On Seasonality and Cointegration

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Preface

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Finally, I am grateful to Annika for love and support and to my parents, Sigrid and Per-Anders, who have always been there when needed.

List of Papers

This thesis is based on the following papers, which are referred to by their respecitve numerals.

1. Löf, M. and Lyhagen, J. (1999), 'Forecasting performance of seasonal cointegration models', *International Journal of Forecasting,* forthcoming.

2. Löf, M. and Franses, P. H. (2000), 'On forecasting cointegrated seasonal time series', *International Journal of Forecasting,* forthcoming.

3. Löf, M. (2001), 'Size and power of the likelihood ratio test for seasonal cointegration in small samples: A Monte Carlo study', *Working Paper Series in Economics and Finance No. 439,* Stockholm School of Economics, 2001.

4. Lyhagen, J. and Löf, M. (2000), 'On seasonal error correction when the processes include different numbers of unit roots', *Working Paper Series in Economics and Finance No. 418,* Stockholm School of Economics, 2000.

Part I

Thesis Summary

Chapter 1

Introduction

In the last decades there has been a renewed interest in modelling seasonally nonadjusted time series. Many econometric studies indicate that it may be worthwhile to study seasonal fluctuations in their own right. The reason for this is the fact that the seasonal pattern dominates the variation in many somehow detrended quarterly or monthly observed economic time series (see for example Miron (1996), where growth rates of the time series are regressed on seasonal dummies). Also, seasonal fluctuations may convey information on the behaviour of economic agents. Recent documentations should provide a motivation to consider time series models that explicitly incorporate a description of seasonal variation. For example, Wallis (1974) shows that the use of seasonally adjusted data may distort the actual underlying economic relation between variables, and Ericsson *et al.* (1994) point out that seasonally adjusted data may alter weak exogeneity properties in multivariate long-run modelling. Ghysels *et al.* (1995) find evidence that the much used seasonal adjustment method X-11 may generate adjusted time series with nonlinear properties, which the original time series did not possess. In Ghysels and Perron (1996) it is shown that the X-11 procedure may disguise (smooth away) structural instability, and thus effect tests for structural change. Furthermore, Maravall (1994) show that seasonally adjusted time series can be described by non-invertible moving average processes, and this in turn may lead to difficulties when constructing, for example, finite order *Vector Auto Regressive* (VAR) models.

The first section in this chapter present univariate seasonal unit root and periodic models, which are tools to describe changing seasonality. These models are used or mentioned in the included papers but are not presented thoroughly there. As this thesis focus on seasonal and periodic cointegration models the second section will give a brief introduction to these multivariate approaches. The last chapter in this part provides a summary of the included papers.

1.1 Changing seasonal patterns

Substantial empirical evidence makes it safe to state that a large part of the seasonal movement in many economic time series is far from constant over time, see for example Öller (1978). Still applied researchers often assume a deterministic seasonal pattern, trying to capture these variations using dummies. Such an approach will lead to dynamically misspecified models if the constancy assumption is not supported by data. A number of univariate and multivariate time series procedures have been introduced that, in one way or another, focus on testing for, or modelling, nonstationary seasonal variation.

Evidence is often found that seasonal unit roots are present in economic time series with a changing seasonal pattern over time. Let y_t be a quarterly observed time series. Further, let L be the lag operator, i.e. $L^d y_t = y_{t-d}$, and let Δ_d denote the differencing operator, i.e. $\Delta_d y_t = (1 - L^d)y_t$. If a seasonal difference filter, Δ_4, is required to transform y_t to yield stationarity, then this time series is said to be seasonally integrated, i.e. $y_t \sim \mathrm{SI}(1)$. The Δ_4 filter implies four unit roots. This can be seen from the following factorization:

$$\Delta_4 = (1 - L^4) = (1 - L)(1 + L)(1 + iL)(1 - iL), \tag{1.1}$$

where $i = \sqrt{-1}$. The $(1 - L)$ part corresponds to the zero frequency and a non-seasonal unit root, while the three roots of $(1 + L)(1 + iL)(1 - iL)$, namely -1 and $\pm\ i$, represent the seasonal frequencies. The -1 root is often referred to as the *biannual* root and the two complex conjugate roots as the *annual* roots. Hylleberg *et al.* (1990) [HEGY] propose a test for unit roots at seasonal frequencies. The test procedure investigates whether the seasonal difference filter is the appropriate one, as compared to other nested filters. The test uses the following auxiliary regression:

$$\Delta_4 y_t = \mu_t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{4,t-1} + \varepsilon_t, \tag{1.2}$$

which is estimated using OLS, and where the following filters remove all unit roots except those at the zero, biannual, and annual frequencies, respectively.

$$\begin{aligned} y_{1,t} &= (1 + L + L^2 + L^3)y_t, \\ y_{2,t} &= -(1 - L + L^2 - L^3)y_t, \\ y_{3,t} &= -(L - L^3)y_t, \\ y_{4,t} &= -(1 - L^2)y_t. \end{aligned} \tag{1.3}$$

The parameter μ_t in (1.2) can include deterministic components such as a constant, a linear trend, and seasonal dummies. To get residuals that can be regarded as generated by a white noise process, the regression (1.2) is often augmented by adding lags of the dependent variable. Unit roots at the zero and biannual frequencies imply $\pi_1 = 0$ and $\pi_2 = 0$, respectively, which is tested against the alternative that $\pi_i < 0$, $i = 1, 2$, using one-sided t-tests. If the F-test for the hypothesis $\pi_3 = \pi_4 = 0$ cannot be rejected there is evidence of two complex conjugate unit roots at the annual frequency. Simulated critical values of the t-statistics for π_1 and π_2 and the F-statistic for $\pi_3 = \pi_4 = 0$, are tabulated in HEGY. The procedure above, designed for quarterly data, has been extended to monthly data, see for example Beaulieu and Miron (1993). See also Franses and Taylor (1997), where multiple non-seasonal and seasonal unit roots are considered.

The HEGY testing approach has been applied in many studies by now, and one main finding is that many macroeconomic time series seem to contain seasonal unit roots, but not necessarily all the roots implied by $1 - L^s$, where s is the number of seasons. It should be mentioned that the HEGY approach, like most other tests for stochastic non-stationarity, has low power against alternatives close to the null hypothesis. This implies that, in practice, one may find evidence of too many roots using this method. It has also been found that the HEGY-test procedure is not very robust to structural mean shifts, see Franses and Vogelsang (1995). One notable observation by Clements and Hendry (1997) is that the seasonal differencing filter (Δ_4) may generate more accurate forecasts than models where a smaller number of roots (suggested by the HEGY-test) are imposed (see also Osborn *et al.* (1999) where similar results were found). Other methods by which one can test for seasonal unit roots have been presented by, for example, Canova and Hansen (1995), Dickey *et al.* (1984), and Osborn *et al.* (1988). See also a procedure below, proposed by Franses (1994), which considers seasonal integration in a so called *periodic* model framework.

Both the seasonal adjustment procedures and the HEGY approach rest on the assumption that it is possible to separate the seasonal component from other (non-seasonal) variations in economic time series. However, recent work shows that it is not always the case that seasonal and other sources of variation, including what is popularly called "the business cycle" variation, can be separated in an obvious way (see for example Ooms and Franses (1997) who show that the use of seasonally adjusted data may lead to biased information about changes in unemployment). Given that the seasonal and non-seasonal components can be correlated over time it seems useful to consider models in which the structural parameters vary according to season. A class of models which allows for this is *periodic autoregressive* (PAR) time series models, and especially periodically integrated time series models. A PAR model of order p for a quarterly observed time series y_t, where $t = 1, 2, ..., n$, may be written as

$$y_t = \sum_{s=1}^{4} \mu_s D_{s,t} + \sum_{i=1}^{p}\sum_{s=1}^{4} \phi_{is} D_{s,t} y_{t-i} + \varepsilon_t, \tag{1.4}$$

where $D_{1,t}, ..., D_{4,t}$ are seasonal dummy variables and ε_t is a standard white noise process. The autoregressive parameters $\phi_{1s}, ..., \phi_{ps}$ vary with the season, s. When studying the unit root concept in periodic autoregressions it is convenient to rewrite the PAR model for y_t in a *vector of quarters* (VQ) representation. Consider below a PAR model of order 1, PAR(1), written in VQ format:

$$\Phi_0 Y_T = \Phi_1 Y_{T-1} + \varepsilon_T. \tag{1.5}$$

Y_T is a (4×1) vector process $(Y_{1,T}, Y_{2,T}, Y_{3,T}, Y_{4,T})'$, where $Y_{s,T}$ is the observation on y_t in season s in year T. The quarterly time series y_t is observed for N years, i.e. $n/4 = N$. $T = 1, 2, ..., N$, ε_T is a (4×1) vector process of the form $(\varepsilon_{1,T}, \varepsilon_{2,T}, \varepsilon_{3,T}, \varepsilon_{4,T})'$ and Φ_0, Φ_1 are (4×4) parameter matrices defined as

$$\Phi_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{bmatrix}, \quad \Phi_1 = \begin{bmatrix} 0 & 0 & 0 & \phi_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{1.6}$$

Unit roots in periodic autoregressive models imply certain cointegration relationships between the variables in the vector process Y_T. The rank (r) of the Π matrix

$$\Pi = (\Phi_0^{-1}\Phi_1 - I_4) = \begin{bmatrix} -1 & 0 & 0 & \phi_1 \\ 0 & -1 & 0 & \phi_2\phi_1 \\ 0 & 0 & -1 & \phi_2\phi_3\phi_1 \\ 0 & 0 & 0 & \phi_2\phi_3\phi_4\phi_1 - 1 \end{bmatrix} \tag{1.7}$$

in the following rewritten form of (1.5):

$$\Delta Y_T = \Pi Y_{T-1} + \Phi_0^{-1}\varepsilon_T, \tag{1.8}$$

is three, when $\phi_1\phi_2\phi_3\phi_4 = 1$, if it is assumed that each $Y_{s,T}$ is at most $I(1)$.

This implies that $\Pi = \alpha\beta'$ and hence that ΠY_{T-1} can be written as follows:

$$\alpha\beta' Y_{T-1} = \begin{bmatrix} \frac{1}{\phi_2} & \frac{1}{\phi_2\phi_3} & \phi_1 \\ 0 & \frac{1}{\phi_3} & \phi_2\phi_1 \\ 0 & 0 & \phi_2\phi_3\phi_1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{bmatrix} \begin{bmatrix} Y_{1,T-1} \\ Y_{2,T-1} \\ Y_{3,T-1} \\ Y_{4,T-1} \end{bmatrix}. \tag{1.9}$$

It can be seen from (1.9) that the following relations are stationary:

$$\begin{aligned} &Y_{2,T} - \phi_2 Y_{1,T}, \\ &Y_{3,T} - \phi_3 Y_{2,T}, \\ &Y_{4,T} - \phi_4 Y_{3,T}. \end{aligned} \tag{1.10}$$

Given these relations it follows that $Y_{4,T} - \phi_4\phi_3\phi_2 Y_{1,T}$, and hence that $Y_{1,T} - \phi_1 Y_{4,T}$ with $\phi_1 = 1/\phi_4\phi_3\phi_2$ are stationary variables. By using the differencing filter $(1 - \phi_s L)$, for $s = 1, 2, 3, 4$, the time series can thus be transformed into a stationary process.

Franses (1994) develops a method to test whether a time series contains nonseasonal and seasonal unit roots or whether it seems to be periodically integrated. This method uses a maximum-likelihood cointegration procedure in a model like (1.8). The fourth difference filter implies that $r = 0$ in Π. No difference filter is needed for y_t if $r = 4$. Finally, if $0 < r < 4$ the matrix Π can be written as $\alpha\beta'$. For $r = 3$ the following restrictions on the rows of β' in (1.9) can be tested:

$$H_0^1 : \beta' = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad H_0^2 : \beta' = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{1.11}$$

If H_0^1 in (1.11) cannot be rejected there is evidence of a zero frequency (nonseasonal) unit root in the time series, while if H_0^2 is supported, y_t contains a biannual root. On the other hand, if both these hypotheses about the cointegrating vectors are rejected there is evidence that the time series is periodically integrated, and the nature of the periodically varying parameters (ϕ_s) can also be tested in this framework. The *Likelihood Ratio* test statistic is given by

$$LR = T \sum_{i=1}^{r} \log \left[\frac{(1 - \widetilde{\lambda}_i)}{(1 - \widehat{\lambda}_i)} \right], \tag{1.12}$$

where $\widetilde{\lambda}_i$ and $\widehat{\lambda}_i$ are the eigenvalues maximizing the likelihood function for the restricted and unrestricted models, respectively (see for example Johansen

1995). In each case LR is asymptotically χ^2 distributed with $r(p-q)$ degrees of freedom, where $p = 4$ in the quarterly case and where q depends on the number of restrictions. Similar hypotheses can be tested for $r = 1$ or $r = 2$.

Model (1.8) runs the risk of being over-parameterized. To avoid this, Boswijk and Franses (1996) propose various likelihood ratio tests to detect periodic integration in a single equation setting. These tests can be used when there is indication of at most one unit root in the PAR(p) process, see also an extension to multiple unit roots in Boswijk *et al.* (1997). The vector process Y_T in (1.5) is stationary if the solution to the characteristic equation $|\Phi_0 - \Phi_1 z| = 1 - (\phi_1\phi_2\phi_3\phi_4)z = 0$ exceeds one, that is $\phi_1\phi_2\phi_3\phi_4 < 1$, and the same process contains a unit root if $\phi_1\phi_2\phi_3\phi_4 = 1$ holds. If $\phi_s = 1$ for all s the process y_t is said to be integrated of order one, i.e. $y_t \sim \mathrm{I}(1)$, whereas if some or all $\phi_s \neq 1$ the process is called periodically integrated of order 1, i.e. $y_t \sim \mathrm{PI}(1)$. The null hypothesis $\phi_1\phi_2\phi_3\phi_4 = 1$ is tested by imposing $\phi_4 = 1/\phi_1\phi_2\phi_3$ in the following restricted version of the PAR(p) model in (1.4), which can be estimated using nonlinear least squares,

$$y_t = \sum_{s=1}^{4} \mu_s D_{s,t} + \sum_{s=1}^{4} \phi_s y_{t-1} + \sum_{i=1}^{p-1}\sum_{s=1}^{4} \beta_{is} D_{s,t}(y_{t-i} - \phi_{s-j} y_{t-i-1}) + \varepsilon_t. \quad (1.13)$$

The null hypothesis is then tested using the statistic $LR = n\ln(\mathrm{RSS}_0/\mathrm{RSS}_1)$, where RSS_0 is the residual sum of squares from the unrestricted model (1.4) and RSS_1 is the residual sum of squares from the restricted model (1.13). If the null hypothesis cannot be rejected, then the time series is periodically integrated. Since periodic integration nests the situation when $\phi_s = 1$ and $\phi_s = -1$ for $s = 1, 2, 3$, it is also useful, as a next step, to investigate the null hypotheses:

$$\begin{aligned} H_0 &: \phi_s = 1 \quad \text{for } s = 1,2,3 \\ H_0 &: \phi_s = -1 \quad \text{for } s = 1,2,3 \end{aligned} \quad (1.14)$$

which obviously are equivalent with the hypotheses tested using (1.11). Finally, the fact that stochastic trends and seasonal variations are related under periodic integration can be shown by calculating the impact of the accumulation of shocks in ε_t, see Franses (1996). Several studies on the forecasting performance of PAR and PIAR models have appeared in the literature showing mixed results (see for example Franses and Paap (1996) and Herwartz (1999)).

Before turning to multivariate approaches to periodic and seasonal unit root models it should be mentioned that other univariate models for changing seasonal variation have been developed and applied in the literature such as conventional seasonal ARIMA models, see Box and Jenkins (1970), and structural time series models, see Harvey (1989). Another example is variants of the *smooth transition autoregression* (STAR) model, see Granger and Teräsvirta (1993). This nonlinear model class assumes a smooth transition between different regimes. Franses (1998) analyzes if seasonality in unemployment rates changes with the business cycle, using a seasonal STAR (SEASTAR) model. The results for quarterly data in 10 countries suggest that this indeed is the case. This agrees with results in earlier studies, but an advantage with the SEASTAR model is that the switching behavior between regimes (contractions and expansions) is estimated from the data. The earlier studies, which used other time series models, relied on pre-determined business cycle chronologies (see Canova and Ghysels (1994) and Franses (1995b)). Van Dijk *et al.* (2001) apply a time-varying smooth transition autoregressive (TV-STAR) model to study the sources of time variation in seasonal patterns using quarterly industrial production series from a number of countries. Their aim is to compare two different explanations to changes in the seasonal patterns over time. The first one is the one already mentioned: seasonal patterns change over time owing to business cycle fluctuation. The other potential explanation is that gradual technological and institutional change affects the seasonal pattern in time series. The results indicate that technological change, changes in institutions and "other" unspecified reasons seem to be the main cause for changing seasonal variation. The results of van Dijk *et al.* thus contradict the previous studies that attribute the change to cyclical fluctuations in the economy.

1.2 Seasonal and periodic cointegration

Apart from a marked seasonal pattern, many economic time series are upward trending over time and these trends appear often best to be described as being stochastic, i.e. they are trend processes that display random walk behavior. This in turn implies that first differencing filters are required to remove the stochastic trends, and thus that they are integrated of order 1. Moreover, several such macroeconomic time series tend to have similar trending patterns (parallel movements over time), that is; they may have at least one common stochastic trend or in other words, they may be cointegrated. Both seasonal unit root and periodic integration models can be extended to include cointegration between seasonally unadjusted time series and procedures are available for both cases.

Engle *et al.* (1993) [EGHL] propose a two-step method for testing for the presence of seasonal and non-seasonal cointegration relations. The stationary linear combinations, involving two variables y_t and x_t, implied by cointegration at the zero, biannual, and the annual frequency are, respectively:

$$\begin{aligned} u_t &= y_t + y_{t-1} + y_{t-2} + y_{t-3} - \alpha_1(x_t + x_{t-1} + x_{t-2} + x_{t-3}), \\ v_t &= -y_t + y_{t-1} - y_{t-2} + y_{t-3} - \alpha_1(-x_t + x_{t-1} - x_{t-2} + x_{t-3}), \\ w_t &= -y_t + y_{t-2} - \alpha_3(-x_t + x_{t-2}) - \alpha_4(-y_{t-1} + y_{t-3}) - \alpha_5(-x_{t-1} + x_{t-3}). \end{aligned}$$

In the first step, one estimates α_1 to α_5 above using OLS, treating u_t, v_t, and w_t as the error terms. The regressions may include deterministic components such as a constant, a trend, and seasonal dummies. Having obtained estimates $\widehat{u}_t$, $\widehat{v}_t$, and $\widehat{w}_t$, non-cointegration at the zero and seasonal frequencies can be tested by using the following auxiliary regressions:

$$\begin{aligned} (1-B)\widehat{u}_t &= \pi_1\widehat{u}_{t-1} + \sum_{i=1}^{k_1}\gamma_i(1-B)\widehat{u}_{1,t-i} + \varepsilon_t, \qquad (1.15) \\ (1+B)\widehat{v}_t &= \pi_2(-\widehat{v}_{t-1}) + \sum_{i=1}^{k_2}\gamma_i(1+B)\widehat{v}_{t-i} + \varepsilon_t, \\ (1+B^2)\widehat{w}_t &= \pi_3(-\widehat{w}_{t-2}) + \pi_4(-\widehat{w}_{t-1}) + \sum_{i=1}^{k_3}\gamma_i(1+B^2)\widehat{w}_{t-i} + \varepsilon_t. \end{aligned}$$

Non-cointegration at the zero and biannual frequencies implies that $\pi_i < 0$, $i = 1, 2$, and it is tested using one-sided t-tests, where the t-statistics follows the 'Dickey-Fuller' distribution. Finally, if the joint F-test $\pi_3 = \pi_4 = 0$ cannot be rejected there is evidence of cointegration at the annual frequency. Critical values for the F-statistic are tabulated in EGHL. Wells (1997) uses this method to test whether some long-run relations suggested by the neoclassical growth theory extends to seasonal frequencies. Finally, in EGHL an application on Japanese consumption and income is presented. The authors cannot reject non-cointegration at the zero and biannual frequencies. However, they find weak evidence of cointegration at the annual frequency and they conclude:

> "In short, if a slightly impatient borrowing-constrained consumer has the habit of using his bonus payments to replace his worn out clothes, furniture, etc. when the payment occurs, one may expect cointegration at the annual frequency."

Lee (1992) suggests a testing procedure for seasonal cointegration among time series. The proposed maximum-likelihood estimator extends the approach summarized in Johansen (1995). Assume that Y_t is a $(k \times 1)$ vector of time series, which are all seasonally integrated $Y_t \sim \text{SI}(1)$. The following seasonal error correction model [SECM] is considered:

$$\Delta_4 Y_t = \sum_{i=1}^{4} \Pi_i Z_{i,t} + \sum_{j=1}^{p} \Gamma_j \Delta_4 Y_{t-j} + \Phi D_t + \varepsilon_t, \tag{1.16}$$

where $Z_{1,t}, ..., Z_{4,t}$ are the transformed variables used in the HEGY-test regression (1.3), with all unit roots removed, except those at the zero, biannual, and annual frequencies for $i = 1, 2, (3, 4,)$ respectively. The variables D_t are deterministic components, such as seasonal dummies and trends, and ε_t is i.i.d. $N_k(0, \Omega)$. There is evidence of seasonal cointegration if at least one of the Π_i matrices for $i = 2, 3, 4$ has reduced, but non-zero, rank. However, Lee only developed inference in a special case, i.e. when $\Pi_4 = 0$. If this restriction is included and if one assumes that the regressors $Z_{i,t}$ are asymptotically uncorrelated, that is

$$T^{-2} \sum_{t=1}^{T} Z_{i,t} Z'_{j,t} \xrightarrow{P} 0, \quad i \neq j,$$

this implies that the cointegration vectors (β_i) and the adjustment coefficients (α_i) can be found by removing the reduced rank restriction on the other frequencies by concentrating out the associated regressors. Tests for reduced rank (r) of Π_i can then be performed using the *trace* statistic:

$$-2\log(H(r)|H(k)) = -T \sum_{i=r+1}^{P} \log(1 - \widehat{\lambda}_i), \tag{1.17}$$

where $H(r)$ is the null hypothesis and full rank, i.e. $H(k)$, is the alternative hypothesis. The eigenvalues, $1 > \widehat{\lambda}_1 > ... > \widehat{\lambda}_p$, which maximize the likelihood function, are obtained by solving eigenvalue problems based on residual vectors from the concentration step at the non-seasonal and seasonal frequencies. The null hypothesis for this test is that there are at most r cointegrating vectors. Franses and Kunst (1999) suggest that the seasonal dummy variables, often included unrestrictedly in (1.16), should be confined to the seasonal cointegrating relations instead. Finally, Johansen and Schaumburg (1998) completed the analysis for the general case, where the restriction $\Pi_4 = 0$ was relaxed. Several studies have used the Lee (1992) model in empirical applications, see e.g. Lee and Siklos (1993), Kunst (1993a), Ermini and Chang (1996), and Bohl (1998). Forecasting with the same model is considered, for example, in Kunst (1993b) and Reimers (1997). Finally, forecasting and restricted seasonal intercepts in (1.16) is considered in Kunst and Franses (1998).

A single equation periodic cointegration model was proposed by Boswijk and Franses (1995) and is written as:

$$\Delta_4 y_t = \sum_{s=1}^{4} \mu_s D_{s,t} + \sum_{s=1}^{4} \tau_s D_{s,t}(y_{t-4} - \theta_s' X_{t-4}) + \varepsilon_t, \qquad (1.18)$$

where y_t again is a quarterly observed variable and where X_t is a vector of explanatory variables. $D_{1,t}, ..., D_{4,t}$ are seasonal dummy variables and ε_t is a standard white noise process. The model could be augmented by adding lagged terms of the dependent and independent differenced variables, where $\Delta_4 X_t$ can be replaced by $\Delta_1 X_t$ if appropriate. Periodic cointegration was first considered by Birchenhall *et al.* (1989), who estimated a single equation model for real non-durables consumption expenditure. They found evidence that the long-run income elasticity of consumption varies with the season, but also that the adjustment towards equilibrium seems to do so. In terms of the model parameters in (1.18), both τ_s and θ_s would then be seasonally varying. To quote the authors:

> "Such variability is consistent with the hypothesis that consumers have seasonal preferences and exhibit seasonally varying degrees of habit persistence; see Miron (1986) and Osborn (1988)."

Full periodic cointegration in (1.18) implies that there is adjustment towards a long-run equilibrium in each season, whereas if the variables are only partially cointegrated there is no adjustment in some quarters. Boswijk and Franses (1995) analyze how to undertake inference in (1.18) and propose a Wald-test for cointegration. They also suggest tests for various parameter restrictions, when there is evidence of cointegration in two or more quarters. These include tests for equality of the adjustment parameters and of the parameters concerning the long-run relationships across seasons. Finally, model (1.18) is sometimes called a periodic error correction model (PECM), when only the adjustment parameters are periodic. For an empirical application on consumption and income in Sweden, see Boswijk and Franses (1995). Herwartz (1997) considers the forecasting performance of (1.18) as compared to various non-periodic versions of it.

Several methods for testing the presence of periodic cointegration in multiple equations have been suggested. One amounts to applying the maximum-likelihood approach, summarized in Johansen (1995), directly to each season separately, see Franses and Paap (1995). A drawback with this method is that it allows the number of cointegrating vectors to vary over the seasons, which may be unrealistic from an economic theory perspective. In another approach, which avoids this problem, one extracts the stochastic trends from the individual series by using a method proposed in Gonzalo and Granger (1995), in a first step. In the next step one can test for cointegration among the new variables, see Franses and Paap (1995). A third approach, presented by Kleibergen and Franses (1999), also avoids the problem concerning different numbers of cointegrating vectors across seasons. Consider the following periodic VAR model, for k quarterly observed time series:

$$Y_t = \varphi_s Y_{t-1} + u_t, \tag{1.19}$$

where $s = 1, ..., 4$, $t = 1, ..., T$. The vector of disturbances, u_t, is assumed to be i.i.d. $(0, \Omega_s)$. The covariance matrices Ω_s and the parameter matrices φ_s are seasonally varying, which allows the process to have different short-run properties across quarters. However, Kleibergen and Franses (1999) show that specification (1.19) implies the same long-run properties at each season. Hence there can be $0 < r < k$ cointegrating relations between the k series. If we determine the relationship between Y_t and Y_{t-4}, where the latter series concerns the same season in the previous year, and rewrite the resulting expression in its non-seasonal annual form, see Tiao and Grupe (1980) and Osborn (1991), the model becomes

$$\Delta_1 Y_{s,n} = \Pi_s Y_{s,n-1} + \varepsilon_{s,n}, \qquad n = 1, ..., N = \frac{T}{4}, \tag{1.20}$$

with Δ_1 for annual data corresponding to Δ_4 for quarterly data, and where $Y_{s,n}$ is the observation in season s in year n. Notice that (1.19) and (1.20) implies that

$$\Pi_s = \left(\prod_{i=1}^{s} \varphi_{s-i+1} \prod_{i=1}^{S-s} \varphi_{S-i+1} \right) - I_k. \tag{1.21}$$

If there is cointegration among the elements in Y_t then

$$\begin{aligned} \Pi_s &= \alpha_s \beta_s' \Longleftrightarrow \\ \varphi_{s+1} \alpha_s \beta_s' \varphi_{s+1}^{-1} &= \alpha_{s+1} \beta_{s+1}'. \end{aligned} \tag{1.22}$$

The result in (1.22) implies that the spaces spanned by α_{s+1} and β_{s+1} are identical to the spaces spanned by $\varphi_{s+1}\alpha_s$ and $\varphi_{s+1}^{-1\prime}\beta_s$, and this shows that the PVAR model as represented in (1.22) assumes $0 < r < k$ cointegration relations among the elements in Y_t. Estimation of the cointegration parameters in Π_s is not straightforward and implies non-linear restrictions on the φ_s parameters. Kleibergen and Franses (1999) therefore suggest a method, which amounts to optimizing a log-likelihood based objective function. This function can also be used to test for the number of cointegration relations. The PVAR(1) model can be extended to include unrestricted seasonally varying intercepts and trends, as well as higher order dynamics.

Methods to choose between seasonal and periodic cointegration models have been proposed in the literature, see Franses (1993) and Franses (1995a). Osborn (2001) analyzes various cointegration possibilities in the bivariate case. The quarterly observed variables (y_t and x_t) can be either I(1), PI(1), or SI(1) and the specific kind of integratedness can be different in y_t and x_t. For example, consider model (1.18) and let $y_t \sim$ SI(1). Furthermore, let the vector of explanatory variables, denoted X_t now be a single variable, which is either I(1) or PI(1). It is shown that a cointegrating relationship can only appear in one of the four quarters in this situation. Moreover, the richest set of cointegration possibilities arises when both variables are seasonally integrated and that a selection between seasonal or periodic cointegration models only has relevance in this specific case.

Chapter 2

Summary and main results of the papers

2.1 Paper 1

In this paper forecasts from two different seasonal cointegration specifications are compared in an empirical forecasting example and in a Monte Carlo study. The two specifications are the one proposed by Lee (1992), with a parameter restriction included at the annual frequency, and the model proposed by Johansen and Schaumburg (1998), with a general specification for the complex root frequency, respectively. In the empirical forecasting example we also include a standard cointegration model based on first differences and seasonal dummies and analyze the effects of restricting or not restricting seasonal dummies in the seasonal cointegration models. We use macroeconomic data sets for Austria, Germany and the United Kingdom, comprising gross domestic product (Y), private consumption (C), gross fixed investment (I), goods exports (X) and real wages (W), all transformed into natural logarithms. The real interest rates (R) are also included and are given in percentage points. These three data sets have previously been used in Kunst and Franses (1998), and the motivation for using these series is that neoclassical growth theory suggests various long-run relations among them. In the Monte Carlo study we analyze systems of three variables. While the Monte Carlo results favor the specification suggested by Johansen and Schaumburg, and definitely so if larger sample sizes are considered, we do not find such clear cut evidence in the empirical example.

2.2 Paper 2

In this paper we examine in an empirical forecasting study the relevance of taking care of changing seasonality within multivariate methods for cointegrated seasonal time series. We evaluate three different approaches, see Franses and McAleer (1998), i.e. nonseasonal cointegration models, seasonal cointegration models and periodic cointegration models. As our empirical study concerns bivariate time series, we also consider single equation methods, and compare these with system methods. A VAR model in first differences, with and without cointegration, and a VAR model in annual differences were used as benchmarks. Our empirical results indicate that the benchmark VAR model in annual differences is often preferred, except for one-step ahead forecasts where the VAR model in first differences, without cointegration, offers the lowest RMSPEs. The finding that the VAR model in annual differences yields the lowest RMSPEs forecasts can be viewed as extending the results found in Clements and Hendry (1997), where similar results are obtained for univariate data. The *seasonal* cointegration models yield the best forecasts among cointegrated models, even though the periodic models seem to be a better choice for some specific data sets. Finally, there is no clear indication that multiple equations methods improve on single equation methods.

2.3 Paper 3

In the original Lee (1992) specification of the seasonal error correction model [henceforth SECM] a certain restriction at the complex root frequency is suggested, assuming the absence of so called *non-synchronous* seasonal cycles. With the restriction imposed the testing procedure for the number of cointegrating (CI) vectors at frequency $\pi/2$ becomes the same as for the zero and biannual frequencies, see e.g. Kunst (1993b), Franses and Kunst (1999) and Kunst and Franses (1998). Johansen and Schaumburg (1998) argue that this restriction is too strong and not justified from a theoretical point of view. They consider the general case, which results in a less straightforward testing and estimation procedure at the annual frequency. The purpose of this paper is to explore how well the likelihood ratio (LR) test for the cointegrating rank works in the SECM, assuming quarterly data and small samples. The paper sheds some light on the following issues:

1. Are there any differences across frequencies, i.e. is there any evidence that the LR-test of the rank is less powerful at either of the two seasonal frequencies than at the zero frequency?

2. Can the restricted version be a useful tool even in cases where non-synchronous seasonal cycles are present?

3. How well does the Johansen and Schaumburg (1998) specification work in cases when the restriction is valid?

4. How does the number of CI relations at the zero and the biannual frequencies affect the test for the rank at the annual frequency, and vice versa in small samples?

The results indicate that the likelihood ratio test for the rank has uniform power across frequencies. This is most evident when test results from the restricted version of the SECM are analyzed in situations where the restrictions are satisfied. The restricted version of the SECM has poor size properties in cases where non-synchronous cointegration clearly should play a role. This result indicates a risk of finding 'evidence' of too many cointegrating vectors at the annual frequency when using this specification. On the other hand, if the restriction is almost satisfied, the general specification looses power at least in smaller samples, while tests using the restricted version have good properties. Furthermore, the number of true CI relations at a certain frequency seem to affect the test for the rank at other frequencies in small samples.

2.4 Paper 4

The purpose of this paper is to show how the more general SECM, proposed by Johansen an Schaumburg (1998), could be specified in the case where the quarterly observed variables contain different numbers of unit roots, which is a common situation when working with real world data. Furthermore, we assume that the interest of an empirical study is:

1. to test for the number of cointegrating vectors and estimate these at the seasonal and nonseasonal frequencies, and

2. to forecast.

A Monte Carlo simulation is carried out to investigate the consequences of specifying a SECM which assumes four unit roots in each process and where the variables are transformed to yield stationarity accordingly, i.e. applying the annual difference filter. This specification is compared to the correctly specified model with different filters for each variable. We also consider pre-testing for the number of seasonal unit roots in the univariate time series and specify models suggested by these tests. The two seasonal unit root tests are the familiar HEGY test and the seasonal KPSS [SKPSS] tests of Lyhagen (2000), respectively. Since the HEGY test has the null hypothesis of nonstationarity while the SKPSS has the opposite null hypothesis (of no roots) it is interesting to compare them in small samples. For comparison, we also include a VAR model in annual differences in the forecasting exercise. The forecasting mean squared error and the mean squared error of the estimated cointegrating relations indicate that, in practice, a cointegration model, where all variables are transformed using the annual difference filter, is more robust than one obtained by pre-testing for a smaller number of unit roots. The second best choice, when the true model is not known and when the aim is to forecast, is a VAR model, also in annual differences, again corroborating the results by Clements and Hendry (1997), see summary of paper 1.

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Part II

Included Papers

Paper 1

Forecasting performance of seasonal cointegration models

1.1 Introduction

It is common in applied work to assume an approximately constant seasonal pattern, modelled by seasonal dummies. Recently the unit root and cointegration analysis at the zero frequency have been extended to seasonal frequencies. Nonzero frequency unit roots imply a nonstationary seasonal pattern. Assuming a deterministic seasonal pattern in a multivariate setting, when it is in fact stochastically evolving over time, could lead to inappropriate inference about short and long run dynamics in the system. These negative effects may be serious when the model is used in forecasting.

Similar to the approach suggested by Johansen (1988) and Johansen (1995) for the zero frequency, Lee (1992) presents a maximum likelihood estimator for seasonally cointegrating relations. The seasonal error correction model [$SECM$], which allows for stochastic seasonality is, however, only partially correct when it comes to testing for the cointegrating rank at the annual frequency. A certain parameter restriction has been used in the literature to overcome this problem. If this restriction is relaxed one cannot apply estimation methods based on canonical correlations, as suggested by Lee.

Johansen and Schaumburg (1998) argue that this is a peculiar restriction as it constrains all coefficients at the annual frequency. They present another estimation procedure for the parameters, corresponding to the annual frequency and introduce a general asymptotic theory for the seasonal cointegration model.

Kunst and Franses (1998) argue that deterministic seasonal dummy variables, which are often included unrestrictedly in the $SECM$, should be confined to the seasonal cointegrating relations instead. If cointegration at seasonal frequencies exists, an inclusion of unrestricted seasonal intercepts then implies a growing amplitude in the seasonal cycles, which may not be a realistic assumption in most cases.

In the present study, various seasonal cointegration specifications will be contrasted to the model suggested by Johansen and Schaumburg (1998) [henceforth JS]. We have found three studies on forecasting and seasonal cointegration: Kunst (1993a), Reimers (1997) and Kunst and Franses (1998). Kunst (1993a) contrasts the $SECM$, where an intercept is included and the restriction on the annual frequency is imposed, to a vector error correction model [$VECM$] and also a VAR model in first differences and seasonal dummies. Two examples based on empirical data and a Monte Carlo experiment show that the benefits from accounting for seasonal cointegration are quite limited. Reimers (1997) uses the conventional seasonal cointegration model suggested by Lee, with another simplifying restriction on the annual frequency and considers a simulated two variable seasonal cointegration model, with a fixed lag length and no seasonal intercepts. The $SECM$ is compared to a traditional $VECM$ in first differences with seasonal dummies. The main conclusion is that models in first differences produce smaller forecast errors for short horizons, but when longer forecasting periods are considered the seasonal cointegration model appears preferable. Kunst and Franses (1998) investigate the forecasting effects of first deleting then either restricting or not restricting the seasonal intercepts as discussed above. Using three empirical data sets and various forecasting periods they show that the suggested restricted seasonal dummy approach yields better forecasts in most cases.

In the present study we show results from an empirical forecasting example, but we also conduct a Monte Carlo study where seasonally cointegrated data generating processes [DGPs], with different parameter constellations, are investigated. In the empirical forecasting part we use macroeconomic data sets for Austria, Germany and the United Kingdom, which are also used in Kunst and Franses (1998). In the Monte Carlo study we analyze systems including three variables, whereas six variables are used in the empirical example. The forecasting performance of the more general specification for the annual frequency, recently proposed by JS is evaluated. We compare that specification with the original version of the seasonal cointegration model, proposed by Lee (1992). In the empirical forecasting example we also compare these two seasonal cointegration specifications with a $VECM$ in first differences.

The remainder of this paper is organized as follows. Section 1.2 presents specifications of the *SECM*, while Section 1.3 describes the data and the estimation results for the models to be used in the empirical forecasting example. Section 1.4 presents forecast performance when using the empirical data sets. In Section 1.5 the Monte Carlo setup is discussed. A comparison of model forecasts from the Monte Carlo study is carried out in Section 1.6. The final section contains conclusions.

1.2 Seasonal cointegration

Lee (1992) suggests a maximum likelihood estimator for seasonal cointegration relations. This procedure extends the maximum likelihood approach, which is summarized in Johansen (1995). Assuming quarterly data and that $\Delta_4 Y_t$ is stationary, a seasonal *VECM* of order p of the following form is considered:

$$\Delta_4 Y_t = \sum_{i=1}^{3} \alpha_i \beta_i' Z_{i,t} + \sum_{j=1}^{p} \Gamma_j \Delta_4 Y_{t-j} + \Phi D_t + \varepsilon_t, \tag{1.1}$$

where the D_t are deterministic components and where ε_t is i.i.d. $N_n(0,\Omega)$. The process $\Delta_4 Y_t$ above is said to be seasonally cointegrated if and only if at least one of the $\alpha_i \beta_i'$ matrices for $i = 2, 3$ on the right hand side has reduced, non-zero rank. The linear filters which in (1.1) remove all unit roots except those on the zero, biannual and annual frequencies, respectively are:

$$\begin{aligned} Z_{1,t} &= (L + L^2 + L^3 + L^4) Y_t, \\ Z_{2,t} &= (L - L^2 + L^3 - L^4) Y_t, \\ Z_{3,t} &= (L^2 - L^4) Y_t. \end{aligned}$$

These filters are the vector equivalents of the univariate transformations used in the so called HEGY-test for seasonal unit roots, see Hylleberg et *al.* (1990) [HEGY]. Furthermore, the regressors $Z_{i,t}$ are asymptotically uncorrelated:

$$T^{-2} \sum_{t=1}^{T} Z_{i,t} Z_{j,t}' \xrightarrow{P} 0, i \neq j,$$

implying that the cointegration vectors and adjustment coefficients can be found by removing the reduced rank restriction on the other frequencies by concentrating out the associated regressors.

One variable that appears in the auxiliary regression of the HEGY-test at the complex frequency but is not present in (1.1) is $Z_{4,t} = (L - L^3) Y_t$. By imposing the restriction that this filtered variable has no influence on $\Delta_4 Y_t$

one assumes the absence of so called non-synchronous seasonal cycles. If this restriction [henceforth denoted as the annual restriction] is relaxed one cannot apply the estimation method that uses canonical correlations. There is also some evidence in the literature that the absence of non-synchronous seasonal cycles should have little effect on the test for cointegration at frequency $\pi/2$, see Lee (1992).

JS, who argue that the above mentioned restriction is peculiar and not justified from a theoretical point of view, refine the theory for seasonal cointegration in the general case. They propose the following error correction model for quarterly data:

$$\begin{aligned}\Delta_4 Y_t &= \sum_{i=1}^{2} \alpha_i \beta_i' X_{i,t} + 2(\alpha_R \beta_R' + \alpha_I \beta_I') X_{R,t} + 2(\alpha_R \beta_I' - \alpha_I \beta_R') X_{I,t} \\ &\quad + \sum_{j=1}^{p} \Gamma_j \Delta_4 Y_{t-j} + \Phi D_t + \varepsilon_t, \end{aligned} \tag{1.2}$$

where the processes $X_{1,t}, \ldots, X_{I,t}$ are:

$$\begin{aligned} X_{1,t} &= \frac{1}{4} Z_{1,t}, \\ X_{2,t} &= -\frac{1}{4} Z_{2,t}, \\ X_{R,t} &= \frac{1}{4} Z_{3,t}, \\ X_{I,t} &= -\frac{1}{4} Z_{4,t}, \end{aligned}$$

respectively. It can be seen that the annual restriction would imply that $\alpha_R \beta_I' - \alpha_I \beta_R' = 0$ in (1.2), which is a strong restriction on the coefficients at the complex root frequency.

The estimation of $\alpha_i \beta_i'$ for $i = 1, 2$ uses canonical correlations in analogy to the Johansen procedure and hence does not require any detailed explanation here. However the estimation of β_R and β_I is nonstandard.

JS propose the following estimation strategy. The first step involves regressing $\Delta_4 Y_t$, X_{1t}, X_{2t}, X_{Rt} and X_{It} on lagged values of $\Delta_4 Y_t$ and D_t, if these are present in the model, and defining the residuals as R_{0t}, R_{1t}, R_{2t}, R_{Rt} and R_{It}, respectively. If no lags or deterministic variables are present, $X_{jt} = R_{jt}$ for $j = 1, 2, R, I$. The restriction of reduced rank on $\alpha_i \beta_i'$ for $i = 1, 2$ is removed by regressing R_{0t}, R_{Rt} and R_{It} on R_{1t} and R_{2t}. It is shown in JS that the resulting residuals, defined as U_{0t}, U_{Rt} and U_{It}, satisfy the following equation:

$$\begin{aligned} U_{0t} &= 2(\alpha_R\beta_R' + \alpha_I\beta_I')U_{Rt} + 2(\alpha_R\beta_I' - \alpha_I\beta_R')U_{It} + U_{\varepsilon t} \\ &= 2\left(\alpha_R - \alpha_I\right)\begin{pmatrix} \beta_R & -\beta_I \\ \beta_I & \beta_R \end{pmatrix}'\begin{pmatrix} U_{Rt} \\ U_{It} \end{pmatrix} + U_{\varepsilon t} \\ &= \widetilde{\alpha}\boldsymbol{\beta}' U_{1t} + U_{\varepsilon t}, \end{aligned} \tag{1.3}$$

asymptotically. Defining the product moments as $S_{ij} = (1/T)\Sigma_{t=1}^T U_{it}U_{jt}'$ for $i,j = 0,1$, we have that for fixed values of $\boldsymbol{\beta}$ the concentrated likelihood function with respect to $\widetilde{\alpha}$ and hence the variance-covariance matrix Ω take the form, apart from a constant:

$$L_{\max}^{-\frac{2}{T}}(\boldsymbol{\beta}) = |S_{00}|\frac{|\boldsymbol{\beta}'(S_{11} - S_{10}S_{00}^{-1}S_{01})\boldsymbol{\beta}|}{|\boldsymbol{\beta}' S_{11}\boldsymbol{\beta}|}. \tag{1.4}$$

The minimization of (1.4) cannot be solved as an eigenvalue problem as in the zero and biannual frequency cases since $\boldsymbol{\beta}$ itself has complex structure while the product matrices $S_{11} - S_{10}S_{00}^{-1}S_{01}$ and S_{11} do not. JS use a switching algorithm proposed by Boswijk (1995) where the maximum likelihood estimator of $\boldsymbol{\beta}$ is calculated iteratively: Isolate β_R and β_I by using a normalized form ($\Omega^{-1/2}U_{0t} = \widetilde{U}_{0t}$) of U_{0t}, namely:

$$\begin{aligned} \widetilde{U}_{0t} &= 2\Omega^{-1/2}(\alpha_R\beta_R' + \alpha_I\beta_I')U_{Rt} \\ &\quad + 2\Omega^{-1/2}(\alpha_R\beta_I' - \alpha_I\beta_R')U_{I,t} + \Omega^{-1/2}U_{\varepsilon t} \\ &= \alpha_R^N\beta_R'U_{Rt} - \alpha_I^N\beta_R'U_{It} + \alpha_R^N\beta_I'U_{It} - \alpha_I^N\beta_I'U_{Rt} + \widetilde{U}_{\varepsilon t}, \end{aligned} \tag{1.5}$$

and then vectorize (1.5) by using:

$$\text{vec(ABC)} = (\text{C}' \otimes \text{A})\text{vec(B)}$$

Since $\widetilde{U}_{0t}$ is a vector and hence $\text{vec}(\widetilde{U}_{0t}) = \widetilde{U}_{0t}$, this yields:

$$\widetilde{U}_{0t} = U_{2t}\begin{bmatrix} \text{vec}(\beta_R') \\ \text{vec}(\beta_I') \end{bmatrix} + \widetilde{U}_{\varepsilon t} \tag{1.6}$$

where $U_{2t} = \left[(U_{Rt}' \otimes \alpha_R^N) - (U_{It}' \otimes \alpha_I^N) \quad (U_{It}' \otimes \alpha_R^N) + (U_{Rt}' \otimes \alpha_I^N)\right]$. The parameters in β_R and β_I can now be found from:

$$\begin{bmatrix} \text{vec}(\beta_R') \\ \text{vec}(\beta_I') \end{bmatrix} = \left(\sum_{t=1}^{T} U_{2t}U_{2t}'\right)^{-1}\sum_{t=1}^{T} U_{2t}\widetilde{U}_{0t}'. \tag{1.7}$$

For a given value of $\boldsymbol{\beta}$, which is generated randomly in the first iteration, estimates of $\widetilde{\alpha} = (S_{01}\boldsymbol{\beta}(\boldsymbol{\beta}' S_{11}^{-1}\boldsymbol{\beta})^{-1})/2$ and $\widehat{\Omega} = S_{00} - S_{01}\boldsymbol{\beta}(\boldsymbol{\beta}' S_{11}^{-1}\boldsymbol{\beta})^{-1})\boldsymbol{\beta}' S_{10}$ are computed. In a second step U_{0t} is normalized and vectorized as described above, yielding a new estimate of $\boldsymbol{\beta}$ for which we can compute a new likelihood function. This procedure is repeated until a suitable convergence criterion is satisfied.

Kunst and Franses (1998) argue that deterministic seasonal dummy variables, which are often included unrestrictedly in (1.1) to handle the deterministic part of seasonality, should be confined to the seasonal cointegrating relations instead. This is because unrestricted seasonal intercepts in the *SECM* may lead to diverging seasonal trends, which is unlikely in most cases. We restrict the seasonal dummy variables in a similar way in the JS specification which leads to the extended variables $\widetilde{X}_{2,t} = (X_{2,t}, \cos \pi(t))$, $\widetilde{X}_{R,t} = (X_{R,t}, \cos \frac{\pi}{2}(t))$ and $\widetilde{X}_{I,t} = (X_{I,t}, -\sin \frac{\pi}{2}(t))$. This gives the following augmented $\boldsymbol{\beta}$ matrix in (1.3):

$$\begin{pmatrix} \beta_R & -\beta_I \\ \rho_R & -\rho_I \\ \beta_I & \beta_R \\ \rho_I & \rho_R \end{pmatrix}$$

where ρ_R and ρ_I are the parameters corresponding to $\cos \frac{\pi}{2}(t)$ and $-\sin \frac{\pi}{2}(t)$, respectively.

As in Kunst and Franses (1998) we also include an unrestricted intercept in the model. Henceforth we will denote the Lee specification with unrestricted seasonal intercepts and the annual restriction imposed as SCM1, which means seasonal cointegration model of type 1. The same model with restricted seasonal intercepts, as proposed by Kunst and Franses (1998), will be denoted SCM2. Finally, SCM3 denote the *SECM* proposed by JS with restricted seasonal intercepts.

1.3 Tests for integration and cointegration

Gross domestic product (Y), private consumption (C), gross fixed investment (I), goods exports (X) and real wages (W) are transformed into natural logarithms. The real interest rates (R) are given in percentage points. The three data sets were previously used in Kunst and Franses (1998), and the motivation for using these series is that the neoclassical growth theory suggests various long-run relations among them. One example is the difference of output and consumption, in logarithms, which should be stationary according to the theory. Another example is the logarithmic difference of output and investments, see Kunst and Neusser (1990). However, it is still an open question how these series should be related at seasonal frequencies, see Wells (1997) for a discussion of these issues. The data set for Austria covers the time period 1964:1 to 1994:4, whereas the time series for Germany run from 1960:1 to 1988:1 and for the UK from 1957:1 to 1994:1. Unit roots in the univariate time series are tested using the HEGY-test. The test procedure investigates whether the seasonal difference $(1 - B^4)$, which assumes the presence of four unit roots, is the appropriate filter compared with other nested filters.

Results of the tests for seasonal and nonseasonal unit roots appear in Table 1.1. All auxiliary regressions include an intercept (I), seasonal dummies (D) and a deterministic trend (T), except those for the real interest rates where an exclusion of the trend seems to be a more appropriate specification. All variables seem to contain unit roots at the zero frequency, except UK consumption and the real interest rate in Austria. The results are more mixed at the seasonal frequencies. All roots at the biannual frequency, except for consumption, are rejected for Austria. On the other hand we find evidence of four unit roots at this frequency for both Germany and UK. Turning to the annual frequency we find evidence of three, one and four stationary variables for Austria, Germany and UK, respectively. Since unit roots are most frequent in the German data set it is sensible to conclude that cointegration at all frequencies is most likely to be found there.

Kunst (1993b), who analyze the variables presented above for Austria, Finland, Germany and the UK, mention that evidence of stochastic seasonality is quite weak in the UK series, while the evidence is rather strong in the German data set. Finally, Kunst and Neusser (1990) find similar unit root results as here, for Austria.

The ranks of the matrices $\alpha_i\beta_i'$ $(i = 1, 2)$ and $\widetilde{\alpha}\beta'$ are determined using the trace test, where the null hypothesis is: at most r cointegrating vectors against the alternative of full rank. Table 1.2 summarizes the result using the three different model specifications SCM1, SCM2 and SCM3. We use the same lag lengths as Kunst and Franses (1998) for SCM1 and SCM2, indicated by p, and we use the same lag lengths for SCM3 as well. A* indicate significance at the 5% level. Critical values for columns two, three and five are based on our own calculations with 100000 replicates and with $T = 400$. Lee and Siklos (1995) present critical values for these cases, but they only consider smaller systems. Critical values for columns four and six are from Tables 1a-1f in Franses and Kunst (1999). For the last column we use the critical values from Table 2 in JS. Notice that the results are identical for the three model specifications at the zero frequency and for SCM2 and SCM3 at the biannual frequency. Three long-run cointegrating vectors are found at the zero frequency for Austria and the UK and two for Germany. On frequency π we find no evidence of unit roots for Austria but identify two cointegrating vectors for Germany. For the UK we find no evidence of cointegrating vectors at frequency π using SCM2 or SCM3, but we identify one if SCM1 is used. The results are more mixed at frequency $\pi/2$. One observation is that SCM3 suggests more cointegrating vectors. Based on these results, where the biannual frequency for Austria has full rank and the rather weak evidence of cointegration at the seasonal frequencies in the UK data set, only the German data set is used in Section 1.4.

Having chosen the rank for SCM3 at the different frequencies we test for further reduction of the model. A hypothesis that simplifies the cointegration analysis on frequency $\pi/2$ is the one implying real structure (H_{REAL}), i.e. $\beta_I = 0$ in (1.2). However this hypothesis is strongly rejected with LR-test statistics 43.5, 72.5 and 22.4 for Austria, Germany and the UK respectively.

1.4 An empirical forecasting example

In this section we investigate the forecasting performance of the models presented in previous sections, namely SCM1, SCM2, SCM3. We also include a *VECM* in first differences, denoted CM. For the seasonal cointegration models we use the same lag lengths as in Section 1.3 and we choose $p = 3$ for CM according to the Schwarz Criterion, in addition to equation by equation diagnostic tests. The empirical data set for Germany, which covers the time period 1960:1 - 1988:1 is used. We impose the ranks according to the results found in Section 1.3 for the different models. All forecast errors correspond to level variables. In the first step we save eight observations at the end to be forecasted ex ante. In the next step, the estimation period is extended by one quarter and we reestimate the parameters in the models. However, we do not change lag orders or the chosen cointegration ranks. These extensions of the sample are through 1987:4, where we generate the last one-step ahead forecast error. Hence there are eight one-step ahead forecast errors, seven two-step ahead forecast errors and so on. The eight-step ahead forecasts are then based on a single observation for each equation. In total we have 36 forecasts for each model and data set. The results are summarized by the RMSE statistic (root mean squared error) for 1, 2 and 4-step ahead forecasts. In Table 1.3 we also consider the RMSE based on all the 36 forecasts from each model, denoted All. Finally, we present the average performance rank of the four model specifications at the different forecast horizons considered, denoted AR.

Comparing the four cointegration specifications we see that the result is quite clear, when looking at the average rank of the models. The version of the seasonal cointegration model with the annual restriction and unrestricted seasonal dummies included (SCM1) is better than the other three specifications if one step ahead forecasts are considered. However, for longer forecast horizons the two versions of the *SECM* with restricted seasonal dummies seems to be a better choice. This result is in line with that found in Kunst and Franses (1998). Comparing SCM2 and SCM3 one can see that SCM2 is clearly better for forecast horizons shorter than four steps. The model in first differences (CM) generates better forecasts than the seasonal cointegration models at all horizons for goods exports (X). Note that the result of the HEGY-tests indicate no seasonal unit roots for goods exports in this data set, which may explain why the model in first differences generates the best forecasts for this time series. However, CM generates the poorest forecasts on average. If the forecast accuracies are evaluated using the mean absolute error (MAE) instead, we come to the same conclusions (not shown here).

1.5 The Monte Carlo setup

The seasonal cointegration specifications we compare are those of Lee (SCM1) and JS (SCM3), respectively and we set $p = 0$ for both specifications. We compare the forecast accuracy for the seasonal cointegration models when seasonality is viewed as being purely stochastic, i.e. no deterministic variables are included. The systems include three variables in each case. Four different DGPs are investigated, see Table 1.4, and they are denoted DGP1, DGP2, DGP3 and DGP4. The matrices α_1 and β_1 concern the zero frequency and include the adjustment parameters and the cointegrating vectors, respectively. The matrices α_2 and β_2 concerns the biannual frequency, whereas α_R, β_R and α_I, β_I concerns the annual frequency. The cointegrating vectors and adjustment parameters are different across frequencies for all DGPs and the adjustment parameters are always different across DGPs for the annual frequency. We let the columns in all the adjustment parameter matrices for a certain DGP sum to the same absolute value, just to have a similar parameter structure across frequencies. For example, the first column sum is 0.9 in absolute value across frequencies in DGP1, whereas the second column sum is 0.5. The adjustment parameters are lower in DGP2 than in DGP1. In DGP3 the values in α_R and α_I lead to lower parameter values in the $\Pi_I = (\alpha_R\beta_I' - \alpha_I\beta_R')$ matrix, so we may expect a lower impact of non-synchronous cointegration. Finally, in DGP4 the filter X_{It} should play an even smaller role, when forecasting.

We investigate the following true rank cases:

$$
\begin{aligned}
1 &: r_0 = 1,\ r_\pi = 1,\ r_{\pi/2} = 1, \qquad 2 : r_0 = 2,\ r_\pi = 2,\ r_{\pi/2} = 1, \\
3 &: r_0 = 1,\ r_\pi = 1,\ r_{\pi/2} = 2, \qquad 4 : r_0 = 2,\ r_\pi = 2,\ r_{\pi/2} = 2.
\end{aligned}
$$

Three different variance-covariance matrices are used in each case (see Table 1.5) and four different simple sizes, namely $T = 40, 80, 120$ and 200. Furthermore, 100 presample observations and 12 postsample observations are generated. These last 12 observations are saved and used for ex-ante forecasting. In each case 5000 replications (N) are generated. The data is generated in the following way:

$$X_t = X_{t-4} + \Pi_1 X_{1,t} + \Pi_2 X_{2,t} + \Pi_R X_{R,t} + \Pi_I X_{I,t} + \varepsilon_t,$$

where $\varepsilon_t \sim N_3(0, \Sigma_{1,2,3})$ and where the different Π matrices are changed according to which rank is chosen at a particular frequency. For example Π_1 equals the first r_0 columns in α_1 times the first r_0 rows in β_1'. The matrices at the biannual and the annual frequency are constructed in the same manner, but the matrices Π_R and Π_I at the annual frequency equal $(\alpha_R \beta_R' + \alpha_I \beta_I')$ and $(\alpha_R \beta_I' - \alpha_I \beta_R')$, respectively.

For both models we calculate squared forecast errors (e_{jk}^2) for each horizon, $k = 1,...,12$, and for each equation, $j = 1$, 2 and 3. Then we take the equation by equation average of these over the number of replications (R). We then weigh the resulting mean squared forecast errors with a variance estimate of the 12 first observations generated from each equation, denoted $\widehat{V}_{jk}$. The variances are calculated after 5000 replications and they are useful when we, in the last step, average the mean squared forecast errors over the three equations. With this relative measure we avoid to give high weight to variables with large variances. To summarize, for each model and horizon we have the following mean squared error [MSE] measure:

$$\text{MSE} = \frac{1}{3} \sum_{j=1}^{3} \left[\left(\frac{1}{R} \sum_{r=1}^{R} e_{jk}^2 \right) \Big/ \widehat{V}_{jk} \right]. \tag{1.8}$$

1.6 Monte Carlo results

In total we have 192 combinations to consider if we look at all sample sizes, true rank cases, DGPs and variance-covariance matrices. However, the results are not different depending on which variance-covariance matrix is used. Because of that only results based on Σ_1 are considered. We choose not to report any results graphically when DGP1 is used, because SCM3 gives the best forecasts in this case, regardless of which sample size or true rank is used. Moreover, the true rank cases 2 and 3 produce similar results as compared to cases 1 and 4, respectively, so we restrict the presentation to the latter two cases (rank = 1 and 2 at all frequencies). Furthermore, if the sample size is larger than or equal to 80 SCM3 always generates lower forecast errors, at all horizons, than SCM1, if DGP2 is used. As the result for DGP1 the difference between the two specifications is sometimes substantial, when using DGP2.

Note that there seems to be a seasonal pattern in the forecast results, see Figures 1.1 and 1.2. This is due to the estimated variances used in (1.8) but also because the results are for data in levels, hence transformed from fourth differences. SCM3 produces poorer forecasts for smaller sample sizes if DGP3 and DGP4 are considered, see for example (c), (d), (e) and (f) in Figure 1.1. This is because the values in α_R and α_I result in a lower impact of so called non-synchronous seasonal cycles. Some results indicate that SCM3 may yield worse forecasts if more cointegrating relations are included in the model. One example is (a) and (b) in Figure 1.1, for DGP2 and $T = 40$. Looking at the rest of the figures increasing sample size works in favor of SCM3, so that for $T =$ 200 SCM3 dominate SCM1. We also find in a smaller complimentary study (not all 192 combinations considered) that these results hold when various lag lengths are selected, using the Akaike Information Criterion.

1.7 Conclusions

The forecasting performance of the seasonally cointegrated model of Johansen and Schaumburg (1998) is compared to a related specification, with a restriction at the annual frequency. In the empirical forecasting part, we also include a model in first differences with cointegration restrictions. We examine data sets from Austria, Germany and the UK, each containing six variables: gross domestic product, private consumption, gross fixed investment, goods exports, real wages and the real interest rate. We examine the integration and cointegration properties for the three data sets and consider the effect of including restricted or unrestricted seasonal dummies in the seasonal cointegration models. The biannual frequency for Austria seems to have full rank and we find rather weak evidence of cointegration at the seasonal frequencies in the UK data set, so that only the German data set is used in the empirical forecasting example.

The seasonal cointegration model with the annual restriction and unrestricted seasonal dummies included is better than the other three specifications if one step ahead forecasts are considered. However, for longer forecast horizons the two versions of the seasonal cointegration model with restricted seasonal dummies seems to be a better choice. When comparing the two restricted seasonal dummy specifications we find that the version with the annual restriction is better for forecast horizons shorter than four steps. The cointegration model in first differences generates the poorest forecasts on average.

In the Monte Carlo study we analyze four different DGPs and four different sample sizes, namely $T = 40, 80, 120$ and 200 when using a three variate system. We do not include any deterministic variables and seasonality is thus generated as being purely stochastic and we do not consider any model in first differences. When the smaller sample sizes are considered, we find some evidence that the specification proposed by Johansen and Schaumburg (1998) may yield worse forecasts if more cointegrating relations are included in the model. Increasing sample size works in favor of this model, so that for $T = 200$ it dominate the seasonal cointegration model with the annual restriction included. This sample size effect may explain the result in the empirical example which is based on fewer observations.

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Tables and figures

Table 1.1 Seasonal integration tests.

	Austria			Germany			UK		
Var	$t_{\hat{\pi}_1}$	$t_{\hat{\pi}_2}$	$F_{\hat{\pi}_{3,4}}$	$t_{\hat{\pi}_1}$	$t_{\hat{\pi}_2}$	$F_{\hat{\pi}_{3,4}}$	$t_{\hat{\pi}_1}$	$t_{\hat{\pi}_2}$	$F_{\hat{\pi}_{3,4}}$
Y	-1.78	-3.47*	1.81	-1.52	-2.83*	3.29	-3.21	-1.38	4.43
C	-1.28	-1.83	3.14	-1.21	-1.45	3.41	-3.49*	-1.85	3.73
I	-2.29	-4.96*	9.79*	-2.83	-2.68	6.21	-2.68	-2.53	13.3*
X	-1.49	-5.18*	48.6*	-0.96	-5.13*	39.7*	-1.12	-2.12	6.82*
W	-1.69	-3.31*	2.58	-1.18	-1.62	2.18	-2.16	-3.05*	19.9*
R	-3.01*	-2.95*	7.20*	-2.77	-2.03	4.73	-1.94	-5.38*	22.3*

Lags of dependent variable and additional regressors (Φ).

	Lags	Φ	Lags	Φ	Lags	Φ
Y	1-4	I,D,T	1	I,D,T	1-5	I,D,T
C	1-4	I,D,T	1-4	I,D,T	1-5	I,D,T
I	1-2	I,D,T	1-3	I,D,T	1	I,D,T
X	-	I,D,T	-	I,D,T	1-4	I,D,T
W	1-6	I,D,T	1-4	I,D,T	1	I,D,T
R	1-3	I,D	1	I,D	1-5	I,D

* Significant at the 5% level. I, D and T denote an intercept, seasonal dummies and a trend, respectively.

Table 1.2 Tests for seasonal cointegration, LR-test statistics. A * denotes significance at the 5% level and p denotes the lag length.

	Frequency					
	0	π		$\pi/2$		
Austria, p=0	SCM1,2,3	SCM1	SCM2,3	SCM1	SCM2	SCM3
rank						
0	161.3*	181.0*	161.4*	200.1*	187.9*	284.7*
1	88.1*	116.5*	113.1*	122.8*	111.1*	173.7*
2	48.8*	77.2*	73.8*	77.8*	67.2*	106.0*
3	22.5	46.1*	42.7*	40.1*	41.7	64.5*
4	6.8	21.0*	20.9*	18.9	21.3	29.7
5	1.5	9.7*	9.7*	6.2	7.8	8.3
Germany, p=0						
rank						
0	132.2*	140.8*	128.9*	174.5*	171.2*	287.3*
1	68.8*	94.1*	82.7*	95.1*	93.6*	179.2*
2	38.2	52.3*	40.8	54.5	56.8	107.6*
3	17.2	24.3	16.8	29.8	32.0	56.9
4	8.4	7.9	8.6	15.2	17.4	27.5
5	1.9	3.2	4.1	4.0	5.9	10.6
UK, p=5						
rank						
0	140.7*	104.0*	95.9	130.7*	130.2	218.7*
1	73.6*	64.5	58.2	71.7	71.5	143.4*
2	47.7*	42.0	39.4	40.6	40.4	92.0
3	25.7	25.3	22.7	22.4	22.1	56.3
4	8.7	10.5	11.3	7.7	7.6	27.8
5	0.8	2.8	3.2	1.7	1.6	6.5

Table 1.3 Forecast performance. The ranks at different frequencies according to trace test result in Table 1.2 for the German data set.

Horizon:	1	2	4	All	Horizon:	1	2	4	All
		SCM1					SCM2		
Y	<u>1.09</u>	1.05	1.42	<u>1.16</u>	Y	1.33	<u>0.97</u>	<u>1.29</u>	1.24
C	<u>0.84</u>	0.99	0.90	0.84	C	0.91	<u>0.80</u>	<u>0.84</u>	<u>0.77</u>
I	<u>3.94</u>	<u>2.87</u>	3.62	<u>3.57</u>	I	4.07	3.03	3.64	3.73
X	2.56	3.17	4.76	4.04	X	2.65	3.02	4.61	3.97
W	<u>1.49</u>	1.70	1.93	2.18	W	1.61	<u>1.58</u>	1.79	2.14
R	<u>1.35</u>	1.54	1.84	1.52	R	1.40	<u>1.40</u>	1.72	<u>1.45</u>
AR	<u>1.33</u>	2.17	3.00	2.17	AR	2.67	<u>1.50</u>	<u>2.00</u>	<u>2.00</u>
		SCM3					CM		
Y	1.30	1.20	1.39	1.27	Y	1.30	1.75	1.79	1.60
C	1.23	1.12	0.86	0.98	C	1.19	1.47	1.29	1.27
I	4.28	3.26	<u>3.37</u>	3.58	I	4.74	4.55	3.77	4.08
X	2.29	2.53	4.35	3.73	X	<u>1.85</u>	<u>2.46</u>	<u>4.14</u>	<u>3.55</u>
W	1.61	1.77	<u>1.74</u>	<u>2.09</u>	W	1.86	2.25	2.09	2.48
R	1.94	1.93	1.89	1.88	R	2.39	2.55	<u>1.49</u>	2.03
AR	2.83	2.83	<u>2.00</u>	2.33	AR	3.17	3.50	3.00	3.50

Forecast errors evaluated with RMSE*100. Unrestricted constant and seasonal intercepts included. All models includes two vectors at the zero frequency. SCM2 and SCM3 includes two cointegration vectors at the biannual frequency, while SCM1 include three. Models SCM1 and SCM2 includes two vectors at the annual frequency, while SCM3 include three. AR denote the average rank of the models, at different forecast horizons, and the smallest is underlined.

Table 1.4 Cointegrating vectors (β) and adjustment parameters (α) used in the Monte Carlo study.

	DGP1		DGP2		DGP3		DGP4	
	1.00	0.00	-1.00	0.00	1.00	0.00	1.00	0.00
β_1	-1.00	1.00	-1.00	1.00	-0.80	1.00	-0.80	1.00
	0.00	0.50	0.00	0.50	0.00	0.30	0.00	2.30
	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
β_2	-0.90	1.00	-0.90	1.00	-0.70	1.00	-0.70	1.00
	0.00	0.30	0.00	0.30	0.00	0.60	0.00	0.60
	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
β_R	-0.70	1.00	-0.70	1.00	-0.90	1.00	-0.90	1.00
	0.00	0.60	0.00	0.60	0.00	0.30	0.00	0.30
	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
β_I	-0.80	1.00	-0.80	1.00	-1.00	1.00	-1.00	1.00
	0.00	0.30	0.00	0.30	0.00	0.50	0.00	0.50
	-0.80	0.00	-0.40	0.00	-0.40	0.00	-0.80	0.00
α_1	-0.10	-0.30	-0.05	-0.15	-0.20	-0.20	-0.10	-0.30
	0.00	-0.20	0.00	-0.10	0.00	-0.10	0.00	-0.20
	-0.60	0.00	-0.30	0.00	-0.35	0.00	-0.60	0.00
α_2	-0.30	-0.40	-0.15	-0.20	-0.25	-0.15	-0.30	-0.40
	0.00	-0.10	0.00	-0.05	0.00	-0.15	0.00	-0.10
	0.10	0.00	0.05	0.00	0.50	0.00	0.80	0.00
α_R	0.80	0.10	0.40	0.05	0.10	0.10	0.10	0.40
	0.00	0.40	0.00	0.20	0.00	0.20	0.00	0.10
	0.80	0.00	0.40	0.00	0.40	0.00	0.85	0.00
α_I	0.10	0.40	0.05	0.20	0.20	0.25	0.05	0.42
	0.00	0.10	0.00	0.05	0.00	0.05	0.00	0.08

Table 1.5 Diagonal and non-diagonal variance-covariance matrices used in the Monte Carlo study. An identity matrix is also included in the analysis, denoted Σ_3

$$\Sigma_1 = \begin{bmatrix} 0.7 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 \\ 0.0 & 0.0 & 0.5 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 0.7 & 0.3 & -0.2 \\ 0.3 & 0.8 & -0.1 \\ -0.2 & -0.1 & 0.5 \end{bmatrix}$$

Figure 1.1 Mean squared errors weighted with variances. Dashed line: SCM1, solid line SCM3. Rank concerns all frequencies, T=40 and 80.

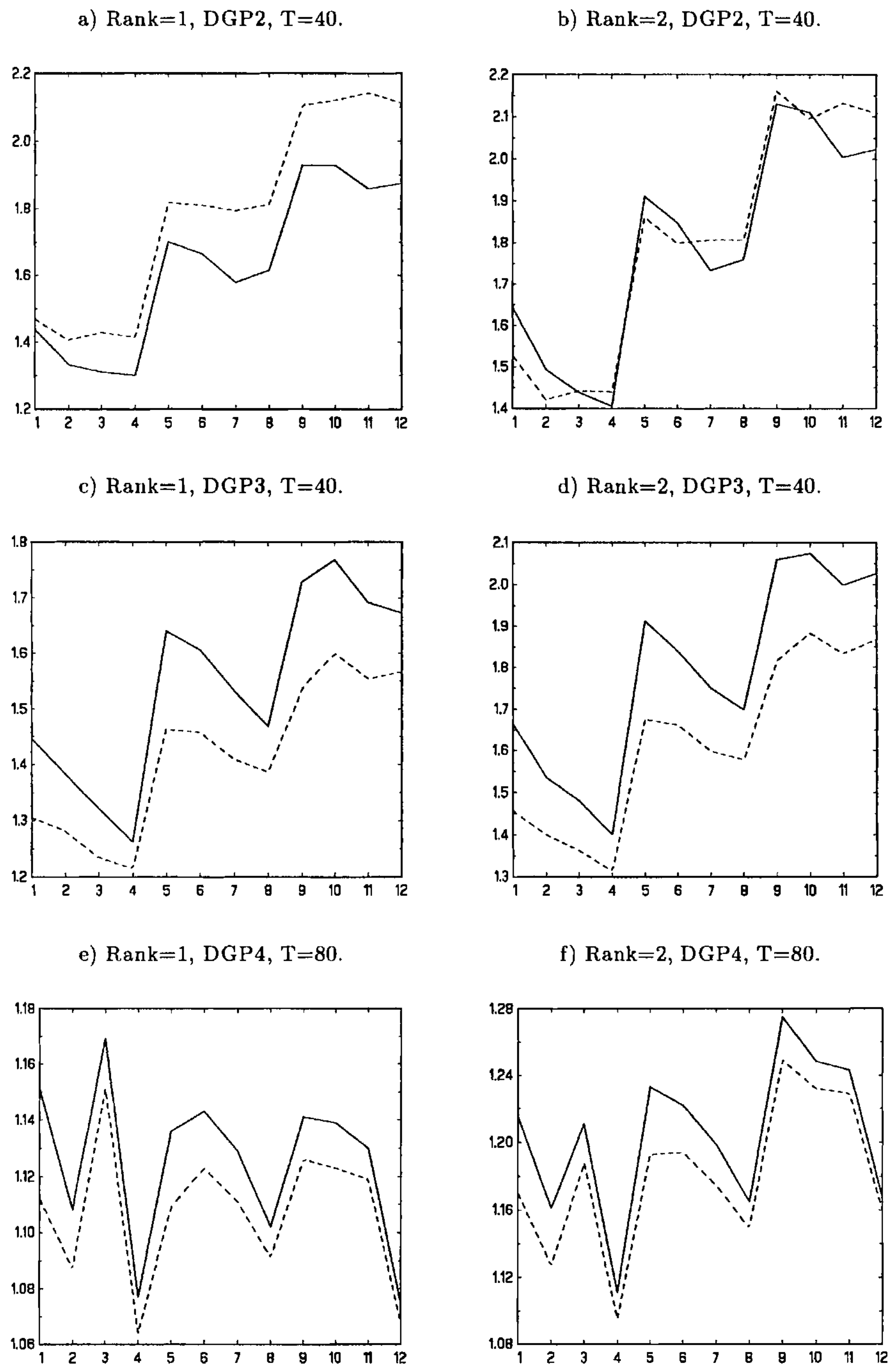
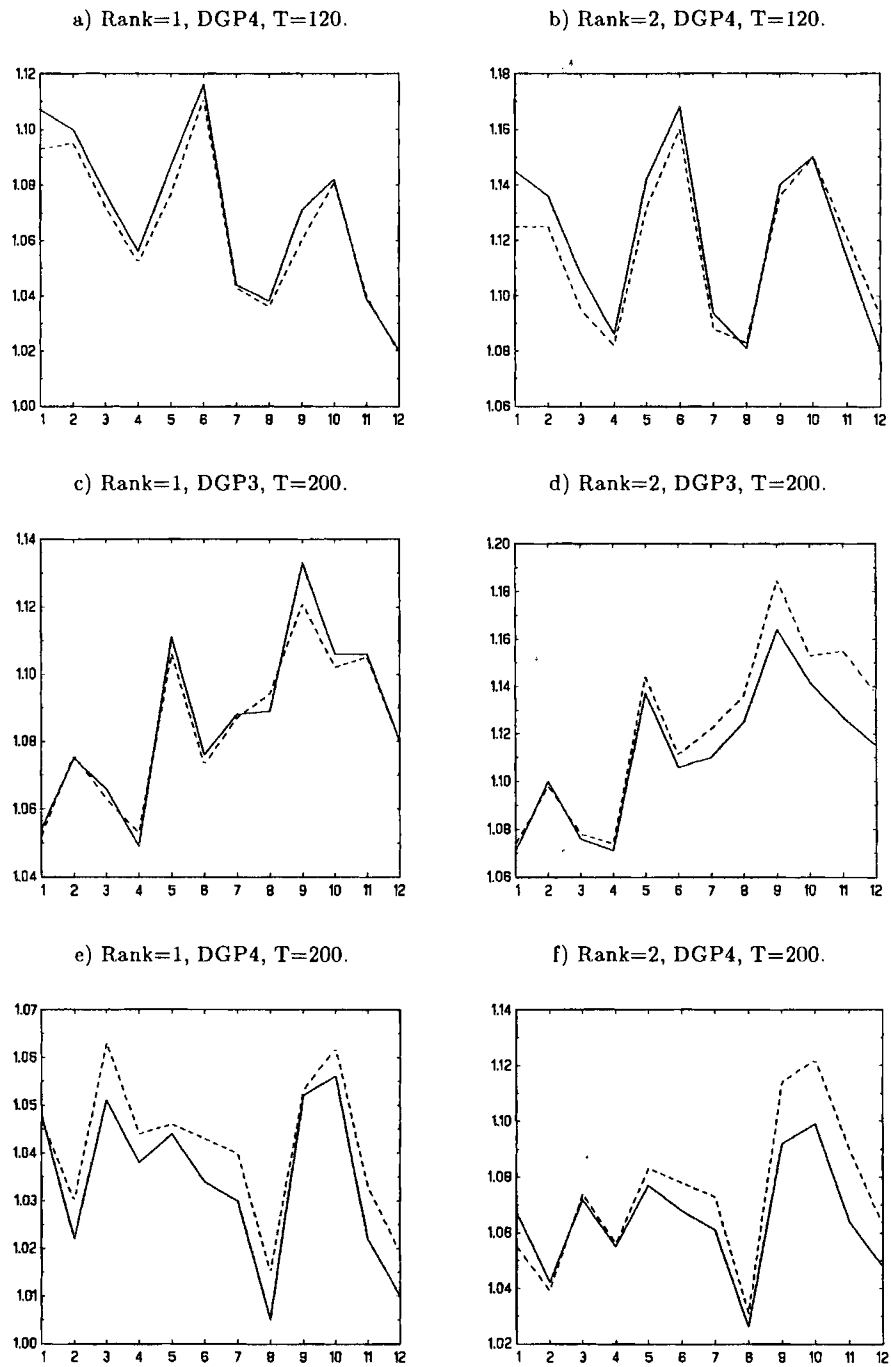


Figure 1.2 Mean squared errors weighted with variances. Dashed line: SCM1, solid line SCM3. Rank concerns all frequencies, T=120 and 200.



Paper 2

On forecasting cointegrated seasonal time series

2.1 Introduction

In recent years several methods for cointegration analysis of nonadjusted seasonal time series have been developed. If one is willing to assume that the seasonal pattern is approximately constant over time, the vector autoregressive error-correction model [VECM] for first differenced series with constant seasonal dummy parameters can be used. However, tests for various types of changing seasonality often find evidence of a stochastic or changing seasonal pattern over time, see Hylleberg *et al.* (1990) [HEGY], Franses (1996), among many others.

In this paper we examine in an empirical forecasting study the relevance of taking care of changing seasonality within multivariate methods for cointegrated seasonal time series. We evaluate three different approaches, see Franses and McAleer (1998), that is nonseasonal cointegration models, seasonal cointegration models and periodic cointegration models. As our empirical study concerns bivariate time series, we also consider single equation methods, and compare these with system methods.

Testing and estimating seasonal cointegration relations can be accomplished in at least two ways. Engle *et al.* (1993) [EGHL] propose a two-step procedure, which is an extension of the Engle and Granger test for cointegration, whereas Lee (1992) suggests a maximum likelihood method for seasonal cointegration using a seasonal error correction model. Johansen and Schaumburg (1998) introduce a general asymptotic theory for the latter cointegration approach. While cointegration at the zero frequency can be interpreted as evidence of parallel long-run movements among the time series considered, cointegration at the biannual or annual frequencies is viewed as evidence of parallel movements across the corresponding seasonal components of the time series.

An alternative model for changing seasonality in multivariate data extends the periodic integration model, see Franses (1996) and Boswijk, Franses and Haldrup (1997), among others. When the individual time series display periodic features, one may want to consider periodic cointegration. A useful single equation method is proposed in Boswijk and Franses (1995), and it is an extension of the cointegration test approach by Boswijk (1994). A multiple equations method is proposed by Kleibergen and Franses (1999), who consider cointegration in periodic VAR models [PVAR].

The forecasting performance of seasonal cointegration models has been analyzed in Kunst (1993) and in Reimers (1997). Two examples based on real data and a Monte Carlo experiment in Kunst (1993) indicate that the benefits from accounting for seasonal cointegration are quite limited as compared to vector error correction models in first differences with deterministic seasonal dummies included. The main conclusion in Reimers (1997) is that models in first differences produce smaller forecast errors for short horizons, but when longer forecasting periods are considered the seasonal cointegration model appears preferable. Finally, the forecasting performance using different specifications of the single equation periodic cointegration model has been examined in Herwartz (1997), where it is found not to be very successful.

The purpose of the present paper is to compare the two model classes in an empirical forecasting study, which involves seven sets of bivariate quarterly time series, which one may expect to be somehow cointegrated. We aim to shed light on the following issues. Do multiple equations methods for seasonal and periodic cointegration generate better forecasts as compared to their single equation counterparts? And, is one of the two model classes, that is seasonal versus periodic cointegration, preferable in terms of forecasting? We also compare these periodic and seasonal cointegration models with a VECM in first differences with constant seasonal dummy parameters and with an estimated long-run relation included. Additionally, in the analysis we also include a VAR model in first differences with constant seasonal dummy parameters and a VAR model in fourth differences with an intercept included, that is, two models without cointegration. The last model is included as its univariate counterpart has been found to be rather successful in terms of out-of-sample forecasting, even when in sample tests indicate that other models are to be preferred, see Clements and Hendry (1997) and Osborn, Heravi and

Birchenhall (1999).

The remainder of this paper is organized as follows. Section 2.2 gives a brief discussion of the various cointegration approaches. In Section 2.3, we present the data and Section 2.4 contains the estimation and forecasting results. The final section presents some concluding remarks.

2.2 Cointegration methods

In this section we review four different approaches to cointegration analysis of quarterly time series. Later on, we will contrast these approaches with a VECM with constant seasonal dummy parameters, where the long-run relation is estimated from the data for the bivariate series at hand and with VAR models in first and fourth differences without cointegration.

Sometimes it may be considered as useful to examine the properties of the univariate time series first. For this purpose one can use for example the HEGY (1990) testing approach for seasonal unit roots, and the Boswijk, Franses and Haldrup (1997) testing approach for unit roots in periodic models. In this paper we adopt the strategy that we straightaway consider multivariate models.

We now turn to the various cointegration methods, where we deal with periodic models first and then with seasonal cointegration models. Within each class, we first deal with the single equation approach and then with the multiple equation method.

2.2.1 Periodic cointegration - single equation

The Boswijk and Franses (1995) approach is an extension of the cointegration test approach in Boswijk (1994). They consider the following single equation periodic cointegration model [PCM]

$$\begin{aligned}\Delta_4 y_t &= \sum_{s=1}^{4}\mu_s D_{s,t} + \sum_{s=1}^{4}\lambda_s D_{s,t}(y_{t-4} - \theta_s' X_{t-4}) \\ &\quad + \sum_{j=1}^{p}\beta_i \Delta_4 y_{t-j} + \sum_{i=0}^{p}\tau_i' \Delta_4 X_{t-i} + \varepsilon_t, \qquad (2.1)\end{aligned}$$

where y_t, $t = 1, 2, ..., T$, is the quarterly observed univariate variable of specific interest and where $\Delta_4 y_t = (1 - B^4)y_t = y_t - y_{t-4}$. The error term ε_t is assumed to be a standard white noise process. Moreover, it is assumed that the vector of explanatory variables X_t is weakly exogenous. $D_{s,t}$ denotes the usual seasonal dummy variable, taking a value 1 in season s and a value 0 otherwise. If needed in (2.1), the lags of $\Delta_4 X_t$ can be replaced by lags of $\Delta_1 X_t$. The parameters λ_s and θ_s in equation (2.1) are seasonally varying adjustment and long-run parameters, respectively. Adjustment can be easier to achieve in some quarters, or economic agents may want to correct disequilibria faster in some seasons. In a consumption model context, the target relations may reflect seasonally varying preferences or seasonally varying availability of goods and services, see Birchenhall *et al.* (1989). Periodic cointegration requires that the λ_s parameters are negative. Full periodic cointegration in (2.1) implies that there is adjustment towards a long-run relationship in all four quarters, whereas partial periodic cointegration implies that there is no adjustment in some quarters.

Boswijk and Franses (1995) propose a Wald test for cointegration in the PCM. Consider the following, slightly rewritten form of (2.1), that is,

$$\begin{aligned}\Delta_4 y_t &= \sum_{s=1}^{4}\mu_s D_{s,t} + \sum_{s=1}^{4}(\delta_{1s} D_{s,t} y_{t-4} + \delta_{2s}' D_{s,t} X_{t-4}) \\ &+ \sum_{j=1}^{p}\beta_i \Delta_4 y_{t-j} + \sum_{i=0}^{p}\tau_i' \Delta_4 X_{t-i} + \varepsilon_t, \end{aligned} \qquad (2.2)$$

where $\delta_{1s} = \lambda_s$ and $\delta_{2s} = -\theta_s'\lambda_s$ in (2.1). Writing $\delta_s = (\delta_{1s}, \delta_{2s}')'$, the null hypothesis of no cointegration in season s is $H_{0s} : \delta_s = 0$, whereas the alternative is $H_{1s} : \delta_s \neq 0$ for some s, respectively. Writing $\delta = (\delta_1', ..., \delta_4')'$, the null hypothesis of no cointegration in any season is $H_0 : \delta = 0$, whereas the alternative is $H_1 : \delta \neq 0$, respectively. The two null hypotheses can be tested using *Wald* statistics, given by

$$\begin{aligned}Wald_s &= (T-m)((RSS_{0s} - RSS_1)/RSS_1), \\ Wald &= (T-m)((RSS_0 - RSS_1)/RSS_1), \end{aligned} \qquad (2.3)$$

where m is the number of parameters in equation (2.2). RSS_1 is the OLS residual sum of squares from the unrestricted model, while RSS_{0s} and RSS_0 are the residual sums of squares under H_{0s} and H_0, respectively. The relevant critical values of the *Wald* test statistics are given in Boswijk and Franses (1995). They also propose tests for various parameter restrictions when there is evidence of cointegration in two or more quarters, like tests for equality of the adjustment parameters and of the parameters concerning the long-run relationships across seasons. In the empirical part below we will refer to this method as the P-SE (periodic single equation) approach.

2.2.2 Periodic cointegration - multiple equations

A periodic VAR model of order 1 [PVAR(1)] of a k dimensional quarterly observed vector of time series Y_t can be written as (where we save on notation by deleting the seasonal dummies $D_{s,t}$):

$$Y_t = \varphi_s Y_{t-1} + u_t, \tag{2.4}$$

where $s = 1,...,S$, $t = 1,...,T$ and u_t is a vector of i.i.d. disturbances with mean zero and $E(u_t u_t') = \Omega_s$. The seasonally varying $(k \times k)$ parameter matrices φ_s are of full rank and the covariance matrices Ω_s imply that the PVAR(1) process can have different short-run properties across quarters. Even though all parameters are allowed to vary across the quarters, Kleibergen and Franses (1999) show that specification (2.4) imposes the same long-run properties for each season. Hence there can be $0 < r < k$ cointegrating relations between the k series.

If we determine the relationship between Y_t and Y_{t-S}, where the second series concerns the same season in the previous year, and rewrite the resulting expression in its nonseasonal annual form, see Tiao and Grupe (1980) and Osborn (1991), the model becomes

$$\Delta_1 Y_{s,n} = \Pi_s Y_{s,n-1} + \varepsilon_{s,n}, \qquad n = 1,...,N = \frac{T}{4}, \tag{2.5}$$

with Δ_1 for annual data corresponding with Δ_4 for quarterly data, and where $Y_{s,n}$ is the observation in season s in year n. Notice that (2.4) implies that

$$\Pi_s = \left(\prod_{i=1}^{s} \varphi_{s-i+1} \prod_{i=1}^{S-s} \varphi_{S-i+1} \right) - I_k. \tag{2.6}$$

If there is cointegration amongst the elements in Y_t then

$$\begin{aligned} \Pi_s &= \alpha_s \beta_s' \Longleftrightarrow \\ \varphi_{s+1} \alpha_s \beta_s' \varphi_{s+1}^{-1} &= \alpha_{s+1} \beta_{s+1}'. \end{aligned} \tag{2.7}$$

The result in (2.7) implies that the spaces spanned by α_{s+1} and β_{s+1} are identical to the spaces spanned by $\varphi_{s+1}\alpha_s$ and $\varphi_{s+1}^{-1'}\beta_s$, and this shows that the PVAR model as represented in (2.4) assumes $0 < r < k$ cointegration relations amongst the elements in Y_t.

Estimation of the cointegration parameters in Π_s is not straightforward as it implies nonlinear restrictions on the φ_s parameters. Kleibergen and Franses (1999) therefore suggest an alternative method, which amounts to optimizing a log-likelihood based objective function. This objective function can also be used to test for the number of cointegration relations. The resulting test statistic can be compared to the relevant critical values of the Johansen trace statistic, at least asymptotically. The PVAR(1) model can be extended to include unrestricted seasonally varying intercepts and trends. In the empirical part below we will denote the model with seasonally varying intercepts included as P-ME-1 (periodic multiple equations of type 1) and the model with both seasonally varying intercepts and trends as P-ME-2.

Finally, the results for the PVAR(1) model hold for higher order dynamics if we assume the following lag structure of a PVAR(p) model (for quarterly data)

$$Y_t = \varphi_{1,s}Y_{t-1} + \varphi_{2,s}Y_{t-5} + ... + \varphi_{p,s}Y_{t-(p-1)S-1} + u_t \tag{2.8}$$

In Kleibergen and Franses (1999) it is argued that this particular representation allows for rather straightforward inference on the cointegration relations. This is due to the fact that one considers S different models for annual data.

2.2.3 Seasonal cointegration - single equation

Engle *et al.* (1993) EGHL propose a two-step test method for seasonal and nonseasonal cointegration, which is similar to the Engle and Granger (1987) test procedure for zero frequency cointegration. In the case of cointegration at all frequencies for bivariate time series involving y_t and x_t, the following linear combinations will be stationary processes:

$$\begin{aligned} z_{1,t} &= y_{1,t} - \alpha_1 x_{1,t}, \\ z_{2,t} &= y_{2,t} - \alpha_2 x_{2,t}, \\ z_{3,t} &= y_{3,t} - \alpha_3 x_{3,t} - \alpha_4 y_{3,t-1} - \alpha_5 x_{3,t-1}, \end{aligned} \tag{2.9}$$

where $w_{1,t} = (1 + B + B^2 + B^3)w_t$, $w_{2,t} = -(1 - B + B^2 - B^3)w_t$ and $w_{3,t} = -(1 - B^2)w_t$, for $w_t = y_t$ and x_t.

The first step involves estimating α_1 to α_5 in (2.9) using OLS, where deterministic components such as a constant, a trend and seasonal dummies may be included in these cointegration regressions. The second step amounts to checking whether the resulting estimated residuals $\widehat{z}_{1,t}$ to $\widehat{z}_{3,t}$ are stationary, using the following auxiliary regressions:

$$
\begin{aligned}
(1-B)\widehat{z}_{1,t} &= \pi_1 \widehat{z}_{1,t-1} + \sum_{i=1}^{l_1} \gamma_i (1-B)\widehat{z}_{1,t-i} + \varepsilon_t, \qquad (2.10)\\
(1+B)\widehat{z}_{2,t} &= \pi_2(-\widehat{z}_{2,t-1}) + \sum_{i=1}^{l_2} \gamma_i (1+B)\widehat{z}_{2,t-i} + \varepsilon_t,\\
(1+B^2)\widehat{z}_{3,t} &= \pi_3(-\widehat{z}_{3,t-2}) + \pi_4(-\widehat{z}_{3,t-1}) + \sum_{i=1}^{l_3} \gamma_i (1+B^2)\widehat{z}_{3,t-i} + \varepsilon_t,
\end{aligned}
$$

Cointegration at the zero and biannual frequencies implies $\pi_1 = 0$ and $\pi_2 = 0$, respectively, which is tested against the alternative that $\pi_i < 0$, $i = 1, 2$ with one-sided t-tests. If the F-test for the hypothesis $\pi_3 = \pi_4 = 0$ cannot be rejected there is evidence of cointegration at the annual frequency. The t-statistics for π_1 and π_2 follow the familiar Dickey-Fuller distribution and critical values can be found in Phillips and Ouliaris (1990). Critical values for the F-statistic are tabulated in EGHL.

If there is cointegration at all frequencies and x_t is weakly exogenous, a final seasonal cointegration equation for y_t reads as

$$
\begin{aligned}
\Delta_4 y_t &= \sum_{j=0}^{q} \delta_j \Delta_4 x_{t-j} + \sum_{i=1}^{p} \beta_i \Delta_4 y_{t-i}\\
&\quad + \gamma_{11} z_{1,t-1} + \gamma_{12} z_{2,t-1} - \gamma_{13} z_{3,t-2} - \gamma_{14} z_{3,t-3} + \varepsilon_t. \qquad (2.11)
\end{aligned}
$$

For further reference, we call this model the S-SE model.

2.2.4 Seasonal cointegration - multiple equations

Lee (1992) suggests a maximum likelihood estimator for seasonal cointegration relations, based on a fully specified VAR model. This procedure extends the approach summarized in Johansen (1995). Assuming that Y_t is seasonally integrated of order 1, where Y_t again denotes a $(k \times 1)$ vector of variables, a seasonal error correction model [SECM] of the following form is considered:

$$
\Delta_4 Y_t = \sum_{i=1}^{4} \alpha_i \beta_i' Z_{i,t} + \sum_{j=1}^{p} \Gamma_j \Delta_4 Y_{t-j} + \Phi D_t + \varepsilon_t, \qquad (2.12)
$$

where $Z_{1,t} = (B + B^2 + B^3 + B^4)Y_t$, $Z_{2,t} = (B - B^2 + B^3 - B^4)Y_t$, $Z_{3,t} = (B^2 - B^4)Y_t$ and $Z_{4,t} = (B - B^3)Y_t$. D_t are deterministic components, possibly including seasonal dummies and trends, and ε_t is i.i.d. $N_k(0, \Omega)$. There is seasonal cointegration if at least one of the $\alpha_i \beta_i'$ matrices for $i = 2, 3, 4$ on the right-hand side has reduced, but non-zero, rank. The linear filters $Z_{i,t}$ in (2.12) remove all unit roots except those at the zero, biannual and annual frequencies for $i = 1, 2, (3, 4)$, respectively. If the matrices $\alpha_i \beta_i'$ have reduced rank, $\beta_i' Z_{i,t}$ is stationary even though the processes $Z_{i,t}$ are nonstationary. Furthermore, the regressors $Z_{i,t}$ are asymptotically uncorrelated, that is

$$T^{-2} \sum_{t=1}^{T} Z_{i,t} Z_{j,t}' \xrightarrow{P} 0, i \neq j,$$

implying that the cointegration vectors and adjustment coefficients can be found by removing the reduced rank restriction on the other frequencies by concentrating out the associated regressors. Lee (1992) suggests the restriction $\alpha_4 \beta_4' = 0$, and we label this method, with unrestricted seasonal dummies included, as S-ME-1.

Franses and Kunst (1999) argue that deterministic seasonal dummy variables, which are often included unrestrictedly in (2.12) to handle the deterministic part of seasonality, should be confined to the seasonal cointegrating relations instead. This is because unrestricted seasonal intercepts in the SECM may lead to diverging seasonal trends, which can be unlikely in certain practical cases. This restricted seasonal dummies case is denoted as S-ME-2.

Finally, Johansen and Schaumburg (1998) argue that the restriction $\alpha_4 \beta_4' = 0$ is very strong and not justified from a theoretical point of view. They refine the asymptotic theory for the multivariate seasonal cointegration model and propose an alternative estimation procedure for the parameters corresponding to the annual frequency. In the forecasting study below we label this third method, where we again include restricted seasonal dummies, as S-ME-3.

2.3 Data

In our forecasting study we consider the logs of quarterly observed time series on consumption and income in the United Kingdom, Sweden, (Western-) Germany, Japan, Italy and the US. The data set for UK covers the time period 1955:1 to 1989:4 (consumption on non-durables and disposable income), whereas the time series for Sweden ranges from the period 1963:1 to 1988:4 (consumption on non-durables and disposable income) and for Germany from 1960:1 to 1988:4 (consumption and disposable income). The data set for Japan covers the period 1961:1 to 1987:4 (total consumption and disposable income) and the data set for Italy the period 1970:1 to 1996:1 (consumption on non-durables and services, and GDP). The data set for the US covers the period

1947:1 to 1991:4 (consumption on non-durables and GNP) [henceforth the US1 data set]. We also examine the nominal money stock and real GNP in the US for the period 1948:1 to 1985:4 [henceforth the US2 data set]. We denote the consumption and money stock series as y_t and the income series as x_t

These bivariate time series have been analyzed previously in Boswijk and Franses (1995), Cubadda (1999), Engle *et al.* (1993), Franses and Paap (1995), Hylleberg *et al.* (1990), Lee and Siklos (1997) and Wells (1997). Note that we discard 16 observations at the end of each sample in order to evaluate the out-of sample forecasting performance of the various models discussed in the previous section.

2.4 Empirical results

In this section we first discuss the in-sample estimation results and then we turn to the out-of-sample forecasting results. To save space we mainly show the test results for cointegration and the forecasting results. Other results can be obtained from the authors upon request. Also, we always consider the 5% significance level.

2.4.1 Estimation results

At first we have a quick look at the results obtained from applying HEGY tests for seasonal unit roots in the univariate series. The auxiliary regressions include an intercept, seasonal dummies and a deterministic trend in each case. We find that all variables seem to contain a unit root at the zero frequency. The results are more mixed at the seasonal frequencies. The annual difference filter seems to be rejected for income in some data sets. In general there is substantial evidence of changing seasonal patterns, and it seems of interest to see if the series with changing seasonality are somehow cointegrated.

Periodic cointegration models

Results of the Wald-tests for periodic cointegration in the single equation case appear in Table 2.1. All the dependent variables (the consumption series and the money stock series) are transformed into fourth differences. We also choose the fourth difference filter for income in all equations. We find no evidence of periodic cointegration in the US1 and the US2 data, but the Wald-test statistics suggest cointegration in the first quarter in Germany, in the second quarter for Sweden and the UK and in the third quarter for Italy. Although the evidence of cointegration may be viewed as weak for Japan, we also proceed with the estimation of a partial PCM with cointegration in the first quarter for Japan.

The relevant estimated adjustment or error-correction terms $\widehat{\lambda}_s$ and the coefficients for the long run relations $\widehat{\theta}_s$ are all significant at the 5% level. Comparisons with previous analysis of similar time series can be done for most cases, although we use a shorter sample period here because sixteen observations are discarded. The results for Germany are in line with the results found in Franses and Paap (1995), where they, using other cointegration methods, find evidence of cointegration between consumption and income in the first quarter only. Boswijk and Franses (1995) examine the same data series for Sweden and find cointegration in the second and fourth quarter.

Finally, we also test for weak exogeneity. Adding the estimated cointegration relations to AR(p) models of annual differenced income variables suggests that the assumption of weak exogeneity seems to be valid in all the examples.

Results using the Kleibergen and Franses (1999) approach to multiple equations periodic models are summarized in Table 2.2. Again, the lag lengths are chosen according to the AIC and BIC criteria, in addition to equation by equation diagnostic tests. We include seasonally varying intercepts (P-ME-1) or, alternatively, seasonally varying intercepts and trends (P-ME-2). In the first case we find evidence of periodic cointegration in the German and the US1 data sets only, as the bivariate series seem stationary in the other cases. When we include seasonally varying trends in the equations there is evidence of cointegration for Japan and the US2 data set as well. Critical values are based on our own simulations for small samples.

Seasonal cointegration models

The tests for seasonal cointegration using the two-step procedure proposed by EGHL are summarized in Table 2.3, where a constant and a trend are included in the cointegration regression if the zero frequency is considered and a constant and seasonal dummies if the seasonal frequencies are considered. There is no evidence of cointegration at the zero frequency, except between money and GNP in the US. We find no evidence of cointegration at the biannual frequency, but for the annual frequency we do so for Germany, Japan and Sweden.

EGHL analyze the consumption and income data for Japan using the same techniques but again with 16 more observations. They argue that the absence of cointegration at the zero and biannual frequencies cannot be rejected and that a question of whether there is cointegration at the annual frequency could be answered with a weak 'maybe'. Lee and Siklos (1997) use the two-step approach to M1 and real GNP in the US, but their results are quite different from ours. Instead of finding evidence of cointegration at the zero and annual frequencies, as we do, they only reject the null hypothesis of no-cointegration at the biannual frequency.

When estimating the final EGHL equations (2.11) with the cointegrating relations at the annual frequency included, only the coefficients for $z_{3,t-2}$ appear to be significant at the 5%-level. Finally, when testing for weak exogeneity, we add the estimated cointegration relations to univariate AR models for $\Delta_4 y_t$. The assumption of weak exogeneity seems to be valid for Germany and Sweden, but not for Japan.

Results of the maximum likelihood seasonal cointegration approach can be found in Table 2.4. The lag lengths are again chosen using the AIC and BIC, in addition to equation by equation diagnostic tests. Lee and Siklos (1995) present critical values for all frequencies in the unrestricted seasonal intercepts case (S-ME-1). Critical values for the restricted seasonal intercept case (S-ME-2) are taken from Tables 1a-1f in Franses and Kunst (1999). Finally, Johansen and Schaumburg (1998) present critical values for the annual frequency in the restricted intercept case (S-ME-3). Note that for the zero frequency case, we use the critical values tabulated in Lee and Siklos (1995) for all model specifications. For the biannual frequency case, we use the critical values tabulated in Franses and Kunst (1999) for specifications S-ME-2 and S-ME-3.

Starting with the results when using unrestricted intercepts, there is evidence of cointegration at the zero frequency for the Italian, Swedish and the US1 data sets. The results further suggest one cointegration vector at the biannual frequency in Germany, UK and again in the US1 data set. For all countries, except for Italy, there is evidence of stationary vectors at the annual frequency, while the results for the UK even suggest two vectors at this frequency.

Turning to the case with restricted seasonal intercepts we see that there is now no evidence of cointegration at frequencies π and $\pi/2$ in the German data set, which is also the case when using the method proposed by Johansen and Schaumburg (1998). The single cointegrating vector at the annual frequency in Sweden is not significant when using restricted seasonal intercepts in the Lee specification. The reverse result is true for the cointegrating vector at $\pi/2$ for Japan.

If we compare these findings with the results obtained using the two-step EGHL procedure, the results are quite different. We find almost no evidence of cointegration at the zero frequency and at the biannual frequency, using the EGHL approach, but we do so using multiple equation methods. However, for the annual frequency we find similar results across the two approaches.

Nonseasonal models

Finally, we include in our forecasting study three nonseasonal models. The first is a vector error correction model [VECM] in first differences, where the cointegration relationship is estimated using the familiar Johansen maximum likelihood method. Table 2.5 summarizes the test results and we observe that zero frequency cointegration is found for the Italian, Swedish and US1 data sets. For further reference, we will denote these models as N-ME-1. We also consider VAR models in first differences and these will be denoted as N-ME-2, that is, in this model no cointegration is assumed. In both models we include deterministic seasonal dummies. Finally, we include VAR models in annual differences with intercepts included, denoted N-ME-3.

2.4.2 Forecasting

We now turn to a discussion of the out-of-sample forecasting exercise, where it is our aim to forecast consumption and money, given past observations on these two variables and on income.

Method

In general, if we do not find any evidence of cointegration using a particular model type for a specific data set, where we base our decision on the 5% significance level, we do not generate any forecasts. Only for the VAR models in first and annual differences without cointegration, that is N-ME-2 and N-ME-3, we always generate forecasts.

To be able to compare the single equation methods with the system approaches when it comes to forecasting, we use the weak exogeneity test equations for income, but now without the cointegration relations added. For the P-SE and S-SE methods, this approach results in a two-equation system.

For the periodic VAR model, we forecast from (2.8) in levels with seasonally varying deterministic terms included, when there is evidence of full rank. When there is evidence of a reduced rank, we include the seasonally varying Π_s-matrices and generate forecasts from the resulting model in fourth differences, and then transform these to levels.

If we do not find any evidence of cointegration at one or two frequencies using the three versions of the multiple equation seasonal cointegration approach, we set the rank of corresponding $\Pi-$ matrices equal to zero and generate forecasts from the resulting model.

We start with forecasting the relevant time series sixteen steps ahead. Next, the estimation period is extended by one quarter and all the equations are reestimated. We do however not reestimate the lag orders and the cointegration ranks, and hence we keep the model structure fixed. This procedure is repeated fifteen times until we generate the last one-step ahead forecast error for each model and data set. This procedure leads to sixteen one-step ahead forecast errors for each model and data set, fifteen two-step ahead forecast errors and so on. In total we have 136 forecasts for each model and data set. The results are summarized by presenting values of the root mean squared prediction errors RMSPE (times 100) for one, four and eight steps ahead forecasts in Tables 2.6, 2.7 and 2.8, respectively. In Table 2.9, we summarize all forecast errors by considering the RMSPE concerning all 136 forecasts. In Table 2.10 we summarize all results by giving the average ranks of the ten different approaches across the seven data sets. Finally, in Table 2.11 we present the average ranks when only the cointegration models are considered.

Results

The results in Table 2.6 (for one-step ahead forecast errors) and in the first column of Table 2.10 are quite clear. The VAR model without cointegration for the data in first differences clearly gives the best one-step ahead forecasts. The average rank of N-ME-2 is 2.0, and its closest competitor is a VAR model in annual differences with average rank 2.9. The N-ME-2 model offers the lowest RMSPE values for 4 out of 7 data sets In the first column of Table 2.11 we observe that the seasonal cointegration models yield the best one-step ahead forecasts and that S-ME-3 offers the lowest average rank.

When we consider the results in Table 2.7 and those in the second column of Table 2.10, we observe that the results for one-step-ahead forecasts extend to the case of four-step ahead forecasts. Again the VAR models without any cointegration yield the best forecasts, but now it is the VAR model in annual differences which yields the best forecasts. The VAR model in first differences comes as a good second. Among the cointegration models (see the second column of Table 2.11) we see that one of the multiple equations periodic cointegration models (P-ME-2) now yields the lowest average rank.

The results in Table 2.8 and the third column of Table 2.10 indicate that for longer forecast horizons, the VAR model in annual differences again improves upon other models and its closest competitor is the VAR model in first differences. S-ME-3 offers the lowest average rank among the cointegration models, see the third column of Table 2.11.

When we average over all forecasts as in Table 2.9, and in the final column of Table 2.10, we find similar results. Interestingly the differences across the close competitors seem negligible (3 models with average rank 4.0).

When we compare the performance of the single equation methods with that of multiple equations methods, we do not find clear-cut signs that one approach consistently outperforms the other.

In sum, our empirical findings suggest that the VAR model in annual differences is preferred, except for one-step ahead forecasts where the VAR model in first differences offers the lowest average rank. The finding that the VAR model in annual differences yield the lowest RMSPEs forecasts can be viewed as extending the results found by Clements and Hendry (1997) where they obtain similar results for univariate data.

It also seems that the two multiple equations seasonal cointegration models with restricted seasonal intercepts included, that is S-ME-2 and S-ME-3, in most cases improve upon the model with unrestricted seasonal intercepts (S-ME-1). This result is in line with that found in Kunst and Franses (1998). Finally, looking again at Table 2.11, we observe that the P-ME-2 and S-ME-2(3) models often are close competitors.

2.5 Concluding remarks

We have analyzed periodic and seasonal cointegration models for bivariate quarterly observed time series in an empirical forecasting study. We included both single equation and multiple equations methods. A VAR model in first differences, with and without cointegration, and a VAR model in annual differences were also included in the analysis, where they served as benchmarks. Our empirical results indicate that the VAR model in annual differences is often preferred, except for one-step ahead forecasts where the VAR model in first differences, without cointegration, offers the lowest RMSPEs. The seasonal cointegration models yield the best forecasts in general if one only considers the cointegrated models, even though the periodic models seem to be a better choice for various specific data sets. Finally, there is no clear indication that multiple equations methods improve on single equation methods.

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Tables

Table **2.1** Testing for cointegration in P-SE.

	GER	ITA	JAP	SWE	UK	US1	US2
$Wald_1$	17.00*	2.98	10.40	6.41	1.26	7.47	1.96
$Wald_2$	4.82	7.03	9.07	17.52*	12.11*	3.51	5.21
$Wald_3$	5.89	12.49*	9.12	3.03	5.47	5.81	5.23
$Wald_4$	3.11	1.91	2.06	7.82	0.45	4.26	0.76
Wald	19.64	16.10	21.05	19.35	16.94	16.75	9.60
p							
$\Delta_4 y_t$	1-5	1-3	1,4	1,4	1,5	1,4	1,4-5
$\Delta_4 x_t$	1	-	3,4	-	1-3	1,3-5	-

* Significant at the 5% level. An intercept, $\Delta_4 x_t$ and seasonal dummies are included in each test equation. The test equation is (2.2). The lag length is established using AIC and BIC as well as diagnostic tests for residual autocorrelation.

Table **2.2** Testing for cointegration in P-ME-1 and P-ME-2.

	GER	ITA	JAP	SWE	UK	US1	US2
H_0:	P-ME-1						
$r = 0$	41.46*	877.0*	43.28*	328.7*	78062*	56.51*	20.70
$r \leq 1$	1.72	7.56*	9.19*	30.48*	17.85*	3.25	7.73
H_0:	P-ME-2						
$r = 0$	40.33	96.99*	73.84*	62029*	38346*	233.9*	79.55*
$r \leq 1$	1.29	7.25*	0.70	9.68*	751*	2.71	1.60
p							
P-ME-1(2)	2	2	2	3	3	4	3

* Significant at the 5% level. P-ME-1 indicates a PVAR model with seasonally varying intercept included, while P-ME-2 indicates a PVAR model with both seasonally varying intercept and trend included. Critical values are based on our own simulations. Lags correspond to the order p of the PVAR model, as given in (2.8).

Table 2.3 Testing for cointegration in S-SE.

	GER	ITA	JAP	SWE	UK	US1	US2
$t_{\hat{\pi}_1}$	-2.18	-1.93	-1.83	-3.33	-2.74	-2.64	-3.95*
$t_{\hat{\pi}_2}$	-0.76	-1.17	-1.41	-2.09	-1.62	-2.10	-1.87
$F_{\hat{\pi}_3 \cap \hat{\pi}_4}$	13.3*	5.41	17.60*	14.90*	7.67	4.78	3.74
l_1	1,3-5	1-2	1,4-5	1-2	1-4	1-2,4-5	1,4-5
l_2	1-4	1-3	1-4,7	1-2,4	1-2,4-6	1-5	1,4-6,9
l_3	1,4	1-3	1-3	3-4	1,3-4	1-2,4-5	1-7

* Significant at the 5% level. Intercept and trend are included in the cointegration regression for the zero frequency. Intercept and seasonal dummies are included in the cointegration regression for the seasonal frequencies. Lags are selected using AIC and BIC as well as diagnostic tests for residual autocorrelation. The test regressions as displayed in (2.10).

Table 2.4 Testing for cointegration in S-ME-1, S-ME-2 and S-ME-3.

Model	H_0:	GER $p=2$	ITA $p=2$	JAP $p=3$	SWE $p=0$	UK $p=0$	US1 $p=1$	US2 $p=2$
				Zero frequency				
S-ME-1	$r=0$	6.56	27.73*	14.60	19.11*	17.60	28.52*	10.56
	$r\leq 1$	2.32	4.67	0.86	4.35	0.01	2.25	3.03
S-ME-2	$r=0$	6.56	27.73*	14.60	19.11*	17.60	28.52*	10.56
	$r\leq 1$	2.32	4.67	0.86	4.35	0.01	2.25	3.03
S-ME-3	$r=0$	6.56	27.73*	14.60	19.11*	17.60	28.52*	10.56
	$r\leq 1$	2.32	4.67	0.86	4.35	0.01	2.25	3.03
				Biannual frequency				
S-ME-1	$r=0$	32.75*	11.99	10.34	13.71	28.51*	21.05*	15.97
	$r\leq 1$	3.77	5.08	1.60	3.44	8.10	7.43	5.62
S-ME-2	$r=0$	15.04	9.56	4.80	13.50	28.41*	20.91*	14.94
	$r\leq 1$	3.98	2.06	0.93	3.63	8.20	7.25	4.21
S-ME-3	$r=0$	15.04	9.56	4.80	13.50	28.41*	20.91*	14.94
	$r\leq 1$	3.98	2.06	0.93	3.63	8.20	7.25	4.21
				Annual frequency				
S-ME-1	$r=0$	22.78*	12.36	35.14*	24.96*	75.37*	36.75*	28.70*
	$r\leq 1$	6.25	0.64	5.74	7.39	21.09*	4.34	4.80
S-ME-2	$r=0$	21.83	12.69	27.75*	25.29	75.29*	36.71*	28.92*
	$r\leq 1$	5.45	1.88	5.74	7.38	21.03*	4.43	5.04
S-ME-3	$r=0$	30.76	40.42*	32.69	38.00*	108.4*	84.35*	61.03*
	$r\leq 1$	11.27	23.48*	7.13	13.18	26.73*	35.91*	22.37*

* Significant at the 5% level. S-ME-1 denotes the Lee (1992) specification with unrestricted seasonal intercepts, S-ME-2 denotes the same specification with restricted seasonal intercepts, S-ME-3 denotes the Johansen and Schaumburg (1998) specification with restricted seasonal intercepts.

Table 2.5 Nonseasonal models for first differences (N-ME-1 and N-ME-2) and for annual differences (N-ME-3).

	GER	ITA	JAP	SWE	UK	US1	US2
H_0:	N-ME-1						
$r=0$	11.70	27.73*	10.32	19.11*	17.60	21.07*	10.83
$r \leq 1$	1.94	4.67	0.39	4.35	0.01	4.35	1.14
p							
N-ME-1(2)	4	5	5	3	3	5	4
N-ME-3	5	3	5	2	3	5	2

* Significant at the 5% level. Lags are selected using AIC and BIC and diagnostic tests for residual autocorrelation. An intercept and seasonal dummies are included in N-ME-1 and N-ME-2. For N-ME-3 we only include an intercept. We use the familiar Johansen trace test statistic for zero frequency cointegration.

Table 2.6 Root mean squared prediction errors (RMSPE*100): one-step ahead forecasts. The smallest RMSPE for each data set is underlined.

	GER	ITA	JAP	SWE	UK	US1	US2
P-SE	1.248	0.448	1.049	1.616	2.720	-	-
P-ME-1	2.250	0.566	0.766	2.311	2.036	1.250	-
P-ME-2	-	0.547	0.977	1.372	2.028	1.217	1.779
S-SE	1.294	-	0.940	1.540	-	-	-
S-ME-1	1.211	-	0.659	1.739	2.306	0.962	1.488
S-ME-2	-	-	0.741	-	2.292	0.966	1.463
S-ME-3	-	0.384	-	1.753	1.889	1.000	1.328
N-ME-1	-	<u>0.373</u>	-	1.816	-	1.001	-
N-ME-2	<u>1.101</u>	0.401	<u>0.640</u>	1.654	1.806	<u>0.791</u>	<u>1.222</u>
N-ME-3	1.285	0.404	0.750	<u>1.178</u>	<u>1.405</u>	0.814	1.472

Table 2.7 Root mean squared prediction errors (RMSPE*100): four-step ahead forecasts. The smallest RMSPE for each data set is underlined.

	GER	ITA	JAP	SWE	UK	US1	US2
P-SE	2.094	1.588	<u>0.898</u>	1.690	4.858	-	-
P-ME-1	2.447	<u>1.155</u>	1.218	3.398	3.668	1.903	-
P-ME-2	-	1.122	1.898	2.182	3.556	1.139	<u>3.557</u>
S-SE	2.107	-	1.584	1.301	-	-	-
S-ME-1	<u>1.668</u>	-	1.253	1.603	3.859	1.343	3.922
S-ME-2	-	-	1.219	-	3.871	1.344	3.932
S-ME-3	-	1.311	-	1.552	3.741	1.555	3.834
N-ME-1	-	1.270	-	1.762	-	1.734	-
N-ME-2	2.013	1.568	1.349	1.411	3.665	1.273	3.800
N-ME-3	2.101	1.273	1.420	<u>1.229</u>	<u>2.468</u>	<u>1.117</u>	3.999

Table 2.8 Root mean squared prediction errors (RMSPE*100): eight-step ahead forecasts. The smallest RMSPE for each data set is underlined.

	GER	ITA	JAP	SWE	UK	US1	US2
P-SE	3.873	2.559	1.516	2.494	8.064	-	-
P-ME-1	3.674	<u>1.598</u>	<u>1.448</u>	4.337	8.030	3.367	-
P-ME-2	-	1.889	3.795	3.027	8.019	<u>1.721</u>	<u>5.234</u>
S-SE	4.081	-	2.952	2.005	-	-	-
S-ME-1	<u>3.278</u>	-	2.569	2.788	7.561	2.700	5.498
S-ME-2	-	-	2.490	-	7.575	2.781	5.508
S-ME-3	-	2.165	-	2.697	7.412	3.145	5.454
N-ME-1	-	2.035	-	3.076	-	3.475	-
N-ME-2	3.893	3.261	3.043	2.009	7.324	2.137	5.351
N-ME-3	4.167	2.051	2.981	<u>1.817</u>	<u>5.714</u>	2.004	5.588

Table 2.9 Root mean squared prediction errors (RMSPE*100): all forecasts. The smallest RMSPE for each data set is underlined.

	GER	ITA	JAP	SWE	UK	US1	US2
P-SE	3.746	2.588	1.591	2.571	7.629	-	-
P-ME-1	3.479	<u>1.801</u>	<u>1.337</u>	4.104	7.502	3.002	-
P-ME-2	-	2.060	3.649	2.783	6.797	<u>1.504</u>	<u>4.052</u>
S-SE	3.967	-	2.841	1.957	-	-	-
S-ME-1	<u>3.234</u>	-	2.623	2.940	6.580	2.655	5.824
S-ME-2	-	-	2.590	-	6.572	2.668	5.817
S-ME-3	-	2.051	-	2.947	6.419	2.922	5.886
N-ME-1	-	1.925	-	3.090	-	3.206	-
N-ME-2	3.861	2.941	3.018	2.018	6.298	1.887	5.799
N-ME-3	4.093	2.174	3.015	<u>1.506</u>	<u>5.278</u>	1.791	5.874

Table 2.10 Average ranks. All models included. Averaged over data sets. The smallest rank is underlined.

Method	1-step	4-step	8-step	All steps
P-SE	5.6	5.0	4.6	4.4
P-ME-1	6.7	5.0	4.5	4.7
P-ME-2	5.3	3.8	4.2	4.2
S-SE	4.7	4.7	4.0	4.0
S-ME-1	4.2	4.0	3.8	4.0
S-ME-2	4.0	5.0	4.5	3.8
S-ME-3	3.8	4.6	4.4	4.8
N-ME-1	5.0	5.7	6.3	6.0
N-ME-2	2.0	3.4	4.0	4.0
N-ME-3	2.9	3.3	3.7	3.9

Table 2.11 Average ranks. Only cointegration models included. Averaged over data sets. The smallest rank is underlined.

Method	1-step	4-step	8-step	All steps
P-SE	3.8	3.6	3.6	3.4
P-ME-1	4.7	3.7	3.5	3.7
P-ME-2	3.7	2.8	3.2	3.2
S-SE	3.0	3.0	3.3	3.3
S-ME-1	3.0	3.4	3.2	3.4
S-ME-2	2.5	3.8	3.3	2.5
S-ME-3	2.4	3.0	2.8	3.4
N-ME-1	3.7	4.3	5.0	4.7

Paper 3

Size and power of the likelihood ratio test for seasonal cointegration in small samples: A Monte Carlo study

3.1 Introduction

Testing procedures for seasonal unit roots have now been available for quite some time and the empirical evidence for macroeconomic time series often suggests the presence of unit roots at both the zero and the seasonal frequencies, implying a changing seasonal pattern over time. Testing for seasonal cointegration can be accomplished in at least two ways. Engle *et al.* (1993) propose a two-step procedure, which is similar to the Engle and Granger (1987) test for zero frequency cointegration. Lee (1992) suggest a maximum likelihood procedure, and the analysis here will follow that route.

In the original Lee (1992) specification of the seasonal error correction model [henceforth SECM] a certain restriction at the complex root frequency (see discussion below) are suggested. The restriction at frequency $\pi/2$, assumes the absence of so called *non-synchronous* seasonal cycles but the testing procedure for the number of cointegrating (CI) vectors at frequency $\pi/2$ becomes the same as for the zero and biannual frequencies, see for example Kunst (1993b), Franses and Kunst (1999) and Kunst and Franses (1998). Johansen and Schaumburg (1998) argue that this restriction is too strong and not justified from a theoretical point of view. They consider the general case, which results in a less straightforward testing and estimation procedure at the annual frequency. Here, the restricted (RS) and the general (GS) specifications will be compared.

The purpose of this paper is to explore how well the likelihood ratio (LR) test for the cointegrating rank works in the SECM, assuming quarterly data and small samples. The paper sheds some light on the following issues: 1) Are there any differences across frequencies, i.e. is there any evidence that the LR test for the rank is less powerful at either of the two seasonal frequencies than at the zero frequency? 2) Can RS be a useful tool even in cases where non-synchronous seasonal cycles are present? 3) How well does GS work in cases where the restriction is valid? 4) How does the number of CI relations at the zero and the biannual frequencies affect the test for the rank at the annual frequency and vice versa in small samples?

The paper is organized as follows: Section 3.2 presents the two specifications of the SECM together with a brief discussion on tests for cointegration. Section 3.3 introduces the Monte Carlo setup and clarifies the choice of the various data generating processes (DGP). Section 3.4 presents the results, while Section 3.5 concludes.

3.2 The model and the LR test.

Lee (1992), presented a maximum likelihood (ML) procedure for testing for seasonal cointegration in the quarterly case, analogous to the ordinary zero frequency case (see e.g. Johansen (1995)). The following model was suggested:

$$\Delta_4 X_t = \sum_{i=1}^{3} \Pi_i X_{it} + \varepsilon_t, \tag{3.1}$$

where:

$$\begin{aligned} X_{1t} &= (L + L^2 + L^3 + L^4)X_t \\ X_{2t} &= (L - L^2 + L^3 - L^4)X_t \\ X_{3t} &= (L^2 - L^4)X_t. \end{aligned} \tag{3.2}$$

One filter that appears in the auxiliary regression of the HEGY-test at frequency $\pi/2$, but not in the model above is $X_{4t} = (L - L^3)X_t$. If X_{4t} is dropped, one assumes the absence of *non-synchronous* seasonal cycles. In the particular case of only synchronous seasonal cycles at the annual frequency, inference can be made in the same way as in the zero frequency case. Lee (1992) writes:

> "We can further assume that $\Pi_4 = 0$ with little effect on the test for seasonal cointegration at the annual frequency when cointegration is contemporaneous."

Furthermore, Kunst (1993a) writes:

> "Synchronous annual fluctuations are generally preferred to asynchronous (non-synchronous) ones. Empirical evidence tends to support this preference, and freely estimated Π_4 is often close to 0. If Π_3 and Π_4 are estimated without restrictions, so called polynomial cointegrating vectors complicate the analysis."

The above restriction, i.e. that X_{4t} has no influence on $\Delta_4 X_t$, has been criticized by Johansen and Schaumburg (1998) [henceforth JS]. They argue that it is a peculiar restriction on *all* coefficients at the annual frequency; also it is difficult to interpret. They suggest the following SECM for the case when unit roots are present at ± 1 and $\pm i$:

$$\Delta_4 X_t = \sum_{i=1}^{2} \alpha_i \beta_i' \widetilde{X}_{it} + \alpha_3 \overline{\beta}_3' \widetilde{X}_{3t} + \alpha_4 \overline{\beta}' \widetilde{X}_{4t} + \varepsilon_t, \tag{3.3}$$

where the processes $\widetilde{X}_{1t}, ..., \widetilde{X}_{4t}$ are given by:

$$\begin{aligned} \widetilde{X}_{1t} &= \frac{1}{4} X_{1t}, \\ \widetilde{X}_{2t} &= -\frac{1}{4} X_{2t}, \\ \widetilde{X}_{3t} &= -\frac{1}{4} X_{3t} - \xi_t, \\ \widetilde{X}_{4t} &= -\frac{1}{4} X_{3t} + \xi_t, \end{aligned} \tag{3.4}$$

where $\xi_t = i\frac{1}{4}(L - L^3)X_t$ and where $i = \sqrt{-1}$. Furthermore, in (3.3), $\beta_3 = (\beta_R + i\beta_I)$, $\alpha_3 = (\alpha_R + i\alpha_I)$, while $\overline{\beta}_4$ and $\overline{\alpha}_4$ are their complex conjugates, i.e. $(\beta_R - i\beta_I)$ and $(\alpha_R - i\alpha_I)$, respectively. Model (3.3) can now be rewritten in a form that contains only real terms, if one lets $\widetilde{X}_{3t} = X_{Rt} + iX_{It}$ and $\widetilde{X}_{4t} = X_{Rt} - iX_{It}$. The annual frequency part of (3.3), i.e. $\alpha_3 \overline{\beta}_3' \widetilde{X}_{3t} + \alpha_4 \overline{\beta}_4' \widetilde{X}_{4t}$, can be written:

$$\begin{aligned} & (\alpha_R + i\alpha_I)(\beta_R - i\beta_I)'(X_{Rt} + iX_{It}) + (\alpha_R - i\alpha_I)(\beta_R + i\beta_I)'(X_{Rt} - iX_{It}) \\ = \; & 2(\alpha_R \beta_R' + \alpha_I \beta_I')X_{Rt} + 2(\alpha_R \beta_I' - \alpha_I \beta_R')X_{It}. \end{aligned}$$

Solving for X_{Rt} and X_{It}, by using $\widetilde{X}_{3t}$ and $\widetilde{X}_{4t}$ in (3.4), gives the following annual components:

$$\begin{aligned} X_{Rt} &= -\frac{1}{4}(L^2 - L^4)X_t \\ X_{It} &= -\frac{1}{4}(L - L^3)X_t, \end{aligned} \tag{3.5}$$

and the SECM then becomes

$$\Delta_4 X_t = \sum_{i=1}^{2} \alpha_i \beta_i' X_{it} + 2(\alpha_R \beta_R' + \alpha_I \beta_I')X_{Rt} + 2(\alpha_R \beta_I' - \alpha_I \beta_R')X_{It} + \varepsilon_t. \tag{3.6}$$

The estimation procedure of the CI vectors at the zero and biannual frequencies uses canonical correlations and hence does not require any explanation here (see Johansen (1985)). However the estimation of β_R and β_I is nonstandard. JS suggests an algorithm providing estimators which are asymptotically ML. First, the regressors X_{jt} $j = 1, 2, R, I$ in (3.6) are asymptotically pairwise uncorrelated:

$$T^{-2} \sum_{t=1}^{T} X_{it} X_{jt}' \xrightarrow{P} 0, i \neq j, \tag{3.7}$$

so when focusing on the annual frequency, one can concentrate out the regressors corresponding to the zero and biannual frequencies by removing the reduced rank restriction on frequencies zero and π. In a first step, regress $\Delta_4 X_t$ and X_{jt}, $j = 1, 2, R, I$ on lagged values of $\Delta_4 X_t$ and on deterministic variables, if such are present in (3.6). Define the resulting residuals as R_{0t}, R_{jt}, respectively. If no lags or deterministic variables are present, $X_{j,t} = R_{jt}$. The restriction of reduced rank on $\alpha_i \beta_i'$ for $i = 1, 2$ is then removed by regressing R_{0t}, R_{Rt} and R_{It} on R_{1t} and R_{2t}, respectively. It is shown in JS that the resulting residuals (U_{0t}, U_{Rt} and U_{It}) asymptotically satisfy the following equation:

$$\begin{aligned} U_{0t} &= 2(\alpha_R \beta_R' + \alpha_I \beta_I')U_{Rt} + 2(\alpha_R \beta_I' - \alpha_I \beta_R')U_{It} + U_{\varepsilon t} \\ &= 2\left(\alpha_R - \alpha_I\right) \begin{pmatrix} \beta_R & -\beta_I \\ \beta_I & \beta_R \end{pmatrix}' \begin{pmatrix} U_{Rt} \\ U_{It} \end{pmatrix} + U_{\varepsilon t} \\ &= \widetilde{\alpha} \beta' \begin{pmatrix} U_{Rt} \\ U_{It} \end{pmatrix} + U_{\varepsilon t}. \end{aligned} \tag{3.8}$$

Define the product moments as $S_{ij} = (1/T)\Sigma_{t=1}^{T} U_{it}U'_{jt}$ for $i, j = 0, 1$. For fixed values of $\boldsymbol{\beta}$ one can concentrate the log likelihood function with respect to $\widetilde{\alpha}$ and Ω. The likelihood function, apart from a constant, then reads:

$$\begin{aligned} L_{\max}^{-\frac{2}{T}}(\boldsymbol{\beta}) &= \left|S_{00} - S_{01}\boldsymbol{\beta}(\boldsymbol{\beta}'S_{11}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}S_{01}\right| \\ &= |S_{00}|\frac{|\boldsymbol{\beta}'(S_{11} - S_{10}S_{00}^{-1}S_{01})\boldsymbol{\beta}|}{|\boldsymbol{\beta}'S_{11}\boldsymbol{\beta}|}. \end{aligned} \tag{3.9}$$

The minimization of (3.9) cannot be done as an eigenvalue problem as in the zero and biannual frequency cases, since $\boldsymbol{\beta}$ itself has a complex structure, while the product matrices $S_{11} - S_{10}S_{00}^{-1}S_{01}$ and S_{11} are real valued. JS suggest a switching algorithm proposed by Boswijk (1995) where the ML estimator of $\boldsymbol{\beta}$ is calculated iteratively: Isolate β_R and β_I by using a normalized form of (3.8), i.e. pre-multiplied by $\Omega^{-1/2}$:

$$\widetilde{U}_{0t} = \alpha_R^N \beta_R' U_{Rt} - \alpha_I^N \beta_R' U_{It} + \alpha_R^N \beta_I' U_{It} - \alpha_I^N \beta_I' U_{Rt} + \widetilde{U}_{\varepsilon t}, \tag{3.10}$$

and then vectorize (3.10) by using the fact that $\text{vec}(\alpha_i^N \beta_i' U_{it}) = (U_{it}' \otimes \alpha_i^N)\text{vec}(\beta_i')$. Since $\widetilde{U}_{0t}$ is a vector, $\text{vec}(\widetilde{U}_{0t}) = \widetilde{U}_{0t}$. This yields:

$$\widetilde{U}_{0t} = U_{2t} \begin{bmatrix} \text{vec}(\beta_R') \\ \text{vec}(\beta_I') \end{bmatrix} + \widetilde{U}_{\varepsilon t}, \tag{3.11}$$

where $U_{2t} = \left[(U_{Rt}' \otimes \alpha_R^N) - (U_{It}' \otimes \alpha_I^N) \quad (U_{It}' \otimes \alpha_R^N) + (U_{Rt}' \otimes \alpha_I^N)\right]$. β_R and β_I can now be found from:

$$\begin{bmatrix} \text{vec}(\beta_R') \\ \text{vec}(\beta_I') \end{bmatrix} = \left(\sum_{t=1}^{T} U_{2t}U_{2t}'\right)^{-1} \sum_{t=1}^{T} U_{2t}\widetilde{U}_{0t}'. \tag{3.12}$$

The concentrated likelihood is minimized using the following steps. For a given value of $\boldsymbol{\beta}$, which is generated randomly in the first iteration, an estimate of $\widetilde{\alpha}$ can be computed as:

$$\widetilde{\alpha} = \frac{1}{2}(S_{01}\boldsymbol{\beta}(\boldsymbol{\beta}'S_{11}\boldsymbol{\beta})^{-1}), \tag{3.13}$$

and an estimate of Ω can be found from:

$$\widehat{\Omega} = S_{00} - S_{01}\boldsymbol{\beta}(\boldsymbol{\beta}'S_{11}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}S_{01} \tag{3.14}$$

Then, normalize U_{0t} to obtain (3.10), and vectorize. Given this, a new estimate of β can be calculated from (3.12). The new likelihood function can be computed from (3.9). If the new likelihood value exceeds the old one by some critical amount, a new iteration is done, otherwise the switching procedure stops.

When a maximum is reached at rank r, one continues with $r+1$, and so on. For each r, this likelihood value is compared with that given under the unrestricted VAR assumption, i.e. rank $= p$, which reads:

$$|S_{00}|\frac{|(S_{11}-S_{10}S_{00}^{-1}S_{01})|}{|S_{11}|}. \tag{3.15}$$

Hence the LR statistic is:

$$-2\log Q(H(r)|H(p)) = T\times\log\left(\frac{|\beta'(S_{11}-S_{10}S_{00}^{-1}S_{01})\beta||S_{11}|}{|\beta' S_{11}\beta||(S_{11}-S_{10}S_{00}^{-1}S_{01})|}\right) \tag{3.16}$$

Note that the rank tests at the zero and at the biannual frequencies are the same for RS and for GS.

3.3 The Monte Carlo setup

The two systems in the Monte Carlo studies have three and four variables, respectively. In the three variate system, six different *data generated processes* (DGPs) are investigated, numbered DGP1 to DGP6. In the four variable system, three DGPs generate the data. The are DGP7, DGP8 and DGP9. Three different variance-covariance matrices are used in each case, see Table 3.3. The following true rank cases at the different frequencies are studied for both the three variable and four variable systems:

$$\begin{aligned}
\text{Case 1} &: r_0=1,\ r_\pi=1,\ r_{\pi/2}=1,\\
\text{Case 2} &: r_0=1,\ r_\pi=1,\ r_{\pi/2}=2,\\
\text{Case 3} &: r_0=2,\ r_\pi=2,\ r_{\pi/2}=1,\\
\text{Case 4} &: r_0=2,\ r_\pi=2,\ r_{\pi/2}=2.
\end{aligned}$$

The 5% critical values at the zero and biannual frequencies, for both RS and GS, are based on own calculations with 100 000 replicates and with $T=400$. So are the critical values at the annual frequency for RS, whereas critical values for GS, at the annual frequency, are from JS (Table 1).

The DGPs in the three variable case are presented in Table 3.1. The matrices α_1 and β_1' operate at the zero frequency, α_2 and β_2' at the biannual frequency and α_R, β'_R and α_I, β'_I at the annual frequency. The cointegrating relationships are similar across DGPs, while the adjustment parameters are different across frequencies for DGP1 to DGP5 in both the three and four variable cases. However, the columns in all the adjustment parameter matrices, for a certain DGP, sum to the same absolute value, so as to preserve a similar parameter structure across frequencies. For example, the columns sum to 0.9 in absolute value across frequencies in DGP1, whereas the columns sum to 0.45 in DGP2.

The adjustment parameters are lower in DGP2, DGP4 and DGP5, so the tests are expected to be less powerful at all frequencies when using these DGPs. DGP3 and DGP4 simulate a situation where the non-synchronous seasonal cycles are less pronounced, so one may expect that the test for cointegration at the annual frequency using GS will be less powerful. For DGP5, $\Pi_I = 0$, i.e., the restriction for non-synchronous seasonal cycles is included at frequency $\pi/2$ and GS is clearly overparameterized. Finally, $\beta_I = 0$ in DGP6, which is a restriction JS suggests one should test for. DGP7 considers the four variable system (see Table 3.2) and simulates a situation where non-synchronous seasonal cycles are pronounced, while DGP8 simulates the opposite situation in the four variable system. Finally the last DGP also considers the four variable case and here $\Pi_I = 0$, i.e. the restriction for non-synchronous seasonal cycles is included. Note that regardless of which DGP is used, specifications (3.1) and (3.6) are estimated without additional deterministic variables or lags and without further restrictions included, apart from the annual restriction in RS.

Figure 3.1 shows an example of the cointegrating relations at different frequencies, when DGP6 and true rank case 1 is used. The data are generated in the following way:

$$X_t = X_{t-4} + \Pi_1 X_{1,t} + \Pi_2 X_{2,t} + \Pi_R X_{R,t} + \Pi_I X_{I,t} + \varepsilon_t$$

where $\varepsilon_t \sim N_k(0, \Sigma_{ij})$ and where the Π matrices are changed according to the rank chosen at a particular frequency. For example, Π_1 consists of the first r_0 columns in α_0 times the first r_0 rows in β'_1. The matrices Π_R and Π_I are $(\alpha_R\beta'_R + \alpha_I\beta'_I)$ and $(\alpha_R\beta'_I - \alpha_I\beta'_R)$, respectively and are constructed in the same manner. Sample series of lengths $T^* = \{180,\ 240,\ 300\}$ are generated but the first 100 observations are discarded in each replicate, so the effective sample sizes are $T = \{80,\ 140,\ 200\}$. The reported size is calculated as the number of rejections of the true rank divided by the number of replicates, whereas power is calculated as the sum of rejections of the hypothesized rank, again divided by the number of replicates. Note that this is an evaluation of the individual tests and not of the rank test sequence. If the convergence criterion is not satisfied in 300 iterations for rank r, a new data set for replicate i is generated. The convergence criterion used in Section 3.4 for a maximum of the likelihood function under the assumption of reduced rank (r) at the annual frequency is:

$$\left| T \times \log\left(L_{\max}^{-\frac{2}{T}}(\beta^{\text{old}}) \right) - T \times \log\left(L_{\max}^{-\frac{2}{T}}(\beta^{\text{new}}) \right) \right| < 0.001. \tag{3.17}$$

In each case a total of 10 000 replicates are used.

3.4 Monte Carlo results

First, the convergence criterion used in the Monte Carlo study for GS is almost always satisfied before 300 iterations are conducted, regardless of which DGP is used. The results are similar across variance-covariance matrices and hence only results based on Σ_{13} and Σ_{14} are presented. The results for the three variable case can be found in Tables 3.4 to 3.7, while those for the four variable case are presented in Tables 3.8 to 3.11. The last number in each column (for each DGP) reports the size of the test, whereas other numbers concern power. The size properties are generally good even in small samples and the performance of the test improves as the number of observations increases, see for example Table 3.8. for DGP8 or DGP9. However, this is not true for RS, if the data come from DGP1, DGP2 or DGP7. These DGPs simulate situations where non-synchronous seasonal cointegration clearly should play a role. In these cases, size actually increases with sample size, thus indicating a risk of finding evidence of too many CI vectors at the annual frequency. Note that this result does not hold true for DGP1 and DGP2 in true rank cases 2 and 4, but these are very special cases since the alternative then implies stationary variables. Some results indicate that the specification suggested by JS also has poor size properties in the cases mentioned above, see for example true rank case 1 for DGP7, but the size steadily decreases with an increasing sample size using GS. Note that RS has good size properties if Π_I is close to zero as in DGP3, DGP4 and DGP8. This implies that if the empirical findings on real world data are correct, that Π_I is often zero or close to zero when estimated freely, then RS is a good choice.

It is evident that DGP2, DGP4, DGP5 and DGP6, where the adjustment parameters were given low values, yield poorer power properties than DGP1 and DGP3. This is most obvious when the smallest sample size is considered and this holds across frequencies. There is no evidence that the LR test for the rank is less powerful at the biannual frequency as compared to its performance at the zero frequency in small samples. The results for DGP5 and DGP9, under which the assumption of absent non-synchronous seasonal cycles is valid, show that this also holds true for tests at the annual frequency (RS). Some results indicate that the general specification (GS) looses power, at least in small samples, when the annual restriction is almost satisfied, see results for DGP3, DGP4 and DGP8. The results for RS and GS indicate that it could be useful to work with the two specifications in pairs, when testing for the number of cointegrating vectors at the annual frequency.

The number of cointegration relations at a certain frequency seems to have some effect on the test results for the rank at other frequencies. One example is the size results for RS if DGP7 is used and true rank cases 1 and 3 are compared. This suggests a possible gain in efficiency, when testing at a certain frequency, by concentrating out the "correct" number of vectors at the other frequencies.

3.5 Concluding remarks

There is no evidence that the likelihood ratio test for the rank is less powerful at a certain frequency as compared to other frequencies in small samples. This is most evident when test results from the restricted version of the SECM is analyzed in situations where the restrictions are satisfied. The restricted version of the SECM has poor size properties in cases where non-synchronous cointegration clearly should play a role. This indicates a risk of finding 'evidence' of too many cointegrating vectors at the annual frequency when using this specification. On the other hand, if the restriction is almost satisfied, the general specification looses power at least in smaller samples, while tests in RS form have good properties. Furthermore, the number of true CI relations at a certain frequency affect the test for the rank at other frequencies in small samples. This result suggests a possible gain in efficiency, when testing at a certain frequency, from concentrating out the 'correct' number of vectors at the other frequencies.

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Tables and figures

Table 3.1 Cointegrating vectors (a) and adjustment parameters (b) used in the Monte Carlo study. The *three* variable case.

	DGP1		DGP2		DGP3		DGP4*	
(a)								
β_1	1.00	0.00	-1.00	0.00	1.00	0.00	1.00	0.00
	-1.00	1.00	-1.00	1.00	-0.80	1.00	-0.80	1.00
	0.00	0.50	0.00	0.50	0.00	0.30	0.00	2.30
β_2	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	-0.90	1.00	-0.90	1.00	-0.70	1.00	-0.70	1.00
	0.00	0.30	0.00	0.30	0.00	0.60	0.00	0.60
β_R	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	-0.70	1.00	-0.70	1.00	-0.90	1.00	-0.90	1.00
	0.00	0.60	0.00	0.60	0.00	0.30	0.00	0.30
β_I	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	-0.80	1.00	-0.80	1.00	-1.00	1.00	-1.00	1.00
	0.00	0.30	0.00	0.30	0.00	0.50	0.00	0.50
(b)								
α_1	-0.80	0.00	-0.40	0.00	-0.80	0.00	-0.40	0.00
	-0.10	-0.70	-0.05	-0.35	-0.10	-0.70	-0.05	-0.35
	0.00	-0.20	0.00	-0.10	0.00	-0.20	0.00	-0.10
α_2	-0.60	0.00	-0.30	0.00	-0.60	0.00	-0.30	0.00
	-0.30	-0.60	-0.15	-0.30	-0.30	-0.60	-0.15	-0.30
	0.00	-0.30	0.00	-0.15	0.00	-0.30	0.00	-0.15
α_R	-0.10	0.00	-0.05	0.00	-0.80	0.00	-0.38	0.00
	-0.80	-0.30	-0.40	-0.15	-0.10	-0.60	-0.07	-0.27
	0.00	-0.60	0.00	-0.30	0.00	-0.30	0.00	-0.18
α_I	-0.80	0.00	-0.40	0.00	-0.75	0.00	-0.40	0.00
	-0.10	-0.60	-0.05	-0.30	-0.15	-0.55	-0.05	-0.30
	0.00	-0.30	0.00	-0.15	0.00	-0.35	0.00	-0.15

* The parameters in DGP5 are equal to those in DGP4 at the zero and biannual frequency but $\alpha_I = \alpha_{R\,(\mathrm{DGP4})}$ and $\beta_I = \beta_{R\,(\mathrm{DGP4})}$, which leads to $\Pi_I = 0$. DGP6 = DGP5 except that $\beta_I = 0$, in DGP6.

Table 3.2 Cointegrating vectors (a) and adjustment parameters (b) used in the Monte Carlo study. The *four* variable case.

	DGP7		DGP8		DGP9	
a)						
β_1	1.00	0.00	1.00	0.00	1.00	0.00
	-0.90	1.00	-1.00	1.00	-1.00	1.00
	0.00	-0.60	0.00	-0.50	0.00	-0.50
	-0.20	-0.20	-0.20	-0.20	-0.20	-0.20
β_2	1.00	0.00	1.00	0.00	1.00	0.00
	-1.00	1.00	-0.90	1.00	-0.90	1.00
	0.00	-0.50	0.00	-0.60	0.00	-0.60
	-0.20	-0.20	-0.20	-0.20	-0.20	-0.20
β_R	1.00	0.00	1.00	0.00	1.00	0.00
	-0.90	1.00	-0.90	1.00	-0.90	1.00
	0.00	-0.50	0.00	-0.50	0.00	-0.50
	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10
β_I	1.00	0.00	1.00	0.00	1.00	0.00
	-0.80	1.00	-0.80	1.00	-0.90	1.00
	0.00	-0.60	0.00	-0.60	0.00	-0.50
	-0.20	-0.30	-0.20	-0.30	-0.10	-0.10
b)						
α_1	-0.60	0.00	-0.65	0.00	-0.60	0.00
	-0.20	-0.60	-0.15	-0.70	-0.20	-0.60
	0.00	-0.20	0.00	-0.10	0.00	-0.20
	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10
α_2	-0.65	0.00	-0.60	0.00	-0.65	0.00
	-0.15	-0.70	-0.20	-0.60	-0.15	-0.70
	0.00	-0.10	0.00	-0.20	0.00	-0.10
	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10
α_R	-0.75	0.00	-0.75	0.00	-0.75	0.00
	-0.05	-0.75	-0.05	-0.75	-0.05	-0.75
	0.00	-0.10	0.00	-0.10	0.00	-0.10
	-0.10	-0.05	-0.10	-0.05	-0.10	-0.05
α_I	-0.05	0.00	-0.65	0.00	-0.75	0.00
	-0.65	-0.15	-0.10	-0.65	-0.05	-0.75
	0.00	-0.65	0.00	-0.15	0.00	-0.10
	-0.20	-0.10	-0.15	-0.10	-0.10	-0.05

Table 3.3 Diagonal and non-diagonal variance-covariance matrices used in the Monte Carlo study. Two identity matrices are also used, denoted Σ_{33} and Σ_{34} for the three and six variable systems, respectively.

$$\Sigma_{13} = \begin{bmatrix} 0.7 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 \\ 0.0 & 0.0 & 0.5 \end{bmatrix}$$

$$\Sigma_{23} = \begin{bmatrix} 0.7 & 0.2 & -0.2 \\ 0.2 & 0.8 & -0.3 \\ -0.2 & -0.3 & 0.5 \end{bmatrix}$$

$$\Sigma_{14} = \begin{bmatrix} 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.1 \end{bmatrix}$$

$$\Sigma_{24} = \begin{bmatrix} 0.7 & 0.2 & -0.2 & 0.5 \\ 0.2 & 0.8 & -0.3 & 0.1 \\ -0.2 & -0.3 & 0.5 & -0.1 \\ 0.5 & 0.1 & -0.1 & 1.1 \end{bmatrix}$$

Table 3.4 True rank case 1. Size and power of the test at different frequencies. *Three* variables.

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP1												
r=0	0.74	0.95	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.06	0.09	0.27	0.09	0.06	0.08	0.30	0.06	0.06	0.07	0.31	0.06
r=2	-	-	-	-	-	-	-	-	-	-	-	-
DGP2												
r=0	0.37	0.59	0.95	0.95	0.67	0.85	1.00	1.00	0.90	0.97	1.00	1.00
r=1	0.04	0.08	0.32	0.12	0.06	0.08	0.37	0.08	0.06	0.08	0.39	0.07
r=2	-	-	-	-	-	-	-	-	-	-	-	-
DGP3												
r=0	0.88	0.89	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.06	0.08	0.06	0.08	0.06	0.07	0.06	0.06	0.06	0.07	0.05	0.06
r=2	-	-	-	-	-	-	-	-	-	-	-	-
DGP4												
r=0	0.35	0.44	0.83	0.60	0.67	0.74	1.00	0.94	0.90	0.92	1.00	1.00
r=1	0.03	0.06	0.06	0.07	0.06	0.06	0.05	0.07	0.06	0.07	0.05	0.07
r=2	-	-	-	-	-	-	-	-	-	-	-	-
DGP5												
r=0	0.36	0.44	0.83	0.58	0.66	0.73	1.00	0.92	0.90	0.92	1.00	1.00
r=1	0.04	0.06	0.06	0.07	0.05	0.07	0.06	0.07	0.05	0.07	0.05	0.07
r=2	-	-	-	-	-	-	-	-	-	-	-	-
DGP6												
r=0	0.30	0.45	0.43	0.53	0.57	0.72	0.72	0.85	0.82	0.91	0.92	0.98
r=1	0.04	0.06	0.04	0.07	0.05	0.07	0.04	0.07	0.06	0.07	0.05	0.07
r=2	-	-	-	-	-	-	-	-	-	-	-	-

The last number in each column for each DGP is size, wheras other numbers indicate power of the test. An hyphon means that this rank is greater than the true rank and that no tests are conducted. Note that this is an evaluation of the individual tests and not of the test sequence.

Table 3.5 True rank case 2. Size and power of the test at different frequencies. *Three* variables. See note to Table 3.4.

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP1												
r=0	0.76	0.98	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.06	0.10	0.76	0.69	0.06	0.08	0.89	0.99	0.06	0.07	0.92	1.00
r=2	-	-	0.05	0.06	-	-	0.04	0.05	-	-	0.04	0.05
DGP2												
r=0	0.38	0.63	0.88	0.92	0.69	0.89	1.00	1,00	0.91	0.98	1.00	1.00
r=1	0.04	0.08	0.43	0.24	0.06	0.09	0.66	0.53	0.06	0.08	0.76	0.81
r=2	-	-	0.05	0.04	-	-	0.05	0.05	-	-	0.06	0.05
DGP3												
r=0	0.92	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.07	0.08	0.69	0.59	0.06	0.07	0.91	0.98	0.06	0.07	0.95	1.00
r=2	-	-	0.05	0.05	-	-	0.05	0.05	-	-	0.05	0.05
DGP4												
r=0	0.37	0.47	0.91	0.60	0.71	0.77	1.00	0.98	0.92	0.94	1.00	1.00
r=1	0.04	0.06	0.26	0.12	0.06	0.07	0.55	0.41	0.06	0.07	0.74	0.73
r=2	-	-	0.03	0.03	-	-	0.05	0.04	-	-	0.05	0.04
DGP5												
r=0	0.38	0.47	0.93	0.60	0.70	0.76	1.00	0.97	0.92	0.93	1.00	1.00
r=1	0.04	0.06	0.34	0.14	0.05	0.07	0.79	0.43	0.06	0.07	0.97	0.75
r=2	-	-	0.04	0.03	-	-	0.05	0.04	-	-	0.05	0.04
DGP6												
r=0	0.28	0.48	0.72	0.72	0.56	0.76	0.97	0.97	0.82	0.93	1.00	1.00
r=1	0.03	0.06	0.30	0.16	0.05	0.08	0.61	0.36	0.05	0.07	0.84	0.62
r=2	-	-	0.04	0.03	-	-	0.05	0.04	-	-	0.05	0.04

Table 3.6 True rank case 3. Size and power of the test at different frequencies. *Three* variables. See note to Table 3.4

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP1												
$r=0$	0.87	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r=1$	0.38	0.53	0.39	0.09	0.86	0.93	0.44	0.07	0.99	1.00	0.46	0.06
$r=2$	0.04	0.05	-	-	0.05	0.05	-	-	0.05	0.05	-	-
DGP2												
$r=0$	0.35	0.42	0.96	0.97	0.76	0.79	1.00	1.00	0.97	0.97	1.00	1.00
$r=1$	0.07	0.11	0.39	0.12	0.24	0.30	0.45	0.07	0.50	0.58	0.48	0.07
$r=2$	0.02	0.02	-	-	0.04	0.04	-	-	0.04	0.05	-	-
DGP3												
$r=0$	0.95	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r=1$	0.38	0.47	0.06	0.09	0.84	0.93	0.06	0.07	0.99	1.00	0.05	0.07
$r=2$	0.05	0.05	-	-	0.05	0.06	-	-	0.05	0.05	-	-
DGP4												
$r=0$	0.36	0.36	0.86	0.64	0.78	0.73	1.00	0.96	0.98	0.95	1.00	1.00
$r=1$	0.07	0.07	0.06	0.07	0.23	0.26	0.06	0.07	0.46	0.56	0.05	0.07
$r=2$	0.02	0.03	-	-	0.04	0.05	-	-	0.05	0.05	-	-
DGP5												
$r=0$	0.36	0.34	0.86	0.62	0.78	0.73	1.00	0.95	0.97	0.95	1.00	1.00
$r=1$	0.07	0.07	0.06	0.07	0.22	0.26	0.06	0.07	0.47	0.55	0.05	0.07
$r=2$	0.02	0.02	-	-	0.04	0.04	-	-	0.05	0.05	-	-
DGP6												
$r=0$	0.30	0.35	0.46	0.57	0.69	0.72	0.77	0.88	0.95	0.95	0.94	0.99
$r=1$	0.05	0.08	0.04	0.07	0.19	0.27	0.05	0.08	0.43	0.56	0.05	0.07
$r=2$	0.02	0.02	-	-	0.03	0.04	-	-	0.04	0.05	-	-

Table 3.7 True rank case 4. Size and power of the test at different frequencies. *Three* variables. See note to Table 3.4

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP1												
r=0	0.92	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.48	0.64	0.86	0.78	0.92	0.97	0.95	0.99	1.00	1.00	0.97	1.00
r=2	0.05	0.05	0.04	0.06	0.05	0.05	0.04	0.06	0.05	0.05	0.03	0.06
DGP2												
r=0	0.39	0.46	0.92	0.95	0.80	0.84	1.00	1.00	0.98	0.98	1.00	1.00
r=1	0.09	0.12	0.48	0.28	0.28	0.34	0.71	0.59	0.56	0.62	0.81	0.85
r=2	0.03	0.03	0.05	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05
DGP3												
r=0	0.99	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.52	0.61	0.82	0.75	0.93	0.97	0.96	0.99	1.00	1.00	0.98	1.00
r=2	0.05	0.05	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05
DGP4												
r=0	0.41	0.40	0.94	0.67	0.83	0.78	1.00	0.99	0.99	0.97	1.00	1.00
r=1	0.08	0.09	0.32	0.15	0.27	0.31	0.63	0.48	0.53	0.62	0.80	0.79
r=2	0.02	0.03	0.04	0.03	0.04	0.05	0.05	0.04	0.05	0.05	0.05	0.04
DGP5												
r=0	0.41	0.38	0.95	0.67	0.83	0.77	1.00	0.98	0.99	0.97	1.00	1.00
r=1	0.08	0.09	0.41	0.17	0.27	0.31	0.84	0.50	0.53	0.62	0.98	0.81
r=2	0.02	0.03	0.04	0.03	0.04	0.04	0.05	0.04	0.05	0.05	0.05	0.05
DGP6												
r=0	0.31	0.39	0.78	0.77	0.70	0.78	0.98	0.98	0.96	0.97	1.00	1.00
r=1	0.05	0.10	0.34	0.18	0.19	0.31	0.67	0.40	0.43	0.62	0.88	0.68
r=2	0.02	0.02	0.04	0.04	0.03	0.04	0.06	0.05	0.04	0.05	0.05	0.05

Table 3.8 True rank case 1. Size and power of the test at different frequencies. *Four* variables. See note to Table 3.4

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP7												
$r=0$	0.81	0.61	1.00	1.00	0.99	0.91	1.00	1.00	1.00	0.99	1.00	1.00
$r=1$	0.10	0.08	0.43	0.14	0.09	0.08	0.50	0.08	0.07	0.08	0.54	0.07
$r=2$	-	-	-	-	-	-	-	-	-	-	-	-
DGP8												
$r=0$	0.83	0.81	0.98	0.85	0.99	0.98	1.00	1.00	1.00	1.00	1.00	1.00
$r=1$	0.09	0.10	0.07	0.10	0.08	0.08	0.06	0.08	0.08	0.08	0.06	0.07
$r=2$	-	-	-	-	-	-	-	-	-	-	-	-
DGP9												
$r=0$	0.87	0.79	0.99	0.87	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00
$r=1$	0.10	0.09	0.07	0.10	0.09	0.07	0.06	0.08	0.08	0.07	0.06	0.07
$r=2$	-	-	-	-	-	-	-	-	-	-	-	-

Table 3.9 True rank case 2. Size and power of the test at different frequencies. *Four* variables. See note to Table 3.4

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP7												
$r=0$	0.91	0.64	1.00	1.00	1.00	0.93	1.00	1.00	1.00	0.99	1.00	1.00
$r=1$	0.12	0.08	0.98	0.95	0.09	0.08	1.00	1.00	0.08	0.08	1.00	1.00
$r=2$	-	-	0.30	0.21	-	-	0.35	0.15	-	-	0.36	0.12
DGP8												
$r=0$	0.91	0.88	1.00	0.97	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00
$r=1$	0.11	0.11	0.68	0.55	0.09	0.09	0.89	0.95	0.08	0.09	0.94	1.00
$r=2$	-	-	0.04	0.06	-	-	0.04	0.07	-	-	0.04	0.06
DGP9												
$r=0$	0.93	0.88	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r=1$	0.12	0.10	0.77	0.45	0.09	0.08	1.00	0.90	0.08	0.08	1.00	1.00
$r=2$	-	-	0.05	0.05	-	-	0.05	0.06	-	-	0.05	0.06

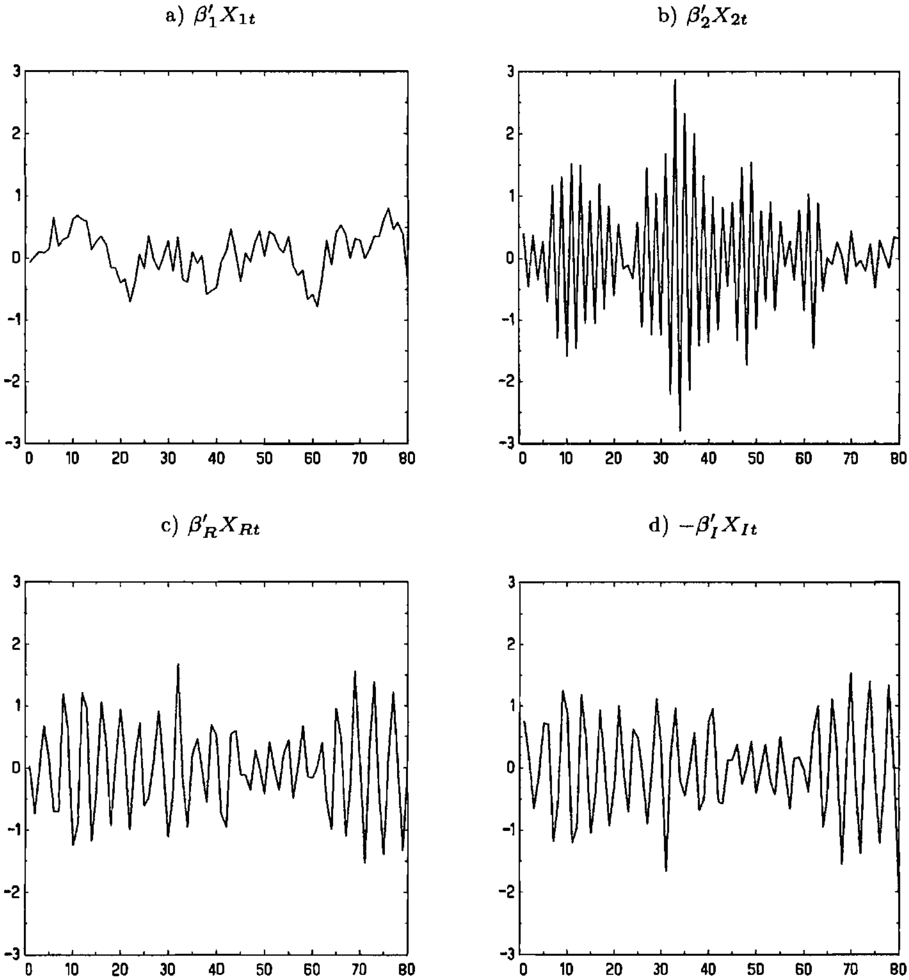
Table 3.10 True rank case 3. Size and power of the test at different frequencies. *Four* variables. See note to Table 3.4

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP7												
r=0	0.93	0.76	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.52	0.31	0.64	0.14	0.90	0.74	0.75	0.08	0.99	0.97	0.80	0.07
r=2	0.06	0.04	-	-	0.06	0.06	-	-	0.06	0.06	-	-
DGP8												
r=0	0.91	0.92	0.99	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.37	0.49	0.07	0.12	0.77	0.88	0.07	0.09	0.97	0.99	0.06	0.08
r=2	0.04	0.06	-	-	0.05	0.06	-	-	0.06	0.07	-	-
DGP9												
r=0	0.94	0.90	1.00	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.49	0.39	0.07	0.11	0.86	0.82	0.06	0.09	0.99	0.98	0.06	0.07
r=2	0.06	0.05	-	-	0.06	0.06	-	-	0.06	0.06	-	-

Table 3.11 True rank case 4. Size and power of the test at different frequencies. *Four* variables. See note to Table 3.4

T	80				140				200			
Freq.	0	π	$\pi/2$		0	π	$\pi/2$		0	π	$\pi/2$	
			RS	GS			RS	GS			RS	GS
H_0:												
DGP7												
r=0	0.97	0.76	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.69	0.30	1.00	0.96	0.97	0.71	1.00	1.00	1.00	0.95	1.00	1.00
r=2	0.08	0.04	0.37	0.21	0.07	0.06	0.43	0.15	0.06	0.06	0.46	0.12
DGP8												
r=0	0.97	0.97	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.54	0.63	0.81	0.70	0.90	0.95	0.94	0.98	0.99	1.00	0.96	1.00
r=2	0.60	0.07	0.05	0.07	0.06	0.06	0.05	0.07	0.06	0.07	0.04	0.07
DGP9												
r=0	0.98	0.96	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
r=1	0.66	0.54	0.88	0.60	0.95	0.91	1.00	0.96	1.00	0.99	1.00	1.00
r=2	0.07	0.06	0.05	0.06	0.06	0.06	0.05	0.06	0.06	0.06	0.05	0.06

Figure 3.1 Cointegrating relations at different frequencies for DGP6, T=80 and true rank case 1.



Paper 4

On seasonal error correction when the processes include different numbers of unit roots

4.1 Introduction

Cointegration was a major break through when introduced by Engle and Granger (1987). They treat cointegration as stable long-run relationships between a set of nonstationary time series processes, which includes unit roots at the nonseasonal, or zero, frequency. One of the main attractions of the idea is that the cointegrating vectors may be interpreted as equilibrium relations between the variables. It is often assumed that the seasonal patterns are constant over time and many applied researchers try to model these variations using deterministic seasonal dummies. However, substantial empirical evidence makes it safe to conclude that the seasonal pattern in many economic time series is far from constant. To the best of our knowledge, there are two major routes to cointegration in the case of changing seasonal variation, namely *periodic* and *seasonal* cointegration. Periodic cointegration models, see for example Boswijk and Franses (1995), consider long-run relationships season by season, whereas seasonal cointegration models are based on the idea of the unit roots (zero and seasonal) implied by the annual difference filter, see Hylleberg *et al.* (1990) [HEGY]. In the present paper the latter model class for quarterly data is considered, a class to which Engle *et al.* (1993) [EGHL], Lee (1992) and Johansen and Schaumburg (1998) have made important contributions. EGHL propose a two-step approach to test for the presence of seasonal and nonseasonal cointegration relationships whereas Lee (1992) suggests a multiple equation *seasonal error correction model* [SECM], which extends the maximum likelihood approach to the nonseasonal case, summarized in Johansen (1995). Finally, Johansen and Schaumburg (1998) refine the asymptotic theory for SECM and propose a general estimation procedure for the parameters corresponding to the annual frequency. The specification of the SECM is straightforward if all the included variables contain roots at the same, but not necessarily at all, frequencies.

One purpose of this paper is to show how the more general SECM, proposed by Johansen an Schaumburg (1998), could be specified in the case where the quarterly observed variables contain different numbers of unit roots, which is a common situation when working with real world data. We assume that the interest of an empirical study is

1. to test for the number of cointegrating vectors and estimate these at the nonseasonal and seasonal frequencies, and

2. to forecast.

A Monte Carlo simulation is carried out to investigate the consequences of specifying a SECM which assumes four unit roots in each process and where the variables are transformed to yield stationarity accordingly, i.e. applying the annual difference filter. This specification is compared to the correctly specified model, attaching a different filter to each variable. Furthermore, we consider pre-testing for the number of seasonal unit roots in the univariate time series and specify models suggested by these tests. The two seasonal unit root tests are the familiar HEGY test and the seasonal KPSS [SKPSS] tests of Lyhagen (2000), respectively. It has been shown that the HEGY test has poor power against alternatives close to the null hypothesis. This implies that one may find evidence of too many roots, in practice, using this method. It is therefore interesting to include the SKPSS, which has the null hypothesis of no unit roots. In the forecasting exercise we also include a VAR model in annual differences.

The outline of the paper is as follows. Section 4.2 describes cointegration and seasonal cointegration, while Section 4.3 discusses the model for variables with different numbers of seasonal unit roots. The Monte Carlo setup is given in Section 4.4 and the results are analyzed in Section 4.5. Some conclusions end the paper.

4.2 Cointegration and seasonal cointegration

Let L be the lag operator, i.e. $L^d Y_t = Y_{t-d}$ and define the first difference filter as $\Delta = (1-L)$. Furthermore, let $\Delta_d = (1-L^d)$. Consider a quarterly observed p-dimensional autoregressive process Y_t. The *vector error correction model* [VECM] can now be written as:

$$\Delta Y_t = \Pi Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + ... + \Gamma_p \Delta Y_{t-p} + \Phi D_t + \varepsilon_t, \qquad (4.1)$$

where $\Pi = \alpha\beta'$ is of reduced rank r and where $\Gamma_1, ..., \Gamma_p$ are lag matrices, see Johansen (1995). The deterministic terms may contain a constant and seasonal dummies. One crucial assumptions is that the first difference filter should remove all unit roots in the process that generates Y_t. However, economic time series possessing a changing seasonal pattern over time often include seasonal unit roots, in addition to the zero frequency root. If Δ_4 filters, henceforth annual difference filters, are required to transform the vector Y_t to yield stationarity, the time series are said to be seasonally integrated i.e. $Y_t \sim \text{SI}(1)$. This filter assumes four unit roots, all of which lie on the unit circle. This can be seen from the following factorization:

$$\Delta_4 = \left(1 - L^4\right) = (1-L)(1+L)(1+iL)(1-iL) \qquad (4.2)$$

where $i = \sqrt{-1}$. Now, the $(1-L)$ part correspond to the zero frequency or nonseasonal unit root, so the first difference filter only removes one of the roots. The $(1+L)(1+iL)(1-iL)$ part corresponds to the three seasonal unit roots, namely -1 and $\pm\, i$. The -1 root is often called the biannual root while the two complex conjugate roots, $\pm\, i$, are called the annual frequency roots. Johansen and Schaumburg (1998), henceforth JS, show that the following transformed processes are needed in the SECM, when the variables in Y_t includes the roots at ± 1 and $\pm i$:

$$\begin{aligned}
Z_{1t} &= \frac{(1+L)(1+iL)(1-iL)L}{4} Y_t, \\
Z_{2t} &= -\frac{(1-L)(1+iL)(1-iL)L}{4} Y_t, \\
Z_{3t} &= \frac{(1-L)(1+L)(1-iL)L}{4i} Y_t, \\
Z_{4t} &= -\frac{(1-L)(1+L)(1+iL)L}{4i} Y_t.
\end{aligned} \qquad (4.3)$$

The above filters can be found in the first rows of Z_{1t}, Z_{2t}, Z_{3t} and Z_{4t}, respectively in the Appendix. Furthermore, Z_{mt} for $m = 1, ..., 4$ are asymptotically pairwise uncorrelated:

$$T^{-2}\sum_{t=1}^{T} Z_{it}Z_{jt}' \xrightarrow{P} 0, i \neq j, \tag{4.4}$$

implying that the cointegration vectors and adjustment coefficients can be found by removing the reduced rank restriction on the other frequencies by concentrating out the associated regressors in a regression. JS propose the following SECM:

$$\Delta_4 Y_t = \sum_{i=1}^{2} \alpha_i \beta_i' Z_{it} + \alpha_3 \beta_3^* Z_{3t} + \alpha_4 \beta_4^* Z_{4t} + \sum_{j=1}^{p} \Gamma_j \Delta_4 Y_{t-j} + \varepsilon_t \tag{4.5}$$

where $\alpha_3 = \alpha_R + i\alpha_I$, $\alpha_4 = \alpha_R - i\alpha_I, \beta_3 = \beta_R + i\beta_I$ and $\beta_4 = \beta_R - i\beta_I$. Here β_i^* denotes the complex conjugate of β_i. The Model above can now be rewritten in a form that contains only real terms, if one lets $Z_{3t} = Z_{Rt} + iZ_{It}$ and $Z_{4t} = Z_{Rt} - iZ_{It}$. The annual frequency part of, i.e. $\alpha_3\beta_3^* Z_{3t} + \alpha_4\beta_4^* Z_{4t}$, can now be written:

$$\begin{aligned} & (\alpha_R + i\alpha_I)(\beta_R - i\beta_I)'(Z_{Rt} + iZ_{It}) + (\alpha_R - i\alpha_I)(\beta_R + i\beta_I)'(Z_{Rt} - iZ_{It}) \\ = \; & 2(\alpha_R\beta_R' + \alpha_I\beta_I')Z_{Rt} + 2(\alpha_R\beta_I' - \alpha_I\beta_R')Z_{It}. \end{aligned}$$

Note again that this is the appropriate specification if all roots are assumed to be present. It can be shown that the filters for the annual frequency equal:

$$\begin{aligned} Z_{Rt} &= -\frac{1}{4}(L^2 - L^4)Y_t \\ Z_{It} &= -\frac{1}{4}(L - L^3)Y_t \end{aligned} \tag{4.6}$$

The asymptotic properties of the estimators and the LR-test for the number of cointegrating vectors for the different frequencies are given in JS. Tests for reduced rank (r) at the zero and biannual frequencies can be performed using the *trace* statistic:

$$-2\log(H(r)|H(p)) - T\sum_{i=r+1}^{P} \log(1 - \widehat{\lambda}_i), \tag{4.7}$$

where $H(r)$ is the null hypothesis and $H(p)$ is the alternative hypothesis of full rank. The eigenvalues, $1 > \widehat{\lambda}_1 > ... > \widehat{\lambda}_p$, which maximize the likelihood function, are obtained by solving eigenvalue problems based on residual vectors. The null hypothesis for this test is that there are at most r cointegrating vectors. Inference on the reduced rank matrix $\Pi_2 = \alpha_2\beta_2'$ follows the usual non-seasonal case, i.e. for Π in (4.1), see Johansen (1995). Tests for the number of cointegrating vectors are similar at the annual frequency. However, it should be noted that estimation of the cointegrating vectors at this frequency is not that straightforward and includes a rather involved iterative procedure. Here we use a switching algorithm suggested in JS, which provides estimators for β_R and β_I that are asymptotically equivalent to the maximum likelihood estimators, see also Löf and Lyhagen (1999).

4.3 Different orders of integration

In this section we study variables appearing in the same model, but containing different numbers of unit roots. Let $z_1, ..., z_s$ be complex numbers and let z_m be a root of $|A(z)| = 0$, where $A(z)$ is the characteristic polynomial of (4.1), expressed in levels. We want to present the model in error correction form, although the number of unit roots varies among the included variables. No series should be overdifferenced or be nonstationary at any frequency. This is achieved by relying on the following results in JS. Let:

$$\begin{aligned} p(z) &= \prod_{m=1}^{s} (1 - \bar{z}_m z) \\ p_a(z) &= \prod_{m \in a} (1 - \bar{z}_m z) = \frac{p(z)}{\prod_{m \notin a} (1 - \bar{z}_m z)}, \quad z \notin a, \end{aligned} \tag{4.8}$$

where $\bar{z}_m = z_m^{-1}$. If a_i denotes the set of unit roots for variable i then $p_{a_i}(z)$ cancels them. For example, if variable 1 includes unit roots at the nonseasonal and the biannual frequencies then:

$$\begin{aligned} p(z) &= \left(1 - \frac{z}{1}\right)\left(1 - \frac{z}{-1}\right)\left(1 - \frac{z}{i}\right)\left(1 - \frac{z}{-i}\right) = 1 - z^4 \\ p_{a_1}(z) &= \frac{\left(1 - \frac{z}{1}\right)\left(1 - \frac{z}{-1}\right)\left(1 - \frac{z}{i}\right)\left(1 - \frac{z}{-i}\right)}{\left(1 - \frac{z}{i}\right)\left(1 - \frac{z}{-i}\right)} = (1 - z)(1 + z). \end{aligned}$$

Let $P(z)$ be a diagonal matrix with diagonal elements such that the element in entry (i, i) cancels the roots of the ith variable, i.e.

$$P(z) = \begin{bmatrix} p_{a_1}(z) & 0 & \cdots & 0 \\ 0 & p_{a_2}(z) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & p_{a_p}(z) \end{bmatrix}. \tag{4.9}$$

It is shown in JS that the error correction formulation is a consequence of a Lagrange expansion of $A(z)$ around the points $z = 0,\ z_1,\ z_2, \ldots,\ z_s$ as follows

$$A(z) = P(z) + \sum_{m=1}^{s} A(z_m) \frac{P_m(z)\, z}{P_m(z_m)\, z_m} + P(z)\, z A_0(z), \tag{4.10}$$

where $P_m(z)$ is the matrix consisting of $p_{a_i}(z) / (1 - \bar{z}_m z)$, which is zero if $(1 - \bar{z}_m z)$ does not belong to $p_{a_i}(z)$. If we again consider variable 1 and let $m = 1$ (zero frequency), then $z_m = z_1 = 1$ and:

$$\begin{aligned} \frac{p_{a_1}(z)}{(1 - \bar{z}_1 z)} &= \frac{(1-z)(1+z)}{\left(1 - \frac{z}{1}\right)} = 1 + z \\ \frac{P_1(z)\, z}{P_1(z_1)\, z_1} &= \frac{(1+z)z}{(1+1)1} = \frac{(1+z)\, z}{2} \end{aligned}$$

The proof of (4.10) is a generalization of the one in JS. Each entry of

$$A(z) - P(z) - \sum_{m=1}^{s} A(z_m) \frac{P_m(z)\, z}{P_m(z_m)\, z_m} \tag{4.11}$$

is zero for $z = 0,\ z_1,\ z_2, \ldots,\ z_s$. Hence, the difference can be written as $P(z)\, z A_0(z)$ for some matrix polynomial $A_0(z)$. Another consequence is the following. Let $z_1,\ z_2, \ldots,\ z_s$ be the unit roots of $|A(z)| = 0$, such that the matrices $A(z_m)$ are of reduced rank. Then Y_t has an error correction representation

$$\begin{aligned} P(L)\, Y_t &= \sum_{m=1}^{s} \alpha_m \beta_m^* \frac{P_m(L)\, L}{P_m(z_m)\, z_m} Y_t - P(L)\, A_0(L)\, L Y_t + \varepsilon_t \\ &= \sum_{m=1}^{s} \alpha_m \beta_m^* Z_{mt} - P(L)\, A_0(L)\, L Y_t + \varepsilon_t. \end{aligned} \tag{4.12}$$

The filters Z_{mt} in (4.12) equal those of (4.3) if the processes includes the roots ± 1 and $\pm i$. As an example, consider a situation where the variables may have the roots at the nonseasonal and at the biannual frequency. Let Y_t be a $T \times p$ vector, where the variables are ordered such that only the last k variables include both roots. That is, let $Y_t = [Y_{1t}, Y_{2t}]'$ where Y_{1t} includes the nonseasonal, or zero, frequency root, while the Y_{2t} variables includes both roots. The most important matrices in (4.12) are:

$$
\begin{aligned}
P(L) &= \begin{bmatrix} (1-L) & 0 \\ 0 & (1-L)(1+L) \end{bmatrix} \\
\frac{P_1(L)L}{P_1(1)1} &= \begin{bmatrix} L & 0 \\ 0 & \frac{(1+L)L}{2} \end{bmatrix} \\
\frac{P_2(L)L}{P_2(-1)-1} &= \begin{bmatrix} 0 & 0 \\ 0 & -\frac{(1-L)L}{2} \end{bmatrix}
\end{aligned}
\tag{4.13}
$$

Using these, it is easily shown that the error correction model for Y_t is

$$
\begin{aligned}
&\begin{bmatrix} (1-L) & 0 \\ 0 & (1-L)(1+L) \end{bmatrix} \begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} \\
= \quad & \alpha_1\beta_1' \begin{bmatrix} L & 0 \\ 0 & \frac{(1+L)L}{2} \end{bmatrix} \begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} \\
& -\alpha_2\beta_2' \begin{bmatrix} 0 & 0 \\ 0 & \frac{(1-L)L}{2} \end{bmatrix} \begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} + \varepsilon_t .
\end{aligned}
\tag{4.14}
$$

The Appendix presents the appropriate transformations of the variables in this situation. These can be found in the second and last rows of the $P(L)$ matrix, respectively. Moreover, the filtered variables, used for the zero frequency in (4.14), can be found in the second and last rows of Z_{1t}, respectively. Finally, the appropriate filters, for the variables in Y_{2t}, at the biannual frequency can be found in the second row of Z_{2t}. One implication is that the k last variables, having the roots 1 and -1, can not be just a single variable. That is, one must require $k > 1$ to be able test for cointegration at the biannual frequency.

Modeling is usually done in steps, starting from pre-testing for seasonal unit roots, using e.g. the HEGY-test. Cointegration is then tested using the corresponding adjusted model. A problem with the HEGY approach is that, asymptotically, there is a chance , ε, of rejecting a true null of a unit root, where ε is the chosen significance level. Hence, the method is not asymptotically efficient. A test which has the null of no unit root, proposed by Lyhagen (2000), may be a better choice because it asymptotically rejects a *false* null and a tendency to accept more rather than less unit roots may lead to more robust models. It has also been shown that the HEGY test has low power against alternatives, close to the null hypothesis. This implies that one tends to find evidence of nonstationarity when the process is in fact stationary at a certain frequency. So, when considering small samples, it is an open question weather one should use the HEGY approach or SKPSS.

4.4 Monte Carlo

Seven bivariate DGPs are considered, where the first variable y_{1t} always includes all unit roots. The set of true unit roots of y_{2t} is different in each DGP:

$$\begin{array}{ll} \text{DGP1:} & 1,\ -1,\ i,\ -i \\ \text{DGP2:} & 1,\ -1 \\ \text{DGP3:} & 1,\ i,\ -i \\ \text{DGP4:} & -1,\ i,\ -i \\ \text{DGP5:} & -1 \\ \text{DGP6:} & i,\ -i \\ \text{DGP7:} & 1 \end{array} \tag{4.15}$$

Cointegration prevails if y_{2t} is nonstationary at a certain frequency, because y_{1t} includes all possible unit roots. For example, cointegration can apply at all frequencies if DGP1 is considered. On the other hand, cointegration is only possible at the zero and biannual frequency if DGP2 is used. We consider sample sizes of $T = 40$, 80 and 120. The number of observations is close to the ones often found in empirical applications. The number of replicates is 10 000. The significance level is 5% throughout. The seven DGPs are based on estimates on income and consumption data for Japan (in logs), previously used in EGHL. To achieve well behaved DGPs, a constant and a time trend are before estimation extracted from the variables. All the DGPs have eigenvalues of the companion matrix either on or inside the unit circle, see Table 4.1.

Four model specifications are compared when cointegration is considered. In the forecasting exercise we include a fifth model (Model 5) which is a *vector autoregressive model* [VAR] in annual differences. When Models 2 and 5 are used, all variables are transformed using the annual difference directly. In Model 1 we use the annual difference filter for y_{1t} and difference y_{2t} according to the true number of unit roots, see $P(L)$ in the Appendix for the seven DGP cases. Finally, in Models 3 and 4 we apply the annual difference filter for y_{1t}, without pre-testing for the number of unit roots, while y_{2t} is differenced according to the unit root test results. We summarize the different models below:

Table A: Models used in the Monte Carlo study.

			Pre-testing	
Model		Cointegration	y_{1t}	y_{2t}
1	True SECM	Yes	No	No
2	Δ_4 SECM	Yes	No	No
3	HEGY $\Rightarrow$ SECM	Yes	No	Yes
4	SKPSS $\Rightarrow$ SECM	Yes	No	Yes
5	Δ_4 VAR	No	No	No

For Models 1 to 4, tests for rank zero against full rank (rank two) and tests for rank one against full rank are considered at all frequencies. Size and power properties are evaluated by estimating the proportion of rejections for each frequency. To evaluate the estimates of the parameters in position i, j of the Π matrices we use the mean of the *mean squared error* [MSE]:

$$\text{MSE} = \sum_{r=1}^{R} \frac{\sum_i \sum_j \left(\hat{\Pi}_{ij} - \Pi_{ij}\right)^2 /4}{R} \tag{4.16}$$

where R equals the number of replicates. We also forecast 12 periods ahead to compare the model specifications. As noted above we include Model 5 in this case. Forecasting accuracy is measured using the determinant (Det_k) and the trace (Tr_k) of the mean squared error matrix, respectively:

$$\begin{aligned} \text{Det}_k &= \left| \sum_{r=1}^{R} \frac{(x_{t+k} - f_{t+k})' (x_{t+k} - f_{t+k})}{R} \right| \\ \text{Tr}_k &= \text{trace} \sum_{r=1}^{R} \frac{\left[(x_{t+k} - f_{t+k})' (x_{t+k} - f_{t+k})\right]}{R} \end{aligned} \tag{4.17}$$

where again R equals the number of replicates.

4.5 Monte Carlo results

The results for size and power are displayed in Tables 4.2 to 4.7 and the MSE of the parameter estimates in Tables 4.8 to 4.11. We also show parts of the forecasting exercise in Table 4.12. The first thing to note is that the model which is transformed according to the true number of roots generally performs better than the other models, concerning inference as well as estimation and forecasting.

Its closest competitor, if one wants to reject a false null of $r = 0$, is the model which assumes the presence of all roots (Model 2), see for example the columns of DGP2 and DGP3 in Table 4.2 except for $T = 40$. Note the results for DGP2 and DGP4 in Table 4.3 for the biannual frequency tests or DGP3, DGP4 and DGP6 in Table 4.4 if the annual frequency is considered. Model 2 is also likely to reject a true null hypothesis of $r = 0$ in some cases. One example is DGP4, which includes the roots -1 and $\pm i$, and when the test concern cointegration at the zero frequency, see Table 4.2. Another example is DGP6, which only includes the two complex roots, when the biannual frequency is considered, see Table 4.3. Similar results for Model 2 can be found when DGP2, DGP5 and DGP7 are used an when the tests concern the annual frequency. These results are due to the fact that y_{2t} constitutes a stationary 'relation' by itself in these cases. So, it is quite logical that the null hypothesis of $r = 0$ is rejected. Note that cointegration should not apply, since y_{1t} is a nonstationary variable. These cases are underlined in Tables 4.2 to 4.4 and in Tables 4.8 to 4.11.

SKPSS (Model 4) is to prefer in some cases, whereas pre-testing with HEGY (Model 3) seems to be a better strategy in other cases. For example, HEGY fails if DGP7 is used and if the test concerns zero frequency cointegration (Table 4.2), whereas SKPSS fails if DGP1 is used and the objective is to test for cointegration at the biannual frequency (Table 4.3). SKPSS is slightly better in the cases when the root for which cointegrating is tested for is not present in y_{2t}, while HEGY seems to have better power when there in fact should be cointegration.

The size for Model 1 and 2, when testing the true null hypothesis of $r = 1$, tends to the nominal with sample size and is fairly close to the nominal. Pre-testing with SKPSS seems to work except in DGP1 where it is greatly oversized (zero and annual frequencies, Tables 4.5 and 4.7) or very undersized (biannual frequency, Table 4.6). The size when pre-testing with HEGY is diverging from the nominal for DGP2 and DGP7 at the zero frequency, while it works better for other DGPs, see Table 4.5. Similar results can be found in the columns for DGP2, DGP4 and DGP5 if the biannual frequency is considered, see Table 4.6, but also at the annual frequency in some cases.

For Models 1 and 2 the MSE of the parameter estimates decreases uniformly with sample size. This is also true for SKPSS, except for Π_3 when using DGP1, see Table 4.10. Pre-testing with the HEGY, on the other hand, either increases MSE with sample size or slowly decreases it. The use of Model

2 sometimes results in much higher MSE as compared to the other methods, see for example the MSE for Π_2 when DGP3 and DGP6 are considered in Table 4.9. This last result depends on the estimation of a nonzero rank matrix, where y_{2t} is stationary, see discussion above.

The results of the forecasting exercise are almost always the same, regardless if the determinant or the trace of the MSE matrix is used. In Table 4.12 we present the ranking of the models after averaging over the 12 forecast periods, and the measure used is the determinant of the MSE matrix. The ranking is quite stable over sample sizes and DGPs. The model that allows for all unit roots (Model 2) comes as a good second after the true model (Model 1) in most cases. Model 5 i.e. the VAR model in annual differences seems to be a better choice than using models resulting from the two pre-testing methods in most cases. Model 5 has actually a better performance than Model 2 in some cases when $T = 40$. This is most likely due to the parameter uncertainty when estimating Model 2 in small samples. Models resulting from SKPSS tests performs better than those based on HEGY tests, except for DGP1 and DGP5 with $T = 80$ or $T = 120$. Forecast MSE often decreases with sample size for Model 1, Model 2 and Model 4, while the opposite is true for Model 3 and sometimes for Model 5 (not shown here). This is still valid when looking at the trace of the MSE matrix, but for both types of measurements it is within the Monte Carlo error bounds.

4.6 Conclusions

We propose a seasonal cointegration model for quarterly data which includes variables with different numbers of unit roots and thus need to be transformed in different ways to yield stationarity. A Monte Carlo simulation is carried out to investigate the consequences of specifying a SECM in annual differences in this situation. We compare the true model and the model where annually differenced variables are included to model specifications suggested by pre-tests for the numbers of unit roots. We consider two seasonal unit root tests in this analysis. One is the so called HEGY test, proposed by Hylleberg *et al.* (1990) and the other is the seasonal KPSS tests, proposed by Lyhagen (2000). We use seven different DGPs which all are based on estimates of quarterly observed income and consumption data for Japan, previously used in Engle *et al.* (1993). The True SECM, the SECM in annual differences and the two specifications suggested by the HEGY and the SKPSS tests are compared when the aim is to tests for cointegration. We consider tests for rank zero against full rank (rank two) and tests for rank one against full rank at all frequencies. Size and power properties are evaluated by estimating the proportion of rejections for each frequency. To evaluate the estimates of the parameters in the Π matrices we use the mean of the *mean squared error* [MSE] for the same models. In the forecasting exercise we include a VAR model in annual differences. The results indicate that, in all practical cases where the true model is not known, a seasonal error correction model in annual differences may be a better choice, than relying on models which are specified according to seasonal unit root tests. This result holds true for both inference, as well as for estimation and forecasting. The second best choice when the true model is not known and when the aim is to forecast, is a VAR model in annual differences. These results extend those in Clements and Hendry (1997) where it was found that univariate models in annual differences may generate more accurate forecasts than models transformed according to HEGY test results.

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Appendix

If we consider the seven cases presented in Section 4.4 the true filters for the second variable in the DGPs are:

$$P(L)=\begin{bmatrix} (1-L)(1+L)(1+iL)(1-iL)\,y_{2t} \\ (1-L)(1+L)\,y_{2t} \\ (1-L)(1-iL)(1+iL)\,y_{2t} \\ (1+L)(1-iL)(1+iL)\,y_{2t} \\ (1+L)\,y_{2t} \\ (1-iL)(1+iL)\,y_{2t} \\ (1-L)\,y_{2t} \end{bmatrix}$$

Note that $P(L)$ is not as defined in (4.9). Here it only concerns y_{2t} and the various filters are ordered after the DGP cases in (4.15). To test for cointegration at the zero and at the biannual frequency we use the following the true filters, for y_{2t}, found from:

$$Z_{mt}=\frac{P_m(L)L}{P_m(z_m)z_m}y_{2t},$$

$$Z_{1t}=\begin{bmatrix} \frac{(1+L)(1+iL)(1-iL)L}{4}y_{2t} \\ \frac{(1+L)L}{2}y_{2t} \\ \frac{(1+iL)(1-iL)L}{2}y_{2t} \\ - \\ - \\ - \\ Ly_{2t} \end{bmatrix} \qquad Z_{2t}=\begin{bmatrix} -\frac{(1-L)(1+iL)(1-iL)L}{4}y_{2t} \\ -\frac{(1-L)L}{2}y_{2t} \\ - \\ -\frac{(1+iL)(1-iL)L}{2}y_{2t} \\ -Ly_{2t} \\ - \\ - \end{bmatrix}$$

Finally, the true filters for variable two at the annual frequency are:

$$Z_{3t}=\begin{bmatrix} \frac{(1-L)(1+L)(1-iL)L}{4i}y_{2t} \\ - \\ \frac{(1-L)(1-iL)L}{2(i+1)}y_{2t} \\ \frac{(1+L)(1-iL)L}{2(i-1)}y_{2t} \\ - \\ \frac{(1-iL)L}{2i}y_{2t} \\ - \end{bmatrix} \qquad Z_{4t}=\begin{bmatrix} -\frac{(1-L)(1+L)(1+iL)L}{4i}y_{2t} \\ - \\ -\frac{(1-L)(1+iL)L}{2(i-1)}y_{2t} \\ -\frac{(1+L)(1+iL)L}{2(i+1)}y_{2t} \\ - \\ -\frac{(1+iL)L}{2i}y_{2t} \\ - \end{bmatrix}$$

Tables

Table 4.1 Roots inside the unit circle of the companion matrix.

DGP 1	2	3	4	5	6	7
0.92	0.23+0.22i	0.65	0.53	0.96	0.97	0.25
-0.19+0.16i	0.23-0.22i	0.46	0.16+0.18i	0	0.21	0
-0.19-0.16i	0	-0.09	0.16-0.18i	0	0	0
0.38	0	0	0	0	0	0

Table 4.2 Probability of rejecting H_0: $r = 0$ at the zero frequency.

T	DGP: / Model	[1]	[2]	[3]	4	5	6	[7]
40	1	0.58	0.82	0.98	-	-	-	0.97
	2	0.58	0.57	0.98	<u>0.44</u>	<u>0.10</u>	<u>0.29</u>	0.63
	3	0.53	0.71	0.93	0.09	0.05	0.34	0.04
	4	0.83	0.70	0.79	0.00	0.16	0.21	0.87
80	1	0.90	1.00	1.00	-	-	-	1.00
	2	0.90	0.98	1.00	<u>0.95</u>	<u>0.08</u>	<u>0.51</u>	0.99
	3	0.84	0.82	0.95	0.12	0.04	0.57	0.00
	4	0.91	0.85	0.86	0.00	0.17	0.36	0.95
120	1	0.99	1.00	1.00	-	-	-	1.00
	2	0.99	1.00	1.00	<u>1.00</u>	<u>0.10</u>	<u>0.72</u>	1.00
	3	0.90	0.79	0.95	0.14	0.04	0.77	0.00
	4	0.95	0.89	0.90	0.00	0.21	0.47	0.97

A unit root at the zero frequency exists for y_{2t} in DGP 1, 2, 3 and 7. These DGPs are within brackets. *A hyphon indicate that no Π matrix exists for Model 1 at this frequency. Underlined numbers, for Model 2, indicate that y_{2t} is a stationary variable in the DGP at this frequency. See a summary of the various model specifications in Section 4.4.

Table 4.3 Probability of rejecting H_0: $r = 0$ at the biannual frequency.

T	DGP: / Model	[1]	[2]	3	[4]	[5]	6	7
40	1	0.79	1.00	-	1.00	1.00	-	-
	2	0.79	0.76	<u>1.00</u>	0.97	0.86	<u>1.00</u>	<u>0.60</u>
	3	0.70	0.94	0.00	0.92	0.89	0.00	0.05
	4	0.11	0.66	0.00	0.39	0.52	0.00	0.00
80	1	1.00	1.00	-	1.00	1.00	-	-
	2	1.00	1.00	<u>1.00</u>	1.00	1.00	<u>1.00</u>	<u>0.99</u>
	3	0.77	0.92	0.00	0.88	0.83	0.00	0.00
	4	0.20	0.75	0.00	0.52	0.65	0.00	0.00
120	1	1.00	1.00	-	1.00	1.00	-	-
	2	1.00	1.00	<u>1.00</u>	1.00	1.00	<u>1.00</u>	<u>1.00</u>
	3	0.72	0.90	0.00	0.85	0.80	0.00	0.00
	4	0.25	0.80	0.00	0.59	0.73	0.00	0.00

A unit root at the biannual frequency exists for y_{2t} in DGP 1, 2, 4 and 5. These DGPs are within brackets. See * in Table 4.2.

Table 4.4 Probability of rejecting H_0: $r = 0$ at the annual frequency.

T	DGP: / Model	[1]	2	[3]	[4]	5	[6]	7
40	1	1.00	-	1.00	1.00	-	1.00	-
	2	1.00	<u>0.84</u>	1.00	1.00	<u>0.98</u>	1.00	<u>0.81</u>
	3	0.92	0.01	0.88	0.83	0.01	0.88	0.10
	4	0.45	0.00	0.70	0.96	0.00	0.93	0.00
80	1	1.00	-	1.00	1.00	-	1.00	-
	2	1.00	1.00	1.00	1.00	<u>1.00</u>	1.00	<u>1.00</u>
	3	0.88	0.01	0.90	0.79	0.03	0.85	0.15
	4	0.58	0.00	0.81	0.98	0.00	0.97	0.00
120	1	1.00	-	1.00	1.00	-	1.00	-
	2	1.00	<u>1.00</u>	1.00	1.00	<u>1.00</u>	1.00	<u>1.00</u>
	3	0.85	0.01	0.92	0.78	0.05	0.81	0.18
	4	0.66	0.00	0.87	0.99	0.00	0.98	0.00

Unit roots at the annual frequency exists for y_{2t} in DGP 1, 3, 4 and 6. These DGPs are within brackets. See * in Table 4.2.

Table 4.5 Probability of rejecting H_0: $r = 1$ at the zero frequency.

T	DGP: / Model	[1]	[2]	[3]	4	5	6	[7]
40	1	0.12	0.06	0.10	-	-	-	0.06
	2	0.12	0.06	0.11	0.07	0.02	0.08	0.07
	3	0.08	0.04	0.10	0.02	0.01	0.08	0.00
	4	0.22	0.05	0.12	0.00	0.02	0.07	0.05
80	1	0.10	0.06	0.08	-	-	-	0.06
	2	0.10	0.06	0.08	0.07	0.02	0.09	0.06
	3	0.07	0.02	0.08	0.02	0.01	0.09	0.00
	4	0.17	0.05	0.09	0.00	0.02	0.06	0.05
120	1	0.09	0.06	0.07	-	-	-	0.06
	2	0.09	0.06	0.07	0.06	0.03	0.09	0.06
	3	0.08	0.02	0.07	0.03	0.01	0.10	0.00
	4	0.16	0.05	0.07	0.00	0.03	0.06	0.05

A unit root at the zero frequency exists for y_{2t} in DGP 1, 2, 3 and 7. These DGPs are within brackets. See * in Table 4.2.

Table 4.6 Probability of rejecting H_0: $r = 1$ at the biannual frequency.

T	DGP: / Model	[1]	[2]	3	[4]	[5]	6	7
40	1	0.06	0.07	-	0.07	0.07	-	-
	2	0.06	0.07	0.07	0.07	0.07	0.08	0.07
	3	0.04	0.04	0.00	0.04	0.04	0.00	0.00
	4	0.00	0.03	0.00	0.02	0.03	0.00	0.00
80	1	0.06	0.06	-	0.07	0.06	-	-
	2	0.06	0.06	0.07	0.07	0.07	0.07	0.06
	3	0.03	0.02	0.00	0.03	0.02	0.00	0.00
	4	0.01	0.03	0.00	0.02	0.03	0.00	0.00
120	1	0.06	0.06	-	0.06	0.06	-	-
	2	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	3	0.04	0.02	0.00	0.02	0.02	0.00	0.00
	4	0.01	0.03	0.00	0.02	0.02	0.00	0.00

A unit root at the biannual zero frequency exists for y_{2t} in DGP 1, 2, 4 and 5. These DGPs are within brackets. See * in Table 4.2.

Table 4.7 Probability of rejecting H_0: $r = 1$ at the annual frequency.

T	DGP: Model	[1]	2	[3]	[4]	5	[6]	7
40	1	0.07	-	0.07	0.07	-	0.07	-
	2	0.07	0.06	0.07	0.07	0.07	0.07	0.06
	3	0.05	0.00	0.03	0.01	0.00	0.03	0.01
	4	0.03	0.00	0.05	0.04	0.00	0.06	0.00
80	1	0.06	-	0.06	0.06	-	0.05	-
	2	0.06	0.06	0.06	0.06	0.06	0.06	0.05
	3	0.07	0.00	0.02	0.01	0.00	0.02	0.01
	4	0.09	0.00	0.06	0.05	0.00	0.05	0.00
120	1	0.06	-	0.06	0.05	-	0.05	-
	2	0.06	0.05	0.05	0.05	0.05	0.06	0.05
	3	0.10	0.00	0.02	0.01	0.00	0.02	0.01
	4	0.15	0.00	0.06	0.05	0.00	0.05	0.00

Unit roots at the annual frequency exists for y_{2t} in DGP 1, 3, 4 and 6. These DGPs are within brackets. See * in Table 4.2.

Table 4.8 Mean of MSE, Π_1.

T	DGP: Model	[1]	[2]	[3]	4	5	6	[7]
40	1	0.06	0.07	0.02	-	-	-	0.04
	2	0.06	0.12	0.03	<u>0.47</u>	<u>0.06</u>	<u>0.02</u>	0.11
	3	0.18	0.11	0.04	0.04	0.05	0.04	0.33
	4	1.04	0.22	0.09	0.00	0.15	0.02	0.07
80	1	0.01	0.03	0.01	-	-	-	0.02
	2	0.01	0.04	0.01	<u>0.37</u>	<u>0.01</u>	<u>0.01</u>	0.04
	3	0.22	0.09	0.03	0.03	0.01	0.03	0.35
	4	1.03	0.15	0.05	0.00	0.03	0.00	0.03
120	1	0.01	0.02	0.01	-	-	-	0.01
	2	0.01	0.03	0.01	<u>0.34</u>	<u>0.01</u>	<u>0.00</u>	0.02
	3	0.30	0.09	0.02	0.04	0.00	0.04	0.35
	4	1.01	0.12	0.04	0.00	0.02	0.00	0.02

A Π_1 matrix does not not exists for Model 1 when DGP 4, 5 and 6 are used. Other DGPs are within brackets. See * in Table 4.2.

Table 4.9 Mean of MSE, Π_2.

T	DGP: Model	[1]	[2]	3	[4]	[5]	6	7
40	1	0.15	0.09	-	0.08	0.01	-	-
	2	0.15	0.22	2.15	0.27	0.20	1.67	0.60
	3	0.84	0.17	0.00	0.37	0.22	0.00	0.02
	4	1.03	0.53	0.00	1.67	0.71	0.00	0.00
80	1	0.06	0.04	-	0.04	0.00	-	-
	2	0.06	0.09	2.09	0.11	0.08	1.63	0.50
	3	0.70	0.16	0.00	0.43	0.27	0.00	0.00
	4	0.95	0.38	0.00	1.31	0.51	0.00	0.00
120	1	0.04	0.02	-	0.02	0.00	-	-
	2	0.04	0.06	2.06	0.07	0.05	1.62	0.46
	3	0.66	0.17	0.00	0.48	0.29	0.00	0.00
	4	0.87	0.30	0.00	1.11	0.40	0.00	0.00

A Π_2 matrix does not not exists for Model 1 when DGP 3, 6 and 7 are used. Other DGPs are within brackets. See * in Table 4.2.

Table 4.10 Mean of MSE, Π_3.

T	DGP: Model	[1]	2	[3]	[4]	5	[6]	7
40	1	0.07	-	0.04	0.07	-	0.18	-
	2	0.07	0.29	0.07	0.08	0.31	0.30	0.32
	3	0.36	0.00	0.16	0.24	0.00	0.28	0.03
	4	1.65	0.00	0.35	0.28	0.00	0.24	0.00
80	1	0.03	-	0.02	0.03	-	0.08	-
	2	0.03	0.26	0.03	0.03	0.28	0.12	0.28
	3	0.49	0.01	0.12	0.25	0.02	0.19	0.05
	4	1.82	0.00	0.21	0.18	0.00	0.10	0.00
120	1	0.02	-	0.01	0.02	-	0.05	-
	2	0.02	0.25	0.02	0.02	0.27	0.08	0.27
	3	0.68	0.01	0.09	0.26	0.03	0.19	0.07
	4	1.99	0.00	0.15	0.15	0.00	0.06	0.00

A Π_3 matrix does not not exists for Model 1 when DGP 2, 5 and 7 are used. Other DGPs are within brackets. See * in Table 4.2.

Table 4.11 Mean of MSE, Π_4.

T	DGP: Model	[1]	2	[3]	[4]	5	[6]	7
40	1	0.06	-	0.06	0.07	-	0.02	-
	2	0.06	0.07	0.06	0.09	0.24	0.10	0.05
	3	0.11	0.00	0.18	0.14	0.00	0.45	0.01
	4	0.71	0.00	1.27	0.51	0.00	0.26	0.00
80	1	0.03	-	0.03	0.03	-	0.01	-
	2	0.03	0.05	0.03	0.04	0.23	0.04	0.03
	3	0.09	0.00	0.12	0.12	0.00	0.50	0.01
	4	0.52	0.00	0.96	0.38	0.00	0.12	0.00
120	1	0.02	-	0.02	0.02	-	0.01	-
	2	0.02	0.05	0.02	0.02	0.23	0.03	0.03
	3	0.10	0.00	0.09	0.12	0.00	0.62	0.01
	4	0.37	0.00	0.72	0.32	0.00	0.07	0.00

A Π_4 matrix does not not exists for Model 1 when DGP 2, 5 and 7 are used. Other DGPs are within brackets. See * in Table 4.2.

Table 4.12 Ranking of forecasting performance (average over 12 periods). The measure is the determinant of the MSE marix.

	T=40					T=80					T=120				
Mod.	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
DGP															
1	1.3	1.3	3.9	5.0	2.6	1.0	1.0	4.0	5.0	3.0	1.0	1.0	4.5	4.5	3.0
2	1.0	2.0	3.6	4.1	4.3	1.0	2.0	5.0	3.3	3.8	1.0	2.0	5.0	3.3	3.8
3	1.0	2.0	4.9	4.1	3.0	1.0	2.0	5.0	4.0	3.0	1.0	2.0	5.0	4.0	3.0
4	1.3	2.5	5.0	3.7	2.5	1.3	2.3	5.0	3.7	2.8	1.1	2.3	5.0	3.7	3.0
5	1.3	3.9	3.2	4.4	2.2	1.0	2.6	3.9	4.5	3.0	1.0	2.3	3.9	4.1	3.8
6	1.5	3.0	5.0	3.4	2.1	1.0	2.0	5.0	3.3	3.7	1.0	2.0	5.0	3.2	3.0
7	1.0	3.0	5.0	2.0	4.0	1.0	2.1	5.0	2.9	4.0	1.0	2.0	5.0	3.0	4.0

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