Monetary Policy under Uncertainty

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to Helena

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Acknowledgments

IF THERE IS ONE THING I HAVE LEARNED from spending ten years studying economics and eventually writing a doctoral thesis, it is this: the more you study economics, the more things you realize that you do not fully understand. It is therefore not obvious that I walk out of the Stockholm School of Economics ten years later with a clearer view of the workings of the economy. What is clear, however, is that on the long road from being a young, confused undergraduate student in the late 1980s to being an older, possibly more confused, prospective Ph.D. in the late 1990s, I have had the great fortune to meet a number of interesting people, all of whom have influenced me in one way or another.

During my undergraduate years, the decision to specialize in economics was crucially influenced by Lars Bergman and Lars Jonung at the Stockholm School of Economics, and Stefano Zamagni at the Università Bocconi in Milan. Although they probably did not realize it themselves, their interest in the field of economics certainly spurred my interest and fascination with the subject.

When considering writing a thesis in the field of macroeconomics, I was lucky to run into an equally interested fellow student, Alexis Stenfors, who became my co-author, and I believe we both gained many insights into the world of economic research. In particular, we learned how important it is to have a good advisor: luckily we found one in Anders Vredin, who has since remained my mentor and a constant source of support and constructive criticism. I am now very much looking forward to continuing our collaboration at my new place of work, Sveriges Riksbank.

Having spent my first two years at the graduate program of the Stockholm School of Economics under the influence of, for example, Jörgen Weibull and Lars Svensson, I received a grant to spend the academic year of 1995/96 at the Massachusetts Institute of Technology. There I had the opportunity to study for, *inter alia*, Daron Acemoglu, Olivier Blanchard, Bengt Holmström, and Robert Solow, all of whom have contributed to my current knowledge of economics.

Back in Stockholm, I spent the summer of 1996 as a visiting student at the economics department of Sveriges Riksbank. I am most grateful to the bank, and especially to Claes Berg and Peter Sellin, for making this a very enjoyable experience.

Having shortly despaired about ever finding a suitable topic for my thesis, in the autumn of 1996 I was approached by Tore Ellingsen, who needed someone to work out an idea he had about monetary policy (Tore being too busy to do it himself). One thing led to another, and one day we had finished two papers, which were later merged into one and became the first chapter of this thesis. Tore soon accepted to be my advisor for the rest of the thesis work, and he has done his job in an admirable way, due to his profound interest in every field of economics. Tore has always been available; for my many questions about the thesis, as a career advisor, and as a tennis partner.

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Apart from economics and monetary policy, I have long had a passion for classical music, which has influenced me also in my professional life. Outside of school, I have been fortunate to spend much time in various constellations of amateur music, and I owe much gratitude to the members of the *S:t Matteus* Symphony Orchestra, the Orchestra *Filialen*, *Bergslagens Kammarsymfoniker*, the MIT Chamber Music Society (in particular my coaches David Deveau and Lynn Chang), and the Archi String Quartet.

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Stockholm, March 1999

Introduction and summary

IN A WORLD WITHOUT UNCERTAINTY, monetary policy would be a simple task. Every week, policymakers would listen to their staff's reports about the current state of the economy, and then quickly decide about the appropriate measures to take: whether to increase or decrease the rate of growth of the money stock (by adjusting their interest rate instrument). In fact, in a world of complete certainty, the job of central bankers could easily be done by a computer, programmed to respond in a suitable way to any new developments of the economy.

Fortunately for central bankers, the world is substantially more complicated than this. Since economic agents (that is, people) respond in more or less unpredictable ways to changing circumstances, monetary policy decisions are made under considerable amounts of uncertainty, both about the current state of the economy and about the effects of monetary policy on the economy. Therefore monetary policymaking is often described as an art rather than a science (if there is a contradiction between the two).

William Poole, a distinguished economist and President of the Federal Reserve Bank of St. Louis, summarizes the uncertainty facing monetary policymakers under five headings (Poole, 1998): uncertainty concerning (1) the data; (2) future events, shocks and disturbances; (3) how the economy works; (4) market reactions to monetary policy; and (5) market anticipations of monetary policy. This thesis deals with issues falling within the last three of these categories.

The first chapter, *Monetary policy and market interest rates*, written with my advisor Tore Ellingsen, deals with financial market reactions to policy. In this chapter, we investigate how interest rates set on financial markets respond to changes in the monetary policy stance. It is well established that market interest rates on average are positively related to the short interest rate instrument of central banks, so that interest rates of all maturities in general go up as the central bank rate is increased. It is equally well known, however, that there are many exceptions to this rule, so that long-term interest rates often fall when the central bank tightens policy. Also, economists disagree about what should be the normal response of interest rates to policy changes: some argue that market rates of long maturities should be positively related to the central bank rate, whereas others argue that a negative relationship should be expected. In a theoretical model, we show that this relationship can be positive or negative, depending on market participants' interpretation of the factors underlying the central bank's policy adjustment. If monetary policy reveals information about economic developments, interest rates of all maturities move in the same direction in response to a policy innovation. If, on the other hand, monetary policy reveals information about the central bank's policy preferences, short and long interest rates move in opposite directions. Using newspaper reports from the *Wall Street Journal* to classify monetary policy moves in the United States from 1988 to 1997, we find empirical support for the theoretical predictions.

The second chapter, *Predicting monetary policy using federal funds futures prices*, examines market anticipations of monetary policy. Using prices on federal funds futures, traded at the Chicago Board of Trade since October 1988, I extract market expectations about near-term changes in the Federal Reserve's main policy instrument, the federal funds rate, and examine how well these market expectations predict actual changes in policy. It turns out that the extracted expectations are too noisy to predict day-to-day changes in policy; partly because of time aggregation problems, partly because they are affected by factors other than monetary policy considerations. Nevertheless, from 1994 onwards they perform quite well in predicting the target level that will prevail after the next policy meeting of the Fed, especially when taking market regularities across months and trading days into account.

Chapters 3 and 4 both deal with uncertainty about the workings of the economy. In Chapter 3, *Monetary policy with uncertain parameters*, I investigate the effects of uncertainty about a model's parameters on optimal monetary policy. Accepted wisdom says that introducing multiplicative parameter uncertainty into a model leads to optimal policy that is less aggressive than under certainty. In this chapter, however, I show that this presumption is not necessarily correct: under certain parameter configurations, increasing uncertainty instead leads to *more* aggressive policy. Thus, the effects of parameter uncertainty on policy seem less clear-cut than previously recognized.

The final chapter, *Should central banks be more aggressive?*, tries to explain why some simple models of monetary policy typically imply more aggressive policy behavior than what is observed in reality. Interpreting a simple monetary policy model as a restricted version of a more general model, the model restrictions are formally tested, and the optimal policy response to shocks in the restricted and the unrestricted models are confronted with the empirically observed policy response in the United States. It is shown that the restrictions of the simple model lead to excessively aggressive policy behavior compared with observed policy, whereas the policy response from the unrestricted model is quite close to actual policy, especially when incorporating parameter uncertainty into the model. Consequently, the general setup of a central bank maximizing a quadratic utility function seems to be a reasonable approximation of actual central bank behavior, but the restrictions introduced in some models have counterfactual implications for the prescribed policy strategy.

Sammanfattning på svenska

Centralbankers huvudsakliga uppgift består i att kontrollera mängden pengar i omlopp i en ekonomi. En av de få saker nationalekonomer är överens om, är att en ökning av penningmängden leder till högre priser, medan en reduktion av penningmängden leder till (temporärt) lägre produktion. Eftersom både inflation och avvikelser från normal produktionsnivå medför samhällsekonomiska kostnader, koncentrerar sig centralbanker på att hålla utbudet av pengar på en "lagom" nivå.

Dock präglas penningpolitiska beslut av en stor portion osäkerhet, både om hur ekonomin utvecklas och om vilka effekter penningpolitik har på ekonomin. De fyra uppsatser som tillsammans utgör denna avhandling behandlar tre olika aspekter av sådan osäkerhet: om hur finansiella marknader reagerar på penningpolitiska beslut (kapitel 1), om vilka förväntningar aktörer på de finansiella marknaderna har om penningpolitiska beslut (kapitel 2), samt osäkerhet om hur ekonomin egentligen fungerar (kapitel 3 och 4).

Det första kapitlet, som är skrivet tillsammans med min handledare Tore Ellingsen, undersöker hur finansiella marknader reagerar när centralbanker justerar sin styrränta. I en teoretisk modell visar vi att marknadsräntor reagerar olika beroende på marknadsaktörernas tolkning av skälen bakom räntejusteringen. Om en justering anses avspegla ny information om ekonomins utveckling, rör sig alla räntor åt samma håll. Om däremot justeringen tolkas som att penningpolitikens allmänna inriktning har förändrats, reagerar räntor av lång och kort löptid åt olika håll. Dessa teoretiska resultat finner stöd i en empirisk studie av de amerikanska finansiella marknaderna.

Det andra kapitlet visar hur man från priser på finansiella instrument kan utläsa marknadsaktörers förväntningar om framtida styrräntejusteringar i USA. De erhållna förväntningsmåtten är mycket användbara för att förutspå kommande förändringar i den amerikanska styrräntan, även om de bör justeras något under vissa kalendermånader och under de sista dagarna i varje månad.

Det tredje kapitlet undersöker hur osäkerhet om den ekonomiska omgivningen (eller mer exakt, osäkerhet om de exakta parametrarna i en ekonomisk modell) påverkar den penningpolitiska strategin. I motsats till tidigare resultat visas att ökad osäkerhet kan leda både till mer eller mindre aggressivt beteende hos centralbanken.

Det fjärde kapitlet, slutligen, undersöker den optimala penningpolitiska strategin i en enkel makroekonomisk modell, och jämför denna med strategin från en mer komplett modell och med den strategi som följs av den amerikanska centralbanken, Federal Reserve. Den enkla modellen leder till avsevärt större ränteförändringar än vad som observeras i verkligheten, medan den mer kompletta modellen kommer mycket nära den observerade strategin, i synnerhet när man tar hänsyn till osäkerhet om den ekonomiska omgivningen.

Reference

Poole, William, "A policymaker confronts uncertainty," Federal Reserve Bank of St. Louis *Review* 80 (5), September/October 1998, 3–8.

Chapter 1

Monetary policy and market interest rates

with Tore Ellingsen

Abstract:
We investigate, theoretically and empirically, the relationship between monetary policy and the term structure of interest rates. In particular, we show in a dynamic macroeconomic model that if monetary policy reveals information about economic developments, interest rates of all maturities move in the same direction in response to a policy innovation. If, on the other hand, monetary policy reveals information about the central bank's policy preferences, short and long interest rates move in opposite directions. In the empirical section, we provide direct measures of endogenous and exogenous monetary policy innovations in the U.S. by analyzing the reaction of financial market participants to Federal Reserve policy moves. The empirical findings support the theoretical predictions.

[0]We are grateful for helpful comments from Annika Alexius, Pierluigi Balduzzi, Hans Dillén, Øyvind Eitrheim, Torsten Persson, Weshah Razzak, Asbjørn Rødseth, Glenn Rudebusch, Patrik Säfvenblad, Peter Sellin, Paul Söderlind, Lars Svensson, Jouko Vilmunen, and Anders Vredin; seminar participants at the Stockholm School of Economics, the Institute for International Economic Studies, the Norwegian School of Economics and Business, Norges Bank, and Sveriges Riksbank; and to Gisela Waisman for research assistance.

1.1 Introduction

Understanding the relationship between monetary policy and market interest rates is of utmost importance to bond traders and central bankers alike. Unanticipated changes in monetary policy strongly affect interest rates of almost all maturities, representing recurrent opportunities for traders to win or lose money. All serious bond analysts have their own quantitative model of the past relationship between policy moves and the yield curve. Policymakers on the other hand carefully watch the yield curve for news about market expectations. Academic economists are interested too: the effect of monetary policy on the real economy is one of our discipline's more controversial topics.

Given these efforts, our understanding of yield curve movements remains remarkably incomplete. True, there are some statistical regularities. It is empirically well established that monetary policy affects market interest rates, and that on average this relationship is positive; an increase in the central bank rate leads to an increase in interest rates of all maturities. It is also well known, however, that there are many exceptions to the rule.[1] For example, on a number of occasions in 1994 when the Federal Reserve announced an increase in its target rate, interest rates of long maturities fell. As noted by Skinner and Zettelmeyer (1995), who study the interest rate response to monetary policy over long periods in four major economies, the fraction of such 'abnormal' responses is considerable in all countries.

At the moment, there is no coherent theory which tells us whether the yield curve will shift or rotate after a policy change. Some argue that the curve should always shift. For example, Cook and Hahn (1989), who first firmly established the positive empirical relationship between target rates and long rates, interpret their finding as supportive of the expectations theory of the term structure.[2] The expectations theory says that a long interest rate should be equal to the sum of short interest rates over the same period of time plus a term premium; thus an increase in the first couple of short rates should drive up the long rate too, but by less. Romer and Romer (1996) disagree. To them, the positive movement in the long rate is inconsistent with standard monetary theory—a puzzle. According to received theory, they claim, an increase in short rates should reduce inflation, and

[1]The classic study which documents the positive relationship for the United States is by Cook and Hahn (1989). We refer to similar studies for other countries below. The fact that there are many exceptions to the rule has been discussed extensively by central bankers; see, e.g., Roley and Sellon (1995).

[2]This view is echoed by, e.g., Mehra (1996) and Skinner and Zettelmeyer (1995).

hence reduce the level of sufficiently long rates. Romer and Romer suggest that the puzzle can be resolved if the central bank has access to private information about economic fundamentals, but they do not develop their argument formally.

In this chapter, we provide a model within which each of the three mechanisms captured by the 'standard' theory, the expectations hypothesis, and Romer and Romer (1996), respectively, are all at play. This model is rich enough to allow a wide variety of market reactions to monetary policy, yet structured enough to allow a simple empirical evaluation. Our argument centers around the presumption that a change in monetary policy can come about for two distinct reasons: either the monetary authorities respond to new and possibly private knowledge about the economy, or their policy preferences change. In the first case, policy is essentially endogenous, reflecting new input into a given objective function; in the second case, policy is exogenous, in the sense that the input is the same but the objective function is new. After an endogenous policy action, our model predicts that interest rates of all maturities move in the same direction as the policy innovation. After an exogenous policy action, on the other hand, short and long interest rates should move in opposite directions. To test this model empirically, it is necessary to classify policy events according to whether they are exogenous or endogenous. We do this by interpreting newspaper reports immediately before and after each event.

Let us now describe our approach in a little more detail. Our theoretical model is taken from Svensson (1997a,b), and is quite simple, with reduced-form relationships for output and inflation. Key features of the economy are that shocks to output and inflation are persistent, and that monetary policy affects output and inflation with a lag. To this model we add an equation describing the term structure of interest rates. The central bank is assumed to control the one-period interest rate and to minimize a loss function which is quadratic in deviations of output and inflation from target. The simplified treatment of the economy allows us to derive the central bank's reaction function endogenously and to obtain a closed-form expression for the yield curve.

Assuming that the expectations hypothesis of the term structure holds, the model yields the following set of predictions. Suppose the central bank's objective function is known and stable. Whenever an economic shock is symmetrically observed by all agents, market interest rates respond immediately, and the change in the central bank rate is fully anticipated. In this case, all interest rates move in the same direction (Proposition 1). *Unanticipated* changes in the central bank rate can occur for two separate reasons. First, the central bank may have private (i.e., advance) information about exogenous shocks to output and prices. In this case, an increase in the short interest rate could be interpreted by market participants as an indication of increased inflation, and as the central bank acts to squeeze inflation out of the economy, interest rates of all maturities go up (Proposition 3).[3] The existence of central bank private information in the United States has been documented by Romer and Romer (1996), and it is also supported by our event studies. Second, the central bank's preferences may change. The policy preferences of the central bank are captured by the parameter λ_t, which indicates the current weight on output stabilization relative to price stabilization in the bank's objective function. Thus, if the short interest rate is increased, and bond traders are confident that there has been no unanticipated change in the fundamentals, then they will typically infer that price stabilization has moved higher on the central bank's agenda. In this case, we show that sufficiently long interest rates will move in the opposite direction, because average inflation is reduced (Proposition 4). We also note that λ_t determines the magnitude of the interest rate response to fundamental shocks. For a given shock, short rates respond less and long rates more as we increase λ_t (Proposition 2).

In this chapter, we concentrate on testing Propositions 3 and 4. To do so, we examine monetary policy in the United States from October 1988 until May 1997. During this sample period, the Federal Reserve has targeted the federal funds rate very strictly, so that changes in the target are much easier to observe than in preceding periods. Most target changes in this period are observed immediately by market participants. Using the commentaries in the *Wall Street Journal*, we are thus able to extract the reactions of market participants in a fairly consistent way.

The empirical results are encouraging. Policy innovations, measured as the change in the three-month rate on the day the funds rate target is adjusted, have different impact on interest rates depending on the bond market's interpretation of the move. Endogenous policy, driven by economic developments, moves long rates in the same direction as the policy innovation. Truly exogenous policy, driven by central bank preference shifts, moves ten- and thirty-year rates in the opposite direction to the policy innovation.

The chapter is organized as follows. We proceed in Section 1.2 by presenting our theoretical model, and Section 1.3 develops the main theoretical predictions. In Section 1.4, we present the methodology behind, and the results from, our classific-

[3] However, it is not necessarily true that all future short rates go up. Because the initial increase in the short rate creates a reduction in output, it may have to be offset by future interest rate reductions.

ation of Federal Reserve policy actions, which we use to study the response of U.S. interest rates to monetary policy shocks. Finally, the importance of the theoretical and empirical results and some possible extensions are discussed in Section 1.5.

1.2 The model

The model we use is taken from Svensson (1997a,b), and is a dynamic version of a simple aggregate supply-aggregate demand model, where we add an equation for the term structure of interest rates. Monetary policy does not affect the inflation rate directly, but only through the level of aggregate demand. An important feature is the introduction of 'control lags' in the response of the economy to monetary policy: policy affects aggregate demand after a lag of one period, and aggregate demand in turn affects the inflation rate in the subsequent period. This feature is consistent with the stylized facts about the response of output and inflation to monetary policy (see, e.g., Bernanke and Gertler, 1995).

1.2.1 Setup

Let π_t and y_t be the percentage deviations at time t of inflation and real output from their 'natural' levels. The inflation process (the aggregate supply relationship) is governed by an accelerationist Phillips curve relation: the change in the inflation gap is positively related to the output gap according to

$$\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1}, \tag{1.1}$$

where $\alpha > 0$ and ε_t is an i.i.d. supply shock with mean zero.

The output gap (or aggregate demand) is mean-reverting and negatively related to the ex-post real short interest rate following

$$y_{t+1} = \beta y_t - \gamma (i_t - \pi_t) + \eta_{t+1}, \tag{1.2}$$

where i_t is the deviation of the short interest rate (set by the central bank) from its long-term equilibrium level; $0 < \beta < 1$; $\gamma > 0$; and η_t is an i.i.d. demand shock with mean zero.

Our own contribution is to append a yield curve to this model. Bonds of different maturities are seen as imperfect substitutes, so the interest rate of maturity n at time t is set as an average of expected future short interest rates during the time to maturity plus a term premium,

$$i_t^n = \frac{1}{n} \sum_{s=0}^{n-1} i_{t+s|t} + \xi_t^n, \tag{1.3}$$

where $i_{t+s|t}$ denotes the expectation as of period t of the short interest rate s periods ahead, and ξ_t^n is the term premium at time t for maturity n. Thus, in determining long rates, market participants will form (rational) expectations about the future path of the short central bank rate.[4]

Our choice of model requires some justification. Of course, relations (1.1) and (1.2) represent a highly stylized, and in some respects unrealistic, view of the macroeconomy.[5] However, at this low level of complexity, it appears to be a close approximation to monetary policymakers' view of the world (see, e.g., Blinder, 1997). Also, the model fits the macroeconomic facts rather well (Rudebusch and Svensson, 1998). Finally, we are quite confident that our main insights are robust to reasonable extensions, most of which would entail the considerable cost of having to give up analytical methods for numerical analysis (Svensson, 1997a).

1.2.2 The central bank problem

At each instant, the central bank is assumed to select the short interest rate i_t to minimize the intertemporal loss function

$$\mathcal{L}_t = E_t \sum_{s=0}^{\infty} \delta^s L\left(\pi_{t+s}, y_{t+s}, \lambda_{t+s}\right), \tag{1.4}$$

where δ is a discount factor and the period loss function $L(\cdot)$ is quadratic in deviations of the inflation and output gaps from their zero targets,

$$L\left(\pi_t, y_t, \lambda_t\right) = \frac{1}{2}\left[\pi_t^2 + \lambda_t y_t^2\right]. \tag{1.5}$$

The parameter $\lambda_t \geq 0$ is the weight of output stabilization relative to inflation fighting at time t. The preferences of the central bank are assumed to be time-variant, following a martingale. Consequently, the expected value as of time t of the preference parameter at any future period is equal to its current value;

$$\lambda_{t+s|t} = \lambda_t \text{ for all } s \geq 0, \tag{1.6}$$

so any change in the preferences is seen as permanent.[6] Since there is a one-to-one relationship between output and the short interest rate from equation (1.2), we

[4]In a similar fashion, Mellin (1997) adds a yield curve to a dynamic macroeconomic model to study the behavior of market interest rates. His basic model and purposes are different from ours, however.

[5]Natural extensions would be to include the long-term ex-ante real interest rate in the aggregate demand relationship instead of the short ex-post real rate; include forward-looking behavior; or consider time-varying parameters or target levels for inflation and output. (See also Svensson, 1997a.)

[6]The martingale assumption could possibly lead to negative realizations of λ_t. Assuming that its variance vanishes as λ_t approaches zero, we can rule out such behavior.

follow Svensson (1997a,b) in treating the expected output gap $y_{t+1|t}$ as the control variable and let the central bank solve the equivalent control problem

$$V\left(\pi_{t+1|t};\lambda_t\right) = \min_{y_{t+1|t}} \left\{ \frac{1}{2}\left[\pi_{t+1|t}^2 + \lambda_t y_{t+1|t}^2\right] + \delta E_t V\left(\pi_{t+2|t+1};\lambda_{t+1}\right)\right\}, \tag{1.7}$$

subject to

$$\begin{aligned} \pi_{t+2|t+1} &= \pi_{t+1} + \alpha y_{t+1} \\ &= \pi_{t+1|t} + \varepsilon_{t+1} + \alpha\left(y_{t+1|t} + \eta_{t+1}\right). \end{aligned} \tag{1.8}$$

Since λ_t is an exogenous stochastic process with $\lambda_{t+s|t} = \lambda_t$, expected future values of the value function will be a function of λ_t only, so

$$E_t V\left(\pi_{t+2|t+1};\lambda_{t+1}\right) = E_t V\left(\pi_{t+2|t+1};\lambda_t\right). \tag{1.9}$$

Therefore, at every period t we can treat λ_t as a given constant in the value function. After solving the control problem (1.7) subject to (1.8), the optimal short interest rate is backed out from the relationship

$$y_{t+1|t} = \beta y_t - \gamma\left(i_t - \pi_t\right). \tag{1.10}$$

The first-order condition associated with (1.7) and (1.8) is

$$\lambda_t y_{t+1|t} + \alpha\delta E_t V_\pi\left(\pi_{t+2|t+1};\lambda_t\right) = 0. \tag{1.11}$$

Using the fact that the value function in (1.7) will be of the form

$$V\left(\pi_{t+1|t};\lambda_t\right) = k_0 + \frac{1}{2}k_t\pi_{t+1|t}^2, \tag{1.12}$$

where $k_t = k(\lambda_t)$ is given at t, together with equation (1.8) and the law of iterated expectations, yields the optimal expected output gap as a function of the expected inflation rate two periods ahead,

$$y_{t+1|t} = -\frac{\alpha\delta k_t}{\lambda_t}\pi_{t+2|t}, \tag{1.13}$$

where the unique positive solution for k_t is given by

$$k_t = \frac{1}{2}\left[1 - \frac{\lambda_t(1-\delta)}{\alpha^2\delta} + \sqrt{\left(1 + \frac{\lambda_t(1-\delta)}{\alpha^2\delta}\right)^2 + \frac{4\lambda_t}{\alpha^2}}\right] \geq 1. \tag{1.14}$$

Details are given in Appendix 1.A, following Svensson (1997a,b).

Given the optimal $y_{t+1|t}$ from (1.13), we can use (1.10) to back out the optimal interest rate as

$$\begin{aligned} i_t - \pi_t &= -\frac{1}{\gamma} y_{t+1|t} + \frac{\beta}{\gamma} y_t \\ &= \frac{\alpha \delta k_t}{\gamma \lambda_t} \pi_{t+2|t} + \frac{\beta}{\gamma} y_t. \end{aligned} \tag{1.15}$$

Leading (1.1) two periods and taking expectations gives

$$\begin{aligned} \pi_{t+2|t} &= \pi_{t+1|t} + \alpha y_{t+1|t} \\ &= \pi_t + \alpha (1+\beta) y_t - \alpha \gamma (i_t - \pi_t). \end{aligned} \tag{1.16}$$

From (1.15) and (1.16) we then have

$$\begin{aligned} i_t - \pi_t &= \frac{\alpha \delta k_t}{\gamma \lambda_t} \pi_t + \frac{\beta \lambda_t + \alpha^2 \delta k_t (1+\beta)}{\gamma \lambda_t} y_t - \frac{\alpha^2 \delta k_t}{\lambda_t} (i_t - \pi_t) \\ &= A_t \pi_t + B_t y_t, \end{aligned} \tag{1.17}$$

where

$$A_t = A(\lambda_t) = \frac{\alpha \delta k_t}{\gamma (\lambda_t + \alpha^2 \delta k_t)} > 0, \tag{1.18}$$

$$B_t = B(\lambda_t) = \frac{\beta}{\gamma} + \alpha A_t > 0. \tag{1.19}$$

Thus, the optimal interest rate for the central bank is an increasing function of the current inflation and output gaps,

$$i_t = (1 + A_t) \pi_t + B_t y_t, \tag{1.20}$$

so the central bank follows a rule similar to that proposed by Taylor (1993).

Some features of the model and of the optimal policy response to supply and demand shocks may need some further consideration at this point. First, the model is formulated in deviations of inflation and output from their natural levels (normalized to zero for convenience), and so is the interest rate in equation (1.20). Therefore a negative shock to inflation or output will lead to negative values of the short interest rate. Second, since monetary policy affects inflation via output, and with a lag of two periods, the way to dampen the inflationary effects of a positive shock is to create a recession. In Appendix 1.B we show that the response of the central bank to both inflation and output shocks is decreasing in the preference parameter λ_t. Thus, a central bank more prone to output stabilization will respond less to any shock. In particular, after a positive shock a central bank with a higher λ_t will choose to create a smaller recession, regardless of whether the initial shock is to inflation or output.

1.2.3 The term structure of interest rates

Knowing the short rate at each point in time, it is now relatively straightforward to compute the economy's yield curve. The n-period interest rate is set as an average of future short rates, plus a term premium,

$$i_t^n = \frac{1}{n}\sum_{s=0}^{n-1} i_{t+s|t} + \xi_t^n, \tag{1.21}$$

so we first need to find the expected path of future short rates in order to evaluate rates of longer maturities. Leading the interest rate rule (1.20) s periods and taking expectations gives

$$i_{t+s|t} = (1 + A_t)\,\pi_{t+s|t} + B_t y_{t+s|t}, \tag{1.22}$$

since A_t and B_t are given at t. The expected output process $s \geq 1$ periods from now is obtained by leading (1.1) and (1.2), taking expectations, and using (1.22),

$$\begin{aligned} y_{t+s|t} &= \beta y_{t+s-1|t} - \gamma\left(i_{t+s-1|t} - \pi_{t+s-1|t}\right) \\ &= -\gamma A_t \pi_{t+s-1|t} + (\beta - \gamma B_t)\, y_{t+s-1|t} \\ &= -\gamma A_t \pi_{t+s|t}. \end{aligned} \tag{1.23}$$

The expected path of inflation for $s \geq 1$ periods into the future is then

$$\begin{aligned} \pi_{t+s|t} &= \pi_{t+s-1|t} + \alpha y_{t+s-1|t} \\ &= (1 - \alpha\gamma A_t)\,\pi_{t+s-1|t}, \end{aligned} \tag{1.24}$$

and it is easily established by repeated substitution that expected inflation and output will follow the geometric series

$$\pi_{t+s|t} = (1 - \alpha\gamma A_t)^{s-1}\left[\pi_t + \alpha y_t\right] \tag{1.25}$$

and

$$y_{t+s|t} = -\gamma A_t\,(1 - \alpha\gamma A_t)^{s-1}\left[\pi_t + \alpha y_t\right]. \tag{1.26}$$

Using these relations in (1.22), the expected future short interest rate s periods ahead is given by

$$i_{t+s|t} = \left[1 + A_t\,(1 - \gamma B_t)\right](1 - \alpha\gamma A_t)^{s-1}\left[\pi_t + \alpha y_t\right], \tag{1.27}$$

and its sum is obtained, using the formula for geometric series, as

$$\sum_{s=1}^{n-1} i_{t+s|t} = [1 + A_t (1 - \gamma B_t)] X_t^n [\pi_t + \alpha y_t], \tag{1.28}$$

where

$$X_t^n = \frac{1 - (1 - \alpha\gamma A_t)^{n-1}}{\alpha\gamma A_t}. \tag{1.29}$$

Finally, using the interest rate rule (1.20) and the sum (1.28) in the definition (1.21), the market interest rate of maturity n is given by

$$i_t^n = \frac{1}{n} \{(1 + A_t) \pi_t + B_t y_t + [1 + A_t (1 - \gamma B_t)] X_t^n [\pi_t + \alpha y_t]\} + \xi_t^n. \tag{1.30}$$

As promised, this is our closed-form expression for the economy's yield curve.

1.3 Policy and the term structure of interest rates

We are now ready to examine how the term structure of interest rates is affected by monetary policy actions. From the central bank reaction function (1.20), we see that current monetary policy is entirely determined by current inflation, output, and the preferences of the central bank. Consequently, it is straightforward to separate endogenous monetary policy, responding to the development of inflation and output, from exogenous policy moves, due to shifts in the preference parameter λ_t.

In a first scenario, we examine how market interest rates vary when all parameters and shocks are symmetrically observed by all agents. In this scenario, interest rates respond to supply and demand shocks directly, with the magnitude depending on the central bank's preference parameter, since the response of the monetary authorities is perfectly predicted by market participants. The actual policy actions of the central bank then add no new information, and so will not affect the term structure of interest rates.

We next turn to a scenario where the central bank has access to advance information about either the supply or demand shock, or about its own preferences. In this case, the central bank's policy actions contain information about the unobservable variable. Consequently, interest rates will react to the actual policy moves, as market participants use this information to revise their beliefs about future monetary policy. Most importantly, the reaction of interest rates to endogenous policy is markedly different from the reaction to exogenous policy moves.[7]

All along, we will assume that the term premium is independent of all relevant variables, that is, that the expectations hypothesis of the term structure holds.[8] This simplifying assumption serves to streamline the results below. In the empirical study of Section 1.4 we will see that certain policy moves in the U.S. have been followed by large shifts in the term premium, so we need to consider these cases separately.

1.3.1 Symmetrically observed shocks

When all variables are publicly observable, we see directly from equation (1.30) how market interest rates are affected by supply and demand shocks as well as by shifts in the preference parameter λ_t.

Differentiating equation (1.30) with respect to ε_t, the interest rate of maturity n will respond to a supply shock according to

$$\frac{di_t^n}{d\varepsilon_t} = \frac{1}{n}\left\{1 + A_t + \left[1 + A_t\left(1 - \gamma B_t\right)\right] X_t^n\right\}. \tag{1.31}$$

Likewise, the interest rate will respond to a demand shock η_t according to

$$\frac{di_t^n}{d\eta_t} = \frac{1}{n}\left\{B_t + \alpha\left[1 + A_t(1 - \gamma B_t)\right] X_t^n\right\}. \tag{1.32}$$

Our first result is that these two derivatives are positive. When an inflationary shock (to supply or demand) hits the economy, the optimal response for the central bank is to increase its interest rate to squeeze out the effects on inflation and output. Since a monetary tightening reduces inflation by depressing output, the optimal response for the central bank is to only partially neutralize the shock in the first instant. Due to the persistence in the output and inflation processes, the economy will then be away from optimum for some time in the future. Hence, the expected path of future short rates is also revised upwards, but with declining magnitude. Under perfect information, this behavior of the central bank is accurately predicted by market

[7]Note that in this private information setting, market interest rates respond only to the unanticipated component of monetary policy. Our terminology may be slightly confusing: endogenous and exogenous policy moves do *not* coincide with anticipated and unanticipated policy, respectively. We refer to endogenous policy as responding to information (possibly private) about the economy, and exogenous policy as independent of the economic development and due to central bank preference shifts.

[8]While we agree that the term premium could vary in a systematic way with inflation, output, or the monetary policy stance, it is noteworthy that a noisy term premium coupled with active monetary policy may account for some of the alleged empirical failures of the expectations hypothesis (see Mankiw and Miron, 1986, and McCallum, 1994).

participants, and interest rates of all maturities increase as a response to a positive supply or demand shock.

This is the intuition underlying our first result:

Proposition 1 *Interest rates of all maturities are positively related to both supply and demand shocks, with the magnitude diminishing with maturity. Thus all interest rates (including the central bank rate) move in the same direction in response to a shock.*

Proof See Appendix 1.C.1.

This result seems quite intuitive, but it turns out not to be as straightforward as it looks. One would expect the central bank to react to an inflationary shock by raising the current and all future interest rates, letting the effect die out as the future gets more distant. This turns out not to be the case, however. From (1.27), future short interest rates are given by

$$i_{t+s|t} = \left[1 + A_t\left(1 - \gamma B_t\right)\right]\left(1 - \alpha\gamma A_t\right)^{s-1}\left[\pi_t + \alpha y_t\right], \tag{1.33}$$

so since $0 < \alpha\gamma A_t \leq 1$, the direction of the reaction of future short rates to a shock is determined by the term $\left[1 + A_t\left(1 - \gamma B_t\right)\right]$. This expression is not necessarily positive; for a sufficiently large value of A_t (i.e., a small value of λ_t, see Appendix 1.B) it could be negative, depending on parameter values. If so, a sufficiently inflation-averse central bank will react very strongly to any shock, creating a large recession to wipe out the inflationary effects of the shock (since the effect on inflation goes via output). In future periods, when the inflation rate is back to more normal levels, the central bank will turn its attention to the output gap, and will lower the interest rate to a level *below* the initial rate, and then slowly raise the rate back toward the initial level. Nevertheless, despite this anomalous response of the central bank, long rates will always react positively to the initial policy action, since the large response in the first period will dominate the negative response in future periods.[9]

A second implication of the model is that the response of all interest rates to a shock is linear, since the terms on the right-hand sides of (1.31) and (1.32) are constant, for a given n. Consequently, the relationship between any two interest rates will also be linear.

It is interesting to see how the magnitude of the preference parameter λ_t affects the response of interest rates to a given shock. As λ_t increases, the central bank becomes less inflation-averse, and more prone to stabilizing output. For a given shock,

[9]Of course, if the central bank is also concerned with smoothing interest rates, or takes parameter uncertainty into account, such odd policy responses could be excluded. (See, e.g., Söderström, 1999b.)

the optimal interest rate policy is less fierce, and the central bank rate is changed by a smaller amount, since both A_t and B_t are decreasing in λ_t (see Appendix 1.B). In the long run, however, a given shock will remain for longer in the economy, so future short rates are expected to be higher than if the central bank had neutralized a larger portion of the shock in the initial move. Therefore, central banks with a larger value of λ_t will see a larger effect on long rates for a given shock, since the central bank rate is expected to differ from the initial level for a longer period of time.

This mechanism lies behind our second result:

Proposition 2 *With a higher value of λ_t, short interest rates respond less and long interest rates respond more to a given shock. Consequently, long rates respond more to a given change in short rates.*

Proof See Appendix 1.C.2.

We can now summarize our first set of results. When all shocks are observable to all agents, all interest rates move in the same direction in response to a shock that leads the public to revise their expectations of future monetary policy. For a more inflation-averse central bank, short rates will respond more, but long rates less to a given shock.

Note that in this scenario, market interest rates do not respond to the monetary policy actions *per se*, since these are perfectly anticipated, and thus already priced into the market. The way we have chosen to model it, the central bank responds instantaneously to new information, and the above distinction is purely notional. In a more realistic setting, the central bank would respond to new information with a lag, and possibly at certain fixed intervals. Then the distinction between interest rate reactions to shocks and the reaction to policy actions becomes important, especially when interpreting the theoretical results empirically.

1.3.2 Asymmetric information

For efficient bond markets to respond to the actual policy moves of the central bank, these moves must contain some information not previously available to market participants. Or, in other words, the central bank must have access to private information about relevant variables in the economy. In our model, this information can be of two kinds: information about shocks to the inflation or output paths, or information about the central bank's preferences. We will study the two kinds of central bank private information separately, to see how the presence of private information affects the determination of interest rates.

We begin by considering the case where the central bank has private (or advance) information about the current realization of the supply or the demand shock.[10] If only one of the shocks is unobservable at a time, the realization of this shock is easily inferred by market participants after observing the reaction of the central bank by inverting the policy rule (1.20).[11] Thus, when the current realization of the supply shock ε_t is unobservable, it is inferred as

$$\hat{\varepsilon}_t(i_t) = \frac{1}{1+A_t} i_t - (\pi_{t-1} + \alpha y_{t-1}) - \frac{B_t}{1+A_t} y_t, \tag{1.34}$$

where all variables on the right-hand side are observable at time t. Similarly, when the central bank has private information about the demand shock η_t, its current realization is inferred as

$$\hat{\eta}_t(i_t) = \frac{1}{B_t} i_t - \frac{1+A_t}{B_t} \pi_t - \left[\beta y_{t-1} - \gamma\left(i_{t-1} - \pi_{t-1}\right)\right]. \tag{1.35}$$

In this simplistic setup, when the realization of the unobservable shock is perfectly inferred by bond markets, the results from the previous section remain. Now, however, market interest rates will react to the policy actions of the central bank, since these reveal information about the realized shocks, and thus about the future path of monetary policy. Consequently, although the results below are simple corollaries of Propositions 1 and 2 above, they have quite distinct interpretations for the response of interest rates to monetary policy.

First, when the supply or demand shock is unobservable to the public, Proposition 1 implies that all interest rates will move in the same direction as the central bank rate, as market participants infer the realization of the unobservable shock:

Proposition 3 *When the central bank has private information about either the supply or the demand shock, market interest rates will be positively related to the central bank rate. This relationship becomes weaker as the interest rate's maturity increases.*

[10]This may not be an innocent assumption, and deserves some closer attention. Recently, Romer and Romer (1996) have suggested the presence of central bank private information as an explanation for the positive relationship between the central bank rate and long-term interest rates. In an empirical test, they found strong support for their hypothesis: the Federal Reserve's inflation forecasts are quite superior to those of private forecasters, and private forecast errors can be explained to a large part by the Fed's own forecasts. Also, there are some signs that private forecasters use Fed policy to revise their own forecasts. The authors conclude that the Federal Reserve has access to more (or better) information than the public, although this could be due to better and more extensive data processing on the part of the Fed.

[11]We thus do not have a proper signal extraction problem for private agents. We choose to concentrate on the simple perfect-inference case here, to illustrate our mechanisms in a transparent way.

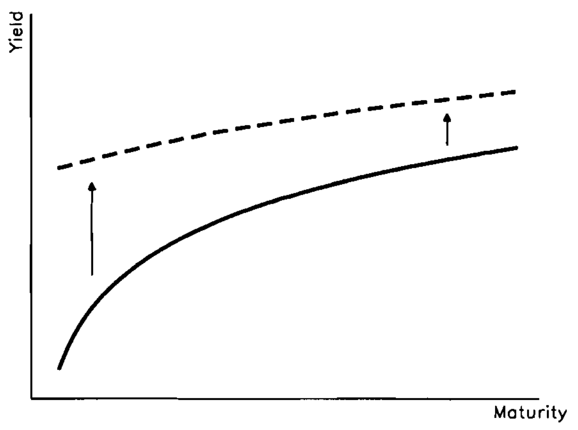

Figure 1.1: Yield curve response to an endogenous policy contraction.

Proof Follows immediately from Proposition 1.

A graphical representation of this result is given in Figure 1.1. A monetary tightening leads the public to infer that a positive inflationary shock has hit the economy, and the entire yield curve shifts upwards, with the reaction decreasing with maturity. For a surprise expansion of policy, the reaction is the opposite.

Most interesting, however, is the response of interest rates to an unexpected shift in the preferences of the central bank. We now assume that all shocks are observable, but that the current value of the preference parameter λ_t is known only to the central bank itself. After a given shock has hit the economy, the public expects the central bank to act according to the rule (1.20), given their belief about the parameter λ_t. Any unexpected policy response is then interpreted as a (permanent) change in λ_t, leading the public to revise their expectations about the future path of the central bank rate.

Since a central bank with a lower value of λ_t will set a higher interest rate (in absolute terms) for a given shock, but keep the interest rate away from the initial level for a shorter period of time, an unexpectedly large tightening leading to a revision downwards in the public's perception of λ_t will lead to rising short rates but falling long interest rates. This is the basic intuition behind our final result:

Proposition 4 *When the central bank's preferences are unobservable to the public, long interest rates will move in the opposite direction to the innovation in the central bank rate. Thus, the yield curve will tilt as a response to unexpected monetary policy:*

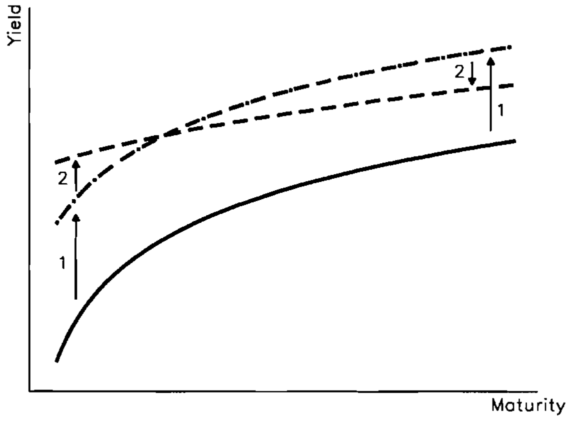

Figure 1.2: Yield curve response to an exogenous policy contraction.

an unexpectedly high central bank rate tilts the yield curve clockwise; an unexpectedly low rate tilts it counter-clockwise.

Proof See Appendix 1.C.3.

This response is shown in Figure 1.2. When a positive shock realizes, the yield curve shifts up in anticipation of the central bank's response (1). If the central bank acts as expected, market interest rates will not move at all when the central bank rate is adjusted. If, however, the central bank sets a higher interest rate than was expected, the public realizes that the bank has become more inflation averse (i.e., λ_t has decreased). Then short rates rise, but longer rates fall, leading to a clockwise tilt of the yield curve (2). Similarly, if the central bank responds with a lower rate than expected, the yield curve tilts counter-clockwise.

1.3.3 Empirical interpretation

In the model, the central bank adjusts its interest rate in every period, as new information about the economy is revealed. In reality, central banks adjust their monetary policy stance at discrete intervals, after accumulating a sufficient amount of information. Consequently, when translating our results to empirically testable hypotheses, we need to separate days on which the central bank does not intervene from days on which it does.

On days when the central bank rate is left unchanged, the information revealed predominantly concerns the state of the economy, and since no information is revealed from the central bank's policy moves, this information is symmetrically observed. Consequently, Propositions 1 and 2 should be expected to hold on days when the central bank does not intervene: interest rates should move in the same direction (Proposition 1), and more inflation-averse central banks should see short interest rates respond more but long rates less to new information, so that the relationship between long and short rates should be weaker (Proposition 2).

On days when the central bank does act to change its interest rate instrument, however, its private information may be revealed. Then Proposition 3 predicts that if the central bank action reveals information about the economy, all interest rates should move in the same direction, with long rates reacting less than short rates, while Proposition 4 implies that on occasions when the central bank move reveals information about the bank's preferences, short and long rates should move in opposite directions.

In their study of the 1974–79 funds rate targeting regime in the U.S., Cook and Hahn (1989) show that when the Fed moved its target level for the federal funds rate, interest rates of all maturities on average moved in the same direction as the target. Interpreting this finding, and similar results for other countries,[12] in the light of our model indicates that monetary policy actions are driven more often by economic developments than by preference shifts. Skinner and Zettelmeyer (1995) present results that lend support to Proposition 2: long interest rates respond more to short rates in the U.S. and the U.K. than in Germany and France. Accepting the hypothesis that the central banks of Germany and France are more inflation-averse than the Federal Reserve and the Bank of England, this is exactly what our model would predict.

In the following section, we will complement these results with our own empirical evidence, testing our theoretical implications more directly.

1.4 The response of interest rates to monetary policy

To test our theoretical predictions, we need to separate policy shifts driven by new information (endogenous policy) from shifts driven by changes in the central bank's preferences (exogenous policy). Here, we are primarily interested in bond markets' perception of monetary policy, since interest rates set on financial markets reflect

[12]See Battelino et al. (1997) for Australia; Buttiglione et al. (1997) for Italy; Lindberg et al. (1997) for Sweden; and Skinner and Zettelmeyer (1995) for France, Germany, the U.K., and the U.S.

investors' perceptions about central bank policy rather than the central bank's 'true' policy strategies.

We attempt to extract such monetary policy perceptions from U.S. bond markets by studying the commentaries in the 'Credit Markets' column of the *Wall Street Journal* on days surrounding changes in the Federal Reserve's target level for the federal funds rate in the period from October 1988 to May 1997. On any day, the *Wall Street Journal* interviews a number of bond traders, analysts, and economists for comments about important events concerning the bond markets. A sample of these comments, along with the journalist's own analysis, is then reported in the *Journal*. Since Fed policy moves are crucial for the development of financial markets, and especially for the bond market, the news of a change in the monetary policy stance typically dominates the commentaries on days following a Fed move.

Even though the comments after a policy move by the Fed are surprisingly homogeneous, any move will typically be interpreted as revealing information both about the economic development and about the Fed's preferences. For simplicity, we will concentrate on finding the dominant factor behind each move; thus our classification is a rough description of the faceted interpretations of the policy adjustments.[13]

We then proceed by analyzing the response of market interest rates to monetary policy, as measured by the one-day change in the 3-month treasury bill rate. The 3-month rate is sufficiently short to be mainly determined by current and expected future policy actions, but of sufficiently long maturity to avoid noise from expectation errors due to the exact timing of Fed actions.[14] On trading days when the Fed leaves its target level for the federal funds rate unchanged, the change in the 3-month rate is interpreted as a measure of expected future changes in the Fed's policy stance in response to new information on that day. On days when the funds rate target level is adjusted, any movement in the 3-month rate is, as a first approximation, interpreted as the surprise element of the policy action, that is, the policy innovation. Thus we can compare the response of market interest rates to policy

[13]This problem of mixed events is likely to be most serious for events classified as exogenous, since on these days, some information about economic developments is also likely to be released. Therefore we will attempt to distill the interest rate response to the 'true' exogenous component from these policy shifts by controlling for the typical non-policy event.

[14]Using a shorter rate as a measure of policy (e.g., the innovation in the funds rate target) is problematic if bond markets anticipate the size of a policy move correctly, but not the actual timing of policy. The measured policy innovation then overestimates the true innovation. Harvey and Huang (1994) present evidence that markets are better at predicting the direction of Fed actions than their timing. Also, as shown by Söderström (1999a), market expectations of Fed policy extracted from the federal funds futures market vary systematically across months and trading days, and thus are less reliable as measures of the expected component of policy moves.

innovations on exogenous and endogenous policy days, and also compare with days when the Fed has left its funds rate target unchanged, but new information has led bond markets to update their expectations of Fed policy.

The length of the sample period is due to changes in the operating procedures of U.S. monetary policy during the 1980s. Although the Federal Reserve returned to targeting the federal funds rate in late 1982, not until late 1988 was the targeting sufficiently strict for financial market participants to rely on funds rate observations to identify changes in the monetary policy stance. Target changes before 1988 were hardly ever noticed by market participants, unless accompanied by a change in the published discount rate.[15]

1.4.1 Classification of monetary policy events

From October 1988 to May 1997 the Federal Reserve changed its target level for the federal funds rate on 47 occasions, as reported by Rudebusch (1995) for 1988–92 and the Federal Reserve Bank of New York for 1993–97.[16] Since our methodology of classifying monetary policy events is new, we need to explain in more detail the criteria used.

Typical comments in the *Wall Street Journal* of cases being interpreted as exogenous, or based on a change in preferences, are: "...there was some disappointment that the Federal Reserve didn't signal a larger cut in the rate," from December 20, 1990, or: "'This rate cut says the Fed is likely to be more aggressive cutting rates than people thought'..." in the commentary of February 2, 1996. An especially clear report comes after the target cut of April 30, 1991, when the *Journal* reports that: "...[the move] didn't follow any major economic report...," indicating that the cut was not based on any new information, but continues by quoting an analyst saying that the move "...smacks of some political pressure on the Fed," since it had come shortly after the Bush administration had argued for global interest-rate cuts.

As for the events interpreted as endogenous responses to the economic development, typical comments are: "The U.S. Federal Reserve's latest move to cut interest rates reflects its uneasiness about the slow growth of money supply and the disappointingly torpid economic recovery," from September 16, 1991, or: "...the Fed's

[15] Below we will see that also during 1988 and 1989 many of the changes in the funds rate target passed unnoticed by financial market participants.

[16] Roley and Sellon (1996) argue that some of the target changes reported by Rudebusch do not correspond to actual decisions to change policy. Since some of these cases were apparently noticed by market participants (see the full classification in Appendix 1.D), we choose to use the Rudebusch series.

decision to cut rates... came primarily for concerns about recent contractions in the U.S. money supply," on April 10, 1992. On some occasions, mostly during the later period of our sample, the Fed announced its target change, accompanied by its own comments about the factors underlying the change. An example is December 20, 1995, when the *Journal* writes: "The Fed said that 'inflation has been somewhat more favorable than anticipated...'." Unless there are other signs of the opposite, these events are also classified as endogenous. Finally, a peculiar, but for our purposes very encouraging, case is July 7, 1995, when the *Journal* speculates that the Fed had access to information in the employment report before the report was published: "... the Fed's willingness to ease ahead of Friday's data suggests that the central bank is looking for a weak employment report."

In ten cases, mostly during 1988 and 1989, the *Journal* makes no mention of the policy move, leading us to conclude that market participants never noticed the change in the funds rate target. These cases are omitted from the sample of target changes, and treated as non-policy days.[17] On seven occasions, the monthly employment report from the Bureau of Labor Statistics was released on the same day as the policy move, so we cannot separate the effects on financial markets of the information release from the effects of policy. Consequently, these cases are also treated as non-policy days.[18]

Of the remaining 30 events of policy changes, on two occasions (January 9, 1991, and October 31, 1991) the change in the funds rate target was noticed by financial market participants on the day *before* the actual target change reported by Rudebusch. On these occasions we choose to use the interest rate response of the day preceding the reported target change, when the information seems to have reached the markets.

Of these 30 events, 19 were classified as endogenous responses to the state of the economy, and 11 as caused by exogenous changes of the Fed's preferences. Table 1.1

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[17]During this early part of the sample, the Fed did not target the funds rate very closely. From 1990 on, target changes reported by the Federal Reserve Bank of New York are always attributed to one particular day. During 1988 and 1989, however, the Fed often reports gradual changes in the target, over several weeks or months. It is then not surprising that many of these changes were not noticed by market participants on the exact day reported by Rudebusch (1995).

[18]Naturally, there is some information in the data for these days also. The problem is that when estimating the policy innovation with the 3-month rate, there is always some measurement error, and on days when other significant information is released on the same day as monetary policy is adjusted, this measurement error is expected to be very large. Therefore we choose not to use these observations. That the employment report is important for the conduct of monetary policy is obvious from the newspaper commentaries. For some empirical evidence, see Cook and Korn (1991) or Balduzzi et al. (1997).

Table 1.1: Summary of classification

Endogenous	Exogenous	Report	Unnoticed
Dec 15, 1988	Jan 5, 1989	Dec 7, 1990	Oct 20, 1988
Feb 23, 1989	Feb 14, 1989	Feb 1, 1991	Nov 17, 1988
Jun 6, 1989	Feb 24, 1989	Mar 8, 1991	Nov 22, 1988
Jul 7, 1989	Dec 20, 1989	Dec 6, 1991	Dec 29, 1988
Jul 27, 1989	Jul 13, 1990	Jul 2, 1992	Feb 9, 1989
Oct 29, 1990	Dec 19, 1990	Sep 4, 1992	May 4, 1989
Jan 8, 1991	Apr 30, 1991	Feb 4, 1994	Aug 10, 1989
Aug 6, 1991	May 17, 1994		Oct 18, 1989
Sep 13, 1991	Aug 16, 1994		Nov 6, 1989
Oct 30, 1991	Nov 15, 1994		Nov 14, 1990
Nov 6, 1991	Jan 31, 1996		
Dec 20, 1991			
Apr 9, 1992			
Mar 22, 1994			
Apr 18, 1994			
Feb 1, 1995			
Jul 6, 1995			
Dec 19, 1995			
Mar 25, 1997			

Classification of 47 changes in the federal funds rate target October 3, 1988–May 30, 1997.

summarizes the classification. A detailed description of all events, with the relevant quotes from the *Wall Street Journal*, is found in Appendix 1.D.

We end this section by stressing that the classification presented here should be seen as tentative. Due to data collection costs, we have limited ourselves to one source of information, and although we believe the *Wall Street Journal* to be one of the most natural places to begin, the information collected is by no means complete. We therefore welcome any efforts to improve upon our classification.

1.4.2 Empirical results

Daily data on interest rates from October 3, 1988, to May 30, 1997, are taken from the FRED database of the Federal Reserve Bank of St. Louis. Short-term interest rates (3-month, 6-month, and 1-year rates) are treasury bill rates from the secondary market, and long-term interest rates (of 2, 3, 5, 7, 10, and 30 years' maturity) are treasury bond rates of constant maturity. The data for the 47 policy days are reported in Appendix 1.E.

Using these data, we want to estimate how market interest rates move both in response to actual Fed policy moves and in anticipation of Fed reactions to new information. We thus want to estimate a regression like

$$\Delta i_t^n = \alpha_n + \left(\beta_n^{\mathrm{NP}} d_t^{\mathrm{NP}} + \beta_n^{\mathrm{End}} d_t^{\mathrm{End}} + \beta_n^{\mathrm{Ex}} d_t^{\mathrm{Ex}}\right) \Delta i_t^{3\mathrm{m}} + v_t^n, \tag{1.36}$$

where Δi_t^n is the change in the n-maturity interest rate on day t; $\Delta i_t^{3\mathrm{m}}$ is the corresponding change in the 3-month rate, that is, our measure of policy innovations; and d_t^j is a dummy taking the value one if day t belongs to group j and zero otherwise.

To the group *NP* (non-policy) belong all days when the Fed has left its funds rate target unchanged. On these days, the 3-month rate moves in anticipation of future Fed policy reactions to information released on day t, and longer interest rates may respond to this policy innovation. The group *End* corresponds to policy days classified as endogenous, and *Ex* are exogenous policy days. The obtained estimates of β_n^j are thus the estimated responses of the n-maturity interest rate to a policy innovation of type j.

According to our theoretical analysis, equation (1.36) is the correct empirical specification given that the term premium ξ_t^n and the taste parameter λ_t are constant. Of course, both of these vary in practice. To take account of variations in the term premium, we have looked in our case material for statements concerning changes in interest rate uncertainty. As it happens, two events stand out; May 17 and August 16, 1994. On these occasions, the reports from the *Wall Street Journal* make clear that the Fed's actions considerably reduced the uncertainty concerning the future path of policy. In other words, these moves seem to have been followed by large reductions in the term premium.[19] To control for these movements in the term premium, we include an intercept dummy for each of these events.

Permanent changes in the taste parameter λ_t are more difficult to handle. Since our model allows shocks to λ_t, and we identify such shocks empirically, there might in principle be a time subscript on each of our slope parameters in equation (1.36). Given the small number of policy events in our sample, we have chosen to ignore this issue.

Before resorting to statistical methods, let us eyeball some of the data. Figures 1.3–1.5 show scatter plots of the change in the 10-year rate against the change in the 3-month rate on policy days. Figure 1.3 shows the relationship for all 30 policy events, and Figures 1.4 and 1.5 break up the relationship into endogenous and exogenous events. In Figure 1.3 there is a clear positive relationship between the long rate response and the policy innovation, although there are some odd observations. For the endogenous events in Figure 1.4, the positive correlation is obvious,

[19] This is also consistent with other analyses of Fed policy during 1994, e.g., Campbell (1995).

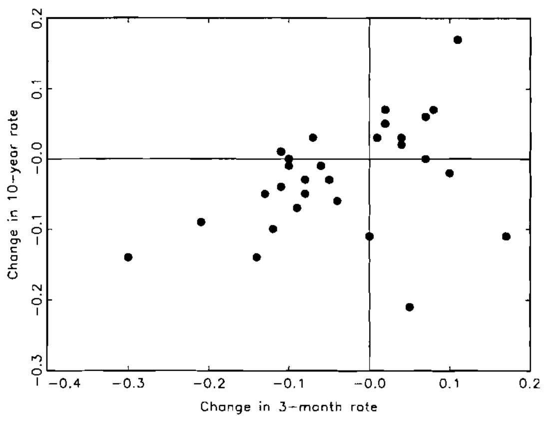

Figure 1.3: Response of the 10-year interest rate to a change in the 3-month rate: 30 classified policy events

whereas the exogenous events in Figure 1.5 show a more ambiguous picture. The two observations from May and August 1994 also stand out clearly in the scatter plots.

The regression we end up estimating then is

$$\begin{aligned}\Delta i_t^n &= \alpha_n + \left(\beta_n^{\mathrm{NP}} d_t^{\mathrm{NP}} + \beta_n^{\mathrm{End}} d_t^{\mathrm{End}} + \beta_n^{\mathrm{Ex}} d_t^{\mathrm{Ex}}\right) \Delta i_t^{3\mathrm{m}} \\ &+ \gamma_n^{9405} d_t^{9405} + \gamma_n^{9408} d_t^{9408} + v_t^n, \qquad (1.37)\end{aligned}$$

where d_t^{9405} and d_t^{9408} are intercept dummies for the events of May and August 1994. The main hypothesis to be examined is that long-term interest rates respond positively to endogenous policy moves but negatively to exogenous moves:

Hypothesis 1 *For large n,* $\beta_n^{\mathrm{Ex}} < 0 < \beta_n^{\mathrm{End}}$.

The discussion in Section 1.3.3 also leads us to test the hypothesis that all rates respond similarly (positively) to endogenous policy innovations as to the information released on non-policy days:

Hypothesis 2 $\beta_n^{\mathrm{NP}} = \beta_n^{\mathrm{End}} > 0$ *for all n.*

And finally, our theoretical model predicts that for all maturities, the response falls with maturity:

Hypothesis 3 β_n^j *is decreasing in n for all j.*

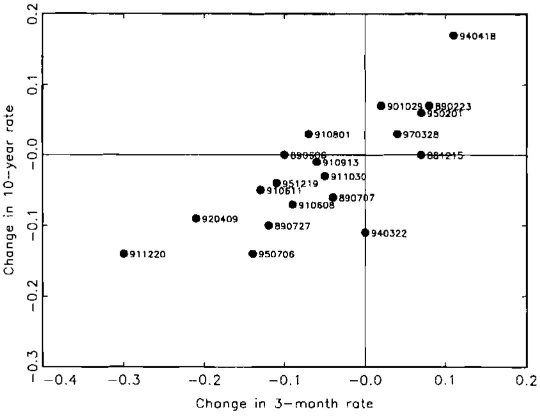

Figure 1.4: Response of the 10-year interest rate to a change in the 3-month rate: endogenous policy events

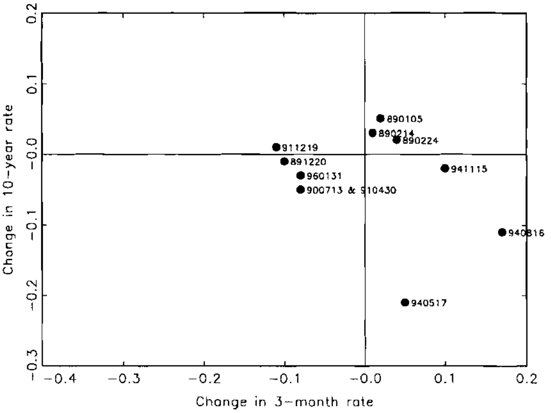

Figure 1.5: Response of the 10-year interest rate to a change in the 3-month rate: exogenous policy events

Table 1.2: Interest rate response to a policy innovation

	6 months	1 year	2 years	3 years	5 years	7 years	10 years	30 years
α_n	0.000	0.000	0.000	0.000	0.000	−0.000	−0.000	−0.000
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
β_n^{NP}	0.808**	0.773**	0.832**	0.787**	0.722**	0.621**	0.557**	0.426**
	(0.024)	(0.032)	(0.039)	(0.040)	(0.039)	(0.037)	(0.035)	(0.032)
β_n^{End}	0.971**	0.953**	0.945**	0.860**	0.730**	0.616**	0.554**	0.419**
	(0.055)	(0.074)	(0.099)	(0.112)	(0.105)	(0.094)	(0.080)	(0.069)
β_n^{Ex}	1.150**	0.967**	0.838**	0.542**	0.465**	0.316*	0.194	0.002
	(0.133)	(0.138)	(0.116)	(0.105)	(0.096)	(0.134)	(0.127)	(0.137)
γ_n^{9405}	−0.108**	−0.158**	−0.192**	−0.197**	−0.213**	−0.246**	−0.219**	−0.190**
	(0.007)	(0.007)	(0.006)	(0.005)	(0.005)	(0.007)	(0.007)	(0.007)
γ_n^{9408}	−0.176**	−0.175**	−0.193**	−0.152**	−0.179**	−0.174**	−0.143**	−0.120**
	(0.023)	(0.023)	(0.020)	(0.018)	(0.016)	(0.023)	(0.022)	(0.023)
$\bar{R}^2$	0.643	0.491	0.372	0.327	0.278	0.226	0.201	0.151
$\beta_n^{NP} = \beta_n^{End}$	7.446**	4.941*	1.144	0.373	0.005	0.003	0.002	0.009
$\beta_n^{End} = \beta_n^{Ex}$	1.561	0.008	0.498	4.302*	3.501°	3.397°	5.802*	7.438**

OLS estimation of equation (1.37) on daily observations from October 3, 1988, to May 30, 1997. Wald-tests (χ^2) with 1 degree of freedom. White (1980) standard errors in parentheses, **/*/° denote significance at the 1%-, 5%-, and 10%-level, respectively.

Table 1.2 reports OLS estimates from equation (1.37).[20] The estimated intercept term is virtually zero, as expected, and our last two hypotheses are clearly confirmed: the slope coefficients for the non-policy and endogenous policy events are large and strongly significant for all maturities (the two responses cannot be statistically separated for maturities of two years and above), and for all groups, the response falls with maturity.[21]

We then turn to our main hypothesis. For the exogenous events, the estimated slope coefficients are positive, but not significantly different from zero for the longest maturities, and virtually zero for the 30-year rate. The reported Wald statistics reject the hypothesis of equal coefficients for the endogenous and exogenous events at the 10%-level for maturities of three years and above. Long interest rates thus

[20]To test the econometric specification, we also estimated regressions including squared independent variables. The squared change in the 3-month rate is occasionally significant, but adds nothing to the explanatory power of the model. The model easily passes a number of other specification tests: when the term premium dummies are included, error terms are normally distributed and autocorrelation is not a problem. The test statistics are omitted for brevity.

[21]The very longest maturities respond surprisingly strongly to policy innovations. On non-policy days and endogenous policy days, above 40% of the movement in the 3-month rate is transmitted to the 30-year rate, a phenomenon that is at odds with our model. Possibly this could be due to perceived changes in the Fed's inflation target: if the Fed is believed to adjust its target for inflation when the economy is hit by shocks, then this adjustment should be transmitted one-for-one to long interest rates. This policy strategy, named the 'opportunistic approach,' has recently been discussed by, e.g., Orphanides and Wilcox (1996) and could possibly be incorporated into a model such as ours, but such work is beyond the scope of this chapter.

respond significantly differently to exogenous policy innovations as compared to endogenous policy, but the estimated response to policy moves is always positive.

To analyze the response of long rates on policy days in more detail, Table 1.3 presents the results for the 10- and 30-year rates from estimating four different regressions. We first estimate the average response for all 30 observations, with and without dummies for the term premium shifts. We then separate the endogenous and exogenous events, again both with and without term premium dummies.

When separating the groups, the explained variance in the interest rate response increases, especially when we do not control for the term premium shifts. In these regressions, the estimated slope coefficients for the exogenous events are negative. This negative coefficient, and most of the increase in explanatory power, disappears when controlling for the events of May and August 1994, but the classification still increases adjusted R^2 from 0.61 to 0.63 for the 10-year rate and from 0.55 to 0.60 for the 30-year rate. Thus, by classifying the Fed's policy moves, and controlling for two exceptional events, we explain about 60% of the variance in the longest interest rates in response to monetary policy actions.[22]

Although we can significantly separate the response of long rates to endogenous and exogenous events, the response of long rates to exogenous events is still positive. This result appears to contradict the first part of Hypothesis 1. However, as we shall now show, this result may well be due to the noise contained in daily data. On most days there will be some new information about the economy, creating a positive relationship between short and long interest rates, according to the results for non-policy days. Therefore the estimated slope coefficients for the exogenous events are biased upwards. We attempt to adjust this bias by calculating the implied slope coefficients for the true exogenous component from the hypothetical regression

$$\Delta i_t^n = \alpha_n + \beta_n^{\text{Ex}*} \Delta i_t^{\text{Ex}*} + \beta_n^{\text{NP}*} \Delta i_t^{\text{NP}*} + \varepsilon_t^n \tag{1.38}$$

on the 9 exogenous observations which remain after we have excluded the events of May and August 1994. Here, $\Delta i_t^{\text{Ex}*}$ is the part of the policy innovation which is truly exogenous, due to a perceived change in the Fed's preferences, and $\Delta i_t^{\text{NP}*}$ is the 'non-policy event,' due to new information released on the policy day. Assuming that these non-policy events on exogenous policy days behave as on any non-policy day and are independent of the true exogenous component, we can calculate the

[22] Note that both estimated slope coefficients and standard errors differ between Tables 1.2 and 1.3. This is due to the constant term, which differs substantially between the two regressions, although it is always very small.

Table 1.3: Response of long-term interest rates (10 and 30 years) on policy days

	(i)		*(ii)*		*(iii)*		*(iv)*	
	10 years	30 years	10 years	30 years	10 years	30 years	10 years	30 years
α_n	−0.014	−0.015	0.008	0.005	−0.009	−0.010	0.007	0.004
	(0.017)	(0.015)	(0.012)	(0.010)	(0.013)	(0.011)	(0.011)	(0.009)
β_n	0.317*	0.181	0.528**	0.369**				
	(0.152)	(0.133)	(0.097)	(0.083)				
β_n^{End}					0.521**	0.382**	0.584**	0.435**
					(0.109)	(0.089)	(0.096)	(0.079)
β_n^{Ex}					−0.228	−0.357*	0.234	0.023
					(0.198)	(0.169)	(0.160)	(0.156)
γ_n^{9405}			−0.245**	−0.213**			−0.229**	−0.195**
			(0.015)	(0.013)			(0.016)	(0.014)
γ_n^{9408}			−0.208**	−0.187**			−0.157**	−0.128**
			(0.026)	(0.022)			(0.033)	(0.031)
$\bar{R}^2$	0.146	0.047	0.613	0.553	0.330	0.305	0.634	0.605
$\beta_n^{\text{End}} = \beta_n^{\text{Ex}}$					13.315**	17.788**	4.640*	6.567*
$\beta_n^{\text{Ex}*}$							−0.017	−0.270°
							(0.169)	(0.131)
$\beta_n^{\text{NP}*}$							0.557*	0.426*
							(0.219)	(0.169)
$\beta_n^{\text{Ex}*} = \beta_n^{\text{NP}*}$							4.312°	10.597*

OLS estimation of

$$(i)\quad \Delta i_t^n = \alpha_n + \beta_n \Delta i_t^{3m} + v_t^n$$

$$(ii)\quad \Delta i_t^n = \alpha_n + \beta_n \Delta i_t^{3m} + \gamma_n^{9405} d_t^{9405} + \gamma_n^{9408} d_t^{9408} + v_t^n$$

$$(iii)\quad \Delta i_t^n = \alpha_n + \left(\beta_n^{\text{End}} d_t^{\text{End}} + \beta_n^{\text{Ex}} d_t^{\text{Ex}}\right) \Delta i_t^{3m} + v_t^n$$

$$(iv)\quad \Delta i_t^n = \alpha_n + \left(\beta_n^{\text{End}} d_t^{\text{End}} + \beta_n^{\text{Ex}} d_t^{\text{Ex}}\right) \Delta i_t^{3m} + \gamma_n^{9405} d_t^{9405} + \gamma_n^{9408} d_t^{9408} + v_t^n$$

on 30 policy events from October 3, 1988, to May 30, 1997. Wald-test (χ^2) of $\beta_n^{\text{End}} = \beta_n^{\text{Ex}}$ with 1 degree of freedom. White (1980) standard errors in parentheses, **/*/° denote significance at the 1%-, 5%-, and 10%-level, respectively. $\beta_n^{\text{Ex}*}$ and $\beta_n^{\text{NP}*}$ are the estimated coefficients from the hypothetical regression (1.38) on 9 exogenous policy events. (Hypothetical standard errors in parentheses not adjusted for heteroskedasticity. See Appendix 1.F for details.)

implied slope coefficient from equation (1.38) as

$$\beta_n^{\text{Ex}*} = \frac{\text{Cov}(\Delta i_t^{\text{Ex}*}, \Delta i_t^n)}{\text{Var}(\Delta i_t^{\text{Ex}*})}$$
$$= \frac{\text{Cov}(\Delta i_t^{3\text{m}}, \Delta i_t^n) - \text{Cov}(\Delta i_t^{\text{NP}*}, \Delta i_t^n)}{\text{Var}(\Delta i_t^{3\text{m}}) - \text{Var}(\Delta i_t^{\text{NP}*})}, \tag{1.39}$$

where $\text{Var}(\Delta i_t^{\text{NP}*})$ and $\text{Cov}(\Delta i_t^{\text{NP}*}, \Delta i_t^n)$ are calculated from the large sample of non-policy days. The resulting coefficients and estimated standard errors (not adjusted for heteroskedasticity) for the 10- and 30-year rates are reported in the lower panel of Table 1.3, along with a test of the restriction $\beta_n^{\text{Ex}*} = \beta_n^{\text{NP}*}$ (see Appendix 1.F for details). After distilling the truly exogenous component, both the 10-year and the 30-year rates respond negatively to exogenous policy innovations, and the latter response is significantly different from zero at the 10%-level.

1.5 Final remarks

As mentioned in the Introduction, there is some confusion in the literature as to what should be the 'normal' response of long interest rates to monetary policy. Some authors argue that long rates should increase as monetary policy is tightened, mainly via the expectations hypothesis of the term structure. Others support the hypothesis that a monetary tightening should increase short rates but decrease long rates, as inflation expectations fall. Our results suggest that these differing views are two sides of the same coin. When long rates are determined via the expectations hypothesis, they may rise or fall after a policy tightening, depending on market participants' interpretation of the reasons behind the policy move.

An objection to our methodology concerns our classification. Since the story we want to convey is commonly heard in financial markets, it is conceivable that traders and analysts have our mechanism in mind when explaining the reaction of financial markets to monetary policy. Then our classification could be a result of the behavior of interest rates, and our empirical results only confirm this correspondence. However, this objection appears to be based on the presumption that interest rates are determined by fundamentals which could be unobservable to traders. Given the vast amount of 'speculative' trade, which is bound to dominate reactions in the short run, we are inclined to think that daily changes in interest rates are essentially determined by traders' beliefs. If so, causality is not a problem (unless traders jointly conspire to fool the readers of the *Wall Street Journal*).

Finally, we would like to put our work in a broader perspective. Apart from the response of interest rates to monetary policy, we also believe that our model has some interesting implications for the empirical literature on the effects of monetary policy on output; see, for example, Bernanke and Blinder (1992), Christiano, Eichenbaum and Evans (1996), Gordon and Leeper (1994), or Sims (1992). For this literature to be of much value, monetary policy should not be entirely endogenous; if the observed monetary policy actions are driven exclusively by developments in the economy, we cannot infer from these regressions what would have been the effect of a different monetary policy. Conventionally, modelers have derived the exogenous component of monetary policy from the econometric model itself. This approach has recently been challenged by Rudebusch (1998), who compares these VAR shocks to monetary policy shocks obtained from data on federal funds futures contracts. Since the two series of estimated shocks are quite dissimilar, Rudebusch concludes that "it would be surprising if VARs could provide even approximately correct answers to structural questions about the monetary transmission mechanism" (page 19). In a commentary, Sims (1998) points out that even if forecasts from the futures market have smaller errors than forecasts from a VAR, the estimated response to the VAR shocks may still be a good measure of the effects of monetary policy, something which is supported by the results of Brunner (1996) and Bagliano and Favero (1998).

We distinguish endogenous and exogenous changes in interest rates directly, by recording how bond traders interpret each movement in the federal funds rate target. In our view, this procedure delivers rather more credible estimates of exogenous policy shifts, which cannot be directly identified either by statistical methods or from futures data. It is conceivable that our data could be of use in settling the debate on the effects of exogenous policy shocks on the real economy. This issue, and many others, are left for future research.

1.A Determining $k(\lambda_t)$

We want to determine $k_t = k(\lambda_t)$ in (1.12), which means determining $\pi_{t+2|t+1}(\pi_{t+1|t})$. Substitute for $\pi_{t+2|t}$ in (1.13), and solve for $y_{t+1|t}$:

$$\begin{aligned} y_{t+1|t} &= -\frac{\alpha\delta k_t}{\lambda_t}\pi_{t+2|t} \\ &= -\frac{\alpha\delta k_t}{\lambda_t}\left(\pi_{t+1|t} + \alpha y_{t+1|t}\right) \\ &= -\frac{\alpha\delta k_t}{\lambda_t + \alpha^2\delta k_t}\pi_{t+1|t}. \end{aligned} \tag{1.40}$$

Then

$$\begin{aligned} \pi_{t+2|t} &= \pi_{t+1|t} + \alpha y_{t+1|t} \\ &= \frac{\lambda_t}{\lambda_t + \alpha^2\delta k_t}\pi_{t+1|t}. \end{aligned} \tag{1.41}$$

Use (1.12), apply the envelope theorem on the Bellman equation (1.7), use the law of iterated expectations, and substitute for $\pi_{t+2|t}$ from (1.41) to get

$$\begin{aligned} V_\pi(\pi_{t+1|t};\lambda_t) &= k_t\pi_{t+1|t} \\ &= \pi_{t+1|t} + \delta k_t\pi_{t+2|t} \\ &= \left(1 + \frac{\delta\lambda_t k_t}{\lambda_t + \alpha^2\delta k_t}\right)\pi_{t+1|t}. \end{aligned} \tag{1.42}$$

Thus,

$$k_t = 1 + \frac{\delta\lambda_t k_t}{\lambda_t + \alpha^2\delta k_t}, \tag{1.43}$$

$$k_t^2 + \left(\frac{\lambda_t(1-\delta)}{\alpha^2\delta} - 1\right)k_t - \frac{\lambda_t}{\alpha^2\delta} = 0 \tag{1.44}$$

gives

$$k_t = \frac{1}{2}\left[1 - \frac{\lambda_t(1-\delta)}{\alpha^2\delta} \pm \sqrt{\left(1 - \frac{\lambda_t(1-\delta)}{\alpha^2\delta}\right)^2 + \frac{4\lambda_t}{\alpha^2\delta}}\right], \tag{1.45}$$

and the unique positive solution for k_t is given by

$$k_t = \frac{1}{2}\left[1 - \frac{\lambda_t(1-\delta)}{\alpha^2\delta} + \sqrt{\left(1 + \frac{\lambda_t(1-\delta)}{\alpha^2\delta}\right)^2 + \frac{4\lambda_t}{\alpha^2}}\right], \tag{1.46}$$

following Svensson (1997b, p. 1141f).

1.B Evaluating $dA_t/d\lambda_t$

Following Svensson (1997b, p. 1143), to evaluate the derivative

$$\frac{dA_t}{d\lambda_t} = \frac{d}{d\lambda_t}\frac{\alpha\delta k_t}{\gamma\left(\lambda_t + \alpha^2\delta k_t\right)}, \tag{1.47}$$

consider the ratio

$$\frac{k_t}{\lambda_t} = \frac{1}{2}\left[\frac{1}{\lambda_t} - \frac{(1-\delta)}{\alpha^2\delta} + \sqrt{\left(\frac{1}{\lambda_t} + \frac{(1-\delta)}{\alpha^2\delta}\right)^2 + \frac{4}{\alpha^2\lambda_t}}\right], \tag{1.48}$$

using (1.14). This expression is clearly decreasing in λ_t, so the inverse of A_t,

$$\frac{1}{A_t} = \frac{\gamma\left(\lambda_t + \alpha^2\delta k_t\right)}{\alpha\delta k_t} = \frac{\gamma\lambda_t}{\alpha\delta k_t} + \alpha\gamma, \tag{1.49}$$

is increasing in λ_t. Consequently A_t will be a decreasing function of λ_t. Also, since $B_t = \beta/\gamma + \alpha A_t$, B_t is also decreasing in λ_t.

1.C Proofs

1.C.1 Proof of Proposition 1

(i) $di_t^n/d\varepsilon_t$ *and* $di_t^n/d\eta_t > 0$. For a supply shock, the expression in curly brackets in equation (1.31) is

$$\begin{aligned} &1 + A_t + \left[1 + A_t\left(1 - \gamma B_t\right)\right] X_t^n \\ &= 1 + A_t + X_t^n + \left(1 - \beta - \alpha\gamma A_t\right) A_t X_t^n. \end{aligned} \tag{1.50}$$

Note that

$$0 < \alpha\gamma A_t = \frac{\alpha^2\delta k_t}{\lambda_t + \alpha^2\delta k_t} \leq 1, \tag{1.51}$$

which implies that

$$0 < \alpha\gamma A_t X_t^n = 1 - \left(1 - \alpha\gamma A_t\right)^{n-1} \leq 1 \tag{1.52}$$

for all n. Consequently,

$$\alpha\gamma A_t^2 X_t^n \leq A_t, \tag{1.53}$$

which, since $\beta < 1$, implies that the right-hand side of equation (1.50) and thus the derivative (1.31) are positive. Similarly, for a demand shock in (1.32), the expression in curly brackets

$$B_t + \alpha \left[1 + A_t (1 - \gamma B_t)\right] X_t^n$$
$$= \frac{\beta}{\gamma} + \alpha \left[A_t + X_t^n + (1 - \beta - \alpha\gamma A_t) A_t X_t^n\right], \tag{1.54}$$

is, by the same argument, also positive.

(*ii*) *$di_t^n / d\varepsilon_t$ and $di_t^n / d\eta_t$ fall with maturity n.* From equation (1.27), note that

$$\frac{di_{t+s|t}}{d\varepsilon_t} = (1 - \alpha\gamma A_t) \frac{di_{t+s-1|t}}{d\varepsilon_t} \tag{1.55}$$

and

$$\frac{di_{t+s|t}}{d\eta_t} = (1 - \alpha\gamma A_t) \frac{di_{t+s-1|t}}{d\eta_t}. \tag{1.56}$$

Since $0 < \alpha\gamma A_t \leq 1$, the response of expected future short rates to a current shock is non-increasing over time (in absolute terms). Since long rates are an average of expected short rates, and every new term will be smaller than the average, the entire average will decrease with maturity n. □

1.C.2 Proof of Proposition 2

Recall that the long rate is given by

$$i_t^n = \frac{1}{n} \left\{(1 + A_t) \pi_t + B_t y_t + \left[1 + A_t (1 - \gamma B_t)\right] X_t^n \left[\pi_t + \alpha y_t\right]\right\} + \xi_t^n, \tag{1.57}$$

where

$$X_t^n = \frac{1 - (1 - \alpha\gamma A_t)^{n-1}}{\alpha\gamma A_t}, \tag{1.58}$$

and ξ_t^n is a term premium.

That the short end of the yield curve responds less to a given shock as λ_t increases follows from the optimal interest rate rule

$$i_t = (1 + A_t) \pi_t + B_t y_t, \tag{1.59}$$

where A_t and B_t are decreasing in λ_t (see Appendix 1.B above).

Showing that the long end responds more to a given shock with a higher λ_t is more complicated. After a supply shock, the interest rate of maturity n reacts according to

$$\frac{di_t^n}{d\varepsilon_t} = \frac{1}{n}\left\{1 + A_t + \left[1 + A_t\left(1 - \gamma B_t\right)\right] X_t^n\right\}. \tag{1.60}$$

As the central bank preference parameter λ_t changes, this reaction changes by

$$\frac{d}{d\lambda_t}\left[\frac{di_t^n}{d\varepsilon_t}\right] = \frac{d}{dA_t}\frac{1}{n}\left\{1 + A_t + X_t^n + \left[1 - \alpha\gamma A_t - \beta\right] A_t X_t^n\right\}\frac{dA_t}{d\lambda_t}$$

$$= \frac{1}{n}\left\{1 + \frac{dX_t^n}{dA_t} - \alpha\gamma A_t X_t^n + \left[1 - \alpha\gamma A_t - \beta\right]\frac{d(A_t X_t^n)}{dA_t}\right\}\frac{dA_t}{d\lambda_t}. \tag{1.61}$$

Define $\rho_t = 1 - \alpha\gamma A_t$, implying that

$$X_t = \frac{1 - \rho_t^{n-1}}{1 - \rho_t}, \tag{1.62}$$

$$A_t X_t^n = \frac{1 - \rho_t^{n-1}}{\alpha\gamma}, \tag{1.63}$$

$$\alpha\gamma A_t X_t^n = 1 - \rho_t^{n-1}, \tag{1.64}$$

and

$$\begin{aligned} \frac{dX_t}{dA_t} &= \frac{dX_t}{d\rho_t}\frac{d\rho_t}{dA_t} \\ &= -\alpha\gamma\frac{-(n-1)(1-\rho_t)\rho_t^{n-2} + (1 - \rho_t^{n-1})}{(1-\rho_t)^2} \\ &= \frac{\rho_t^{n-2}\left[(n-1)(1-\rho_t) + \rho_t\right] - 1}{(1-\rho_t)A_t}. \end{aligned} \tag{1.65}$$

Then

$$\frac{d}{d\lambda_t}\left[\frac{di_t^n}{d\varepsilon_t}\right] = \frac{1}{n}\left\{1 + \frac{dX_t^n}{dA_t} - \alpha\gamma A_t X_t^n + (\rho_t - \beta)\frac{d\left(A_t X_t^n\right)}{dA_t}\right\}\frac{dA_t}{d\lambda_t} \tag{1.66}$$

$$= \frac{1}{n}\left\{\rho_t^{n-1} + \frac{\rho_t^{n-2}\left[(n-1)(1-\rho_t) + \rho_t\right] - 1}{(1-\rho_t)A_t} + (\rho_t - \beta)(n-1)\rho_t^{n-2}\right\}\frac{dA_t}{d\lambda_t}.$$

Multiplying by $(1-\rho_t)A_t \geq 0$ and rearranging, the term in curly brackets is

$$\rho_t^{n-2}(n-1)(1-\rho_t)\left[A_t(\rho_t - \beta) + 1\right] + \rho_t^{n-1}\left[(1-\rho_t)A_t + 1\right] - 1. \tag{1.67}$$

As n increases indefinitely, both ρ_t^{n-1} and $(n-1)\rho_t^{n-2}$ tend to zero, making the term in (1.67) negative. Since $dA_t/d\lambda_t$ is negative, the entire derivative (1.66) is then positive for a sufficiently large n.

After a demand shock, the reaction of long rates is

$$\begin{aligned} \frac{di_t^n}{d\eta_t} &= \frac{1}{n}\left\{B_t + \alpha\left[1 + A_t\left(1 - \gamma B_t\right)\right] X_t^n\right\} \\ &= \frac{1}{n}\left\{\frac{\beta}{\gamma} + \alpha\left[A_t + \left(1 + A_t(1 - \gamma B_t)\right) X_t^n\right]\right\}. \end{aligned} \tag{1.68}$$

Consequently

$$\frac{d}{d\lambda_t}\left[\frac{di_t^n}{d\eta_t}\right] = \alpha\frac{d}{d\lambda_t}\left[\frac{di_t^n}{d\varepsilon_t}\right], \tag{1.69}$$

so the reaction of long rates to a given demand shock is thus affected by changes in λ_t in the same direction as the reaction to a supply shock. □

1.C.3 Proof of Proposition 4

For a new shock, the proof follows directly from the proof of Proposition 2 in Appendix 1.C.2. For an old shock being worked out by the central bank, note that (1.22) implies that the sensitivity of the central bank rate in period $t+s$ to a supply shock in period t is

$$\frac{di_{t+s}}{d\varepsilon_t} = \left(1 + A_{t+s}\right)\frac{d\pi_{t+s}}{d\varepsilon_t} + B_{t+s}\frac{dy_{t+s}}{d\varepsilon_t}. \tag{1.70}$$

Since $d\pi_{t+s}/d\varepsilon_t$ and $dy_{t+s}/d\varepsilon_t$ depend only on the initial λ_t, and so are not affected by the preference shift at $t+s$, and since $dB_t/d\lambda_t = \alpha dA_t/d\lambda_t$, the derivative of $di_{t+s}/d\varepsilon_t$ with respect to λ_{t+s} is, using (1.25) and (1.26),

$$\begin{aligned} \frac{d}{d\lambda_{t+s}}\left[\frac{di_{t+s}}{d\varepsilon_t}\right] &= \left[\frac{d\pi_{t+s}}{d\varepsilon_t} + \alpha\frac{dy_{t+s}}{d\varepsilon_t}\right]\frac{dA_{t+s}}{d\lambda_{t+s}} \\ &= \left(1 - \alpha\gamma A_t\right)^s \frac{dA_{t+s}}{d\lambda_{t+s}} \\ &= \left(1 - \alpha\gamma A_t\right)^s \frac{d}{d\lambda_{t+s}}\left[\frac{di_t}{d\varepsilon_t}\right]. \end{aligned} \tag{1.71}$$

After s periods, only a fraction $\left(1 - \alpha\gamma A_t\right)^s$ of the shock from time t remains in the system. Thus, the qualitative effects of a preference shift in period $t+s$ are the same as a change in period t, and the same applies to all long rates. □

1.D Classification of Federal Reserve actions

Classification:

End	Endogenous; based on new economic information
Ex	Exogenous; based on preference shifts
R	Employment report released on same day
U	Action unnoticed

Event	Date	Adj (%)	Description of event	Class
1	Oct 20, 1988	+0.125	"...the Federal Reserve provided a hint that it isn't tightening credit."	U
2	Nov 17, 1988[1]	+0.0625	"Investment managers worry that the dollar's weakness soon will lead to even higher interest rates."	U
3	Nov 22, 1988	+0.0625	No mention of monetary policy.	U
4	Dec 15, 1988	+0.3125	"Several recent economic reports have indicated robust economic growth that aroused inflation jitters."	End
5	Dec 29, 1988[1]	+0.0625	"...the federal funds rate rose again, largely reflecting what traders refer to as 'year-end window dressing'."	U
6	Jan 5, 1989	+0.25	"...the Fed's aggressive moves might encourage bond investors by convincing them of the central bank's determination to keep inflation under control."	Ex
7	Feb 9, 1989[1]	+0.0625	"Some analysts predict the Fed... will raise rates Friday or early next week."	U
8	Feb 14, 1989	+0.25	"Fed officials are tightening their credit clamp further in an effort to rein in on inflation." *Before:* " 'If, as we expect, the Fed gradually nudges the federal funds rate towards 9 1/2%, market participants may regain faith that containing inflation remains a top priority for the monetary authorities.' "	Ex
9	Feb 23, 1989[1]	+0.25	"The Federal Reserve, trying to calm inflation worries, drove up short-term interest rates."	End
10	Feb 24, 1989[2]	+0.1875	"The Fed's long-awaited discount-rate increase is too small and too late to help calm inflation fears..."	Ex
11	May 4, 1989[1]	+0.0625	No mention of monetary policy.	U
12	Jun 6, 1989	−0.25	"The U.S. Federal Reserve apparently has eased its grip on credit, reflecting the belief of many Fed officials that the economy has slowed..."	End

Event	Date	Adj (%)	Description of event	Class
13	Jul 7, 1989	–0.25	"...for several weeks now, strong signs of economic weakness have convinced Fed officials to ease instead."	End
14	Jul 27, 1989	–0.25	"...it became clear that the Federal Reserve is easing credit and that the economy is growing weaker."	End
15	Aug 10, 1989[1]	–0.0625	No mention of monetary policy.	U
16	Oct 18, 1989	–0.25	No mention of monetary policy.	U
17	Nov 6, 1989	–0.25	No mention of monetary policy.	U
18	Dec 20, 1989	–0.25	"'Coming right after an FOMC meeting, they would not have entered the market unless they wanted to send a clear signal that policy had changed.'"	Ex
19	Jul 13, 1990	–0.25	"Several investment managers fear that the Fed pulled the trigger too soon..." "'If you're looking to the Fed as a bulwark against inflation, then this doesn't support that case.'"	Ex
20	Oct 29, 1990	–0.25	"...widely anticipated move..." *Before:* "...further signs of U.S. economic weakness..."	End
21	Nov 14, 1990	–0.25	"...few investors are willing to participate in the market until they see clear signs that the Federal Reserve has eased monetary policy."	U
22	Dec 7, 1990	–0.25	"...[the Fed's] move came shortly after the U.S. Labor Department reported a surge in the November U.S. employment and sharp declines in jobs."	R
23	Dec 19, 1990[2]	–0.25	"...some disappointment that the Federal Reserve didn't signal a larger cut in the rate."	Ex
24	Jan 8, 1991[3]	–0.25	"After yesterday's easing move, the new level for the rate is believed to be 6 3/4%."	End
25	Feb 1, 1991[2]	–0.5	"Prices of U.S. government bonds soared Friday in response to a surprisingly weak U.S. employment report and a cut in the discount rate by the Federal Reserve."	R
26	Mar 8, 1991	–0.25	"...they ignored the Department of Labor's report that the unemployment rate rose to 6.5% from 6.2%..."	R
27	Apr 30, 1991[2]	–0.25	"...the central bank surprised the market by pushing rates another notch lower." "...[the move] didn't follow any major economic report..." "...'smacks of some political pressure on the Fed.'"	Ex
28	Aug 6, 1991	–0.25	"'On any kind of economic basis, the Fed move was entirely justified'..."	End

Event	Date	Adj (%)	Description of event	Class
29	Sep 13, 1991[2]	−0.25	"The U.S. Federal Reserve's latest move to cut interest rates reflects its uneasiness about the slow growth of money supply and the disappointingly torpid economic recovery."	End
30	Oct 30, 1991[3]	−0.25	"...by late afternoon, the Fed had eased at least 25 basis points..." *Before:* "Evidence the recovery is wilting and inflation is waning..."	End
31	Nov 6, 1991[2]	−0.25	"...the Federal Reserve Bank's surprise announcement of a discount rate cut."	End
32	Dec 6, 1991	−0.25	"...news from the U.S. Labor Department that non-farm payrolls shrank 241.000 in November."	R
33	Dec 20, 1991[2]	−0.5	"A still-faltering economy and slower inflation is likely to cause U.S. interest rates to fall even further..." "...following the Federal Reserve's surprisingly aggressive move on Friday..."	End
34	Apr 9, 1992	−0.25	"...the Fed's decision to cut rates...came primarily for concerns about recent contractions in the U.S. money supply."	End
35	Jul 2, 1992[2]	−0.5	"...a stunningly weak employment report, which unlocked the door for lower interest rates."	R
36	Sep 4, 1992	−0.25	"...in the wake of Friday's extraordinarily weak employment report."	R
37	Feb 4, 1994	+0.25	"The tightening came about three hours after a weaker-than-expected January employment report."	R
38	Mar 22, 1994	+0.25	*Before:* "Some studies show that inflationary pressures are building..." "...traders and investors had been expecting such a move for some time..."	End
39	Apr 18, 1994	+0.25	*Before:* "...fear that we are going to see an acceleration of inflation." "...disappointment that the Fed didn't raise interest rates by a larger margin."	End
40	May 17, 1994[2]	+0.5	"...analysts said the Fed has indicated it will sit tight for a little while..." "...the action cleared the air of uncertainty that had been restraining investors for months."	Ex
41	Aug 16, 1994[2]	+0.5	"'...a clear signal that the Fed intends to fight inflation pressures,'" "...improvement in inflation psychology..."	Ex
42	Nov 15, 1994[2]	+0.75	"...bigger-than-expected boost in interest rates by the U.S. Federal Reserve." "...market participants view the Fed as doing well in its effort to contain inflation."	Ex

Event	Date	Adj (%)	Description of event	Class
43	Feb 1, 1995[2]	+0.5	"...the US Federal Reserve raised short-term rates and indicated that there are only tentative signs the economy is slowing."	End
44	Jul 6, 1995	−0.25	"...the Fed's willingness to ease ahead of Friday's data suggests that the central bank is looking for a weak employment report."	End
45	Dec 19, 1995	−0.25	"'...inflation has been somewhat more favorable than anticipated...'"	End
46	Jan 31, 1996[2]	−0.25	"'This rate cut says the Fed is likely to be more aggressive cutting rates than people thought'..."	Ex
47	Mar 25, 1997	+0.25	"...'the risk of inflation is increasing'..."	End

[1]No actual policy decision, according to Roley and Sellon (1996).
[2]Also discount rate change.
[3]Target change noticed one day before official target change.

1.E Interest rate data

Date	Target	Change	3m	6m	1y	2y	3y	5y	7y	10y	30y
881020	8.25	+0.1250	+0.02	−0.01	−0.01	+0.00	+0.00	−0.01	−0.02	−0.03	−0.06
881117	8.3125	+0.0625	−0.02	−0.02	−0.02	+0.00	+0.03	+0.03	+0.05	+0.06	+0.04
881122	8.3750	+0.0625	+0.01	+0.08	+0.07	+0.07	+0.08	+0.07	+0.05	+0.05	+0.01
881215	8.6875	+0.3125	+0.07	−0.08	+0.05	+0.00	+0.01	+0.02	+0.00	+0.00	+0.01
881229	8.75	+0.0625	−0.13	−0.11	−0.03	−0.05	−0.03	−0.03	−0.05	−0.03	+0.00
890105	9.00	+0.25	+0.02	+0.09	+0.07	+0.07	+0.08	+0.08	+0.06	+0.05	+0.02
890209	9.0625	+0.0625	−0.05	−0.01	+0.00	+0.07	+0.08	+0.15	+0.19	+0.18	+0.17
890214	9.3125	+0.25	+0.01	+0.05	+0.04	+0.04	+0.02	+0.05	+0.04	+0.03	+0.04
890223	9.5625	+0.25	+0.08	+0.08	+0.12	+0.12	+0.11	+0.04	+0.09	+0.07	+0.05
890224	9.75	+0.1875	+0.04	+0.13	+0.03	+0.06	+0.02	+0.02	+0.01	+0.02	+0.01
890504	9.8125	+0.0625	+0.00	+0.02	−0.01	+0.00	+0.00	−0.01	+0.00	+0.00	+0.04
890606	9.5625	−0.25	−0.10	−0.12	+0.02	+0.01	+0.01	+0.02	+0.00	+0.00	−0.03
890707	9.3125	−0.25	−0.04	−0.04	−0.06	−0.11	−0.07	−0.05	−0.07	−0.06	−0.06
890727	9.0625	−0.25	−0.12	−0.08	−0.12	−0.10	−0.13	−0.12	−0.09	−0.10	−0.08
890810	9.00	−0.0625	−0.05	−0.01	−0.02	+0.00	−0.01	+0.00	+0.00	−0.03	−0.04
891018	8.75	−0.25	+0.07	+0.02	−0.01	+0.00	+0.00	+0.00	+0.03	+0.01	+0.02
891106	8.50	−0.25	+0.03	+0.04	+0.04	+0.03	+0.05	+0.06	+0.06	+0.04	+0.05
891220	8.25	−0.25	−0.10	−0.09	−0.08	−0.08	−0.03	−0.02	−0.03	−0.01	−0.01
900713	8.00	−0.25	−0.08	−0.06	−0.07	−0.07	−0.06	−0.05	−0.06	−0.05	−0.04
901029	7.75	−0.25	+0.02	+0.03	+0.02	+0.02	+0.04	+0.04	+0.06	+0.07	+0.08
901114	7.50	−0.25	+0.03	−0.02	+0.01	−0.02	+0.00	+0.00	−0.01	−0.01	−0.01
901207	7.25	−0.25	−0.11	−0.13	−0.13	−0.15	−0.15	−0.15	−0.15	−0.15	−0.16
901219	7.00	−0.25	−0.11	−0.13	−0.12	−0.07	−0.03	−0.03	−0.01	+0.01	+0.04
910108	6.75	−0.25	−0.07	−0.10	−0.08	−0.05	−0.04	−0.01	+0.02	+0.03	+0.05
910201	6.25	−0.50	−0.19	−0.23	−0.22	−0.22	−0.20	−0.17	−0.14	−0.12	−0.12
910308	6.00	−0.25	−0.10	−0.10	−0.09	−0.04	+0.01	+0.04	+0.04	+0.06	+0.07
910430	5.75	−0.25	−0.08	−0.14	−0.14	−0.10	−0.07	−0.07	−0.05	−0.05	−0.03
910806	5.50	−0.25	−0.09	−0.11	−0.12	−0.12	−0.14	−0.08	−0.07	−0.07	−0.06
910913	5.25	−0.25	−0.06	−0.05	−0.03	−0.02	−0.02	−0.02	−0.04	−0.01	−0.01
911030	5.00	−0.25	−0.05	−0.06	−0.05	−0.05	−0.02	−0.03	−0.01	−0.03	−0.01
911106	4.75	−0.25	−0.13	−0.11	−0.08	−0.07	−0.07	−0.07	−0.05	−0.05	−0.01
911206	4.50	−0.25	−0.07	−0.09	−0.07	−0.03	−0.02	+0.01	+0.03	+0.05	−0.08
911220	4.00	−0.50	−0.30	−0.29	−0.26	−0.25	−0.20	−0.17	−0.14	−0.14	−0.09
920409	3.75	−0.25	−0.21	−0.22	−0.23	−0.24	−0.22	−0.17	−0.12	−0.09	−0.09
920702	3.25	−0.50	−0.31	−0.29	−0.32	−0.27	−0.23	−0.25	−0.20	−0.17	−0.13
920904	3.00	−0.25	−0.22	−0.23	−0.23	−0.20	−0.21	−0.20	−0.17	−0.14	−0.08
940204	3.25	+0.25	+0.10	+0.11	+0.17	+0.14	+0.15	+0.15	+0.14	+0.13	+0.06
940322	3.50	+0.25	+0.00	−0.06	−0.07	−0.06	−0.11	−0.11	−0.12	−0.11	−0.09
940418	3.75	+0.25	+0.11	+0.14	+0.16	+0.19	+0.20	+0.20	+0.20	+0.17	+0.12
940517	4.25	+0.50	+0.05	−0.05	−0.11	−0.15	−0.17	−0.19	−0.23	−0.21	−0.19
940816	4.75	+0.50	+0.17	+0.02	−0.01	−0.05	−0.06	−0.10	−0.12	−0.11	−0.12
941115	5.50	+0.75	+0.10	+0.09	+0.05	+0.04	+0.04	+0.03	−0.02	−0.02	−0.04
950201	6.00	+0.50	+0.07	+0.08	+0.11	+0.07	+0.05	+0.02	+0.07	+0.06	+0.04
950706	5.75	−0.25	−0.14	−0.17	−0.17	−0.22	−0.20	−0.20	−0.15	−0.14	−0.10
951219	5.50	−0.25	−0.11	−0.09	−0.09	−0.07	−0.06	−0.08	−0.06	−0.04	−0.06
960131	5.25	−0.25	−0.08	−0.07	−0.07	−0.09	−0.07	−0.05	−0.05	−0.03	−0.01
970325	5.50	+0.25	+0.04	+0.02	+0.03	+0.04	+0.03	+0.05	+0.03	+0.03	+0.01

New level and adjustment of the federal funds rate target, one-day changes in market interest rates, and classification of 47 policy events October 1988–May 1997. Sources: Funds rate target 1988–92, Rudebusch (1995); Funds rate target 1993–97, Federal Reserve Bank of New York; Market interest rates, Federal Reserve Bank of St. Louis.

1.F Calculating the imputed coefficients and standard errors

1.F.1 General case

Before analyzing our special case, let us consider a more general problem. Suppose we would like to estimate the regression

$$y_t = \alpha + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \varepsilon_t, \tag{1.72}$$

where x_1 and x_2 are independent variables. Recall that the least-squares estimate of β_i, $i = 1, 2$ is given by

$$b_i = \frac{\mathrm{Cov}(x_i, y)}{\mathrm{Var}(x_i)}, \tag{1.73}$$

and its variance is

$$\mathrm{Var}(b_i) = \frac{\sigma^2}{(N-1)\mathrm{Var}(x_i)}, \tag{1.74}$$

where the residual variance σ^2 is estimated as

$$\hat{\sigma}^2 = \frac{\sum_h e_h^2}{N-k}. \tag{1.75}$$

The parameter N is the number of observations in regression (1.72), and k is the number of explanatory variables (here $k = 3$).

Suppose we cannot observe x_1 and x_2 directly, but only their sum $x = x_1 + x_2$. Thus, equation (1.72) cannot be estimated. However, if we have estimates from other sources of $\mathrm{Var}(x_1)$ and $\mathrm{Cov}(x_1, y)$, then we can calculate $\mathrm{Var}(x_2)$ and $\mathrm{Cov}(x_2, y)$ as

$$\mathrm{Var}(x_2) = \mathrm{Var}(x) - \mathrm{Var}(x_1) \tag{1.76}$$

and

$$\mathrm{Cov}(x_2, y) = \mathrm{Cov}(x, y) - \mathrm{Cov}(x_1, y), \tag{1.77}$$

since x_1 and x_2 are independent, and $\mathrm{Var}(x)$ and $\mathrm{Cov}(x, y)$ are known. Consequently, we can calculate the least-squares estimates of the slope coefficients for the hypothetical regression (1.72) from equation (1.73).

As for the variance of the slope coefficients, we need an expression for the hypothetical residual sum of squares. This sum can be computed as[23]

$$\sum_h e_h^2 = (N-1)\left[\mathrm{Var}(y) - b_1 \mathrm{Cov}(x_1, y) - b_2 \mathrm{Cov}(x_2, y)\right]. \tag{1.78}$$

The estimated variance of b_i is then calculated from equations (1.74) and (1.75). Finally, given the residual sum of squares, we can compute the measures of fit as

$$R^2 = 1 - \frac{\sum_h e_h^2}{\sum_h y_h^2} \tag{1.79}$$

and

$$\bar{R}^2 = 1 - \frac{\sum_h e_h^2/(N-k)}{\sum_h y_h^2/(N-1)}. \tag{1.80}$$

1.F.2 Our case

To translate these results into our setting, we would like to estimate

$$\Delta i_t^n = \alpha_n + \beta_n^{\mathrm{Ex}^*} \Delta i_t^{\mathrm{Ex}^*} + \beta_n^{\mathrm{NP}^*} \Delta i_t^{\mathrm{NP}^*} + \varepsilon_t^n, \tag{1.81}$$

on our 9 exogenous policy events, where $\Delta i_t^{\mathrm{Ex}^*}$ is the truly exogenous component of the policy innovation at t and $\Delta i_t^{\mathrm{NP}^*}$ is the non-policy event of exogenous policy days. We cannot observe $\Delta i_t^{\mathrm{Ex}^*}$ and $\Delta i_t^{\mathrm{NP}^*}$ directly, however, but we can observe the total policy innovation

$$\Delta i_t^{\mathrm{3m}} = \Delta i_t^{\mathrm{Ex}^*} + \Delta i_t^{\mathrm{NP}^*}. \tag{1.82}$$

Thus we can estimate the regression

$$\Delta i_t^n = \tilde{\alpha}_n + \beta_n \Delta i_t^{\mathrm{3m}} + \tilde{\varepsilon}_t^n, \tag{1.83}$$

and the results are reported in Table 1.4.

Assuming that the non-policy event of exogenous policy days behaves like on any non-policy day, we can approximate its variance and covariance with the dependent

[23]Note that

$$\begin{aligned}
\sum_h e_h^2 &= \sum_h e_h \left[y_h - a - b_1 x_{1h} - b_2 x_{2h}\right] \\
&= \sum_h (y_h - \bar{y}) \left[(y_h - \bar{y}) - b_1(x_{1h} - \bar{x}_1) - b_2(x_{2h} - \bar{x}_2)\right] \\
&= \sum_h (y_h - \bar{y})^2 - \sum_h b_1 (y_h - \bar{y})(x_{1h} - \bar{x}_1) - \sum_h b_2 (y_h - \bar{y})(x_{2h} - \bar{x}_2),
\end{aligned}$$

giving the expression in equation (1.78). See, for example, Gujarati (1988, section 7A.3).

Table 1.4: Original regression results

	6 months	1 year	2 years	3 years	5 years	7 years	10 years	30 years
β_n	1.283**	0.953**	0.861**	0.577**	0.527*	0.302	0.198	−0.009
	(0.205)	(0.180)	(0.164)	(0.160)	(0.168)	(0.178)	(0.163)	(0.159)
$\sum_h (e_h^n)^2$	0.01321	0.01019	0.00843	0.00801	0.00881	0.00989	0.00827	0.00795
$\hat{\sigma}^2$	0.00189	0.00146	0.00120	0.00114	0.00126	0.00141	0.00118	0.00114
R^2	0.848	0.799	0.797	0.650	0.585	0.291	0.175	0.001
$\bar{R}^2$	0.826	0.771	0.768	0.600	0.526	0.190	0.057	−0.142

OLS estimation of equation (1.83) on 9 exogenous policy days. Constant terms not reported, standard errors in parentheses. **/* denote significance at the 1%-/5%-level.

variable Δi_t^n by those calculated over the 2,135 non-policy days. Also, the variance of the policy innovation Δi_t^{3m} and its covariance with the interest rate response Δi_t^n on the 9 exogenous events are known.

Thus, assuming that the truly exogenous component and the non-policy event are independent, we can compute the variances of the truly exogenous component and its covariance with the dependent variable on the exogenous events as

$$\mathrm{Var}(\Delta i_t^{\mathrm{Ex}^*}) = \mathrm{Var}(\Delta i_t^{3\mathrm{m}}) - \mathrm{Var}(\Delta i_t^{\mathrm{NP}^*}) \tag{1.84}$$

and

$$\mathrm{Cov}(\Delta i_t^{\mathrm{Ex}^*}, \Delta i_t^n) = \mathrm{Cov}(\Delta i_t^{3\mathrm{m}}, \Delta i_t^n) - \mathrm{Cov}(\Delta i_t^{\mathrm{NP}^*}, \Delta i_t^n). \tag{1.85}$$

The resulting variances and covariance for all maturities are reported in Table 1.5.

Following the general discussion above, we are then able to calculate the least-squares estimates from the hypothetical regression (1.81) as

$$b_n^j = \frac{\mathrm{Cov}(\Delta i_t^j, \Delta i_t^n)}{\mathrm{Var}(\Delta i_t^j)} \tag{1.86}$$

for $j = Ex*, NP*$. To calculate the estimated variance of b_n^j, we first calculate the residual sum of squares as

$$\begin{aligned} \sum_h (e_h^n)^2 &= (N-1) \\ &\times \left[\mathrm{Var}(\Delta i_t^n) - b_n^{\mathrm{Ex}^*}\mathrm{Cov}(\Delta i_t^{\mathrm{Ex}^*}, \Delta i_t^n) - b_n^{\mathrm{NP}^*}\mathrm{Cov}(\Delta i_t^{\mathrm{NP}^*}, \Delta i_t^n)\right], \end{aligned} \tag{1.87}$$

and the variance of b_n^j is given by

$$\mathrm{Var}(b_n^j) = \frac{\sum_h (e_h^n)^2 / (N-k)}{(N-1)\mathrm{Var}(\Delta i_t^j)}, \tag{1.88}$$

and R^2 and $\bar{R}^2$ are calculated from equations (1.79) and (1.80). The results from the hypothetical regression (1.81) are presented in Table 1.6.

Table 1.5: Variances and covariances

	Policy innovation							
$\text{Var}(\Delta i_t^{3\text{m}})$	0.00559							
$\text{Var}(\Delta i_t^{\text{NP}*})$	0.00209							
$\text{Var}(\Delta i_t^{\text{Ex}*})$	0.00349							
	6 months	1 year	2 years	3 years	5 years	7 years	10 years	30 years
$\text{Var}(\Delta i_t^n)$	0.01085	0.00634	0.00519	0.00286	0.00265	0.00174	0.00125	0.00099
$\text{Cov}(\Delta i_t^{3\text{m}}, \Delta i_t^n)$	0.00717	0.00532	0.00481	0.00322	0.00294	0.00168	0.00111	−0.00005
$\text{Cov}(\Delta i_t^{\text{NP}*}, \Delta i_t^n)$	0.00169	0.00162	0.00174	0.00165	0.00151	0.00130	0.00117	0.00089
$\text{Cov}(\Delta i_t^{\text{Ex}*}, \Delta i_t^n)$	0.00548	0.00370	0.00307	0.00158	0.00143	0.00038	−0.00006	−0.00094

$\text{Var}(\Delta i_t^{3\text{m}})$, $\text{Var}(\Delta i_t^n)$, and $\text{Cov}(\Delta i_t^{3\text{m}}, \Delta i_t^n)$ are calculated over 9 exogenous policy days; $\text{Var}(\Delta i_t^{\text{NP}*})$ and $\text{Cov}(\Delta i_t^{\text{NP}*}, \Delta i_t^n)$ are calculated over 2,135 non-policy days; $\text{Var}(\Delta i_t^{\text{Ex}*})$ and $\text{Cov}(\Delta i_t^{\text{Ex}*}, \Delta i_t^n)$ are computed according to equations (1.84) and (1.85).

Table 1.6: Hypothetical regression results

	6 months	1 year	2 years	3 years	5 years	7 years	10 years	30 years
$\beta_n^{\text{Ex}*}$	1.568**	1.060**	0.879**	0.451°	0.410	0.110	−0.017	−0.270°
	(0.207)	(0.236)	(0.224)	(0.202)	(0.215)	(0.207)	(0.169)	(0.131)
$\beta_n^{\text{NP}*}$	0.808*	0.773*	0.832*	0.787*	0.722*	0.621°	0.557*	0.426*
	(0.267)	(0.305)	(0.289)	(0.261)	(0.278)	(0.267)	(0.219)	(0.169)
$\sum_h \left(e_h^n\right)^2$	0.00716	0.00933	0.00840	0.00683	0.00779	0.00715	0.00481	0.00287
$\hat{\sigma}^2$	0.00119	0.00155	0.00140	0.00114	0.00130	0.00119	0.00080	0.00048
$\sum_h \left(\Delta i_h^n\right)^2$	0.08870	0.06010	0.04600	0.02400	0.02140	0.01530	0.01030	0.00800
R^2	0.919	0.845	0.817	0.716	0.636	0.532	0.533	0.641
$\bar{R}^2$	0.892	0.793	0.756	0.621	0.515	0.377	0.377	0.521
$\beta_n^{\text{Ex}*} = \beta_n^{\text{NP}*}$	5.075°	0.554	0.016	1.039	0.786	2.296	4.312°	10.597*

Hypothetical OLS estimation of equation (1.81) on 9 exogenous observations. Constant terms not reported, standard errors in parentheses. **/*/° denote significance at the 1%-/5%-/10%-level.

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Chapter 2

Predicting monetary policy using federal funds futures prices

Abstract:
In theory, prices of current-month federal funds futures contracts should reflect market expectations of near-term movements in the Federal Reserve's target level for the federal funds rate. However, empirical results show that such measures of market expectations are too noisy to predict day-to-day changes in the funds rate target; partly due to time aggregation problems, partly because they are affected by funds rate movements not directly related to monetary policy considerations. In particular, the futures market shows a large amount of systematic variation across months and trading days, variation that needs to be taken into account when predicting policy moves or extracting policy expectations. For the period from January 1994 to February 1998, the extracted expectations perform fairly well in predicting the target level that will prevail after the next meeting of the Federal Open Market Committee, especially when adjusting for market regularities.

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2.1 Introduction

SINCE THEIR INTRODUCTION in October 1988, prices of federal funds futures contracts have become very popular as a simple way of measuring market expectations about the future path of monetary policy and trying to predict future policy moves. Since these contracts are based on the monthly average of the federal funds rate, which is the main policy instrument of the Federal Reserve, efficient futures markets should set prices to reflect the expected path of Fed policy. The usefulness of federal funds futures contracts in predicting monetary policy moves one to three months ahead has been demonstrated by, for example, Carlson et al. (1995), Krueger and Kuttner (1996), Robertson and Thornton (1997), and Rudebusch (1998).

Existing studies have concentrated on using monthly averages or end-of-the-month observations on futures contracts of one to three months' maturity to predict policy moves on that horizon. However, in contrast to many other futures contracts, federal funds futures are also traded during the contract month, when past observations of the funds rate are known. Therefore it should be possible to extract even more precise market expectations about the average funds rate for the rest of the current contract month. Since the federal funds rate on average follows the target set by the Federal Reserve, such measures could be interpreted as the expected average of the federal funds rate target for the remaining days of the month, and, in particular on the day before a meeting of the Federal Open Market Committee from 1994 onwards, as the level of the funds rate target expected to prevail after the meeting, since policy moves after 1994 have been made almost exclusively at FOMC meetings.[1]

The purpose of this chapter is to examine the use of current-month futures prices to measure monetary policy expectations in the very short run (over the period from the next day to the end of the month, when the contract matures). This is done for the period from the introduction of the futures contracts in October 1988 until March 1998. The main question to examine is whether the expected funds rate series calculated from the futures prices is a good predictor of near-term movements in the target level for the federal funds rate.

It turns out that the extracted expectations perform very poorly when predicting day-to-day changes in the federal funds rate target; partly because the market may have been expecting a policy move later in the month, partly because of noise

[1] See Pakko and Wheelock (1996) for such an exercise over the period from 1994 to 1996.

coming from the federal funds cash market and other regularities. Nevertheless, expectations from the day before meetings of the Federal Open Market Committee in the period from 1994 to 1998 are quite successful in predicting the target level which will prevail after the meeting. Even on those occasions, however, the funds rate expectations display some systematic variation across trading days and calendar months. The first of these regularities can be ascribed to the behavior of the actual federal funds rate, which tends to increase on the last days of each month, possibly due to banks engaging in balance sheet 'window dressing.' The second regularity is more puzzling, since no corresponding movement in the federal funds cash market can be observed, although it could be due to increases in the perceived riskiness of futures contracts in these months.

Adjusting for the monthly variation of the expected funds rate series, its predictive value improves dramatically. In an out-of-sample test, the extracted expectations are shown to have predicted the target change in September 1998 very well, and they also improve on expectations taken from the financial press.

As a final exercise, using average monthly futures data to predict the average funds rate and funds rate target one to three months ahead (following, e.g., Carlson et al., 1995, and Krueger and Kuttner, 1996), the monthly variation in futures prices remains. Consequently, these regularities of the futures market are an important factor to take into account when extracting market expectations or predicting policy moves.

The chapter proceeds as follows. In Section 2.2, the federal funds futures market and the extraction of funds rate expectations from futures prices is described, and the relation between the federal funds futures and cash markets is discussed. Section 2.3 presents the empirical results from predicting the funds rate target using the expected funds rate series obtained in the previous section, and Section 2.4 presents some alternative tests of the estimates, by using a case study of the policy move of September 1998, and by comparing the estimates with market expectations from the financial press. Finally, after briefly considering average monthly data in Section 2.5, Section 2.6 concludes.

2.2 The federal funds futures market

2.2.1 The futures contract

The 30-day federal funds futures contract, traded on the Chicago Board of Trade since October 3, 1988, calls for delivery of the interest paid on a principal amount of $5 million in overnight federal funds held for the contract month. The settlement price is calculated as 100 minus the average effective federal funds rate for the contract month, and at maturity, the contract is cash-settled against the monthly average of daily effective federal funds rates, including weekends and holidays, as calculated and reported each business day by the Federal Reserve Bank of New York.[2]

Thus a buyer of a federal funds futures contract will pay (or receive from) the seller an amount corresponding to the interest on $5 million held for the contract month, with the interest rate determined by the difference between the average funds rate for the month and the futures rate negotiated at the trade. Instead of paying the entire sum at maturity, the contract is marked-to-market daily, so payments are made each day as the futures price changes, using a constant tick size of $41.67 (one hundredth of a percent of $5 million over one month). If during a trading day the futures price falls by two basis points (e.g., from 94.53 to 94.51, so that the implied funds rate increases from 5.47% to 5.49%), the buyer pays the seller $2 \times \$41.67 = \$83.34$ per contract. In total, a buyer of a futures contract at a price of 95.50 will, if the futures price settles at 95.00, have paid the seller $50 \times 41.67 = \$2,083.50$ at maturity, equal to the difference between a 5% and a 4.50% interest on $5 million held for 30 days.[3]

As with most futures markets, the federal funds futures market is mainly used by two groups of traders: hedgers and speculators. To see how the futures market can be used for hedging purposes, consider the following example, adapted from the Chicago Board of Trade (1997b). A bank consistently buying $75 million per month in federal funds is worried that the funds rate will increase from the current rate of 5.25%. By selling 15 futures contracts ($15 \times \$5$ million $= \$75$ million), any losses incurred from increases in the funds rate will be offset by gains from the futures position. If the price of the futures contracts is 94.75, implying an expected funds rate of 5.25%, and the average funds rate for the contract month subsequently increases to 5.45%, the monthly interest expense on $75 million is $5.45\% \times 30/360 \times \$75$ million $= \$340,625$. At the same time, however, if the futures price has converged to 94.55 at maturity,[4] the bank gains $15 \times 20 \times \$41.67 = \$12,501$, so the net cost is $328,124, implying an effective cost of funds of 5.25%.

[2]The effective federal funds rate is a weighted average of the rates on those overnight federal funds transactions arranged through New York brokers.

[3]See the Chicago Board of Trade (1997a) for details.

[4]Carlson et al. (1995) show that the futures price does converge to the average funds rate at maturity.

In contrast to the federal funds cash market, which is open only to those depository institutions required to hold reserves with Federal Reserve Banks (Goodfriend and Whelpley, 1993), the federal funds futures market is open to anyone who can satisfy margin requirements (Carlson et al., 1995). Thus, traders and 'Fed watchers' can use futures contracts to speculate on the future path of the federal funds rate. In the example above, a Fed watcher expecting the funds rate to increase from 5.25% to 5.45% when the futures contract sells for 94.75, could, by selling a number of contracts, make a profit of 20 × \$41.67 = \$833.40 per contract as the futures price falls to 94.55.

Such speculation should drive the futures price to the level consistent with market participants' expectations of the average federal funds rate, plus a hedging premium as speculators must be compensated for bearing the risk of hedgers. Since the federal funds rate is closely monitored by the Federal Reserve, and used as their primary policy tool, expected shifts in the monetary policy stance should therefore be priced into the futures market. Consequently, the prices of futures contracts can be used to estimate the expected path of monetary policy over the near future.

An important feature of federal funds futures contracts is that they are traded also during the contract month, to offer more flexible management of interest rate exposure (Chicago Board of Trade, 1997a). During this month, past observations of the funds rate are publicly known, so efficient futures markets should adjust prices to reflect the observed path of the funds rate. Thus, the price of the current-month futures contract contains information about the expected path of the funds rate for the rest of the month, information that should get more precise as the contract gets closer to maturity. Therefore, while most existing studies have used average monthly futures prices to predict coming policy moves, using current-month contracts is a promising way of predicting policy moves in the very near-term future.

2.2.2 Extracting market expectations from futures prices

Since the futures settlement price is calculated as 100 minus the average effective federal funds rate for the contract month, the implied futures rate at day t for the mth month ahead, $i^f_{m,t}$, is given by

$$i^f_{m,t} = 100 - p^f_{m,t}, \tag{2.1}$$

where $p^f_{m,t}$ is the price at day t of a futures contract maturing m months from now. For coming months, the implied futures rate is simply equal to the expected average effective funds rate for the contract month, so

$$i^f_{m,t} = \sum_{\tau=1}^{n_m} \frac{1}{n_m} E_t i^{ff}_{m,\tau}, \tag{2.2}$$

where $i^{ff}_{m,\tau}$ is the effective funds rate on day τ in the mth month from now, and n_m is the number of days in the mth month, including weekends and holidays.

For the current month, market participants have observed the effective funds rate up to the previous trading day (the effective funds rate for a trading day is calculated by the Federal Reserve Bank of New York and published on the following morning), so the expected average funds rate for the entire contract month can be divided into two parts: the observed rates so far in the month and the expected rates for the remaining days. Thus,

$$i^f_t = \frac{1}{n}\left[\sum_{\tau=1}^{t-1} i^{ff}_\tau + \sum_{\tau=t}^{n} E_t i^{ff}_\tau\right], \tag{2.3}$$

where the month subscript m has been skipped for the current month. Defining i^a_{t-1} as the average funds rate up to day $t-1$ in the month,

$$i^a_{t-1} = \frac{1}{t-1}\sum_{\tau=1}^{t-1} i^{ff}_\tau \tag{2.4}$$

and i^e_t as the average expected funds rate for the rest of the month, including day t,

$$i^e_t = \frac{1}{n-t+1}\sum_{\tau=t}^{n} E_t i^{ff}_\tau, \tag{2.5}$$

we can express the current-month futures rate as

$$i^f_t = \frac{1}{n}\left[(t-1)i^a_{t-1} + (n-t+1)i^e_t\right]. \tag{2.6}$$

Then it is straightforward to solve for the expected average funds rate for the rest of the month as

$$i^e_t = \frac{n i^f_t - (t-1)i^a_{t-1}}{n-t+1}. \tag{2.7}$$

Strictly interpreted, the calculated i^e_t is the expected average effective federal funds rate from day t until the end of the current month. Since the Federal Reserve uses the federal funds rate as its primary policy instrument by setting a target for the funds rate and performing open market operations to steer is towards the target level, the average funds rate could be interpreted as a measure of the current

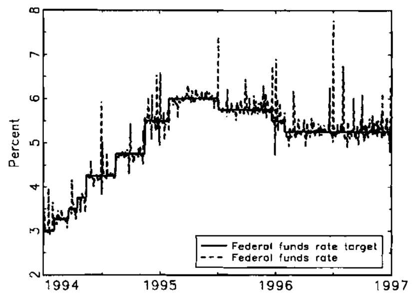

Figure 2.1: Federal funds rate and funds rate target, 1994–96

monetary policy stance. The expected average funds rate is then a measure of the expected path of monetary policy for the rest of the month. Since 1994 the Federal Reserve has adjusted its policy stance almost exclusively directly after a meeting of its main policy body, the Federal Open Market Committee (FOMC), and the funds rate target has never been changed twice during the same month. Consequently, during this period, on the day preceding an FOMC meeting, the expected average funds rate for the rest of the month can be interpreted as a measure of the funds rate target expected to prevail after the meeting.[5]

2.2.3 Relation to the federal funds cash market

Although the federal funds rate in the long run is largely determined by movements in the Federal Reserve's funds rate target, in the short run there may be significant deviations between the funds rate and the target, seen as temporary by the Fed and therefore not offset through open market operations. Figure 2.1 shows the funds rate target and the daily effective funds rate for the period from 1994 to 1996. As can be seen, the funds rate tends to fluctuate around the target, occasionally with large deviations, but in the long run it always returns to the target level. Some of these movements in the funds rate are due to Federal Reserve regulations of depository institutions, and as such are predictable, while other movements are more difficult to predict in advance.

The method used by the Fed to compute and maintain the reserves kept by

[5]Interpreting the extracted series as the expected funds rate (target) from the next day on also relieves the identification problem inherent in the series. Given a measure of the average funds rate expected to prevail over a certain period of time, one cannot separately identify the expected magnitude and the expected timing of a monetary policy move. See Robertson and Thornton (1997) for details.

depository institutions has been demonstrated to lead to predictable movements in the funds rate (see, e.g., Griffiths and Winters, 1995; Hamilton, 1996; Ho and Saunders, 1985; Saunders and Urich, 1988; and Spindt and Hoffmeister, 1988). Under the current system of reserve accounting, required reserves are computed as fractions of daily average deposit levels, which are computed over a two-week period beginning every other Tuesday (Meulendyke, 1998, p. 71ff). Daily average reserves must then be close to computed required reserves over a two-week period (the reserve maintenance period) beginning two days after the start of the reserve computation period. Thus not until the last two days of the reserve maintenance period do banks know the exact level of reserves they need to satisfy, so there is much volatility on the reserves (or federal funds) market on the last days of the period, especially on the very last day, the so-called settlement Wednesday. Since the Federal Reserve has a hard time trying to predict the demand for reserves in the market around the end of the reserve maintenance period, large movements in the federal funds rate are common on these days.

Another phenomenon affecting the fed funds market is the so-called balance sheet 'window dressing.' At the end of each quarter and year, banks (and other corporations) have their balance sheets evaluated by regulators and investors. Therefore, bank managers may have incentives to 'window dress' their balance sheets before reporting the data, that is, to undertake temporary asset and liability transactions to manipulate the accounting values around the report date. Allen and Saunders (1992) find strong evidence of systematic upward window dressing adjustment on the last day of each quarter over the period from 1978 to 1986. Such window dressing is often conducted using federal funds, both on the asset and on the liability side, since these provide low transaction cost financing. Therefore, if window dressing is important among federal funds market participants, and if such behavior is difficult to predict by the Federal Reserve, the federal funds rate should be expected to exhibit more volatility around the end of each quarter.

Movements in the federal funds rate such as these, if they affect the average monthly funds rate, will tend to introduce noise into the estimates of the expected funds rate target acquired from the futures market, since they affect futures prices but are not related to actual policy adjustments of the Federal Reserve. After examining how well the expectations derived from the futures market predict movements in the funds rate target, the following section will try to evaluate the importance of such noise.

2.3 Empirical results

Given the daily estimates of the expected average federal funds rate for the rest of the month, it is time to see how well these predict movements in the Federal Reserve's target level for the funds rate. The Fed conducts monetary policy by affecting the cost of federal funds via open market operations, and although the specific targeting procedure has changed somewhat over the sample period, the actual procedure of the Fed for the entire sample has been one of effective funds rate targeting (Meulendyke, 1998).

Daily data on federal funds futures prices, volumes, and open interest (the number of outstanding contracts) were obtained from the Chicago Board of Trade; data on the effective federal funds rate were downloaded from the FRED database of the Federal Reserve Bank of St. Louis;[6] and data on the federal funds rate target are from Rudebusch (1995) for 1988–92 and the Federal Reserve Bank of New York for 1993–98. The sample period used is from the introduction of federal funds futures on October 3, 1988, to March 6, 1998.

2.3.1 Predicting the funds rate target

To see how well the deviation of the calculated expected funds rate from target predicts the actual target change on the next day, the following regression is estimated:

$$\Delta i_{t+1}^T = \alpha + \beta \left(i_t^e - i_t^T \right) + \varepsilon_{t+1}, \tag{2.8}$$

where $\Delta i_{t+1}^T = i_{t+1}^T - i_t^T$ is the change in the funds rate target from day t to day $t+1$, and i_t^e is the expected funds rate for the rest of the month, as given by equation (2.7) above.[7] In the case where expected funds rate deviations predict target changes well, we would expect the intercept α to be zero and the slope coefficient β to be close to unity.[8]

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[6] *http://www.stls.frb.org/fred/*.

[7] On the last day of each month, the futures rate for the next month's contract is used as a measure of the expected funds rate. Also, I choose to subtract the current level of the target from both sides of equation (2.8) to control for the general level of the funds rate. An alternative would be to predict the level of the target using the level of the expected funds rate, which would tend to capture long-run movements in the target level. Since the focus here is on short-run movements, I choose to concentrate on the specification of equation (2.8).

[8] An estimate of β below unity could be interpreted as market participants not being perfectly informed about the Federal Reserve's policy motivations, but assigning a positive probability (although less than unity) to the possibility of a change. If market participants are completely ignorant about future policy moves, β should be close to zero, whereas $\beta = 1$ implies that markets

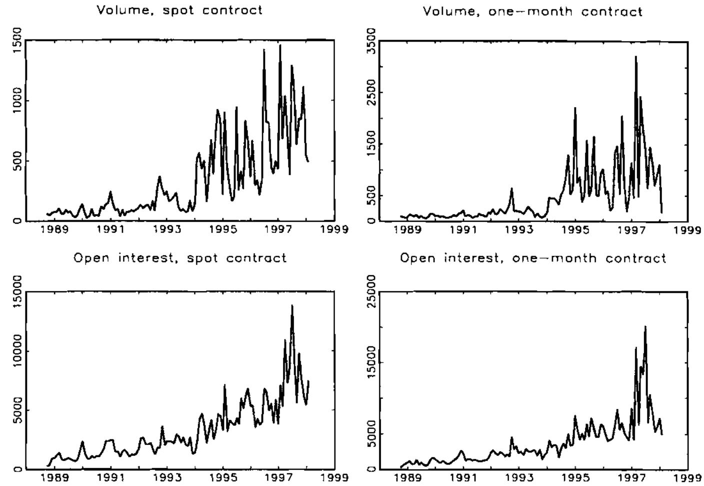

Figure 2.2: Volume and open interest on futures market, monthly averages of daily data

The estimation of equation (2.8) is done for the entire sample, and for two subsamples: 1988–93 and 1994–98. The separation into subsamples is done for several reasons. First, trading volumes in the futures market were rather small during the first years of the sample, but have since grown substantially. The upper graphs in Figure 2.2 show monthly averages of trading volume and the lower graphs show average open interest for the current-month (spot) contract and the one-month contract. From 1988 until 1993, both volume and open interest were fairly low, although steadily growing. From 1994 on, market activity increases significantly, but also becomes more volatile between months. Thus, there is reason to believe that the estimates from the late part of the sample are more reliable measures of market expectations than those from the early part of the sample.

Second, after being secretive about its policy decisions during the early part of the sample, when policy changes were more or less unpredictable, the Federal Reserve began announcing changes in the funds rate target at the February 1994 meeting of the FOMC, a procedure that was formalized in February 1995 (Thornton, 1996). Also, since 1994, it has been a deliberate policy of the FOMC to change

are perfectly informed (or always guess correctly) about future moves. Alternatively, if there are movements over time in the perceived risk of the futures contracts, we would expect the estimated coefficient for β to be biased downwards (see, e.g., Söderlind and Svensson, 1997).

the target almost exclusively at policy meetings.[9] As a consequence, 10 out of 11 target changes in the sample since 1994 have occurred on days of an FOMC meeting.[10] Furthermore, along with the move towards less secrecy in monetary policymaking, financial market participants have become better informed about the Fed's motivations, and nowadays eagerly await the next meeting of the FOMC amid vivid speculation about the probability of a change in the policy stance.

A related third reason to concentrate on the 1994–98 period is the lack of consensus about the number or exact dates of target changes in the early part of the sample. Although the series of target changes compiled by Rudebusch (1995) is often used for similar purposes, some authors (e.g., Roley and Sellon, 1996) argue that some of the target changes reported by Rudebusch do not correspond to actual decisions to change policy. However, Ellingsen and Söderström (1998) show, from reading newspaper reports in the *Wall Street Journal*, that market participants did notice some of these alleged non-changes in policy. On the other hand, Ellingsen and Söderström also show that on two occasions (January 8 and October 30, 1991) market participants noticed the policy change on the day preceding that reported by Rudebusch. Also, in the early part of the sample (from 1988 to 1990), ten of the policy changes were not noticed by market participants.

The results from estimating equation (2.8) with ordinary least squares[11] on the 2,376 daily observations from October 3, 1988, to March 6, 1998, are presented in panel (*a*) of Table 2.1. Since the expected funds rate series measures the expected funds rate for the rest of the month, and the sample interval is daily, we are likely to have serial correlation in the error term. Therefore, standard errors are adjusted following Newey and West (1987), using 20 lags (the maximum number of overlapping observations). Apparently, the expected funds rate performs very poorly in predicting changes in the funds rate target on a daily basis; both adjusted R^2 and the slope coefficient are virtually zero. The results for the two subsamples are not much different, although the fit is slightly better for the period from 1994 to 1998. That daily prediction of policy moves is not successful should not come as a surprise, since the actual funds rate target was changed only 47 times during the sample

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[9] This procedure was adopted after committee members complained to Chairman Greenspan that they were not fully part of the policy decisions (Beckner, 1996, p. 348). That this policy is still very much in effect can be seen from the financial market turmoil following the unexpected policy move in between meetings on October 15, 1998.

[10] The exception is April 18, 1994.

[11] Attempts to capture the probability of target changes through probit modeling were not very successful, since the futures expectations are very noisy, see below.

Table 2.1: Predicting target changes using the expected funds rate

Sample	Intercept	Slope	$\bar{R}^2$	Observations
(*a*) *All trading days*				
1988–98	−0.004**	0.038**	0.019	2,376
	(0.001)	(0.013)		
1988–93	−0.006**	0.025*	0.011	1,326
	(0.001)	(0.010)		
1994–98	−0.004*	0.090*	0.054	1,050
	(0.002)	(0.036)		
(*b*) *Target changes*				
1988–98	−0.132**	0.607**†	0.331	47
	(0.031)	(0.204)		
1988–93	−0.176**	0.379**	0.246	36
	(0.031)	(0.119)		
1994–98	−0.119	1.433**	0.734	11
	(0.073)	(0.144)		
(*c*) *FOMC meetings*				
1988–98	−0.025*	0.478**	0.287	75
	(0.012)	(0.173)		
1988–93	−0.012	0.081	0.095	42
	(0.010)	(0.076)		
1994–98	−0.069**	0.869**‡	0.460	33
	(0.026)	(0.238)		

OLS estimation of equation (2.8) on various samples of daily data from October 3, 1988, to March 6, 1998. Newey-West (1987) standard errors with (*a*) 20, (*b*) 3, and (*c*) 0 lags in parentheses. **/* denote coefficient significantly different from zero at the 1%-/5%-level, ‡/† denote coefficient *not* significantly different from 1 at the 10%-/5%-level.

period of 2,376 observations, and the expected funds rate series is affected by other things than the funds rate target, and thus moves around day by day. Also, if the market expects a policy adjustment later in the month, these expectations will be priced into the futures market, but not captured by the estimation of equation (2.8).

Instead, a more interesting test is to see how well the funds rate deviation predicts target changes on certain occasions, when the noise from other funds rate movements is dominated by monetary policy expectations. In panels (*b*) and (*c*) of Table 2.1, equation (2.8) is estimated for two groups of observations, when the market might have been able to foresee target changes: all actual target changes, and meetings of the FOMC.[12]

[12] In the regressions of panels (*b*) and (*c*), standard errors are adjusted using 3 and 0 lags, respectively.

Panel (*b*) shows the results for all days of actual target changes. Now the expected funds rate deviation from target performs fairly well in predicting policy moves. Adjusted R^2 is 0.331 for the entire sample, 0.246 for the first part, and 0.734 for the second subsample, which is surprisingly high, although slope coefficients are not very close to unity (only the coefficient for the entire sample is *not* significantly different from unity).

Predicting actual target changes may seem rather *ad hoc*, however, since these dates are not known to market participants *ex ante*, especially during the early subsample. Instead panel (*c*) shows the results from predicting the target after each FOMC meeting. This is especially interesting for the late subsample from 1994 to 1998, since, as mentioned above, during this period, the focus when predicting target changes has shifted almost entirely towards these meetings. For the whole sample, adjusted R^2 is 0.287, and the slope coefficient is 0.478, which is well below unity. This result is completely dominated by the late subsample, however. For the early period, the expected funds rate deviation contains virtually no information about future target changes: adjusted R^2 is 0.095, and the slope coefficient is not even significantly different from zero. For the late subsample, on the other hand, $\bar{R}^2$ is 0.460, and the slope coefficient is 0.869, which is not significantly different from unity at the 10%-level (the marginal significance level of the χ^2-statistic is 0.581).

That the period 1994–98 performs so well is encouraging, and the results for the period 1988–93 are not very surprising. During this period, the target was changed 36 times, but only once (on June 6, 1989) at an FOMC meeting. Thus, the meetings did not attract much attention from people predicting immediate target changes; in fact, they were not very different from any other day. Since the beginning of 1994, the focus is completely concentrated on the policy meetings, so that the market is more successful in predicting the direction of policy moves around these days.

2.3.2 Other movements in the expected funds rate series

As is obvious from the first regression reported in Table 2.1, there is a lot of movement in the expected funds rate series that is not related to changes in the funds rate target, and presumably reflects something other than expectations of target changes. From the discussion of Section 2.2.3 and from Figure 2.1, it is clear that the funds rate does not follow the target very closely in the short run, and deviations of the funds rate from target could possibly be ascribed to predictable factors such as reserve accounting and balance sheet window dressing. Insofar as these movements in the funds rate affect the average funds rate for the rest of the month, they

Table 2.2: Mean deviation of actual and expected funds rate from target

Sample	Actual funds rate		Expected funds rate	
	Mean deviation	Mean absolute deviation	Mean deviation	Mean absolute deviation
All days				
1988–98	0.0573	0.1361	0.0729	0.0949
1988–93	0.0708	0.1473	0.0783	0.1101
1994–98	0.0404	0.1220	0.0660	0.0757
FOMC meetings				
1988–98	0.0465	0.1383	0.0816	0.1179
1988–93	0.0712	0.1218	0.0751	0.1082
1994–98	0.0152	0.1594	0.0899	0.1303

Means and means of absolute values of $(i_t^{ff} - i_t^T)$ and $(i_t^e - i_{t+1}^T)$ over subsamples of daily data from October 3, 1988, to March 6, 1998.

should also have an effect on the expected funds rate estimates.

Table 2.2 shows the mean deviation and the mean absolute deviation of the actual and the expected funds rate from the funds rate target. As can be seen, both the actual and the expected funds rate are above target on average; on all days the actual funds rate is on average 4–7 basis points above target and the expected funds rate is 6–8 basis points above, depending on the sample period. The expected funds rate deviation is smaller for the second part of the sample, indicating that the measures of market expectations are more reliable for this period, and/or that market participants were better informed about the Fed's policy motivations. On days with FOMC meetings, the mean deviation is not very different from that on regular trading days.

To get an idea of the relative importance of the different regularities for the deviation of the actual and the expected funds rate from target, I calculate the mean deviation and its standard error across groups by estimating a simple dummy regression for each potential regularity. For example, to analyze the actual funds rate deviation from target across trading days, I estimate

$$i_t^{ff} - i_t^T = \sum_{j=1}^{20} \alpha_j d_j^D + \varepsilon_t, \tag{2.9}$$

where d_j^D is a dummy for the trading day j days before maturity. Similar regressions are estimated across the days of the reserve maintenance period and across calendar months, both for the deviation of the actual funds rate from target and for the deviation of the expected funds rate from next day's target.

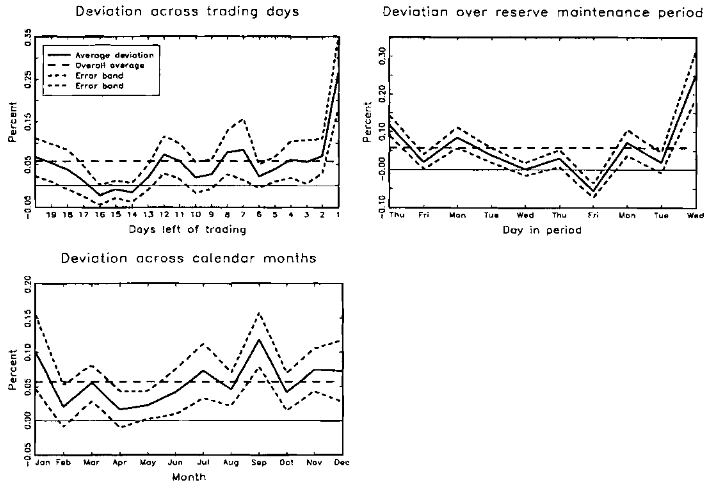

Figure 2.3: Average deviation of federal funds rate from target

Figures 2.3 and 2.4 show the estimated means with 5% confidence intervals (± 1.96 standard errors) for the actual and the expected deviation from target, along with the overall mean for the entire sample. As is clear from Figure 2.3, the actual funds rate deviation from target varies substantially across trading days and over the reserve maintenance period. On the last day of trading, the funds rate is on average 26 basis points above target, whereas the overall average deviation is only 6 basis points. This behavior is probably due to balance sheet window dressing, as described by Allen and Saunders (1992). Likewise, on settlement Wednesdays, the funds rate deviation is on average considerably larger than on other days (25 basis points), confirming the results of, for example, Griffiths and Winters (1995) and Hamilton (1996). Also across months there is some variation, for example, in September the deviation is typically larger than in other months, but this variation is less significant statistically, judging from the wide confidence intervals.

To the extent that these regularities on the federal funds cash market affect the average funds rate for the rest of the month, they should also, if they are predictable, affect the expected funds rate series. As can be seen from Figure 2.4, there is a lot of variation in the expected funds rate's deviation from target, especially across trading days and calendar months. The average deviation of the expected funds rate from target increases steadily as the month passes, and reaches 20 basis points

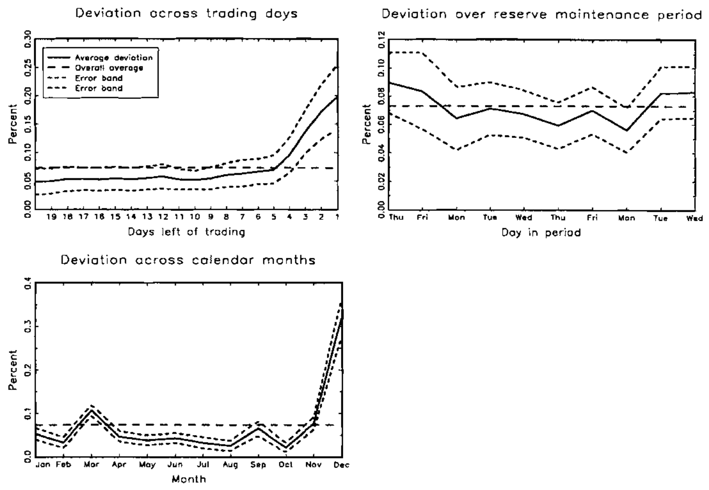

Figure 2.4: Average deviation of expected funds rate from next day's target

on the last trading day of the month, since the increase of the actual funds rate on the last days of the month becomes increasingly important for the average as the number of trading days left falls. Across the reserve maintenance period, there is less variation, and confidence intervals are very wide, which is not surprising, since the funds rate variation across the reserve maintenance period is unlikely to systematically affect the monthly average funds rate. Most surprising is the variation of the expected funds rate deviation across calendar months, since there seems to be no corresponding variation in the actual funds rate. The expected funds rate is on average 32 basis points above target in December, as compared to 7.3 basis points overall.

To get an idea of the reasons behind the monthly regularities, Table 2.3 shows the average daily change and the squared change (as a measure of volatility) in the effective funds rate across months. Here we see that there is considerably more volatility in the funds rate in January, July, and December than in other months, indicating that the monthly regularities in the futures market are likely to be derived from movements in the risk premium of futures contracts.

Table 2.3: Average daily change and volatility of federal funds rate

Month	Change	Volatility	Month	Change	Volatility
January	0.0247	0.2241	July	−0.0220	0.1728
February	0.0004	0.0766	August	−0.0082	0.0414
March	0.0084	0.0530	September	0.0233	0.1116
April	−0.0060	0.0600	October	−0.0133	0.0551
May	0.0058	0.0306	November	−0.0063	0.0923
June	0.0205	0.0671	December	−0.0233	0.1505

Monthly averages of $(i_t^{ff} - i_{t-1}^{ff})$ and $(i_t^{ff} - i_{t-1}^{ff})^2$ over 2,393 daily observations from October 3, 1988, to March 31, 1998.

2.3.3 Extended predictions

Can the regularities documented in the previous section be used to improve on the policy predictions? As suggested by Figure 2.4, the important regularities to take into account when using the expected funds rate series to predict monetary policy moves are across calendar months and trading days. Therefore, these are included in the prediction regression (2.8), so I estimate[13]

$$\Delta i_{t+1}^T = \beta \left(i_t^e - i_t^T \right) + \sum_{j=1}^{12} \gamma_j d_j^M + \sum_{j=1}^{3} \delta_j d_j^D + \nu_{t+1}, \tag{2.10}$$

where d_j^M is the intercept dummy for calendar month j and d_j^D is the dummy for the trading day j days from maturity. Thus intercepts are allowed to vary across months and across the last three days of trading in each month.

The results from estimating equation (2.10), first on all trading days and then on the dates of FOMC meetings from 1994 to 1998, are presented in Table 2.4. Columns (i)–(iii) show the results for all trading days in the sample, and columns (iv) and (v) those for FOMC meetings in the late subsample. The first column in each group—(i) and (iv)—repeats the results from Table 2.1, where the systematic variation over months and trading days is not taken into account. The second column—(ii) and (v)—shows the results when including only monthly dummies, and the third column for the daily observations (iii) shows the results when also including trading day dummies.[14]

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[13]Several different configurations of interaction dummies were tested, but never proved significant.

[14]Since only two FOMC meetings (January 31, 1996, and September 30, 1997) were held on the last day of the month, and none on the second or third to last day, trading day dummies are not included when predicting the target level after policy meetings. Also, since only one FOMC meeting was held in January, it is excluded from the sample of the regression in column (v).

Table 2.4: Predicting target changes, including intercept dummies

	All days			FOMC meetings	
	(i)	*(ii)*	*(iii)*	*(iv)*	*(v)*
Slope	0.0384**	0.0583**	0.0661**	0.8686**‡	0.9820**‡
	(0.0129)	(0.0148)	(0.0161)	(0.2380)	(0.1508)
Intercept	−0.0039**			−0.0691**	
	(0.0009)			(0.0264)	
January		−0.0042*	−0.0033		
		(0.0020)	(0.0019)		
February		0.0030	0.0042		0.0357
		(0.0046)	(0.0047)		(0.0326)
March		−0.0051	−0.0046		−0.1026
		(0.0027)	(0.0028)		(0.0540)
April		−0.0039	−0.0029		
		(0.0024)	(0.0023)		
May		0.0006	0.0016		−0.0014
		(0.0019)	(0.0019)		(0.0463)
June		−0.0037**	−0.0027*		
		(0.0013)	(0.0012)		
July		−0.0093**	−0.0081**		−0.1107**
		(0.0028)	(0.0029)		(0.0384)
August		−0.0006	0.0005		0.0308
		(0.0024)	(0.0025)		(0.0394)
September		−0.0064**	−0.0055*		−0.2473**
		(0.0022)	(0.0021)		(0.0859)
October		−0.0040*	−0.0028		
		(0.0017)	(0.0016)		
November		−0.0038	−0.0030		0.0174
		(0.0033)	(0.0034)		(0.0475)
December		−0.0248**	−0.0259**		−0.2295**
		(0.0057)	(0.0058)		(0.0461)
Last			−0.0127**		
			(0.0040)		
2nd last			−0.0084**		
			(0.0030)		
3rd last			−0.0072		
			(0.0037)		
$\bar{R}^2$	0.0194	0.0359	0.0408	0.4600	0.7062
Observations	2,376	2,376	2,376	33	32

OLS estimation of equation (2.10) on 2,376 daily observations from October 3, 1988, to March 6, 1998, and 33 FOMC meeting dates from January 1994 to February 1998, respectively (meeting of January 31, 1996, excluded in regression (*v*)). Newey-West (1987) standard errors with 20 and 0 lags, respectively, in parentheses. **/* denote coefficient significantly different from 0 at the 1%-/5%-level, ‡/† denote coefficient *not* significantly different from 1 at the 10%-/5%-level.

For the regressions including all 2,376 observations in columns (i)–(iii), intercepts vary considerably across months, from around 0.3 basis points in February to around -2.5 in December, regardless of whether or not we also adjust for trading days. The intercepts are smallest (and significantly negative) in June, July, September, and December, where consequently the expected funds rate deviation from target is unusually large relative to the actual target change. On the last three days of trading in column (iii), the intercepts fall further, by 0.7, 0.8, and 1.3 basis points, respectively, where the latter two effects are significant.

Predicting target changes after the next day's meeting of the FOMC in the late part of the sample is also more successful when taking market regularities into account, as seen in column (v). The intercept again varies across months, and considerably more than for the whole sample, with July, September, and December being strongly negative, and significantly different from zero. In September and December the intercept is around -25 basis points, which is considerably more than the average intercept of -7 points in column (iv). Introducing the monthly dummies increases the slope coefficient from 0.87 to 0.98, neither of which can be statistically separated from unity at the 10%-level, and adjusted R^2 increases from 0.46 to 0.71.

Consequently, variation across months and trading days is important on the futures market, and taking the regularities into account substantially improves the predictions of the target to prevail after the next FOMC meeting.

2.4 Additional tests

Although the expected funds rate series has been demonstrated to give a good prediction of the target level that will prevail after FOMC meetings from 1994 to 1998, especially when taking the systematic monthly variation into account, it is less clear how useful the estimates are for specific occasions. The results from Table 2.4 indicate that there is a large amount of time variation in the predictions, so the uncertainty is still large. Therefore this section presents two alternative tests of the estimates of market expectations extracted from futures prices. First, the policy move of September 29, 1998, is examined to see how the model performs out-of-sample. Second, the estimates are compared to market expectations of policy moves taken from the *Financial Times* on the day preceding each FOMC meeting from January 1994 to February 1998.

2.4.1 September 29, 1998

On September 29, 1998, the Federal Open Market Committee announced that it had decided to "ease the stance of monetary policy slightly, expecting the federal funds rate to decline 1/4 percentage point to around 5-1/4 percent" (Federal Reserve Board, 1998). This change in the target level for the federal funds rate had been widely expected by market participants, after several hints by Chairman Alan Greenspan and by William McDonough, President of the Federal Reserve Bank of New York and Vice Chairman of the FOMC. Discussion in the financial press and among Fed watchers circled around whether the cut would be 25 or 50 basis points, rather than whether there would be a cut at all.

On September 28, the September futures contract closed at 94.505 and the October contract at 94.820. The average level of the effective funds rate from September 1 to September 27 had been 5.49%, so the spot futures rate of 5.495% implies that the expected average funds rate for the rest of the month was

$$\begin{aligned} i_t^e &= \frac{n i_t^f - (t-1) i_{t-1}^a}{n - t + 1} \\ &= \frac{30 \times 5.495 - 27 \times 5.49}{30 - 28 + 1} \\ &= 5.54\%, \end{aligned} \tag{2.11}$$

and the expected average funds rate for October was $100 - 94.820 = 5.18\%$.

Since the level of the funds rate target on September 28 was 5.50%, a quick look at the expected funds rate for the rest of the month would suggest that the market did not expect the target to be changed on September 29. On the other hand, since there was no FOMC meeting scheduled for October, looking at the one-month October contract would lead to the conclusion that the market expected a large rate cut of between 25 and 50 basis points at the September 29 meeting, or possibly a 25 point cut on September 29, followed by a second cut in October, in between meetings.[15]

To explain why the estimates from the spot contract and the one-month contract seem to contradict each other, we need to recall that futures prices tend to fall on the last trading days of each month, so the futures rate tends to increase, and that there are large variations across months.[16] Because of the small number of observations,

[15]Note that this last scenario is what actually happened: the FOMC decided to cut the funds rate target by 25 basis points on September 29, and then surprised markets with a second cut of 25 points on October 15.

[16]Note also that September is also the last month of the third quarter, so the results of Allen

we have no estimates of the effects on futures prices on the last days of the month, but we can adjust the estimate for the month of September. Using the results in column (v) of Table 2.4, a gap of 0.04% between the expected funds rate and the target level on the day before the September meeting implies an average expected target change of $-0.2473 + 0.9820 \times 0.04 = -0.2080\%$, which is close to the actual change of -0.25%, without adjusting for the end-of-month/-quarter effect, which would probably have moved the estimate even closer to or beyond the actual target change.

Consequently, the example of the target change on September 29, 1998, illustrates very well how funds rate expectations extracted from the current month futures contract can be used to predict target changes. Taking the expectation directly does not capture the 'true' market expectation, but adjusting the estimate for the variation of futures pricing over calendar months we get very close to the actual target change, which was very well anticipated by market participants.[17]

2.4.2 Estimates from the financial press

As a second test of the market expectations extracted from futures prices, these are compared to expectations taken from the *Financial Times* on the day preceding each meeting of the Federal Open Market Committee from January 1994 to February 1998. On the day preceding an FOMC meeting, the financial press typically interviews a number of traders and analysts to see what outcome the market expects from the meeting. These reports have been collected from the *Financial Times* and processed to yield a measure of the expected target change for the 33 dates of FOMC meetings.

Table 2.5 reports the federal funds rate target before and after each meeting, the predicted target change from the futures data (both the simple measure and the measure adjusted for monthly variation in Table 2.4), and the data collected from the *Financial Times*. When collecting these data, a problem of calculating

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and Saunders (1992) lead us to predict large movements in the funds rate due to end-of-quarter window dressing by banks.

[17]Although there was some 'hope' in the financial press for a 50 point cut in the funds rate target on September 29, and some talk afterwards of financial markets being disappointed with the small magnitude of the cut, many serious Fed watchers did not find such a large move likely. For example, Steven Beckner of *Market News International* wrote on September 28 that "the rate cut is likely to take the form of a 25 basis point reduction in the key funds rate... Some are calling for a larger rate cut of 50 basis points, but while not out of the question, it would be unusual for the Fed to make a change of this magnitude as its first move in a different direction..." (available at *http://www.economeister.com*).

Table 2.5: Actual and predicted target changes around FOMC meetings

Date	Old target	New target	Target change	Futures predictions: Simple	Futures predictions: Adjusted	*Financial Times*: Range	*Financial Times*: Weights†	*Financial Times*: Average
940204	3.00	3.25	0.25	0.162	0.194	+0.25	100	+0.25
940322	3.25	3.50	0.25	0.295	0.188	+0.25/+0.50	75/25	+0.3125
940517	3.75*	4.25	0.50	0.361	0.353	+0.25	100	+0.25
940706	4.25	4.25	0.00	0.247	0.132	0	100	0.00
940816	4.25	4.75	0.50	0.345	0.370	+0.25/+0.50	50/50	+0.375
940927	4.75	4.75	0.00	0.246	−0.006	0/+0.25	50/50	+0.125
941115	4.75	5.50	0.75	0.588	0.594	+0.50	100	+0.50
941220	5.50	5.50	0.00	0.364	0.128	0/+0.25	75/25	+0.0625
950201	5.50	6.00	0.50	0.460	0.487	+0.50	100	+0.50
950328	6.00	6.00	0.00	0.112	0.007	0	100‡	0.00
950523	6.00	6.00	0.00	0.031	0.029	0	100	0.00
950706	6.00	5.75	−0.25	−0.161	−0.269	0/-0.25	50/50	−0.125
950822	5.75	5.75	0.00	−0.015	0.017	0	100	0.00
950926	5.75	5.75	0.00	0.040	−0.208	0/-0.25	75/25	−0.0625
951115	5.75	5.75	0.00	−0.034	−0.016	0	100	0.00
951219	5.75	5.50	−0.25	0.001	−0.228	0	100	0.00
960131	5.50	5.25	−0.25	0.198	NA	0/-0.25/-0.50	25/50/25	−0.25
960326	5.25	5.25	0.00	0.242	0.135	0	100	0.00
960521	5.25	5.25	0.00	0.011	0.010	0	100	0.00
960703	5.25	5.25	0.00	0.060	−0.052	0/+0.25	75/25	+0.0625
960820	5.25	5.25	0.00	0.052	0.082	0	100‡	0.00
960924	5.25	5.25	0.00	0.186	−0.064	0/+0.25/+0.50	44/48/8‡	+0.16
961113	5.25	5.25	0.00	0.066	0.083	0/-0.25	90/10	−0.025
961217	5.25	5.25	0.00	0.148	−0.084	0	100	0.00
970205	5.25	5.25	0.00	0.031	0.066	0	100	0.00
970325	5.25	5.50	0.25	0.278	0.170	0/+0.25/+0.50	10/80/10	+0.25
970520	5.50	5.50	0.00	0.111	0.108	0/+0.25	54/46‡	+0.115
970702	5.50	5.50	0.00	0.050	−0.062	0	100	0.00
970819	5.50	5.50	0.00	0.001	0.032	0	100	0.00
970930	5.50	5.50	0.00	0.535	0.278	0	100	0.00
971112	5.50	5.50	0.00	0.072	0.088	0	100	0.00
971216	5.50	5.50	0.00	0.167	−0.065	0	100	0.00
980204	5.50	5.50	0.00	−0.034	0.002	0	100	0.00

*Target changed 940418 from 3.50 to 3.75. †Author's subjective estimate, unless marked by ‡, when based on poll results reported in the *Financial Times*. Observation of 960131 excluded from calculation of adjusted expected funds rate, see Table 2.4. Sources: Rudebusch (1995), Federal Reserve Bank of New York, Federal Reserve Bank of St. Louis, *Financial Times*, own calculations.

the average expectations from the newspaper reports arises. Typically the *Financial Times* reports a number of possible outcomes suggested by market participants, so a probability distribution must be assigned to these outcomes to calculate the average market expectation. Sometimes, such a distribution is given by poll results, but often it is necessary to assign a probability distribution by studying the newspaper report carefully. (The resulting probability distribution is reported in Table 2.5 as 'weights.') Consequently, an element of arbitrariness in the measurement of market expectations from the newspaper reports is inevitable.

Also, market participants in the poll are well aware that the Fed typically changes its target for the funds rate in steps of 25 or 50 basis points, if at all, and they take this discrete character of monetary policy changes into account. The resulting expectations are often of no change in the target, and on only one occasion (December 19, 1995) was a zero expectation followed by a change in the target, so that market participants were completely caught off guard. The sign of target changes is always correctly predicted, and only twice (May 17 and November 15, 1994) did the actual target change fall outside the range of market predictions, these changes being unusually large (50 and 75 basis points, respectively).

The question is whether the expectations from the financial press perform better than those extracted from the futures market when predicting monetary policy moves at FOMC meetings. And additionally, do the futures estimates improve on the expectations from the newspaper reports?

The first of these questions is answered by estimating the same simple regression as before (equation (2.8)), using the expected target change from the *Financial Times*, $i_t^{FT} - i_t^T$, as the independent variable;

$$\Delta i_{t+1}^T = \alpha + \beta \left(i_t^{FT} - i_t^T \right) + \varepsilon_{t+1}. \tag{2.12}$$

The results are presented in Table 2.6, column (*ii*). For reference, column (*i*) repeats the results when using the expected target change from the futures market as the independent variable in equation (2.8). The *Financial Times* estimates clearly outperform the estimates from the futures market: adjusted R^2 is 0.832 compared with 0.460, when not adjusting for the monthly variation (including monthly dummies, the futures expectations reached an $\bar{R}^2$ of 0.706, see Table 2.4, column (*v*)). Thus the expectations from the financial press seem to be a better source of information if one is to predict the target level after the next day's FOMC meeting.

To see if the information in the financial press is completely superior to that on futures markets, I estimate the regressions

$$\Delta i_{t+1}^T = \alpha + \beta \left(i_t^{FT} - i_t^T \right) + \gamma \left(i_t^e - i_t^T \right) + \nu_{t+1} \tag{2.13}$$

Table 2.6: Expectations from the *Financial Times* versus futures estimates

	(i)	*(ii)*	*(iii)*	*(iv)*
Intercept	−0.069**	−0.023	−0.042*	−0.026*
	(0.026)	(0.014)	(0.018)	(0.013)
Expectations				
Financial Times		1.207**†	1.080**‡	0.771**‡
		(0.117)	(0.095)	(0.142)
Simple futures	0.869**‡		0.182	
	(0.238)		(0.093)	
Adjusted futures				0.481**
				(0.147)
$\bar{R}^2$	0.460	0.832	0.839	0.884
Observations	33	33	33	32

OLS estimation of equations (2.8), (2.12), (2.13), and (2.14), respectively, on 33 FOMC meeting dates from January 1994 to February 1998. White (1980) standard errors in parentheses. **/* denote coefficient significantly different from 0 at the 1%-/5%-level, ‡/† denote coefficient *not* significantly different from 1 at the 10%-/5%-level.

and

$$\Delta i_{t+1}^{T} = \alpha + \beta \left(i_t^{FT} - i_t^{T} \right) + \gamma \left(\hat{i}_t^{e} - i_t^{T} \right) + \eta_{t+1}, \tag{2.14}$$

where $\hat{i}_t^e$ is the fitted value from the regression including monthly dummies. The results from estimating these regressions are presented in columns *(iii)* and *(iv)* of Table 2.6. Interestingly, both the simple expectations in column *(iii)* and the adjusted expectations in column *(iv)* improve on the newspaper reports when predicting the funds rate target: adjusted R^2 increases to 0.839 and 0.884, respectively, although only the coefficient on $\left(\hat{i}_t^e - i_t^T \right)$ is significantly different from zero at the 5%-level.[18]

Consequently, although the expectations of target changes reported in the *Financial Times* outperform the expectations from the futures market, the two types of measures do not contain the same information. Adding information from the futures market improves on the predictions from the newspaper reports.

2.5 Using monthly data

Most previous studies have concentrated on monthly averages of futures data, predicting policy moves one to three months ahead. Although there are significant

[18]The coefficient on $(i_t^e - i_t^T)$ is significantly different from zero at the 10%-level, however.

problems with time aggregation using monthly data (see Evans and Kuttner, 1998), such data are still a convenient way of measuring policy expectations, since the daily noise in the futures market tends to cancel out. Also, because of the futures prices' simple conversion into the expected funds rate, monthly data are often used by market analysts when predicting future policy moves.

Carlson et al. (1995) show that monthly averages of futures rate of up to five months' maturity yield better predictions of the average effective federal funds rate for the contract month in terms of mean squared errors than do a naive random walk model and an estimated univariate model. Krueger and Kuttner (1996) perform out-of-sample forecasts of future monetary policy based on one- and two-month futures prices, and conclude that predictable changes in the funds rate are rationally forecast by the futures market, and that the inclusion of other information only marginally improves on the futures-based forecasts. However, none of these studies take into account the monthly variation of futures prices. Therefore, it is natural to ask whether these variations are still important when using monthly data.

To analyze the predictive power of monthly futures data, I estimate regressions similar to equations (2.8) and (2.10), but using the monthly averages of the federal funds rate, the funds rate target, and the one- to three-month futures rates. Here I choose to predict changes not only in the average funds rate target, but also in the average funds rate, since the futures contracts are based on the average funds rate for the contract month.

Consequently, I estimate

$$\Delta\bar{\imath}_{t+m} = \alpha + \beta\left(\bar{\imath}^{f}_{m,t} - \bar{\imath}_t\right) + \varepsilon_{t+1}, \tag{2.15}$$

and

$$\Delta\bar{\imath}_{t+m} = \sum_{j=1}^{12}\gamma_j d^M_{m,j} + \delta\left(\bar{\imath}^{f}_{m,t} - \bar{\imath}_t\right) + \nu_{t+1}, \tag{2.16}$$

where $\Delta\bar{\imath}_{t+m} = \bar{\imath}_{t+m} - \bar{\imath}_t$ is the change in the average funds rate or funds rate target from month t to $t+m$, and $\bar{\imath}^{f}_{m,t}$ is the average m-month futures rate in month t. The results using monthly data from October 1988 to March 1998 are presented in Tables 2.7 and 2.8.

From Table 2.7 it is clear that the average monthly futures prices have considerable predictive power for changes in the average funds rate target until the contract matures. The results when predicting changes in the average funds rate are very similar, since the average funds rate in a given month follows the average funds rate target very closely. When allowing the intercepts to vary across trading months in

Table 2.7: Predicting monthly changes in the funds rate and target

	Funds rate target			Funds rate		
	1-month contract	2-month contract	3-month contract	1-month contract	2-month contract	3-month contract
Intercept	−0.069**	−0.118**	−0.172*	−0.045*	−0.089*	−0.145*
	(0.018)	(0.040)	(0.067)	(0.018)	(0.039)	(0.065)
Slope	0.809**	0.970**‡	0.976**‡	0.847**‡	0.989**‡	1.002**‡
	(0.083)	(0.133)	(0.169)	(0.111)	(0.145)	(0.173)
$\bar{R}^2$	0.462	0.501	0.446	0.459	0.505	0.459
Observations	113	112	111	113	112	111

OLS estimation of equation (2.15) on 114 monthly observations from October 1988 to March 1998. Newey-West (1987) standard errors with 1, 2, and 3 lags, respectively, in parentheses. **/* denote coefficient significantly different from 0 at the 1%-/5%-level, ‡/† denote coefficient *not* significantly different from 1 at the 10%-/5%-level.

Table 2.8, we see that futures prices do vary considerably across months. Although in most months the intercept is not significantly different from zero, and quite close to the overall intercept, the months of January, July, August, and December have significantly negative intercepts (up to −30 basis points), indicating that the futures rates are unusually large relative to the expected funds rate in these months. Figures 2.5 and 2.6 show the average deviation of the futures rate from the contract month funds rate and target across calendar months. For the spot contract, December clearly stands out, and for the one- to three-month contracts, January, July, and August also have unusually large deviations. As indicated by Table 2.3, these variations are probably due to increases in the perceived riskiness of futures contracts in these months.

These results clearly show that there are strong monthly variations in prices on the federal funds futures market, which definitely need to be taken into account if one is to use futures prices to extract market expectations or predict policy moves, also when using monthly futures data.

2.6 Conclusions

Because of their simple interpretation, prices of federal funds futures contracts one to three months into the future are often used to extract market expectations of the path of monetary policy. In the current month, futures prices should contain even more precise information about the near-term path of monetary policy, since market participants have already observed part of the federal funds rate path that determines the price of the futures contract at maturity. Thus one should in theory

Table 2.8: Predicting monthly changes, including monthly intercept dummies

	Funds rate target			Funds rate		
	1-month contract	2-month contract	3-month contract	1-month contract	2-month contract	3-month contract
January	−0.185**	−0.255**	−0.271	−0.141**	−0.207*	−0.223
	(0.046)	(0.098)	(0.141)	(0.041)	(0.094)	(0.143)
February	−0.029	−0.135	−0.170	−0.031	−0.135	−0.169
	(0.053)	(0.091)	(0.148)	(0.057)	(0.095)	(0.146)
March	−0.002	−0.011	−0.119	0.032	0.025	−0.087
	(0.035)	(0.088)	(0.140)	(0.039)	(0.089)	(0.144)
April	−0.074	0.019	−0.018	−0.095*	0.002	−0.037
	(0.044)	(0.047)	(0.117)	(0.044)	(0.054)	(0.124)
May	−0.051	−0.118	0.007	−0.047	−0.115	0.010
	(0.029)	(0.077)	(0.086)	(0.031)	(0.074)	(0.082)
June	−0.039*	−0.110*	−0.178	−0.015	−0.088	−0.158
	(0.016)	(0.048)	(0.106)	(0.030)	(0.057)	(0.109)
July	−0.127**	−0.200**	−0.296**	−0.093*	−0.167**	−0.267**
	(0.040)	(0.045)	(0.078)	(0.047)	(0.060)	(0.096)
August	−0.053	−0.175**	−0.277**	−0.024	−0.143**	−0.249**
	(0.034)	(0.046)	(0.051)	(0.029)	(0.040)	(0.050)
September	−0.017	−0.081	−0.214*	0.062	0.000	−0.134
	(0.039)	(0.084)	(0.097)	(0.037)	(0.081)	(0.086)
October	−0.027	−0.034	−0.121	−0.013	−0.015	−0.104
	(0.025)	(0.053)	(0.109)	(0.036)	(0.059)	(0.112)
November	−0.073	−0.109	−0.135	−0.009	−0.039	−0.066
	(0.042)	(0.065)	(0.080)	(0.044)	(0.072)	(0.086)
December	−0.207**	−0.224*	−0.276*	−0.186**	−0.195*	−0.266*
	(0.063)	(0.096)	(0.128)	(0.052)	(0.085)	(0.106)
Slope	0.910**‡	1.002**‡	0.987**‡	0.947**‡	1.031**‡	1.017**‡
	(0.095)	(0.147)	(0.177)	(0.111)	(0.154)	(0.179)
$\bar{R}^2$	0.527	0.513	0.430	0.530	0.509	0.436
Observations	113	112	111	113	112	111

OLS estimation of equation (2.16) on 114 monthly observations from October 1988 to March 1998. Newey-West (1987) standard errors with 1, 2, and 3 lags, respectively, in parentheses. **/* denote coefficient significantly different from 0 at the 1%-/5%-level, ‡/† denote coefficient *not* significantly different from 1 at the 10%-/5%-level.

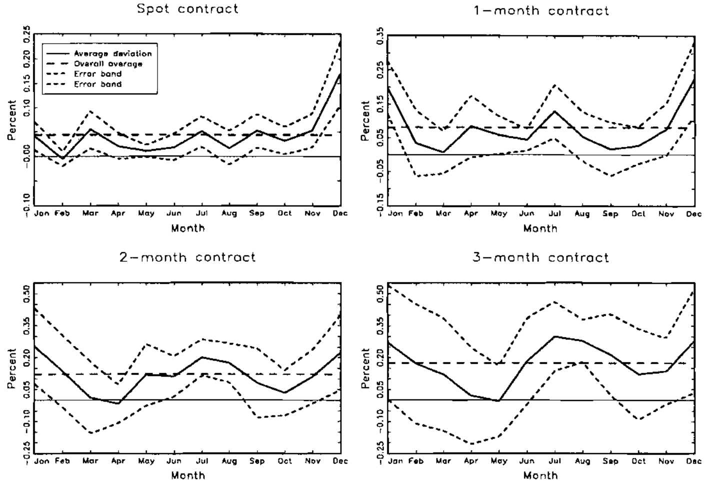

Figure 2.5: Average futures deviation from funds rate target across months, monthly data

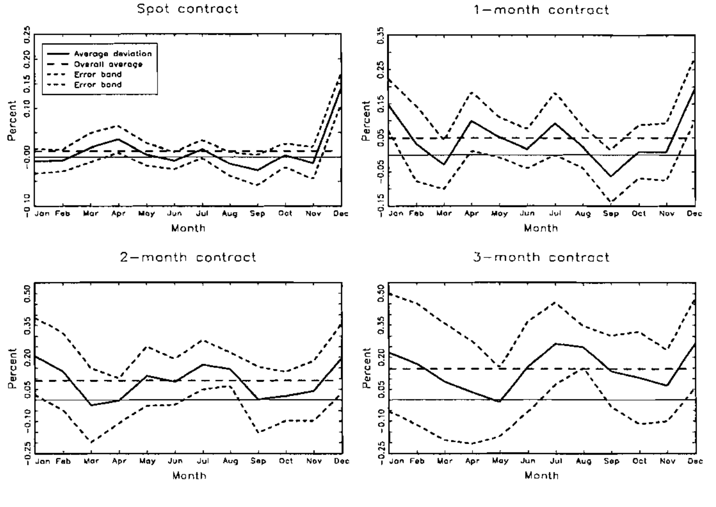

Figure 2.6: Average futures deviation from funds rate across months, monthly data

be able to extract fairly exact measures of the expected federal funds rate—and consequently of its target—from the current month's contract, especially as the contract gets close to maturity.

This study has shown that things are not that straightforward in reality. Even though the extracted expected funds rate from the day before meetings of the Federal Open Market Committee from January 1994 to February 1998 performs fairly well in predicting the target level that will prevail after the meeting, there are large systematic variations in the funds rate expectations, especially across calendar months, and possibly across trading days in the contract month (although the number of FOMC meetings is too small to verify this last claim). The monthly variation is probably due to increased volatility in the underlying federal funds cash market in the relevant months, leading to increased risk premia on the futures market.

Adjusting the funds rate expectation for monthly variation substantially improves the prediction of target changes. Additional tests have shown that the extracted expectations were successful in predicting the (widely anticipated) policy move in September 1998, and that they improve on market expectations taken from newspaper reports on the days preceding the FOMC meetings.

Consequently, the expectations of near-term changes in the federal funds rate target extracted from the federal funds futures market seem to be useful as measures of market expectations, although a simple adjustment for systematic monthly variations is recommended.

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Chapter 3

Monetary policy with uncertain parameters

Abstract:
In a simple dynamic macroeconomic model, it is shown that uncertainty about structural parameters does not necessarily lead to more cautious monetary policy, refining the accepted wisdom concerning the effects of parameter uncertainty on optimal policy. In particular, when there is uncertainty about the persistence of inflation, it is optimal for the central bank to respond more aggressively to shocks than if the parameter were known with certainty, since the central bank wants to avoid bad outcomes in the future. Uncertainty about other parameters, in contrast, acts to dampen the policy response.

[0] I am indebted to Tore Ellingsen, Lars Ljungqvist, Anders Paalzow, Glenn Rudebusch, Anders Vredin, and workshop participants at the Stockholm School of Economics for comments.

3.1 Introduction

IT IS WIDELY ACCEPTED that policymakers facing uncertainty about the structure of the economy should be more cautious when implementing policy than if acting under complete certainty (or certainty equivalence). The attractiveness of this result, named the 'Brainard conservatism principle' by Alan Blinder (1997, 1998) after the original analysis of William Brainard (1967), lies in both the simplicity of the original argument and in the underlying intuition: when you are uncertain about the effects of policy, it makes sense for policymakers to move more cautiously in the response to economic shocks.[1]

Recently, Svensson (1997a) has shown this result to hold also in a dynamic macroeconomic model, often used to analyze issues in monetary policy. When there is uncertainty about some of the structural parameters, the optimal policy response to current inflation and output (i.e., the coefficients in the policymaker's reaction function) are shown to get smaller as the amount of uncertainty increases.[2] Due to the complexity of the model with parameter uncertainty, however, Svensson chooses to analyze a special case, where only inflation (and no measure of output) enters the central bank's objective function.

The purpose of the present chapter is to analyze the effects of multiplicative parameter uncertainty in a more general setting of the same model, where all structural parameters are allowed to be uncertain, and where the preferences of the central bank in the choice between stabilizing output and inflation are allowed to vary. In addition to the initial response of policy, the time path of policy after a shock is examined.

Surprisingly, the results show that parameter uncertainty does not necessarily dampen the policy response, but may actually make policy more aggressive than under certainty equivalence. In particular, uncertainty about the persistence of inflation increases the optimal reaction function coefficients, whereas uncertainty about other parameters dampens the response. In the special case analyzed by Svensson, when the weight on output stabilization in the central bank's objective function is zero, uncertainty about the persistence of inflation does not affect the policy response. For positive weights on output, however, the policy response is increasing in the variance of the persistence parameter, so policy becomes more

[1] That this principle is well understood and used by central bankers in the practical policy process is made clear by Blinder (1998) and Goodhart (1998).

[2] Similar results have been reached by, e.g., Estrella and Mishkin (1998), Sack (1998a), and Wieland (1998).

aggressive as the amount of uncertainty increases.

This result may seem counter-intuitive at first glance, but is less puzzling after careful examination. The possibility that the inflation rate moves away from target by itself leads the central bank to take precautionary steps, to avoid paying the price of larger interest rate (and output) volatility later. Without any costs of output volatility, this is not important for the central bank, but as the bank cares more about stabilizing output, it gets more important to keep inflation at bay, so as to avoid output fluctuations in later periods. As such, the results are similar to those of Craine (1979), who shows that uncertainty about the impact effect of policy leads to less aggressive policy behavior, but uncertainty about the dynamics of the economy leads to more aggressive policy. Also, Sargent (1998a,b) argues that uncertainty leads to more cautious policy, but that 'caution' could mean that the policymaker tries to avoid bad outcomes in the future by responding more aggressively to shocks today.

Perhaps less surprisingly, when parameter uncertainty does act to dampen the current policy response, it is optimal for the central bank to return to a neutral policy stance later than if all parameters were known with certainty. This is due to the persistence of inflation and output: a smaller initial response leads to larger deviations of the goal variables from target in future periods, so policy needs to be away from neutral for a longer period to get inflation and output back on track. Thus, parameter uncertainty can lead to a smoother policy path in response to shocks, an issue analyzed in more detail by Sack (1998a) and Söderström (1999).

The chapter is organized as follows. In Section 3.2 the theoretical framework is presented, and the optimal policy of the central bank is derived in a dynamic economy with stochastic parameters. Since analytical solutions of the model are difficult, if not impossible, to find, Section 3.3 presents numerical solutions for different configurations of uncertainty, to establish the effects of parameter uncertainty on the optimal policy response to output and inflation shocks. Finally, the results are discussed and conclusions are drawn in Section 3.4.

3.2 The model

3.2.1 Setup

The basic model used in the analysis is the dynamic aggregate supply-aggregate demand framework developed by Lars Svensson (1997a,b) and used by, for example, Ellingsen and Söderström (1999), Rudebusch and Svensson (1998), and

Rudebusch (1998). This model is similar to many other models used for monetary policy analysis, for example by Ball (1997), Cecchetti (1998), Taylor (1994), and Wieland (1998), and consists of two equations relating the output gap (the percentage deviation of output from its 'natural' level) and the inflation rate to each other and to a monetary policy instrument, the short interest rate. Assuming a quadratic objective function for the central bank, one can solve for the optimal decision rule as a function of current output and inflation, similar to a Taylor (1993) rule.

Important features of the model are the inclusion of control lags in the monetary transmission mechanism, and the fact that monetary policy only affects the rate of inflation indirectly, via the output gap. Monetary policy is assumed to affect the output gap with a lag of one period, which in turn affects inflation in the subsequent period.[3] Policymakers thus control the inflation rate with a lag of two periods. In the simplest version, including only one lag,[4] the output gap in period $t+1$ (or rather the deviation of the output gap from its long-run mean), y_{t+1}, is related to past output and the ex-post real interest rate in the previous period, $i_t - \pi_t$, by the IS-relationship

$$y_{t+1} = \alpha_{t+1} y_t - \beta_{t+1}(i_t - \pi_t) + \varepsilon^y_{t+1}, \tag{3.1}$$

where ε^y_{t+1} is an i.i.d. demand shock with mean zero and constant variance σ^2_y. The rate of inflation between periods t and $t+1$ (or its deviation from the long-run mean), π_{t+1}, depends on past inflation and the output gap in the previous period according to the Phillips curve relation

$$\pi_{t+1} = \delta_{t+1}\pi_t + \gamma_{t+1} y_t + \varepsilon^\pi_{t+1}, \tag{3.2}$$

where ε^π_{t+1} is an i.i.d. supply shock with zero mean and variance σ^2_π.

In the model presented here, there are two important modifications to the original Svensson framework: the persistence parameter of the inflation process, δ_{t+1}, is allowed to take values different from unity; and the parameters of the model are stochastic, and therefore time-varying. When the central bank sets its interest rate instrument at time t, it is assumed to know all realizations of the parameters up

[3]In the simple one-lag model used here, one period can be thought of as equal to one year. The short interest rate could then be interpreted as the central bank's interest rate instrument, e.g., the federal funds rate target in the U.S., assumed to be held constant for a year at a time. See Svensson (1997a).

[4]Rudebusch and Svensson (1998), Rudebusch (1998), and Söderström (1999) use a version of the model including four lags in each relationship, and estimate it on quarterly U.S. data. Söderström (1999) also formally tests the restrictions imposed by Svensson (1997a,b).

to and including period t, but it does not know their future realizations, and thus cannot be certain about the effects of policy on the economy.[5] For simplicity, assume that each parameter is given by a constant mean plus a random shock. Thus, for example, the persistence parameter of the output process, α_{t+1}, is given by

$$\alpha_{t+1} = \alpha + \nu_{t+1}^{\alpha}, \tag{3.3}$$

where ν_{t+1}^{j}, $j = \alpha, \beta, \gamma, \delta$, are i.i.d. shocks with mean zero and constant variance σ_j^2. The parameters are assumed to be independent of each other and of the structural shocks ε_t^{π} and ε_t^{y}.[6] Furthermore, the realizations of the parameters are drawn from the same distribution in each period, so issues of learning and experimentation are disregarded in the analysis.[7]

3.2.2 Optimal policy

To determine the optimal path for the interest rate over the entire future, contingent on the development of the economy, the central bank is assumed to minimize the expected discounted sum of future values of a loss function, which is quadratic in output and inflation deviations from target (here normalized to zero). Thus, the central bank solves the optimization problem

$$\min_{\{i_{t+\tau}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \phi^{\tau} L(y_{t+\tau}, \pi_{t+\tau}), \tag{3.4}$$

subject to (3.1)–(3.3), where in each period the loss function $L(y_t, \pi_t)$ is given by

$$L(y_t, \pi_t) = \pi_t^2 + \lambda y_t^2, \tag{3.5}$$

[5] That policymakers do not have complete information about the structural parameters in an economy is clearly not an unrealistic assumption. Holly and Hughes Hallett (1989) point to three reasons why a model's parameters may be seen as stochastic: (1) they are genuinely random; (2) they are really fixed, but are impossible to estimate precisely, due to the sampling variability in a finite data set; and (3) they vary according to some well-defined but imperfectly known scheme, e.g., because the model is a linearization around a trajectory of uncertain exogenous variables. Blinder (1997, 1998), Goodhart (1998), and Poole (1998) all stress the relevance of uncertainty for practical monetary policy.

[6] The assumption of independence is convenient for the derivation of optimal policy, and may be realistic if the model equations (3.1) and (3.2) are interpreted as structural relationships. If, on the other hand, one interprets the model as reduced-form relations derived from microeconomic foundations, the parameters might well be correlated if they are derived from the same micro relations.

[7] See Sack (1998b) or Wieland (1998) for similar models of monetary policy including learning and experimentation; or Balvers and Cosimano (1994), Başar and Salmon (1990), and Bertocchi and Spagat (1993) for models in slightly different contexts.

and where ϕ is the central bank's discount factor.[8] The parameter $\lambda \geq 0$ specifies the relative weight of output stabilization to inflation fighting, and is assumed to be known and constant.[9] In the simple case when parameters are non-stochastic, it is relatively straightforward to find an analytical solution for the optimization problem (3.4), as shown by Svensson (1997a,b). When parameters are stochastic, however, finding an analytical solution is prohibitively difficult, so I shall here focus on numerical solutions.[10]

To solve the central bank's optimization problem it is convenient to rewrite the model (3.1)–(3.2) in state-space form as

$$x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + \varepsilon_{t+1}, \tag{3.6}$$

where $x_{t+1} = [\ y_{t+1} \quad \pi_{t+1}\ ]'$ is a state vector, and $\varepsilon_{t+1} = [\ \varepsilon^y_{t+1} \quad \varepsilon^\pi_{t+1}\ ]'$ is a vector of structural shocks. The parameter matrices A_{t+1} and B_{t+1} are then stochastic with means

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}, \quad B = \begin{bmatrix} -\beta \\ 0 \end{bmatrix}, \tag{3.7}$$

and variance-covariance matrices

$$\Sigma_A = \begin{bmatrix} \sigma^2_\alpha & 0 & 0 & 0 \\ 0 & \sigma^2_\beta & 0 & 0 \\ 0 & 0 & \sigma^2_\gamma & 0 \\ 0 & 0 & 0 & \sigma^2_\delta \end{bmatrix}, \quad \Sigma_B = \begin{bmatrix} \sigma^2_\beta & 0 \\ 0 & 0 \end{bmatrix}, \quad \Sigma_{AB} = \begin{bmatrix} 0 & 0 \\ -\sigma^2_\beta & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{3.8}$$

Using the state-space formulation, the central bank's optimization problem can be written as the control problem

$$J(x_t) = \min_{i_t} \left[x_t'Qx_t + \phi E_t J(x_{t+1})\right], \tag{3.9}$$

[8]The quadratic specification of the objective function is very common in the literature. Some authors, e.g., Rudebusch and Svensson (1998) and Rudebusch (1998), include an interest rate smoothing objective in the loss function to capture the apparent preference of central banks for small persistent changes in the instrument. As shown by Sack (1998a) and Söderström (1999), however, such an *ad hoc* smoothing objective is not necessary to mimic policy behavior in the U.S., at least not in an unrestricted VAR framework.

[9]Typically, λ is positive also in regimes of inflation targeting, since central banks want to stabilize short-term fluctuations in output even when their main goal is price stability. See Svensson (1998) for a discussion of 'strict' versus 'flexible' inflation targeting, and Fischer (1996) for a critique of central banks' tendency to only acknowledge price stability and not output stabilization as the goal of monetary policy.

[10]Svensson (1997a) analytically solves a very simple case of parameter uncertainty, where δ is non-stochastic and always equal to unity, and where $\lambda = 0$. Since the most interesting results are obtained when $\lambda > 0$ and δ is stochastic, I shall not follow his route.

subject to (3.6), where Q is a (2×2) preference matrix of the central bank, with λ and 1 on the diagonal and zeros elsewhere. The loss function will in this framework be quadratic, so

$$J(x_{t+1}) = x'_{t+1} V x_{t+1} + w, \tag{3.10}$$

where the matrix V remains to be determined.

When parameters are non-stochastic, so that there is only additive uncertainty in the linear-quadratic model, it is well known that optimal policy is certainty-equivalent, that is, only the expected value of the state vector x_{t+1} matters for optimal policy. In that case, the expected value of the value function (3.10) is simply

$$E_t J(x_{t+1}) = (E_t x_{t+1})' V (E_t x_{t+1}) + w. \tag{3.11}$$

When parameters are stochastic, however, certainty equivalence no longer holds, since the variance of the vector x_{t+1} also matters for policy. In mathematical terms, the difference from the certainty equivalence case is that the expected value of the value function is now

$$E_t J(x_{t+1}) = (E_t x_{t+1})' V (E_t x_{t+1}) + \mathrm{tr}(V \Sigma_{t+1|t}) + w, \tag{3.12}$$

where $\Sigma_{t+1|t}$ is the variance-covariance matrix of x_{t+1}, evaluated at time t, and the notation 'tr' denotes the trace operator. Consequently, the variance-covariance matrix of x_{t+1}, containing the parameter variances, will affect the optimal policy rule.

Appendix 3.A shows that the optimal decision rule for the central bank is to set the short interest rate as a linear function of the state vector in each period, that is

$$i_t = f x_t, \tag{3.13}$$

where

$$f = -\left[B'\left(V + V'\right) B + 2v_{11}\Sigma_B^{11}\right]^{-1}\left[B'\left(V + V'\right) A + 2v_{11}\Sigma_{AB}^{11\,\prime}\right]. \tag{3.14}$$

Here Σ_{AB}^{ij} denotes the covariance matrix of the ith row of A_{t+1} with the jth row of B_{t+1}, and v_{ij} denotes element (i, j) of the matrix V, which is given by iterating on the Ricatti equation

$$\begin{aligned} V &= Q + \phi(A + Bf)'V(A + Bf) \\ &+ \phi v_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11} f + f'\Sigma_B^{11} f\right) + \phi v_{22}\Sigma_A^{22}. \end{aligned} \tag{3.15}$$

As is clear from equations (3.14) and (3.15), the optimal policy rule depends on the variances (and covariances) of the parameters in the economy, so certainty equivalence ceases to hold when the parameters are stochastic. To obtain an analytical solution for this problem, one would need to solve equation (3.15) for the fixed-point value of V. For some simple configurations, for example, in the non-stochastic case, this is manageable (although tedious), since the system of equations obtained is relatively straightforward to solve. In this setup of multiplicative parameter uncertainty, however, the system of equations is highly non-linear and far too complicated to yield a usable solution. Therefore I proceed by numerical methods to analyze the optimal behavior of the central bank in this setting.

3.3 The effects of parameter uncertainty on optimal policy

Having derived the optimal policy rule (3.13) for the central bank, this section will analyze how the rule, and the resulting path of the short interest rate, depends on the degree of uncertainty in the economy. I therefore choose some values for the parameters $\alpha, \beta, \gamma, \delta$, and ϕ, and then examine how optimal policy behaves for different configurations of the parameter variances $\sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2$, and σ_δ^2, and of the preference parameter λ.

Shocks to output and inflation in equations (3.1) and (3.2) will affect monetary policy on two different, but related, levels. First, there is an initial effect, as policy is adjusted to respond to current shocks. This effect is given by the vector f in the decision rule (3.13). Second, there is a dynamic effect of shocks, since these will not be completely offset in the initial period, but will partly be transmitted to subsequent periods through the dynamics of the economy. Thus policy will also need to respond to past shocks, as these remain in the economy. I will distinguish between these two effects, and begin by analyzing the initial response of policy in Section 3.3.1, followed by an analysis of the dynamic response over time in Section 3.3.2.

The exact parameter values used for this numerical exercise are chosen so as to best illustrate the results, but are also consistent with empirical studies of the monetary transmission mechanism in the U.S. The reported results will not depend on the exact configuration of parameter values, but hold for many different plausible and implausible configurations.

The mean of the persistence parameter of the output gap, α, is given a value of 0.85, taken from Cooley and Hansen (1995, Table 7.1). This value is the autocorrelation coefficient of the observed detrended output process, and as such would

Table 3.1: Numerical values of parameter means and variances

	Stochastic parameters					Non-stochastic parameters	
	Mean	Variance					Value
α	0.85	{0.01,	0.00,	0.01,	0.01}	ϕ	0.95
β	0.35	{0.01,	0.00,	0.01,	0.01}	λ	[0,2]
γ	0.4	{0.01,	0.00,	0.01,	0.01}		
δ	1.0	{0.00,	0.10,	0.10,	0.20}		

tend to overestimate the true persistence of the output gap, unaffected by active stabilization policy. To the parameter β, the elasticity of output with respect to the real interest rate, a mean value of 0.35 is assigned, taken from Fuhrer's (1994, Table 3) estimate of output's sensitivity to the long real interest rate for the U.S. from 1966 to 1993. The mean of the persistence parameter of the Phillips curve, δ, is assigned a value of unity, leading to a standard accelerationist Phillips curve, on average. Finally, for γ, the inflation rate's sensitivity to the output gap, I assign a value of 0.4, which is approximately what Romer (1996, Table 2) finds for the U.S. economy for the period 1952–73, and which is also consistent with the correlation coefficient reported by Cooley and Hansen (1995, Table 7.1).

Since uncertainty concerning α, β, and γ has similar effects on policy, but uncertainty concerning δ has very different implications, the analysis will concentrate on three different configurations of uncertainty: (1) when α, β, and γ are stochastic, but δ is not (so $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.01$ and $\sigma_\delta^2 = 0$); (2) when δ is stochastic, but α, β, and γ are constant ($\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0$ and $\sigma_\delta^2 = 0.1$); and (3) when all four parameters are stochastic ($\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.01$ and $\sigma_\delta^2 = 0.1$ and 0.2). For simplicity, I shall call the first of these the case of 'impact uncertainty,' since the parameters α, β, and γ are all part of the direct impact of policy on output and inflation (via output). The second is a case of 'adjustment uncertainty,' since the parameter δ mainly determines the adjustment dynamics of the model; and the third case is a combination of impact and adjustment uncertainty. In each case, optimal policy will be compared to the certainty equivalence case, when all parameters are constant and equal to their means. The actual degree of uncertainty assigned through the parameter variances is chosen to make clear the effects of parameter uncertainty on policy. The qualitative results remain irrespective of the actual size of the parameter variances. The resulting values for the means and variances of the stochastic parameters are given in the left-hand panel of Table 3.1.

As shown in the right-hand panel of Table 3.1, the discount factor ϕ is assigned a value of 0.95, implying a discount rate of 5% per period. Finally, since the effects of uncertainty on policy depend crucially on the value of the preference parameter λ, this will be allowed to take values varying from 0, that is, 'strict inflation targeting,' to 2, with a larger weight on stabilizing output than on fighting inflation.

3.3.1 The initial policy response

The two top graphs of Figure 3.1 show the initial policy response to current output and inflation shocks for different values of λ in the case of certainty and in the case of impact uncertainty, that is, when there is some uncertainty about α, β, and γ, but δ is non-stochastic. The left-hand graph shows the response to output (or demand shocks) and the right-hand graph the response to inflation (supply shocks), with the solid line representing the certainty case, and the dashed line representing the response under uncertainty.

For the case of impact uncertainty, the response coincides well with the accepted wisdom formalized by Brainard (1967) and stressed by Blinder (1997, 1998). When there is uncertainty about α, β, and/or γ, it is optimal for the central bank to be more cautious and respond less fiercely to any shocks to output and inflation.[11] Increasing the variance of either α, β, or γ will weaken the optimal response of the central bank, and in the limit, as the variances tend to infinity, the optimal response is to do nothing.[12]

However, in the case of adjustment uncertainty, when the persistence parameter of inflation, δ, is stochastic, but α, β, and γ are constant, the effects of uncertainty on policy are dramatically altered, as seen in the two bottom graphs of Figure 3.1. Now, when $\lambda = 0$, so that the central bank cares only about stabilizing inflation, uncertainty about δ does not affect the optimal response to output or inflation. When $\lambda > 0$, however, the pattern goes against the Brainard conservatism principle: the optimal policy under parameter uncertainty is *more* aggressive than under certainty equivalence, so that the initial central bank response is stronger, not weaker.

Finally, consider the case when there is uncertainty about all four parameters, shown in Figure 3.2. Now we have two different possibilities: when λ is low, optimal policy under uncertainty is more cautious than under certainty, since the uncertainty about α, β, and γ dampens the response, but the uncertainty about δ has no or little effect. As λ increases, the uncertainty about δ starts to affect the response positively,

[11] In his original analysis, Brainard (1967) shows that large covariances between the instrument and exogenous variables may overturn his conservatism result (see also Blinder, 1998). Since all parameters and shocks are assumed independent here, such situations are not considered.

[12] The special case analyzed by Svensson (1997a), when $\lambda = 0$ and δ is non-stochastic, is represented along the vertical axes of the top graphs of Figure 3.1.

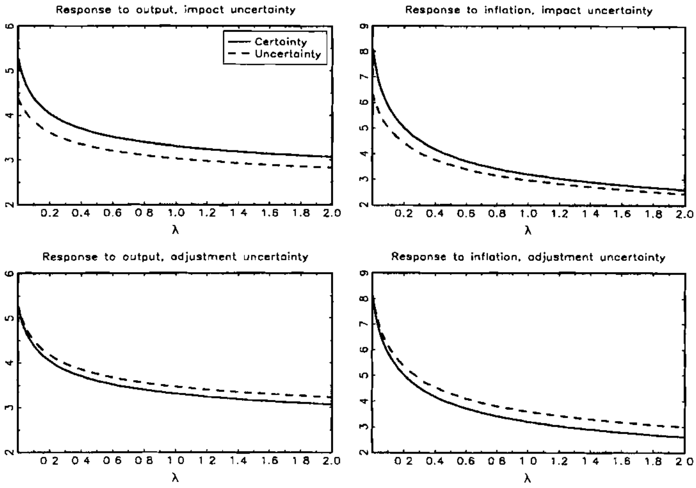

Figure 3.1: Initial response to current output and inflation, impact uncertainty and adjustment uncertainty

and eventually the response under uncertainty is stronger than under certainty. For a given λ, whether the initial response is more or less aggressive under uncertainty depends on the relative variances of α, β, and γ on the one hand and δ on the other. When the degree of adjustment uncertainty is relatively small ($\sigma_\delta^2 = 0.1$) in the top graphs of Figure 3.2, the response to inflation shocks is larger under uncertainty for $\lambda \geq 0.58$, whereas the response to output shocks is always smaller under uncertainty.[13] When adjustment uncertainty gets relatively more important, however, in the lower part of Figure 3.2 (where $\sigma_\delta^2 = 0.2$), policy is more likely to be more aggressive under uncertainty; the corresponding cutoff values are now $\lambda \geq 0.66$ for output shocks and $\lambda \geq 0.22$ for inflation shocks .

Since the above results may be counterintuitive at first glance, they may need some further explanation. The model used here differs from that of Brainard (1967) in two respects: it is dynamic rather than static, and it incorporates uncertainty concerning not only the impact effect of policy, but also concerning the dynamic development of the economy. Craine (1979) comes to a similar conclusion, using a dynamic model with one target variable, encompassing the Brainard result as a special case. In the formulation of Holly and Hughes Hallett (1989), let z_t be the

[13]For these parameter values, this is true for all λ at least up to 50,000.

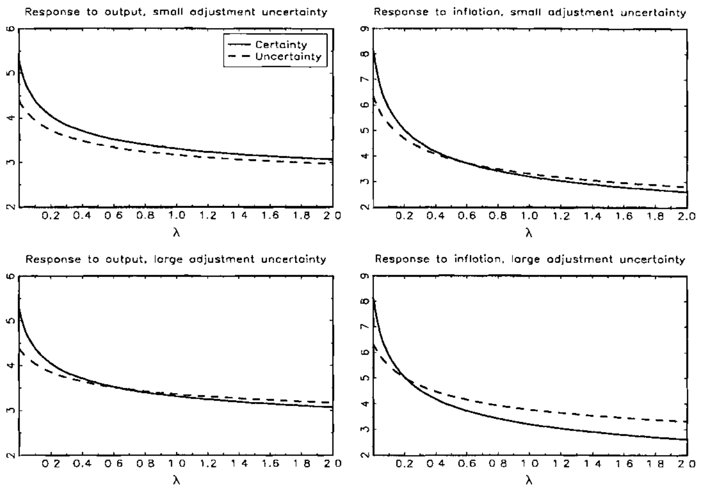

Figure 3.2: Initial response to current output and inflation, combinations of impact and adjustment uncertainty

target variable, p_t a policy variable, and e_t an exogenous variable, and let them be related by the equation

$$z_t = a_t z_{t-1} + b_t p_t + c_t e_t + \varepsilon_t^z. \tag{3.16}$$

Using a quadratic objective function, impact uncertainty (concerning b_t) can be shown to lead to less aggressive policy in response to shocks, but adjustment uncertainty (concerning a_t) leads to more aggressive policy. Naturally, since dynamics are necessary to model adjustment uncertainty, a dynamic formulation is crucial for the latter result.

In the Holly and Hughes Hallett setup, it is straightforward to separate impact from adjustment uncertainty, but in the Svensson model, this separation is less clear-cut. The analysis above shows that the Craine (1979) result is valid also in the Svensson setup, and the results of Holly and Hughes Hallet (1989) imply that this does not depend crucially on the assumptions that policy affects one target variable only via the other.

Craine's result can be understood by realizing that adjustment uncertainty implies that shocks hitting the economy eventually lead to fluctuations so large that the discounted sum of the variance of target variables is unbounded. In the two-target setup, where policy affects inflation only via output, this intuition needs to be modified. Now, 'adjustment uncertainty' leads to potentially large variability in one of the target variables, the inflation rate. As long as $\lambda < +\infty$, the central bank is concerned that inflation might move away from target by itself, that is, that $\delta_{t+\tau} > 1$. If this happens, the bank must adjust the interest rate to move inflation closer to target, which in turn will move output away from target. If $\lambda = 0$, the cost of the extra output variability is zero, so the central bank would gladly adjust output to keep inflation at bay. If $\lambda > 0$, however, the extra adjustment of the interest rate and output if inflation moves away is costly. Since its loss function is quadratic, the central bank does not want to take the bet that inflation stays under control, so instead the optimal policy is to move more aggressively in response to any shock, to minimize the expected cost of future adjustment.[14] Consequently, when the central bank is uncertain about the workings of the economy, it may be optimal to respond more aggressively to shocks, so as to avoid bad outcomes in the future.[15]

3.3.2 The time path of policy

The introduction of multiplicative parameter uncertainty also has interesting implications for the dynamic response of monetary policy, that is, the response of policy to past shocks to output and inflation. Figures 3.3 and 3.4 show the response of monetary policy to supply and demand shocks over the first ten periods following a shock, for $\lambda = 0$ and $\lambda = 1$. Figure 3.3 illustrates the case where there is both impact and adjustment uncertainty, with $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.01$, and $\sigma_\delta^2 = 0.2$, and Figure 3.4 illustrates the case of impact uncertainty only, with $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.05$, and $\sigma_\delta^2 = 0$.

As noted by Ellingsen and Söderström (1999), in the simple Svensson model under certainty equivalence, the response of monetary policy over time varies substantially with the preference parameter λ. In particular, for small values of λ, the optimal policy response to an inflationary shock under certain parameter configurations is to raise the interest rate instrument in the first period, but then lower it below the initial level and move gradually back to neutral policy. This is shown by

[14]It should be noted that the qualitative effects of uncertainty concerning δ do not hinge on its mean value being equal to unity. For smaller values of the mean, uncertainty still makes policy more aggressive, although quantitatively the effects get smaller as the probability of a realization above unity gets small.

[15]Onetski and Stock (1998) and Sargent (1998a) use robust control theory, where the policymaker chooses policy to minimize the risk of bad outcomes under model uncertainty, to show that particular configurations of uncertainty lead to more aggressive policy than under certainty equivalence. Intuitively, 'cautious' policy can also mean that bad future outcomes are avoided by acting more aggressively today.

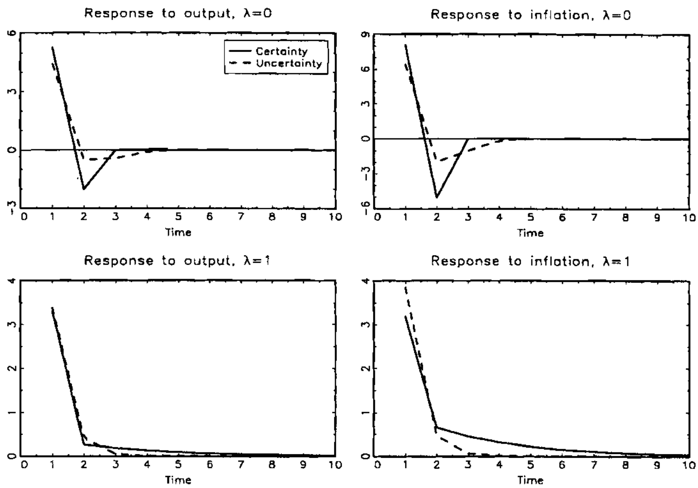

Figure 3.3: Response to inflation over time, combination of impact and adjustment uncertainty

the solid lines in the top two graphs of Figures 3.3 and 3.4.

When parameters are uncertain, this behavior can be mitigated or magnified, depending on whether the initial response is dampened or strengthened. When, as in the bottom graphs of Figure 3.3, uncertainty about δ dominates (since $\lambda = 1$), so that the initial policy response is more aggressive under uncertainty, policy in later periods is closer to neutral, since the strong initial move has neutralized a larger part of the shock. If, on the other hand, uncertainty about α, β, and γ dominates, as in Figure 3.4, so that the policy response is initially dampened, policy stays away from neutral longer, to compensate for the weaker initial response.

Thus, as is clear from Figure 3.4, parameter uncertainty can lead to smoother paths of the interest rate than under certainty equivalence, without introducing an explicit smoothing objective into the central bank's loss function. Casual observation suggests that central banks tend to respond to shocks by first slowly moving the interest rate in one direction, and then gradually moving back to a more neutral stance. When parameters are certain, the model suggests a large initial move, and then a quick return to the original level, unless λ is very large. Under certain configurations of parameter uncertainty, however, the central bank behaves in a more gradual way: although the initial response is always the largest, it is more modest under these cases of uncertainty, and the policy move is drawn out longer

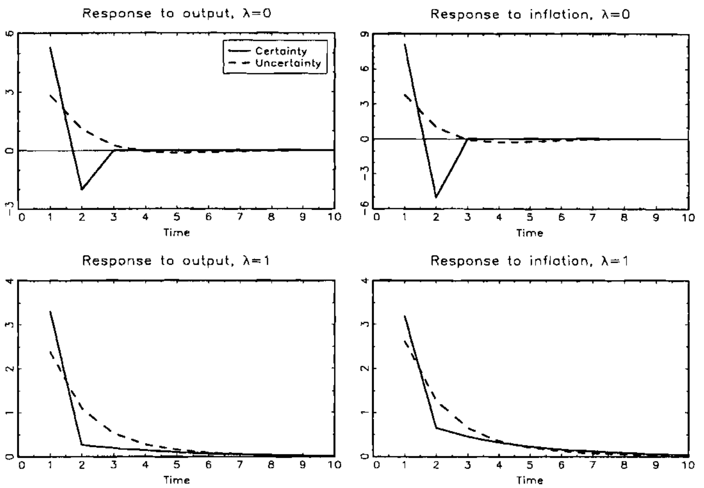

Figure 3.4: Response to inflation over time, impact uncertainty

over time. In particular, the tendency of the bank to 'whipsaw' the market by creating large swings in the interest rate is mitigated.[16]

3.4 Concluding remarks

The purpose of this chapter has been to illustrate how uncertainty about parameters in a dynamic macroeconomic model can lead the central bank to pursue *more* aggressive monetary policy, providing a counterexample to the results of Brainard (1967). When a policymaker is uncertain about the adjustment dynamics of the economy—in the context of this chapter, the persistence parameter of the inflation process—he might find it optimal to move more aggressively in response to shocks, so as to avoid bad outcomes in the future. Uncertainty about the impact effect of policy still leads to less aggressive policy, in accordance with Brainard's original analysis.

It should be stressed that the model and the examples used are highly stylized and may not be entirely satisfactory from an empirical point of view, so any serious

[16]This issue of parameter uncertainty leading to more plausible paths of policy is examined more carefully by Sack (1998a) and Söderström (1999). The latter shows, however, that the Svensson model always implies excessive volatility of the policy instrument, whereas optimal policy from an unrestricted VAR model comes very close to mimicking the actual behavior of the Federal Reserve.

implications for policy are difficult to estimate. However, the qualitative points obtained from this simple model are also present in the more general empirical framework of Rudebusch and Svensson (1998), and are likely to remain also in models incorporating forward-looking behavior.

It is possible that configurations of uncertainty in the real world are such that the Brainard result is always valid, or to quote Blinder (1998, p. 12), "My intuition tells me that this finding is more general—or at least more wise—in the real world than the mathematics will support." Using the standard errors of econometric parameter estimates as proxies for the degree of uncertainty concerning each parameter in a more complete econometric formulation of the Svensson model, Söderström (1999) shows that in the resulting configuration of variances, uncertainty about α, β, and γ dominates uncertainty about δ, so parameter uncertainty does act to dampen policy. Also, Rudebusch (1998) argues that multiplicative parameter uncertainty is not a very important source of cautious behavior of the Federal Reserve. Nevertheless, the main point in this chapter is that the effects on policy of parameter uncertainty may be less clear-cut than previously recognized. Determining the relevance of this result for actual policy should be an interesting topic for future research.

3.A Solving the control problem

First, the state vector x_{t+1} has expected value

$$E_t x_{t+1} = Ax_t + Bi_t, \tag{3.17}$$

and covariance matrix

$$\Sigma_{t+1|t} = \begin{bmatrix} \Sigma^{y}_{t+1|t} & \Sigma^{y,\pi}_{t+1|t} \\ \Sigma^{\pi,y}_{t+1|t} & \Sigma^{\pi}_{t+1|t} \end{bmatrix}, \tag{3.18}$$

evaluated at t. Since all parameters are assumed independent, the off-diagonal elements of $\Sigma_{t+1|t}$ are zero. The diagonal elements are

$$\begin{aligned} \Sigma^{y}_{t+1|t} &= \mathrm{Var}_t[\alpha_{t+1} y_t - \beta_{t+1}(i_t - \pi_t) + \varepsilon^{y}_{t+1}] \\ &= x_t' \Sigma^{11}_A x_t + 2x_t' \Sigma^{11}_{AB} i_t + i_t' \Sigma^{11}_B i_t + \Sigma^{11}_{\varepsilon}, \end{aligned} \tag{3.19}$$

and

$$\begin{aligned} \Sigma^{\pi}_{t+1|t} &= \mathrm{Var}_t[\delta_{t+1}\pi_t + \gamma_{t+1} y_t + \varepsilon^{\pi}_{t+1}] \\ &= x_t' \Sigma^{22}_A x_t + \Sigma^{22}_{\varepsilon}, \end{aligned} \tag{3.20}$$

where Σ^{ij}_{AB} is the covariance matrix of the ith row of A_{t+1} with the jth row of B_{t+1}, that is,

$$\Sigma^{11}_A = \begin{bmatrix} \sigma^2_\alpha & 0 \\ 0 & \sigma^2_\beta \end{bmatrix}, \quad \Sigma^{22}_A = \begin{bmatrix} \sigma^2_\gamma & 0 \\ 0 & \sigma^2_\delta \end{bmatrix}, \tag{3.21}$$

$$\Sigma^{11}_B = \sigma^2_\beta, \quad \Sigma^{11}_{AB} = \begin{bmatrix} 0 \\ -\sigma^2_\beta \end{bmatrix}, \tag{3.22}$$

and

$$\Sigma^{11}_{\varepsilon} = \sigma^2_y, \quad \Sigma^{22}_{\varepsilon} = \sigma^2_\pi. \tag{3.23}$$

The extra term to take into account in equation (3.12) is then

$$\begin{aligned} \mathrm{tr}(V\Sigma_{t+1|t}) &= v_{11}\left(x_t' \Sigma^{11}_A x_t + 2x_t' \Sigma^{11}_{AB} i_t + i_t' \Sigma^{11}_B i_t + \Sigma^{11}_{\varepsilon}\right) \\ &+ v_{22}\left(x_t' \Sigma^{22}_A x_t + \Sigma^{22}_{\varepsilon}\right), \end{aligned} \tag{3.24}$$

where v_{11} and v_{22} are the diagonal elements of the matrix V.

Using equations (3.10), (3.12), and (3.17) in the control problem (3.9), we can express the Bellman equation as

$$x_t'Vx_t + w$$
$$= \min_{i_t} \left\{ x_t'Qx_t + \phi\left(Ax_t + Bi_t\right)' V \left(Ax_t + Bi_t\right) + \phi \mathrm{tr}(V\Sigma_{t+1|t}) + \phi w \right\}, \quad (3.25)$$

which gives the necessary first-order condition as[17]

$$\phi \left[B'(V+V')Ax_t + B'(V+V')Bi_t + \frac{d\mathrm{tr}(V\Sigma_{t+1|t})}{di_t} \right] = 0, \quad (3.26)$$

where, from (3.24),

$$\frac{d\mathrm{tr}(V\Sigma_{t+1|t})}{di_t} = 2v_{11}\left(\Sigma_{AB}^{11\,\prime} x_t + \Sigma_B^{11} i_t\right). \quad (3.27)$$

Thus we get the optimal policy rule

$$\begin{aligned} i_t &= -\left[B'\left(V+V'\right)B + 2v_{11}\Sigma_B^{11}\right]^{-1}\left[B'\left(V+V'\right)A + 2v_{11}\Sigma_{AB}^{11\,\prime}\right]x_t \\ &= fx_t. \end{aligned} \quad (3.28)$$

Finally, using equation (3.24) and the policy rule (3.28) in the Bellman equation (3.25) gives

$$\begin{aligned} x_t'Vx_t + w &= x_t'Qx_t + \phi\left[(Ax_t + Bfx_t)'V(Ax_t + Bfx_t) + w\right] \\ &+ \phi v_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}fx_t + x_t'f_t'\Sigma_B^{11}fx_t + \Sigma_\varepsilon^{11}\right) \\ &+ \phi v_{22}\left(x_t'\Sigma_A^{22}x_t + \Sigma_\varepsilon^{22}\right) \\ &= x_t'\left[\begin{array}{l} Q + \phi(A+Bf)'V(A+Bf) \\ +\phi v_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11}f + f_t'\Sigma_B^{11}f\right) + \phi v_{22}\Sigma_A^{22} \end{array}\right]x_t \\ &+ \phi\left[w + v_{11}\Sigma_\varepsilon^{11} + v_{22}\Sigma_\varepsilon^{22}\right], \end{aligned} \quad (3.29)$$

so the matrix V is determined by

$$\begin{aligned} V &= Q + \phi(A+Bf)'V(A+Bf) \\ &+ \phi v_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11}f + f'\Sigma_B^{11}f\right) + \phi v_{22}\Sigma_A^{22}. \end{aligned} \quad (3.30)$$

See also Chow (1975).

[17]Use the rules $\partial x'Ax/\partial x = (A+A')x$, $\partial y'Bz/\partial y = Bz$, and $\partial y'Bz/\partial z = B'y$, see, e.g., Ljungqvist and Sargent (1997). Note also that V is not necessarily symmetric in this setup with multiplicative uncertainty.

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Chapter 4

Should central banks be more aggressive?

Abstract:
Simple models of monetary policy often imply optimal policy behavior that is considerably more aggressive than what is commonly observed. This chapter argues that such counterfactual implications are due to model restrictions and a failure to account for multiplicative parameter uncertainty, rather than to policymakers being too cautious in their implementation of policy. Comparing a restricted and an unrestricted version of the same empirical model, the unrestricted version leads to less volatility in optimal policy, and, taking parameter uncertainty into account, to policy paths very close to actual Federal Reserve policy.

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4.1 Introduction

IT IS A COMMON OBSERVATION that central banks implement monetary policy in a gradual manner. As documented by Rudebusch (1995) and the Bank for International Settlements (1998), central banks tend to adjust their interest rate instrument in small, persistent steps, moving the interest rate several times in the same direction before reversing policy. To understand such behavior of policy-makers, we need to develop theoretical models that are consistent with the empirical evidence.

Many simple models designed for monetary policy analysis, such as those used by Ball (1997), Cecchetti (1998), Svensson (1997a,b), and Wieland (1998), have the attractive property that the optimal monetary policy rule is a simple linear function of the state of the economy, similar to a Taylor (1993) rule. In a dynamic setting, the central bank acts to minimize the variation over time of the goal variables from their targets, so when facing a shock, the policy instrument is moved away from the initial position, and then gradually returned towards a neutral stance (see, e.g., Ellingsen and Söderström, 1998).

It has been noted that these models often imply considerably more aggressive policy than what is empirically observed. For example, Rudebusch (1998a) and Rudebusch and Svensson (1998) show that the restricted reaction function from an empirical version of the Svensson (1997a,b) model has considerably larger coefficients than those shown by Taylor (1993) to match the behavior of the Federal Reserve.[1] Also, Ellingsen and Söderström (1998) show that the simple Svensson model implies excessive volatility and 'whip-sawing' behavior of the short interest rate for reasonable parameter values. Therefore, to match the observed behavior, it is common to introduce an explicit interest rate smoothing motive into the objective function of the central bank (see, e.g., Rudebusch and Svensson, 1998).

However, although such a smoothing objective might be motivated by central banks' concern about financial market stability (see Goodfriend, 1989, or Cukierman, 1991) or uncertainty about the economic environment (Blinder, 1998; Bank for International Settlements, 1998), if the basic model is misspecified, we should be wary about its policy predictions. Also, as shown by Sack (1998a), an interest rate smoothing objective is not necessary to match the policy path of the Federal Reserve using a standard vector autoregression (VAR) model. Instead, multiplicative parameter uncertainty acts to dampen optimal policy, leading to paths for the

[1]The restricted reaction function allows policy to respond only to current output and inflation.

federal funds rate which are very close to those actually observed for the period from 1983 to 1996.

This chapter further analyzes the properties of optimal monetary policy in the model developed by Svensson (1997a,b) by estimating a version of the model on U.S. data, and comparing the obtained estimates with results from an unrestricted VAR model of the same variables. The analysis shows that the optimal policy in both the restricted and the unrestricted model is more aggressive than observed policy, implying more volatility in the short interest rate than is observed in reality. However, policy in the restricted model is more aggressive than in the unrestricted model, pointing to the importance of the model's restrictions. Introducing multiplicative parameter uncertainty makes policy less aggressive in both models, following the result of Brainard (1967), but the restricted model still implies far too volatile interest rates to match the data. The unrestricted model, on the other hand, leads to policy that is very close to observed policy, in parallel with the results of Sack (1998a). These results indicate that the general setup with an optimizing central bank is a good approximation of actual policy behavior, whereas the restrictions imposed in the Svensson model are at odds with the data.

Rudebusch and Svensson (1998) estimate a similar version of the model on similar data, without examining the dynamic policy response to shocks, and conclude, on the basis of statistical information criteria, that the model restrictions are *not* at odds with the data. As is shown below, however, formal hypothesis tests of the restrictions leads one to reject the restricted model in favor of the unrestricted model. Rudebusch (1998a) introduces several types of uncertainty into the Svensson model in an attempt to make the coefficients of the optimal restricted Taylor rule match the empirical rule for the U.S. Taking the model setup as given, he finds that combinations of data and parameter uncertainty lead to more reasonable reaction functions.

The present chapter is organized as follows. Section 4.2 presents the dynamic model introduced by Svensson (1997a,b), relates that model to an unrestricted VAR model, and estimates the two models on quarterly U.S. data. In Section 4.3, optimal policy rules for the models are derived, and the resulting reaction functions and policy responses over time are compared with actual Federal Reserve behavior. Section 4.4 introduces parameter uncertainty into the models, and discusses the consequences for optimal policy, and Section 4.5 compares the implied path of the federal funds rate from the models with the actual funds rate path. Finally, Section 4.6 offers some concluding remarks.

4.2 A dynamic framework

4.2.1 The Svensson model

The monetary policy model to be analyzed is the dynamic framework developed by Lars Svensson (1997a,b). This framework, which has been primarily used to study issues of inflation targeting, contains the important aspects that the policymaker has imperfect control over the inflation rate, and that policy, implemented through an interest rate instrument, affects the economy with a lag. Most importantly, the policymaker cannot affect the inflation rate directly, but only via the output gap, and with an extra control lag. Thus, monetary policy affects the output gap with a one-period lag and the inflation rate with a lag of two periods. As shown below, this feature, designed to be consistent with the stylized facts of the monetary transmission mechanism, has important implications for the behavior of monetary policy when responding to innovations to inflation and output.

The model consists of two relationships between inflation, output (or the output gap), and a short (one-period) interest rate, controlled by the central bank. In a general formulation, with an unspecified number of lags, the output gap in period $t+1$ is determined by the IS-relationship

$$y_{t+1} = \alpha(L)y_t + \beta(L)\left(i_t - \pi_t\right) + \varepsilon^y_{t+1}, \tag{4.1}$$

where y_t is the percentage deviation of output from its trend (or 'potential') level; i_t is the central bank's interest rate instrument (or its deviation from the long-run mean) at an annualized rate; π_t is the annualized inflation rate, in percentage points (also its deviation from its long-run mean, or target); and ε^y_{t+1} is an i.i.d. demand shock, with zero mean and constant variance. The output gap is thus assumed to depend on past values of itself and past realizations of the ex-post short real interest rate, or the 'pseudo-real' interest rate (Svensson, 1997a). The inflation rate is assumed to follow an accelerationist-type Phillips curve;

$$\pi_{t+1} = \delta(L)\pi_t + \gamma(L)y_t + \varepsilon^\pi_{t+1}, \tag{4.2}$$

thus being determined by past inflation, past values of the output gap, and an i.i.d. supply shock ε^π_{t+1}, also with zero mean and constant variance. To close the model, a quadratic loss function is assigned to the central bank, and then the bank's optimal control problem is solved to obtain a decision rule for the short interest rate, contingent on the development of output and inflation.[2]

[2]Note that the model is formulated in deviations from targets or long-run means, so negative values of all variables are allowed.

This setup is clearly a severe simplification of the true economy, but it could be interpreted as reduced-form relationships from a more complete model including sticky prices and some kind of transmission mechanism of monetary policy, such as the standard interest rate channel or a credit channel. Under this reduced-form interpretation, any policy experiments in this model are clearly at odds with the Lucas (1976) critique. However, Fuhrer (1995) argues that Phillips curve specifications like equation (4.2) are very close to being 'structural' relationships, since they do not seem to change much over time.

The model is also subject to criticism for not incorporating forward-looking behavior of agents. In particular, the Phillips curve (4.2) does not include inflation expectations, except in the adaptive form of a distributed lag of past inflation rates. The IS-specification (4.1) includes an ex-post real interest rate instead of an ex-ante real rate, which arguably is more important for investment behavior, or credit market considerations.[3] Again, however, Fuhrer (1997) shows that expectations of future prices are not very important in determining price and inflation behavior: backward-looking price specifications are actually favored by the data. On the other hand, backward-looking models exhibit long-run dynamics which are less consistent with existing evidence. Accepting equation (4.2) as a reasonable specification for the inflation rate, Svensson (1997b) shows how an IS-equation with an ex-ante real interest rate is easily transformed into an IS-equation like equation (4.1).[4]

Finally, Estrella and Fuhrer (1998) argue that many dynamic models incorporating rational expectations and optimizing behavior have the counterfactual implication that the inflation rate (or real spending) jumps in response to shocks, making them unsuitable for short-run monetary policy analysis. A version of the Svensson setup, with partially forward-looking behavior, is shown to be more consistent with the data.

4.2.2 A VAR interpretation

As pointed out by Rudebusch and Svensson (1998) and Rudebusch (1998a), the model (4.1)–(4.2) can be interpreted as restrictions on the first two equations of a

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[3] Eijffinger et al. (1998) include an ex-ante long real interest rate in the specification of the IS-curve, and find that optimal policy becomes more aggressive than in the original formulation with the short real rate.

[4] A third criticism of the model is that it does not strictly obey the natural-rate hypothesis, since the central bank could increase output indefinitely by accepting accelerating inflation. Given the loss function assigned to the central bank (see below), such behavior will never be optimal (Svensson, 1997b).

trivariate vector autoregression (VAR) model containing the output gap, the inflation rate, and the short interest rate.[5] Writing out the three equations, and assuming that the central bank responds to current output and inflation when setting the interest rate, but that policy has no contemporary effects on the economy, such an unrestricted VAR system is given by

$$y_t = \sum_{s=1}^{L} A_s^y y_{t-s} + \sum_{s=1}^{L} B_s^y \pi_{t-s} + \sum_{s=1}^{L} C_s^y i_{t-s} + \xi_t^y, \tag{4.3}$$

$$\pi_t = \sum_{s=1}^{L} A_s^\pi y_{t-s} + \sum_{s=1}^{L} B_s^\pi \pi_{t-s} + \sum_{s=1}^{L} C_s^\pi i_{t-s} + \xi_t^\pi, \tag{4.4}$$

$$i_t = \sum_{s=0}^{L} A_s^i y_{t-s} + \sum_{s=0}^{L} B_s^i \pi_{t-s} + \sum_{s=1}^{L} C_s^i i_{t-s} + \xi_t^i. \tag{4.5}$$

The Svensson model then puts restrictions on the parameters in the first two equations, and assumes that the parameters of the third equation are obtained from the central bank's optimization problem. The parameter restrictions imply that $B_s^y = -C_s^y$ and $C_s^\pi = 0$ for all s.

Although these restrictions may seem plausible, it is conceivable that they are not consistent with the true transmission mechanism of monetary policy. If, for example, output were affected by the ex-ante real interest rate (or even the long real rate), and inflation expectations were not directly related to past inflation, the restriction on the output equation would be rejected. Also, one could argue that the restricted inflation equation is likely to be at odds with the data: although Phillips curve relationships like (4.2) seem to hold empirically (see, e.g., Fuhrer, 1997, or Blanchard and Katz, 1997), monetary policy could possibly affect inflation without affecting the level of output first, for example, if there were bottlenecks in the economy. In that case, a monetary easing would create excess demand, that could not be satisfied directly with increased output. Then inflation would increase *before* output, leading to a direct link from monetary policy to inflation.

Following Rudebusch and Svensson (1998), a first test of the Svensson model is to estimate the restricted equations (4.1)–(4.2) on quarterly U.S. data, and compare the results with those obtained from estimating the unrestricted VAR model (4.3)–(4.5).[6]

[5]That the methodology behind VAR models is not entirely uncontroversial can be seen from the debate between Rudebusch (1998b,c) and Sims (1998).

[6]When estimating the Svensson model, Rudebusch and Svensson (1998) use four lags of inflation and one lag of output in the inflation equation, and two lags of output and one lag of the average real interest rate for the last four quarters in the output equation. Also, the sum of the B_s^π coefficients

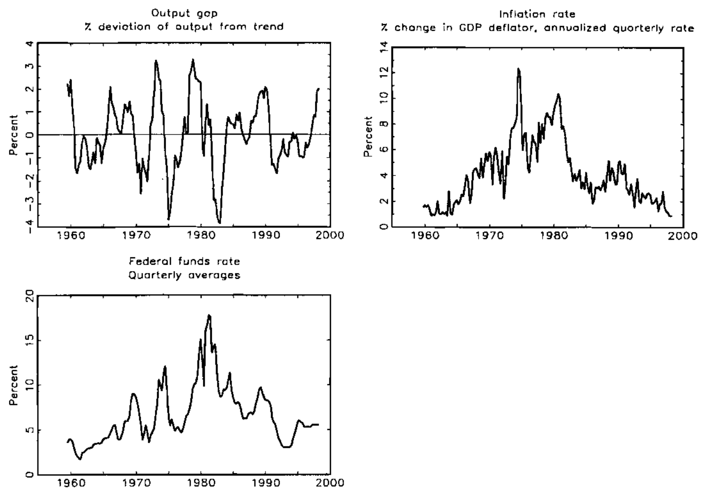

Figure 4.1: Data series 1959:3–1998:2

4.2.3 Data

The two models are estimated on quarterly U.S. data from 1959:3 to 1998:2; graphs of the data series are shown in Figure 4.1. The output series used is real GDP, measured in billions of fixed 1992 dollars and seasonally adjusted. The output gap is defined as the percentage deviation of output from trend, where the trend is calculated using a Hodrick-Prescott filter. The price series is the implicit GDP deflator, seasonally adjusted, and the inflation rate is the quarterly percentage change in the price index, at an annual rate. Both of these series are from the Bureau of Economic Analysis at the U.S. Department of Commerce. The interest rate used is the quarterly average of the effective federal funds rate, taken from the Board of Governors of the Federal Reserve.[7] All data have been downloaded from the FRED

is not significantly different from unity, so the authors impose that restriction in the estimation. Thus, a third restriction of $B^y_j = B^y_h$ and $C^y_j = C^y_h$ for $j, h = 1, \dots, 4$, and a fourth restriction of $\sum_s B^\pi_s = 1$ are imposed. Since the Rudebusch-Svensson setup leads to extreme volatility (and sometimes exploding paths) in the optimal interest rate, I choose not to impose these additional restrictions here, and instead concentrate on the restrictions from the original Svensson model.

[7]During this sample period, the Federal Reserve has occasionally changed its policy instrument, most notably during the experiment of non-borrowed reserves targeting from 1979 to 1982. Although the preferred choice of policy indicator varies across researchers, Bernanke and Mihov (1998), while concluding that no simple measure of policy is appropriate for the entire period from 1965 to 1996, show that a federal funds rate targeting model marginally outperforms models of borrowed reserves and non-borrowed reserves targeting for the whole sample period.

database of the Federal Reserve Bank of St. Louis at *http://www.stls.frb.org/fred/*.[8] Since the Svensson model is formulated in deviations from long-run means or targets, all variables are de-meaned before estimation, so no constants will appear in the regressions.

4.2.4 Estimation and hypothesis tests

Table 4.1 shows the results from estimating the unrestricted VAR model and the restricted model on quarterly data, using four lags.[9] Since the independent variables are likely to be highly multicollinear, it does not make much sense to discuss the significance of individual coefficients. Note, however, that the coefficients on inflation in the unrestricted model's output equation are not very close to the negative of the interest rate coefficients.

Table 4.2 shows some simple criteria for model selection. Depending on how strongly the different criteria penalize extra explanatory variables, one or the other model is selected. Using the most common criterion, adjusted R^2, leads to a preference for the unrestricted model, for both the output and the inflation equation. As criteria are chosen to punish extra right-hand variables more heavily, there is a gradual shift towards the restricted model. The Akaike information criterion chooses the unrestricted output equation but the restricted inflation equation, whereas using the Schwarz information criterion, the restricted model is preferred (recall that a smaller value of the information criteria is preferred to a larger). Consequently, as noted by Rudebusch and Svensson (1998), the simple criteria give a split decision as to which model to choose.

For formal hypothesis tests, Table 4.3 shows the results from univariate F-tests of each restriction separately (in the upper panel) and bivariate likelihood ratio tests for the two-equation system (in the lower panel), both on each restriction separately and jointly on both restrictions. The univariate and the bivariate tests of the separate hypotheses give very similar results: at the 5% confidence level we reject the hypothesis of $B_s^y = -C_s^y$ in the output equation, but we cannot reject the

[8]Many authors, for example, McCallum (1993), Orphanides (1998), Rudebusch (1998a), and Ghysels et al. (1998), stress the importance of data uncertainty for economic modeling; using final (revised) data in econometric estimation, and in particular in policy rules, is highly inappropriate, since these data are typically not available at the time of the policy decisions. On the other hand, although data on GDP and prices are only available with a delay, central banks do have access to a number of indicators of output and prices, which they use when formulating policy.

[9]Likelihood ratio tests on the VAR model reject the hypotheses of two and three lags in favor of four lags, but do not reject the hypothesis of four lags against five lags. See Hamilton (1994) for details.

Table 4.1: Estimated coefficients in restricted and unrestricted models

	Restricted		Unrestricted		
	y_t	π_t	y_t	π_t	i_t
y_t					0.465**
					(0.114)
y_{t-1}	1.070**	0.213	1.050**	0.077	−0.005
	(0.085)	(0.127)	(0.089)	(0.139)	(0.168)
y_{t-2}	−0.023	−0.002	0.005	0.074	−0.146
	(0.123)	(0.185)	(0.124)	(0.195)	(0.167)
y_{t-3}	−0.175	0.128	−0.177	0.206	0.046
	(0.121)	(0.183)	(0.120)	(0.188)	(0.162)
y_{t-4}	−0.061	−0.050	−0.056	−0.081	−0.069
	(0.085)	(0.127)	(0.085)	(0.134)	(0.114)
π_t					0.086
					(0.072)
π_{t-1}	0.045	0.579**	0.084	0.564**	−0.010
	(0.042)	(0.083)	(0.053)	(0.084)	(0.083)
π_{t-2}	0.063	0.006	−0.051	0.042	0.122
	(0.051)	(0.095)	(0.061)	(0.096)	(0.082)
π_{t-3}	−0.093	0.201*	−0.058	0.185	0.003
	(0.050)	(0.095)	(0.061)	(0.096)	(0.083)
π_{t-4}	0.027	0.142	0.053	0.180*	−0.102
	(0.043)	(0.082)	(0.055)	(0.086)	(0.075)
i_{t-1}	−0.045		0.051	0.162	0.929**
	(0.042)		(0.063)	(0.099)	(0.086)
i_{t-2}	−0.063		−0.277**	−0.215	−0.291*
	(0.051)		(0.085)	(0.133)	(0.119)
i_{t-3}	0.093		0.260**	−0.034	0.290*
	(0.050)		(0.087)	(0.136)	(0.120)
i_{t-4}	−0.027		−0.079	0.036	−0.007
	(0.043)		(0.064)	(0.100)	(0.086)
$\bar{R}^2$	0.799	0.835	0.807	0.838	0.930

Coefficient estimates from quarterly restricted Svensson model and unrestricted VAR model, 151 observations 1960:4 to 1998:2. Standard errors in parentheses, **/* denote significance at the 1%-/5%-level. In the y_t regression of the restricted model, the coefficients on i_{t-s} are restricted to be the negative of those on π_{t-s}.

Table 4.2: Simple criteria for model selection

	Output equation		Inflation equation	
	Restricted	Unrestricted	Restricted	Unrestricted
$\bar{R}^2$	0.799	0.807	0.835	0.838
Akaike	641.150	637.997	773.356	774.309
Schwarz	665.289	674.204	797.494	810.516

Adjusted R^2, Akaike, and Schwarz information criteria for the restricted and the unrestricted model.

Table 4.3: Hypothesis tests

Null	Test statistic	Distribution	Significance level
Univariate F-tests			
$B_s^y = -C_s^y$	2.664	$F(4,139)$	0.035
$C_s^\pi = 0$	1.660	$F(4,139)$	0.163
Bivariate LR-tests			
$B_s^y = -C_s^y$	10.268	$\chi^2(4)$	0.036
$C_s^\pi = 0$	6.488	$\chi^2(4)$	0.166
Joint hypothesis	16.732	$\chi^2(8)$	0.033

Hypothesis tests of restrictions in the model (4.3)–(4.4).

hypothesis of $C_s^\pi = 0$ in the inflation equation. The joint hypothesis is nevertheless rejected at the 5%-level. Thus, using formal hypothesis tests, we lean towards a rejection of the restricted Svensson model in favor of the unrestricted VAR model, and both hypothesis tests and the information criteria hint that the restriction on the output equation is more severe than that on the inflation equation.

4.3 Optimal policy

In the previous section we have seen that the restrictions of the simple Svensson model do not find very strong support in the data. In the remainder of this chapter, we shall see how important these restrictions are for the optimal path of monetary policy. Assigning a loss function to the central bank, it is straightforward to calculate the bank's optimal decision rule for both the restricted and the unrestricted model. Since the Svensson model is a special case of the unrestricted VAR model, let us derive the optimal policy rule for the unrestricted model, and then apply the rule to both models.

The central bank is assumed to minimize the expected discounted sum of future values of a loss function, which is quadratic in output and inflation deviations from target (here normalized to zero). Thus, the central bank solves the optimization problem

$$\min_{\{i_{t+\tau}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \phi^\tau L(y_{t+\tau}, \pi_{t+\tau}), \tag{4.6}$$

subject to (4.3)–(4.4), where in each period the loss function $L(\cdot)$ is given by

$$L(y_t, \pi_t) = \pi_t^2 + \lambda y_t^2, \tag{4.7}$$

and where $\lambda \geq 0$ is the weight of output stabilization relative to inflation fighting.[10] The parameter ϕ is the central bank's discount factor, set to 0.987 per quarter, implying an annual discount rate of around 5%.

To calculate the optimal policy rule, it is convenient to rewrite the general model (4.3)–(4.4) in state-space form as

$$x_{t+1} = Ax_t + Bi_t + \varepsilon_{t+1}. \tag{4.8}$$

Here x_t is an (11×1) state vector, given by current and lagged values of y_t and π_t, and lags of i_t,

$$x_t = \{y_t, \ldots, y_{t-3}, \pi_t, \ldots, \pi_{t-3}, i_{t-1}, \ldots, i_{t-3}\}; \tag{4.9}$$

the (11×11) matrix A has its first and fifth rows filled with the parameters from the VAR according to

$$A_1 = \begin{bmatrix} A_1^y & A_2^y & A_3^y & A_4^y & B_1^y & B_2^y & B_3^y & B_4^y & C_2^y & C_2^y & C_3^y \end{bmatrix} \tag{4.10}$$

$$A_5 = \begin{bmatrix} A_1^\pi & A_2^\pi & A_3^\pi & A_4^\pi & B_1^\pi & B_2^\pi & B_3^\pi & B_4^\pi & C_2^\pi & C_2^\pi & C_3^\pi \end{bmatrix}, \tag{4.11}$$

and occasional ones on the other rows, to complete the identities; and the (11×1) vector B has zeros everywhere except for the first and fifth elements, which correspond to C_1^y and C_1^π, and the ninth element, which is 1.[11]

---

[10]This formulation of the central bank objective function brings to mind at least three comments. First, it is widely accepted that all modern central banks put some weight on output stabilization, even when their ascribed goal only includes inflation or price stability (see Svensson, 1998). Fischer (1996) criticizes the tendency of central banks to only acknowledge price stability as their objective. Second, note that the loss function is formulated in terms of the quarterly inflation rate, and not the yearly rate, which would be the average rate of the last four quarters. Targeting the yearly inflation rate often makes it optimal (if λ is small enough) to move the instrument in four-period cycles in response to shocks. Therefore the quarterly inflation rate is chosen in the loss function. Third, as mentioned in the Introduction, Rudebusch and Svensson (1998) choose to include an interest rate smoothing motive in the loss function. Since such a motive seems warranted only to make the model fit the data, and since Sack (1998a) finds that a dynamic model which takes parameter uncertainty into account leads to optimal policy that fits the actual data very well, I choose to not include such an objective.

[11]As above, the Svensson restrictions imply that $B_s^y = -C_s^y$ and $C_s^\pi = 0$.

The loss function (4.7) can then be written as

$$L_t = x_t' Q x_t, \tag{4.12}$$

where the preference matrix Q has λ as element $(1,1)$, 1 as element $(5,5)$, and zeros elsewhere. The central bank solves the control problem

$$J(x_t) = \min_{i_t} \{x_t' Q x_t + \phi E_t J(x_{t+1})\}, \tag{4.13}$$

subject to (4.8), and Appendix 4.A shows that the optimal interest rate is given by

$$i_t = f x_t, \tag{4.14}$$

where the decision vector f is given by

$$f = -(B'VB)^{-1} B'VA, \tag{4.15}$$

and the matrix V is determined by the Ricatti equation

$$V = Q + \phi (A + Bf)' V (A + Bf). \tag{4.16}$$

(See also Chow, 1975 or Sargent, 1987, ch. 1.) Consequently, it is optimal for the central bank to set the interest rate instrument in each period as a function of current and lagged values of the output gap and the inflation rate, and lagged values of the instrument itself.

4.3.1 Reaction functions

Using the parameter values obtained from the unrestricted VAR model and the restricted model in Table 4.1 in the A-matrix and the B-vector, we can calculate the optimal policy rule from (4.14)–(4.16) numerically for the two models, for different values of the preference parameter λ. Table 4.4 shows the policy rules, or reaction functions, obtained for the two models with $\lambda = 0$ and $\lambda = 1$, along with the empirical estimates of the reaction function from the VAR model from equation (4.5) and Table 4.1.

As has been noted elsewhere, the coefficients in the optimal reaction functions are typically larger (in absolute value) than the empirical estimates. This is true for both models, although the restricted model has even larger coefficients than the unrestricted model, leading to more aggressive policy in the restricted model. This is especially striking for the response to current output and inflation, where the coefficients in the restricted model are extremely large. When $\lambda = 1$, that is, with equal weights on inflation and output stabilization, the coefficients are typically smaller than when $\lambda = 0$, but not for all variables. The optimal rules thus imply more aggressive policy than the empirical rule in the last column, which also seems more persistent, with a larger coefficient on the lagged interest rate.

Table 4.4: Optimal reaction functions

	Restricted		Unrestricted		Empirical
	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	
y_t	20.071	11.848	3.110	3.926	0.465
y_{t-1}	2.131	−0.719	0.392	−0.194	−0.005
y_{t-2}	−1.529	−1.843	−0.760	−0.838	−0.146
y_{t-3}	−1.623	−0.765	−0.178	−0.240	0.046
y_{t-4}					−0.069
π_t	16.405	4.018	1.487	1.173	0.086
π_{t-1}	11.559	1.426	1.224	0.598	−0.010
π_{t-2}	8.052	0.388	0.849	0.562	0.122
π_{t-3}	3.017	0.717	0.223	0.295	0.003
π_{t-4}					−0.102
i_{t-1}	−0.189	−0.091	−0.489	−0.314	0.929
i_{t-2}	0.874	0.779	0.638	0.684	−0.291
i_{t-3}	−0.298	−0.273	−0.168	−0.238	0.290
i_{t-4}					−0.007

Optimal reaction function (the vector f in equation (4.14)) from restricted and unrestricted models, and estimated empirical reaction function from Table 4.1.

4.3.2 The policy response over time

Using the calculated reaction functions and the transition dynamics of the models, we can calculate how policy responds over time to shocks to output and inflation by conducting the following experiment. In the first period, the economy is hit by a shock, either to output (ξ_t^y) or to inflation (ξ_t^π). This shock is then transmitted through the economy by equation (4.8), and the central bank responds optimally in each period according to its reaction function (4.14). Proceeding for a number of periods, we can trace the dynamic effects of a shock on policy by calculating how the central bank reacts in each period. This policy response is then similar to the impulse response function obtained from the VAR model, and thus the optimal response from the two models can be compared with the empirical response to shocks.

To calculate the impulse response functions of the VAR, however, we need to make some identifying assumptions concerning the structural relationships between the variables. A convenient method to identify the dynamic effects on one variable of a shock to another variable in the VAR is to assume that there is a causal ordering between the variables. A reasonable assumption in this particular model is that monetary policy is affected by current values of the output gap and the inflation

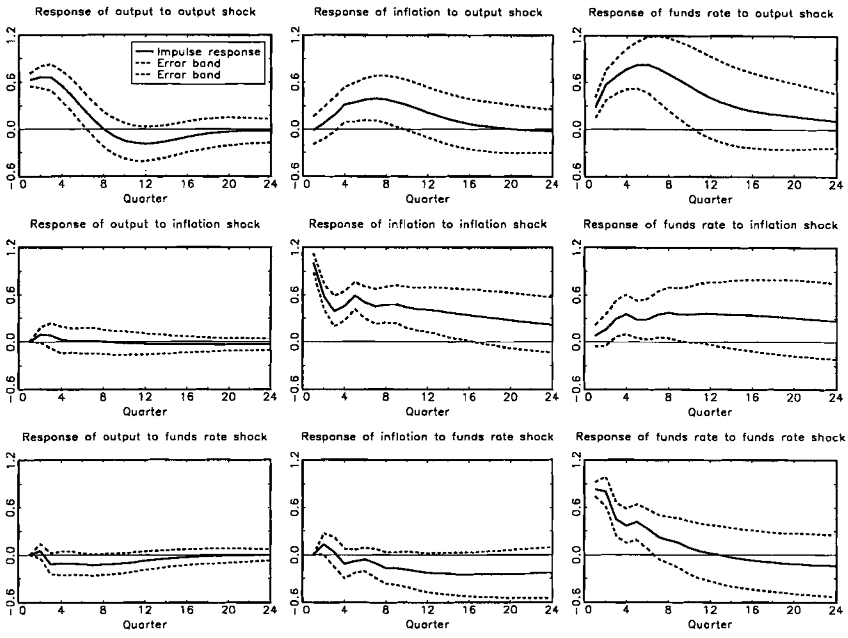

Figure 4.2: Impulse responses from VAR

rate, but that these do not respond to contemporaneous policy.[12] To identify the response of monetary policy to shocks to output and inflation, we need a further assumption, and following Rudebusch and Svensson (1998), Sack (1998a), Bagliano and Favero (1998), and many others, I shall assume that the inflation rate is affected by contemporaneous output, but not vice versa. Consequently, we end up with the ordering (y_t, π_t, i_t), and identification can be achieved through a Choleski decomposition.[13]

The resulting impulse responses are graphed in Figure 4.2 as the response of each variable to a unit shock to an orthogonalized innovation in another variable. The solid line is the estimated impulse response, and the dotted lines are confidence intervals of two standard deviations, calculated with Monte Carlo simulations. The impulse responses are consistent with the conventional view of the monetary

[12]This recursive assumption is very common in the VAR literature, see Christiano et al. (1998) and references therein.

[13]The alternative ordering, with inflation before output, leads to very similar impulse responses.

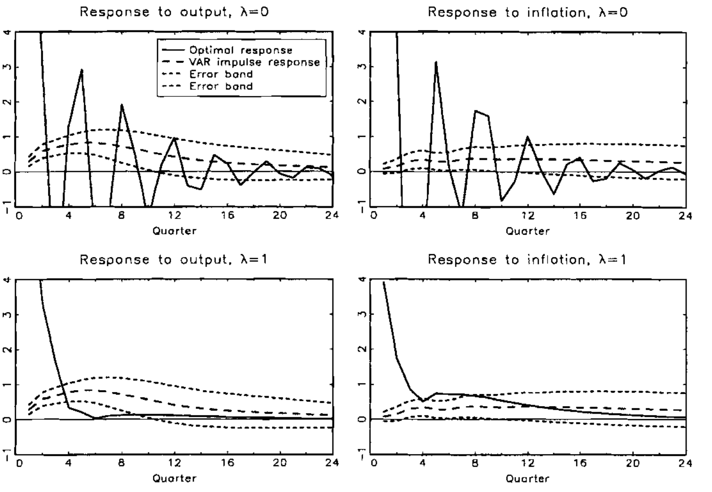

Figure 4.3: Estimated and optimal policy response to shocks, restricted model

transmission mechanism from a number of VAR studies: after a funds rate shock, there is a sustained decline in output and inflation, and output reaches its minimum after four to eight quarters (these responses are not significant, however).[14] The funds rate response to output and inflation shocks is positive and persistent, and significant for the first ten quarters.

Letting the dynamic systems of the restricted and unrestricted models be hit by a shock of comparable size to that in the impulse responses,[15] we can trace the optimal policy response over time and compare it with the empirical impulse responses.[16] Figure 4.3 shows the optimal response of monetary policy in the restricted model along with the empirical impulse responses from the VAR including the two-standard deviation confidence intervals. The two left-hand graphs show the re-

[14]Note that there is a tendency to a 'price puzzle' and an 'output puzzle,' i.e., that inflation and output increase slightly before falling after a monetary contraction (see the third row of columns 1 and 2), indicating that the VAR model is misspecified. The standard method of solving the price puzzle is to include commodity prices in the VAR as a leading indicator of inflation. See Christiano et al. (1998) for a discussion.

[15]A unit shock to the orthogonalized innovations in output and inflation corresponds to a 0.621 shock to output, and a 0.976 shock to inflation, respectively.

[16]Rudebusch and Svensson (1998) calculate the impulse response using the dynamics of their estimated restricted model, but including an estimated reaction function from a VAR model. Since the resulting impulse responses are not far from those of the VAR, they conclude that the model restrictions do not significantly alter the dynamics of the model relative to the unrestricted VAR.

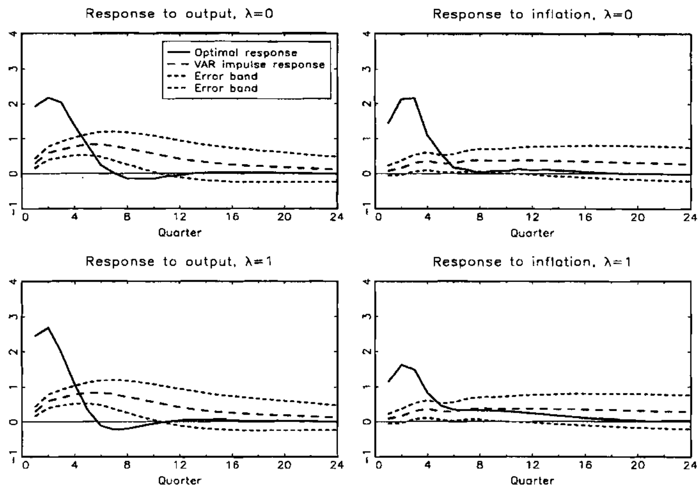

Figure 4.4: Estimated and optimal policy response to shocks, unrestricted model

sponse to output shocks and the right-hand graphs the response to inflation shocks, with 'strict inflation targeting' ($\lambda = 0$) at the top and 'flexible inflation targeting' ($\lambda = 1$) at the bottom.

In the top row, where $\lambda = 0$, the restricted model implies an extremely volatile response to shocks, with large fluctuations in the central bank instrument.[17] When $\lambda = 1$, the response is less volatile, and more reasonable. Still, the initial response to both output and inflation shocks is very strong, whereas the impulse response is weak at first, and then increases somewhat before reverting back to zero. Consequently, the restricted Svensson model leads to substantially more aggressive policy behavior than what seems to be observed in practice.

Figure 4.4 shows the optimal response from the unrestricted model. As was clear from the reaction functions, optimal policy is less aggressive in the unrestricted model, and the response is much closer to the empirical impulse responses, even if the initial response is always too aggressive. As compared with the restricted model, the unrestricted policy response is more intuitively attractive, since it implies more interest rate smoothing, in the sense that policy is adjusted in the same direction at least twice before returning towards zero. In the restricted model, at least for

[17]The response in the first periods falls outside the graphs, with the response to output being 12.48 in the first period and -4.04 in the third period, and the response to inflation being 16.01 and -7.43, respectively.

$\lambda = 1$, it is always optimal to make a large initial adjustment and then quickly return towards zero.

From this experiment, we can conclude that the restrictions of the Svensson model have serious counterfactual implications not only for the coefficients of the optimal reaction function, but also for the path of monetary policy over time. Still, however, we are far away from a reasonable model of monetary policy, since the unrestricted model also implies considerably more interest rate volatility than what is empirically observed. In an attempt to add some realistic features to the models, the next section will evaluate the consequences of multiplicative parameter uncertainty for the optimal response of policy.

4.4 Parameter uncertainty

The assumption of additive uncertainty in macroeconomic modeling is very convenient when deriving optimal policy rules, since, coupled with a quadratic loss function, the optimal policy rule depends only on the first moments of the goal variables (so 'certainty equivalence' holds). It has long been known that multiplicative uncertainty, for example uncertainty about the parameters in a model, has important implications for the optimal behavior of policymakers. The analysis of Brainard (1967) shows that a policymaker who is uncertain about the multiplier of policy should be less aggressive in his policy moves, at least if covariances are small.[18] This result has recently been stressed by Blinder (1997, 1998) and Goodhart (1998) as having a major relevance for practical policymaking within the Federal Reserve and the Bank of England. Also, Sack (1998a) has shown that allowing for parameter uncertainty makes the optimal policy path from a standard unrestricted VAR model very similar to the actual path of Federal Reserve policy.[19]

One can think of a number of reasons why policymakers are not certain about the

[18]Contributions by Craine (1979) and Söderström (1999) show that the Brainard result does not apply to all types of multiplicative parameter uncertainty: uncertainty about the impact of policy leads to less aggressive policy, whereas uncertainty about the adjustment dynamics of the economy leads to more aggressive policy than under certainty equivalence.

[19]Apart from uncertainty about model parameters, one can also imagine other sources of uncertainty that complicate the policymaker's situation. Rudebusch (1998) investigates the effects of several sources of uncertainty in the same model framework: multiplicative parameter uncertainty, uncertainty about the quality of incoming data, and uncertainty about the means of parameters. In doing so, he does not use the standard methods of dynamic optimization, but instead simulates the economy a number of times for each configuration of decision rules to find the optimal restricted Taylor rule. This method is more flexible than the optimization techniques used in this chapter, but also more time-demanding.

parameters in a model of the economy (see, e.g., Holly and Hughes Hallett, 1989). Parameters could be genuinely random, as agents adjust their behavior over time. The source of such randomness would then need to be found in more complete models of, for example, price-setting and investment behavior, that is, in the underlying equations of a reduced-form system. Alternatively, the parameters could be fixed in reality, but estimated by policymakers over finite samples, thus leading to randomness in point estimates. Finally, the model could be a linear approximation of a non-linear model, so that parameters vary in a well-defined but imperfectly known manner.

In this section, the second type of parameter uncertainty will be assumed, so the parameter matrices A and B vary stochastically over time, with known means and variances, but I disregard issues of learning and experimentation by assuming that the realizations of parameters are drawn from the same known distribution over time.[20] Consequently, I will continue to use the econometric estimates from Table 4.1, which were obtained assuming that parameters are non-stochastic.

The state-space formulation of the general model is then

$$x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + \varepsilon_{t+1}, \tag{4.17}$$

where A_{t+1} and B_{t+1} are stochastic, with means A and B, variance matrices Σ_A and Σ_B, and covariance matrix Σ_{AB}. It is assumed that all parameters are independent of each other, so in the unrestricted model Σ_{AB} is zero, whereas in the restricted model, it is non-zero, since $B_1^y = -C_1^y$.

The central bank faces the same control problem

$$J(x_t) = \min_{i_t} \{x_t'Qx_t + \phi E_t J(x_{t+1})\} \tag{4.18}$$

but now subject to (4.17), leading to the policy rule

$$i_t = \tilde{f}x_t. \tag{4.19}$$

Now, however, the reaction function depends not only on the parameter means, but also on their variances. Appendix 4.B shows that the solution to the central bank's problem is given by

$$\begin{aligned} \tilde{f} &= -\left[B'(\tilde{V}+\tilde{V}')B + 2\tilde{v}_{11}\Sigma_B^{11} + 2\tilde{v}_{55}\Sigma_B^{55}\right]^{-1} \\ &\times \left[B'(\tilde{V}+\tilde{V}')A + 2\tilde{v}_{11}\Sigma_{AB}^{11}{}'\right], \end{aligned} \tag{4.20}$$

[20]See Sack (1998b) or Wieland (1998) for analyses of learning and experimentation in models of monetary policy. However, to quote former Vice-Chairman of the Federal Reserve Board, Alan Blinder (1998, p. 11), "You don't conduct experiments on a real economy solely to sharpen your econometric estimates." See also Sargent (1998) for a discussion of this issue.

Table 4.5: Optimal reaction functions under parameter uncertainty

	Restricted		Unrestricted		Empirical
	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	
y_t	4.801	4.615	1.288	1.339	0.465
y_{t-1}	−0.661	−0.679	−0.106	−0.149	−0.005
y_{t-2}	−0.826	−0.871	−0.251	−0.267	−0.146
y_{t-3}	−0.372	−0.329	−0.107	−0.108	0.046
y_{t-4}					−0.069
π_t	2.377	1.601	0.565	0.510	0.086
π_{t-1}	0.795	0.454	0.214	0.159	−0.010
π_{t-2}	0.419	0.119	0.234	0.206	0.122
π_{t-3}	0.414	0.296	0.151	0.148	0.003
π_{t-4}					−0.102
i_{t-1}	−0.011	−0.006	−0.191	−0.167	0.929
i_{t-2}	0.311	0.312	0.233	0.237	−0.291
i_{t-3}	−0.120	−0.120	−0.078	−0.085	0.290
i_{t-4}					−0.007

Optimal reaction function (the vector $\tilde{f}$ in equation (4.19)) from restricted and unrestricted models under multiplicative parameter uncertainty, and estimated empirical reaction function from Table 4.1.

where

$$\begin{aligned}\tilde{V} &= Q + \phi(A + B\tilde{f})'\tilde{V}(A + B\tilde{f}) \\ &+ \phi\tilde{v}_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11}\tilde{f} + \tilde{f}'\Sigma_B^{11}\tilde{f}\right) + \phi\tilde{v}_{55}\left(\Sigma_A^{55} + \tilde{f}'\Sigma_B^{55}\tilde{f}\right),\end{aligned} \tag{4.21}$$

and where Σ_{AB}^{ij} is the covariance matrix of the ith row of A with the jth row of B.

Using the estimated parameter standard errors from the different models as a measure of the uncertainty concerning individual parameters, but assuming all covariances across parameters to be zero, we can plug in the parameter mean and variance estimates from Table 4.1 into the modified reaction function (4.19)–(4.21). The resulting reaction functions are given in Table 4.5. Comparing with the certainty equivalence case in Table 4.4, the coefficients under multiplicative parameter uncertainty are considerably smaller, leading to less aggressive policy, following the Brainard intuition. Policy is still more aggressive in the restricted than in the unrestricted model, which, in turn, is more aggressive than the empirical policy behavior.

The optimal responses of policy over time under parameter uncertainty are shown in Figures 4.5 and 4.6. In the restricted model of Figure 4.5, parameter uncertainty makes optimal policy much less volatile in response to a shock, especially for the case where $\lambda = 0$. At least for the first periods, however, the optimal response is

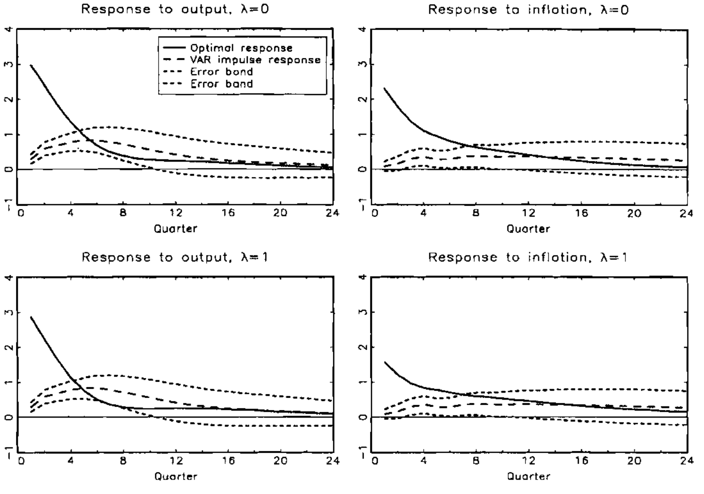

Figure 4.5: Estimated and optimal policy response to shocks in restricted model under parameter uncertainty

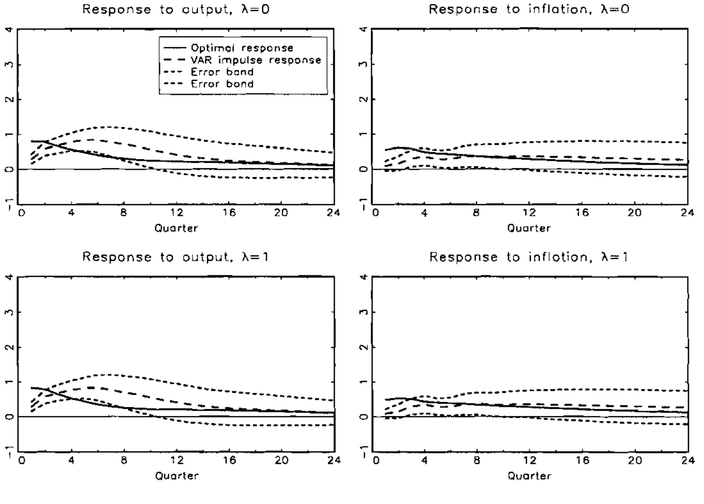

Figure 4.6: Estimated and optimal policy response to shocks in unrestricted model under parameter uncertainty

considerably stronger than the empirical impulse response. The unrestricted model in Figure 4.6 is also less volatile than under certainty equivalence, and now implies an optimal response which is very similar to the observed response. The optimal response lies outside the confidence bands of the impulse response functions only during the first periods; in later periods, it is very close to the observed behavior.

Consequently, taking parameter uncertainty into account, at least in this configuration of uncertainty, leads to less aggressive policy for both models.[21] The unrestricted policy response is now very close to the empirically observed response, whereas the restricted model still implies too aggressive behavior as compared with the empirical impulse responses.

4.5 The implied path of the funds rate

As a final experiment, we can calculate the implied optimal path of the federal funds rate over the sample period by applying the different reaction functions to the actual data for the U.S. economy. Comparing the resulting path with the actual path of the funds rate gives a further illustration of the results of previous sections.

Letting the central bank respond in an 'optimal' manner to output, inflation, and past values of the funds rate, assuming that the weights of output and inflation in the loss function are equal (so $\lambda = 1$), the implied paths of the funds rate from 1959 to 1998 are shown in Figure 4.7. The two top graphs show the implied paths from the restricted and unrestricted models under certainty equivalence and the actual funds rate path, and the two bottom graphs show the paths under parameter uncertainty. The standard deviations of the funds rate in the different models and in the actual path are shown in Table 4.6, along with the mean squared deviation of the optimal path from the actual funds rate path.

It is immediately clear, from both Figure 4.7 and Table 4.6, that the restricted model implies considerably more interest rate volatility than the unrestricted model, especially in the certainty equivalence case (note that the scales on the vertical axes in Figure 4.7 differ across graphs).[22] The unrestricted model under certainty equivalence and the restricted model under parameter uncertainty are remarkably

[21] In some configurations of uncertainty in the restricted model, with much emphasis on uncertainty concerning the B_s^{π}-coefficients, the optimal policy under parameter uncertainty is actually *more* aggressive than under certainty equivalence. See Söderström (1999) for a discussion of this result within a simpler one-lag version of the Svensson model.

[22] A serious flaw of the methodology applied is also obvious from Figure 4.7: it allows for negative values of the nominal interest rate. For models taking the zero-bound on nominal interest rates into account, see, e.g., Fuhrer and Madigan (1997) or Orphanides and Wieland (1998).

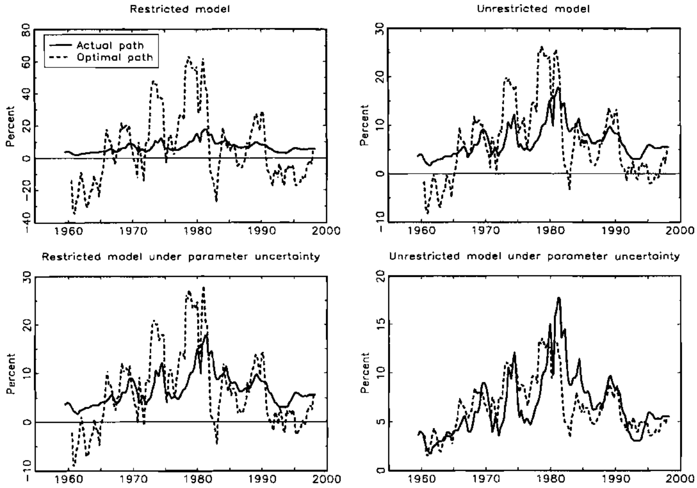

Figure 4.7: Actual and optimal interest rate paths, 1959–98

similar in their policy paths, although they are far from the actual path. The only reasonable approximation of the true policy path comes from the unrestricted model under parameter uncertainty. As seen in the bottom right-hand graph of Figure 4.7, the implied optimal path of policy is very similar to the actual path; according to Table 4.6, optimal policy is even less volatile than actual policy, although it has a tendency to lead actual policy in the response to macroeconomic developments.

It is remarkable how close we can get to mimicking the actual behavior of the Federal Reserve by introducing parameter uncertainty into an unrestricted optimizing model, without including an interest rate smoothing objective into the central bank's loss function. As noted by Sack (1998a), such an assumption of interest rate smoothing does not seem to be warranted solely because of the apparent propensity of central banks to smooth their interest rate instrument. Instead, such behavior can equally plausibly be the result of simple optimizing behavior of the central bank, taking into account the dynamic properties of the economy and the effects of uncertainty on policy.

4.6 Final remarks

The results of this chapter indicate that the restrictions introduced in the simple macroeconomic model of Svensson (1997a,b) are responsible for the model's failure

Table 4.6: Comparison of optimal and actual funds rate paths

	Certainty equivalence		Parameter uncertainty
Standard deviation: levels			
Actual		3.265	
Restricted model	22.426		8.158
Unrestricted model	7.998		2.801
Standard deviation: differences			
Actual		1.058	
Restricted model	9.201		3.569
Unrestricted model	2.899		1.018
Mean squared deviation from actual			
Restricted model	415.116		41.914
Unrestricted model	38.484		6.347

Standard deviations of optimal and actual funds rate and mean squared deviations of optimal from actual funds rate. In the derivation of the optimal funds rate, $\lambda = 1$.

to match the observed policy behavior. The coefficients of the optimal decision rule are considerably larger than those of empirical reaction functions, leading to excessive interest rate variability in response to shocks. In contrast, an unrestricted VAR model leads to less volatility in the policy instrument, and, taking parameter uncertainty into account, policy predictions which are very close to the observed behavior of the Federal Reserve, as suggested by Sack (1998a).

From the formal hypothesis tests, the restriction on the output equation seems more at fault than that on the inflation equation. However, additional experiments indicate that the extreme interest rate volatility emanates from the inflation restrictions rather than the output restrictions. In any case, more work on the exact specification seems warranted if one is to come up with a model that better fits the empirical facts.

4.A Solving the control problem

From equation (4.13), the central bank solves the problem

$$J(x_t) = \min_{i_t} \{x_t' Q x_t + \phi E_t J(x_{t+1})\} \tag{4.22}$$

subject to

$$x_{t+1} = A x_t + B i_t + \varepsilon_{t+1}. \tag{4.23}$$

Since the objective function is quadratic and the constraint linear, the value function will be of the form

$$J(x_t) = x_t' V x_t + w. \tag{4.24}$$

Using the transition law to eliminate the next period's state, the Bellman equation is

$$x_t' V x_t + w = \min_{i_t} \{x_t' Q x_t + \phi(A x_t + B i_t)' V (A x_t + B i_t) + \phi w\}. \tag{4.25}$$

The first-order condition for the minimization problem is then[23]

$$B' V B i_t = -B' V A x_t, \tag{4.26}$$

leading to the optimal interest rate

$$\begin{aligned} i_t &= -\left(B' V B\right)^{-1} B' V A x_t \\ &= f x_t. \end{aligned} \tag{4.27}$$

Substituting the decision rule into the Bellman equation (4.25), we get

$$\begin{aligned} x_t' V x_t + w &= x_t' Q x_t + \phi\left[(A x_t + B f x_t)' V (A x_t + B f x_t) + w\right] \\ &= x_t' \left[Q + \phi(A + Bf)' V (A + Bf)\right] x_t + \phi w. \end{aligned} \tag{4.28}$$

Thus V is determined by the Ricatti equation

$$V = Q + \phi(A + Bf)' V (A + Bf), \tag{4.29}$$

where

$$f = -\left(B' V B\right)^{-1} B' V A. \tag{4.30}$$

[23]Use the rules $\partial x' A x / \partial x = (A + A')x$, $\partial y' B z / \partial y = Bz$, and $\partial y' B z / \partial z = B' y$, and the fact that V is symmetric. See, e.g., Ljungqvist and Sargent (1997).

4.B The stochastic control problem

From (4.18), the bank's problem under parameter uncertainty is

$$J(x_t) = \min_{i_t} \{x_t' Q x_t + \phi E_t J(x_{t+1})\} \tag{4.31}$$

subject to

$$x_{t+1} = A_{t+1} x_t + B_{t+1} i_t + \varepsilon_{t+1}. \tag{4.32}$$

The value function will still be

$$J(x_t) = x_t' \tilde{V} x_t + \tilde{w}, \tag{4.33}$$

but now with expected value

$$E_t J(x_{t+1}) = (E_t x_{t+1})' \tilde{V} (E_t x_{t+1}) + \operatorname{tr}\left(\tilde{V} \Sigma_{t+1|t}\right) + \tilde{w}, \tag{4.34}$$

where the expected value of x_{t+1} is given by

$$E_t x_{t+1} = A x_t + B i_t, \tag{4.35}$$

and where $\Sigma_{t+1|t}$ is the covariance matrix of x_{t+1}, evaluated at t, and 'tr' denotes the trace operator.

Following Holly and Hughes Hallet (1989), the (i, j)th element of $\Sigma_{t+1|t}$ is given by

$$\Sigma_{t+1|t}^{ij} = x_t' \Sigma_A^{ij} x_t + 2 x_t' \Sigma_{AB}^{ij} i_t + i_t' \Sigma_B^{ij} i_t + \Sigma_\varepsilon^{ij}, \tag{4.36}$$

where Σ_{AB}^{ij} is the covariance matrix of the ith row of A with the jth row of B. Since at t, y_{t+1} and π_{t+1} are the only stochastic variables in x_{t+1}, and these are assumed independent of each other, the only non-zero entries of $\Sigma_{t+1|t}$ are the matrices $\Sigma_{t+1|t}^{11}$ and $\Sigma_{t+1|t}^{55}$.

The (11×11) matrix Σ_A^{11} has diagonal elements

$$\left[\begin{array}{ccccccccccc} \sigma^2_{A_1^y} & \sigma^2_{A_2^y} & \sigma^2_{A_3^y} & \sigma^2_{A_4^y} & \sigma^2_{B_1^y} & \sigma^2_{B_2^y} & \sigma^2_{B_3^y} & \sigma^2_{B_4^y} & \sigma^2_{C_2^y} & \sigma^2_{C_3^y} & \sigma^2_{C_4^y} \end{array}\right], \tag{4.37}$$

and other elements equal to zero, and, likewise, the diagonal of Σ_A^{55} is

$$\left[\begin{array}{ccccccccccc} \sigma^2_{A_1^\pi} & \sigma^2_{A_2^\pi} & \sigma^2_{A_3^\pi} & \sigma^2_{A_4^\pi} & \sigma^2_{B_1^\pi} & \sigma^2_{B_2^\pi} & \sigma^2_{B_3^\pi} & \sigma^2_{B_4^\pi} & \sigma^2_{C_2^\pi} & \sigma^2_{C_3^\pi} & \sigma^2_{C_4^\pi} \end{array}\right]. \tag{4.38}$$

The variances Σ_B^{11} and Σ_B^{55} are simply $\sigma^2_{C_1^y}$ and $\sigma^2_{C_1^\pi}$, and both Σ_{AB}^{11} and Σ_{AB}^{55} are zero in the general setup, assuming parameters are uncorrelated with each other. In the

Svensson model, however, the restriction $B_1^y = -C_1^y$ implies that Σ_{AB}^{11} is an (11×1) vector given by

$$\Sigma_{AB}^{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & -\sigma_{C_1^y}^2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}', \tag{4.39}$$

whereas Σ_{AB}^{55} is still a vector of zeros. Finally, the covariances of the shocks are given by $\Sigma_\varepsilon^{11} = \sigma_y^2$ and $\Sigma_\varepsilon^{55} = \sigma_\pi^2$.

The only non-zero elements of $\Sigma_{t+1|t}$ are then

$$\begin{aligned} \Sigma_{t+1|t}^{11} &= \operatorname{Var}_t(y_{t+1}) \\ &= x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}i_t + i_t'\Sigma_B^{11}i_t + \Sigma_\varepsilon^{11} \end{aligned} \tag{4.40}$$

and

$$\begin{aligned} \Sigma_{t+1|t}^{55} &= \operatorname{Var}_t(\pi_{t+1}) \\ &= x_t'\Sigma_A^{55}x_t + i_t'\Sigma_B^{55}i_t + \Sigma_\varepsilon^{55}. \end{aligned} \tag{4.41}$$

Consequently

$$\begin{aligned} \operatorname{tr}\left(\tilde{V}\Sigma_{t+1|t}\right) &= \tilde{v}_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}i_t + i_t'\Sigma_B^{11}i_t + \Sigma_\varepsilon^{11}\right) \\ &+ \tilde{v}_{55}\left(x_t'\Sigma_A^{55}x_t + i_t'\Sigma_B^{55}i_t + \Sigma_\varepsilon^{55}\right), \end{aligned} \tag{4.42}$$

where $\tilde{v}_{ij}$ is the (i,j)th element of $\tilde{V}$.

Using (4.33)–(4.35) and (4.42) in (4.31), the Bellman equation is

$$\begin{aligned} x_t'\tilde{V}x_t + \tilde{w} &= \min_{i_t}\left\{x_t'Qx_t + \phi(Ax_t + Bi_t)'\tilde{V}(Ax_t + Bi_t)\right. \\ &+ \phi\tilde{v}_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}i_t + i_t'\Sigma_B^{11}i_t + \Sigma_\varepsilon^{11}\right) \\ &+ \left.\phi\tilde{v}_{55}\left(x_t'\Sigma_A^{55}x_t + i_t'\Sigma_B^{55}i_t + \Sigma_\varepsilon^{55}\right) + \phi\tilde{w}\right\}, \end{aligned} \tag{4.43}$$

so the first-order condition is[24]

$$B'(\tilde{V} + \tilde{V}')(Ax_t + Bi_t) + 2\tilde{v}_{11}\left(\Sigma_{AB}^{11}{}'x_t + \Sigma_B^{11}i_t\right) + 2\tilde{v}_{55}\Sigma_B^{55}i_t = 0, \tag{4.44}$$

leading to the optimal interest rate

$$i_t = \tilde{f}x_t, \tag{4.45}$$

[24]Note that in the setup with multiplicative parameter uncertainty, $\tilde{V}$ is not necessarily symmetric.

where

$$\begin{aligned}\tilde{f} &= -\left[B'(\tilde{V}+\tilde{V}')B + 2\tilde{v}_{11}\Sigma_B^{11} + 2\tilde{v}_{55}\Sigma_B^{55}\right]^{-1} \\ &\times \left[B'(\tilde{V}+\tilde{V}')A + 2\tilde{v}_{11}\Sigma_{AB}^{11}{}'\right].\end{aligned} \tag{4.46}$$

Substituting back into the Bellman equation (4.43), we get

$$\begin{aligned}x_t'\tilde{V}x_t + \tilde{w} &= x_t'Qx_t + \phi\left[(Ax_t + B\tilde{f}x_t)'\tilde{V}(Ax_t + B\tilde{f}x_t)\right] \\ &+ \phi\tilde{v}_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}\tilde{f}x_t + x_t'\tilde{f}'\Sigma_B^{11}\tilde{f}x_t + \Sigma_\varepsilon^{11}\right) \\ &+ \phi\tilde{v}_{55}\left(x_t'\Sigma_A^{55}x_t + x_t'\tilde{f}'\Sigma_B^{55}\tilde{f}x_t + \Sigma_\varepsilon^{55}\right) + \phi\tilde{w},\end{aligned} \tag{4.47}$$

and it can be established that $\tilde{V}$ is determined by the Ricatti equation

$$\begin{aligned}\tilde{V} &= Q + \phi(A+B\tilde{f})'\tilde{V}(A+B\tilde{f}) \\ &+ \phi\tilde{v}_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11}\tilde{f} + \tilde{f}'\Sigma_B^{11}\tilde{f}\right) + \phi\tilde{v}_{55}\left(\Sigma_A^{55} + \tilde{f}'\Sigma_B^{55}\tilde{f}\right).\end{aligned} \tag{4.48}$$

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