

# ESSAYS ON ENTRY EXTERNALITIES AND MARKET SEGMENTATION

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# Essays on Entry Externalities and Market Segmentation



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STOCKHOLM SCHOOL OF ECONOMICS  
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*for my father Johan Martensen*





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## PREFACE

While I passed through the undergraduate courses in economics, I doubted that the knowledge of economics would provide me anything over and above my daily bread in the form of a paycheck. The first courses brought more confusion than clarity – how could one possibly try to analyze an economy with several disparate models, not all (any?) of them consistent with the other. Coming from a background in mathematics, I saw no structure.

The first light came when Jörgen W. Weibull agreed to be my undergraduate thesis supervisor. He showed me how to build a model that was internally consistent – by that time, perhaps already indoctrinated by my undergraduate years, I had since long stopped thinking of consistency across models. Jörgen – a great pedagogue, showed us graduate students how to climb the theoretical mountain to gain a perspective. This was a main inspiration.

The first two years of graduate studies changed fundamentally the way I view the world, by giving me a framework and the tools to understand “everyday life” in a new perspective. I think that this is the personal gift that Science bestows on its followers, although it came as an unexpected but welcome surprise to me. Then came the harder but more interesting part – pushing the research frontier forward.

My deepest felt thanks goes to Tore Ellingsen, my supervisor at the Stockholm School of Economics. Tore, with his outstandingly clear intuition and breadth, showed me how to actually push the frontier forward. Thank you for all the encouragement and support.

Johan Stennek and Jörgen were also helpful, particularly with my Licentiate paper. Karl Wärneryd, always with an open door and at-the-point comments, which significantly improved my papers, deserves a special thanks. Many thanks also to Richard Friberg, who is co-author on two of the papers. I learned a lot from working with Richard. In particular, he showed me how to find what really are the economically interesting problems, and further set the mark for me on how to write in a concise and lucid manner.

A part of my graduate program was located at the Department of Economics at Harvard University, which was a valuable experience, and I thank Jörgen for getting me there. I would also like to thank Tore, Jörgen and especially Renato Fragelli Cardoso and the Fundação Getulio Vargas (FGV) for the opportunity to do research at FGV. There, I am very thankful to Marcos de Barros Lisboa for generously sharing his advice. The excellent handling of administrative matters by Clemente Gonzaga Leite and his staff is gratefully acknowledged.

At Harvard, the presence of my friend Magnus Allgulín from Sweden made the stay so much more enjoyable. At FGV, the fellow Ph.D. students and, in particular Mônica Viegas Andrade, showed me a real Carioca hospitality and friendship. Further, Johan Kejerfors was very generous with his help and friendship.

Here, at the Stockholm School of Economics, I was privileged to have Mats Ekelund, Niklas Strand, Rickard Eriksson, Sven Skallsjö, Jianying Liu-Wijkander, Katariina Hakkala, Pernilla Sjöquist, Björn Persson, Chloe Le Coq, Arvid Nilsson and every one at Saltmätargatan 11, as friends and fellow Ph.D. students. Time is a scarce commodity, and I would like to thank them all for taking their time to comment on my research and just being great friends.

I was very fortunate with roommates over the years. In particular, sharing room with Mats Ekelund has been a positive experience; almost everything imaginable has been subject to economic analysis in interesting discussions. I am happy to count him as one of my closest friends.

The Stockholm School of Economics has provided the complementarities necessary for research and an excellent environment. I would like to thank Bengt Jönsson and Lars Bergman and all faculty members for the creation of this ambience. Further, administrative matters have been handled with great diligence by Pirjo Furtenbach, Kerstin Niklasson, Ritva Kiviharju, Ingrid Nilsson, Britt-Marie Eisler, Britt-Marie Östling, Titti Unckel and Azad Saleh, and I owe them my gratitude.

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Last, but not definitely not least, a great thanks to my family and friends for putting up with me during these years.

I dedicate this dissertation to my father Johan Martensen who passed away in December 1996. Pappa, du var mitt starkaste stöd och närmast mitt hjärta. Jag kommer alltid att älska dig.

Stockholm, April 2001.

Kaj Martensen

# SUMMARY OF THE ESSAYS

Essays on Entry Externalities and Market Segmentation consists of four papers. The first two essays of this doctoral dissertation deal with entry externalities, the third studies the Law of One Price (LOP), while the last essay examines average profits for a monopolist under uncertainty.

## Essays I and II

The first two essays consider strategic interaction under entry externalities. When a firm enters a market, it often imposes externalities on existing firms and/or future potential entrants. If products are substitutes, these externalities are typically negative; if products are complements, the externalities are typically positive. Externalities related to substitution or complementarities between products are called payoff externalities, since entry by one firm has a direct effect on the other firms' payoff. Another type of externality arises when firms have private information about the profitability of entry. In this case, the entry decision of one firm potentially reveals that firm's private information.

The two essays analyze these externalities and generally find that strategic interaction hinder firms from realizing what would be optimal from a welfare perspective. Of course, a firm's objective is to maximize its own profits, rather than the welfare of society. However, it is often the case that the firms would be better off individually, if an uninformed planner intervened. The scope for intervention is examined, and I give conditions for when intervention by a "bumbling bureaucrat" (uninformed social planner) is possible.

In the first essay, coordination under incomplete information and weak payoff complementarities is studied. Specifically, potential entrants can endogenously decide on their time of entry. Further, each entrant has private information about market profitability. Entry is assumed risky and private information is then crucial for the entry decision. Entrants then have an incentive to delay entry in order to learn other firms' private signals. However, since there are positive payoff externalities, that is, the profit of one firm increases with entry by other firms, there is also an incentive to enter early in order to signal that market conditions are profitable. If a firm signals that the entry is profitable, then it might attract other entrants, which, in turn, will increase its profits. I show that there is insufficient entry in equilibrium. The main finding is that an uninformed social planner can always increase welfare by subsidizing early entrants. However, the scope for intervention decreases with payoff complementarities. Further, I show that lump sum subsidies can impede learning and that welfare is a non-monotonic function of subsidies. I also show that it can be justifiable to bail out early entrants that have been left stranded. Hence, this fits well with the observation that subsidies are given to firms entering markets that have for a long time been considered as unprofitable.

Continuing on this theme, the second essay studies the same type of problem, but instead takes the time of entry as fixed, while generalizing the analysis of payoff externalities also to the case of negative payoff externalities. I find that under symmetric information, there can be inefficient entry only if there are negative payoff externalities (i.e. if entry decisions are substitutes), and hence, insufficient entry never takes place.

Under asymmetric information, there are several distortions coming from payoff and information externalities in a non-separable way. In particular, insufficient entry is now a possibility both for strong positive and negative payoff externalities. However, I find that entry subsidies can be justified only when entry decisions are weakly complementary. Again, also in the case of sequential entry, saving a stranded early entrant can increase welfare.

### Essay III

In the third essay, written jointly with Richard Friberg, deviations from the LOP are studied in the presence of transport costs and further, under the assumption that firms can endogenously choose to segment markets in order to prevent arbitrage by consumers.

To the surprise of many, price deviations between markets characterized by imperfect competition have often been little affected by lower transport costs. For example, price differentials on cars between European countries have been remarkably unaffected by lower border barriers. After all, if consumers can choose where to shop, we should expect the price differential to equal only the consumers' cost of transporting the good from one country to the other. If not, the country with the higher price would lose all its customers who can find a cheaper good in the country with a lower price.

We show that if a firm's decision to segment markets is endogenous, then lower transport costs are, in many cases, associated with a greater deviation from the law of one price. This result extends from a general monopoly setting to the case of Cournot competition. The intuition is that lower transport costs, by facilitating arbitrage, place a tighter restriction on the maximization problem and a firm is willing to take a greater cost in order to segment. We examine how the resulting equilibria depend on transport costs, product differentiation and costs of segmenting. Further, examining the implications of country asymmetries, we find that when goods are differentiated, the firm from the poorer market has a greater incentive to segment markets, whereas when goods are homogeneous, the firm from the richer market has the greater incentive.

### Essay IV

In the last essay, written jointly with Richard Friberg, we study the problem facing a monopolist when the cost of inputs is uncertain. The article is inspired by the seminal article by Walter Oi (*Econometrica* 29, 1961). Oi shows that the average profits of a price taker are increasing in the variability of the output price. We show that, for the same reason, the average profits of the price taker are increasing in the variability of the price of inputs. We then proceed to establish that the same holds for a firm with a downward sloping demand curve. Unless the inverse demand curve of the firm with market power is very convex, the profit function of the price taker forms an upper limit for the convexity of profit (assuming constant curvature of costs). In deriving the above results, we specified no specific demand or cost function, and thus, Oi's result does generalize to a considerable extent.

## CHAPTER 1

# **Informational Externalities, Technological Complementarities and Industrial Policy**





# Informational Externalities, Technological Complementarities and Industrial Policy\*

Kaj Martensen

## Abstract

This paper studies coordination under incomplete information and payoff complementarities. Specifically, potential entrants can decide on their time of entry. Each entrant has private information about market profitability. The main finding is that an uninformed social planner can always increase welfare by subsidizing early entrants. However, the scope for intervention decreases with payoff complementarities. Further, we show that lump sum subsidies can impede learning and that welfare is a non-monotonic function of subsidies. We also show that it can be justifiable to bail out early entrants that have been left stranded.

## 1 Introduction

Payoff complementarities have long been considered as the main reason for industrial policy. Nevertheless, planning in the 1950s and 1960s targeted at realizing positive payoff externalities by selective investments and transfers failed; “Such planning failed in part because it attempted to concentrate all relevant information in the government planning bureau. This was simply impossible; information is too diffuse, too complex.” (World Bank, 1993, p. 92). It is therefore important to examine the scope for intervention under the assumption that information is private to firms.

Our contribution is that we allow firms to learn about market conditions from each other, that is, the firms have the option to delay in order to learn from others’ entry, and we will offer a theory of an optimal decentralized solution where firms themselves decide on the merit of entry in a sector. In particular, we will look at whether a “bumbling bureaucrat” (uninformed planner) can increase welfare, and if so, how.

In a simple model, we show that there is insufficient entry given these entry externalities; firms that initially have received private information in favor of entry, delay too much in equilibrium, which gives insufficient entry. The main finding is that an uninformed planner always can improve on welfare by offering entry subsidies. Empirically, the entry pattern predicted by our model is well established; IMF finds uncertainty to

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\*I would like to thank Richard Friberg, Chloe Le Coq and especially Tore Ellingsen for valuable comments. I am grateful to the Wallander and Hedelius Foundation for financial support.

be the main reason for delay in investment in their analysis of developing countries. Further, "...since firms have incomplete information and may in practice gauge investment prospects partly by observing competitors' and partners' behavior, the economy may get locked into a low-investment and low-growth equilibrium." (IMF, 1996, p. 39).

This paper is related to the literature on intervention in the presence of complementarities, which we extend by allowing for information externalities. The unifying idea of this large body of literature is that the private and social benefit of entry can diverge, because entry by a firm may create non-appropriable spillovers. Typically, this leads to insufficient investment from a welfare perspective. Sources of positive payoff externalities are notably "learning by doing" (Arrow 1962) and network externalities (see, for example, Katz and Shapiro 1985). This line of thought potentially justifies Big Push intervention (see Murphy, Shleifer and Vishny, 1989) and has also been used to motivate protection of infant industries (see Corden, 1997, Ch. 8 for an overview).

Our work is of course also related to the literature on information externalities. Previous work by Chamley and Gale (1994) and Alexander et al. (1998), among others, have shown that if firms have the incentive to free ride on other firms' informative entry, then firms enter "too slowly" and, from a welfare perspective, there will not be enough entry.<sup>1</sup> Largely, this literature does not allow for payoff externalities. In particular, in the few cases both types of externalities are considered, the scope for intervention is not the focus.

Thus, our purpose is to investigate how positive payoff externalities and private information interact to determine entry into a market and to compare the equilibrium entry pattern to the socially optimal one. To fix ideas, we consider an economy with a benevolent but uninformed social planner and a number of privately informed potential entrants in a new sector. Entry by one firm in this sector benefits the other firms; there are complementarities in entry and further, entry by one firm conveys that this firm has an optimistic view of the market, in effect the entrant's private information. Since there is then an incentive for firms to wait and learn, we allow firms to choose their time of entry. Entry is assumed to be risky and private information is crucial in determining the profitability of entry.

We find that an uninformed social planner can always increase welfare by offering entry subsidies. Nevertheless, the scope for intervention decreases with complementarities. There is insufficient entry in equilibrium and even optimistic firms will sometimes never enter. However, we show that subsidies that ensure entry by optimistic firms with certainty are never optimal when entry is risky. Instead, a subsidy must be chosen to induce a subsidized mixed equilibrium. The intuition is that an optimal subsidy must balance the risk of entry taking place when the state is bad versus the risk of insufficient entry when the state is good. A high subsidy, set to induce early entry, also has the effect that few firms wait and learn. On the other hand, with a low subsidy, there are too few early entrants which results in insufficient entry. Further, the unsubsidized symmetric equilibria have the same (low) welfare properties as a game where no learning takes place. Surprisingly, information *is* transmitted efficiently in equilibrium; given the first period

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<sup>1</sup>The paper by Chamley and Gale is related to earlier papers on pure information externalities; see Banerjee (1992) or Bikhchandani et al. (1992).

entry pattern, firms can infer whether entry is profitable in the next period. Instead, the amount of positive payoff externalities is too low in equilibrium – there are too few early entrants providing payoff externalities. With subsidies to early entry, the efficient amount of externalities will be provided in equilibrium.

We also consider the efficiency of “bailing out” an entrant that has been left stranded. Consider the situation where a leader enters in the first period and there is no subsequent entry. This means that the leader was the only firm with an optimistic signal and that the expected industry profit is negative. However, in some cases, the expected industry profits might increase if other firms enter to provide complementarities. We show that this can be profitably achieved through a subsidy to late entrants given that there is a single entrant in period one. Hence, this fits well with the observation that subsidies are given to firms entering markets that have for a long time been considered as unprofitable. Note that the subsidy is welfare improving even if it sometimes goes to firms that would enter in the second period even without a subsidy. To minimize the risk of firms using this informational rent, the optimal speed of entry is now higher than in the game with only a first period subsidy. Finally, we find that these results depend crucially on incomplete information; the less noise in the firms’ private information, the less inefficiency. Specifically, under complete information, firms will enter efficiently.

Our setup takes its main motivation from the quote below (World Bank, 1993, page 92), and will use it to further discuss our assumption that information and payoff externalities are crucial to the understanding of insufficient entry in new markets.

...if a steel plant and a steel-using industry are needed concurrently, it does not pay to develop the steel-using industry unless there is a plant. If each awaits the other, nothing happens.

This paper aims at understanding this interaction by assuming private information to be an important determinant for firms’ investment decisions. Lack of information has, for example, been given as an explanation for the difficulty in achieving growth in the new Eastern European market. An OECD study (1995, p. 20) states “One of the greatest negative factors inhibiting the attraction of FDI is the uncertainty that surrounds doing business in the region ...”. Further, in their poll of firms, Reuben et al. (1973) find that firms regard information about market and product conditions in the host country as the most important problem when deciding on foreign investment.

In contrast to our approach, there is a strand of literature taking the view that the government has superior information about market conditions and tries to signal its information via taxes and tariffs; see Raff and Srinivasan (1998) and Bond and Samuelson (1986). However, to quote Corden (1997, p. 142):

Why should the private firm (or state enterprise) concerned have less information about the prospects for its own cost curves than a central state authority? The planners or civil servants may be more optimistic than private firms and more ready to speculate about the future because they will not personally have to meet the losses if the risks do not come off.

The example given by the World Bank also illustrates that investors take a wait-and-see attitude.<sup>2</sup> It is then important to allow for firms to choose their time of entry in a setting with strategic interaction. Generally, waiting games will be inefficient from a welfare perspective, since costs of delay dissipate profits; see, for instance, Chamley and Gale (1994) or Choi (1997) in the context of information externalities. Another example is Bolton and Farrell (1990) who compare the benefit of a centralized solution to a decentralized solution in a situation where firms with privately known costs consider entering into a natural monopoly. The authors show that there is scope for an uninformed social planner to increase welfare, if firms wait too long before investing. In contrast, we will assume that there is no cost of delay and thus, our results on inefficiency will be stronger in the sense that delay does not dissipate profits.

In this context, it is often asserted that insufficient entry can be remedied by subsidies to entering firms. For instance, Thimann and Thum (1998) state in their conclusions that “Subsidizing early investment in the first period can be justified...”, in the context of subsidizing a first entrant to provide a positive information externality.<sup>3</sup> However, so far, the literature has generally failed to recognize that subsidies can impede the transmission of information. In particular, we show that welfare can be a non-monotonic function of lump sum subsidies, without invoking adverse selection or distortions related to the financing of subsidies. As mentioned, our work is related to the literature on information externalities, where Chamley and Gale (1994) and Alexander et al. (1998), among others, have shown that firms enter too slowly from a welfare perspective. Largely, this literature does not allow for payoff externalities. The exceptions are Alexander et al. (1998), Choi (1997), Martensen (see Ch. 2) and Dasgupta (2000). Alexander et al. do not focus on intervention and welfare, which is the topic of this paper. Choi, Martensen and Dasgupta focus on sequential entry while we examine the interaction of timing and transmission of information.<sup>4</sup> Further, Rob (1996) and Creane (1996) study information aggregation in competitive markets where entry resolves uncertainty about the size of the market. In their models, entry is sequential and there is no strategic interaction. The papers most relevant for our analysis is Chamley and Gale (1994) and Alexander et al. (1998), and we will relate our results to theirs in the appropriate sections of this paper.

Note that in the theory of industrial organization, entry under asymmetric information is by and large treated as a capacity, pricing or quantity signal problem for the incumbent; see, for example, Milgrom and Roberts (1982). Our approach differs in that it studies the informational aspects of entry in itself; we do not allow “investments in disinformation”.

Information externalities and search theory have also been used to explain FDI pat-

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<sup>2</sup>For an additional discussion, see World Bank, 1994, p. 154.

<sup>3</sup>Other examples include Chamley and Gale (1994). Our results are related to theirs in Section 4.2.1. There are cases where a subsidy does not have this conflicting role. For instance, Rob (1991) recognizes that early entry should be subsidized to increase the amount of information available to late entrants. This does not create distortions in his model since the number of entrants does not add up; regardless of the number of entrants in the first period, there is always the same number of potential entrants in later periods.

<sup>4</sup>Choi briefly examines the waiting game induced by endogenous timing. We relate our results to his in Section 4.2.1.

terns. Firms have incentives to free ride on other firms' costly search for investment opportunities, since an entry decision has a positive information externality; see Thi-  
mann and Thum (1998) or Huang and Shirai (1994). These papers do not consider  
payoff externalities, however. Finally, asymmetric information has been given as a rea-  
son for intervening and protecting infant industries. These approaches focus on adverse  
selection and moral hazard of firms entering product markets; see Grossman and Horn  
(1988) and Flam and Staiger (1989). They differ from us in that they do not consider  
the dynamic effects of learning. Most importantly, our results do not depend on adverse  
selection.

The model is presented in Section 2. The analysis is carried out in Section 3, where  
we derive the equilibria of the entry game. The scope for intervention is examined in  
Section 4, and we conclude in Section 5.

## 2 Model

To formally capture the linkage between information and payoff externalities, we study  
two risk neutral expected profit maximizing firms, A and B. Both firms receive a private  
signal about the value of an entry opportunity and chooses whether to enter (and take  
a sunk cost) or stay out in period one or two, respectively. The private "market infor-  
mation" can, for example, be information on the size of the market, or more generally,  
anything influencing the value of the entry opportunity. Thus, two firms, A and B, both  
consider entering into a new market. Entry is irreversible and associated with the fixed  
cost  $c$ . Capital markets are assumed to be incomplete, so that no firm can bear the cost  
 $c$  twice. The timing and order of entry is assumed to be endogenous in the two time  
periods,  $t = 1, 2$ . Specifically, the firm can choose whether to enter in period one or two  
but once a firm has entered, it will stay in the market for the remainder of the game.  
Entry is assumed to be observable with a time lag so that a firm can only learn if it delays  
entry to period two in order to observe entry in period one.

Profits to entry depend both on the state of the market and the number of entrants.  
There are two states of the market,  $H$  and  $L$ , and the two firms share the common  
prior belief that both states are equally likely so that  $\Pr(H) = \Pr(L) = 0.5$ . The payoff  
externality is captured by the parameter  $k$  which relates the ex post payoff for a single  
entrant to the profit it would obtain if two firms entered. There are two signals,  $\sigma^A$  and  
 $\sigma^B$ , on the true state of the market where,  $\sigma^A, \sigma^B \in \Sigma = \{\sigma_H, \sigma_L\}$ .<sup>5</sup> The signals are  
drawn independently from the conditional Bernoulli distribution

$$\begin{aligned}\Pr(\sigma_H|H) &= \Pr(\sigma_L|L) = p, \\ \Pr(\sigma_L|H) &= \Pr(\sigma_H|L) = 1 - p,\end{aligned}$$

where  $p > 0.5$ . The informativeness of the signals is thus given by the parameter  $p$  and

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<sup>5</sup>With a slight abuse of notation, we let  $\sigma$  denote both the stochastic variable and its realizations.  
Which applies will be clear from the context.

the signals are informative and of equal precision.<sup>6</sup> The signal  $\sigma_H$  will be referred to as a *high signal*, since the state it is more likely to be high, given this signal. Correspondingly, the signal  $\sigma_L$  will be referred to as a *low signal*. Further, a firm receiving the signal  $\sigma_H$  will be referred to as an *optimistic firm* and similarly, a firm receiving the signal  $\sigma_L$  will be referred to as a *pessimistic firm*.

The timing is as follows: In period  $t = 0$ , Nature chooses the state of market  $x \in \{H, L\}$  and the firms receive their respective private signal. At periods  $t = 1, 2$ , firms are free to enter or stay out.

Thus, a firm's strategy for the first period is  $s_1 : \Sigma \rightarrow [0, 1]$ , i.e. a mapping from its signal to the probability of entry in the first period. Similarly, a firm's strategy for the second period is a mapping  $s_2 : \Sigma \times \{0, 1\} \rightarrow [0, 1]$  from its signal and its observation,  $y \in \{0, 1, 2\}$  of the number of entrants in the first period to the probability of entry in the second period. Since a firm can only enter once, we have  $s_2 = 0$  given  $y = 2$ .

The ex post revenue  $R(x, n)$  to a firm depends on the state  $x$ , and the number of entering firms  $n$ . The high state gives a positive revenue and the low state gives zero revenue to the entering firm. Thus,

$$R(H, n) > 0 \text{ for } n = 1, 2.$$

To formalize the payoff interdependence, define

$$k = \frac{R(H, 2)}{R(H, 1)}.$$

Further, we will say that a strategy profile is an equilibrium if it is a perfect Bayesian equilibrium.

**Assumption A1:**  $k > 1$ .

We assume that  $k > 1$  throughout, so that entry decisions are complements. Nevertheless, we will sometimes find it useful to examine the case  $k = 1$ , but then this will be clearly stated. By the statement "entry decisions are complements", we mean that the expected profit of entry for one firm increases in the entry of the other firm, holding the strategies and information available to the firms fixed. That is, effects of complementarity will refer to payoff interdependence and not to strategic complementarity.

Without loss of generality we can normalize the ex post revenue for a single firm under the high state.

**Assumption A2:**  $R(H, 1) = 1$ .

The complementarity gains can not be unlimited; for there to be a value of information in this model, entry is assumed unprofitable a priori.

**Assumption A3:**  $\Pr(H) R(H, 2) - c < 0$ .

To show that insufficient entry can occur even under weak conditions we assume that the

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<sup>6</sup>If  $p < 1/2$ , this would only mean that the firms reversed the meaning they attach to their signal and the same results would obtain.

entry opportunity is profitable for a single entrant given one high signal.

**Assumption A4:**  $\Pr(H|\sigma_H) R(H, 1) - c > 0$ .

In terms of the parameters  $p, k$  and  $c$ , Assumption A3 can be written as

$$k < 2c. \quad (1)$$

Further, Assumption A4 is equivalent to

$$p - c > 0. \quad (2)$$

Since  $k > 1$ , there is then a restriction on  $c$

$$c > \frac{1}{2}. \quad (3)$$

By equations (3) and (2), we then have  $c \in (1/2, 1)$ .

It is now convenient to define some expressions that will frequently occur in the following analysis.

**Definition 1** Let the constants  $D, N \in R_{++}$  be defined by<sup>7</sup>

$$\begin{aligned} N &= p^2 k + p(1 - p) - c, \\ D &= p^2 k - c(p^2 + (1 - p)^2). \end{aligned}$$

### 3 Analysis

In this section, we analyze the equilibria of the game. Note that we only need to analyze the decision problem for firms with high signals, since firms with low signals never consider entry. Entry is thus informative; a firm observing that another firm enters can infer that the firm has an optimistic view of the market or, in other words, that the entrant has a high signal. We will focus on symmetric equilibria, where optimistic firms pursue the same strategy, since a firm never wants to assume the role of a first entrant in an asymmetric equilibrium; the late entrant is always better off in terms of expected profits given entry, since it can learn the early entrant's signal.<sup>8</sup>

To put the below analysis in perspective, it is useful to first consider the implications of our assumption of the value of information versus the value of complementarities. Consider the case when Assumption A3 of the value of information does not hold. If A3 does not hold, this, in particular, means that

$$\Pr(H|\sigma_H, \sigma_L) k - c > 0,$$

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<sup>7</sup>It is easy to see that  $D, N \in R_{++}$ . We have  $N > 0$ , since  $p^2 k + p(1 - p) - c > p^2 k + p(1 - p) - p = p^2(k - 1) > 0$ , where the first inequality follows from Assumption A1, and the second inequality from A4. Further,  $D > 0$  since  $N > 0$  and  $D - N = p(1 - p)(2c - 1) > 0$  by Eq (3) and  $p > 1/2$ .

<sup>8</sup>For completeness, we examine also the existence of asymmetric equilibria, where optimistic firms pursue different strategies. The analysis is given in the Appendix, Section 6.1.

since the conflicting signals cancel out so that  $\Pr(H|\sigma_H, \sigma_L) = \Pr(H)$ . Thus, even if a firm with a high signal knows that the other entrant has a pessimistic view of the market, the expected profit from entry is positive. Then, an optimistic firm will enter in the first period, since it knows that upon observing entry, the pessimistic firm will also find it profitable to enter. Hence, if information is not important, then firms will enter efficiently. This reasoning easily extends to the case where complementarities are sufficiently strong to make entry profitable, even when both firms take a pessimistic view of the market. Thus, if information is relatively unimportant, then we would not expect to find waiting in equilibrium. However, the wait-and-see behavior of investors given incomplete information is strongly supported by empirical evidence; for instance, as mentioned earlier, IMF finds that uncertainty is the main reason for delay in investment in their analysis of developing countries (IMF 1996).

### 3.1 Symmetric equilibria

In this section, we study the symmetric equilibria of our model. The main focus will be on the equilibrium in mixed strategies, since we will show that the equilibrium in pure strategies is not robust to a small perturbation in the form of a cost of delay. There exists an equilibrium in mixed strategies, since an optimistic firm faces two potentially profitable choices; by early entry, a firm with a high signal can signal that it has an optimistic view of the market which can, in turn, induce complementary entry. On the other hand, by delaying entry, a firm can reduce the risk of entry, since it might learn that the other firm has a pessimistic view of the market, which makes entry unprofitable. Thus, it is the interaction between incentives to signal, and incentives to learn that determines the equilibrium.

For equilibria in mixed strategies, we know that if entry takes place in period one, any remaining non-entrant with a high signal will enter in period two. The more interesting case is when no entry takes place in period one, and its consequence for entry in the second period. We will refer to this case as the *second period continuation game* (SPCG) hereafter. In the SPCG, the inference drawn by the event “no entry in period one” will depend directly on the probability of first period entry; if, for instance, this probability is low, the event is more likely to have taken place even if entrants are optimistic, which makes the informational content low.

We will show that there exist two symmetric equilibria in mixed strategies. These equilibria differ *only* in the probability of second period entry. Then, in particular, the inference drawn by the event “no entry in period one” will be the same, since the first period entry probabilities in the two equilibria are equal. Nevertheless, even if the inferences are the same, the learning aspects of the equilibria are fundamentally different; the firms must use the revealed information in only one of the equilibria, while in the other equilibrium, revealed information can be ignored. Thus, it is important to make a distinction between learning just as a consequence of inference, and learning in the sense that information is not ignored. We will take learning to mean the latter henceforth.



**Definition 2** We will say that a firm “learns”, if each of its optimal second period decisions are contingent on revealed information.

### 3.1.1 Pure strategy equilibria

First, we prove the existence of a symmetric equilibrium in pure strategies. It is instructive for the following analysis to do a longer proof than strictly necessary.

**Claim 1** *There is a unique symmetric equilibrium in pure strategies. In this equilibrium, firms learn nothing.*

**Proof.** Assume that a firm A, given a high signal, enters in the first period. If firm B has a high signal, its expected profit of entering in period one is<sup>9</sup>

$$\begin{aligned}\pi_1 &= \Pr(\sigma_H|\sigma_H) \Pr(H|\sigma_H, \sigma_H) k + \Pr(\sigma_L|\sigma_H) \Pr(H|\sigma_L, \sigma_H) - c \\ &= N.\end{aligned}$$

If firm B instead waits until period two, it can infer firm A’s signal by its entry decision. Since entry is not optimal given that firm A has a low signal, firm B will only enter if A entered. The expected profit is then

$$\pi_2 = \Pr(\sigma_H|\sigma_H) (\Pr(H|\sigma_H, \sigma_H) k - c) = D.$$

Then, B will always find it optimal to wait until the second period, since

$$\pi_2 - \pi_1 = p(2c - 1)(1 - p) > 0,$$

where the inequality follows from Eq. (3). On the other hand, given that B always waits until the last period, A can learn nothing by delaying entry. Firm A is then indifferent between entry in period one and period two. Thus, the only symmetric equilibrium is for both firms to enter in the last period, where none of them can learn from the other. ■

Note that if the equilibrium strategies are such that firms enter simultaneously in the last period, then the two interpretations of learning coincide, since if nothing is inferred, nothing can be learned. Further, note that this equilibrium is not robust to the introduction of an arbitrarily small cost of delay. If there is a small cost of delay, then there is no symmetric equilibrium in pure strategies; both firms would then strictly prefer to enter in the first period instead, but this is not an equilibrium, since given that one firm enters in the first period, the other prefers to wait – an asymmetric equilibrium.

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<sup>9</sup>Since the signals are independent given the state, we have  $\Pr(\sigma_H^B, \sigma_H^A|H) = p^2$  and  $\Pr(\sigma_H^B, \sigma_H^A|L) = (1-p)^2$ . Thus,  $\Pr(\sigma_H^B|\sigma_H^A) = \Pr(\sigma_H^B, \sigma_H^A)/\Pr(\sigma_H^A) = 2\Pr(\sigma_H^B, \sigma_H^A) = 2(\Pr(\sigma_H^B, \sigma_H^A|H)\Pr(H) + \Pr(\sigma_H^B, \sigma_H^A|L)\Pr(L)) = \Pr(\sigma_H^B, \sigma_H^A|H) + \Pr(\sigma_H^B, \sigma_H^A|L) = p^2 + (1-p)^2$ . Further,  $\Pr(H|\sigma_H^A, \sigma_H^B) = \Pr(H, \sigma_H^A, \sigma_H^B)/\Pr(\sigma_H^A, \sigma_H^B) = \Pr(\sigma_H^A, \sigma_H^B|H)\Pr(H)/\Pr(\sigma_H^A, \sigma_H^B) = p^2/(p^2 + (1-p)^2)$  by the previous derivation.

### 3.1.2 Mixed strategy equilibria

Given that entry takes place in period one, any remaining non-entrant with a high signal will enter in period two. Thus, we only need to analyze the equilibrium in period two given that no entry took place in period one – the SPCG. Let  $\lambda$  be the probability of entry in the first period for an optimistic firm and  $\mu$  the probability of entry in the SPCG.

**Period two of the SPCG** In order to perform an equilibrium analysis, we first need the second period expected profits. Since a firm with a low signal will not enter, we analyze the problem for a firm with a high signal, say firm A. Define

$$\begin{aligned}\lambda &= \Pr(En1|\sigma_H^A), \\ \mu &= \Pr(En2|\sigma_H^A, \sim En1),\end{aligned}$$

where  $En1$  is the event “firm B enters in period one”,  $En2$  is the event “firm B enters in period two” and  $\sim En1$  is the complement of  $En1$ . Given  $\mu$  and  $\lambda$ , firm A’s expected profit from entry is (see Appendix, Section 6.2 for a derivation of the probabilities),

$$\begin{aligned}\pi_2^{Enter}(\lambda, \mu) &= \Pr(H, En2|\sigma_H^A, \sim En1)k + \Pr(H, \sim En2|\sigma_H^A, \sim En1) - c \\ &= \frac{p^2\mu(1-\lambda)k + p^2(1-\mu)(1-\lambda) + p(1-p)}{(p^2 + (1-p)^2)(1-\lambda) + 2p(1-p)} - c,\end{aligned}$$

where  $\sim En2$  is the event that “firm B does not enter in period two”. The profit of staying out is 0. To be indifferent between entry and non entry in period two, given no entry in period one, it must hold that

$$\pi_2^{Enter}(\lambda, \mu) = \pi_2^{NotEnter} = 0. \quad (4)$$

Thus, for firm A to be indifferent,  $\mu$  must solve

$$\pi_2^{Enter}(\lambda, \mu) = 0,$$

which gives

$$\mu = \frac{1}{p(k-1)} \left( \frac{(2c-1)(1-p)}{(1-\lambda)} - \frac{p^2 - c(p^2 + (1-p)^2)}{p} \right),$$

for  $k > 1$ . For pure information externalities, we have

$$\pi_2^{Enter}(\lambda, \mu)|_{k=1} = \frac{p(1-p\lambda)}{(p^2 + (1-p)^2)(1-\lambda) + 2p(1-p)} - c,$$

where  $\mu$  does not enter. This is intuitive, since there is no payoff externality and then no reason for a firm to condition its second period entry on the entry of other firms.

**Period one** From the last subsection, we know that the expected profit in the SPCG is zero. For the expected profit of not entering in period one, we thus only have to take into account the case where firm B enters in period one. The expected profit is then

$$\begin{aligned}\pi_1^{NotEnter}(\lambda) &= \Pr(\sigma_H|\sigma_H) \lambda (\Pr(H|\sigma_H, \sigma_H) k - c) \\ &= \lambda (p^2 k - c (p^2 + (1-p)^2)) \\ &= \lambda D.\end{aligned}$$

The expected profit of entry in period one is

$$\begin{aligned}\pi_1^{Enter} &= \Pr(\sigma_H|\sigma_H) (\Pr(H|\sigma_H, \sigma_H) k - c) + \Pr(\sigma_L|\sigma_H) (\Pr(H|\sigma_H, \sigma_L) - c) \\ &= p^2 k - c (p^2 + (1-p)^2) + 2p(1-p) \left( \frac{1}{2} - c \right) \\ &= N,\end{aligned}$$

where we used the fact that entry by A will make firm B enter, given that B has a high signal. For a firm to be indifferent between entry and non entry in period one, it must hold that

$$\pi_1^{Enter} = \pi_1^{NotEnter}(\lambda),$$

or, equivalently

$$N = \lambda D. \quad (5)$$

In a mixed equilibrium,  $\lambda$  must then satisfy

$$\lambda_M = \frac{p^2 k + p(1-p) - c}{p^2 k - c(p^2 + (1-p)^2)} = \frac{N}{D}. \quad (6)$$

Note that in the case  $k = 1$ , i.e. when there are no payoff externalities, we have

$$\lambda_{M|k=1} = \frac{p - c}{p^2 - c(p^2 + (1-p)^2)},$$

which is also the  $\lambda$  that solves  $\pi_2^{Enter}(\lambda, \mu)|_{k=1} = 0$ . Thus, if there are no payoff externalities, any rate of entry in the SPCG will give zero expected profit.

**The equilibria** Given  $\lambda_M$ , we can write

$$\pi_2^{Enter}(\lambda_M, \mu) = (\mu - 1) \frac{p^2(k-1)(2c-1)}{2p^2(k-1) + 2p-1}. \quad (7)$$

Then,  $\pi_2^{Enter}(\lambda_M, \mu) = \pi_2^{NotEnter} = 0$  holds if and only if  $\mu = 1$ . Hence,  $(\lambda, \mu) = (\lambda_M, 1)$  is an equilibrium. Further,  $\pi_2^{Enter}(\lambda_M, \mu) < 0$  for  $\mu < 1$ . Thus, given no entry in period one, a firm becomes so pessimistic (i.e. infers that the probability that the other firm has

a high signal is low), that it can obtain an expected profit of zero only if optimistic firms enter with certainty in the continuation game.

Now, Eq. (4) is only a sufficient condition for second period profits to be zero. By definition, a firm that never enters in period two also obtains zero profits. If, for example, B never enters in period two, given no entry in period one, then  $\pi_2^{Enter}(\lambda_M, \mu) < 0$  and firm A will stay out as well. Thus,  $\mu = 0$  is an equilibrium in the continuation game that gives zero profits to the firms given  $\lambda_M$ . Further,  $\lambda_M$  was derived under the assumption that firms make zero expected profits in the continuation game. Hence,  $(\lambda, \mu) = (\lambda_M, 0)$  is also an equilibrium and we have the following result.

**Proposition 1** *There are two equilibria in mixed strategies,*

$$(\lambda, \mu) = (\lambda_M, 1),$$

*and*

$$(\lambda, \mu) = (\lambda_M, 0),$$

*respectively.*

Now, given Definition 2, the informational aspects of the equilibria can be characterized; firms learn nothing in the equilibrium where  $(\lambda, \mu) = (\lambda_M, 1)$ , since entry in the second period is not conditional on entry in the first period. In contrast, for the equilibrium where  $(\lambda, \mu) = (\lambda_M, 0)$ , period one entry cannot be ignored; if there was entry in period one, any remaining entrant would want to enter in period two. Thus, firms learn in this equilibrium.

The equilibrium  $(\lambda, \mu) = (\lambda_M, 0)$ , will henceforth be referred to as the *Market Equilibrium* and we will mainly focus on this equilibrium. However, the equilibrium selection will not affect the welfare analysis (see Footnote 12 and Lemma 1).

## 4 Welfare and Intervention

The multiplicity of symmetric equilibria can potentially cause problems for a welfare analysis. In the previous section, we have showed that there is one symmetric equilibrium in pure strategies and two symmetric equilibria in mixed strategies. Although it will be shown that they all share the low welfare properties of a one-shot game absent of intervention, any intervention might give equilibria with different welfare properties. Thus, before proceeding with the welfare analysis, we need to know how the equilibria are affected by changes in the speed of entry. By “speed of entry”, we will mean the magnitude of  $\lambda$ ; a high  $\lambda$  makes it more likely that entry takes place early.

**Lemma 1** *An exogenous arbitrarily small increase in the equilibrium speed of entry will make the Market Equilibrium the unique symmetric equilibrium.*

**Proof.** First, consider the equilibrium in mixed strategies  $(\lambda, \mu) = (\lambda_M, 1)$  and study  $\mu$  as a function of  $\lambda$ . We have  $d\mu(\lambda)/d\lambda > 0$ . Thus, for  $\lambda > \lambda_M$  we must have  $\mu > 1$ , which cannot hold. Then, the only way of obtaining zero profits in equilibrium is to not enter, that is  $\mu = 0$ . Thus, in contrast, the perturbed Market Equilibrium is still an equilibrium for  $\lambda > \lambda_M$  with  $(\lambda, \mu) = (\lambda, 0)$ .

Next, consider the equilibrium in pure strategies where optimistic firms enter in the second period. Given that, say, firm B follows the equilibrium strategy, consider the problem for firm A. Since A can learn nothing from B's entry, A is indifferent between entry in the first and second period. Now, an exogenous increase in the speed of entry, must mean that first period entry is now relatively more attractive than second period entry. If first period entry is more attractive, then A will optimally enter in the first period, since A was indifferent before. Given that A enters first, firm B will prefer to enter in the second period, given that the relative attractiveness of the first period is small. Thus, the firms will instead play an asymmetric equilibrium. ■

This result facilitates the analysis to come. First, we will analyze the outcome of the symmetric equilibria from a welfare perspective. Next, intervention in the form of subsidies to the firms is considered. We only allow for interventions that are non-discriminatory; specifically, intervention is only contingent on entry. Since entry restrictions are never optimal, we thus limit the set of interventions to subsidies to firms that are only contingent on whether a firm enters in period one or two, respectively. The welfare measure used to compare outcomes, is the expected industry profit given only the priors. Thus, it is assumed throughout that the planner has no private information. Finally, we will briefly discuss some extensions of our model given our results on intervention.

## 4.1 Welfare

First, let us consider an appropriate benchmark to compare the outcome of the entry game. Optimally, all firms would enter if the state is high and stay out if the state is low. We will, however, use a more modest benchmark based on the firms' private information. Assume for a moment that the firms learn each others' signals. Then, the firms will enter only if both firms have received high signals. Under this assumption, the expected industry profit is

$$\begin{aligned} I_W &= 2 \Pr(\sigma_H, \sigma_H) (\Pr(H|\sigma_H, \sigma_H) k - c) \\ &= p^2 k - c (p^2 + (1-p)^2) \\ &= D. \end{aligned}$$

We will take  $I_W$  as the relevant benchmark hereafter. Firms would be able to attain this benchmark if they pooled their information. The benchmark thus stresses the informational aspects of entry. One weakness of our model is that we do not allow for communication. Since the firms interests are aligned we might think that allowing for communication would instead give efficiency. However, assume that each firm's signal

changes by an *arbitrarily small* probability from period one to period two.<sup>10</sup> We can equally think of this as each firm's signal being a signal of a true signal. Thus, the model remains largely unchanged but for the fact that the signals have more noise.<sup>11</sup> Nonetheless, the firms no longer have an incentive to always tell the truth; a firm with a low signal will always state that it received a favorable signal, since the other firm might then enter which, in turn, would benefit the pessimistic firm, should it receive a high signal later on.

We begin by deriving the ex ante industry profit of the symmetric equilibria in mixed strategies. Then, we will compare the outcome with a one shot game, that is, a game where there is no opportunity for learning.

The ex ante industry profit deriving from firms with high signals is

$$2 \Pr(\sigma_H, \sigma_H) (\lambda^2 + 2\lambda(1 - \lambda)) (\Pr(H|\sigma_H, \sigma_H) k - c),$$

where  $\lambda^2$  is the probability that the two firms enter in the first period and  $2\lambda(1 - \lambda)$  is the probability that one of the firms enters in the first period.<sup>12</sup> The ex ante industry profit deriving from firms with mixed signals is

$$\Pr(\sigma_H, \sigma_L) \lambda (\Pr(H|\sigma_H, \sigma_L) - c).$$

Since firms with low signals do not enter, the expected industry profit is the sum of the expected profits given above. The expected industry profit in the endogenous game is thus

$$I_M = p^2 k + p(1 - p) - c = N,$$

where we have used the  $\lambda_M$  given by Eq. (6). First, note that  $I_M > 0$  and that  $I_M$  is increasing in  $k$  as expected. If we look at our previous results and compare this with the profits calculated in the proof of Claim 1, we see that this is, in fact,  $\pi_1$ . Thus, under endogenous timing, the expected industry profit is exactly the expected profit of *one* firm with a high signal given that any firm with a high signal enters in period one. It is thus natural to examine what happens in a one period version of the game. In a one period game, all firms with a high signal enter, since entry is a dominant strategy.

**Proposition 2** *The expected industry profits of the symmetric equilibria are equal to the expected industry profit of a one-shot game.*

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<sup>10</sup>If the states of nature in our model reflect a “good investment climate” and a “bad investment climate” respectively, where a good investment climate might mean low taxes, the ability to repatriate profits and so on, then the state and thus the signal are likely to change over time. Such uncertainty over future policy is detrimental for investors; see Rodrik 1991 for a model of policy uncertainty (but no private information) and a survey of empirical evidence.

<sup>11</sup>We model noise explicitly in our model by the parameter  $p$ . Thus any additional noise on the true (but still noisy) signal could equally well be modeled by decreasing  $p$ .

<sup>12</sup>Note that we have omitted the expected industry profit in the case where firms do not enter in period one. In the Market Equilibrium  $(\lambda, \mu) = (\lambda_M, 0)$ , this expected profit is obviously zero. In the equilibrium  $(\lambda, \mu) = (\lambda_M, 1)$  it is also zero by the law of iterated expectations and the fact that the expected profit to the firms, given their respective signals, is zero.

**Proof.** See Appendix, Section 6.3. ■

Thus, it is as if nothing is learned in equilibrium. For the symmetric equilibrium in pure strategies and further, the equilibrium in mixed strategies where optimistic firms enter with certainty in the period two, the intuition is clear, since no learning takes place. The result is more surprising if we consider the Market Equilibrium. After all, entry is fully informative and the event “no entry in period one” is also very informative in the sense that it contains sufficient information for a firm to conclude that entry is not profitable in period two. The answer is that in the Market Equilibrium, there is lower probability of entry than in the one-shot game. Thus, the effects of complementarities are lower in the Market Equilibrium, while, on the other hand, there is also learning. In equilibrium, these effects offset each other, and profits are the same as in the one-shot game.

Proposition 2 seems counter-intuitive for the case of pure information externalities, since we noted that  $(\lambda, \mu) = (\lambda_{M|k=1}, \mu)$  is an equilibrium for any rate of entry  $\mu$  in the second period; given, for example  $\mu = 0$ , the firms will surely enter less than in the one-shot game and expected profits seem then likely to be lower. Nevertheless, we can show the following:<sup>13</sup>

**Proposition 3** *For pure information externalities, the expected industry profit is equal to the expected profit of a one-shot game for any rate of entry in the SPCG.*

**Proof.** See Appendix, Section 6.4. ■

Thus, Proposition 2 extends to the case of pure information externalities. The intuition is that although firms infer something by the event “no entry in period one”, there is no learning in any of the equilibria  $(\lambda, \mu) = (\lambda_{M|k=1}, \mu)$ ; entry is not conditional on what is inferred, since firms can enter with any probability in the second period.

A comparison with  $I_W$  shows that

$$I_W - I_M = p(1 - p)(2c - 1) > 0.$$

As a corollary to Proposition 2, we also have the following result:

**Corollary 1** *Entry is efficient under complete information.*

The corollary follows directly from the comparison of  $I_W$  and  $I_M$  above (and is only another way of expressing that our welfare benchmark is an informational one).

## 4.2 Intervention

A subsidy can potentially encourage entry in the case of insufficient entry. Here we discuss whether subsidies are ex ante profitable, where “ex ante profitable” refers to expected profits given no private information. We assume that it is possible to give a subsidy contingent only on whether a firm enters in the first or second period.

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<sup>13</sup>Chamley and Gale (1994) show a similar result in the case of pure information externalities. Further, Choi (1997) gives a related result in a model where agents fail to learn which of two technologies is the better.

**Proposition 4** *It is not efficient to subsidize so much that entry by optimistic firms takes place with certainty.*

**Proof.** A first period subsidy that induces firms to enter with certainty in the first period would give expected industry profits equal to those of the one-shot game analyzed above, minus the costs of subsidies which is not efficient. ■

The proposition highlights that subsidies have an informational role.

**Corollary 2** *The optimal subsidy induces an equilibrium in mixed strategies.*

We will consider subsidies only to entrants in the first period and next, two subsidies of different size targeted at entrants in the first and second period, respectively. Since subsidies induce a mixed strategy equilibrium, we will refer to the first type of game as a “first period subsidy game” (FPS) and the second as a “both period subsidy game” (BPS). We start by analyzing the efficiency of giving subsidies to entrants in the first period. The other type of subsidy (BPS), instead tries to bail out existing firms by adding cluster effects. To analyze this, we study the effects of a second period subsidy in the case a first period entrant has been left stranded.

Before proceeding with the analysis of subsidies, we give a result that simplifies the analysis.

**Lemma 2** *It is never optimal to reduce the speed of entry, as reflected by  $\lambda$ , lower than  $\lambda_M$ .*

**Proof.** See Appendix, Section 6.5. ■

The intuition is that a lower  $\lambda$  implies two things. First, it potentially introduces noise for inference in the event “no entry in period one”. Second, there will be lower gains from complementarities.

#### 4.2.1 First period subsidies

A subsidy to first period entrants makes early entry relatively more attractive. Thus, the timing of entry and hence, the entry rate  $\lambda$ , will change. First, let us consider how the expected industry profit is affected by a change in  $\lambda$ . Remember that if  $\lambda$  changes, then the inference based on “no entry in period one” will change. Nevertheless,  $\lambda_M$  is fully informative in the sense that firms learn in the Market Equilibrium. Thus, the inference changes, but what is learned (“put in use”, see Definition 2), does not change; if we raise  $\lambda$ , then no entry is even more informative and there will be no subsequent entry. By Lemma 2, we can restrict ourselves to this type of intervention. Thus, let us study the effects of raising  $\lambda$  by a subsidy  $s_1$  for first period entry. The entry probability given a first period subsidy  $s_1$  is

$$\lambda = \frac{N + s_1}{D}.$$



The expected industry profit in the subsidy game excluding the transfers to the firms is ( $s_1$  then enters into the expected profits only via its effect on  $\lambda$ )<sup>14</sup>

$$\begin{aligned} I &= N + s_1 \frac{p(1-p)(2c-1) - s_1}{D} \\ &= I_M + s_1 \frac{p(1-p)(2c-1) - s_1}{D}. \end{aligned}$$

It is easily shown that the last term and thus  $I$  are concave in  $s_1$ . Further, the constraint  $\lambda < 1$  implies that  $s_1$  can take values up to  $p(1-p)(2c-1)$ . Thus,  $I$  is not only concave in  $s_1$  but the following result can also be showed directly.

**Proposition 5** *Expected industry profits are a non-monotonic function of the first period subsidy.*

The result is interesting, since the subsidies have no distortionary costs. The intuition is that there are two ways of obtaining the (low) expected industry profits of a one-shot game. As previously shown, a subsidy  $s_1 = 0$  would give  $I = I_M = O$ , the one-shot game expected industrial profit. On the other hand, a great enough subsidy would induce every optimistic firm to enter in the first period. Again, this is the same outcome as in a one-shot game.

To find the optimal  $s_1$ , consider the first-order condition

$$0 = \frac{\partial}{\partial s_1} \left( s_1 \frac{p(1-p)(2c-1) - s_1}{p^2k - c(p^2 + (1-p)^2)} \right),$$

which gives

$$s_1 = \frac{p(1-p)(2c-1)}{2}.$$

The optimal subsidy is thus a compensation for the risk of being the first entrant; it is higher for less informative signals and for higher costs of entry.<sup>15</sup> Note that the optimal subsidy does not depend on complementarities  $k$ . To see the intuition, consider the asymmetric equilibria, where the first entrant takes a greater risk than the second entrant. The second entrant has an expected profit of  $p(1-p)(2c-1)$  greater than that of the first entrant (see the proof of Claim 1), which is independent of  $k$ . Hence, the option value of delay is independent of  $k$ . Then, given that a firm takes the decision to enter before another firm, its decision is independent of  $k$ . Thus, the subsidy that affects the timing of entry should also be independent.

<sup>14</sup>See Appendix, Section 6.6, for a derivation of this expression.

<sup>15</sup>Note that the subsidy is not high enough to induce entry by pessimistic firms. Entry in period one would give a firm with a low signal an expected profit of  $\pi_1^{Enter} = s_1 + \Pr(\sigma_H|\sigma_L)(\Pr(H|\sigma_H, \sigma_L)k - c) + \Pr(\sigma_L|\sigma_L)(\Pr(H|\sigma_H, \sigma_L) - c) = p(2p - 1 + 2k - 2c - 2pk)/2$ , which is negative if  $k < (2c - 2p + 1)/(2(1 - p))$ . The right hand side of the inequality is greater than  $2c$ . Thus, the inequality is guaranteed to hold and there will be no incentive for the pessimistic firm to enter.

Given the optimal subsidy, the entry probability in the first period is

$$\lambda_{FPS} = \frac{p^2 k - c + p(1-p) + p(1-p)(2c-1)/2}{p^2 k - c(p^2 + (1-p)^2)}.$$

Rewriting this in terms of expectations we have

$$\lambda_{FPS} = 1 - \frac{-\frac{1}{2}E[I \mid \text{Entry takes place only if firms have mixed signals}]}{E[I \mid \text{Entry takes place only if both firms have optimistic signals}]},$$

where  $E[\cdot]$  is the expectation of industry profits using only priors.<sup>16</sup> Thus, optimally, firms should enter with certainty in the first period but for a penalty related to the risk of sub-optimal entry in the case that they have mixed signals. Using  $\lambda_{FPS}$ , we immediately have the next result.

**Proposition 6** *An uninformed planner can increase welfare by a first period subsidy. The highest attainable expected industry profit is then*

$$I_{FPS} = I_M + \frac{1}{4} \frac{p^2 (1-p)^2 (2c-1)^2}{D}.$$

Note that  $I_{FPS} > I_M$  and that the difference is increasing in  $c$  and decreasing in  $p$ .<sup>17</sup> Given  $I_{FPS}$ , the next result follows immediately.

**Corollary 3** *The scope for intervention is decreasing in  $k$ .*

**Proof.** The welfare increase due to intervention is

$$I_{FPS} - I_M = p^2 (1-p)^2 (2c-1)^2 / (4(p^2 k - c(p^2 + (1-p)^2))),$$

which is decreasing in  $k$ . ■

To understand why an uninformed planner can increase welfare, it is worthwhile to revisit the unsubsidized case. Consider again the Market Equilibrium; too few firms enter in equilibrium, and the gains of complementarities are not fully realized. By a subsidy to first period entry, early entrants are compensated for providing information and payoff externalities, which makes first period entry less risky. On the other hand, the non-monotonicity result comes from a countervailing force; as first period entry becomes relatively more attractive, fewer firms will delay and learn. The following figure gives private ( $\pi$ ) and industry profits ( $I$ ) accruing from entry in period one and two, respectively, given  $c = 3/4$ ,  $k = 11/10$  and  $p = 7/9$ .

<sup>16</sup>E.g.

$E[I \mid \text{Entry takes place only if both firms have optimistic signals}] = 2 \Pr(\sigma_H, \sigma_H) (\Pr(H \mid \sigma_H, \sigma_H) k - c)$  is the expected industry profit given that firms only enter when both have high signals.

<sup>17</sup>To see this, define  $m = p^2 (1-p)^2 (2c-1)^2 / D$ . Then

$\partial m / \partial c = p^2 (p-1)^2 (2c-1) \left( \left[ (2p^2 (k-1) + 2p-1) / D^2 \right] + 2/D \right) > 0$ . Further,  $\partial m / \partial p = -2p(1-p)(2c-1)^2 \left( \left[ cp^2 (1-p)^2 / (p^2 D^2) \right] + p/D \right) < 0$ .

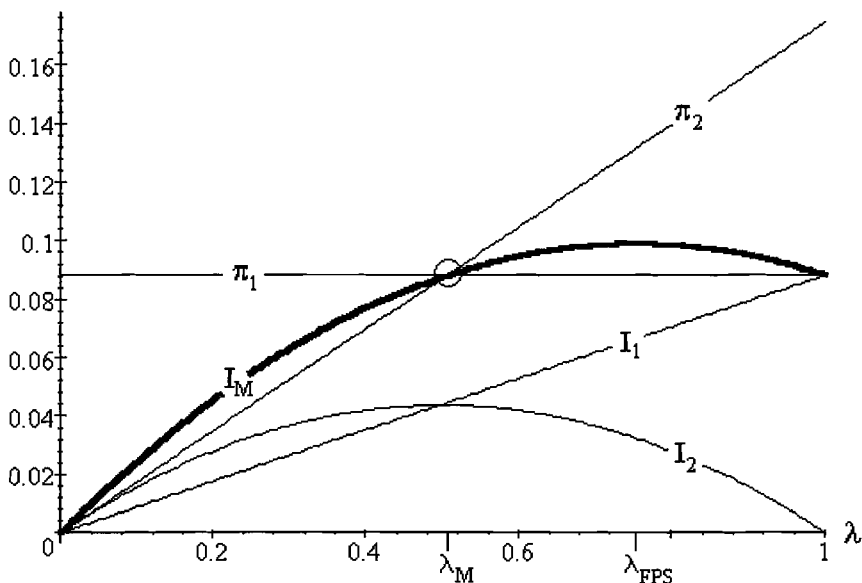


Figure 1. Profits from entry in period one and two, respectively.

In equilibrium, the private profit of entry in the first period  $\pi_1$ , must be equal to the private profit of waiting to period two  $\pi_2$  (where  $\pi_1 = N$  and  $\pi_2 = \lambda D$ ). The intersection (circled) thus gives the equilibrium entry rate  $\lambda_M$ . The curve  $I_2$ , giving the industry profit accruing from entry in period two is concave, which, in turn, makes total industry profits  $I_M$ , a concave function.<sup>18</sup> We see that total industry profits are increasing in  $\lambda$  at  $\lambda = \lambda_M$  as stated by Lemma 2. By the subsidy  $s_1$ , the curve  $\pi_1$  is shifted upwards so that the new equilibrium is at  $\lambda = \lambda_{FPS}$ .

At first sight, the result that a subsidized equilibrium in mixed strategies is optimal, seems to depend strongly on the assumption that if all optimistic firms enter early, there will be no entrants left in later periods that can make use of the revealed information. After all, the number of firms is very limited in our model. Could it be that if only a limited number of firms had private information but there were a great number of potential entrants without any private information, then it would be optimal to give high subsidies so that information were quickly revealed to the benefit of the uninformed? First, note that if uninformed firms find it profitable to enter in the first period, then early entry will lose its informational role. Thus, a first period subsidy must be set sufficiently low to not induce uninformed entry in the first period. Or, in other words, the option value of delay (OVD) must be positive for uninformed entrants. Hence, in this setting, the subsidy must be set to create a low or negative OVD for informed firms and

<sup>18</sup>Formally,  $I_2 = (2\lambda(1-\lambda))D/2$  and  $I_1 = (2\lambda^2 + 2\lambda(1-\lambda))D/2 + 2p(1-p)\lambda(1/2 - c)$ .

a positive OVD for uninformed firms. Given the conflicting constraints on the subsidy, it is not obvious that welfare increases by a subsidy or even the existence of a subsidy that fulfills the constraints.

Chamley and Gale (1994) show that there can be an inefficient delay in entry when agents have the incentive to free ride on the informative entry of others. They also find that, in equilibrium, there is no learning and that a subsidy should be set in order to enhance learning. According to them, the optimal subsidy should be set by  $s = OVD - \varepsilon$ , where  $\varepsilon$  is a small positive number (there is no formal proof in their paper).<sup>19</sup> Thus, they argue that the subsidy should almost remove the incentive for firms to delay in order to induce early entry. Again, this fails to recognize that moving entry forward can result in less learning, since there are fewer firms left that can learn in later periods. In particular, we have showed that the optimal subsidy is  $s = OVD/2$ .<sup>20</sup> Hence, their  $\varepsilon$  must be  $OVD/2$  to induce the optimal rate of entry. This equals the subsidy itself and it is questionable whether  $\varepsilon$  can then be regarded as small.

Alexander et al. (1998) study entry under information and positive payoff externalities, and show that there is insufficient entry in equilibrium. They claim, without a formal proof, that efficiency can be attained by transfers to early entrants. The main difference between our results and theirs, is that aggregate information does not matter; entry by one firm is fully informative about the state in their model, since signals have no noise.<sup>21</sup>

In the literature on information and payoff externalities, another related type of intervention has been suggested. In the model of Choi (1997), efficiency can be attained if entrants ignore their private information and enter collectively in the first period. A potential first entrant is driven by the fear that late entrants will use the information revealed by entry against him, which gives an incentive to opt for a safe but inefficient choice. Collective ignorance gives efficiency in his model, since there are then no firms that can act against the interest of the first entrants and moreover, there will be no delay that dissipates profits. In contrast to Choi's model, information revealed by entry will never be used against the entrant's interest in our model and further, there is no cost

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<sup>19</sup> Chamley and Gale state that a first period subsidy  $S$  should be set by (equation given on p. 1079 in their paper)  $\sum_n v(n) g(n) + S = \sum_n \max\{v(n), 0\} g(n) - \varepsilon$ , where  $g(n)$  is the posterior probability (i.e. given private information in the form of an investment option) that  $n$  investors have received investment options,  $v(n)$  is the payoff of investing given that  $n$  investors have options and  $\varepsilon$  is a small positive number. Thus, the left-hand side is the expected payoff of entry in period one for an investor with an investment option plus the first period subsidy. The right-hand side is the expected payoff for a privately informed investor that will know the true  $n$  in the next period. Hence, the right-hand side is the value of delay given that  $n$  is revealed in the first period. In terms of our model,  $n = 1, 2$  is the number of optimistic firms (zero is excluded since the calculation is done by an optimistic firm). Further,  $g(1) = \Pr(\sigma_L|\sigma_H)$  and  $g(2) = \Pr(\sigma_H|\sigma_H)$  are the posterior probabilities. Finally  $v(1) = \Pr(H|\sigma_L, \sigma_H) - c$  and  $v(2) = \Pr(H|\sigma_H, \sigma_H) k - c$ . Thus, their left-hand side is equal to  $\pi_1$  as given in the proof of Claim 1, and further, their right-hand side is  $\pi_2$ , since  $\pi_2$  was derived under the assumption that firm A's signal was revealed to firm B.

<sup>20</sup> Formally, by comparing with the proof of Claim 1, we see that  $s_1 = p(1-p)(2c-1)/2 = (D-N)/2 = OVD/2$ .

<sup>21</sup> Further, this implies that entry in period two increases with non-entry in period one in their model, since a firm that knows that the state is good with certainty will not revise his beliefs based on first period entry.

of delay in our model. Collective ignorance can also increase welfare in the context of pure information externalities. In the model of Banerjee (1992), agents' choices will, in equilibrium, sometimes not reveal payoff relevant information, since, from a welfare perspective, agents take too much of the information generated by predecessors' choices into account. Collective ignorance is here efficient, since the cost of mistakes by some agents is low compared to the social gain of revealing the optimal choice. The main difference is that agents choose sequentially in his model. Thus, the policy maker will never face the issues of timing which are central to this paper. In our setting, non-discriminatory collective ignorance would mean that all firms entered in the first period, which is not efficient.<sup>22</sup>

**Temporary or permanent intervention** There is a common opinion in the literature on infant industry protection that if external economies are always present, then long-lasting protection is the best policy. Consider the following quote (Corden (1997), p. 146), "If external economies are being continually created and assistance can be given in the form of direct subsidies, then subsidization should be permanent ...". Externalities are continuously created in our model in the sense that both early and late entrants create positive payoff externalities. In this section we show that this connection between the persistence of protection and the persistence of externalities is questionable. Consider an implementation of the optimal rate of entry (as reflected by  $\lambda_{FPS}$ ) by instead a subsidy for both periods. An indiscriminating subsidy  $s$  that gives an optimal rate of entry (as reflected by  $\lambda_{FPS}$ ) must fulfill

$$\frac{p^2k + p(1-p) - c + s}{p^2k - (c-s)(p^2 + (1-p)^2)} = \lambda_{FPS}.$$

The subsidy  $s$  is then<sup>23</sup>

$$s = s_1 \frac{D}{D - (p^2 + (1-p)^2)(N + s_1)} > s_1.$$

The expected number of times that  $s$  must be given to the firms is

$$C_s = 2(p^2 + (1-p)^2)(\lambda^2 + 2\lambda(1-\lambda)) + 2p(1-p)\lambda,$$

while the expected number of times that  $s_1$  must be given to the firms is

$$C_{s_1} = \lambda.$$

Since  $C_s - C_{s_1} = \lambda(1-\lambda)(p^2 + (1-p)^2) > 0$ , we then know that an indiscriminate subsidy is more costly and further, must be paid out with higher frequency. Thus, an

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<sup>22</sup>If the planner would force the two firms to enter regardless of signals, then the expected industry profits is  $k - 2c < 0$ . If entry was voluntary, then the outcome would be that of a first period game, which is not efficient.

<sup>23</sup>The inequality follows from  $D - (p^2 + (1-p)^2)(N + s_1) \geq D - (N + s_1) = p(1-p)(2c-1) - s_1 > 0$ .

indiscriminate subsidy cannot be optimal. A simpler way of seeing this is to note that if the planner sets the same subsidy in both periods, then the planner solves a constrained optimization problem that is also more costly.

#### 4.2.2 Both period subsidies

The previous section shows that a subsidy can increase welfare by speeding up entry. One inefficiency remains, however. Consider the case where there was only a single entrant in the first period. This could mean two things; either both firms are optimistic but one firm did not enter in the first period or, firms have mixed signals which means that entry is not profitable. Nevertheless, in the case of mixed signals, entry by the pessimistic firm can still be socially desirable. The intuition is that although the industry will still make a loss on average, the complementarities given by additional entry increase industry profits. To study the effects of bailing out a stranded first period entrant, we now introduce a second period subsidy that is sufficient to make a pessimistic firm enter. This subsidy is given only if there was a single entrant in the first period. First, we need to examine whether this is welfare enhancing at all. Consider the planner's problem. Given that a single firm entered in the first period, an uninformed planner can calculate the following probability

$$\alpha = \Pr(\sigma_H^B | \sigma_H^A, Ne1B),$$

where the non entrant is labeled as firm B and the first period entrant as firm A. Let us now assume that the planner wants any firm to enter given entry in the first period. Then, the second period subsidy must be set so as to induce a firm with a low signal to enter. The subsidy  $b$  that makes the pessimistic firm indifferent in the second period is

$$b = c - \frac{1}{2}k = -(\Pr(H|\sigma_L^B, \sigma_H^A)k - c).$$

Since this subsidy must be paid to any entrant in the second period, the *additional* expected industry profit (not including subsidies) is

$$(1 - \alpha) \left( 2 \left( \frac{1}{2}k - c \right) - \left( \frac{1}{2} - c \right) \right),$$

where the first term is the expected profit of two entrants with mixed signals. The second term is the cost avoided, which is the expected profit of a single entrant in period one given mixed signals. In the second period, a planner thus wants to give a second period subsidy if<sup>24</sup>

$$k - c > \frac{1}{2}. \tag{8}$$

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<sup>24</sup>We already know that  $pk - c > 0$  from Eq. (2). This implies that Eq. (8) holds if  $1/2 - k(1 - p) > 0$ . If, for example, the precision of the signals are high, then this holds.

This condition reflects that saving a stranded entrant is only welfare increasing if complementarities are strong. Now, let us assume that Eq. (8) holds and look at the properties of this subsidized game. If there is a subsidy  $a$  in the first period and a second period subsidy  $b$ , then the equilibrium entry probability in the first period is

$$\lambda_{BPS} = \frac{N + a}{D + b(p^2 + (1 - p)^2)}.$$

Now, by Lemma 2, we know that optimally  $\lambda_{BPS} > \lambda_M$ . Remember that for  $\lambda > \lambda_M$ , there is no entry in period two, given that no firm entered in period one. Denote the expected industry profits in the subsidy game as  $I$ .<sup>25</sup> Taking the FOC of the expected industry profits

$$\frac{\partial I}{\partial a} = 0,$$

gives the optimal first period subsidy

$$a = \frac{(p^2 + (1 - p)^2)(pk - c)}{D} b + p(1 - p)(k - 1).$$

The optimal entry rate is then given by<sup>26</sup>

$$\lambda_{BPS} = \frac{pk - c}{D}.$$

Comparing with the non-subsidized entry rate gives

$$\lambda_{BPS} - \lambda_M = \frac{p(1 - p)(k - 1)}{D} > 0.$$

Further,

$$\lambda_{BPS} - \lambda_{FPS} = \left(k - c - \frac{1}{2}\right) \frac{p(1 - p)}{D}. \quad (9)$$

So  $\lambda_{BPS}$  is greater than  $\lambda_{FPS}$  exactly when we want to give a second period subsidy (by Eq. (8)). This gives a more rapid entrance.

The expected industry profit is

$$I_{BPS} = \frac{(pk - c)^2}{D}.$$

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<sup>25</sup>See Appendix, Section 6.7, for a derivation of this expression.

<sup>26</sup> $\lambda_{BPS} < 1$  requires  $p^2k - c(p^2 + (1 - p)^2) - (pk - c) > 0$ , or equivalently  $p(1 - p)(2c - k) > 0$ , which is true.

The industry profits derived so far, can be rewritten in terms of our welfare benchmark, which gives

$$I_{BPS} = \lambda_{BPS}^2 I_W, \quad (10)$$

$$I_{FPS} = \lambda_{FPS}^2 I_W, \quad (11)$$

$$I_M = \lambda I_W. \quad (12)$$

Ideally, only firms with high signals enter as reflected by the benchmark  $I_W$ . The first factors in equations (10) and (11) are then ideally 1. The first factor  $\lambda^2$  is then a measure of how close the equilibrium comes to the ideal. A comparison of the relative merits of subsidies in the first and both period is given by the following result.

**Proposition 7** *If complementarities are sufficiently strong to raise expected industry profits given a stranded early entrant, then the optimal policy is to give subsidies in both periods.*

The proof is simple; we have  $I_{BPS} > I_{FPS}$  precisely when we want to give a first period subsidy (by equations (8) and (9)).

Given Propositions 6 and 7, we thus know that if Eq. (8) holds, then

$$I_{BPS} > I_{FPS} > I_M.$$

Taking as an example  $c = 3/4$ ,  $k = 11/8$  and  $p = 7/8$ , we have

$$\begin{aligned} \lambda_{FPS}^2 &\approx 0.89, \\ \lambda_{BPS}^2 &\approx 0.942, \\ \lambda_M &\approx 0.88. \end{aligned}$$

The expected industry profit of the FPS game is then roughly 1% better than the equilibrium outcome, while the BPS game increases welfare by 6%.

### 4.2.3 Extensions

Throughout, we have assumed that there is no exogenous cost of delaying actions. This makes the equilibrium characterization of welfare interesting, since the results are given for the case when delay cannot dissipate profits. If a cost of delay is introduced, then the Market Equilibrium is the unique symmetric equilibrium (see also Lemma 1), since firms would not want to incur a cost of delay of doing what they equally well could do in the first period. The scope for welfare analysis changes, since firms will have an incentive to enter earlier and  $\lambda_M$  will be higher, which increases welfare. At the same time, late entry will cost more, and information externalities are then more costly to realize.

The analysis was carried out assuming that there was only two time periods. This was made to facilitate the exposition only; if there is a cost of delay, the game will end after two periods, even if there is a (countable) infinite number of periods. The intuition



is that when delay is costly, then firms will only delay if they learn something by waiting (see also Definition 2). If there is no entry in period one, then firms learn and stay out for the remainder of the game. On the other hand, if there was entry in period one, then there is no reason to delay further for any optimistic non-entrant. Thus, assuming that there is a cost of delay, the analysis does not change if we increase the number of time periods. Since we have shown that the optimal intervention implicitly imposes a cost of delay by making early entry more attractive, the results could equally well have been shown for the infinite number of periods case.

## 5 Conclusions

The present paper shows how private information and positive payoff externalities distort entry into new markets. Firms try to free ride on other firms' informative entry and as a result, there is insufficient entry in equilibrium. We offer a decentralized solution where firms are induced to learn from each other. The main result of the paper is that an uninformed social planner can improve welfare by subsidizing entry. However, we show that subsidies must be carefully constructed, since subsidies can impede the flow of information. The theory is applicable to several areas of economics, ranging from the formation of clusters in the IT industry to industrial policy in developing countries.

Although information was modelled as crucial for the firms' decisions, we strikingly found that the equilibrium payoffs were equal to those of a game where firms learn nothing. The intuition is that in equilibrium, there are too few entrants, and the gains from complementarity are not fully realized.

We also examined the relative profitability of intervention, and found the welfare gain from subsidies to be decreasing in the strength of complementarity. That complementarities decrease the effectiveness of intervention is interesting; in interventions owing to Big Push arguments, the "leading sectors" that were to be subsidized were in part identified due to the existence of strong externalities. On the other hand, if complementarities are strong enough, private information plays no role and firms will enter efficiently.

Subsidies can, to some extent, also overcome a second form of distortion. Even if the aggregate of private information shows that entry is not profitable, a firm with an optimistic signal might still have entered in the first period. It will then be left stranded without additional investments (entry) that give complementarities. In some cases, a social planner can then intervene to provide complementarities by also offering a second period subsidy in the case a single firm entered in the first period. We show that this distortion gives more leverage to intervention when payoff externalities are relatively high.

An interesting extension would be to model the signal space as continuous.<sup>27</sup> Less information will then be revealed by entry in period one. A subsidy to early entrants might have the effect that less optimistic entrants consider entry in the first period, which, in turn, would decrease the informative content of early entry; see Chapter 2, Section 5.1.1 of this dissertation, for an example of this when firms enter sequentially.

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<sup>27</sup>See the survey by Gale (1996), where he considers the relative importance of making different assumptions on timing and signals.

## 6 Appendix

### 6.1 Asymmetric equilibria

In this section we show that there exists an asymmetric equilibrium. As an example, consider the case where, say, firm A is the first to enter. Firm A then enters only if it has received a high signal. Since firm A's entry decision then conveys the private signal, firm B will enter in period two only if its private signal is high and A has already entered.

**Claim 2** *There is an asymmetric equilibrium where optimistic firms enter sequentially.*

**Proof.** The proof is given by the proof of Claim 1, noting that firm A was indifferent between entry in period one and two. ■

Note that formally, there exist two such equilibria, where either A or B has the first entry opportunity. Since we know that two optimistic firms always enter in the sequential game, the expected industry profit is

$$\begin{aligned} I_{AE, \sigma_H, \sigma_H} &= 2 \Pr(\sigma_H, \sigma_H) (\Pr(H|\sigma_H, \sigma_H) k - c) \\ &= D. \end{aligned}$$

Further, since we know that entry by the optimistic firm, when the firms receive mixed signals, occurs, on average, half the time in the asymmetric equilibrium (assuming that there is a 50% chance that either firm A, or firm B will be the leader), the expected industry profit is

$$\begin{aligned} I_{AE, \sigma_H, \sigma_L} &= \frac{1}{2} \Pr(\sigma_H, \sigma_L) (\Pr(H|\sigma_H, \sigma_L) - c) \\ &= p(1-p) \left( \frac{1}{2} - c \right). \end{aligned}$$

Adding up gives

$$I_{AE} = D + p(1-p) \left( \frac{1}{2} - c \right).$$

### 6.2 Mixed strategy equilibria - derivation of probabilities

The probabilities used are

$$\begin{aligned} \Pr(H, En2|\sigma_H^A, \sim En1) &= \frac{p^2 \mu (1 - \lambda)}{(p^2 + (1 - p)^2) (1 - \lambda) + 2p(1 - p)}, \\ \Pr(H, \sim En2|\sigma_H^A, \sim En1) &= \frac{p^2 (1 - \mu) (1 - \lambda) + p(1 - p)}{(p^2 + (1 - p)^2) (1 - \lambda) + 2p(1 - p)}, \\ \Pr(L|\sigma_H^A, \sim En1) &= \frac{(1 - p)^2 (1 - \lambda) + p(1 - p)}{(p^2 + (1 - p)^2) (1 - \lambda) + 2p(1 - p)}. \end{aligned}$$

Let  $\beta = \Pr(H, En2 | \sigma_H^A, \sim En1)$ . Then, for example,  $\beta$  is calculated as

$$\begin{aligned}\beta &= \frac{\Pr(H, En2, \sigma_H^A, \sigma_H^B, \sim En1) + \Pr(H, En2, \sigma_H^A, \sigma_L^B, \sim En1)}{\Pr(\sigma_H^A, \sigma_H^B, \sim En1) + \Pr(\sigma_H^A, \sigma_L^B, \sim En1)} \\ &= \frac{\Pr(H, \sigma_H^A | En2, \sigma_H^B, \sim En1) \Pr(En2, \sigma_H^B, \sim En1)}{\Pr(\sigma_H^A | \sigma_H^B, \sim En1) \Pr(\sigma_H^B, \sim En1) + \Pr(\sigma_H^A | \sigma_L^B, \sim En1) \Pr(\sigma_L^B, \sim En1)} + \\ &\quad + \frac{\Pr(H, En2 | \sigma_H^A, \sigma_L^B, \sim En1) \Pr(\sigma_H^A, \sigma_L^B, \sim En1)}{\Pr(\sigma_H^A | \sigma_H^B, \sim En1) \Pr(\sigma_H^B, \sim En1) + \Pr(\sigma_H^A | \sigma_L^B, \sim En1) \Pr(\sigma_L^B, \sim En1)}.\end{aligned}$$

The equality follows from the definition of conditioned probability and the fact that the signals partition the state space. Now, use that  $\Pr(H, En2 | \sigma_H^A, \sigma_L^B, \sim En1) = 0$ , since a pessimistic firm B never enters. Further,  $\Pr(\sigma_H^A | \sigma_H^B, \sim En1) = \Pr(\sigma_H^A | \sigma_H^B)$ , since the event “no entry in period one” is only a noisy signal of  $\sigma_H^B$ . The same argument shows that  $\Pr(H, \sigma_H^A | En2, \sigma_H^B, \sim En1) = \Pr(H, \sigma_H^A | \sigma_H^B)$ . Then,

$$\begin{aligned}\beta &= \frac{\Pr(H, \sigma_H^A | \sigma_H^B) \Pr(En2, \sigma_H^B, \sim En1)}{\Pr(\sigma_H^A | \sigma_H^B) \Pr(\sigma_H^B, \sim En1) + \Pr(\sigma_H^A | \sigma_L^B) \Pr(\sigma_L^B, \sim En1)} \\ &= \frac{\Pr(H, \sigma_H^A | \sigma_H^B) \Pr(En2 | \sigma_H^B, \sim En1) \Pr(\sim En1 | \sigma_H^B) \Pr(\sigma_H^B)}{\Pr(\sigma_H^A | \sigma_H^B) \Pr(\sim En1 | \sigma_H^B) \Pr(\sigma_H^B) + \Pr(\sigma_H^A | \sigma_L^B) \Pr(\sim En1 | \sigma_L^B) \Pr(\sigma_L^B)} \\ &= \frac{\Pr(H, \sigma_H^A | \sigma_H^B) \Pr(En2 | \sigma_H^B, \sim En1) \Pr(\sim En1 | \sigma_H^B)}{\Pr(\sigma_H^A | \sigma_H^B) \Pr(\sim En1 | \sigma_H^B) + \Pr(\sigma_H^A | \sigma_L^B)},\end{aligned}$$

where the last equality follows from  $\Pr(\sigma_H^B) = \Pr(\sigma_L^B) = 1/2$  and  $\Pr(\sim En1 | \sigma_L^B) = 1$ . Hence,

$$\beta = \frac{p^2 \mu (1 - \lambda)}{(p^2 + (1 - p)^2) (1 - \lambda) + 2p(1 - p)}.$$

### 6.3 Proof of Proposition 2

**Proof.** Given

$$\lambda_M = \frac{N}{D} = \frac{p^2 k + p(1 - p) - c}{p^2 k - c(p^2 + (1 - p)^2)},$$

the expected industry profit in the symmetric equilibrium in mixed strategies is

$$\begin{aligned}I_M &= 2 \Pr(\sigma_H, \sigma_H) (\lambda_M^2 + 2\lambda_M (1 - \lambda_M)) (\Pr(H | \sigma_H, \sigma_H) k - c) + \\ &\quad + \Pr(\sigma_H, \sigma_L) \lambda_M (\Pr(H | \sigma_H, \sigma_L) - c) \\ &= 2 \left( \frac{1}{2} (p^2 + (1 - p)^2) \right) (\lambda_M^2 + 2\lambda_M (1 - \lambda_M)) \left( \frac{p^2 k}{p^2 + (1 - p)^2} - c \right) + \\ &\quad + 2p(1 - p) \lambda_M \left( \frac{1}{2} - c \right) \\ &= p^2 k - c + p(1 - p).\end{aligned}$$

Similarly, for a one-shot game, we have

$$\begin{aligned}
O &= 2 \Pr(\sigma_H, \sigma_H) (\Pr(H|\sigma_H, \sigma_H) k - c) + \Pr(\sigma_H, \sigma_L) (\Pr(H|\sigma_H, \sigma_L) - c) \\
&= 2 \Pr(\sigma_H, \sigma_H) \left( \frac{p^2 k}{p^2 + (1-p)^2} - c \right) + \Pr(\sigma_H, \sigma_L) \left( \frac{1}{2} - c \right) = p^2 k - c + p(1-p) \\
&= I_M.
\end{aligned}$$

Further, as noted in Section 3.1.1, the outcome of a symmetric equilibrium in pure strategies is the same as that of a one-shot game. ■

## 6.4 Proof of Proposition 3

In the case  $k = 1$ , i.e. when there are no payoff externalities, we have

$$\lambda = \frac{p - c}{p^2 - c(p^2 + (1-p)^2)}$$

which is also the  $\lambda$  that solves  $\pi_2^{Enter}(\lambda, \mu)|_{k=1} = 0$ . Thus, if there are no payoff externalities, any rate of entry in the SPCG will give zero expected profit. To formally show that the expected industry profit is equal to the expected industry profit of a one shot game for any  $\mu$ , consider the expected industry profits for firms with high signals

$$\begin{aligned}
\pi_{\sigma_H, \sigma_H} &= \Pr(\sigma_H, \sigma_H) (2\lambda^2 + 4\lambda(1-\lambda) + (1-\lambda)^2 (2\mu^2 + 2\mu(1-\mu))) * \\
&\quad * (\Pr(H|\sigma_H, \sigma_H) - c) \\
&= \frac{1}{2} (p^2 + (1-p)^2) (2\lambda^2 + 4\lambda(1-\lambda) + (1-\lambda)^2 (2\mu^2 + 2\mu(1-\mu))) * \\
&\quad * \left( \frac{p^2}{(p^2 + (1-p)^2)} - c \right),
\end{aligned}$$

where  $(1-\lambda)^2$  is the probability that no entry takes place in period one,  $\mu^2$  is the probability of two entrants in the SPCG (therefore the coefficient 2 in the calculation) and finally,  $2\mu(1-\mu)$  is the probability that one firm enters in the SPCG.

The same calculation for firms with mixed signals gives

$$\begin{aligned}
\pi_{\sigma_H, \sigma_L} &= \Pr(\sigma_H, \sigma_L) (\lambda + (1-\lambda)\mu) (\Pr(H|\sigma_H, \sigma_L) - c) \\
&= 2p(1-p) (\lambda + (1-\lambda)\mu) \left( \frac{1}{2} - c \right).
\end{aligned}$$

Adding up gives

$$\pi_{\sigma_H, \sigma_H} + \pi_{\sigma_H, \sigma_L} = p - c,$$

regardless of  $\mu$ . Further, as calculated in the proof of Proposition 2,  $O = p^2 k - c + p(1-p) = p - c$  given  $k = 1$ . Thus, the expected industry profits are equal.

## 6.5 Proof of Lemma 2

**Proof.** Any  $\lambda_{low} < \lambda_M$  makes it more likely that the event “no entry in period one” occurs. Since any optimistic firm earns zero profits given its signal in this continuation game, it follows from the law of iterated expectations that the expected industry profit, given only priors, is also zero. The marginal change in welfare of a change in  $\lambda$  is thus

$$\begin{aligned} I_\lambda &= \frac{\partial}{\partial \lambda} ((\lambda^2 + 2\lambda(1-\lambda)) D + p(1-p)\lambda(1-2c)) = -2D\lambda + 2D - (D - N) \\ &= -2D\lambda + D + N. \end{aligned}$$

(compare with the derivation in Section 4.1), since entry is lower in period one. The gain is that there is a lower chance that an optimistic firm enters when firms have mixed signals. Evaluated at  $\lambda_M = N/D$ , we have  $I_\lambda = D - N > 0$  so that industry profits are increasing with  $\lambda$  at  $\lambda_M$ . ■

## 6.6 First period subsidies

Let  $\lambda$  be given by

$$\lambda = \frac{p^2 k + p(1-p) - c + s_1}{p^2 k - c(p^2 + (1-p)^2)}.$$

Then, the expected industry profit is

$$\begin{aligned} I_{FPS} &= 2 \Pr(\sigma_H, \sigma_H) (\lambda^2 + 2\lambda(1-\lambda)) (\Pr(H|\sigma_H, \sigma_H) k - c) + \\ &\quad + \Pr(\sigma_H, \sigma_L) \lambda (\Pr(H|\sigma_H, \sigma_L) - c) \\ &= (p^2 + (1-p)^2) (\lambda^2 + 2\lambda(1-\lambda)) \left( \frac{p^2 k}{p^2 + (1-p)^2} - c \right) + 2p(1-p)\lambda \left( \frac{1}{2} - c \right) \\ &= p^2 k + p(1-p) - c + s_1 \frac{p(1-p)(2c-1) - s_1}{p^2 k - c(p^2 + (1-p)^2)}. \end{aligned}$$

The FOC

$$\frac{\partial}{\partial s_1} ((p(1-p)(2c-1) - s_1) s_1) = 0,$$

then gives  $s_1 = p(1-p)(2c-1)/2$ . The second-order condition is clearly satisfied, since

$$\frac{\partial^2 I_{FPS}}{\partial s_1^2} = -\frac{2}{p^2 k - c(p^2 + (1-p)^2)} < 0.$$

## 6.7 Both period subsidies

Let  $\lambda$  be given by

$$\lambda = \frac{p^2 k + p(1-p) - c + a}{p^2 k - (c - b)(p^2 + (1-p)^2)}.$$

Probability  $\alpha$  is then given by

$$\begin{aligned} \alpha &= \Pr(\sigma_H^B | \sigma_H^A, Ne1B) = \frac{\Pr(\sigma_H^B, \sigma_H^A, Ne1B)}{\Pr(\sigma_H^A, Ne1B)} \\ &= 2 \frac{\Pr(\sigma_H^A | \sigma_H^B, Ne1B) \Pr(Ne1B | \sigma_H^B) \Pr(\sigma_H^B)}{(1-\lambda)(p^2 + (1-p)^2) + (1)(2p(1-p))} \\ &= 2 \frac{\Pr(\sigma_H^A | \sigma_H^B) \Pr(Ne1B | \sigma_H^B) \Pr(\sigma_H^B)}{(1-\lambda)(p^2 + (1-p)^2) + (2p(1-p))} = \frac{(1-\lambda)(p^2 + (1-p)^2)}{(1-\lambda)(p^2 + (1-p)^2) + (2p(1-p))}, \end{aligned}$$

where  $Ne1B$  denotes the event that firm B does not enter in period one.

Now, given expected industry profits (see the derivation  $I_{FPS}$  in the previous section), and  $b = (c - \frac{1}{2}k)$ , the first-order condition

$$\frac{\partial}{\partial a} I_{FPS} = 0,$$

gives

$$a = \frac{(p^2 + (1-p)^2)(pk - c)}{p^2 k - c(p^2 + (1-p)^2)} b + p(1-p)(k-1).$$

Further, the second-order condition is clearly satisfied, since

$$\frac{\partial^2 I_{FPS}}{\partial a^2} = -8 \frac{p^2 k - c(p^2 + (1-p)^2)}{(2p-1)^2 k^2}.$$

Given  $\lambda$ ,  $a$  and  $b$ , we then have

$$I_{FPS} = \frac{(pk - c)^2}{p^2 k - c(p^2 + (1-p)^2)}.$$

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## CHAPTER 2

### Inefficient Sequential Entry



# Inefficient Sequential Entry\*

Kaj Martensen

## Abstract

This paper studies sequential entry decisions when two potential entrants have private information about market profitability. Under symmetric information, there can be inefficient entry only if there are negative payoff externalities (i.e. if entry decisions are substitutes), and hence, insufficient entry never takes place. Under asymmetric information, there are several distortions coming from payoff and information externalities in a non-separable way. In particular, insufficient entry is now a possibility both for strong positive and negative payoff externalities. However, entry subsidies can be justified only when entry decisions are weakly complementary.

## 1 Introduction

When a firm enters a market, it often imposes externalities on existing firms and/or future potential entrants. If products are substitutes, these externalities are typically negative; if products are complements, the externalities are typically positive. Externalities related to substitution or complementarities between products are called payoff externalities, since entry by one firm has a direct effect on the other firms' payoff. Another type of externality arises when firms are asymmetrically informed about the profitability of entry. In this case, the entry decision of one firm potentially reveals that firm's private information.

The purpose of this paper is to investigate how payoff externalities and private information interact to determine sequential entry into a new market and to compare the equilibrium entry pattern to the socially optimal one. There are many "new markets" in a developing country and the problems of information accumulation are generally severe. "Diffuse externalities", i.e. where there are multiple linkages of information and payoff externalities between firms constitute an important explanation of distortions to entry in developing economies (World Bank, 1993, p. 92). Capital market imperfections exacerbate the problems, since they prevent entrepreneurs from accruing the capital required for entry into several complementary markets.

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The paper studies entry into new markets where both the pioneer and the follower's main uncertainty concerns the profitability of entering. The difficulty in achieving growth in the new Eastern European market has, for example, been attributed to lack of information. An OECD study (1995, p. 20) states that "One of the greatest negative factors inhibiting the attraction of FDI is the uncertainty that surrounds doing business in the region ...".<sup>1</sup> If we believe that such uncertainty is not, or cannot be successfully resolved by the government (perhaps because the government does not have the relevant knowledge or the correct incentives), the importance of the role of entry and information becomes clear.

To formally capture the linkage between information and payoff externalities, we study two risk neutral and rational firms, A and B. In the case of asymmetric information, firm A receives a private signal about the value of an entry opportunity and chooses whether to enter or not, knowing that firm B's choice will affect its own payoff. Firm B observes firm A's choice, receives a private signal, and decides whether to enter.

As an example, consider the case where both an aluminum plant and an aluminum using industry are needed. There will be no entry by the aluminum producer if the probability that it will be followed by the entry of the aluminum using industry is low. Similarly, the aluminum using firm will not enter if there is no aluminum production. The entry will naturally depend on the expectations of the state of the market for aluminum products. If the prospects are low for the aluminum producer, it might abstain from entering although its entry can be socially beneficial. Further, the information that could be generated by entry is lost to society.

We find that under symmetric information, equilibrium entry departs from socially optimal entry only when the entry of one firm creates a negative payoff externality on the other, and in this case, there is sometimes excess entry. For positive payoff externalities on the other hand, entry decisions are always efficient. When information is asymmetric, the above result no longer holds. There are several distortions, some of which give insufficient entry, and some of which give excess entry. Further, pioneering entry is not always informative. When complementarities are strong enough to make uninformed entry profitable, two distinct distortions may create inefficient entry.

The first distortion is that initial entry decisions may fail to reveal information, in which case there can be an "entry cascade" driven by the information externality. The second potential distortion is that a leader with a pessimistic signal fails to enter, although expected industry profits are positive. The reason here is that the leader takes all the losses if there is no subsequent entry, whereas followers share in the gain if they choose to enter. Surprisingly, the probability of entry of a follower can then be *decreasing* in the

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<sup>1</sup> Apart from local entry, the inflow of foreign direct investment is important for a developing country. As a further illustration of the informational issues, consider General Motor's decision to locate its Asian center in Thailand: "... the fact that 11 car manufacturers already operate in Thailand was a sign that the country's infamous physical infrastructure and labor bottlenecks could be overcome." (Bardacke 1996). Although multinationals do not suffer from a lack of capital, there are often other exogenous factors that prevent a multinational from entering on the desired scale (legislation etc.). Under such conditions, the results of this paper also apply to foreign direct investment. See Section 4.6 for the case of perfect capital markets.

entry of the leader, even under strong complementarities, a mechanism here termed the “isolation effect”. The insufficient entry created by the isolation effect seems to be the problem the World Bank has in mind. However, when uninformed entry is profitable, we show that it is very hard to design policies to deal with the inefficiencies, without introducing sizable new distortions.

On the other hand, when uninformed entry is not profitable, new distortions appear. We show that these distortions arise as a consequence of the leader not being compensated for the positive informational externality of entry.<sup>2</sup> This extends the earlier information externality literature, since it shows that information externalities are important also when there are negative payoff externalities. Here, we show that the scope for intervention is greater. Subsidies are sometimes efficient in the case of insufficient entry. Further, entry barriers are also welfare improving in some cases of excess entry.

In the theory of industrial organization, entry under asymmetric information is by and large treated as a capacity, pricing or quantity signal problem for the incumbent; see for example Milgrom and Roberts (1982). Our approach differs in that it studies the informational aspects of entry in itself; we do not allow “investments in disinformation”. There is a small number of related papers; information externalities and entry have been studied by, for example, Rob (1996) and Creane (1996), who focus on information externalities of entry in competitive markets. There is no strategic interaction in their models and information and payoff externalities operate only on an aggregate level.<sup>3</sup> Choi (1997) finds a mechanism similar to the isolation effect, but studies the adoption of technical standards (the choice between two network goods rather than entry decisions). His approach differs from ours in that a first adopter directly reveals the quality of the adopted good, whereas entry in our model does not generate information – it conveys information. Moreover, the information conveyed by entry is determined in equilibrium in our model. If the leader enters regardless of signal for example, the entry decision has no informational content. Alexander et al. (1998) study entry under increasing returns to scale when firms are privately informed of market profitability. As in Choi’s model, entry is always informative. Distortions to entry can also occur under pure information externalities. Chamley and Gale (1994) show that there can be an inefficient delay in entry when agents have the incentive to free ride on the informative entry of others.<sup>4</sup> Information externalities and search theory have also been used to explain FDI patterns. Firms have incentives to free ride on other firms’ costly search for investment opportunities, since an entry decision has a positive information externality; see Thimann and Thum (1998) or Huang and Shirai (1994). These papers do not consider payoff externalities, however. Information aggregation in developing economies has also been studied in other contexts. In particular, Acemoglu and Zilibotti (1999) show how entry into new markets

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<sup>2</sup>More precisely, the inefficiencies come from the leader not being compensated for conveying information. In some cases, it is socially desirable that the leader conveys his information without entering. However, in these cases, the leader has no incentive to communicate his information; see Section 4.5.

<sup>3</sup>A recent (unpublished) paper by Dasgupta (2000) models payoff and information externalities with strategic interaction. His results are related to ours in the concluding section.

<sup>4</sup>The paper by Chamley and Gale is related to earlier papers on pure information externalities; see Banerjee (1992) or Bikhchandani et al. (1992).

can ameliorate principal agent distortions in a developing economy through the effects of entry on information accumulation.

The model is presented in Section 2. Symmetric information is treated in Section 3 and the asymmetric case in Section 4. The scope for intervention is examined in Section 5 and we conclude in Section 6.

## 2 The Model

Two firms, A and B, each with one unit to invest, both consider entering a new market. The timing and order of entry is assumed to be exogenous. Capital markets are assumed to be incomplete, so that no firm can enter with two units of capital.<sup>5</sup> Profits to entry depend both on the state of the market and the number of entrants. There are two states of the market,  $H$  and  $L$ , and the two firms share the common prior belief that both states are equally likely, so that  $\Pr(H) = \Pr(L) = 0.5$ . The payoff externality is captured by the parameter  $t$  which relates the ex post payoff of one single entrant to the profit it would obtain if two firms entered. There are two signals,  $\sigma^A$  and  $\sigma^B$ , about the true state of the market, where  $\sigma^A, \sigma^B \in \Sigma = \{\sigma_H, \sigma_L\}$ .<sup>6</sup> The signals are drawn independently from the conditional Bernoulli distribution

$$\begin{aligned}\Pr(\sigma_H|H) &= \Pr(\sigma_L|L) = p, \\ \Pr(\sigma_L|H) &= \Pr(\sigma_H|L) = 1 - p,\end{aligned}$$

where  $p > 0.5$ . The informativeness of the signals are thus given by the parameter  $p$  and the signals are informative and of equal precision.<sup>7</sup> Let  $\sigma = (\sigma^A, \sigma^B)$ .

In the case of symmetric information, the timing is as follows: At date 0, Nature chooses the state of the market,  $x \in \{H, L\}$ . At date 1, firm A receives  $\sigma$  and chooses whether to enter or to stay out (the outside option is assumed to be of zero value to the firms). At date 2, firm B receives  $\sigma$ , observes A's choice and, contingent on this information, decides whether to enter. Finally, at date 3, the profits are realized.

In the asymmetric information model, the firms are only assumed to receive a private signal before their respective entry opportunity (firm B still observes A's entry decision). Hence, at date 1, firm A receives the private signal  $\sigma^A$  and at date 2, firm B receives the private signal  $\sigma^B$ .

Thus, under asymmetric information, firm A's (pure) strategy  $s^A : \Sigma \rightarrow \{0, 1\}$  is a mapping from its signal to an entry decision, where 1 is taken to mean entry. Similarly, firm B's (pure) strategy is a mapping  $s^B : \Sigma \times \{0, 1\} \rightarrow \{0, 1\}$  from its signal and its

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<sup>5</sup>Capital markets are assumed to be incomplete, so that the market participants cannot amass the capital to benefit from complementarities, or, when entry decisions are substitutes, preempt entry by entering on a larger scale. Since the outcome is worse if imitation and larger scale entry are permitted (see Section 4.6), the inefficiencies are shown to hold under weaker conditions in one respect.

<sup>6</sup>With a slight abuse of notation, we let  $\sigma$  denote both the stochastic variable and its realizations; which applies will be clear from the context.

<sup>7</sup>If  $p < 1/2$  this would only mean that the firms reversed the meaning they attach to their signal and the same results would be obtained.

observation,  $y \in \{0, 1\}$  of A's decision. The strategies under symmetric information are similarly defined.<sup>8</sup>

The firms are assumed to enter if the expected profit is strictly positive. The ex post payoff  $\pi(x, n)$  for a firm depends on the state  $x$ , and the number  $n$ , of entering firms. The high state is profitable and the low state gives a loss to the entering firm. Thus,

$$\pi(L, n) < 0 < \pi(H, n) \text{ for } n = 1, 2.$$

We assume that when entering is "bad" (the low state is realized), then it is equally bad notwithstanding the number of entrants, i.e.

$$\pi(L) = \pi(L, 2) = \pi(L, 1).$$

To formalize the payoff interdependence, define

$$t = \frac{\pi(H, 2)}{\pi(H, 1)}.$$

Then,  $t > 1$  signifies that entry decisions are complements while  $t < 1$  means that they are substitutes. Hence, when  $t = 1$ , there is no payoff interdependence between the firms' entry decisions. By the statement "entry decisions are complements", we mean that the expected profit of entry for one firm increases in the entry of the other firm, holding the strategies and information available to the firms fixed. That is, the effects of e.g. complementarity will refer to payoff interdependence and not to strategic complementarity. Similarly, the statement "entry decisions are complements on the industry level", means that, *ceteris paribus*, the sum of the firms' expected profits is increasing in entry. When entry decisions are weak substitutes for the firms, the industry profit may still be increasing in entry, since the gain from additional entry can outweigh the effects of the negative payoff externality.

The paper focuses on information and insufficient entry. Thus, we study the case of

$$\pi(L) \leq -\pi(H, 1),$$

so that there is a "downside" to entry that potentially forces the firms to use their signals. We let the parameter  $d \geq 1$  capture this downside. Thus we have

$$\pi(L) = -d\pi(H, 1).$$

In the asymmetric case, let  $\pi^A(s^A, s^B; \sigma^A)$  be firm A's expected value of  $\pi(x, n)$  given its signal and the strategy profile. Similarly, firm B's payoff is  $\pi^B(s^A, s^B; \sigma^B, y)$  where it also takes into account its signal (observation)  $y$ . The payoff functions under symmetric information are similarly defined.

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<sup>8</sup>Note that the action space is "fine enough" to be a sufficient statistic for the signal. That is, entry is a binary decision that can potentially convey the binary signal; potentially, since the informational content of entry is determined in equilibrium. If, for example, A entered regardless of its signal, no information would be conveyed by entry. In contrast, informational cascades can be exogenously introduced if the there are less actions than signals; see Lee (1993).

Formally, under symmetric information we define the game  $G_{p,t,d}$  as the triplet  $G_{p,t,d} = (N, S, \pi)$ , where  $\pi = (\pi^A(s^A, s^B; \sigma), \pi^B(s^A, s^B; \sigma, y))$ ,  $N = \{1, 2\}$  and  $S = S^A \times S^B$ . Each triple  $(p, t, d)$  thus gives a game, but our interest is in the set of games

$$G = \{G_{p,t,d} | p \in (0.5, 1], t \in R_{++}, d \in [1, \infty)\}.$$

The set of games under asymmetric information,  $\Gamma$ , is similarly defined. Although  $G$  and  $\Gamma$  are sets of games, they will be casually referred to as games.

To avoid cumbersome expressions later, the following definitions are useful.

**Definition 1** *For the symmetric information model, let*

$$\begin{aligned} f_1 &= dp^2 / (1-p)^2, \\ f_2 &= d, \\ f_3 &= \begin{cases} d(1-p)^2 / p^2, & \text{for } p \leq \sqrt{d} / (\sqrt{d} + 1); \\ (p^2 + d(1-p)^2) / 2p^2, & \text{otherwise,} \end{cases} \\ g_1 &= dp^2 / (1-p)^2, \\ g_2 &= d, \\ g_3 &= (1-p)^2 / p^2. \end{aligned}$$

**Definition 2** *For the asymmetric information model, let*

$$\begin{aligned} \phi_1 &= \begin{cases} dp / (1-p), & \text{if } p < (d + 1 + \sqrt{d(d-1)}) / (3d + 1); \\ ((1-p)^2 + dp^2) / (2(1-p)^2), & \text{otherwise,} \end{cases} \\ \phi_2 &= \begin{cases} dp / (1-p), & \text{if } p < (d + 1 + \sqrt{d(d-1)}) / (3d + 1); \\ (d - (1-p)^2(d+1)) / (2p(1-p)), & \text{otherwise,} \end{cases} \\ \phi_3 &= \begin{cases} d(1-p) / p, & \text{if } p < 2d / (3d + 1); \\ (1+d) / 2, & \text{otherwise,} \end{cases} \\ \phi_4 &= \begin{cases} d(1-p) / p, & \text{if } p < 2d / (3d + 1); \\ (1-p)(d(2-p) - p) / (2p^2), & \text{if } p \in [2d / (3d + 1), \sqrt{d} / (\sqrt{d} + 1)]; \\ (p(2p-1) + d(1-p)) / (2p^2), & \text{otherwise,} \end{cases} \\ \gamma_1 &= dp / (1-p), \\ \gamma_2 &= d, \\ \gamma_3 &= \begin{cases} d, & \text{if } p < (\sqrt{((d-1)^2 + 4d^2)} - (d+1)) / (2(d-1)); \\ (d-p)(1-p) / p^2, & \text{otherwise,} \end{cases} \\ \gamma_4 &= \begin{cases} \gamma_3(p), & \text{if } p \leq d / (d+1); \\ d(1-p)^2 / p^2, & \text{otherwise.} \end{cases} \end{aligned}$$



Given a fixed parameter pair  $(p, d)$ , the above expressions are definitions of real numbers. However, we will use them as functions of the parameters. The functions will be used to partition the parameter space. All functions have domain  $p \in (0.5, 1]$ , and we will, for instance, write  $f_1(p)$ , to denote that we are using  $f_1$  as a function of  $p$  (the dependency on  $d$  is suppressed).<sup>9</sup> Functions written in Roman letters will refer to either first best ( $f$ -functions) or equilibria ( $g$ -functions) of  $G$ , that is, the game under symmetric information. Similarly, Greek letters will refer to first best ( $\phi$ -functions) or equilibria ( $\gamma$ -functions) of  $\Gamma$ , the game under asymmetric information.

**Remark 1** *For a fixed pair  $(p, d)$ , the functions given the same letter are ordered, e.g.  $g_1(p) \geq g_2(p)$ ,  $\phi_1(p) \geq \phi_2(p) \geq \phi_3(p)$  etc., for all  $d \geq 1$  and  $p \in (0.5, 1]$ .*

**Proof.** See Appendix, Section 7.1. ■

### 3 Symmetric Information

We will begin by finding entry that maximizes expected industry profits conditional on both signals. The optimal level of entry is shown to be increasing in the strength of complementarities, but sometimes decreasing in the informativeness of the signals.

Next, the equilibrium level of entry is derived and the resulting inefficiency is characterized. To facilitate reading, a strategy where e.g. firm B enters if  $\sigma = (\sigma_H, \sigma_H)$  or  $y = 1$ , will be written  $s^B = \text{"Enter if A did or high signals"}$ .

Since no informational distortions are present under symmetric information, it is the difference between complementarities on the firm and industry level, respectively, that distorts the firms' entry from the socially optimal level.

#### 3.1 First best

As a benchmark, consider the decisions which maximize expected industry profits, conditional on both signals. Consider first the case  $\sigma = (\sigma_H, \sigma_H)$ . The expected profit to one entrant, given two high signals, is positive if

$$\Pr(H|\sigma_H, \sigma_H) \pi(H, 1) + \Pr(L|\sigma_H, \sigma_H) \pi(L) > 0,$$

or equivalently

$$p^2 - d(1-p)^2 > 0.$$

---

<sup>9</sup>Formally,  $\gamma_3$  is not well-defined for  $d = 1$ . However, the function is well-behaved in the sense that the limit  $\lim_{d \rightarrow 1} \left( \sqrt{(d-1)^2 + 4d^2} - (d+1) \right) / (2(d-1))$  exists and is finite. Functions  $f_1$ ,  $g_1$  and  $\gamma_1$  are not well-defined at  $p = 1$ , but this has no practical importance in what follows.

This holds if  $p > \sqrt{d}/(\sqrt{d} + 1)$ . First consider the case  $p > \sqrt{d}/(\sqrt{d} + 1)$ .<sup>10</sup> Then, at least one firm ought to enter. It is optimal that both firms enter if

$$2(\Pr(H|\sigma_H, \sigma_H)t - d\Pr(L|\sigma_H, \sigma_H)) > \Pr(H|\sigma_H, \sigma_H) - d\Pr(L|\sigma_H, \sigma_H).$$

Thus, two firms enter if

$$t > \frac{p^2 + d(1-p)^2}{2p^2},$$

and one firm otherwise.

Next, consider the case  $p \leq \sqrt{d}/(\sqrt{d} + 1)$ .<sup>11</sup> Single entry is not profitable, but two firms should enter if

$$2(\Pr(H|\sigma_H, \sigma_H)t - d\Pr(L|\sigma_H, \sigma_H)) > 0.$$

That is, if

$$t > d \frac{(1-p)^2}{p^2}.$$

Thus, for high signals, both firms enter for  $t > f_3(p)$ .

Entry by a single firm is not profitable if the firms receive low or mixed signals, since

$$\Pr(H|\sigma_H, \sigma_L) - d\Pr(L|\sigma_H, \sigma_L) = \frac{1}{2}(1-d) \leq 0.$$

Now, consider the case of mixed signals. Given  $\sigma = (\sigma_H, \sigma_L)$  or  $\sigma = (\sigma_L, \sigma_H)$ , both firms should enter if  $t > f_2(p)$ , and none otherwise. Similarly, if  $\sigma = (\sigma_L, \sigma_L)$  is obtained, they should both enter if  $t > f_1(p)$ , and none otherwise. The figure below gives the number of entrants in the first best solution for  $p \in (0.5, 1]$  (the horizontal axis),  $t \in [0, 4]$  (the vertical axis) and  $d = 2$ .

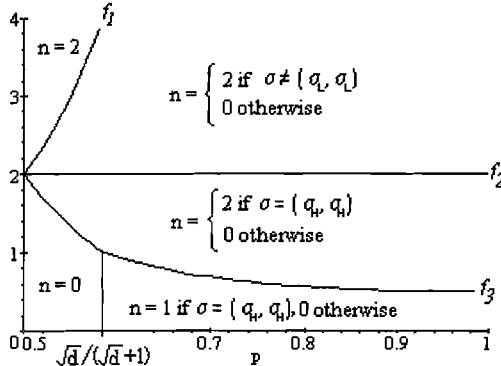


Figure 1. First best entry under symmetric information.

<sup>10</sup>Since  $\sqrt{d}/(\sqrt{d} + 1) < 1$  for all  $d \in [1, \infty)$ , there is always a  $p$  such that single entry is profitable.

<sup>11</sup>If  $d = 1$ , this case is not valid.

From a first best perspective, it is thus sometimes optimal for both firms to enter even when entry decisions are (weak) substitutes ( $t \in (f_3(p), 1)$ ). Here, entry decisions are complements on the industry level, since the gain from additional entry outweighs the negative payoff externality.

When complementarities outweigh the potential downside ( $t > f_2(p)$ ), entry can be decreasing in the informativeness of the signals. The intuition is that when complementarities are strong, the presumption is to enter. However, if the signals are very informative, then low signals make entry unprofitable.

### 3.2 Equilibria

We vary the parameters  $p$  and  $t$ , and examine the existence of subgame perfect equilibria of  $G$ .

**Proposition 1** *The subgame perfect equilibria of  $G$  are characterized by:*

- (1): *If  $t > g_1(p)$ , the unique equilibrium is*  
*("Always enter", "Enter if A did or high signals").*
- (2): *If  $t \in (g_2(p), g_1(p)]$ , the unique equilibrium is*  
*("Enter for mixed or high signals", "Enter for high signals or mixed signals only if A did").*
- (3): *If  $t \in (g_3(p), g_2(p)]$ , the unique equilibrium is*  
*("Enter for high signals", "Enter for high signals").*
- (4): *If  $t < g_3(p)$  and  $p > \sqrt{d}/(\sqrt{d} + 1)$ , the unique equilibrium is*  
*("Enter for high signals", "Never enter if A entered, else enter for high signals").*
- (5): *If  $t < g_3(p)$  and  $p \leq \sqrt{d}/(\sqrt{d} + 1)$ , the unique equilibrium is*  
*("Never enter", "Never enter").*

**Proof.** First look at B's best reply, given the signals and A's entry decision. Then, given B's best reply, A's best reply can be derived. There are three cases to consider. Here, the proof is given for the case of low signals. The other cases are similar.

**Case  $\sigma = (\sigma_L, \sigma_L)$ :** Note that it is not profitable to be a sole entrant since

$$\Pr(H|\sigma_L, \sigma_L) - d\Pr(L|\sigma_L, \sigma_L) = ((1-p)^2 - dp^2) / (p^2 + (1-p)^2) < 0.$$

If A entered, B enters if

$$((1-p)^2 t - dp^2) / (p^2 + (1-p)^2) > 0.$$

That is, B enters if

$$t > dp^2 / (1-p)^2 = g_1(p).$$

Given that B only enters if  $t > g_1(p)$  and that the expected profit of entering for two entrants is positive in this region, A also enters for  $t > g_1(p)$ .

Since the expected profit is increasing in the signals, we can conclude that both firms will enter also for mixed or high signals when  $t > g_1(p)$ . In this region, entry is thus not conditional on the signals. ■

The number of entrants for each equilibrium is given by the following figure (again  $d = 2$  is chosen for illustrative purposes).

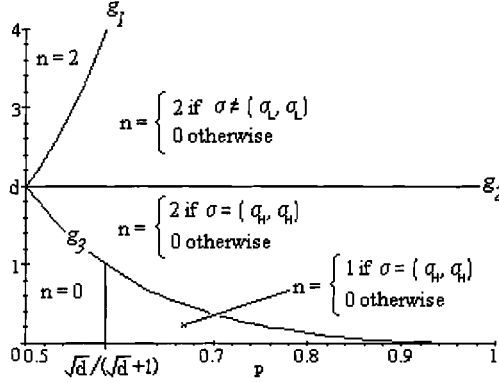


Figure 2. Entry under symmetric information.

As expected, entry is increasing in complementarities  $t$ , and decreasing in the downside  $d$ . However, if  $t > d$ , entry is decreasing in the informativeness of the signals. The same intuition as in the first best applies; when complementarities outweighs the risk, as reflected by  $d$ , then the presumption is to enter, even for low signals. Only if the low signals are very informative firms stay out. On the other hand, if  $t < d$ , then entry is increasing in  $p$ ; since the risk is high ( $d$  is high relative to  $t$ ), the presumption is not to enter, even at favorable signals. Only if these favorable signals are very informative it is profitable for firms to enter.

### 3.3 Distortions

Comparing the equilibria with the first best reveals that entry is efficient when entry decisions are complements. The intuition behind this result can be found by considering a simultaneous game. If the signals are mixed, entry is only optimal when both firms enter. In the simultaneous game, there is thus a coordination problem; either both should enter or both stay out. On the other hand, in the sequential game, the leader can coordinate on the Pareto dominant equilibrium. There can be inefficient entry only when entry decisions are substitutes on the industry level. The figure below and the following proposition characterizes this result.

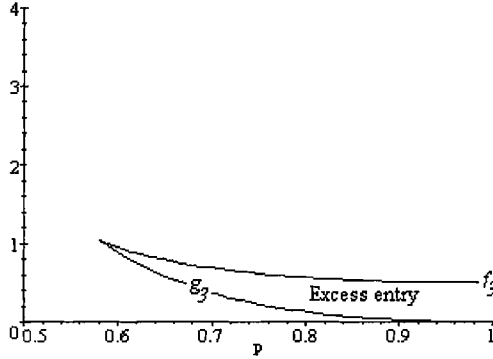


Figure 3. Equilibria versus first best under symmetric information.

**Proposition 2** *Under symmetric information, the only distortion comes from the negative externality that a follower's entry imposes on the leader when entry decisions are substitutes.*

In particular, insufficient entry never occurs under symmetric information. Thus, the symmetric information model can not explain growth problems related to under-investment. We now turn to an analysis of the case of asymmetric information.

## 4 Asymmetric Information

We proceed with the model under asymmetric information. As before, we begin with a derivation of the socially optimal level of entry. Next, the equilibria are characterized and contrasted to the first best. Distortions now come from both payoff and information externalities in a non-separable way.

To study the robustness of the model to the underlying assumptions, three further extensions are considered. First, endogenous timing is discussed. Next, we allow for the leader to communicate its signal to the follower to examine whether this can ameliorate the observed distortions. Finally, we consider what would happen if the leader could imitate a potential follower.

### 4.1 First best

Consider now the entry problem a single owner of both firms would face if information is sequentially revealed. Let each firm maximize expected industry profits conditional on its information. After A has entered, it shares its signal with firm B. Thus, in period one, A receives the signal  $\sigma^A$  and chooses whether to enter. Subsequently, in period two, B receives  $\sigma = (\sigma^A, \sigma^B)$  and decides whether to enter or not.

Let  $Y = \{0, 1\}$  and  $\Omega = \{\sigma_L, \sigma_H\}$ . Optimality requires that firm B, for each history  $h \in Y \times \Omega$  and signal  $\sigma^B$ , compares the expected profit of entering with the outside

opportunity. Given the optimal choice conditioned on  $h$  and  $\sigma^B$  in period two, firm A's expected profit of utilizing the first entry opportunity given  $\sigma^A$ , can be compared with the value of the outside opportunity. The firms' entry as a function of the parameters  $p$ ,  $t$  and  $d$  (see Appendix, Section 7.2 for calculations), are described by the figure below for  $p \in (0.5, 1]$ ,  $t \in [0, 5]$  and  $d = 2$ .

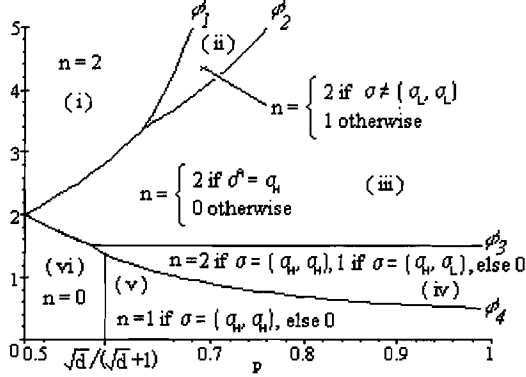


Figure 4. First best under asymmetric information.

Under strong complementarities (region i), entry by both firms is optimal, regardless of signals. With lower complementarities, the firms are successively more selective (regions ii to iii). In region ii, B will defer from entry in period two, should two low signals be obtained.

When entry decisions are substitutes on the firm level but complements on industry level (in a sub region of region iv), the first best is similar to the case of symmetric information; entry by both firms is sometimes optimal. Finally, when entry decisions are substitutes on the industry level (region v), only one firm should enter.

## 4.2 Equilibria

We vary the parameters  $p$  and  $t$  and examine the existence of perfect Bayesian equilibria of  $\Gamma$ .<sup>12</sup>

**Proposition 3** *The perfect Bayesian equilibria of  $\Gamma$  are characterized by:*

(I): *If  $t > \gamma_1(p)$ , the unique equilibrium is*

*(“Always enter”, “Enter if A did, if A did not enter, enter if  $\sigma^B = \sigma_H$  and  $p > d/(1+d)$ ”).*

<sup>12</sup>When B's beliefs do not play a strategic role, we have assumed passive conjectures. In the proof of Proposition 3, we show that B's beliefs do not have a strategic importance for equilibrium I. Thus, if firm A abstains out of equilibrium, then B enters given a high signal and  $p > d/(1+d)$ . In case A enters out of equilibrium, beliefs have a strategic role. However, the assumption that a firm only enters if the expected profit from entry is positive, restricts beliefs to  $\Pr(\sigma^A = \sigma_H | \text{Entry}) = 1$  when entry is not ex ante profitable (see Section 7.3.2).

- (II): If  $t \in (\gamma_2(p), \gamma_1(p)]$ , the unique equilibrium is ("Enter if  $\sigma^A = \sigma_H$ ", "Enter if A did").
- (III): If  $t \in (\gamma_3(p), \gamma_2(p)]$ , the unique equilibrium is ("Enter if  $\sigma^A = \sigma_H$ ", "Enter if A did and  $\sigma^B = \sigma_H$ ").
- (IV): If  $t \in (\gamma_4(p), \gamma_3(p)]$  and  $p > d/(1+d)$ , the unique equilibrium is ("Never enter", "Enter if  $\sigma^B = \sigma_H$ ").
- (V): If  $t < \gamma_4(p)$  and  $p > d/(1+d)$ , the unique equilibrium is, ("Enter if  $\sigma^A = \sigma_H$ ", "Never enter").
- (VI): If  $t < \gamma_3(p)$  and  $p \leq d/(1+d)$ , the unique equilibrium is ("Never enter", "Never enter").

**Proof.** See Appendix, Section 7.3. ■

Note that for all equilibria except IV, the equilibria can also interpreted as coming from a game where the follower discovers the entry opportunity only if entry has taken place. This also gives another possible interpretation of B's signal: It can be thought of as public information generated by A's entry.

The number of entrants in equilibrium is given by the following figure (again  $d = 2$ ).

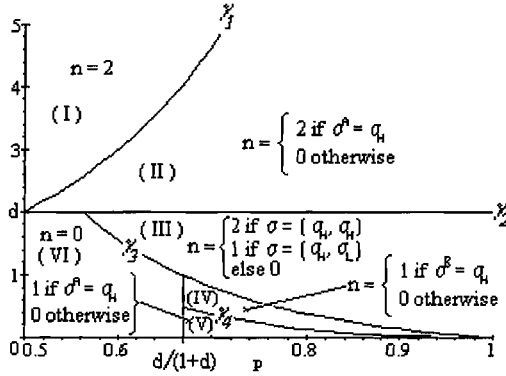


Figure 5. Entry under asymmetric information.

Let us compare the equilibria with the equilibria under symmetric information (see Figure 2). We order the regions in Figure 5 such that region I, for example, means the region in Figure 5 where equilibrium I holds, namely  $t > \gamma_1$ . Now, as signals becomes more informative (as  $p \rightarrow 1$ ), we expect firms to behave as under symmetric information. Comparing the relevant regions (II and III in Figure 5, and corresponding in Figure 2), we find that when uninformed entry is profitable ( $t > d$ ), the entry pattern is the same, except that no entry takes place under asymmetric information when  $\sigma = (\sigma_L^A, \sigma_H^B)$ . Further, when uninformed entry is not profitable, we again find the same entry pattern, except that firms enter also for  $\sigma = (\sigma_H^A, \sigma_L^B)$  under asymmetric information. The intuition is that, as  $p \rightarrow 1$ , the probability of firms receiving mixed signals approaches zero. Thus,

the entry pattern will converge to that of symmetric information, as signals become more informative.

When uninformed entry is not profitable, there are several differences between the symmetric and asymmetric equilibria. First, the region of no entry increases in size. The reason is that when entry is profitable for one firm only, a firm with more information (i.e. two signals in the symmetric case) can find an entry opportunity profitable even for low informativeness of the signals. Next, when single entry is profitable given one high signal, there are new strategic effects. In equilibrium V, firm A has a first mover advantage. The substitution effect is so strong that if A enters given a high signal, B will not find it optimal to enter given two high signals. On the other hand, in equilibrium IV, firm B has a second mover advantage. The substitution effect is not so strong, so if A's entry conveys information, B will find it optimal to enter given two high signals. That is, given two high signals, B finds the expected profit sufficiently great to allow for two firms on the market, in spite of the substitution effects. However, given only one high signal, firm A's expected profit of entry is negative; either does B have a low signal in which case entry is not profitable or, B has a high signal in which case entry is not very profitable since B will enter. Thus, A will abstain regardless of signal and B will enter for a high signal.

On the other hand, when uninformed entry is profitable ( $t > d$ ), firms efficiently coordinate their entry, and enter for all but two pessimistic signals under symmetric information. Under asymmetric information, the firms enter regardless of signals for a larger set of values of  $p$  and  $t$  (since  $g_1 > \gamma_1$ ). Further, there will be less entry when information is asymmetric – the leader will not enter for a low signal in region II.

Note that for all equilibria except V, the leader is more cautious than the follower in the sense of weakly requiring a higher signal to enter than the follower.

Taking the first best case as benchmark, we can characterize the entry made by the two firms. First, consider the case when the ex ante gain from single entry is equal to the ex ante loss ( $d = 1$ ).<sup>13</sup> The following figure compares the first best entry to the equilibrium entry for  $d = 1$ .

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<sup>13</sup>That is, when  $d = 1$ , then  $\Pr(H) \pi(H, 1) + \pi(L, 1) \Pr(L) = 0$ , so that single entry is not profitable for an uninformed firm.



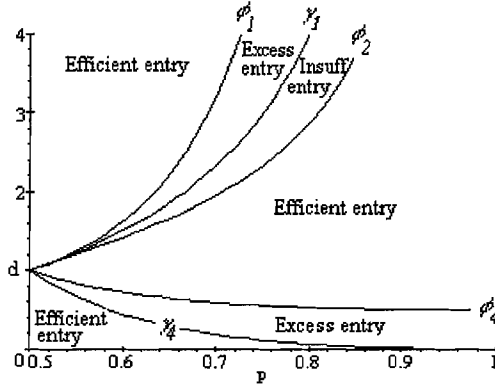


Figure 6. Equilibria versus first best under asymmetric information ( $d = 1$ ).

Next, a figure for the case  $d = 2$ . Here, we have only plotted the appropriate pieces of the functions.

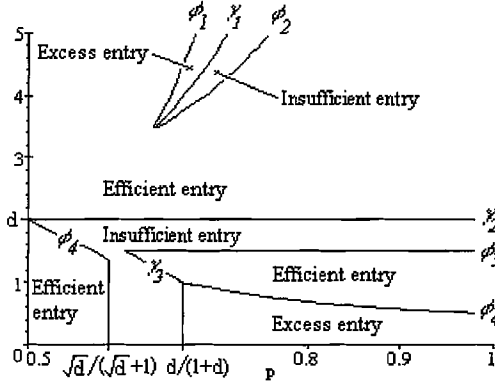


Figure 7. Equilibria versus first best under asymmetric information ( $d = 2$ ).

Thus, for  $d = 2$ , there are some new distortions if uninformed entry is not profitable, which is why our analysis is carried out for  $d > 1$ . Does Figure 7 give a correct characterization of the distortions for all  $d$ ?

**Remark 2** For  $d > 1$ , we have  $\phi_1(p) > \gamma_1(p) > \phi_2(p)$ ,  $\gamma_2(p) > \phi_3(p)$  whenever the functions do not coincide. Further, the functions do not coincide everywhere for  $d > 1$ . Finally  $\phi_4(p) > 0$ .

**Proof.** See Appendix, Section 7.4. ■

Thus, these inefficiencies do not disappear for any downside  $d > 1$ . The next section proceeds with a detailed analysis of the distortions.

### 4.3 Distortions

We have seen that entry under payoff and information externalities can lead to inefficiencies. The task is now to separate the effects of payoffs and information under asymmetric information. We focus on the case  $d > 1$ , since the distortions for  $d = 1$  are subsumed by this case (by Remark 2). Three distinct distortions that can give insufficient entry are identified. Further, two separate distortions that can cause excess entry are identified. Moreover, the payoff externality that created excess entry under symmetric information is still present. To relate the distortions to the parameters for which they occur, the regions of the equilibria versus the first best are enumerated as follows:

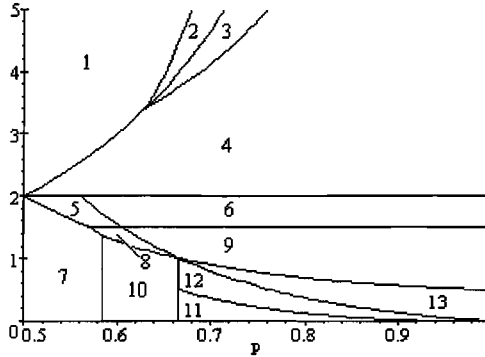


Figure 8. First best and equilibrium regions ( $d = 2$ ).

#### 4.3.1 Insufficient entry

**The isolation effect** Insufficient entry can occur even though complementarities are fairly strong (region 3 in Figure 8). To understand this, consider a situation where entry decisions are complements and the entry of a follower is uncertain. By entering, the leader gives the follower a positive externality – the opportunity to benefit from complementarities. If the follower does not enter, the leader is not compensated for giving this opportunity to the follower and, because of that risk, the leader will reduce entry.

Farrell and Saloner (1985) show that there can be excess inertia in the adoption of a network good under incomplete information. The inefficiency is similar: The fear of being the sole adopter of a network good with little value in isolation reduces initial adoption. In their model, if a consumer adopted in the first period, additional adoption would follow with high probability.

Our isolation effect differs, since the expected profitability of entry is endogenously determined. The leader's decision conveys information. Hence, if the leader enters for a larger subset of signals, the informative content of her decision is diminished. This effectively forces a follower to reduce entry by demanding that entry to a greater extent

is conditional on his information. The probability of entry of a follower can thus be decreasing in the entry of the leader.

In terms of our model, if firm A always entered in region II of Figure 5, firm B would only enter if it received a high signal. Then, firm A would be a sole entrant, should B receive a low signal. This risk reduces the profitability of entering for A. Thus, firm A has to be more cautious by requiring a high signal before entering. For sufficiently high complementarities (region 3 of Figure 8), the expected industry profits rise if A is compensated for its risk. The expected profit, given an initial low signal, is zero in equilibrium II, while the expected industry profit in the first best is

$$\bar{\pi}_3 = 2 \left( \Pr(H, \sigma_H^B | \sigma_L^A) t - d \Pr(L, \sigma_H^B | \sigma_L^A) \right) + \Pr(H, \sigma_L^B | \sigma_L^A) - d \Pr(L, \sigma_L^B | \sigma_L^A) .$$

This expression is greater than zero if

$$t > (2p(1-p)d - (1-p)^2 + dp^2) / (2p(1-p)) .$$

But,

$$t > \phi_2(p) > (2p(1-p)d - (1-p)^2 + dp^2) / (2p(1-p)) ,$$

in region 3, so profits are on average increased by compensating the leader for its risk.

### **The leader is not compensated for the positive information externality of entry**

In equilibrium, A enters for a high signal in region 6, and given entry by A, firm B enters for a high signal. However, industry profits are maximized if B enters also for a low signal, given entry by A. It might be argued that this is “throwing good money after bad”, since entry by both for mixed signals does not give positive expected profits, though given entry by A, entry by B raises expected industry profits. Due to complementarities, it is cheaper to compensate A by making B enter than by a subsidy for example. Of course, there is no such solidarity from B, which leads to insufficient entry. Note that the efficient amount of information is transmitted in equilibrium, since firm A only enters for a high signal. Thus, it seems like this is not an informational problem. Nevertheless, comparing with the case of symmetric information, the distortion disappears. The intuition is that under symmetric information, firm A can avoid entry in the case signals are mixed. Under asymmetric information, firm A finds it profitable to enter given a high signal, but firm B will not follow given that it receives a low signal. In this sense, firm A is sometimes not compensated for its provision of externalities, although the inefficiency comes from firm B’s failure to internalize the complementary effect of entry. Hence, this distortion is similar to the isolation effect.

The same “lack of solidarity” from B occurs in region 5, but now the low informativeness of the signals makes expected profitability so low that in equilibrium, A abstains from entry. Moreover, there will be no single entry, since  $p < d/(d+1)$ . Here, the lack of solidarity effect is stronger in the sense that now, too little information is transmitted and used in equilibrium.

In region 8, the complementarities are weak, and industry profits do not increase with entry should B receive a low signal. Nevertheless, the expected industry profits, given an initial high signal, are great enough to make initial entry profitable in the first best. In equilibrium, A fails to internalize the complementary effect of entry, since the expected profit of entry is negative (given that B would only follow for a high signal). There is a lack of solidarity effect in the sense that A would prefer that B always entered. If A entered for a high signal, and B entered given entry by A and a high signal, then the expected profits to A is

$$\pi^A = \Pr(H, \sigma_H | \sigma_H) t + \Pr(H, \sigma_L | \sigma_H) - d \Pr(L | \sigma_H) = p^2 t + p(1 - p) - d(1 - p),$$

while the profit to A if B always entered is

$$\hat{\pi}^A = \Pr(H | \sigma_H) t - d \Pr(H | \sigma_H) = pt - d(1 - p).$$

Then  $\hat{\pi}^A > \pi^A$  if  $t > 1$ , which holds in region 8.<sup>14</sup>

**The leader does not give information to the follower** In region 10, the leader does not convey his information to the follower. In this region, only single entry given two high signals gives positive expected profits. Since A only has access to one signal, A abstains. Abstaining does not convey any information, and thus, B has access to only one signal and also abstains. The mechanism differs from the one in the previous subsection, for here, entry by A can not be profitably compensated: Since only entry by one firm given two high signals gives positive expected profits here, B can not enter to give any additional complementarities. On the other hand, in the first best, A abstains and shares its information with B. Hence, in the first best, B can enter for two high signals, which is profitable. The firms would thus be better off sharing their information and profits.

### 4.3.2 Excess entry

**Entry cascades** In region 2, both firms enter for  $\sigma = (\sigma_L, \sigma_L)$ , while positive expected profits are only obtained for mixed or high signals. Since A always enters, its decision does not convey any information. Given only its own signal, strong complementarities make it optimal for B to always enter. In the first best, A's information is conveyed and B can avoid entry under two low signals. Two low signals reveal that the outlook is so bad that additional investment is wasteful.

**Opportunism** Regions 11 and 12 are characterized by excess entry with regard to information. That is, entry is made on the basis of one high signal and not two. However, the firms are not always better off individually, sharing their information and profits. Formally, the expected profit for an entrant given one high signal is

$$\pi_{\sigma_H} = \Pr(H | \sigma_H) - d \Pr(L | \sigma_H) = p - d(1 - p),$$

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<sup>14</sup>However, even if B entered with certainty, it would not always be the case that A would enter. If, for instance,  $t = 1.32$ , then  $pt - d(1 - p) < 0$ , and A would not enter.

while the expected profit given that entry only takes place given two high signals is

$$\pi_{\sigma_H, \sigma_H} = \Pr(H|\sigma_H, \sigma_H) - d \Pr(L|\sigma_H, \sigma_H) = \frac{p^2 - d(1-p)^2}{p^2 + (1-p)^2}.$$

If, for example,  $p = 0.85$  and  $d = 2$ , then  $\pi_{\sigma_H, \sigma_H}/2 \approx 0.45$ , while  $\pi_{\sigma_H} = 0.55$ . Thus, the entrant (A in region 11, and B in region 12), would then never enter voluntarily in to a risk- and information-sharing agreement (assuming that half of the profits would be given to the entrant).

**Payoff externalities** As in the case of symmetric information, excess entry can be driven by pure payoff externalities. In region 13, both firms enter given high signals when only one should. Information is conveyed by A's entry and the excess entry is thus driven by pure payoff externalities.

## 4.4 Endogenous timing

When the time of entry is decided by each firm individually, there is a possibility of learning. If the time of entry of a firm is dependent on its signal, the other firm might infer the first firm's signal. Both firms can thus have an incentive to free ride on the other firm's informative entry, which effectively delays entry. Endogenous timing can improve welfare if firms can tell which signal a firm received from its entry decision. We will here only consider the case when uninformed entry is profitable, that is, when  $t > d$ . If there is no cost of delaying actions and the time of entry is endogenous, there are no inefficiencies. The intuition is that when entry is optimal given mixed signals, a firm with a low signal can wait for a firm with a high signal to enter. The firm with a high signal finds it optimal to enter in the first period, since it is certain of being followed in the next. This gives the (efficient) equilibria of the game under symmetric information.

The case of lower complementarities is studied in Chapter 1 of this dissertation. There we show that insufficient entry can occur in a region similar to that of region 6 in Figure 8. In the endogenous case, the risk of being the first entrant, makes firms delay in equilibrium. In equilibrium, the loss of payoff complementarities due to insufficient entry in the first period is offset by the positive effect of learning. From a welfare perspective, the expected industry profit is then equal to that of a game where no learning takes place.

## 4.5 Asymmetric information and communication

In the first best benchmark under asymmetric information, the leader was allowed to communicate its signal to the follower. In this section, we examine the leader's incentive to send a costless verifiable message to the follower and whether this can ameliorate the observed distortions. We will show that allowing the leader to communicate does not change the outcome. If the leader does not know the identity of the follower, the message could, for example, be in the form of a published report.

First, consider the entry cascade when entry decisions are complements. If A has a low signal, B would abstain given an additional low signal. Given that A has a sunk investment, the expected profit for A is lower when the signal is made public. Hence, A has no incentive to send a (truthful) message and the entry cascade remains. It is in the interest of both firms to avoid entry when low signals are received, but given that firm A's investment is sunk, it prefers that the second firm enters regardless of signal. If firm A could communicate its signal before sinking its investment, the results would not change. Entry by A is still profitable regardless of signal, and it will thus make the investment.

Second, consider insufficient entry due to the isolation effect. In equilibrium, A's signal is revealed and communicating A's signal does thus not have any effects, regardless if it is done before or after entry by A.

Finally, when entry decisions are substitutes, it is only profitable for A to enter given a high signal. Thus, A's entry decision conveys information and communication adds nothing. Next, if A never enters, it can never profit from communication.

Thus, allowing A to communicate its signal will not improve on efficiency. In contrast, the World Bank notes that information-sharing was important in resolving problems of under-investment in the Asian economies (World Bank, 1993, p. 93). If we consider the equilibria under symmetric information as the outcome of an information-sharing agreement, then our results are consistent with the reported efficiency. Our point is instead, if entry is sequential, then the leader has no incentive to share information to the follower, given that the follower does not share information. If firm B's private information is generated by firm A's entry (the other interpretation of our model), then firm B cannot share information, and our result follows.

## 4.6 Imitation

We have not allowed the leader to "imitate" a potential follower by assuming capital markets to be incomplete. In this section, we relax this assumption and examine the leader's incentive to imitate. As before, we assume that the leader only has access to his private signal.

Consider first the case of firm level complementarities. Since the leader is basically worse off the more a potential follower uses information (see Section 4.3.1), the leader could find it profitable to imitate a potential follower and reap the profits, albeit it does not have the other firm's information. However, consider complementarities that come from the leader and the follower offering goods consumed as a composite good. Then, the leader's expertise might not be in the same field as the follower's, so that imitation can not be done at low cost. Thus, the assumption of no imitation is not so restrictive when complementarities stem from composite goods. If there is imitation, it is bad from a welfare point of view, since excess entry by the leader takes place for a greater range of parameter values.

Next, when entry decisions are substitutes on the firm level, but complements on the industry level, the leader might find it profitable to imitate a potential follower to preempt further entry and take the gain from industry level complementarities. Again,

the assumption of no imitation is not so restrictive if the products offered by the firms differ somewhat, so that the leader can not easily imitate the follower.

Note that imitation is bad from a welfare point of view only when the imitator has little information (one signal). If the leader had access to both signals as in the first best, welfare is improved.

## 5 Intervention

The effects of externalities on welfare were examined in the previous section. Here, we examine whether, and in what form, intervention can remove the inefficiencies. Before proceeding with the feasibility of direct interventions, we give a result on a more indirect way of intervention.

**Proposition 4** *Increasing the informativeness of the signals has a non-monotonic effect on welfare.*

A plausible conjecture is that more information is always better. If the social planner is able to affect  $p$ , for instance by publishing reports about the inherent traits of a technology, it immediately follows from Figure 6 that this should be done carefully. For instance, a move from region I (efficient entry) to region II (excess entry) or III (insufficient entry) in Figure 6, due to an increase in  $p$ , would be socially detrimental, i.e. such an intervention can only be successful if the planner knows the exact parameters,  $t$ ,  $p$  and  $d$ , of the environment.<sup>15</sup>

Typically, there are two types of direct interventions – subsidies and entry barriers. Further, the firm(s) could be state owned, which makes large-scale investment possible and further internalizes externalities. This however, leaves aside the question of whether one should consider the state to be a good entrepreneur. The distortions show that there can be welfare gains when firms are owned by the same owner. If this is the case, it might instead be in the interest of the state to provide loan guarantees or otherwise make large-scale investment feasible.

We now proceed with an examination of whether subsidies to entry can ameliorate the distortions giving rise to insufficient entry. Thereafter, we study whether barriers to entry can be socially beneficial in the case of excess entry.

### 5.1 Subsidies

A subsidy can potentially make entry efficient in the case of insufficient entry. Here we discuss whether subsidies are ex ante profitable, where “ex ante profitable” refers to expected profits given no private information. We assume it to be possible to give a subsidy exclusively to a leader or a follower, respectively.

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<sup>15</sup>Note that, ceteris paribus, an increase in complementarity, as reflected by  $t$ , also has a non-monotonic effect on welfare.

### 5.1.1 A subsidy is inefficient

**Proposition 5** *There is no subsidy that provides efficiency when there is an isolation effect.*

**Proof.** The isolation effect makes the leader enter only for a high signal. Thus, *any* entry subsidy that makes the leader enter regardless of signal has a negative information externality. This will force the follower to enter only for a high signal. Formally, the profit of entry for firm B, given a low signal, is

$$\Pr(H|\sigma_L)t - d\Pr(L|\sigma_L) = (1-p)t - dp,$$

which is greater than zero if

$$t > d \frac{p}{1-p}.$$

However, in region 3,  $t < \gamma_1(p) = dp/(1-p)$ , which implies that firm B will not enter given a low signal. The outcome is then the same as in the game, that is, the firms will, on average, only enter half the time for mixed signals. ■

This result is interesting since it shows why it is important to study payoff and information externalities jointly; correcting for the payoff externality means destroying the informational aspect of entry in this case. Further, if *both* the leader and the follower are subsidized to enter regardless of signals, the ex ante industry profits are less than the ex ante profits of the game. As an application, consider for example a monopolists sale of a good of uncertain quality where two customers are privately informed about the quality of the good. If there are scale advantages, the monopolist might consider subsidizing the sale to the first consumer in order to attract more orders. Again, this might destroy the informational role of consumption by the first consumer, which might lead to less sales later on.

A subsidy is also inefficient if the informativeness of the signals is low, and entry decisions are substitutes or weakly complementary; then single entry is only profitable given two high signals (region 10). Since a subsidy can not reveal the leader's signal without entry, there is no subsidy that can give efficient entry.

### 5.1.2 A subsidy is efficient

When there is no entry at all, and entry by both firms is socially optimal given high signals (regions 5 and 8), a subsidy to the leader that makes it profitable to enter for a high signal can give efficient entry and, at the same time, be ex ante profitable. In region 8, such a subsidy is profitable while for region 5, this is only true for a sub region.

When entry decisions are more (but still weakly) complementary (region 6), a subsidy that makes the follower enter given entry by the leader and regardless of signal is profitable.<sup>16</sup> Since the follower enters if it receives a high signal even without the subsidy,

<sup>16</sup>Formally, the ex ante industry profits given a subsidy but not including the costs of the subsidy is  $\pi^S = 2(\Pr(H, \sigma_H^A)t - d\Pr(L, \sigma_H^A)) = pt - d(1-p)$ . Similarly, the industry profit given no subsidy is  $\pi = 2(\Pr(H, \sigma_H^A, \sigma_H^B)t - d\Pr(L, \sigma_H^A, \sigma_H^B)) + \Pr(H, \sigma_H^A, \sigma_L^B) - d\Pr(L, \sigma_H^A, \sigma_L^B) = p^2t - ((1-p)(2d - dp - p)/2)$ . Then,  $\pi^S > \pi$  if  $t > (d+1)/2$ , which holds in region 6.



the follower in this case obtains an “informational rent” from its private information.

## 5.2 Entry barriers

A barrier to entry can potentially be efficient when there is excess entry. The (excess) entry driven by the information cascade can not be profitably reduced by an entry barrier. Such a barrier would exclude entry by both firms, given mixed or two high signals, which does not outweigh the gain from reducing entry given two low signals.

In region III (equilibrium III of Figure 5), the leader enters for a high signal and the follower enters subsequently given entry by the leader and a high signal. As the first best shows, this is not socially desirable when entry decisions are stronger substitutes. In a sub region of region III (given by the intersection of  $(d(1-p) - p(1-p))/p^2$  and  $(2(1-p)^2 + p^2)/2p^2$ ), it is ex ante profitable to restrict entry to one entrant only. In the first best, the planner reduces entry for even weaker substitution effects (namely below  $\phi_4$ ). The entry barrier requires stronger substitution effects to be efficient, since the expected profits are lower for the leader, given a barrier, than in the first best case (where firm B has access to both signals).

## 6 Conclusions

The present paper shows how private information and payoff externalities distort entry into new markets. We show that there can be both excess and insufficient entry and that the effects of payoffs and information are non-separable. Surprisingly, insufficient entry can occur even though complementarities are fairly strong; there is an “isolation effect” that makes the first entrant reduce entry. Further, we show that the isolation effect cannot be overcome by subsidizing entry. There is a general lesson to be learned here: Subsidizing entry means that its informative content can be lost to society. In the case of excess entry driven by an entry cascade, we show that an entry barrier can not improve on welfare. Further, by Proposition 4, a social planner must be well informed if she wants to intervene by changing the parameters of the game. All in all, this suggests that a “hands off” approach is the best option for a social planner when entry decisions are ex ante profitable to the firms.

When entry decisions are not ex ante profitable, the scope for intervention is greater. As for the case of ex ante profitable entry, there can be both insufficient and excess entry. A subsidy preserves the informational content of the leader’s entry and does not create additional distortions. Excess entry can also be curbed with more efficiency. An entry barrier can be more selectively targeted and will not, as in the case of ex ante profitable entry, be indiscriminating.

There is one recent, independently developed paper, that is close to ours. The paper by Dasgupta (2000) studies investment with payoff and information externalities under sequential timing. The paper is more general than ours in terms of the number of agents and stochastic properties of their private signals but more restrictive in terms of payoffs.<sup>17</sup>

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<sup>17</sup>Further, the signals are drawn from a subset of the real line while the investment decision is binary

In particular, he only models a very strong form of positive payoff externalities, where investors receive negative payoffs in the case that at least one agent abstains from investment. Given some further assumptions, Dasgupta derives an “investment equilibrium” which shares some features with the asymmetric information equilibria under complementarities in this paper. The main difference between his paper and ours is that we show how entry and its welfare consequences depend in a non-trivial way on the size of complementarities and the informativeness of the signals. Further, his model does not fit well with entry, the focus of this paper, since firms always earn negative profits should at least one firm abstain. Dasgupta obtains some results similar to ours, however. There is an isolation effect in equilibrium and the earlier an investor has to enter, the more cautious the investor is.

One limitation of our model is that the timing and order of entry is exogenous. However, the equilibria under asymmetric information would also be the outcome of a game where the follower only discovered the entry opportunity if the leader entered.<sup>18</sup> Thus, the assumption of sequential timing is not crucial considering that one firm could have an advantage over another in identifying a new market.

From a more theoretical perspective, this paper contributes to the theory of information externalities by showing that these externalities can give distortions also when there are negative payoff externalities. Our results further provide new explanations for phenomena found in the literature on network externalities (although “entry” is not a pure network good in our model, since it is valuable even if consumed in isolation). Farrell and Saloner (1985) show that “excess inertia” and “excess momentum” can occur in the adoption of new standards (new techniques). We give a new mechanism for inertia when showing that under-investment can occur when products are complements. Further, we can also give an alternative explanation to excess momentum, since excess entry can also occur when products are complements.

## 7 Appendix

Sometimes, we will find it convenient to use  $\sigma_H^A$ , for example, to denote that firm A has received a high signal. As before, we will suppress the dependency of functions on  $d$ .

### 7.1 Proof of Remark 1

The proof is given for  $\phi_1(p) \geq \phi_2(p)$  and  $\phi_3(p) \geq \phi_4(p)$ . The other cases are trivial.

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in his model. When the signal space is larger than the action space, then information externalities are, in a sense, exogenously introduced since the action can not fully convey the signal; see Lee (1993).

<sup>18</sup>This interpretation of our model is correct for all equilibria except for the equilibrium where the follower has a second-mover advantage (equilibrium IV).

### 7.1.1 Proof of $\phi_1(p) \geq \phi_2(p)$

Fix a pair  $(p, d) \in (0.5, 1] \times [1, \infty)$ . The functions  $\phi_1(p)$  and  $\phi_2(p)$  coincide for

$$p < p_I = \left( d + 1 + \sqrt{d(d-1)} \right) / (3d + 1),$$

so we need only look at the case  $p \geq p_I$ . We have

$$\phi_1(p) - \phi_2(p) = ((3p - 2)dp + (1 - p)^2) / (2p(1 - p)^2).$$

Thus,  $\phi_1(p) \geq \phi_2(p)$  holds if  $(3p - 2)dp + (1 - p)^2 \geq 0$ . Note that this expression is increasing in  $p$  since

$$\frac{\partial}{\partial p} ((3p - 2)dp + (1 - p)^2) = 2(2d(2p - 1) + d(1 - p) - (1 - p)) > 0.$$

Hence, it is sufficient to evaluate  $(3p - 2)dp + (1 - p)^2$  at  $p = p_I$ . This yields 0. Thus, the inequality is proved and  $\phi_1(p) \geq \phi_2(p)$  for all pairs  $(p, d) \in (0.5, 1] \times [1, \infty)$ .

### 7.1.2 Proof of $\phi_3(p) \geq \phi_4(p)$

Define the functions

$$\begin{aligned} a_1(p) &= (1 + d) / 2, \\ a_2(p) &= (1 - p)(d(2 - p) - p) / (2p^2), \\ a_3(p) &= (p(2p - 1) + d(1 - p)) / (2p^2). \end{aligned}$$

We have

$$\begin{aligned} \phi_3(p) &= \begin{cases} d(1 - p) / p, & \text{if } p < 2d / (3d + 1); \\ a_1(p), & \text{otherwise,} \end{cases} \\ \phi_4(p) &= \begin{cases} d(1 - p) / p, & \text{if } p < 2d / (3d + 1); \\ a_2(p), & \text{if } 2d / (3d + 1) \leq p < \sqrt{d} / (\sqrt{d} + 1); \\ a_3(p), & \text{otherwise.} \end{cases} \end{aligned}$$

Fix a pair  $(p, d) \in (0.5, 1] \times [1, \infty)$ . Note that  $\left( 2d / (3d + 1), \sqrt{d} / (\sqrt{d} + 1) \right)$  is a non-empty subset of  $R$  for  $d > 1$  since

$$\frac{\sqrt{d}}{\sqrt{d} + 1} - 2 \frac{d}{3d + 1} = \frac{\sqrt{d}(\sqrt{d} - 1)^2}{(\sqrt{d} + 1)(3d + 1)} > 0.$$

First, we will show that  $a_1(p) \geq a_2(p)$  for  $p \geq 2d/(3d+1)$ . We have  $a_1(p) - a_2(p) = ((3d+1)p - 2d)/(2p^2)$ . But then,  $a_1(p) - a_2(p) \geq 0$  for  $p \geq 2d/(3d+1)$ , which is what we wanted to show. Hence, the inequality is proved.

Next, we show that  $a_1(p) \geq a_3(p)$ . We have

$$a_1(p) - a_3(p) = \frac{1}{2} \frac{p^2(d-1) - d(1-p) + p}{p^2}.$$

The nominator is increasing in  $p$  since

$$\frac{\partial}{\partial p} (dp^2 - d + dp + p - p^2) = 2p(d-1) + d + 1 > 0.$$

Thus,  $a_1(p) > a_3(p)$  if the nominator is positive for  $\bar{p} = \sqrt{d}/(\sqrt{d}+1)$ . Evaluating the nominator at  $\bar{p}$  gives

$$\sqrt{d} \frac{(\sqrt{d}-1)^2}{(\sqrt{d}+1)},$$

which is non-negative for  $d \geq 1$ . Hence, the inequality is proved.

Thus,  $\phi_3(p) \geq \phi_4(p)$  for all pairs  $(p, d) \in (0.5, 1] \times [1, \infty)$ .

## 7.2 First best under asymmetric information

To find the optimal contingent plan, we use backward induction and begin by determining the optimal period two entry, given history  $h \in Y \times \Omega$ . Then, given the second period entry, we can calculate the expected profit of entry in period one.

### 7.2.1 B's entry: Optimal period two action given history $h = (y, \sigma^A)$

Case  $h = (1, \sigma_H)$

Case  $\sigma^B = \sigma_H$ : Enter if

$$2(\Pr(H|\sigma_H, \sigma_H)t - d\Pr(L|\sigma_H, \sigma_H)) > \Pr(H|\sigma_H, \sigma_H) - d\Pr(L|\sigma_H, \sigma_H).$$

That is, if

$$t > \frac{p^2 + d(1-p)^2}{2p^2}.$$

Case  $\sigma^B = \sigma_L$ : Enter if

$$2(\Pr(H|\sigma_H, \sigma_L)t - d\Pr(L|\sigma_H, \sigma_L)) > \Pr(H|\sigma_H, \sigma_L) - d\Pr(L|\sigma_H, \sigma_L).$$

That is, if

$$t > \frac{1+d}{2}.$$

**Case  $h = (1, \sigma_L)$**

**Case  $\sigma^B = \sigma_H$ :** Enter if

$$2(\Pr(H|\sigma_H, \sigma_L)t - d\Pr(L|\sigma_H, \sigma_L)) > \Pr(H|\sigma_H, \sigma_L) - d\Pr(L|\sigma_H, \sigma_L).$$

That is, if

$$t > \frac{1+d}{2}.$$

**Case  $\sigma^B = \sigma_L$ :** Enter if

$$2(\Pr(H|\sigma_L, \sigma_L)t - d\Pr(L|\sigma_L, \sigma_L)) > \Pr(H|\sigma_L, \sigma_L) - d\Pr(L|\sigma_L, \sigma_L).$$

That is, if

$$t > \frac{(1-p)^2 + dp^2}{2(1-p)^2}.$$

**Case  $h = (0, \sigma_H)$**

**Case  $\sigma^B = \sigma_H$ :** Enter if

$$\Pr(H|\sigma_H, \sigma_H) - d\Pr(L|\sigma_H, \sigma_H) > 0.$$

That is, if

$$p^2 - d(1-p)^2 > 0,$$

i.e. if

$$p > \sqrt{d}/(\sqrt{d} + 1).$$

**Case  $\sigma^B = \sigma_L$ :** Never enter.

**Case  $h = (0, \sigma_L)$ :** Never enter.

### 7.2.2 A's entry: Optimal period one action given optimal period two action

First we derive A's entry, given that A receives a high signal. Firm A then knows that entry will make B's decision contingent on the history  $h = (1, \sigma_H)$ . For this history, A knows for which regions  $R_i, i = 1, 2$  and 3, B will enter or not, and whether B's entry will be conditional on its signal (the regions are given by the results in the previous subsection). For each of these regions, A's entry depends on some conditions. First, we check if entry should be made, given what B will do subsequently in this region. This will

give one restriction of the form  $t > u(p)$ , where  $u$  is some real valued function. Next, if applicable, we check if the expected profit if both enters is positive. The reason is that B enters if this *increases* expected industry profits, which is different from entering when entry makes the expected industry profits positive. Again, this will give a restriction of the form  $t > v(p)$  for some real valued function  $v$ . Given these conditions (in terms of functions  $u$  and  $v$ ), the next thing to check is whether one of these conditions subsumes the other. This consists of checking, for each  $p$  and  $d$ , when and where, one function is greater than the other. The result will be the  $\phi$  - functions as defined in Definition 2.

Finally, the same procedure is carried out in case A receives a low signal.

**Case: A receives a high signal,  $\sigma^A = \sigma_H$**

It is convenient to define  $\bar{p} = \sqrt{d}/(\sqrt{d} + 1)$  in the following cases. Remember that for  $p > \bar{p}$ , the expected profit of single entry given two high signals is positive.

**Case  $t \in R_1$  :** Region  $R_1$  is defined by  $R_1 = (\tilde{r}_1(p), \infty)$ , where  $\tilde{r}_1(p) = (1 + d)/2$ .

**Sub case  $p > \bar{p}$**

A knows that B will always enter if A enters now. If A does not enter, B will enter in period two given a high signal. A enters if

$$2(\Pr(H|\sigma_H)t - d\Pr(L|\sigma_H)) > \Pr(H, \sigma_H|\sigma_H) - d\Pr(L, \sigma_H|\sigma_H).$$

That is, if

$$t > \frac{p^2 + d(1 - p^2)}{2p}.$$

Define the expression on the right-hand side of the inequality as the function  $u_1$ .

Further, the expected profits must be positive:

$$2(\Pr(H|\sigma_H)t - d\Pr(L|\sigma_H)) > 0,$$

i.e., we must have

$$t > d\frac{1 - p}{p}.$$

Define the expression on the right-hand side of the inequality as the function  $v_1$ .

Thus, A enters for  $t$  greater than the maximum of  $\tilde{r}_1(p)$ ,  $u_1(p)$  and  $v_1(p)$ . By “maximum” we will mean which function value is greatest for each applicable  $p$ . Thus, we want to find the upper envelope of the functions as  $p$  varies. We have

$$v_1(p) - u_1(p) = \frac{d(1 - p)^2 - p^2}{2p}.$$

The nominator  $d(1 - p)^2 - p^2$  has a root at  $p = \bar{p}$  (the other root is not applicable). Further, we have

$$\frac{\partial}{\partial p} (d(1-p)^2 - p^2) = 2(p-1)d - 2p < 0. \quad (1)$$

So, for  $p < \bar{p}$  we have  $v_1(p) > u_1(p)$  and for  $p > \bar{p}$  (the current case), we have  $v_1(p) < u_1(p)$ . Next, we have

$$u_1(p) - \tilde{r}_1(p) = \frac{1}{2} \frac{p^2 + d - dp^2 - p - dp}{p}.$$

The nominator is decreasing in  $p$ , since

$$\frac{\partial}{\partial p} (p^2 + d - dp^2 - p - dp) = -2p(d-1) - 1 - d < 0.$$

If  $p^2 + d - dp^2 - p - dp$  is evaluated at  $p = \bar{p}$ , we get

$$-\sqrt{d} \frac{(\sqrt{d}-1)^2}{(\sqrt{d}+1)},$$

which is negative for  $d > 1$  and zero for  $d = 1$ . Thus,  $u_1(p) - \tilde{r}_1(p) \leq 0$  at  $p = \bar{p}$  and as we increase  $p$ ,  $u_1(p) - \tilde{r}_1(p)$  becomes more negative. Hence  $\tilde{r}_1(p) \geq u_1(p) > v_1(p)$ , for  $p > \bar{p}$ . Thus, when  $t > \tilde{r}_1(p)$ , A will enter given  $p > \bar{p}$ .

**Sub case  $p \leq \bar{p}$**

A knows that B will always enter if it enters now. If A does not enter, B will not enter. Thus A enters if

$$2(\Pr(H|\sigma_H)t - d\Pr(L|\sigma_H)) > 0,$$

i.e., we must have

$$t > d \frac{1-p}{p} = v_1(p).$$

Now,

$$v_1(p) - \tilde{r}_1(p) = \frac{2d - (3d+1)p}{2p}.$$

Thus  $v_1(p) > \tilde{r}_1(p)$  when  $p < 2d/(3d+1)$ .

Consequently, the following function characterizes the entry decision

$$\phi_3(p) = \begin{cases} v_1(p), & \text{if } p < 2d/(3d+1); \\ \tilde{r}_1(p), & \text{otherwise.} \end{cases}$$

For  $t > \phi_3(p)$ , A enters given a high signal. B enters given entry by A.

**Case  $t \in R_2$  :** Region  $R_2$  is defined by  $R_2 = (\tilde{r}_2(p), \hat{r}_2(p)]$ , where  $\tilde{r}_2(p) = (p^2 + d(1-p)^2) / (2p^2)$  and  $\hat{r}_2(p) = (1+d)/2$ .

**Sub case  $p > \bar{p}$**

A knows that B enters only for a high signal, regardless of entry in the first period. Thus, A enters if

$$2(\Pr(H, \sigma_H | \sigma_H) t - d \Pr(L, \sigma_H | \sigma_H)) + \Pr(H, \sigma_L | \sigma_H) - d \Pr(L, \sigma_L | \sigma_H) > \\ > \Pr(H, \sigma_H | \sigma_H) - d \Pr(L, \sigma_H | \sigma_H).$$

That is, if

$$t > \frac{p(2p-1) + d(1-p)}{2p^2}.$$

Define the expression on the right-hand side of the inequality as the function  $u_2$ . Further, the profit from two entrants must be positive

$$2(\Pr(H, \sigma_H | \sigma_H) t - d \Pr(L, \sigma_H | \sigma_H)) + \Pr(H, \sigma_L | \sigma_H) - d \Pr(L, \sigma_L | \sigma_H) > 0,$$

which holds if

$$t > \frac{(1-p)(d(2-p) - p)}{2p^2}.$$

Define the expression on the right-hand side of the inequality as the function  $v_2$ . Thus, A will enter for  $t \in \bar{R}_2 = (\max\{\tilde{r}_2(p), u_2(p), v_2(p)\}, \hat{r}_2(p)]$ , where the maximum should be interpreted as taken for each  $p \in (0.5, 1]$ . We proceed by deriving the maximum of these functions. We have  $v_2(p) - u_2(p) = (d(1-p)^2 - p^2) / (2p^2)$ , which is positive when  $d(1-p)^2 - p^2 > 0$ , hence by Eq. (1),  $u_2(p) > v_2(p)$  when  $p > \bar{p}$  and otherwise the reversed inequality holds. Further,  $u_2(p) - \tilde{r}_2(p) = (1-p)(d-1)/2p \geq 0$ . Thus,  $u_2(p) \geq \tilde{r}_2(p)$  is always true. Hence,  $\max\{\tilde{r}_2(p), u_2(p), v_2(p)\} = u_2(p)$ . It now remains to check that the interval  $\bar{R}_2$  is well defined, i.e. we need to check where  $u_2(p) < \hat{r}_2(p)$ . We have

$$\hat{r}_2(p) - u_2(p) = \frac{p(1-p) + dp(p+1) - d}{2p^2}.$$

Since

$$\frac{\partial}{\partial p} (p(1-p) + dp(p+1) - d) = 1 + d + 2p(d-1) > 0,$$

we know that the nominator of  $\hat{r}_2(p) - u_2(p)$  is increasing in  $p$ . Evaluating  $\hat{r}_2(p) - u_2(p)$  at  $p = \bar{p}$  gives

$$\hat{r}_2(p) - u_2(p) = \frac{1}{2} (\sqrt{d} + 1) \frac{(\sqrt{d} - 1)^2}{\sqrt{d}} \geq 0.$$



Thus,  $\hat{r}_2(p) \geq u_2(p)$  for  $p > \bar{p}$ . Consequently, A will enter for  $t \in \bar{R}_2 = (u_2(p), \hat{r}_2(p)]$  when  $p > \bar{p}$ .

**Sub case  $p \leq \bar{p}$**

A knows that if it enters, then B enters only for a high signal. If A does not enter, neither will B. Thus A enters if

$$2(\Pr(H, \sigma_H | \sigma_H)t - d\Pr(L, \sigma_H | \sigma_H)) + \Pr(H, \sigma_L | \sigma_H) - d\Pr(L, \sigma_L | \sigma_H) > 0,$$

which holds if

$$t > \frac{2d(1-p)^2 + p(d-1)(1-p)}{2p^2} = v_2(p).$$

Thus, A will enter for  $t \in \underline{R}_2 = (v_2(p), \hat{r}_2(p)]$ . It now remains to check that the interval  $\underline{R}_2$  is well defined, i.e. we need to check where  $\tilde{r}_2(p) < v_2(p) < \hat{r}_2(p)$ . We have

$$\hat{r}_2(p) - v_2(p) = \frac{1}{2} \frac{(3d+1)p - 2d}{p^2}.$$

For  $\hat{r}_2(p) - v_2(p) > 0$  to hold, we thus need  $p > 2d/(3d+1)$ . But we know  $\bar{p} > 2d/(3d+1)$  by Remark 1. Thus, for  $p \in (2d/(3d+1), \bar{p}]$ , A will enter if  $t > v_2(p)$ . For  $p \in (0.5, 2d/(3d+1)]$ , we have that  $v_2(p) > \hat{r}_2(p)$ . If  $t > \hat{r}_2(p)$ , i.e. when we are outside of region  $R_2$ , another criterium must be used to ensure positive profits, since B's behavior changes. The appropriate condition from the previous case (region  $R_1$ ) is  $t > d(1-p)/p$ .

We further have

$$v_2(p) - \tilde{r}_2(p) = \frac{-(p - d(1-p))}{2p^2},$$

Formally,  $p - d(1-p)$  is the expected profit from single entry given one high signal. However, single entry given *two* high signals is not profitable when  $p \leq \bar{p}$  (the sub-case under study). Thus, single entry given one high signal can not be profitable and  $p - d(1-p) < 0$  so that  $v_2(p) > \tilde{r}_2(p)$ .

Thus, the following function characterizes entry

$$\phi_4(p) = \begin{cases} d(1-p)/p, & \text{if } p \leq 2d/(3d+1); \\ v_2(p), & \text{if } p \in (2d/(3d+1), \sqrt{d}/(\sqrt{d}+1)]; \\ u_2(p), & \text{otherwise.} \end{cases}$$

For  $t > \phi_4(p)$ , A enters given a high signal and B enters if it receives a high signal.

**Case  $t \in R_3$ :** Region  $R_3$  is defined by  $R_3 = [0, \hat{r}_3(p)]$ , where  $\hat{r}_3(p) = (p^2 + d(1-p)^2)/(2p^2)$ . A knows that if it enters, there will be no entry in the second period. If does not enter, B will enter for another high signal and  $p > \bar{p}$ .

First, single entry is profitable for A if

$$\Pr(H|\sigma_H) - d\Pr(L|\sigma_H) > 0.$$

That is, if

$$p > \frac{d}{1+d}.$$

**Sub case**  $p > d(1+d)$

A should enter if

$$\Pr(H|\sigma_H) - d\Pr(L|\sigma_H) > \Pr(H, \sigma_H|\sigma_H) - d\Pr(L, \sigma_H|\sigma_H),$$

i.e. if

$$p(1-p)(1-d) > 0,$$

which does not hold. So A should not enter.

**Sub case**  $p \leq d(1+d)$

It is never profitable for A to be the sole entrant so A should not enter.

Thus, for  $t \in R_3$ , A never enters. This is intuitive; it is better to wait for more information given that only one firm should enter.

**Case: A receives a low signal,  $\sigma^A = \sigma_L$**

Now, we turn to the case where A receives a low signal. Similar to the above reasoning, we can partition the parameter space into regions  $R_i$ ,  $i = 4, 5$  and 6, where B's behavior is known, given history  $h = (\cdot, \sigma_L^A)$ .

**Case**  $t \in R_4$ : Region  $R_4$  is defined by  $R_4 = (\check{r}_4(p), \infty)$ , where  $\check{r}_4(p) = ((1-p)^2 + dp^2)/2(1-p)^2$ . A knows that if it enters, it will always be followed. If does not enter, there will be no entry at all.

Thus, A enters if

$$2(\Pr(H|\sigma_L)t - d\Pr(L|\sigma_L)) > 0,$$

or equivalently

$$t > d\frac{p}{1-p}.$$

Define the expression on the right-hand side of the inequality as the function  $u_4$ . Thus, A enters for  $t$  greater than the maximum of  $\check{r}_4(p)$  and  $u_4(p)$ .

The intersection is given by the solution to

$$\check{r}_4(p) = u_4(p),$$

which is  $p_I = \left( d + 1 + \sqrt{d(d-1)} \right) / (3d + 1)$ . Further,

$$u_4(p) - \tilde{r}_4(p) = \frac{-((1-p)^2 + pd(3p-2))}{2(1-p)^2}.$$

Taking the derivative of the nominator shows

$$\frac{\partial}{\partial p} (-(1-p)^2 - pd(3p-2)) = -((4p-2)d + 2(p(1+d) - 1)) < 0,$$

where the inequality follows, since  $p(1+d) > 1$ . Thus, the nominator of  $u_4(p) - \tilde{r}_4(p)$  decreases as  $p$  increases. Since  $u_4(p) - \tilde{r}_4(p)$  evaluated at  $p_I$  gives zero, we then know that  $u_4(p) < \tilde{r}_4(p)$  for  $p > p_I$  and  $u_4(p) > \tilde{r}_4(p)$  for  $p < p_I$ . Thus, entry is described by the function

$$\phi_1(p) = \begin{cases} u_4(p), & \text{if } p < \left( d + 1 + \sqrt{d(d-1)} \right) / (3d + 1); \\ \tilde{r}_4(p), & \text{otherwise.} \end{cases}$$

For  $t > \phi_1(p)$ , A enters for a low signal (and for a high signal), and B will always enter given entry by A.

**Case  $t \in R_5$ :** Region  $R_5$  is defined by  $R_5 = (\tilde{r}_5(p), \hat{r}_5(p)]$ , where  $\tilde{r}_5(p) = (1+d)/2$  and  $\hat{r}_5(p) = ((1-p)^2 + dp^2) / (2(1-p)^2)$ .

A knows that if it enters, it will be followed only if B receives a high signal. If A does not enter, there will be no entry at all. Thus, A enters if

$$2(\Pr(H, \sigma_H | \sigma_L) t - d \Pr(L, \sigma_H | \sigma_L)) + \Pr(H, \sigma_L | \sigma_L) - d \Pr(L, \sigma_L | \sigma_L) > 0,$$

i.e., if

$$t > \frac{dp^2 + 2dp(1-p) - (1-p)^2}{2p(1-p)}.$$

Define the expression on the right-hand side of the inequality as the function  $u_5$ .

Next, we need to check when the condition  $t > u_5(p)$  is consistent with region  $R_5$ . The intersection with the upper region limit is given by

$$\hat{r}_5(p) = u_5(p).$$

This gives the intersection  $p_I = \left( d + 1 + \sqrt{d(d-1)} \right) / (3d + 1)$ . Further,

$$\hat{r}_5(p) - u_5(p) = \frac{(1-p)^2 + pd(3p-2)}{2p(1-p)^2}.$$

Thus,  $\hat{r}_5(p) - u_5(p)$  is positive when  $p(3p - 2)d + (1 - p)^2$  is positive. Note (as in the previous case) that

$$\frac{\partial}{\partial p} (p(3p - 2)d + (1 - p)^2) = (4p - 2)d + 2p(1 + d) - 2 > 0.$$

Then, an evaluation of  $p(3p - 2)d + (1 - p)^2$  at  $p_I$  gives 0, which shows that the  $\hat{r}_5(p)$  is greater than  $u_5(p)$  for  $p > p_I$ . The function  $u_5$  thus characterizes A's entry for  $p > p_I$ . When  $p \leq p_I$  on the other hand, A will not enter in region  $R_5$  since we must then have  $t > \hat{r}_5(p)$ , which is outside the current region ( $R_5$ ). To define  $\phi_2(p)$  for the case  $p \leq p_I$ , we need the appropriate function from region  $R_4$  (see the previous case), since entry is only optimal for  $t$  greater than what is obtained in  $R_5$ . The correct condition for  $p \leq p_I$  is found to be  $dp/(1 - p)$ . One further consistency check is thus necessary. We need

$$u_5(p) - \tilde{r}_5(p) = \frac{1}{2} \frac{dp - 1 + p}{p(1 - p)} > 0.$$

Hence,  $u_5(p) > \tilde{r}_5(p)$  holds if  $p > 1/(d + 1)$ . But  $p > 0.5 \geq 1/(d + 1)$  for  $d \geq 1$ , so this condition is satisfied. Entry can thus be characterized by

$$\phi_2(p) = \begin{cases} dp/(1 - p), & \text{if } p \leq \left(d + 1 + \sqrt{d(d - 1)}\right) / (3d + 1); \\ u_5(p), & \text{otherwise.} \end{cases}$$

For  $t > \phi_2(p)$ , A enters for a low signal (and for a high signal), firm B follows given entry by A if both signals are not low.

**Case  $t \in R_6$ :** Region  $R_6$  is defined by  $R_6 = [0, \hat{r}_6(p)]$ , where  $\hat{r}_6(p) = (1 + d)/2$ .

A will not be followed if it enters and single entry is not profitable either (since  $\hat{r}_6(p) \leq \bar{p}$ ). Thus, there is no entry in this region.

## 7.3 Equilibria under asymmetric information (proof of Proposition 3)

It is convenient to divide the equilibria into “pooling equilibria” and “separating equilibria” respectively. We define a separating equilibrium as an equilibrium where A takes different actions for different private signals. Similarly, we define a pooling equilibrium as an equilibrium where A takes the same action regardless of the private signal.

### 7.3.1 Separating equilibria

Since the “contrarian” strategy  $s^A =$  “Enter for a low signal, abstain for a high signal” is never optimal, attention is restricted to the strategy  $s^A =$  “Enter for a high signal, abstain for a low signal”.

First, we derive B's best reply given its signal and entry by A. Next, given B's best reply, firm A's best reply is derived.

**B's best reply** In the separating equilibria, B can infer that  $\sigma^A = \sigma_H$  if A enters, and  $\sigma^A = \sigma_L$  if A abstains.

**Case:**  $y = 1, \sigma^B = \sigma_H$ : Firm B enters if

$$\Pr(H|\sigma_H, \sigma_H) t - d \Pr(L|\sigma_H, \sigma_H) > 0,$$

i.e. firm B enters if

$$t > d \frac{(1-p)^2}{p^2}.$$

**Case:**  $y = 1, \sigma^B = \sigma_L$ : By similar computation, B enters if

$$t > d.$$

For the remaining cases where  $y = 0$ , firm B will not enter.

**A's best reply** There are three cases to consider given  $s^A =$  "Enter for a high signal, abstain for a low signal". From the calculation of firm B's best reply, firm A knows that for  $t > d$ , B will always enter. For  $t \in (d(1-p)^2/p^2, d]$ , B enters only for a high signal. And finally, for  $t \leq d(1-p)^2/p^2$ , B never enters.

**Case**  $t > d$ : A enters if

$$\Pr(H|\sigma_H) t - d \Pr(L|\sigma_H) > 0,$$

i.e. if

$$t > d \frac{1-p}{p}.$$

To maintain  $s^A$ , it must not be profitable for A to always enter. This holds if

$$\Pr(H|\sigma_L) t - d \Pr(L|\sigma_L) \leq 0,$$

i.e. if

$$t \leq d \frac{p}{1-p}.$$

Since  $t > d$  implies  $t > d(1-p)/p$ , we conclude that

$s^A =$  "Enter for a high signal, abstain for a low signal",  $s^B =$  "Enter always",

is a perfect Bayesian equilibrium for  $t \in (d, dp/(1-p)]$ .

**Case**  $t \in (d(1-p)^2/p^2, d]$  : A enters if

$$\Pr(H, \sigma_H | \sigma_H) t + \Pr(H, \sigma_L | \sigma_H) - d \Pr(L | \sigma_H) > 0,$$

i.e. if

$$t > \frac{d(1-p) - p(1-p)}{p^2}.$$

To maintain  $s^A$ , it must not be profitable for A to always enter. This holds if

$$\Pr(H, \sigma_H | \sigma_L) t + \Pr(H, \sigma_L | \sigma_L) - d \Pr(L | \sigma_L) \leq 0,$$

i.e. if

$$t \leq \frac{dp - (1-p)^2}{(1-p)p}.$$

Thus, the equilibrium we are after lies in the intersection of the case studied, namely  $t \in (d(1-p)^2/p^2, d]$ , and the two inequalities derived above. Now,

$$\frac{d(1-p) - p(1-p)}{p^2} - \frac{d(1-p)^2}{p^2} = \frac{(1-p)(d-1)}{p} \geq 0,$$

thus  $(d(1-p) - p(1-p))/p^2$  is the lower binding restriction. Further,

$$\frac{dp - (1-p)^2}{(1-p)p} - d = \frac{dp^2 - (1-p)^2}{p(1-p)} > 0.$$

Hence  $t \leq d$  is a binding upper restriction. However, the binding lower restriction intersects with the binding upper restriction. The equation

$$d = \frac{d(1-p) - p(1-p)}{p^2},$$

determines the intersection to

$$p_I = \left( \sqrt{(d-1)^2 + 4d^2} - (d+1) \right) / (2(d-1)).$$

Hence,

$$s^A = \text{"Enter only if high"}, s^B = \text{"Enter only if high"},$$

for  $p \geq p_I$  and  $t \in ((d(1-p) - p(1-p))/p^2, d]$ , is the unique perfect Bayesian equilibrium where A uses strategy  $s^A$ .

**Case**  $t \leq d(1-p)^2/p^2$ : A enters if

$$\Pr(H|\sigma_H) - d\Pr(L|\sigma_H) > 0,$$

i.e. if

$$p > \frac{d}{d+1}.$$

Hence,

$$s^A = \text{"Enter only if high"}, s^B = \text{"Never enter"},$$

for  $t \leq d(1-p)^2/p^2$ , is the unique perfect Bayesian equilibrium where A uses the strategy  $s^A$ .

### 7.3.2 Pooling equilibria

There are two types of potential pooling equilibria in this game. One where A always enters, and one where A never enters. In the pooling equilibria, strategies and beliefs must be specified for out of equilibrium actions. Since "threats with beliefs" might play a role, we must investigate when B has an incentive to threaten with beliefs.

**Case:**  $s^A = \text{"Always enter"}$

**Lemma 1**  $s^A$  can only be played in equilibrium when  $t > dp/(1-p)$ . Further, B's beliefs do not influence A.

**Proof.** Note that double entry, given access to one low signal, is not profitable when  $t \leq dp/(1-p)$ . Moreover, single entry is never optimal given one low signal. Thus, entry by A for a low signal requires  $t > dp/(1-p)$ .

Further, the only type of deviation A can make from  $s^A$  is to abstain from entry for a low signal or always abstain. Abstaining (whether only for a low signal or totally) gives zero payoff to A in each case. Further, it removes the payoff interdependence. Thus, B's beliefs concerning A's out of equilibrium actions does not affect A in this case. ■

Thus, when considering equilibria where A always enters, there is no need to examine strategic effects from B's beliefs. B's beliefs will only affect its own entry given out of equilibrium behavior by A.

Now, if A always enters for  $t > dp/(1-p)$ , B also finds it optimal to enter regardless of signal. Hence,  $s^A = \text{"Always enter"}$ ,  $s^B = \text{"Always enter if A did, enter given beliefs if A did not enter"}$ , for  $t > dp/(1-p)$ , is the only type of perfect Bayesian equilibrium where A uses the pooling strategy  $s^A$ . Since B's beliefs have no strategic importance, we can, without loss of generality, assume that B makes the passive conjecture  $\Pr(\sigma_H^A | A \text{ does not enter}) = 0.5$ . The unique perfect Bayesian equilibrium is thus

$$s^A = \text{"Always enter"}, s^B = \text{"Always enter if } y = 1, \text{ enter for } t > d/(d+1) \text{ if } y = 0",$$

for  $t > \gamma_1 = dp/(1-p)$ .

**Case:**  $s^A = \text{"Never enter"}$

**Case**  $p > d/(d+1)$  : First we study the case when single entry given one high signal is profitable, that is, when  $p > d/(d+1)$ .

**Remark 3** *If there are substitution effects, the only possible out of equilibrium deviation from  $s^A$  is "Enter if high signal".*

**Proof.** When there are substitution effects, we know that entry by A given a low signal is not profitable for A whatever strategy B uses. Since we have assumed that entry will only be made if it gives positive expected profits, the only possible deviation is to enter for a high signal. ■

Hence, we can restrict the attention to the strategic effects of A making the out of equilibrium move "enter for a high signal" if there are substitution effects.

**Lemma 2** *Strategy  $s^A$  is an equilibrium strategy only if conditions (i), (ii) and (iii) hold:*

- (i) *There are substitution effects.*
- (ii) *B enters for a high signal in equilibrium.*
- (iii) *Entry given a high signal is not profitable for A, given that B enters for a high signal.*

**Proof.** Strategy  $s^A$  can only be profitable when there are substitution effects. This is because if there are complementarities, A's decision to not enter will make entry by B profitable given a high signal. This, in turn, will make entry by A profitable for a high signal. Thus, we can focus on the case of substitution. Remember that we are studying the case where single entry given access to one high signal is profitable. If B never enters if A did, entry by A is then profitable given a high signal. Thus,  $s^A = \text{"Never enter"}$ , will only be used if there is entry by B. If there is entry by B, it is either conditional on B's signal or regardless of B's signal. That is, B follows either  $s_1^B = \text{"Enter only for a high signal if A did not enter, or enter given } \sigma_H^A \text{ and } \sigma^B \text{ if A did enter"}$  or  $s_2^B = \text{"Enter if A did not enter, or enter given } \sigma_H^A \text{ and } \sigma^B \text{ if A did enter"}$  (from Remark 3, we know that we now can restrict the attention to the strategic effects of A making the out of equilibrium move "enter for a high signal"). Note that the second strategy does not give positive expected profits for B in equilibrium, since entry given access to one low signal is never optimal when there are substitution effects. Hence, we can restrict the attention to the case where B only enters for a high signal in equilibrium. Finally, given these results,  $s^A$  can only be an equilibrium strategy when entry by A given a high signal is not profitable given that B enters for a high signal. ■

For condition (iii) to hold, we must have

$$\Pr(H, \sigma_H^B | \sigma_H^A) t + \Pr(H, \sigma_L^B | \sigma_H^A) - d \Pr(L | \sigma_H^A) \leq 0,$$

or equivalently

$$t \leq \frac{(d-p)(1-p)}{p^2}.$$



Now, consider B's profit from entry given a high signal, if A enters for a high signal

$$\Pr(H|\sigma_H^B, \sigma_H^A) t - d \Pr(L|\sigma_H^B, \sigma_H^A) = \frac{p^2 t - d(1-p)^2}{p^2 + (1-p)^2}.$$

The profit is negative if

$$t < d \frac{(1-p)^2}{p^2}.$$

Thus, when  $t \leq d(1-p)^2/p^2$ , A can profitably deviate, since even the most optimistic belief  $\Pr(\sigma_H^A|A \text{ enters}) = 1$ , will make B abstain. On the other hand, if

$$t \in \left( d \frac{(1-p)^2}{p^2}, \frac{(d-p)(1-p)}{p^2} \right],$$

then B's expected profit given A's out of equilibrium move depends on its beliefs. However, by Remark 3, B holds only the most optimistic belief and entry given entry by A is thus profitable for B given a high signal. Further, given that A's equilibrium action does not convey information and that we are considering the case where single entry given a high signal is optimal, we know that B will enter for a high signal given no entry by A.

Hence,

$$s^A = \text{"Never enter"}, s^B = \text{"Enter for a high signal"},$$

for  $t \in (d(1-p)^2/p^2, (d-p)(1-p)/p^2]$  and  $p > d/(d+1)$ , is the unique perfect Bayesian equilibrium where A uses the pooling strategy  $s^A$ .<sup>19</sup>

**Case  $p \leq d/(d+1)$ :** Next, we study the case when single entry is not profitable, that is, when  $p \leq d/(d+1)$ . A will only play  $s^A = \text{"Never enter"}$  if there is no entry by B. Hence, we can restrict the attention to deviations from the equilibrium candidate where A and B never enter. If A is to profitably deviate from  $s^A$ , then it must be that the deviation makes B enter. From the section on separating equilibria, we know that if  $t \in (((d-p)(1-p))/p^2, d]$ , and

$$p > \left( \sqrt{((d-1)^2 + 4d^2)} - (d+1) \right) / (2(d-1)),$$

then it is optimal for A to deviate and enter for a high signal since this gives a separating equilibrium. Further, if  $t > d$ , we know that also here it is profitable for A to deviate,

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<sup>19</sup>Function  $\gamma_4$  (as defined in Section 2), is the lower border of this region in the parameter space. As this equilibrium only holds when  $p > d/(d+1)$  and we have, as a matter of convenience, defined all functions for all  $p \in (0.5, 1]$ , the remaining piece of  $\gamma_4$  (for  $p \leq d/(d+1)$ ) is taken to coincide with  $\gamma_3$ . Since  $\gamma_3$  is the upper border for this region, the lower and upper border then coincide for  $p \leq d/(d+1)$ . This correctly makes the equilibrium non-existent for  $p \leq d/(d+1)$ .

since the complementarities are strong enough to make B enter for a low signal. However, when the parameters are not in these regions, entry by A will not make B enter for any beliefs. Hence,

$$s^A = \text{"Never enter"}, s^B = \text{"Never enter"},$$

is the unique perfect Bayesian equilibrium for  $p \leq d/(d+1)$  and

$$t < \gamma_3(p) = \begin{cases} d, & \text{if } p < \left( \sqrt{((d-1)^2 + 4d^2)} - (d+1) \right) / (2(d-1)); \\ (d-p)(1-p)/p^2, & \text{otherwise.} \end{cases}$$

## 7.4 Proof of Remark 2

Here, we prove  $\phi_1(p) > \gamma_1(p) > \phi_2(p)$  except when the functions coincide. The proof that  $\phi_4(p) > 0$  for  $d > 1$  follows directly from the definition of  $\phi_4$ . The proof of  $\gamma_2(p) > \phi_3(p)$  is also trivial.

### 7.4.1 Proof of $\phi_1(p) \geq \gamma_1(p)$

Since  $\gamma_1$  and  $\phi_1$  coincide when  $p < p_I = \left( d+1 + \sqrt{d(d-1)} \right) / (3d+1)$ , we study the case when  $p \geq p_I$ . Then,

$$\phi_1(p) - \gamma_1(p) = \frac{1}{2} \frac{(1-p)^2 + dp(3p-2)}{(1-p)^2},$$

which is positive if the nominator  $(1-p)^2 + dp(3p-2)$  is positive. Now,

$$\frac{\partial}{\partial p} ((1-p)^2 + dp(3p-2)) = (4p-2)d + 2((d+1)p-1) > 0.$$

The nominator evaluated at the lowest  $p$ , namely  $p_I$ , gives the nominator equal to zero. Thus, the nominator is positive for  $p > p_I$  and the inequality is proved.

### 7.4.2 Proof of $\gamma_1(p) \geq \phi_2(p)$

Since  $\gamma_1$  and  $\phi_2$  coincide when  $p < p_I = \left( d+1 + \sqrt{d(d-1)} \right) / (3d+1)$ , we study the case when  $p \geq p_I$ . Then,

$$\gamma_1(p) - \phi_2(p) = \frac{1}{2} \frac{(1-p)^2 + dp(3p-2)}{p(1-p)}.$$

The denominator is positive and the nominator is exactly the same as in the proof of  $\phi_1(p) \geq \gamma_1(p)$  and pertains to the same values of  $p$ . Thus,  $\gamma_1(p) \geq \phi_2(p)$  follows from the proof in the previous subsection.

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## CHAPTER 3

# Endogenous Market Segmentation and the Law of One Price



# Endogenous Market Segmentation and the Law of One Price\*

Richard Friberg and Kaj Martensen

## Abstract

To the surprise of many, price deviations between markets characterized by imperfect competition have often been little affected by lower transport costs. We show that if a firm's decision to segment markets is endogenous, then lower transport costs are, in many cases, associated with a greater deviation from the law of one price. This result extends from a general monopoly setting to the case of Cournot competition. The intuition is that lower transport costs, by facilitating arbitrage, place a tighter restriction on the maximization problem and a firm is willing to take a greater cost in order to segment. We examine how the resulting equilibria depend on transport costs, product differentiation and costs of segmenting. Examining the implications of country asymmetries, we find that when goods are differentiated, the firm from the poorer market has a greater incentive to segment markets, whereas when goods are homogeneous, the firm from the richer market has the greater incentive.

Decreasing transport costs are associated with lower price differentials between markets for commodities such as wheat or gold – which follows from the underlying logic of arbitrage and is well established empirically (see Goodwin, 1992 or O'Rourke and Williamson, 1999). The deviation from the law of one price (LOP), defined as the price differential between two markets, in this type of markets is equal to the costs of shipping the good and costs arising because of formal trade barriers (tariffs). However, price deviations between markets characterized by imperfect competition have often been little affected by the dismantling of various barriers to trade (see Engel and Rogers, 1996, Goldberg and Knetter, 1997, Obstfeld and Rogoff, 2000), which is quite surprising. The answers to many questions in international economics hinge on whether markets are seen as segmented or integrated, and therefore, it is interesting to try to deepen our understanding of the frictions that segment markets.<sup>1</sup>

The simple idea pursued in this paper is to study how the price differences between markets respond to changes in transport costs when the decision to segment markets is

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<sup>1</sup>For instance, market delineations for anti-trust cases and the constraints faced by monetary and fiscal policy in an open economy.

endogenous.<sup>2</sup> By controlling distribution, marketing and product design, a firm can affect the price differential needed to make arbitrage attractive. Greater ability to segment markets translates into greater ability to price discriminate, and thus the potential to increase profits. We examine the implications of a fixed cost for segmenting markets both in a simple general formulation of a monopolist's problem and in a Cournot duopoly. A number of implications follow from the assumption that segmentation is endogenous. The main insight is that as transport costs between markets decrease, arbitrage places a tighter constraint on the maximization problem – as a result, the incentives for a firm to segment markets increase. This is the driving mechanism behind our result that as transport costs fall, the deviation from LOP can actually increase. For the same reason, the deviation from LOP does not converge to 0 as transport costs go to 0 (given sufficiently low costs of segmenting). All in all, how deviations from LOP respond to changes in transport costs will depend on the characteristics of demand and the costs of segmenting, rather than just being determined by a simple arbitrage condition.

By making the decision to segment markets endogenous, we extend a large body of literature originating with Brander (1981) and Brander and Krugman (1983). Their "segmented markets" model has become a standard model of trade under imperfect competition (see, for instance, Baldwin and Venables, 1995, for a discussion). Markets are assumed to be exogenously segmented in the sense that decisions on quantities are made separately for each market. An alternative is to analyze games where capacity is first set at a global level, whereafter competition under various assumptions is analyzed – this is explored in Venables (1990, segmented and integrated markets, differentiated goods) and Ben-Zvi and Helpman (1992, price competition in homogenous goods). In the aforementioned analysis, the countries are mainly assumed to be symmetric – optimal prices are therefore equal and arbitrage will not pose binding constraints on the optimizing decisions of firms.<sup>3</sup> The focus of the analysis has been welfare consequences of trade and whether or not we observe cross-hauling, i.e. two-way trade in identical goods. In contrast, the current paper focuses on understanding the interaction between transport costs and LOP.

The ability to segment customer groups with differing willingness to pay (or markets where marginal costs differ), increases profits for a monopolist by allowing for third degree price discrimination. By considering the incentives to segment markets under oligopoly, we relate to a small literature examining price discrimination under imperfect competition (Holmes, 1989, and Corts, 1998). Different mechanisms allowing for (some degree of) segmentation are vertical restraints (see, for instance, Gould, 1977), having

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<sup>2</sup>In principle anything that segments markets can be defined as a transport cost. In this paper we will take transport costs to mean actual costs of shipment and tariffs. By endogenous segmentation we mean that the firm raises the cost of arbitrage to the extent that even when transport costs, narrowly defined, are zero, arbitrage is unprofitable. As discussed below, this includes bundling with non-traded goods, vertical integration and using different brand names in different locations. See Hummels (1999) for a recent discussion of transport costs and an attempt to measure these.

<sup>3</sup>Transport costs are typically assumed to be positive for the export market and 0 for the domestic market. We therefore observe price discrimination ("dumping") even if LOP holds, since markups differ between the home and export market. Anderson et al (1995) provide an analysis of issues related to those in the present paper – in a first stage, governments decide in a non-cooperative game whether to impose antidumping laws or not.



different brand names in different locations, bundling with non-traded goods (explored in Horn and Shy, 1996) and adulteration of the good such as having different technical standards in different regions or countries (see Carlton and Perloff, 2000, p. 278, for an overview). Rather than focusing on any specific way of segmenting markets we note that a firm can, in a number of ways, deter arbitrage and that this is likely to be associated with some cost.<sup>4</sup>

We now turn to a description of the model and an analysis of the monopoly case before proceeding with the Cournot model.

## 1 The Model

Examine the decision problem of a firm located in Home who sells its goods on two markets, Home and Foreign. Let  $x$  denote quantity,  $p(x)$  the downward sloping inverse demand function,  $c$  the fixed marginal cost and  $t$  the cost of transporting a unit of the good between Home and Foreign (the firm and the consumers face the same  $t$ ). Variables in lower case letters denote the Home market and capital letters denote the Foreign market. Let the firm have the ability to invest in market segmentation such that by spending an amount  $K$ , arbitrage is ruled out and consumers are then not able to benefit from a lower price abroad. The way we think about these fixed cost is that they rule out arbitrage by increasing the total cost of arbitrage to consumers to exceed  $t$  to such an extent, that arbitrage is unprofitable also when  $t = 0$ . If the firm has not paid the fixed cost of market segmentation, profit maximization will be subject to the constraint that the differences in price are less than or equal to the transport cost. As a consequence, the maximization problem under integrated markets is given by

$$\begin{aligned}\Pi &= \max_{x, X} xp(x) + XP(X) - c(x + X) - tX, \\ \text{s.t. } & |P - p| \leq t,\end{aligned}\tag{1}$$

and under segmented markets by

$$\Pi^S - K = \max_{x, X} \underbrace{xp(x) + XP(X) - c(x + X) - tX}_{\Pi^S} - K.\tag{2}$$

We denote the optimal quantities under integrated markets with a superscript  $*$ , and optimal quantities under segmented markets with superscript  $s$ . To examine how the deviation from the LOP depends on  $t$ , we need a formal definition of LOP.

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<sup>4</sup>The mechanisms which help segmenting markets may clearly also be motivated on other grounds, vertical restraints for instance may be motivated by several factors (see Martin, 2001). We merely note that the optimal usage of these mechanisms will be different if market segmentation is also an issue. The assumption of fixed costs of segmenting is also used in Friberg (2001) which examines how sunk costs of segmenting may create an option value of segmenting when there is real exchange rate variability. Related to our analysis is also Anderson and Ginsburgh (1999) who examine how search costs differing across individuals can introduce elements of second-degree price discrimination into international price setting.

**Definition 1** *The deviation from LOP is defined as  $P(X) - p(x)$ .*

If there is a possibility for arbitrage, then  $P(X) - p(x) = t$  must hold, and we thus say that the deviation from LOP is  $t$ . To analyze the monopolist's problem, we first need to determine for what values of  $t$  and  $K$  the monopolist will find it profitable to segment markets. We restrict the attention to the case where demand is such that it would be optimal to have a higher price for the Foreign market:  $P(X^S) > p(x^S)$  at  $t = 0$ . The decision problem is then only interesting when the constraint  $P(X^S) \leq p(x^S) + t$  is binding, which is the region we examine.

## 1.1 Integrated markets

Maximize Eq. (1) subject to the constraint that  $P(X) = p(x) + t$ . Using the Inverse Function Theorem we may solve this expression for the quantity  $X$ ,<sup>5</sup>

$$X = P^{-1}(p(x) + t).$$

Since the inverse of the inverse demand function is simply the demand function, we may write  $X(p(x) + t)$ . Substituting the constraint into the maximization problem, we obtain

$$\max_x (p(x) - c)(x + X(p(x) + t)), \quad (3)$$

with the first-order condition for profit maximization given by

$$(p(x) - c) \left( 1 + \frac{dX(p(x) + t)}{dx} \right) + \frac{dp(x)}{dx} (x + X(p(x) + t)) = 0, \quad (4)$$

which yields an optimal quantity  $x = x^*(t)$  (given, as usual, that the second-order condition for profit maximization is fulfilled). Thus, after maximization,

$$\Pi = \Pi(x^*(t), t). \quad (5)$$

## 1.2 Segmented markets

Maximize Eq. (2) with the following first-order conditions

$$p(x) + x \frac{dp(x)}{dx} - c = 0, \quad (6)$$

and

$$P(X) + X \frac{dP(X)}{dX} - c - t = 0. \quad (7)$$

So after maximization

$$\Pi^S = \Pi^S(x^S, X^S(t), t). \quad (8)$$

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<sup>5</sup>Note that  $p(x) + t$  is a monotonic function of  $x$ , for a fixed  $t$ .

### 1.3 Difference in operating profits

The decision whether to segment markets will depend on the difference in operating profits between segmented and integrated markets. The unconstrained profit cannot be dominated by the constrained profit and, as a consequence,  $\Pi^S - \Pi \geq 0$ . The firm will segment markets if  $\Pi^S - \Pi > K$  and the difference between  $\Pi^S$  and  $\Pi$  will, in turn, depend on  $t$ . Some manipulation (in Appendix A) yields the following result:

**Proposition 2** *A sufficient condition for  $\Pi^S - \Pi$  to be decreasing in  $t$ , is that the inverse demand function on the Foreign market is concave, linear or not too convex.*

**Proof.** See Appendix A. ■

As long as the inverse demand curve is concave, linear or not too convex, the firm will segment markets when transport costs are low and integrate when transport costs are high. Note that second-order conditions for profit maximization place restrictions on the degree of convexity of the demand function. The intuition for the decreasing difference is simple: as transport costs increase, there are two effects on profits – first, it becomes more costly to export which has a negative impact on profits both under segmented and integrated markets. Second, the constraint that the deviation from LOP be no greater than  $t$  should hold under integrated markets, is relaxed, which decreases the difference in operating profits between the integrated and the segmented case. Since the quantity shipped differs between the integrated and segmented cases, the curvature of the demand curve will determine the relative responses to changes in quantity.

### 1.4 Deviations from the law of one price

Under integrated markets, the deviation from LOP ( $P - p$ ) will equal  $t$ . Under segmented markets the difference between optimal profits will be determined by the first order conditions on the respective markets. Define demand elasticities  $\varepsilon = - (dx^S/dp) (p/x^S)$  and  $E = - (dX^S/dP) (P/X^S)$ . The optimal prices are given by

$$\begin{aligned} p \left( 1 - \frac{1}{\varepsilon} \right) &= c, \\ P \left( 1 - \frac{1}{E} \right) &= c + t. \end{aligned} \tag{9}$$

Under segmented markets (and constant marginal costs), the optimal price on the Home market is independent of the transport cost. Changes in  $t$  will be passed through into the Foreign price however, to what extent depends on the curvature of the demand function.

**Remark 3** *i) When markets are segmented the deviation from LOP will depend on the curvature of the Foreign demand function. If the demand curve is less convex than a constant elastic demand curve, the deviation from LOP will decrease less than proportionately to decreases in  $t$ . ii) When markets are segmented the deviation from LOP be positive at  $t = 0$  if  $E < e$ .*

The first observation is an application of a result regarding the pass-through of tariffs or exchange rates (see, for instance, Feenstra, 1989).<sup>6</sup> The more convex the demand curve, the greater the pass-through elasticity. A case of special interest is that of a linear demand curve, for which pass-through equals one half – thus, the deviation from LOP decreases by half the change in transport costs (less than half in terms of elasticities since  $(d(P - p)/dt) (t / (P - p)) = -t / (2(P - p))$ ). In contrast, when markets are integrated, the deviation from LOP clearly decreases one-to-one with transport costs (less than 1 in terms of elasticities). The second observation follows trivially from Eq. (9). Figure 1 illustrates the case where the condition in Proposition 2 holds.

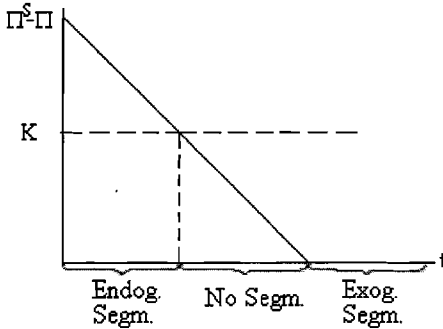


Figure 1a. Difference in profits.

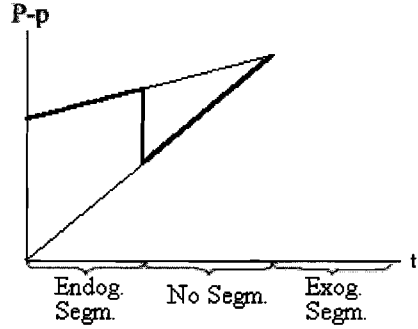


Figure 1b. Difference in prices.

The behavior of the deviation from LOP implied under endogenous segmentation may at first glance be surprising – lower transport costs may actually be associated with a larger deviation from LOP because of the discrete increase in the deviation when markets are segmented. Also, if demand elasticities differ between markets the deviation from LOP will not converge to zero as  $t$  approaches zero. With such a model in mind, the limited impact on price differentials of lower transport cost, tariffs and non-tariff barriers comes as no surprise. The cost of segmenting ( $K$ ) also affects the deviation from LOP, and this effect is discrete. As the costs of segmenting markets increase (if, for instance, governments take a more restrictive view on exclusive territories), this will induce some firms to integrate markets, with substantial reductions in deviations from LOP as a result – whereas for other firms, the deviation from LOP will be unaffected. Having pointed out these fundamental differences in the development of the deviation from LOP under monopoly with endogenous segmentation, compared to the standard case where the price differential always decreases one-for-one with transport costs, let us now take a closer look at endogenous segmentation under oligopoly.

<sup>6</sup>The factors that determine pass-through of transport costs are the same as the determinants of exchange rate pass-through (if we disregard dynamic aspects, see Feenstra, 1989). The stylized fact of less than proportional pass-through of exchange rate changes into import prices (Goldberg and Knetter, 1997), fits well with the small impact of transport costs on LOP.

## 2 A Cournot example

### 2.1 Model

As before, there are two countries, Home and Foreign, with one firm located in each country. Let  $x$  represent the quantity produced by the Home firm for the Home market and  $X$  the quantity it produces for the Foreign market. Similarly, let  $y$  be the Foreign firm's export to the Home market and let  $Y$  be the production of the Foreign firm for its domestic market. As before, let prices be denoted by  $p$  in the Home country and  $P$  in the Foreign country.

The degree of product differentiation is given exogenously by  $\gamma \in [0, 1)$ . In the case of homogenous goods ( $\gamma = 1$ ), there will be multiple equilibria and we will analyze a limiting case. We assume linear demands for each market with inverse demand curves given by the following matrix,

	Home country	Foreign country
Home product	$p(x) = 1 - x - \gamma y$	$P(X) = A - X - \gamma Y$
Foreign product	$p(y) = 1 - y - \gamma x$	$P(Y) = A - Y - \gamma X$

We confine the attention to the case where  $A \geq 1$  which means that we can think of the Foreign market as the rich market. As before, there is a per-unit transportation cost  $t$ , and a cost of segmenting the markets  $K$ .

Now, consider the market structure, given different transport costs. First, for very high transport costs, each firm will have a local monopoly. Next, with intermediate transport cost, it will be profitable for firms to enter the other market, but transport costs will still be sufficiently high to make arbitrage unprofitable for consumers. Finally, for low transport costs, arbitrage possibilities will restrict the firms' profit maximization problems, unless the firms take the cost of segmenting markets. We limit attention to the case where arbitrage poses a binding constraint for both firms. Under the assumption that both firms segment, neither of the conditions  $P(Y) - p(y) < t$  and  $P(X) - p(x) < t$  hold if<sup>7</sup>

$$t < (A - 1) \frac{2 - \gamma}{(\gamma + 2)(3 - 2\gamma)} \approx \frac{A - 1}{3}. \quad (10)$$

Thus, arbitrage by consumers poses a potential problem for firms when transport costs are low in relation to country asymmetries; the larger the difference between willingness to pay, the larger is the potential difference in prices and the larger is the range of  $t$  for which consumers find arbitrage profitable. On the other hand, if the countries are identical ( $A = 1$ ), then we are in the Brander-Krugman world where arbitrage is not an issue, since optimal prices are equal on identical markets with identical firms.

<sup>7</sup>The constraint is fairly unsensitive to  $\gamma$ . A series expansion shows that  $(2 - \gamma) / ((\gamma + 2)(3 - 2\gamma)) = 1/3 - \gamma/9 + 5\gamma^2/54 + O(\gamma^3) \approx 1/3$ . Further, the constraint is non-linear in  $\gamma$ , with maxima at  $\gamma$  equal to 0 and 1, where the condition becomes  $t < (A - 1)/3$ . The constraint is derived at the end of Appendix B.

We assume that the firms simultaneously choose quantities and whether they want to segment in a one-shot game. Suppressing quantities, a strategy profile will, for instance, be written as  $(S, N)$  when the Home firm segments ( $S$ ) and the Foreign firm does not ( $N$ ). A strategy profile is an equilibrium if the strategy profile is a Nash equilibrium. A firm that does not segment will have to optimize its profit under a constraint of the form  $|P - p| \leq t$  to prevent customer arbitrage. When a firm integrates, it sets a larger quantity on the rich market and smaller quantity on the poor market than it would under segmented markets, thereby driving down the price difference between markets. In one sense one could think of this as a commitment to more aggressive play on the rich market. We assume that a segmenting firm commits itself strongly not to uphold LOP (i.e. ignores commitment to aggressive play by an integrating firm) and further, that an integrating firm only considers the price deviation for its own good. In doing this, we largely avoid problems of multiplicity of equilibria, but we lose some strategic interaction effects.<sup>8</sup> This is a crucial assumption and we will relate to it throughout the analysis.

Let  $\Pi$  denote the profit of a firm. To simplify, we assume the marginal cost of production to be zero. The firms' maximization problems (not including constraints) are:

$$\Pi^H - I_s^H K = \max_{x, X} \underbrace{xp(x) + XP(X) - tX}_{\Pi^H} - I_s^H K,$$

for the Home firm and the Foreign firm's problem is

$$\Pi^F - I_s^F K = \max_{y, Y} \underbrace{yp(y) + YP(Y) - ty}_{\Pi^F} - I_s^F K,$$

where  $I_s$  is an indicator function for the case that the firm segments. Solving for the optimal quantity in each case under the appropriate constraints, gives functions  $\Pi^H(A, t)$  and  $\Pi^F(A, t)$ . By,  $\Pi_{N,S}^F$ , for example, we will denote the operating profit for the Foreign firm (super index  $F$ ), given the strategy profile  $(N, S)$ . The payoff matrix is then given by:

		Foreign	
		Segment	Not Segment
Home	Segment	$\Pi_{S,S}^H - K, \Pi_{S,S}^F - K$	$\Pi_{S,N}^H - K, \Pi_{S,N}^F$
	Not Segment	$\Pi_{N,S}^H, \Pi_{N,S}^F - K$	$\Pi_{N,N}^H, \Pi_{N,N}^F$

(11)

The profits given in the matrix are calculated in Appendix B. To isolate the effects of market asymmetries, we first study the case where  $\gamma = 0$ . Then, there is no strategic interaction since no firm takes market shares from another firm.

<sup>8</sup>If the segmenting firm incorporated the constraint, that the deviation from LOP should equals  $t$  for the other firm, then in one sense the segmenting firm is also responsible for fulfilling the constraint. There will potentially be multiple equilibria unless the relative responsibility for making the price differential equal  $t$ , is exogenously specified. Our assumption amounts to such an exogenous specification that highlights non-strategic effects.

## 2.2 Country effects absent of strategic interaction

When the products are perfectly differentiated ( $\gamma = 0$ ), the relative benefit from segmentation compared to integration for a firm is independent of the strategy of the other firm. In Figure 2, we plot the difference in profits between the integrated and segmented markets case, as a function of transport costs (for  $A = 2$ ). The fat line gives the gain in revenue from operating under segmented markets rather than under integrated markets for the Foreign firm, and the thin line is the equivalent for the Home firm.<sup>9</sup> Each firm will segment if the gain from segmenting outweighs the cost of segmenting.

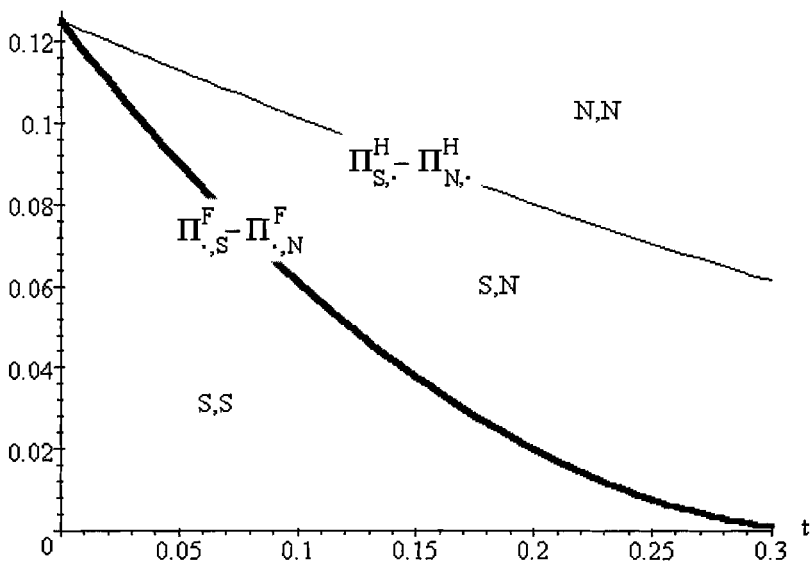


Figure 2. Integration vs. Segmentation under perfectly differentiated goods ( $\gamma = 0$ ).

First, for low transport costs (such as  $t = 0.05$ ), and intermediate costs of segmentation (such as  $K = 0.06$ ), the gain from segmenting outweighs the cost and both firms segment. This is true for the whole region below the fat line, which is marked with S,S, denoting that both firms segment in equilibrium.<sup>10</sup> Now, for values of  $t$  and  $K$  such that we are above both lines, neither of the firms will find it beneficial to segment. Finally, we note that in the region above the fat line, but below the thin line, the Home firm

<sup>9</sup>Since payoffs are independent of what the other firm does,  $\Pi_{S,S}^H = \Pi_{S,N}^H$  and  $\Pi_{N,S}^H = \Pi_{N,N}^H$ .

<sup>10</sup>Note that  $K$  and  $t$  have the same magnitude in Figure 2. This might seem surprising as  $t$  is a per-item cost and  $K$  is a fixed cost, which we think is large. However, if both firms segment, and  $t = 0.05$ , then, for instance,  $y = 0.475$ , so that the total cost of exports is roughly equal to 0.02 for the Foreign firm. Then,  $K = 0.06$  is three times the size of total export costs for the Foreign firm, which is in line with what we expect.

segments whereas the Foreign firm integrates. The intuition for this comes from the following observation.

**Proposition 4** *Given that there is no strategic interaction and no consumer arbitrage, the firm from the poor market sets prices such that the price difference between countries is higher than what the firm from the rich market sets.*

**Proof.** See Appendix C. ■

We will call this the *country effect* henceforth. Optimal prices are higher on the Foreign market, since that is the rich market. For the Home firm, transport costs serve to further increase the Foreign price (since that is its export market). On the other hand, for the Foreign, rich country firm, transport costs raise the price on the cheaper market, and therefore serve to decrease price differentials. In the Figure below, we illustrate the optimal prices under segmented markets for the two firms.

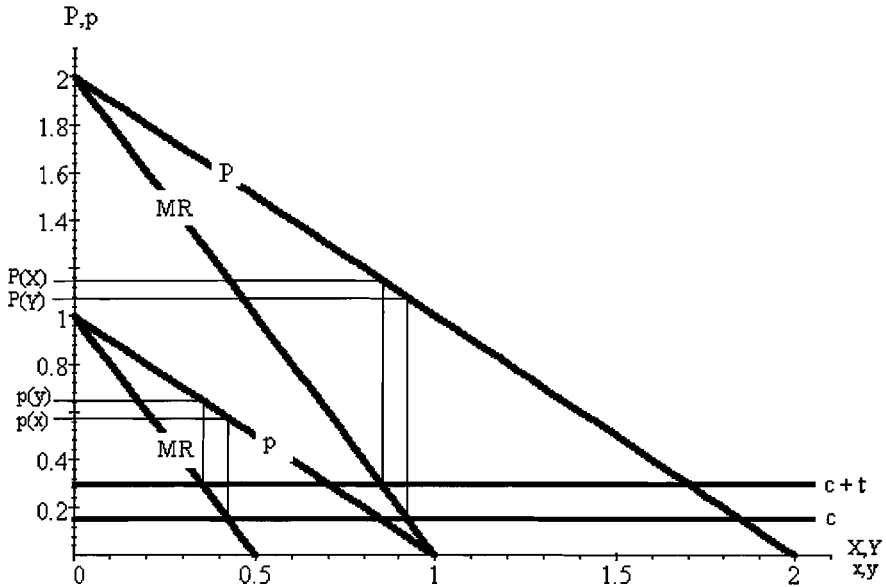


Figure 3. Price differences for perfectly differentiated goods ( $\gamma = 0$ ).

It is clear from the figure that  $|P(X) - p(x)| > |P(Y) - p(y)|$ . Since the difference between optimal prices is greater for the Home firm, it is willing to bear a higher cost to avoid the constraint that the deviation from LOP should equal  $t$ . In Figure 2, this country effect explains why the Home firm segments for a higher cost of segmentation (in the region between the thin and the fat line).



## 2.3 Cournot Equilibria

To find the equilibria of the game when goods are substitutes, we need to compare the relative benefits of strategies, that is, compare the difference in profits between segmenting and integrating for the cases where the rival firm segments and integrates, respectively. The difference between payoffs will depend on  $t$ , and we wish to explore how the resulting equilibria then depend on  $t$ . It is useful to define the following functions:

$$\begin{aligned}\Delta_S^H(A, \gamma, t) &= \Pi_{S,S}^H(A, \gamma, t) - \Pi_{N,S}^H(A, \gamma, t), \\ \Delta_N^H(A, \gamma, t) &= \Pi_{S,N}^H(A, \gamma, t) - \Pi_{N,N}^H(A, \gamma, t),\end{aligned}$$

and

$$\begin{aligned}\Delta_S^F(A, \gamma, t) &= \Pi_{S,S}^F(A, \gamma, t) - \Pi_{S,N}^F(A, \gamma, t), \\ \Delta_N^F(A, \gamma, t) &= \Pi_{N,S}^F(A, \gamma, t) - \Pi_{N,N}^F(A, \gamma, t).\end{aligned}$$

For example,  $\Delta_S^H$  gives the gain in revenue for the Home firm (super index  $H$ ) of segmenting compared to not segmenting, given that the Foreign firm segments (sub index  $S$ ). Then, if, for instance,  $\Delta_S^H > K$  and  $\Delta_N^H > K$  (we suppress the dependency on parameters hereafter), this means that segmenting is a dominant strategy for the Home firm. The gain in profit from segmenting outweighs the cost of segmenting, notwithstanding the actions of the Foreign firm. On the other hand, the game would be a coordination game if  $\Delta_S^H > K$ ,  $\Delta_N^H < K$ ,  $\Delta_S^F > K$  and  $\Delta_N^F < K$ . To find the equilibria of this game, we plot the  $\Delta$ - functions. Which equilibria prevail will depend on the parameter values.

First, we will analyze the case of homogenous goods ( $\gamma = 1$ ) and next, the case of differentiated goods ( $\gamma = 1/2$ ). We set  $A = 2$  in all cases. In general, when products are more similar (as  $\gamma \rightarrow 1$ ), the gain from segmentation is lower. The intuition is that firms segment in order to produce less on the large market and thereby raise profits. If products are similar, the price increase following from the quantity reduction will make some customers choose the competitor's product.<sup>11</sup> Thus, in general, we expect less segmentation in the oligopoly case ( $\gamma > 0$ ).

### 2.3.1 Homogeneous goods – Strategic effects

If there is no difference between the firms' products ( $\gamma = 1$ ), then the firms play a standard Cournot game when markets are segmented. However, when  $\gamma = 1$  we will have multiple equilibria in the case where both firms integrate, both firms then have to fulfill the *same* LOP constraint and we are not able to uniquely determine the optimal quantities of the firms. In consequence this section relies on quantities calculated for the limiting case as  $\gamma \rightarrow 1$ . Figure 4 below illustrates the resulting equilibria as a function of  $K$  and  $t$ . The Foreign firm's  $\Delta^F$  - functions are plotted with fat lines, while the Home firm's functions are plotted with thin lines. Differences in profits are measured on the

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<sup>11</sup>The literature examining price discrimination under oligopoly is so far very limited. Holmes (1989) analyzes this latter effect by examining oligopolistic firms that set prices on differentiated products. Holmes examines symmetric firms and does not analyze the case of transport costs.

vertical axis and are a function of transport costs, measured along the horizontal axis. The figure is given for  $t < 0.3$ , which is the limit below which arbitrage will place a restriction on the unsegmented maximization problem following Eq. (10).

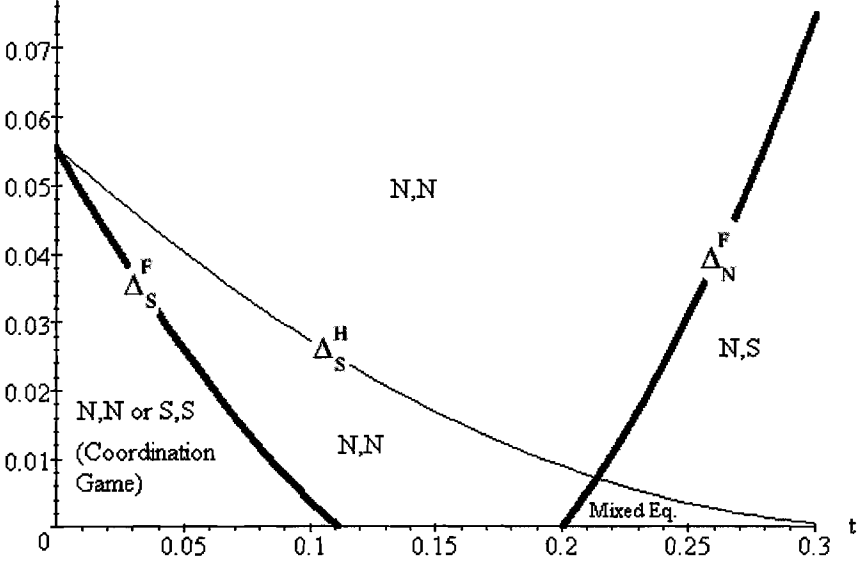


Figure 4. Equilibria for  $\gamma \rightarrow 1$ .

Consider, for example, the cost  $K = 0.03$  of segmentation. If the transport cost is  $t = 0.15$ , then we are in the top-most region of Figure 4, which means that  $\Delta_S^H < K$ ,  $\Delta_N^H < K$ ,  $\Delta_S^F < K$  and  $\Delta_N^F < K$ . Using these relations (compare with matrix (11)), we can then establish that it is a dominant strategy for each of the firms to integrate markets. A comparison with the equilibria of Figure 2 shows that there is less segmentation when producers sell identical goods; now, there is no region where segmentation is a dominant strategy or unique Nash equilibrium.<sup>12</sup> For a broad range of values of  $K$  and  $t$ , the unique equilibrium is that neither firm segments. Now, the interdependence of the firms' decisions is apparent in the case where both transport and segmentation costs are low – we then have a coordination game and there are two equilibria – either both firms segment or both integrate.<sup>13</sup>

<sup>12</sup>Note that we are not really doing a comparative statics exercise here; we are holding the intercepts of the demand function fixed as we increase  $\gamma$ .

<sup>13</sup>In a model of Bertrand competition in differentiated goods, Corts (1998) examines the incentives for firms to set a uniform price (integrate) or price discriminate (segment) in vertically segmented markets. Similar to our approach, each firm has a relative advantage in a specific market. He finds that when the decision to segment or integrate is simultaneous to the price setting decision, then firms segment

We also see that for intermediate values of  $K$  and high transport costs, the Foreign firm now segments while the Home firm does not – this can be compared to the monopoly case where the region under which the Home firm segmented was larger than that under which the Foreign segmented. Since products are homogenous, there is only one price in each country and it is sufficient that one firm segments for the price differential to equal  $t$ . Then, what is meant by "segmentation" by the Foreign firm in this case? The Foreign firm's reaction functions are given by the segmented markets case, whereas the Home firm's reaction functions are connected by the condition that the price differential should equal  $t$ . This a direct result of our assumption that a segmenting firm commits itself to not uphold the restriction on price differentials imposed by arbitrage.

To understand how transport costs affect the equilibrium and the relative gains of segmenting, it is useful to turn to an analysis of the reaction functions under segmented markets.

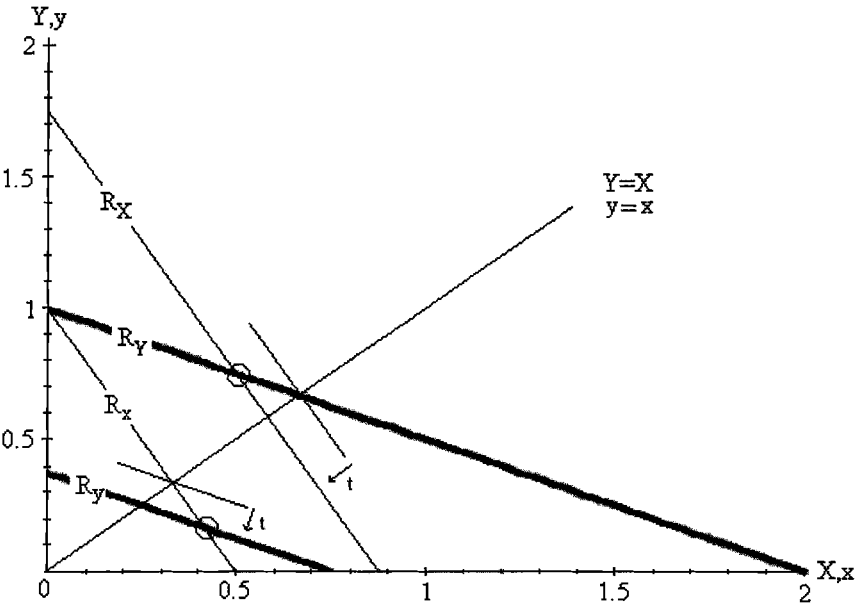


Figure 5. Reaction functions given that both firms segment markets and  $\gamma = 1$ .

In Figure 5,  $R_Y$  is the reaction function of the Foreign firm on the Foreign market, and  $R_y$  is its reaction function on the Home market. Similarly,  $R_X$  and  $R_x$  are the reaction functions of the Home firm. The short lines show the export market reaction functions  $R_X$

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in equilibrium, since each firm is unilaterally better off by being unconstrained in its maximization problem. Note that we can not simply interpret the transport cost in our model as the relative distance in preference for high and low quality goods in his model, since the transport costs directly affect the firms' costs in our model.

and  $R_y$  absent of transport costs. Thus, the equilibrium quantities, absent transport costs, are given by the intersections of the short lines with the local market reaction function in each country, the firms then split each market equally (the equilibrium points are on the 45-degree line). Transport costs shift the equilibrium quantities in each market from equal shares to local advantage (the figure is drawn for  $t = 1/4$ ), by shifting the firms' reaction functions on their export markets inward. Further, as  $t$  increases,  $|X - x|$  decreases and  $|Y - y|$  increases. Since the equilibrium quantities absent transport costs are on the 45-degree line, we know that  $X = Y$  and  $x = y$ , which implies that  $|Y - y| = |X - x| = 0$  for  $t = 0$ . Thus,  $|Y - y| > |X - x|$  for  $t > 0$ . Hence, the Foreign firm is more constrained by consumer arbitrage, since its quantity choices absent arbitrage would give higher price differences between countries – henceforth we call this the *strategic effect*. This leads to the following observation.

**Remark 5** *In contrast to the monopoly case (Proposition 4), the firm from the rich country now has the greatest incentive to segment markets.*

**Deviation from LOP** Now, turn to the resulting deviation from LOP. For low transport costs and low costs of segmenting, there are two equilibria and thus a potential that the deviation from LOP will be greater than  $t$ . Apart from this case (and a small area involving mixed equilibria for very low segmentation costs and high transport costs), the unique equilibrium implies that at least one firm integrates markets, as seen in Figure 4. As a consequence the deviation from LOP will typically equal  $t$  when goods are homogeneous.

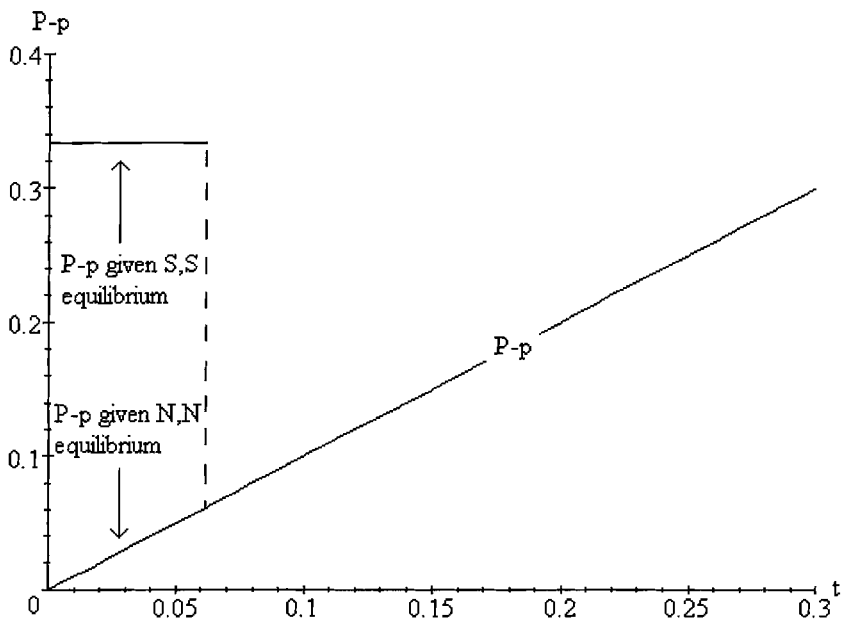


Figure 6. LOP as  $\gamma \rightarrow 1$ .

However, if both firms segment, we note that the deviation from LOP is independent of  $t$ . This stand-out feature depends crucially on the assumption that the demand curves in the two countries have the same slope. As the reaction function of firms' are shifted in their respective export markets, the resulting quantity changes are traced out along each local firm's reaction functions ( $R_Y$  and  $R_x$ , respectively). In each case, the perceived change in a firm's own quantity is read along its own axis, where the perceived slopes are *equal*. A change in  $t$  thus changes local quantities by a factor of  $1/2$ , so that when  $t$  increases,  $Y$  increases and  $x$  increases, each with a factor of  $1/2$ . The same reasoning applies to each firms' exports, where each firm's quantity decreases by a factor of 2. On each market, increases in  $t$  lead to a greater market share for the local firm. An increase in  $t$  will thus reduce aggregate quantities on each market ( $X + x$  and  $Y + y$ ) *by the same amount*, since the decrease in exported quantities is the same for both countries (the change in  $X$  equals the change in  $y$ , and the change in  $Y$  equals the change in  $x$ ). Thus, given that the demand functions in each country have the same slope, we have the following result.

**Remark 6** *Under homogenous goods, if both firms segment, the price difference  $P - p$  does not depend on  $t$ .*

Now, let us consider the impact of our assumption that a firm ignores the information whether the other firm segments or not. Consider the asymmetric equilibria where one

firm segments and the other integrates. If instead a firm that segments firms acknowledges that the other firm integrates, then the segmenting firm must incorporate the integrating firm's LOP constraint in its own maximization problem. If products are homogenous, then there is only one LOP constraint and there will be multiple equilibria. If the segmenting firm must fully incorporate (uphold) the LOP constraint, then segmentation makes no sense and thus, the asymmetric equilibria seem implausible. In this case we would expect to see only coordination between N,N and S,S. Note that the result given in Remark 6 still holds, since the analysis does not change if both firms segment markets in equilibrium. It would be interesting to derive sufficient conditions for an S,S equilibrium to exist but the analysis is complicated by the multiplicity of N,N equilibria (which exist by reasoning similar to the one for asymmetric equilibria).

This concludes our analysis of the homogeneous goods case. After studying the two polar cases of homogeneous goods and independent demands, we now focus on the intermediate case of imperfect substitutes.

### 2.3.2 Differentiated products

We now set  $\gamma$  equal to  $1/2$ . This case is the richest from an analytical perspective, since both the country effect and the strategic effect will be affecting outcomes. The resulting equilibria are given in Figure 7, whose layout is analogous to that of Figure 4

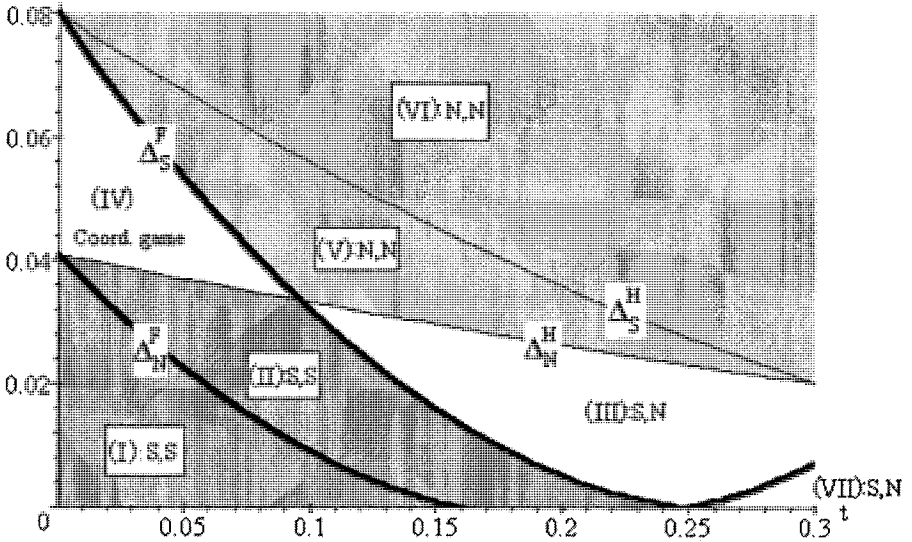


Figure 7. Equilibria when goods are differentiated ( $\gamma = 1/2$ ).

Consider, for example, the cost  $K = 0.06$  of segmentation. If transport costs are intermediate, then we are in region V in Figure 7. This means that  $\Delta_S^H > K$ ,  $\Delta_N^H < K$ ,  $\Delta_S^F < K$  and  $\Delta_N^F < K$ . Only  $\Delta_S^H$  is larger than  $K$ . Using these relations (compare with matrix (11)), we can then establish that the unique Nash equilibrium is that neither of the firms' segment markets. The regions where neither of the firms segment are shaded in light grey. Intuitively, these regions are associated with relatively high fixed segmentation costs and relatively high transport costs. Similarly, the regions where both firms segment (I and II) are shaded in dark grey and characterized by a low segmentation cost and low transport costs.

In regions III and VII, the unique equilibrium is that the Home firm segments, while the Foreign firm does not, which suggests that the country effect dominates the strategic effect as described above. Given that goods are more differentiated, the behavior of competitors is relatively less important, and our result will be less sensitive to the assumption that firms only consider their own constraints imposed by arbitrage.

The only region with two equilibria is region IV, where there is a coordination game. Both firms either segment or not. So, for intermediate levels of segmentation costs and low transport costs, the interdependence of firms' decisions is especially pronounced in the sense of our having a coordination game.

**Deviations from LOP** Given the equilibria in Figure 7, we are interested in the price differences between the two countries as  $t$  changes. Set  $K = 0.02$ , then, as  $t$  decreases from 0.3, we move from region III to regions II and I. The Home firm will segment for all  $t$ , while for  $t = 0.13$ , the Foreign firm starts to segment as  $t$  decreases. The difference in prices for the Foreign good,  $P(Y) - p(y)$  is then not a continuous function. As Figure 8 below shows, the deviation from LOP is equal to  $t$  for  $t > 0.13$ , while for lower  $t$ , the price difference is greater, the lower the transport costs.

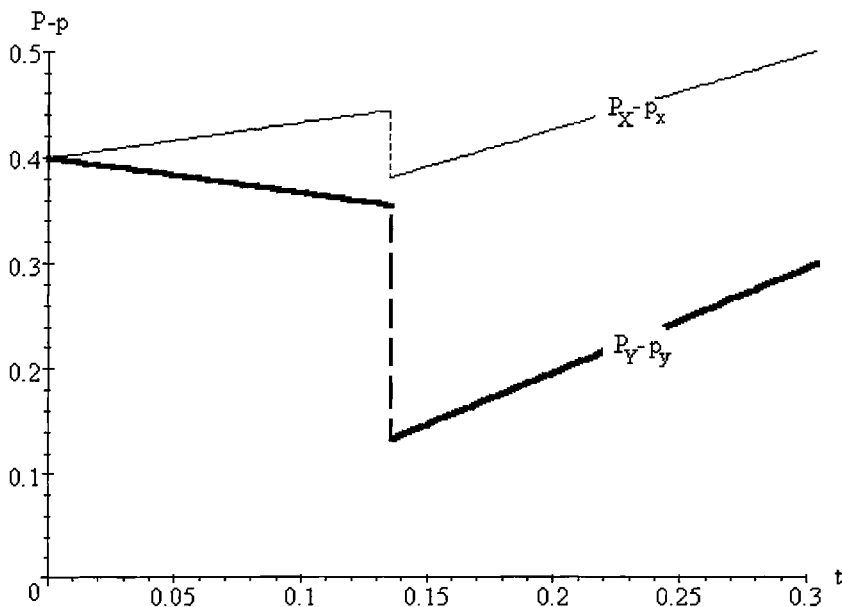


Figure 8. LOP when goods are differentiated ( $\gamma = 1/2$ ).

Also, note that LOP will not hold for any of the goods at zero transport costs. Since the firms segment, and demand on the Home and Foreign markets, the optimal quantities and prices differ even though the costs are the same. A prominent feature is that the deviation from LOP for the Foreign producer's good increases as transport costs fall; not only is there a discrete increase as the Foreign firm starts to segment, but also a continuing increase as transport costs fall further. To explain this at first surprising result, we study how the Foreign firm's prices depend on  $t$  on the Home and Foreign market, respectively. On the Foreign market

$$\frac{\partial}{\partial t} (P(Y)) = \frac{\gamma}{4 - \gamma^2} > 0,$$

and on the Home market

$$\frac{\partial}{\partial t} (p(y)) = \frac{2 - \gamma^2}{4 - \gamma^2} > 0.$$

Thus, both prices increase if  $t$  increases; on the Home (export) market, simply because transport costs are passed through (incompletely) onto the price that Home consumers face. On the Foreign market the effect of transport costs on prices is only indirect through the lower quantities of imports from Home. For the Home firm, the corresponding difference in prices comes from a comparison of its export price minus the price it sets



on the Home market. As transport costs increase, the Home firm's export price increases more than does the price on the Home market and therefore, the deviation from LOP is a mirror image of the Foreign firm's. Further, as the Foreign firm starts to segment, and thereby reduces its quantity on the Foreign market, this is associated with a discrete increase in the price charged by the Home firm on the Foreign market and thus with a discrete jump in the deviation from LOP, albeit smaller than for the Foreign firm.

### 3 Discussion

We have explored the implications of endogenous segmentation in a simple Cournot framework. Considering the cases in this paper, one might conclude that anything can happen – the deviation from LOP can increase as transport costs fall (both in the form of a discrete jump and a continued increase), it can decrease (one for one with the transport cost, or at a slower rate) or, it can be totally unaffected by transport costs. That any response of endogenous variables to changes in exogenous ones can be rationalized, is typically not a hallmark of good theory, since one might ask how the model could be tested. One response to this is that it emphasizes a fundamental point of the paper; we should not expect deviations from LOP to respond to changes in transport costs in markets for differentiated goods in the same way as in commodity markets. It should be noted that our analysis was done given an assumption that minimizes the multiplicity of equilibria, a richer model is therefore likely to further underscore this point.

Another response is that the outcome will partly be determined in an intuitive way by observable variables: We are more likely to observe endogenous segmentation, the greater the asymmetry of markets, the lower the transport costs, and for differentiated goods. That markets with different characteristics respond differently to reduced transport costs, is also in line with the thrust of our arguments.<sup>14</sup> Finally, it deserves to be pointed out that this richness of results comes from using the standard workhorse model of international oligopoly with a simple addition of endogenous segmentation.

The segmented markets model analyzed here was originally applied to understand how monopoly power could be a driving force of trade and welfare aspects of this. It has also been of much use in strategic trade policy; see Brander (1995) for an overview. We here note that it also offers a large potential for understanding deviations from LOP and issues of market segmentation on the forefront of today's policy discussion. The analysis in this paper can be seen as a response to the call by Baldwin and Venables (1995, p. 1612): "European experience suggests that the removal of tariffs is not sufficient to create a 'single market'...merely comparing the outcomes of different games leaves the analysis incomplete, as it leaves open the more difficult question of how different degrees of market segmentation or integration could arise. Ultimately we wish to know what

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<sup>14</sup>Somewhat more worrying is that the outcome will also depend on the costs of segmenting which are not readily observable. In reality they are likely to depend on a large number of issues and one might expect the cost of segmenting to increase as transport costs decrease. In terms of the previous analysis, the  $K$ -line would be upward sloping and we would be more likely to observe integrated markets when transport costs are low.

policy instruments might be used to change the degree of market integration”.

Our model can be used to gain some insight into the European car markets, for instance. As Goldberg and Verboven (1998) show, price differentials on cars between European countries have been remarkably unaffected by lower border barriers. The actual cost of transporting a car from one European country to another is low, so in terms of our model,  $t$  is low. Goldberg and Verboven observe that car manufacturers have actively been trying to segment markets by, for instance, working to maintain the selectivity of the distribution system.<sup>15</sup> The present paper provides a rationale for focusing on exclusive territories, pursuing cases of price discrimination to court and other mechanisms that can be considered as raising the cost of segmentation, rather than a narrow focus on transport costs and trade barriers (if the goal of policy makers is to lower price differentials between markets).

## 4 Appendix A

Examine

$$\frac{d}{dt}(\Pi^S(x^S, X^S(t), t) - \Pi(x(t), t)).$$

The segmented market has

$$\begin{aligned} \frac{d}{dt}\Pi^S(x^S, X^S(t), t) &= \frac{\partial \Pi^S}{\partial x^S} \frac{dx^S}{dt} + \frac{\partial \Pi^S}{\partial X^S} \frac{dX^S}{dt} + \frac{\partial \Pi^S}{\partial t} \\ &= 0 \times 0 + 0 \times \frac{dX^S}{dt} - X^S \\ &= -X^S, \end{aligned}$$

where the first two terms are zero by the monopolist’s first order conditions, and the last term is  $-X^S$ , as easily seen by Eq. (2). The first order condition for  $X$  under segmentation, Eq. (7), gives

$$X^S = \frac{P - c - t}{-dP/dX^S},$$

so that

$$\frac{d}{dt}\Pi^S = \frac{P - c - t}{dP/dX^S}.$$

When markets are integrated, then

$$\Pi = \Pi(x^*(t), t).$$

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<sup>15</sup>And in some cases refusing to sell to foreign customers and pursuing retailers that do. See, for instance, the 1998 court case against Volkswagen for punishing Italian dealers selling to German and Austrian customers.

Thus, similarly,

$$\begin{aligned}\frac{d}{dt}\Pi(x^*(t), t) &= \frac{\partial \Pi}{\partial x^*} \frac{dx^*}{dt} + \frac{\partial \Pi}{\partial t} = 0 \times \frac{dx^*}{dt} + \frac{\partial \Pi}{\partial t} \\ &= \frac{\partial \Pi}{\partial t}.\end{aligned}$$

Now, by Eq. (3), we have

$$\begin{aligned}\frac{\partial \Pi}{\partial t} &= \frac{\partial}{\partial t}(p(x^*) - c)(x^* + X(p(x^*) + t)) \\ &= (p(x^*) - c) \frac{1}{P'(p(x^*) + t)}.\end{aligned}$$

This is negative, since  $P$  is downward sloping. Hence,

$$\frac{d}{dt}(\Pi^S - \Pi) < 0,$$

if

$$\frac{P(X^S) - c - t}{dP/dX^S} - \frac{p(x^*) - c}{dP/dX} < 0.$$

This is true if

$$\frac{p(x^*) - c}{P(X^S) - c - t} < \frac{dP/dX}{dP/dX^S}.$$

The left-hand side is less than 1 if  $p(x^*) - c < P(X^S) - c - t$ . Denote  $X^* = X(p(x^*) + t)$ . A binding LOP constraint implies that  $p(x^*) - P(X^*) + t = 0$ . By assumption  $P(X^S) > P(X^*)$  (and thus  $X^S < X^*$ ). This, in turn, implies  $p(x^*) - P(X^S) + t < 0$ , and hence,  $p(x^*) - c < P(X^S) - c - t$ , and the left-hand side is thus less than 1. Hence, we have shown the sufficient condition, that  $d(\Pi^S - \Pi)/dt < 0$ , if

$$\frac{dP/dX^*}{dP/dX^S} > 1. \quad (12)$$

Consider the left-hand side. This is a comparison of slopes of inverse demand functions. In particular, we know that

$$X^S < X \Rightarrow \frac{dP(X^*)/dX^*}{dP(X^S)/dX^S} > 1,$$

if  $P(X)$  is concave ( $= 1$  if  $P(X)$  is linear and  $< 1$  if  $P(X)$  is convex). Since Eq. (12) is a sufficient but not necessary condition, Proposition 2 then follows.

## 5 Appendix B

In the following sections we calculate profits, prices and quantities for each strategy profile.

### 5.1 Both segments

Assume that both firms segment, then the Home firm's problem is

$$\Pi_{S,S}^H = \max_{x,X} xp(x) + XP(X) - tX - K,$$

and the Foreign firm's problem is

$$\Pi_{S,S}^F = \max_{y,Y} yp(y) + YP(Y) - ty - K.$$

#### 5.1.1 Home country FOCs

The first-order conditions are

$$\frac{\partial}{\partial x} (xp(x) + XP(X) - tX - K) = 1 - 2x - \gamma y = 0,$$

which gives

$$x = \frac{1}{2} - \frac{1}{2}\gamma y,$$

and

$$\frac{\partial}{\partial y} (yp(y) + YP(Y) - ty - K) = 1 - 2y - \gamma x - t = 0,$$

which gives

$$y = \frac{1}{2} - \frac{1}{2}\gamma x - \frac{1}{2}t.$$

Combining the FOCs gives

$$\begin{aligned} y &= \frac{2(1-t) + \gamma(-1)}{4 - \gamma^2}, \\ x &= \frac{2 + \gamma t + \gamma(-1)}{4 - \gamma^2}. \end{aligned}$$

Thus, the Home firm's price in the Home country is  $p(x) = \frac{2-\gamma+\gamma t}{(2-\gamma)(\gamma+2)}$ , and the Foreign firm's price in the Home country is  $p(y) = \frac{2-\gamma+2t-\gamma^2 t}{(2-\gamma)(\gamma+2)}$ .

### 5.1.2 Foreign country FOCs

The first-order conditions are

$$\frac{\partial}{\partial X} (x(1-x-\gamma y) + X(A-X-\gamma Y) - tX - K) = A - 2X - \gamma Y - t = 0,$$

which gives

$$X = \frac{1}{2}A - \frac{1}{2}\gamma Y - \frac{1}{2}t,$$

and

$$\frac{\partial}{\partial Y} (y(1-y-\gamma x) + Y(A-Y-\gamma X) - ty - K) = A - 2Y - \gamma X = 0,$$

which gives

$$Y = \frac{1}{2}A - \frac{1}{2}\gamma X.$$

Combining the FOCs gives

$$\begin{aligned} X &= \frac{2A - \gamma A - 2t}{4 - \gamma^2}, \\ Y &= \frac{2A - \gamma A + \gamma t}{4 - \gamma^2}. \end{aligned}$$

Thus, the Home firm's price in the Foreign country is  $P(X) = \frac{2A - \gamma A - \gamma^2 t + 2t}{(2-\gamma)(\gamma+2)}$ , and the Foreign firm's price in the Foreign Country is  $P(Y) = \frac{2A - \gamma A + \gamma t}{(2-\gamma)(\gamma+2)}$ .

### 5.1.3 The profits

The profits to the firms are then

$$\begin{aligned} \Pi_{S,S}^H &= x(1-x-\gamma y) + X(A-X-\gamma Y) - tX - K \\ &= \frac{1}{2}t \frac{-2(A-1) - \gamma t - 2t}{(\gamma+2)(2-\gamma)} + \\ &\quad + \left( \frac{1}{4} \frac{(2A-t)^2}{(\gamma+2)^2} + \frac{1}{2} \frac{t^2}{(\gamma-2)^2} + \frac{1}{4} \frac{(t-2)^2}{(\gamma+2)^2} + \frac{1}{2} \frac{t^2}{2-\gamma} - K \right), \end{aligned}$$

and

$$\begin{aligned}
\Pi_{S,S}^F &= y(1 - y - \gamma x) + Y(A - Y - \gamma X) - ty - K \\
&= \frac{1}{2}t \frac{2(A - 1) - \gamma t - 2t}{(\gamma + 2)(2 - \gamma)} + \\
&\quad + \left( \frac{1}{4} \frac{(2A - t)^2}{(\gamma + 2)^2} + \frac{1}{2} \frac{t^2}{(\gamma - 2)^2} + \frac{1}{4} \frac{(t - 2)^2}{(\gamma + 2)^2} + \frac{1}{2} \frac{t^2}{2 - \gamma} - K \right).
\end{aligned}$$

Further, we have (compare with Remark 5)

$$\Pi_{S,S}^F - \Pi_{S,S}^H = 2t \frac{A - 1}{4 - \gamma^2} > 0.$$

## 5.2 Home segments / Foreign doesn't

Let us now assume that the Home firm segments and the Foreign firm does not segment. The Home firm's problem is

$$\Pi_{S,N}^H = \max_{x,X} xp(x) + XP(X) - tX - K,$$

and the Foreign firm's problem is

$$\begin{aligned}
\Pi_{S,N}^F &= \max_{y,Y} yp(y) + YP(Y) - ty, \\
\text{s.t. } &|p(y) - P(Y)| \leq t.
\end{aligned}$$

Remember that the Home firm has paid its Segmentation fee  $K$  and does not need to worry about fulfilling the constraint  $P(Y) - p(y) < t$  (although its products enter into this inequality). Thus, given our assumptions, the Home firm optimizes without restrictions and ignores the knowledge how the Foreign firm sets quantities. The Foreign firm optimizes under the restriction  $P(Y) - p(y) = t$ . This effectively decides one of the two quantities for the Foreign firm and the Foreign firm will thus only have one FOC to consider.

### 5.2.1 Home *firm* FOCs

The FOCs give two equations

$$\begin{aligned}
\frac{\partial}{\partial x} (x(1 - x - \gamma y) + X(A - X - \gamma Y) - tX - K) &= 1 - 2x - \gamma y = 0, \\
\frac{\partial}{\partial X} (x(1 - x - \gamma y) + X(A - X - \gamma Y) - tX - K) &= A - 2X - \gamma Y - t = 0.
\end{aligned}$$

### 5.2.2 Foreign firm FOCs

Given that the Foreign firm must uphold LOP on its market, the firm can only choose one of its quantities freely. The LOP constraint

$$t = P(Y) - p(y) = A - Y - \gamma X - 1 + y + \gamma x,$$

gives  $y$  as a function of  $Y$ ,

$$y = -A + Y + \gamma X + 1 - \gamma x + t.$$

The Foreign firm's problem is then

$$\max_Y y p(y) + Y P(Y) - ty,$$

given that  $y$  is a function of  $Y$ . The FOC is then (given restriction on  $y$ )

$$\frac{\partial}{\partial Y} (y(1 - y - \gamma x) + Y(A - Y - \gamma X) - ty) = 3A - 4Y - 3\gamma X - 3t - 1 + \gamma x = 0.$$

### 5.2.3 Equilibrium

There are now four equations that have to be fulfilled. Setting up the problem

$$\begin{pmatrix} -2 & 0 & -\gamma & 0 \\ 0 & -2 & 0 & -\gamma \\ -\gamma & \gamma & -1 & 1 \\ \gamma & -3\gamma & 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ X \\ y \\ Y \end{pmatrix} = \begin{pmatrix} -1 \\ t - A \\ A - 1 - t \\ 3t + 1 - 3A \end{pmatrix},$$

where the first row is the Home firm's FOC w.r.t.  $x$ , the second row w.r.t.  $X$ . Further, the third row is the constraint implied by LOP for the Foreign firm and finally, the fourth row is the Foreign firm's FOC w.r.t.  $Y$ . The determinant of the matrix is  $D = 2(\gamma - 2)(\gamma + 2)(-2 + \gamma^2)$ , which is non-zero for  $\gamma \in [0, 1]$ .

The solution to this linear equation system is

$$\begin{aligned} Y &= \frac{-A\gamma^2 + \gamma^2 t - 1 + 3A - 3t}{(\gamma + 2)(2 - \gamma^2)}, \\ y &= \frac{3 - \gamma^2 - A + t}{(\gamma + 2)(2 - \gamma^2)}, \\ X &= \frac{1 - A(2\gamma^2 + \gamma - 4) + \gamma + \gamma t + 2\gamma^2 t - 4t}{2(\gamma + 2)(2 - \gamma^2)}, \\ x &= \frac{1 - 2\gamma^2 - \gamma + 4 + \gamma A - \gamma t}{2(\gamma + 2)(2 - \gamma^2)}. \end{aligned}$$

Price differences are then

$$\begin{aligned} P(X) - p(x) &= t + \frac{(A - 1 - t)(1 - \gamma)}{2 - \gamma^2}, \\ P(Y) - p(y) &= t. \end{aligned}$$

#### 5.2.4 Profits

The Home firm's profit is

$$\begin{aligned} \Pi_{S,N}^H &= x(1 - x - \gamma y) + X(A - X - \gamma Y) - tX - K \\ &= \frac{1}{2} \frac{(A - t + 1)^2}{(\gamma + 2)^2} + \frac{1}{2} (A - 1 - t)^2 \frac{(\gamma - 1)^2}{(2 - \gamma^2)^2} - K. \end{aligned}$$

The Foreign firm's profit is

$$\begin{aligned} \Pi_{S,N}^F &= y(1 - y - \gamma x) + Y(A - Y - \gamma X) - ty \\ &= \frac{1}{2} \frac{(A - t + 1)^2}{(\gamma + 2)^2} + t(A - 1 - t) \frac{2 - \gamma}{2 - \gamma^2}. \end{aligned}$$

### 5.3 Foreign segments / Home doesn't

Let us now assume that the Foreign firm segments and the Home firm does not segment. The Home firm's problem is

$$\begin{aligned} \Pi_{N,S}^H &= \max_{x,X} xp(x) + XP(X) - c(x + X) - tX, \\ \text{s.t. } &|p(x) - P(X)| \leq t, \end{aligned}$$

and the Foreign firm's problem is

$$\Pi_{N,S}^F = \max_{y,Y} yp(y) + YP(Y) - c(y + Y) - ty - K.$$

Similar to the previous case, the Foreign firm optimizes without restrictions, while the Home firm optimizes under the restriction  $P(X) - p(x) = t$ . This effectively decides one of the two quantities for the Home firm and the Home firm will thus only have one FOC to consider.

#### 5.3.1 Home firm's FOCs

Given that the Home firm must uphold LOP on its market, the firm can only choose one of its quantities freely. The LOP constraint

$$t = P(X) - p(x) = A - X - \gamma Y - 1 + x + \gamma y,$$



gives  $x$  as a function of  $X$ ,

$$x = -A + X + \gamma Y + 1 - \gamma y + t.$$

The FOC is then (given that  $x$  is a function of  $X$ )

$$\frac{\partial}{\partial X} (x(1 - x - \gamma y) + X(A - X - \gamma Y) - tX) = 3A - 4X - 3\gamma Y - 3t - 1 + \gamma y = 0.$$

### 5.3.2 Foreign firm's FOCs

The problem is

$$\max_{y, Y} y p(y) + Y P(Y) - ty - K.$$

The FOCs are

$$\begin{aligned} \frac{\partial}{\partial Y} (y(1 - y - \gamma x) + Y(A - Y - \gamma X) - ty - K) &= A - 2Y - \gamma X = 0, \\ \frac{\partial}{\partial y} (y(1 - y - \gamma x) + Y(A - Y - \gamma X) - ty - K) &= 1 - 2y - \gamma x - t = 0. \end{aligned}$$

### 5.3.3 Equilibrium

There are now four equations that have to be fulfilled. Setting up the problem

$$\begin{pmatrix} -1 & 1 & -\gamma & \gamma \\ 0 & -4 & \gamma & -3\gamma \\ 0 & -\gamma & 0 & -2 \\ -\gamma & 0 & -2 & 0 \end{pmatrix} \begin{pmatrix} x \\ X \\ y \\ Y \end{pmatrix} = \begin{pmatrix} A - 1 - t \\ -3A + 3t + 1 \\ -A \\ t - 1 \end{pmatrix},$$

where the first row is the LOP constraint for the Home firm and the second row is the FOC w.r.t.  $X$  for the Home firm. Further, the third row is the FOC w.r.t.  $Y$  and finally, the fourth row is the FOC w.r.t.  $y$  for the Foreign firm. The determinant of the matrix is  $D = -2(\gamma - 2)(\gamma + 2)(-2 + \gamma^2)$ , which is non-zero for  $\gamma \in [0, 1]$ .

The solution to this linear equation system is

$$\begin{aligned} x &= \frac{3 - \gamma^2 - A + 2\gamma t + t + \gamma^2 t}{(\gamma + 2)(2 - \gamma^2)}, \\ X &= \frac{3A - 2\gamma t - \gamma^2 A - 1 - 3t}{(\gamma + 2)(2 - \gamma^2)}, \\ y &= \frac{1}{2} \frac{4 - \gamma + \gamma A - 3\gamma t - 2\gamma^2 - 4t}{(\gamma + 2)(2 - \gamma^2)}, \\ Y &= \frac{1}{2} \frac{4A - 2\gamma^2 A - \gamma A + 2\gamma^2 t + \gamma + 3\gamma t}{(\gamma + 2)(2 - \gamma^2)}. \end{aligned}$$

Price differences are then

$$\begin{aligned} P(Y) - p(y) &= t \frac{\gamma^2 + \gamma - 1}{2 - \gamma^2} + \frac{(A - 1)(1 - \gamma)}{2 - \gamma^2}, \\ P(X) - p(x) &= t. \end{aligned}$$

### 5.3.4 Profits

The Home firm's profit is

$$\begin{aligned} \Pi_{N,S}^H &= x(1 - x - \gamma y) + X(A - X - \gamma Y) - tX \\ &= \frac{1}{2} \frac{(t - A - 1)^2}{(\gamma + 2)^2}. \end{aligned}$$

The Foreign firm's profit is

$$\begin{aligned} \Pi_{N,S}^F &= y(1 - y - \gamma x) + Y(A - Y - \gamma X) - ty - K \\ &= \frac{1}{2} \frac{(t - A - 1)^2}{(\gamma + 2)^2} + \frac{1}{2} \frac{((A - 1)(1 - \gamma) + \gamma t + t)^2}{(2 - \gamma^2)^2} - K. \end{aligned}$$

## 5.4 No Segmentation

Let us now assume that no firm segments. The Home firm's problem is

$$\begin{aligned} \Pi_{N,N}^H &= \max_{x,X} xp(x) + XP(X) - c(x + X) - tX, \\ \text{s.t. } &|p(x) - P(X)| \leq t. \end{aligned}$$

and the Foreign firm's problem is

$$\begin{aligned} \Pi_{N,N}^F &= \max_{y,Y} yp(y) + YP(Y) - c(y + Y) - ty, \\ \text{s.t. } &|p(y) - P(Y)| \leq t. \end{aligned}$$

### 5.4.1 FOCs

Here we can take the relevant constraints and first-order conditions from the above sections where firms segment. From the section on the case Home segments / Foreign doesn't, we have the constraint

$$y = -A + Y + \gamma X + 1 - \gamma x + t,$$

and FOC is then

$$\frac{\partial}{\partial Y} (y(1 - y - \gamma x) + Y(A - Y - \gamma X) - ty) = 3A - 4Y - 3\gamma X - 3t - 1 + \gamma x = 0.$$

Further, from the section on the case Foreign segments / Home doesn't, we have the constraint

$$x = -A + X + \gamma Y + 1 - \gamma y + t,$$

and the FOC is then

$$\frac{\partial}{\partial X} (x(1 - x - \gamma y) + X(A - X - \gamma Y) - tX) = 3A - 4X - 3\gamma Y - 3t - 1 + \gamma y = 0.$$

### 5.4.2 Equilibrium

There are now four equations that have to be fulfilled,

$$\begin{aligned} 0 &= -\gamma x + \gamma X - y + Y - A + 1 + t : y - \text{constraint} \\ 0 &= \gamma x - 3\gamma X + 0 * y - 4Y + 3A - 3t - 1 : Y - \text{FOC} \\ 0 &= -x + X - \gamma y + \gamma Y - A + 1 + t : x - \text{constraint} \\ 0 &= 0 * x - 4X + \gamma y - 3\gamma Y + 3A - 3t - 1 : X - \text{FOC} \end{aligned}$$

As a linear system

$$\begin{pmatrix} -\gamma & \gamma & -1 & 1 \\ \gamma & -3\gamma & 0 & -4 \\ -1 & 1 & -\gamma & \gamma \\ 0 & -4 & \gamma & -3\gamma \end{pmatrix} \begin{pmatrix} x \\ X \\ y \\ Y \end{pmatrix} = \begin{pmatrix} A - 1 - t \\ -3A + 3t + 1 \\ A - 1 - t \\ -3A + 3t + 1 \end{pmatrix}.$$

The determinant of the matrix is  $D = 4(\gamma - 1)(\gamma - 2)(\gamma + 2)(\gamma + 1)$ , which is non-zero for  $\gamma \in [0, 1)$ . For  $\gamma = 1$ , there is a problem, since there is only one single LOP constraint and the firms then both optimize under the same constraint.

The solution to this linear equation system for  $\gamma \neq 1$  is

$$\begin{aligned} Y &= \frac{1}{2} \frac{2\gamma A - 2\gamma t - 3t - 1 + 3A}{(\gamma + 2)(\gamma + 1)}, \\ x &= \frac{1}{2} \frac{2\gamma + 3 - A + t}{(\gamma + 2)(\gamma + 1)}, \\ y &= \frac{1}{2} \frac{2\gamma + 3 - A + t}{(\gamma + 2)(\gamma + 1)}, \\ X &= \frac{1}{2} \frac{2\gamma A - 2\gamma t - 3t - 1 + 3A}{(\gamma + 2)(\gamma + 1)}. \end{aligned}$$

Price differences are then

$$\begin{aligned} P(Y) - p(y) &= t, \\ P(X) - p(x) &= t, \end{aligned}$$

as we would expect.

### 5.4.3 The profits

The Home firm's profit is

$$\begin{aligned}\Pi_{N,N}^H &= x(1-x-\gamma y) + X(A-X-\gamma Y) - tX \\ &= \frac{1}{2} \frac{(A-t+1)^2}{(\gamma+2)^2},\end{aligned}$$

and the Foreign firm's profit is

$$\begin{aligned}\Pi_{N,N}^F &= y(1-y-\gamma x) + Y(A-Y-\gamma X) - ty \\ &= \frac{1}{2} \frac{(A-t+1)^2}{(\gamma+2)^2} + t \frac{A-1-t}{\gamma+1}.\end{aligned}$$

Then,  $\Pi_{N,N}^F - \Pi_{N,N}^H = t \frac{(A-1-t)}{\gamma+1} > 0$ .

### 5.5 Derivation of $\Delta$ -functions

Given the profits derived in the previous sections we have

$$\begin{aligned}\Delta_S^H(A, \gamma, t) &= \Pi_{S,S}^H(A, t, K) - \Pi_{N,S}^H(A, t) + K \\ &= \frac{1}{4} t \frac{A-1}{-2+\gamma} + \frac{1}{2} \frac{t^2}{(-2+\gamma)^2} - \frac{1}{4} t \frac{A-1}{\gamma+2} + \frac{1}{2} \frac{A^2+1-2A}{(\gamma+2)^2},\end{aligned}$$

$$\begin{aligned}\Delta_N^H(A, \gamma, t) &= \Pi_{S,N}^H(A, t, K) - \Pi_{N,N}^H(A, t) + K \\ &= \frac{1}{2} \frac{(\gamma-1)^2}{(\gamma^2-2)^2} A^2 - (\gamma-1)^2 \frac{t+1}{(\gamma^2-2)^2} A + \frac{1}{2} (t+1)^2 \frac{(\gamma-1)^2}{(\gamma^2-2)^2}\end{aligned}$$

$$\begin{aligned}\Delta_S^F(A, \gamma, t) &= \Pi_{S,S}^F(A, t, K) - \Pi_{S,N}^F(A, t) + K \\ &= -\frac{1}{4} t \frac{A-1}{-2+\gamma} + \frac{1}{2} \frac{t^2}{(-2+\gamma)^2} + \frac{1}{4} t \frac{A-1}{\gamma+2} + \frac{1}{2} \frac{A^2+1-2A}{(\gamma+2)^2} - \\ &\quad - t \frac{-2A+2+2t+\gamma A-\gamma-\gamma t}{\gamma^2-2}\end{aligned}$$

$$\begin{aligned}\Delta_N^F(A, \gamma, t) &= \Pi_{N,S}^F(A, t, K) - \Pi_{N,N}^F(A, t) + K \\ &= \frac{1}{2} \frac{(A-1-t)^2}{\gamma^2-2} - t \frac{A-1-t}{\gamma+1} - \\ &\quad - \frac{1-2(t-1+A)(t+1-A)\gamma-3(A-1)^2-t(-2A+2+3t)}{2(\gamma^2-2)^2}\end{aligned}$$

## 5.6 The region of interest

In Eq. 10, we state that, in terms of our model, none of the conditions  $P(Y) - p(y) < t$  and  $P(X) - p(x) < t$ , hold if

$$t < (A - 1) \frac{2 - \gamma}{(\gamma + 2)(3 - 2\gamma)} \approx \frac{A - 1}{3}.$$

This constraint is derived by looking at the equilibrium prices

$$\begin{aligned} P(X) &= \frac{2A - \gamma A - \gamma^2 t + 2t}{(2 - \gamma)(\gamma + 2)}, \\ p(x) &= \frac{2 - \gamma + \gamma t}{(2 - \gamma)(\gamma + 2)}, \\ P(Y) &= \frac{2A - \gamma A + \gamma t}{(2 - \gamma)(\gamma + 2)}, \\ p(y) &= \frac{2 - \gamma + 2t - \gamma^2 t}{(2 - \gamma)(\gamma + 2)}, \end{aligned}$$

when both firms segment market. If both firms segment, then both firms have the incentive to set prices such that price differences are high. Now,

$$P(X) - p(x) = \frac{1 - \gamma}{2 - \gamma} t + \frac{A - 1}{\gamma + 2},$$

and

$$P(Y) - p(y) = -\frac{1 - \gamma}{2 - \gamma} t + \frac{A - 1}{\gamma + 2}.$$

Then,  $P(X) - p(x) > t$ , if

$$t < (A - 1) \frac{2 - \gamma}{\gamma + 2},$$

and  $P(Y) - p(y) > t$ , if

$$t < (A - 1) \frac{2 - \gamma}{(\gamma + 2)(3 - 2\gamma)},$$

which is the tightest condition, since

$$(A - 1) \frac{2 - \gamma}{(\gamma + 2)(3 - 2\gamma)} < (A - 1) \frac{2 - \gamma}{\gamma + 2}.$$

Thus, if

$$t < (A - 1) \frac{2 - \gamma}{(\gamma + 2)(3 - 2\gamma)},$$

then  $P(Y) - p(y) > t$ , and  $P(X) - p(x) > t$ .

## 6 Appendix C

To prove  $|P(X) - p(x)| > |P(Y) - p(y)|$ , it is sufficient to prove  $P(X) > P(Y)$  and  $p(y) > p(x)$ . We will prove  $p(y) > p(x)$  for  $c = 0$ , since the remaining case,  $P(X) > P(Y)$ , is similar. Now, the first-order condition for the Foreign firm gives  $p(y) = t - p'(y)y$ , so that  $y = y(t)$ . Let  $f(t) = p(y(t)) - p(x^*)$ , where  $y(t)$  and  $x^*$  are the optimal quantities given by the respective first-order condition. Note that  $y(0) = x^*$ . Taking the derivative of  $p(y(t))$ , as given by the Foreign firm's FOC, gives  $y'(t) = 1/SOC < 0$ . Thus,  $f'(t) = p'(y(t))y'(t) > 0$ , since the demand curve is downward sloping. Then, since  $f$  is an increasing function with  $f(0) = 0$ , we know that  $f(t) > 0$  for all  $t > 0$ . Hence,  $p(y) > p(x)$ .

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## CHAPTER 4

### **Variability and Average profits – Does Oi's Result Generalize?**



# Variability and Average profits – Does Oi’s Result Generalize?\*

Richard Friberg and Kaj Martensen

## Abstract

The average profits of a price taker are increasing in the variability of the output price (Oi, 1961). We show that, for the same reason, the average profits of the price taker are increasing in the variability of the price of inputs. We then proceed to establish that the same holds for a firm with a downward sloping demand curve. Unless the inverse demand curve of the firm with market power is very convex, the profit function of the price taker forms an upper limit for the convexity of profit (assuming constant a curvature of costs).

## 1 Introduction

Dealing with variability in underlying conditions such as inflation, interest rates, real exchange rates and commodity prices is crucial to many lines of business. Variability presents problems for risk averse firms but also offers opportunities for increasing the average profits. The seminal article by Walter Oi (1961) demonstrates that the average profits of a price taker are increasing in the variability of the output price. A natural question is: Does Oi’s result generalize to firms facing downward sloping demand curves?

In one sense, the answer is trivially no, since price is an exogenous stochastic variable only to the price taker.<sup>1</sup> In this paper, we first establish that the profits of the price taker are strictly convex in the variability of input costs. Is this also true for firms facing a downward sloping demand curve? It could be conjectured that the ability of the price taker to expand and contract quantities without affecting the price is central for being flexible enough to benefit from variability. Despite the prominent place given to Oi’s (1961) result in the literature (see, for instance, Varian, 1992) and the importance of risk for firms, it has, to the best of our knowledge, not been proved whether these results generalize.

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<sup>1</sup>There is one more sense in which the answer is trivially no – all else equal, profits will be concave in changes in the price for a price setting firm, otherwise the second order condition for profit maximization would not hold.

In the next section, we examine how average profits depend on a shock to costs. The average profits of both a price-taking firm and a firm faced with downward sloping demand are shown to be increasing in the variability of the cost shock. We proceed to establish the conditions under which the profits of the price taker have a greater degree of convexity in the cost shock than those of a firm with market power (under some additional conditions).

## 2 What is the curvature of profits?

The timing used throughout the paper is as follows: the firm first observes the cost or price shift and then decides what quantity to produce. We confine the attention to internal solutions and assume cost and demand functions (for the firm with market power) to be twice continuously differentiable.

### 2.1 The price taker

Let us first establish Oi's result as a reference point. Let a variable  $\theta' > 0$  affect the market price faced by a price-taking firm. Using Oi's notation, denote the market price in a competitive industry with  $P$  and the quantity of the price taker with  $x$ . Production costs are given by  $c(x)$  with  $c_x > 0$ ,  $c_{xx} > 0$ ; subindexes denote partial derivatives throughout. Strictly convex marginal costs are necessary to ensure existence of an optimal quantity. Thus, the profit maximization problem of the firm is Eq. (1), which yields an optimal quantity, denoted  $x^*$

$$\Pi(\theta') = \max_{x>0} [\theta' P x - c(x)]. \quad (1)$$

**Proposition 1** (*Oi, 1961*) *Let a variable  $\theta' > 0$  affect the market price facing a price taking firm. Average profits are increasing in the variability of  $\theta'$ .*

**Proof.** Average profits are increasing in the variability of  $\theta'$  if and only if  $d^2\Pi(x^*(\theta'))/d\theta'^2 > 0$ . Twice totally differentiating profits yield

$$\frac{d^2\Pi(x^*(\theta'))}{d\theta'^2} = \Pi_{xx} \left( \frac{dx^*}{d\theta'} \right)^2 + 2\Pi_{x\theta'} \frac{dx^*}{d\theta'} + \Pi_x \frac{d^2x^*}{d\theta'^2} + \Pi_{\theta'\theta'}.$$

Totally differentiating the first-order condition establishes that  $dx^*/d\theta' = P/c_{xx}$ . Further, use  $\Pi_{x\theta'} = P$ ,  $\Pi_{\theta'\theta'} = 0$ ,  $\Pi_x = 0$  around the optimum and  $c_{xx} > 0$  to establish that

$$\frac{d^2\Pi(x^*(\theta'))}{d\theta'^2} = \frac{P^2}{c_{xx}} > 0.$$

■

Oi (1961) established the result graphically.<sup>2</sup>

Now, denote the profits of the competitive firm under cost variability by  $\Pi^c$  and let the variable  $\theta > 0$  affect the vector of input prices  $w$  such that the maximization problem faced by the firm is

$$\Pi^c(\theta) = \max_{x>0} [Px - c(x, \theta w)].$$

Since the cost function is homogeneous of degree 1 in input costs, given competitively supplied inputs, we can write the maximization problem as

$$\Pi^c(\theta) = \max_{x>0} [Px - \theta c(x, w)].$$

**Proposition 2** *Let a variable  $\theta > 0$  affect the price of inputs for the price-taking firm. Average profits are increasing in the variability of  $\theta$ .*

**Proof.** The proof is analogous to the proof of Proposition 1,

$$\frac{d^2\Pi^c(x^*(\theta))}{d\theta^2} = \Pi_{xx}^c \left( \frac{dx^*}{d\theta} \right)^2 + 2\Pi_{x\theta}^c \frac{dx^*}{d\theta} + \Pi_x^c \frac{dx^{*2}}{d\theta^2} + \Pi_{\theta\theta}^c.$$

Totally differentiating the first-order condition establishes that  $dx^*/d\theta = c_x/\Pi_{xx}^c$ . Use  $\Pi_{x\theta}^c = -c_x$ ,  $\Pi_{\theta\theta}^c = 0$ ,  $\Pi_x^c = 0$  around the optimum and that  $\Pi_{xx}^c < 0$  by the second-order conditions for profit maximization to establish that

$$\frac{d^2\Pi^c(x^*(\theta))}{d\theta^2} = -\frac{(c_x)^2}{\Pi_{xx}^c} > 0.$$

■

Shephard (1970), among others, provides a proof that with competitively supplied inputs, the cost function is concave in input costs. Since strict concavity is not shown, this is a necessary but not a sufficient condition for the Proposition to hold.

Profits are strictly convex in the cost of inputs for the same reason as they are strictly convex in the price of output. The first-order conditions for profit maximization are  $\theta'P = c_x$  and  $P = \theta c_x$ , respectively. In both cases, quantity is set optimally – by making the best of favorable conditions and cutting back in less favorable ones, profits are increasing in variability.

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<sup>2</sup>A formal proof for the profit function of a price taker being convex in  $P$  is found in, for instance, Shepard (1970) and many graduate textbooks. Despite claims to the contrary by, for instance, Varian (1992), this only proves that average profits are non-decreasing in the variability of the price shock, not that they are increasing.

## 2.2 The firm with market power

Now, turn to the issue of how variability affects profits of firms facing downward sloping demand. Here, the case where there is no strategic interaction, no price discrimination and where the firm faces a non stochastic demand curve is considered. Denote the profits of this firm with  $\pi$ , quantity with  $q$ , costs with  $c(q, \theta w)$  and inverse demand with  $p(q)$ , where  $p_q < 0$ . The firm's maximization problem is

$$\pi(\theta) = \max_{q>0} [p(q)q - c(q, \theta w)],$$

with the first-order condition (using homogeneity of degree 1 in costs)

$$p_q q + p - \theta c_q = 0.$$

**Proposition 3** *Let a variable  $\theta > 0$  affect the price of inputs for a firm with (stable) downward sloping demand. Average profits of the firm are increasing in the variability of  $\theta$ .*

**Proof.** The proof is analogous to the proof of Proposition 1,

$$\frac{d^2 \pi(q^*(\theta))}{d\theta^2} = \pi_{qq} \left( \frac{dq^*}{d\theta} \right)^2 + 2\pi_{q\theta} \frac{dq^*}{d\theta} + \pi_q \frac{d^2 q^*}{d\theta^2} + \pi_{\theta\theta}.$$

Totally differentiating the first-order condition yields  $dq^*/d\theta = c_q/\pi_{qq}$ . Further, use that  $\pi_{q\theta} = -c_q$ ,  $\pi_{\theta\theta} = 0$ ,  $\pi_q = 0$  around optimum and that  $\pi_{qq} < 0$  by second-order conditions for profit maximization to establish that

$$\frac{d^2 \pi(q^*(\theta))}{d\theta^2} = -\frac{(c_q)^2}{\pi_{qq}} > 0.$$

■

Note the very general nature of the result – no specific functional forms are assumed. For a firm faced with cost shocks and a stable downward sloping demand curve, average profits will be increasing in the variability of the cost of inputs. In this sense, the result of Oi (1961) generalizes – not only price takers benefit from variability. Our initial interest was motivated by the feeling that although it is possible that all firms may improve their average profits as a result of variability, the ability of a price taker to change quantity without affecting the price should put it in a superior position for benefiting from variability. Therefore, we proceed with a comparison of the curvature of profits.

## 2.3 Which profit function has the greater curvature?

Is the price taker more “flexible” than the firm with market power and can it therefore achieve higher average profits? By choosing for example the degree of differentiation, a

firm influences the way the price will be affected by quantity changes – and thus chooses how profits will respond to shocks in the underlying environment.<sup>3</sup>

For a comparison, assume that both the price taker and the firm with downward sloping demand have the same cost function and that the third derivative of the cost function is zero.<sup>4</sup> The profit of the competitive firm is more convex than the firm with market power if

$$\frac{d^2\Pi^c}{d\theta^2} > \frac{d^2\pi}{d\theta^2}, \quad (2)$$

where the expressions are evaluated at their optimal levels of  $x$  and  $q$ , respectively. Then, for Eq. (2) to hold it must be true that

$$\frac{-(c_x)^2}{\Pi_{xx}^c} > \frac{-(c_q)^2}{\pi_{qq}}.$$

Inserting the expressions for the second-order conditions and marginal costs yields

$$\frac{p_{qq}q + 2p_q - \theta c_{qq}}{-\theta c_{xx}} > \left(\frac{c_q}{c_x}\right)^2. \quad (3)$$

By standard theory  $q < x$  so that  $c_q < c_x$  by convexity and consequently, the right-hand side of Eq. (3) is less than 1. Use that  $c_{qq} = c_{xx}$  and rewrite to establish that

$$\frac{d^2\Pi^c}{d\theta^2} > \frac{d^2\pi}{d\theta^2}$$

$\Leftrightarrow$

$$\frac{p_{qq}q + 2p_q}{-\theta c_{qq}} > \left(\frac{c_q}{c_x}\right)^2 - 1. \quad (4)$$

The ratio between the curvature of revenue and the curvature of costs is on the left hand side of Eq. (4), while the right-hand side is negative. Clearly, this condition is satisfied for a concave demand function ( $p_{qq} \leq 0$ ). It will also hold if  $p_{qq}$  is positive but not too large. It will not hold for sufficiently large degree of convexity, however, since by the second-order condition,  $p_{qq}q + 2p_q$  is allowed to be as great as  $\theta c_{qq}$  which reverses the above inequality.

**Proposition 4** *A competitive firm gains more from variability in the price of inputs than a firm with market power, as long as demand is concave or not too convex (assuming identical cost functions). For sufficient convexity of demand, the firm with market power gains more.*

<sup>3</sup>Flexibility is thus also of importance on the demand side. Most attention on flexibility has focused on costs, see Carlsson (1989) for a survey or Athey and Schmutzler (1995), for a recent contribution.

<sup>4</sup>The last assumption amounts to making a second-order Taylor approximation of the cost function.

**Proof.** The second-order condition requires  $p_{qq}q < \theta c_{qq} - 2p_q$  but (4) requires  $p_{qq}q < \theta c_{qq} \left(1 - \left(\frac{c_q}{c_x}\right)^2\right) - 2p_q$ , which is a tighter condition. Thus, when

$$p_{qq}q \in \left(-\infty, \theta c_{qq} \left(1 - \left(\frac{c_q}{c_x}\right)^2\right) - 2p_q\right),$$

the competitive firm gains more from variability and when

$$p_{qq}q \in \left[\theta c_{qq} \left(1 - \left(\frac{c_q}{c_x}\right)^2\right) - 2p_q, \theta c_{qq} - 2p_q\right),$$

the opposite holds. ■

Consider a firm faced with the choice of selling its product on a competitive world market or differentiating and selling as a local monopoly (with linear demand). For small enough  $\pi(\bar{\theta}) - \Pi^c(\bar{\theta}) > 0$  ( $\bar{\theta}$  denoting the average  $\theta$ ) and high enough variability, the average profits of the price taker will be larger than those of the monopolist.<sup>5</sup> A related result is found in Chang and Harrington (1996) who show that under a linear demand duopoly with constant marginal costs and asymmetric cost shocks, the degree of product differentiation maximizing industry profits is reduced by cost variability. The reason is precisely the above – the better substitutes the products are, the more quantity can be expanded after a beneficial shock. Proposition 4 shows that the results of Chang and Harrington depend on the assumption that demand is linear. To see the intuition why the curvature of demand matters, consider the case when the inverse demand curve is very convex. When quantity is expanded, the price effect is small, and thus, in a similar manner as the price taker, the firm is thus able to expand quantities to benefit from decreased costs without much affect on the price. Conversely, when costs rise and quantity is being cut down, this is associated with a large increase in the price that the monopolist obtains, if demand is very convex.

### 3 Discussion

It is worth emphasizing that in deriving the above results, we specified no specific demand or cost function, and thus, Oi's result does generalize to a considerable extent. In line with recent work on envelope theorems (see Milgrom and Roberts, 1996 or Milgrom and Segal, 2000), a topic for future research is to examine if the results generalize to settings where, for instance, output is not continuous and there is strategic interaction. Envelope theorems also provide an alternative way of proving the Propositions in this paper — under some assumptions it can be shown that the maximum value function of an unconstrained maximization problem is strictly more convex than that of a corresponding constrained maximization problem (see Silberberg, 1971). The simplicity of the present

<sup>5</sup>It deserves to be pointed out, though, that if there is free entry into the "price taking" industry, the expected profits will be 0. This is explored in Sheshinski and Drèze (1976).



proof and the straightforward manner in which it allows us to compare who benefits the most from variability (Proposition 4) serve as motivation for the path chosen. Let us also stress that we have not attempted to provide an analysis of whether society as a whole benefits from variability.<sup>6</sup> Finally, it is important to realize that financial markets do not make the current issue irrelevant. A financial hedge affects utility by creating an offsetting exposure, and not by affecting the curvature of the profit function.

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<sup>6</sup>Oi's paper did indeed spur several investigations of welfare implications of commodity price variability and price stabilizing schemes. See Samuelsson (1972) or Turnovski et al. (1984).



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