

Essays on the Effects of Fiscal and Monetary Policy

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To Therese and Jonathan

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Preface

Early fall 1990: I took my first course in macroeconomics at the University College of Örebro. The lecturer *Hans Bylund* tried to convince me and other undergraduate students that Sweden was entering a very deep recession in the economy in general, and a severe banking crisis in particular. These lectures and subsequent informal discussions with Hans convinced me, *a priori* quite determined to become an accountant, that I should study economics instead, and I am grateful to him for his advice.

I would also like to take the opportunity to thank my high-school teacher *Curt Karlsson* who induced me to end up at the University College of Örebro in the first place instead of at the closed and compact “Kumla Bunkern” (a high-security prison in the Örebro neighborhood). Curt, with his military discipline and time-consistent teaching style, certainly set my life on the right track.

The work on my MSc thesis in the spring of 1994, supervised by my excellent advisors *Walter Stervander* and *Jan B Gajda*, inspired me to apply to the Ph.D. program at the Stockholm School of Economics. In May 1994, *Villy Bergström* employed me as a research assistant at FIEF (the Trade Union Institute for Economic Research) to work on the *SNEPQ* macroeconometric model of the Swedish economy. My work at FIEF with Villy as boss was very inspiring, and prepared me for the Ph.D. program. I am also grateful to Villy for persuading (?) *Lennart Berg*, *Reinhold Bergström* and *Christian Nilsson* to print my name on the resulting book, otherwise my list of publications would have been empty.

Towards the end of July, *Anders Vredin* and *Sune Karlsson* interviewed me for the Ph.D. program. During this interview, Anders asked me if I had ever heard of the Lucas critique. I still remember that my answer was not very illuminating, but it was somehow

sufficient; later on Sune phoned to tell me that I had been accepted into the Ph.D. program. I thank Sune not only for this phone call, but also for all subsequent help when the computer and I did not cointegrate.

During the first year, when the studies were extremely time consuming (especially in the opinion of my wife-to-be), I was on my way out of academia. I got an interesting job offer, and told *Karl Jungenfeldt* that I had decided to quit the Ph.D. program. Karl promptly told me that he thought that I was making a huge mistake which I would regret later on, and persuaded me not to accept the job offer. As always, Karl was perfectly right.

Well into the program, I managed to convince *Anders Vredin* to be the head advisor for my thesis. Anders helped me cross the bridge between taking courses and formulating as well as executing research ideas. He guided me in setting up a project, after which I could apply for and receive funds from the *Tore Browaldhs stiftelse för samhällsvetenskaplig forskning och undervisning* (which I hereby gratefully acknowledge). Although Anders left the school in January 1997, he has continued to support me as if he was still my head advisor (although he formally became co-advisor) with very constructive comments on my papers, help with conferences and general encouragement. I am deeply grateful to Anders for helping me enter and remain on the saddle path towards the completion of this dissertation.

Paul Söderlind became my head advisor as of February 1997. This contributed to further improving the quality of my thesis and my skills in economics and econometrics in a (statistically?) significant way. Paul has shown me repeatedly how to strengthen research ideas and has provided invaluable comments on my papers. This was also done in an extremely efficient way. While many Ph.D. students have to wait a long time for comments from their advisors, Paul delivers his insights and comments in a few days. Paul's advising style is indeed a "workhorse" model which others should follow. Moreover, with Paul as advisor, the need for Sune's help with computer problems was suddenly the empty set. Paul always has one or two efficient GAUSS procedures, algorithms, or even fast working computers up his sleeve. I am indebted to Paul for all his help in completing this dissertation, and I am confident that I could not have found a better head advisor

elsewhere in academia.

I have had the privilege of writing a paper with *Martin Flodén*. From the outset of the Ph.D. program, our discussions have been very beneficial. In addition, Martin convinced me that $i = i + 1$ holds in general (in mathematical computer programs, given that i exists) and taught me how to program my first loop in GAUSS. Our duels on the badminton court also deserve appreciative recognition.

This thesis has benefited from constructive suggestions by participants in the macro workshop, in particular *Lars Ljungqvist*. Special thanks also to Lars for providing me with funds from the *Jan Wallander and Tom Hedelius stiftelse* (which I hereby gratefully acknowledge) to complete the thesis. Indeed, seminars in general make it worthwhile to stay in academia. Over the years, I have heard some wonderful comments, such as, “Well Lars, welcome to reality” (Paul after a comment from Lars that he could not imagine any theoretical model which could mimic an observed pattern in the data); “Indeed, this is the worst cointegrating vector I have even seen” (*David F Hendry* to Anders); and “Indeed, this model is worse than useless” (Hendry again, but luckily enough not to Anders).

I would also like to thank my former room-mates at *Saltmätargatan*: *Magnus Hyll* for always curing my headaches over various mathematical problems, *Ingela Ternström* for letting me stay at the computer instead of answering all the phone calls, *Kasper Roszbach* for making me popular with the secretaries although I had to answer a lot more phone calls with him than with Ingela ..., and *Zhang Gang* for allowing me to complete my thesis more efficiently alone by putting up a wall in our room. Many thanks also to *Marcus Asplund* for providing hardrock music and whiskey on a train to the 1998 EEA conference in Berlin, *Ulf Söderström* for generous help with \LaTeX codes and for providing classical music when Markus was not at “home”, *Arvid Wallgren* and *Mats Ekelund* for providing a good mood and laughs when teaching at the undergraduate level became somewhat boring, *Torbjörn Becker* for help with my first paper and for encouraging e-mails (“have you finished your thesis yet Jesper?”), and other colleagues at *Saltmätargatan* for making it such a friendly place.

Among the staff at the Department of Economics, thanks are due in particular to *Anders Paalzow*, *Pirjo Furtenbach*, *Kerstin Niklasson*, *Britt-Marie Eisler* (my “Älskling”)

and *Ritva Kiviharju* for all their efficient help with various matters over the years.

I wish to thank my mother *Gyno* and father *Göran*, brother *Jakob* and sisters *Johanna* and *Josefin* for all their generous support. In particular, my beloved mother has always stood by my side during my life and encouraged me to continue with my studies in all possible ways. I guess I was born very lucky.

Finally, I wish to express my deepest gratitude to my wife *Therese* and son *Jonathan* for giving me the chance to receive this degree. During the work on this thesis, they have always provided me with the greatest love and inspiration, although they have had to endure an absent-minded and preoccupied husband and father. To them, I dedicate this dissertation.

Stockholm, March 1999

Jesper Lindé

Summary of the Essays

This dissertation consists of four essays in the field of applied/empirical macroeconomics. Although not directly related from either an economic or a methodological point of view, the essays reflect my diversified interest in macroeconomics, and my belief that it is the specific economic issue which should govern the choice of model and not vice versa. A common theme, however, in essays I, II and III, is that they deal with the real effects of fiscal policy in various ways, while Essay IV is more concerned with monetary policy. The purpose of this summary is primarily to make the main ideas and results in the essays accessible to researchers who are not specialists in the fields addressed. In addition, I also attempt to provide the interested non-economist with some insights.

In **essay I**, I investigate the relative importance of foreign and domestic shocks for Swedish postwar business cycles. A common view in Sweden is - or at least has been - that business cycles are mainly caused by foreign shocks, such as fluctuations in foreign demand for Swedish exports (see, for example, Lindbeck, 1975). The validity of this view is examined using two different approaches.

One approach to investigating the sources of business cycles is that of vector autoregressions (VARs). This approach has been used by, for example, Mellander et al. (1992) and Englund et al. (1994). In general, these studies have found that foreign and domestic shocks contribute about equally to fluctuations in output in Sweden.

Another approach, following Lucas (1975, 1977) and Kydland and Prescott (1982) among others, is to use fully specified equilibrium models. The only study which has applied this approach to investigate the sources of Swedish business cycles is Lundvik (1992). In an equilibrium small open economy model, Lundvik finds that substantial fluctuations in Swedish macroeconomic variables seem to be due to foreign shocks; how-

ever, his model is statistically rejected by the data on all reasonable significance levels. In a general equilibrium closed economy setting, Jonsson and Klein (1996) find that the introduction of stochastic fiscal policy can account for some of the key features of Swedish postwar business cycles. But as a consequence of the closed economy setting, they do not account for the presence of foreign shocks.

The ideas behind essay I are then to introduce stochastic fiscal policy into a modified version of Lundvik's model, first, to test whether introducing fiscal policy significantly improves the empirical fit of the model in an open economy framework, and second, to investigate whether either foreign shocks or domestic shocks have a greater effect on Swedish postwar business cycles.

The results show that the introduction of fiscal policy improves the empirical fit of the model, but not significantly so. Foreign and domestic shocks are shown to be approximately equally important for fluctuations in output, while foreign shocks have a decisive effect on fluctuations in the real exchange rate and the current account. Among the domestic shocks, the conclusion is that shocks in technology are the most important for postwar business cycles, while innovations in fiscal policy are found to have relatively little impact. In this respect, the results in Jonsson and Klein thus seem to be sensitive to model specification.

In **essay II**, I investigate the classical issue of whether large budget deficits give rise to higher interest rates in a small open economy framework. This issue was widely debated in Sweden when the budget deficit peaked at some 20 percent of GDP in the early 1990s and nominal (and real) interest rates were exceptionally high.

Theoretically, it is well known that the effects of changes in fiscal policy on the term structure of interest rates are ambiguous. The Ricardian equivalence theorem states that changes in government budget deficits do not affect interest rates under a certain set of assumptions, but according to the more conventional view (that is, Keynesian or non-Ricardian models) in macroeconomics, large budget deficits normally increase interest rates.

So far, however, the empirical studies undertaken, mostly utilizing data for the United States, have not been able to supply either view with convincing evidence. For instance,

Evans (1985, 1987a, 1987b, 1988) and Plosser (1982, 1987) present evidence consistent with Ricardian equivalence, while, for example, Allen (1990, 1992) and Correia-Nunes and Stemitsiotis (1995) find support for the conventional view. Since the resolution of this issue is important for the design of macroeconomic policy, there is a need for more research in the field.

In this paper, I utilize data for Sweden. In an international comparison with the other OECD countries (and in particular against the U.S.), Sweden has experienced very large fluctuations in government budget deficits and short- and long-term nominal interest rates since the beginning of the 1980s. Consequently, this paper provides a high powered empirical test compared to previous studies in the field.

The approach is as follows: First, I survey the results in the previous literature and try to draw some important lessons for the investigation in this paper. Second, I set up a conventional stochastic small open economy macro model, in which the yield curve of nominal interest rates is determined in terms of different policy variables, and use this model to study the effects of fiscal policy. In addition to providing a framework for the empirical research in this paper, this approach also turns out to offer several important insights. For example, it is found that large budget deficits do not automatically produce higher interest rates in the small open economy framework.

In the empirical part of the paper, I estimate the implied nominal interest rate regression equations on Swedish data, taking the lessons from the survey and the conventional model into account. The empirical results suggest that large budget deficits lead to higher nominal interest rates and thus provide evidence inconsistent with the Ricardian view.

In **essay III**, written jointly with Martin Flodén, we investigate whether the higher level of government insurance and redistribution schemes in Sweden as compared to the U.S. can be motivated by differences in individual uninsurable income-specific risks faced by agents in these two countries. Two important motivations for government taxation are that it provides insurance against individual-specific income variations if private insurance markets are absent, and that it redistributes wealth from those who were born lucky to those who were not. Since all feasible tax systems are to some extent distortionary, there is a trade-off between insurance and redistribution on the one hand and efficiency on the

other.

To examine this question, we begin by estimating the degree of individual-specific wage risk in the U.S. and Sweden on micro data, decomposed into a permanent and a transitory part. We then use the estimated processes to parameterize a general equilibrium model with heterogeneous agents where the agents choose their labor supply endogenously and are subject to a nonborrowing constraint as in Aiyagari (1994). In the model, we assume that the government uses proportional labor income taxes to redistribute income among agents, and that the government wishes to maximize the *ex ante* utility of agents. We also abstract from aggregate uncertainty, since a number of studies suggest that aggregate uncertainty is important in this setting (see İmrohoroglu (1989) and Krusell and Smith, 1999). Moreover, the results in Heaton and Lucas (1996) suggest that aggregate shocks only seem to account for a few percent of the variability in household income.

The estimation results show that wage uncertainty in the U.S. seems to dominate that in Sweden by any measure. The variability in both the permanent and transitory part of the wage processes is found to be higher in the U.S. than in Sweden. Consequently, according to our approach, it will be always be optimal with a higher level of government insurance and redistribution in the U.S. than in Sweden, which is at odds with the data.

When we calibrate the model with the estimated wage processes, distortions are found to be significant. The optimal size of government insurance programs varies between 21 and 38 percent in the U.S., and between 0 and 14 percent in Sweden depending on the specification of the utility function.

In **essay IV**, I investigate whether the statistical test used to identify the relevance or nonrelevance of the Lucas critique in data works satisfactorily in small samples. In brief, the Lucas critique (see Lucas, 1976) is that reduced form econometric models cannot provide any useful information about the actual consequences of alternative policies because the structure of the economy will change when policy changes, thereby making the estimated parameters in the reduced form econometric models nonconstant.

The observation underlying this essay is that although most economists regard the Lucas critique as important, it has received very little empirical support; see Ericsson and Irons (1995) for a complete survey. Two natural questions then arise. First, can this

observation be due to the fact that the Lucas critique is not quantitatively important in our models in a statistically significant way in small samples? Or, second and more likely, is it the case that the statistical test used to examine the validity of the Lucas critique in practice, the super exogeneity test developed by Engle et al. (1983), has poor small sample properties?

In this essay, I try to answer these two questions using a modified version of Cooley and Hansen's (1995) real business cycle model with money. The difference is that the model here includes government expenditures and a Taylor (1993) inspired policy rule for nominal money supply. This policy rule is then estimated on U.S. data for the recent Federal Reserve chairmanships of Burns, Volcker, and Greenspan. As reported by Judd and Rudebusch (1998), the conduct of monetary policy has varied systematically with these three chairmanships.

By calibrating the theoretical model with these estimated monetary policy regimes, I verify that the properties of the simulated model economy actually change significantly in a statistical sense when there is a monetary regime shift. I thus have a model where the super exogeneity test should be able to identify the relevance of the Lucas critique. Despite this, it is found that the super exogeneity test far too often accepts the false null hypothesis that the Lucas critique does not apply when there is a regime shift in the conduct of monetary policy between the chairmanships of Burns, Volcker and Greenspan. This lack of power exhibited by the super exogeneity test may then be a possible explanation as to why the Lucas critique has not been found in real-world data. Consequently, more research on the properties of the super exogeneity test is warranted.

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Included Essays

Essay I

Have Swedish Postwar Business Cycles Been Generated Abroad or at Home?*

1 Introduction

What is the relative importance of domestic and foreign shocks for the Swedish postwar business cycle? In Sweden, it is a common view that business cycles are mainly caused by foreign shocks, such as fluctuations in foreign demand for Swedish exports.¹ The validity of this view has been investigated with two types of approach. One, more empirical, approach to the investigation of the sources of business cycles is that of vector autoregressions (VARs). This approach has been used by, for example, Mellander et al. (1992) and Englund et al. (1994). In general, these studies have found that foreign and domestic shocks contribute about equally to fluctuations in output. Another, more theoretical, approach is the use of fully specified equilibrium models, following Lucas (1975, 1977), Kydland and Prescott (1982) and others. The only study which has used this approach

*I would like to thank Magnus Jonsson, Paul Söderlind and Anders Vredin for helpful comments and discussions; Magnus Forsells at Sveriges Riksbank (Bank of Sweden) for data; and Paul Klein for providing a fast algorithm for non-optimal economies. I have also benefitted from comments by participants in seminars at the Stockholm School of Economics and the European Economic Association Meeting 1998 in Berlin. Part of the work on this paper was carried out while the author was a visiting Ph D student at Sveriges Riksbank, and the author is grateful to Sveriges Riksbank for its hospitality.

¹ See, for example, Lindbeck (1975).

is that of Lundvik (1992). In an equilibrium small open economy model, Lundvik (1992) finds, on the one hand, that substantial fluctuations in Swedish macroeconomic variables seem to be due to foreign shocks, but, on the other hand, that the model is statistically rejected by the data on all reasonable significance levels. Notably, Lundvik (1992) introduced only one type of domestic shock into his model: innovations in total factor productivity.

Recent research has provided evidence that fluctuations in fiscal policy seem to matter for the business cycle; see for instance Braun (1994), McGrattan (1994) and Jonsson and Klein (1996). In a general equilibrium closed economy setting, Jonsson and Klein (1996) find that the introduction of stochastic fiscal policy can account for some of the key features of the Swedish postwar business cycle. They demonstrate that the empirical fit of the basic neoclassical stochastic growth model is significantly improved when allowing for fiscal policy shocks. But as a consequence of the closed economy setting, they do not account for the presence of foreign shocks. Figure 1, which depicts export and import and the current account as ratios of GDP at factor costs during the postwar period, clearly demonstrates that Sweden is better characterized as a open economy than a closed economy during the postwar period.

The ideas of this paper are, then, to set up an equilibrium small open economy model with stochastic fiscal policy incorporated, first, to test whether the introduction of fiscal policy significantly improves the empirical fit of the model in an open economy framework, and second, to investigate whether either foreign shocks or domestic shocks are most important for the Swedish postwar business cycle.

The equilibrium model used draws on the small open economy model in Lundvik (1992), and incorporates fiscal policy in the same spirit as Jonsson and Klein (1996). I follow the strategy used by Lundvik (1992) and Jonsson and Klein (1996) and estimate the model with Simulated Method of Moments (SMM) with and without fiscal policy, to see whether the model without fiscal policy is significantly outperformed by the model with fiscal policy in a small open economy framework. To quantify the contribution of foreign and domestic shocks to fluctuations in Swedish key macroeconomic variables, the simulated volatilities are decomposed into fractions explained by domestic and foreign

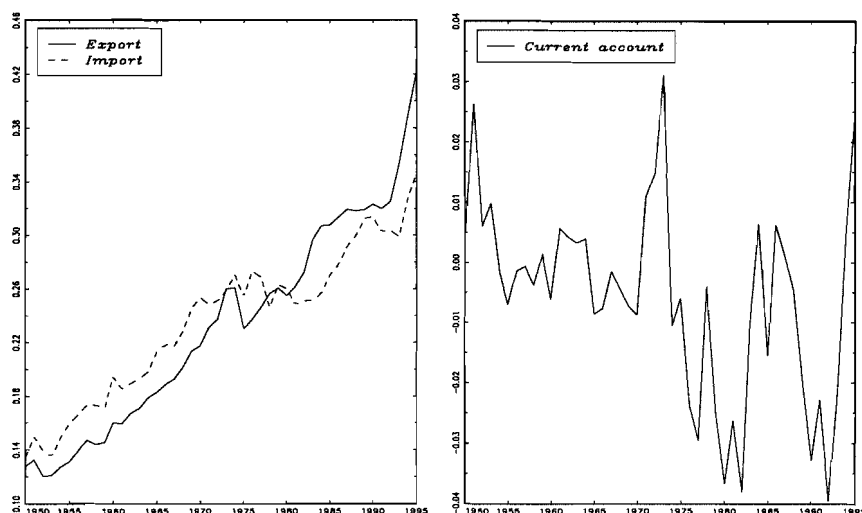


Figure 1: Export, import and the current account as ratios of GNP.

innovations.

The main results in the paper are as follows. First, I find that the introduction of fiscal policy improves the empirical fit of the small open economy model, but not significantly in contrast to the findings of Jonsson and Klein (1992). Both versions of the model are rejected by the data. One possible explanation for this failure is that the model is not capable of capturing the increased standard deviations in private consumption, private investment and employment, caused by the deep recession in Sweden during the beginning of the 1990s.

Second, by decomposing the simulated volatility in output per capita, the real exchange and the current account attributed to various foreign and domestic shocks, I find that foreign shocks contribute with 43 percent to the volatility in output in the long run. This figure compares well with the findings of the studies which have used VARs; see Mellander et al. (1992) and Englund et al. (1994). Of the domestic shocks, innovations in total factor productivity are most important, and contribute to 57 and 45 percent of the fluctuations in output in the short and long run respectively. Innovations in fiscal

policy only account for around 10 percent of the fluctuations in output, in both the short and long run. For fluctuations in the real exchange rate and the current account, foreign shocks are of the utmost importance, in both the short and long run. The latter result is nice since it is an identifying assumption in the VAR study by Mellander et al. (1992).

The structure of the paper is as follows. In Section 2, I present the equilibrium model. The data set and some basic stylized facts for the Swedish postwar period are reported in Section 3. In Section 4, I discuss various issues regarding calibration and estimation of the deep parameters and processes for the exogenous variables in the model. The SMM estimation results and the variance decompositions are then reported in Section 5. Finally, Section 6 concludes.

2 The equilibrium model

In this section, I construct a neoclassical stochastic growth model for a small open economy with infinitely many identical agents. This setup implies that each agent takes all aggregate variables as given. There are two goods in the model, one domestic good which can be used for private and public consumption, investment and export, and one foreign good which can be used for consumption and as an intermediate input in production. The price of the foreign good in terms of the domestic good, the real exchange rate, is endogenously determined in the economy. The foreign demand of the domestically produced good is determined by the exogenous foreign income level and the real exchange rate. Foreign income is assumed to be exogenous since Sweden can be characterized as a small open economy. The individuals have access to an international market for one-period real bonds with an exogenously given real interest rate. However, as in Lundvik (1992), the economy as a whole is assumed to affect the interest rate which the country faces via a risk premium. There are two main sources of domestic disturbances in the economy: fluctuations in fiscal policy and the technology level. As in the “best” version of the models with fiscal policy in Jonsson and Klein (1996), I account for three types of fiscal policy disturbances.² On the income side I have shocks to payroll and consumption taxes,

² “Best” in the sense that it mimicked the Swedish postwar business cycle best.

which, in 1996, covered over 50 percent of the public sector incomes. I do not include income taxes in the model, since good data do not exist on (marginal) labor income taxes and the capital stock in Sweden during the whole postwar period. On the expenditure side, I model public consumption expenditures exogenously, while public transfers to the agents are endogenous and equal to the difference between tax incomes and public consumption expenditures.³ By this procedure, then, the public debt is always zero in every time period.

In the model, I abstract from population growth and represent all variables in per capita terms.

Finally, a notational comment: in the following, capital letters denote economy-wide averages which the agent takes as given and small letters individual specific values which the agent internalizes.

2.1 An equilibrium model for a small open economy

Infinitely many identical infinitely lived agents maximize expected utility given by

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, c_t^*, h_t), \\ u(c_t, c_t^*, h_t) \equiv \frac{\left[(c_t^\eta (c_t^*)^{1-\eta})^\alpha (1-h_t)^{1-\alpha} \right]^{1-\sigma} - 1}{1-\sigma} \end{aligned} \quad (1)$$

where c_t is consumption of the domestically produced good in time period t , c_t^* is consumption of the foreign good and h_t is the share of available time spent in employment. In (1), $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution between consumption and leisure and β the subjective discount factor, while α and η reflect the trade-off between consumption and leisure, and the foreign and domestic consumption good respectively.

The flow budget constraint facing the agent is

$$(1 + \tau_t^c)(c_t + Q_t c_t^*) + i_t + Q_t b_{t+1}^* = \frac{W_t}{1 + \tau_t^w} h_t + R_t^K k_t + Q_t (1 + R_t^{B^*}) b_t^* + TR_t \quad (2)$$

³ Note that Jonsson and Klein (1996) considered the ratio of public consumption expenditures to GDP, denoted g_t , to be exogenous. This assumption may be valid in the long run, but in a business cycles analysis it seems more reasonable to assume that g_t is endogenous and the level of public consumption expenditures exogenous.

where τ_t^c and τ_t^w are the exogenous consumption and payroll tax, both formally paid by the households.⁴ Thus, W_t is interpreted as a gross wage. Q_t denotes the real exchange rate; a higher value means a real depreciation, or equivalently, that the terms of trade are worsened. i_t is the agent's investment and R_t^K , given by (6), is the gross real return on the capital stock k_t . b_{t+1}^* denotes the agent's holding of the foreign bond at the beginning of time period $t+1$ bought in time period t , and $R_t^{B^*}$, given by (14) below, the real interest rate received on the stock of foreign bonds bought in t . From (2), it is clear that b_t^* is not a risk-free asset; the return on it will fluctuate because of changes in Q_t and $R_t^{B^*}$. Finally, TR_t is the lump-sum transfer from the public sector to the agent.

The production function is assumed to have constant returns to scale and to be of the Cobb-Douglas type

$$Y_t = e^{\ln Z_t} \left(IM_t^{\theta_{IM}} K_t^{1-\theta_{IM}} \right)^{\theta} (T_t^H H_t)^{1-\theta} \quad (3)$$

where Y_t is output, IM_t intermediate input, Z_t the technology level and T_t^H the deterministic labor-augmenting technological level which follows

$$T_t^H = (1 + \gamma) T_{t-1}^H = (1 + \gamma)^t. \quad (4)$$

In (4), γ is the deterministic labor-augmenting rate of technological change. Accordingly, the perfect competition zero profit maximizing conditions for the representative company are

$$W_t = (1 - \theta) e^{\ln Z_t} \left(\frac{IM_t^{\theta_{IM}} K_t^{1-\theta_{IM}}}{T_t^H H_t} \right)^{\theta} T_t^H, \quad (5)$$

$$R_t^K = \theta (1 - \theta_{IM}) e^{\ln Z_t} \left(\frac{IM_t^{\theta_{IM}} K_t^{1-\theta_{IM}}}{T_t^H H_t} \right)^{\theta} \frac{T_t^H H_t}{K_t} \quad (6)$$

and

$$Q_t = \theta \theta_{IM} e^{\ln Z_t} \left(\frac{IM_t^{\theta_{IM}} K_t^{1-\theta_{IM}}}{T_t^H H_t} \right)^{\theta} \frac{T_t^H H_t}{IM_t}. \quad (7)$$

The technology level is assumed to be exogenous and the natural log of it to follow a stationary AR(1)-process

$$\ln Z_{t+1} = \rho^{\ln Z} \ln Z_t + \varepsilon_{t+1}^{\ln Z}, \quad \varepsilon_t^{\ln Z} \sim i.i.d. \ N(0, \sigma_{\ln Z}^2). \quad (8)$$

⁴ Equivalently, I could let the firms pay the payroll tax.

Individual and aggregate investment in period t produces productive capital in period $t + 1$ according to

$$k_{t+1} = (1 - \delta) k_t + i_t \quad (9)$$

and

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (10)$$

where δ is the rate of capital depreciation.

The government's budget constraint is

$$\tau_t^c (C_t + Q_t C_t^*) + \frac{\tau_t^w W_t}{1 + \tau_t^w} H_t = G_t + TR_t \quad (11)$$

where G_t is exogenous public consumption expenditures. Since τ_t^c and τ_t^w are exogenous too, TR_t can have the interpretation of government budget deficit. Consequently, the government debt is always zero in this economy.⁵

As noted earlier, domestic production can either be used for private and public consumption, investment or export. Foreign demand for the domestically produced good, denoted X_t , is assumed to be determined by foreign income and the real exchange rate according to

$$X_t = Y_t^* Q_t^{\epsilon_X} \quad (12)$$

where Y_t^* denotes the exogenous foreign income level, assumed to grow at the same rate as domestic output in steady state.⁶ In (12), the income elasticity is assumed to be equal to unity and ϵ_X denotes the price elasticity. The specification (12) can be derived in an optimizing framework; see Armington (1969). The stochastic part of Y_t^* in natural logs, \tilde{Y}_t^* , is assumed to evolve according to

$$\tilde{Y}_{t+1}^* = \rho \tilde{Y}_t^* + \varepsilon_{t+1}^{\tilde{Y}^*}, \quad \varepsilon_{t+1}^{\tilde{Y}^*} \sim i.i.d. \ N(0, \sigma_{\tilde{Y}^*}^2), \quad -1 < \rho^{Y^*} < 1. \quad (13)$$

As in Lundvik (1992), it is assumed that there is only one foreign one-period real bond denominated in the foreign good, which pays a given world interest rate R_t^* . In order to get a stationary solution for the foreign bondholdings and the real exchange rate,

⁵ Note that, given sequences for $\{G_t\}_{t=1}^\infty$, $\{\tau_t^c\}_{t=1}^\infty$ and $\{\tau_t^w\}_{t=1}^\infty$, Ricardian equivalence holds in this economy, since households and the government pay the same interest rate, $R_t^{B^*}$. Therefore, it does not matter whether the government has a debt or not.

⁶ Formally, as noted below, this implies that $Y_t^* = T_t^H \exp(\tilde{Y}_t^*)$.

I adopt the assumption in Lundvik (1992), and specify exogenously an economy-specific risk premium on the given world interest rate as

$$R_t^{B^*} = \frac{\omega_0 \bar{Y}_t + \omega_1 B_t^*}{\omega_0 \bar{Y}_t + B_t^*} R_t^* \quad (14)$$

where $\omega_1 < 1$, B_t^* denotes aggregate foreign bondholdings and \bar{Y}_t the steady state value of GDP in time period t . The motivation for this specification is that the larger the aggregate debt as a percentage of long run expected GDP, the larger the risk premium that all the agents in the economy must pay.⁷ $R_t^{B^*}$ is then interpreted as a risk adjusted interest rate.⁸ The world real interest rate is assumed to be exogenously given by the stationary process

$$R_{t+1}^* = (1 - \rho^{R^*}) \bar{R}^* + \rho^{R^*} R_t^* + \varepsilon_{t+1}^{R^*}, \varepsilon^{R^*} \sim i.i.d.N(0, \sigma_{R^*}^2)$$

which is a standard small open economy assumption.

On the aggregate level, the change in real foreign bond holdings in domestic terms is given by

$$Q_t (B_{t+1}^* - B_t^*) = X_t - Q_t (C_t^* + IM_t) + Q_t R_t^{B^*} B_t^* \quad (15)$$

where $Q_t (C_t^* + IM_t)$ is aggregate import of the foreign good. It is then natural to think of $X_t - Q_t (C_t^* + IM_t)$ as the trade balance and $Q_t (B_{t+1}^* - B_t^*)$ as the current account.

The aggregate resource constraint

$$Y_t = C_t + I_t + G_t + X_t \quad (16)$$

also holds in every period

As stated in the introduction, the model will be solved with and without fiscal policy being incorporated. If the notation $\tau_t \equiv [\tilde{G}_t \tau_t^c \tau_t^w]^T$ is introduced, where \tilde{G}_t denotes the stochastic part of public expenditures in natural logs, the model without fiscal policy is straightforward, and is obtained by setting $\tau_t = [-\infty \ 0 \ 0]^T$ for all time periods t .

⁷ Another, less economic, motivation for this assumption is that, without it, the decision rules would have to be calculated again in every time period, which is an extremely time consuming procedure when the parameters in the model are estimated with SMM.

⁸ (14) implies that, if $B_t^* = 0$, then $R_t^{B^*} = R_t^*$, but if $B_t^* \rightarrow \infty$, then $R_t^{B^*} = \omega_1 R_t^* < R_t^*$. Finally, if $B_t^* \rightarrow -\omega_0 \bar{Y}_t$, then $R_t^{B^*} \rightarrow \infty$. As in Lundvik (1992), I set the parameters governing the risk premium, ω_0 and ω_1 , to 5 and 0.99 respectively.

In the version of the model with fiscal policy, fiscal policy is treated as an exogenous VAR(p) model

$$\tau_t = v + \sum_{i=1}^p \varphi_i \tau_{t-i} + \varepsilon_t^\tau, \varepsilon_t^\tau \sim i.i.d. N(0, \Sigma) \quad (17)$$

as in Jonsson and Klein (1996).

2.2 Computation of equilibrium

A well-known feature of the solution to the model is that output, consumption, investment, public expenditures, export, import, capital stock and foreign bondholdings grow at the deterministic rate γ in steady state, while the share of available time spent in paid employment, the real exchange rate and the current account are constant over time; see Hansen and Prescott (1995). In this paper, I have followed the convention in the literature and growth-adjusted the model by dividing the following variables with the constant growth factor:

$$\begin{aligned} \hat{Y}_t &\equiv \frac{Y_t}{T_t^H}, \hat{C}_t = \frac{c_t}{T_t^H}, \hat{c}_t = \frac{c_t}{T_t^H}, \hat{C}_t^* = \frac{c_t^*}{T_t^H}, \hat{c}_t^* = \frac{c_t^*}{T_t^H}, \hat{I}_t = \frac{I_t}{T_t^H}, \\ \hat{i}_t &= \frac{i_t}{T_t^H}, \hat{IM}_t = \frac{IM_t}{T_t^H}, \hat{K}_t = \frac{K_t}{T_t^H}, \hat{k}_t = \frac{k_t}{T_t^H}, \hat{B}_t^* = \frac{B_t}{T_t^H} \text{ and } \hat{b}_t^* = \frac{b_t}{T_t^H}. \end{aligned}$$

We then interpret the new defined variables as per efficiency units of labor.

Following Hansen and Prescott (1995), the representative agent's optimization problem can then be expressed as the recursive dynamic programming problem:

$$V(\mathbf{S}_t, \hat{K}_t, \hat{B}_t^*, \hat{k}_t, \hat{b}_t^*) \equiv \max_{\{\hat{c}_t, \hat{c}_t^*, \hat{h}_t, \hat{k}_{t+1}, \hat{b}_{t+1}^*\}} \left[u(\hat{c}_t, \hat{c}_t^*, \hat{h}_t) + \tilde{\beta} E_t V(\mathbf{S}_{t+1}, \hat{K}_{t+1}, \hat{B}_{t+1}^*, \hat{k}_{t+1}, \hat{b}_{t+1}^*) \right] \quad (18)$$

$s.t.$

$$\hat{c}_t = -Q_t \hat{c}_t^* + \frac{1}{1 + \tau_t^c} \left[\frac{W_t}{1 + \tau_t^w} h_t + (1 + R_t^K - \delta) \hat{k}_t - (1 + \gamma) \hat{k}_{t+1} + TR_t + Q_t (1 + R_t^{B^*}) \hat{b}_t^* - Q_t \hat{b}_{t+1}^* \right],$$

$$\mathbf{S}_{t+1} = \mathbf{A} \mathbf{S}_t + \varepsilon_{t+1}^S,$$

$$(1 + \gamma) \hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \hat{I}_t, (1 + \gamma) \hat{k}_{t+1} = (1 - \delta) \hat{k}_t + \hat{i}_t,$$

$$\hat{K}_{t+1} = \hat{K}(\hat{K}_t, \hat{B}_t^*, \mathbf{S}_t), \hat{B}_{t+1}^* = \hat{B}^*(\hat{K}_t, \hat{B}_t^*, \mathbf{S}_t), H_t = H(\hat{K}_t, \hat{B}_t^*, \mathbf{S}_t).$$

In (18), $\tilde{\beta} \equiv \beta(1 + \gamma)^{\alpha(1-\sigma)}$ is the effective subjective discounted factor and \mathbf{S}_t is a vector which contains all the exogenous aggregate state variables; for instance, if the VAR model

in (17) is of order one, then \mathbf{S}_t could be $\left[\ln Z_t, \tilde{Y}_t^*, R_t^*, \tilde{G}_t, \tau_t^c, \tau_t^w \right]^T$.⁹ In the maximizing of (18), the agent takes the economy-wide average variables $W_t, R_t^K, R_t^{B^*}, Q_t$ and TR_t as given. The functions \hat{K}, \hat{B}^* and H describe the relationship perceived by agents between the aggregate decision variables and the state of the economy. As the solution to the problem in (18), we have the individual agent's decision rules $\hat{k}_{t+1} = \hat{k}(\hat{K}_t, \hat{k}_t, \hat{B}_t^*, \hat{b}_t^*, \mathbf{S}_t)$, $\hat{b}_{t+1}^* = \hat{b}^*(\hat{K}_t, \hat{k}_t, \hat{B}_t^*, \hat{b}_t^*, \mathbf{S}_t)$ and $h_t = h(\hat{K}_t, \hat{k}_t, \hat{B}_t^*, \hat{b}_t^*, \mathbf{S}_t)$. The competitive equilibrium is obtained when the individual and average decision rules coincide for $\hat{k}_t = \hat{K}_t$ and $\hat{b}_t^* = \hat{B}_t^*$.

Since it is impossible to derive the decision rules analytically, I have used the conventional method of calculating the decision rules numerically by approximating the original problem with a second order Taylor expansion around the constant steady state values in the growth-adjusted economy. As a consequence of this approximation, the method produces linear decision rules. The algorithm utilized is documented in Klein (1994). I have also followed the convention in the literature and solved for the decision rules in natural logs for \hat{k} and h . Hence, the competitive equilibrium is computed by solving a fixed point problem where each agent's decision rules must be optimal given the aggregate decision rules in the economy.

3 Data

In this section, I present the annual data set and some stylized facts for the Swedish postwar business cycle from 1950 to 1995. By including the most recent data, I cover the deep recession in the Swedish economy in the beginning of the 1990s.¹⁰

⁹ As a consequence of the growth adjustment, the production function reads $\hat{Y}_t = e^{\ln Z_t} \left(\widehat{IM}_t^{\theta_{IM}} \hat{K}_t^{1-\theta_{IM}} \right)^{\theta} H_t^{1-\theta}$. Hence, the wage rate, real rental price of capital and real exchange rate are now redefined as $W_t = (1-\theta) e^{\ln Z_t} \left(\widehat{IM}_t^{\theta_{IM}} \hat{K}_t^{1-\theta_{IM}} H_t^{-1} \right)^{\theta}$, $R_t^K = \theta(1-\theta_{IM}) e^{\ln Z_t} \left(\widehat{IM}_t^{\theta_{IM}} \hat{K}_t^{1-\theta_{IM}} H_t^{-1} \right)^{\theta} (H_t/\hat{K}_t)$ and $Q_t = \theta\theta_{IM} e^{\ln Z_t} \left(\widehat{IM}_t^{\theta_{IM}} \hat{K}_t^{1-\theta_{IM}} H_t^{-1} \right)^{\theta} (H_t/\widehat{IM}_t)$. Similarly, we have $R_t^{B^*} = \frac{\omega_0 \bar{Y} + \omega_1 \bar{B}_t^*}{\omega_0 \bar{Y} + \bar{B}_t^*} R_t^*$.

¹⁰ I use only postwar data, since Hassler et al. (1994) have found some instability in Swedish business cycles in connection with World War I and World War II. To provide results that can be compared to the results in the VAR studies, the analysis is therefore restricted to the postwar period. In fact, this is also an additional reason to redo the calculations in Lundvik (1992), since he uses data for the period

3.1 Basic definitions

A major part of the data set are the GDP identities in nominal and real per capita terms. In addition, I have total employment in hours, the total nominal gross wage sum, nominal social insurance contributions, the nominal current account in the data set plus GDP deflators, nominal exchange rates vis-a-vis the Swedish krona and real GDPs at market prices for a number of OECD countries.¹¹ The measure of output, Y , is nominal GDP at factor costs per capita divided by the deflator for GDP at market prices.¹² The series for private consumption per capita, C , includes durable goods but excludes the net of indirect taxes and subsidies (deflated with the deflator for GDP at market prices). Two reasons for this procedure are, first, that durable goods are subject to consumption taxes and, second, that I want the GDP identity to hold in real terms up to a measurement error in the national accounts. For the latter reason, I have also included inventory investments in the series for private investment per capita, I . Real public expenditures, G , includes both real public consumption and investment. The consumption tax, τ^c , is calculated as the net of nominal indirect taxes and subsidies divided by nominal consumption expenditures, while the payroll tax, τ^w , is calculated as nominal social insurance fees divided by the net of the total nominal gross wage sum and social insurance fees. The share of available time spent in employment, H , is measured as the average total number of hours worked per capita. Foreign demand, Y^* , is calculated as a TCW-weighted average of foreign real GDPs per capita in Swedish kronor.¹³ Finally, the real exchange rate (inverse of the terms-of-trade), Q , and the current account, CA , are calculated as the import deflator divided by the export deflator and the nominal CA divided by the nominal GDP at factor costs respectively.

Figure 2 depicts the variables generated from the raw data set. The variables Y , C , G , I , X , M , H and Y^* are depicted in natural logarithms. Although there is a strong

1871-1988.

¹¹ See Appendix A for sources of the data set and exact definitions of composite variables.

¹² One shortcoming in the national accounts in Sweden is that there are no deflators for GDP at factor costs, or for indirect taxes and subsidies. Therefore, I have had to accept the deflator for GDP at market prices as a proxy for the deflators of GDP at factor costs, and for indirect taxes and subsidies. This has some quantitative importance which is discussed in Appendix A in greater detail.

¹³ TCW is a totally competitive trading weighted currency basket used by Sveriges Riksbank (Bank of Sweden), and the weights for the countries included to calculate \tilde{Y}^* sum up to 86.5 percent of the basket.

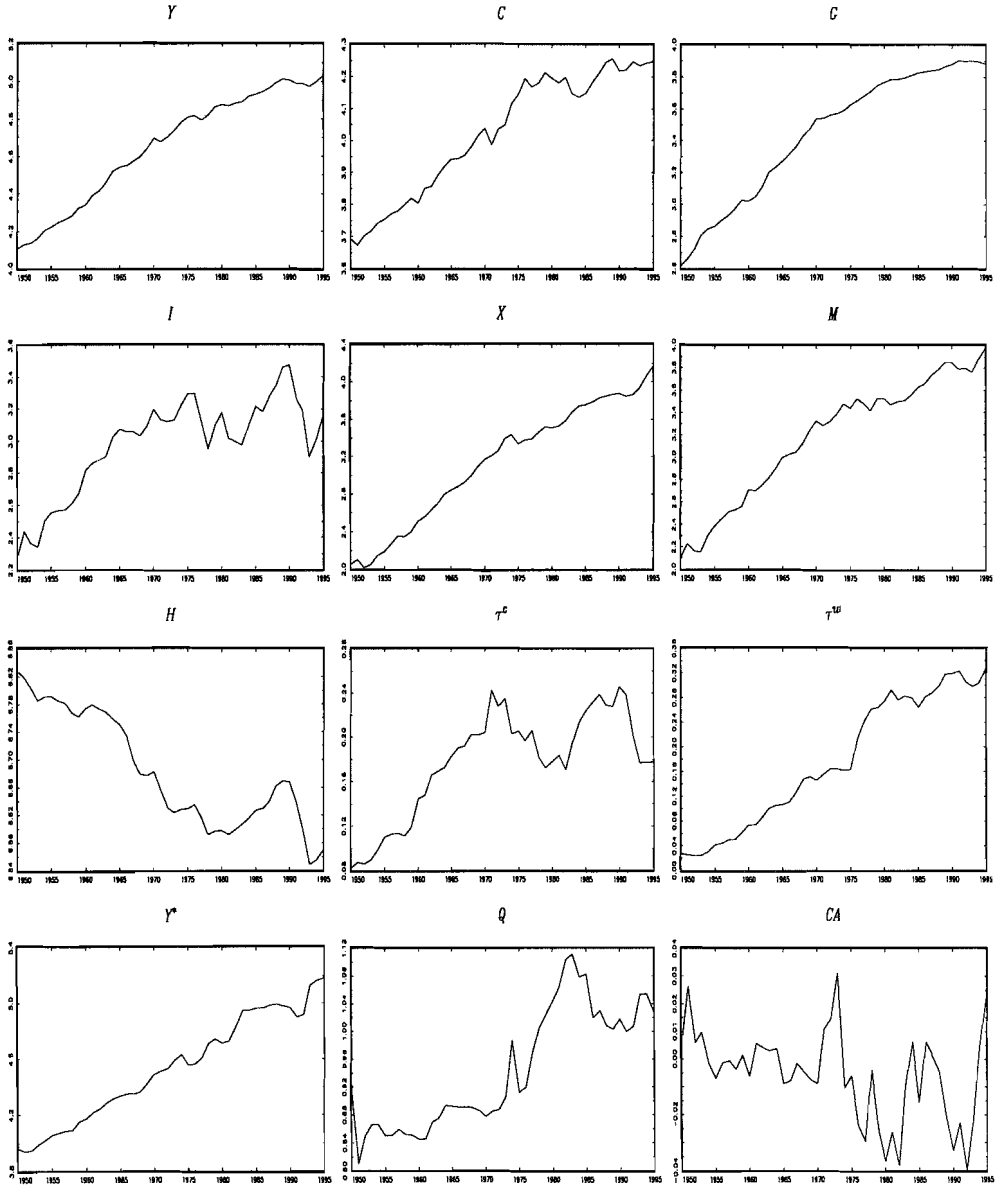


Figure 2: The data.

trend in most of the series, the figure clearly demonstrates the deep recession in the 1990s with a large fall in I . Most notable is also the strong declining trend in H over time.

3.2 Filtering and some stylized facts

From Figure 2, it is clear that most of the variables follow, perhaps stochastic, trends. Since the purpose of the paper is to study business cycle fluctuations, not trends, it is desirable to filter the data to extract the business cycle component of the series. There are different filtering methods available and worth considering for this purpose. For ease of comparison, and since Englund et al. (1992) and Hassler et al. (1994) have found that business cycle regularities in Swedish data do not seem to be sensitive to the filtering method, I have followed the convention in the real business cycle literature and applied the Hodrick-Prescott (1997) (H-P) filter on the data in natural logs with the smoothness coefficient λ set to 100.¹⁴ This choice of λ captures what we normally mean by business cycles; that is, it produces cycles with a periodicity of 3-8 years. In the following, I denote a variable which has been H-P filtered with a tilde; for example, H-P filtered output is denoted \tilde{Y} . On a priori grounds, and in accordance with the model, I have not H-P filtered τ^c , τ^w , CA and Q .¹⁵ The basic features of the Swedish postwar business cycle are reported in Table 1.

The figures in Table 1 compare well with Englund et al. (1992) and Hassler et al. (1994) in several respects, but there are some exceptions. First of all, while the variability in \tilde{Y} is unchanged, the variability of \tilde{C} , \tilde{I} and \tilde{H} relative to \tilde{Y} is larger than previously reported due to the deep recession in the 1990s. The variability in the measure of foreign demand, \tilde{Y}^* , is also higher than reported in Hassler et al. (1994) due to the adjustment for the real exchange rate, but has the usual procyclical properties and leads the business cycle. Unlike Hassler et al. (1994), we find evidence that exports, \tilde{X} , and the current

¹⁴ Note that the growth part of the model in Section 2.1 suggests filtering the data by removing the same log-linear trend from the variables Y , C , I , G , X , M , and Y^* above; but since the growth part of the model is a very crude approximation of reality, I do not consider this to be a good alternative. Another filtering method is simply to take first differences of the variables in natural logs, which is an appropriate method if the variables follow stochastic trends. Both these filters, and the H-P filter, work in the time domain. It is also possible to use the so-called band-pass filter which works in the frequency domain, see Hassler et al. (1992).

¹⁵ Note that the share of available time spent in employment per capita, H , does not follow a trend in the model. But since the trend in the Swedish postwar data for this variable is so strong, as can be seen in Figure 2, I have chosen to apply the H-P filter on it.

account, CA , are strongly procyclical and lead the business cycle while \tilde{H} lags the business cycle by including the most recent experience.

Table 1: Basic stylized business cycle facts for Sweden 1950-1995.

	Std. dev. in %	Std. dev. to \tilde{Y}	Correlation of variable with									
			itself in t at					\tilde{Y}_t at				
			$t-2$	$t-1$	t	$t+1$	$t+2$	$t-2$	$t-1$	t	$t+1$	$t+2$
\tilde{Y}	1.56	1	-0.20	0.38	1.00	0.38	-0.20	-0.20	0.38	1.00	0.38	-0.20
\tilde{C}	2.25	1.44	0.14	0.46	1.00	0.46	0.14	-0.21	0.12	0.49	0.17	0.13
\tilde{I}	9.39	6.01	-0.06	0.48	1.00	0.48	-0.06	-0.24	0.12	0.76	0.64	0.06
\tilde{G}	1.88	1.20	0.06	0.56	1.00	0.56	0.06	-0.22	-0.08	0.04	0.05	0.15
\tilde{X}	4.41	2.82	-0.16	0.48	1.00	0.48	-0.16	0.28	0.49	0.41	-0.16	-0.55
\tilde{M}	4.61	2.95	-0.17	0.33	1.00	0.33	-0.17	-0.16	0.20	0.67	0.41	-0.16
\tilde{H}	1.68	1.08	0.17	0.68	1.00	0.68	0.17	-0.16	0.17	0.64	0.62	0.17
\tilde{Y}^*	4.43	2.83	-0.14	0.50	1.00	0.50	-0.14	0.45	0.45	0.06	-0.43	-0.59
Q	8.54	5.46	0.90	0.94	1.00	0.94	0.90	0.05	0.03	-0.06	-0.13	-0.09
CA	1.61	1.03	0.32	0.58	1.00	0.58	0.32	0.39	0.38	0.03	-0.29	-0.31

Note: \tilde{Y} , \tilde{C} , \tilde{I} , \tilde{G} , \tilde{X} , \tilde{M} , \tilde{H} and \tilde{Y}^* are the business cycle components of the H-P filtered series Y , C , I , G , X , M , H and Y^* in natural logs with the smoothness parameter, λ , set to 100. See Appendix A for a detailed description of the data set and exact definitions of variables.

4 Estimation and calibration

The parameters in the model are estimated in two ways. Most of the parameters are estimated with SMM. The rest of the parameters are chosen so that the estimated models' steady state properties are consistent with the data (growth facts). Parameters which fall within the second category are assumed to be independent of whether fiscal policy shocks are considered or not.¹⁶ Since no good data are available on the domestic technology level Z_t and the world real interest rate R_t^* , the parameters in the AR(1) processes for these shocks are included in the SMM estimations. However, as mentioned in Section 3.1, it has been possible to construct a measure of the business cycle component of foreign demand \tilde{Y}_t^* . Therefore, I estimate the AR(1) process for \tilde{Y}_t^* with maximum likelihood.¹⁷ The fiscal policy VAR(p) model is also estimated with maximum likelihood. To sum up,

¹⁶ Formally, these parameters are regarded as part of the null hypothesis in the SMM estimations, that a certain subset of the (true) moments in data and the considered model coincide.

¹⁷ The estimation results for $\rho^{\tilde{Y}^*}$ and $\sigma_{\tilde{Y}^*}^2$ are 0.5057 and 0.00148 respectively. I have checked for normality and (absence of) first order autocorrelation of the residuals.

the basic reason for estimating some of the parameters in the models with SMM is that there exist neither any good information about them nor good data to calibrate them. In addition, since the answer to the question this paper addresses is heavily dependent on the values for these parameters, SMM has the advantage that it provides us with an “objective” set of parameters.

4.1 Calibration

The deterministic steady state growth rate for output, γ , is set to 0.021, which is the average growth rate for Y in the sample. The share of gross labor income to output, θ , is set equal to 0.355, which is the average gross wage sum as a fraction of Y in the data. To calibrate δ , I exploited the law of motion for capital in the steady state written as $\delta = \frac{\bar{I}}{\bar{Y}} \frac{\bar{Y}}{\bar{K}} - \gamma$, and used the estimated capital stock in Hansson (1991), GDP at factor costs and gross investment (private as well as public) for the time period 1960-1988 to compute $\delta = 0.122$ on average. The utility function parameter α , which typically determines the steady state share of available time spent in market activities, \bar{H} , is set to 0.33 for reasons discussed in Kydland (1995). The steady state value for public expenditures, \bar{G} , is calibrated so that steady state public expenditures as a fraction of output, \bar{g} , equal 0.299, which is the sample mean of g_t . τ_t^c and τ_t^w are in steady state set to 0.177 and 0.176, which are their sample means. Finally, foreign income is by construction equal to the value of exports.¹⁸

4.2 The fiscal policy VAR(p) model

In the estimation of the fiscal policy VAR(p) model, I followed the strategy in Jonsson and Klein (1996), by demeaning the variables in (17) prior to estimation. Different specification tests and information criteria suggested setting $p = 1$.¹⁹ The estimate of $\hat{\varphi}_1$ in (17) shows that the autocorrelations for τ_t^c and τ_t^w are high, 0.933 and 0.975 respectively, while the autocorrelation for H-P filtered public expenditures in natural logs, \tilde{G}_t , is con-

¹⁸ This means that I consider a steady state where $\bar{B}^* = 0$ and $\bar{Q} = 1$.

¹⁹ One problem worth mentioning in the determination of a good value for p was the lack of normality for the estimated residuals in the equation for τ_t^w , which was impossible to cure unless very large values of p were considered.

siderably lower and equal to 0.568. The notably high autocorrelations for τ_t^c and τ_t^w are of course a consequence of not detrending them, although they exhibit strong trends as can be seen in Figure 2.²⁰

4.3 The SMM estimation

How SMM works

Broadly speaking, the SMM estimator chooses estimates of the unknown parameters so as to make the chosen moments in the model mimic the corresponding moments in the data. The method uses the complete representation of the stochastic general equilibrium model. Under some certain conditions, provided in Lee and Ingram (1991), the SMM estimator is asymptotically normal and one can therefore use a goodness-of-fit test statistic based on the χ^2 -distribution. Let \mathbf{m}_T denote a $j \times 1$ vector with j sample moments in data and $\mathbf{m}_N(\hat{\beta})$ the corresponding simulated moments in the model, where T denotes the number of observations in the data, N the considered number of simulated observations in the model and $\hat{\beta}$ the $k \times 1$ vector with estimated parameters. Then, the SMM estimator minimizes the loss-function

$$T * \left(\mathbf{m}_T - \mathbf{m}_N(\hat{\beta}) \right)' \mathbf{W}_T * \left(\mathbf{m}_T - \mathbf{m}_N(\hat{\beta}) \right) \quad (19)$$

which is χ^2 -distributed with $j - k$ degrees of freedom if \mathbf{W}_T in (19) is a positive definite weighting matrix chosen to give the minimum asymptotic variance of $\hat{\beta}$.²¹

Choice of moment vector

I have chosen to use moments which highlight three dimensions in the model: the volatility dimension, the contemporaneous correlation dimension, and the autocorrelation dimension. I have also followed the convention in the literature and related volatilities and

²⁰ For a constant steady state to exist in the model, it is required that all eigenvalues z satisfy $\det(\mathbf{I}_3 z - \hat{\varphi}_1) \in (-1, 1)$. This condition is met for $\hat{\varphi}_1$ in (17).

²¹ Since I have implemented SMM the same way as Jonsson and Klein (1996), see their excellent summary of SMM in Appendix B for more technical details. However, there are two things worth mentioning in addition to their exposition there. The first thing is that I have followed Newey and West's (1994) recommendation in setting the bandwidth $p = 4(T/100)^{2/9}$ for the Bartlett kernel in the calculation of the variance-covariance matrix. Second, I have simulated the model 400 times and skipped the first 100 numbers in each simulation to get a stochastic initial state. Thus $N = 300$ and $T = 46$.

contemporaneous correlations for the different variables to output. Since the main interesting variables in the study are considered to be output, private consumption, private investment, hours worked, the real exchange rate and the current account (as a fraction of output), the following 17 moments have been included in the SMM estimation

$$\mathbf{m} = \begin{bmatrix} \hat{\sigma}_{\tilde{Y}_t}, \hat{\sigma}_{\tilde{C}_t}/\hat{\sigma}_{\tilde{Y}_t}, \hat{\sigma}_{\tilde{I}_t}/\hat{\sigma}_{\tilde{Y}_t}, \hat{\sigma}_{\tilde{H}_t}/\hat{\sigma}_{\tilde{Y}_t}, \hat{\sigma}_{Q_t}/\hat{\sigma}_{\tilde{Y}_t}, \hat{\sigma}_{CA_t}/\hat{\sigma}_{\tilde{Y}_t}, \\ \hat{\rho}_{\tilde{C}_t, \tilde{Y}_t}, \hat{\rho}_{\tilde{I}_t, \tilde{Y}_t}, \hat{\rho}_{\tilde{H}_t, \tilde{Y}_t}, \hat{\rho}_{Q_t, \tilde{Y}_t}, \hat{\rho}_{CA_t, \tilde{Y}_t}, \\ \hat{\rho}_{\tilde{Y}_t, \tilde{Y}_{t-1}}, \hat{\rho}_{\tilde{C}_t, \tilde{C}_{t-1}}, \hat{\rho}_{\tilde{I}_t, \tilde{I}_{t-1}}, \hat{\rho}_{\tilde{H}_t, \tilde{H}_{t-1}}, \hat{\rho}_{Q_t, Q_{t-1}}, \hat{\rho}_{CA_t, CA_{t-1}} \end{bmatrix}'. \quad (20)$$

In (20), the first “row” contains the standard deviation of output, \tilde{Y} , and the standard deviations of the other variables relative to \tilde{Y} , the second the contemporaneous correlations of the other variables with \tilde{Y} and the third the autocorrelation coefficients one year backwards for all variables.²² By this choice of moment set, I capture all three dimensions.²³

The procedure in the estimation

In practice, minimization of the concave loss function (19) is done by a grid search.²⁴ Since there are $k = 9$ parameters to estimate, it is only computationally possible to consider 3 values for each parameter in the final grid. To obtain the final grid, I considered large variations at a time for the parameters θ_{IM} , η , ϵ^X , σ , β , ρ^{R*} , σ_{R*}^2 , $\rho^{\ln Z}$ and $\sigma_{\ln Z}^2$ in that order, updating the parameter values recursively, to find the centre and the steps of the final grid.²⁵

²² The reason for not including the corresponding moments for export, \tilde{X} , import, \tilde{M} , and foreign demand \tilde{Y}^* in the SMM estimation is that there must be as many shocks as endogenous variables in the model, otherwise the model is singular and impossible to estimate, as demonstrated by Ingram et al (1994).

²³ Of course, it would be of interest to extend the moment set for the model with fiscal policy with the variables \tilde{G}_t , τ_t^c and τ_t^w to be able to ensure that the propagation mechanisms in the model are correct. But since they are not the target variables here, and it is of interest to compare the goodness-of-fit criterion in the model with fiscal policy with the one without, I have chosen not to include them.

²⁴ Since the loss function is not sufficiently smooth in the parameters θ_{IM} , η , ϵ^X , σ and β , it has not been possible to use a simple optimization algorithm, such as the steepest descent, to find the SMM estimates.

²⁵ Since the loss function is globally concave in each parameter, the estimation ordering of θ_{IM} , η , ϵ^X , σ , β , ρ^{R*} , σ_{R*}^2 , $\rho^{\ln Z}$ and $\sigma_{\ln Z}^2$ to obtain the final grid does not matter.

5 Empirical results

5.1 SMM estimation of the models

The results of the SMM estimation are reported in Tables 2 and 3.

Table 2: SMM estimates.

In the model	SMM point estimate of								
	θ_{IM}	η	ϵ_X	σ	β	ρ^{R^*}	$\sigma_{R^*}^2$	$\rho^{\ln Z}$	$\sigma_{\ln Z}^2$
Without F.P.	0.016	0.702	0.590	1.70	0.987	0.245	0.0016	0.950	0.00010
With F.P.	0.025	0.570	0.928	2.45	0.979	0.460	0.0020	0.910	0.00012

Note: F.P. is shorthand notation for fiscal policy. See Appendix A for a detailed description of the data set and exact definitions of variables.

On the whole, the point estimates in Table 2 are reasonable and could be the outcome of an ordinary calibration procedure, at least for the version of the model with fiscal policy. However, comparison of the models with and without fiscal policy reveals some important differences. First of all, the estimated price elasticity for export demand, $\hat{\epsilon}_X$, is rather low in the model without fiscal policy. This will tend to lower the export fluctuations, and thereby also the volatility in output. Second, the estimated inverse of intertemporal elasticity of substitution, $\hat{\sigma}$, in the model without fiscal policy is lower. This will tend to raise the relative volatility of consumption to output in that version of the model compared to the fiscal policy one. Third, $\hat{\rho}^{R^*}$ and $\hat{\sigma}_{R^*}^2$ are higher in the model with fiscal policy, implying that interest rate shocks are more important in that model compared to the model without fiscal policy.

Turning to Table 3, we see that both versions of the model underestimate the standard deviation in \tilde{Y} and the relative volatilities for private investment and hours worked to \tilde{Y} in the data, although the version with fiscal policy does so to a lesser extent. However, both models reproduce the relative volatilities in private consumption, the real exchange rate and current account to \tilde{Y} remarkably well. Most surprisingly, the relative volatility in \tilde{C} to \tilde{Y} is as high as 1.27 in the model without fiscal policy. This means that, although fluctuating consumption taxes and the higher variance for the foreign real interest rate tend to drive up the relative volatility of consumption to output in the fiscal policy version of the model, they just compensate for the higher estimate of σ reported in Table 2. Both models exaggerate the contemporaneous correlation between \tilde{Y} and Q but track the rather

Table 3: SMM estimated moments and goodness-of-fit statistics.

Moment	Model with		Empirical
	No fiscal policy	Fiscal policy	
$\hat{\sigma}_{\tilde{Y}_t} * 100$	1.01	1.26	1.56
$\hat{\sigma}_{\tilde{C}_t} / \hat{\sigma}_{\tilde{Y}_t}$	1.27	1.29	1.44
$\hat{\sigma}_{\tilde{I}_t} / \hat{\sigma}_{\tilde{Y}_t}$	1.91	2.95	6.03
$\hat{\sigma}_{\tilde{H}_t} / \hat{\sigma}_{\tilde{Y}_t}$	0.39	0.77	1.08
$\hat{\sigma}_{Q_t} / \hat{\sigma}_{\tilde{Y}_t}$	7.02	6.22	5.49
$\hat{\sigma}_{CA_t} / \hat{\sigma}_{\tilde{Y}_t}$	1.08	0.76	1.03
$\hat{\rho}_{\tilde{C}_t, \tilde{Y}_t}$	0.72	0.73	0.49
$\hat{\rho}_{\tilde{I}_t, \tilde{Y}_t}$	0.55	0.62	0.77
$\hat{\rho}_{\tilde{H}_t, \tilde{Y}_t}$	0.19	0.59	0.64
$\hat{\rho}_{Q_t, \tilde{Y}_t}$	0.08	0.24	-0.06
$\hat{\rho}_{CA_t, \tilde{Y}_t}$	0.02	0.08	0.02
$\hat{\rho}_{\tilde{Y}_t, \tilde{Y}_{t-1}}$	0.46	0.49	0.38
$\hat{\rho}_{\tilde{C}_t, \tilde{C}_{t-1}}$	0.38	0.46	0.46
$\hat{\rho}_{\tilde{I}_t, \tilde{I}_{t-1}}$	0.33	0.34	0.48
$\hat{\rho}_{\tilde{H}_t, \tilde{H}_{t-1}}$	0.40	0.45	0.68
$\hat{\rho}_{Q_t, Q_{t-1}}$	0.80	0.84	0.94
$\hat{\rho}_{CA_t, CA_{t-1}}$	0.58	0.50	0.58
χ^2_{obs}	172.62	63.54	
p -value	0.00	0.00	

Note: The standard deviation in \tilde{Y} and the relative standard deviations in \tilde{C} , \tilde{I} , \tilde{H} , Q and CA to \tilde{Y} are somewhat different than the one reported in Table 1, since the first year has been left out here for estimation technical reasons. The same explanation applies for the contemporaneous correlations between these variables.

low correlation between \tilde{Y} and CA well. The estimated correlation between \tilde{H} and \tilde{Y} is too low in the model without fiscal policy. In general, the autocorrelations are slightly better tracked by the model with fiscal policy. It is remarkable how well the very high autocorrelation in Q is tracked in the model with fiscal policy.

Finally, a few comments upon the χ^2 -statistics in Table 3 are in order. As in Jonsson and Klein (1996), Table 3 reveals that the introduction of stochastic fiscal policy improves the empirical fit of the model. But in contrast to their findings, it does not enter in any significant way when one considers a broader set of moments which also contains open economy variables. Both models are strongly rejected by the data using asymptotic significance levels.²⁶ In light of the most recent experience in the Swedish economy, this

²⁶ Of course, it is very likely that the small sample distribution of the χ^2 -statistic deviates significantly from its asymptotic distribution, but since it takes about 1 day to estimate the model on a very fast computer, it is not computationally possible to investigate this issue.

is not very surprising; Table 1 showed that the standard deviations in many variables are much higher when including the deep recession in the 1990s. Of course, all models are in some sense a crude approximation of the economy; they do better in some aspects than others. Here, the model with fiscal policy above does a fairly good job for the variables \tilde{Y} , Q and CA , which is a good thing since we are particularly interested in investigating what the forces are behind the fluctuations in these variables. For the variables \tilde{C} , \tilde{I} and \tilde{H} , the model performs less well; but before rejecting the properties of the model in this sense, the impact of the deep recession should be kept in mind.

5.2 Variance decomposition of the volatilities

To investigate the relative importance of foreign and domestic shocks for the key macro variables \tilde{Y} , Q and CA , which were reasonably well tracked by the model, I follow Sims (1980) and use the variance decomposition method. By not including more variables than shocks in the SMM estimation of the model, I avoid the critique against the variance decomposition method raised by Ingram et al. (1994). The variance decomposition method is attractive, since it measures the fraction of simulated volatility in a variable k years ahead accounted for by different shocks in a very precise way. However, it has one drawback: the identifying assumptions are, in general, of substantial importance for the results obtained. In the setting here, it is the ordering of the shocks that matters since the estimated disturbance vector is orthogonalized by a Cholesky decomposition. But since the innovations in the processes for R^* , \tilde{Y}^* and $\ln Z$ are uncorrelated with other shocks, the ordering of these variables does not matter. Thus only the ordering of the fiscal policy variables matters for the results here. Since the focus of the paper is to investigate the relative importance between foreign and domestic shocks, and thus the fiscal policy innovations as a whole rather than the relative importance among them, the effects of different ordering of the innovations have no importance here. Completely arbitrarily, then, I chose the following order: R^* , \tilde{Y}^* , $\ln Z$, \tilde{G} , τ^c and τ^w .

In Table 4, I present the results for different horizons. The short run impact of a shock is captured by k equal to 1, 5 and 10, and the long run by k equal to 50 and ∞ . Before turning to the results reported in Table 4 below, it should be emphasized that the figures

presented there are point estimates, which are sensitive to the parameterization of the model. Therefore, one should interpret the exact figures with a grain of salt, but take the main features in the table more seriously. From Table 4, we see that domestic shocks account for most of the volatility in output per capita, \tilde{Y} . In the short run, innovations in productivity are most important, and account for over 50 percent of the volatility in \tilde{Y} . But in the long run, fiscal policy shocks, in particular shocks to τ^w , become the most important source of output fluctuations.²⁷ Fluctuations in the real exchange rate, Q , are to a large extent caused by foreign shocks in the short run, but in the long run, domestic shocks become more important. For the current account, we find, essentially, that only fluctuations in foreign variables matter.²⁸

Table 4: Variance decomposition k years ahead.

Due to	Fraction of simulated volatility k years ahead in variable														
	\tilde{Y}					Q					CA				
	k					k					k				
	1	5	10	50	∞	1	5	10	50	∞	1	5	10	50	∞
R^*	0.09	0.10	0.08	0.05	0.05	0.93	0.78	0.70	0.57	0.56	0.47	0.45	0.45	0.45	0.45
\tilde{Y}^*	0.09	0.03	0.02	0.01	0.02	0.05	0.13	0.15	0.16	0.16	0.53	0.54	0.54	0.54	0.54
$\ln Z$	0.52	0.52	0.50	0.38	0.37	0.01	0.05	0.08	0.09	0.09	0.00	0.00	0.00	0.00	0.00
\tilde{G}	0.04	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
τ^c	0.14	0.16	0.17	0.17	0.16	0.00	0.02	0.03	0.07	0.08	0.00	0.01	0.01	0.01	0.01
τ^w	0.12	0.17	0.22	0.38	0.39	0.00	0.02	0.04	0.11	0.11	0.00	0.00	0.00	0.00	0.00
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: The variance decomposition has been made in natural logs for \tilde{Y} around its trend and in natural numbers for the stationary variables Q and CA . The fractions reported above have been calculated by analyzing the effects of a one standard deviation increase in each of the shocks at a time.

The results in Table 4 are in line with Jonsson and Klein (1996) in the sense that it appears as though innovations in fiscal policy are very important for the business cycle, especially in the long run. However, they differ in one important aspect. Jonsson and Klein found that it was innovations in the exogenous ratio of government expenditures to

²⁷ Although not reported in Table 4, I have performed variance decompositions for \tilde{C} , \tilde{I} and \tilde{H} . Quite naturally, innovations in foreign demand and, in particular, the foreign real interest rate are most important for fluctuations in \tilde{I} , and together they account for over 70 percent of the fluctuations in both the short and long run. For \tilde{C} , innovations in τ^c account for approximately 46 of the fluctuations in the short run, and for \tilde{H} innovations in τ^w are most important.

²⁸ I have tested the sensitivity of the results in Table 4 w.r.t. the ordering of the fiscal policy variables. It turned out that the relative importance of innovations in \tilde{G} , τ^c and τ^w crucially depended on the ordering of these variables, in contrast to Jonsson and Klein (1996). For instance, if τ^w changes place with \tilde{G} , it becomes the only important source of fluctuations among the fiscal policy variables.

output, g , which contributed to most of the fluctuations in output in the long run among the fiscal policy variables. Here, I find, with the same ordering of the fiscal policy variables, that innovations in the level of government expenditures are unimportant in the long run. There are two possible explanations for this inconsistency. First, it might be because I keep the level of government expenditures exogenous, rather than g as Jonsson and Klein. But when I reestimated the model with SMM with \tilde{G} in (17) replaced with H-P filtered $\ln g$, denoted \tilde{g} , as exogenous variable and decomposed the simulated variance again, the results in Table 4 were practically unchanged.²⁹ The remaining possible explanation is that Jonsson and Klein do not H-P filter g ; a priori and in accordance with the model they treat it as a stationary variable together with τ^c and τ^w , while I H-P filter the level of exogenous government expenditures here. A closer look at Table 4 reveals that this seems to be correct. In the table, we see that innovations in τ^c and τ^w become more and more important relative to \tilde{G} , and the other shocks too, when k increases, since the autocorrelations are so high for these variables (as noted in section 4.2) due to the strong trends in them (see Figure 2).

But this, then, raises doubts about the result that fiscal policy is very important for the business cycle, since it may be the case that this result crucially depends on the assumption of treating τ^c and τ^w as stationary variables, thereby not removing the strong trends in them with the H-P filter. In the next section, I therefore test to what extent the results change if one relaxes this assumption and H-P filters them.

5.3 Sensitivity analysis

In this section, I reestimate the model with SMM when τ^c and τ^w in (17), together with Q and CA , are H-P filtered to examine the robustness of the results in Table 4.³⁰ Before turning to the results of the variance decompositions in Table 5, I want to comment briefly on the SMM estimation results. Again, all the SMM estimated parameters are reasonable,

²⁹ I have also reestimated the model with SMM with \tilde{G} in (17) replaced with $\ln g$ as exogenous variable. Shocks to $\ln g$ are then, as in Jonsson and Klein, found to be the outstanding source of business cycles among the fiscal policy variables. For instance, shocks to $\ln g$ then account for 71 and 48 percent of the fluctuations in \tilde{Y} and Q respectively in the long run.

³⁰ I have also H-P filtered Q and CA in the reestimation of the model. Thus, in this section, all variables used in the SMM estimation are H-P filtered.

except $\hat{\rho}^{\ln Z}$ which equals 0.57, a rather low estimate (corresponds to 0.87 on quarterly data). The computed chi-square statistic is 103.04. Thus, the empirical fit of the model is clearly worsened. However, it is still the case that the version of the model with fiscal policy outperforms the one without, but not significantly so, using asymptotic critical values.

Turning to Table 5, we see that the variance decomposition results change completely. Innovations in fiscal policy are now only of moderate importance for output fluctuations. Instead, productivity shocks are of greater importance for fluctuations in \tilde{Y} . Foreign shocks are also more important for fluctuations in \tilde{Y} and the real exchange rate, in both the short and long run. As in Table 4, it is still the case that foreign shocks are the only source of fluctuations in the current account.³¹

Table 5: Variance decomposition k years ahead; all variables H-P filtered.

Due to	Fraction of simulated volatility k years ahead in variable														
	\tilde{Y}					Q					CA				
	k					k					k				
	1	5	10	50	∞	1	5	10	50	∞	1	5	10	50	∞
R^*	0.06	0.10	0.10	0.10	0.10	0.75	0.51	0.45	0.38	0.31	0.12	0.11	0.11	0.11	0.11
\tilde{Y}^*	0.27	0.20	0.23	0.28	0.33	0.23	0.43	0.49	0.57	0.65	0.88	0.89	0.89	0.89	0.89
$\ln Z$	0.57	0.57	0.53	0.49	0.45	0.01	0.04	0.04	0.03	0.03	0.00	0.00	0.00	0.00	0.00
\tilde{G}	0.02	0.07	0.08	0.08	0.07	0.01	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00
$\tilde{\tau}^c$	0.04	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\tilde{\tau}^w$	0.04	0.04	0.04	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: The variance decomposition has been made in natural logs for \tilde{Y} around its trend and in natural numbers for the stationary variables Q and CA . The fractions reported above have been calculated by analyzing the effects of a one standard deviation increase in each of the shocks at a time.

So what are the general impressions of Table 5? First of all, the figures compare well qualitatively with the results of Lundvik (1992), although it appears like foreign shocks are slightly more important here. In the short run, we see that domestic shocks are, quite naturally, slightly more important, and account for 67 percent of the fluctuations in output. The results also compare well with the papers which have exploited vector autoregressions (VARs) to investigate the sources of business cycles. Englund et al. (1994) find that foreign shocks account for 13 percent of the fluctuations in output in the short

³¹ Foreign shocks also now account for more than 70 percent of the fluctuations in \tilde{I} and \tilde{H} , and 36 percent of the fluctuations in \tilde{C} .

run and 47 percent in the long run. For the long run, Mellander et al. (1992) find that foreign shocks account for 57 percent of output fluctuations. Mellander et al. (1992) also find that foreign shocks account for all the variability in the real exchange rate in the long run.

6 Concluding remarks

In Sweden, it is, or at least has been, a common view that most of the business cycles are caused by foreign shocks, such as shocks to foreign demand for Swedish exports. In this paper, I have investigated the validity of this view by computing the relative importance of foreign and domestic shocks for the Swedish postwar business cycle 1950-1995 using a stochastic growth model designed for a small open economy. I have extended previous work by Lundvik (1992) by allowing for stochastic fiscal policy, since recent research by Jonsson and Klein (1996) suggests that innovations in fiscal policy might be important for postwar business cycles in Sweden. By using data up to 1995, this is the first study which takes into account the deep recession in the beginning of the 1990s. I have also tested whether the introduction of stochastic fiscal policy improves the empirical fit of the model in a significant way, by estimating most of the parameters in the model with SMM.

The main results in the paper are as follows. First, it is shown that fiscal policy shocks improve the empirical fit of the model, but not significantly so using asymptotic critical values. The main reason for rejecting the model is the increased standard deviations in private consumption, private investment and employment per capita due to the deep recession in the 1990s. It also turns out that the empirical improvement of the model with fiscal policy is dependent on whether the fiscal policy variables are H-P filtered or not. If they are H-P filtered, the fit is clearly worsened in comparison to if they are not. The reason why this result is obtained is that the fiscal policy variables contain strong trends during the postwar period, due to the public sector expansion in Sweden. This finding casts some doubts as to whether the results provided in Jonsson and Klein (1996) are robust with respect to detrending and choice of sample period.

Second, by decomposing the simulated volatility in output per capita, the real exchange rate and the current account attributed to various foreign and domestic shocks, I find that the relative importance of foreign and domestic shocks, and in particular the importance of fiscal policy shocks, for fluctuations in output are heavily dependent on whether the fiscal policy variables are filtered with the H-P filter or not. If the fiscal policy variables are left unfiltered, as in Jonsson and Klein (1996), then innovations in fiscal policy are found to be very important for output fluctuations, and account for over 50 percent of the fluctuations in output in the long run. But if the fiscal policy variables are filtered, fiscal policy shocks only account for 12 percent of the fluctuations in output in the long run. In this case, foreign shocks account for 43 percent of the output fluctuations, while domestic productivity shocks account for 45 percent. Regardless of whether or not the fiscal policy variables are H-P filtered, foreign shocks are found to be of decisive importance for fluctuations in the real exchange rate and, in particular, the current account. The reason why these differences occur for output fluctuations is that the fiscal policy variables contain strong trends during the postwar period, due to the public sector expansion in Sweden. Therefore, it is the latter results that are valid, and these results compare well with the findings of the VAR studies in the field.

To conclude, it seems to be a robust finding, in both the empirical and theoretical literature, that foreign and domestic shocks contribute about equally to output fluctuations, and that foreign shocks contribute by far the most to fluctuations in the real exchange rate and the current account.

What about the limitations of the paper? As in all other papers in the previous literature, fiscal policy is treated as an exogenous, non-optimal, process since it is not an easy task to attach a stable loss-function to the government. It would be an interesting extension to consider the effects of endogenous, optimal, fiscal policy on the business cycle within a certain regime. However, for the purpose of parameter estimation (with SMM perhaps) and hypotheses testing, one must develop an algorithm to solve the government's and households' problem simultaneously, because an iterative procedure which (i) calculates household decision rules given a fiscal policy decision rule, (ii) checks how the government's loss-function can be improved on the margin by changing parameters in the

fiscal policy decision rule, and (iii) repeats (i) and (ii) until convergence would be far too slow. In a recent paper, Söderlind (1998) shows how this can be done. More specifically, Söderlind (1998) shows how to solve and estimate a linear rational expectations model with optimal policy on data using the Kalman filter.

Another possible limitation is that there are no money shocks in the model. The reason for this is that only real variables are of interest here, and unless one imposes (*ad hoc*) some rigidities, money shocks seems to be unimportant for real variables in both the short and the long run. Even if one imposes some rigidities, money only seems to have a limited effect in the short run and be neutral in the long run. See Cooley and Hansen (1995) for further details. However, one extension of the work here would be to consider the short-run effects of money shocks on the real exchange rate and the current account in a model with nominal price and/or wage rigidities.

Appendix A Data sources and definitions

In this appendix, I present exact sources for the data set and exact definitions of the composite variables used. Table A.1 displays the exact sources of the data. In Table A.2, I provide exact definitions of composite variables, following the notation in Section 2.1.

To give a motivation for the definitions of the variables in the national account block, I start out from the nominal

$$P_t^{YM} Y_t^M = P_t^{CP} CP_t + P_t^{CG} CG_t + P_t^{IG} IG_t + P_t^{IP} IB_t + P_t^{II} II_t + P_t^X X_t - P_t^M M_t \quad (\text{A.1})$$

and real

$$Y_t^M = CP_t + CG_t + IG_t + IB_t + II_t + X_t - M_t \quad (\text{A.2})$$

GDP identities where the nominal holds for the whole sample period while the real only holds from 1990 to 1995. In addition, it is the case that

$$P_t^{YM} Y_t^M = P_t^{Y^F} Y_t^F + P_t^{T^I} T_t^I - P_t^{T^S} T_t^S \quad (\text{A.3})$$

holds in the data from 1950 to 1995. Unfortunately, no data on $P_t^{Y^F}$, Y_t^F , $P_t^{T^I}$, T_t^I , $P_t^{T^S}$ and T_t^S exist. It is only possible to acquire data on $P_t^{Y^F} Y_t^F$, $P_t^{T^I} T_t^I$ and $P_t^{T^S} T_t^S$. This creates a data problem since the most adequate measure of production in the model, Y_t , is Y_t^F in the data. As noted in Hassler et al. (1994) and Englund et al. (1992), the way one handles this problem is also of quantitative importance. Here, I have followed the strategy in Hassler et al. (1992) and accepted the GDP-deflator at market prices, P_t^{YM} , as proxy for $P_t^{Y^F}$, $P_t^{T^I}$ and $P_t^{T^S}$. By combining (A.2) and (A.3), I then obtain

$$\frac{P_t^{Y^F} Y_t^F}{P_t^{YM}} = CP_t - \frac{P_t^{T^I} T_t^I - P_t^{T^S} T_t^S}{P_t^{YM}} + CG_t + IG_t + IB_t + II_t + X_t - M_t \quad (\text{A.4})$$

which forms the basis for the definitions of many variables used.³²

³² However, it should be emphasized that by using (A.4), one still has a considerable measurement error between 1950 and 1989 due to the measurement error for (A.2) in the Swedish national accounts. An alternative way then to get rid of the measurement error would be to combine (A.1) and (A.3) and divide through with P_t^{YM} as in Englund et al. (1992).

Table A.1: The data set.

Variables	Sample period	Source
GDPMN and GDPM	1950-1995	SCB TSDB
GDPFN	1950-1995	SCB TSDB
TINDN and TSUBN	1950-1995	SCB TSDB
CPN and CP	1950-1995	SCB TSDB
CGN, IGN, CG and IG	1950-1995	SCB TSDB
IBN, IIN, IB and II	1950-1995	SCB TSDB
XN, MN, X and M	1950-1995	SCB TSDB
MFMT	1950-1995	SCB TSDB
HWT	1950-1969	Jonsson and Klein (1996)
HWT	1970-1979	SCB, N10 SM 8901 Table H:5 last row
HWT	1980-1995	SCB, N10 SM 9601 Table 6 last row
WSISIFN and SIFN	1950-1969	SCB, N 1971:11 Table 5 rows 1, 14
WSISIFN and SIFN	1970-1979	SCB, N10 SM 8601 Table H:12 rows 1, 19
WSISIFN and SIFN	1980-1995	SCB, N10 SM 9601 Table 7, rows 1, 19
CAN	1950-1995	Sveriges Riksbank, Fredrika Röckert
PFOR	1950-1995	OECD MEI
ESWEFOR	1960-1995	OECD MEI
GDPFOR	1950-1995	OECD MEI

Note: All real macroeconomic variables are measured in 1991 prices in millions. SCB stands for Statistics Sweden, TSDB for SCB's time series database, Sveriges Riksbank for Bank of Sweden. Abbreviations; GDPMN and GDPM denote GDP at market prices in nominal and real terms; GDPFN nominal GDP at factor prices; TINDN and TSUBN nominal indirect tax revenues and various subsidies; CPN and CP nominal and real private consumption expenditures; CGN, IGN, CG and IG public consumption and investment in nominal and real terms; IBN, IIN, IB and II nominal and real business and inventory investments; XN, MN, X and M nominal and real exports and imports; MFMT average population in thousands; HWT total hours worked in millions; WSISIFN and SIFN total nominal wage sum including social insurance fees and nominal social insurance fees respectively; CAN the nominal current account; **PFOR**, **ESWEFOR** and **GDPFOR** are 11×1 vectors with GDP deflators, nominal exchange rates vis-a-vis the Swedish krona and real GDPs per capita for the U.S., U.K., Germany, France, Japan, Italy, Finland, Norway, Denmark, Belgium and Canada, respectively.

Table A.2: Generation of composite data series.

Variable	Calculation formula
Y	$GDPFN/(GDPMN/GDPM)/MFMT$
C	$CP/MFMT-(TINDN-TSUBN)/(GDPMN/GDPM)/MFMT$
G	$(CG+IG)/MFMT$
g	$(CG+IG)/(GDPFN/(GDPMN/GDPM))$
I	$(IB+II)/MFMT$
X	$X/MFMT$
M	$M/MFMT$
H	$HWT*1000/MFMT$
Y^*	$\sum_{i=1}^{11} \omega_i [PFOR_i * GDPFOR_i * ESWEFOR_i / (GDPMN/GDPM)]$
CA	$CAN/GDPFN$
Q	$(ZN/Z)/(XN/X)$
τ^c	$(TINDN-TSUBN)/CPN$
τ^w	$SIFN/(WSISIFN-SIFN)$

Note: Y , C , G , I , X , M , H and Y^* are then subject to Hodrick-Prescott filtering in natural logs, as described in Section 3.2. The ω_i weights from the TCW currency basket index are normalized so that they sum to 1.

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Essay II

Fiscal Policy and the Yield Curve in a Small Open Economy^{*}

1 Introduction

In this paper, I address the question of whether larger budget deficits produce higher interest rates. Theoretically, it is well known that the effects of changes in fiscal policy on the term structure of interest rates are ambiguous. The Ricardian equivalence theorem states that, for a given path of government consumption expenditures, individuals view budget deficits as postponed tax-liabilities. Therefore budget deficits do not alter wealth, desired consumption paths or interest rates. According to the more conventional view in macroeconomics, on the other hand, individuals do not fully internalize the future tax-liabilities, which implies that changes in government debt add to private wealth, influencing desired consumption paths and thus interest rates.¹ However, the empirical studies undertaken to date, mostly utilizing data for the United States, have not been able to supply either view with convincing evidence. Since the resolution of this issue is important for the design of macroeconomic policy, there is a need for more research in the field.

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¹ When the term “conventional” is used, reference is made to Keynesian or other non-Ricardian models with rational expectations.

In this paper, I utilize data for Sweden to provide a good empirical answer to the question posed above. The reason why Sweden is an interesting case is that the country has experienced very large fluctuations in the government budget deficits and short- and long term nominal interest rates since the beginning of the 1980s. Consequently, this paper provides a high-powered empirical test compared to previous studies in the field.

The empirical approach in this investigation is close to that of Evans (1985, 1987a, 1987b, 1988). First, I make a survey of the results in the previous literature and try to draw some important lessons for the investigation in this paper. Second, I set up a conventional stochastic macro model, in which the term structure of nominal interest rates is determined in terms of different policy variables, and use this model to study the effects of fiscal policy. Since Sweden is best characterized as a small open economy, a conventional stochastic macro model for a small open economy is constructed. The reason for spending time on this is that there has been little attention paid to the effects of fiscal policy on the term structure in a small open economy.² In addition to providing a framework for the empirical research in this paper, this approach may also offer several important insights about this issue. For example, is it automatically the case that larger budget deficits produce higher interest rates even in a conventional model designed for a small open economy? Finally, I estimate the implied nominal interest rate regression equations on Swedish data, taking the lessons from the survey and the conventional model into account.

The empirical results suggest that larger budget deficits spell higher nominal interest rates. According to the empirical evidence, an increase in the budget deficit as a percentage of GDP by one percent leads to increases in the domestic short- and long term interest rates of approximately 0.20 percentage points after a period of two years.

The structure of the paper is as follows. The next section is a survey of different approaches used and empirical results obtained in the earlier literature. In Section 3, the

² In a closed economy setting, Turnovsky (1989), develops and uses a stochastic macro model to study the effects of changes in macroeconomic policy on the term structure of real and nominal interest rates. A closely related paper is McCafferty (1986). Grinols and Turnovsky (1994) use stochastic calculus to study the interaction between exchange rates and interest rates in a small open economy, but without a term structure of interest rates explicitly incorporated. Finally, the seminal paper by Cox, Ingersoll and Ross (1985) contains the most general stochastic utility maximizing approach to the term structure of interest rates, but they do not explicitly consider changes in macroeconomic policy.

model is developed and solved for the nominal interest rates. The quarterly and monthly data set are discussed in Section 4. In Section 5, some empirical issues are discussed and the empirical results for Sweden are presented. Some tentative conclusions are then finally drawn in Section 6.

2 Previous related studies

Previous results in the literature have been obtained within three types of approach. The first is denoted the conventional one, since it encompasses stochastic macro models, Keynesian or non-Ricardian, where agents are assumed to form their expectations rationally. The papers by Allen (1990, 1992) and Evans (1985, 1987a, 1987b, 1988) fall into this category.

Evans (1987a) develops a stochastic rational expectations model to study the effects of macroeconomic policy on real and nominal interest rates in a closed economy setting. In particular, he focuses on the validity of the proposition that larger budget deficits are associated with higher interest rates.³ For a long sample period from the United States, he provides evidence inconsistent with this proposition. That is, larger present or expected government budget deficits do not significantly push up either nominal or real interest rates. The same conclusion is reached from a data set containing six countries (Evans, 1987b).⁴ Finally, Evans (1988) investigates whether forward rates in the United States during the second world war were an increasing function of government debt. In the empirical tests, no evidence for such a positive relationship can be found; rather, there is a negative relationship.

Allen (1990) estimates a reduced form IS-LM-AS model using quarterly data on various measures of the federal debt in the United States between 1961 and 1985, and finds that there is a positive and statistically significant linkage between government debt and a tax-adjusted short-term real interest rate. Allen (1992) models first differences in order to control for autocorrelation and intercept instability, and provides more empirical evidence

³ Evans (1985) investigates the empirical relationship between nominal and real interest rates and current and past government budget deficits in the United States, and finds no positive association.

⁴ The six countries are: Canada, France, West Germany, Japan, United Kingdom and the United States.

of a positive and statistically significant relationship. There are several possible explanations for the different results obtained by Allen and Evans. Allen chooses not to model a reduced form for the inflation expectations. Instead he uses proxies in the estimated equation. Moreover, Allen primarily considers alternative measures of debt, while Evans focuses on different measures of deficits.

The second type of model attempts to test the Ricardian equivalence theorem more directly. The papers by Plosser (1982, 1987) are perhaps the most well known examples in a closed economy setting. In neither of his papers does Plosser find any statistically significant relation between deficits and interest rates in the United States. He interprets these findings as indirect evidence for the Ricardian view. Boothe and Reid (1989) extend the work of Plosser to the Canadian case, which they consider to be a small open economy. The empirical results of Boothe and Reid are also consistent with the previous studies undertaken by Evans and Plosser. In a political economy setting, Minford (1988) provides theoretical arguments against the Ricardian view. In brief, the argument is that different political parties tend to pursue policies designed to favor their own electorate. For instance, the “left” wing monetary policy will be more inflationary than the “right” wing, since the “right” electorate’s nominal government bond holdings can be expropriated through unanticipated inflation. This will lead to a risk premium on nominal government bonds, which will be an increasing function of the size of the bond financed deficit. Minford then provides empirical evidence consistent with the predictions from the model, using annual data for the United Kingdom between 1920-1982.⁵

The third type of model considered in the literature is the so called “loanable funds” model. This type of model, which models interest rates as equilibrium responses to the demand and supply in the loan markets, is used, for example, by Cebula et al. (1988), de Haan and Zelhorst (1990), Cebula et al. (1990), Cebula and Rhodd (1993), Correia-Nunes and Stemitsiotis (1995) and Miller and Russek (1996). The estimated equations in this literature are very similar; some nominal long-term interest rate is linearly related to a set of explanatory variables, including some measures of the expected inflation rate and

⁵ It is notable that the parameterization in the model considered is not as parsimonious as in the other studies. He also includes a measure of inflation expectations, dummies for the second world war and the Korean war etc. in his regression model.

government deficits and debts. Another characteristic of these studies is that they use annual data. The empirical evidence provided in this setting points in one direction: the level of nominal interest rates is positively related to government budget deficits.

So, which theoretical view is supported by the empirical evidence? Although the empirical results presented by Boothe and Reid, Evans, and Plosser are consistent with Ricardian equivalence, their investigations do not constitute a direct test. This stems from the fact that some of the assumptions underlying the theorem can be violated simultaneously, but work in different directions, so that even if Ricardian equivalence is not rejected by the data, one should only interpret the empirical results supporting Ricardian equivalence as a crude approximation of reality.⁶ On the other hand, the papers which test loanable funds models, and the papers by Allen and Minford, seem to point in another direction, namely that government deficits and debts have a significant impact on nominal and real interest rates. Therefore, these papers provide some evidence for the conventional model, and against the Ricardian view.

What are the tentative conclusions as to why these discrepancies have occurred in the empirical evidence reported? The analyses summarized above suggest two important factors, which may account for the different empirical results. First of all, the data frequency seems to be important. In studies which have exploited lower frequency data, the evidence is more in favor of the conventional view and against the Ricardian view. Some economists have also argued that misleading estimates can result from fitting econometric models to data too finely disaggregated over time.⁷ Secondly, the treatment of the expected inflation rate seems to be of considerable importance. In the studies surveyed, the results tend to be more supportive of the conventional model when a proxy has been used to account for the expected inflation rate, rather than a reduced form.

⁶ See Becker (1995) for a deeper discussion of this problem.

⁷ For a discussion of the reasons, see Evans (1987a) and the references therein.

3 The yield curve in a conventional small open economy model

In this section, I construct and use a conventional stochastic macro model to illustrate the effects of fiscal policy on the term structure of nominal interest rates in a small open economy. The model is a straightforward small open economy extension of the models presented by Turnovsky (1989) and Rogoff (1985). For ease of exposition, no dynamics are explicitly considered, but of course, in an empirical analysis of real world data, dynamics are important. Therefore, one can view the parameters in the theoretical model below as stationary polynomials in the lag operator.

3.1 The model

The aggregate supply function, where output, measured as a deviation around its natural rate, depends upon the unanticipated change in the domestic price level is given by:

$$y_t = \beta (p_t - E_{t-1}p_t) + \varepsilon_t^{AS}, \quad (1)$$

where y_t denotes real output gap in natural logs in time period t , p_t the price of y in natural logs and $E_{t-1}p_t$ the conditional expectation of the price level in t conditional on all available information in $t - 1$. In (1), ε^{AS} is interpreted as an exogenous white noise productivity shock.

Aggregate demand in the model is described by the IS-LM equations. The IS curve is given by

$$\begin{aligned} y_t &= -\lambda_1 r_t^l + \lambda_2 g_t + \lambda_3 D_t + \lambda_4 (s_t + p_t^* - p_t) + \varepsilon_t^{IS} \\ &\equiv -\lambda_1 r_t^l + X_t + \lambda_4 (s_t - p_t) + \varepsilon_t^{IS} \end{aligned} \quad (2)$$

where r^l denotes the domestic long-term real interest rate in natural units, g real government spending in natural logs, D real government budget deficit in natural units, s the nominal spot exchange rate in

also captures the conventional mechanism that government budget deficits add to private wealth, influencing desired consumption paths and thus output and interest rates for a given exchange rate and a given domestic price level.

Money market equilibrium is described by the LM curve

$$m_t - p_t = \alpha y_t - \gamma i_t^s + \varepsilon_t^{LM}, \quad (3)$$

where m denotes the nominal money supply in natural logs and i^s the domestic nominal short-term interest rate. Thus, as in Turnovsky (1989), the demand for money is assumed to depend on the domestic short-term nominal interest rate. In (2) and (3), ε^{IS} and ε^{LM} are interpreted as real demand and money demand shocks, respectively. It is assumed that the parameters in (1), (2) and (3), denoted $\alpha, \beta, \gamma, \lambda_1, \lambda_2, \lambda_3$ and λ_4 , are all positive, which is standard in conventional macro models.

The financial part of the model involves the relationships between the domestic and foreign short- and long-term real and nominal interest rates.⁸

The Fisher equations which relate domestic nominal and real interest rates are

$$i_t^s = r_t^s + (E_t p_{t+1} - p_t) \quad (4)$$

and

$$i_t^l = r_t^l + \frac{1}{2} (E_t p_{t+2} - p_t) \quad (5)$$

where r^s = domestic short-term real interest rate in natural units and i^l = domestic long-term nominal interest rate in natural units.

The equations which describe the real and nominal term structures of interest rates are given by

$$r_t^l = \frac{1}{2} (r_t^s + E_t r_{t+1}^s) \quad (6)$$

and

$$i_t^l = \frac{1}{2} (i_t^s + E_t i_{t+1}^s). \quad (7)$$

The uncovered interest parity, UIP, condition, which relates the domestic short- and long-term nominal interest rates to their foreign counterparts, denoted i^{s*} and i^{l*} , and

⁸ In accordance with Turnovsky (1989), it is assumed that there exist two types of domestic and foreign (zero coupon) assets with one and two periods to maturity. It is then straightforward to derive (6), (7), (8) and (9) up to a constant risk-premium as simple asset pricing relationships.

the expected one and two period changes in the nominal exchange rate are

$$i_t^s - i_t^{s*} = \Delta E_t s_{t+1} \quad (8)$$

and

$$i_t^l - i_t^{l*} = \frac{1}{2} (E_t s_{t+2} - s_t). \quad (9)$$

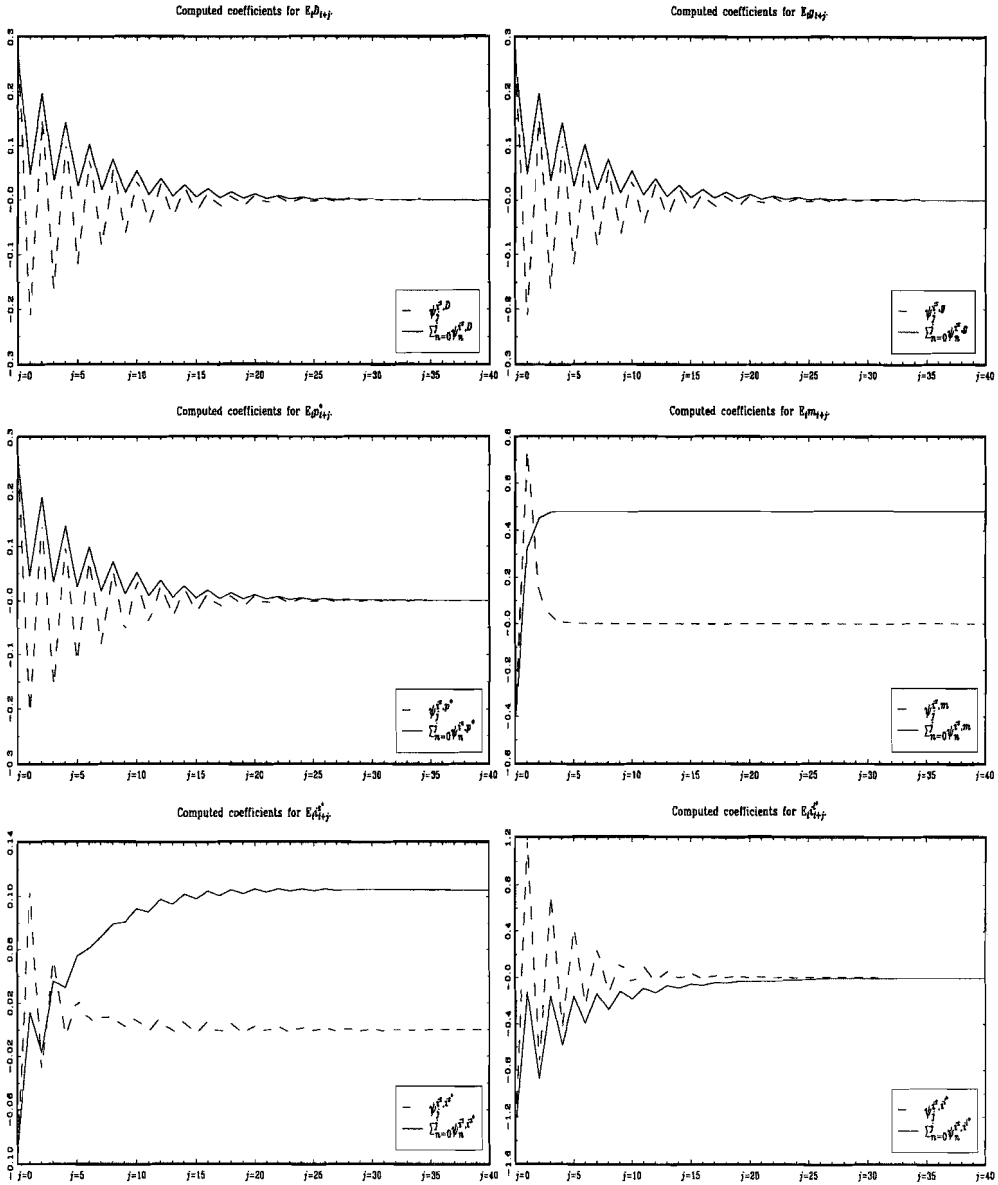
In order to close the model, we need to make some additional assumptions. First, in conventional macro models, g and D are normally considered to be exogenous. We will adopt this approach throughout the theoretical analysis in this paper. Second, p^* , i^{s*} and i^{l*} will also be treated as exogenous to the domestic economy, which is quite natural in a small open economy framework. Finally, we need to specify a policy rule for m . In theoretical analysis, it is standard to assume that m is independent of the other exogenous variables. Since the main interest in this paper is the interaction between fiscal policy and the term structure, we adopt the conventional view in the theoretical part of the paper. However, in an analysis of real world data, this strategy may lead to problems, since, for instance, the monetary policy rule is unlikely to be independent of g and D . Therefore, the first and third assumptions are relaxed in the empirical analysis in this paper.

3.2 Determination of nominal interest rates

To derive analytical solutions for the endogenous variables i^s and i^l in terms of current and expected future values of the exogenous variables g , D , m , p^* , i^{s*} and i^{l*} , we proceed by first determining price level expectations, and then substitute the resulting expressions back into the system to solve for s_t . Finally, the solution for s_t can then be used in the UIP conditions to get the solutions for the short- and long-term domestic nominal interest rate differentials $i_t^s - i_t^{s*}$ and $i_t^l - i_t^{l*}$.⁹

By this procedure, the short- and long-term interest rate differential depends both on the as of period t and $t - 1$ expected discounted sum of nominal money supplies, government expenditures and deficits, foreign price level and short-term nominal interest rates and the as of t expected discounted sum of foreign long-term nominal interest rates.

⁹ All the derivations of the equations informally presented and analyzed in this section are provided in Appendix A.

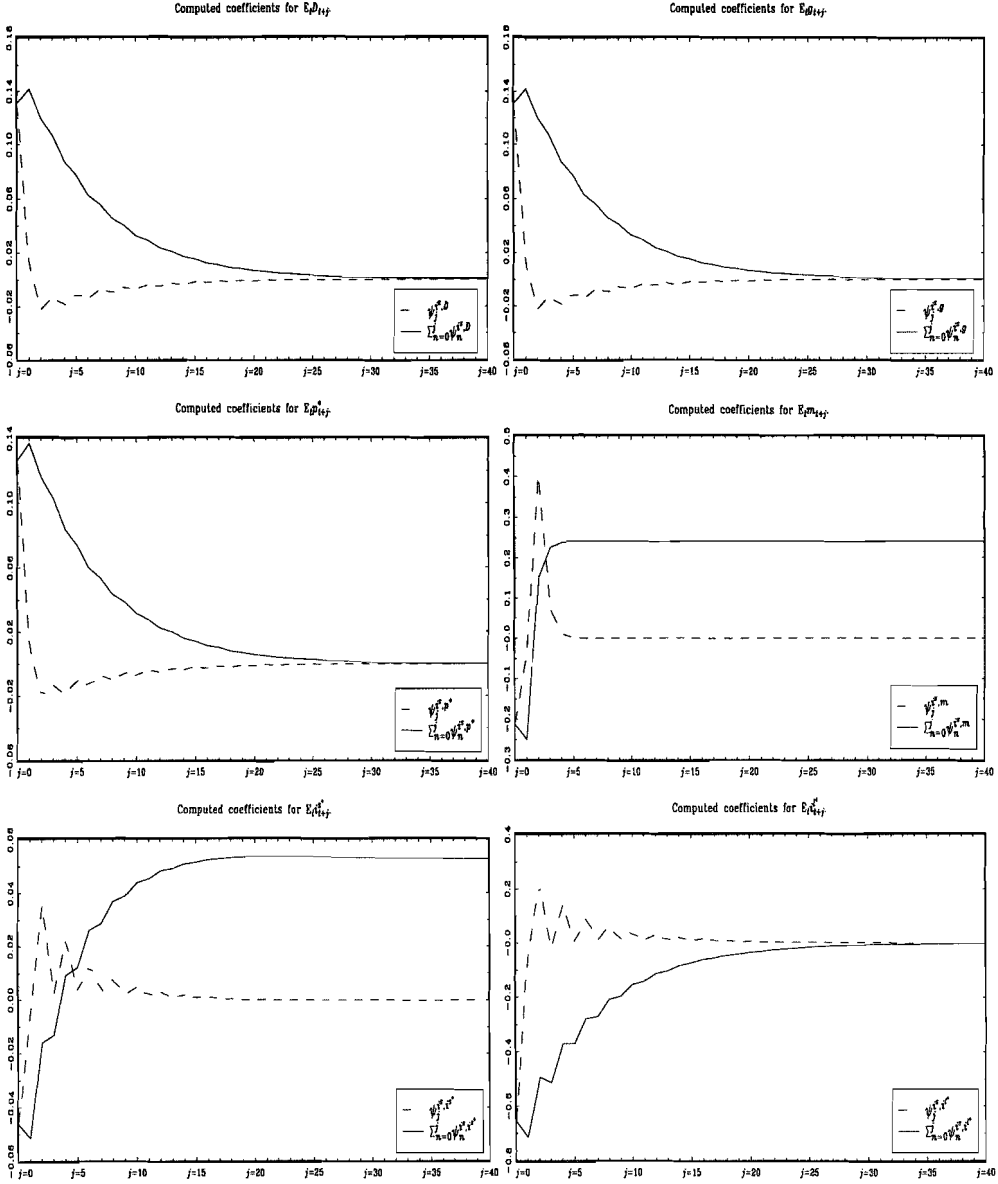
Figure 1: Effects on $i_t^s - i_t^*$ of exogenous variables j periods ahead.

More formally, let $\psi_j^{i^s,D}$ and $\psi_j^{i^l,D}$ measure the effects of (as of period t , unknown in period $t - 1$) expected budget deficits $j = 0, 1, 2, \dots$ periods ahead, $E_t D_{t+j}$, on the short- and long-term interest rate differentials respectively (analogous notation for the other variables g, p^*, m, i^{s*} and i^{l*} as well). Since the $\psi_j^{i^s}$ and $\psi_j^{i^l}$ coefficients are quite messy to evaluate analytically for $j > 1$, I have made simulations conditional on some reasonable values for $\alpha, \beta, \gamma, \lambda_1$ and λ_4 in order to get a feeling for the size and magnitude of the paths for them.¹⁰ For simplicity, it is assumed that $\lambda_2 = \lambda_3 = 1$, so that $\psi_j^{i^s,g} = \psi_j^{i^s,D}$ and $\psi_j^{i^l,g} = \psi_j^{i^l,D}$ for all j . The resulting paths for $j = 0, 1, 2, \dots, 40$ are depicted in Figures 1 and 2 for $i_t^s - i_t^{s*}$ and $i_t^l - i_t^{l*}$ respectively. In Figures 1 and 2, the dashed lines refer to the $\psi_j^{i^s}$ and $\psi_j^{i^l}$ coefficients, while the solid lines refer to the accumulated effects in period $t + j$, measured as $\sum_{n=0}^j \psi_n^{i^s}$ and $\sum_{n=0}^j \psi_n^{i^l}$.

As can be seen from Figure 1, all the $\psi_j^{i^s}$ coefficients range from positive to negative values for $j = 0, \dots, 40$; for $j = 0$, $\psi_0^{i^s,g}$, $\psi_0^{i^s,D}$ and $\psi_0^{i^s,p^*}$ are positive while $\psi_0^{i^s,m}$, $\psi_0^{i^s,s^*}$ and $\psi_0^{i^s,i^*}$ are negative, which can be demonstrated analytically. The coefficients for the foreign price level are very similar to those of government expenditures and budget deficit since the numerical value for λ_4 is close to 1. Turning to Figure 2, we find (as can also be shown analytically) that the coefficients now range from positive to negative and negative to positive values after the second period. It is interesting to note that the simulated paths for the budget deficit in Figures 1 and 2 compare well qualitatively with the closed economy results in Turnovsky (1989), in the sense that the $\psi_{t+j}^{i^s,D}$ and $\psi_{t+j}^{i^l,D}$ coefficients are both positive and negative, but poorly with Evans (1987a), where all the corresponding $\psi_j^{i^s,D}$ and $\psi_j^{i^l,D}$ coefficients are found to be greater than zero. It can be shown that this result is due to the introduction of a term structure within the model; see Turnovsky (1989) for a deeper discussion about the intuition.

From Figures 1 and 2, we also see that the accumulated effects of an as of t permanent change in g, D, p^* and i^{l*} go towards zero when j increases. This result is due to the small open economy assumption; for example, after an as of t permanent change (unknown in period $t - 1$) in the budget deficit, the nominal exchange rate today, s_t , and the as of t

¹⁰ The values for α and γ are taken from the empirical study by Goldfeld and Sichel (1990) and set to 0.6179 and 0.2170 respectively. λ_1 and β are taken from Söderlind (1997) and set to 5 and 500 respectively. λ_4 is taken from Hansson (1993) and set to 0.9644.

Figure 2: Effects on $i_t^l - i_t^{l*}$ of exogenous variables j periods ahead.

expected exchange rates in period $t + 1$ and $t + 2$, $E_t s_{t+1}$ and $E_t s_{t+2}$, change by the same amount. Via the UIP conditions (8) and (9), the effects on the short- and long-term interest rate differentials are then zero. This result is important since it suggests that the effects of budget deficits on interest rates in a small open economy framework are negligible if budget deficits can be characterized as (or close to) random walks.

Finally, the long-run accumulated effects of an as of t increase in money supply and the foreign short-term nominal interest rate are positive and exactly half as large on the long-term interest rate differential compared to the short-term interest differential. The intuition behind this result is that the as of $t - 1$ expected price level in period t , $E_{t-1} p_t$, is unaffected by an increase in as of t variables, so that s_t is unaffected in this respect by a permanent increase in m_t and i_t^{s*} . But permanent increases in m_t and i_t^{s*} increase the as of t expected future price level in the periods $t + 1$ and $t + 2$, $E_t p_{t+1}$ and $E_t p_{t+2}$, in this respect and thereby also $E_t s_{t+1}$ and $E_t s_{t+2}$. Via the UIP conditions (8) and (9), we then get increases in $i_t^s - i_t^{s*}$ and $i_t^l - i_t^{l*}$.¹¹

3.3 Empirical implementation of the model

In order to generate empirical testable implications for the nominal short- and long-term interest rate differentials, we need to make some assumptions regarding the stochastic processes for the exogenous variables, and thus how about the expectations for these variables are formed.

Here, it is assumed that the exogenous vector of variables $\mathbf{z}_t^T \equiv [p^* \ i^{s*} \ i^{l*} \ g \ D \ m]$ evolves according to a VAR(p) process

$$\mathbf{z}_{t+1} = \boldsymbol{\rho}^z(L) \mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}^z, \quad (10)$$

where $\boldsymbol{\rho}^z(L) \equiv \sum_{i=0}^p \boldsymbol{\rho}_i^z L^i$ and the errors in $\boldsymbol{\varepsilon}^z$ are normally distributed and serially uncorrelated with $E_t \boldsymbol{\varepsilon}_{t+j}^z = \mathbf{0}$ for all $j > 0$ with a positive definite covariance matrix. This is a conventional assumption in empirical analysis, and for instance Evans (1987a) uses an assumption similar to (10). The specification in (10) relaxes the earlier assumption

¹¹ However, this result is sensitive to the parameterization of the model. With the numerical assumptions about α , λ_1 and λ_4 here, it is the case that $\alpha(\lambda_1 + 2\lambda_4) - 2 > 0$. But if $\alpha(\lambda_1 + 2\lambda_4) - 2 < 0$, permanent increases in m_t and i_t^{s*} have negative accumulated effects on $i_t^s - i_t^{s*}$ and $i_t^l - i_t^{l*}$.

of independently distributed exogenous variables. For example, it allows for money supply to be determined by some policy function of the other exogenous variables.¹²

Using (10), it can be shown that the solutions for the interest rate differentials are of the following general form

$$i_t^r - i_t^{r*} = \delta_0^r + \delta^r(L)z_t + \nu_t^r \quad (11)$$

where $\delta^r(L) \equiv [\delta^{r,g}(L), \delta^{r,D}(L), \delta^{r,p^*}(L), \delta^{r,m}(L), \delta^{r,i^{**}}(L), \delta^{r,i^*}(L)]$ for $r = s, l$.¹³ However, in this general case, nothing can be said about the sums of the individual parameters in the lag polynomials in δ^r ; the sign and size of these sums will ultimately depend on the coefficients in $\rho^z(L)$, about which we know very little. In this sense, it is fair to say that it is essentially an empirical question whether larger government budget deficits lead to higher interest rates; that is, whether the sums of the coefficients in the lag polynomials $\delta^{s,D}(L)$ and $\delta^{l,D}(L)$, equal to $\sum \delta_i^{s,D} L^i$ and $\sum \delta_i^{l,D} L^i$ respectively, are positive or negative.

However, if we make the simplifying assumption that $\rho^z(L) \equiv \rho^z$, where ρ^z is a diagonal matrix with the elements $[\rho^{p^*} \rho^{s^*} \rho^{l^*} \rho^g \rho^D \rho^m]$ in the diagonal, it is possible to draw further conclusions. In this case, the solution for the interest rate differentials is

$$\begin{aligned} i_t^r - i_t^{r*} = & \delta_0^r + \delta_1^r(L)g_t + \delta_2^r(L)D_t + \delta_3^r(L)p_t^* - \delta_4^r(L)m_t - \delta_5^r(L)i_t^{s*} - \\ & \delta_6^r(L)i_t^{l*} + \delta_7^r(L)\Delta g_t + \delta_8^r(L)\Delta D_t + \delta_9^r(L)\Delta p_t^* + \delta_{10}^r(L)\Delta m_t + \\ & \delta_{11}^r(L)\Delta i_t^{s*} + \nu_t^r. \end{aligned} \quad (12)$$

for $r = s, l$. By introducing the notation

$$\begin{aligned} \delta^r(L) & \equiv \begin{bmatrix} \delta_1^r(L) + \delta_7^r(L)(1-L), \delta_2^r(L) + \delta_8^r(L)(1-L), \delta_3^r(L) + \delta_9^r(L)(1-L), \\ -\delta_4^r(L) + \delta_{10}^r(L)(1-L), -\delta_5^r(L) + \delta_{11}^r(L)(1-L), -\delta_6^r(L) \end{bmatrix} \\ & \equiv [\delta^{r,g}(L), \delta^{r,D}(L), \delta^{r,p^*}(L), \delta^{r,m}(L), \delta^{r,i^{**}}(L), \delta^{r,i^*}(L)] \end{aligned}$$

the solution can be written in the general form considered in (11). With these restrictive assumptions, the model has some nice implications. It is now the case that all

¹² Note also that from now on, dynamics are explicitly considered in the model. That is, we use the implicit assumption that all the parameters are stationary lag polynomials, i.e. $\alpha \equiv \alpha(L)$, $\beta \equiv \beta(L)$ and so forth.

¹³ All the derivations of the equations presented in this section are provided in Appendix A.

the parameters in the lag polynomials $\delta_i^r(L)$ for $i = 1, \dots, 6$ are positive provided that $\{\rho^g, \rho^D, \rho^{p^*}, \rho^m, \rho^{i^{s^*}}, \rho^{i^{l^*}}\} \in [0, 1)$. This implies that the sums of all the parameters in each of the polynomials $\delta^{r,g}(L)$, $\delta^{r,D}(L)$, $\delta^{r,p^*}(L)$, $\delta^{r,m}(L)$, $\delta^{r,i^{s^*}}(L)$ and $\delta^{r,i^{l^*}}(L)$ are also positive. However, except for $\delta^{r,i^{l^*}}(L)$, the same conclusion cannot be made for all the individual parameters in these polynomials; the sign of them can alternate over time. The reason is that changes in the exogenous variables have effects on the interest rate differentials via the lag polynomials $\delta_i^r(L)$ for $i = 7, \dots, 11$, and that the signs of the parameters in these lag polynomials are ambiguous. Indeed, these theoretical predictions are different from those of Allen (1990, 1992) and Evans (1985, 1987a, 1987b), since their models did not imply these ambiguities for the individual parameters. The reason why these differences occur is that I have an aggregate supply function in the model, which makes it possible to explicitly solve for the price level expectations. The fact that the model considered includes a term structure of interest rates, and is designed for a small open economy, does not matter for this result.

In addition, if $\rho^z = \mathbf{I}_6$, then the variables g , D , p^* and i^{l^*} do not have any effects either on the short- or long-term interest rate differentials, and in this case, only changes in money supply and the foreign nominal short-term interest rate influence $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ via changes in the expected price level. The intuition behind this result is straightforward. Consider, for example, an increase in the budget deficit in period t . If ρ^D is equal to one, then the nominal exchange rate today, s_t , and the expected exchange rates in the periods $t + 1$ and $t + 2$, $E_t s_{t+1}$ and $E_t s_{t+2}$, will be fully adjusted downwards by the same amount (s_t appreciates), thus leaving $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ unaffected via the UIP conditions. But if ρ^D is less than one, then the nominal exchange rate s_t is still fully adjusted, while $E_t s_{t+1}$ and $E_t s_{t+2}$ are only partially adjusted downwards, thus increasing $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ via the UIP conditions since $\Delta E_t s_{t+1}$ and $E_t s_{t+2} - s_t$ become positive. Moreover, since it is plausible to assume that all the parameters ρ^g , ρ^{p^*} , ρ^m , $\rho^{i^{s^*}}$ and $\rho^{i^{l^*}}$ are equal or very close to one, we do not expect any of these variables to have any large level effects on the short- and long-term interest rate differentials. It also seems reasonable to assume that ρ^D is high, but slightly less than one, which implies that an increase in the budget deficit will increase $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ by a relatively small amount. Thus, if ρ^D is sufficiently

close to one, the effect of changes in the budget deficit on $i_t^s - i_t^{s*}$ and $i_t^l - i_t^{l*}$ will be almost zero, independently of the monetary policy rule. However, it should be emphasized that these last results are due to the small open economy feature of the model.

The most striking implication of the derivations above is that a simple conventional macroeconomic model may offer a possible explanation for the lack of empirical relationship between government budget deficits and interest rates. That is, when empirical analyses based on (11) are carried out, one might readily obtain “wrong” results, because, as argued above, even in the simplest case when the exogenous variables are assumed to follow univariate autoregressive processes, it may very well be the case that the sum of the elements in the lag polynomials for D_t is very close to zero.

(11) provides the framework for the empirical investigation that follows below, and it should therefore be noted that the exogenous shocks ν_t^r are very likely to be serially correlated over time.¹⁴

4 Data

Since the Swedish financial markets were heavily regulated until the beginning of the 1980s, it is hard to acquire good interest rate data for long samples for Sweden. In this paper, a three-month government Treasury bill and a five- to ten-year government Treasury bond are used as measures of i^s and i^l (both expressed as effective yields), and data of good quality on these two series are only available from January 1982 and the middle of February 1984 respectively.¹⁵ Accordingly, the data frequencies which can be exploited in the analysis must be rather high, in order to get a sufficient number of observations. Monthly frequency until June 1996 then gives 174 observations, while quarterly frequency gives at the most only 58. This means that the monthly frequency is desirable, and almost every data series that is needed is also available on monthly frequency. Unfortunately the highest frequencies for g and y are quarterly.¹⁶ Hence, in

¹⁴ Since I consider the parameters to be stationary lag polynomials, i.e. $\alpha \equiv \alpha(L)$, $\beta \equiv \beta(L)$ and so forth, the model implies that the error terms in (11), ν_t^s and ν_t^l , are moving average (MA) terms.

¹⁵ To get an indication of the robustness with respect to the choice of maturity for the short-term Treasury bill for the empirical investigation, other Treasury bills with one, six and twelve months to maturity have been examined, and since the results were unaffected they are not reported.

¹⁶ From now on, y denotes the gross domestic product, GDP, and not the log of the output gap.

order to be able to use monthly data in the regressions, some kind of interpolation for these two variables is necessary. In this paper, it is assumed that: (i) the quarterly values of g are uniformly distributed over the months within each quarter; (ii) the monthly distribution of y within the quarters follows the private industrial production, denoted x , for which data are available on a monthly basis, according to the scheme $y_{m,t} = \kappa_t x_{m,t}$ where $\kappa_t \equiv \frac{y_{q,j(t)}}{x_{q,j(t)}}$ and $j(t) = 1$ for all $t = 1, 2, 3$, $j(t) = 2$ for all $t = 4, 5, 6$ etc.¹⁷ Therefore, in order to get a feeling for the validity of the interpolation, both monthly and quarterly data are used in this paper. Another justification for using both quarterly and monthly data is that the survey of earlier empirical literature suggested that a different choice of data frequency has been important for different empirical evidence. By using both frequencies here, we take this aspect into account. There are two principal reasons for the need to use y . First, we want to detrend the data series and only consider the business cycle component of the variables g , D and m . A very natural way to accomplish this is to divide the relevant variables by y . Second, the presumption is made that the Swedish economy has the property of homogeneity; that is, doubling government consumption and deficits and nominal money supply and the size of the economy leaves the interest rate differentials unaffected. Evans (1987a) uses the same approach.

Since Sweden had a fixed exchange rate regime between 1982 and November 1992, and thus for the greater part of the sample period, "currency-basket" weighted foreign short- and long-term interest rates (both expressed as effective yields) and price levels have been constructed to obtain measures of i^{s*} , i^{l*} and p^* during the whole sample period. When there has been no possibility of acquiring interest rate data for certain countries during limited periods in the sample, the "currency-basket" weights have been normalized to one.¹⁸ Moreover, since the foreign long-term interest rates were only available as monthly averages, averages have been utilized for the other interest rates as well.¹⁹ The calculated

¹⁷ To test the sensitivity of these assumptions for the analysis, an alternative method suggested by Litterman (1983) with better properties from a statistical viewpoint has been tested to generate g and y on monthly frequencies, but since the qualitative conclusions were unaffected, they are not reported.

¹⁸ This has not been a significant problem though; countries which together make up at least 67.10 percent in the beginning and up to 100 percent at the end of the sample period are included in the calculation of the foreign long-term interest rate. The corresponding figures for the short-term interest rate are 100 percent until November 1992, and thereafter between 79.9 to 97.54 percent.

¹⁹ Note that the macroeconomic variables g , D , m and p^* have been subjected to seasonal adjustment. Since there seemed to be tendencies of changing seasonal pattern in most of the data series, the X11-method was used to deseasonalize the data. However, since the large and changing monthly seasonal

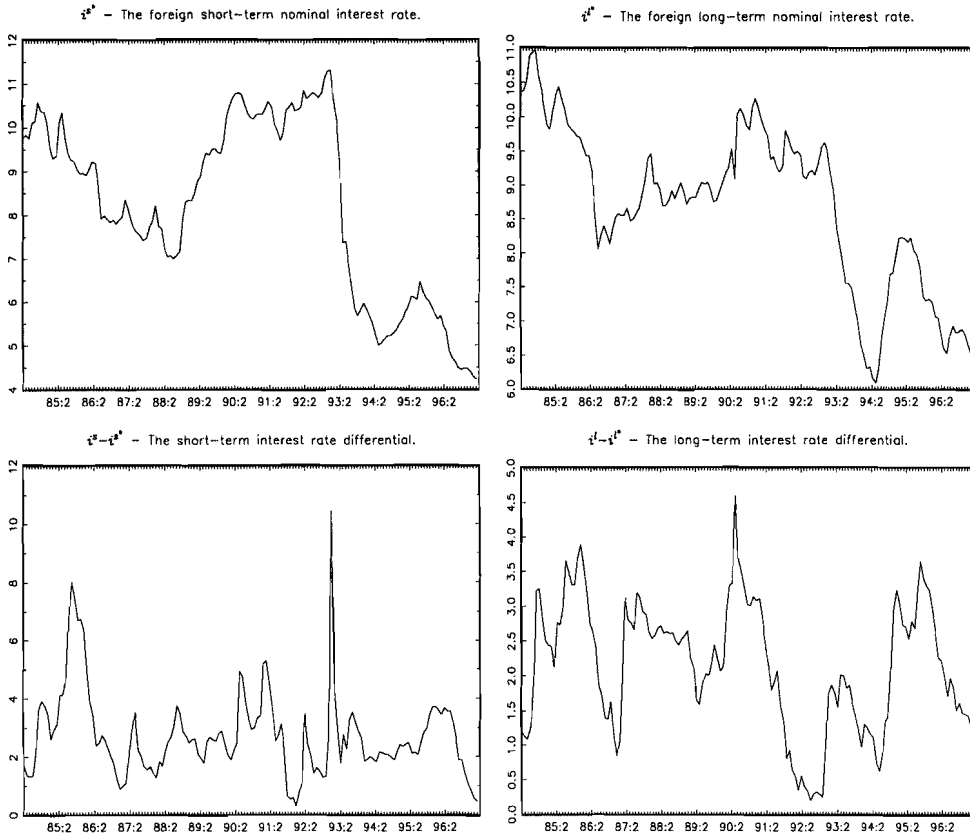


Figure 3: Monthly data on nominal interest rates.

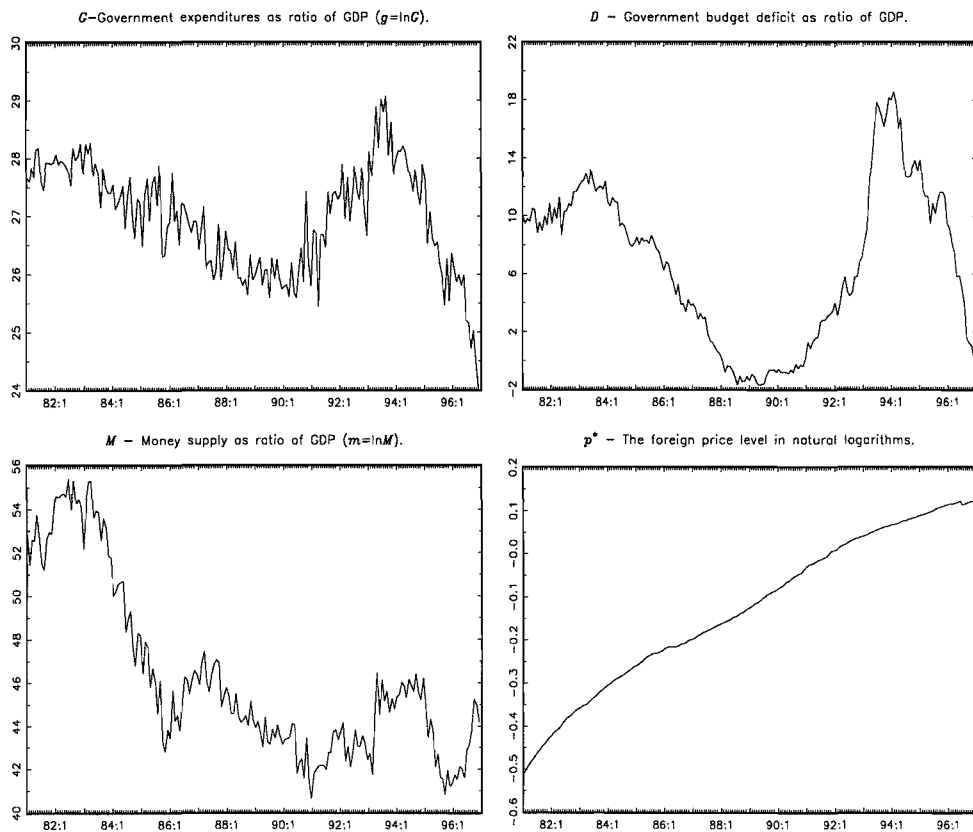


Figure 4: Monthly data on seasonally adjusted macrovariables.

series for i^{s*} , i^{l*} , $i^s - i^{s*}$ and $i^l - i^{l*}$, and g , D , m and p^* are depicted in Figures 3 and 4 respectively.

Summary statistics for quarterly and monthly data are given in Tables 1 and 2 respectively.²⁰ In general, Tables 1 and 2 show that the sample autocorrelations are very high and taper off very slowly over time, with the possible exception of i^s , i^{s*} , i^l , i^{l*} , $i^s - i^{s*}$, $i^l - i^{l*}$ and D . This pattern is normally an indication that the variables may be non-stationary. They reveal that the premium has on average been higher on $i^s - i^{s*}$ compared to $i^l - i^{l*}$. The variance is also higher for the bills, both Swedish and foreign, than the bonds. Among the macroeconomic variables, the variance in D is much higher than the variability in g and m . This is a remarkable fact, since Plosser (1987) reports variabilities in g and m which exceed the variability in D by large amounts for the United States. For instance, Plosser (1987) reports that the ratios between the standard deviation in g to D and in m to D are 7.3 and 2.1 on the monthly frequency. In this data set, the corresponding figures are 0.35 and 0.81. Of course, the high volatility in D reflects the dramatic swings in the Swedish budget deficits, which can be seen in Figure 4.

Table 1: Summary statistics for quarterly data.

Variable	Mean	Std.dev.	Sample autocorrelations					
			$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_8$	$\hat{\rho}_{12}$
i^s	10.92	2.68	0.82	0.64	0.50	0.33	0.12	-0.12
i^{s*}	8.60	2.36	0.89	0.73	0.61	0.49	0.22	-0.06
$i^s - i^{s*}$	2.65	1.29	0.56	0.23	0.03	-0.23	-0.15	0.03
i^l	10.80	1.71	0.84	0.62	0.39	0.21	0.20	0.14
i^{l*}	9.13	1.69	0.94	0.85	0.76	0.65	0.40	0.28
$i^l - i^{l*}$	2.13	0.91	0.75	0.43	0.18	-0.07	0.03	-0.09
g	3.26	0.05	0.94	0.89	0.85	0.80	0.58	0.37
D	5.53	4.55	0.95	0.88	0.81	0.71	0.35	-0.05
m	3.85	0.12	0.97	0.94	0.91	0.88	0.75	0.65
p^*	-0.15	0.20	0.95	0.90	0.85	0.80	0.61	0.45

Note: g , D , m and p^* have been subjected to seasonal adjustment. g , m and p^* are in natural logs.

To summarize, the data set utilized in this paper implies that the regressions for the short- and long-term interest rate differentials on the government budget deficit will have

variation in the private industrial production (x_m), used to generate a measure of monthly GDP (y_m), could not be sufficiently well deseasonalized with the X11-method, I used deseasonalized raw data on y_q and x_m to calculate y_m .

²⁰ Exact definitions and sources of all the variables used are given in Appendix B.

Table 2: Summary statistics for monthly data.

Variable	Mean	Std.dev.	Sample autocorrelations						
			$\hat{\rho}_1$	$\hat{\rho}_6$	$\hat{\rho}_{12}$	$\hat{\rho}_{18}$	$\hat{\rho}_{24}$	$\hat{\rho}_{30}$	$\hat{\rho}_{36}$
i^s	10.92	2.76	0.91	0.59	0.30	0.18	0.12	0.04	-0.11
i^{s*}	8.60	2.36	0.97	0.73	0.48	0.32	0.22	0.11	-0.06
$i^s - i^{s*}$	2.65	1.46	0.76	0.16	-0.20	-0.24	-0.10	0.05	0.02
i^l	10.81	1.71	0.96	0.59	0.18	0.08	0.20	0.30	0.11
i^{l*}	9.35	1.67	0.98	0.84	0.64	0.48	0.40	0.31	0.18
$i^l - i^{l*}$	2.15	0.93	0.92	0.41	-0.08	-0.15	-0.01	0.14	-0.10
g	3.27	0.06	0.80	0.77	0.71	0.59	0.47	0.42	0.32
D	5.63	4.74	0.98	0.89	0.72	0.56	0.37	0.17	-0.02
m	3.91	0.12	0.95	0.91	0.85	0.78	0.74	0.71	0.67
p^*	-0.15	0.20	0.98	0.90	0.80	0.70	0.61	0.53	0.45

Note: g , D , m and p^* have been subjected to seasonal adjustment. g , m and p^* are in natural logs.

high power compared to previous studies.

5 Estimation and empirical results

This section deals with the problem of how to estimate (11) in an appropriate way and then reports the results of the regressions.

5.1 Non-stationarity and cointegration

As already noted in section 4, one striking feature of the sample autocorrelations is that they start at very high values and then taper off very gradually, possible exceptions being $i^s - i^{s*}$, $i^l - i^{l*}$ and D . This pattern is generally an indication that the time series are non-stationary. Banerjee et al. (1993) discuss the properties of the regression estimates obtained when some of the variables are integrated, and find that what is often called balance in the regression is an important property. This means that when the dependent variable is stationary, the explanatory variables should also be integrated of order zero or cointegrated.²¹ Consequently, there is a need to test the integration order of the variables involved in the regressions.

To test the integration order of the individual series, I have used the augmented

²¹ Note: If a variable needs to be differentiated exactly k times to achieve stationarity, then the variable is $I(k)$, integrated of order k . It follows that a stationary variable is $I(0)$. If for a particular variable $k > 0$, where k is a positive integer, then it is said to be non-stationary.

Dickey-Fuller (ADF) procedure and applied the practical guidelines proposed in Hamilton (1994), which means that a constant and/or linear trend is included in the regression if the variable displays a non-zero mean and/or sign of linear trend in the observed sample. In the ADF procedure, H_0 is the hypothesis that the series under consideration is non-stationary, which in practice implies an estimated $\hat{\phi}$ in Tables 3 and 4 not significantly lower than zero. The ADF test results for the variables in level form are reported in Table 3.

As seen from Table 3, the null hypothesis that the variables are non-stationary can only be rejected for $i^s - i^{s*}$, $i^l - i^{l*}$ and D on reasonable significance levels. Although we can reject the hypothesis that D is non-stationary, the estimated autoregressive coefficients are close to one, implying that our empirical estimate of ρ^D is also close to one. These findings are consistent with what we expected a priori, and discussed in Section 3.2. Thus, according to the theoretical model (11), the effects of government budget deficits on the nominal interest rate differentials are likely to be relatively small.

Table 3: Augmented Dickey-Fuller tests of integration order on levels.

Variable	Quarterly frequency				Monthly frequency			
	T	p	$\hat{\phi}$	t -value	T	p	$\hat{\phi}$	t -value
i^s	51	4	-0.105	-1.465	158	9	-0.060	-1.581
i^{s*}	56	3	-0.056	-1.116	166	13	-0.012	-1.063
$i^s - i^{s*}$	48	7	-0.794	-3.218**	157	10	-0.306	-3.588***
i^l	49	1	-0.175	-2.508	147	6	-0.039	-1.965
i^{l*}	60	5	-0.050	-1.574	185	13	-0.016	-1.554
$i^l - i^{l*}$	47	3	-0.375	-3.475**	143	10	-0.120	-3.173**
g	98	7	-0.067	-2.176	293	24	-0.065	-2.210
D	93	8	-0.097	-3.208**	297	20	-0.032	-3.142***
p^*	62	4	-0.087	-1.149	196	5	-0.010	-2.342
m	92	13	-0.127	-1.827	294	23	-0.110	-2.539

Note: g , D , p^* and m have been subjected to seasonal adjustment as described in Appendix B. T is the number of observations included in the test. *(**) [***] indicates that $H_0: Z \sim I(k)$ where $k > 0$ is rejected at the 10 (5) [1] percent significance level. McKinnon critical values are used.

In order to determine the integration order for the other variables, we proceed to the $I(1)$ tests. With quarterly and monthly frequencies, both first differences on seasonally adjusted data or annual changes on seasonal unadjusted data can be utilized in the tests. One of the aims of using annual changes is to eliminate most of the seasonal variability prior to estimation. In addition, the series obtained are often easier to interpret than first

difference series, where the seasonal variability often completely swamps the remaining variability.²² Thus, for i^{s*} and i^{l*} , the tests are based on first differences, and for g , p^* and m on annual changes of seasonally unadjusted data.²³

The overall impression from Table 4 is that the null hypothesis is firmly rejected, and together with the test results in Table 3, it is concluded that i^{s*} , i^{l*} , g , p^* and m are non-stationary and integrated of order one. It is a relief to note the resemblance of the results for the different frequencies.

Table 4: Augmented Dickey-Fuller tests of integration order on differences.

Variable	Quarterly frequency				Monthly frequency			
	T	p	$\hat{\phi}$	t -value	T	p	$\hat{\phi}$	t -value
Δi^{s*}	58	0	-0.598	-4.938***	166	12	-0.545	-3.649***
Δi^{l*}	60	4	-1.185	-6.138***	184	13	-0.805	-4.770***
Δg	100	1	-0.263	-3.099***	283	22	-0.239	-1.952**
Δp^*	59	4	-0.073	-2.964**	177	12	-0.028	-3.360***
Δm	90	11	-0.431	-3.445**	283	22	-0.414	-3.865***

Note: The tests are performed on first differences for i^{s*} and i^{l*} , and on annual changes on seasonally unadjusted data for g , p^* and m . T is the number of observations included in the test. *(**)[***] indicates that $H_0: Z \sim I(k)$ where $k > 1$ is rejected at the 10 (5) [1] percent significance level. McKinnon critical values are used.

The ADF tests above have shown that the dependent variables involved in the regression (11) are stationary, but that every explanatory variable except D is non-stationary. Consequently, we have the undesirable unbalanced regression case, where some variables involved are stationary and some non-stationary. But, if one trusts in (11), then it is also a fact that changes in a set of $I(1)$ variables cannot explain changes in a $I(0)$ variable in the long run, unless the $I(1)$ variables cointegrate, meaning that certain linear combinations of the non-stationary explanatory variables are indeed stationary. Thus in order to remedy the unbalanced regression problems prior to estimation, we first seek to find the stationary cointegrating relationships between p^* , i^{s*} , i^{l*} , g and m , the error-correction terms, which can be included in the regressions in (11) to represent the long run dynamics of the different variables. Secondly, in order to incorporate elements that describe the short-run development, the stationary differences of these variables are also included in

²² Furthermore, note that $1 - L^4 = (1 - L)(1 + L + L^2 + L^3)$, which shows that an analysis based on annual changes can be regarded as an analysis based on first differences on seasonally adjusted data.

²³ The $I(1)$ tests are not executed for i^s and i^l , since these variables are not individually involved in the regressions.

the regressions.

Table 5: Test results for the number of cointegrating vectors.

<i>r</i>	Trace test				Maximum eigenvalue test			
	Test statistic		Critical values		Test statistic		Critical values	
	Quarterly	Monthly	95%	99%	Quarterly	Monthly	95%	99%
0	125.54	105.81	69.98	77.91	58.28	50.90	33.26	38.86
1	67.26	54.90	48.42	55.55	36.96	32.93	27.34	32.62
2	30.30	21.98	31.26	37.29	17.37	15.16	21.28	26.15
3	12.93	6.82	17.84	21.96	8.91	5.67	14.60	18.78
4	4.02	1.15	8.08	11.58	4.02	1.15	8.08	11.58

Note: The samples consist of 53 and 166 observations respectively. The critical values are taken from Table A2 in Johansen and Juselius (1990).

A methodology to test for the number of cointegrating relationships between the variables p^* , i^{s*} , i^{l*} , g and m has been developed by Johansen (1988). If we define a VAR(p) model with p^* , i^{s*} , i^{l*} , g and m included, the analysis of the number and the shape of the cointegrating vectors in the Johansen procedure is based on the solution of a certain eigenvalue problem in the model. However, the first part of the analysis consists of determining the best value of p , the number of lags in the model. Based on the properties of the estimated residuals (normality and lack of serial correlation) in the different VAR equations, $p = 4$ and $p = 7$ were chosen for quarterly and monthly data frequencies respectively. A sensitivity analysis showed that other values of p did not lead to greatly different results for either quarterly or monthly data.

The overall impression from Table 5, using the two standard tests discussed in more detail by Johansen and Juselius (1990), is that two cointegration vectors exist. That is, $r = 2$ on both the quarterly and monthly frequency. The estimated normalized quarterly error correction terms are

$$\hat{u}_{1,t}^Q = 5.169 + p_t^* + 0.524i_t^{l*} + 4.494g_t - 6.556m_t,$$
$$\hat{u}_{2,t}^Q = 33.020 + i_t^{s*} - 2.325i_t^{l*} - 52.808g_t + 40.852m_t,$$

and the corresponding monthly terms are

$$\hat{u}_{1,t}^M = 914.741 + p_t^* + 18.338i_t^{l*} + 60.263g_t - 334.933m_t,$$
$$\hat{u}_{2,t}^M = -329.102 + i_t^{s*} - 8.459i_t^{l*} - 45.373g_t + 143.087m_t.$$

In the estimation of the cointegrating vectors, the error-correction terms, I have followed the practical guidelines proposed in Hamilton (1994) and included a constant term since the variables considered follow linear trends and/or have means different from zero.²⁴ The estimated coefficients in the quarterly and monthly error-correction terms have the same signs and structure, which is good. However, as noted in Section 3.3, the theoretical model in (11) does not provide any guidance as to what the estimated coefficients should be, and it is therefore pointless to discuss them here.

5.2 Econometric Issues

The estimated regression equations for the short- and long-term interest rate differentials include the stationary error correction terms as defined above, a lag polynomial in the stationary variable D , and lag polynomials in the stationary differences for the $I(1)$ variables. Since the coefficient sums on the regressors in (11) have appropriate probability limits only if enough lagged values are included in the regressions, one should not be too parsimonious. On the other hand, the more extraneous regressors included, the less power there is to test hypotheses. These two competing considerations have been balanced by including lagged values up to 3 years. The insignificant lagged values of each variable were then removed so that the most important dynamics were captured in the final estimated equations.²⁵ To give an indication of the estimated model's goodness of fit, the adjusted sample coefficients of determination, \bar{R}^2 , are provided. In general, the econometric methodology employed here is in line with the "general to specific" approach in econometrics suggested by David F. Hendry.

Before turning to the estimation results presented in Tables 6 and 7, a comment on the method used in the estimations is in order. First, estimation with OLS is based on the assumptions that the error term is uncorrelated with the regressors and that the regressors are weakly exogenous with respect to the dependent variables. However, it is easy to argue

²⁴ According to Hamilton, one could also consider a linear trend in the cointegrating vector. But when I included a linear trend, it did not change r and the results reported in Section 5.3 were unaffected. Therefore, and in accordance with the model in (11), the specification without a trend was finally chosen.

²⁵ In the estimations, I use Almon lags with no end point restrictions and allow for a third degree polynomial. The lag in effect may, by this procedure, be distributed as a straight line, a parabola or an "s-curve".

that aggregate money, demand and supply shocks, contained in the residual ν_t^r , also have contemporaneous effects on the regressors. For example, consider a positive aggregate demand shock. Nominal interest rates and output rise simultaneously. The increased output reduces some components of government spending, increases tax revenue and thus lessens the budget deficit. Furthermore, the monetary authorities may accommodate some of the increased money demand that the higher spending induces. As a result, $i^s - i^{s*}$ and $i^l - i^{l*}$ rise while g and D are falling and m is rising endogenously. As a consequence of this, the OLS estimates of the coefficient sums in (11) are very likely to be inconsistent. The inconsistencies can be serious and of either sign, depending on how important each source of endogeneity is. In this paper, I have overcome this problem in three ways. First, for the sample period considered, I think it is reasonably fair to say that exogenous influences have been important for g and D . Second, relatively high frequency data have been used so that the endogeneity effects from the shocks in the residual on the regressors are likely to be relatively small. Moreover, the regressions have been estimated with the Two-Stage Least Squares (2SLS) method with correction for serial correlation suggested by Fair (1970). Fair shows that consistent estimates can be obtained when the residuals are serially correlated, if lagged values of the regressors and the dependent variable are used as instruments and the estimated residual is explicitly modelled as an ARMA(p, q) process. The reason not to use 2SLS without serial correction is, as discussed in Section 3.3, that we expect the residuals to be serially correlated. Therefore, an augmented ARMA(p, q)-process $\nu_t^r = \rho_1^{\nu^r} \nu_{t-1}^r + \dots + \rho_p^{\nu^r} \nu_{t-p}^r + \varepsilon_t^{\nu^r} + \theta_1^{\nu^r} \varepsilon_{t-1}^{\nu^r} + \dots + \theta_q^{\nu^r} \varepsilon_{t-q}^{\nu^r}$ was included in the 2SLS estimations of (11), until the Ljung-Box (LB) statistic indicated absence of serial correlation in the residuals.²⁶

Finally, since I have a limited number of observations in the regressions, I have simulated the critical values reported in Tables 6 and 7 below to get the correct small sample significance levels. In the simulations, I first estimated and then simulated (10) on quarterly and monthly data to get a sample of the same size as used in the regressions reported

²⁶ Another estimation method which produces consistent estimates of the coefficient sums in (11) is the so called Two-Step Two-Stage Least Squares (2S2SLS) method proposed by Cumby et al. (1983). However, when I tested 2S2SLS and the 2SLS method with correction for serial correction, I found that the results were very similar. But since the latter method was much simpler to implement, due to the fact that coefficient sums were estimated with Almon lags, it was used in the final regressions.

in Tables 6 and 7 for the independent variables, then which were used to generate $i^s - i^{s*}$ and $i^l - i^{l*}$. I then used the simulated dependent and independent variables to estimate the regressions in the Tables 6 and 7. To get small sample distributions for the coefficient sums, I repeated this procedure until the simulated distributions converged in mean and variance.²⁷ To get a feeling for the importance of the small sample significance levels, the asymptotic t -statistics are also provided in parentheses.

5.3 Results

Table 6 reports that the coefficient sums for D are indeed positive and strongly statistically significant on the quarterly frequency. The estimated coefficient sums are 0.19 and 0.24, suggesting that a one percentage unit increase in the government budget deficit as a ratio of GDP leads to an increase in the short- and long-term nominal interest rate differentials by 0.19 and 0.24 percentage points respectively after two years' time. These figures are close to point estimates reported by Corriea-Nunes and Stemitsiotis (1995) for Japan (0.21), Germany (0.22) and Ireland (0.22), but are lower than their estimate for the United States (0.79) using yearly data.

Among the other regressors, the short-run dynamics for g and p^* are most important, although their estimated parameters have opposite signs. The error correction terms are highly significant in the regression for $i^s - i^{s*}$, but not in the regression for $i^l - i^{l*}$. A dummy variable has also been included in the regression for the short-term interest rate differential to capture the effects of the interventions of Sveriges Riksbank (Bank of Sweden) on the market for short-term bills in September to November 1992. This "intervention effect" is easily seen in Figure 3.

On the monthly frequency, as seen from Table 7, the estimated coefficient sums for D are still positive and highly significant in the regression equation for $i^s - i^{s*}$. The coefficient sums for D in the regressions for $i^s - i^{s*}$ on quarterly and monthly data are also relatively similar. But, in the $i^l - i^{l*}$ regression, the coefficient sum for D is not statistically different from zero, and is thus significantly lower than on quarterly data.

²⁷ In practice, it took approximately 1000 repetitions on both the monthly and quarterly frequency for the simulated distributions to converge according to the mean-variance criteria.

Table 6: Quarterly 2SLS with correction for serial correlation regressions.

	$i^s - i^{s*}$		$i^l - i^{l*}$	
	Coefficient sum	Lag length	Coefficient sum	Lag length
Δg	- 0.292*** (-3.95)	0	- 0.129*** (-2.68)	0
Δp^*	1.246*** (3.93)	0	0.814** (3.93)	10
Δm	- 0.190 (-3.25)	12	0.043* (1.37)	0
Δi^{s*}	- 0.770 (-0.62)	12	2.400*** (2.47)	9
Δi^{l*}	0.227* (0.88)	0	- 10.669*** (-5.16)	10
D	0.188** (4.35)	8	0.236*** (6.09)	8
$\hat{u}_{1,-12}^M$	2.517*** (4.22)		0.381 (0.46)	
$\hat{u}_{2,-12}^M$	0.828** (5.88)		0.132 (1.01)	
c	- 3.244*** (-2.58)		- 1.851 (-2.30)	
<i>Dummy</i>	1.623*** (4.07)			
p, q	1,4		0,0	
\bar{R}^2	0.92		0.88	

Note: Simulated critical limits. c denotes the constant term and *Dummy* is a dummy variable equal to 1 1992:3 - 1992:4 and 0 otherwise. * (**) [***] indicates that the coefficient is statistically significant at the 10 (5) [1] percent level according to the simulated distribution. Asymptotic t -statistics within parentheses. The samples consist of 44 and 47 observations, respectively. Lagged dependent and explanatory variables have been used as instruments. p and q denote the order of the ARMA(p, q) process for the residual in the estimations.

However, the null hypothesis that the coefficient sum for D is less than zero can be statistically rejected at the ten percent level.

Comparison of the Tables 6 and 7 also reveals that the lag length effect of D is the same in both the quarterly and monthly regressions. For the other variables, the most pronounced difference is that the estimated coefficient for g is positive, in contrast to the quarterly regressions. This may be an indication that our interpolation measure of G is flawed on the monthly frequency. Unlike the quarterly regressions, the estimated coefficient sums for m are now always negative and statistically significant. We also see that i^{s*} and i^{l*} now become highly significant in the regression for $i^s - i^{s*}$, while the short-run dynamics for p^* are not statistically significant. The estimated parameters for the error correction terms have opposite signs, but are not significant. Finally, the estimated parameter for the dummy variable is higher, since the variable can be defined in a more appropriate way with monthly data.

Table 7: Monthly 2SLS with correction for serial correlation regressions.

	$i^s - i^{s*}$		$i^l - i^{l*}$	
	Coefficient sum	Lag length	Coefficient sum	Lag length
Δg	0.117*** (3.14)	0	0.010*** (1.26)	0
Δp^*	0.259 (1.06)	11	- 0.116 (-0.66)	24
Δm	- 0.182*** (-0.89)	4	- 0.187** (-2.67)	22
Δi^{s*}	8.545*** (3.60)	24	4.179** (1.87)	24
Δi^{l*}	- 9.773*** (-2.86)	16	- 18.342*** (-3.67)	34
D	0.134*** (2.98)	24	0.070 (1.76)	24
$\hat{u}_{1,-12}^M$	- 0.056 (-0.60)		0.038 (1.22)	
$\hat{u}_{2,-12}^M$	- 0.126 (-0.56)		0.102 (1.38)	
c	1.084 (1.22)		1.945* (2.81)	
<i>Dummy</i>	3.680*** (7.36)			
p, q	0,1		0,3	
\bar{R}^2	0.76		0.89	

Note: Simulated critical limits. c denotes the constant term and *Dummy* is a dummy variable equal to 1 1992:09 and 1992:11, 0 otherwise. * (**) [***] indicates that the coefficient is statistically significant at the 10 (5) [1] percent level according to the simulated distribution. Asymptotic t -statistics within parentheses. The samples consist of 148 in both regressions. Lagged dependent and explanatory variables have been used as instruments. p and q denote the order of the ARMA(p, q) process for the residual in the estimations.

What is then the general impression of the estimation results of the conventional macro model reported in Tables 6 and 7? First, we notice that the model in (11) did not suggest sign uniqueness of the coefficient sums estimated above. Consequently, one cannot reject the conventional macro model either on the basis that the estimated coefficient sums for the variables which capture the short run dynamics were not statistically significant or because they have the “wrong” sign. Second, if the parameters ρ^{p*} , $\rho^{i^{s*}}$, $\rho^{i^{l*}}$, ρ^g and ρ^m were equal to one, the simplified version of the conventional model, summarized by (12), would suggest that the variables p^* , i^{s*} , i^{l*} , g and m should not have any long-run influence on $i^s - i^{s*}$ and $i^l - i^{l*}$. In the ADF tests reported above, we could not reject the null hypothesis that these parameters were integrated of order one.²⁸ Thus, the fact that the error correction terms in most cases turned out to be insignificant does not form

²⁸ Formally, the fact that these variables were found to be integrated of order one does not imply that they are random walks. But if we consider ρ^{p*} , $\rho^{i^{s*}}$, $\rho^{i^{l*}}$, ρ^g and ρ^m to be lag polynomials in the lag operator with $\rho^{p*}(1) = 1, \dots, \rho^m(1) = 1$, the interpretations made in the random walk case are still valid.

a basis for rejection of the conventional model either. Rather, I would like to argue that the goodness of fit criterion for the model should be used to evaluate the model as a whole. The \bar{R}^2 values reported in Tables 6 and 7 are high, but we need a comparison with an alternative model, in order to get a measure of the model's within-sample forecasting accuracy. Here, I followed the approach in Meese and Rogoff (1983), and used the \bar{R}^2 values generated by a random walk with a drift to form a basis for a comparison. The corresponding \bar{R}^2 values for the short- and long-term interest differentials on quarterly and monthly frequency were $\{0.17, 0.58\}$ and $\{0.50, 0.86\}$ respectively. In all cases they are lower than the ones in Tables 6 and 7. Thus, it is tempting to argue that the empirical evidence presented here also supports the conventional model in general; at least, the empirical results are not obviously inconsistent with the predictions of the conventional model.

In accordance with many other countries, Sweden went from a fixed to a managed floating exchange rate regime in November 1992. In this paper, I have used data from both the fixed and floating regimes to get sufficiently many observations in the regressions. Therefore, it is desirable to test whether the structures of the regressions reported in Tables 6 and 7 are the same after the regime shift. The standard test available for this purpose is the Chow test, the basic idea of which can be described as comparing the results of separate estimation in the two subperiods, fixed and floating regime periods, and on the basis of the complete period; in the latter case assuming that the structure of the model is unchanged.²⁹

As can be seen from Table 8, we can only reject the null hypothesis of an unchanged structure in one out of four regressions at reasonable significance levels. In no cases is the test statistic very large, indicating rather moderate changes. On the whole therefore, it

²⁹ Since the original Chow-test is impossible to use in our case due to the short floating exchange rate regime period, I have used a modification of the test, sometimes called the Chow forecast test. If the

model structure is unchanged, the statistic $F^{obs} = \frac{\hat{\sigma}_T^2 + (\hat{\sigma}_T^2 - \hat{\sigma}_{f_i}^2) \frac{(n_{f_i} - k)}{n_{f_i}}}{\hat{\sigma}_{f_i}^2}$ follows the F -distribution with n_{f_i} and $n_{f_i} - k$ degrees of freedom, where n_{f_i} = number of observations in the fixed exchange rate regime period, n_{f_i} = number of observations in the managed floating exchange rate regime period, k = number of estimated parameters, $\hat{\sigma}_T^2$ = estimated residual variance in the complete period and $\hat{\sigma}_{f_i}^2$ = estimated residual variance in the fixed exchange rate regime period. If the residual variance is unchanged, the value of F^{obs} is 1. A change in structure should lead to a large residual variance for the complete period, with a consequent F^{obs} that is larger than 1.

Table 8: Chow test for structural stability.

Test statistic	Quarterly regressions		Monthly regressions	
	$i^s - i^{s*}$	$i^l - i^{l*}$	$i^s - i^{s*}$	$i^l - i^{l*}$
F^{obs}	2.215	1.425	1.207	1.556
p -value	0.116	0.272	0.232	0.045

Note: n_{fi} is equal to 14 (1993Q1 – 1996Q2) on quarterly data and 43 (1992:12 – 1996:06) on monthly data, while n_{fi} is equal to 30 (1985Q3 – 1992Q4) and 33 (1984Q4 – 1992Q4) on quarterly data and 105 (1984:03 – 1992:11) on monthly data for $i^s - i^{s*}$ and $i^l - i^{l*}$ respectively.

seems as though the results reported in Tables 6 and 7 are fairly robust with respect to the exchange rate regime shift in Sweden.

6 Concluding remarks

In this paper, I have tried to shed light upon the empirical relation between nominal interest rates and government budget deficits. The strategy employed is similar to that of Evans (1985, 1987a, 1987b, 1988) in the sense that I have used a conventional macro model as my point of departure for the empirical investigation. But on the basis of a survey, I have also taken into account what seem to be the most important lessons from the previous empirical literature.

The survey suggests two factors that may account for the different empirical results in the previous literature. First of all, the treatment of the expected inflation rate seems to be of considerable importance; the results tend to be more supportive for the conventional view when a proxy is used to account for the expected inflation rate, rather than a reduced form. Second, the data frequency seems to be important. In studies which have exploited lower frequency data, the evidence is more in favor of the conventional than the Ricardian view. In order to control for the first factor, I have constructed and used a conventional model in which it is possible to solve for the rational inflation expectations analytically. To take the latter factor into proper account, I have used both quarterly and monthly data in the estimations.

The theoretical analysis shows that the conventional macro model developed here, in contrast to the findings in the previous literature, does not imply sign uniqueness for the sum of the elements in the parameter polynomials for the budget deficit in the regression

equations for the short- and long-term nominal interest rates. However, in the special case when the budget deficit is assumed to follow an $AR(p)$ process, the model implies that the coefficient sum of the elements in the parameter polynomials for the budget deficit should be positive, although individual elements may be negative, given that the budget deficit is a stationary process. Thus, the model offers three important insights for empirical investigations of this issue. First, it stresses the importance of a careful determination of the number of lags for the regressors; if an insufficient number of lags for the government budget deficit are included in the regressions, the estimated coefficient sums may be close to zero. Second, if the persistence in the budget deficit is sufficiently high, the estimated coefficient sums for the budget deficit will be close to zero regardless of how many lags one includes in the estimations; however, it should be emphasized that this is a (small) open economy result. Third, it is essentially an empirical question as to whether larger budget deficits are associated with higher interest rates or not. Consequently, the lack of a robust finding between budget deficits and interest rates should not necessarily be interpreted as evidence against the conventional view and indirect support for the Ricardian equivalence theorem, as claimed, for instance, by Evans (1987a) and Plosser (1987).

The empirical study utilizes data for Sweden, a small open economy with extremely high sample variability for the government budget deficit compared to previous studies. Thus, the empirical results here ought to be more reliable than those of previous studies. The results presented in the paper, which seem to be fairly robust over time, provide evidence for the conventional view in macroeconomics: larger government budget deficits produce higher nominal interest rates!

Appendix A Theoretical derivations

A.1 Derivation of the expected price level

Leading (2) one and two periods, taking E_t and differences, one derives

$$\lambda_1 \Delta E_t r_{t+2}^l = \lambda_4 \Delta E_t s_{t+2} + \Delta E_t X_{t+2} - \Delta E_t p_{t+2}$$

since $E_t y_{t+j} = 0$ for all $j = 1, 2, \dots$. Substituting (6), (4), (3) and (8) into the expression above, yields the following difference equation in the expected price level

$$\begin{aligned} E_t p_{t+1} = & -d_1 \Delta E_t X_{t+2} + d_1 \lambda_4 E_t i_{t+1}^{s*} + d_2 E_t m_{t+1} - \\ & \frac{d_2 \lambda_1}{\lambda_1 + 2\lambda_4} E_t m_{t+3} + d_2 \gamma E_t p_{t+2} + \frac{d_1 \lambda_1}{2d_2 \gamma} E_t p_{t+3} - \frac{d_1 \lambda_1}{2} E_t p_{t+4} \end{aligned} \quad (A.1)$$

where by definition $d_1 \equiv \frac{2\gamma}{(1+\gamma)(\lambda_1+2\lambda_4)} > 0$ and $d_2 \equiv \frac{1}{1+\gamma} > 0$. Since γ , λ_1 and λ_4 are assumed to be positive, (A.1) converges forward if speculative bubbles are ruled out. It is then straightforward to show that the stable solution for the expected price level is given by

$$E_t p_{t+1} = \sum_{j=0}^{\infty} \psi_j^{p,X} E_t X_{t+1+j} + \frac{1}{1+\gamma} \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_t m_{t+1+j} + \sum_{j=0}^{\infty} \psi_j^{p,s*} E_t i_{t+1+j}^{s*} \quad (A.2)$$

where

$$\begin{aligned} \psi_0^{p,X} &\equiv d_1 > 0, \psi_1^{p,X} \equiv d_2 \gamma \psi_0^{p,X} - \psi_0^{p,X} < 0, \psi_2^{p,X} \equiv d_2 \gamma \psi_1^{p,X} + \frac{d_1 \lambda_1}{2d_2 \gamma} \psi_0^{p,X} \geq 0, \\ \psi_3^{p,X} &\equiv d_2 \gamma \psi_2^{p,X} + \frac{d_1 \lambda_1}{2d_2 \gamma} \psi_1^{p,X} - \frac{d_1 \lambda_1}{2} \psi_0^{p,X}, \dots, \psi_j^{p,X} = d_2 \gamma \psi_{j-1}^{p,X} + \frac{d_1 \lambda_1}{2d_2 \gamma} \psi_{j-2}^{p,X} - \frac{d_1 \lambda_1}{2} \psi_{j-3}^{p,X}, \\ \psi_0^{p,s*} &\equiv d_1 \lambda_4 > 0, \psi_1^{p,s*} \equiv d_2 \gamma \psi_0^{p,s*} > 0, \psi_2^{p,s*} \equiv d_2 \gamma \psi_1^{p,s*} + \frac{d_1 \lambda_1}{2d_2 \gamma} \psi_0^{p,s*} > 0, \\ \psi_3^{p,s*} &\equiv d_2 \gamma \psi_2^{p,s*} + \frac{d_1 \lambda_1}{2d_2 \gamma} \psi_1^{p,s*} - \frac{d_1 \lambda_1}{2} \psi_0^{p,s*}, \dots, \psi_j^{p,s*} = d_2 \gamma \psi_{j-1}^{p,s*} + \frac{d_1 \lambda_1}{2d_2 \gamma} \psi_{j-2}^{p,s*} - \frac{d_1 \lambda_1}{2} \psi_{j-3}^{p,s*}. \end{aligned}$$

A.2 Solution for the nominal exchange rate

Insert (5), (9) and (1) into (2) for r_t^l and p_t to get

$$\begin{aligned} y_t = & \frac{\beta(\lambda_1 + 2\lambda_4)}{\lambda_1 + 2(\beta + \lambda_4)} s_t + \frac{2\beta}{\lambda_1 + 2(\beta + \lambda_4)} (X_t - \lambda_1 i_t^{l*}) - \\ & \frac{\beta(\lambda_1 + 2\lambda_4)}{\lambda_1 + 2(\beta + \lambda_4)} E_{t-1} p_t + \frac{\beta \lambda_1}{\lambda_1 + 2(\beta + \lambda_4)} E_t p_{t+2} + \\ & \frac{\lambda_1 + 2\lambda_4}{\lambda_1 + 2(\beta + \lambda_4)} \varepsilon_t^{AS} + \frac{2\beta}{\lambda_1 + 2(\beta + \lambda_4)} \varepsilon_t^{IS} - \frac{\beta \lambda_1}{\lambda_1 + 2(\beta + \lambda_4)} E_t s_{t+2}. \end{aligned}$$

By substituting (8) and (1) for i_t^s and p_t into (3) and rearranging, we have

$$y_t = -\frac{\beta\gamma}{1+\alpha\beta}s_t + \frac{\beta}{1+\alpha\beta}(m_t - E_{t-1}p_t) + \frac{\beta\gamma}{1+\alpha\beta}(i_t^{s*} + E_t s_{t+1}) + \frac{\beta}{1+\alpha\beta}\left(\frac{\varepsilon_t^{AS}}{\beta} - \varepsilon_t^{LM}\right).$$

Combining the two expressions above gives the solution for s_t

$$\begin{aligned} s_t = & -\frac{2(1+\alpha\beta)}{d_3}X_t + \frac{\lambda_1+2(\beta+\lambda_4)}{d_3}m_t + \frac{\gamma(\lambda_1+2(\beta+\lambda_4))}{d_3}i_t^{s*} + \\ & \frac{2(1+\alpha\beta)\lambda_1}{d_3}i_t^{l*} + \frac{(\alpha(\lambda_1+2\lambda_4)-2)\beta}{d_3}E_{t-1}p_t - \frac{(1+\alpha\beta)\lambda_1}{d_3}E_t p_{t+2} + \frac{2-\alpha(\lambda_1+2\lambda_4)}{d_3}\varepsilon_t^{AS} - \frac{2(1+\alpha\beta)}{d_3}\varepsilon_t^{IS} - \\ & \frac{\lambda_1+2(\beta+\lambda_4)}{d_3}\varepsilon_t^{LM} + \underbrace{\frac{\gamma(\lambda_1+2(\beta+\lambda_4))}{d_3}E_t s_{t+1}}_{\equiv \phi_1^s > 0} + \underbrace{\frac{(1+\alpha\beta)\lambda_1}{d_3}E_t s_{t+2}}_{\equiv \phi_2^s > 0} \end{aligned} \quad (A.3)$$

as a second order difference equation where $d_3 \equiv (1+\alpha\beta+\gamma)(\lambda_1+2\lambda_4)+2\gamma\beta$. By ruling out speculative bubbles, it can be verified that (A.3) always converges forward provided that $\beta \geq 1$ and $0 < \alpha \leq 5.5$ for all $\{\gamma, \lambda_1, \lambda_4\} \in R_{++}^3$. Straightforward recursions on (A.3), using $E_t \varepsilon_{t+j}^{AS} = E_t \varepsilon_{t+j}^{IS} = E_t \varepsilon_{t+j}^{LM} = 0$ for all $j > 0$, then gives the stable solution for s_t as

$$\begin{aligned} s_t = & k - \sum_{j=0}^{\infty} \psi_j^{s,X} E_t X_{t+j} + \sum_{j=0}^{\infty} \psi_j^{s,m} E_t m_{t+j} + \gamma \sum_{j=0}^{\infty} \psi_j^{s,m} E_t i_{t+j}^{s*} + \\ & \lambda_1 \sum_{j=0}^{\infty} \psi_j^{s,X} E_t i_{t+j}^{l*} + \psi_0^{s,p} E_{t-1} p_t + \sum_{j=1}^{\infty} \psi_j^{s,p} E_t p_{t+j} - \frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} - \psi_0^{s,X} \varepsilon_t^{IS} - \psi_0^{s,m} \varepsilon_t^{LM} \end{aligned} \quad (A.4)$$

where by construction

$$\psi_0^{s,X} \equiv \frac{2(1+\alpha\beta)}{d_3} > 0, \psi_1^{s,X} \equiv \phi_1^s \psi_0^{s,X} > 0, \psi_2^{s,X} \equiv \phi_1^s \psi_1^{s,X} + \phi_2^s \psi_0^{s,X} > 0, \dots,$$

$$\psi_j^{s,X} = \phi_1^s \psi_{j-1}^{s,X} + \phi_2^s \psi_{j-2}^{s,X} > 0 \quad \forall j \geq 2,$$

$$\psi_0^{s,m} \equiv \frac{\lambda_1+2(\beta+\lambda_4)}{d_3} > 0, \psi_1^{s,m} \equiv \phi_1^s \psi_0^{s,m} > 0, \psi_2^{s,m} \equiv \phi_1^s \psi_1^{s,m} + \phi_2^s \psi_0^{s,m} > 0, \dots,$$

$$\psi_j^{s,m} = \phi_1^s \psi_{j-1}^{s,m} + \phi_2^s \psi_{j-2}^{s,m} > 0 \quad \forall j \geq 2,$$

$$\psi_0^{s,p} \equiv \frac{(\alpha(\lambda_1+2\lambda_4)-2)\beta}{d_3} \geq 0, \psi_1^{s,p} \equiv \phi_1^s \psi_0^{s,p} \geq 0, \psi_2^{s,p} \equiv \phi_1^s \psi_1^{s,p} - \frac{(1+\alpha\beta)\lambda_1}{d_3} + \phi_2^s \psi_0^{s,p} \geq 0,$$

$$\psi_3^{s,p} = \phi_1^s \psi_2^{s,p} + \phi_2^s \psi_1^{s,p} \geq 0, \dots, \psi_j^{s,p} = \phi_1^s \psi_{j-1}^{s,p} + \phi_2^s \psi_{j-2}^{s,p} \geq 0 \quad \forall j \geq 3.$$

Combining (A.4) and (A.2) together with the definitions above gives

$$s_t = -\sum_{j=0}^{\infty} \psi_j^{s,X} E_t X_{t+j} + \sum_{n=1}^{\infty} \psi_n^{s,p} \sum_{j=0}^{\infty} \psi_j^X E_t X_{t+n+j} + \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^X E_{t-1} X_{t+j} + \quad (A.5)$$

$$\begin{aligned}
& \sum_{j=0}^{\infty} \psi_j^{s,m} E_t m_{t+j} + d_2 \sum_{n=1}^{\infty} \psi_n^{s,p} \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_t m_{t+n+j} + d_2 \psi_0^{s,p} \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_{t-1} m_{t+j} + \\
& \gamma \sum_{j=0}^{\infty} \psi_j^{s,m} E_t i_{t+j}^{s*} + \sum_{n=1}^{\infty} \psi_n^{s,p} \sum_{j=0}^{\infty} \psi_j^{s*} E_t i_{t+n+j}^{s*} + \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{s*} E_{t-1} i_{t+j}^{s*} + \\
& \lambda_1 \sum_{j=0}^{\infty} \psi_j^{s,X} E_t i_{t+j}^{l*} - \left(\frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s,X} \varepsilon_t^{IS} + \psi_0^{s,m} \varepsilon_t^{LM} \right).
\end{aligned}$$

the solution for the nominal exchange rate in (A.5). Thus, similar to (A.2), the nominal exchange rate in period t is the discounted sum of all as of t and $t-1$ expected future nominal money supplies, foreign price levels and short and long-term nominal interest rates, government expenditures and budget deficits plus some current disturbances.

A.3 Derivation of the short- and long-term interest rate differentials

From (A.4), the one period expected change in the nominal exchange rate is

$$\begin{aligned}
\Delta E_t s_{t+1} = & - \sum_{j=0}^{\infty} \psi_j^{s,X} \Delta E_t X_{t+1+j} + \sum_{j=0}^{\infty} \psi_j^{s,m} \Delta E_t m_{t+1+j} + \\
& \gamma \sum_{j=0}^{\infty} \psi_j^{s,m} \Delta E_t i_{t+1+j}^{s*} + \lambda_1 \sum_{j=0}^{\infty} \psi_j^{s,X} \Delta E_t i_{t+1+j}^{l*} + \sum_{j=1}^{\infty} \psi_j^{s,p} \Delta E_t p_{t+1+j} + \\
& \psi_0^{s,p} (E_t p_{t+1} - E_{t-1} p_t) + \frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s,X} \varepsilon_t^{IS} + \psi_0^{s,m} \varepsilon_t^{LM}.
\end{aligned} \tag{A.6}$$

Using (A.2) to substitute for $\Delta E_t p_{t+1+j}$, $E_t p_{t+1}$ and $E_{t-1} p_t$ gives

$$\begin{aligned}
\Delta E_t s_{t+1} = & - \sum_{j=0}^{\infty} \psi_j^{s,X} \Delta E_t X_{t+1+j} + \sum_{j=1}^{\infty} \psi_j^{s,p} \sum_{n=0}^{\infty} \psi_n^{p,X} \Delta E_t X_{t+1+j+n} + \\
& \sum_{j=0}^{\infty} \psi_j^{s,m} \Delta E_t m_{t+1+j} + \sum_{j=1}^{\infty} \psi_j^{s,p} d_2 \sum_{n=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^n \Delta E_t m_{t+1+j+n} + \\
& \gamma \sum_{j=0}^{\infty} \psi_j^{s,m} \Delta E_t i_{t+1+j}^{s*} + \sum_{j=1}^{\infty} \psi_j^{s,p} \sum_{n=0}^{\infty} \psi_n^{p,s*} \Delta E_t i_{t+1+j+n}^{s*} + \\
& \lambda_1 \sum_{j=0}^{\infty} \psi_j^{s,X} \Delta E_t i_{t+1+j}^{l*} + \\
& \psi_0^{s,p} \left(\sum_{j=0}^{\infty} \psi_j^{p,X} E_t X_{t+1+j} + \sum_{j=0}^{\infty} \psi_j^{p,s*} E_t i_{t+1+j}^{s*} + d_2 \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_t m_{t+1+j} \right) -
\end{aligned}$$

$$\psi_0^{s,p} \left(\sum_{j=0}^{\infty} \psi_j^{p,X} E_{t-1} X_{t+j} + \sum_{j=0}^{\infty} \psi_j^{p,s^*} E_{t-1} i_{t+j}^* + d_2 \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_{t-1} m_{t+j} \right) + \frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s,X} \varepsilon_t^{IS} + \psi_0^{s,m} \varepsilon_t^{LM},$$

which after some algebraic manipulations can be rewritten as

$$\begin{aligned} \Delta E_t s_{t+1} = & \lambda_2 \sum_{j=0}^{\infty} \psi_j^{i^s,X} E_t g_{t+j} - \lambda_2 \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{p,X} E_{t-1} g_{t+j} + \\ & \lambda_3 \sum_{j=0}^{\infty} \psi_j^{i^s,X} E_t D_{t+j} - \lambda_3 \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{p,X} E_{t-1} D_{t+j} + \\ & \lambda_4 \sum_{j=0}^{\infty} \psi_j^{i^s,X} E_t p_{t+j}^* - \lambda_4 \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{p,X} E_{t-1} p_{t+j}^* + \\ & \sum_{j=0}^{\infty} \psi_j^{i^s,m} E_t m_{t+j} - d_2 \psi_0^{s,p} \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_{t-1} m_{t+j} + \\ & \sum_{j=0}^{\infty} \psi_j^{i^s,s^*} E_t i_{t+j}^* - \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{p,s^*} E_{t-1} i_{t+j}^* + \\ & \sum_{j=0}^{\infty} \psi_j^{i^s,l^*} E_t i_{t+j}^{l^*} + \frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s,X} \varepsilon_t^{IS} + \psi_0^{s,m} \varepsilon_t^{LM} \end{aligned} \quad (A.7)$$

where by definition the ψ^{i^s} -coefficients are

$$\begin{aligned} \psi_0^{i^s,X} &\equiv \psi_0^{s,X} > 0, \psi_1^{i^s,X} \equiv \left(\psi_1^{s,X} - \psi_0^{s,X} \right) - \psi_1^{s,p} \psi_0^{p,X} + \psi_0^{s,p} \psi_0^{p,X}, \\ \psi_2^{i^s,X} &\equiv \left(\psi_2^{s,X} - \psi_1^{s,X} \right) + \left(\psi_1^{s,p} \psi_0^{p,X} - \left(\psi_1^{s,p} \psi_1^{p,X} + \psi_2^{s,p} \psi_0^{p,X} \right) \right) + \psi_0^{s,p} \psi_1^{p,X}, \\ \psi_3^{i^s,X} &\equiv \left(\psi_3^{s,X} - \psi_2^{s,X} \right) + \left(\begin{aligned} & \left(\psi_1^{s,p} \psi_1^{p,X} + \psi_2^{s,p} \psi_0^{p,X} \right) - \\ & \left(\psi_1^{s,p} \psi_2^{p,X} + \psi_2^{s,p} \psi_1^{p,X} + \psi_3^{s,p} \psi_0^{p,X} \right) \end{aligned} \right) + \psi_0^{s,p} \psi_2^{p,X}, \dots, \\ \psi_j^{i^s,X} &= \left(\psi_j^{s,X} - \psi_{j-1}^{s,X} \right) + \left(\psi_1^{s,p} \psi_{j-2}^{p,X} + \psi_2^{s,p} \psi_{j-3}^{p,X} + \dots + \psi_{j-2}^{s,p} \psi_1^{p,X} + \psi_{j-1}^{s,p} \psi_0^{p,X} \right) - \\ & \left(\psi_1^{s,p} \psi_{j-1}^{p,X} + \psi_2^{s,p} \psi_{j-2}^{p,X} + \dots + \psi_{j-1}^{s,p} \psi_1^{p,X} + \psi_j^{s,p} \psi_0^{p,X} \right) + \psi_0^{s,p} \psi_{j-1}^{p,X}, \end{aligned}$$

and

$$\begin{aligned} \psi_0^{i^s,m} &\equiv -\psi_0^{s,m} < 0, \psi_1^{i^s,m} \equiv -\left[\left(\psi_1^{s,m} - \psi_0^{s,m} \right) + d_2 \psi_1^{s,p} - d_2 \psi_0^{s,p} \right], \\ \psi_2^{i^s,m} &\equiv -\left[\left(\psi_2^{s,m} - \psi_1^{s,m} \right) + d_2 \left(\left(\psi_1^{s,p} \frac{\gamma}{1+\gamma} + \psi_2^{s,p} \right) - \psi_1^{s,p} \right) - d_2 \psi_0^{s,p} \frac{\gamma}{1+\gamma} \right], \\ \psi_3^{i^s,m} &\equiv -\left[\left(\psi_3^{s,m} - \psi_2^{s,m} \right) + d_2 \left(\psi_1^{s,p} \left(\frac{\gamma}{1+\gamma} \right)^2 + \psi_2^{s,p} \frac{\gamma}{1+\gamma} + \psi_3^{s,p} \right) - \right. \\ & \quad \left. d_2 \left(\psi_1^{s,p} \frac{\gamma}{1+\gamma} + \psi_2^{s,p} \right) - d_2 \psi_0^{s,p} \left(\frac{\gamma}{1+\gamma} \right)^2 \right], \dots, \end{aligned}$$

$$\psi_j^{i^s, m} = - \left[\begin{aligned} & (\psi_j^{s, m} - \psi_{j-1}^{s, m}) + d_2 \left(\psi_1^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-1} + \psi_2^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-2} + \dots + \psi_{j-1}^{s, p} \left(\frac{\gamma}{1+\gamma} \right) + \psi_j^{s, p} \right) - \\ & d_2 \left(\psi_1^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-2} + \psi_2^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-3} + \dots + \psi_{j-2}^{s, p} \left(\frac{\gamma}{1+\gamma} \right) + \psi_{j-1}^{s, p} \right) - d_2 \psi_0^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-1} \end{aligned} \right]$$

and

$$\begin{aligned} \psi_0^{i^s, s^*} &\equiv -\gamma \psi_0^{s, m} < 0, \psi_1^{i^s, s^*} \equiv - \left[\gamma (\psi_1^{s, m} - \psi_0^{s, m}) + \psi_1^{s, p} \psi_0^{p, s^*} - \psi_0^{s, p} \psi_0^{p, s^*} \right], \\ \psi_2^{i^s, s^*} &\equiv - \left[\gamma (\psi_2^{s, m} - \psi_1^{s, m}) + \left(\psi_1^{s, p} \psi_1^{p, s^*} + \psi_2^{s, p} \psi_0^{p, s^*} \right) - \psi_1^{s, p} \psi_0^{p, s^*} - \psi_0^{s, p} \psi_1^{p, s^*} \right], \\ \psi_3^{i^s, s^*} &\equiv - \left[\gamma (\psi_3^{s, m} - \psi_2^{s, m}) + \left(\psi_1^{s, p} \psi_2^{p, s^*} + \psi_2^{s, p} \psi_1^{p, s^*} + \psi_3^{s, p} \psi_0^{p, s^*} \right) - \right. \\ &\quad \left. \left(\psi_1^{s, p} \psi_1^{p, s^*} + \psi_2^{s, p} \psi_0^{p, s^*} \right) - \psi_0^{s, p} \psi_2^{p, s^*} \right], \dots, \\ \psi_j^{i^s, s^*} &= - \left[\gamma (\psi_j^{s, m} - \psi_{j-1}^{s, m}) + \left(\psi_1^{s, p} \psi_{j-1}^{p, s^*} + \psi_2^{s, p} \psi_{j-2}^{p, s^*} + \dots + \psi_{j-1}^{s, p} \psi_1^{p, s^*} + \psi_j^{s, p} \psi_0^{p, s^*} \right) \right. \\ &\quad \left. - \left(\psi_1^{s, p} \psi_{j-2}^{p, s^*} + \psi_2^{s, p} \psi_{j-3}^{p, s^*} + \dots + \psi_{j-2}^{s, p} \psi_1^{p, s^*} + \psi_{j-1}^{s, p} \psi_0^{p, s^*} \right) + \psi_0^{s, p} \psi_{j-1}^{p, s^*} \right], \end{aligned}$$

and finally

$$\psi_0^{i^s, l^*} \equiv -\lambda_1 \psi_0^{s, X} < 0, \psi_1^{i^s, l^*} \equiv -\lambda_1 \left(\psi_1^{s, X} - \psi_0^{s, X} \right), \dots, \psi_j^{i^s, l^*} \equiv -\lambda_1 \left(\psi_j^{s, X} - \psi_{j-1}^{s, X} \right)$$

for all $j \geq 3$. If (A.7) is substituted into (8), utilizing the definitions

$$\psi_j^{i^s, g} \equiv \lambda_2 \psi_j^{i^s, X}, \psi_j^{i^s, D} \equiv \lambda_3 \psi_j^{i^s, X}, \psi_j^{i^s, p^*} \equiv \lambda_4 \psi_j^{i^s, X},$$

we have the solution for the short-term interest rate differential as

$$\begin{aligned} i_t^s - i_t^{s^*} &= \sum_{j=0}^{\infty} \psi_j^{i^s, g} E_t g_{t+j} - \lambda_2 \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} g_{t+j} + \\ &\quad \sum_{j=0}^{\infty} \psi_j^{i^s, D} E_t D_{t+j} - \lambda_3 \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} D_{t+j} + \\ &\quad \sum_{j=0}^{\infty} \psi_j^{i^s, p^*} E_t p_{t+j}^* - \lambda_4 \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} p_{t+j}^* + \\ &\quad \sum_{j=0}^{\infty} \psi_j^{i^s, m} E_t m_{t+j} - d_2 \psi_0^{s, p} \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_{t-1} m_{t+j} + \\ &\quad \sum_{j=0}^{\infty} \psi_j^{i^s, s^*} E_t i_{t+j}^{s^*} - \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, s^*} E_{t-1} i_{t+j}^{s^*} + \\ &\quad \sum_{j=0}^{\infty} \psi_j^{i^s, l^*} E_t i_{t+j}^{l^*} + \frac{\psi_0^{s, p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s, X} \varepsilon_t^{IS} + \psi_0^{s, m} \varepsilon_t^{LM}. \end{aligned} \tag{A.8}$$

Repeating the procedure above for $E_t s_{t+2} - s_t$ gives

$$\begin{aligned}
 E_t s_{t+2} - s_t &= \lambda_2 \sum_{j=0}^{\infty} \psi_j^{i^l, X} E_t g_{t+j} - \lambda_2 \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} g_{t+j} + \\
 &\quad \lambda_3 \sum_{j=0}^{\infty} \psi_j^{i^l, X} E_t D_{t+j} - \lambda_3 \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} D_{t+j} + \\
 &\quad \lambda_4 \sum_{j=0}^{\infty} \psi_j^{i^l, X} E_t P_{t+j}^* - \lambda_4 \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} P_{t+j}^* + \\
 &\quad \sum_{j=0}^{\infty} \psi_j^{i^l, m} E_t m_{t+j} - d_2 \psi_0^{s, p} \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_{t-1} m_{t+j} + \\
 &\quad \sum_{j=0}^{\infty} \psi_j^{i^l, s^*} E_t i_{t+j}^* - d_2 \psi_0^{s, p} \sum_{j=0}^{\infty} \psi_j^{p, s^*} E_{t-1} i_{t+j}^* + \\
 &\quad \sum_{j=0}^{\infty} \psi_j^{i^l, l^*} E_t l_{t+j}^* + \left(\frac{\psi_0^{s, p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s, X} \varepsilon_t^{IS} + \psi_0^{s, m} \varepsilon_t^{LM} \right),
 \end{aligned} \tag{A.9}$$

where

$$\begin{aligned}
 \psi_0^{i^l, X} &\equiv \psi_0^{s, X} = \psi_0^{i^s, X} > 0, \psi_1^{i^l, X} \equiv \psi_1^{s, X} - \psi_1^{s, p} \psi_0^{p, X}, \\
 \psi_2^{i^l, X} &\equiv \left(\psi_2^{s, X} - \psi_0^{s, X} \right) - \left(\psi_1^{s, p} \psi_1^{p, X} + \psi_2^{s, p} \psi_0^{p, X} \right) + \psi_0^{s, p} \psi_0^{p, X}, \\
 \psi_3^{i^l, X} &\equiv \left(\psi_3^{s, X} - \psi_1^{s, X} \right) + \left(\psi_1^{s, p} \psi_0^{p, X} - \left(\psi_1^{s, p} \psi_2^{p, X} + \psi_2^{s, p} \psi_1^{p, X} + \psi_3^{s, p} \psi_0^{p, X} \right) \right) + \psi_0^{s, p} \psi_1^{p, X}, \dots, \\
 \psi_j^{i^l, X} &= \left(\psi_j^{s, X} - \psi_{j-2}^{s, X} \right) + \left(\psi_1^{s, p} \psi_{j-3}^{p, X} + \psi_2^{s, p} \psi_{j-4}^{p, X} + \dots + \psi_{j-3}^{s, p} \psi_1^{p, X} + \psi_{j-2}^{s, p} \psi_0^{p, X} \right) - \\
 &\quad \left(\psi_1^{s, p} \psi_{j-1}^{p, X} + \psi_2^{s, p} \psi_{j-2}^{p, X} + \dots + \psi_{j-1}^{s, p} \psi_1^{p, X} + \psi_j^{s, p} \psi_0^{p, X} \right) + \psi_0^{s, p} \psi_{j-2}^{p, X},
 \end{aligned}$$

and

$$\begin{aligned}
 \psi_0^{i^l, m} &\equiv -\psi_0^{s, m} = \psi_0^{i^s, m} < 0, \psi_1^{i^l, m} \equiv -[\psi_1^{s, m} + d_2 \psi_1^{s, p}], \\
 \psi_2^{i^l, m} &\equiv -\left[\left(\psi_2^{s, m} - \psi_0^{s, m} \right) + d_2 \left(\psi_1^{s, p} \frac{\gamma}{1+\gamma} + \psi_2^{s, p} \right) - d_2 \psi_0^{s, p} \right], \\
 \psi_3^{i^l, m} &\equiv -\left[\left(\psi_3^{s, m} - \psi_1^{s, m} \right) + d_2 \left(\left(\psi_1^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^2 + \psi_2^{s, p} \frac{\gamma}{1+\gamma} + \psi_3^{s, p} \right) - \psi_1^{s, p} \right) - d_2 \psi_0^{s, p} \frac{\gamma}{1+\gamma} \right], \dots, \\
 \psi_j^{i^l, m} &= -\left[\begin{aligned} &(\psi_j^{s, m} - \psi_{j-2}^{s, m}) - d_2 \psi_0^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-2} + \\ &d_2 \left(\psi_1^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-1} + \psi_2^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-2} + \dots + \psi_{j-1}^{s, p} \left(\frac{\gamma}{1+\gamma} \right) + \psi_j^{s, p} \right) - \\ &d_2 \left(\psi_1^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-3} + \psi_2^{s, p} \left(\frac{\gamma}{1+\gamma} \right)^{j-4} + \dots + \psi_{j-3}^{s, p} \left(\frac{\gamma}{1+\gamma} \right) + \psi_{j-2}^{s, p} \right) \end{aligned} \right],
 \end{aligned}$$

and

$$\psi_0^{i^l, s^*} \equiv -\gamma \psi_0^{s, m} = \psi_0^{i^s, s^*} < 0, \psi_1^{i^l, s^*} \equiv -\left[\gamma \psi_1^{s, m} + \psi_1^{s, p} \psi_0^{p, s^*} \right],$$

$$\begin{aligned}
\psi_2^{i^l, s^*} &\equiv - \left[\gamma (\psi_2^{s, m} - \psi_0^{s, m}) + \left(\psi_1^{s, p} \psi_1^{p, s^*} + \psi_2^{s, p} \psi_0^{p, s^*} \right) - \psi_0^{s, p} \psi_0^{p, s^*} \right], \\
\psi_3^{i^l, s^*} &\equiv - \left[\gamma (\psi_3^{s, m} - \psi_1^{s, m}) + \left(\psi_1^{s, p} \psi_2^{p, s^*} + \psi_2^{s, p} \psi_1^{p, s^*} + \psi_3^{s, p} \psi_0^{p, s^*} \right) - \psi_1^{s, p} \psi_0^{p, s^*} - \psi_0^{s, p} \psi_1^{p, s^*} \right], \dots, \\
\psi_j^{i^l, s^*} &= - \left[\gamma (\psi_j^{s, m} - \psi_{j-2}^{s, m}) + \left(\psi_1^{s, p} \psi_{j-1}^{p, s^*} + \psi_2^{s, p} \psi_{j-2}^{p, s^*} + \dots + \psi_{j-1}^{s, p} \psi_1^{p, s^*} + \psi_j^{s, p} \psi_0^{p, s^*} \right) - \right. \\
&\quad \left. \left(\psi_1^{s, p} \psi_{j-3}^{p, s^*} + \psi_2^{s, p} \psi_{j-4}^{p, s^*} + \dots + \psi_{j-3}^{s, p} \psi_1^{p, s^*} + \psi_{j-2}^{s, p} \psi_0^{p, s^*} \right) - \psi_0^{s, p} \psi_{j-2}^{p, s^*} \right],
\end{aligned}$$

and finally

$$\begin{aligned}
\psi_0^{i^l, l^*} &\equiv -\lambda_1 \psi_0^{s, X} = \psi_0^{i^s, l^*} < 0, \psi_1^{i^l, l^*} \equiv -\lambda_1 \psi_1^{s, X}, \psi_2^{i^l, l^*} \equiv -\lambda_1 (\psi_2^{s, X} - \psi_0^{s, X}), \dots, \\
\psi_j^{i^l, l^*} &\equiv -\lambda_1 (\psi_j^{s, X} - \psi_{j-2}^{s, X}),
\end{aligned}$$

for all $j \geq 4$. If (A.9) is substituted into (9), utilizing the definitions

$$\psi_j^{i^l, g} \equiv \frac{\lambda_2}{2} \psi_j^{i^l, X}, \psi_j^{i^l, D} \equiv \frac{\lambda_3}{2} \psi_j^{i^l, X}, \psi_j^{i^l, p^*} \equiv \frac{\lambda_4}{2} \psi_j^{i^l, X},$$

we have the solution for the long-term interest rate differential as

$$\begin{aligned}
i_t^l - i_t^{l^*} &= \sum_{j=0}^{\infty} \psi_j^{i^l, g} E_t g_{t+j} - \frac{\lambda_2 \psi_0^{s, p}}{2} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} g_{t+j} + \\
&\quad \sum_{j=0}^{\infty} \psi_j^{i^l, D} E_t D_{t+j} - \frac{\lambda_3 \psi_0^{s, p}}{2} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} D_{t+j} + \\
&\quad \sum_{j=0}^{\infty} \psi_j^{i^l, p^*} E_t p_{t+j}^* - \frac{\lambda_4 \psi_0^{s, p}}{2} \sum_{j=0}^{\infty} \psi_j^{p, X} E_{t-1} p_{t+j}^* + \\
&\quad \sum_{j=0}^{\infty} \psi_j^{i^l, m} E_t m_{t+j} - \frac{d_2 \psi_0^{s, p}}{2} \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^j E_{t-1} m_{t+j} + \\
&\quad \sum_{j=0}^{\infty} \psi_j^{i^l, s^*} E_t i_{t+j}^{s^*} - \frac{d_2 \psi_0^{s, p}}{2} \sum_{j=0}^{\infty} \psi_j^{p, s^*} E_{t-1} i_{t+j}^{s^*} + \\
&\quad \sum_{j=0}^{\infty} \psi_j^{i^l, l^*} E_t i_{t+j}^{l^*} + \frac{1}{2} \left(\frac{\psi_0^{s, p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s, X} \varepsilon_t^{IS} + \psi_0^{s, m} \varepsilon_t^{LM} \right).
\end{aligned} \tag{A.10}$$

A.4 Proof of equation (11)

Follows by induction from the derivations in the next section. ■

A.5 Derivation of equation (12)

Inserting (10) (under the simplifying assumption that $\rho^z(L) \equiv \rho^z$, where ρ^z is a diagonal matrix with the elements $[\rho^{p^*} \rho^{s^*} \rho^{l^*} \rho^g \rho^D \rho^m]$ in the diagonal) in (A.2), recognizing that

$E_t \varepsilon_{t+s}^z = 0 \forall s > 0$, gives the solution for the expected price level j periods in the future as

$$E_t p_{t+j} = \frac{2\gamma(1-\rho^X)}{(\lambda_1(1-(\rho^X)^2)+2\lambda_4)(1+\gamma(1-\rho^X))} (\rho^X)^j X_t + \frac{1}{1+\gamma(1-\rho^m)} (\rho^m)^j m_t + \frac{2\gamma\lambda_4}{(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4)(1+\gamma(1-\rho^{s*}))} (\rho^{s*})^j i_t^{s*}. \quad (A.11)$$

Now, after some considerable algebra, it can be shown that (A.11), (10) and (A.5) imply that

$$\begin{aligned} s_t &= -d_4 X_t + d_5 m_t + d_6 i_t^{s*} + d_7 i_t^{l*} + \frac{(\alpha(\lambda_1+2\lambda_4)-2)\beta}{d_3} E_{t-1} p_t \\ &\quad - \psi_0^{s,p} \varepsilon_t^{AS} - \psi_0^{s,X} \varepsilon_t^{IS} - \psi_0^{s,m} \varepsilon_t^{LM} \\ &= -d_4 X_t + d_5 m_t + d_6 i_t^{s*} + d_7 i_t^{l*} + \underbrace{\frac{2(\alpha(\lambda_1+2\lambda_4)-2)\beta\gamma\rho^X(1-\rho^X)}{d_3(\lambda_1(1-(\rho^X)^2)+2\lambda_4)(1+\gamma(1-\rho^X))}}_{\equiv d_8} X_{t-1} + \\ &\quad \underbrace{\frac{(\alpha(\lambda_1+2\lambda_4)-2)\beta\rho^m}{d_3(1+\gamma(1-\rho^m))}}_{\equiv d_9} m_{t-1} + \underbrace{\frac{2(\alpha(\lambda_1+2\lambda_4)-2)\beta\gamma\lambda_4\rho^{s*}}{d_3(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4)(1+\gamma(1-\rho^{s*}))}}_{\equiv d_{10}} i_{t-1}^{s*} - \\ &\quad \frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} - \psi_0^{s,X} \varepsilon_t^{IS} - \psi_0^{s,m} \varepsilon_t^{LM} \end{aligned} \quad (A.12)$$

where by definition

$$\begin{aligned} d_4 &\equiv \frac{2\gamma^2(\lambda_1+2(\beta+\lambda_4))(1-\rho^X)[(1+\alpha\beta(1-\rho^X))(\lambda_1+2\lambda_4)+2\beta\rho^X]}{d_3(\lambda_1(1-(\rho^X)^2)+2\lambda_4)(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^X)+(1+\alpha\beta)(\lambda_1(1-(\rho^X)^2)+2\lambda_4))(1+\gamma(1-\rho^X))} + \\ &\quad \frac{2(1+\alpha\beta)\gamma[(1-\rho^X)(\lambda_1+2\lambda_4)+(1+\alpha\beta\gamma(1-\rho^X))(\lambda_1(1-(\rho^X)^2)+2\lambda_4)(\lambda_1+2\lambda_4)+2\beta(\lambda_1(1-(\rho^X)^3)+2\lambda_4)]}{d_3(\lambda_1(1-(\rho^X)^2)+2\lambda_4)(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^X)+(1+\alpha\beta)(\lambda_1(1-(\rho^X)^2)+2\lambda_4))(1+\gamma(1-\rho^X))}, \\ d_5 &\equiv \frac{(\lambda_1+2(\beta+\lambda_4))[\gamma(1-\rho^m)(d_3+2\beta)+\alpha\beta\gamma\rho^m(\lambda_1(1-\rho^m)+2\lambda_4)+(1+\alpha\beta\gamma)(\lambda_1(1-(\rho^m)^2)+2\lambda_4)]}{d_3(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^m)+(1+\alpha\beta)(\lambda_1(1-(\rho^m)^2)+2\lambda_4))(1+\gamma(1-\rho^m))}, \\ d_6 &\equiv \frac{\gamma^2(\lambda_1+2(\beta+\lambda_4))[d_3(1-\rho^{s*})(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4)+2\beta\lambda_4(2(1-\rho^{s*})+\alpha\rho^{s*}(\lambda_1(1-\rho^{s*})+2\lambda_4))]}{d_3(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4)(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^{s*})+(1+\alpha\beta)(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4))(1+\gamma(1-\rho^{s*}))} + \\ &\quad \frac{2\gamma\lambda_4[2d_3\lambda_4+(1+\alpha\beta)\beta\lambda_4(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4)]}{d_3(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4)(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^{s*})+(1+\alpha\beta)(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4))(1+\gamma(1-\rho^{s*}))} \end{aligned}$$

and

$$d_7 \equiv \frac{2(1+\alpha\beta)\lambda_1}{(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^{s*})+(1+\alpha\beta)(\lambda_1(1-(\rho^{s*})^2)+2\lambda_4))}.$$

From the definitions of d_4 , d_5 , d_6 and d_7 , it follows that they are all positive as long as the elements in ρ^z lie between 0 and 1. However, the signs of d_8 , d_9 and d_{10} are ambiguous,

and depend on whether $\alpha(\lambda_1 + 2\lambda_4) - 2 \gtrless 0$. If $\alpha(\lambda_1 + 2\lambda_4) - 2 > 0$, then they are all positive and vice versa. (A.12) implies that

$$\begin{aligned} \Delta E_t s_{t+1} = & d_4 (1 - \rho^X) X_t - d_5 (1 - \rho^m) m_t - d_6 (1 - \rho^{s*}) i_t^{s*} - \\ & d_7 (1 - \rho^{l*}) i_t^{l*} + d_8 \Delta X_t + d_9 \Delta m_t + d_{10} \Delta i_t^{s*} + \\ & \frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s,X} \varepsilon_t^{IS} + \psi_0^{s,m} \varepsilon_t^{LM} \end{aligned} \quad (\text{A.13})$$

and

$$\begin{aligned} E_t s_{t+2} - s_t = & d_4 \left(1 - (\rho^X)^2\right) X_t - d_5 \left(1 - (\rho^m)^2\right) m_t - \\ & d_6 \left(1 - (\rho^{s*})^2\right) i_t^{s*} - d_7 \left(1 - (\rho^{l*})^2\right) i_t^{l*} + d_8 \rho^X \Delta X_t - d_8 (1 - \rho^X) X_{t-1} + \\ & d_9 \rho^m \Delta m_t - d_9 (1 - \rho^m) m_{t-1} + d_{10} \rho^{s*} \Delta i_t^{s*} - d_{10} (1 - \rho^{s*}) i_{t-1}^{s*} + \\ & \frac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s,X} \varepsilon_t^{IS} + \psi_0^{s,m} \varepsilon_t^{LM}. \end{aligned} \quad (\text{A.14})$$

If we use the implicit assumption that all the parameters are stationary lag polynomials, i.e. $\alpha \equiv \alpha(L)$, $\beta \equiv \beta(L)$ and so forth, together with the definition of X , (A.13) and (A.14) can be rewritten in the following form

$$\begin{aligned} \Delta E_t s_{t+1} = & \delta_1^s(L) g_t + \delta_2^s(L) D_t + \delta_3^s(L) p_t^* - \delta_4^s(L) m_t - \delta_5^s(L) i_t^{s*} - \\ & \delta_6^s(L) i_t^{l*} + \delta_7^s(L) \Delta g_t + \delta_8^s(L) \Delta D_t + \delta_9^s(L) \Delta p_t^* + \delta_{10}^s(L) \Delta m_t + \delta_{11}^s(L) \Delta i_t^{s*} + \\ & \psi_0^{s,p}(L) (\beta(L))^{-1} \varepsilon_t^{AS} + \psi_0^{s,X}(L) \varepsilon_t^{IS} + \psi_0^{s,m}(L) \varepsilon_t^{LM} \end{aligned} \quad (\text{A.15})$$

and

$$\begin{aligned} E_t s_{t+2} - s_t = & \delta_1^l(L) g_t + \delta_2^l(L) D_t + \delta_3^l(L) p_t^* - \delta_4^l(L) m_t - \delta_5^l(L) i_t^{s*} - \\ & \delta_6^l(L) i_t^{l*} + \delta_7^l(L) \Delta g_t + \delta_8^l(L) \Delta D_t + \delta_9^l(L) \Delta p_t^* + \delta_{10}^l(L) \Delta m_t + \delta_{11}^l(L) \Delta i_t^{s*} + \\ & \psi_0^{s,p}(L) (\beta(L))^{-1} \varepsilon_t^{AS} + \psi_0^{s,X}(L) \varepsilon_t^{IS} + \psi_0^{s,m}(L) \varepsilon_t^{LM} \end{aligned} \quad (\text{A.16})$$

where

$$\begin{aligned} \delta_1^s(L) & \equiv d_4(L) (1 - \rho^X) = d_{4,0} (1 - \rho^X) + d_{4,1} (1 - \rho^X) L + \dots + d_{4,p} (1 - \rho^X) L^p, \dots, \\ \delta_{11}^s(L) & \equiv d_{10}(L) = d_{10,0} + d_{10,1} L + \dots + d_{10,p} L^p \end{aligned}$$

and

$$\begin{aligned}
 \delta_1^l(L) &\equiv \underbrace{d_{4,0} \left(1 - (\rho^X)^2\right)}_{\equiv \delta_{1,0}^l} + \left[d_{4,1} \left(1 - (\rho^X)^2\right) - d_{8,0} (1 - \rho^X) \right] L + \\
 &\quad \left[d_{4,2} \left(1 - (\rho^X)^2\right) - d_{8,1} (1 - \rho^X) \right] L^2 + \dots + \\
 &\quad \underbrace{\left[d_{4,p} \left(1 - (\rho^X)^2\right) - d_{8,p-1} (1 - \rho^X) \right] L^p}_{\equiv \delta_{1,p}^l} - \underbrace{d_{8,p} (1 - \rho^X) L^{p+1}}_{\equiv \delta_{1,p+1}^l}, \dots, \\
 \delta_{11}^l &\equiv d_{10}(L) \rho^{s*} = d_{10,0} \rho^{s*} + d_{10,1} \rho^{s*} L + \dots + d_{10,p} \rho^{s*} L^p.
 \end{aligned}$$

Clearly, all the elements in the $\delta_1^s(L), \dots, \delta_6^s(L)$ polynomials in (A.15) are positive if $\{\rho^g, \rho^D, \rho^{p*}, \rho^m, \rho^{i^{s*}}, \rho^{i^{**}}\} \in [0, 1]$ since the $d_{i,j}$ for all $i = 1, \dots, 6$ and $j = 0, \dots, p$ are positive. But, the signs of $\delta_7^s(L), \dots, \delta_{11}^s(L)$ are not uniquely determined and the $\delta_{i,j}^s$ s for all $i = 7, \dots, 11$ and $j = 0, \dots, p$ can be both positive or negative even if $\{\rho^g, \rho^D, \rho^{p*}, \rho^m, \rho^{i^{s*}}\} \in [0, 1]$. The story about the lag polynomials in (A.16) is somewhat different, although it still holds that $\text{sign}(\delta_i^l(L)) = \text{sign}(\delta_i^s(L))$ for all $i = 1, \dots, 11$. The difference stems from the fact that it is no longer certain that all the elements in the $\delta_1^l(L), \delta_2^l(L), \delta_3^l(L), \delta_4^l(L)$ and $\delta_5^l(L)$ polynomials for $L \geq 1$ are positive, which can be directly seen from the definitions of $\delta_1^l(L), \delta_2^l(L), \delta_3^l(L), \delta_4^l(L)$ and $\delta_5^l(L)$ above.

Finally, combining (8) with (A.15) and (9) with (A.16), introducing the definitions $\delta_0^s \equiv \frac{\sigma^2}{2}$, $\delta_0^l \equiv k_t$, $\nu_t^s \equiv \psi_0^{s,p}(L)(\beta(L))^{-1} \varepsilon_t^{AS} + \psi_0^{s,X}(L) \varepsilon_t^{IS} + \psi_0^{s,m}(L) \varepsilon_t^{LM}$ and $\nu_t^l \equiv \frac{1}{2}(\psi_0^{s,p}(L)(\beta(L))^{-1} \varepsilon_t^{AS} + \psi_0^{s,X}(L) \varepsilon_t^{IS} + \psi_0^{s,m}(L) \varepsilon_t^{LM})$, establishes (12) for $r = s, l$.

Appendix B Data sources and definitions

This appendix contains a comprehensive description of the data set utilized in the paper. Below, Table B.1 describes the raw data series and Tables B.2 and B.3 the generation of composite variables on a monthly and quarterly basis.

Table B.1: The raw data set.

Variable	Sample period	Frequency	Source
1,3,6,12 month STB	1982:01-1996:12	daily	Sveriges Riksbank
1,3,6,12 month FEB	1982:01-1992:11	daily	Lindberg and Söderlind (1994)
1,3,6,12 month FEB	1992:12-1996:12	daily	O.c., Sveriges Riksbank
SGB5Y	1984:02-1986:12	monthly	Sveriges Riksbank
SGB10Y	1987:01-1996:12	monthly	OECD MEI
FGB520Y	1980:01-1996:10	monthly	O.c., OECD MEI
CPI	1970:01-1996:10	monthly	IFS
FCPI	1980:01-1996:10	monthly	O.c., Findata
IIP	1960:01-1996:10	monthly	OECD MEI
IIPSA	1960:01-1996:10	monthly	OECD MEI
GDP	1970:1-1979:4	quarterly	SNEPQ-database
GDP	1980:1-1996:2	quarterly	OECD MEI
GDPSA	1980:1-1996:2	quarterly	OECD MEI
PGDP	1970:1-1979:4	quarterly	SNEPQ-database
PGDP	1980:1-1996:2	quarterly	OECD MEI
GC	1970:1-1979:4	quarterly	SNEPQ-database
GC	1980:1-1996:2	quarterly	OECD MEI
GDEBT	1950:01-1996:09	monthly	Swedish National Debt Office
M3	1960:01-1996:10	monthly	OECD MEI

Note: All real macroeconomic variables are measured in 1991 prices. O.c. stands for own calculations. Abbreviations: STB=Yield on Swedish Treasury bills, FEB=Yield on SEK "basket" weighted foreign ECU bills, SGB5Y=Average yields on 5-year Swedish government bonds, SGB10Y=Average yields on 10-year Swedish government bonds, FGB520Y=Average yields on SEK "basket" weighted 5- to 20-year foreign government bonds, CPI=Swedish consumer price index, FCPI=SEK "basket" weighted foreign consumer price index, IIP=Swedish private industrial production index, IIPSA=Seasonally adjusted IIP, GDP=Swedish gross domestic product, GDPSA=Seasonally adjusted GDP, PGDP= Swedish Implicit GDP deflator, GC=Swedish government consumption (investments not included), GDEBT=Nominal value of the Swedish government debt in SEK, M3=Nominal Swedish M3.

Table B.2: Generation of composite data series on quarterly frequency.

Variable	Sample period	Calculation formula
$i^s - i^{s*}$	1982:1 - 1996:4	Average 3 months STB-FEB
$i^l - i^{l*}$	1984:1 - 1986:4	SGB5Y-FGB520Y
$i^l - i^{l*}$	1987:1 - 1996:3	SGB10Y-FGB520Y
g	1970:1 - 1996:2	$\ln((CG/GDP)*100)$
d	1970:1 - 1996:2	$((GDEBT-GDEBT(-4))/(GDP*PGDP))*100$
p^*	1980:1 - 1996:3	$\ln(FPCI)$
m	1970:1 - 1996:2	$\ln((M3/(GDP*PGDP))*100)$

Note: g , d and m are then subject to seasonal adjustment with the X11-method, as described in

Section 4.

Table B.3: Generation of composite data series on monthly frequency.

Variable	Sample period	Calculation formula
$i^s - i^{s*}$	1982:01 - 1996:12	Average 3 months STB-FEB
$i^l - i^{l*}$	1984:02 - 1986:12	SGB5Y-FGB520Y
$i^l - i^{l*}$	1987:01 - 1996:10	SGB10Y-FGB520Y
g	1970:01 - 1996:06	$\ln(((CGMSA)/GDPMSA)*100)$
d	1970:01 - 1996:06	$((GDEBT-GDEBT(-12))/CPI)/(GDPMSA*3))*100$
p^*	1980:01 - 1996:10	$\ln(FPCI)$
m	1970:01 - 1996:06	$\ln(((M3SA/CPI)/(GDPMSA*3))*100)$

Note: CGSA and M3SA denote CG and M3 seasonally adjusted with the X11-method respectively. For the period 1970:1 - 1979:4, GDPMSA are GDP seasonally adjusted with the X11-method and GDPMSA then denotes the monthly GDPMSA figures, generated as described in Section 4. Since the X11-method can only adjust monthly data up to the length of 20 years, the seasonal adjustment of CG and M3 has been divided into the subperiods 1970:01 - 1979:12 and 1980:01 - 1996:06 respectively.

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Essay III

Idiosyncratic Risk in the U.S. and Sweden: Is There a Role for Government Insurance?*

1 Introduction

Two important motivations for government taxation are that it provides insurance of individual specific income variations if private insurance markets are absent, and that it redistributes wealth from those who were born lucky to those who were not. As all feasible tax systems are to some extent distortionary, there is a trade-off between insurance and redistribution on the one hand and efficiency on the other. In some countries, such as Sweden, taxes are considerably higher than in other countries, for example the U.S.; tax receipts are approximately 60 percent of GDP in Sweden and 30 percent of GDP in the U.S. Can these differences in tax levels be motivated by differences in income distributions and income risks? Obviously, there are other reasons for government taxation than those mentioned. A more interesting question is how much government taxation is motivated by insurance and redistribution arguments.

There are two main purposes of this paper. The first is to estimate the degree of

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individual specific income risk in Sweden and the U.S., and the second is to investigate to what extent government insurance via taxes and transfers should be provided. To quantify the degree of idiosyncratic risk in the respective countries, we use micro data on wages and hours worked. The estimated wage processes are then used to parameterize a general equilibrium model, in which labor supply is endogenous and agents are subject to a no-borrowing constraint. We assume that the government uses proportional labor income taxes to redistribute income among agents, and that the government wishes to maximize the ex ante utility of agents.

The wage processes are found to be highly persistent in both countries, especially in the U.S. The variance of temporary as well as permanent wage shocks is also higher in the U.S. Consequently, the wage uncertainty in the U.S. seems to dominate that in Sweden by any measure.

In the absence of tax distortions, it would be optimal for the government to redistribute almost all income equally across agents. However, we find that distortions are significant. When we calibrate the model with the estimated wage processes, the optimal size of government insurance programs amounts to labor income tax rates of 3 percent in Sweden, and 27 percent in the U.S. for our baseline calibration. The welfare loss of having no government insurance programs instead of the optimal level are 0.1 percent of annual consumption in Sweden and 5.6 percent in the U.S. The results are sensitive to the parameterization of the utility function. For the alternatives we consider, the optimal tax rate varies between 0 and 14 percent in Sweden and between 21 and 38 percent in the U.S.

The calibrated models also imply Laffer curves. These curves are of separate interest since there may be reasons for taxation in addition to the insurance motive, for example the provision of public goods. We find that the Laffer curves peak when tax rates on labor income are high, approximately 60 percent or more. As a fraction of total production, taxes levied are then around 40 percent. The shape of the Laffer curve depends on the labor-supply elasticity but seems to be invariant to a variety of other changes in parameter values and specifications of the model.

Our paper is closely related to that of Aiyagari and McGrattan (1997). They consider

the welfare effects of government debt in a model where agents face idiosyncratic and uninsurable wage uncertainty and are subject to a no-borrowing constraint. Government debt increases the liquidity in the economy and effectively loosens the borrowing constraint for individuals, but it also has negative side effects. Distortive taxation is needed to finance interest payments, and debt crowds out accumulation of real capital and hence lowers production in the economy. For their benchmark calibration of the model, Aiyagari and McGrattan find the optimal level of government debt in the U.S. to be $2/3$ of GDP. The income tax rate needed to sustain this debt is approximately 8 percentage points higher than in the economy with no debt. However, the welfare loss of having no debt at all instead of the optimal level would be less than 0.1 percent of consumption. Recent work by Flodén (1999) confirms that, in providing insurance, government debt is a weak instrument compared to direct transfers. In addition to allowing for government debt and not considering variations in the transfer level, the key difference from our paper is that Aiyagari and McGrattan use a considerably less persistent and slightly less volatile wage process than what we found in U.S. data.

The persistence and magnitude of wage shocks is indeed central for our results. Much previous work, for example Heaton and Lucas (1996), Aiyagari (1994), Aiyagari and McGrattan (1997), and Krusell and Smith (1997, 1998), has built on less volatile income or wage processes. In these papers, the effects of uninsurable idiosyncratic risk are in most cases small. When the persistence of shocks is low, a small buffer of wealth offers good insurance against bad outcomes, and most agents are able to build up such a buffer. Heaton and Lucas estimate the AR(1) coefficient to be 0.53 in annual U.S. income. Our estimation, and similar estimations by Card (1991) and Storesletten, Telmer, and Yaron (1997), result in higher persistence – we estimate a coefficient of 0.91 for the U.S. and 0.81 for Sweden. There are mainly two reasons for why this difference has occurred.. First, Heaton and Lucas do not allow for measurement error in wage and income data. If measurement errors exist but are neglected, as in Heaton and Lucas's paper, the estimated persistence will be downward biased. Second, Heaton and Lucas allow for an unobservable permanent individual specific wage component in the estimation of the persistence coefficient, while we estimate permanent wage differences based on observable character-

istics such as education, occupation, and age prior to the estimation of the persistence component. This will tend to lower the estimated persistence coefficient in Heaton and Lucas's paper compared to ours since their permanent wage component will contain a part of our estimated persistence component.

Hansen and İmrohoroğlu (1992) explored the potential benefits of unemployment insurance in an economy where agents are subject to unemployment risk. The capital stock is exogenous, as is the working time of the employed. They find that unemployment insurance has positive welfare effects, as long as unemployed can be forced to accept all job offers. The result in that setting is not surprising since neither taxes nor unemployment benefits have any distortionary effects. Hansen and İmrohoroğlu also consider the case where unemployed can, with some probability, reject job offers but still keep their benefits. Allowing for these moral hazard considerations, the welfare gains of unemployment insurance become small or even negative.

Our paper is different from Hansen and İmrohoroğlu's in several ways. First, the uncertainty and heterogeneity we consider is richer and more important. Hansen and İmrohoroğlu only allow for two different income states, employment and unemployment, and the amount of persistence in this process is negligible. For example, the expected earnings in a six-week period one year from now for an unemployed worker is 99.7 percent of the expected earnings of an agent who is employed today, if unemployed agents accept all job offers. Second, the possible distortions from the insurance programs in the two papers are quite different. In our paper, since the capital stock is endogenous, government policy will affect the return agents get on their private savings. As the insurance program becomes more extensive, savings fall and the interest rate increases. The cost of self insurance is then effectively reduced. Moreover, we allow for taxes and transfers having effects on labor supply, not only on the extensive margin, but also on hours worked for those who work. On the other hand, we do not allow for an explicit unemployment state, as Hansen and İmrohoroğlu do.

Some important assumptions underlying our study are worth commenting on. We abstract from aggregate uncertainty. The motivation for doing so is that a number of studies, for example İmrohoroğlu (1989), and Krusell and Smith (1999), indicate that

aggregate uncertainty is negligible in this setting. Also, the estimation results in Heaton and Lucas (1996) indicate that aggregate shocks only seem to account for a few percent of the variability in household income.

We rule out private insurance contracts by assumption.¹ This market failure can be motivated by assuming that agents cannot observe each others' income. The government, on the other hand, is assumed to observe agents' income but not their productivity. Moreover, the government, contrary to private institutions, can force agents to participate in programs that have negative expected value for specific individuals. It should also be pointed out that the intention of this paper is not to look for efficient contracts and redistribution schemes. It is, for example, possible that it would be more efficient to condition tax rates and transfers on the income that agents have. Thus when we use the phrase "optimal tax", we do not mean this in a strict sense.

We do not explicitly allow for unemployment when estimating the wage process. Instead, we assume that log productivity (that is, the log of the relative wage) follows an AR(1) process, but we have in mind that individuals with low productivity are unemployed. However, unemployed workers need not be completely unproductive. There are, for example, opportunities for home production or informal services. Consequently, we believe that an "unemployed" person with no accumulated wealth and no or very low guaranteed income will spend much of his time on some kind of working activity.

The structure of the paper is as follows. In the next section, we outline the model, describe how to parameterize it, and how to compute the equilibrium. The data and the strategy used to estimate the wage processes in Sweden and the U.S. are then presented in Section 3, together with the results of these estimations. In Section 4, we present results for the optimal tax level, Laffer curves, and asset distributions implied by the model. In Section 5, we try to assess how sensitive the results are to parameter choices. We also consider some changes in the specification of the model. Finally, Section 6 concludes.

¹ There is a significant literature studying such contracts in models with information asymmetries. Recent contributions are Atkeson and Lucas (1995) and Cole and Kocherlakota (1998).

2 The Model

Consider an economy with a continuum of ex ante identical agents. Each year a fraction γ of the agents dies and new agents with no asset holdings enter the economy. Each agent is endowed with a level of productivity, $q_t^i = e^{\psi^i + z_t^i}$, where ψ^i is a permanent component and z_t^i a temporary component. The temporary component evolves stochastically over time according to the process

$$z_t^i = \rho z_{t-1}^i + \varepsilon_t^i, \quad (1)$$

where ρ determines the degree of persistence in the temporary productivity shocks. The permanent component ψ^i and the temporary shock ε^i are both assumed to be *i.i.d.* normally distributed with mean zero and variance σ_ψ^2 and σ_ε^2 respectively. Hence, the lower bound of the possible realizations of the productivity level is zero.

Each agent is also endowed with one unit of time, which is divided between labor, h , and leisure, l . There is no aggregate uncertainty in the economy. The interest rate, the wage rate, and the aggregate labor supply and capital stock will therefore be constant. The government insures agents by transferring b to each agent in every period.² These transfers are financed by a proportional tax on labor income. An agent's disposable resources are then

$$y_t^i = b + (1 - \tau) w q_t^i h_t^i + (1 + r) a_t^i,$$

where τ is the tax rate and $(1 + r) a_t^i$ is the agent's asset holdings in the beginning of the period. The agent's budget constraint is

$$c_t^i \leq y_t^i - \hat{a}_{t+1}^i, \quad (2)$$

where \hat{a}_{t+1}^i is the assets the agent chooses to hold for the next period.

In the beginning of a period, after new agents are born, a fraction γ of the population is randomly picked to be heirs to the deceased agents. The wealth of the deceased agents is then evenly distributed among the heirs.³ Let g_t^i denote agent i 's received bequests in period t , and let \bar{a} denote the average wealth of an agent. Then $g_t^i = \bar{a}$ with probability γ and $g_t^i = 0$ with probability $1 - \gamma$.

² A more efficient redistribution scheme would condition transfers on agents' productivity level, but we assume that q is unobservable to the government.

³ This is similar to Huggett's (1996) "accidental bequests".

A crucial assumption in the model is that agents are subject to a no-borrowing constraint, i.e. that $\hat{a}_t \geq 0$. This assumption is not entirely ad hoc. If government transfers cannot be used as a security for loans, the lower bound on the present value of future incomes is zero.⁴ In that case there is no positive debt which an agent can repay for sure.⁵

Let s_t^i denote the exogenous productivity state of agent i , $s_t^i = (\psi^i, z_t^i) \in \mathbf{S}$. The agents' asset holdings are restricted to be in the interval $[0, \bar{A}] = \mathbf{A}$, where \bar{A} is chosen high enough to never be a binding restriction. Further, let $\lambda(a, s)$ be the measure of agents, and normalize the mass of agents to unity.

Agents maximize their expected life-time utility,

$$U_0 = E_0 \sum_{t=0}^{\infty} (1 - \gamma)^t \beta^t u(c_t^i, l_t^i),$$

where β is the time discount rate. The Bellman equation to the consumer's problem is then

$$v(a_t^i, s_t^i) = \max_{\{\hat{a}_{t+1}^i, h_t^i\}} u(c_t^i, l_t^i) + (1 - \gamma) \beta E[v(a_{t+1}^i, s_{t+1}^i) | \hat{a}_{t+1}^i, s_t^i] \quad (3)$$

subject to (2), and

$$h_t^i + l_t^i = 1,$$

$$a_t^i = \hat{a}_t^i + g_t^i,$$

$$h_t^i \geq 0,$$

$$\hat{a}_{t+1}^i \geq 0.$$

Each period the government has tax incomes given by

$$T(\tau, b) = \int_{\mathbf{A} \times \mathbf{S}} \tau w q(s) h(a, s) d\lambda,$$

where h is the agent's decision rule for labor supply, and $q(s)$ is the productivity level associated with state s . The government makes a lump sum transfer, b , to all agents. Its per period expenses are thus

$$G(b) = b.$$

⁴ Alternatively, the transfer can be in a nontradable form.

⁵ See Aiyagari (1994) for a discussion of this.

There is a continuum of firms which have Cobb-Douglas production functions and behave competitively in product and factor markets. Let K denote the aggregate capital stock and H the aggregate labor supply in efficiency units, i.e. $H = \int q(s) h(a, s) d\lambda$. Aggregate production is then given by

$$F(K, H) = K^\theta H^{1-\theta}.$$

Finally, let δ denote the depreciation rate of capital.

2.1 Equilibrium

A stationary equilibrium of this economy is given by (i) a constant tax rate τ and level of transfers b , (ii) a constant interest rate r and wage rate w , (iii) time invariant decision rules for agents' asset holdings, $\hat{a}_{t+1}^i = \hat{a}'(a_t^i, s_t^i; \tau, b, r, w)$, and hours worked, $h_t^i = h(a_t^i, s_t^i; \tau, b, r, w)$, (iv) a measure of agents over the state space, $\lambda(a, s)$, (v) aggregate values for asset holdings, $A(\tau, b, r, w) = \int \hat{a}'(a, s) d\lambda$, and for the number of efficiency hours worked, $H(\tau, b, r, w) = \int q(s) h(a, s) d\lambda$, such that the following equilibrium conditions are fulfilled:

- The decision rules solve agents' maximization problem, given by (3).
- Tax revenues equal government expenses,

$$T(\tau, b, r, w) = G(\tau, b, r, w).$$

- Factor markets clear,

$$r = F_K(K, H) - \delta,$$

$$w = F_H(K, H).$$

- Aggregate supply of savings is equal to firms' demand for capital,

$$(1 + \gamma) A(\tau, b, r, w) = K(\tau, b, r, w).$$

- The measure of agents over the state space is invariant, i.e.

$$\lambda(\mathbf{a}, \mathbf{s}) = \int_{\mathbf{A} \times \mathbf{S}} P(a, s, \mathbf{a}, \mathbf{s}) d\lambda,$$

for all $\mathbf{a} \times \mathbf{s} \subseteq \mathbf{A} \times \mathbf{S}$. The transition function P is the probability that an agent with state (a, s) will have a state belonging to $\mathbf{a} \times \mathbf{s}$ next period,

$$P(a, s, \mathbf{a}, \mathbf{s}) = \int_{\mathbf{s}} \{ (1 - \gamma)^2 \mathcal{I}[\hat{a}'(a, s) \in \mathbf{a}] + (1 - \gamma) \gamma \mathcal{I}[\hat{a}'(a, s) + \bar{a} \in \mathbf{a}] + \gamma (1 - \gamma) \mathcal{I}[0 \in \mathbf{a}] + \gamma^2 \mathcal{I}[\bar{a} \in \mathbf{a}] \} \Gamma(s, ds'),$$

where \mathcal{I} is an indicator function and $\Gamma(s, s')$ is the probability that the exogenous state next period belongs to $s' \subseteq \mathbf{S}$, given that it is s today.

2.2 Computation of equilibrium

To find the agent's decision rules for saving and labor supply, we discretize the state space and make a piecewise linear approximation of agents' decision rules over this.⁶ To solve for the equilibrium, we use an algorithm inspired by Huggett (1993) and Aiyagari (1994). The algorithm consists of the following steps: Fix the tax rate, τ , and guess an interest rate, r , and the average efficiency hours of labor supply, \hat{H} . Then solve for the wage per efficiency unit of labor as a function of r and \hat{H} , and calculate the transfer level implied by government budget balance, by setting $b = \tau \hat{H} w$. The agents' decision rules are then solved for and average asset holdings and efficiency hours worked are calculated from simulations.⁷ If the implied aggregate saving of agents does not equal firms' demand for capital, or if the implied labor supply is different than the guess, then make new guesses and start over. If both equalities hold, the equilibrium of the economy with tax rate τ has been found.

⁶ More precisely, we solve the Euler equation by fitting a cubic spline between gridpoints. In the simulations, the decision rules for asset holdings are approximated with piecewise linear functions. Consumption and labor decisions are solved as functions of asset choices and are therefore allowed to be nonlinear between gridpoints. The state space is approximated by a grid consisting of 50 values for asset holdings, one high and one low value for the permanent shock, and 11 values for the temporary productivity level. The AR(1) process for productivity is approximated with the algorithm by Tauchen (1986). We use a spread of $\pm 3\sigma_\varepsilon / (1 - \rho^2)^{1/2}$ for the productivity grid. The step size in the grid for asset holdings is increasing in wealth.

⁷ We simulate an economy populated by 100 agents with low permanent productivity and 100 agents with high permanent productivity for 1500 periods. When one agent dies, he is replaced by a new agent with no accumulated wealth. The initial productivity of this agent is drawn from the stationary distribution of the productivity process. We discard the first 500 periods and use the remaining 200,000 observations to calculate statistics for the economy.

2.3 Parameterization

For our baseline calibration, the agents' utility function is assumed to be in the class of CES utility functions with unit elasticity of substitution between consumption and leisure, i.e.

$$U(c_t, l_t) = \frac{(c_t^\alpha l_t^{1-\alpha})^{1-\mu}}{1-\mu}.$$

This utility function has been extensively used in the real business cycle literature and it is consistent with the observation that hours worked have remained more or less constant although real wages have increased sharply the last century. However, evidence from micro studies (for example, MaCurdy, 1981, and Altonji, 1986) indicates that the intertemporal elasticity of labor supply is smaller than what is implied by this specification of utility. When doing sensitivity analysis, we consider a utility function with less elastic labor supply.

The parameter α is set to 0.50. This implies that the average time an agent spends in market activities is close to 0.50 if there is no income taxation. For positive tax rates, the agent will on average choose to work less. The time discount rate, β , is set to 0.9796, and the death probability, γ , to 2 percent. Hence, the average length of an agent's work life is 50 years and the effective time discount rate is 0.96. The inverse of the intertemporal elasticity of substitution, μ , is set to 2.

On the production side of the economy, the capital share, θ , is set to 0.36 and the depreciation rate of physical capital, δ , is set to 8 percent per year.

The parameters ρ , σ_ψ^2 , and σ_ϵ^2 in the productivity process are estimated in the next section.

3 Data and estimation

In this section, we discuss the data sets for the U.S. and Sweden, and how we estimate the productivity processes in (1) on the data for the two countries. Our measure of productivity, which captures the degree of individual specific risk in the model, is an agent's hourly wage rate relative to all other agents.

3.1 Data

We use the Panel Study of Income Dynamics (PSID) data set for 1988 to 1992 to estimate ρ , σ_ψ^2 , and σ_ε^2 for the U.S.⁸ For Sweden we use the Household Income Survey (HINK) for the years 1989, 1990 and 1992. HINK is a two-year overlapping household panel collected by Statistics Sweden, but in 1992 the collected panel is partly the same as in 1989 and 1990.

Our measure of productivity is a worker's hourly wage rate relative to all other agents. To obtain these data, we proceed as follows: For the U.S., we only look at individuals who were heads of the same household in the 1988 to 1992 surveys, and who were in the labor force (working, unemployed or temporarily laid off) all of these years. To avoid problems with oversampling of poor people in the PSID data set, we exclude people stemming from the Survey of Economic Opportunity sample. We also exclude people for whom relevant data on labor supply and earnings are of poor quality (major assignments or top-coding have been done). For Sweden, we look at adults who remained in the same household and who were in the labor force all of these years.⁹

The measure of the hourly wage which interests us is one which will hold for a wide range of hours worked for a specific individual. For example, someone who was unemployed 1000 hours in a year and worked 900 hours at the wage rate 8 dollars per hour is not assigned a wage of 8 dollars per hour but rather $8 \times 900/1900 = 3.79$ dollars per hour. Of course, unemployment is to some extent voluntary since most people could get some job at some small but positive wage rate. We will not control for this problem of inference when estimating the wage process. To avoid some of the worst problems, however, we assume that nobody has a wage rate less than ten percent of the average wage. This assumption also captures our belief that all agents have some productivity, although some activities are unobservable in data.

For the U.S., we calculate work hours supplied as the sum of the variables hours

⁸ The reason for not using a longer period is that the sample size becomes considerably smaller. The period 1988-1992 is chosen to match the Swedish data period.

⁹ We include all adults for Sweden, and not only the heads of households, since there is no good definition of "heads" in the HINK database and since it is very common in Sweden that both men and women in a household participate in the regular labor market. Consequently, the share of women is higher in the Swedish sample.

worked, hours in unemployment and work hours lost due to illness. These are directly observable in the PSID. For Sweden, we calculate work hours supplied as the sum of the variables hours worked and work hours lost due to illness, which are directly observable in the HINK. To this sum, we then add the estimated time in unemployment, since time spent in unemployment is not directly observable in the HINK.¹⁰

For people spending most of their time out of the labor force, it is difficult to infer the wage they would get if working more. Therefore, all agents with less than 1000 work hours supplied are excluded from the sample. The hourly wage rates in a year for the 1789 and 2856 persons remaining in the sample for the U.S. and Sweden respectively are then computed as the wage sums divided by the total work hours supplied.¹¹

Table 1: Descriptive statistics for relative wages.

Statistic	U.S.					Sweden		
	1988	1989	1990	1991	1992	1989	1990	1992
\bar{W}	12.48	13.31	14.11	14.79	15.60	71.20	81.42	88.83
$\text{Std}(w^i)$	0.64	0.62	0.65	0.66	0.71	0.40	0.40	0.45
$\text{Max}(w^i)$	8.18	5.53	8.27	10.11	12.14	4.63	4.53	4.82
$\text{Min}(w^i)$	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10

Note: \bar{W} is the average hourly wage in USD and SEK respectively. w^i is the relative wage, $\text{Std}(w^i)$ the standard deviation in w^i and $\text{Max}(w^i)$ and $\text{Min}(w^i)$ the maximum and minimum relative wage in the constructed relative wage series in a given year.

We are only interested in fluctuations in relative wages. Therefore, we remove year effects in the data by expressing agent i 's hourly wage rate as a fraction of the average hourly wage rate in that year, and we denote this by w_t^i .

Descriptive statistics for the constructed relative hourly wages are reported in Table 1. For information, we also include the average hourly wage \bar{W} in USD for the U.S. and in Swedish Kronor (SEK) for Sweden in the Table. We see that the variability in the relative wage series is larger in the U.S. than in Sweden, and slightly increasing over time in both countries. The minimum relative wage is 0.10 for all years as a consequence of our assumption that no individual has a wage lower than ten percent of the average. However,

¹⁰ The estimated time in unemployment is an increasing function of the unemployment benefits such that the total sum of hours worked for an individual who has received unemployment benefits is set equal to the stipulated work time in Sweden, which presently is 2080 hours per year.

¹¹ All the definitions of variables and the data programs are provided in an appendix which is available on request from the authors. However, the HINK data set is not available upon request without a permission from Statistics Sweden.

it should be noted that this adjustment has been made for very few individuals.¹²

3.2 Estimation

Taking logarithms of the data, we now observe $x_t^i \equiv \ln w_t^i$ for $t = 1988$ to 1992 in the U.S. and $t = 1989, 1990$ and 1992 for Sweden. We want to estimate the process

$$\begin{aligned} x_t^i &= \psi^i + z_t^i + \xi_t^i, \\ z_t^i &= \rho z_{t-1}^i + \varepsilon_t^i. \end{aligned} \quad (4)$$

where we allow for a measurement error ξ and where $\psi^i + z^i$ is the logarithm of the true, but unobservable, wage rate for agent i , relative to all other agents. Both ε and ξ are assumed to be identically and independently distributed over time and across individuals.

Since our data series are short, we do not try to estimate ψ^i directly from each individual's data. Instead, we assume that the permanent wage differences can be captured by individual specific characteristics such as age, education and occupation. Hence, we estimate

$$x_{1988}^i = \varphi_1 + \varphi_2 AGE_i + \varphi_3 (AGE_i)^2 + \varphi_4 DMALE_i + \varphi_5 EDUC_i + \varphi_O OCC_i + v_{1988}^i \quad (5)$$

for the U.S. with OLS where AGE is the individual's age, $DMALE$ is a dummy for the individual's gender, $EDUC$ is the agent's number of years spent in school, and $OCC_i = [OCC_{1,i} \dots OCC_{8,i}]^T$ are occupation dummies.

For Sweden, we estimate

$$\begin{aligned} x_{1989}^i &= \varphi_1 + \varphi_2 AGE_i + \varphi_3 (AGE_i)^2 + \varphi_4 DMALE_i + \varphi_E EDUC_i \\ &\quad + \varphi_O OCC_i + v_{1989}^i \end{aligned} \quad (6)$$

where $EDUC_i = [EDUC_{1,i} \dots EDUC_{3,i}]^T$ is a vector containing dummies for the agent's education level, and $OCC_i = [OCC_{1,i} \dots OCC_{4,i}]^T$ is a vector containing occupation dummies. The variables considered in the regressions above are similar to those used by, for example, Blau and Kahn (1995) and Edin and Holmlund (1995). The estimation results for (5) and (6) are reported in Table 2.

¹² In the U.S., X^i was adjusted upwards to 0.10 for 19, 18, 20, 31 and 28 individuals in 1988, 1989, 1990, 1991 and 1992 respectively. For Sweden, X^i was set to 0.10 for 6, 10, and 26 individuals in 1989, 1990 and 1992. Changing the minimum relative wage assumption to 0.05, say, has no impact on the results.

Table 2: OLS estimation results for the initial relative wage level.

U.S. - 1988			Sweden - 1989		
Variable	Estimate	p-value	Variable	Estimate	p-value
<i>CONSTANT</i>	-3.330	0.000	<i>CONSTANT</i>	-1.079	0.000
<i>AGE</i>	0.076	0.000	<i>AGE</i>	0.033	0.000
<i>AGE</i> ² /100	-0.077	0.000	<i>AGE</i> ² /100	-0.035	0.000
<i>DMALE</i>	0.272	0.000	<i>DMALE</i>	0.194	0.000
<i>EDUC</i>	0.074	0.000	<i>EDUC</i> ₁	0.099	0.000
<i>OCC</i> ₁	0.421	0.000	<i>EDUC</i> ₂	0.218	0.000
<i>OCC</i> ₂	0.320	0.000	<i>EDUC</i> ₃	0.475	0.000
<i>OCC</i> ₃	0.277	0.001	<i>OCC</i> ₁	0.061	0.000
<i>OCC</i> ₄	0.231	0.042	<i>OCC</i> ₂	0.068	0.013
<i>OCC</i> ₅	0.257	0.000	<i>OCC</i> ₃	0.055	0.006
<i>OCC</i> ₆	0.171	0.017	<i>OCC</i> ₄	0.083	0.002
<i>OCC</i> ₇	-0.558	0.000			
<i>OCC</i> ₈	0.076	0.233			
<i>F</i>	59.166	0.000		120.238	0.000
\bar{R}^2	0.281			0.295	
<i>N</i>	1789			2856	

Note: Dependent variables are the ratio between the hourly wage and average hourly wage in the U.S. 1988 and Sweden 1989 in natural logarithms. For the U.S., *EDUC* is the number of years spent in school, *OCC*₁, ..., *OCC*₈ are dummy variables equal to 1 if the individual is a professional or technical worker, manager, sales worker, clerical worker, craftsman, operative, farm worker, or service worker, respectively and 0 otherwise. A dummy for unclassified occupations is excluded in the regression. For Sweden, *EDUC*₁, ..., *EDUC*₃ are dummy variables equal to 1 if the individual has between 2 – 3, 3 – 6 and over 6 years education after primary school respectively and 0 otherwise. A dummy for those with less than 2 years education after primary school is excluded. *OCC*₁, ..., *OCC*₄ are occupation dummies equal to 1 if the individual works in the private industry, building industry, sales sector and the communication and transport sector. A dummy variable for those who work in the public sector and in banks is excluded. Finally, *DMALE* is a dummy variable equal to 1 if the individual's gender is male and 0 otherwise.

As seen from Table 2, most of the variables are highly significant and the *F*-statistics are satisfactory both for the U.S. and for Sweden. The adjusted r-squares are reasonably high and similar for both countries. All the estimated parameter values are also reasonable. The point estimates for gender and age in Sweden are of the same magnitudes as the ones presented in Edin and Holmlund's (1995) wage regressions.

We use the regression results from Table 2 to calculate estimates of the permanent wage component as the unconditional mean for each individual, $\hat{\psi}^i = \hat{x}_{1988}^i$ in the U.S. and $\hat{\psi}^i = \hat{x}_{1989}^i$ in Sweden, and then to compute the variance of $\hat{\psi}^i$. For the U.S., we get $\hat{\sigma}_{\psi}^2 = 0.1175$, and for Sweden we get $\hat{\sigma}_{\psi}^2 = 0.0467$. Hence, there is more wage inequality in the U.S. than in Sweden in the sense that permanent wage differences between individuals are larger.

Table 3: Descriptive statistics for transformed relative wages.

Statistic	U.S.					Sweden		
	1988	1989	1990	1991	1992	1989	1990	1992
Std(\tilde{w}^i)	0.57	0.56	0.59	0.59	0.64	0.31	0.30	0.37
Max(\tilde{w}^i)	5.73	4.37	5.79	7.09	8.51	2.95	3.27	4.10
Min(\tilde{w}^i)	0.09	0.08	0.08	0.08	0.08	0.10	0.07	0.07

Note: $\tilde{w}^i \equiv \exp(\tilde{x}^i)$, that is, the relative wage where the estimated systematic component due to permanent differences between individuals in the sample has been removed. Std(\tilde{w}^i) the standard deviation in \tilde{w}^i and Max(\tilde{w}^i) and Min(\tilde{w}^i) the maximum and minimum relative wage in the constructed relative wage series in a given year.

To extract the risk which remains for individuals in the U.S. after permanent differences have been removed, we construct the variable $\tilde{x}_t^i \equiv x_t^i - \hat{\psi}^i$ for $t = 1988, \dots, 1992$. For Sweden, we construct the variable $\tilde{x}_t^i \equiv x_t^i - \hat{\psi}^i$ for $t = 1989, 1990$ and 1992. Summary statistics for the transformed relative wage variables are reported in Table 3. A comparison of the figures reported in Table 1 and Table 3 reveals that the variability in the data, quite naturally, becomes lower for both countries after the systematic factors have been removed from the data. We also see that there still is a slight increase in wage variability over time.

Finally, we use \tilde{x}_t^i in (4) to construct the following unconditional moment conditions

$$\begin{aligned} E \left[(\tilde{x}_t^i)^2 \right] - \frac{\sigma_\varepsilon^2}{1 - \rho^2} - \sigma_\xi^2 &= 0, \\ E [\tilde{x}_t^i \tilde{x}_{t-s}^i] - \rho^s \frac{\sigma_\varepsilon^2}{1 - \rho^2} &= 0 \end{aligned} \quad (7)$$

in order to estimate ρ , σ_ε^2 , and σ_ξ^2 for the U.S. and Sweden with the general method of moments. Since we have observations from five periods in the U.S., (7) implies that we can use 15 moments. For Sweden, (7) implies that we can use 6 moments. Since we have more moments than estimated parameters, the model is overidentified, and we use Hansen's (1982) χ^2 -test to test the overidentifying restrictions. However, it is well known that Hansen's test may fail (see Newey, 1985). Therefore, the p -values for Hansen's test, reported in Table 4, were generated with a Monte Carlo simulation.¹³

The GMM estimation results are reported in Table 4. We see that the relative hourly wage series are highly persistent, especially in the U.S. Moreover, the variance of tempo-

¹³ In the Monte Carlo study, we have simulated the process $x_t^i = z_t^i + \xi_t^i$ where $z_t^i = \rho z_t^i + \varepsilon_t^i$, using $\hat{\rho}$, $\hat{\sigma}_\varepsilon^2$ and $\hat{\sigma}_\xi^2$ from Table 4, and calculated χ^2 from these simulated series.

Table 4: GMM estimation results for the productivity process.

Parameter	U.S.		Sweden	
	Estimate	Standard error	Estimate	Standard error
ρ	0.9136	0.0090	0.8139	0.0268
σ_ε^2	0.0426	0.0048	0.0326	0.0059
σ_ξ^2	0.0421	0.0039	0.0251	0.0046
χ_{obs}^2	23.45		46.35	
p -value	0.051		0.000	

Note: White's heteroskedasticity consistent standard errors. The p -values are simulated probabilities of obtaining a χ^2 higher than χ_{obs}^2 when the model is correctly specified.

rary shocks is considerably higher in the U.S. than in Sweden. Consequently, the wage risk that agents face after having observed their permanent productivity level is higher in the U.S. The estimates of ρ and σ_ε^2 are precise for both countries. As indicated by the simulated p -values, one possible shortcoming is that the overidentifying restrictions do not seem to hold, in particular not for Sweden. One reason for this result might be that the estimated AR(1)-process for the agent's productivity process is a too crude approximation of reality.¹⁴

To sum up, we have found that individuals in the U.S. are subject to more wage inequality as well as more wage uncertainty. The estimated variance of permanent log wage differences is 0.1175 in the U.S. and 0.0467 in Sweden. The estimated variance of temporary log wage shocks is 0.0426 in the U.S. and 0.0326 in Sweden, and temporary shocks are more persistent in the U.S., with the estimate of ρ equal to 0.9136 against 0.8139 in Sweden.

The findings for the U.S. wage process resemble those in Card (1991). He estimated a similar wage process for the U.S. based on men in the PSID from 1969 to 1979. The estimated persistence was 0.886 while the estimates of variances were 0.124, 0.027, and 0.039, for permanent shocks, temporary shocks, and measurement errors, respectively.

¹⁴ However, if we assume that all unemployment is voluntary (which here in practice means that we do not add time in unemployment to hours worked in the calculation of hourly wages), the estimated σ_ψ^2 , ρ and σ_ε^2 are practically unchanged ([0.1075, 0.9165, 0.0379] and [0.0421, 0.8545, 0.0227] for the U.S. and Sweden respectively). But the χ^2 -statistics are now changed to 15.02 and 12.12 with p -values 0.27 and 0.03 respectively. Thus, we can no longer clearly reject the model. We therefore conclude that the estimates for the parameters σ_ψ^2 , ρ and σ_ε^2 seem to be robust, but that Hansen's χ^2 -statistic seems to be sensitive to the data generation.

4 Optimal tax level, Laffer curves and asset distributions

4.1 Optimal tax level and Laffer curves

To find the optimal tax level, we solve the model for tax rates between 0 and 65 percent, with increments of 1 percentage point, and look for the tax rate that maximizes the average utility of agents in the economy, \bar{u} . Equilibrium outcomes for some selected tax rates are shown in Tables 5a and 5b. As a reference, we also report the outcome we would get if agents were provided with full insurance at zero tax rates.¹⁵

For the baseline calibration, we find the optimal tax rate to be 27 percent for the U.S. and 3 percent for Sweden. This result is visualized in Figure 1, where the average utility is increasing up to $\tau = 0.27$ in the U.S. but decreasing in Sweden for all τ larger than 0.03. The relatively large differences between the U.S. and Sweden are not surprising, given the estimated wage processes. Some experiments show that both the differences in variances of wage shocks and the difference in persistence of these shocks are quantitatively important. The optimal tax rate in the U.S. falls to 16 percent if ρ is set to the value estimated from Swedish data, and it falls to 23 and 21 percent, respectively, if σ_ϵ^2 and σ_ψ^2 are set to the Swedish counterparts (while leaving the other parameters unchanged).

In the U.S., the welfare gain of having the optimal level of government insurance instead of no insurance at all is large, while the same welfare gain for Sweden is negligible. We measure these gains as the percentage increase in consumption needed for agents in the no-insurance world to get the same average utility as agents in the optimal-insurance world. When doing these calculations, we fix decision rules and prices in the no-insurance world.¹⁶ The welfare gain is 5.6 percent of yearly consumption in the U.S. and 0.1 percent in Sweden.

As seen in Tables 5a and 5b and Figure 2, the Laffer curve has its maximum at very high values of τ . The Laffer curve peaks at a tax rate of 60 percent in the U.S. and 59

¹⁵ By full insurance, we mean that all agents insure before observing their first productivity level. The insurance then yields the same marginal utility of total expenditure in each state.

¹⁶ By allowing agents to reoptimize or by increasing transfers instead of consumption, the measured welfare effects would be slightly lower.

Table 5a: Results for different tax rates - U.S.

τ	\bar{u}	r	K	H	Y	C	\bar{h}	T	T/Y
0.00	-1.820	2.42	3.28	0.472	0.949	0.685	0.427	0.000	0.000
0.05	-1.804	2.55	3.12	0.458	0.915	0.664	0.409	0.029	0.032
0.10	-1.790	2.67	2.97	0.444	0.881	0.642	0.389	0.056	0.064
0.15	-1.781	2.79	2.82	0.430	0.847	0.620	0.369	0.081	0.096
0.20	-1.774	2.90	2.68	0.415	0.813	0.598	0.349	0.104	0.128
0.25	-1.771	3.02	2.54	0.399	0.777	0.573	0.328	0.124	0.160
0.30	-1.772	3.12	2.40	0.382	0.741	0.548	0.306	0.142	0.192
0.35	-1.777	3.23	2.26	0.366	0.701	0.523	0.285	0.158	0.225
0.40	-1.787	3.34	2.12	0.348	0.667	0.497	0.263	0.171	0.256
0.45	-1.801	3.44	1.98	0.330	0.630	0.470	0.241	0.181	0.287
0.50	-1.822	3.54	1.84	0.311	0.589	0.441	0.219	0.189	0.321
0.55	-1.849	3.65	1.70	0.291	0.549	0.412	0.197	0.193	0.352
0.60	-1.886	3.75	1.55	0.270	0.507	0.382	0.176	0.195	0.385
0.65	-1.934	3.85	1.41	0.248	0.463	0.350	0.154	0.193	0.417
FI	-1.598	4.17	2.46	0.451	0.831	0.634	0.303	0.000	0.000

Note: \bar{u} = average utility, r = real interest rate, K = aggregate capital stock, H = aggregate efficiency units of hours worked, Y = aggregate output, C = aggregate consumption, \bar{h} = average hours worked, T = total tax revenues, and FI = outcome under full insurance.

Table 5b: Results for different tax rates - Sweden.

τ	\bar{u}	r	K	H	Y	C	\bar{h}	T	T/Y
0.00	-1.736	3.63	2.69	0.460	0.868	0.652	0.436	0.000	0.000
0.05	-1.736	3.67	2.62	0.447	0.842	0.633	0.421	0.027	0.032
0.10	-1.737	3.73	2.50	0.433	0.814	0.613	0.406	0.052	0.064
0.15	-1.740	3.77	2.40	0.419	0.786	0.592	0.390	0.076	0.097
0.20	-1.746	3.82	2.30	0.405	0.751	0.571	0.373	0.097	0.129
0.25	-1.754	3.87	2.20	0.389	0.726	0.548	0.356	0.116	0.160
0.30	-1.765	3.91	2.10	0.373	0.695	0.525	0.338	0.133	0.191
0.35	-1.779	3.95	1.99	0.356	0.662	0.501	0.319	0.148	0.224
0.40	-1.798	3.99	1.88	0.338	0.627	0.475	0.299	0.161	0.257
0.45	-1.821	4.03	1.77	0.319	0.591	0.448	0.278	0.170	0.288
0.50	-1.851	4.07	1.65	0.300	0.554	0.420	0.257	0.177	0.319
0.55	-1.889	4.12	1.53	0.279	0.514	0.391	0.235	0.181	0.352
0.60	-1.934	4.16	1.40	0.256	0.471	0.360	0.211	0.181	0.384
0.65	-1.997	4.16	1.28	0.235	0.432	0.326	0.189	0.177	0.410
FI	-1.651	4.17	2.47	0.454	0.836	0.638	0.391	0.000	0.000

Note: See Table 5a.

percent in Sweden. Although the optimal tax rates differ significantly between Sweden and the U.S., the Laffer curves are similar. The distortive effects of income taxes do not seem to be sensitive to the amount of risk that agents face.

A couple of other features in Tables 5a and 5b are also worth noting. In an economy with a higher degree of idiosyncratic risk, aggregate output reacts more to changes in the tax rate. We see this in Figure 3, where the difference in aggregate output between the U.S. and Sweden becomes smaller as τ increases. The intuition behind this result is that an increase in the transfer level has larger insurance effects in a country with much idiosyncratic risk than in a country with little risk. As b increases, agents in the U.S. therefore reduce their holding of precautionary wealth more than agents in Sweden do.

We also note that a high degree of idiosyncratic risk is “good” for the agents if they can insure themselves against periods with low productivity and “bad” if they cannot. This result can be seen by comparing the full insurance rows and the first rows in Tables 5a and 5b. The explanations behind this result are twofold. When agents are fully insured, they are able to smooth consumption by borrowing and lending. The agents can then choose to work more when their productivity is high and less when productivity is low, and the higher the degree of idiosyncratic risk, the more agents can increase their utility by working when their productivity is high and staying at home when it is low.¹⁷ But when asset markets are incomplete, agents can no longer smooth consumption and leisure independently. If they have little wealth and low productivity, agents must work to be able to consume. Because of the concavity of the utility function, productivity fluctuations will decrease agents’ utility. Therefore, the average utility, \bar{u} , is higher in the U.S. than in Sweden under full insurance, but lower when asset markets are incomplete and no government insurance is provided.

When looking for the optimal tax rate, we have taken a utilitarian approach and put equal weight on every agent’s utility. To understand for which agents government transfers really matter, when considering the stationary distribution of agents, we have computed optimal tax rates for different percentile agents in this distribution. The main value of the experiment is that it gives a picture of inequality and a sense of which agents experience

¹⁷ This mechanism is most clearly seen from the FI rows in Tables 5a and 5b, where H is significantly higher than \bar{h} .

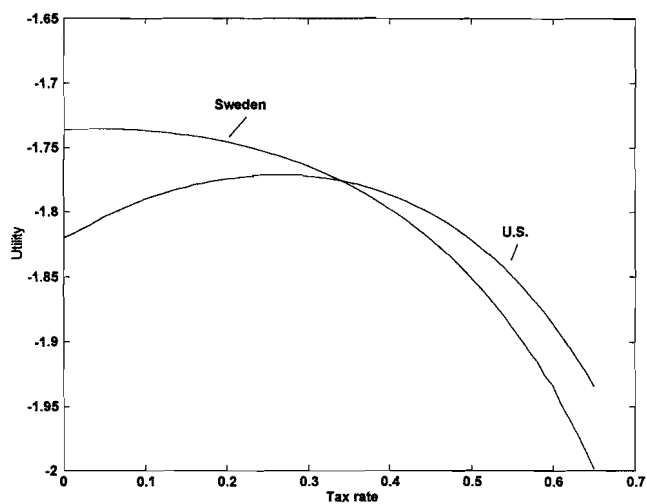


Figure 1: Average utility

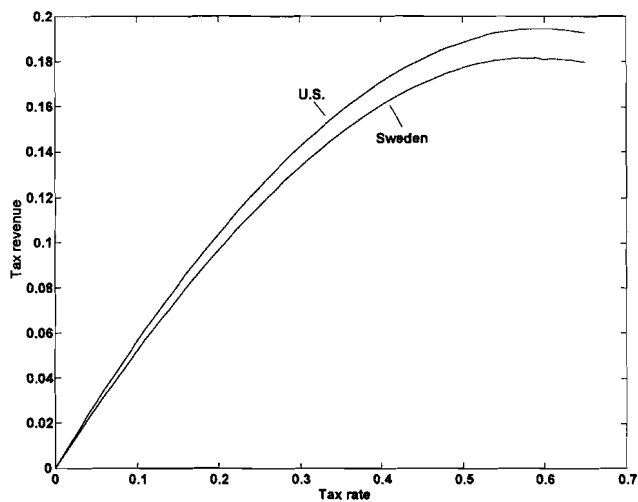


Figure 2: Laffer curves

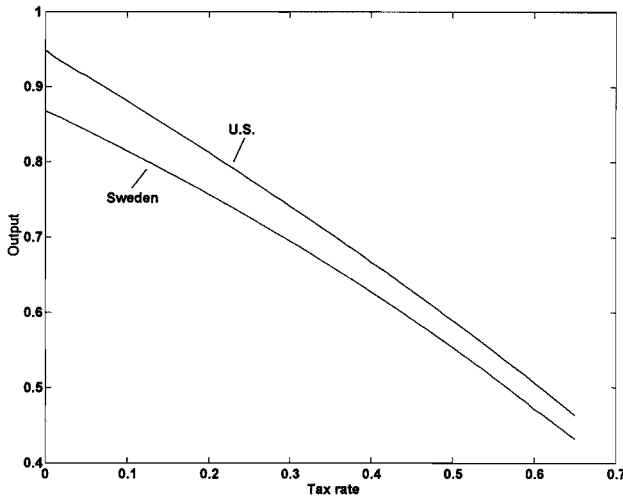


Figure 3: Output per capita

that social security really matters. The results show that government transfers, at the level suggested by the previous analysis, benefit the lowest 30 percentiles in the utility distribution. The median utility in both countries is maximized when tax rates are close to zero.

4.2 Asset distributions

To investigate the empirical validity of the calibrated model, we present distributional implications for the U.S. and Sweden in Tables 6a and 6b, respectively. It is a well known fact that models with plausible parameterizations of income processes and risk aversion have problems in generating asset and income distributions which are as skewed as in the U.S. data. This is documented in e.g. Quadrini and Ríos-Rull (1997).¹⁸ The wealth distributions implied by our model are skewed, but not as skewed as the actual Swedish and U.S. distributions. In particular, the model cannot generate wealth holdings that are

¹⁸ Examples of such studies are Aiyagari (1994) and Huggett (1996). A recent exception is the paper by Castañeda, Díaz-Giménez, and Ríos-Rull (1998). They calibrate the underlying productivity process so that asset and income distributions are matched.

as extreme as for the top few percent of households in the data. However, for the question we are interested in, we argue that it is most important to capture the asset and income distributions of the poor agents, because it is for these that social security really matters. The model does fairly well in this respect.

For the U.S., we report distributions for two tax rates, $\tau = 0.15$ and $\tau = 0.30$. The lower tax level implies that transfers are approximately at the U.S. postwar average of 8 percent of GDP reported in Aiyagari and McGrattan, while the higher tax rate is closer to the U.S. labor income tax rate.¹⁹ For Sweden, we use the tax rates $\tau = 0.30$ and $\tau = 0.50$.

The tables show that asset holdings are unequally distributed, with Gini coefficients around 0.60, but still not as skewed as in the actual economies. In particular, the wealthiest agents (households) in the model are not at all as wealthy as in Sweden and the U.S. The richest one percent of agents hold 8 percent of aggregate wealth in the model, but in the U.S. they hold 29 percent of all wealth. For Sweden, the richest one percent hold 5 percent of all wealth in the model and 13 percent of all wealth in the data.

The asset distribution for the poorest agents is better matched by the model. The bottom 40 percent of agents (households) in the wealth distribution hold approximately 1 percent of the U.S. wealth in the data and 2 percent in the model. In Swedish data they hold -6 percent of all wealth and 5 percent in the model. According to Domeij and Klein (1998) there are two main reasons for the frequent measures of negative wealth holdings for Swedish households. First, the value of privately owned apartments is approximated by the taxable value, which is considerably lower than the market value. Second, students' loans are measured at the full value but human capital is not included in wealth. Considering these data problems, we think the model gives a satisfactory fit of the poor agents in the asset distribution.

The earnings and income distributions for Sweden are well captured by the model, both for those in the bottom and those in the top of the distributions. The model generates too compressed distributions for the U.S., however. For example, the bottom 40 percent in the earnings distribution have only 3 percent of earnings in the data but around 10 percent in the model. In the U.S. data, entrepreneurs who report losses significantly contribute to

¹⁹ In the benchmark model, all tax income is used for transfers, but of course this is not the case in reality.

Table 6a: Distributional implications - U.S.

		Percent of total			
	Gini	Bottom 40%	Top 20%	Top 10%	Top 1%
<i>Wealth</i>					
Actual U.S. Data	0.78	1.4	79.5	66.1	29.5
Model, $\tau = 0.15$	0.63	2.6	63.6	42.6	8.0
Model, $\tau = 0.30$	0.64	2.2	64.4	43.2	8.0
<i>Earnings</i>					
Actual U.S. Data	0.63	2.8	61.4	43.5	14.8
Model, $\tau = 0.15$	0.48	10.9	51.3	33.2	6.1
Model, $\tau = 0.30$	0.53	8.0	54.9	35.9	6.7
<i>Total income</i>					
Actual U.S. Data	0.57	8.8	59.9	45.2	18.6
Model, $\tau = 0.15$	0.39	16.3	45.8	29.3	5.3
Model, $\tau = 0.30$	0.39	17.0	45.4	29.0	5.2

Note: U.S. data adapted from Díaz-Giménez, Quadrini, and Ríos-Rull (1997). Earnings is defined as net labor income before taxes. Total income is defined as net factor income plus transfers but before taxes. Note that U.S. data refer to households while the income process in the model is calibrated to match individual wage processes.

Table 6b: Distributional implications - Sweden.

		Percent of total			
	Gini	Bottom 40%	Top 20%	Top 10%	Top 1%
<i>Wealth</i>					
Actual Swedish Data	0.79	-6	72	49	13
Model, $\tau = 0.30$	0.56	5	56	35	5
Model, $\tau = 0.50$	0.57	5	56	35	5
<i>Earnings</i>					
Actual Swedish Data	0.48	8	47	29	5
Model, $\tau = 0.30$	0.38	16	42	25	4
Model, $\tau = 0.50$	0.46	11	47	29	5
<i>Total income</i>					
Actual Swedish Data	0.33	19	37	14	5
Model, $\tau = 0.30$	0.26	23	36	21	3
Model, $\tau = 0.50$	0.27	23	37	22	3

Note: Swedish data adapted from Domeij and Klein (1998). Earnings is defined as net labor income before taxes. Total income is defined as net factor income plus transfers but before taxes. Note that Swedish data refer to households while the income process in the model is calibrated to match individual wage processes.

the low earnings for the bottom percentiles in the distribution. In the model, wage rates are observable in the beginning of a period, and we do not allow for negative wages.

Maybe surprisingly, changes in tax rates have negligible effects on wealth distributions. For both countries, an increase in taxes actually increases Gini coefficients. When transfers increase, there is less need for poor agents to save for bad times, and in bad times they do not need to work as hard as when there are no transfers.

5 Sensitivity to parameter choice and model specification

In this section, we examine how sensitive the results are with respect to the most important parameters and some specific model assumptions. The results are summarized in Table 7. Since the estimated confidence intervals for ρ and σ_ε^2 were small, we do not make any sensitivity analysis for these parameters, but some of the effects of changes in ρ and σ_ε^2 were shown in Section 4.1.

5.1 Labor-supply elasticity and intertemporal elasticity

To get plausible values for hours worked we chose to set α , the weight on consumption relative to leisure, to 0.50 in the baseline calibration. We examined the effects of setting α to the more common value 0.33, although this value yields a counterfactually low labor supply in the current model. A lower α also implies a higher labor-supply elasticity. Taxes therefore become more distortive when α is decreased. The results in Table 7 show that this effect is quantitatively small. With the lower α , the optimal tax rates fall to 23 and 1 percent in the U.S. and Sweden respectively. The peaks of the Laffer curves shift slightly to the left.

Plausible values for the intertemporal elasticity of substitution are often claimed to be in the interval $[0.2, 1]$. We considered the extreme values, $\mu = 5$, and $\mu = 1$. Not surprisingly, the chosen value for μ is important for the results obtained. If μ is increased from 1 to 5, the optimal tax rate increases from 21 to 36 percent in the U.S., and from 0

to 14 percent in Sweden.

Table 7: Sensitivity analysis results.

Considered cases	U.S.						Sweden					
	τ^*	$(\frac{T}{Y})^*$	r	$\frac{\bar{c}(\tau^*)}{\bar{c}(0)}$	τ^{\max}	$\frac{T^{\max}}{Y}$	τ^*	$(\frac{T}{Y})^*$	r	$\frac{\bar{c}(\tau^*)}{\bar{c}(0)}$	τ^{\max}	$\frac{T^{\max}}{Y}$
<i>Parameters</i>												
μ												
2.00	0.50	0.02	0.27	0.17	3.05	0.82	0.60	0.38	0.03	0.02	3.66	0.98
1.00	0.50	0.02	0.21	0.13	3.07	0.87	0.60	0.38	0.00	0.00	3.64	1.00
5.00	0.50	0.02	0.36	0.23	2.78	0.73	0.59	0.38	0.14	0.09	3.54	0.91
2.00	0.33	0.02	0.23	0.15	2.76	0.82	0.58	0.37	0.01	0.01	3.47	0.99
2.00	0.50	0.00	0.23	0.15	2.60	0.85	0.60	0.38	0.02	0.01	3.39	0.99
<i>Utility function</i>			0.38	0.24	3.37	0.89			0.10	0.06	3.76	0.98
<i>Temporary risk</i>			0.18	0.12	2.87	0.87	0.58	0.37	0.00	0.00	3.65	1.00
<i>10% base tax^a</i>			0.18	0.18	3.02	0.87	0.62	0.40	0.00	0.00	3.71	1.00
<i>Open economy</i>									0.06	0.04	3.12	0.97

Note: τ^* = optimal tax rate, $\frac{\bar{c}(\tau^*)}{\bar{c}(0)}$ = average consumption when taxes are τ^* as a fraction of average consumption when there is no taxation, τ^{\max} is the tax rate which maximizes government tax income. The row ‘utility function’ refers to the case where instantaneous utility is $(c^{1-\mu} - 1) / (1 - \mu) + \Lambda (l^{1-\lambda} - 1) / (1 - \lambda)$, $\mu = 2$, and $\lambda = 2.5$. ^a The base tax is included in τ^{\max} but not in τ^* .

5.2 Infinitely lived agents

In the baseline calibration of the model, agents live 50 years on average, bequests are random over the life cycle, and newly born agents have no wealth. We think that this is a good way to describe reality in a parsimonious way, but the assumptions are non-standard. One might suspect that our results hinge on the poor situation for newly born agents who have not had time to accumulate a buffer of wealth. However, if we assume that agents have infinite lives ($\gamma = 0$, but the effective discount rate unchanged, $\beta = 0.96$), the optimal tax rate only falls slightly, to 23 percent and 2 percent in the U.S. and Sweden respectively.

We are a bit surprised by this small effect of changes in γ . With $\gamma = 0$, agents live for ever and hence have time to accumulate some wealth to self insure against bad times. There are then few agents who have both very little wealth and low productivity, the state which agents want to avoid almost at any cost. However, the accumulation of individual buffer stocks is inefficient in itself, and although government redistribution schemes distort labor supply, they seem to provide better insurance than private savings.

5.3 The utility function

Estimates of the wage elasticity of labor supply vary widely between studies. However, most estimates of the elasticity are less than 0.5 for men, and the estimated elasticity for women is typically higher than that for men – see for example MaCurdy (1981) and Altonji (1986) for estimates on U.S. data, and Flood and MaCurdy (1992) and Aronsson and Palme (1998) for Swedish estimates. As mentioned earlier, the labor-supply elasticity implied by the Cobb-Douglas utility function is higher than what was found in these studies. To allow for a less elastic labor supply, we consider the utility function

$$u(c, l) = \frac{c^{1-\mu} - 1}{1-\mu} + \Lambda \frac{l^{1-\lambda} - 1}{1-\lambda},$$

where $1/\lambda$ is the labor-supply elasticity. When this utility function is parameterized with $(\mu, \lambda) = (2, 2.5)$, τ^* increases to 38 percent in the U.S. and 10 percent in Sweden.

Although labor supply seems inelastic, microdata display considerable variability in hours worked. The evidence reported in Altonji and Paxson (1985), Abowd and Card (1989), and Card (1991) suggest that the coefficient of variation for hours worked, conditional on hours being positive, is between 0.25 and 0.40 in the U.S. Aronsson and Palme (1998) report coefficients of variation of 0.14 and 0.41 for married Swedish men and women respectively. Both utility functions considered here are consistent with these facts. For the baseline specification of the utility function, the standard deviation of changes in log hours worked is 0.32 in the U.S. when there is no government sector, and 0.46 when tax rates are 30 percent. With the new utility function these figures drop to 0.19 and 0.23 respectively. For the Swedish setup of the model, the values are 0.27 and 0.40 for the baseline calibration and 0.13 and 0.14 with the alternative utility function.

5.4 Only temporary risk

The U.S. wage process displays more temporary risk as well as more permanent inequality than the Swedish process. Which of these differences is most important for our results? Although we prefer to think of both the permanent wage differences and the temporary fluctuations as risks for which the government can provide insurance, in daily life transfers because of the former would usually be thought of as redistribution.

By ignoring the permanent wage differences in the calibration of the wage process, we get an impression of which source of risk is driving our results. We find that with only temporary wage uncertainty, the optimal tax rate is 18 percent in the U.S. while no redistribution is motivated in Sweden.

5.5 Government spending

There are other reasons for the government to levy taxes than the insurance and redistribution motive. If the government has a fixed amount of spending on public goods to finance, the distortive effects of increased taxes will be more severe since the tax base for public goods is eroded. We have tried to quantify such effects in the following way. Assume that spending on public goods is not valued by the agents, or equivalently that the utility is additively separable in private and public consumption. Fix government spending at the level it would be if all taxes levied with 10 percent income taxes were used for public consumption. Any amount of tax income the government raises above that amount is transferred in lump sums to the agents as previously.

With this assumption, we obtain lower optimal redistribution levels than in the baseline solution. For Sweden, no redistribution is motivated, and for the U.S. the optimal tax rate is 28 percent including the 10 percent base tax. Note that more than 10 of the 28 percentage points of the tax rate are used to finance government expenditures. When including the base tax in tax income, the Laffer curves shift slightly to the right. This is partly because of a wealth effect, making all agents work more when resources are wasted on government consumption, and partly because of an insurance effect - poor agents get less transfers at a given tax rate and must work harder to get a sufficient income.

5.6 Open economy

Sweden is often thought of as a small, open economy which faces a given world interest rate, but until now we have assumed that both Sweden and the U.S. are closed economies. In Table 5b, we saw that the equilibrium capital stock in Sweden is decreasing in the tax rate. Does this mean that distortions are less important when the world capital stock is

given? We conducted some experiments to answer this question. We solved the model for Sweden with the interest rate fixed at 3.12 percent, which is the equilibrium interest rate for the U.S. at a 30 percent labor income tax.²⁰

The results for this scenario are similar to what we found with the original specification. The optimal tax rate increases to 6 percent and the Laffer curve peaks at tax rates close to 60 percent. The reason for the small change in the optimal insurance level is that the interest rate is not the sole determinant of the capital stock. More important is the supply of efficiency units of labor, and this supply is very sensitive to tax rates. So, although the world interest rate is given and capital is totally mobile, the equilibrium capital input in Swedish production is sensitive to changes in the tax rate.

6 Concluding remarks

We want to stress the main findings of the paper. Wage inequality and wage fluctuations seem to be important features of the economies studied, but more severe in the U.S. than in Sweden, and it seems as if agents, at least in the U.S., are willing to give up a significant amount of consumption in order to insure against this uncertainty.

One possible explanation for the results is that agents in the U.S. are less risk averse than agents in Sweden, and choose higher average wages at the price of higher wage fluctuations. This interpretation is consistent with the fact that GDP per capita is higher in the U.S. than in Sweden.

For all the specifications we have considered, the Laffer curve has peaked when tax rates on labor income have been 58 percent or higher. In our experiments, only changes in the labor-supply elasticity matter for the shape of the Laffer curve. To claim that the Laffer curve peaks at lower tax rates, one has to believe that the elasticity of labor supply is considerably higher than what is typically estimated from data.

There are also some caveats we want to point out to the reader. First, although we look at wages before taxes and transfers, the relatively low degree of wage risk in Sweden may

²⁰ This approach could have been invalid if the Swedish interest rates in autarky had been lower than the U.S. interest rate. People in Sweden might then want to hold much wealth when the high world interest rate prevails. Consequently, even if the Swedish population is small, it could have a significant impact on capital formation.

be a result of the big government sector. For example, a large fraction of the population work in the government sector and wage setting there seems to imply a significant amount of risk sharing. Also, many old persons who become unemployed go into early retirement and hence fall out of the labor force and our sample. Moreover, we take labor market and wage setting institutions as given. That is, we do not try to understand or explain why wage processes are different in different countries. Arguably, some of these differences are a result of government policy. If, for example, wages are a result of bargaining between unions and firms, the bargaining position of low income groups may improve relative to that of high income groups if transfers are increased. We abstract from such issues.

Second, a lot has happened in Sweden after the period examined. Unemployment has increased drastically and in particular employment in the government sector has fallen. It is therefore possible that the income risk in Sweden has increased. Third, we believe that our modeling of idiosyncratic risk is more appropriate for the U.S. than for Sweden. In Europe, the risk that agents face seems to be mainly unemployment risk, not wage risk.

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Essay IV

Testing for the Lucas Critique: A Quantitative Investigation^{*}

1 Introduction

In a very influential article, Lucas (1976) raised serious critique against existing econometric models that were used for policy evaluation. In brief, the Lucas critique is that reduced form econometric models cannot provide any useful information about the actual consequences of alternative policies since the structure of the economy will change when policy changes, making the estimated parameters in the reduced form econometric models non-constant.

Instead, Lucas (1975, 1977), Kydland and Prescott (1982) and others initiated a new research program, often named the real business cycle or equilibrium business cycle approach, in which the models used for policy analysis are immune against the Lucas critique since they are equilibrium models with forward looking behavior. At the same time, researchers who were interested in the applicability of the Lucas critique in practice started to work on how the Lucas critique could be tested on data. In an important paper, Engle et al. (1983) introduced the concept of super exogeneity, and subsequent papers (for instance Engle and Hendry, 1993) have shown how this concept can be used to test whether

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the Lucas critique is important in practice or not.

Surprisingly, most, not to say all, of the many papers which used the concept of super exogeneity to test for the Lucas critique empirically, have found no evidence in favor of the proposition; see the survey by Ericsson and Irons (1995). Two natural questions then arise. Is it obvious that the Lucas critique is quantitatively important in theory in a statistical sense? Even if the Lucas critique is theoretically important quantitatively, it might not be so in a statistical sense.¹ Secondly, given that the answer to the first question is yes (or no), does the test for super exogeneity work in the sense that it really delivers the presence (or absence) of the Lucas critique in observed data?

In this paper, I investigate these two questions in greater detail. My aim is to try to “solve” the puzzle why the Lucas critique, believed to be very important by many researchers, does not seem to be important in studies of real world data. To accomplish this, I set up a slightly modified version of Cooley and Hansen’s (1995) real business cycle model with money. The difference is that the model here includes government expenditures and a Taylor inspired policy rule (see Taylor, 1993) for nominal money supply. This policy rule is then estimated on U.S. data for the recent Federal Reserve’s chairmen Burns, Volcker, and Greenspan office periods, since Judd and Rudebusch (1998) report that the conduct of monetary policy has varied systematically with these Federal Reserve’s chairmen office periods.² By calibrating the theoretical model with these different estimated monetary policy regimes, it is possible first to verify that the properties of the simulated model economy actually change significantly in a statistical sense when there is a monetary regime shift, and secondly, to apply a test of super exogeneity by means of a Monte-Carlo simulation to check whether the test actually discovers the relevance/nonrelevance of the Lucas critique in the model economy. I consider two applications of the super exogeneity tests in the model, associated with the money demand and consumption functions. The reason for focusing on money demand and consumption, is that most of the empirical studies in the field have applied the super exogeneity test on these two relations, see

¹ See Leeper’s (1995) comments on the paper by Ericsson and Irons (1995) for a discussion along this line.

² Judd and Rudebusch (1998) start out by noting that there is instability in the Fed reaction function. Then they find support for the hypothesis that the Fed monetary policy rule has varied systematically with the different Fed chairmen Burns, Volcker, and Greenspan office periods.

Ericsson and Irons (1995).

The results in the paper are as follows. First, with a standard parameterization of the model and by considering the estimated Federal Reserve monetary policy rules for nominal money growth during Burns, Volcker and Greenspan office periods following Judd and Rudebusch (1998), it is found that the Lucas critique is theoretically important in a statistically significant way. The properties of the simulated model economy change significantly when the parameters in the estimated Taylor inspired policy rule change. I thus have a model where the super exogeneity test should be able to identify the relevance of the Lucas critique. Despite this, it is found that the super exogeneity test far too often accepts the false null hypothesis that the Lucas critique does not apply when we have a change in the conduct of monetary policy between Burns, Volcker and Greenspan office periods. This lack of power for the super exogeneity test is then, quite naturally, a possible explanation why the Lucas critique has not been found in the data. Consequently, more research on the properties of the super exogeneity test is warranted.

The structure of the paper is as follows. In the next section, I present and discuss the theoretical model, indicate how to compute the competitive equilibrium. Estimation and calibration issues are discussed in section 3. In Section 4, I investigate whether the Lucas critique is significantly important in a statistical sense in the equilibrium business cycle model by computing impulse response functions, simulating the model and a χ^2 -test for moment stability between the different estimated monetary policy regimes. The super exogeneity test is briefly presented in Section 5, together with the results of the Monte-Carlo simulations used to see whether the super exogeneity test works satisfactory on simulated data from the model. Finally, Section 6 concludes.

2 The equilibrium model

In this section, I describe and solve a slightly modified version of Cooley and Hansen's (1989, 1995) monetary equilibrium business cycle model. The model is a standard real business cycle model with some additional features. A stochastic nominal money supply interacts with a cash-in-advance technology and one-period nominal wage contracts, which

creates short run real effects of nominal money supply shocks. Originally, Cooley and Hansen (1995) calibrated the model to quarterly data. Here, one period is one year. The reason for this is that I think it is reasonable that real effects on monetary policy within the model last for a year rather than a quarter, which is an implication of the one-period nominal wage contract setting.³

The difference between the model in this paper and the one in Cooley and Hansen (1995) is that the central bank is here assumed to use a “Taylor rule” when it decides on the nominal money supply growth in each period. More specifically, the growth rate in nominal money supply in period t is assumed to depend on the output gap, the difference between actual and targeted inflation rate (hereafter named inflation gap), an uncontrollable shock, and the growth rate in nominal money in period $t - 1$. This specification is supposed to capture the real world phenomenon that central banks often use money supply to affect inflation and output gaps, although they act gradually and do not have perfect control of the process.⁴

In the model I abstract from population and technological growth and represent all variables in per capita terms.

Finally, a notational comment; in the following, capital letters denote economy wide averages which the agent takes as given and small letters individual specific values which the agent internalizes.

2.1 An equilibrium monetary business cycle model

Infinitely many identical infinitely lived agents maximize expected utility with preferences summarized by

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, h_t), \\ u(c_{1t}, c_{2t}, h_t) \equiv \alpha \ln(c_{1t}) + (1 - \alpha) \ln(c_{2t}) - \gamma h_t \end{aligned} \tag{1}$$

³ I would like to emphasize that the qualitative aspects of the results in the paper are not at all dependent on whether I calibrate the model to match quarterly or yearly data.

⁴ Note that the Taylor-rule does not imply that monetary policy is “optimal” in any sense in this model. See Flodén (1998) for a version of Cooley and Hansen’s (1995) model with “optimal” monetary policy given a quadratic loss function in the inflation and output gaps.

where c_{1t} is consumption of the “cash good” in period t , c_{2t} is consumption of the “credit good,” and h_t is the share of available time spent in employment which enters linearly in (1) because of the “indivisible labor” assumption (see Hansen, 1985). In (1), β is the subjective discount factor, γ the disutility the agent gets from working, while α reflects the trade-off between consumption of the cash and credit goods.

The flow budget constraint facing the agent is

$$c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{P_t} = \left(\frac{W_t^c}{P_t} \right) h_t + R_t^K k_t + \frac{m_t}{P_t} + (1 + R_{t-1}) \frac{b_t}{P_t} + \frac{TR_t}{P_t} \quad (2)$$

where i_t denotes the agent’s investment, m_{t+1} and b_{t+1} the agent’s holdings of nominal money and government bonds in the end of period t , P_t the aggregate price level, W_t^c the contracted nominal wage, R_t^K the gross real return on the capital stock k_t , R_{t-1} the nominal interest rate on government bonds in between period $t-1$ and t , and TR_t nominal lump-sum transfers (or taxes if negative) from the government.

The agent has the following cash-in-advance constraint for the cash-good c_{1t} ,

$$P_t c_{1t} = m_t + (1 + R_{t-1}) b_t + TR_t - b_{t+1} \quad (3)$$

which always holds with equality since the nominal interest rate will always be positive in this model.

The government’s budget constraint is

$$P_t G_t + TR_t = M_{t+1} - M_t + B_{t+1} - (1 + R_{t-1}) B_t \quad (4)$$

where G is exogenous public consumption expenditures, and M and B aggregate nominal money supply and government bonds. As in Cooley and Hansen (1995), I will assume that $B_t = 0$ for $t \geq 0$ and only use it to compute the nominal interest rate in the economy. It can be shown that the nominal interest rate in equilibrium is given by

$$R_t = \frac{\alpha}{1 - \alpha} \frac{C_{2t}}{C_{1t}} - 1 \quad (5)$$

where C_{1t} and C_{2t} are aggregate consumption of the cash and credit goods, respectively.

Government consumption, G , in (4) is assumed to be generated by the following stationary AR(1)-process,

$$\ln G_{t+1} = (1 - \rho^{\ln G}) \ln \tilde{G} + \rho^{\ln G} \ln G_t + \varepsilon_{t+1}^{\ln G}, \quad 0 < \rho^{\ln G} < 1, \quad \varepsilon^{\ln G} \sim i.i.d. \ N(0, \sigma_{\ln G}^2). \quad (6)$$

Aggregate nominal money supply is assumed to evolve according to

$$M_{t+1} = e^{\mu_t} M_t \quad (7)$$

where the growth rate in nominal money supply in period t , defined as $\Delta \ln M_{t+1}$ and denoted μ_t , is assumed to be determined by

$$\begin{aligned} \mu_t &= \eta \mu_{t-1} - \lambda_\pi (\pi_t - \pi^*) - \lambda_Y (\ln Y_t - \ln Y^*) + \xi_t, \quad 0 < \eta < 1, \\ \xi &\sim i.i.d. \text{ Log Normal}, \quad E[\xi] = (1 - \eta) \bar{\mu}, \quad \text{Var}(\xi) = \sigma_\xi^2 \end{aligned} \quad (8)$$

where π_t is defined as $\ln P_t - \ln P_{t-1}$, and λ_π and λ_Y measure how the central bank reacts to deviations in the inflation ($\pi_t - \pi^*$) and the output gap ($\ln Y_t - \ln Y^*$).⁵ The implicit assumption underlying the specification in (8) is that the central bank tries to stabilize inflation and/or output, and one might think of (8) as an implied monetary policy rule for a central bank which has been attached a conventional quadratic loss function in the inflation and output gaps. For simplicity, we will also set π^* and $\ln Y^*$ in (8) equal to steady state nominal money supply growth ($\bar{\mu}$) and log of output ($\ln \bar{Y}$), respectively. By introducing the error term ξ and the persistence component $\eta \mu_{t-1}$, I implicitly assume that the central bank does not control μ_t perfectly, and that it reacts gradually to shocks which hits the economy.⁶

The production function is assumed to have constant returns to scale and be of Cobb-Douglas type

$$Y_t = e^{\ln Z_t} K_t^\theta H_t^{1-\theta} \quad (9)$$

where K_t and H_t are aggregate (average) capital stock and hours worked, respectively, and Z_t is the technology level. The perfect competition zero profit maximizing conditions

⁵ Although we assume that ξ is log normally distributed, we require that ξ has mean $(1 - \eta) \bar{\mu}$, and variance σ_ξ^2 as seen in (8). By using that $E[\xi] = e^{E[\ln \xi] + \frac{1}{2} \text{Var}(\ln \xi)}$ and that $\text{Var}(\xi) = E\{(\xi - E[\xi])^2\} = E[\xi^2] - [(1 - \eta) \bar{\mu}]^2 = e^{2E[\ln \xi] + \text{Var}(\ln \xi)} - [(1 - \eta) \bar{\mu}]^2$ since ξ is log-normally distributed, one can pin down the mean and the variance for $\ln \xi$ as $-\frac{1}{2} \ln \left(\sigma_\xi^2 + [(1 - \eta) \bar{\mu}]^2 \right) + 2 \ln ((1 - \eta) \bar{\mu})$ and $\ln \left(\sigma_\xi^2 + [(1 - \eta) \bar{\mu}]^2 \right) - 2 \ln ((1 - \eta) \bar{\mu})$ respectively.

⁶ Normally, the Taylor rule is specified in terms of the nominal interest rate, see for instance Taylor (1998). Under the assumption of conditional joint log-normality for C_{1t} and P_t , we can, ignoring government expenditures ($G_t = 0$), derive the following Taylor rule for the nominal interest rate R within the model $\ln(1 + R_t) = (1 - \eta) ((1 - \eta) \bar{\mu} - \ln \beta - \text{var}_t(\mu_{t+1})) + \eta \ln(1 + R_{t-1}) - \lambda_\pi (\pi_t - \pi^*) - \lambda_Y (\ln Y_t - \ln Y^*) + \varepsilon_t^R$ where $\varepsilon_t^R \equiv \lambda_\pi [\eta E_{t-1} \pi_t - E_t \pi_{t+1} + (1 - \eta) \pi^*] + \lambda_Y [\eta E_{t-1} \ln Y_t - E_t \ln Y_{t+1} + (1 - \eta) \ln Y^*] + \eta \xi_t$. Thus, it is possible to transform the Taylor rule for μ to a standard Taylor rule for R in the model. However, one may also note that it is most likely the case that R_t and μ_t is positively correlated in the model, at odds with the U.S. data.

for the representative firm are

$$W_t^c = (1 - \theta) e^{\ln Z_t} \left(\frac{K_t}{H_t} \right)^\theta P_t \quad (10)$$

and

$$R_t^K = \theta e^{\ln Z_t} \left(\frac{K_t}{H_t} \right)^{\theta-1}. \quad (11)$$

The nominal wage W_t^c is assumed to be set in period $t-1$ (see Cooley and Hansen (1995) for further details on the nominal wage arrangement) as

$$W_t^c = (1 - \theta) e^{E_{t-1} \ln Z_t} \left(\frac{K_t}{E_{t-1} H_t} \right)^\theta E_{t-1} P_t \quad (12)$$

where the capital stock in period t is known in $t-1$. If we combine (10) and (12) in natural logarithms, using (15) below, we obtain

$$\ln H_t = E_{t-1} \ln H_t + \frac{1}{\theta} (\ln P_t - E_{t-1} \ln P_t) + \frac{1}{\theta} \varepsilon_t^{\ln Z}. \quad (13)$$

Similarly, one realizes that the natural logarithm of h_t for an agent in equilibrium is given by

$$\ln h_t = E_{t-1} \ln H_t + \frac{1}{\theta} (\ln P_t - E_{t-1} \ln P_t) + \frac{1}{\theta} \varepsilon_t^{\ln Z}. \quad (14)$$

In (9), the technology level is assumed to be exogenous and the natural log of it to follow a stationary AR(1)-process

$$\ln Z_{t+1} = \rho^{\ln Z} \ln Z_t + \varepsilon_{t+1}^{\ln Z}, \quad \varepsilon^{\ln Z} \sim i.i.d. \quad N(0, \sigma_{\ln Z}^2). \quad (15)$$

Individual and aggregate investment in period t produces productive capital in period $t+1$ according to

$$k_{t+1} = (1 - \delta) k_t + i_t \quad (16)$$

and

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (17)$$

where δ is the rate of capital depreciation.

The aggregate resource constraint

$$Y_t = C_{1t} + C_{2t} + I_t + G_t \equiv C_t + I_t + G_t \quad (18)$$

also holds in every period where C_t is total consumption.

2.2 Equilibrium in the model

The equilibrium in the model consists of a set of decision rules for the agents $\ln k_{t+1} = k(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$, $\ln \hat{m}_{t+1} = \hat{m}(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$ and $\ln h_t = h(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$, and a set of aggregate decision rules $\ln K_{t+1} = K(\mathbf{S}_t)$, $\ln H_t = H(\mathbf{S}_t)$, $\ln \hat{P}_t = \hat{P}(\mathbf{S}_t)$ where $\mathbf{S}_t = [\ln Z_{t-1}, \varepsilon_t^{\ln Z}, \mu_{t-1}, \xi_t, \ln G_t, \ln K_t, \ln \hat{P}_{t-1}]'$ such that; (i) agents optimize utility, (ii) firms maximize profits, and (iii), individual decision rules are consistent with aggregate outcomes. Equilibrium condition (iii) implies that $k(\mathbf{S}_t, \ln K_t, 1) = K(\mathbf{S}_t)$, $\hat{m}(\mathbf{S}_t, \ln K_t, 1) = 1$, and $h(\mathbf{S}_t, \ln K_t, 1) = H(\mathbf{S}_t)$ for all \mathbf{S}_t .

In Appendix A, I describe how to compute the equilibrium in this model.

3 Estimation and calibration

The parameters in the model are determined in two ways. About half of the parameters (η , $\bar{\mu}$, σ_ξ^2 , λ_π , λ_Y , $\rho^{\ln G}$, $\sigma_{\ln G}^2$ and $\bar{g} \equiv \frac{\bar{G}}{\bar{Y}}$) are estimated on yearly U.S. data 1960-1997 with Instrumental Variables method (IV) and Maximum Likelihood (ML). The other half of the parameters (α , β , δ , γ , θ , $\rho^{\ln Z}$ and $\sigma_{\ln Z}^2$) are adapted from Cooley and Hansen (1995), and chosen so that the model's steady state properties are consistent with U.S. growth facts.

To estimate the parameters η , $\bar{\mu}$, σ_ξ^2 , λ_π , and λ_Y in (8) for different Fed chairmen periods, I collected quarterly data on real gross national product per capita in natural logarithms ($\ln Y_t$), growth rate in nominal money supply (μ_t) and the inflation rate in the consumer price index (π_t), and filtered the series for output with the Hodrick-Prescott (H-P) filter (see Hodrick and Prescott, 1997) to get a measure of $\ln Y_t - \ln Y^*$.⁷ To get a measure of $\pi_t - \pi^*$, I simply demeaned the series for π_t . The reason for collecting quarterly rather than yearly data is to get more observations in each subsample. I then estimated (8) with IV for the whole sample period (1960Q2 – 1997Q4), for chairman Burn's office period (1970Q1 – 1978Q1), chairman Volcker's office period (1979Q3 – 1987Q2), chairman Greenspan's office period (1987Q3 – 1997Q4), and omit chairman Miller as in Judd and

⁷ I use the common value 1600 (quarterly data) for the smoothness coefficient λ in the H-P filter. See also Appendix B for a detailed description of the raw data and data transformations.

Rudebusch (1998) because of his short tenure. The reason for estimating with IV rather than Ordinary Least Squares (OLS), is that OLS is likely to be an biased and inconsistent estimator due to the fact that we may have contemporaneous correlation between the error term and the regressors in (8). In terms of the theoretical model used in this paper, there will, via the equilibrium decision rules, be a positive correlation between the error term ξ_t and the regressors π_t and $\ln Y_t$ in (8). As instruments in the estimation, I therefore use $\ln Y_{t-1} - \ln Y^*$, μ_{t-1} and $\pi_{t-1} - \pi^*$ which are uncorrelated with the error term ξ_t in (8). The results of the estimations are reported in Table 1 (a constant is included in the regressions but not spelled out).

Table 1: IV estimation of (8) on different periods.

Estimation period	Estimation output								
	$\hat{\eta}$	$\hat{\lambda}_\pi$	$\hat{\lambda}_Y$	$\hat{\sigma}_\xi$	\bar{R}^2	D-W	B-G $\chi^2(4)$	J-B	T
Whole	0.947 (0.029)	0.014 (0.038)	0.084 (0.085)	0.0132	0.88	1.31	40.73 (0.000)	0.676 (0.713)	151
Burns	0.593 (0.162)	0.114 (0.069)	-0.156 (0.154)	0.0075	0.72	1.97	11.97 (0.018)	1.540 (0.463)	33
Volcker	0.701 (0.122)	0.185 (0.094)	-0.003 (0.228)	0.0166	0.70	1.57	12.58 (0.014)	1.003 (0.605)	32
Greenspan	0.931 (0.053)	-0.126 (0.260)	0.148 (0.271)	0.0158	0.91	0.99	13.00 (0.011)	0.655 (0.721)	42

Note: Standard errors in parenthesis for $\hat{\eta}$, $\hat{\lambda}_\pi$ and $\hat{\lambda}_Y$, and p -values in parenthesis for the Breusch-Godfrey autocorrelation test (null hypothesis no autocorrelation up to 4 lags) and the Jarque-Bera normality test (null hypothesis normally distributed residuals). A constant, $\ln Y_{t-1} - \ln Y^*$, μ_{t-1} and $\pi_{t-1} - \pi^*$ have been used as instruments. T denote the number of observations in the regressions.

From Table 1, the implied yearly estimates of η for the different periods are approximately 0.804 (0.947⁴), 0.123, 0.242 and 0.752 respectively. All the regressions show signs of positive autocorrelation, so it is therefore difficult to say anything about the significance levels of these estimates. It should, however, be emphasized that the estimates are not sensitive to this autocorrelation. When I augmented the regressions with lags on the dependent variables until the autocorrelation was removed, the estimates of λ_π and λ_Y and the implied estimate of η changed very little. Somewhat surprisingly, we get the lowest/highest estimate of λ_π/λ_Y during chairman Greenspan's office period, although the average inflation has been lowest during that period (see Judd and Rudebusch, 1998). It is, however, interesting to note that, when I reestimated (8) on H-P filtered data for all the variables involved in the regression, the estimates of λ_π changed to 0.309, 0.143, 0.177 and 2.035 respectively for the different periods, which means that Fed has reacted most

strongly to deviations in the inflation gap under chairman Greenspan. Consequently, the qualitative aspects of the results in Table 1 are dependent on the filtering method, but for the purposes of this study, this “problem” is not of decisive importance since the primary purpose of the paper not is to match “common belief” about the conduct of monetary policy, but rather to get reasonable benchmark estimates to study the effects of changes in policy parameters.

To estimate $\rho^{\ln G}$ and $\sigma_{\ln G}^2$ in (6), I collected yearly data series on real government expenditures on consumption and investment per capita in natural logarithms, and filtered the series with the Hodrick-Prescott (H-P) filter (see Hodrick and Prescott, 1997) to get a measure of $\ln G_t$.⁸ I then estimated (6) with ML with the result (standard error in parenthesis)

$$\ln G_t = \underset{(0.1040)}{0.7706} \ln G_{t-1} + \varepsilon_t^{\ln G}, \hat{\sigma}_{\ln G} = 0.01747, D-W = 0.94, T = 37, \bar{R}^2 = 0.60. \quad (19)$$

(19) shows tendencies of positive autocorrelation, but when I augmented the estimation with more lags on the dependent variable to remove this autocorrelation, I found that the estimated parameters were not much affected.

To compute values for $\bar{\mu}$ and \bar{g} , I took averages of nominal money growth and the ratio of government expenditures to gross national product to get 0.05345 and 0.21038 respectively.

Since Cooley and Hansen (1995) calibrated their model on quarterly data, I have mapped some of their parameters to yearly growth facts. More specifically; β is set to 0.9567 (0.989⁴) instead of 0.989; δ is set to 0.07386 ($1 - (1 - 0.019)^4$) instead of 0.019; $\rho^{\ln Z}$ is set to 0.8145 (0.95⁴) instead of 0.95; γ is set so that hours worked as share of available time in steady state, \bar{H} , equal 0.30.⁹ The parameter values for α , θ and $\sigma_{\ln Z}$ are set to 0.84, 0.40 and 0.00721 respectively as in Cooley and Hansen (1995).

⁸ I use the common value 100 (yearly data) for the smoothness coefficient λ in the H-P filter. See also Appendix B for a detailed description of the raw data and data transformations.

⁹ Formally, we have that $\gamma = \frac{(1-\theta)(\alpha\beta\frac{1}{\varepsilon^L} + 1 - \alpha)(\frac{1-\beta(1-\delta)}{\beta\theta})}{\bar{H}((1-\bar{g})(\frac{1-\beta(1-\delta)}{\beta\theta}) - \delta)}$, which can be used to compute $\gamma = 3.404$ given the values for the other parameters. This value is higher than Cooley and Hansen's value (2.53) since I have government expenditures in the model.

4 The theoretical importance of the Lucas critique

In this section, I will investigate whether the Lucas critique seems to be significant in a statistical sense in the equilibrium model by simulating the model for the different estimated “Taylor rules” for nominal money growth. Throughout this section, I will focus on output (per capita in natural logarithms), $\ln Y$, the inflation rate (π), the nominal interest rate (R), nominal money growth (μ), private consumption per capita in natural logarithms ($\ln C$), and the real money stock in natural logarithms (at the end of period t), denoted $\ln M^r$, since these variables are directly involved in the super exogeneity tests in the next section.¹⁰

4.1 Model simulations

To examine whether the different monetary policy regimes result in changes in volatility, cross correlation and autocorrelation patterns that are statistically significant, I have simulated the model 1000 times for 1000 periods for the different estimated “Taylor rules” and computed statistics for $\ln Y$, π , R , μ , $\ln C$ and $\ln M^r$ which reflect three dimensions in the model; volatility, contemporaneous correlation, and autocorrelation. The results are reported in Table 2.¹¹

From Table 2, we see that by using different monetary policy regimes in the model, it can replicate most moments in the U.S. economy fairly well, except for the standard deviation in $\ln M^r$ and the correlations for π and R with μ .¹² The negative correlations present in the data for these variables cannot be replicated by the model, and in this respect the model clearly fails. The strong positive correlations between these variables are partly a result of the cash-in-advance constraint technology and the simple shock structure in the model.

¹⁰ Note that M_t^r in the model is generated as $\frac{M_{t+1}}{P_t}$, which corresponds to $\frac{M_t}{P_t}$ in the data (which are measured at the end of the year), since M_{t+1} in the model is determined at the end of period t .

¹¹ The simulations are made in the GAUSS programming language, using the random number generator RDND with RDNDSEED set to $159425 + \text{iter}$ for $\text{iter} = 1, 2, \dots, 1000$. To get a stochastic initial state in each simulation, the model is simulated for additional 100 periods, which are then discarded.

¹² An alternative way of estimating the different policy rules would be to estimate the parameters in (8) with Simulated Method of Moments (SMM) on a given set of moments for the different monetary policy rule regimes'. One would then be able to test (see Lee and Ingram, 1991), whether the model can replicate moments in the data in a significant way.

Table 2: Summary statistics in the model economy and the U.S. economy.

Statistic	Estimated rule for monetary policy				U.S.
	Whole Sample	Burns	Volcker	Greenspan	
$\text{std}(\ln Y_t) * 100$	2.240 (0.228)	1.874 (0.045)	2.282 (0.069)	2.768 (0.258)	1.926 (0.046)
$\text{std}(\pi_t) * 100$	1.300 (0.148)	0.785 (0.020)	1.317 (0.046)	1.650 (0.154)	1.660 (0.049)
$\text{std}(R_t) * 100$	2.073 (2.138)	0.504 (0.030)	0.872 (0.090)	2.856 (1.450)	1.263 (0.026)
$\text{std}(\mu_t) * 100$	1.073 (0.142)	0.809 (0.020)	1.281 (0.047)	1.395 (0.145)	2.497 (0.110)
$\text{std}(\ln C_t) * 100$	0.634 (0.118)	0.585 (0.022)	0.632 (0.024)	0.764 (0.099)	1.243 (0.018)
$\text{std}(M_t^r) * 100$	0.579 (0.024)	0.490 (0.018)	0.535 (0.019)	0.693 (0.030)	4.703 (0.310)
$\text{corr}(\ln Y_t, \pi_t)$	0.431 (0.059)	0.492 (0.024)	0.697 (0.022)	0.313 (0.053)	-0.061 (0.032)
$\text{corr}(\ln Y_t, R_t)$	0.589 (0.058)	0.545 (0.049)	0.644 (0.031)	0.493 (0.078)	-0.142 (0.025)
$\text{corr}(\ln Y_t, \mu_t)$	0.473 (0.036)	0.790 (0.012)	0.834 (0.012)	0.314 (0.042)	-0.155 (0.029)
$\text{corr}(\ln Y_t, \ln C_t)$	0.344 (0.077)	0.490 (0.017)	0.317 (0.024)	0.432 (0.073)	0.806 (0.184)
$\text{corr}(\ln Y_t, \ln M_t^r)$	0.116 (0.072)	0.435 (0.023)	0.217 (0.031)	0.172 (0.051)	0.288 (0.018)
$\text{corr}(\pi_t, R_t)$	0.788 (0.033)	0.228 (0.041)	0.646 (0.027)	0.793 (0.048)	0.601 (0.098)
$\text{corr}(\pi_t, \mu_t)$	0.908 (0.016)	0.829 (0.010)	0.924 (0.006)	0.940 (0.007)	-0.321 (0.103)
$\text{corr}(R_t, \mu_t)$	0.833 (0.054)	0.451 (0.044)	0.724 (0.025)	0.809 (0.062)	-0.548 (0.068)
$\text{corr}(\ln Y_t, \ln Y_{t-1})$	-0.004 (0.042)	0.066 (0.033)	-0.011 (0.032)	-0.004 (0.040)	0.573 (0.084)
$\text{corr}(\pi_t, \pi_{t-1})$	0.231 (0.033)	-0.131 (0.030)	-0.098 (0.021)	0.355 (0.030)	0.513 (0.053)
$\text{corr}(R_t, R_{t-1})$	0.117 (0.080)	0.059 (0.042)	-0.062 (0.033)	0.148 (0.089)	0.549 (0.084)
$\text{corr}(\mu_t, \mu_{t-1})$	0.435 (0.027)	-0.029 (0.030)	0.028 (0.030)	0.505 (0.027)	0.477 (0.032)
$\text{corr}(\ln C_t, \ln C_{t-1})$	0.609 (0.094)	0.615 (0.023)	0.621 (0.021)	0.692 (0.011)	0.629 (0.088)
$\text{corr}(\ln M_t^r, \ln M_{t-1}^r)$	0.551 (0.027)	0.546 (0.027)	0.549 (0.026)	0.649 (0.023)	0.688 (0.112)

Note: Standard errors within parenthesis. Before the statistics in the table have been calculated, all the variables have been detrended with the H-P filter with the smoothness coefficient λ set to 100. The statistics for the U.S. economy are based on yearly data for the sample period 1960-1997. See Appendix B for sources and exact definitions of the U.S. data.

But what is most important for the purpose of this paper, is that Table 2 displays that the behavior of both nominal and real variables seems to change significantly, as indicated by the standard errors in parenthesis, when the central bank changes the policy rule. To test this proposition more formally, I have simulated the model 1000 times for 200 years and used a χ^2 -statistic to test simultaneously whether the 20 moments in Table 2 change when the monetary policy rule changes.¹³ The results are reported in Table 3.

Table 3: χ^2 - test for moment stability in the model economy.

Benchmark regime	Comparison regime			
	Whole sample	Burns	Volcker	Greenspan
Whole sample	NC	7273.9***	3035.6***	2110.4***
Burns	15200.8***	NC	485.6***	12984.7***
Volcker	3269.7***	546.8***	NC	4640.2***
Greenspan	1378.2***	8955.0***	2996.0***	NC

Note: NC is shorthand notation for not computed. $*(**)[***]$ indicates that the statistic is significant at the 10 (5) [1] percent level where the critical values are 28.412, 31.410 and 37.566 respectively. If we let \mathbf{m}_{bv} (\mathbf{m}_{cv}) denote a $j \times 1$ vector with moments generated with the benchmark (comparison) values and \mathbf{W}_{bv} a positive definite variance-covariance matrix for the benchmark moments, the χ^2 -statistic is computed as $T * (\mathbf{m}_{bv} - \mathbf{m}_{cv})' \mathbf{W}_{bv}^{-1} (\mathbf{m}_{bv} - \mathbf{m}_{cv})$ which is asymptotically distributed with j (here, $j = 20$) degrees of freedom under the null hypothesis that $\mathbf{m}_{bv} - \mathbf{m}_{cv} = 0$. The χ^2 -figures reported are averages from the 1000 simulations.

From Table 3 we see that the computed χ^2 -statistics are highly statistically significant in every case, implying that the structure of the economy changes in a statistically significant way when the policy rule changes. Between two chairman office periods, Burns and Volcker and vice versa, the changes in χ^2 -statistics are more moderate, and have a p -value only slightly less than 0.01 on the one percent significance level.

To sum up, the results in this section suggest that the Lucas critique is quantitatively important in this typical equilibrium model with a standard parameterization in the sense that the structure of the economy, and thus how the economy reacts to different policies, changes in a statistically significant way when the monetary policy rule changes.

¹³ Of course, if one simulates the model for an infinite number of periods, one can compute true moments, which will change when the policy rule change. Here, the purpose is rather to see whether the model structure changes in a statistically significant way for an upper bound sample size considered in empirical work.

5 Does the super exogeneity test detect the Lucas critique in practice?

In the previous section, we verified that the Lucas critique is quantitatively important in a statistically significant way in a typical equilibrium model with a standard parameterization when we moved between different monetary policy rule regimes. A natural question then arises: Why has, in principle, every paper that has tried to test whether the Lucas critique is significantly important in practice, using tests for super exogeneity, rejected the Lucas critique in practice? One possible explanation, investigated in this section, is that the test for super exogeneity is not able to measure the relevance of the Lucas critique in data sufficiently well.

The structure of this section is the following. First, following Engle and Hendry (1993), and Ericsson and Irons (1995) closely, I give a short introduction to the concept of super exogeneity, discuss how one can test for it, and for the sake of clarity also provide a formal definition of it. Then I use the theoretical model, where the Lucas critique is applicable, to investigate whether the super exogeneity test has enough power in small samples on simulated data. I present results for both the money demand equation and the consumption function in the model.

5.1 Super exogeneity: concept, testing and a formal definition

Consider the following simple (presented in reduced form) forward looking model in the spirit of Lucas (1977):

$$\begin{aligned} x_t &= \theta E_t \sum_{j=0}^{\infty} z_{t+j} + \varepsilon_t, \quad E_t \varepsilon_{t+j} = 0, \\ z_t &= \phi z_{t-1} + v_t, \quad -1 < \phi < 1, \quad E_t v_{t+j} = 0 \end{aligned} \tag{20}$$

In (20), z_t is the policy variable and x_t the target variable. Solving (20) for x_t as function of z_t , we obtain

$$x_t = \beta_z z_t + \varepsilon_t \tag{21}$$

where $\beta_z \equiv \theta/(1-\phi)$. If the econometrician estimates (21), ignoring the dependency of β_z on ϕ , then policy simulations based on the estimate $\hat{\beta}_z$ for alternative paths of $\{z_{t+j}\}_{j=0}^{\infty}$

(treating v_{t+j} as a fixed exogenous shock), and thus for alternative paths of ϕ , will give misleading results.

Testing for the constancy/non-constancy of ϕ and β_z in (20) and (21) by estimating these equations then provides a simple way of testing for the Lucas critique; if β_z is constant but ϕ is not, then the Lucas critique cannot apply. z_t is then, loosely speaking, said to be super exogenous to β_z . The testing procedure can generate three other combinations of constancy/non-constancy for ϕ and β_z , but those combinations can arise from other sources (that is, changes in other policy variables) and thus not constitute evidence for or against the Lucas critique in practice.¹⁴

Now, let us consider a general case and give a formal definition of super exogeneity. Formally, the joint distribution of x_t and z_t conditional on the sigma field, denoted \mathcal{F}_t , consisting of x_{t-1}, x_{t-2}, \dots and z_{t-1}, z_{t-2}, \dots , and the current and past observations on all other valid conditioning variables, can be written

$$D(x_t, z_t | \mathcal{F}_t, \lambda_t) = D_{x|z}(x_t | z_t, \mathcal{F}_t, \lambda_{1t}) D_z(z_t | \mathcal{F}_t, \lambda_{2t})$$

where D , $D_{x|z}$ and D_z denote the joint density, the conditional density of x_t given z_t , and the marginal density of z_t , respectively, and λ_t , λ_{1t} and λ_{2t} the corresponding parameters. Engle et al. (1983) define z_t as weakly exogenous for a set of parameters of interest θ if: (i) θ is a function of the parameters λ_{1t} alone, and (ii), λ_{1t} and the parameters λ_{2t} of the marginal model for z_t are variation free.¹⁵ Finally, Engle et al. (1983) define z_t as super exogenous for θ if z_t is weakly exogenous for θ and λ_1 is invariant to changes in λ_2 (that is, changes in λ_2 do not imply changes in λ_1).

5.2 Application 1: The money demand equation

In this subsection, I will investigate if the test for super exogeneity is able to judge the relevance of the Lucas critique in small samples by using simulated data from the

¹⁴ In the context of the theoretical equilibrium model, it may be that the processes for the other exogenous shocks $\ln Z_t$ and/or $\ln G_t$ have changed.

¹⁵ With the term "variation free" Engle et al (1983) mean that over periods of constant λ_{2t} , there is no information in λ_2 that would help estimating λ_1 . Note that variation free and invariance are different concepts since if $\lambda_{1t} = \phi \lambda_{2t}$ then λ_1 is said to be variation free but not invariant to λ_2 , but if the relation instead is $\lambda_1 = \phi_t \lambda_{2t} \forall t$, then λ_1 is said to be both variation free and invariant to λ_2 .

equilibrium model for a money demand equation and the Taylor rule for nominal money growth. We might think of money demand as x_t and monetary policy as z_t in (20).

By using the cash-in-advance constraint (3) in equilibrium, $C_{1t} = \frac{M_{t+1}}{P_t} - G_t$ (log linearized around steady state), (5) (log linearized around steady state, where I have used the approximation $\ln(1 + R_t) \approx R_t$ since R_t is a small number), and the resource constraint (log linearized around steady state) (18), it is possible to derive the following “true” money demand equation

$$\ln \left(\frac{M_{t+1}}{P_t Y_t} \right) + \frac{\bar{P}\bar{G}}{1-\bar{P}\bar{G}} \ln \left(\frac{M_{t+1}}{P_t G_t} \right) = \kappa_0 + \delta \kappa_1 (\ln Y_t - \ln I_t) + \kappa_2 (\ln Y_t - \ln G_t) - \kappa_3 R_t \quad (22)$$

where a bar over a variable denotes steady state, κ_1 denotes the capital-consumption ratio in steady state, κ_2 the government expenditure-consumption ratio in steady state, and κ_3 is shorthand notation for $\frac{\bar{C}-\bar{C}_1}{\bar{C}} = \frac{\bar{C}_2}{\bar{C}} > 0$. Note that M_{t+1} in (22) corresponds to M_t in the data (M_t in the data is the money stock at the end of period t) since M_{t+1} is determined at the end of period t .

Since $\ln I_t$ and $\ln G_t$ normally do not enter in the estimation of a money demand equation, (22) is approximated with the following traditional money demand equation

$$\ln \left(\frac{M_{t+1}}{P_t} \right) = \beta_0 + \beta_1 \ln Y_t + \beta_2 R_t + \beta_3 \ln \left(\frac{M_t}{P_{t-1}} \right) + \varepsilon_t^{MD}. \quad (23)$$

The purpose of approximating the true money demand equation in (22) with (23), is that this kind of misspecification could very well be made in practical work. It should be emphasized that an OLS estimation of (23) also produces “normal” results; $\beta_1 > 0$ (≈ 0.10), $\beta_2 < 0$ (≈ -0.15) and $\beta_3 > 0$ (≈ 0.80), the hypothesis that ε_t^{MD} is normally distributed and serially uncorrelated over time cannot be rejected, and the adjusted r -square is satisfactory (≈ 0.95).

To apply the test of super exogeneity on (23) together with the nominal money growth policy rule, (8), the procedure has been as follows:

1. Simulate the model for T periods for a given monetary policy regime (whole sample, Burns, Volcker or Greenspan).¹⁶

¹⁶ The simulations are made in the GAUSS programming language, using the random number generator RDND with RDNDSEED set to 159425 + $iter$ for $iter = 1, 2, \dots, N$. To get a stochastic initial state in each simulation, the model is simulated for $T + 100$ periods, where the first 100 are then discarded in the OLS estimations.

2. Estimate (23) and (8) with OLS on the simulated data. Denote the estimated parameter vectors $\hat{\beta}_{MD}$ and $\hat{\beta}_{TR}$ respectively.
3. Simulate the model for T periods again, using the same stochastic shocks for the exogenous processes as in step 1, under the assumption that the monetary policy rule changes from one regime to another (for example, from Burns to Volcker and Burns to Greenspan) completely unexpectedly after $T/2$ periods.
4. Estimate (23) and (8) with OLS again on the new data. Denote these estimated parameter vectors $\hat{\alpha}_{MD}$ and $\hat{\alpha}_{TR}$ respectively.
5. Use the F -test to examine if the null hypothesis $\hat{\alpha}_{MD} = \hat{\beta}_{MD}$ is maintained while the null $\hat{\alpha}_{TR} = \hat{\beta}_{TR}$ can be rejected on the ten, five and one percent significance level.¹⁷
6. Repeat step 1 to step 5 many (N) times to compute probabilities for how often the null hypothesis $\hat{\alpha}_{TR} = \hat{\beta}_{TR}$ is rejected while the null hypothesis $\hat{\alpha}_{MD} = \hat{\beta}_{MD}$ is maintained for the different significance levels.

If the computed probabilities in step 6 of rejecting parameter stability in (8) while accepting parameter stability in (23) at the same time are lower/higher than the given significance levels, the Lucas critique is/is not relevant according to the super exogeneity test. But since we know that the Lucas critique actually is relevant here, this provides evidence of that the super exogeneity test has sufficiently/not sufficiently high power in small samples.

The critical assumptions in step 1 to 6 are clearly made in step 3, and I would like to briefly comment on them. First, I have chosen to change monetary policy regime in the middle of the sample. The motivation behind this procedure is that it gives the super exogeneity test the highest possible power in the small sample simulations. Secondly, I have chosen to model the once and for all change in monetary policy regime as a completely unexpected shift in the estimated monetary policy rule where I in the simulations let the economy bring the state vector from the last period in the previous regime to the first

¹⁷ See, for instance, Greene (1993) pp. 203-206 for how to compute the F -statistic.

period in the new regime. By this procedure, I implicitly assume a first order Markov chain for the different monetary policy regimes where I let the diagonal elements in the transition matrix approach unity. This assumption is very convenient since it allows me to use the same decision rules for the first $T/2$ periods and then change to new decision rules only once in the beginning of period $T/2 + 1$ for the remaining $T/2$ periods.

The results of this exercise for the different estimated monetary policy rules, and sample sizes $T = 100$ and $T = 200$, are provided in Table 4.¹⁸ Since only simulations when the F -statistic for (8) exceed the critical value on the given significance level can be used, I also report in Table 4 the number of simulations used to compute the probabilities in parenthesis.¹⁹

The general message from Table 4 is striking: the probability of accepting the null hypothesis that the Lucas critique does not apply, and thus that the parameters in the money demand function are super exogenous to the Taylor rule for money growth, is far too high regardless of chosen significance level and sample size. This is a clear indication that the test for super exogeneity too often tends to accept a false null hypothesis, and hence seems to have weak power in small samples.²⁰

Three further comments on specific features in Table 4 are warranted. First, we see that a larger sample size improves the properties of the super exogeneity test, except when the economy change from Greenspan to "Whole sample" monetary policy rule. The finding regarding the change from Greenspan to "Whole sample" regime is somewhat puzzling, so I repeated the exercise in Table 4 for $T = 1000$ and $T = 10000$ to confirm that the super exogeneity test has enough power in general on these sample sizes, and improved

¹⁸ I use $T = 100$ and $T = 200$ in the simulations since $T = 100$ seems to be an upper bound on observations in the studies which have applied the super exogeneity test to assess the practical importance of the Lucas critique on yearly data.

¹⁹ To get sufficiently many simulations when parameter stability in (8) is rejected, I have simulated the model $N = 1000000$ times.

²⁰ I have also tested the properties of the super exogeneity test under the alternative hypothesis that no change in the monetary policy regime has occurred as follows. First, I estimate (for every regime; whole sample, Burns, Volcker and Greenspan) the parameters in (23) and (8) with OLS by simulating the model 10000000 times for $T = 100$ and $T = 200$ periods and regard the average values of these OLS estimates respectively as "true" parameters. Second, by using the "true" parameters, I simulate the model again 10000000 times to test how often the hypothesis " H_0 : Estimated parameters in (23) are equal to the true parameters while the estimated parameters in (8) are not equal to the true parameters" is (falsely) maintained for every regime. The results with this procedure are the same as with the procedure used in the main text; the probabilities of accepting H_0 are too high - indicating that the super exogeneity test does not seem to have enough power in small samples.

**Table 4: Small sample properties of the super exogeneity test:
F-test probabilities of accepting stability in money demand
while rejecting stability in nominal money growth.**

Benchmark regime	Comparison regime							
	Whole sample		Burns		Volcker		Greenspan	
	$T = 100$				$T = 200$			
	10 percent significance level							
Whole Sample	NC	0.666 (841554)	0.924 (990957)	0.798 (447713)	NC	0.457 (978645)	0.875 (999519)	0.737 (718860)
Burns	0.010 (999964)	NC	0.324 (861794)	0.001 (10 ⁶)	0.000 (10 ⁶)	NC	0.053 (999456)	0.000 (10 ⁶)
Volcker	0.754 (862424)	0.877 (4792)	NC	0.300 (996969)	0.666 (998502)	0.726 (116845)	NC	0.138 (999999)
Greenspan	0.757 (16035)	0.524 (608289)	0.792 (965419)	NC	0.875 (28461)	0.372 (917534)	0.710 (999598)	NC
	5 percent significance level							
Whole Sample	NC	0.736 (74857)	0.949 (98425)	0.831 (36845)	NC	0.559 (958035)	0.916 (999059)	0.795 (642845)
Burns	0.015 (99991)	NC	0.400 (75272)	0.001 (10 ⁶)	0.000 (10 ⁶)	NC	0.094 (997572)	0.000 (10 ⁶)
Volcker	0.806 (86326)	0.927 (467)	NC	0.370 (99701)	0.745 (995969)	0.818 (49920)	NC	0.203 (10 ⁶)
Greenspan	0.766 (1649)	0.597 (60815)	0.848 (96577)	NC	0.886 (14355)	0.458 (849403)	0.791 (999005)	NC
	1 percent significance level							
Whole Sample	NC	0.837 (517897)	0.976 (954199)	0.875 (242394)	NC	0.723 (875465)	0.959 (997189)	0.873 (496924)
Burns	0.031 (999501)	NC	0.529 (477313)	0.003 (999999)	0.000 (10 ⁶)	NC	0.207 (978562)	0.000 (10 ⁶)
Volcker	0.872 (732423)	0.977 (298)	NC	0.496 (991870)	0.851 (982050)	0.930 (6429)	NC	0.341 (999982)
Greenspan	0.769 (5753)	0.718 (396309)	0.917 (886188)	NC	0.873 (3732)	0.605 (649308)	0.892 (995228)	NC

Note: NC is shorthand notation for not computed. The probability in each entry is formally defined as $\Pr(F_{MD} < F_{0,XX} \mid F_{TR} > F_{0,XX})$ where F_{TR} denotes the computed F -statistic for the monetary policy rule, F_{MD} the computed F -statistic for money demand, and XX the i 'th percentile in the given distribution (90, 95 or 99'th percentile). The probability in each entry is formally defined to be significant on the $(1 - 0.XX) * 100$ percent level if $\Pr(F_{MD} < F_{0,XX} \mid F_{TR} > F_{0,XX}) > 1 - 0.XX$. Under the null hypothesis, the F -statistic follows the F -distribution with J, T degrees of freedom where J is the number of parameter restrictions (here, $J = 4$) and T the number of observations. In parenthesis, the number of cases when $F_{TR} > F_{0,XX}$ in the simulations is provided. The reported figures are based on 1000000 simulations.

power for the change from Greenspan to “Whole sample” regime when T was increased from 1000 to 10000. Secondly, we note that the figures in parenthesis (the number of cases when structural stability in the monetary policy rule is rejected out of 1000000) in general increase when T increase. This is natural since the number of observations when the economy is exposed to the two different regimes increases, which makes it easier for the F -test to reject structural stability in the monetary policy rule. Finally, we see that structural stability in the monetary policy rule is rejected in few cases when we move from the Volcker to Burns, and Greenspan to “Whole sample” regimes. This is natural since the estimates of η , λ_π and λ_Y in the monetary policy rules for these regime pairs are similar (see Table 1) and the estimated standard errors of the residuals, $\hat{\sigma}_\xi^2$, are highest for the regimes (Volcker and Greenspan) which initialize the first $T/2$ periods in the simulations, and therefore make it hard for the F -test to identify the structural instability between these regime pairs.

5.3 Application 2: The consumption function

In this subsection, I will investigate the properties of the test for super exogeneity by using simulated data from the equilibrium model to estimate a consumption function together with the Taylor rule for nominal money growth. We might think of consumption as x_t and monetary policy as z_t in (20).

By using the same equations and approximations as in the previous subsection (which were used to derive the “true” money demand equation), it is possible to derive the following “true” consumption function

$$\ln C_t = \kappa_4 - \frac{\bar{P}}{1 - \bar{P}\bar{G} - \bar{P}\bar{C}} (\bar{Y} \ln Y_t - \delta \bar{K} \ln I_t) + \frac{1 - \bar{P}\bar{G}}{1 - \bar{P}\bar{G} - \bar{P}\bar{C}} \kappa_3 R_t - \frac{1}{1 - \bar{P}\bar{G} - \bar{P}\bar{C}} \ln \left(\frac{M_{t+1}}{P_t} \right) \quad (24)$$

where a bar over a variable denotes steady state, and $\bar{Y} = \bar{C} + \delta \bar{K} + \bar{G}$. Note that M_{t+1} in (24) corresponds to M_t in the data since M_{t+1} is determined in period t . It is possible to show that $1 - \bar{P}\bar{G} - \bar{P}\bar{C} < 0$, so that an increase in output/nominal interest rate increases/decreases consumption.

Since $\ln I_t$ and $\ln \left(\frac{M_{t+1}}{P_t} \right)$ normally do not enter in the estimation of a traditional consumption function, (24) is approximated with the following “keynesian flavored” con-

sumption function

$$\ln C_t = \beta_0 + \beta_1 \ln Y_t + \beta_2 R_t + \beta_3 \ln C_{t-1} + \varepsilon_t^{CF}. \quad (25)$$

Again, the purpose of approximating the true consumption function in (24) with (25), is that this kind of misspecification could very well be made in practical work. Again, it should be emphasized that an OLS estimation of (25) also produces reasonable results; $\beta_1 > 0$ (≈ 0.15), $\beta_2 < 0$ (≈ -0.20) and $\beta_3 > 0$ (≈ 0.80), the hypothesis that ε^{CF} is normally distributed and serially uncorrelated over time cannot be rejected, and the adjusted r-square is satisfactory (≈ 0.96).

The procedure in the testing for super exogeneity here is exactly the same as in the previous subsection, except for that the consumption function in (25) is used instead of money demand in (23), and thus not repeated here. The results for consumption are reported in Table 5.²¹

From Table 5, we again see that the probabilities of accepting the null hypothesis that the Lucas critique is not relevant, are far too high (in fact, they are actually higher than those in Table 4 on average). Only in one case, the shift from the Burns to the Greenspan regime, the super exogeneity test works reasonably properly. So also in this application, we find that the super exogeneity test does not have enough power in small samples.²²

To sum up this section, it seems to be a robust finding in this model that the test for super exogeneity does not have enough power in small samples, and hence is not able to shed light on the practical importance of the Lucas critique in small samples.²³

²¹ Since the cases when structural stability in nominal money growth is rejected are the same as those in Table 4, they are for ease of exposition not provided in Table 5.

²² I have also tested the properties of the super exogeneity test on consumption and money growth under the alternative approach described in footnote 20. The results with this alternative approach are the same as those in Table 5; the probabilities of accepting H_0 are too high - indicating that the super exogeneity test does not have enough power in small samples.

²³ Since monetary policy rules nowadays normally are estimated on nominal interest rates, I have also investigated this combination by estimating $R_t = \beta_0 + \beta_1 (\pi_t - \pi^*) + \beta_2 (\ln Y_t - \ln Y^*) + \beta_3 R_{t-1} + \varepsilon_t^R$ and applied the super exogeneity test on this relation together with (23)/(25). But since the results were qualitatively the same as those reported in Table 4 and 5 (in fact, the properties of the super exogeneity test were even worsened in this setting), they are not reported.

**Table 5: Small sample properties of the super exogeneity test:
F-test probabilities of accepting stability in consumption
while rejecting stability in nominal money growth.**

Benchmark regime	Comparison regime							
	Whole sample				Burns			
	$T = 100$				$T = 200$			
	10 percent significance level							
Whole sample	NC	0.582	0.910	0.628	NC	0.376	0.868	0.486
Burns	0.284	NC	0.894	0.062	0.111	NC	0.690	0.014
Volcker	0.619	0.934	NC	0.203	0.556	0.871	NC	0.106
Greenspan	0.711	0.520	0.802	NC	0.828	0.411	0.768	NC
	5 percent significance level							
Whole sample	NC	0.660	0.940	0.679	NC	0.479	0.913	0.561
Burns	0.357	NC	0.939	0.087	0.165	NC	0.828	0.022
Volcker	0.671	0.958	NC	0.243	0.627	0.929	NC	0.137
Greenspan	0.715	0.593	0.852	NC	0.830	0.498	0.829	NC
	1 percent significance level							
Whole sample	NC	0.778	0.972	0.752	NC	0.656	0.958	0.686
Burns	0.496	NC	0.974	0.147	0.293	NC	0.954	0.045
Volcker	0.748	0.983	NC	0.317	0.730	0.980	NC	0.197
Greenspan	0.706	0.714	0.914	NC	0.807	0.640	0.904	NC

Note: NC is shorthand notation for not computed. The probability in each entry is formally defined as $\Pr(F_{CF} < F_{0,XX} \mid F_{TR} > F_{0,XX})$ where F_{TR} denotes the computed *F*-statistic for the monetary policy rule, F_{CF} the computed *F*-statistic for the consumption function, and XX the *i*'th percentile in the given distribution (90, 95 or 99'th percentile). The probability in each entry is formally defined to be significant on the $(1 - 0.XX) \times 100$ percent level if $\Pr(F_{CF} < F_{0,XX} \mid F_{TR} > F_{0,XX}) > 1 - 0.XX$. Under the null hypothesis, the *F*-statistic follows the *F*-distribution with *J*, *T* degrees of freedom where *J* is the number of parameter restrictions (here, *J* = 4) and *T* the number of observations. The reported figures are based on 1000000 simulations.

6 Concluding remarks

In this paper, I have tried to shed new light on why the Lucas critique, although believed to be very important by most economists, has received very little empirical support (see Ericsson and Irons, 1995). Using Cooley and Hansen's (1995) standard real business cycle model with a Taylor inspired rule for monetary policy calibrated to different Federal Reserve regimes (Chairmen Burns, Volcker and Greenspan, and the period 1960-1997) following Judd and Rudebusch (1998), I have found, first, that the Lucas critique is quantitatively important in theoretical model in a statistical sense, and secondly, that the super exogeneity test, which is used to identify the relevance/nonrelevance of the Lucas critique in practice, does not have enough power in small samples. Consequently, a possible reason why the Lucas critique has received very little empirical support, is that the super exogeneity test is not able of detecting the applicability/nonapplicability of the Lucas critique in practice.

One thing which I think might be important for the obtained results, is that I have approximated "true" (in the model) money demand and consumption functions with traditional IS-LM functional forms for these equations, and used the approximated relationships together with the monetary policy rule in the investigation of the properties of the super exogeneity test. I would like to stress, however, that the estimated approximated money demand and consumption functions from a statistic point of view are fully acceptable: the estimated coefficients are very reasonable from an economic perspective and similar to those obtained elsewhere in the literature, and the estimated regressions pass a battery of statistical tests for autocorrelation, normality of residuals and so forth. To which extent are then my results driven by this misspecification? On one hand, I think that this deserves to be investigated further, but, on the other hand, since no researcher really knows the true money demand and consumption function in the real world, this type of error could have been made in the empirical studies of the Lucas critique just as well.

What could I have done differently? I have chosen to follow Judd and Rudebusch's (1998) empirical classification of monetary policy regimes for the U.S. An alternative

approach would have been to follow a more theoretical approach, as in Rudebusch and Svensson (1998), by considering monetary policy rules that are consistent with some kind of optimizing behavior of a central bank. I would, however, be very surprised if this alternative approach would change the qualitative conclusions regarding the properties of the super exogeneity test in small samples.²⁴ Obviously, I could also have investigated other relations than money demand and consumption together with the policy rule for nominal money growth. But I do not think that this would affect the results at all within the framework adopted here.

What about the limitations of the study? In this paper, I have implicitly assumed that the institutional design of the economy (that is, the one period nominal wage contacting assumption in the model) is unaffected by the monetary policy regime shifts. This assumption can be motivated by the real world observation that institutions, for example labor market arrangements, change very slowly over time. Consequently, the effects of institutional changes when testing for the Lucas critique in practice should be of “second order”, while the effects of monetary policy regime shift examined in this paper should be of “first order”.²⁵

To sum up, the results in this paper suggest that more research about the properties of the super exogeneity test is warranted before one may draw the conclusion that the Lucas critique is not empirically relevant.

²⁴ I have done some experiments with a quadratic loss function of the form $L_t = \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda (\ln Y_t - \ln Y^*)^2 \right]$ for the central bank for values of λ equal to 0, 0.3, 1 and 5 as in Svensson and Rudebusch (1998) (Barro and Broadbent (1997) have estimated $\lambda \approx 0.3$ on U.S. data) to find that the qualitative conclusions were unaffected.

²⁵ If a sufficiently long period is considered (100 years or so) the effects of institutional changes may be larger, see for instance Fregert and Jonung (1998). However, when I think of the effects of the Lucas critique here, I have in mind a shorter period (5 to 10 years or so).

Appendix A Computation of equilibrium

In order to make all variables in the deterministic version of the model above converge to a steady state, I transform the nominal variables by dividing m_{t+1} and P_t with M_{t+1} , and m_t with M_t . If we introduce the notation

$$\hat{m}_{t+s} \equiv \frac{m_{t+s}}{M_{t+s}} \text{ and } \hat{P}_{t+s} \equiv \frac{P_{t+s}}{M_{t+s+1}}$$

and use the transformations to rewrite the equations (2), (3), (4), (8), (13) and (14), the representative agent's optimization problem can, following Hansen and Prescott (1995), be expressed as the recursive dynamic programming problem:

$$V(\mathbf{S}_t, \hat{m}_t, k_t) \equiv \max_{\{\hat{m}_{t+1}, h_t, k_{t+1}\}} [\alpha \ln(c_{1t}) + (1 - \alpha) \ln(c_{2t}) - \gamma h_t + \beta E_t V(\mathbf{S}_{t+1}, \hat{m}_{t+1}, k_{t+1})] \quad s.t. \quad (A.1)$$

$$\begin{aligned} c_{1t} &= \frac{\hat{m}_t + e^{\mu_t} - 1}{e^{\mu_t} \hat{P}_t} - G_t, \\ c_{2t} &= \frac{W_t^c / M_{t+1}}{\hat{P}_t} h_t + (1 + R_t^K - \delta) k_t - k_{t+1} - \frac{\hat{m}_{t+1}}{\hat{P}_t}, \\ \mu_t &= \frac{\eta}{1+\lambda_\pi} \mu_{t-1} - \frac{\lambda_\pi}{1+\lambda_\pi} \left(\ln \hat{P}_t - \ln \hat{P}_{t-1} - \pi^* \right) - \frac{\lambda_Y}{1+\lambda_\pi} (\ln Y_t - \ln Y^*) + \frac{1}{1+\lambda_\pi} \xi_t, \\ H_t - E_{t-1} \ln H_t &= \frac{1}{\theta(1+\lambda_\pi) + (1-\theta)\lambda_Y} \left(\ln \hat{P}_t - E_{t-1} \ln \hat{P}_t \right) + \frac{1+\lambda_\pi - \lambda_Y}{\theta(1+\lambda_\pi) + (1-\theta)\lambda_Y} \varepsilon_t^{\ln Z} + \frac{\xi_t - (1-\eta)\bar{\mu}}{\theta(1+\lambda_\pi) + (1-\theta)\lambda_Y}, \\ h_t - E_{t-1} \ln h_t &= \frac{1}{\theta(1+\lambda_\pi) + (1-\theta)\lambda_Y} \left(\ln \hat{P}_t - E_{t-1} \ln \hat{P}_t \right) + \frac{1+\lambda_\pi - \lambda_Y}{\theta(1+\lambda_\pi) + (1-\theta)\lambda_Y} \varepsilon_t^{\ln Z} + \frac{\xi_t - (1-\eta)\bar{\mu}}{\theta(1+\lambda_\pi) + (1-\theta)\lambda_Y}, \\ \ln K_{t+1} &= K(\mathbf{S}_t), \ln H_t = H(\mathbf{S}_t), \ln \hat{P}_t = \hat{P}(\mathbf{S}_t). \end{aligned}$$

In (A.1), \mathbf{S}_t is a 1×8 row vector which contains all the aggregate state variables $\ln Z_{t-1}$, $\varepsilon_t^{\ln Z}$, μ_{t-1} , ξ_t , $\ln G_t$, $\ln K_t$, $\ln \hat{P}_{t-1}$ and a constant term. If $\lambda_\pi = 0$, then $\ln \hat{P}_{t-1}$ vanishes in \mathbf{S}_t . In maximization of (A.1), the agent takes the economy-wide aggregate (average) variables as given. The functions K , \hat{P} and H describe the relationship perceived by agents between the aggregate decision variables and the state of the economy. As the solution to the problem in (A.1), we have the agent's decision rules $\ln k_{t+1} = k(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$, $\ln \hat{m}_{t+1} = \hat{m}(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$ and $\ln h_t = h(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$. The competitive equilibrium is obtained when the individual and average decision rules coincide for $\ln k_t = \ln K_t$ and $\ln \hat{m}_{t+1} = \ln \hat{m}_t = 0$.

Since it is impossible to derive the decision rules analytically, I have used the same method as Cooley and Hansen (1995) and computed the decision rules numerically by approximating the original problem with a second order Taylor expansion around the constant steady state values in the nominal-growth adjusted economy. As a consequence of this approximation, the method produces linear decision rules (in natural logarithms for K_{t+1} , H_t and \hat{P}_t). The algorithm utilized is described in detail in Hansen and Prescott (1995).

Appendix B Data sources and definitions

In this appendix, I provide the sources of the data collected in Table B.1 below.

Table B.1: The data set.

Variables	Sample period	Source
GNP	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St Louis
CNDS	1960:1 - 1997:12	FRED database, Federal Reserve Bank of St Louis
GEC	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St Louis
M1	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St Louis
POP	1960-1996	OECD Main Economic Indicators
CPI	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St Louis
TB1Y	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St Louis

Note: All real macroeconomic variables are measured in 1992 billion U.S. dollars. Abbreviations; GNP denotes real (fixed, seasonally adjusted) gross national product; CNDS real (chained, seasonally adjusted) personal expenditures on nondurable goods and services; GEC real (chained, seasonally adjusted) government consumption and investment; M1 (not seasonally adjusted) nominal money supply 1; CPI (not seasonally adjusted) consumer price index; POP average U.S. population (for 1997, POP is set equal to average gross growth rate times the value for 1996); TB1Y (ultimo) maturity nominal interest rate on a one year U.S. treasury bill.

The transformations made to generate the variables used in Table 1 are displayed in Table B.2.

Table B.2: Generation of composite quaterly data series.

Variable	Sample period	Calculation formula
$\ln Y$	1960Q1-1997Q1	$\ln (\text{GNP}/\text{POP})$
μ	1960Q1-1997Q4	$\ln (\text{M1}_t/\text{M1}_{t-4})$
$\pi - \pi^*$	1960Q1-1997Q4	$\ln (\text{CPI}_t/\text{CPI}_{t-4}) - \frac{1}{38} \sum_{t=1960Q1}^{1997Q4} \ln (\text{CPI}_t/\text{CPI}_{t-4})$

Note: To get a measure of $\ln Y - \ln Y^*$, $\ln Y$ is then subject to Hodrick-Prescott filtering with the smoothness coefficient λ set to 1600, as described in section 3.

To transform the quarterly and monthly data in Table B.1 to yearly data, which are

used in (19) and to compute summary statistics for the U.S. in Table 2, I added up all the quarterly and monthly observations within a year to get a yearly observation. The final transformations of the as of now yearly data are reported in Table B.3.

Table B.3: Generation of composite yearly data series.

Variable	Sample period	Calculation formula
$\ln Y$	1960-1997	$\ln (\text{GNP}/\text{POP})$
$\ln C$	1960-1997	$\ln (\text{CNDS}/\text{POP})$
$\ln G$	1960-1997	$\ln (\text{GEC}/\text{POP})$
\bar{g}	1960-1997	$\frac{1}{38} \sum_{t=1960}^{1997} (\text{GEC}_t/\text{GNP}_t)$
$\ln M^r$	1960-1997	$\ln (\text{M1}/\text{CPI}/\text{POP})$
μ	1960-1997	$\ln (\text{M1}_t/\text{M1}_{t-1})$
$\bar{\mu}$	1960-1997	$\frac{1}{38} \sum_{t=1960}^{1997} \mu_t$
π	1960-1997	$\ln (\text{CPI}_t/\text{CPI}_{t-1})$
R	1960-1997	TB1Y

Note: $\ln Y$, $\ln C$, $\ln G$, $\ln M^r$, μ , π and R are then subject to Hodrick-Prescott filtering with the smoothness coefficient λ set to 100, as described in section 4.

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