Essays in Public Finance

Tatiana Damjanovic

AKADEMISK AVHANDLING

som för avläggande av Filosofie doktorsexamen
vid Handelshögskolan I Stockholm
framläggs offentlig granskning
fredagen den 24 may 2002, kl 13.15 i sal Ragnar,
Handelshögskolan, Sveavägen 65,
Stockholm
Essays in Public Finance
EFI Mission
EFI, the Economic Research Institute at the Stockholm School of Economics, is a scientific institution which works independently of economic, political and sectional interests. It conducts theoretical and empirical research in management and economic sciences, including selected related disciplines. The Institute encourages and assists in the publication and distribution of its research findings and is also involved in the doctoral education at the Stockholm School of Economics.
EFI selects its projects based on the need for theoretical or practical development of a research domain, on methodological interests, and on the generality of a problem.

Research Organization
The research activities are organized in twenty Research Centers within eight Research Areas. Center Directors are professors at the Stockholm School of Economics.

ORGANIZATION AND MANAGEMENT
Management and Organisation; (A) Prof Sven-Erik Sjöstrand
Center for Ethics and Economics; (CEE) Adj Prof Hans de Geer
Center for Entrepreneurship and Business Creation; (E) Prof Carin Holmquist
Public Management; (F) Prof Nils Brunsson
Information Management; (I) Prof Mats Lundeborg
Center for People and Organization; (PMO) Prof Jan Löwstedt
Center for Innovation and Operations Management; (T) Prof Christer Karlsson

ECONOMIC PSYCHOLOGY
Center for Risk Research; (CFR) Prof Lennart Sjöberg
Economic Psychology; (P) Prof Guje Sev6n

MARKETING
Center for Information and Communication Research; (CIC) Adj Prof Bertil Thorngren
Center for Consumer Marketing; (CCM) Acting Prof Magnus Söderlund
Marketing, Distribution and Industrial Dynamics; (D) Prof Lars-Gunnar Mattsson

ACCOUNTING, CONTROL AND CORPORATE FINANCE
Accounting and Managerial Finance; (B) Prof Lars Östman
Managerial Economics; (C) Prof Peter Jennergren

FINANCE
Finance; (F1) Prof Clas Bergström

ECONOMICS
Center for Health Economics; (CHE) Prof Bengt Jönsson
International Economics and Geography; (IEG) Prof Mats Lundahl
Economics; (S) Prof Lars Bergman

ECONOMICS STATISTICS
Economic Statistics; (ES) Prof Anders Westlund

LAW
Law; (RV) Prof Erik Nerep

Chairman of the Board: Prof Håkan Lindgren. Director: Associate Prof Bo Sellstedt.

Adress
EFI, Box 6501, S-113 83 Stockholm, Sweden • Internet: www.hhs.se/efi/
Telephone: +46(0)8-736 90 00 • Fax: +46(0)8-31 62 70 • E-mail efi@hhs.se
Essays in Public Finance

Tatiana Damjanovic

STOCKHOLM SCHOOL OF ECONOMICS
EFI, THE ECONOMIC RESEARCH INSTITUTE
Keywords:
Pareto-improving pension reform, heterogeneity, optimal transition, Initial distortions and the duration of transition periods, tax avoidance, optimal taxation, inequality, tax base broadening, regional policy, secession

Printed by

Distributed by EFI, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden.
To Ljubica
Acknowledgement

First of all, I wish to express my sincere gratitude to my supervisor, Lars Ljungqvist, for his invaluable guidance, numerous inspiring discussions and helpful suggestions.

I am also indebted to Kjetil Storesletten, Jonathan Heathcote and Martin Flodén for helpful comments and suggestions.

I am grateful to all my teachers from the Stockholm School of Economics, the New Economic School (Moscow), and the Computer Academy and Mathematics Department of Moscow State University.

Special thanks to Jochen Wennuth and other colleagues from the Economic Expert Group at the Ministry of Finance of the Russian Federation, who influenced the formation of my economic intuition and helped me obtain a work experience inspiring this thesis. I am also grateful to Gonzalo Pastor for a pleasant work experience during my summer internship at IMF, and John S. Earle who gave me the possibility to improve my technical tools by working in the Labor Project under his supervision.

I also want to thank all the people who have commented on the essays, such as Erik Berglof, Guido Friebel, Fabrizio Zilibotti, and seminar participants at SITE, IIES, IUI, FIEF and the European Network on Industrial Policy (EUNIP) conference.

I am indebted to Gun Malmquist, Ritva Kiviharju, Pirjo Furtenbach, Anders Paalzow, Elisabet Tilert, Kerstin Niklason, Ingrid Nilsson, Britt-Marie Eisler and Carin Blanksvärd, who all helped me with administrative and other matters during these years of study and writing. Special thanks to Christina Lönnblad for her fine editing.

Financial support from the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged.

The largest thanks go to my family, Vladislav and Ljubica, who put up with me throughout these years. Without Vladislav's support, this thesis would not have been finished.

Stockholm, May 2002

Tatiana Damjanovic
# Contents

## Introduction and Summary

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Part I: The Possibility and Duration of Pareto-Improving Pension Reforms</td>
<td></td>
</tr>
<tr>
<td>1.1 Essay I: The Possibility of a Pareto-Improving Pension Reform in a Heterogeneous Economy</td>
<td></td>
</tr>
<tr>
<td>1.2 Essay II: Designing Optimal Pareto-Improving Pension Reforms: A More Distorted Economy Can be Reformed Faster</td>
<td></td>
</tr>
<tr>
<td>2 Part 2: An Economy with Clever Tax Avoidance Providers</td>
<td></td>
</tr>
<tr>
<td>2.1 Essay III: The Importance of Income Distribution for the Price of the Tax Avoidance Service</td>
<td></td>
</tr>
<tr>
<td>2.2 Essay IV: Tax Avoidance as a Reason for Secession</td>
<td></td>
</tr>
</tbody>
</table>

## I The Possibility and Duration of Pareto-Improving Pension Reforms

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Introduction</td>
<td>3</td>
</tr>
<tr>
<td>4 On the existence of a Pareto-efficient transition in a representative agent's economy</td>
<td>4</td>
</tr>
<tr>
<td>4.1 Background</td>
<td>4</td>
</tr>
<tr>
<td>4.2 Classification and extension</td>
<td>5</td>
</tr>
<tr>
<td>5 Technical framework</td>
<td>8</td>
</tr>
<tr>
<td>6 Heterogeneity is not an obstacle in implementing a Pareto-Improving transition</td>
<td>10</td>
</tr>
<tr>
<td>6.1 Differences in productivity or preferences still allow the Pareto-Improving pension reform to be complete by using homogeneous tax rates</td>
<td>11</td>
</tr>
<tr>
<td>6.2 Redistributive and insurance role of a pay-as-you-go system</td>
<td>15</td>
</tr>
<tr>
<td>7 Concluding summary</td>
<td>16</td>
</tr>
</tbody>
</table>

## Designing Optimal Pareto-Improving Pension Reforms: A More Distorted Economy Can be Reformed Faster

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 Introductions</td>
<td>19</td>
</tr>
<tr>
<td>9 Technical description</td>
<td>20</td>
</tr>
<tr>
<td>9.1 General technical complications</td>
<td>20</td>
</tr>
<tr>
<td>9.2 Solution method</td>
<td>21</td>
</tr>
<tr>
<td>10 The results</td>
<td>22</td>
</tr>
<tr>
<td>11 An Alternative Explanation of the Chilean Success</td>
<td>24</td>
</tr>
<tr>
<td>12 Concluding remarks</td>
<td>24</td>
</tr>
</tbody>
</table>

## II An Economy with Clever Tax Avoidance Providers

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 Introduction</td>
<td>31</td>
</tr>
<tr>
<td>14 General Model</td>
<td>33</td>
</tr>
<tr>
<td>14.1 Defence of the Main Assumptions</td>
<td>33</td>
</tr>
<tr>
<td>14.2 Formalization</td>
<td>34</td>
</tr>
<tr>
<td>14.3 Risk Neutrality</td>
<td>36</td>
</tr>
<tr>
<td>14.4 Solution Under Different Restrictions</td>
<td>37</td>
</tr>
<tr>
<td>14.5 A Discrete Case</td>
<td>39</td>
</tr>
<tr>
<td>15 Inequality and Tax Revenue</td>
<td>39</td>
</tr>
</tbody>
</table>
CONTENTS

15.1 The Possibility of Positive Correlation Between Tax Revenue and Inequality Under Stable Tax Code .................................................. 40
15.2 When The Government Maximizes Its Revenue ........................................... 41
15.3 When the Government Can Collect More than Target .................................. 45
16 When an Avoidance Supply is Costly .......................................................... 47
17 Applications ......................................................................................... 48
  17.1 Tax Base Broadening as a Measure of Increasing Revenue ....................... 48
  17.2 Immigration Policy ........................................................................... 52
  17.3 Regional Policy ................................................................................. 52
  17.4 Should Growth in Inequality Reduce Tax Progressivity? ......................... 53
18 Conclusion ............................................................................................ 53
19 Appendix .............................................................................................. 53
20 Appendix: Calculation for Uniform Distribution ........................................ 55

Tax Avoidance as a Reason for Secession .................................................. 59
21 Introduction ......................................................................................... 59
22 Tax Avoidance Model: a Short Description .............................................. 60
23 A Two Agent Economy ......................................................................... 60
  23.1 Immigration as an example of a change in the poor to rich ratio. Who will benefit? 61
  23.2 A unification of two regions distinguished by the proportions of poor individuals 61
24 An Example with Uniform Distribution: All but the Poorest have Incentives for Separation 62
  24.1 The Solution of the Tax Avoidance Model for Uniform Income Distribution 62
  24.2 When individuals are risk averse ......................................................... 66
  24.3 Inefficient Government and Intentions to Secede in the Union of Poor and Rich Regions 67
25 Democracy and the Movement for Independence ..................................... 67
26 Concluding summary ............................................................................. 69
Introduction and Summary

I have carried out research in two main areas, tax avoidance and pension reform. The choice of both topics has been motivated by their great importance.

1 Part I: The Possibility and Duration of Pareto-Improvement Pension Reforms

The first part of my research is related to pension reforms. Many countries face large problems in supporting current pay-as-you-go systems arising from demographic changes: fertility rates are decreasing, while life expectancies continue to grow. This leads to an enlargement of the old age dependency ratio and makes current pay-as-you-go systems unsustainable. To support such systems in the near future, governments must either increase the payroll tax rate or reduce pension benefits. Moreover, a pay-as-you-go system is even very inefficient at present, because the returns on capital exceed population growth. Inefficiency arises from implicit taxation on pension contributions by the tax rate equaling the difference between interest rate and payroll growth. In an open economy, this tax is thrown away without being used for government spending. In a closed economy, it is even more destructive due to its negative impact on savings and capital accumulation.

Although the transition to a fully funded system is a great improvement of the economy, its implementation entails several difficulties if Pareto-efficiency is required. The transition dilemma can be formulated in the following way. On the one hand, the old generation must be compensated for the contributions already made to a pay-as-you-go system and therefore, the young generations must pay "twice": charged with taxes for pensions for the old, they must contribute to the accumulation of their own pension funds. On the other hand, a reduction in payroll taxes leads to higher wages, and labor and capital supplies, thereby providing some financing for a transition. The main purpose of my research is to investigate whether and when those funds are sufficient for a Pareto-improving reform. In particular, I investigate whether a Pareto-improving transition is possible and how long the society should wait for the realizations of the gains.

1.1 Essay I: The Possibility of a Pareto-Improving Pension Reform in a Heterogeneous Economy

This article consists of two main parts. First, I show that Pareto-improving transitions from pay-as-you-go to fully funded pension systems nearly always exist. To demonstrate this, I classify theoretical results according to the type of reformed economy, the form of pension benefits, and the policy instruments used during the transition. Along the way, I analyze various economies not considered in the existing literature. Then, I show the possibility of a Pareto-improving transition for an economy with heterogeneous agents within generations. In other words, I state that intergenerational heterogeneity should no longer be considered as an obstacle when implementing Pareto-improving policy reforms. To maintain redistributive or insurance mechanisms supported by pay-as-you-go systems, I propose to replace inefficient social security with redistributive tax and transfer payments inside one generation. This would save the economy from the inefficiency related to the implicit taxes on pension contributions imposed by pay-as-you-go systems.

1.2 Essay II: Designing Optimal Pareto-Improving Pension Reforms: A More Distorted Economy Can be Reformed Faster

In this article, I investigate the optimal Pareto-improving debt-financed transition from pay-as-you-go to fully funded pension systems. In particular, I examine the relationship between key parameter values characterizing the preferences, the technology and the size of the initial system, and the necessary time for a Pareto-improving transition. My finding is that a more distorted economy can be reformed faster. This result gives an additional
explanation to the success of the Chilean reform, where an initial pay-as-you-go system was the largest and, at the same time, the most distorting.

2 Part 2: An Economy with Clever Tax Avoidance Providers.

Tax avoidance is a problem of great importance. Taxes do not collect themselves. People have incentives for hiding their incomes from the government; in the US; for example, about 17% of the income tax revenue are not paid. The government spends about 10% of the total tax revenue on tax enforcement. Furthermore, tax avoidance leads to an increase in deadweight loss. Thus, according to Feldstein, deadweight loss from income tax is more than ten times as large as would otherwise be the case. This inefficiency is probably even larger in other countries. Since rich people usually use the avoidance practice, the government must tax the poor more heavily, which leads to large social costs. Tax collection is an even more important problem for developing countries, for example the government’s inability to collect sufficient tax revenue led to the Russian financial crisis in 1998.

My particular research has been motivated by my observations of the Russian government trying to improve tax collection in 1996-1997. According to the relevant budget laws, the tax revenue should be about 30% of GDP while, in reality, it was only 13%. The government has tried to broaden the tax bases, to introduce new income sources as part of the tax base and, finally, a new emergency committee to enforce tax collection has been established. All these efforts have been without result, however, which motivated me to consider a model where avoidance technology changes with tax avoidance demand.


In this paper, I design a model with a clever tax avoidance provider, who maximizes a profit by setting the price for the tax avoidance services. Therefore, the price for the tax avoidance service is endogenously defined. In that setup, the change in income distribution is not less important than changes in the tax code, which together are responsible for the tax avoidance demand. My paper introduces the distribution component of a price for tax avoidance service for the first time, in contrast to the entire previous tax avoidance literature, which neglects income distribution as an important factor for price setting. In other words, my model captures the fact that the part of income a householder pays for tax avoidance should depend on the demand for such services in the rest of society.

My model has a wide range of applications. In particular, I analyze the relation between inequality and a government’s ability to collect tax revenue. It might be that maximum revenue grows with inequality, while there is a reduction in the inefficiency from avoidance purchase. I also consider tax base broadening, providing examples where this leads to a reduction in tax revenue.

2.2 Essay IV: Tax Avoidance as a Reason for Secession

In this essay, I provide an additional explanation for the intentions to secede related to expected changes in the tax avoidance activities after the “break down of a nation”. To demonstrate my points, I use a tax avoidance model designed in Essay III, where active tax avoidance providers make a decision about the price and quantity of their services. Secession gives the avoidance provider the option of setting different prices in separate regions. As a consequence, the price for the tax avoidance service may fall in the poorer region and the elite of this region would be able to avoid the tax, which is impossible in union. Moreover, regional separation may lead to tremendous changes in the shape of income distribution. Facing these changes, new governments would be forced to change the tax codes. Thus, the government of the richer region may reduce tax rates in order to enlarge the tax revenue collection. This decision would not only increase private consumption for tax payers but also for rich tax avoiders, since the reduction of tax duties leads to a decline in the price for the tax avoidance service. The sum of tax revenues in both regions would fall significantly after the secession, however, which means that governments would provide less public goods. If the value of public goods were low for the citizens, however, many households would be better off after the secession. And thus, the model predicts a higher secession tension in economies with less efficient governments, which explains the increase in the popularity of secession ideas at the beginning of transitions, when the population of the restructuring countries negatively evaluated the provision of public goods. To avoid a breakdown of the state, the government should reduce inefficient spending and tax duties. Promoting democracy or increasing the political influence of poor households may reduce the tendency to separate.
Bibliography


Part I

The Possibility and Duration of Pareto-Improving Pension Reforms
The Possibility of a Pareto-Improving Pension Reform in a Heterogeneous Economy

Abstract

In this article, I show that Pareto-improving transitions from pay-as-you-go to fully funded pension systems nearly always exist. To demonstrate this, I classify theoretical results according to the type of reformed economy, the form of pension benefits, and the policy instruments used during the transition. Along the way, I analyze various economies not considered in the existing literature. Then, I show the possibility of a Pareto-improving transition for an economy where agents are heterogeneous within generations. I state that intergenerational heterogeneity should no longer be considered an obstacle when implementing Pareto-improving pension reforms. To maintain redistributive or insurance mechanisms supported by pay-as-you-go systems, I propose to replace inefficient social security with redistributive tax and transfer payments inside one generation. This would save the economy from the inefficiency related to the implicit taxes on pension contributions imposed by pay-as-you-go systems.

Keywords: Pension reform, Pareto-improving transition, Heterogeneous population, Redistribution and insurance

JEL-codes: H55

3 Introduction

The privatization of social security is one of the most important economic issues today. In the last thirty years, many countries have faced drastic demographic changes leading to a dramatic increase in dependency ratio and a reduction in payroll growth. Economists and policy makers worldwide predict that this tendency will continue for at least 50 years\(^1\), which will make pay-as-you-go systems even more inefficient. For a more detailed discussion, see various World Bank publications, for example, World Bank (1994) and, more recently, Fox and Palmer (2001). A continuation of the current systems will force governments to either increase taxes or reduce the benefits for future generations, which would cause a decrease in welfare levels, as compared to those guaranteed by the present pension schemes. Thus, Feldstein and Samwick (2000) predict that the U.S. 'current pure pay-as-you-go system can only maintain the benefits specified in current law by raising the payroll tax rate from 12.4 percent today to more than 17 percent by 2037 and nearly 19 percent by the end of the actuaries' 75 year forecast period.'

Pay-as-you-go systems are inefficient even in the present demographic environment, since the rates of capital return have already exceeded the growth of labor income\(^2\). These conclusions were first drawn by Samuelson (1975). The inefficiency arises from the indirect taxation on pension contributions imposed by pay-as-you-go systems. This inefficiency is fueled by the distortions to the labor market due to social security payroll taxes. According to Feldstein (1996), the relevant deadweight loss equals approximately one percent of GDP.

---

\(^{1}\)For demographic projections see, for example, the World Bank data base "World Development Indicators".

\(^{2}\)According to Feldstein and Samwick (2000), the real rate of return will exceed the growth of the average real wage by at least 4.5% annually.
in the U.S. In closed economies, the damage from pay-as-you-go systems is even larger due to the reduction in savings. An increase in capital accumulation, caused by the pension reform will also increase pre-tax wages. The transition to a fully funded system might also cause larger economic growth in the presence of endogenous technical progress, as discussed in Wiedmer (1996).

The question considered in this paper is the one of how to introduce a better pension system without anybody suffering. On the one hand, the old generations must be compensated for their contributions to a pay-as-you-go system, and therefore, the young generations have to pay "twice": taxed for pensions for the old, they also have to contribute to their own pension accumulation fund. On the other hand, a reduction in payroll taxes leads to higher wages and larger capital supply, thereby providing some financing for a transition. This paper investigates whether and when those funds are sufficient for a Pareto-improving reform, with the purpose of convincing the readers that a Pareto-efficient transition can nearly always be implemented.

The paper is organized as follows. I start section 4 by providing a systematic classification of earlier studies related to the possibility of a Pareto-improving pension reform in a representative agent economy. Then, I discuss the particular features of various types of economies and the difficulties the reform may meet. Along the way, I analyze various economies not considered in the existing literature. In section 5, I describe the technical framework I used for my analysis. Then, in section 6, I reject the intragenerational heterogeneity as an argument against the possibility of a Pareto-improving transition. In particular, I provide simulations of Pareto-improving transitions for economies with two types of agents, distinguished by productivity levels and preference parameterization. To maintain the pre-reform welfare level of less productive households, I propose to make the redistribution within one generation. This mechanism endures the redistributive and insurance functions of a pay-as-you-go system, but saves the economy from the inefficiency related to the implicit taxes on pension contributions. Section 7 concludes the paper.

4 On the existence of a Pareto-efficient transition in a representative agent’s economy.

Studying the recent literature on the existence of a Pareto-efficient pension reform, I realize that researchers and policy makers are sometimes not aware of each others’ results. I have also found that some results stating the impossibility of a Pareto-efficient transition are not robust to the choice of policy mechanisms implemented during a transition, while others are incorrectly quoted, i.e., neglecting the assumptions for the economy for which the result were conducted. This motivated me to write the present section. First, I review previous literature and systemize it by the characteristics of an initial economy. Then, I discuss some features of transition in each class of economy, based on both the existing literature and my own simulation results.

4.1 Background

The early literature considers a pay-as-you-go system with a lump-sum form of pension benefits. Thus, Breyer (1989) proved the impossibility of an efficient debt-financed transition for both closed and open economies, using a simple two-period overlapping generations model (OLG2) and assuming the constancy of labor supply. Then, Homburg (1990) showed that a Pareto-improving transition is possible for an open economy with endogenous labor supply. Finally, Breyer and Straub (1995) proved the existence of a Pareto-efficient transition for a closed economy, where labor is endogenously chosen. Although the authors documented the possibility of an efficient transition, they did not provide a policy mechanism.

Later, researchers have investigated different policies to find out whether their implementations allow for intergenerational-efficient pension reforms. The first bulk of papers considers different debt-tax policies for reforming a lump-sum benefit pension system. Raffelhuschen (1993) provides an efficient policy, which uses both a public debt issue and lump-sum taxes and transfers for compensating transition generations. Kotlikoff (1995) also provides the simulations of Pareto-improved transitions using a lump-sum redistributive authority mechanism for the compensation of transition generations. Brunner (1994, 1996) rejected the use of lump-sum taxes for compensating losers as hardly being implemented in an economy with heterogeneity inside one generation. However, Hire and Weber (1997) simulated an efficient transition without using lump-sum taxes. They demonstrated transitions implemented by debt issue to compensate for the first years of reform, and then consumption or income taxes for debt and interest repayments. Demmel and Keuschnigg (1999) proved

3 This paper does not consider the impact of capital growth on technology.
4 Oresag and Stiglitz (2001) (p. 24), for example, characterize the transition cost as unavoidable. "If the economy is dynamically efficient, one cannot improve the welfare of later generations without making intervening generations worse off." To support this point, the authors provide Breyer's (1989) argument for an open economy with exogenous labor supply, ignoring, however, the entire later literature.
the feasibility of a debt-financed Pareto-improving transition from a system with a lump-sum form of benefits, where the government gradually reduces payroll taxes.

The next group of papers investigated the feasibility of a Pareto-improving pension reform, when benefits are proportional to contributions. Fenge (1995) opened the discussion. He proved the Pareto-efficiency of a pay-as-you-go system in an open economy with elastic labor supply, where benefits are proportional to contributions. In other words, he demonstrated that a Pareto-improving transition could not be implemented by changing payroll tax rates and the implicit return of contribution rates, combined with debt-financing. Wrede (1998) extended this result for a three-period model, doubting the simulation results provided by Hirte and Weber (1997), where the authors simulated a Pareto-efficient transition for both extreme cases of linkage between benefits and contributions. However, Hirte and Weber's results do not contradict Fenge's findings, since they investigate a closed economy. They also compared two transition paths starting from steady states, distinguished by the degree of benefits to compensations linkage, and found that transition from the lump-sum scheme can more easily be implemented. Kotlikoff (1995) confirmed this result.

Valdes-Prieto (1997) proposed a new interesting transition policy, implemented by a gradual increase in the rate on pension contributions and a simultaneous reduction in the use of payroll tax rate. This policy allows the government to carry out an efficient pension reform in an open economy with constant labor supply and stochastic income.

Belan, Michel and Pestieau (1998) introduced a saving subsidy form of pension benefits, in addition to generally considered lump-sum benefits and proportional to contribution forms for the closed economy. Assuming a constant labor supply and lump-sum benefits in an initial steady state, the authors showed that switching to saving subsidies improves the welfare of both current and future generations, which allows a pension reform to be implemented in one period. (See also Wigger (1998)).

Feldstein (1996) proposed the policy mechanism, which is the perfect opposite to debt-financed strategy. In particular, he suggested increasing the payroll tax rate and investing additional revenue, which would allow the government to accumulate the necessary for a transition fund. Although such a policy does not in itself allow the implementation of a Pareto-improving transition itself, it is an excellent addition to the policy schemes, which do not use debt but generate an additional welfare gain of the first generation in a transition, for example when a government increases the degree of linkage between benefits and contributions. I will refer to such a policy as "prefunding".

### 4.2 Classification and extension

To better understand all the results on the existence of a Pareto-improving pension reform, I classify the literature by four characteristics of reformed economies and transition policies. In other words, the existence of a Pareto-efficient pension reform depends on the answers to the following questions:

1) Is the economy open or closed?

2) Is labor supply elastic to the payroll tax?

3) What is the form of the pension benefits: lump-sum or proportional to the contribution?

4) Which policy mechanisms are implemented during the transition?

Table 1 summarizes the existing literature according to my classification.

---

5This article considers deterministic income only. For more information about stochastic income and Social Security see De Nardi, Imrohoroglu and Sargent (1999) and Storesletten, Telmer and Yaron (1999).
In the remainder of this subsection, I will discuss peculiarities associated with the reform of each type of economy in the proposed classification. In this review, I will use the results from previous literature as well as my own conclusions based on the simulations described in subsection 5.

4.2.1 Open vs. closed economy.

The differences between open and closed economies are important for investigating a pension system for the following reasons. First, in the case of a closed economy, savings equal investments and, therefore, contribute as an important intergenerational pecuniary externality. In an open economy, a pay-as-you-go system can simply be considered as tax on pension contributions, where the tax rate equals the difference between the interest rate and the wage fund growth. (See Valders-Prieto, 1997). Those taxes are thrown away without being used for financing government expenditures. In a closed economy, a pay-as-you-go system does not only create this wasteful tax on pension contributions, but also reduces the capital stock. Second, in the case of a closed economy, a capital provision, made by the old, has an impact on the wage rate earned by the young which, in turn, causes the changes in payroll tax revenue. Finally, government borrowing has more complicated consequences in a closed economy. For example, it does not only increase government liabilities, but also enlargers the interest rate, thereby causing the growth of savings and increase in welfare of lending generations. From the above, I can conclude that pay-as-you-go systems are less destructive in open economies, but a closed economy can more easily be reformed.

Thus, if labor supply is independent of pay-roll tax rate, a Pareto-improving transition from a pay-as-you-go system with lump-sum benefits exists in a closed economy and can be implemented by saving subsidizing (Belan, Michel and Pestieau (1998)). This is impossible in an open economy, however (Breyer (1989)). Moreover, a debt-financing Pareto-improving transition from a pay-as-you-go system with proportional benefits is possible for a closed economy but impossible for an open one, as proved by Fenge (1995).

Summarizing, I would like to emphasize that a Pareto-improving transition exists in a closed economy regardless of the elasticity of labor or the form of pension benefits. A Pareto-improving transition is also possible for the world consisted of many open and one closed economies. To complete it the general government can firstly reform the closed economy and then use the gains to reform open ones. Moreover, the terms "closed" and "small open" represent two extreme theoretical structures. Everything in the world is located somewhere in between, containing some degree of "closeness". In other words, national savings are more or less important for national investment as well as for the wage rates in any country. Thus, let me conclude that a Pareto-improving reform is possible and could be more easily implemented for the economy that is "more closed" in

---

Table 1. Classification of the existing literature

<table>
<thead>
<tr>
<th>References and comments</th>
<th>Type of economy</th>
<th>Initial benefits</th>
<th>Labor supply</th>
<th>Transition policy</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breyer 1989</td>
<td>Open</td>
<td>Lump sum</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Homburg 1990</td>
<td>Closed</td>
<td>Lump sum</td>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Breyer and Straub 1993</td>
<td></td>
<td>Proportional</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaffelhusen 1991</td>
<td></td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Krilhoff 1992</td>
<td></td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fenge 1995</td>
<td></td>
<td></td>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrede 1998</td>
<td></td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hirte and Weber 1997</td>
<td></td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demmel and Kranching 1999</td>
<td></td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belan, Michel and Pestieau 1998</td>
<td></td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6The result is "yes" if the corresponding paper proves the existence of or provides a simulation of a Pareto-improving pension reform. The result is "No" if the paper proves the impossibility of implementing a Pareto-improving reform of the corresponding pay-as-you-go system in a corresponding economy by using the corresponding transition policy.
an initial steady state. The last sentence should not be used as an argument for increasing capital controls, it just reflects the fact that pay-as-you-go systems create more harm in more closed economies.

4.2.2 The elasticity of labor supply.

Once a pay-as-you-go system is financed by labor income taxes, the elasticity of labor supply is important for both the existence and the duration of a Pareto-improving reform. Indeed, a pay-as-you-go system financed by payroll taxes in a deterministic open economy with exogenous labor supply is efficient, as shown by Breyer (1989). But tax on labor is not the most important source of the inefficiency in a closed economy: the bad pecuniary intergenerational externalities created by insufficient savings seem to be more important. Moreover, the degree of elasticity of labor supply plays an essential role in the choice of transition policy. Thus, Belan, Michel and Pestieau (1998) propose to reform a closed economy with constant labor in one period by substituting lump-sum pension benefits with saving subsidies. This is impossible for an economy with a large elasticity of labor supply and lump-sum benefits, but such an economy can be efficiently reformed by increasing the degree of linkage between benefits and contributions.

4.2.3 The degree of linkage between benefits and contributions.

The third dimension of my classification is related to the form of the initial pension system. If benefits are proportional to contributions, the negative impact of a payroll tax on labor supply is much lower than in the case of lump-sum pensions. For example, labor supply does not depend on the contribution rate in an economy with logarithmic preferences when benefits are proportional to contributions. The linkage between benefits and contributions is even more important for the existence of a Pareto-improving pension reform. Thus, for the open economy, Penge (1995) showed that an efficient transition is impossible if the benefits are proportional to the contributions, while its feasibility for the lump-sum form of benefits was proved by Homburg (1990). The general conclusion is that although pension systems with a lower linkage between benefits and contributions more significantly destroy the economy, they can more easily be reformed. In the real world, almost all pension systems have a lump-sum component, which allows a Pareto-improving reform even in a perfectly open economy.

4.2.4 The choice of transition policy.

The existence and speed of a Pareto-improving pension reform do not only depend on the initial structure of the economy, but also on the instruments used by a government during a transition. The existing literature considers several instruments: switching to another tax base, debt issue, an increase in the degree of linkage between benefits and contributions, the introduction of a saving subsidy. Below, I discuss all these measures in more detail.

The choice of tax base. According to Auerbach and Kotlikoff (1987), consumption tax is less destructive than income, capital income or wage income taxes, and Kotlikoff (1996) repeats this result. The authors do not set Pareto-efficiency as a goal in either of these papers. To define the consumption tax rate, they use a revenue-neutral condition. However, for the welfare-neutral tax, the government should increase the tax rate up to the amount that allows compensating the old for having paid their consumption taxes. Such a large rate has too large a destructive effect on the economy and, therefore, switching to consumption tax in the first period does not help find additional financial sources for an efficient transition. Nevertheless, it can be used in later stages to reach a better steady state in an economy where a government must finance expenditures non-related to social securities. In other words, the implementation of a pension reform constitutes the financing of a global Pareto-improving tax reform, when a more distorting income tax can be replaced by a less distorting consumption tax. It also allows the repayment of part of the government debt and reduces interest expenditure which, in turn, leads to a further reduction in the tax burden.

Increasing linkage between benefits and contributions. Increasing the degree of linkage between benefits and contributions has been widely proposed as an improvement of pension systems. This measure causes a significant improvement even in the first period of reform for two reasons. First, the growth of labor supply will exceed capital growth, thereby shifting the interest rate upward, which would make the lending generation better off. Second, although the growth in labor supply reduces wages, the total payroll is growing, which allows the government to collect additional revenue without changing the tax rate. Maintaining pension on the promised level, the government can invest additional revenue in order to generate means for further reform. This investment will also create a favorable pecuniary externality, providing the economy with a
larger capital stock. My simulations show that an efficient transition can be implemented by a pure increasing linkage between benefits and contributions if a government has the option to reduce promised pensions while maintaining utility levels. Otherwise, if the government must commit on pension promises, the pure policy of increasing in linkage between benefits and contribution may not allow a Pareto-improving transition. However a combination with prefunding makes such a reform feasible.

Saving subsidy in a closed economy. The policy of a saving subsidy seems to be the most efficient in a closed economy, since the main source of inefficiency in the current pay-as-you-go systems is a capital underaccumulation due to low returns on pension contributions. The saving subsidy form of pension benefits has been proposed relatively recently by Belan, Michel and Pestieau (1998). In a closed economy, a saving subsidy increases capital accumulation and significantly improves the welfare of working generations. A larger capital stock increases the wages of the next generation, which allows a reduction in the payroll tax rate without loss in tax revenue. Moreover, pensions paid in the form of saving subsidies create lower distortions for the labor market than lump-sum pensions, which will be show later in the example of logarithmic preference7. Belan, Michel and Pestieau (1998) show that the government can make the efficient reform of an economy with constant labor supply in one period by using savings subsidies instead of lump-sum pensions. I find that savings subsidies are not sufficient in themselves for an economy with endogenous labor supply, if the government has obligations in term of the size of pensions. Nevertheless, a savings subsidy in combination with government prefunding makes it possible to complete a Pareto-improving reform when labor is elastic to the tax rate. This is also possible for savings subsidies if government obligations are expressed in terms of welfare. A savings subsidy can improve the debt financing strategy for the reform of pension systems characterized by a proportional relation between benefits and contributions.

Government pre funding There is another transition policy which I will call government prefunding8. This policy is the exact opposite of debt-financing and is realized via additional government savings, allowing the generation of a sufficient budget surplus to finance a reform. This policy can only be Pareto-improving in combination with another policy, which does not include the running of additional debt and allows the generation of additional revenue in the first period of transition. An example of such policies is the increase in the share of the pension in proportion to contributions and savings subsidies. Those transition methods can be significantly improved by prefunding. To complete the prefunding policy in isolation, the government must raise a payroll tax or reduce expenditures, thereby breaking the Pareto-improving constraint9.

5 Technical framework

I conduct my research by experimenting with listed transition methods and applying them to different types of economies. For this purpose, I use a two period overlapping generations model.

Formula (1) describes a household's saving decision made under budget constraint (2).

\[
\max_{c_y, c_o} U_t = \ln(c_y) + \beta \ln(c_{o,t+1}) + \gamma \ln(1 - l_t); \quad (1)
\]

\[
\text{st. : } c_y(1 + t_c) + \frac{c_{o,t+1}(1 + t_{c,t+1})}{(1 + r_{t+1} + \Phi_{t+1})} = \frac{w_t l_t (1 - t_a z_t) + \Omega_{t+1}(1 - t_a z_t)w_t l_t + P e_{t+1}}{1 + r_{t+1} + \Phi_{t+1}}. \quad (2)
\]

Here, \(c_y, c_o\) constitute the consumption of young and old households in period \(t\), respectively, and \(l_t\) is labor. Terms \(r\) and \(w\) represent the interest rate and the wage. \(\beta\) and \(\gamma\) are test parameters; in particular, \(\beta\) is a time discount factor, while \(\gamma\) indicates the intensity of household preferences for leisure relative to consumption. I consider a combination of three types of pension benefit forms: lump-sum, proportional to contributions, and proportional to savings; using the following notation:

7See formula (6) p. 9
8Although the authors propose to place the means in the Individual Retirement Accounts, these savings are mandatory, and the householders have no access to their accounts before their retirement, when accumulated resources become withdrawable according to the design of the government pension plan. In the simple framework of my model, this operation is equivalent to a government investment.
9In earlier papers (1995, 1996, 1997), Feldstein and Samwick propose an increase in a payroll tax rate, leaving pension obligations unchanged, while later on (1999, 2000), they propose to use the exogenous gain generated by the US economy. (See also Martin Feldstein "Don't Waste the Budget Surplus", The Wall Street Journal, November 4, 1997 http://www.nber.org/feldstein/wj10797.html) This source is not generally available.
5. TECHNICAL FRAMEWORK

$P_{en_{t+1}}$ - a lump-sum pension benefit;

$\Omega_{t+1}$ - the coefficient of return on after tax labor income;\(^{10}\)

$\Phi_{t+1}$ - a savings subsidy rate.

Notations $tax_t$ and $tc_t$ represent payroll and consumption tax rates, which the government can set up to finance its expenditures.

The solution to problems (1), (2) is given by equations (3)-(7).

\[
\frac{1}{\lambda} = \frac{w_t(1-tax_t)\{1 + r_{t+1} + \Phi_{t+1} + \Omega_{t+1} + Pen_{t+1}\}}{(1 + \beta + \gamma)\{1 + r_{t+1} + \Phi_{t+1}\}},
\]

\[
c_{pt} = \frac{1}{\lambda(1 + tc_{t+1})},
\]

\[
c_{ot_{t+1}} = \frac{\beta(1 + r_{t+1} + \Phi_{t+1})}{\lambda(1 + tc_{t+1})},
\]

\[
l_t = \frac{1 + \beta}{1 + \beta + \gamma} \frac{\gamma Pen_{t+1}}{(1 + \beta + \gamma)\{1 + r_{t+1} + \Phi_{t+1} + \Omega_{t+1}\} w_t(1-tax_t),}
\]

\[
s_t = \frac{w_t(1-tax_t)l_t - c_{pt}(1 + tc_t)}{1 + \beta + \gamma} \left[ \beta - \frac{\Omega_{t+1}}{(1 + r_{t+1} + \Phi_{t+1})} \right] - \frac{Pen_{t+1}}{(1 + \beta + \gamma)\{1 + r_{t+1} + \Phi_{t+1} + \Omega_{t+1}\}} - \frac{1}{(1 + r_{t+1} + \Phi_{t+1})}.
\]

Considering a closed economy, I assume that the production function has a Cobb-Douglas form (38), relating output to capital $k_t$, and labor $l_t$.

\[Y_t = k_t^\alpha l_t^{1-\alpha}.
\]

Profit maximization by representative firms in the economy and competitiveness imply the following expressions for wage and capital price:

\[w_t = (1 - \alpha)k_t^\alpha l_t^{-\alpha},
\]

\[r_t = \alpha k_t^{\alpha-1} l_t^{-\alpha},
\]

where $k_{t+1}$ is capital per worker in the next period, defined as current workers' savings $s_t$ minus government debt in formula (41) in the assumption of zero population growth

\[k_{t+1} = s_t - debt_t,
\]

where $debt_{t+1}$ equals government debt.

In an open economy, the wage and the interest rate are constant.

For simplicity, I assume that the only government function is to support a pension system. Therefore, the government has the next budget constraint:

\[debt_{t+1} + T_t = P_t + (1 + r_t) debt_{t-1},
\]

\[T_t = tax_t w_t l_t + c_{pt}(c_{ot} + c_{pt}),
\]

\[P_t = Pen_t + l_{t-1} w_{t-1}(1-tax_{t-1}) \Omega_t + \Phi_t s_t,
\]

where $T_t$ and $P_t$ represent total tax revenues and total payments to the old, respectively. Equations (1)- (14) provide a complete description of the simulated model.

In this framework, the government sets policy parameters ($Pen_t$, $\Omega_t$, $\Phi_t$, $tax_t$, $tc_t$, $debt_t$) in any time period $t$ satisfying budget constraint (42). The Pareto-improving condition implies that for any time period $t$,

\[U_t \geq U_{ss},
\]

\(^{10}\) $\Omega_{t+1} = \Omega_{t+1} \frac{\gamma}{1 - \gamma}$, where $\Omega_{t+1}$ is the coefficient of returns to contribution, which should be equal to GDP growth in steady state in a zero debt economy, or to 1 according to my simple specification. I choose this notation for simplicity.
where $U_t$ is a lifetime utility of the individual born at period $t$, while $U_{ss}$ is the utility level of the individual living in an initial steady state. This puts additional restrictions on government policy.

I use a variation of the solution method described in Auerbach and Kotlikoff (1987). Following these authors, I make the calculation in three stages: (1) solving for the initial steady state, (2) solving for the final steady state, (3) choosing the parameterization and solving for the transition. In stage (3), I choose policy parameters so that restriction (15) is satisfied. I made my simulations assuming that in period 100, the economy has already converged to new steady states. This is more than enough; for comparison, the Auerbach and Kotlikoff (1987) simulation model provides the economy with 150 years to reach a new steady state in a fifty-five period OLG model.

For the welfare analysis, I use a wealth equivalent defined as the proportion by which a householder living in an initial steady state needs to increase his consumption in every period, in order to reach the lifetime utility value of currently young households. Expression (16) formalizes the definition

$$\text{wel}_t = \exp \left( \frac{U_t - U_{ss}}{1 + \beta} \right) - 1,$$

where $U_{ss}$ is equal to the householder's utility in the initial steady state.

### 6 Heterogeneity is not an obstacle in implementing a Pareto-improving transition

The main goal of this section is to convince the readers that heterogeneity inside one generation is not an obstacle to completing an efficient pension reform. The first time heterogeneity was provided as an argument against a pension reform for an open economy in Brunner (1994) and for a closed one in Brunner (1996). In particular, the author introduced two types of householders with identical preferences but different productivity levels and concluded that if there is a great difference in earning abilities, the Pareto-improving transition is impossible. The main reason is that to maintain the same level of utility as in the initial steady state, the government might have to use different tax-transfer rates for different individuals. In the case of imperfect information, it might be difficult to define what tax rate must be associated with a particular agent. An incorrect tax level, in turn, might make one householder worse off. I will refer to this argument as a "coordination problem". Subsection 6.1 disputes this by providing Pareto-improving transitions in the economy with two agents distinguished by productivity levels or preference parameters. In those transitions, the government only uses flat payroll taxes to compensate transition generations.

Brunner (1994, 1996) correctly rejected lump-sum taxation as a policy instrument, but his main finding was based on incorrect assumptions. For example, Brunner (1996) assumed the impossibility of a Pareto-improving debt-financed transition for a closed economy. This assumption was theoretically refuted by Demmel and Keuschnigg (1999) by demonstrating that a transition to a funded system might be made welfare improving for all generations through a proper use of public debt and proportional payroll taxes to compensate any potential losers. Before that, Hirte and Weber (1997) simulated Pareto-improving debt-financed transition without using lump-sum tax. Despite the incorrectness of some assumptions, Brunner's argument is very important and must be carefully investigated. The importance of the argument of intra-cohort diversity is supported by resent research attention. Thus, Kotlikoff (1996) uses a Smetters and Walliser modification of the Auerbach-Kotlikoff Dynamic Life-Cycle Model, which incorporates twelve householder groups distinguished by productivity level. Discount rate heterogeneity is considered by Samwick (1998), where the author proposes a transition tax/benefit menu, investigating the dependence of a chosen option on a householder's time discount rate. Those papers do not pretend to achieve a Pareto-efficiency, however.

Brunner's argument is very often referred to as an obstacle in the implementation of a pension reform. This is not correct and must be rejected.

The second argument against pension reforms in a heterogeneous economy relates to the redistributive or the insurance function of a pay-as-you-go system. The simple abolishment of social security in this case can lead to the welfare loss of poor householders or general worthiness in case of very risky income and in the absence of another insurance mechanism. These arguments were provided by Fenge and Schwager (1995) and have been used by other researchers. Thus, Conesa and Krueger (1999) find "that the role of a pay as you go social security system as a partial insurance and redistributive device significantly reduces political support for a transition to an economy with a fully funded system". Storesletten, Telmer and Yaron (1998) attach...
importance to the risk-sharing aspect of social security and the fundamental trade-off of distortion versus risk sharing. I find these arguments to be weak since redistributive or insurance mechanisms can be supported without running inefficient pay-as-you-go systems, which I propose to replace by taxes and transfers within one generation. This policy allows the government to maintain its redistributive or insurance function, while it helps us get rid of the inefficiencies related to indirect taxing of the pension contribution. Subsection 6.2 discusses this in more detail.

6.0.5 Technical modifications for the simulation of a heterogeneous model

Here, I list the additional formulas needed to simulate a two-agent model. The average efficient labor supply equals weighted average of efficient labor provided by all households

\[ l_t = \sum q^i a^i l^i_t. \]

Here, \( a^i \) is the productivity level of an individual from group \( i = 1, 2 \); \( q^i \) is a proportion of the corresponding group; while \( l^i_t \) is the working time of the representative of group \( i \). In other words, \( l^i_t \) is a solution of problems (1) and (2) where the wage is substituted by the efficient wage

\[
\begin{align*}
\max_{c^i_{yt}, c^{i+1}_{yt}, l^i_t} & \quad U^i_t = \ln(c^i_{yt}) + \beta^i \ln(c^{i+1}_{yt+1}) + \gamma^i \ln(1 - l^i_t); \\
\text{st.} & \quad c^i_{yt} + c^{i+1}_{yt+1} = w_t a^i l^i_t (1 - \text{tax}_t) + \frac{Pen^i_{t+1}}{1 + r^{i+1}_t}. 
\end{align*}
\]

(18) (19)

The following equation defines savings

\[ s^i_t = w_t a^i (1 - \text{tax}_t) l^i_t - c^i_{yt}. \]

The savings per efficient worker equal the weighted average savings

\[ s_t = \sum q^i s^i_t. \]

The formula defined output, the wage per efficient unit, the interest rate and the capital supply (38)-(41) remain unchanged. Equations (13)-(14) should be slightly modified

\[
\begin{align*}
T_t & = \sum q^i \text{tax}_t w_t a^i l^i_t; \\
P_t & = \sum q^i \text{Pen}^i_t. 
\end{align*}
\]

(20) (21)

Further, I assume that society splits equally between two groups of individuals, or that \( q^i = 1/2 \). A complete sensitive analysis has been done for all simulations provided in this section.

6.1 Differences in productivity or preferences still allow the Pareto-improving pension reform to be complete by using homogeneous tax rates.

In the present section, I will disprove the "coordination" argument by providing numerical simulations for economies with householders distinguished not only by productivity level but also by preferences. For this purpose, I will consider a closed economy with endogenous labor supply, where an initial pay-as-you-go system has a lump-sum pension benefit, but different groups of individuals receive different pensions proportional to the payroll tax payed by the total group. By using this construction, I avoid redistribution inside one generation, but capture all distortions related to low linkage between benefits and contributions. Such a pension system appears when the government pays a benefit that is not proportional to individual contribution, but dependent on the contributions made by the part of society with the same occupation, professional qualification, regional location, union or other memberships. The elements of such pension systems were present in the Former Soviet Union where benefits were independent of individual efforts, but differed across regions and occupations. According to Edvards (1996), the pre-reform Chilean system had the same characteristics. Considering such a pension system allows me to separate the effects related to the coordination problem of a pension reform from the ones related to redistribution.
6.1.1 The role of differences in productivity

In this subsection, I argue that at least for the logarithmic utility function, the difference in productivity levels is not important since the economy might be replaced by an equivalent representative consumer model, providing the same outcome of capital and labor supply as well as the welfare dynamics. Then, I discuss simulation results for the additive constant relative risk-averse (CRRA) preferences.

The solution to problems (18) and (19) is represented by formulas (22)-(25)

\[ c^i_{pt} = \frac{\rho c^i_{pt-1} + w_t a^i(1 - taX_t)}{1 + \beta + \gamma}; \]

\[ c^i_{pt+1} = c^i_{pt} \beta(1 + r_t+1); \]

\[ l^i_t = 1 - \frac{c^i_{pt} \gamma}{w_t a^i(1 - taX_t)}; \]

\[ s^i_t = w_t a^i(1 - taX_t) l^i_t - c^i_{pt}. \]

As already mentioned, I will consider an economy with a zero intergenerational redistributive effect. In my economy, householders pay a proportional labor income tax, and receive different lump-sum benefits, proportional to the contribution of the total group with the same productivity. Therefore, pensions are proportional to productivity levels and, as a result, both householders work the same number of hours and consume proportionally to the productivity level or to efficient labor supply. Formally, if \((e, l)\) is the solution to the problem for parameter \(a_i = 1\), then \((a_i e, l)\) will be the solution to the general problem. Moreover, individuals' utilities are satisfied to relation (26):

\[ U_t^2 = U_t^1 + (1 + \beta) \ln \left( \frac{a^i}{a^2} \right), \] (26)

which is true for all periods, \(t\). This means that the transition policy has the same impact on individuals' welfare dynamics. Since efficient labor supply and savings are proportional to productivity levels, the problem is equivalent to that of a representative agent with a productivity level equal to the weighted sum of the productivity of heterogeneous agents

\[ a = \sum q^i a^i, \]

where \(q^i\) is a proportion of individuals with productivity \(a^i\).

Therefore, if preferences are logarithmic, the introduction of different productivity levels does not principally change the model, even if the differences are very large.

The difference in productivity is important for other preferences, but nevertheless, a Pareto-efficient debt-financed transition exists. I consider an efficient transition for the CRRA utility function, given in formula (27)

\[ U_t = \frac{c^i_{pt}^{1-1/\rho} - 1}{1 - 1/\rho} + \beta \frac{c^i_{pt+1}^{1-1/\rho} - 1}{1 - 1/\rho} + \gamma \frac{(1 - l_t)^{1-1/\rho} - 1}{1 - 1/\rho} \] (27)

where \(\rho > 0\) is a parameter determining how responsive individual labor supply is to changes in the wage rate. If \(\rho > 1\), a householder works more when wages increase, while if \(\rho < 1\), a growing wage causes increasing leisure. Logarithmic preference is a special case when \(\rho = 1\), and the choice of labor supply does not depend on the wage.

Tables 2 and 3 provide efficient transition paths for \(\rho = 1.67\) and 0.5, respectively.
Table 2.
Different Productivity Levels and CRRA Preference ($\rho > 1$)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\theta$</th>
<th>productivity</th>
<th>$\rho_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.01</td>
<td>0.001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>debt</th>
<th>tax rate</th>
<th>welfare1</th>
<th>welfare2</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$r$</th>
<th>$w$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\rho_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 2 is consistent with Kotlikoff's (1995) paper, where the author states that the welfare gains for the poorer income group exceed those of the richer group. Moreover, a low-productive class starts to benefit earlier than high-productive ones. This occurs as soon as the net tax wage reaches an initial level, which must be reduced in the first periods of debt-financed transition, following the reduction in capital. A decreasing wage is compensated by payroll tax reductions; these two factors together create a positive impact on the growth of labor supply. These results held for all $\rho > 1$.

Table 3 presents an efficient transition for a small $\rho = 0.5$. As expected, employment is lower in a new steady state but households benefit from a higher capital stock and wages. The gain from a transition is larger for the more productive group.

Table 3.
Different Productivity Levels and CRRA Preference ($\rho = 0.5$)
6.1.2 Difference in preferences should not be considered a barrier to a pension reform

Motivated by Brunner's idea on heterogeneity in each generation, I investigate an economy with two types of individuals distinguished by preferences. Simulating numerically an economy where two householders have different time discount and leisure to consumption preferences, I find that Pareto-improving debt-financed transitions exist. In my experiment, I set different $\beta$ ($\beta_1 = 0.7$; $\beta_2 = 0.5$), different $\gamma$ ($\gamma_1 = 2.5$; $\gamma_2 = 5$) and different $\rho$ ($\rho_1 = 0.7$, $\rho_2 = 1.11$). The results are presented in tables 4, 5 and 6, respectively.

### Table 4.

**Different Preferences for Leisure Relative to Consumption**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>alpha</th>
<th>beta</th>
<th>gamma1</th>
<th>gamma2</th>
<th>theta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>2.5</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$r$</th>
<th>$w$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$\pi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 4 presents an efficient transition path for an economy with two individuals distinguished by the parameter of the preference for leisure to consumption. A welfare analysis shows that the householder with a larger preference for leisure starts to enjoy a pension reform earlier and his gain in a new steady state is larger.

### Table 5.

**Different Time Discount Rates**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>alpha</th>
<th>beta1</th>
<th>beta2</th>
<th>gamma</th>
<th>theta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.8</td>
<td>0.5</td>
<td>2.5</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t$</th>
<th>debt</th>
<th>tax rate</th>
<th>welfare1</th>
<th>welfare2</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$r$</th>
<th>$w$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$\pi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 5 provides a simulation result for an economy where householders have different time discount rates. At the beginning of a transition, when interest rates should necessarily increase due to government borrowing, an individual with a smaller time discount rate will have a somewhat larger welfare gain, while later, when the interest rate falls to a lower level as compared to that in an initial steady state, a householder representing
the other type will obtain more in terms of the welfare measure presented by formula (16).

Table 6.
Different CRRA parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CRRA</th>
<th>Other</th>
<th>CRRA</th>
<th>Other</th>
<th>CRRA</th>
<th>Other</th>
<th>CRRA</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>rho 0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>rho 0.7</td>
<td>0.7</td>
<td>1.1</td>
<td>0.7</td>
<td>1.1</td>
<td>0.7</td>
<td>1.1</td>
<td>0.7</td>
<td>1.1</td>
</tr>
<tr>
<td>1</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6 provides an efficient debt financed transition for an economy with two types of householders, distinguished by the concavity of the preference function. At the beginning of the transition, when the after tax wage is lower as compared to an initial steady state, a government should take more care about a householder whose labor supply is more elastic with respect to wage, which means that $\rho$ is larger. At this stage, representatives for the other type are better off, while later on, when the net of tax wage grows, the individual with larger $\rho$ gains more.

6.2 Redistributive and insurance role of a pay-as-you-go system

When linkages between contributions and benefits are low, a pay-as-you-go system receives additional functions of redistributive and insurance mechanisms. Therefore, the simple abolishment of social security in this case can lead to a welfare loss for poor householders. This argument against pension reform is provided by Fenge and Schwager (1995), where the authors pay attention to the redistributive effect of a pay-as-you-go system across one generation. Similar arguments were used by other researchers until recently. Nevertheless, a Pareto-improvement is possible if a pay-as-you-go system is replaced by other mechanisms maintaining a sufficient level of redistribution. Such replacement would maintain the utility welfare level of the poor and insure agents against an idiosyncratic productivity shock. However, an inefficient tax on pension contributions related to interest rate dominance over population growth would be eliminated. The best policy would be to tax the oldest and pay transfers to the youngest. In this way, the government would exploit the difference between the population growth rate and the rate on capital returns. If such a policy is not feasible for political reasons, the government can still achieve a welfare progress for everybody by intragenerational redistribution; i.e. a transition to an economy with a progressive wage income tax or a proportional payroll tax and lump-sum transfers within one generation.

To consider the described policy, I must make some modifications. Thus, equation (19) is modified to (28)

\[
\text{st. } c_{t+1}^0 + \frac{c_{t+1}^0}{1 + r_{t+1}} = w_t a_t I_t^r (1 - (t_{pen_t} + t_{tt})) + \text{transfer}_t^r + \frac{\text{Pen}_{t+1}^r}{1 + r_{t+1}}. \tag{28}
\]

In a simulated economy, payroll taxes are used for three purposes. The government pays pensions to the old, transfers to the young and execute debt service. I assume that the government sets payroll taxes at the rate $t_{pen_t}$ and issues new debt for expenditures related to the pension system and debt running, while $tt_t$ is the share of payroll only used for transfers to the young.

Under these assumptions, equations (42)-(14) should be replaced by (29)-(31)

\[
\text{Pen}_t^r = \frac{1}{2} (\text{debt}_t + \text{Pen}_t^r \text{Tax}_t - (1 + r_t) \text{debt}_{t-1}). \tag{29}
\]
The equations for saving should also be modified

\[ s_i^t = w_i a (1 - (t_{pen} + t_{tt}) + t_{transfer}^t) - c_{yt}. \]

7 Concluding summary

The purpose of this paper is to demonstrate that Pareto-improving transitions from pay-as-you-go to fully funded pension systems nearly always exist. Based on the review of existent literature as well as my own simulations, I find that such a transition is possible for all reasonable assumptions on reformed economies. I also discuss some details related to the execution of such transitions. In particular, the policy used in transition should be chosen depending on the economic characteristics of the initial pension system. Thus, for the closed economy saving subsidizing is more efficient when labor supply is exogenous, while a strengthening of the linkage between benefits and contributions is preferable when the coefficient of elasticity between leisure and consumption is large. The answer to the existence question depends drastically on the policy instruments chosen by the government.

My second finding is that intragenerational heterogeneity in productivity or preferences does not prevent the execution of a Pareto-improving transition to a fully funded system with redistribution within one generation.
Bibliography


Designing Optimal Pareto-Improving Pension Reforms: A More Distorted Economy Can be Reformed Faster

Abstract

In this article, I investigate the optimal Pareto-improving debt-financed transition from pay-as-you-go to fully funded pension systems. In particular, I examine the relationship between key parameter values and the time necessary for a Pareto-improving transition. My finding is that a more distorted economy can be reformed faster. This result gives an additional explanation for the success of the Chilean reform, where the initial pay-as-you-go system was the largest and, at the same time, the most distorting.

Keywords: Pension reform, Pareto-improving transition, The duration of optimal reform.

JEL-codes: H55

8 Introductions

In recent years, many countries have experienced an increasing demographic pressure on their pay-as-you-go pension systems. Decreasing fertility rates and increasing life expectancy have increased the old age dependency ratio, tendency that will continue for at least 50 years. The current systems will force governments to either increase taxes or reduce the benefits for future generations, which will decrease the welfare levels as compared to the ones guaranteed in the present pension plans. Thus, Feldstein and Samwick (2000) predict that the U.S. "current pure pay-as-you-go system can only maintain the benefits specified in current law by raising the payroll tax rate from 12.4 percent today to more than 17 percent by 2037."

Moreover, current pay-as-you-go systems are very inefficient at present time, because the rates of capital return exceed the growth of payroll, as demonstrated by Samuelson (1975). The inefficiency arises from the indirect taxes on pension contributions imposed by pay-as-you-go systems. Even in an open economy with constant labor supply, the loss from running a pay as you go system is equivalent to taxation at a rate equal to the difference between interest rate and population growth. (See Valders-Prieto, 1997). Those taxes are thrown away without being used for financing government expenditures. In an economy with elastic labor, this inefficiency is fueled by distortions in the labor market, due to social security payroll taxes. According to Feldstein (1996), the relevant deadweight loss equals approximately one percent of GDP in the U.S. The damage from a pay-as-you-go system is much larger in closed than in open economies, because the reduction in savings causes capital underaccumulation and a decline in pre-tax wages. Transition to a fully funded system might also cause larger economic growth in the presence of endogenous technical progress, as discussed in Wiedmer (1996).

To avoid pension crises and improve their economy, some countries have implemented fundamental reforms of their pension systems, with different degree of success. Others continue to wait. This delay might be...
explained by the uncertainty of the short-run consequences of reform, and the high evaluation of the cost of transition. The transition dilemma is formulated as follows. On the one hand, the old generations must be compensated for their contributions to a pay-as-you-go system and therefore, the young generations must pay "twice": taxed for pensions for the old, they must also contribute to their own pension accumulation fund. On the other hand, a reduction in payroll taxes leads to higher wages and larger capital supply, thereby providing financing, which may be sufficient for a gradual transition with an appropriate design. Despite the fact that a Pareto-improving transition was proved to exist, the optimal policy parameters and the duration of such a reform have not yet been studied.

This paper applies optimization tools to the transition problem. In particular, I compare the duration of Pareto-improving reforms in two different countries distinguished by either the initial size of pay-as-you-go systems, or other key economic parameters. The results help us understand why some countries were more successful in implementing pension reforms than others.

I conduct my research using numerical simulations of a two-period overlapping generations model for a closed economy. For comparison, I use the duration of the optimal transition, defined as a debt-financed pension reform which maximizes the government objective function under the restriction of Pareto-improvement. The most interesting result is that a more distorted economy can be reformed faster. This finding gives an additional explanation as to why the Chilean pension reform was so extremely successful, even though the administrative costs were extraordinarily high.14

Besides studying the relationship between the size of a pay-as-you-go system and minimum reform duration, I examine the impact of other economic parameters related to preferences and technology on the duration of a Pareto-improving pension reform. My research is of great importance for the policy issue. First, the results encourage countries with a large pay-as-you-go system to start a reform, alarming the other countries about the larger difficulties they might face during a transition. This should not avert these countries from starting a reform, however, it should only give some warning about the length of the period before the gains are realized.

The rest of the paper is structured as follows. Section 8 provides a technical framework and discusses technical problems related to a comparison of the transition in two different economies. Section 10 investigates how the parameters of the initial economy might explain the duration of the shortest Pareto-improving pension reform. Section 11 discusses the Chilean case in more detail, providing some facts in support of the Chilean pension system being the most distorting, and its pension reform Pareto-improving. Section 12 concludes.

9 Technical description

In this subsection, I discuss the techniques used to obtain the results reported in subsection 10. I simulate transitions from pay-as-you-go to fully-funded pension systems in a standard two-period overlapping generations model of closed economies. Furthermore, I assume that the initial pension system has a lump-sum form of benefits. In particular, I consider Pareto-improving debt-financed transitions, where the government only sets two policy parameters, debt stock and payroll tax rate, for each time period.

9.1 General technical complications

Numerical simulation is an effective method of disproving false statements by providing readers with counterexamples. For example, it is easy to reject the statement that an efficient debt-financial transition does not exist in a closed economy by providing a simulation of a Pareto-improving pension reform. However, comparing two different policies, like two alternative ways of taxation, or the success of the particular policy applied to economies distinguished by economic characteristics, such as preferences or technology, is a much more difficult task for numerical simulation techniques. The difficulties arise from the large numbers of policy parameters, as well as very complicated non-linear restrictions. The existing literature usually provides two simulation results which are then compared. Such an approach has a significant lack of rigidity: it might be that the authors use almost optimal parameters for the first policy, while worse than average ones for the second. Such a choice might lead to a conclusion opposite to the true one, obtained when two of the best sets of parameters are compared.

Finding the optimal parameters is a very complicated problem, even for modern computers. For example, a simple debt-financed transition requires two sets of policy parameters: payroll tax rates and the value of public

9. TECHNICAL DESCRIPTION

debt which, under the assumption of 6 periods of active policy in transition\textsuperscript{15} and 15 period of convergence, require the solution of a 12-parameter optimization problem with at least 30 nonlinear resections. Researchers must thus find six optimal values for the government debt stock and payroll tax rates, respectively, accounting for the positiveness of the pension benefits and the sufficient level of welfare for all transitional generations.

9.2 Solution method

I use a variation of the solution method described in Auerbach and Kotlikoff (1987). Following these authors, I proceed with the calculation in three stages: (1) solving for the initial steady state, (2) solving for the final steady state and (3) choosing the parametrization and solving for the transition.

Stage (3) is the most important and worth to be explained in detail. I choose policy parameters maximizing the government objective function under given restrictions. Running a large number of simulations, I find that a higher value of the objective function can be obtained when the period of active transition policy, or the period when either the debt per capita or the payroll tax rate differs from the ones in a final steady state, is shorter. Therefore, I first make an assumption about the duration of this period and then run over all the parameters, thereby maximizing the objective function. Another helpful finding, supported by intuition, is that a small reduction in welfare gains for generations living at the beginning of the transition implies a much larger improvement in the objective function and a decrease in the necessary duration of active transition policy. Setting zero welfare gains for the \( n \) first periods of transition, I obtain the indirect expression for the corresponding \( n \) debt\textsuperscript{16} values for given \( n + 1 \) tax rates.

9.2.1 Objective function

To compare two policies, I need to define a government objective function. Generally, a government is assumed to live infinitely, maximizing the discounted utilities of all householders. (Atkeson, Chari, Kehoe (1999))

\[
U_g = \sum \beta^{t}_{gov} (U_t - U_0).
\]  

(32)

Formula (32) represents an objective function, where \( U_t \) and \( U_0 \) are life-time utilities for individuals born in period \( t \) and the initial steady state, respectively. \( \beta_{gov} \) is a government discount rate. If \( \beta_{gov} \) is low, the final steady state might be characterized by large debt and a tax revenue spent on interest payment. The higher is \( \beta_{gov} \), the more the government accounts for future generations, and the smaller would its debt be in a final steady state. Theoretically, I cannot set \( \beta^{t}_{gov} = 1 \), since in this case, formula (32) will represent a non-convergent row. Still, this allows me to compare the values of objective function calculated on the two paths converging to the same final steady states. It also has an interesting interpretation as "a perfect democracy", where the interests of future generations are considered to be of the same importance as those of the current generation. Moreover, maximizing such an objective function, I will find the fastest transition to the best steady state. Once I assume a transition to be completed after 100 time periods, the government objective function in my numerical simulations has the form described in formula (33)

\[
U_g = \sum^{100} (U_t - U_0).
\]  

(33)

I use this formula, assuming that in period 100, the economy has already converged to a new steady state. To ensure this, I restrict the government not to implement the policy different from one in a new steady state longer than during the first 20 periods. I obtain the assumption of the sufficiency of 100 time periods for convergence by running the simulations. My experiments show that 15 periods of unchanged policy are enough for convergence in a framework of a two-period overlapping generations model; or, in other words, the differences between the values of economic variables in the fifteenth period and in a final steady state are comparable to machine null. For comparison, Auerbach and Kotlikoff’s (1987) simulation model provides the economy with 150 years to reach a new steady in a fifty-five-period OLG model.

9.2.2 Policy and unknown parameters

I will consider the simplest transition policy, a debt-financed pension reform, when only the payroll tax is available. I assume that a government actively operates in the first 10 periods of transition, and that the convergence to a new steady state is not longer than 100 time periods. This means that I must find 9 parameters of a government debt stock \( (\text{debt}_1, \ldots, \text{debt}_9) \) and 10 parameters of a payroll tax rate \( (\text{tax}_1, \ldots) \).

\textsuperscript{15}I define the period when the policy parameters differ from the ones in a final steady state as "a period of active transition"

\textsuperscript{16}I choose debt because the expression of this parameter is easy and guarantees a fast convergence.
INITIAL DISTORTION AND DURATION

It might be interesting to investigate the best transition allowing consumption tax, saving subsidies, and a linkage between benefits and contribution during transition, but I leave this exercise as a topic for future research.

9.2.3 Restrictions

Since I have very complicated, non-linear restrictions, I decide to use a penalty functions method, described in Karmanov (1989).

Pareto improvement I would like to consider the only intergenerational efficient transitions. A pension reform should not hurt anybody, which means that life-time utility should not be less than one for generations in an initial steady state. For each generation, I calculate a loss function as in formula (34)

\[ V_t = \min(0, U_t - U_0). \] (34)

Then, I sum up the losses, defining the first penalty function (35)

\[ V_\text{pen} = \sum_{t=1}^{100} V_t. \] (35)

Non-negative pension benefit I restrict a policy set by claiming non-negativity of the pension benefits. Although redistributing from the old to the young leads to Pareto-improvement in a dynamically efficient economy\(^{17}\), a negative pension is politically unpopular. Another reason for this restriction is that a negative lump-sum pension looks like a perfect non-distortion lump-sum tax, hardly achieved in a real world.

10 The results

In this subsection, I will report the result of the research on how the parameter values are related to transition speed. This investigation should help policy makers compare their economy with others where pension reforms have already been started, encouraging or warning about higher difficulties or the longer duration of active transition.

For my research, I use a simple two-period overlapping generations model with logarithmic preferences. The lifetime utility of an individual born in period \(t\) is represented in formula (36)

\[ U_t = \log c_y + \beta \log c_{o,t+1} + \gamma \log (1 - l_t), \] (36)

where, \(c_y\), \(c_o\) constitute the consumption of young and old households in period \(t\), respectively, and \(l_t\) is labor. A household makes his consumption decision by maximizing lifetime utility under budget constraint (37)

\[ \text{st. : } c_y + \frac{c_{o,t+1}}{(1 + r_{t+1})} = w_t l_t (1 - taz_t) + \frac{Pen_{t+1}}{1 + r_{t+1}}, \] (37)

where \(r_t\) and \(w_t\) are interest rate and wage, while \(tax_t\) and \(Pen_t\) are payroll tax rate and lump-sum pension benefit, respectively. I assume that the production function has a Cobb-Douglass form (38), relating output to capital \(k_t\), and labor \(l_t\)

\[ Y_t = k_t^{\alpha} l_t^{1-\alpha}. \] (38)

Profit maximization by representative firms in the economy and competitiveness imply the following expressions for wage and capital price:

\[ w_t = (1 - \alpha) k_t^{\alpha-1} l_t^{-\alpha}, \] (39)

\[ r_t = \alpha k_t^{-\alpha-1} l_t^{\alpha}, \] (40)

\(^{17}\)This can partly explain a bequest institution which, in the framework of a simple OLG model, can be considered as a tax on the old, serving as a transfer to the young.
where $k_{t+1}$ is capital per worker in the next period, defined as current workers' savings $s_t$ minus government debt in formula (41) in the assumption of zero population growth

$$k_{t+1} = s_t - debt_t,$$

(41)

where $debt_{t+1}$ equals government debt.

For simplicity, I assume that the only government functions are to support a pension system and serve the debt. Therefore, the government has the budget constraint (42)

$$debt_t + tax_t\nu_t = Pen_t + (1 + \tau_t)debt_{t-1}.$$  

(42)

10.0.4 Benchmark

My benchmark economy has the following parameter values:

$$Tax_0 = 0.124, \quad \beta = 0.6, \quad \alpha = 0.3, \quad \gamma = 1.5.$$  

I choose the initial payroll tax rate, $Tax_0$, as estimated by Feldstein (1995) in the US. To calculate $\beta$, I use a 1.5% annual discount rate as in Auerbach and Kotlikoff (1987) and the assumption that one time period equals 35 years. I take the intensity of the household preference for leisure relative to consumption, $\gamma$, from Auerbach and Kotlikoff (1987). The share of capital income $\alpha$ is assumed to equal 2.5 in Auerbach and Kotlikoff (1987) and 0.4 in Hansen and Prescott (1998). The relations between the minimum duration of the active transition period and the parameter values are robust to sensitive analysis.

In a benchmark economy, I obtain a Pareto-improving path with an active transition during the period $t = 5$.

10.0.5 Time discount rate

The term $\beta$ indicates the degree to which, everything else equal, a householder would prefer more leisure and consumption when young rather than old. If $\beta$ is smaller, on the one hand, people would save less and put more strict constraints on government borrowing but, on the other hand, a government would need to borrow less to compensate for the pension fund reduction when cutting a payroll tax rate. The second effect dominates. Decreasing $\beta$ to $\beta = 0.3$, I find the possible transition in 4 periods: $t = 4$, the first period tax reduction being smaller in this case.

**Capital intensity.** The term $\alpha$ is the measure of the intensity of use of capital in production. If $\alpha$ is smaller, we have a larger distortion caused by lower labor supply, but a slower capital adjustment when this distortion is eliminated. The second effect dominates. Increasing capital intensity $\alpha = 0.6$, I find that the transition might be faster. The new transition period is $t = 4$. For $\alpha = 0.1$, I find that the optimal transition has 6 periods. It is interesting that in the case of low capital intensity, the optimal transition policy is to sharply reduce a payroll tax rate in the first and second periods and then let it remain stable, or even increase in order to repay the debt which might exceed half the initial capital stock. Conversely, in the case of a large $\alpha$, the first period tax reduction must be very modest.

Although $0.3$ is a standard assumption on the value of capital intensity in advanced countries, it varies across industries and years and therefore can also differ across countries.

**Elasticity of leisure.** Shifting parameter $\gamma$ upwards would mean increasing the elasticity of leisure to consumption in preference functions. If labor supply is more elastic, a reduction of the payroll tax will lead to a larger gain, due to a higher reaction in employment. For $\gamma = 12$, I receive a shorter efficient transition path, $t = 3$. Moreover, higher elasticity of leisure to consumption allows me to reduce the payroll tax rate more sharply.

**The initial size of a pay-as-you-go system** It is interesting that a lower level of initial payroll tax rate does not mean an easier transition. My simulations results show that a larger initial pay-as-you-go system will allow a faster reform. This relationship seems to be robust to changes in parametrization. The fact that the larger pay-as-you-go system can be reformed faster explains why the pension reform in Chile has been so successful, despite the cost of fund management being extremely high.
11 An Alternative Explanation of the Chilean Success

This section focuses on the Chilean experience and defends an alternative explanation of the success of the Chilean pension reform in relation to the large size of the initial pension system.

Some authors connect the Chilean success with political economy, supposing that it might be easier to conduct a pension reform in a country with dictatorship than in a democracy. For example, Ghilarducci and Liebana (2000) describe the role of Argentinian Unions in resisting the Chilean model. Brooks and James (2001) underline the negative impact of political diversity: "The structure of a political party system, particularly the number of parties it contains, affects the range of interests represented in the reform process and, as a result, the extent of bargaining necessary to build a legislative majority." Even the Chilean dictatorship faced strong resistance when deciding to implement the reform. Thus, Edwards (1996) mentions that the implementation had to be postponed for almost a full year due to staff opposition. "The General himself, however, was not fully successful in convincing his military colleagues. The armed forces did not join the new system - an option not available to any other group in the country". This strong resistance, however, should rather be explained by myopia related to the novelty of the project: Chile was the first country daring to conduct a pension reform. Even the reform designers were not able to foresee all benefits from the reform. According to Edwards, the Chilean authorities barely referred to the potential effects of a new system on domestic savings in explaining the reform.

In spite of strong social resistance, the Chilean reform was Pareto-improving. The government has not only committed to its pension obligations to current pensioners, but it has also issued recognition bonds for those worker having already made contributions. Those bonds yielded a 4% return in real terms. Although this yield was below the ex-post market return, it significantly exceeded the Chilean population growth rate, equal to 1.5%. Therefore, all participants received much larger benefits than they were guaranteed by the pre-reform pension system and a pension system crisis caused by further increasing the old age dependency ratio was successfully avoided. Edwards also writes about the increase in reform popularity immediately after the reform was started, which indicates that a majority has benefited from the reform and that the reform policy would even be chosen in a democratic society, if the citizens were better informed on the future reform realization.

Although it is easier to start the transition in a dictatorship, the success of the Chilean reform should be explained by other reasons, which are shown in this paper. I found that the larger the size of the initial pay-as-you-go system, the faster a Pareto-improving transition can be implemented. The Chilean experience supports my conclusion, because Chile is the most successful country as concerns the implementation of pension reform, but it also had one of the largest initial pay-as-you-go systems. In the early 1970s, the implicit debt of the system exceeded 100 percent of GDP, and social security spending was more than 18 percent of GDP. According to Kritzer (1996), the social security deficit rose to about 25 percent of GDP in 1975, while Diamond (1993) claims that a contribution rate including health was in the range of 51-59% in this period. For comparison, the average pension spending to the GDP ratio equals 9.2% in OECD countries, 8.0% in Eastern Europe and only 2.0% in Latin America and the Caribbean (World Bank 1994).

12 Concluding remarks

In conclusion, I should underline that a more distorted economy can be reformed faster, when intergenerational Pareto-improvement is claimed. This is true if the larger distortions are caused by:

(i) an economy being "more closed" and the shortage of savings leading to a lower level of capital;
(ii) benefits being less connected to contributions and a pay-as-you-go system causes lower labor supply;
(iii) leisure being more elastic to consumption and the payroll tax being more destructive to labor;
(iv) production technology being more capital intensive and capital underaccumulation thereby causes larger damage to output;
(v) the payroll tax rate being higher in an initial steady state;
(vi) the time discount rate being lower and the tax paid when young has a larger value relative to the pension received when old.

18 The share of the Chilean population older than 65 has grown from 5.5% in 1980 to 6.9% in 1998.
However, if the reason for the larger distortion is a worse demographic situation, the less distorted economy can be reformed faster. In other words, an economy with a lower old-age dependency rate can be reformed faster.

These results are important for policy designers: they encourage countries with a more distorting pay-as-you-go system to start a reform, but alarm other countries about the larger difficulties they might face during a transition. However, my finding should not discourage those countries from starting a reform, it should only give some warning about the longer period before the realization of the gains from a Pareto-improving transition.
Bibliography


Part II

An Economy with Clever Tax Avoidance Providers
The Importance of Income Distribution for the Price of the Tax Avoidance Service

Abstract

The traditional avoidance literature undeservedly neglects tax base distribution as a factor affecting the avoidance price, and generally assumed to be equal to the avoidance cost. In reality, avoidance providers are usually either high-skilled specialists or insiders. The strong collusion thus, naturally seems to be an assumption of the behavior of avoidance providers. Within such a framework, income distribution, which forms an avoidance demand together with tax codes, plays a very essential roll for the outcome of both avoidance price and quantity. My article models an economy with a monopolistic avoidance provider and imperfect information, and illustrates possible consequences of tax base changes. The paper examines the relationship between inequality and a government's ability to collect tax revenue, and also considers the possible outcome of a tax base broadening. Furthermore, it provides an additional explanation for the secession decision.

Journal of Economic Literature Classification: C72; D31; D42; D43; D69; D82; E61; E65; F15; G28; G29; H21; H24; H25; H31; H32; J61; K34; L12; O17

Keywords: Tax avoidance, optimal taxation, income distribution, endogenous prices, inequality, tax base broadening, secession.

13 Introduction

Tax avoidance is a problem of great importance for many countries. Taxes do not collect themselves and taxpayers may try to avoid their legal liabilities\(^{19}\). Thus, according to Slemrod and Yitzhaki (2000), the U.S. government spends about 10% of the total tax revenue on tax enforcement. Furthermore, the Internal Revenue Service estimates that about 17% of income tax are not paid. The figure for most other countries is probably even higher. The government's inability to collect a sufficient tax revenue was the main reason for the Russian financial crisis in 1998. The tax avoidance practice forces the government to increase the tax burden on non-avoiding tax payers, which also leads to an increase in deadweight loss. For example, Feldstein (1995) estimates that due to tax avoidance, the deadweight loss from income tax in the U.S. is more than ten times larger than would otherwise be the case. Since rich individuals usually use avoidance practice, the government levies heavier taxes on the poor who have larger marginal utility. This also slows down the development

\(^{14}\)I wish to thank Lars Ljungqvist for his invaluable guidance. I have benefited from comments given by Kjetil Storesletten, who discussed this paper in my licentiate seminar at the Stockholm School of Economics. I am also indebted to Jonathan Heathcote, and Guido Friebel for helpful comments and suggestions. Special thanks go to Christina Lonnblad for her fine editing. Financial support from the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged. Remaining errors are my own responsibility.

\(^{19}\)In this paper, I use the term "avoidance" to denote all transactions motivated by the desire not to pay tax. They might be illegal, but non-revealable or non-punishable due to imperfections in the legal system.
of small business and economic growth. Moreover, avoidance behavior may cause shifts in the real economy, thereby affecting occupational choice, human capital investment and labor supply20.

My particular research has been motivated by observing the Russian government’s attempts to improve tax collection in 1996-1997. The government broadened the tax bases, introduced new sources as subject to tax, and finally, organized a new emergency committee to enforce tax collection. The efforts gave no results, however, which motivated me to design a model where the tax avoidance supply is adjusted to changes in tax avoidance demand. In particular, I investigate an economy where the avoidance provider endogenously defines the price for the tax avoidance service. An avoidance provider’s ability to react not only to a new tax code, but also to the changes in the tax base distribution, creates an additional problem for the government. My paper introduces the distribution component of price for the tax avoidance services, for the first time.

Studying the existing avoidance literature, I find that it neglects the tax base distribution as a factor affecting the avoidance price. The authors assume either perfect competition between the provider’s of the tax avoidance service or the ability to reveal the clients’ income by an avoidance provider. In the first case, the price a householder pays for the avoidance service equals the cost, while in the second case, the avoidance provider receives an ability for the perfect price discrimination. In both cases, aggregate demand for the tax avoidance service has no impact on the avoidance price formation. Cross and Shaw (1992), Atkinson and Stiglitz (1980) and Slemrod (1998) are representative examples of the first class of the literature. In Cross and Shaw (1992), the tax avoider pays a fixed exogenous price. In Atkinson and Stiglitz (1980), the avoidance price depends on the avoidance amount and the government’s effort to enforce tax collection. Slemrod (1998) proposes that initially richer individuals pay a lower price for the same amount of income saved from taxation. Hindriks, Keen and Muthoo’s (1999) paper belongs to the second class of the literature. This paper provides the model where a tax inspector audits the true individual’s income, but can be bribed by the householder to misreport the audit result to the government. In this model, the tax inspector is considered to be a monopolistic avoidance provider with perfect monitoring ability. Although, the price he sets for providing the tax evasion service is defined endogenously, it does not depend on the aggregate demand for tax avoidance. Demand becomes important for price setting after the introduction of an imperfect information structure, which reduces the possibility to monitor and price discriminate.

Some degree of market power seems to be a reasonable assumption for avoidance provision. In reality, the tax avoidance service is provided by high-skilled intermediaries, such as accountants or legal consultants, who could also be insiders with access to specific information. In developing countries, tax avoidance services are often provided by the firms enjoying especially favorable legal treatment, or by corrupted officials. Therefore, avoidance provision should not be considered as a business easily entered into, and a high degree of monopolistic power is rather a rational assumption. Together with the assumption of imperfect information the latter gives avoidance demand a more important role in price setting. In turn, the demand for tax avoidance depends on tax base distribution to at least the same extent as on the tax code set by the government. Hence, changes in the tax base distribution should not be neglected in a tax avoiding economy, when the government decides to broaden a tax base or implement a new immigration policy, or considers regional issues.

To show certain possible consequences of various policies related to the changes in the tax base distribution, I make my model as illustrative as possible. The model assumes monopolistic power of the agency, or a strong collusion among the institutions providing tax avoidance services. Another assumption is imperfect information: both the government and the agency know the distribution of a tax base but not the amount owned by any particular individual. The avoidance service is indivisible, the agency either helps hide the entire income, or none at all. Moreover, the householder’s income is not revealed to the agency. Therefore, the only option for the agency is to set a fixed price for its service. I make such strong assumptions to simplify the model, the main purpose of which is to illustrate the possible consequences of avoidance price reactions to changes in demand. All conclusion can hold for an economy characterized by a lower degree of market power and some ability to price discriminate.

Due to the prevalence of tax avoidance, my model has a very wide range of applications, and it considers several of those which I find to be among the most interesting. In particular, I analyze the relation between inequality and a government’s ability to collect tax revenue. It might be that maximum revenue grows with inequality, while inefficiency due to avoidance purchase falls. Hence, this paper contributes to the dilemma of a trade-off between the advantages of equity and the disadvantages related to the unpleasantness that will occur due to a government’s disability to collect taxes when inequality is small. Even when the government can collect a target revenue, a higher degree of equality will increase inefficiency due to higher avoidance payments. The government can only operate effectively in an economy with a small inequality if an agency’s cost, or the cost of the access to the avoidance service (the avoidance cost), is high enough. Moreover, the

20 For a detailed overview of the main problems related to tax compliance see Andreoni, Erard and Feinstein (1998). For examples of popular ways of carrying out tax avoidance in developed countries, see Slemrod (1996).
cost of additional tax revenue collection in terms of the funds flowed to the avoidance provider decreases with inequality.

Another important application is the analysis of a broadening of the tax base. On the one hand, it might be an efficient measure of the increase in the tax avoidance cost when the avoidance technology reclassifies a particular business to a class subject to tax exemption. In this case, the elimination of tax exemption would force the agency to spend additional time and money on developing and realizing new tax avoidance schemes: an effect which has most likely been observed after the implementation of the Tax Reform Act (TRA 86). On the other hand, a tax base broadening might also lead to a reduction in revenue collection, due to the change in the tax base distribution and the ensuing adjustment of the avoidance price.

I also investigate the issues of unification and secession, since these acts lead to crucial changes in the shape of income distributions. Secession is favorable for the tax avoidance providers because it gives them an additional possibility for price discrimination. Maximizing the profit tax, avoidance providers may set the lower tax avoidance cost for the poorer country, which creates the possibility of tax avoidance for the regional elite and provides an important reason for secession of the poorer region. On the other hand, the elite in the wealthier region could expect less progressiveness and therefore smaller avoidance expenditures after the secession. Despite the desistance of secession incentives, the government operates more efficiently in a unified economy: collecting the same tax revenue it can set a tax code allowing a larger average after tax/avoidance income.

The paper is organized as follows. Section 14 presents a general model and provides technical solutions for a specific case of a risk neutral householder’s utility function. Section 15 investigates the relationship between inequality and tax collection. Section 16 extends the set-up of the model by introducing an agency cost or the cost of access to the avoidance service. Section 17 considers a wide range of applications including tax base broadening, immigration and unification issues. Section 12 concludes.

14 General Model

There are three players in the modeled economy: householders, the government and the agency providing tax avoidance services. There is a continuum of householders, normalized to 1. Each householder exogenously obtains some economic item referred to as "income", which the government taxes in order to collect its revenue. \( F(x) \) - is a cumulative distribution of \( x \), known to everybody, while the particular \( x \) belonging to each householder is private information. The government sets up a tax code on the level \( \tau(x) \) in order to finance its target expenditure, \( \mathcal{Y} \). If the government is not able to collect \( \mathcal{Y} \), it simply maximizes a revenue\(^{21}\). Otherwise, it collects revenue \( \mathcal{Y} \) by setting a tax code maximizing the social welfare function \( W \) equal to householders’ average utility. The agency helps a householder avoid a tax by hiding his personal income \( x \), and it has a monopoly on tax avoidance services. Since the agency is unable to monitor individual income, before or after providing assistance, it sets a fixed price \( T \) for its service. The shape of income distribution \( F(x) \) is known to the agency.

14.1 Defence of the Main Assumptions

My model underlines, for the first time, the importance of income distribution for the price of the tax avoidance service. Although, this idea is very simple and actually reflects the fact that price should depend on demand, I would like to discuss why this is applicable to the tax avoidance market in particular. The previous tax avoidance literature ignores this relation by either assuming that the tax avoidance market is perfectly competitive\(^{22}\) or giving the avoidance provider the ability to costlessly monitor household income\(^{23}\). In both setups, the price a household pays for the tax avoidance service does not depend on the neighbors tax duties, or in other words, on aggregate demand. I consider both these assumptions to be too strong and they should thus be weakened. To show certain possible consequences of various policies related to changes in the tax base distribution, I make my model as illustrative as possible. It assumes monopolistic power of the agency, or a strong collusion among the institutions providing tax avoidance services. Another assumption is the impossibility to make any price discrimination inside one region. I make such strong assumptions to\(^{21}\)

\(^{21}\) It is a question of what actions a Government should take if it is not able to collect the target revenue. Further, I will show that maximizing revenue collection might be very inefficient, due to the high level of the marginal rate of avoidance expenditure. This assumption is motivated by Russian events. The Russian parliament has set up inexecutable revenue targets during 1996-1999 and then, the executive authorities have tried to do their best in collecting taxes. The attempt to implement expenditure sequestering in 1997 was unsuccessful.

\(^{22}\) See, for example, Slemrod (1998), where the price of the tax avoidance service is assumed to equal the cost.

\(^{23}\) See Hindriks, Keen and Muthoo’s (1999), where a tax inspector audits the true individual’s income, but can be bribed to misreport the audit result to the government.
simplify the model, the main purpose of which is to illustrate the possible consequences of avoidance price reactions to changes in demand.

14.1.1 The Positive Market Power of Tax Avoidance Providers

Some degree of market power seems to be a reasonable assumption for avoidance provision. In reality, the tax avoidance service is provided by high-skilled intermediaries, such as accountants or legal consultants, who could also be insiders with access to specific information. In developing countries, tax avoidance services are often provided by firms enjoying especially favorable legal treatment, or corrupted officials. Therefore, avoidance provision should not be considered as a business easily entered into, and should not be competitive.

Another argument supporting the assumption of market power can be taken from some observations of price fluctuations, which may be reflected in the wage of highly skilled professionals in consulting services, such as lawyers and accountants. If the market were perfectly competitive, a decrease in demand would cause a decrease in supply, but not a reduction in price. The consequences of the Russian financial crisis in 1998 have turned out to be the opposite. Thus, the fall in the wages of professional accountants was much larger than the reduction in average wages in both real terms and the dollar equivalent.

14.1.2 Impossibility of Price Discrimination

Price discrimination is possible only if the provided services cannot be resold and if the provider can distinguish its consumers by some sign correlated with demand. For perfect discrimination, this correlation should also be perfect. In this particular case, the agency should monitor the individual's income in order to reveal her demand for tax avoidance.

Privacy Right The assumption of not revealing one's income to the agency is probably too strong, but it is supported by evidence. For instance, some regions (Panama, the Bahamas, British Virgin Islands, etc.), declared as off-shore zones, set up special rules for non-resident international business companies. According to these regulations, a firm registered in these regions pays no regional taxes and is not required to present an annual report, it only pays the registration and fixed annual fees. Why does the agency provide such services without demanding a statement of income? One of the reasons might be confidentiality: by providing financial privacy, the agency can attract a larger number of clients. Once the agency can reveal a household's income, it has the option to blackmail the householder by threatening to report both his income and his avoidance practice to the government. This option may distract a large number of potential clients.

The Cost of Monitoring Moreover, monitoring is always costly. This special case may include the cost of time and the cost of inefficient matching. For example, it may be more efficient to cut off potential customers by setting a high price than spending time auditing a poor consumer. Moreover, it might be the case, that the tax avoidance service is provided before the income can be observed by an avoidance provider. Unrestricted tax grants for particular firms belongs to this class. Those reasons explain the fixed bribe fees for many services in Russia. The same motives clarify why the government sometimes prefers licence and lumpsum taxes. For example, in Yugoslavia, the profit tax for a restaurant owner is substituted by a business fee, proportional to the number of tables, and taxi-drivers pay lump-sum tax.

The Possibility to Resell Some tax avoidance technologies can not be protected from resale. For this reason the housing sellers in tax heaves cannot discriminate their clients, for example.

Although I modeled the simple and extreme case, a tax base distribution would remain important for the formation of the avoidance price if my assumptions about either the monopolistic power or the disability of the agency to discriminate clients were weaker.

14.2 Formalization

Let me consider the case where the agency has a zero cost per client and access to its service is free. After the government has set up a tax rule $r(x)$, the agency chooses a price $T$ for its service. Then, each individual decides either to declare his income $x$ and pay $T(x)$ to the government or use an avoidance service, paying $T$ to the agency and nothing to the government. Since an individual maximizes his after tax/avoidance earnings, he prefers to pay tax if $T(x) < T$.

If $F(x)$ is a continuous function, strictly-monotonic on the set $\{x, 0 < F(x) < 1\}$, and householders are either risk-averse or risk-neutral, then $T(x)$ can be selected from non-decreasing functions without loss of generality. This is formulated in more detail in the next proposition:
Proposition 1 If $F(x)$ is a continuous function, strictly monotonic on the set $\{x, 0 < F(x) < 1\}$ and householders are risk-averse or risk-neutral,

then $\forall \tau(.)$, the government can choose another tax code $\tau_1(.)$ such that:

1) $\tau_1(x)$ is non-decreasing;
2) the government will collect the same amount of revenue;
3) the welfare function, which is equal to the householder's average utility function, is not smaller than under the initial tax code $W(\tau(.)) \leq W(\tau_1(.))$;

The formal proof is given in Appendix 19. The intuition is very simple: if a richer individual pays less tax than a particular householder, the government can simply exchange their tax duties, which would lead to the same revenue collection and the same tax avoidance, but might improve the social objective function.

Based on proposition 1, I will further consider the only non-decreasing tax codes for continuous monotonic distributions. When the government and the agency have made their decision, all householders are separated into three groups: those preferring to be taxed, those who are indifferent and tax avoiders. For simplicity, I assume that indifferent individuals would avoid taxation (since the agency has the option of decreasing its price by an infinitely small amount, thereby attracting all indifferent individuals).

Let $\bar{x}$ be a solution of equation (86)

$$\bar{x} = \inf_{x} \{\tau(x) = T\}. \quad (43)$$

Then, taxpayers whose income is larger than or equal to $\bar{x}$ will use the agency's service, while the others will prefer to declare their income and pay tax $\tau(x)$ to the government. Further, I will call $\bar{x}$ an income "breakdown". Thus, an agency's revenue is calculated by formula (87)

$$T(1 - F(\bar{x})) = \tau(\bar{x})(1 - F(\bar{x})). \quad (44)$$

The agency maximizes its profit $m$ by solving problem (88):

$$m \overset{d}{=} \max \{\tau(x)(1 - F(x))\}; \quad (45)$$

where $\tau(x)$ is a non-decreasing function, defining a tax level. There should exist some $\bar{x}$ so that $\lim_{\bar{x} \to \bar{x}} \tau(x)(1 - F(x)) = m$. Solving this problem, the government would set price $T = \tau(\bar{x}) = \frac{m}{1 - F(\bar{x})}$.

The solution of the agency's problem is illustrated in figure 21, where a thin line represents an agency's isoprofit curve $\left(\frac{m}{1 - F(\bar{x})}\right)$, expressing the largest possible profit $m$ under the given tax code $\tau(x)$, drawn as a solid line. Income breakdown, $\tilde{x}$, is defined as the maximum of a set of incomes, where the best available agency's isoprofit curve equals the tax code $\tau(\tilde{x})$.

Once $\bar{x}$ has been chosen, government revenue is calculated by formula (89):

$$\int_{\bar{x}}^{\bar{x}} \tau(x)dF(x) = g. \quad (46)$$

The government needs to collect a target revenue $g$ setting $\tau(x)$, which maximizes householders' average utility, depending on the after-tax/avoidance income.

Generally, the problem can be reformulated in the following way:

$$\max_{\tau(.)} W(\tau(.)) = \int_{0}^{\bar{x}} u(x - \tau(x))dF(x) + \int_{\bar{x}}^{1} u(x - T)dF(x); \quad (47)$$

$$\bar{x} = \arg \max \{\tau(\bar{x})(1 - F(\bar{x}))\}; \quad (48)$$

$$\tau(\tilde{x}) = T; \quad (49)$$

$$\int_{0}^{\bar{x}} \tau(x)dF(x) = g; \quad (50)$$

$$\tau(x) \leq x \text{ for any } x \in [0, 1], \quad (51)$$

\footnote{To simplify the presentation, I assume the agency to be government friendly and in case of indifference, the agency chooses the income "breakdown" that is the best for the government. Otherwise, the government can enforce this by slightly reducing a tax code.}
where formula (90) represents the government objective function. Householders whose income $x$ is less than $\hat{x}$ pay tax $\tau(x)$ consuming $x - \tau(x)$, while richer tax payers purchase a tax avoidance service paying price $T$ and consuming $x - T$ good. Expressions (91-94) represent the budget constraints. Formulas (91) and (92) describe the mechanism through which the agency sets its price, while equation (93) expresses the necessity to collect a certain amount of government revenue, and condition (94) shows that the tax level cannot be higher than income.

I am particularly interested in finding an upper limit for the government’s ability to collect revenue. The maximum tax revenue $G$ can be found in equations (52-54) and does not depend on householders’ preferences.

\[
G = \max_{\tau(.)} g(\tau(.)) = \int_0^{\hat{x}} \tau(x)dF(x); \quad (52)
\]
\[
st. \quad \hat{x} = \arg \max_{x} \tau(x)(1 - F(x)); \quad (53)
\]
\[
\tau(x) \leq x, \text{ for any } x \in [0,1] \quad (54)
\]

### 14.3 Risk Neutrality

In this subsection, I will consider a risk neutral utility function, $u(x) = x$. Since this assumption does not only simplify the model but also allows me to catch the effects non-related to risk aversion, it is worth considering as an extreme and interesting case.

If the government sets $\tau(x)$ to maximize the average utility function, which is proportional to after-tax income, formula (90) can be simplified as (95)

\[
\max_{\tau(.)} W(x) = \int_0^{\hat{x}} (x - \tau(x))dF(x) + \int_{\hat{x}}^1 (x - T)dF(x) = \int_0^1 x dF(x) - \tau - T(1 - F(\hat{x})). \quad (55)
\]

Expression (55) shows that maximizing the social welfare function is equivalent to minimizing agency revenue. Therefore, problem (90) can be rewritten as problem (96)
14. GENERAL MODEL

\[ \min_{\tau(.)} T(1 - F(\tau)) \]  
(56)

while constraints (91-94) remain unchanged.

14.4 Solution Under Different Restrictions

This section provides the solution to problems (96, 91 - 94).

14.4.1 No Restrictions

It might be easier to consider a dual problem, where the government first decides how large a profit, \( m \), it wants to give to the agency, and then maximizes its revenue choosing the optimal tax code \( \tau(X, m) \) from the set of tax codes, thereby providing the agency with revenue \( m \).

\[ \tau(., m) = \arg\max_{\tau(., m)} g(\tau) \]  
(57)

Solving equation (97), I construct \( g(m) = g(\tau(., m)) \). For any level of \( g \), I find the minimum value of \( m \), such that \( g(m) = g \). Those \( m \), \( \tau(., m) \), \( \bar{z} \) would represent the solution to an original problem (96, 91 - 94).

Now, I will describe the solution to a dual problem in more detail. According to conditions (91) and (92), an agency's isoprofit curve is represented by the expression \( \frac{m}{1 - F(\bar{z})} \), where \( m \) is an agency's profit. Accounting for restriction (94), the government should thus set up \( \tau(x) = \min(x, \frac{m}{1 - F(\bar{z})}) \) (see Figure 1). The agency is indifferent to which point \( x \) to choose on its isoprofit curve, while the government prefers the largest \( \bar{z} \).

As a widely applied example, let me consider a case when the isoprofit curve does not cross a 45 degree line more than twice.\(^{25}\) Let \( (x_1, x_2) \) be all solutions of equation \( x = \frac{m}{1 - F(\bar{z})} \) and \( x_1 < x_2 \). Then, the highest value of the government revenue equals \( g(m) = \int_{x_1}^{x_2} xdF(x) + \int_{x_1}^{x_2} \frac{m}{1 - F(\bar{z})} dF(x) \); and it might be simplified as formula (98).

\[ g(m) = \int_{0}^{x_2} xdF(x) + m \ln \left( \frac{1 - F(x_1)}{1 - F(x_2)} \right) \]  
(59)

Figure 22 represents solution (98) graphically.

Therefore, the optimal tax code in the assumption of risk neutrality is \( \tau(x) = \min(x, \frac{m}{1 - F(\bar{z})}) \); where \( m(\bar{z}) = \min_{g(m)=g} m \).

Additional restrictions do not principally change the algorithm, they only modify function \( g(m) \) by adding the corresponding restriction to formula (97).

Figure 22 shows that a non-restricted government would tax the poorest individuals very heavily. A tax code shaped like that in Figure 22 could be observed in Russia, where target revenue was too high in 1995 – 1999, when the Russian income tax code had five different brackets. Although formally progressive, the frequent usage of wage and pension arrears could be interpreted as a heavier tax for the poor.

14.4.2 Progressiveness and Marginal Tax Restrictions

All the logics in this subsection is applicable under the assumption of convexity of agency's isoprofit curve. I will now consider the case when the government is forced to set a progressive tax for some political reason.

Under this restriction, the government should use a proportional tax for householders whose income is less than \( x_t \), where \( x_t \)\(^{26}\) is the income level characterized by equivalence between marginal and average tax rates.

\(^{25}\)This is true for a wide class of distribution functions. For example; if \( F'' + 2F' - F'F \geq 0 \),
(58)

the agency's isoprofit curve is convex and it either does not cross, tangents or crosses a 45 degree line at two points. Although an isoprofit curve is not convex for the lognormal distribution, it nevertheless crosses a 45 degree line at two points.

\(^{26}\)Here, I should mention an interesting property of \( x_t \), which would be a "breakdown" point for any proportional tax code independent in the tax rate. It means that if avoidance is costless, a change of the rate in the proportional tax code would not affect avoidance behavior, but the avoidance price, and the loss for the economy would be proportional to the tax rate and tax revenue.
For those people whose income is larger than \( x_t \), the tax level should be the same as in the previous case: 

\[
\tau(x) = \min(x, \frac{m}{1-F(x)})
\]

The value of \( x_t \) is shown in the next calculation. Let \( \alpha \) be the average tax rate for people whose income is less than \( x_t \),

\[
\alpha = \frac{m}{1-F(x_t)}
\]

At the same time, \( \alpha \) is equal to a marginal tax rate and presents a slope of the agency's isoprofit curve at point \( x_t \).

\[
\alpha = \frac{m}{(1-F(x_t))^2} * F'(x_t).
\]

Equations (60) and (61) define \( x_t \)

\[
x_t F'(x_t) = 1 - F(x_t).
\]

If a householder has the option to throw away his income, then the government has an additional restriction: the marginal tax rate should not exceed 1. Further, I will consider this restriction as given. Now, the breakdown point of income is equal to \( x_{mr} \), found by the condition that marginal tax is equal to one, in other words, \( x_{mr} \) is a point where the agency’s isoprofit curve has a 45 degree slope

\[
\frac{d}{dx} \left( \frac{m}{1-F(x_{mr})} \right) = \frac{mF'(x_{mr})}{(1-F(x_{mr}))^2} = 1.
\]

Given both restrictions, a government revenue is calculated as 

\[
g(m) = \frac{m}{x_t (1-F(x_t))} \int_0^{x_t} xdF(x) + \int_{x_t}^{x_{mr}} \frac{m}{1-F(x_{mr})} dF(x);
\]

more simply:

\[
g(m) = \frac{m}{x_t (1-F(x_t))} \int_0^{x_t} xdF(x) + m \ln \left( \frac{1-F(x_t)}{1-F(x_{mr})} \right). \tag{64}
\]

Figure 3 gives a graphical illustration.

Appendix 20 provides a detailed solution for a uniform distribution under different restrictions.

---

\(^{27}\) Although, such \( x_t \) may neither exist nor be unique in general, but all the further reasoning are true for the case of logarithmic distribution or when the agency’s isoprofit curve is convex on the interval \([x_1; x_2]\). The convexity of a cumulative distribution function on this interval is sufficient condition for the convexity of an isoprofit curve. This follows from condition (58).
15. INEQUALITY AND TAX REVENUE

14.5 A Discrete Case

In reality, the information about income distribution has never been collected with perfect precision. Income statistics separate society into about 20 groups, distinguished by their respective income levels. Such approximation gives a reason for considering a discrete income economy.

14.5.1 Formalization

There are $N$ income groups in an economy, where $x_i$, $\tau_i$, $n_i$ represent the income, the tax level and the size of group $i$, respectively.

Agency problem. Let the groups be sorted by a tax level. The optimal policy for the agency is to choose a price $T = \tau_{i^*}$.

$$
\tau^* = \max \left\{ i : \arg\max \left( \tau_i \cdot \sum_{j \geq i} n_j \right) \right\}. \tag{65}
$$

Government revenue. Formula (65) defines a government revenue calculated by expression (66)

$$
g = \sum_{j < i^*} n_j \tau_j. \tag{66}
$$

Distribution $F(x)$ is no longer continuous - and the assumptions of Proposition 1 are violated. It might be the case that the optimal tax code is not non-decreasing.

Example 1 "When a non-monotonic tax code must be implemented to increase a revenue".

An economy consists of 3 groups with sizes 4, 2 and 1 and income levels 0.3, 0.4 and 1. The maximal government revenue can be collected with tax levels 1/5, 1/7, and 1.

15 Inequality and Tax Revenue

In this section I investigate the relationship between inequality and tax revenue collection. First, I show that tax revenue as a percentage of total income grows with inequality in assumption of lognormal distribution if the government sets probabilistic tax code. Then I consider the economy where government is unable to collect target revenue and tries to collect as much as possible. Along the way, I show the inefficiency of such a policy: small decrease in target revenue compare to the maximum available allows the householders to save...
Stylized facts: US
before and after the Economic Recovery Tax Act 
imposed in 1982

Figure 4:

large income part from being paid for the tax avoidance services. Finally, I demonstrate that higher inequality lower "cost of funds". In other words, if inequality is bigger, increasing in target revenue leads to the lower increasing in householders' loses due to paying for tax avoidance services.

15.1 The Possibility of Positive Correlation Between Tax Revenue and Inequality 
Under Stable Tax Code.

First, let me provide stylized facts observed in the US economy.

Figure 4 shows a positive correlation between the Gini coefficient and tax revenue before and after the Economic Recovery Tax Act imposed in 1982. Such a relation might be explained by the willingness of the government to redistribute more when inequality grows. This explanation requires a positive correlation between inequality and non-interest government expenditure, which has been observed during 1972-1982. The relation has then been negative since the government has used revenue to reduce outstanding debt. My paper provides an additional explanation, showing that tax revenue as a percentage of GDP might grow with inequality when the tax code is not significantly changed. To show this, let me consider a flat tax rate in an economy with lognormal income distribution.

Figure 5 illustrates the growing government revenue, and the decreasing agency profit with inequality under the stable flat tax rate.

28The data for the Gini coefficient are taken from the United Nation University date base. http://www.wider.unu.edu/wiid/wiid.htm. I choose Family as the Reference Unit, and Brandolini 1998 as the source. The data represent the inequality of monetary income.

IMF: Government Financial Statistics is a source for budget indicators.
15.2 When The Government Maximizes Its Revenue

Now, I will show that the government could not survive in an economy with small inequality if tax avoidance is costless. This is intuitively understandable; whatever tax the government sets, the agency would ask for a slightly lower price $T$ for its service, and all householders would avoid paying the tax. In an egalitarian world with costless avoidance, the government should charge no tax at all, since householders would suffer from a loss of income equal to the tax level, but they would not benefit from the consumption of any public goods provided by the government, the revenue of which would be zero. If the economy is egalitarian and the agency has zero costs, all attempts by the government to increase the tax revenue would only lead to larger agency profits. Although the arguments I use for an egalitarian case can also be applied to an economy with perfect concentration, there are two particular distributions for which I find that the government is able to collect large revenues when the Gini index is huge, while tax collection remains negligible if inequality is small.

When an economy has a small inequality, the government can collect a small tax revenue, giving some profit to the agency. This profit depends on the shape of the distribution function $F(x)$ and the government target revenue, $\tilde{g}$. Figure 6 represents the "Agency profit/government revenue" graph for lognormal and $F(x) = x^r$ distributions with Gini=0.01.

If the government tries to collect as much revenue as possible, facing income distribution $F(x) = x^r$, the agency profit would be nearly 87% of total income. In the case of lognormal distribution, it would be quite low (3.8%). The maximum government revenue is small in both cases: 0.9% for power distribution and 1.8% for lognormal distribution (see figures 6 and 7). Figure 7 supports the idea that maximizing the collected revenue might be a very irrational policy, since the government could save half the national income from going to the agency by reducing the target tax revenue by less than 0.05%.

15.2.1 The Distribution $F(x) = x^r$ ("power" distribution)

This subsection will consider an economy with a cumulative distribution $F(x) = x^r$, normalized in such a way that the income of the richest person equals 1. Further, I will call this a "power" distribution. Parameter $\gamma$ is the measure of inequality for "power" distribution. The lower is $\gamma$, the larger is income concentration in the economy. If $\gamma = 0$, almost all individuals have zero income but the richest one has an income equal to 1, which means perfect concentration. If $\gamma = \infty$, almost all individuals have an income equal to 1, which means equality. The correspondent Gini coefficient is represented by formula (67).
Lognormal distribution, Gini=0.01

Figure 6:

Power distribution, Gini=0.01

Figure 7:
The government's ability to collect a tax grows with the Gini index. Solving the problem numerically, I found that the maximum amount the government can collect grows with inequality; see figure 8.

Inefficiency related to avoidance decreases with inequality. I have found that agency profit decreases with inequality, when the government maximizes revenue, as shown by figure 9. A progressiveness restriction increases agency profit if the target government revenue remains unchanged, and it also reduces the maximum amount the government is able to collect.

Householders' after tax/avoidance income is larger when the government is not restricted by progressiveness. The total amount that householders should pay both to the government and the agency is lower when the government is not restricted by progressiveness, as shown in figure 10.

\[
Gini = 1 - 2 \int_{0}^{1} L(n)dn = 1 - \frac{2}{2 + \frac{1}{\gamma}} = \frac{1}{1 + 2\gamma}. \quad (67)
\]
15.2.2 Lognormal Distribution

Lognormality is the most common assumption for income distribution and is therefore of interest. If \( x \) has a lognormal distribution, \( x = \exp(z) \), where \( z \) is normally distributed: \( z \sim N(\alpha, \sigma) \); then the Gini coefficient could be calculated in formula (68)

\[
Gini = 1 - 2 \int_{-\infty}^{\infty} N(0,1) = \int_{-\infty}^{\infty} N(0,1).
\]  

(68)

Figure 11 demonstrates the relationship between government revenue and agency profit for a lognormal distribution with \( Gini = 0.33 \).

When the government maximizes a revenue, its collection increases with inequality as in the case of "power" distribution, while agency profit has a principally different shape: it is not strictly decreasing, after achieving a maximum level when the Gini index is equal to 24%, it falls.

The total amounts collected from householders by both the agency and the government grow with inequality, but they are lower when the government has a progressiveness restriction. This is due to the fact that such a restriction raises agency profit by relatively small amounts, compared to the reduction in government revenue collection. Under lognormality, the agency receives less profit if the Gini index is smaller than under "power" distribution.
15. INEQUALITY AND TAX REVENUE

When the Government Can Collect More than Target.

When the government can collect more than its target, inefficiency related to avoidance falls drastically. Figures 13 and 14 show what happens when the government becomes able to collect a target revenue under some rate of inequality. The transition from the state when the government collects as much as possible to the situation when a target revenue could be achieved, dramatically decreases the profit of an agency, which is considered as a loss for the economy. The crucial turns to decreasing trends correspond to that transition point.

Figures 13 and 14 also show that it is not only important to know the Gini index, but also the form of distribution, in order to estimate the total loss for an economy when the government is claimed to set a progressive tax. In the case of "power" distribution, the loss is larger than in lognormal one. Moreover, higher inequality implies lower marginal cost of additional revenue. In other words, government’s decision to increase revenue is less profitable for the agency when inequality is high. Figure 15 demonstrates this.

To conclude, I want to repeat that a high level of inequality is profitable for the government and unprofitable for the agency. In a risk-neutral world, inequality thus makes society better off, because disparity allows the government to collect more revenue or give the agency less profit under the same revenue level.
INCOME DISTRIBUTION AND TAX AVOIDANCE

Figure 14:

Agency Profit: Power distribution

Gini index

Figure 15:

The growth of agency's profit when government increases a revenue target from 1% of GDP to 2% of GDP
16 When an Avoidance Supply is Costly

I will now extend my analysis by introducing a positive cost $c$ per individual to the agency's technology, which may also describe the costly access to an avoidance service. Following the logic of Section 14.4, it can be found that an agency's isoprofit curve is illustrated by formula (69):

$$\frac{m}{1 - F(x)} + c. \quad (69)$$

Another option: no inefficiency due to avoidance. If the avoidance cost, $c$, is relatively large compared to the target revenue $\bar{y}$, the government could set a tax level lower than the agency cost for all householders. In that case, an economy would operate without loss, since the agency could not make any profit. The maximum amount the government could collect, when giving nothing to the agency, corresponds to the tax code in formula (70):

$$\tau(x) = \begin{cases} x & \text{if } x < c \\ c & \text{if } x \geq c \end{cases}. \quad (70)$$

In this case, government revenue is calculated by formula (71):

$$g_c = \int_0^c x dF(x) + (1 - F(c))c. \quad (71)$$

If $g_c > \bar{y}$, the government does not need to give anything to the agency, while in case of a small cost, the government might prefer to collect more revenue by taxing somebody by more than the avoidance cost. In the latter case, the optimal tax code is presented in formula (72).

$$\tau(x) = \min(x, \frac{m}{1 - F(x)} + c) \quad (72)$$

Two alternative tax codes, given in formula (72) and formula (70), are illustrated in figure 16.

![Two tax codes when the cost is positive](image)

Figure 16:

The government might prefer the first choice only if area $A$, measured by $dF$, is larger than area $B$.

In a low-cost case, $B$ is smaller than $A$ and the shape of the tax code (72) looks very similar to a zero-cost case, but the government is able to collect more tax, giving the agency the same revenue, since the agency's isoprofit curve is more favorable for the government.

When an Avoidance Cost depends on Government enforcement. Following Atkinson and Stiglitz (?) and Slemrod (?), I will introduce a positive relation between an agency cost $c = c(e)$ and government expenditures on enforcement, $e$. Extending the government problem (90 - 94), the following formulas (73 - 75) are obtained:
\[
\max_{\tau(.), e} W(\tau(.)) = \int_0^\infty u(x - \tau(x))dF(x) + \int_0^1 u(x - T)dF(x); \quad \text{st :} \quad (73)
\]

\[
\int_0^\infty \tau(x)dF(x) = \bar{y} + e; \quad (74)
\]

\[
\bar{x} = \left\{ \begin{array}{l}
\arg\max [\tau(x) - c(e)](1 - F(x)); \text{ if } \exists x, \tau(x) > c(e)
\\
1, \text{ otherwise}
\end{array} \right\} \quad (75)
\]

If the government is not efficient (\(c(e)\) does not grow quickly enough), the optimal policy might be not to struggle against the agency at all (\(e = 0\)).

17 Applications

In this section, I will demonstrate that despite its simplicity, the model could be applied to a wide range of economic issues, such as tax reform and immigrational and regional policies.

17.1 Tax Base Broadening as a Measure of Increasing Revenue

This subsection examines if, and when, tax base broadening leads to improved tax revenue collection. There is a large amount of literature discussing tax base broadening versus tax rate increases. In particular, Baer and Silvani (*) argue that the simplification of a tax system, including a base broadening and the reduction of exemptions, makes administration much easier and increases the level of compliance. Moreover, according to Pillarisetti (?), the complexity of tax codes and high tax rates could also result in corruption.

Some other studies argue against tax base broadening. For example, Piggott and Whalley (?) show that the Canadian VAT base broadening on commodities and services, which are difficult to tax when provided by small scale suppliers, induced substitution into relatively inefficient household production, and stimulated underground activities.

My model considers the effects of tax base broadening related to the change in base distribution and the adjustment of the avoidance price. I will illustrate that tax base broadening has at least two indirect effects. The first, related to the change in the tax base distribution function, could cause both growth and reduction in collecting revenue. The second relates to an avoidance cost which might increase after the tax base broadening.

17.1.1 When an Avoidance Cost does not Depend on Tax Exemptions

Tax base broadening might lead to a change in the tax base distribution function, causing a reduction in the government’s ability to collect a tax revenue.

**A new income source as a tax subject.** The first type of tax base broadening is carried out by including a new income source in the tax base. Example 2 presents a case where such a policy leads to decreasing revenue.

**Example 2** "When the tax of a new income source leads to decreasing tax revenue"  
Let me consider an economy with two sources of income: farming \(x\) and fishing \(y\), which might be interdependent; the extreme case is when \(y_i = 1 - x_i\) for any householder \(i\). Initially, the government only taxed farming income \(x_i\), thereby granting fishing income \(y_i\) a tax exemption. Tax base broadening means that the government decides to tax total income \(z_i = x_i + y_i = 1\). In this particular case, tax base broadening leads to complete tax avoidance and zero tax revenue.

**A new tax payer.** The second type of tax base broadening appears when the government adds another, new household, as a tax subject. This involves no risk as long as the government can distinguished him by income level. Otherwise, a new tax subject might cause a decrease in tax revenue.
Example 3 "When a new tax payer causes a reduction in tax revenue"

Let an initial economy consist of eight householders, with incomes 1, 1/4, 1/4, 1/4, 1/8, 1/8, 1/8, and 1/8, respectively. The government can set a tax code exactly equal to the income. In this case, only the richest householder would avoid tax, if spending his total income in payment for avoidance service. Total government revenue would be $\frac{1}{4} + \frac{1}{4} = \frac{1}{2} = 25\%$. Let a new individual with income 1/4 be included as a tax subject. The government could not tax the poorest at more than 1/9, otherwise 1/9 would be an avoidance price and all householders would avoid paying tax. For the same reason, the government could not tax medium-income householders at more than 1/5, which means that total government revenue could not exceed $\frac{3}{5} + \frac{1}{5} = 45\%$, which is lower than previously.

Tax rate reduction as a necessary complement to tax base broadening. Example 3 also demonstrates the necessity to decrease the tax levels together with tax base broadening. Although this example seems very complicated, there are plenty of simple examples showing that tax base broadening must be accompanied by a reduction in tax rates, to sustain the same revenue level as before tax base broadening took place.

Example 4 "When tax base broadening may require a tax rate reduction"

An initial economy consists of two householders, with incomes 1 and 0.8 respectively. The government sets tax codes 1 and 0.5, collecting 0.5 as a revenue. Let a new household earning 0.8 be included to the tax base. If the tax remains unchanged, the agency will set a price of 0.5 and everybody will avoid paying tax. For a positive revenue collection, a new tax level for the poorer individuals should not exceed 1/3.

17. APPLICATIONS

17.1.2 If Avoidance Technology Uses Tax Exemptions

In many cases, a tax avoidance technology includes a reclassification of income. For example, the Russian government decision to cancel VAT for goods imported from Belarus in late 1996 led to a tremendous growth in imports from Belarus, while Ukrainian imports declined by approximately the same amount. Since the changes in export activities did not lead to considerable variations in real economies, they can be explained by a tax avoidance practice through reexports of Ukrainian goods as described in the IMF publication.

Example 4 "When tax base broadening may require a tax rate reduction"

Another example of tax avoidance by using a tax grant was observed in the early 1990's when the Russian government granted tax exemptions instead of financing the activity of some non-profit organization. As a result, these organizations started to share tax grants with other firms, thereby reducing the tax base.

In both cases, tax avoidance occurs due to the existence of tax exemptions in the Russian tax code. Eliminating these would thus induce an increasing avoidance cost, due to the necessity of an avoidance technology adjustment. If the cost is positively correlated with tax exemptions, a tax base broadening entails an additional component, causing a tax revenue increase. Accounting for the above, my model confirms that it might be better to abolish tax exemptions and give financial support to non-profitable organizations, rather than providing them with tax grants. This point was clarified in detail by IMF(1995) Policy Analysis and Assessments paper, which refers to the experience of Albania, Poland, Romania, Russia, Kazakhstan, and Uzbekistan.

More Evidence from Russia. Tax collection continues to be a question of survival for the Russian government. All economic reforms can be stopped because of the shortage of government revenues. The main reason for unsuccessful tax collection is high tax rates and consequently, a high disposition for tax avoidance. Pavel Kuznetsov, Gregory Gorobetz and Alexander Fominykh provide some concrete examples of the popular tax evasion schemes implemented by financial industrial groups in Russia. Main financial transactions between the groups are conducted through off-shore accounts which are hidden, or not taxable, while the accounts of the industrial plants are always empty and thus, not taxable. There were no instruments for punishing such financial industrial groups since the links are not officially registered: the accounts of an offshore company are not public, neither can they be monitored or punished, while the implementation of bankruptcy legislation on industrial plants with empty account has a very high political cost. According to Ustinov, the Russian government was thus not able to collect even half the prescribed revenue expressed as a percentage of GDP. High tax rates and avoidance practices create further distortions, such as inter-enterprise and tax arrears. Because an economy provides an avoidance service for the rich, the government levies heavier taxes on the poor in order to collect the necessary tax revenue, which, in turn, slows down small business development and economic growth. Wages, pensions and other widespread budget arrears in Russia.

According to the IMF Direction of Trade Statistics, Russian imports from Belarus and Ukraine amounted to USD 2795 mn. and USD 6256 mn. in 1996. In 1997, exports from Ukraine amounted to USD 3981 mn. only, while Belarus exports rose tremendously and reached USD 4627 mn.
are an informal way of taxing the poor heavily as predicted by the model. In addition, the model shows the inefficiency of the excessive revenue target set by the Russian budget in the last 5 years and, as a consequence, the government tried to collect as much as possible but failed. Social welfare would improve a great deal if the government set the target revenue somewhat lower than the maximum possible target, because of the significant reduction in resources wasted on payment for avoidance services. This is illustrated by figures 7, 6, 11, 13 and 14.

The consequences of the introduction of a new 13% income tax might be ambiguous. Figure 17 demonstrates two possible cases. $D$ is a breakdown point for a flat tax code or $x_1$ defined in formula (62), and $X$ is a corresponding tax level. $A$ and $A'$ represent the tax payment by the richest household in minimal tax brackets, defined by an initial tax code. $C$ and $C'$ correspond to the incomes of the richest non-avoiding householders, while $B$ and $B'$ illustrate their tax duties. Lines $OAC$ and $OAC'$ correspond to two possible initial tax codes. If the Russian tax code was initially similar to line $OAC$, more precise, $A$ was allocated to the right side of $X$, the introduction of a flat tax would lead to a loss in tax revenue equal to $DXABC$. However, if the householder with income $D$ initially paid more than 13 percent, the government would lose $AIBIY$, but obtain $YXDCI$. The avoidance price would fall in case $OAC$ and change ambiguously in case $OAC'$ depending on the relation between $ICIBI$ and $IDXI$. In both cases, the agency profit would be lower. I should say that in the real world, the agency might need some time to learn and react. In this case, the Government would lose $ABM$ and receive $MKLC$ immediately after the introduction of a new code, but then breakdown income $L$ will drift to $D$, and the maximum level of paying tax $K$ will drift toward $X$.

The Tax Reform Act of 1986 and the Model The model will now attempt to describe the consequences of the Tax Reform Act of 1986 (TRA86) in the US. I do not wish to question other explanations, however, I only want to provide an additional one.

TRA86 reduced the number of marginal tax rate brackets and compressed the marginal rate structure so that the sharpest decline in marginal tax rates was experienced by high-income individuals, whose rate fell from 50% to 28%. TRA86 dramatically reduced the top individual and corporate tax rates and broadened the tax base at both the individual and corporate levels. Reported incomes of high-income taxpayers began to increase rapidly at the time of TRA86. The phenomenon was summarized by Feenberg and Poterba (1), who calculated the share of adjusted gross income reported by the top 0.5 percent of the tax payers, ordered by adjusted gross income. After remaining almost flat at about 6% from 1970 to 1981, it began to increase continuously in 1982 and reached 7.7% percent in 1985, and then jumped sharply to 9.2% in 1986. There was a slight increase to 9.5% in 1987, then another sharp rise to 12.1% in 1988. After 1988, there was a decline to 11.2 % in 1989 and 10.9% in 1990. Such changes might be explained by the impact of a tax code modification on an avoidance and evasion behavior, rather than by the movement of a real economy causing the change in inequality, since the considered time period is very short.

---

30 A very detail overview of the related literature is provided by Auerbach and Slemrod in (7).
The possible explanation of the above dynamics of reported income after the implementation of TRA86 is illustrated by figure 18.

Once TRA86 reduced the income margin tax rate for everyone, it might have led to the shift from a tax rule represented by line A to a tax rule drawn as line BC. If this were the only change, the avoidance breakdown point A would be shifted to point B. Even this change could lead to an increase in the maximum reported income and augment the number of individuals who prefer paying tax, rather than using an avoidance technology, which could lead to an increase in the reported income concentration. (This could happen but might not necessary be the case. If a new tax code were much more regressive compared to the old one, this would lead to tax avoidance by some people paying taxes under the old tax rule. But agency revenue would be lower in any case; see Figure 19). The second impact of TRA86 on an economy is a broadening of the tax base, which can be interpreted as the increase in an agency's cost, as described in subsection 17.1.2. After the implementation of TRA86, the agency must spend more time and money on uncovering and effecting the legal reduction in taxable income as well as the outright evasion of tax liability, and the expenses for camouflaging this. The effect of such a change in taxes is shown by the shift of the "breakdown" point from B to C in figure 18. In the following years, tax avoidance technology has been developed and the tax avoidance cost has decreased. This has led to the drift of a "breakdown" point from C toward B along the tax rule line, but B has not been achieved.
17.2 Immigration Policy

Another important issue causing the change in income distribution is the immigration policy. Example 3 can also be applied to immigration. Although that case illustrates the loss of revenue due to the entrance of a new immigrant, there are plenty of examples illustrating growth of revenue when the government uses an immigration policy together with tax rate reduction; see example 5.

Example 5 "Immigration and increasing in tax progressivity" Let an economy initially consist of two householder, with incomes $1$ and $2/5$. The government can set a tax code $4/5$ and $2/5$ respectively and collect the maximum possible revenue $2/5$ from the poorest householder. In this case only the richest householder would avoid tax, spending $4/5$ on payment for the avoidance service. Now, a new immigrant with income $2/5$ arrives. The government could set tax levels equal to $1$, $1/3$ and $1/3$ and collect $2/3$ from the two poorer householder while the richer householder would avoid paying his entire income.

Example 5 provides the case when a tax base broadening increases both the ability to collect revenue and social welfare, but this policy redistributes after tax/avoidance incomes in such a way that the rich suffer. Therefore, an immigration could be blocked if the rich have larger bargaining power. The example ones more illustrates that tax base broadening must sometimes be implemented together with decreasing tax levels for the poor and perhaps increasing levels for the rich. If the government does not change the tax code $4/5$, $2/5$, $2/5$, the agency would lower its price to $2/5$, thereby causing complete avoidance.

17.3 Regional Policy.

The appearance of new independent states and the European unification attracts an increasing research interest in the secession/unification problem. The distribution of income inside the united regions, as well as a tax policy, plays an essential role for break-up or integration decision. For example, Bolton and Roland (1997) by investigating a democracy where the median voter takes a decision about a flat tax rate, show that "a region with very low income inequality may want to break away from a nation with high income inequality and high tax rates in order to impose lower tax rates, and vice versa a region with high income inequality may want to separate in order to impose more redistribution than in the unified country". In this section, I will emphasis some regional issues related to tax avoidance.

17.3.1 Unification Could both Reduce and Increase the Government’s Ability to Collect a Tax Revenue.

The effect of a change in the tax base distribution would appear after the unification of two states. Generally, unification improves the government ability to collect taxes, since it revokes the agency's option in price discrimination. Unification might also increase the efficiency of anti-avoidance measures, since the new government will use the anti-avoidance skills of both regions. But the change in income distribution could contribute negatively, as shown in example 3, which could be applied to the unification issue, if we consider a new tax payer as a new region collecting no tax before the unification.

17.3.2 The Elite of the Poorer Region Can Gain From a Secession.

Although the average tax burden might be lower after unification, rich householders in the poorer region might have considerable incentives to stay separate. Before unification, or after secession, they could enjoy an avoidance ability, or a lower tax level than in the united economy. Incentives are larger if the unified government has a higher target revenue.

17.3.3 The Rich in the Richer Country Might Have Incentives for Separation.

On the other hand, the rich in the richer region might also be interested in secession, especially if a unified government sets a high target revenue. In a union, a smaller proportion of rich might cause a more progressive tax. Example 6 shows that in an economy with two types of agents, the rich gain if their proportion is extended. Collecting tax from the poor, the government sets a minimum tax for avoiding rich just to insure necessary agency revenue to draw the agency away from the poor. That means that the government is only concerned with the sum of the tax duties of the rich, and that the individual tax is inversely related to the number of rich. Thus, the rich in the region with a higher proportion of rich would be interested in secession.

Example 6 "Two householders, economy" It is interesting to consider an economy with two types of individuals as the simplest of heterogeneous agents. Let us assume that there are $n_1$ poor and $n_2$ rich
householders earning an income of \( x \) and 1, respectively. If the government sets a tax level \( \tau_1 < \tau_2 \), then the agency will set

\[
T = \begin{cases} 
\tau_2; & \text{if } \tau_1(n_1 + n_2) \leq \tau_2n_2 \\
\tau_1; & \text{otherwise}
\end{cases}
\] (76)

and the optimal tax code will be

\[
\tau_1 = \frac{\tau_2n_2}{n_1 + n_2}.
\] (77)

The government will collect \( \tau_1n_1 = \bar{y} \) if \( \bar{y} \leq \min \left( n_1x; \frac{n_1n_2}{n_1 + n_2} \right) \); otherwise the government maximizing its revenue will set \( \tau_2 = 1 \). Householders' consumption will be \( x - \tau_1 \) and \( 1 - \tau_2 \) for the poor and the rich respectively. Social welfare is represented in formula (78)

\[
W = n_1(x - \tau_1) + n_2(1 - \tau_2) = I - \bar{y} \left( 2 + \frac{n_2}{n_1} \right).
\] (78)

Using the same logic for a decreasing tax code, it might be concluded that if \( n_1 > n_2 \), the government will choose an increasing tax code, otherwise \( \tau_2 \) could be less than \( \tau_1 \), if \( \bar{y} \leq \frac{n_1n_2}{n_1 + n_2}x \). Table 1 provides a summary solution for all possible cases of the parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>( T )</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
<th>( x - \frac{n_1n_2}{n_1 + n_2} )</th>
<th>( 1 - \bar{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{n_1n_2}{n_1 + n_2} )</td>
<td>( n_1x )</td>
<td>( \frac{n_1n_2}{n_1} )</td>
<td>( x )</td>
<td>( 1 - \frac{n_2}{n_1} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{n_1n_2}{n_1 + n_2} )</td>
<td>( n_1x )</td>
<td>( \frac{n_1n_2}{n_1} )</td>
<td>( x )</td>
<td>( 1 - \frac{n_2}{n_1} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{n_1n_2}{n_1 + n_2} )</td>
<td>( n_1x )</td>
<td>( \frac{n_1n_2}{n_1} )</td>
<td>( x )</td>
<td>( 1 - \frac{n_2}{n_1} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{n_1n_2}{n_1 + n_2} )</td>
<td>( n_1x )</td>
<td>( \frac{n_1n_2}{n_1} )</td>
<td>( x )</td>
<td>( 1 - \frac{n_2}{n_1} )</td>
</tr>
</tbody>
</table>

where case 1 represents the only situation when the government decides to set a decreasing tax rate, that is when \( n_1 < n_2 \) and \( \bar{y} \leq \frac{n_1n_2}{n_1 + n_2}x \). Case 2 corresponds to a situation when the government is able to collect a target tax \( \bar{y} \leq \min \left( \frac{n_1n_2}{n_1 + n_2}, n_1x \right) \) but when the conditions of case 1 are not satisfied. Cases 3 and 4 agree with the events when the government is not able to collect a target revenue, due to the lack of income of the poor (case 3, \( x \leq \frac{n_2}{n_1 + n_2} \)) or the rich (case 4).

17.4 Should Growth in Inequality Reduce Tax Progressivity?

Slemrod and Bakija (2000) analyze the links between increasing inequality in pre-tax incomes and decreasing tax progressivity. The authors provide an overview of the optimal tax literature and consider a possibility of different causalities, including the likelihood that both trends are caused by the same third factor. My model provides an additional factor contributing to the considered phenomena. By changing parameters \((x, n_1)\) in Example 6, both a positive and a negative correlation between inequality and progressivity can be reached.

| \( n_1 \) | 50 | 150 | 150 |
| \( n_2 \) | 50 | 50 | 50 |
| \( x \) | 0.3 | 0.15 | 0.6 |
| inequality | benchmark | larger | smaller |
| progressivity = \( \frac{x}{\tau_2} \) | 2 | \( \frac{3}{5} \) smaller | \( \frac{2}{3} \) smaller |

## 18 Conclusion

The most essential property of the model is its ability to endogenously solve an avoidance price, depending on a tax code and a tax base distribution. Although the model neglects a large number of issues, rather considering the specific case of exogenous income, it is still capable of capturing many real world properties. It illustrates the necessity to account for changes in a tax base distribution and the possibility of decreasing the avoidance price when the government sets a tax code, makes decisions about tax base broadening, implements an immigration policy, or solves regional problems.

31 I choose the parameters so that the Lorenz curves do not intersect. According to Atkinson (1983) such a construction guarantees that the order of inequality does not depend on the measure used.
19 Appendix

In this section, I will prove Proposition 1 on page 35.

Proof. I will prove Proposition 1 by providing the algorithm of construction \( \tau_1(.) \), giving \( \tau(.) \). Simple intuition is shown in figure 20. A new tax code is simply constructed by shifting a high tax burden from poor to rich tax payers, so that for any tax level \( T \), there is equality \( \int_{A_1(T)} dF = \int_{A(T)} dF \); where \( A(T) = \{ x, x \in X, \tau(x) \geq T \} \); \( A_1(T) = \{ x, x \in X, \tau_1(x) \geq T \} \); \( \tau(x) \) — original tax code, \( \tau_1(x) \) — new tax code. Moreover, \( A_1(T) \) is interval and \( F(\sup A_1(T)) = 1 \); I will now give the formal construction and proof. Let \( X \) be the domain of \( \tau(.) \) or the support of income. For any \( y \in \tau(X) \), \( \exists x \) such that \( 1 - F(x) = \int_{A(y)} dF \) because \( F(x) \) is a continuous function. Such \( x \) is unique, because \( F(x) \) is strictly monotone. If \( y_1 < y_2 \), then \( A(y_1) \supseteq A(y_2) \) and \( \int_{A(y_1)} dF \geq \int_{A(y_2)} dF \); so that I can define function \( x = h(y) = F^{-1}(1 - \int_{A(y)} dF) \) for any \( y \in \tau(X) \); where \( h(y) \) is non-decreasing. It means that \( \forall x \in X, \exists y \in \tau(X), \) such that \( \forall \epsilon > 0, h(y + \epsilon) \geq x; h(y - \epsilon) \leq x \). Such \( y \) is the new tax level for individuals with income \( x \). Now I will prove that \( \tau_1(x) \) is non-decreasing; Let \( x_1, x_2 \in X; x_1 \leq x_2 \); Let \( y_1 = \tau_1(x_1); y_2 = \tau_1(x_2) \); then \( \forall \epsilon > 0, h(y_2 + \epsilon) \geq x_2 \geq \tau_1(x_1) \geq h(y_1 - \epsilon) \); Because \( h(y) \) is non-decreasing, \( y_2 + \epsilon \geq y_1 - \epsilon \), or \( y_1 - y_2 \leq 2\epsilon \), which means that \( y_1 \leq y_2 \) and \( \tau_1(x) \) is non-decreasing; 2) the government will collect the same amount of revenue; 3) the agency will choose the same price for its service \( T(\tau) = T(\tau_1) \); the agency’s profit is \( T * F(\tau, T) \), where \( \tau, T \) are tax level and tax code. By the construction \( F(\tau, T) = F(\tau_1, T) \) for any \( T \). So, \( \arg \max T * F(\tau, T) = \arg \max T * F(\tau_1, T) \). Government revenue equals \( g(\tau) = \int_{B(\tau, T)} \tau(x) dF \); where \( T \) is the avoidance price and \( B(\tau, T) = \{ x, x \in X, \tau(x) \leq y \} \); \( g(\tau) = \int_{B(\tau, T)} \tau(x) dF(x) = \lim_{n \to \infty} \sum_{m=0}^{n} F(\tau, \frac{mT}{n}) \). By the construction \( F(\tau, T) = F(\tau_1, T) \) for any \( T \); So, \( g(\tau) = g(\tau_1) \). 3) The value of a welfare function, which equals average utility, will not be lower than previous \( W(\tau(.) \leq W(\tau_1(.)); W(\tau(.)) = \int_{B(\tau, T)} u(x - \tau(x)) dF(x) + \int_{A(\tau, T)} u(x - T) dF(x) \); This expression can be simplified by introducing a new function \( \tau(x) = \max(T, \tau(x)) \). Then, \( W(\tau(.)) = \int_X u(x - \tau(x)) dF(x) \); From above we know that the government would collect the same amount of revenue \( \int_X \tau(x) dF(x) = \int_X \tau_1(x) dF(x) \); and a new tax code \( \tau_1(x) \) is a non-decreasing function. Now, I will prove the lemma I need for continuing:
Lemma 2 Let:

1. \((x_1 < x_2 < ... < x_n)\);

2. \(\tilde{F}(x_i)\) be a non-decreasing function, constrained from \(\tilde{F}(x_i)\) such that \(\forall i, i \in I_n, \exists\) unique \(k_i \in I_n\), such that \(\tilde{F}(x_i) = \tilde{F}(x_{k_i})\); where \(I_n = \{1, 2, 3, ..., N\}\);

3. \(\sum_{i=1}^{N} u(x_i - \tilde{F}(x_i)) \leq \sum_{i=1}^{N} u(x_i - \tilde{F}(x_i))\);

**Proof.** prove the Lemma by induction on \(n\). If \(N = 1\) then \(\tilde{F}(x_1) = \tilde{F}(x_1)\); and the lemma is obviously true. Let the lemma be true for \(N - 1\) and let \(x_k = \arg\max_{i \leq N} \tilde{F}(x_i)\); which means that \(\tilde{F}(x_{k_N}) = \tilde{F}(x_{k_N})\) by definition of \(I_n = \{1, 2, 3, ..., N\}\);

\[
\sum_{i=1}^{N} u(x_i - \tilde{F}(x_i)) \leq \sum_{i=1}^{N} u(x_i - \tilde{F}(x_i))
\]

because \(u(x)\) is concave \(u(x) \leq u(x_n) - u(x)\) and because \(u(x)\) is an increasing function \(u(x_k) - \tilde{F}(x_k) \leq u(x_n) - \tilde{F}(x_n)\) and because \(u(x)\) is an increasing function \(u(x_k) - \tilde{F}(x_k) \leq u(x_n) - \tilde{F}(x_n)\) and because \(u(x)\) is an increasing function \(u(x_k) - \tilde{F}(x_k) \leq u(x_n) - \tilde{F}(x_n)\) and because \(u(x)\) is an increasing function \(u(x_k) = \arg\max_{i \leq N} \tilde{F}(x_i)\) constrained from \(r(x_i)\) by the next way:

\[
\tilde{F}(x_{k_N}) = \arg\min_{r(x_i) \leq x} \tilde{F}(x_i) if x_{n+1} = \inf_x \tilde{F}(x_i).
\]

By the lemma, proved above, \(\frac{1}{N} \sum_{i=1}^{N} u(x_i - \tilde{F}(x_i)) \leq \frac{1}{N} \sum_{i=1}^{N} u(x_i - \tilde{F}(x_i))\); By the construction \(\lim_{N \to \infty} \tilde{F}^N(x) = \tilde{F}_1(x)\), because \(\tilde{F}^N(x)\) is non-decreasing and \(\lim_{N \to \infty} F(A(\tilde{F}^N, T)) = F(A(\tilde{F}, T))\) for any \(T\), but \(\tilde{F}_1(x)\) is the only function, with such properties. \(\int_X u(x - \tilde{F}(x))dF(x) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} u(x_i - \tilde{F}(x_i)) \leq \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} u(x_i - \tilde{F}(x_i)) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} u(x_i - \tilde{F}(x_i)) = \int_X u(x - \tilde{F}_1(x))dF(x)\); End of proof.

20 Appendix. Calculation for Uniform Distribution

**Example 7** Uniform distribution. \(F(x) = x\); no restrictions. The agency's isoprofit curve is \(\alpha = 4m\); the solution of equation \(x = l - x\) or \(x = \frac{l}{2} + \frac{1}{2}\sqrt{1 - 4m}\) is \(x_1 = \frac{l}{2} + \frac{1}{2}\sqrt{1 - 4m}\); using the general formula (98), I obtain the next expression: \(g(m) = \frac{1}{4} - \frac{1}{2}\sqrt{1 - 4m} - \frac{m}{2} + m \ln \left(\frac{1 + \sqrt{1 - 4m}}{2l/2}\right)\). In the case of uniform distribution, \(g(m)\) is concave and has the following quantitative characteristics: \(\max g(m) = 0.25\); \(\arg \max g(m) = 0.16\).

**Example 8** Uniform distribution. \(F(x) = x\); progressive tax. Following the algorithm described above, I find that \(x_1 = \frac{l}{2}; \alpha = 4m\); \(x_2 = \frac{l}{2} + \frac{1}{2}\sqrt{1 - 4m}\). Applying formula (100), I get: \(g(m) = 4m \int_0^{1/2} xdx + m \left(\ln \frac{1}{2} - \ln \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4m}\right)\); \(g(m) = \frac{m}{2} - m \ln (1 - \sqrt{1 - 4m})\).

**Example 9** Uniform distribution. \(F(x) = x\); restriction on marginal tax. \(x = 1 - \sqrt{m}\); \(x_1 = \frac{l}{2}\). From formula 100: \(g(m) = \frac{1}{4} \left(1 - \sqrt{1 - 4m}\right) - \frac{m}{2} + m \ln \left(\frac{1 + \sqrt{1 - 4m}}{2l/2}\right)\).

**Example 10** Uniform distribution \(F(x) = x\); progressiveness and restriction on marginal tax. \(x = 1 - \sqrt{m}\); \(x_1 = \frac{l}{2}\). From formula 100: \(g(m) = m \left(\frac{1}{4} + \ln \left(\frac{1}{2}\right)\right)\).
Figure 20 summarizes all four cases of restrictions.

![Graph showing uniform distribution of income distribution and tax avoidance](image_url)
Bibliography


Tax Avoidance as a Reason for Secession

Abstract

In this paper, I provide an additional explanation to the intentions to secede related to expected changes in the tax avoidance activities after the "break down of a nation". To demonstrate my points, I use a tax avoidance model where active avoidance providers make a decision about the price and quantity of their services. Secession gives the avoidance provider the option of setting different prices in separate regions. As a consequence, the price for the tax avoidance service may fall in the poorer region and the elite of this region would be able to avoid the tax, which is impossible in union. Moreover, regional separation may lead to tremendous changes in the shape of income distribution. Facing these changes, new governments would be forced to change the tax codes. Thus, the government of the richer region may reduce tax rates in order to enlarge the tax revenue collection. This decision would not only increase private consumption for tax payers but also for rich tax avoiders, since the reduction of tax duties leads to a decline in the price for the tax avoidance service. The sum of tax revenues in both regions would fall significantly after the secession, however, which means that governments would provide less public goods. If the value of public goods is low for the citizens, however, many households would be better off after the secession. And thus, the model predicts a higher secession tension in economies with less efficient governments. This explains the increase in the popularity of secession ideas at the beginning of transitions, when the population of the restructuring countries negatively evaluated the provision of public goods. To avoid a breakdown of the state, the government should reduce inefficient spending and tax duties. The promotion of democracy or the political influence of poor households may reduce the tendency to separate.

Keywords: political integration, tax avoidance, public expenditure.

JEL classification: F15, H26, D72

21 Introduction

The emergence of new independent states and the European unification attract an increasing research interest in the secession-unification problem, which is of great importance. All over the world, some regions are looking for independence, sometimes leading to sanguinary conflicts, thereby affecting thousands of people. The national costs of those tensions in developing countries cannot be overestimated. In developed countries, the antiglobalization attitude damages cooperation and prevents a further increase in economic growth.

The objective of this paper is to contribute economic considerations to this problem. In particular, I focus on the reasons for the intention to separate that are related to tax avoidance. Secession changes the shape of income distribution, which may force new governments to review the tax code. This may happen because the government must account for the tax avoidance activities, when a clever tax avoidance provider reacts to the changes in demand for tax avoidance by setting the price for the tax avoidance service. Secession gives the avoidance provider a new option for price discrimination, which can create a reduction in the tax avoidance price in the poorer region, thereby giving an additional benefit to the rich. To demonstrate these effects, I use an enriched tax avoidance model proposed in Damjanovic (2001), where I introduce the provider of the tax avoidance service, which sets its fee to maximize the revenue, taking as given both the income distribution and

\[31\] I wish to thank Lars Ljungqvist for his invaluable guidance. Financial support by the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged. Special thanks goes to Christina Lonnblad for her fine editing. Remaining errors are my own responsibility.
the announced tax schedule. At the same time, a government seeks to raise a certain amount of tax revenue in the least wasteful way, i.e. by maximizing the social welfare function. Although the paper briefly discusses several implementations related to changes in income distribution, the importance of the question of political integration motivates me to dedicate a separate article to this problem.

The impact of income distribution inside the united regions, as well as a tax policy, on break-up or integration decisions has already been studied. For example, Bolton and Roland (1997) explain secession by the willingness of the region with larger inequality to redistribute more. The authors assume a flat tax code, where the rate is set by democratic voting. I will also exploit the change in income distribution, but in another framework. In particular, I will examine an economy where the government sets the tax code, but citizens are allowed to vote for the separation.

My model shows that the sum of tax revenues after separation would be significantly lower, while the tax burden imposed on the poor would become larger. The former means that governments would provide less public goods. This disadvantage, however, can be dominated by a willingness to increase private consumption, if the evaluation of the provided public goods is low. To avoid a secession, the government must reduce inefficient spending.

The rest of the paper is structured as follows. Section 22 describes the model on an intuitive level. Section 23 considers a simple example of two economies populated by two types of individuals, where the regions making decisions on unification are distinguished by the ratio of poor individuals. Section 24 provides the formal description of the model and solves it for the case of uniform income distribution. Section 25 provides a short discussion on the relationship between democracy and the possibility to preserve the union. Section 25 concludes.

22 Tax Avoidance Model: a Short Description

Three different groups of economic agents operate in an economy: a government setting the tax code, an agency providing the tax avoidance service; and households maximizing after tax or avoidance income. Each householder exogenously obtains some economic item referred to as "income", which the government taxes in order to collect its revenue. $F(x)$—is a cumulative distribution of $x$, known to everybody, while the particular $x$ belonging to each householder is private information. The model has the following dynamics. In the first stage, the government sets up a tax code on the level $\tau(x)$ in order to finance its target expenditure, $g$. If the government is not able to collect $g$, it simply maximizes a revenue. Otherwise, it collects revenue $\bar{g}$ by setting a tax code maximizing the social welfare function $W$ and anticipating the activities of the other economic agents. In the second stage, the monopolistic agency sets its fee in order to maximize its profit, taking as given both income distribution and the announced tax schedule. The agency helps a householder avoid a tax by hiding his personal income $x$, and it has a monopoly on tax avoidance services. I will assume that the agency is unable to price discriminate and therefore, it sets a fixed price $T$ for its service, accounting for the demand composed by income distribution $F(x)$ and tax code $\tau(x)$. For simplicity, I will consider the case where the agency has zero costs per client and access to its service is free. Neither the agency nor the tax authority can observe an individual's income prior to the decision on whether to pay the fee, though both know the income distribution in the population. After the secession, an avoidance provider obtains the ability to price discriminate across regions. Using this option, a tax avoidance provider may set lower prices for the poor region. In a final stage, each individual decides either to declare his income $x$ and pay $\tau(x)$ to the government or use an avoidance service, paying $T$ to the agency and nothing to the government. Since an individual maximizes his after tax/avoidance earnings, he prefers to pay tax if $\tau(x) < T$.

23 A Two Agent Economy

First, I would like to demonstrate how my model works by considering the simplest economy with two individuals, distinguished by income. Let the poorer individual receive income $x < 1$, while the income of the richer individual is normalized to $1$. A government defines a tax code $(\tau_1, \tau_2)$ for poor and rich, respectively. Then, the agency sets the price for the tax avoidance service, $T$. If $T$ is larger than $\tau_2$, the agency will be out of business. If $\tau_1 < T < \tau_2$, the rich individual will avoid the tax and the agency will receive a profit equal to $T$. If $T \leq \tau_1$, everybody will avoid tax duties and the agency will receive $2T$ as a profit. Therefore, the best agency's strategy is to set price $T$ as in formula (79).

$$T = \begin{cases} \tau_1, & \text{if } 2\tau_1 > \tau_2 \\ \tau_2, & \text{otherwise} \end{cases}.$$ (79)
The government anticipating the agency's action should never allow complete tax avoidance. Therefore, it should ensure the condition of "non-avoidance of the poor" represented in formula (80)

\[ \tau_1 \leq \frac{\tau_2}{2}, \]

which would allow the government to collect \( \tau_1 \) as a tax revenue. Once \( \tau_2 \leq 1 \), \( \tau_1 \) should always be less that 0.5. Taking this restriction as given, the government chooses \( \tau_1 = \min(x, g, 0.5) \); and \( \tau_2 = 2\tau_1 \), which is equal to the price for the tax avoidance service, \( T = \tau_2 - 2\tau_1 \). Although the rich individual never pays his tax duty, the increase in the tax burden for the poor automatically leads to an increase in the expenditure on tax avoidance. Therefore, a high target tax revenue is not so good for the rich as may be concluded from the classical "tax avoidance for exogenous cost" models\(^{32}\).

23.1 Immigration as an example of a change in the poor to rich ratio. Who will benefit?

Let me now consider some possible consequences of immigration. I will assume the government to be hostile and maximizing tax revenue.

23.1.1 Poor immigrant

Case 1: \( x < \frac{1}{2} \)

Before accepting an immigrant, the government had the tax code \((x; 2x)\), with tax revenue \( x \). After the immigration of an individual with income \( x \), the optimal tax code becomes \((x, 3x)\) and the revenue equals \( 2x \). As a consequence, the spending on tax avoidance by the rich increases from \( 2x \) to \( 3x \). This is an example when the immigration of a poor individual allows an increase in revenue, but leads to more progressive taxation and enlarges the tax avoidance expenditures of the rich.

Case 2: \( x > \frac{1}{2} \)

Before immigration, the tax code and the revenue were \((\frac{1}{2}, 1)\) and \( \frac{1}{2} \), while afterwards they were \((\frac{3}{2}, 1)\) and \( \frac{3}{2} \). This case demonstrates the situation when the acceptance of a poor immigrant reduces the per capita tax revenue, but enforces the government to reduce the tax burden for poor people.

Both cases show that the acceptance of poor immigrants is favorable for the poor.

23.1.2 Immigration of rich individuals

Case 3: \( x < \frac{1}{2} \)

The pre-immigration tax code and revenue were \((x, 2x)\) and \( x \), respectively. After the arrival of a rich immigrant with income 1, the government is still unable to collect more than \( x \), since rich householders will still avoid their tax duties. But governments can make the tax codes less progressive and save some of the income of the rich from being paid to the agency. The optimal tax code after immigration is \((x, \frac{1}{2}x)\). Government revenue per capita decreases as in case 2, but the gain for the rich due to an increase in after avoidance income may exceed the loss in public consumption, if the government is inefficient.

Case 4: \( x > \frac{1}{2} \)

The pre-immigration tax code and revenue were \((\frac{1}{2}, 1)\) and \( \frac{1}{2} \), while the highest revenue can be collected with tax codes \((x, \frac{3}{2}x)\) and \( x \) after the rich immigrant has entered the country. In other words, the government can increase the tax duties for the poor, should it have such objectives, while the tax avoidance expenditure for the rich is expected to be reduced.

Summarizing all four cases, I can say that an increase in the share of the same type group may lead to a reduction in tax duties for the poor or tax avoidance expenditures for the rich and, therefore, to higher private consumption. An increase in proportion of the other group can have the opposite effect, however. The negative effect on individual consumption may be relaxed by an increase in the provision of public goods. If this is not the case, immigration may be strongly restricted by the people belonging to the opposite type.

23.2 A unification of two regions distinguished by the proportions of poor individuals

Now, I will switch to the main topic. Let me consider two regions populated by two types of individuals and distinguished by different proportions of poor individuals.

\(^{32}\)See, for example, Rouine (2000).
Let me assume that there are $N_1$ poor and $N_2$ rich householders, earning an income of $x$ and $1$, respectively. I will consider the case when $N_1 > N_2$, a condition reflecting the real situation and ensuring an increase in the "optimal" tax code.

If the government sets a tax level $\tau_1 < \tau_2$, then the agency will set

$$T = \tau_2; \text{ if } \tau_1(N_1 + N_2) \leq \tau_2 N_2$$

$$\tau_1; \text{ otherwise.}$$

Therefore, the condition of the non-avoidance of the poor in this economy is represented by formula (82)

$$\tau_1 \leq \frac{\tau_2 N_2}{N_1 + N_2} \leq \frac{N_2}{N_1 + N_2}.$$

If I define

$$n_1 = \frac{N_1}{N_1 + N_2} = \frac{N_1}{N},$$

$$\tau_1 \leq \tau_2 (1 - n_1) \leq 1 - n_1,$$

the government will collect $\tau_1 N_1$. Therefore, a target revenue per capita $\bar{y}$ is achieved if satisfying the next two conditions

$$\bar{y} \leq n_1 x$$

$$\bar{y} \leq n_1 (1 - n_1).$$

Expression (84) should be imposed since the government can only collect taxes from the poor, due to the avoidance of the rich. Condition (85) follows from (83). The government strategy is to set $\tau_1 = \min(x, \frac{\bar{y}}{n_1}, 1 - n_1)$; $\tau_2 = \tau_1/(1 - n_1)$. The householders' consumption will be $x - \tau_1$ and $1 - \tau_2$ for the poor and the rich, respectively.

Now, I would like to see what might occur if another country with a different proportion of poor households $n_1^U$ is united with the considered country. The union will have a $n_1^U$ proportion of poor individuals, which is equal to the average weighted proportions of poor in the regions in autarchy

$$n_1^U = \frac{n_1 N + n_1^* N^*}{N + N^*}.$$

Without loss of generality, I can assume that $n_1 < n_1^U < n_1^*$.

Following the logic of the previous subsection, I can show that the poor in the poorer region would consume less private goods after the unification, due to the increasing tax burden. In a union, the proportion of poor would be lower, all the rich would avoid tax duties and therefore, the government may increase the tax rate of the poor. The rich in the richer region also have the incentive to live in autarchy, since unification may lead to an increase in progressivity and, as a consequence, to a higher price for the tax avoidance service. I should underline that the resisting group has a larger political influence in its region than the group of the same type in the other region, which may explain the popularity of the secession issue. The effect is stronger, the higher is the government tax revenue target and the smaller is the value of the public goods it provides.

24 An Example with Uniform Distribution: All but the Poorest have Incentives for Separation

24.1 The Solution of the Tax Avoidance Model for Uniform Income Distribution

The setup and solution of the model is provided in Damjanovic (2001). In this section, I will briefly demonstrate how the model works in a continuous case by considering an economy with uniformly distributed income ($F(x) = x$). As shown in Damjanovic (2001), the optimal tax code should be increasing. The intuition is very simple: if a richer individual pays less tax than a particular householder, the government can simply exchange their tax duties, which would lead to the same revenue collection and the same tax avoidance activities, but might improve the social objective function. After the agency has decided on the price for the tax avoidance service, all householders are separated into two groups, consisting of tax avoiders and tax payers, respectively.\footnote{For simplicity, I assume that indifferent individuals would avoid taxation since the agency has the option of decreasing its price by an infinitely small amount, thereby attracting all indifferent individuals.}
Let $\tilde{x}$ be a solution to equation (86)

$$\tilde{x} = \inf_x \{\tau(x) = T\}. \tag{86}$$

Then, taxpayers whose income is larger than or equal to $\tilde{x}$ will use the agency's service, while the others will prefer to declare their income and pay tax $\tau(x)$ to the government. Further, I will call $\tilde{x}$ an income "breakdown". Thus, an agency's revenue is calculated by formula (87)

$$T(1 - F(\tilde{x})) = \tau(\tilde{x})(1 - F(\tilde{x})). \tag{87}$$

The agency maximizes its profit $m$ by solving problem (88):

$$m = \max_\tilde{x} \{\tau(x)(1 - F(x))\} \tag{88}$$

where $\tau(x)$ is a non-decreasing function, defining a tax level. There should exist some $\tilde{x}$ so that $\tilde{x} = \max\{x, \tau(x)(1 - F(x)) = m\}^{34}$. Solving this problem, the agency would set the price $T = \tau(\tilde{x}) = \frac{m}{(1 - F(\tilde{x}))}$.

The solution of the agency's problem is illustrated in figure 21, where a thin line represents an agency's isoprofit curve $\tilde{x}$ expressing the largest possible profit $m$ under the given tax code $\tau(x)$, drawn as a solid line. Income breakdown, $\tilde{x}$, is defined as the maximum of a set of incomes, where the best available agency's isoprofit curve equals the tax code.

Once $\tilde{x}$ has been chosen, government revenue is calculated by formula (89):

$$\int_0^{\tilde{x}} \tau(x) dF(x) = g. \tag{89}$$

The government needs to collect a target revenue $g$ setting $\tau(x)$, which maximizes the social welfare function. Generally, the problem can be reformulated in the following way:

$$\max_{\tau(.)} W(\tau(.) = \alpha \int_0^{\tilde{x}} u(x - \tau(x)) dF(x) + \beta \int_{\tilde{x}}^1 u(x - T) dF(x) + \gamma T(1 - F(\tilde{x}));$$

34To simplify the presentation, I assume the agency to be government friendly and in case of indifference, it chooses the income "breakdown" that is the best for the government. Otherwise, the government can enforce this by slightly reducing a tax code.
where formula (90) represents the government objective function. This formula can be interpreted as egalitarian social welfare, where \( u \) is individual utility. Alternatively, \( u \) may represent a function measuring the degree of government care for poor households. Householders whose income \( x \) is less than \( \bar{x} \) pay tax \( \tau(x) \) consuming \( x - \tau(x) \), while richer tax payers purchase a tax avoidance service paying price \( T \) and consuming \( x - T \). Good. \( T(1 - F(\bar{x})) \) represent the profit of the provider of the tax avoidance service. Parameters \( \alpha, \beta \) and \( \gamma \) represent the degree of government consideration paid to a particular group of economic agents. Further, I will consider the case when the government gives the same care to the utilities of each household, but is not at all interested in the profit gained by the agency \( (\alpha = \beta = 0) \). Expressions (91-94) represent the budget constraint. Formulas (91) and (92) describe the mechanism through which the agency sets its price, while equation (93) expresses the necessity to collect a certain amount of government revenue, and condition (94) shows that the tax level cannot be higher than income.

### 24.1.1 Risk Neutrality

In this subsection, I will consider a risk neutral utility function, \( u(x) = x \), or the situation when the government maximizes the average households’ private consumption. Since this assumption does not only simplify the model but also allows me to catch the effects non-related to risk aversion, it is worth considering as an extreme and interesting case.

If the government sets \( \tau(x) \) to maximize the average utility function, which is proportional to after-tax income, formula (90) can be simplified as (95)

\[
\max_{\tau(\cdot)} W(x) = \int_0^{\bar{x}} (x - \tau(x))dF(x) + \int_{\bar{x}}^1 (x - T)dF(x) = \int_0^1 xdF(x) - \bar{g} - T(1 - F(\bar{x})).
\]

Expression (95) shows that maximizing the social welfare function is equivalent to minimizing agency revenue. Therefore, problem (90) can be rewritten as problem (96)

\[
\min_{\tau(\cdot)} T(1 - F(\bar{x})),
\]

while constraints (91-94) remain unchanged.

### 24.1.2 Solution For the Risk Neutral Economy

This section provides the solution to problems (96, 91 – 94).

No restrictions It might be easier to consider a dual problem, where the government first decides how large a profit, \( m \), it wants to give to the agency, and then maximizes its revenue choosing the optimal tax code \( \tau(x, m) \) from the set of tax codes, thereby providing the agency with revenue \( m \)

\[
\tau(\cdot, m) = \arg\max_{\tau(\cdot), m(\tau) = m} g(\tau).
\]

Solving equation (97), I construct \( g(m) = g(\tau, m) \). For any level of \( \bar{g} \), I find the minimum value of \( m \), such that \( \bar{g} = g(\tau) \). Those \( m, \tau(\cdot, m), \bar{g} \) would represent the solution to an original problem (96, 91 – 94).

Now, I will describe the solution to a dual problem in more detail. According to conditions (91) and (92), an agency’s isoprofit curve is represented by the expression \( \frac{m}{1 - F(\bar{x})} \), where \( m \) is an agency’s profit. Accounting
for restriction (94), the government should thus set up \( \tau(x) = \min(x, \frac{m}{1 - F(x)}) \) (see Figure 1). The agency is indifferent to which point \( \tilde{x} \) to choose on its isoprofit curve, while the government prefers the largest \( \tilde{x} \).

As a widely applied example, let me consider a case when the isoprofit curve does not cross a 45 degree line more than twice \(^{35}\). Let \((x_1, x_2)\) be all solutions to equation \( x = \frac{m}{1 - F(x)} \) and \( x_1 < x_2 \). Then, the highest value of the government revenue equals \( g(m) = \int_0^{x_1} x dF(x) + \int_{x_1}^{x_2} \frac{m}{1 - F(x)} dF(x) \); and it might be simplified as formula (98)

\[
g(m) = \int_0^{x_1} x dF(x) + m \ln \left( \frac{1 - F(x_1)}{1 - F(x_2)} \right).
\]

Figure 22 represents solution (98) graphically. It shows that an unrestricted government would tax the poorest individuals very heavily. A tax code shaped like that in Figure 22 could be observed in Russia, where the target revenue was too high in 1995 - 1999, when the Russian income tax code had 5 different brackets. Although formally progressive, the frequent usage of wage and pension arrears could be interpreted as a heavier tax for the poor.

Marginal Tax Restrictions If an individual has the option of throwing away his income, then the government has an additional restriction: the marginal tax rate should not exceed 1. Further, I will consider this restriction as given. Now, the breakdown point of income \( \tilde{x} \) is equal to \( x_{mr} \), found by the condition that the marginal tax is equal to one, in other words, \( x_{mr} \) is a point where the agency's isoprofit curve has a 45 degree slope.

\(^{35}\)This is true for a wide class of distribution functions; for example; if \( F'' + 2F'^2 - F'F \geq 0 \), then the agency's isoprofit curve is convex and it does not cross, tangents or crosses a 45 degree line at two points. Although an isoprofit curve is not convex for the lognormal distribution, it nevertheless crosses a 45 degree line at two points.
EXAMPLE: UNIFORM DISTRIBUTION OF INCOME

\[ \frac{d}{dx} \left( \frac{m}{1 - F(x_{mr})} \right) = \frac{mF'(x_{mr})}{(1 - F(x_{mr}))^2} = 1. \] (99)

Given the marginal tax, the government revenue is calculated as

\[ g(m) = \int_0^x zdF(x) + \int_{x_1}^{x_{mr}} \frac{m}{1 - F(x)} dF(x); \]

or more simply as:

\[ g(m) = \int_0^x zdF(x) + m \ln \left( \frac{1 - F(x_1)}{1 - F(x_{mr})} \right). \] (100)

**Example 12** Uniform distribution. \( F(x) = \frac{x}{x_{mr}} \); restriction on marginal tax.

\( x_{mr} \) should be found from equation

\[ \frac{mF'(x_{mr})}{(1 - F(x_{mr}))^2} = \frac{m}{(1 - x_{mr})^2} = 1; \quad x_{mr} = 1 - \sqrt{m}; \quad x_1 = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4m}; \]

From formula (100): \( g(m) = \frac{1}{4} \left( 1 - \sqrt{1 - 4m} \right) - \frac{m}{2} + m \ln \left( \frac{1 + \sqrt{1 - 4m}}{2\sqrt{m}} \right) \)

Figure 23 gives a graphical illustration.

**24.2 When individuals are risk averse**

In the economy with a risk averse population, the government will try to equalize after-tax-avoidance income. Tax-avoidance activities impose additional problems for the government, however. The government faces a trade-off between the inefficiency related to outflow of income as a payment for the avoidance service, and the minimum amount it leaves to each household in order to maximize the social welfare function.

Figure 24 demonstrates an optimal tax code for a risk-aversion economy. The government should thus set up \( \tau(x) = \min(0, (x - X_{min}) \frac{m}{1 - F(x))} \). In this case, two parameters, the minimal nontaxable income \( X_{min} \) and the agency's profit \( m \), define the optimal tax code. The government will not tax households with an income less than \( X_{min} \), leaving \( X_{min} \) to individuals earning less than \( x_1^* \). The tax code will go along the Agency's isoprofit curve for citizens whose income is above \( x_1^* \), but lower than \( x_{mr} \). Here, \( x_1^* \) is a solution to equation (101), while formula (102) calculates the corresponding government revenue.

\[ x - X_{min} = \frac{m}{1 - F(x)} \] (101)

\[ g(m, X_{min}) = \int_{X_{min}}^{x_1^*} (x - X_{min}) dF(x) + m \ln \left( \frac{1 - F(x_1^*)}{1 - F(x_{mr})} \right). \] (102)
25. DEMOCRACY AND THE MOVEMENT FOR INDEPENDENCE.

When households are risk averse

The choice of \( m \) and \( X_{\text{min}} \) depends on the target revenue and the concavity of function \( u \), which may represent risk aversion. The more risk-averse the economy, the larger the after tax income the government will leave to the poor. On the other hand, this forces the government to allow a higher agency profit, as shown in figure 25. The bold black line represents the agency's profit and government revenue in the economy with higher risk aversion. In this case, the government collects the same revenue, but the loss related to tax avoidance is higher. Both rich and medium-income householders are worse off in a more risk-averse economy. The price for the tax avoidance service is higher and more households are involved in tax avoidance activities.

24.3 Inefficient Government and Intentions to Secede in the Union of Poor and Rich Regions

Now, I will consider a simple case where the break-down of a union with a uniform distribution of income occurs. For simplicity, let all individuals with an income below the union-median live in a region deciding on separation. Figure 26 provides the example when secession leads to an increase in households' private consumption in both regions. Government revenue and agency profit in a united economy are represented by the bold line. After the secession, the agency will become more powerful, since it will be able to price discriminate by regions. The agency may set a lower price for the poor region, thus allowing some rich individuals to save a larger share of their incomes. This would force the new government of the poor region to tax the poorest heavily, thereby reducing the minimum untaxed income. On the other hand, the government of the rich region will reduce taxes. As a result, all households in the rich region would benefit, either from the reduction of tax duties or the price for the tax avoidance service.

After the secession, the total revenue collected by the governments will fall, but private consumption in both regions will grow. Therefore, the total incentives for secession are higher, the lower households evaluate public goods. To avoid a secession, the government of the Union must reduce the target revenue, and the government can save the union and maintain the relevant benefits, like, for example, the economy of scale. Moreover, the government can ensure households' private consumption to be on the same level as after the secession, but collect a revenue significantly larger than the sum of revenues collected by both governments after the break up of the nation.

25 Democracy and the Movement for Independence.

It is obvious that not all countries should be considered as democracies. The votes of different individuals may be calculated with different weights representing political influence. If these weights are equal, then this political structure is a democracy. If only one person has a positive weight, this is perfect totalitarianism.
Figure 25: Different degrees of risk aversion

Figure 26: Before and after the secession: uniform income distribution.
The correction of vote weights can explain the violation of the Median Voter Theorem demonstrated in recent empirical works (Milanovic 2000). Voter weights may depend directly or indirectly on different factors. For example, political influence was directly related to race or gender in the United States in the 18th century. Nationality or ethnicity can be important for political influence nowadays. The rent-seeking model reflects the situation where individuals can change the political influence function by investing some effort, providing military service, for example. Thus, in the Soviet Union, political weights were attached to the position in the communist party’s hierarchy. Unequal distribution of political influence across regions may create additional reasons for secession. To avoid this, the Government of the Soviet Union has always selected the representative of the national majority for the highest party post, while the representatives of the other large ethnicity groups were appointed to deputy positions. Violation of this principle led to the first large nationalist demonstration in the capital of Kazakhstan at the end of 1986. The changes in the distribution of political influence may explain the difference in the outcomes of the referendums on the independency of the Soviet republics, which took place at the beginning and the end of 1991,

Although direct political discrimination is forbidden in many countries, indirect discrimination is still observed. Public choice may even be manipulated via the proposed default or “yes” option in a referendum. Such manipulation was observed in the referendums on the preservation of the USSR in 1991. Thus, in all-Soviet Union referendum in March 1991, voting “yes” meant supporting preservation, while in the referendums in the republics in late 1991, positive voting was equivalent to endorsing the separation. Indirect political influence can also be executed by managing the public opinion via mass media. In many countries, this control can be bought, which explains the necessity for large spending on pre-election campaigns. Bassett et al. (1999) propose a positive relation between income and political influence, which means that in more democratic countries, the outcome of choice is more profitable for the poor. I will take this assumption and discuss how the degree of democracy can have an impact on the union’s durability.

It may be the case that independence is very profitable for rich people in a poor region and deteriorates the situation of the poor, as shown in section 24. Under the assumption that the degree of political influence is positively related to individual income, an increase in democracy would give poor people more political power and reduce the attempts of secession in the poor region. The other example with two types of individuals, considered in section 23.2, predicts that the higher degree of democracy would increase the intentions to secede in the region with a larger proportion of poor individuals, but reduce such attitudes in the other region, in case households do not account for the decline in consumption of public goods. The public goods are more highly evaluated by poorer individuals, however, an thus, an increase in democracy should increase the degree of regional cooperation in general.

### 26 Concluding summary

I provide an additional dimension explaining the intention to secede, which relates to tax avoidance activities. The change in income distribution after a regional split may lead to a significant change in the price of the tax avoidance service and the tax codes of new states, which may create an increase in private consumption. My model predicts that the possibility of secession is higher, the higher is the tax burden in the union and the lower is the value of the provided public good in united regions.

The elite of the poorer region can gain from a decline in the tax avoidance price. Although the average tax burden might be lower after unification, rich householders in the poorer region might have considerable incentives to remain separate. Before unification, or after secession, they could enjoy a tax avoidance ability, or a lower tax level than in the united economy. On the other hand, the poor in the poor region might suffer, since the government would impose heavier tax duties. Separation incentives for the entire society are larger if the unified government has a higher target revenue.

The rich in the richer and the poor in the poorer regions might also have incentives for separation. My model predicts that in an economy with two types of agents, the household gains when the proportion of its own type grows. Thus, the rich in the region with a higher proportion of rich would be interested in secession, since a smaller proportion of rich might cause a more progressive tax and a higher price for the tax avoidance service. On the other hand, the poor in the poorer region might also be interested in

---

36 For details and an alternative explanation, see Austin (1994). The author explains the change in the result of the referendum to the opposite by a reduction in indirect subsidies via internal low prices, under which Russia provided raw materials to the other republics of the former Soviet Union. I think that the change in political influence is a more powerful explanation. The fact that in 1991, prices were not adjusted to world prices makes Austin’s points even weaker.

37 Sheshinski (2002) refers to social studies showing that people tend to choose the “default” alternative.
secession, since they carry the entire tax burden and the reduction of the proportion of poor would increase the individual tax duty.

The incentives for secession are lower if the government provides more highly evaluated public goods. After the secession, the tax avoidance provider receives additional power of price discrimination across regions. This puts the governments in a worse position, without an increase in the tax burdens, the governments would not be able to collect a pre-secession revenue. Moreover, separation leads to dyseconomy of scale, and the quantity and quality of public goods would become significantly lower. Anticipating these reductions in the provision of public goods, rational individuals may not wish to split the country, if evaluating the government service highly.
Bibliography


71
EFI
The Economic Research Institute

Reports since 1997
A complete publication list can be found at www.hhs.se/efi

Published in the language indicated by the title

2002
Barinaga, E., Levelling Vagueness – A study of cultural diversity in an international project group.
Mähring, M., IT Project Governance.
Tullberg, J., Reciprocitet – Etiska normer och praktiskt samarbete.

2001
Adolfson, M., Monetary Policy and Exchange Rates – Breakthrough of Pass-Through.
Andersson, P., Expertise in Credit Granting: Studies on Judgment and Decision-Making behavior.
Björklund, C., Work Motivation - Studies of its Determinants and Outcomes.
Charpentier, C., Uppfoljning av kultur- och fritidsförvaltningen efter stadsdelsnämndsreformen.
Ekelund, M., Competition and Innovation in the Swedish Pharmaceutical Market.
Engström, S., Success Factors in Asset Management.
Ericsson, D., Kreativitetsmysteriet – Ledtrådar till arbetslivets kreativisering och skrivandets metafysik.
Eriksson, R., Price Responses to Changes in Costs and Demand.
Frisell, L., Information and Politics.
Giordani, P., Essays in Monetary Economics and Applied Econometrics.
Gustavsson, P., Essays on Trade, Growth and Applied Econometrics.
Hedlund, A., Konsumentens erfarenhet – och dess inverkan på livsmedelsinköp på Internet.
Hvenmark, J., Varför socknar elden? Om utbrändhet bland chefer i ideella organisationer.
Hägglund, P.B., Företaget som investeringsobjekt – Hur placerare och analytiker arbetar med att ta fram ett investeringsobjekt.
Höök, P., Stridspiloter i vida kjolar, om ledarutveckling och jämställdhet.
Johansson, C., Styrning för samordning.
Josephson, J., Evolution and Learning in Games.
Kjellberg, H., Organising Distribution - Hakonbolaget and the efforts to rationalise food distribution, 1940-1960.
Lange, F. och Wahlund, R., Category Management – När konsumenten är manager.
Liljenberg, A., Customer-geared competition – A socio-Austrian explanation of Tertius Gaudens.
Lindkvist, B., Kunskapsöverföring mellan produktutvecklingsprojekt.
Ljunggren, U., Nyckeltal i grundskolan i Stockholms stad före och efter stadsdelsnämndsreformen.
Löf, M., On Seasonality and Cointegration.
Martensen, K., Essays on Entry Externalities and Market Segmentation.
Matros, A., Stochastic Stability and Equilibrium Selection in Games.
Mårtensson, P., Management Processes – An Information Perspective on Managerial Work.
Nilsson, A., Market Transparency.
Norberg, P., Finansmarknadens moralitet och det kalvinska kyrkorummet – En studie i ekonomisk mentalitet och etik.
Persson, B., Essays on Altruism and Health Care Markets.
Rech, G., Modelling and Forecasting Economic Time Series with Single Hidden-layer Feedforward Autoregressive Artificial Neural Networks.
Skoglund, J., Essays on Random Effects Models and GARCH.
Strand, N., Empirical Studies of Pricing.
Thorén, B., Stadsdelsnämndsforsvaret och det ekonomiska styrsystemet - Om budgetavvikeln.

**2000**
Brodin, B., Lundkvist, L., Sjöstrand, S-E., Östman, L., Koncernchefen och ägarna.
Bornefalk, A., Essays on Social Conflict and Reform.
Edman, J., Information Use and Decision Making in Groups.
Emling, E., Svenskt familjeföretagande.
Ericson, M., Strategi, kalkyl, känsla.
Hyll, M., Essays on the Term Structure of Interest Rates.
Håkansson, P., Beyond Private Label – The Strategic View on Distributor Own Brands.
I huvudet på kunden. Söderlund, M., (red).
Karlsson Stider, A., Familjen och firman.
Ljunggren, U., Styrning av grundskolan i Stockholms stad före och efter stadsdelsnämndsförsvar – Resultat från intervjuer och enkät.
Ludvigsen, J., The International Networking between European Logistical Operators.
Nittmar, H., Produktutveckling i samarbete – Strukturförändring vid införande av nya Informationssystem.
Stenström, E., Konstiga företag.
Sweet, S., Industrial Change Towards Environmental Sustainability – The Case of Replacing Chlorofluorocarbons.
Tamm Hallström, K., Kampen för auktoritet – standardiseringsorganisationer i arbete.

**1999**
Adler, N., Managing Complex Product Development.
Allgulin, M., Supervision and Monetary Incentives.

Andersson, P., Experto Credite: Three Papers on Experienced Decision Makers.

Ekman, G., Från text till batong – Om poliser, busar och svennar.

Eliasson, A-C., Smooth Transitions in Macroeconomic Relationships.


Hamrefores, S., Spontaneous Environmental Scanning.

Helgesson, C-F., Making a Natural Monopoly: The Configuration of a Techno-Economic Order in Swedish Telecommunications.


Lindé, J., Essays on the Effects of Fiscal and Monetary Policy.


Ljunggren, U., En utvärdering av metoder för att mäta produktivitet och effektivitet i skolan – Med tillämpning i Stockholms stads grundskolor.

Lundbergh, S., Modelling Economic High-Frequency Time Series.


Nilsson, K., Ledtider för ledningsinformation.


Sandström, M., Evaluating the Benefits and Effectiveness of Public Policy.

Skalin, J., Modelling Macroeconomic Time Series with Smooth Transition Autoregressions.


Strauss, T., Governance and Structural Adjustment Programs: Effects on Investment, Growth and Income Distribution.


Söderström, U., Monetary Policy under Uncertainty.


Wijkström, F., Svenskt organisationsliv – Framväxten av en ideell sektor.

1998


Berg-Suurwee, U., Styrning av kultur- och fritidsförvaltning innan stadsdelsnämndsreformen.

Berg-Suurwee, U., Nyckeltal avseende kultur- och fritidsförvaltning innan stadsdelsnämndsreformen.

Bergström, F., Essays on the Political Economy of Industrial Policy.

Bild, M., Valuation of Takeovers.


Gredenhoff, M., Bootstrap Inference in Time Series Econometrics.
Ioannidis, D., I nationens tjänst? Strategisk handling i politisk miljö – en nationell teleoperatörs interorganisatoriska, strategiska utveckling.

Johansson, S., Savings Investment, and Economic Reforms in Developing Countries.

Levin, J., Essays in Company Valuation.

Ljunggren, U., Styrning av grundskolan i Stockholms stad innan stadsdelsnämnders reformen.

Mattsson, S., Från stat till marknad – effekter på nätverksrelationer vid en bolagiseringsreform.

Nyberg, A., Innovation in Distribution Channels – An Evolutionary Approach.


Reneby, J., Pricing Corporate Securities.

Roszbach, K., Essays on Banking Credit and Interest Rates.

Runsten, M., The Association Between Accounting Information and Stock Prices. Model development and empirical tests based on Swedish Data.


Sjögren, A., Perspectives on Human Capital: Economic Growth, Occupational Choice and Intergenerational Mobility.

Studier i kostnadsintäktsanalyse. Jennergren, P., (red)

Söderholm, J., Målstyrning av decentraliserade organisationer. Styrning mot finansiella och icke-finansiella mål.

Thorburn, K., Cash Auction Bankruptcy and Corporate Restructuring

Wijkström, F., Different Faces of Civil Society.


1997

Alexius, A., Essays on Exchange Rates, Prices and Interest Rates.

Andersson, B., Essays on the Swedish Electricity Market.

Berggren, N., Essays in Constitutional Economics.


Charpentier, C., Budgeteringens roller, aktörer och effekter. En studie av budgetprocesserna i en offentlig organisation.


Friberg, R., Prices, Profits and Exchange Rates.

Från optionsprissättning till konkurslagstiftning. Bergström, C., Björk, T., (red)

Hagerud, G.E., A New Non-Linear GARCH Model.


He, C., Statistical Properties of Garch Processes.

Holmgren, M., Datorbaserat kontrollrum inom processindustrin; erfarenheter i ett tidsperspektiv.


Lange, F., Wahlund, R., Planerade och oplanerade köp - Konsumenternas planering och köp av dagligvaror.

Löthgren, M., Essays on Efficiency and Productivity; Contributions on Bootstrap, DEA and Stochastic Frontier Models.

Sjöberg, L., Ramsberg, J., En analys av en samhällsekonomisk bedömning av ändrade säkerhetsföreskrifter rörande heta arbeten.
Säfvenblad, P., Price Formation in Multi-Asset Securities Markets.
Sällström, S., On the Dynamics of Price Quality.
Södergren, B., På väg mot en horisontell organisation? Erfarenheter från näringslivet av decentralisering och därefter.
Thorén, B., Berg-Surwee, U., Områdesarbete i Östra Hökarängen - ett försök att studera effekter av decentralisering.
Ählström, P., Sequences in the Profess of Adopting Lean Production.
Åkesson, G., Företagsledning i strategiskt vakuum. Om aktörer och förändringsprocesser.
Åsbrink, S., Nonlinearities and Regime Shifts in Financial Time Series.