

Modelling macroeconomic time series
with smooth transition autoregressions



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Modelling macroeconomic time series with smooth transition autoregressions

Joakim Skalin



STOCKHOLM SCHOOL OF ECONOMICS
EFI, THE ECONOMIC RESEARCH INSTITUTE

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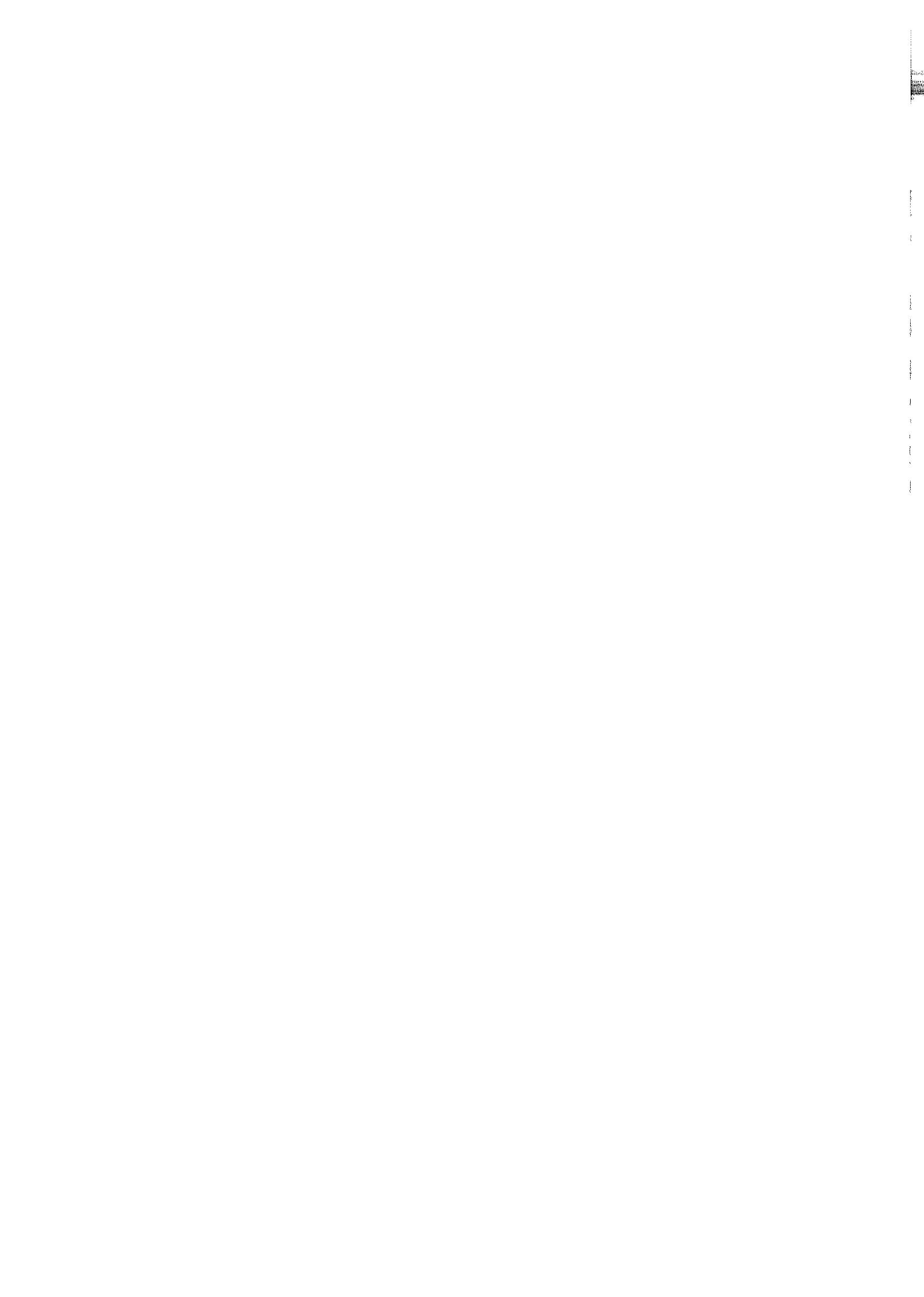
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INTRODUCTION AND SUMMARY

Background

The use of nonlinear parametric statistical models in time series analysis and econometrics has attracted attention among both theoretical and applied econometricians for some time. Recently, new models and new methods for inference have been developed and the decreased costs of computation have made an increasing number of these models feasible for practical application. Thus given increased access to nonlinear models, the practitioner faces new problems in model selection and specification.

Once the possibility of leaving the linear framework is considered, the number of different models available is virtually infinite and economic theory often of little help when it comes to the detailed specification of the model. For example, economic theory often has little to say about the dynamics. Despite the developments in numerical estimation methods, estimating a nonlinear model is always at least as complicated as estimating a linear one. This favours choosing a linear model. Moreover, if the true data generating process is indeed linear, estimation is sometimes not even statistically feasible since the nonlinear model may not be identified. A linear model may therefore be preferred if the variable or relationship under study can be satisfactorily characterised using a linear statistical model, at least as a local

approximation, even in cases where theory may support a nonlinear functional form. For these reasons, linearity should be tested before any nonlinear modelling is attempted.

Among the parametric nonlinear time series model families, the smooth transition regression (STR) model has recently received attention in the literature. The considerations in this dissertation focus on the univariate special case of this model, the smooth transition autoregression (STAR) model, although large parts of the discussion can be easily generalised to the more general STR case. Many nonlinear univariate time series models can be described as consisting of a number of regimes, each one corresponding to a linear autoregressive parametrisation, between which the process switches. In STAR models, as opposed to certain other popular models involving multiple regimes, the transition between the extreme regimes is smooth and assumed to be characterised by a bounded continuous function of a transition variable. The transition variable, in turn, may be a lagged value of the variable in the model, or another stochastic or deterministic observable variable. A number of other commonly discussed nonlinear autoregressive models can be viewed as special or limiting cases of the STAR model. The applications presented in the first two chapters of this dissertation make use of certain desirable properties of STAR models that will be further discussed below.

As noted above, it is important to test the null hypothesis of linearity against a nonlinear alternative before specifying and estimating a nonlinear model. One of the advantages of using STAR models is that a modelling cycle comprising linearity testing, model specification, estimation, and model evaluation has been developed,

studied and applied in the literature. The steps of this modelling approach are presented, for univariate models, by Teräsvirta (1994) and Eitrheim and Teräsvirta (1996). A recent survey focusing on STR models and including an application and references to others can be found in Teräsvirta (1998).

Teräsvirta (1994) and Eitrheim and Teräsvirta (1996) thoroughly discuss the modelling process up to and including the evaluation of the estimated model by means of various misspecification tests, but they provide a somewhat less comprehensive discussion of different means of describing, illustrating and interpreting estimated STAR models and their dynamic properties. Since only a few of the estimated parameters in these models are usually of interest *per se*, these issues are important in applied modelling. The choice of methods used to present, interpret, and discuss an estimated model and its properties is at least partly dependent on what dynamic aspects of the estimated model the investigator is primarily interested in. The two studies in applied univariate time series modelling forming the two first chapters of this dissertation are concerned with these issues. In these two studies, STAR models are used to provide insights into dynamic properties of the time series which cannot be properly characterised by linear time series models, and which thereby may be obscured by estimating only a linear model in cases where linearity would be rejected if tested. The applications being of interest in their own right, an important common objective of these two chapters is also to develop, suggest, and give examples of various methods that may be of use in discussing the dynamic properties of estimated STAR models in general. The third chapter reports the results of a small simulation

study considering a new test of linearity against STAR based on bootstrap methodology.

The rest of this introduction contains a summary of the contents and main results of the three chapters in the dissertation.

Chapter I: Business cycles in Swedish macroeconomic time series, 1861-1988

The first chapter discusses cycles in long macroeconomic time series. The data consists of approximately 130 years of annual observations on a set of key time series describing the Swedish economy from the mid-nineteenth century to present time. While these series have previously been examined by use of conventional linear univariate time series techniques (see Englund, Persson and Svensson 1992), the chapter sets out to investigate whether it is possible to gain further insight into the existence and nature of business cycles in these time series through the application of nonlinear modelling. Specifically, it is suggested that a STAR model, that can be interpreted as a set of locally linear autoregressive models representing different dynamic regimes, may be able of giving a somewhat more detailed picture of the business cycle properties of these long series than a single linear model estimated for this extended and rather turbulent period of economic history as a whole. The issue of asymmetry in the business cycle is also discussed.

When the null hypothesis of linearity is tested against a STAR specification for the first differences of the logarithmic series, it is rejected for seven out of the nine

variables investigated. For these seven series, the detailed results from specifying, estimating and evaluating STAR models are reported and the dynamic properties of the estimated models are discussed in the first chapter. This discussion is supported in part by new techniques, including the so-called 'sliced' spectrum which provides an overview of the varying local nonexplosive dynamics of the STAR model. The impact of shocks on the process is illustrated by generalised impulse-response (GIR) functions (see Koop, Pesaran and Potter, 1996), and the estimated GIR values for a set of time horizons are visualised using highest density region graphs (see Hyndman 1995 and 1996).

Interactions between series are also of interest from the business cycle point of view. Although the time series analysis in the chapter is basically univariate, the consequences of nonlinear modelling for the concept of Granger-causality is also discussed. The fact that linearity is rejected for an individual time series suggests that a simple test of Granger-noncausality between two series should be performed within the chosen nonlinear framework instead of within a linear one. A test of (pairwise) Granger-noncausality based on a STAR model is therefore developed and applied, and the results are compared to those obtained from customary linear pairwise Granger-noncausality tests.

According to the analysis of the estimated STAR models, cyclical variation at business cycle frequencies does not seem to be constant over time for all series, and it is difficult to find a single 'Swedish business cycle'. Generally the estimated nonlinear models reveal more of individual behaviour in the different series than would be discerned using only linear models for the whole period. In some series the cyclical

behaviour is weak, in some it is pronounced, in others yet the cyclicity varies over time. Only some of the series may be regarded as having genuinely asymmetric cyclical variation. The results of the pairwise Granger noncausality tests suggest the existence of strong temporal interactions between series and indicate that the assumption of functional form (linear or STAR) strongly affects the outcome of these tests.

Chapter II: Asymmetries and moving equilibria in unemployment rate time series

In the second chapter, unemployment rates for a number of OECD countries are studied using nonlinear univariate time series models. Persistence being a key issue in the discussion of unemployment series of many industrialised countries, empirical studies involving these series frequently start with applying a unit root test to the series. The unit root hypothesis is most often not rejected (see, e.g., Røed, 1997). However, since the unemployment rate is bounded, a unit root representation can only be seen as an approximate description of these series. There are also other implications of linear models that may make them less well suited for unemployment rates. One important stylised fact frequently observed in OECD unemployment rate series is that the increases tend to be more rapid than the decreases. This observation may explain the fact that the unit root hypothesis is frequently not rejected, but since linear univariate time series models preclude asymmetric behaviour, it is suggested in this chapter that the unemployment rate series could be regarded as generated by stationary but possibly nonlinear processes permitting asymmetric realisations. If the obser-

ved series are asymmetric and thus nonlinear, the analysis of persistence should preferably be carried out within a model that is capable of characterising such asymmetry.

The linearity of the investigated series is therefore tested against a nonlinear alternative. The alternative model can be viewed as an unemployment rate level model, parametrised as a variant of the standard logistic STAR model for the first difference of the unemployment rate. A lagged seasonal difference of the unemployment rate is used for transition variable and a lagged level term is included. This makes the model capable of generating and describing asymmetries of the type often found in empirical unemployment data. The behaviour of the suggested model is first discussed using an artificial example. The performance of standard unit root tests when applied to realisations from this nonlinear model is investigated through a small simulation study.

Turning to the modelling of real unemployment series, linearity is again first tested. If it is rejected, the customary modelling cycle consisting of specification, estimation, and misspecification testing, is adopted to the present special form of STAR model. Since seasonally unadjusted quarterly data are used, and since the issue of changes in seasonal unemployment patterns over time as well as with the business cycle has received attention in the literature, the tests of constant parameters are particularly interesting in this application.

Linearity is rejected in favour of the STAR-type asymmetric alternative for a group of series mainly comprising European countries, whereas the tests for the US, Canadian,

Japanese, and Norwegian series do not reject linearity. For a number of the series that are deemed nonlinear, models of the form outlined above are successfully estimated, in some cases after including one or more variable parameters.

The dynamic properties of the estimated models are illustrated in various ways, and the issues of asymmetry and persistence are discussed. Indeed, some of the estimated models exhibit pronounced asymmetries combined with persistence. The conclusions regarding persistence thus accord with those from a linear analysis, but the nonlinear model also allows the asymmetry to be accounted for. The analyses starting with unit root testing usually do not consider asymmetry as an alternative.

Finally, the series for which symmetry (linearity) cannot be rejected (or meaningful STAR models cannot be estimated) are reconsidered by testing linearity against a simplified STAR model. The alternative model in this test is capable of describing moving equilibria in unemployment rates by allowing level and seasonal parameters to vary as smooth functions of time. Linearity is rejected for all but the US series against this alternative, and nonlinear models for these countries are specified, estimated, and evaluated. The results regarding the moving levels are found to be largely comparable to those obtained by other authors using Markov-switching models for this purpose, but the models presented here also explicitly take the possibility of varying seasonality into account.

Chapter III: Bootstrap linearity testing

This chapter considers the following statistical problem. When testing the null hypothesis of linearity of a time series against a STAR alternative, standard asymptotic distribution results for the classical test statistics do not apply since nuisance parameters in the model are unidentified under the null hypothesis. Several solutions to this problem, which appears in a number of other testing situations as well, have been suggested in the literature. The derivation of a commonly applied test of linearity against STAR, due to Luukkonen, Saikkonen and Teräsvirta (1988), circumvents the identification problem by an appropriate linearisation. More specifically, the transition function is replaced by a low-order Taylor series approximation around the null hypothesis. This linearisation can be viewed as an approximation of the likelihood function.

Although this test has been shown to possess good power properties in previous simulation studies, the power of the test may be a cause for concern since information about the nonlinear structure under the alternative is lost through the approximation. Therefore, it may be of interest to consider other tests and compare their performance to the linearisation based one. The successful candidate should have at least as good power properties as the prevailing test, and the computational costs should be reasonable. A number of alternative test procedures have been suggested in the literature, but while not guaranteeing superior power, some of them are computationally very demanding. Recent developments in bootstrap testing, prompted by an expanding theoretical literature and increased access to inexpensive computer power, make it

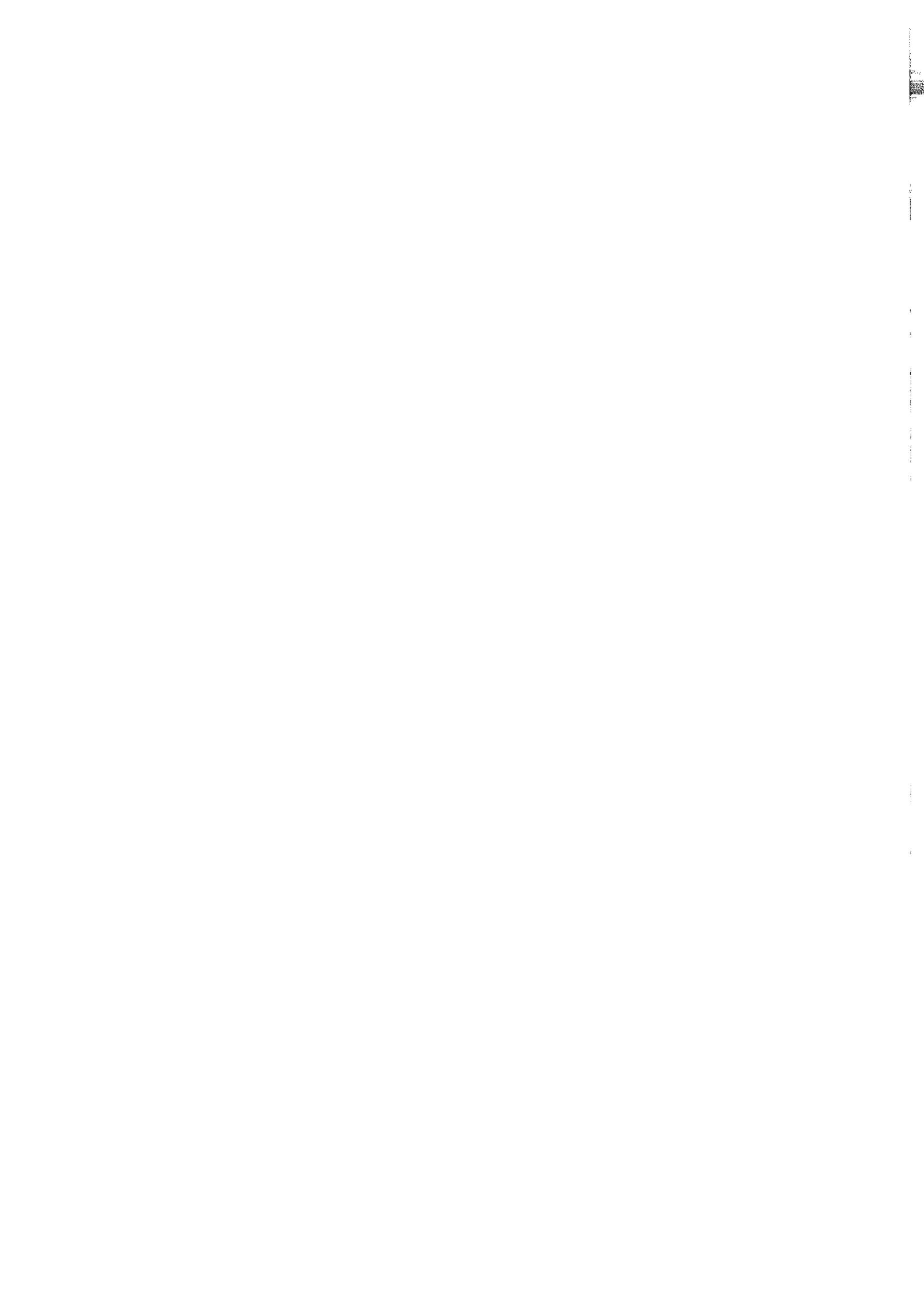
natural to consider a bootstrap test procedure for this situation. Applying a bootstrap test to the linearity testing problem can be seen as replacing the approximation of the likelihood function discussed above with a numerical approximation of the finite-sample distribution of the test statistic.

In this chapter a bootstrap test of linearity against STAR is presented, and its size and power properties are studied and compared to those of the Taylor series approximation based test by a simulation experiment. Since the bootstrap test itself involves repeated instances of numerical estimation, a simulation study comprising a large number of replicates becomes very time consuming. The present simulation study is therefore limited in both size and scope. Thus only a few data generating processes and a small number of replicates are used. The results suggest that the bootstrap test is well-sized but generally less powerful than the test based on the Taylor series approximation. The simulation results regarding the latter test accord with those obtained in previous studies.

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CHAPTER I

ANOTHER LOOK AT SWEDISH BUSINESS CYCLES, 1861-1988

1. INTRODUCTION

In a recent paper, Englund, Persson and Svensson (1992), henceforth referred to as EPS, studied Swedish business cycles ('cyclical comovements between important macroeconomic variables with periods of around five years') using nine long Swedish macroeconomic time series extending from 1861 to 1988. After defining business cycle variation as that found in the spectrum of a stationary series between frequencies corresponding to a cycle length (period) of 3 to 8 years the authors filtered the series to achieve stationarity, estimated the spectral density and applied a band-pass filter (Priestley, 1981, p. 274-275) to remove all variation at frequencies other than those of interest. An inverse transformation was applied to transform the filtered series back to the time domain. Having done that, the authors computed contemporaneous correlations between GDP and the other series and observed that most of them were positive. The same analysis performed for moving subperiods indicated that the correlations were reasonably stable over time. EPS also computed the standard deviations of the filtered series over time and noted that the ratios of the standard deviations of the individual series to that of the GDP had been fairly constant over time. The main

conclusion of EPS was that 'the Swedish business cycle seems to be uniform across very different epochs of Swedish economic history'.

Even after the findings of EPS, there may still be interesting issues to consider using the same macroeconomic data set. First, although EPS concluded that the Swedish business cycle has been uniform over time it also appears from the paper that the estimated spectra of the differenced logarithmic series at 'business cycle frequencies' corresponding to 3 to 8 years are not alike. The cyclical comovements appearing in the definition of business cycles thus do not seem to occur in complete harmony, and it would be interesting to know more about differences between them. Second, the estimated spectra in EPS contain another interesting feature: a fairly marked peak at some frequency corresponding to a period longer than 10 years. Since this peak lies outside the band of business cycle frequencies, EPS did not discuss it in detail. Nevertheless, its prominence makes it an interesting object of study. Third, the issue of business cycle asymmetry has received considerable attention in the literature (see, for example, Mitchell, 1927, Neftçi, 1984, Stock, 1987, Luukkonen and Teräsvirta, 1991, Sichel, 1993, and references therein). The methodology applied in this paper provides an opportunity to consider the possible asymmetry in the Swedish business cycle. Finally, a discussion of temporal dynamic relationships between the variables was beyond the scope of the EPS paper. Causal links between variables, such as the existence or otherwise of a driving variable for the cycle, are an interesting object of study. In this paper this aspect is investigated using Granger-causality as the econometric tool, and as most of the series turn out to be nonlinear the standard non-causality test has to be extended to cover this situation.

Long macroeconomic time series have been rather extensively analysed in the econometric literature. One interesting issue has been to investigate if such long series have a unit root as opposed to the alternative that the series are stationary around a linear trend (see Nelson and Plosser, 1982). When the alternative has been generalised to allow a single structural break in the trend at a known or unknown point (see, e.g. Perron, 1989, Raj, 1992, Zivot and Andrews, 1992) several unit root results have been reversed. For yet another alternative to the unit root hypothesis, see McCabe and Tremayne (1995). The Swedish macroeconomic time series considered by EPS are almost 130 years long and extend through a rather turbulent period in history. Allowing just a single break in a linear trend in 130 years may seem a rather restrictive assumption. This study therefore follows Teräsvirta (1995) by assuming that after taking first differences, the series are stationary and ergodic but may be nonlinear. This extension considerably increases the flexibility in modelling the series. Furthermore, nonlinearity does not seem an implausible assumption in view of all the dramatic events that have affected the Swedish economy, such as the two world wars and the Great Depression of the 1930s. Although Sweden was not a belligerent country in these wars, it was and still is a small open economy, dependent on foreign trade. These events thus represented major exogenous shocks to the economy, and the response to those shocks may have been nonlinear. Therefore the hypothesis of linearity is tested against a nonlinear alternative. In cases where the null hypothesis is rejected, nonlinear models are estimated for the series, the results are interpreted, and the properties of the estimated models are compared. The estimated models also form a basis for tests of Granger noncausality.

Instead of assuming that very large shocks to the Swedish economy have had nonlinear dynamic effects it is possible to view them as single isolated events. The response of the economy to such shocks might then lack a systematic pattern. As a result, unusual observations could rather be taken as outliers. In the following, this view is not adopted, an important reason being that unusual observations may be quite informative about causal relationships between the variables. If these observations are treated as outliers in each series separately the results of Granger noncausality tests may be distorted.

The results discussed in detail in the following sections indicate that most of the nine time series are nonlinear. The estimated nonlinear models suggest that the dynamic properties of the series vary considerably from one series to the other. Granger noncausality tests indicate strong temporal links between many of the series. The plan of the paper is as follows. Section 2 presents the data set. Section 3 introduces the main econometric tool, the smooth transition autoregressive (STAR) model, and testing linearity against STAR. In Section 4 the results of fitting STAR models to the data are reported, the estimated models interpreted and the findings discussed. Section 5 is devoted to testing Granger noncausality in a nonlinear framework and considering the results. Section 6 concludes.

2. DATA

The variables under study in this paper are the same as in EPS: gross domestic product, industrial production (value added in manufacturing and mining), private consumption, investment, exports, imports, employment (hours worked in manufacturing and mining), real wages, and productivity (industrial production divided by hours worked). For most of the variables, data is available for the period 1861-1988. The productivity and employment series start from 1870. Brief definitions of the variables and a discussion of the quality of the data are provided in EPS, whereas the raw data, detailed definitions and a description of the primary data sources can be found in Hassler, Lundvik, Persson and Söderlind (1992).¹ The logarithmic series appear in Figure 2.1 and their first differences in Figure 2.2. All series have a positive trend, but the trend in employment levels off and starts bending downwards in the 1960s. Some series like exports and wages contain remarkably large perturbations as seen from Figure 2.2. In general, the period between 1910 and 1950 has been more turbulent than the remaining parts in most of the series. An exception to this rule is the investment series in which the largest fluctuations occur in the beginning, from 1861 to 1875. Industrial production also shows large fluctuations already in the 1880s.

A brief characterisation of the economic development during the period may be helpful in reading the graphs. The sample contains subperiods characterised by markedly different economic conditions. The years between 1875 and 1890 constitute a defla-

¹ Most of the series used in this paper have been taken directly from the data set in Hassler, Lundvik, Persson and Söderlind (1992). Some of the series have been computed by the present author from the same raw data in accordance with the definitions provided in EPS and in appendix A in Hassler et al (1992).

tionary period between two cyclical peaks, with extensive layoffs and increased productivity. The First World War (associated with industrial growth until 1916, when the war restrained foreign trade) is followed by the 1920-1925 crisis (with a short but deep fall in employment) and a deep depression in 1929-1933. The expansion of the late thirties ends with the outbreak of the Second World War. The war leads to decreased private consumption and foreign trade but, on the other hand, many industries meet a relatively stable domestic demand. Finally, the stable growth during the 1950s and 1960s is succeeded by the oil crisis and slow growth in the 1970s. For a detailed description of the historical developments during the period, see, for example, Larsson (1993).

3. THE STAR MODEL AND LINEARITY TESTING

3.1 The model

In this section the possibility that the first differences of the logarithmic series are nonlinear is considered. Moreover, it is assumed that if this is the case these nonlinear series can be adequately characterised by smooth transition autoregressive models. The objective is to obtain a useful characterisation of the dynamics of the series. It is not argued that this is the only way to model nonlinearity. For instance, nonparametric alternatives could be considered, although the series seem somewhat short for that provided the strong turbulence observed. However, the STAR family of models does have some useful properties. First, it is suitable for modelling series with asymmetric

cyclical variations and turbulent periods (see for instance Teräsvirta and Anderson, 1992, and Teräsvirta, 1995). A second advantage is that the estimated, locally linear, models can be easily interpreted. Finally, a modelling cycle comprising specification, estimation and evaluation stages already exists (Teräsvirta, 1994; Eitrheim and Teräsvirta, 1996) and will be applied here.

A STAR model is defined as

$$y_t = \pi_{10} + \pi_1' w_t + (\pi_{20} + \pi_2' w_t) F(y_{t-d}) + u_t, \quad (3.1)$$

where $\pi_j = (\pi_{j1}, \dots, \pi_{jp})'$, $j = 1, 2$, $w_t = (y_{t-1}, \dots, y_{t-p})'$, $u_t \sim \text{nid}(0, \sigma_u^2)$. The transition function $F(y_{t-d})$ is defined to be either a logistic function

$$F(y_{t-d}) = \left(1 + \exp\{-\gamma_L (y_{t-d} - c_L)\}\right)^{-1}, \quad \gamma_L > 0 \quad (3.2)$$

or an exponential function

$$F(y_{t-d}) = 1 - \exp\{-\gamma_E (y_{t-d} - c_E)^2\}, \quad \gamma_E > 0. \quad (3.3)$$

In the following, model (3.1) with transition function (3.2) will be referred to as a logistic STAR model of order p , LSTAR(p), whereas (3.1) with (3.3) is called an exponential STAR model of order p , ESTAR(p). It may be noted that other models, nonlinear as well as linear, appear as special cases of the STAR specifications. The LSTAR model approaches a two-regime threshold autoregressive model (see Tong,

1990) when $\gamma_L \rightarrow \infty$, since (3.2) in the limit is a step function of y_{t-d} the value of which changes from zero to unity at c_L . When $\gamma_L \rightarrow 0$, the LSTAR model approaches a linear AR(p) model. Turning to the ESTAR model, equation (3.1) approaches a linear model both as $\gamma_E \rightarrow 0$ and (with probability one) as $\gamma_E \rightarrow \infty$. If $c_E = \pi_{20} = 0$, the ESTAR model is identical to the exponential autoregressive model of Haggan and Ozaki (1981).

The role of the transition function in (3.1) is that it allows the coefficients for lagged values of y_t , $[\pi_1 + \pi_2 F(y_{t-d})]'$, and the intercept, $\pi_{10} + \pi_{20} F(y_{t-d})$, to change smoothly with y_{t-d} . This means that the local dynamics of the model change with y_{t-d} . This idea works differently for the two STAR models. The LSTAR model allows the local dynamics to be different for high and low values of the transition variable, y_{t-d} . The modelling of local dynamics as a function of a lagged value of y makes it possible to model nonlinear effects of a shock. For instance, if a negative shock pushes a characterisation away from a locally stable regime (F close to unity, say), the subsequent change in the value of F changes the local dynamics (F now close to zero, say). If this regime contains a pair of explosive complex roots, y may be returned to the previous level a lot more quickly than would be the case if it followed a linear AR process.

In contrast to the LSTAR case, the ESTAR transition function is symmetric about c_E in the sense that the local dynamics are the same for high as for low values of y_{t-d} whereas the mid-range behaviour of the variable (values close to c_E) is different. (It should be noted that the mid-regime does not necessarily have to be locally stable.) For instance, with the exponential transition function, it is possible for y to move rapidly between very small and very large values for which local dynamics are stable.

The test described in the next subsection is applied to test linearity and to select the form of the STAR model to be estimated if linearity is rejected. Diagnostic tests will reveal whether a STAR model offers an adequate characterisation of the data or not.

3.2 Testing linearity

The modelling cycle for building STAR models is discussed in Teräsvirta (1994) and Eitrheim and Teräsvirta (1996). Testing linearity against STAR constitutes the first step of the model specification stage. First, a linear autoregressive model for the series with apparently no autocorrelation in the residuals is selected. This can be done by applying an appropriate model selection criterion such as the AIC. The selected model is the null model. For details of the test, which has power against both LSTAR and ESTAR, see Teräsvirta (1994). The test is carried out for different values of the unknown delay parameter d , and the value of d associated with the test with the smallest p -value is selected. If none of the p -values is sufficiently small, linearity is not rejected. It may be noted that if testing linearity were the main point of the whole investigation d could be assumed to be unknown and the test carried out starting from that assumption as in Luukkonen, Saikkonen and Teräsvirta (1988). That way the investigator would control the overall significance level of the test. In this paper model selection, including the choice of d , is an important part of the work. Therefore linearity is tested conditionally on d and the results are also used to select the delay. Thus the overall significance level of the procedure is not controlled, but an erroneous

rejection of linearity is likely to be discovered at the evaluation stage of the modelling cycle.

A summary of the results of the tests can be found in Table 3.1. The table contains the maximum lag length of the autoregressive model, the smallest p -value of the tests, and the corresponding delay. It also contains the results of the model selection test sequence for choosing between ESTAR and LSTAR (p -values p_{F_4} , p_{F_3} , p_{F_2} ; see Teräsvirta, 1994) and the chosen model family. In most cases linearity is rejected against STAR, and in many cases the rejection is extremely strong as indicated by the very low p -values. The only exception is GDP. The lag length selected by the AIC is two, and applying that lag length in the tests yields a minimum p -value of about 0.02. If the lag length is increased to four, however, the evidence against linearity is weaker. An LSTAR model for GDP has been estimated, but in accordance with the test results the improvement in the fit compared to the linear model was small, hence in the following GDP is treated as a linear process. As for the investment series, it is seen to be very turbulent in the beginning of the period. Excluding the years 1861-1875 from the sample, the evidence against linearity practically vanishes. Instead of trying to fit a STAR model just to accommodate those most distant observations, in the following these observations are excluded and the remaining part of the series is treated as being linear. The other seven series are deemed nonlinear and STAR models are fitted to them. In cases where the decision between the ESTAR and the LSTAR is not clear-cut, both types of models are fitted to the data and the final decision is taken at the model evaluation stage. The modelling of the individual series will be discussed in Sections 4.2-4.7 below.

4. MODELLING THE SERIES AND INTERPRETING THE RESULTS

4.1. Evaluation of STAR models

In this section the results from the estimation and evaluation of STAR models for five of the seven nonlinear series are reported, and the dynamic properties of the models are considered. For the two remaining series the most interesting results are briefly outlined. The objective is to shed light on the cyclical properties of the series. Every estimated model is evaluated by a series of tests, the results of which are reported in Table 4.1-4.3. The assumption of no error autocorrelation is tested using the Lagrange multiplier test derived for this purpose by Eitrheim and Teräsvirta (1996). Their paper also contains two other tests. One is for testing the hypothesis of no remaining non-linearity. In this test the alternative hypothesis is that the data-generating process is an additive STAR model with two 'STAR components' instead of a single one as in (3.1). Finally, the constancy of the parameter vectors $(\pi_{j0}, \pi_j)'$, $j = 1, 2$, is tested against the hypothesis that the parameters change smoothly over time. Three tests are carried out. The first one, LM1, assumes that the parameters change monotonically over time, the second one, LM2, that the change is symmetric with respect to an unknown point in time, and the third one, LM3, that the change is possibly non-monotonic but not necessarily symmetric. All the tests are carried out by auxiliary regressions. For details, see Eitrheim and Teräsvirta (1996).

Each estimated STAR model equation is reported together with a number of statistics: AIC is the Akaike information criterion, SBIC is the Rissanen-Schwarz information

criterion, s is the estimated standard deviation of the residuals, LJB is the Lomnicki-Jarque-Bera test of normality, LM is an LM statistic of no ARCH (Engle, 1982) based on two lags, and V.R. is the variance ratio s^2/s_{AR}^2 , where s_{AR}^2 is the estimated variance of the residuals from the linear autoregressive model used as a basis for linearity testing. The ratio gives an idea of the relative gain in the fit made from applying a STAR model instead of a linear AR model. Numbers in parentheses following values of test statistics are p -values, whereas those below the coefficient estimates are asymptotic standard errors of the estimates. The various graphs and other descriptive techniques used to characterise the dynamic properties of the estimated models are introduced and explained in Section 4.2 using the model for industrial production as an example.

4.2 Industrial production

The first series to be considered is industrial production, an important component of GDP. The test sequence in Table 3.1 suggests an ESTAR model, and the finally estimated model has the form

$$\begin{aligned}
 y_t = & 2.87y_{t-1} + 1.30y_{t-2} + 2.00y_{t-3} + 0.41y_{t-4} - 0.89y_{t-5} - 0.56y_{t-6} & (4.1) \\
 & (1.44) \quad (0.62) \quad (1.38) \quad (0.40) \quad (0.40) \quad (0.32) \\
 & + \left[\begin{array}{ccccccc} 0.073 & -2.87y_{t-1} & -1.30y_{t-2} & -2.25y_{t-3} & -0.41y_{t-4} & +0.89y_{t-5} & +0.56y_{t-6} & -0.30y_{t-7} \end{array} \right] \\
 & (0.010) \quad (1.44) \quad (0.62) \quad (1.38) \quad (0.40) \quad (0.40) \quad (0.32) \quad (0.093) \\
 & \times \left[\begin{array}{cc} 1 - \exp \{ -1.82 (y_{t-3} + 0.089)^2 / \hat{\sigma}^2(y) \} & \end{array} \right] + \hat{u}_t \\
 & (0.63) \quad (0.013)
 \end{aligned}$$

$s = 0.065$	skewness = 0.22	excess kurtosis = 3.1
LJB = 50 (1×10^{-11})	AIC = -5.37	SBIC = -5.11
LM = 0.14 (0.87)	V.R. = 0.85	$R^2 = 0.23$.

The standardisation of the exponent of F through division by $\hat{\sigma}^2(y)$, the sample variance of y_t , is introduced to make γ scale-free and thus facilitate the interpretation of its estimate.

The maximum lag (seven) in (4.1) seems long but is not different from that of the corresponding linear AR model. As seen from Table 4.1, containing the results of the tests of no error autocorrelation, the null hypothesis is not rejected. Table 4.2 contains the results of the tests of no remaining nonlinearity. The tests are based on the third-order Taylor expansion of the second transition function; see Eitrheim and Teräsvirta (1996). The smallest p -value is 0.034, corresponding to a delay of $d = 4$. Given the number of tests this is not very strong evidence against the model. When the constancy of all parameters is tested simultaneously the model passes the test. In contrast, when the null hypothesis only concerns the nonlinear intercept, assuming that the other parameters are constant even under the alternative, the test result suggests that this intercept may not have been constant over time. This result may have to do with the slowdown in the growth rate in the 1970s but it has not been pursued further here.

The estimated equation (4.1) contains a number of parameter restrictions of the type $\pi_{1j} = -\pi_{2j}$. These restrictions exclude the 'combined parameter' $\pi_{1j} + \pi_{2j}F$ for $F = 1$ and are supported by the data. (The restriction $\pi_{1j} = 0$ does the same for $F = 0$.) The

equation has a low location parameter value, $\hat{c}_E = -0.089$. In Figure 4.3 the shape of the estimated transition function is shown, with one dot for every observation in the sample so that it is easily seen which values the transition function has obtained and how frequently. The same information ordered over time is found in Figure 4.2. The two figures together show that the transition function normally has been close to unity. It has moved further away from unity mainly in the 1880s, from 1910 to the early 1920s, in the mid-1930s, and in the early 1980s. Comparing the fit of the linear AR(7) model and the ESTAR model (4.1) (see Figure 4.1), it is seen that the latter fits much better in the 1880s and early 1920s than the former. It also improves the explanation of the high growth rates following the depression in the early 1930s. It can be concluded that the nonlinear model describes the most turbulent periods in the data better than the linear autoregressive model. The fit is not good for the last years of the sample for either of the two models. In fact, as already indicated, the rejection of the constancy of the intercept in the ESTAR model may have something to do with this.

The dynamic behaviour of the model can be characterised in different ways. Trying to interpret individual parameter estimates or the delay d does not provide much useful information (with the exceptions of $\hat{\gamma}$ and \hat{c}). It is more instructive to compute the roots of the characteristic polynomial of (4.1) at given values of the transition function F as in Teräsvirta (1994). The extreme values $F = 0$ and $F = 1$ are particularly interesting. The dominant roots of the characteristic polynomial of (4.1) given $F = 0$ and $F = 1$, respectively, can be found in Table 4.4. For $F = 1$ (lowest and high values of the series), all roots are stationary. For $F = 0$, there exists a real root greater than unity

that is needed to describe the sawtooth movements of the growth rate; it is not possible to explain this behaviour by a purely linear autoregressive model.

The situation for other values of F could be considered, but there exists another, more economic way of characterising local dynamics. To this end, the ‘local’ or ‘sliced’ spectrum of the STAR model is defined as follows:

$$f_{yy}(\omega; y_{t-d}) = (1/2\pi) \left[\left\{ 1 - \sum_{j=1}^p (\pi_{1j} + \pi_{2j}F) e^{-ij\omega} \right\} \left\{ 1 - \sum_{j=1}^p (\pi_{1j} + \pi_{2j}F) e^{ij\omega} \right\} \right]^{-1} \quad (4.2)$$

for $-\pi \leq \omega \leq \pi$, see, for example, Priestley (1981, Section 4.12). As is seen from (4.2), this spectrum is a function of F and thus of y_{t-d} . It is defined for those values of

F for which the roots of the lag polynomial $1 - \sum_{j=1}^p (\pi_{1j} + \pi_{2j}F) B^j$ lie outside the unit

circle, i.e., for which the estimated STAR model is locally stationary. Note that (4.2)

is not standardised; integrated from zero to π , the function does not integrate to one.

Thus the estimated local spectra have to be standardised to be comparable. Figure 4.4

contains the standardised local spectra for model (4.1). In this figure, each curve

represents a local spectrum and corresponds to a single observation of the transition

variable. Most of these relate to values of F close to unity, as Figures 4.2 and 4.3

already suggest. An interesting fact conveyed by Figure 4.4 is that there exists a local

peak corresponding to a period of about 13-14 years. This peak is not very prominent

for F close to unity. It increases in size, however, with increasing distance from $F = 1$,

and it can thus to a large extent be ascribed to the most turbulent periods in the data.

For values of F close to unity, a conspicuous (local) peak appears at ‘business cycle

frequencies' corresponding to a period length between 4 and 5 years. The peak is visible in every local spectrum appearing in the figure, indicating that the corresponding cycle is an important underlying characteristic of the process. Another very distinct feature is the high-frequency tail of the spectrum due to large short-term swings in the growth rate.

The set of local spectra as defined in (4.2) thus can be used to illustrate the local dynamics of the estimated STAR model. However, it does not characterise the global dynamics of the model and should not be interpreted that way. Global dynamics are better illustrated by a 'model' spectrum; see Priestley (1981, pp. 268-9) for the linear case. Because the spectrum of a STAR model cannot be computed analytically, it is done by simulation¹. The dominant peak in the model spectrum for industrial production in Figure 4.5 appears at a period of 13-14 years. Keeping in mind the information in Figure 4.4, this is another indication of the fact that nonlinearity in (4.1) characterises the most turbulent periods in the data. The conclusion is that there exists a distinct cyclical component in the industrial production series at the business cycle frequencies. The long 'cycle' is more of an artefact, because its appearance is merely due to the most turbulent parts of the series and their timing.

Thus, in this case the traditional nonparametric sample spectrum, the STAR-based model spectrum, and the local spectra complement each other. In particular, the

¹ For every STAR model, 99 realizations of 300 observations are generated using the estimated coefficients and a sequence of error terms, $u_t \sim \text{nid}(0, s^2)$. The first 150 observations of each one of the 99 series are dropped, the spectral densities for the series are estimated using customary nonparametric methods. The "model" spectrum is the average of these 99 densities.

inspection of the local spectra together with the estimated transition function sheds light on the appearance of the 13-14-year cycle visible in the global spectral densities.

Finally, generalised impulse response (GIR) functions as defined in Koop, Pesaran and Pottter (1996) are used for characterising the dynamic properties of the models, the persistence of shocks in particular. The procedure of estimating the GIR functions is described in the Appendix. To illustrate the asymmetric effects of shocks the functions are computed separately for sufficiently large positive and negative shocks. For ease of exposition, especially as the estimated GIR densities may be multimodal, the GIR functions are presented as graphs of highest density regions following suggestions in Hyndman (1995, 1996). The highest density regions for the estimated GIR functions for industrial production appear in Figure 4.6 a-c. The shocks seem quite persistent as the tails of the GIR densities are quite heavy even at long lags. Negative shocks are more persistent than positive ones.

4.3 Exports

Next, the exports series, which has been strongly affected by events exogenous to the Swedish economy, is considered. As seen from Table 3.1, linearity is rejected very strongly. The model specification test sequence suggests $d = 3$ and an LSTAR model. The estimated model is

$$\begin{aligned}
y_t = & 0.51y_{t-1} - 0.92y_{t-2} - 0.37y_{t-3} + 0.47y_{t-4} - 0.85y_{t-5} - 0.23y_{t-6} - 0.42y_{t-7} + 0.17y_{t-9} & (4.3) \\
& (0.18) \quad (0.23) \quad (0.14) \quad (0.15) \quad (0.22) \quad (0.077) \quad (0.17) \quad (0.079) \\
& + \left[\begin{array}{cccccccc} 0.05 & -0.51y_{t-1} & +1.13y_{t-2} & +0.27y_{t-3} & -0.47y_{t-4} & +0.85y_{t-5} & +0.42y_{t-7} & -0.098y_{t-8} \end{array} \right] \\
& (0.017) \quad (0.18) \quad (0.27) \quad (0.19) \quad (0.15) \quad (0.22) \quad (0.17) \quad (0.075) \\
& \times \left[1 + \exp \{ -6.79 (y_{t-3} + 0.047) \} / \hat{\sigma}(y) \right]^{-1} + \hat{u}_t \\
& (3.46) \quad (0.014)
\end{aligned}$$

$s = 0.099$	skewness = -0.14	excess kurtosis = 3.5
LJB = 61 (6×10^{-14})	AIC = -4.52	SBIC = -4.19
LM = 0.30 (0.74)	V.R. = 0.65	$R^2 = 0.48$

The estimated model does not seem to have autocorrelated errors (Table 4.1). On the other hand, (4.3) does not adequately characterise the nonlinearity in the data (Table 4.2). This may not be surprising given the irregularity of the series and the very strong rejection of linearity at the outset. The p -values of the tests of no remaining nonlinearity are nevertheless remarkably higher than those of the linearity tests. Parameter constancy tests do not indicate any nonconstancy (Table 4.3). Although model (4.3) does not account for all nonlinearity in the series its properties are discussed below.

The transition function has varied most widely between 1900 and 1950. Compared with the linear model, the LSTAR model makes a contribution in explaining the developments during the First World War and the export boom following the Second World War; see Figure 4.8. It also predicts a drop in the exports at the end of 1970s which did not occur. To understand the dynamic properties of (4.3) it may be instructive to begin with the roots of the characteristic polynomial as functions of F . From (4.3) it is noted that $\hat{c}_L = -0.047$, and Figure 4.10 shows that the transition function gets close to zero roughly for $y_{t-3} < -0.1$. When $F = 0$ there exist two pairs of explosive complex roots (Table 4.4). The corresponding period lengths are 4.4 and 9.3

years. For $F = 1$, all roots are stationary, and the largest pair has the period 2.3 years. When F decreases this root becomes real and strongly negative.

The model spectrum in Figure 4.12 is dominated by the 4 and 10.5 year peaks. However, from the local spectra in Figure 4.11, these peaks are seen to be mainly associated with relatively low values of the transition function, whereas the short-term dynamics dominate for transition function values close to unity. This information can be combined with that in Figure 4.9, showing that the estimated transition function takes values close to unity much more frequently after than before 1950. It can thus be concluded that the cyclical fluctuations have not been constant over time. The 4 and 10.5 year peaks mainly characterise the earlier, more turbulent, period of the series. From 1950 onwards, short-run fluctuations (length of period less than 3 years) have dominated and there has been little business cycle variation. This does not necessarily show in the spectrum estimated from the whole sample, and the local and the global spectra thus again neatly complement each other. The highest density region graphs of the generalised impulse response functions in Figure 4.7 a-c indicate that the model is less persistent and asymmetric than the model for industrial production.

4.4 Imports

Like exports, imports are also greatly affected by exogenous events. Linearity is rejected extremely strongly against STAR (Table 3.1). The smallest p -value occurs at $d = 1$, and the specification test sequence suggests an ESTAR model. The estimated model has the form

$$\begin{aligned}
y_t = & 0.57y_{t-1} - 0.42y_{t-2} + 0.29y_{t-3} + 0.19y_{t-4} + 0.15y_{t-5} + 0.37y_{t-6} & (4.4) \\
& (0.19) \quad (0.080) \quad (0.093) \quad (0.072) \quad (0.11) \quad (0.12) \\
& + [-1.55y_{t-1} - 1.19y_{t-3} - 0.93y_{t-5} - 2.46y_{t-6} - 1.41y_{t-7}] \\
& \quad (0.34) \quad (0.33) \quad (0.31) \quad (0.46) \quad (0.47) \\
& \times [1 - \exp \{ -0.32 (y_{t-1} - 0.0030)^2 / \hat{\sigma}^2(y) \}] + \hat{u}_t \\
& \quad (0.17) \quad (0.016)
\end{aligned}$$

$s = 0.12$	skewness = -1.34	excess kurtosis = 3.4
LJB = 94 (5×10^{-21})	AIC = -4.17	SBIC = -3.87
LM = 0.77 (0.47)	V.R. = 0.51	$R^2 = 0.61$

There does not seem to be any autocorrelation in the errors (Table 4.1). The hypothesis of no remaining nonlinearity is rejected (Table 4.2) but, as in the case of the exports series, the p -values of the tests are a few magnitudes higher than in the linearity tests. The hypothesis of parameter stability cannot be rejected (Table 4.3). Thus, the properties of (4.4) are considered. However, there is a caveat: this inference is not valid because the model is not stable. This point will be further discussed shortly.

Figure 4.13 indicates that the two world wars and their aftermaths constitute the most turbulent periods in the series. That is where the ESTAR model contributes most to the explanation (see Figure 4.14). In particular, it captures the extremely sharp peak in the growth rate after the end of the Second World War had ended. The roots of the characteristic polynomial in Table 4.4 reflect the very large fluctuations in the imports. It is perhaps not surprising that there exist several explosive roots for $F = 1$. But there is also an explosive real root for $F = 0$; the corresponding value of the transition variable y_{t-1} is practically zero. This explosive band about zero is very narrow but enhances the sharpness of the fluctuations. The local spectra in Figure 4.16

accord well with the information in the roots. The dominating feature is the ridge representing the period of 4-5 years. As in the case of the industrial output, this 4-5 year cycle seems an important characteristic of the series. The estimation of the model spectrum failed because some of the simulated characterisations diverged. This implies that (4.4) is not a stable model, rendering the standard statistical inference invalid. The instability is also reflected in the estimated generalised impulse response functions depicted in Figure 4.17 a-c. The model may thus be seen only as a local (in time) approximation to the true data-generating process. The problems in finding a stable representation of imports have their root in the extremely large fluctuations in the growth rate of the series in connection with the two world wars.

4.5. Employment

The most distinct features of this series can be seen from Figure 4.19. There was a large drop in employment in the years following the First World War and another one followed by a rapid recovery in the 1930s. Linearity of the series is strongly rejected against STAR, the rejection being strongest when $d = 1$. The model specification tests point at the ESTAR family, but an LSTAR model could also be considered a possibility. The estimated ESTAR model is

$$\begin{aligned}
y_t = & -0.98 - 3.53y_{t-1} - 9.65y_{t-2} + 6.16y_{t-3} + 3.52y_{t-4} & (4.5) \\
& (0.73) \quad (2.25) \quad (5.55) \quad (5.26) \quad (4.16) \\
& + \left[\begin{array}{ccccc} 1.01 & + 3.53y_{t-1} & + 9.65y_{t-2} & - 6.16y_{t-3} & - 3.52y_{t-4} \\ (0.73) & (2.25) & (5.55) & (5.26) & (4.16) \end{array} \right] \\
& \times \left[1 - \exp \left\{ -0.32 (y_{t-1} + 0.17)^2 / \hat{\sigma}^2(y) \right\} \right] + \hat{u}_t \\
& \quad (0.17) \quad (0.057)
\end{aligned}$$

$s = 0.038$	skewness = -0.62	excess kurtosis = 2.4
LJB = 36 (2×10^{-8})	AIC = -6.49	SBIC = -6.30
LM = 0.22 (0.80)	V.R. = 0.56	$R^2 = 0.50$

The estimated coefficients of the longest lags seem insignificant but removing the lags has an adverse effect on the fit. There does not seem to be any error autocorrelation in the model (Table 4.1). It appears from Table 4.2 that (4.5) has captured almost all nonlinearity in the data. There is some, but not very strong, evidence of parameter instability (Table 4.3). An important thing to notice in (4.5) is that $\hat{c}_E = -0.17$. Thus almost all observations belong to the right-hand tail of the transition function as is also seen from Figure 4.21. In this case the LSTAR model would therefore fulfill almost the same function as the ESTAR model (see the discussion in Teräsvirta, 1994). Such a model has also been estimated, but because the ESTAR model has a slightly better fit, it is reported here. Another special characteristic of (4.5) is that because of the parameter restrictions $\pi_{1j} = -\pi_{2j}$, $j = 1, \dots, 4$, the model is locally white noise with a positive mean (0.03) when $F = 1$. Figure 4.20 shows that values of the transition function have remained close to unity most of the time. The ESTAR model makes two major contributions. First, it explains the big decrease in employment in the beginning of the 1920s better than the linear AR model. Second, it tracks the data well from the mid-1960s onwards where the linear model fails.

The characteristic polynomial for $F = 0$ contains a pair of complex roots with a very large modulus (cf Table 4.4). This pair of roots remains explosive for considerably large values of F . The model spectrum (see Figure 4.23) has a broad peak corresponding to cycle length of 3.5 years and another peak at the zero frequency. Figure 4.22 depicting local spectra allows another view. For F very close to unity the local spectrum is flat because the process is locally white noise with drift for $F = 1$. A peak corresponding to the period of 3.5 years emerges rather quickly when F decreases. Thus when employment has been growing (see Figure 4.19) at a steady rate, the series has shown little 'business cycle' or other regular cyclical variation. On the other hand, when the employment has been declining as from the 1960s onwards, very regular cyclical variation has appeared in the series. By combining the information in Figure 4.19 with the graph of the transition function it can be concluded that the cyclical variation has been asymmetric: the troughs have had a tendency of being sharper than the peaks, and as indicated by the explosive pair of roots commented on above, the recovery of the growth rate from a deep trough has always been quick. In conclusion, whereas the series up to the 1960s displays little or no cyclical movements, roughly the last quarter of the century is characterised by a prominent asymmetric cycle with a rather short period. In contrast to the case of industrial production, a steady 4-5 year 'business cycle' component is not a property of the employment series.

The estimated GIR functions in Figure 4.18 a-c show that the asymmetry of the effects of the shocks is even more pronounced than in the case of industrial production. Negative shocks are remarkably persistent. This result is due to the effects of the very

large negative shocks in the sample that have caused a permanent fall in the level of the series.

4.6. Private consumption

Figure 4.24 shows that the growth rate of the logarithmic private consumption fluctuates most in the 1910s due to the First World War. There is also a drop in the growth rate in the 1930s and another one followed by a rapid recovery in the early 1940s. When linearity is tested against STAR it is rejected (Table 3.1). The model specification procedure indicates a slight preference for an LSTAR model, but it turns out that an ESTAR model has a somewhat better fit. It has the form

$$\begin{aligned}
 y_t = & 0.033 + 0.36y_{t-3} - 0.71y_{t-4} - 0.23y_{t-6} & (4.6) \\
 & (0.0081) \quad (0.15) \quad (0.14) \quad (0.12) \\
 & + \left[-0.016 + 0.21y_{t-1} - 0.20y_{t-2} - 0.36y_{t-3} + 1.28y_{t-4} - 0.44y_{t-5} + 0.23y_{t-6} + 0.29y_{t-9} \right] \\
 & \quad (0.013) \quad (0.091) \quad (0.091) \quad (0.15) \quad (0.20) \quad (0.11) \quad (0.12) \quad (0.12) \\
 & \times \left[1 - \exp \left\{ -1.67 (y_{t-1} - 0.059)^2 / \hat{\sigma}^2(y) \right\} \right] + \hat{u}_t \\
 & \quad (0.52) \quad (0.0041)
 \end{aligned}$$

$s = 0.032$	skewness = -0.36	excess kurtosis = 0.2
LJB = 3 (0.3)	AIC = -6.78	SBIC = -6.50
LM = 2.66 (0.08)	V.R. = 0.79	R2 = 0.44

Model (4.6) is the only model whose residuals do not contain outliers (the Lomnicki-Jarque-Bera test does not reject the normality assumption). There is no error autocorrelation, but the model does not adequately describe all the nonlinearity. However, the p -values of the tests are a few magnitudes higher than those of the linearity tests,

and the residual variance is just 4/5 of that of the linear model. No stability test rejects parameter constancy. The model is also different from the preceding ones in the sense that while it fits better than the linear one, the fit of (4.6) is not vastly superior to that of the AR model anywhere in the sample. This can be seen from Figure 4.24. Figure 4.25 indicates that the transition function fluctuates between zero and one during the whole sample period. Thus, unlike all the other series considered here the consumption series seems inherently nonlinear. Model (4.6) is locally stationary everywhere. There is a complex pair of roots with modulus 0.99 and period 2.7 years and another one with modulus 0.91 and period 9.8 years at $F = 0$ (for this value, $\hat{c}_E = 0.06$).

The model spectrum in Figure 4.28 has a large and flat peak at a frequency corresponding to a period of 3 to 4 years, but there is also a very distinct 10-year peak. The local spectra in Figure 4.27 are interesting. For values of F close to unity there exists a strong peak at a cycle length of about 4 years. When F decreases, this peak moves to the right. This indicates a certain asymmetry: at high rates of growth (around 6 per cent) the cyclical fluctuations tend to be more peaked than at lower rates of growth. This asymmetry accords well with the flat peak of the model spectrum. Naturally, a univariate analysis does not provide an explanation of asymmetric cyclical behaviour of consumption, it merely establishes the fact. When F approaches zero, a 10-year peak visible for all values of F grows stronger. This peak seems mainly due to the distance between the end of the World War I turbulence and the Great Depression on the one hand, and that between this recession and the effects of World War II on consumption on the other. Its emergence can also be expected from the roots of the

characteristic polynomial for $F = 0$ discussed above. Again, the same peak appeared in the model spectrum. Thus, also for this apparently inherently nonlinear series, the information in the model spectrum can be usefully complemented by the set of estimated sliced spectra. The estimated GIR functions are depicted in Figure 4.29 a-c. While there seems to be cyclical asymmetry in the process it can be seen that the effects of the shocks on consumption are symmetric. In terms of the growth rate, the shocks are also much less persistent than in the cases of industrial production and employment.

4.7. Other series

The other two series modelled by STAR models are productivity and wages. The productivity series is industrial production divided by employment, and the estimated model shares characteristics of the employment model. For this series, the rejection of linearity is not overwhelming ($p_{min} = 0.0248$). The estimated model is not reported here, but Figure 4.31 shows the local spectra. The local spectrum is completely flat for $F = 1$, and a peak at the cycle length 4 years emerges as F decreases. Figure 4.30 shows that the model fit is not good everywhere. The substantial productivity increases after the First World War remain unexplained. This was mainly due to a rapid fall in employment. On the other hand, the turbulence of the 1880s due to large fluctuations in the industrial production is captured reasonably well. Furthermore, the test of no additive nonlinearity indicates that the model does pick up all nonlinearity there is in the data.

Linearity is rejected very strongly for the wages series. The estimated LSTAR model explains a part of the turbulence due to the First World War, see Figures 4.32 and 4.33. The transition function differs from unity only on two occasions. However, the test of no remaining nonlinearity reveals that the STAR model gives a far from adequate description of the data, hence the model has not been pursued further. Detailed modelling results for the wages series are available from the author upon request.

4.8. Linear series

As seen from Table 3.1, there is not much evidence against linearity in the GDP series. The same is true for the investment series after omitting the first 15 observations. Model spectra have nevertheless been computed for both series, see Figures 4.34 and 4.35. Because of linearity, this can be done by assuming $F \equiv 0$ and applying (4.2). It is seen that the cyclical variation parametrised by an AR(4) model is not strong. The spectrum has a rather broad peak at a frequency corresponding to a period of about 3 years. The AR model for investment suggests that the most prominent feature in the data, ignoring the years 1861-1875, is a cycle with a period of about 6 years.

4.9 Business cycle fluctuations, asymmetries, and the adequacy of the STAR models

One of the purposes of this paper is to complement the findings of EPS regarding the characteristics of the Swedish business cycle. The results indicate that two of the series investigated, industrial production and imports, contain fluctuations at business cycle frequencies corresponding to a period of 4-5 years. Consumption, the only inherently nonlinear process among the ones considered, also shows fluctuations within the 3-8 year band, but their frequency is higher (period 3-4 years). For exports and employment, the business cycle is a somewhat elusive concept. The exports series shows cyclical variation before 1950 but since then, short-term variation has been dominant in the series. As for employment, the early part of the period is characterised by strong steady growth interrupted by occasional collapses of the growth rate. Regular 'business cycle variation' (period 3.5 years) has appeared only in the descending part of the series, from 1960s onwards. Among the linear series, investment seems to have a six-year cyclical component. For GDP, the evidence of a regular cyclical component is weak. The techniques applied thus yield a somewhat more detailed picture of the cyclical variation in long Swedish macroeconomic series than that of EPS.

As noted above, linearity is strongly rejected for many of the series. This does not, however, automatically imply asymmetry. In fact, the discussion above suggests that the observed nonlinearity is mainly due to the large exogenous shocks that the Swedish economy has been subjected to during the last 130 years. Consumption

appears to be one variable whose growth rate is inherently nonlinear and whose cyclical fluctuations show signs of asymmetry. In this case, troughs on the average are sharper than peaks, except for some very extreme peaks. Another one is employment. While the cyclical frequency is constant over values of F , the peak in the spectrum grows in importance for descending values of F . This indicates that troughs even in this case are sharper than peaks.

Summarising, STAR models offer a reasonable explanation of fluctuations in three series out of seven: industrial production, employment, and productivity. They explain some but not all of the movements in the most turbulent periods in consumption, exports, and imports. At least for exports and imports, which have been directly subjected to a few very large shocks, one might consider an outlier approach as in Balke and Fomby (1994) or van Dijk, Franses and Lucas (1996). The same consideration applies to the wages series whose dynamic behaviour a STAR model has not been able to capture.

5. TESTING THE NONCAUSALITY HYPOTHESIS

5.1 General

The estimated STAR models are of interest *per se*, but they are also important in investigating temporal relationships between the series. Considering these relations would be necessary, for instance, if one wanted to know whether or not there is a

driving variable behind the cyclical variation and what kind of dynamic interactions there may have existed between the variables. EPS studied correlations between filtered variables but did not discuss dynamic aspects. In this study, the number of variables is relatively large (nine) whereas the sample size is only moderate (fewer than 130 observations). This fact, combined with strong observed nonlinearities, makes a system approach to these problems a less feasible alternative. A more modest beginning may be made by investigating pairwise causal relationships between variables. This is normally done by testing the null hypothesis of Granger noncausality (Granger, 1969) between two variables. There is a large literature on this topic; see Geweke (1984) for a survey. However, most of the testing is carried out in a completely linear framework. More recently, Baek and Brock (1992), Hiemstra and Jones (1994), and Bell, Kay and Malley (1996), have proposed nonparametric tests of noncausality.

The linear framework is not applicable in this case because most of the Swedish macroeconomic series under study are nonlinear. On the other hand, since parametric nonlinear STAR models have already been constructed for the variables it appears natural to continue using parametric tests. Such tests do not seem to exist for nonlinear series, but below a test based on smooth transition regression (STR) is proposed and applied to the series. The results are compared to what is obtained by ignoring nonlinearity and applying corresponding tests based on linear equations to the same series.

5.2 An STR-based test of Granger-noncausality

A simple way of testing the null hypothesis that an observed series x_t does not (linearly) Granger cause another series, y_t , in a single-equation framework is to test the null hypothesis $\beta_1 = \dots = \beta_{q^*} = 0$ in

$$y_t = \theta + \alpha_1 y_{t-1} + \dots + \alpha_{p^*} y_{t-p^*} + \beta_1 x_{t-1} + \dots + \beta_{q^*} x_{t-q^*} + \varepsilon_t \quad (5.1)$$

where $\varepsilon_t \sim \text{iid}(0, \sigma_\varepsilon^2)$ under H_0 . Analogously when y_t (under the null hypothesis of noncausality) is generated by the STAR model (3.1), a noncausality test can be performed by testing a null hypothesis of non-existent predictive power of lagged values of another variable, x_t . The sequence $\{x_t\}$ is assumed to be stationary and ergodic and the nonlinear impact of x on y is characterised by an additive smooth transition component. The following *additive* smooth transition regression model is considered

$$y_t = \pi_{10} + \pi_1' w_t + (\pi_{20} + \pi_2' w_t) F(y_{t-d}) + \delta_1' v_t + (\delta_{20} + \delta_2' v_t) G(x_{t-e}) + u_t \quad (5.2)$$

where $\delta_j = (\delta_{j1}, \dots, \delta_{jq})'$, $j = 1, 2$, $v_t = (x_{t-1}, \dots, x_{t-q})'$, $G(\cdot)$ is a transition function similar to the ones defined in (3.2-3.3), and e is an unknown delay. The noncausality hypothesis is $H_0: G \equiv 0$ and $\delta_{1i} = 0, i = 1, \dots, q$. Furthermore, under H_0 , it is assumed that $u_t \sim \text{iid}(0, \sigma_u^2)$ and that the parameters of (5.2) can be consistently estimated by nonlinear least squares under this null hypothesis.

The test is a modification of the test of no additive nonlinearity in Eitrheim and Teräsvirta (1996). The identification problem that is due to the fact that (5.2) is not identified under H_0 is again circumvented by approximating the second transition function G by its Taylor approximation. However, in this case one wants to control the overall significance level of the test while retaining the assumption that the delay e is unknown. This can be achieved by proceeding as in Luukkonen, Saikkonen and Teräsvirta (1988). Choosing the ‘economy version’ (S3) of their test, the relevant approximation to (5.2) has the form

$$y_t = \tilde{\pi}_{10} + \tilde{\pi}'_1 w_t + (\pi_{20} + \pi'_2 w_t) F(y_{t-d}) + \kappa' v_t + \sum_{i=1}^q \sum_{j=i}^q \phi_{ij} x_{t-i} x_{t-j} + \sum_{i=1}^q \psi_i x_{t-i}^3 + r_t \quad (5.3)$$

where $\kappa = (\kappa_1, \dots, \kappa_q)'$. The null hypothesis is $H_0: \kappa_i = 0, \phi_{ij} = 0, \psi_i = 0, i = 1, \dots, q, j = i, \dots, q$. Under $H_0, r_t = u_t$ and the resulting test statistic has an asymptotic χ^2 -distribution with $q(q+1)/2 + 2q$ degrees of freedom, assuming that all necessary moments and cross-moments exist. The test may be carried out by regressing the residuals from (5.2) estimated under the null hypothesis on the gradient vector of the null model, $x_{t-i}, i = 1, \dots, q, x_{t-i} x_{t-j}, i = 1, \dots, q, j = i, \dots, q,$ and $x_{t-i}^3, i = 1, \dots, q$. The degrees of freedom in the numerator of the F -variant of the test are $q(q+1)/2 + 2q$ and in the denominator $T - n - q(q+1)/2 - 2q$ where n is the dimension of the gradient vector. The test may be carried out in the same way even if $\{y_t\}$ is a linear sequence. The only modification is that $F \equiv 0$ in (5.2).

The size of the test in samples like the present one may be a cause for concern. Because of rather long lags, in some of the tests the number of degrees of freedom in the numerator relative to that in the denominator of the statistic is rather large, which might give rise to size distortion. A small simulation study has been carried out to check this potential problem (details available from the author) but the results did not give rise to concern.

It may be argued that the assumption that the second additive component in (5.2) is of the STR type is unduly restrictive. The more general approach, not assuming a particular functional form for this component, as discussed in Eitrheim and Teräsvirta (1996), can straightforwardly be adapted to this situation. For large values of q , however, the dimension of the null hypothesis may be so large that in small samples it often leads to a substantial loss of power.

5.3 Results

Table 5.1 contains the results from the nonlinear tests of Granger-noncausality. The tests are based on the STAR models presented in Section 4 for all variables except GDP and investments. To give an idea of the effect of the lag length on the results, the tests were performed for $q = 5, \dots, 10$ in (5.2). Interpreting the results requires care because a rejection of the null hypothesis does not imply a direct causal link between a pair of variables. The tests are bivariate, and changing the information set may change conclusions. Nevertheless, the STR-based tests suggest a large number of interactions between the variables. In particular, employment and productivity are

strongly linked with a majority of the other variables. The results may not directly point out a driving variable but industrial production appears much more often as a causing than as a caused variable. Another conspicuous observation is that the variables representing links between the Swedish economy and the rest of the world, exports and imports, appear clearly more often as causing than as caused variables. (Between the two, exports seem to Granger-cause imports rather than the other way round. It should be kept in mind that the model of imports was not stable so that the p -values are merely indicative.) In particular, causality running from these variables (but also from consumption) to wages is very strong. This is in accord with Sweden being a small open economy dependent on foreign trade. On the other hand, industrial production has a role similar to imports and exports. Finally, one should note that the analysis focuses on short-run connections between the series. Possible long-run relationships between the Swedish macroeconomic variables are not discussed here.

For comparison, the noncausality hypothesis was also tested linearly using (5.1). The results can be found in Table 5.2. Comparing these results with those of Table 5.1, the outcomes of the two forms of noncausality tests are seen to differ in a number of ways. In some cases, the nonlinear test indicates two-way causality between variables where the linear test finds no causality in any direction or one-way causality. In other cases, the direction of causality may differ between the two tests.

A general observation is that the linear tests find much fewer causal links between the series than the nonlinear tests do. The employment and productivity series serve as illustrative examples. According to the STR-based test they are linked with almost all the other series. The linear tests suggest both fewer and weaker links. The industrial

production also is remarkably weakly linked with the other variables. A general conclusion is that assumptions concerning the functional form play an important role in testing the null hypothesis of Granger noncausality. On the other hand, it should be kept in mind that the analysis is bivariate. A third variable shocking the two variables in the test with different delays may therefore cause a rejection of a noncausality hypothesis. Furthermore, observed nonlinearities in at least some models may be due to responses to a few unusually large shocks. It is thus possible that these occasions strongly affect the results of the tests and that their influence is not the same in linear and nonlinear tests. This being the case, treating responses to large shocks as outliers and proceeding accordingly could lead to another set of noncausality test results. That possibility has not been considered any further in this paper.

6. CONCLUSIONS

The results of the paper show that behind the uniform business cycle found by EPS from analysing long Swedish macroeconomic time series there is considerable individual behaviour in the series. Some series, such as GDP, show only weak cyclical behaviour. In some other ones cyclical variation is elusive; sometimes it may be present, sometimes not. For series with relatively strong cyclical components the 'business cycle frequencies' with peaks in the spectra are not necessarily the same. In some series, cyclical variation seems asymmetric, in others it does not. It seems that the sometimes strong peaks in the spectra of these series at low frequencies can be ascribed to rather few exceptional observations or periods in the series.

The study also indicates that there are many temporal links between the variables. There may not exist a clear driving variable among the ones considered, industrial production being the closest candidate. On the methodological side the study has demonstrated that the functional form strongly affects the results of Granger non-causality tests. That this may be the case is well known, see, for example, the discussion in Hendry (1995, p. 175-176). Empirical examples of this phenomenon, however, have not been many, which may be due to the fact that tests of noncausality allowing nonlinearity have been developed rather recently. The STR-based test in this paper is obviously one of the first parametric ones.

Finally, it may be concluded that STAR models are useful alternatives to linear models as research tools in this work. Although it has not been possible to find an adequate STAR representation for every nonlinear series, the STAR models that have been fitted to the data provide insight into characteristic features of the long Swedish macroeconomic time series considered. With the help of these models, the analysis and conclusions of EPS on the many properties of Swedish business cycles from 1860s to late 1980s have been complemented and enriched.

APPENDIX. GENERALISED IMPULSE RESPONSE FUNCTIONS AND HIGHEST DENSITY REGIONS

This appendix briefly describes the estimation of the generalised impulse response (GIR) functions for the estimated STAR models. The reader is referred to Koop, Pesaran and Potter (1996) for a general definition of the function and a comprehensive discussion. In estimating GIRs for the variables in the present study, all available observations (i.e., all possible vectors of the necessary lags) of the time series are used once (without sampling) as ‘histories’. For each history, 100 initial shocks are drawn randomly with replacement from a subset of the residuals from the estimated STAR model, formed either of all residuals (panel (d) in Figures 2.1-2.3), of all residuals greater than one residual standard error (panel (e)), or of all residuals less than one negative residual standard error (panel (f)). For each combination of history and initial shock, 800 replicates of a 21-step prediction sequence (0,1,...,20 step ahead) are generated with and without the selected initial shock in the first step, and using randomly drawn residuals as noise everywhere else. For every horizon, the means over the 800 replicates of the two prediction sequences are computed, and the vector of 21 differences between the two means forms an observation of the GIR.

The observations of the GIR form the basis of the highest density regions used for a graphical representation of the generalised impulse response densities. For each one of the 21 horizons, the response density is estimated with a kernel algorithm. The points used in the kernel algorithm are a random sample from the GIR. For instance, in the

case of industrial production, with 120 'histories', this is a random sample from 12000 GIR values for each horizon. The 50% and 95% highest density regions are then estimated using the density quantile method as described in Hyndman (1996).

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TABLES

Table 3.1

Results of linearity tests: number of lags in the linear AR model (p), minimum p -value ($\min[p_{FL}]$) over delays $d = 1, \dots, p$; corresponding delay (d), p -values of the tests in the model selection sequence (p_{F_4} , p_{F_3} , p_{F_2}), the selected model family (E = ESTAR, L = LSTAR).

Variable	p	$\min[p_{FL}]$	d	p_{F_4}	p_{F_3}	p_{F_2}	STAR
Industrial production	7	0.031	3	0.070	0.028	0.63	E
Imports	7	2.1×10^{-13}	1	0.026	7.7×10^{-10}	6.74×10^{-6}	E
Exports	9	2.1×10^{-8}	3	0.00025	0.0039	0.00013	L
Productivity	2	0.025	1	0.045	0.036	0.47	E
Real wage	4	2.7×10^{-10}	1	1.4×10^{-6}	6.9×10^{-5}	0.013	L
Investments	4	1.8×10^{-7}	1	0.0061	0.00027	0.00062	L
Consumption	9	4.2×10^{-5}	1	0.0030	0.0048	0.054	E
GDP	2	0.019	2	0.0017	0.64	0.54	L
	4	0.065					
Employment	4	3.2×10^{-10}	1	0.0050	6.26×10^{-6}	2.8×10^{-5}	E

Note: For details of the test sequence for choosing between ESTAR and LSTAR models see Teräsvirta (1994).

Table 4.1

p -values of LM tests of no error autocorrelation.

Variable	Number of lags							
	1	2	3	4	5	6	7	8
Industrial production	0.70	0.91	0.98	0.94	0.97	0.49	0.23	0.28
Imports	0.22	0.49	0.44	0.32	0.11	0.12	0.10	0.091
Exports	0.66	0.56	0.68	0.75	0.47	0.49	0.61	0.68
Productivity	0.77	0.93	0.96	0.99	0.92	0.96	0.68	0.37
Real wage	0.23	0.32	0.46	0.59	0.72	0.82	0.88	0.61
Consumption	0.40	0.54	0.69	0.82	0.93	0.95	0.93	0.96
Employment	0.22	0.069	0.091	0.069	0.084	0.18	0.38	0.37

Note: For details of the test see Eitrheim and Teräsvirta (1996).

Table 4.2
p-values of tests of no remaining nonlinearity.

Variable	Delay								
	1	2	3	4	5	6	7	8	9
Ind. prod.	0.10	0.33	0.22	0.034	0.84	0.14	0.15		
Imports	0.0057	0.0027	0.0021	0.030	0.14	0.72	0.11		
Exports	0.00094	0.00053	0.028	6.1×10^{-5}	0.0052	0.0031	0.0038	0.0055	0.026
Productivity	0.15	0.76							
Real wage	7.7×10^{-9}	0.00032	9.0×10^{-5}	0.024					
Consumption	0.0083	0.019	0.057	0.082	0.053	0.087	0.53	0.092	0.24
Employment	0.40	0.035	0.30	0.072					

Note: The tests are based on the third-order Taylor expansion of the second transition function; for details see Eitrheim and Teräsvirta (1996).

Table 4.3
p-values of LM tests of parameter constancy: LM_1 , LM_2 , LM_3 (*F* tests).

Variable	Parameters	LM_1	LM_2	LM_3
Industrial production	Intercepts and lags	0.41	0.41	0.82
	Intercepts only	0.016	0.033	0.066
Imports	Intercepts and lags	0.58	0.44	0.47
	Intercepts only	0.94	0.61	0.28
Exports	Intercepts and lags	0.72	0.92	0.98
	Intercepts only	0.049	0.13	0.31
Productivity	Intercepts and lags	0.80	0.26	0.50
	Intercepts only	0.23	0.34	0.45
Consumption	Intercepts and lags	0.47	0.81	0.59
	Intercepts only	0.31	0.029	0.049
Employment	Intercepts and lags	0.28	0.10	0.15
	Intercepts only			

Note: For details of the test see Eitrheim and Teräsvirta (1996).

Table 4.4

Roots of characteristic polynomials for various values of the transition function F .
Only roots with modulus ≥ 0.90 are displayed.

Industrial production (ESTAR)	Mid-regime ($F = 0$)			Outer regime ($F = 1$)		
	Root	Modulus (half-life)	Period	Root	Modulus (half-life)	Period
	3.42 $-0.10 \pm 0.91i$	3.42 0.92 (8.9)	3.7	-0.90	0.90	
Imports (ESTAR)	Mid-regime ($F = 0$)			Outer regime ($F = 1$)		
	Root	Modulus (half-life)	Period	Root	Modulus (half-life)	Period
	$0.38 \pm 0.811i$ 1.04	0.90 (7.2) 1.04	5.5	$0.89 \pm 0.64i$ $-0.05 \pm 1.19i$ -1.29	1.09 1.19 1.29	10.0 3.9
Exports (LSTAR)	Lower regime ($F = 0$)			Upper regime ($F = 1$)		
	Root	Modulus (half-life)	Period	Root	Modulus (half-life)	Period
	$0.17 \pm 1.142i$ $0.79 \pm 0.632i$ -0.90	1.16 1.01 0.90	4.4 9.3	$-0.86 \pm 0.33i$	0.92 (9.4)	2.3
Productivity (ESTAR)	Mid-regime ($F = 0$)					
	Root	Modulus	Period			
	$0 \pm 1.69i$	1.69	4			
Consumption (ESTAR)	Mid-regime ($F = 0$)			Outer regime ($F = 1$)		
	Root	Modulus (half-life)	Period	Root	Modulus (half-life)	Period
	$0.75 \pm 0.53i$ $-0.70 \pm 0.70i$	0.91 (8.6) 0.99 (69.4)	10.3 2.7	$0.03 \pm 0.94i$	0.94 (12.4)	4.1
Employment (ESTAR)	Mid-regime ($F = 0$)					
	Root	Modulus	Period			
	$-1.98 \pm 2.78i$	3.41	2.9			
	Regime $F = 0.7$			Regime $F = 0.95$		
	Root	Modulus	Period	Root	Modulus	Period
	$-0.72 \pm 1.792i$	1.93	3.2	$-0.22 \pm 0.88i$	0.90 (7.7)	3.5

Table 5.1

Nonlinear Granger noncausality tests.

The numbers denote the lag orders of the polynomial in the causing variable ($q = 5, \dots, (1)0$).Asterisks indicate p -values: *, $p \leq 0.05$; **, $p \leq 0.01$; ***, $p \leq 0.001$; ****, $p \leq 0.0001$.

Caused variable	Causing variable								
	Ind. prod	GDP	Imp.	Exp.	Prod.	Real wages	Inv.	Cons.	Empl.
Industrial production		5	5	5	5*	5	5*	5	5*
		6	6	6	6**	6	6	6	6*
		7	7	7	7****	7	7*	7	7
		8**	8	8	8****	8	8*	8	8*
		9	9	9	9****	9	9	9	9
	0	0	0	0****	0	0*	0	0***	
GDP	5		5*	5*	5*	5**	5	5	5
	6		6*	6*	6	6**	6	6	6
	7		7	7	7	7*	7	7	7*
	8		8	8	8**	8*	8	8	8**
	9**		9	9*	9***	9	9	9	9**
0**		0*	0	0***	0**	0	0	0**	
Imports	5	5		5****	5	5*	5	5*	5
	6	6		6***	6	6	6	6*	6**
	7	7		7***	7	7*	7*	7*	7*
	8	8		8**	8	8*	8*	8*	8*
	9**	9		9****	9**	9**	9**	9*	9***
0***	0		0****	0**	0*	0*	0	0**	
Exports	5	5**	5*		5	5	5	5**	5
	6	6*	6*		6	6	6	6***	6**
	7	7	7*		7	7	7	7***	7**
	8*	8	8		8*	8	8	8**	8**
	9*	9*	9		9**	9	9*	9**	9***
0***	0	0		0*	0	0**	0	0***	
Productivity	5*	5***	5**	5*		5**	5	5***	5***
	6*	6**	6**	6**		6*	6*	6***	6***
	7***	7*	7*	7*		7*	7	7**	7***
	8**	8*	8*	8***		8	8**	8**	8**
	9***	9*	9**	9***		9*	9**	9****	9*
0****	0**	0**	0**		0**	0**	0**	0****	
Real wages	5	5*	5****	5****	5		5	5****	5
	6	6*	6****	6****	6		6	6****	6**
	7	7*	7****	7****	7		7	7****	7**
	8	8*	8***	8****	8		8	8***	8**
	9	9*	9***	9****	9***		9	9***	9
0	0*	0***	0***	0*		0	0**	0	
Investment [†]	5**	5	5	5	5**	5		5	5
	6**	6	6	6	6**	6		6	6
	7**	7	7*	7	7*	7		7	7
	8**	8	8	8	8*	8		8	8
	9***	9	9	9	9*	9		9	9*
0**	0	0	0	0***	0		0	0***	
Consumption	5	5**	5**	5**	5	5	5		5
	6	6***	6**	6****	6	6	6		6
	7*	7**	7**	7****	7*	7*	7*		7
	8*	8**	8**	8***	8*	8**	8		8
	9***	9*	9*	9**	9	9**	9		9
0****	0**	0	0*	0	0**	0		0	
Employment	5	5***	5***	5	5*	5	5	5	
	6*	6***	6***	6**	6***	6	6	6	
	7**	7**	7**	7**	7*	7	7	7	
	8***	8*	8**	8***	8**	8	8	8	
	9***	9	9*	9**	9**	9	9	9	
0****	0	0**	0*	0	0	0**	0		

[†]Computed from observations for 1876-1988

Table 5.2

Linear Granger noncausality tests.

The numbers denote the lag orders of the polynomial in the causing variable ($q = 5, \dots, (1)0$).Asterisks indicate p -values: *, $p \leq 0.05$; **, $p \leq 0.01$; ***, $p \leq 0.001$; ****, $p \leq 0.0001$.

Caused variable	Causing variable								
	Ind. prod	GDP	Imp.	Exp.	Prod.	Real wages	Inv.	Cons.	Empl.
Industrial production		5	5	5	5	5	5	5	5
		6	6	6	6	6	6	6	6
		7	7	7	7	7	7	7	7
		8	8	8	8	8	8	8	8
		9	9	9	9	9	9	9	9
		0	0	0	0	0*	0	0	0
GDP	5		5	5**	5*	5**	5	5	5
	6		6	6**	6	6**	6	6	6
	7		7	7**	7	7*	7	7	7
	8		8	8*	8	8**	8	8	8
	9		9	9*	9	9*	9	9	9
	0		0	0	0	0**	0	0	0
Imports	5	5		5***	5	5*	5	5*	5
	6	6		6***	6	6*	6	6	6
	7	7		7***	7	7**	7	7	7
	8	8		8*	8	8**	8	8	8
	9	9		9*	9	9**	9	9	9
	0	0		0*	0	0**	0	0	0
Exports	5	5*	5		5*	5***	5	5	5
	6	6	6		6**	6***	6	6	6**
	7	7	7		7*	7****	7	7	7**
	8	8	8		8*	8***	8	8	8**
	9	9	9		9*	9***	9	9*	9*
	0	0	0		0*	0**	0	0	0*
Productivity	5*	5	5	5		5***	5	5*	5*
	6*	6	6	6		6***	6	6*	6*
	7*	7	7	7		7***	7	7	7*
	8**	8	8	8		8**	8	8	8**
	9*	9	9	9		9**	9	9	9*
	0*	0	0	0		0**	0	0	0*
Real wages	5	5*	5	5	5		5	5*	5
	6	6*	6**	6	6		6	6	6
	7	7*	7***	7	7		7	7*	7
	8	8*	8****	8	8		8	8**	8
	9	9	9****	9	9		9	9*	9
	0	0	0***	0	0		0	0*	0
Investment [†]	5*	5	5*	5*	5*	5		5	5
	6***	6	6	6*	6*	6		6*	6
	7***	7	7	7*	7*	7		7	7
	8***	8	8	8*	8*	8		8**	8
	9***	9	9	9*	9*	9		9**	9
	0***	0	0	0*	0*	0		0*	0
Consumption	5	5	5*	5**	5	5*	5		5
	6	6	6	6****	6	6*	6		6
	7	7	7	7***	7	7*	7		7*
	8	8	8	8***	8	8*	8		8
	9	9	9	9***	9	9	9		9*
	0	0	0	0***	0	0	0		0*
Employment	5	5**	5	5	5	5**	5	5**	
	6	6**	6	6*	6	6**	6	6*	
	7	7*	7	7	7	7**	7	7*	
	8	8*	8	8*	8	8**	8	8*	
	9	9*	9	9*	9	9*	9	9*	
	0	0	0	0	0	0*	0	0*	

[†] Computed from observations for 1876-1988

FIGURES

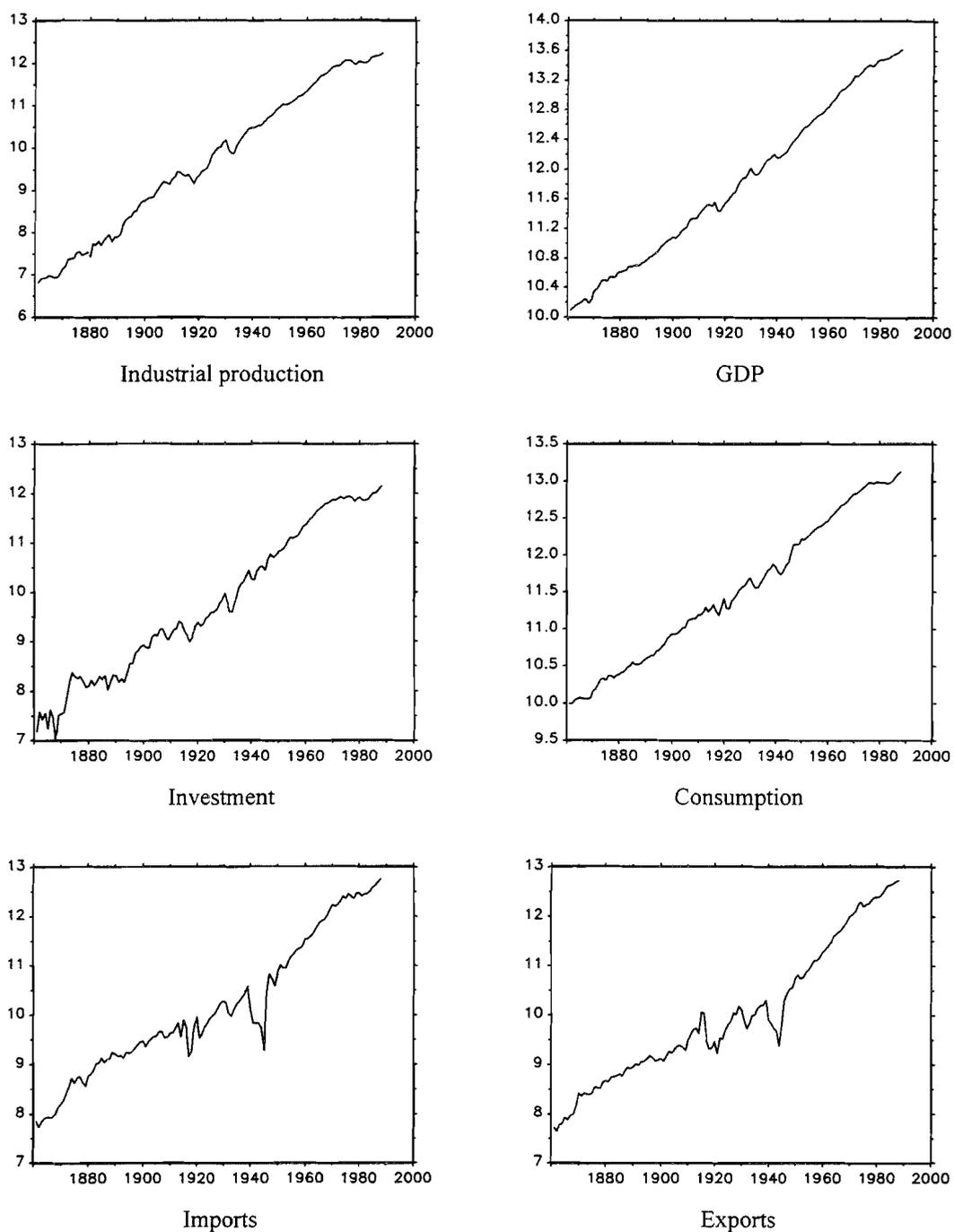
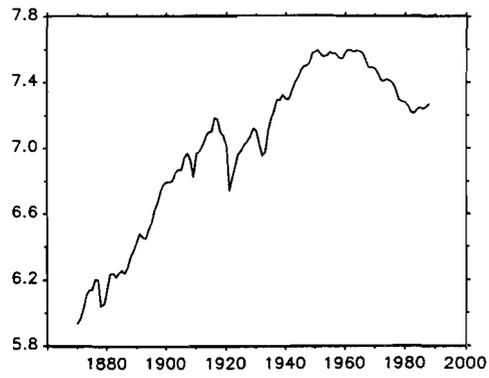
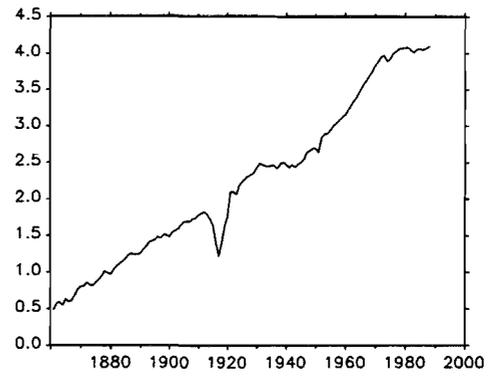


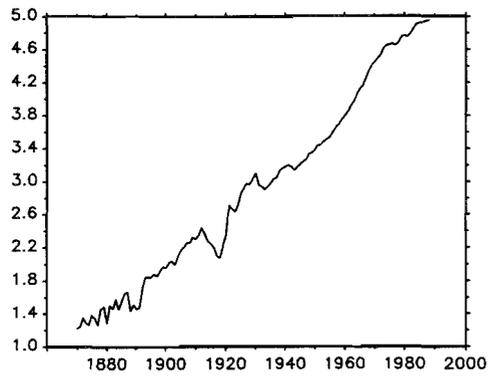
Figure 2.1 Logarithms of the series

(Figure 2.1, continued)

Employment



Real wages



Productivity

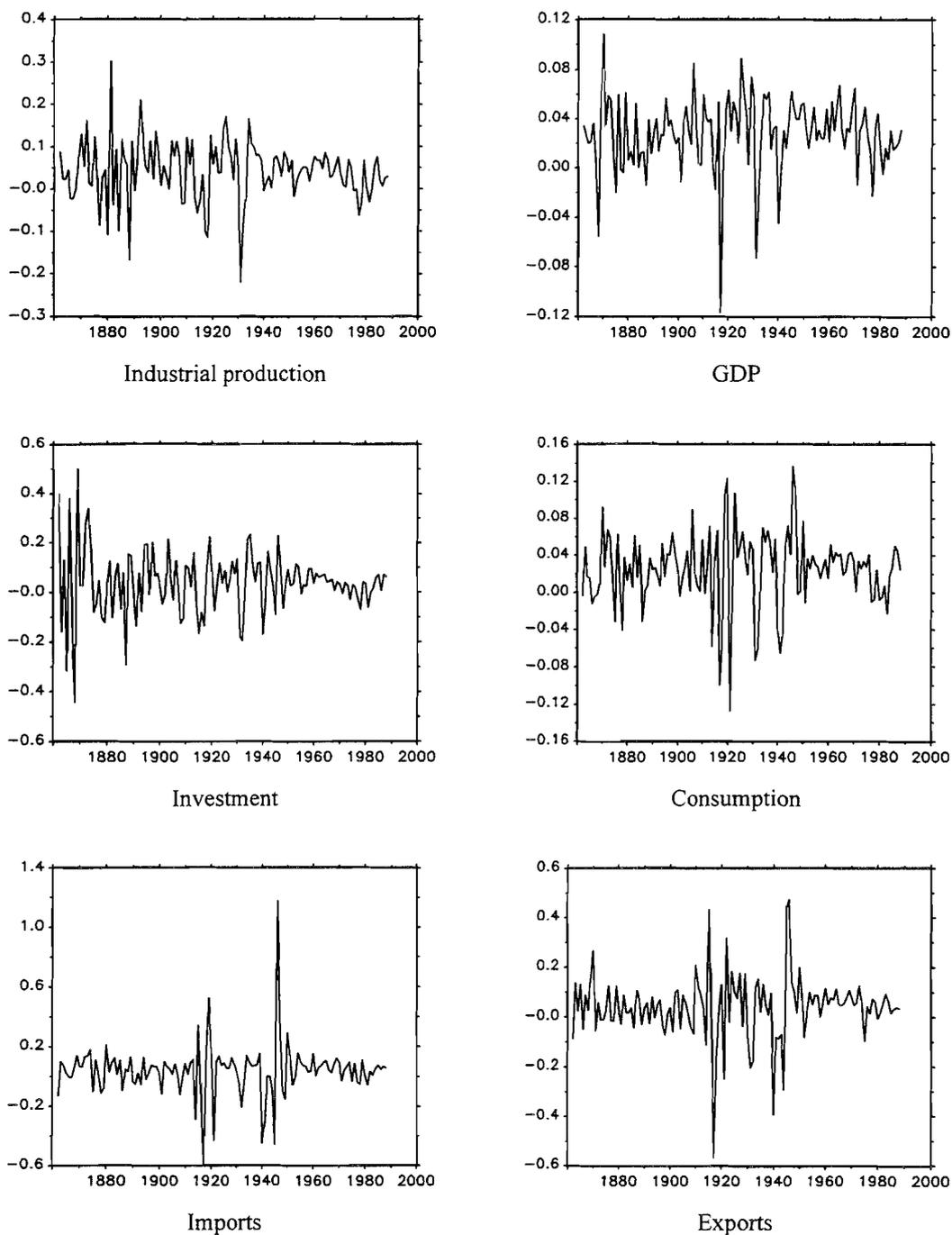
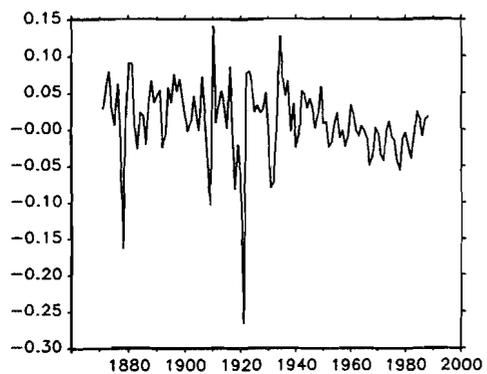
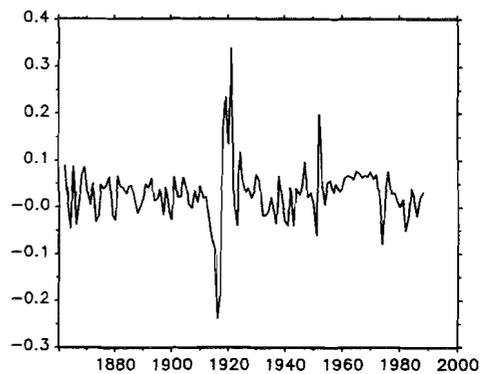


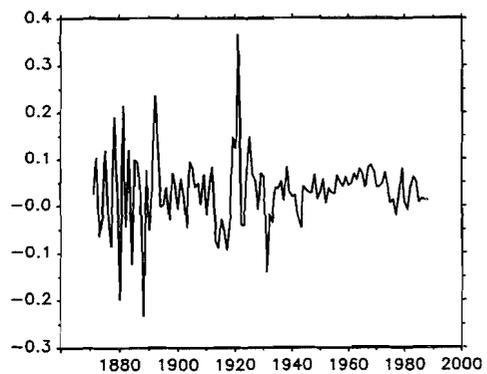
Figure 2.2 First differences of the logarithms of the series

(Figure 2.2, continued)

Employment



Real wages



Productivity

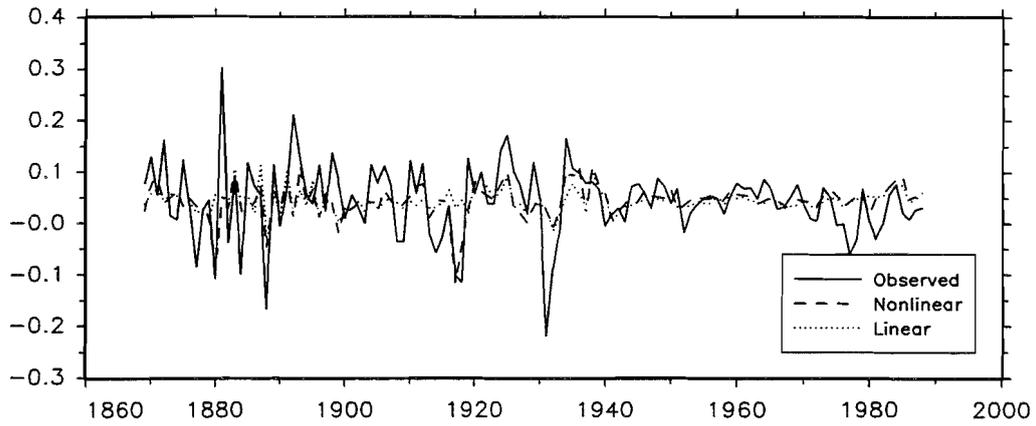


Figure 4.1. Industrial production (first difference of logarithm). Observed values, predictions from STAR model, and predictions from AR model.

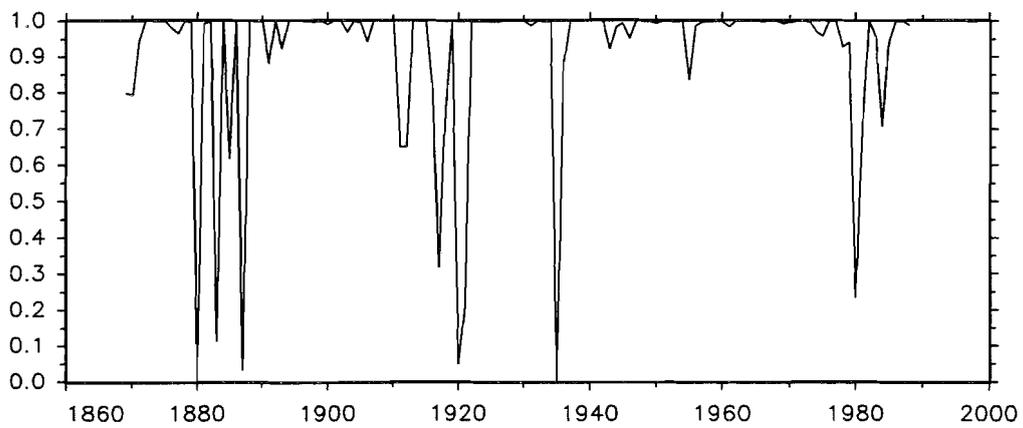


Figure 4.2. Industrial production (first difference of logarithm). Estimated transition function over time.

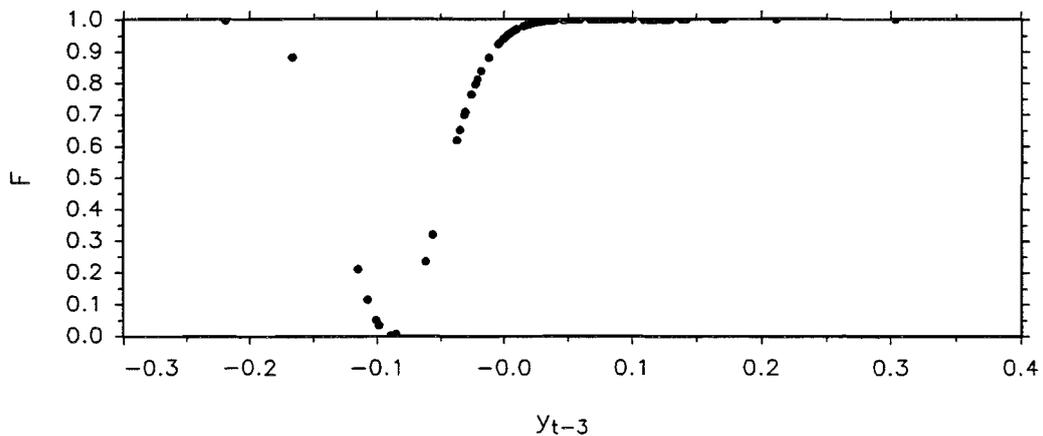


Figure 4.3. Industrial production (first difference of logarithm). Estimated transition function vs the transition variable.

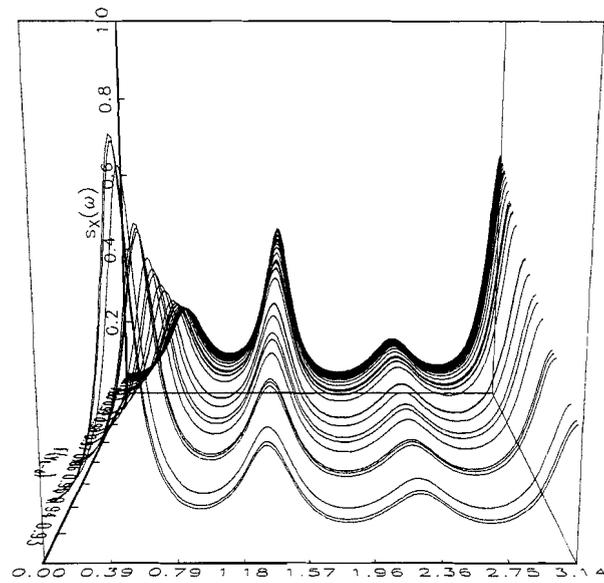


Figure 4.4. Industrial production (first difference of logarithm). 'Sliced' spectra.

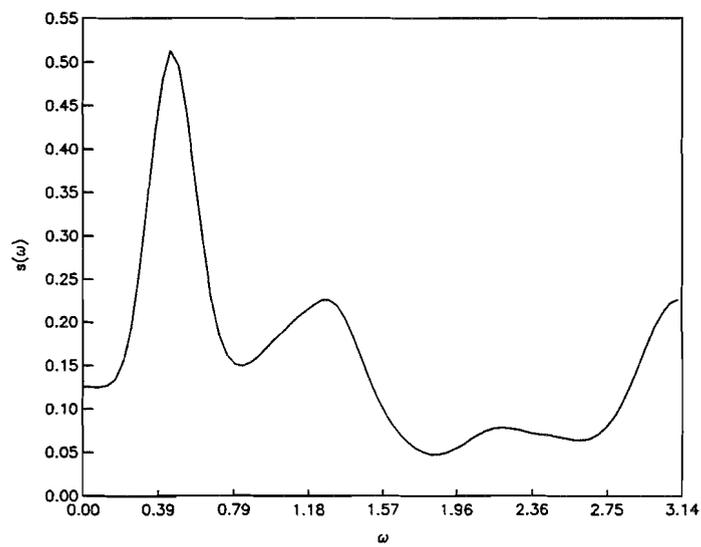
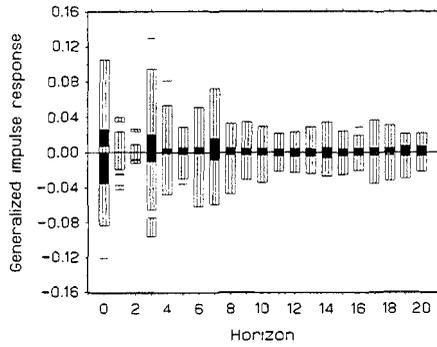
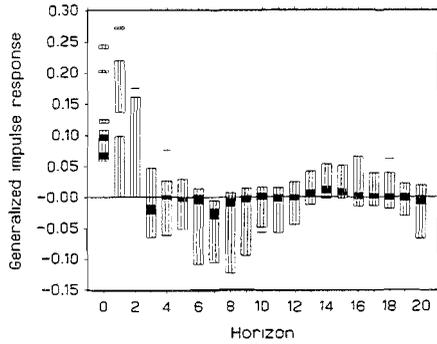


Figure 4.5. Industrial production (first difference of logarithm). Model spectrum.

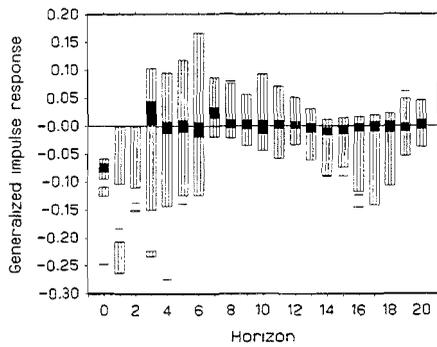
Figure 4.6.
Industrial production (first difference of logarithm). 50% (black) and 95% (hatched) highest density regions for the generalised impulse response to shocks



a. Response to symmetric shocks

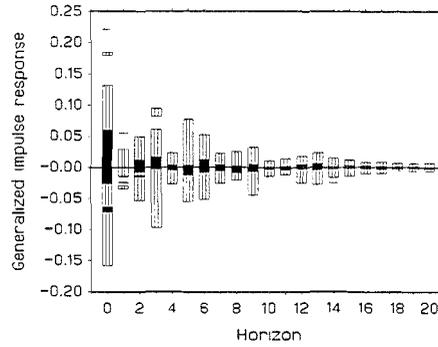


b. Response to positive shocks, greater than 1 error s.d.

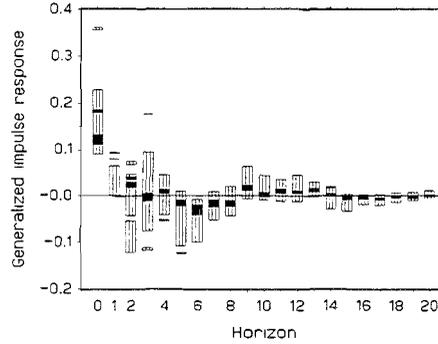


c. Response to negative shocks, smaller than -1 error s.d.

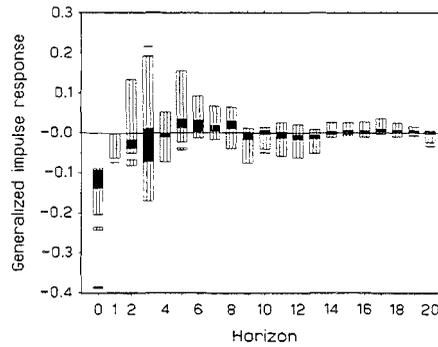
Figure 4.7.
Exports (first difference of logarithm). 50% (black) and 95% (hatched) highest density regions for the generalised impulse response to shocks



a. Response to symmetric shocks



b. Response to positive shocks, greater than 1 error s.d.



c. Response to negative shocks, smaller than -1 error s.d.

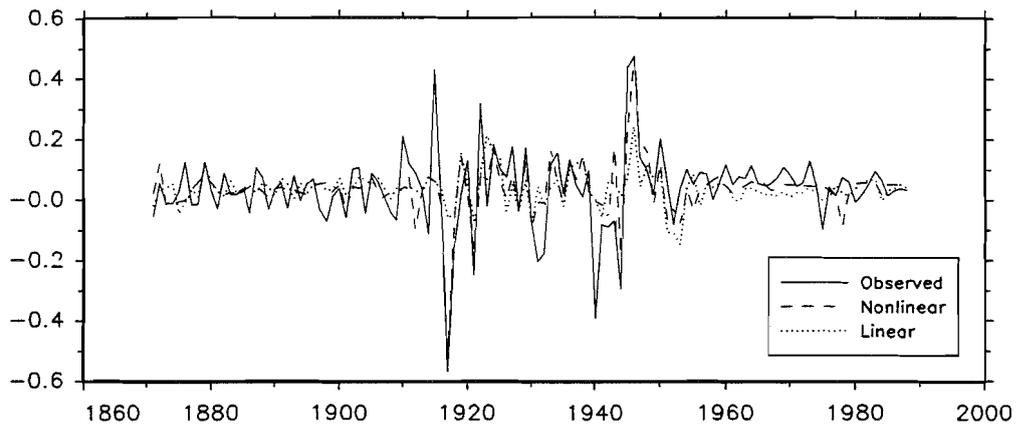


Figure 4.8. Exports (first difference of logarithm). Observed values, predictions from STAR model, and predictions from AR model.

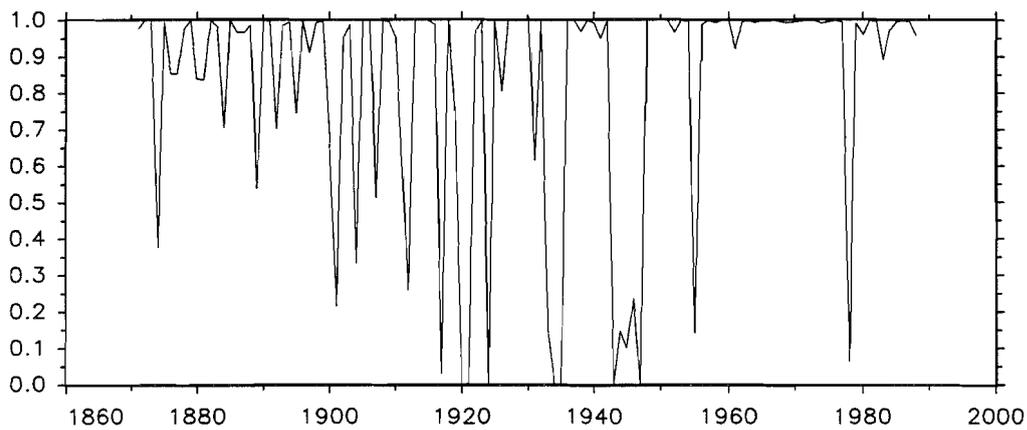


Figure 4.9. Exports (first difference of logarithm). Estimated transition function over time.

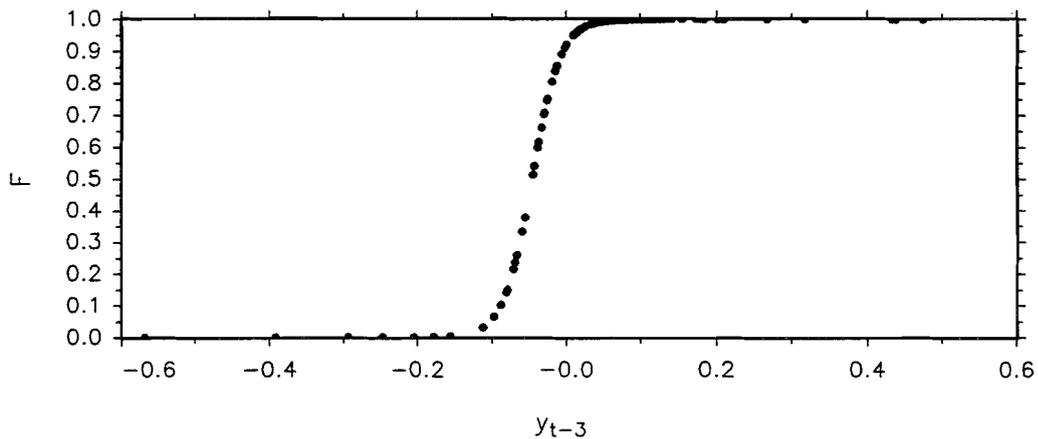


Figure 4.10. Exports (first difference of logarithm). Estimated transition function vs the transition variable.

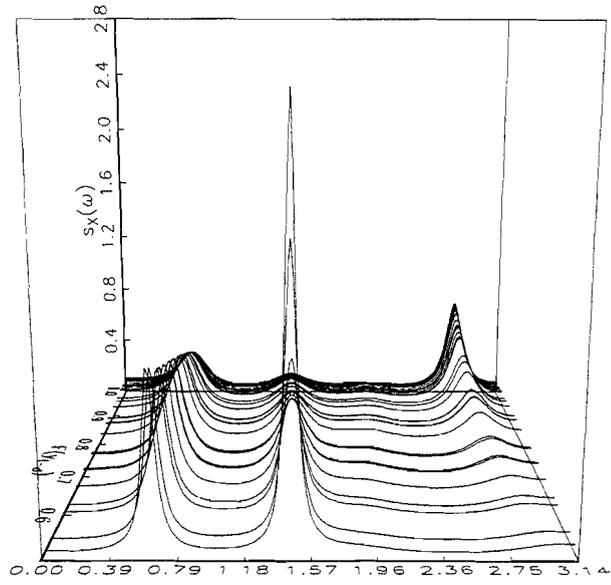


Figure 4.11. Exports (first difference of logarithm). 'Sliced' spectra.

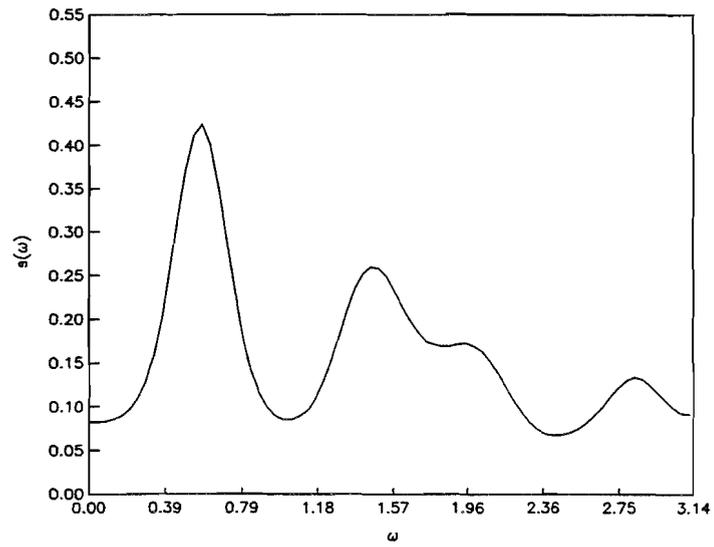


Figure 4.12. Exports (first difference of logarithm). Model spectrum.

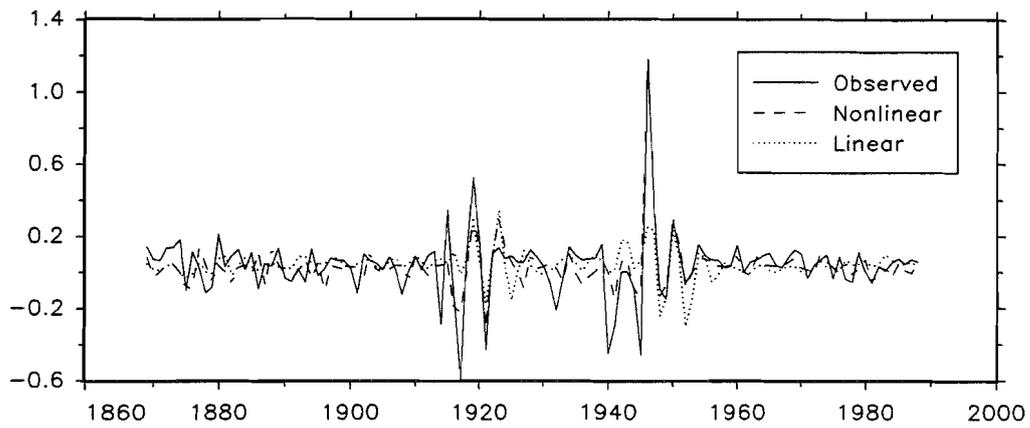


Figure 4.13. Imports (first difference of logarithm). Observed values, predictions from STAR model, and predictions from AR model.

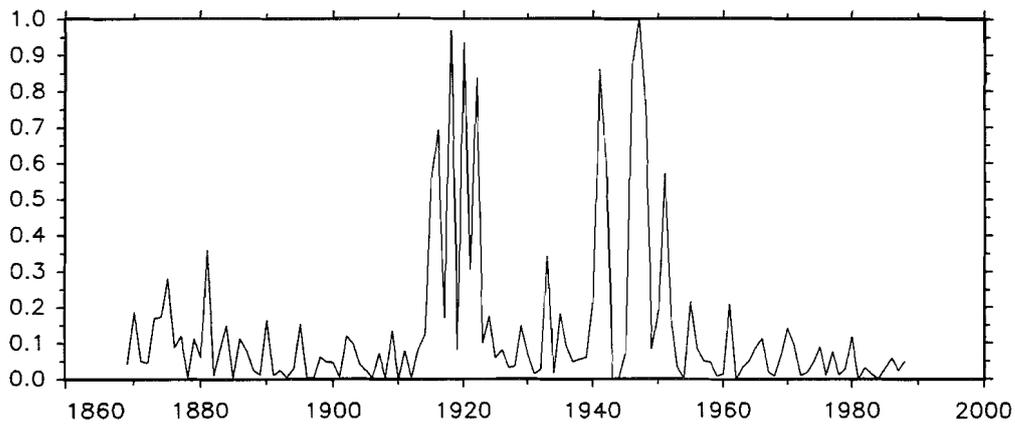


Figure 4.14. Imports (first difference of logarithm). Estimated transition function over time.

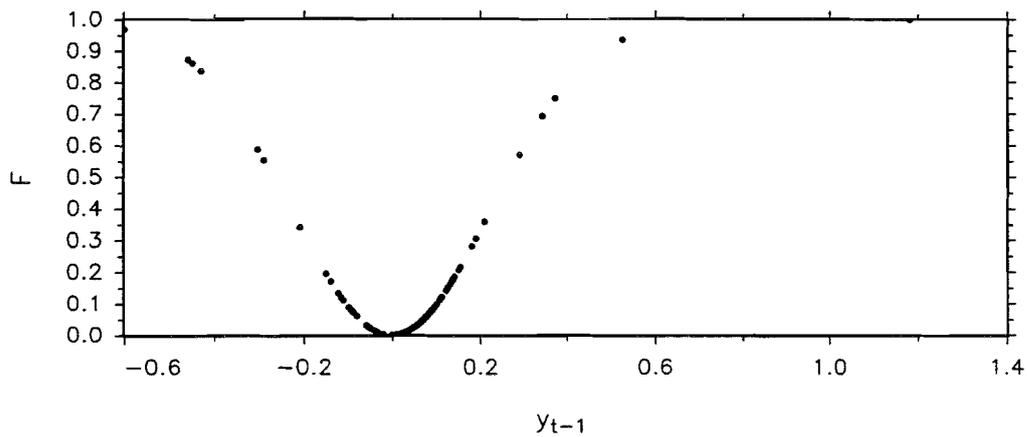


Figure 4.15. Imports (first difference of logarithm). Estimated transition function vs the transition variable.

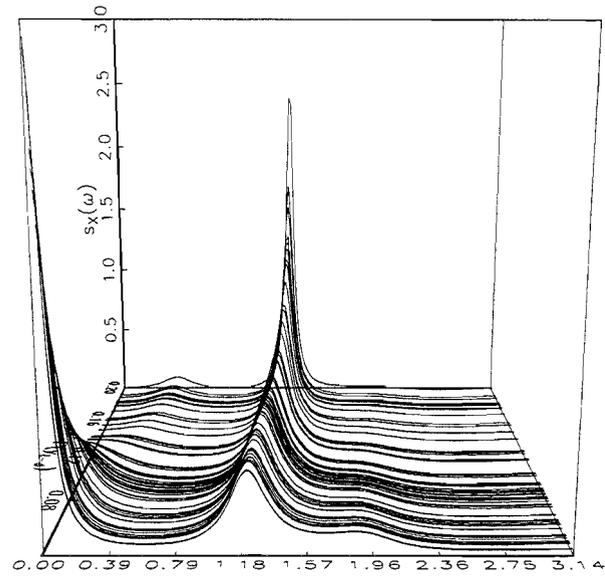
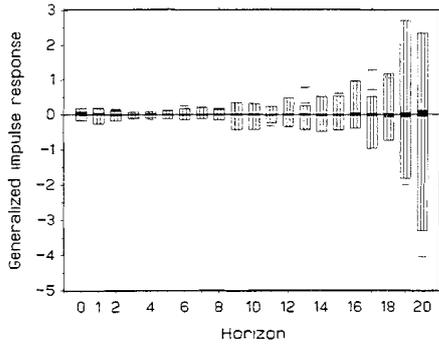
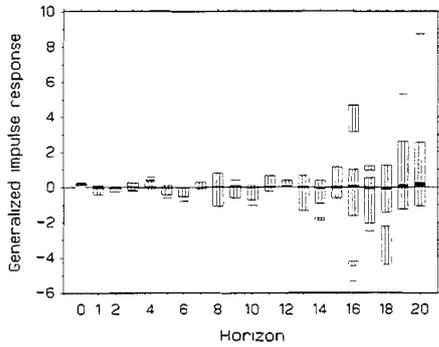


Figure 4.16. Imports (first difference of logarithm). 'Sliced' spectra.

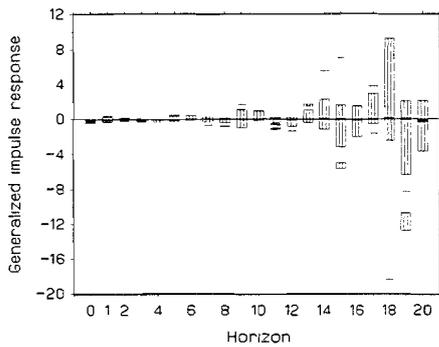
Figure 4.17.
Imports (first difference of logarithm). 50% (black) and 95% (hatched) highest density regions for the generalised impulse response to shocks



a. Response to symmetric shocks

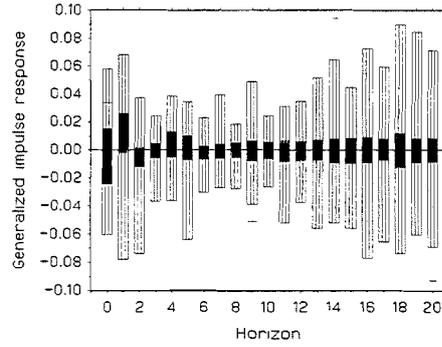


b. Response to positive shocks, greater than 1 error s.d.

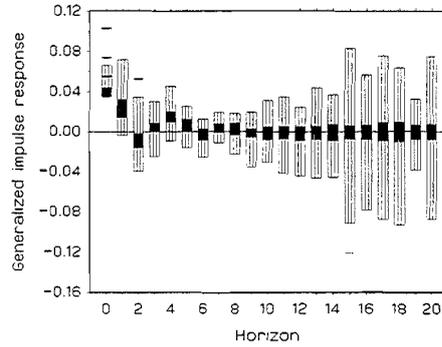


c. Response to negative shocks, smaller than -1 error s.d.

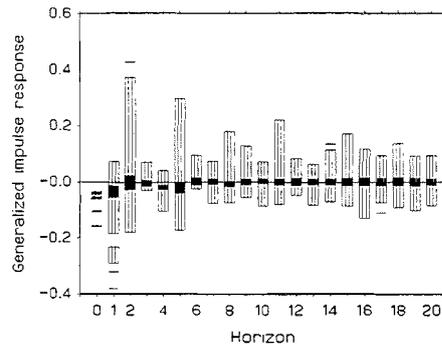
Figure 4.18.
Employment (first difference of logarithm). 50% (black) and 95% (hatched) highest density regions for the generalised impulse response to shocks



a. Response to symmetric shocks



b. Response to positive shocks, greater than 1 error s.d.



c. Response to negative shocks, smaller than -1 error s.d.

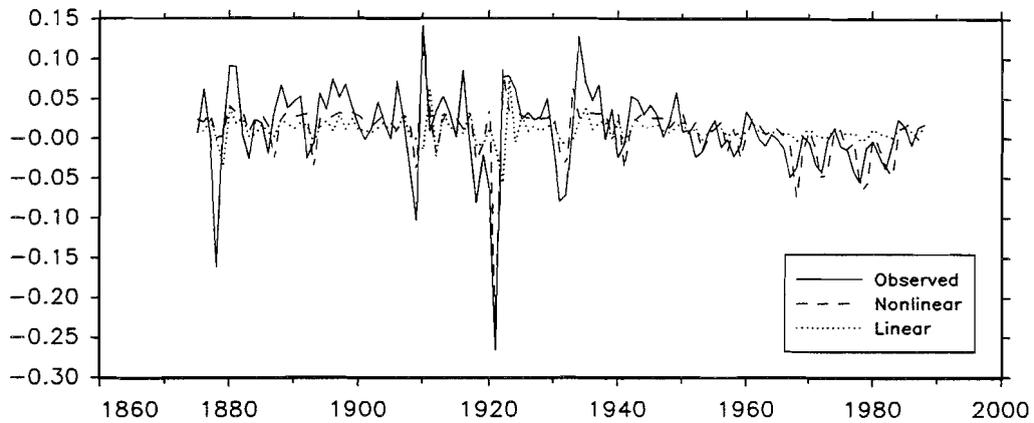


Figure 4.19. Employment (first difference of logarithm). Observed values, predictions from STAR model, and predictions from AR model.

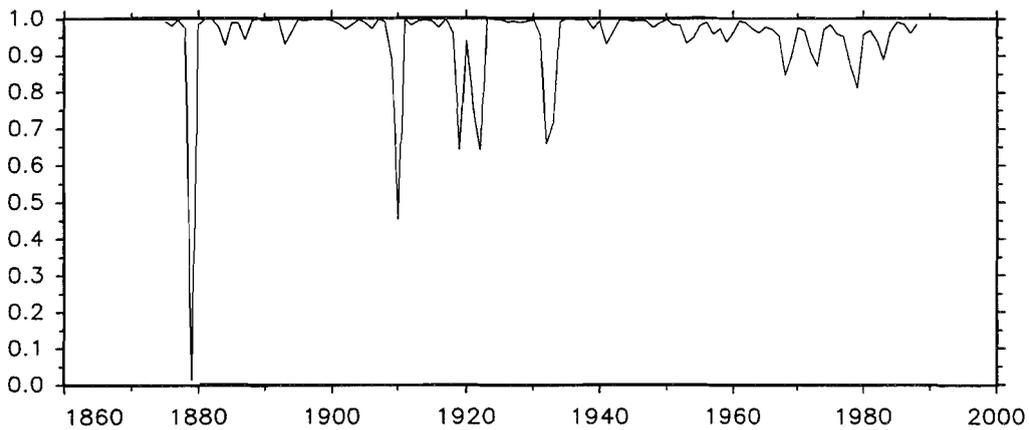


Figure 4.20. Employment (first difference of logarithm). Estimated transition function over time.

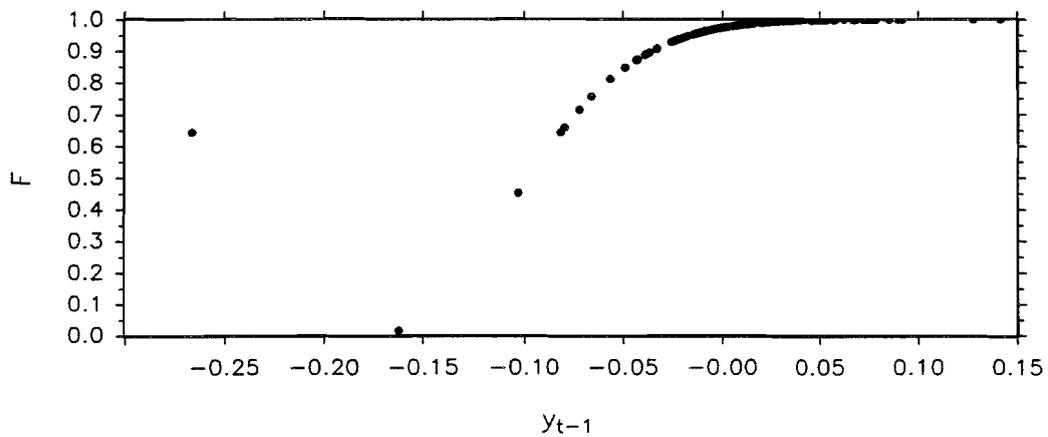


Figure 4.21. Employment (first difference of logarithm). Estimated transition function vs the transition variable.

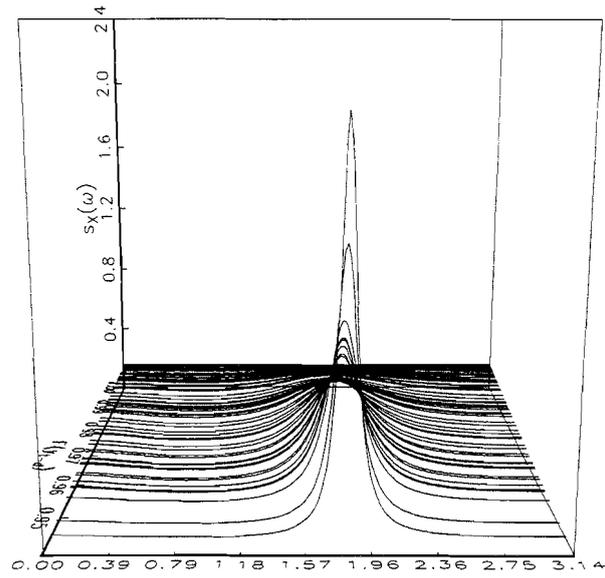


Figure 4.22. Employment (first difference of logarithm). 'Sliced' spectra.

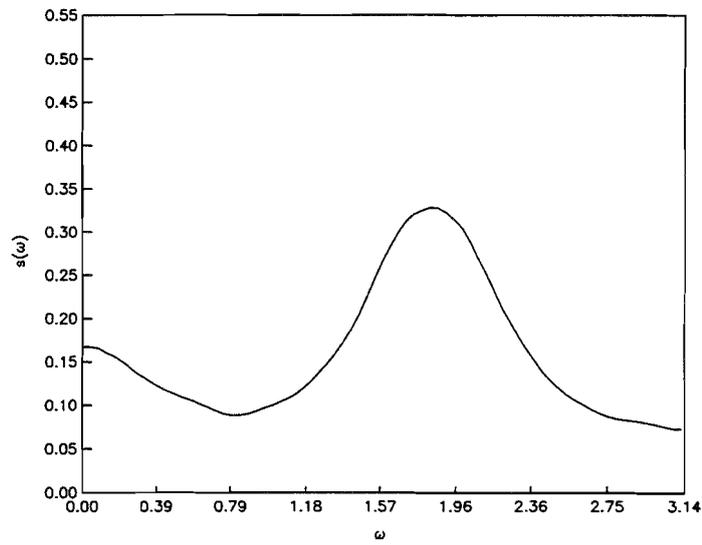


Figure 4.23 Employment (first difference of logarithm). Model spectrum.

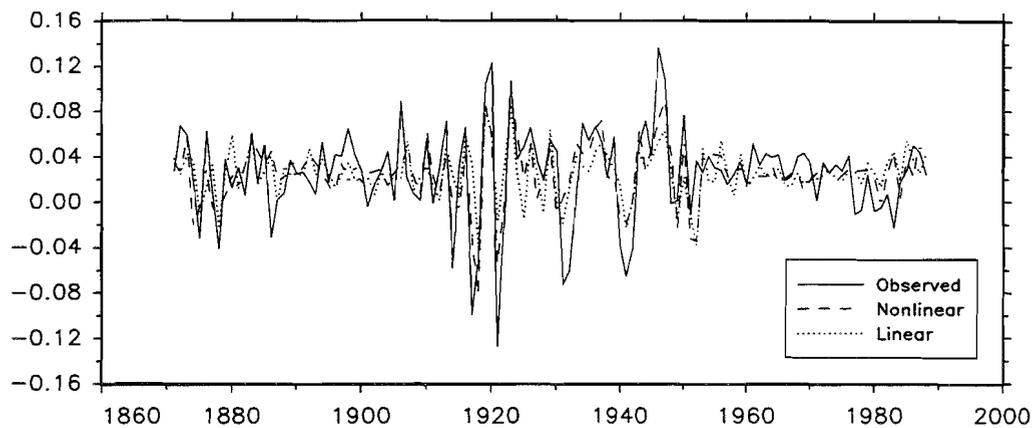


Figure 4.24. Consumption (first difference of logarithm). Observed values, predictions from STAR model, and predictions from AR model.

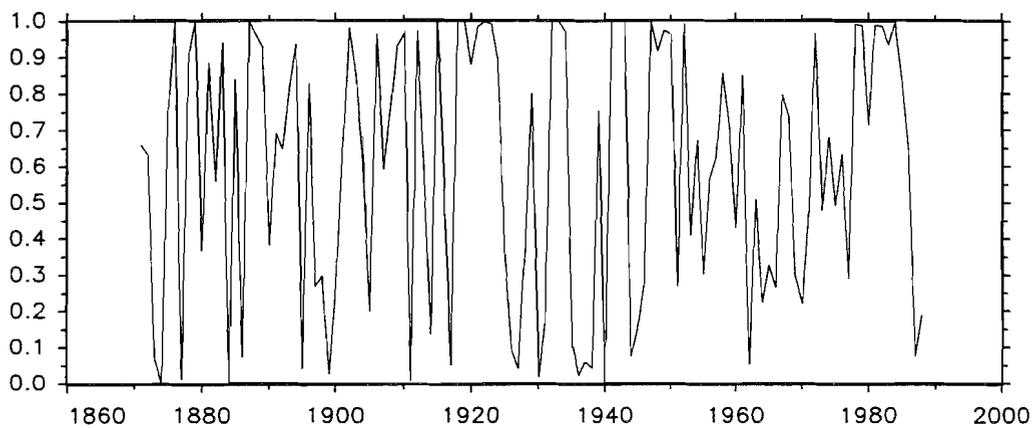


Figure 4.25. Consumption (first difference of logarithm). Estimated transition function over time.

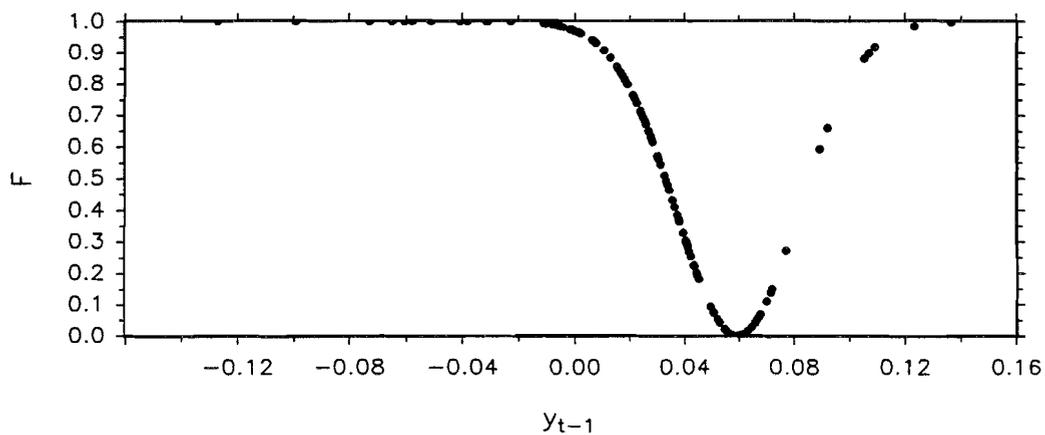


Figure 4.26. Consumption (first difference of logarithm). Estimated transition function vs the transition variable.

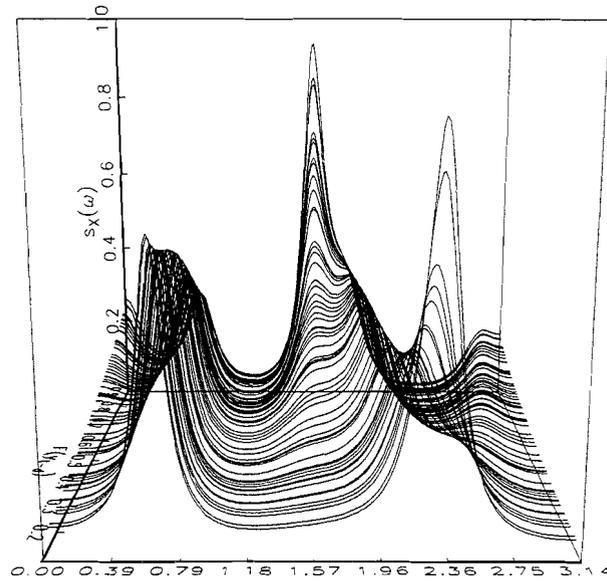


Figure 4.27. Consumption (first difference of logarithm). 'Sliced' spectra.

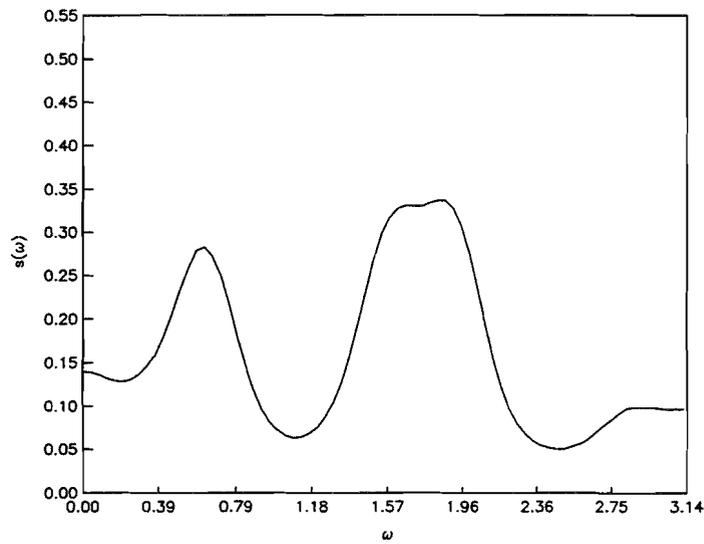
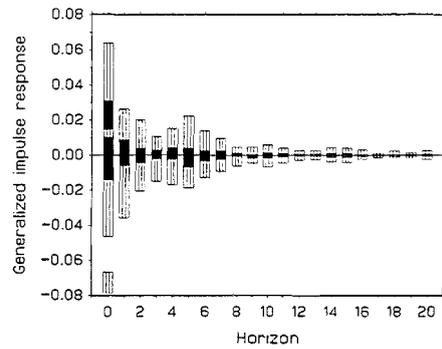


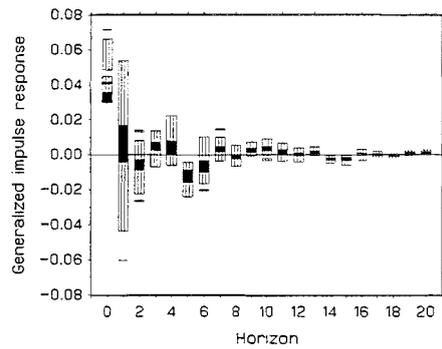
Figure 4.28. Consumption (first difference of logarithm). Model spectrum.

Figure 4.29.

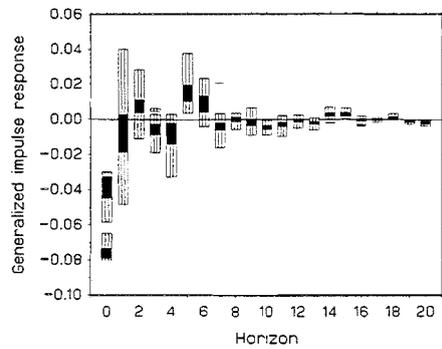
Consumption (first difference of logarithm). 50% (black) and 95% (hatched) highest density regions for the generalised impulse response to shocks



a. Response to symmetric shocks



b. Response to positive shocks, greater than 1 error s.d.



c. Response to negative shocks, smaller than -1 error s.d.

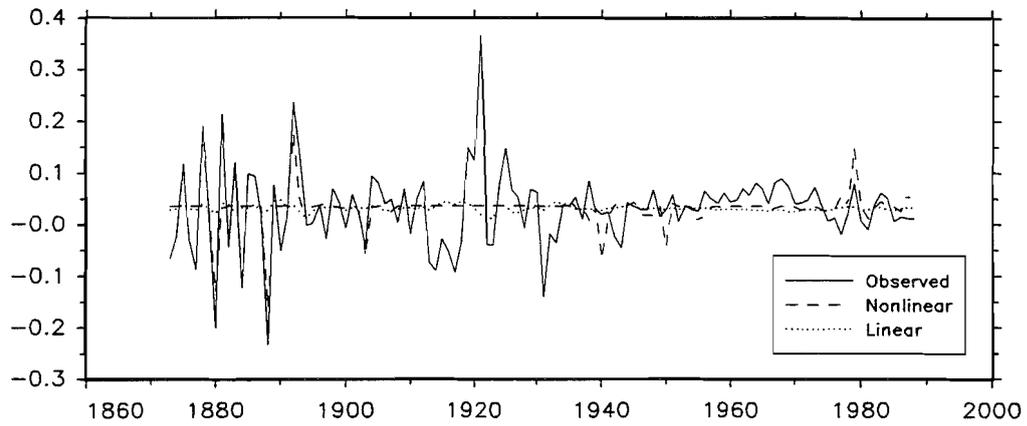


Figure 4.30. Productivity (first difference of logarithm). Observed values, predictions from STAR model, and predictions from AR model.

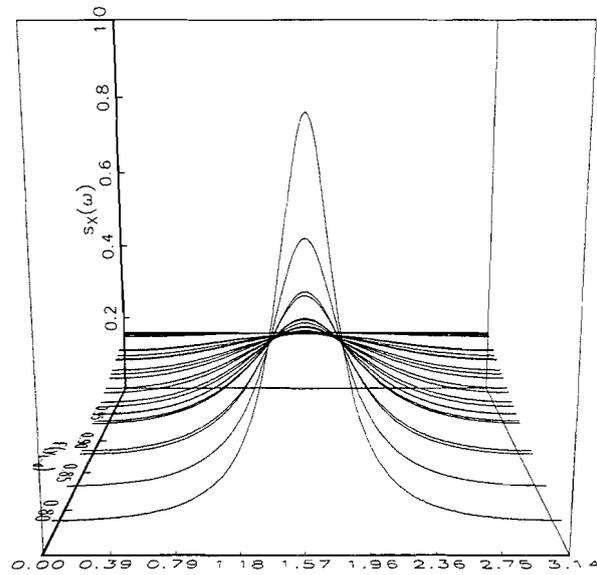


Figure 4.31. Productivity (first difference of logarithm). 'Sliced' spectra.

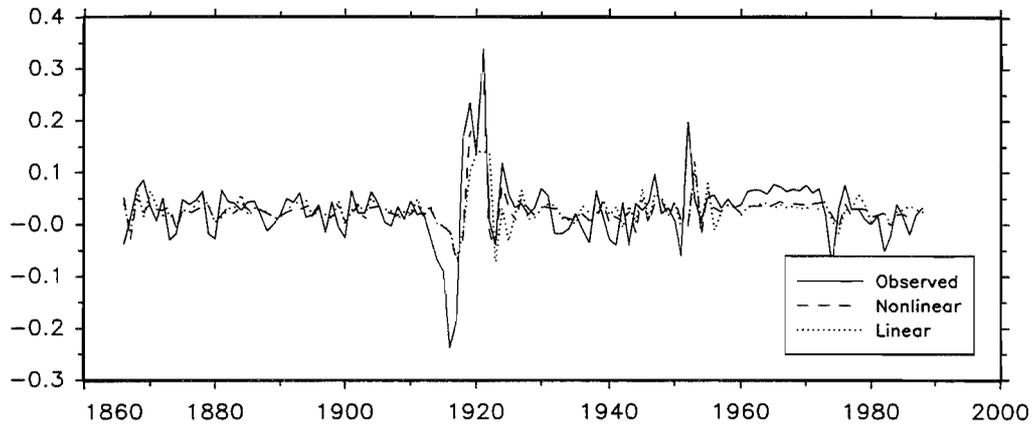


Figure 4.32. Real wage (first difference of logarithm). Observed values, predictions from STAR model, and predictions from AR model.

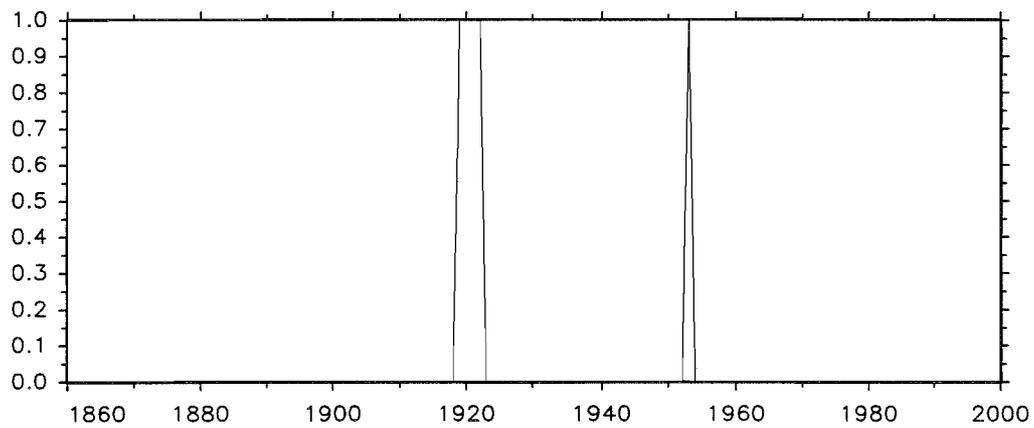


Figure 4.33. Real wage (first difference of logarithm). Estimated transition function over time.

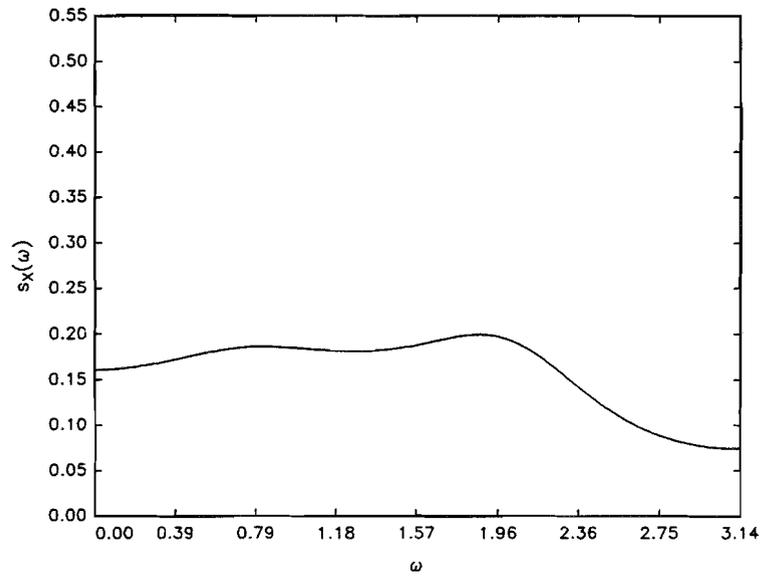


Figure 4.34. GDP (first difference of logarithm). Model spectrum, AR(4) model.

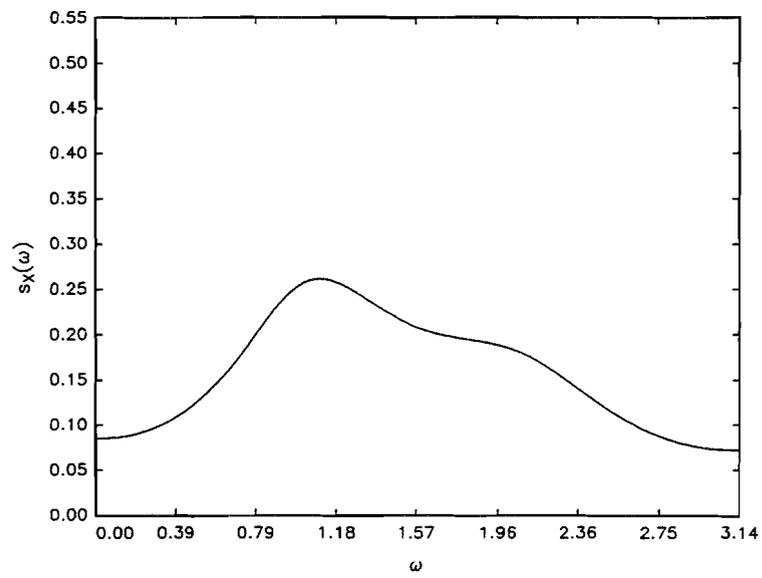


Figure 4.35. Investments (first difference of logarithm). Model spectrum, AR(4) model.

CHAPTER II

MODELLING ASYMMETRIES AND MOVING EQUILIBRIA IN UNEMPLOYMENT RATES

1. INTRODUCTION

In most OECD countries unemployment rates have increased markedly since the 1960s. Moreover, the unemployment rates of many of these countries appear to be asymmetric in the sense that they tend to increase rapidly in economic contractions but decrease slowly in periods of expansion. Whereas the increased level of unemployment has long been a main concern of economic models in the field, the feature of asymmetry has been explicitly discussed mainly in rather recent work. A large portion of the empirical work regarding European unemployment is related either to NAIRU (or 'natural rate') models of unemployment or to models assuming (long-run) hysteresis. Simply put, according to the natural rate hypothesis there is a unique long-run equilibrium unemployment level, conditional on the institutional structure of the labour market, from which the actual level may deviate only temporarily as a result of a shock. In the long run the unemployment rate will revert to the equilibrium. Models exhibiting hysteresis, on the other hand, imply the existence of infinitely many equilibrium rates; since every shock to the actual unemployment level has a

permanent effect every unemployment rate is an equilibrium rate. Blanchard and Summers (1987) demonstrated that a simple 'insider model' (see Lindbeck and Snower (1988a) for the insider-outsider theory of employment) leads to employment following a random walk with an error. They also showed that, under certain conditions, this result is obtained if the wage pressure from the outsiders (unemployed) does not depend on total unemployment but only on expected short-term unemployment. A popular way of investigating hysteresis or full persistence in unemployment rates has consequently been to test the null hypothesis that the unemployment rate has a unit root. In models characterised by persistence, but not hysteresis, the unemployment rate eventually reverts after a shock, but only very slowly.

Recently, a number of authors have suggested models that are capable of generating multiple (but finitely many) equilibrium unemployment levels associated with different welfare outcomes. In most of these models small and large shocks have different effects on the equilibrium unemployment level. Small shocks cause temporary deviations leaving the equilibrium unchanged, so that the unemployment level still fluctuates around the same level in the short run, whereas large shocks (whether structural or originating from aggregate demand) may cause a shift to a new equilibrium unemployment rate. The search and match model of Diamond (1982) is an example of a theory model that would cause employment to switch between multiple equilibria. According to that model, shocks to the economy may cause employment to get stuck in an equilibrium that is not maximising employment and consumption. A time series characterisation of such behaviour would require a nonlinear model.

Akram (1998) provides a discussion and more references on multiple equilibrium models.

If asymmetry is taken to mean that the unemployment rate increases faster than it decreases then neither hysteresis nor multiple equilibria as such imply asymmetry in unemployment rates. On the other hand, as noted above, at least some of the OECD unemployment rate series visually appear to be asymmetric in this sense. This stylised fact calls for an explanation, and various models have been suggested. Asymmetric adjustment costs of labour provide one such explanation. Several micro studies have shown that the costs of hiring and firing are not symmetric; see Hamermesh and Pfann (1996) for a survey. However, as Hamermesh and Pfann pointed out, at the moment it is not clear that asymmetry at the firm level implies asymmetry in the aggregates. On the other hand, Bentolila and Bertola (1990) argued that a microeconomic model with an asymmetric labour cost adjustment function explains much of the developments in the European unemployment rates after the first oil price shock in 1973. Another theory that might explain asymmetries is that of recessions as cleansing periods; see, for example, Caballero and Hammour (1994) and references therein. Empirical studies have shown that job destruction is highly asymmetric over the business cycle: jobs disappear at a higher rate during recessions than expansions. As this is not compensated by asymmetry in job creation, the result is asymmetry in employment. This seems to accord with the insider-outsider theory. Lindbeck and Snower (1988b) pointed out that under certain conditions, incumbent workers, i.e., insiders, would be able to prevent employment from rising during expansions. Furthermore, capital

destruction has been discussed as a constraint creating asymmetries; see, for example, Bean (1989).

Modelling asymmetry would require nonlinear time series models. Various nonlinear models have been fitted to a number of unemployment rates or their differences or functions of them for varying reasons. Neftçi (1984) used unemployment series as a business cycle indicator when he investigated asymmetry of business cycles using a two-state Markov chain; see also Sichel (1989) and Rothman (1991), and Pfann (1993) for a survey. Parker and Rothman (1997), Rothman (1998), and Montgomery, Zarnowitz, Tsay and Tiao (1998) considered forecasting US unemployment with nonlinear models. A number of authors have illustrated new statistical theory by applying it to unemployment series. Hansen (1997) developed statistical inference for the threshold parameter in a threshold autoregressive (TAR) model and fitted a TAR model to a US unemployment rate series. Brännäs and Ohlsson (1998) derived analytical temporal aggregation formulas for autoregressive asymmetric moving average (ARasMA) models and fitted an ARasMA model to monthly and quarterly Swedish unemployment rates. Tschernig (1996) checked his nonparametric model selection and estimation techniques by applying them to a set of German unemployment rates (original ratios and differences; seasonally unadjusted and adjusted).

Some work has also been done to test economic theories. Stock (1989) used US and UK data to test the time deformation idea: the dynamic properties of unemployment rate series are a function of the level of the series. With a suitable set of parameters, it could be demonstrated how a series may get stuck at a high level as the work of Diamond (1982) would suggest. This leads to models that resemble TAR and switch-

ing regression models. Franses (1998) considered the hypothesis that seasonality varies with the business cycle using unemployment rates. Bianchi and Zoega (1998) discussed the issue of multiple equilibria and studied their existence in 15 OECD unemployment series using a variant of the switching regression model with Markov switching introduced by Lindgren (1978). In their model, only the intercept is switching, the parametrisation being identical to that in Hamilton (1989). Multivariate work includes the time-varying adjustment cost model of Burgess (1992a,b) and the work of Acemoglu and Scott (1994).

This paper focuses on asymmetry and moving equilibria, both studied within a unified nonlinear framework. The primary objective is to find a satisfactory family of models suitable for parametrising the type of asymmetry seen in a number of the series, test symmetry against this model and, upon rejection of the null hypothesis, model the series using the alternative specification, thereby providing new insights into the unemployment rate dynamics. In cases where asymmetry is not found the existence and characteristics of moving levels (cf. Bianchi and Zoega, 1998) that could be interpreted as multiple equilibria are discussed. The plan of the paper is as follows: In Section 2 the model is introduced and some of its important properties are illustrated, and in Section 3 the testing and modelling procedure is outlined described. The presentation of the data set in Section 4 is followed by the empirical results in Sections 5 and 6. Section 7 concludes.

2. THE MODEL

2.1 General

As discussed in the Introduction testing the null hypothesis of a unit root has been a common way of analysing unemployment rate series. Testing the null hypothesis of a unit root is tantamount to having hysteresis or full persistence as the null hypothesis. In a recent survey, Røed (1997) reported that the unit root hypothesis has rarely been rejected for any country or unemployment series. Some of the seasonally adjusted US monthly or quarterly unemployment rates seem to constitute the only exceptions to this pattern. The implication of this result is that the unemployment rate is fully persistent. In other words, in a univariate setting, excluding the possibility of a drift, any unemployment rate is an equilibrium rate. Besides, as the tests assume linearity, at any level, be it 2% or 22%, say, the rate is as likely to go up as it is to move down from that level.

The evidence collected by Røed (1997) indicates that empirical modelling of an unemployment rate series starting with a test of hysteresis is unlikely to take asymmetry into account at any stage of the modelling process. Therefore, the analysis in the present paper has a different starting-point: the unemployment rate is assumed to be a stationary process. Since the rate is a bounded variable this assumption may not be an unrealistic one. Furthermore, from the outset, the process is assumed to be linear. The linearity assumption is tested against an alternative of nonlinearity parametrised in such a way that asymmetric behaviour of the unemployment rate is

permitted. If linearity is rejected against this alternative, a nonlinear model for the unemployment rate is specified, estimated, and evaluated, and the properties and implications of the estimated model, e.g., concerning persistence, are discussed. The possibility of multiple equilibria will also receive attention.

2.2 An artificial example

The approach to modelling asymmetries in unemployment rates suggested in this paper can be illustrated by an example. Figure 2.1 depicts a stylised ‘unemployment series’, though without seasonality. The series grows rapidly and decreases rather slowly, and a relatively long decrease is followed by another rapid upsurge. The time and magnitude scales are arbitrary. In fact, the series has been generated by the following simple nonlinear autoregressive model

$$\begin{aligned} \Delta y_t = & \mu_1 + \alpha_1 y_{t-1} + (\mu_2 + \alpha_2 y_{t-1}) \\ & \times \left(1 + \exp\{-\gamma(\Delta y_{t-1} - c)\} \right)^{-1} + u_t, \end{aligned} \quad (2.1)$$

where $u_t \sim \text{nid}(0, \sigma_u^2)$, and $\mu_1=0$, $\alpha_1=-0.01$, $\mu_2=0.32$, $\alpha_2=-0.20$, $\gamma=1000$, $c=0.05$, and $\sigma_u^2=0.02^2$. As can be seen, the realisation displays many features that are characteristic of asymmetric unemployment series.

Although parametrised using first differences, equation (2.1) is seen to be a level equation. The process behaves as follows. When the value of the process (y_t) is close

to zero the next observation (y_{t+1}) tends to be near zero as well, unless a sufficiently large positive shock arrives. In the latter case, in the next period the large value increases the value of the logistic transition function (which is a function of the first difference of y_t). It can be noted that the value of the steepness parameter γ is so high that the value of the logistic function changes rapidly from zero to unity around the location parameter c as a function of Δy_{t-1} . Supposing that the next value is unity the next observation is generated by

$$\Delta y_{t+1} = 0.32 - 0.21y_t + u_{t+1} \quad (2.2)$$

which means that it is likely to be clearly higher than the previous one because the process is still far below the mean of (2.2) which is about 1.5. The (positive) mean is determined by $\alpha_1 + \alpha_2 < 0$ and $\mu_1 + \mu_2 > 0$. It is essential that $\alpha_2 < 0$ and $\mu_2 > 0$. Thus the growth continues until the realisation approaches the mean of the above regime. Then any sufficiently large negative shock will slow down the process, and this slowdown shows in the transition function in the next period. The process then may switch back to

$$\Delta y_{t+1} = -0.01y_t + u_{t+1} \quad (2.3)$$

corresponding to the value zero of the transition function. The drift towards the original zero level is weak, however, as indicated by the near-zero negative coefficient of y_t in (2.3). Thus the coefficient α_1 controls the speed of adjustment to the lower level. The parameter c determines the magnitude of the shock that will trigger growth

in the realisation when the process is decreasing or fluctuating at the bottom level. It should be noted that (2.1) with the parameter values used in the example does not imply the existence of multiple equilibria.

More lags may be added to (2.1) but the basic idea remains the same. The key is that the variable in the transition function is a difference while the process itself is expressed in levels and remains bounded in probability. In the next section a more general version of the model will be presented and discussed more formally.

2.3 The LSTAR model

The model is a generalisation of (2.1) and can be written as

$$\begin{aligned} \Delta y_t = & \mu_1^0 + \mu_1^1 H_k(t^*; \gamma_1, \mathbf{c}_1) + \alpha_1 y_{t-1} + \sum_{j=1}^p \beta_{1j} \Delta y_{t-j} + \sum_{i=1}^{s-1} (\delta_{1i}^0 + \delta_{1i}^1 H_k(t^*; \gamma_1, \mathbf{c}_1)) d_{it} + \\ & + \left\{ \mu_2^0 + \mu_2^1 H_k(t^*; \gamma_1, \mathbf{c}_1) + \alpha_2 y_{t-1} + \sum_{j=1}^p \beta_{2j} \Delta y_{t-j} + \sum_{i=1}^{s-1} (\delta_{2i}^0 + \delta_{2i}^1 H_k(t^*; \gamma_1, \mathbf{c}_1)) d_{it} \right\} \\ & \times G(\Delta_s y_{t-d}; \gamma, c) + u_t \end{aligned} \quad (2.4)$$

where y_t is the unemployment rate in percent, d_{it} denote seasonal dummies, $i=1,2,\dots,s-1$, $s=4$ for quarterly data, and $u_t \sim \text{nid}(0, \sigma_u^2)$. The function $G(\Delta_s y_{t-d}; \gamma, c)$ is a logistic transition function of $\Delta_s y_{t-d}$ given by

$$G(\Delta_s y_{t-d}; \gamma, c) = \left[1 + \exp\left\{-\gamma(\Delta_s y_{t-d} - c) / \sigma(\Delta_s y_t)\right\} \right]^{-1}, \gamma > 0 \quad (2.5)$$

where $\sigma(\Delta_s y_t)$ is the standard deviation of $\Delta_s y_t$ and $\gamma > 0$ is an identifying restriction. The value of the delay parameter d in (2.5) is generally unknown and has to be specified from the data. Furthermore,

$$H_k(t^*; \gamma_1, \mathbf{c}_1) = \left(1 + \exp\left\{-\gamma_1 \prod_{j=1}^k (t^* - c_{1j})\right\} \right)^{-1}, \quad k = 1, 2, 3 \quad (2.6)$$

where $t^* = t/T$ (i.e., time in the transition function is scaled between 0 and 1), $\gamma_1 > 0$ and with $\mathbf{c}_1 = (c_{11}, \dots, c_{1k})'$, for $k = 1, 2, 3$ where $c_{1i} \leq c_{1j}$ for $i < j$. The function (2.6) with $k=1$ describes a monotonic parameter change over time that becomes a single break when $\gamma_1 \rightarrow \infty$, whereas $k=2$ allows symmetric change around $(c_{11} + c_{12})/2$ with a structural break and a counterbreak as the limiting case as $\gamma_1 \rightarrow \infty$. Finally, $k=3$ permits nonmonotonic and nonsymmetric change, but H_3 may also be a monotonically increasing function of t .

Model (2.4) is an example of the well-known logistic smooth transition autoregressive (LSTAR) model; see, for example, Granger and Teräsvirta (1993) or Teräsvirta (1994, 1998). However, it has been modified and extended slightly. First, the transition variable in G is a difference. This is essential as demonstrated by the example of the previous section. Enders and Granger (1998) discussed a similar construction for a threshold autoregressive (TAR) model and called their model the Momentum TAR model. (The two-regime TAR model is a special case of the LSTAR model obtained

as $\gamma \rightarrow \infty$ in G .) Second, the model contains another transition function H_k as in the multiple regime STAR model of van Dijk and Franses (1997a); see also Eitrheim and Teräsvirta (1996). This function accounts for the possibility that the seasonal pattern of unemployment rates may change over time, which is not uncommon. Since it also enables the model to handle smooth (or abrupt) level changes, (2.4) is capable of characterising moving and multiple equilibria. It should be noted that the seasonal pattern can also change with the growth rate of unemployment, allowing a multiple effect from the two sources; cf. the discussion of Franses (1998). The model could be defined as a general multiple regime STAR model but that has not been necessary.

A crucial property of this model in the present context is that it allows asymmetric behaviour by permitting 'local' nonstationarity in a model that yet may be globally stable. It should be noted that using the seasonal rather than the first difference as the transition variable is important, because the asymmetries of primary interest here are those pertaining to the unemployment cycle, not those of the annual seasonality cycle. Even without seasonality, a relatively long difference would normally be preferable to the first difference, since if the transition variable is rather noisy then a first difference does not represent variations in the unemployment cycle and smoothing in the form of a longer difference becomes necessary.

2.4 Unit root and asymmetry

As mentioned before, testing the hypothesis of a unit root has been common in unemployment series modelling, and the null hypothesis has usually been accepted. Therefore, it might be of interest to see what happens if a unit root test is performed on a series like the artificial one in the example above. Would the null hypothesis normally be rejected? To find out, a small simulation experiment was carried out applying a standard augmented Dickey-Fuller test. The data were generated both according to the LSTAR model (2.1) and according to the linear part of the same model, using the same random numbers in both cases. Thus, the linear generating process was simply

$$\Delta y_t = \alpha_1 y_{t-1} + u_t \quad (2.7)$$

where $u_t \sim \text{nid}(0, 0.02^2)$. The lag-level parameters α_1 and α_2 were varied, using the values -0.01, -0.02, -0.05, -0.1 for α_1 and 0, -0.01, -0.1, and -0.2 for α_2 . In order to generate realisations with peaks of approximately the same magnitude over the different combinations of α_1 and α_2 , the value of μ_2 was adjusted from one experiment to the other. Figure 2.2 gives an example of the realisations produced by the different parameter combinations, using the sequence of random numbers that generated the realisation in Figure 2.1. Series of 200, 500 and 1000 observations were generated. The lag length used in the augmented Dickey-Fuller test was determined by adding lags until the Ljung-Box test statistic no longer rejected the null hypothesis at the significance level 0.05. In practice, the resulting lag length was almost always equal to

one or two. The augmented Dickey-Fuller tests were computed with and without a constant and a trend.

The results of the simulations based on 10000 replicates each can be found in Tables 2.1-2.3. The figures in the tables are the relative rejection frequencies at the 5% level. The rightmost column of each table refers to the linear case, in which the data have been generated by assuming $\alpha_2 = \mu_2 = 0$ in (2.1).

For the smallest sample size, $T=200$ (Table 2.1), it is seen that the test version without a constant and a trend has the highest frequency of null hypothesis rejections, and for this test statistic the rejection frequencies for the nonlinear models are of roughly the same magnitude as for the linear ones. The differences between the test results for nonlinear and linear series using this test statistic become somewhat more pronounced when the sample size increases. For the top rows, representing the asymmetric series, the rejection frequencies for the nonlinear realisations are below those for the linear ones for most values of α_2 when $T=500$ and $T=1000$ (Tables 2.2 and 2.3). For the two other panels, i.e., the ADF tests with a constant and possibly a trend, the evidence for $T=500$ and $T=1000$ is more mixed. Comparing the results for $T=500$ and $T=1000$ it seems that the ADF tests may not be consistent against the LSTAR model. Taken together, the results appear to suggest that at sample sizes often encountered in applied work, asymmetry as defined by (2.1) does not provide additional evidence against the unit root hypothesis compared to the corresponding linear model. Finding a unit root on the one hand and nonlinearity (asymmetry) on the other using the same data thus need not be a contradiction.

3. TESTING LINEARITY AND MODELLING NONLINEAR SERIES

A practical advantage of considering models of the smooth transition regression (STR) or autoregression (STAR) type in this context is that there already exists an established modelling procedure for STR and STAR models that can be applied to the unemployment series. The modelling cycle, fully described in Teräsvirta (1998), comprises three stages: testing linearity against STAR, if linearity is rejected specifying and estimating the STAR model, and evaluating the estimated model. In this case the modelling does not, however, start directly with the full model (2.4) but rather in stages. As noted in the Introduction, the possible asymmetry expressed through the transition function G is of primary interest and is treated first. Thus initially it is assumed that seasonality is constant over time and that there exists just a single equilibrium, i.e., $H_k \equiv 0$. The specification and estimation of the LSTAR model is carried out under this assumption, and the assumption is tested when the estimated model is evaluated. If it is rejected, the model is augmented as indicated by (2.4) and the parameters of the augmented model are estimated.

Taking into account the restrictions mentioned in the previous paragraph, the modelling begins with the following LSTAR model:

$$\begin{aligned}
\Delta y_t = & \mu_1 + \alpha_1 y_{t-1} + \sum_{j=1}^p \beta_{1j} \Delta y_{t-j} + \sum_{j=1}^{s-1} \delta_{1j} d_{j,t} + \\
& + \left(\mu_2 + \alpha_2 y_{t-1} + \sum_{j=1}^p \beta_{2j} \Delta y_{t-j} + \sum_{j=1}^{s-1} \delta_{2j} d_{j,t} \right) \\
& \times \left(1 + \exp \left\{ -\gamma (\Delta_s y_{t-d} - c) / \sigma (\Delta_s y_{t-d}) \right\} \right)^{-1} + u_t, \gamma > 0
\end{aligned} \tag{3.1}$$

where $u_t \sim \text{nid}(0, \sigma_u^2)$. The linearity test used here is the LM-type test described in Granger and Teräsvirta (1993, ch. 6) and Teräsvirta (1994, 1998). A linear model with a lag structure such that the errors can be assumed to be free from autocorrelation is first selected. To this end, Δy_t is regressed on $y_{t-1}, \Delta y_{t-1}, \dots, \Delta y_{t-p}$, seasonal dummies, and a constant, for various values of p , and the value minimising an information criterion (AIC) is selected to be the null model. Linearity is tested against (3.1) with and without seasonal dummies in the nonlinear part of the model. This is done separately for different lags (d) of the seasonal difference of unemployment as the transition variable. If linearity is rejected for more than one value of d , the lag yielding the smallest p -value is selected. (See Teräsvirta (1994) for the motivation behind this selection rule.) If the p -values for a number of different values of d are of similar magnitude, more than one model may be tentatively estimated and the final choice left to the evaluation stage. After rejecting linearity and choosing d the LSTAR model is estimated by nonlinear least squares. At this stage, the size of the model may be reduced by imposing exclusion restrictions whenever appropriate.

Finally, the adequacy of the estimated model is evaluated. An LM test of serially uncorrelated errors is performed as well as the test of no autoregressive conditional heteroskedasticity. The hypothesis of no remaining nonlinearity is tested against an alternative of additional nonlinearity of the STR type. Furthermore, since constancy of

parameters is a crucial assumption underlying the estimation of the model, this hypothesis is tested against an alternative of smoothly changing parameters. This is done for the case where the alternative covers all parameters except γ and c , but testing against (2.4) is of course of particular interest. If that null hypothesis is rejected then (2.4) is estimated. The results of the different statistics from the three parameter constancy tests corresponding to $k=1,2,3$ in (2.6) help select k . For details, see Teräsvirta (1998), Eitrheim and Teräsvirta (1996), and Jansen and Teräsvirta (1996). Rejecting the general hypothesis may be taken to indicate misspecification and should in that case lead to a respecification of the model.

4. DATA

The data consists of quarterly, seasonally unadjusted unemployment rate series from a number of OECD countries. The selection of series has been determined by the availability of seasonally unadjusted unemployment figures starting no later than in the early 1970s. Seasonally adjusted series are not used here because the effects of heavy two-sided filtering on asymmetric series are not known. The countries included are USA, Canada, Japan, Australia, (West) Germany, Austria, Italy, Denmark, Finland, Norway, and Sweden. The German series ends in 1991 to avoid the effects of the German unification and the resulting structural break in the series. The main data source is the *OECD Main Economic Indicators*, but for some countries data of higher quality have been obtained from other sources. (The complete data set is available from the author.) The series are depicted on a comparable scale in Figure 4.1. It

should be mentioned that none of the three ADF tests (no constant; constant, no trend; constant and trend) rejects the unit root hypothesis for any of the series at the 10% significance level).

5. RESULTS: ASYMMETRY

5.1 Linearity tests

As noted above, the modelling begins with linearity tests, the main results of which are summarised in Table 5.1. Linearity is not rejected against (3.1) for either the US, Japanese, Norwegian, or Canadian series. For all of the European unemployment series except the Norwegian one, and for the Australian one, linearity is rejected, and nonlinear model building is attempted for these series. For the Austrian and Italian series, however, the LSTAR model does not seem to be an appropriate alternative to linearity. In the Italian case, it has not been possible to estimate a reasonable LSTAR model for the series. The model suggested by the specification procedure for the Austrian series can be estimated in the sense that convergence is achieved, but parameter estimates, tests against misspecification, and other statistics indicate that the estimated model is not a meaningful one. In fact, Figure 4.1 already conveys the impression that the form of asymmetry that is the main concern of this study is not present in the Austrian series. In Section 6, these two series will be discussed further, along with the ones for which linearity was not rejected at all.

The next subsection presents the modelling results for the (West) German series in some detail, thereby also introducing the different descriptive techniques used to highlight interesting features of the estimated models and to discuss their dynamic properties. The main results and characteristics of the other successful models are then summarised more briefly in the following subsection.

5.2 Germany

As linearity is rejected for the German series an LSTAR model of order 8, with $d=1$, and with seasonal dummies in both the linear and the nonlinear part of the model is estimated. The residuals from the estimated model exhibit no serial correlation. However, the tests of no additional nonlinearity indicate misspecification, and the null hypothesis of parameter constancy is strongly rejected in favour of an alternative with a time-varying intercept in the nonlinear part of the model, i.e., a model where, under the alternative, μ_2 is replaced by $\mu_2^0 + \mu_2^1 H_k(t^*; \gamma_1, \mathbf{c}_1)$. Since the test result in this case gives little information as to which one of the three alternative transition function specifications to choose, all three models have been estimated. The most parsimonious specification, with $k = 1$, turns out adequately to capture the parameter variability detected by the test and the finally estimated model has the form

$$\begin{aligned}
\Delta y_t = & -0.013y_{t-1} + 0.51\Delta y_{t-4} - 0.31\Delta y_{t-5} + 0.20\Delta y_{t-8} & (5.1) \\
& (0.0081) \quad (0.14) \quad (0.080) \quad (0.080) \\
& + 0.20d_1 + 0.17d_2 - 0.29d_3 \\
& (0.12) \quad (0.17) \quad (0.11) \\
& + \left[0.58 + 1.27 \left(1 + \exp \{ -2.74(t^* - 0.50) / \hat{\sigma}(t^*) \} \right)^{-1} \right. \\
& \quad (0.21) \quad (0.61) \quad (1.54) \quad (0.072) \\
& \left. - 0.15y_{t-1} + 0.58\Delta y_{t-1} - 0.36\Delta y_{t-4} + 0.024d_1 - 1.66d_2 + 0.098d_3 \right] \\
& \quad (0.051) \quad (0.14) \quad (0.22) \quad (0.22) \quad (0.39) \quad (0.24) \\
& \times \left[1 + \exp \{ -4.29(\Delta_4 y_{t-1} - 0.31) / \hat{\sigma}(\Delta_4 y_t) \} \right]^{-1} + \hat{u}_t \\
& \quad (2.18) \quad (0.12)
\end{aligned}$$

$s = 0.23$	skewness = 0.86	excess kurtosis = 2.29
LJB = 40.6 (1.5×10^{-9})	AIC = -2.78	SBIC = -2.34
LM = 0.15 (0.86)	SDR = 0.81	$R^2 = 0.92$

In this and following equations, figures in parentheses below parameter estimates denote estimated standard errors, otherwise they are p -values of tests. Here, s is the estimated standard deviation of the residuals, LM denotes the LM statistic of no ARCH (Engle, 1982) computed with two lags, SDR denotes the ratio s / s_{LIN} , where $s_{LIN} = \sqrt{\frac{1}{T-k_{LIN}} \sum \hat{\varepsilon}_t^2}$, $\hat{\varepsilon}_t$ are the residuals from a regression of Δy_t on a constant, y_{t-1} , Δy_{t-j} , $j = 1, \dots, p$, and seasonal dummies, and k_{LIN} is the number of parameters in this linear model. The Lomnicki-Jarque-Bera (LJB) test rejects normality due to a small number of large residuals. The standardisation of the transition function exponent by dividing it by $\hat{\sigma}(\Delta_4 y_t)$ makes γ scale-free.

Containing 19 parameters, equation (5.1) does not appear to be parsimonious. However, eight (almost one half) of the parameters are directly related to modelling seasonality: the coefficients of the seasonal dummy variables and those of Δy_{t-4} . The modelling of the smooth level shift in the peak of the unemployment rate accounts for

another four parameters. In comparison, already the linear AR(8) model with seasonal dummy variables contained 13 parameters. It should be noted that $\hat{\alpha}_2 < 0$ and, in this case, $\hat{\mu}_2^0 + \hat{\mu}_2^1 \hat{H}_1 > 0$, as may have been expected from the example in Section 2.2. As for misspecification tests, the test statistics of no error autocorrelation are not significant and none of the tests of no additive nonlinearity with seasonal dummies included now rejects at the 5% level. The tests of parameter constancy now all have clearly higher p -values than the previous tests, but there are still indications of parameter nonconstancy. Since the remaining parameter variation is not easily captured by including more parameters into the time-varying subset of parameters in (5.1), the model is retained for the moment despite the fact that it is not entirely satisfactory. (All the results mentioned here are available from the author upon request.)

Figure 5.1 shows the observed unemployment level (upper panel) and the two estimated transition functions plotted over time (lower panel): the transition function for the model, taking values at or close to one during the upsurges, and the transition function of time for the varying intercept of the nonlinear part of the model, increasing smoothly over time allowing the peaks of the unemployment rate to increase over the observation period. In Figure 5.3 the estimated transition function is plotted against the transition variable with one dot for every observation in the sample (dots may overlap so that a single dot may represent more than one observation). The transition between the two extreme regimes is seen to be smooth. Figure 5.2 depicts the residuals from the nonlinear model together with the residuals from the linear model used as a basis for linearity testing. The time series plot indicates that the major contributions

of the nonlinear model are obtained where the unemployment rate is growing. Elsewhere in the sample, the gains from fitting a nonlinear model seem to be minor.

To illustrate the dynamic properties of model (5.1), the persistence of shocks in particular, generalised impulse response (GIR) functions have been estimated for this model. The reader is referred to Koop, Pesaran and Potter (1996) for a general definition and discussion of GIR functions and to Appendix 1 for the computational details in this particular application. The estimated GIR functions are presented graphically in Figure 5.4. As in Chapter I, highest density regions (Hyndman 1995, 1996) are used to show the distribution of functions up to 20 quarters ahead. Three GIR functions are shown in Figure 5.4. The first one is a general GIR function based on all shocks, the second one is based on negative shocks greater than one residual standard deviation in absolute value and the third one on corresponding positive shocks. The general GIR function shows that a shock to the unemployment rate can be quite persistent. After 20 quarters the density function still has not shrunk even close to a point. The other two figures reveal the important role of the positive shocks in explaining this observation. A sufficiently large positive shock at a crucial moment may trigger a strong increase in the unemployment rate, which shows as pronounced multimodality in the GIR function. These shocks are very persistent. The corresponding graph for the negative shocks is not a mirror image of the one for positive shocks, which is another indication of the asymmetry pertaining to the model. While the general conclusion about persistence is similar to the one that may be drawn from a unit root test, the other implications of this analysis are not.

The persistence of the level of unemployment (hysteresis) may be illustrated by considering the deterministic extrapolation of the model. Figure 5.5 shows the trajectory obtained by extrapolating the estimated model starting from the most recent values without adding noise. It turns out that the estimated model contains a limit cycle. The trajectory is a self-repeating asymmetric cycle fluctuating (ignoring seasonal variation) between 6 and 10 per cent. Extrapolations starting from earlier points in time, in the 1960s and 1970s, also exhibit limit-cycle behaviour, but the amplitude of the cycle is less than from the 1980s onwards. Figure 5.6 illustrates how these projections change over time when the extrapolation starts from different points of time in the sample period. Deterministic projections using the estimated model (5.1) are computed, starting from $t=t_0, t_0+1, \dots, T$, where t_0 is the first possible starting point in the sample allowing for the necessary lags, and T is the last observation in the sample. All projections end at the same point in time, i.e., in this case 50 years beyond the end of the sample as described in the previous paragraph. The minimum, maximum and average over the last m observations in the predicted sequence obtained for every starting point t are recorded and assigned to the time t . The series of these averages, minima, and maxima over time, using $m=60$ (15 years) in order to let the the emerging cyclical behaviour be reflected by the two bordering curves, are plotted in Figure 5.6. Peel and Speight (1998) found a limit cycle in a somewhat different STAR model for the seasonally adjusted UK unemployment series. While a limit cycle in (5.1) may be interpreted as suggesting the presence of endogenous cycles in the (West) German economy, a more prudent interpretation is that the unemployment rate of the country has been a strongly persistent variable.

5.3 Other countries

The other countries for which informative unemployment rate STAR models have been estimated are Australia, Denmark, Finland, and Sweden. This subsection discusses the modelling of the individual series concluding with some general comments.

Finland

For the Finnish series, both a model with $d=1$ and a model with $d=2$ are initially estimated, and the parameter estimates are similar. The latter model, having slightly better information criteria, is selected. The finally estimated model is

$$\begin{aligned}
 \Delta y_t = & -0.0031y_{t-1} + 0.32\Delta y_{t-3} + 0.35\Delta y_{t-5} - 0.26\Delta y_{t-6} & (5.2) \\
 & (0.012) \quad (0.074) \quad (0.13) \quad (0.084) \\
 & + 0.85d_1 - 0.59d_2 - 0.0049d_3 \\
 & (0.12) \quad (0.11) \quad (0.12) \\
 & + \left[-0.017y_{t-1} + 0.67\Delta y_{t-1} + 0.50\Delta y_{t-4} - 0.90\Delta y_{t-5} + 0.26\Delta y_{t-6} \right] \\
 & (0.023) \quad (0.093) \quad (0.11) \quad (0.18) \quad (0.084) \\
 & \times \left[1 + \exp \left\{ -2.87 (\Delta_4 y_{t-2} + 0.060) / \hat{\sigma}(\Delta_4 y_t) \right\} \right]^{-1} + \hat{u}_t \\
 & (1.57) \quad (0.30)
 \end{aligned}$$

$s = 0.33$	skewness = 0.013	excess kurtosis = -0.11
LJB = 0.078 (0.96)	AIC = -2.13	SBIC = -1.86
LM = 5.85 (0.0037)	SDR = 0.91	$R^2 = 0.86$

using the same notation as in the German case. The tests of parameter constancy and no error autocorrelation are not significant. The test of no remaining nonlinearity is non-significant for alternatives with $d=2$ (but is significant (p -value=0.0073) for an alternative with $\Delta_4 y_{t-1}$ as the transition variable). The graph of the estimated transition

function in Figure 5.11c illustrates that there are relatively few observations for small values of the transition variable but that the estimated function is smooth.

Figure 5.7 shows the observed unemployment level (upper panel) and the values of the estimated transition function (lower panel). The latter is seen to assume values close to one during three periods in the sample, corresponding to the rapid increases in the unemployment level. Figure 5.8, depicting the residuals from the nonlinear model together with the residuals from the linear AR model used as the basis for linearity testing, shows that the largest gains in terms of fit from the introduction of the nonlinear model do appear during the same periods, with the dramatic period in the 1990s being the most important one. The highest density region graphs of the estimated generalised impulse response functions in Figure 5.12 are multimodal, asymmetric and do not tend to a point within the 20-period horizon, suggesting that the response to shocks is asymmetric and persistent.

Figure 5.9 depicts a deterministic extrapolation of the estimated model (cf Figure 5.5) for a 50 year horizon starting from the last observed value. The extrapolated sequence returns very slowly from the extreme increase in the end of the sample and appears to converge to an asymptotic value around 6% (averaging over the seasons). If the series of the minimum, maximum and average of the last four observations in the predicted sequence obtained for every starting point t in the sample, with predictions ending 50 years beyond the end of the sample period, as in Figure 5.9, the resulting average prediction for the Finnish model is an almost constant level, i.e., the extrapolated unemployment level for this distant future is virtually unaffected by the starting point of the prediction. If an extrapolation end point closer to the end of the sample is chosen, the

result is different. Figure 5.10 shows the graph of the average, minimum and maximum taken over the last four observations of prediction sequences ending 10 years after the sample period. The predicted unemployment level of the model for this horizon is seen to lie near 6% until the dramatic increase in unemployment in the early 1990s. If the prediction asymptotes are perceived as some sort of ‘equilibrium’ levels implied by the model, this ‘equilibrium’ is affected by the upsurge of the 1990s only for the short run, whereas, in the long run it is seen to return to a level of approximately 6% though the decrease is slow.

Denmark

The model specification for the Danish data is given by

$$\begin{aligned}
 \Delta y_t = & -0.023y_{t-1} + 0.47\Delta y_{t-1} + 0.54\Delta y_{t-4} - 0.23\Delta y_{t-5} - 0.17\Delta y_{t-6} + 0.20\Delta y_{t-8} & (5.3) \\
 & (0.011) \quad (0.11) \quad (0.10) \quad (0.12) \quad (0.076) \quad (0.088) \\
 & + 0.40d_1 - 0.20d_2 + 0.70d_3 \\
 & (0.16) \quad (0.22) \quad (0.24) \\
 & + \left[\begin{array}{ccccccc} 1.18 & -0.11y_{t-1} & +0.39\Delta y_{t-1} & -0.95\Delta y_{t-2} & -0.71\Delta y_{t-5} & +0.17\Delta y_{t-6} & -0.68\Delta y_{t-8} \end{array} \right] \\
 & (0.44) \quad (0.055) \quad (0.16) \quad (0.17) \quad (0.19) \quad (0.076) \quad (0.23) \\
 & \times \left[1 + \exp \left\{ -13.37 \left(\frac{\Delta_4 y_{t-1} - 1.22}{\hat{\sigma}(\Delta_4 y_t)} \right) \right\} \right]^{-1} + \hat{u}_t \\
 & (10.33) \quad (0.099)
 \end{aligned}$$

$s = 0.36$	skewness = -0.23	excess kurtosis = 0.52
LJB = 1.85 (0.40)	AIC = -1.89	SBIC = -1.44
LM = 6.35 (0.0026)	SDR = 0.82	$R^2 = 0.91$

The residuals from (5.3) pass the tests of parameter constancy, no error autocorrelation and no additional nonlinearity. As can be seen from Figure 5.14, the estimated transition function is rather close to a step function in this case, assuming the value one during the three sharpest increases in the unemployment level (see also

Figure 5.11b). These periods are also the ones with the largest difference between the residuals from the nonlinear model and a linear reference model (Figure 5.15). The long run extrapolation (Figure 5.16) is close to the 10% level (and does not change substantially with the starting point). The estimated GIRs in Figure 5.13 indicate that shocks are persistent. The asymmetry may be somewhat less obvious in this case.

Sweden

The Swedish series, featuring a dramatic level change at the end of the sample with very little data after the increase, proves to be a difficult case to model. The linearity test suggests selecting $p=6$ and $d=1$. The p -values of both versions of the linearity test, with and without seasonal dummies in the nonlinear part, are small. Both versions of the alternative model are tried, but the larger model is obviously overparametrised and thus the version with seasonal dummies only in the linear part is selected. This model is also difficult to estimate, correlations between the parameter estimates are high, and both the linearity and the parameter constancy hypotheses are strongly rejected. The results of the latter tests suggest the parametrisation of variability over time in the seasonal dummies. If one of the seasonal dummy coefficients is modelled as varying according to an LSTR1 transition function of time, point estimates of the parameters are obtained, but the standard errors are high and the parameter estimates are highly correlated. In this model, the location parameter of the transition function for the time-varying part of the third quarter dummy coefficient is estimated to $\hat{c}_1 = 2.82$, indicating that only a very small range of small values of this transition function is actually supported by the data. (The time variable used as the transition variable is scaled as t/T .) If only a small part of the transition function is observed, it is obviously

difficult to estimate its parameters. Since the estimators of γ , c , and the dummy parameter δ_{13}^1 in this situation are very strongly correlated, setting one of them to a fixed value can solve the estimation problem. The obvious suggestion in this case is to estimate the model with fixed $\hat{c}_1 \equiv 1$, which yields the following model.

$$\begin{aligned} \Delta y_t = & -0.034y_{t-1} - 0.35\Delta y_{t-1} + 0.27\Delta y_{t-4} - 0.22\Delta y_{t-6} + 0.28d_1 - 0.16d_2 + 0.15d_3 & (5.4) \\ & (0.014) \quad (0.11) \quad (0.083) \quad (0.069) \quad (0.076) \quad (0.062) \quad (0.098) \\ & + 1.75d_3 \times \left[1 + \exp \left\{ -4.18(t^* - 1) / \hat{\sigma}(t^*) \right\} \right]^{-1} \\ & (0.56) \quad (3.39) \\ & + \left[2.02 - 0.42y_{t-1} + 1.31\Delta y_{t-1} + 1.82\Delta y_{t-6} \right] \\ & (0.83) \quad (0.12) \quad (0.28) \quad (0.44) \\ & \times \left[1 + \exp \left\{ -1.95(\Delta_4 y_{t-1} - 1.75) / \hat{\sigma}(\Delta_4 y_t) \right\} \right]^{-1} + \hat{u}_t \\ & (0.54) \quad (0.26) \end{aligned}$$

Note: The model is estimated with fixed $\hat{c}_1 \equiv 1$.

$s = 0.25$	skewness = 0.54	excess kurtosis = 0.58
LJB = 8.23 (0.016)	AIC = -2.67	SBIC = -2.34
LM = 2.90 (0.059)	SDR = 0.82	$R^2 = 0.75$

The null hypotheses of parameter constancy and linearity are strongly rejected. Nevertheless, it might be instructive to provide the same set of graphs as for the other models. The fact that only a small range of the transition function for the time-varying parameter is estimated is seen from Figure 5.19. Figure 5.11d, showing the model transition function plotted against the transition variable, $\Delta_4 y_{t-1}$, shows that the sample contains little information about large ranges of this function as well. As expected, comparing the residuals from the nonlinear model to those from the linear reference model in Figure 5.20 shows that most of the contribution from the nonlinear modelling is attributable to the 1990s, the period during and after the most dramatic

increase in the unemployment level. The long run extrapolation illustrated by Figure 5.17 shows that the model does not imply a return from the high level encountered in the end of the estimation period. The problems of estimating a feasible model for the Swedish series are all likely to arise because of the very limited amount of data available after the unprecedented level shift close to the end of the sample. The estimated generalised impulse response function densities for shocks of mixed signs and for negative shocks shown in Figure 5.21 are extremely peaked. The 50% highest density region is a very short interval, close to being a single point, whereas the 75% one is wider. (The third set of densities, relating to positive shocks, could not be computed for numerical reasons.)

Australia

When a model with constant parameters, $p=6$, $d=1$, and seasonal dummies only in the linear part of the model, as indicated by the linearity test, is estimated for the Australian series, all but the first of the parameters of the lagged first differences in the linear part of the model are insignificantly different from zero, and are omitted. When the residuals from the estimated model are examined, the null hypothesis of parameter constancy is rejected in favour of an alternative with time-varying dummies and intercepts. After omitting the time-varying parts of the parameters for the linear intercept and the second quarter dummy variable, since these turned out to be insignificant, and a number of parameters of the lagged first differences for the same reason, model (5.5) is obtained. It is notable that after introducing the restrictions mentioned, only the lagged level terms and the intercepts and dummies are left in the model.

$$\begin{aligned}
\Delta y_t = & 0.36 - 0.08y_{t-1} + 0.16d_{1t} - 0.56d_{2t} - 0.51d_{3t} & (5.5) \\
& (0.12) \quad (0.016) \quad (0.20) \quad (0.089) \quad (0.18) \\
& + (0.96d_{1t} + 0.51d_{3t}) \left(1 + \exp \{ -2.48(t^* - 0.28) / \hat{\sigma}(t^*) \} \right)^{-1} \\
& \quad (0.25) \quad (0.23) \quad (0.43) \quad (0.019) \\
& + \left[-0.30y_{t-1} + 3.67 \times \left(1 + \exp \{ -2.48(t^* - 0.28) / \hat{\sigma}(t^*) \} \right)^{-1} \right] \\
& \quad (0.085) \quad (0.86) \quad (0.43) \quad (0.019) \\
& \times \left(1 + \exp \{ -3.36(\Delta_4 y_{t-1} - 0.74) / \hat{\sigma}(\Delta_4 y_t) \} \right)^{-1} + \hat{u}_t \\
& \quad (1.29) \quad (0.20)
\end{aligned}$$

$s = 0.32$	skewness = 0.48	excess kurtosis = 0.20
LJB = 4.39 (0.11)	AIC = -2.17	SBIC = -1.85
LM = 2.67 (0.07)	SDR = 0.85	R2 = 0.82

The estimated model passes all the three tests against misspecification. The levels, residuals and the two transition functions are displayed in Figures 5.23 and 5.24. The lower panel of Figure 5.18 shows the time series of the total of the intercepts and seasonal dummy coefficients multiplied by the respective transition function values at every point in time, illustrating the total effect of the changing parameters in the model.

Figure 5.25 shows the results of long-run extrapolations with the model and Figure 5.26 the corresponding average, maximum, and minimum curves. As in the case of the German model, the Australian model produces a cycle when the extrapolation starts near the upsurge in the early 1980s or later. The GIR function graphs in Figure 5.22 suggest strong asymmetry but less persistence than for the other countries.

Conclusions

Table 5.2 summarises some of the key properties of the estimated models. It is seen that four of the five models have the same general pattern: $\hat{\alpha}_2 < 0$ and $\hat{\mu}_2 > 0$,

Finland constituting the only exception. In the Finnish case the dynamics of the process are mainly characterised by the lagged first differences. In addition to Germany, Australia also has a model indicating that the expected rate of unemployment has been moving smoothly from a lower to a higher equilibrium over time. The models for Denmark and Finland have constant parameters which could be interpreted as being equivalent to a single equilibrium whereas seasonality in the unemployment rate seems to have changed over time in Sweden. A detailed study of the estimated equation would show that the change started in the mid-1980s. (As noted above, the model for the Swedish series does not pass the tests of parameter constancy and linearity.) In general, the estimated equations are different special cases of the general model (3.1). As seen from the estimated transition functions in Figures 5.11a-d, another common feature is that the transition from the one extreme regime to the other is smooth. (Denmark is closest to being an exception.)

The GIR graphs illustrating the degree of persistence of shocks in Figures 5.12-5.13 and 5.21-5.22 show that the unemployment rates in the three Scandinavian countries are quite persistent. Shocks to the Australian unemployment rate seem to be somewhat less persistent than those for the other four countries. On the other hand, the Australian LSTAR model contains a limit cycle; see Figure 5.25. However, these two observations are not necessarily contradictory. Extrapolating the deterministic counterpart ('skeleton'; Tong, 1990) of the stochastic model and shocking the stochastic one and following the effects of the shock(s) over time are two procedures that emphasise different dynamic aspects of the model.

6. RESULTS: MOVING EQUILIBRIA

As reported in Section 5.1, symmetry (linearity) is not rejected when tested against the asymmetric STAR model for the US, Japanese, Norwegian, or Canadian series. For Austria and Italy, linearity is rejected, but it is not possible to estimate meaningful asymmetry models for these two series. In this section, these series are reconsidered focusing on the second objective of the paper, investigating the possibility that the unemployment rate has been moving between different equilibria under the observation period. As noted above, this alternative can be investigated within the same framework as before using a simplified model. The alternative model now is the STAR model

$$\Delta y_t = \mu_1^0 + \mu_1^1 H_k(t^*; \gamma_1, \mathbf{c}_1) + \alpha_1 y_{t-1} + \sum_{j=1}^p \beta_{1j} \Delta y_{t-j} + \sum_{i=1}^{s-1} (\delta_{1i}^0 + \delta_{1i}^1 H_k(t^*; \gamma_1, \mathbf{c}_1)) d_{1i} + u_t \quad (6.1)$$

which may also be seen as a linear AR model with a time-varying intercept and seasonal parameters. If $\mu_1^1 \neq 0$ and $H_k(t^*; \gamma_1, \mathbf{c}_1)$ is not constant in (6.1) then the process moves smoothly between equilibria as described by the transition function. If $\delta_{1i}^1 \neq 0$ while $H_k(t^*; \gamma_1, \mathbf{c}_1)$ is not constant then the seasonal pattern changes over time. Such a change is quite common in practice and is often due to (usually slowly) changing institutions. Both types of transitions (level and seasonality) may be present simultaneously in the series.

The remaining series are modelled by first testing the linearity against (6.1); see Lin and Teräsvirta (1994) or Teräsvirta (1998). The test statistics are the ones called LM_k , $k=1,2,3$, in these two references and are linearity tests against (6.1) with $k=1,2,3$, respectively, in $H_k(t^*; \gamma_1, \mathbf{c}_1)$. If linearity (parameter constancy) is rejected then a STAR model of the form (6.1) is fitted to the series. The parameter k is chosen by looking at the test results and by comparing estimated models in cases it is considered necessary to estimate them for different values of $k(\leq 3)$.

Table 6.1 provides the results of testing the constancy of the intercept and the constancy of the coefficients of the seasonal dummy variables separately. It is seen that the only country for which neither null hypothesis is rejected at the 5% level of significance is the United States. For Austria and Canada there is evidence of the intercept not being constant over time. As for Italy, Japan, and Norway, the constancy of the seasonal pattern is rejected very strongly, but there is also evidence for change in the intercept.

Consequently, STAR models are estimated for all series except the US unemployment rate. The estimated models are presented in Appendix 2. The results of the misspecification tests do not in all cases indicate model adequacy, but the p -values have improved for all series for which modelling was previously attempted. The estimated models are best interpreted by considering the graphs of the moving parameters multiplied by the seasonal dummies,

$$\hat{\mu}_1^0 + \hat{\mu}_1^1 H_k(t^*; \gamma_1, \mathbf{c}_1) + \sum_{i=1}^{s-1} (\hat{\delta}_{1i}^0 + \hat{\delta}_{1i}^1 H_k(t^*; \gamma_1, \mathbf{c}_1)) d_{1i} \quad (6.2)$$

over time. These can be found in Figures 6.1-6.5. The Austrian and Canadian models display a smooth transition in unemployment level over time. For Japan, there is a very slow climb in the level, but the most striking feature in the graph is the strong decrease in the amplitude of the seasonal variation from the early 1960s to the 1990s. The Italian results indicate that the unemployment rate has slowly moved from an equilibrium level in the early 1970s to a higher equilibrium by the late 1980s. At the same time, the seasonal pattern has undergone a considerable change. In the Norwegian series there is a very rapid rise in the level of the unemployment rate in 1988 accompanied by a sharp increase in the amplitude of the seasonal variation.

It is interesting to compare these results with those in Bianchi and Zoega (1998), BZ for short; see their Figure 2. For Austria BZ discover five different equilibrium levels, each higher than the previous one with one exception, the period 1988(2)-1990(3), for which a lower equilibrium prevailed. This accords reasonably well with the results from a smooth transition autoregression. For Canada the results of BZ indicate a switch to a higher mean level in the early 1980s. This finding can be compared with Figure 6.2 according to which a smooth transition to a higher equilibrium is completed by that time. (BZ also find a temporary drop in the equilibrium rate in 1987-1990.) According to BZ, the Japanese unemployment series has had four equilibrium levels since 1970, each higher than the previous one. Again, this accords well with the monotonically increasing level in the STAR model. As BZ use seasonally adjusted series the temporal development in the seasonal patterns picked up by the STAR

model is not captured by their model. This is of course true also for Norway. But then, while the results for Norway of the present study indicate a rapid level change in 1988, those in BZ also indicate a single switch in the equilibrium level at the same time. This also agrees with the findings of Akram (1998) who, however, claims another period of 'high' unemployment in 1982(4)-1984(3). Finally, BZ do not estimate any local mean level shift for the US which again is in line with the test results in this study. For Italy, the results are different. Originally BZ find two equilibria of which the latter one is higher (their sample period is 1970(1)-1995(3)) with a break in 1983(2). Next, assuming a break in this quarter, and testing the constancy of the intercept in a linear AR model with a Chow test against this break, BZ do not reject the null hypothesis of no break. This may not be surprising, however, because the Chow test against a single break is not a powerful test against smooth structural change. A break in 1983 may not be a convincing finding, which BZ acknowledge. Figure 6.5 clearly shows that the Italian unemployment rate began a steady climb already in the mid-1970s and that the growth continued to the late 1980s. The results of the present section correspond to this visual impression.

Thus, with the exception of Italy, the results of the STAR model look rather similar to those in BZ. One may argue, however, that the similarities are superficial because the models are completely different and, besides, the series are not the same. The approach of BZ appears to accord well with the idea that large shocks may cause shifts in the equilibrium rate of unemployment. The approach of this paper may be viewed rather as a way of presenting stylised facts because the transitions are described with deterministic functions of time. Nevertheless, it may be argued that

what is seen in Figures 6.1-6.5 does have a 'shock interpretation'. A smooth transition in the level may be a result of a series of shocks of the same sign. It may also be that while only large shocks cause switches in the equilibrium level, the transition from the old level to a new one initiated by such a shock may take time and be smooth. Viewed through a hidden Markov model, such a transition may appear as a sequence of discrete equilibria of either increasing (as is the case here) or decreasing order.

The results also accord well with those of the standard unit root tests. It is well known that structural breaks or, for that matter, smooth structural change, may bias test results towards accepting the null hypothesis (see, for example, Perron (1989) or Hendry and Neale (1991) for discussion and Monte Carlo evidence). As seen already in Section 2.4 asymmetry of the type defined in (2.1) may make it difficult to reject the unit root hypothesis. As neither of the hypotheses, a unit root or (6.1), can be taken as the 'true model', the unit root results and the idea of smooth structural change just characterise the same reality from two different angles.

7. DISCUSSION

The results of Section 5 indicate that many unemployment series are asymmetric and can be adequately characterised by an LSTAR model of the form (3.1). The fact that the same model fits several series is encouraging as it reduces the risk that the findings are spurious. The estimated equations also indicate that shocks to the system are rather persistent. These findings are in accordance with the result of the unit root tests, but

accepting the unit root hypothesis also has other implications, including symmetry, that are quite different from what the present approach leads to. For the remaining series discussed in Section 6, the results are also in harmony with those of the unit root tests. On the other hand, they yield interesting information about how the equilibrium level of unemployment has moved over the years. Even there, many countries exhibit similar results and the STAR model is successfully fitted to all series for which linearity is rejected (although the models do not capture all the nonlinearity in the data). As these models characterise the unemployment rate as stationary around a nonlinear trend they also underline the high persistence of these rates.

As is at least implicitly clear from previous sections, the information set used for the modelling is the history of the processes. This excludes any extraneous information and naturally shapes up the results. On the other hand, it may be argued that, for example, the large increases in the unemployment rates in Sweden and Finland in the beginning of the 1990s have been caused by extraordinary, or even unique events. Because these increases constitute the single most dominating events in both series, this information may lead to treating them as outlying observations which do not fit into the general pattern. Adopting this view and modelling the corresponding observations as outliers could have an impact on the results and the conclusions; for an illuminating discussion of this issue see Teräsvirta (1997) and van Dijk and Franses (1997b). It should be noted, however, that neutralising the dominating observations by dummy variables is a remarkably strong assumption. It implies that the presumed unique events only affect the level of the process and leave the dynamics intact.

In this paper, only univariate models have been applied. Often, however, the interest lies in modelling relationships between economic variables, including unemployment. A rather standard linear analysis begins by testing the unit root hypothesis in the unemployment rate, accepting it, and finding a cointegrating relationship between the unemployment rate and a set of $I(1)$ variables such as real wages, real output and the like. This approach frequently ignores important stylised facts of the unemployment rate series such as asymmetry. On the other hand, it should be possible to extend the univariate model by including relevant economic variables and relationships and adding equations. The standard cointegration-VAR approach assumes away any asymmetry already at the start of the analysis. This may be appropriate if a linear approximation is an adequate description to the unknown system that is to be modelled. If this is not known at the outset, however, then respecting the stylised facts and starting from the properties found in univariate series could be a preferable approach. Even if a linear model ultimately provides an adequate description of the relationships of interest it may be better to find that out by starting out with the asymmetry assumption when appropriate. In addition to parametrising stylised facts in unemployment series, the univariate analysis presented in this study is intended as a starting-point for such an approach.

APPENDIX 1. GENERALISED IMPULSE RESPONSE FUNCTIONS AND HIGHEST DENSITY REGIONS

As in Chapter I the reader is referred to Koop, Pesaran and Potter (1996) for a general definition and a discussion of the generalised impulse response (GIR) function. The actual estimation of the GIR functions is carried out in the following way (see also the Chapter I Appendix): All available observations of the unemployment series are used once (without sampling) as 'histories'. For each history, 100 initial shocks are drawn randomly with replacement from a subset of the residuals from the estimated STAR model, formed either of all residuals, of all residuals greater than one residual standard error, or of all residuals less than one negative residual standard error. For each combination of history and initial shock, 800 replicates of a 21-step prediction sequence (0,1,...,20 step ahead) are generated with and without the selected initial shock in the first step, and using randomly drawn residuals as noise everywhere else. For every horizon, the means over the 800 replicates of the two prediction sequences are computed, and the vector of 21 differences between the two means forms an observation of the GIR. For each one of the 21 horizons, the response density is estimated with a kernel algorithm. The points used in the kernel algorithm are a random sample from the GIR. For instance, in the case of Australia, with 111 'histories', this is a random sample from 11100 GIR values for each horizon.

APPENDIX 2. ESTIMATED MOVING EQUILIBRIA MODELS

Austria

$$\begin{aligned} \Delta y_t = & -0.29 - 0.17y_{t-1} - 0.14\Delta y_{t-2} + 0.67\Delta y_{t-4} + 0.12\Delta y_{t-10} + 0.13d_1 - 0.60d_2 + 0.51d_3 \quad (\text{A2.1}) \\ & (0.22) \quad (0.041) \quad (0.058) \quad (0.047) \quad (0.054) \quad (0.11) \quad (0.21) \quad (0.17) \\ & + 1.70 \times \left[1 + \exp \left\{ -2.23 (t^* - 0.29)^2 / \hat{\sigma}(t^*) \right\} \right]^{-1} + \hat{u}_t \\ & (0.46) \quad (0.68) \quad (0.026) \end{aligned}$$

$s = 0.25$	skewness = 0.38	excess kurtosis = 0.69
LJB = 5.67 (0.059)	AIC = -2.70	SBIC = -2.45
LM = 6.77 (0.0016)	SDR = 0.95	$R^2 = 0.97$

Italy

$$\begin{aligned} \Delta y_t = & -0.011 - 0.41y_{t-1} + 0.083\Delta y_{t-2} + 0.29\Delta y_{t-4} + 0.19\Delta y_{t-5} + 0.27\Delta y_{t-8} \quad (\text{A2.2}) \\ & (0.27) \quad (0.064) \quad (0.077) \quad (0.076) \quad (0.072) \quad (0.075) \\ & + 0.33d_1 - 0.28d_2 + 0.67d_3 + (4.74 - 0.40d_1 + 0.11d_2 - 0.86d_3) \\ & (0.38) \quad (0.36) \quad (0.39) \quad (0.76) \quad (0.48) \quad (0.46) \quad (0.47) \\ & \times \left[1 + \exp \left\{ -11.56 (t^* - 0.27)^3 / \hat{\sigma}(t^*) \right\} \right]^{-1} + \hat{u}_t \\ & (1.64) \quad (0.017) \end{aligned}$$

$s = 0.42$	skewness = -0.60	excess kurtosis = 2.22
LJB = 35.49 (1.96×10^{-8})	AIC = -1.64	SBIC = -1.32
LM = 10.041 (8.85×10^{-5})	SDR = 0.91	$R^2 = 0.66$

Canada

$$\begin{aligned}
\Delta y_t = & 0.71 - 0.11y_{t-1} + 0.49\Delta y_{t-1} + 0.21\Delta y_{t-4} - 0.17\Delta y_{t-15} & (A2.3) \\
& (0.30) \quad (0.029) \quad (0.081) \quad (0.081) \quad (0.078) \\
& + 0.24\Delta y_{t-16} + 0.16\Delta y_{t-19} - 0.24\Delta y_{t-20} + 0.71d_1 - 1.70d_2 - 0.31d_3 \\
& (0.074) \quad (0.063) \quad (0.064) \quad (0.23) \quad (0.25) \quad (0.16) \\
& + 0.70 \times \left[1 + \exp \left\{ -2.56(t^* - 0.30) / \hat{\sigma}(t^*) \right\} \right]^{-1} + \hat{u}_t \\
& (0.34) \quad (1.80) \quad (0.13)
\end{aligned}$$

$s = 0.35$

$\text{skewness} = 0.82$

$\text{excess kurtosis} = 1.53$

$\text{LJB} = 26.26 (1.99 \times 10^{-6})$

$\text{AIC} = -1.99$

$\text{SBIC} = -1.67$

$\text{LM} = 0.31 (0.73)$

$\text{SDR} = 0.94$

$R^2 = 0.89$

Japan

$$\begin{aligned}
\Delta y_t = & -0.13 - 0.076y_{t-1} + 0.16\Delta y_{t-3} + 0.17\Delta y_{t-4} & (A2.4) \\
& (0.19) \quad (0.037) \quad (0.089) \quad (0.089) \\
& + 0.78d_1 - 0.24d_2 + 0.15d_3 + (0.29 - 0.51d_1 + 0.24d_2 - 0.12d_3) \\
& (0.34) \quad (0.21) \quad (0.11) \quad (0.24) \quad (0.48) \quad (0.27) \quad (0.14) \\
& \times \left[1 + \exp \left\{ -1.24(t^* - 0.37) / \hat{\sigma}(t^*) \right\} \right]^{-1} + \hat{u}_t \\
& (1.55) \quad (0.21)
\end{aligned}$$

$s = 0.11$

$\text{skewness} = 0.035$

$\text{excess kurtosis} = -0.38$

$\text{LJB} = 0.88 (0.64)$

$\text{AIC} = -4.35$

$\text{SBIC} = -4.074$

$\text{LM} = 2.79 (0.065)$

$\text{SDR} = 0.90$

$R^2 = 0.86$

Norway

$$\begin{aligned}
 \Delta y_t = & 0.0012 - 0.22y_{t-1} + 0.26\Delta y_{t-2} + 0.52d_1 + 0.62d_2 + 0.76d_3 & (A2.5) \\
 & (0.16) \quad (0.053) \quad (0.098) \quad (0.14) \quad (0.14) \quad (0.13) \\
 & + (0.64 + 0.79d_1 - 0.18d_2 - 0.55d_3) \\
 & (0.22) \quad (0.21) \quad (0.21) \quad (0.23) \\
 & \times \left[1 + \exp \left\{ -93.98 (t^* - 0.59) / \hat{\sigma}(t^*) \right\} \right]^{-1} + \hat{u}_t \\
 & (440.41) \quad (0.011)
 \end{aligned}$$

 $s = 0.34$

LJB = 0.59 (0.74)

LM = 0.11 (0.89)

skewness = -0.19

AIC = -2.01

SDR = 0.87

excess kurtosis = 0.10

SBIC = -1.69

 $R^2 = 0.68$

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TABLES

Table 2.1. ADF tests on data generated with nonlinear model: Relative rejection frequencies at the 5% level. $T=200$, 10000 replicates

a. No constant, no trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.15	0.16	0.17	0.17	0.14
-0.02	0.19	0.19	0.18	0.17	0.26
-0.05	0.56	0.58	0.66	0.82	0.76
-0.10	0.91	0.92	0.95	0.99	0.99
b. Constant, no trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.07	0.07	0.07	0.08	0.06
-0.02	0.12	0.13	0.13	0.13	0.09
-0.05	0.24	0.24	0.28	0.35	0.31
-0.10	0.64	0.65	0.75	0.89	0.81
c. Constant and trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.09	0.09	0.08	0.07	0.05
-0.02	0.08	0.08	0.07	0.07	0.07
-0.05	0.14	0.14	0.15	0.16	0.18
-0.10	0.36	0.37	0.52	0.63	0.58

Table 2.2. ADF tests on data generated with nonlinear model: Relative rejection frequencies at the 5% level. $T=500$, 10000 replicates

a. No constant, no trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.11	0.11	0.09	0.09	0.34
-0.02	0.57	0.61	0.76	0.75	0.76
-0.05	0.86	0.85	0.92	0.99	0.99
-0.10	0.77	0.80	0.91	1.00	0.99
b. Constant, no trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.09	0.10	0.18	0.19	0.12
-0.02	0.30	0.36	0.50	0.50	0.31
-0.05	0.87	0.87	0.93	0.98	0.95
-0.10	0.81	0.84	0.91	1.00	0.99
c. Constant and trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.06	0.06	0.08	0.08	0.08
-0.02	0.13	0.15	0.23	0.20	0.18
-0.05	0.75	0.76	0.86	0.91	0.79
-0.10	0.81	0.83	0.90	0.99	0.99

Table 2.3. ADF tests on data generated with nonlinear model: Relative rejection frequencies at the 5% level. $T=1000$, 10000 replicates

a. No constant, no trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.47	0.53	0.63	0.72	0.77
-0.02	0.91	0.87	0.92	0.99	0.99
-0.05	0.71	0.67	0.80	0.99	0.99
-0.10	0.48	0.54	0.74	0.99	0.99
b. Constant, no trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.43	0.55	0.75	0.83	0.30
-0.02	0.90	0.90	0.94	0.99	0.84
-0.05	0.83	0.79	0.83	0.99	0.99
-0.10	0.58	0.62	0.74	0.99	0.99
c. Constant and trend					
α_1	α_2				
	0	-0.01	-0.10	-0.20	Linear
-0.01	0.19	0.29	0.47	0.55	0.18
-0.02	0.77	0.81	0.89	0.96	0.59
-0.05	0.84	0.79	0.83	0.99	0.99
-0.10	0.58	0.62	0.74	0.99	0.99

Table 5.1. Smallest p -values of linearity tests for various values of delay d against logistic smooth transition autoregression, 11 unemployment rate series

Series	Order of AR model	Linearity test <i>without</i> seasonal dummies in the nonlinear part		Linearity test <i>with</i> seasonal dummies in the nonlinear part	
		d	p -value	d	p -value
Sweden 1962:1-1996:3	6	1	1.4×10^{-7}	1	3.8×10^{-7}
Germany 1960:1-1991:4	8	1	0.0074	1	0.00037
Finland 1960:1-1996:3	6	1	0.0032	1	0.0062
		2	0.0027	2	0.0027
Denmark 1970:1-1995:4	8	1	0.017	1	0.046
Austria 1960:1-1995:4	6	1	3.5×10^{-6}	1	1.9×10^{-6}
Italy 1960:1-1995:4	9	3	0.025	3	0.011
Australia 1966:3-1995:4	5	Linearity not rejected*		Linearity not rejected	
Norway 1972:1-1997:3	8	Linearity not rejected		Linearity not rejected	
USA 1960:1-1995:4	6	Linearity not rejected		Linearity not rejected	
Canada 1960:1-1995:4	20	Linearity not rejected		Linearity not rejected	
Japan 1960:1-1995:4	4	Linearity not rejected		Linearity not rejected	

Note: The results are based on the first order Taylor series approximation of the transition function. See Teräsvirta (1998) for details of the test. The significance level 0.05 is used as a criterion for rejecting linearity.

* Since the smallest p -value for the test statistic based on the third order Taylor series approximation is small (p -value=0.008) for Australia, linearity is rejected and a nonlinear model specified and estimated although none of the p -values for the first-order tests is less than 0.05.

Table 5.2. Estimated intercepts and coefficients of the lagged level variable in the nonlinear part of the model, and the estimated location parameter c , in the five LSTAR models

Series	$\hat{\alpha}_2$	$\hat{\mu}_2$	\hat{c}	Limit cycle	Transition
Australia	-0.30 (0.085)	$3.67 \hat{H}_1$ (0.86)	0.74 (0.20)	yes	smooth
Denmark	-0.11 (0.055)	1.18 (0.44)	1.22 (0.099)	no	smooth
Finland	-0.017 (0.023)		-0.06 (0.30)	no	smooth
Germany	-0.15 (0.051)	$0.58 + 1.27 \hat{H}_1$ (0.21) (0.61)	0.31 (0.12)	yes	smooth
Sweden	-0.42 (0.12)	2.02 (0.83)	1.75 (0.26)	no	smooth

Table 6.1. p -values of the parameter constancy tests against smooth structural change (a) in the intercept, (b) in the coefficients of the seasonal dummy variables in linear autoregressive models for series for which linearity is not rejected

Series	AR order	Test results, p -values, alternative with time-varying					
		(a) intercept			(b) seasonal dummy coefficients		
		p	$LM1$	$LM2$	$LM3$	$LM1$	$LM2$
Austria	6	0.048	0.020	0.009	0.88	0.82	0.046
	13	0.034	0.014	0.017	0.86	0.96	0.71
Italy	9	0.011	0.038	0.022	0.29	0.031	0.00015
USA	6	0.25	0.072	0.099	0.63	0.23	0.41
Canada	20	0.073	0.010	0.019	0.20	0.37	0.39
Japan	4	0.012	0.042	0.092	1.3×10^{-5}	0.00012	0.00034
Norway	8	0.054	0.16	0.029	0.0033	0.0027	0.00017

Note: For a detailed description of the tests, see Lin and Teräsvirta (1994) or Teräsvirta (1998)

FIGURES

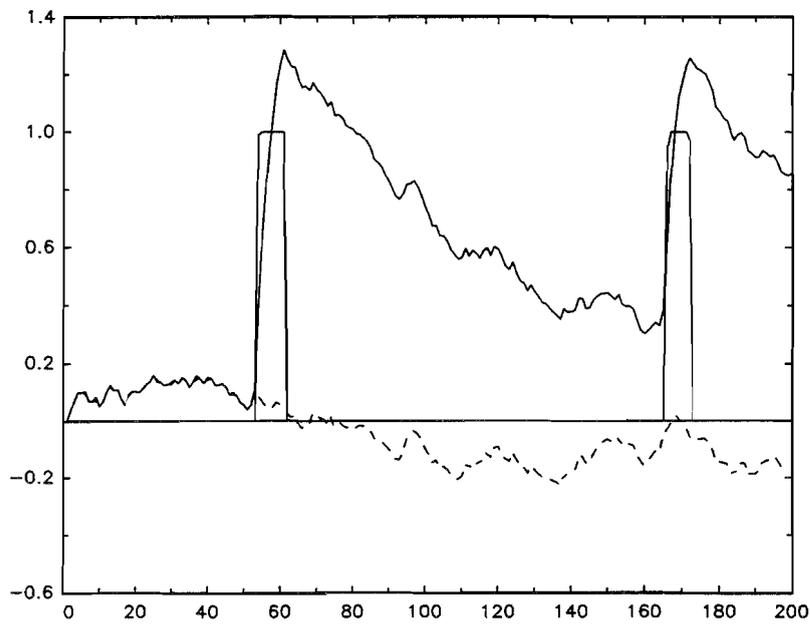


Figure 2.1. One realization of the process (2.1) (solid); the corresponding realization of the linear model (2.3) (dashed); and the transition function (solid)

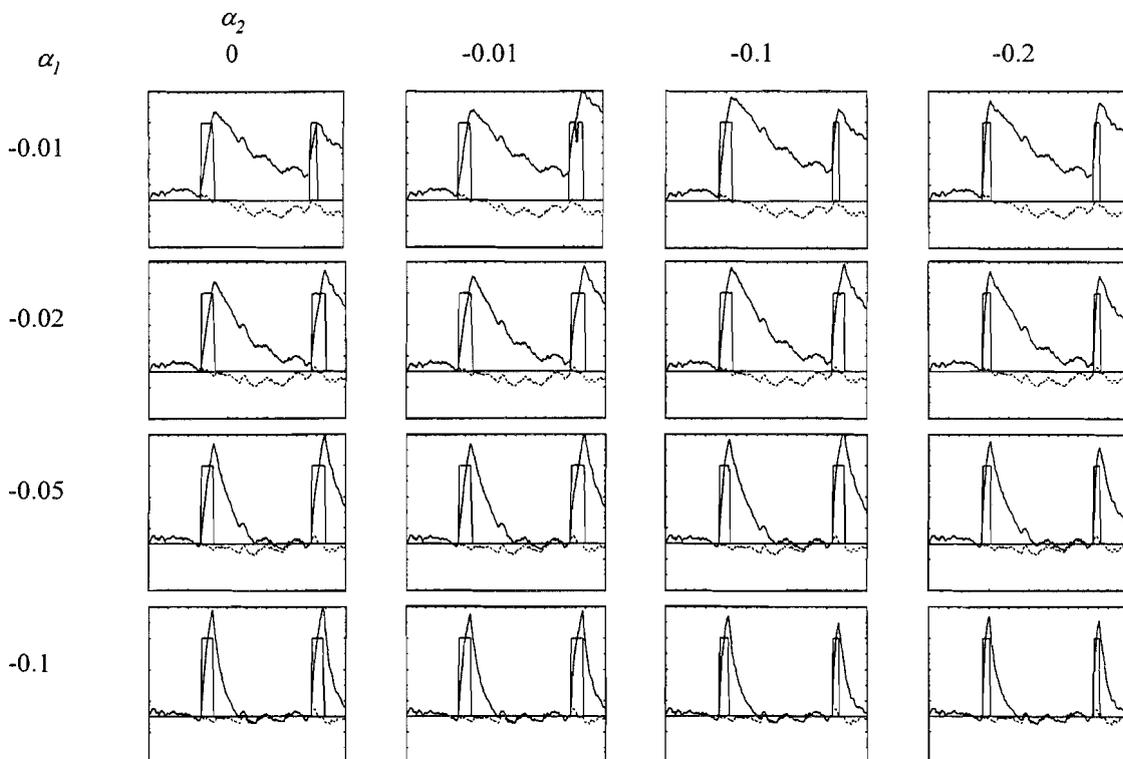
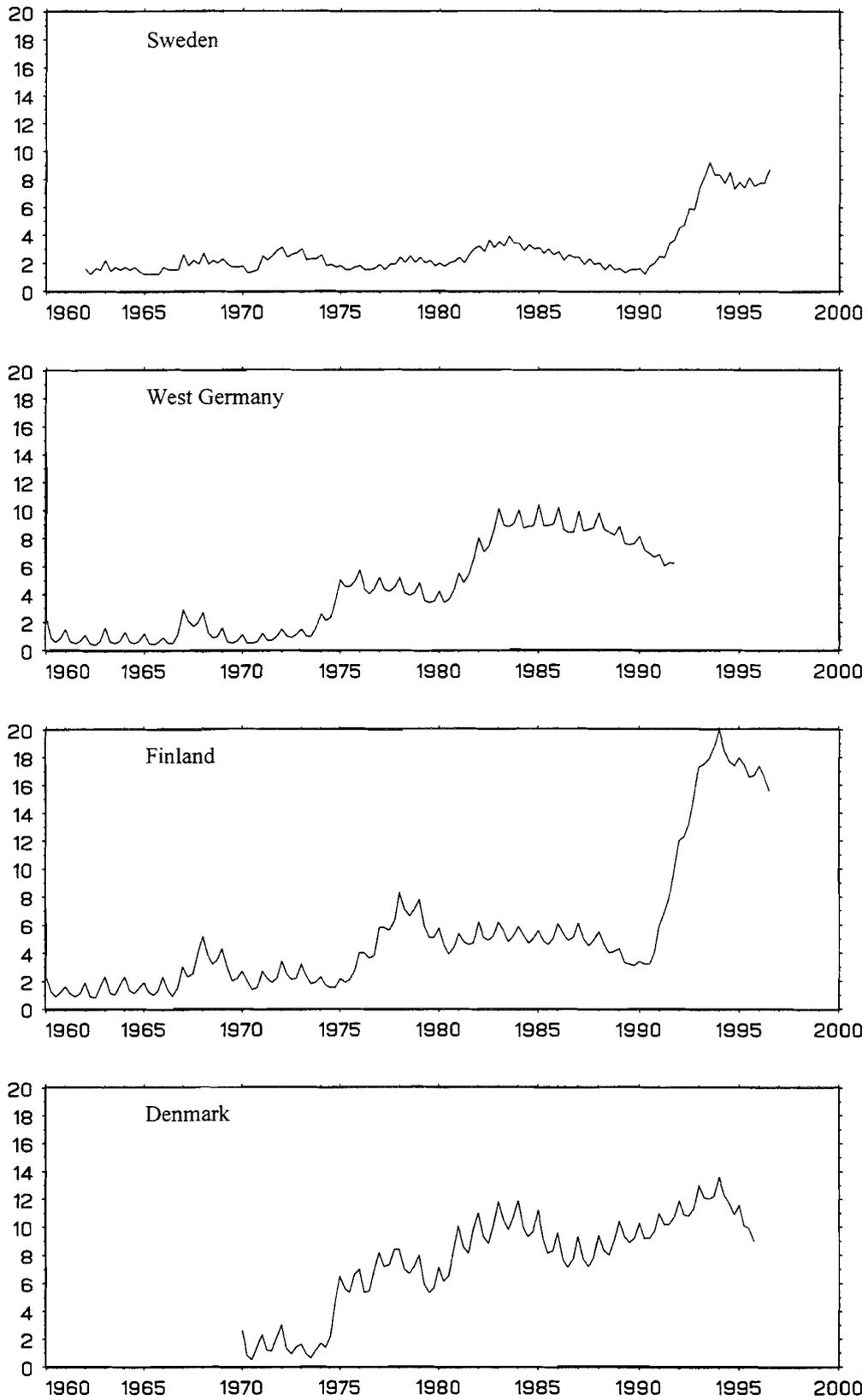
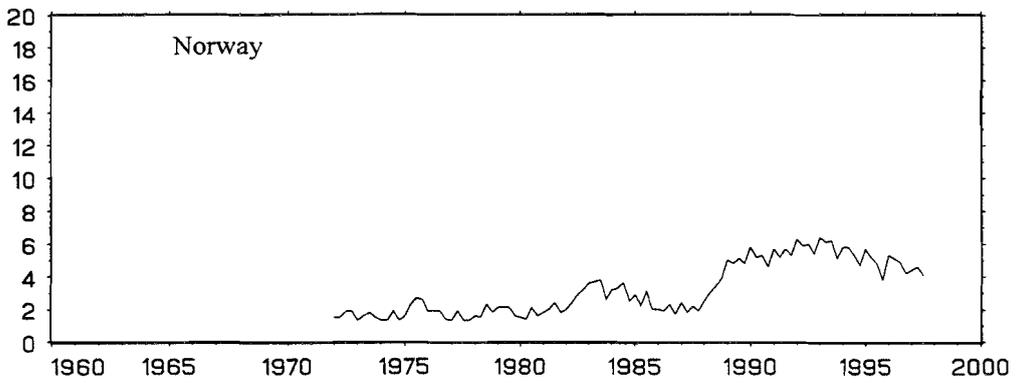
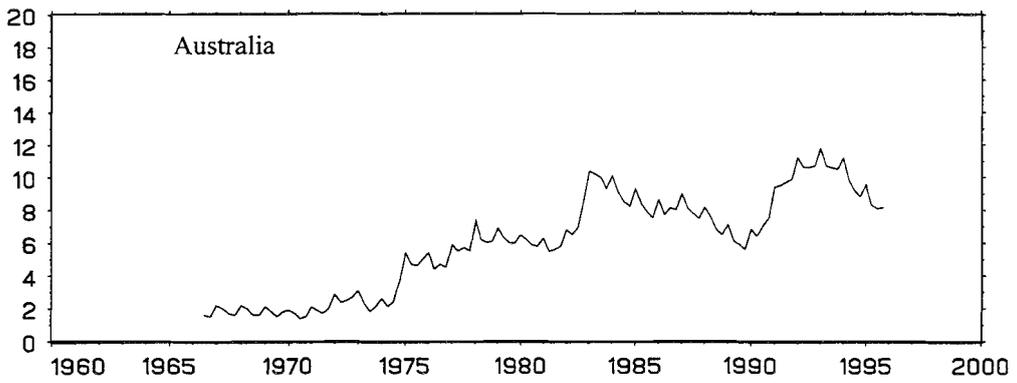
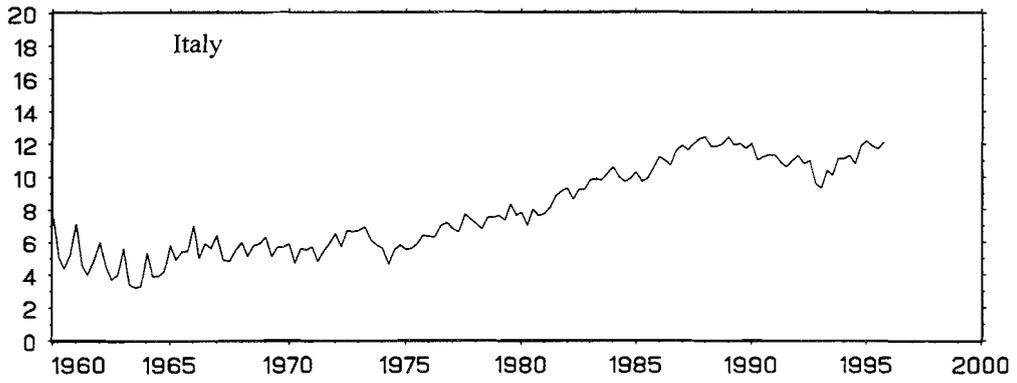
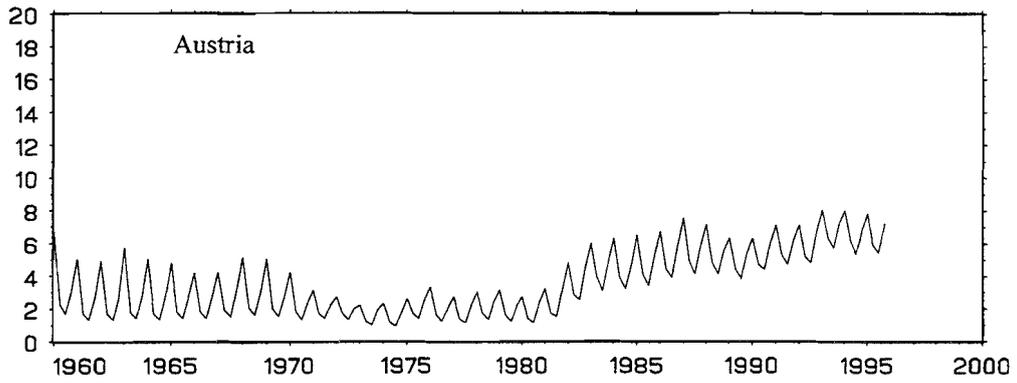
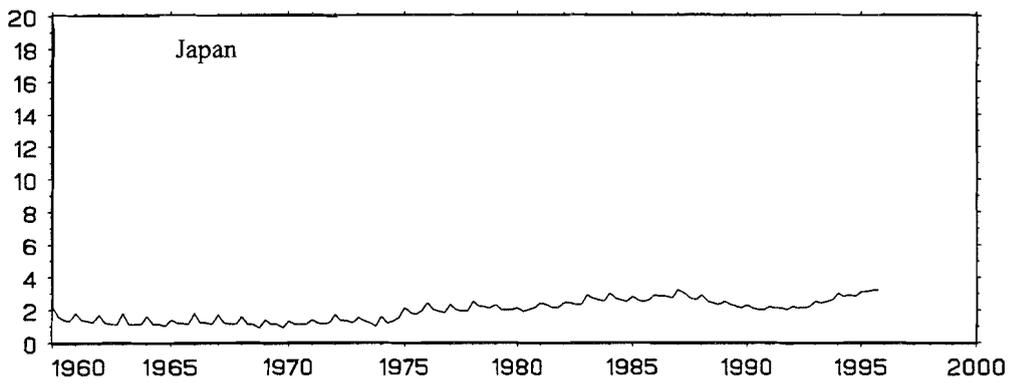
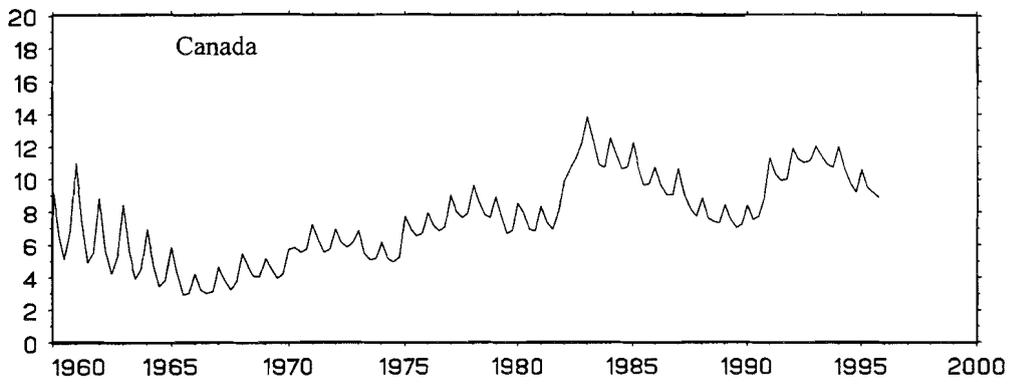
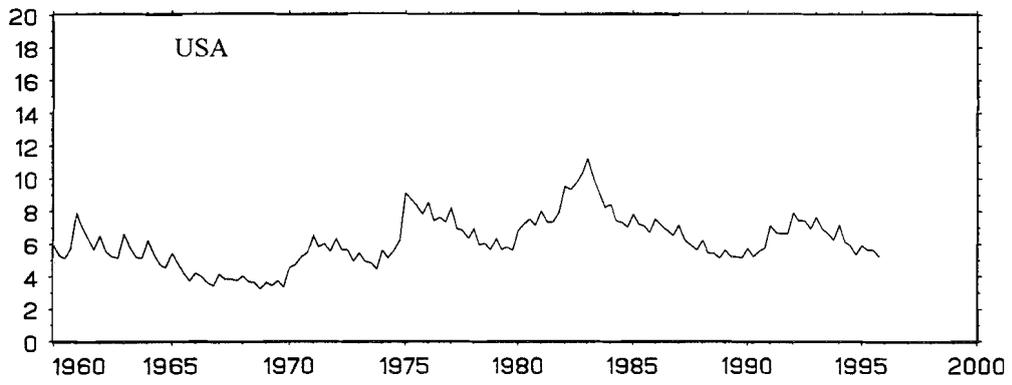


Figure 2.2. Examples of realizations generated with (2.1) (solid) for various values of α_1 and α_2 , examples of realizations generated with the linear model (2.7) (dashed), and the transition function (solid)

Figure 4.1. Unemployment rates by country







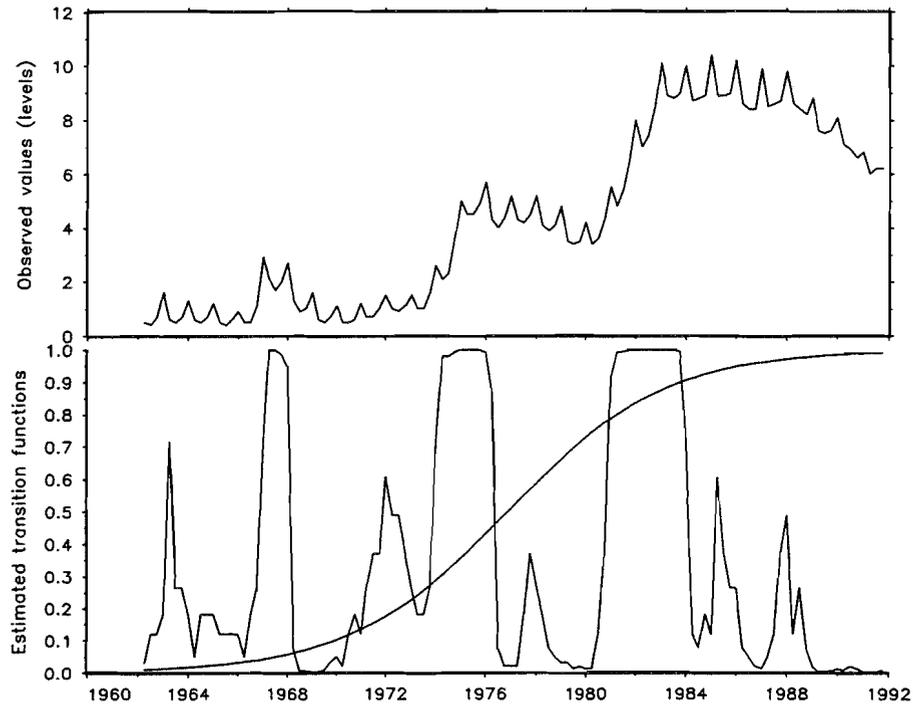


Figure 5.1. Germany. Observed values (upper panel). Estimated transition functions over time (lower panel): (a) transition function \hat{G} over time (irregular curve), (b) transition function \hat{H} (logistic curve)

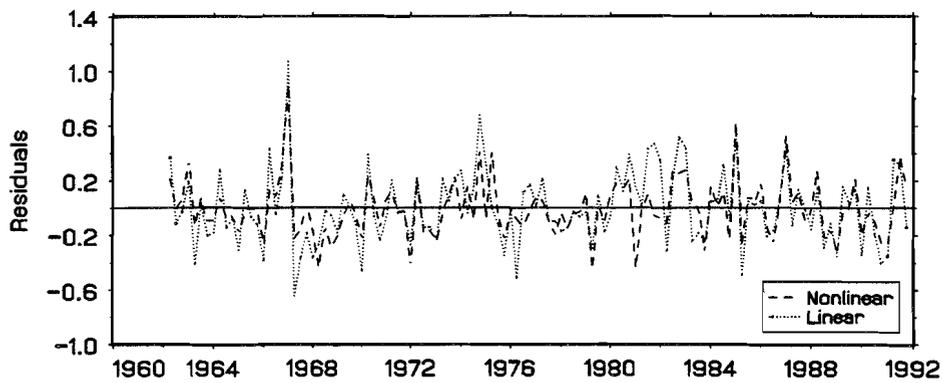


Figure 5.2. Germany. Residuals of the estimated LSTAR model (dashed line) and the corresponding linear AR model (dotted line).

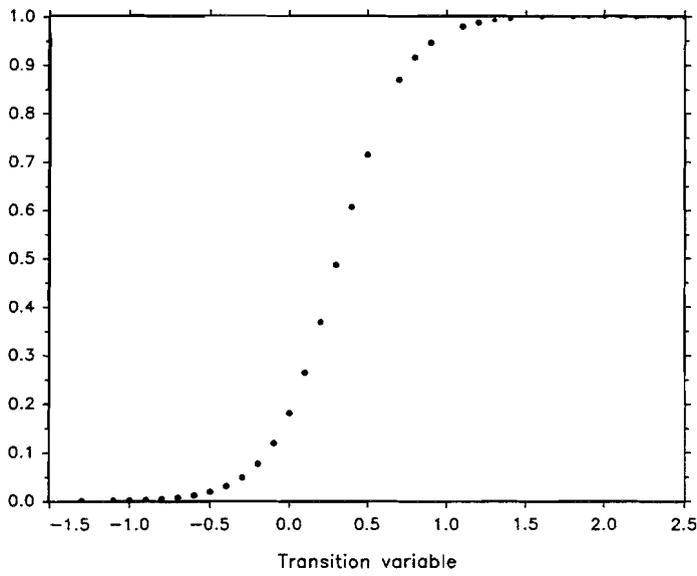
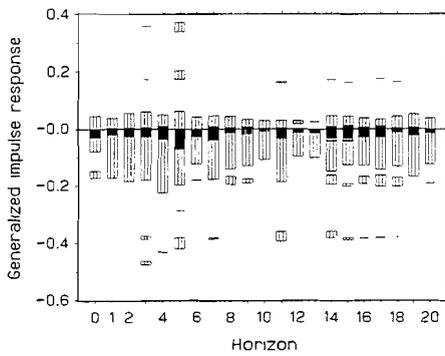
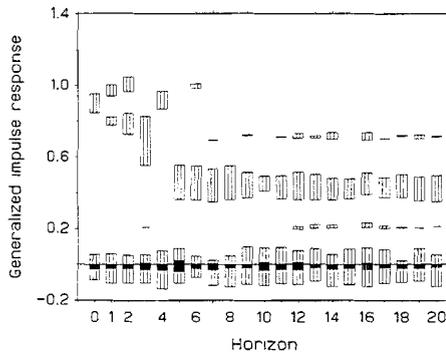


Figure 5.3. Germany. Estimated transition function, \hat{G} , as a function of the transition variable. Each dot represents at least one observation.

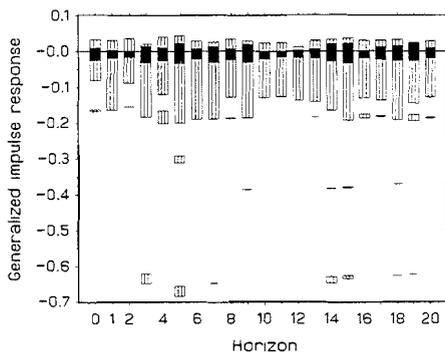
Figure 5.4. Germany: 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks



Response to symmetric shocks



Response to positive shocks, greater than 1 error s.d.



Response to negative shocks, less than -1 error s.d.

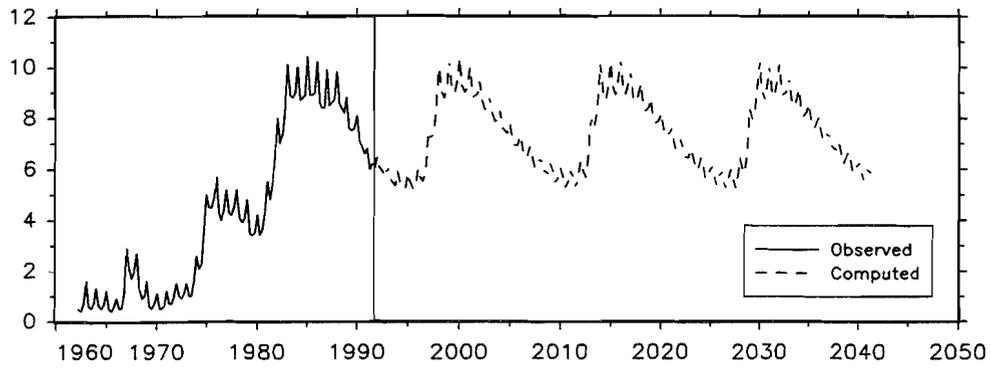


Figure 5.5. Germany. Deterministic extrapolation of the estimated model (horizon 50 years)

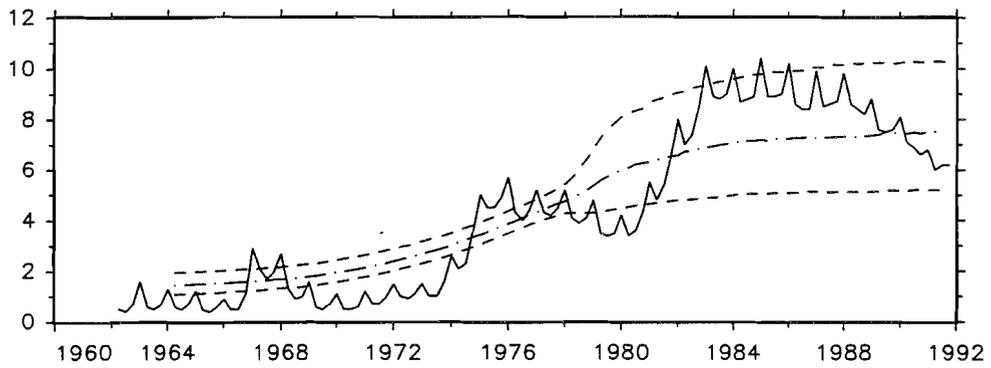


Figure 5.6. Germany. Average, max., and min. over the last 60 observations of consecutive deterministic extrapolations (horizon 50 years).

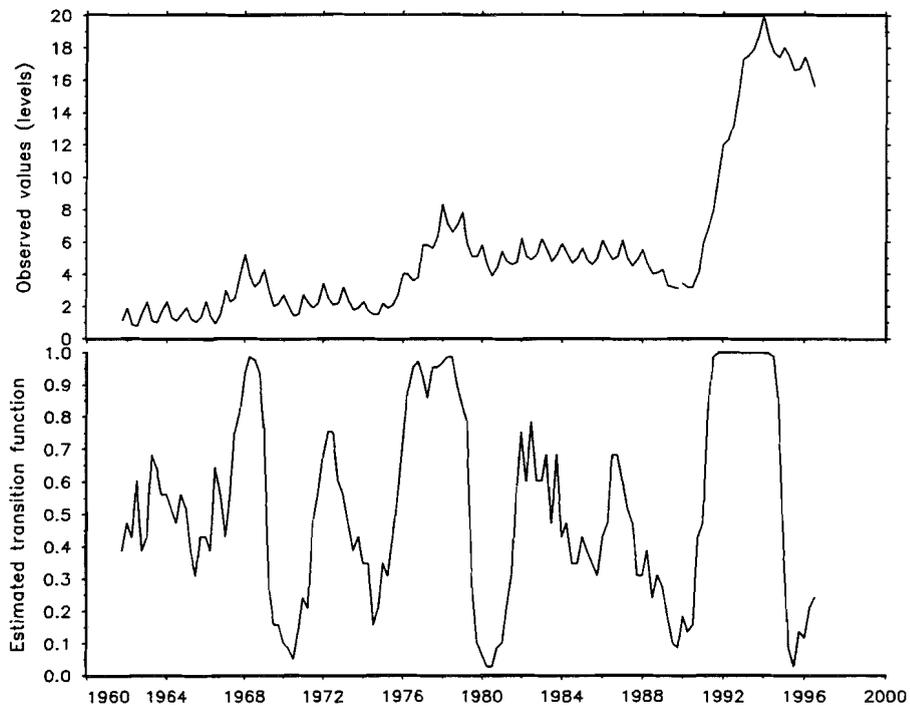


Figure 5.7. Finland. Observed values (upper panel). Estimated transition function (lower panel).

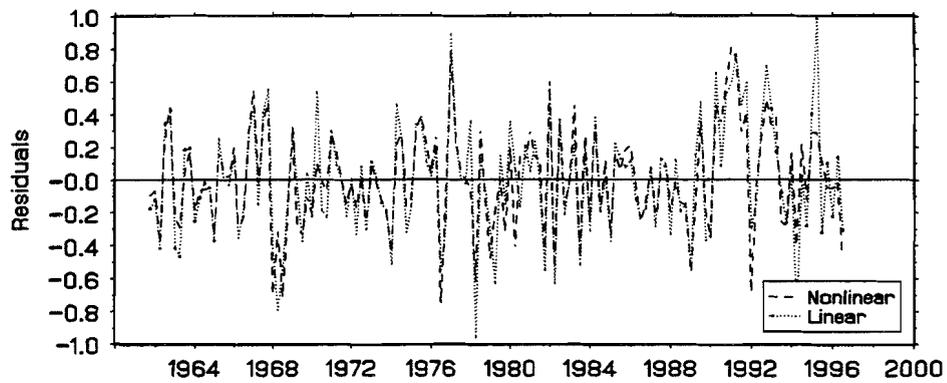


Figure 5.8. Finland. Residuals of the estimated LSTAR model (dashed line) and the corresponding linear AR model (dotted line).

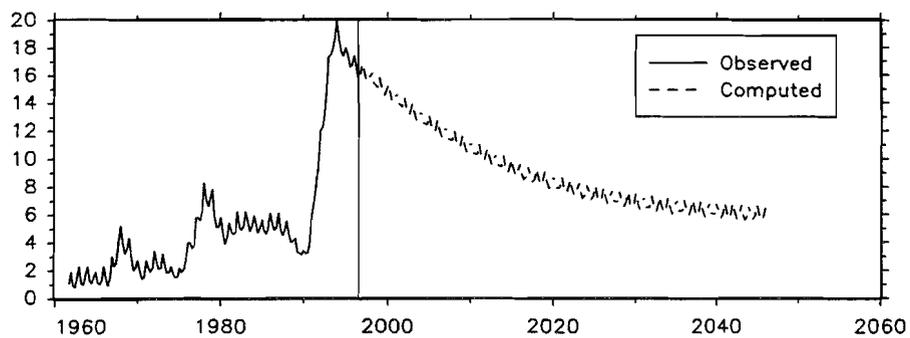


Figure 5.9. Finland. Deterministic extrapolation of the estimated model (horizon 50 years).

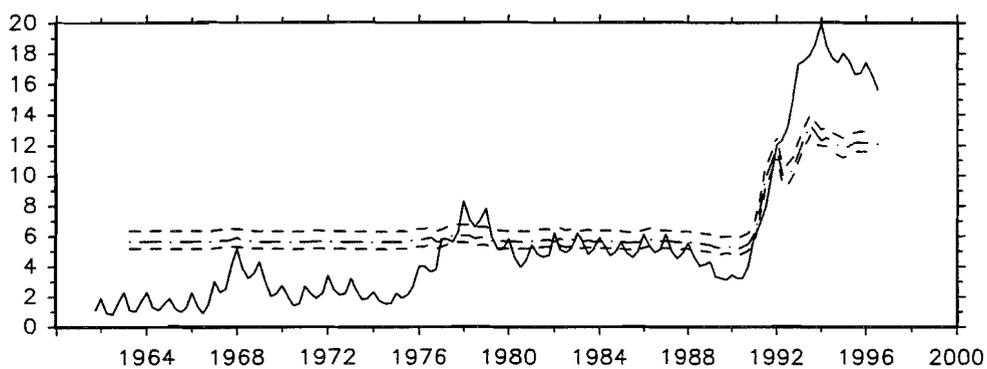
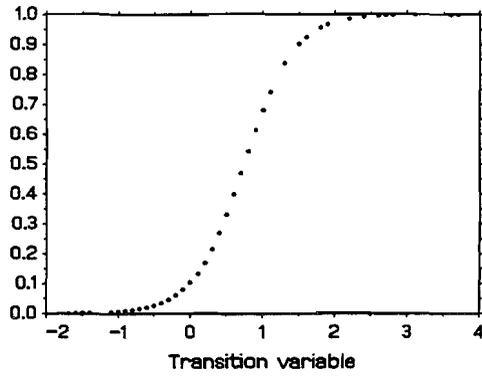
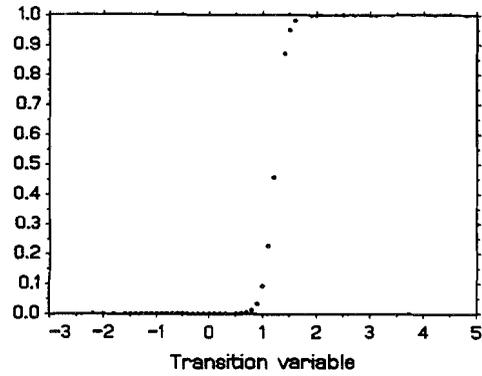


Figure 5.10. Finland. Average, max., and min. over the last four observations of consecutive deterministic extrapolations (horizon ten years).

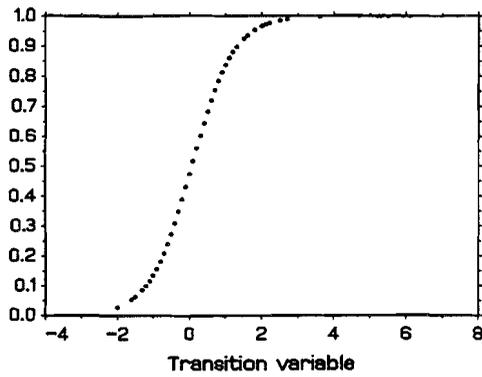
Figure 5.11. Estimated transition functions, \hat{G} , as functions of the transition variables. Each dot represents at least one observation



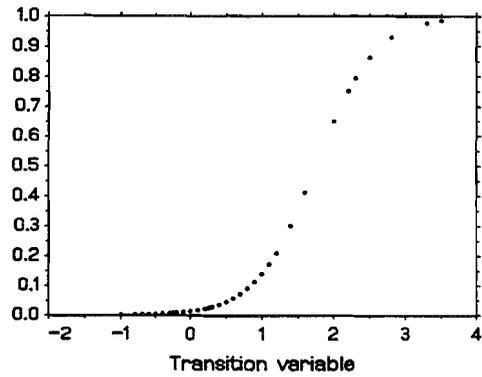
a. Australia



b. Denmark



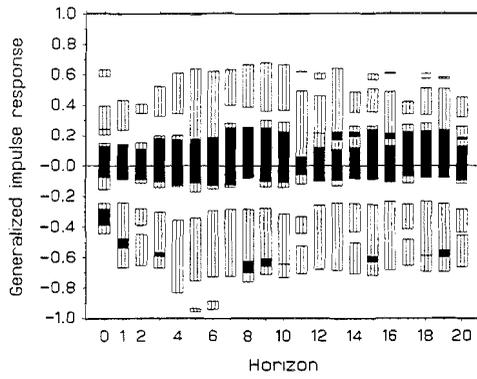
c. Finland



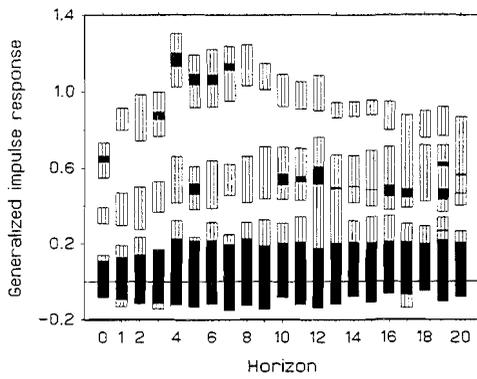
d. Sweden

Figure 5.12.

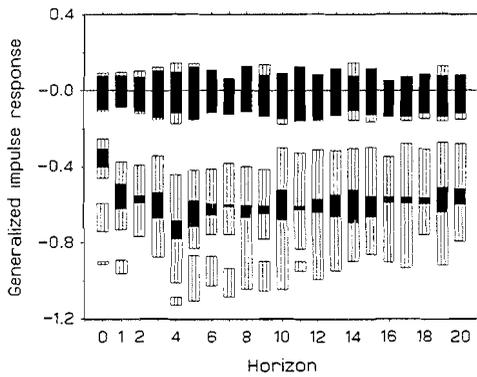
Finland. 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks



Response to symmetric shocks



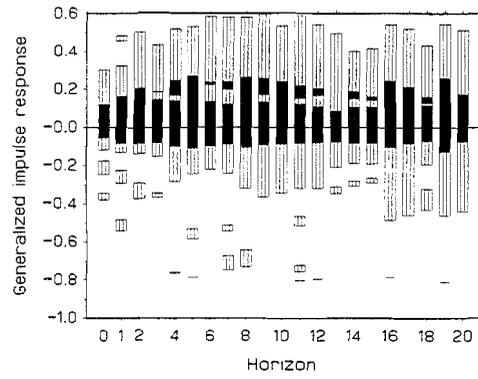
Response to positive shocks, greater than 1 error s.d.



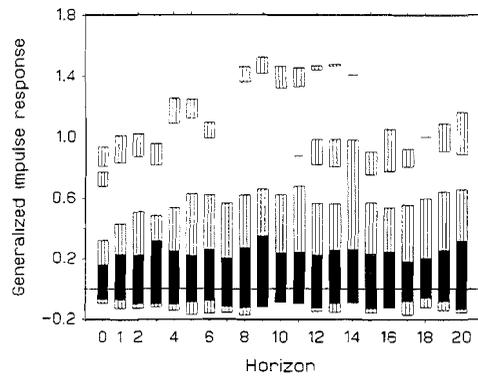
Response to negative shocks, less than -1 error s.d.

Figure 5.13.

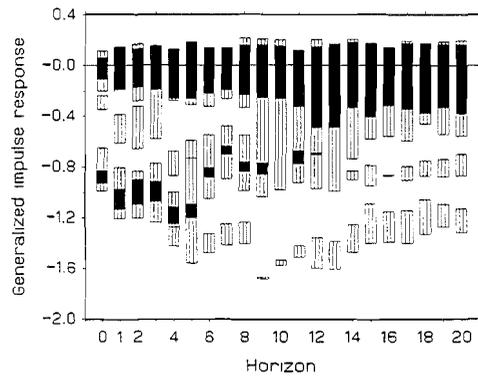
Denmark. 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks



Response to symmetric shocks



Response to positive shocks, greater than 1 error s.d.



Response to negative shocks, less than -1 error s.d.

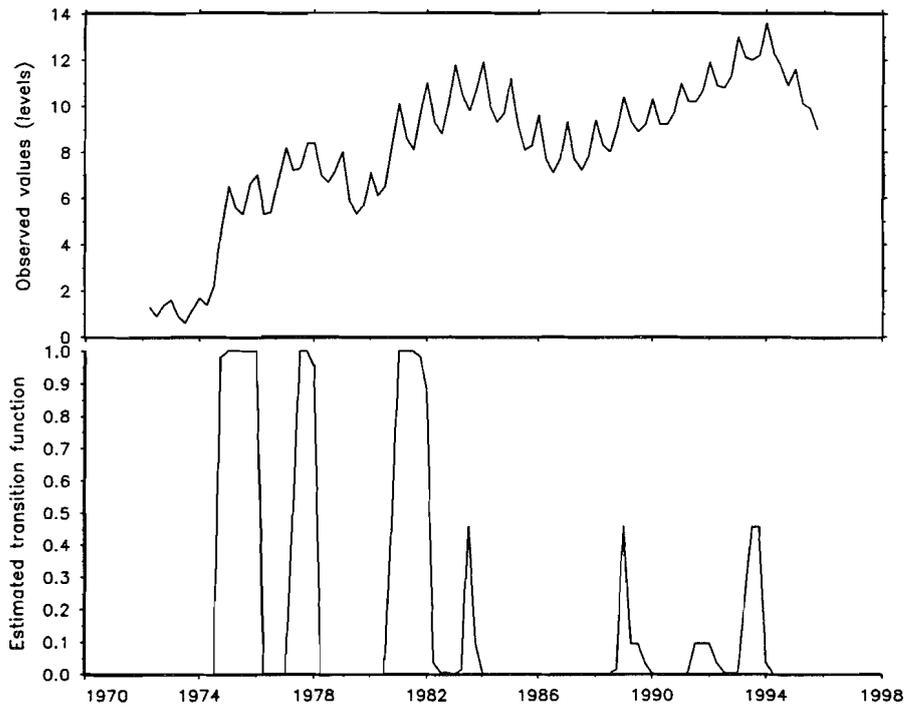


Figure 5.14. Denmark. Observed values (upper panel). Estimated transition function (lower panel).

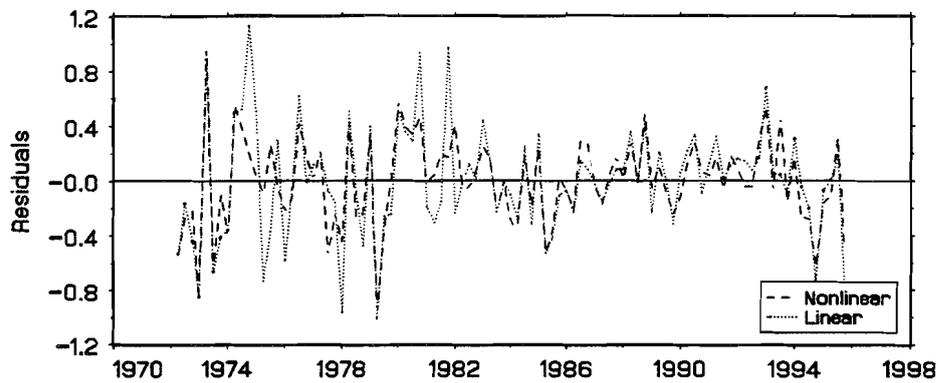


Figure 5.15. Denmark. Residuals of the estimated LSTAR model (dashed line) and the corresponding linear AR model (dotted line).

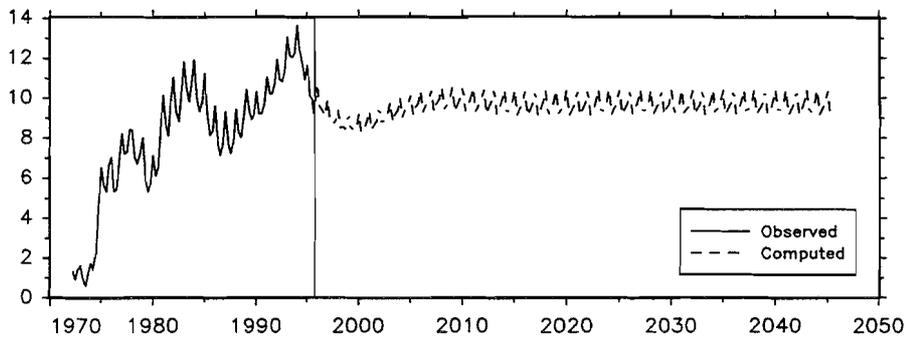


Figure 5.16. Denmark. Deterministic extrapolation of the estimated model (horizon 50 years).

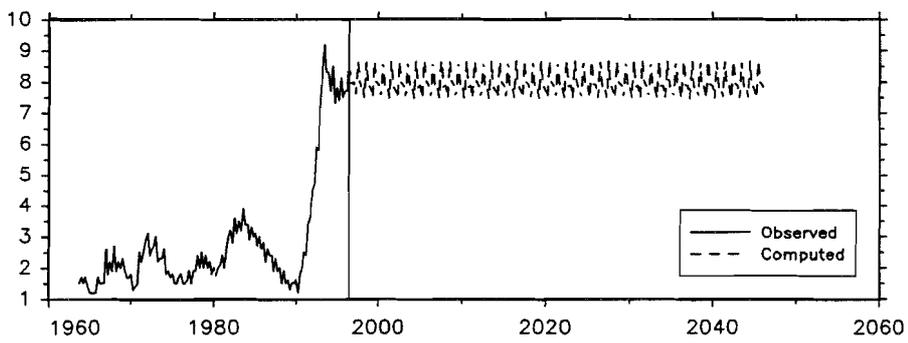


Figure 5.17. Sweden. Deterministic extrapolation of the estimated model (horizon 50 years).

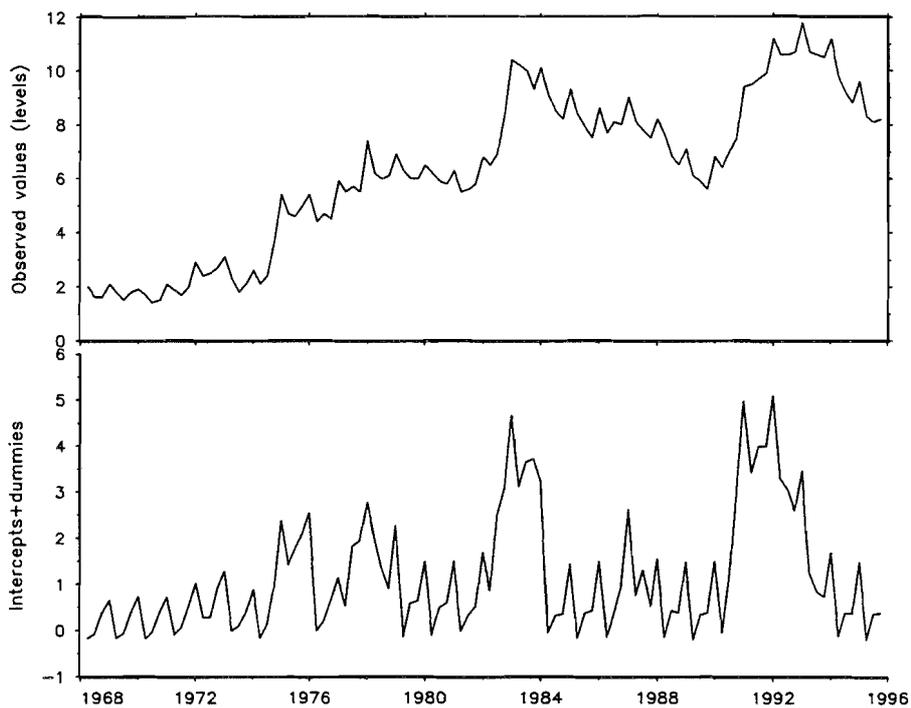


Figure 5.18. Australia. Time series of the total of estimated intercepts and dummies multiplied by transition functions.

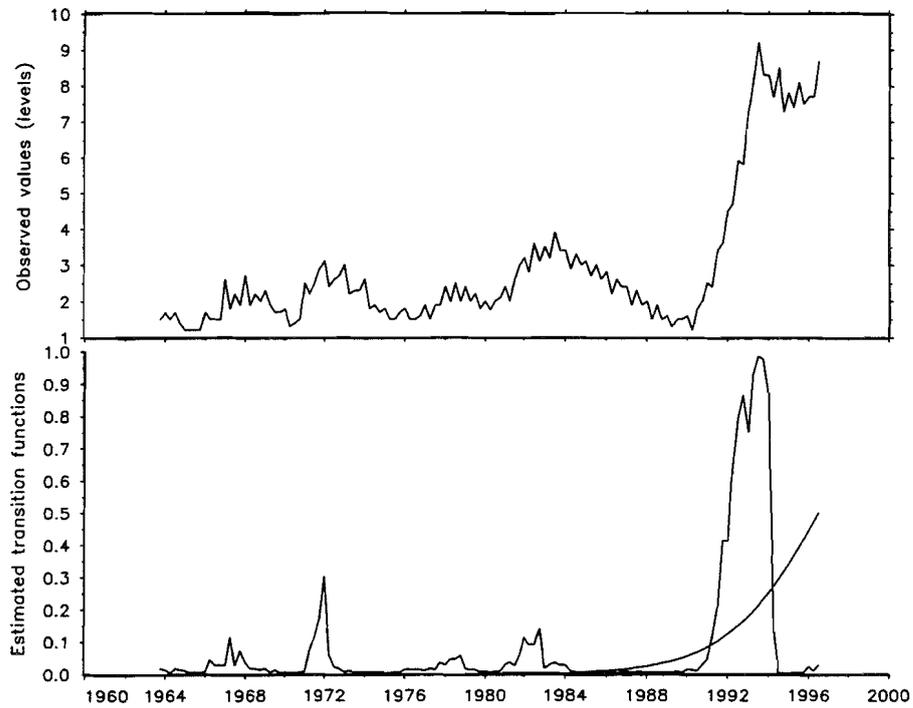


Figure 5.19. Sweden. Observed values (upper panel). Estimated transition functions over time (lower panel): (a) transition function \hat{G} over time (irregular curve), (b) transition function \hat{H} (smooth curve).

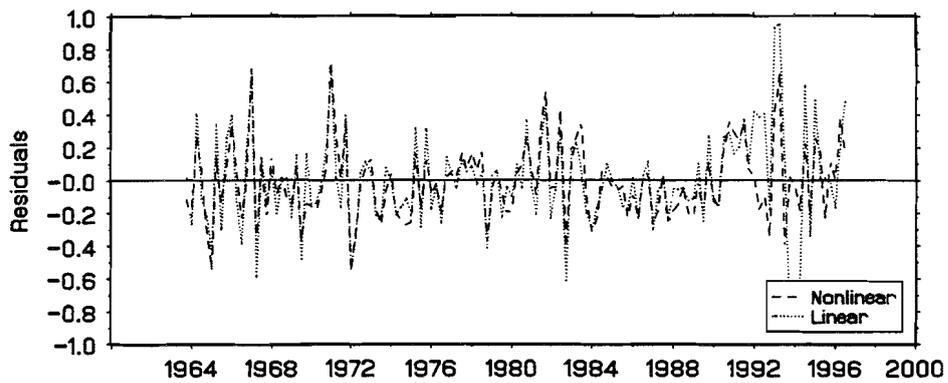
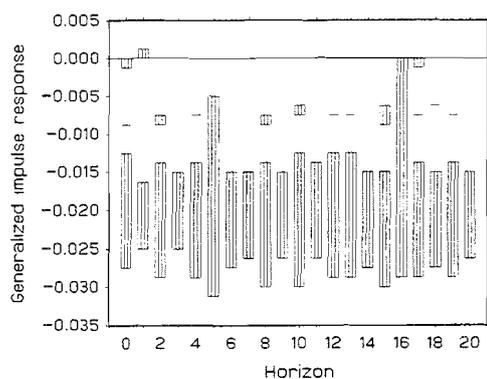


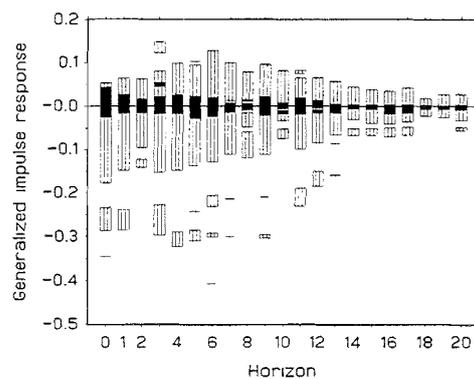
Figure 5.20. Sweden. Residuals of the estimated LSTAR model (dashed line) and the corresponding linear AR model (dotted line).

Figure 5.21.
Sweden. 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks

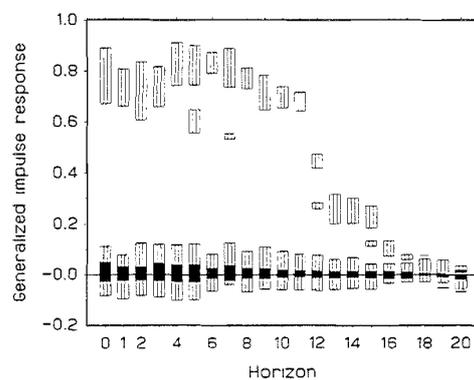


Response to symmetric shocks

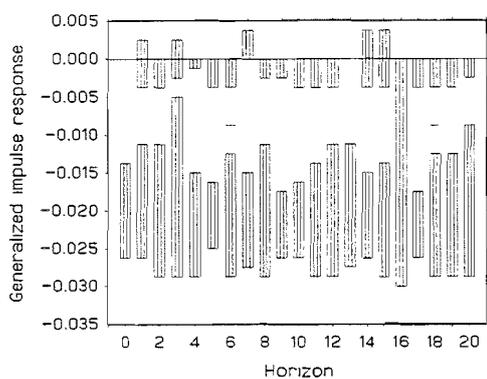
Figure 5.22.
Australia. 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks



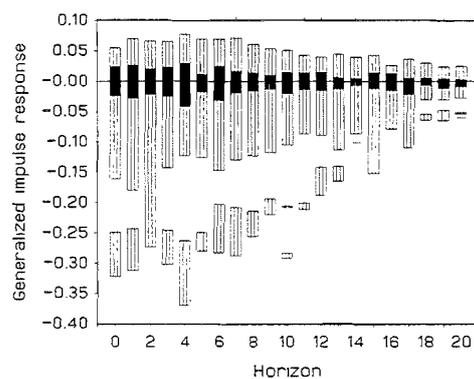
Response to symmetric shocks



Response to positive shocks, greater than 1 error s.d.



Response to negative shocks, less than -1 error s.d.



Response to negative shocks, less than -1 error s.d.

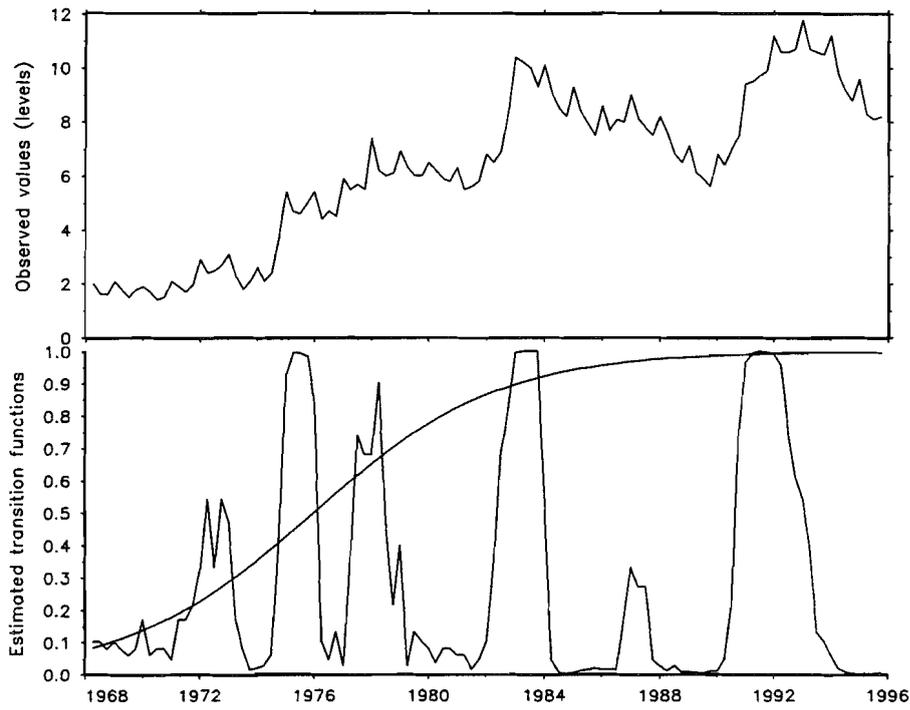


Figure 5.23. Australia. Observed values (upper panel). Estimated transition functions over time (lower panel): (a) transition function \hat{G} over time (irregular curve), (b) transition function \hat{H} (smooth curve).

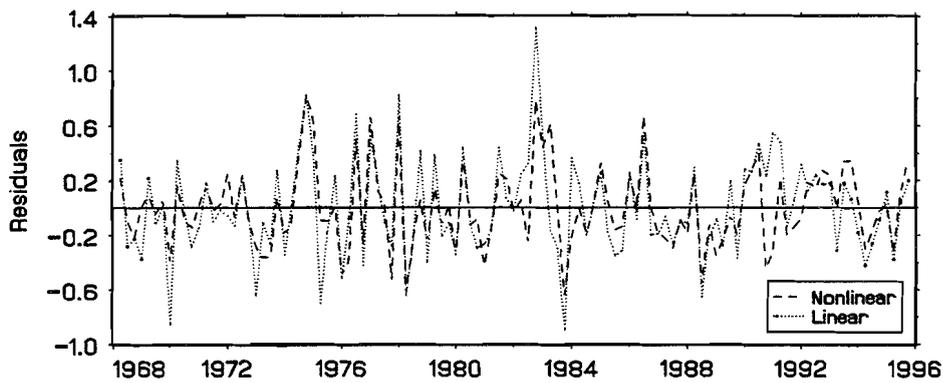


Figure 5.24. Australia. Residuals of the estimated LSTAR model (dashed line) and the corresponding linear AR model (dotted line).

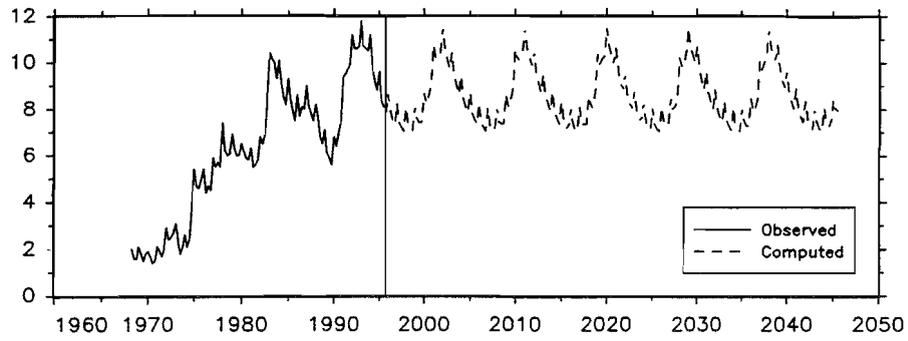


Figure 5.25. Australia. Deterministic extrapolation of the estimated model (horizon 50 years).

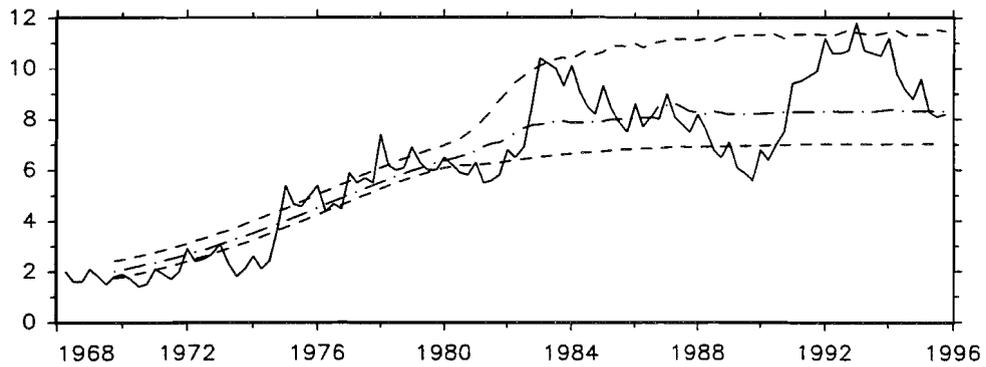


Figure 5.26. Australia. Average, max., and min. over the last 60 observations of consecutive deterministic extrapolations (horizon 50 years).

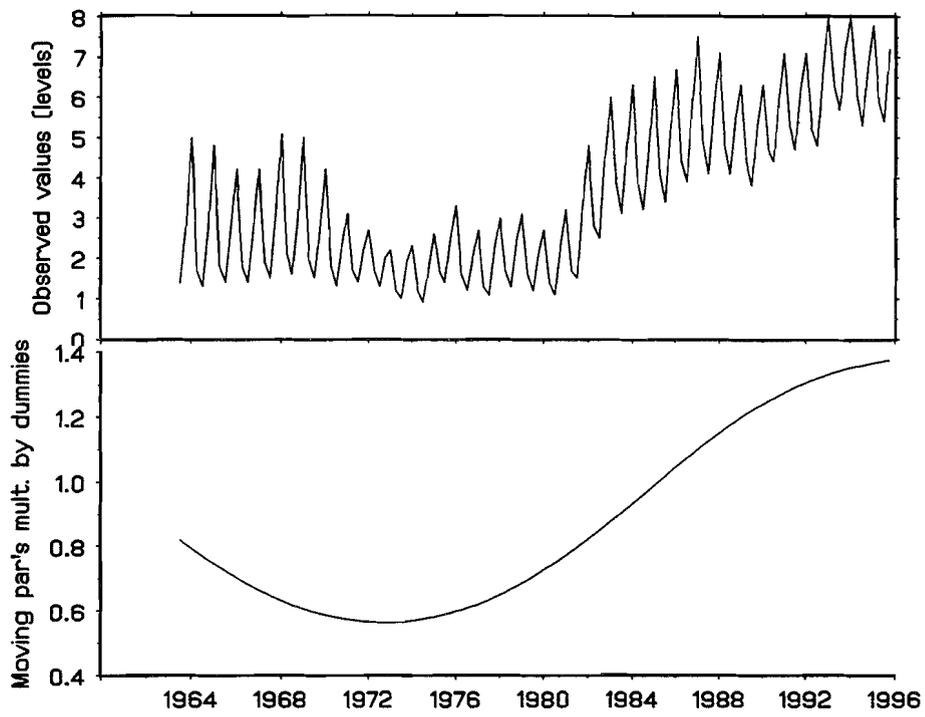


Figure 6.1 Austria. Observed values (upper panel) and moving intercept (lower panel)

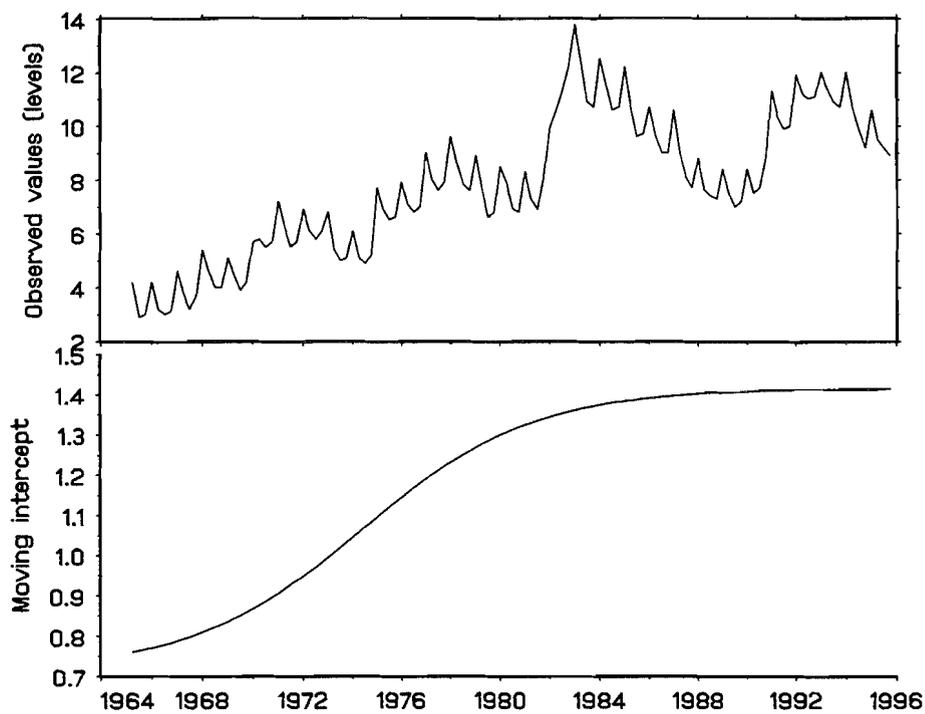


Figure 6.2 Canada. Observed values (upper panel) and moving intercept (lower panel)

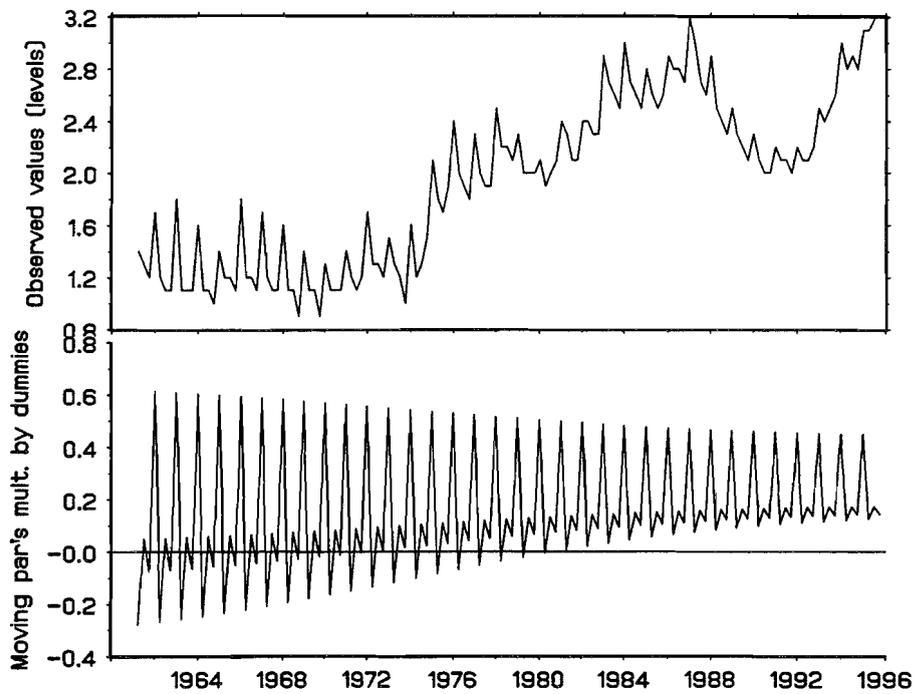


Figure 6.3 Japan. Observed values (upper panel) and moving parameters multiplied by seasonal dummies (lower panel)

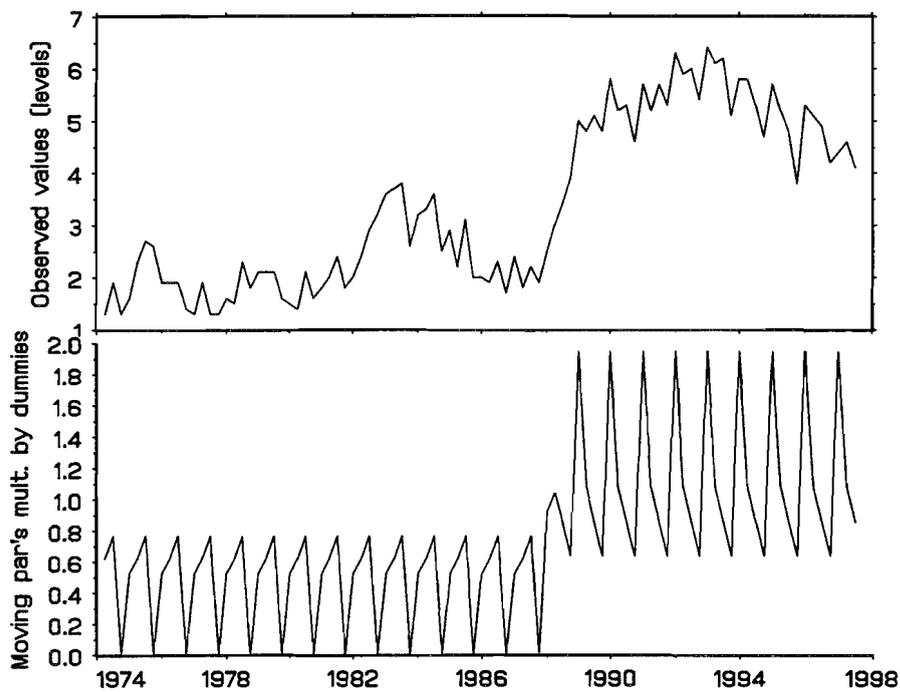


Figure 6.4 Norway. Observed values (upper panel) and moving parameters multiplied by seasonal dummies (lower panel)

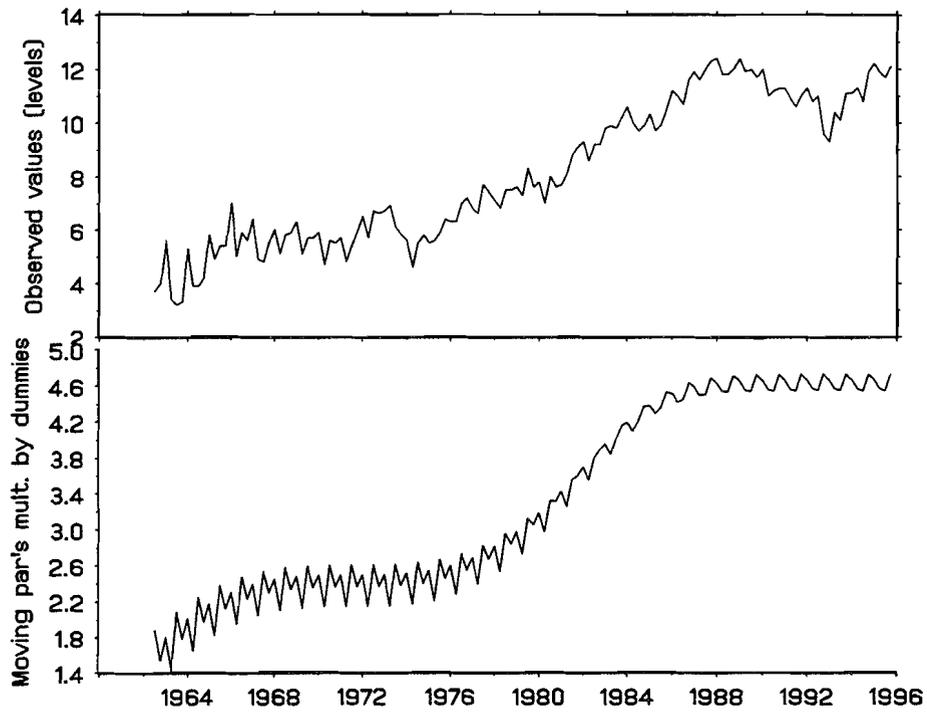


Figure 6.5 Italy. Observed values (upper panel) and moving parameters multiplied by seasonal dummies (lower panel)



CHAPTER III

TESTING LINEARITY AGAINST SMOOTH TRANSITION AUTOREGRESSION USING A PARAMETRIC BOOTSTRAP

1 Introduction

Testing hypotheses about parameters in econometric models when one or more nuisance parameters are not identified under the null hypothesis requires non-standard methods, since the standard distribution results for the classical test statistics (the likelihood ratio, Lagrange multiplier, and Wald tests) do not apply. The problem arises in many situations in applied work. For instance, when the null of linearity of a time series is tested against a smooth transition autoregressive (STAR) model (or against any one of a number of other nonlinear alternative specifications) some of the parameters are unidentified under the null hypothesis.

Among the different solutions to this general statistical problem suggested in the literature, the method proposed by Teräsvirta (1994) is the most commonly utilised one in STAR modelling. However, since this test procedure circumvents the identification problem by an appropriate linearisation (see, e.g., Luukkonen, Saikkonen and Teräsvirta (1988) for details) information about the nonlinear structure under the alternative is lost and the power may be adversely affected. If so, it might be worthwhile to consider other tests. While this test is based on standard asymptotic distribution theory, solutions suggested by other authors in practice rely on computational statistical methods for establishing critical values for the test statistics. In this spirit, Hansen (1996) developed a technique for establishing critical values for test statistics derived by Andrews and Ploberger (1994) but his method is computationally very demanding.

Desirable properties of any competing method include reasonable computational costs and, above all, at least as good power properties as the linearisation-based procedure mentioned above. The present paper is a first attempt at investigating whether bootstrapping a likelihood-ratio test of linearity against STAR meets these requirements and therefore merits further research.

The rest of the paper is organised as follows: In Section 2 the STAR model and the linearity test are introduced, the identification problem is discussed, and some previously suggested general solutions are briefly reviewed. Section 3 describes the bootstrap test and Section 4 the simulation study. Section 5 concludes.

2 Testing linearity against STAR

2.1 The STAR model

A smooth transition autoregression, STAR, model of order p is defined as

$$y_t = \theta' w_t + (\pi' w_t) F(y_{t-d}; \gamma, c) + u_t, \quad (1)$$

where $u_t \sim \text{nid}(0, \sigma_u^2)$, $\theta = (\theta_0, \theta_1, \dots, \theta_p)'$, $\pi = (\pi_0, \pi_1, \dots, \pi_p)'$, and $w_t = (1, y_{t-1}, \dots, y_{t-p})'$. Model (1) is called a logistic smooth transition autoregression, LSTAR, model, if the transition function $F(y_{t-d}; \gamma, c)$ is defined as

$$F(y_{t-d}; \gamma, c) = (1 + \exp\{-\gamma(y_{t-d} - c)\})^{-1}, \gamma > 0 \quad (2)$$

and an exponential, ESTAR, model, if

$$F(y_{t-d}; \gamma, c) = 1 - \exp\{-\gamma(y_{t-d} - c)^2\}, \gamma > 0. \quad (3)$$

Early discussions of smooth transition (auto)regression models can be found in Bacon and Watts (1971), Goldfeld and Quandt (1972) and Maddala (1977, p. 396). More recently, STAR models have received attention in both theoretical and applied work. Teräsvirta (1994) and Eitrheim and Teräsvirta (1996) present a modelling cycle for STAR models including testing linearity against STAR, specifying and estimating a STAR model if linearity is rejected, and evaluating the estimated model by a number of diagnostic tests. An overview of the modelling cycle, as well as applications and references, can be found in Teräsvirta (1998).

The first step of the modelling cycle, testing the null hypothesis of linearity against the proposed nonlinear model, is crucial; clearly, nonlinear modelling should not be attempted if a linear model is an adequate representation of the data. However, devising a test of linearity against the STAR model is complicated by the fact that (1) is not identified under the null hypothesis. The next subsection describes the problem of hypothesis testing when nuisance parameters are not identified under the null hypothesis.

2.2 The identification problem

Testing linearity against STAR in (1) with (2) or (3) amounts to testing $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$. However, under the null hypothesis, both π and c are nuisance parameters that can take any value without affecting the likelihood. Thus the model is identified only under the alternative but not under the null hypothesis and the standard χ^2 asymptotic theory of the three classical tests does not hold. The presence of an identification problem is also seen by noticing the fact that the testing problem could equally well be formulated in terms of the lag parameters of the 'nonlinear part' of the model, i.e., testing $H_0 : \pi = 0$ against $H_1 : \pi \neq 0$.

The general problem of hypothesis testing when a nuisance parameter vector, β , is not identified under the null hypothesis was first discussed by Davies (1977, 1987), who suggested that a test statistic, $S(\beta)$, be first derived under the assumption that the nuisance parameter vector is fixed, $\beta = \beta^*$. The actual test statistic is then defined as $\sup_{\beta^* \in \mathcal{B}} S(\beta^*)$ (assuming a right-tailed

test). Covering the 'worst case', this procedure is clearly conservative. The first econometric application of this idea was Watson and Engle (1981). The asymptotic null distribution of the supremum test statistic is not generally known analytically.

Luukkonen, Saikkonen and Teräsvirta (1988) and Saikkonen and Luukkonen (1988) suggested a technique that may be viewed as being similar in spirit to the solution of Davies (1977). Here, the transition function is replaced by a Taylor approximation around H_0 and the model is reparameterised in order to obtain an auxiliary linear regression where certain parameters equal zero if $\gamma = 0$, thereby circumventing the identification problem. This test has been used in a number of applied studies (see Teräsvirta (1998) for examples). In a comparative simulation study in Hansen (1996) the Taylor approximation based test performed well against a self-exciting threshold autoregression (SETAR) model.

Andrews and Ploberger (1994) derived optimal versions of the three classical tests for a situation where one of the nuisance parameters affects the likelihood only under the alternative hypothesis. However, these average exponential tests require that the investigator specifies a weight function over the possible values of the unidentified nuisance parameter. From a Bayesian point of view the weight function may be seen as a prior, and the tests are asymptotically equivalent to Bayesian posterior odds ratios. Andrews and Ploberger do not discuss how critical values for the test statistics should be obtained in practice.

Hansen (1996) developed a procedure for computing simulated critical values for, e.g., the Davies and the Andrews and Ploberger test statistics. If the nuisance parameter that is not identified under the null belongs to a continuous parameter space, the investigator in practice has to select a number of discrete values for the parameter. In every replicate of the simulation a test statistic is then calculated given each one of these parameter values, and one of the methods suggested by Davies or Andrews and Ploberger is applied to arrive at a single test statistic for the replicate. The decision is then based upon the comparison of the original test statistic, in the form of a p -value, to the simulated distribution. The combination of simulating the distribution and the need for selecting a set of values for the nuisance parameter makes the method computationally intensive. Furthermore, Hansen (1996) explicitly discussed only the case where *one* nuisance parameter is unidentified under the null hypothesis. In the STAR case, whichever way the test is parameterised, the testing problem generally involves at least two such parameters. In practice, a generalisation to more than a single nuisance parameter would considerably increase the already substantial computational burden. It should also be noted that the power of the test is dependent on the selected set of values for the nuisance parameter (or parameters).

Given the statistical problems briefly described above and the increasing interest in simulation-based methods, reflected in part by the works cited above, it seems natural to consider using a bootstrap procedure to establish the empirical distribution of a linearity test. In the next section a bootstrap test that should be less computationally intensive than the one described in the previous paragraph is outlined, and its size and power properties are discussed in the subsequent section.

3 The bootstrap test

It is possible to construct a parametric bootstrap likelihood ratio test of linearity against STAR. The empirical distribution for the test statistic is established through resampling from time series generated using the estimates of the parameters under the null hypothesis and normally distributed random errors whose variance equals the estimated residual variance of the model. For a general discussion of bootstrap tests with examples and applications the reader is referred to, e.g., Davidson and MacKinnon (1996).

In order to obtain an empirical distribution of the test statistic, model (1) is assumed to be completely specified under the null and under the alternative, i.e., the parameters p (the autoregressive order), the form of the transition function F (LSTAR (2) or ESTAR (3)), and d (the delay parameter of the transition function) are assumed to be known and only the values of the parameters in the vector $\phi = [\theta', \pi', \gamma, c]'$, and σ_u^2 , are unknown. In practice, neither the form of the transition function nor the value of the delay parameter d is typically known and has to be selected using the data. However, before considering the test in this more complicated situation, its performance in the simplest case has to be investigated. The sample data set is denoted $y = [y_1, \dots, y_T]'$. The test procedure consists of the following steps:

1. Estimate the model under H_0 , i.e. a linear AR model. The estimates are denoted $\tilde{\theta}$ and $\tilde{\sigma}_u^2$ and the value of the likelihood function is L_{max}^0 .
2. Estimate the model under H_1 , i.e. the STAR model. The estimates are denoted $\hat{\phi} = [\hat{\theta}', \hat{\pi}', \hat{\gamma}, \hat{c}]'$ and $\hat{\sigma}_u^2$. The value of the likelihood function is L_{max}^1 .
3. Compute the value of the LR statistic $\hat{\tau} = -2 \log (L_{max}^0 / L_{max}^1)$.
4. Generate $T_B R$ pseudo random numbers, $u_{r,t}^* \sim N(0, \tilde{\sigma}_u^2)$, $t = 1, \dots, T_B$, $r = 1, \dots, R$, $T_B < T$.
5. Generate R time series of length T_B using the estimated model under H_0 , $y_{r,t}^* = \tilde{\theta}' w_t^* + u_{r,t}^*$.
6. For each time series, $r = 1, \dots, R$, estimate the model under H_0 , i.e. a linear AR model. The estimates are denoted $\tilde{\theta}^*$ and $\tilde{\sigma}_{r,u}^{*2}$, and the value of the likelihood function is $L_{r,max}^{*0}$.
7. For each time series, $r = 1, \dots, R$, estimate the model under H_1 , i.e. the STAR model. The estimates are denoted $\hat{\phi}_r^* = [\hat{\theta}_r^{*'}, \hat{\pi}_r^{*'}, \hat{\gamma}_r^*, \hat{c}_r^*]'$ and $\hat{\sigma}_{r,u}^{*2}$. The value of the likelihood function is $L_{r,max}^{*1}$.
8. For each time series, $r = 1, \dots, R$, compute the value of the LR statistic $\tau_r^* = -2 \log (L_{r,max}^{*0} / L_{r,max}^{*1})$.
9. Compute the estimated bootstrap p -value function (see, e.g., Davidson and MacKinnon, 1996) as the ratio

$$p^*(\hat{\tau}) = \frac{\#(\tau_r^* > \hat{\tau})}{R}$$

where $\#(\tau_r^* > \hat{\tau})$ is the number of times over the R series that $\tau_r^* > \hat{\tau}$, $r = 1, \dots, R$.

10. If $p^*(\hat{\tau}) < \alpha$, where α is the selected significance level, the null hypothesis of linearity is rejected.

Note that $T_B < T$, i.e. the bootstrap replicate series is shorter than the original time series. This choice is based on the theoretical considerations in Bickel, Götze and van Zwet (1997); asymptotically, $T_B/T \rightarrow 0$ as $T_B, T \rightarrow \infty$. (See also Rydén, Teräsvirta, and Åsbrink (1998) for a discussion and an application.)

In practice it may happen that the nonlinear least squares estimation of the STAR model in step 2 or 7 above converges to a local optimum rather than the global, or simply fails to converge at all. Convergence to a local optimum due to a flat likelihood function cannot be controlled, but since only the estimated value of the likelihood function, and not the estimated parameter vector, enters the test statistic, the effects of a flat likelihood should not be too severe. If the numerical optimisation frequently fails to converge at all, however, the results become difficult to interpret. To mitigate this problem, the optimisation is restarted with a new set of initial values for the parameters whenever the first attempt fails to converge within a certain limit on computing time or on the number of iterations. Only if optimisation starting from, say, three or four different initial parameter vectors fails to yield convergence the simulation replicate (if the problem appears at step 2) or the bootstrap replicate (if the problem occurs at step 7) is dropped. In the simulations reported below, the problems of non-convergence turn out to be very small.

4 Size and power properties

In this section, the power and size properties of the proposed procedure are illustrated by means of a small Monte Carlo study. Since the nonlinear least squares estimation of the STAR model in the second and seventh steps of the test procedure described in the previous section is rather time-consuming, and given scarce computing resources, only a very limited simulation study has been possible at this stage, and the results should be regarded as indicative.

4.1 The simulation setup

In the size simulations, the data are generated by

$$y_t = 1.3y_{t-1} - 0.5y_{t-2} + u_t \quad (4)$$

where u_t is a pseudo random number, $u_t \sim \text{nid}(0, \sigma_u^2)$, with $\sigma_u^2 = 0.0004$ or $\sigma_u^2 = 0.001$. In the power simulations, data are generated either by the LSTAR model

$$y_t = 1.8y_{t-1} - 1.06y_{t-2} + (0.02 - 0.9y_{t-1} + 0.795y_{t-2}) \times (1 + \exp\{-\gamma(y_{t-1} - 0.02)\})^{-1} + u_t \quad (5)$$

where $\gamma = 20$ or $\gamma = 100$ (cf Teräsvirta 1994, eq. 4.1-4.2) or by the ESTAR model

$$y_t = 1.8y_{t-1} - 1.06y_{t-2} + (0.02 - 0.9y_{t-1} + 0.795y_{t-2}) \times \left(1 - \exp\left\{-\gamma(y_{t-1} - 0.02)^2\right\}\right) + u_t \quad (6)$$

with $\gamma = 100$ or $\gamma = 1000$ (cf Teräsvirta 1994, eq. 4.1 and 4.6). In both cases, $u_t \sim \text{nid}(0, \sigma_u^2)$, with $\sigma_u^2 = 0.0004$. Two different sample sizes are used, $T = 100$ (with $T_B = 90$) and $T = 500$ (with $T_B = 450$). As noted above, due to computation time constraints, the simulation study is rather limited in scope: $R = 200$ bootstrap replicates are used throughout, and the simulations are all based on 500 replicates. For comparison, the empirical size and power of the linearity test of Teräsvirta (1994) have been calculated for the same data generating processes. In these simulations, however, 10000 replicates were used. A nominal significance level of $\alpha = 0.05$ is applied throughout.

The simulation study is programmed in *Gauss for Windows* (using the *Optimum* package for all numerical optimisation) and executed on personal computers. Since the equipment used varies considerably with respect to, e.g., processor type and speed, no systematic evaluation of the computing time is made, but run times for one simulation experiment range approximately from 30 hours to 200 hours (on PCs with frequencies between 200 and 350 MHz).

4.2 Simulation results

The results of the simulations are given in Tables 1-4. Tables 1 and 2 indicate that the bootstrap test is fairly well-sized, making a power simulation meaningful. The Taylor approximation based test is slightly conservative. The empirical power of the bootstrap test is lower than or equal to that of the Taylor approximation based test in all investigated cases except for the ESTAR model with the sharper transition where the bootstrap test has better power. On the other hand, the ESTAR model with the smoother transition is the case that favours the auxiliary regression based test the most. As $\gamma \rightarrow \infty$ the ESTAR model approaches a linear model (with probability one) and the ESTAR realisations become more difficult to distinguish from linear ones since the 'nonlinearity' involves a very small number of observations in the sample. This is illustrated by the power results for the ESTAR data generating process with $\gamma = 1000$; with a very sharp transition, there is power to be gained by not omitting information about the nonlinear structure through linearisation. The problems of non-convergence seem to be manageable in the present setting.

5 Conclusions

The simulation study of this paper suggests that the bootstrap likelihood ratio test of linearity against STAR has good size properties but is generally less powerful than the auxiliary regression based test suggested by Luukkonen, Saikkonen and Teräsvirta (1988). The simulation results concerning the latter test accord well with previous studies. Thus the simulations do not indicate that the latter should be abandoned in favour of the bootstrap test. Furthermore, it should be noted that only testing against a fully known alternative model has

been considered here. The auxiliary regression based test has the advantage that it is generally powerful against LSTAR and ESTAR simultaneously if the order of the Taylor expansion is at least two. This property is a useful one in STAR model building; see, for example, Teräsvirta (1994).

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Table 1. Empirical size for sample size = 100, and numbers of simulation replicates discarded due to non-convergent estimation (in square brackets)

Data generating process		Bootstrap test		Approximation based test
		Alternative model LSTAR (eq. 5)	Alternative model ESTAR (eq. 6)	
AR (eq. 4)	$\sigma_u^2 = 0.0004$	0.058 [0]	0.038 [0]	0.039
	$\sigma_u^2 = 0.01$	0.052 [6]	0.052 [6]	0.039

Table 2. Empirical size for sample size = 500, and numbers of simulation replicates discarded due to non-convergent estimation (in square brackets)

Data generating process		Bootstrap test		Approximation based test
		Alternative model LSTAR (eq. 5)	Alternative model ESTAR (eq. 6)	
AR (eq. 4)	$\sigma_u^2 = 0.0004$	0.050 [0]	0.056 [0]	0.041
	$\sigma_u^2 = 0.01$	0.048 [2]	0.054 [11]	0.041

Table 3. Empirical power for sample size = 100

Data generating process		Alternative model			
		LSTAR (eq. 5)		ESTAR (eq. 6)	
		Bootstrap test	Approx. based test	Bootstrap test	Approx. based test
LSTAR (eq. 5)	$\gamma = 20$	0.328	0.391		
	$\gamma = 100$	0.848	0.947		
ESTAR (eq. 6)	$\gamma = 100$			0.754	0.907
	$\gamma = 1000$			0.942	0.873

Table 4. Empirical power for sample size = 500

Data generating process		Alternative model			
		LSTAR (eq. 5)		ESTAR (eq. 6)	
		Bootstrap test	Approx. based test	Bootstrap test	Approx. based test
LSTAR (eq. 5)	$\gamma = 20$	0.802	0.994		
	$\gamma = 100$	0.978	1.000		
ESTAR (eq. 6)	$\gamma = 100$			0.996	1.000
	$\gamma = 1000$			1.000	1.000

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