receiving income in that period. When spending resources on government consumption and bailouts in period t, the government's budget constraint is

$$g_t + \zeta_t xRB_t = Q_{t-1} + \zeta_{t-1} PxB_{t-1}, \tag{6}$$

where $g_t \geq 0 \ \forall t$. The left-hand side of equation (6) shows that resources are spent on government consumption in every period. If there is a financial crisis in the period, the government also spends resources on pursuing bailouts according to the bailout policy, x. The first term on the right-hand side of equation (6) says that in every period, the lump-sum tax levied in the previous period contributes to government resources in the current period. Similarly, the second term states that any repayment of bailouts received in period $t-1$ contributes to the government resources in period t. Note that we have assumed that the government must run a balanced budget. The government is subject to a revenue lag, but has no other possibility of saving resources to build a buffer against future bailout costs. For expositional reasons, it is useful to define government revenues in period t as

$$T_t = \begin{cases} Q_t, & \text{if } \zeta_t = 0 \\ Q_t + PxB_t, & \text{if } \zeta_t = 1 \end{cases}, \ t \geq 0. \tag{7}$$

Because consumers are competitive, we need to distinguish between the individual decisions k_{t+1} and b_{t+1} and the aggregate values K_{t+1} and B_{t+1}. Consumers should not think that their individual actions affect the aggregate state in the next period, thereby affecting prices or the government's actions. In equilibrium, because all consumers are identical, $k_{t+1} = K_{t+1}$ and $b_{t+1} = B_{t+1}$.[4]

Since a fraction of the individual's capital stock must sometimes be liquidated in the model, we need to enhance the notation we have used so far. Let k_{t+1} denote the

[4]The initial endowments are such that $k_0 = K_0$ and $b_0 = B_0$.

consumers' choice in period t and let κ_{t+1} denote the part of the capital stock actually used in production in period $t+1$. The law of motion for κ_{t+1} depends on the bailout policy in the following way:

$$\kappa_{t+1} = \kappa(x, k_{t+1}, \zeta_{t+1}) = \begin{cases} k_{t+1} & \text{if } \zeta_{t+1} = 0 \\ x k_{t+1} & \text{if } \zeta_{t+1} = 1 \end{cases} . \tag{8}$$

The first line of equation (8) simply states that if a crisis does not occur in period $t+1$, there is no liquidation. The second line says that for a given bailout policy x, consumers can keep xk_{t+1} in production if a crisis occurs in period $t+1$.

2.4 The Timing

The timing of actions within period $t = 0$ differs from subsequent periods, since the government chooses its bailout policy only in the first period. In subsequent periods, the actions within a period depend on whether a crisis occurs; the government pursues bailouts and investments are liquidated only if a crisis occurs in that period. The timing of actions within a period is the following:

1. The variable ζ_t is realized and the aggregate state is $S_t = (K_t, B_t, T_{t-1}, \zeta_t)$.
2. If $t = 0$, the government commits to a bailout policy x.

If a crisis does not occur, $\zeta_t = 0$*,*

3. The government provides g_t and decides Q_t.
4. Production takes place, the endowment is realized and taxes are paid.
5. Each consumer repays his international loans, amounting to Rb_t.
6. Each consumer chooses c_t, k_{t+1} and b_{t+1}, taking x, P and Q_t as given.

If a crisis occurs, $\zeta_t = 1$*,*

3. Each consumer is asked to repay Rb_t early.
4. The government provides g_t, decides Q_t, and spends xRB_t on bailouts.
5. Each consumer must liquidate part of his invested capital, $(1-x)k_t$, and use it to repay part of his international loans. Since capital can be liquidated with a return of $1/\beta = R$, and since $k_t = b_t$, the repayment is worth $(1-x)Rb_t$ to the international lenders.
6. Using the part of the capital which is left in production, xk_t, production takes place, the endowment is realized, taxes are paid and bailouts are repaid to the government with the amount PxB_t.

7. Each consumer chooses c_t, k_{t+1} and b_{t+1}, taking x, P and Q_t as given.

An important feature of the timing is that the government in a period must spend resources on government consumption and bailouts *before* it receives any revenues in that period. The government carries over any revenues from one period to the next. If loans need to be repaid early, the government disposes of revenues from the previous period and can therefore bail out the consumers, who are illiquid at the beginning of each period.

3 Equilibrium in period $t = 0$

Within each period, the aggregate state $S_t = (K_t, B_t, T_{t-1}, \zeta_t)$, the bailout policy x, the government's choice of Q_t, the consumers' choices c_t, k_{t+1} and b_{t+1}, the interest rate on international loans R and the price of bailouts P determine the equilibrium. Since we want to analyze the government's optimal bailout policy, we will focus on the equilibrium in period $t = 0$, which is the only period where x is a choice variable for the government.[5]

To define a recursive equilibrium, we first present the individual consumer's problem, which takes x and Q_t as given. Next, we present the government's problem, which takes into account that the consumer's choices will depend on the bailout policy x and the lump-sum tax, Q_t.

The solution of an agent's maximization problem is given by the value function providing the maximum attainable value of the agent's utility function given his state, and by policy functions providing the maximizing choices of the agent's choice variables in the current period, given his state. In equilibrium, agents solve their own problems by correctly predicting other agents' policies.

When an individual consumer acts, he knows the bailout policy x, the size of the lump-sum tax Q_t, the aggregate state S_t, his individual levels of k_t and b_t, the interest rate R and the price of bailouts, P. To save on notation, let $h_t = (k_t, b_t, S_t)$. The individual consumer's value function is defined by the following functional equation:

$$V^C(x, h_t) = \max_{\{c_t, k_{t+1}, b_{t+1}\}} \left\{c_t + v(g_t) + \beta E_t V^C(x, h_{t+1})\right\} \tag{9}$$

$$\begin{aligned} s.t \quad c_t + k_{t+1} + (1-\zeta_t)Rb_t + \zeta_t Pxb_t + Q_t &\leq f(\kappa_t) + b_{t+1} + \omega \\ k_{t+1} &= b_{t+1} \\ k_{t+1} &\geq 0 \end{aligned}$$

[5] In section 7, we show that the optimal bailout policy in this equilibrium is identical to the optimal policy in a model where the government in every period must commit to a bailout policy for the next period.

$$
\begin{aligned}
\zeta_{t+1} &= \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } \pi \end{cases} \\
Q_t &= Q(x, S_t) \\
K_{t+1} &= K(x, S_t) \\
B_{t+1} &= B(x, S_t) \\
\kappa_t &= \kappa\left(x, k_t, \zeta_t\right) \; \forall t \\
T_t &= Q_t + \zeta_t P x B_t \\
& \quad x, h_t \text{ given,}
\end{aligned}
$$

where the functions $Q(.)$, $K(.)$ and $B(.)$ will subsequently be defined and where $\kappa(.)$ was defined in equation (8). The consumer's policy functions are $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$.

When the government chooses its bailout policy x in period $t = 0$, it knows the initial aggregate state $S_0 = (K_0, B_0, T_{-1}, \zeta_0)$, the interest rate R and the price of bailouts P. The government realizes that it can affect the consumers' decisions through its choices of x and Q_t. When the government in any period chooses Q_t, it also knows the bailout policy x. Let $H_t = (K_t, B_t, S_t)$. The government's value function in period $t = 0$ is defined by the following functional equation:

$$
V_0^G(S_0) = \max_{\{x, Q_0\}} \left\{ c(x, H_0) + v(g_0) + \beta E_0 V^G(x, S_1) \right\} \tag{10}
$$

where

$$
V^G(x, S_t) = \max_{\{Q_t\}} \left\{ c(x, H_t) + v(g_t) + \beta E_t V^G(x, S_{t+1}) \right\}, \; t > 0
$$

$$
\begin{aligned}
s.t \qquad g_t + \zeta_t x R B_t &= T_{t-1} \\
g_t &\geq 0 \\
x &\in [0, 1] \\
\zeta_{t+1} &= \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } \pi \end{cases} \\
K_{t+1} &= K(x, S_t) \\
B_{t+1} &= B(x, S_t) \\
T_t &= Q_t + \zeta_t P x B_t \\
& \quad S_0 \text{ given.}
\end{aligned}
$$

The government's policy function for the lump-sum tax is $Q(x, S_t)$, and the set of optimal bailout policies is X^*.

Having developed these concepts, we can now define an equilibrium for the first period in our model economy.

Definition of equilibrium in period $t = 0$. An equilibrium in period $t = 0$ is a list of value functions V^C for individual consumers and V_0^G for the government; policy functions $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$ for the consumers; a policy function $Q(x, S_t)$ and an optimal bailout policy $x^* \in X^*$ for the government; an interest rate R; a price of bailouts P; and laws of motion for the aggregate state variables, $K(x, S_t)$, and $B(x, S_t)$, such that the following conditions hold.

1. Given R, P, x and $Q(x, S_t)$, V^C is the value function for the solution to the consumer's problem and $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$ are the maximizing choices.

2. Given R, P, $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$, V_0^G is the value function for the solution to the government's problem, and x^* and $Q(x, S_t)$ are the maximizing choices.

3. $K(x, S_t) = k(x, H_t)$ and $B(x, S_t) = b(x, H_t)$.

4 The Market Response to Bailouts

To characterize the equilibrium in period $t = 0$, we start by presenting the individual consumer's optimal behavior in response to a given bailout policy, a given lump-sum tax and a given price of bailouts. Then, we characterize the government's optimal choice for lump-sum taxation, still for a given bailout policy.

To find the consumer's optimal response to a given bailout policy, we can use the investment constraint in equation (4) to substitute for b_{t+1} in the maximization program presented in (9). The fact that $f'(0) = \infty$ ensures that the non-negativity constraint on borrowed capital never binds in equilibrium. Using the consumer's budget constraint to substitute for c_t, the form of the function $\kappa(.)$ and the fact that $R = 1/\beta$, we obtain the following optimality condition for the consumer's investment decision:

$$(1 - \pi) f'(k_{t+1}) + \pi x f'(x k_{t+1}) = (1 - \pi)\frac{1}{\beta} + \pi P x. \tag{11}$$

The details of the derivation are presented in appendix A. The left-hand side of (11) contains the expected marginal return on investment, which is the weighted sum of the marginal product of borrowed capital in a period without and with a crisis, respectively. The right-hand side represents the consumer's expected marginal cost of capital. Note that for crisis periods, the consumer need only consider the marginal return and cost of the part of the capital stock that will be left in production after government bailouts, xk_{t+1}.

In equation (11), we see that the optimal level of borrowed capital is independent of the bailout policy x, if the probability of crisis is zero. This is natural, since bailouts never occur if crises never happen. Since our interest lies in analyzing the optimal bailout policy in an environment where crises can occur, we will focus on equilibria with a positive probability of crisis in the rest of the paper.

Note that for a given $\pi > 0$, the consumer's optimal investment decision will only depend on the bailout policy x, not on the individual consumer's state h_t. In equilibrium, the consumer therefore makes the same investment decisions in every period. To simplify the exposition, we will henceforth denote the consumer's policy function for borrowed capital as $k(x)$.

Since the individual consumer is risk neutral, the optimal rule for consumption is simply to consume whatever resources are left in each period, once investments, tax payments, loan repayments or bailout repayments have been made.

To find the government's optimal lump-sum tax, Q_t, for a given bailout policy x, we first use equation (7) to recast the government's value function (10) as a maximization problem in terms of government revenues. This can be done, since we know from equation (11) that the consumer's choice of k_{t+1} is independent of the lump-sum tax. Note also that, since $v(0) = -\infty$, the non-negativity constraint on government consumption never binds in equilibrium.

Using the fact that $k_{t+1} = K_{t+1}$ and $b_{t+1} = B_{t+1}$, the investment constraint in equation (4) further enables us to substitute for B_{t+1} in the maximization program presented in equation (10). Substituting for $R = 1/\beta$, the optimality condition for government revenues, T_t, is given by

$$(1-\pi)\, v'\left(T_t\right) + \pi v'\left(T_t - \frac{xk(x)}{\beta}\right) = \frac{1}{\beta}. \tag{12}$$

The details of the derivation are presented in appendix A. Note that equation (12) implies that the equilibrium level of T_t is constant in the model, since the optimal level of revenues only depends on x and the consumer's (constant) equilibrium investment decision. For simplicity, we henceforth denote the optimal level of government revenues for a given bailout policy by $T(x)$. The optimality condition in equation (12) can be understood by remembering that government revenues can only be used for government consumption with a one-period lag. The government optimally equates the discounted expected marginal utility from public consumption tomorrow to the marginal utility from private consumption today which, due to linearity, is constant and equal to 1 (in equation (12), we have divided both sides by β).

Given the constant level of $T(x)$, the optimal lump-sum tax, $Q(x, S_t)$, can be obtained as a residual from equation (7). For a given bailout policy x, the optimal tax is smaller in periods of financial crisis, since the government in such periods receives additional revenues from the repayments of bailouts.

The fact that utility is linear in private consumption implies that the optimal level of private consumption, $c(x, H_t)$, may be negative in response to a particular bailout policy x and the associated level of government revenues. If the lump-sum tax exceeds the endowment, ω, consumption in period $t = 0$ will, for example, be negative. Negative consumption can be avoided in the model if the endowment in each period is sufficiently large.

5 The Social Planner Solution

Before proceeding to analyze the government's optimal bailout policy, it is instructive to consider the solution of a centralized economy where a social planner maximizes consumer utility, subject to the resource constraints of the economy. This solution will serve as a useful benchmark in interpreting the decentralized equilibrium defined in section 3.

To facilitate comparisons between the social optimum and the market economy, we assume that the social planner must also commit to a bailout policy x at the beginning of period $t = 0$ and adhere to it in all subsequent periods.[6] Apart from choosing x in period $t = 0$, the social planner will act at two points in time within each period. At the beginning of a period, the social planner knows S_t and delivers g_t, subject to the budget constraint

$$g_t = T_{t-1} - \zeta_t x R B_t, \tag{13}$$

where $R = 1/\beta$ as in the decentralized equilibrium, and T_{t-1} is the amount of resources transferred from the consumers to the public sector in period $t - 1$.

Later in the period, the social planner makes decisions for c_t, K_{t+1}, B_{t+1} and T_t. The value function when the social planner acts for the second time in a period is defined by the following functional equation

$$V^{SP}(x, S_t) = \max_{\{c_t, K_{t+1}, B_{t+1}, T_t\}} \left\{c_t + v(g_t) + \beta E_t V^{SP}(x, S_{t+1})\right\} \tag{14}$$

$$s.t \quad c_t + K_{t+1} + (1 - \zeta_t) R B_t + T_t \leq f(\kappa(x, K_t, \zeta_t)) + B_{t+1} + \omega$$

$$\begin{aligned} g_t &= T_{t-1} - \zeta_t x R B_t \\ g_t &\geq 0 \\ K_{t+1} &= B_{t+1} \\ K_{t+1} &\geq 0 \\ \zeta_{t+1} &= \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } (\pi) \end{cases} \\ & x, S_t \text{ given,} \end{aligned} \tag{15}$$

[6] The social planner's problem can be formulated without defining bailout policies or assuming commitment, as discussed in Section 7.

where the function $\kappa(.)$ was defined in equation (8). The policy functions associated with V^{SP} are $c_{SP}(x, S_t)$, $K_{SP}(x, S_t)$, $B_{SP}(x, S_t)$ and $T_{SP}(x, S_t)$.

The value function for the centralized problem in period $t = 0$ can now be specified as

$$
\begin{aligned}
V_0^{SP}(S_0) &= \max_{x \in [0,1]} \left\{ c_{SP}(x, S_0) + v(T_{-1}) + \beta E_0 V^{SP}(x, S_1) \right\} \\
& \qquad S_0 \text{ given.}
\end{aligned}
\tag{16}
$$

The set of optimal bailout policies is X_{SP}^{*}.

For a given probability of crisis, π, and a given bailout policy, x, the socially optimal choices of c_t, K_{t+1}, B_{t+1} and T_t can be found by using the budget constraints for private and government consumption to substitute for c_t and g_t, and the investment constraint in (15) to substitute for B_{t+1} in the maximization program in equation (14). Since $v(0) = -\infty$ and $f'(0) = \infty$, the non-negativity constraints will never bind at the social optimum. Substituting for $R = 1/\beta$, the optimality conditions for K_{t+1} and T_t are, respectively, given by

$$
(1-\pi) f'(K_{t+1}) + \pi x f'(x K_{t+1}) = (1-\pi)\frac{1}{\beta} + \pi \frac{x}{\beta} v'\left(T_t - \frac{x K_{t+1}}{\beta}\right) \tag{17}
$$

$$
(1-\pi) v'(T_t) + \pi v'\left(T_t - \frac{x K_{t+1}}{\beta}\right) = \frac{1}{\beta}. \tag{18}
$$

A more detailed derivation is presented in appendix A. Note that the first-order conditions in (17) and (18) for a given bailout policy are independent of the aggregate state variables. For a given bailout policy, x, the optimal investment and transfer decisions in the centralized economy will therefore be constant over time. To simplify the exposition, we henceforth denote the social planner's policy functions for borrowed capital and the transfer as $K_{SP}(x)$, and $T_{SP}(x)$.

We now proceed to analyze the optimal bailout policy in the centralized model economy. Since the investment decisions and the optimal tax are state independent for a given bailout policy, the economy can be in one of two states only after the initial period. Depending on the realization of the random variable ζ_t, the economy is either in a crisis or in a period of no crisis. The stationary nature of the equilibrium considerably simplifies the recursive value functions. Letting superscripts n and cr denote consumption in periods of no crisis and crisis, respectively, the value function of the centralized economy in period $t = 0$ in equation (16) can be written as

$$V_0^{SP}(S_0) = \max_{x \in [0,1]} \left\{ c_0(x) + v(T_{-1}) + \beta E_0 V^{SP}(x, S_1) \right\} \tag{19}$$

where

$$E_0 V^{SP}(x, S_1) = \frac{1}{1-\beta} \Big((1-\pi) \left[c^n(x) + v(g^n(x)) \right] + \pi \left[c^{cr}(x) + v(g^{cr}(x)) \right] \Big)$$

$$\begin{aligned} s.t. \qquad c_0(x) &= \omega - T_{SP}(x) \\ c^n(x) &= f(K_{SP}(x)) + \omega - RK_{SP}(x) - T_{SP}(x) \\ g^n(x) &= T_{SP}(x) \\ c^{cr}(x) &= f(xK_{SP}(x)) + \omega - T_{SP}(x) \\ g^{cr}(x) &= T_{SP}(x) - xRK_{SP}(x), \end{aligned}$$

where we have used the investment constraint in equation (15) to substitute for $B_{SP}(x)$.

In the centralized economy, we can use the Envelope Theorem to find the optimal bailout policy since the policy rules $K_{SP}(x)$ and $T_{SP}(x)$ attain the social optimum for any given bailout policy, x. Substituting for private and government consumption in the value function, the derivative w.r.t. x of the expected utility function in equation (19) is given by

$$\begin{aligned} &\frac{d}{dx} \left\{ c_0(x) + v(T_{-1}) + \beta E_0 V^{SP}(x, S_1) \right\} = \\ &\qquad \frac{\beta}{1-\beta} \pi K_{SP}(x) \left[f'(xK_{SP}(x)) - \frac{1}{\beta} v' \left(T_{SP}(x) - \frac{xK_{SP}(x)}{\beta} \right) \right], \end{aligned} \tag{20}$$

where we have used the equilibrium value of $R = 1/\beta$.

Proposition 1 *For any positive crisis probability, the optimal bailout policy in the centralized economy lies in the interior of the policy space and only partially protects investment against liquidation. Formally,* $X_{SP}^* \subset (0, 1)$. *Furthermore, for any positive crisis probability, the social planner's optimal bailout policy is unique.*

Proof. See appendix B. ■

According to Proposition 1, the optimality condition for the bailout policy in the centralized economy can be written as

$$f'(x_{SP}^* K_{SP}(x_{SP}^*)) = \frac{1}{\beta} v' \left(T_{SP}(x_{SP}^*) - \frac{x_{SP}^* K_{SP}(x_{SP}^*)}{\beta} \right), \tag{21}$$

where x_{SP}^* is the unique optimal bailout policy, and $K_{SP}(x_{SP}^*)$ and $T_{SP}(x_{SP}^*)$ are determined by the optimality conditions for borrowed capital and the transfer in equations

(17) and (18). Equation (21) tells us that the optimal bailout policy in the centralized economy must trade off the benefit of bailouts against the cost they incur. The left hand side is associated with the benefit, i.e. that bailouts help to avoid inefficient liquidation of investment in crises. The right hand side of equation (21) is associated with the cost, i.e. that bailouts cause government consumption to fluctuate, since resources that are spent on bailouts cannot be used for government consumption.

Proposition 1 tells us that at $x = 0$, the benefit of reducing inefficient liquidation outweighs the cost of increased volatility in government consumption. At $x = 1$, the volatility of government consumption should instead be reduced at the expense of some inefficient liquidation.

The model also enables us to analyze how the optimal bailout policy varies across countries with different probabilities of experiencing a crisis. The social planner's optimality conditions in equations (17), (18) and (21) implicitly define the optimal bailout policy, x_{SP}^*, as a function of the crisis probability, π.

Proposition 2 *For a higher probability of crisis, the social planner should optimally commit to bailing out a larger fraction of the borrowed capital in the economy. Formally, for any π_1 and π_2, such that $\pi_1 < \pi_2$, it is the case that $x_{SP}^*(\pi_2) > x_{SP}^*(\pi_1)$.*

Proof. See appendix B. ■

The result in proposition 2 can be understood by jointly considering the three optimality conditions in equations (17), (18) and (21). We start by noting that in the social optimum, for any probability of crisis, the social planner always chooses the same level of borrowed capital, at which the marginal product of capital in periods of no crisis is equal to the world interest rate, $1/\beta$.[7] Cost-benefit considerations for capital in crisis periods can be ignored, since in the social optimum, the levels of bailouts and transfers are chosen so that the marginal product of borrowed capital always equals its marginal cost in a crisis according to equation (21).

When there is an increase in the probability of a crisis, *ceteris paribus*, the social planner will, according to equation (18), need to transfer more resources to the public sector to guard against low government consumption in crisis periods. The social planner must further trade off the benefit of bailouts against their costs according to equation (21). For a higher level of public resources in a crisis, the social planner should optimally spend more resources on both the provision of public consumption and protection against liquidation. For a higher probability of crisis, the social planner therefore transfers more to the public sector *and* spends more resources on bailouts.[8]

[7] This can be seen by plugging equation (21) into equation (17) and evaluating at x_{SP}^*.

[8] Note that with the level of borrowed capital being constant, equation (21) implies that a higher value of T must be associated with a higher value of x_{SP}^*.

6 The government's optimal bailout policy

The government's value function in period $t = 0$, given by (10), can be written on exactly the same form as the social planner's value function in equation (19). The only differences stem from the decision rules for borrowed capital and the government revenues entering the value function

$$V_0^G(S_0) = \max_{x\in[0,1]} \left\{c_0(x) + v(T_{-1}) + \beta E_0 V^G(x, S_1)\right\} \tag{22}$$

where

$$E_0 V^G(x, S_1) = \frac{1}{1-\beta}\left((1-\pi)\left[c^n(x) + v(g^n(x))\right] + \pi\left[c^{cr}(x) + v(g^{cr}(x))\right]\right)$$

$$\begin{aligned} s.t. \qquad c_0(x) &= \omega - T(x) \\ c^n(x) &= f(k(x)) + \omega - Rk(x) - T(x) \\ g^n(x) &= T(x) \\ c^{cr}(x) &= f(xk(x)) + \omega - T(x) \\ g^{cr}(x) &= T(x) - xRk(x), \end{aligned}$$

The optimal bailout policy can be derived from the government value function in (22). The Envelope Theorem does not hold in the decentralized economy, since the consumer's decision rule for borrowed capital, $k(x)$, is not socially optimal. In appendix A, we show that the derivative of the government's expected utility function in (22) w.r.t. x is given by

$$\begin{aligned} &\frac{d}{dx}\left\{c_0(x) + v(T_{-1}) + \beta E_0 V^G(x, S_1)\right\} = \qquad (23) \\ &\qquad \frac{\beta}{1-\beta}\pi\left(k(x)\left[f'(xk(x)) - \frac{1}{\beta}v'\left(T(x) - \frac{xk(x)}{\beta}\right)\right] + xk'(x)D(x)\right), \end{aligned}$$

where we have used the equilibrium value of $R = 1/\beta$, and $D(x)$ is defined as

$$D(x) = P - \frac{1}{\beta}v'\left(T(x) - \frac{xk(x)}{\beta}\right). \tag{24}$$

6.1 Optimally priced bailouts

Comparing the optimality conditions in the decentralized economy for borrowed capital and government revenues in equations (11) and (12) with the corresponding conditions in the social planner's solution in equations (17) and (18), the only difference is one term in the condition for borrowed capital. While the price of borrowed capital actually used in production for the atomistic consumers in crisis periods is P, the social planner takes into account that loans through bailouts reduce government consumption in a crisis.

If the price of bailouts, P, is equal to

$$P^{*}=\frac{1}{\beta}v'\left(T_{SP}(x_{SP}^{*})-\frac{x_{SP}^{*}K_{SP}(x_{SP}^{*})}{\beta}\right), \tag{25}$$

the optimality conditions for borrowed capital in equations (11) and (17) coincide for $x=x_{SP}^{*}$, which implies that $k\left(x_{SP}^{*}\right)=K_{SP}(x_{SP}^{*})$ and $T\left(x_{SP}^{*}\right)=T_{SP}(x_{SP}^{*})$. When $P=P^{*}$, the derivative of the expected utility w.r.t. x therefore coincides in the decentralized and centralized economies for $x=x_{SP}^{*}$, which can be seen by comparing equations (23) and (20). The government's optimal bailout policy is to set $x=x_{SP}^{*}$, which enables the government to achieve the social optimum since P^{*} is the price of bailouts that makes the atomistic consumers internalize the costs associated with bailouts in the social optimum.

6.2 Suboptimally priced bailouts

For any positive probability of crisis, equation (18) implies that

$$v'\left(T_{SP}(x)-\frac{xK_{SP}(x)}{\beta}\right)>\frac{1}{\beta}, \tag{26}$$

whenever $x>0$. Given equation (25), we can therefore conclude that $P^{*}>1/\beta^{2}$ for any positive probability of crisis. The price of bailouts associated with the social optimum is thus higher for individual consumers, for a loan with a maturity of less than a period, than the one-period world interest rate.

It is interesting to analyze economies where the price of bailouts is lower than P^{*}, since the available empirical evidence suggests that governments in emerging markets actually lose resources when trying to help the financial system in a crisis (Dziobek and Pazarbasioglu (1997), Kaminsky and Reinhart (1999) and Eichengreen and Rose (1997)).

If the price of bailouts is such that $P\leq 1/\beta^{2}$, the consumers will not internalize the full social costs of using risky borrowed capital in production. For any positive level of bailout guarantees, this will lead the consumers to choose a level of borrowed capital that is too high compared to the social optimum. This is formally stated in proposition 3.[9]

Proposition 3 *For a given positive probability of crisis and $P\leq 1/\beta^{2}$, $k(x)\geq K_{SP}(x)$, with equality iff $x=0$.*

Proof. See appendix B. ■

[9]We omit the analysis of the cases when P lies between $1/\beta^{2}$ and P^{*}, in order to focus on prices of bailouts that are empirically relevant. For $1/\beta^{2}<P<P^{*}$, propositions 3 and 4 do not hold, but it is the case that $k(x_{SP}^{*})>K_{SP}(x_{SP}^{*})$ and $T(x_{SP}^{*})>T_{SP}(x_{SP}^{*})$.

From the atomistic consumer's perspective, a bailout guarantee at a price lower than $1/\beta^2$ increases the expected return on borrowed capital, without appropriately increasing the perceived cost of taking loans. Only for $x = 0$, i.e. a policy of *No Bailout*, do the investment allocations in the centralized and decentralized solutions coincide. This can easily be seen by comparing the consumer's optimality condition for borrowed capital in equation (11) with the social first-order condition in equation (17).

A consequence of the distorted investment decisions of the individual consumer is that to optimally smooth government consumption for a given bailout policy x, the government must set the lump-sum tax so that government revenues are higher than in the centralized solution.

Proposition 4 *For a given positive probability of crisis and $P \leq 1/\beta^2$, $T(x) \geq T_{SP}(x)$, with equality iff $x = 0$.*

Proof. See appendix B. ■

The extreme case of suboptimally priced bailouts is a model economy where bailouts are provided free of charge, so that their price is $P = 0$. In such an environment, the level of borrowed capital will be monotonically increasing in the level of bailout guarantees, since for a higher level of bailouts, the consumer can keep a larger part of his borrowed capital in production during a crisis, without having to pay for it.

Proposition 5 *For a given positive probability of crisis and $P = 0$, the optimal level of borrowed capital, $k(x)$, and the optimal level of government revenues, $T(x)$, are increasing in the level of bailout guarantees. Formally, $k'(x) > 0$ and $T'(x) > 0$.*

Proof. See appendix B. ■

The result concerning government revenues in proposition 5 can be understood by considering that the government needs more resources if it is to pursue a higher level of bailouts. The fact that the individual consumer takes larger loans for a higher level of bailouts further increases the need for resources in order to smooth government consumption.

When the government is restricted to providing bailouts for free, the policy considerations differ from the case when bailouts are optimally priced. Free bailouts induce consumers to choose higher levels of borrowed capital, which aggravates the social cost of providing the bailout guarantees in the first place. However, Proposition 6 states that in such an environment, the government should still provide a positive level of bailout guarantees.

Proposition 6 *For any positive crisis probability and $P = 0$, the government should optimally choose a bailout policy in the interior of the policy space, and should thereby* *only partially protect private investment against liquidation. Formally, $X^* \subset (0,1)$. Furthermore, for any positive crisis probability, the government's optimal bailout policy is unique.*

Proof. See appendix B. ■

According to Proposition 6, the government should optimally set the derivative in equation (23) equal to zero, which implies that its optimality condition for the bailout policy can be written as

$$f'\left(x^* k(x^*)\right) = \frac{1}{\beta} v'\left(T(x^*) - \frac{x^* k(x^*)}{\beta}\right)\left[1 + \frac{x^* k'(x^*)}{k(x^*)}\right], \tag{27}$$

where we have substituted for $P = 0$ and $D(x)$ in equation (23) and x^* is the optimal bailout policy. The right-hand side of equation (27) captures the fact that the cost of bailouts is larger in the decentralized economy with free bailouts than in the centralized economy. Compared to the optimality condition in the centralized economy in equation (21), the right-hand side of equation (27) contains an additional cost term associated with the distortion of investment decisions. In addition to reducing government consumption as in the centralized economy, bailout guarantees induce the individual consumers to choose suboptimally high levels of borrowed capital, which aggravates the volatility of government consumption in the decentralized economy. The left-hand side of equation (27) shows that when bailouts are provided for free, their social benefit is the same as in the centralized economy, i.e. bailouts help avoiding inefficient liquidation in a crisis.

Proposition 6 tells us that, as in the centralized economy, the government chooses a bailout policy in between the extremes of *No Bailout* and *Full Bailout*, to optimally weigh the benefits against the costs of bailouts. Uniqueness of the optimal bailout policy is ascertained by the fact that the benefit on the left-hand side of equation (27) is decreasing in x, while the cost on the right-hand side increases with the bailout policy.

In an environment where bailouts must be provided for free, the government must choose a level of bailout guarantees addressing the problem of consumers' overinvestment. The distorted investment incentives thus imply that the government's optimal bailout policy must deviate from the bailout policy in the social optimum.

Proposition 7 *For any positive crisis probability, the optimal level of bailout guarantees is lower in the decentralized economy with $P = 0$ than in the centralized economy. Formally, $x^* < x^*_{SP}$.*

Proof. See appendix B. ■

The reason for the result in Proposition 7 is that the cost associated with bailouts is higher in the decentralized economy, whereas the benefit of bailouts is the same as in the centralized economy. The distortion leads to larger fluctuations in government consumption for a given bailout policy. Since the marginal benefit of reducing inefficient liquidation decreases with the level of bailouts, the government must choose a lower bailout policy to make the benefit equal the cost.

Just as in the centralized economy, the model enables us to analyze how the optimal bailout policy varies across countries with different probabilities of experiencing a crisis. When bailouts are provided for free, the optimality condition in equation (27) implicitly defines the optimal bailout policy, x^*, as a function of the crisis probability, π, since the decision rules $k(x)$ and $T(x)$ depend on π.

Proposition 8 *For a higher probability of crisis and $P = 0$, the government should optimally commit to bailing out a smaller fraction of the borrowed capital in the economy. Formally, for any π_1 and π_2 such that $\pi_1 < \pi_2$, it is the case that $x^*(\pi_1) > x^*(\pi_2)$.*

Proof. See appendix B. ∎

The result in proposition 8 stands in stark contrast to the centralized economy, where a higher probability of crisis implied a higher optimal level of bailouts. When bailouts are provided for free, the consumer chooses a higher level of borrowed capital when the probability of crisis is higher, for a given bailout policy x. From the atomistic consumer's perspective, a higher probability of crisis increases the expected net return to borrowed capital, since it is more likely that he gets to keep part of the return of the investment, without having to pay for it. For a given bailout policy, a higher level of borrowed capital decreases the social benefit and increases the social cost of bailouts. For a higher probability of crisis, the government must therefore choose a lower level of bailout guarantees to make the benefit equal the cost.

7 Relaxing the commitment assumption in the model

The formal analysis and the policy conclusions of the paper would remain unchanged if in each period, the government were allowed to commit to a bailout policy for the subsequent period. In the centralized economy, the analysis and the results are robust to relaxing the commitment assumption altogether and allowing the social planner to reconsider the level of bailouts, once a crisis has occurred.

As noted in Section 5, the social planner's problem can be formulated without bailout policies and policy functions for a given level of bailouts. In a centralized economy without the artificial assumption of commitment, the social planner would simply invest foreign capital in production until the marginal product equalled $1/\beta$, and transfer the optimal amount of resources to the public sector in the next period, knowing how much resources he would want to spend on productive capital and government consumption in the event

Parameter	Value	Description
P	0	Price of bailouts in the decentralized economy
β	0.95	Discount factor
α	1/3	Capital income share
A	1	Productivity
ω	3.00	Domestic endowment of consumption goods
γ	1.05	Relative weight of government consumption in utility
π	$[0, 1/2]$	Probability of crisis

Table 1: Parameter values used in the numerical example

of a crisis. In the actual event of a crisis in the next period, the social planner would not want to act differently than what he foresaw one period ago.

In the market economy, the government's optimal bailout policy depends on the consumer's policy function for the level of borrowed capital, which is state independent as noted in the discussion of equation (11). In each period, the government would therefore optimally choose the same level of bailout guarantees for the subsequent period. This level would be identical to the optimal bailout under commitment in period $t = 0$, since the relative magnitudes of the costs and benefits associated with bailouts would be unchanged. In such an environment, the equilibrium for period $t = 0$, which we defined in Section 3, would be replaced by a definition of equilibrium under one-period commitment, which would be valid for any period t.

8 A numerical example

In this section, we present a numerical example to illustrate the model outcomes for the centralized economy and an economy where bailouts are provided for free. This example is not intended to reveal any new results on the optimal bailout policy, but rather to convey a sense of the relative magnitudes of the variables in the model for a reasonable parameterization. The graphs presented in figures 1 and 2 quantify the effects of bailouts that have so far only been analytically investigated in the paper.

We assume the utility function for government consumption to be $v(g) = \gamma \ln(g)$, where γ represents the relative weight of government consumption in the consumers' utility. As stated in Section 2, the production function is assumed to be of Cobb-Douglas form, $f(k) = Ak^{\alpha}$.

The parameter values used in the numerical example are presented in Table 1. We assume the length of a period to be one year. Using a standard value in the literature, we set $\beta = 0.95$, which implies a world interest rate of about 5 percent. Gollin (2002) shows the capital income share, α, to roughly equal $1/3$ for a large number of countries around the world. The TFP parameter, A, is normalized to unity.

The value for the domestic endowment, ω, should ideally be obtained by matching the model's ratio of borrowed capital to total output, $k/(Ak^{\alpha}+\omega)$, with the ratio of short-term foreign debt to GDP in the data. Rodrik and Velasco (1999) consider 16 episodes of financial crisis emerging markets between 1990 and 1998, and find that the average ratio of capital outflows to GDP was 0.09. Interpreting this observed ratio in terms of our model is problematic, however, since the level of borrowed capital in the model depends on the probability of a crisis, π, and the bailout policy, x. However, as shown in section 4, the level of borrowed capital in the model is independent of the bailout policy, if the crisis probability is zero, which might be the case for the largest economies in the industrialized world. Assuming the ratio of short-term foreign debt to GDP to be lower in these economies than in emerging markets, we set the ratio of short-term debt to GDP to 0.05,

$$\frac{k}{\omega+Ak^{\alpha}}=0.05, \tag{28}$$

in a country where $\pi=0$. This enables us to find a value of ω, since when $\pi=0$,

$$k=(A\beta\alpha)^{\frac{1}{1-\alpha}}, \tag{29}$$

according to equation (11).

The value for γ is also chosen for a country where $\pi=0$. In this case, the size of the equilibrium government revenues can be expressed as

$$T=\beta\gamma, \tag{30}$$

according to equation (12). The value of γ is set so that the ratio of government revenues to total output in the model equals the average ratio of government spending to GDP in the G7 countries between 1990 and 1992. According to Rodrik (1998), the G7 average is 0.28, so that

$$\frac{T}{Ak^{\alpha}+\omega}=0.28, \tag{31}$$

and

$$\gamma=\frac{0.28}{\beta}\left(Ak^{\alpha}+\omega\right). \tag{32}$$

When presenting the solutions to the model, we only consider crisis probabilities up to 1/2, since it is hard to imagine countries for which the crisis probability would exceed 1/2 for a prolonged period of time.

For a given probability of crisis, figure 1.a shows how the consumer's optimal choice of borrowed capital varies with the level of bailout guarantees in the decentralized economy with free bailouts. In line with proposition 5, we see that the level of borrowed capital is increasing in the level of bailout guarantees. As stated in proposition 3, we also see that, for a given $\pi>0$ and a given $x>0$, the level of borrowed capital is higher than in figure 1.b, which presents the level of borrowed capital in the centralized economy.

Interestingly, figure 1.b reveals that the social planner optimally decreases the level of foreign capital for higher levels of bailouts to reduce the social cost of taking loans.

Analogously, figure 1.c shows that in the decentralized economy, the optimal level of government revenues increases in the level of bailout guarantees for a given probability of crisis, which is in line with proposition 5. Comparing figures 1.c and 1.d, we also see that, for a given $\pi > 0$ and a given $x > 0$, the level of government revenues is higher in the market economy than in the centralized economy.

For a given probability of crisis, figure 2.a shows the expected utility attained by the government by committing to different levels of bailout guarantees in the environment where bailouts must be provided for free. At least for higher levels of π, we can see that Proposition 6 holds, since the expected utility function reaches a unique maximum in the interior of the policy space. Single peakedness and interior solutions are harder to discern in figure 2.b, which shows the expected utility function in the centralized economy. However, figure 2.c numerically verifies proposition 1, by showing that for all crisis probabilities, the socially optimal bailout policy is indeed unique and interior in our numerical example. The same figure also illustrates propositions 2,7 and 8. We see that the socially optimal bailout policy is increasing in the probability of crisis, that the optimal bailout policy in the decentralized economy is lower than in the social optimum for any positive crisis probability, and that the optimal bailout policy in the decentralized economy is decreasing in the probability of crisis. In figure 2.c, we cannot plot the optimal bailout policies for $\pi = 0$, since the bailout policy is irrelevant if crises never occur.

Finally, in figure 2.d, we compare the expected welfare attained in period $t = 0$ under the optimal bailout policy in the decentralized economy to the social optimum. We see that the value attainable to the government in the environment of free bailouts lies below the social optimum for positive probabilities of crisis, and that the economy is worse off for a higher probability of crisis.

For all crisis probabilities considered in the numerical example, private consumption is positive in the entire policy space.

9 Concluding remarks

The model presented in this paper provides a framework for analyzing the effects of bailout policies in a general equilibrium environment including both benefits and costs of bailouts. Considering both aspects of bailout guarantees, the model provides a beginning to bridging the gap between the two strands in the literature treating bailouts as a 'good' or a 'bad' policy.

We showed that committing to a partial bailout of borrowed capital is always socially optimal in the centralized economy. The extreme policy of *No Bailout* is inferior, since the benefit of reducing the inefficient liquidation associated with such a policy outweighs the cost of increased volatility in government consumption. Similarly, a policy of *Full Bailout* can be improved on by reducing the associated volatility of government consumption at the expense of some inefficient liquidation.

In the centralized economy, it was further shown that the social planner optimally chooses a higher level of bailout guarantees and public sector revenues in countries with a higher probability of crisis. For a higher probability of crisis, the social planner provides more protection against inefficient liquidation and provides more resources to the public sector to guard against low government consumption in crisis periods, at the expense of private sector consumption.

In a decentralized economy, the conclusions of the model depend on the price of bailouts that the government charges the private sector. Empirical evidence indicates that the economically relevant price of bailouts in a decentralized economy is below the price associated with the social optimum in the model. With a sub-optimally low price of bailouts, the decentralized model economy exhibits higher levels of foreign borrowing than what is socially optimal, since the private sector can enjoy the benefits of foreign borrowing without paying its full social cost. In such an environment, bailout guarantees lead to overinvestment in the emerging market.

In the extreme case when bailouts must be provided for free, we showed that, analogously to the centralized economy, the government should commit to a bailout policy only partially protecting private investment against liquidation. The optimal level of bailout guarantees in such an environment will always be lower than in the social optimum, however. Due to the investment distortions induced by bailouts, a given bailout policy leads to larger fluctuations in government consumption in the decentralized economy. In equilibrium, the government must counter the higher cost of bailouts by choosing a lower level of bailout guarantees.

We also showed that, in contrast to the results for the centralized economy, the government optimally bails out a smaller fraction of private investments for a higher probability of crisis. When crises occur more frequently, the government finds it optimal to expose the private sector more to the negative effects of crises, to make the consumers internalize more of the social costs of foreign borrowing.

In this paper, we have restricted our analysis to the optimal bailout policy under commitment and an exogenous price of bailouts. If the government could choose the price of bailouts, the endogenous price would be P^*, which is the price of bailouts associated with the social optimum. In such a model, the optimal bailout policy would be time consistent. It would be an interesting topic for future research to investigate what the government should optimally do if it cannot commit nor control the price of bailouts. We believe that a version of our model incorporating a reputational mechanism for the government could provide an appropriate framework for starting to analyze the optimal credible bailout policy.

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A Derivations of first-order conditions

A.1 Derivation of the consumer's first-order condition with respect to k_{t+1}

Using the investment constraint in equation (4) to substitute for b_{t+1}, the consumer's budget constraint to substitute for c_t, the form of the function $\kappa(.)$ and the fact that $R = 1/\beta$, the value function in equation (9) can be written as

$$\begin{aligned} V^C(x, h_t) = \max_{k_{t+1}} \Bigg\{ & f(\kappa_t) + \omega - (1-\zeta_t)\frac{1}{\beta} b_t - \zeta_t P \kappa_t - Q_t + v(g_t) + \\ & +\beta(1-\pi)\left(f(k_{t+1}) + \omega - \frac{1}{\beta} k_{t+1} - Q(x, S_{t+1,0}) + v(g_{t+1,0}) + \beta E_{t+1}\left[V^C(x, h_{t+2}) | \zeta_{t+1} = 0\right]\right) \\ & +\beta\pi \left(f(x k_{t+1}) + \omega - P x k_{t+1} - Q(x, S_{t+1,1}) + v(g_{t+1,1}) + \beta E_{t+1}\left[V^C(x, h_{t+2}) | \zeta_{t+1} = 1\right]\right) \Bigg\} \\ & s.t. \quad k_{t+1} \geq 0, \end{aligned} \tag{33}$$

where $S_{t+1,0} = (K_{t+1}, B_{t+1}, T_t, 0)$, $S_{t+1,1} = (K_{t+1}, B_{t+1}, T_t, 1)$, $g_{t+1,0} = T_t$ and $g_{t+1,1} = T_t - \frac{x}{\beta} B_{t+1}$. The first-order condition of (33) with respect to k_{t+1} is given by equation (11).

A.2 Derivation of the government's first-order condition with respect to T_t

Using equation (7), we recast the government's problem in equation (10) as a maximization with respect to bailouts and government revenues. Using the fact that $k_{t+1} = K_{t+1}$ and $b_{t+1} = B_{t+1}$, the investment constraint in equation (4) enables us to substitute b_{t+1} and B_{t+1} with $k(x)$. Employing the consumer's optimal consumption rule, the government's budget constraint and substituting for $R = 1/\beta$, the government's second value function in equation (10) can be written as

$$\begin{aligned} V^G(x, S_t) = \max_{T_t} \Bigg\{ & f(\kappa(x, K_t, \zeta_t)) + \omega - (1-\zeta_t)\frac{1}{\beta} B_t - T_t + v(g_t) \\ & +\beta(1-\pi)\left(f(k(x)) + \omega - \frac{1}{\beta} k(x) - T_{t+1} + v(T_t) + \beta E_{t+1}\left[V^G(x, S_{t+2}) | \zeta_{t+1} = 0\right]\right) \\ & +\beta\pi \left(f(x k(x)) + \omega - T_{t+1} + v\left(T_t - \frac{x k(x)}{\beta}\right) + \beta E_{t+1}\left[V^G(x, S_{t+2}) | \zeta_{t+1} = 1\right]\right) \Bigg\} \\ & s.t. \quad T_t - \frac{x k(x)}{\beta} \geq 0. \end{aligned} \tag{34}$$

The first-order condition of (34) with respect to T_t is given by equation (12).

A.3 Derivation of the Social Planner's first-order conditions with respect to K_{t+1} and T_t

Using the budget constraint for private and government consumption to substitute for c_t and g_t, the investment constraint to substitute for B_{t+1}, the form of the function $\kappa(.)$ and the fact that $R = 1/\beta$, the social planner's value function in equation (14) can be written as

$$
\begin{aligned}
V^{SP}(x, S_t) = \max_{\{K_{t+1}, T_t\}} & \Big\{ f\left(\kappa(x, K_t, \zeta_t)\right) + \omega - (1-\zeta_t)\frac{1}{\beta}B_t - T_t + v(g_t) \qquad (35) \\
+\beta(1-\pi) & \left(f\left(K_{t+1}\right) + \omega - \frac{1}{\beta}K_{t+1} - T_{t+1} + v(T_t) + \beta E_{t+1}\left[V^{SP}(x, S_{t+2}) | \zeta_{t+1} = 0\right] \right) \\
+\beta\pi & \left(f\left(xK_{t+1}\right) + \omega - T_{t+1} + v\left(T_t - \frac{xK_{t+1}}{\beta}\right) + \beta E_{t+1}\left[V^{SP}(x, S_{t+2}) | \zeta_{t+1} = 1\right] \right) \Big\} \\
& s.t. \qquad K_{t+1}, T_t - \frac{xK_{t+1}}{\beta} \geq 0.
\end{aligned}
$$

The first-order conditions of (35) with respect to K_{t+1} and T_t are given by equations (17) and (18).

A.4 Derivation of the government's first-order condition with respect to x

Using the Implicit Function Theorem, it can be shown that equations (11) and (12) implicitly define the policy functions $k(x)$ and $T(x)$ as continuously differentiable in x. This, in turn, implies that the government's expected utility function in (22) is continuously differentiable in x. Since x must be an element of the compact set $[0, 1]$, we know by the Weierstrass Theorem that the government's set of optimal bailout policies, X^*, is non-empty and compact.

Substituting for private and government consumption in (22), and differentiating the government's expected utility function w.r.t. x, we obtain

$$
\begin{aligned}
& \frac{d}{dx}\left\{c_0(x) + v\left(T_{-1}\right) + \beta E_0 V^G(x, S_1)\right\} = \\
& \quad -T'(x) + \frac{\beta(1-\pi)}{1-\beta}\left[f'\left(k(x)\right)k'(x) - \frac{1}{\beta}k'(x) - T'(x) + v'(T(x))T'(x)\right] \\
& \quad + \frac{\beta\pi}{1-\beta}\left[f'\left(xk(x)\right)\left[k(x) + xk'(x)\right] - T'(x)\right] \qquad (36) \\
& \quad + \frac{\beta\pi}{1-\beta}v'\left(T(x) - \frac{xk(x)}{\beta}\right)\left[T'(x) - \frac{k(x)}{\beta} - \frac{xk'(x)}{\beta}\right].
\end{aligned}
$$

Rearranging terms and adding and subtracting $\frac{\beta}{1-\beta}k'(x)\pi Px$, we obtain

$$\frac{d}{dx}\left\{c_0(x)+v\left(T_{-1}\right)+\beta E_0 V^G(x,S_1)\right\}=$$

$$\frac{\beta k'(x)}{1-\beta}\left[\underbrace{(1-\pi)\,f'\left(k(x)\right)+\pi x f'\left(xk(x)\right)-\frac{(1-\pi)}{\beta}-\pi Px}_{C_1(x)}+\pi Px-\frac{\pi x}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]$$

$$+\frac{\beta T'(x)}{1-\beta}\underbrace{\left[(1-\pi)\,v'\left(T(x)\right)+\pi v'\left(T(x)-\frac{xk(x)}{\beta}\right)-\frac{1}{\beta}\right]}_{C_2(x)}$$

$$+\frac{\beta\pi k(x)}{1-\beta}\left[f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]. \tag{37}$$

Now, $C_1(x)=0$ by equation (11) and $C_2(x)=0$ by equation (12), which implies that

$$\frac{d}{dx}\left\{c_0(x)+v\left(T_{-1}\right)+\beta E_0 V^G(x,S_1)\right\}= \tag{38}$$

$$=\frac{\beta}{1-\beta}\pi\left(k(x)\left[f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]+xk'(x)D(x)\right),$$

where $D(x)$ is defined as

$$D(x)=P-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right).$$

B Proofs of propositions

B.1 Proof of Proposition 1

Using the Implicit Function Theorem, it can be shown that equations (17) and (18) implicitly define the policy functions $K_{SP}(x)$ and $T_{SP}(x)$ as continuously differentiable in x. This, in turn, implies that the expected utility function in equation (19) is continuously differentiable in x.

The derivative of the expected utility function w.r.t. x can be found by applying the Envelope Theorem, since the policy functions, $K_{SP}(x)$ and $T_{SP}(x)$, have been defined as the socially optimal choices for each given bailout policy x.

To see that a policy of *No Bailout* is never optimal in the social planner's solution, consider the limit of the derivative of the expected utility function in equation (20),

$$\lim_{x\to 0}\left\{\frac{\beta}{1-\beta}\pi K_{SP}(x)\left[f'\left(xK_{SP}(x)\right)-\frac{1}{\beta}v'\left(T_{SP}(x)-\frac{xK_{SP}(x)}{\beta}\right)\right]\right\}=$$
$$\frac{\beta}{1-\beta}\pi K_{SP}(0)\left[f'\left(0\right)-\frac{1}{\beta}v'\left(T_{SP}(0)\right)\right]. \tag{39}$$

The limit value in equation (39) is clearly positive when $\pi > 0$, since $K_{SP}(0) > 0$, $f'(0) = \infty$ and $v'(T_{SP}(0)) = 1/\beta$ by equation (18). By continuity of the expected utility function, we can conclude that a policy of *No Bailout* is never optimal. Formally, $0 \notin X^*_{SP}$.

To see that a policy of *Full Bailout* is never optimal in the social planner's solution, consider the limit

$$\lim_{x\to 1}\left\{\frac{\beta}{1-\beta}\pi K_{SP}(x)\left[f'\left(xK_{SP}(x)\right)-\frac{1}{\beta}v'\left(T_{SP}(x)-\frac{xK_{SP}(x)}{\beta}\right)\right]\right\}=$$
$$\frac{\beta}{1-\beta}\pi K_{SP}(1)\left[\frac{(1-\pi)}{\beta}-\frac{(1-\pi)}{\beta}v'\left(T_{SP}(1)-\frac{K_{SP}(1)}{\beta}\right)\right], \tag{40}$$

where we have used the fact that

$$f'\left(K_{SP}(1)\right)=(1-\pi)\frac{1}{\beta}+\pi v'\left(T_{SP}(1)-\frac{K_{SP}(1)}{\beta}\right)\frac{1}{\beta}, \tag{41}$$

according to equation (17). The derivative of the expected utility function in equation (20) is negative in the upper limit when $\pi > 0$, since by equation (18),

$$v'\left(T_{SP}(x)-\frac{x}{\beta}K_{SP}(x)\right)\geq\frac{1}{\beta}.$$

By continuity of the expected utility function, we can therefore conclude that a policy of *Full Bailout* is never optimal in the social planner's solution. Formally, $1 \notin X^*_{SP}$.

To prove that the social planner's optimal bailout policy is unique, first note that in equilibrium, the equilibrium values of $K_{SP}(x), T_{SP}(x)$ and X^*_{SP} are determined by a system of the three following equations

$$(1-\pi)\,f'\left(K_{SP}(x^*_{SP})\right)+\pi x^*_{SP}f'\left(x^*_{SP}K_{SP}(x^*_{SP})\right)=$$
$$(1-\pi)\frac{1}{\beta}+\pi\frac{x^*_{SP}}{\beta}v'\left(T_{SP}(x^*_{SP})-\frac{x^*_{SP}K_{SP}(x^*_{SP})}{\beta}\right) \tag{42a}$$
$$(1-\pi)\,v'\left(T_{SP}(x^*_{SP})\right)+\pi v'\left(T_{SP}(x^*_{SP})-\frac{x^*_{SP}K_{SP}(x^*_{SP})}{\beta}\right)=\frac{1}{\beta} \tag{42b}$$
$$f'\left(x^*_{SP}K_{SP}(x^*_{SP})\right)=\frac{1}{\beta}v'\left(T_{SP}(x^*_{SP})-\frac{x^*_{SP}K_{SP}(x^*_{SP})}{\beta}\right), \tag{42c}$$

where equation (42c) holds because the optimal policy must be interior. Substituting (42c) into (42a) leads to

$$\pi x_{SP}^* f'\left(x_{SP}^* K_{SP}(x_{SP}^*)\right) = (1-\pi) f'\left(K_{SP}(x_{SP}^*)\right) + \pi x_{SP}^* f'\left(x_{SP}^* K_{SP}(x_{SP}^*)\right) - (1-\pi)\frac{1}{\beta},$$

which implies that

$$\begin{aligned} f'\left(K_{SP}(x_{SP}^*)\right) &= \frac{1}{\beta} \\ K_{SP}(x_{SP}^*) &= f'^{-1}\left(\frac{1}{\beta}\right). \end{aligned} \tag{43}$$

Thus, in equilibrium, the social planner's optimal level of investments is constant. We denote it by K_{SP}^*.

Next, by expressing $T_{SP}(x_{SP}^*)$ from (42c), we obtain

$$T_{SP}(x_{SP}^*) = w\left(\beta f'\left(x_{SP}^* K_{SP}^*\right)\right) + \frac{x_{SP}^* K_{SP}^*}{\beta}, \tag{44}$$

where $w = (v')^{-1}$. The value of x_{SP}^* can then be determined from equation (42b). After substituting the expression for $T_{SP}(x_{SP}^*)$ into (42b) and rearranging it, we obtain

$$(1-\pi) v'\left(w\left(\beta f'\left(x_{SP}^* K_{SP}^*\right)\right) + \frac{x_{SP}^* K_{SP}^*}{\beta}\right) = \frac{1}{\beta} - \pi\beta f'\left(x_{SP}^* K_{SP}^*\right). \tag{45}$$

For uniqueness of x_{SP}^*, it needs to be demonstrated that the right-hand side of (45) is increasing in x, while the left-hand side of (45) is decreasing in x. We have that

$$\frac{\partial}{\partial x}\left\{\frac{1}{\beta} - \pi\beta f'\left(x K_{SP}^*\right)\right\} = -\pi\beta f''(x K_{SP}^*) K_{SP}^*,$$

which is positive since $f''(.) < 0$, and

$$\frac{\partial}{\partial x}\left\{(1-\pi) v'\left(w\left(\beta f'\left(x K_{SP}^*\right)\right) + \frac{x K_{SP}^*}{\beta}\right)\right\} =$$
$$(1-\pi) v''\left(w\left(\beta f'\left(x K_{SP}^*\right)\right) + \frac{x K_{SP}^*}{\beta}\right)\left(w'(\beta f'\left(x K_{SP}^*\right))\beta f''(x K_{SP}^*) K_{SP}^* + \frac{K_{SP}^*}{\beta}\right),$$

which is negative, since $v''(.) < 0, w'(.) < 0$ and $f''(.) < 0$.■

B.2 Proof of Proposition 2

According to equation (42b), the socially optimal bailout policy satisfies

$$(1-\pi) v'\left(T_{SP}(x_{SP}^*)\right) + \pi v'\left(T_{SP}(x_{SP}^*) - \frac{x_{SP}^* K_{SP}^*}{\beta}\right) = \frac{1}{\beta}, \tag{46}$$

where K^*_{SP} is given by (43) and $T_{SP}(x^*_{SP})$ is given by (44). Implicitly differentiating (46) w.r.t. π, and rearranging, we obtain

$$\frac{\partial x^*_{SP}}{\partial \pi} = \frac{v'\left(T_{SP}(x^*_{SP})\right) - v'\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K^*_{SP}}{\beta}\right)}{(1-\pi)\,v''\left(T_{SP}(x^*_{SP})\right)T'_{SP}(x^*_{SP}) + \pi v''\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K^*_{SP}}{\beta}\right)\left(T'_{SP}(x^*_{SP}) - \frac{K^*_{SP}}{\beta}\right)}. \tag{47}$$

Next, according to equation (44), the derivative of $T_{SP}(x^*_{SP})$ w.r.t. x^*_{SP} is

$$T'_{SP}(x^*_{SP}) = \frac{K^*_{SP}}{\beta} + w'(\beta f'\left(x^*_{SP}K^*_{SP}\right))\beta f''(x^*_{SP}K^*_{SP})K^*_{SP}, \tag{48}$$

which is positive, since $w'(.) < 0$ and $f''(.) < 0$. We conclude that $\frac{\partial x^*_{SP}}{\partial \pi} > 0$, since

$$v'\left(T_{SP}(x^*_{SP})\right) - v'\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K_{SP}}{\beta}\right) < 0,$$

$v''(.) < 0$, $T'_{SP}(x^*_{SP}) > 0$ and since $T'_{SP}(x^*_{SP}) - \frac{K^*_{SP}}{\beta} > 0$, according to equation (48).■

B.3 Proof of Proposition 3

For any given bailout policy, x, and any given probability of crisis, $\pi > 0$, compare the first-order conditions for borrowed capital in the decentralized and centralized economies, respectively

$$\begin{aligned} (1-\pi)\,f'\left(k(x)\right) + \pi x f'\left(xk(x)\right) &= (1-\pi)\frac{1}{\beta} + \pi x P & (49)\\ (1-\pi)\,f'\left(K_{SP}(x)\right) + \pi x f'\left(xK_{SP}(x)\right) &= (1-\pi)\,\frac{1}{\beta} + \pi x \frac{1}{\beta} v'\left(T_t - \frac{xK_{SP}(x)}{\beta}\right) . & (50)\end{aligned}$$

For $P \leq 1/\beta^2$, the left-hand side of equation (49) must be smaller than the left-hand side of equation (50) if $x > 0$. This, together with the concavity of the production function, implies that $k(x) \geq K_{SP}(x)$, with equality if and only if $x = 0$.■

B.4 Proof of Proposition 4

For any given bailout policy, x, and any given probability of crisis, $\pi > 0$, compare the first-order conditions for the lump-sum tax in the decentralized and centralized economies, respectively

$$\begin{aligned} (1-\pi)\,v'\left(T(x)\right) + \pi v'\left(T(x) - \frac{xk(x)}{\beta}\right) &= \frac{1}{\beta} & (51)\\ (1-\pi)\,v'\left(T_{SP}(x)\right) + \pi v'\left(T_{SP}(x) - \frac{xK_{SP}(x)}{\beta}\right) &= \frac{1}{\beta}. & (52)\end{aligned}$$

By Proposition 3, $k(x) \geq K_{SP}(x)$, when $P \leq 1/\beta^2$, with equality iff $x = 0$. Together with strict concavity of the utility function $v(.)$, this implies that $T(x) \geq T_{SP}(x)$, with equality if and only if $x = 0$.■

B.5 Proof of Proposition 5

With $P = 0$, the optimality condition for investments in the decentralized economy in equation (11) becomes

$$(1-\pi) f'(k_{t+1}) + \pi x f'(xk_{t+1}) = (1-\pi)\frac{1}{\beta}. \tag{53}$$

Implicitly differentiating (53) w.r.t. x, and rearranging, we obtain

$$k'(x) = -\frac{\pi \left[f'(xk(x)) + xk(x) f''(xk(x)) \right]}{(1-\pi) f''(k(x)) + x^2 \pi f''(xk(x))}, \tag{54}$$

which is positive when $\pi > 0$, since $f''(.) < 0$ and $f'(i) + i f''(i) > 0$ for any Cobb-Douglas production function.

Implicitly differentiating the first-order condition in equation (12) w.r.t. x, and rearranging, we obtain

$$T'(x) = \frac{\pi v''\left(T(x) - \frac{x}{\beta}k(x)\right)\left[k(x) + xk'(x)\right]}{\beta\left[(1-\pi)v''(T(x)) + \pi v''\left(T(x) - \frac{x}{\beta}k(x)\right)\right]}, \tag{55}$$

which is positive when $\pi > 0$, since $v''(.) < 0$ and $k'(x) > 0$ according to equation (54).■

B.6 Proof of Proposition 6

To see that the government never finds it optimal to choose a policy of *No Bailout*, consider the limit of (23) when $P = 0$ and x approaches zero,

$$\lim_{x \to 0} \left\{ \frac{\beta}{1-\beta} \pi \left(k(x) \left[f'(xk(x)) - \frac{1}{\beta} v'\left(T(x) - \frac{xk(x)}{\beta} \right) \right] + xk'(x) D(x) \right) \right\} =$$
$$\frac{\beta}{1-\beta} \pi k(0) \left[f'(0) - \frac{1}{\beta} v'(T(0)) \right]. \tag{56}$$

The limit value in equation (56) is clearly positive when $\pi > 0$, since $k(0) > 0$, $f'(0) = \infty$ and $v'(T(0)) = 1/\beta$ by equation (12). By continuity of the objective function, we can conclude that the government never finds a policy of *No Bailout* optimal. Formally, $0 \notin X^*$.

To see that the government never finds it optimal to choose a policy of *Full Bailout* when $P = 0$, consider the limit

$$\lim_{x\to 1}\left\{\frac{\beta}{1-\beta}\pi\left(k(x)\left[f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]+xk'(x)D(x)\right)\right\}=$$
$$\frac{\beta}{1-\beta}\pi\left(k(1)\left[(1-\pi)\frac{1}{\beta}-\frac{1}{\beta}v'\left(T(1)-\frac{k(1)}{\beta}\right)\right]-k'(1)v'\left(T(1)-\frac{k(1)}{\beta}\right)\frac{1}{\beta}\right), \quad (57)$$

where we have used the fact that $f'\left(k(1)\right) = (1-\pi)/\beta$ according to equation (11) when $P = 0$. The derivative of the expected utility function is negative in the upper limit when $\pi > 0$, since $k'(.) > 0$ according to proposition 5, $v'(.) > 0$ and since, by equation (12),

$$v'\left(T(x)-\frac{x}{\beta}k(x)\right)\geq\frac{1}{\beta}.$$

By continuity of the objective function, we can therefore conclude that the government never finds a policy of *Full Bailout* optimal. Formally, $1 \notin X^*$.

The fact that the optimal bailout policy must lie in the interior of the policy space implies that any optimal bailout policy satisfies

$$f'\left(x^*k(x^*)\right)=\frac{1}{\beta}v'\left(T(x^*)-\frac{x^*k(x^*)}{\beta}\right)\left[1+\frac{x^*k'(x^*)}{k(x^*)}\right], \quad (58)$$

where we have substituted $P = 0$ into equation (23) and rearranged under the condition that the derivative w.r.t. x equals zero.

To prove that the optimal bailout policy is unique, we now proceed to show that the left-hand side of equation (58) is a decreasing function of x, and that the right-hand side is an increasing function of x.

The derivative of the left-hand side of equation (58) is

$$\frac{d}{dx}\left\{f'\left(xk(x)\right)\right\}=f''(xk(x)\left[k(x)+xk'(x)\right], \quad (59)$$

which is negative, since $f''(.) < 0$ and $k'(x) > 0$ according to equation (54).

Let $p(x)$ denote the second factor on the right-hand side of equation (58). The derivative of $p(x)$ is

$$\begin{aligned}p'(x)&=\frac{d}{dx}\left\{v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right\} \quad (60)\\ &=v''\left(T(x)-\frac{xk(x)}{\beta}\right)\left[T'(x)-\frac{k(x)}{\beta}-\frac{xk'(x)}{\beta}\right],\end{aligned}$$

which is positive, since $v''(.)$ is negative and since

$$T'(x)<\frac{1}{\beta}\left[k(x)+xk'(x)\right], \quad (61)$$

according to equation (55).

For any production function of the form $f(k) = Ak^{\alpha}$, $\alpha < 1$, the optimality condition in equation (53) implies that the consumer's policy function for borrowed capital is

$$k(x) = \left[\frac{A\alpha\beta}{(1-\pi)}\right]^{\frac{1}{1-\alpha}} \left[1 - \pi + \pi x^{\alpha}\right]^{\frac{1}{1-\alpha}}, \tag{62}$$

in an environment where $P = 0$. This, in turn, implies that the third factor on the right-hand side of equation (58), which we denote $q(x)$, can be written as

$$\begin{aligned} q(x) &= 1 + \frac{xk'(x)}{k(x)} \\ &= 1 + \frac{\alpha}{1-\alpha}\left[1 - \pi + \pi x^{\alpha}\right]^{-1} \pi x^{\alpha}. \end{aligned} \tag{63}$$

The derivative of $q(x)$ is

$$q'(x) = \frac{\alpha^2}{(1-\alpha)} \frac{\pi(1-\pi)x^{\alpha-1}}{(1-\pi+\pi x^{\alpha})^2}, \tag{64}$$

which is positive.

The derivative of the right-hand side of equation (58) can now be expressed as

$$\frac{d}{dx}\left\{\frac{1}{\beta}p(x)q(x)\right\} = \frac{1}{\beta}\left[p'(x)q(x) + p(x)q'(x)\right], \tag{65}$$

which is positive, since $p'(x) > 0$, $q'(x) > 0$ and $v'(.) > 0$.■

B.7 Proof of Proposition 7

To construct the proof, we start by noting that in both the decentralized and the centralized economies, the equilibrium level of resources transferred from the private to the public sector can be written as a function of the optimal level of borrowed capital. According to the first-order conditions in equations (12) and (18), $T(x) = j(k(x))$ and $T_{SP}(x) = j(K_{SP}(x))$. For a given x, the partial derivative of $j(.)$ w.r.t. k is given by

$$\frac{\partial j(k)}{\partial k} = \frac{x\pi v''\left(j(k) - \frac{xk}{\beta}\right)}{\beta\left[(1-\pi)v''(j(k)) + \pi v''\left(j(k) - \frac{xk}{\beta}\right)\right]}. \tag{66}$$

Now, consider the bailout policy x^*_{SP}, which according to Proposition 1 must be such that

$$f'\left(x^*_{SP}K_{SP}(x^*_{SP})\right) - \frac{1}{\beta}v'\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K_{SP}(x^*_{SP})}{\beta}\right) = 0. \tag{67}$$

Using the fact that when $P = 0$ and $\pi > 0$, by Proposition 4, $k(x) > K_{SP}(x)$ for $x > 0$, we next proceed to show that evaluated at any such bailout policy x_{SP}^*, the derivative of the government's expected utility function in (23) is negative, i.e. that

$$k(x)\left[f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]+xk'(x)\left[P-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]<0, \tag{68}$$

for $x = x_{SP}^*$.

Notice that, when $P = 0$, the last term on the left-hand side in inequality (68) is negative, since $k'(x) > 0$ according to proposition 5 and since $v'(.) > 0$. The derivative of the government's expected utility function will therefore be negative whenever the term in the first square brackets in (68) is negative.

Let this term be denoted by Z, so that

$$Z(k(x)) = f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(j(k(x))-\frac{xk(x)}{\beta}\right). \tag{69}$$

For a given x, the partial derivative of $Z(.)$ w.r.t. k is given by

$$\frac{\partial Z(k)}{\partial k} = f''(xk)x-\frac{1}{\beta}v''\left(j(k)-\frac{xk}{\beta}\right)\left(\frac{\partial j(k)}{\partial k}-\frac{x}{\beta}\right), \tag{70}$$

which is negative, since $f''(.) < 0$, $v''(.) < 0$ and since, by equation (66),

$$\frac{\partial j(k)}{\partial k} < \frac{x}{\beta}.$$

Since $Z(K_{SP}(x_{SP}^*)) = 0$ and $k(x_{SP}^*) > K_{SP}(x_{SP}^*)$, the fact that the partial derivative of $Z(.)$ w.r.t. k is negative enables us to conclude that $Z(k(x_{SP}^*)) < 0$ and hence, that the derivative of the government's expected utility function is negative at x_{SP}^*.

The fact that the government's expected utility function is continuously differentiable and has a positive slope as x approaches zero, together with the result that there is a unique bailout policy which satisfies the first-order condition in equation (27), allow us to conclude that it must be the case that $x^* < x_{SP}^*$.■

B.8 Proof of Proposition 8

We start by showing that for a given bailout policy x and $P = 0$, the level of borrowed capital, $k(x)$, is an increasing function of π. For a given bailout policy, the partial derivative of $k(x)$ w.r.t. to π is given by

$$\begin{aligned}\frac{\partial k(x)}{\partial \pi} &= \frac{\partial}{\partial \pi}\left\{\left[\frac{A\alpha\beta}{(1-\pi)}\right]^{\frac{1}{1-\alpha}}\left[(1-\pi)+\pi x^{\alpha}\right]^{\frac{1}{1-\alpha}}\right\} \qquad (71)\\ &= \frac{1}{(1-\alpha)}\left[\frac{A\alpha\beta}{(1-\pi)}\right]^{\frac{1}{1-\alpha}}\left[(1-\pi)+\pi x^{\alpha}\right]^{\frac{1}{1-\alpha}}\left[\frac{1}{(1-\pi)}-\frac{(1-x^{\alpha})}{[(1-\pi)+\pi x^{\alpha}]}\right],\end{aligned}$$

which is positive.

To complete the proof, we next show that for a given bailout policy, the left-hand side of equation (58) is a decreasing function of π and the right-hand side is an increasing function of π.

For a given bailout policy x, the partial derivative of the left-hand side of equation (58) w.r.t. π is given by

$$\frac{\partial}{\partial \pi}\left\{f'\left(xk(x)\right)\right\} = f''(xk(x))x\frac{\partial k(x)}{\partial \pi}, \qquad (72)$$

which is negative, since $f''(.) < 0$. Put differently, the social benefit associated with a given bailout level shifts downwards for a higher π.

For a given bailout policy x, the partial derivative of the right-hand side of equation (58) w.r.t. π is given by

$$\frac{\partial}{\partial \pi}\left\{\frac{1}{\beta}p(x)q(x)\right\} = \frac{1}{\beta}\left[\frac{\partial p(x)}{\partial \pi}q(x) + p(x)\frac{\partial q(x)}{\partial \pi}\right], \qquad (73)$$

where $p(x)$ and $q(x)$ were defined in equations (60) and (63), respectively. To see that the derivative in equation (73) is positive, first consider

$$\frac{\partial p(x)}{\partial \pi} = v''\left(T(x) - \frac{xk(x)}{\beta}\right)\left[\frac{\partial j(k)}{\partial k} - \frac{x}{\beta}\right]\frac{\partial k(x)}{\partial \pi}, \qquad (74)$$

which is positive, since $v''(.) < 0$ and since, by equation (66),

$$\frac{\partial j(k)}{\partial k} < \frac{x}{\beta}.$$

Second, consider the partial derivative of $q(x)$ w.r.t. π,

$$\frac{\partial q(x)}{\partial \pi} = \frac{\alpha x^{\alpha}}{(1-\alpha)}\left[(1-\pi)+\pi x^{\alpha}\right]^{-2}, \qquad (75)$$

which is also positive. Put differently, the social cost associated with a given bailout level shifts upwards for a higher π.

The fact that for a given bailout policy, x, the benefit of bailouts becomes lower and the cost becomes higher when π increases, implies that the government must choose a lower level of bailout guarantees for a higher probability of crisis. As shown in the proof of Proposition 6, the reason is that for a given probability of crisis, a lower level of bailout guarantees increases the marginal benefit and reduces the marginal cost of bailouts. Formally, we have shown that for any π_1 and π_2 such that $\pi_1 < \pi_2$, it is the case that $x^*(\pi_1) > x^*(\pi_2)$.■

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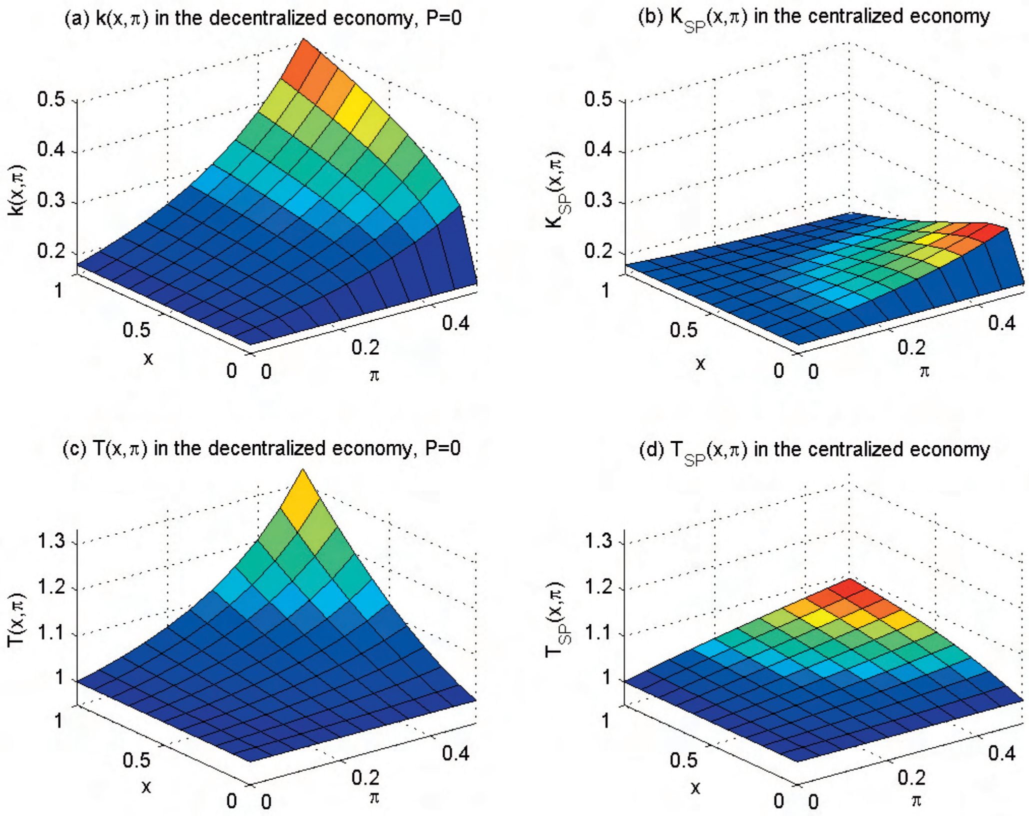

Figure 1: The optimal decision rules for the level of borrowed capital and government revenues in the decentralized and centralized economies

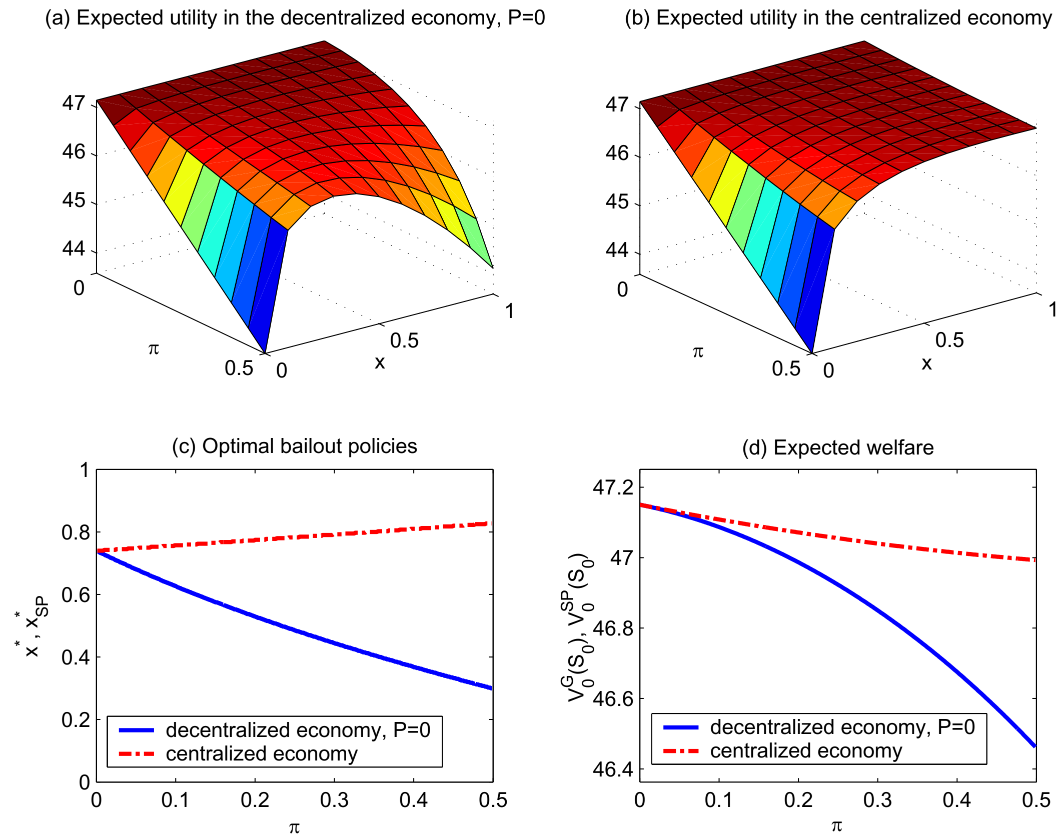

Figure 2: Expected utility functions, optimal bailout policies and expected welfare in the decentralized and centralized economies