Information Use
and
Decision Making in Groups

Jan Edman

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A Study of an
Experimental Oligopoly Market
with the Use of a Business Game

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After I graduated I always thought that one of these days I would make a comeback to academia to study some more. When working with information-systems development, I got so interested in the use of the information in these systems that I eventually decided to study information use in decision making. Although it was from the beginning not a clear-cut choice for me to study information use in a business game, it turned out to be as interesting as I could have hoped for.

I have been most fortunate to have Professor Ingolf Ståhl as my thesis advisor. He has read all kinds of materials with both cheering and critical comments. He has spent a lot of time and effort on guiding me through this work. I thank him for all his inspiring ideas and his enormous endurance. Hopefully, I have learned a little from his wisdom and wit.

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Jan Edman

Contents

1 Introduction

1.1 Information use in decision making

As a starting point, many theories use the assumption that all information is available for the decision makers in a decision making situation. Hayek (1945) started one of his papers with the following (here assembled into a listing):

+	If we can start out from a given system of preferences
+	If we posses all the relevant information
+	If we command complete knowledge of available means
=	The problem which remains is purely one of logic

Table 1.1. Relevant information when decision making.

Here we assume that the problem we want to solve is to find out which decisions are best according to our preferences. Once we have found out the decisions that best suit our preferences, we can make our preferred decisions. We can call these decisions our best decisions or even our optimal decisions.

The decision making situation could, however, be a complex situation. A number of decisions might have to be made at the same time. The decisions might also have to be made repeatedly, where earlier decisions might have an effect on subsequent decisions. The problem is still one of logic, but of complex logic.

In fact, many situations in business life are situations of such complexity that there is no means to solve the problem of finding optimal decisions. Therefore, it does not seem realistic to assume that the optimal decisions can always be found by the decision makers in these situations.

Furthermore the assumption above, that we possess all relevant information when making decisions, does not seem realistic for many situations in business life. In fact, many situations in business life are situations where the relevant information for decision making is limited. There might be limitations on which decision alternatives are available. There might also be limitations on the results of different decision alternatives.

These limitations on relevant information could also make it difficult, or even impossible, to make the best possible decisions. This could apply in complex decision making situations, but also in situations with minor complexity, where the optimal decisions could not be found due to limitations on information.

Some time after decisions have been made, information about the results of the decisions often becomes available. However, there are often limitations on the information about the results of the other decision alternatives. Thus, it can be difficult to verify that the results of the decisions made are the best possible ones. It can even be difficult to verify that the decisions made are better than other decision alternatives. Therefore, due to the limitations of information about the results of different decision alternatives, many decisions are unverifiable as best decisions or even as better decisions compared to other decisions.

As mentioned above, decisions might have to be made repeatedly. In such situations, information about the results of previous decisions can be used when making subsequent decisions. Moreover, decisions can be made with the purpose of obtaining information about the results of different decision alternatives. This information can then be used to make better subsequent decisions.

However, the future decision making situation might differ from the present one. The information about the present decision making situation might not then be entirely relevant in a future decision making situation.

The reasoning above on decision making in decision making situations in business life points to the following:

- Many situations are complex.
- In many situations there are limitations on relevant information.
- Optimal decisions cannot be determined.

Nevertheless, it seems reasonable to assume that some relevant information is available in most decision making situations, and that this information is used for decision making. Moreover, most decision makers request relevant information for their decision making. Presumably, this requested information is used to improve decisions, i.e. to make their decisions closer to the optimal decisions.

However, it is not fully understood how information is used in decision making. March (1994, p. 226) gives the following description of information use: "Decision makers gather information and do not use it; ask for more and ignore it; make decisions first and look for relevant information afterwards; gather and process a great deal of information that has little or no direct relevance to decisions".

The purpose of this study is to take some small steps toward a better understanding of how information is used in decision making on an experimental oligopoly market.

1.2 The use of a business game as an experiment

It might be difficult to observe what information is available and what information is used for decision making in business life. Furthermore, situations in business life might also have specific characteristics that cannot be neglected when examining the available information and the decisions made.

Therefore, a constructed situation can offer a good alternative for examining information use in decision making. In a constructed situation, both the information available for decision making and the decision making situation can be reasonably controlled. One such constructed situation is the playing of a business game.

For this study on information use in decision making, my thesis advisor, I. Ståhl, offered in 1996 to let me use the decisions and results from roughly fifty sessions of a business game, called MINIMAX, which he had designed and used for a number of years (Ståhl, 1986). The business game had been used in education on a regular basis, and it had also been played extensively by professionals for a number of years.

At the start of this study, decisions and results were available from game sessions in 1991–1996. I was also offered the opportunity to use this business game for experimental research in 1997–1999. The game provided a controlled and observable situation for my study on information use in decision making.

The business game was designed to be a reasonably realistic model of an oligopoly market in business life, where five firms compete on the market to maximize equity at the end of the game. The firms are producing and selling similar, but not identical, storable products on the same market.

The firms in the game received information in different reports during the playing of the game. Ståhl regarded this information as relevant for the decision making in the game. However, the game was originally played with limitations on the availability of information about the market model and the decisions and results of the competitors.

For research purposes, I could for example:

- Alter the information conditions in the game by increasing the available information for the firms and then make comparisons between decisions made by firms with different information conditions.
- Determine optimal solutions according to oligopoly theory, since I had access to the market model of the game.
- Compare optimal solutions to decisions made when the game was played.
- Use a questionnaire to capture some opinions of the participants.
- Observe the decision making when the game was played.
- Compare decisions made by students with decisions made by professionals, since professionals had also had played the game, e.g. in 1994 and in 1996.

Since the business game was used for an educational purpose, consideration had to be given to the educational situation, when alterations of the game were made for experimental purposes. Thus, there were limitations on the alterations I could make in the business game. There were also strong limitations on the funding of rewards. Referring to Ståhl (1994), I call the experimental use of the business game a *poor man's experiment*. I will discuss the implications of this below.

The general idea in this study is to use the business game as an experiment. By increasing available information for the firms when playing the game, the use of the additional information in decision making is studied.

1.3 A short review of the contents

1	Introduction
2	Research approach
3	Literature review
4	Description of the market model
5	Optimal solutions in the market model
6	Procedures when the business game was used as an experiment
7	Comparisons between the optimal solutions and the decisions in the game sessions
8	Comparisons between decisions made by firms with different information conditions
9	Use of additional information
10	Adaptiveness to information
11	Decision making in groups
12	Effect of group compositions on decisions
13	Experimental limitations
14	Summary and future research

Table 1.2. Contents of the study.

In Chapter 2, the research approach is described in connection with the research question, the research design, the research process, the research considerations and the research purpose.

Chapter 3 reviews literature from related fields of research. The fields are: Industrial organization and game theory, economics of information, decision making, groups as decision makers, and experimental economics and gaming.

In Chapter 4, the market model of the business game is described. The mathematical notation for the market model is formulated and the profit is derived.

In Chapter 5, the optimal solutions are determined. Some simplifications are made to the market model. An analytical method and several numerical methods are used to determine the optimal solutions in the market model. The solutions obtained from the different methods are presented and compared. The decisions of the optimal solutions are to be used for comparisons with decisions made when the game was played.

Chapter 6 describes how the business game was played before alterations were made. It also reviews the alterations made when the game was played.

In Chapter 7, comparisons are made between the optimal solutions of the market model and the decisions made when the game is played.

In Chapter 8, comparisons are made between decisions made by firms with different information conditions. Further comparisons are also made between the optimal solutions and decisions made by firms with different information conditions.

Chapter 9 contains an analysis of what particular information have effect on the decisions made by the firms. Comparisons are made between the equities at the end of the game of firms with different information conditions.

Chapter 10 examines the adaptiveness to information. The dispersion among the decisions made by firms with the same information conditions is studied. Furthermore, adjustments and adaptations of decisions during the game, and the equity at the end of the game is studied.

Chapter 11 deals with some aspects of decision making in groups, such as motivation when playing the game, what decision alternatives are discussed and what expectations are formed when making decisions, and also the decision making process.

Chapter 12 examines the effect of compositions of groups on decisions.

Chapter 13 examines some limitations when the business game is used as an experiment. These are: The number of repetitions of playing the game and the number of alteration at the same time, the rewards, the information about the game before playing it and the complexity of the market model in the business game.

Chapter 14 contains a summary of the study and future research is discussed.

2 Research approach

2.1 Research question

Due to my earlier working experience, I had an interest in the use of information in decision making. In the place where I worked for a number of years, some decision makers requested more information for their decision making, others less. One reason why some of the decisions makers did not request as much information as others could be that the acquisition and use of information could be costly. For example the following activities can incur costs:

1. Searching for information
2. Obtaining information
3. Processing information

It seems plausible to assume that, if the decision makers incurred costs for acquiring and using additional information, they would acquire additional information, as long as the benefits of improved decision making exceeded the costs of acquiring information. The value of additional information would then be the difference between the benefits of the additional acquired information and the costs of this information.

My observations on information use were perhaps not as pessimistic as the observations that March (1994) made, described in section 1.1. Still, I started to wonder how information is used in decision making. Therefore, the problem of interest in this study is how information is used in decision making. As mentioned above, I was given the opportunity to alter the information conditions, by giving the firms in the business game additional information during the play of the game.

I shall here introduce some notions on information:

- When firms have some additional information, and make decisions that differ from those made by other firms that lack this additional information, the additional information is regarded as *relevant* for the decision making.
- When firms have some additional information available, and make the same decisions as other firms that do not have this additional information, the additional information is regarded as *redundant* for the decision making.
- When firms that have the same amount of available information make different decisions, I use the notion of *ambiguous* information for this information.

In this study, comparisons will be made between decisions and results of firms playing the game with different information conditions. The definition of the use of additional information is as follows: Additional information is used in the decision making, if firms with this additional information *make different decisions* compared to firms that do not have this additional information.

I am interested in finding what information is used the decision making, i.e. what information is relevant. I am also interested in finding out what additional information is not used in decision making, i.e. what information is redundant.

There are theories of optimal solutions that might give good descriptions of the decisions made when the game is played. I would like to know if these theories do in fact give us good descriptions of the decisions made when the game is played. I will therefore try to determine optimal solutions in the business game. I can then make comparisons between the optimal solutions and the decisions made when the game is played. Furthermore, I will investigate if decisions and results are closer to the optimal decisions when additional relevant information is made available for the decision making.

I will examine the conventional wisdom, that decisions can be improved when additional information is made available for decision making. The research question for this study is:

- Are better decisions made when additional information is available?

By better decisions I mean decisions that give better results. I assume above that additional relevant information will improve the decisions and give better results.

As mentioned above, when the benefits of better decisions exceed the costs of acquiring and using information, the additional information has a *positive value*. When the benefits do not exceed the costs of acquiring and using information, the additional information does not have has not a positive value. In fact, the additional information can for example have a *negative value*, if the results do not improve and there are costs for obtaining and using information.

In the business game there are no costs for obtaining and using information, but the value of additional information can be either positive or negative depending on the results of the decisions made with relevant information.

However, there might also be other explanations for the decisions made in the business game, apart from information conditions. I will therefore additionally examine the effect of group compositions on the decisions made and I will also monitor and examine the decision making when the game is played.

2.2 Research design

As my research design I shall make alterations of information conditions when the business game is played. I shall study how the firms' use of information when they are making decisions in the business game.

The business game consists of a computer program, the rules of the game (Appendix A) and an interest table (Appendix B). When the business game is used as an experiment, we will distinguish between the market model of the game, which is defined by the computer program, and the procedures when the business game is played, which consist of information at the briefing (Appendix C), decision form (Appendix D) and reports (Appendix E). I will hereafter use the notions of *market model* and *procedures* of the game.

The figure below shows the relationships between the market model and the procedures. It also shows the relationship between the decisions and results in the game.

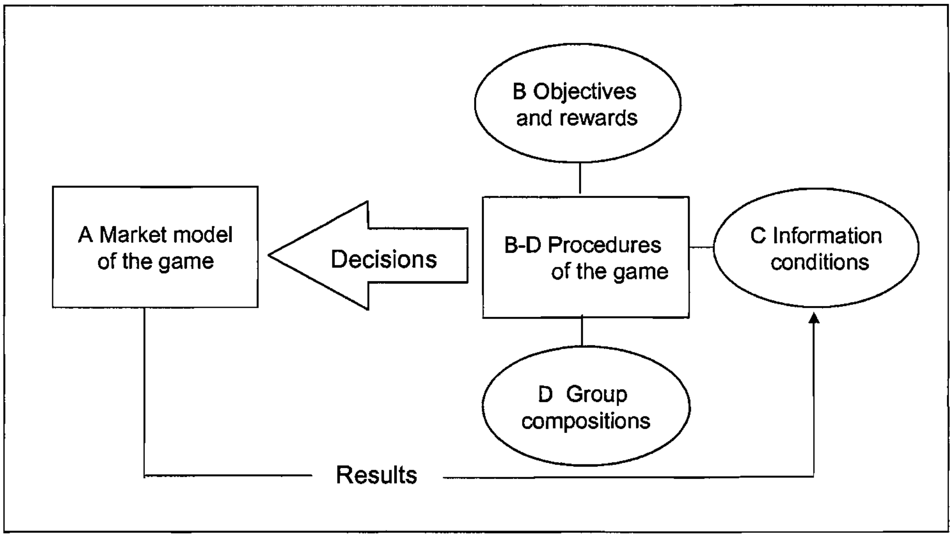

Figure 2.1. Research design.

In the figure above, we see the labels A–D, which are the components in the research design. Later these labels are used for descriptions of the research design and also when altering the procedures of the game.

The market model is labeled A. The market model of the game was not altered (A) in this study. This model is described in Chapter 4. The optimal decisions in the market model are calculated in Chapter 5.

The procedures of the game are labeled B–D. The research design is primarily used to study what effect different information conditions (C) have on the decisions made when the game is played. This means that the procedures of information conditions are altered as the game is played, and that the main interest is to study these alterations (see Chapters 8–9).

The procedures and the alterations of the procedures when the game was used as an experiment are described in Chapter 6. Comparisons between the optimal decision and the decisions made when the game is played are made in Chapters 7–8.

Some alterations of the procedures of the game, apart from the information conditions, were made to improve the control of the experimental situation, when using the business game as an experiment. Since the objectives and rewards (B) and the compositions of groups (D) could have an effect on the decisions when the game is played, these procedures were also altered (see section 6.2).

2.3 Research process

In the 1980's, Ståhl designed a business game, which was so small that it could be played within 3–4 hours. The requirement was that, although the game should be simple, it should also have a resemblance to a real market (Ståhl, 1986). Another requirement was that the computer program of the game could be played on personal computers, and it should be able to give quick and safe feedback. At that time, most business games were quite large and many of them were played for days and they required mainframe computers. Ståhl made amendments to the business game for a number of years in the 1980s. The game was also ported from one computer system to several others. In the 1990s only minor amendments have been made.

As we shall see in the literature review, many experiments have been made with simple market models. The realism of these market models compared to business life is debatable (Edman & Ståhl, 1998). The market model of the business game is a more realistic description of markets in business life. The market model includes the effect of advertising, it has loans with variable interest rates and the game includes balance sheets. It is also of importance that the model is dynamic in the sense that the asset structure has effect for more than one period, so that decisions in one period have effects in subsequent periods.

I. Ståhl, as my thesis advisor, encouraged me to participate in one of the game sessions played in the course of introductory managerial economics, in 1996. Thereafter, I also observed another game session in the same year. The impression I got then was that the students had some information about the game before participating, but very few of them knew the game in detail. The participants did not at that time seem to have any strong opinion about what decisions should be made when the game was played, not even after they had played the game! Nor did I. The participants seemed quite motivated when they played the game. The motivation to make better decisions seemed to increase after some periods of the game had been played. At the debriefing, after the game, the students were eager to know the results.

The business game seemed suitable for me not only for a study of information use, but also of decision making when firms make decisions to compete on markets. Decisions in the game were made in groups, which made it possible to observe the decision making, when the alternatives were discussed, "out in the open", compared to when individuals make decisions. Many decisions in business life are, if not made in groups, at least discussed in groups.

The "bias" I had from my working experience was that the differences in decisions and results between decision makers, who acquired less or more information for their decision making, were not apparent. When I was working with the development of a new information system, I started to wonder about how much information was needed for decision making. My interest in the question grew and I eventually started this research about information use in decision making.

This study was made with the use of experiments, which are quite different from the situation where I worked. I think it is fair to say that I did not, at the start, have any expectations about what the outcome would be when using the business game as an experiment. I shall here mention that I was at the time inspired by Simon's (1991) discussion on the use of experiments in research.

When I started this study, I did not expect to obtain complete descriptions of how information is used in decision making and how much information is needed in decision making. A better understanding of information use in decision making would be a sufficient reward.

I spent considerable time reading about and reflecting on the connections between theory, experiments and business life. I think that using experiments could be a helpful activity to test the validity of theories and to generate hypotheses in general. I have also spent some time on definitions. For example, a decision maker can also be referred to as actor, player or firm, and decisions can be termed behavior, strategy and so forth. I have tried to be consistent, but I still believe that there are notations and definitions that could be refined.

If comparisons were to be made between the decisions in the optimal solutions and the decisions made when the game was played, I had to determine the optimal solutions. However, this was not easy, due to the relative complexity of the market model. We would like to know which of the optimal solutions gives the best description of the decisions made when the game is played.

As earlier mentioned, already at the start of this study the business game had the feature of allowing the information available to the decision makers to be altered. I was cautioned by earlier studies of Swedish researchers (Hägg *et al.*, 1970 and Hägg & Johansson, 1975), as regards the possibility of using a business game as an experiment. They found that only big differences in available information would cause differences in decisions.

We end this section by pinpointing some activities in the research process. Some of them led to findings during the research process, which in turn had impact on the remaining research process.

In chronological order, some of the activities and findings were:

- There were big differences between decisions and results among firms playing the game with the same information conditions (1996).
- Additional information, in the form of more reports, was given to the firms playing the game (1997).
- The compositions of groups were examined (1997)[1]. If the compositions of groups matter, the effect of altering of information conditions could be minor.
- When the decision making was monitored, indications were found of relations between decision alternatives and expectations, and the decisions made in the game (1997, 1998).

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[1] A paper on this topic was presented, together with Patric Andersson, Stockholm School of Economics, at the VIII International Conference on the Foundations & Applications of Utility, Risk and Decision Theory, in Mons in Belgium, 1997. The paper was then called: "On the information of the firm: Group Composition".

- As we shall also see below, there were only minor differences in decisions and results when altering the information conditions from 1996 to 1997. This led me to alter the game in 1998, so that it was played with asymmetric information, where firms competing on the same market had different amounts of information available. This can be seen as a critical test. The decisions and results showed that with different information conditions, it was possible to obtain differences in decisions when the business game was played[2].
- Information about the demand on the market was made available to the firms in a demand table. The information had effect on the decisions made in the game (1998).
- I had almost given up on the possibility of obtaining solutions in the market model for six periods (1998). In 1999, however, I found methods that could give reasonably good estimates of the optimal decisions.
- Since optimal solutions could be approximately determined, it was of interest to play the game again in 1999. The theories of optimal decisions assume complete information for the decision making. The information conditions were altered in the game so that it would be played with information conditions similar to conditions with complete information. Comparisons could then be made between decisions in the optimal solutions and the decisions when the game was played with complete information (1999).

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[2] A paper on this topic was presented, together with Patric Andersson, Stockholm School of Economics, at the conference on "Decision making in theory and practice", in Oxford in Great Britain, 1998. The paper was then called: "Amount of Information used in decision making – Some experimental and empirical results".

2.4 Research considerations

The advantages when studying information use in decision making using an experiment, compared to studying information use in business life, are:

E1 Hypotheses on information use can be generated in a controlled situation.
E2 Hypotheses on information use can be tested and verified through repetitions of experiments.
E3 Hypotheses on the validity of optimal solutions can be examined using comparisons with decisions made when the game is played.

Admittedly it is possible to observe information use and generate hypotheses in business life. Although there are experimental limitations when using the business game, the decision making situation is more controlled than in business life (E1).

It is also possible to repeat opportunities of observations in business life, but there are usually limitations to the number of observations that can be made (E2). In this study more than eighty markets are studied, with five firms on each market, giving a total of four hundred firms[3]. Furthermore, in business life it is almost impossible to determine optimal solutions, due to the complexity and limitations on relevant information in many decision making situations (E3).

Some of the limitations when using a business game for the study are:

L1 The business game can have limitations when it is used as an experiment.
L2 The number of alterations of procedures that can be made in this study is limited.
L3 The study concerns a business game and not business life.

In Chapter 13, we will look at some of the limitations when the business game is used as an experiment (L1).

[3] See Chapters 6 and 13, and also Appendix H for further details.

Although I can study more than eighty game sessions when the game was played with different procedures, some of the alterations that I would have liked to do have not been made. In Chapter 14, I present some further alterations of the procedures that seem to be of interest (L2).

An important limitation of this study is that it concerns a decision making situation in a business game and not in business life (L3). The limitation implies that:

A	The market model supposedly differs from markets in business life.
B–D	The procedures in the game differ from procedures in business life.

With these differences in the market model and in the procedures when the business game is played, it is apparent that the conclusions in this study do not necessarily apply in business life. The conclusions from this study need an assumption of *ceteris paribus* to business life, if they are to apply in business life. This means that the market model and the procedures of the game may need to be similar to business life if the conclusions are to apply.

Presumably, this similarity does not exist. Even if it did, some further examination of information use in business life would still be desirable. Thus, an important question is how much we can generalize the conclusions in this study. Although the conclusions of this study do not necessarily apply in business life, they can still be of interest when studying information use in decision making in business life, e.g. by providing hypotheses to be tested.

An alternative method would be to study information use in business life. In that case we need to have access to a substantial number of firms in both similar and different situations. Many managers would hesitate to have studies carried out on information use in their firms, and those managers who did permit access for such studies might be a biased sample. Furthermore, theories and hypotheses inferred from studies on observations of some situations in business life do not necessarily apply to other situations in business life. The reason for this could be that the characteristics of decision making situations in business life also differ from each other in some respects.

Another alternative is to perform interviews or use questionnaires. Blinder (1991) performed a number of interviews with firms, asking about their decision making on prices. Blinder compared the results from the interviews with theories of price decisions, but he was met by skepticism. One critique was that the interviewees would not reveal any information about price collusion, since price collusion is illegal under most conditions in business life. Therefore the interviewees may not have answered the questions accurately. The same may apply for answers in questionnaires.

With this reasoning I only want to show that there are advantages and disadvantages with alternative methods, as well as with experiments. There have been many other studies carried out in business life, and I argue here that comparisons of outcomes from these studies and from this study are of interest.

A completely different approach, compared to examining business life, is to continue the line of research in experimental economics. There are numerous experiments in the field with simple market models, such as the models of Cournot and of Bertrand (Holt, 1995). The business game that I use in this study is more complex than these experiments. This study can therefore be seen as one of the first steps towards research in a field, which lies between studies of business life and of experiments with simple market models. Thus, this study can provide us with some insights into a new field of research, where much work can be done in the future.

Not all conditions in business life can be properly represented in experiments, but this should not necessarily be seen as a critique of the use of experiments in general. Experiments might not be useful in all possible situations; they could be useful for testing hypotheses from theories in general, but less useful when testing hypotheses where specific conditions matter.

This means that the generality of theories or hypotheses generated from both deductive studies and empirical studies, such as for example information use in decision making in business life, can be tested in experiments for generality. Testing theories or hypotheses in experiments could therefore be seen as *critical tests* – or even so-called acid tests, where theories or hypotheses which are not valid in experiments might not have general validity either.

An important question concerns the generality of theories and hypotheses. If theories and hypotheses were valid in general situations, they would be valid in an experiment. If the theories and hypothesis tested in experiments are not confirmed, the generality of the theory or the hypothesis is, at least, not valid for the experiment. This might be explained by the procedures of the experiment where the conditions differ from what is specified by a theory or a hypothesis. However, it can also be the case that the theories and hypotheses need to be reviewed.

2.5 Research purpose

For the purpose of this study, I will here refer to Rubinstein (1998, p. 190) where he discusses the profession of economist and the interpretation of economic theory with H. Simon.

One of the following might be the intended purpose for the use of models in economics:

M1 They aim at predicting behavior.

M2 They are normative, in the sense that they are supposed to guide the economist in giving advice to economic agents about what to do.

M3 They are exercises intended to sharpen economists' intuitions when dealing with complicated situations. Even if the models do not fully correspond to reality, dealing with such models is an indirect, yet helpful activity.

M4 They are meant to establish "linkages" between the concepts and statements that appear in our daily thinking on economic situations.

I think the research purpose of this study falls primarily into category M3 above. This is in line with my argument that the use of a business game as an experiment is of interest for the study of information use. One purpose is to find out how information is used in decision making in this business game. Thus, this study can provide us with intuitions on information use in decision making. Hypotheses generated in this study can be used for modeling information use and for tests in other experiments and tests in business life (M3).

This study can also provide us with some linkage between the statement that "better decisions are made when more relevant information is available for decision making" and the concept of optimal solutions with all relevant information (M4). That is, the explanation for the statement would be that the more relevant information that is available, the closer the decisions are to the optimal solution.

The purpose is also to find out what information is relevant in decision making. We might even be able to make predictions about what information is used and to predict behavior, i.e. decisions and results when the game is played as an experiment (M1). Furthermore, the purpose is also to investigate the use of a business game as an experiment. Future studies using business games as experiments can consider the methodology used in this study (M2).

3 Literature review

In order to investigate the research topic for this study, it is useful to review prior research in some related fields. I shall here point out that the review is limited to matters that are dealt with in this study. The references below are mostly related to research described in literature that deals with decision making about competition on markets.

The use of business games is a part of the field of *gaming*, which is mainly concerned with designing, creating, and using games for education and for operational research. However, the business game itself touches upon some basic issues in economics and in business administration.

In the table below, some fields of research related to the research topic are presented. The table also shows how the fields can contribute to this study and it shows the section where each field is described.

Related field of research	Contributions to the study	Section
Industrial organization and Game theory	Market models and optimal solutions	3.1
Economics of information	Impact of different information conditions in economic models	3.2
Decision making	Descriptions of decision making	3.3
Groups as decision makers	Decisions made by groups	3.4
Experimental economics and gaming	The use of economic models for experiments and the outcomes of these experiments	3.5

Table 3.1. Related fields of research and contributions of the fields.

In Chapter 14, potential contributions of this study to these related fields will be described.

3.1 Industrial organization and game theory

Industrial organization is the field in economics that is concerned with the study of the structure of markets, firms and their interaction. Thus it hence concerns the functioning of markets and the competition between firms on markets.

Theories of the field stem back to the 19th century. Cournot's model in 1838 and Bertrand's model in 1883 of oligopolies have been widely used and are still used as basic market models by many researchers.[1]

The field of industrial organization can be said to hold two paradigms. One of the paradigms is the structure-conduct-performance paradigm of e.g. Mason in 1939 and 1949 and Bain in 1959, which uses empirical data to study markets[2]. I will not comment on this paradigm here. I will only consider the other paradigm, which uses models of markets to gain insight into competition on markets. This paradigm uses game theory to find optimal solutions, here also called optimal decisions, in the market models. The standard textbook describing this paradigm is from the 1980s (Tirole, 1988). There are also a vast number of articles on the subject.

One basic assumption is that the firms on the markets make their decisions to maximize their own profits. Another basic assumption is that of *complete* information, which means that the firms have all relevant information about the market model.

When I defined the notions of the research design in the previous chapter, the market model was denoted A. The decisions in the market models in the field are usually assumed to be made with the following two basic assumptions:

B	Profit maximization
C	Complete information

[1] See for example Tirole (1988) for references on Cournot and Bertrand, and also on Bowley, Hotelling and Stackelberg that will be discussed below.
[2] See for example Scherer & Ross (1990) for further details.

3.1.1 Solutions of interest

One of the interests in this field is directed to deriving optimal solutions in market models. In this study I will primarily focus on two solution concepts; Stigler (1964), Friedman (1983), Shubik (1959), Shubik & Levitan (1980), Tirole (1988) and Shapiro (1989):

- Cooperative solution (also called joint-maximum solution)
- Non-cooperative solution (also called Nash equilibrium)

The *cooperative solution* means that we want to maximize the sum of the profits of all firms in the market model. Thus, the firms' profits are jointly maximized.

The literature distinguishes between two types of cooperative solution, depending on whether or not side payments are possible. Side payments are payments from some firms to compensate other firms, in order for them to agree to the cooperative solution. The cooperative solution *with* side payments is often a solution that will lead to very high profits for one firm and low profits for the others before the side payments are distributed, but a more even distribution after the side payments. In the cooperative solution *without* side payments, the firms commit themselves to decisions that will give each firm the highest profit it can get without any side payments taking place.

In the game sessions studied below it appeared impossible to carry out side payments, and at no time were there any indications that side payments had taking place. Consequently we shall limit our discussion below to the cooperative solution without side payments. Hence when we use the notion of cooperative solution below, we refer to the solution without side payments.

For the cooperative solution without side payments, it is important to distinguish between the cases when the market model is symmetric and when it is not symmetric. The model is symmetric when all firms on the same market face the same demand, have the same costs and have the same starting values for the state variables, e.g. cash and machine capacity.

In the case of a symmetric market model, all firms on the market will make identical optimal decisions and the cooperate solution, if unique, is the same for all firms. The deduction that in a symmetric market model all firms make the same decisions can be based on an axiom that is basic for game theoretic rationality, namely that the optimal solution should be independent of the names of the firms (see e.g. the Nash bargaining model[3]). Hence, if all firms on the same market are identical in every respect, except for their names, then they will also make identical optimal decisions.

The validity of the cooperative solution depends on many things. The timing of decisions is important. If the firms make the decisions sequentially, the last firm to make decisions can have *perfect* information, i.e. the firm can have information available about decisions made by the other firms, and it can take this information into account when making its own decisions.

Thus if all firms but one on the market have made cooperative decisions, when the last firm has information available about the other firms' decisions, it can take these decisions into account when making its own decisions. The last firm can then make the same cooperative decisions as the other firms, to maximize the *joint* profit. However, as an alternative, the last firm can instead decide to maximize its *own* profit. The decisions that maximize the profit of a firm, when the decisions of the other firms on the market are given, are called the *best reply* to the decisions of the other firms.

The best reply generally differs from the cooperative decisions. The best reply decisions may not only increase the profit of the firm that makes its best reply, but they usually decrease the profits of the other firms. The last firm to make its decisions with perfect information can make its best reply, rather than the optimal cooperative decisions, since the best reply gives the firm higher profit. In fact, the last firm needs to make its best reply in order to maximize its own profit. It seems plausible that the last firm should make these decisions, if no binding agreements have been reached about what decisions to make. We have here a conflict of interest, since the best reply may, as mentioned, decrease the profits of the other firms.

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[3] See Ståhl (1972), and also Fudenberg & Tirole (1991) and Osborne & Rubinstein (1994) for details.

We use the notion *reaction function*, when we determine the best reply for a firm that prefers to optimize its decisions, for each possible alternative of the decisions of the other firms. A reaction function is a locus of best replies. Thus, reaction functions give the best reply for each firm, given the decisions of the other firms.

The firms that have to make their decisions before the last firm have imperfect information about the decisions of the last firm. If the firms that make their decisions before the last firm assume that the last firm will use its best reply, they can make this an assumption, and take it into account when making their decisions, as modeled by von Stackelberg in 1934. That is, the firm that makes its decisions first can use the other firms' reaction functions when making its own decisions.

When firms believe that their decisions will affect the decisions of the other firms, this is called *conjectural variations*, as modeled by Bowley in 1924. The firm which made its decisions first then assumes that its decisions will affect the decisions of the other firms.

If all firms make their decisions at the same time, simultaneously, without having information about the other firms' decisions, all firms have *imperfect* information about the decisions of the other firms when making their own decisions.

The firms can exchange information about what decisions they will make, before they make their decisions. However, the firms can make other decisions than those that they informed the other firms they would make. Therefore, agreements are needed about what decisions the firms should make. These agreements need to be enforceable, i.e. they need to be arranged so that none of the firms can make an additional profit by not keeping to the agreement.

Agreements are a way to solve the conflict of interest, but such agreements have to be allowed. If agreements are not allowed, the firms will have difficulties in coordinating their decisions. Presumably the firms prefer to maximize their own profits, and they will therefore each make their best replies.

The reaction functions give the best reply for each firm, given the decisions of the other firms. However, in the case of imperfect information the firms do not, however, have information about the other firms' decisions. Hence, the firms cannot make their decisions with the use of their reaction functions by regarding the decisions of the other firms as given with certainty.

Reaction functions are, however, of interest in a situation where the firms make decisions repeatedly, and the market model is the same from one time to the next. After the firms have made some arbitrary decisions, they receive information about the other firms' decisions.

If we postulate, as Cournot and Bertrand did, that the firms assume that the other firms will make the same decisions as they did in the previous period, a solution can be based on the reaction functions. Then there is a lag of one period, but we still have a situation where each firm makes its best reply to the other firms' decisions. Eventually, after a number of repetitions of decisions, each firm will make its best reply to the other firms' best replies, provided that the reaction functions fulfill certain criteria so that they will uniquely intersect (Friedman, 1983). It should be noted that in this case the firms do not need to know the reaction functions of the other firms.

Nash generalized the solution concepts of Cournot and Bertrand as the so-called *Nash equilibrium*. This applies to both the cases of the decision alternatives being on a continuum and to the discrete case of a limited number of decision alternatives. It also applies to the case of a single period where the decisions are made with imperfect information. The reasoning behind the Nash equilibrium is then perhaps less intuitively clear. There is much discussion in the literature on this (De Bornier, 1992).

One situation when a Nash equilibrium would be compelling even in the case of a single period of decisions, on e.g. price, is the following: The firms, e.g. in an experimental situation, can overhear each others' ongoing discussions of the decision alternatives temporarily contemplated. Then each firm can in its decision making take the decision alternatives contemplated by the other firms as given, when in turn contemplating its best reply. Eventually, after a number of contemplated decision alternatives, all firms make their best replies to the other firms' best replies.

For this kind of situation we use the notion *non-cooperative solution*. This means that each firm maximizes its own profit in equilibrium, by using its best reply to the best reply of the other firms. This equilibrium is called a Nash equilibrium.

The Nash equilibrium is the basic solution concept in game theory. Decisions are in a Nash equilibrium if, given the decisions of the other firms, a firm cannot increase its profit by making decisions other than its

Nash equilibrium decisions. This applies for all firms in the Nash equilibrium. When looking for a Nash equilibrium, we are concerned with the existence and the uniqueness of the equilibrium.

As mentioned above for the cooperative solution, a special case is when symmetry exists. This is when firms on the same market have the same demand and the same costs, and the same starting values for the state variables. The firms then have identical reaction functions and the same best replies, and all firms make the same non-cooperative decisions in the Nash equilibrium. Hence, the best reply of one of the firm consists of the decisions that are the same best reply, i.e. the same decisions, for the other firms. We can say that *the best reply is the best reply to itself.* This non-cooperative solution often, but not always, depends on how many firms there are on the market[4].

As in the example above, a market model can consist of repeated decision making, as the model consists of more than one period. The number of periods in a market model is important, since the number of periods can make a difference to the optimal solutions. It is important to distinguish between market models with a finite number of periods and models with an infinite number of periods.

Furthermore, decisions can be made in each period to optimize the profit in the present period, so-called *myopic decisions* (M), or decisions can be made to optimize total profits over of a number of periods, so-called *dynamic decisions* (D). We shall use (C) for cooperative solutions and (N) for non-cooperatives solutions. Thus, we have four optimal solutions of interest in this study:

MC	Myopic cooperative solution
DC	Dynamic cooperative solution
MN	Myopic non-cooperative solution
DN	Dynamic non-cooperative solution

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[4] The best reply can, however, in the symmetric case be derived by studying the best reply of one of the firms to the identical or mean decisions of the other firms. If the decisions of the other firms are the best replies to the decisions of the firm studied, none of the other firms will alter its decisions. The best reply of the studied firm is, in this symmetric case, the same best reply as of any of the other firms, and hence no single firm will alter its decisions.

3.1.2 Basic market models

The basic market models primarily have the following variables:

- Prices
- Costs
- Quantities

The models consist of the relations between these variables, where price and quantities are related to each other through a demand function, and where quantities and costs are also related. The basic market models in the game theoretical paradigm usually have the following properties:

Code	Property
A1	1 decision variable (price or quantity)
A2	0 dynamic state variables
A3	2 firms competing on the same market

Table 3.2. Basic market model in industrial organization.

The optimal solutions in these models are usually found analytically.

3.1.3 Extensions of the basic market models

When a certain aspect is studied, the basic market models are extended to cover a certain phenomenon. Mostly these phenomena are studied one at a time. For example, we have entry and exit from markets, studied by Bain in 1956, product differentiation, studied by Hotelling in 1929, advertising and assets such as capacity, stocks and capital, studied by Shubik & Levitan (1980) and Friedman (1983).

Market models can be extended to have dynamic properties, for example on asset structures. Assets are then carried over from one period to another. Thus, decisions in one period can have an effect on decisions in subsequent periods. There can even be bounds on decisions in subsequent periods due to decisions in earlier periods. This is called *inertia*, and can be part of the market model.

To sum up, most models in industrial organization in the paradigm of game theory are simple representations of markets in business life.

3.2 Economics of information

As mentioned in the previous section, one basic assumption for many models in industrial organization is the assumption of complete information, i.e. that the firms on the market have information about the market model. This assumption also applies for many other models in other fields of economics. I do not mean to say that the field of economics does not acknowledge that real markets might be markets with incomplete information. Instead, it seems reasonable to start to study market models with an assumption of complete information. We shall here keep in mind that firms with complete information (about the market model) might not have perfect information, i.e. information about the decisions of the other firms, when making their own decisions.

Assumptions of incomplete information have been made in economic models over the years. The reason for the interest in decisions made with incomplete information is that this assumption is more realistic in business life, where there are no exactly specified market models. Rather, there are reports on prices, advertising, sales, profits and assets of the firms on the market.

The interest in the field of *economics of information* is to study the impact of different information conditions on models of economic theory and their solutions. One basic assumption about the information in the field is that on many occasions the information is incomplete. Still, the objective is to maximize profits. Thus, we have models in the field of economics of information, which consist of:

B	Profit maximization
C	Incomplete information

Levine & Lippman (1995) are editors of two volumes of papers on the topic. They claim that the field started with two articles by Vickery (1961) and Stigler (1961). Stigler's classical article was called Economics of information and deals with search for information. It brings up the topic of advertising as a way for a firm to give or share information to potential customers and competitors. Although Arrow (1973), Rothschild (1973) and Stiglitz (1985) wrote about the importance of the field, and it has found its way into textbooks in microeconomics, the field itself has only a few textbooks (Molho, 1997).

Exchange of information between competitors in business life will supposedly facilitate collusion, with higher prices and profits for the firms (Joskow, 1975 and Shubik, 1975). In order to improve their profits, the firms could make information about their own costs, sales, prices and assets available to their competitors (Gal-Or, 1985 and Shapiro, 1986). The assumption here is that additional information would make firms more cooperative. The additional information would then have a positive value for the firms. In business life, there are restrictions on legal agreements and on the exchange of price information between competitors on the same market. These limitations on available information are supposed to make decisions closer to the non-cooperative solution, compared to the cooperative solution.

Here I shall also mention three important contributors on incomplete information: Harsanyi (1967) and Aumann *et al.* (1995). Harsanyi introduced an assumption of a "draw of the nature", which transforms a model of incomplete information into a model of imperfect information. However, this involves a demanding assumption about *mutual expectations* of decision makers when deriving their optimal solutions.

This can be shown by the following example with two firms, called firm 1 and firm 2, on the same market. Firm 1 has unit costs of 10. Firm 2 does not have information about the costs of firm 1. When making its decisions, firm 2 forms the expectation that firm 1 has costs of either 10, 20, 30 or 40, with a probability of ¼ for each alternative. The demanding assumption of mutual expectations is here that firm 1, with costs of 10, in turn knows that firm 2 has expectations that its cost will be either 10, 20, 30 or 40, with a probability of ¼ for each alternative. This seems to be a more demanding assumption than that firm 2 has expectations that the unit costs of firm 1 will be 10.

3.3 Decision making

There is an extensive literature on decision making. One of the interests in the field is to study how decisions are made, or "How decisions happen", as March (1994) puts it.

Some of the literature on decision making is in business administration, while other literature is in the field of psychology. Simon's notion of *bounded rationality* from the 1950s seems to connect these two fields. Bounded rationality (Simon, 1955, 1956 and 1982) implies that there are bounds on the cognition and computational capabilities of the decision maker. There can also be bounds on, that is limitations on available information, as assumed by incomplete information. Simon called decisions made with bounded rationality *satisficing* decisions, since the optimal decisions are not made. Instead the decisions made are satisfactory.

Decisions are supposedly made with some rationality, where rational decisions would be the alternatives which give the most preferred results.

Simon and March distinguish between rationality of two types:

- Substantive rationality, i.e. "a style of behavior that is appropriate to the achievement of a given goal, within the limits imposed by given conditions and constraints".
- Procedural rationality, i.e. "a behavior that is the outcome of appropriate deliberation".

March calls these two procedures of rationality the *logic of consequence* and the *logic of appropriateness*, respectively. He describes four components of decision making in the logic of consequence as follows:

- Preferences
- Alternatives
- Expectations
- Decision rules

The logic of consequence can be explained as follows: There are a number of possible decision alternatives available for the decisions makers. The decision makers have to form expectations of the results of these decision alternatives. The decision makers then use decision rules to decide on the decision alternative that gives the (expected) result that they prefer. We see here some resemblance to Hayek (1945) in section 1.1, where he used "means" instead of decision alternatives. He used "all relevant information" instead of expectations, and in the case of perfect information, we can say that the decision maker has correct expectations of the results of the decision alternatives.

For completeness, we shall also mention the logic of appropriateness. March thinks that the components of logic of appropriateness might be more useful when studying decision making than the components of the logic of consequence. The following components are then of interest, when studying decision making:

Components	**Reasoning**
Recognition; decision making situation	What kind of decision making situation is this?
Identity; decision maker	What kind of decision maker am I?
Rules; appropriate decision	What does a decision maker such as I do in a situation like this?

Table 3.3. Logic of appropriateness.

As we can see in the table above, the logic of appropriateness is concerned with the characteristics of decision making situations, types of decision makers, and also what decisions are expected by the decision makers. Thus the characteristics of the decision making situation are important for the decisions made.

Holmström (1989) claims that as regards the field of industrial organization, "while substantial progress has been made as regards the description and analysis of market performance, firm behavior and organization have remained poorly understood". By firm behavior we understand decision making in the firms. Many of the theories in economics treat the decision making as a black box. This means that there exist descriptions of firms' decisions, but there is a lack of descriptions on how firms their make decisions. The question is if we can accept theories of, for example, firms' decisions, without having any descriptions of the decision making in the firms. This problem is well acknowledged and it has been debated for some decades (Friedman, 1953). Theories that predict decisions with accuracy might not give good descriptions and explanations for the decisions made.

However, much effort has been made over the years to obtain descriptions of decision making. Two examples of methods for obtaining such descriptions of decision making, that I have come across in my research process, are:

- Birts *et al.* (1997) used a method of so-called *think-aloud-protocols*. Individuals were asked to tell observers about their thoughts when decision making.
- Hennig-Schmidt (1997) used a method where she videotaped decision making in groups.

3.4 Groups as decision makers

In the literature, we have considered the market model (A), the objectives (B) and the information conditions (C). The identity of the decision maker has so far not been of importance. On most occasions, the identity of the decision makers is not part of the assumptions in economic theory. However, in most models the decision maker is thought of as a single individual. Nevertheless, the decision maker does not have to be a single individual. The decision makers are often groups of individuals (D). In fact, in business life many decisions are made by groups. Groups that are common in business life are boards of directors, executive teams, working groups and so on.

The basic entity in game theory is the player. Osborne & Rubinstein (1994) mention that a player in game theory can be interpreted as an individual or as a group of individuals who make the decisions. Tirole (1988) makes a note on group behavior, but relates its behavior to incentives rather than to characteristics of groups and their compositions.

Two classical books on groups are Homans (1950) and Brown (1965). Much research has been carried out in the field of groups. Some research on groups shows that groups can have some characteristics that make them better decision makers than individuals. For instance, comparisons between individuals and groups indicate that, since groups are more capable of checking and correcting errors than individuals, groups respond better to information and consequently make better decisions (Tindale, 1989 and Davis & Harless, 1996).

Henry (1995) found that groups that have one high performing member are likely to perform well. The larger the group, the greater the probability that the group has a high performing member. Hill (1982) found that groups which are heterogeneous with regard to individual characteristics could be more capable of analyzing a situation.

There is nevertheless ambiguity about the importance of group composition (Levine & Moreland, 1990 and Guzzo & Shea, 1992). Group compositions can be considered as an important determinant of the performance of a group. However, group compositions can merely be seen as a determinant of the resources available to a group working on a task, where for example larger groups have more resources, but they do not necessarily make better decisions than smaller groups. Thus, the question is whether we can distinguish certain compositions of groups that make better decisions than other groups.

Ball & Cech (1996) reviewed the research on groups in experimental economics. They claim that there has not been any formal systematic examination of groups as decision makers in experiments. Consequently, the perspective of groups as decision makers, i.e. players, deserves more attention and the question of the effect of group composition is open to more research.

3.5 Experimental economics and gaming

Models in economic theory are representations of situations in business life. The models are however less complex than business life. This relative simplicity makes the models useful when studying certain aspects of real markets. Normally, specific characteristics of business life are not described in the theories and the models. The market, the product and so forth are not specified. The interest is to test theories and hypotheses without specific characteristics. Hence, the model studied might differ from real business life and the model could be a poor representation of a decision making situation on a real market.

Over 50 years ago, researchers came up with the idea of using market models in experiments. Some of the pioneer articles were those by Chamberlin (1948), Hoggatt (1959), who used a business game, and Fouraker & Siegel (1963), who altered information conditions in their experiments. Three review articles on experiments of oligopoly markets are Friedman (1969), Plott (1989) and Holt (1995). Textbooks have set standards for conducting and reporting experiments. Apart from two books by Smith (1989, 1991), four textbooks describe the field extensively; Hey (1991), Davis & Holt (1993), Friedman (1994) and Kagel & Roth (1995).

The field of experimental economics has grown during the last decades. In the database Econlit there were in the year 2000 over 5000 articles where "experiment" was mentioned. Plott (1990) and many others express a great optimism about the future of the field. Two of the reasons are that experimental evidence has been found to support some of the theories, and that the possibility of verifying outcomes of experiments has made the field "scientific". However, it remains to verify whether the theoretical and experimental outcomes are valid outcomes in similar situations in business life.

For this study, the literature on experimental oligopoly markets is in focus. There are a number of articles on experiments on duopoly markets. For these experimental markets, with two competing firms, there is substantial evidence, perhaps even in a majority of the studies, that firms make decisions closer to the cooperative solution than to the non-cooperative solution (e.g. Ståhl, 1994). It is of interest to note that the firms make cooperative decisions despite the fact that the firms cannot exchange any information and that they hence cannot make any binding agreements. When we come to experiments with more than two firms, there is much less literature. Some examples where cooperative decisions were noticed with two firms competing on the same market, and where non-cooperative decisions were noticed when three firms competed on the same market are: Fouraker & Siegel (1963), Muren & Pyddoke (1999) and Dufwenberg & Gneezy (2000). Thus, experiments conducted with more than two firms point at a more favorable situation for the non-cooperative solution. However, more research is needed to determine the issue more definitely.

We will now turn to business games. Business games often consist of models of oligopoly markets, with a few firms competing on the same market. As mentioned, experiments in experimental economics use theories in economics as models, while models in business games are constructed to be more realistic representations of markets in business life. The figure below is made to illustrate this difference, using a scale ranging from business life to economic theory.

```
 Business life                                Economic theory
-|----------|-----------------|---|-------------
            Market models in   Basic market models in
            business games     experimental economics
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Figure 3.1. Difference between market models in business games and basic market models in experimental economics.

I mentioned above that the field of experimental economics is quite large. The field of business games and gaming is also substantial. As we shall see below, the number of business games is large, and so is the potential for using them for research.

Business games are designed to be models that have similarities with situations in business life. Business games in general can be used with several purposes in mind; Shubik (1975) and Ståhl (1983, 1988):

- Education of students
- Training of professionals
- Analyses of specific market situations (operational gaming)
- Experiments for research

A business game that is constructed primarily for one of these purposes can be used later for another purpose.

Business games have been widely used for education and for training for at least the last four decades. Shubik & Brewer (1972) made a survey in the USA, which identified about 450 models, simulations and games. A more recent survey was done by Faria (1987), who reported on the use of business games in academia and in business life in the USA. He reported that about 2000 colleges used them in 3000 courses with over 9000 business instructors. In business life, more than 6000 firms used business games in management training programs. The commercial side of the use of business games might make firms reluctant to reveal information on their use of games and data from the game sessions. Nevertheless, business games are also in frequent use in business life.

Here it should be mentioned that the models in business games represent different markets and that the games are of different sizes. Some games are played for days and even weeks, while other games are played for half a day or a whole day. Most of the games are used for educational purposes or the training of professionals.

Some of the books found on business games and gaming are Berglund & Grubbström (1969), Shubik (1975), Ståhl (1983) and Elgood (1984). Keys (1990) has assembled a bibliography of business games. The journal Simulation and Gaming (1979–) has a vast number of articles on business games. In the literature on business games and gaming, relatively little is written in a systematic way on the use of business games for research. Among others, Hägg *et al.* (1970), Hägg & Johansson (1975) and Keys & Wolfe (1990) have published articles on the use of business games in research.

As has been argued above, business games have a potential for research purposes. However, it is, important that the research should fulfil some requirements. For example, the standards in experimental economics could be considered when using business games as an experiment. It is important to consider, and if possible to avoid, the potential failures of experimental economics given in the table below (Davis & Holt, 1993, p. 33). The two first failures should be considered when altering the research design. For the last three failures I have used upper case letters within parenthesis to refer to the research design in this study.

–	Failure to include a baseline control treatment that calibrates outcome.
–	Failure to restrict focus to a few control treatments of interest that do not change too many things at once.
–	Failure to use salient rewards (B).
–	Failure to use complete and unbiased instructions (C).
–	Failure to choose the degree of institutional complexity appropriate to the problem being investigated (A).

Table 3.4. Potential failures in experimental economies.

I will discuss these potential failures in Chapters 6 and 13, when using a business game for experiments.

4 Description of the market model

Above, we made a distinction between the market model (A) of the business game and the procedures (B–D) when the game is played. The market model consists of relations between the decisions of the firms and the results. In this chapter, the market model of the game is described and the model is also formulated in mathematical form. The rules of the game are presented in Appendix A.

4.1 The market model

The business game used for this study models five firms competing on an oligopoly market, producing and selling similar, but not identical, storable products. The firms have to decide upon the following four decision variables in each period, as the firms produce to sell their products on the market (shown as decisions 1 to 4 in the decision form in Appendix D):

- Investments in machinery
- Production quantity
- Advertising
- Prices

One unit of machinery costs \$ 30. The machines depreciate each period by 10 percent, both physically and in accounting terms. One unit of machine capacity can produce one unit of the product in each period of the game. The cost of producing one unit is \$10. Products not sold in one period go into stocks and can be sold in subsequent periods.

The game has dynamic properties, since the following four state variables are carried over from one period to the next:

- Machine capacity
- Stocks
- Checking account
- Marketing effect

These variables above represent the state of each firm in the different periods of the game.

Equity is calculated as the balance on the checking account plus machine capacity (valued at $30 per unit) plus stocks (valued at $10 per unit). The marketing effect is not included in the equity, as presented in the balance sheet of the game[1]. All firms start with the same amount of equity, consisting of $ 200 in cash on the checking account. Initially the firms have no machine capacities, no units in stocks and no marketing effect.

The marketing effect consists of advertising in the period, plus a part of the marketing effect from the previous period, and the sales in the previous period, which also have some effect on the marketing effect. This cumulative advertising is here called *marketing effect*, to be distinguished from advertising in a period. In period 1, the marketing effect is defined only by the decision on advertising in that period.

The results of the decisions are:

- Sales
- Profits
- Equity

The demand on the market is determined by a demand function. The demand for a firm's products in a period is dependent not only on the price and the marketing effect of that firm, but also on the prices and the marketing effects of the other four firms competing on the same market. The market model in the game has the characteristics of an oligopoly market, where one firm's decision on prices and advertising affects the results of the other four firms. This feature of an oligopoly market can be called *interdependence* among the firms.

In this connection it should be mentioned that there are no random factors involved in the game, not even when the demand for the products is calculated. Thus, the state of the firms and the decisions of the firms in a period completely determine the results of the decisions for the firms in

[1] According to Ståhl there were several reasons not to include the marketing effect in the equity, to be shown in the reports on the balance sheets to the firms, although from a theoretical point of view it could be regarded as part of the equity. The main reason was one of pedagogical realism. The marketing effect is seldom known exactly, especially not by competitors. Furthermore, this type of continuously updated “goodwill” is in reality seldom included in the balance sheet. Finally, when the game is ended after a certain number of periods, the value of the marketing effect at the end of the last period might be 0.

that period. This does not mean that identical decisions of all firms in a period necessarily yield the same result. The results of the identical decisions of the firms can differ in a period, if the values of the state variables of the firms differ.

The firms each have a checking account. If the balance on the checking account is negative, e.g. due to outlays on investment, production and advertising, the firms can borrow money. The interest rate depends on the size of the balance on the checking account and the equity of the firm. The more negative the balance and the smaller the equity, the higher the interest rate. The interest rate on a loan varies between 13 percent and 28 percent. If a firm has a positive balance on the checking account, i.e. cash instead of loan, it earns a flat interest rate of 10 percent on the cash balance (see also the interest table in Appendix B).

If a firm has equity below zero, it goes into bankruptcy. The firm can then receive a money grant from the government that increases its equity. If a firm goes into bankruptcy repeatedly, the game leader may omit it from the game.

The game also has the feature of the possibility of an importer, who makes an entry if the prices of the firms in the previous period were above a certain level. The feature of an importer was used in a few game sessions played before 1997, but it was not used in any of the game sessions played in 1997–1999.

The market model in the business game is summarized in the table below.

Code	Property
A1	4 decision variables (investments, production, prices, advertising)
A2	4 dynamic state variables (check.acc./loan, stocks, capacity, marketing effect)
A3	5 firms competing on the same market

Table 4.1. Summary of the market model of the business game.

It should be noted that all five firms in the market model have the same state variables at the start, the same costs and the same interest function, and they also face the same demand function. Hence, the market model is symmetric for all five firms. Finally, it should be pointed out that the market model used in this study is more complex than the basic market models normally used in the field of industrial organization (see section 13.5 for a comparison).

4.2 Mathematical description of the market model

In this section, the market model is described in detail using mathematical notations. The main reason for this mathematical description is to form a basis when determining optimal solutions. A general description, using parameters and symbols etc. instead of numerical values, also lays the ground for the improvement of the game, e.g. for future use of the game for research.

The mathematical description below is linked to the decision form used when the game is played, presented in Appendix D, and the reports, presented in Appendix E. The variables in the decision form and in the sample report are numbered with brackets. These numbers correspond to the numbers in brackets used in the mathematical description below.

There are variables of two types: *decision variables*, denoted with lower case characters, and *state variables*, denoted with upper case characters. Furthermore, there are a number of parameters which are denoted with Greek characters. Each of the firms is indexed with a j. The total number of firms is denoted by N. Each period is also indexed, but with a t. The total number of periods is denoted by T.

The firm index and the time index are both placed at the bottom right of the variables, but they are only used one at a time. The firm index is omitted from a variable when an equation is identical for all firms. The time index is omitted from a variable, when the index for the time t applies to all variables in the same equation.

I will here use dots to represent different states of state variables in the game, for example $K, \dot{K}, \ddot{K}$. This will be explained in more detail below. Please note that the use of dots in this study is different to the use of dots in other studies, where the dot is often used to indicate time derivatives.

The market model is described in the following ten steps:

4.2.1 Variables at the start of a period
4.2.2 Decision variables in a period
4.2.3 Intermediate variables in a period
4.2.4 Demand, sales, revenue, costs and profit
4.2.5 Variables at the end of a period
4.2.6 Deriving the profit from the balance sheet
4.2.7 Interest function
4.2.8 Demand function
4.2.9 Demand table
4.2.10 Summary of the market model

4.2.1 Variables at the start of a period

We start the description with a table of the variables at the start of a period.

State variable at the start of a period	**Parameter**[2]	**Restriction**	[3]
K = Capacity, units of machines	σ	$K \geq 0$	[2]
S = Stocks, in units	χ	$S \geq 0$	[3]
C = Checking account balance			[1]
E = Equity		$E \geq 0$	
M = Marketing effect	μ, η	$M \geq 0$	

Table 4.2. State variables at start of a period

The four basic state variables, K, S, C, M, are carried over from one period to the next. The state variables can have three different states:

- Variables at start of period are denoted in plain letters, e.g. K.
- Intermediate variables in a period are denoted with one dot, e.g. $\dot{K}$.
- Variables at end of period are denoted with two dots, e.g. $\ddot{K}$.

---

[2] The parameters σ and χ are respectively, the cost of one unit of machinery and the cost of production of one unit. Further details regarding the values of the parameters are specified in table 4.6 on page 65.

[3] The numbers within brackets in this column, and in the text below, correspond to the numbers within brackets in the decision form in Appendix D, which is used when the game is played.

Equity is calculated as a combination of three of the four basic state variables (i.e. excluding M^4), and the two parameters, σ, χ:

(1) $$E = \sigma K + \chi S + C \geq 0$$

σK is the value of capacity, χS is the value of stocks and C is the balance on the checking account. A positive balance implies a cash surplus and a negative balance implies a loan. A firm can have negative cash as part of its equity, as long as the equity $E \geq 0$.

The state variables, as well as the decisions, are shown in the table below, which shows how the variables are positioned during a period:

Start of period	**Decisions**	**Intermediate**	**End of period**
K	i	$\dot{K}$	$\ddot{K}$
S	o	$\dot{S}$	$\ddot{S}$
C		$\dot{C}$	$\ddot{C}$
M	a	$\dot{M}$	$\ddot{M}$
	p		
E			$\ddot{E}$
--- | --------- | --------- | ----- ^^^^ ------ | -------			
		"long time"	

Table 4.3. Variables in the market model.

4.2.2 Decision variables in a period

In each period, the firms have to make four decisions. A general restriction is that all decision variables must be non-negative. There are some specific restrictions shown in Table 4.8.

Decision variable			**Restriction**	**No.**
i	=	investments in units of machinery	$i \geq 0$	[4]
o	=	production output in units	$\dot{K} \geq o \geq 0$	[6]
a	=	advertising	$a^{MAX} \geq a \geq a^{MIN}$	[7]
p	=	price	$p^{MAX} > p > \chi$	[11]

Table 4.4. Decision variables in a period.

4 See page 50, especially footnote 1.

4.2.3 Intermediate variables in a period

At the time the decisions are made, the model has an intermediate state, where the basic state variables are transformed to what are called here *intermediate state variables*. The intermediate state variables have, as mentioned, the same capital letters as the state variables, but they are denoted with one dot on top of the letter.

The intermediate capacity consists of the capacity at the start of a period plus the investments in the period.

$$\dot{K} = K + i \tag{2a}$$ [5]

As shown in Table 4.4, the intermediate capacity poses a restriction for the production, as the production is limited to the capacity, i.e.:

$$o \le \dot{K} \tag{2b}$$

The payments of a firm in a period for investments, production and advertising are denoted P where:

$$P = \sigma i + \chi o + a \tag{3}$$ [8]

The balance on the intermediate checking account is denoted $\dot{C}$. It is an intermediate state depending on the checking account balance at the start of the period minus the payments, due to decisions made in the period.

$$\dot{C} = C - P \quad \text{i.e.} \quad \dot{C} = C - \sigma i - \chi o - a \tag{4}$$ [9]

When the balance on the intermediate checking account $\dot{C}$ is positive, the firms are said to have cash. When $\dot{C}$ is negative, a loan is automatically taken up. There is a restriction on the loan, $\dot{C} \ge C^{MAX}$ where C^{MAX} is the lowest negative balance allowed in the checking account[5]. The interest rate is calculated according to the interest function presented in section 4.2.7, also shown in Appendix B.

[5] In the game sessions studied, $C^{MAX} = -1000$, restricting loans to 1000.

The supply of products that can be sold on the market from the intermediate stocks is denoted $\dot{S}$. It consists of unsold units of the product from earlier periods plus the amount of units produced in the period.

(5) $\dot{S} = S + o$ [10]

M is the marketing effect at the start of the present period. The advertising, a, in the present period is added to this market effect, M. Together they form the intermediate state marketing effect, $\dot{M}$.

(6) $\dot{M} = M + a$

4.2.4 Demand, sales, revenue, costs and profit

When all firms have made their decisions in a period, the demand, d, is calculated using the following multiplicative demand function:

(7) $d = d(p, p_{\overline{N}}, \dot{M}, \dot{M}_{\overline{N}}) = d_1(p)d_2(\dot{M})d_3(p, p_{\overline{N}})d_4(\dot{M}, \dot{M}_{\overline{N}})$

where mean price, $p_{\overline{N}}$, and mean marketing effect, $\dot{M}_{\overline{N}}$, refer to the mean for all N firms on the market, i.e.

(8a) $p_{\overline{N}} = (\sum_{j=1}^{N} p_j)/N$ and (8b) $\dot{M}_{\overline{N}} = (\sum_{j=1}^{N} \dot{M}_j)/N$

Sales, denoted q, is the smallest of demand and supply.

(9) $q = \min[d, \dot{S}]$ (sales) [12]

The revenue for each firm is calculated as:

(10) pq (revenue) [13]

The cost components are the following:

(11) χq (cost of sold units) [14]

where χ is the unit cost

(12) a (cost of advertising) [7]

(13) $\delta\sigma\dot{K}$ (depreciation) [16]

where δ is the fraction of depreciation[6] and σ the unit price of machines.

The interest payment can be either a cost component or a revenue component, depending on the balance on the intermediate checking account:

(14) $r\dot{C}$ (interest payment) [15]

where r is the interest rate of the period.

The profits for the period for each firm are the revenue minus the cost components plus the interest payment:

(15) $$\Pi = pq - \chi q - a - \delta\sigma\dot{K} + r\dot{C}$$ [17]

4.2.5 Variables at the end of a period

The four state variables are updated at the end of the period.

The checking account balance is affected by the revenue and interest on cash. The interest payment, $r\dot{C}$, is negative, if the intermediate balance on the checking account is negative, i.e. a loan is taken, and otherwise positive.

(16) $$\ddot{C} = \dot{C} + pq + r\dot{C}$$ [20]

[6] If, as like in the game session studied, we have 10 percent depreciation per period, $\delta = 0.1$.

Intermediate capacity is decreased by depreciation

(17) $\ddot{K} = (1-\delta)\dot{K}$ [18]

Intermediate stocks is decreased by the number of units sold

(18) $\ddot{S} = \dot{S} - q$ [19]

The marketing effect is also changed. A fraction, μ, of intermediate marketing effect, $\dot{M}$, remains and to this is added the marketing effect, e.g. in the form of "mouth-to-mouth advertising", of having sold q units, such that each unit sold has the same effect as η spent on advertising at the end of the period.

(19) $\ddot{M} = \mu\dot{M} + \eta q$ marketing effect

When we move into the next period, from t to $t+1$, the closing balances of period t become the opening balances of period $t+1$, i.e.:

(20a) $K_{t+1} = \ddot{K}_t$

(20b) $S_{t+1} = \ddot{S}_t$

(20c) $C_{t+1} = \ddot{C}_t$

(20d) $M_{t+1} = \ddot{M}_t$

4.2.6 Deriving the profit from the balance sheet

As a control on the profit (15), we also derive the profit from the balance sheet. The profit of a firm consists of the difference between the equity at the end of the period and the equity at the start of the period.

(21a) $\Pi = \ddot{E} - E$

The equity at the end of a period is

(21b) $\ddot{E} = \ddot{C} + \sigma\ddot{K} + \chi\ddot{S}$ [21]

Using first (16), (17) and (18) and then (1), (2a), (3), (4) and (5) we get

(21c) $\ddot{E} = \dot{C} + pq + r\dot{C} + \sigma(1-\delta)\dot{K} + \chi(\dot{S} - q) =$

$C - P + pq + r\dot{C} + \sigma(1-\delta)(K + i) + \chi(S + o - q) =$

$C - \sigma i - \chi o - a + pq + r\dot{C} + \sigma K + \sigma i - \sigma\delta(K + i) + \chi S + \chi o - \chi q =$

$pq - \chi q - a + r\dot{C} - \sigma\delta\dot{K} + C + \chi S + \sigma K =$

$pq - \chi q - a + r\dot{C} - \sigma\delta\dot{K} + E$

Hence,

(21d) $\Pi = pq - \chi q - a - \sigma\delta\dot{K} + r\dot{C}$

i.e. the same as above in (15).

We note that we can also write this as

(22a) $\Pi = pq - \chi q - a + rC - r\sigma i - r\chi o - ra - \sigma\delta K - \sigma\delta i =$

(22b) $pq - \chi q - r\chi o - a - ra - r\sigma i - \sigma\delta i + rC - \sigma\delta K =$

(22c) $q(p - \chi) - r\chi o - a(1 + r) - i(r\sigma + \sigma\delta) + \Gamma$,
where $\Gamma = rC - \sigma\delta K$, which can be regarded as a constant at the time of the decisions.

We see that the profit depends on the four decisions: investment (i), production (o), advertising (a) and price (p). The profit also depends on the values of C and K at the start of a period, the parameters χ, σ, δ and the interest rate, r. All variables, constants and parameters are zero or non-negative, except the balance on the checking account, which is restricted by $\dot{C} \geq C^{MIN}$.

4.2.7 Interest function

The variable interest rate, r, depends on $\dot{C}$ and E, more specifically the quotient $\dot{C}/E$. Hence, r is a function of $\dot{C}$ and E:

(23) $r(\dot{C},E)$

If the checking account balance $\dot{C}$ is positive, the interest rate is given by the flat rate $r^{CASH} > 0$. If the balance is negative, a loan is taken up automatically. The interest rate is then variable in the interval $r^{MIN} \leq r \leq r^{MAX}$. The smaller the quotient $\dot{C}/E$, i.e. the higher absolute value of $\dot{C}/E$, the higher the interest rate. The firms can at the most borrow C^{MAX}.

The interest function r is not continuous and not differentiable in the interval. When $\dot{C}$ is very close to 0, r jumps from r^{CASH} to r^{MIN}. Figure 4.1 below illustrates the interest function with the parameters used in the game sessions studied below.

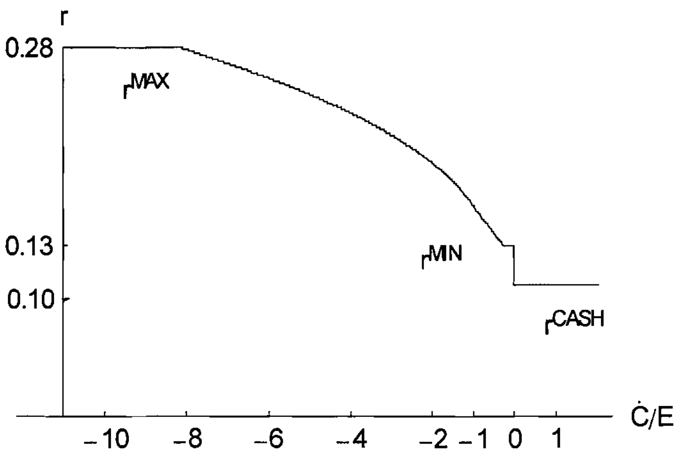

Figure 4.1. The interest function. The quotient $\dot{C}/E$ is on the x-axis and interest rate on the y-axis.

The interest function is specified in detail in Appendix B. This appendix also contains the interest table, used in the game sessions studied. However, the interest table shows only rounded off interest rates.

4.2.8 Demand function

The core of the market model is the demand function. The function is denoted *d*, which is the demand for the products on the market. The demand for a firm's product in a period depends on a combination of the price and marketing effect of the firm, and the mean price and the mean marketing effect of all firms on the market.

The demand function (7) is this time presented with three parameters: the size of the market, α, the price elasticity, β, and a parameter for marketing effect, ς[7],

(7) $$d(p, p_{\bar{N}}, \dot{M}, \dot{M}_{\bar{N}}) = d_1(p) d_2(\dot{M}) d_3(p, p_{\bar{N}}) d_4(\dot{M}, \dot{M}_{\bar{N}})$$

where

(24a) $d_1(p) = \alpha p^{-\beta}$ (24b) $d_2(\dot{M}) = \ln(\dot{M} + \varsigma)$

Function (24a) is a function of constant price elasticity.

(24c) $d_3(p, p_{\bar{N}}) = e^{f(p, p_{\bar{N}})}$, where $f(p, p_{\bar{N}}) = (p_{\bar{N}} - p)/p_{\bar{N}} = 1 - p/p_{\bar{N}}$,
i.e. $f(p, p_{\bar{N}})$ is the relative difference in price.

(24d) $$d_4(\dot{M}, \dot{M}_{\bar{N}}) = 1 + (\dot{M} - \dot{M}_{\bar{N}})/\dot{M}_{\bar{N}} = 1 + \dot{M}/\dot{M}_{\bar{N}} - 1 = \dot{M}/\dot{M}_{\bar{N}}$$

As regards the parameters we assume that

(24e) $\beta > 1$ and $\alpha, \varsigma > 0$.

We notice that the functions $d_1(p)$ and $d_2(\dot{M})$ depend only on the price, p, and the marketing effect, $\dot{M}$, for one firm. The functions $d_3(p, p_{\bar{N}})$ and $d_4(\dot{M}, \dot{M}_{\bar{N}})$ depend on these variables too, but also on the mean of all firms on the market as regards these variables.

---

[7] The parameter ς is used in (24b); this is meant to ensure that demand is defined when $\dot{M} = 0$.

The figure below illustrates the demand function in the market model. The numerical values of the parameters in the demand function are presented below in section 4.2.10.

The curves are so-called *iso-curves*, contouring all the points that refer to the demand for a given number of units, as given in the figure at the right hand end of the curve. All firms here have the same price and same marketing effect, i.e. we have symmetric decisions.

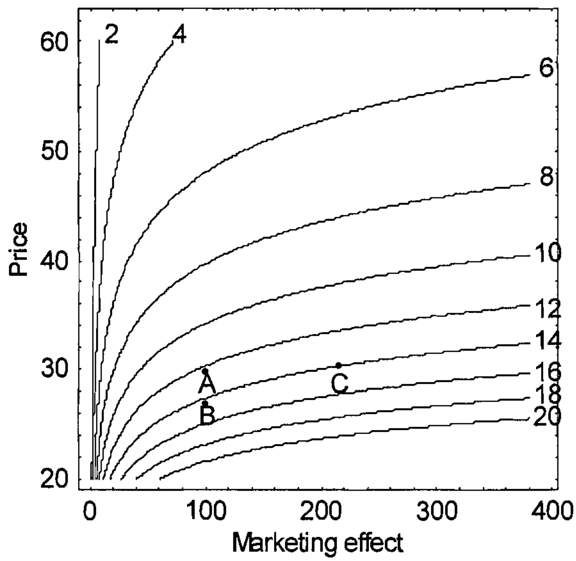

Figure 4.2. Demand function. Shows the demand for symmetric decisions.

As we can see in the figure above, an increase in the marketing effect over for example 100 has a moderate effect on the demand. In the figure, there are three points which show the sensitivity of demand. The points are on the iso-curves for 12 and 14 units:

A	12 units with price 30.28 and marketing effect 100.00
B	14 units with price 27.33 and marketing effect 100.00
C	14 units with price 30.00 and marketing effect 215.23

We see here that a decrease in price of about 3 increases the demand by 2 units, from 12 to 14. To have a similar increase of the demand, from 12 to 14 units, at the price of 30, the marketing effect has to be increased by about 115.

4.2.9 Demand table

Sections 4.2.1–4.2.8 consisted of my mathematical formulation of the market model as defined in the MINIMAX computer program.

For game sessions in 1998 and 1999, I constructed a demand table on the basis of the demand function in section 4.2.8. The purpose of the demand table (presented in Appendix F) was to make information about the demand on the market easily available to the firms (see also section 6.2.4). On the left side of the demand table, we have combinations of prices and marketing effects for one of the firms. At the top of the table, we have similar combinations for the remaining firms on the market. We shall call these firms *the other firms*.

We have called the number of all firms on the market N. Now we will call the number of the other firms n. Hence, $n = N - 1$. We denote the mean of the decisions of the n other firms by $\bar{n}$. Hence, at the top of the demand table we have the mean $p_{\bar{n}}$ and $\dot{M}_{\bar{n}}$ of the other firms. To calculate the values in the table, we first used the following relationships:

(25) $p_{\bar{N}} = (p + np_{\bar{n}})/N$ and (26) $\dot{M}_{\bar{N}} = (\dot{M} + n\dot{M}_{\bar{n}})/N$

On the basis of (24c) and (24d) we can formulate

(27a) $d_3(p, p_{\bar{N}}) = e^{f(p, p_{\bar{n}})}$, where

(27b) $f(p, p_{\bar{n}}) = 1 - p/\big((p + np_{\bar{n}})/N\big) = 1 - Np/\big(p + np_{\bar{n}}\big)$ and

(28) $d_4(\dot{M}, \dot{M}_{\bar{n}}) = \dot{M}/\big((\dot{M} + n\dot{M}_{\bar{n}})/N\big) = N\dot{M}/(\dot{M} + n\dot{M}_{\bar{n}})$

We can now redefine the demand function as

(29) $d(p, p_{\bar{n}}, \dot{M}, \dot{M}_{\bar{n}}) = d_1(p)d_2(\dot{M})d_3(p, p_{\bar{n}})d_4(\dot{M}, \dot{M}_{\bar{n}})$

The demand table shows the demand for only some of all possible combinations of prices and marketing effects. For values in between these combinations, interpolations can be used to estimate demand.

4.2.10 Summary of the market model

We end this chapter with three tables that summarize the market model.

Variable	Description	Relation
K	Capacity at start of period	
S	Stocks at start of period	
C	Check. acc. at start of period	
E	Equity at start of period	$E = \sigma K + \chi S + C$
M	Marketing eff. at start of period	
p	Price in period	*Decision*
$p_{\bar{n}}$	Mean of prices of the other firms in period	$p_{\bar{n}} = \left(\sum_{j=1}^{n} p_j\right)/n$
a	Advertising in period	*Decision*
$\dot{M}$	Marketing eff. in period	$\dot{M} = M + a$
$\dot{M}_{\bar{n}}$	Mean of marketing eff. of the other firms in period	$\dot{M}_{\bar{n}} = \left(\sum_{j=1}^{n} \dot{M}_j\right)/n$
d	Demand in period	$d = d(p, p_{\bar{n}}, \dot{M}, \dot{M}_{\bar{n}})$
q	Sales in period	$q = \min[d, \dot{S}]$
o	Production in period	*Decision*
$\dot{S}$	Supply in period	$\dot{S} = S + o$
i	Investments in period	*Decision*
$\dot{K}$	Capacity in period	$\dot{K} = K + i$
P	Payments in period	$P = \sigma i + \chi o + a$
$\dot{C}$	Check. acc. after payments	$\dot{C} = C - P$
r	Interest rate	See Appendix B
$\ddot{K}$	Capacity at end of period	$\ddot{K} = (1-\delta)\dot{K}$
$\ddot{S}$	Stocks at end of period	$\ddot{S} = \dot{S} - q$
$\ddot{C}$	Check. acc. at end of period	$\ddot{C} = \dot{C} + pq + r\dot{C}$
$\ddot{E}$	Equity at end of period	$\ddot{E} = \ddot{C} + \sigma\ddot{K} + \chi\ddot{S}$
$\ddot{M}$	Marketing eff. at end of period	$\ddot{M} = \mu\dot{M} + \eta q$
Π	Profit of the period	$\Pi = \ddot{E} - E$

Table 4.5. Summary of variables in the market model.

This table is also presented at the end of the book.

Below is a table showing the values of the parameters in the market model, for all game sessions studied.

Parameter		**Value**	**Description**
α	=	434.3	Market size
β	=	1.5	Price elasticity of demand
χ	=	10.0	Cost of production for one unit of the product
δ	=	0.1	Depreciation of capacity of machines
σ	=	30.0	Cost of one machine unit
μ	=	0.6	Part of marketing effect remaining in next period
η	=	1.0	Effect of one unit of sales on marketing effect in next period
ς	=	0.1	Parameter to avoid ln(0)
N	=	5	Number of firms on the market

Table 4.6. Parameters in the market model.

Below is a table showing the values of the state variables at the start of the game for all game sessions studied.

State variable at start		**Value**	**Description**
K	=	0	Capacity, units of machines
S	=	0	Stocks, unsold units
C	=	200	Checking account balance
E	=	200	Equity
M	=	0	Marketing effect

Table 4.7. State variables at the start.

The values of the parameters or the values of the state variables at the start of the game were not altered for the game sessions studied.

Finally, there is a table of values used in the game sessions showing the restrictions on some of the variables.

Restriction		Value	Variable
p^{MAX}	=	300.00	Maximum price, p
p^{MIN}	=	10.01	Minimum price, p, as $p > \chi$
a^{MAX}	=	425.00	Maximum advertising, a
a^{MIN}	=	0	Minimum advertising, a
r^{MAX}	=	0.28	Maximum interest on loan, r
r^{MIN}	=	0.13	Minimum interest on loan, r
r^{CASH}	=	0.10	Interest on cash r
C^{MAX}	=	1000.00	Maximum negative balance on checking account, $\dot{C}$

Table 4.8. Restrictions on price, advertising, interest rate and loans.

5 Optimal solutions in the market model

5.1 Introduction

In this chapter, we shall try to find four optimal solutions in the market model. As mentioned earlier, the following solutions are of interest in this study:

MC	Myopic cooperative solution
DC	Dynamic cooperative solution
MN	Myopic non-cooperative solution
DN	Dynamic non-cooperative solution

The purpose of determining these solutions is to compare the optimal decisions in the solutions with the decision made when the game is played. We can then see which of the solutions above gives the best description of the decisions made when the game is played.

We assume, when determining optimal solutions, that all firms have complete information about the market model when maximizing their equities, i.e. the firms have the information as described in Chapter 4. This means, in particular, that the firms have information about the demand function, as described in section 4.2.8.

We also assume that the firms have information about how many periods they make decisions for. For the myopic solutions, this information is redundant, since the firms optimize their decisions in the present period. However, for dynamic decisions this information is crucial.

In section 5.2, we make some simplifications by reducing the four decision variables (investment, production, advertising and price) to two decision variables, namely price and advertising. In sections 5.3 and 5.4, we then try to determine the optimal solutions. As we shall see, it is not a simple task to determine the optimal solutions in the market model.

The preferred solution method would be to derive the optimal solutions analytically. The analytical solutions have the properties that the optimal decisions are expressed in closed form, depending only on the parameters in the market model. The analytical solutions are of special interest, since they show relations between the optimal decisions and these parameters.

In section 5.3, we shall try to derive analytical myopic solutions, i.e. solutions that are optimal for one period if this period is the last one in the game. Due to the complexity of interest function in the market model, we use a simplification in the form of a fixed interest rate. The solutions obtained with this simplification are seen as approximates of the real solutions.

In section 5.4, we describe how several numerical methods were used, as part of the research process, in order to find optimal solutions.

Finally, in section 5.5, I present the optimal solutions obtained by the different methods. When finding the optimal solution we have mainly three kinds of problem:

- The numerical solutions will to some extent depend on factors such as starting values and the convergence criterion for an iterative search. We have to establish that the solutions obtained are to a reasonable extent independent of these factors.
- For both the analytical and the numerical solutions, we have the problem that we are mainly looking for local optima. We have to make it probable that these local optima are global optima[1].
- For the non-cooperative solution there can also be the problem that, although local best replies are all global best replies, there might be several equilibria. Hence we have to try to find out whether the equilibrium found is unique.

We shall discuss these problems as we try to determine optimal solutions.

[1] The task of verifying that local optima are global is not trivial. For example, the designers of one of the numerical methods (Fylstra *et al.*, 1998) say for instance that because of the possibility of multiple locally optimal points, there is no known way to determine with certainty that optimal solution is the "global optimum".

5.2 Simplification in case of complete information

We shall now show that we can reduce the four decision variables (investment, production, advertising and price) to only two, namely price and advertising. In section 4.2.6 we have:

(22c) $\Pi = q(p-\chi) - r\chi o - a(1+r) - i(r\sigma + \sigma\delta) + \Gamma$,
where $\Gamma = rC - \sigma\delta K$.

We see that the profit, Π, depends on four decision variables. We use this expression for the reduction of decisions. We start with two restrictions in the market model. The first restriction is (9) $q = \min[d, S]$. The second restriction is on production, where production can only be less than or equal to capacity in a period, (2b) $o \le \dot{K}$ where $\dot{K} = K + i$.

We shall make this reduction in three steps, where we assert the following, when we are maximizing profit and equity with complete information:

Step 1 Production cannot be larger than sales
Step 2 Production cannot be less than demand
Step 3 Capacity cannot be larger than production

Step 1 (Production cannot be larger than sales). The simplifications are as follows: We start by looking at a period where $K = 0$ and $S = 0$ (e.g. in period 1). Then $\dot{S} = S + o = o$ and $q = \min[d, \dot{S}]$ becomes $q = \min[d, o]$, i.e. $q \le d$ and $q \le o$. We note that in any such period with $S = 0$, i.e. no stock at the start of the period, there is a reason *not* to end the period with any stock. Hence we can set $o = q$. The reason is as follows: With the value of q given, due to complete information, and $o \ge q$, it is enough to rule out the alternative $o > q$. With q given, the only reason to produce in the present period rather than in subsequent periods would be that the cost would be lower later. Since we have constant positive unit cost, χ, which is the same in all periods, and we a have positive interest rate, there is no reason to produce in this period for sales in a later period. Applying this for $t = 1..T$ periods repeatedly, we have $q = o$ in every period, and hence $S = 0$ and $\dot{S} = o$ in every period. Hence, (9) $q = \min[d, \dot{S}]$ becomes $q = \min[d, o]$.

Step 2 (Production cannot be less than demand). $q = \min[d,o]$ implies with $q = o$ that $q \le d$. With complete information of demand, d, we can rule out the alternative of $q < d$. With $p > \chi$ (see Table 4.4), every unit sold at the given price and advertising resulting in d would lead to a profit. Hence we get $q = d$.

Step 3 (Capacity cannot be larger than production). The restriction that (2a), $\dot{K} = K + i \ge o$, i.e. that $i \ge o - K$, can be written as $i = o - K$, since we can rule out the alternative of $i > o - K$. We are not going to invest in a capacity that we are not going to use in the present period. The reason is that we have the same fixed costs, as well as production characteristics, for investments in any period and with $r > 0$, it is better to invest later rather than now.

With $o = q$, $q = d$ and $i = o - K$, and hence $i = d - K$, we can for period 1 write (22c) as:

(30a) $\Pi = d(p-\chi) - r\chi d - a(1+r) - (d-K)(r\sigma + \sigma\delta) + \Gamma =$
where $\Gamma = rC - \sigma\delta K$

(30b) $= d(p - \chi - \sigma\delta - r\chi - r\sigma) - a(1+r) + K(r\sigma + \sigma\delta) + \Gamma =$

(30c) $= d(p - \mathrm{X}) - a(1+r) + \Gamma'$,
where $\mathrm{X} = \chi + \sigma\delta + r\chi + r\sigma$
and $\Gamma' = K(r\sigma + \sigma\delta) + \Gamma = K(r\sigma + \sigma\delta) + rC - \sigma\delta K = rC + Kr\sigma$

We here note that, since $\chi, \delta, \sigma, r > 0$ we also have:

(30d) $\mathrm{X} > 0$

We use this simplification of decisions on production and on investment, when determining optimal solutions. After demand, d, is determined we have:

(31) $i = d - K$, where $i \ge 0$ and

(32) $o = d$

We then determine investments and production as in (31) and (32).

Before I describe in the following two sections how the optimal solutions are determined, I denote the optimal decisions on price and on advertising, as shown in the table below. (This table is also on the inner cover at the back of the book.)

Optimal solution	**Price**	**Advertising**
MC – Myopic cooperative solution	p^{MC}	a^{MC}
DC – Dynamic cooperative solution	p^{DC}	a^{DC}
MN – Myopic non-cooperative solution	p^{MN}	a^{MN}
DN – Dynamic non-cooperative solution	p^{DN}	a^{DN}

Table 5.1. Optimal solutions and decisions on price and on advertising.

5.3 Determining analytical myopic solutions

In this section we shall derive analytical myopic solutions, i.e. the myopic cooperative solution (MC) and the myopic non-cooperative solution (MN). These solutions are optimal when the game is ended after this period.

A necessary simplification is that a fixed interest rate $\hat{r}$ will be used instead of the complex interest function (23) $r(\dot{C},E)$. It should be mentioned that I have done some work with a variable interest rate, i.e. with the interest function, depending on $\dot{C}$ and E. However, this work gave expressions that were so complicated that it was impossible for me to obtain any analytical solutions.

In an earlier stage of the research process, I also made efforts to derive analytical dynamic solutions for two periods with a fixed interest rate. Again, I ended up with complex equations without analytical solutions in closed forms. Therefore, I settled for deriving analytically myopic solutions with a fixed interest rate.

We will here refrain from the work of deriving the second order derivatives. We here make an assumption that the extreme point found is a maximum. In a later section, 5.5, we shall try to check that this really is the case. We also assume that the optimum is within the bounds, given the restrictions in Table 4.8. This is also checked in section 5.5. Furthermore, we initially only search for local optimums defined in closed forms. If a local optimum is uniquely defined in closed form, it is also the global optimum.

We shall use the analytical solutions only as a complement to other methods, when determining numerical values of the solutions. In section 5.5, we will make comparisons between the analytical solutions that we obtain in this section and the numerical solutions obtained in the next section, section 5.4.

This section has the following subsections:

5.3.1 Derivatives of the profit function

In order to determine analytical myopic solutions, MC and DC, we have to find values on p and a, for which the derivatives of the profit function with respect to price and advertising equal zero, i.e. $\Pi'(p)=0$ and $\Pi'(a)=0$.

The optimal solutions are found on the basis of (30c), where we here use a fixed interest rate, $\hat{r}$, as an approximation of the variable interest rate, r.

(30c) $$\Pi(p,a)=d(p-\mathrm{X})-a(1+\hat{r})+\Gamma'$$

The derivatives of the profit, Π, with respect to price and advertising, p and a, are:

(33) $$\Pi'(p)=d'(p)(p-\mathrm{X})+d$$

(34) $$\Pi'(a)=d'(a)(p-\mathrm{X})-(1+\hat{r})$$

5.3.2 Derivatives of the demand function

A starting point of finding $\Pi'(p)=0$ and $\Pi'(a)=0$ is to find the derivatives of the demand function,

(29) $$d = d_1(p)d_2(\dot{M})d_3(p, p_{\bar{n}})d_4(\dot{M}, \dot{M}_{\bar{n}})$$

with respect to price and to advertising, denoted $d'(p)$ and $d'(a)$. Since $\dot{M} = M + a$ is a function of a, we can write $d_2 = d_2(\dot{M})$ as $d_2(a)$ and $d_4 = d_4(\dot{M}, \dot{M}_{\bar{n}})$ as $d_4(a)$. We then obtain the following derivatives of the demand function with respect to p and a:

(35) $$d'(p) = d_2 d_4\left(d_1'(p)d_3(p) + d_1 d_3'(p)\right)$$

(36) $$d'(a) = d_1 d_3\left(d_2'(a)d_4 + d_2 d_4'(a)\right)$$

We shall next determine the derivatives of the four parts of the demand function:

1. Since (24a) $d_1(p) = \alpha p^{-\beta}$

(37) $$d_1'(p) = -\beta\alpha p^{-\beta-1} = -\beta d_1 / p$$

Since $\alpha, p > 0$ and $\beta > 1$, $d_1'(p) < 0$.

2. Since (24b) $d_2(a) = \ln\left(\dot{M} + \varsigma\right) = \ln\left(M + a + \varsigma\right)$

(38) $$d_2'(a) = 1/\left(M + a + \varsigma\right)$$

Since $M, a \geq 0$ and $\varsigma > 0$, $d_2'(a) > 0$.

3. From (27a) in section 4.2.9, we have $d_3(p, p_{\bar{N}}) = e^{f(p, p_{\bar{N}})} = e^{f(p)}$, where (27b) $f(p) = 1 - Np/(p + np_{\bar{n}})$.

We first establish that

$$f'(p) = -N/\left(p + np_{\bar{n}}\right) + Np/(p + np_{\bar{n}})^2 = -N\left(p + np_{\bar{n}} - p\right)/(p + np_{\bar{n}})^2 = - Nnp_{\bar{n}}/(p + np_{\bar{n}})^2$$

Hence

(39a) $d_3'(p) = f'(p)d_3(p) = -d_3(p)Nnp_{\bar{n}}/(p + np_{\bar{n}})^2$

In the particular case when $p_{\bar{n}} = p$ we get

$f(p) = 1 - Np/(p + np_{\bar{n}}) = 1 - Np/Np = 1 - 1 = 0$, which gives

(39b) $d_3(p) = e^0 = 1$ and $d_3'(p) = 0$.

With $N, n, p, p_{\bar{n}}, e^{f(p,p_{\bar{N}})} > 0$, $d_3'(p) < 0$. Since both $d_1'(p) < 0$ and $d_3'(p) < 0$, demand will decrease monotonically, when p increases.

4. From (28) in section 4.2.9 we have, since $\dot{M} = M + a$,

$$d_4(a) = N\dot{M}/(\dot{M} + n\dot{M}_{\bar{n}}) = N(M + a)/(M + a + n\dot{M}_{\bar{n}})$$

which gives

$$d_4' = N/(M + a + n\dot{M}_{\bar{n}}) - N(M + a)/(M + a + n\dot{M}_{\bar{n}})^2 =$$
$$N((M + a + n\dot{M}_{\bar{n}}) - (M + a))/(M + a + n\dot{M}_{\bar{n}})^2 =$$

(40a) $$Nn\dot{M}_{\bar{n}}/(M + a + n\dot{M}_{\bar{n}})^2$$

In the particular case when $\dot{M}_{\bar{n}} = M + a$ we get

$$d_4(a) = N(M + a)/((M + a) + n(M + a)) =$$

(40b) $N(M + a)/N(M + a) = 1$ and $d_4'(a) = 0$.

With $M, \dot{M}_{\bar{n}} \geq 0$ and $N, n > 0$, $d_4'(a) > 0$. Since both $d_2'(a) > 0$ and $d_4'(a) > 0$, demand will increase monotonically, when a increases.

5.3.3 Determining the myopic cooperative solution

As we have symmetry, all firms will in every period decide on the same price p and on the same advertising a [1]. Hence the price, p, is not only the price for one of the firms, but also the price for each of the other firms, i.e. $p_{\bar{n}} = p$. The same applies for advertising, i.e. the advertising for one of the firms is the same as the advertising for the other firms, i.e. $a_{\bar{n}} = a$.

If all firms start with the same value of marketing effect, M, in one period, and all spend a in period 1, then $\dot{M}_{\bar{n}} = M + a$. With the same rate of decrease of marketing effect for all firms (19), all firms will have the same M at the start of the next period. From this it follows, since, $M = 0$ for all firms in period 1, that the marketing effect M will be the same for all firms in any period, and $\dot{M}_{\bar{n}} = M + a$ is valid for all periods. Furthermore, the derivatives of the profit function with respect to price and advertising can be simplified due to symmetry, since:

(41a) $\sum_{j=1}^{N} \Pi_j'(p) = N\Pi'(p)$ and (41b) $\sum_{j=1}^{N} \Pi_j'(a) = N\Pi'(a)$

Determining the myopic cooperative price

When determining the optimal price using (37), (38), (39b) and (40b), we have for the cooperative solution that

$d = d_1 d_2$, since $d_3 = 1$ and $d_4 = 1$

Hence $d'(p) = d_2 d_1'(p)$ and $d'(a) = d_1 d_2'(a)$

(33) $\Pi'(p)$ then becomes $\Pi'(p) = d_2 d_1'(p)(p - \mathrm{X}) + d_1 d_2$

[1] See the discussion on page 34.

Using (37) gives

(42) $\Pi'(p) = d(-\beta / p)(p - \mathrm{X}) + d = d\big((-\beta / p)(p - \mathrm{X}) + 1\big)$

Since $d > 0$, we then get when setting $\Pi'(p) = 0$, that

$$(-\beta / p)(p - \mathrm{X}) + 1 = 0 \rightarrow p - \mathrm{X} = p / \beta \rightarrow \beta(p - \mathrm{X}) = p \rightarrow$$
$$\rightarrow p\beta - \mathrm{X}\beta = p \rightarrow p(\beta - 1) = \mathrm{X}\beta \rightarrow p = \mathrm{X}\beta / (\beta - 1)$$

(43a) $p^{MC} = \mathrm{X}\beta / (\beta - 1)$

This can also be written as[2]

(43b) $p^{MC} = \mathrm{X} + \mathrm{X} / (\beta - 1)$

Hence the optimal price, p^{MC}, in the cooperative solution only depends on the parameters β and X. Since $\mathrm{X} > 0$ and $\beta > 1$, we have $p^{MC} > \mathrm{X} > 0$ and the optimal price, p^{MC}, is unique and therefore the global optimum[3].

Determining the myopic cooperative advertising

The derivative of profit (34) with respect to advertising, a, becomes $\Pi'(a) = d_1 d_2'(a)(p - \mathrm{X}) - (1 + \hat{r})$

Setting $\Pi'(a) = 0$ and using (38), we get

$$d_1 (p - \mathrm{X}) / (M + a + \varsigma) = 1 + \hat{r} \rightarrow d_1 (p - \mathrm{X}) / (1 + \hat{r}) = M + a + \varsigma \rightarrow$$

$$a = d_1 (p - \mathrm{X}) / (1 + \hat{r}) - M - \varsigma$$

---

[2] Further elaboration of the expression (43a) gives
$p = (\beta\mathrm{X} + \mathrm{X} - \mathrm{X}) / (\beta - 1) = \big(\mathrm{X}(\beta - 1) + \mathrm{X}\big) / (\beta - 1) = \mathrm{X} + \mathrm{X} / (\beta - 1)$

[3] For discussion on whether this extreme point is a maximum, and not a minimum or an inflection point, see page 120.

The optimal advertising, a^{MC}, in the cooperative solution is, using (24a),

(44a) $$a^{MC} = \alpha p^{-\beta}(p - \mathrm{X})/(1+\hat{r}) - M - \varsigma$$

Using (43b) for the optimal price, i.e. $p = \mathrm{X} + \mathrm{X}/(\beta - 1)$, we get

$$a^{MC} = \alpha p^{-\beta}(\mathrm{X} + \mathrm{X}/(\beta-1) - \mathrm{X})/(1+\hat{r}) - M - \varsigma =$$
$$\alpha p^{-\beta}\mathrm{X}/((\beta-1)(1+\hat{r})) - M - \varsigma, \text{ i.e.}$$

(44b) $$a^{MC} = \alpha(\mathrm{X}\beta/(\beta-1))^{-\beta}\mathrm{X}/((\beta-1)(1+\hat{r})) - M - \varsigma$$

Hence the optimal advertising, a^{MC}, depends on parameters $\alpha, \beta, \mathrm{X}, \hat{r}$ and ς, and on the state variable marketing effect, M. Since $\alpha, \mathrm{X}, \hat{r}, \varsigma > 0$ and $\beta > 1$, a^{MC} is unique.

We can see the profit and the cooperative solution in the figure below for period 1, where $M = M_{\bar{n}} = 0$, and with the parameters in Table 4.6. The values of the iso-profit are shown at the top, ranging from 0 - 125. The small point at profit 125 shows the myopic cooperative solution in period 1.

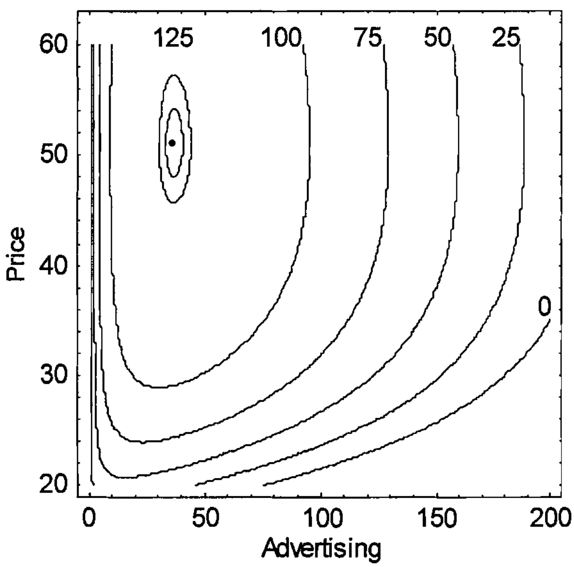

Figure 5.1. Profit in case of symmetric decisions, when $p = p_{\bar{n}}$ and $a = a_{\bar{n}}$.

5.3.4 Determining the reaction functions

We shall next derive the reaction functions that can be used to find the optimal decisions for one firm, when the optimal decisions depend on the decisions of the other firms. As mentioned on page 36, they consist of the best replies for one of the firms, given the decisions of the n other firms, i.e. $p_{\bar{n}}$ and $a_{\bar{n}}$.

Determining the reaction functions for price

We first derive the reaction function for prices. Using (37) and (39a) for d_1' and d_3' we obtain:

$$\begin{aligned}(45)\qquad & d'(p) = d_2 d_4 \left(d_1'(p) d_3(p) + d_1 d_3'(p)\right) = \\ & d_2 d_4 \left(\left(-\beta d_1 / p\right) d_3 - d_1 d_3 N n p_{\bar{n}} / \left(p + n p_{\bar{n}}\right)^2\right) = \\ & d_1 d_2 d_3 d_4 \left(-\beta / p - N n p_{\bar{n}} / \left(p + n p_{\bar{n}}\right)^2\right) = \\ & d\left(-\beta / p - N n p_{\bar{n}} / \left(p + n p_{\bar{n}}\right)^2\right)\end{aligned}$$

On the basis of this we can determine

$$\begin{aligned}(33)\qquad & \Pi'(p) = d'(p)(p - \mathrm{X}) + d = \\ & d\left(-\beta / p - N n p_{\bar{n}} / \left(p + n p_{\bar{n}}\right)^2\right)\left(p - \mathrm{X}\right) + d = \\ & -d\left(\left(\beta / p + N n p_{\bar{n}} / \left(p + n p_{\bar{n}}\right)^2\right)\left(p - \mathrm{X}\right) - 1\right)\end{aligned}$$

To get the best reply, p, to $p_{\bar{n}}$ we set $\Pi'(p) = 0$, i.e.

$$(46)\qquad \left(\beta / p + N n p_{\bar{n}} / \left(p + n p_{\bar{n}}\right)^2\right)\left(p - \mathrm{X}\right) = 1$$

This is an implicit function for how the optimal price, p, varies with the mean price of the other firms, $p_{\bar{n}}$. Hence this function, the reaction function, defines the best reply, p, to each possible value of $p_{\bar{n}}$. We illustrate this reaction function with the figure below, where we use the parameter values given in Table 4.6. The reaction functions shown are those for period 1. Recall that we use a fixed interest rate, $\hat{r}$. Here we use the value 0.1738 of the fixed interest rate, i.e. $\hat{r} = 17.38\%$[4].

The horizontal line in the figure shows the best reply on price, p, for one of the firms against the mean of decisions on price of the other firms. The vertical line shows the mean of the best reply for the other firms, $p_{\bar{n}}$, against the one firm, p. The point where the two lines intersect is the myopic non-cooperative solution for the optimal price, p^{MN} and $p_{\bar{n}}^{MN}$.

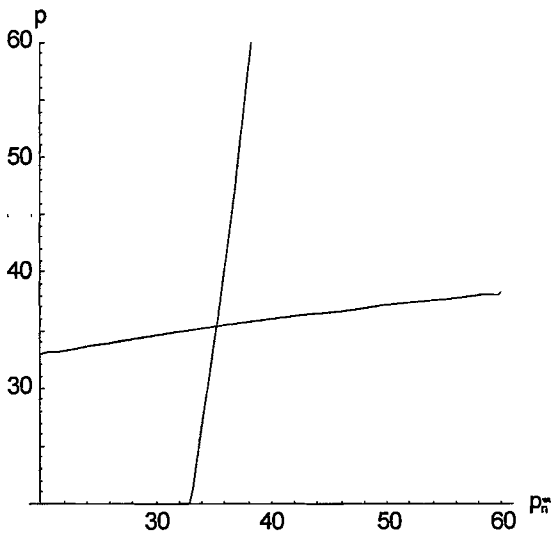

Figure 5.2. Reaction functions for decisions on prices.

Determining the reaction functions for advertising

We next determine the corresponding reaction function for advertising. Using (36), (38) and (40a) we get:

$$d'(a) = d_1 d_3 \left(d_2'(a) d_4 + d_2 d_4'(a)\right) =$$

$$d_1 d_3 \left(d_4 / \left(M + a + \varsigma\right) + d_2 N n \dot{M}_{\bar{n}} / \left(M + a + n\dot{M}_{\bar{n}}\right)^2\right) \quad (47)$$

[4] This is the same value for the interest rate as we obtain for the myopic non-cooperative solution in period 1 with variable interest, r.

Setting (34) to $\Pi'(a) = d'(a)(p - X) - (1 + \hat{r}) = 0$, we get:

$$(48) \qquad d_1 d_3 \left(d_4 /(M + a + \varsigma) + d_2 N n \dot{M}_{\bar{n}} / \left(M + a + n\dot{M}_{\bar{n}}\right)^2 \right)(p - X) = 1 + \hat{r}$$

We get an implicit function, as a reaction function, for the optimal advertising, a^N, which varies with the mean marketing effect of the other firms, $\dot{M}_{\bar{n}}$. Since $\dot{M}_{\bar{n}} = M_{\bar{n}} + a_{\bar{n}}$, the function gives the best reply, a, to each possible mean advertising of the other firms, $a_{\bar{n}}$.

5.3.5 Determining the myopic non-cooperative solution

Determining the myopic non-cooperative price

We look for the non-cooperative prices p^{MN} and $p_{\bar{n}}^{MN}$, as the p that is the best reply to $p_{\bar{n}}$. Since we look for symmetric solutions, the optimal price for each of the firms must be the same as the mean price of the other firms. We want the price to be the best reply to this price itself. Hence we start by setting $p = p_{\bar{n}}$ in the reaction function (46) and get:

$$\left(\beta / p + nNp / (p + np)^2\right)(p - X) = 1$$

Since $p + np = (n+1)p = Np$, we get:

$$\begin{aligned}
&(\beta / p + n / Np)(p - X) = 1 \rightarrow (N\beta + n)(p - X) = Np \rightarrow \\
&\rightarrow p(N\beta + n) - X(N\beta + n) = Np \rightarrow p(N\beta - N + n) = X(N\beta + n) \rightarrow \\
&\rightarrow p(N\beta - 1) = X(N\beta + n) \rightarrow p = X(N\beta + n)/(N\beta - 1)
\end{aligned}$$

We can also write this as

$$p^{MN} = X(N\beta - 1 + 1 + n)/(N\beta - 1) = X + NX/(N\beta - 1)$$

Since $N, \mathrm{X} > 0$ and $\beta > 1$ we have a unique solution:

(49) $$p^{MN} = \mathrm{X} + \frac{N\mathrm{X}}{N\beta - 1} = \mathrm{X} + \frac{N\mathrm{X}}{N(\beta - 1/N)} = \mathrm{X} + \frac{\mathrm{X}}{\beta - 1/N}$$

The solution varies with N. When N=1, $p^{MN} = \mathrm{X} + \mathrm{X}/(\beta - 1)$, which is the same solution as the cooperative solution p^{MC}. When N=5, $p^{MN} = \mathrm{X} + \mathrm{X}/(\beta - 1/5)$. As $N \to \infty$, $p^{MN} \to \mathrm{X} + \mathrm{X}/\beta$. With $\beta > 1$ we find that even for $N \to \infty$ $p^{MN} > \mathrm{X}$, i.e. there will always be a contribution.

Determining the myopic non-cooperative advertising

We next establish the equivalent non-cooperative solution for advertising. We rewrite the reaction function for advertising (48) as

(50) $$d_4 /(M + a + \varsigma) + d_2 Nn\dot{M}_{\bar{n}} /(M + a + n\dot{M}_{\bar{n}})^2 = (1 + \hat{r})/((p - \mathrm{X})d_1 d_3)$$

Since a^{MN} is the best reply to $a_{\bar{n}}^{MN}$ and, due to symmetry, a^{MN} the best reply to itself, we set $a = a_{\bar{n}}$ and hence

$M + a = M_{\bar{n}} + a_{\bar{n}}$, i.e. $\dot{M} = \dot{M}_{\bar{n}}$, i.e. we have $d_4(a) = \dot{M} / \dot{M}_{\bar{n}} = 1$.

Furthermore, with $M + a = \dot{M}$, we have $M + a + n\dot{M}_{\bar{n}} = N\dot{M}$ and we write (50) as

(51) $$1/(M + a + \varsigma) + d_2 nN\dot{M} /(N\dot{M})^2 = (1 + \hat{r})/((p - \mathrm{X})d_1 d_3)$$

From we have (24a) $d_1 = \alpha p^{-\beta}$. In this non-cooperative solution, with symmetric decisions, we have $p = p_{\bar{n}}$, implying that (27b)

$f(p) = 1 - p / p_{\bar{n}} = 1 - 1 = 0$. Hence $d_3 = e^{f(p)} = e^0 = 1$.

Finally, $d_2 = \ln(M + a + \varsigma)$ and $\dot{M} = M + a$. Hence we obtain:

(52a) $\quad 1/(M + a + \varsigma) + n\ln(M + a + \varsigma)/\big(N(M + a)\big) = (1 + \hat{r})/\big((p - \mathrm{X})\alpha p^{-\beta}\big)$

We shall use Φ to denote $(1 + \hat{r})/\big((p - \mathrm{X})\alpha p^{-\beta}\big)$ on the right hand side of (51). Since $p = \mathrm{X} + \mathrm{X}/(\beta - 1/N)$, Φ is constant. We obtain:

(52b) $\quad 1/(M + a + \varsigma) + n\ln(M + a + \varsigma)/\big(N(M + a)\big) = \Phi$

This equation, involving a *ln-term* of a, $\ln(a)$, and a in other parts of the expression, does not have any analytical solution. We therefore formulate the following function of advertising, a, since $n = N - 1$:

(52c) $\quad f(a) = 1/(M + a + \varsigma) + (N - 1)\ln(M + a + \varsigma)/\big(N(M + a)\big) - \Phi$

The non-cooperative solution for advertising, a^{MN}, is found when $f(a) = 0$. This equation has to be solved numerically by an iterative method, where one first gives a starting value of a, and a is then changed until $f(a)$ is close to 0. If one gets the same solution a^{MN} independent of the starting value of a, this solution a^{MN} is unique. This is discussed further on page 84.

This section can be summed up as follows:

- A fixed interest rate, $\hat{r}$, was used as an approximation of the variable interest rate, r.
- The myopic prices, p^{MC}, for the cooperative solution and p^{MN} for the non-cooperative solution were uniquely determined analytically. The optimal price in both solutions depends on the parameters β and X, where $\mathrm{X} = \chi + \sigma\delta + \hat{r}\chi + \hat{r}\sigma$. The non-cooperative solution also depends on the number of firms N.
- The myopic cooperative solution of advertising, a^{MC}, was uniquely determined analytically. The myopic non-cooperative solutions of advertising, a^{MN}, could only be represented implicitly and would have to be solved numerically. The optimal advertising depends on the same parameters as the corresponding optimal price, but also on α, ς and M.

5.4 Description of the numerical methods used to find optimal solutions

5.4.1 Introduction

In this section, we shall study the numerical methods used to find optimal solutions in the market model. Due to complete information, we shall use simplifications for investments and production (31) and (32). Thus, we shall determine optimal decisions on prices and advertising, and we shall then use these decisions to calculate demand, production and investments, and also profit and equity.

In the previous section, we determined the myopic solutions for a fixed interest rate $\hat{r}$ as an approximation for r. In this section, we shall use a variable interest rate $r(\dot{C},E)$ as in (23) when we search for both myopic and dynamic solutions. The notation used for the all numerical methods are presented in section 5.4.2.

In the research process, I started by using the software *Mathematica* (Wolfram, 1996) to find optimal solutions. This software has functions for determining derivatives and for solving equations both analytically and numerically. It was quite straightforward to use the software for determining the myopic solutions with a fixed interest rate, $\hat{r}$. However, I did not succeed in finding the desired *dynamic solutions* with this software, even with a fixed interest rate. Therefore, I will not give any further description here of my use of this software.

I and Ståhl then used the *Grid search method* for preliminary calculations (Edman & Ståhl, 1998). We used parts of the computer program of the business game to calculate demand, profit and equity. Below we use this as our main method for obtaining *myopic solutions*. However, the Grid search method has severe limitations, due to its requirements on the number of combinations of decisions needed to be searched for finding optimal solutions; hence it has not been very useful for finding the non-cooperative solution directly. The method has, however, been used on and off during the research process, when trying to verify dynamic solutions obtained by the other methods. The Grid search method is described in section 5.4.3.

I also did some tests in 1996 with the *Solver* program that was then available in spreadsheets. At that time I did not succeed in finding any optimal solutions. When I tried the Solver program again in 1999, with the Solver program available in Microsoft Excel 97, I found to my great surprise that this version of the Solver program could actually calculate *dynamic solutions* in the market model. There was still a problem in finding the exact solutions, however. In section 5.4.4, we will see how the Solver program was used to find optimal solutions.

When I was working with Mathematica in 1998, I came across a method called the *Random search method* (Zhigljavsky, 1991)[1]. Since I had only been able to find solutions with a fixed interest rate when using Mathematica, I decided to try the Random search method. After only a few tests, the method seemed promising, as it found values of decisions for dynamic cooperative solutions, with different start values of decisions. I used the Random search method before I started to use the newer Solver program of Excel 97. Here we shall use the Random search method to verify solutions obtained by the Solver program and vice versa. It should be mentioned that the Random search method could be used for problems where the Solver program might not work. The Random search method is described in section 5.4.5.

There are numerical restrictions on decision alternatives for prices and advertising (see Table 4.8). Thus there are also bounds on the optimal solutions. As mentioned on page 37, we are concerned with the existence and uniqueness of the solutions obtained. We stated on page 71, for the analytical myopic solutions, that if a local optimum is unique, it is also a global optimum. As also mentioned above, it is not trivial to verify that a solution is unique with numerical methods. We shall use different start values of decision alternatives as we are searching for the optimal decisions. If, regardless of start values of decisions we obtain the same optimal solution, then we have made it *probable* that the optimal solution obtained is unique.

---

[1] Many thanks to Anders Carlsson for the suggestion of the method.

The different numerical methods presented below can be seen as complements, where cooperative solutions and non-cooperative solutions can be compared between the different methods. We will also compare the non-cooperative solutions with the best reply to the non-cooperative solutions.

It should here be mentioned that the computer program of the game rounded off the values of the demand, d, to one decimal in most years, so as to give consistent values in the reports, presented with one decimal. This computer program also rounded off the values of the decisions made when the game was played to two decimals. Since both the Solver program and the Random search method work better with more decimals on demand, prices and advertising, I choose to use the maximum precision allowed by Excel.

5.4.2 Mathematical notation for optimal solutions

In this section I present the mathematical notations for the optimal solutions. Below we will distinguish between myopic and dynamic solutions.

Mathematical notation for myopic solutions

As mentioned in section 5.2, we use p^{MC} and a^{MC} to denote the myopic cooperative decisions for price and advertising, and we use p^{MN} and a^{MN} to denote the myopic non-cooperative decisions, respectively. We will for the notation here omit the index t for all decisions, except when describing the numerical methods used to obtain myopic solutions in sections 5.4.3 – 5.4.5.

The *myopic cooperative decisions* on price, p^{MC}, and advertising, a^{MC}, in a period are decisions that maximize the sum of the profits of all firms on the market. As mentioned on page 34, we want to find, symmetric decisions in the cooperative solution, i.e. all firms make the same decisions where $p = p_{\bar{n}}$ and $a = a_{\bar{n}}$.

Hence, based on (39b) and (40b), we can write $\Pi(p,a,p_{\bar{n}},a_{\bar{n}})$ as $\Pi(p,a)$. Thus, for all possible decisions, p and a, we have the cooperative decisions, p^{MC} and a^{MC}, such that

(53) $\Pi(p^{MC},a^{MC}) \geq \Pi(p,a)$

We denote the *myopic best reply* of the firm studied by MB, so that for optimal price we write p^{MB}, and for optimal advertising a^{MB}. This best reply maximizes the profit of the firm studied, given the mean decisions, $p_{\bar{n}}$ and $a_{\bar{n}}$, of the n other firms. Thus for the firm studied, for all possible decisions, p and a, we have best replies p^{MB} and a^{MB}, so that for all possible decisions, p and a:

(54) $\Pi(p^{MB},a^{MB},p_{\bar{n}},a_{\bar{n}}) \geq \Pi(p,a,p_{\bar{n}},a_{\bar{n}})$

In line with the definition on page 34, we denote the best reply of price, p^{MB}, and of advertising, a^{MB}, given the mean decisions, $p_{\bar{n}}$ and $a_{\bar{n}}$, of the other firms, as:

(55a) $p^{MB}(p_{\bar{n}},a_{\bar{n}})$ and (55b) $a^{MB}(p_{\bar{n}},a_{\bar{n}})$

The *myopic non-cooperative decisions* on price, p^{MN}, and on advertising, a^{MN}, are decisions that maximize the profit of the firm studied when the other firms make their non-cooperative decisions, $p_{\bar{n}}^{MN}$ and $a_{\bar{n}}^{MN}$.

For all possible decisions, p and a, the firm studied makes its best reply, which is also its non-cooperative decisions, i.e. $p^{MN} = p^{MB}$ and $a^{MN} = a^{MB}$, when:

(56) $\Pi(p^{MB},a^{MB},p_{\bar{n}}^{MN},a_{\bar{n}}^{MN}) \geq \Pi(p,a,p_{\bar{n}}^{MN},a_{\bar{n}}^{MN})$

We also have, as mentioned on page 37, symmetric decisions in the non-cooperative solution, i.e. all firms make the same decisions where $p = p_{\bar{n}}$ and $a = a_{\bar{n}}$, which means that $p^{MN} = p_{\bar{n}}^{MN}$ and $a^{MN} = a_{\bar{n}}^{MN}$.

In the case when we have obtained *tentative* non-cooperative solutions (MN), we can verify that these solutions, p^{MN} and a^{MN}, really constitute the best reply to itself. We have to find best replies for the firm studied so that:

(57a) $\quad p^{MB}(p_{\bar{n}}^{MN}, a_{\bar{n}}^{MN}) = p^{MN}$ and (57b) $\quad a^{MB}(p_{\bar{n}}^{MN}, a_{\bar{n}}^{MN}) = a^{MN}$

Decisions can be optimized myopically for a number of periods. This means that in each period the firms make their decisions to optimize the profit in the present period. As mentioned, the state variables are carried over from one period to the next period. On page 70, we made a simplification due to complete information, implying that the stock is zero in all periods, i.e. $S = 0$. With the other state variables K, C, M at the start of a period and the optimal decisions, on price and on advertising, we can calculate the intermediate state, $\dot{K}, \dot{C}, \dot{M}$, and also the state variables at the end of a period, $\ddot{K}, \ddot{C}, \ddot{M}$.

The values on state variables at the end of period t, are in turn the opening balances in period $t+1$, i.e. $K_{t+1}, C_{t+1}, M_{t+1}$ (see also 20a – 20d). For each firm, the decisions p_t and a_t together with the state variables for each firm K_t, C_t, M_t determine $K_{t+1}, C_{t+1}, M_{t+1}$.

We denote the transformation of the state variables from the start of one period to the start of the next period for the myopic cooperative decisions (MC) as:

(58a) $\quad (K_{t+1}, C_{t+1}, M_{t+1}) \leftarrow (K_t, C_t, M_t, p_t^{MC}, a_t^{MC})$

and for the corresponding myopic non-cooperative decisions (MN) as:

(58b) $\quad (K_{t+1}, C_{t+1}, M_{t+1}) \leftarrow (K_t, C_t, M_t, p_t^{MN}, a_t^{MN})$

Mathematical notation for dynamic solutions

We define dynamic solutions as solutions where the equity is maximized at the end of period T, where $T > 1$. The equity is given by $\ddot{E} = \sigma\ddot{K} + \chi\ddot{S} + \ddot{C}$, at the end of each period. For the next period we have $E_{t+1} = \ddot{E}_t$. We denote the sequence of decisions on prices p_t and advertising a_t in all T periods, by $\vec{p}$ and $\vec{a}$, i.e.:

(59a) $\vec{p} = p_1, p_2 ... p_T$ and (59b) $\vec{a} = a_1, a_2 ... a_T$

With the initial values on K_1, C_1, M_1 and the decisions of the other firms, $\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}}$, we can, determine $\ddot{E}_1, \ddot{E}_2 ... \ddot{E}_T$, based on the decisions $\vec{p}$ and $\vec{a}$, period for period. We can hence denote the equity at the end of period T as:

(60) $\ddot{E}_T(\vec{p}, \vec{a}, \vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$

Corresponding to (53), for the dynamic cooperative solution (DC) we have for all possible decisions $\vec{p}$ and $\vec{a}$:

(61) $\ddot{E}_T(\vec{p}^{\,DC}, \vec{a}^{\,DC}) \geq \ddot{E}_T(\vec{p}, \vec{a})$

For the *best reply*, corresponding to (54), we have for all possible decisions $\vec{p}$ and $\vec{a}$:

(62) $\ddot{E}_T(\vec{p}^{\,DB}, \vec{a}^{\,DB}, \vec{p}_{\bar{n}}, \vec{a}_{\bar{n}}) \geq \ddot{E}_T(\vec{p}\ , \vec{a}\ , \vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$

We want to find the best reply for price, $\vec{p}^{\,DB}$, and advertising, $\vec{a}^{\,DB}$, over T periods for the firm studied, given the mean decisions of the other firms, $p_{\bar{n}}$ and $a_{\bar{n}}$:

(63a) $\vec{p}^{\,DB}(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$ and (63b) $\vec{a}^{\,DB}(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$

For the decisions in the *non-cooperative solution* of the firm studied we have, corresponding to (56), that for all possible decisions $\vec{p}$ and $\vec{a}$, the firm studied makes its best reply to the optimal decisions of the other firms, which also is its non-cooperative decision:

$$(64) \qquad \ddot{E}_T\left(\vec{p}^{\,DB},\vec{a}^{\,DB},\vec{p}_{\bar{n}}^{\,DN},\vec{a}_{\bar{n}}^{\,DN}\right)\geq \ddot{E}_T\left(\vec{p}\ ,\vec{a}\ ,\vec{p}_{\bar{n}}^{\,DN},\vec{a}_{\bar{n}}^{\,DN}\right)$$

As described earlier in (57a) and (57b), for the symmetric market model we have the requirement that the non-cooperative decisions must be symmetric, i.e. all firms make the same decisions where $\vec{p} = \vec{p}_{\bar{n}}$ and $\vec{a} = \vec{a}_{\bar{n}}$, which means that we also have $\vec{p}^{\,DN} = \vec{p}_{\bar{n}}^{\,DN}$ and $\vec{a}^{\,DN} = \vec{a}_{\bar{n}}^{\,DN}$.

Just as for the myopic non-cooperative solutions, we can verify for the dynamic non-cooperative solutions that the *tentative* symmetric non-cooperative solutions, $\vec{p}^{\,DN}$ and $\vec{a}^{\,DN}$, really constitute the best reply to itself. We have to find best replies for the firm studied so that:

$$(65a) \qquad \vec{p}^{\,DB}(\vec{p}_{\bar{n}}^{\,DN},\vec{a}_{\bar{n}}^{\,DN}) = \vec{p}^{\,DN} \ \text{ and } \ (65b)\ \vec{a}^{\,DB}(\vec{p}_{\bar{n}}^{\,DN},\vec{a}_{\bar{n}}^{\,DN}) = \vec{a}^{\,DN}.$$

5.4.3 Description of the Grid search method

In brief, the Grid search method calculates the profit (or the equity) of a number of combinations of prices and advertising in T periods. The word grid refers to the specification of the number of alternatives searched for each decision. Each combination of decisions, on price and on advertising, can be seen as a candidate for the highest profit. We search all the specified combinations in the grid, to find the combination that gives the highest profit.

The computer program of the game rounds off the values of the decisions on price and on advertising in the game to two decimals. There are upper and lower bounds on price, $p^{MAX} > p > \chi$, and on advertising, $a^{MAX} > a \geq a^{MIN}$, as shown in Table 4.8. With the minimum interval of 0.01 for decisions on price and advertising, when the game is played, there is a maximum number of combinations of decisions, $Z^{\max}$.

We can therefore theoretically claim that the optimal decisions, of price and of advertising, can be found with the Grid search method.

Here we use the market model as it was programmed in the computer program of the business game. However, we only use the part of the program that consists of the market model and we exclude other routines (for example for printing decision forms and reports). We use the following start values of the state variables, according to Table 5.8:

(66) $\quad (K_1, C_1, M_1) \leftarrow start\_values \quad$ (i.e. $K_1 = 0, C_1 = 200, M_1 = 0$)

We added a dialogue to the computer program, which allows us to specify the grid as follows: The lowest decisions on prices and on advertising in each period, the number of decision alternatives and the size of the interval between decision alternatives.

In the table below, we see the specification of the grid with its notation:

Specification of grid	**Notation**
Lowest decisions ($t = 1..T$)	$p_t^{\min}$, $a_t^{\min}$
Number of decision alternatives (of each decision p and a)	k
Size of interval between decision alternatives	Δp, Δa

Table 5.2. Specification of the grid.

The lowest decisions are the start decisions, $p_t^{\min}$ and $a_t^{\min}$ for the search. These are the first decisions for which the profit is calculated. We use an index, z, for each combination, where Z is the total number of combinations and where $z = 1..Z$. This value of Z is different for the myopic procedures and the dynamic procedures, as will be explained below and can also be different in different periods.

Below there are descriptions of three procedures for myopic solutions and for dynamic solutions, respectively. We use the abbreviations from before, but we also use the notations MB and DB for the procedure of verifying the non-cooperative solution. The procedures are, in order, for establishing:

- Cooperative solutions (MC and DC)
- Non-cooperative solutions (MN and DN)
- Best reply to a tentative non-cooperative solution (MB and DB)

As I describe the procedures below, I use the mathematical notation for the optimal solutions in the previous section, section 5.4.2. Furthermore, I use a so-called *meta-code*, which has a resemblance to the VISUAL BASIC code. I use IF...THEN...ELSE for conditions and we use FOR...NEXT or DO...UNTIL to define loops. I use PRINT to print a message and TERMINATE to stop a procedure. I use the symbol "←" to assign values to variables. I also use CALCULATE, to calculate state variables from the start of one period to the start of the next period.

Myopic solutions with the Grid search method

We shall first show how, for myopic decisions, the Z alternatives for each period are generated. For this myopic cooperative solution there are two FOR...NEXT loops, one loop for price and one loop for advertising. Together the loops generate the grid, Z, of k^2 combinations of decisions.

```
z = 0
FOR p = p_t^min TO p_t^min + (k − 1)Δp STEP Δp
    FOR a = a_t^min TO a_t^min + (k − 1)Δa STEP Δa
        z = z + 1
        CALCULATE AND COMPARE PROFIT OR EQUITY
    NEXT a
NEXT p
Z = z
```

Figure 5.3. Part of procedures for myopic solutions, used for a generating a grid for one period.

MC Myopic cooperative solution

The search for the best combination, i.e. the combination which gives the highest profit, consists of two loops: one loop for the number of periods and one loop that is used for the search of the Z combinations. We denote the combination which gives the highest profit so far by $(\tilde{p}_t, \tilde{a}_t)$. At the start, the first combination, when $z = 1$, the highest profit so far is obtained by this first combination$(p_{z=1}, a_{z=1})$. When a combination (p_z, a_z) is found to give a higher profit, this combination is stored as $(\tilde{p}_t, \tilde{a}_t)$ and used for comparisons of profits with all remaining combinations.

When all Z combinations in a period have been searched, the combination which gives the highest profit for this period is declared the myopic cooperative solution of the period out of the combinations searched, and this combination is printed. Before the combinations of decisions are searched in the next period, the state variables based on the values of the solution are calculated and used at the start of the next period as in (58a). Thus we obtain the combinations, (p_t^{MC}, a_t^{MC}), for T periods which give the highest equity.

```
FOR t = 1 to T
    (p̃_t, ã_t) ← (p_{z=1}, a_{z=1})
    FOR z = 1 TO Z
        IF Π_t(p_z, a_z) > Π_t(p̃_t, ã_t)
            THEN (p̃_t, ã_t) ← (p_z, a_z)
    NEXT z
    (p_t^MC, a_t^MC) ← (p̃_t, ã_t) AND PRINT (p_t^MC, a_t^MC)
    (K_{t+1}, C_{t+1}, M_{t+1}) ← CALCULATE (K_t, C_t, M_t, p_t^MC, a_t^MC)
NEXT t
```

Figure 5.4. Procedure for the myopic cooperative solution.

The number of combinations in each period is $Z = k^2$ and the number for all T periods is hence Tk^2.

It should be mentioned that in the actual implementation we have added a test of whether $\Pi_t(p_z, a_z) = \Pi_t(\tilde{p}_t, \tilde{a}_t)$ to the procedure above, to check for the possibility of multiple optimal solutions. If there were any decisions for which the profit was equal to that of the temporary best decisions, a message was printed with this profit. When the procedure ended, we compared the highest profit obtained with the profits printed. If they were equal, we have obtained multiple solutions. Otherwise the solution is unique.

MN Myopic non-cooperative solution

This search consists of three loops, i.e. one more loop than for the cooperative myopic solution. The third loop consists of a combination of the decisions of the other firms $(p_{\bar{n}}, a_{\bar{n}})$. These decisions are searched the same number of times as (p, a), i.e. for Z combinations. We call the combinations of these decisions, $(p_{\bar{n}}, a_{\bar{n}})$, of the n other firms $z_{\bar{n}}$ and they are searched for $Z_{\bar{n}}$ combinations. The optimal solution is obtained for a combination of the other firms' decisions, $(p_{\bar{n}}, a_{\bar{n}})$, if the temporary best decisions of all combinations, $(\tilde{p}_t, \tilde{a}_t)$, are the same as the combination of decisions of the other firms, i.e. as in line with (56), $(\tilde{p}_t, \tilde{a}_t) = (p_{\bar{n}}, a_{\bar{n}})$. This implies that the tentative $(p_{\bar{n}}, a_{\bar{n}})$ constitutes the best reply to $p_{\bar{n}}$ and $a_{\bar{n}}$, and hence it is the best reply to itself.

Before the combinations of decisions are searched in the next period, the state variables based on the values of the solution are calculated and used at the start of the next period as in (58b).

$$
\begin{array}{l}
\text{FOR t} = 1 \text{ to T} \\
\quad \text{FOR } z_{\bar{n}} = 1 \text{ TO } Z_{\bar{n}} \\
\quad\quad (\tilde{p}_t, \tilde{a}_t) \leftarrow (p_{z=1}, a_{z=1}) \\
\quad\quad \text{FOR z} = 1 \text{ TO Z} \\
\quad\quad\quad \text{IF } \Pi_t(p_z, a_z, p_{\bar{n}}, a_{\bar{n}}) > \Pi_t(\tilde{p}_t, \tilde{a}_t, p_{\bar{n}}, a_{\bar{n}}) \\
\quad\quad\quad\quad \text{THEN } (\tilde{p}_t, \tilde{a}_t) \leftarrow (p_z, a_z) \\
\quad\quad \text{NEXT z} \\
\quad\quad \text{IF } (\tilde{p}_t, \tilde{a}_t) = (p_{\bar{n}}, a_{\bar{n}}) \\
\quad\quad\quad \text{THEN } (p_t^{MN}, a_t^{MN}) \leftarrow (\tilde{p}_t, \tilde{a}_t) \text{ AND PRINT } (p_t^{MN}, a_t^{MN}) \\
\quad \text{NEXT } z_{\bar{n}} \\
\quad (K_{t+1}, C_{t+1}, M_{t+1}) \leftarrow \text{CALCULATE } (K_t, C_t, M_t, p_t^{MN}, a_t^{MN}) \\
\text{NEXT t}
\end{array}
$$

Figure 5.5. Procedure for the myopic non-cooperative solutions.

The number of combinations for the inner loop for the firm studied is $Z = k^2$. We have the same number of combinations for the other firms, i.e. $Z_{\bar{n}} = k^2$. This implies k^4 calculations in each period and hence a total of Tk^4 for T periods.

MB Method for verifying the myopic non-cooperative solution

Next we study the procedure for verifying myopic non-cooperative solutions for T periods, where the tentative solution to be verified is denoted $\left(p_t^{MN}, a_t^{MN}\right)$ for each period t. The combinations we use to verify the solutions must include the tentative non-cooperative solution to be verified. We search the Z combinations in each period. If we find that that the temporary best combination $\left(\tilde{p}_t, \tilde{a}_t\right)$ after having searched all Z combinations is the best reply $\left(p_t^{MN}, a_t^{MN}\right)$, as in (57a) and (57b), we have then verified this tentative non-cooperative solution for period t. However, the procedure is terminated if all combinations in one period have been searched and the best reply is *not* the same as the non-cooperative solution, i.e. $\left(\tilde{p}_t, \tilde{a}_t\right) \neq \left(p_t^{MN}, a_t^{MN}\right)$.

FOR t = 1 to T

$\quad\left(\tilde{p}_t, \tilde{a}_t\right) \leftarrow \left(p_{z=1}, a_{z=1}\right)$

$\quad$FOR z = 1 TO Z

$\quad\quad$IF $\Pi_t\left(p_z, a_z, p_{\bar{n}}^{MN}, a_{\bar{n}}^{MN}\right) > \Pi_t\left(\tilde{p}_t, \tilde{a}_t, p_{\bar{n}}^{MN}, a_{\bar{n}}^{MN}\right)$

$\quad\quad\quad$THEN $\left(\tilde{p}_t, \tilde{a}_t\right) \leftarrow \left(p_z, a_z\right)$

$\quad$NEXT z

$\quad$IF $\left(\tilde{p}_t, \tilde{a}_t\right) \neq \left(p_t^{MN}, a_t^{MN}\right)$

$\quad\quad$THEN TERMINATE

$\quad\left(K_{t+1}, C_{t+1}, M_{t+1}\right) \leftarrow$ CALCULATE $\left(K_t, C_t, M_t, p_t^{MN}, a_t^{MN}\right)$

$\quad$NEXT $z_{\bar{n}}$

NEXT t

PRINT ("MYOPIC NON-COOPERATIVE SOLUTION VERIFIED")

Figure 5.6. Procedure for verifying the myopic non-cooperative solution.

The number of combinations searched in each period $Z = k^2$. Hence, the total number of combinations searched in T periods is $Z = Tk^2$, which is the same as for the dynamic cooperative solution presented below.

Dynamic solutions with the Grid search method

We shall next study how the Grid search method is used to find dynamic solutions. As mentioned above, for the dynamic solutions in T periods, where $T > 1$, we use the notations for price, (59a) $\vec{p} = (p_1..p_T)$, and for advertising, (59b) $\vec{a} = (a_1..a_T)$. When taking the decision of the other firms, $(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$, into account, we denote the equity after T periods for the firm studied (60) $\ddot{E}_T(\vec{p}, \vec{a}, \vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$.

For the dynamic solutions we will redefine Z to be the total number of combinations of all T periods. For myopic solutions, we needed two loops, one for price and one for advertising, which generate the combinations of decisions for one period. For dynamic solutions, we have two loops, one for price and one for advertising, for *each* period t of the T periods. In the example below, combinations of dynamic decisions are generated for two periods, period t and period $t+1$:

```
z = 0
FOR p_t = p_t^min TO p_t^min + (k − 1)Δp STEP Δp
   FOR a_t = a^min TO a_t^min + (k − 1)Δa STEP Δa
      FOR p_{t+1} = p_{t+1}^min TO p_{t+1}^min + (k − 1)Δp STEP Δp
         FOR a_{t+1} = a_{t+1}^min TO a_{t+1}^min + (k − 1)Δa STEP Δa
            z = z + 1
            CALCULATE AND COMPARE PROFIT OR EQUITY
         NEXT a_{t+1}
      NEXT p_{t+1}
   NEXT a_t
 NEXT p_t
 Z = z
```

Figure 5.7. Part of procedure for dynamic solutions, used for generating a grid for two periods.

The procedure above generates combinations for two periods. If we want to have combinations for a third period, $t+2$, we add two loops. For each additional period for which we want to generate combinations of decisions, we need to add two more loops. The number of combinations, Z, depends on the number of decision alternatives, k, and the number of periods, T. For one period we have k^2 combinations, for two periods we have k^4 combinations and generally for T periods we have $Z = k^{2T}$ combinations.

DC Dynamic cooperative solution

We assign the start combination $(\vec{p}_{z=1}, \vec{a}_{z=1})$ to our best combination of decisions so far, $\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right)$. We calculate the equity, $\ddot{E}_T$, at the end of period T, for each specified combination and compare it with the equity obtained by the equity of the best combination of decisions so far, $\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right)$. When all combinations have been searched, we have obtained the combination that gives the highest equity, (61) $\ddot{E}_T$, of all specified combinations. This is then the dynamic cooperative solution, $\left(\vec{p}^{\,DC}, \vec{a}^{\,DC}\right)$, according to (61).

```
(p̃⃗, ã⃗) ← (p⃗_{z=1}, a⃗_{z=1})
FOR z = 1 TO Z
    IF Ë_T(p⃗_z, a⃗_z) > Ë_T(p̃⃗, ã⃗)
        THEN (p̃⃗, ã⃗) ← (p⃗_z, a⃗_z)
NEXT z
(p⃗^DC, a⃗^DC) ← (p̃⃗, ã⃗) AND PRINT (p⃗^DC, a⃗^DC)
```

Figure 5.8. Procedure for the dynamic cooperative solution.

The total number of combinations is $Z = k^{2T}$.

DN Dynamic non-cooperative solution

Here we have two loops, one for the firm studied and one for the other firms. The same combinations of decisions are used for both the firm studied and the other firms. For each combination of the other firm, $(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$, every combination $(\vec{p}_z, \vec{a}_z)$ of the firm studied is compared. Again, if at the end of the search of all combinations have found that the temporary best decisions $(\tilde{\vec{p}}, \tilde{\vec{a}})$ are equal to $(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$, we have obtained a dynamic non-cooperative solution according to (64).

FOR $z_{\bar{n}} = 1$ TO $Z_{\bar{n}}$

$\quad(\tilde{\vec{p}}, \tilde{\vec{a}}) \leftarrow (\vec{p}_{z=1}, \vec{a}_{z=1})$

$\quad$FOR z = 1 TO Z

$\quad\quad$IF $\ddot{E}_T(\vec{p}_z, \vec{a}_z, \vec{p}_{z_{\bar{n}}}, \vec{a}_{z_{\bar{n}}}) > \ddot{E}_T(\tilde{\vec{p}}, \tilde{\vec{a}}, \vec{p}_{z_{\bar{n}}}, \vec{a}_{z_{\bar{n}}})$

$\quad\quad\quad$THEN $(\tilde{\vec{p}}, \tilde{\vec{a}}) \leftarrow (\vec{p}_z, \vec{a}_z)$

$\quad$NEXT z

$\quad$IF $(\tilde{\vec{p}}, \tilde{\vec{a}}) = (\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$

$\quad\quad$THEN $(\vec{p}^{DN}, \vec{a}^{DN}) \leftarrow (\tilde{\vec{p}}, \tilde{\vec{a}})$ AND PRINT $(\vec{p}^{DN}, \vec{a}^{DN})$

NEXT $z_{\bar{n}}$

Figure 5.9. Procedure for the dynamic non-cooperative solution.

The number of combinations for the firm studied is $Z = k^{2T}$ and for the other firms it is $Z_{\bar{n}} = k^{2T}$, implying that the total number of combinations is k^{4T}.

DB Method for verifying the dynamic non-cooperative solution

Here we verify that the temporary best decisions, $\left(\tilde{\vec{p}},\tilde{\vec{a}}\right)$, are the best decisions against the earlier computed tentative non-cooperative decisions of the other firms, $\left(\vec{p}^{DN},\vec{a}^{DN}\right)$, according to (65a) and (65b).

$\left(\tilde{\vec{p}},\tilde{\vec{a}}\right) \leftarrow \left(\vec{p}_{z=1},\vec{a}_{z=1}\right)$
FOR z = 1 TO Z
    IF $\ddot{E}_T\left(\vec{p}_z,\vec{a}_z,\vec{p}^{DN},\vec{a}^{DN}\right) > \ddot{E}_T\left(\tilde{\vec{p}},\tilde{\vec{a}},\vec{p}^{DN},\vec{a}^{DN}\right)$
        THEN $\left(\tilde{\vec{p}},\tilde{\vec{a}}\right) \leftarrow \left(\vec{p}_z,\vec{a}_z\right)$
NEXT z
IF $\left(\tilde{\vec{p}},\tilde{\vec{a}}\right) = \left(\vec{p}^{DN},\vec{a}^{DN}\right)$
    THEN PRINT ("DYNAMIC NON-COOPERATIVE SOLUTION VERIFIED")

Figure 5.10. Procedure for verifying the dynamic non-cooperative solution.

The total number of combinations is $Z = k^{2T}$, which is the same as for the dynamic cooperative solution.

Limitations of the Grid search method

At the start of this section, we claimed that the Grid search method will eventually find all solutions in the market model, if we have the minimum size of interval between decision alternatives set to 0.01. Since the number of combinations is finite, the task of finding optimal solutions could be seen as a matter of computing time. For the dynamic cooperative solution (DC) in six periods, $T = 6$, i.e. when $p^{MAX} = 300$, $p^{MIN} = 10$, $a^{MAX} = 425$ and $a^{MIN} = 0$ (as in Table 4.8) and when with the size of intervals between decision alternatives set to 0.01, i.e. $\Delta p = 0.01$ and $\Delta a = 0.01$, the total number of combinations, Z, is more than 10^{54}.

This number of combinations is so big that it will never be executed on any normal computer, since the computing time will exceed 10^{30} years (see Ståhl, 1972).

Thus the Grid search method is not useful for dynamic solutions of the market model in the business game. Even for five decision alternatives, $k = 5$, in six periods, $T = 6$, the number of combinations is $Z = k^{2T} = 5^{12} = 244{,}140{,}625$ for the dynamic cooperative solution (DC). For verifying non-cooperative dynamic solutions with best reply (DB), also for five decision alternatives in six periods, the number of combinations is the same. At the time of writing, it takes about 24 hours to search this number of combinations using an ordinary personal computer. An increase from five to six decision alternatives in the two examples above would need a computer with nine times the speed, as $6^{12}/5^{12} = 8.916$. With the same number of combinations as in the two examples above, 5^{12}, the dynamic non-cooperative solution (DN) could be computed for only three periods, $T = 3$, since we then have $Z = k^{4 \cdot T} = 5^{12}$.

For the myopic non-cooperative solution (MN) on the other hand, and with the same number of combinations, 5^{12}, for six periods, i.e. $T = 6$, we can use about 80 decision alternatives, i.e. $k = 80$, since the number of combinations, Tk^4, is $6 \cdot 80^4 \approx 5^{12}$.

As we use the Grid search method to find solutions, we can alter the specification of the Grid and use the Grid search method repeatedly. We can use a fixed number of decision alternatives, k, but we alter the values for the lowest decision alternatives and we also alter the size of the interval between decision alternatives, Δp and Δa.

We start with a wide search, with values for decisions close to the lower and upper bounds on price and on advertising (as in Table 4.8). We choose the values for the lowest decision alternatives, $p_t^{\min}$ and $a_t^{\min}$, close to the minimum values, p_t^{MIN} and a_t^{MIN}, and fairly large values for the size of the intervals between decision alternatives, Δp and Δa. After this first search has ended, we have obtained the values of the temporary best combinations, $\tilde{p}_t$ and $\tilde{a}_t$, which give the highest profit of the combinations thus specified in the grid.

Before we make the next search, we decrease the size of the intervals between decision alternatives, Δp and Δa. We also alter the lowest decisions, $p_t^{\min}$ and $a_t^{\min}$, so that the temporary best combinations, $\tilde{p}_t$ and $\tilde{a}_t$, are in the middle of the grid, e.g. $p_t^{\min} = \tilde{p}_t - ((k-1)/2) \cdot \Delta p$ and $a_t^{\min} = \tilde{a}_t - ((k-1)/2) \cdot \Delta a$, respectively. As we do these searches repeatedly, we successively narrow the specification of the grid to the optimal solutions.

5.4.4 Description of the use of the Solver program

As mentioned above, I have used the Solver program of Excel to find optimal solutions of the market model. The Solver program uses start values as it iteratively searches for a local maximum, by successively finding better decisions that give higher profit. Since there might exist more than one local maximum, each time I searched for a solution I used the Solver program with different randomly selected start values for price and advertising within the specified bounds.

The Solver program uses a number of numerical methods. For nonlinear problems it uses an algorithm called *the Generalized Reduced Gradient* (GRG2), which is a nonlinear optimization code. I refer to Smith & Lasdon (1992) and Fylstra *et al.* (1998) for descriptions of this algorithm and on how the optimization is done by the Solver program.

I implemented the market model, as it is described in Chapter 4 in Excel (see Appendix G). The model also consists of the parameters in Table 4.6 and the state variables at the start of the game in Table 4.7. Furthermore, the two functions, the demand function (7) and the interest function (Appendix B), were also implemented.

For the cooperative solutions, we assumed symmetry as described on page 85, and for the myopic and dynamic solutions we determined the optimal decisions for one of the firms according to (53) or (61), respectively. For the non-cooperative solutions, as in (56) and (64), and for verifying the non-cooperative solutions with best reply, as in (57), (64) and (65), the decisions of the other firms needed to be included in the model in Excel. Therefore, the decisions on price, advertising and marketing of the other firms are included just below the model shown in Appendix G. The decisions of the other firms are used in the demand function.

The Solver program has two so-called *dialog boxes* in Excel:

- Solver Parameters
- Solver Options

The first dialog box, *Solver Parameters*, was filled in as follows, when determining optimal solutions in the market model:

Solver Parameter	**Parameter chosen**
Set target Cell	Profit, Π_t or equity, $\ddot{E}_T$
Equal to	Maximum
By Changing Cells	Decisions on price, p, and advertising, a
Subjects to the Constraints	See Table 4.4 and Table 4.8.

Table 5.3. Solver parameters used when searching for solutions in the market model.

The explanation for the parameters chosen in the table above is as follows: The profit or the equity is maximized by adjusting prices and advertising, with bounds on investments and production (as in Table 4.4) and with bounds on prices, advertising, checking accounts and equities (as in Table 4.8).

In the second dialog box, *Solver options*, the values are filled in with defaults when the Solver program is opened. I have modified these default values to the values shown in the table below. It should be noted here that I have not made a complete test of the effects of different options. There are three groupings of Solver options in the dialog box. The values I used are given within brackets and the options I chose were specified below:

Solver Option and option chosen
Max time [100], Iterations [10000], Precision [0.0001], Tolerance [0.0001%] , Convergence [0.00001]
Assume Non-negative
Quadratic, Central, Newton

Table 5.4. Solver options used when searching for optimal solutions in the market model.

The Solver program uses the current values of the decisions to be adjusted as its start values, i.e. the values currently specified for price and for advertising in the spreadsheet.

When the Solver program is used with the same start values for decisions, the values of the optimal solution obtained are exactly the same each time. However, when the Solver program is used with different start values for decisions, the values of the optimal solution obtained can differ because:

- There are multiple optimal solutions, i.e. there is no unique optimal solution.
- There is a unique solution, but the precision of the Solver program is not sufficient.

As we search for optimal solutions, we will randomize the start values of decisions within the bounds of the decisions. We use a random generator, to generate random numbers uniformly[1]. The random numbers are so-called *pseudo-random numbers*, as they are generated by a specified algorithm. We use a seed to generate the first random number. The

[1] The random generator we use is the multiplicative congruential generator (MCG) introduced by Lehmer. A sequence of random numbers $U_1, U_2, \ldots$ is defined by the algorithm: $U_z = aU_{z-1} \bmod m$. The multiplier, *a*, is 16807 and *m* is $2^{31}-1$. The choice of multiplier was dictated by the 15-digit precision of Excel. The seeds, U_0, are the same as described in Law & Kelton (1991, p 450). The random numbers U_z generated are then divided by *m*, to give normalized random numbers in the interval $0 \le u_z \le 1$.

random number generated from this seed is in turn used to generate the next random number, and so on. Thus we generate a sequence of random numbers from one seed. The sequence of random numbers is used to calculate randomized start values for decisions.

We denote the randomized decisions (p^u, a^u) and we specify the generation of them as follows:

Specification of randomized decisions	**Notation**
Mid of interval of randomized decisions	p_0, a_0
Interval of randomized decisions	Δp_0, Δa_0
Uniformly distributed random numbers $(0 \leq u \leq 1)$	u
Randomized values of decisions	p^u, a^u

Table 5.5. Specification of randomized decisions.

We calculate the randomized decisions, p^u and a^u, as:

(67a) $\quad p^u = p_0 - \Delta p_0 / 2 + u \cdot \Delta p_0 \quad$ (67b) $\quad a^u = a_0 - \Delta a_0 / 2 + u \cdot \Delta a_0$

This is illustrated by the figure below.

Randomized price, p^u			Randomized advertising, a^u		
	$\longleftarrow - - - - - p^u - - - - - \longrightarrow$			$\longleftarrow - - - - - a^u - - - - - \longrightarrow$	
--|-------	-------|-------	|--	--|-------	-------|-------	|--
$p_0 - \Delta p_0 / 2$	p_0	$p_0 + \Delta p_0 / 2$	$a_0 - \Delta a_0 / 2$	a_0	$a_0 + \Delta a_0 / 2$
	$\longleftarrow \Delta p_0 \longrightarrow$			$\longleftarrow \Delta a_0 \longrightarrow$	

Figure 5.11. Randomized values for price and advertising.

More specifically, we denote the randomized myopic decisions p_t^u and a_t^u and the randomized dynamic decisions $\vec{p}^u$ and $\vec{a}^u$.

Below I will, just as for the Grid search method, describe three procedures for *myopic* solutions (MC, MD, MB) and three procedures for *dynamic* solutions (DC, DN and DB). For the procedures below we will use the start values for the state variable (66) K, C, M.

Myopic solutions with the MAXIMIZE function

The Solver program which is used for maximizing the myopic profits in the market model will be denoted by a function called *MAXIMIZE*. This function uses as input the profit, Π_t, and the start values for the Solver program on price and on advertising.

MAXIMIZE searches iteratively for a local maximum by adjusting price and advertising. When it finds one local maximum, the function gives two variables as output: the decisions on price and advertising that lead to a local maximum of profit.

MC Myopic cooperative solution

We use the function *MAXIMIZE* with randomized start values of decisions, (p_t^u, a_t^u), to find a myopic cooperative solution in each period t, for T periods according to (53), as Π_t depends only on (p_t, a_t). The decisions on price and on advertising, (p_t^{MC}, a_t^{MC}), give a maximum of the profit. When a solution is found, it is printed and the values of the state variables are calculated for the next period, $t+1$ according to (58a).

FOR t = 1 to T

$\quad (p_t^{MC}, a_t^{MC}) \leftarrow MAXIMIZE(\Pi_t(p_t, a_t), p_t^u, a_t^u)$

$\quad$ PRINT (p_t^{MC}, a_t^{MC})

$\quad (K_{t+1}, C_{t+1}, M_{t+1}) \leftarrow CALCULATE(K_t, C_t, M_t, p_t^{MC}, a_t^{MC})$

NEXT t

Figure 5.12. Procedure for the myopic cooperative solution.

MN Myopic non-cooperative solution

The procedure described below differs from the procedure for non-cooperative solutions used for the Grid search method. In the Grid search method, all combinations of decisions of the firm studied were compared with the same combinations of decisions of the other firms. Here instead we use a procedure which has some resemblance to the example on page 36, where each firm in its decision making can take into account the decision alternatives contemplated by the other firms, when in turn contemplating its decision.

The firm studied makes its temporary best decisions, $(\tilde{p}_t, \tilde{a}_t)$, against the decisions of the other firms, $(p_{\bar{n}}, a_{\bar{n}})$. The other firms then use these best decisions as their own decisions, i.e. $(\tilde{p}_t, \tilde{a}_t)$ are assigned to $(p_{\bar{n}}, a_{\bar{n}})$. The firm studied again makes its best decision to the other firms' decisions, which are its own earlier best decisions.

Each time the firm studied has found its best decisions, we compare them to the decisions of the other firms. We do this by calculating the sum of the absolute differences between the decisions of the firm studied and the decisions of the other firms. If the sum of the absolute differences is less than a specified value ε, the convergence criterion below is met and we stop the search considering that we have found a solution[2]:

(68) $$\left|\tilde{p}_t - p_{\bar{n}}\right| + \left|\tilde{a}_t - a_{\bar{n}}\right| < \varepsilon$$

The procedure in the figure below starts by assigning randomized values of decisions, (p_t^u, a_t^u), to the temporary best decisions of the firm studied $(\tilde{p}_t, \tilde{a}_t)$. The same values are also assigned to the decisions of the other firms, $(\tilde{p}_{\bar{n}}, \tilde{a}_{\bar{n}})$. We use the function *MAXIMIZE* and a DO....UNTIL loop to find the non-cooperative solution until the convergence criterion (68) is met. We have then obtained a symmetric non-cooperative solution according to (56) for period t. Before the decisions are searched in the next period, the state variables based on the

[2] ε can have different values in the different procedures in this section.

values of the solution are calculated and used at the start of the next period as in (58a), as Π_t depends on all four of $p_t, a_t, p_{\bar{n}}, a_{\bar{n}}$. Thus we obtain the combinations, $(\tilde{p}_t, \tilde{a}_t)$, for T periods which give the highest profit.

```
FOR t = 1 TO T
```

$$(\tilde{p}_t, \tilde{a}_t) \leftarrow (p_t^u, a_t^u)$$

DO

$$(\bar{p}_{\bar{n}}, \bar{a}_{\bar{n}}) \leftarrow (\tilde{p}_t, \tilde{a}_t)$$

$$(\tilde{p}_t, \tilde{a}_t) \leftarrow MAXIMIZE(\Pi_t(p_t, a_t, p_{\bar{n}}, a_{\bar{n}}), \tilde{p}_t, \tilde{a}_t)$$

UNTIL $|\tilde{p}_t - p_{\bar{n}}| + |\tilde{a}_t - a_{\bar{n}}| < \varepsilon$

$(p_t^{MN}, a_t^{MN}) \leftarrow (\tilde{p}_t, \tilde{a}_t)$ AND PRINT (p_t^{MN}, a_t^{MN})

$(K_{t+1}, C_{t+1}, M_{t+1}) \leftarrow$ CALCULATE $(K_t, C_t, M_t, p_t^{MN}, a_t^{MN})$

NEXT t

Figure 5.13. Procedure for myopic non-cooperative solution.

MB Procedure for verifying the myopic non-cooperative solution

When verifying the tentative non-cooperative myopic solution (MN), we also use randomized values of the start decisions, (p_t^u, a_t^u). For each period t, we find the best reply, $(\tilde{p}_t, \tilde{a}_t)$, to the non-cooperative myopic decisions of the other firms, $(p_{\bar{n}}^{MN}, a_{\bar{n}}^{MN})$, according to (57a) and (57b). If the sum of the absolute difference between this best reply to the non-cooperative myopic decisions is larger than a small value of ε, the procedure is terminated. Otherwise, the state variables based on the values of the best decisions are calculated and used at the start of the next period as in (58a). If the best replies are found to be close to the non-cooperative solutions in all T periods, i.e. the sum of the two absolute differences is smaller than ε, a message is printed that the myopic non-cooperative solutions are verified.

FOR t = 1 TO T

$\left(\tilde{p}_t, \tilde{a}_t\right) \leftarrow MAXIMIZE\left(\Pi_t\left(p_t, a_t, p_{\bar{n}}^{MN}, a_{\bar{n}}^{MN}\right), p_t^u, a_t^u\right)$

IF $\left|\tilde{p}_t - p_{\bar{n}}^{MN}\right| + \left|\tilde{a}_t - a_{\bar{n}}^{MN}\right| > \varepsilon$

THEN TERMINATE

$\left(K_{t+1}, C_{t+1}, M_{t+1}\right) \leftarrow$ CALCULATE $\left(K_t, C_t, M_t, p_t^{MN}, a_t^{MN}\right)$

NEXT t

PRINT ("MYOPIC NON-COOPERATIVE SOLUTION VERIFIED")

Figure 5.14. Procedure for the verifying the myopic non-cooperative solution.

Dynamic solutions with the MAXIMIZE function

We also use the Solver program, *MAXIMIZE*, as a function to maximize equity, $\ddot{E}_T$, for T periods, where $T>1$. The difference between this function and the myopic function is that this function now maximizes the equity with decisions for a number of periods.

The function uses as input the equity and the start values for prices and advertising. The start values for prices and advertising, $\left(\vec{p}^u, \vec{a}^u\right)$, are selected at random. MAXIMIZE searches iteratively for a local maximum by adjusting prices and advertising, $\left(\vec{p}, \vec{a}\right)$. When it finds one local maximum, the function gives two variables as output: the decisions on prices and on advertising in the T periods, $\left(\vec{p}, \vec{a}\right)$, that give a local maximum of equity.

DC Dynamic cooperative solution

This procedure finds, according to (61), the dynamic cooperative solutions, $\left(\vec{p}^{DC}, \vec{a}^{DC}\right)$, as we maximize the equity, $\ddot{E}_T$, for the firm studied in T periods. It uses randomized start values for price and advertising, $\left(\vec{p}^u, \vec{a}^u\right)$.

$$\left(\vec{p}^{\,DC}, \vec{a}^{\,DC}\right) \leftarrow MAXIMIZE\left(\ddot{E}_T(\vec{p}, \vec{a}), \vec{p}^{\,u}, \vec{a}^{\,u}\right)$$

Figure 5.15. Procedure for the dynamic cooperative solution.

DN Dynamic non-cooperative solution

This procedure finds, according to (64), the dynamic non-cooperative solution, $\left(\vec{p}^{\,DN}, \vec{a}^{\,DN}\right)$, in T periods. That is, the firm studied makes its best decision to the decisions of the other firms repeatedly, and the other firms adjust their decisions to the best decision of the firm studied in a manner similar to that described for the myopic non-cooperative solution on page 106.

The convergence criterion consists of the sum of the absolute differences for price and for advertising in T periods, between the temporary best decisions, $\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right)$ and the decisions of the other firms, $\left(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}}\right)$. Since $\vec{p} = p_1, p_2 \ldots p_T$ and $\vec{a} = a_1, a_2 \ldots a_T$, the convergence criterion for dynamic non-cooperative solution is:

(69) $$\sum_{t=1}^{T}\left(\left|\tilde{p}_t - p_{\bar{n}}\right| + \left|\tilde{a}_t - a_{\bar{n}}\right|\right) < \varepsilon$$

$$\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right) \leftarrow \left(\vec{p}^{\,u}, \vec{a}^{\,u}\right)$$

DO

$$\left(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}}\right) \leftarrow \left(\tilde{\vec{p}}, \tilde{\vec{a}}\right)$$

$$\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right) \leftarrow MAXIMIZE\left(\ddot{E}_T\left(\vec{p}, \vec{a}, \vec{p}_{\bar{n}}, \vec{a}_{\bar{n}}\right), \tilde{\vec{p}}, \tilde{\vec{a}}\right)$$

UNTIL $\sum_{t=1}^{T}\left(\left|\tilde{p}_t - p_{\bar{n}}\right| + \left|\tilde{a}_t - a_{\bar{n}}\right|\right) < \varepsilon$

$\left(\vec{p}^{\,DN}, \vec{a}^{\,DN}\right) \leftarrow \left(\tilde{\vec{p}}, \tilde{\vec{a}}\right)$ AND PRINT $\left(\vec{p}^{\,DN}, \vec{a}^{\,DN}\right)$

Figure 5.16. Procedure for the dynamic non-cooperative solution.

DB Procedure for verifying the dynamic non-cooperative solution

When we have calculated optimal decisions of the dynamic non-cooperative solution, $\left(\vec{p}^{DN}, \vec{a}^{DN}\right)$, we try to find the best replies, $\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right)$, to verify this tentative non-cooperative solution. If the sum of the absolute differences for price and for advertising in T periods between the best replies and the non-cooperative decisions is less than the value ε, a message is printed that the non-cooperative solution is verified.

$\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right) \leftarrow MAXIMIZE\left(\ddot{E}_T\left(\vec{p}, \vec{a}, \vec{p}^{DN}, \vec{a}^{DN}\right), \vec{p}^{u}, \vec{a}^{u}\right)$
IF $\sum_{t=1}^{T}\left(\lvert\tilde{p}_t - p_{\bar{n}}\rvert + \lvert\tilde{a}_t - a_{\bar{n}}\rvert\right) < \varepsilon$
THEN PRINT "DYNAMIC NON-COOPERATIVE SOLUTION VERIFIED "

Figure 5.17. Procedure for verifying the dynamic non-cooperative solution.

Finding optimal solutions with the Solver program

In order to determine optimal solutions we have repeated each of the six procedures described above 100 times with the 100 seeds given in the textbook Law & Kelton (1991, p. 450). We used the same specified values of mid of the interval of randomized decisions, p_0 and a_0, and also the same interval of randomized decisions, Δp_0 and Δa_0, for 100 repetitions (see page 123). We can choose to have big intervals of the randomized decisions, so that local solutions are searched within most of the bounds on price and on advertising. By doing this we make it probable that we will find the optimal local solutions that exist in the market model.

As the 100 optimal solutions with the same procedure were obtained, we made comparisons between these values of the optimal solutions. The values found for the optimal solutions were similar for each of the six procedures, but they are not exactly the same. However, the precision of the Solver program could be improved. There is a possibility to adjust the options in the second dialog box, *Solver Options*, to improve the precision of the optimal solutions. For the non-cooperative solutions we can also adjust the value of the convergence criterion, ε.

It was not possible to obtain sufficient precision for the values of the optimal solution in the Solver program to rule out the possibility that there can exist many solutions within a small range of values for prices and advertising in each period. However, for our purpose, to make comparisons between optimal solutions and the decisions made when playing the game, the precision of the solutions is sufficient.

5.4.5 Description of the use of the Random search method

When I started to use the Random search method to find dynamic cooperative solutions in the market model, I obtained similar values for the optimal decisions with different start values. The Random search method therefore seemed useful for finding optimal solutions of the market model. Here we will mainly use the method to obtain solutions for use in comparisons with the dynamic solutions obtained by the Solver program. Therefore, the description of the Random search method below is restricted to the procedures for finding and verifying *dynamic solutions* (DC, DN and DB). Compared to the Solver program, the Random search method:

- Uses a different method to find the local maximum, as will shown below.
- Is "open" in the sense that there exist simple and complete descriptions of the method.

It should be mentioned that there are a number of ways to do random search described in the literature. For refinements of the Random search method, see for example Zhigljavskij (1991) and Andradóttir (1998). I have briefly tried some refinements of the method. However, since my interest has primarily been directed towards finding values of the optimal solutions to be used for comparisons with the decisions in the game sessions, I will here limit the description of the Random search method to a so-called *crude version* of the method.

When we use the method, we rely on an assumption that it is possible to find a local maximum of the market model by iteratively making randomized adjustments of some arbitrary start decisions. First, as with the Solver program, we will start the search with randomized values of the start decisions, $(\vec{p}^{\,u}, \vec{a}^{\,u})$. We then assign these randomized decisions to the temporary best decisions, $(\tilde{\vec{p}}, \tilde{\vec{a}})$. Thereafter we randomize the adjustments to these temporary best decisions.

If some randomized adjustments of the temporary best decisions lead to a higher equity than the temporary best decisions do, the randomized adjustments of the temporary best decisions are made to the temporary best decisions. Hence we iteratively make randomized adjustments of temporary best decisions in order to find a local maximum.

The Random search method used here starts its search from randomized start values, and it searches along a path of randomized adjustments in a specified region around the temporary best decisions. The method differs from the Solver program, which searches along a path of gradient values. The Solver program may skip over a discontinuity or it may never encounter a region where discontinues occur. As described on page 60, the interest function in the market model is not continuous and not differentiable. Thus the Solver program may not find all local maximums in the market model. Therefore, the Random search method is used as a complement in order to make it more probable that all local maximums are found.

We denote the randomized adjustments on price and on advertising as Δp^u and Δa^u, respectively. The generation of randomized adjustments is made with random numbers (as described on page 103). A sequence of random numbers is generated from one seed. This sequence of random numbers is used both to calculate randomized start values of decisions, as for the Solver program, and for the randomized adjustments which are obtained within specified intervals denoted Δp and Δa.

We randomize adjustments, Δp_t^u and Δa_t^u, for each period as:

(70a) $\Delta p_t^u = -\Delta p/2 + u \cdot \Delta p$ (70b) $\Delta a_t^u = -\Delta a/2 + u \cdot \Delta a$

We denote the ordered set (for t=1..T) of dynamic randomized adjustments as $\Delta\vec{p}^u$ and $\Delta\vec{a}^u$. The randomized adjustments of decisions of temporary best decisions are $\tilde{\vec{p}} + \Delta\vec{p}^u$ and $\tilde{\vec{a}} + \Delta\vec{a}^u$. We use the bounds on decisions as defined in Table 4.8. If any of the randomized adjustments of decisions are outside these bounds, we set them to the border of the bound of the decisions. For example, if a randomized adjustment of price in a period is out of bound, i.e. $\tilde{p}_t + \Delta p_t^u < p^{MIN}$, we set the adjusted price in this period to p^{MIN}.

We use a counter, w, for the number of iterations. This counter w is set to zero at the start of the procedure and it is also set to zero when we have randomized adjustments of decisions that lead to higher equity than the temporary best decisions. However, if the randomized adjustments of decisions do not give higher equity than the temporary best decisions, the counter is increased by one. At the start we specify the maximum number of iterations allowed without finding any randomized adjustments of decisions. We will denote this as W.

Dynamic solutions with Random search method

DC Dynamic cooperative solution

The procedure starts by assigning the randomized values $(\vec{p}^u, \vec{a}^u)$ to the temporary best decisions, $(\tilde{\vec{p}}, \tilde{\vec{a}})$. The procedure then searches for randomized adjustments of temporary best decisions that give higher equity, $\ddot{E}_T$, than these temporary best decisions. When such decisions are found, the randomized adjustments of decisions are assigned to the temporary best decisions.

The procedure consists of one DO...UNTIL loop. When the specified number of iterations, W, of randomized adjustments of decisions has been searched and no randomized adjustments of decisions have been found to give higher equity than the temporary best decisions, the temporary best decisions, $(\tilde{\vec{p}},\tilde{\vec{a}})$, are assigned to the dynamic cooperative decisions $(\vec{p}^{DC},\vec{a}^{DC})$ as shown in (61).

$$
\begin{array}{l}
(\tilde{\vec{p}},\tilde{\vec{a}}) \leftarrow (\vec{p}^{u},\vec{a}^{u}) \ AND \ w=0 \\
\text{DO} \\
\quad \text{IF } \ddot{E}_T(\tilde{\vec{p}}+\Delta\vec{p}^{u},\tilde{\vec{a}}+\Delta\vec{a}^{u}) \geq \ddot{E}_T(\tilde{\vec{p}},\tilde{\vec{a}}) \\
\qquad \text{THEN } (\tilde{\vec{p}},\tilde{\vec{a}}) \leftarrow (\tilde{\vec{p}}+\Delta\vec{p}^{u},\tilde{\vec{a}}+\Delta\vec{a}^{u}) \text{ AND } \mathrm{w}=0 \\
\qquad \text{ELSE } w \leftarrow w+1 \\
\text{UNTIL } w=W \\
(\vec{p}^{DC},\vec{a}^{DC}) \leftarrow (\tilde{\vec{p}},\tilde{\vec{a}})
\end{array}
$$

Figure 5.18. Dynamic cooperative solution.

DN Dynamic non-cooperative solution

The procedure starts by assigning the randomized values of decisions, $(\vec{p}^{u},\vec{a}^{u})$, to the temporary best decisions, $(\tilde{\vec{p}},\tilde{\vec{a}})$. These randomized decisions are also assigned to the other firms, $(\vec{p}_{\bar{n}},\vec{a}_{\bar{n}})$. The procedure then searches for randomized adjustments of decisions that give the highest equity, $\ddot{E}_T$. The procedure consists of two DO...UNTIL loops. The first loop continues until the convergence criterion (69) is met. The second loop continues until W randomized adjustments have been made without finding a higher equity than the one that temporary best decisions lead to.

The temporary best decisions found are then compared to the decisions of the other firms as in (64). We specify a value of the convergence parameter ε. If the convergence criterion (69) is met, the temporary best decisions, $(\tilde{\vec{p}},\tilde{\vec{a}})$, are assigned to the dynamic non-cooperative solution $(\vec{p}^{DN},\vec{a}^{DN})$. The solution is then printed and the procedure is stopped. If the convergence criterion is not met, the temporary best decisions, $(\tilde{\vec{p}},\tilde{\vec{a}})$, are assigned to the decisions of the other firms, $(\vec{p}_{\bar{n}},\vec{a}_{\bar{n}})$. The search continues until the convergence criterion is met[3].

$(\tilde{\vec{p}},\tilde{\vec{a}}) \leftarrow (\vec{p}^{u},\vec{a}^{u})$ AND $w=0$
DO
    $(\vec{p}_{\bar{n}},\vec{a}_{\bar{n}}) \leftarrow (\tilde{\vec{p}},\tilde{\vec{a}})$
    DO
        IF $\ddot{E}_T(\tilde{\vec{p}}+\Delta\vec{p}^{u},\tilde{\vec{a}}+\Delta\vec{a}^{u},\vec{p}_{\bar{n}},\vec{a}_{\bar{n}}) \geq \ddot{E}_T(\tilde{\vec{p}},\tilde{\vec{a}},\vec{p}_{\bar{n}},\vec{a}_{\bar{n}})$
            THEN $(\tilde{\vec{p}},\tilde{\vec{a}}) \leftarrow (\tilde{\vec{p}}+\Delta\vec{p}^{u},\tilde{\vec{a}}+\Delta\vec{a}^{u})$ AND $w=0$
            ELSE $w \leftarrow w+1$
    UNTIL $w=W$
UNTIL $\sum_{t=1}^{T}\left(|\tilde{p}_t - p_{\bar{n}}| + |\tilde{a}_t - a_{\bar{n}}|\right) < \varepsilon$
$(\vec{p}^{DN},\vec{a}^{DN}) \leftarrow (\tilde{\vec{p}},\tilde{\vec{a}})$ AND PRINT $(\vec{p}^{DN},\vec{a}^{DN})$

Figure 5.19. Procedure for the dynamic non-cooperative solution.

DB Procedure for verifying the dynamic non-cooperative solution

When we have found values of the optimal decisions of the dynamic non-cooperative solution, $(\vec{p}^{DN},\vec{a}^{DN})$, we will try to find the dynamic best replies, $(\vec{p}^{DB},\vec{a}^{DB})$, to these decisions to verify this tentative non-cooperative solution as in (65a) and (65b). The decisions of the other

[3] If the convergence criterion is not met within a reasonable time limit, the search is manually terminated.

firm, $(\vec{p}_{\bar{n}}, \vec{a}_{\bar{n}})$, are assigned to the non-cooperative solution, i.e. $\left(\vec{p}_{\bar{n}}^{\,DN}, \vec{a}_{\bar{n}}^{\,DN}\right)$.

As in the procedure for the dynamic non-cooperative solution above, we specify the maximum number of iterations W to be made without finding any better reply and we also specify the convergence parameter ε. If the sum of the absolute differences for price and for advertising in T periods between the best reply and the non-cooperative decisions is less than this value ε, then a message is printed that the non-cooperative solution is verified.

$\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right) \leftarrow \left(\vec{p}^{\,u}, \vec{a}^{\,u}\right)$
DO
  IF $\ddot{E}_T\left(\tilde{\vec{p}} + \Delta\vec{p}^{\,u}, \tilde{\vec{a}} + \Delta\vec{a}^{\,u}, \vec{p}_{\bar{n}}^{\,DN}, \vec{a}_{\bar{n}}^{\,DN}\right) \geq \ddot{E}_T\left(\tilde{\vec{p}}, \tilde{\vec{a}}, \vec{p}_{\bar{n}}^{\,DN}, \vec{a}_{\bar{n}}^{\,DN}\right)$
    THEN $\left(\tilde{\vec{p}}, \tilde{\vec{a}}\right) \leftarrow \left(\tilde{\vec{p}} + \Delta\vec{p}^{\,u}, \tilde{\vec{a}} + \Delta\vec{a}^{\,u}\right)$ AND $\mathrm{w} = 0$
    ELSE $w \leftarrow w + 1$
UNTIL $w = W$
IF $\sum_{t=1}^{T} \left(\left|\tilde{p}_t - p_{\bar{n}}\right| + \left|\tilde{a}_t - a_{\bar{n}}\right|\right) < \varepsilon$
    THEN PRINT "DYNAMIC NON-COOPERATIVE SOLUTION VERIFIED"

Figure 5.20. Procedure for verifying the dynamic non-cooperative solution.

Finding dynamic solutions with the Random search method

We use the same market model in Excel as was used for the Solver program (Appendix G). Also as for the Solver program, we use the procedures described above repeatedly with Random search, e.g. 100 times, with 100 different seeds.

When searching for optimal solutions with the Random search method we need to specify the following:

Specification to use the Random search method	**Notation**
Mid of interval of randomized decisions	p_0 , a_0
Interval of randomized decisions	Δp_0 , Δa_0
Interval of randomized adjustments	Δp , Δa
Specified number of iterations	W
Value of convergence criterion	ε

Table 5.6. Specification of randomized decisions.

The Random search method is sensitive to the interval of randomized adjustments of decisions, Δp and Δa, and also to the specified number of iterations, W.

In section 5.5, the values of the optimal solutions are presented. As we shall then see, the dispersion is bigger for the values obtained by the Random search method than for the values obtained with the Solver program. However, the difference between the mean of the values of the solutions obtained by 100 runs of the Random search method and 100 runs of the Solver program is as we shall see in section 5.5.2 small.

5.5 Presentation of optimal solutions

In this section, the values of the optimal solutions of the market model are presented. Since most, but not all, game sessions were played for six periods, we shall limit the optimal solutions to six periods, both for the myopic and for the dynamic solutions. Below we shall make comparisons between the values obtained by the different methods described in sections 5.3 and 5.4.

In section 5.5.1, we shall make comparisons for myopic solutions between the values obtained analytically and the values obtained with the Grid search method. In section 5.5.2, we shall make comparisons for dynamic solutions between the values of obtained with the Solver program and the values obtained with the Random search method. In section 5.5.3, we shall briefly discuss some similarities and differences between the optimal solutions.

5.5.1 Comparison between myopic solutions determined analytically and with the Grid search method

In section 5.3, we determined the myopic solutions analytically, with a fixed interest rate, $\hat{r}$. We obtained expressions for optimal prices and advertising, (43b), (44b), (49), (52c). We will now use these expressions to calculate the values of the myopic decisions in six periods. The optimal price depended on the parameters β and X. The optimal advertising depended on $\alpha, \varsigma, M, \hat{r}$ and on the optimal price (i.e. in turn on β and X). The unit cost, X, was defined by (30c) $\mathrm{X} = \chi + \sigma\delta + r\chi + r\sigma$. Furthermore, the non-cooperative solution depends on the number of firms N.

We use the values of these parameters in Table 4.6 and the values of the state variables in Table 4.7 to calculate the values of the myopic solutions. However, we also need a value for the interest rate. The problem is that the interest rate in the market model is determined by the decisions on price and on advertising, but that in the analytical method these decisions are determined by the interest rate. Hence, we must use an iterative approach as follows:

Step 0 Start with a fixed interest rate, $\hat{r}$, chosen at random in the interval between 10% and up to 28%.

Step 1 Calculate value for prices with (43b) p^{MC} and (49) p^{MN}.

Step 2 Calculate values for advertising with (44b) a^{MC} and (52c) a^{MN}.

Step 3 Calculate the intermediate checking accounts $\dot{C} = C - P$ (where $P = \sigma i + \chi o + a$), resulting from prices and advertising.

Step 4 Calculate resulting variable interest rates, (23) $r(\dot{C}, E)$.

Step 5 If the fixed interest rate, $\hat{r}$, differs from the variable interest rate, r, i.e. $\hat{r} \neq r$, the variable interest rate is assigned to the fixed interest rate, i.e. $\hat{r} \leftarrow r$ and go back to Step 1. Otherwise the solutions are the values calculated in Steps 2–3.

We will call the values of the decisions obtained with the procedure above, $\hat{r}$-*solutions*.

Values of the myopic cooperative solution

As it turns out, the procedure above cannot help us to obtain $\hat{r}$-solutions for the myopic cooperative solution in period 1, since we, as shown below in (71), get 13% as resulting interest rate if we start with 10% and, as shown in (72), get 10% as resulting interest rate if we start with 13%.

(71) $\hat{r} = 10.00\% \rightarrow p^{MC} = 51.00, a^{MC} = 36.76 \rightarrow r = 13.00\%$

(72) $\hat{r} = 13.00\% \rightarrow p^{MC} = 54.60, a^{MC} = 34.58 \rightarrow r = 10.00\%$

In Table 5.7 below, we mark the values in period 1 with the symbol ‡, to denote that these values on the analytical myopic cooperative solution (MC) are not unique. Here we have chosen to use the fixed interest rate, $\hat{r} = 10\%$, to calculate the values of the myopic cooperative solution in period 1. The reason is that the solution obtained with the Grid search method gives an intermediate checking account, $\dot{C}$, of zero, i.e. no loan is taken, and the interest rate is then 10%. Both the fixed interest rate and the variable interest rate are 10% from period 2 to period 6, since the intermediate checking account is positive.

The values for the myopic solution are found with the Grid search method using a variable interest rate, r. The method is used repeatedly with alteration of the specification of the grid as described in section 5.4.2. The final size of the intervals between decision alternatives was set to $\Delta p = 0.01$ and $\Delta a = 0.01$, respectively. There was only one local solution found in each period. Hence the values for these solutions are unique and globally optimal. The following table shows the values for the two methods for a myopic cooperative solution:

Myopic cooperative solution (MC)	Period					
	1	2	3	4	5	6
Price – Analytical	51.00‡	51.00	51.00	51.00	51.00	51.00
Price – Grid search	52.40	51.00	51.00	51.00	51.00	51.00
Price – Abs. difference	*1.40*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*
Adv. – Analytical	36.76‡	10.00	11.00	10.40	10.40	10.40
Adv. – Grid search	35.88	11.13	10.40	10.40	10.40	10.40
Adv. – Abs. difference	*0.88*	*1.13*	*0.60*	*0.00*	*0.00*	*0.00*

Table 5.7. Values of the myopic cooperative solution (MC) obtained analytically and with the Grid search method, and differences between the values from the two methods. The symbol ‡ shows that the solution is not unique.

In the table above, we find only four differences between the solution methods. The biggest difference is in period 1 (1.40 for price). When we determined the analytical solutions in section 5.3, we did not rule of the possibility that the myopic solutions could be a minimum or an inflection point. We see in the table above, that the Grid search method which searches for the maximum value of the profit in one period have found values on decisions that are close to the values of decisions on obtained with the analytical myopic solution. This rules of the possibility that the optimal analytical decisions give the minimum profit, since we know that the profit can be negative. Furthermore, the similarities in values of the two solutions make it likely that the analytical solutions are maxima and not inflection points.

Values of the myopic non-cooperative solution

Next we study the values of the myopic non-cooperative solution (MN) obtained with the two methods. For the analytical myopic non-cooperative solutions (MN), we obtain unique values in periods 1–3[1].

[1] In period 1, for the analytical solution, the interest rate $\hat{r}$ determined by the methods described above is 18.27%. This gives a value of $X = \chi + \delta\sigma + r(\chi + \sigma) = 13 + 40r = 13 + 7.31 \approx 20.31$. Since with $N = 5$, (49) $p = X + X/(\beta - 1/5)$ and with $\beta = 1.5$, $p \approx 1.769X \approx 35.93$.

In period 4 we experience the same problem as for the myopic dynamic cooperative solution (MC) in period 1, i.e. that the interest rate is either 10% or 13%. We again choose to use the values obtained with a fixed interest rate of 10 %, as the values of the solution with the Grid search are 10%. We also have different values in period 5[2]. In period 6 we obtain the same unique values.

It should be mentioned that, for the Grid search method more than one solution was found in some of the periods. The values of decisions in these solutions differed by at most 0.01. Below we have chosen the values that give the highest profit for the period.

Myopic non-cooperative solution (MN)	Period					
	1	2	3	4	5	6
Price – Analytical	35.93	34.37	32.57	30.08‡	30.08‡	30.08
Price – Grid search	38.60	35.74	33.01	31.08	30.08	30.08
Price – Abs. difference	*2.67*	*1.36*	*0.44*	*1.00*	*0.00*	*0.00*
Adv. – Analytical	130.41	48.88	52.46	59.31‡	49.90‡	49.90
Adv. – Grid search	120.33	50.31	54.51	56.02	53.50	49.90
Adv – Abs. difference	*10.09*	*1.46*	*2.04*	*3.29*	*3.60*	*0.00*

Table 5.8. Values of the myopic non-cooperative solution (MN) obtained analytically and with the Grid search method, and differences between the values between the two methods. The symbol ‡ shows that the solution is not unique. For some of the values, the second decimals are approximates.

In the table above, we notice substantial differences between the two methods. The biggest differences are in period 1, where the difference in price is 2.67 and the difference in advertising is 10.09. The differences are smaller in periods 2–5. In period 6 the values of the solutions are the same. Hence the analytical method is not suitable for estimating the myopic non-cooperative solution.

[2] There is also a solution in period 4 for price = 32.20 and advertising = 48.24, which in turn gives a solution in period 5 for price = 30.08 and advertising = 58.01.

5.5.2 Comparison between values of dynamic solutions obtained with the Solver program and those obtained with the Random search method

For both the Solver program and the Random search method, we want to specify the randomization of start decisions so that the whole bound of the decisions on price and on advertising is searched for local solutions (see Table 4.8). However, the equities in all periods need to be non-negative, $\ddot{E}_t \geq 0$. Low start values for prices and high start values for advertising can give negative equities. We therefore specify smaller intervals for the randomization of the start decisions than for the bounds on the decisions. For the randomization of the start values of decisions, we use the mid values of the intervals as $p_0 = 40$ and $a_0 = 100$, and the values of the intervals as $p_0 = 40$ and $a_0 = 200$. For the Solver program, we use the Solver parameters and options as specified in section 5.4.4.

Values for the dynamic cooperative solution

For the Random search method, we set the interval of the randomized adjustments for the dynamic cooperative solution to $\Delta p = 0.05$ and $\Delta a = 0.10$, and the specified number of iterations was $W = 1000$.

Both the Solver program and the Random search method give slightly different values for the optimal decisions depending on the start values of decisions. The values for the dynamic cooperative decisions (DC) in the following table are the mean values of decisions, price and advertising, out of 100 calculations with randomized start values for decisions, that give the highest equity at the end of period 6.

Dynamic cooperative solution (DC)	Period					
	1	2	3	4	5	6
Price – Solver	53.43	48.00	48.00	48.00	49.52	49.81
Price – Random	53.45	48.02	48.01	48.40	49.48	49.77
Price – Abs. difference	*0.02*	*0.02*	*0.01*	*0.40*	*0.04*	*0.04*
Adv. – Solver	72.56	35.17	27.59	27.70	11.40	0.00
Adv. – Random	72.60	34.96	28.26	27.11	11.36	0.00
Adv – Abs. difference	*0.04*	*0.21*	*0.67*	*0.59*	*0.04*	*0.00*

Table 5.9. Mean values of dynamic cooperative solution (DC) obtained with the Solver program and with the Random search method, and differences between the values between the two methods.

In the table above, there are only small differences between the mean values from the two methods. The biggest difference between the two methods is 0.67 for advertising in period 3. It should also be noted that the difference in the resulting equity at the end of period 6 between the two methods is only 0.002. Finally, it should be noted for both methods that the dispersion, measured in standard deviations, among the 100 values of optimal decisions in each period is less than 1.

Values for the dynamic non-cooperative solution

For the Random search method, I have tested a number of different intervals of the randomized adjustments to price and advertising, Δp and Δa, and a also number of iterations W. For very small values of the randomized adjustments and for a large number of iterations, the procedure does not find any optimal values[3]. After testing with different values, I eventually used the randomized adjustments $\Delta p = 0.5$ and $\Delta a = 16$, and the maximum number of iterations $W = 100$, which give the values presented in the table below. I set the value of the convergence parameter as $\varepsilon = 0.1$.

[3] As mentioned on page 115, if the convergence criterion is not met within a reasonable time limit, the search is manually terminated.

Again, the Solver program and the Random search method give different values of the optimal decisions depending on the start values of decisions. The values of the dynamic non-cooperative decisions (DN) in the following table are the mean values of decisions, on price and on advertising, for 100 calculations with randomized start values of decisions.

Dynamic non-cooperative solution (DN)	Period					
	1	2	3	4	5	6
Price – Solver	38.85	39.40	36.89	34.81	32.79	30.17
Price – Random	38.87	39.26	36.84	34.74	32.78	30.41
Price – Abs. difference	*0.02*	*0.14*	*0.05*	*0.07*	*0.01*	*0.24*
Adv. – Solver	232.14	87.66	116.80	128.04	95.67	0.00
Adv. – Random	232.09	86.51	116.58	127.37	93.72	0.01
Adv – Abs. difference	*0.05*	*1.15*	*0.22*	*0.67*	*1.95*	*0.01*

Table 5.10. Mean values of dynamic cooperative non-solution (DN) obtained with the Solver program and with the Random search method, and differences from the values between the two methods.

For the dynamic non-cooperative solution (DN), we see in the table above only small differences between the mean values of decisions on prices using the two methods. For advertising the difference is somewhat larger, since the biggest difference between the two methods is 1.95, in period 5.

The largest dispersion among the values of optimal decisions for the Solver Program, measured in standard deviations, among the 100 values of the optimal decisions in each period is 0.05 for price and 0.90 for advertising. The largest dispersion for the Random search, among the 100 values of the optimal decisions, is 0.40 for price and 8.70 for advertising.

5.5.3 Tables and figures of optimal decisions

We shall next summarize in tables and figures the optimal solutions to be used for comparisons with the decisions in the game sessions. In order not to give the impression of a higher precision of the values of the optimal solutions than really was possible with the methods, I present below a table with these values as integers.

For the myopic solutions we use the values (rounded off to integers) obtained with the Grid search method shown in Table 5.7 and 5.8. As regards the dynamic solutions there are limitations in the precision of the values obtained by both the Solver program and the Random search method, and we also saw above that the mean values of the solutions of the two methods differed. We use the following principle when obtaining the integer values of the dynamic solutions: We choose the integer that minimizes the greatest distance between this integer and the values of the two solution methods[4]. The (absolute) difference between these integer values and the values in Table 5.9 and 5.10 is at most 1.28.

Values of optimal decisions	*Period*					
	1	*2*	*3*	*4*	*5*	*6*
Price, p^{MC}	52	51	51	51	51	51
Advertising, a^{MC}	36	11	10	10	10	10
Price, p^{MN}	39	36	33	31	30	30
Advertising, a^{MN}	120	50	55	56	54	50
Price, p^{DC}	53	48	48	48	49	50
Advertising, a^{DC}	73	35	28	28	11	0
Price, p^{DN}	39	39	37	35	33	30
Advertising, a^{DN}	232	87	116	128	95	0

Table 5.11. Values of the optimal solutions in the market model[5].

[4] For the cooperative price in period 5, both 49 and 50 can be chosen, but we choose the higher integer, 50, since it gives higher equity at the end of period 6.

[5] On page 128, I will verify the original decimal values of the dynamic non-cooperative solution with the Solver program and the Random search method, but it shall here be mentioned that I have attempted to verify the values above using the Grid method. Because of the limitations mentioned on pages 99-100, only five values on each decision variable were used in each period to obtain best replies. I then found that the values of the non-cooperative prices were the same as the values of the best reply, while the values of the non-cooperative advertising and the best reply differed at the most 6 (in period 3), which is about 5 percent.

The values to be used for comparisons are also presented in the following two figures.

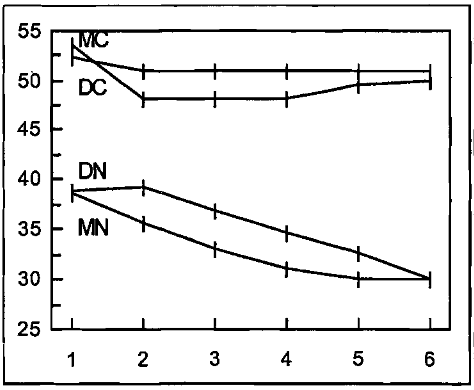

Figure 5.21. Prices in the optimal solutions.

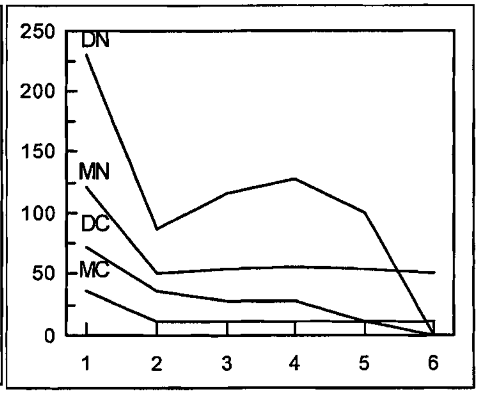

Figure 5.22. Advertising in the optimal solutions.

As we see in the figures above, optimal prices in the two cooperative solutions (MC and DC) are quite similar, as they are both in the region of 50 for all six periods. The non-cooperative prices (MN and DN) are lower than the prices for the cooperative solutions. They also quite similar, decreasing from about 39 to 30 from period 1 to period 6.

The four solutions have their highest values for advertising in period 1. There is a big difference between the values in period 1, as the lowest advertising is 36 and the highest is 232. From period 2 to period 5, the advertising in the dynamic non-cooperative solution differs the most from the other three solutions. In period 6, the advertising is zero for both dynamic solutions (DC) and (DN).

We end the presentation of the values of the optimal solutions with a table showing the equity at the end of period 6, based on the values in Table 5.11.

Equity at the end of period 6	*MC*	*DC*	*MN*	*DN*
Equity, $\ddot{E}_6$	1345	1414	1135	787

Table 5.12. Equity at the end of period 6 (including the 200 in cash at the start of the game), based on the values obtained for the optimal decisions.

The equity at the end of period 6 is, as expected, highest for the dynamic cooperative solution (DC). The second highest equity is for the myopic cooperative solution (DN). Perhaps not expected, however, is the relatively high equity for the myopic non-cooperative solution (MN). The lowest equity is in the dynamic non-cooperative solution (MN), which is about half of the equity of the cooperative solutions. Still, it is considerably higher than the equity of 200 (in cash) at the start of the game.

5.5.4 Verifying the values of the optimal solutions

In order to estimate the errors of the values for the optimal solutions presented in the tables above, we use the procedures denoted *DB* in sections 5.4.3 – 5.4.5.

Verifying the myopic non-cooperative solution

We use the Grid search method with the incremental steps $\Delta p = 0.01$ and $\Delta a = 0.01$, the number of decisions $k = 300$ and the lowest decisions, $p_t^{\min}$ and $a_t^{\min}$, which are specified to be 1.50 below, to verify the values for the Grid search method in Table 5.8.

Verifying the myopic non-cooperative solution (MB)	Period					
	1	2	3	4	5	6
Price – optimal	38.60	35.74	33.01	31.08	30.08	30.08
Price, p^{MB} – Grid	38.60	35.74	33.01	31.04	30.08	30.08
Price – Abs. Difference	0.00	0.00	0.00	0.04	0.00	0.00
Adv. – optimal	120.33	50.31	54.51	56.02	53.50	49.90
Adv., a^{MB} – Grid	120.33	50.32	54.52	55.67	53.76	49.93
Adv. – Abs. difference	0.00	0.01	0.01	0.35	0.26	0.03

Table 5.13. Verifying values of the myopic cooperative non-solution (MN) obtained with the Grid search method with best replies.

We notice that the biggest difference is 0.35 for advertising in period 4.

Verifying the dynamic non-cooperative solution

When we verify the values of the dynamic non-cooperative solutions we use the same randomization of start values as before, i.e. the mid values of the intervals as $p_0 = 40$ and $a_0 = 100$, and the values of the intervals as $p_0 = 40$ and $a_0 = 200$. For randomized adjustments in the Random search method we use $\Delta p = 0.5$ and $\Delta a = 16$, and the maximum number of iterations is increased to $W = 1000$, which gives the values presented in the table below. We again set the value of the convergence parameter as $\varepsilon = 0.1$. We make 100 calculations with 100 start values. The means of these values are presented in the table below.

Verifying the dynamic non-cooperative solution (DB)	Period					
	1	2	3	4	5	6
Price – optimal	39	39	37	35	33	30
Price, p^{DB} – Solver	38.88	39.25	36.88	34.84	32.85	30.13
Price – Abs. difference	0.12	0.25	0.12	0.16	0.15	0.13
Price, p^{DB} – Random	38.86	39.22	36.88	34.87	32.76	30.32
Price – Abs. difference	0.14	0.22	0.12	0.13	0.24	0.32
Adv. – optimal	232	87	116	128	95	0
Adv., a^{DB} – Solver	232.72	84.96	117.40	129.13	96.23	0.00
Adv. – Abs. difference	0.72	2.04	0.40	1.13	1.23	0.00
Adv., a^{DB} – Random	232.89	84.03	118.35	129.13	94.46	0.00
Adv. – Abs. difference	0.89	2.97	1.35	1.13	0.54	0.00

Table 5.14. Verifying values of the dynamic cooperative non-solution (DN) obtained with the Solver program and with the Random search method with best replies.

We recall that the biggest difference between the integer values on optimal decisions was 1.28. In the table above we see that the biggest difference is 2.97, for advertising in period 2. The biggest difference between the Solver program and the Random search method is 1.77 for advertising in period 5. We have a similar dispersion among the values in the table above as for the calculations of the non-cooperative solutions.

6 Procedures when the business game was used as an experiment

In this study, we distinguish between the market model (A) of the business game and the procedures (B–D) when the business game is played. In this chapter, the procedures are presented.

All students in the first year at the Stockholm School of Economics (about 300 students each year from 1991 to 1999) have been required to participate once in the business game, as part of an introductory course in managerial economics. The students played the game for educational purposes. In Chapters 7–11 we will study the decisions they made with what I here call the *original procedures* of the game used in 1991–1996, and also the decisions they made when the procedures were altered in 1997–1999.

In section 6.1, I describe the original procedures of the business game. In section 6.2, we shall study the alterations of the procedures as well as the purposes of the alterations.

The procedures have been given codes, B–D. I will give each of these procedures number codes. The original procedures in section 6.1 are given the digit "1" after the dot. For example, objectives and rewards are given the codes B1.1 and B2.1, respectively. Similar codes and numbers are used for the alterations of the procedures described in section 6.2. The alterations are given numbers higher than "1"; for example, alterations to objectives are given codes such as B1.2 and B1.3. Together, the codes represent all procedures used in the game sessions examined in this study. In section 6.2.6, a table sums up all procedures with their corresponding codes and in Appendix H, there is a table of the alterations in chronological order.

As mentioned, the business game has also been played by what I call *professionals*. In Chapter 12 we shall compare their decisions to those decisions made by students. When professionals play the game, instead of students, we shall consider it to be an alteration of the procedures. Therefore a code is given to this alteration. When students play the game I use the code D1.1 and when professionals play the game I use D1.2.

6.1 The original procedures

The business game when played at the Stockholm School of Economics in 1991–1996 was usually played by the students (D1.1) during three consecutive weeks, three evenings each week, in a total of nine separate game sessions. In earlier years, the students played the game during the first semester (1991–1993). In more recent years (1995–1996), the students had a background of half a year of studies at the school in various topics when playing the game.

The game took altogether about 3–4 hours to play. The game sessions started with an introductory briefing for half an hour. Upon arrival at the game session, the participants received the rules of the game (Appendix A) together with the interest table (Appendix B) and a decision form (Appendix D). The introduction was made by using overheads (Appendix C), including one overhead with an example of how to calculate the costs of decision alternatives. Ståhl was normally the game leader and the administrator of the game.

The time for actually playing the game was about 2½–3 hours. The briefing before the game started and the debriefing at the end of the game took together about 1 hour. The game was divided into periods, where the decisions of all firms were made simultaneously. The participants could ask questions about the game at the briefing and also during the game session. For instance, they sometimes needed help to understand the reports. However, no answers were given to questions about specific decision alternatives or how many periods the game would be played.

The groups of participants, acting as firms, consisted of two to nine participants. The groups, as firms, were seated around separate tables to discuss alternatives for their decisions. Some of the participants used calculators. The participants had decision forms on which they could calculate the results of different decision alternatives. Usually each group was given only one copy of the reports in each period.

The time for decision making in each period was between 10 and 20 minutes. At the start of the game, the time for decision making was about 20 minutes. In the later periods of the game, this time decreased. Some firms handed in their decisions before the specified time limit for the decision. Usually at least two of the firms were ready with their decisions when the other firms were requested to submit in their decisions.

The game was stopped without further notice for pedagogical reasons, for example to avoid boredom, or because of time limitations. When the firms made the decisions in the period that came to be the last period of the game, they did not have exact information about how many periods the game would be played.

At the end of the game, the participants were called together for the debriefing for about 20 minutes. The results from the game were presented, using computer-based graphics, displayed on a screen. The participants were usually given a questionnaire that they answered (Appendix M), while the game leader arranged for the display of the final results.

Below we will study the procedures (B–D) of the original procedures of the game, in the following sections:

6.1.1 Objectives and rewards for playing the game (B)
6.1.2 Information conditions (C)
6.1.3 Composition of groups (D)

6.1.1 Objectives and rewards for playing the game (B)

The objective for firms playing the game (B1.1), as stated in the rules of the game, was to "maximize total equity at the end of the game".

The firm with the highest equity of all game sessions in the same year was given the title "champion of the year of the game". The reward (B2.1) was a diploma with the title printed on it and some small gifts to each participant in this firm.

B1.1	Maximize total equity at the end of the game
B2.1	The title "champion of the year of the game", diploma and small gifts

6.1.2 Information conditions (C)

The participants received the rules of the game (Appendix A) at the start of the briefing. The rules were then presented with overheads (Appendix C). The rules and the information given to the participants at the briefing were the same in all game sessions.

In the rules and at the briefing, the firms received the following specific information about the demand for their products in period 1: "If all firms have the price $ 25 and advertising of $ 50, all firms will sell 15 units". We call this information the original *reference point of demand* (C1.1).

The game started with one test period, where the firms could try their decisions before the real game started (C2.1). Decisions and results from the test period were discarded and the game was then started from scratch again in period 1.

A computer program was used to calculate the results of the decisions. After the decisions were made in each period of the game, the decisions were fed into the computer, calculations were made, and reports were printed and given to each firm. The reports consisted of each firm's own decisions, results and balance sheet, as well as varied information about the decisions and states of the other firms (Appendix E).

The following table shows the reports given to the firms during the game.

Reports available from periods	**Period**						**Variables**
	Test	1	2	3	4	5	
Prices of each firm	x	x	x	x	x	x	p
Total sales	x		x	x	x	x	Σq
Total advertising	x		x	x	x	x	Σa
Sales, market shares of each firm	x		x			x	q
Advertising of each firm	x					x	a
Cash, stock, capacity of each firm	x			x			C, K, S

Table 6.1. Reports during the game as part of the original information conditions.

The reports from the test period were available at the time of making decisions for period 1. The reports from period 1 were available for decisions in period 2, and so on. We see in the table above that the firms received all reports after the test period. From period 1 to period 5, the firms received varied reports (C3.1).

In this connection it should be mentioned that the different firms were not allowed to exchange information with each other during the game. Thus, the reports were the only way for the firms to receive information about the market and the decisions and results of the other firms.

The firms were informed at the briefing that the game would be played for 5–10 periods, but they did not have information about exactly how many periods the game would be played (C4.1). The number of periods varied, as mentioned, in different game sessions.

The firms played the game with symmetric information conditions (C5.1), i.e. all firms received the same information about the other firms in the report, and they did not have any demand table (C6.1).

We sum up the original information conditions as follows:

C1.1	One original reference point of demand
C2.1	One test period
C3.1	Varied reports during the game
C4.1	Number of periods specified within an interval, 5–10 periods
C5.1	Symmetric information conditions for all firms
C6.1	No demand table

6.1.3 Composition of groups (D)

The students (D1.1) were divided into nine game sessions according to their student registration numbers (D2.1). The number of participants in each game session normally ranged from 15 to 35, with a mean close to 30. The composition of the groups of students, as firms, was not formally randomized. It was achieved in an informal way at the end of the briefing, just before the game was started (D3.1). Students sitting next to each other in the classroom were asked to form groups, often implying that students well acquainted with each other formed the groups. The first firm usually consisted of participants sitting in the front rows, and so on. The size of the groups ranged from 2–9 participants in each group.

D1.1	Groups consisted of students
D2.1	Game sessions composed according to with students' registration numbers
D3.1	Groups composed informally

6.1.4 The original procedures in 1991–1996

The original procedures are summarized in the following table:

Code	Procedure
B1.1	Maximize total equity at the end of the game
B2.1	The title "champion of the year of the game", diploma and small gifts
C1.1	One original reference point of demand
C2.1	One test period
C3.1	Varied reports during the game
C4.1	Number of periods specified within an interval, 5–10 periods
C5.1	Symmetric information conditions for all firms
C6.1	No demand table
D1.1	Groups consisted of students
D2.1	Game sessions composed according to with students' registration numbers
D3.1	Groups composed informally

Table 6.2. Original procedures when the game was played 1991–1996.

When I started this study, game sessions where students at Stockholm School of Economics played the game were available from each of the years 1991, 1993, 1995 and 1996.

There is some uncertainty about the exact information conditions when the game was played in 1991. I will therefore exclude them from this study. Thus, we will study the game sessions played by students with the original procedures in 1993, 1995 and 1996. These game sessions can be seen as so-called *replications* when used as an experiment, as they were all played with the same procedures, the original procedures.

6.2 Alterations of the procedures

In 1997–1999 there were altogether five occasions when the procedures of the game (B–D) were altered, as the students at Stockholm School of Economics played the game. Three of these occasions were so-called *regular* game sessions when the business game was used for educational purposes at the school in the course on introductory managerial economics.

I and Ståhl arranged for two more occasions when the game was played, one in 1997 and one in 1998. We call these game sessions *playoffs*, since the firms played three semifinals and one final each year. Firms performing well in the regular game sessions in each of these two years were qualified to play the game again. The playoffs were mainly used for explorative purposes and they were used as means to capture data from game sessions when participants played the game for a second and third time. Furthermore, the decision making was monitored in the playoffs (see sections 11.2 and 11.3 for details). In the description below, I have tried to distinguish between the alterations in the semifinals and in the finals. We will study the decisions in the semifinals, but we will exclude the two finals, since we only had one final each year.

Altogether, I made alterations to the procedures on five occasions presented in the table below.

Year	Type of game session	Number of game sessions	Section
1997	Regular	9	6.2.1
1997	Playoffs	3+1	6.2.2
1998	Regular	9	6.2.3
1998	Playoffs	3+1	6.2.4
1999	Regular	9	6.2.5

Table 6.3. Occasions for alterations.

The purpose of making alterations in the procedures was to study the effects of the alterations. I was primarily interested in information use when making decisions in the business game. Therefore, my main focus was on altering the information conditions. When we make alterations in the information conditions in the business game we assume, as mentioned on page 11, that relevant information is used for decision making in the business game. Before making alterations we might have some expectations about the effect of alterations of information conditions, but still, in line with what has been discussed in Chapter 2, we do not know for sure. As described on page 18, we are interested to see what information is relevant for the decision making in the game and how this information is used.

As we shall see below, I did not make alterations only to the information conditions in the game, but I also made some alterations to objectives and rewards (B) and to the composition of groups (D). In sections 6.2.1–6.2.5, the alterations of the procedures are described. In each of the five sections I will start by describing the alterations to the information conditions (C), since they are of most interest. I then will describe the objectives and rewards for playing the game (B) and thereafter the altering of the group composition (D). In section 6.2.6, there is a table of all procedures, and in Appendix H there is a table of these procedures in chronological order. In section 6.2.7, I will comment on the considerations when altering the procedures.

Before I describe the alterations to the procedures, I will make some remarks on all the game sessions that we will study:

- Ståhl was the game leader in all but one of the game sessions, when I was the game leader.
- Ståhl usually introduced me at the briefing of each game sessions, and I then walked around among the tables to observe the decision making. This could have had effects on the decision making.
- There were limitations on the alterations I was allowed to make, since the game was mainly played for educational purposes. We referred to this earlier as a *poor man's experiment.*
- We did not decide in advance on how many occasions alterations could be made.

6.2.1 Alterations in the regular game sessions in 1997

I made the first alterations to the procedures of the game in 1997, when the students played the game in regular game sessions. Since the reports during the game were varied as part of the original information conditions, it seemed natural that the first alteration should be to give the firms *all reports in all periods* during the game (C3.2). Comparisons could then be made between the decisions of firms who in earlier years had received varied reports during the game and the decisions of firms who now received all reports during the game.

I had noticed that firms in the earlier years made decisions in period 1 on both price and advertising that were higher than those given in the original reference point. Presumably, the firms were influenced by the reference point when they made their decisions. However, at the time it seemed adequate to alter the *reference point* so that it was closer to the decisions that had been made when the game was played earlier. I therefore altered the reference point of demand (C1.2) to: "If all firms have the price $ 30 and advertising of $ 100 all firms will sell 12 units".

As mentioned earlier, playoffs were introduced in 1997 to obtain more occasions when the procedures could be altered. The objectives for playing the game were therefore altered slightly in the nine regular game sessions. The nine winning firms in each of the nine game sessions plus six additional firms played three semifinals, to qualify for participation in the playoffs in 1997. The objective was still to maximize the equity at the end of the game. However, some firms could have played the game to win their game sessions in the hope of qualifying for the semifinals, rather than to maximize their equities at the end of the game to be compared with all game sessions in the same year (B1.2)[1].

Code	Procedure
B1.2	Qualify for semifinals
C1.2	Alteration of the reference point
C3.2	All reports in all periods during the game

Table 6.4. Alterations in the regular game sessions in 1997.

[1] The criteria for qualifying for the semifinal were not completely clear for the firms in 1997, when they played the game in the regular game sessions.

The decisions made by the firms in the regular game sessions in 1997 are described in section 8.2. Comparisons are made between decisions in these game sessions and decisions made in the game sessions from earlier years.

6.2.2 Alterations in playoffs in 1997

In the playoffs in 1997, no reference point (C1.3) was given at the briefing and no test period (C2.2) was played. In order to further increase the information, the reports were enhanced with the information about so-called *lost sales* (C3.3)[2]. Furthermore, the firms were informed that the game would be played for exactly six periods (C4.2). This alteration was mainly made due to time limitations, since all game sessions in the playoffs in 1997 were played on one evening. It was also of interest to see if this alteration had any effect on the decisions in the last period, i.e. in period 6.

The winning firms in each semifinal plus two of the firms in second place with the highest equity qualified for the final (B1.3). The firms in the final played the game to win the game and be the champion of the year (B1.4). In the final there was a monetary reward, US $ 250, given to the firms, in proportion to their equity at the end of the final (B2.2).

The firms were deliberately arranged in the semifinals with respect to their answers in the questionnaire, regarding what price and advertising the participants would choose if they were to play the game again (D2.2).

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[2] Lost sales is the number of units of the product that a firm could have sold due to the demand, if the firm had had more units in supply. While a firm receives an exact value of the demand for its products, when its supply is bigger than the demand and some products go into stocks, the information lost sales gives a firm an exact value of the demand for its products also when the demand is bigger than the supply.

Code	Procedure
B1.3	Qualify for the final
B1.4	Win the final
B2.2	Monetary reward in the final
C1.3	No reference point
C2.2	No test period
C3.3	All reports in all periods during the game including lost sales
C4.2	Number of periods specified exactly, 6 periods
D2.2	Game sessions were deliberately composed

Table 6.5. Alterations in the playoffs in 1997.

The decisions made by the firms in the semifinals in 1997 are described in section 11.2.

6.2.3 Alterations in the regular game sessions in 1998

As already mentioned in section 2.3, there were no big differences between the decisions by the firms that received varied reports and the firms that received all reports in all periods in the regular game sessions in 1997. Against this background, the alteration in 1998 was made as a critical test. I wanted then to know whether differences in the reports really made any difference to the decisions. I therefore altered the information conditions in the game sessions so that the game sessions were played with *asymmetric* information.

Three firms in each game session played the game with varied reports (C3.4), in the same way as in the original information conditions (see page 132), and two firms received all reports in all periods during the game (C3.5). The participants were informed at the briefing that there would be a difference in the information conditions when the game was played (C5.2).

The firms could qualify for the playoffs in 1998 depending on their equity at the end of the game (B1.2). Nine of the twenty-seven firms which played the game with the varied reports and six of the eighteen firms which played the game with all reports in all periods qualified for the playoffs.

Furthermore, the participants were randomly assigned into the groups, acting as firms in the game (D3.2). The purpose was to reduce the possibility that differences in decisions between firms would depend on the group composition, rather than on the difference in reports.

Code	Procedure
B1.2	Qualify for semifinals
C3.4	Three firms received varied reports during the game
C3.5	Two firms received all reports in all periods during the game
C5.2	Information about asymmetric information
D3.2	Randomization of the group composition

Table 6.6. Alterations in the regular game sessions in 1998.

The decisions made by the firms in the regular game sessions in 1998 are described in section 8.3. There we will make comparisons between decisions made by firms which received varied reports and firms which received all reports during the game.

6.2.4 Alterations in the playoffs in 1998

Up to this time, the firms had had *incomplete* information about the market model when they played the game. In order to investigate the effect of information more thoroughly, I was interested to see the effects on decisions if the firms had *complete* information about the market model.

So far, i.e. in the game sessions of 1993 – 1998, we could not determine if the decisions in the game sessions were made due to limitations on information or if similar decisions would also be made with complete information. Therefore, alterations were made in the playoffs in 1998 to give the firms information about the market model.

This alteration can be seen as a so-called *baseline control treatment* for decisions made in the game sessions with incomplete information.

One alternative would have been to give the participants in the firms the market model as it is presented in Chapter 4. However, the model seemed too complex to be helpful for the participants when playing the game. With inspiration from Fouraker & Siegel (1963), I created a demand table. The demand table is a simplification of the market model presented in Chapter 4. The demand table was discussed in section 4.2.8 (see also Appendix F). The demand table was presented to the participants at the briefings before the game started. They were also given information about how they could calculate the marketing effect. The firms could use the demand table when they played the game (C6.2).

Again, as in the playoffs in 1997, there was no reference point given (C1.2) and no test period (C2.2). The firms received all reports in all periods during the game, including lost sales (C3.3), since in this case I really wanted the information to be as complete as possible. In the semifinals the firms were also informed that there would be exactly 6 periods played in the game (C4.2). The alterations to the information conditions, with the demand table and the exact information about how many periods the game would be played, were made to be similar to what I will call *complete* information.[3]

Four of the fifteen firms with the highest equity at the end of the three semifinals qualified for the final (B1.3). One firm qualified for the final because it had more correct expectations (see section 11.3 for details). The firms in the final played the game to win the game session, and be the champion of the year (B1.4). As in 1997, a monetary reward, of around US $ 250, was given to the firms in the final, in proportion to their equity at the end of the final (B2.2).

The firms were arranged into semifinals based on how they stated their motivation in the questionnaire when they played the game for the first time (D2.2).

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[3] In the final, the ending of the game was randomized with 1/3 chance that the game would end after each of the periods 6, 7 and 8. This alteration is given the code (C4.3).

Code	Procedure
B1.3	Qualify for the final
B1.4	Win the final
B2.2	Monetary reward in the final
C1.2	No reference point
C2.2	No test period
C3.3	All reports in all periods during the game including lost sales
C4.2	Number of periods specified exactly (semifinals)
C4.3	Randomized ending (the final)
C6.2	Demand table
D2.2	Game sessions were deliberately composed

Table 6.7. Alterations in the playoffs in 1998.

The decisions made by the firms in the semifinals in 1998 are described in section 11.3.

6.2.5 Alterations in regular the game sessions in 1999

The decisions in the playoffs in 1998 differed from those made in the playoffs in 1997. The explanations for this would presumably be the availability of the demand table in the playoffs in 1998. Hence, it was of interest to use this alteration again in all nine regular game sessions in 1999. We could then see if the effect on decisions found in the playoffs in 1998, when the participants had already played the game once, also occurred when the participants played the game for the first time in 1999. Furthermore, there were nine game sessions that could be used for statistical tests in 1999, while there were only three game sessions in the semifinals in 1998.

At the briefing all participants were introduced to the demand table (C6.2), which they could use when playing the game. The firms received all reports in all periods during the game, including lost sales (C3.3). The firms had information that the game would be played for exactly six periods (C4.2). Again, the information conditions were made to be similar to what I here will call *complete* information.

The rewards for playing the game were increased. At the debriefings one of the five firms in each game session was selected by randomization to win its equity at the end of period 6, minus the 200 in cash the firms had at the start of the game, in Swedish crowns (B2.3)[4].

As in the regular game sessions in 1998, the participants were again randomized into their groups acting as firms (D3.2).

Code	Procedure
B2.3	Monetary reward by lottery in each game session
C3.3	All reports in all periods during the game including lost sales
C4.2	Number of periods specified exactly, 6 periods
C6.2	Demand table
D3.2	Randomization of the group composition

Table 6.8. Alterations in the regular game sessions in 1999.

The decisions made by the firms in the regular game sessions in 1999 are described in section 8.4. Comparisons are made between decisions in these game sessions and decisions made in the game sessions in 1993–1998, i.e. comparisons are made between firms with complete information and firms with incomplete information.

6.2.6 Summary of the alterations to the procedures

All procedures (B–D) used when the game has been played in this study are summarized in the table below. The original procedures are on the left in the table and the alterations to the procedures are on the right in the table (this table is also on the inner cover at the back of the book).

[4] One Swedish crown was at the time about fifteen US cents.

Code	Objectives/reward	Code	Alteration
B1.1	Maximize equity	B1.2	Qualify for semifinal
		B1.3	Qualify for final
		B1.4	Win the final
B2.1	Champions of the year, diploma and small gifts	B2.2	Monetary rewards (finals)
		B2.3	Monetary rewards (lottery)
	Information Condition		**Alteration**
C1.1	Original reference point	C1.2	Altered reference point
		C1.3	No reference point
C2.1	One test period	C2.2	No test period
C3.1	Varied reports	C3.2	All reports
		C3.3	All reports incl. lost sales
		C3.4	Three firms varied reports
		C3.5	Two firms all reports
C4.1	Game played 5–10 periods	C4.2	Game played 6 periods
		C4.3	Randomized ending
C5.1	Symmetric	C5.2	Asymmetric
C6.1	No demand table	C6.2	Demand table
	Group composition		**Alteration**
D1.1	Students	D1.2	Professionals
D2.1	Game sessions composed according to registration numbers	D2.2	Game sessions deliberately composed
D3.1	Groups composed informally	D3.2	Groups randomized

Table 6.9. Summary of the procedures when the game was played.

As we can see in the table above, there have been a number of alterations on the five occasions given for alterations. We will next discuss the number of alterations.

6.2.7 Considerations when making alterations to the procedures

As noted above, many alterations to the procedures were made on the five occasions in 1997–1999. The alterations of the procedures were part of my research and learning process. After making alterations and studying the effects of alterations, I have learned something about using experiments and making alterations that can be useful when making other experiments.

I will here point out that I had no assurances, when I started to make alterations to the procedures, that any alterations would give any significant effects on decisions. If the decisions were the same before and after alterations had been made, the alterations made would probably not have had any effects on the decisions. However, it could also be possible that the business game is not appropriate for use as an experiment. It was therefore important for me to try to make alterations which would give significant effects. The alterations I have made should be seen in this perspective. It can be seen as the start of a research area, where more complex market models, for example business games, are used for experiments. We will discuss this further in Chapter 14.

It is important to mention that in order to be able to reach a final conclusion on the effect of an alteration to the procedures, we should only make one alteration at the time. If we make more than one alteration at the time, implying so-called *confounded alterations* (Friedman, 1994), and we find effects of the alterations, we have difficulties in determining which of the alterations had an effect.

On the other hand, if we make many alterations at the same time and we do not obtain any effects, we could conclude that either:

- The alterations had no effect.
- The alterations had effects, but they were counteractive.

In the regular game sessions I made the following numbers of alterations to the procedures: 3 alterations in 1997, 5 alterations in 1998, and 5 alterations in 1999. For especially explorative purposes I made as many as 8 alterations in the playoffs in 1997 and 10 alterations to the playoffs in 1998, compared to the original procedures.

It seemed appropriate, when making the first alterations to the procedures of the game, to try several alterations so that significant effects on the decisions would be obtained. When I made the alterations to the procedure of the game in 1997, I had almost no idea whether or not the effects of the alterations would be significant. Therefore, I decided to make more than one alteration the first time. If there had been significant effects of the three alterations in the regular game sessions in 1997, I would have chosen to make only one of the three alterations made in 1997, when the game was played in 1998. Thus, there was an option to reduce the number of alterations when the game was played in subsequent years.

However, the effects of the three alterations in 1997 were minor. I had doubts about whether the alterations of reports would affect the decisions in the game. Therefore a critical test was made in 1998 to determine the effects of the information in the reports. Thus, the alterations in the regular game sessions in 1998 were mainly made to enable comparisons between the firms in the sessions that specific year. Furthermore, I chose to randomize the group composition, a so-called *indirect control* of the experimental procedure, to reduce the possibility that differences in decisions would depend on the group composition, rather than the information condition.

In the playoffs in 1997, I noticed differences in decisions in period 6, which could perhaps be due to the information that exactly six periods would be played (C4.2). In order to see if this effect was consistent, I made the same alteration in the playoffs in 1998. In the playoffs in 1998, I noticed differences which could perhaps also depend on the demand table (C6.2). At the time, I had also obtained some preliminary values of the optimal solutions. As earlier mentioned, I observed some similarities between the decisions made by the firms in the playoffs in 1998 and the values of decisions in the non-cooperative solutions. Therefore, the alterations to the procedures in the nine regular game sessions in 1999 were made with the purpose to verifying the findings from the playoffs in 1998. Furthermore, the higher number of firms in 1999 was more suitable for statistical tests.

7 Comparisons between the optimal solutions and the decisions in the game sessions

There were 54 regular game sessions, from 1993 to 1999, available for this study. Most game sessions were played for six periods. A few game sessions were played for seven or eight periods. We shall only study the decisions made in periods 1–6. We shall exclude 3 of the 54 game sessions, since they were played for less than six periods. We shall also exclude one more of the game sessions, since all five firms went into bankruptcy in this game session. After the exclusion of these four game sessions, there are 50 regular game sessions left for us to study, with a total of 250 firms.

We use the median value of the decisions to represent the aggregate of the decisions by the 250 firms. An alternative would be to use the mean. However, there are a few extreme decisions made by some firms, on high prices and advertising that affect the mean values of decisions considerably[1]. Therefore, the median is chosen to represent the decisions made by the firms (Appendix I). It should be mentioned here that there is notable dispersion among the decisions made by the firms studied. For details of the dispersion (measured in standard deviations), I refer to Appendix L and also to section 10.1.

In section 3.1, we discussed four optimal solutions of interest. In this chapter we shall compare the values of these solutions, calculated in Chapter 5, and the decisions made by the firms in the regular game sessions. In this chapter we shall pose the following question:

- Which of the optimal solutions (MC, DC, MN, and DN) gives the best description of the decisions made in the game sessions?

Here we must keep in mind that the firms studied have been playing under different experimental conditions. Later, in the Chapter 8, we will study similarities and differences between the decisions of the firms with respect to the information conditions in the game sessions.

---

[1]Skewness characterizes the degree of asymmetry of a distribution around its mean. Positive skewness indicates a distribution with an asymmetric tail extending toward more positive values. The decisions on price and advertising in the game sessions have a so-called *positive skewness* between 0.25 – 1.49.

In the comparisons below we shall use the values of optimal prices and advertising presented in Table 5.11. The curves for optimal prices, $p^{MC}, p^{DC}, p^{MN}, p^{DN}$, and for optimal advertising, $a^{MC}, a^{DC}, a^{MN}, a^{DN}$, are denoted MC, DC, MN and DN, respectively in the figures below. We start with the figure below showing optimal prices in the solutions and median prices of the firms in the game sessions.

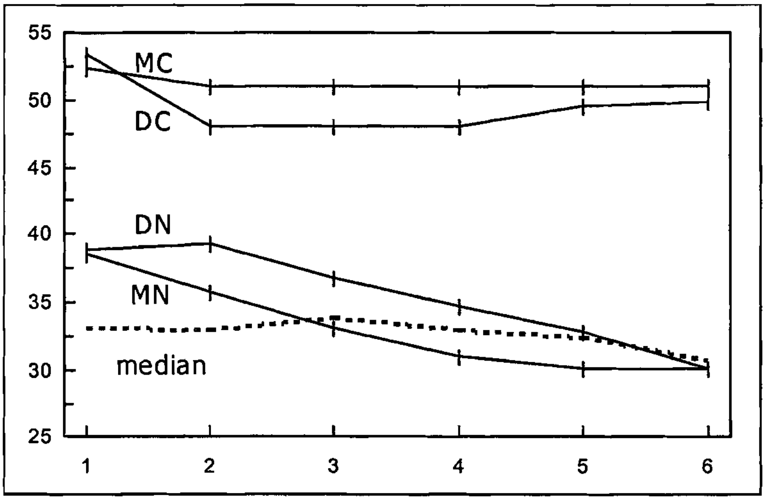

Figure 7.1. Optimal prices and median decisions on prices.

As we can see clearly in the figure above, the median prices are closer to the non-cooperative prices, p^{MN} and p^{DN}, than to the cooperative prices, p^{MC} and p^{DC}, in all six periods. From period 1 to period 3, the median prices are closer to the myopic non-cooperative prices, p^{MN}. From period 4 to period 6, the median prices are closer to the dynamic non-cooperative prices, p^{DN}.

Next, we look at the figure below which shows the optimal advertising in the solutions and the median advertising of the firms in the game sessions.

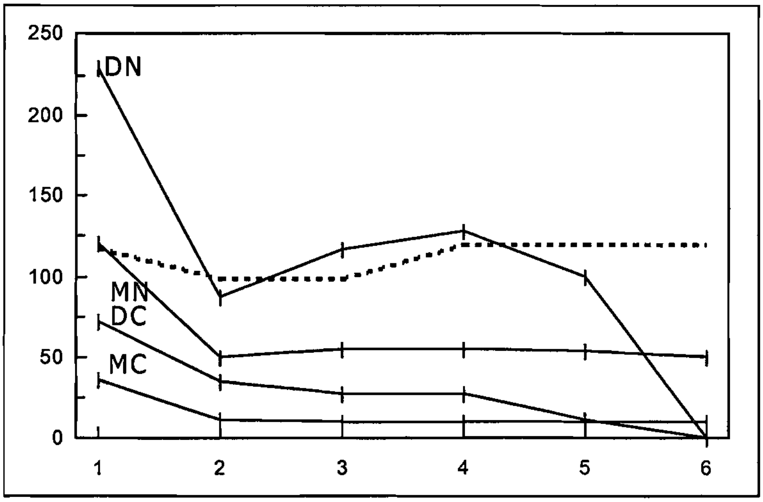

Figure 7.2. Optimal advertising and median decisions on advertising.

The median advertising is also closer to the non-cooperative advertising, a^{MN} and a^{DN}, than to the cooperative advertising, a^{MC} and a^{DC}, in all six periods. In periods 1 and 6, the median advertising is closer to the myopic non-cooperative advertising, a^{MN}, and in periods 2–5 the median advertising is closer to the dynamic non-cooperative advertising, a^{DN}.

Thus the median decisions of the firms on prices and advertising seem to be closer to the non-cooperative solutions (MN and DN). It should be noted that the median decisions on prices and advertising are quite stable from period 1 to period 6, as the median prices are in the range 31–34 and median advertising is in the range 100–120.

Next we shall measure the distance between the optimal solutions and the decisions made by the firms. For example, for the myopic cooperative solution (MC), we calculate the absolute difference between the optimal prices p^{MC}, and the decisions made by each firm, i.e. $\left|p_t^{MC} - p_t\right|$. We shall do this for all optimal decisions on prices and on advertising and for the decisions of each of the 250 firms in six periods.

Here we shall here use the *median distances* of these values for each period. It should be pointed here out that the median of distances between the optimal solutions and the decisions made by the firms might differ from the difference between the optimal decisions and the median decisions of the firms[2]. The difference between the two measurements is bigger when the optimal decisions are closer to the median of the decisions. Hence, when the difference between the optimal decisions and the median decisions is small, the median distance can be quite large.

We start with the medians of distances for optimal prices and the prices of all 250 firms, in the figure below. A lower value of the distance measured on the y-axis means a smaller distance between the optimal prices and the prices of the firms.

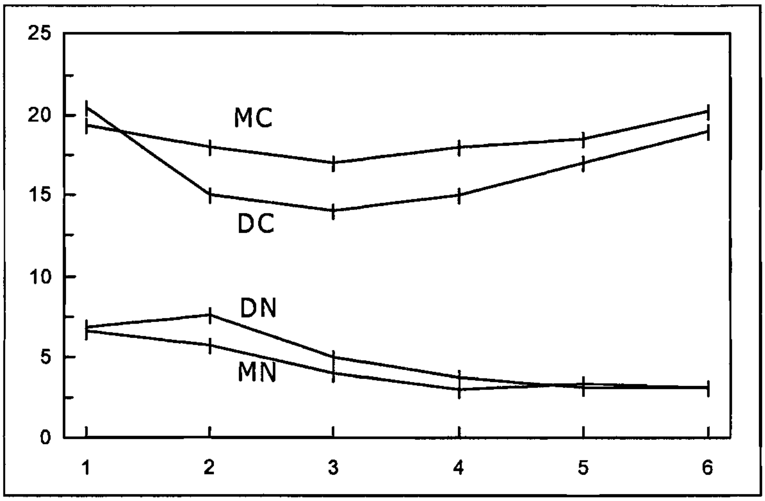

Figure 7.3. Median distance between optimal prices and decisions on prices.

We notice that the median distance between optimal prices and the decisions on prices lies between 3 and 20. The biggest median differences are for cooperative prices, p^{MC} and p^{DC}, in all six periods.

[2] An example is when we make comparisons between the 10 numbers 1,2...10 and to a point with value 5. The difference between the point and the median (which is 5.5 for the 10 numbers) is 0.5, but the median of absolute distance is 2.5.

The median distances for myopic non-cooperative prices, p^{MN}, and for the dynamic non-cooperative prices, p^{MN}, are the smallest and are also almost the same, though they are somewhat smaller for myopic non-cooperative prices, p^{MN}.

Next we study the figure showing the medians of the distances between the value of optimal advertising and the decisions on advertising made by the firms.

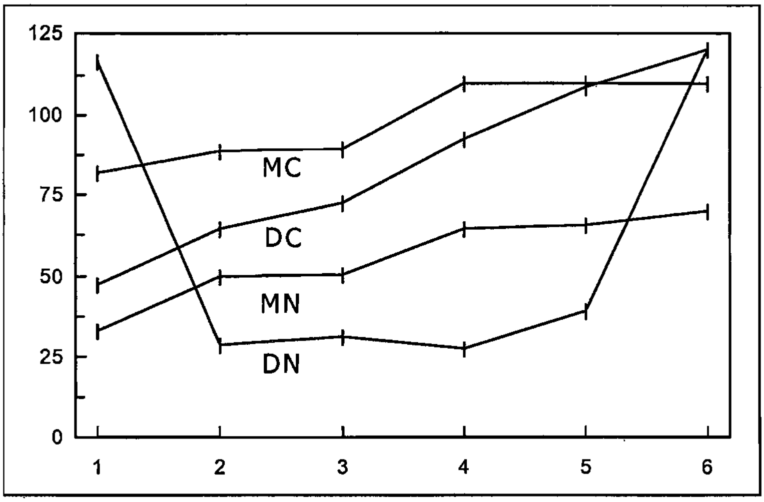

Figure 7.4. Median distance between optimal advertising and decisions on advertising.

We notice that the median distances between optimal advertising and the decisions on advertising are between 25 and 120. The biggest median differences are for cooperative advertising, a^{MC} and a^{DC}, except in periods 1 and 6 where the median distance for dynamic non-cooperative advertising, a^{MN}, is equally big or bigger. However, the median distance is the smallest for the dynamic non-cooperative advertising, a^{MN}, from period 2 to period 5. In section 8.6, we will study this big difference in distance for dynamic non-cooperative advertising, a^{MN}, more thoroughly.

A Kruskal-Wallis test (Siegel & Castellan, 1988) shows that the distances between the optimal solutions and the actual decisions are significantly ($p \leq 0.05$) closer to the non-cooperative solutions than to the cooperative solutions. Thus, we can rule out the possibility that the cooperative solutions, MC and DC, give the best descriptions of the decisions made by the firms studied.

The statistical test also shows that the myopic non-cooperative solution (MN) is significantly closer to decisions on prices in periods 2 and 3, and on advertising in periods 1 and 6. However, the dynamic non-cooperative solution (DN) is significantly closer to decisions on advertising in periods 2–5.

Summing up, the table below shows the median differences in distances for all six periods, i.e. the median for all six periods of the absolute difference between the optimal solution and the decisions in the game sessions. Thus, the median distances are based on 1,500 values, i.e. the median distance between the decisions of 250 firms in 6 periods.

Median distance	*MC*	*DC*	*MN*	*DN*
Prices, p	18.40	16.50	4.20	4.75
Advertising, a	94.11	82.36	50.30	50.00

Table 7.1. Median absolute distance, for period 1 to period 6, between the optimal decision in the solution and the decisions made by the firms in the game sessions.

We see that the values in the table above correspond quite well to the two figures (Figure 7.3 and 7.4) above. We see again that the non-cooperative decisions, p^{MN}, p^{DN}, a^{MN} and a^{DN} in all six periods give equally good descriptions of decisions on prices and on advertising. Hence we conclude the following:

- Of the optimal solutions (MC, DC, MN and DN) compared with the decisions in the game sessions, the myopic non-cooperative solution (MN) and the dynamic non-cooperative solution (DN) give the best description of the decisions made.

One possible reason for this is that the number of firms competing on the same market is five in the business game studied. As mentioned in section 3.5, there is experimental evidence on cooperation on markets with only two, and sometimes also three, firms on the same market. Possibly a decrease, from five firms on the same market to two or three firms, could cause firms to make cooperative decisions. Playing the business game with only two or three firms will be left for future research.

8 Comparisons between decisions made by firms with different information conditions

In Chapter 2, we defined *use of additional information* in the decision making as follows: Additional information is used, if different decisions are made when this additional information is available, compared to when it is not available. In this chapter we shall make comparisons between decisions made by firms with additional information, in some cases in the form of more reports (C3) and in other cases by having a demand table (C6) and information about the exact number of periods in the game (C4). The question we pose is:

- Did firms with additional information make different decisions compared to firms without this additional information?

In order to make such comparisons, we separate the 250 firms studied in Chapter 7 into types according to what information they had in the game sessions. It is important to point out that when we make these comparisons on information use, in this and the following two chapters we shall only consider the alterations made to the information conditions (C). However, when drawing conclusions me must bear in mind that other alterations of the procedures (B and D) may also have affected the decisions. We will discuss this more in Chapter 13.

8.1 Different information conditions

In Chapter 6, the different information conditions in the game sessions were described. Below, we sum up the information conditions of the six regular game sessions (1993, 1995, 1996, 1997, 1998 and 1999).

Before 1997, all firms (C5.1) received varied reports during the game (C3.1). We call these firms with *less* information. In 1997, all firms (C5.1) received all reports in all periods during the game (C3.2). We call these firms with *more* information. We refer to this information condition as *incomplete and symmetric* information.

In 1998, firms on the *same* market received different amounts of information (C5.2), in the form of either less (C3.4) or more information (C3.5). We refer to this information condition as *incomplete and asymmetric* information.

In 1999, all firms (C5.1) had demand tables (C6.2), and all reports included lost sales (C3.3) and information that the game would be played for exactly six periods (C4.2). We refer to these firms as firms with *complete* information and we refer to firms in all of the regular game sessions before game session 1999 as firms with *incomplete* information.

The following table shows the different information conditions in the regular game sessions:

Original information	Less information	Incomplete Information	Symmetric/ Asymmetric information
Additional information	More information		
	Complete information		Symmetric information

Figure 8.1. Information conditions in the regular game sessions.

We shall call the information conditions with less, more and complete information *different types of information*. Furthermore we shall use codes, consisting of one character and one digit, to distinguish between the information type and the year the firms played the game. The character represents the type of information, either *less* (L), *more* (M) or *complete* (C). The digit is the last number of the year in which the firms played the game. This means that the firms with code "L3" had less information, "L", and that the firms played the game in 1993, which we denote "3". Each code represents a category of firms.

The information conditions for the firms in the regular game sessions are presented in Table 8.1 below. The first column contains the information conditions described above. The second column contains codes for the firms. The third column contains the number of firms playing the game with different information conditions and in different years. The fourth column (labeled "bankr.") contains the number of these firms that went into bankruptcy. The fifth column contains the number of the sections of this chapter where the particular game sessions are studied.

Information condition	Code	Firms	Bankr.	Section
Incomplete and symmetric information	L3	45	5	8.2
	L5	40	10	
	L6	40	2	
	M7	40	1	
Incomplete and asymmetric information	L8	27		8.3
	M8	18	1	
Complete information	C9	40	8	8.4

Table 8.1. Information conditions for the firms in the regular game sessions.

As mentioned above, we shall make comparisons between decisions made by firms with additional information and decisions made by firms without this additional information.

In section 8.2, comparisons are made between decisions by firms of M7 (N=40) with more information and decisions by all firms of L3, L5 and L6 with less information, denoted below L3–L6 (N=125).

In section 8.3, comparisons are made between decisions by firms of M8 with more information (N=18) and decisions by firms of L8 with less information (N=27) playing in the same game sessions.

In section 8.4, comparisons are made between decisions by firms of C9 (N=40) with complete information and decisions by firms of L3, L5, L6, M7, L8 and M8, with incomplete information, denoted below L3–M8 (N=210).

In Appendix I, there are two tables, for prices and advertising, with the median decisions of the seven categories of firms rounded off to integers. In sections 8.2–8.4 the decisions on prices and on advertising are illustrated with figures based on these medians of decisions. We use a Kruskal-Wallis test (Siegel & Castellan, 1988) to make comparisons in pairs between decisions by firms with different information conditions[1]. We call a difference when $\rho \leq 0.05$ *significant*, and we call a difference when $0.05 < \rho \leq 0.10$ *nearly significant*.

In section 8.5, we make some further comparisons between decisions made by firms with different information conditions and playing in different years, i.e. firms of different categories. We use the Kruskal-Wallis test to clarify differences and similarities of decisions made by firms with different codes.

In Chapter 7, we ruled out the possibility that the cooperative solutions (MC and DC) gave the best descriptions of the decisions made by the 250 firms studied. We also saw that the non-cooperative decisions gave equally good descriptions of the decisions made on price and advertising for the 250 firms studied. In section 8.6, we shall therefore study if we can establish which of the two non-cooperative solutions (MN and DN) gives the best description of the decisions made for each one of the three information types, less, more and complete information.

[1] The question at the beginning of this chapter can be formulated into the following hypothesis: Firms with additional information made decisions that differ from decisions made by firms without this additional information. We formulate from this hypothesis the corresponding null hypothesis (Davis & Holt 1993, p. 518) as: Firms with additional information did not make decisions that differ from decisions by firms without this additional information.

8.2 Incomplete and symmetric information

In this section, we make comparisons between decisions made by firms with more information, M7, and decisions made by firms with less information, L3–L6. The question we pose is:

- Did the decisions differ between firms with *more* information and firms with *less* information?

Below we can study two figures showing median decisions on prices and advertising. The median decisions of firms with more information (N=40) are represented by solid lines. The median decisions of firms with less information (N=125) are represented by dotted lines.

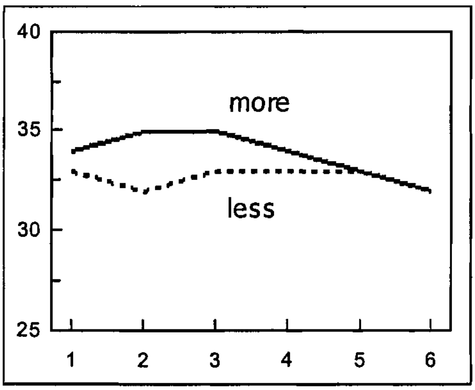

Figure 8.2. Median prices of firms with more and less information.

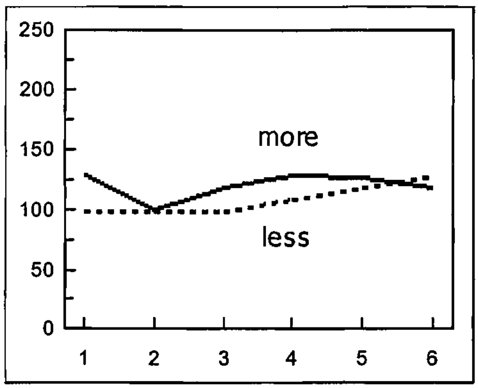

Figure 8.3. Median advertising of firms with more and less information.

In the figure to the left above we see that the median decisions on prices by firms with more information are higher than the medians of prices of firms with less information in periods 1–4. In periods 5 and 6, the medians of prices are roughly the same. The statistical test, Table 1 in Appendix J, shows a significant difference in period 2 and a nearly significant difference in period 3.

In the figure to the right above we see that the median decisions on advertising of firms with more information are higher than the medians of advertising of firms with less information in most of the periods. The statistical test shows significant differences in decisions on advertising for all of the first four periods.

Thus, we have found some significant differences in both prices and advertising between firms with more and less information. We conclude the following as regards the question we posed at the start of this section:

- There is some evidence that the decisions differ between firms with *more* information and firms with *less* information.

We must remember that there was one more alteration of the information condition in the regular game sessions in 1997 (see page 138). Firms with more information played the game with an altered reference point (C1.2), while firms with less information played the game with what we called the original reference point of demand (C1.1).

The differences in decisions can therefore be explained either by the differences in the reference points (C1.1 and C1.2) or by the differences in reports (C3.1 and C3.2), or by both. We shall discuss this further in section 9.1.

8.3 Incomplete and asymmetric information

In this section we make comparisons between decisions made by firms with more information, M8, and decisions made by firms with less information, L8, when the firms competed on the same market. The question we pose is:

- Did the decisions differ between firms with *more* information and firms with *less* information, when the firms competed on the same market?

Below we can study two figures showing median decisions on prices and advertising. The median decisions of firms with more information (N=18) are represented by solid lines. The median decisions of firms with less information (N=27) are represented by dotted lines.

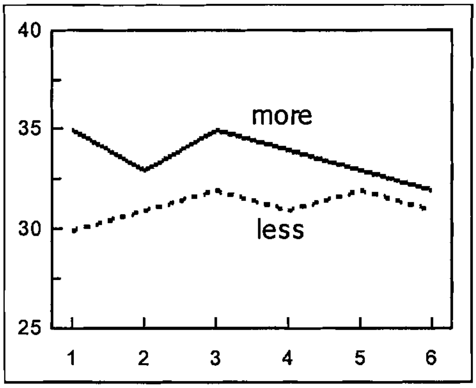

Figure 8.4. Median prices of firms with more and less information.

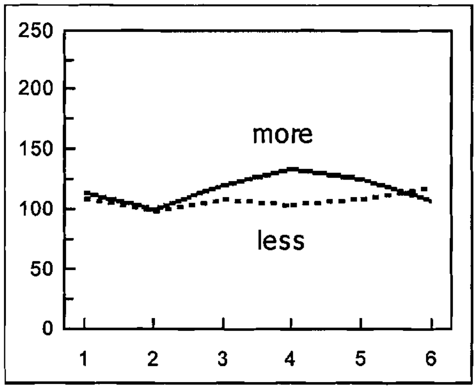

Figure 8.5. Median advertising of firms with more and less information.

The median decisions on prices by firms with more information are in all periods higher than the medians of firms with less information. The statistical test, Table 2 in Appendix J, shows that the prices of firms with more information were nearly significantly higher in period 3 and significantly higher in period 4 than prices of firms with less information.

Since the firms received the same information from the test period, the difference in median price in period 1 can possibly be explained by the alteration that the firms have received information about the different information conditions (C5.2). The finding that decision makers make different decisions due to information about different information conditions has been described in the literature by, for example, Oskamp (1965). We call this phenomenon *increased confidence* in decision making, due to the information about having additional information available for decision making.

The median decisions on advertising by the firms are similar in periods 1–3. In periods 4 and 5, the medians of advertising are higher for firms with more information than for firms with less information. The statistical test shows that the only period where the difference is nearly significant is period 5.

We conclude the following:

- There is some evidence that the decisions differ between firms with *more* information and firms with *less* information when the firms competed on the same market.

We see the same tendency here as in the previous section, i.e. firms with more information make decisions on higher prices and higher advertising, compared to firms with less information. However, we noticed fewer significant differences between decisions by firms with more and less information here than in the previous section.

The difference in prices in period 1 may depend on the information about asymmetric information conditions (C5.2). The differences in decisions in the other periods may also depend on this (C5.2) or on the differences in reports (C3.4 and C3.5), or on both.

8.4 Incomplete and complete information

In this section, we make comparisons between decisions made by firms with complete information, C9, and decisions made by firms with incomplete information, i.e. firms of L3–M8. The question we pose is:

- Did the decisions differ between firms with *complete* information and firms with *incomplete* information?

Below we can study two figures showing median decisions on prices and advertising. The median decisions of firms with complete information (N=40) are represented by solid lines. The median decisions of firms with incomplete information (N=210) are represented by dotted lines.

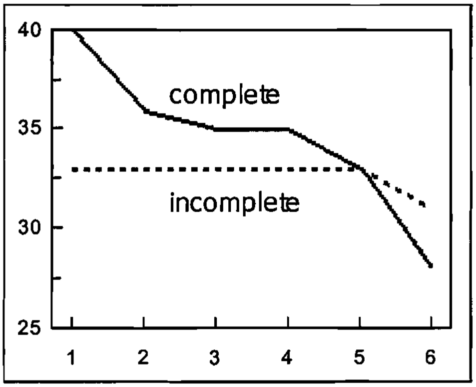

Figure 8.6. Median prices of firms with complete and incomplete information.

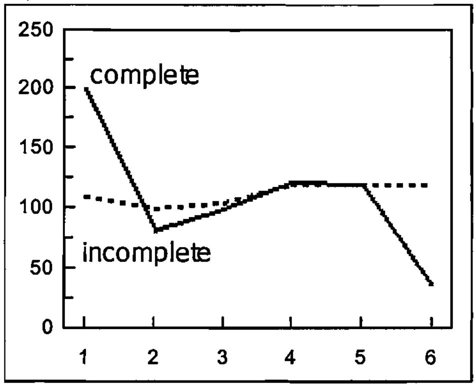

Figure 8.7. Median advertising of firms with complete and incomplete information.

We notice that the median decisions on prices by firms with complete information are higher than those of firms with incomplete information in all periods, except in period 6. The statistical test, Table 3 in Appendix J, shows significant differences in periods 1–3, a nearly significant difference in period 4 and a significant difference in period 6. It should be remembered here that complete information consisted of the alteration in the information condition that the game would end after period 6.

We notice large differences also as regards the median decisions on advertising in period 1 and period 6. However, in periods 3–5 the medians on advertising are quite similar. The statistical test shows significant differences on advertising in periods 1–2 and also in period 6.

To sum up, there are significant differences on decisions on both prices and advertising in periods 1–2 and period 6. We conclude the following:

- There is some evidence that the decisions differ between firms with *complete* information and firms with *incomplete* information.

8.5 Discussion of similarities and differences in decisions

In the comparisons in sections 8.2–8.4, we found the following:

- Firms with *more* information made significantly different decisions in some periods compared to firms with *less information*.
- Firms with *complete* information made significantly different decisions in some periods compared to firms with *incomplete* information.

The first conclusion above was based on statistical tests of the decisions by all 125 firms with less information, denoted L3–L6, compared to decisions by the 40 firms with more information, denoted M7. The second conclusion above was based on the decisions of all 210 firms with incomplete information, denoted L3–M8, compared to the 40 firms with complete information, denoted C9.

We shall next make some comparisons between decisions of firms from *each* separate category, i.e. L3, L5, L6, M7, L8, M8 and C9. In Appendix K, we make what can be called a *technical analysis*, with the Kruskal-Wallis test, based on comparisons of differences in the mean ranks between pairs of categories[2]. We compare categories in pairs as follows:

(i) Firms in categories L3, L5 and L6 (less information) on the one hand and firms in category M7 (more information) on the other hand.

(ii) Firms in categories L3, L5, L6, M7, L8 and M8 (incomplete information) on the one hand and firms in category C9 (complete information) on the other hand.

---

[2] The technical analysis in Appendix K consists of altogether four comparisons, (i)–(iv). Comparisons (i) and (ii) are used here, comparison (iii) is used in Chapter 12 and comparison (iv) is used in Chapter 13.

We see in comparison (i) in the technical analysis in Appendix K that we have evidence in conflict with the findings in section 8.2, i.e. that firms with more information (M7) in some periods made significantly higher decisions on prices and advertising compared to firms with less information (L3–L6). We find in the analysis that there were no significant differences between the decisions of L6 firms and M7 firms in any of the periods 1–6.

Furthermore, in comparison (ii) in Appendix K, we have evidence in conflict with the findings in section 8.4. We there compare decisions, on price and advertising, of each of the categories of firms of L3, L5, L6, M7, L8 and M8 to firms of C9. We find that there were no significant differences in decisions between the periods 2–5, but that there were significant differences in periods 1 and 6.

Although we have found some evidence in conflict with some of the significant differences in decisions in sections 8.2–8.4, we have found significant evidence that firms with additional information did make decisions that differ from decisions made by firms without this additional information. The evidence consists of the following:

- Firms with *complete* information (C9) compared to firms with *incomplete* information (L3–M8) made significantly different decisions in period 1 and in period 6.

Although we in comparison (i) did not find any significant differences between decisions by one of the categories of firms with less information (L6) and firms with more information (M7), we still found differences in decisions between the other categories of firms with less information (L3 and L5) and firms with more information (M7). Furthermore, the critical test in 1998, described in section 8.3, when firms with more and less information (M8 and L8) competed on the same market, showed that there were some significant differences between decisions with these information conditions, i.e. between firms with more and less information. Therefore, we conclude that in general:

- *More* information compared to *less* information seems likely to have some effect on the decisions.

8.6 Comparison between non-cooperative solutions and decisions made with different information conditions

In Chapter 7 we ruled out the possibility that the cooperative solutions (MC and DC) gave the best description of the decisions made by the 250 firms studied. Hence we can concentrate on determining which of the two non-cooperative solutions (MN and DN) gives the best description of the decisions in the game sessions.

We saw that the non-cooperative decisions gave equally good descriptions of the decisions made on price and advertising for the 250 firms studied. In this section, we shall study if we can establish the following:

- Which of the non-cooperative solutions (MN and DN) gives the best description of the decisions made by firms with *less, more* and *complete* information?

We shall study this question in the three following sections for each of these three information types. In section 8.6.1, we study the distances between decisions made by firms with less information (L3–L6) and the non-cooperative solutions. In section 8.6.2, we study these distances for firms with more information (M7) and in section 8.6.3, the distances for firms with complete information (C9). As in Chapter 7, we shall use two methods when we study these distances. We shall study the *distance* between the median of the decisions of the firms and the non-cooperative solutions, for each type of firm. Furthermore, we shall also use the measurement, the *median distance* between the decisions of the firms and the non-cooperative solutions, described on pages 149–150.

In each period of the game, we shall use the median distances of decisions to the non-cooperative solutions to determine if the decisions made by the firms are *closer* to the myopic non-cooperative solution (MN) or if the decisions are *closer* to the dynamic non-cooperative solution (DN). As we shall see below, the differences in median distances to the two non-cooperative prices of decisions on prices are small in some periods. If this difference in median distance of decisions is 0.5 or less in a period, we use the notion *equally close*, i.e. the decisions in this period are determined to be equally close to both non-cooperative solutions.

On the basis of this, we shall consider that one of the two non-cooperative solutions gives the *best description* of the decisions in the game sessions, if the decisions, on both prices and advertising, are closer or equally close to this solution in all six periods. However, as we shall also see below, in some periods the decisions can be closer to the myopic non-cooperative decisions, while in other periods the decisions can be closer to the dynamic non-cooperative decisions. In this case, we cannot determine which of the two non-cooperative solutions gives the best description of the decisions.

8.6.1 Comparison between non-cooperative solutions and decisions made by firms with less information

In Figures 7.1 and 7.2 in Chapter 7, we saw the optimal solutions and the medians of all 250 firms in the regular game sessions. 125 of these firms were firms with less information, L3–L6. The medians between all 250 firms and the 125 firms with less information are quite similar, as they differ by at most 1 on prices and 18 on advertising. Therefore, the conclusion is the same for the 125 firms with less information as for all 250 firms, i.e. neither of the two non-cooperative solutions (MN and DN) gives the best description of the median decisions of firms with less information, L3–L6.

In order to study more closely the non-cooperative solutions and the decisions made by the firms with less information, L3–L6, we shall also study the median distance between the non-cooperative solutions (MN and DN) and the decisions of these 125 firms.

The two figures below show the median distances between the non-cooperative solutions and the decisions of 125 L3–L6 firms with less information.

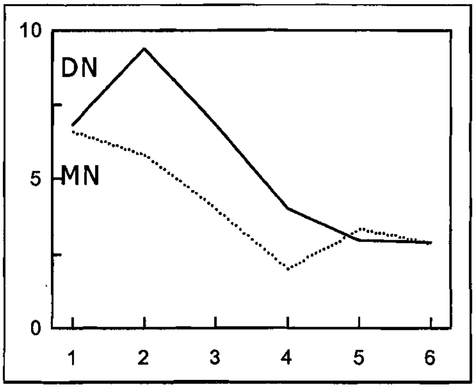

Figure 8.8. Median distance between decisions on prices made by firms with less information, (N=125), L3–L6, and the non-cooperative prices (MN and DN).

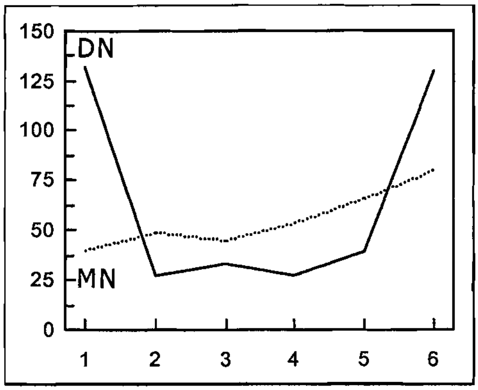

Figure 8.9. Median distance between decisions on advertising made by firms with less information, (N=125), L3–L6, and the non-cooperative advertising (MN and DN).

We see in the figure to the left above that the decisions on prices are closer to the myopic non-cooperative prices in periods 2–4. In periods 1, 5 and 6, the decisions are equally close to the two non-cooperative solutions, i.e. the absolute difference is less than 0.5. We also notice in the figure to the right above that the decisions on advertising are closer to the myopic non-cooperative advertising in periods 1 and 6, while the decisions are closer to the dynamic non-cooperative advertising in periods 2–5. The differences in distances of the decisions to the two non-cooperative solutions are significant in periods 2–4 for prices and in all periods for advertising.

To sum up, for the firms with less information, the decisions on prices are closer or equally close to the myopic non-cooperative prices, but the decisions on advertising are closer to the myopic in two periods and closer to the dynamic non-cooperative advertising in four periods. We therefore conclude that:

- None of the non-cooperative solutions (MN and DN) gives the best description of the decisions made by firms with *less* information (L3–L6).

8.6.2 Comparison between non-cooperative solutions and decisions made by firms with more information

The medians of all 250 firms, in Figures 7.1 and 7.2 in Chapter 7, and the median of the 40 firms with more information, M7, are quite similar as they differ by at most 2 on prices and 18 on advertising. Therefore, the conclusion is the same for the 40 firms with more information as for all 250 firms, i.e. neither of the two non-cooperative solutions (MN and DN) gives the best description of the median decisions of firms with more information, C9.

Below we shall study the median distance between the non-cooperative solutions (MN and DN) and the decisions of the 40 firms with more information. The following two figures show these median distances.

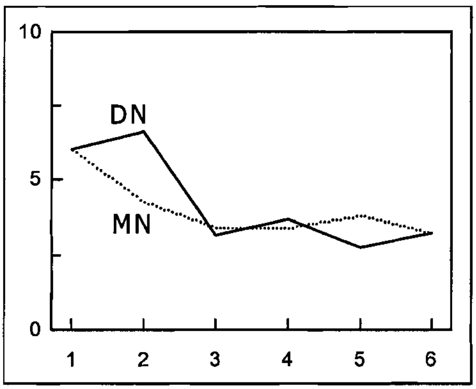

Figure 8.10. Median distance between decisions on prices made by firms with more information (N=40), M7, and the non-cooperative prices (MN and DN).

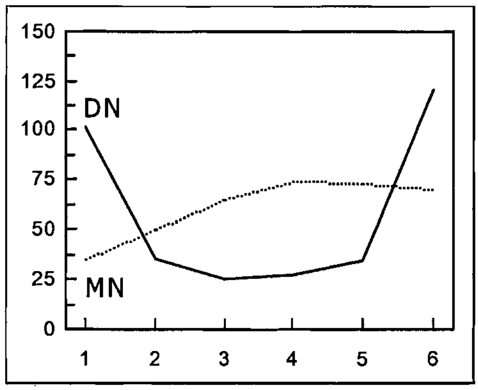

Figure 8.11. Median distance between decisions on advertising made by firms with more information (N=40), M7, and the non-cooperative advertising (MN and DN).

We see in the figure to the left above that the decisions on price are closer to the myopic non-cooperative prices in period 2, but that the decisions are closer to the dynamic non-cooperative prices in period 5. In the other periods the decisions are equally close to the two non-cooperative solutions. We also notice in the figure to the right above that the decisions on advertising are closer to the myopic non-cooperative advertising in periods 1 and 6, while the decisions are closer to the dynamic non-cooperative advertising in periods 2–5.

The differences in distances of the decisions to the two non-cooperative solutions (MN and DN) are significant in period 2 for prices and in all periods for advertising.

To sum up for firms with more information, the decisions on prices are closer to the myopic non-cooperative price in one period, closer to the dynamic non-cooperative price in another period, and equally close to the two solutions in four periods. Furthermore, the decisions on advertising are closer to the myopic in two periods and closer to the dynamic non-cooperative advertising in four periods. We therefore conclude that:

- Neither of the non-cooperative solutions (MN or DN) gives the best description of the decisions of firm with *more* information (M7).

8.6.3 Comparison between non-cooperative solutions and decisions made by firms with complete information

In section 8.5 we found some significant differences in decisions between firms with complete information, C9, and firms with incomplete information, L3–M8. We therefore start with two figures showing the medians of decisions of firms with complete information, where we also see the non-cooperative decisions (MN and DN).

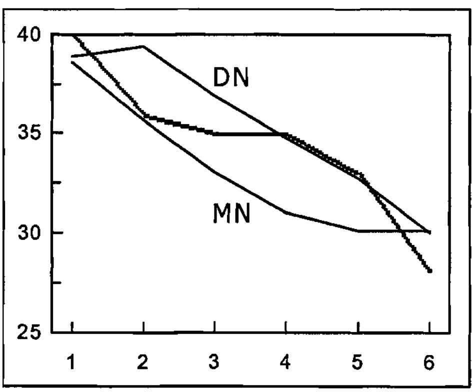

Figure 8.12. Medians of decisions on prices made by firms with complete information (N=40), C9, and the non-cooperative prices (MN and DN).

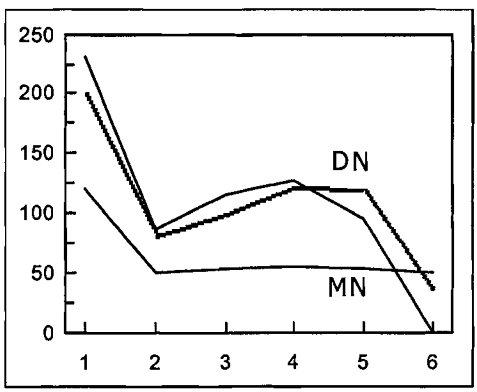

Figure 8.13. Medians of decisions on advertising made by firms with complete information (N=40), C9, and the non-cooperative advertising (MN and DN).

In Figure 8.12 to the left above we see that the medians of prices for firms with complete information are closer or equally close to the dynamic non-cooperative prices in all periods, except in period 2, compared to the myopic cooperative prices. In Figure 8.13 to the right above we see that the medians on advertising is closer to the dynamic non-cooperative advertising in all periods, except in period 6, compared to the myopic non-cooperative advertising.

Thus we cannot conclude that the dynamic non-cooperative solutions (DN) give the best description of the median decisions in *every period.* However, in Figures 8.12 and 8.13 we see that the median decisions of firms with complete information, C9, are quite close to the dynamic non-cooperative solution, especially on advertising, in all periods. This finding is in line with, for example, Morrison & Kamarei (1990).

We proceed, as in the previous two sections, to study the differences between the median distances between the decisions of the 40 firms with complete information and the non-cooperative solutions (MN and DN). The following two figures show the median distances, in this case between the non-cooperative solutions (MN and DN) and the decisions of the 40 firms.

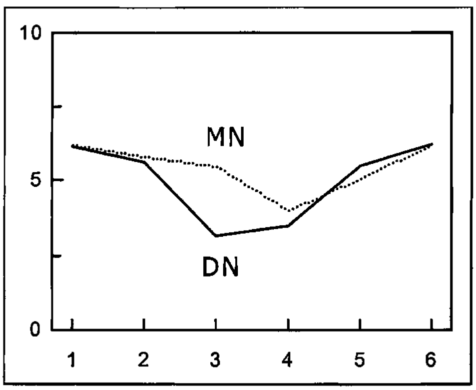

Figure 8.14. Median distance between decisions on prices made by firms with complete information (N=40), C9, and the non-cooperative prices (MN and DN).

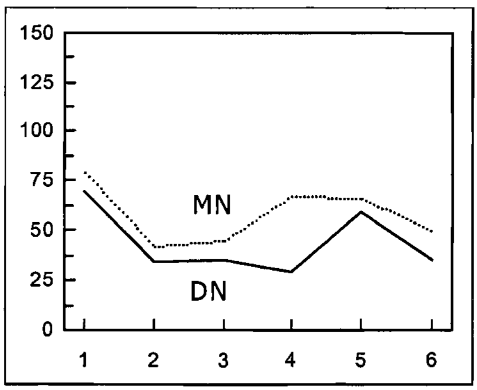

Figure 8.15. Median distance between decisions on advertising made by firms with complete information (N=40), C9, and the non-cooperative advertising (MN and DN).

In the figure to the left above we see that the decisions on prices are closer, or equally close, to the dynamic non-cooperative prices in all periods compared to the myopic non-cooperative prices. In the figure to the right above we also see that the decisions on advertising are closer to the dynamic non-cooperative in all periods, compared to the myopic non-cooperative advertising. However, the statistical test shows that there is only a significant difference for advertising in period 4. Although the other differences are not significant, we state with the hypothesis that:

- The dynamic non-cooperative solutions (DN) give the best description of the decisions of firms with *complete* information (C9).

Thus, on the basis of both methods used for comparisons in this section between the dynamic non-cooperative solution (DN) and the decisions made by the firms with complete information, I think we have an interesting finding.

9 Use of additional information

In Chapter 8, we noticed some significant differences between decisions made by firms with additional information and firms without this additional information. In Chapter 2, we defined the notions relevant, redundant and value of information. In this chapter, we shall study the use of additional information with respect to these notions.

In section 9.1, we shall try to determine which of the additional information was *relevant* and which information was *redundant* in the decision making in the game sessions. In section 9.2, we shall try to estimate the *value* of information as regards the additional relevant information. In section 9.3, we shall answer the research question, “Are better decisions made when additional information is available?”.

When we study the use of additional information we shall study the answers to the questions in the two questionnaires (Appendix M) to see if they give further support for the findings.

9.1 Relevant and redundant information

In this section we pose the following question regarding the use of information (C1–C6):

- Which additional information was used?

When we study which information was relevant and which was redundant, we divide the information that the firms received during the game into three parts, depending on the time in the game. In section 9.1.1, we shall study the use of information at the start of the game. In section 9.1.2, we shall study the information during the game. In section 9.1.3, we shall study the information about the end of the game. In section 9.1.4, we summarize the additional information as relevant or redundant in a table.

9.1.1 Information at the start of the game

In 1997–1999, I collected the decisions made in the test period. The following table shows the median decisions made by firms M7, L8, M8 and C9, in the test period and in period 1.

Test period – period 1	*M7*	*L8*	*M8*	*C9*	*All*
Prices – test period	34	30	35	40	35
Prices – period 1	34	30	35	40	35
Advertising – test period	100	100	100	190	100
Advertising – period 1	130	110	116	200	130

Table 9.1. Medians of decisions in the test period and in period 1.

We shall use the table above, as we study the use of the information at the start of the game.

Information from the test period

As mentioned earlier, the game starts with a test period where the firms can try out the decisions they plan to make in period 1. The firms receive all reports from this test period and they can use the information in these reports when they make decisions in period 1. The game is re-started from "scratch" again in period 1.

In Table 9.1 above, we see that the medians of decisions on prices made by firms of M7, L8, M8 and C9 are the same in the test period and in period 1, for each code of firms. We notice, however, that the medians on advertising were increased, in the range 10–30, from the test period to period 1. A Wilcoxon Signed Rank test (Siegel & Castellan, 1988) does not show any significant differences in prices from the test period to period 1, but it shows a significant increase in advertising, for each of the codes of firms.

Thus the firms used the information from the test period to increase their decisions on advertising. Here we shall keep in mind that the firms had information in the rules of the game that the game would be restarted after the test period. The firms could possibly have disguised the decisions they planned to make in period 1, by making other decisions in the test period.

A Pearson correlation test (Newbold, 1995) is used to make a comparison between the decisions in the test period and those in period 1, of M7, L8, M8 and C9, and shows the following:

Test period – period 1	*M7*	*L8*	*M8*	*C9*	*All*
Prices	.73**	.74**	.61**	.87**	.70**
Advertising	.68**	.75**	.86**	.70**	.76**

Table 9.2. Pearson Correlation coefficients of initial decisions as in the test period and in period 1. The symbols ** show significance of $p \leq 0.01$.

We notice high and significant correlation coefficients between the adjustments of the decisions of firms from the test period to period 1. This means that firms with the same code made decisions in the test period that were strongly related to the decisions in period 1. Due to these relations between decisions, we may have some doubts concerning the use of the information in the reports from the test period.

We cannot assess the information in the reports from the test period as redundant, since there are no decisions available from when the game was played without a test period. Furthermore, we have seen that the firms significantly increased their advertising from the test period to period 1. However, as we shall try to determine whether the information is relevant or redundant, we shall here use the notion *possibly redundant*, where we mean that there is a possibility that the same or similar decisions can be reached with or without this additional information.

This does not rule out the possibility that some of the firms made considerable adjustments of their decisions from the test period to period 1. One of the purposes of playing a test period is that firms that go bankrupt or make poor decisions in the test period can adjust their decisions in period 1, when the real game starts. For example, in 1997–1999 2 of the 125 firms went bankrupt in the test period, and they then altered their decisions in period 1 so that they did not go bankrupt in period 1.

However, our main conclusion is:

- The information in the reports from the test period (C2) was possibly redundant.

Reference point of demand

We shall here study the effect of the alteration of the reference point on decisions at the start of the game. The original reference point of demand was price 25 and advertising 50. All regular game sessions were played with this reference point, with the exception of the game sessions in 1997, i.e. firms of M7. As described in section 6.2.1, the reference point was then altered to price 30 and advertising 100. Hence firms of L8 and M8 played with the original reference point.

When we compare the median decisions between firms of M7, L8 and M8 in Table 9.1, we see that there are differences between the median decisions of these firms, on price and on advertising, both in the test period and in period 1. However, a statistical test shows no significant difference between the decisions of firms of M7, L8 and M8. Hence, we conclude that:

- The alteration of the reference point of demand (C1) did not have any significant effect on the decisions.

The demand table

We shall next try to determine the use of the demand table. As described in section 6.2.5, firms with complete information, C9, had a demand table that they could use when they made decisions. These firms were given the demand table at the start of the game and they could use it in the test period and in all periods during the game[1].

In Table 9.1 we see that the medians, on price and advertising, were higher for firms of C9 than for firms of M7, L8 and M8, both in the test period and in period 1. A statistical test shows significant differences in decisions between the firms of C9 and the firms in the other categories. We therefore establish these differences to be indications that firms of C9 used the demand table in the test period and in period 1.

[1] We study the use of the demand table under the heading "Information at the start of the game", since on page 163 we concluded that firms with *complete information* (C9) compared to firms with *incomplete information* (L3–M8) made significantly different decisions in period 1.

In connection with the use of the demand table, we shall also study the answers to question Q7:

Q7	Try to estimate how much the table of market demand influenced the decision making in your firm.
	Answers to the question were on a scale from 0–100, where 0 is no use of the demand table, 50 medium use and 100 is much use.

The median stated use of the demand table was 58 (N=225). The question above concerns the use of the demand table in all periods of the game. Thus we cannot connect the stated use of the demand table specifically to the test period and period 1. However, the answers to question Q7 are not in conflict with the indications established above, that the demand table was used in these periods. Hence, we here conclude that:

- The additional information in the demand table (C6) was relevant information, at least at the start of the game.

9.1.2 Information during the game

As we have seen above, firms with complete information, C9, had a demand table which they used, as concluded above, when making decisions at the start of the game. Here we shall study if we can find support for the use of the demand table also during the game. Firms with incomplete information, L3–M8, played the game without this demand table.

The information during the game consisted of the demand table plus information in the reports (see Appendix E and also Table 6.1). All firms received all reports in the test period and all firms received income statements and balance sheets for their own firms in all periods.

The information in the reports differed as follows:

- Firms with less information, L3–L6 and L8, received varied reports during the game (C3.1 or C3.4).
- Firms with more information, M7 and M8, received all reports, without lost sales, in all periods during the game (C3.2 or C3.5).
- Firms with complete information, C9, received all reports, including lost sales, in all periods during the game (C3.3.).

We should recall here that in section 8.5 we concluded that more information, compared to less information, seemed likely to have some effect on the decisions.

We shall use the answers to the two questions, Q0 and Q2, in the questionnaires (Appendix M) to see if we can find some further support for the use of the additional information consisting of the demand table (C6) and the reports (C3) during the game.

Considerations when making decisions on price

We start with the question, Q0, about what information was taken into consideration when making decisions on price:

Q0	What did the participants consider when making decisions on price?

The question was asked of the participants in firms with less information L3–L6 and the participants in firms with complete information, C9. Each participant was asked, in question Q0, to state 3 of the 17 alternatives[2].

[2] The questionnaire with question Q0 was used for participants playing the game in 1993–1996, i.e. L3–L6. About 40%–50% of the participants answered question Q0 in these three years. The questionnaire was also used in 1999, i.e. C9, where about 95% of the participants answered the question.

The differences in answers between firms with complete information and firms with less information depended on either the use of the demand table (C6) or the use of additional information in the reports (C3) during the game, or both. The following two alternatives were answered significantly more by firms with complete information, C9, than by firms without this additional information.

(e)	What prices our competitors could be expected to decide in the present period.
(g)	How much machine capacity our firm had.

We can connect alternative (e) to the use of the demand table as each firm needs to form expectations of the other firms' decisions on price in order to be able to use the demand table for determining the demand for its product. We can also possibly connect alternative (g) to the use of the demand table, since when the firms have determined the demand, they might consider their capacity to make investments to match the determined demand.

We notice that there was no significant difference in the answers about the machine capacity of the competitors, i.e. alternative (h), which was included in the additional information in the reports. Here we might have expected that firms with complete information would have stated this alternative more often. The same applies for alternative (b), stocks of the competitors, and alternative (k), advertising of the competitors. We might expect that firms with complete information would have stated these alternatives more often. Thus we do not find support, in the answers to question Q0, for the use of additional information in the reports about the competitors. We therefore conclude that:

- There was further support for the conclusion that the demand table (C6) was relevant information, when considering decisions on price, but there was no support for the use of the additional information in the reports (C3).

It should be pointed out that question Q0 explicitly refers to decisions on price. Hence, there is a possibility that the answers could have been different if the question had dealt with decisions on, for example, advertising.

Expectations of the other firms' decisions

Next we shall study the answers to question Q2, which was answered by participants in firms of M7, L8, M8 and C9.

Q2	Which option do you think will best describe how your firm used the information you got about your competitors during the game?
(a)	we did not discuss our competitors′ decisions much
(b)	we did discuss our competitors′ decisions, but we did not have correct expectations of their decisions
(c)	we did discuss our competitors′ decisions, and we had correct expectations of their decisions

We would expect that firms with additional information in the reports, M7, M8 and C9, would state that they had more correct expectations than those in L8. The figure below shows the answers given by participants in M7, L8, M8 and C9.

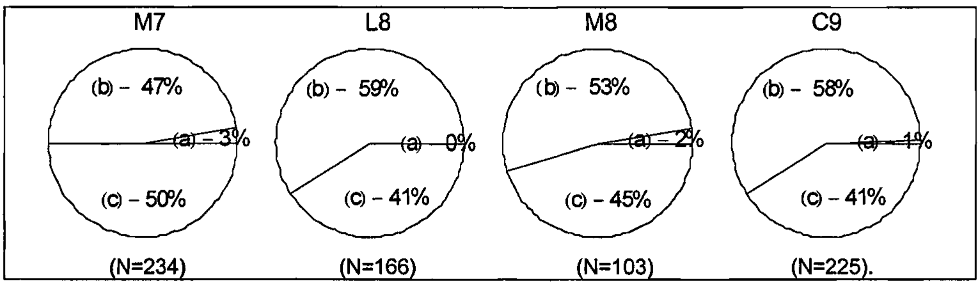

Figure 9.1. Stated correctness of expectations. Chi-square=9.9, df=6, Asymp. Sig=.13.

We notice that 47%–59% of the answers indicated that their firms did not have correct expectations, alternative (b), while 41%–50% of the answers indicated that their firms had correct expectations, alternative (c). There are no significant differences between the answers of firms with different codes. Thus we conclude that:

- There was no support for the hypothesis that the additional information during the game, in the reports (C3) and in the demand table (C6), would lead to the participants stating that they had more correct expectations of the competitors′ decisions.

The answers to the questions Q0 and Q2 did not indicate that the additional information in the reports was used. Therefore we again use the notion possibly redundant, since there is a possibility that the same or similar decisions could have been made with or without the additional information in the reports.

9.1.3 Information about the end of the game

We recall from section 8.4 that firms with complete information significantly decreased their prices and advertising in period 6, while firms with incomplete information made about the same decisions in periods 5 and 6.

Therefore, we shall study the answers to question Q6, which concerns the decisions made at the end of the game:

Q6	Did your firm make decisions in the last periods, or what ended up as the last period, with respect to the assumption that the game would end?
(a)	we did not discuss the ending of the game very much
(b)	we did discuss the end of the game, but we did not decide any differently
(c)	we did discuss the end of the game, and decided accordingly

In the figure below we see the answers to question Q6 in percentages of the three alternatives.

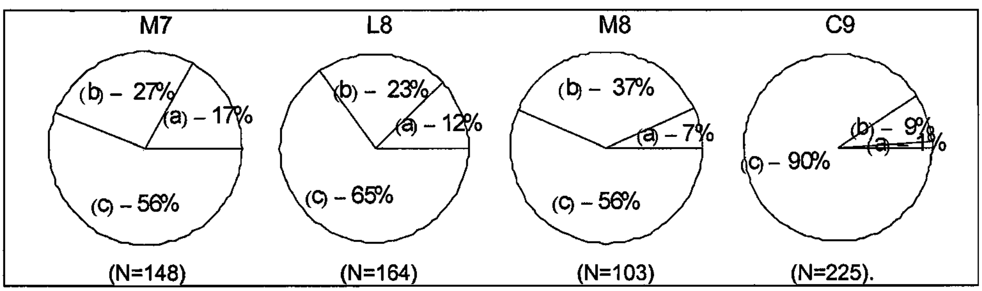

Figure 9.2. Stated correctness of expectations. Df=6, Asymp. Sig=.13.

In the figure above we see that 90% of the participants playing the game in firms with complete information, i.e. firms of C9, stated alternative (c). This is significantly more than the participants in firms with incomplete information, M7, L8 and M8. Hence, we have support for:

- The additional information about exactly how many periods the game would be played (C4) was relevant.

However, 56%–65% of the participants in firms of M7, L8 and M8 also stated alternative (c), but these firms did not decrease price or advertising as much as firms of C9. Here we should remember that the game was sometimes played for seven or eight periods by these categories. Hence one possible explanation for these answers in the questionnaire is that the firms of M7, L8 and M8 were not certain that period 6 was the last period when they made their decision in period 6, and they therefore took this into account when they made their decisions.

9.1.4 Summary of relevant and redundant information

Above we have studied which information is relevant and which is redundant, i.e. which information was used and not used when making decisions in the game.

The following table summarizes the findings on information use:

Code	Information use	Finding	Section
C1	Reference point	No effect of alteration	
C2	Test period	Possibly redundant	9.1.1
C3	Reports	Possibly redundant	9.1.2
C4	Number of periods	Relevant	9.1.3
C5	Information about asymmetric information conditions	Relevant	8.3
C6	Demand table	Relevant	9.1.1

Table 9.3. Findings on information use based on of the alterations of the information conditions.

We cannot generally assess the reference point of demand (C1) as being redundant information, since we do not have any decisions to study that are based on a major alteration of the reference point.

The notion possibly redundant for reports in the test period and during the game is in conflict with our conclusion in section 8.5, where we concluded that more information compared to less information seems likely to have some effect on the decisions. Further experimentation is needed to determine whether the additional information in the reports is relevant for the decision making in the game.

9.2 Value of additional information

In Chapter 2, we discussed the value of information. We determined that the value of additional information was the difference between the benefits of the additional information and the costs of such information. There were no costs for obtaining additional information in the game sessions. Therefore, the value of the additional information is the difference in equity between firms with additional information and without this additional information. If additional information has a positive value in the game, firms with additional information have higher equity than firms without this additional equity. The question we pose is:

- What was the value of the additional information?

In section 9.2.1 we shall study the value of information in the game sessions and in section 9.2.2 we shall study some hypothetical values of information in order to gain some further insights.

9.2.1 Value of information in the game sessions

The following figure shows bars of median equity at the end of period 6:

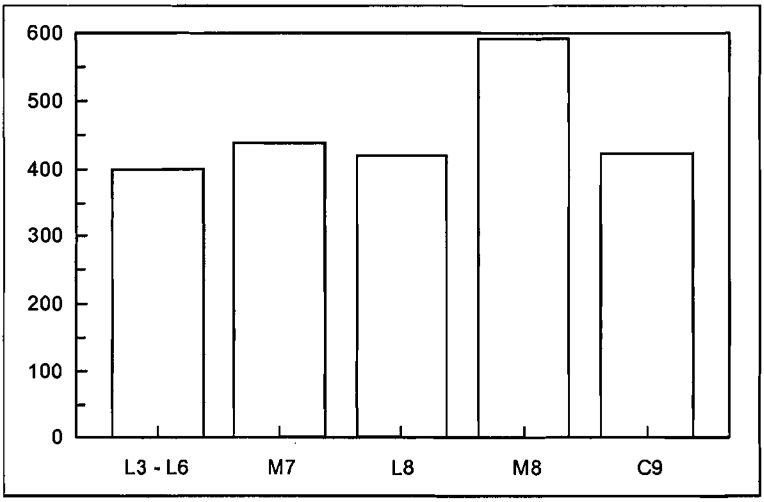

Figure 9.3. Median equity at the end of period 6.

In the analysis below we shall distinguish between the case when firms have symmetric information and the case when firms have asymmetric information.

We first compare the equities at the end of period 6 for firms with symmetric information, i.e. L3–L6, M7 and C9. We notice no big differences between the median equities for these firms in the figure above (equities are in the range 400–438) and a statistical test shows no significant differences between the equities of these firms. Thus, we conclude:

- The additional information did not have a positive value on markets with symmetric information.

Thus the additional information (C3.2, C4.2 and C6.2) did not have a positive value, when the firms received this additional information symmetrically.

We then compare the equities at the end of period 6 for firms with asymmetric information, i.e. firms with more information, M8, and firms with less information, L8. In the figure above we notice that the median equity is higher for firms with more information, M8, when competing on the same market as firms with less information, L8. This difference in equity at the end of period 6 is nearly significant.

- The additional information had a positive value on markets with asymmetric information.

If we compare the median equity of firms M8, 590, and of firms L8, 420, we get a difference of 590–420 = 170. Thus the additional information in the reports (C3.4 and C3.5) and the information about the asymmetric information condition (C5.2) can be assessed to have a value of around 170.

9.2.2 Hypothetical value of asymmetric information

In the previous section, we concluded that firms competing in the same market with different types of information, i.e. with asymmetric information conditions, have different equities. The only evidence of this conclusion is from the game sessions in 1998, where firms with less and more information (M8 and L8) competed in the same game sessions.

In section 8.5, we concluded that there were some significant differences between firms with complete information and firms with incomplete information. In section 8.6, we saw that the median decisions of firms with complete information were quite near the non-cooperative solution (DN) in all periods. Furthermore, we concluded that the dynamic non-cooperative solutions gave the best description of the decisions of firms with complete information.

Firms with complete information have not competed in the same game sessions as firms with less or more information. However, in order to assess the value of asymmetric information, i.e. complete information compared to more information, it is of interest to calculate the effect on equity of the differences in decisions between these types of firms. We therefore make some comparisons between equities based on median decisions when firms compete in the same market with different information conditions. We shall use the median decisions of the firms of

L8, M8 and C9 to calculate some hypothetical equities at the end of period 6.

We make the following three assumptions for these hypothetical calculations:

- The firms make the median decisions of either L8, M8 and C9 in all periods, i.e. the decisions of the firms in all periods are determined before the game starts and they are not adjusted during the game.
- There are correct estimations of demand as assumed in (31) and (32) on page 70 in section 5.2, i.e. the firms invest and produce according to the demand incurred by median price and advertising. Thus there are no products in stock between periods and there are no lost sales (demand bigger than supply).
- There are five firms competing in the same market, where three firms are of the same information type and two firms are of another information type.

Here it should be pointed out that these comparisons do not include the best reply to the medians of these decisions, since the purpose is to make assessments of the medians of the decisions actually reached in the game. However, we shall compare the median decisions of firms with complete information, C9, with the theoretical dynamic non-cooperative decisions (DN). We shall study the following three cases:

(i) Two firms make the median decisions of firms of M8 and three firms make the median decisions of firms of L8.

(ii) Two firms make the median decisions of firms of C9 and three firms make the median decisions of firms of M8.

(iii) Two firms follow the dynamic non-cooperative solution (DN) and three firms make the median decisions of C9.

	N	**Firms of**	$\ddot{E}_6$	N	**Firms of**	$\ddot{E}_6$	**Difference**
(i)	2	M8	764	3	L8	707	57
(ii)	2	C9	868	3	M8	735	134
(iii)	2	DN	829	3	C9	782	47

Table 9.4. Hypothetical equity at the end of period 6 with asymmetric information.

Compared to the values of the equities of the optimal solutions, which were in the range 783–1414 (in Table 5.12 on page 126), the range of all equities in the table above can be considered as quite small, as they are in the range 707–868.

I shall finally give some comments on the hypothetical values of information.

(i) The difference in equity between the median decisions of two firms of M8 and the median decisions of three firms of L8 is 57. This hypothetical value of the additional information (C3.5 and C5.2) is less than the value of 170 for the additional information noted on page 183, when the firms played the game. This difference can be explained by the fact that firms with both more and less information made correct estimations of demand in the hypothetical calculations (as we assumed above), while firms with more information, M8, made better estimation of demand than firms with less information, L8, in the game sessions.

(ii) The median decisions of two firms with complete information, C9, give higher equity than the median decisions of three firms with more information, M8. The hypothetical value of the additional information (C3.3, C4.2 and C6.2) of firms of C9, compared to the information of firms of M8 (C3.1, C4.1 and C6.1), is 134.

(iii) The hypothetical value of full computational capability and "exact information" of the demand function, as described in Chapter 4, is 47, since this is the difference between the of hypothetical equities of two firms which follow the dynamic non-cooperative solutions (DN) and three firms which make the median decisions of firms with complete information (C9).

9.3 Use of additional information to make better decisions

In Chapter 2, we posed the research question, "Are better decisions made when additional information is available?". In this chapter, we have studied which additional information was used and we have also determined values of the additional information.

Next we need to describe what we mean by better decisions. The purpose of playing the game was stated in the rules of the game (Appendix A), namely to maximize the equity at the end of the game. As discussed in section 3.1, we can argue from two perspectives when maximizing the equity at the end of the game:

- Better decisions would be for *all firms* to make optimal cooperative decisions, primarily dynamic cooperative decisions (DC), as we have seen in Table 5.11.
- Better decisions would be for *one of the firms* to make its best reply to the decisions of the other firms.

In section 9.2, we concluded that differences in decisions between firms with different information (L3–L6, M7 and C9) did not lead to any significant differences in equity when firms had symmetric information in the same game session.

However, the differences in decisions between firms with more information and less information (M8 and L8) and the differences in the hypothetical medians of decisions with more and complete information (M8 and C9) lead to higher equity for firms with additional information, if the firms with the additional information had this information asymmetrically. Hence, we can conclude that:

- Better decisions are made, when additional information is available, since decisions made with additional information give higher equity when this information is available asymmetrically, i.e. to some, but not all, firms, and decisions made with additional information give roughly the same equity when this information is available symmetrically, i.e. to all firms.

10 Adaptiveness to information

In Chapters 8–9, our attention was mainly directed to the differences between decisions made by firms with different information. In this chapter we shall also distinguish between firms with different information as we study their adaptiveness to information. However, the main focus here is directed to some of the similarities when firms with different information used the available information make decisions. The general question for this chapter is:

- How did the firms use the available information?

In section 10.1, we discuss the dispersion among the decisions of the firms. In section 10.2, we study the adjustments of decisions, from one period to the next. In section 10.3, we study the adaptations of decisions to the mean decisions in the previous period or to the decisions of the leader in the previous period. In section 10.4, we compare the actual decisions made by the firms in period 1 with the decisions which the participants stated in a questionnaire after the game that they would make if they were to play the game again. In section 10.5, finally, we study what determines the equity at the end of period 6.

10.1 Dispersions of decisions

In Chapter 2, we defined the notion *ambiguous* information, referring to the situation when firms with the same amount of available information make different decisions. In this section, we shall study if firms with the same information make different decisions. We call these differences between decisions *dispersions* of decisions.

I would like to state two reasonable hypotheses about the relation between adaptiveness to information and dispersion of decisions:

- Firms with additional information have less dispersion among their decisions than firms without this information.
- As more information becomes available during the periods of the game the dispersion among decisions decrease, i.e. as more information becomes available, the firms adjust their decisions so that they become more similar.

We do not set any exact limits to define whether or not the decisions in the game session should be considered to be dispersed or not. We shall use quartiles and standard deviations as our measurements to describe the dispersion among decisions.

We first use quartiles in the two figures below to illustrate the dispersion among decisions of firms with complete information, C9. 25% of the decisions are below or on the lines marked 25%, 50% of the decisions are below or on the lines marked 50%, i.e. coinciding with the medians, and 75% of the decisions are below or on the lines marked 75%. This means that about 50% of the decisions are between the 25% and 75% lines, but it also means that the remaining 50% of the decisions are below the 25% lines or above the 75% lines.

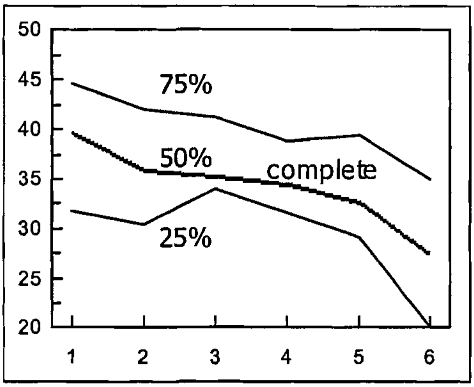

Figure 10.1. Quartiles of prices for firms with complete information, C9 (N=40).

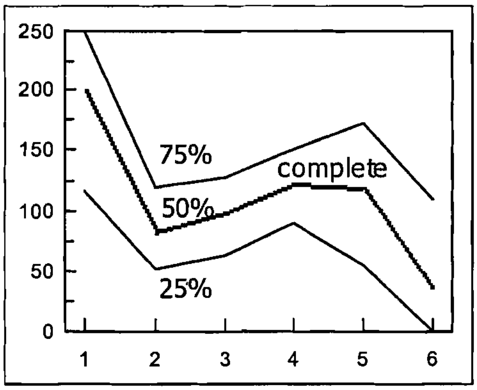

Figure 10.2. Quartiles of adverting for firms with complete information, C9 (N=40).

We see in the two figures above that the lines of 25% and 75% for firms with complete information, C9, were at considerable distances from the intervals of the medians (50%).

For example, the median decision on price was 39 in period 1, but 50% of the 40 firms, i.e. 20 of the 40 firms, made decisions on prices in period 1 which were either higher than 44 or lower than 32. Hence, the two figures above with quartiles of decisions show considerable dispersion among the decisions of firms with complete information, C9.

We shall next use the standard deviations of decisions as measurement for dispersions of decisions. It should be mentioned here that standard deviation relies on the mean of decisions. Below we shall study two figures showing the standard deviations of firms with complete information, C9, and firms with incomplete information, L3–M8[1].

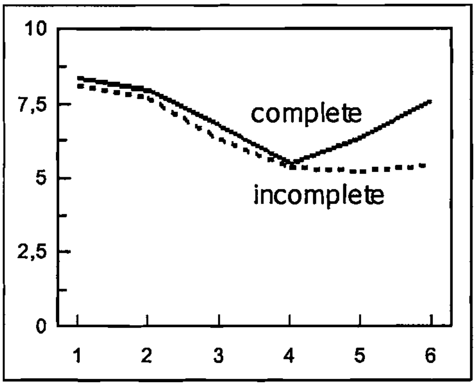

Figure 10.3. Standard deviation of prices for firms with complete information, C9 (N=40) and firms with incomplete information, L3–M8 (N=210).

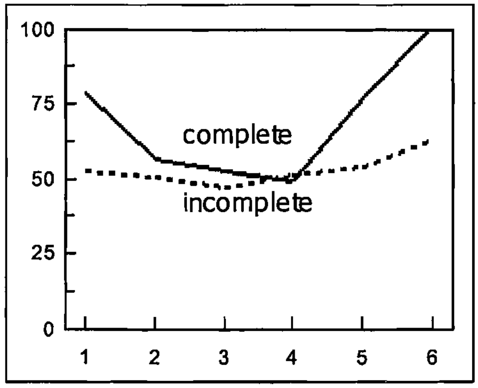

Figure 10.4. Standard deviation of advertising for firms with complete information, C9 (N=40) and firms with incomplete information, L3–M8 (N=210).

In both the figures above we see that the dispersion is higher or the same for firms with complete information compared to firms with incomplete information. This contradicts my hypothesis above that the dispersion among decisions of firms with additional information, i.e. firms with complete information, would be less than for firms without this information, i.e. firms with incomplete information.

We can also use the two figures above to study how the standard deviation changed during the game. In the figure to the left above we see that the standard deviation decreased in periods 1–4 for firms with both incomplete and complete information. However, in periods 5–6 the standard deviation increased for firms with complete information. In the figure to the right above, we see that the standard deviation of advertising decreased in periods 1–4 for firms with complete information and that it increased in periods 5–6. The standard deviation on advertising for firms with incomplete information did not change very much over time.

[1] In Appendix L, there are two tables of the standard deviations of decisions, prices and advertising in periods 1–6 for all seven categories of firms in the regular game sessions. In these tables we can see that the standard deviation of the seven categories of firms was in the range 3–11 for price and 29–102 for advertising.

Thus, we see at least some initial tendencies that the dispersion among decisions decreased somewhat or was about the same during the game for firms with incomplete information. However, we also see that the dispersion increased in periods 5–6 for firms with complete information. This increase in the dispersion of decisions may be due to the information that in this case the game would end after exactly six periods (C4.2).

On the basis of the two measurements above we conclude that:

- Firms with the same amount of available information used their information to make different decisions.

Furthermore, since we defined the notion *ambiguous* information to describe the situation when firms with the same amount of available information make different decisions, we can consider the information in the game to be ambiguous, at least to some extent.

10.2 Adjustments of decisions

The firms can use the available information when they make adjustments of their decisions. Here we shall study what adjustments of decisions the firms made from one period to the next.

We measure the adjustments of decisions using the absolute difference between decisions, on price, Δp_t, and advertising, Δa_t:

(73a) $\Delta p_t = |p_{t-1} - p_t|$ (73b) $\Delta a_t = |a_{t-1} - a_t|$

where the number of the period is $t = 2..6$:

For the firms in each of the seven categories in the regular game sessions, we calculate absolute differences of decisions from one period to the next. The medians of adjustments are based on all adjustments by all firms of the same category. The table below shows the medians of adjustments for period 2 to period 6, i.e. in five periods. For example, the median adjustments of prices for the five periods by firms of L3 (N=45) are based on $5 \cdot 45 = 225$ adjustments.

Adjustments	*L3*	*L5*	*L6*	*M7*	*L8*	*M8*	*C9*	*All*
Price, Δp	2	1	1	2	1	1	3	2
Advertising, Δa	25	20	20	20	20	20	50	26

Table 10.1. Median adjustments of prices and advertising from one period to the next.

In the table above, we see that the median adjustments of all firms were 2 for price and 26 for advertising. With prices of about 30 and advertising of about 100, we see that the firms adjusted their prices from one period to the next by about 7%, while advertising was adjusted about 26%. The median adjustments, 2 for price and 26 for advertising, can be considered as quite small compared to the dispersion among decisions, as the standard deviations were in the range 3–11 for price and 29–102 for advertising.

In the table above we also see that the median adjustments of firms with more information (M7 and M8) are about the same as for the firms with less information (L3, L5, L6, and L8). Thus the differences in the reports (C3.2 to C3.1) between firms did not have any effect on the size of the adjustments.

However, the median adjustments of firms with complete information, C9, are noticeably higher than for firms with incomplete information, L3–M8. A statistical test shows that firms with complete information made significantly or nearly significantly bigger adjustments than firms with incomplete information to prices in all but one period (from period 3 to period 4) and to advertising in all periods.

This shows that the additional information, principally the demand table (C6.2) but also the exact information about when the game would end (C4.2), was used to make bigger adjustments of decisions, especially on advertising. Thus we conclude the following:

- The firms adjusted their decisions more on advertising, both absolutely and relatively, than on prices. Firms with complete information adjusted their decisions more than firms with incomplete information.

10.3 Adaptations of decisions

As the firms receive information during the game about the decisions on prices and advertising of the other firms, they can adjust their decisions to make decisions closer to the decisions of the other firms in the previous period. We shall call this *adaptations* of decisions. In this section we shall study if the firms use the available information to adapt their decisions to the decisions of the other firms.

We use two types of adaptations of decisions to the decisions of the other firms in the same game sessions:

(i) adaptation to the mean of decisions.
(ii) adaptation to the decisions of the leader, i.e. the firm with the highest equity in the previous period.

We shall define the firms' adaptations of decisions as follows: The decisions of each firm on price and advertising in the previous period, are denoted p_{t-1} and a_{t-1}, respectively, and the decisions of each firm in the present period are denoted p_t and a_t. The adaptations of decisions are determined from period 2 to period 6, where the number of the period is *t= 2..6.*

(i) There is an adaptation to the mean price in the previous period, $\overline{p}_{t-1}$, if

(74a) $$\left|p_{t-1}-\overline{p}_{t-1}\right|>\left|p_t-\overline{p}_{t-1}\right|$$

and there is an adaptation to the mean advertising in the previous period, $\overline{a}_{t-1}$, if

(74b) $$\left|a_{t-1}-\overline{a}_{t-1}\right|>\left|a_t-\overline{a}_{t-1}\right|$$

(ii) There is an adaptation to the decision on price of the leader in the previous period, p_{t-1}^{leader}, if

(75a) $$\left|p_{t-1}-p_{t-1}^{leader}\right|>\left|p_t-p_{t-1}^{leader}\right|$$

and there is an adaptation to the decision on advertising of the leader in the previous period, a_{t-1}^{leader}, if

$$(75b)\qquad \left|a_{t-1} - a_{t-1}^{leader}\right| > \left|a_t - a_{t-1}^{leader}\right|$$

We first determine the adaptations according to the definitions above, from period 2 to period 6, for all firms in the regular game sessions. For adaptation to the leader we exclude the decisions of the firm that was the leader in the previous period. We then sum the number of adaptations separately, on price and advertising, for the firms in each of the seven categories.

Since the number of firms varies in the seven categories, we divide the sum of adaptations of each category by the number of firms in the category (the number of leaders is excluded when calculating the adaptations to the leader) and thereafter we also divide by the number of periods when adaptation is possible, which is five, to obtain a percentage of adaptations to a certain decision.

Next we see a table with the percentages, rounded off to integers, of the adaptations.

Adaptation in percent to	*L3*	*L5*	*L6*	*M7*	*L8*	*M8*	*C9*	*All*
(i) Mean price	36	34	32	38	44	40	36	37
(i) Mean advertising	31	40	33	32	37	28	30	33
(ii) Leader price	27	28	29	35	36	30	37	31
(ii) Leader advertising	27	43	29	30	36	26	39	32

Table 10.2. Percentage of adaptations of decisions to mean or to leader.

In the table above we see that the firms are adaptive in the range 27%–44%. An adaptation level of 40% shows that each firm adapted its decisions in two out of the five periods. Randomized decisions would be adaptive with the probability of 50%. Thus the adaptation is less than randomized decisions. In the table above we also notice that the firms adapted their decisions somewhat more to the mean of decisions than to the decisions of the leaders. We conclude that:

- The firms did not use the available information to adapt their decisions to the mean decisions or to the decisions of the leader in the previous period.

10.4 Relations between decisions in period 1 and decisions stated after the game has ended

As mentioned earlier, the participants played the game for several hours. During this time the participants received information about the game, discussed a number of decision alternatives and received information about the results of their own decisions and information, although sometimes to a lesser extent, about the decisions and the results of the other firms.

Above we have studied the firms' adaptiveness to information from one period to the next. Here we shall try to study the "total adaptiveness" from the start of the game to the very end of the game. We shall do this by making comparisons between the decisions made by the firms in period 1 and the decisions that the participants stated, after the game had ended, that they would make if they were to play the game again. In the years 1997–1999, the participants were asked the following two questions, Q9 and Q10, after they played the game[2]:

Q9 – Q10	If you were to participate once more, what price and advertising would you decide in period 1?

The following table shows median decisions of firms in period 1 and the median decisions on price and on advertising stated by the participants after the game for the four categories of firms: M7, L8, M8 and C9.

Decisions in period 1 and after the game	*M7*	*L8*	*M8*	*C9*
Prices – period 1	34	30	35	40
Price – after the game	35	32	34	36
Advertising – period 1	130	110	116	200
Advertising – after the game	130	120	120	200

Table 10.3. Medians of decisions in period 1 and of stated decisions after the game.

[2] The question was not completely hypothetical for all participants. Participants in 15 of the 45 firms in both 1997 and 1998 played the game more than once. The decisions made by participants who played the game for a second time are shown in Chapter 11.

We see in the table above that the medians of decisions in period 1, on price and advertising, are quite similar to the stated decisions after the game for firms of the same category. We also see that median decisions of firms of C9 are higher both in period 1 and after the game than those of the other firms of M7, L8 and M8. A statistical test (Kruskal-Wallis test) shows that these differences are significant. Thus participants in firms, regardless of amounts of available information, stated decisions after they had played the game that were similar to the decisions of their firms in period 1.

We shall next use the Pearson correlation test between the decisions the firms made in period 1 and the answers the participants stated in the questionnaire, for the four categories of firms and also for all firms.

Stated price and advertising after the game (Q9 and Q10)	*M7* N=227	*L8* N=163	*M8* N=102	*C9* N=204	*All* N=696
Price – period 1	.39**	.43**	.49**	.41**	.48**
Advertising – period 1	.29**	.49**	.70**	.33**	.50**

Table 10.4. Pearson correlation coefficients for decisions in period 1 and decisions stated in a questionnaire after the game. The symbols ** show significance of $p \leq 0.01$.

In the table above we see that there are significant relations between the decisions the firms made in period 1 and decisions stated after the game had ended, for all four categories of firms.

It should be emphasized that there were usually more than 2 hours between the time when firms made their decisions in period 1 and when the participants answered the questionnaire at the debriefing after the game had ended. Still, there are significant relations between the decisions the firms made in period 1 and the decisions stated by the participants after the game had ended.

A statistical test with only firms not performing so well, i.e. firms in third to fifth places in each game session based on equity at the end of period 6, show similar relations between decisions the firms made in period 1 and the decisions stated by the participants after the game had ended[3]. This indicates that participants also in these firms would make similar decisions although they had been less successful.

[3] The statistical test, with N=407, show price (.45**) and advertising (.58**).

In section 10.1, we saw that there was dispersion among decisions of the firms during the periods of the game. The dispersion decreased somewhat for prices during the game. We might therefore expect that the decisions of the firms in period 1 would be more dispersed than the decisions the participants stated after the game. We shall use the standard deviation to compare the dispersion among the decisions in period 1 and the decisions stated after the game.

Period 1 and after the game	*M7*	*L8*	*M8*	*C9*
Prices – period 1	8	6	7	8
Price – after the game	5	5	7	7
Advertising – period 1	51	41	68	79
Advertising – after the game	52	38	46	89

Table 10.5. Standard deviations of decisions in period 1 and of stated decisions after the game.

In the table above, we notice that the standard deviations of prices stated after the game was the same or somewhat less, for all four categories of firms, than in period 1. Similarly, the standard deviations of advertising stated after the game was about the same or somewhat less than in period 1 for firms of M7, L8 and M8. However, we notice that the standard deviations of advertising increased for firms of C9.

To conclude, the three measurements of medians, correlations and standard deviations of decisions together show that:

- The participants used the available information to keep the decisions of their firms in period 1 when they stated their decisions after they had played the game, also when they had been less successful.

We can call this *commitment to earlier decisions*, which has been discussed in the literature by for example March (1994).

10.5 Equity at the end of period 6

In section 9.2, we studied the median equities at the end of period 6 for firms with different information. We found, on the one hand, that asymmetric information could explain the differences in equity at the end of period 6 between the firms with less and more information (L8 and M8). We did not, on the other hand, find any significant differences in these equities between firms with symmetric information i.e. firms of L3–L6, M7 and C9. Hence there is a reason to study the equities of these firms more closely.

We first separate the firms with symmetric information into three different information types: less (L3–L6), more (M7) and complete (C9), and we divide each type of firms into quartiles based on equity at the end of period 6.

Equities – quartiles	*L3–L6*	*M7*	*C9*	*All*
25%	246	295	237	287
50% (Median)	400	438	422	406
75%	554	600	671	592

Table 10.6. Quartiles of equities at the end of period 6 for firms with less (L3–L6), more (M7) and complete information (C9).

In the table above we see that 25% of the 205 firms had equity less than or equal to 287. This is only 87 more than the equity of 200 that all firms had at the start of period 1. We also notice that 25% of all firms had equity more than or equal to 592. Thus somewhat less than 25% of the firms had tripled their equity by the end of period 6.

As mentioned earlier, the median equities at the end of period 6 for firms with the three types of information were quite similar, being in the range 400–438. In the table above we can see that the quartiles are also quite similar between the firms of the three different types.

However, we should notice that the biggest difference between the values of the 25% quartile and the 75% quartile was for firms with complete information, C9, where the difference was 671–237 = 434. Thus, the information about when the game would end (C4.2) and the demand table (C6.2) increased the dispersion in equity at the end of period 6.

Since there was considerable dispersion among the firms with each of the three types of information we conclude that:

- Firms with the same amount of available information had different equities.

In the table above we saw big spreads between the quartiles of equity at the end of period 6. We shall see if we can establish what determines this dispersion. A possible explanation could be the equity at the end of period 1.

We can use the following figure to illustrate the equities at the end of period 1 (on the y-axis) and at the end of period 6 (on the x-axis) for 205 firms with symmetric information. We exclude 11 of these firms, since they had negative equity either at the end of period 1 or at the end of period 6, and also one firm that had 1035 in equity at the end of period 6. We include a so-called *trendline* in the figure below. This is calculated with linear regression analysis between equity at the end of period 1 and equity at the end of period 6.

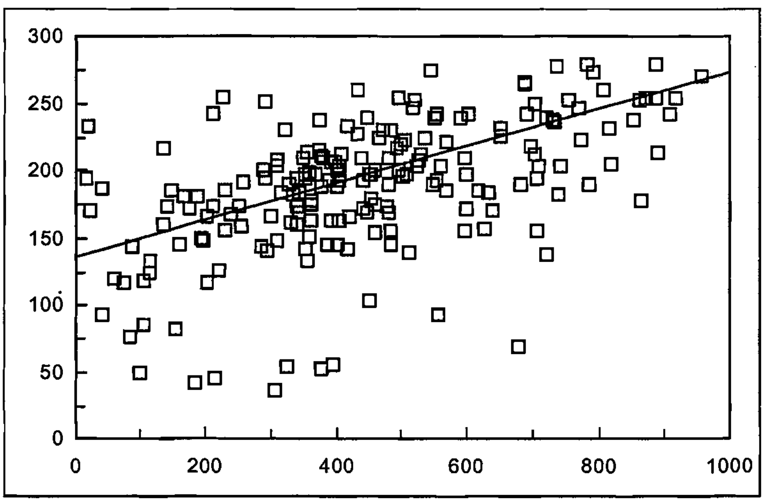

Figure 10.5. Median equity at the end of period 1 (on y-axis) and at the end of period 6 (on x-axis) for firms of L3–L6, M7 and C9.

In the figure above we notice big differences in the equities of the firms at the end of period 1 (y-axis) and even bigger differences in equities in period 6 (x-axis). We also notice that many of the equities in period 1 and in period 6 are close to the trendline.

We use the Pearson correlation test to establish the relations between the equity at the end of period 1 and at the end of period 6.

Equities	*L3–L6*	*M7*	*C9*	*All*
Periods 1 and 6	.54**	.88**	.60**	.47**

Table 10.7. Pearson correlation coefficients of equity at the end of period 1 and at the end of period 6. The symbol ** shows significance of $p \leq 0.01$.

In the table above we notice significant relations between the equity of the firms in periods 1 and 6 for each of the three types of firms and also for all 205 firms. This means that relatively high equities in period 1 were related to relatively high equities in period 6. This relation can partly be explained by the fact that the dispersion among the decisions of the firms was bigger than the adjustment of decisions from one period to the next.

We conclude that:

- The firms kept their relative levels of equity from the end of period 1 to the end of period 6.

11 Decision making in groups

In the literature review in section 3.3, we noted that descriptions existed about *what* decisions firms make in business life, but there was a lack of descriptions about *how* firms make these decisions. From this perspective, I shall present some observations in this chapter on how the decisions were made in the groups in the business game. This chapter can be seen as more of a start to describing the decision making than a complete description of the decision making in the game. We pose the following question:

- How were the decisions made in the groups?

In the literature review, two procedures were mentioned as descriptions for rationality in decision making. We shall use one of them, the *logic of consequence*, to describe the decision making in the groups. It should be mentioned here that although the logic of consequence has been seen as a valid description of decision making over the years, doubts have been put forward about the usefulness of the logic of consequence as a general description of decision making (March, 1994). Therefore we are also concerned about the usefulness of the logic of consequence as a description of decision making in the firms when playing the game.

As described in the review on page 41, the logic of consequence consists of four components: preferences, alternatives, expectations and decision rules. Here we understand *preferences* as being the objective when playing the game, *alternatives* as the possible values of the decision variables, *expectations* as the expectations of the adjustments of the decisions by the competitors and *decision rules* as how the decisions were reached and also who made the decisions in the groups (here called the *decision process*).

In section 11.1, we compare the motivation that the participants stated in a questionnaire with the objective when playing the game. In section 11.2, we compare the decision alternatives discussed by the groups with their actual decisions. In section 11.3, we compare the expectations of the groups with their actual decisions. In section 11.4, we shall study what the participants stated in a questionnaire about how the decisions were reached in groups.

In this chapter, we shall study the answers to three questions in the questionnaires. We shall also study the two methods that I designed to capture which decision alternatives were discussed and what expectations were held about the decisions of the other firms. The description of the decision making in the groups also relies on observations I have made when the groups made decisions during the game sessions. For the sections on alternatives (section 11.2) and on expectations (section 11.3), I should say that the methods are perhaps of greater interest than the conclusions, since the amount of data collected is limited.

11.1 Motivation and objective

In this section we shall study the stated motivation for realizing the objective when playing the game. The motivation can be seen as *how great* the desire is to accomplish an objective. The objective in the business game was stated in the rules as: "A suitable goal might be to maximize equity at end of the game". As before, we shall consider period 6 to be the end of the game.

First, we shall study the answers to two questions in the questionnaire where participants stated their motivation during the game, Q4, and also their change of motivation during the game, Q5. Then we shall compare the stated motivation and change of motivation with the equity at the end of period 6.

We start with question Q4.

Q4	Try to estimate your motivation when participating in the game
	Answers to the question were on a scale from 0–100, where 0 was low motivation, 50 medium motivation and 100 high motivation

The participants answered question Q4 as shown in the following table.

Stated motivation	*M7*	*L8*	*M8*	*C9*	*All*
Median	74	72	70	70	72
Standard deviation	16	18	17	18	17

Table 11.1. Stated motivation.

In the table above we see that for all four categories M7, L8, M8 and C9, the median motivation was in the range 70–74 and the standard deviation was in the range 16–18. Hence, the participants stated that they were fairly motivated when making decisions in the game. We shall discuss the motivation when playing the game further in section 13.3, as we discuss the rewards for playing the game.

We shall next study the answers to question Q5.

Q5	Did the motivation in your firm change during the game?
	(a) motivation increased (b) unchanged (c) motivation decreased

The stated change of motivation, question Q5, is shown in the following figure:

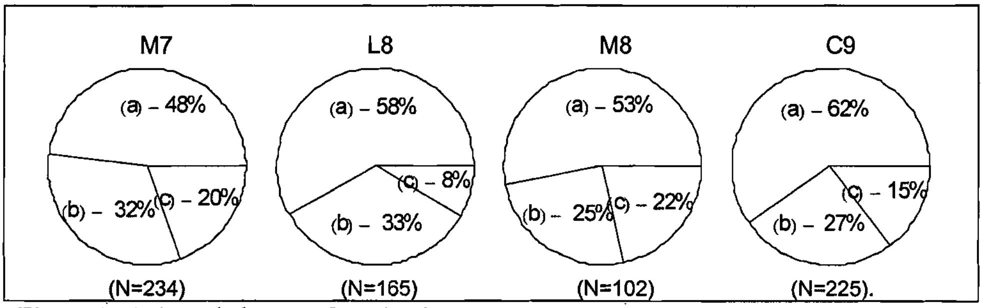

Figure 11.1. Stated change of motivation.

In the figure above we notice that 48%–62% of the participants stated alternative (a), i.e. the motivation in their firms increased during the game. 25%–32% of the participants stated alternative (b), i.e. that the motivation was unchanged in their firms, while 15%–22% of the participants stated alternative (c), i.e. the motivation decreased. Hence, most participants stated increased or unchanged motivation in their firms during the game.

We can next use the Pearson correlation test to compare the stated motivation, Q4, and the change of motivation, Q5, to the equity at the end of period 6. We then need to allocate numbers to the three alternatives in question Q5. For the sake of simplicity, we give them the numbers from 1 to 3. Alternative (a), increased motivation, is given number 3, alternative (b), unchanged motivation, is given number 2 and alternative (c), decreased motivation, is given number 1.

The following table shows the Pearson correlation test between the answers to questions Q4 and Q5 and equity at the end of period 6.

Equity at the end of period 6	*M7*	*L8*	*M8*	*C9*	*All*
Motivation (Q4)	.18**	.22**	.30**	.18**	.20**
Change of motivation (Q5)	.38**	.22**	.53**	.28**	.33**

Table 11.2. Pearson correlation coefficients for stated motivation, C4, stated change of motivation, C5, and equity at the end of period 6.

In the table above we see that there were significant relations between both motivation, Q4, and change of motivation, Q5, and the equity at the end of period 6. The correlation coefficients are low, which indicates relatively weak relations, but we see that the relations apply for all four categories of firms, M7, L8, M8 and C9, i.e. in all three years 1997–1999.

There might be two possible explanations for the relation between the high motivation stated in answer to question Q4, and high equity at the end of period 6.

(i) Participants who stated a high motivation were highly motivated during the game, and due to their high motivation they made their decisions so that their firms got a high equity at the end of period 6.

(ii) Participants in firms that had high equity at the end of period 6 stated a high motivation since they played in firms that were better able to achieve the objective for playing the game.

To determine which of these two possible explanations seems most likely, we consider the answers to question Q5, i.e. change of motivation in the firms during the game. Since there are positive correlation coefficients between answers to question Q5, stated change of motivation and the equity at the end of period 6, implying that participants in firms with high equity towards the end of the game were increasingly motivated, we infer that (ii) the participants in firms that had high equity at the end of period 6 stated a high motivation. It should, however, be pointed out here that the participants answered the questions in the questionnaires before they were presented with the equity at the end of the game. In section 13.3, we shall discuss rewards with respect to motivation when playing the game.

11.2 Decision alternatives discussed

When I observed the decision making by the groups during the game sessions in 1996, I became interested in the decision alternatives discussed. I heard that a number of alternatives were discussed by each group in each period, but it was difficult see if there was any relation between the decisions discussed and the decision actually reached. Therefore, I in 1997 designed a method to capture the decision alternatives discussed in the groups. The question in which I am interested is:

- Was there a relation between the decision alternatives discussed and the decisions actually made?

I shall first describe the method I used to capture the decision alternatives discussed by the groups, and I shall then compare these alternatives with the decisions actually made by the groups. Below I shall refer to the "decision alternatives discussed" simply as the "alternatives".

The method for capturing the alternatives in the groups was used in the three semifinals in 1997. Hence, the study on alternatives is limited to the 15 firms that were invited to play the game for a second time at that time. Since the groups were seated around tables when making their decisions, the decision alternatives discussed in each group could easily be heard. I therefore used observers to manually capture the alternatives.

The participants were told at the briefing before the semifinals that there would be observers seated at their tables, but they were not told what the observers captured. One observation I made at the time was that the participants, to my surprise, did not seem to take much notice of the observers when making their decisions.

The observers used forms as shown in Appendix N, where for each alternative in each period they noted:

- The order of the numbers of the alternatives, i.e. the first alternative was given number 1, the second alternative was given number 2 and so forth.
- The level of the alternative (see the table below for the intervals)

The levels and intervals for price and advertising discussed are shown in the following table:

Level	Price	Advertising	Code
High	> 35	> 130	3
Medium	30–35	70–130	2
Low	< 30	< 70	1

Table 11.3. Decision alternatives discussed, levels, intervals and codes.

In the first column in this table we see that there were three levels each for price and advertising: high, medium and low alternatives. The second column contains the intervals for the levels on prices and in the third column the intervals for the levels on advertising. The fourth column contains the codes from 1 to 3 that we shall use below when comparing alternatives discussed and decisions actually made. A higher code represents a higher alternative.

Next we shall study four figures showing the decision alternatives discussed, for price and advertising, and the median decisions of the 15 firms in the semifinals in 1997 in each of the six periods[1].

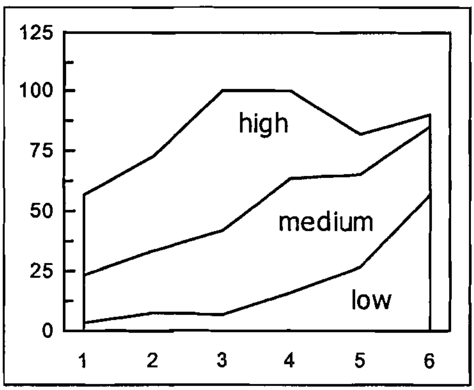

Figure 11.2. Total number of alternatives discussed in three price levels in the semifinals in 1997.

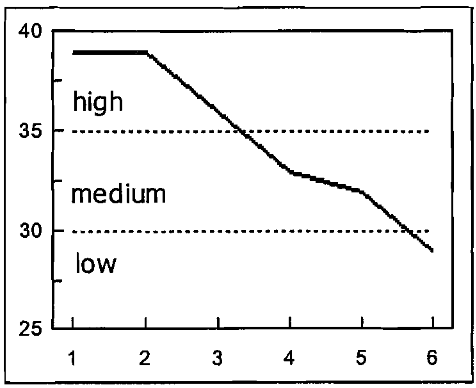

Figure 11.3. Median price in the semifinals in 1997.

[1] The procedures used in the semifinals in 1997 are described in section 6.2.2. The median decisions of the firms in semifinals in 1997 (S7) are presented in Appendix I.

In Figure 11.2 above we see that the alternatives discussed for prices were mostly in the "high level" in periods 1–3. The alternatives for prices were mostly in the "medium level" in periods 4–5, and alternatives for prices were mostly in the "low level" in period 6.

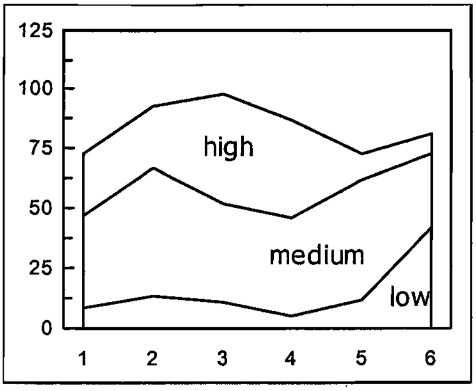

Figure 11.4. Total number of alternatives discussed in three advertising levels in the semifinals in 1997.

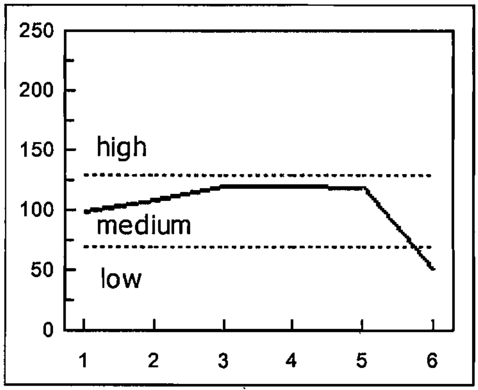

Figure 11.5. Median advertising in the semifinals in 1997.

In Figure 11.4 we see that the alternatives for advertising are mostly in the "medium level". In periods 3–4 there are several alternatives in the "high level" and in period 6 there are several alternatives in the "low level".

We shall next study the question posed above, i.e. if there was a relation between the decision alternatives discussed and the decisions actually made. We use the codes from Table 11.2 to calculate the mean level of alternatives discussed by each firm in periods 1–6, i.e. for each firm, the codes of all alternatives discussed are summarized and divided by the total number of alternatives. We also calculate the mean decisions of each firm in periods 1–6. We use the Pearson correlation test to compare, on the one hand, the mean level of alternatives discussed on price and advertising, and on the other hand, the mean price and the mean advertising in periods 1–6 for each firm.

We then find the following significant relations:

- The mean decisions on price were significantly related to the mean level of alternatives for prices (.82**).
- The mean decisions on advertising were significantly related to the mean level of alternatives for advertising (.66**).

Hence, this statistical test with aggregated data shows that in periods 1–6 the aggregates of alternatives discussed by each firm were related to the decisions actually made by each firm.

We shall also study the number of alternatives discussed. In Figures 11.2 and 11.4 we see that the total number of alternatives discussed by the 15 firms in each period was in the range 60–100. The lower number of alternatives in period 1 can possibly be explained by the fact that participants in the firms could have met and discussed their decisions in period 1 before the semifinals started, as they had already played the game once and knew that they would play the game together in the same firm in the semifinal. The number of alternatives is highest in periods 3 and 4. On average the 15 firms discussed 5–7 decision alternatives for price and the same number of alternatives on advertising in each period.

The number of alternatives discussed differed between periods, but also between the firms. The range of number of alternatives discussed per period by the firms was 1–14 for price and 2–18 for advertising. This means for example that one of the firms discussed only one alternative for price in one of the periods, while one of the firms discussed as many as 18 alternatives for advertising in one of the periods.

We shall use a Pearson statistical test to study the relations between the number of alternatives discussed by a firm in one of the periods and the total number of alternatives discussed by the firm in the other five periods.

Alternatives	*Period 1*	*Period 2*	*Period 3*	*Period 4*	*Period 5*	*Period 6*
Other periods	.34	.54	.53**	.86**	.61*	.66**

Table 11.4. Pearson correlation between the number of decision alternatives in each of the six periods and the number of decision alternatives in the other five periods.

In the table above we see that the number of alternatives for each of the firms in one period was related to the number of alternatives in “Other periods” for the same firm. Hence, a firm that discussed many alternatives most likely did so in all periods and a firm that discussed few alternatives most likely did so in all periods.

For each firm we shall also compare the total number of discussed alternatives for price to the total number for advertising, using the Pearson correlation test. It shows that the total number of alternatives for price was related to the total number for advertising (correlation coefficient r=.55*). Hence, firms that discussed many decision alternatives for prices also discussed many alternatives for advertising.

The conclusion from studying the decision alternatives above is:

- The decision alternatives that the firms discussed were related to the decisions the firms actually made. Furthermore, the firms seemed to develop firm-specific ways to discuss decision alternatives, as some firms discussed many alternatives and other firms discussed few alternatives in all periods of the game.

11.3 Expectations about competitors' decisions

When I observed the decision making in the groups, I noticed that the groups discussed the decisions of their competitors. In section 9.1.2, on page 178, we saw that almost all participants (97%–100%) stated that they discussed the decisions of their competitors. It was, however, difficult to see if there was any relation between the expectations about the decisions of the other firms and the decisions that the firms actually made. Therefore, I designed a method to capture the expectations in the groups in order to answer the following question:

- Was there a relation between the expectations and the decisions actually made by the groups?

I shall first describe the method used to capture the expectations and then compare the expectations with the decisions actually made.

The method for capturing the expectations in the groups was used in the three semifinals in 1998. Hence, the study on expectations is limited to the 15 firms that were invited to play the game for a second time at that time. The procedures used in the semifinals in 1998 were described in section 6.2.4. When playing the game, the firms had a demand table (C6.2) as part of their information. In order to use this table to determine the demand for their products, the firms had to form expectations of the other firms' prices and advertising.

The time for decision making was limited to 15 minutes. The firms were therefore asked to fill in their expectations of the adjustments of the decisions of the other firms (i.e. not the actual decisions of the other firms). Below we shall refer to these adjustments as the "expectations".

The expectations were captured in forms, see Appendix O, that the firms filled in themselves. The expectations could be either of the three categories: increased, unchanged or decreased decisions. Unchanged decisions were defined as an absolute adjustment of a decision from one period to the next that was less than 1.

Each of the 15 firms was asked to fill in their expectations in periods 2–5 at the same time as they filled in the decision forms. The reason for not capturing expectations in period 6 was that we wanted the firms to use their limited time for decision making to concentrate on their own decisions in period 6, which was the last period in the semifinals in 1998 (C4.2). Hence, the expectations were captured four times, that is, for the first time in period 2, regarding adjustments from period 1 to period 2, and the fourth and the last time in period 5, regarding adjustments from period 4 to period 5.

The incentive for the firms to form correct expectations of the competitors was that the firms with the highest number of correct expectations was awarded a place in the final in 1998[2].

As a background, we shall study the median decisions of the firms that played in the semifinals in 1998[3]. The lines represent the median decisions of the 15 firms in the semifinals in 1998.

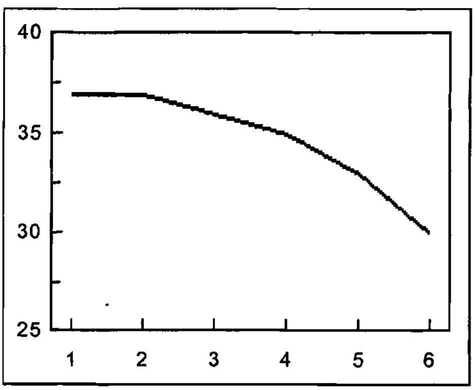

Figure 11.6. Median price in the semifinals in 1998.

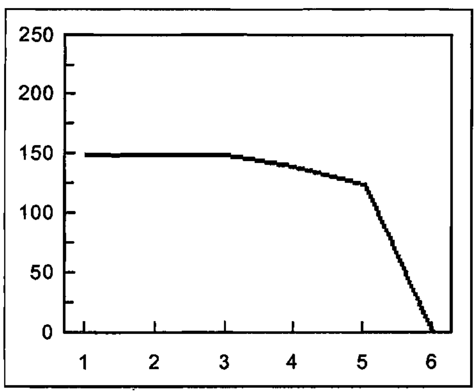

Figure 11.7. Median advertising in the semifinals in 1998.

[2] After the game sessions had ended, the expectations that the firms had formed about adjustments of decisions were compared with the firms' actual adjustments of decisions.

[3] The median decisions of the firms in the semifinals in 1998 (S8) are presented in Appendix I.

In the figures above we notice that the median price was unchanged (an absolute difference less than 1) from period 1 to period 2 and that the median price decreased in the other periods. The median advertising was unchanged in periods 2 and 3 and it decreased from period 4 to period 5.

Next we can study Figure 11.8, showing the percentage of the expectations about adjustments and also the actual adjustments of all 15 firms, where (i) is increased, (u) is unchanged and (d) is decreased.

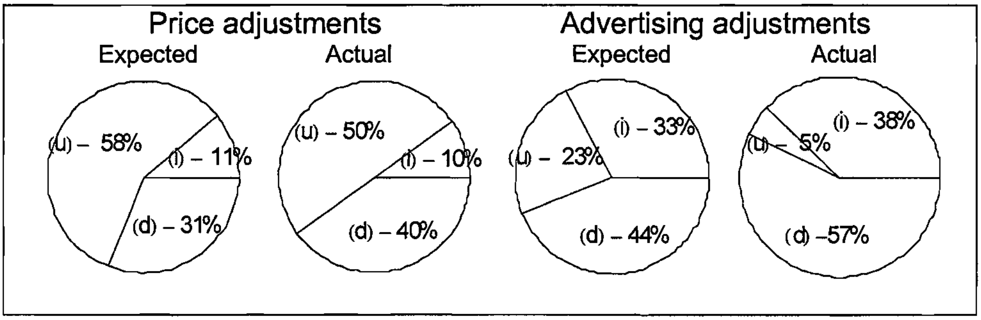

Figure 11.8. Percentage of expected (N=60) and actual (N=15) adjustments of decisions, on price and advertising, in periods 2–5 in the semifinals in 1998.

In the circle to the left in the figure above, we see that a majority of 58% expected prices to remain unchanged (u), 31% expected prices to decrease (d) and 11% expected prices to increase (i). The corresponding actual adjustments of prices, presented in the second circle from the left, show resemblance to these expectations, as 50% of the actual prices were unchanged (u), 40% were decreased (d) and 10% were increased. However, compared to the median decisions on price in periods 1–5 in Figure 11.6 above, the firms did not adjust their decisions as much as the median decisions might give the impression of.

In the third circle from the left above we see that 44% expected the advertising to decrease (d), 33% expected advertising to increase (i) and 23% expected advertising to be unchanged (u). These expectations differ from the corresponding actual adjustments of advertising, presented in the fourth circle from the left above, since 57% of the actual decisions on advertising were decreased (d), 38% increased (i) and only 5% were left unchanged (u). The firms had expectations of more unchanged advertising than the firms actually decided. Compared to the median decisions on advertising in periods 1–5 in Figure 11.7 above, the firms adjusted their decisions more than the medians might give the impression of. We notice that more than 1/3 of the actual adjustments of advertising were in the category increased (i).

We return to the question posed above, i.e. whether there was a relation between the expectations and the decisions actually made. We use codes between 1–3 for the three categories of adjustment, where expectations of increased (i) are given the code "3", of unchanged decisions (u) the code "2", and of decreased (d) is given the code "1". For each firm, the codes of all expectations, for price and advertising separately, are summarized and divided by the number of periods, which was 4.

In periods 2–5 we compare on the one hand the mean level of the expectations about decisions, price and advertising, and on the other hand the mean of each firm's decisions. The Pearson correlation test shows the following significant relations:

- The mean level of expectations about adjustment on price was related to the mean decisions on price (.66**)
- The mean level of expectations about adjustment on advertising was related to the mean decisions on advertising (.52*)

Hence, this statistical test with aggregated data shows that the expectations of each firm, in periods 2–5, were related to the decisions actually made by each firm.

On page 178, we saw that 41%–50% of the participants stated that they had correct expectations of the decisions of their competitors. For this reason, we shall also study the correctness of expectations of each separate firm.

Each firm had to form expectations about the price and advertising of the four other firms in the same game session. In order to be counted as correct, the expectation needed to be correct for each of the four firms. Since there were 15 firms, a total of 60 expectations were formed each on price and on advertising in each period. The following table shows the number of correct expectations.

Correct expectations	*Period 2*	*Period 3*	*Period 4*	*Period 5*
Adjustments of price	27	26	32	37
Adjustments of advertising	32	31	34	21

Table 11.5. Number of correct expectations of the competitors' adjustments of decisions in the semifinals in 1998.

The table above shows that the number of correct expectations lies in the range 21–37. Since there were three categories of adjustments, increased, unchanged and decreased adjustment, randomized expectations would give 20 correct expectations about price and advertising in each period. Thus the number of correct expectations was higher than a randomized decision would give in all periods. If the number of correct expectations is 28 or more, they are (according to a Chi-square test) significantly different from randomized decisions. Thus, in two of the four periods there were significantly correct expectations about price and in three of the four periods there were significantly correct expectations about advertising. The conclusion from above is:

- The decisions that the firms made were related to the expectations they had about the adjustments of decisions of the other firms. Furthermore, the firms were fairly correct in their expectations of the adjustments of decisions of the other firms.

11.4 The decision process

For an outside observer, it was not obvious how the decisions were made and who in the groups made the decisions. Therefore a question, Q3, was posed in the questionnaire concerning what the participants stated regarding the decision making process in their groups:

Q3	Which option is the best description for how your firm made decisions?
(a)	discussions; decisions approved by all members
(b)	decisions of majority; like voting
(c)	someone or some of the members made the decisions

The answers are presented in the following figure:

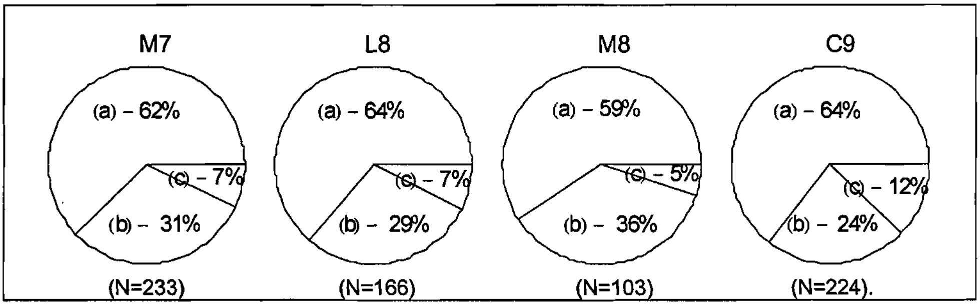

Figure 11.9. Stated characteristics of decision making in groups.

We see above that a majority, between 59%–64%, stated that the decision making was of type (a) – "discussions; decisions approved by all members". Between 24%–36% of the participants stated that there were such different opinions among the participants that the decisions were made according to alternative (b) – "decisions of majority; like voting". Only 5%–12% of the participants stated alternative (c) – "someone or some of the members made the decisions". Hence, it seems that the decisions were made following with discussions in the groups.

We shall further study the groups where one or more participants stated that decision making was not mainly in the form of discussions, i.e. the case of alternative (b) or (c). We count the number of groups where at least one of the participants stated alternative (b) or (c). In 65 of the 125 groups, at least one of the participants stated alternative (b). In 24 of 125 groups at least one of the participants stated alternative (c). Hence, in 71% of the groups (89 of the 125 groups) at least one of the participants stated that the decisions were not approved by all members of the group.

We shall not exaggerate the differences in the opinions stated by the participants in the groups. Most of participants stated that the decision making was approved by all members. However, since in a majority of the groups at least one of the participants stated the other alternatives for decision making, we have some indications of different opinions when the decisions were made in the groups.

We shall use some of the conclusions from Chapter 10 to give a possible explanation for these differences. In section 10.1 we noticed that there was dispersion among the decisions of the firms with the same information condition. Hence, there were different opinions among the firms about what decisions to make. It therefore seems likely that there could also have been different opinions among the participants in the same groups about what decisions to make. In this case the participants could have made their decisions according to alternative (b) – "decisions of majority; like voting" or alternative (c) – "someone or some of the members made the decisions". It should, however, be mentioned that alternative (c) could also have been stated when some participants chose not to participate in the decision making in the groups, e.g. due to lack of motivation.

In section 10.4, however, we noticed that the participants in the firms largely retained their decisions from period 1, as they stated their decisions in the questionnaire if they were to play the game again. Together with my observations of the decision making[4], I put forward the hypothesis that the differences in opinions within the groups about what decisions to make, at least to some extent, concerned minor differences in decisions.

To conclude, we can describe the decision making in the groups as follows:

- The decisions were made mostly following discussions in the groups. However, there were some indications of differences in opinions within the groups about what decisions to make. These differences in opinions may have been about minor adjustments of decisions.

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[4] This is also in line with the observations made by I. Ståhl.

12 Effect of group compositions on decisions

In the literature review, in section 3.4, we discussed the possibility that the compositions of groups can have an effect on decisions. In this chapter we shall study the decisions made by different compositions of groups, acting as firms, in the business game. In section 12.1, we study the effect of different compositions of groups of students on decisions made during the game and on equity at the end of period 6. In section 12.2, we compare decisions made by students to decisions made by professionals.

12.1 Decisions and equities of groups of students with different compositions

In section 10.1, we noticed that there was dispersion among both the decisions during the game and the equities at the end of period 6, even when firms played the game with the same information conditions. Based on some observations in the literature, one might try to explain this dispersion by differences in the compositions of the groups. In this section we shall see if it is possible to relate certain decisions to certain compositions of groups. We pose the question:

- Are better decisions made by certain compositions of groups?

The criterion for the data to be used for our study on group compositions was that the data was generally and readily available. Our study on group composition can then be compared with other studies.

We study game sessions from 1996–1999. Data was available on the group sizes and on the gender of students in the groups, in the lists that the students filled in to confirm their participation in the game. We searched for some data on the capabilities of the students. With help of the central student office, we obtained the grades on which the students were admitted to the school and the age of the students[1]. Furthermore, we could also obtain data on the scores in the exam for the course of which the game formed a part at the school.

[1] Many thanks to Kerstin Johansson and Rita Dubbelmann Berg for their assistance.

The following table shows the variations of groups and participants in the game sessions in 1996–1999. As in Chapters 7–10, we exclude 3 of the 36 regular game sessions in these four years. Furthermore, to avoid extreme groups, we exclude the only group with 2 participants and the only two groups with 9 participants. Thus in this section we study a total of 861 participants in 162 groups and 33 game sessions for the four years.

Characteristics	**Range**	**Median**	**St. dev.**
Sizes of groups	3–8	6	1.3
Number of females	0–5	2	1.2
Age	18–40	21	2.3
Grade	2.7–5.1	4.9	0.4
Score in exam	37–200	136	31

Table 12.1. Variation of groups and participants playing the business game.

We shall briefly study the characteristics of the students to see if there are some relations between these characteristics, which could be of interest when we study the relations between the group compositions and the decisions and equities of the groups, acting as firms in the game.

A Pearson correlation test shows for each of the four years that there are significant differences between male and female students. Female students were significantly younger, they had significantly higher grades and they had significantly lower scores in the exam than males[2]. However, there is also a weak positive relation between grade and score in exam. We shall return to these relations when we discuss the conclusions on group compositions and decisions in the game sessions.

On the basis of a review literature on group composition, Tindale (1989), Hill (1982), Henry (1995) and Davis & Harlees (1996), we hypothesize that larger groups, with more decision making capacity, and groups with higher heterogeneity of gender, in this case groups with more females, make better decisions. Also on the basis of the literature, we hypothesize that the *best member* in each group has a significantly positive effect on the decisions made by the group.

[2] The difference in age and grade can be partly explained by the fact that some male students had done military service before admission to the school.

If the hypothesis stated above is valid for the best member, the decisions made by a group are related to the best member in the group. From this, it may follow that the decisions made by a group would be different when including or excluding the best member from a group.

We define the *best member* in a group as a participant who either is older or has higher grades or has higher scores in the exam than the other participants in the group. Thus, for each group we shall use the age of the oldest participant, the grade of the participant with the highest grade and the score of the participant with the highest score in exam, when we study the relations to the decisions and the equity of each firm. We pose the following five questions:

Question	**Hypothesis**
Are better decisions made by larger groups?	Groups with more participants have more capacity to use information, e.g. consider more decision alternatives at the same time.
Are better decisions made by groups with more females?	Groups with *more females*, i.e. heterogeneity of gender, will consider a greater variety of decision alternatives.
Are better decisions made by groups with one participant with a higher age?	Groups where the oldest participant is *older* than the oldest participants in other groups have more experience of decision making and of business life.
Are better decisions made by groups with one participant with higher grade?	Groups with a participant with a *higher grade* have a better decision making capacity.
Are better decisions made by groups with one participant with a higher score in the exam?	Groups with a participant with a *higher score* in the exam have a better decision making capacity.

Table 12.2. Questions and hypotheses on the effect of compositions of groups.

The table above relies on the assumption that the best member in each group has a significantly positive effect on the decisions made by the group.

It should be pointed out that probabilistically it is more likely that larger groups will have more females and also that they will have the oldest participant, a participant with a high grade and a participant a with high score in the exam. Thus there might be relations between the size of groups and the other characteristics of the groups. When we use the Pearson correlation test of all 162 groups to test these possible relations, we find the following two significant relations, with weak correlation coefficients: The number of participants and the number of females in the groups ($r = 0.23$) and the number of participants and the maximum age of a participant in a group ($r = 0.18$).

As a complement to the analysis with maximum values, we shall also study the relation between the median age, median grade and median score in exams of the participants in each group, and the decisions made by the groups and their equity at the end of period 6.

Thus, the composition of a group is characterized by: the number of participants in the group, the number of females in the group and also both the maximum and the median age, the grade and the exam score in the group. We shall study the relations between these characteristics and the following:

- The mean price in periods 1–6 of each firm
- The mean advertising in periods 1–6 of each firm
- The equity at the end of period 6 of each firm

Before we make the comparisons, I should mention that I thought there would (at least) be a significant relation between the score in the exam and equity at the end of period 6.

We first use the Pearson correlation test to compare two characteristics of group composition, on the one hand the size of groups and the number of females, and on the other hand the decisions and the equities of the 162 firms in *all* four years. We then find the following two weak significant negative correlations between:

- Number of participants in a group and mean of advertising ($r = -0.18$)
- Number of females in a group and mean of advertising ($r = -0.20$)

When we make the same comparisons for *each* of the four years, we do not obtain any significant relations in 1997 or in 1998. Furthermore, only one of the two relations is significant in each of the years 1996 and 1999. Thus the relations do not apply for every one of the four years.

We next use the Pearson correlation test to compare the three characteristics age, grade and score in the exam (with maximums and medians) on the one hand, and the decisions and the equity of the groups on the other hand for *all* four years. We then find two nearly significant weak negative correlations between:

- Best member with highest grade and mean of price ($r = -0.14$)
- Median score in exam and mean of price ($r = -0.14$)

When we then make the same comparisons for each of the four years, we do not obtain any significant relations in 1996 or in 1998. Furthermore, only one of the two relations is significant in each of the years 1997 and 1999. Thus the relations do not apply over the four years.

The significant relations obtained for all four years are considered as *spurious*, since they do not apply in each of the four years.

Two more points should be mentioned in this context. Female students had significantly lower scores in the exam than males, but as regards the playing of the business game, we did not find that groups with more females made different decisions or had either more or less equity than groups with fewer females.

In 1997, a significant relation was found through some further statistical tests. Due to the informal composition of the groups (D3.1), participants seated in the front row at the briefing had significantly less equity at the end of the game. However, this was not found in the other years and therefore it can also be considered as a spurious correlation.

To sum up, we have not found any significant relations between group composition and the mean of decisions made by the firms in the game sessions. Furthermore, we have not found any significant relations between group composition and the equity at the end of period 6. Hence we conclude that:

- No composition of a group was found to make better decisions.

This means that the groups could be either small or large, and that the other characteristics do not matter for the decisions or the equity when playing the game. The dispersion among decisions and equity could not be explained in the terms of the characteristics of groups studied above.

It should, however, be pointed out that the sample of participants was limited to students at the Stockholm School of Economics. These students had been selected for admission to the school on the basis of their grades or a general admission test. Consequently the sample of students was relatively homogeneous in these data and also, presumably, in their interest in business administration and economics.

12.2 A comparison between decisions made by students and by professionals

Another question regarding group composition is if students make similar decisions to professionals. This is a frequent question in the experimental literature, since students are often used as decision makers in experiments. The answer is of interest for relevance for the so-called *parallelism* (Smith, 1982) of the conclusions for this study, i.e. whether we can expect professionals to make similar decisions to students in a situation that is similar to the business game.

Since both students and professionals have played the game, we can compare the decisions made by students to the decisions made by professionals. A reasonable starting hypothesis is that decisions of professionals differ from the decisions of students, since professionals generally have more experience of markets. The question we pose in this section is:

- Do professionals make better decisions than students?

Professionals have played the game on several occasions. Here we shall limit our study to game sessions played by professionals in England, on two occasions, in 1994 and in 1996. The professionals played the game as part of two different conferences held by their corporations. The professionals had various kinds of positions in their corporations.

Six game sessions were played simultaneously on each of the two occasions. The games were on these occasions organized and led by Dr. John Handley and Karen Valverde, Cranfield School of Management together with Ståhl. The game sessions were played as competitions where the participants in the firm with the highest equity (B1.1) of the six game sessions on each occasion received small gifts (B2.1). The game was played with the original information conditions (C1.1, C2.1, C3.1, C4.1, C5.1 and C6.1), i.e. incomplete and symmetric information. The professionals (D1.2) were arranged into their groups, acting as firms, in a manner outside the control of the game leader (D2.1 and D3.1).

Due to many bankruptcies in 2 of the 12 game sessions played by the professionals, we exclude these 2 game sessions. The remaining 50 firms in 10 game sessions include 8 firms who went into bankruptcy. We make comparisons between the decisions made by these professionals and the decisions made by the 125 firms played by students at the Stockholm School of Economics in 1993 – 1996 (L3, L5 and L6)[3].

All game sessions were played with the same information condition (the original information condition). However, the game sessions played by professionals in 1996 were only played for five periods. Therefore, the comparisons between decisions and equities of students and professionals are limited to five periods of the game.

We study the two figures below with median prices and median advertising. The solid lines represent the median decisions of professionals (N=50) and the dotted lines represent the median of decisions of students (N=125).

[3] Earlier we excluded 2 of 27 game sessions played by students (firms of L3, L5 and L6). The proportion of firms that went into bankruptcy for professionals and for students is about the same (as 8 of the 50 firms played by professionals and 17 of the 125 firms played by students went into bankruptcy).

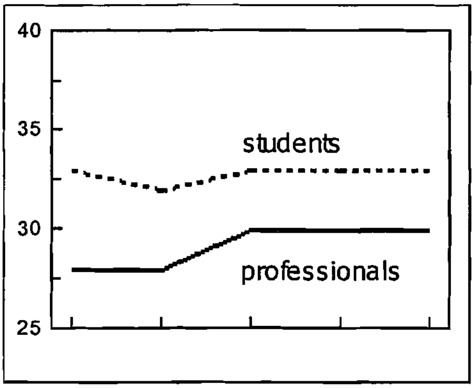

Figure 12.1. Median prices of firms played by professionals and by students (L3–L6).

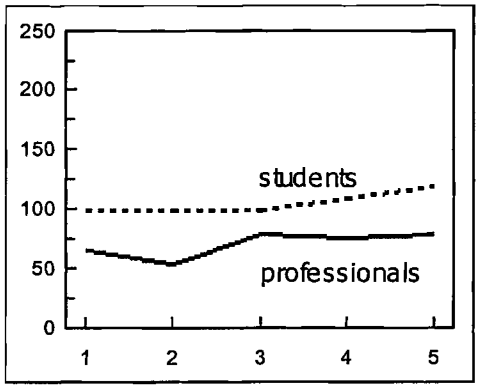

Figure 12.2. Median advertising of firms played by professionals and by students (L3–L6).

In the figure to the left above we see that the median decisions on prices by professionals are lower in all five periods than the medians of prices for students. In the figure to the right above we also see that the medians of advertising for professionals are lower in all five periods than the median advertising for students.

A statistical test, Table 4 in Appendix J, shows that the differences between professionals and students in decisions on price and on advertising are significant in all five periods (L3–L6). Despite evidence in conflict with differences in prices between professionals and the students in category L5, we have significant differences between decisions made by students and professionals[4].

Since the original reference point of demand stated in the rules of the game was a price of 25 and advertising of 50 (C1.1), a possible explanation for the decisions made by professionals, at least in the first periods of the game, is that they made decisions close to the reference point. The logic of appropriateness (March 1994), mentioned in section 3.3, may be used as a description for decisions made close to the reference point.

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[4] We can see in Appendix K in comparison (iii) that we find evidence which is in conflict with significantly different decisions on prices between professionals and one of the categories of firms played by students (L5).

We shall next determine who make better decisions, professionals or students. When we compare the median equities in period 5 we find that the median of students (L3–L6) is 352, while for professionals it is 515, which is a difference of 163 between the median equities. A statistical test shows that this difference is significant. Thus, professionals made better decisions than students with respect to the equity at the end of period 5. We therefore conclude that:

- Professionals made better decisions than students did, as they had higher equity at the end of period 5.

To give some perspective on the decisions made by professionals and students, we can calculate the hypothetical values of equities at the end of period 5, just as in section 9.2.2. If in the same game session, 3 firms make the median decisions of professionals and 2 firms make the median decisions of students (L3–L6), the firms with professionals would have an equity of 602 while the firms with the students would have as much as an equity of 862[5]. Hence, it is not inconceivable that students would make better decisions than professionals, if they were to compete in the same game session.

[5] If instead we calculate the hypothetical equities of two firms making the median decisions of professionals and three firms making the median decisions of students, the firms with the professionals would have an equity of 543 and the firms with the students would have an equity of 750.

13 Experimental limitations

The conclusions in this study rely on the decisions made by 330 firms in 66 game sessions, involving about 2000 participants who played the business game in 1993–1999[1]. In the three years 1993, 1995 and 1996, the same procedures were used. We denote this *replications over the years*. The procedures were altered in the years 1997–1999. In each of these three years, 9 game sessions were played with the same procedures. We denote this *replications in the same year*. Moreover, the questionnaires with question Q0 have been used for four years and the questions Q1–Q10 have been used for three years. Thus, the conclusions are based a considerable amount of data, where the playing of the game has been replicated a number of times.

The conclusions may, however, depend on some limitations when the business game was used as an experiment. On the basis of the listing of potential failures in experimental economics (Table 3.4 in section 3.5), we shall discuss the following question:

- Which experimental limitations can have some effect on the conclusions?

In section 13.1, we discuss the notion baseline control treatment and we discuss the number of replications. In section 13.2, we discuss the effects of more than one alteration of the procedures at the same time. In section 13.3, we discuss rewards. In section 13.4, we discuss the possibility that the participants had information about the game before playing the game. In section 13.5, finally, we discuss if the complexity of the market model in the business game limits the conclusions that can be drawn from when using the game as an experiment.

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[1] The total number of game sessions in played by students and by professionals were 72 (see Appendix H). 6 game sessions were excluded and the remaining 66 game sessions were studied.

13.1 Replications of the baseline control treatment

The firms in 1999 played the game with what we called in section 6.2.4 a *baseline control treatment,* as these firms had complete information, while the firms in the regular game sessions in 1993–1998 had incomplete information. By making alterations to the information conditions (C), from incomplete to complete information, we were able to draw conclusions regarding decisions made with both incomplete and complete information.

Before these alterations were made, we could describe decisions made by firms with incomplete information, but we could not compare them with decisions made by firms with complete information. Hence, one of the potential failures in experimental economies has been avoided, because the requirement of a baseline control treatment is fulfilled, since firms with complete information have also played the game. This is of importance since one of the main conclusions in the study is the difference between decisions made by firms with complete information and decisions made by firms with incomplete information. Furthermore, we could compare decisions made by firms with both complete information and incomplete information with the decisions in the optimal solutions.

However, one experimental limitation related to the baseline control treatment is that the game has been played with this treatment in regular game sessions in only one year, i.e. 9 game sessions. Replications over the years, i.e. repetitions of the playing of the game over the years, could possibly lead to other conclusions about information use.

One example of differences in decisions made by firms with the same experimental procedures in different years is shown by the firms that played the game in 1993, 1995 and 1996. In the technical analysis in Appendix K, comparison (iv), we can see that there were significant differences between the decisions in periods 1–5 by firms of categories L3, L5 and L6. These differences in decisions show the importance of making replications over the years.

Hence, similar differences in decisions can appear between years for firms with complete information, C9. Although we have nine game sessions played by firms with complete information, C9, we have an experimental limitation that can have an effect on the conclusions, since there are no replications made over the years of firms with complete information, C9.

It is also of interest to replicate the procedures used in 1998 where some firms played with more (M8) and some with less information (L8) in the same game session, since the conclusions about the differences between these firms were drawn from only 27 and 18 firms, respectively.

13.2 More than one alteration of the procedures at the same time

In Chapter 6, we discussed the fact that when we make more than one alteration at the same time, so-called *confounded alterations*, and we find effects of the alterations, we have difficulties in determining which of the alterations had an effect. The reason why we made more than one alteration at the same time was that the number of occasions when the business game could be used as an experiment at the school was limited. However, we consider it to be an experimental limitation to make confounded alterations. Here we shall discuss the effects of these alterations for the conclusions when:

- More than one alteration was made to the information conditions at the same time (section 13.2.1)
- Rewards were altered at the same time as the information conditions (section 13.2.2)
- Group compositions were altered at the same time as the information conditions (section 13.2.3)

13.2.1 Alterations of the information conditions (C)

Earlier in section 6.2.7, it was mentioned that confounded alterations of the information conditions in 1997–1999 were made in order to obtain significant differences in decisions. This can be seen in the figure below, which shows the information conditions for firms with symmetric information conditions in 1993–1996, 1997 and 1999.

Incomplete Information	1993–1996	Less information	L3–L6	C1.1, C3.1, C4.1, C6.1
	1997	More information	M7	C1.2, C3.2, C4.1, C6.1
	1999	Complete information	C9	C1.1, C3.3, C4.2, C6.2

Figure 13.1. Symmetric information conditions in the regular game sessions.

In this figure we see that in 1997, two alterations were made to the information conditions in 1993–1996 (C1 and C3). In section 8.5 we compared the decisions of firms with more information, denoted M7, and firms with less information, denoted L3–L6, and concluded that more information compared to less information seemed likely to have some effect on the decisions. In section 9.1, we reached the conclusion that the alteration of the reference point (from C1.1 to C1.2) did not have any effect on decisions and that the reports consisted of possibly redundant information. However, we could not, completely determine the effect of the alteration of the reports (from C3.1 to C3.2), since we altered the reference point at the same time[2].

In Figure 13.1 we also see that in 1999 three alterations were made compared to the information conditions in 1993–1996 (C3, C4 and C6) and four alterations compared to the information conditions in 1997 (C1, C3, C4 and C6). In section 8.5, we reached the conclusion that there were differences between firms with complete information, denoted C9, and firms with incomplete information, denoted L3–M8. In section 9.1, we reached the conclusions that the differences in decisions in period 1 could be explained by the demand table (C6.1 to C6.2) and that the differences in decisions in period 6 could be explained by the information about how many periods the game would be played (C4.1 to C4.2). However, we could not completely determine the effect of the alteration of reports (C3.1 and C3.2 to C3.3), since we altered other information conditions at the same time.

[2] In the spring of 2000, the business game was played with the alteration of the reference point of demand (C1.2), but with varied reports during the game (C3.1). Hence, there was only one single alteration compared to L3–L6. The analysis of these decisions is left for future research.

In 1998, the firms played the game with asymmetric information, some with more information (C3.4), denoted M8, and some with less information (C3.5), denoted L8. For firms with asymmetric information we had the problem that the participants had to be informed at the briefing that there would be a difference in the information conditions (C5.2) for the firms, since we did not want to have uncertainty about the information conditions when the game was played. As mentioned in section 8.3, we could not determine if the differences in decisions were due to the asymmetric information condition itself or due to the information about the asymmetric information condition.

13.2.2 Alterations of rewards (B)

The reward for playing the game in 1993–1996 was to become "the champion of the year of the business game". The participants in the winning firm also received some small gifts.

The rewards in the regular game sessions were altered in 1997–1999 as follows:

- In 1997 and in 1998, the firms could qualify for the semifinals.
- In 1999, the firms could win by lottery (with a 1/5 chance) their equity at the end of the game minus the equity at the start of the game.

Since we altered the reward (B) at the same time as the information conditions (C), we have difficulties in determining the effect of each alteration. We cannot rule out the possibility that the alteration of rewards could have had an effect on decisions. In fact, the purpose of increasing the rewards in 1999, when the firms played the game with complete information, was to improve the experimental situation (see section 13.3 below). Nevertheless, we have here an experimental limitation that can have an effect on the conclusions.

13.2.3 Alterations of the composition of groups (D)

Since we altered the group composition (D) at the same time as the information conditions (C) we have difficulties in determining the effect of the alterations. Before 1998 the groups were informally composed (D3.1). In 1998 and in 1999, the composition of groups (D3.2) was altered and was done using randomization. So-called *indirect control* of the composition of groups was used to reduce the possibility of a systematic bias of the groups' compositions. Hence differences in decisions in 1993–1997, compared to 1998–1999, can depend on the alteration of the group composition.

As mentioned earlier, differences in decisions between firms with more and less information in 1993–1998 were observed both between firms of L3–L6 and M7, and between firms of L8 and M8. The procedures of group compositions were different in 1997 and in 1998, but similar differences between firms with more and less information were found in both years. Hence, it is likely that the alteration of group composition did not affect the decisions. Furthermore, since the groups were randomized in both 1998 and 1999, the differences in decisions between firms with incomplete (L8 and M8) and complete information (C9) cannot be explained by the alteration of group composition.

Here we also shall consider that we studied the effects of different group compositions on decisions and equity at the end of period 6 in section 12.1, but we did not find any differences with respect to differences in group compositions.

Summing up, it does not seem likely that the differences in group compositions have any effect on the conclusions.

13.3 Rewards

Generally, rewards in experiments are used to motivate the participants to pursue the objective for participation in the experiment. According to the literature (e.g. Smith, 1982, Hey, 1991 and Friedman, 1994), the ideal rewards should be:

- In proportion to the degree of fulfillment of the objective.
- On a level that makes the participants motivated to pursue the objective.

Earlier in this study we have referred to the use of the business game as an experiment as a *poor man's experiment*[3]. Due to financial restrictions, I have not been able to conduct the experiments exactly as I would have liked to do them. During the game sessions in 1993–1996, I had no influence. In 1997–1999, there was a compromise between the pedagogical aims at the school and my interest in using the business game as an experiment.

Since the rewards in the regular game session 1993–1998 were not proportional, the firms might have preferred to win the game session in which they participated instead of pursuing the objective of the game, to maximize the equity at the end of the game. They could have preferred to have a higher chance of winning their game session with a lower expected equity, instead of having a lower chance of winning their game session but with a higher expected equity.

Furthermore, firms with lower equities than the other firms and firms that went bankrupt may have had little motivation to pursue the objective of playing the game well. In the answers to question Q4 in section 11.1, we saw that participants in firms with low equity at the end of period 6 stated lower motivation than other participants.

As mentioned above, in 1999 the rewards were proportional to the equity at the end of the game, since the firms could earn the equity at the end of the game with a 1/5 chance. However, the rewards might not have been high enough to motivate the participants so that their sole aim was to maximize the equity at the end of period 6. In the answers to question Q4 on page 202, we saw that participants in firms in 1999 did not state higher motivation than participants in other years.

Hence, we have an experimental limitation that can have an effect on the conclusions, since the rewards have not been proportional and not on a level that motivated all participants to pursue the objective for playing the game.

Referring to the initial remarks above about ideal rewards, we can say that the rewards should be proportional to the equity at the end of the game, so that the participants would state unchanged motivation as their answer to question Q5 (see page 203). Furthermore, the rewards should be so high that the participants would state high motivation as their answer to question Q4.

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[3] Up to 1997, there was very little money for rewards, US $50–$100, in the regular game sessions. In 1997 and in 1998, the total rewards were around US $250, but only in the two finals.

Finally, we should point out that "correct" rewards would have been very expensive. As the game has been played with about 250 participants each year, we must multiply the magnitude of reward that would motivate a participant by 250. For example, if a participant is to have a fair chance of winning e.g. US $ 40 for playing the game in 3–4 hours[4], the total sum of rewards in one year is US $ 10,000. This can be compared to the total reward used in 1999 of about US $ 500.

13.4 Uncontrolled information about the business game

The information in the business game consisted of: the rules of the game (Appendix A), the interest table (Appendix B), the introduction at the briefing before the game (see overheads in Appendix C), the reports that the firms received during the game (Appendix E) and in some sessions also the demand table (Appendix F). After the game, the decisions and results were presented at a debriefing.

As the business game was played as part of a course at the Stockholm School of Economics in 1993–1999, participants who were about to play the game could also have heard about the game from participants who had previously played the game:

- In the current year (except the first game session in each year).
- In previous years.

Since the business game was mainly used for educational purposes and had to be played on nine different occasions each year[5], the information about the game could not be *controlled.* However, at the debriefings the participants were discouraged from discussing the game with participants who had not yet played the game. Moreover, the fact that the business game was played as a competition among the firms in all the game sessions in the same year could have restrained participants from discussing the game with each other.

Question Q1 in the questionnaire (Appendix M) asked about the information about the game before playing the game.

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[4] Based on discussions on "correct" rewards in a course in Experimental economics held by Professor Peter Bohm at Stockholm University.

[5] There were limitations on classroom and teaching staff capacity in the course.

Q1	To what extent did you and the other participants in your firm have information about game sessions played earlier (MiniMax)?
(a)	we did not have any information at all
(b)	we had some information, by rumors about earlier game sessions
(c)	we had information about prices and advertising from (some) earlier game sessions

The question was asked in the last three regular game sessions in 1997 (85 participants in 15 firms) and in all regular game sessions in 1998 and in 1999. The following figure shows the answers to question Q1.

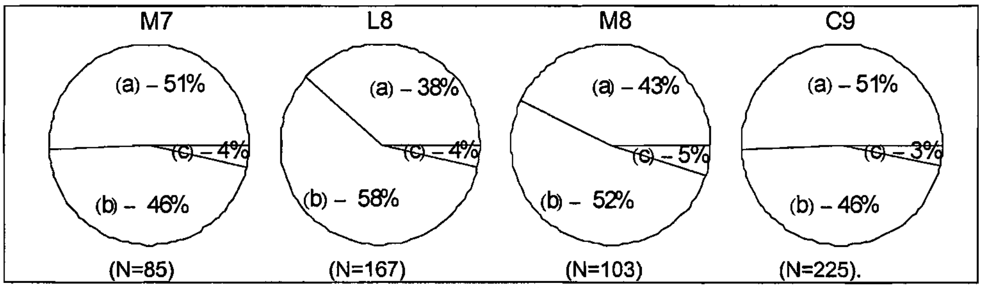

Figure 13.2. Stated information about the game before playing it. Chi-square=9.9, df=6, Asymp. Sig=.59.

38%–51% of the participants stated alternative (a), i.e. that they did not have any information at all about the game before playing it. 46%–58% of the participants stated alternative (b), i.e. that they had some information via rumors about earlier game sessions. This information could possibly have been limited to what the game was about, if it was fun to play etc. Only 3–5% of the participants stated that they had information about prices and advertising from earlier game sessions, alternative (c).

No data exist from game sessions where students are certain to have played the game without any information about the game before playing it. However, the professionals presumably played the game without any information about the game before playing the game, since the participants of each of the two corporations played the game at the same time. Hence, some of the differences between decisions of students and professionals might be explained by the differences in information about the game before playing the game.

13.5 Complexity of the market model in the business game

Finally, we shall discuss the complexity of the market model in the business game in order to judge the validity of our experiments. The complexity of the market model can be discussed from two perspectives, as shown in Figure 3.1 on page 46:

- Basic market models in experimental economics.
- Business life.

First we shall discuss differences between the market model in the business game and basic market models commonly used in experimental economics. In Chapter 3, a basic market model in the field of industrial organization was described and in Chapter 4 the market model in the business game was described. The table below shows the differences between these models.

Code	**Property**	**Market model**	
		Basic	This business game
A1	Decision variables	1	4
A2	Dynamic state variables	–	4
A3	Number of firms on market	2	5

Table 13.1. Properties of a basic market model and the market model in the business game.

In the table above we see that the market model in the business game has more variables and also more firms on the same market than a basic market model. In this sense, the market model in the business game is more complex than the basic market model.

In the four previous Chapters (8–11), we have studied decisions made with different information conditions C1–C6. The main difference with respect to the complexity of the market model compared to the basic models is the information in the reports (C3), as the reports in the business game studied are more complex than the reports in a basic model.

Despite the warning from some economists (e.g. Hey, 1991 and Holt, 1995), I would like to argue here that the complexity of the market model compared to a basic model is not an experimental limitation, since the purpose was to study the use of information in decision making. Furthermore, since the optimal solutions were not easily obtained due to the relative complexity of the market model, the participants could not have had any information about these solutions. Hence, it was perhaps even more interesting to make comparisons between the decisions made in the business game and the optimal solutions in this perspective.

In this connection it is also important to discuss the similarity of the market model in the business game with the markets in business life. As mentioned in Chapter 1, the business game was designed to be a reasonably realistic model of an oligopoly market in business life. Nevertheless, compared to many situations in business life the business game is less complex.

The following question was posed in 1998 (L8 and M8) and in 1999 (C9) to get some insights regarding the participants' thoughts about the realism of the game:

Q8	Try to estimate the similarity of the game with a real market.
	The answers were on a scale from 0–100, where 0 is little similarity, 50 medium and 100 is great similarity.

There were no significant differences between the answers of participants in firms of L8, M8 and C9, despite the fact that the firms had different information conditions when they played the game. Hence, the different information conditions in 1998 and in 1999 did not affect the stated similarity between the business game and a real market. Differences in decisions between firms with different information conditions could have been made depending on the stated similarity.

However, the median of the answers to question Q8 was 46, i.e. about medium similarity. One possible explanation for the participants only stating medium similarity is that the market model is a simplification of a real situation in business life[6]. It is not an experimental limitation *per se* to use simpler models of markets. However, the so-called *parallelism*, between conclusions on the use of information in the business game and information use in business life, may rely on the similarity between the market model and a real market.

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[6] For example, there is only one product on one market, the firms have the same assets at the start of the game ("symmetry"), the decisions cannot be made continuously, the game ends with certainty after 5–10 periods and the time for making decisions is limited to 10–20 minutes.

14 Summary and future research

As a starting point, many theories use the assumption that all relevant information is available for the decision makers in a decision making situation. However, in many decision making situations in business life the available information is limited. Furthermore, many decision making situations in business life are complex. These realities of business life can make it impossible for decision makers to determine the optimal decisions. Furthermore, it could even be difficult for them to determine whether one decision alternative is better than another alternative.

It seems reasonable to assume that additional information can be used to improve decisions, that is, the amount of information available has an effect on what decisions are made.

The purpose of the study was to take some small steps toward a better understanding of how information is used in decision making. The research question posed in this study was: "Are better decisions made when additional information is available?"

Since it might be difficult to observe what information is available and what information is used for decision making in business life, a constructed situation offered an alternative. Business games are usually designed to be reasonably realistic models of business life. In this study I used a business game that consisted of five firms competing on an dynamic oligopoly market for experimental purposes. The decision makers, i.e. the firms in the business game, consisted of groups of participants. The business game was used on a regular yearly basis for educational purposes at the Stockholm School of Economics. On several occasions the business game had also been played by professionals.

I first studied decisions made in game sessions in 1991 – 1996. I then altered the information conditions for the firms as the business game was played in 1997–1999 by giving the firms additional information and I studied the decisions made in these game sessions. Hence, I generated, tested and verified hypotheses on information use through alterations of the information conditions and repetitions of playing of the game at the same time as the business game was used for educational purposes. My use of the business game as a reasonably controlled situation, i.e. as an experiment, has a connection to the growing field of experimental economics.

I was interested in comparing the decisions made by the firms in the game sessions with the optimal decisions in the market model. I therefore formalized the market model in the business game, which was available only in a computer program, by using mathematical notation. In particular, I used symbols instead of numerical values for parameters. Thus, I first described the market model in detail and I then used this description to determine the optimal solutions in the market model.

The solutions of interest were the cooperative and the non-cooperative solutions in game theory. These solutions could be either myopic, i.e. with the assumption that the game would end after the present period (the period for which the solution is determined), or dynamic, i.e. with the assumption that the game would continue after the present period. Hence, the decisions made by the firms in the game sessions were to be compared to four solutions: the myopic and dynamic cooperative solutions and the myopic and dynamic non-cooperative solutions.

The market model in the business game was more complex than the basic market models commonly used in the field of industrial organization. The business game had more decision variables (price, advertising, investment and production), and the market model also had dynamic properties (capacity, stocks, cash/loans and long term advertising effect). Due to this relative complexity of the market model, myopic solutions could be obtained analytically only after simplifications of the market model. Even with these simplifications, the myopic solutions could partly be determined analytically.

All dynamic solutions in the market model had to be determined numerically. The numerical solutions were not easily obtained. I used three numerical methods: the Grid search method, the Solver Program in Excel and the Random search method. I partly developed these methods and used them for finding and verifying the approximate values of the optimal decisions. I have described the analytical and the three numerical methods in detail, since these methods are also of general interest, as they can be used to complement each other to determine optimal solutions in other market models.

When I used the business game as an experiment, I deliberately made a number of alterations to the information conditions for the firms, at the same time, in order to increase the possibility of obtaining differences in decisions between firms with different amounts of available information. The most interesting alterations of the information conditions were made from incomplete information (limited information about the demand on the market and unspecific information about when the game would end) to complete information (information about the demand on the market in the form of a demand table and specific information about when the game would end). Also of great interest were the alterations in the information conditions where three of the five firms on the same market received a varied number of reports during the game, while the other two firms on the same market received all reports during the game.

When I compared decisions of the firms in the game sessions to the optimal decisions, I found that the firms did not make cooperative decisions (i.e. the decisions were not made according to the myopic or the dynamic cooperative solution). An explanation for this could be that there were five firms competing on the same market and cooperative decisions are made in experiments with two, or at the most three, firms on the same market. Furthermore, I found that the decisions made by firms with complete information were best described by the dynamic non-cooperative decisions. In particular, the median decisions of these firms were found to be remarkably close to the dynamic non-cooperative decisions. I did not expect this at the start of this study.

I also found differences in decisions between firms with complete and incomplete information. These differences could be explained by information about the demand on the market and the exact information about the end of the game.

Finally, I found differences in decisions when firms with different information conditions competed on the same market. These differences in decisions could be explained by either the information about different information conditions or the differences in reports, or both. It seemed that information about differences in information conditions has an effect on the decisions, in particular in the first period, where firms that are told that they will receive more information than other firms are more confident.

However, when I compared the decisions made when all firms on the same market received all reports during the game with the decisions made when all firms on the same market received varied reports during the game, I could not determine that the effect of the differences in reports led to significantly different decisions.

The objective for the firms playing the game was to maximize the equity at the end of the game. The alteration of the information conditions, from incomplete information to complete information, did not affect the equity at the end, when all five firms on the same market had the same information condition. However, when firms on the same market had different information conditions, the firms with additional information had a higher equity at the end of the game. Hence, I could answer the research question mentioned above as follows: "Better decisions are made, when additional information is available to some, but not all, of the firms".

I also studied how the firms adapted to information. The dispersion among the decisions of the firms did not decrease when additional information became available. Furthermore, I found that the firms were not adaptive to the decisions of the other firms on the same market. Instead, I found an interesting relation between the decisions made by the firms at the start of the game, and the decisions the participants stated they would make at the start of the game, if they were to participate once more. The participants seemed to be committed to the decisions that their firms made at the start of the game even after the game had ended, also when the decisions of their firm had been less successful. I found a relation between the equities of the firms at the start of the game and the equities at the end of the game. I also found considerable differences in equity at the end of the game between firms with the same information conditions.

I designed two methods for capturing the decision making in the groups. These methods should only be seen as a start for making descriptions of the decision making in the groups, acting as firms in the game. The decision making process consisted mainly of discussions, but possibly the discussions in the groups were only about minor differences in the decisions. The participants were fairly motivated, but they related their motivation to the equity of their firms towards the end of the game.

When I studied the effect of composition of groups on decisions, I found no relations between, on one hand, the firms' group composition (size, number of females, as well as age, grade and score in exam of some of participants) and, on the other hand, the firms' decisions and equity. However, I found differences in decisions between firms played by students and professionals. An explanation for these differences could be that professionals made decisions closer to the price and advertising levels stated in the rules of the game.

The following table sums up the main conclusions of this study:

1. The firms did not make cooperative decisions.
2. The decisions made by firms with complete information are best described as the dynamic non-cooperative decisions.
3. There were differences in decisions between firms with complete and firms with incomplete information.
4. There were differences in decisions when firms with different information conditions competed on the same market.
5. No substantial effects of the use of the information in the reports could be determined.
6. Firms on markets where all five firms had complete information did not have higher equities at the end than firms on markets where all five firms had incomplete information.
7. When three firms that had varied reports during the game competed on the same market as two firms that had all reports during the game, firms with all reports had higher equity.
8. There was a considerable dispersion among decisions made by firms with incomplete information, but the dispersion was even bigger among decisions made by firms with complete information.
9. The firms were not adaptive to the decisions of the other firms.
10. The participants seemed to be committed to the decisions that their firms made at the start of the game, even after the game had ended, also when these decisions had been less successful.
11. There was considerable dispersion among the equities of firms playing the game with the same information condition.
12. There was a relation between the equities of the firms at the start of the game and the equities of the firms at the end of the game.
13. The participants related their motivation to the equity of their firms towards the end of the game.
14. Group compositions had no effect on decisions depending on group compositions when students played the game.
15. Professionals made different decisions compared to students.

Table 14.1. Main conclusions of the study.

Since the business game that I used for experimental purposes was also used for educational purposes at the same time, one experimental limitation was that the information about the game before the participants played the game was uncontrolled. Another experimental limitation was the strong financial restrictions on rewards. A third limitation was that the experimental design had an explorative purpose focus, leading to more than one alteration being made at the same time, rather than a strict focus on testing hypotheses, which would have allowed only one alteration at the same time. Despite the fact that this study relied on decisions made by a great number of firms, a fourth limitation was that the number of game sessions studied was limited.

It cannot be ruled out that the experimental limitations described can have an effect on some of the conclusions in this study. Therefore, the above list of conclusions from this study should foremost be seen as a list of hypotheses for future research.

For future research, it will be of interest to use the business game to test the hypotheses listed above. If possible, the game should then be used as an experiment without the limitations listed above, in order to make critical tests of these hypotheses. Furthermore, this study can be seen as one of the first steps towards research in a field which lies between studies of business life and of experiments with basic market models.

Future studies using experiments can build on parts of the methodology for making alterations to the business game. For example, the alterations of information conditions made in this study can be tested on other experimental markets. Another variant not tested in this study, but also of interest, would be deliberately composed groups acting as firms in the game, i.e. participants would be ordered into groups depending on a particular characteristic. It should also be mentioned that the effect on differences of the time available for decision making has not been studied here, but should be included in future research using this business game.

This study also provides several tools to be used in future research. Among these tools are the mathematical description of the market model, the analytical method for finding the myopic solutions, the numerical methods for finding both the myopic and dynamic solutions as well as a table of the demand on the market, the capturing of the decision making and the questionnaires for following the decision processes in the firms.

The market model description could form the basis for variations in the business game, by allowing alterations not only in the values of the parameters of the costs and the demand function, but also in the functional form of the demand function. The change in the functional form, e.g. of the function for the effect of the advertising of each firm and the interest function, could make it possible to have a model for which one could obtain a closed form of the non-cooperative analytical solution for the myopic case, a desideratum of some experimental economists.

The number of firms competing on the same market is also a parameter in the market model, which could be altered. The playing of the game with two, three or four firms would be of interest to answer a question that is fundamental in experimental game theory: "At what number of firms will the cooperative solutions become a better description than non-cooperative solutions?". Furthermore, it is of interest to study the decision making of one single firm in the game, i.e. to study what decisions a firm in a monopoly would make.

The numerical methods developed here for finding both the myopic and dynamic solutions could be used for a great number of market models that could differ fairly substantially from the market model of the game in this study. The capturing of the decision making and the questionnaires for following the decision processes could be used for studying the decision processes. The dispersions of decisions and also the commitment to earlier decisions are of interest to study more. Finally, the hypotheses generated regarding the effect of information in the reports could hopefully also be used to study the effect of such reports in business life.

Appendix

Appendix A–F and M–O have been translated from Swedish into English.

Appendix A and C are shown according to the original procedures of the game in 1991–1996. As the procedures of the game were altered in 1997–1999, Appendix A and C were altered correspondingly.

Appendix A – The rules of the MiniMax business game

You compete on a market with four other corporations, producing a similar, but not identical, storable product. In each period, you make four decisions:

1. Number of machines to be purchased at $ 30 per unit
2. Number of units to be produced at $ 10 per unit
3. Amount of money (in $) to be spent on advertising
4. Price of the product (in $)

You can use any numbers, e.g. purchase 11.25 machines. One machine can produce one product unit. Total machine capacity sets the limit on total production. Thus, in period 1 you can only produce as much as the capacity you buy.

Machinery is subject to a 10% real depreciation; 1/10 breaks down each period. At the start, each player has $ 200 in cash, 0 machines and 0 stock.

If there is not enough cash to cover investments, cost of production and advertising, money will automatically be borrowed. Interest rates rise with the borrowed amount as well as increased debts in relation to equity. The rate is given in the attached table. If equity (at the beginning of the period) is $20. then interest on a $ 200 loan is 16%, on a $ 400 loan 19 % and on a $ 800 loan 23 %. Surplus cash is invested at 10 %.

Sales in a period depend on price and advertising policies, mainly those of the period, but to some extent also those of earlier periods. Market research indicates: If all corporations charge $ 25 and spend $ 50 on advertising, each corporation sells roughly 15 units.

A suitable goal might be to maximize equity at the end of the game, where Equity = Cash+Capacity*$30+Stock*$10.

In the case that a corporation obtains a negative equity, it will go bankrupt. In that case the game leader can either decide upon the corporation ceasing operations, i.e. leaving the game, or being taken over by the government.

At the end of every period, each corporation obtains a report on its own results and balance sheet, as well different types of reports on the other firms in the different periods of the game. The game starts with a test period when all reports are given. The game is then restarted from period 1 again, so that the test period will not have any effect on the "real game" played over 6–10 periods.

Appendix B – Interest function and interest table

$$l = \max[-\dot{C},1] \;;\; e = \max[E,0.5] \;;\; s = l/e$$

$$\text{if } s \le 1 \text{ then } \tilde{r} = 0.12 + 0.04s \text{ else } \tilde{r} = 0.1 + 0.04(\ln(2) + \ln(2s)) + 0.005s$$

$$r = \min\left[\max[0.13, \tilde{r}]\ ,\ 0.28\right]$$

Interest function $r(\dot{C}, E)$.

Interest rates and interest payments

	Equity					
Loan	50	100	150	200	250	300
50	16% (8)	14% (7)	13% (7)	13% (7)	13% (7)	13% (7)
100	19% (19)	16% (16)	15% (15)	14% (14)	14% (14)	13% (13)
150	21% (32)	18% (27)	16% (24)	15% (23)	14% (21)	14% (21)
200	23% (46)	19% (38)	17% (34)	16% (32)	15% (30)	15% (30)
250	24% (60)	20% (50)	18% (45)	17% (43)	16% (40)	15% (38)
300	26% (78)	21% (63)	19% (57)	18% (54)	17% (51)	16% (48)
350	27% (95)	22% (77)	20% (70)	19% (67)	18% (63)	17% (60)
400	28% (112)	23% (92)	21% (84)	19% (76)	18% (72)	17% (68)
450	28% (126)	24% (108)	21% (95)	20% (90)	19% (86)	18% (81)
500	28% (140)	24% (120)	22% (110)	20% (100)	19% (95)	18% (90)
550	28% (154)	25% (138)	23% (127)	21% (116)	20% (110)	19% (105)
600	28% (168)	26% (156)	23% (138)	21% (126)	20% (120)	19% (114)
650	28% (182)	26% (169)	24% (156)	22% (143)	21% (137)	20% (130)
700	28% (196)	27% (189)	24% (168)	22% (154)	21% (147)	20% (140)
750	28% (210)	27% (203)	24% (180)	23% (173)	21% (158)	20% (150)
800	28% (224)	28% (224)	25% (200)	23% (184)	22% (176)	21% (168)
850	28% (238)	28% (238)	25% (213)	23% (196)	22% (187)	21% (179)
900	28% (252)	28% (252)	26% (234)	24% (216)	22% (198)	21% (189)
950	28% (266)	28% (266)	26% (247)	24% (228)	23% (219)	22% (209)
1000	28% (280)	28% (280)	26% (260)	24% (240)	23% (230)	22% (220)

Figures within parentheses refer to interest payments in $. These payments are rounded to the nearest integer in the table, but **not** in the game.

Interest table.

Appendix C – Overheads at the briefing

Welcome to
the MINIMAX business game

MINI

- Played in less than 4 hours
- Only the most important decisions
- Very simple figures
- SSE first year students

MAX

- Maximize learning effect/hour

Most important aspects included:

- Relationship investments, production, advertising, price and financing
- Effect of competitors
- Used in executive education in Sweden, England, France, Balticum, USA and Russia

Cost/Revenue analysis: Match revenues and costs, cost calculation

Marketing: Analysis of effects of decisions on advertising and price

Competition: Analysis of competitors

Accounting: Analysis of balance sheets

Finance: Importance of loan decision

Strategy: What type of firm?

Administration: Division of work under time pressure

5 corporations on the same market

Similar, but not identical, storable products

5–10 periods with 4 decisions in each period:

1. Number of new machines ($ 30 each)
2. Number of units produced ($ 10 each)
3. Advertising budget in $
4. Price of product in $

Integers not necessary; 11.25 units permitted

1 machine can produce 1 unit

Machines subject to 10 percent real depreciation each period

Start with :
$ 200 cash + 0 machines + 0 stocks

Negative cash = loan

13–28 % interest on loan depending on size of loan and equity at start of period

Surplus cash invested at 10 %
Money *out* at period start
money *in* at period end

Sales depend on price + relative price + advertising + relative level of advertising

Advertising + sales have certain effects also in later periods

Decreasing effect of last $ on advertising

In period 1: If all firms charge $ 25
and spend $ 50 on advertising,
each sells roughly 15 units

Goal: Maximize accumulated profits = maximize equity:

Cash + Capacity*$ 30 + Stocks*$ 10
(–Loan+ Capacity*$ 30 + Stocks*$10)

Negative equity = Bankruptcy:
Government takes over or firm leaves market

Reports in all periods:

- Own results + own balance sheet
-
- All prices

Reports in some periods:

- Market shares (+ all advertising)
- All balance sheets

Winner of the evening: Largest equity

More important: Compete with the firms of all 9 game sessions (evenings)

SSE MINIMAX Champion of the year

Each team at assigned table until end
Always one person there

Start with a test period

10–20 minutes for decision making

Advice: Test real strategy

After reports start again from scratch with real game

Learn from test period:
Change bad strategy

In every period: Fill out computer made decision form, always 4 boxes, legibly. Other lines also for planning, at least in first periods

Appendix D – Decision form

Decision form

Firm number
Period t

	Machine Capacity	Production and stocks	Payments	Cash (or loans)	
Cash (if negative = loan)				$\$\ C$	[1]
Production capacity	K units				[2]
Stocks		S units			[3]
Decision 1 New investment in capacity $\$\ \sigma$ per unit	$\boxed{i}$ units		$\$\ \sigma i$		[4]
Total available capacity (rows 2 + 4)	$\dot{K}=K+i$ units				[5]
Decision 2 Production at $\$\ \chi$ per unit (may not exceed row 5)		$\boxed{o}$	χo		[6]
Decision 3 Advertising			$\$\ \boxed{a}$		[7]
Total payments (rows 4 + 6 + 7)			$\$\ P=\sigma i+\chi o+a$		[8]
Remaining cash row 1 - row 8) if negative = loan)			$\$\ \dot{C}=C-P$		[9]
Quantity available for sales (rows 3 + 6)		$\dot{S}=S+o$			[10]
Decision 4 Price	$\$\ \boxed{p}$				[11]

Appendix E – Reports

Firm number				Period t
Sales (units)		q		[12]
Revenues	\$	pq		[13]
Cost of goods sold	\$	χq		[14]
Advertising costs	\$	a		[7]
Interests	\$	$r\dot{C}$	(r depends on $\dot{C}$)	[15]
Depreciation	\$	$\delta\sigma\dot{K}$		[16]
Profit Π	\$	$pq - \chi q - a - r\dot{C} - \delta\sigma\dot{K}$		[17]
Machine capacity	\$	$\sigma\ddot{K}$	($\ddot{K}$ units)	[18]
Stocks ¤	\$	$\chi\ddot{S}$	($\ddot{S}$ units)	[19]
Cash	\$	$\ddot{C}$		[20]
Equity	\$	$\ddot{E}$		[23]

Firm 1	p	
...		
Firm N	p	
Total	$\sum q$	$\sum a$

	Units Sold	Market share-	Advertising in \$
Firm 1	q	$q/\sum q$	a
...			
Firm N	q	$q/\sum q$	a

	Cash -=Loan \$	Machine units	Stock units	Equity \$
Firm 1	$\ddot{C}$	$\ddot{K}$	$\ddot{S}$	$\ddot{E}$
...				
Firm N	$\ddot{C}$	$\ddot{K}$	$\ddot{S}$	$\ddot{E}$

Lost sales $d - \dot{S}$ units

Appendix F – Demand table

Mean decisions of the other firms $(n = 4)$ on price, $p_{\bar{n}}$, and marketing effect, $\dot{M}_{\bar{n}}$, where $\dot{M}_{\bar{n}} = M_{\bar{n}} + a_{\bar{n}}$

Decisions of a firm on price, p, and marketing effect, $\dot{M}$, where $\dot{M} = M + a$

	Price	25	30	35	40	50	25	30	35	40	50	25	30	35	40	50	25	30	35	40	50	25	30	35	40	50	25	30	35	40	50
Price	Mark. eff.	50	50	50	50	50	100	100	100	100	100	150	150	150	150	150	200	200	200	200	200	300	300	300	300	300	400	400	400	400	400
25	50	13.6	15.6	17.3	18.8	21.2	7.6	8.7	9.6	10.4	11.8	5.2	6.0	6.7	7.2	8.2	4.0	4.6	5.1	5.5	6.2	2.7	3.1	3.5	3.8	4.2	2.1	2.4	2.6	2.8	3.2
30	50	8.9	10.3	11.6	12.8	14.6	4.9	5.7	6.5	7.1	8.1	3.4	4.0	4.5	4.9	5.6	2.6	3.0	3.4	3.8	4.3	1.8	2.1	2.3	2.6	2.9	1.3	1.6	1.8	1.9	2.2
35	50	6.1	7.2	8.2	9.1	10.6	3.4	4.0	4.6	5.1	5.9	2.3	2.8	3.2	3.5	4.1	1.8	2.1	2.4	2.7	3.1	1.2	1.4	1.6	1.8	2.1	0.9	1.1	1.2	1.4	1.6
40	50	4.4	5.2	6.0	6.7	7.9	2.4	2.9	3.3	3.7	4.4	1.7	2.0	2.3	2.6	3.1	1.3	1.5	1.8	2.0	2.3	0.9	1.0	1.2	1.3	1.6	0.7	0.8	0.9	1.0	1.2
50	50	2.5	3.0	3.5	4.0	4.8	1.4	1.7	1.9	2.2	2.7	0.9	1.2	1.3	1.5	1.8	0.7	0.9	1.0	1.2	1.4	0.5	0.6	0.7	0.8	1.0	0.4	0.5	0.5	0.6	0.7
25	100	26.7	30.6	34.0	36.9	41.6	16.0	18.4	20.4	22.1	25.0	11.4	13.1	14.6	15.8	17.8	8.9	10.2	11.3	12.3	13.9	6.2	7.1	7.8	8.5	9.6	4.7	5.4	6.0	6.5	7.3
30	100	17.4	20.3	22.8	25.0	28.7	10.4	12.2	13.7	15.0	17.2	7.5	8.7	9.8	10.7	12.3	5.8	6.8	7.6	8.3	9.6	4.0	4.7	5.3	5.8	6.6	3.1	3.6	4.0	4.4	5.1
35	100	12.0	14.1	16.1	17.8	20.8	7.2	8.5	9.7	10.7	12.5	5.1	6.1	6.9	7.6	8.9	4.0	4.7	5.4	5.9	6.9	2.8	3.3	3.7	4.1	4.8	2.1	2.5	2.8	3.1	3.7
40	100	8.6	10.3	11.8	13.2	15.6	5.2	6.2	7.1	7.9	9.3	3.7	4.4	5.1	5.6	6.7	2.9	3.4	3.9	4.4	5.2	2.0	2.4	2.7	3.0	3.6	1.5	1.8	2.1	2.3	2.7
50	100	4.8	5.9	6.9	7.8	9.4	2.9	3.5	4.1	4.7	5.7	2.1	2.5	2.9	3.3	4.0	1.6	2.0	2.3	2.6	3.1	1.1	1.4	1.6	1.8	2.2	0.9	1.0	1.2	1.4	1.7
25	150	37.3	42.8	47.5	51.6	58.2	23.7	27.3	30.3	32.8	37.0	17.4	20.0	22.2	24.1	27.2	13.7	15.8	17.5	19.0	21.4	9.7	11.1	12.3	13.4	15.1	7.5	8.6	9.5	10.3	11.6
30	150	24.3	28.4	31.9	35.0	40.2	15.5	18.1	20.3	22.3	25.6	11.4	13.2	14.9	16.3	18.8	9.0	10.5	11.8	12.9	14.8	6.3	7.4	8.3	9.1	10.4	4.9	5.7	6.4	7.0	8.0
35	150	16.7	19.8	22.5	25.0	29.1	10.7	12.6	14.3	15.9	18.5	7.8	9.2	10.5	11.6	13.6	6.2	7.3	8.3	9.2	10.7	4.3	5.1	5.8	6.5	7.5	3.3	4.0	4.5	5.0	5.8
40	150	12.0	14.4	16.5	18.4	21.8	7.6	9.1	10.5	11.7	13.9	5.6	6.7	7.7	8.6	10.2	4.4	5.3	6.1	6.8	8.0	3.1	3.7	4.3	4.8	5.6	2.4	2.9	3.3	3.7	4.4
50	150	6.8	8.2	9.6	10.9	13.2	4.3	5.2	6.1	6.9	8.4	3.2	3.8	4.5	5.1	6.2	2.5	3.0	3.5	4.0	4.9	1.8	2.1	2.5	2.8	3.4	1.4	1.6	1.9	2.2	2.6
25	200	46.0	52.8	58.6	63.7	71.8	30.7	35.2	39.1	42.4	47.9	23.0	26.4	29.3	31.8	35.9	18.4	21.1	23.5	25.5	28.7	13.1	15.1	16.8	18.2	20.5	10.2	11.7	13.0	14.1	16.0
30	200	30.0	35.0	39.4	43.2	49.6	20.0	23.3	26.3	28.8	33.0	15.0	17.5	19.7	21.6	24.8	12.0	14.0	15.8	17.3	19.8	8.6	10.0	11.3	12.3	14.2	6.7	7.8	8.8	9.6	11.0
35	200	20.7	24.4	27.8	30.8	35.9	13.8	16.3	18.5	20.5	23.9	10.3	12.2	13.9	15.4	17.9	8.3	9.8	11.1	12.3	14.3	5.9	7.0	7.9	8.8	10.2	4.6	5.4	6.2	6.8	8.0
40	200	14.8	17.7	20.3	22.7	26.9	9.9	11.8	13.6	15.2	17.9	7.4	8.9	10.2	11.4	13.4	5.9	7.1	8.1	9.1	10.7	4.2	5.1	5.8	6.5	7.7	3.3	3.9	4.5	5.1	6.0
50	200	8.4	10.2	11.9	13.4	16.3	5.6	6.8	7.9	9.0	10.8	4.2	5.1	5.9	6.7	8.1	3.3	4.1	4.7	5.4	6.5	2.4	2.9	3.4	3.8	4.6	1.9	2.3	2.6	3.0	3.6
25	300	59.5	68.2	75.8	82.2	92.7	42.5	48.7	54.1	58.7	66.2	33.0	37.9	42.1	45.7	51.5	27.0	31.0	34.4	37.4	42.1	19.8	22.7	25.3	27.4	30.9	15.6	18.0	19.9	21.6	24.4
30	300	38.8	45.2	50.9	55.8	64.0	27.7	32.3	36.3	39.9	45.7	21.5	25.1	28.3	31.0	35.6	17.6	20.6	23.1	25.4	29.1	12.9	15.1	17.0	18.6	21.3	10.2	11.9	13.4	14.7	16.9
35	300	26.7	31.5	35.9	39.8	46.3	19.1	22.5	25.6	28.4	33.1	14.8	17.5	19.9	22.1	25.7	12.1	14.3	16.3	18.1	21.1	8.9	10.5	12.0	13.3	15.4	7.0	8.3	9.4	10.5	12.2
40	300	19.1	22.9	26.3	29.4	34.7	13.7	16.3	18.8	21.0	24.8	10.6	12.7	14.6	16.3	19.3	8.7	10.4	11.9	13.4	15.8	6.4	7.6	8.8	9.8	11.6	5.0	6.0	6.9	7.7	9.1
50	300	10.8	13.1	15.3	17.4	21.0	7.7	9.4	10.9	12.4	15.0	6.0	7.3	8.5	9.7	11.7	4.9	6.0	7.0	7.9	9.6	3.6	4.4	5.1	5.8	7.0	2.8	3.5	4.0	4.6	5.5
25	400	69.4	79.7	88.4	96.0	108.2	52.0	59.7	66.3	72.0	81.2	41.6	47.8	53.1	57.6	64.9	34.7	39.8	44.2	48.0	54.1	26.0	29.9	33.2	36.0	40.6	20.8	23.9	26.5	28.8	32.5
30	400	45.3	52.8	59.4	65.2	74.7	33.9	39.6	44.5	48.9	56.1	27.2	31.7	35.6	39.1	44.8	22.6	26.4	29.7	32.6	37.4	17.0	19.8	22.3	24.4	28.0	13.6	15.8	17.8	19.5	22.4
35	400	31.1	36.8	41.9	46.4	54.1	23.4	27.6	31.4	34.8	40.6	18.7	22.1	25.1	27.8	32.4	15.6	18.4	20.9	23.2	27.0	11.7	13.8	15.7	17.4	20.3	9.3	11.0	12.6	13.9	16.2
40	400	22.3	26.7	30.7	34.3	40.5	16.8	20.0	23.0	25.7	30.4	13.4	16.0	18.4	20.6	24.3	11.2	13.4	15.3	17.1	20.3	8.4	10.0	11.5	12.9	15.2	6.7	8.0	9.2	10.3	12.2
50	400	12.6	15.3	17.9	20.3	24.5	9.4	11.5	13.4	15.2	18.4	7.6	9.2	10.7	12.2	14.7	6.3	7.7	8.9	10.1	12.3	4.7	5.7	6.7	7.6	9.2	3.8	4.6	5.4	6.1	7.4

Appendix G – The market model in Excel

This is an example of the market model in Excel used for the Solver Program and the Random Search method. The formulas used in Excel are described in Table 4.5, with exception of simplifications (31) and (32) described on page 70. The parameters are described in Table 4.6. The state variables at the start are described in Table 4.7. The restrictions of variables are described in Table. 4.8. Calculations are shown for the dynamic non-cooperative solutions (DN) presented in Table 5.11.

N	*Number of firms*	5			*Parameters*		
				α	Market size		434.3
	State variables at the start of the game			β	Price elasticity		-1.5
K	Capacity	0		χ	Cost of production		10.0
S	Stocks	0		δ	Deprecation		0.1
C	Check. Acc.	200		σ	Cost of machine unit		30.0
E	Equity	200		μ	Part of advertising		0.6
M	Marketing effect	0		η	Effect of sales		1.0
		Period 1	*Period 2*	*Period 3*	*Period 4*	*Period 5*	*Period 6*
K	Capacity	0.00	8.74	8.77	9.71	10.76	11.67
S	Stocks	0.00	0.00	0.00	0.00	0.00	0.00
C	Check. acc.	200.00	-124.01	-27.38	33.33	93.12	202.80
E	Equity	200.00	138.25	235.69	324.51	415.99	552.78
M	Marketing effect	0.00	148.91	151.29	171.16	191.45	184.83
p	Price	39	39	37	35	33	30
a	Advertising	232	87	116	128	95	0
$\dot{M}$	Marketing effect	232.00	235.91	267.29	299.16	286.45	184.83
d	Demand	9.71	9.74	10.78	11.96	12.96	13.80
q	Sales	9.71	9.74	10.78	11.96	12.96	13.80
o	Production	9.71	9.74	10.78	11.96	12.96	13.80
$\dot{S}$	Supply	9.71	9.74	10.78	11.96	12.96	13.80
i	Investment	9.71	1.00	2.02	2.25	2.20	2.13
$\dot{K}$	Capacity	9.71	9.74	10.78	11.96	12.96	13.80
P	Payments	620.53	214.46	284.32	315.14	290.62	201.89
$\dot{C}$	Check account	-420.53	-338.47	-311.69	-281.81	-197.50	0.91
r	Interest rate	0.20	0.20	0.17	0.15	0.14	0.10
$\ddot{K}$	Capacity	8.74	8.77	9.71	10.76	11.67	12.42
$\ddot{S}$	Stocks	0.00	0.00	0.00	0.00	0.00	0.00
$\ddot{C}$	Check. acc.	-124.01	-27.38	33.33	93.12	202.80	414.90
$\ddot{E}$	Equity	138.25	235.69	324.51	415.99	552.78	787.42
$\ddot{M}$	Marketing effect	148.91	151.29	171.16	191.45	184.83	124.70
Π	Profit	-61.75	97.44	88.83	91.47	136.79	234.64

The market model in Excel.

Appendix H – Game sessions examined in this study

Code[1]	Year	#Game sessions	B1 Objective	B2 Monetary reward	D1 Participants	D2/D3 Group composition
L3	1993	9	Max.	No	Students	Informally
Prof.	1994	6	Max.	No	Professionals	Informally
L5	1995	9	Max.	No	Students	Informally
L6	1996	9	Max.	No	Students	Informally
Prof.	1996	6	Max.	No	Professionals	Informally
M7	1997	9	Qual. [2]	No	Students	Informally
S7	1997	3	Qual.	No	Students	Deliberately [4]
L8 M8	1998[1] 1998[1]	9	Qual. [2] Qual. [2]	No No	Students Students	Randomized Randomized
S8	1998	3	Qual.	No	Students	Deliberately [4]
C9	1999	9	Max. [3]	Yes	Students	Randomized

1) See section 8.1 for description of these codes.
2) The participants were informed that the firms could qualify for a so-called playoffs.
3) The firms could with 1/5 chance win their aggregated profits in Swedish currency.
4) The game sessions were deliberately arranged with firms depending on the answers of their participants to questions Q4 , Q9–Q10 in the questionnaire (Appendix M).

Code[1]	C1	C2	C3	C4	C5	C6
	Asymmetric information[5]	**Demand table**	**Reference point**[6]	**Test period**	**Reports**	**# Periods**
L3	No	No	25/50/15	Yes	Varied	5–10
Prof.	No	No	25/50/15	Yes	Varied	5–10
L5	No	No	25/50/15	Yes	Varied	5–10
L6	No	No	25/50/15	Yes	Varied	5–10
Prof.	No	No	25/50/15	Yes	Varied	5–10
M7	No	No	30/100/12	Yes	All	5–10
S7	No	No	-	No	All[7]	6
L8	Yes	No	25/50/15	Yes	Varied	5–10
M8	Yes	No	25/50/15	Yes	All	5–10
S8	No	Yes	-	No	All[7]	6
C9	No	Yes	25/50/15	Yes	All[7]	6

1) See section 8.1 for description of these codes.
5) Asymmetric information means that the firms were informed at the briefing of the game session that two of the firms would receive "all" information about the other firms on the same market, while three of the five firms would receive varied information of the other firms. Five firms played in the same game session.
6) The original reference point was for example: "If all firms have the price $ 25 and advertising of $ 50 all firms will sell 15 units".
7) Lost sales were included in the reports (see appendix E).

Appendix I – Medians of decisions

Information condition	**Code**	*Period*					
		1	*2*	*3*	*4*	*5*	*6*
Less	L3	33	32	32	35	34	33
Less	L5	30	30	30	32	31	31
Less	L6	35	34	35	33	32	32
Less	*L3–L6*	*33*	*32*	*33*	*33*	*33*	*32*
More	M7	34	35	35	34	33	32
Less	L8	30	31	32	31	32	31
More	M8	35	33	35	34	33	32
Incomplete	*L3–M8*	*33*	*33*	*33*	*33*	*33*	*31*
Complete	C9	40	36	35	35	33	28
All	*L3–C9*	*33*	*33*	*34*	*33*	*33*	*31*
Semifinals	S7	39	39	36	33	32	29
Semifinals	S8	37	37	36	35	33	30
Professionals	Prof.	28	29	30	30	30	30

Table 1. Medians of decisions on prices of firms.

Information condition	**Code**	*Period*					
		1	*2*	*3*	*4*	*5*	*6*
Less	L3	80	80	95	100	130	130
Less	L5	115	100	100	100	100	128
Less	L6	120	100	120	120	123	130
Less	*L3–L6*	*100*	*100*	*100*	*110*	*120*	*130*
More	M7	130	101	120	130	128	120
Less	L8	110	100	110	105	111	120
More	M8	116	103	123	135	128	109
Incomplete	*L3–M8*	*110*	*100*	*105*	*120*	*120*	*120*
Complete	C9	200	83	100	123	120	35
All	*L3–C9*	*118*	*100*	*100*	*120*	*120*	*120*
Semifinals	S7	101	110	121	121	120	50
Semifinals	S8	150	150	150	140	125	0
Professionals	Prof.	55	50	73	75	78	75

Table 2. Medians of decisions on advertising of firms.

Appendix J – Statistical tests for differences of decisions

The Kruskal-Wallis test is used for determining differences of decisions between firms with different information conditions (Tables 1–3) or between different categories of participants (Table 4). The asymptotic significance, here denoted ρ, of the tests is presented in the four tables below. We call a difference "significant" when $\rho \leq 0.05$ and we call a difference "nearly significant" when $0.05 < \rho \leq 0.10$. The Kruskal-Wallis test is the same as the Mann-Whitney U test, when it is used to make comparisons between pairs.

Comparison between types of information	*Period*					
L3 – L6, M7	*1*	*2*	*3*	*4*	*5*	*6*
Price	.17	.03	.08	.81	.88	.97
Advertising	.00	.04	.01	.04	.75	.15

Table 1. Asymptotic significance in Kruskal-Wallis test on decisions on prices and advertising of firms with less information, L3–L6, and firms with more information, M7.

Comparison between types of information	*Period*					
L8, M8	*1*	*2*	*3*	*4*	*5*	*6*
Price	.12	.15	.06	.04	.20	.46
Advertising	.90	.46	.21	.19	.08	.88

Table 2. Asymptotic significance in Kruskal-Wallis test on decisions on prices and advertising of firms with less information, L8, and firms with more information, M8.

Comparison between types of information	*Period*					
L3–M8, C9	*1*	*2*	*3*	*4*	*5*	*6*
Price	.00	.01	.00	.10	.78	.00
Advertising	.00	.03	.18	.98	.61	.00

Table 3. Asymptotic significance in Kruskal-Wallis test on decisions on prices and advertising of firms with incomplete information, L3–M8, and firms with complete information, C9.

Comparison between students and professionals	*Period*					
L3–L6, Prof.	*1*	*2*	*3*	*4*	*5*	*6*
Price	.01	.01	.00	.00	.01	.02
Advertising	.00	.00	.00	.00	.00	.00

Table 4. Asymptotic significance in Kruskal-Wallis test on decisions on prices and advertising of firms played by students (L3–L6) and firms played by professionals (Prof.).

Appendix K – Technical analysis with Kruskal-Wallis test

We shall make what is called here a technical analysis of decisions made by firms with different codes, i.e. firms that played the game with different information conditions, firms that played the game in different year or firms with different participants. Based on the Kruskal-Wallis test (Siegel & Castellan 1988), we make comparisons between decisions made by firms with different codes with using mean ranks. We use the mean ranks as an approximate test for differences and similarities between decisions of pairs of firms with different codes. In order for a difference in decisions between firms of two different codes to be significant, the absolute difference between the mean ranks of a pair of firms (two different codes) needs to be bigger than a minimum critical value, here denoted $R_{\min}$.

First I shall describe how the mean ranks for the decisions of the firms are calculated and I shall then describe how the minimum critical value, $R_{\min}$, is calculated. Then we make comparisons between absolute differences in mean ranks between firms with different codes and minimum critical values, $R_{\min}$.

The decisions in each period of all firms are given ordered rank numbers depending on the values of the decisions. The rank numbers are given from 1 to the total number of firms of all codes. The highest decision of all firms is given the highest rank number, i.e. the same as the total number of all firms, and the lowest decision of all firms is given the rank number 1. To obtain the mean rank for each code of firms, the rank numbers are summarized for each code of firms and divided by the number of firms with this code.

We calculate the mean ranks of the decisions made by the 250 firms in the regular game sessions plus the decisions made by the 15 firms in the semifinals in 1997, denoted S7, plus the 15 firms in the semifinals in 1998, denoted S8, plus the decisions made by firms played by professionals, denoted "Prof.". Altogether there are 10 codes of firms and the total number of firms is 330. Comparisons can be made in pairs between the mean ranks of firms with all these 10 codes. We limit the comparisons to the following:

(i) Comparisons of mean ranks between firms with each of the codes L3, L5 and L6 and firms with code M7.
(ii) Comparisons of mean ranks between firms with each of the codes L3, L5, L6, M7, L8 and M8 and firms with code C9.
(iii) Comparisons of mean ranks between firms with each of the codes L3, L5 and L6 played by students and firms with code "Prof.".
(iv) Comparisons of mean ranks between firms with each of the codes L3, L5 and L6.

The following two tables, for use with the Kruskal-Wallis test, show the number of firms and the mean ranks of firms with different codes.

Information condition	**Code**	*Number of firms*	*Period*					
			1	*2*	*3*	*4*	*5*	*6*
Less	L3	45	155	153	153	182	194	180
Less	L5	40	130	131	139	150	155	170
Less	L6	40	184	181	190	180	171	175
More	M7	40	179	191	191	177	178	176
Less	L8	27	135	139	137	130	143	150
More	M8	18	170	175	178	181	177	163
Complete	C9	40	221	203	210	193	175	107
Semifinals	S7	15	229	231	196	185	169	132
Semifinals	S8	15	211	218	208	195	165	135
Professionals	Prof.	50	116	117	112	118	133	

Table 1. Mean ranks of prices of firms.

Information condition	**Code**	*Number of firms*	*Period*					
			1	*2*	*3*	*4*	*5*	*6*
Less	L3	45	122	133	134	154	192	196
Less	L5	40	169	184	163	152	154	186
Less	L6	40	178	201	193	193	189	193
More	M7	40	202	204	204	202	186	174
Less	L8	27	171	166	170	165	153	168
More	M8	18	168	183	200	203	194	165
Complete	C9	40	249	142	153	175	166	95
Semifinals	S7	15	167	204	216	210	174	85
Semifinals	S8	15	249	243	244	197	168	57
Professionals	Prof.	50	67	97	99	92	108	

Table 2. Mean ranks of advertising of firms.

I illustrate a comparison of absolute differences in mean ranks between firms with different codes and minimum critical values, $R_{\min}$, with an example. I shall compare the decisions on prices of firms L3 and firms L5 in period 1. In Table 1, we see that the mean rank of prices of firms with code L3 in period 1 is 155 and that the mean rank of prices of firms with code L5 in period 1 is 130. The absolute difference between the mean ranks is $|155-130|=25$ If the minimal critical value, $R_{\min}$, is less than 25, i.e. $R_{\min} < 25$, the difference in the decisions between the firms of L3 and L6 is not significant. Next we shall see how the minimal critical value, $R_{\min}$, is calculated.

The minimum critical value of the absolute difference in mean ranks, $R_{\min}$, depends on the total number of firms, denoted N, the number of firms of each of the two codes of firms whose decisions are compared, R_{N1} and R_{N2}, and the value of z which is the significance level for normal distribution. The formula below is used to calculate the minimum critical value, $R_{\min}$.

$$R_{\min} = z\sqrt{\frac{N(N+1)}{12} + \left(\frac{1}{R_{N1}} + \frac{1}{R_{N2}}\right)}$$

The total number of firms is $N = 330$ and the significance level for "nearly significant", i.e. when $\rho \leq 0.10$, of the normal distribution is $z = 1.645$. We calculate the minimum critical value of $R_{\min}$ for the comparisons between the decisions of firms with code L3 and with code L5 above using the formula above. In Table 1 we see that there are 45 firms of L3 and 40 firms of L5. This means that we have $R_{N1} = 45$ and $R_{N2} = 40$, and we get:

$$R_{\min} = 1.645\sqrt{\frac{330(330+1)}{12}\left(\frac{1}{45} + \frac{1}{40}\right)} \approx 34$$

According to the calculation of $R_{\min}$ above, the absolute difference between mean ranks in the example above should be rejected as significant if it was below 34. Since the absolute difference was 25 we can reject the possibility that there is a significant difference between decisions in period 1 for firms of L3 and L6.

The following table shows minimum critical values, $R_{\min}$. The numbers of firms, R_{N1} and R_{N2}, are defined in rows and columns in the table.

Number of firms	*15*	*18*	*27*	*40*	*45*	*50*
15	57	55	51	48	47	46
18	55	52	48	45	44	43
27	51	48	43	39	38	37
40	48	45	39	35	34	33
45	47	44	38	34	33	32
50	46	43	37	33	32	31

Table 3. Minimum critical values, $R_{\min}$, for significant differences.

For example, row 5 $(R_{N1} = 45)$ and column 4 $(R_{N2} = 40)$ give $R_{\min} = 34$.

(i) Comparisons of mean ranks between each of firms with codes L3, L5, L6 to firms with code M7

The statistical test in Table 1 in appendix J showed significant or nearly significant differences of decisions between firms of L3–L6 and firms of M7, on prices in periods 2–3, and advertising in periods 1–4.

We limit the comparisons to the mean ranks of firms of L6 and M7, in Tables 1 and 2. The biggest absolute difference of mean ranks on prices and on advertising between firms of L6 and M7 in these periods, is 10 on price in period 2 and 23 on advertising 24 in period 1.

The minimum critical value, R_{min}, is 35 (according to Table 3, for 40 firms of L6 and 40 firms of M7).

Since the biggest absolute differences in mean ranks above, 10 and 23, are less than the minimum critical value, 35, there are no significant differences between the decisions of firms of L6 and M7 in any of these periods.

(ii) Comparisons of mean ranks between each of firms with codes L3, L5, L6, M7, L8 and M8 to firms with code C9.

The statistical test in Table 3 in appendix J showed significant or nearly significant differences of decisions between L3–M8 and C9, on price in periods 1–4 and period 6, and advertising in periods 1–2 and period 6.

We first compare mean ranks in periods 1 and 6. We see for these two periods that the smallest absolute differences between each of the firms of L3–M8 and C9 is 37 on prices (L6 to C9 in period 1) and 43 on advertising, in Tables 1 and 2. We then also compare mean ranks in periods 2–4. The biggest absolute difference in mean ranks in periods 2–4 is 19.

The minimum critical values, R_{min}, is in the range of 34–45 (according to Table 3, for 45 firms of L3, 40 firms of L5, L6, M7, 27 firms of L8,18 firms of M8 and 40 firms of C9).

Since the smallest absolute difference in mean ranks in periods 1 and 6 is 37, which is in the range of the minimum critical values of 34–45, there are significant or nearly significant differences in these two periods. However, the biggest absolute difference in mean ranks in periods 2–4 is 19, which is less than the range of the minimum critical values of 34–45, Therefore there are no significant differences in these periods.

(iii) Comparisons of mean ranks between each of firms with codes L3–L6 to firms with code Prof. (professionals).

The statistical test in Table 4 in appendix J showed significant differences of decisions between firms of L3–L6 and firms of professionals in periods 1–5 on both prices and on advertising.

We compare the mean ranks of firms of L3–L6 and firms of professionals, in Tables 1 and 2. The biggest absolute difference of mean ranks on prices of firms of L3–L6 and of professionals in these periods, is 32 on price in period 4. The smallest absolute difference of mean ranks on advertising of firms of L3–L6 and of professionals in these periods, is 35 on advertising in period 4.

The minimum critical value, $R_{\min}$, is 32 or 33 (according to Table 3, for 45 firms of L3, 40 firms of L5–L6 and 50 firms of professionals).

Since the biggest absolute differences in mean ranks for prices is 32, which is less or equal to the minimum critical value of 32 or 33, there are no significant differences between the decisions of firms of L3–L6 and professionals on prices. However, the smallest absolute difference of mean ranks for advertising is 35, which is bigger than the minimum critical value of 32 or 33. Therefore there are significant differences between the decisions of firms of L3–L6 and professionals on advertising.

(iv) Comparisons of mean ranks between each of firms with codes L3, L5 and L6.

We compare the maximum values on mean ranks with the minimum values of L3, L5 and L6 in periods 1–6, in Tables 1 and 2.

In periods 1–5, the smallest differences of mean ranks on price and advertising is in the range of 32–68 in periods 1–5. In period 6, the biggest difference is 10.

The minimum critical value, $R_{\min}$, is 34 or 35 (according to Table 3, for 45 firms of L3 and for 40 firms of L5–L6).

Since the absolute differences in mean ranks in periods 1–5 is in the range 32–68 which is similar or bigger than the minimum critical value of 34 or 35, there are significant or nearly significant differences between the decisions in period 1–5 of firms of L3, L5 and L6.

Appendix L – Standard deviations of decisions

Information condition	**Code**	*Period*					
		1	*2*	*3*	*4*	*5*	*6*
Less	L3	6	5	6	5	5	6
Less	L5	9	9	8	7	6	7
Less	L6	11	10	7	6	6	6
Less	*L3–L6*	*9*	*9*	*7*	*6*	*6*	*6*
More	M7	8	6	5	4	4	5
Less	L8	6	6	6	5	4	3
More	M8	7	6	4	4	6	4
Incomplete	*L3–M8*	*8*	*8*	*6*	*5*	*5*	*5*
Complete	C9	8	8	7	6	6	8
All	*L3–C9*	*8*	*8*	*7*	*5*	*5*	*6*
Semifinals	S7	5	3	4	4	3	4
Semifinals	S8	2	2	3	3	3	6
Professionals	Prof.	12	10	8	8	8	6

Table 1. Standard deviations of decisions on prices by firms.

Information condition	**Code**	*Period*					
		1	*2*	*3*	*4*	*5*	*6*
Less	L3	50	49	41	40	50	69
Less	L5	61	64	60	66	60	62
Less	L6	44	40	47	56	71	87
Less	*L3–L6*	*53*	*53*	*51*	*55*	*60*	*73*
More	M7	51	54	44	42	40	48
Less	L8	41	40	43	55	43	38
More	M8	68	29	31	40	53	48
Incomplete	*L3–M8*	*53*	*51*	*47*	*52*	*54*	*64*
Complete	C9	79	57	53	50	78	102
All	*L3–C9*	*63*	*52*	*48*	*51*	*59*	*74*
Semifinals	S7	31	36	33	23	29	45
Semifinals	S8	46	53	47	49	66	57
Professionals	Prof.	36	46	38	38	38	51

Table 2. Standard deviations of decisions on advertising by firms.

Appendix M – The use of questionnaires

As mentioned in Chapter 6, the participants received a questionnaire that they answered while the game leader arranged for the display of the final results. Since the answers to the questions in the questionnaires are studied in several chapters (Chapters 9–11 and 13), I shall comment on the use of these answers below.

We shall limit the use of the answers to seeing whether they support our findings on information use based on comparisons of decisions, or if they are in conflict with these findings. I must emphasize that the following concerns arise with regards to the answers to the questions in the questionnaires:

- Did the participants understand the questions?
- Did the participants answer the questions truthfully?
- Would the participants answer the questions differently later?

Two questionnaires were used. One questionnaire, question Q0, was designed by Ståhl, and used in 1993, 1995, 1996 and 1999. The other questionnaire, questions Q1–Q10, was designed by me and it was used in 1997–1999. It should be mentioned that I altered some of the questions between these years.

Since both questionnaires were used for a number of years, we can make comparisons between the answers to the questions between different years. It should be mentioned that the participants did not have information about the final results when they answered the questions.

In Chapter 9, we study the answers to questions Q0, Q2, Q6 and Q7. In Chapter 10, we study the answers to questions Q9 and Q10. In Chapter 11, we study the answers to questions, Q3, Q4 and Q5 and in Chapter 13, we study the answers to questions Q1 and Q8.

The questions Q1–Q10 are listed on the next three pages.

Q0 When in our firm we made a decision concerning the price in a certain period we considered...

Please, draw a circle around those letters that belong to the three statements that you think are the ones that most appropriately represent the decision making in your firm:

(a) how large the stocks of our firm were
(b) how large the stocks of our competitors were
(c) what prices our competitors decided in the previous period
(d) what prices our competitors decided in all previous periods
(e) what prices our competitors could be expected to decide in the present period
(f) what prices our competitors could be expected to believe that our firm would decide in the present period
(g) how much machine capacity our firm had
(h) how much machine capacity our competitors had
(i) how much our firm would spend on advertising in the present period
(j) how much our firm had spent on advertising in the previous periods
(k) how much our competitors had spent on advertising in the previous periods
(l) how large production quantity our firm would decide on in the present period
(m) how much money our firm had borrowed
(n) how our firm could decrease the interest charged
(o) that all costs should be covered
(p) that our firm should get as large a surplus per unit as possible
(q) that our firm should get a large market share

Q1 To what extent did you and the other participants in your firm have information about games played earlier (MiniMax)?

(a) we did not have any information at all
(b) we had some information, by rumors about earlier games
(c) we had information about prices and advertising from (some) earlier games

Q2 Which option do you think will best describe how your firm used the information you got about your competitors during the game?

(a) we did not discuss our competitors' decisions much
(b) we did discuss our competitors' decisions, but we did not have correct expectations of their decisions
(c) we did discuss our competitors' decisions, and we had correct expectations of their decisions

Q3 Which option is the best description for how your firm made decisions?

(a) discussions; decisions approved by all members
(b) decisions of majority; like voting
(c) someone or some of the members made the decisions

Q4 Try to estimate your motivation when participating in the game.

very motivated	medium	little motivated
----|---------------------	---|---	---------------------|----

Q5 Did the motivation in your firm change during the game?

(a) motivation increased (b) unchanged (c) motivation decreased

Q6 Did your firm make decisions in the last periods or what came to be the last period, with respect to the assumption that the game would end?

(a) we did not discuss the ending of the game very much
(b) we did discuss the end of the game, but we did not decide any differently
(c) we did discuss the end of the game, and decided accordingly

Q7 Try to estimate how much the table of market demand influenced the decision making in your firm?

much use	medium	little use

----|------------------------|------------------------|----

Q8 Try to estimate the similarity of the game with a real market.

much similarity	medium	little similarity

----|------------------------|------------------------|----

If you were to participate once more, what price and advertising would you decide in period 1?

Q9 Price $ _____ **Q10** Advertising $ _____

Appendix N – Form for decision alternatives discussed

The form used for capturing the decision alternatives discussed in each period is shown below with an example. Alternatives discussed for price and advertising were arranged into the following three levels: High, medium and low. Each alternative discussed was given an ordered number. In the example below, the first alternative discussed (1) was on *medium price* and the last alternative discussed (17) was on *medium advertising*.

Game___ Firm___ Period___	**Price**	**Advertising**
High Price > $35 Advertising > $130	8, 13	
Medium Price $30 - $35 Advertising $70 - $130	1, 3, 11, 12, 14, 15	2, 4, 5, 6, 7, 9, 16, 17
Low Price < $30 Advertising < $70		11

Appendix O – Form for expectations about competitors' decisions

Form used for capturing expectations of the adjustments of the decisions of the competitors.

Form expectations about the adjustments of prices and advertising by the other four firms in the game. That is, form expectations about the adjustments of prices and advertising as increased, unchanged or decreased from one period to the next. The firm with the highest total number of correct expectations will earn a place in the final. Unchanged price and unchanged advertising are adjustments to price or advertising of less than $ 1.

Adjustments in this period compared to the previous period:

Firm 1	increased price increased adv.	unchanged price unchanged adv.	decreased price decreased adv.
...			
Firm 5	increased price increased adv.	unchanged price unchanged adv.	decreased price decreased adv.

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Table 4.5 from page 64.

Variable	Description	Relation
K	Capacity at start of period	
S	Stocks at start of period	
C	Check. acc. at start of period	
E	Equity at start of period	$E = \sigma K + \chi S + C$
M	Marketing eff. at start of period	
p	Price in period	*Decision*
$p_{\bar{n}}$	Mean of prices of the other firms in period	$p_{\bar{n}} = \left(\sum_{j=1}^{n} p_j \right) / n$
a	Advertising in period	*Decision*
$\dot{M}$	Marketing eff. in period	$\dot{M} = M + a$
$\dot{M}_{\bar{n}}$	Mean of marketing eff. of the other firms in period	$\dot{M}_{\bar{n}} = \left(\sum_{j=1}^{n} \dot{M}_j \right) / n$
d	Demand in period	$d = d(p, p_{\bar{n}}, \dot{M}, \dot{M}_{\bar{n}})$
q	Sales in period	$q = \min[d, \dot{S}]$
o	Production in period	*Decision*
$\dot{S}$	Supply in period	$\dot{S} = S + o$
i	Investments in period	*Decision*
$\dot{K}$	Capacity in period	$\dot{K} = K + i$
P	Payments in period	$P = \sigma i + \chi o + a$
$\dot{C}$	Check. acc. after payments	$\dot{C} = C - P$
r	Interest rate	See Appendix B
$\ddot{K}$	Capacity at end of period	$\ddot{K} = (1-\delta)\dot{K}$
$\ddot{S}$	Stocks at end of period	$\ddot{S} = \dot{S} - q$
$\ddot{C}$	Check. acc. at end of period	$\ddot{C} = \dot{C} + pq + r\dot{C}$
$\ddot{E}$	Equity at end of period	$\ddot{E} = \ddot{C} + \sigma\ddot{K} + \chi\ddot{S}$
$\ddot{M}$	Marketing eff. at end of period	$\ddot{M} = \mu\dot{M} + \eta q$
Π	Profit of the period	$\Pi = \ddot{E} - E$

Table 4.5. Summary of variables in the market model.

Table 5.1 from page 71.

Optimal solution	Price	Advertising
MC – Myopic cooperative solution	p^{MC}	a^{MC}
DC – Dynamic cooperative solution	p^{DC}	a^{DC}
MN – Myopic non-cooperative solution	p^{MN}	a^{MN}
DN – Dynamic non-cooperative solution	p^{DN}	a^{DN}

Table 5.1. Optimal solutions and decisions on price and on advertising.

Table 6.9 from page 144.

Code	Objectives/reward	Code	Alteration
B1.1	Maximize equity	B1.2 B1.3 B1.4	Qualify for semifinal Qualify for final Win the final
B2.1	Champions of the year, diploma and small gifts	B2.2 B2.3	Monetary rewards (finals) Monetary rewards (lottery)
	Information Condition		**Alteration**
C1.1	Original reference point	C1.2 C1.3	Altered reference point No reference point
C2.1	One test period	C2.2	No test period
C3.1	Varied reports	C3.2 C3.3 C3.4 C3.5	All reports All reports incl. lost sales Three firms varied reports Two firms all reports
C4.1	Game played 5–10 periods	C4.2 C4.3	Game played 6 periods Randomized ending
C5.1	Symmetric	C5.2	Asymmetric
C6.1	No demand table	C6.2	Demand table
	Group composition		**Alteration**
D1.1	Students	D1.2	Professionals
D2.1	Game sessions composed according to registration numbers	D2.2	Game sessions deliberately composed
D3.1	Groups composed informally	D3.2	Groups randomized

Table 6.9. Summary of the procedures when the game was played.