Smooth transitions in macroeconomic relationships

Ann-Charlotte Eliasson

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Preface

A number of people have contributed to this thesis in various ways. I would like to express my sincere gratitude to my family, friends, colleagues and others for your never failing support and your ability to always be there for me. Passing on this long list of names I hope that those concerned will feel included.

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Ann-Charlotte Eliasson

Summary of the Essays

The purpose of this thesis is to explore the possibilities and advantages of describing macroeconomic relationships with a certain well-defined class of parametric nonlinear econometric models. The assumption of linearity has long dominated in macroeconometric modelling, and usually proved very successful. Nevertheless, situations arise where a more flexible specification is necessary and econometric modelling has to reflect that fact. In this thesis smooth transition regressions (STR) are applied to modelling a number of macroeconomic relationships. For an introduction to STR models see, e.g., Granger and Teräsvirta (1993) and Teräsvirta (1998a). An STR model is a flexible nonlinear model with a continuum of regimes. It is locally linear and transitions from one extreme regime to another are determined by a function of a continuous variable, called transition variable. Besides, certain discrete regime models such as a switching regression model with two regimes and an observable switching variable are nested in the STR class of models. The smooth transition between two locally linear extreme regimes is an attractive parameterisation when dealing with economic data since it often allows for straightforward interpretations of the estimated parameters.

This dissertation consists of four essays and each of them is shortly reviewed below.

Essay I, "*Smooth transitions in a UK consumption function*", reconsiders an equilibrium correction model of UK nondurable consumption by Davidson, Hendry, Srba and Yeo (1978), DHSY for short. Several studies have concluded that the DHSY model fails outside the original sample period, and this essay explores whether neglected nonlinearities or time-varying parameters can explain this failure. The DHSY model is re-estimated for two different samples; the original time period and an extended one. For each sample

two consumption functions are specified; one in first differences and the other in four-quarter differences. There are substantial differences between the four final nonlinear consumption functions. One reason is due to the fact that the two samples consists of different vintages of data which are not even cointegrated, see Hendry (1994). Another reason is that the choice of specifying the models in four-quarter or first differences does not seem to be innocuous.

The estimated models possess several interesting features. The assumption of the equilibrium correction term being stationary is not satisfied within the original observation period, which makes the DHSY model unbalanced. However, specifying an STR model yields a balanced equation since the estimated model is just locally equilibrium correcting. When the DHSY model is specified for first differences the STR equation is also balanced whereas the linear equation is unbalanced.

Estimating the DHSY consumption function for the extended sample both the model in four-quarter and the one in first differences give a similar result: the parameters are not constant. The implications of the model based on first differences accord with the findings of Harvey and Scott (1994) who claimed that the reason for the failure of the DHSY model is time-varying seasonality. The four-quarter difference model suggest a nonconstant equilibrium correction term where the equilibrium correction is assumed to include inflation with a time-varying coefficient. Putting these together, the findings of this essay support the idea that time-varying parameters are a possible explanation for the failure of the DHSY model rather than neglected nonlinearity.

Essay II, "*Nonlinear equilibrium correction and the UK demand for broad money, 1878-1997*" reconsiders a nonlinear equilibrium correction model of UK money demand presented in Ericsson, Hendry and Prestwich (1998), EHP for short. The results of this essay give further support to the idea that a nonlinear equilibrium correction mechanism is needed when modelling the demand for broad money in the UK with annual data. EHP, who advocated this notion, based their formulation of the equilibrium correction mechanism on the work of Escribano (1985). The present considerations indicate that this alternative may be generalized in at least two ways. Viewing the Escribano-type

equilibrium correction as an approximation to a specific STR-type parameterization as Teräsvirta (1998b) suggested leads to an STR model which encompasses the EHP equation.

On the other hand, modelling the UK money demand with the same variables as those EHP used but adopting a more general nonlinear approach leads to another STR model. According to this model the growth rate of national income is the transition variable. This implies that money demand is described by one linear equation when the economy faces an expansion and another when the economy is contracting and there is a smooth transition between the two extreme regimes. This model variance dominates the others but does not encompass them. In fact, both the EHP model and the first STR model seem to contain information that the more general model does not convey. These results may suggest a combination of the two STR models into an additive STR specification with two nonlinear components. On the other hand, the number of observations speak against a further extension of the model. Moreover, the second STR model pass all the standard misspecification test and may therefore be considered an appropriate specification within the family of single STR models in this application.

In Essay III, "*Detecting equilibrium correction with smoothly time-varying strength*" Monte Carlo simulations are used to explore the empirical power of the Johansen's cointegration test; see Johansen (1995) and the Lin and Teräsvirta parameter constancy test; see Lin and Teräsvirta (1994). This is done in a situation where the equilibrium correcting mechanism present in the model is nonlinear in the sense that the strength of the equilibrium correction varies smoothly over time. The purpose of the simulation study is to find out whether the existing tests when applied in succession are able to detect time-varying equilibrium correction. The cointegrating combination of variables is assumed to remain constant over time. The null hypothesis is defined as no cointegration and parameter constancy. However, there are no tests for testing both subhypotheses jointly, and a two step procedure is applied instead. First the hypothesis of no cointegration is tested and, if rejected, the test of parameter constancy is carried out. When testing the hypothesis of parameter constancy three different scenarios are considered. In the first case the cointegration relationship is assumed to be unknown and therefore estimated. In

the second case the cointegration relationship is assumed to be known. In the third case the cointegrated variables enter the specification without any restrictions. That is, the cointegrating relationship is not specified in advance, instead the cointegrated variables enter the model in levels with unknown coefficients. An exciting result is the fact that the power of the parameter constancy tests strongly improves when the cointegrated variables enter without restrictions compared to the case where the cointegrating relationship is estimated separately. The power becomes almost as high as it is when the cointegrating vector is assumed to be known. This result may have important implications to empirical work.

From the results of the essay it seems fair to conclude that the chances of a researcher to detect time-varying equilibrium correction when using the general modelling strategy of testing for cointegration, estimating a cointegrating vector and finally, testing constancy of the coefficient of the equilibrium correction term in an equilibrium correcting equation are rather small. This calls for a new modelling approach which is based on simultaneously testing the joint hypothesis of no cointegration and parameter constancy. Developing such an approach is, however, beyond the scope of the present work.

Essay IV is entitled "*Is the Phillips curve nonlinear? Empirical evidence for Australia, Sweden and the US*". Strong theoretical support for a nonlinear Phillips curve can be found in the literature, for a review see, e.g., Dupasquier and Rickets (1998). Different economic theories suggest different kinds of nonlinear relationships. In this essay a nonlinear specification is allowed for when estimating the Phillips curve for Australia, Sweden and the United States. The STR methodology makes it possible to estimate a nonlinear model without any a priori restrictions other than those implied by the assumption of the STR structure. When the STR model is estimated it is compared to various economic theories leading to nonlinear Phillips curves. The STR is applied to simple expectations-augmented Phillips curves.

The nonlinear Phillips curve of Australia has very interesting features. According to this specification the relationship between inflation and unemployment is negative most of the time but turns positive for large increases in the unemployment rate. This suggests that the empirically observed failure of the Phillips curve might be the result of a nonlinear

relationship and only mirrors a shift in the Phillips curve to a new level where the usual negative relationship is valid. The model also indicates that the NAIRU varies over time.

The Swedish Phillips curve appears nonlinear as well. The rate of change of inflation expectations is the key variable in this specification. It is more influential in describing the rate of change of inflation within the estimated model than lagged values of the endogenous variables. It is also the transition variable, which decides which regime is most adequate in describing the rate of change of inflation for each observation. The specification emphasize the importance of inflation expectations when monetary policy are committed to an inflation target.

The Phillips curve of the United States is characterized by strong inertia in the inflation process. In the final model specification the estimate of the intercept is not statistically significant which is surprising since the intercept can be viewed as a measure of the NAIRU. There is no evidence of nonlinearity or time-varying parameters and the Phillips curve appears to be linear in the United States, at least when the alternative to linearity is the STR model.

Interpreting the estimated nonlinear Phillips curves in the light of the economic theories which suggests a nonlinear relationship the two models support different theories. The asymmetry of the Australian Phillips curve is ruled by changes in unemployment which supports the theories where changes in aggregate demand is the reason for the nonlinearities. The nonlinearities of the Swedish Phillips curve are ruled by changes in inflation expectations but none of the theories consider asymmetries due to inflation expectations. However, by assuming that the firms have rational expectations and are forward looking changes in inflation expectations should result in changes in inflation. In that case the estimated Swedish Phillips curve could be viewed as supporting the theories where the asymmetries are due to changes in inflation.

The essays of this dissertation emphasize the importance of allowing for a flexible functional form when dealing with macroeconomic relationships. It may be worthwile to point out that the present class of nonlinear specifications chosen here forms just one of many potential alternatives. It does not follow that the other conceivable nonlinear specifications would necessarily be inferior to the STR model. On the other hand, choosing

a well-specified family as a basis for nonlinear modelling reduces the risk of spurious findings. Such findings may be more easily obtained if no restrictions on the parametric functional form are imposed a priori. But then, as the results of Essay III indicate, extending a standard linear analysis by considering nonlinear specifications may also lead to new challenging statistical problems for which adequate solutions still have to be worked out.

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Included Essays

Essay I

Smooth transitions in a UK consumption function*

1 Introduction

During the last twenty years equilibrium correction models (ECM) have successfully been applied to describing macroeconomic relationships. The advantage of the methodology is that it makes it possible to model short-run as well as long-run dynamics of integrated economic variables in a single model without running into problems of spurious regressions. In order for this to be true the integrated variables have to be cointegrated, see Engle and Granger (1987). A pioneering work in equilibrium correction modelling is Sargan's (1964) study of wages and prices in the UK, but the best-known example is the ECM of nondurable consumption in the UK presented in a seminal paper by Davidson, Hendry, Srba and Yeo (1978), DHSY for short.

Later evaluations of the DHSY model have shown that it fails outside the original observation period and various explanations of the failure have been suggested. Harvey and Scott (1994) claim that changing seasonality is the reason for the failure whereas Carruth and Henley (1990) and Gausden and Brice (1995) suggest time-varying parameters and neglected nonlinearities. This paper will reconsider the above explanations

*I am grateful to Neil Ericsson, David Hendry, Joakim Skalin and Timo Teräsvirta for insightful comments. I am also indebted to David Hendry and Luigi Ermini for providing the two data sets used in the paper. The responsibility for any errors or shortcomings remains mine.

by taking nonlinearity and time-varying parameters into account simultaneously. This is done by using the methodology of smooth transition regressions (STR), see Granger and Teräsvirta (1993). In that framework time-varying parameters are just another type of nonlinearity.

The analysis will be carried out for two different samples. The first sample is the one used in the DHSY study, 1958(2)-1970(4), and the second is an extended sample, 1958(2)-1992(2). It should be noted that the longer sample is not a simple extension of the original sample. The two data sets belong to different revisions of data and some variables have been redefined, see Hendry (1994). This will have consequences for the estimated consumption functions, and simple comparisons between the models will not be straightforward.

In the DHSY model the short-run dynamics are specified as annual changes (fourth differences). This might be too restrictive in the case of time-varying seasonality and this paper will also use an alternative specification of quarterly changes (first differences). Testing reveals a nonlinear relationship between the explanatory variables when the model is specified in fourth differences using the original sample period. But when it is defined for quarterly changes the model appears to be linear with time-varying parameters instead. For the second sample both the model specified in fourth differences and the one in first differences have time-varying parameters. The resulting nonlinear models variance dominate and encompass their linear equivalents for each sample.

There are large differences between the four final nonlinear consumption functions presented in this paper. This emphasizes the sensitivity to data transformations and choice of data set when specifying a UK consumption model. Considering this, it is not surprising that there are a wide variety of explanations to the DHSY model failure.

The outline of the paper is as follows. In Section 2 the DHSY model and some of the criticism against it is presented. Section 3 views the data that is used in the analysis and in Section 4 there is a short introduction to STR models. Section 5 suggests a specification of the consumption behaviour during 1958(2)-1970(4) and in Section 6, the period 1958(2)-1992(2) is analysed. Section 7 concludes.

2 Some previous consumption studies

There are numerous descriptions of UK nondurable consumers' expenditures. The choice of methodology is highly varied, see e.g. the DHSY model mentioned in Section 1, HUS-models introduced by Hendry and von Ungern-Sternberg (1981), periodic autoregressive models (PAR) used by Osborn (1988) and "structural models" estimated by Harvey and Scott (1994). In this study the DHSY model will be reconsidered and some of the criticisms are reviewed in this section.

In DHSY the authors start by comparing previously published empirical models of UK nondurable consumption. They point out inconsistencies and provide possible explanations for the discrepancies between the models. Finally they estimate an ECM for 1958(2)-1970(4). The short-run dynamics are specified as annual changes since the authors find this to be a sensible choice for capturing the consumption behaviour in different quarters. The long-run dynamics are captured by the fourth lag of the equilibrium correction term. The resulting model encompasses the models discussed earlier and it performs well in the forecasting period, 1971(1)-1975(4). The DHSY model has the form

$$\begin{aligned}\Delta_4 c_t &= \alpha_1 \Delta_4 y_t + \alpha_2 \Delta_1 \Delta_4 y_t + \alpha_3 (c-y)_{t-4} + \\ &\quad \alpha_4 \Delta_4 D_t^0 + \alpha_5 \Delta_4 p_t + \alpha_6 \Delta_1 \Delta_4 p_t + \varepsilon_t\end{aligned}$$

where c_t is consumption of goods and services, y_t is real personal disposable income, $\Delta_1\Delta_4 y_t$ is the first difference of the annual change of income, $(c-y)_{t-4}$ is the equilibrium correction term, D_t^0 is a dummy variable[1], $\Delta_4 p_t$ is the level of inflation and $\Delta_1\Delta_4 p_t$ the rate of change of inflation. Furthermore, Δ_j is the difference operator, $\Delta_j x_t = x_t - x_{t-j}$, and lower-case letters denote logarithms.

As mentioned in Section 1 the DHSY model fails outside the original sample, that is after 1976(2). According to Bollerslev and Hylleberg (1985) the DHSY model underpredicts consumers' expenditures over the period 1977(3)-1980(2); meanwhile Drobny and Hall (1989) find a distinct tendency for DHSY to overpredict consumption over the pe-

[1] $D_t^0 = D_{t-1} - D_t$. D_t is a dummy variable which assumes unity 1968(ii) and zero elsewere. It is included to capture a change in consumption as a result of information of a purchase tax increase in the 1968 budget. For a discussion of the dummy variable, see Davidson *et al.* (1978).

riod 1980-1986. Carruth and Henley (1990) claim that the DHSY model underpredicts consumption during 1985-1987. Although it is a commonly accepted view that the DHSY model breaks down outside the original observation period, the reason for the failure is still open to debate. A few comments are listed below.

- Hendry (1994) stresses the importance of data revisions and points out that recent data vintages are not cointegrated with earlier vintages. Hence, if the DHSY were estimated using present data vintages a different model would be chosen. The different data vintages are shown and compared in Section 3.

- Hendry, Muellbauer and Murphy (1990) in turn find that consumption, income and inflation are cointegrated, 1957-1976. This should imply a unit root in $\Delta_4 p_t$ and $(c-y)_{t-4}$. The unit root tests in Appendix A show that $(c-y)_{t-4}$ has a unit root 1958(2)-1970(4) whereas $\Delta_4 p_t$ contains a unit root only when the estimation period is extended to 1976(2). For the period 1958(2)-1970(4) the unit-root hypothesis is rejected.

- Carruth and Henley (1990) check whether or not the existing consumption models, such as HUS and DHSY, are adequate for describing quarterly UK nondurable consumption behaviour after the original sample periods. They estimate the DHSY model for 1969(2)-1984(4) and find that the (Chow) parameter constancy test fails. Since the purpose of their study is to evaluate existing models rather than to suggest a new model with time-varying parameters, they settle by concluding that a majority of the models they evaluate are inadequate for forecasting UK consumption.

- Harvey and Scott (1994) claim that seasonality varies over time and that this is the main reason for the failure of the DHSY model. In order to account for this effect the authors estimate a "structural model" both for the original data set (1958(2)-1970(4)) and for the extended one (1958(2)-1992(2)). The "structural model" is specified for quarterly changes instead of annual in the short-run dynamics and dummy variables are used to capture the seasonality. The authors find that the impact of seasonality changes over time in both samples and model this by introducing a stochastic seasonality component into the ECM specification. The "structural

model" out-performs the DHSY model: there is no error autocorrelation, which is a severe problem in the extended DHSY model, and the standard deviation of the structural model is 74% of that of the DHSY for the sample 1958(2)-1992(2).

- Gausden and Brice (1995) examine the performance of the DHSY model in predicting aggregate consumers' expenditures in UK during 1956(2)-1984(4) for seasonally adjusted and seasonally unadjusted data. They perform their study for both consumption of nondurable goods and services and total expenditures. According to their results it is the impact of income growth that changes over time making a more flexible specification necessary and, hence, they allow for time-varying parameters. To begin with they compute sub-period estimates of the income growth coefficient and find that it is not constant over time. Using a "variable-coefficient" model improves the goodness-of-fit within the sample compared to a "fixed-coefficient" applied to seasonally adjusted data for both total and nondurable consumers' expenditures.

- Song (1995) re-estimates the DHSY and the HUS consumption functions using time-varying parameters in order to improve the forecasting performance of the models, using quarterly data over the period 1961-1991. The time-variation is modelled using a Kalman filter where the parameters are assumed to be a function of their value in the last period as well as of lagged wealth and inflation. The model with time-varying parameters gives better forecasts than the original DHSY.

In four of the studies mentioned above neglected nonlinearities are suggested as the reason for the failure of the DHSY model. However, it is not obvious whether the problem lies in time-varying seasonality or nonlinear relationships between the variables. The analyses in Sections 5 and 6 will therefore consider both types of nonlinearities simultaneously. The main interest of this study lies in the dynamic properties of DHSY, the effect of the functional form on the fit and the conclusions from the model. The choice of variables is the same as in the DHSY specification. This means that other suggested equations for explaining consumer behaviour, such as those including, for example, wealth and interest

rate variables, credit constraints and demographic factors[2] are ruled out. This does not imply that these variables are superfluous in describing nondurable consumption, but it is beyond the scope of this paper to address other consumption functions than the DHSY.

3 The Data

This section presents some stylized facts of the quarterly observations of logarithmic nondurable consumers' expenditures (c_t) and logarithmic real personal disposable income (y_t), for the United Kingdom. The analysis will be carried out for two different time periods. The first period is the one used in the DHSY article, and the corresponding time series will be referred to as the original data. This time series, 1958(2)-1976(2), is from Economic Trends (1976 Annual Supplement) and is quarterly unadjusted in £ million at 1970 prices. The second time period is 1958(2)-1992(2), and is quarterly unadjusted in £ million at 1955 prices. It will be referred to as the extended data set. The two series belong to different vintages of data revisions and will therefore be shown in separate graphs.

The logarithms of nondurable consumption (c_t) and disposable income (y_t) can be found in Figures 3.1 and 3.2. The salient features of the data are a distinct seasonal pattern and strong positive trends in both c_t and y_t. In 1990 the growth slows down because of a recession. When the series are transformed into annual changes (four-quarter differences) the seasonality disappears from sight, see Figures 3.3 and 3.4. Consumption is smoother than income until 1985 while it is the other way around during 1985(3)-1988(3). From 1988(4) onwards the stylized fact of consumption being smoother than income seems to be valid again.

In Figure 3.5 the equilibrium correction term, $(c-y)_t$, is shown for both samples during 1958(2)-1976(2). The difference between the data vintages is conspicuous, especially from 1970 and onwards. The equilibrium correction term of the original sample is greater in absolute value than that of the extended sample based on a later vintage of data. The time series of the two vintages are not cointegrated according to results presented in Hendry

[2]See for example Hendry and von Ungern-Sternberg (1981), Hendry, Muellbauer, Murphy (1990).

(1994), which implies that there is no linear long-run relationship between the values of the variables for the different vintages.

4 Nonlinear modelling

In this section, modelling with smooth transition regression (STR) models is discussed. When nonlinearities in the conditional mean cannot be excluded a priori, a flexible nonlinear model specification is necessary. A large number of parametric and nonparametric nonlinear models exist in the literature. In modelling macroeconomic relationships the limited amount of data which is usually available sets a practical limit to the number of alternative models and techniques available. Smooth transition regression (STR) models enable a rich parameterisation of the conditional mean once linearity has been rejected, and these models may be applied to rather short time series since there exists a modelling strategy that does not necessarily require very long time series, Teräsvirta (1994). STR models have been successfully applied in macroeconometric modelling, see for example Lütkepohl, Teräsvirta and Wolters (1999), Michael, Peel and Taylor (1997) and Teräsvirta and Eliasson (1998) who model asymmetries in money demand functions, Michael, Nobay and Peel (1997) who describe nonlinear adjustment in real exchange rates, van Dijk and Franses (1999) who estimate a nonlinear relationship for interest rates, Teräsvirta (1998) who describes house prices and Johansen (1999) who models wage equations.

4.1 Smooth transition regression models (STR)

The exposition below is a brief introduction to STR models. For a more comprehensive presentation, see Granger and Teräsvirta (1993) or Teräsvirta (1998).

Start by considering the following STR model,

$$y_t = x_t'\varphi + x_t'\theta F(s_t) + u_t. \qquad t = 1, .., T. \tag{1}$$

The model consists of a linear and a nonlinear component. The linear component is $x_t'\varphi$, where $x_t = (1, x_{1t}, .., x_{mt})' = (1, y_{t-1}, .., y_{t-p}, z_{1t}, .., z_{qt})'$ is a $((m+1) \times 1)$ vector of explanatory variables where $m = p + q$. Furthermore, $\varphi = (\varphi_0, .., \varphi_m)'$ and $\theta = (\theta_0, .., \theta_m)'$

are parameter vectors. The nonlinear component is specified as a linear combination of variables multiplied with a nonlinear transition function $F(s_t)$ which is continuous and bounded. The error sequence $\{u_t\}$ consists of independent, identically distributed irregular terms such that $Eu_t = 0$, $Ex_t u_t = 0$, $Es_t u_t = 0$. It is customary to bound F between zero and unity; hence the model will change from $E(y_t|x_t) = x_t'\varphi$ to $E(y_t|x_t) = x_t'(\varphi + \theta)$ with the transition variable s_t. This feature makes it possible to capture different dynamics in different states of the economy.

The transition function of a kth order logistic smooth transition regression, LSTR(k), model is

$$F(s_t) = F(\gamma, c; s_t) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k} (s_t - c_i)\right\}\right)^{-1}, \ \gamma > 0, c_1 \leq c_2 \leq ... \leq c_k \quad (2)$$

where the slope parameter (γ) determines how rapid the transition is and the vector of location parameters (c) decides where the transitions occur. The restrictions $\gamma > 0$ and $c_1 \leq c_2 \leq ... \leq c_k$ are identifying restrictions. If $\gamma = 0$ the nonlinear STR model (1) becomes linear and if $\gamma \to \infty$ the STR model (1) becomes a step function. For $k = 2$ and $c_1 = c_2$ the LSTR(2) model closely approximates the exponential STR (ESTR) model. The ESTR model is specified as (1) with

$$F(s_t) = F(\gamma, c; s_t) = 1 - \exp\left\{-\gamma (s_t - c)^2\right\}, \ \gamma > 0.$$

In what follows the ESTR will be regarded as a special case of LSTR(2). Moreover, note that when $s_t = t$ the STR model (1) may be interpreted as a linear model with time varying parameters. This makes it possible to use the STR model to investigate the claims of unstable parameters in the DHSY model.

4.2 The modelling cycle for STR models

Following Teräsvirta (1998) the modelling cycle contains four stages.

1. Estimate a linear model with no error autocorrelation.

2. Perform the tests of linearity and parameter constancy in this model against an STR alternative. If either of the two hypotheses are rejected choose an STR model for further consideration.

3. Estimate the parameters of the STR model and reduce the size of the model if necessary.

4. Evaluate the final STR model using misspecification tests of no error autocorrelation, parameter constancy and no additive nonlinearity.

In the second step the null hypothesis of linearity is tested. The alternative is specified using several different variables as potential transition variables. If more than one of the null hypotheses are rejected, one starts by specifying an STR model using the transition variable which was used in the most forcefully rejected null hypothesis. The rationale behind this decision rule is discussed in Teräsvirta (1994). The initial choices of potential transition variables are provided by economic theory.

The third step consists of estimating the STR model. This will be done by nonlinear least squares (NLS) which is equivalent to conditional maximum likelihood estimation in the case of normal homoscedastic errors. The estimation is sensitive to the choice of starting values. The slope parameter γ is therefore standardized through division by the standard deviation of the transition variable for an LSTR(1) model and by the variance for an LSTR(2). It may be mentioned that when γ is large its estimate easily gets an inflated standard error estimate in small and even moderate samples. The reason for this is that an accurate estimation requires a sufficient number of observations of s_t in a small neighborhood of the location parameter, see e.g. Teräsvirta (1998). The parameters in the estimated full model will be reduced according to their explanatory power using the t-values as a guidance, except for $\hat{\gamma}$ and $\hat{c}$.

The last step of the modelling consists of model evaluation. The assumptions of no error autocorrelation and parameter constancy used in the parameter estimation are tested. Furthermore, it is interesting to know whether or not all nonlinearity of STR type is captured by the estimated model, so this should be checked as well. A test of no additive nonlinearity is available for this purpose. The misspecification tests are described in Section 4.4. If the model fails the test of no error autocorrelation, respecification of the model is the only solution. If the STR model fails the test of no additional nonlinearity or parameter constancy there are two different solutions. Respecification is again one

of them; the other alternative is to introduce another nonlinear component of the type that the alternative hypothesis indicated. However, this can only be considered for large samples.

4.3 Testing linearity

If the true data generating process is nonlinear and a linear model is used for describing the nonlinear process, the estimated model will be underidentified and the dynamics will be misspecified. On the other hand, if the true data generating process is linear and a nonlinear parameterisation is applied then the nonlinear model will overfit the data in the sample. In the post-sample period the model might perform badly, even if it satisfies all the diagnostic tests and variance dominates the linear model, see Granger and Teräsvirta (1992). Testing linearity before fitting a model helps to avoid such misspecifications.

The linearity test presented below follows Teräsvirta (1994). The hypothesis of linearity is tested against an LSTR model of order k. In order to derive the test some additional assumptions are necessary. The stochastic variables $x_{1t}, .., x_{mt}$ are assumed to be stationary. The transition variable s_t is also assumed to be stationary or to be a time trend (t), and all cross-moments $Ex_{it}x_{jt}$, $Ex_{it}s_t^m$, $Ey_{t-i}s_t^m$ and $Ey_{t-i}x_{jt}$, $m \leq 3$ are assumed to exist. The linearity test is derived under the assumption of constant variance and is not robust against conditional heteroscedasticity.

The LSTR(k) model becomes linear if $\gamma = 0$ so that the transition function (F) equals $\frac{1}{2}$. For notational simplicity the transition function in equation (1) is replaced by $\tilde{F}(\gamma, c; s_t) = F(\gamma, c; s_t) - \frac{1}{2}$. This implies $\tilde{F}(0, c; s_t) = 0$, and $H_0 : \gamma = 0$ is a natural hypothesis of linearity. Assuming normality of u_t, $t = 1, .., T$, the conditional log likelihood becomes

$$L = \sum_{t=1}^{T} \ell(\varphi, \theta, c; y_t | x_t, s_t) = -\frac{1}{2}\left(T \ln(2\pi) + T \ln \sigma^2 + \frac{1}{\sigma^2} \sum_{t=1}^{T} u_t^2\right). \tag{3}$$

There is a small caveat though: the STR model is not identified under the null hypothesis due to the nuisance parameters θ and c. That is, under the null θ and c can take on any values without affecting the value of the log-likelihood (3). In order to circumvent this difficulty the nonlinear component of (1) is Taylor-expanded around $\gamma = 0$. Using a kth

order Taylor expansion leads, after rearranging terms, to the STR model

$$y_t = x_t'\beta_0 + \sum_{j=1}^{k} \left(\tilde{x}_t s_t^j\right)' \beta_j + v_t \tag{4}$$

where $\tilde{x}_t = (x_{1t}, ..., x_{mt})'$ and $v_t = u_t + R(\gamma, \theta, c; x_t)$. The parameter vector β_j is a function of γ such that $\beta_j = 0$, $j = 1, .., k$, when $\gamma = 0$. The new null hypothesis becomes $H_0' : \beta_j = 0$, $j = 1, .., k$. The remainder term $R(\gamma, \theta, c; x_t)$ equals zero under the null hypothesis and does not affect the asymptotic distribution theory when the test is based on the LM-principle. The LM-type test will have an asymptotic χ^2 distribution with $k * \dim(\tilde{x}_t)$ degrees of freedom under the null. In small samples an F-version of the test is preferable to the χ^2 variant since it has good size and power properties; see for example Granger and Teräsvirta (1993, Chapter 7). The test can be carried out in stages:

1. Regress y_t on x_t and compute the residual sum of squares $SSR_0 = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t^2$.
2. Regress $\hat{u}_t$ (or y_t) on x_t, $\tilde{x}_t s_t$, $\tilde{x}_t s_t^2$ and $\tilde{x}_t s_t^3$ and compute $SSR_3 = \frac{1}{T}\sum_{t=1}^{T} \hat{w}_t^2$.
3. Compute the F statistic $F = \frac{(SSR_0 - SSR_3)/3m}{SSR_3/(T-4m-1)} \overset{approx}{\sim} F(3m, (T-4m-1))$ under H_0.

If H_0' is rejected the conditional mean is assumed to be nonlinear and an STR model is specified and estimated. If the null hypothesis of linearity cannot be rejected the conditional mean is assumed to be linear and no further modelling will be done.

A specification procedure for STR models is outlined in Teräsvirta (1994). After rejecting the hypothesis of linearity (H_0') the order of the LSTR model is selected. In this paper $k \leq 2$, hence LSTR(1) and LSTR(2) are the two parameterisations that will be used. The testing sequence is defined within equation (4) for $k = 3$:

$$\begin{aligned} H_{04} &: \beta_3 = 0, \\ H_{03} &: \beta_2 = 0 | \beta_3 = 0, \\ H_{02} &: \beta_1 = 0 | \beta_3 = \beta_2 = 0. \end{aligned}$$

Independently of whether a hypothesis is accepted or rejected it is still vital to carry out the whole sequence, since the choice between an LSTR(1) and an LSTR(2) specification

which are the main alternatives is based on p-values, Teräsvirta (1994). If the rejection of H_{03} is strongest in terms of p-values an LSTR(2) model is selected to describe the nonlinearity. Consequently, if H_{04} or H_{02} has the lowest p-value an LSTR(1) model is the final choice. The reason for including $\beta_3 = 0$ in the testing procedure when the choice stands between an LSTR(1) and an LSTR(2) model is to manage the situation where the only nonlinear element is the intercept. Consider the case when s_t is an element of x_t and $\theta = (\theta_0, 0, ., 0)'$, $\theta_0 \neq 0$; then $\beta_1 = 0$ even under the alternative and the test has no power. To remedy this situation $\beta_3 = 0$ is included in the testing sequence.

A special case of nonlinearity is when the transition variable is a time trend (t). This can be viewed as a model with time-varying parameters. Because of changes in institutions it is reasonable to be prepared for the possibility that some parameters change over time when dealing with macroeconomic variables. A standard assumption in many parameter constancy tests is that if a parameter changes it will only change once, as is the case in the Chow breakpoint test. Lin and Teräsvirta (1994) applied the STR framework to obtain a more general test by allowing the alternative to parameter constancy to be a smooth deterministic change. The special case of a discrete structural break is nested within the procedure. The transition function equals (2) with t as transition variable. Following Lin and Teräsvirta (1994) $k = 3$ is chosen as the order of the LSTR(k) model. This yields a flexible non-monotonous transition function. As for the linearity test $\tilde{F}(t, \gamma) = F(t, \gamma) - \frac{1}{2}$ will be used to simplify the notation and $H_0 : \gamma = 0$ in equation (2) is the hypothesis for parameter constancy; the alternative is $H_1 : \gamma > 0$. Testing for parameter constancy is similar to testing for linearity, see Lin and Teräsvirta (1994). The Taylor approximation is used to circumvent the problem with unidentified parameters (γ, c) under the null hypothesis. The Taylor expansion is applied to the STR model and after rearranging terms the auxiliary regression becomes

$$y_t = \delta_0' x_t + \delta_1' x_t t + \delta_2' x_t t^2 + \delta_3' x_t t^3 + v_t$$

The testing sequence is

$$LM3 \;\; : \;\; \delta_1 = \delta_2 = \delta_3 = 0$$

$$LM2 \;:\; \delta_1 = \delta_2 = 0 \mid \delta_3 = 0$$
$$LM1 \;:\; \delta_1 = 0 \mid \delta_2 = \delta_3 = 0.$$

As in the case of testing for linearity it is important to perform all tests before specifying a model. If the rejection of LM1 is strongest (measured by the p-value) an LSTR(1) model is selected, and if LM2 is the strongest rejected hypothesis an LSTR(2) model is the most appropriate. When dealing with time-varying parameters an even more flexible specification may be needed and thus if LM3 is the strongest rejected hypothesis an LSTR(3) model will be estimated.

4.4 Misspecification tests

The misspecification tests to be discussed here are those of (i) no error autocorrelation, (ii) constant parameters, and (iii) no remaining nonlinearity. These tests are presented in Eitrheim and Teräsvirta (1996) and Teräsvirta (1998). The model, under the alternative hypothesis, is specified as

$$y_t = G(\psi; w_t) + A(a; v_t) + u_t \qquad t = 1, .., T \tag{5}$$

where $G(\psi; w_t)$ is the estimated STR model and $\psi = (\varphi, \theta)'$ and $\{u_t\}$ is a sequence of independent standard normal variables. Different parameterisations of $A(a; v_t)$ yield the alternative hypotheses for the various types of misspecification tests that will be explored. The function $A(a; v_t)$ is assumed to be at least twice continuously differentiable for all a. The maximum likelihood estimates of ψ are assumed to be consistent and asymptotically normal under any null hypothesis to be considered. The LM test can be applied for testing the null hypothesis of no misspecification, that is $H_0 : a = 0$, but the F-statistic is preferable because of its superior size properties.

Test of no error autocorrelation

To test the hypothesis of no serial dependence in the conditional mean, the alternative is stated as a remaining serial dependence of order q. This gives the extended model (5) with $A(a; v_t) = a'v_t$, where $a = (a_1, ..., a_q)'$ and $v_t = (u_{t-1}, ..., u_{t-q})'$. The null hypothesis

of no error autocorrelation is equivalent to testing $H_0 : a = 0$. As mentioned in Section 4.4 the F-statistic is used. Under the null hypothesis the F-statistic is asymptotically distributed with q and $(T - \dim(\psi) - q)$ degrees of freedom.

Test of parameter constancy

The hypothesis of constant parameters is tested against the alternative of time-varying parameters of STR type. Under the alternative hypothesis the STR model becomes

$$y_t = \bar{x}_t'\tilde{\varphi}(t) + \tilde{x}_t'\tilde{\theta}(t)\,\tilde{F}(\gamma, c; s_t) + u_t \tag{6}$$

where $\tilde{\varphi}(t) = \tilde{\varphi} + \lambda_1 \tilde{F}_1(\gamma_1, c_1; t)$ and $\tilde{\theta}(t) = \tilde{\theta} + \lambda_2 \tilde{F}_1(\gamma_1, c_1; t)$. That is, all time varying variables are assumed to have the same structure. The transition function $\tilde{F}_1$ is defined as in (2) assuming $k = 3$ and $s_t = t$, and the null hypothesis of parameter constancy becomes $H_0 : \gamma_1 = 0$. The function $\tilde{F}_1(\gamma_1, c_1; t)$ is Taylor-expanded around $\gamma_1 = 0$ to circumvent the identification problem. Substituting it into equation (6) and rearranging terms yields

$$\begin{aligned} y_t &= \bar{x}_t'\beta_0 + (\bar{x}_t t)'\beta_1 + \left(\bar{x}_t t^2\right)'\beta_2 + \left(\bar{x}_t t^3\right)'\beta_3 + \\ &\quad \left[\tilde{x}_t'\beta_4 + (\tilde{x}_t t)'\beta_5 + \left(\tilde{x}_t t^2\right)'\beta_6 + \left(\tilde{x}_t t^3\right)'\beta_7\right]\tilde{F}(\gamma, c; s_t) + u_t^* \end{aligned} \tag{7}$$

where $u_t^* = u_t + (\bar{x}_t'\varphi + \tilde{x}_t'\theta)\,R(\gamma_1, c_1; t)$. The remainder term is equal to zero under the null and does not affect the asymptotic distribution theory. Model (7) can be regarded as a special case of model (5) with $A(a; v_t) = a'v_t$, where $a = (\beta_1, \beta_2, \beta_3, \beta_5, \beta_6, \beta_7)'$ and $\hat{v}_t = \left\{\bar{x}_t' t, \bar{x}_t' t^2, \bar{x}_t' t^3, (\tilde{x}_t' t, \tilde{x}_t' t^2, \tilde{x}_t' t^3)\,\tilde{F}(\gamma, c; s_t)\right\}'$. The null hypothesis can be rewritten as $H_0 : a = 0$ and the F-statistic is applied with $3\dim(a)$ and $(T - 4\dim(a) - 1)$ degrees of freedom.

Test of no remaining nonlinearity

The third misspecification test checks if all nonlinearity is captured against the alternative of additive nonlinearity of STR type. Under the alternative the model becomes

$$y_t = G(\psi; w_t) + \left(w_t'\psi\right) F^*(r_t; \gamma^*, c^*) + u_t \tag{8}$$

This STR model has two additive nonlinear components: one included in the original LSTR model is denoted $G(\psi; w_t)$ and the other is $(w_t'\psi) F^*(r_t; \gamma^*, c^*)$. The transition function F^* may be defined as equation (2) with $k = 1, 2$. For notational simplicity it is assumed that the transition variable r_t is an element of $\tilde{x}_t$. The function $F^*(r_t; \gamma^*, c^*)$ is Taylor-expanded around $\gamma^* = 0$ to circumvent the identification problem. Substituting the Taylor-expansion into equation (8) gives, after rearranging terms,

$$y_t = G(\psi; w_t) + (\tilde{x}_t r_t)' \beta_1 + (\tilde{x}_t r_t^2)' \beta_2 + (\tilde{x}_t r_t^3)' \beta_3 + u_t^* \tag{9}$$

where $u_t^* = u_t + R(\gamma_r, c_r; r_t)$ and $u_t^* = u_t$ under the null. Model (9) is a special case of model (5) with $A(a; v_t) = a'v_t$ where $a = (\beta_1', .., \beta_3')'$ and $v_t = (\tilde{x}_t' r_t, \tilde{x}_t' r_t^2, \tilde{x}_t' r_t^3)'$. The null hypothesis of parameter constancy becomes $H_0 : a = 0$. The F-statistic will have $3dim(a)$ and $(T - 4dim(a) - 1)$ degrees of freedom.

4.5 Encompassing

After specifying an STR model that passes all misspecification tests, it is necessary to find out if the model really is an improvement over the linear or previous nonlinear ones. The theory of encompassing seeks to resolve the problem of selecting among competing models that describe the same economic phenomenon. To find out if the STR model encompasses the linear model amounts to testing whether the former can explain the results of the latter. The reverse has to be checked too. If the models are not nested, a practical way of testing encompassing restrictions is to define a minimal nesting model (MNM; Mizon and Richard, 1986) that nests the competing models. If the STR model can explain the results of the MNM while the linear cannot, the former is said to encompass ($\mathcal{E}$) the latter. For a more comprehensive discussion of the theory of encompassing, see Hendry (1995, Chapter 14).

Here is an example of the encompassing test exploring whether an STR model encompasses a linear model and vice versa. The MNM is constructed by linearly completing the STR model with the variables that are included in the linear model (L) but do not enter the STR model. The encompassing test STR $\mathcal{E}$ L consists of testing the null hypothesis that these variables have zero coefficients in the MNM. A likelihood ratio test, see Teräsvirta

(1998), or an F-test can be used to compare the rival models. The null hypothesis is "the variables in the MNM not included in the STR model have zero coefficients".

As discussed above, one may also check whether the linear model can explain the results of the STR. In this case the MNM is not identified under the null hypothesis: L $\mathcal{E}$ STR. This problem is solved as before using a Taylor series expansion to approximate the transition function and thus the MNM. The encompassing test is equivalent to a linearity test of the type discussed in Section 4.3.

5 UK consumption behaviour, 1958(2)-1970(4)

In this section the original DHSY model and a variant in first differences will be considered applying the STR framework. Seasonal dummies are used to capture the seasonality in the latter model. This transformation makes it possible to explicitly model time-varying seasonality as in Harvey and Scott (1994). The modelling procedure is as follows. One starts by specifying a linear model and performing the linearity and parameter constancy tests. If either of the hypotheses is rejected an STR model is estimated and evaluated.

5.1 Annual changes

When modelling annual changes, i.e. four-quarter differences of consumption, the starting-point is equation (45) in DHSY. It has the form

$$\begin{aligned}
\Delta_4 c_t &= \underset{(.0032)}{.007}\ \Delta_4 D_t^0 + \underset{(.038)}{.47}\ \Delta_4 y_t - \underset{(.048)}{.21}\ \Delta_1\Delta_4 y_t \\
&\quad - \underset{(.021)}{.097}\ (c-y)_{t-4} - \underset{(.070)}{.13}\ \Delta_4 p_t - \underset{(.13)}{.30}\ \Delta_1\Delta_4 p_t + \hat{u}_t \qquad (10)\\
T &= 51, R^2 = .77, \hat{\sigma}_l = .0059, LJB = .06\,(.97)\,, AIC = -10.16, \\
A(2) &= 1.25\,(.94)\,, A\,(4) = .91\,(.46)
\end{aligned}$$

where the standard deviations of the parameter estimates appear in parentheses, T is the number of observations, R^2 the coefficient of determination, $\hat{\sigma}_l$ the estimated residual standard deviation and AIC the Akaike Information Criterion. The errors of equation (10) seem to be normal (LJB is the Lomnicki-Jarque-Bera test statistic of normality; the p-value is in parenthesis), and the LM tests of no ARCH against ARCH of at most order

q, denoted $A(q)$, do not indicate misspecification. There is a small discrepancy between the estimates in equation (10) and the original DHSY regarding the coefficient of the rate of change of inflation, which Harvey and Scott (1994) also obtained. Except for this the estimated values are the same as in DHSY.

In linearity tests against STR, all explanatory variables in equation (10), except the dummy variable $\Delta_4 D_t^0$, appear as transition variables. The results can be found in Table 5.1 and $\Delta_4 p_t$ seems to be the preferred transition variable since the hypothesis of linearity is most heavily rejected using $\Delta_4 p_t$ as the transition variable, see Section 4.3 for details. As mentioned in Section 2, the results of the unit-root tests in Appendix A reveal that not all variables of equation (10) are stationary. A unit root cannot be rejected for $(c-y)_{t-4}$ during the observation period. Hence, the variables on the left- and right-hand sides of equation (10) are of different orders of integration and equation (10) is said to be unbalanced. Because of this, there is a small caveat concerning p-values since the tests require stationary variables under H_0. Nevertheless, the smallest p-value is so small that the apparent lack of balance hardly affects any conclusions. The results of the model specification test sequence point at the LSTR(1) model. Furthermore, it should be noted that the parameters of equation (10) seem to be constant. Estimation of an LSTR(1) model for $\Delta_4 c_t$ yields

$$\begin{aligned}
\Delta_4 c_t &= \underset{(.0025)}{.009}\,\Delta_4 D_t^0 + \underset{(.038)}{.34}\,\Delta_4 y_t - \underset{(.034)}{.15}\,\Delta_1\Delta_4 y_t \\
&\quad - \underset{(.063)}{.18}\,(c-y)_{t-4} - \underset{(.076)}{.11}\,\Delta_4 p_t - \underset{(.20)}{.34}\,\Delta_1\Delta_4 p_t \\
&\quad + \left(\underset{(.0038)}{.016} + \underset{(.063)}{.18}\,(c-y)_{t-4} + \underset{(.20)}{.34}\,\Delta_1\Delta_4 p_t\right) \\
&\quad \left[1+\exp\left(\underset{(11.7)}{-12.4}\left(\Delta_4 p_t - \underset{(.0017)}{.022}\right)/\sigma_{(\Delta_4 p_t)}\right)\right]^{-1} + \hat{u}_t \qquad (11) \\
T &= 51, R^2 = .85, \hat{\sigma}_{nl} = .0049, \hat{\sigma}_{nl}/\hat{\sigma}_l = .84, LJB = .78\,(.68), \\
AIC &= -10.47, A(2) = .97\,(.39), A(4) = .57\,(.69).
\end{aligned}$$

where $\hat{\sigma}_{nl}$ is the estimated residual standard deviation of the model. The nonlinear model (11) variance dominates the linear model (10); its standard deviation is 84% of that of equation (10). In equation (11), the linear and nonlinear coefficients of $(c-y)_{t-4}$

and $\Delta_1\Delta_4 p_t$ are restricted to having the same size but opposite signs. The data strongly support these restrictions. They make the impact of equilibrium correction and the change in inflation approach zero as the value of the transition function approaches unity. The linear intercept is restricted to zero as in the DHSY specification while the nonlinear intercept enters significantly. The model survives a battery of misspecification tests. The Lomnicki-Jarque-Bera test of normality of the errors does not reject normality. There is no evidence of autoregressive conditional heteroscedasticity. Furthermore, equation (11) satisfies the assumptions of no error autocorrelation, no remaining nonlinearity and constant parameters according to the results of the misspecification tests in Table 5.2.

The speed of transition from one extreme regime to the other is rather rapid ($\gamma = 12$). This is seen from Figure 5.1 where the values of the estimated transition function are plotted against its argument, $\Delta_4 p_t$. Each dot represents an observation. The transition function attains value unity when $\Delta_4 p_t$ is large ($\geqslant 3\%$) and zero when $\Delta_4 p_t$ is small ($\leqslant 1\%$). Figure 5.2 shows the time path of the transition function.

The next step is to find out whether the nonlinear model (11) can explain the empirical findings of its linear competitor (10), that is, find out whether (11) $\mathcal{E}$ (10). Since all variables of the linear model (10) enter linearly in the STR model (11), (11) $\mathcal{E}$ (10), see Section 4.5. The opposite has to be checked too, that is whether (11) $\mathcal{E}$ (10). Testing the null hypothesis of linearity of equation (10) against the alternative of an LSTR(1) model with a constant, $(c-y)_{t-4}$, $\Delta_1\Delta_4 p_t$ in the nonlinear component and $\Delta_4 p_t$ as transition variable is rejected (p-value $= 0.0096$). Hence, (11) $\mathcal{E}$ (10) while the reverse is not true.

The functioning of the model is best seen from Figure 5.3 which shows the contribution of the equilibrium correction to $\Delta_4 c_t$ over time (solid line). In model (11), the equilibrium correcting term equals

$$ec_t = -0.18(c-y)_{t-4} + \{0.016 + 0.18(c-y)_{t-4}\}\, F\left(\Delta_4 p_t; \hat{\gamma}, \hat{c}\right) \tag{12}$$

where the combined intercept increases from zero to 0.016 when the value of the transition function changes from zero to unity. It thus compensates for the diminishing effect of the equilibrium correction as a function of $\Delta_4 p_t$. The figure shows how the mechanism equilibrium corrects when the inflation rate is low but does not (ec_t remains constant)

when the inflation rate is sufficiently high. The dotted line depicts the equilibrium correcting component of the original DHSY specification (10) where $ec_t = -0.097(c-y)_{t-4}$. This component appears to be trending and thus nonstationary, as the augmented Dickey-Fuller unit root test indicates; see Appendix A. On the other hand, equation (12) does not have a trend so that, at least visually, the LSTR specification seems a balanced equation.

The equilibrium correction mechanism may be given the following interpretation. The assumption of $(c-y)_t$ being stationary within the observation period is not correct. If the analysis is extended to cover data until 1976(2) then it appears that $\Delta_4 p_t$ is nonstationary and cointegrated with $(c-y)_t$. For the shorter period until 1970(4) $\Delta_4 p_t$ is stationary. However, c_t and y_t can be thought of as being "locally cointegrated" with coefficients 1 and -1 when the rate of inflation is low. The relationship disintegrates when the inflation rate is high. This may be compared with the discussion in Stewart (1998). In that paper it is pointed out that adding an intercept into model (10) renders the equilibrium correction term insignificant. Adding an intercept may therefore be viewed as another way of achieving a balanced equation, but at the cost of "losing" the long run. It also turns out that testing linearity with a model containing both an intercept and the equilibrium correction variable weakens the evidence for nonlinearity in the data. The p-value corresponding to 0.0041 in Table 5.1 becomes 0.067. However, estimating the corresponding fully parameterized LSTR(1) model and reducing its size inevitably leads to equation (11). The locally equilibrium correcting LSTR(1) model (11) is thus an intermediate specification between the unbalanced DHSY with global equilibrium correction and a linear model containing an intercept but no equilibrium correction or long-run equilibrium. The LSTR(1) model may also be given an economic interpretation: it suggests that the long-run savings ratio should be a nonlinear function of the current inflation rate. Nevertheless, the above, mainly statistical, interpretation may be the most plausible one. The small number of observations in the estimation period does caution against strong economic interpretations of equation (11).

5.2 Quarterly changes and seasonal dummies

DHSY wrote that their choice of differencing was based on the fact that $\Delta_4 c_t$ "represents a sensible decision variable when different commodities are being purchased in different quarters of the year." Harvey and Scott (1994), who focused on modelling evolving seasonal patterns, used $\Delta_1 c_t$ as the regressand. In this section their example is followed. The idea is to find out how that change affects the description of the equilibrium correction mechanism. $\Delta_4 y_t$ is also replaced by $\Delta_1 y_t$ but the annual inflation rate $\Delta_4 p_t$ will be retained. This is partly because it seems to be the measure of inflation which most agents use in practice, but also because the annual inflation rate is a potential I(1) variable; see Appendix A. It turns out that a linear model for the period 1958(2)-1970(4) requires extra dynamics in the form of two lags of $\Delta_1 c_t$. Incorporating these into the equation and estimating the parameters yields

$$\begin{aligned}
\Delta_1 c_t &= \underset{(.011)}{.067} + \underset{(.0033)}{.007}\,\Delta_4 D_t^0 - \underset{(.0067)}{.14}\,S_1 - \underset{(.017)}{.035}\,S_2 - \underset{(.019)}{.073}\,S_3 + \underset{(.059)}{.20}\,\Delta_1 y_t \\
&\quad - \underset{(.073)}{.23}\,\Delta_4 p_t - \underset{(.054)}{.082}\,(c-y)_{t-1} - \underset{(.13)}{.36}\,\Delta_1 c_{t-1} - \underset{(.13)}{.41}\,\Delta_1 c_{t-2} + \hat{u}_t \\
T &= 50, R^2 = .99, \hat{\sigma}_l = .0067, LJB = .46\,(.80)\,, AIC = -9.84, \qquad (13) \\
A(2) &= 0.51\,(.60)\,, A(4) = .40\,(.81)
\end{aligned}$$

While R^2 is high (not comparable with that of equation (10)), the residual standard error is 10% higher than that of model (10). Equation (13) is unbalanced for the same reason as equation (10), that is $(c-y)_{t-1}$ consists of a unit root while the other variables are stationary. It should be noted that model (13) contains a strongly significant intercept whereas the coefficient estimate of the equilibrium correction term seems insignificant (with a caveat for the interpretation of the t-ratio). This accords with the consumption model in fourth differences in Stewart (1998), where the inclusion of a constant in the original DHSY specification makes the equilibrium correction term insignificant.

In testing linearity with model (13) the choice of potential transition variables is affected by the fact that the first differences are strongly seasonal. The aim is not to consider possible nonlinearity of the intra-year seasonal pattern. The main interest lies, as before, in nonseasonal nonlinearity. Thus, annual differences $\Delta_4 c_{t-1}$ and $\Delta_4 y_t$ are used

as potential transition variables instead of $\Delta_1 c_{t-1}$ and $\Delta_1 y_t$.

The results of linearity and parameter constancy tests are found in Table 5.3. It is shown that linearity cannot be rejected for any potential transition variable whereas parameter constancy is strongly rejected. Further testing indicates that the rejection is due to nonconstancy of the coefficients of the stochastic explanatory variables. On the other hand, while the estimation results of Harvey and Scott (1994), see their Table 1, indicate stochastic variation in the seasonal pattern even in the original DHSY data, the parameter constancy tests here do not reject the null hypothesis of stable seasonal parameters. But then, their specification is not the same as the one used here.

The threesome of tests (LM1, LM2 and LM3) in Table 5.3 for testing the constancy of the coefficients of other than dummy variables point at an LSTR(2) model. Estimating this model yields an LSTR(2) model with $c_1 = c_2$. For ease of interpretation an ESTR model will be applied instead of the LSTR(2) (the transition function of an ESTR model is bounded between zero and one while in the LSTR(2) model it is bounded between d and unity; $0 \leq d \leq 1/2$). The estimated model has the form

$$\begin{aligned}
\Delta_1 c_t &= \underset{(.0092)}{.065} + \underset{(.0027)}{.0066}\,\Delta_1 D_t^0 - \underset{(.0055)}{.14}\,S_1 - \underset{(.013)}{.044}\,S_2 - \underset{(.016)}{.093}\,S_3 \\
&\quad + \underset{(.047)}{.21}\,\Delta_1 y_t - \underset{(.046)}{.16}\,(c-y)_{t-1} - \underset{(.071)}{.37}\,\Delta_4 p_t - \underset{(.10)}{.39}\,\Delta_1 c_{t-1} \\
&\quad - \underset{(.11)}{.62}\,\Delta_1 c_{t-2} + \left\{ \underset{(.075)}{.16}\,\Delta_4 p_t + \underset{(.036)}{.15}\,\Delta_1 c_{t-2} \right\} \\
&\quad \left[1 - \exp\left\{ -\underset{(0.74)}{1.38} \left(t/T - \underset{(.022)}{.45} \right)^2 / \sigma^2_{t/T} \right\} \right] + \hat{u}_t. \qquad (14) \\
T &= 50, R^2 = .99, \hat{\sigma}_{nl} = .0055, \hat{\sigma}_{nl}/\hat{\sigma}_l = .82, LJB = 0.99\,(.61)\,, \\
AIC &= -10.19, A\,(2) = 0.07\,(.93)\,, A\,(4) = 0.21\,(.93)\,.
\end{aligned}$$

where $\sigma^2_{t/T} = (t/T - \bar{t}/T)^2 / n - 1$ and $\bar{t} = \sum_{t=1}^{T} t/T$. The standard deviation of the ESTR model (14) is 82% of that of the linear model (13). The residuals seem to be normal and the hypothesis of no autoregressive conditional heteroscedasticity cannot be rejected. Equation (14) passes most misspecification tests; see Table 5.4. The test of parameter constancy for all parameters rejects the null hypothesis against another LSTR(2) component. However, the other tests do not give a strong indication about the source of the problem so that model (14) is tentatively accepted as an adequate specification for $\Delta_1 c_t$.

The transition function of the ESTR model (14) plotted against time is shown in Figure 5.6. The residuals in Figure 5.7 show that the nonlinear model (14) outperforms the linear (13) during the period when the transition function is close to zero.

The next step is to check whether or not the nonlinear ESTR model (14) can explain the empirical findings of the linear equation (13) and vice versa. In the former case the MNM equals the ESTR model (14) since all variables of the linear equation (13) are included in the linear part of equation (14). Thus (14) $\mathcal{E}$ (13). To see whether the linear model (13) $\mathcal{E}$ the ESTR model (14), a parameter constancy test against the alternative of an ESTR is applied. The nonlinear part consists solely of $\Delta_4 p_t$ and $\Delta_1 c_{t-2}$ as in equation (14). This null hypothesis is rejected (p-value $= 0.0080$); hence the linear model (13) does not encompass the ESTR model (14).

Equation (14) contains several interesting features. First, both the intercept and the equilibrium correction term have significant coefficient estimates. Second, the equilibrium correction term enters the equation linearly, which was not the case in (11). Third, the negative effect of the inflation rate on the consumption seems to have changed over time, being strongest in the beginning and at the end of the period. To obtain an idea of the statistical implications of this result, the extended "nonlinear equilibrium correction" has to be considered

$$\begin{aligned} ec_t &= -0.16\,(c-y)_{t-1} - 0.37\Delta_4 p_t + \{0.16\Delta_4 p_t\} \\ &\quad \left[1 - \exp\left\{-\underset{(0.74)}{1.38}\left(t/T - \underset{(.022)}{.45}\right)^2 / \sigma^2_{t/T}\right\}\right] \end{aligned} \tag{15}$$

The graph of variable (15) appears in Figure 5.8 together with that of $ec_t = -\underset{(.054)}{.082}\,(c-y)_{t-1} - \underset{(.073)}{.23}\,\Delta_4 p_t$. It seems that variable (15) is stationary, a conclusion supported by a unit root test of ec_t, see Table A.1. The nonlinear component thus neutralizes the nonstationarity of the equilibrium correction term $(c-y)_{t-1}$.

The common feature in equations (11) and (14) is that in each case, STR-type nonlinearity balances the equation. A plausible conclusion from both equations may be that the relationship between consumption, income and the annual rate of inflation in 1958-1970 appears nonlinear. One has to add, however, that the actual form of the relationship is still open to debate precisely because the two specifications are so different. It seems that

the number of observations in the estimation period is sufficiently small to leave room for different specifications.

6 UK consumption behaviour, 1958(2)-1992(2)

In this section the consumption models are estimated using the extended data set, 1958(2)-1992(2). Unfortunately the consumption models in this section are not directly comparable with the ones in the previous section since the data sets belong to different vintages. Some of the variables have been redefined and the variables of the two data sets are not even cointegrated, see Hendry (1994). Therefore, any comparisons between models estimated from the two samples have to be made with great caution.

As in Section 5 the consumption function will first be specified for annual changes, as in the original DHSY specification, and then for quarterly changes with dummy variables capturing the seasonality.

6.1 Annual changes

A linear model of nondurable consumption is estimated for the period 1958(2)-1992(2). The short-run dynamics are specified as annual changes (fourth differences) and the set-up is identical to the one in the original DHSY model. This specification results in a model with strong error autocorrelation and lagged values of the dependent variable are included, making the problem of autocorrelated errors vanish. The dummy D_t^0 captures the effects of changes in VAT in 1968, 1973 and 1979. The parsimonious linear model becomes

$$\begin{aligned}
\Delta_4 c_t &= \underset{(.0027)}{.014}\ \Delta_4 D_t^0 + \underset{(.035)}{.20}\ \Delta_4 y_t - \underset{(.012)}{.026}\ (c-y)_{t-4} - \underset{(.019)}{.041}\ \Delta_4 p_t \\
&\quad - \underset{(.068)}{.31}\ \Delta_1 \Delta_4 p_t + \underset{(.057)}{.70}\ \Delta_4 c_{t-1} + \hat{u}_t. \qquad (16)\\
T &= 137, R^2 = .85, \hat{\sigma}_l = .0085, LJB = 1.69\,(.43)\,, AIC = -9.50, \\
LMA\,(2) &= .87\,(.42)\,, LMA\,(4) = .81\,(.52)\,.
\end{aligned}$$

The residuals of equation (16) seem to be normal and there is no evidence of autoregressive heteroscedasticity in the error process. It can be noted that equation (16) variance dominates the "structural model" presented in Harvey and Scott (1994) in the sense that its residual standard deviation is 96% of that of the latter model when the models are estimated from the same sample, that is 1958(2)-1992(2).

A comparison between the linear consumption functions (10) and (16) reveals no obvious similarities but several discrepancies. The inclusion of lagged consumption ($\Delta_4 c_{t-1}$) in equation (16) makes some of the other variables redundant, and the magnitudes of the estimated coefficients of variables included in both models are quite different. Furthermore, it is not only $(c-y)_{t-4}$ that is a unit root process in equation (16) but also $\Delta_4 p_t$; see Appendix A. The unit root test indicates nonstationarity for $\Delta_4 c_t$ too. A closer look shows that when excluding the last six observations in the sample, thus only including data until 1990(4), the hypothesis of a unit root in $\Delta_4 c_t$ is only rejected at a 10% significance level, and thus the series is treated as stationary. Because of the cointegrating relationship between $(c-y)_{t-4}$ and $\Delta_4 p_t$, see Table A.3, and the fact that $\Delta_4 c_t$ is stationary, equation (16) can be regarded as a balanced equation.

In the next step linearity and parameter constancy tests are performed. The results of the tests can be found in Table 6.1. It turns out that the linearity test cannot be forcefully rejected for any of the transition variables, while there is evidence of time-varying parameters. The estimated LSTR(1) model with t/T as transition variable has the form

$$\begin{aligned}
\Delta_4 c_t &= \underset{(.0025)}{.012}\ \Delta_4 D_t^0 + \underset{(.033)}{.25}\ \Delta_4 y_t - \underset{(.012)}{.050}\ (c-y)_{t-4} - \underset{(.064)}{.26}\ \Delta_1 \Delta_4 p_t \\
&\quad + \underset{(.081)}{.45}\ \Delta_4 c_{t-1} + \left\{ - \underset{(.13)}{.23}\ \Delta_4 p_t + \underset{(.20)}{.37}\ \Delta_4 c_{t-1} \right\} \\
&\quad \left[1 + \exp\left\{ - \underset{(1.09)}{1.81} \left(t/T - \underset{(.18)}{.72} \right) / \sigma_{t/T} \right\} \right]^{-1} + \hat{u}_t. \qquad (17) \\
T &= 137, R^2 = .87, \hat{\sigma}_{nl} = .0081, \hat{\sigma}_{nl}/\hat{\sigma}_l = .95, LJB = .56\,(.76)\,, \\
AIC &= -9.58, LMA\,(2) = 2.14\,(.12)\,, LMA\,(4) = 1.35\,(.25)\,.
\end{aligned}$$

The standard deviation of the LSTR(1) model (17) is 95% of that of the linear model

(16) and 92% of that of the "structural model" in Harvey and Scott (1994). The residuals seem to be normal and the hypothesis of no autoregressive heteroscedasticity cannot be rejected. According to the results of the misspecification tests in Table 6.2, equation (17) satisfies the assumptions of no additive nonlinearity and parameter constancy. However, the hypothesis of no error autocorrelation is rejected against fourth-order autocorrelation at the 5% significance level. The reason for the misspecification might be the transformation into fourth difference which implicitly assumes constant seasonals. Harvey and Scott (1994) strongly reject constant seasonality, which at least implicitly makes modelling four-quarter differences a plausible idea in the present context.

The transition function of model (17) plotted against its argument t/T is shown in Figure 6.1. The transition is very smooth and it does not attain unity during the sample period. The residuals of equations (16) and (17) are shown in Figure 6.2. The graph shows the improved fit of the LSTR(1) model (17) during the 1990s compared to the linear equation (16). Except for this, the differences between the models are negligible.

There are few similarities between the nonlinear consumption functions (11) and (17). First and foremost, equation (11) is a nonlinear model whereas model (17) has time-varying parameters. Furhtermore, the system is equilibrium correcting in all periods in model (17) while it is locally equilibrium correcting in model (11). However, it is impossible to say whether the difference between the models depends on a different dynamic structure or is a result of data revisions.

Also, one would like to find out whether or not the LSTR(1) model (17) can explain the outcomes of the linear model (16). The MNM is created by including $\Delta_4 p_t$ linearly in equation (17). Under the null hypothesis the estimated coefficient of $\Delta_4 p_t$ is equal to zero. The F statistic $= 0.44$ (p-value $= 0.51$) and the hypothesis that (17) $\mathcal{E}$ (16) cannot be rejected. In order to find out if (16) $\mathcal{E}$ (17) a parameter constancy test is performed. The variables included in the nonlinear part are $\Delta_4 p_t$ and $\Delta_4 c_{t-1}$; when estimating the LM1 tests it turns out that the hypothesis can be rejected (p-value $= 0.0014$). Hence, (17) $\mathcal{E}$ (16) while the reverse is not true.

The effect of the time-varying coefficients of $\Delta_4 p_t$ and $\Delta_4 c_{t-1}$ in the LSTR(1) model (17) needs further discussion. In the nonlinear specification (17) $\Delta_4 p_t$ and $\Delta_4 c_{t-1}$ enter

nonlinearly and the coefficients are increasing over time (in absolute value). The time-varying coefficient of $\Delta_4 p_t$ has a mean value of -0.071 and it varies between -0.0028 and -0.20. Comparing it to the coefficient of the linear model (16) which equals -0.041, it turns out that the mean of the time-varying coefficient is larger in absolute value. Hence, the negative impact on consumption due to $\Delta_4 p_t$ is stronger in the nonlinear model. Moreover, the positive coefficient of $\Delta_4 c_{t-1}$ is also increasing over time, reducing the negative impact of $\Delta_4 p_t$ on consumption. It has a mean of 0.56, and it varies between $(0.45; 0.76)$. The coefficient of $\Delta_4 c_{t-1}$ in the linear equation (16) equals 0.70, making the positive impact of $\Delta_4 c_{t-1}$ smaller in the nonlinear model (17). These two effects yield a larger overall negative effect on consumption in equation (17) which is compensated for by stronger equilibrium correction.

In the nonlinear specification (17) $\Delta_4 p_t$ only enters nonlinearly and the absolute value of the coefficient is increasing over time. However, neither the specification with a constant coefficient of $\Delta_4 p_t$, equation (16), nor the one with a time-varying coefficient, equation (17) yields a stationary cointegrating relationship for the observation period. Furthermore, the results of the Johansen cointegrating test, Table A.3, reveal that the linear cointegrating relationship suggested by Hendry *et al.* (1990) ceases to exist when the sample is extended beyond 1990(4).

Also, neither of the models (16) and (17) has an intercept. This corresponds to the specification of the original DHSY model but there are two major differences. By including an intercept in equations (16) or (17) the significance of the equilibrium correction term is not affected and the intercept does not enter significantly.

Finally, because of the different features of the consumption functions of the two samples it seems hazardous to draw any conclusions about consumption behaviour in the UK.

6.2 Quarterly changes and seasonal dummies

In this section the quarterly changes of consumption 1958(2)-1992(2) are modelled. As in Section 5.2 seasonal dummies are used to capture the seasonality, and only inflation enters the model as an annual change to complete the cointegrating relationship. As

mentioned above, Harvey and Scott (1994) concluded that time-varying seasonality is the major reason for the failure of the DHSY model when using this sample. It is therefore interesting to see whether or not the STR approach leads to a similar conclusion. When estimating the DHSY model there is strong error autocorrelation and to remedy this extra dynamics, in the form of $\Delta_1 c_{t-4}$, have to be included in the specification. The inclusion of $\Delta_1 c_{t-4}$ renders some of the variables redundant and the linear model becomes, after some reduction,

$$\begin{aligned}
\Delta_1 c_t &= \underset{(.0024)}{.0060}\,\Delta_1 D_t^0 - \underset{(.0071)}{.032}\,S_1 - \underset{(.0027)}{.0048}\,S_2 - \underset{(.0026)}{.0052}\,S_3 \\
&\quad + \underset{(.043)}{.16}\,\Delta_1 y_t - \underset{(.020)}{.082}\,(c-y)_{t-1} - \underset{(.020)}{.047}\,\Delta_4 p_t \\
&\quad - \underset{(.072)}{.40}\,\Delta_1\Delta_4 p_t + \underset{(.057)}{.71}\,\Delta_1 c_{t-4} + \hat{u}_t. \qquad (18)\\
T &= 137, R^2 = .98, \hat{\sigma}_l = .0095, LJB = 2.17\,(.34)\,, \\
AIC &= -9.24, LMA\,(2) = .17\,(.85)\,, LMA\,(4) = .26\,(.90)\,.
\end{aligned}$$

According to the diagnostic tests the residuals seem to be normal and there is no evidence of autoregressive conditional heteroscedasticity.

There are several differences between equations (13) and (18). Because its estimate was not significant the intercept is excluded from equation (18). Furthermore, the lag structure is different in the two equations. The differences in the specifications also result in discrepancies regarding the magnitude of the coefficients for the variables that are included in both models. As concluded in Section 6.1, it is not possible to determine whether these differences are due to altered dynamics in consumption or if they are a consequence of the different data vintages.

The next step is to test for linearity and parameter constancy of equation (18). The choice of transition variables is made exactly as in Section 5.2. The results appear in Table 6.3. Both linearity and parameter constancy can be rejected. Parameter constancy for the seasonals is the most strongly rejected hypothesis and the test sequence points at an LSTR(1) specification. The estimated model becomes

$$
\begin{aligned}
\Delta_1 c_t &= \underset{(.0022)}{.0063}\,\Delta_1 D_t^0 - \underset{(.0095)}{.055}\,S_1 + \underset{(.0096)}{.0054}\,S_2 - \underset{(.0027)}{.012}\,S_3 + \\
&\quad \underset{(.037)}{.19}\,\Delta_1 y_t - \underset{(.022)}{.16}\,(c-y)_{t-1} - \underset{(.019)}{.056}\,\Delta_4 p_t - \underset{(.064)}{.31}\,\Delta_1\Delta_4 p_t \\
&\quad + \underset{(.064)}{.46}\,\Delta_1 c_{t-4} + \left\{ - \underset{(.0089)}{.017}\,S_1 - \underset{(.0089)}{.026}\,S_2 \right\} \\
&\quad \left[1 + \exp\left\{ - \underset{(1.82)}{2.38}\left(t/T - \underset{(.11)}{.44} \right) / \sigma_t \right\} \right]^{-1} + \hat{u}_t \qquad (19) \\
T &= 137, R^2 = .98, \hat{\sigma}_{nl} = .0087, \hat{\sigma}_{nl}/\hat{\sigma}_l = .91, LJB = 1.55\,(.46)\,, \\
AIC &= -9.39, LMA\,(2) = .70\,(.50)\,, LMA\,(4) = .58\,(.68)\,.
\end{aligned}
$$

The residual standard deviation of the LSTR(1) model (19) is 91% of that of the linear model (18) and 99% of that of the "structural model" presented in Harvey and Scott (1994). The residuals seem to be well behaved in the sense that they pass the tests of normality and no autoregressive conditional heteroscedasticity. The results of the misspecification tests can be found in Table 6.4. Equation (19) satisfies the assumption of no error autocorrelation but the seasonals are still time-varying according to the parameter constancy tests. An LSTR(3) model would also be possible since the LM3 test in Table 6.2 is also heavily rejected. Estimating this yields a model where two of the location parameters get out of range, and excluding them yields the LSTR(1) model (19). Nevertheless, comparing the significance levels of the rejection of parameter constancy in Table 6.3 and Table 6.4, there is a distinct improvement. The hypothesis of no additional nonlinearity can be rejected at the 5% significance level when $\Delta_4 y_t$ is used as the transition variable. Except for this the nonlinearity seems to be captured by the nonlinear components of equation (19).

Figure 6.3 shows the transition function of model (19) plotted against time; the amplitude of the seasonal fluctuations is increasing over time as is shown in Figure 6.4. The residuals appear in Figure 6.5. It can be seen that the STR model (19) has a better fit than the linear model (18) overall and not just at some selected quarters.

In order to find out whether the LSTR(1) model (19) can explain the empirical results of the linear model (18) and vice versa, all variables of the linear equation (18) are also included linearly in equation (19) so that the specifications are nested, that is (19) $\mathcal{E}$ (18).

The reverse has to be checked as well, that is to find out whether or not (18) $\mathcal{E}$ (19). This is done by testing if the parameters of the linear model (18) are constant against the alternative of time-variation of the LSTR(1) specification. The null hypothesis can be rejected and, hence, (19) $\mathcal{E}$ (18), while the reverse is not true.

Comparing the consumption functions specified for quarterly changes in equations (14) and (19), there are more differences than similarities. Equation (14) is characterised by time-varying parameters, while on the other hand equation (19) is characterised by time-varying seasonality. Except for this the lag structure is different, making a direct comparison between the parameters difficult.

The most interesting feature of the nonlinear equation (19) is that the impact of seasonality has been increasing over time. Consumption in the first and second quarters has decreased compared to consumption in the third and fourth quarters. In the fourth quarter consumption is higher than in any other quarter. The results probably mirror the increase in spending around Christmas during the observed period. According to Figure 6.4 the time-variation of the coefficients wears off by the middle of the 1980s and seems to stabilise in the beginning of the 1990s. When allowing for time-varying seasonals, the negative impact of the first quarter is larger than it was in the linear model (18), which implies a systematic lower consumption if it was not compensated for by stronger equilibrium correction in all periods in equation (19).

Thus a general conclusion is that model (19) accords with the results of Harvey and Scott (1994), who also found evidence of time-varying seasonality when estimating the DHSY consumption function using the same sample.

7 Final remarks

In this paper four nonlinear specifications of the DHSY consumption function have been presented. When estimated for the original sample and four-quarter differences the DHSY model is nonlinear with a nonlinear equilibrium correction term. When data is specified in first differences the model is characterised by time-varying parameters and a constant equilibrium correction term. But, when specifying the DHSY consumption function for

the extended sample, both the model in four-quarter and the one in first differences give similar results: the parameters are not constant. The model based on first differences accords with the findings of Harvey and Scott (1994), who claimed that the reason for the failure of the DHSY model is time-varying seasonality. The four-quarter difference model suggests a nonconstant equilibrium correction term when the equilibrium correction is assumed to include inflation which is changing over time. Putting these together, the findings of this paper support the idea that time-varying parameters are a possible explanation for the failure of the DHSY model.

The differences between the final model specifications are striking. One reason is probably due to the fact that the data used are from two different vintages which are not even cointegrated, see Hendry (1994). Another reason is that the choice of specifying the models in four-quarter or first differences does not seem to be innocuous. The features of the final models, when estimated for the same sample, are conspicuous.

The assumption of the equilibrium correction term being stationary is not satisfied within the original observation period, making the DHSY model unbalanced. However, specifying an STR model yields a balanced equation. This is also true for the ESTR model specified for first differences. The LSTR(1) model defined for four-quarter differences is locally equilibrium correcting, that is, when the inflation rate is low the mechanism equilibrium corrects but when the inflation rate is sufficiently high, making the equation stationary, it does not. When the linear DHSY model is specified for first differences the equilibrium correction term does not enter significantly but it is still kept in the model because of theoretic arguments. In the corresponding STR model, in which the time-varying coefficients are parameterised, the equilibrium correction term enters linearly and significantly. The inclusion of the equilibrium correction term could in principle make the model unbalanced but in the ESTR model the "nonlinear equilibrium correction" is stationary and, hence, the model is balanced. Again the STR specification transforms the unbalanced linear equation into a balanced one.

Finally, the choice of STR models is just one out of many nonlinear alternatives. This does not imply inferiority of other nonlinear specifications and there may be other nonlinear models that also shed light on consumption. However, in the STR framework one is able to investigate both nonlinearity and parameter constancy simultaneously.

Appendix A: Unit root tests

The results of the Augmented Dickey-Fuller (ADF) tests for a unit root are shown below. A constant and a deterministic trend are included for all variables. The main criteria for determining the number of lags included are the Ljung-Box Q-statistic and the Akaike Information Criterion. The results of the tests can be found in Table A.1 for the original data set and Table A.2 for the extended data set. According to Table A.1 it is only the equilibrium correction term $(c-y)_t$ that seems to be characterized by a unit root 1958(2)-1970(4). However, when the sample is extended to include data until 1976(2) the hypothesis of a unit root cannot be rejected for $\Delta_4 p_t$ either. This supports the results of Hendry, Muellbauer and Murphy (1990) where they find a cointegrating relationship between the equilibrium correction term and inflation, $(c-\alpha_1 y-\alpha_2\Delta_4 p)_t$. In Table A.3 the results of the Johansen cointegration tests are shown for different samples.

According to Table A.2. the values of the ADF test statistics differ from those in Table A.1 for 1958(2)-1970(4), but the order of integration remains the same so the discrepancies in the test statistics do not change any conclusions. However, extending the sample up to 1992(2) a unit root cannot be rejected for $\Delta_4 c_t$ at the 10% significance level. The null hypothesis is rejected at a 10% significance level for $\Delta_4 y_t$ but not on 5%. Apart from this, $(c-y)_t$ and $\Delta_4 p_t$ are both nonstationary.

The results of the cointegration tests in Table A.3 show that there is a cointegrating relationship between $(c-y)_t$ for the whole sample and between $(c-y-\Delta_4 p)_t$ until 1990(4).

Exogeneity tests of the variables will not be performed. Davidson and Hendry (1981) examine the exogeneity of income and find that it cannot be rejected. Hendry, Muellbauer and Murphy (1990) test and reject weak exogeneity defined as in Engle, Hendry and Richard (1983).

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Graphs

Figure 3.1: Logarithms of consumption and income 1958(2)-1976(2).

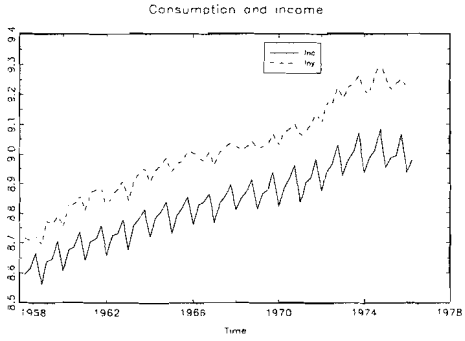


Figure 3.2: Logarithms of consumption and income 1958(2)-1992(2)

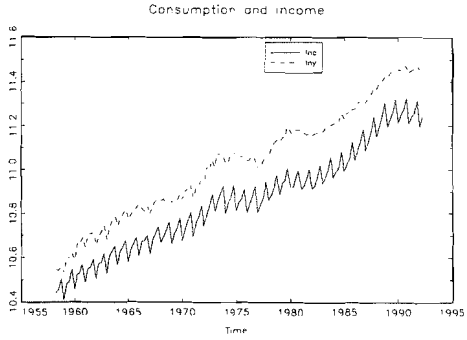


Figure 3.3: Annual changes of the logarithms of consumption and income 1958(2)-1976(2).

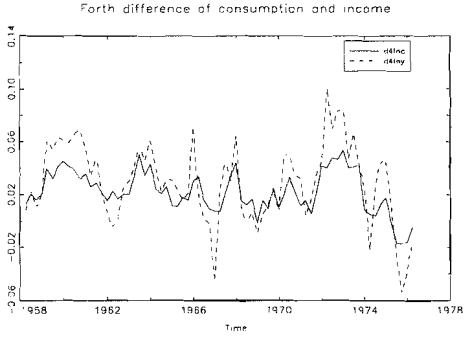


Figure 3.4: Annual changes of the logarithms Of consumption and income 1958(2)-1992(2).

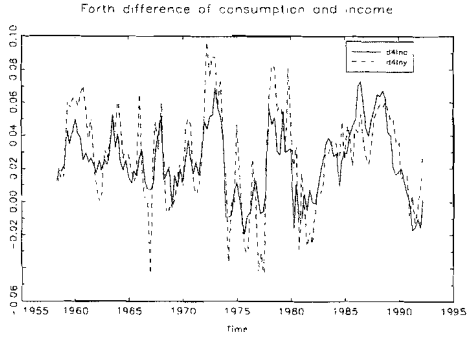


Figure 3.5: The equilibrium correction term for the two different data vintages, 1958(2)-1976(2).

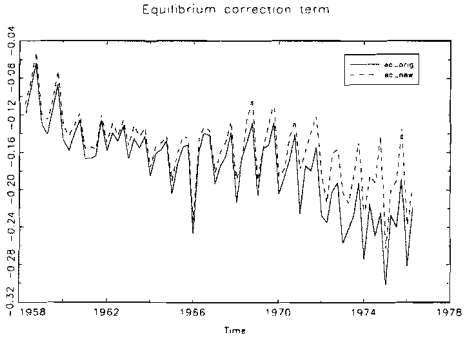


Figure 5.1: Values of the transition function of (5.2) plotted against the transiti variable $\Delta_1 p_t$.

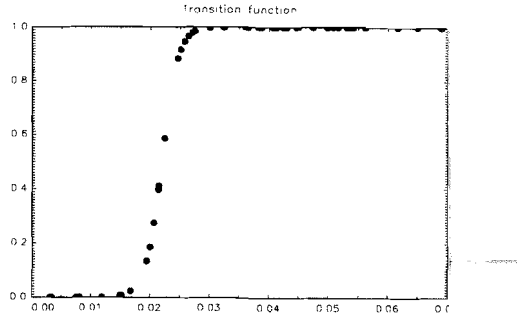


Figure 5.2: Transition function of (5.2) plotted against time, 1958(2)-1970(4).

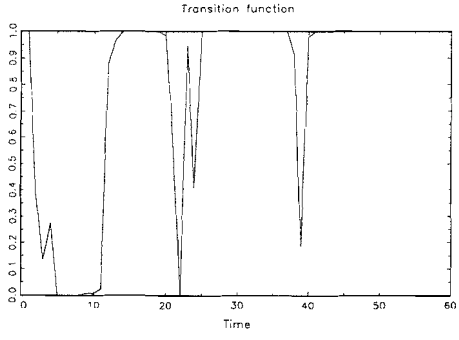


Figure 5.3: Linear and nonlinear equilibriu correction of (5.1) and (5.2) plotted against time, 1958(2)-1970(4).

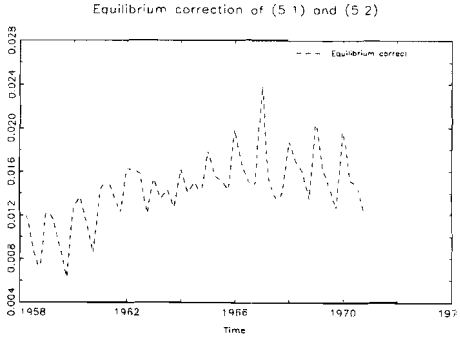


Figure 5.4: Residuals of (5.1) and (5.2), plotted against time, 1958(2)-1970(4).

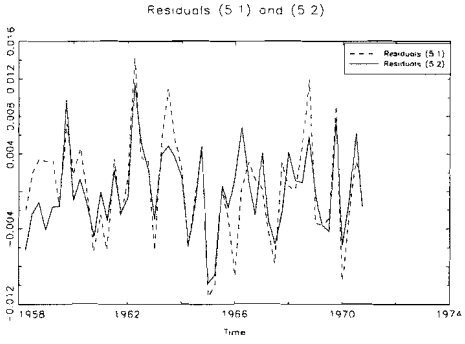


Figure 5.5: Differences in consumption betw (5.1) and (5.2), 1958(2)-1970(4). The constan the nonlinear part replaces the joint effect of $(c-y)_{t-4}$ and $\Delta_1\Delta_4 p_t$.
Diff $= -.18 * (c-y)_{t-4} - .34 * \Delta_1\Delta_4 p_t - .016$

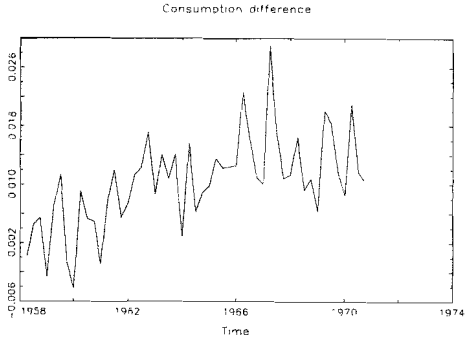


Figure 5.6: The transition function of the ESTR model (5.5), 1958(3)-1970(4).

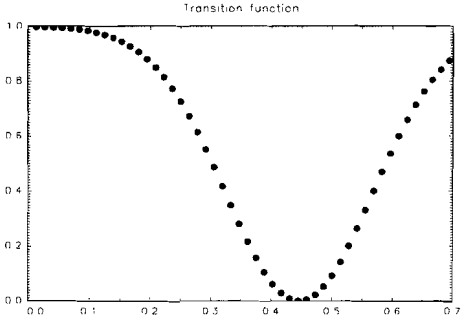


Figure 5.7: Residuals of (5.4) and (5.5), 1958(3)-1970(4).

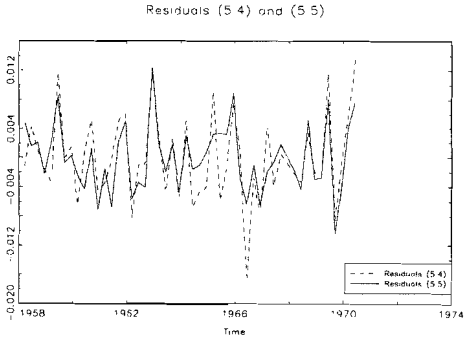


Figure 5.8: Linear and nonlinear equilibrium correction of (5.4) and (5.5) plotted against time, 1958(3)-1970(4).

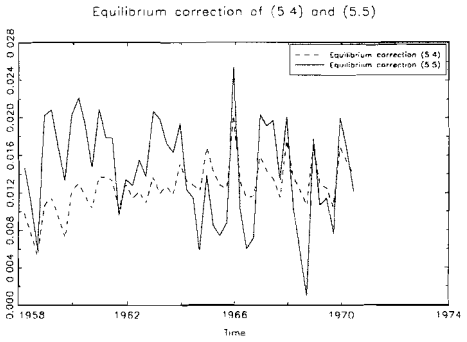


Figure 6.1: Transition function of (6.2) plotted against the transition variable $\Delta_4 y$ 1958(2)-1992(2).

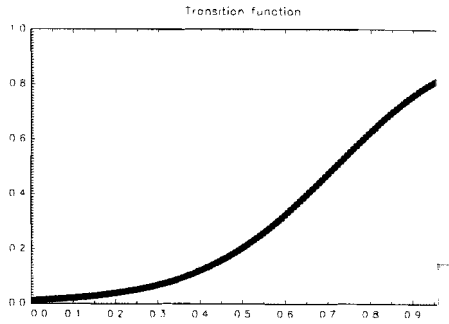


Figure 6.2: Residuals of consumption functions (6.1) and (6.2), 1958(2)-1992(4).

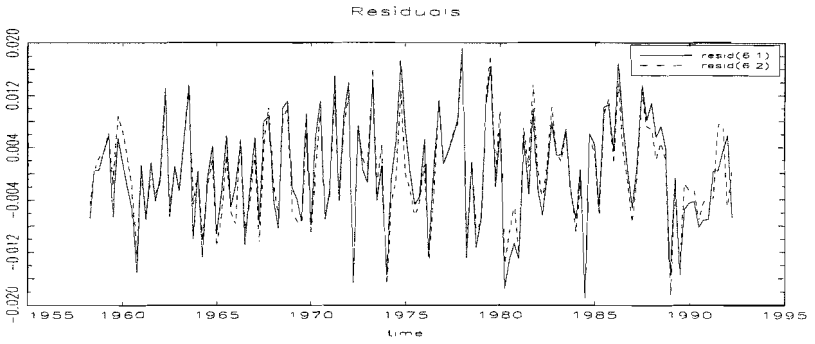


Figure 6.3: Transition function of the non-linear part of (6.4), where t/T is transition variable, 1958(2)-1992(2).

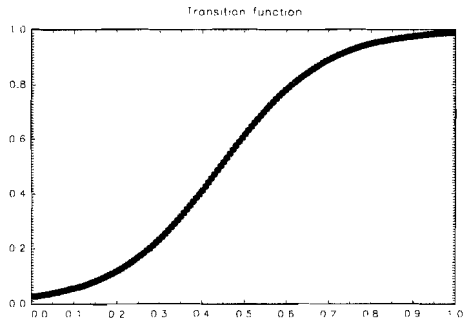


Figure 6.4: The time-varying seasonal pattern in the STR-model (6.4), 1958(2) -1992(2). (-.017s1-.026*s2)*tf.

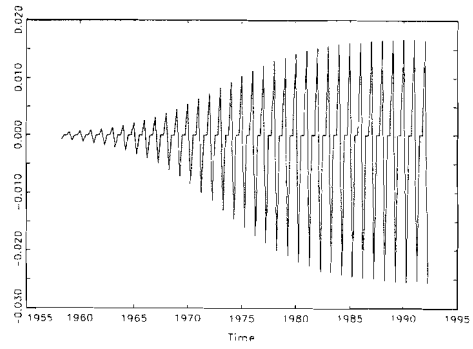
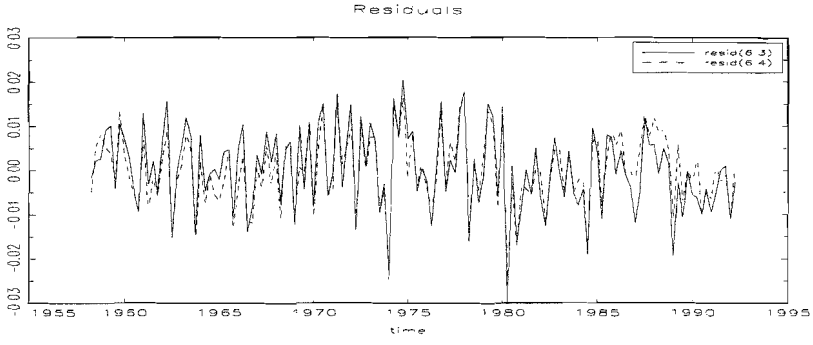


Figure 6.5: The residuals of (6.3) and (6.4), 1958(2)-1992(2).



Tables

Table 5.1: *p*-values of the LM tests of linearity and parameter constancy of the DHSY consumption function (5.1), 1958(2)-1970(4). The hypotheses are presented in Section 4.3.

Linearity test	Transition variables $\Delta_4 y_t$	$\Delta_1\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$
F_0	0.28	0.83	0.21	0.17
F_{04}	0.19	0.91	0.83	0.37
F_{03}	0.68	0.35	0.88	0.21
F_{02}	0.22	0.70	0.0041	0.16

Parameter constancy test	Null hypothesis (1)
LM3	0.42
LM2	0.42
LM1	0.25

The null hypothesis is:

(1): "All parameters except the coefficient of the dummy variable are constant."

The remaining parameters not under test are assumed constant in each case.

Table 5.2: *p*-values of the misspecification tests of the LSTR(1) consumption function (5.2), 1958(2)-1970(4).

LM test of no error autocorrelation against an AR(q) and MA(q) error process in the LSTR(1) model (5.2).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.62	0.56	0.56	0.60	0.62	0.56

p-values of tests of no additive nonlinearity in the LSTR(1) model (5.2) for a set of transition variables.

Linearity test	Transition variables			
	$\Delta_4 y_t$	$\Delta_1\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$
F_0	0.63	0.89	0.76	0.86
F_{02}	0.68	0.63	0.94	0.82

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 5.2 cont. *p*-values of parameter constancy tests of the LSTR(1) model (5.2) against STR type nonconstancy.

Tests of parameter constancy	Null hypothesis		
Test	(1)	(2)	(3)
LM3	0.35	0.78	0.68
LM2	0.63	0.89	0.85
LM1	0.91	0.70	0.83

The null hypotheses are:

(1): "All parameters except the coefficient of the dummy variable are constant."
(2): "All parameters of the linear part except the coefficient of the dummy variable are constant."
(3): "All parameters of the nonlinear part are constant."

The remaining parameters not under test are assumed constant in each case.

Table 5.3: *p*-values of the LM tests of linearity and parameter constancy of the linear consumption function (5.4), 1958(3)-1970(4). The hypotheses are presented in Section 4.3.

Linearity test	Transition variables $\Delta_4 y_t$	$\Delta_1\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$	$\Delta_4 c_{t-1}$
F_0	0.46	0.85	0.34	0.27	0.56
F_{04}	0.40	0.84	0.71	0.29	0.68
F_{03}	0.68	0.32	0.15	0.15	0.33
F_{02}	0.25	0.90	0.19	0.78	0.39

Tests of parameter constancy	Null hypothesis		
Test	(1)	(2)	(3)
LM3	0.0092	0.88	0.049
LM2	0.030	0.81	0.027
LM1	0.40	0.44	0.46

The null hypotheses are:

(1): "All parameters except the coefficient of the dummy variable are constant."
(2): "The parameters of the seasonals are constant."
(3): "All parameters of the exogenous variables are constant."

The remaining parameters not under test are assumed constant in each case.

Table 5.4. *p*-values of the misspecification tests of the nonlinear consumption function (5.5), 1958(3)-1970(4).

LM test of no error autocorrelation against an AR(q) and MA(q) error process in the nonlinear model (5.5).

Test	Maximum lag	q				
	1	2	3	4	5	6
No error autocorrelation	0.96	0.74	0.90	0.92	0.85	0.62

p-values of tests of no additive nonlinearity in (5.5) for a set of transition variables.

Linearity test	Transition variables				
	$\Delta_4 y_t$	$\Delta_1\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$	$\Delta_4 c_{t-1}$
F_0	0.86	0.38	0.58	0.70	0.76
F_{02}	0.58	0.46	0.96	0.62	0.93

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 5.4 cont: p-values of parameter constancy tests of the nonlinear model (5.5) against STR type nonconstancy.

Tests of parameter constancy	Null hypothesis			
Test	(1)	(2)	(3)	(4)
LM3	0.11	0.30	0.35	0.65
LM2	0.017	0.048	0.18	0.81
LM1	0.22	0.23	0.77	0.57

The null hypotheses are:

(1): "All parameters except the coefficient of the dummy are constant."
(2): "The linear parameters except the coefficient of the dummy are constant."
(3): "All linear seasonal parameters are constant."
(4): "All nonlinear parameters are constant."

Table 6.1: *p*-values of the LM tests of linearity and parameter constancy of the linear consumption function (6.1), 1958(2)-1992(2). The hypotheses are presented in Section 4.3.

Linearity test	Transition variables $\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$	$\Delta_4 c_{t-1}$
F_0	0.25	0.37	0.31	0.065
F_{04}	0.32	0.69	0.60	0.73
F_{03}	0.065	0.25	0.10	0.0089
F_{02}	0.88	0.25	0.50	0.30

Parameter constancy test	Null hypothesis (1)
LM3	0.049
LM2	0.027
LM1	0.012

The null hypothesis is:

(1): "All parameters except the coefficient of the dummy variable are constant."

The remaining parameters not under test are assumed constant in each case.

Table 6.2: *p*-values of the misspecification tests of the LSTR(1) consumption function (6.2), 1958(2)-1992(2).

LM test of no error autocorrelation against an AR(q) and MA(q) error process in the LSTR(1) model (6.2).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.59	0.77	0.77	0.029	0.056	0.10

p-values of tests of no additive nonlinearity in the LSTR(1) model (6.2) for a set of transition variables.

Linearity test	Transition variables			
	$\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$	$\Delta_4 c_{t-1}$
F_0	0.43	0.37	0.47	0.29
F_{02}	0.98	0.83	0.13	0.59

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 6.2 cont. *p*-values of parameter constancy tests of the nonlinear model (6.2) against STR type nonconstancy.

Tests of parameter constancy	Null hypothesis		
Test	(1)	(2)	(3)
LM3	0.69	0.81	0.39
LM2	0.33	0.59	0.29
LM1	0.54	0.95	0.11

The null hypotheses are:

(1): "All parameters except the coefficient of the dummy variable are constant."
(2): "All parameters of the linear part except the coefficient of the dummy variable are constant."
(3): "All parameters of the nonlinear part are constant."

The remaining parameters not under test are assumed constant in each case.

Table 6.3. *p*-values of the LM tests of linearity and parameter constancy of the linear consumption function (6.3), 1958(2)-1992(2). The hypotheses are presented in Section 4.3.

Linearity test	Transition variables $\Delta_4 y_t$	$\Delta_1\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$	$\Delta_4 c_{t-1}$
F_0	0.043	0.024	0.0034	0.014	0.024
F_{04}	0.38	0.30	0.46	0.0071	0.30
F_{03}	0.24	0.0029	0.13	0.20	0.0029
F_{02}	0.015	0.59	0.00045	0.35	0.59

Tests of parameter constancy	Null hypothesis		
Test	(1)	(2)	(3)
LM3	0.00057	0.000061	0.26
LM2	0.0023	0.00019	0.13
LM1	0.00020	0.000017	0.020

The null hypotheses are:

(1): "All parameters except the coefficient of the dummy variable are constant."
(2): "The parameters of the seasonals are constant."
(3): "All parameters of the exogenous variables except seasonals are constant."

The remaining parameters not under test are assumed constant in each case.

Table 6.4. *p*-values of the misspecification tests of the nonlinear consumption function (6.4), 1958(2)-1992(2).

LM test of no error autocorrelation against an AR(q) and MA(q) error process in the nonlinear model (6.4).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.45	0.26	0.44	0.61	0.63	0.75

p-values of tests of no additive nonlinearity in (6.4) for a set of transition variables.

Linearity test	Transition variables				
	$\Delta_4 y_t$	$\Delta_1\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta_1\Delta_4 p_t$	$\Delta_4 c_{t-1}$
F_0	0.044	0.31	0.74	0.15	0.18
F_{02}	0.019	0.96	0.83	0.14	0.14

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 6.4. cont. *p*-values of parameter constancy tests of the nonlinear model (6.4) against STR type nonconstancy.

Tests of parameter constancy	Null hypothesis			
	(1)	(2)	(3)	(4)
Test				
LM3	0.039	0.16	0.10	0.0076
LM2	0.018	0.047	0.14	0.0038
LM1	0.15	0.16	0.089	0.58

The null hypotheses are:

(1): "All parameters except the coefficient of the dummy are constant."
(2): "The linear parameters except the coefficient of the dummy are constant."
(3): "All linear seasonal parameters are constant."
(4): "All nonlinear seasonal parameters are constant."

Table A.1: Unit root tests for all variables, 1958(2)-1970(4) and 1958(2)-1976(2) using the DHSY data. *, ** and *** indicates 10%, 5% and 1% rejection levels.

	1958(2)-1970(4)		1958(2)-1976(2)	
Variable	F-value	Lag length	F-value	Lag length
$\Delta_4 c_t$	-5.16***	3	-3.58**	3
$\Delta_1 c_t$	-3.90***	3	-3.26*	3
$\Delta_4 y_t$	-5.15***	3	-4.09**	3
$\Delta_1 y_t$	-3.92***	3	-3.93***	3
$(c-y)_t$	-1.19	4	-1.33	4
$\Delta_4 p_t$	-3.34*	4	-2.65	4
$\Delta_1 \Delta_4 p_t$	-4.46***	4	-3.59**	3
(5.6)	-4.57***		0.69	

Table A.2: Unit root tests for all variables, 1958(2)-1970(4) and 1958(2)-1992(2). *, ** and *** indicates 10%, 5% and 1% rejection levels.

	1958(2)-1970(4)		1958(2)-1992(2)	
Variable	F-value	Lag length	F-value	Lag length
$\Delta_4 c_t$	-4.43***	3	-3.07	5
$\Delta_1 c_t$	-4.31***	6	-3.87**	10
$\Delta_1\Delta_4 c_t$	-5.35***	3	-8.81***	3
$\Delta_4 y_t$	-4.27***	3	-3.42*	4
$\Delta_1 y_t$	-3.76**	3	-5.36***	3
$(c-y)_t$	-1.55	4	-2.43	4
$\Delta_1(c-y)_t$	-3.90**	4	-5.26***	4
$\Delta_4 p_t$	-3.21*	4	-1.65	8
$\Delta_1\Delta_4 p_t$	-5.36***	3	-4.83***	4

Critical values are
1958(2)-1970(4): -4.15***, -3.50**, -3.18*
1958(2)-1976(2): -4.09***, -3.47**, -3.16*
1958(2)-1992(2): -4.02***, -3.44**, -3.15*

Table A.3: Cointegration tests for (c_t, y_t) and (c_t, y_t, $\Delta_4 p_t$) for different sample periods. The null hypothesis is no cointegration and, *, ** and *** indicates 10%, 5% and 1% rejection levels.

Time period	**H_A: c_t and y_t are cointegrated**	**H_A: c_t, y_t and $\Delta_4 p_t$ are cointegrated**
1958(2)-1970(4)	LR = 28.29***	---
1958(2)-1975(4)	LR = 21.69***	LR = 35.21***
1958(2)-1980(4)	LR = 22.63***	LR = 45.09***
1958(2)-1985(4)	LR = 20.05***	LR = 39.23***
1958(2)-1990(4)	LR = 15.14**	LR = 29.95**
1958(2)-1992(2)	LR = 13.28**	LR = 23.00

Essay II

Nonlinear equilibrium correction and the UK demand for broad money, 1878-1997*

1 Introduction

Long macroeconomic time series form a challenge for those who argue that certain basic economic relationships remain unchanged over time and want to find support for their arguments with constant-parameter models. Hendry and Ericsson (1991) developed such a model for the UK money demand in 1878-1975. Ericsson, Hendry and Prestwich (1998), henceforth EHP, recently remarkably successfully extended the model to cover the years 1878-1993. Their model is a single-equation error-correction model in which short-term dynamics are built around a long-run theory-based equilibrium relationship. The model is not, however, a standard error-correction model. It contains a nonlinear error-correction mechanism which is specified following Escribano (1985). This type of model involves higher powers of the attractor or error-correction term; see EHP. It turns out that the nonlinear error-correction is a prerequisite for parameter constancy. If it is replaced by an ordinary linear error-correction mechanism, other things equal, the hypothesis of pa-

*I am grateful to Neil Ericsson, David Hendry who generously allowed me to use their data set. I also wish to thank both for stimulating discussions. The responsibility for any errors or shortcomings remains mine.

rameter constancy, when tested by an appropriate stability test, is rejected. Nonlinearity thus plays a key role in the modelling effort of EHP.

In this paper we reconsider the nonlinear error-correction in EHP. Teräsvirta (1998a) recently suggested that the Escribano-type error-correction in that paper may be seen as a first-order approximation to an error correction characterized by smooth transition regression (STR); see, for example, Granger and Teräsvirta (1993) and Teräsvirta (1998b). We shall depart from this possibility by generalizing the error-correction mechanism in EHP directly using the STR framework. Sarno (1998) recently applied a similar idea when modelling Italian money demand using long annual (1861-1991) time series. Alternatively, and this we shall do first, one may start from another step back and model the nonlinearity in the data with an STR model, applying the modelling cycle described in Teräsvirta (1998b). This involves finding the appropriate transition variable by a specification search and leads to an error-correction mechanism that differs from that in EHP. The results of the two approaches are compared with each other and with the Escribano-type model in EHP. Encompassing tests, among other things, are applied for this purpose. It appears from the results that the STR-based error-correction is an improvement over the specification in EHP.

The outline of the paper is as follows. Section 2 reviews the economic theory EHP applied and reminds the reader of their dummy variables. Section 3 discusses the actual modelling including the results, and Section 4 concludes.

2 Economic theory and data

EHP pointed out that in a modern economy, money may be demanded as an inventory to smooth differences between income and expenditure streams but it also forms an asset in a multi-asset portfolio. These demands yield a classical long-run specification money demand specification

$$M^d = g(P, I, \Delta P, \mathbf{R}) \tag{1}$$

where M^d is the nominal money demanded, P is the price level, ΔP is inflation and $\mathbf{R}$ is a vector of rates of returns of a set of assets. In econometric work, (1) is often log-

linear, except that interest rates enter in levels. EHP specified (lowercase letters denote logarithms) the equation

$$m^d - p = \beta_0 + \beta_1 i + \beta_2 \Delta p + \beta_3 R^{own} + \beta_4 R^{out} \tag{2}$$

where R^{own} and R^{out} are the own and the outside rates of interest. Equation (2) defines the error-correction relationship whose linearity is not called into question. Nonlinearity in this paper has to do with the strength of the attraction, not the form of the attractor itself.

In this paper we use the same data set as EHP. For descriptions of data, including the dummy variables, and discussion on the quality of observations we refer to that paper. Dummy variables include D_1 and D_3 (they are assumed to have the same coefficient) for the two World Wars, and D_c for financial deregulation starting 1971. Finally, there is D_4 which attains value unity between 1971-1975 and only appears together with the short-term interest rate as $D_4 \Delta_1 rs_t$.

3 Reconsidering the error-correction mechanism

3.1 Testing linearity and specifying an STR model

As mentioned in Introduction, one possibility in reconsidering the money demand equation of EHP is to follow the modelling strategy outlined in Teräsvirta (1998b). In order to do that we begin with a linear equation containing, with a single exception, the same variables as EHP included. We test linearity of that model against STR and, if rejected, specify and estimate an error-correcting STR model for the money demand. The natural starting-point is equation (20), henceforth called (ehp), in EHP. For comparison, we report results of misspecification tests of (ehp); see Tables 3.1 and 3.2. In particular, we test the parameter constancy because EHP does not contain tests against smoothly changing parameters that are applied in this paper. It is seen that while (ehp) has constant parameters there appears to be some autocorrelation left in the errors.

Prior to any modelling, equation (ehp) is modified in two ways. First the term $\Delta_1^2 (m-p)_{t-2}$ is replaced by the straightforward lag $\Delta_1 (m-p)_{t-2}$ because that slightly

improves the fit. Second, to obtain a linear model to serve as a baseline equation for linearity tests we substitute the error-correction term $\tilde{u}_{t-1}$ for $(\tilde{u}_{t-1} - .2)\,\tilde{u}_{t-1}^2$ which represents the nonlinear error-correction. Note that the error-correction term $\tilde{u}_{t-1}$ is estimated directly from (2) : we use the same estimated values as EHP.

To complete the notation, let rn_t be the opportunity cost[1] whereas rl_t is the long-term nominal interest rate. Estimation of the perfectly linear model yields

$$\begin{aligned}
\Delta_1 (m-p)_t &= \underset{(.059)}{.51}\ \Delta_1 (m-p)_{t-1} - \underset{(.046)}{.12}\ \Delta_1 (m-p)_{t-2} - \underset{(.044)}{.64}\ \Delta_1 p_t \\
&\quad + \underset{(.048)}{.44}\ \Delta_1 p_{t-1} - \underset{(.0062)}{.020}\ \Delta_1 rn_t - \underset{(.016)}{.057}\ \Delta_2 rl_t - \underset{(.015)}{.069}\ \tilde{u}_{t-1} + \underset{(.0023)}{.011} \\
&\quad + \underset{(.0057)}{.034}\ (D_1 + D_3)_t + \underset{(.0076)}{.052}\ D_{ct} + \underset{(.028)}{.11}\ D_4 \Delta_1 rs_t + \hat{e}_t \qquad (3) \\
T[1878-1993] &= 116, R^2 = .85, \hat{\sigma}_l = .017, AIC = -8.03, LJB = .15(.93), \\
A(1) &= .99\,(.32)\,, A(4) = .68\,(.60)\,, RESET : F(1,104) = .0057\,(.94)
\end{aligned}$$

The errors of (3) are normal (LJB is the Lomnicki-Jarque-Bera test of normality; the p-value in parentheses). The LM tests of no ARCH against ARCH of at most order q, denoted $A\,(q)$, do not indicate misspecification, neither does the second order RESET. Table 3.3 shows that the errors are not serially correlated. However, in contrast to (ehp), (3) does not have constant parameters. The rejection of parameter constancy against smoothly changing parameters (Teräsvirta, 1998b) is very strong, as the results in Table 3.4 indicate. Nonlinear error-correction clearly is a prerequisite for parameter stability.

Linearity tests in Table 3.5 show that linearity is rejected against STR for three potential transition variables. One of them is $\tilde{u}_{t-1}$, which given the results of EHP is not surprising, and $\tilde{u}_{t-1}^2$ causes a rejection as well. Furthermore, the specification test sequence with $\tilde{u}_{t-1}$ as the transition variable leads to an LSTR(2) model as H_{03} is rejected more strongly than H_{02} and H_{04}. (See the Appendix for definitions of models and hypotheses.) In accord with this outcome the same sequence with $\tilde{u}_{t-1}^2$ as the transition variable suggests an LSTR(1) model. Nevertheless, the rejection of linearity is strongest for $\Delta_1 i_t$, so that we first construct an LSTR model for $\Delta_1 (m-p)_t$ with $\Delta_1 i_t$ as the tran-

[1]The opportunity cost is defined as: $Rn = (H^a/M^a)\,RS$, where H is high-powered money, RS is short-term interest rate and ^{a} denotes actual values.

sition variable. The specification test sequence in this case does not unambiguously point at either one of LSTR(1) and LSTR(2), and we tentatively select the higher-order one of the two.

Estimation of an LSTR(2) model yields, after a parameter reduction,

$$\begin{aligned}
\Delta_1 (m-p)_t &= \underset{(.10)}{.83}\, \Delta_1 (m-p)_{t-1} - \underset{(.039)}{.16}\, \Delta_1 (m-p)_{t-2} - \underset{(.040)}{.62}\, \Delta_1 p_t \\
&+ \underset{(.069)}{.64}\, \Delta_1 p_{t-1} - \underset{(.0053)}{.015}\, \Delta_1 rn_t + \underset{(.046)}{.092}\, \tilde{u}_{t-1} \\
&+ \underset{(.0047)}{.035}\, (D_1 + D_3)_t + \underset{(.0070)}{.071}\, D_{ct} + \underset{(.022)}{.12}\, D_4 \Delta_1 rs_t \\
&+ \left\{ \underset{(.0024)}{.016} - \underset{(.11)}{.49}\, \Delta_1 (m-p)_{t-1} - \underset{(.065)}{.34}\, \Delta_1 p_{t-1} - \underset{(.018)}{.072}\, \Delta_2 rl_t - \underset{(.048)}{.19}\, \tilde{u}_{t-1} \right\} \\
&\times \left[1 + \exp\left\{ - \underset{(.98)}{2.68} \left(\Delta_1 i_t + \underset{(.0068)}{.052} \right) \left(\Delta_1 i_t - \underset{(.0051)}{.011} \right) / \hat{\sigma}^2_{\Delta_1 i_t} \right\} \right]^{-1} \\
&+ \hat{e}_t \qquad (4) \\
T[1878-1993] &= 116, R^2 = .90, \hat{\sigma}_{nl} = .014, \hat{\sigma}_{nl}/\hat{\sigma}_l = .82, AIC = -8.36, \\
LJB &= .67(.72), A(2) = 1.00\,(.32)\,, A(4) = .80\,(.53)\,.
\end{aligned}$$

where $\hat{\sigma}^2_{\Delta_1 i_t}$ is the sample variance of the transition variable. The residual standard deviation of (4) is 82% of that of (3) while the corresponding ratio for (ehp) is 93%. The errors of (4) appear normal and the LM tests of no conditional heteroskedasticity do not cause any alarm. Results of the standard misspecification tests (Eitrheim and Teräsvirta, 1996; Teräsvirta, 1998b) can be found in Tables 3.6, 3.7 and 3.8. It is seen that the errors of (4) are free from autocorrelation and that the model has constant parameters. The tests of no additive nonlinearity indicate that this model adequately characterizes all nonlinearity present in the data.

The transition function of (4) as a function of the transition variable $\Delta_1 i_t$ is depicted in Figure 3.1. The transition between the extreme regimes is rather smooth. Broadly speaking, the error correction mechanism is only operating when the economy (real income) is growing. There are, however, some very low (< -0.04) values of $\Delta_1 i_t$ for which the local dynamics of the system are the same as for high growth rates. These correspond to 1931 and the rather exceptional years following the end of World War I. Figure 3.1 suggests that downweighting those observations while specifying the STR model is likely to lead to

an LSTR(1) model. Against this background it is not surprising that the model selection test sequence does not offer a clear choice between LSTR(1) and LSTR(2) alternatives. According to (4) the short-term dynamic behaviour of the real money is also a nonlinear function of the growth rate of the economy. The money demand in the short run depends more strongly on its own past when the error-correction is not operating (the transition function has a small absolute value) than is the case when the system is error-correcting. Furthermore, the impact of inflation on real money increases with the growth in nominal income as does the negative impact of the long interest rate.

3.2 Direct generalization of the error-correction component in EHP

Since the income variable also appears in the error-correction term $\tilde{u}_{t-1}$, interpreting its role in (4) is not straightforward. At a first glance, the strength of the error-correction would seem to increase as $\Delta_1 i_t$ grows (or becomes strongly negative) but the situation may be more complex than that. We shall return to this question shortly. Before doing so we consider the possibility of modelling the error-correction mechanism by a straightforward generalization of (ehp). As mentioned above, EHP formulated their nonlinear error-correction mechanism following Escribano (1985). This formulation may be seen as a first-order approximation to an STR-type parameterization in which the error-correcting variable $\tilde{u}_{t-1}$ itself is the transition variable; see Teräsvirta (1998a). Indeed, the results of linearity tests in Table 3.4 show that linearity is also rejected against STR when $\tilde{u}_{t-1}$ is the transition variable.

We specify and estimate a corresponding STR model for $\Delta_1 (m-p)_t$. It is seen from Table 3.4 that the specification sequence points at an LSTR(2) model, as one might expect from EHP. However, the estimation yields a model in which the estimate of c_1 is so small and far outside the observed range of $\tilde{u}_{t-1}$ that the observed transition function has value zero for the lowest value of the transition variable (for LSTR(2), the value of the transition function approaches unity as the value of the transition variable becomes sufficiently small). Thus we estimate an LSTR(1) model which has the form

$$
\begin{aligned}
\Delta_1 (m-p)_t &= \underset{(.051)}{.52}\ \Delta_1 (m-p)_{t-1} - \underset{(.040)}{.15}\ \Delta_1 (m-p)_{t-2} - \underset{(.044)}{.66}\ \Delta_1 p_t \\
&+ \underset{(.042)}{.46}\ \Delta_1 p_{t-1} - \underset{(.0051)}{.022}\ \Delta_1 rn_t - \underset{(.022)}{.17}\ \tilde{u}_{t-1} \\
&+ \underset{(.0050)}{.037}\ (D_1 + D_3)_t + \underset{(.0066)}{.061}\ D_{ct} + \underset{(.024)}{.070}\ D_4 \Delta_1 rs_t \\
&+ \left\{ \underset{(.069)}{.21} + \underset{(.18)}{.38}\ \Delta_1 p_t - \underset{(.081)}{.24}\ \Delta_2 rl_t - \underset{(.23)}{.62}\ \tilde{u}_{t-1} \right\} \\
&\times \left[1 + \exp\left\{ - \underset{(.48)}{2.54} \left(\tilde{u}_{t-1} - \underset{(.028)}{.19} \right) / \hat{\sigma}_{\tilde{u}_{t-1}} \right\} \right]^{-1} + \hat{e}_t \qquad (5) \\
&T[1878-1993] = 116, R^2 = .90, \hat{\sigma}_{nl} = .015, \hat{\sigma}_{nl}/\hat{\sigma}_l = .84, \\
AIC &= -8.31, LJB = 6.11\ (.047), A(2) = 0.83\,(.44)\,, A(4) = .39\,(.82)\,.
\end{aligned}
$$

where $\hat{\sigma}_{\tilde{u}_{t-1}}$ is the standard deviation of the transition variable. The residual standard deviation of (5) is 84% of that of (3), so that (5) fits better than (ehp). Normality of the errors is close to being rejected due to observations 1936 and 1981. (If these are dummied out, we have $LJB = 2.2$ (0.34).) The tests of no ARCH do not indicate any model misspecification. The errors are not autocorrelated, which is an improvement upon (ehp). The hypothesis of parameter constancy is accepted. But then, the test of no additive nonlinearity rejects the null hypothesis when $\Delta_1 i_t$ is transition variable. This suggests that the previous equation characterizes the nonlinearity of the error-correction mechanism better than (5). On the other hand, the nonlinear structure of (5) is quite simple involving only $\tilde{u}_{t-1}$, $\Delta_1 p_t$ and $\Delta_2 rl_t$. The short-term dynamic structure of the model closely resembles that of (ehp). The only major difference is the negative impact of the long interest rate on the change in money demand: it is nonlinear even according to (5), almost non-existing for low values of the transition function and increasing in strength with $\tilde{u}_{t-1}$. The graph of the transition function of (5) in Figure 3.2 shows that the transition is smooth.

As (5) offers yet another way of characterizing the error-correction mechanism in the UK demand for money it is interesting to look at differences between the equations. As mentioned above, interpreting the workings of the error-correction term in (4) is not straightforward as the error-correcting combination contains the level of income while its annual change is the transition variable. To obtain an idea of the contribution of the error-correction term to the dependent variable we have graphed the values of the

error-correction component over time for (ehp), (4) and (5) in Figure 3.3. Note that in both (4) and (5), the intercept in the nonlinear part of the equation is included in the error-correction mechanism as it acts as a counterbalance to the change in the coefficient of $\tilde{u}_{t-1}$ as a function of $\Delta_1 i_t$ (equation (4)) or $\tilde{u}_{t-1}$ itself (equation (5)). It is seen that in the beginning of the observation period (4) error-corrects more than the other two models. Between 1920 and 1960 the error-correction components in (ehp) and (5) remain practically constant. The largest differences between the models occur after 1973 when the estimation period in Hendry and Ericsson (1991) comes to an end. The wide amplitude of the error-correction component in (ehp) is the most conspicuous feature of Figure 3.3. The amplitude of the error correction component in (4) during that period is just about one-third of that of (ehp). The same component of (5) has a large negative value in 1975, otherwise it follows that of (4). Should the error-correction be considerably stronger from 1973 onward than elsewhere in the sample is a question without a definite answer as the true model is not known. It may be difficult, however, to find a convincing reason for extra strong deviations from the equilibrium during that period, in particular as the financial deregulation is accounted for by other means.

The residuals from the three models appear in Figure 3.4. They indicate that starting from the end of the 1960s, (ehp) does fit the data somewhat less well than (4) whereas model (5) lies somewhere between (ehp) and (4). On the other hand, while all models offer a good description of the exceptional post-World War I period, (5) fits slightly less well there than the other two models.

3.3 Results of the encompassing tests

The LSTR(2) model (4) has the best fit of the three nonlinear error-correction models for the demand for money in the UK considered here. It would therefore be interesting to see if (4) wholly explains the results obtained by the other two models. To do that we first have to investigate whether or not (4) which is an extension of (ehp) encompasses the latter model. In the present context it is natural to study encompassing using the Minimal Nesting Model (MNM; Mizon and Richard, 1986) and applying a simplification encompassing test (SET). The MNM is a model that nests the two models included in

our pairwise comparison.

Appendix B shows the structure of the MNM in this application and the appropriate null hypotheses. When we consider (ehp) and (4) we assume, for technical reasons, that the intercept in (4) is unrestricted. In that case, the MNM is (4) completed linearly by the error-correction term $(\tilde{u}_{t-1} - .2)\,\tilde{u}_{t-1}^2$, $\Delta_2 rl_t$ and $\Delta_1^2\,(m-p)_{t-2}$. Accepting the null hypothesis that these variables have zero coefficients is equivalent to concluding that (4) encompasses (ehp). Next, in comparing (4) and (5) the MNM is an additive STR model (Eitrheim and Teräsvirta, 1996; Teräsvirta 1998b). The nonlinear components are those of (4) and (5), and the structure of the encompassing hypothesis becomes clear from Appendix B. The test is a combination of a test of no additive nonlinearity with (4) and a customary significance test. Finally, we may also compare (5) and (ehp). In that case the MNM consists of (5) augmented linearly by the same variables as in the comparison between (4) and (ehp). Equation (5) encompassing (ehp) is equivalent to the coefficients of these three variables being zero in the MNM. These three cases together comprise the encompassing hypotheses in which the model assumed to encompass the other one under the null hypothesis also satisfies the variance (or AIC) dominance condition of encompassing; see Hendry (1995, p. 518) for discussion.

The results of all possible SETs, those ignoring the variance dominance condition included, can be found in Table 3.12. It is seen that the only model that encompasses another is (5) which encompasses (ehp). Except from that neither of the models encompasses the other. Many rejections are remarkably strong: however, (4) does come close to encompassing (5) at the 5% level ($p = 0.047$) if the order of the LSTR model ($= 1$) is assumed known.

4 Final remarks

The results of this paper give further support to the idea that a nonlinear error-correction mechanism is needed when modelling the demand for broad money in the UK with annual data. EHP, who advocated this notion, based their formulation of the error-correction mechanism on the work of Escribano (1985). The present considerations indicate that this

alternative may be generalized in at least two ways. Viewing the Escribano-type error-correction as an approximation to a specific STR-type parameterization as Teräsvirta (1998a) suggested leads to the STR model (5) which encompasses the EHP equation.

On the other hand, modelling the UK money demand with the same variables as those EHP used but adopting a more general nonlinear approach leads to another STR model (4). This model variance dominates the others but does not encompass them. In fact, both (ehp) and (5) seem to contain information that the more general model does not convey.

These results may suggest combining (4) and (5) to an additive STR model with two nonlinear components. On the other hand, the number of observations available may speak against a further extension of the model. Furthermore, although (4) fails the Simplification Encompassing Test against (5), it does pass the standard (Teräsvirta, 1998b) misspecification tests. It may therefore be considered an appropriate specification within the family of single STR models in this application.

Finally, we should like to point out that the present choice of nonlinearity is just one of many potential alternatives. It does not follow that all the other nonlinear specifications would necessarily be inferior to the STR model. It may be concluded, however, that in this application, STR-based specifications provide a useful way of refining previous models of the demand for broad money in the UK.

Appendix A: Smooth transition regression model

Consider the following nonlinear model,

$$y_t = x_t'\varphi + x_t'\theta F(s_t) + e_t. \qquad t = 1, .., T. \tag{A.1}$$

where $e_t \sim \text{nid}(0, \sigma^2)$, $x_t = (1, y_{t-1}, ..., y_{t-k}; z_{1t}, ...z_{mt})' = (1, \tilde{x}_t')'$ with $p = k + m$ is the vector of stationary explanatory variables, some of which may be linear combinations of nonstationary variables. Furthermore, $\varphi = (\varphi_0, ..., \varphi_p)'$ and $\theta = (\theta_0, ..., \theta_p)'$ are parameter vectors. $F(s_t; \gamma, c)$ is the transition function which is continuous in s_t and bounded between zero and unity. The transition variable s_t is either stationary or a time trend (t). The transition function of a kth order logistic smooth transition regression, LSTR(k), model is

$$F(s_t; \gamma, c) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k} (s_t - c_i)\right\}\right)^{-1}, \ \gamma > 0, \ c_1 \leq .... \leq c_k \tag{A.2}$$

where $k = 1$ yields the LSTR(1) and $k = 2$ the LSTR(2) model. These are the two parameterisations of transition functions that are considered in this paper. The slope parameter γ determines how rapid the transition from one extreme to the other is and the vector of location parameters, c termines the location of the transition. For derivations of the tests and an exhaustive description of STR models we refer to Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998b).

Testing linearity against the alternative of an LSTR(k) model amounts to testing if $\gamma = 0$ in $(A.2)$. The model is not identified under the null hypothesis due to the nuisance parameters θ and c. A Taylor series approximation about $\gamma = 0$ is used as a substitute to circumvent this problem, and the tests are based on this transformed equation:

$$y_t = x_t'\beta_0 + \sum_{j=1}^{k} \left(\tilde{x}_t s_t^j\right)' \beta_j + e_t^*, \ k = 1, 2. \tag{A.3}$$

where $e_t^* = e_t + \tilde{x}_t'\tilde{\theta} R(\gamma, c)$, but $e_t^* = e_t$ under H_0. The null hypothesis $H_0 : \gamma = 0$ in $(A.1)$ implies

$$H_0' : \beta_1 = \beta_2 = \beta_3 = 0 \tag{A.4}$$

within $(A.3)$ because $\beta_j = \gamma\tilde{\beta}_j$ where $\tilde{\beta}$ is a function of the parameters in the original STR specification. In order to decide between $k = 1$ and $k = 2$, one continues by carrying out the following test sequence within $(A.3)$:

$$H_{04} \ : \ \beta_3 = 0, \tag{A.5}$$

$$H_{03} \ : \ \beta_2 = 0|\beta_3 = 0, \tag{A.6}$$

$$H_{02} \ : \ \beta_1 = 0|\beta_2 = \beta_3 = 0. \tag{A.7}$$

If the rejection of H_{03} is strongest (measured by the p-value of the test), the rule is to choose $k = 2$, otherwise one selects $k = 1$; for the reasoning behind this rule see Teräsvirta (1994). It has been shown that in small samples an F approximation to the LM-test statistic is preferable to the asymptotic χ^2-distribution because it has good size properties.

Tests of parameter constancy against smoothly changing parameters are based on the same idea by assuming $s_t = t$; see Lin and Teräsvirta (1994). Following Lin and Teräsvirta (1994), the tests of

$$H_{04} \ : \ \beta_3 = 0, \tag{A.8}$$

$$H_{03} \ : \ \beta_2 = 0|\beta_3 = 0, \tag{A.9}$$

$$H_{02} \ : \ \beta_1 = 0|\beta_2 = \beta_3 = 0. \tag{A.10}$$

in $(A.5)$ are called LM_3, LM_2 and LM_1, respectively.

Appendix B: Encompassing tests using the Minimal Nesting model (MNM)

B.1 Comparing (ehp) with either (4) or (5)

We have

$$y_t = \alpha_1^{'} x_{1t} + \alpha_2^{'} x_{2t} + u_{1t} \tag{B.1}$$

and

$$y_t = \alpha_1^{'} x_{1t} + \alpha_3^{'} x_{3t} + \alpha_4^{'} x_{4t} G_1 (s_t; \gamma_1, c_1) + u_{2t} \tag{B.2}$$

where the x_{1t}, x_{2t} and x_{3t} do not contain common elements. The MNM becomes

$$y_t = \alpha_1^{'} x_{1t} + \alpha_2^{'} x_{2t} + \alpha_3^{'} x_{3t} + \alpha_4^{'} x_{4t} G_1 (s_t; \gamma_1, c_1) + u_{3t}.$$

The null hypotheses are:

$$\begin{aligned} H_{01}\,[(3.2),\ \text{alternatively}\ (3.3)\,,\ \text{encompasses (ehp)}] &\ :\ \alpha_2 = 0 \\ H_{02}\,[(\text{ehp}),\ \text{encompasses}\ (3.2),\ \text{alternatively}\ (3.3)] &\ :\ \alpha_3 = 0,\ \gamma_1 = 0. \end{aligned}$$

B.2 Comparing (4) with (5)

Let (B.2) represent (4), and write

$$y_t = \alpha_1^{'} x_{1t} + \alpha_5^{'} x_{5t} + \alpha_6^{'} x_{6t} G_2 (r_t; \gamma_2, c_2) + u_{4t}$$

for (5). The MNM becomes

$$
\begin{aligned}
y_t &= \alpha_1^{'} x_{1t} + \alpha_3^{'} x_{3t} + \alpha_5^{'} x_{5t} + \alpha_4^{'} x_{4t} G_1 \left(s_t; \gamma_1, c_1\right) \\
&\quad + \alpha_6^{'} x_{6t} G_2 \left(r_t; \gamma_2, c_2\right) + u_{5t}
\end{aligned}
$$

The null hypotheses are:

$$
\begin{aligned}
H_{03} \left[(3.3) \text{ encompasses } (3.2)\right] &: \alpha_3 = 0, \ \gamma_1 = 0 \\
H_{04} \left[(3.2) \text{ encompasses } (3.3)\right] &: \alpha_5 = 0, \ \gamma_2 = 0.
\end{aligned}
$$

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Graphs

Figure 3.1: Transition function of the LSTR(2) model (3.2), 1878-1993.

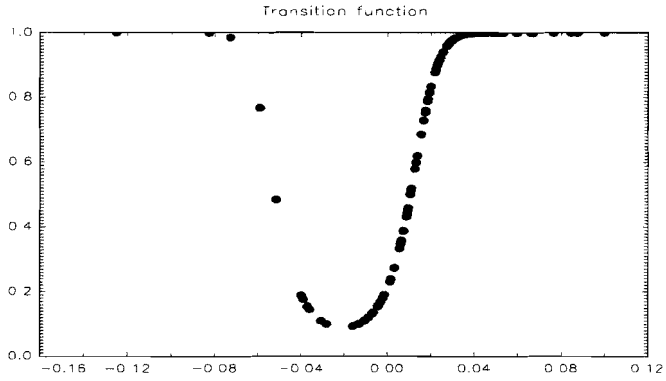


Figure 3.2: Transition function of the LSTR(1) model (3.3), 1878-1993.

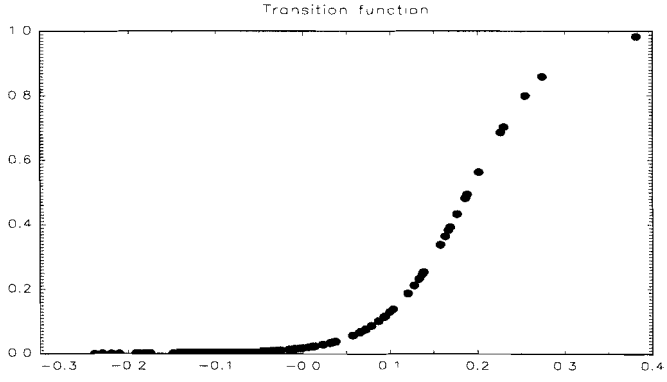


Figure 3.3: The long run properties of (ehp), (3.1) and (3.2), 1878-1993, where ecm(ehp) $= -2.26\,(\tilde{u}_{t-1} - .2)\,\tilde{u}_{t-1}^2$,
ecm(3.2) $= .092\tilde{u}_{t-1} + (.016 - .19\tilde{u}_{t-1})\,F\,(\Delta_1 i_t; \hat{\gamma}, \hat{c})$,
ecm(3.3) $= -.17\tilde{u}_{t-1} + (.21 - .62\tilde{u}_{t-1})\,F\,(\tilde{u}_{t-1}; \hat{\gamma}, \hat{c})$.

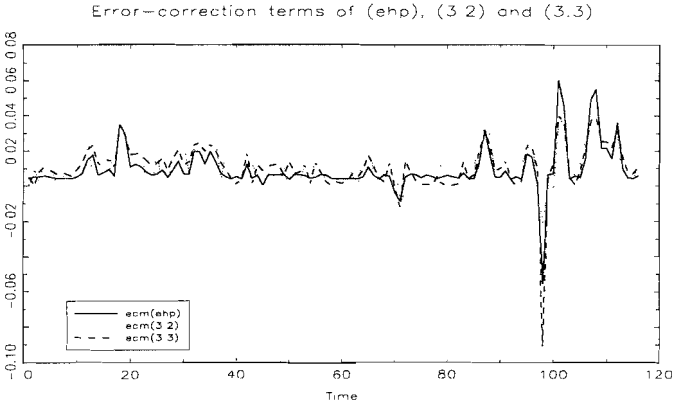
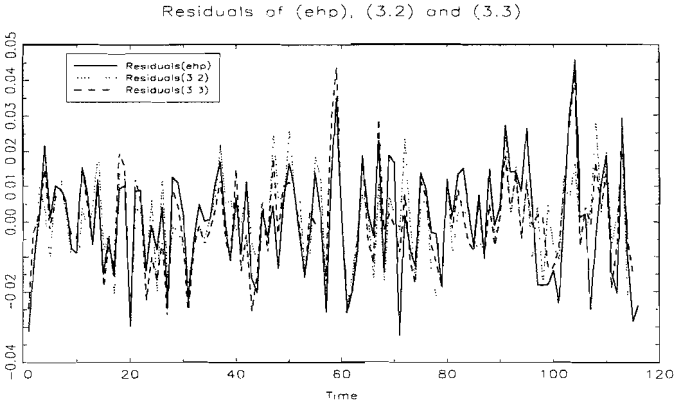


Figure 3.4: Residuals of (ehp), (3.2) and (3.3), 1878-1993.



Tables

Table 3.1. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the nonlinear model (ehp) for the UK money demand, 1878-1993.

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.15	0.037	0.079	0.12	0.20	0.012

Table 3.2. *p*-values of parameter constancy tests of the nonlinear model (ehp) against STR type nonconstancy for the UK money demand, 1878-1993.

Parameter constancy test	Null hypothesis (1)	(2)	(3)
LM_3	0.21	0.20	0.40
LM_2	0.10	0.14	0.67
LM_1	0.25	0.21	0.61

Notes: The null hypotheses are
(1): "All parameters except the coefficients of the dummy variables are constant"
(2): "Coefficient of the error-correction component is constant"
(3): "The intercept is constant"

The remaining parameters not under test are assumed constant in each case.

Table 3.3. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the linear model (3.1) for the UK money demand, 1878-1993.

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.11	0.16	0.30	0.45	0.60	0.17

Table 3.4. *p*-values of parameter constancy tests of the linear model (3.1) against STR type nonconstancy for the UK money demand, 1878-1993.

Parameter constancy test	Null hypothesis (1)	(2)
LM_3	0.00046	0.097
LM_2	0.00031	0.053
LM_1	0.034	0.039

Notes: The null hypothesis are
(1): "All parameters except the coefficients of the dummy variables are constant"
(2): "The intercept is constant"
The remaining parameters not under test are assumed constant in each case.

Table 3.5. *p*-values of linearity tests in the linear model (3.1) for a set of transition variables for the UK money demand, 1878-1993.

Linearity test	Transition variable								
	$\Delta_1(m\text{-}p)_{t-1}$	$\Delta_1(m\text{-}p)_{t-2}$	$\Delta_1 p_t$	$\Delta_1 p_{t-1}$	$\Delta_1 rn_t$	$\Delta_2 rl_t$	u_{t-1}	u^2_{t-1}	$\Delta_1 i_t$
F_0	0.40	0.53	0.20	0.059	0.23	0.17	0.0025	0.0023	0.00097
F_{04}	0.26	0.87	0.54	0.46	0.32	0.58	0.37	0.027	0.0093
F_{03}	0.46	0.083	0.17	0.021	0.52	0.083	0.0030	0.45	0.0075
F_{02}	0.51	0.76	0.17	0.25	0.12	0.25	0.032	0.0025	0.30

Table 3.6. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process the LSTR(2) model (3.2) for the UK money demand, 1878-1993.

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.71	0.74	0.90	0.92	0.89	0.16

Table 3.7. *p*-values of tests of no additive nonlinearity in the LSTR(2) model (3.2) for a set of transiti variables for the UK money demand, 1878-1993.

Linearity test	Transition variables							
	$\Delta_1(m\text{-}p)_{t-1}$	$\Delta_1(m\text{-}p)_{t-2}$	$\Delta_1 p_t$	$\Delta_1 p_{t-1}$	$\Delta_1 rn_t$	$\Delta_2 rl_t$	u_{t-1}	$\Delta_1 i_t$
F_0	0.42	0.95	0.85	0.33	0.10	0.82	0.22	0.26
F_{02}	0.78	0.89	0.39	0.72	0.18	0.85	0.083	0.47

Test of linear restriction
"Linear" coefficient of $\Delta_2 rl_t$=0: *p*-value=0.67

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 3.8. *p*-values of parameter constancy tests of the LSTR(2) model (3.2) against STR type nonconstancy for the UK money demand, 1878-1993.

Tests of parameter constancy	Null hypothesis (1)	(2)	(3)	(4)	(5)
LM_3	0.11	0.056	0.10	0.051	0.33
LM_2	0.33	0.15	0.027	0.12	0.19
LM_1	0.70	0.86	0.24	0.25	0.20

Notes: The null hypotheses are:
(1): "All parameters except the coefficients of the dummy variables are constant."
(2): "All parameters of the linear part except the coefficients of the dummy variables are constant."
(3): "All parameters of the nonlinear part are constant."
(4): "Coefficient of the error-correction terms are constant."
(5): "Nonlinear intercept is constant."
The remaining parameters not under test are assumed constant in each case.

Table 3.9. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the LSTR(1) model (3.3) for the UK money demand, 1878-1993.

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.30	0.24	0.32	0.47	0.56	0.18

Table 3.10. *p*-values of tests of no additive nonlinearity in the LSTR(1) model (3.3) for a set of transition variables for the UK money demand, 1878-1993.

Linearity test	Transition variable							
	$\Delta_1(m\text{-}p)_{t-1}$	$\Delta_1(m\text{-}p)_{t-2}$	$\Delta_1 p_t$	$\Delta_1 p_{t-1}$	$\Delta_1 rn_t$	$\Delta_2 rl_t$	u_{t-1}	$\Delta_1 i_t$
F_0	0.40	0.079	0.16	0.42	0.41	0.77	0.30	0.013
F_{02}	0.57	0.71	0.36	0.25	0.10	0.31	0.32	0.27

Test of linear restriction
"Linear" coefficient of $\Delta_2 rl_t$=0: *p*-value=0.19

Notes: Linearity tests: *F* is the *F*-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 3.11. *p*-values of parameter constancy tests of the LSTR(1) model (3.3) against STR type nonconstancy for the UK money demand, 1878-1993.

Tests of parameter constancy	Null hypothesis (1)	(2)	(3)	(4)	(5)
LM_3	0.39	0.17	0.57	0.86	0.73
LM_2	0.49	0.53	0.65	0.69	0.83
LM_1	0.62	0.41	0.78	0.91	0.67

Notes: The null hypotheses are:
(1): "All parameters except the coefficients of the dummy variables are constant."
(2): "All parameters of the linear part except the coefficients of the dummy variables are constant."
(3): "All parameters of the nonlinear part are constant."
(4): "Coefficient of the error-correction terms are constant."
(5): "Nonlinear intercept is constant."

The remaining parameters are not under test assumed constant in each case.

Table 3.12. *p*-values of simplification encompassing test based on the Minimal Nesting Model for (ehp), LSTR(1) model (3.3) and LSTR(2) model (3.2) for the UK money demand, 1878-1993.

B \ A	"A encompasses B" (ehp)	("yes" : p-value >0.05 "no" : 0.01< *p*-value <0.05 "NO" : p-value < 0.01) (3.3)	(3.2)
(ehp)	o	yes 0.35	NO 0.0077
(3.3)	NO 0.037 (0.0010)	o	no 0.018 (0.047)
(3.2)	NO 0.00021	NO 0.0098	o

Notes:

(1) The auxiliary regression (when needed in testing) is based on the third-order Taylor approximation of the transition function; results based on a first-order Taylor approximation appear in parentheses where appropriate, that is, when the model to be encompassed is the LSTR(1) model (3.3).

(2) Conclusion "yes/no" ignores the variance dominance condition.

Essay III

Detecting equilibrium correction with smoothly time-varying strength*

1 Introduction

The concept of cointegration is based on the assumption that a pair of integrated economic variables are linked by a long-run stationary equilibrium relationship, see Engle and Granger (1987). It is implicitly assumed that the variables are cointegrated at all time periods and that the rate of adjustment towards the long-run equilibrium is constant over the sample. Recently, there have been several empirical studies where the equilibrium correction term enters the model nonlinearly, see e.g. Michael *et al.* (1996), Ericsson *et al.* (1998) and Teräsvirta and Eliasson (1998). This raises the question of the reliability of the existing tests for detecting cointegration and nonlinearities in a nonlinear equilibrium correction model. Two recent papers have studied similar questions. Balke and Fomby (1997) compare the performance of the Engle-Granger and the Johansen cointegration tests when the adjustment follows a threshold cointegration model, and van Dijk and Franses (1999) perform a study where the adjustment is of the smooth transition regression (STR) type. In the former study the cointegrating relationship is locally coin-

*This paper has benefitted from comments by Graham Elliot, Clive Granger, David Hendry, Johan Lyhagen and Timo Teräsvirta. The responsibility for any errors or shortcomings remains mine.

tegrating, that is, the system may not be equilibrium correcting in all time periods. In the latter study the simulations are equilibrium correcting in all periods although the strength of the equilibrium correction varies. These studies conclude that both the Engle-Granger and the Johansen cointegration tests work well when the equilibrium correction is nonlinear.

This paper will consider the probability of detecting time-varying equilibrium correction by applying the existing tests of no cointegration and parameter constancy. The simulated series are locally cointegrated and the time-varying equilibrium correction is characterised by an STR model. There are no tests for simultaneously testing the joint hypotheses of no cointegration and constant parameters, and a two-step procedure will be applied. The investigator is assumed to follow a standard modelling procedure. First, the null of no cointegration is tested. The series are generated according to a two-dimensional equation system, and the Johansen cointegration test (Johansen, (1995)) is applied to test the hypothesis of no cointegration. If this is rejected, the cointegrating relationship is estimated. A lagged estimated relationship forms the equilibrium correction (EC) term, and from here on a single equation is used. The constancy of the coefficient of the EC term is tested. This is done as in Lin and Teräsvirta (1994), so that the alternative to parameter constancy is a smoothly changing parameter. The purpose of the paper is to find out how often the investigator reaches the correct conclusion that there exists a cointegrating relationship whose strength varies over time. Furthermore, the aim is to find out which factors affect the probability of arriving at this conclusion.

The results of the simulations show that the investigator rarely reaches the correct conclusion of a time-varying equilibrium correction. Considering the case where the sample size is large and the coefficient of the equilibrium correction term is high (both of which have positive effects on the power of both tests), the joint power of the two tests is still low and varies a lot, depending on the other parameter values. The power is at its lowest when the short-run dynamics are very strong since this makes it difficult to estimate the cointegrating relationship, and the estimated equilibrium correction term often seems redundant. The low significance level of the cointegrating term makes it difficult for the parameter constancy test to detect the time variation. However, by not

estimating the cointegrating relationship and instead including the cointegrated variables in levels, the power of the equilibrium correction equation without a priori restrictions strongly increases and becomes almost as high as it is when the cointegrating relationship is assumed to be completely known a priori.

The outline of this paper is as follows. Section 2 reviews the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test. The simulation setup is also presented. In Section 3 the testing procedure and the results of the Monte Carlo simulations appear, and Section 4 concludes.

2 Methodology

2.1 Johansen cointegration test

A vector time series $\{y_t\}$ is said to be cointegrated of order b if each of the series taken individually is integrated of order d, $I(d)$, $d \geqslant 1$, whereas a linear combination of the series, $\beta' y$, is $I(d-b)$, $b > 0$, where β is the cointegrating vector. The vector is not unique since it can be multiplied by any non-zero scalar and still satisfy the cointegration condition.

There are several tests for no cointegration. The Johansen cointegration test, see Johansen (1995, Chapter 6) is one of the most popular tests in empirical studies and it is therefore chosen for this study. The testing procedure will be reviewed briefly. Start by considering the structural VAR(p) model,

$$y_t = \sum_{i=1}^{p} \Pi_i y_{t-i} + \varepsilon_t, \quad t = 1, ..., T \tag{1}$$

where y_t is an $(n \times 1)$ vector, Π_i are $(n \times n)$ matrices and $\varepsilon_t \sim \text{nid}(0, \Omega)$. Each individual variable in y_t is assumed to be $I(1)$. The reduced form of the equilibrium correction model becomes

$$\Delta y_t = \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t, \quad t = 1, ..., T \tag{2}$$

where $\Gamma_1 = \Pi_1 - I_n$, $\Gamma_2 = \Pi_2 + \Gamma_1, ...,$ and $\Pi = I - \sum_{i=1}^{p-1} \Pi_i$. Under the null hypothesis $H(h)$ there are assumed to be exactly h linear combinations of y which are $I(0)$. Consequently,

Π can be rewritten as $\alpha\beta'$ where α and β are $(n \times h)$. The alternative hypothesis is that there are n cointegrating relations, where n is the number of elements of y_t, which would imply that every linear combination of y_t is stationary and no restrictions would be imposed on Π. The likelihood ratio test of $H(h)$ against $H(n)$ is given by

$$\mathcal{L}_n - \mathcal{L}_h = -\frac{T}{2} \sum_{i=h+1}^{n} \ln\left(1 - \hat{\lambda}_i\right). \tag{3}$$

The eigenvalues $\hat{\lambda}_i$ of Π can be found by performing the first steps of Johansen's algorithm for the maximum likelihood estimator of β in $H(h) : \Pi = \alpha\beta'$. If equation (??) consisted only of stationary variables the test statistic would be distributed as a χ^2 asymptotically. However, under the null hypothesis the test statistic will depend on $(n-h)$ random walks, and the critical values are not the standard ones but can be found in Johansen (1995), Chapter 15.

2.2 Lin and Teräsvirta parameter constancy test

Consider the following single-equation STR model,

$$y_t = x_t'\varphi + x_t'\theta F(s_t) + u_t. \qquad t = 1, .., T. \tag{4}$$

where $\varphi = (\varphi_0, \varphi_1, .., \varphi_m)'$ is the parameter vector for the linear component and $x_t = (1, x_{1t}, .., x_{mt})' = (1, y_{t-1}, .., y_{t-p}, z_{1t}, .., z_{qt})'$ is the corresponding vector of stationary and ergodic explanatory variables. The nonlinear component is specified as a linear component multiplied by a nonlinear function, $x_t'\theta F(s_t)$ and $u_t \sim \text{nid}(0, \sigma^2)$ for simplicity. The nonlinear function $F(s_t)$ is the transition function which is continuous and bounded. It is customary to bound F between zero and unity. Hence, the model will change locally from $E(y_t|x_t) = x_t'\varphi$ for $F = 0$ to $E(y_t|x_t) = x_t'(\varphi + \theta)$ for $F = 1$ with the transition variable s_t. When $s_t = t$ the STR model can be interpreted as a linear model with time-varying parameters, which is the case that will be considered here. For elaborate discussions on STR models see Granger and Teräsvirta (1993) and Teräsvirta (1998).

The transition function of a kth order logistic smooth transition regression model,

LSTR(k), has the form

$$F(t) = F(\gamma, c; t) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k} (t - c_i)\right\}\right)^{-1}, \ \gamma > 0 \tag{5}$$

when a time trend (t) is used as the transition variable. The slope parameter (γ) determines how rapid the transition is and the vector of location parameters $c = (c_1, ..., c_k)'$ decides where the transitions occur. The STR model (4) becomes linear when $\gamma = 0$ so that the transition function $F(t) \equiv \frac{1}{2}$. For notational simplicity the transition function in (4) is replaced by $\tilde{F}(t, \gamma) = F(t, \gamma) - \frac{1}{2}$. This implies $\tilde{F}(t, 0) = 0$ and $H_0 : \gamma = 0$ becomes a natural hypothesis of parameter constancy, see Lin and Teräsvirta (1994). The alternative hypothesis is $H_1 : \gamma > 0$. There is one caveat though; the STR model is not identified under the null hypothesis because of the nuisance parameters θ and c. Thus the classical asymptotic distribution theory for testing the null hypothesis $\gamma = 0$ does not work. In order to circumvent this problem a Taylor series approximation to the transition function is used to obtain an appropriate test. Equation (6) shows the auxiliary regression for the tests when $k = 2$. In equations (7) and (8) the null hypotheses of the parameter constancy tests are presented.

$$y_t = \delta_0' x_t + \delta_1' x_t t + \delta_2' x_t t^2 + v_t \tag{6}$$

$$LM_2 : \delta_1 = \delta_2 = 0 \tag{7}$$

$$LM_1 : \delta_1 = 0 \mid \delta_2 = 0. \tag{8}$$

If $k = 1$, the test to be applied is LM_1; if $k = 2$ it is LM_2. Thus, size and power of the LM_1 statistic are reported when an LSTR(1) model is used for simulating the series and LM_2 is used in simulations with the LSTR(2) model.

Testing the hypotheses of parameter constancy, a Lagrange multiplier (LM) type test statistic can be obtained by using the linear auxiliary regression (6). The test statistic will have an asymptotic χ^2 distribution under the null hypotheses. In practice, an F test is preferred to the χ^2-test since it has better small sample properties, see Lin and Teräsvirta (1994).

2.3 Simulation setup

The following two-dimensional system is used in the experiment:

$$\Delta y_t = \theta \Delta y_{t-1} + \delta \left(y_{t-1} - z_{t-1} \right) F(t) + u_t \tag{9}$$

$$\Delta z_t = \omega_t \tag{10}$$

where $F(t)$ is the transition function, $u_t \sim \text{nid}(0, 0.25)$, $\omega_t \sim \text{nid}(0, 1)$, u_t and ω_t are mutually independent. Equation (9) represents a time-varying equilibrium correction model and equation (10) generates a random walk. The magnitude of the short-run dynamics is allowed to vary, $\theta = \{0.2, 0.4, 0.9\}$ and so is the coefficient of the cointegration relationship, $\delta = \{-0.1, -0.4, -0.8\}$. In order to reduce the importance of the starting values, the first third of the observations are excluded from the reported results. That is, $T + \frac{1}{2}T$ observations are generated and the sample sizes used are $T = 100$, 200. Each test is performed $N = 10000$ times.

The simulated series are all generated by the time-varying equilibrium correction model (9) and (10). The transition function $F(t)$ is given by equation (5) with either $k = 1$ or $k = 2$. In the former case one has an LSTR(1) model and in the latter one has an LSTR(2) model.

The LSTR(1) model will be generated for different speeds of transition. When $\gamma = 1$ the transition is very smooth, when $\gamma = 10$ the transition function has the typical S-shape and for $\gamma = 100$ the transition is rather quick, see Figures 2.1-2.3. Setting $\gamma = 1$, the strength of attraction increases almost linearly over time and cointegration will be present in the whole observation period, see Figure 2.1. The location parameter c is also allowed to vary and the series are simulated for $c = \{0.25, 0.50, 0.75\}$. When $\gamma = 10$ or $\gamma = 100$ the length of the observation period including cointegration will vary with c. For larger values of c the observation period including cointegration ($F > 0$) will be shorter, see Figures 2.2 and 2.3.

Generating an LSTR(2) model, the speed of transition is always high, $\gamma = 100$, see Figure 2.4. There are two location parameters in the LSTR(2) model, see equation (5) $k = 2$, and they are allowed to vary too. The series are generated with $c_1 = \{0.25, 1/3\}$

and $c_2 = 1 - c_1$. For the LSTR(2) model cointegration will be present in the beginning and the end of the sample. Hence, there will not be any cointegration ($F = 0$) in the middle of the sample, see Figure 2.4. The situation resembles that in threshold cointegration; the difference is that in this experiment time is the transition variable. One may order the observations in a threshold cointegration model according to the threshold variable and draw a graph of the strength of attraction. If this is done it turns out that the figure is analogous to that corresponding to an LSTR(2) model where $\gamma \to \infty$.

3 Monte Carlo simulations

In this section the testing procedure and the results of the simulations are presented. The purpose of the simulations is to find out how often a hypothetical researcher would arrive at the correct conclusion, that there exists a cointegrating relationship whose strength varies over time. As mentioned in the introduction, there are no tests for simultaneously testing the hypothesis of no cointegration and parameter constancy; hence two tests will be applied: the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test. The results of the simulations will be presented for the two tests separately, but also jointly since this is the major concern. For the sake of simplicity, the rejection frequencies of the tests will be referred to as "power" throughout the paper.

3.1 The testing procedure

The testing procedure starts by estimating the null hypothesis of no cointegration. If it is rejected, the procedure continues by exploring the constancy of the cointegrating parameter using the Lin and Teräsvirta parameter constancy test. It is also tested whether the cointegration term enters the linear equilibrium correction equation significantly. This will be referred to as the significance test.

Three different scenarios are considered when testing for parameter constancy: In Case 1 the cointegrating vector is assumed to be unknown, in Case 2 it is known and in Case 3 no cointegration relationship is defined. In addition, the size of the Johansen cointegration test and the parameter constancy test are explored.

A more detailed description of the testing procedure is as follows:

- **Size of Johansen cointegration test and the Lin and Teräsvirta parameter constancy test**. When the size of the first test is investigated the series are generated according to equations (9) and (10), keeping the coefficient of the cointegrating relationship δ equal to zero in (9). The results are reported as "J-size" in Table 3.1. This yields the empirical size of the Johansen test for the sample sizes $T = 100$ and $T = 200$. When the size of the parameter constancy test is examined the series are generated according to equations (9) and (10) assuming that the transition function $F(t)$ is equal to unity. Hence the model is linear. The hypotheses of linearity are tested and the size results for Case 1 are reported as "LT-size" in Tables 3.2-3.5. This yields the empirical size of the parameter constancy test for Case 1. The selected nominal size is 5% for both tests.

- **The Johansen cointegration test.** The numbers of rejections of the null hypothesis will be referred to as the power of the test and are reported as "Johansen test" in Tables 3.2-3.5. If the null hypothesis of no cointegration is rejected, and only then, the testing procedure continues with the parameter constancy test for three different scenarios named Case 1, 2 and 3 as follows.

- **Case 1: The cointegrating vector is unknown**. After testing and rejecting the hypothesis of no cointegration the cointegrating vector is estimated. The maximum likelihood estimator is given by the eigenvector corresponding to the h largest eigenvalues $\hat{\lambda}_1$, see Johansen (1995, Chapter 6), and is included in the linear model subject to testing which becomes

$$\Delta y_t = \theta \Delta y_{t-1} + \delta \left(y_{t-1} - \hat{\xi} z_{t-1} \right) + u_{1t}.$$

  Existence and parameter constancy of the estimated cointegration coefficient $\hat{\delta}$ are tested; note that this is done only if the null of no cointegration is rejected. The number of times a false null hypothesis is rejected (power) is reported under "Case 1" in Tables 3.2-3.5. The only test results that will be reported for the parameter constancy tests are LM_1 for the LSTR(1) model and LM_2 for the LSTR(2) model.

- **Case 2: The cointegrating vector is known.** In this case the real cointegrating relationship is assumed to be known. This assumption is not realistic but the purpose is to create a case which can be used for comparisons with the more realistic scenarios. By doing this it is easier to detect where the major weaknesses in the testing procedure occur. The linear model subject to tests becomes

$$\Delta y_t = \theta \Delta y_{t-1} + \delta \left(y_{t-1} - z_{t-1} \right) + u_{2t}.$$

  The power of the tests are reported under "Case 2" in Tables 3.2-3.5.

- **Case 3: Unrestricted cointegration.** In the third case the integrated variables are included without any restriction in the linear model. That is, the variables are still cointegrated but the cointegrating vector is not estimated. The estimated linear model becomes

$$\Delta y_t = \theta \Delta y_{t-1} + \delta_1 y_{t-1} + \delta_2 z_{t-1} + u_{3t}.$$

  The former cointegrating coefficient will be divided into two separate estimates, one for each integrated variable. The estimated coefficients $\hat{\delta}_1$ and $\hat{\delta}_2$ ought to satisfy $\hat{\delta}_1 \approx -\hat{\delta}_2$ since data are generated according to equation (9). The hypothesis $\delta_1 = \delta_2 = 0$ and that of parameter constancy are tested. Note that the assumptions of the parameter constancy tests are violated, as the test requires stationarity of the regressors. But since the $I(1)$ variables are cointegrated, the combination of the two is still stationary. The power of the tests are reported under "Case 3" in Tables 3.2-3.5.

3.2 Simulation results

The simulation results of the LSTR(1) model can be found in Tables 3.2-3.4 and the results of the LSTR(2) model are shown in Table 3.5. Tables 3.2.A-C present the results when the coefficient of the lagged first difference Δy_{t-1} is small ($\theta = 0.2$), in Tables 3.3.A-C the coefficient is higher ($\theta = 0.4$), and in Tables 3.4.A-C the coefficient of Δy_{t-1} is $\theta = 0.9$. The capital letters in the tables' titles refer to the speed of transition. Thus, A denotes the smoothest transition $\gamma = 1$, B corresponds to $\gamma = 10$ and C is for a transition function close to a step function, $\gamma = 100$, see Figures 2.1-2.4.

The size and power results of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test are discussed below. The results of the latter test will be considered separately for each of the three different cases. The two tests are initially discussed separately for simplicity of exposure since some parameter changes affect the power of the two tests in opposite ways. The joint performance of the tests will be discussed in Section 3.2.5.

The size of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test

Before considering the power of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy tests (LM_1 and LM_2), the size of the tests are examined. It turns out that the size of the Johansen test varies between 3.3% and 5.7% and is usually slightly below the chosen significance level of 5%, see Table 3.1. The size of the Lin and Teräsvirta parameter constancy test, on the other hand, is in the close neighborhood of the nominal significance level of 5%. The smallest value equals 4.33% and the largest 5.40%, see Tables 3.2.A-C - 3.5. The size is also good when the basic assumption of stationarity is violated: Case 3, where two of the variables are characterised by a unit root. Obviously, this is because when the two I(1) variables are cointegrated a combination of the two variables is still stationary. This may affect the outcome even though the parameters of the cointegrating vector are not restricted. The basic models for the LM_1 and LM_2 tests are equations (4) and (5) with $k = 1, 2$.

The power of the Johansen cointegration test

The results of the simulations show that the power of the Johansen cointegration test is usually quite high when there is time-varying equilibrium correction. However, it is sensitive to some of the parameter changes and in this section the effect of parameter changes on the power of the Johansen cointegration test will be discussed in more detail.

When the magnitude of the cointegrating coefficient δ increases (in absolute value) the power increases for both the LSTR(1) and LSTR(2) models, see e.g. Tables 3.2.A and 3.5. An increase in δ makes the cointegrating relationship more distinct and therefore

easier for the test to detect, and hence the power increases. For the LSTR(1) models the power of the Johansen test is also high when the speed of transition is low ($\gamma = 1$), which is not surprising considering that the test is developed for a linear model. Besides, the parameter change is almost a linear function, making the equilibrium correction present during the whole observation period. For a more rapid transition speed ($\gamma = 10, 100$) the cointegration only enters when $F > 0$ and the observation period during which the system equilibrium corrects is only a subperiod of the original one. Because of this, the power also depends on the location parameter c. A time series generated with a higher c will have lower power when $\gamma = 10, 100$ since there will be fewer observations available with information about the cointegration relationship, see e.g. Table 3.2.B. The same result also appears when the observations are simulated according to an LSTR(2) model, see Table 3.5. When the location parameter $c_1 = 1/3$ the period including equilibrium correction will be longer than when $c_1 = 0.25$, since it is assumed that $c_2 = 1 - c_1$, and the power of the test is higher. The power is also positively affected by the sample size for both the LSTR(1) and LSTR(2) models; hence the tests appear consistent.

More surprisingly, the power of the cointegration test increases when the strength of the short-run dynamics (θ) is high. This is especially noticeable when $\gamma = 100$, see Tables 3.2.C and 3.4.C. The power is at least three times as high in the latter table as in the former. A reason for this might be that the change in the strength of the equilibrium correction disturbs the test less when the relative weight of the cointegrating relationship is small. Besides, a previous experiment comparing the power of the Johansen and the Engel-Granger cointegration tests showed that the power of the Johansen test was higher when data was generated according to equations (4) and (5) instead of $\Delta y_t = \theta \Delta z_{t-1} + \delta (y_{t-1} - z_{t-1}) F(t) + u_t$ and (5). Hence, the Johansen test performs better when the short-run dynamics are given by lagged values of the dependent variable than when they are given by an exogenous variable. Keeping this in mind, the improved power of the cointegration test when $\theta = 0.9$ is less surprising.

The power of the Lin and Teräsvirta parameter constancy test for Case 1: The cointegrating vector is unknown

The performance of the Lin and Teräsvirta parameter constancy test is strongly dependent on the choice of parameter values. For most setups the power is fair but when the short-run dynamics are very strong ($\theta = 0.9$) it is close to zero. A more detailed discussion of how the parameter changes affect the power is given below.

When the model is generated by an LSTR(1) model an increase (in absolute value) in the coefficient of the cointegrating relationship (δ) improves the power, see e.g. Table 3.2.B. When the speed of transition increases ($\gamma > 1$) the nonlinearity becomes more distinct and the power of the test increases, see Table 3.2.A-C. However, there seems to be a peak when $\gamma = 10$ since the power is often slightly weaker when $\gamma = 100$. This is due to the fact that the test is conditional on the rejection of the hypothesis of no cointegration. Moreover, changing the location of the transition (c) does not generally seem to affect the power much. But in some cases the power is slightly smaller when $c = 0.75$ and γ is high, see Table 3.2.C. In this case, the period during which the equilibrium correction is operating is short. This might make it hard to estimate the parameters of the cointegrating vector and it becomes difficult for the test to detect the time-variation. An increase in the sample size improves the power, especially when δ is small, see e.g. 3.2.B.

Finally, increasing the coefficient of the short-run dynamics from $\theta = 0.2$ to $\theta = 0.9$ reduces the power sharply, see Table 3.3.A-C. The reason for the poor performance when $\theta = 0.9$ is that the dominating short-run dynamics make the cointegrating vector difficult to estimate. The power of the significance test is also very low for this scenario and if the cointegrating relationship does not enter significantly it is hardly surprising that the parameter constancy test has low power. When data is generated according to an LSTR(2) model the behaviour is similar. A stronger coefficient of the cointegrating relationship and a larger sample size improve the power of the parameter constancy test while it seems to be unaffected by the value of the location parameter, see Table 3.5.

The power of the Lin and Teräsvirta parameter constancy test for Case 2: The cointegrating vector is known

In this case the cointegrating vector is known and the overall performance of the Lin and Teräsvirta parameter constancy test is better than for Case 1. The power of the significance test also improves considerably. This demonstrates the importance of high quality estimates of the cointegrating vector for the second test.

Generating the series according to an LSTR(1) model, the power of the parameter constancy test increases for a higher coefficient of the cointegrating relationship, a larger sample size and when the speed of transition is high ($\gamma > 1$). The power is usually not affected by changes in the location parameter c. But, as in Case 1, the power is slightly smaller when $c = 0.75$ and γ is high, see e.g. Table 3.3.C. Moreover, an increase in the short-run dynamics from $\theta = 0.2$ to $\theta = 0.9$ really improves the power, which is quite puzzling. However, when the cointegrating relationship is known and the short-run dynamics are strong it might be easier for the test to separate the dynamics of the two components and the time variation is more easily found. This can be compared to the results of Case 1 where the power was close to zero for strong short-run dynamics, pointing at the importance of a correct estimate of the cointegrating vector when trying to detect the time-varying coefficient.

When data is simulated according to an LSTR(2) model the power properties are similar to those of the LSTR(1) model. A higher coefficient of the equilibrium correction term and a larger sample size has a positive effect on the power. The performance of the test when changing the location parameter (c_1) is puzzling. Because it is assumed that $c_2 = 1 - c_1$, increasing c_1 implies a longer operating period for the equilibrium correction. But, as is seen from Table 3.5, the power is not higher for higher values of c_1. This is due to the selection bias, since the test is only carried out if the null of no cointegration is rejected.

The power of the Lin and Teräsvirta parameter constancy test for Case 3: Unrestricted cointegration

In this scenario the cointegrating vector is not estimated and the long-run variables enter unrestricted into the linear model. Hence, since the cointegrating vector will not be estimated, there are two coefficients for the long-run variables. The power is measured as the number of times parameter constancy can be rejected for either one of the two parameters.

Considering the LSTR(1) model, the power properties for Case 3 are very similar to those in Case 2. Hence, the power increases when the cointegrating coefficient is stronger and the speed of transition increases. The power is quite sensitive to large values of the location parameter which, as discussed, makes the observation period with an equilibrium correction short. An increase in the sample size improves the power and very strong short-run dynamics have a positive effect on the power. The LSTR(2) model has very similar features to those of Case 2, also in the sense that an increase in the location parameter, which makes the period including the equilibrium correction longer, has a negative effect on the test.

How often will the conclusion be a time-varying equilibrium correction?

The overall power of the two tests, Johansen cointegration test and Lin and Teräsvirta parameter constancy test, is rather low. This might be surprising at first since both tests perform well separately. But since some parameter changes affect the power of the tests in opposite ways, the joint power will not be as high as for each test separately and it varies a lot depending on the parameter choices.

When the series are generated according to an LSTR(1) model, the highest joint power occurs when the transition speed is quite fast ($\gamma = 10$), the coefficient of $\Delta_1 y_{t-1}$ is moderate ($\theta = 0.2, 0.4$), the magnitude of the cointegrating coefficient is strong ($\delta < -0.1$), the location parameter is small ($c = 0.25$) and the sample size is large. However, the joint power deteriorates when any of the parameter values change. It is especially sensitive to changes in γ, c and θ. That is, when $\gamma = 1$ the power of the parameter constancy test is strongly negatively affected and when $\gamma = 100$ the performance of the Johansen

cointegration test deteriorates and, hence, a change in either direction results in a decrease in the joint power. Moreover, a higher c implies a shorter period, including the cointegrating relationship which results in a sharp fall of the power of the Johansen test. Another variable that severely affects the joint power is a high θ, when considering Case 1. However, it turns out that by not estimating the cointegrating relationship, Case 3, and only including the cointegrated variables (in levels) in the equation, the joint power will almost be as high as for Case 2, where the cointegrating vector is assumed to be known.

When generating the series according to an LSTR(2) model the joint power is also generally low. In this case the power is most strongly affected by the value of the location parameter c. A high c implies a longer period, including the cointegrating relationship, and the Johansen cointegration test is especially sensitive to this, which makes the joint power smaller.

4 Conclusions

In this paper the probability of detecting a time-varying equilibrium correction by using the Johansen cointegration test and Lin and Teräsvirta parameter constancy test is explored. It turns out that the joint power of the two tests is quite low and the power varies a lot, depending on the choice of parameter values. The most interesting result is perhaps the fact that the power of the parameter constancy tests strongly improves when the cointegrated variables enter without restrictions. This improves the power, which becomes almost as high as it is when the cointegrating vector is assumed to be known. This result may have important implications for empirical work.

Finally, based on the results above, it seems fair to conclude that the probability for a potential researcher to detect a time-varying equilibrium correction when using the general modelling strategy of testing for cointegration, estimating a cointegrating vector and detecting time variation of the equilibrium correction term, is low. This calls for a new test which simultaneously tests the joint hypothesis of no cointegration and parameter constancy, but that is beyond the scope of this paper.

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Graphs

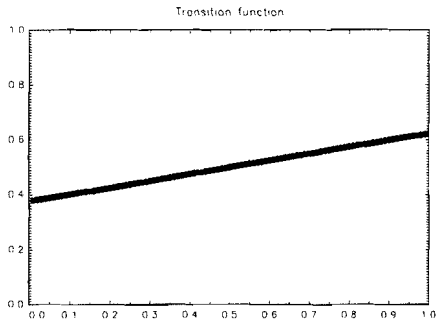

Figure 2.1: Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 1$.

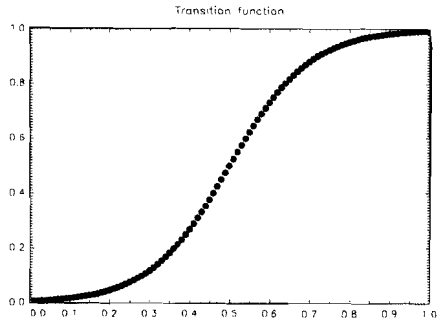

Figure 2.2: Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 10$.

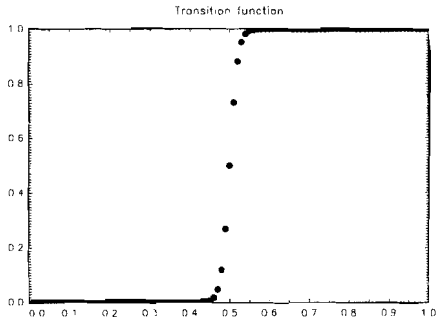

Figure 2.3: Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 100$.

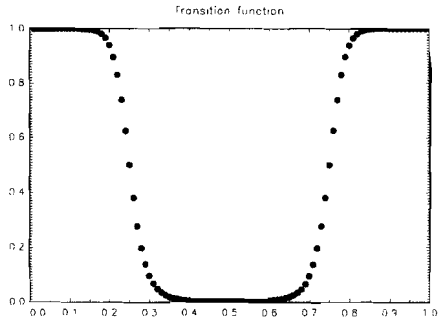

Figure 2.4: Transition function of an LSTR(2) model where $c_1 = 0.25$, $c_2 = 0.75$ and $\gamma = 100$.

Tables

Table 3.1 Size of the Johansen cointegration test. Rejection frequencies of the null hypothesis are presented in the table. N=10000.

T=100	**θ=0.2**	**θ=0.4**	**θ=0.9**
J-size	3.68	3.46	5.70
T=200	**θ=0.2**	**θ=0.4**	**θ=0.9**
J-size	3.54	3.33	4.67

Table 3.2.a: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.2, γ =1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	δ = **–0.1**			δ = **–0.4**			δ = **–0. 8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	54.62	48.28	40.16	100.0	99.94	99.76	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	6.52	6.48	6.53	11.98	13.70	14.38	16.89	19.81	22.42
H_0: $\delta = 0$	88.50	89.23	89.74	74.22	75.77	78.08	51.76	57.39	62.36
Case 2:									
LM_1	6.74	7.33	6.80	14.50	16.11	17.00	30.34	32.12	33.66
H_0: $\delta = 0$	99.56	99.38	99.08	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.67	10.34	10.28	12.55	13.71	14.02	23.03	24.80	26.33
H_0: $\delta_1 = \delta_2 = 0$	71.84	67.77	64.42	99.98	99.95	99.79	100.0	100.0	100.0
LM-size	4.59	4.79	4.56	4.73	5.04	4.84	4.81	4.58	4.71
T = 200	δ = **–0.1**			δ = **–0.4**			δ = **–0..8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	96.96	93.41	88.30	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	8.19	8.71	9.17	21.24	22.94	24.49	28.98	34.68	38.33
H_0: $\delta = 0$	86.17	87.22	88.57	74.22	76.51	77.83	50.05	56.46	62.12
Case 2:									
LM_1	8.67	9.06	10.08	26.91	29.08	30.07	54.23	59.08	60.29
H_0: $\delta = 0$	99.99	99.98	99.90	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.86	10.28	10.55	21.66	22.40	23.64	44.61	48.12	50.08
H_0: $\delta_1 = \delta_2 = 0$	95.12	92.41	89.24	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.78	4.90	4.77	4.75	4.88	4.95	4.85	4.89	5.02

Table 3.2.b: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.2, γ=10. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	$\delta = -0.1$			$\delta = -0.4$			$\delta = -0.8$		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	56.68	22.46	14.61	95.89	31.62	13.30	99.22	31.64	11.28
Parameter constancy test									
Case 1:									
LM_1	34.33	24.00	10.81	79.32	75.96	50.98	62.62	69.12	64.63
H_0: $\delta = 0$	91.95	89.76	88.36	81.08	81.63	85.41	61.37	69.43	77.30
Case 2:									
LM_1	48.71	61.98	70.77	96.73	99.34	93.53	99.90	100.0	99.82
H_0: $\delta = 0$	99.70	97.33	87.13	100.0	99.78	94.89	100.0	99.87	96.28
Case 3:									
LM_1	38.64	49.29	60.37	91.76	97.15	90.38	99.79	100.0	99.02
H_0: $\delta_1 = \delta_2 = 0$	75.72	52.05	49.08	99.76	87.98	59.62	100.0	92.95	77.04
LM-size:	4.87	4.48	5.08	5.04	4.77	5.23	4.60	4.87	5.19
T = 200	$\delta = -0.1$			$\delta = -0.4$			$\delta = -0.8$		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	91.41	40.59	21.06	99.99	51.05	21.94	100.0	52.94	17.69
Parameter constancy test									
Case 1:									
LM_1	70.44	58.17	21.70	84.09	83.80	64.04	59.98	76.79	71.40
H_0: $\delta = 0$	95.26	93.27	90.50	83.31	86.33	87.33	58.65	76.20	83.15
Case 2:									
LM_1	80.10	89.53	79.77	99.89	100.0	99.32	100.0	100.0	100.0
H_0: $\delta = 0$	100.0	99.24	91.98	100.0	99.94	98.27	100.0	99.98	99.04
Case 3:									
LM_1	75.52	84.60	68.47	99.77	100.0	98.72	100.0	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	94.44	74.92	60.97	100.0	95.75	77.44	100.0	97.15	89.49
LM-size:	4.90	4.77	4.88	5.03	5.01	4.81	5.15	4 .57	4.72

Table 3.2.c: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ =0.2, γ =100. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	$\delta = -0.1$			$\delta = -0.4$			$\delta = -0.8$		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	32.44	11.55	6.33	29.91	8.82	3.26	23.41	6.40	3.06
Parameter constancy test									
Case 1:									
LM_1	28.61	26.75	18.00	63.66	64.06	38.04	39.85	46.09	42.16
H_0: $\delta = 0$	88.22	83.20	73.93	65.86	68.03	54.60	40.62	44.53	44.44
Case 2:									
LM_1	44.88	59.05	54.82	97.06	99.77	73.31	99.96	100.0	87.58
H_0: $\delta = 0$	99.32	92.29	71.41	99.67	95.80	70.55	99.32	95.16	67.65
Case 3:									
LM_1	41.25	47.79	33.18	96.05	99.09	61.04	99.87	99.84	79.08
H_0: $\delta_1 = \delta_2 = 0$	69.27	45.54	30.96	95.29	75.85	34.05	93.21	79.38	47.39
LM-size:	5.03	4.57	4.84	4.85	5.10	4.81	4.76	5.03	4.93
T = 200	$\delta = -0.1$			$\delta = -0.4$			$\delta = -0.8$		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	45.41	16.39	7.58	32.15	9.26	3.87	24.63	6.59	2.54
Parameter constancy test									
Case 1:									
LM_1	64.52	51.68	22.82	71.32	69.33	51.42	41.74	51.44	51.18
H_0: $\delta = 0$	91.96	87.49	81.00	71.51	74.08	63.82	41.58	50.22	53.15
Case 2:									
LM_1	83.97	88.90	65.17	100.0	100.0	89.95	100.0	100.0	92.52
H_0: $\delta = 0$	99.85	95.42	78.63	99.88	96.76	74.94	99.27	95.76	69.69
Case 3:									
LM_1	80.33	80.54	42.35	99.97	100.0	74.16	100.0	100.0	86.61
H_0: $\delta_1 = \delta_2 = 0$	85.40	63.09	40.63	96.98	84.88	46.77	94.48	82.25	46.46
LM-size:	4.85	4.67	4.68	4.78	4.66	5.16	4.71	5.27	5.21

Table 3.3.a: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.4, γ =1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = –0.1**			**δ = –0.4**			**δ = –0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	52.43	43.60	36.04	100.0	99.99	99.88	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	6.29	6.08	6.60	7.88	9.56	9.92	6.82	8.84	10.31
H_0: $\delta = 0$	75.22	75.34	76.44	47.08	50.90	54.77	16.35	22.56	28.47
Case 2:									
LM_1	6.28	6.01	5.85	14.59	15.89	16.84	31.00	33.64	34.48
H_0: $\delta = 0$	99.31	99.54	98.97	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	8.98	8.62	9.46	12.52	13.74	14.20	24.13	26.61	26.56
H_0: $\delta_1 = \delta_2 = 0$	67.92	61.81	56.83	100.0	99.94	99.83	100.0	100.0	100.0
LM-size:	4.59	4.87	4.70	4.80	4.97	5.06	4.48	5.03	4.84
T = 200	**δ = –0.1**			**δ = –0.4**			**δ = –0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	97.55	93.74	87.84	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	7.36	8.05	8.06	12.62	15.69	17.38	8.51	13.69	17.35
H_0: $\delta = 0$	73.45	74.39	76.71	45.06	50.00	53.57	13.73	21.08	26.15
Case 2:									
LM_1	8.39	9.58	9.06	26.13	29.24	31.27	56.51	60.75	63.14
H_0: $\delta = 0$	100.0	99.99	99.94	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.48	10.36	9.56	20.73	22.62	24.34	46.17	51.21	52.52
H_0: $\delta_1 = \delta_2 = 0$	95.40	92.76	88.50	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.61	4.76	4.37	4.56	4.74	5.16	4.83	5.10	5.26

Table 3.3.b: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.4, γ =10. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	50.80	16.62	8.13	97.20	30.71	10.85	99.93	37.53	11.02
Parameter constancy test									
Case 1:									
LM_1	30.06	22.32	8.86	48.60	50.21	39.91	26.28	34.85	44.19
H_0: $\delta = 0$	75.93	76.47	73.92	48.57	52.33	60.18	26.13	34.51	48.00
Case 2:									
LM_1	45.33	53.67	49.08	96.69	99.67	90.14	99.92	100.0	99.46
H_0: $\delta = 0$	99.74	96.15	78.60	100.0	99.80	92.17	100.0	99.87	95.64
Case 3:									
LM_1	42.15	45.67	34.81	94.87	99.19	84.06	99.87	100.0	99.46
H_0: $\delta_1 = \delta_2 = 0$	70.63	43.32	28.91	99.80	88.70	57.97	100.0	92.97	81.03
LM-size:	4.60	5.10	4.51	4.74	4.65	4.81	4.93	5.17	5.16
T = 200	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	88.21	31.09	13.00	99.98	47.95	16.96	100.0	58.93	16.08
Parameter constancy test									
Case 1:									
LM_1	61.46	50.08	21.62	49.64	57.88	56.72	17.11	41.80	47.57
H_0: $\delta = 0$	79.75	78.39	81.77	48.74	56.87	67.39	16.77	41.08	49.56
Case 2:									
LM_1	80.18	85.46	70.62	99.90	100.0	99.46	100.0	100.0	100.0
H_0: $\delta = 0$	100.0	99.16	90.46	100.0	99.98	97.46	100.0	100.0	98.32
Case 3:									
LM_1	75.31	79.22	57.92	99.78	100.0	98.76	100.0	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	93.55	67.39	43.69	100.0	95.43	77.95	100.0	96.95	86.32
LM-size:	4.77	4.95	5.10	5.48	5.01	4.97	4.95	5.02	4.62

Table 3.3.c: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.4, γ =100. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	26.73	8.98	4.41	32.73	10.82	4.57	32.24	9.80	4.50
Parameter constancy test									
Case 1:									
LM_1	25.63	20.60	11.79	31.01	36.23	26.91	14.14	27.76	28.00
H_0: $\delta = 0$	70.37	67.15	56.92	31.38	35.03	35.23	15.79	21.33	24.22
Case 2:									
LM_1	42.76	52.56	31.52	97.68	99.72	76.59	100.0	100.0	90.89
H_0: $\delta = 0$	99.51	90.31	60.32	99.60	94.55	66.74	99.32	91.02	62.00
Case 3:									
LM_1	37.75	41.09	23.81	97.22	98.89	58.64	100.0	99.90	75.56
H_0: $\delta_1 = \delta_2 = 0$	66.55	39.64	24.04	94.29	76.34	29.76	92.56	76.12	42.22
LM-size:	4.87	4.96	4.55	4.89	4.71	4.81	4.64	5.13	4.79
T = 200	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	38.37	12.46	4.64	33.34	10.48	3.81	30.86	9.07	3.08
Parameter constancy test									
Case 1:									
LM_1	54.96	44.94	18.97	32.63	41.41	30.71	13.90	26.24	32.79
H_0: $\delta = 0$	72.58	70.55	62.07	31.97	41.22	41.21	16.14	23.59	30.52
Case 2:									
LM_1	83.89	87.40	48.06	100.0	100.0	77.43	100.0	100.0	87.34
H_0: $\delta = 0$	99.79	94.54	72.41	99.67	96.56	70.08	99.58	93.16	58.44
Case 3:									
LM_1	80.43	78.41	32.33	100.0	99.81	69.03	100.0	99.89	84.09
H_0: $\delta_1 = \delta_2 = 0$	84.15	59.95	26.50	95.71	83.78	39.37	94.49	80.50	42.53
LM-size:	4.90	4.37	4.77	4.72	4.58	5.28	4.94	4.63	5.23

Table 3.4.a: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.9, γ =1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	98.58	96.93	94.25	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	3.04	3.19	2.98	1.43	1.51	1.89	1.02	0.97	0.94
H_0: $\delta = 0$	2.51	2.40	2.53	0.28	0.38	0.51	0.03	0.07	0.06
Case 2:									
LM_1	9.80	10.60	10.90	39.64	42.05	43.27	79.06	81.49	82.18
H_0: $\delta = 0$	99.98	99.98	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.65	10.74	11.05	31.49	33.69	34.58	71.42	74.15	74.35
H_0: $\delta_1 = \delta_2 = 0$	97.18	95.90	92.89	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.33	4.78	4.60	5.04	5.01	4.82	4.85	4.52	4.87
T = 200	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	100.0	100.0	99.97	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	1.08	1.00	1.10	0.39	0.50	0.48	0.35	0.32	0.36
H_0: $\delta = 0$	0.80	0.92	1.22	0.03	0.03	0.03	0.00	0.00	0.00
Case 2:									
LM_1	16.02	18.74	18.42	68.17	71.87	72.50	97.55	98.42	98.26
H_0: $\delta = 0$	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	13.32	15.86	14.71	58.59	62.93	64.03	95.58	96.85	96.73
H_0: $\delta_1 = \delta_2 = 0$	100.0	100.0	99.97	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.88	4.51	4.68	4.95	5.09	5.23	5.00	5.40	5.09

Table 3.4.b: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.9, γ =10. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	97.79	45.96	20.26	100.0	74.29	28.26	100.0	87.21	25.75
Parameter constancy test									
Case 1:									
LM_1	3.82	3.76	5.82	1.07	2.53	4.21	0.58	2.00	3.81
H_0: $\delta = 0$	1.17	2.59	4.64	0.16	1.10	3.22	0.03	0.80	2.60
Case 2:									
LM_1	87.79	95.56	85.93	99.73	100.0	99.65	99.99	100.0	100.0
H_0: $\delta = 0$	100.0	99.56	92.20	100.0	99.97	96.89	100.0	100.0	97.71
Case 3:									
LM_1	85.09	92.93	80.45	99.57	100.0	99.50	99.99	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	95.46	65.67	47.14	100.0	77.90	62.81	100.0	84.53	65.86
LM-size:	5.11	4.61	4.70	5.07	4.72	4.61	4.88	4.93	4.76
T = 200	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	99.96	53.91	18.90	100.0	92.46	30.29	100.0	99.23	31.25
Parameter constancy test									
Case 1:									
LM_1	1.41	2.15	2.96	0.35	1.46	2.64	0.11	1.16	3.55
H_0: $\delta = 0$	0.49	1.67	3.49	0.02	0.61	1.85	0.00	0.35	2.24
Case 2:									
LM_1	97.91	99.91	94.23	100.0	100.0	100.0	100.0	100.0	100.0
H_0: $\delta = 0$	100.0	99.87	94.60	100.0	100.0	98.28	100.0	100.0	99.10
Case 3:									
LM_1	96.75	99.80	91.43	100.0	100.0	99.97	100.0	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	99.89	76.63	56.72	100.0	92.70	70.55	100.0	99.00	74.66
LM-size:	4.86	4.93	4.68	5.04	4.81	4.69	4.64	4.97	4.93

Table 3.4.c: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.9, γ =100. Numbers of repetitions N = 10000.

T = 100	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	93.58	69.58	54.53	94.59	59.80	49.33	84.90	43.15	30.98
Parameter constancy test									
Case 1:									
LM_1	1.70	1.16	2.59	0.34	0.72	1.48	0.31	1.02	2.84
H_0: $\delta = 0$	0.56	0.82	1.82	0.25	0.48	0.75	0.26	0.76	1.71
Case 2:									
LM_1	95.91	99.17	97.21	99.95	100.0	99.74	100.0	100.0	99.71
H_0: $\delta = 0$	100.0	99.57	97.41	99.99	99.38	94.65	100.0	98.22	86.67
Case 3:									
LM_1	95.03	98.58	95.47	99.94	99.98	99.70	100.0	100.0	99.55
H_0: $\delta_1 = \delta_2 = 0$	91.12	79.72	72.82	88.56	73.51	70.18	77.13	65.17	54.84
LM-size:	4.50	4.58	4.84	5.21	4.99	4.72	5.31	5.04	5.08
T = 200	δ = **–0.1**			δ = **–0.4**			δ = **–0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	89.49	59.39	48.20	78.43	40.01	27.79	57.49	25.31	12.90
Parameter constancy test									
Case 1:									
LM_1	0.77	0.96	1.45	0.32	1.32	2.37	0.37	2.92	4.96
H_0: $\delta = 0$	0.27	0.61	1.22	0.26	0.67	1.84	0.57	2.02	3.33
Case 2:									
LM_1	99.83	99.93	97.26	100.0	100.0	98.92	100.0	100.0	99.30
H_0: $\delta = 0$	99.99	99.29	95.35	100.0	97.88	87.51	99.98	94.19	75.47
Case 3:									
LM_1	99.68	99.76	95.75	100.0	99.93	98.38	100.0	99.92	97.21
H_0: $\delta_1 = \delta_2 = 0$	88.97	77.18	73.42	79.05	66.73	55.31	77.42	61.36	40.85
LM-size:	5.23	4.41	4.65	4.67	4.96	4.54	5.03	4.51	5.18

Table 3.5: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(2), θ=0.2, γ=100, c_2 =1- c_1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	$\delta = -0.1$		$\delta = -0.4$		$\delta = -0.8$	
Test	c_1=0.25	c_1=1/3	c_1=0.25	c_1=1/3	c_1=0.25	c_1=1/3
Johansen test						
H_0: No cointegration	19.93	43.27	22.73	74.19	18.40	78.10
Parameter constancy test						
Case 1:						
LM_1	18.41	14.93	60.36	60.97	54.57	48.35
H_0: $\delta = 0$	80.63	82.20	66.74	67.52	54.57	47.63
Case 2:						
LM_1	48.86	28.15	99.16	94.33	100.0	99.92
H_0: $\delta = 0$	96.09	99.19	99.43	100.0	99.84	100.0
Case 3:						
LM_1	37.43	24.66	97.22	90.25	100.0	99.77
H_0: $\delta_1 = \delta_2 = 0$	51.73	74.79	89.75	99.77	97.34	100.0
LM-size:	4.37	4.86	4.37	4.88	5.03	4.77
T = 200	$\delta = -0.1$		$\delta = -0.4$		$\delta = -0.8$	
Test	c_1=0.25	c_1=1/3	c_1=0.25	c_1=1/3	c_1=0.25	c_1=1/3
Johansen test						
H_0: No cointegration	34.91	76.79	31.66	96.11	24.79	98.41
Parameter constancy test						
Case 1:						
LM_1	45.92	40.19	72.33	71.11	59.06	50.56
H_0: $\delta = 0$	82.15	83.28	73.12	69.59	57.68	49.47
Case 2:						
LM_1	81.18	65.24	100.0	99.94	100.0	100.0
H_0: $\delta = 0$	98.85	99.95	99.84	100.0	100.0	100.0
Case 3:						
LM_1	71.50	56.75	100.0	99.77	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	74.25	92.84	94.32	100.0	98.95	100.0
LM-size:	4.76	4.97	4.72	4.71	4.76	4.96

Essay IV

Is the short-run Phillips curve nonlinear? Empirical evidence for Australia, Sweden and the United States*

1 Introduction

The shape of the Phillips curve is central to the conduct of monetary policy. The Phillips curve has generally been estimated in a linear framework, see e.g. Gordon (1970, 1975, 1977, 1983, 1997) but recent studies have allowed for a nonlinear relationship, see e.g. Laxton *et al.* (1998), Callen and Laxton (1998), Debelle and Vickery (1997). In all these studies the nonlinearity is imposed on the Phillips curve without prior econometric testing. If the true relationship is nonlinear this should be reflected in the econometric specification since it has important implications for the effects of monetary policy, see e.g. Isard *et al.* (1998) and Dupasquier and Rickets (1998a). On the other hand, if the true relationship is linear introducing nonlinearities yields an overparameterised model which will break down outside the sample period, see Granger and Teräsvirta (1992).

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In this paper the expectations-augmented short-run Phillips curve will be reconsidered. The focus is on investigating whether the relationship between inflation and unemployment is nonlinear, using the modelling strategy for smooth transition regression (STR) models outlined in Teräsvirta (1994, 1998). There are many theories suggesting a nonlinear relationship but the different theories yield different shapes, and the STR methodology makes it possible to test for linearity and estimate a nonlinear model without making any a priori assumptions about the shape of the nonlinear relationship. The implications of the final model specification can be compared to the economic theories. It turns out that the Phillips curves for Australia and Sweden are nonlinear while the one for the United States appears to be linear.

The paper is organised as follows. After a historical and theoretical background in Section 2 the STR models are introduced in Section 3. Expectations-augmented short-run Phillips curves for Australia, Sweden and the United States are estimated and the results discussed in Sections 4, 5 and 6 respectively. Section 7 contains conclusions.

2 Background

In a seminal paper, Phillips (1958) found a negative relationship between unemployment and wage inflation in the United Kingdom, 1861-1957. Subsequent research found a similar relationship between unemployment and price inflation. This relationship has since been known as the Phillips curve and at the time there was strong empirical support for a stable inflation-unemployment trade-off. In the late 1960s this hypothesis was severely criticized from the theoretical side by Friedman (1968) and Phelps (1968), who argued that it was unreasonable to assume that nominal variables could affect real variables and that a shift by policy makers to expansionary policies would eventually change the way prices and wages are set. In the early 1970s the first empirical failures of the Phillips curve occurred when both inflation and unemployment increased simultaneously, primarily due to the oil-price shocks. The critique prompted the formulation of the expectations-augmented Phillips curve. According to this specification no policy can permanently lower unemployment below its natural rate unless expectations are highly irrational.

The Phillips curve is regaining interest after a period in neglect and there has been considerable theoretical work suggesting a nonlinear relationship between inflation and unemployment. The so called "new Phillips curves", see Galí and Gertler (1998), are based on early studies of Taylor (1980) and Calvo (1983) using staggered nominal wage contracts and price setting by forward looking individuals and firms. There are other microeconomic theories that also give rise to an asymmetric relationship such as the theories dealing with, for example, capacity constraints, menu costs and nominal rigidities. However, the shape of the nonlinearity is ambiguous since the different theories yield different nonlinear relationships, see Section 2.2. The focus in empirical work has also shifted towards issues concerning the shape of the relationship, see e.g. Yates (1998), Gruen *et al.* (1998) and Dupasquier and Rickets (1998b) for reviews on empirical studies of nonlinear Phillips curves.

2.1 The expectations-augmented short-run Phillips curve

The linear expectations-augmented short-run Phillips curve is typically assumed to be of the following form

$$\pi_t = \pi_t^e + \gamma\left(u^* - u_t\right) + \varepsilon_t \tag{1}$$

where π_t is inflation, π_t^e is inflation expectations, u^* is the non-accelerating inflation rate of unemployment, NAIRU, u_t is the unemployment rate and ε_t is the error term. There are two main difficulties when estimating this model empirically. They are due to measurement problems regarding inflation expectations and the NAIRU. Neither of these variables are directly observable and they have to be approximated. In some studies inflation expectations are modelled as a simple weighted average of past inflation rates. But since expectation formations are sensitive to monetary policy, inflation expectations based on past inflation rates might be inappropriate, and survey measures of inflation expectations are becoming more popular for capturing the forward-looking component. However, in empirical work inflation expectations are generally assumed to be a combination of backward- and forward-looking components. This can be viewed as a mixture between the traditional and the "new" Phillips curve according to Galí and Gertler (1998). Thus,

$$\pi_t^e = \lambda_1 \pi_t^f + \lambda_2 \pi_t^{imp} + (1 - \lambda_1 - \lambda_2) A(L) \pi_{t-i} \tag{2}$$

where π_t^f is the forward-looking component and $A(L)$ is a lag polynomial. The backward-looking component reflects the inertia in the inflation process and the forward-looking component mirrors public expectations. Equation (2) is extended to include inflation in imported goods, π_t^{imp}, which may be an influential component when modelling Phillips curves for small open economies. The other major difficulty when estimating empirical Phillips curves is due to the NAIRU. By assuming that u^* is constant, model (1) can be rewritten as

$$\pi_t = \alpha + \pi_t^e + \gamma u_t + \varepsilon_t \tag{3}$$

where $\alpha = \gamma u^*$ in (3). This assumption is not innocuous. Several studies estimate the unobservable NAIRU by using the Kalman filter and they often conclude that the NAIRU is not stable over time, see e.g. Apel and Jansson (1997), Gruen *et al.* (1998), Laxton *et al.* (1998) for studies on Sweden, Australia and the United States respectively. But the uncertainty concerning the estimates is considerable and the results are also highly dependent on the initial setup, see Staiger *et al.* (1996). Hence, the characteristics of the NAIRU will not be modelled in this paper and in the following the NAIRU is assumed to be constant.

2.2 Microfoundations for a nonlinear Phillips curve relationship

According to Section 2.1 the short-run trade-off between unemployment and inflation is assumed to be constant over time. However, many theoretical models of price-setting behaviour predict the slope of the Phillips curve to be a function of macroeconomic conditions. There are several theories that may give rise to an asymmetric relationship but the precise form of the nonlinearity is ambiguous. A comprehensive review of the microfoundations can be found in Dupasquier and Rickets (1998a) and Yates (1998). Below is a brief review:

- The *capacity constraint model* is based on the assumption that increasing marginal costs and a fixed capacity in the short run are making it costly for the firms to

increase output and employment in the short run. Thus, inflation becomes more sensitive to output in times of excess demand and the short-run Phillips curve has a convex shape.

- The *signal extraction model*, Lucas (1972, 1973), suggests that the relationship between output and inflation arises because agents are unable to distinguish between aggregate and relative price shocks. The shocks are not directly observable and the relationship between output and inflation will depend on the variance of inflation. The more volatile the aggregate prices, the less a given price change will be attributed to a change in relative prices, and thus the smaller will be the output response. The short-run Phillips curve is linear but its slope will vary positively with the volatility of inflation.

- The *costly adjustment model*, see e.g. Ball *et al.* (1988), implies that the relationship between output and inflation varies with the level of inflation. In the presence of menu costs only some firms will change their prices in response to a demand shock. But, the greater the number of firms that change their prices the more responsive will the aggregate price level be to demand shocks. The firms will increase the frequency and size of price adjustment as inflation rises so that aggregate demand shocks will have less effect on output and more effect on the price level. Another example of the costly adjustment model occurs when the wage contracts between firms and workers have long duration. The short-run Phillips curve is a convex function that becomes linear as inflation approaches zero.

- The *downward nominal wage rigidity model*, see e.g. Stiglitz (1986), Fisher (1989), suggests that workers are more reluctant to accept a decrease in their nominal wages than a decrease in their real wages because of money illusion, institutional or behavioural factors. And, hence, a low inflation environment is more likely to create allocation inefficiencies. Provided that full adjustment to individual demand shocks eventually occurs this model has two implications for the shape of the short-run Phillips curve. First, the effects of nominal wage floors are more likely to be important at low rates of inflation. Second, if this is true then excess supply will have

less effect on inflation than excess demand, resulting in asymmetries with respect to the output gap.

- The *monopolistically competitive model,* see e.g. Stiglitz (1984), refers to the strategic pricing behaviour of firms in monopolistically competitive or oligopolistic markets. Producers might lower prices quickly in order to avoid being undercut by rivals. Furthermore, they might be reluctant to raise prices even in the face of generally rising prices, hoping to keep out potential new competitors. In this case the short-run Phillips curve will be concave.

2.3 Monetary policy and the Phillips curve

The main emphasis in monetary policy will differ depending on which of the above theories is the "right" one. The question of whether the Phillips curve is convex, concave or linear yields different effects of monetary policy. Yates (1998) and Dupasquier and Rickets (1998a) discuss the policy implications of the different theories thoroughly and the arguments will not be repeated here. The most important issue when dealing with asymmetric relationships is the importance of timing of the policy. If the central bank allows the economy to deviate from the target for some time a larger change in the monetary instrument may be necessary to achieve the desired effect. In a linear environment (setting credibility issues aside) it does not matter if there is one large or several small changes, whereas in a nonlinear environment the effect of several small changes might be very different from that of a single large one.

3 Smooth transition regression model

Smooth transition regression (STR) models will be applied to describe the empirical expectations-augmented short-run Phillips curves of Australia, Sweden and the United States. This makes it possible to test for linarity and estimate a nonlinear model without making any a priori assumptions about the shape of the nonlinear relationship. In this section some basic features of the STR models are introduced and the modelling cycle

will be reviewed together with the tests of linearity and parameter constancy.

Start by considering the following nonlinear model,

$$y_t = x_t'\varphi + x_t'\theta F(s_t) + e_t. \qquad t = 1, ..., T. \tag{4}$$

where $e_t \sim \text{nid}(0, \sigma^2)$, and $x_t = (1, y_{t-1}, ..., y_{t-k}; z_{1t}, ..z_{mt})' = (1, \tilde{x}_t')'$ with $p = k+m$ is the vector of stationary explanatory variables, some of which may be linear combinations of nonstationary variables. Furthermore, $\varphi = (\varphi_0, ..., \varphi_p)'$ and $\theta = (\theta_0, ..., \theta_p)'$ are parameter vectors. $F(s_t)$ is the transition function which is continuous in s_t and bounded between zero and unity. The transition variable s_t is either stationary or a time trend (t). The transition function of a kth order logistic smooth transition regression, LSTR(k) model is

$$F(s_t) = F(s_t; \gamma, c) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k} (s_t - c_i)\right\}\right)^{-1}, \ \gamma > 0, \ c_1 \leq .... \leq c_k \tag{5}$$

where $k = 1$ yields the LSTR(1) model and $k = 2$ the LSTR(2), which are the two parameterisations of transition functions that will be considered in this paper. The slope parameter γ determines the speed of transition between the two extreme regimes and the vector of location parameters c determines the location of the transition. For derivations of the tests and an exhaustive description of STR models, see Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998).

3.1 The modelling cycle for STR models

Following Teräsvirta (1994) the modelling cycle consists of four steps. Initially a linear model with no error autocorrelation is estimated. The tests of linearity and parameter constancy are performed in this model against the alternative of an STR parameterisation. If either of the hypotheses are rejected an STR model is estimated. The preferred STR specification is evaluated using misspecification tests of no error autocorrelation, parameter constancy and no additive nonlinearity, see Eitrheim and Teräsvirta (1996).

When testing for linearity the alternative is specified using several different variables as the potential transition variable. If more than one of the null hypotheses are rejected the STR model is specified for the transition variable which was used in the most forcefully

rejected hypothesis. The initial choice of possible transition variables is based on economic theory.

3.2 Testing linearity and parameter constancy

Testing linearity against the alternative of an LSTR(k) model amounts to testing if $\gamma = 0$ in equation (5). The model is not identified under the null hypothesis due to the nuisance parameters θ and c. A Taylor series approximation about $\gamma = 0$ is used as a substitute to circumvent this problem, and the tests are based on this transformed equation:

$$y_t = x_t'\beta_0 + \sum_{j=1}^{k} \left(\tilde{x}_t s_t^j\right)' \beta_j + e_t^* \tag{6}$$

where $e_t^* = e_t + \tilde{x}_t'\tilde{\theta} R(\gamma, c)$, but $e_t^* = e_t$ under H_0. The null hypothesis $H_0 : \gamma = 0$ in equation (5) where $k = 3$ implies

$$H_0' : \beta_1 = \beta_2 = \beta_3 = 0 \tag{7}$$

within (6) because $\beta_j = \gamma\tilde{\beta}_j$, where $\tilde{\beta}$ is a function of the parameters in the original STR specification. In order to decide between $k = 1$ and $k = 2$, one continues by carrying out the following test sequence within (6) :

$$H_{04} \ : \ \beta_3 = 0, \tag{8}$$

$$H_{03} \ : \ \beta_2 = 0 | \beta_3 = 0, \tag{9}$$

$$H_{02} \ : \ \beta_1 = 0 | \beta_2 = \beta_3 = 0. \tag{10}$$

If the rejection of H_{03} is strongest (measured by the p-value of the test), the rule is to choose $k = 2$, otherwise one selects $k = 1$; for the reasoning behind this rule see Teräsvirta (1994). Furthermore, it is shown that in small samples an F approximation to the LM-test statistic is preferable to the asymptotic χ^2-distribution because it has good size properties.

A special case of nonlinearity is when the transition variable is a time trend (t). The STR model can be viewed as a linear model with time-varying parameters, see Lin and

Teräsvirta (1994). Following Lin and Teräsvirta (1994), the testing procedure becomes

$$\begin{aligned} LM_3 &: \beta_1 = \beta_2 = \beta_3 = 0, \\ LM_2 &: \beta_1 = \beta_2 = 0 | \beta_3 = 0, \\ LM_1 &: \beta_1 = 0 | \beta_2 = \beta_3 = 0. \end{aligned}$$

in (7). If the rejection of LM_1 is strongest (measured by the p-value) an LSTR(1) model is preferred and if LM_2 is the strongest rejected hypothesis an LSTR(2) model is the most appropriate. When dealing with time-varying parameters a more flexible specification may be needed, and if LM_3 is the strongest rejected hypothesis the non-monotonous LSTR(3) model will be estimated.

In the following, the Phillips curve for each of Australia, Sweden and the United States is estimated and evaluated. A linear model is specified for each country and in a second step linearity is tested against a nonlinear alternative of smooth transition regression (STR) type. STR is not the only possible nonlinear alternative but an advantage of STR models is that there exists a well established specification and modelling cycle complete with linearity and misspecification tests, see Teräsvirta (1994). The tests have good small sample properties, which makes them useful for analysing macroeconomic relationships since the time series are often short. The STR modelling procedure makes it possible to simultaneously test for linearity and parameter constancy. Time-varying parameters may be an issue, especially since the NAIRU is assumed to be constant and due to large changes in monetary policies. If any of the linearity or parameter constancy hypotheses are rejected the corresponding STR model will be estimated and evaluated.

4 Australia

There have been numerous studies of the Australian Phillips curve starting with Phillips (1959). A review of old and recent studies of the Australian Phillips curve can be found in Gruen *et al.* (1998). The older studies usually estimate a linear relationship while the more recent ones often allow for either a time-varying NAIRU or a nonlinear specification. The common theme of the studies that allow for a nonlinear relationship is that nonlinearities

are introduced without any initial tests of linearity. For example, Debelle and Vickery (1997) estimate and compare a linear and a nonlinear short-run Phillips curve with forward looking expectations using quarterly data from 1959(3) to 1997(1). They find that the nonlinear specification is more accurate than the linear since it has higher explanatory power. This does not necessarily imply that the true relationship is nonlinear although this possibility cannot be excluded. The reason is that the authors do not perform any initial test of linearity before estimating the nonlinear model. The true relationship may still be linear and the estimated nonlinear specification will in this case be over-parameterised.

In this section a linear Phillips curve will be estimated and the tests of linearity and parameter constancy will be performed against the alternative of an STR model. If any of the hypotheses are rejected a nonlinear model will be specified.

4.1 Data

Quarterly seasonally unadjusted series for 1977(1)-1997(4) are used for the estimation. The Phillips curve is defined for the annual inflation rate, that is, $\pi_t = \Delta_4 p_t$ where p_t is the consumer price index in logarithms, $\Delta_i = (1 - L^i)$ is the difference operator and L is the lag operator. There are several measures of inflation available. The underlying inflation series is used for the Australian study since this is the inflation measure currently targeted by the Reserve Bank of Australia. The underlying inflation rate is calculated as the total inflation net of fresh fruit and vegetables, mortgage interest and consumer credit charges, automotive fuel and health services. In the rest of Section 4, the term inflation always refers to underlying inflation unless otherwise explicitly stated. The inflation expectation series used here is derived from bond yields by Debelle and Vickery (1997).

The CPI inflation and the underlying inflation are plotted against time in Figure 4.1. Inflation has been diminishing over the sample. It was very high in the 1980s but has decreased sharply in the beginning of the 1990s. One possible explanation is that the Reserve Bank of Australia announced an explicit inflation target of 2-3 % in December 1993 in targeting the underlying inflation rate. Inflation expectations are graphed together with the CPI inflation in Figure 4.2, and with underlying inflation in Figure 4.3. The most striking feature in both figures is that the inflation expectations series is systematically

higher than both the actual and the underlying inflation rate. This can be interpreted as lack of credibility of the monetary policy in Australia. The systematic difference persists when the levels of both inflation and inflation expectations decrease.

Since stationarity is a prerequisite for the modelling techniques used, see Section 3, it is necessary to determine the order of integration of the variables involved. The augmented Dickey-Fuller tests are applied to test the hypothesis of a unit root and the results can be found in Table 4.1. The hypothesis of a unit root is rejected for all series at the 1 % level of significance and, hence, the variables appear to be stationary.

4.2 Empirical results

The estimation procedure starts by specifying a linear expectations-augmented short-run Phillips curve for 1977(1)-1997(4). The initial setup is consistent with the theoretical arguments in Section 2. Economic theory does not give any guidance regarding the dynamics and all variables are initially included with several lags in order to achieve a model without error autocorrelation. Variables with poor explanatory power are excluded from the final specification using the t-values as a guidance. The parsimonious equation becomes

$$\begin{aligned}
\pi_t &= \underset{(0.0065)}{0.019} + \underset{(0.058)}{1.04}\,\pi_{t-1} - \underset{(0.099)}{0.49}\,\pi_{t-4} + \underset{(0.079)}{0.29}\,\pi_{t-5} + \underset{(0.025)}{0.091}\,\pi_t^f \\
&\quad + \underset{(0.0099)}{0.050}\,\pi_{t-1}^{imp} - \underset{(0.011)}{0.023}\,\pi_{t-2}^{imp} - \underset{(0.0084)}{0.030}\,u_{t-1} + \underset{(0.0088)}{0.021}\,u_{t-2} + \hat{e}_t \qquad (11) \\
T &= 84, R^2 = 0.99, \sigma_{lin} = 0.0035, AIC = -11.19, \\
LJB &= 0.22(0.90), A(1) = 0.18(0.67), A(4) = 0.88(0.48).
\end{aligned}$$

where π_t denotes the annual inflation rate, π_t^f is the expected annual inflation rate, π_t^{imp} is the annual inflation rate in import prices and u_t denotes the log of unemployment. The standard deviations of the coefficients appear in parentheses, T is the number of observations, R^2 is the coefficient of determination, σ_{lin} is the standard deviation of the linear model, and AIC is Akaike Information Criterion. The Lomnicki-Jarque-Bera (LJB) test does not reject normality of the error process; the corresponding p-value appear in parenthesis, and the LM test of no ARCH of order j, $A(j)$, does not indicate

a misspecified model. The results of the test of no error autocorrelation can be found in Table 4.2. The hypothesis of no error autocorrelation against error autocorrelation of order one is rejected at the 5 % significance level (p-value = 4.7 %). However, since the hypothesis of no error autocorrelation against error autocorrelation of a higher order than one cannot be rejected, and the rejection level of no error autocorrelation of order one is close to the chosen significance level of 5 %, the linear model (11) will be treated as if it fulfills the assumption of no error autocorrelation.

The linear specification (11) is estimated without any a priori restrictions of long-run neutrality of money. The coefficients of the inflation variables sum to 0.958 which is close to unity, and imposing the restriction of long-run neutrality does not change any conclusions. The sum of the coefficients of the unemployment variables is negative, indicating a negative relationship between inflation and unemployment. The intercept can be interpreted as a measure of the NAIRU and the linear equation (11) seems to be consistent with economic theory.

The tests of linearity and parameter constancy are performed and the results are shown in Table 4.3. Parameter constancy is the most strongly rejected hypothesis, pointing at an LSTR(3) model. Estimating this yields a model with strong error autocorrelation and the null hypothesis of no additional nonlinearity is rejected for two of the transition variables. Moreover, the specification does not capture all time-variation in the parameters. Taking all these features together, the model does not seem to perform well and will not be considered further. According to Table 4.3 linearity is also rejected. It is most strongly rejected when the change in contemporary unemployment $\Delta_1 u_t$ is used as transition variable and the testing procedure indicates an LSTR(2) model. Estimating this yields a model with non-normal residuals and time-varying parameters. Moreover, the hypothesis of no additional nonlinearity is also rejected. But since there are very few observations that make the second transition necessary an LSTR(1) model is estimated instead. Excluding the variables with poor explanatory power the model becomes

$$
\begin{aligned}
\pi_t &= \underset{(0.0069)}{0.031} + \underset{(0.051)}{0.92}\,\pi_{t-1} - \underset{(0.092)}{0.35}\,\pi_{t-4} + \underset{(0.076)}{0.17}\,\pi_{t-5} + \underset{(0.022)}{0.12}\,\pi_t^f \\
&\quad + \underset{(0.0064)}{0.043}\,\pi_{t-1}^{imp} - \underset{(0.0090)}{0.039}\,u_{t-1} + \underset{(0.0094)}{0.026}\,u_{t-2} \\
&\quad + \left\{ - \underset{(0.042)}{0.13} - \underset{(0.33)}{1.08}\,\pi_{t-4} + \underset{(0.35)}{1.33}\,\pi_{t-5} + \underset{(0.057)}{0.11}\,\pi_t^f + \underset{(0.020)}{0.053}\,u_{t-2} \right\} \\
&\quad \times \left[1 + \exp\left\{ - \underset{(1.49)}{6.61} \left(\Delta_1 u_t - \underset{(0.0095)}{0.053} \right) / \hat{\sigma}_{\Delta_1 u_t} \right\} \right]^{-1} + \hat{e}_t \qquad (12) \\
T &= 84, R^2 = 0.99, \sigma_{nl} = 0.0032, \sigma_{nl}/\sigma_{lin} = 0.90, AIC = -11.35, \\
LJB &= 1.62(0.45), A(1) = 0.24(0.62), A(4) = 1.38(0.25).
\end{aligned}
$$

where σ_{nl} denotes the standard deviation of the nonlinear model (12). With a residual standard deviation that is 10 % less than that of (11), the LSTR(1) model (12) variance dominates the linear model (11). The assumptions of normality and of no ARCH are satisfied according to the diagnostic tests shown below the equation. There is no evidence of error autocorrelation, which was close to being rejected in the linear specification (11), see Table 4.4. The hypothesis of no additional nonlinearity is rejected (p-value = 0.031) when using π_t^f as the transition variable, see Table 4.4. However, linearity cannot be rejected for any of the other transition variables, which is an improvement compared to the results of the initial linearity tests in Table 4.2. Parameter constancy of the whole set of parameters is still rejected (p-value = 0.0029) but equation (12) captures a major part of the time variation compared to the parameter constancy tests presented in Table 4.3.

The transition function of equation (12) is plotted against its argument ($\Delta_1 u_t$) in Figure 4.4. When the transition function equals zero the linear component of equation (12) adequately describes the relationship, and when the transition function equals one the complete model is necessary for capturing the features of the short-run Phillips curve. In Figure 4.5 the transition function is plotted against time. This graph shows when the complete model is working (the transition function equals one) and when only the linear part is necessary to describe the inflation in Australia. The residuals of equations (11) and (12) are plotted in Figure 4.6 and the nonlinear specification (12) outperforms the linear (11) for the observations where the complete nonlinear model (12) is working.

In order to find out whether the nonlinear model (12) explains the empirical results

of the linear model (11) and vice versa, the encompassing tests are being computed. An introduction to encompassing tests can be found in Hendry (1995), and for applications of encompassing tests in STR models see Teräsvirta (1998) and Teräsvirta and Eliasson (1998). A brief review can be found in Appendix A. Testing the hypothesis of whether the linear model (11) encompasses the nonlinear (12) amounts to testing the null hypothesis that the nonlinear part of (12) does not enter the linear equation (11). The hypothesis is strongly rejected ($F = 4.15$, p-value $= 0.000038$) which implies that the linear model (11) does not encompass the nonlinear (12). The next step is to find out if equation (12) encompasses (11), that is, whether the explanatory power of equation (12) is significantly improved by linearly including the explanatory variables of model (11) that do not enter equation (12) linearly. The null hypothesis is that the variables are superfluous and the hypothesis cannot be rejected ($F = 0.16$, p-value$= 0.67$). Hence, model (12) encompasses (11) while the reverse is not true.

The estimated nonlinear Phillips curve of Australia displays very interesting features. The unemployment variable plays a central role and it enters equation (12) both in levels and in first differences. Since $\Delta_1 u_t$ is used as the transition variable it determines which regime is most adequate in describing the Phillips curve for different observations. According to model (12) the relationship between inflation and unemployment is negative most of the time, but for large increases in the unemployment rate the relationship turns positive. This suggests that the empirically observed failure of the Phillips curve might be a result of a nonlinear relationship and only mirrors a shift in the Phillips curve to a new level where the usual negative relationship is again valid. Moreover, equation (12) indicates that the NAIRU varies over time but the nonlinear regime does not prevail sufficiently long to permit any conclusions about the movements in the NAIRU.

The next issue is to find out whether equation (12) supports any of the theories of Section 2.2. The interpretation of the model is not straightforward since the nonlinearity of equation (12) is ruled by the rate of change of unemployment instead of the level of unemployment. The theories where the nonlinearity originates from demand factors are defined for excess demand or supply which can translated to unemployment or full employment but not as changes in unemployment. The capacity constraint model states that a low

demand, or high unemployment, makes inflation less sensitive to unemployment. This is supported by the nonlinear equation (12) since the negative relationship between inflation and unemployment decreases (turns positive) in times of increasing unemployment.

The downward nominal rigidity model suggests that excess supply will have less effect on inflation than excess demand. According to equation (12) the relationship appears to be the opposite. The coefficient of unemployment is large (and positive) in times of large increases in unemployment, but the number of observations in the nonlinear regime are few and the size of the coefficient might change with a larger sample.

5 Sweden

In November 1992, Sveriges Riksbank, the Central Bank of Sweden, announced an inflation target regime of an annual inflation rate of 2 % with a tolerance band of 1 % from 1995 and onwards. Initially there was a severe credibility problem for the new target since inflation expectations exceeded the upper limit of 3 % and it seemed as if the target would be missed for 1995 and 1996; see Svensson (1995). However, the policy quickly gained credibility and was in the neighborhood of the target of 2 % by the end of the sample, see Figure 5.1.

There are few empirical studies of the Swedish Phillips curve allowing for a nonlinear trade-off. Yates (1998) investigates, using Swedish annual data for 1864-1938, whether or not the slope of the Phillips curve varies according to whether the prices are rising or falling. There is no significant evidence of any variation in the slope indicating asymmetries. Furthermore, he explores whether or not there is a kink in the Phillips curve which could indicate downward nominal rigidity. The conclusion is that there is no evidence of a kink.

In this section a Swedish linear Phillips curve will be estimated and evaluated with respect to linearity and parameter constancy where the alternative is specified as an STR model. If any of the hypotheses are rejected a nonlinear model will be specified.

5.1 Data

The time series consists of quarterly seasonally unadjusted data for the period 1979(3)-1997(4). The inflation measure used is annual changes in total CPI, which is the inflation measure currently targeted by Sveriges Riksbank. The forward-looking component of the inflation expectations is approximated by a survey of the households' inflation expectations one year ahead gathered by the National Institute of Economic Research. The time series of inflation and inflation expectations can be found in Figure 5.1. The series generally move closely together and there are no systematic differences.

Unit root tests are performed for all variables and the results are shown in Table 5.1. All variables seem to be integrated of order one, and taking the first difference yields stationary variables[1]. Because of this, the Phillips curve will be estimated in differences instead of levels as in the original specification shown in Section 2. The rate of change of inflation is graphed against time in Figure 5.2. The salient features of the series are the oil-price shock in the late 1970s and the turbulence in the early 1990s.

5.2 Empirical analysis

Estimating a linear expectations-augmented short-run Phillips curve for Sweden 1979(3)-1997(4), an eight order lag structure is initially applied for all variables to capture the inertia in the inflation process. German inflation is also included as one of the explanatory variables in the original setup. This is done partly to capture inflation in imports, Germany being one of Sweden's major trading partners, and partly because of its leading role in the ERM. In addition, Juselius (1992) shows that German inflation is influential in describing the Danish inflation; however, it turns out that this is not the case for Sweden. The variables with poor explanatory power are excluded from the equation and the parsimonious linear model becomes

[1] Skalin and Teräsvirta (1999) show that the unemployment series for Sweden is not a unit-root process. It is the asymmetries of the series that make it appear like one. However, in this paper the series will be treated as an I(1).

$$
\begin{aligned}
\Delta_1\pi_t &= \underset{(0.088)}{0.31}\,\Delta_1\pi_{t-2} - \underset{(0.089)}{0.46}\,\Delta_1\pi_{t-4} + \underset{(0.15)}{0.49}\,\Delta_1\pi_t^f \\
&\quad + \underset{(0.14)}{0.57}\,\Delta_1\pi_{t-1}^f - \underset{(0.0076)}{0.011}\,\Delta_1 u_{t-1} + \hat{e}_t \qquad (13)\\
T &= 74, R^2 = 0.47, \sigma_{lin} = 0.80, AIC = -0.38, \\
LJB &= 17.36(2\times 10^{-4}), A(1) = 0.23\,(0.63)\,, A(4) = 0.077\,(0.99)\,.
\end{aligned}
$$

The assumption of normality is rejected while the assumption of no ARCH is satisfied according to the diagnostic tests shown below equation (13). One way to deal with the lack of normality is to introduce dummies capturing the large changes in inflation due to the turbulence in early 1990s. But by including dummy variables, information about the dynamics of the process would be disregarded in these observations and, hence, dummy variables are not introduced at this stage. There is no evidence of any error autocorrelation according to the results in Table 5.2.

The most striking feature of the linear model (13) is the strong dependence on inflation expectations. Note that the variables of inflation expectations are more influential than the variables of lagged inflation. The result stresses the important role of inflation expectations when the monetary policy is devoted to an inflation target. The relationship between inflation and unemployment is negative and there is no intercept in the model. The exclusion of the intercept is reasonable if the assumption of a constant NAIRU is true since equation (13) is specified for first differences.

The results of the linearity and parameter constancy tests can be found in Table 5.3. It is seen that linearity is most strongly rejected using either $\Delta_1\pi_{t-2}$ or $\Delta_1\pi_{t-1}^f$ as transition variable. Both models are estimated and there are many similarities between the two specifications. However, the LSTR(1) model where $\Delta_1\pi_{t-1}^f$ is used as the transition variable performs slightly better. It becomes

$$
\begin{aligned}
\Delta_1\pi_t &= \underset{(0.14)}{0.49}\,\Delta_1\pi_{t-2} - \underset{(0.080)}{0.46}\,\Delta_1\pi_{t-4} + \underset{(0.13)}{0.62}\,\Delta_1\pi_{t-1}^f - \underset{(0.018)}{0.039}\,\Delta_1 u_{t-1} \\
&\quad + \left\{ -\underset{(0.18)}{0.30}\,\Delta_1\pi_{t-2} + \underset{(0.17)}{0.67}\,\Delta_1\pi_t^f - \underset{(0.019)}{0.032}\,\Delta_1 u_{t-1} \right\} \\
&\quad \times \left[1 + \exp\left\{ -\underset{(28065)}{134.35}\left(\Delta_1\pi_{t-1}^f + \underset{(3.88)}{0.42}\right)/\hat{\sigma}_{\Delta_1\pi_t^f} \right\} \right]^{-1} + \hat{e}_t \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
T &= 74, R^2 = 0.54, \sigma_{nl} = 0.77, \sigma_{nl}/\sigma_{lin} = 0.96, AIC = -0.42, \\
LJB &= 3.23(0.20), A(1) = 0.028\,(0.87)\,, A(4) = 0.60\,(0.66)\,.
\end{aligned}
$$

The nonlinear LSTR(1) model (14) variance dominates its linear equivalent (13) in the sense that its standard deviation is 96% of the latter one. The assumption of normality is satisfied in equation (14), which was a problem for model (13), and there is no indication of ARCH. The results of the misspecification tests can be found in Table 5.4. Equation (14) satisfies the assumptions of no error autocorrelation and parameter constancy. Linearity is still rejected using $\Delta_1\pi_{t-2}$ as transition variable but the number of observations is not large enough to consider two nonlinear components, and the remaining nonlinearity will not be considered further.

The transition function of equation (14) is plotted against its argument $\left(\Delta_1\pi_t^f\right)$ in Figure 5.3. It is shown that the transition is very fast ($\gamma = 134$) and that the transition function is equal to either unity or zero during this sample. In Figure 5.4 the transition function is graphed against time, showing when the complete model is necessary to describe the Phillips curve relationship (the transition function equals unity). The full specification is necessary most of the time but for some observations the linear component is adequate for capturing the features of the rate of change of inflation. The residuals of equations (13) and (14) can be found in Figure 5.5. The residuals appear to be similar except in the early 1990s, where the nonlinear model (14) performs better than the linear (13).

The next step is to find out whether the nonlinear model (14) encompasses the linear (13) and vice versa. Testing the hypothesis of whether equation (13) encompasses (14) the MNM, see Appendix A, is achieved by including the nonlinear part of equation (14) in model (13). The hypothesis is rejected ($F = 2.43$, p-value $= 0.019$) which implies that the explanatory power of model (13) would be improved by including the nonlinear component. The linear model (13) does not encompass the nonlinear (14). To test whether the nonlinear model (14) encompasses the linear (13), equation (14) is completed linearly with the variables included in (13) that do not enter the linear component of (14). The results appear in Table 5.4 as the "test of linear restrictions". The null hypothesis cannot be rejected, and hence model (14) encompasses (13) while the reverse is not true.

In the nonlinear Phillips curve for Sweden (14) the central variable is $\Delta_1 \pi_t^f$, which not only enters the model with a high coefficient but also rules the transition between the extreme regimes. According to equation (14) the rate of change of inflation depends more strongly on inflation expectations than on lagged values of inflation. The full model is usually needed for capturing the dynamic features of the rate of change of inflation. The coefficients of the inflation variables sum to 1.02 and the unemployment variables enter with a negative coefficient. However, for very large negative changes $\left(\Delta_1 \pi_t^f < -0.42\right)$ the linear component is sufficient for describing inflation and the influence of both the inflation variables and the unemployment variable is smaller. Note that there is no intercept in equation (14). This may be interpreted as if the assumption of a constant NAIRU is true since the transformation into first difference would make the intercept vanish. However, the observed time period is quite short and any strong conclusions of the NAIRU has to be done with great caution.

None of the theories of Section 2.2 considers asymmetries due to inflation expectations. However, the costly adjustment model states that higher inflation makes it more costly for the firms to keep their prices fixed, resulting in an overall higher price level. By assuming that the firms are forward-looking, higher inflation expectations should result in a price increase.

6 United States

Many of the empirical models of the Phillips curve for the United States are heavily influenced by the work of Gordon, see e.g. Gordon (1970, 1975, 1977, 1983, 1997). His preferred Phillips curve specification is linear with backward-looking inflation expectations. It incorporates a long lag structure and there are several dummy variables included in the specification. In Gordon (1996) he allows for a kinked functional form and finds no significant evidence of nonlinearity and therefore concludes that the Phillips curve is resolutely linear. Despite this, there have been many nonlinear Phillips curves estimated for the United States. Clark *et al.* (1996) find significant nonlinearity when estimating the Phillips curve for 1964(1)-1990(4) allowing for a kinked functional form. Debelle and

Laxton (1997) find that a nonlinear model fits data better when estimated for 1971(2)-1995(2), but they do not perform any test of linearity before estimating the models. There are many more studies for the United States and a review of empirical nonlinear Phillips curve studies can be found in Yates (1998) or Dupasquier and Rickets (1998b).

In this section an expectations-augmented short-run Phillips curve will be specified for the United States and the hypotheses of linearity and parameter constancy will be tested against a nonlinear alternative of STR type.

6.1 Data

The time series are quarterly seasonally unadjusted quarterly data covering the period 1978(1)-1997(4). The inflation measure used is annual changes in total CPI and the inflation expectations are given by the Michigan survey measure of expected inflation over the next year held at period t. Monetary policy in the United States differs from that in Australia and Sweden in the sense that the Federal Reserve has not chosen to announce an explicit inflation target or to apply one. The Federal Reserve formally opened its disinflationary policy in October 1979 and inflation has been reversed and stabilized after the peak in 1981, see Figure 6.1. The time series move closely together during the whole sample and there are no systematic discrepancies. A striking feature of the inflation series is the change in the level of inflation before and after 1982, when the disinflation policies became efficient and the inflation stabilized.

The results of the augmented Dickey-Fuller tests can be found in Table 6.1. The null hypothesis of a unit root is forcefully rejected for all the variables and the Phillips curve will be specified in levels.

6.2 Empirical analysis

Estimating a linear Phillips curve for the United States for the period 1978(1)-1997(4), a twelfth-order lag structure is initially applied. Excluding the variables with poor explanatory power using the t-values as guidance, the linear Phillips curve becomes,

$$
\begin{aligned}
\pi_t &= \underset{(0.0064)}{0.0062} + \underset{(0.10)}{1.00}\,\pi_{t-1} - \underset{(0.15)}{0.39}\,\pi_{t-2} + \underset{(0.15)}{0.53}\,\pi_{t-3} - \underset{(0.16)}{1.11}\,\pi_{t-4} \\
&\quad + \underset{(0.17)}{0.87}\,\pi_{t-5} - \underset{(0.16)}{0.27}\,\pi_{t-6} + \underset{(0.15)}{0.35}\,\pi_{t-7} - \underset{(0.14)}{0.63}\,\pi_{t-8} \\
&\quad + \underset{(0.078)}{0.35}\,\pi_{t-9} + \underset{(0.056)}{0.44}\,\pi_t^f - \underset{(0.056)}{0.61}\,u_{t-1} + \hat{e}_t \qquad (15) \\
T &= 79, R^2 = 0.99, \sigma_{lin} = 0.0033, AIC = -11.27, \\
LJB &= 4.64(0.098), A(1) = 1.23\,(0.27)\,, A(4) = 0.61\,(0.66)\,.
\end{aligned}
$$

The errors appear normally distributed, and the tests of no ARCH do not indicate misspecification. The null hypothesis of no error autocorrelation cannot be rejected, see Table 6.2.

The linear Phillips curve of the United States (15) is characterized by strong inertia, which supports earlier studies. The coefficient of inflation expectations is quite large and there is a negative relationship between inflation and unemployment. The intercept is not significant (p-value $= 0.33$) but it is still included for theoretic arguments since it is assumed to capture the NAIRU, see Section 2.1.

The results of the linearity and parameter constancy test can be found in Table 6.3. There is no sign of either nonlinearities or time-varying parameters and, hence, the United States Phillips curve appears to be linear when tested against an STR model.

7 Conclusions

The results of the paper support the assumption of a nonlinear Phillips curve in Australia and in Sweden while the Phillips curve in the United States appears to be linear. The empirical models agree with the original Phillips curve setup despite the differences in the final specifications. This is not so strange considering that economic theory is not explicit about the dynamics and the only thing that can be judged is the sign and to some extent the magnitude of the coefficients. In the nonlinear models, when estimated, variance dominates and encompasses its linear equivalents.

The nonlinear Phillips curve of Australia has very interesting features. According to this specification the relationship between inflation and unemployment is negative most

of the time but turns positive for large increases in the unemployment rate. This suggests that the empirically observed "failure" of the Phillips curve might be the result of a nonlinear relationship and only mirrors a shift in the Phillips curve to a new level where the usual negative relationship is valid. The model also indicates that the NAIRU varies over time.

The Swedish Phillips curve also appears to be nonlinear. The rate of change of inflation expectations seems to be the key variable in this model since it rules the nonlinearity, and it is more influential in the estimated model than lagged values of the inflation variable itself. The specification emphasizes the importance of inflation expectations, which will have important implications for monetary policy.

The Phillips curve of the United States is characterized by strong inertia in the inflation process. The intercept is not significantly included in the final specification which is surprising since the intercept can be viewed as a measure of the NAIRU. There is no evidence of nonlinearity or parameter constancy in the final model specification, and the Phillips curve appears to be linear in the United States when the alternative is defined as an STR model.

According to the results of this paper the Phillips curve appears to be nonlinear in both Australia and Sweden. It would be interesting to find out if similar conclusions could be reached by other nonlinear model specifications. Another topic for further research would be to define the Phillips curve with output gaps instead of unemployment variables. This is however beyond the scope of this paper and will be left for future research.

Appendix A: Encompassing tests using the Minimal Nesting Model (MNM)

A.1 Comparing the linear and nonlinear equations

Consider

$$y_t = \alpha_1^{'} x_{1t} + u_{1t} \tag{A.1}$$

and

$$y_t = \alpha_2^{'} x_{2t} + \alpha_3^{'} x_{3t} G_1 \left(s_t; \gamma_1, c_1\right) + u_{2t} \tag{A.2}$$

where the x_{1t}, x_{2t}, x_{3t} do not contain common elements. The MNM becomes

$$y_t = \alpha_1^{'} x_{1t} + \alpha_2^{'} x_{2t} + \alpha_3^{'} x_{3t} G_1 \left(s_t; \gamma_1, c_1\right) + u_{3t}. \tag{A.3}$$

The null hypotheses are:

$$H_{01}\left[(A.1) \text{ encompasses } (A.2)\right] \ : \ \alpha_2 = 0 \text{ and } \gamma_1 = 0 \text{ in } (A.3)$$

$$H_{02}\left[(A.2) \text{ encompasses } (A.1)\right] \ : \ \alpha_1 = 0 \text{ in } (A.3)$$

The model is not identified under the null hypothesis due to the nuisance parameters α_2 and γ_1. As with the linearity tests in Section 3.2, this can be circumvented by using a Taylor series approximation about $\gamma_1 = 0$.

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Graphs

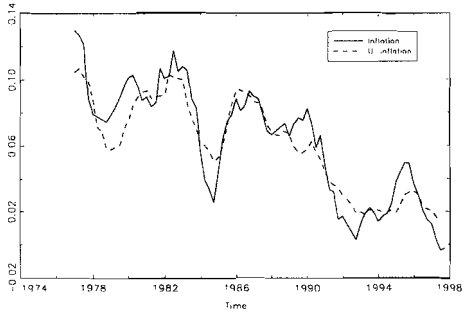

Figure 4.1: Inflation and underlying inflation for Australia, 1977(1)-1997(4).

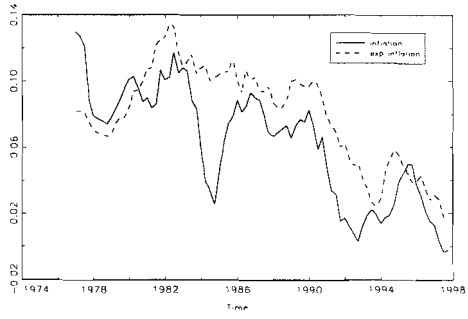

Figure 4.2: Inflation and inflation expectati for Australia, 1977(1)-1997(4).

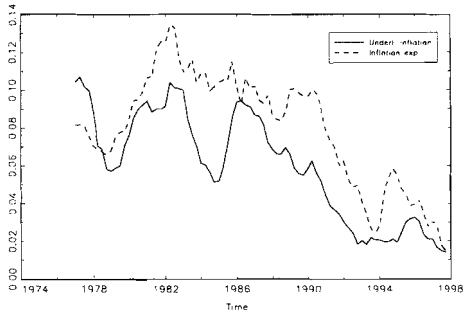

Figure 4.3: Underlying inflation and inflation expectations for Australia, 1977(1)-1997(4).

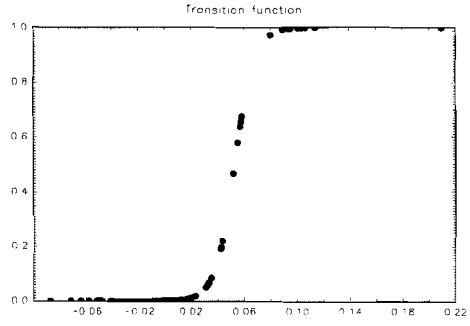

Figure 4.4: Transition function of (4.2) plott against its argument, 1977(1)-1997(4).

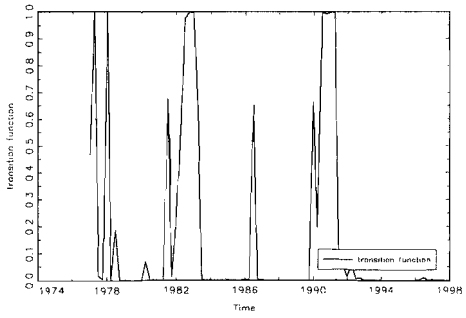

Figure 4.5: Transition function of (4.2) plotted against time, 1977(1)-1997(4).

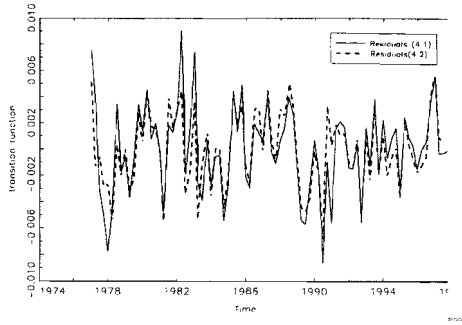

Figure 4.6: Residuals of (4.1) and (4.2), 1977(1)-1997(4).

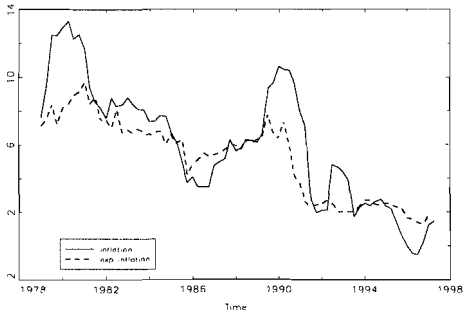

Figure 5.1: Inflation and inflation expectations for Sweden, 1979(2)-1997(4).

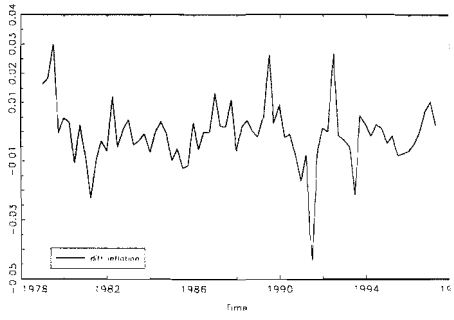

Figure 5.2: Rate of change of inflation for Sweden, 1979(2)-1997(4).

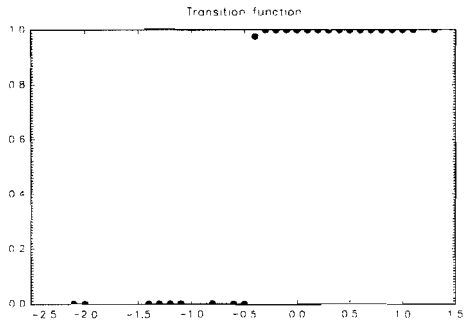

Figure 5.3: Transition function of (5.2) plotted against its argument, π_t^f, 1979(3)-1997(4).

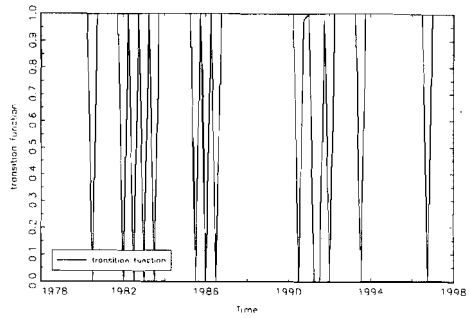

Figure 5.4: Transition function of (5.2) plott against time, 1979(3)-1997(4).

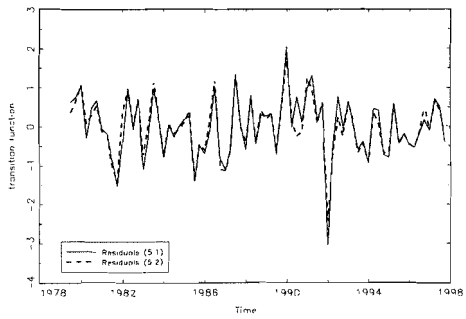

Figure 5.5: Residuals of (5.1) and (5.2), 1979(3)-1997(4).

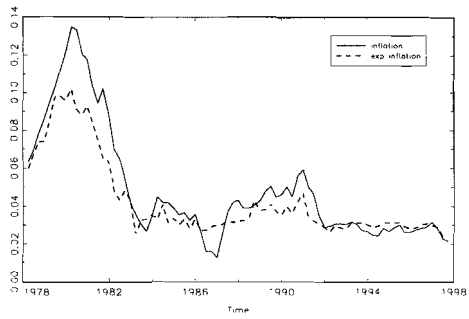

Figure 6.1: Inflation and inflation expectati for the U.S, 1978(1)-1997(4).

Tables

Table 4.1: Augmented Dickey-Fuller unit-root tests for Australian data 1977(1) - 1997(4)
H_0: The series has a unit root. Against the alternative of a stationary process.
*, **, *** indicates the 10%, 5%, 1% significance level of the rejection

	1977(1) - 1997(4)		
Variables	**Lag length**	**constant & trend**	**ADF**
Annual underlying inflation	10	c & t	-4.12^{***}
Expectation of annual inflation rate	0	---	-7.63^{***}
Annual inflation rate of import prices	11	c & t	-4.10^{***}
Log unemployment	8	---	-3.33^{***}

Table 4.2: *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the linear model (4.1) for the Australian inflation 1977(1)-1997(4).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.047	0.13	0.24	0.062	0.063	0.080

Table 4.3. *p*-values of tests of linearity in the linear model (4.1) for a set of transition variables estimating the Australian inflation 1977(1)-1997(4).

Linearity test	Transition variables									
	π_{t-1}	π_{t-4}	π_{t-5}	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^{imp}$	$\Delta_1\pi_{t-2}^{imp}$	u_{t-1}	u_{t-2}	$\Delta_1 u_t$	$\Delta_1\pi_{t-1}$
F	---	---	---	---	0.50	0.28	---	---	0.0014	0.77
F_{04}	---	---	---	---	0.68	0.59	---	---	0.74	0.59
F_{03}	---	---	---	---	0.89	0.71	---	---	0.00003	0.68
F_{02}	0.11	0.52	0.66	0.018	0.046	0.024	0.060	0.027	0.11	0.63

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_{02} is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of the linear model (4.1) against STR type nonconstancy for the Australian inflation, 1977(1)-1997(4).

Tests of parameter constancy	Null hypothesis (1)
LM3	0.000028
LM2	0.0028
LM1	0.030

Notes: The null hypotheses are:
(1): "All parameters are constant."

The remaining parameters not under test are assumed constant in each case.

Table 4.4. Misspecification tests for the LSTR(1) model (4.2) for the Australian inflation 1977(1)-1997(4).

p-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.34	0.42	0.48	0.11	0.18	0.18

p-values of tests of no additive nonlinearity in (4.2) for a set of transition variables

Linearity test	Transition variable							
	π_{t-1}	π_{t-4}	π_{t-5}	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^{imp}$	$\Delta_1\pi_{t-2}^{imp}$	u_{t-1}	u_{t-2}
F	---	---	---	---	0.41	0.38	---	---
F_{02}	0.30	0.61	0.66	0.031	0.16	0.057	0.080	0.18

Test of linear restriction
"Linear" coefficient of $\Delta_1\pi_{t-2}^{imp} = 0$, *p*-value: 0.67

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 4.4. cont. *p*-values of parameter constancy tests of the LSTR(1) model (4.2) against STR type nonconstancy.

Tests of parameter constancy	Null hypotheses		
	(1)	(2)	(3)
LM3	0.0029	0.0090	0.84
LM2	0.0029	0.0095	0.75
LM1	0.030	0.16	0.76

Notes: The null hypotheses are:
(1): "All parameters of the linear part are constant."
(2): "All parameters of the nonlinear part except the intercept are constant."
(3): "The intercept in the nonlinear part is constant."

The remaining parameters not under test are assumed constant in each case.

Table 5.1: Augmented Dickey-Fuller unit-root tests for Swedish data 1979(1) - 1997(4)
H_0: The series has a unit root. Against the alternative of a stationary process.
*,**,*** indicates the 10%, 5%, 1% significance level of the rejection

	1979(1) - 1997(4)		
Variables	**Lag length**	**constant & trend**	**ADF**
Annual inflation rate	4	c & t	–2.67
First difference	3	---	–6.43***
Expectation of annual inflation rate	4	c & t	–3.15
First difference	4	---	–3.60***
Annual inflation rate of German prices	7	c & t	–2.79
First difference	3	---	–4.66***
Log unemployment (u_t)	7	---	–0.22
First difference	6	---	–2.56**

Table 5.2. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the linear model (5.1) for the Swedish inflation 1979(3)-1997(4).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.42	0.18	0.34	0.33	0.21	0.20

Table 5.3. *p*-values of tests of no additive nonlinearity in the linear model (5.1) for a set of transition variables estimating the Swedish inflation 1979(3)-1997(4).

Linearity test	Transition variables				
	$\Delta_1\pi_{t-2}$	$\Delta_1\pi_{t-4}$	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^f$	$\Delta_1 u_{t-3}$
F	0.0079	0.82	0.40	0.0062	0.24
F_{04}	0.029	0.71	0.55	0.13	0.15
F_{03}	0.011	0.49	0.33	0.015	0.11
F_{02}	0.66	0.77	0.29	0.090	0.99

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of the linear model (5.1) against STR type nonconstancy for the Swedish inflation, 1979(3)-1997(4).

Tests of parameter constancy	Null hypothesis
	(1)
LM_3	0.71
LM_2	0.74
LM_1	0.97

Notes: The null hypotheses are:
(1): "All parameters are constant."

The remaining parameters not under test are assumed constant in each case

Table 5.4. Misspecification tests for the LSTR(2) model (5.2) for the Swedish inflation 1979(3)-1997(4).

p-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error.

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.31	0.082	0.17	0.17	0.11	0.12

p-values of tests of no additive nonlinearity in (5.2) for a set of transition variables

Linearity test	Transition variables				
	$\Delta_1\pi_{t-2}$	$\Delta_1\pi_{t-4}$	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^f$	$\Delta_1 u_{t-3}$
F	0.0053	0.37	0.65	0.057	0.13
F_{02}	0.037	0.32	0.16	0.34	0.55

Test of linear restriction
"Linear" coefficient of $\Delta_1\pi_t^f = 0$, *p*-value: 0.29

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

Table 5.4. cont. *p*-values of parameter constancy tests of (5.2) against STR type nonconstancy

Tests of parameter constancy			Null hypotheses
Test	(1)	(2)	(3)
LM_3	0.35	0.39	0.94
LM_2	0.90	0.72	0.87
LM_1	0.92	0.87	0.72

Notes: The null hypotheses are:
(1): "All parameters are constant."
(2): "All parameters of the linear part are constant."
(3): "All parameters of the nonlinear part are constant."

The remaining parameters not under test are assumed constant in each case.

Table 6.1: Augmented Dickey-Fuller unit-root tests for US data 1978(1) - 1997(4)
H_0: The series has a unit root. Against the alternative of a stationary process.
*,**,*** indicates the 10%, 5%, 1% significance level of the rejection

	1978(1) - 1997(4)		
Variables	**Lag length**	**constant & trend**	**ADF**
Annual inflation rate	7	---	-3.78^{***}
Expectation of annual inflation rate	5	c	-3.24^{***}
Log unemployment	4	---	-3.84^{***}

Table 6.2. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the linear model (6.1) for the U.S. inflation 1978(1)-1997(3).

Test	Maximu lag	q				
	1	2	3	4	5	6
No error autocorrelation	0.53	0.70	0.80	0.85	0.93	0.97

Table 6.3. *p*-values of tests of no additive nonlinearity in the linear model (6.1) for a set of transition variables estimating the U.S. inflation 1978(1)-1997(3).

Linearity test	Transition variables										
	π_{t-1}	π_{t-2}	π_{t-3}	π_{t-4}	π_{t-5}	π_{t-6}	π_{t-7}	π_{t-8}	π_{t-9}	π_t^f	u_{t-1}
F	---	---	---	---	---	---	---	---	---	---	---
F_{04}	---	---	---	---	---	---	---	---	---	---	---
F_{03}	---	---	---	---	---	---	---	---	---	---	---
F_{02}	0.27	0.14	0.077	0.091	0.85	0.069	0.074	0.058	0.054	0.053	---

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of the linear model (6.1) against STR type nonconstancy for the U.S. inflation, 1978(1)-1997(3).

Tests of parameter constancy	Null hypothesis			
	(1)	(2)	(3)	(4)
LM_3	0.059	0.78	0.055	0.061
LM_2	0.034	0.48	0.027	0.028
LM_1	0.78	0.55	0.72	0.53

Notes: The null hypotheses are:
(1): "The coefficient of inflation expectations is constant."
(2): "All parameters of inflation variables are constant."
(3): "The coefficient of the unemployment variable is constant."
(4): "The intercept is constant."

The remaining parameters not under test are assumed constant in each case

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