Studies in Company Valuation

Per Olsson

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Preface and summary

How do we best measure value creation in companies, and how do we best summarize our expectations about future value creation into an estimate of equity value? These are the questions this dissertation tries to address. Put in somewhat less grand terms, the theme of this dissertation is company valuation, or more precisely equity valuation. In particular, models derived from the neoclassical definition of equity value – that the value of a share is equal to the present value of future expected dividends – will be investigated. Such valuation models, in particular the free cash flow model but also the abnormal earnings (or residual income) model, have received an enormous interest in later years, both in academic circles and among practitioners. (This family of valuation models also includes other popular approaches, such as the EVA model – Economic Value Added – and the APV model – Adjusted Present Value. These approaches are not explicitly considered in this dissertation, since they are closely related to the aforementioned models).

So why is this a relevant problem area? Why can't we be satisfied with the dividend model? Why not just take a stream of for many firms readily available dividend forecasts, discount them, calculate the estimated equity value, and be done with it? While this is sometimes made, especially in some academic papers in the economics field, practitioners have long recognized that, as a practical matter, an equity valuation model involving only expected dividends is difficult or even impossible to implement. Without going into details at this stage, one may think about a company such as Microsoft, which at present has a zero dividend policy. If one believes that this policy will persist for the foreseeable future, then the value of the Microsoft share according to a standard dividend valuation model would be zero. This is obviously nonsensical, and there are different suggestions 'out there' as how to handle this problem. Before turning to such suggestions, one should note that there is often some confusion about this issue. What about Miller and Modigliani's [1961] dividend policy irrelevance? Does that not hold? The answer is no. Not here. For the shareholder, the Miller and Modigliani argument basically implies that (under certain assumptions) what is not paid out as dividends will become capital gains instead and the owner of a share will receive the money anyhow – and then dividend policy is without consequence. The problem with the standard dividend model, however, is that it leaves no room for capital gains. It typically assumes an infinite series of dividends (as in the Gordon model) and then capital gains will never be realized. So by abstracting from capital gains, the dividend model reintroduces dividend policy as a variable of primary importance. Since we know empirically (at least since Lintner [1956]) that most companies are careful and conservative when setting dividends, the dividend model will likely underestimate the value of most companies.

This thesis can be said to deal with valuation models designed to circumvent the practical problems with standard dividend valuation, thus adhering to 'sound valuation principles' such as dividend policy irrelevance. The different valuation models have a broader rationale (and perhaps appeal) than that, however: a selling argument one often hears for the free cash flow model, for example, is that it concentrates on the value-creating side of the firm: its operations. Thus, the free cash flow model should *measure value* better than other models. Advocates for the abnormal earnings (residual income) model also focus on measurement issues: the accrual accounting system is designed to measure value in a way that matches revenues with associated costs, and thus a valuation model based on accrual accounting income should be able to measure value more adequately. There are many more arguments made for and against different models – the above are only meant as examples.

There is an inherent paradox in all this. These valuation models are formally equivalent, and yet there is an often fierce debate about which model is superior. One answer to this apparent paradox is, I think, that the implementation of the different models requires that different items be estimated. Some items might be much more difficult to estimate than others. Furthermore, one typically makes different simplifying assumptions along the road when implementing the different models – and different assumptions may cause quite substantial differences in the resulting value estimates. The reader may note that all of these arguments revolve around practical issues, such as estimations and assumptions.

This thesis tries to deal with these and related issues. The first study *Comparing the Accuracy and Explainability of Dividend, Free Cash Flow and Abnormal Earnings Equity Valuation Models* (with J. Francis and D. Oswald) compares the three valuation models using what is meant to be standard implementations. Using a large sample of analysts' forecasts of the elements valued by these models, the study establishes that there are substantial differences between the models, thus confirming that the choice of valuation model actually matters. Specifically, relative to both dividend and free cash flow based value estimates, abnormal earnings based value estimates are more accurate (i.e., they have smaller absolute deviations from observed security prices) and explain more of the variation in observed security prices. While the free cash flow model on average prices shares correctly, it has a much higher variability than the abnormal earnings model and the study concludes that this is to a large extent driven by the inclusion of book value of equity in the abnormal earnings model, which substantially reduces the overall estimation uncertainty.

The second study, *Discount Rates in Equity Valuation*, continues the focus on estimation uncertainty and its implications for standard implementations of valuation models, but this time concentrating on discount rate estimation, in particular estimation of the cost of equity capital. The question is once again a practical one: are common asset pricing models such as the CAPM *useful* as bases for discount rate estimation in an equity valuation context? Valuation textbooks assume that they are, presumably because asset pricing models such as the CAPM in ex-post empirical tests have proven to have significant explanatory power. The situation in a company valuation situation is quite different, however. In short, we have to estimate future values of all relevant parameters and variables. To continue the example with the CAPM, we have to come up with estimates of the risk-free rate, beta and the market risk premium. There may be a trade-off between exposure to more estimation uncertainty and 'theoretical correctness' when moving from a simple discount rate estimation method to a more advanced asset pricing model, such as the CAPM. The study contrasts different standard techniques for estimating discount rates (the cost of equity capital) in a company (equity) valuation setting. The use of cross-sectionally constant discount rates is shown to provide more reliable value estimates for individual stocks than do standard CAPM implementations, which in turn dominate the Fama-French three-factor model. The reason, it is argued, is that estimation uncertainty plagues the more advanced discount rate estimation techniques to such an extent that it overshadows any potential gains from using what may be more 'correct' models.

In study 3, *Looking Beyond the Horizon and Other Issues in Company Valuation* (with J. Levin), the focus shifts. Instead of replicating standard implementations of different models to investigate the differential properties, this study takes the opposite approach to the paradox described above – that the models are theoretically equivalent, yet they yield different results. Here the question is asked what is required to achieve consistency, both the theoretical details and practical implementation routines. The study focuses on the free cash flow model described in Copeland, Koller and Murrin [1994], since this particular model is arguably the most popular one in academic education today. For a full-scale valuation there are a number of questions to keep track of, and this study goes into quite some detail with many of them. A large part of the study is devoted to problems connected with the horizon value. The primary focus is on clarifying the steady state assumption that underlies the use of horizon values, and the conditions necessary to make this assumption operational. The objective is to suggest a systematic approach, making explicit use of properties of the accounting system. In particular, we note that the time series of forecasted financial statements can be seen as a system of difference equations. Seen in that light, the steady state concept can be made operational by an analysis of difference equations, where all conditions necessary for a steady state can be derived as initial value conditions on the system of equations. The main result is of a normative nature: Flows in the first year after the horizon should be decided such that corresponding stocks grow at the (assumed constant) revenue growth rate. This rule ensures that the company remains qualitatively similar throughout the post-horizon period, which is the main implication of the steady state concept. Although this result is quite simple (and perhaps obvious), the analysis can be viewed as providing a methodology which can be applied to larger and more complicated models as well.

Then follows the equivalence issue: the purpose is to find a discounting procedure that ensures that free cash flow valuation is consistent with the principle that the equity value equals the present value of expected dividends. Complete valuations are analyzed in a model based on explicit forecasts of financial statements. The study then discusses parameter estimation empirically, exemplifying with individual companies but also reporting data on the industry level. Finally, a full-scale implementation of the valuation model is carried out using results from the previous analysis. The idea is to highlight many of the practical problems involved in a valuation of this type and to suggest ways of dealing with them.

Study 4, *Company Valuation with a Periodically Adjusted Cost of Capital* (with J. Levin), in a way continues Study 3. There, the equivalence between the dividend model and the free cash flow model was achieved by periodically adjusting the weights in the weighted average cost of capital for anticipated changes in capital structure. This was shown to be sufficient. Varying capital structure may also have consequences for the cost of equity and the cost of debt, however. Concentrating on the cost of equity, this study extends the analysis in Study 3 by showing how the cost of equity and the weighted average cost of capital can be simultaneously adjusted to reflect a varying capital structure. Different cost of capital settings are introduced and the underlying assumptions of the different settings are related to specific valuation situations, with particular reference to different assumptions of financing policy and the valuation of interest tax shields. Another main purpose is to develop a method for implementing the periodical cost of capital adjustment (and discounting) procedure under the different cost of capital settings.

List of Studies

Study 1: Francis, J., P. Olsson and D. Oswald [1997]: *Comparing the Accuracy and Explainability of Dividend, Free Cash Flow and Abnormal Earnings Equity Valuation Models.*

Study 2: Olsson, P. [1998]: *Discount Rates in Equity Valuation*

Study 3: Levin, J. and P. Olsson [1995]: *Looking Beyond the Horizon and Other Issues in Company Valuation.*

Study 4: Levin, J. and P. Olsson [1998]: *Company Valuation with a Periodically Adjusted Cost of Capital.*

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Copeland, T., T. Koller and J. Murrin [1994]: *Valuation: Measuring and Managing the Value of Companies* (2nd ed.). New York: Wiley & Son.

Lintner, J. [1956]: *Distribution of Incomes of Corporations among Dividends, Retained Earnings, and Taxes.* American Economic Review 46, 97-113.

Miller, M.H., Modigliani, F. (1961), Dividend Policy, Growth, and the Valuation of Shares, *Journal of Business* 34, 411-433.

Study 1:

Comparing the Accuracy and Explainability of Dividend, Free Cash Flow and Abnormal Earnings Equity Valuation Models

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Abstract

This paper compares the reliability of value estimates from the discounted dividend model, the discounted free cash flow model and the discounted abnormal earnings model. Using a large sample of Value Line 5-year horizon forecasts of the elements valued by these models, we show that the discounted abnormal earnings model outperforms the other models. Specifically, relative to both dividend and free cash flow based value estimates, abnormal earnings based value estimates are more accurate (i.e., they have smaller absolute deviations from observed security prices) and explain more of the variation in observed security prices. These results hold even for firms with book values which are expected to understate intrinsic values and for firms with the most flexibility to manage earnings. Further analyses suggest that the superiority of the abnormal earnings model is likely driven by the sufficiency of book value of equity as a measure of intrinsic value and by the greater precision and predictability of (abnormal) earnings forecasts.

1. Introduction

This study provides empirical evidence on the reliability of intrinsic value estimates derived from three theoretically-equivalent valuation models: the discounted dividend (*DIV*) model, the discounted free cash flow (*FCF*) model and the discounted abnormal earnings (*AE*) model. We use Value Line (VL) annual forecasts of the elements in these models to calculate value estimates for a sample of publicly-traded firms followed by Value Line during 1989-1993.[1] We contrast the reliability of value estimates in terms of their accuracy (defined as the absolute price scaled difference between the value estimate and the current security price) and in terms of their explainability (defined as the ability of value estimates to explain cross sectional variation in current security prices). To our knowledge, this is the first study to provide large-sample empirical evidence on the relative performance of these models.[2] Although in theory the models yield identical estimates of intrinsic value, in practice, value estimates may differ across models. One reason differences may exist relates to the fact that the *AE* model contains both a stock component (book value of equity at the valuation date) as well as a flow component (forecasts of abnormal earnings); in contrast, the *DIV* and *FCF* models are pure flow-based models. *AE* value estimates may be superior to *DIV* and *FCF* value estimates when errors in forecasting flows and errors in measuring discount rates and growth rates are more severe than any distortions in book values resulting from accounting procedures (such as the immediate expensing rather than capitalization of R&D investments) and accounting choices (such as a firm's accrual practices). In our sample, the magnitude of this stock versus flow effect is potentially quite large, as indicated both by the high proportion of *AE* value estimates represented by book value of equity (70% on average), and the high

[1] We collect third-quarter annual forecast data over a five-year forecast horizon for all December year-end firms followed by Value Line in each of the years 1989-1993. After excluding firms for which Value Line does not forecast the necessary data to compute a forecast of free cash flows, the final sample contains between 554 and 607 firms per year (2,907 observations in the pooled sample).

[2] As discussed in section 2, Penman and Sougiannis [1997] provide empirical evaluations of these models for a large sample of firms, but their value estimates are based on realized, not forecasted, attributes. This distinction is important because realizations contain unpredictable components which may confound comparisons of the valuation models. Penman and Sougiannis attempt to average out these unpredictable components of the valuation errors by constructing portfolios. Whether that averaging process allows one to interpret average realizations as ex-ante expectations remains an empirical question which we do not attempt to answer in this paper.

proportion of *FCF* and *DIV* value estimates represented by the terminal value calculations of these models (82% and 65%, on average).[3]

A second reason the models may provide differentially reliable value estimates is that the precision and the predictability of the fundamental attributes valued by the models may differ. We define precision as the absolute difference between the predicted value of an attribute and its realization, scaled by share price at the valuation date. We define predictability as the ease with which market participants can forecast the attribute, and we measure this construct as the standard deviation of historical year-to-year percentage changes in the attribute. All things equal, more precise and more predictable attributes should result in more reliable value estimates.

The results show that the most accurate value estimates also explain the most variation in contemporaneously observed prices. The widest gap in the performance of the models exists for the accuracy measure, where (under a perpetuity-based terminal value calculation) the median absolute prediction error for the *AE* model is about three-quarters that of the *FCF* model (30% versus 40%) and less than one-half that of the *DIV* model (30% versus 69%). *AE* and *DIV* estimates consistently explain more of the variation in current stock prices than do *FCF* estimates. For rank regressions, the difference between the *AE* (or *DIV*) model R^2 and the *FCF* model R^2 is modest (between 5-13% additional explanatory power depending on the model specification). For OLS regressions, the difference is significantly larger and is especially pronounced for the perpetuity-based terminal value specification where *AE* (*DIV*) estimates explain 71% (51%) of the variation in current prices compared to 35% for *FCF* estimates.

There are, however, significant differences in the accuracy and explainability of value estimates depending on the calculation of terminal values. All three models perform extremely well when the Value Line 3-5 year ahead price-earnings forecast is used to impute share price at the end of year 5; for example, each of the value estimates based on this terminal value specification explains a significant portion of the variation in contemporaneous prices (77-96%). Because long-term price forecasts are not available for most publicly-traded

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[3] We focus on the *FCF* and *DIV* terminal value calculations because they are likely the most noisy component of the value estimates, reflecting errors in forecasting the attribute itself, the growth rate and the discount factor.

firms, terminal values are usually calculated. When terminal values are based on a 4% growth-in-perpetuity formula, the performance of the models declines sharply. For example, the median absolute prediction error of the *DIV* (*FCF* or *AE*) model increases roughly four-fold (two-fold) over the median absolute prediction error computed using VL inferred prices.

We perform several additional analyses to investigate the potential sources of the superiority of the *AE* model over the *FCF* and *DIV* models. Briefly, these results indicate that the greater reliability of *AE* value estimates is driven both by the sufficiency of book value as a measure of intrinsic value and by the greater precision and predictability of analysts' earnings forecasts. Further tests indicate that neither accounting discretion nor accounting conservatism has a significant impact on the reliability of *AE* value estimates, suggesting that the superiority of the *AE* measure is robust to differences in firms' accounting practices and policies.

In summary, we find substantial differences in the performance of value estimates across valuation models. In terms of the "best" fundamental attribute, the results indicate that abnormal earnings perform at least as well as, or significantly better than, dividends or free cash flows. This result holds even for firms with book values that exclude internally-developed intangible assets and for firms with the most flexibility to manage earnings. We conclude there is little, if anything, to gain from algebraic manipulations of readily-available earnings forecasts, such as is needed to convert forecasted accounting data into free cash flows.

The rest of the paper is organized as follows. Section 2 describes the three valuation models and reviews the results of prior studies' investigations of estimates derived from these models. Section 3 describes the sample and data and presents the formulations of the *DIV*, *FCF* and *AE* models we estimate in this paper. The empirical tests and results are reported in section 4. In section 5, we investigate the stock versus flow distinction and the precision and the predictability of forecast elements; we also explore the effects of R&D spending and accounting discretion on the performance of the valuation models. Section 6 summarizes the results and concludes.

2. Valuation Methods

2.1 Models

The three equity valuation techniques considered in this paper build on the notion that the market value of a share is the discounted value of the expected future payoffs generated by the share. Although the three models differ with respect to the payoff attribute considered, it can be shown that (under certain conditions) the models yield theoretically-equivalent measures of intrinsic value.

The *discounted dividend* model, attributed to Williams [1938], equates the value of a firm's equity with the sum of the discounted expected dividend payments to shareholders over the life of the firm, with the terminal value equal to the liquidating dividend:

$$(1) \qquad V_F^{DIV} = \sum_{t=1}^{T} \frac{DIV_t}{(1+r_E)^t}$$

where: V_F^{DIV} = market value of equity at time F;
F = valuation date;
DIV_t = forecasted dividends for year t;
r_E = cost of equity capital;
T = expected end of life of the firm (often $T \rightarrow \infty$).

(For ease of notation, firm subscripts and expectation operators are suppressed. All variables are to be interpreted as time F expectations for firm j.)

The *discounted free cash flow* model substitutes free cash flows for dividends, based on the assumption that free cash flows provide a better representation of value added over a short horizon. Free cash flows equal the cash available to the firm's providers of capital after all required investments. In this paper, we follow the *FCF* model specified by Copeland, Koller and Murrin [1994]:[4]

(2) $$V_F^{FCF} = \sum_{t=1}^{T} \frac{FCF_t}{(1+r_{WACC})^t} + ECMS_F - D_F - PS_F$$

(2a) $$FCF_t = (SALES_t - OPEXP_t - DEPEXP_t)(1-\tau) + DEPEXP_t - \Delta WC_t - CAPEXP_t$$

(2b) $$r_{WACC} = w_D(1-\tau)r_D + w_{PS}r_{PS} + w_E r_E$$

where V_F^{FCF} = market value of equity at time F;
$SALES_t$ = sales revenues for year t;
$OPEXP_t$ = operating expenses for year t;
$DEPEXP_t$ = depreciation expense for year t;
ΔWC_t = change in working capital in year t;
$CAPEXP_t$ = capital expenditures in year t;
$ECMS_t$ = excess cash and marketable securities at time t;[5]
D_t = market value of debt at time t;
PS_t = market value of preferred stock at time t;
r_{WACC} = weighted average cost of capital;
r_D = cost of debt;
r_{PS} = cost of preferred stock;
r_E = cost of equity capital;
w_D = proportion of debt in target capital structure;
w_{PS} = proportion of preferred stock in target capital structure;
w_E = proportion of equity in target capital structure;
τ = corporate tax rate.

[4] The *FCF* measure specified in equation (2a) is similar to Copeland, Koller, and Murrin's [1994] specification except we omit the change in deferred taxes because VL does not forecast this item.

[5] Excess cash and marketable securities (ECMS) are the short-term cash and investments that the company holds over and above its target cash balances.

The *discounted abnormal earnings* model is based on valuation techniques introduced by Preinrich [1938] and Edwards and Bell [1961], and further developed by Ohlson [1995]. The *AE* model assumes an accounting identity – the clean surplus relation – to express equity values as a function of book values and abnormal earnings:[6]

(3) $$V_F^{AE} = B_F + \sum_{t=1}^{T} \frac{AE_t}{(1+r_E)^t}$$

(3a) $$AE_t = X_t - r_E B_{t-1}$$

(3b) $$B_t = B_{t-1} + X_t - DIV_t$$

where: V_F^{AE} = market value of equity at time F;
AE_t = abnormal earnings in year t;
B_t = book value of equity at end of year t;
X_t = earnings in year t;
DIV_t = dividends for year t net of capital contributions;
r_E = cost of equity capital;

2.2 Prior research comparing estimates of intrinsic values

Several studies investigate the ability of one or more of these valuation methods to generate reasonable estimates of market values. Kaplan and Ruback [1995] provide evidence on the ability of discounted cash flow (DCF) estimates to explain transaction values for a sample of 51 firms engaged in high leverage transactions.[7] Their results indicate that the median DCF value estimate is within 10% of the market price. They also report that DCF estimates

[6] Clean surplus requires that any increase in book value must flow through earnings. The exception is dividends, which are defined net of capital contributions.

[7] Transaction value equals the sum of the market value of common stock and preferred stock, book value of debt not repaid as part of the transaction, repayment value of debt for debt repaid and transaction fees; less cash balances and marketable securities.

significantly outperform estimates based on comparables or multiples approaches. Frankel and Lee [1995; 1996] compare *AE* estimates with value estimates based on earnings, book values, or a combination of the two, and find that the *AE* estimates explain a significantly larger portion of the variation in security prices than estimates based on these other accounting attributes.

In addition to these horse races (which pit one theoretically-based value estimates against one or more atheoretically-based, but perhaps best practice, value estimates), there are at least two studies which contrast the elements of, or the value estimates from, the *DIV*, *FCF* and/or *AE* models. Bernard [1995] compares the ability of forecasted dividends and forecasted abnormal earnings to explain variation in current security prices. Specifically, he regresses current stock price on current year, 1-year ahead and the average of the 3-5 year ahead forecasted dividends, and contrasts the explanatory power of this model with the explanatory power of the regression of current stock price on current book value and current year, 1-year ahead and the average of 3-5 year ahead abnormal earnings forecasts. He finds that dividends explain 29% of the variation in stock prices, compared to 68% for the combination of current book value and abnormal earnings forecasts.

Penman and Sougiannis [1997] also compare dividend, cash flow and abnormal earnings based value estimates using infinite life assumptions. Using realizations of the payoff attributes as proxies for expected values at the valuation date, they estimate intrinsic values for horizons of T=1 to T=10 years, accounting for the value of the firm after time T using a terminal value calculation. Regardless of the length of the horizon, Penman and Sougiannis find that *AE* estimates have significantly lower average errors than do *FCF* estimates, with *DIV* estimates falling in between.[8]

Our study extends previous investigations by comparing *DIV*, *FCF* and *AE* value estimates for a large sample of publicly-traded firms. In contrast to Penman and Sougiannis [1997], but similar to Bernard [1995] and Kaplan and Ruback [1995], we use forecasts rather than realizations. In addition to evaluating value estimates in terms of their accuracy (absolute deviation between the value estimate and market price at the valuation date, scaled by the

[8] The average prediction error for Penman and Sougiannis' *AE* estimate is 6.1%, for the *DIV* estimate it is -16.7%, and for the *FCF* estimate it is -76.5%. These prediction errors are for a 4-year forecast horizon and a terminal value calculation with a 4% growth rate.

latter), we contrast their ability to explain cross sectional variation in current market prices. Both metrics assume that forecasts reflect all available information and that valuation date securities prices are efficient with respect to these forecasts. Under the accuracy metric, the model which generates estimates with the smallest absolute forecast errors is the most reliable. The explainability tests, which compare value estimates in terms of their ability to explain cross sectional variation in current market prices, control for systematic over- or under-estimation by the valuation models.[9]

3. Data and Model Specification

3.1 Descriptive information

Our analyses require data on historical book values, market prices and proxies for the market's expectations of the fundamental attributes. We obtain information on book values from Compustat, and take security prices from CRSP. We use VL forecasts of accounting data as inputs to the valuation models; we assume these forecasts reflect all available information at time F. VL data are preferred to other analyst forecast data sources (such as IBES or Zacks) because Value Line Investment Survey reports contain a broader set of accounting data forecast over longer horizons than the typical data provided by sell-side analysts. In particular, VL reports dividend, earnings, book value, revenues, operating margins, capital expenditures, working capital and income tax rate forecasts for the current year, the following year, and "3-5 years ahead".[10] VL also reports a 3-5 year ahead price-earnings ratio which we use to infer a terminal value estimate at the end of year 5. Because the valuation models require projected attributes for *each* period in the forecast horizon, we assume that 3-5 year forecasts apply to all years in that interval.[11] Also, because VL does not

[9] In the OLS regression, bias is captured both by the inclusion of an intercept and by allowing the coefficient relating the value estimate to current market price to deviate from a theoretical value of one (bias which is correlated with the value estimate itself). Rank regressions implicitly control for bias by using the ranks of the variables rather than the values of the variables.

[10] In contrast, IBES and Zacks contain, at most, analysts' current year and one-year ahead earnings forecasts (annual and quarterly) and an earnings growth rate.

[11] The results are not sensitive to this assumption.

report 2-year ahead forecasts, we set year 2 forecasts equal to the average of the 1-year ahead and the 3-year ahead forecast. We use data from third-quarter VL because this is the first time all data are reported for the complete 5-year forecast horizon; third quarter VL reports have calendar dates ranging from F = July 1 to September 30 (incrementing weekly) for each sample year, 1989-1993. Finally, we restrict our analysis to December 31 year end firms to simplify calculations.

VL publishes reports on about 1,700 firms every 13 weeks; 800-900 of these firms have December year ends. Because VL does not forecast all of the inputs to the three valuation models for all firms (e.g., they do not forecast capital expenditures for firms in the retail industry) the sample is reduced to those firms with a complete set of forecasts. This requirement excludes about 250-300 firms each year, leaving a pooled sample of 3,085 firm-year observations (a security appears at most once each year). Missing Compustat and CRSP data reduce the sample further, to 2,907 firm-year observations, ranging from 554 to 607 firms annually.

Table 1 presents descriptive information for the pooled sample and the five yearly samples. Given the reliance on VL data, it is not surprising that the sample firms are large, with a mean market capitalization of $2.6 billion and a mean beta of 0.97. Most of the sample firms are listed on either the NYSE or the AMSE (82%), with the remainder trading on the NASDAQ. In comparison, during the sample period only 40% of publicly traded firms traded on either the NYSE or AMSE.

Table 1: Descriptive information on the market characteristics of 2,907 firm-year observations with Value Line forecasts available during 1989-1993[a]

	Sample Year					
	1989	1990	1991	1992	1993	Pooled
# observations	568	554	578	600	607	2907
mean market capitalization[b]	2008	2467	2370	2997	3134	2607
mean beta[c]	0.98	0.92	0.96	1.00	1.00	0.97
# firms traded on:						
NYSE	423	415	428	451	457	2174
AMSE	41	38	43	43	40	205
NASDAQ	104	101	107	106	110	528

[a] The sample consists of the common equity securities of December year-end firms with the following information available for any year t=1989-1993: third-quarter Value Line forecasts of all fundamental values; Compustat data on the book value of common equity for year t-1; and CRSP security prices.

[b] In millions of dollars, as at December 31, year t-1.

[c] Betas are calculated using daily stock returns over the 12-months ending December 31, year t-1.

3.2 Model specification

For each valuation model, we discount the forecasted fundamental attributes back to the date of the VL report. Because we use third-quarter reports, we adjust both for the horizon of the forecast (e.g., three years for a 3-year ahead forecast) and for a part year factor, f, to bring the current year estimate back to the forecast date (f equals the number of days between F and December 31, divided by 365).

In addition to forecasts of the fundamental attributes valued by each model, we require estimates of discount rates and terminal values. We calculate industry cost of equity as follows:[12]

$$(4) \qquad r_E = r_f + \beta(E(r_m) - r_f)$$

where: r_E = industry-specific discount rate;

r_f = intermediate-term Treasury bond yield minus the historical premium on Treasury bonds over Treasury bills (Ibbotson and Sinquefield [1993]);

β = estimate of the systematic risk for the industry to which firm j belongs. Industry betas are calculated by averaging the firm-specific betas of all sample firms in each 2-digit SIC code. Firm-specific betas are calculated using daily returns over fiscal year t-1;

$E(r_m) - r_f$ = market risk premium. We set this equal to 6%.[13]

For a given firm and valuation date, we assume r_E (r_{WACC} for the *FCF* model) is constant across the forecast horizon. If the estimated cost of equity is less than the risk free rate, we set r_E - r_f. The average cost of equity for the pooled sample is about 13%.

In the *FCF* model, the calculation of r_{WACC} requires estimates of the firm's capital structure as well as estimates of the cost of debt and the cost of equity. The cost of debt is proxied by the ratio of the VL reported interest on long-term debt to the book value of long-term debt; the cost of preferred stock is proxied by the VL reported preferred dividends divided by the book value of preferred stock.[14] We set the pre-tax upper bound on the cost of debt and the cost of preferred stock equal to the industry cost of equity, and we set the pre-tax lower bound equal

[12] Fama and French [1997] argue that industry costs of equity are more precise (albeit still quite noisy) than firm-specific costs of equity. We replicate the analyses using firm-specific discount rates. The results (not reported) yield similar inferences as those drawn from the industry discount rates.

[13] Six percent is roughly the geometric mean market risk premium; it is advocated by Stewart [1991] and is similar to the 5-6% recommended by Copeland et al. [1994]. We obtain qualitatively similar results using the arithmetic average market risk premium.

[14] VL reports book values of long-term debt and preferred stock as of the end of quarter 1. The results are not affected if we use Compustat data on book values of debt and preferred stock at the end of quarter 2. In theory, we should use the market values of debt and preferred stock.

to the risk-free rate.[15] Following Copeland et al. [1994, pp. 241-242] we develop long-term target capital weights for the r_{WACC} formula rather than use the weights implied by the capital structure at the valuation date.[16] For the pooled sample, the mean cost of debt is 9.3%, the mean cost of preferred stock is 10.3% and the mean weighted average cost of capital is 11.8%. The *FCF* model also requires an estimate of excess cash and marketable securities (*ECMS*). Based on Copeland et al's [1994, p. 161] suggestion that short-term cash and investments above 0.5%-2% of sales revenues are not necessary to support operations, we define *ECMS* as cash and marketable securities in excess of 2% of revenues.

We compute three estimates of terminal values for each valuation model, TV_i^{FUND}, where i=1,2,3 and *FUND* = *DIV*, *FCF* or *AE*. The first estimate (i=1) equals the VL 3-5 year ahead price-earnings ratio, $(P/X)_5$, multiplied by the firm's 3-5 year ahead forecasted earnings (X_5).[17] This method essentially infers VL's estimate of security price 3-5 years hence, and uses this number as a measure of the liquidating dividend to shareholders at the end of year T=5; we term TV_1 the *VL inferred terminal value.* Because long-term price forecasts are not available in many situations, we also consider terminal value estimates which discount into perpetuity the stream of forecasted fundamentals which occur after T=5:

$$(5) \qquad TV_T^{FUND} = \frac{FUND_T\,(1+g)}{r-g}$$

[15] The results are not sensitive to these boundary conditions.

[16] Specifically, we use Value Line's long-term (3-5 years ahead) predictions to infer the long-term capital structure. We use the long-term price-earnings ratio multiplied by the long-term earnings prediction to calculate the implied market value of equity 5 years hence. For debt, we use VL's long-term prediction of the book value of debt. For preferred stock, we assume that it remains unchanged from the valuation date. The equity weight in the WACC formula, w_E, is then given by: w_E = implied equity value / (implied equity value + forecasted debt + current book value of preferred stock). The debt and preferred stock weights are calculated similarly.

[17] For the *AE* model, book value of equity must be subtracted from the inferred future stock price because B_T is already incorporated in the value estimate through expression (3). For the *FCF* model, capital other than equity must be added, since the sum of future free cash flows equals *total* company value rather than just equity value. We adjust for the latter by: (1) adding to the VL inferred price forecast the ratio of the long-term VL earnings prediction, X_5, by VL's forecasted "percentage earned on total capital" $(X/C)_5$ (this calculation essentially yields a prediction of total capital in book value terms); and (2) subtracting the book value of equity at the end of year 5 (this calculation essentially means that we use the predicted market value of equity and the book value predictions of debt as proxies for market values).

where TV_i^{FUND} = terminal value of the firm at time T;

$FUND_T$ = predicted fundamental for year T;

r = discount rate (equal to r_E for the *DIV* and *AE* models, and to r_{WACC} for the *FCF* model);

g = growth rate.

The second terminal value estimate (i=2) assumes a zero growth rate of the fundamental attribute after year T, and the third estimate (i=3) allows the fundamental in year T+1 to grow by a constant 4% each year.[18] We refer to both TV_2 and TV_3 as *perpetuity-based terminal values*. If the forecasted T=5 fundamental is negative, we set the second and third estimates of terminal value to zero, based on the assumption that the firm will not survive if it continues to generate negative cash flows or negative abnormal earnings (dividends cannot be less than zero). Because we draw similar inferences from the results based on the no-growth and the 4% growth assumptions, we discuss only the latter in the paper; however, we report both sets of results in the tables.

Table 2: Summary Statistics on the Valuation Model Inputs for the Yearly and Pooled Samples[a]
(Amounts in $ per share except for % discount rates)

Panel A: Discounted Dividend Model

	Sample Year					
	1989	1990	1991	1992	1993	Pooled
Dividend forecast for year t (DIV_t)						
t=0 (half year)	0.39	0.39	0.36	0.33	0.32	0.36
t=1	0.85	0.82	0.76	0.71	0.68	0.76
t=2	1.00	0.97	0.91	0.85	0.82	0.91
t=3-5	1.14	1.11	1.06	0.99	0.95	1.05
Terminal value estimate (TV_i^{DIV})[b]						
TV_1^{DIV}	56.71	52.95	53.24	52.60	51.22	53.31
TV_2^{DIV}	8.80	8.57	8.49	7.64	7.77	8.24
TV_3^{DIV}	13.32	13.00	13.28	11.65	12.23	12.68
Cost of capital						
r_E (%)	13.05	13.08	12.84	13.19	12.64	12.96

[18] The growth rate is often assumed to equal the rate of inflation. Consistent with Kaplan and Ruback [1995] and Penman and Sougiannis [1997], we use a 4% growth rate.

Panel B: Discounted Free Cash Flow Model

	Sample Year					
	1989	1990	1991	1992	1993	Pooled
Free cash flow estimate for year *t* (FCF_t)						
t=0 (half year)	0.48	0.59	0.63	0.60	0.55	0.57
t=1	1.62	1.86	1.64	1.29	1.44	1.56
t=2	1.18	1.10	1.03	0.86	0.83	0.99
t=3	1.98	1.98	1.84	1.65	1.59	1.80
t=4,5	4.25	4.06	4.05	3.82	3.74	3.98
Terminal value estimate (TV_i^{FCF})[b]						
TV_1^{FCF}	63.58	60.34	61.08	59.36	57.84	60.40
TV_2^{FCF}	36.27	35.14	35.73	32.60	33.40	34.59
TV_3^{FCF}	57.73	56.12	58.21	52.15	55.32	55.86
Weighted average cost of capital						
r_D (%)	9.68	9.68	9.41	9.09	8.44	9.25
r_{PS} (%)	9.81	10.35	10.59	10.58	10.23	10.33
r_E (%)	13.05	13.08	12.84	13.19	12.64	12.96
r_{WACC} (%)	12.00	11.97	11.71	12.00	11.54	11.84

Panel C: Discounted Abnormal Earnings Model

	Sample Year					
	1989	1990	1991	1992	1993	Pooled
Earnings forecast for year *t* (X_t)						
t=0 (half year)	1.30	1.09	0.88	0.93	0.86	0.99
t=1	2.94	2.52	2.36	2.20	2.16	2.43
t=2	3.62	3.28	3.17	2.98	2.89	3.18
t=3-5	4.31	4.04	3.98	3.76	3.62	3.94
Book value forecast at end of year *t* (BV_t)						
t=Q2	17.09	16.70	16.68	15.39	14.18	15.98
t=0	17.58	16.71	16.93	15.75	14.43	16.25
t=1	19.57	18.30	18.27	17.12	15.92	17.80
t=2	23.75	21.95	21.90	20.64	19.37	21.48
t=3-5	27.93	25.59	25.52	24.16	22.82	25.16
Abnormal earnings estimate for year *t* (AE_t)						
t=0 (half year)	0.18	-0.01	-0.20	-0.19	-0.04	-0.05
t=1	0.62	0.33	0.19	0.12	0.35	0.32
t=2	1.05	0.88	0.82	0.71	0.89	0.86
t=3	1.19	1.15	1.16	1.02	1.18	1.14
t=4,5	0.64	0.67	0.69	0.55	0.74	0.66
Terminal value estimate (TV_i^{AE})[b]						
TV_1^{AE}	28.78	27.36	27.72	28.44	28.40	28.15
TV_2^{AE}	6.66	6.82	7.43	6.01	7.43	6.87
TV_3^{AE}	10.14	10.55	11.87	9.31	11.83	10.74
Cost of capital						
r_E (%)	13.05	13.08	12.84	13.19	12.64	12.96

[a] The sample securities are for December year-end firms with the following information available for any year t = 1989-1993: third-quarter Value Line forecasts of all fundamental values; Compustat data on the book value of common equity for year t-1; and CRSP security prices.

[b] We compute three terminal value calculations for each model. The first (i=1) sets the terminal value equal to the Value Line forecasted price (equal to the forecasted price/earnings ratio multiplied by the forecasted earnings per share for year 5). (See note 17 for adjustments made for the *FCF* and the *AE* models.) The second (i=2) equals the present value of the year 5 fundamental remaining constant into perpetuity. The third (i=3) equals the present value of the year 5 fundamental growing at 4% in perpetuity.

Discounted dividend model specification:

(6) $$V_F^{DIV} = (1+r_E)^{-f}\,.5DIV_0 + \sum_{t=1}^{5}(1+r_E)^{-(t+f)}\,DIV_t + (1+r_E)^{-(5+f)}\,TV_i^{DIV}$$

where: F = publication date of the Value Line forecast;
f = number of days between F and December 31, divided by 365;
0 = index for the current year forecast;
t = 1,…,5 subscripts one-year ahead through 5-year ahead forecasts;
TV_i^{DIV} = estimate of discounted dividend terminal value, i = 1,2,3;
TV_1^{DIV} = $(P/X)_5 \times X_5$;
TV_2^{DIV} = DIV_5 / r_E;
TV_3^{DIV} = $DIV_5(1+g)/(r_E - g)$.

Table 2, Panel A reports summary information on the forecasted inputs to the discounted dividend model. For the pooled sample, the average forecasted dividends for the second half of the current year and the next five years are, on average, \$0.36, \$0.76, \$0.91, \$1.05, \$1.05, and \$1.05. The mean terminal value estimates for the pooled sample are \$53.31, \$8.24, and \$12.68, for the first, second and third TV specifications, respectively.

Discounted free cash flow specification:

(7) $$V_F^{FCF} = (1+r_{WACC})^{-f}\,.5FCF_0 + \sum_{t=1}^{5}(1+r_{WACC})^{-(t+f)}\,FCF_t$$
$$+ (1+r_{WACC})^{-(5+f)}\,TV_i^{FCF} + ECMS_0 - D_0 - PS_0$$

where: TV_i^{FCF} = estimate of discounted FCF terminal value, i = 1,2,3;
TV_1^{FCF} = $(P/X)_5 \times X_5 + X_5/(X/C)_5 - B_5$; (see note 17)
TV_2^{FCF} = FCF_5 / r_{WACC};
TV_3^{FCF} = $FCF_5(1+g)/(r_{WACC} - g)$.

Table 2, Panel B reports summary information on the forecasted inputs to the discounted free cash flow model. (Recall that *FCF* forecasts are calculated using expression (2a).) The mean estimates of free cash flows for the remaining half of the current year and the next five years for the pooled sample are \$0.57, \$1.56, \$0.99, \$1.80, \$3.98, and \$3.98.[19] The average terminal values for the pooled sample are TV_1^{FCF} = \$60.40, TV_2^{FCF} = \$34.59 and TV_3^{FCF} = \$55.86.

Discounted abnormal earnings model specification:

$$(8) \qquad V_F^{AE} = B_{Q2} + (1+r_E)^{-f}\,.5(X_0 - r_E \times B_{Q2}) + \sum_{t=1}^{5}(1+r_E)^{-(t+f)}(X_t - r_E \times B_{t-1}) + (1+r_E)^{-(5+f)}\,TV_i^{AE}$$

where: B_{Q2} = reported book value of equity at the end of quarter 2 of year 0;[20]

TV_i^{AE} = estimate of discounted abnormal earnings terminal value, i = 1,2,3;

$TV_1^{AE} = (P/X)_5 \times X_5 - B_5$; (see note 17)

$TV_2^{AE} = (X_5 - r_E B_4)/r_E$;

$TV_3^{AE} = (X_5 - r_E B_4)(1+g)/(r_E - g)$.

Table 2, Panel C provides summary information on the forecasted inputs to the discounted abnormal earnings model. For the pooled sample, the average forecasted abnormal earnings for the remainder of the current year and the next five years are \$-0.05, \$0.32, \$0.86, \$1.14,

[19] The *FCF* estimate for year three is different from years four and five. For t=3 the change in working capital is based on the estimate of working capital in *t*=2. For *t*=4 and *t*=5, the change in working capital is zero because working capital forecasts are equal across *t*=3, *t*=4 and *t*=5 (recall that we assume that VL 3-5 year forecasts apply to *each* year in that interval). This causes the *FCF* forecasts for years 4 and 5 to exceed the *FCF* forecast for year 3.

[20] We obtain similar results using book value at the end of year -1.

$0.66 and $0.66.[21] The three terminal value estimates for the pooled sample are, on average, $28.15, $6.87 and $10.74.

4. Empirical Results

For all analyses we set negative value estimates to zero (since security prices cannot be negative). This truncation causes the sample distributions of value estimates to be right skewed; in addition, a few extreme observations accentuate the positive skewness. To mitigate the concern that a few observations drive the results, we report the results of both nonparametric tests (which retain all observations in ranked form) as well as parametric tests performed after excluding extreme observations. For pairwise comparisons of absolute prediction errors, we define extreme as the top 1% of the sample distribution of the variable being examined; for ordinary least squares regressions we define extreme as observations with studentized residuals greater than two in absolute value.

Table 3 reports mean and median security prices at the valuation date and value estimates for each of the nine value estimates for the pooled sample.[22] We also show information on mean and median signed prediction errors. Although the mean and median prediction errors are significantly different from zero for nearly all specifications, they are substantially smaller in economic terms for value estimates using VL inferred terminal values (e.g., median prediction errors range from 2% for the *FCF* model to 8% for the *AE* model) than they are for value estimates which use perpetuity-based terminal values (e.g., median prediction errors range from -69% for the *DIV* model to -9% for the *FCF* model). The frequency and magnitude of the underestimation is most severe for *DIV* value estimates which are less than price 99% of the time (not reported in Table 3) and have mean and median prediction errors of about -70%.

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[21] The abnormal earnings estimate for year 3 is a function of the estimated book value at the end of year 2. Hence, the estimate of abnormal earnings for year 3 differs from the estimate of abnormal earnings for years 4 and 5 (which are a function of the constant book value estimate for years 3-5).

[22] We repeat all analyses for each of the five sample years; individual year results are similar to the pooled sample results and are not reported. To ensure that our results are not driven by the concern that investors have not fully impounded the information in VL analysts' forecasts made at time *F*, we repeat the analyses using security prices five trading days after the valuation date; results are similar and are not reported.

Table 3: Descriptive statistics on the pooled sample distribution of contemporaneous security prices, intrinsic value estimates and signed prediction errors[a]

	mean value[b]	mean difference[b]	α-level difference=0[b]	median value[b]	median difference[b]	α-level difference=0[b]
Current share price	31.27	n/a	n/a	25.12	n/a	n/a
Value estimate:						
V_1^{DIV}	31.11	0.0790	0.0001	25.86	0.0271	0.0001
V_1^{FCF}	31.16	0.0514	0.0001	25.57	0.0187	0.0001
V_1^{AE}	32.97	0.1429	0.0001	27.44	0.0791	0.0001
V_2^{DIV}	7.84	-0.7626	0.0001	5.78	-0.7575	0.0001
V_2^{FCF}	18.40	-0.3591	0.0001	13.79	-0.4271	0.0001
V_2^{AE}	22.04	-0.2167	0.0001	17.91	-0.2825	0.0001
V_3^{DIV}	10.21	-0.6921	0.0001	7.44	-0.6871	0.0001
V_3^{FCF}	30.02	0.1048	0.0001	22.93	-0.0878	0.0678
V_3^{AE}	24.15	-0.1485	0.0001	19.37	-0.2290	0.0001

[a] The sample securities are for December year-end firms with the following information available for any year t = 1989-1993: third-quarter Value Line forecasts of all fundamental values; Compustat data on the book value of common equity for year t-1; and CRSP security prices. $P_{j,F}$ = observed share price of security j on the Value Line forecast date; $V_{j,i}^{FUND}$ = security j's estimate of intrinsic value calculated using the i'th terminal value specification of the valuation model based on *FUND* = dividends (*DIV*), free cash flows (*FCF*) or abnormal earnings (*AE*). We compute three terminal value calculations for each model. The first (i=1) sets the terminal value equal to the Value Line forecasted price (see note 17 for adjustments made for the *FCF* and the *AE* models). The second (i=2) equals the present value of the year 5 fundamental remaining constant into perpetuity. The third (i=3) equals the present value of the year 5 fundamental growing at 4% in perpetuity.

[b] The columns labeled mean (median) difference show the mean (median) signed prediction error, equal to $(V_{j,i}^{FUND} - P_{j,F})/P_{j,F}$. We also report the significance level associated with the t-statistics (sign rank statistic) of whether the mean (median) prediction error equals zero.

Tests of the accuracy of the value estimates, reported in table 4, show median absolute prediction errors for each specification along with the significance levels associated with pairwise comparisons of the accuracy of the value estimates. In order to highlight the effects of terminal value specification versus valuation model, we first compare accuracy measures across terminal value specifications, holding the valuation model constant (panel A), and then compare accuracy measures across valuation models holding the terminal value specification constant (panel B). The results in panel A show the powerful effect of terminal value specification. For each valuation model, median absolute prediction errors are smallest using VL inferred terminal values: the TV_1 specification is over four times as accurate as the TV_3 specification for *DIV*-based value estimates, and about twice as accurate as value estimates based on either the *FCF* model or the *AE* model.[23]

The far right column of table 4 contains a measure of the central tendency of the value estimate distribution; following Kaplan and Ruback [1995] we define central tendency as the percent of observations where the value estimate is within 15% of the observed security price. Results based on this metric are similar to those based on the median absolute prediction errors. In particular, we find that holding constant the terminal value specification, *AE* estimates are more concentrated around observed prices than are *FCF* estimates; for example, about 22% of perpetuity-based *AE* value estimates are within 15% of observed price compared to 18% of *FCF* estimates. *FCF* estimates, in turn, exhibit substantially greater central tendency than do *DIV* estimates; fewer than 2% of the perpetuity-based *DIV* estimates are within 15% of observed price.

Comparisons of absolute prediction errors conditional on the terminal value calculation show that when VL inferred terminal values are used, the *DIV* model is most accurate and has the largest mass near share price (median absolute forecast error of 16% and central tendency of 47%), followed closely by the *AE* model (18% accuracy and 43% central tendency) and then the *FCF* model (24% accuracy and 33% central tendency), with the difference in accuracy between each pair of models significant at the .00 level. In contrast, when perpetuity-based terminal values are used, we find that the *AE* model yields significantly more accurate estimates than the *FCF* model (median absolute prediction error of 30% versus 40%) and the *DIV* model (with a median absolute prediction error of 69%). The difference between the accuracy of the *FCF* and *DIV* value estimates is also significant at the .00 level.

[23] For the *DIV* and *AE* models, the third specification generates significantly (at the .002 level or better) more accurate value estimates than the second specification.

Table 4: Median Absolute Prediction Errors for the Pooled Sample[a]

Panel A: Comparison of value estimates, holding constant the fundamental attribute[b]

	median	vs V_2^{FUND}	vs V_3^{FUND}	central tendency[c]
V_1^{DIV}	0.1623	0.0001	0.0001	0.4702
V_2^{DIV}	0.7545		0.0001	0.0093
V_3^{DIV}	0.6871			0.0172
V_1^{FCF}	0.2387	0.0001	0.0001	0.3347
V_2^{FCF}	0.4822		0.0001	0.1324
V_3^{FCF}	0.4045			0.1844
V_1^{AE}	0.1768	0.0001	0.0001	0.4314
V_2^{AE}	0.3284		0.0015	0.2019
V_3^{AE}	0.3003			0.2250

Panel B: Comparison of value estimates, holding constant the terminal value specification

	median	vs V_i^{FCF}	vs V_i^{AE}	central tendency
V_1^{DIV}	0.1623	0.0001	0.0001	0.4702
V_1^{FCF}	0.2387		0.0001	0.3347
V_1^{AE}	0.1768			0.4314
V_2^{DIV}	0.7545	0.0001	0.0001	0.0093
V_2^{FCF}	0.4822		0.0001	0.1324
V_2^{AE}	0.3284			0.2019
V_3^{DIV}	0.6871	0.0001	0.0001	0.0172
V_3^{FCF}	0.4045		0.0001	0.1844
V_3^{AE}	0.3003			0.2250

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values. The absolute prediction error for security j for value estimate $V_{j,i}^{FUND}$ is $|V_{j,i}^{FUND} - P_{j,F}|/P_{j,F}$.

[b] The third and fourth columns report the significance levels for Wilcoxon tests comparing the pooled sample median absolute prediction errors for the noted row-column combination. For example, in panel A (row 8, column 4), we show that the median difference between V_2^{AE} and V_3^{AE} (.3284 vs .3003) is significant at the .0015 level.

[c] Central tendency is the percent of observations with value estimates within 15% of observed security price.

We draw the following conclusions from the results in tables 3 and 4. When terminal values are inferred from VL price forecasts, all of the models perform reasonably well insofar as they yield estimates which are typically within 16-24% of observed security prices. Although we observe statistically significant differences in the accuracy of the *DIV*, *FCF* and *AE* estimates based on VL inferred terminal values, the differences do not appear to be economically meaningful. When perpetuity-based terminal values are used, all of the models tend to underestimate security prices, with the discounted dividend model being especially biased. In this scenario, we observe the *AE* model generating statistically and economically more accurate value estimates than the *FCF* model (median absolute prediction errors are about 35% larger for *FCF* estimates than for *AE* estimates), which are themselves statistically and economically more accurate than value estimates from the *DIV* model (median absolute prediction errors are over 70% larger for *DIV* estimates than *FCF* estimates).

Our second test examines the ability of the value estimates to explain cross sectional variation in valuation date securities prices. We begin by assessing the explained variability of OLS and rank regressions of market price on each value estimate:[24]

(9) $$P_{j,F} = \lambda_0 + \lambda_1 V_{j,i}^{FUND} + \varepsilon_j$$

where: $P_{j,F}$ = market price of security j at valuation date F.

Pooled sample coefficient estimates and R^2s are reported in table 5, panel A.[25] Using VL inferred terminal values, the results show that each valuation estimate explains a substantial portion of the variability in security prices, ranging from 77% to 96% depending on the model and the form of the regression. Using the perpetuity-based TV, the results show that the explained variability of the rank regressions remains relatively high (between 77% and 90%)

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[24] OLS regressions include an intercept; rank regressions do not.

[25] If the value estimates are unbiased predictors of market security prices, we expect the intercept in expression (9) will equal zero and the slope coefficient will equal one. In almost all cases, statistical tests reject the joint hypothesis that λ_0=0 and λ_1=1. Because rejections of this hypothesis may arise from heteroscedasticity (we conduct White [1980] tests and generally reject the hypothesis that the variance of the disturbance term is constant across observations), we repeat all analyses after transforming the data, using the procedure described in the Appendix. The transformed results (not reported) show small changes in the parameter estimates; in all cases the results are qualitatively similar to the untransformed results reported in the tables.

for all three valuation models. The OLS regressions show, however, much greater variation in explanatory power – between 35% and 71%. Focusing on the OLS results, we observe that the *FCF* model performs substantially worse than either the *AE* model or the *DIV* model in explaining variation in price, with *FCF* value estimates explaining about one-half (two-thirds) of the variation in price explained by *AE* (*DIV*) estimates.

Table 5 also presents the results of multiple regressions of price on the three value estimates for a given TV specification (panel B):

$$(10) \qquad P_{j,F} = \rho_0 + \rho_1 V_{j,i}^{DIV} + \rho_2 V_{j,i}^{FCF} + \rho_3 V_{j,i}^{AE} + \varepsilon_j$$

Expression (10) allows us to evaluate the incremental contribution of each value estimate, holding constant the other two.[26] To calibrate the economic importance of the incremental t-statistics, we decompose the explanatory power of the model containing all three value estimates as independent variables into the portion explained by each value estimate controlling for the other two. For example, the incremental explanatory power of V_1^{DIV} equals the adjusted R^2 from estimating expression (10) minus the adjusted R^2 from the regression of price on V_1^{FCF} and V_1^{AE}.[27]

Under the first terminal value specification, the large White [1980] adjusted t-statistics for the *DIV* value estimate suggest *DIV* dominates *FCF* and *AE* estimates in explaining prices. However, the R^2-decomposition shows that V_1^{DIV} adds little (1-3%) in terms of explanatory power beyond that captured by V_1^{FCF} and V_1^{AE}. In fact, the incremental R^2 tests suggest that

[26] For expression (10), we test the joint hypothesis that the intercept equals zero and the sum of the slope coefficients equal one. We are unable to reject this hypothesis for value estimates based on the first terminal value specification, and we reject at the .00 and .08 levels for the second and third specifications, respectively.

[27] For comparison with Frankel and Lee [1995], we also examine the incremental contribution of each value estimate to regressions of price on earnings (measured before extraordinary items for the most recent four quarters) and book value of equity (measured as of June 30). The results (not reported) show that value estimates provide significant additional explanatory power over and above earnings and book value in all nine specifications. Earnings is significant in all cases and book value is significant in 8 of the 9 regressions. Excluding value estimates, we find that earnings and book values jointly explain 55% (89%) of the OLS (rank) explained variation in price. When the value estimates are included in the earnings and book value regressions, the OLS (rank) explanatory power increases to between 56% and 91% (89% to 96%) depending on the value specification.

none of the value estimates adds much in the way of explanatory power beyond the other two. As with the accuracy results in table 4, we find that the relative superiority of the models changes when perpetuity-based terminal values are used. Controlling for the information in the *FCF* and the *DIV* value estimates, *AE* value estimates add 14% explanatory power for the OLS regressions and 4% for the rank regressions. In contrast, neither the *FCF* nor the *DIV* value estimates add much beyond the other two (0-1% incremental adjusted R^2) in explaining variation in security prices.

Table 5: Results of Pooled Sample Regressions of Contemporaneous Stock Prices on Intrinsic Value Estimates[a]

Panel A: Univariate regressions of price on value estimate[b]

	TV specification: i=1			TV specification: i=2			TV specification: i=3		
	V_1^{DIV}	V_1^{FCF}	V_1^{AE}	V_2^{DIV}	V_2^{FCF}	V_2^{AE}	V_3^{DIV}	V_3^{FCF}	V_3^{AE}
OLS coefficient	1.01	0.79	0.94	1.75	0.76	1.23	1.30	0.46	1.09
OLS R^2	0.91	0.77	0.88	0.53	0.40	0.73	0.51	0.35	0.71
Rank R^2	0.96	0.90	0.95	0.84	0.77	0.90	0.84	0.77	0.90

Panel B: Multivariate regressions of price on value estimates[c]

	TV specification: i=1			TV specification: i=2			TV specification: i=3		
	V_1^{DIV}	V_1^{FCF}	V_1^{AE}	V_2^{DIV}	V_2^{FCF}	V_2^{AE}	V_3^{DIV}	V_3^{FCF}	V_3^{AE}
OLS coefficient	0.92	0.00	0.08	0.17	0.11	1.04	0.04	-0.01	1.06
t-stat: OLS coef=0	(27.23)	(0.13)	(2.47)	(2.13)	(5.01)	(22.17)	(0.54)	(-1.11)	(22.17)
t:stat: Rank coef=0	(21.54)	(4.99)	(-0.65)	(10.99)	(9.71)	(33.97)	(11.19)	(4.01)	(33.83)
model OLS R^2	0.90			0.73			0.71		
model Rank R^2	0.96			0.91			0.90		
incremental OLS R^2	0.03	0.00	0.00	0.01	0.00	0.12	0.00	0.00	0.14
incremental Rank R^2	0.01	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.04

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] Panel A reports results of estimating the following regression: $P_{j,F} = \lambda_0 + \lambda_1 V_{j,i}^{FUND} + \varepsilon_{j,t}$, where: $P_{j,F}$ = observed share price of security j on the Value Line forecast date; $V_{j,i}^{FUND}$ = security *j*'s estimate of intrinsic value calculated using the *i*'th terminal value specification of the valuation model based on *FUND* = dividends (*DIV*), free cash flows (*FCF*) or abnormal earnings (*AE*). OLS results exclude observations with studentized residuals greater than two in absolute magnitude.

[c] Panel B shows results of estimating the following regression: $P_{j,F} = \rho_0 + \rho_1 V_{j,i}^{DIV} + \rho_2 V_{j,i}^{FCF} + \rho_3 V_{j,i}^{AE} + \varepsilon_{j,t}$. The last two rows in panel B show the incremental adjusted R^2 provided by the noted value estimate, beyond that provided by the other two value estimates. For the OLS regression, we report White [1980] adjusted t-statistics. The incremental adjusted R^2 is the difference between the adjusted R^2 for the OLS (rank) regression containing all three value estimates and the adjusted R^2 for the OLS (rank) regression which excludes the value estimate in the noted column.

In summary, the results in tables 4 and 5 suggest the following conclusions about the choice of fundamental attribute and terminal value specification. When terminal values are based on VL inferred prices, there are only slight differences in the performance of the value estimates. In this scenario, we find that both the *DIV* model and the *AE* model perform similarly (16-18% accuracy and 88%-96% explainability), with the *FCF* model performing slightly worse (23% accuracy and 77%-90% explainability). In the more common scenario where terminal values are calculated based on a perpetuity formula, we find that *AE* estimates are significantly more accurate and have significantly greater explanatory power than *DIV* and *FCF* value estimates.

5. Extensions

In this section we perform several analyses to shed light on the potential sources of the superiority of the *AE* model over the *FCF* and *DIV* models. Specifically, we provide evidence on the stock-and-flow feature of the *AE* model versus the pure-flow feature of the *DIV* and *FCF* models (section 5.1) as well as the effects of the precision and the predictability of the attributes being valued (section 5.2). In both sections, we provide both across-model tests (which compare the performance of value estimates across different models) and within-model tests (which contrast the performance of same-model value estimates across sample partitions).

5.1. Stock versus flow explanation

One reason we may observe differences in the reliability of the value estimates stems from the fact that the *AE* model contains both a stock component (book value of equity at the valuation date) and a flow component (forecasts of abnormal earnings), whereas the *DIV* and *FCF* models are pure flow-based models. *AE* value estimates will be superior to *DIV* and *FCF* value estimates when forecast errors and measurement errors in estimating discount rates and growth rates are more severe than biases introduced in book values as a result of accounting procedures (such as immediate expensing rather than capitalization of R&D investments) and accounting choices (such as a firm's accrual practices). As an indication of the potential severity of this issue, we note that book value of equity represents 72% of the average *AE* value estimate and that perpetuity-based terminal values represent 65% and 82% of the average *DIV* value estimate and *FCF* value estimate, respectively – thus, significant biases in measuring book values or significant errors in forecasting flows and estimating discount rates and growth rates could have a substantial impact on the reliability of the value estimates. Tables 4 and 5 document the poor performance of value estimates based on perpetuity-based terminal values relative to value estimates based on VL inferred terminal values; these results suggest that TV_2 and TV_3 are error laden. These results also show that the *AE* model significantly outperforms the *DIV* and *FCF* models when terminal values are calculated. Together, these results suggest that pure flow-based *DIV* and *FCF* models are at a disadvantage relative to the stock-and-flow *AE* model. We provide more direct evidence on this assertion by examining the incremental explanatory power of the components of each value estimate: [28]

$$(11) \quad P_{j,F} = \omega_0 + \omega_1 PV_j^{DIV} + \omega_2 DTV_{j,i}^{DIV} + \varepsilon_j$$

$$(12) \quad P_{j,F} = \omega_0 + \omega_1 NFA_j + \omega_2 PV_j^{FCF} + \omega_3 DTV_{j,i}^{FCF} + \varepsilon_j$$

$$(13) \quad P_{j,F} = \omega_0 + \omega_1 B_j + \omega_2 PV_j^{AE} + \omega_3 DTV_{j,i}^{AE} + \varepsilon_j$$

where PV_j^{FUND} = the present value of the 5-year stream of the forecasted attribute;

DTV_j^{FUND} = the discounted (to time *F*) value of the terminal value for the *i*'th TV specification;

NFA_j = net financial assets at time *F* = *ECMS* – *D* - *PS*.

[28] The terminal value component equals the present value, at the valuation date, of the estimated terminal value 5 years hence. To be consistent with the *FCF* model specified by equation (2), we include net financial assets (*NFA*), equal to excess cash and marketable securities minus debt minus preferred stock, as a component in the *FCF* model.

Table 6 reports the results of estimating expressions (11), (12) and (13) for each terminal value specification. For each regression, we report White-adjusted t-statistics of whether the estimate differs from its theoretical value of one, the adjusted R^2 for each equation and model and the additional explanatory power added by each component of the model holding constant the other component(s). Turning first to the coefficient estimates, we note that VL inferred terminal values are well-specified for all three models, with the coefficient estimate on DTV_1^{FUND} indistinguishable from its theoretical value of one. In contrast, the perpetuity-based terminal values are not well-specified; for all three models, we strongly reject the null hypothesis that the coefficient relating DTV_1^{FUND} to price equals one. For the *AE* model, the results also reject the hypothesis that the coefficient relating price to book value is one, although in all cases, ω_1 is positive.

Model R^2s support the results shown in panel A of table 5. Specifically, we find that the VL inferred terminal values work well for all models (explaining 83-96% of the variation in observed prices). For the perpetuity-based terminal value calculations, we find that the *AE* model (with R^2s of 74-91%) outperforms the *FCF* and *DIV* models (with R^2s ranging from 32-84%).

Most importantly, table 6 reveals the relative importance of stock versus flow components in explaining security prices when perpetuity-based terminal values are used to compute value estimates. The results for the *AE* model, shown in panel C, indicate that book value of equity adds significant incremental explanatory power -- 45% for OLS regressions and 15% for rank regressions. The present value of abnormal earnings over the forecast horizon add 14%, while the discounted terminal value adds nothing. We note, too, that even when terminal value is well-specified (as it is in TV_1), book value of equity adds materially to explaining variation in security prices; in fact, the incremental explanatory power provided by book value exceeds that provided by the VL inferred terminal values (18% versus 14% OLS, and 7% versus 3% rank). For the *DIV* model, the results (panel A) show that the present value of dividends over the forecast horizon offer 10% explanatory power beyond that provided by terminal values. For the *FCF* model (panel B), OLS results indicate that the present value of *FCFs* over the forecast horizon add 27% in explanatory power, controlling for net financial assets and terminal values; note, however, that the overall explanatory power of this model is low (32%).

Table 6: Results of pooled sample regressions of contemporaneous stock prices on the components of value estimates[a]

Panel A: Discounted dividend model[b]

	TV specification: *i*=1			TV specification: *i*=2			TV specification: *i*=3		
		$PV_{i,F}^{DIV}$	DTV_1^{DIV}		$PV_{i,F}^{DIV}$	DTV_2^{DIV}		PV_i^{DIV}	DTV_i^{DIV}
OLS coefficient		0.98	1.01		6.47	-2.04		5.64	-0.87
t-stat: OLS coef=1		(-0.28)	(0.84)		(15.41)	(-10.66)		(21.91)	(-18.45)
t:stat: Rank coef=0		(22.08)	(89.87)		(9.52)	(-0.34)		(12.86)	(-0.80)
model OLS R^2	0.91			0.57			0.57		
model Rank R^2	0.96			0.84			0.84		
incremental OLS R^2		0.00	0.34		0.05	0.00		0.10	0.01
incremental Rank R^2		0.01	0.11		0.00	0.00		0.01	0.00

Panel B: Discounted free cash flow model[c]

	TV specification: *i*=1			TV specification: *i*=2			TV specification: *i*=3		
	$NFA_{j,F}$	$PV_{i,F}^{FCF}$	DTV_1^{FCF}	$NFA_{j,F}$	$PV_{i,F}^{FCF}$	DTV_2^{FCF}	$NFA_{j,F}$	PV_i^{FCF}	DTV_i^{FCF}
OLS coefficient	0.29	0.23	1.01	0.26	6.47	0.71	0.19	0.84	0.21
t-stat: OLS coef=1	(-35.65)	(-0.28)	(0.84)	(-21.30)	(-6.48)	(-4.87)	(-22.97)	(-1.59)	(-20.73)
t:stat: Rank coef=0	(21.97)	(22.08)	(89.87)	(26.94)	(10.00)	(15.61)	(27.03)	(13.31)	(12.98)
model OLS R^2	0.83			0.35			0.32		
model Rank R^2	0.93			0.82			0.82		
incremental OLS R^2	0.07	0.01	0.30	0.04	0.01	0.05	0.04	0.27	0.03
incremental Rank R^2	0.01	0.00	0.24	0.04	0.01	0.12	0.05	0.01	0.12

Panel C: Discounted abnormal earnings model[d]

	TV specification: *i*=1			TV specification: *i*=2			TV specification: *i*=3		
	$B_{j,F}$	$PV_{j,F}^{AE}$	DTV_1^{AE}	$B_{j,F}$	$PV_{j,F}^{AE}$	DTV_2^{AE}	$B_{j,F}$	PV_i^{AE}	DTV_i^{AE}
OLS coefficient	0.94	0.70	1.00	1.24	6.47	-0.48	1.24	0.84	-0.31
t-stat: OLS coef=1	(-3.44)	(-5.45)	(0.06)	(12.36)	(19.96)	(-15.82)	(12.51)	(22.73)	(-28.03)
t:stat: Rank coef=0	(56.56)	(4.50)	(40.94)	(66.84)	(22.64)	(-4.95)	(67.03)	(23.75)	(-5.52)
model OLS R^2	0.89			0.74			0.74		
model Rank R^2	0.94			0.91			0.91		
incremental OLS R^2	0.18	0.01	0.14	0.45	0.11	0.00	0.45	0.14	0.00
incremental Rank R^2	0.07	0.00	0.03	0.15	0.02	0.00	0.15	0.02	0.00

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] Panel A reports coefficient estimates and White-adjusted t-statistics for the following regression: $P_{j,F} = \omega_0 + \omega_1 PV_{j,F}^{DIV} + \omega_2 DTV_{j,F}^{DIV} + \varepsilon_{j,t}$, where: $P_{j,F}$ = observed share price of security j on the Value Line forecast date; $PV_{j,i}^{DIV}$ = the present value of the 5-year stream of forecasted dividends; $DTV_{j,i}^{DIV}$ = discounted (to time *F*) value of the terminal value for the noted specification. OLS results exclude observations with studentized residuals greater than two in absolute magnitude.

[c] Panel B reports coefficient estimates and White-adjusted t-statistics for the following regression: $P_{j,F} = \omega_0 + \omega_1 NFA_{j,F}^{FCF} + \omega_2 PV_{j,F}^{FCF} + \omega_3 DTV_{j,F}^{FCF} + \varepsilon_{j,t}$, where: $P_{j,F}$ = observed share price of security j on the Value Line forecast date; $NFA_{j,F}$ = net financial assets at the valuation date (excess cash and marketable securities - debt - preferred stock); $PV_{j,i}^{FCF}$ = the present value of the 5-year stream of forecasted free cash flows; $DTV_{j,i}^{DIV}$ = discounted (to time F) value of the terminal value for the noted specification. OLS results exclude observations with studentized residuals greater than two in absolute magnitude.

[d] Panel C reports coefficient estimates and White-adjusted t-statistics for the following regression: $P_{j,F} = \omega_0 + \omega_1 B_{j,F} + \omega_2 PV_{j,F}^{AE} + \omega_3 DTV_{j,F}^{AE} + \varepsilon_{j,t}$, where: $P_{j,F}$ = observed share price of security j on the Value Line forecast date; $B_{j,F}$ = book value of equity at the valuation date; $PV_{j,i}^{AE}$ = the present value of the 5-year stream of forecasted dividends; $DTV_{j,i}^{AE}$ = discounted (to time F) value of the terminal value for the noted specification. OLS results exclude observations with studentized residuals greater than two in absolute magnitude.

Overall, the evidence in table 6 supports the view that, despite conservatism in the measurement of book value, the stock component of the *AE* model explains a significant portion of the variation in observed prices. The results indicate that book value of equity explains more of the variation in prices than either the present value of *AE* forecasts over the forecast horizon or the discounted terminal value. Further, the incremental explanatory power provided by the present value of *DIV* and *FCF* forecasts over the forecast horizon (0-27%) is substantially less than the explanatory power provided by book value alone in the *AE* model (18-45%).

Our second analysis of the stock versus flow explanation focuses on situations where we might expect accounting practices to result in book value estimates which are biased estimates of book value. On the one hand, we expect that when the current book value of equity does a good job of recording the intrinsic value of the firm, the *AE* model produces more reliable value estimates – relative to the *FCF* or *DIV* models – because more of intrinsic value is included in the forecast horizon (and therefore less is included in the terminal value calculation). On the other hand, even if book values exclude value relevant assets, the *AE* model's articulation of the balance sheet and the income statement will link lower book values today with larger abnormal earnings in future periods. For example, if the net R&D payoff component of earnings is stable through time (as we expect in equilibrium), then the sum of current book value of equity and the discounted stream of abnormal earnings will result in the same estimate of intrinsic value as would be provided by a scenario in which R&D investments were not expensed in the period incurred, but were capitalized at their net present values (Bernard [1995, note 9] makes a similar point with respect to accounting distortions which result in overstatements of book values).

Table 7: Comparison of the Performance of Value Estimates for High and Low R&D Samples[a]

Panel A: Accuracy (measured as the median absolute prediction error)

	High R&D Sample[b]			Low R&D Sample[b]			Difference[c]
	Median	vs V_i^{FCF}	vs V_i^{AE}	Median	vs V_i^{FCF}	vs V_i^{AE}	High vs Low
V_1^{DIV}	0.1495	0.0001	0.0025	0.1723	0.0001	0.0028	0.0046
V_1^{FCF}	0.2090		0.0001	0.2561		0.0001	0.0001
V_1^{AE}	0.1688			0.1875			0.0089
V_2^{DIV}	0.7399	0.0001	0.0001	0.7819	0.0001	0.0001	0.0016
V_2^{FCF}	0.4827		0.0001	0.4977		0.0001	0.0518
V_2^{AE}	0.3234			0.3371			0.1620
V_3^{DIV}	0.6718	0.0001	0.0001	0.7182	0.0001	0.0001	0.0041
V_3^{FCF}	0.4060		0.0001	0.5413		0.0001	0.0001
V_3^{AE}	0.2945			0.3165			0.0277

Panel B: Explainability (R^2 from univariate regression of price on value estimate)

	TV specification: i=1			TV specification: i=2			TV specification: i=3		
	V_1^{DIV}	V_1^{FCF}	V_1^{AE}	V_2^{DIV}	V_2^{FCF}	V_2^{AE}	V_3^{DIV}	V_3^{FCF}	V_3^{AE}
High R&D Sample									
OLS coefficient	0.99	0.84	0.91	1.93	0.65	1.17	1.50	0.36	1.07
OLS R^2	0.89	0.79	0.87	0.67	0.41	0.77	0.66	0.31	0.76
Rank R^2	0.96	0.91	0.95	0.87	0.80	0.92	0.87	0.79	0.92
Low R&D Sample									
OLS coefficient	1.02	0.86	0.97	1.34	0.71	1.34	0.90	0.21	1.06
OLS R^2	0.91	0.86	0.91	0.32	0.36	0.71	0.28	0.11	0.62
Rank R^2	0.95	0.86	0.94	0.81	0.70	0.88	0.81	0.67	0.88

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] The High (Low) R&D sample consists of the firms with (without) disclosed research and development expenses in year t-1.

[c] We report the significance level for the Wilcoxon test of whether the median absolute prediction error of the High R&D sample differs from that of the Low R&D sample.

We test whether the *AE* model performs differently for firms with high R&D spending than for firms with low or no R&D spending. We identify a sample of High R&D firms by first ranking the sample firms based on the ratio of R&D spending in year *t*-1 to total assets at the beginning of year *t*-1; data on R&D spending and total assets are from Compustat. About 48% (1,390 firm-year observations) of the sample disclose no, or immaterial amounts of, R&D expenditures (the Low R&D sample); the remaining firms (the High R&D sample) have mean annual R&D spending of 4.1% of total assets and a median of 2.7%.

We replicate the accuracy and explainability tests for the High R&D and Low R&D samples; results are shown in table 7. For the High R&D sample, Panel A shows no evidence that the *AE* model produces larger absolute prediction errors than either the *DIV* or the *FCF* models. In fact, using perpetuity-based terminal values, we find that *AE* value estimates are significantly *more* accurate than *DIV* and *FCF* estimates. We also compare the median absolute prediction errors of the *AE* value estimates between the High and Low R&D samples (far right column of panel A). The results indicate that the *AE* value estimates for High R&D firms are significantly *more* accurate than the *AE* value estimates for Low R&D firms. Panel B summarizes results of tests of the explainability of the value estimates for the two samples. Within the High R&D sample, we do not find that *AE* estimates perform worse than *DIV* and *FCF* estimates in explaining variability in current market prices; in fact, for the perpetuity-based TV calculations, *AE* estimates perform best. Differences in R^2s between the High and Low R&D samples also show no pattern indicating that the *AE* model performs substantially worse for High R&D firms.

Our third analysis focuses on the ability of firms to manipulate the flow component of the *AE* model. Unlike free cash flows and dividends, management can affect the timing of abnormal earnings (and therefore *AE* value estimates) by exercising more or less discretion in their accrual practices. Whether such discretion leads to *AE* value estimates being more or less reliable measures of market prices depends on whether management uses accounting discretion to clarify or obfuscate value relevant information. Because we have no a priori reason for believing that one effect dominates the other in explaining the accrual behavior of the sample firms, we do not predict whether *AE* value estimates perform better or worse for firms with high accounting discretion. We identify samples of firms with relatively high and low levels of accounting discretion, and examine whether *DIV*, *FCF* and *AE* estimates perform similarly within the high discretion sample; we also investigate whether *AE* estimates for the high discretion sample perform significantly differently from *AE* estimates for the low discretion sample.

We partition firms based on the level of accounting discretion available to firms, as proxied by the ratio of total accruals to total assets. Consistent with prior studies (Healy [1985]) we compute total accruals (*TA*) as ΔCurrent assets - ΔCurrent liabilities - ΔCash + ΔShort term debt - Depreciation.[29] We compute total accruals for each firm-year observation using financial information for year t-1, and then rank observations based on the absolute value of total accruals as a percent of beginning of year total assets. Securities in the top (bottom) quartile of the ranked distribution are assigned to the High (Low) Accruals sample. The mean (median) ratio of accruals to assets for the High Accruals sample is 14% (12%); for the Low Accruals sample the mean and the median value is 1.5%.

Table 8 summarizes the performance of the value estimates for the accruals sub-samples. For the High Accruals sample, panel A shows that the *AE* model generally produces significantly smaller absolute prediction errors than the *DIV* and the *FCF* models. Comparisons of the median absolute prediction errors of the perpetuity-based *AE* value estimates between the High and Low Accrual samples (reported in the far right column of panel A) show no significant differences in accuracy. Panel B summarizes results of tests of the explainability of the value estimates for the two samples. Within the High Accruals sample, *AE* estimates perform best in explaining variability in current market prices. Comparisons of the explainability of *AE* estimates between the two samples indicate that the model R^2s for the High Accruals sample are generally equal to or greater than those for the Low Accruals sample.

In summary, we find no evidence that the *AE* model provides less reliable value estimates for firms where book values poorly reflect intrinsic values (firms with high R&D spending) or for firms where there is scope for managing earnings (firms with high accruals). The results suggest the opposite: both within-sample and across-sample tests indicate that, if anything,

[29] The relevant Compustat data items are: current assets (#4), current liabilities (#5), cash (#1), short term debt (#34), depreciation and amortization expense (#14) and total assets (#6).

high R&D spending and high accounting discretion are associated with more reliable *AE* value estimates.

Table 8: Comparison of the Performance of Value Estimates for High and Low Accrual Samples[a]

Panel A: Accuracy (measured as the median absolute prediction error)

	High Accrual Sample[b]			Low Accrual Sample[b]			Difference[c]
	Median	vs V_i^{FCF}	vs V_i^{AE}	Median	vs V_i^{FCF}	vs V_i^{AE}	High vs Low
V_1^{DIV}	0.1809	0.0001	0.0360	0.1480	0.0001	0.0192	0.0062
V_1^{FCF}	0.2641		0.0001	0.2215		0.0001	0.0006
V_1^{AE}	0.1940			0.1758			0.0139
V_2^{DIV}	0.8002	0.0001	0.0001	0.7688	0.0001	0.0001	0.0072
V_2^{FCF}	0.4827		0.0001	0.4751		0.0001	0.6313
V_2^{AE}	0.3282			0.3517			0.3125
V_3^{DIV}	0.7441	0.0001	0.0001	0.7102	0.0001	0.0001	0.0532
V_3^{FCF}	0.4273		0.0001	0.3903		0.0001	0.1267
V_3^{AE}	0.2965			0.3212			0.8524

Panel B: Explainability (R^2 from univariate regression of price on value estimate)

	TV specification: *i*=1			TV specification: *i*=2			TV specification: *i*=3		
	V_1^{DIV}	V_1^{FCF}	V_1^{AE}	V_2^{DIV}	V_2^{FCF}	V_2^{AE}	V_3^{DIV}	V_3^{FCF}	V_3^{AE}
High Accrual Sample									
OLS coefficient	1.01	0.81	0.92	2.06	0.73	1.24	1.58	0.43	1.08
OLS R^2	0.90	0.81	0.86	0.61	0.40	0.75	0.60	0.31	0.72
Rank R^2	0.95	0.91	0.95	0.86	0.77	0.91	0.82	0.77	0.90
Low Accrual Sample									
OLS coefficient	1.09	0.81	0.93	1.40	0.75	1.19	0.92	0.48	1.02
OLS R^2	0.95	0.79	0.87	0.42	0.41	0.65	0.35	0.35	0.64
Rank R^2	0.96	0.92	0.95	0.83	0.81	0.89	0.83	0.79	0.89

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] The High (Low) Accrual sample consists of the top (bottom) quartile of firms ranked on the absolute value of total accruals as a percentage of total assets, measured in year *t*-1.

[c] We report the significance level for the Wilcoxon test of whether the median absolute prediction error of the High Accrual Sample differs from that of the Low Accrual Sample.

5.2. Precision and predictability of attributes

A second explanation for differences in the reliability of *DIV*, *FCF* and *AE* value estimates is that the precision and predictability of the fundamental attributes themselves may differ. We measure precision as the absolute difference between the predicted value of an attribute and its realization, scaled by share price at the valuation date. We also examine the bias in the fundamental attributes, measured as the signed price-scaled difference between the predicted value and its realization. We define predictability as the ease with which market participants can forecast the attribute, and we measure this construct as the standard deviation of historical year-to-year changes in the attribute. All things equal, more precise and more predictable attributes should result in more accurate value estimates which explain a greater portion of the variation in observed prices. To provide information on the importance of these forecast properties in explaining the relative performance of the models, we first compare the precision and the predictability of the valuation attributes; we then construct within-model partitions (as we did with R&D spending and accruals) to investigate whether these properties explain differential accuracy and explainability, holding constant the attribute being valued.

We compute precision and bias statistics for each of the current year, one-year ahead and 3-5 year ahead forecasted attributes.[30] The realized dividend for year t equals the total amount of common stock dividends declared in year t (Compustat data item #21). The realized free cash flow per share in year t equals the net cash flow from operating activities (#308) minus the change in working capital (#179) minus capital expenditures (#128). The realized abnormal earnings for year t equals earnings per share after extraordinary items (#53) minus the estimated discount rate multiplied by the book value of common equity in year t-1 (#60). To ensure consistency across models, we scale all variables by the number of shares used to calculate primary earnings per share (#54), and we delete observations with missing data for dividends, free cash flows or abnormal earnings.

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[30] We use the 3-year ahead realization as the benchmark for measuring the bias in the 3-5 year ahead forecast of each attribute. Because our sample ends in 1992 and because we have Computstat data through 1995, this convention provides a full sample of observations to measure bias.

Table 9: Comparison of Bias and Precision of Forecast Attributes, Holding Constant the Forecast Horizon[a]

Panel A: Bias (forecast attribute minus realized value of the attribute, scaled by share price at the valuation date)

		α-level: t-tests[b]			α-level:Wilcoxon tests[b]	
	mean	vs *FCF*	vs *AE*	median	vs *FCF*	vs *AE*
current yr						
DIV	-0.0031	0.0001	0.0001	0.0000	0.0001	0.0001
FCF	0.0088		0.0001	0.0088		0.0001
AE	0.0479			0.0045		
1-yr ahead						
DIV	0.0009	0.0001	0.0001	0.0000	0.0001	0.0001
FCF	0.0378		0.0001	0.0174		0.0001
AE	0.0589			0.0186		
3-yr ahead						
DIV	0.0081	0.0001	0.0001	0.0054	0.0001	0.0001
FCF	0.0627		0.0001	0.0338		0.0001
AE	0.0863			0.0498		

Panel B: Precision (absolute value of the difference between the forecast attribute minus realized value of the attribute, scaled by share price at the valuation date)

		α-level: t-tests[b]			α-level:Wilcoxon tests[b]	
	mean	vs *FCF*	vs *AE*	median	vs *FCF*	vs *AE*
current yr						
DIV	0.0042	0.0001	0.0001	0.0000	0.0001	0.0001
FCF	0.1186		0.0001	0.0510		0.0001
AE	0.0696			0.0204		
1-yr ahead						
DIV	0.0052	0.0001	0.0001	0.0008	0.0001	0.0001
FCF	0.1340		0.0001	0.0604		0.0001
AE	0.0783			0.0269		
3-yr ahead						
DIV	0.0131	0.0001	0.0001	0.0060	0.0001	0.0001
FCF	0.1412		0.0001	0.0746		0.0001
AE	0.0988			0.0536		

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] We report mean and median bias and precision measures as well as the significance levels for t-tests and Wilcoxon tests comparing the pooled sample mean and median bias and precision measures across models.

Mean and median bias and precision statistics are reported in table 9 for each of the three forecasts. Bias measures, shown in panel A, indicate that the typical current year *AE* (*FCF*) forecast overstates realized abnormal earnings by about 0.45% (0.88%) of security price, with current year *DIV* forecasts showing no bias. For all attributes, forecast optimism increases with the forecast horizon: the typical one-year ahead *AE* (*FCF*) forecast overstates its realization by 1.9% (1.7%) compared to 5% (3.3%) for the 3-5 year ahead *AE* (*FCF*) forecasts. Panel A also reports the results of tests comparing bias across attributes. With one exception, these results show that abnormal earnings forecasts are significantly more optimistic than *FCF* forecasts, which are themselves significantly more optimistic than *DIV* forecasts.

Although *AE* forecasts are significantly more optimistic than *FCF* forecasts, panel B shows that *AE* forecasts are more precise for all forecast horizons. For example, current year *AE* forecasts have a mean (median) precision of 7% (2%) compared to 12% (5%) for *FCF* forecasts. *DIV* forecasts are the most precise; this result is to be expected given firms' reluctance to alter dividend policies.[31] As with the bias measure, we note that forecasts are less precise the longer the forecast horizon.

For each fundamental attribute, we average the precision of current year, 1-year ahead and 3-year ahead forecasts, and then identify those observations in the top quartile of average precision (i.e., those with the largest differences between forecasts and realizations) as the Low Precision sample, and those observations in the bottom quartile of average precision as the High Precision sample. Comparisons of the reliability of the value estimates for each of the within-model precision partitions are shown in table 10. Value estimates based on VL inferred terminal values generally support the prediction that more precise forecasts lead to more reliable value estimates. For example, the median prediction error for *FCF* value estimates for the High Precision partition is less than half that of the Low Precision partition (19% versus 39%); *FCF* value estimates for the High Precision sample also explain a larger portion of the variation in observed prices than do *FCF* value estimates for the Low Precision sample (80% versus 57% for OLS, and 92% versus 86% for rank). We observe similar results for *AE* value estimates, but not for *DIV* value estimates.

[31] For example, firms which have a policy of paying no dividends will appear highly accurate.

Table 10: Comparison of the Performance of Value Estimates for High and Low Precision Samples[a]

Panel A: Accuracy (measured as the median absolute prediction error)

	High Precision Sample[b]			Low Precision Sample[b]			Difference[c]
	Median	vs V_i^{FCF}	vs V_i^{AE}	Median	vs V_i^{FCF}	vs V_i^{AE}	High vs Low
V_1^{DIV}	0.1972	0.2981	0.0001	0.1746	0.0001	0.0001	0.0874
V_1^{FCF}	0.1943		0.0001	0.3860		0.0093	0.0001
V_1^{AE}	0.1384			0.3248			0.0001
V_2^{DIV}	1.0000	0.0001	0.0001	0.6676	0.0001	0.0001	0.0001
V_2^{FCF}	0.5060		0.0001	0.4819		0.0001	0.9914
V_2^{AE}	0.4342			0.2372			0.0001
V_3^{DIV}	1.0000	0.0001	0.0001	0.5831	0.0004	0.0001	0.0001
V_3^{FCF}	0.3523		0.5691	0.6179		0.0001	0.0001
V_3^{AE}	0.3842			0.2448			0.0001

Panel B: Explainability (R^2 from univariate regression of price on value estimate)

	TV specification: i=1			TV specification: i=2			TV specification: i=3		
	V_1^{DIV}	V_1^{FCF}	V_1^{AE}	V_2^{DIV}	V_2^{FCF}	V_2^{AE}	V_3^{DIV}	V_3^{FCF}	V_3^{AE}
High Precision Sample									
OLS coefficient	0.99	0.84	1.01	1.53	1.24	1.45	1.13	0.76	1.13
OLS R^2	0.90	0.80	0.91	0.29	0.76	0.74	0.28	0.49	0.64
Rank R^2	0.96	0.92	0.95	0.72	0.78	0.91	0.72	0.79	0.90
Low Precision Sample									
OLS coefficient	1.01	0.52	0.77	2.08	0.37	0.93	1.54	0.20	0.81
OLS R^2	0.89	0.57	0.86	0.65	0.27	0.79	0.61	0.23	0.77
Rank R^2	0.96	0.86	0.94	0.91	0.75	0.93	0.91	0.76	0.92

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] The Low (High) Precision sample consists of the top (bottom) quartile of firms ranked on the average precision of the current year, 1-year ahead and 3-year ahead forecasts of the attribute valued by the noted model. Precision equals the absolute value of the difference between the forecast attribute and its realization, scaled by share price at the valuation date.

[c] We report the significance level for the Wilcoxon test of whether the median absolute prediction error of the High Precision sample differs from that of the Low Precision sample.

For the *FCF* value estimates, the finding that more precise forecasts result in more reliable value estimates is also apparent using perpetuity-based terminal values. However, neither *DIV* nor *AE* estimates show the expected relation using perpetuity-based terminal values; table 10 shows that *DIV* and *AE* value estimates are more accurate, and explain more of the variation in observed prices, when the forecasts are *less* precise. For the *DIV* model, this anomalous result is not unexpected, since for a large portion of the sample firms, *DIV* value estimates understate intrinsic values (as shown in table 3);[32] in these cases, the slight average optimism in dividend forecasts (documented in table 9) improves the accuracy of *DIV* value estimates. Similarly, the optimism in abnormal earnings forecasts (table 9) may compensate for the underestimation of *AE* value estimates (table 3), resulting in more reliable *AE* value estimates for the Low Precision partition than the High Precision partition.

We also partition the sample based on the predictability of the attribute, measured as the standard deviation of percentage yearly changes in realized values. In order to be included in these tests, we require each firm to have a minimum of 10 yearly changes in realized dividends, free cash flows and abnormal earnings. Realized dividends equal the total amount of dividends declared in year t (#21). Because the statement of cash flows changed over the years preceding the sample period, we calculate realized free cash flow for each year as earnings before extraordinary items (#18), plus after tax interest expense (#15),[33] minus the change in total assets (#6), plus the change in total liabilities (#181), minus the change in long term debt (#9), minus the change in short term debt (#34). Realized abnormal earnings equal earnings before extraordinary (#18) minus the estimated discount rate times the book value of equity in year t-1 (#60).

Table 11 reports mean and median values of the predictability measure for each model; we also report comparisons of predictability between each pair of models. Consistent with firms making few changes in dividend payments and policies, we find that dividends are highly predictable. Of more interest (we believe) are the results comparing the predictability of free cash flows and abnormal earnings. Nonparametric tests indicate that the *AE* series is

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[32] This is certainly true for the 20% of firms in our sample who do not pay dividends. Even for dividend paying firms, *DIV* estimates likely understate value because our terminal value calculations do not include a liquidating dividend.

[33] Interest expense is item #15. We use statutory tax rates to proxy for the firm's realized tax rate.

significantly more predictable than the *FCF* series; however, we note no difference in the mean predictability of *FCFs* and *AEs*.

Table 11: Comparison of the Predictability of the Forecast Attributes[a,b]

	α-level: t-tests[c]			χ-level:Wilcoxon tests[c]		
Attribute	mean	vs *FCF*	vs *AE*	median	vs *FCF*	vs *AE*
DIV	0.71	0.0001	0.0001	0.23	0.0001	0.0001
FCF	25.98		0.8284	7.72		0.0001
AE	24.71			3.86		

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] Predictability is measured as the standard deviation of percentage yearly changes in the historical realized values of the attribute valued by each model.

[c] We report mean and median predictability measures as well as the significance levels for t-tests and Wilcoxon tests comparing the pooled sample predictability measures across models.

To assess the importance of predictability on the reliability of the value estimates, we partition the data as follows. For each attribute, we rank observations based on the magnitude of the predictability measure. Observations in the bottom quartile of the standard deviation of percentage yearly changes are labeled the High Predictability sample, while those in the top quartile of the distribution are labeled the Low Predictability sample. Table 12 summarizes the accuracy and explainability of the value estimates for each of the within-model predictability partitions. In general, the results show no strong evidence that a more predictable series leads to significantly more reliable value estimates. For example, using perpetuity-based TVs, securities with the smallest standard deviations of changes in *FCFs* or changes in *AEs* have less accurate value estimates, which explain less of the variation in observed prices, than securities with the largest standard deviations of these attributes. However, for *DIV* value estimates we find the opposite.

Table 12 Comparison of the Performance of the Value Estimates for the High and Low Predictability Samples[a]

Panel A: Accuracy (measured as the median absolute prediction error)

	High Predictability Sample[b]			Low Predictability Sample[b]			Difference[c]
	Median	vs V_i^{FCF}	vs V_i^{AE}	Median	vs V_i^{FCF}	vs V_i^{AE}	High vs Low
V_1^{DIV}	0.1233	0.0001	0.0055	0.1699	0.0001	0.4508	0.0002
V_1^{FCF}	0.2608		0.0001	0.2278		0.0001	0.6647
V_1^{AE}	0.1529			0.1666			0.0262
V_2^{DIV}	0.6876	0.0001	0.0001	0.7218	0.0001	0.0001	0.0001
V_2^{FCF}	0.4546		0.0001	0.4564		0.0001	0.3132
V_2^{AE}	0.3641			0.3149			0.0003
V_3^{DIV}	0.5994	0.0001	0.0001	0.6342	0.0001	0.0001	0.0001
V_3^{FCF}	0.4095		0.0001	0.3665		0.0001	0.4340
V_3^{AE}	0.3141			0.2920			0.3120

Panel B: Explainability (R^2 from univariate regression of price on value estimate)

	TV specification: i=1			TV specification: i=2			TV specification: i=3		
	V_1^{DIV}	V_1^{FCF}	V_1^{AE}	V_2^{DIV}	V_2^{FCF}	V_2^{AE}	V_3^{DIV}	V_3^{FCF}	V_3^{AE}
High Predictability									
OLS coefficient	1.00	0.73	0.95	2.04	0.58	1.40	1.57	0.33	1.20
OLS R^2	0.86	0.78	0.88	0.62	0.45	0.72	0.60	0.37	0.71
Rank R^2	0.95	0.90	0.96	0.86	0.78	0.92	0.86	0.79	0.92
Low Predictability									
OLS coefficient	1.02	0.70	0.89	1.32	0.93	1.02	0.93	0.66	0.96
OLS R^2	0.95	0.63	0.87	0.32	0.62	0.73	0.31	0.58	0.74
Rank R^2	0.95	0.90	0.95	0.80	0.78	0.90	0.80	0.76	0.90

[a] See note a to table 3 for a description of the sample and the calculations of value estimates and terminal values.

[b] The High (Low) Predictability sample consists of the bottom (top) quartile of firms ranked on the average predictability of the attribute valued by the noted model. Predictability is measured as the standard deviation of percentage yearly changes in the historical realized values of the attribute valued by each model.

[c] We report the significance level for the Wilcoxon test of whether the median absolute prediction error of the High Predictability sample differs from that of the Low Predictability sample.

Overall, the results in tables 9-12 provide mixed evidence on whether the precision and the predictability of the attribute valued by each model are important determinants of the reliability of the value estimates. Consistent with the *AE* model's relative superiority over the *FCF* model, across-model tests generally indicate that *AE* forecasts are more precise and more predictable than *FCF* forecasts. However, within-model tests show no consistent evidence that securities with the most precise or the most predictable forecasts have more reliable value estimates than do securities with the least precise or the least predictable forecasts.

6. Summary and Conclusions

This paper compares the reliability of value estimates from the discounted dividend model, the discounted free cash flow model and the discounted abnormal earnings model. We use Value Line forecasts of expected fundamentals for a forecast horizon of five years, collected for a sample of 2,907 firm-year observations over 1989-1993. We compare value estimates in terms of their absolute percentage deviations from current market prices and their ability to explain cross-sectional variation in contemporaneous stock prices.

When Value Line price-earnings ratios are used to infer stock price at the end of the forecast horizon, all of the valuation models perform very well. *DIV* and *AE* value estimates have median absolute prediction errors of 16-18% (with 43-47% of observations within 15% of share price) and explain 88-96% of the variation in current market prices. *FCF* value estimates have a median accuracy of 23% (33% of observations within 15% of share price) and a model R^2 of 77-89%. Relative to value estimates using perpetuity-based terminal values, value estimates based on inferred VL price forecasts are two to four times more accurate and explain 25%-220% more of the variation in current stock prices. Taking the VL price forecast as the ideal measure of terminal value, these results suggest ample scope for improving the performance of all of the valuation models by better specification of terminal values.

Because it is uncommon in practice to have estimates of future prices to proxy for terminal values, we believe that in determining which valuation model is best, more weight should be placed on the results from perpetuity-based terminal value calculations. In these cases, the results consistently show that the *AE* value estimates are more accurate and explain more of the variation in security prices than do estimates from the *FCF* and *DIV* models. Our explorations of the sources of the relative superiority of the *AE* model show that the greater reliability of *AE* value estimates is likely driven by the sufficiency of book value of equity as a measure of intrinsic value and perhaps by the greater precision and predictability with which analysts forecast earnings (from which abnormal earnings are calculated).

We also investigate whether accounting discretion enhances or detracts from the performance of the *AE* model. These results indicate no difference in the accuracy of *AE* estimates between firms exercising high versus low accounting discretion, although there is some evidence that *AE* value estimates explain more of the variation in current market prices for high discretion firms than for low discretion firms. Together these findings indicate no empirical basis for concerns that accounting practices (such as immediate expensing of R&D or the flexibility afforded by accruals) result in inferior estimates of market equity value. Our results are more consistent with the argument that the articulation of clean surplus financial statements ensures that value estimates are unaffected by conservatism or manipulations (provided that analysts are aware of these distortions).

In summary, we find that when differences in performance exist across value estimates, the *AE* model is usually superior to the other models. Thus, there is little to gain – and if anything something to lose – from selecting dividends or free cash flows over abnormal earnings as the fundamental attribute to be valued. Together with the fact that earnings is by far the most consistently forecasted attribute, our results suggest little basis for manipulating accounting data (for example, to generate estimates of free cash flows) when earnings forecasts and book values are available.

Appendix

White [1980] tests, as well as visual inspections of residual plots, indicate significant heteroscedasticity in the residuals from equation (9). To alleviate the heteroscedasticity and to provide a check on the robustness of the results, we attempt to discern the functional form of the heteroscedasticity using Glejser [1969] tests. In our context, these tests require that we investigate various specifications relating the absolute value of the OLS residuals with the value estimates, choosing the functional form which best fits the data. The results of these examinations (not reported) led us to select the specification described by equation A1; for this regression, the value of b_1 is close to 1 and the estimate of b_0 is statistically indistinguishable from zero.[34]

(A1) $$\left|e_j\right| = b_0 + b_1\sqrt{V_{j,F}^{FUND}} + v_j$$

We use this result to transform the dependent and independent variables in expression (9); we then estimate the following transformed regression:

(A2) $$\frac{P_{j,F}}{\sqrt{V_{j,F}^{FUND}}} = \delta_0 \frac{1}{\sqrt{V_{j,F}^{FUND}}} + \delta_1\sqrt{V_{j,F}^{FUND}} + u_j$$

Residual plots show no obvious heteroscedasticity concerns with the residuals from regressions of (A2). For the first TV specification of the *AE* model, White tests cannot reject (at the .05 level) the null of equal variances. For the other models and TV specifications, White tests continue to reject (at the .05 level) equal variances; however, the heteroscedasticity problems are clearly less severe for (A2) than they are in the untransformed

[34] These estimates are for the *AE* model using the first terminal value specification; results are qualitatively similar for the other models and terminal value specifications.

regression or in any other transformations we examined.[35] We also note that the parameter estimates from the transformed regressions are very close to the estimates obtained from the untransformed regressions. Thus, the conclusions based on the untransformed OLS regressions are robust to the more elaborate procedures used to address (but unfortunately not eliminate in most cases) heteroscedasticity problems.

[35] The heteroscedasticity problem is smaller for the *AE* model than for the *FCF* or the *DIV* model, irrespective of the terminal value specification (the null of equal variances cannot be rejected at the .01 level for any of the *AE* terminal value specifications).

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Study 2:

Discount Rates in Equity Valuation

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Abstract

This study contrasts different standard techniques for estimating discount rates (the cost of equity capital) in a company (equity) valuation setting. The aim is to provide indications on the usefulness of different asset pricing models for this particular purpose. The empirical study finds that using cross-sectionally constant discount rates provides more reliable value estimates for individual stocks than does using standard CAPM implementations, which in turn dominate the so-called three-factor model. The likely reason is that estimation uncertainty plagues the more elaborate discount rate estimation techniques (the CAPM and the three-factor model) to such an extent that it overshadows any potential gains from using what may be more 'correct' models of market equilibrium.

Discount rates and company valuation

1. Introduction

This paper takes a hands-on approach to the question of discount rate estimation in equity (company) valuation models. The aim is pragmatic and to some extent normative: to show some practically feasible alternatives for estimating the cost of equity, and give a feeling for how the different alternatives behave relative to each other. The latter aspect is visualized through an empirical study. Of particular interest is the question of how estimation problems affects the implementation of asset pricing models as bases for discount rate estimates.

Students in business schools are generally recommended to use the Sharpe-Lintner Capital Asset Pricing Model (Sharpe [1964], Lintner [1965]) to determine the cost of equity capital. The Sharpe-Lintner CAPM is also advocated in most corporate finance textbooks (e.g., Copeland and Weston [1988], Brealey and Myers [1996]). It is arguably the most widely used asset pricing model also among practitioners for determining discount rates. While an analytically tractable model, empirical investigations of the Sharpe-Lintner CAPM have rejected it as a model that is completely descriptive of average returns.[1] The empirically based critique against the CAPM reached its height with Fama and French [1992], which led the Wall Street Journal to proclaim that 'beta is dead'. One should note, however, that such criticism does not necessarily imply that the CAPM is 'dead' for practical purposes. The

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[1] Even the earliest tests, e.g. Friend and Blume [1970], Black, Jensen and Scholes [1972], Fama and MacBeth [1973], generally reject the Sharpe-Lintner version of the CAPM. Later, a number of tests have called the validity of both the Black [1972] and the Sharpe-Lintner versions of the CAPM into question. In particular, the many 'anomalies' discovered during the late 1970s and 1980s can be interpreted as rejections of the CAPM in the sense that they show that, as an empirical matter, beta does not suffice to explain average returns (e.g., Ball [1978], Banz [1981], Basu [1983], DeBondt and Thaler [1985]). The issue is further complicated by the fact that the empirical rejections in the asset-pricing literature do not necessarily mean that the CAPM is false. An argument often used by CAPM advocates is that asset-pricing tests generally have questionable proxies for the market portfolio, and that a rejection of CAPM may be attributable to an inadequate choice of market portfolio rather than an indication that the CAPM as such is a bad model. Since, however, practical applications of the CAPM generally use market portfolio proxies similar to the ones used in the empirical tests (e.g., a country's stock exchanges), the empirically based CAPM critique potentially spells problems for the practitioner even if it is true that the CAPM in its 'ideal form' is still valid.

empirical investigations generally show that while the CAPM does not *suffice* to explain average returns, it still retains significant explanatory power, and thus it is potentially useful as basis for discount rate estimation.

Multi-factor models, in particular the Fama and French [1993] three-factor model, are mentioned in later editions of valuation books (although not used, not even in examples). This comes from the many asset pricing articles in the finance literature showing that the three-factor model explains average returns better than the CAPM. While there is considerable debate about the Fama and French interpretations, even the most critical articles seem to agree on the empirical results: that the ratio of book equity to market equity and market capitalization provides significant explanatory power over and above the CAPM market factor. (The interpretation issues are discussed in section 4.3.) For practical purposes the interpretation debate may be something of a side issue, though: if there is an empirical regularity involving book-to-market and market capitalization, then the three-factor model is potentially useful to base discount rate estimates upon.

When trying to implement an asset pricing model in a practical application, one also cannot escape the question of estimation problems. Fama and French [1997] highlight this issue. They contrast the CAPM and the three-factor model and conclude that estimation problems may be even *more* important than the question of which of the two asset pricing models is the more correct one. Estimation problems are severe both when estimating factor loadings (e.g., beta in the CAPM) and when trying to determine the associated risk premia (in the CAPM: $r_M - r_f$). Thus there are two main problems facing the practitioner: identifying the correct asset pricing model and estimating the necessary parameters. On top of all this, there is an additional difficulty when working with valuation models. In virtually all more advanced valuation models one uses expectations of *future* outcomes of some valuation attribute. An example is the dividend discount model where one calculates the equity value as the present value of future expected dividends. The discount rate enters into this in the present value calculations. If one then, for example, uses the CAPM to determine the discount rate, one would use historical data to estimate beta. There is an implicit assumption in this, however, namely that beta remains stable into the future. Looking at realized betas over time, this does not seem descriptive of reality. So we have the added question of parameter stability when looking forward in time. All in all there are several things to keep track of, and it may be helpful with a short summary of the problems:

When implementing a company valuation model we need discount rate estimates. These are normally based on some model of market equilibrium, some asset pricing model. The obvious first question is then which asset pricing model is the correct one, correct in the sense that it is descriptive of average returns. The asset pricing literature provides some guidance here. The second question concerns the implementation. Even if we can identify the correct model, can we implement it? Is it possible to estimate the parameters with any reasonable precision? The third question (very much related to the second) is that even if we can estimate the parameters reasonably well based on historical data, is it reasonable to assume that they remain stable in the future? These main questions can be combined into one in the context of company valuation:

> *Which asset pricing model seems most useful for estimation of discount rates to use in company valuation models?*

The CAPM holds a rather firm grasp on the valuation literature. Widely used corporate finance textbooks like Brealey and Myers [1996] do not really question it. Practitioner-oriented valuation books like Stewart [1991] and Copeland, Koller and Murrin [1994] use it in all their examples. The underlying assumption must be that the CAPM works fairly well in real-world applications – or at the very least that at present we lack better alternatives that are practically feasible. As mentioned above, some newer valuation texts also mention multi-factor models, thus implicitly assuming that these models are useful for discount rate estimation in a company valuation context. Is this true? This paper will try to address explicitly such implicit claims.

Three (classes of) asset pricing models will be considered here. The first one is a simple, naïve model, namely that all companies have the same expected return, here set equal to 12%. This is the long-run average realized return on equities in the United States. This model will serve as a benchmark against which the results from the more elaborate models will be contrasted. The second class of models includes different versions of the CAPM. The third model is the so-called three-factor model, as identified by Fama and French [1993].

The models will be evaluated both in terms of how easy they are to implement and how they behave. The idea is a simple and practical one: the naïve model is *very* easy to implement, so the use of a more complicated model can only be motivated by superior performance. As performance indicators the results from a large-sample empirical investigation will be used, where analysts' forecasts of earnings and book values are used as proxies for market expectations to derive value estimates for each stock. Different discount rates will yield different value estimates. By comparing these different value estimates to market prices on the basis of *bias* (average percentage forecast error), *accuracy* (absolute percentage forecast error) and *explainability* (ability to explain cross-sectional variation in market prices), a ranking on the 'performance' of the different ways to determine discount rates can be achieved. The performance indicators thus assume market efficiency and that analysts' forecasts include all information available at the valuation date.

It should be stressed that this paper seeks to provide evidence on the usefulness of standard implementations of the asset pricing models. For the CAPM I have tried to stay true to the implementation 'prescriptions' in standard textbooks, such as Brealey and Myers [1996] and other similar texts. Even in standard books there are different opinions on some issues such as the best way to estimate beta (e.g., daily or monthly data), the appropriate way to estimate the market risk premium (e.g., arithmetic or geometric mean). I try to present the different popular alternatives. For the three-factor model there is less choice. The Fama and French implementation (but not necessarily interpretation) is, I think, dominating in the debate, and I have followed their implementation fairly meticulously. I do *not* attempt to 'improve' (should that be possible) on these standard implementations; rather the purpose is to take them as representative of what we teach in business schools, and of what presumably is common practice. To summarize this paragraph: This paper does *not* seek provide evidence on asset pricing models. What it does provide are *indications on the usefulness* of standard implementations of different asset pricing models as determinants of discount rates in valuation models. As already mentioned, poor performance of an asset-pricing model in a practical valuation situation can also be caused by estimation problems – it need not be a bad model as such. But when all is said and done – in a practical situation we may not really care about the reasons as long as we have something that works.

This paper also implicitly illustrates the question of whether discount rate estimation really is a first-order effect in equity valuation. Is there a big difference between using different ways of estimating discount rates? This is also important for the classroom situation. Given the limited number of lectures in a typical valuation course, how much time should we spend on the technical aspects of discount rate estimation?

The next section describes the valuation model and the sample. Section 3 details the evaluation criteria. Section 4 describes the different ways of estimating discount rates: the underlying asset pricing models as well as different possible implementation choices. The actual results are also presented and discussed in this section. Section 5 concludes.

2. The valuation model and the sample

2.1 The valuation model

To provide an illustration to the problems and to evaluate I use the so-called *discounted abnormal earnings* equity valuation model (sometimes referred to as the *residual income* model). Like the popular free cash flow (FCF) model, the discounted abnormal earnings (DAE) model is an algebraic reformulation of the discounted dividends model. As an empirical matter, however, the DAE model seems to dominate other commonly used equity valuation models in terms of accuracy and ability to explain cross-sectional variation in market prices (see, e.g., Bernard [1995], Frankel and Lee [1995, 1996], Penman and Sougiannis [1997], Francis, Olsson and Oswald [1997]). One may note that this evidence is unanimous. A further advantage with the DAE model for the purpose of this paper is that it directly uses the *cost of equity capital* as discount rate.

The DAE model is based on valuation techniques introduced by Preinreich [1938] and Edwards and Bell [1961], and further popularized by Ohlson [1995]. The DAE model assumes the validity of an accounting identity – the clean surplus relation – to express equity value as a function of book values and abnormal earnings:[2]

(1) $$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{AE_t}{(1+k_E)^t}$$

(2) $$AE_t = X_t - k_E B_{t-1}$$

(3) $$B_t = B_{t-1} + X_t - DIV_t$$

where V_0 = market value of equity at time *0*;
AE_t = abnormal earnings in year *t*;
B_t = book value of equity at *t*;
X_t = earnings in year *t*;
k_E = cost of equity;
DIV_t = dividends in year t net of capital contributions/withdrawals.

The *cost of equity*, k_E, thus has two roles in the valuation formula (1): it is used both as discount rate and as the determinant of the required capital charge each period.

2.2 The sample

The sample consists of 2907 firm-years between 1989 and 1993 for firms with December 31 fiscal year ends, i.e. between 500 and 600 firms per year for five years.[3] Historical book values are taken from Compustat and security prices are taken from CRSP (Center for Research in Security Prices, Chicago). Value Line Investment Survey (VL) forecasts of book values and earnings are used to calculate forecasts of future abnormal earnings. It is thus assumed that the Value Line analysts' forecasts incorporate all available information available to the market. In other words analysts' forecasts are used to proxy for market expectations of

[2] Clean surplus requires that any change in book value must flow through earnings. The exception is dividends, which are defined net of capital contributions. See expression (3).

[3] The sample firms are the same as in Francis, Olsson and Oswald [1997].

these future attributes. Prior research has generally shown that analysts' forecasts are more accurate than time-series based forecasts, and it is quite common in the research literature to see analysts' forecasts as proxies for market expectations.[4]

Value Line gives forecasts of accounting data for the current year, one year ahead and "3-5 years ahead". Following Bernard [1995], it is assumed that the 3-5 year ahead forecast applies to each year in that interval. Forecasts for year 2 are obtained by averaging year 1 and year 3 forecasts. The forecasts are made during the third quarter; the calendar dates of the valuations range from July 1 to September 30 for each sample year, 1989-1993.

Some descriptive statistics on the sample can be found in table 1. It is obvious that the sample consists of rather large firms. This is so probably because larger companies attract analyst following to a much larger extent than smaller companies.

Table 1: Descriptive information on the market characteristics of 2,907 firm-year observations with Value Line forecasts available during 1989-1993[a]

	Sample Year					
	1989	1990	1991	1992	1993	Pooled
# observations	568	554	578	600	607	2907
mean market capitalization[b]	2008	2467	2370	2997	3134	2607
mean beta[c]	0.98	0.92	0.96	1.00	1.00	0.97
# firms traded on:						
NYSE	423	415	428	451	457	2174
AMSE	41	38	43	43	40	205
NASDAQ	104	101	107	106	110	528

[a] The sample consists of the common equity securities of December year-end firms with the following information available for any year t=1989-1993: third-quarter Value Line forecasts of all fundamental values; Compustat data on the book value of common equity for year t-1; and CRSP security prices.

[b] In millions of dollars, as at December 31, year t-1.

[c] Betas are calculated using daily stock returns over the 12-months ending December 31, year t-1.

[4] See further Schipper [1991] for an overview of issues involved in using analysts forecasts to proxy for market expectations. A more detailed description/examination of Value Line forecasts can be found in Philbrick and Ricks [1991]. Furthermore, one may note that Value Line analysts seem to be viewed as typical analysts, since Value Line is often chosen as the particular object under study when the research question pertains to analysts in general. A good example is Abarbanell and Bernard [1992].

Given the specifics of the sample (in particular the valuation dates) the valuation formulas (1-3) are made operational in the following manner:

$$(4) \qquad V_S = B_{Q2} + \frac{.5\left(X_0 - k_E \cdot B_{Q2}\right)}{\left(1 + k_E\right)^s} + \sum_{t=1}^{5} \frac{\left(X_t - k_E \cdot B_{t-1}\right)}{\left(1 + k_E\right)^{t+s}}$$

$$+ \frac{1.04\left(X_5 - k_E \cdot B_4\right)/\left(k_E - .04\right)}{\left(1 + k_E\right)^{5+s}}$$

where: S = publication date of the Value Line forecast;
s = the number of days between S and December 31, divided by 365;
0 = index for the current year forecast;
t=1,…,5 subscripts one-year ahead through 5-year ahead forecasts;
$Q2$ = quarter 2 of year 0;

The first term in (4) is the book value of equity at the end of quarter two in year zero (the year in which the valuation is made). The second term is the discounted abnormal earnings for year zero.[5] The third term (the sum) are the discounted abnormal earnings over the explicit forecast period. The fourth term, finally, is the discounted terminal value (horizon value). The terminal value is calculated as the last explicit year's abnormal earnings growing at 4% in perpetuity.

The choice of terminal value is meant to reflect a typical 'textbook-like' terminal value technique. The underlying assumption is that the valuation attribute (here: abnormal earnings) should have settled down to a steady state by the end of the forecast horizon, and only grow by expected inflation thereafter. Following Kaplan and Ruback [1995], 4% is taken as a rough proxy for expected inflation.[6] One question is, of course, if it is reasonable to use a forecast horizon of only five years. The evidence in Barker [1998] seems to suggest that analysts and fund managers are reluctant to use a horizon of more than two years (for

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[5] Note that the abnormal earnings model is derived from a dividend valuation model and that half of year zero's dividends have been paid out at valuation date. Hence, only half of the abnormal earnings for year zero should be included in the valuation.

[6] The results are similar using growth rates of 2, 6, 8, or 10%.

some particular industries longer horizons are possible, but never more than five years), so the length of the horizon here is probably indicative of practical reality. Should the terminal value be negative, then it is set to zero.[7] This captures the intuition that a company will not perpetually engage in operations that earn less than the cost of capital.

The forecasts of accounting data (and also historical book values) will remain the same in all tests, using the Value Line sample. What will change are the discount rates – the estimates of the cost of equity – thus providing the indications on which asset pricing model is most useful as basis for discount rate estimation in company valuation. It should further be noted from equation (4) that the discount rate is assumed constant through time. This is almost universally so in applied valuation.[8] Within a given asset pricing framework, it simply means that the expected one-period simple return on a security is constant.

3. Evaluation criteria

All evaluation criteria use market price as benchmark. Behind this lies the assumption of market efficiency, i.e. that market prices incorporate all available information. The vast finance and accounting literature on the subject seem to indicate that market efficiency is indeed a reasonable assumption in the US market with respect to accounting data (and forecasts thereof). The one deviation from market efficiency that stands up to scrutiny is the so-called post-earnings announcement drift, i.e., the market's underreaction to the information inherent in quarterly earnings announcements,[9] but that problem is not material here (since it would mainly affect quarterly earnings forecasts up to three or four quarters after the announcement and this paper uses annual forecasts of up to five years). One should further

[7] This affects 16 observations (of 2,907). The results are not sensitive to this assumption.

[8] See, however, Levin and Olsson [1998] for ways of updating the cost of equity to reflect a changing capital structure.

[9] There is still some discussion, but the evidence is mounting in favor of market efficiency. See Fama and French [1996] for a thorough discussion and investigation of the different alleged inefficiencies. Bernard, Thomas and Wahlen [1997] also investigate the different accounting-based stock price 'anomalies'. They find that all alleged anomalies except the post-earnings announcement drift can be explained by risk, and are thus not indications of market inefficiency.

note that this is a large-sample study with almost 3,000 observations, so even if there is an occasional mispricing, it is unlikely to materially affect the study here.

Three evaluation criteria will be considered. The first one is *bias*, defined as the percentage forecast error:

$$(5) \qquad bias = \frac{value\ estimate\ -\ actual\ price}{actual\ price}$$

Bias may be less serious, since one may (conceivably) be able to correct for it. Furthermore, aggregate statistics on the bias measure, such as mean or median bias, in a way cancel out positive and negative forecast errors.

A more reliable evaluation measure may be *accuracy*, defined as the absolute percentage forecast error:

$$(6) \qquad accuracy = \frac{|\ value\ estimate\ -\ actual\ price\ |}{actual\ price}$$

Even in aggregate, accuracy takes both negative and positive forecast errors into consideration and weights them equally (i.e., an assumed symmetric loss function). Note that both bias and accuracy are negatively defined evaluation metrics – the lower, the better. *Central tendency* is another measure of accuracy. Following Kaplan and Ruback [1995], it is defined as the fraction of the sample that lies within 15% of observed market price.

The third evaluation measure is *explainability* – the ability of the value estimates to explain cross-sectional variation in market prices. It is defined as the R^2 from a regression of market price on the value estimate:

(7) $$P_{i,S} = \alpha_0 + \alpha_1 V_{i,S} + \varepsilon_i$$

where: $P_{i,S}$ = market price of security i at valuation date S
$V_{i,S}$ = estimated value of security i, valuation date S

Explainability is perhaps the evaluation measure that is most relevant here. If an asset-pricing model identifies the relevant risk measure(s) better than the naïve 12%-model, then it should also explain the cross-section of observed security prices better. The explainability measure also in a way controls for bias (by including an intercept and allowing the coefficient to vary). This is important since we already have a bias evaluation measure. Abstracting from bias also really tests what is perhaps the most important feature of an asset pricing model in a valuation context: whether it can discriminate among individual firms on the basis of risk.

4. Asset pricing models

We have now arrived at the key variable in this investigation: the different asset pricing models on which one can base the discount rate estimates.

4.1 The benchmark model – the naïve model

This model is very simple. It says that the expected return for every security is equal to the long run average return on equities: in the US 12% (1926-1988; i.e., up to the year before the sample period). It is admittedly a naïve approach in the sense that it does not attempt to differentiate among stocks on the basis of risk. It is, however, robust in the sense that it leaves little scope for estimation problems. Another plus is that it is extremely easy to implement. Whether it is, say, 11% or 12% is not the big issue. Rather, the important thing is that it is constant across firms.

This model can be viewed as a sort of base-case, against which one can measure the other asset-pricing models' usefulness as determinants of discount rates. More complex models should at a minimum improve on this naïve model. The usefulness perspective should be stressed once again. Even if it is true that, say, the CAPM is the correct model, it is still quite possible that estimation problems are so severe that they overshadow the potential gains from using a more correct model.

4.2 CAPM based models

As already mentioned, the CAPM is by far the most widely used asset pricing model. The theoretical underpinnings are well known, and can be found in any intermediate finance book. Some will count both the Black (1972) model as well as the Sharpe-Lintner version (Sharpe [1964], Lintner [1965]) as CAPM versions, although most texts mean the Sharpe-Lintner model when they refer to the CAPM. This paper will also adhere to this convention.

According to the CAPM, the expected return on any security i is:

(8) $$E(r_i) = r_f + \beta_{iM}\left(E(r_M) - r_f\right)$$

where $E(r_i)$ = the expected return on security i

r_f = risk-free rate of interest

β_{iM} = the 'systematic risk' of security i

$E(r_M) - r_f$ = the market risk premium

The second term in (8), $\beta_{iM}\left(E(r_M) - r_f\right)$, is often referred to as the *risk premium* in the relation between the expected return on security i and its risk in the market portfolio. Note further that β_{iM} is the only measure of risk that appears in (8), hence it is the only measure of risk needed in order to explain differences among the expected returns on securities.

Going from asset-pricing model to discount rate estimate will for practical purposes mean a simple switch in notation, and the *cost of equity*, $k_{E,i}$, is thus given by:

$$k_{E,i} = r_f + \beta_{iM}\left(r_M - r_f\right) \tag{9}$$

There are some implementation choices for (9) that must be made. The first one is what to use as proxy for the risk-free rate of return. Since the valuation of a firm almost by definition means an intermediate or long-term investment horizon, it is reasonable to use intermediate or long-term bonds rather than the t-bill (one month) rate. This seems to be the universal recommendation in valuation texts (see, e.g., Brealey and Myers [1996], Copeland, Koller and Murrin [1994], Palepu, Bernard and Healy [1996]). One should deduct the maturity premium relative to t-bills, however.[10] This will then yield an estimate of the intermediate (or long-term) risk free rate of return.

Beta (β) is an estimate of the systematic risk of the firm. Several beta estimates, suggested in the literature, will be considered. The first one is an ordinary, OLS-estimated, firm-specific, beta. It is taken from the CRSP tapes, where it is calculated using daily returns over the fiscal year preceding the valuation date. To account for possible non-synchronous trading, the Scholes-Williams estimation routine is applied (Scholes and Williams [1977]).

Using the preceding year's data for estimation should give the most 'up-to-date' estimate of risk (i.e., risk as proxied by beta). Many valuation texts recommend a longer estimation period, however, then using monthly data. The estimation interval then typically covers a minimum of five years, i.e. 60 monthly observations. The length of the estimation period is a potentially important issue. Using even longer estimation periods will reduce estimation uncertainty given that beta is stable through time. In reality, betas are not that stable over time, so for practical purposes there is a trade-off between increased estimation precision and potential violation of the stability assumption when increasing the length of the estimation period. Fama and French [1997] compare a five-year estimation period with a longer period (from 1963 and onwards) and find that the longer estimation period provides somewhat lower

[10] See, e.g., Brealey and Myers [1996], footnote 14, p. 218. Basically, we want today's expectations of future t-bill rates. 'Today's expectations' are clearly given by using longer term securities, such as bonds. However, a long-term investment in a bond means investing in a security with a fixed nominal pay-off and this carries an extra 'risk', reflected in the maturity premium.

prediction errors than rolling three-, four- and five-year regressions when looking at a five-year forecast horizon. The second set of betas in this paper is based on monthly data and a long estimation period, from July 1963 to December the year preceding the valuation date. The returns are taken from CRSP, and the betas are estimated using OLS regression.[11]

Concerns are sometimes voiced that OLS-estimated betas are too noisy because of estimation problems. Different ways to mitigate this problem have been suggested. Industry betas are sometimes used. The basic claim of asset-pricing models such as the CAPM, however, is that they discriminate *among individual firms* on the basis of risk, hence it is questionable whether it really can be labelled a CAPM implementation when averaging over industries. Another possibility that still retains some of the 'firm specific' character of the CAPM is to use some sort of shrinkage technique. A very simple and easy-to-implement one is used by Merrill Lynch:[12] $\hat{\beta}_{shrunk} = 0.35 + 0.65\hat{\beta}_{OLS}$. This basically pulls OLS-estimated betas towards one, which of course is the market beta. Results based on this simple shrinkage technique will also be reported here. More elaborate Bayesian techniques can be applied, but they come at the sacrifice of user-friendliness.

All of the above beta estimates are easily obtained. Firm-specific betas can be calculated in standard regression packages (given that one has access to returns data), or one can simply use any of the numerous 'store-bought' betas available in financial publications. Summary statistics on betas used in this paper are presented in Table 2. It can be noted that using the long estimation period produces substantially higher betas.

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[11] Sometimes, outliers are deleted in these estimations. The analysis was re-run deleting observations with studentized residuals greater than 2, but this made no difference for the overall results.

[12] Source: Holthausen and Zmijewski [1996].

Table 2 : Mean and median beta estimates for the 2,907 firm-year observations [a]

	Estimation period	
	preceding year, daily data [b]	long period, monthly data [c]
mean	0.97	1.19
median	0.95	1.18

[a] The sample firms are described in Table 1.
[b] Taken from the CRSP tapes.
[c] Calculated using OLS regression, monthly returns. The estimation period starts in July 1963 and ends in December the year preceding the valuation. If a firm did not exist or was not listed in 1963, then the estimation period starts when the firm first appears on the CRSP tapes.

The next implementation choice concerns the market risk premium. It is usually estimated using a historical arithmetic average. Most texts seem to advocate as long an estimation period as possible. The theoretical foundation for this is an assumption that the true market risk premium is constant through time. Using the full period 1926-1988,[13] this yields an annual market risk premium estimate of 8.50%. One should exercise caution, however, since the market risk premium empirically varies considerably over time. In fact, the standard error for the full period 1926-1988 is 2.69%, so a common 95% confidence interval of roughly plus/minus two standard errors will include market risk premia from 3% to 14%. Given this massive uncertainty, one may be justified in doubting the wisdom of using historical data for estimation purposes.

An alternative approach is to just use something 'that works'. Stewart [1991, p. 438] suggests using 6%:

"Is there any fundamental reason why the market risk premium should be 6%? Not that I can figure. The question is a little like asking why did God make pi the number 3.14159... Don't ask. Just memorize it, and then head out to recess."

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[13] CRSP data are available starting from 1926.

Such atheoretical approaches can of course be criticized for being unscientific, but one should keep in mind that what we are after is the *expected* market risk premium, and using something suggested by a practitioner-oriented book may very well come *closer* to the market's expectation than a highly uncertain historical average. In fact, if the 6% figure 'works', then the reason for that may very well be that 6% is fairly close to the market's expectation.

A third possibility is to use the geometric mean (estimated over the period 1926-1988 it was 6.3%). The geometric mean is advocated by, e.g., Copeland, Koller and Murrin [1994]; however, its theoretical validity in a CAPM world is questioned by many, and most academics seem to favor the arithmetic mean. Ibbotson and Sinquefield [1993] explain in some detail why the arithmetic mean is the correct one to use. Fama [1996a] provides a thorough discussion on related theoretical issues.

To visualize the market risk premium issue, results are reported based on both the (full-period) arithmetic average, the 6% market risk premium and the (full-period) geometric mean. In the reported tables, estimates are made using historical data from 1926 up until December 31 the year preceding the valuation date.

The mean and median value estimates based on the naïve 12% model as well as on the different CAPM implementations are presented in Table 3. The bias metric is also presented. It is obvious from Table 3 that regardless of discount rate, the valuation model fails to capture the average market price. The estimated value is always lower than market price. One may note that in terms of unbiasedness the naïve 12% model dominates all versions of the CAPM, and the CAPM model that comes nearest is the model with a 6% market risk premium, probably because 6% is lower than both the geometric and arithmetic historic mean.

Table 3 in a way illustrates why bias is far too simplistic to be really useful as an evaluation measure. The 12% model may 'win' simply because it yields the lowest discount rates and consequently the smallest average underestimation of market price. The main claim of the CAPM, however, is that it distinguishes *among* stocks on the basis of risk. On this important point median and mean bias is silent, since negative and positive errors to a large extent cancel out, and thus there is no (implicit) loss function attached to variability.

Table 3 – Value estimates, bias: Mean and median of the actual share price at valuation date, of the value estimates based on a constant discount rate of 12%, and of the value estimates based on different CAPM implementations. Mean and median bias of the different value estimates.

	Value ($ per share)		Bias[a]	
	Mean	Median	Mean	Median
Actual share price at valuation date	31.27	25.12	n/a	n/a
Discount rate = 12%	26.49	21.46	-7.1%	-13.1%
CAPM (daily data from preceding year)				
- hist. arithm. mean market risk premium	20.63	16.26	-25.4%	-36.7%
- 6% risk premium	22.00	20.19	-8.4%	-20.2%
- hist. geom. mean market risk premium	24.55	19.53	-11.7%	-23.5%
CAPM (long est. period, monthly data)				
- hist. arithm. mean market risk premium	17.09	13.46	-40.6%	-45.2%
- 6% risk premium	21.56	16.98	-25.3%	-31.1%
- hist. geom. mean market risk premium	20.68	16.32	-28.3%	-33.7%
CAPM (shrunk beta estimates based on daily data from preceding year)[b]				
- hist. avg. risk premium	18.99	15.47	-31.9%	-38.3%
- 6% risk premium	23.91	19.47	-14.8%	-21.8%
- hist. geom. mean market risk premium	22.94	18.64	-18.2%	-25.1%
CAPM (shrunk beta estimates based on monthly data, long estimation period)[b]				
- hist. avg. risk premium	17.29	13.83	-39.5%	-43.9%
- 6% risk premium	18.22	17.47	-23.8%	-29.3%
- hist. geom. mean market risk premium	20.96	16.68	-27.0%	-32.1%

[a] Bias is defined as the signed prediction error, i.e. (value estimate – observed price) / observed price.
[b] The shrunk beta estimates are calculated as $0.35 + 0.65\hat{\beta}_{OLS}$.

Evaluation metrics that also take variability into consideration are presented in tables 4 and 5. Table 4 gives accuracy figures (= absolute forecast errors) as well as the measure of central tendency (the fraction of estimated values within 15% of observed market price).

Table 4 – Accuracy: Mean and median absolute forecast errors and central tendency for the value estimates based on a constant 12% discount rate and for the value estimates based on different CAPM implementations.

	Abs. forecast error [a]		Central tendency [b]
	Mean	Median	
Discount rate = 12%	29.2%	24.5%	30.9%
CAPM (daily data from preceding year)			
- hist. arithm. mean market risk premium	42.9%	42.7%	14.8%
- 6% risk premium	38.9%	34.2%	21.6%
- hist. geom. mean market risk premium	39.3%	35.7%	20.3%
CAPM (long est. period, monthly data)			
- hist. arithm. mean market risk premium	45.0%	46.3%	8.8%
- 6% risk premium	35.7%	34.6%	19.1%
- hist. geom. mean market risk premium	37.2%	36.8%	16.7%
CAPM (shrunk beta estimates based on daily data from preceding year) [c]			
- hist. avg. risk premium	40.1%	40.9%	14.3%
- 6% risk premium	33.5%	31.2%	24.0%
- hist. geom. mean market risk premium	34.3%	32.9%	23.3%
CAPM (shrunk beta estimates based on monthly data, long estimation period) [c]			
- hist. avg. risk premium	43.7%	44.8%	9.6%
- 6% risk premium	34.4%	32.8%	19.3%
- hist. geom. mean market risk premium	35.8%	34.8%	17.7%

[a] | (value estimate – observed price) | / observed price.
[b] The proportion of value estimates within 15% of observed price.
[c] The shrunk beta estimates are calculated as $0.35 + 0.65\hat{\beta}_{OLS}$.

Table 4 provides a clear picture. Using CAPM discount rates means a considerable loss in accuracy. Mean and median absolute forecast errors are substantially larger with the CAPM than with the naïve 12% model. This is true regardless of which beta estimation procedure is used and regardless of which estimate for the market risk premium one uses, but the difference is most marked for the arithmetic average risk premium. It is interesting to note that the shrinkage method helps somewhat, but also the estimates based on the shrunk betas are inferior to the naïve model. The same picture emerges when looking at central tendency:

the naïve model has the highest fraction of estimated values within 15% of observed price and clearly dominates all CAPM-related approaches. The differences are substantial enough to have significant economic meaning – an 'extra' mis-valuation of 10-20% of share price when using the CAPM based discount rate estimates is certainly no trivial matter.

Still, although mean and median absolute forecast errors do not cancel out positive and negative errors, the bias that was apparent in Table 3 will also have an influence on accuracy. To try to further isolate the main 'usefulness' claim of the CAPM – that it distinguishes *among* shares on the basis of risk – the regression approach described under *explainability* in Section 3 is utilized in Table 5, below. When using a regression, then bias is in a way controlled for, both by the inclusion of an intercept and by allowing the coefficient relating the value estimate to observed market price to deviate from its theoretical value of one (bias correlated with the value estimate itself). The measure of explainability, R^2, should thus be a decent indication of whether the inclusion of beta adds any 'risk-based' explanatory power when deciding on discount rates for company valuation.

The naïve 12% model has a fairly high explanatory power: R^2=77%. Furthermore, the coefficient is close to one (in fact, the null hypotheses that the coefficient is equal to one cannot be rejected at the 5% risk level). The CAPM based discount rates produce inferior value estimates: the R^2s range from around 50% to 70% and the coefficients are not equal to one. There is an interesting difference, however, between the CAPM alternatives. The R^2s from the long estimation period (using monthly returns) are much higher than the R^2s from the shorter estimation period (using daily returns). Using the shrinkage helps the short estimation period results substantially (R^2s go up from around 50% to around 65%), but it has a very small effect on the long estimation period results.

Table 5 – Explainability: Coefficient estimates and R^2s from univariate regressions of price on the value estimate based on a constant 12% discount rate and of price on the value estimates based on different CAPM implementations.

	coefficient	(std. error)	R^2
Discount rate = 12%	1.0187	(.0106)	.77
CAPM (daily data from preceding year)			
- hist. arithm. mean market risk premium	.9660	(.0185)	.49
- 6% risk premium	.8442	(.0147)	.54
- hist. geom. mean market risk premium	.8720	(.0153)	.54
CAPM (long est. period, monthly data)			
- hist. arithm. mean market risk premium	1.3837	(.0176)	.69
- 6% risk premium	1.1173	(.0135)	.71
- hist. geom. mean market risk premium	1.1562	(.0141)	.70
CAPM (shrunk beta estimates based on daily data from preceding year)[a]			
- hist. avg. risk premium	1.2532	(.0183)	.63
- 6% risk premium	1.0477	(.0139)	.67
- hist. geom. mean market risk premium	1.0833	(.0146)	.66
CAPM (shrunk beta estimates based on monthly data, long estimation period)[a]			
- hist. avg. risk premium	1.4291	(.0173)	.71
- 6% risk premium	1.1496	(.0132)	.73
- hist. geom. mean market risk premium	1.1562	(.0141)	.70

[a] The shrunk beta estimates are calculated as $0.35 + 0.65\hat{\beta}_{OLS}$.

The main result so far is that all CAPM based discount rate estimates perform worse than the naïve 12% model. This is so regardless of evaluation metric (bias, accuracy or explainability). In short: using betas to estimate discount rates not only doesn't add anything, it actually introduces variation that makes the value estimates more biased and less accurate, and it means a considerable loss in explainability. Add to this fact that the naïve model is much easier to implement, and the indication from tables 3, 4, and 5 is clear: the CAPM is not useful as a basis for discount rate estimation in a company valuation context.

It should once again be stressed that this paper seeks to provide an indication on the *usefulness* of the CAPM for discount rate estimation in company valuation. It doesn't really say anything about the CAPM per se, and whether it is the true model of market equilibrium.

The bad results could also be due to estimation problems. Another possibility is of course that the implementation in this paper is less successful, so the CAPM has not been given its 'fair chance'. Although the purpose of this paper is to comment on the overall usefulness of the CAPM, it may be of some interest to comment on which reason seems more probable: (i) the bad model explanation, (ii) the estimation problem explanation, or (iii) the bad implementation explanation.

One may note that even though CAPM betas in asset pricing tests generally are not sufficient to explain average returns, betas do have significant explanatory power. One would therefore a priori think that they would be useful in discount rate estimation unless estimation problems are too severe. This argument would speak for the estimation problem explanation. There is, however, a possible counter-argument to this. Fama and French [1992, p. 427] conclude that "when the tests allow for variation in β that is unrelated to size, the relation between market β and average return is flat, even when β is the only explanatory variable." It should be noted that the sample used in this paper consists of firms followed by Value Line analysts, and these are mainly companies with a fairly high market capitalization,[14] i.e. large size companies ('size' defined as market equity – note that this size definition is not uncontroversial since it abstracts from differences in capital structure). Hence there is limited variation in market capitalization, and if the Fama and French result is applicable here, then we would not expect beta to be very helpful in describing average returns for this sample, and hence nor to be useful in estimating discount rates. Furthermore, cross-sectional variation *is* introduced by using betas, and if this variation is 'unnecessary', then it may actually destroy explanatory power. This is what we observe, so this point may speak for the 'bad model explanation'.

One may further note from tables 4 and 5 that the shrinkage (which is used to mitigate estimation problems) makes the CAPM based estimates look less bad. Substantially so for the estimates based on the short estimation period betas; somewhat so for the estimates based on the long period betas. Discount rates based on shrunk betas are still inferior to the naïve model, however, so this particular variance reduction technique does not alter the main result. There might still be problems with the risk-free rate and the market risk premium, of course, but especially the explainability measure (R^2) in Table 5 should be fairly robust to such

[14] See Table 1.

problems, since bias is controlled for. The comparison of the R^2 of the naïve model with the R^2s of the CAPM-based models clearly indicates that introducing risk-based variation in discount rates through CAPM betas is not a good idea.

The bad implementation argument is trickier. In a narrow sense, any empirical investigation really only tests the particular implementation used in the investigation, so the problem is the same here as in any other empirical study. I have tried to stay close to recommendations made in standard valuation texts, and, when such recommendations differ, to try the different alternatives. How well I succeed in this and how representative the results in this paper are is ultimately for the reader to decide. At a minimum, the results *illustrate* the problems encountered when trying to implement CAPM based discount rates in a company valuation situation. I believe the results are more generally indicative, for the following reasons: (i) it is a large sample study (almost 3,000 observations); (ii) the forecasts of fundamental data (earnings and book values) are taken from Value Line, which is a large and well renowned analyst service, and – more importantly – the properties of Value Line forecasts have been investigated rather extensively in earlier research (see the discussion in Section 2.2, in particular footnote 6; see also section 4.4.). A further indication on the generalizability of the results is how they relate to earlier studies. To my knowledge there exists no similar study; however one can look at the results implicit from the sensitivity analysis and footnotes in a couple of earlier papers. (The papers cited below are the only papers I have found which, even indirectly, deal with the issue.)

Kaplan and Ruback [1995] have access to cash flow forecasts from 51 firms engaged in high leverage transactions (HLTs). They use these in what they call a "compressed APV" valuation,[15] and then compare their value estimates to the actual transaction values. Kaplan and Ruback have three more or less CAPM based methods using (i) individual firm betas, (ii) industry betas, and (iii) the market beta. An industry beta is the average over the two digit SIC code, so all firms within an industry is assumed to have the same operating risk. The market beta is (obviously) the same for all firms. Using the same beta for all firms (the market beta) could be interpreted as implicitly saying that although the CAPM might be a valid model, it is *not useful* for providing estimates of discount rates for valuations of

[15] Rather close to a free cash flow valuation technique.

individual companies. Kaplan and Ruback find that the estimates based on the market beta have lower average bias than the firm-specific CAPM model (2.5% vs. 6.0%).[16] The market beta based estimates are also more accurate (mean absolute forecast error 16.7% vs. 21.1% for the firm-specific; mean squared error 5.1% vs. 8.4%), and have better explainability: they explain a higher fraction of the variation in the actual transaction values (R^2=39% vs. R^2=24% in an univariate regression of transaction value on value estimate; R^2=39% vs. R^2=33% when using log transformations). The Kaplan and Ruback results thus indicate that using firm-specific betas is not a good idea. Their sample is somewhat special (high leverage transactions) and not very large (51 firms), but the results from their analysis concur with the results in this study.

Sougiannis and Yaekura (1997) use three different specifications of the DAE valuation model and in addition analyze an earnings capitalization model in a large-sample study. They compare their value estimates, obtained using CAPM discount rates and analyst earnings forecasts, to observed security prices on the basis of bias and accuracy. In footnote 9 they report that they also try using cross-sectionally constant discount rates and that this produced lower valuation errors. Thus, it seems that the Sougiannis and Yaekura study also – implicitly – find that the CAPM is less useful than just using constant discount rates.

It was argued in the beginning of this paper that the emphasis on the CAPM in the valuation literature is indicative of an – perhaps implicit – assumption that the CAPM works fairly well in practical situations or that at the very least it is the best alternative that is still practically feasible; in short, that the CAPM is the most *useful* way to estimate discount rates. The cited studies, Kaplan and Ruback [1995] and Sougiannis and Yaekura [1997], are examples of this: neither study questions the usefulness of the CAPM, even though both studies find that constant discount rates across firms yield better results.[17]

One may, of course, interpret this as evidence of extreme mean reversion rather than evidence that the CAPM is a bad model to use. This is a question of labelling rather than content,

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[16] See Kaplan and Ruback [1995], tables II and IV.

[17] This is not meant as a criticism. Neither of the two studies is primarily interested in discount rate estimation.

however. The main indication remains: the CAPM does not seem useful for the purpose of distinguishing among firms on the basis of risk in a company valuation situation.

4.3 The three-factor model

The next step is to see whether the positive results in the asset pricing literature on multi-factor models can be put into practical use in a valuation context. So far we have not been able to address head-on the question of whether the negative CAPM results are due to the fact that the CAPM really is a bad model, or whether it is just too difficult to estimate the parameters ex ante. Multi-factor models have not generally been disproved as 'bad models' in asset pricing tests, and implementing multi-factor models requires estimation procedures similar to those used for the CAPM. Hence, if a multi-factor model actually works here, this would indicate that the bad CAPM results in the preceding section are because the CAPM is a bad model, not because it is impossible to estimate. If, however, a multi-factor implementation does not work either, then this might indicate that estimation problems are too severe to make any of these models, single- or multi-factor, useful as a basis for discount rate estimation. Indeed, if estimation uncertainty is the first-order problem, then we would a priori expect the opposite: that multi-factor models are less useful than the single-factor CAPM, simply because multi-factor models require more estimations.

Multi-factor models received their first boost with the so-called Arbitrage Pricing Theory, APT (Ross [1976]). The APT basically assumes that the covariation of returns on securities is captured by a small number of common, non-diversifiable, factors in returns. Though theoretically interesting, the APT may – even a priori – be of little help to the practitioner. It is derived using an argument that applies to perfectly diversified portfolios, but does not say anything about individual securities.[18] This theoretical problem aside, the model does not identify the relevant factors. Some factors have been suggested, e.g., industrial production, inflation, etc. The empirical validity of such proposed factors remains an open question, however. Unless one for some reason a priori believes in such proposed factors, one is forced

[18] Connor [1984] develops an APT model based on utility maximization rather than arbitrage statements that is applicable to individual securities. It still assumes that the market portfolio is perfectly diversified, though.

to perform some sort of factor analysis to form portfolios that will capture common variation in returns.

Another way of looking at multi-factor models is to view them as a discrete time version of Merton's [1973] intertemporal CAPM. Fama [1996b] develops this framework. This way, the intuition from the CAPM is maintained and mean-variance efficiency is replaced by multi-factor efficiency. The discrete-time version of the intertemporal CAPM (henceforth: ICAPM) produces exact statements on the expected return of individual securities. This is what we need for our purposes, and furthermore the logical structure of the ICAPM is similar to the CAPM. Hence, it may be easier to discuss the multi-factor models in terms of the ICAPM. The same problem as in the APT regarding the identification of relevant factors still apply, however.

So which risk factors do we need to consider? The asset pricing literature does provide some guidance on this issue. The long series of articles on 'CAPM anomalies' in the 1970s and '80s was mentioned in the introduction to this paper (the 'size effect', 'dividend yield effect', 'price-earnings effect', etc.). These articles share the feature that something other than beta helps in explaining average returns. Hence, such an anomaly might indicate that the anomalous factor represents (or proxies for) an omitted risk factor. Before concluding that all of the anomalies represent distinct risk factors, one should keep Ball's [1978] argument in mind, that scaled price variables will always proxy for omitted risk factors, so different factors may be proxying for the same omitted risk.

Fama and French [1992] conclude based on empirical investigations that size (defined as market capitalization) and book-to-market (B/M, book equity value over market capitalization) seem to explain cross-sectional variation in returns. They have continued with a series of articles. Fama and French [1993] develops the so-called three-factor model, a version of the ICAPM where the relevant factors for expected stock returns are a market factor (as in the CAPM), a size factor and a book-to-market factor. The size and book-to-market factors are made operational as state-variable mimicking portfolios, i.e., portfolios proxying for state variables that carry special risk premiums that cannot be explained by the CAPM (details follow below). Fama and French [1996] continue to show that most of the so-called CAPM anomalies disappear when applying the three-factor model (the one exception is the continuation of short-term earnings, see Jegadeesh and Titman [1993]).

There has been substantial criticism of the Fama and French interpretations. To mention but one example, Berk [1995] argues that market equity is not a good measure of firm size. He reasons that, for a given set of cash flows, low market equity firms are firms with high discount rates, i.e. high risk firms. If this is so, then it may not be surprising that 'small' firms (small market equity firms) have high average returns. He then goes on to argue that other definitions of size do not demonstrate this relation with average returns.

Criticisms such as the one cited in the preceding paragraph are, however, more conceptual than practical. The market equity effect is there, and could potentially be exploited in discount rate estimation, whether or not one labels it a measure of size. So for practical purposes such conceptual criticisms may be less relevant. (For ease of notation and comparison, I will stick with the definition of 'size' as market equity for the rest of this paper.) One may further note that although there is considerable debate about the Fama and French interpretations, even the most critical articles agree on the empirical results – only the interpretation differs. Lakonishok, Shleifer and Vishny [1994], for example, claim that the book-to-market and size effects represent exploitable market inefficiencies rather than separate risk factors. They basically identify the same phenomena as Fama and French, but the labelling is different.[19] For our purposes the distinction between 'predictable market inefficiency' and 'risk' is not really that important. We want an estimate of the expected return for a stock to use as discount rate, and whether this estimate is based on three distinct risk factors or on one risk factor and two predictable inefficiencies may not matter very much: the key word is *predictability*, and since the identified (and hence predictable) risk factors or predictable market inefficiencies are the same we can leave the asset pricing debate – satisfied about which factors we need to care about.

According to the three-factor model the expected return on a security is given by

$$E(r_i) = r_f + b_i\left(E(r_M) - r_f\right) + s_i E(SMB) + h_i E(HML) \qquad (10)$$

[19] See, however, the Fama and French [1996] rebuttal of Lakonishok, Shleifer and Vishny, and Bernard, Thomas and Wahlen [1997], whose results also corroborate the Fama and French story.

The expected return in excess of the risk-free rate is hence given by the sensitivity of the stock's return to three-factors: (i) the difference between the return on a market index and the risk-free rate (cf. the market risk premium in the CAPM); (ii) SMB – small minus big – the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks; and (iii) HML – high minus low – the difference between the return on a portfolio of high book-to-market stocks and the return on a portfolio of low book-to-market stocks.

The two portfolios SMB and HML are meant to mimic risk factors in addition to the market factor. The loadings, b_i, s_i and h_i, are estimated as the slopes in the regression:

$$(11) \qquad r_i - r_f = a_i + b_i\left(r_M - r_f\right) + s_i SMB + h_i HML + e_i$$

The estimation procedure is thus quite similar to CAPM calculations.

The returns data are taken from CRSP. I use the factors as calculated by Fama and French. The SMB factor is made operational by dividing stocks (in this case on the combined US stock exchanges) on the basis of market capitalization. *Small* is defined as stocks below the median market capitalization on the NYSE. *Big* stocks are above the median. The HML factor is made operational by dividing stocks on the basis of book-to-market (book equity / market equity). Three book-to-market groups are constructed: High (the highest 30%), Medium (the medium 40%) and Low (the bottom 30%). Combining the two partitionings (the size dimension and the B/M dimension) thus yields six portfolios.

SMB (small minus big) is the difference, each month, between the average of the returns on the three small-stock portfolios and the three big-stock portfolios. HML (high minus low) is the difference between the returns on the two high B/M portfolios and the two low B/M portfolios.

The factor loadings b_i, s_i and h_i are estimated using a long estimation period (from July 1963 to December the year preceding the valuation date). Like in the CAPM estimations in the preceding section, this choice is based on the finding in Fama and French (1997) that the 5-year forecast errors of returns are slightly lower using a long estimation period. The loadings are estimated using expression (11). Mean and median loadings as well as the mean and median values of the factors are presented in table 6.

Table 6 – descriptives on the implementation of the three-factor model:

Panel A: Mean and median loadings for the 2,907 firm-year observations [a, b]

	b_i	s_i	h_i
mean	1.08	0.70	-0.01
median	1.07	0.63	0.08

Panel B: Mean and median factors, monthly data starting in 1963 [c]

	$r_M - r_f$	SMB	HML
mean	0.0582	0.0355	0.0489
median	0.0612	0.0126	0.0510

[a] The sample firms are described in Table 1.
[b] Calculated using OLS regression, monthly returns. The estimation period starts in July 1963 and ends in December the year preceding the valuation. If a firm did not exist or was not listed in 1963, then the estimation period starts when the firm first appears on the CRSP tapes.
[c] The factors are described in the text.

The firm-year specific discount rate estimates are calculated according to expression (12):

$$(12) \qquad k_{E,i} = r_f + b_i\left(\overline{r_M - r_f} \right) + s_i\,\overline{SMB} + h_i\,\overline{HML}$$

where the bars indicate the mean values of these factors (the mean taken from July 1963 to December the year preceding the valuation date). We now have discount rate estimates based on the three-factor model and can thus calculate value estimates for each stock. Tables 7, 8 and 9 contains the figures for the estimates based on the three-factor model compared to the estimates based on the naïve 12% model. To facilitate comparisons, the estimates based on the 'best' CAPM version (long estimation period) are also repeated in tables 7, 8 and 9.

Table 7 – Value estimates, bias: Mean and median of the actual share price at valuation date, of the value estimates based on a constant discount rate of 12%, of the value estimates based on the three-factor model , and of the value estimates based on the (long estimation period) CAPM.

	Value ($ per share)		Bias[a]	
	Mean	Median	Mean	Median
Actual share price at valuation date	31.27	25.12	n/a	n/a
Discount rate = 12%	26.49	21.46	-7.1%	-13.1%
Three-factor model	24.66	16.31	-20.0%	-33.6%
CAPM (long est. period, monthly data)				
- hist. arithm. mean market risk premium	17.09	13.46	-40.6%	-45.2%
- 6% risk premium	21.56	16.98	-25.3%	-31.1%
- hist. geom. mean market risk premium	20.68	16.32	-28.3%	-33.7%

[a] Bias is defined as the signed prediction error, i.e. (value estimate – observed price)/ observed price.

Table 8 – Accuracy: Mean and median absolute forecast errors and central tendency for the value estimates based on a constant 12% discount rate, for the value estimates based on the three-factor model, and for the value estimates based on the (long estimation period) CAPM.

	Abs. forecast error[a]		Central tendency[b]
	Mean	Median	
Discount rate = 12%	29.2%	24.5%	30.9%
Three-factor model	45.1%	40.7%	15.1%
CAPM (long est. period, monthly data)			
- hist. arithm. mean market risk premium	45.0%	46.3%	8.8%
- 6% risk premium	35.7%	34.6%	19.1%
- hist. geom. mean market risk premium	37.2%	36.8%	16.7%

[a] | (value estimate – observed price) | / observed price.
[b] The proportion of value estimates within 15% of observed price.

Table 9 – Explainability: Coefficient estimates and R^2s from univariate regressions of price on the value estimate based on a constant 12% discount rate, on the value estimates based on the three-factor model, and on the value estimates based on the (long estimation period) CAPM.

	coefficient	(std. error)	R^2
Discount rate = 12%	1.0187	(.0106)	.77
Three-factor model	0.6367	(.0115)	.52
CAPM (long est. period, monthly data)			
- hist. arithm. mean market risk premium	1.3837	(.0176)	.69
- 6% risk premium	1.1173	(.0135)	.71
- hist. geom. mean market risk premium	1.1562	(.0141)	.70

The main result from tables 7-9 is the following: the three-factor model is *not useful* as a basis for discount rate estimation for individual firms. Its performance is inferior to the naïve 12% model. The 12% model has lower bias (Table 7), better accuracy (Table 8) and better explainability (Table 9). Furthermore, as perhaps is obvious from the description in the text, the implementation of the three-factor model is rather tedious.

The results for the three-factor model are interesting because they give an indication on which is the first order-effect when deciding on discount rates in company valuation: the choice of asset pricing model or estimation problems. We know from the asset pricing literature that the three-factor model is empirically a fairly correct model of market equilibrium (correct in the sense that it describes average returns well). Yet it seems less useful for discount rate estimation. This indicates that estimation problems overshadow any potential gains from using a 'correct' model.

What about the CAPM vs. the three-factor model? – Depending on choice of market risk factor for the CAPM, sometimes the three-factor model dominates and sometimes the CAPM dominates when looking at either bias or accuracy (tables 7 or 8). However, as was argued earlier the most important evaluation criterion is probably explainability since this measure most directly evaluates the main claim of an asset pricing model: distinguishing among firms on the basis of risk. And looking at explainability, the CAPM dominates the three-factor model – but both are still dominated by the naïve 12% model.

Comparing all three models – the naïve 12% model, the CAPM, and the three-factor model – gives further indications about which is the more important: the choice of model or estimation problems. As mentioned earlier, if estimation problems are the dominating factor, then we would expect the model with the fewest estimations to have the superior performance and the model with the most things to estimate to be the least successful. The naïve model has by its very nature only one thing to estimate: which level the constant discount rate should have. The CAPM requires more estimations: the riskfree rate, beta, and the market risk premium. The three-factor model even more: the riskfree rate, the three loadings, and the three-factors. And this is also the 'ranking' we see in table 9: the naïve model dominates the CAPM which in turn dominates the three-factor model. Given that the asset pricing literature has identified the *reverse* ranking in terms of which model is more 'correct', it seems we can conclude that for practical purposes minimizing estimation problems is more important than the choice of asset pricing model.

4.4 The Hardwiring Argument – Endogeneity Concerns

In all studies of this type, the skeptic may (and perhaps should) raise the endogeneity question. And while generally impossible to refute, I will argue why I do not think that this is driving the results. The particular version of endogeneity that would be a concern here is if Value Line analysts make forecasts of earnings, book values, etc., then formulate an abnormal earnings valuation model using *a particular discount rate estimation method*, then compare the resulting value estimate with market price, and then, if the difference is deemed unreasonable, go back and adjust the forecasts of accounting items to better reflect current market price. If this were the case, then a study such as this would by definition be biased towards favoring the particular discount rate estimation method that is used by Value Line. If this discount rate method is different from what the market uses/expects, then the indications in this study would not be generally valid outside this sample. Kaplan and Ruback [1995] label the danger that the forecasts are adjusted to yield a certain market price the potential hardwiring effect.

So, why would this be less likely to cause the results in this study? Note that the hardwiring argument requires two things: (i) that Value Line analysts do this using constant rates, and (ii) that constant rates are not indicative of the market as a whole.

Other studies

As has already been mentioned, other studies also find that cross-sectionally constant discount rates provide more reliable value estimates than CAPM based rates. Both Kaplan and Ruback [1995] and Sougiannis and Yaekura [1997] report this in their sensitivity analyses. The former study uses firm-internal forecasts of fundamental accounting data, the latter uses IBES forecasts (i.e., analysts' consensus forecasts). So, if hardwiring is going on, it would not be unique to Value Line, rather it would be going on within firms (the Kaplan and Ruback sample) and it would be going on in the entire analyst community (the Sougiannis and Yaekura sample) and it would be going on using different valuation models. This might still be so. But if both firms and the entire analyst community use this technique of calibrating forecasts to market value using a constant discount rate, then it is a rather strong indication that constant discount rates are indeed indicative of 'the market', and that the finding in this study is not spurious and not caused by Value Line analysts hardwiring their forecasts of particular future accounting fundamentals.

Then we have the valuation model used by Value Line analysts. While one cannot know with certainty which valuation model individual VL analysts use, there is no reason to believe that they are any different from other analysts, and so they probably use several models. Barker [1998] rank (on frequency) valuation models used by analysts, and the abnormal earnings model is not up there among the top ten models. Given this, it would be hard to believe that analysts hardwire their forecasts to accommodate specifically an abnormal earnings valuation model. The sample period in this study (1989-1993) I think further reinforces this argument, since the abnormal earnings valuation model did not gain its present popularity until the very last few years (much thanks to Ohlson [1995]).

The above arguments are, of course, just arguments. One would perhaps like a more direct test based on this sample. Basically we want to test whether the Value Line forecasts are good proxies for market expectations. The problem is, of course, that market expectations are unobservable. In some finance and economics studies, a rational expectations argument is made for using realizations to proxy for market expectations. If one accepts this as valid, then this opens up a way of testing the possible claim that the finding in this study is spurious and caused by VL analysts hardwiring their forecasts, and that in reality a CAPM (or three-factor model) implementation would yield more reliable value estimates than the naïve, constant 12% model.

Testing ex-post forecast accuracy

If hardwiring is going on within this sample, then it is likely that it has differential impact on different observations. Some forecasts will have been changed more substantially, whereas others will have been more or less untouched. If the VL forecasts are hardwired using a cross-sectionally constant 12% rate, and the market's expectations are instead based on, say, a standard CAPM model, then the set of particular forecasts that has been hardwired by VL should be further away from market expectations than the set of observations that is less affected by this operation. This supposition is impossible to test directly, since the ex-ante accuracy of forecasts (i.e., proximity to market expectations) is unobservable. Relying instead on ex-post realizations of abnormal earnings to gauge the ex-ante accuracy of the forecasts one would expect that the more accurate the AE forecasts, the less likely it is that they have been adjusted using a constant discount rate method even though the market's expectation is more in line with one of the more advanced asset-pricing models (and vice versa).

Following this line of reasoning, I measure ex-post forecast accuracy as the absolute difference between the forecasted and realized abnormal earnings, scaled by price at valuation date. This forecast accuracy measure is then, for each firm, averaged over the forecasted years (i.e., over the explicit forecast period): the resulting measure is labelled precision. Since I only have access to realized accounting data up until 1996 (from Compustat), this operation is possible to make only on the forecasts made in 1989, 1990, and 1991 (since five years of realizations are required). This yields a sample size of 2331. I then split the sample into quartiles based on the precision measure, and the 'worst' quartile is labelled the low precision sample, whereas the best quartile is the high precision sample. The low precision sample should then include the hardwired forecasts whereas the high precision sample should have the relatively 'correct' forecasts. If it is true that a standard CAPM implementation in fact yields more reliable value estimates than a constant 12% discount rate (and if so, the results to the contrary in this study only reflects VL hardwiring their forecasts using a constant 12% rate) then one would expect the CAPM-based estimates to fare better in the high precision sample than in the low precision sample (since the forecasts in the high precision sample should be less hardwired). The same argument would apply to the three-factor model. This is not what one finds, however. Both accuracy and explainability is always *worse* in the high precision sample than in the low precision sample – for the CAPM implementations as well as for the three-factor model.[20] Thus the test does not provide any evidence that hardwiring is causing this study to spuriously reject the usefulness of the CAPM and of the three-factor model.

To summarize the endogeneity concerns: Such concerns are always valid in empirical studies of this type, and while it can never be ruled out that hardwiring is driving the results, the test speaks against this. Furthermore, evidence from other studies suggest that the finding in this study is not unique to this sample.

5. Concluding remarks

The choice of discount rate can have a large impact in equity (company) valuation. Practically oriented valuation texts generally recommend the Sharpe-Lintner capital asset pricing model to estimate the cost of equity. Some newer texts also mention multi-factor models, in particular the three-factor model, inspired by the CAPM criticism in the asset pricing literature.

The asset pricing literature has generally found the three-factor model to be more descriptive of average returns than the single-factor CAPM, which in turn explains average returns better than a cross-sectionally constant discount rate. All of these tests are ex-post, however, thereby not taking the added uncertainty into account that comes from having to estimate

[20] R-squares (explainability) for CAPM and three factor versions range from 0.66 to 0.81 for the low precision sample; from 0.41 to 0.71 for the high precision sample.

future risk premia and loadings. In a practical valuation situation there is no escaping this added uncertainty.

This study attempts to make standard, textbook-like implementations of the CAPM and the three-factor model to see whether they are useful to determine discount rates. A model is deemed 'useful' if it produces discount rate estimates that lead to better (according to a number of performance measures) company valuations than just using the same discount rate for all firms. The general indication from this study is that the fewer things one has to estimate, the better. Consequently, the naïve approach of using the same discount rate for all firms produces the best results, the CAPM comes in second, and the three-factor model third. Hence, the model selection problem seems to be subordinate to the question of estimation problems.

The CAPM is arguably still the most popular asset-pricing model. The three-factor model is, according to some, about to overtake the CAPM as the front-runner asset pricing model – at least in terms of ex-post empirical validity. Be that as it may, there has always been a general assumption in valuation books that the 'state of the art' asset pricing model can be used in practical situations – that one can use this model to estimate discount rates. This, however, neglects the question of estimation uncertainty, something one cannot neglect in a practical situation. The statistical behavior of factor loadings and risk premia in popular asset pricing models has been known for a long time. There is massive uncertainty. This makes the use of any historical average treacherous. Still, there has been an implicit assumption in equity valuation texts that standard implementations of asset pricing models are still useful for discount rate estimation. This study indicates otherwise. Estimation uncertainty is a first-order problem for standard implementations of these models, and it is – it seems – so severe that it might be better to use a more robust approach. The one used here – and shown superior – being a cross-sectionally constant discount rate. It is an important task for future research in this area to develop implementation routines that deliver robust discount rate estimates that still manage to distinguish among firms on the basis of risk.

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Study 3:

Looking Beyond the Horizon and Other Issues in Company Valuation

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Abstract. This essay deals with a number of issues concerning company valuation models with special reference to the so-called McKinsey valuation model, presented in *Valuation: Measuring and Managing the Value of Companies* by Tom Copeland, Tim Koller, and Jack Murrin. Special attention is given to horizon value problems, and we investigate the many hidden assumptions inherent in continuing value formulas. Different equity valuation approaches are also commented upon, and we develop a procedure that yields equivalence between the discounted free cash flow approach and the discounted dividend approach. The procedure is shown to hold under quite general conditions, also when the capital structure is non-constant. Finally, the traditional free cash flow approach, our modified free cash flow approach and the dividend valuation approach are all applied to a real-world company, and it is shown that the latter two yield exactly the same value.

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0. Introduction

Company valuation with discounted free cash flow models has received much attention during the last few years. One of the most prominent examples is the McKinsey valuation model, described in *Valuation: Measuring and Managing the Value of Companies* by Tom Copeland, Tim Koller, and Jack Murrin.[1] The book has sold more than 50,000 copies and is used as textbook in many leading business schools - favoured more by finance teachers than by their colleagues in accounting, however. Letting the question whether free cash flow valuation is superior to other valuation approaches remain unanswered, we make the observation that the popularity of the McKinsey model in academic education will make it even more widespread among practitioners once today's students graduate and start to work. This observation is the pragmatic reason for our work: an in-depth look at many of the questions connected with free cash flow models in general and the McKinsey model in particular.

The title of this report - Looking Beyond the Horizon - alludes to the concept of horizon value, common in valuation models, and also present in the McKinsey model. The approach is explained in Brealey & Myers (1991): "The value of a business is usually computed as the discounted value of free cash flows out to a **valuation horizon** (*H*), plus the forecasted value of the business at the horizon, also discounted back to present value. That is,

$$PV = \frac{FCF_1}{1+r} + \frac{FCF_2}{(1+r)^2} + \ldots + \frac{FCF_H}{(1+r)^H} + \frac{PV_H}{(1+r)^H}$$

Of course, the [...] business will continue after the horizon, but it's not practical to forecast free cash flow year by year to infinity."[2] The horizon value is almost always calculated as a continuing value, using for instance the Gordon formula.[3]

The fact that the horizon value is calculated using a very simplifying formula can in no way be taken as evidence that it is somehow unimportant. On the contrary, Copeland et al report

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[1] The book and the model will be referred to alternately as the *McKinsey book/model* and *Copeland et al.*

[2] Brealey & Myers (1991), p. 64

[3] See, e.g., Brealey & Myers (1991), p. 34.

typical values for some industries: for a company in the tobacco industry the horizon value accounts for 56% of the total company value, in the sporting goods industry it is 81%, for the typical skin care business the figure is 100% and for a high tech company 125% (the figures are calculated using a horizon eight years into the future).[4] Furthermore, practitioners like horizon values. A colleague told us that when he had discussed the McKinsey book with corporate analysts at Swedish investment banks, they typically wanted to use a horizon value after not more than five years, some even after only two or three years.

A large part of this report is devoted to problems connected with the horizon value. We particularly want to uncover the many hidden assumptions inherent in continuing value formulas. This also makes the reverse possible, namely to say what conditions must be fulfilled for a continuing value calculation to be admissible - or at least create an awareness about the problems connected with it. The rest of the report covers a range of subjects that we feel are insufficiently dealt with in Copeland et al, and that are of general interest in company valuation. The findings in the report also lend themselves to conclusions regarding the choice of discounting method, a problem treated rather sketchily in many practically oriented textbooks.

The methodology we use in much of the report is to investigate the structure of difference equations that govern the different versions of the valuation model. This technique makes it possible to see exactly what restrictions are implied by the different proposed modelling approaches - and only when such restrictions are stated explicitly can one judge whether they are reasonable or not.

It should be noted that the McKinsey model does not include any stochastic elements. The free cash flows, which are discounted, could be interpreted as expected free cash flows. The stochastic processes that generate them are left out. In this report we adhere to this mode of presentation.

The first section in Chapter 1 contains a short description of the McKinsey model. This is meant more as an introduction for the reader not fully acquainted with the particulars of the Copeland et al approach. The second section introduces the alternative specifications of the

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[4] Copeland et al, p. 275

property, plant and equipment (PPE) items in the McKinsey model. In Chapter 2, 3 and 4 the actual methods and results of this report are developed. Chapter 5 is a "How to do it"-guide, where we use the results in a step by step implementation of the model on the Swedish company Eldon AB. Chapter 6, finally, contains the concluding remarks.

1. The Modelling Framework

1.1 The model's structure

The model contains two sections: one historical and one for the future. In the historical section, balance sheets and income statements from a number of years are inserted into the model in order to calculate several financial ratios. These historical ratios can then be used as a basis for forecasting the corresponding ratios in the future, which in turn are used for calculating future balance sheets and income statements. From these, it is possible to derive the free cash flow (FCF), the net profit (NP) and the dividends (DIV) for each future year. The company value can then be calculated:[5]

Definitions:

> *The free cash flow (FCF) valuation approach* is a model where the forecasted free cash flows are discounted by the weighted average cost of capital (WACC). The sum of the discounted free cash flows is then the *total company value*. Deducting the market value of the debt yields the *equity value*.
>
> *The dividend (DIV) valuation approach* is a model where the forecasted future dividends are discounted by the cost of equity capital (k_e). The sum of the discounted dividends makes up the *equity value*, and by adding the market value of the debt, one arrives at the *total company value*.

[5] The terminology is not standardised in the literature: "Company value" sometimes stands for the equity value, other times it refers to the total value of the company, i.e. equity value plus debt value. In this report we will use the terminology *equity value* and *total company value* where it is necessary to distinguish between the two.

The net profit (NP) valuation approach is a family of models where the forecasted future net profits are calculated and capitalised, often after deducting a charge for the use of capital.

In the original version, the book by Copeland et al advocates the free cash flow valuation method. This method has also become fashionable with many financial economists, and we will also concentrate on it. Some consideration will also be given to the dividend valuation approach. Lately, many academics in the accounting field have begun advocating earnings, or net profits, as the relevant valuation measure. We do not share their enthusiasm. However, as there is a demand for explicit net profit figures, we show how these can be derived from the same modelling framework as we use to derive free cash flows and dividends. In section 3.2 we discuss the earnings issue, and explain why we are somewhat sceptical towards it.

Since it is very difficult to make detailed forecasts for periods in the distant future, the valuation can be divided into two periods: the explicit forecast period and a continuing value for the time after this period.[6] This so-called horizon value is often calculated with the Gordon formula,[7] but there are a number of interesting problems connected with the horizon value, which will be dealt with in coming chapters.

For a hypothetical company, the "McKay" company (adapted from a teaching note by L. Peter Jennergren (1994) and similar to the Preston company in Copeland et al), historical income statements and balance sheets are set up in order to calculate the historical values of a number of financial ratios that are later used as forecast assumptions, e.g. revenue growth, operating expenditures as a percentage of revenues, etc. An example from 1986 and 1987 can be seen in Table 1, below (note that it is necessary to present more than one year since some of the financial ratios involve figures from previous years). The whole model is in Appendix 1. For ease of understanding some simplifications from the Copeland et al version have been made, which do not affect the issues we are concerned with here.[8] The valuation is as of Jan. 1, 1993.

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[6] See Copeland et al, p. 207.
[7] The Gordon model is described in Brealey & Myers (1991), p. 34.

From the figures in Table 1 financial ratios are easily calculated, some examples are given in Table 2. Most ratios are self-explanatory, but some are accompanied by comments. Items and ratios refer to the current year unless they have a subscript that indicates otherwise (thus *gross PPE* means gross property, plant and equipment for the year in question, whereas $GPPE_{t-1}$ refers to the same item the preceding year).

Income statement	1986	1987
Revenues	197.6	222.3
- Operating expenses	-175.4	-206.9
- Depreciation expense	-12.8	-9.3
	------	------
Operating income	9.4	6.1
Interest income	0.0	0.0
- Interest expense	-0.1	-0.4
	------	------
Earnings before taxes	9.3	5.7
- Taxes	-5.3	-2.4
	------	------
Net profit	4.0	3.3
	======	======

Statement of retained earnings	1986	1987
Beginning retained earnings	60.6	62.5
Net profit	4.0	3.3
- Common dividends	-2.1	-2.1
	------	------
Ending retained earnings	62.5	63.7
	======	======

	1986	1987
Invested capital	70.7	88.9
	======	======

Balance sheet	1986	1987
Operating cash	4.0	4.4
Excess marketable securities	10.9	3.0
Trade receivables	17.9	24.4
Other receivables	1.5	2.0
Inventories	1.9	2.1
Prepaid expenses	4.3	5.1
	------	------
Current assets	40.5	41.0
Gross property, plant and equipment	100.0	117.7
- Accumulated depreciation	-37.7	-42.3
	------	------
Net property, plant and equipment	62.3	75.4
	------	------
Total assets	102.8	116.4
	======	======
Short-term debt	0.3	0.8
Accounts payable	7.3	11.0
Other current liabilities	13.9	13.5
	------	------
Total current liabilities	21.5	25.3
Long-term debt	5.5	11.5
Deferred income taxes	8.7	11.0
Common stock	4.6	4.9
Retained earnings	62.5	63.7
	------	------
Total common equity	67.1	68.6
	------	------
Total liabilities and equity	102.8	116.4
	======	======

Table 1 - The McKay Company in 1986 and 1987

[8] McKay has no capitalised leases or goodwill. The capital structure consists only of equity and debt.

	1986	1987	
Operations			
Revenue growth		12.5%	
Operating exp. / rev.	88.8%	93.1%	
Working capital / revenues			
Operating cash / rev.	2.0%	2.0%	
Trade receivables / rev.	9.1%	11.0%	
Other receivables / rev.	0.8%	0.9%	
Inventories / rev.	1.0%	0.9%	
Prepaid expenses / rev.	2.2%	2.3%	
Accounts payable / rev.	3.7%	4.9%	
Other current liab. / rev.	7.0%	6.1%	
Gross Property, Plant and Equipment (Gross PPE)			Note: Specification A (see section 1.2)
Gross PPE / revenues	50.6%	52.9%	
Depr. / gross PPE_{t-1}		9.3%	Depreciation as percentage of *preceding year's* gross PPE
Ret. / gross PPE_{t-1}		4.7%	Retirements as percentage of *preceding year's* gross PPE
Taxes			
Tax rate	49.0%	49.0%	
Increase in deferred taxes / gross PPE		2.0%	

Table 2 - Some financial ratios for the McKay Company in 1986 and 1987

The historical ratios should be calculated for several years before they are used as a base for the estimation of future ratios. The expected future values of the ratios then determine the (forecasted) future performance of the company and thus the total company value. See the following tables for an example of how this works, the example is for 1994. In this case, the forecast period starts in 1993, so 1994 is the second prediction year.[9] Note the large number of items in the income statement and balance sheet that are *ratio driven*, i.e. that are decided by one of the financial ratios described in Table 2 and Table 3.

[9] The entire explicit forecast period can be found in the appendix.

Forecast assumptions	1994	
Operations		
Real growth	12.0%	Forecast
Inflation	3.0%	Forecast
Revenue growth	15.4%	[1 + Real growth] × [1 + Inflation] - 1
Operating exp. / revenues	91.0%	Forecasted ratio
Working capital / revenues		
Operating cash / rev.	2.0%	Forecasted ratio
Trade receivables / rev.	11.9%	Forecasted ratio
Other receivables / rev.	1.4%	Forecasted ratio
Inventories / rev.	2.5%	Forecasted ratio
Prepaid expenses / rev.	0.9%	Forecasted ratio
Accounts payable / rev.	3.9%	Forecasted ratio
Other curr. liab. / rev.	6.1%	Forecasted ratio
Gross Property, Plant and Equipment (Gross PPE)		Note: Specification A
Gross PPE / revenues	54.6%	Forecasted ratio
Depr. / gross PPE_{t-1}	9.7%	Forecasted ratio
Ret. / gross PPE_{t-1}	4.1%	Forecasted ratio
Taxes		
Tax rate	39.0%	Forecast (or perhaps known)
Increase in deferred taxes / gross PPE	0.8%	Forecasted ratio
Other		
Borrowing rate	9.0%	Forecast
Current year short term debt / preceding year's long term debt	20.0%	Forecasted ratio

Table 3 - Forecast assumptions for the McKay Company in 1994

Income statement	1994	
Revenues	690.6	Revenues 1993 + revenue growth
- Operating expenses	-628.4	Ratio driven: 91% of revenues.
- Depreciation expense	-31.9	Ratio driven: 9.7% of gross PPE in 1993

Operating income	30.2	
Interest income	0.0	Direct forecast
- Interest expense	-12.2	Borrowing rate × Debt (at beginning of year, i.e. $Debt_{t-1}$)

Earnings before taxes	18.0	
- Taxes	-7.0	Tax rate × Earnings before taxes

Net profit	11.0	
	======	

Statement of retained earnings	1994	
Beginning retained earnings	77.5	Preceding year's "Ending retained earnings"
Net profit	11.0	From income statement
- Common dividends	0.0	Direct forecast

Ending retained earnings	88.5	
	======	

Table 4 - Forecasted income statement and statement of retained earnings for the McKay Company in 1994

Balance sheet	1994	
Operating cash	13.8	Ratio driven: 2.0% of revenues
Excess marketable securities	0.0	Direct forecast
Trade receivables	82.2	Ratio driven: 11.9% of revenues
Other receivables	9.7	Ratio driven: 1.4% of revenues
Inventories	17.3	Ratio driven: 2.5% of revenues
Prepaid expenses	6.2	Ratio driven: 0.9% of revenues

Current assets	129.1	
Gross property, plant and equipm. (Gross PPE)	377.1	Ratio driven: 54.6% of revenues
- Accumulated depr.	-140.1	Ratio driven: Accumulated depreciation in 1993 + + This year's depr. expense - This year's retirements = = Acc. depr. 1993 + Depr. exp. 1994 - 4.1 % of gross PPE in 1993

Net property, plant and equipm. (Net PPE)	237.0	

Total assets	366.2	
	======	
Short-term debt	23.0	Ratio driven: 20% of long-term debt in 1993
Accounts payable	26.9	Ratio driven: 3.9% of revenues
Other current liabilities	42.1	Ratio driven: 6.1% of revenues
	------	
Total current liabilities	92.1	
Long-term debt	136.0	Residual item: Total assets - Total current liabilities - - Deferred income taxes - Total common equity
Deferred income taxes	26.0	Ratio driven: Deferred taxes in 1993 + + 0.8% of this year's gross PPE
Common stock	23.6	Same as preceding year (no changes foreseen)
Retained earnings	88.5	Current year's "Ending retained earnings"

Total common equity	112.1	

Total liabilities and equity	366.2	
	======	

Table 5 - Forecasted balance sheet for the McKay Company in 1994

The free cash flow for 1994 can be derived from the forecasted income statement and balance sheet:

Free cash flow	1994	
Revenues	690.6	From income statement
- Operating expenses	-628.4	From income statement
- Depreciation expense	-31.9	From income statement

Adjusted EBIT	30.2	(EBIT = Earnings before interest and taxes)
- Taxes on EBIT	-11.8	Tax rate × Adjusted EBIT
Change in deferred taxes	3.0	Current year's deferred taxes - Preceding year's deferred taxes
	------	
NOPLAT	21.4	(NOPLAT = Net operating profit less adjusted taxes)
Depreciation expense	31.9	From income statement

Gross cash flow	53.4	
Change in working capital	8.0	(Between current and preceding year) [10]
Capital expenditures	61.2	Current year's net PPE - Preceding year's net PPE + + Current year's depreciation expense
	------	
Gross investment	69.2	

Free cash flow	-15.9	Gross cash flow - Gross investment
	======	

Table 6 - Forecasted free cash flow for the McKay Company in 1994

The free cash flow should correspond to the financial cash flow:

Financial cash flow	1994	
Increase(+) / Decrease(-) in excess marketable securities	0.0	Between current and preceding year
After-tax interest income (-)	0.0	[1–Tax rate] × Interest income (from income statem.)
Increase(-) / Decrease(+) in debt	-23.3	Between current and preceding year
After-tax interest expense (+)	7.5	[1–Tax rate] × Interest expense (from income statem.)
Common dividends (+)	0.0	From statement of retained earnings
Incr.(-) / Decr.(+) in common stock	0	Between current and preceding year
	------	
Financial cash flow	-15.9	
	======	

Table 7 - Forecasted financial cash flow for the McKay Company in 1994

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[10] *Increase* in operating cash, trade receivables, other receivables, inventories, and prepaid expenses; *decrease* in accounts payable and other current liabilities.

Since the model is completely specified by the parameters (the ratios) shown in Table 3, it is of vital importance to get these right. This problem consists of two separate issues. The obvious one is, of course, to have estimation routines that yield reasonable parameter values. Estimation procedures will be discussed in detail in Chapter 4. Notwithstanding the importance of estimation (obviously, the choice of profit margin has an enormous impact) we will also give considerable attention to the other aspect, namely the analytical and empirical feasibility of the ratios as such. For forecasting purposes, it is desirable to have ratios that are fairly stable over time (or at least predictable), since the whole idea of using ratios as driving parameters builds on the notion that there exist a number of relationships in a company that remain predictable over time and business cycles.

1.2 The specification of PPE

This section introduces different specifications of the *property, plant and equipment* related items. These closely related items are the following: *net* and *gross property, plant and equipment (net* and *gross PPE), accumulated depreciation, capital expenditures (CapX), depreciation expense (DepX)* and *retirements (Ret).*

The specifications under consideration in this report are all presented by Copeland et al.: Specification A in the first edition of their book, while the other two (Spec. B and C) are proposed in the second edition. The McKay example earlier in this chapter utilises Specification A.

As mentioned in the previous section, the McKinsey model is driven by ratios. The main difference between the specifications is which item is being driven by a ratio to *revenues.* The three different revenue-related ratios are referred to as the *main driving ratios.* In Specification A the main driving ratio is *gross PPE / revenues*, in Specification B the ratio is *CapX / revenues* and in Specification C it is *net PPE / revenues.* Both *depreciation expense* and *retirements* are on the other hand determined in the same way in all three specifications. The specifications will be presented in full after some matters of notation:

The PPE-items are denoted in the following way:

A_t = Accumulated depreciation at the end of year t

$CapX_t$ = Capital expenditures in year t

$DepX_t$ = Depreciation expense in year t

G_t = Gross PPE at the end of year t

N_t = Net PPE at the end of year t

Ret_t = Retirements in year t

Further, revenues in year t will be denoted R_t.

Now it remains to define the input ratios. The ratios, common to all three specifications, are defined as:

$$d_t \equiv \left[\frac{DepX_t}{G_{t-1}}\right]_t = \text{Depreciation expense as a percentage of preceding year's gross PPE}$$

$$r_t \equiv \left[\frac{Ret_t}{G_{t-1}}\right]_t = \text{Retirements as a percentage of preceding year's gross PPE}$$

The main driving ratios are defined as:

$$b_t \equiv \left[\frac{G_t}{R_t}\right]_t = \text{Gross PPE as a percentage of revenues (Spec. A)}$$

$$e_t \equiv \left[\frac{CapX_t}{R_t}\right]_t = \text{Capital expenditures as a percentage of revenues (Spec. B)}$$

$$n_t \equiv \left[\frac{N_t}{R_t}\right]_t = \text{Net PPE as a percentage of revenues (Spec. C)}$$

The entire specifications of the PPE-items can now be written in the following way:

Specification A:

Items directly determined by ratios :

$$G_t = b_t \cdot R_t$$

$$DepX_t = d_t \cdot G_{t-1}$$

$$Ret_t = r_t \cdot G_{t-1}$$

Items derived indirectly :

$$A_t = A_{t-1} + DepX_t - Ret_t$$

$$N_t = G_t - A_t$$

$$CapX_t = N_t - N_{t-1} + DepX_t = G_t - G_{t-1} + Ret_t$$

Specification B:

Items directly determined by ratios :

$$CapX_t = e_t \cdot R_t$$

$$DepX_t = d_t \cdot G_{t-1}$$

$$Ret_t = r_t \cdot G_{t-1}$$

Items derived indirectly :

$$G_t = G_{t-1} + CapX_t - Ret_t$$

$$A_t = A_{t-1} + DepX_t - Ret_t$$

$$N_t = G_t - A_t$$

Specification C:

Items directly determined by ratios :

$$N_t = n_t \cdot R_t$$

$$DepX_t = d_t \cdot G_{t-1}$$

$$Ret_t = r_t \cdot G_{t-1}$$

Items derived indirectly :

$$CapX_t = N_t - N_{t-1} + DepX_t$$

$$A_t = A_{t-1} + DepX_t - Ret_t$$

$$G_t = N_t + A_t$$

In Chapter 4, the different specifications are analysed, both empirically and theoretically. It turns out that Specification C is very similar to Specification A and does not add any improvements. Accordingly, the horizon value study in Chapter 2 has been concentrated on Specifications A and B.

Appendix 1 - The McKay Company

Table A1:1 - Historical income statement

Income statement	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Revenues	197.6	222.3	272.3	299.5	350.0	418.9	505.4
Operating expenses	-175.4	-206.9	-249.6	-274.7	-327.5	-383.6	-467.4
Depreciation expense	-12.8	-9.3	-11.2	-13.0	-15.0	-17.7	-26.4
	------	------	------	------	------	------	------
Operating income	9.4	6.1	11.5	11.8	7.5	17.6	11.6
Interest income	0.0	0.0	0.0	0.0	0.9	0.7	0.6
Interest expense	-0.1	-0.4	-0.8	-1.0	-3.4	-4.1	-10.1
	------	------	------	------	------	------	------
Earnings before taxes	9.3	5.7	10.7	10.8	5.0	14.2	2.1
Income taxes	-5.3	-2.4	-5.8	-5.2	-1.0	-7.1	-0.7
	------	------	------	------	------	------	------
Net profit	4.0	3.3	4.9	5.6	4.0	7.1	1.4
	======	======	======	======	======	======	======

Statement of retained earnings	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Beginning retained earnings	60.6	62.5	63.7	65.8	68.6	69.8	74.0
Net profit	4.0	3.3	4.9	5.6	4.0	7.1	1.4
Common dividends	-2.1	-2.1	-2.8	-2.8	-2.8	-2.9	-2.9
	------	------	------	------	------	------	------
Ending retained earnings	62.5	63.7	65.8	68.6	69.8	74.0	72.5

Table A1:2 - Historical balance sheets

	1986	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Operating cash	4.0	4.4	5.4	6.0	6.0	8.4	10.1
Excess marketable securities	10.9	3.0	20.5	10.3	0.0	5.8	3.2
Trade receivables	17.9	24.4	33.0	33.3	43.9	49.8	57.7
Other receivables	1.5	2.0	2.7	2.7	6.2	4.9	5.7
Inventories	1.9	2.1	2.8	2.5	9.0	10.9	11.9
Prepaid expenses	4.3	5.1	5.3	6.0	2.4	4.4	5.0
Current assets	40.5	41.0	69.7	60.8	67.5	84.2	93.6
Gross property, plant and equipment	100.0	117.7	128.2	155.6	204.7	272.5	297.6
Accumulated depreciation	-37.7	-42.3	-48.7	-56.5	-71.9	-86.9	-103.4
Net property, plant and equipment	62.3	75.4	79.5	99.1	132.8	185.6	194.2
Total assets	102.8	116.4	149.2	159.9	200.3	269.8	287.8
Short-term debt	0.3	0.8	0.9	1.5	11.5	12.5	20.7
Accounts payable	7.3	11.0	11.9	10.5	14.2	16.2	18.9
Other current liabilities	13.9	13.5	18.2	18.5	21.4	27.8	28.8
Total current liabilities	21.5	25.3	31.0	30.5	47.1	56.5	68.4
Long-term debt	5.5	11.5	16.2	21.7	40.2	90.6	103.0
Deferred income taxes	8.7	11.0	12.6	15.5	19.6	25.1	20.3
Common stock	4.6	4.9	23.6	23.6	23.6	23.6	23.6
Retained earnings	62.5	63.7	65.8	68.6	69.8	74.0	72.5
Total common equity	67.1	68.6	89.4	92.2	93.4	97.6	96.1
Total liabilities and equity	102.8	116.4	149.2	159.9	200.3	269.8	287.8
Invested capital	70.7	88.9	98.6	120.6	164.7	220.0	236.9
Debt/invested cap	8.2%	13.8%	17.3%	19.2%	31.4%	46.9%	52.2%
Debt+deferred taxes/ invested cap	20.5%	26.2%	30.1%	32.1%	43.3%	58.3%	60.8%
NOPLAT/invested cap		6.5%	7.0%	7.5%	5.7%	6.5%	0.8%

Table A1:3 - Historical free cash flow

Free cash flow	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Revenues	222.3	272.3	299.5	350.0	418.9	505.4
Operating expenses	-206.9	-249.6	-274.7	-327.5	-383.6	-467.4
Depreciation expense	-9.3	-11.2	-13.0	-15.0	-17.7	-26.4
	------	------	------	------	------	------
EBIT	6.1	11.5	11.8	7.5	17.6	11.6
Taxes on EBIT	-2.6	-6.2	-5.7	-2.2	-8.8	-5.0
Change in deferred taxes	2.3	1.6	2.9	4.1	5.5	-4.8
	------	------	------	------	------	------
NOPLAT	5.8	6.9	9.0	9.4	14.3	1.8
Depreciation expense	9.3	11.2	13.0	15.0	17.7	26.4
	------	------	------	------	------	------
Gross cash flow	15.1	18.1	22.0	24.4	32.0	28.2
Change in working capital	5.1	5.6	2.4	10.4	2.5	8.3
Capital expenditures	22.4	15.3	32.6	48.7	70.5	35.0
	------	------	------	------	------	------
Gross investment	27.5	20.9	35.0	59.1	73.0	43.3
	------	------	------	------	------	------
Free cash flow	-12.4	-2.8	-13.0	-34.7	-41.0	-15.1
	======	======	======	======	======	======

Financial cash flow	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Incr.(+)/Decr..(-) excess mkt. securities	-7.9	17.5	-10.2	-10.3	5.8	-2.6
After-tax interest income (-)	0.0	0.0	0.0	-0.5	-0.4	-0.3
Incr.(-)/Decr..(+) in short- and long-term debt	-6.5	-4.8	-6.1	-28.5	-51.4	-20.6
After-tax interest expense (+)	0.2	0.4	0.5	1.7	2.1	5.6
Common dividends (+)	2.1	2.8	2.8	2.8	2.9	2.9
Incr.(-)/Decr..(+) in common stock	-0.3	-18.7	0.0	0.0	0.0	0.0
	------	------	------	------	------	------
Financial cash flow	-12.4	-2.8	-13.0	-34.7	-41.0	-15.1

Table A1:4 - Historical ratios for forecast assumptions

	1986	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Operations							
Real growth							
Inflation							
Revenue growth		12.5%	22.5%	10.0%	16.9%	19.7%	20.6%
Operating exp./							
revenues	88.8%	93.1%	91.7%	91.7%	93.6%	91.6%	92.5%
Working cap/revenues							
Operating cash	2.0%	2.0%	2.0%	2.0%	1.7%	2.0%	2.0%
Trade receiv.	9.1%	11.0%	12.1%	11.1%	12.5%	11.9%	11.4%
Other receiv.	0.8%	0.9%	1.0%	0.9%	1.8%	1.2%	1.1%
Inventories	1.0%	0.9%	1.0%	0.8%	2.6%	2.6%	2.4%
Prepaid expenses	2.2%	2.3%	1.9%	2.0%	0.7%	1.1%	1.0%
Accounts payable	3.7%	4.9%	4.4%	3.5%	4.1%	3.9%	3.7%
Other curr. liab's	7.0%	6.1%	6.7%	6.2%	6.1%	6.6%	5.7%
Property.Plant and							
Equipment (PPE)							
Gross PPE/rev's	50.6%	52.9%	47.1%	52.0%	58.5%	65.1%	58.9%
Depr/gross PPE *		9.3%	9.5%	10.1%	9.6%	8.6%	9.7%
Retirements/							
gross PPE *		4.7%	4.1%	4.1%	-0.3%	1.3%	3.6%
Taxes							
Tax rate	49.0%	49.0%	49.0%	49.0%	49.0%	49.0%	45.0%
Incr. in deferred							
tax/gross PPE		2.0%	1.2%	1.9%	2.0%	2.0%	-1.6%

* last year's gross PPE

Table A1:5 - Forecasted income statement

Income statement	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Revenues	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1241.7	1279.0	1317.3
Operating expenses	-550.8	-628.4	-699.1	-770.5	-841.2	-909.7	-974.5	-1033.9	-1086.2	-1130.0	-1163.8	-1198.8
Depreciation expense	-28.9	-31.9	-36.6	-40.4	-44.2	-47.9	-51.4	-54.6	-57.5	-60.0	-61.9	-63.3
	------	------	------	------	------	------	-------	-------	-------	-------	-------	-------
Operating income	19.0	30.2	32.6	35.8	39.0	42.1	45.0	47.6	49.9	51.8	53.2	55.3
Interest income *	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest expense	-11.1	-12.2	-14.3	-15.5	-16.5	-17.3	-17.9	-18.2	-18.1	-17.6	-16.7	-17.4
	------	------	------	------	------	------	-------	-------	-------	-------	-------	-------
Earnings before taxes	8.2	18.0	18.2	20.4	22.6	24.8	27.1	29.5	31.8	34.1	36.5	37.9
Income taxes	-3.2	-7.0	-7.1	-7.9	-8.8	-9.7	-10.6	-11.5	-12.4	-13.3	-14.2	-14.8
	------	------	------	------	------	------	-------	-------	-------	-------	-------	-------
Net profit	5.0	11.0	11.1	12.4	13.8	15.1	16.5	18.0	19.4	20.8	22.2	23.1
	======	======	======	======	======	======	======	======	======	======	======	======

Statement of retained earnings	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Beginning retained earnings	72.5	77.5	88.5	99.6	112.0	125.8	140.9	157.5	175.4	194.8	215.7	215.9
Net profit	5.0	11.0	11.1	12.4	13.8	15.1	16.5	18.0	19.4	20.8	22.2	23.1
Common dividends **	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-22.0	-16.5
	------	------	------	------	------	------	-------	-------	-------	-------	-------	-------
Ending retained earnings	77.5	88.5	99.6	112.0	125.8	140.9	157.5	175.4	194.8	215.7	215.9	222.5

* direct forecast

** direct forecast until 2002. residual thereafter

Table A1:6 - Forecasted balance sheet

	1993	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Operating cash	12.0	13.8	15.4	16.9	18.5	20.0	21.4	22.7	23.9	24.8	25.6	26.3
Excess marketable securities *	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Trade receivables	71.2	82.2	91.4	100.8	110.0	119.0	127.4	135.2	142.0	147.8	152.2	156.8
Other receivables	8.4	9.7	10.8	11.9	12.9	14.0	15.0	15.9	16.7	17.4	17.9	18.4
Inventories	15.0	17.3	19.2	21.2	23.1	25.0	26.8	28.4	29.8	31.0	32.0	32.9
Prepaid expenses	5.4	6.2	6.9	7.6	8.3	9.0	9.6	10.2	10.7	11.2	11.5	11.9
Current assets	111.9	129.1	143.7	158.3	172.9	186.9	200.3	212.5	223.2	232.2	239.2	246.3
Gross property. plant and equipm.	329.3	377.1	416.4	455.5	493.6	529.9	563.3	593.1	618.3	638.2	652.3	671.8
Accumulated depreciation	-121.6	-140.1	-159.4	-178.8	-197.9	-216.1	-233.2	-248.6	-262.0	-273.0	-281.3	-289.8
Net property. plant and equipment	207.7	237.0	256.9	276.7	295.7	313.7	330.1	344.4	356.2	365.2	370.9	382.0
Total assets	319.6	366.2	400.6	435.0	468.6	500.7	530.3	556.9	579.4	597.4	610.1	628.3
Short-term debt	20.6	23.0	27.2	28.9	30.8	32.2	33.2	33.7	33.5	32.5	30.7	32.4
Accounts payable	23.3	26.9	30.0	33.0	36.1	39.0	41.8	44.3	46.6	48.4	49.9	51.4
Other current liabilities	36.5	42.1	46.9	51.6	56.4	61.0	65.3	69.3	72.8	75.7	78.0	80.4
Total current liabilities	80.5	92.1	104.0	113.6	123.2	132.2	140.3	147.3	152.8	156.7	158.6	164.2
Long-term debt	115.2	136.0	144.5	154.0	161.2	166.2	168.4	167.3	162.5	153.6	162.2	166.2
Deferred income taxes	22.9	26.0	28.9	31.8	34.8	37.7	40.5	43.2	45.7	47.9	49.9	51.9
Common stock	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6
Retained earnings	77.5	88.5	99.6	112.0	125.8	140.9	157.5	175.4	194.8	215.7	215.9	222.5
Total common equity	101.1	112.1	123.2	135.6	149.4	164.5	181.1	199.0	218.4	239.3	239.5	246.1
Total liabilities and equity	319.6	366.2	400.6	435.0	468.6	500.7	530.3	556.9	579.4	597.4	610.1	628.3
Invested capital	259.8	297.1	323.8	350.3	376.2	400.7	423.3	443.3	460.1	473.2	482.2	496.6
Debt/invested cap **	52.3%	53.5%	53.0%	52.2%	51.0%	49.5%	47.6%	45.4%	42.6%	39.3%	40.0%	40.0%
Debt+deferred taxes/ invested cap	61.1%	62.3%	61.9%	61.3%	60.3%	58.9%	57.2%	55.1%	52.5%	49.4%	50.3%	50.4%
NOPLAT/invested cap	5.5%	7.2%	7.0%	7.1%	7.1%	7.1%	7.1%	7.2%	7.2%	7.1%	7.1%	7.2%

* direct forecast

** assumption from 2003

Table A1:7 - Forecasted free cash flow

Free cash flow	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Revenues	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1241.7	1279.0	1317.3
Operating expenses	-550.8	-628.4	-699.1	-770.5	-841.2	-909.7	-974.5	-1033.9	-1086.2	-1130.0	-1163.8	-1198.8
Depreciation expense	-28.9	-31.9	-36.6	-40.4	-44.2	-47.9	-51.4	-54.6	-57.5	-60.0	-61.9	-63.3
EBIT	19.0	30.2	32.6	35.8	39.0	42.1	45.0	47.6	49.9	51.8	53.2	55.3
Taxes on EBIT	-7.4	-11.8	-12.7	-14.0	-15.2	-16.4	-17.5	-18.6	-19.5	-20.2	-20.7	-21.6
Change in deferred taxes	2.6	3.0	2.9	3.0	3.0	2.9	2.8	2.7	2.5	2.2	2.0	2.0
NOPLAT	14.2	21.4	22.8	24.8	26.8	28.6	30.3	31.7	32.9	33.8	34.4	35.7
Depreciation expense	28.9	31.9	36.6	40.4	44.2	47.9	51.4	54.6	57.5	60.0	61.9	63.3
Gross cash flow	43.1	53.4	59.4	65.2	70.9	76.5	81.7	86.4	90.4	93.8	96.3	99.0
Change in working capital	9.4	8.0	6.8	6.8	6.8	6.6	6.2	5.7	5.0	4.2	3.2	3.3
Capital expenditures	42.4	61.2	56.5	60.1	63.3	65.8	67.8	69.0	69.4	68.9	67.6	74.4
Gross investment	51.8	69.2	63.3	66.9	70.0	72.4	74.0	74.6	74.4	73.1	70.9	77.7
Free cash flow	-8.6	-15.9	-3.9	-1.7	0.9	4.1	7.7	11.7	16.1	20.7	25.4	21.3

Financial cash flow	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Incr.(+)/Decr..(-) excess mkt. sec'ties	-3.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
After-tax interest income (-)	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Incr.(-)/Decr..(+) in short- and long-term debt	-12.1	-23.3	-12.6	-11.2	-9.1	-6.5	-3.2	0.6	5.0	9.9	-6.8	-5.8
After-tax interest expense (+)	6.8	7.5	8.7	9.4	10.0	10.5	10.9	11.1	11.0	10.8	10.2	10.6
Common dividends (+)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	22.0	16.5
Incr.(-)/Decr..(+) in common stock	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Financial cash flow	-8.6	-15.9	-3.9	-1.7	0.9	4.1	7.7	11.7	16.1	20.7	25.4	21.3

Table A1:8 - Forecast assumptions

	1993	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Operations												
Real growth	15.0%	12.0%	8.0%	7.0%	6.0%	5.0%	4.0%	3.0%	2.0%	1.0%	0.0%	0.0%
Inflation	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
Revenue growth	18.5%	15.4%	11.2%	10.2%	9.2%	8.2%	7.1%	6.1%	5.1%	4.0%	3.0%	3.0%
Operating exp./ revenues	92.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%
Working cap/revenues												
Operating cash	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%
Trade receiv.	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%
Other receiv.	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%
Inventories	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%
Prepaid expenses	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%
Accounts payable	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%
Other curr. liab's	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%
Property.Plant and Equipment (PPE)												
Gross PPE/rev's	55.0%	54.6%	54.2%	53.8%	53.4%	53.0%	52.6%	52.2%	51.8%	51.4%	51.0%	51.0%
Depr/gross PPE *	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%
Retirements/ gross PPE *	3.6%	4.1%	4.6%	5.0%	5.5%	6.0%	6.5%	7.0%	7.4%	7.9%	8.4%	8.4%
Taxes												
Tax rate	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%
Incr. in deferred tax/gross PPE	0.8%	0.8%	0.7%	0.7%	0.6%	0.6%	0.5%	0.5%	0.4%	0.4%	0.3%	0.3%
Other												
Borrowing rate	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%
This year short-term/ last year long-term debt	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%

* last year's gross PPE

2. The Value Beyond the Horizon – *Development in Steady State*

2.0 Introduction

When performing a valuation, one can either make the explicit forecast period infinitely long or make explicit forecasts for a limited number of years only and account for the time after that with a horizon value.

An *infinitely* long explicit forecast period is only a theoretical notion. In practice, one tries to forecast free cash flows for perhaps 100 or 150 years ahead - until the *present value* of the cash flow from a certain year no longer contributes anything (almost) to the company value. This is obviously quite difficult to do: It is not an easy task to estimate how different input parameters will develop in, say, the 2050s.

Instead, the second approach is often used,[11] and the future is divided into two periods: the explicit forecast period and the infinite future after that, accounted for by the horizon value. As mentioned earlier, the horizon value can take up a very large part of the total company value, so it is obviously very important that this part of the valuation is made carefully and with correct calculation techniques. If the horizon value is calculated in an erroneous or sloppy way, it does not matter how precise and sophisticated the estimation procedures for the explicit forecast period are. The issue becomes even more problematic when taking into consideration that practitioners often ask for very short explicit forecast periods - 2 to 5 years - and an incorrect horizon value of course makes the whole valuation totally unreliable.

The use of a horizon value requires the assumption that the company has settled down to a steady state by the beginning of the period after the explicit forecast period. The meaning of steady state might seem somewhat unclear, but it will be explained in more detail below. Generally, the way steady state is achieved is by setting the input parameters constant from

[11] Proposed in, e.g., Copeland et al.

the first year after the explicit forecast period.[12] As we will see later, however, this may not be sufficient.

In order to assess the reasonableness of a steady-state assumption one must know what it implies. Although this type of assumption is often recommended in textbooks, the authors seldom give any deeper answers to this valid question. In this chapter we therefore analyse the concept of steady state thoroughly.

2.0.1 Definitions and problem identification

It is useful, for theoretical and technical reasons, to distinguish between different types of steady states:

Definitions:

A **parametric steady state** (PSS) means that the parameters (of the model) that describe the company's development are constant over the coming years.

This is the most intuitive steady state definition. It means that one assumes, e.g., a constant revenue growth, a constant profit margin, etc. PSS is also the weakest form of steady state. No restrictions are placed on the parameters other than that they remain constant.

A **FCF steady state** (FSS) means that the company's predicted free cash flow grows at a constant rate.

In other words, the free cash flow in any year t+1 will be described by $FCF_{t+1} = (1+g) \cdot FCF_t$ *where g is the constant growth rate. This is supposed to hold for all future years.*

A **NP steady state** (NSS) means that the company's predicted net profit grows at a constant rate.

$NP_{t+1} = (1+g) \cdot NP_t$

A **DIV steady state** (DSS) means that the company's predicted dividends grow at a constant rate.

$DIV_{t+1} = (1+g) \cdot DIV_t$

---

[12] This year will in this essay henceforth be called year 0. Consequently, the last year in the explicit forecast period will be denoted year (-1).

A **Textbook steady state** (TSS) means that the following conditions are fulfilled (according to Copeland et al): "The company earns constant margins, maintains a constant capital turnover, and, therefore, earns a constant return on existing capital. The company grows at a constant rate and invests the same proportion of its gross cash flow in its business each year. The company earns a constant return on all new investments."[13]

Copeland et al claim that *any* continuing-value approach relies on the key assumptions stated above under "Textbook steady state".[14] Less is said about how one practically goes about to ensure TSS. This will therefore be one of the main tasks in this chapter.

The Copeland et al conditions for TSS may need some clarifications to become operational:

Constant margins - this will hold if the company is in PSS, since operating expenditures will then be a constant percentage of revenues.

Constant capital turnover - this will hold if invested capital grows at the same rate as revenues.[15] By PSS, revenues grow at a constant rate (the revenue growth rate is one of the parameters); consequently invested capital will have to grow at the same rate. In all cases in our settings below, invested capital will equal the balance sheet total,[16] and the condition can be stated that the balance sheet total must grow at the same rate as revenues.

Constant return on existing capital - this will hold as a consequence of constant margins and a constant capital turnover, since the return on existing invested capital can be calculated as: *operating margin* × *capital turnover* × [1 - *tax rate*].[17]

The company grows at a constant rate - this can be interpreted in different ways. The first possibility is a constant revenue growth. That would follow immediately from PSS. The second possibility is that the balance sheet total grows at a constant rate. The third and final possibility is that the free cash flow generated by the company grows at a constant rate, i.e. FSS.

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13 Copeland et al p. 290
14 Copeland et al p. 290
15 Capital turnover is the ratio between *revenues* and *invested capital*. See Copeland et al p. 167.
16 In our setting, balance sheet total means the sum of *net* working capital and *net PPE*.

The company invests the same proportion of its gross cash flow in its business each year - this will hold if gross cash flow grows at the same rate as gross investments *or*, equivalently, if gross cash flow grows at the same rate as FCF.[18] If FSS holds the free cash flow will be proportional to preceding year's revenues (see FSS definition above and expression (7) below). In the setting of Specification A, to be discussed later, the gross cash flow also turns out to be be proportional to preceding year's revenues, i.e. *gross cash flow* = *constant* × *preceding year's revenues*. Thus a sufficient condition for a constant relation between investments and cash flows is that FSS is established. This will be shown to hold also for Specification B, since the condition for FSS in Specification B reduces it to Specification A.

Constant return on all new investments - this will hold if the balance sheet total grows at the same rate as revenues.

All of the TSS conditions may seem economically intuitive features of a steady state. The rather long list of conditions can be shortened considerably, however, since many of the requirements are only different ways of saying the same thing, and the above conditions for TSS can hence be summarised:

1. The company is in PSS.
2. It exhibits FSS.
3. The balance sheet total grows at the same rate as revenues.

Are these three conditions reasonable then?

1. PSS

For obvious practical reasons, the concept of parametric steady state will hereafter be treated as a necessary condition: the reason for using a horizon value approach in the first place is to *simplify* the valuation (this is indeed also what Copeland et al do). Without constant input

---

[17] Copeland et al. p. 167
[18] Follows from the fact that FCF equals gross cash flow minus gross investments. See Copeland et al p. 169.

parameters one would be back in the first approach with a very long explicit forecast, and the whole point of introducing a horizon is to avoid that.

2. *FSS*

To calculate a horizon value, a continuing value formula similar to the familiar Gordon-formula[19] is often proposed.[20] However, a first prerequisite for the use of the simple Gordon-formula is that the company is in steady state with respect to the valuation measure. When using the free cash flow approach, the free cash flow must grow at a constant rate in all future years, i.e. FSS. Otherwise one cannot calculate the sum of discounted future free cash flows by using a simple geometric series formula.

3. *Balance sheet total growing at the same rate as revenues*

This third condition may seem less self-evident than the former two, but it is not unrealistic. The growth rate of the balance sheet total and the sales are empirically closely related. This is commented upon by Johansson (1995).[21]

Some other questions are also of importance:

When is steady state established? The parameters are set to be constant from year 0, so PSS will by definition hold from the beginning. But the other conditions? For instance FSS - is it established already in year 0 (so one can use the free cash flow from year 0 in the continuing value calculation) or later?

What if steady state (other than the assumed PSS) is *not* implied, not in year 0 nor in any future year? One must then take the actual future behaviour of the valuation measure (FCF, DIV or NP) into consideration when calculating the horizon value. Will there be a way to do this with a formula or must one resort to the long explicit forecast, and set up financial statements for 100 years or more in the spreadsheet program?

---

[19] See, e.g., Brealey & Myers p. 34.
[20] See, e.g., Copeland et al pp. 274-277.
[21] Johansson (1995), p. 21

It is also of interest to find out how one can ensure that an intuitive development of the company will be implied by the steady state assumption: what is required for the model to give intuitive results? One example is that a more efficient use of property, plant and equipment should increase the company value.[22]

So far, the following problems have been identified:

- Under which conditions on the input parameters does *parametric* steady state imply the other types of steady state (FSS, NSS, DSS, TSS)?
- Is it possible to perform a horizon value calculation if steady state, with respect to the valuation measure, is *not* implied?
- Under which conditions on the input parameters does one obtain intuitive results?
- If steady state is implied, *when* is it established?

2.0.2 Steady state models - framework and assumptions

It is assumed that the company has reached a point in time where the ratios driving the company's balance sheets and income statements have settled to be constant over the years, i.e. the company exhibits a parametric steady state. The company's balance sheet is defined to consist of the following items:

Assets	**Debt and Equity**
(A1) (Net) Working Capital	(D1) Debt
(A2) Net Property, Plant & Equipment	(D2) Deferred Taxes
= Gross PPE - Accumulated Depr.	(D3) Ending equity

[22] In order to answer questions like this, the approach in this report is superior to the "infinitely long" forecast period approach. The latter numerical approach will of course give indications when something counter-intuitive is going on, but it does not allow any explicit analysis on the parameter level regarding what intuitive conditions are being violated.

Two different Specifications of the PPE-related items are considered in this chapter. We define the following parameters, denoting the forecasted values of the different items, which by the PSS assumption are constant over all future years:[23]

a net working capital in % of revenues (sales)

b gross PPE in % of revenues (sales) [Specification A only]

c change in deferred taxes in % of gross PPE

d depreciation in % of preceding year's gross PPE

e capital expenditures in % of revenues (sales) [Specification B only]

g nominal growth rate,[24] revenues (sales)

i interest rate on debt

p operating expenses in % of revenues (sales)

r retirements in % of preceding year's gross PPE

τ tax rate

w debt in % of balance sheet total (book value)

All parameters except a and b are assumed to be greater than 0 but smaller than 1. The parameters a and b are assumed only to be greater than 0. The revenue growth rate, g, can never exceed the discount rate in the perpetuity period since the equity value then would become infinite, which is clearly absurd.

The use of a single parameter to determine *net* working capital differs at a first glance from the way Copeland et al deal with this. However, since we here consider a PSS period, we can without loss of generality or information sum all the different *working capital* to *revenues* ratios to a single parameter. Of course, when performing the forecast of the explicit forecast period, this type of aggregation may lead to information losses.

In order to determine the development of the debt and equity side of the balance sheet it is assumed that the clean surplus relation (CSR) holds and that net book value of equity is the

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[23] To distinguish between the parameters in the PSS period and the corresponding ones in the explicit forecast period, PSS parameters are henceforth denoted **without** any time-index.

[24] This means that the revenues of all years in the PSS period can be calculated as (1+g) times the preceding year's revenues.

residual item of the balance sheet. The clean surplus relation means that the change in net book value of equity equals earnings *minus* dividends (see, e.g., Ohlson (1995)). It should be noted that CSR also holds in the McKinsey model, but we use it for the purpose of establishing dividends as the residual item of the model.[25]

The company debt is assumed to be on market terms, i.e. that the book value of debt is equal to the market value of debt. It should also be noted that the excess marketable securities (EMS) equal zero in the forecast period, which also holds in the Eldon case in Chapter 5. Any ending excess marketable securities of the last historical year are added to the resulting equity value from the model.

Deferred taxes are "a quasi-equity account".[26] For valuation purposes we will treat them as equity. This is also the recommendation in Copeland et al (p. 163).

The first year in the parametric steady state period is denoted year 0. When the case-specific definitions are made in addition, the following state variables can be identified:

Specification A

R_t the revenues (sales) of year *t*,
A_t the accumulated depreciation at the end of year *t*,
T_t deferred taxes at the end of year *t*.

Specification B

R_t revenues (sales) of year *t*,
A_t accumulated depreciation at the end of year *t*,
G_t gross PPE at the end of year *t*,
T_t deferred taxes at the end of year *t*.

[25] This differs from Copeland et al who define the debt as the residual item. Our approach implies that dividends are the residual item in the whole system of equations. The reason is twofold: First, this is more intuitive when the company is in steady state, since the excess capital will then be directly distributed to equity owners (it could otherwise lead to negative debt if the company is profitable). Secondly, it also allows us to explicitly derive an expression for the dividend development which makes it possible to compare different valuation methods.
[26] Copeland et al, p. 162

Utilising the fact that the balance sheet over the years is a set of difference equations, analytical expressions for the state variables are calculated, and other important economic variables' development over time is in turn derived from the state variables.

2.0.3 A numerical example company

In order to visualise the results an example company will be used throughout this chapter. The model input items have been forecasted to be constants and the company is thus assumed to have entered into a parametric steady state, PSS. The parameters take on the following values:

a=5%
b=40%
c=0.3%
d=6%
e=3.428%
g=5%
i=10%
p=90%
r=4%
τ=30%
w=40%

The initial values of the state variables are the following:
R_0=500
A_0=125
G_0=200
T_0=5.4

2.0.4 Outline of the chapter

Each of the two specifications is treated in a section of its own (sections 2.1-2.2). In each section the case under consideration is analysed in order to find solutions to the problems identified above. Most of the derivations and proofs of each section have been brought together in the appendix at the end of the chapter. Still, a few illustrative proofs have been kept in the sections. Section A2.3 of the appendix gives an example of how the general modelling approach considered in this report can be extended to more complicated settings than the basic McKinsey model setting.

2.1 Specification A - Constant *gross PPE* to *revenues* ratio

In this section we consider Specification A of the PPE items:

$$
\begin{aligned}
G_t &= b \cdot R_t \\
DepX_t &= d \cdot G_{t-1} \\
Ret_t &= r \cdot G_{t-1}
\end{aligned}
$$

By the PSS assumption the parameters are constant. Thus the items in the balance sheet can be defined as follows:

$$
\begin{aligned}
&\text{A1}: aR_t \\
&\text{A2}: bR_t - A_t \\
&\qquad \text{where } A_t = \left[(d-r)bR_{t-1} + A_{t-1}\right] \\
&\text{D1}: w\left(aR_t + bR_t - A_t\right) \\
&\text{D2}: T_t = cbR_t + T_{t-1} \\
&\text{D3}: (1-w)\left(aR_t + bR_t - A_t\right) - T_t
\end{aligned}
$$

Note that, for t=0, we have $A_0 = (d-r)b'R_{(-1)} + A_{(-1)}$ where b' may not equal b.

The following entities for $t \geq 1$ can now be derived:

Free cash flow, FCF_t:

$$
\begin{aligned}
&(1-\tau)(R_t - pR_t - dbR_{t-1}) + dbR_{t-1} + T_t - T_{t-1} \\
&- (aR_t - aR_{t-1}) - (bR_t - bR_{t-1} + rbR_{t-1}) = \\
&= R_{t-1}\left[a + (1-r)b + \tau db\right] + R_t\left[(1-\tau)(1-p) - a - b\right] \\
&+ T_t - T_{t-1}
\end{aligned} \tag{1}
$$

Net profit, NP_t:

$$
(1-\tau)\left[R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1})\right] \tag{2}
$$

Dividends (residual item: last year's ending equity plus net profit minus this year's ending equity), DIV_t:

$$
\begin{aligned}
&(1-w)(aR_{t-1} + bR_{t-1} - A_{t-1}) - T_{t-1} \\
&+ (1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1})) \\
&- \left[(1-w)(aR_t + bR_t - A_t) - T_t\right]
\end{aligned} \tag{3}
$$

Three state-variables R_t, A_t and T_t. can also be identified from the balance sheet since the balance sheets over the years is a system of difference equations. The solution to this system is found to be (for $t \geq 1$):[27]

$$
R_t = (1+g)R_{t-1} = (1+g)^t R_0 \tag{4}
$$

$$
A_t = \frac{(1+g)^t - 1}{g} \cdot (d-r)bR_0 + A_0 \tag{5}
$$

$$
\begin{aligned}
T_t &= \frac{(1+g)^t - 1}{g} \cdot c(1+g)bR_0 + T_0 = \\
&= \left[\frac{(1+g)^{t+1} - 1}{g} - 1\right] cbR_0 + T_0
\end{aligned} \tag{6}
$$

---

[27] Note again that derivations are provided in Appendix 2 at the end of this chapter.

These findings will now be used to analyse the parametric steady state behaviour of free cash flow, net profit and dividends.

2.1.1 Free cash flow

Substituting expressions (4 - 6) into the equation for the free cash flow at each point in time (equation 1) yields the following for $t \geq 1$:

$$(7) \quad FCF_t = (1+g)^t R_0 \cdot (m + z_{FCF})$$

where m is the constant after - tax margin $(1-\tau)(1-p)$

and $z_{FCF} = \dfrac{b(c + \tau d - r) - g(b(1-c) + a)}{1+g}$ is a constant.

This is a very neat expression. The *free cash flow* of any year t in the parametric steady state period is given by letting the constant term $R_0(m + z_{FCF})$ grow with $(1+g)^t$. The term $(m + z_{FCF})$ shows that the free cash flow will grow with the after-tax margin of year 0's revenues plus a term involving the ratios *gross PPE / revenues (b), working capital / revenues (a), change in deferred taxes / gross PPE (c), depreciation / preceding year's gross PPE (d)* and *retirements / preceding year's gross PPE (r)* multiplied by revenues.

Proposition 2.1

A company in PSS exhibits FSS without any restrictions on the constant input ratios.

So, by just letting the ratios being constants, a FCF steady state is achieved in the free cash flow development. The proposition follows directly from equation (7).

Properties of the FCF function

Now turning to the question whether the free cash flow develops intuitively, comparative statics can be performed, the results from which make it possible to derive conditions for the intuitive behaviour of the free cash flows. These conditions can be tested numerically when performing a valuation to ensure that the assumptions do not contradict each other or basic economic intuition. The derivations for *FCF* as well as for *NP* and *DIV* in Specification A are presented in the appendix. Since the overall methodology, of deriving properties and suitable conditions, is apparent from the treatment here (i.e. for Spec. A), the corresponding part for Specification B is left out.

In a model like this, comparative statics, i.e. changing one parameter while keeping all other constant, can for some parameters be less realistic. This is especially the case for the *depreciation* and *retirements* parameters (r and d), which are closely related provided one looks at depreciation as a real economic variable and not only something used for tax accounting purposes. These two parameters are therefore not considered here.

- FCF_t is **decreasing** in a, *net working capital / revenues*, without any conditions.

The result is intuitive: the more efficient the use of capital (i.e. the lower the a), the higher the free cash flow. This interpretation is straightforward when considering decreases in items from the asset side of the balance sheet, like, e.g., operating cash. Even if the lower a comes from an increase in one of the debt items included in net working capital, however, it should be interpreted as increased efficiency. For example, larger accounts payable means, all else equal, that the company has negotiated better terms (longer time of payment) with its suppliers.

- FCF_t is **decreasing** in b, *gross PPE / revenues*, if the following inequality is fulfilled:

$$(8) \qquad \tau d - r + (1+g)c < g$$

This should intuitively hold, since a higher b-value means a less efficient use of the company's capital and should therefore yield smaller cash flows. A first glance at the condition seems to indicate that this does not have to be the case. However, when examining the terms of the left hand side for reasonable parameter values, it can be seen that the condition will hold in almost all reasonable situations. First, the difference $\tau d - r$ will be negative in most reasonable cases or at least very small.[28] Further, since the next term c (multiplied by [1+g]) will generally be small in magnitude, the left hand side will be negative or at worst relatively small. The only critical companies are then companies with very low growth. As noted, intuition dictates that the free cash flows be decreasing in b, however, and hence the condition above can be used as a restriction on the parameter c.

- FCF_t is **increasing** in c, *change in deferred taxes / gross PPE.*

The intuition is basic tax-evasion: the more that can be hidden from taxation (by increasing deferred taxes), the better the free cash flow.

- FCF_t is **decreasing** in g, the growth rate, in the *beginning* of the PSS period, but turns to be **increasing** in g after a number of years.

The result can be attributed to two effects:

1. an increase in g lowers the constant z_{FCF}, which thus lowers the constant term $R_0\left(m + z_{FCF}\right)$ (i.e. the initial value of FCF)
2. but g **is** the growth rate, so an increase will eventually raise the free cash flow when t gets large enough.

- FCF_t is **decreasing** in p, *operating expenses / revenues,* without any conditions.

This is trivially intuitive: the higher the operating profit margin (1-p) the higher the free cash flow.

- FCF_t is **decreasing** in τ, *the tax rate*, as long as the *operating profit after depreciation* is positive.

[28] The relationship between d and r will be discussed in Chapter 4.

Also this result is intuitive: the more paid out in taxes, the smaller the free cash flow for a profitable company. The explicit parameter condition for a positive operating profit after depreciation is:

$$(9) \qquad p + \frac{bd}{1+g} < 1$$

2.1.2 Net profit

The net profit in year t is given by:

$$(2) \qquad NP_t = (1-\tau)\left[R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right]$$

Rearranging and substituting expressions (4 - 6) into (2) yields:

$$(10) \qquad NP_t = (1+g)^t R_0 \left(m + z_{NP}\right) - \chi\left(\gamma R_0 - A_0\right)$$

with the following constants: $m = (1-\tau)(1-p), \quad \chi = (1-\tau)iw, \quad \gamma = \frac{(d-r)b}{g}$

$$z_{NP} = -\frac{(1-\tau)db + (1-\tau)iw(a+b) - \frac{(d-r)b}{g}(1-\tau)iw}{1+g}$$

The expression for each year's net profit is more complicated than the corresponding one for the free cash flow, expression (7). The major difference is that in expression (10) the time-dependent growth expression has to be reduced by a constant. This means that the net profit will not grow at a constant rate, unless one or more of the following conditions are fulfilled:

1. All equity financing (or zero-interest rate)
2. $gA_0 = (d - r)bR_0$
3. 100% tax-rate

Obviously, the third condition is totally unrealistic and can be ruled out. The remaining two are thus conditions for a NP steady state (NSS), and one of them has to be fulfilled. In reality most companies have at least some debt, and the interesting condition will be the second one.

Properties of the NP function

- NP_t is **decreasing** in *a, net working capital / revenues,* without any restrictions.

This is intuitive for the same reason as in the free cash flow case; the more efficient the use of capital (i.e. the lower the a), the higher the net profit.

- NP_t is **decreasing** in *b, gross PPE / revenues,* if the sufficient, but not necessary, condition that *net PPE* does not decrease year by year is fulfilled.

As in the free cash flow case, one would intuitively like to have NP_t decreasing in b. This is actually the case as long as *net PPE* does not decrease between years in the parametric steady state period. This should be a reasonable (and in reality non-binding) restriction on the parameters. If this restriction is not posed, *net PPE* can at some point in the future become negative. The condition can in terms of the input parameters be stated as:

(11) $(d - r) \leq g$

The condition is necessary to ensure an intuitive development of the PPE-items, and it can be used as a test in a practical valuation.

- NP_t is **independent** of *c, change in deferred taxes / gross PPE.*

The parameter *c* only affects the distribution of tax-payments (between now and the future) and not the tax on the income statement.

- NP_t is **increasing** in *g, the revenue growth rate,* for large *t*:s.

This case is more complicated than the corresponding one for free cash flows. Effects with bearing on the sign of the derivative come from either the growth rate or from the constant $z_{NP.}$

1. The effect from the growth rate is positive since *g* **is** the growth rate.
2. The sign of the effect from the constant term z_{NP} is not clear and will depend on the values of other input parameters, since it in fact consists of two opposite effects: one in the numerator and one in the denominator.

In the beginning of the parametric steady state period the case is not clear-cut: net profits can be either increasing or decreasing depending on the values of the parameters included in z_{NP}. But as $t \rightarrow \infty$ the growth effect will dominate the constant term effect, which will ensure that after a certain point in time NP_t will be increasing in *g*.

- NP_t is **decreasing** in *i, the interest rate on debt,* for all relevant cases.

Intuitive and trivial.

- NP_t is **decreasing** in *p, operating expenses / revenues,* without any conditions.

This is also trivially intuitive as it was in the free cash flow case.

- NP_t is **decreasing** in τ, *the tax rate,* for all relevant cases.

As shown in the derivation in the appendix, the condition for this to hold is that the pre-tax net profit is positive in the PSS period. Since we are in a steady state period the company under consideration must always be profitable - an eternally loss-making company is hardly conceivable - and the alternative of zero tax for negative income will not have to be modelled explicitly. This implies that one should check whether the pre-tax net profits implied by the parameter assumptions are positive.

2.1.3 Dividends

Now the dividend stream will be considered. Rearranging the clean surplus definition for dividends at year *t*, expression (3), and substituting expressions (4 - 6) yields:

$$(12) \qquad DIV_t = (1+g)^t R_0 \left[m + z_{DIV}\right] - \chi\left[R_0 \cdot \gamma - A_0\right]$$

with the following constants:

$$m = (1-\tau)(1-p), \quad \chi = (1-\tau)iw, \quad \gamma = \frac{(d-r)b}{g}$$

$$z_{DIV} = cb + \frac{b\left[d\tau + w(r-d+g) - r - g - (1-\tau)iw\right] - a\left[g(1-w) + (1-\tau)iw\right] + \dfrac{(d-r)b}{g}(1-\tau)iw}{1+g}$$

Equation (12), like expression (10) for net profit, is more complicated than the corresponding one for the free cash flow, expression (7). The difference is that here (as in the net profit case), the time-dependent growth expression has to be reduced by a constant, which is the same, independent of time.

Properties of the DIV function

- DIV_t is **decreasing** in *a, net working capital / revenues,* without any conditions.

The more efficient the use of capital (i.e. the lower the *a*), the higher the surplus that can be used for dividends.

- DIV_t is **decreasing** in *b, gross PPE / revenues* if and only if the following inequality is fulfilled:

$$(13) \quad d\tau + c(1+g) + \frac{\chi(d-r)}{g} - r - w(d-r) - \chi < g(1-w)$$

Also regarding *b*, the requirement for dividends to be decreasing is more complicated and harder to interpret than in the free cash flow case. Looking at the expression, one can conclude that the negative terms in realistic cases will be large in relation to the positive terms, thus securing that the desired property will hold.

- DIV_t is **increasing** in *c, change in deferred taxes / gross PPE,* without any conditions.

That the dividend payments are increasing in *c* follows from the modelling approach where the book equity value is the residual item of the system and from the clean surplus assumption: Larger deferred taxes reduce book equity. A lower book equity means that a larger part of the earnings has been paid out as dividends. Thus, an increase in deferred taxes increases the dividends paid out since the ending equity is lowered.[29] The book equity should never be negative, and the following two boundary conditions must hold in order to satisfy the *non-negative book equity constraint*:

$$(14) \quad (1-w)(a+b) > \left(\frac{b}{g} - \frac{b}{g(1+g)}\right)[(d-r)(1-w)+c(1+g)] + \frac{(1-w)A_0 + T_0}{R_0\,(1+g)}$$

$$(15) \quad (1-w)(a+b) > \frac{b}{g}[(d-r)(1-w)+c(1+g)]$$

- DIV_t is **decreasing** in *i, the interest rate on debt,* for all relevant cases.

---

[29] Note that the TSS conditions do not necessarily imply a constant relation between book equity and deferred taxes.

The result is trivially intuitive: the more of the company's free cash flow that is paid out to debt-holders, the less can be paid out as dividends.

- DIV_t is **decreasing** in *p, operating expenses / revenues,* without any conditions.

Trivial.

- DIV_t is **decreasing** in τ, *the tax rate,* for all relevant cases.

As in the net profit case, the condition for this to hold is that the pre-tax profits is positive. As argued there, this should always be true in a steady state period.

2.1.4 The transition to steady state - calculational implications

It has so far been seen that FSS will be established from $t \geq 1$ if a parametric steady state is assumed from year 0. For the net profit and dividend cases, NSS and DSS are obtained only under certain conditions. If steady state with respect to the measure used for valuation prevails and in addition the discount rate is constant, a continuing value formula can be used to calculate the value from the future years. Of course, one must then know what year to use as base year for the continuing value calculation. The approach in Copeland et al suggests that the year we call year 0, the first year in the perpetuity period, should be used as base year for calculating the FCF continuing value.

Proposition 2.2

If at least one of the ratios gross PPE / revenues (b) and net working capital / revenues (a) is changed between year (-1) and year 0 (where year 0 is the first year in the period with constant input parameters) year 0 will ***not*** *be a FSS year: the first actual FSS will be year 1. Thus year 1 should be used as base-year in the continuing value calculation in such cases.*

This means that only if these ratios[30] remain constant between the last year of the explicit forecast period and the first parametric steady state year (year 0), the continuing value can be calculated using year 0 as basis. When using free cash flow valuation, the FCF formula then also applies to year 0. Whenever any one of these ratios is changed, however, the continuing value calculation must be moved one year ahead. The FCF equation (7) can then be used to determine the free cash flow of year 1, whereas a spreadsheet model must be used for calculating the free cash flow of year 0, which for discounting purposes can be seen as belonging to the explicit forecast period.[31] For simplicity, year 1 should always be used as base-year for calculating the continuing free cash flow value.

The way Copeland et al deal with this problem is in our opinion less transparent. They suggest one should adjust the forecast of capital expenditures in year 0, which they use as base-year, to "normalise" the free cash flow. In our approach this normalisation falls out automatically by just adding another equivalent column (year) in the spread-sheet model and by using this equivalent year (here: year 1) as base-year for the continuing value. Further, since this approach will give the same value even if the adding of year 1 is unnecessary (and the adding is a very simple operation in itself) one never has to worry about whether adjustments should be made or not. Another advantage is that one explicitly sees what is going on numerically.

The intuition behind Proposition 2.2 is that the free cash flow in year zero depends on the change in working capital and gross PPE between year (-1) and year 0. Since year (-1) does not belong to the PSS period, the change in these items can be different from what would be the case had the company already settled down to a steady state.

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[30] The ratio *net working capital / revenues* used in steady state is of course equal to the sum of the different working capital to revenues ratios used in the explicit forecast period.

[31] Conceptually, the present value of the free cash flow of year 0 plus the present value of the continuing value (for year 1 to infinity) give what is called the *horizon value*.

2.1.5 Capital structure

In the beginning of this chapter an assumption was made about a constant book value debt ratio (in the example company w=40%) in the PSS period. It is assumed that the company has found this to be the optimal capital structure and wishes it to remain at that level. There may exist a number of reasons for this, as is evident from the vast corporate finance literature on the subject. For a further discussion regarding different motives we refer to the major corporate finance books.[32] Here, we will look at the difference between the book value debt ratio and the market value debt ratio. Any differences between the two will be attributable to differences between the book asset value and the market asset value, since, as stated earlier, the book value of debt equals the market value by assumption.[33]

The commitment of the example company to a debt ratio of 40% means that the debt will each year constitute 40% of the balance sheet total. This by no means guarantees that the market value debt ratio will remain constant, however, as is evident from Figure 1 and Table 8 below:

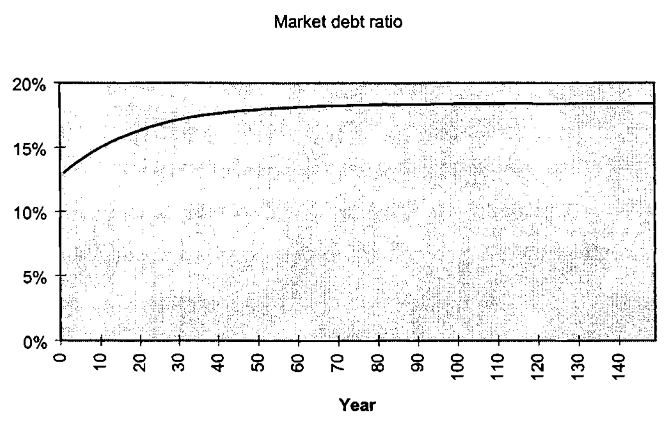

Figure 1 - Market debt ratios in the steady state period using year-to-year WACC

[32] A more specific discussion on the subject can be found in, e.g., Arditti (1973).
[33] The equality between book and market value of debt is implied by assuming that the interest rate on debt, i, is the market rate of debt for the risk class of companies to which the company belongs.

Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)
1	13,04%	26	16,85%	51	17,95%	76	18,27%	101	18,37%	126	18,39%
2	13,30%	27	16,92%	52	17,97%	77	18,28%	102	18,37%	127	18,39%
3	13,55%	28	17,00%	53	17,99%	78	18,28%	103	18,37%	128	18,39%
4	13,79%	29	17,06%	54	18,01%	79	18,29%	104	18,37%	129	18,40%
5	14,01%	30	17,13%	55	18,03%	80	18,29%	105	18,37%	130	18,40%
6	14,22%	31	17,19%	56	18,05%	81	18,30%	106	18,37%	131	18,40%
7	14,43%	32	17,25%	57	18,06%	82	18,30%	107	18,38%	132	18,40%
8	14,62%	33	17,30%	58	18,08%	83	18,31%	108	18,38%	133	18,40%
9	14,80%	34	17,36%	59	18,10%	84	18,31%	109	18,38%	134	18,40%
10	14,98%	35	17,41%	60	18,11%	85	18,32%	110	18,38%	135	18,40%
11	15,14%	36	17,45%	61	18,13%	86	18,32%	111	18,38%	136	18,40%
12	15,30%	37	17,50%	62	18,14%	87	18,33%	112	18,38%	137	18,40%
13	15,45%	38	17,54%	63	18,15%	88	18,33%	113	18,38%	138	18,40%
14	15,59%	39	17,58%	64	18,16%	89	18,33%	114	18,38%	139	18,40%
15	15,73%	40	17,62%	65	18,17%	90	18,34%	115	18,39%	140	18,40%
16	15,86%	41	17,66%	66	18,19%	91	18,34%	116	18,39%	141	18,40%
17	15,98%	42	17,70%	67	18,20%	92	18,34%	117	18,39%	142	18,40%
18	16,10%	43	17,73%	68	18,21%	93	18,35%	118	18,39%	143	18,40%
19	16,21%	44	17,76%	69	18,22%	94	18,35%	119	18,39%	144	18,40%
20	16,32%	45	17,79%	70	18,22%	95	18,35%	120	18,39%	145	18,40%
21	16,42%	46	17,82%	71	18,23%	96	18,35%	121	18,39%	146	18,40%
22	16,51%	47	17,85%	72	18,24%	97	18,36%	122	18,39%	147	18,40%
23	16,60%	48	17,88%	73	18,25%	98	18,36%	123	18,39%	148	18,40%
24	16,69%	49	17,90%	74	18,26%	99	18,36%	124	18,39%	149	18,40%
25	16,77%	50	17,93%	75	18,26%	100	18,36%	125	18,39%	150	18,40%

Table 8 - Market debt ratios in the steady state period using year-to-year WACC

The values in Table 8 are calculated using the balance sheet and income statement for each year 150 years ahead. A market value calculation is then performed each year, starting from the last year, using the particular year's balance sheet and income statement information and the particular year's weighted average cost of capital as discount rate. From the final result in Table 8, it can be seen that it takes quite a number of years for the market value debt ratio to converge towards the steady-state-level in the example company: 18.4 %.

The different behaviour of the debt ratio in book-value terms and in market-value terms is interesting in itself, but it also has implications for the choice of discounting method. With a varying market debt ratio the weighted average cost of capital will also be non-constant over time, since the market debt ratio constitutes the weights in WACC formula:

$$(16) \qquad k_{WACC,t} = \omega_{t-1}(1-\tau)i + (1-\omega_{t-1})k_E$$

where: $k_{WACC,t}$ is the weighted average cost of capital[34]
ω_{t-1} is the entering market debt ratio
τ is the tax rate
i is the borrowing rate
k_E is the cost of equity capital

This presents the analyst with practical as well as conceptual problems. To deal with the latter first, one should note that a varying discount rate is actually the rule rather than the exception. The basic DCF model[35] for valuation of an asset has a time-dependent discount rate, r_t:

$$(17) \qquad PV = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t}$$

A constant discount rate is only a special case - where all r_t are the same. That the discount rate should be varying over time when valuing such a complex asset as a company is nothing to be surprised at. The fact that in practice the discount rate is often assumed to be constant more reflects the computational problems involved when working with time-dependent rates.

Returning to the practical problems, they can be overcome by starting the discounting process sufficiently long into the future, at a time when the market value debt ratio has itself approached a steady state. When this occurs depends on the parameter values; for normal values using about 100 years will make the approximation error negligible.

Table 8 is based upon an explicit modelling of the example company's operations 150 years ahead, where the revenues each year grow at the rate g. All other financial items are calculated via the formulas in section 2.1 (all parameters are constants by the PSS assumption). This gives explicit financial statements for each of the 150 years. The value of the operations at the

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[34] The defintion in expression (16) means that the discount rate applied for a particular year is calculated using entering values for the market debt ratio.

[35] Used to determine the present value (PV) of an asset by discounting all future incremental cash flows, C_t, pertaining to the asset at the appropriate discount rate r_t. See, e.g., Brealey & Myers (1991), p. 30.

end of year 149 is then calculated using the normal *FCF* valuation formula (the one proposed in Copeland et al):

$$(18) \qquad EV_{149} = \frac{FCF_{150}}{k_{WACC,150} - g} - D_{149}$$

(*EV* stands for equity value and *D* for debt value; the subindices mark the year.[36]) The equation is solved iteratively, since the equity value affects the WACC which in turn affects the equity value and so on. The actual market value debt ratio in Table 8, i.e. ω_{149}, is calculated by rearranging expression (16):

$$(19) \qquad \omega_{149} = \frac{k_E - k_{WACC,150}}{k_E - (1-\tau)i}$$

The equity value at the end of year 148 is then calculated as (also using an iterative procedure):

$$(20) \qquad EV_{148} = \frac{FCF_{149} + EV_{149} + D_{149}}{1 + k_{WACC,149}} - D_{148}$$

The market value debt ratio is subsequently calculated in the same way as in expression (19).

The equity value at the end of each year is then calculated as in expression (20) yielding the market value debt ratios in Table 8 above. This may seem (and indeed is) quite complicated and time-consuming, and hence the practice of applying the WACC from year 0 as the discount rate throughout is certainly understandable - but it is an approximation only, and in many cases an approximation too crude for comfort. In Chapter 3, below, the problem is discussed further and a solution is proposed. First, however, we will look into what actually causes the market value debt ratio to change over time even though the book debt ratio (the parameter *w*) remains constant.

[36] End of year to be precise.

The discounting method described above is not in itself the cause of the non-constant capital structure, as can be seen in Figure 2 below, where the market value debt ratio using a constant discount rate (i.e. the WACC from year zero) is plotted:

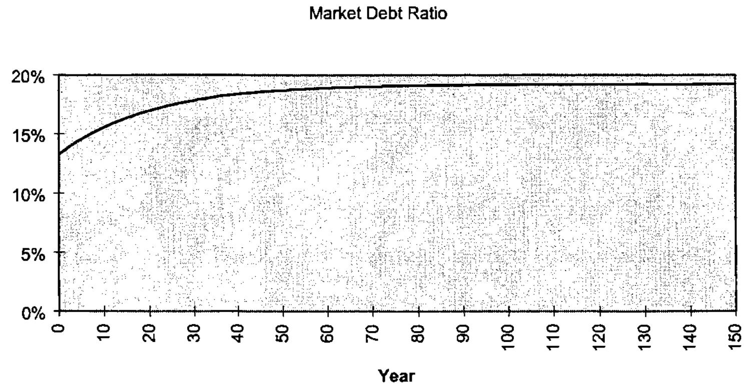

Figure 2 - Market debt ratios in the steady state period using constant (year 0) WACC

In this case the market debt ratio converges towards 19.23%. Using a constant discount rate makes it possible to analyse analytically what is happening: The market value debt ratio is defined as the market value of the debt divided by the market value of the assets. The market value of debt is by assumption equal to the book value and the market value of the assets is the sum of the discounted free cash flows:

$$(21)\qquad \frac{D_t}{Assets_t} = \frac{w(aR_t + bR_t - A_t)}{\dfrac{(1+g)R_t(m + z_{FCF})}{k_{WACC} - g}} = \frac{w(aR_t + bR_t - A_t)(k_{WACC} - g)}{(1+g)R_t(m + z_{FCF})} =$$

$$= \frac{w(a+b)(k_{WACC} - g)}{(1+g)(m + z_{FCF})} - \frac{wA_t(k_{WACC} - g)}{(1+g)R_t(m + z_{FCF})} =$$

(21) $$=\frac{w(a+b)(k_{WACC}-g)}{(1+g)(m+z_{FCF})}-\frac{w\left(\frac{(d-r)b}{g}R_t-\frac{(d-r)b}{g}R_0+A_0\right)(k_{WACC}-g)}{(1+g)R_t(m+z_{FCF})}=$$

$$=\frac{w\left(a+b-\frac{(d-r)b}{g}\right)(k_{WACC}-g)}{(1+g)(m+z_{FCF})}-\frac{w\left(A_0-\frac{(d-r)b}{g}R_0\right)(k_{WACC}-g)}{(1+g)^{t+1}R_0(m+z_{FCF})}$$

The first of the two terms in the last row of equation (21) is the steady-state debt ratio. The second term goes to zero as t becomes very large. From equation (21) it is also possible to derive the general condition that must hold if a constant book value debt ratio shall always imply a constant debt ratio also in market value terms. The technical version of this initial condition is obviously:[37]

(22) $$gA_0 = (d-r)bR_0 .$$

This is perhaps more intuitively stated in Proposition 2.3:

Proposition 2.3

In order for the debt ratio to remain constant over time in book value terms as well as in market value terms, the accumulated depreciation must grow at the same rate as the revenues.

Proof of Proposition 2.3:

From equation (21) clearly gA_0 must equal $(d-r)bR_0$ if the debt ratio is to remain constant each year, and hence:

(22) $$gA_0 = (d-r)bR_0$$

[37] Remember that g is always less than the discount rate.

The expression for the accumulated depreciation is:

(23) $$A_t = \frac{(1+g)^t - 1}{g}(d-r)bR_0 + A_0$$

Substituting equation (22) into equation (23) yields:

(24) $$A_t = (1+g)^t \frac{(d-r)b}{g} R_0 \Leftrightarrow A_t = (1+g)^t A_0$$

and since all parameters are constant by assumption, the growth rate is clearly g, the same as the revenue growth rate. Q.E.D.

Proposition 2.3 also has some interesting implications:

Corollary 2.1

A constant market value debt ratio, attained by letting the initial condition $gA_0 = (d-r)bR_0$ hold, also implies that:

i) the net property plant and equipment will grow at the rate g
ii) the balance sheet total will grow at the rate g
iii) the debt will grow at the rate g
iv) the net profit will grow at the rate g
v) the dividends will grow at the rate g

Proof

i) *Gross PPE* is defined as a constant percentage of revenues (b) in Specification A. Since the revenues grow at the rate g, so will the *gross PPE*. The accumulated depreciation will grow at the rate g according to Proposition 2.3. The *net PPE*, finally, is defined as the difference between *gross PPE* and accumulated depreciation and will thus also have the growth rate g.

ii) The balance sheet total is defined as $aR_t + bR_t - A_t$, and since both revenues and accumulated depreciation grow at the rate g, so will the balance sheet total.

iii) The debt is defined as a constant percentage of the balance sheet total, so the proof for the balance sheet total also holds for the debt.

iv) The expression for the net profit is:

$$(10) \quad NP_t = (1+g)^t R_0 (m + z_{NP}) - \chi\left(\frac{(d-r)b}{g} R_0 - A_0\right)$$

Since $gA_0 = (d-r)bR_0$ by assumption, the second term in the net profit expression will equal zero, and the net profits will also have the constant growth rate g.

v) The expression for the dividends is:

$$(12) \quad DIV_t = (1+g)^t R_0 (m + z_{DIV}) - \chi\left(\frac{(d-r)b}{g} R_0 - A_0\right)$$

Since $gA_0 = (d-r)bR_0$ by assumption, the second term in the dividend expression will equal zero, and the dividends will also have the constant growth rate g. Q.E.D.

From Proposition 2.3 and Corollary 2.1 it is clear that a constant capital structure (in market terms) is a sufficient condition for attaining a steady state with respect to all different concepts under consideration. In particular, Corollary 2.1 shows that the conditions for TSS[38] are fulfilled. When using the model for performing valuations, the constant capital structure has the further advantage that the weighted average cost of capital will remain the same each year (since the weights in the WACC formula remain constant), and thus it is correct to use the year zero WACC as discount rate throughout.

Another interesting implication for valuations is the general irrelevancy of valuation approach implied by this "true steady state" - the constant market value debt ratio:

Proposition 2.4

A constant market value debt ratio, attained by letting the initial condition $gA_0 = (d-r)bR_0$ *hold, implies that the free cash flow valuation approach using a constant WACC as discount rate will yield the same result as the dividend valuation approach.*

Thus, when considering a steady-state valuation, with a constant market value debt ratio one can be certain that the neo-classical way of determining market value, as the present value of dividends, will be equal to the value obtained from a free cash flow valuation à la Copeland et al. Accordingly, in the setting under consideration here, i.e. using the constant WACC from year 0, the constant capital structure case is the only one that ensures the same value. This is also fully in line with the findings by Chambers, Harris & Pringle (1982) for project valuation. They conclude that using a constant WACC approach will give the same value as using the equity residual method (which is the project valuation case's analogue to the dividend valuation approach considered in this report) if and only if the "debt in every period equals a constant fraction of the value of the cash flows yet to be received."[39] These matters will be further discussed in Chapter 3.

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[38] The conditions for TSS are, as stated earlier: 1. The company is in PSS; 2. The company exhibits FSS; 3. The balance sheet total grows at the same rate as revenues.

Proof

Denote the constant market-value debt ratio by ω.[40] From expression (21) and the steady-state condition, ω in the free cash flow case is given by the debt value divided by the asset value at the end of year zero:

$$(25) \qquad \omega = \frac{w\left(aR_0 + bR_0 - A_0\right)}{\dfrac{(1+g)R_0\left(m + z_{FCF}\right)}{k_{WACC} - g}} = \frac{w\left(a + b - \dfrac{(d-r)b}{g}\right)\left(k_{WACC} - g\right)}{(1+g)\left(m + z_{FCF}\right)}$$

The weighted average cost of capital is given by:

$$(26) \qquad k_{WACC} = \omega(1-\tau)i + (1-\omega)k_E$$

Substituting (26) into (25) and rearranging yields:

$$(27) \qquad \omega = \frac{w\left(a + b - \dfrac{(d-r)b}{g}\right)}{\dfrac{(1+g)\left(m + z_{FCF}\right) - w\left(a + b - \dfrac{(d-r)b}{g}\right)\left((1-\tau)i - k_E\right)}{k_E - g}}$$

The constant z_{FCF} is equal to:

$$(28) \qquad z_{FCF} = \frac{b\left(c + \tau d - r\right) - g\left(b(1-c) + a\right)}{1+g}$$

Thus, (27) can equivalently be stated as:

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[39] Chambers, Harris & Pringle (1982), p. 27.

$$(29)\ \omega = \frac{w\left(a+b-\frac{(d-r)b}{g}\right)}{\frac{(1+g)m+(1+g)cb+b\tau d-br-g(a+b)-w\left(a+b-\frac{(d-r)b}{g}\right)((1-\tau)i-k_E)}{k_E-g}}$$

The constant z_{DIV} is given by:

$$(30)\ z_{DIV} = cb + \frac{b(d\tau + w(r-d+g)-r-g-(1-\tau)iw)-a(g(1-w)+(1-\tau)iw)+\frac{(d-r)b}{g}(1-\tau)iw}{1+g}$$

Substituting (30) into (29), rearranging and multiplying numerator as well as denominator by R_0 yields:

$$(31)\qquad \omega = \frac{w(aR_0+bR_0-A_0)}{w(aR_0+bR_0-A_0)+\frac{(1+g)R_0(m+z_{DIV})}{k_E-g}}$$

The denominator in (31) is the market value of debt[41] plus the market value of all possible future dividends.

Comparing equation (25) with equation (31), the following expression is obtained:

$$(32)\qquad \frac{(1+g)R_0(m+z_{FCF})}{k_{WACC}-g} - w(aR_0+bR_0-A_0) = \frac{(1+g)R_0(m+z_{DIV})}{k_E-g}$$

[40] The subindex denoting time is dropped, since the market-value debt ratio is constant over time, i.e. the same for all t.

[41] By assumption equal to the book value.

The left-hand-side is the equity value using the free-cash-flow approach and the right-hand-side is the equity value using the dividend approach. Q.E.D.

It is now possible to summarise the findings about when the different concepts of steady state are achieved and what this implies for the calculation of the horizon value:

Proposition 2.5

If and only if the initial condition $gA_0 = (d - r)bR_0$ *is fulfilled a TSS will be established,*

and consequently,

it is only with a constant capital structure in market terms that one can use a FCF continuing value formula to calculate the horizon value without approximation errors.

The intuition behind this is that a non-constant capital structure means that the weights in the *weighted average cost of capital (WACC)* [42] *formula* will change over time. A further consequence is that the different costs of capital for the company in question are not likely to remain constant if the riskiness of the company changes over time due to the varying capital structure. These issues are discussed further in Chapter 3. The more formal proof of Proposition 2.5 is given below.

One contribution in this report is the derivation of an analytical condition, in terms of forecasted parameter values, to ensure that the underlying assumptions of the continuing value approach are fulfilled. This is missing in Copeland et al; they say verbally what conditions should be fulfilled but they do not give the analyst much guidance as to how this is done in practice.

Proof of Proposition 2.5

In section 2.0.1 it was concluded that TSS is established if

1. the company is in *PSS*,
2. it exhibits *FSS* and
3. the *balance sheet total* grows at the same rate as revenues.

The first condition holds trivially by the basic PSS assumption. The same is true for the FSS condition by Proposition 2.1. Since the balance sheet total is the sum of net working capital and net PPE, and since net working capital will be growing at the same rate, *g*, as revenues (by definition of the model), the balance sheet total will grow at the same rate as revenues if and only if net PPE grows at the rate *g*. But since net PPE is equal to gross PPE minus accumulated depreciation, and since gross PPE by model definition grows at the same rate as revenues, the third condition is equivalent to the condition that accumulated depreciation grow at the rate *g*: $A_t = (1+g)^t A_0$. From equation (24) in the proof to Proposition 2.3 we know that this is true if and only if condition (22) holds. Thus, a TSS is established if and only if condition (22), i.e. $gA_0 = (d-r)bR_0$, holds. Q.E.D.

The constant capital structure is necessary also for the dividend valuation approach.

Corollary 2.2

It is only with a constant capital structure in market terms that one can use a DIV continuing value formula to calculate the horizon value without approximation errors.

[42] The discount rate proposed by Copeland et al, p. 239.

Proof

The equation for the dividends is:

$$(12) \qquad DIV_t = (1+g)^t R_0 (m + z_{DIV}) - (1-\tau) iw \left(\frac{(d-r)b}{g} R_0 - A_0 \right)$$

In order to use a continuing value, the growth rate must be constant. This is the case only when the constant term equals zero. There are four possibilities: 100% tax rate (τ=1), zero interest rate on debt (*i*=0), all-equity financing (*w*=0), and finally when the condition $gA_0 = (d-r)bR_0$ holds. 100% tax rate is absurd as is a zero interest rate. All-equity financing means that dividends will equal free cash flow and the proof of Proposition 2.5 applies. Hence, we are left with the fourth possibility, namely that $gA_0 = (d-r)bR_0$. Q.E.D.

2.2 Specification B - Constant capital expenditures to revenues ratio

If the PPE-items are modelled differently, the system of difference equations also must be modified. Here, it will be shown how the solution is affected when the *gross PPE* at the end of year *t* is derived as the preceding year's *gross PPE* plus capital expenditures made during year *t* minus retirements. The capital expenditures are forecasted as a percentage of revenues, the retirements as a percentage of the preceding year's *gross PPE*. This is what is called Specification B (presented in section 1.2):

(33) $$G_t = G_{t-1} + CapX_t - Ret_t$$

(34) $$CapX_t = eR_t$$

(35) $$Ret_t = rG_{t-1}$$

The balance sheet items are defined as follows:

A1: aR_t

A2: $G_{t-1} + eR_t - rG_{t-1} - A_t = eR_t + (1-r)G_{t-1} - A_t$

where $A_t = (d-r)G_{t-1} + A_{t-1}$

D1: $w(aR_t + G_t - A_t)$

D2: $T_t = cG_t + T_{t-1}$

D3: $(1-w)(aR_t + G_t - A_t) - T_t$

The balance sheet now contains four time-dependent state-variables: revenues (R_t), gross PPE (G_t), accumulated depreciation (A_t), and deferred taxes (T_t). Once again utilising the fact that the development of the balance sheet over the years is a system of difference equations, the expressions for the state-variables are found to be:

(36-39):

$$
\begin{aligned}
R_t &= R_0(1+g)^t \\
G_t &= e\frac{1+g}{g+r}R_0(1+g)^t + \left(G_0 - e\frac{1+g}{g+r}R_0\right)(1-r)^t \\
A_t &= A_0 + \frac{(1+g)^t - 1}{g}(d-r)e\frac{1+g}{g+r}R_0 + \frac{d-r}{r}\left(G_0 - e\frac{1+g}{g+r}R_0\right)\left(1-(1-r)^t\right) \\
T_t &= T_0 + \frac{(1+g)^t - 1}{g}c(1+g)e\frac{1+g}{g+r}R_0 + \frac{c(1-r)}{r}\left(G_0 - e\frac{1+g}{g+r}R_0\right)\left(1-(1-r)^t\right)
\end{aligned}
$$

Now define the constant $\beta = e\dfrac{1+g}{g+r}$ and substitute into the solution:

$$
\begin{aligned}
R_t &= R_0(1+g)^t \\
G_t &= \beta R_0(1+g)^t + (G_0 - \beta R_0)(1-r)^t \\
A_t &= A_0 + \frac{(1+g)^t - 1}{g}(d-r)\beta R_0 + \frac{d-r}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right) \\
T_t &= T_0 + \frac{(1+g)^t - 1}{g}c(1+g)\beta R_0 + \frac{c(1-r)}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right)
\end{aligned}
\tag{40-43}
$$

These expressions can now be compared with the ones obtained when using the previous specification (Specification A) to describe the PPE-development:

Revenues

Spec. A: $R_t = (1+g)^t R_0$

Spec. B: $R_t = (1+g)^t R_0$

Gross property, plant and equipment

Spec. A: $G_t = bR_0(1+g)^t$

Spec. B: $G_t = \beta R_0(1+g)^t + (G_0 - \beta R_0)(1-r)^t$

Accumulated depreciation

Spec. A: $A_t = A_0 + \frac{(1+g)^t - 1}{g}(d-r)bR_0$

Spec. B: $A_t = A_0 + \frac{(1+g)^t - 1}{g}(d-r)\beta R_0 + \frac{d-r}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right)$

Deferred taxes

Spec. A: $T_t = T_0 + \frac{(1+g)^t - 1}{g}c(1+g)bR_0$

Spec. B: $T_t = T_0 + \frac{(1+g)^t - 1}{g}c(1+g)\beta R_0 + \frac{c(1-r)}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right)$

The expressions for the revenue development are the same in the two cases. The *gross PPE* differs by the term $(G_0 - \beta R_0)(1-r)^t$, the conclusion being that only in the special case where β is specified such that $G_0 - \beta R_0 = 0$ are the two specifications the same,[43] i.e. they are equal when $G_0 = \beta R_0$ - but this is exactly the definition of Specification A, when *gross PPE* was assumed to be a certain percentage of revenues in each year, also in year zero.[44] The same condition holds true also for accumulated depreciation as well as for deferred taxes, and hence:

Observation 1

The specification of gross PPE in year t as a fixed percentage of the revenues the same year

(44) $G_t = bR_t$ *("Specification A")*

is only a special case of the more general modelling of G_t *as the preceding year's gross PPE plus capital expenditures minus retirements*

(45) $G_t = G_{t-1} + CapX_t - Ret_t$ *("Specification B")*

where capital expenditures are defined as a percentage of revenues and retirements as a percentage of the preceding year's gross PPE.

[43] The two expressions for the gross PPE development are also the same when $t \to \infty$. This case is never of any practical interest, however, since for large values of *t*, the *discounted* value is (almost) zero.
[44] Remember that the constant β is short for $e(1+g)/(r+g)$.

2.2.1 Free cash flow

The expression for the free cash flow in year t is:

$$(46)\quad \begin{aligned} FCF_t &= (1-\tau)\left(R_t - pR_t - dG_{t-1}\right) + dG_{t-1} + T_t - T_{t-1} \\ &\quad - \left(aR_t - aR_{t-1}\right) - \left(G_t - G_{t-1} + rG_{t-1}\right) \\ &= aR_{t-1} + \left((1-\tau)(1-p) - a\right)R_t + \left(1 - r + \tau d\right)G_{t-1} - G_t + T_t - T_{t-1} \end{aligned}$$

Substituting the expressions for R_t, G_t, and T_t into the free cash flow equation yields eventually the following expression (see the appendix for a more detailed derivation):

$$(47)\quad \begin{aligned} FCF_t &= (1+g)^t R_0 \left(m + \frac{\beta\left(c + \tau d - r\right) - g\left(\beta(1-c) + a\right)}{1+g} \right) \\ &\quad + \left(c + \tau \frac{d}{1-r} \right) \left(G_0 - \beta R_0\right)(1-r)^t \end{aligned}$$

The resulting expression for the free cash flow at year t is:

$$(48)\quad FCF_t = (1+g)^t R_0 \left(m + z_{FCF(2)}\right) + \phi\left(G_0 - \beta R_0\right)(1-r)^t$$

where: $\beta = e\dfrac{1+g}{g+r}$, $\quad \phi = \left(c + \tau \dfrac{d}{1-r} \right)$, $\quad m = (1-\tau)(1-p)$

$$z_{FCF(2)} = \frac{\beta\left(c + \tau d - r\right) - g\left(\beta(1-c) + a\right)}{1+g}$$

This can be compared with the free cash flow expression using Specification A:

Specification A: $\quad FCF_t = (1+g)^t R_0 \left(m + z_{FCF}\right)$

Specification B: $\quad FCF_t = (1+g)^t R_0 \left(m + z_{FCF(2)}\right) + \phi\left(G_0 - \beta R_0\right)(1-r)^t$

It is obvious that the expressions are equal only in the special case where $G_0 = \beta R_0$,[45] which is in line with Observation 1 above. This, however, also has implications for steady state. In Proposition 2.1 it was claimed that *a company in parametric steady state* (PSS) *exhibits FCF steady state* (FSS) *without any restrictions on the constant input ratios*. In the more general Specification B, this proposition must be modified:

Proposition 2.6

A company in parametric steady state exhibits FCF steady state with respect to the free cash flow development if and only if the initial condition

$$e = \frac{G_0 (g + r)}{(1 + g) R_0} \tag{48}$$

is fulfilled.

If the initial condition in Proposition 2.6 is fulfilled Specification B reduces to Specification A, where the ratio *gross PPE / revenues* is constant, as is seen from Observation 1. Thus the only instance in this setting where steady state can be established, allowing for the use of a continuing value formula, is when the parameter values are such that the specification exactly equals that of Specification A. Hence, there exists no reason to use Specification B when specifying the PSS in order to yield a FSS; the simpler Specification A does the trick completely. However, as will be argued in section 4.1, Specification B does have some intuitive features which makes it useful in the explicit forecast period, but when the company is assumed to settle down to a steady state, it is easier to turn to Specification A when determining the property, plant and equipment items.

As the only steady state in Specification B exists when the FCF function of Specification B equals the one of Specification A, the analysis of the FCF function for Specification A is apparently valid also here.

---

[45] This can be restated as $e = \dfrac{G_0(g+r)}{(1+g)R_0}$.

2.2.2 Net profit

The net profit in year t is given by:

$$(50) \qquad NP_t = (1-\tau)\left(R_t - pR_t - dG_{t-1} - iw\left(aR_{t-1} + G_{t-1} - A_{t-1}\right)\right)$$

Substituting the expressions (40-43) into (50) and rearranging finally yields:

$$(51) \qquad NP_t = (1+g)^t R_0\left(m + z_{NP(2)}\right) - \chi\left(\frac{d-r}{g}\beta R_0 - A_0\right)$$

$$- \eta\left(\beta R_0 - G_0\right) + \varphi\left(\beta R_0 - G_0\right)(1-r)^t$$

where: $\beta = e\dfrac{1+g}{g+r}$, $\chi = (1-\tau)iw$, $m = (1-\tau)(1-p)$,

$$\varphi = \chi\frac{d-r}{r(1-r)} + \frac{(1-\tau)(d+iw)}{1-r}, \quad \eta = \chi\frac{d-r}{r} \quad ,$$

$$z_{NP(2)} = -\frac{(1-\tau)d\beta + \chi\left(a + \beta\left(1 - \dfrac{d-r}{g}\right)\right)}{1+g}$$

The net profit will generally not grow at a constant rate. There is one correction term involving accumulated depreciation and another involving gross PPE. Also where net profit is concerned, a comparison between Specification A and Specification B shows that Specification A is only a subcase of Specification B, and Observation 1 still holds:

$$(52) \qquad NP_t = (1+g)^t R_0\left(m + z_{NP(2)}\right) - \chi\left(\frac{d-r}{g}\beta R_0 - A_0\right)$$

$$- \eta\left(\beta R_0 - G_0\right) + \varphi\left(\beta R_0 - G_0\right)(1-r)^t$$

$$(53) \qquad NP_t = (1+g)^t R_0\left(m + z_{NP}\right) - \chi\left(\frac{(d-r)b}{g}R_0 - A_0\right)$$

Consequently, if the initial condition (49) is fulfilled, then (52) reduces to (53), and the *NP* function will be exactly the same as for Specification A.

2.2.3 Dividends

Now, the dividend stream will be analysed using Specification B. The expression for dividends in year t is:

$$
\begin{aligned}
DIV_t = \; & (1-w)\left(aR_{t-1}+G_{t-1}-A_{t-1}\right)-T_{t-1} \\
& +(1-\tau)\left((1-p)R_t-dG_{t-1}-iw\left(aR_{t-1}+G_{t-1}-A_{t-1}\right)\right) \\
& -\left[(1-w)\left(aR_t+G_t-A_t\right)-T_t\right]
\end{aligned}
\tag{54}
$$

When substituting for revenues, *gross PPE*, accumulated depreciation and deferred taxes (expressions 40-43) the following expression is obtained:

$$
\begin{aligned}
DIV_t = \; & (1+g)^t R_0\left(m+z_{DIV(2)}\right)-\chi\left(\frac{d-r}{g}\beta R_0-A_0\right) \\
& -\eta\left(\beta R_0-G_0\right)-\kappa\left(\beta R_0-G_0\right)(1-r)^t
\end{aligned}
\tag{55}
$$

where: $\beta=e\dfrac{1+g}{g+r}$, $\chi=(1-\tau)iw$, $\eta=\chi\dfrac{d-r}{r}$,

$\kappa=c+\dfrac{d}{1-r}\left(\tau-w-\dfrac{\chi}{r}\right)$, $m=(1-\tau)(1-p)$,

$$
z_{DIV(2)}=c\beta+\frac{\beta\left(d\tau+w\left(r-d+g\right)-r-g-\chi\right)-a\left(g(1-w)+\chi\right)+\chi\dfrac{(d-r)}{g}\beta}{1+g}
$$

This dividend expression is quite complicated. It is, however, equal to the Specification A expression in the special case where $G_0=\beta R_0$, which is to be expected, given Observation 1.

Appendix 2

A2.1 Specification A

A2.1.0 Solution to system of difference equations

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ A_t &= (d-r)bR_{t-1} + A_{t-1} \end{cases}$$

In matrix notation:

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1}$$

where: $\mathbf{x}_t = \begin{pmatrix} R_t \\ A_t \end{pmatrix}$ and $\mathbf{A} = \begin{pmatrix} 1+g & 0 \\ (d-r)b & 1 \end{pmatrix}$

The roots of the characteristic equation are $\lambda_1 = 1+g$ and $\lambda_2 = 1$.

A is diagonalised by **P**

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ \dfrac{(d-r)b}{g} & 1 \end{pmatrix}$$

So

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} 1+g & 0 \\ 0 & 1 \end{pmatrix}.$$

Substituting $\mathbf{x}_t = \mathbf{P}\mathbf{u}_t$ and $\mathbf{x}_{t+1} = \mathbf{P}\mathbf{u}_{t+1}$ yields the system $\mathbf{u}_{t+1} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}\mathbf{u}_t$, the solution of which is:

$$\mathbf{u_t} = \begin{pmatrix} c_1(1+g)^t \\ c_2 \end{pmatrix}.$$

Substituting back yields the solution to the original system:

$$\mathbf{x_t} = \mathbf{Pu_t} = \begin{pmatrix} 1 & 0 \\ \frac{(d-r)b}{g} & 1 \end{pmatrix} \begin{pmatrix} c_1(1+g)^t \\ c_2 \end{pmatrix} = \begin{pmatrix} c_1(1+g)^t \\ \frac{(d-r)b}{g} c_1(1+g)^t + c_2 \end{pmatrix},$$

and since the initial values are R_0 and A_0, the complete solution is:

$$\begin{cases} R_t &= (1+g)^t R_0 \\ A_t &= \frac{(1+g)^t - 1}{g}(d-r)bR_0 + A_0 \end{cases}$$

In the same way, the solution for T_t is derived.

A2.1.1a Free cash flow derivation - equation (7)

$$\begin{aligned} FCF_t &= R_{t-1}\left[a + (1-r)b + \tau db\right] + R_t\left[(1-\tau)(1-p) - a - b\right] + T_t - T_{t-1} \\ &= (1+g)^{t-1} R_0\left[a + (1-r)b + \tau db\right] + (1+g)^t R_0\left[(1-\tau)(1-p) - a - b\right] \\ &\quad + \frac{(1+g)^t - 1}{g}(1+g)cbR_0 + T_0 - \frac{(1+g)^{t-1} - 1}{g}(1+g)cbR_0 - T_0 \\ &= (1+g)^t R_0\left[(1-\tau)(1-p) - a - b + \frac{a + (1-r)b + \tau db}{(1+g)}\right] \\ &\quad + \frac{(1+g)^t - 1}{g}\left[(1+g)cbR_0 - cbR_0\right] + cbR_0 \end{aligned}$$

$$= \left(1+g\right)^t R_0 \left[\frac{(1+g)(1-\tau)(1-p)-ag+b(\tau d-r-g)}{1+g}\right]+\left(1+g\right)^t R_0 cb$$

$$= \left(1+g\right)^t R_0 \left[\frac{(1+g)(1-\tau)(1-p)-ag+b(\tau d-r-g)}{1+g}+cb\right]$$

$$= \left(1+g\right)^t R_0 \left[(1-\tau)(1-p)+\frac{g\left(b\left(c-1\right)-a\right)+b\left(c+\tau d-r\right)}{1+g}\right]$$

So, for $t \geq 1$:

$$FCF_t = \left(1+g\right)^t R_0 \left(m+z_{FCF}\right)$$

where m is the constant after - tax margin $(1-\tau)(1-p)$

and $z_{FCF} = \dfrac{b\left(c+\tau d-r\right)-g\left(b\left(1-c\right)+a\right)}{1+g}$ is a constant.

A2.1.1b Derivations of the FCF function properties

- *Net working capital / revenues (a)*

Follows directly from equation (7).

- *Gross PPE / revenues (b)*

The constant b appears only in the term z_{FCF} of equation (7). The marginal effect of a change in b on FCF_t is thus equal to the first derivative of z_{FCF} with respect to b. If z_{FCF} is decreasing in b, then FCF_t is decreasing in b, since FCF_t is increasing in z_{FCF}.

$$\frac{\partial z_{FCF}}{\partial b} = \frac{gc - g + c + \tau d - r}{1+g}$$

Then z_{FCF} is decreasing in b, if and only if $\frac{\partial z_{FCF}}{\partial b} < 0$:

$$\frac{gc - g + c + \tau d - r}{1+g} < 0$$

After rearranging :

$$\tau d - r + (1+g)c < g$$

- *Change in deferred taxes / gross PPE (c)*

Follows directly from equation (7).

- *Growth rate (g)*

Differentiating the free cash flow expression (7) with respect to g yields:

$$\frac{\partial FCF_t}{\partial g} = (1+g)^t \frac{t}{(1+g)} R_0 \left(m + z_{FCF}\right) - (1+g)^t R_0 \left[\frac{(b(1-c)+a) + z_{FCF}}{1+g}\right]$$

There are two opposite effects: the left term strives towards a positive derivative, whereas the right term strives in the opposite direction. For low t:s, the negative term tends to dominate the positive, but since the positive term increases faster with respect to t, it will eventually dominate the negative.

In the original FCF formula, the positive term is attributed to the growth factor (1+g), whereas the negative term is comes from g's lowering effect on the constant term z_{FCF}.

- *Tax rate, τ*

From equation (7) one can see that a change in the tax rate, τ, will affect the after-tax margin, m, and also the constant z_{FCF}. The effects of a change in τ on m and z_{FCF} go in the opposite direction, i.e. a raised τ decreases m and increases z_{FCF}. To compare these effects the partial derivatives of m and z_{FCF} are taken with respect to $(-\tau)$ and (τ) respectively:

$$\frac{\partial m}{\partial(-\tau)} = (1-p) \qquad \frac{\partial z_{FCF}}{\partial \tau} = \frac{bd}{1+g}$$

For FCF_t to be decreasing in τ, the following must hold:

$$(1-p) > \frac{bd}{1+g}$$

Rearranging then gives

$$p + \frac{bd}{1+g} < 1$$

Multiplying both sides with R_t yields:

$$R_t - pR_t - dbR_{t-1} > 0 \iff \textit{Operating income - operating expenses - depreciation} > 0$$

Thus the condition can be phrased: the *operating profit after depreciation* should be positive.

A2.1.2a Derivation of net profit expression - equation (10)

$$\begin{aligned} NP_t &= (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right) \\ &= (1-\tau)\left((1-p)R_t + R_{t-1}\left(-db - iw(a+b)\right) + iwA_{t-1}\right) \\ &= R_t\left((1-\tau)(1-p) - \frac{1-\tau}{1+g}\left(db + iw(a+b)\right)\right) + (1-\tau)iwA_{t-1} \end{aligned}$$

$$= R_t\left((1-\tau)(1-p)-\frac{1-\tau}{1+g}(db+iw(a+b))\right)+(1-\tau)iw\left(A_0+\frac{(d-r)b}{g}(R_{t-1}-R_0)\right)$$

$$= (1+g)^t R_0\left((1-\tau)(1-p)-\frac{(1-\tau)db+(1-\tau)iw(a+b)-\frac{(d-r)b}{g}(1-\tau)iw}{1+g}\right)$$

$$-(1-\tau)iw\left(\frac{(d-r)b}{g}R_0-A_0\right)$$

$$= (1+g)^t R_0\left(m+z_{NP}\right)-\chi\left(\gamma R_0-A_0\right)$$

where: $m=(1-\tau)(1-p)$, $\chi=(1-\tau)iw$, $\gamma=\dfrac{(d-r)b}{g}$

$$z_{NP} = -\frac{(1-\tau)db+(1-\tau)iw(a+b)-\frac{(d-r)b}{g}(1-\tau)iw}{1+g}$$

A2.1.2b Derivations of the NP function properties

- *Net working capital / revenues (a)*

Follows directly from equation (10).

- *Gross PPE / revenues (b)*

Differentiating the net profit expression (10) with respect to b yields:

(i) $$\frac{\partial NP_t}{\partial b}=R_0(1+g)^{t-1}\left(-(1-\tau)d-\chi+\frac{(d-r)}{g}\chi\right)-R_0\frac{(d-r)}{g}\chi$$

Intuitively, net profit should be decreasing in b, since a lower b-value means a more efficient use of company capital. From equation (i) it is clear that this is not necessarily the case. The critical case to examine is when t is very large. Then it must hold that:

(ii) $$-(1-\tau)d - \chi + \frac{(d-r)}{g}\chi < 0$$

Then, a *sufficient* condition for (ii) to hold is that *(d-r)* is smaller than or equal to g:

(iii) $$(d-r) \leq g$$

Multiplying both sides by bR_{t-1} yields:

(iv) $$(d-r)bR_{t-1} \leq gbR_{t-1}$$

Using the definitions of the parameters, it can be seen that this is the same thing as:

(v) $$A_t - A_{t-1} \leq G_t - G_{t-1}$$

Or, alternatively:

(vi) $$N_t - N_{t-1} \geq 0$$

Hence, it is a ***sufficient condition*** *that the net property plant and equipment will not be decreasing between years in the parametric steady state period,* which seems a reasonable restriction on the parameters.

- *Change in deferred taxes / gross PPE (c)*

The parameter c does not appear in the NP function.

- *Growth rate (g)*

$$\frac{\partial NP_t}{\partial g} = R_{t-1}\left[t\cdot\left(m + z_{NP}\right) - \frac{1}{g}\cdot\gamma\cdot\chi - z_{NP}\right] + \frac{R_0}{g}\cdot\gamma\cdot\chi$$

This partial derivative is hard to interpret. However as $t \rightarrow \infty$ it will definitely grow positive since the term $m + z_{NP}$ is positive for all relevant cases in a steady state.

- *Interest rate on debt (i)*

See the same section under *dividends*.

- *Operating expenses / revenues (p)*

Follows trivially from equation (10).

- *Tax rate, τ*

Differentiating equation (2):

$$\frac{\partial NP_t}{\partial \tau} = -\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} - bR_{t-1} - A_{t-1}\right)\right)$$

This is simply the pre-tax income multiplied by minus one, and the somewhat trivial conclusion is that in a profit-making company the net profit is decreasing in τ, i.e. the net profit will be smaller the higher the tax rate.

A2.1.3a Derivation of dividend expression - equation (12)

$$
\begin{aligned}
DIV_t &= (1-w)(aR_{t-1}+bR_{t-1}-A_{t-1})-T_{t-1}+\\
&\quad+(1-\tau)(R_t-pR_t-dbR_{t-1}-iw(aR_{t-1}+bR_{t-1}-A_{t-1}))-\\
&\quad-[(1-w)(aR_t+bR_t-A_t)-T_t]=\\
&= R_{t-1}[(1-w)(a+b)-(1-\tau)(db+iw(a+b))]+\\
&\quad+R_t[m-(1-w)(a+b)]+\\
&\quad+A_{t-1}[(1-\tau)iw-(1-w)]+\\
&\quad+A_t(1-w)+\\
&\quad+T_t-T_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
DIV_t &= (1+g)^{t-1}R_0[(1-w)(a+b)-(1-\tau)(db+iw(a+b))]+\\
&\quad+(1+g)^t R_0[m-(1-w)(a+b)]+\\
&\quad+\frac{(1+g)^{t-1}-1}{g}(d-r)bR_0[(1-\tau)iw-(1-w)]+A_0[(1-\tau)iw-(1-w)]+\\
&\quad+\frac{(1+g)^t-1}{g}(d-r)bR_0(1-w)+A_0(1-w)+\\
&\quad+cbR_0(1+g)^t
\end{aligned}
$$

$$
\begin{aligned}
DIV_t &= (1+g)^t R_0\cdot[m+cb]+(1+g)^{t-1}R_0[(d-r)b(1-w)-(a+b)(1-w)g\\
&\quad-(1-\tau)bd-(1-\tau)iw(a+b)]+\\
&\quad+\frac{(1+g)^{t-1}-1}{g}R_0(1-\tau)iw(d-r)b+A_0(1-\tau)iw\\
&= (1+g)^t R_0\cdot[m+cb]+(1+g)^{t-1}R_0[b[d\tau-w(d-r)-r]\\
&\quad-(a+b)(1-w)g-(a+b)(1-\tau)iw]+\frac{(1+g)^{t-1}R_0(1-\tau)iw(d-r)b}{g}\\
&\quad-R_0\frac{(1-\tau)iw(d-r)b}{g}+A_0(1-\tau)iw
\end{aligned}
$$

$$= \quad (1+g)^t R_0 \left[m + cb + \frac{b[d\tau - w(d-r) - r] - (a+b)(1-w)g - (a+b)(1-\tau)iw}{1+g} \right.$$

$$\left. + \frac{(1-\tau)iw(d-r)b}{g(1+g)} \right] - (1-\tau)iw \left[R_0 \frac{(d-r)b}{g} - A_0 \right]$$

and the following expression is obtained for dividends payed out during year t:

$$DIV_t = (1+g)^t R_0 \left[m + z_{DIV} \right] - \chi \left[R_0 \cdot \gamma - A_0 \right]$$

where :

$$m = (1-\tau)(1-p) \quad \text{(the after - tax profit margin)}$$

$$z_{DIV} = cb + \frac{b[d\tau + w(r-d+g) - r - g - \chi] - a[g(1-w) + \chi]}{1+g} + \frac{(d-r)b\chi}{g(1+g)}$$

$$\chi = (1-\tau)iw$$

$$\gamma = \frac{(d-r)b}{g}$$

A2.1.3b Derivations of the DIV function properties

- *Net working capital / revenues (a)*

$$\frac{\partial DIV_t}{\partial a} = -(1+g)^{t-1} R_0 \left[g(1-w) + (1-\tau)iw \right] < 0$$

- *Gross PPE / revenues (b)*

$$\frac{\partial DIV_t}{\partial b} = (1+g)^t R_0 \left[c + \frac{d\tau + w(r-d+g) - r - g - \chi}{1+g} + \frac{(d-r)\chi}{g(1+g)} \right] - \chi R_0 \frac{d-r}{g}$$

Then the dividend is decreasing in b if and only if $\frac{\partial DIV_t}{\partial b} < 0$, i.e. iff:

$$(1+g)^t R_0 \left[c + \frac{d\tau + w(r-d+g) - r - g - \chi}{1+g} + \frac{(d-r)\chi}{g(1+g)} \right] - \chi R_0 \frac{d-r}{g} < 0$$

Rearranging:

$$c + \frac{d\tau + w(r-d+g) - r - g - \chi}{1+g} < \chi(d-r)\left[\frac{1}{g(1+g)^t} - \frac{1}{g(1+g)} \right]$$

$$c(1+g) + d\tau + w(r-d+g) - r - g - \chi < \chi(d-r)(1+g)\frac{1-(1+g)^{t-1}}{g(1+g)^t}$$

$$c(1+g) + d\tau + w(r-d+g) - r - g - \chi < \frac{\chi(d-r)}{g}\left[\frac{1}{(1+g)^{t-1}} - 1 \right]$$

$$d\tau + wg + c(1+g) + \frac{\chi(d-r)}{g}\left[1 - \frac{1}{(1+g)^{t-1}} \right] - w(d-r) - r - \chi < g$$

The most critical case will be for large t:s. Letting $t \to \infty$ yields the final condition:

$$d\tau + wg + c(1+g) + \frac{\chi(d-r)}{g} - w(d-r) - r - \chi < g$$

- *Change in deferred taxes / gross PPE (c)*

$$\frac{\partial DIV_t}{\partial c} = b \cdot R_0 (1+g)^t > 0$$

The book equity should always be non-negative:

$$[(1-w)(aR_t + bR_t - A_t) - T_t] > 0 \quad \forall t$$

Substituting in the solution to the system of difference equations:

$$(1-w)(a+b)(1+g)^t R_0 - (1-w)\left[\frac{(1+g)^t - 1}{g} bR_0 (d-r) + A_0\right]$$

$$-\left[\frac{(1+g)^t - 1}{g} bR_0 c(1+g) + T_0\right] > 0$$

Rearranging gives:

$$(1-w)(a+b) > \left(\frac{b}{g} - \frac{b}{g(1+g)^t}\right)[(d-r)(1-w) + c(1+g)] + \frac{(1-w)A_0 + T_0}{R_0 (1+g)^t}$$

On the right hand side there are two time-dependent effects with opposite directions. Taking time into consideration, the following two boundary conditions are obtained, for t=1 and $t \to \infty$ respectively:

1. $(1-w)(a+b) > \left(\frac{b}{g} - \frac{b}{g(1+g)}\right)[(d-r)(1-w) + c(1+g)] + \frac{(1-w)A_0 + T_0}{R_0 (1+g)}$

2. $(1-w)(a+b) > \frac{b}{g}[(d-r)(1-w) + c(1+g)]$

- *The interest rate on debt (i)*

$$\frac{\partial DIV_t}{\partial i} = R_0 (1+g)^t \left[\frac{-(a+b)(1-\tau)w + \frac{d-r}{g} b(1-\tau)w}{1+g}\right] -$$

$$-R_0 \frac{d-r}{g} b(1-\tau)w + (1-\tau)wA_0 \qquad \left(= \frac{\partial NP_t}{\partial i}\right)$$

Rearranging:

$$R_0 \left(1+g\right)^{t-1} (1-\tau)w\left[-(a+b)+\frac{d-r}{g}b\right]-R_0 \frac{d-r}{g}b(1-\tau)w+(1-\tau)wA_0$$

Looking at the two boundary cases in order to find the expression for determining the sign:

$(t=1)$:

$$(1-\tau)wR_0\left[-(a+b)+\frac{d-r}{g}b\right]-R_0 \frac{d-r}{g}b(1-\tau)w+(1-\tau)wA_0$$

which, in order to determine the sign, can be simplified to

$$-(a+b)R_0 + A_0 = -\left[\text{balance sheet total}\right] < 0$$

$(t \to \infty)$:

Searching for the condition for a negative derivative:

$$(a+b) > \frac{(d-r)b}{g}$$

$$\left(a+b\right)R_t > \frac{(d-r)b}{g}R_t$$

$$\textit{net working capital}_t + G_t > \frac{d-r}{g}G_t$$

Applying the condition for *net PPE* to be non-decreasing over time, $d-r \leq g$:

$$\textit{net working capital}_t + \{\textit{something non-negative}\} > 0$$

Thus a sufficient condition for dividends to be decreasing in *i* is that *a*>0, i.e. that the net working capital in parametric steady state is positive. In the PSS period the balance sheet total must be positive. Hence, in addition to the non-decreasing *net PPE* condition, one must assume that a, the *net working capital* to *revenues* ratio is larger than zero, and as a consequence DIV_t will always be decreasing in i.

- *The tax rate (τ)*

$$\frac{\partial DIV_t}{\partial \tau} = \left(-1 + p + \frac{db + aiw + biw}{1+g} - \frac{(d-r)b}{g(1+g)} iw \right) (1+g)^t R_0 + iw \left(\frac{(d-r)b}{g} R_0 - A_0 \right)$$

This is exactly the same expression as for net profits with respect to τ. See that section above for the further proof.

A2.1.4 Proof of proposition 2.2

Expression (1) for the free cash flow in year 0 can be rewritten as:

$$\begin{aligned} FCF_0 &= (1-\tau)(R_0 - pR_0 - db'R_{(-1)}) + db'R_{(-1)} + T_0 - T_{(-1)} \\ &\quad - (aR_0 - a'R_{(-1)}) - (bR_0 - b'R_{(-1)} + rb'R_{(-1)}) \\ &= (1-\tau)(R_0 - pR_0 - db'R_{(-1)}) + db'R_{(-1)} + cbR_0 \\ &\quad - (aR_0 - a'R_{(-1)}) - (bR_0 - b'R_{(-1)} + rb'R_{(-1)}) \end{aligned}$$

where a and b are constants for $t \geq 0$, and a' and b' are the corresponding parameters for $t = (-1)$. Note that $R_0 = (1+g)R_{(-1)}$.

It is easy to see that FCF_0 will generally not be the same as $FCF_1 / (1+g)$ if $a \neq a'$ and/or $b \neq b'$. Thus FCF_0 is not the free cash flow that will grow at the rate g for the following years and year 0 is therefore not the first FCF steady state year. Instead, as equation (7) shows, year 1 will be the first FCF steady state year. Q.E.D.

A2.2 Specification B

A.2.2.0 Solution to system of difference equations

Revenues, gross PPE and taxes:

The definitions are given in the text and can be formalised as follows:

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ G_t &= G_{t-1} + eR_t - rG_{t-1} \\ T_t &= cG_t + T_{t-1} \end{cases}$$

or equivalently:

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ G_t &= e(1+g)R_{t-1} + (1-r)G_{t-1} \\ T_t &= ce(1+g)R_{t-1} + c(1-r)G_{t-1} + T_{t-1} \end{cases}$$

In matrix notation:

$$\mathbf{x_t} = \mathbf{A}\mathbf{x_{t-1}}$$

where: $\mathbf{x_t} = \begin{pmatrix} R_t \\ G_t \\ T_t \end{pmatrix}$ and $\mathbf{A} = \begin{pmatrix} 1+g & 0 & 0 \\ e(1+g) & 1-r & 0 \\ ce(1+g) & c(1-r) & 1 \end{pmatrix}$

A is diagonalised by **P**:

$$\mathbf{P} = \begin{pmatrix} \dfrac{g+r}{e(1+g)} & 0 & 0 \\ 1 & 1 & 0 \\ \dfrac{c(1+g)}{g} & -c\dfrac{1-r}{r} & 1 \end{pmatrix}$$

and thus:

$$\mathbf{P}^{-1}\mathbf{AP} = \begin{pmatrix} 1+g & 0 & 0 \\ 0 & 1-r & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Substituting $\mathbf{x}_t = \mathbf{Py}_t$ and $\mathbf{x}_{t+1} = \mathbf{Py}_{t+1}$ yields the system $\mathbf{y}_{t+1} = \mathbf{P}^{-1}\mathbf{APy}_t$, the solution to which is:

$$\mathbf{y}_t = \begin{pmatrix} k_1(1+g)^t \\ k_2(1-r)^t \\ k_3 \end{pmatrix}$$

Substituting back yields:

$$\mathbf{x}_t = \mathbf{Py}_t = \begin{pmatrix} \dfrac{g+r}{e(1+g)} & 0 & 0 \\ 1 & 1 & 0 \\ \dfrac{c(1+g)}{g} & -c\dfrac{1-r}{r} & 1 \end{pmatrix} \begin{pmatrix} k_1(1+g)^t \\ k_2(1-r)^t \\ k_3 \end{pmatrix}$$

$$= \begin{pmatrix} k_1(1+g)^t \dfrac{g+r}{e(1+g)} \\ k_1(1+g)^t + k_2(1-r)^t \\ k_1(1+g)^t \dfrac{c(1+g)}{g} - k_2(1-r)^t c\dfrac{1-r}{r} + k_3 \end{pmatrix}$$

and since the initial values are R_0, G_0 and T_0, the constants k_1, k_2 and k_3 are found to be:

$$\begin{cases} k_1 &= \dfrac{e(1+g)}{g+r} R_0 \\ k_2 &= G_0 - \dfrac{e(1+g)}{g+r} R_0 \\ k_3 &= T_0 - \dfrac{c(1+g)}{g} \dfrac{e(1+g)}{g+r} R_0 + \dfrac{c(1-r)}{r} \left(G_0 - \dfrac{e(1+g)}{g+r} R_0 \right) \end{cases}$$

The complete solution is:

$$\begin{cases} R_t &= R_0 (1+g)^t \\ G_t &= \dfrac{e(1+g)}{g+r} R_0 (1+g)^t + \left(G_0 - \dfrac{e(1+g)}{g+r} R_0 \right) (1-r)^t \\ T_t &= T_0 + \dfrac{(1+g)^t - 1}{g} c(1+g) e \dfrac{(1+g)}{g+r} R_0 + \dfrac{c(1-r)}{r} \left(G_0 - \dfrac{e(1+g)}{g+r} R_0 \right) \left(1 - (1-r)^t\right) \end{cases}$$

The expression for the accumulated depreciation can be derived in a similar way.

A2.2.1 Derivation of FCF expression - equation (48)

The definition of free cash flow in any year t is:

$$\begin{aligned} FCF_t &= (1-\tau)\left(R_t - pR_t - dG_{t-1}\right) + dG_{t-1} + T_t - T_{t-1} \\ &\quad - \left(aR_t - aR_{t-1}\right) - \left(G_t - G_{t-1} + rG_{t-1}\right) \\ &= aR_{t-1} + \left((1-\tau)(1-p) - a\right)R_t + \left(1 - r + \tau d\right)G_{t-1} - G_t + T_t - T_{t-1} \end{aligned}$$

Substituting the solutions for the state-variables yields:

$$FCF_t = a(1+g)^{t-1} R_0 + ((1-\tau)(1-p)-a)(1+g)^t R_0 +$$

$$+(1-r+\tau d)\left[e\frac{1+g}{g+r}(1+g)^{t-1} R_0 + \left(G_0 - e\frac{1+g}{g+r}R_0\right)(1-r)^{t-1}\right] -$$

$$-\left[\frac{e(1+g)}{g+r}R_0(1+g)^t + \left(G_0 - \frac{e(1+g)}{g+r}R_0\right)(1-r)^t\right] +$$

$$+\frac{(1+g)^t-1}{g}c(1+g)e\frac{(1+g)}{g+r}R_0 + \frac{c(1-r)}{r}\left(G_0 - \frac{e(1+g)}{g+r}R_0\right)\left(1-(1-r)^t\right) -$$

$$-\left[\frac{(1+g)^{t-1}-1}{g}c(1+g)e\frac{(1+g)}{g+r}R_0 + \frac{c(1-r)}{r}\left(G_0 - \frac{e(1+g)}{g+r}R_0\right)\left(1-(1-r)^{t-1}\right)\right]$$

Substituting $\beta = e\frac{1+g}{g+r}$ and $m = (1-\tau)(1-p)$, and rearranging gives:

$$FCF_t = (1+g)^t R_0\left(m - \frac{ag}{1+g}\right) + \beta(1+g)^t R_0 \cdot \left(\frac{1-r+\tau d}{1+g} - 1\right) +$$

$$+(1-r)^{t-1}(G_0 - \beta R_0)((1-r+\tau d)-(1-r)) + (1+g)^t R_0 c\beta +$$

$$+\frac{c(1-r)}{r}(G_0 - \beta R_0)\left[(1-r)^{t-1} - (1-r)^t\right]$$

Simplifying further:

$$FCF_t = (1+g)^t R_0\left(m - \frac{ag - \beta(1-r+\tau d)}{1+g} - \beta + c\beta\right)$$

$$+\tau d(G_0 - \beta R_0)(1-r)^{t-1} + (G_0 - \beta R_0)c(1-r)^t$$

and finally:

$$FCF_t = (1+g)^t R_0 \left(m + \frac{\beta(c + \tau d - r) - g(\beta(1-c) + a)}{1+g} \right) + \left(c + \tau \frac{d}{1-r} \right) (G_0 - \beta R_0)(1-r)^t$$

This is expression (47) in section 2.2.1. The final expression (48) is obtained by introducing a couple of new constants.

A2.2.2 Derivation of NP expression - equation (51)

The net profit in year t is given by:

$$NP_t = (1-\tau)(R_t - pR_t - dG_{t-1} - iw(aR_{t-1} + G_{t-1} - A_{t-1}))$$

Rearranging and introducing the constants $m = (1-\tau)(1-p)$ and $\chi = (1-\tau)iw$:

$$NP_t = R_{t-1}\chi(-a) + R_t m + G_{t-1}(1-\tau)(-d - iw) + A_{t-1}\chi$$

Substituting the expressions for *revenues*, *gross PPE* and *accumulated depreciation*, and letting $\beta = e\frac{1+g}{g+r}$ yields:

$$\begin{aligned} NP_t \quad = \quad & (1+g)^{t-1} R_0 \cdot \chi \cdot (-a) + (1+g)^t R_0 m + \\ & + \left[\beta R_0 (1+g)^{t-1} + (G_0 - \beta R_0)(1-r)^{t-1} \right] (1-\tau)(-d - iw) + \\ & + \left[A_0 + \frac{(1+g)^{t-1} - 1}{g} (d-r) \beta R_0 + \frac{d-r}{r} (G_0 - \beta R_0) \left(1 - (1-r)^{t-1} \right) \right] \chi \end{aligned}$$

Factoring out:

$$NP_t = (1+g)^t R_0 \left(m + \frac{\chi \cdot (-a) + (1-\tau)(-d-iw)\beta + \chi \dfrac{(d-r)}{g}\beta}{1+g} \right) +$$

$$+(1-\tau)(-d-iw)(G_0 - \beta R_0)(1-r)^{t-1} - \chi \frac{(d-r)}{g}\beta R_0 + \chi A_0 +$$

$$+\chi\left[\frac{d-r}{r}(G_0 - \beta R_0)\left(1-(1-r)^{t-1}\right)\right]$$

Rearranging:

$$NP_t = (1+g)^t R_0 \left(m + \frac{\chi \cdot (-a) + (1-\tau)(-d-iw)\beta + \chi \dfrac{(d-r)}{g}\beta}{1+g} \right) -$$

$$-\chi\left[\frac{(d-r)}{g}\beta R_0 - A_0\right] - \chi\frac{(d-r)}{r}(\beta R_0 - G_0) +$$

$$+\left[\chi\frac{(d-r)}{r(1-r)} + (1-\tau)\frac{d+iw}{1-r}\right](\beta R_0 - G_0)(1-r)^t$$

Introducing three more constants yields equation (51).

A2.2.3 Derivation of DIV expression - equation (55)

The expression for dividends in year t is:

$$\begin{aligned} DIV_t = {} & (1-w)\left(aR_{t-1} + G_{t-1} - A_{t-1}\right) - T_{t-1} + \\ & + (1-\tau)\left((1-p)R_t - dG_{t-1} - iw\left(aR_{t-1} + G_{t-1} - A_{t-1}\right)\right) - \\ & - \left[(1-w)\left(aR_t + G_t - A_t\right) - T_t\right] \end{aligned}$$

Rearranging and recognising that $T_t - T_{t-1} = cG_t$:

$$\begin{aligned} DIV_t = \; & R_t\left((1-\tau)(1-p)-a(1-w)\right)+R_{t-1}\left(a((1-w)-(1-\tau)iw)\right) \\ & + G_t\left(c-1+w\right)+G_{t-1}\left(1-w-d(1-\tau)-(1-\tau)iw\right) \\ & + A_t\left(1-w\right)+A_{t-1}\left(w-1+(1-\tau)iw\right) \end{aligned}$$

Substituting the expressions for the state variables and simplifying:

$$\begin{aligned} DIV_t = \; & R_0(1+g)^t \cdot \left((1-\tau)(1-p)-a(1-w)\right)+R_0(1+g)^{t-1}\left(a((1-w)-(1-\tau)iw)\right)- \\ & -\left(\beta R_0(1+g)^t+\left(G_0-\beta R_0\right)(1-r)^t\right)(1-c-w)+ \\ & +\left(\beta R_0(1+g)^{t-1}+\left(G_0-\beta R_0\right)(1-r)^{t-1}\right)\left(1+w(r-d)-(1-\tau)iw+\tau d-w-r\right)+ \\ & +\left(A_0+\frac{(1+g)^{t-1}-1}{g}(d-r)\beta R_0+\frac{d-r}{r}\left(G_0-\beta R_0\right)\left(1-(1-r)^{t-1}\right)\right)(1-\tau)iw \end{aligned}$$

Collecting terms and using the constants $m=(1-\tau)(1-p)$ and $\chi=(1-\tau)iw$:

$$\begin{aligned} DIV_t = \; & R_0(1+g)^t\left(m+c\beta+\frac{\beta\left(d\tau+w(r-d+g)-r-g-\chi\right)-a\left(g(1-w)+\chi\right)+\chi\frac{(d-r)}{g}\beta}{1+g}\right) \\ & -\chi\left(\frac{d-r}{g}\beta R_0-A_0\right)-\left(\chi\frac{d-r}{r}\right)\left(\beta R_0-G_0\right)- \\ & -\left(c+\frac{d}{1-r}\left(\tau-w-\frac{\chi}{r}\right)\right)\left(\beta R_0-G_0\right)(1-r)^t \end{aligned}$$

Introducing two more constants gives equation (55).

A2.3 Specification A with capital-based reserve

A2.3.1 A more complex tax-system

In order to show the general usefulness of the modelling approach considered in this report, we will in this appendix apply the methods to a very complicated and specific case.[47] Thereby, we wish to visualise the fact that this modelling approach can be extended and specialised to capture virtually any specific legal system and/or relations between parameters.

In the report, the specification of taxes has been rather rudimentary. The possibility of tax deferrals has been modelled as the difference equation:

$$T_t = cbR_t + T_{t-1}$$

with: T_t deferred taxes, year t

c increase in deferred taxes in % of gross PPE

bR_t gross PPE, year t (using Spec. A)

In a specific case, it is possible to explicitly model the company's tax operations in accordance with the tax system under consideration. A modified system of difference equations is then obtained. An interesting example from Sweden is the *capital-based reserve* (*K-SURV*), which was proposed in a government committee report in 1989 and a variation of it has also been in operation for a few years but is now being gradually abolished. The technical details are taken from the government committee report SOU 1989:34.

[47] All derivations are presented in section A2.3.6.

The *capital-based reserve* is an untaxed reserve, which can maximally be 30% of the capital base, defined as the firm's ending equity plus the capital-based reserve itself. Hence, the expression for the untaxed capital-based reserve, U_t, becomes:[48]

$$
\begin{aligned}
U_t \quad = \quad & s\lfloor a \quad R_{t-1} + bR_{t-1} - A_{t-1} \quad + \\
& + R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) - \\
& - w(a \quad R_t + bR_t - A_t) - \\
& - \tau(R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) - (U_t - U_{t-1}))]
\end{aligned}
$$

where s denotes the (maximal) percentage of the capital base that may be considered an untaxed reserve (in Sweden 30%) and U_t is the untaxed reserve in year t.

Solving the system of difference equations now yields the following result:

$$
\begin{aligned}
R_t &= R_0 (1+g)^t \\
A_t &= A_0 + \frac{(1+g)^t - 1}{g}(d-r)bR_0 \\
U_t &= \theta \quad R_0 (1+g)^t + (U_0 - \theta \quad R_0 - \xi)(-\psi)^t + \xi
\end{aligned}
$$

where *6*, ξ and ψ are constants:

$$
\theta = \frac{s\left((a+b)(1-(1+(1-\tau)i)w - gw) - (1-\tau)db + (d-r)bw + (1+g)(1-p)(1-\tau) - (1-(1+(1-\tau)i)w)\frac{d-r}{g}b\right)}{1+g(1-s\tau)}
$$

$$
\xi = s(1-(1+(1-\tau)i)w)\left(\frac{d-r}{g}bR_0 - A_0\right)
$$

$$
\psi = \frac{s\tau}{1-s\tau}
$$

---

[48] In words, the expression states the following:

$$
\begin{aligned}
U_t \quad = \quad & s \cdot [\text{entering assets} \\
& + \text{this yr's profits before taxes} \\
& - \text{debts} \\
& - \text{this yr's taxes}]
\end{aligned}
$$

The equation for the untaxed reserve may look quite complex, and there is indeed not much insight to be gained from trying to interpret the new parameters *6,* ξ and ψ. Instead, attention will be focused on the development over time of the untaxed-reserves-variable. The constant ψ is different from zero[49] so the untaxed reserves will actually exhibit an oscillating behaviour over time. Since (the absolute value of) ψ is also strictly smaller than one in all reasonable cases, the untaxed reserves will eventually approach $\theta R_0 (1+g)^t + \xi$. This will have implications for the free cash flow, the net profit and the dividends.

A2.3.2 Free cash flow

The expression for the free cash flow in year t is:

$$
\begin{aligned}
FCF_t &= R_t - pR_t - \tau\left(R_t - pR_t - dbR_{t-1} - \left(U_t - U_{t-1}\right)\right) \\
&\quad - \left(aR_t - aR_{t-1}\right) - \left(bR_t - bR_{t-1} + rbR_{t-1}\right) \\
&= R_{t-1}\left(a + (1-r)b + \tau db\right) + R_t\left((1-\tau)(1-p) - a - b\right) + \tau\left(U_t - U_{t-1}\right)
\end{aligned}
$$

Substituting the expressions for R_t and U_t into the free cash flow expression yields the following equation:

$$
FCF_t = (1+g)^t R_0 \left(m + z_{FCF(UR)}\right) + \tau\varsigma(-\psi)^t
$$

where the constants are defined as before, but adding:

$$
z_{FCF(UR)} = \frac{b(\tau d - r) - g(a + b - \tau\theta)}{1+g} \quad \text{and} \quad \varsigma = \frac{\psi+1}{\psi}\left(U_0 - \theta R_0 - \xi\right)
$$

[49] In the system under consideration s=0.3 and τ=0.3, and thus ψ=0.0989

The original free cash flow expression was:

$$(7) \qquad FCF_t = (1+g)^t R_0 (m + z_{FCF})$$

where $z_{FCF} = \dfrac{b(c + \pi d - r) - g(b(1-c)+a)}{1+g}$

There is a slight difference in the two z-constants, but this is due to the different technical specifications of tax deferrals. The other - and more interesting - difference is of a qualitative nature: The free cash flow in the capital-based reserve case will have an oscillating trajectory *around* the straight line given by $(1+g)^t R_0 (m + z_{FCF(UR)})$, whereas in the original case the free cash flow grows at the constant rate g. The growth rate in the capital-based reserve case will, however, approach g as t becomes larger; with reasonable parameters after only a few years. In the following table and graph the development of free cash flow in the example company is shown (g is 5%):[50]

Year	FCF	Growth rate
1	23,34011	
2	22,46945	-3,7303%
3	23,79445	5,8969%
4	24,96424	4,9162%
5	26,21442	5,0079%
6	27,52495	4,9993%
7	28,90121	5,0001%
8	30,34627	5,0000%
9	31,86358	5,0000%
10	33,45676	5,0000%

Table A2.3:1 - Free cash flows and growth rates withexample company characteristics using untaxed capital-based reserve

[50] a=5%, b=40%, d=6%, r=4%, w=40%, g=5%, i=10%, τ=30%, p=90%, R_0=500, A_0=125 as before and adding: s=30%, U_0=18.

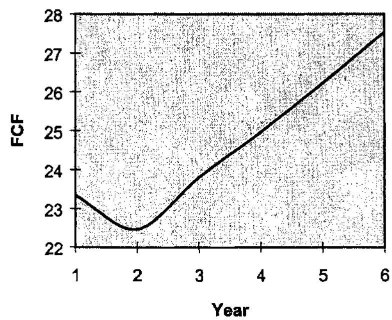

Figure A2.3:1 - Free cash flow with untaxed capital-based reserve

With a tax system of this kind, there is thus a time lag between the *parametric steady state* and the *free cash flow steady state*, i.e. it takes a few years with constant parameters for the firm's cash flow development to settle down to a constant growth (note that mathematically this is strictly true only when $t \to \infty$). In order to calculate the horizon value one must take the discount rate into consideration. In this case the initial condition of Proposition 2.3 and Corollary 2.1 will not be sufficient to ensure a constant capital structure over the PSS period, due to the oscillating behaviour.

In the final value calculation, which in this case only will be an approximation (since the WACC will not be constant), it is necessary to calculate a correction term in addition to the Gordon formula:

$$Equityvalue(FCF) \approx \frac{(1+g)R_0(1+z_{UR})}{k_{WACC}-g} - \tau(U_0 - \theta R_0 - \xi)\frac{1+\psi}{1+\psi+k_{WACC}} - D_0$$

The correction term is quite small for normal parameter values. This is due to the fact that the oscillating behaviour of the free cash flow will be dampened and almost fade after only a few years.

A2.3.3 Net profit

The net profit expression is:

$$NP_t = (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - \left(U_t - U_{t-1}\right)\right)$$

Substituting the expressions for revenues, accumulated depreciation and untaxed reserves, equations and rearranging yields:

$$NP_t = (1+g)^t R_0 \left(m + z_{NP(UR)}\right) - \chi\left(R_0 \frac{(d-r)b}{g} - A_0\right) - (1-\tau)\varsigma(-\psi)^t$$

with: $m = (1-\tau)(1-p)$, $\chi = (1-\tau)iw$, $\varsigma = \dfrac{\psi+1}{\psi}\left(U_0 - \theta R_0 - \xi\right)$

$$z_{NP(UR)} = -\frac{(1-\tau)\left(db + g\theta + iw\left(a + b - \frac{(d-r)b}{g}\right)\right)}{1+g}$$

Thus, net profit will also exhibit an oscillating behaviour, albeit the oscillation quickly becomes negligible using reasonable parameter values. Figure A2.3:2 below is obtained using the example company data:

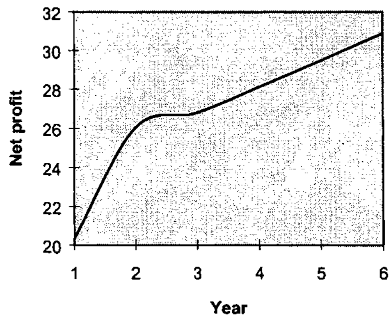

Figure A2.3:2 - Net profit with untaxed capital-based reserve

The oscillation in the net profit is somewhat more pronounced than in the free cash flow, since the tax rate is set at 30% and the net profit oscillation therefore is multiplied by 0.7 whereas the free cash flow oscillation is multiplied by 0.3. The presence of an oscillating term is, however, further obscured in the net profit case by the constant $\chi\left(R_0\left(d-r\right)b/g-A_0\right)$, which has to be deducted each year, thus causing the net profit not to grow constantly even if there were no untaxed reserves.

A2.3.4 Dividends

The dividend expression is similar to the original case (equation 12), but also here with an additional term causing an oscillating behaviour:

$$DIV_t = \left(1+g\right)^t R_0 \left(m + z_{DIV(UR)}\right) - \chi\left(R_0\gamma - A_0\right) + \tau\varsigma\left(-\psi\right)^t$$

with the following constants:

$$m = (1-\tau)(1-p), \quad \chi = (1-\tau)iw, \quad \gamma = \frac{(d-r)b}{g}, \quad \varsigma = \frac{\psi+1}{\psi}\left(U_0 - \theta R_0 - \xi\right)$$

$$z_{DIV(UR)} = \frac{b\left[d\tau + w(r-d+g) - r - g - (1-\tau)iw\right] - a\left[g(1-w) + (1-\tau)iw\right] + \frac{(d-r)b}{g}(1-\tau)iw + \tau\theta g}{1+g}$$

Qualitatively, dividends exhibit the same behaviour as free cash flow: an oscillating term, $(-\psi)^t$, multiplied by the constant $\tau \cdot \varsigma$. Using once again the example company to visualise:

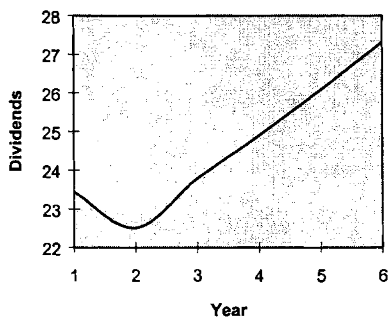

Figure A2.3:3 - Dividends with untaxed capital-based reserve

A2.3.5 Concluding remarks

When introducing capital-based reserves, there will not exist any FCF, DIV or NP steady state, at least not in the strict mathematical sense. The reason for this is that the *FCF, NP,* and *DIV* sequences have an oscillating behaviour. Only as $t \to \infty$ will the functions grow exactly by a constant growth rate. Numerically, however, the different functions will tend towards a steady state in a few years.

A2.3.6 Derivations for A2.3.5

Solution to system of difference equations

The expression for the untaxed reserves is:

$$\begin{aligned} U_t \;=\;& s\lfloor a \quad R_{t-1} + bR_{t-1} - A_{t-1} \quad + \\ +\;& R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) - \\ -\;& w(a \quad R_t + bR_t - A_t) - \\ -\;& \tau(R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) - (U_t - U_{t-1}))\rfloor \end{aligned}$$

After rearranging:

$$\begin{aligned} U_t \;=\;& \frac{s}{1-s\tau}[a \quad R_{t-1} + bR_{t-1} - A_{t-1} \quad + \\ +\;& R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) - \\ -\;& w(a \quad R_t + bR_t - A_t) - \\ -\;& \tau(R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) + U_{t-1})] \end{aligned}$$

Keeping in mind that $A_t = (d-r)bR_{t-1} + A_{t-1}$. U_t can equivalently be written:

$$\begin{aligned} U_t \;=\;& \frac{s}{1-s\tau}[a \quad R_{t-1} + bR_{t-1} - A_{t-1} + \\ +\;& R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) - \\ -\;& w(a \quad R_t + bR_t - (d-r)bR_{t-1} - A_{t-1}) - \\ -\;& \tau(R_t - pR_t - dbR_{t-1} - iw(a \quad R_{t-1} + bR_{t-1} - A_{t-1}) + U_{t-1})] \end{aligned}$$

Now define the following constants:

$$q_1 = a + b - (1-\tau)db - (1-\tau)iw(a+b) + w(d-r)b + (1+g)((1-\tau)(1-p) - w(a+b))$$

$$q_2 = 1 - (1 + (1-\tau)i)w$$

The untaxed reserves will be:

$$U_t = \frac{s}{1-s\tau}\left(q_1 R_{t-1} - q_2 A_{t-1} - \tau U_{t-1}\right)$$

The expression for A_{t-1} is known (from section A2.1):

$$A_{t-1} = \frac{(1+g)^{t-1}-1}{g}(d-r)bR_0 + A_0$$

Define the following constants:

$$q_3 = \frac{s}{1-s\tau}\left(q_1 - q_2\frac{(d-r)b}{g}\right)$$

$$q_4 = \frac{s}{1-s\tau}q_2\left(\frac{(d-r)b}{g}R_0 - A_0\right)$$

$$q_5 = \frac{s\tau}{1-s\tau}$$

The untaxed reserves can now be expressed as:

$$U_t = q_3 R_{t-1} + q_4 - q_5 U_{t-1}$$

The following system of difference equations can now be solved:

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ U_t &= q_3 R_{t-1} - q_5 U_{t-1} + q_4 \end{cases}$$

In matrix notation:

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{Q}$$

where: $\mathbf{x}_t = \begin{pmatrix} R_t \\ U_t \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} 1+g & 0 \\ q_3 & -q_5 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 0 \\ q_4 \end{pmatrix}$

The homogenous equation is:

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1}$$

The roots of the characteristic equation are $\lambda_1 = 1+g$ and $\lambda_2 = -q_5$.

A is diagonalised by **P**:

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ \dfrac{q_3}{q_5 + 1 + g} & 1 \end{pmatrix}$$

And:

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} 1+g & 0 \\ 0 & -q_5 \end{pmatrix}$$

Substituting $\mathbf{x}_t = \mathbf{P}\mathbf{y}_t$ and $\mathbf{x}_{t+1} = \mathbf{P}\mathbf{y}_{t+1}$ yields the system $\mathbf{y}_{t+1} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}\mathbf{y}_t$, the solution to which is:

$$\mathbf{y}_t = \begin{pmatrix} c_1 (1+g)^t \\ c_2 (-q_5)^t \end{pmatrix}$$

Substituting back yields:

$$\mathbf{x_t} = \mathbf{P}\mathbf{y_t} = \begin{pmatrix} 1 & 0 \\ \dfrac{q_3}{q_5+1+g} & 1 \end{pmatrix} \begin{pmatrix} c_1(1+g)^t \\ c_2(-q_5)^t \end{pmatrix} = \begin{pmatrix} c_1(1+g)^t \\ c_1(1+g)^t \dfrac{q_3}{q_5+1+g} + c_2(-q_5)^t \end{pmatrix}$$

A particular solution to the system of equations has to be added. The steady state will serve as particular solution:

$$\mathbf{x}^{SS} = \mathbf{A}\mathbf{x}^{SS} + \mathbf{Q}$$

$$\mathbf{x}^{SS} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{Q} = \begin{pmatrix} -g & 0 \\ -q_3 & 1+q_5 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ q_4 \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{g} & 0 \\ -\dfrac{q_3}{g(q_5+1)} & \dfrac{1}{q_5+1} \end{pmatrix} \begin{pmatrix} 0 \\ q_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \dfrac{q_4}{q_5+1} \end{pmatrix}$$

and since the initial values are R_0 and U_0, the complete solution is:

$$\begin{cases} R_t = (1+g)^t R_0 \\ U_t = \dfrac{q_3}{q_5+1+g}(1+g)^t R_0 + \left(U_0 - \dfrac{q_3}{q_5+1+g} R_0 - \dfrac{q_4}{q_5+1} \right)(-q_5)^t + \dfrac{q_4}{q_5+1} \end{cases}$$

Define the following constants:

$$\theta = \frac{q_3}{q_5+1+g} = \frac{s\left(q_1 - q_2 \dfrac{(d-r)b}{g} \right)}{s\tau + (1+g)(1-s\tau)} =$$

$$= \frac{s\left((a+b)(1-(1+(1-\tau)i)w - gw) - (1-\tau)db + (d-r)bw + (1+g)(1-p)(1-\tau) - (1-(1+(1-\tau)i)w)\dfrac{(d-r)b}{g} \right)}{1+g(1-s\tau)}$$

$$\xi = \frac{q_4}{q_5+1} = sq_2\left(\frac{(d-r)b}{g} R_0 - A_0 \right) = s(1-(1+(1-\tau)i)w)\left(\frac{(d-r)b}{g} R_0 - A_0 \right)$$

$$\psi = q_5 = \frac{s\tau}{1 - s\tau}$$

and the expression for the untaxed reserves will be:

$$U_t = \theta \quad R_0 (1+g)^t + \left(U_0 - \theta \quad R_0 - \xi\right)\left(-\psi\right)^t + \xi$$

Free cash flow

$$\begin{aligned}
FCF_t &= R_t - pR_t - \tau\left(R_t - pR_t - dbR_{t-1} - \left(U_t - U_{t-1}\right)\right) \\
&\quad - \left(aR_t - aR_{t-1}\right) - \left(bR_t - bR_{t-1} + rbR_{t-1}\right) \\
&= R_{t-1}\left(a + (1-r)b + \tau db\right) + R_t\left((1-\tau)(1-p) - a - b\right) + \tau\left(U_t - U_{t-1}\right) \\
&= R_t\left((1-\tau)(1-p) + \frac{b(\tau d - r) - g(a+b)}{1+g}\right) + \tau\left(U_t - U_{t-1}\right) \\
&= R_t\left((1-\tau)(1-p) + \frac{b(\tau d - r) - g(a+b)}{1+g}\right) \\
&\quad + \tau\left(g\theta R_{t-1} + \frac{\psi+1}{\psi}\left(U_0 - \theta \quad R_0 - \xi\right)\left(-\psi\right)^t\right) \\
&= R_t\left((1-\tau)(1-p) + \frac{b(\tau d - r) - g(a+b-\tau\theta)}{1+g}\right) + \tau\frac{\psi+1}{\psi}\left(U_0 - \theta \quad R_0 - \xi\right)\left(-\psi\right)^t
\end{aligned}$$

Define the following constants:

$$m = (1-\tau)(1-p)$$

$$z_{FCF(UR)} = \frac{b(\tau d - r) - g(a+b-\tau\theta)}{1+g}$$

$$\varsigma = \frac{\psi+1}{\psi}\left(U_0 - \theta R_0 - \xi\right)$$

and the free cash flow expression will be:

$$FCF_t = (1+g)^t R_0 \left(m + z_{FCF(UR)}\right) + \tau\varsigma(-\psi)^t$$

Net Profit

$$NP_t = (1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}) - (U_t - U_{t-1}))$$

$$= (1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}) -$$

$$-\left(\frac{g\theta}{1+g}R_t + (U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t\right)\Bigg)$$

$$= (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - \left(A_0 + \frac{(d-r)b}{g}(R_{t-1} - R_0)\right)\right)\right.$$

$$\left.-\left(\frac{g\theta}{1+g}R_t + (U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t\right)\right)$$

$$= R_t\left((1-\tau)(1-p) - \frac{(1-\tau)db + (1-\tau)iw(a+b) - (1-\tau)iw\frac{(d-r)b}{g}}{1+g}\right)$$

$$-(1-\tau)iw\left(\frac{(d-r)b}{g}R_0 - A_0\right) - (1-\tau)\left(\frac{g\theta}{1+g}R_t + (U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t\right)$$

$$= R_t\left((1-\tau)(1-p) - \frac{(1-\tau)db + (1-\tau)g\theta + (1-\tau)iw(a+b) - (1-\tau)iw\frac{(d-r)b}{g}}{1+g}\right)$$

$$-(1-\tau)iw\left(\frac{(d-r)b}{g}R_0 - A_0\right) - (1-\tau)(U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t$$

Define the following constants:

$$m=(1-\tau)(1-p)\ ,\quad \chi=(1-\tau)iw\ ,\quad \varsigma=\ \frac{\psi+1}{\psi}\left(U_0-\theta R_0-\xi\right)$$

$$z_{NP(UR)}\ =-\frac{(1-\tau)\left(db+g\theta+iw\left(a+b-\frac{(d-r)b}{g}\right)\right)}{1+g}$$

and the expression for the net profit will be:

$$NP_t=(1+g)^t R_0\left(m+z_{NP(UR)}\right)-\chi\left(R_0\frac{(d-r)b}{g}-A_0\right)-(1-\tau)\varsigma(-\psi)^t$$

Dividends

$$\begin{aligned}
DIV_t\ &=\ (1-w)\left(aR_{t-1}+bR_{t-1}-A_{t-1}\right)-U_{t-1}+\\
&+(1-\tau)\left(R_t-pR_t-dbR_{t-1}-iw\left(aR_{t-1}+bR_{t-1}-A_{t-1}\right)-\left(U_t-U_{t-1}\right)\right)-\\
&-\left((1-w)\left(aR_t+bR_t-A_t\right)-U_t\right)
\end{aligned}$$

$$\begin{aligned}
=\ &(1-w)\left(aR_{t-1}+bR_{t-1}-A_{t-1}\right)+\\
&+(1-\tau)\left(R_t-pR_t-dbR_{t-1}-iw\left(aR_{t-1}+bR_{t-1}-A_{t-1}\right)\right)-\\
&-\left((1-w)\left(aR_t+bR_t-A_t\right)\right)+\tau\left(U_t-U_{t-1}\right)
\end{aligned}$$

$$\begin{aligned}
=\ &R_{t-1}\left((1-w)(a+b)-(1-\tau)(db+iw(a+b))\right)+\\
&+R_t\left((1-\tau)(1-p)-(1-w)(a+b)\right)+\\
&+A_{t-1}\left((1-\tau)iw-(1-w)\right)+\\
&+A_t(1-w)+\tau\left(U_t-U_{t-1}\right)
\end{aligned}$$

$$
\begin{aligned}
= \quad & (1+g)^{t-1} R_0 \left((1-w)(a+b) - (1-\tau)(db + iw(a+b))\right) + \\
& + (1+g)^t R_0 \left((1-\tau)(1-p) - (1-w)(a+b)\right) + \\
& + \frac{(1+g)^{t-1} - 1}{g} (d-r) b R_0 \left((1-\tau) iw - (1-w)\right) + A_0 \left((1-\tau) iw - (1-w)\right) + \\
& + \frac{(1+g)^t - 1}{g} (d-r) b R_0 (1-w) + A_0 (1-w) + \\
& + \tau (U_t - U_{t-1})
\end{aligned}
$$

$$
\begin{aligned}
= \quad & (1+g)^t R_0 (1-\tau)(1-p) + \\
& + (1+g)^{t-1} R_0 \left((d-r) b (1-w) - (a+b)(1-w) g - (1-\tau) bd - (1-\tau) iw (a+b)\right) + \\
& + \frac{(1+g)^{t-1} - 1}{g} R_0 (1-\tau) iw (d-r) b + A_0 (1-\tau) iw + \\
& + \tau \left(\frac{g\theta}{1+g} (1+g)^t R_0 + (U_0 - \theta R_0 - \xi) \frac{\psi + 1}{\psi} (-\psi)^t \right)
\end{aligned}
$$

Define the following constants:

$$
z_{DIV(UR)} = \frac{b\left[d\tau + w(r-d+g) - r - g - (1-\tau) iw\right] - a\left[g(1-w) + (1-\tau) iw\right] + \frac{(d-r)b}{g}(1-\tau) iw + \tau\theta g}{1+g}
$$

$$
m = (1-\tau)(1-p), \quad \chi = (1-\tau) iw, \quad \gamma = \frac{(d-r)b}{g}, \quad \varsigma = \frac{\psi + 1}{\psi}(U_0 - \theta R_0 - \xi)
$$

and the dividend expression will be:

$$
DIV_t = (1+g)^t R_0 \left(m + z_{DIV(UR)}\right) - \chi (R_0 \gamma - A_0) + \tau \varsigma (-\psi)^t
$$

3. Value Calculation Methods

3.1 Discounting methods

What if the market value debt ratio is not constant according to the predictions and consequently there exists no textbook steady state? Surely, this will very often be the case in practice. A constant capital structure is even more unlikely in the explicit forecast period. As has already been indicated a non-constant market debt ratio has implications for the discounting procedure.

It was established in Proposition 2.5 that it is *only with a constant capital structure in market terms that one can use a continuing value formula to calculate the FCF horizon value without approximation errors.* Such approximation errors normally arise from two causes: 1) the weights in the WACC formula changes over time when the capital structure changes, and 2) the costs of different kinds of capital are likely to change when the capital structure changes, since capital structure influences the risk. This will be formalised in the following way:

Definitions:

Type 1 approximation error is the error introduced into the valuation by neglecting that the weights in the WACC formula change over time when the market debt ratio changes.

Type 2 approximation error is the error introduced into the valuation by neglecting that the risk of the company may very well change over time as the market debt ratio changes. Hence, it is often not correct to assume a constant cost of equity capital, k_E (or for that matter a constant cost of debt).

The task for the practically oriented analyst must then be to minimise such approximation errors. We will here show how one can avoid type 1 errors completely. (In Chapter 5 we show how one can get at least a rough estimate of the severity of type 2 approximation errors.)

One way of removing type 1 errors for the FCF valuation approach was suggested in section 2.1.5: to start from a year long into the future (when the market value debt ratio has converged towards its steady-state level) and work oneself backwards, continuously updating the WACC. This is a rather cumbersome procedure, however, and we have never seen it suggested in the literature.

One obvious way to avoid type 1 errors altogether is to use the dividend valuation approach, i.e. to discount the future dividends by the equity cost of capital, k_E. Many financial economists prefer the free cash flow approach, however, claiming that it is conceptually more valid since it explicitly values the asset side operations of the company, i.e. the actual value-creating process. This is also the view taken by Copeland et al. The discrepancy between practical usefulness and theoretical viability may seem discouraging, but one should then remember that *the two valuation approaches should yield exactly the same value*. This equality has been commented upon by several economists, most notably Miller and Modigliani,[51] and it is also mentioned in Copeland et al.[52]

Discussions regarding the equality of valuation approaches tend to be normative in nature and on a rather high level of abstraction. Copeland et al write that the valuation approaches are equal "as long as the discount rates are selected properly" [53] - but without stating what this "proper" method should look like. The discounting method they employ[54] certainly does not yield equivalence. Financial economists often assume that the capital structure remains constant. This is a rather restrictive assumption, as has been seen in Chapter 2, but if fulfilled the equality holds. The method we suggested in section 2.1.5, to continuously update the WACC, turns out to yield the equivalence without any restricting assumptions. This is formalised in Proposition 3.1, which can be viewed as a fairly substantial extension of Proposition 2.4:

[51] Miller and Modigliani (1961), p. 416 and p. 419. The Miller-Modigliani model is more stylised than the one discussed in this chapter, but the general discussion concerning these issues is clearly of interest.
[52] Copeland et al, p. 132.
[53] Copeland et al, p. 132.
[54] The weighted average cost of capital at the moment of valuation ("year 0") used throughout.

Proposition 3.1

Valuation by discounting the free cash flows at a continuously updated weighted average cost of capital ("year-to-year-WACC") will yield the same value as valuation by discounting the future dividends at the cost of equity capital.

Proof

The free cash flow is - in the general case - defined as follows:[55]

$$
\begin{aligned}
(56) \quad FCF_t \;=\; & \textit{Net profit, } NP_t \\
& + \textit{Net interest payments after taxes, } (1-\tau)iD_{t-1} \\
& + \textit{Increase in deferred taxes, } (T_t - T_{t-1}) \\
& - \textit{Increase in asset side } (AS_t - AS_{t-1})
\end{aligned}
$$

where: τ is the tax rate
i is the interest rate
D_{t-1} is the (net) debt at the end of year t-1 on which interest is paid

The dividends are by the clean surplus relationship defined as:[56]

$$
\begin{aligned}
(57) \quad DIV_t \;=\; & \textit{Net profit, } NP_t \\
& + \textit{Increase in debt, } (D_t - D_{t-1}) \\
& + \textit{Increase in deferred taxes, } (T_t - T_{t-1}) \\
& - \textit{Increase in asset side, } (AS_t - AS_{t-1})
\end{aligned}
$$

Assume there exists an equity value at time T, called EV_T. This value is the same for both the *FCF* and the *DIV* approach and it is calculated after a possible dividend at time T. This means that the total company value at time T, called TCV_T, will be $EV_T + D_T$ (i.e. equity value

[55] Using the McKinsey definition in full: *FCF = Net profit + Net interest payments after taxes + Depreciation expense + Increase in deferred taxes - Increase in working capital - Capital expenditures = Net profit + Net interest payments after taxes + Depreciation expense + Increase in deferred taxes - Increase in working capital - (Current year's net PPE - Preceding year's net PPE + Depreciation expense) = Net profit + Net interest payments after taxes + Increase in deferred taxes - Increase in working capital - Increase in net PPE = Net profit + Net interest payments after taxes + Increase in deferred taxes - Increase in asset side.* Note that excess marketable securities are not present in the forecast period.
[56] As in expression (3)

plus debt value). Valuation by the *FCF* approach will then yield the following total company value at the end of year *T*-1:[57]

$$(58)\quad TCV_{T-1} = \frac{FCF_T + EV_T + D_T}{1 + k_{WACC,T}}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T}{1 + k_{WACC,T}}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T}{1 + \frac{D_{T-1}}{TCV_{T-1}}(1-\tau)i + \left(1 - \frac{D_{T-1}}{TCV_{T-1}}\right)k_E}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T}{\frac{TCV_{T-1} + (1-\tau)iD_{T-1} + TCV_{T-1}k_E - D_{T-1}k_E}{TCV_{T-1}}}$$

Rearranging yields:

$$(59)\quad TCV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T + D_{T-1}k_E}{1 + k_E}$$

The equity value is then obtained by deducting the debt, i.e. by deducting D_{T-1}:

$$(60)\quad EV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T + D_{T-1}k_E}{1 + k_E} - D_{T-1}$$

$$= \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + (D_T - D_{T-1})}{1 + k_E}$$

[57] Note again that the WACC formula, expression (16), implies that the WACC used for discounting during year t is based on the entering market values of debt and equity, i.e. D_{t-1} and EV_{t-1}, and hence also the total company value, TCV_{t-1}.

By inserting (57) into (60) one obtains:

$$(61) \qquad EV_{T-1} = \frac{DIV_T + EV_T}{1 + k_E}$$

which is exactly the valuation formula used when discounting the dividends by the equity cost of capital. Having thus established that the value at T-1 will be the same when employing the different methods, one can go on to time T-2 and so on. Q.E.D.

The proof hinges on the assumption that there exists an equity value at some future date (the same for both the *FCF* and the *DIV* approach) from which the discounting process can start. In the example company of Chapter 2 there is no TSS, and hence such a value does not exist. However, the approximation error can be made arbitrarily small by choosing a starting point sufficiently long into the future. Since the market value debt ratio strives towards its theoretical steady state value as t increases (visualised in Figure 1 in section 2.1.5), the approximation will become better and better with time, and the approximation error becomes zero as $t \to \infty$. Furthermore, since any approximation error will occur at a date far into the future, the discounting process itself will also make the impact of an approximation error on the company value smaller and smaller as t increases.

Having thus established that the *FCF* approach, using the correct (updated) WACC discounting method, is equivalent to the *DIV* approach (using k_E), one can use the dividend valuation as the technical way of calculating the equity value, regardless of approach. If one for theoretical or other reasons prefers the *FCF* approach, then the argument is that one actually uses this approach when modelling (*FCF* statements are set up, etc.), but the actual computations are programmed in a way that makes them equal to the dividend valuation approach. This simplifies the computational work considerably: In the example company instead of performing 150 different iterative calculations one can simply use the dividend valuation formula:[58]

[58] Expression (62) is the simply the sum of the discounted dividend expression (12) taken from now to eternity.

$$EV_0 = \frac{(1+g)R_0(m+z_{DIV})}{k_E - g} - \frac{\chi(\gamma R_0 - A_0)}{k_E} \tag{62}$$

The second term in (62) is a correction term to account for the fact that the company never reaches a (textbook) steady state (see expression (12) for parameter explanations).

So far the discussion has been exemplified by a company in parametric steady state, i.e. the part of the valuation that concerns the horizon value. Proposition 3.1 is quite general, however, and is clearly valid also for the explicit forecast period. For the calculations in the explicit forecast period, one does not have to worry about any asymptotic proofs, since there clearly exists an equity value at time T - the horizon value - and the existence of such a value guarantees that Proposition 3.1 holds. Another suggestion often made in practically oriented textbooks is to limit the life of the company to, say, 100 years and assume zero value thereafter. In this case Proposition 3.1 also holds, since there will exist an exact value at some future date, namely the value zero in year 100.

To conclude this discussion: There is never any reason to resort to using a *constant* weighted average cost of capital when the capital structure, and consequently the weights in the WACC formula, changes over time. It would inevitably introduce type 1 approximation errors into the valuation. The reason often given for using a constant WACC is that it simplifies the computational work considerably, but as we have shown above, any practical problems can be overcome by *calculating* the value by the dividend approach instead. Furthermore, it is often argued that the present value of future dividends is the correct benchmark for any calculation of the equity value.[59]

There remains, however, the other approximation problem: that the cost of equity capital may change when the capital structure (and hence the riskiness) changes. It should be noted that Proposition 3.1 holds even if the cost of equity capital (k_E) is allowed to vary over time. This is easily seen by just adding the subindex t to the cost of equity capital (i.e. $k_{E,t}$) in the proof. If k_E varies, no easy summation formula can be used, but it is still easier to perform dividend

[59] This is for example emphasized by Ohlson (1995), who starts out by the assumption that "value equals the present value of expected dividends" (p. 1).

discounting than *FCF* discounting, since the latter involves tedious and time-consuming iterative calculations. How to address these issues in practice is shown in Chapter 5.

3.2 Earnings capitalisation

During the last few years, a number of papers and articles have begun advocating earnings as valuation measure. Such notions originate in the accounting field, and finance academics are on the whole less enthusiastic about earnings. We also lean more towards free cash flows and dividends as the relevant measures to be used in valuations, but the ardent support for earnings from some accounting researchers has made us include net profit calculations in earlier chapters. The reader will have noticed, however, that we have not used net profits for valuation purposes, and we will here give some indications as to why we are somewhat more sceptical.

In his article *Return to Fundamentals*[60] Stephen Penman discusses the development of the theory behind earnings as a valuation measure. He points out that the present value of expected dividends, P_t, is a "non-controversial description of price" (p. 23). Thus having argued that P_t is a relevant bench-mark, he continues: "Ohlson has shown that

$$[(63)] \quad V_t^T = \left(\rho^T - 1\right)^{-1} E\left[\left(\sum_{\tau=1}^{T} \tilde{X}_{t+\tau} + \sum_{\tau=1}^{T} \left(\rho^{T-\tau} - 1\right)\tilde{d}_{t+\tau}\right) \middle| Z_t\right]$$

(where $\tilde{X}_{t+\tau}$ is earnings in period $t+\tau$) approaches P_t as T, the number of periods ahead, gets large."

[V_t^T is the equity value using formula (63). P_t is the present value of expected dividends; ρ is $1+k$, i.e. 1+ the discount rate; $\tilde{d}_{t+\tau}$ stands for dividends in period $t+\tau$; Z_t is the information set at time t.]

---

[60] Penman (1991), pp. 23-25

Penman continues: "One might be thrown off by the inclusion of future dividends in [(63)]. Does this mean one has to predict future dividends also? No. These appear because, if dividends are paid out in the future, expected subsequent earnings are also reduced. The formula simply puts the dividends into an earning escrow account. Indeed, their presence in the formula is because they are irrelevant for value (but affect future earnings). Again, earnings-based valuation gets rid of the discretionary dividends. One can legitimately forget them and consider expected future earnings as if all dividends are reinvested in the firm."

We find such arguments somewhat hard to swallow. On intuitive grounds, one would suspect that dividends *do* matter (in the sense that they cannot always be zero), and that the reason that V_t^T approaches P_t is that the capitalised earnings sequence actually approaches zero as T gets large.

This supposition can be checked by separately assessing the earnings contribution to value, VX_t^T :

(64) $$VX_t^T = \left(\rho^T - 1\right)^{-1} E\left[\sum_{\tau=1}^{T} \widetilde{X}_{t+\tau} \middle| Z_t\right]$$

The conditional expected earnings sequence has to be specified. In Corollary 2.1, it was shown that with a constant capital structure, the earnings (i.e. net profit) would grow by a constant each year. It turned out that this constant was equal to the revenue growth rate, g. This is a very commonly used assumption, and any valuation formula would have to be able to handle this case.

Hence, given the information set at time t (i.e. the conditioning Z_t), where time t corresponds to year zero in our Chapter 2 model, we would expect:

$$(65)\quad E\left[\sum_{\tau=1}^{T}\widetilde{X}_{t+\tau}\,|Z_t\right]=E\left[\widetilde{X}_{t+1}\,|Z_t\right]+E\left[\widetilde{X}_{t+2}\,|Z_t\right]+\ldots+E\left[\widetilde{X}_{t+T}\,|Z_t\right]=$$

$$=(1+g)x_0+(1+g)^2x_0+\ldots+(1+g)^Tx_0=\sum_{\tau=1}^{T}x_0(1+g)^\tau=x_0\sum_{\tau=1}^{T}(1+g)^\tau$$

where x_0 stands for the earnings (net profit) in year zero.

The test will now be to evaluate expression (64) as $T\to\infty$, using expression (65) as the expected earnings sequence.

$$(66)\ \lim_{T\to\infty}VX_t^T=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}E\left[\sum_{\tau=1}^{T}\widetilde{X}_{t+\tau}\,|Z_t\right]=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}x_0\sum_{\tau=1}^{T}(1+g)^\tau$$

$$=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}x_0\left(\frac{(1+g)^{T+1}}{g}-\frac{(1+g)}{g}\right)=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}x_0\frac{(1+g)}{g}\left((1+g)^T-1\right)$$

$$=x_0\frac{(1+g)}{g}\lim_{T\to\infty}\frac{\left((1+g)^T-1\right)}{\left(\rho^T-1\right)}=x_0\frac{(1+g)}{g}\lim_{T\to\infty}\frac{\left((1+g)^T-1\right)}{\left((1+k)^T-1\right)}$$

$$=\begin{cases}0 & \text{if}\quad g<k\\ x_0\dfrac{(1+g)}{g} & \text{if}\quad g=k\\ \to\infty & \text{if}\quad g>k\end{cases}$$

(k stands for the discount rate; in our earlier models this would be the cost of equity capital, k_E.)

The magnitude of the revenue growth rate, g, is supposed to be less than the discount rate (see Chapter 2). This is surely not a very restricting assumption: The discount rate (in Sweden) is typically somewhere around 13%, and a revenue growth rate as high as that in every year from now to eternity is not realistic when inflation is assumed to be around 3%. This means that we can discard the possibility that $g \geq k$.[61] Miller & Modigliani (1961) also argue that the growth rate in a constant growth scenario must be strictly smaller than the discount rate.[62]

Thus the first possibility in the solution to expression (66) remains: that the growth rate is smaller than the discount rate. This means, however, that the value of the earnings are zero, and hence the entire value stems from the dividend stream. (This can be verified the other way around as well. Specify, e.g., a continuous dividend growth, i.e. $DIV_{t+1} = (1+g)DIV_t$. and the limit of the "dividends part" of formula (63) will be exactly equal to what we get from the Gordon formula, which is the standard "finance" way of calculating the sum of discounted dividends. See Appendix 3.)

If the earnings sequence has no value, then the informational content of the aggregate earnings figure *in itself* should also be zero. This does not mean that earnings are somehow unimportant. The earnings, or net profit, figure is a necessary link between the income statement and the balance sheet, and it is thus necessary to calculate earnings as a way of deriving the development over time of the financial statements.

Two objections to our reasoning immediately present themselves:

- Is it reasonable to use asymptotic proofs, thereby assuming an infinite life of the company?
- Many practitioners seem to use the earnings figures when contemplating an investment, is this not an indication that earnings valuation is reasonable?

The answer to the first objection is short and straightforward: When modelling a finite life, there will generally be a liquidating dividend and retained earnings will serve as one component in the determination of the size of the liquidating dividend. This is, however, a dividend valuation approach, albeit a very special case. The earnings figure serves as a link in determining the size of the dividend, not as a valuation measure itself. Hence any finite life assumption implies a dividend valuation approach.

---

[61] A growth rate larger than the discount rate is never the case in reality. It might be in a single year, but not on average in every year from now to infinity. Since there is a finite value for every company, the market clearly shares this view.

The second objection: Any observer of stock market reporting in the news media will notice the prominence of the earnings figure. The standard answer to this is that while earnings have no direct value to the stock holder, it will give an indication as to future dividends and future capital gains. Quite possibly the earnings figure works well enough as a proxy variable for value, and hence some investors stop there. We do not disagree with this, we are merely pointing out that it is in reality nothing but a proxy variable, giving guidance as to the level of future dividends. As was seen in Chapter 2, the net profit figure combined with *additional* balance sheet information can be translated into free cash flows or dividends. This should be nothing controversial. Accounting academics will only phrase this somewhat differently: that the earnings figure combined with the book value of the company provides value relevant information. And yet it is obviously controversial: Penman spends four pages (pp. 28-31) telling us why the free cash flow approach is inferior, with special reference to Copeland et al.

Ohlson (1995) claims that the dividend approach is equal to the earnings approach if one introduces the concept of "abnormal earnings".[63] We have shown earlier in this report that the dividend approach is equal to the free cash flow approach provided the discounting is done properly. So it seems that one has a choice: abnormal earnings, dividends, or free cash flows. Personally, we remain unconvinced that "abnormal earnings" is a clearer concept than "free cash flows", and, as was demonstrated earlier in this section, "earnings" as such do not work as a valuation measure.

[62] Miller and Modigliani (1961), p. 421. They consider what they call "*the convenient case of constant growth rates*", i.e. the same case that we have investigated in expression (66).

[63] Earnings in excess of the discount rate multiplied by the entering book value.

Appendix 3 - Constant dividend growth

Assume a scenario with constant dividend growth, i.e.

$$\text{(A3:1)}\quad DIV_{\tau} = (1+g)^{\tau} DIV_0$$

where g is the constant dividend growth. The value of the equity is the sum of the discounted future dividends. Assuming an infinite life of the company:

$$\text{(A3:2)}\quad V_{DIV} = \sum_{\tau=1}^{\infty} \frac{DIV_{\tau}}{(1+k_E)^{\tau}} = \sum_{\tau=1}^{\infty} \frac{(1+g)^{\tau} DIV_0}{(1+k_E)^{\tau}}$$

By evaluating the infinite sum and simplifying, the Gordon formula is obtained (provided $g < k_E$):

$$\text{(A3:3)}\quad V_{DIV} = \frac{(1+g)DIV_0}{k_E - g} = \frac{DIV_1}{k_E - g}$$

This definition of equity value is non-controversial. We now want to show that the capitalised dividend sequence in expression (63) equals (A3:3) when using an infinite horizon, i.e. we want to show that:

$$\text{(A3:4)}\quad \lim_{T\to\infty} \left(\rho^{T} - 1\right)^{-1} E\left[\left(\sum_{\tau=1}^{T} \left(\rho^{T-\tau} - 1\right)\tilde{d}_{t+\tau}\right) \| Z_t\right] = \frac{(1+g)DIV_0}{k_E - g}.$$

We denote dividends with *DIV*, Penman uses *d*. ρ is equal to $(1+k_E)$, so the left-hand-side of (A3:4) can equivalently be written:

$$\text{(A3:5)}\quad LHS = \lim_{T\to\infty} \left((1+k_E)^{T} - 1\right)^{-1} E\left[\left(\sum_{\tau=1}^{T} \left((1+k_E)^{T-\tau} - 1\right)\tilde{d}_{t+\tau}\right) \| Z_t\right]$$

The scenario under consideration is constant dividend growth. This means that the conditional expectations are:

(A3:6) $E\left[\left((1+k_E)^{T-\tau}-1\right)\tilde{d}_{t+\tau}\middle|Z_t\right]=\left((1+k_E)^{T-\tau}-1\right)(1+g)^{\tau}d_0$

Hence, expression (A3:5) can be rewritten:

(A3:7) $LHS=\lim_{T\to\infty}\left((1+k_E)^T-1\right)^{-1}\sum_{\tau=1}^{T}\left((1+k_E)^{T-\tau}-1\right)(1+g)^{\tau}d_0$

Rearranging and noting that the limit of a sum is equal to the sum of the limits:

(A3:8) $LHS=\lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left((1+k_E)^{T-\tau}\right)(1+g)^{\tau}d_0}{(1+k_E)^T-1}-\lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{(1+g)^{\tau}d_0}{(1+k_E)^T-1}$

The two terms in (A3:8) can be evaluated separately. Define:

(A3:9) $A\equiv\lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left((1+k_E)^{T-\tau}\right)(1+g)^{\tau}d_0}{(1+k_E)^T-1}$

(A3:10) $B\equiv\lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{(1+g)^{\tau}d_0}{(1+k_E)^T-1}$

Evaluating (A3:9):

(A3:11) $A\equiv\lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left((1+k_E)^{T-\tau}\right)(1+g)^{\tau}d_0}{(1+k_E)^T-1}=$

$$=d_0\lim_{T\to\infty}\left(-\frac{(1+g)^{T+1}}{\left((1+k_E)^T-1\right)(k_E-g)}+\frac{(1+g)(1+k_E)^T}{\left((1+k_E)^T-1\right)(k_E-g)}\right)=$$

$$= \frac{(1+g)d_0}{(k_E-g)} \lim_{T\to\infty}\left(-\frac{(1+g)^T}{\left((1+k_E)^T-1\right)} + \frac{(1+k_E)^T}{\left((1+k_E)^T-1\right)} \right) =$$

$$= \frac{(1+g)d_0}{(k_E-g)} \left(\lim_{T\to\infty}\left(-\frac{(1+g)^T}{\left((1+k_E)^T-1\right)} \right) + \lim_{T\to\infty}\left(\frac{(1+k_E)^T}{\left((1+k_E)^T-1\right)} \right) \right) =$$

$$= /\ g < k_E \text{ by assumption } / = \frac{(1+g)d_0}{(k_E-g)}(-0+1) = \frac{(1+g)d_0}{(k_E-g)}$$

Evaluating (A3:10):

(A3:12) $$B \equiv \lim_{T\to\infty} \sum_{\tau=1}^{T} \frac{(1+g)^\tau d_0}{(1+k_E)^T-1} = d_0 \lim_{T\to\infty}\left(\frac{(1+g)^{T+1}}{\left((1+k_E)^T-1\right)g} - \frac{(1+g)}{\left((1+k_E)^T-1\right)g} \right) =$$

$$= d_0 \frac{1+g}{g} \lim_{T\to\infty} \frac{(1+g)^T-1}{(1+k_E)^T-1} = 0 \qquad /\text{ since } g < k_E \text{ by assumption } /$$

And the entire LHS expression (A3:8) becomes:

(A3:13) $$LHS = A + B = \frac{(1+g)d_0}{(k_E-g)} + 0 = \frac{(1+g)d_0}{(k_E-g)}$$

The LHS of expression (A3:4) equals the RHS. QED.

4. Implementation – Empirical and Theoretical Analysis

In this chapter topics related to the practical implementation of the modelling approach will be discussed. An analysis of the different input parameters is performed, both from an empirical and a theoretical point of view: we look at the parameters themselves and also at their interaction in determining the financial statements from which valuation measures are derived.

In section 4.1 the different specifications of the property, plant and equipment (PPE) items presented by Copeland et al are analysed. Section 4.2 contains the determination of an analytical expression for the steady-state parameter *r, retirements / preceding year's gross PPE*, based on steady state considerations. In section 4.3 a solution is derived for determining the parameter *c, change in deferred taxes / gross PPE*, in steady state, by using assumptions about the *effective* tax rate. In section 4.4 we look at the parameter *p, operating expenses in % of revenues,* before turning to the *working capital* to *revenues* relation in section 4.5.

Some of the model's parameters are very company specific, and there is not much help to be drawn from an analytical treatment or from empirical industry data. In those cases the company's own historical figures together with (the often treacherous) intuition will have to suffice. For other parameters, the company's parameter value may for some reason be correlated to an industry average figure - one example being the profit margin of businesses operating in highly competitive markets. In those cases, industry average figures may be of some guidance. Such figures are presented in tables for the parameters *a, e* and *p*. They all originate from Statistics Sweden (SCB).[64] Since, generally, the information content of accounting data is rapidly decreasing with time we only include figures from 1988 and onwards (the latest available year is 1993). This still enables us to include both ends of the business cycle (such as the top years at the end of the 1980's and the in many businesses almost depression-like year 1992).

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[64] See the appendix at the end of the chapter for a brief discussion about the data quality.

4.1 The determination of PPE

4.1.1 Introduction

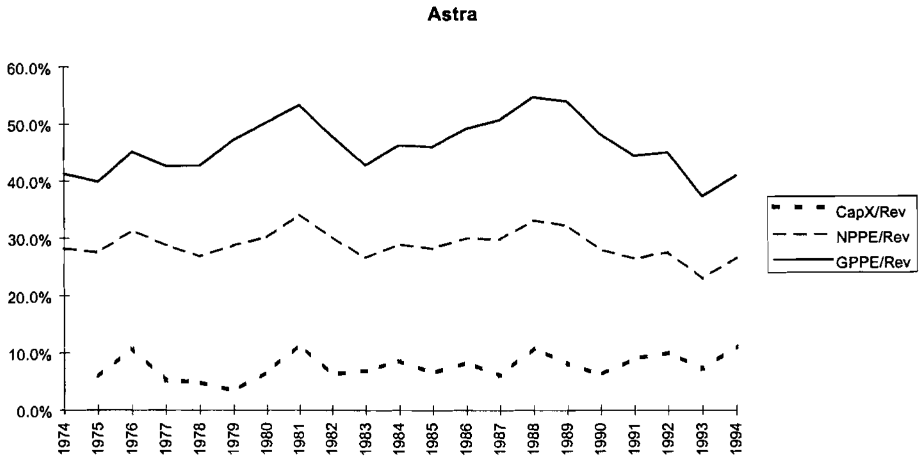
This section contains a discussion about PPE related items. These closely related items are the following: *net* and *gross property, plant and equipment (net* and *gross PPE), accumulated depreciation (A), capital expenditures (CapX), depreciation expense (DepX)* and *retirements (Ret).* Copeland et al give only limited attention to the determination of them.

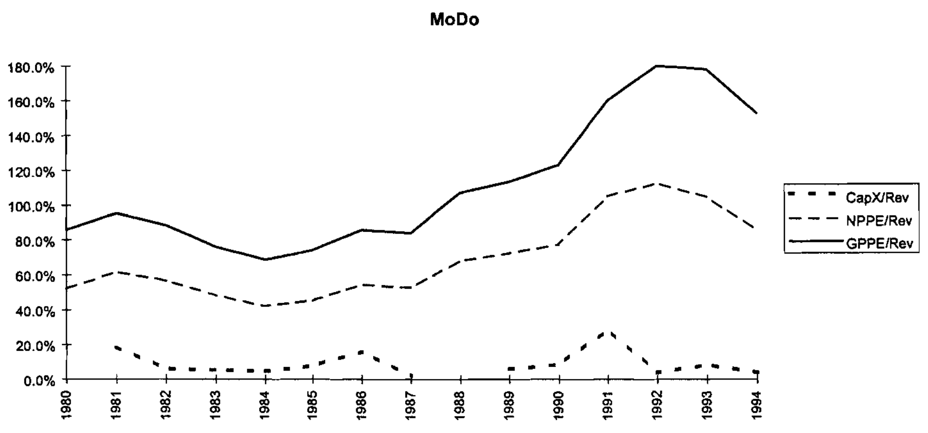
The three different specifications of these items (already presented in section 1.2) will be investigated here in order to assess their usefulness as predictors of the future. This is done by examining the behaviour over time of the different driving ratios (*gross PPE / revenues*, *CapX / revenues*, and *net PPE / revenues*) in three large Swedish companies: Astra, Volvo and MoDo. Although an investigation of only three firms yields more an indication than substantive proof in the statistical sense, we believe the study to be of interest. The companies have been chosen because they represent different kinds of firms and businesses. The pharmaceutical company Astra has experienced tremendous growth in recent years and it is interesting to see how this affects the percentages. Volvo, the largest Swedish car manufacturer, has a much more fluctuating revenue development. MoDo is in the pulp and paper business, which traditionally is a cyclical business. Also, the economic interpretation of the ratios will be discussed.

4.1.2 The main driving ratios - empirical findings and interpretations

For forecasting purposes one would like to have input ratios with a fairly stable development over time. Moreover, the ratio should ideally be some kind of "identity" with a development of its own, so that the forecast of the ratio is unconditional of the revenue forecast. At the very least, it should be easier to predict the ratio than to make a direct forecast of the item determined by the ratio. Otherwise, the whole approach with revenue-related ratios would be meaningless. In Figure 3 the three different ratios are plotted for Astra, MoDo and Volvo.

Note that *net* and *gross PPE* are denoted by *NPPE* and *GPPE* respectively in the table legends throughout this chapter.





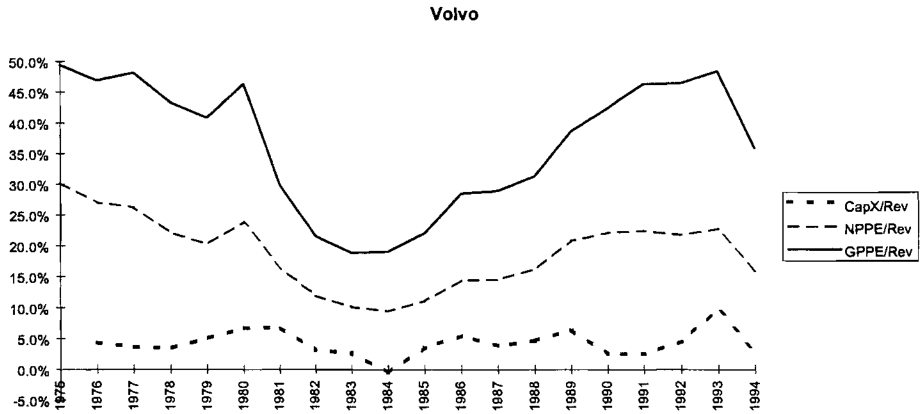


Figure 3 - The historical behaviour of the main driving input ratios in Astra, MoDo and Volvo.

Gross PPE as a percentage of revenues, b_t (Specification A)

In Specification A the main driving ratio is *gross PPE / revenues*. The thought behind this is that a certain amount of property, plant and equipment is required to generate a certain amount of revenues.[65] The assumption conflicts, one suspects, with reality since revenues often tend to be fluctuating whereas *gross PPE* should be more stable. Astra, Volvo and MoDo will provide an indication on whether this is true or not:

[65] This means that a decrease in this percentage can be viewed as a more efficient use of *gross PPE*; less *gross PPE* being required to yield a certain amount of revenues. The reverse, of course, holds for an increase.

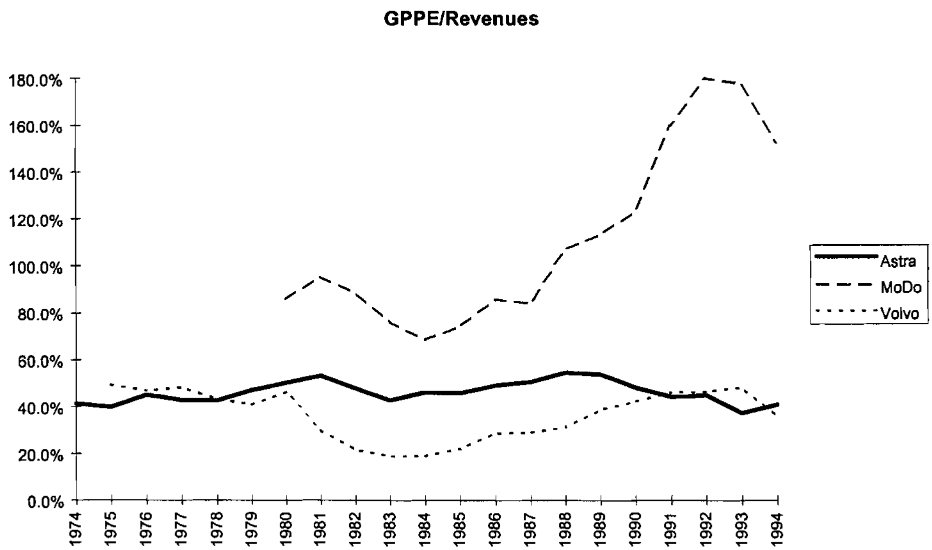


Figure 4 - Ratio between gross PPE and revenues in Astra, MoDo and Volvo.

As can be seen from Figure 4, the *gross PPE* to *revenues* ratio works well in Astra, the company with continuous revenue growth. In Volvo and MoDo, the ratio fluctuates much more. This effect is mainly attributable to the difference in revenue development as is obvious from Figure 5.

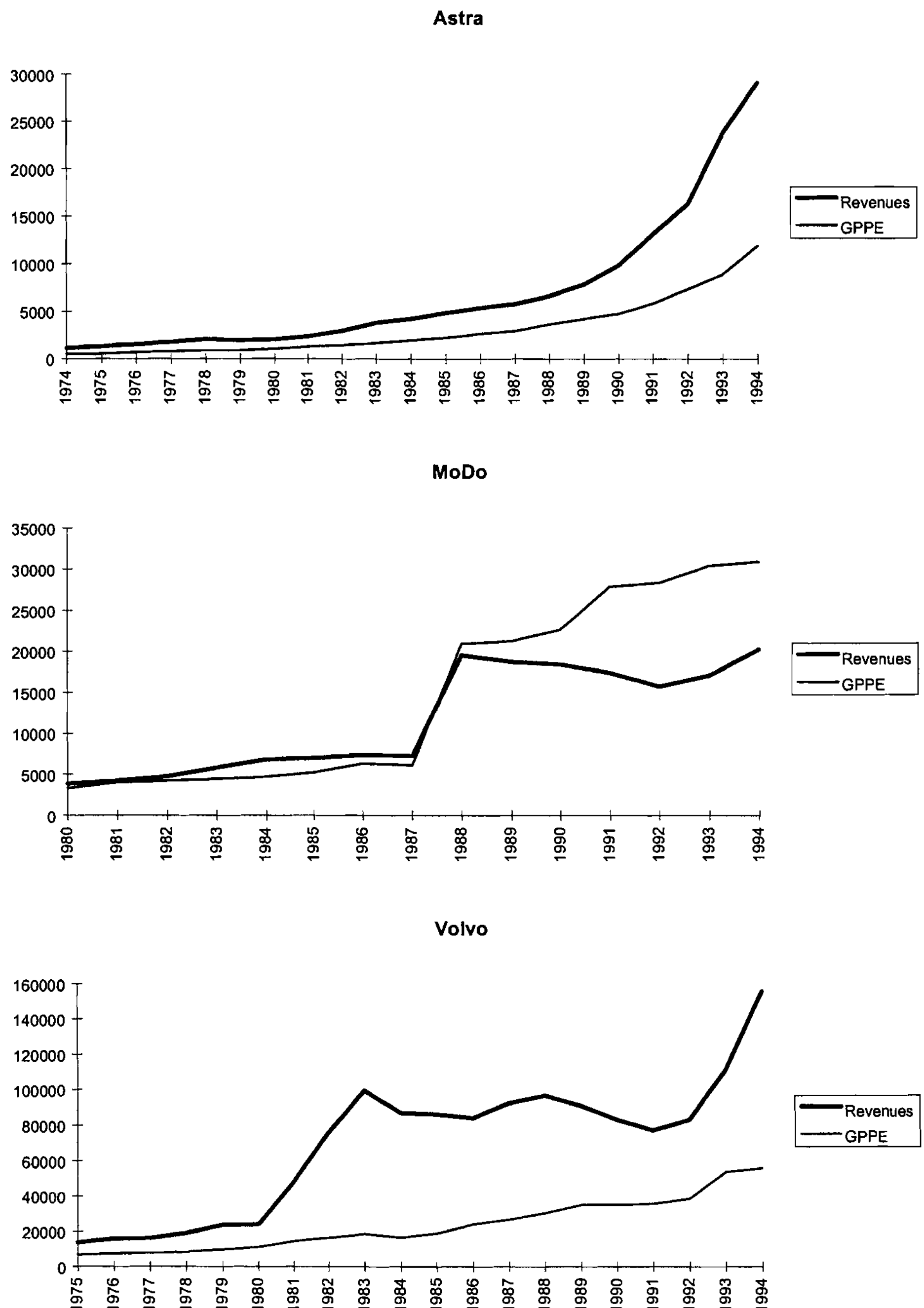


Figure 5 - Revenues and gross PPE in Astra, MoDo and Volvo.

In all three companies, *gross PPE* grows every year (with one exception). This is the case irrespective of revenue development: the revenues are increasing in Astra, fluctuating in Volvo and cyclical in MoDo.[66] The historical behaviour of *gross PPE / revenues* is thus very company specific. In Astra, with a long history of growth, the ratio tends to lie within a specific range. In the other two companies, more fluctuating revenue developments result in fluctuating ratios.

Fluctuating ratios in themselves are not necessarily bad. If the ratio really is an independent economic entity, it could of course have its own fluctuating development, which could be used for forecasting the future development of the ratio, unconditional on the forecast of the denominator. In the case of the *gross PPE* to *revenues* ratio for MoDo and Volvo this ideal does not hold, however, mainly because *gross PPE* tends to increase steadily with time, whereas revenues go up and down. This implies that the ratio's development will be very much determined by the revenue development: the ratio increases whenever revenues are decreasing or at a constant level, and vice versa. Therefore, forecasts of the ratio must be made *conditional* on the forecast of the future revenue growth for Volvo and MoDo. The choice, in such cases, stands between assuming a non-fluctuating revenue development or adjusting the *gross PPE* to *revenues* ratio in order to compensate for the fluctuations in revenues, so that the *gross PPE* obtains a stable development. To conclude: for this type of companies, with fluctuating/cyclic revenue development, the main driving ratio of Specification A clearly works poorly.

Turning to the economic interpretation of the ratio, a first-hand interpretation is: *a certain amount of property, plant and equipment is required to generate a certain amount of revenues*. This implies that the ratio is a kind of efficiency measure. A problem is that the components (i.e. machines, buildings, etc.) of *gross PPE* are valued at their purchase value, while the revenues are "inflated", i.e. the revenues in each calculated ratio come from a specific year, whereas the components of *gross PPE* come from a wide dispersion of years, with different monetary value. If one uses inflation-adjusted accounting the interpretation would indeed be what was suggested above: a certain amount of property, plant and

[66] The peak in both curves in 1987 for MoDo, is attributed to the fact that two other companies were absorbed by MoDo.

equipment required to generate a certain amount of revenues. When not using inflation-adjusted accounting, the validity of this interpretation is not that clear anymore. This is due to the fact that the denominator, *gross PPE*, is then not the actual amount of the property, plant and equipment, but merely some kind of weighted average of PPE-values, with much larger weights on *recently bought* property, plant and equipment. It is desirable to have a measure that takes into account how *many/much* (in physical terms) PPE used, or alternatively a measure of how much money (at today's prices) that has been invested in PPE, neither of which is the case with *gross PPE*. Therefore the interpretation is somewhat unclear.

In conclusion, the main interpretation problem with inflation concerns *gross PPE* and how it develops over time. In times of high inflation the PPE bought recently makes up a larger part of the total PPE, than in times with low inflation. This means that if a company purchases a great deal of new equipment during a high inflation period, *gross PPE* will increase at a faster rate than in years of low inflation, even if *gross PPE* in terms of plants, numbers of machines, etc., is the same.

Capital expenditures as a percentage of revenues, e_t (Specification B)

The parameter e_t, *CapX / revenues,* is used as main driving input ratio in Specification B. The interpretation is straightforward: a certain amount of the revenues is used for buying new property, plant and equipment.

Specification B seems to be more intuitive than Specification A, mainly because the *gross PPE*-development is modelled in a much more stable way. The *gross PPE* is determined by adding this year's *capital expenditures* (minus *retirements*) to the preceding year's *gross PPE*. This ensures that *gross PPE* will have a smooth development.

Empirically the ratio is very unstable for the pulp and paper company MoDo, as can be seen in Figure 6. This is not surprising taking into account the characteristics of machinery and plants in the pulp and paper business, where new investments are large and occur at discrete, distant points in time. This is also obvious from Figure 7.

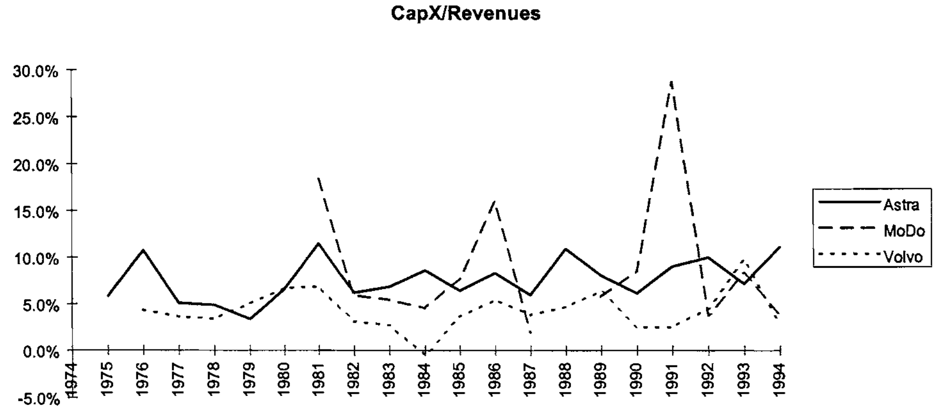
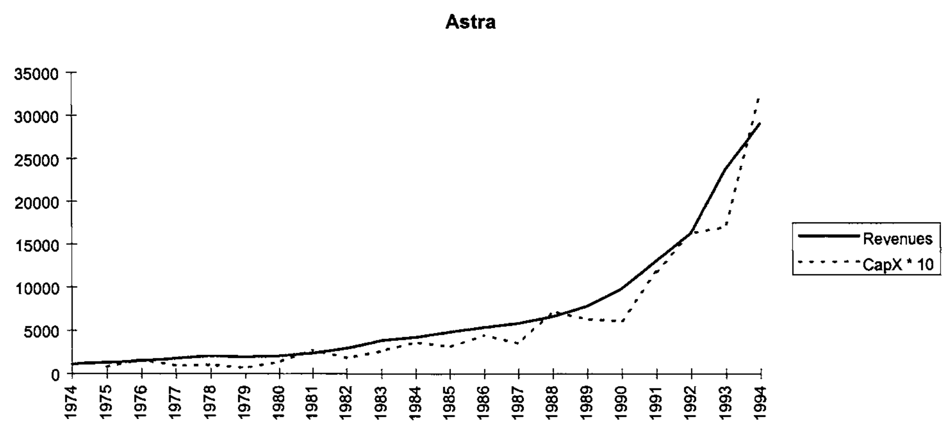
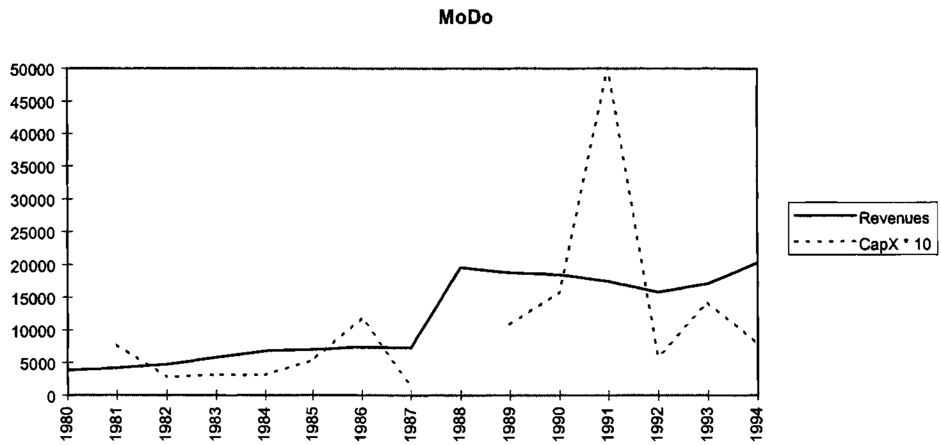


Figure 6 - Ratio between capital expenditures and revenues in Astra, MoDo and Volvo.

On the other hand, it is obvious from Figure 7 that Astra is a company where the interpretation of the *CapX* to *revenues* ratio is applicable; the capital expenditure development closely follows the revenue development. The ratio itself is not *exceptionally* stable, but it tends to fluctuate around 8%. For Volvo, the relation between capital expenditures and revenues are not as obvious as for Astra, but working better than for MoDo.





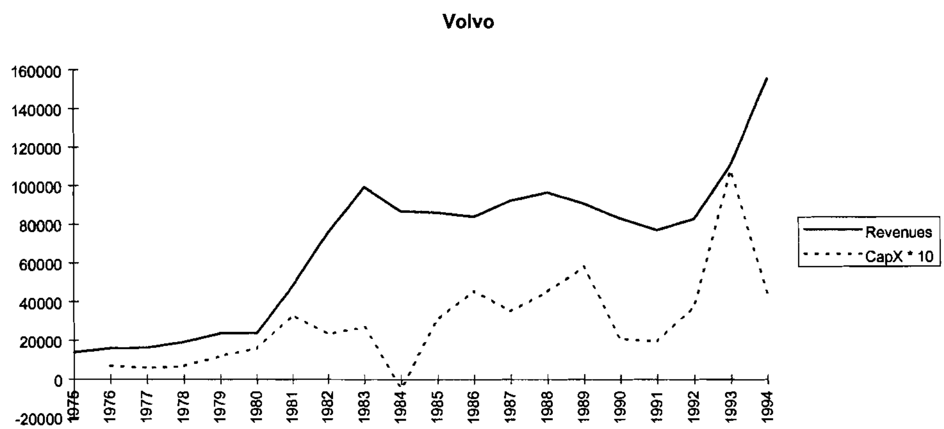


Figure 7 - Capital expenditures (scaled) and revenues in Astra, MoDo and Volvo.

It is important to note that Specification B differs quite a lot from Specification A when it comes to the determination of *gross PPE*. In Specification B it is just the yearly *change* in *gross PPE* that is determined through the ratio, and not the whole stock of *gross PPE* as was the case with Specification A. This means that Specification B provides a way of modelling property, plant and equipment that is more consistent with the empirical finding that *gross PPE* is growing steadily over time.

It is reasonable to believe that the parameter e_r, the *capital expenditures* to *revenues* ratio, can differ between industries. Table 9 shows that this in fact is the case: it is low in trade industries, whereas it is quite high in heavy industries like *mining and quarrying, electricity, gas and water* and *manufacture of pulp and paper products.* A discussion about the data in Tables 9, 10 and 11 can be found in Appendix 4.

Industry	median	min	max	latest value	
Agriculture, hunting, forestry and fishing	4,1%	3,5%	5,0%	3,7%	(1992)
Mining and quarrying	10,8%	8,0%	14,3%	14,3%	
Manufacture of food, beverages and tobacco	3,1%	1,8%	3,5%	3,3%	
Textile, wearing apparel and leather industries	3,4%	2,9%	9,3%	2,9%	
Manufacture of wood and wood products	4,8%	4,0%	6,2%	4,8%	(1992)
Manufacture of pulp, paper products	6,9%	4,0%	9,0%	4,7%	
Printing and publishing	4,4%	-2,0%	6,1%	5,2%	
Manufacture of chemicals, petroleum, coal rubber and plastic products	5,2%	3,5%	6,2%	5,4%	
Manufacture of non-metallic mineral products, except products of petroleum and coal	5,4%	1,7%	7,0%	1,7%	
Basic metal industries	3,6%	2,8%	4,4%	2,8%	
Manufacture of fabricated metal products	4,7%	4,5%	5,4%	4,7%	(1992)
Manufacture of fabricated machinery and equipment	2,9%	1,8%	3,5%	1,8%	(1992)
Manufacture of electromechanical products	2,9%	2,4%	3,9%	2,9%	(1992)
Electricity, gas and water	11,6%	9,1%	15,3%	9,3%	
Construction	2,5%	1,7%	2,8%	1,7%	
Wholesale trade	1,2%	1,1%	1,8%	1,1%	
Retail trade	1,7%	1,5%	2,3%	1,5%	(1992)
Restaurants and hotels	4,2%	2,3%	5,6%	2,7%	
Land transportation	8,4%	6,0%	12,8%	6,4%	
Sea transportation	2,5%	0,2%	6,6%	5,4%	(1992)
Air transportation	11,6%	8,0%	12,9%	8,0%	(1992)
Consulting business	4,1%	3,0%	5,8%	3,0%	(1992)
Social and personal services	4,7%	3,4%	5,8%	3,4%	(1992)

Table 9 - capital expenditures in % of revenues (sales) in Swedish businesses 1988-1993

The observation made earlier that e_t is rather unstable over time is also obvious from these figures.

Net PPE as a percentage of revenues, n_t (Specification C)

Specification C is very much the same as Specification A. The difference is that instead of relating *gross* PPE to revenues, one here relates *net* PPE to revenues. The analogous (to Specification A) interpretation of the ratio *net PPE / revenues* is thus that a certain amount of *net* property, plant and equipment (i.e. the book value of PPE) is required to generate a certain amount of revenues.

Empirically, it can be seen from Figure 3 that the ratio *net PPE / revenues* behaves in almost exactly the same way as *gross PPE / revenues.* In fact, both net *and* gross PPE exhibit much more stable developments than revenues. This means that there will be the same problems with Specification C as was noticed for Specification A above, namely that forecasts of the ratio often must be made *conditional* on the forecast of the future revenue growth, which makes the approach with revenue-related ratios not very practical. Further, since the ratio *net PPE / revenues* is not easier to interpret than the corresponding ratio in Specification A, the modelling will not be enhanced by using Specification C instead of Specification A.

Other revenue-related ratios

In the McKinsey model, there are a lot of ratios that have revenues as denominator. Most of these other ratios tend, theoretically, to be more natural than *gross PPE / revenues,* e.g. *trade receivables / revenues.*[67] But, they can also fluctuate quite a lot in some cases. The reason for this can be that the items driven by these ratios are influenced by changes in the management policy concerning, for example, cash or inventory levels. By assuming constant management policy, these items should be closely tied to the revenue development in the future, however, and are therefore useful as forecasting tools.

[67] "Natural" meaning "more naturally related to the revenue development", i.e. for items, the amount of which depend on the amount of revenues in a specific year.

4.1.3 The retirements and depreciation ratios

Attention will now be turned to the two parameters that are the same in all three specifications: *retirements / preceding year's gross PPE* and *depreciation expense / preceding year's gross PPE*. The problems with the denominator, *gross PPE*, have already been discussed, and the analysis here will focus on the respective numerator.

The ratio *retirements / preceding year's gross PPE* is difficult to interpret since the item *historical retirements* in the McKinsey model, in excess of "real" retirements may contain accounting transactions, common in real world Swedish companies. If "retirements" were indeed real retirements, then the interpretation would be "the fraction of preceding year's *gross PPE* retired the next year." With the Copeland et al definition of retirements, however, the interpretation will be something like "the fraction of preceding year's *gross PPE* that will be the difference between the next year's depreciation expense and the next year's actual[68] change in accumulated depreciation." This is due to the fact that the Copeland et al definition is based on a clean surplus relation (CSR),[69] which empirically seems to be violated. Thus, when retirements are calculated according to Copeland et al's definition, a noise term stemming from the violation of the CSR may be included in the retirement figure. Obviously, *retirements as a percentage of preceding year's gross PPE* is not a very meaningful entity, and is even less meaningful if one also applies the interpretation of *gross PPE* stated above.

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[68] After accounting transactions.

[69] Restating the CSR definition of Feltham & Ohlson (1994) in the terminology used here, one obtains: $N_{t+1} = N_t + CapX_{t+1} - DepX_{t+1}$. Using the Copeland et al definition of *Retirements*, $Ret_{t+1} = DepX_{t+1} - (A_{t+1} - A_t)$, to substitute for depreciation yields: $G_{t+1} = G_t + CapX_{t+1} - Ret_{t+1}$. Since this expression intuitively is consistent with clean surplus accounting, the same must be true for the Copeland et al definition of retirements.

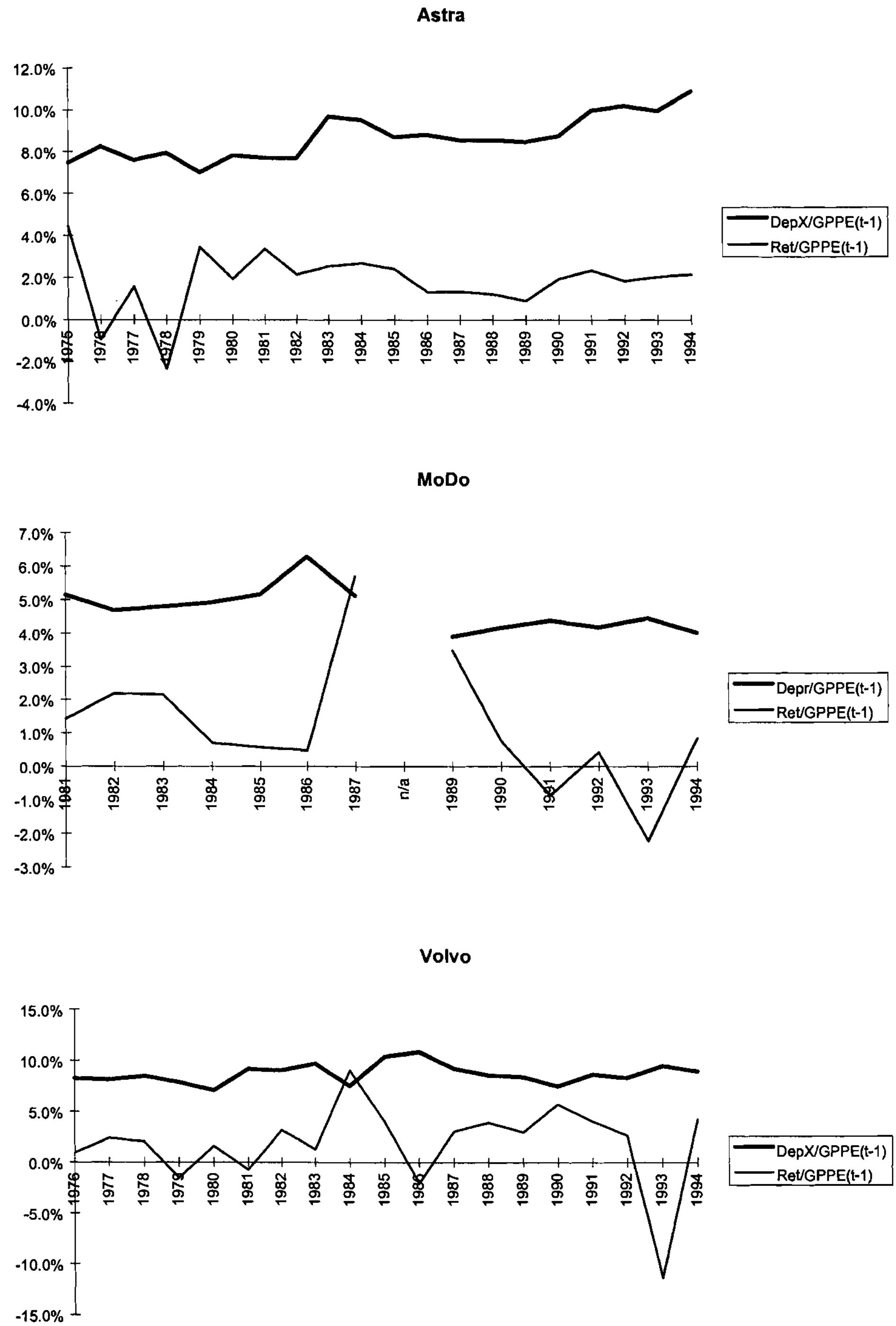

Figure 8 - Depreciation expense and retirements as percentages of preceding year's gross PPE in Astra, MoDo & Volvo.

The real retirements should intuitively be correlated with the size of the *gross PPE*. However, when the determination of historical retirements is done according to a clean surplus relation that historically will not hold, and consequently other things than pure retirements are being measured, the correlation between *"retirements"* and *gross PPE* seems dubious at best. The empirical study of the three companies supports this suspicion.

As can be seen from Figure 8 the *retirements / preceding year's gross PPE* ratio, *r*, is not at all stable in Volvo and MoDo, the two companies with fluctuating and cyclical revenue development. The ratio is more stable in Astra, the growth company. This indicates that *r* might not be a robust predictor. Thus there are three major weaknesses of the *retirements* to *gross PPE* ratio: that it is difficult to interpret, that it indeed can be unstable over time, and that it is not a precise measure of true retirements. Later in this report we point at ways to overcome the forecasting problems by using the steady state concept to determine *r*.

The parameter d_t, *DepX / preceding year's gross PPE*, is intuitively easy to interpret: a fraction of the preceding year's gross PPE will be written off each year. In a normal going concern, this fraction should be fairly constant over time. This is also the case for Astra, MoDo and Volvo as can be seen from Figure 8. Empirically the ratio tends to remain at a specific level for each company, with relatively small variations each year. This makes the ratio quite useful for forecasting.

4.1.4 Concluding remarks on PPE-determination

Specification A of the property, plant and equipment items has one major drawback: for companies with fluctuating revenue development the main driving ratio *gross PPE / revenues* is not working well for forecasting purposes. This problem does not appear when there is stable revenue growth, as in the continuous growth firm Astra. This means that Specification A will work well when modelling a steady state development where revenues grow at a constant rate.

Specification B automatically gives a smoother, and thus more intuitive, development of the property, plant and equipment. Unfortunately, it can be difficult to make forecasts of the main driving ratio based on historical data, especially for companies where new investments in property, plant and equipment take place with large portions at distant points in time. However, since the main driving ratio of Specification B just determines the yearly *change* in *gross PPE*, and not the whole *gross PPE*-stock, the overall sensitivity of the *gross PPE*-determination, with regard to the forecasting problem, will be smaller than for Specification A. In order to yield any steady state (other than PSS), it was shown in Chapter 2 that the parameters have to fulfil a condition that actually reduces Specification B to Specification A.

Specification C is very similar to Specification A and does not add any improvements to Specification A: the forecasting problems are the same. Accordingly, the analysis in previous chapters has been concentrated on Specifications A and B.

A common problem for all three specifications is the determination of *retirements*. The ratio *retirements / preceding year's gross PPE* empirically has a very unstable historical development, which may make forecasts based on historical data impossible.

The determination of *depreciation expense* through the ratio *DepX / preceding year's gross PPE* on the other hand works well for forecasting purposes in all three companies in the study.

4.2 An analytical determination of the *retirements* to *preceding year's gross PPE* ratio

It was noted above that forecasts of the ratio *retirements / preceding year's gross PPE* are almost impossible to infer from the data provided by annual reports, and we will here instead use the steady state concept to determine the retirements by its relation to growth and depreciation.

If the company's new investments in PPE, its depreciation and its retirements each year together have settled to a steady state,[70] a constant fraction of the old PPE will be replaced by new PPE each year. Using this relation, the following can be concluded:

Proposition 4.1

If a company's new investments in PPE, retirements and depreciation together have settled to a steady state, the gross PPE grows with growth rate g, and the economic life equals the depreciation period, then the fraction, r, of preceding year's gross property, plant and equipment that is retired is given by the following relation:

$$(67) \quad \left[{Retirements_t}\Big/{G_{t-1}} \right]_t = r = \frac{g}{(1+g)^{1/d} - 1}$$

where d is the fraction of preceding year's gross PPE that is depreciated.

Proposition 4.1 may seem very convenient when determining the *retirements / gross PPE* ratio in the steady state period. By the steady state assumption this ratio-value automatically falls out once the depreciation rate d and the growth rate g have been determined. Since the company is in steady state these two parameters are constants. The usefulness of Proposition 4.1 is somewhat limited, however, because of the assumptions underlying it. The derivation rests on the assumption that the economic life equals the depreciation period, and that this has been the case for quite some time. This is seldom the case in real world companies. Hence, the r-figure from expression (67) will generally be only an approximate bench-mark figure. In the Eldon valuation in Chapter 5 we use the r-value derived using expression (67) only as a starting point for a more complicated procedure.

Proof of proposition 4.1

Consider a model in continuous time. The assumptions are the following: The company is in steady state where the relations between the company's new investments in PPE, depreciation

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[70] The exact meaning of this is obvious from the proof below.

and retirements are constant. Furthermore, a constant fraction, d, of the *gross PPE* one year ago is depreciated. The *gross PPE* grows at the rate g. All this means that at each point in time, the PPE bought $1/d$ years ago will be retired and replaced by the amount $x(1+g)^{1/d}$ of new PPE. Let r denote the constant fraction of the PPE one year ago that is retired. The *gross PPE* at each point in time only consists of the PPE bought during the last $1/d$ years. All PPE bought earlier has been retired.

Now, suppose that *gross PPE* at time $t+1/d$ in steady state is $G_{t+(1/d)}$. This can be written as (where x denotes the amount spent on PPE at time t):

$$G_{t+(1/d)} = x \int_{t}^{t+(1/d)} (1+g)^{s-t}\, ds = x \cdot \frac{1}{(1+g)^{t} \ln(1+g)} \left[(1+g)^{t+(1/d)} - (1+g)^{t} \right]$$
$$= x \frac{1}{\ln(1+g)} \left[(1+g)^{(1/d)} - 1 \right]$$

At time $t+(1/d)+1$ the depreciation will be:

$$dG_{t+(1/d)} = x \frac{d}{\ln(1+g)} \left[(1+g)^{1/d} - 1 \right]$$

At time $t+1/d$ the PPE bought $1/d$ years ago cost the amount x. To stay in steady state, the amount x of the PPE has to be retired. Also by the steady state assumption the amount retired will grow at the rate g, simply because retirements are constantly related to *gross PPE*, which in turn grows at this rate g. The retirements the coming year will then be:

$$x \int_{t+(1/d)}^{t+(1/d)+1} (1+g)^{s-(t+(1/d))}\, ds = x \frac{1}{(1+g)^{t+(1/d)} \ln(1+g)}$$
$$= \left[(1+g)^{t+(1/d)+1} - (1+g)^{t+(1/d)} \right] = x \frac{1}{\ln(1+g)} \left[(1+g)^{1} - 1 \right] = x \frac{g}{\ln(1+g)}$$

This is by definition equal to $rG_{t+(1/d)}$. The relation between the retirement and depreciation ratios, r and d, then becomes:

$$\frac{r}{d} = \frac{g}{d((1+g)^{1/d} - 1)}$$

This relation between retirements and depreciation must hold in order for the company to be in steady state with respect to its replacement and growth of PPE. From this relation an explicit expression for r can be found:

$$(67) \quad \left[{}^{Ret_t}\!\big/_{G_{t-1}}\right]_t = r = \frac{g}{(1+g)^{1/d} - 1}$$

4.3 Determining the development of deferred taxes

The parameter c, the change in deferred taxes as a percentage of *gross PPE*, is closely related to τ, the corporate tax rate. The nominal corporate tax rate is notoriously unstable over time (during the last ten years in Sweden it has been 52%, 40%, 30%, and 28%), and it is thus hazardous to assume that the present tax rate will remain constant in the future. More stable is the *effective* tax rate, i.e. the tax rate that firms actually face (i.e. pay) when taking all sorts of tax deferrals and tax exemptions into consideration. It can be argued that the government wants to keep the effective tax rate relatively constant, whereas the nominal tax rate is subject to all sorts of political considerations. This is one possible starting-point for the determination of the steady-state value c. In the steady-state period this would mean (using Specification A and denoting the effective tax rate by α):

$$(68) \quad \alpha = \frac{\tau\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right) - \left(T_t - T_{t-1}\right)}{\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right)}$$

Simplifying and substituting for expressions (4 - 6) yields

$$(69) \quad \alpha = \tau - \frac{cb}{1 - p - \frac{db}{1+g} - \frac{iw}{1+g}\left(a + b - \frac{(d-r)b}{g}\right) - \frac{iw}{(1+g)^t R_0}\left(\frac{(d-r)b}{g}R_0 - A_0\right)}$$

It should be noticed that α is time-dependent in the general case, but in the special case of a constant capital structure, $(d-r)b/gR_0$ equals A_0 (see Proposition 2.3 and Corollary 2.1) and the time-dependent part of (69) disappears.

Manipulating (69) yields the expression for *c*:

$$(70) \quad c = \frac{\tau - \alpha}{b}\left(1 - p - \frac{db}{1+g} - \frac{iw}{1+g}\left(a + b - \frac{(d-r)b}{g}\right) - \frac{iw}{(1+g)^t R_0}\left(\frac{(d-r)b}{g}R_0 - A_0\right)\right)$$

If α, the *effective* tax rate, is kept constant, *c* obviously becomes time-dependent except in the constant capital structure case.

4.4 The parameter *p* - operating expenses in % of revenues (sales)

This is the most important parameter in a valuation since it completely determines the profit margin (1-*p*), which is the basic foundation for any corporate value. Effort should thus not be spared on the estimation of the steady-state value for this parameter. Obviously, a thorough analysis of the industry is of the essence. For example, the profit margin is higher (and consequently the value of *p* lower) in monopolistic industries with high barriers of entry than in industries where free competition prevails. Notwithstanding the importance of this subject, we will refrain from pursuing it, as it is a science in itself and lies far beyond the scope of this report. For an excellent introduction we refer to Porter (1980). As a starting-point for an analysis, the averages for a number of Swedish industries are presented in Table 10, below:

Industry	median	min	max	latest value	
Agriculture, hunting, forestry and fishing	92,3%	92,1%	93,5%	92,1%	(1992)
Mining and quarrying	87,7%	81,5%	90,3%	88,9%	
Manufacture of food, beverages and tobacco	93,8%	92,3%	94,6%	92,3%	
Textile, wearing apparel and leather industries	93,6%	91,9%	95,4%	91,9%	
Manufacture of wood and wood products	92,0%	90,4%	95,0%	95,0%	(1992)
Manufacture of pulp, paper products	88,5%	84,2%	92,9%	88,1%	
Printing and publishing	93,5%	92,4%	94,7%	92,4%	
Manufacture of chemicals, petroleum, coal rubber and plastic products	88,0%	83,6%	89,4%	83,6%	
Manufacture of non-metallic mineral products, except products of petroleum and coal	89,3%	86,7%	92,2%	90,8%	
Basic metal industries	93,5%	88,9%	98,5%	91,1%	
Manufacture of fabricated metal products	92,0%	91,4%	93,7%	92,9%	(1992)
Manufacture of fabricated machinery and equipment	94,1%	93,6%	94,8%	94,6%	(1992)
Manufacture of electromechanical products	94,5%	93,2%	96,9%	94,5%	(1992)
Electricity, gas and water	75,6%	72,9%	77,5%	72,9%	
Construction	94,9%	94,5%	95,1%	95,0%	
Wholesale trade	97,0%	96,5%	97,7%	96,5%	
Retail trade	96,6%	95,9%	96,8%	96,8%	(1992)
Restaurants and hotels	94,4%	92,6%	95,5%	93,0%	
Land transportation	89,6%	88,1%	95,3%	88,1%	
Sea transportation	92,0%	89,7%	95,3%	94,6%	
Air transportation	94,2%	90,3%	99,3%	99,3%	
Consulting business	93,5%	92,8%	94,1%	94,1%	(1992)
Social and personal services	91,5%	90,5%	92,7%	91,5%	(1992)

Table 10 - Operating expenditures in % of revenues in Swedish businesses 1988-1993

The percentage is rather stable in most industries, a fact that may be taken as justification for the use of industry averages as benchmarks.

4.5 The parameter *a* - net working capital in % of revenues (sales)

The definition is *(Current assets-Current non-interest bearing liabilities)/Revenues*. This is the empirically most troublesome parameter, since it varies a lot both over time and across businesses. The management policy on different working capital items may be quite company specific, customs regarding the length of payment periods may differ substantially over time, etc. The unstable nature of the parameter is evident from Table 11, below, where industry averages vary substantially over time.

It should be noted once again that in order not to lose valuable information, the aggregate measure should not be used in the explicit forecast period. Instead the different balance sheet items of the working capital should be related to revenues, one by one.

Appendix 4 - Empirical industry figures.

The *industry average* is a concept open for different interpretations. The Swedish empirical figures used in this chapter are derived from treating the entire industry as one company, calculating the different financial ratios from its income statement and balance sheet. Conceptually, this is the same as a weighted mean, where the weights are given by the size of the companies. Another possibility would of course be an unweighted mean, a third would be the median.

There is an inherent difficulty in making empirical observations regarding the stability over time of financial ratios. On the one hand one wants a long time period to be able to draw statistically valid conclusions, on the other hand the underlying relationships may change over a long period of time and the principles of financial reporting may also change, causing additional "definition bias". We do not go further back than 1988 in the tables since the informational content in earlier figures is rather questionable.

The data in the empirical tables are taken from the industry-specific income statements and balance sheets in *Official Statistics of Sweden, Statistics Sweden: Enterprises 1987,*

Industry	median	min	max	latest value	
Agriculture, hunting, forestry and fishing	10,4%	9,7%	12,8%	12,8%	(1992)
Mining and quarrying	40,6%	26,7%	45,2%	45,2%	
Manufacture of food, beverages and tobacco	8,0%	4,8%	9,3%	5,1%	
Textile, wearing apparel and leather industries	24,0%	19,3%	25,6%	19,3%	
Manufacture of wood and wood products	15,5%	13,3%	16,3%	16,3%	(1992)
Manufacture of pulp, paper products	11,6%	-10,1%	21,5%	14,1%	
Printing and publishing	6,7%	2,0%	12,1%	12,1%	
Manufacture of chemicals, petroleum, coal rubber and plastic products	12,2%	4,0%	18,1%	15,7%	
Manufacture of non-metallic mineral products, except products of petroleum and coal	11,7%	4,1%	14,0%	13,0%	
Basic metal industries	14,0%	6,7%	25,9%	14,7%	
Manufacture of fabricated metal products	15,6%	13,8%	20,2%	20,2%	(1992)
Manufacture of fabricated machinery and equipment	16,9%	14,1%	18,4%	17,8%	(1992)
Manufacture of electromechanical products	20,9%	16,0%	22,5%	16,0%	(1992)
Electricity, gas and water	-1,4%	-11,3%	10,7%	1,2%	
Construction	7,5%	1,8%	11,3%	7,8%	
Wholesale trade	9,5%	8,8%	10,1%	8,8%	
Retail trade	9,0%	8,5%	10,0%	8,5%	(1992)
Restaurants and hotels	-1,5%	-6,1%	0,3%	-4,5%	(1992)
Land transportation	-1,2%	-5,1%	0,9%	0,9%	
Sea transportation	11,0%	-7,2%	31,2%	4,6%	
Air transportation	20,6%	15,7%	28,3%	28,3%	
Consulting business	8,0%	5,9%	12,4%	8,4%	(1992)
Social and personal services	7,9%	7,3%	9,1%	7,6%	(1992)

Table 11 - Net working capital in % of revenues in Swedish businesses 1988-1993

Enterprises 1988, ..., Enterprises 1993. The official industry classification (SNI) has changed from 1993. In some cases this makes the figures up until 1992 incompatible with the 1993 figures, so sometimes the 1993 figure has had to be left out. This is marked in the tables.

The many problems and reservations connected with the subject of average industry data means they should be used with caution. We present them merely as a possibility for situations where one can find no other guidance as to reasonable parameter values.

5. Eldon AB

In this chapter it will be shown how one in practice can implement a model of the Copeland et al type on a Swedish company using the results from previous chapters. The idea is to highlight many of the practical problems that arise in a valuation of this type and to suggest ways of dealing with them. The Swedish company Eldon AB will be used as example. In many cases it makes more sense to value the (consolidated) company group rather than the parent company; this is also what will be done here: what we refer to as Eldon is henceforth the consolidated group.

The actual procedure will be dealt with step by step:

5.1 Setting up historical financial statements

5.2 Calculating historical financial ratios, i.e. historical parameter values

5.3 Forecasting future parameter values

5.4 Setting up future (forecasted) financial statements

5.5 Calculating future (forecasted) free cash flows

5.6 Calculating the equity value

It should be noted that the aim here is to construct a working model that is internally consistent and that the numbers plugged into it are for demonstration only. We have chosen to use as a sort of base-case a rather gloomy picture of the company's future with virtually no real growth. This is in no way meant to be a realistic view of this specific company - in reality there are growth opportunities (indicated by the fact that the market value is considerably higher than our "base-case value"). Questions of future growth, operating margin, etc., are more for the corporate analyst, however, and we will concentrate on the actual model building.

5.1 Historical financial statements

The first step in the valuation procedure is to insert historical accounting data into a spreadsheet in order to calculate the historical values of the parameters.

5.1.1 Looking for changes in the structure of financial statements

A natural starting point is to look at the structure of the financial statements (balance sheets and income statements) over the time period of interest in order to find out whether there have been any major structural changes in the company's financial statements and/or accounting principles. In the Eldon case, one important change has occurred: in 1990 the group accounting changed from untaxed-reserves accounting to deferred-taxes accounting. This will have as a consequence that some of the accounting figures from 1989 and 1990 will not be consistent with later years, and hence figures from 1989 and 1990 should be used with caution.

5.1.2 Creating a spreadsheet model for the historical financial statements

Let the first column be reserved for text description of the items and the following columns be reserved for each one of the historical years. Then add the different items of the income statement and the balance sheet in a row each, in column 1. The summarising items (like *operating income*) are indicated in the tables below by a straight line *above* them. Other figures that are calculated (and *not* directly inserted) are indicated by *italics*.

The next step is then to insert the figures of the historical financial statements. All summarising items can be calculated in the spreadsheet instead of inserting them directly from the annual report. They can then be used as checkpoints against the corresponding items in the annual reports in order to ensure that all inserted figures are correct. The historical income statements are presented in Table 12:

INCOME STATEMENT:	1989	1990	1991	1992	1993	1994
Revenues	1,136.6	1,169.4	1,289.4	1,271.8	1,407.2	1,663.9
Operating expenses	-1,027.7	-1,057.9	-1,205.9	-1,182.1	-1,277.0	-1,478.4
Depreciation expense	-35.5	-35.4	-41.6	-44.9	-47.3	-46.5
Operating income	73.4	76.1	41.9	44.8	82.9	139.0
Net financial income	-21.1	-17.8	-29.2	-26.8	-37.4	-26.3
Extraordinary items	0.0	25.8	-10.1	-40.4	10.4	12.6
Earnings before taxes	52.3	84.1	2.6	-22.4	55.9	125.3
Appropriations	32.4	32.1				
Taxes	-32.6	-52.6	-8.1	-6.6	-19.0	-56.6
Net profit	**52.0**	**63.5**	**-5.5**	**-29.0**	**36.9**	**68.7**

Table 12 - Historical income statements, Eldon

The insertion of the balance sheet figures requires some extra considerations, especially when it comes to the property, plant and equipment items, since these are generally presented as net values in Swedish annual reports, whereas most of the models in the Copeland et al book and in this report require that the net PPE is explicitly calculated as gross PPE minus accumulated depreciation. We also need a measure of retirements.

Practically, the insertion of the respective PPE figures is done in the following way: gross PPE in the balance sheet is calculated as the sum of the gross property, gross plant, gross equipment and installations in progress from the PPE calculations table (Table 13); accumulated depreciation and net PPE are calculated in the same manner.[71] The figures in Table 13 come from the depreciation expense note of the balance sheet with the exception for retirements, which are derived by calculating the difference between the depreciation expense (from Table 12) and the change in accumulated depreciation.

[71] Appreciation will be netted against accumulated depreciation in the balance sheet (Table 14).

CALCULATION OF PROPERTY, PLANT & EQUIPMENT ITEMS:	1989	1990	1991	1992	1993	1994
Installation in progress	0.0	4.3	0.3	12.4	0.1	5.0
Property:						
Gross	172.6	197.8	220.5	248.1	257.8	257.4
Acc. depr	-42.0	-37.8	-47.1	-53.9	-58.2	-64.6
Appreciation	4.6	4.4	4.0	3.8	3.0	3.0
Net Property	135.2	164.4	177.4	198.0	202.6	195.8
Equipment:						
Gross	352.3	360.9	420.3	413.7	439.0	459.3
Acc. depr	-187.1	-178.2	-207.6	-215.8	-236.3	-260.7
Net Equipment	165.2	182.8	212.7	197.9	202.7	198.6
Plant:						
Gross	18.9	29.7	31.4	36.4	38.1	38.0
Acc. depr	-2.1	-2.0	-2.3	-2.4	-2.4	-2.6
Net Plant	16.8	27.6	29.1	34.0	35.7	35.4
Depreciation	35.5	35.4	41.6	44.9	47.3	46.5
Property	4.5	4.0	4.8	5.8	6.6	6.8
Plant	0.2	0.2	0.2	0.2	0.2	0.2
Equipment	30.8	31.3	36.6	38.9	40.5	39.5
Retirements	*n/a*	*48.5*	*2.2*	*29.6*	*21.7*	*15.5*
Property		*8.0*	*-4.9*	*-1.2*	*1.5*	*0.4*
Plant		*0.2*	*-0.1*	*0.1*	*0.2*	*0.0*
Equipment		*40.2*	*7.2*	*30.7*	*20.0*	*15.1*

Table 13 - Calculation of PPE, depreciation and retirements, historically, Eldon

With the additional information from Table 13, the balance sheets from the annual reports can be inserted as in Table 14:

BALANCE SHEET:						
	1989	1990	1991	1992	1993	1994
ASSET SIDE						
Operating cash	38.4	78.8	82.5	95.9	122.1	99.7
Trade receivables	166.9	190.0	175.7	175.2	200.8	262.2
Prepaid expenses	10.3	9.3	12.4	14.9	17.3	17.5
Other receivables	15.6	25.0	35.8	47.0	21.5	14.6
Inventories	331.2	344.6	350.1	320.9	336.8	349.0
Current assets	562.5	647.8	656.5	653.9	698.5	743.0
Excess marketable securities	8.8	7.5	7.5	7.8	7.7	0.9
Investment fund	47.4	39.3	9.6	1.2		
Gross property, plant and equipm	*543.8*	*592.6*	*672.5*	*710.6*	*735.0*	*759.7*
Accumulated depreciation	*-226.6*	*-213.6*	*-253.0*	*-268.3*	*-293.9*	*-324.9*
Net property, plant and equipment	317.2	379.1	419.5	442.3	441.1	434.8
Total assets	**935.9**	**1,073.6**	**1,093.1**	**1,105.2**	**1,147.3**	**1,178.7**
Net Total Assets*	739.1	797.6	850.5	874.3	888.6	863.7
* Total assets - Working capital liabilities						
	1989	1990	1991	1992	1993	1994
DEBT & EQUITY SIDE						
Short-term debt	83.1	116.6	104.0	163.4	139.7	91.7
Accounts payable	82.4	130.8	103.9	92.2	118.3	140.0
Accrued expenses	75.7	75.7	89.5	98.8	78.1	91.3
Taxes Payable	17.3	19.9	7.0	3.4	33.0	47.2
Other current liabilities	21.3	49.7	42.2	36.5	29.3	36.5
Total current liabilities	279.8	392.6	346.6	394.3	398.4	406.7
Long-term debt	80.8	92.0	170.0	158.3	172.1	152.6
Used check-credit	30.8	38.6	60.5	66.9	73.3	56.3
Pension funds	51.6	53.2	59.5	61.8	62.4	63.5
Deferred taxes	8.2	5.4	92.6	90.0	65.0	70.5
Total longterm liabilities	171.4	189.3	382.6	377.0	372.8	342.9
Untaxed Reserves	300.3	268.6				
Common stock	43.3	51.9	51.9	51.9	51.9	51.9
Restricted reserves	19.6	13.9	194.8	160.0	151.6	139.6
Non-restr. equity	121.5	157.3	117.2	122.0	172.6	237.6
Total common equity	184.4	223.1	363.9	333.9	376.1	429.1
Total liabilities and equity	**935.9**	**1,073.6**	**1,093.1**	**1,105.2**	**1,147.3**	**1,178.7**

Table 14 - Historical balance sheets, Eldon

In Eldon, equity consists of three categories: common stock, restricted reserves (which together make up restricted equity) and non-restricted equity (basically retained earnings). When deferred-taxes accounting was introduced at Eldon (from 1991) it meant that the restricted reserves were raised dramatically. The untaxed reserves were then divided between deferred taxes and restricted reserves.

5.2 Calculating historical financial ratios (parameters)

We are now ready to calculate the historical performance measures, i.e. the historical values of the model parameters. We also insert the historical inflation in order to make it possible to calculate the real growth in revenues.[72]

Operations:

Operations:	1989	1990	1991	1992	1993	1994
Real growth	1.3%	-6.8%	0.8%	-3.6%	5.7%	15.7%
Inflation	6.4%	10.4%	9.4%	2.3%	4.7%	2.2%
Revenue growth (g)	7.8%	2.9%	10.3%	-1.4%	10.6%	18.2%
Operating exp/revenues (p)	90.4%	90.5%	93.5%	92.9%	90.7%	88.9%

Table 15 - Historical parameter values, operations

Working capital:

Working cap./revenues:	1989	1990	1991	1992	1993	1994
+Operating cash/rev's	3.4%	6.7%	6.4%	7.5%	8.7%	6.0%
+Trade receivables/rev's	14.7%	16.3%	13.6%	13.8%	14.3%	15.8%
+Other receivables/rev's	1.4%	2.1%	2.8%	3.7%	1.5%	0.9%
+Inventories/rev's	29.1%	29.5%	27.2%	25.2%	23.9%	21.0%
+Prepaid expenses/rev's	0.9%	0.8%	1.0%	1.2%	1.2%	1.1%
-Accounts payable/rev's	7.2%	11.2%	8.1%	7.2%	8.4%	8.4%
-Other curr liab's/rev's	1.9%	4.2%	3.3%	2.9%	2.1%	2.2%
-Accrued expenses/rev's	6.7%	6.5%	6.9%	7.8%	5.6%	5.5%
-Taxes payable/rev's	1.5%	1.7%	0.5%	0.3%	2.3%	2.8%
Net working cap./revenues (a)	32.2%	31.8%	32.1%	33.3%	31.3%	25.7%

Table 16 - Historical parameter values, working capital

[72] Real growth = ((1 + revenue growth) / (1 + inflation)) - 1

Property, plant and equipment:

Property,Plant and Equipment (PPE):	1989	1990	1991	1992	1993	1994
CapX/rev's (e)		8.3%	6.7%	5.3%	3.3%	2.4%
Gross PPE/rev's (b)	47.8%	50.7%	52.2%	55.9%	52.2%	45.7%
DepX/prec. year's gross PPE (d)		6.5%	7.0%	6.7%	6.7%	6.3%
Ret/prec. year's gross PPE (r)		8.9%	0.4%	4.4%	3.1%	2.1%

Table 17 - Historical parameter values, PPE

Deferred taxes and debt:

Deferred taxes and debt:	1989	1990	1991	1992	1993	1994
Incr in def. taxes/gross PPE (c)		-0.5%	0.9%	-0.4%	-3.4%	0.7%
Debt/net total assets (w)	33.3%	37.7%	46.3%	51.5%	50.4%	42.2%
Short-term/net total assets	11.2%	14.6%	12.2%	18.7%	15.7%	10.6%
Long-term/net total assets	10.9%	11.5%	20.0%	18.1%	19.4%	17.7%
Used check-cr./net total asset:	4.2%	4.8%	7.1%	7.7%	8.2%	6.5%
Pension funds/net total assets	7.0%	6.7%	7.0%	7.1%	7.0%	7.4%

Table 18 - Historical parameter values, deferred taxes and debt

The figure for *increase in deferred taxes / gross PPE* in 1991 is calculated using the adjusted figure for deferred taxes in 1990, given in the annual report of 1991 (adjusted for change in accounting principles).[73]

5.3 Forecasting future parameter values

The next step in the valuation process is to predict the parameter values for future years, i.e. to specify the forecast assumptions. The future has been divided into a ten-year explicit forecast period and the time after that has been set as a steady state, where all input parameters remain

[73] As mentioned earlier, the Eldon group accounting changed from untaxed-reserves accounting to deferred-taxes accounting in 1991.

constant and, as will be seen further on, we will also have achieved the different steady states discussed in earlier chapters.

The first year in the constant parameter period is 2005. This is *year zero* in the parametric steady state period, as discussed previously. This year is marked by italics in the tables below. Year 1 in the perpetuity period is thus 2006, and in accordance with earlier results 2006 will be the base year for any continuing value calculations.

The revenue growth and the operating margin related parameters are without doubt the most important ones and no effort should be spared in trying to make the forecasts as accurate as possible. As was indicated in previous chapters, this is more a problem for the corporate analyst, however, and our treatment here will be a simple *base-case scenario*, which of course can be altered to incorporate any features deemed reasonable by the analyst.

5.3.1 Operations

Operations:	1995	1996	1997	1998	1999	2000
Real growth	7.0%	5.0%	3.0%	1.5%	0.6%	0.0%
Inflation	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
Revenue growth (g)	10.2%	8.2%	6.1%	4.5%	3.6%	3.0%
Operating exp/revenues (p)	90.0%	90.0%	90.0%	90.0%	90.0%	90.0%
Operations:	2001	2002	2003	2004	*2005*	2006 (perpetuity)
Real growth	0.0%	0.0%	0.0%	0.0%	*0.0%*	0.0%
Inflation	3.0%	3.0%	3.0%	3.0%	*3.0%*	3.0%
Revenue growth (g)	3.0%	3.0%	3.0%	3.0%	*3.0%*	3.0%
Operating exp/revenues (p)	90.0%	90.0%	90.0%	90.0%	*90.0%*	90.0%

Table 19 - Forecasted parameter values, operations

The growth was exceptionally high in 1994, due to the very large depreciation of the Swedish currency in 1993 and 1994. This effect may fade in the coming years and we have set a zero real growth from the year 2000. The official inflation goal, set by the Swedish central bank, is 2 ±1%, and since the upper limit tends to be the rule rather than the exception in these matters, our forecast is 3% throughout.[74] The *operating expenditures to revenues* ratio has revolved around 90% in the historical period, hence the 90%-figure also in the forecast period.

5.3.2 Working capital

The different working capital figures are set so as to achieve a (parametric) steady state from 2005 and onwards. Other than that we make no attempt to justify the figures in Table 20, except that they are reasonable from a historical perspective.

Working cap./revenues:	1995	1996	1997	1998	1999	2000
+Operating cash/rev's	6.0%	5.9%	5.7%	5.6%	5.5%	5.4%
+Trade receivables/rev's	15.5%	15.4%	15.3%	15.3%	15.2%	15.2%
+Other receivables/rev's	1.0%	1.2%	1.3%	1.4%	1.5%	1.6%
+Inventories/rev's	21.5%	22.0%	22.5%	23.0%	23.5%	24.0%
+Prepaid expenses/rev's	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%
-Accounts payable/rev's	8.4%	8.4%	8.4%	8.4%	8.4%	8.4%
-Other curr liab's/rev's	2.3%	2.3%	2.4%	2.4%	2.4%	2.4%
-Accrued expenses/rev's	5.4%	5.6%	5.8%	5.9%	6.0%	6.1%
-Taxes payable/rev's	2.5%	2.4%	2.3%	2.3%	2.2%	2.2%
Net working cap./revenues (a)	26.4%	26.8%	26.9%	27.3%	27.7%	28.1%
Working cap./revenues:	2001	2002	2003	2004	*2005*	2006 (perpetuity)
+Operating cash/rev's	5.3%	5.2%	5.1%	5.0%	*5.0%*	5.0%
+Trade receivables/rev's	15.1%	15.1%	15.0%	15.0%	*15.0%*	15.0%
+Other receivables/rev's	1.7%	1.8%	1.9%	2.0%	*2.0%*	2.0%
+Inventories/rev's	24.5%	24.7%	24.9%	25.0%	*25.0%*	25.0%
+Prepaid expenses/rev's	1.0%	1.0%	1.0%	1.0%	*1.0%*	1.0%
-Accounts payable/rev's	8.4%	8.4%	8.4%	8.4%	*8.4%*	8.4%
-Other curr liab's/rev's	2.5%	2.5%	2.5%	2.5%	*2.5%*	2.5%
-Accrued expenses/rev's	6.2%	6.3%	6.4%	6.5%	*6.5%*	6.5%
-Taxes payable/rev's	2.2%	2.1%	2.0%	2.0%	*2.0%*	2.0%
Net working cap./revenues (a)	28.3%	28.5%	28.6%	28.6%	*28.6%*	28.6%

Table 20 - Forecasted parameter values, working capital

[74] Revenue growth (g) = ((1 + real growth) × (1 + inflation)) - 1

5.3.3 Taxes and debt ratio

The forecasts look as follows:

Taxes and debt:	1995	1996	1997	1998	1999	2000
Tax rate (tau)	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Effective Tax rate						
Incr in def. taxes/gross PPE (c)	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%
Borrowing rate (i)	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
Debt/net total assets (w)	41.8%	41.4%	41.2%	41.0%	40.8%	40.6%
Short-term/net total assets	10.6%	10.5%	10.5%	10.4%	10.4%	10.3%
Long-term/net total assets	17.4%	17.2%	17.1%	17.0%	17.0%	16.9%
Used check-cr./net total assets	6.2%	5.9%	5.6%	5.4%	5.1%	4.9%
Pension funds/net total assets	7.6%	7.8%	8.0%	8.2%	8.3%	8.5%
Taxes and debt:	2001	2002	2003	2004	2005	2006 (perpetuity)
Tax rate (tau)	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Effective Tax rate					27.5%	27.5%
Incr in def. taxes/gross PPE (c)	0.3%	0.3%	0.3%	0.3%	0.318%	0.318%
Borrowing rate (i)	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
Debt/net total assets (w)	40.4%	40.2%	40.1%	40.0%	40.0%	40.0%
Short-term/net total assets	10.3%	10.3%	10.2%	10.2%	10.2%	10.2%
Long-term/net total assets	16.9%	16.8%	16.8%	16.7%	16.7%	16.7%
Used check-cr./net total assets	4.6%	4.4%	4.3%	4.2%	4.2%	4.2%
Pension funds/net total assets	8.6%	8.7%	8.8%	8.9%	8.9%	8.9%

Table 21 - Forecasted parameter values, taxes and debt

Corporate tax rate, effective tax rate and increase in deferred taxes

The corporate tax rate is assumed to be 30%. The increase in deferred taxes is not easy to predict by itself, but in section 4.3 it was shown how the parameter c, the increase in deferred taxes, should be specified in steady state, once the *effective* tax rate had been set - and the effective tax rate is much easier to have an opinion about. In this example we have set it at 27.5% (in the steady state period from 2005 and onwards) which yields an increase in deferred taxes of 0.318% when applying expression (70). It should be stressed once again that this modelling approach is insensitive to the beliefs about the nominal corporate tax rate. Should one believe that the government will tax at 28% instead, the value of c will be adjusted accordingly.

Book value debt ratio (w)

The *debt / net assets* ratio, i.e. the parameter *w*, is set at 40% in the steady state period, assuming that this is the optimal capital structure as the management sees it. It should be noted that this is the book debt ratio and *not* the market debt ratio. The ratio is also decomposed in four different debt item ratios (which are later used for calculating the different debt items of the balance sheet).

5.3.4 Property, plant and equipment (PPE)

In accordance with the findings in Chapter 4, the specification of PPE will be best made using *Specification B*, i.e.:

Specification B:

Items directly determined by ratios :

$$CapX_t = e_t \cdot R_t$$

$$DepX_t = d_t \cdot G_{t-1}$$

$$Ret_t = r_t \cdot G_{t-1}$$

Items derived indirectly :

$$G_t = G_{t-1} + CapX_t - Ret_t$$

$$A_t = A_{t-1} + DepX_t - Ret_t$$

$$N_t = G_t - A_t$$

CapX = Capital expenditures
DepX = Depreciation expense
R = Revenues
Ret = Retirements
G = Gross property, plant and equipment
A = Accumulated depreciation
N = Net property, plant and equipment
e = Capital expenditures as a percentage of revenues
d = Depreciation expense as percentage of preceding year's gross PPE
r = Retirements as a percentage of preceding year's gross PPE
t = time-index denoting year

The forecasted parameter values are presented in Table 22. Note that the parameter *b* (gross PPE as a percentage of revenues) is not meaningful when working with Specification B in the explicit forecast period (until 2004). In the steady state period (from 2005), there are values of *b* presented in Table 22. The rationale for this will be discussed below.

Property,Plant and Equipment (PPE):	1995	1996	1997	1998	1999	2000
CapX/rev's (e)	2.9%	2.9%	3.0%	3.0%	3.0%	3.0%
Gross PPE/rev's (b)	n/m	n/m	n/m	n/m	n/m	n/m
DepX/prec. year's gross PPE (d)	6.5%	6.5%	6.5%	6.5%	6.5%	6.5%
Ret/prec. year's gross PPE (r)	3.2%	3.4%	3.6%	3.7%	3.9%	4.1%
Property,Plant and Equipment (PPE)	2001	2002	2003	2004	*2005*	2006 (perpetuity)
CapX/rev's (e)	3.1%	3.1%	3.1%	3.2%	*3.195%*	3.195%
Gross PPE/rev's (b)	n/m	n/m	n/m	n/m	*41.162%*	41.162%
DepX/prec. year's gross PPE (d)	6.5%	6.5%	6.5%	6.5%	*6.5%*	6.5%
Ret/prec. year's gross PPE (r)	4.3%	4.5%	4.6%	4.8%	*4.995%*	4.995%

Table 22 - Forecasted parameter values, PPE

Depreciation / preceding year's gross PPE (d)

The parameter *d* generally tends to lie on a company specific level as was indicated in section 4.1.4. In Eldon it has historically been around 6.5%. There is no reason to believe that this will change in the future.

Capital expenditures / revenues (e) **and** *retirements / preceding year's gross PPE (r)*

The two remaining PPE-related parameters *e* and *r* are important for the fulfilment of the initial conditions necessary to ensure TSS:

(22) $$gA_0 = (d - r)bR_0$$

(49) $$e = \frac{G_0 (g + r)}{(1 + g)R_0}$$

Condition (22) is derived using Specification A. However, it was shown in section 2.2 that if expression (49) holds, Specification B reduces to Specification A, where the ratio *gross PPE / revenues* (*b*) is constant. This means that in the perpetuity period, i.e. from 2005 and onwards, the only alternative is to use Specification A in order to be able to use a continuing value. One can of course work with the *terminology* of Specification B as long as e is determined by expression (49). The "translation" between Specification A and Specification B will then be:

$$(71) \qquad b = \beta = \frac{e(1+g)}{(g+r)}$$

and condition (22) can, using Specification B terminology, be stated as:

$$(72) \qquad gA_0 = (d-r)e\frac{(1+g)}{(g+r)}R_0$$

Expression (72) can be rearranged as:

$$(73) \qquad r = \frac{de(1+g)R_0 - g^2 A_0}{e(1+g)R_0 + gA_0}$$

As seen in Chapter 4, the historical parameter values of e and r give little guidance in forecasting, hence other procedures have to be employed. As before, the forecast will be divided into two parts: the explicit forecast period and the perpetuity period.

We will make a direct forecast for 1995, the first year in the explicit forecast period. For 2005 the e- and r-values will be given by the initial value conditions (49) and (73) to guarantee textbook steady state in the perpetuity period. The parameter values between 1995 and 2005 are then determined by a linear interpolation.

A problem is that in order to determine the e- and r-values in 2005, we need A_0 and G_0. This is obvious from the initial conditions (49) and (73). A_0 and G_0 will be given from the spreadsheet model only after all parameters up to 2005 have been specified, including the e- and r-values. Thus, a more complex and partly iterative procedure is needed:

1. Make direct forecasts of the e- and r-values in 1995.
2. Set trial values for e and r in 2005.

3. Model values for the years in-between by way of linear interpolation.
4. This yields complete financial statements for the years in question including values in 2005 for accumulated depreciation and gross PPE, i.e. . The initial value conditions (49) and (73) will generally not hold for these values of A_0 and G_0.
5. Calculate *r* in 2005 according to condition (73). Since the 2005 value of *r* is changed, the linearly interpolated *r*-values of earlier years will also be changed. This will give new financial statements and new, updated values of A_0 and G_0.
6. Use the updated G_0 to calculate *e* in 2005 according to condition (49). Since the *e*-value of 2005 is changed, the linearly interpolated *e*-values of earlier years will also be changed. This will give new financial statements and new, updated values of A_0 and G_0.
7. Repeat procedures 5 and 6 until there is convergence.

The direct forecasts of *e* and *r* for 1995 are apparent from Table 22. The first trial values for 2005 were *e*=2.9% and *r*=5.2%.[75] Using these figures the program converged very quickly.

5.3.5 Checks for the steady state period

Having specified all parameter values, their reasonableness should be checked. By inspecting the resulting balance sheets and income statements, it can be verified that the development of the company in the explicit forecast period is reasonable (Tables 23, 24 and 25, below). For the steady state period, the checks derived in section 2.1 can be used, since we are back in Specification A.[76] These checks were the following:

- A higher gross PPE / revenues ratio should lead to lower FCF and vice versa,
- FCF decreasing in the tax rate,
- Net PPE non-decreasing over time,
- Positive pre-tax profits,
- Dividends decreasing in the gross PPE / revenues ratio,
- Non-negative book equity.

---

[75] The first trial value of *r* in 2005 is given by expression (67) in Proposition 4.1. This formula is valid under the assumption that the economic life equals the depreciation period, and although these ideal conditions do not quite hold here, the *r*-value arrived at with this method should be a reasonable first guess. The first trial value of *e* in 2005 is simply set equal to the 1995 value.

1. A higher gross PPE / revenues ratio should lead to lower FCF and vice versa:

The condition to check is the following:

$$(8) \qquad \pi d - r + (1+g)c < g$$

Inserting the predicted values: $0.3 \cdot 0.065 - 0.05 + (1.03) \cdot 0.003 \approx -2.7\% < g = 3\%$

Obviously, the condition is fulfilled by a wide margin.

2. FCF decreasing in the tax rate:

The condition for this property is the following:

$$(9) \qquad p + \frac{bd}{1+g} < 1$$

Inserting the parameter values yields:

$$0.90 + \frac{0.416 \cdot 0.065}{1.03} \approx 92.6\% < 1$$

Also this condition holds without any problems.

3. Net PPE non-decreasing over time:

This condition, sufficient for NP to be decreasing in the ratio *gross PPE / revenues*, is:

$$(11) \qquad (d-r) \leq g$$

[76] Expression (71) should be applied whenever the parameter b is appearing: $b=\beta-e(1+g)/(g+r)$.

Inserting the parameter values:

(0.065-0.05) < 0.03

and, consequently, the condition is fulfilled.

4. Positive pre-tax profits:

From the forecasted income statement of year 2006 one can see directly that this holds.

5. Dividends decreasing in the gross PPE / revenues ratio:

The parameter condition to check here is:

$$(13) \qquad d\tau + wg + c(1+g) + \frac{\chi(d-r)}{g} - w(d-r) - r - \chi < g$$

Inserting the predicted parameter values for 2005 and onwards yields a LHS value of -3.7%, which is clearly less than +0.03.

6. Non-negative book equity:

The following two boundary conditions must be fulfilled:

$$(14) \qquad (1-w)(a+b) > \left(\frac{b}{g} - \frac{b}{g(1+g)}\right)[(d-r)(1-w) + c(1+g)] + \frac{(1-w)A_0 + T_0}{R_0\,(1+g)}$$

$$(15) \qquad (1-w)(a+b) > \frac{b}{g}[(d-r)(1-w) + c(1+g)]$$

The left hand side takes on the value 41.9%. The right hand side (RHS) of condition (14) is equal to 16.2% and for condition (15) the RHS equals 16.9%, so the conditions hold.

5.4 The forecasted financial statements

The forecasted financial statements, finally, are presented below. Most items follow directly from the forecast assumptions (i.e. the parameter values) described in the previous sections. Some additional comments will be made, however.

5.4.1 Income statements

The setting up of income statements is straightforward:

FORECASTED INCOME STATEMENT:	1995	1996	1997	1998	1999	2000
Revenues	1,833.8	1,983.2	2,104.0	2,199.6	2,279.2	2,347.6
Operating expenses	-1,650.4	-1,784.9	-1,893.6	-1,979.7	-2,051.3	-2,112.8
Depreciation expense	-49.4	-51.3	-53.3	-55.5	-57.7	-59.9
Operating income	134.0	147.1	157.1	164.5	170.3	174.9
Net financial income	-40.1	-42.4	-44.5	-46.2	-48.0	-49.7
Extraordinary items	0.0	0.0	0.0	0.0	0.0	0.0
Earnings before taxes	93.9	104.6	112.6	118.3	122.2	125.2
Taxes	-28.2	-31.4	-33.8	-35.5	-36.7	-37.6
Net profit	**65.8**	**73.2**	**78.8**	**82.8**	**85.6**	**87.6**

FORECASTED INCOME STATEMENT:	2001	2002	2003	2004	*2005*	2006
Revenues	2,418.0	2,490.6	2,565.3	2,642.2	*2,721.5*	2,803.2
Operating expenses	-2,176.2	-2,241.5	-2,308.8	-2,378.0	*-2,449.4*	-2,522.8
Depreciation expense	-62.1	-64.2	-66.4	-68.6	*-70.7*	-72.8
Operating income	179.7	184.8	190.1	195.7	*201.5*	207.5
Net financial income	-51.2	-52.6	-54.1	-55.6	*-57.1*	-58.8
Extraordinary items	0.0	0.0	0.0	0.0	*0.0*	0.0
Earnings before taxes	128.5	132.2	136.0	140.1	*144.4*	148.7
Taxes	-38.6	-39.7	-40.8	-42.0	*-43.3*	-44.6
Net profit	**90.0**	**92.5**	**95.2**	**98.0**	***101.0***	**104.1**

Table 23 - Forecasted income statements

5.4.2 Balance sheets

Now consider the balance sheets:

FORECASTED BALANCE SHEET:						
	1995	1996	1997	1998	1999	2000
ASSET SIDE						
Operating cash	110.0	117.0	119.9	123.2	125.4	126.8
Trade receivables	284.2	305.4	321.9	336.5	346.4	356.8
Prepaid expenses	18.3	19.8	21.0	22.0	22.8	23.5
Other receivables	18.3	23.8	27.4	30.8	34.2	37.6
Inventories	394.3	436.3	473.4	505.9	535.6	563.4
Current assets	825.2	902.4	963.6	1,018.4	1,064.4	1,108.1
Excess marketable securities	0.0	0.0	0.0	0.0	0.0	0.0
Investment fund	0.0	0.0	0.0	0.0	0.0	0.0
Gross property,plant and equipm	788.6	820.0	853.1	886.9	921.0	954.8
Accumulated depreciation	-350.0	-374.6	-398.7	-422.3	-445.2	-467.3
Net property, plant and equipment	438.6	445.4	454.4	464.7	475.8	487.5
Total assets	**1,263.8**	**1,347.8**	**1,418.0**	**1,483.1**	**1,540.2**	**1,595.6**
Net Total Assets*	922.7	976.9	1,020.4	1,065.2	1,107.2	1,147.2
* Total assets - Working capital liabilities						

FORECASTED BALANCE SHEET:						
	2001	2002	2003	2004	*2005*	2006
ASSET SIDE						
Operating cash	128.2	129.5	130.8	132.1	*136.1*	140.2
Trade receivables	365.1	376.1	384.8	396.3	*408.2*	420.5
Prepaid expenses	24.2	24.9	25.7	26.4	*27.2*	28.0
Other receivables	41.1	44.8	48.7	52.8	*54.4*	56.1
Inventories	592.4	615.2	638.8	660.6	*680.4*	700.8
Current assets	1,151.0	1,190.5	1,228.8	1,266.3	*1,306.3*	1,345.5
Excess marketable securities	0.0	0.0	0.0	0.0	*0.0*	0.0
Investment fund	0.0	0.0	0.0	0.0	*0.0*	0.0
Gross property,plant and equipm	988.3	1021.7	1054.8	1087.6	*1120.2*	1153.8
Accumulated depreciation	-488.5	-508.7	-527.7	-545.5	*-561.9*	-578.7
Net property, plant and equipment	499.9	513.0	527.0	542.1	*558.4*	575.1
Total assets	**1,650.8**	**1,703.5**	**1,755.8**	**1,810.4**	***1,864.7***	**1,920.6**
Net Total Assets*	1,184.2	1,222.8	1,260.7	1,297.8	*1,336.7*	1,376.8
* Total assets - Working capital liabilities						

Table 24 - Forecasted balance sheets, asset side

	1995	1996	1997	1998	1999	2000
DEBT & EQUITY SIDE						
Short-term debt	97.8	102.6	107.1	110.8	115.1	118.2
Accounts payable	154.0	166.6	176.7	184.8	191.5	197.2
Accrued expenses	99.0	111.1	122.0	129.8	136.8	143.2
Taxes Payable	45.8	47.6	48.4	50.6	50.1	51.6
Other current liabilities	42.2	45.6	50.5	52.8	54.7	56.3
Total current liabilities	438.9	473.4	504.8	528.7	548.2	566.6
Long-term debt	160.6	168.0	174.5	181.1	188.2	193.9
Used check-credit	57.2	57.6	57.1	57.5	56.5	56.2
Pension funds	70.1	76.2	81.6	87.3	91.9	97.5
Deferred taxes	72.9	75.3	77.9	80.5	83.3	86.2
Total longterm liabilities	360.8	377.2	391.1	406.5	419.9	433.8
Common stock	51.9	51.9	51.9	51.9	51.9	51.9
Other equity	412.3	445.3	470.2	496.0	520.2	543.4
Total common equity	464.2	497.2	522.1	547.9	572.1	595.3
Total liabilities and equity	**1,263.8**	**1,347.8**	**1,418.0**	**1,483.1**	**1,540.2**	**1,595.6**

	2001	2002	2003	2004	*2005*	2006
DEBT & EQUITY SIDE						
Short-term debt	122.0	125.9	128.6	132.4	*136.3*	140.4
Accounts payable	203.1	209.2	215.5	221.9	*228.6*	235.5
Accrued expenses	149.9	156.9	164.2	171.7	*176.9*	182.2
Taxes Payable	53.2	52.3	51.3	52.8	*54.4*	56.1
Other current liabilities	60.5	62.3	64.1	66.1	*68.0*	70.1
Total current liabilities	588.6	606.6	623.7	645.0	*664.3*	684.3
Long-term debt	200.1	205.4	211.8	216.7	*223.2*	229.9
Used check-credit	54.5	53.8	54.2	54.5	*56.1*	57.8
Pension funds	101.8	106.4	110.9	115.5	*119.0*	122.5
Deferred taxes	89.1	92.2	95.4	98.6	*102.2*	105.9
Total longterm liabilities	445.6	457.8	472.3	485.4	*500.5*	516.2
Common stock	51.9	51.9	51.9	51.9	*51.9*	51.9
Other equity	564.7	587.1	607.9	628.1	*647.9*	668.3
Total common equity	616.6	639.0	659.8	680.0	*699.8*	720.2
Total liabilities and equity	**1,650.8**	**1,703.5**	**1,755.8**	**1,810.4**	***1,864.7***	**1,920.6**

Table 25 - Forecasted balance sheets, debt and equity side

Excess marketable securities

Excess marketable securities are assumed to be zero. They were very low during the last historical year (1994). The cash required to run the operations is modelled as operating cash. Any excess capital generated by the company will be distributed to the shareholders as dividends (same assumption as in earlier chapters) unless it is needed for investments or for maintaining the desired capital structure.

Equity

No stock issues are foreseen. Hence the *common stock* remains unchanged. The item *other equity* corresponds to retained earnings plus other non-restricted and restricted reserves. *Other equity* is the residual item of the balance sheet (i.e. decided so as to make the asset side equal to the debt and equity side).

5.4.3 Statements of equity

The statement of equity is used to forecast the dividends.

STATEMENT OF EQUITY:						
	1995	1996	1997	1998	1999	2000
COMMON STOCK:						
Beginning common stock	51.9	51.9	51.9	51.9	51.9	51.9
Issues/Repurchases	0.0	0.0	0.0	0.0	0.0	0.0
Ending common stock	51.9	51.9	51.9	51.9	51.9	51.9
OTHER EQUITY:						
Beginning other equity	376.3	412.3	445.3	470.2	496.0	520.2
Net profit	65.8	73.2	78.8	82.8	85.6	87.6
Common dividends	-29.8	-40.2	-53.9	-57.0	-61.3	-64.5
Ending other equity	412.3	445.3	470.2	496.0	520.2	543.4

STATEMENT OF EQUITY:						
	2001	2002	2003	2004	2005	2006
COMMON STOCK:						
Beginning common stock	51.9	51.9	51.9	51.9	51.9	51.9
Issues/Repurchases	0.0	0.0	0.0	0.0	0.0	0.0
Ending common stock	51.9	51.9	51.9	51.9	51.9	51.9
OTHER EQUITY:						
Beginning other equity	543.4	564.7	587.1	607.9	628.1	647.9
Net profit	90.0	92.5	95.2	98.0	101.0	104.1
Common dividends	-68.6	-70.1	-74.5	-77.8	-81.3	-83.7
Ending other equity	564.7	587.1	607.9	628.1	647.9	668.3

Table 26 - Forecasted statements of equity

Beginning other equity is the same as *ending other equity* from the preceding year. *Net profit* comes from the income statement. *Ending other equity* is taken directly from the balance sheet. To balance the entire system, this leaves us with *common dividends* as the residual item. This follows immediately from the clean surplus relationship (the change in net book value of equity equals net profit minus dividends).

The equity/dividend forecasting may seem technically complicated, but conceptually it can be summarised in a very simple rule, namely that the forecasted excess capital not required for internal company use (after rebalancing the debt) is distributed to the share-holders as dividends.

We have not touched on the subject of restricted vs. non-restricted reserves. The subject is somewhat peripheral in this context and is addressed in Appendix 5.

5.5 Free cash flow

All items in the free cash flow calculations can be derived directly from the earlier tables:

FORECASTED FREE CASH FLOW:	1995	1996	1997	1998	1999	2000
Revenues	1833.8	1983.2	2104.0	2199.6	2279.2	2347.6
Operating expenses	-1650.4	-1784.9	-1893.6	-1979.7	-2051.3	-2112.8
Depreciation expense	-49.4	-51.3	-53.3	-55.5	-57.7	-59.9
Adjusted EBIT	134.0	147.1	157.1	164.5	170.3	174.9
Taxes on EBIT	-40.2	-44.1	-47.1	-49.4	-51.1	-52.5
Change in deferred taxes	2.4	2.5	2.6	2.7	2.8	2.9
NOPLAT	96.2	105.4	112.5	117.8	122.0	125.3
Depreciation expense	49.4	51.3	53.3	55.5	57.7	59.9
Gross cash flow	145.5	156.7	165.8	173.3	179.6	185.2
Change in working capital	56.1	47.4	34.5	34.5	30.8	28.3
Capital expenditures	53.2	58.1	62.3	65.7	68.8	71.5
Gross investment (-)	109.3	105.5	96.7	100.3	99.6	99.9
Operating FCF	**36.2**	**51.2**	**69.1**	**73.0**	**80.0**	**85.3**
Non-operating cash flow	0.0	0.0	0.0	0.0	0.0	0.0
Total FCF	**36.2**	**51.2**	**69.1**	**73.0**	**80.0**	**85.3**

Table 27a - Forecasted statements of free cash flow 1995 - 2000

FORECASTED FREE CASH FLOW:	2001	2002	2003	2004	2005	2006
Revenues	2418.0	2490.6	2565.3	2642.2	2721.5	2803.2
Operating expenses	-2176.2	-2241.5	-2308.8	-2378.0	-2449.4	-2522.8
Depreciation expense	-62.1	-64.2	-66.4	-68.6	-70.7	-72.8
Adjusted EBIT	179.7	184.8	190.1	195.7	201.5	207.5
Taxes on EBIT	-53.9	-55.4	-57.0	-58.7	-60.4	-62.3
Change in deferred taxes	3.0	3.1	3.2	3.3	3.6	3.7
NOPLAT	128.8	132.4	136.2	140.2	144.6	148.9
Depreciation expense	62.1	64.2	66.4	68.6	70.7	72.8
Gross cash flow	190.8	196.7	202.7	208.8	215.3	221.7
Change in working capital	24.6	25.5	23.9	22.0	22.7	23.4
Capital expenditures	74.4	77.4	80.5	83.6	87.0	89.6
Gross investment (-)	99.0	102.9	104.3	105.7	109.6	112.9
Operating FCF	**91.8**	**93.8**	**98.3**	**103.1**	**105.7**	**108.8**
Non-operating cash flow	0.0	0.0	0.0	0.0	0.0	0.0
Total FCF	**91.8**	**93.8**	**98.3**	**103.1**	**105.7**	**108.8**

Table 27b - Forecasted statements of free cash flow 2001 - 2006

The free cash flow should correspond to the financial cash flow:

FORECASTED FINANCIAL CASH FLOW:	1995	1996	1997	1998	1999	2000
Incr(+)/Decr(-) exc. mkt sec'ties	0.0	0.0	0.0	0.0	0.0	0.0
Incr(-)/Decr(+) debt, check-cr.	-15.0	-12.7	-10.5	-10.6	-10.4	-8.4
Incr(-)/Decr(+) in pension funds	-6.6	-6.1	-5.4	-5.7	-4.5	-5.6
After-tax net interest exp. (+)	28.0	29.7	31.1	32.4	33.6	34.8
Common dividends (+)	29.8	40.2	53.9	57.0	61.3	64.5
Incr(-)/Decr(+) in common stock	0.0	0.0	0.0	0.0	0.0	0.0
Financial cash flow	**36.2**	**51.2**	**69.1**	**73.0**	**80.0**	**85.3**
FORECASTED FINANCIAL CASH FLOW:	2001	2002	2003	2004	2005	2006
Incr(+)/Decr(-) exc. mkt sec'ties	0.0	0.0	0.0	0.0	0.0	0.0
Incr(-)/Decr(+) debt, check-cr.	-8.3	-8.6	-9.4	-9.0	-12.1	-12.5
Incr(-)/Decr(+) in pension funds	-4.3	-4.5	-4.6	-4.6	-3.5	-3.6
After-tax net interest exp. (+)	35.9	36.8	37.9	38.9	40.0	41.2
Common dividends (+)	68.6	70.1	74.5	77.8	81.3	83.7
Incr(-)/Decr(+) in common stock	0.0	0.0	0.0	0.0	0.0	0.0
Financial cash flow	**91.8**	**93.8**	**98.3**	**103.1**	**105.7**	**108.8**

Table 28 - Forecasted statements of financial cash flow

5.6 Value calculation

We now have access to forecasts of free cash flows and dividends and can hence proceed to the actual equity value calculation. Three different ways have been discussed earlier in this report:

1. Discounting the free cash flows at a constant WACC. Calculating the equity value as the sum of the discounted free cash flows minus the debt value.

2. Discounting the free cash flows at a year-to-year updated WACC. Calculating the equity value as the sum of the discounted free cash flows minus the debt value.

3. Discounting the dividends at the equity cost of capital. Calculating the equity value as the sum of the discounted dividends.

As discussed earlier, methods 2 and 3 yield the same value. For completeness we will present all three valuations.

Cost of equity capital

First, the cost of equity capital must be estimated. This is necessary for all three valuation methods. As suggested by Copeland et al, the capital asset pricing model[77] is used for determining the cost of equity capital, k_E:

$$(74) \qquad k_E = r_f + \beta\left[\mathrm{E}(r_m) - r_f\right]$$

[77] The capital asset pricing model (CAPM) by Sharpe (1964) has been very popular as a way of determining the cost of capital. The discount rate is determined by adding a risk premium to the risk-free interest rate. The risk premium is calculated by multiplying the asset's sensitivity to general market movements (its *beta*) by the market risk premium. A useful practical article on the subject is Dimson & Marsh (1982). However, the empirical validity of CAPM is a matter of debate. Proposed alternatives to the single-factor CAPM include different multi-factor approaches based on Ross' arbitrage pricing model (Ross (1977)). Fama & French (1993) identifies five common risk factors in returns on stocks and bonds that can be used for estimating the cost of capital.

where: r_f is the riskfree rate

β is the beta value

$E(r_m) - r_f$ is the market risk premium

For Eldon, the beta value has been estimated to be 1.08,[78] the market risk premium has been estimated to be 5.7% in Sweden,[79] and consequently the following cost of equity capital is estimated for Eldon:

$$k_E = r_f + \beta\left[\mathrm{E}(r_m) - r_f\right] = 7.0\% + 1.08 \cdot 5.7\% = 13.156\%$$

As was mentioned in Chapter 3, a constant cost of equity capital may not be theoretically viable if the capital structure is not constant (referred to as "type 2 approximation error"). In the Eldon case the capital structure is rather stable, as can be seen from Figure 9, and hence there are no substantial indications that the risk level, and with it k_E, should change.[80]

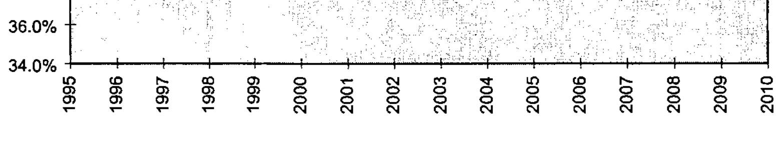

Figure 9 - Forecasted market debt ratio in Eldon

[78] Öhmans Börsguide 1995.
[79] Nyman & Smith (1994).
[80] The market value debt ratio turns out to be very close to the book value debt ratio in the steady state period. As was seen in Chapter 2, this is not always the case.

Thus, we in this example use a constant k_E. Note, however, that it is perfectly possible to use a time-varying cost of equity capital whenever the analyst may find this reasonable (if using either method 2 or method 3).

5.6.1 FCF valuation with constant WACC

This is the model proposed in Copeland et al. As has been shown in Chapter 3, it will suffer from approximation errors if the market debt ratio varies over time.

The state of the company at 1 January 1995, is the basis for the calculation of the WACC. The formula looks as follows:

$$(75) \qquad k_{\text{WACC}} = \frac{D}{D+EV}(1-\tau)i + \left(1 - \frac{D}{D+EV}\right)k_E$$

D is the market debt value (Jan 1, 1995), EV is the market equity value (Jan 1, 1995). The market interest rate on debt, i, is predicted to be 11%. The outstanding debt is assumed to be on market terms.[81] Since k_{WACC} depends on the market value of equity, which in turn depends on k_{WACC}, the value calculation and the calculation of k_{WACC} have to be made simultaneously. This requires a recursive procedure:

a) Use a trial input WACC.
b) Calculate the equity value.
c) Calculate the WACC implied by the equity value from b) by using formula (75). If this resulting WACC value differs from a), the trial input WACC in a) has to be adjusted. The procedure has to be redone until the resulting WACC value (c) equals the trial input value (a). (The procedure can be automated in a modern spreadsheet program.)

The final value calculation from the spreadsheet is presented in Table 29:

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[81] That means that the market interest rate on debt equals the borrowing rate.

Value Calculation: FCF with constant WACC						
Trial Input WACC	10.943%					
Resulting WACC	10.943%					
	1995	1996	1997	1998	1999	2000
Forecasted Free Cash Flow	36.2	51.2	69.1	73.0	80.0	85.3
Present Value as of Jan. 1, 1995	32.7	41.6	50.6	48.2	47.6	45.7
	2001	2002	2003	2004	2005	Perpetuity 2006
Forecasted Free Cash Flow	91.8	93.8	98.3	103.1	105.7	108.8
Present Value as of Jan. 1, 1995	44.4	40.9	38.6	36.5	33.7	437.2
Present Value of Forecasted FCF	897.6					
Excess Marketable Securities	0.9					
-Debt Value	-364.1					
Final Equity Value	534.4					

Table 29 - Value calculation 1 using a constant WACC

The value of the perpetuity period is calculated using the Gordon formula. The market value capital structure is non-constant in the explicit forecast period (evident from Figure 9) and hence using a constant WACC will yield a small approximation error of type 1.

5.6.2 FCF valuation with updated WACC

As argued in Chapter 3, this is in fact the correct discounting method to use when discounting free cash flows since it explicitly incorporates possible variations in the capital structure. The procedure was described in detail in section 2.1.5. The market interest rate on debt, i, is predicted to be 11%. The outstanding debt is assumed to be on market terms:

Value Calculation: FCF with updated WACC						
	1995	1996	1997	1998	1999	2000
Forecasted Free Cash Flow	36.2	51.2	69.1	73.0	80.0	85.3
Present Value of Forecasted FCF at beginning of each year	892.1	953.4	1006.6	1047.8	1089.8	1129.3
WACC	10.929%	10.949%	10.964%	10.967%	10.969%	10.974%
	2001	2002	2003	2004	*2005*	Perpetuity 2006
Forecasted Free Cash Flow	91.8	93.8	98.3	103.1	*105.7*	108.8
Present Value of Forecasted FCF at beginning of each year	1168.0	1204.4	1243.0	1281.3	*1319.2*	1358.7
WACC	10.980%	10.989%	10.998%	11.003%	*11.009%*	11.009%
Present Value of Forecasted FCF	892.1					
Excess Marketable Securities	0.9					
-Debt Value	-364.1					
Final Equity Value	528.9					

Table 30 - Value calculation 2 using updated WACC

The equity value arrived at by using the updated WACC approach does not differ very much from the value given by the constant WACC approach (529 million instead of 534 million). Thus the approximation error introduced by wrongly assuming a constant capital structure is only 1%. The size of the error will depend on how much the capital structure (debt vs. value of future operations) changes over the forecast period. Eldon is an exceptionally stable company, but for many other companies the magnitude of the approximation error would be much larger.

5.6.3 Dividend valuation

Now we can calculate the equity value by discounting the estimated future dividends at the cost of equity capital. As was argued in Chapter 3, this should yield exactly the same value as the updated WACC approach - a proposition that can now be checked:

Value Calculation: Dividend Valuation						
	1995	1996	1997	1998	1999	2000
Forecasted Dividends	29.8	40.2	53.9	57.0	61.3	64.5
Present Value as of Jan. 1, 1995	26.3	31.4	37.2	34.7	33.1	30.7
						Perpetuity
	2001	2002	2003	2004	2005	2006
Forecasted Dividends	68.6	70.1	74.5	77.8	81.3	83.7
Present Value as of Jan. 1, 1995	28.9	26.1	24.5	22.6	20.9	211.6
Excess Marketable Securities	0.9					
Final Equity Value	528.9					

Table 31 - Value calculation 3 using dividends

As can be seen the dividend approach yields the same value as the FCF approach using updated WACC.

5.7 Concluding remarks on Eldon

The base-case scenario, with no real growth after year 2000, yields an equity value of SEK 529 million (using method 2 or 3). This is, as predicted, lower than the observed market value of SEK 722 million. There are clearly growth opportunities, which we have not included here. Neither have we made any thorough analysis of future profit margins, which of course are crucial to any value calculation. The purpose of this chapter has instead been to show how some practical problems can be overcome, also how some of the results derived in earlier chapters can be incorporated into a "real-world" company valuation. We hope also to have conveyed a sense of how flexible these kinds of valuation approaches are. Some items have been modelled/forecasted very carefully whereas others have been treated more sketchily. This shows that one can basically be as explicit and thorough as one deems reasonable when setting up a valuation model of this kind.

Appendix 5 - Restricted reserves

Swedish companies generally have restricted reserves as part of their equity, as well as non-restricted reserves. We have not modelled this explicitly in the Eldon case. However, one has to remember that only non-restricted reserves can be used for paying dividends. Thus, it may be a good idea to check that the dividend forecasts are not violating this requirement, i.e. that dividends only are paid out of non-restricted equity. We suggest the following model:

1. Retrieve the item *other equity* from the balance sheet (i.e. book equity other than common stock).
2. Calculate the *restricted reserves*.
3. Calculate the *non-restricted equity* as the difference between *other equity* and *restricted reserves*.
4. Check that *non-restricted equity* is not negative. Since *non-restricted equity* is after dividends, a negative value would indicate unrealistically large dividends.

We will go through the procedure, and use the year 2005 to visualise:

1. Other equity

Other equity is 647.9 in 2005.

2. Restricted reserves

We propose the following model for calculating the *restricted reserves*:

$$\text{(A5:1)}\quad \text{Restricted reserves} = 0.20 \times \text{common stock} + \frac{1 - \text{tax rate}}{\text{tax rate}} \times \text{deferred taxes}$$

Firms are required by law to keep restricted reserves at least as high as 20% of the common stock[82] (unless losses make this impossible), hence the first term in the right-hand-side of expression (A5:1).

The second term has to do with untaxed reserves. Although deferred-taxes accounting is used in the group accounting, the deferred taxes are *derived* from the untaxed reserves of the different companies that belong to the group. This is a complicated issue, and it is by no means clear how one should address it when valuing the consolidated group. Our suggestion here builds on the following facts:

- Untaxed reserves can be divided into deferred taxes and equity.
- Parent company and subsidiaries have untaxed reserves (which cannot be used for dividends).
- The group is basically a sort of "sum" of the parent company and its subsidiaries.
- The group accounting is done using deferred taxes accounting.
- When adding the untaxed reserves of the different group companies, part of it will be counted as deferred taxes, part of it will be counted as equity.
- The equity part will in reality be part of the untaxed reserves of the different companies belonging to the group, hence it should by definition be labelled *restricted* equity since it cannot be used for paying dividends.

This "split-up" of the untaxed reserves is done according to the following formula:[83]

- To *deferred taxes*: tax rate × untaxed reserves
- To *restricted reserves*: (1 - tax rate) × untaxed reserves

Since the deferred taxes are known (modelled via the parameter c as described in section 5.3.4), it is possible to calculate also the "restricted reserves part" of the untaxed reserves, i.e. the second term in (A5:1). In the year 2005:

[82] Swedish: "reservfond"

[83] It can be noted (see Table 14) that the Eldon group changed from untaxed reserves to deferred taxes accounting in 1990-91, and the split-up of untaxed reserves seems to have been made according to this division formula. This was also the recommended accounting standard (see FAR (1991) pp. 286).

Common stock	51.9
Deferred taxes	102.2

$$\textit{Restricted reserves} = 0.20 \times 51.9 + \frac{1-0.30}{0.30} \times 102.2 \approx 248.8$$

This is obviously a rather heuristic way of estimating the restricted reserves.

3. Calculation of non-restricted equity

In year 2005:

Other equity	647.9
-Restricted reserves	-248.8
Non-restricted equity	399.1

4. Non-restricted equity greater than zero

The non-restricted equity is positive in year 2005. Remember that non-restricted equity (as well as the other equity items) is calculated after dividends. Since the remaining non-restricted equity is positive, the forecasted dividends in 2005 have been fully paid out of non-restricted reserves. Had the *non-restricted equity* item been negative, the forecasted dividends would have been unrealistically high.

6. Concluding Remarks

We have in Chapter 5 shown how a McKinsey-style valuation of Eldon works in practice. We have, however, also made use of some of the improvements derived in earlier chapters of this report. The purpose has not been to show a "recommended" model, rather we have shown the consequences implied when using different versions of the valuation model.

Some more normative remarks may be warranted. Eldon turned out to be quite a stable company in all respects, and approximation errors are of fairly limited magnitude. In more unstable circumstances the problems with approximation errors would be much larger. Small or large - the point is that it is *unnecessary* to introduce approximation errors when there is absolutely nothing to be gained from the approximation in the first place:

In Eldon (and indeed in almost any real-world company), using the original Copeland et al model[84] would introduce approximation errors into the valuation. Both type 1 errors[85] and type 2 errors[86] would be present. Type 1 errors can be avoided by updating the weighted average cost of capital, thus taking into consideration the impact of the changing weights in the WACC formula, implied by the actual forecast. This is a rather cumbersome procedure. There exists a computationally simpler alternative that yields exactly the same result: to discount the forecasted dividends at the equity cost of capital. This in no way alters the approach (free cash flow valuation), it should merely be thought of as a calculational tool.

Type 2 errors may be present, even if the actual value calculation is done by discounting the dividends. The importance of such errors is a qualitative question. A useful diagnostic tool is a graph of the development of the capital structure over time (Figure 9 in Chapter 5). In the Eldon case, the capital structure remained fairly constant over time, and the conclusion was that any type 2 errors are small in magnitude.

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[84] Discounting the free cash flows at a constant WACC.

[85] The calculational errors stemming from the use of a constant weighted average cost of capital although the weights are not constant over time.

Continuing value calculations are popular among practitioners because of their computational simplicity. What is typically done with revenue-driven models, such as the McKinsey model, is to assume a constant revenue growth after, say, ten years. Continuing value calculations do, however, require assumptions much stronger than "continuous revenue growth" in order to yield correct results - assumptions rarely fulfilled in practice, one would suspect. The McKinsey model requires a "Textbook steady state," which means that the company earns constant margins, grows at a constant rate, invest a constant proportion of its gross cash flow each year, and that it earns a constant return on existing capital as well as on all new investments. The detailed analysis in Chapter 2 showed that this boils down to a one or two (depending on specification) initial value conditions: For Specification A it is concluded that a "true" or textbook steady state (TSS) is implied by the general assumption that the model's parameters are constants (PSS) and the initial parameter condition (22) that ensures a constant capital structure. For Specification B, the only possibility to establish a steady state is where the parameters fulfil condition (49) that reduces the more general Specification B setting to the more specific Specification A.

Trying to fulfil these conditions will almost inevitably lead the analyst to assign values to the parameters that are dictated by his spreadsheet model rather than by his expert analysis and knowledge of the company and the industry. Thus, even if the calculations are formally correct, there might be errors of a more qualitative nature stemming from the analyst's wish to make the input numbers fit the modelling requirements. This is evident in the Eldon valuation - many degrees of freedom are lost because of the TSS requirement. Is TSS really necessary then?

TSS is necessary only if one requires that there be no approximation errors whatsoever. But it is generally sufficient that the approximation errors are small in magnitude and that there are no counter-intuitive effects in the valuation. To achieve this, much less stringent assumptions are needed, making the valuation more realistic.

A *valuation schedule* might, with this in mind, look something like this:

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[86] The errors stemming from ignoring that the risk of the company probably changes when the capital structure changes which should have effects on the cost of equity capital.

1. Set up the historical financial statements.
2. Calculate the historical financial ratios.
3. Forecast the financial ratios (the parameters) year by year for, say, the next ten years (i.e. the explicit forecast period).
4. Calculate the explicit forecast period's income statements, statements of equity, balance sheets and *FCF* statements, implied by the forecasted parameter values. Check that everything looks reasonable, otherwise correct the parameter values.
5. Set the parameter values in the perpetuity period. These must be set such that no counter-intuitive effects occur. Such parameter restrictions were derived in Chapter 2. For example, the condition $(d\text{-}r) \leq g$ guarantees that the net PPE will be non-decreasing over time. The restrictions derived in Chapter 2 are appropriate for this model. With other models, other restrictions may apply, but the methodology used in Chapter 2 can be applied to any model. Note that these conditions are in the form of inequalities and hence are much less restrictive than many of the TSS conditions, which completely specify the values of some parameters.
6. Calculate year zero and year one in the perpetuity period (income statement, balance sheet, equity statement and FCF statement).
7. Calculate the horizon value, using the dividends from year one as valuation measure:

$$Horizon\ value = \frac{DIV_1}{k_E - g} - \frac{(1-\tau)iw\left(\dfrac{d-r}{g}bR_0 - A_0\right)}{k_E}$$

   Equivalently, the horizon value can be calculated using a long (100-150 years) explicitly modelled forecast, which is really only a copy-routine in the spreadsheet program since the parameter values are constant. The latter approach makes the valuation more flexible (see step 9 below).
8. Discount the horizon value and the forecasted dividends from the years modelled explicitly. The best way to do this is to start with the last value and "discount backwards" one period at a time. This way of modelling makes it possible to change the discount rate, should the riskiness of the company change (see step 9 below).

9. Check for type 2 approximation errors by constructing a graph of the capital structure over time.[87] If the graph shows substantial deviations from today's capital structure, the analyst may consider changing the equity cost of capital over time to make the discount rate better reflect the changing risk of the company, and then go back to the previous step in the schedule. If one does not want to go that far and make actual adjustments, a look at the capital structure over time will still give the analyst an indication as to whether the valuation is biased upwards or downwards. An increasing debt ratio may indicate an increasing risk which would tell the analyst that the estimated equity value is probably a bit too high.

Using this approach there will be no type 1 approximation errors, possible type 2 approximation errors are explicitly searched for, and there will be a large amount of freedom in specifying parameter values and still no counter-intuitive implicit effects.

This report has in part been very detailed. We will end it on a more general note. The value of an asset is in its most general sense equal to the present value of all future cash flows pertaining to the asset. Both the *free cash flow approach* and the *dividend approach* utilise this basic valuation concept. In the free cash flow approach the "asset" is the company's operations. In the dividend approach the asset is the shares. Intuitively, two valuation approaches stemming from the same basic concept should result in the same value. We have established a procedure that ensures that the equivalence of approaches holds under quite general conditions, even if the capital structure is non-constant. This has also been shown to hold in a real-world case.

What we originally set out to do was to value the equity of a company, i.e. to derive a price for its shares. Subscribing to the wide-held view that the value of a share equals the present value of expected dividends, it is reassuring that the FCF valuation approach can be applied in such a way that the value derived equals the value from the dividend approach.

[87] The graph of the perpetuity period will be much easier to construct if this period is modelled explicitly, as suggested in step 7.

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Study 4:

Company Valuation with a Periodically Adjusted Cost of Capital

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Abstract. This study points out procedures and relations that are useful in applications of company valuation methods using discounted free cash flow valuation models. The study, which can be viewed as an extension of Levin and Olsson (1995) (Study 1 in this dissertation), develops a general discounting procedure where the equity cost of capital and the weighted average cost of capital can be simultaneously adjusted to reflect a varying capital structure under different assumptions about the value of interest tax shields. Implementation issues including programming are also discussed.

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1. Introduction

One of the most popular company valuation techniques is discounted free cash flow valuation. The free cash flows of the unlevered firm (i.e., independent of financing) are calculated and discounted at the weighted average cost of capital. The McKinsey book *Valuation: Measuring and Managing the Value of Companies* (Copeland et al. (1994)) has substantially added to the popularity of such models – indeed, the valuation technique itself is often referred to as "the McKinsey model". The latter fact tends to anger some academics, who recognise that free cash flow valuation is not an invention of McKinsey and Company. While this is certainly true, the point highlights the fact that much of the importance in business related academia lies in the applications and in the possibility of implementing results. In this respect the McKinsey book is an important addition to the literature.

There are, however, some issues left uncommented on in Copeland et al. For example, a number of financial ratios are chosen as parameters in their forecasting model, but the rationale for these choices is not always obvious. Another issue on which the book is silent is the interdependence between the costs of capital and the development of the company. We discuss these problems in Study 1. The particular aspects pursued further here have to do with the latter issue – the relations between the cost(s) of capital and the forecasted development of the company.

The common use of a constant weighted average cost of capital (WACC) as discount rate assumes a constant capital structure (in market value terms). If the projected capital structure varies, then the use of a constant WACC is inappropriate. This "conventional wisdom" seems to have different implications for different people. The prudent university lecturer is quick to point out that with a varying capital structure one should choose a valuation technique that can accommodate such variation. The adjusted present value (APV) method suggests itself as the prime candidate for this role.[1] Another approach is to ignore that the assumption of constant capital structure is being violated, and still use a constant WACC. Copeland et al. (1994) follow the latter path. It makes the exposition clear, but comes at a price, namely approximation errors that may or may not be serious.

[1] See, e.g., Brealey and Myers (1991), p. 458.

Study 1 shows that it is perfectly possible to use the popular and intuitively appealing WACC approach, but continuously adjust the weighted average cost of capital for anticipated changes in capital structure, thereby alleviating the approximation error problems, in particular those stemming from changing weights in the WACC formula.[2] This study extends the analysis by showing how the *equity cost of capital* and the weighted average cost of capital can be simultaneously adjusted to reflect a varying capital structure. The cost of debt is still assumed to be constant (as discussed further in the next section).

1.1 The modelling framework

The company's forecasted future financial statements can be expressed as:

Assets	**Debt and Equity**
(Net) Working Capital, NWC_t	Debt, D_t
Net Property, Plant and Equipment, N_t	Deferred Taxes, T_t
= Gross Property, Plant and Equipm., G_t - Accumulated Depr., A_t	Book Equity, BV_t

Forecasted balance sheet for period t

+Revenues, R_t
-Operating Expenses, OpX_t
-Depreciation Expense, $DepX_t$
-Interest Expense, $IX_t = i\,D_{t-1}$
-Taxes, IT_t
=**Net profit,** NP_t

Forecasted income statement for period t

Free cash flow equals the gross cash flow minus the gross investments. Using the notation from the financial statements above:

$$\begin{aligned} FCF_t &= (1-\tau)(R_t - OpX_t - DepX_t) + (T_t - T_{t-1}) + DepX_t \\ &\quad - (NWC_t - NWC_{t-1}) - (N_t - N_{t-1} + DepX_t) \end{aligned}$$

Invested capital, IC_t, is defined as net working capital plus net property, plant and equipment:

$$IC_t = NWC_t + N_t$$

An alternative way of calculating free cash flow is to start from net profit:

$$FCF_t = NP_t + (1-\tau)IX_t + (T_t - T_{t-1}) - (IC_t - IC_{t-1})$$

Further, the clean surplus relation is supposed to hold (i.e., the change in book equity equals net profit minus dividends):[3]

$$BV_t - BV_{t-1} = NP_t - DIV_t$$

The company's debt is assumed to be on market terms, i.e., the book value of debt is equal to the market value; equivalently, the coupon rate on debt, is assumed to equal the (market) cost of debt for all future periods. Moreover, the coupon rate (and thus the cost of debt) is assumed to be constant over time, and both (equal) rates are denoted by i. These simplifying assumptions are made to avoid cumbersome modelling of the detailed debt structure at each future period. The reader will note that we explicitly abstract from all non-quantifiable costs of debt, since the extreme cases where they may be important are not the primary concern of this study. For example, expected costs of financial distress and agency costs are not

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[2] Specifically, the equity cost of capital and the cost of debt are assumed to be constant.

[3] Dividends, DIV_t, are defined net of capital contributions/withdrawals.

addressed here. Also, we disregard the issue of personal taxes and consider only taxes on the corporate level. Our framework is thus similar to Modigliani and Miller (1963), and consequently, an increase in leverage leads to a lower cost of capital, because of tax consequences. This is obviously a valid approximation only within a limited range of debt ratios. At extreme debt ratios, financial distress and agency costs cannot be disregarded. The reader is referred to the vast corporate finance literature on the subject.

There are no excess marketable securities in the forecast period. Deferred taxes are treated as a quasi-equity account.[4] Conceptually, deferred taxes are really neither assets nor liabilities – they are linked to the firm's operations and one may even think about them as a negative item on the asset side. For valuation purposes they are "important only to the extent that we need them to calculate income tax payments."[5] That means that the net flow to/from the deferred taxes account each period is included in the FCF calculation as an adjustment to the income statement item *Taxes*.

2. A General Discounting Procedure

2.1 The basic discounting procedure

In many ways, this study can be viewed as an extension of Study 1. The particular result of interest here is Proposition 3.1:

Valuation by discounting the free cash flows at a continuously updated weighted average cost of capital will yield the same value as valuation by discounting the future dividends at the cost of equity capital.

The key point is that Proposition 3.1 makes no statement about the cost of equity capital (hereinafter denoted $k_{E,t}$) being constant – indeed, the proposition will hold regardless of

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[4] See Copeland et al. (1994), p. 162.
[5] Holthausen and Zmijewski (1996), Chapter 5B, p. 6.

how $k_{E,t}$ is defined.[6] We use the t-subscript to highlight the fact that the cost of equity capital may vary over time.

The basic idea behind the valuation procedure is the following: Starting at a future point in time where the equity value is either zero (a company with finite life) or can be expressed as a terminal value, one then goes backwards, one period at a time, and calculates the equity value at the beginning of each period. One should use an 'updated' discount rate that reflects the anticipated capital structure at each point. This is repeated until the valuation date is reached. Traditionally, the period length in this type of company valuation models is one year. We will adhere to this convention and refer to periods as years. Note, however, that it is perfectly possible to use any period length in the modelling framework developed in this study.

The discounting procedure is based on the following difference equation (1):

$$(1)\quad EV_t = \frac{FCF_{t+1} + EV_{t+1} + D_{t+1}}{1 + k_{WACC,t+1}} - D_t$$

where EV_t is the market equity value at the end of year t,
FCF_t is the free cash flow in year t,
D_t is the debt at the end of year t,
$k_{WACC,t}$ is the weighted average cost of capital during year t, defined as

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \left(1 - \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}\right)k_{E,t}$$

where i is the market rate on debt,
$k_{E,t}$ is the cost of equity capital during year t (calculated at beginning of year t)
τ is the (corporate) tax rate.

[6] See Appendix 2 for a proof.

Formula (1) thus expresses the equity value at the end of year t-1 as the present value of the next year's free cash flow and the (ex-dividend) total company value at the end of year t minus the value of the debt at t-1.

In practice, one can start the valuation procedure either at the horizon H, and use some terminal value technique to calculate the equity value at that point in time, EV_H, or, if the company has finite life, at the end of its lifetime. One then goes back one year in time, and uses (1) to calculate the equity value at H-1, and then H-2, and so on, until the valuation date is reached.

2.2 Updating the cost of equity

Arguably, the best known way of updating the cost of equity with respect to changes in the capital structure starts from the approach formulated in Modigliani and Miller (1963)[7] and is discussed in popular corporate finance textbooks. The approach is quite straight-forward to implement.[8] It has an important drawback, however, in that it is limited to the following special case:

1) the debt is a fixed dollar amount, i.e., constant for all future years
2) the company is expected to generate the same amount of cash flow in perpetuity

These underlying assumptions make the MM approach less useful in many real world applications,[9] and a more general approach is often needed. Holthausen and Zmijewski (1996) develop a more general analysis for the relations between different cost of capital items. The MM approach arises as a special case in this framework.

[7] Modigliani and Miller (1963) will henceforth be referred to as MM.
[8] See, e.g., Copeland and Weston (1988), p. 451.
[9] Basically, since the original MM setting implies a non-changing financial structure, there is no *need* to update any costs of capital, once the financial structure has been set.

In a case where the company at the beginning of year t is financed with debt and equity only, the general relation between the cost of equity, $k_{E,t}$, the cost of equity for the unlevered firm, k_U, and the cost of debt, i, is (Holthausen and Zmijewski, Chapter 2, pp. 12-14):[10]

$$(2) \qquad k_{E,t} = k_U + (k_U - i) \cdot \frac{(D_{t-1} - PVTS_{t-1,i})}{EV_{t-1}}$$

where D_{t-1} = market value of debt at the beginning of year t
$PVTS_{t-1,i}$ = present value at the beginning of year t of the part of the interest tax shields that is discounted at i during year t
EV_{t-1} = market value of equity at the beginning of year t

The unlevered cost of equity, k_U, depends on the company's operations only and is assumed to be constant over time; $k_{E,t}$ is the cost of equity adjusted for current financing. Note that interest tax shields are either discounted by i or k_U. The present value of all tax shields, $PVTS_{t,all}$, can thus be expressed as $PVTS_{t,i} + PVTS_{t,k_U}$, where the second index refers to the discount rate (i or k_U). Formula (2) is quite general with respect to assumptions about the valuation of interest tax shields: it holds for any case where the tax shields are discounted at k_U or i, and for any case where some part of the tax shields is discounted at one of the rates and the rest is discounted at the other. This also means that one must make an explicit assumption about how interest tax shields should be valued. This assumption may depend on the specifics of the company under consideration and is closely tied to how the company determines its capital structure, since the riskiness of the tax shields (and thus their value) depends on how the company carries out its financing.

If the company has decided its financing plan once and for all, independent of the value of the company, then it is reasonable to regard the interest tax shields as being just as risky as the debt, since the size of the tax shields is completely determined by the pre-specified debt schedule. The proper discount rate for the interest tax shields in this case is the cost of debt, i. Given such *passive debt management* it is straight-forward to use the general formula (2)

[10] See Appendix 4 for a derivation.

directly if one has a forecast of the debt development from year to year. Note, however, that we explicitly have to forecast the value of tax shields at the end of each future year.

This passive debt management view also corresponds to the MM article. However, MM further assume that debt is a fixed dollar amount (determined once and for all). Their assumptions result in the following well-known cost of equity formula for the MM case:

$$(3) \qquad k_{E,t} = k_U + \left(k_U - i\right) \cdot \frac{D_{t-1}}{EV_{t-1}} (1 - \tau)$$

It is sometimes argued that an assumption about passive debt management is unrealistic in many cases, since borrowing decisions may very well be made to approach a target capital structure.[11] Pursuing this line of reasoning, a different picture of the valuation of interest tax shields emerges. With perfect *active debt management*, i.e., if the size of the company's debt is *continuously* adjusted in order to maintain a constant market debt-to-value ratio, the size of the interest tax shields will be directly related to the development of the company's operations. Consequently, the proper discount rate for the interest tax shields in this case would be the unlevered cost of equity, k_U. This seems to be the underlying assumption in the so-called *compressed APV technique* (Kaplan and Ruback (1995)) and results in the following formula for the calculation of the cost of equity:[12]

$$(4) \qquad k_{E,t} = k_U + \left(k_U - i\right) \cdot \frac{D_{t-1}}{EV_{t-1}}$$

In practice, it will be quite problematic to *continuously* adjust the borrowing to maintain a constant capital structure. A common way of handling this problem, introduced by Miles and Ezzell (1980), is that the debt amount is determined at the end of each year and then kept constant until the end of next year. Thus, at the end of each year the company's capital structure equals its target capital structure. This means that the interest tax shield for the coming year is as risky as the outstanding debt, whereas the interest tax shields from all subsequent years will be as risky as the unlevered equity. The tax shield valuation at the

[11] See Fama and French (1997) for empirical evidence on this.
[12] See Appendix 4 for the derivation.

beginning of each year will be performed using the cost of debt as discount rate for the interest tax shield from the immediately following year, whereas the unlevered cost of equity is used as discount rate for the interest tax shield from the years thereafter.

Inserting this valuation principle in the general equation (2) gives the Miles and Ezzell (1980) formula for the cost of equity (Holthausen and Zmijewski (1996), Chapter 2, pp. 15-16):[13]

$$(5) \qquad k_{E,t} = k_U + \left(k_U - i\right) \cdot \frac{D_{t-1}}{EV_{t-1}} \left(1 - \frac{\tau \cdot i}{1+i}\right)$$

This formula is even more general than it may appear at first sight. Assume that the yearly debt adjustment is contingent on the company's market value, but only approximately (the target debt ratio is not reached exactly). The debt is held constant for one year until the next debt adjustment at the end of the following year. The debt being contingent on the company's market value makes it appropriate to assume that subsequent (i.e., *beyond* the next year) interest tax shields be discounted at k_U. The tax shield from the *first* year should be discounted at the cost of debt, *i,* since debt is assumed to be kept constant for one year at a time. Consequently, the crucial assumption for (5) to be the appropriate formula is not that the debt adjustment is made to reach the exact target debt ratio at the end of each year, but that interest tax shields beyond the end of the following year are assumed to have the same risk as the firm's operations and hence should be discounted at k_U.

Finally, the underlying assumptions about debt policy have an impact not only on the perceived riskiness of tax shields but also on the actual forecasts:

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[13] A derivation is provided in Appendix 4.

No debt adjustment (i.e., *passive debt management*) implies that:

- interest tax shields are discounted at the cost of debt:

$$PVTS_{t,all} = PVTS_{t,i} = \sum_{s=t+1}^{\infty} \frac{\tau i D_{s-1}}{(1+i)^{s-t}}$$

  in the MM case this reduces to:

$$PVTS_{t,all} = PVTS_{t,i} = \tau D_t$$

- the debt schedule is predetermined, i.e., fixed once and for all.

Yearly adjusted debt (i.e., *active debt management*) implies that:

- interest tax shields are valued using the Miles and Ezzell (1980) procedure:

$$PVTS_{t,all} = PVTS_{t,i} + PVTS_{t,k_U} \text{ and } \begin{cases} PVTS_{t,i} = \dfrac{\tau i D_t}{1+i} \\ PVTS_{t,k_U} = \dfrac{PVTS_{t+1,all}}{1+k_U} \end{cases}$$

- the debt schedule is contingent on the expected development of the company value.

Continuously adjusted debt (i.e., *perfect active debt management)* implies that:

- interest tax shields are discounted at the unlevered cost of equity, k_U:

$$PVTS_{t,all} = PVTS_{t,k_U} = \sum_{s=t+1}^{\infty} \frac{\tau \cdot i \cdot D_{s-1}}{(1+k_U)^{s-t}}$$

- the debt schedule is contingent on the expected development of the company value.

In the passive debt management case the debt schedule is fixed. In the two active cases, the debt amount in each period is variable, it is tied to the expected company value through the target capital structure. The passive and active methods will generally not yield the same forecasted debt schedule.

2.3 Simultaneous updating of the WACC and the cost of equity

Before turning to the issue of how to combine the cost of equity updating with the basic discounting procedure, some problems of a technical nature must be addressed.

The situation is the following: the equity value at the end of (the horizon) year H, EV_H, has been computed and forecasts of FCF_H, D_H, and D_{H-1} have been obtained. Equation (1) can in principle be used to calculate EV_{H-1}, the equity value at the end of year *H*-1.

In order to compute EV_{H-1}, however, one must know WACC during year *H*, but to calculate WACC both EV_{H-1} and the cost of equity, $k_{E,H}$, must be known (and the latter is itself a function of EV_{H-1}). An iterative numerical procedure must be used, one that eventually will make EV_{H-1}, $k_{E,H}$, and $k_{WACC,H}$ converge to their mutually consistent values.

The simultaneity problem is solved in part by inserting the general formula (2) for the cost of equity updating into the standard WACC definition, and the general WACC formula (6) is obtained:[14]

$$(6)\quad k_{WACC,t} = k_U \left(1 - \frac{PVTS_{t-1,i}}{D_{t-1} + EV_{t-1}} \right) + i \left(\frac{PVTS_{t-1,i} - \tau D_{t-1}}{D_{t-1} + EV_{t-1}} \right)$$

Formula (6) can be simplified in some of the cases discussed above:

i) for the MM case:

$$(7) \qquad k_{WACC,t} = k_U \left(1 - \frac{\tau D_{t-1}}{D_{t-1} + EV_{t-1}} \right)$$

ii) for the active debt management case (with yearly debt adjustments, including the Miles and Ezzell (1980) approach):

$$(8) \qquad k_{WACC,t} = k_U - \tau i \frac{D_{t-1}}{D_{t-1} + EV_{t-1}} \left(\frac{1 + k_U}{1 + i} \right)$$

iii) for the perfect active debt management case (with continuously adjusted debt):

$$(9) \qquad k_{WACC,t} = k_U - \tau i \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}$$

Thus, the yearly iterative procedure will only have to consider the interdependence between the equity value and the weighted average cost of capital, since the appropriate WACC formula already includes the correct specification of the updated cost of equity. To complete the valuation the iterative procedure must be carried out for years *H*, *H*-1, *H*-2, etc., until the valuation date is reached.

The general WACC formula (6) as well as the three special cases ((7), (8) and (9)) also contain the unlevered cost of equity k_U, which is a measure of the riskiness of the company's operations. We assume that this parameter is constant through time. A problem is that k_U may be unobservable. Intuitively, the value of k_U may be regarded as an industry specific number – firms in a certain industry can be assumed to have equally risky operations. If this assumption is made, then k_U can be inferred from industry data using some quantitative

[14] The derivation of equations (6), (8) and (9) is in Appendix 4. Equation (6) is taken from Holthausen and Zmijewski (1996, Ch. 2, p. 20). Equation (8) is from Holthausen and Zmijewski (1996, Ch. 2, p. 21).

method.[15] Once an estimate of k_U has been obtained the yearly iterative procedure, *Procedure Y*, is uncomplicated:

<u>*Procedure Y*</u>

Y1 Assign a trial value to the WACC, $k_{WACC,t+1}$.

Y2 Calculate the equity value at t through equation (1) (or by using a terminal value formula if at the horizon, *H*).

Y3 Compute the implied resulting WACC by inserting the equity value from Y2 into the appropriate WACC formula ((6), (7), (8) or (9)).

Y4 Compare the implied resulting $k_{WACC,t+1}$ with the trial value. If equal, the equity value is correct, and one can go on to the preceding year. If the resulting WACC differs from the trial value, go to Y1 again, where the resulting WACC from Y3 can be used as a new trial value.

One starts at the forecast horizon (year *H*) with *Procedure Y*. The procedure is then repeated every forecasted year until the valuation date is reached. More detailed implementation issues will be discussed in the next section.

3. Implementing the General Discounting Procedure

To visualise the implementation of the general discounting procedure we will use the stylised (fictitious) company XMPL as illustration. XMPL is the same company that was referred to as *the example company* in Chapter 2 of Study 1. Only the years in the *parametric steady state* (PSS) – the period after the explicit forecast period when all parameters are assumed constant – were considered there, however. Here, we make a full-scale valuation including nine explicitly forecasted years. This will further highlight the problems introduced by a non-constant capital structure.

[15] See, e.g., Jennergren and Näslund (1996) pp. 68-69 for an implementation of a CAPM related approach.

The XMPL company follows a pre-set financing plan in the explicit forecast period, i.e., in the first nine years. This will exemplify the passive debt management case discussed in section 2.2. The company will enter into a parametric steady state in year 10. The expected market debt ratio will remain non-constant, however, until it asymptotically approaches its steady state value. The PPE items are forecasted as in the first edition of Copeland et al. (1990) and *Specification A* in Study 1.

It may well be argued that following MM and viewing the debt level as pre-determined is unrealistic, especially over longer time periods. Indeed, Modigliani (1988) acknowledges this point: "It seems much more reasonable to suppose that the leverage policy of the representative firm can be described as aiming at maintaining the debt in a stable relation to the scale of the firm as seen at any given date" (Modigliani (1988), p. 152). Even so, the influence of MM on mainstream corporate finance makes it interesting to see what is involved when making the passive debt management assumption operational in a full-scale company valuation.

In the period after the explicit forecast period we assume that the debt management will be related to the development of the company. In particular, the (assumed) debt policy is to keep debt as a constant fraction of the balance sheet. So whereas the first nine years exemplify *passive debt management*, the period after that will visualise *active debt management.*

Interest tax shields in the explicit forecast period will be discounted at the cost of debt, *i.* As argued in section 2.2, this is appropriate when the debt levels are pre-set and unrelated to the company's development. In this case the debt is not a fixed dollar amount, however, so we cannot use the MM formulas. Instead the cost of equity can be updated using formula (2) and the WACC can be updated using formula (6). In the steady state period, the debt level is explicitly tied to the company's operations in that it is forecasted as a percentage of the balance sheet, and the cost of equity will be updated using formula (5) and the weighted average cost of capital using formula (8). The unlevered cost of equity, k_U , is assumed to be 12%. Year 0 will denote the last historical year, years 1 to 9 are the explicitly forecasted years, and the parametric steady state period starts in year 10. Parametric steady state (PSS) means that the parameters in the forecasting model are expected to be constant. In our framework, PSS also means that the forecasted accounting items for any year can be derived directly through the formulas in Study 1, Ch. 2. For a description of how the corresponding accounting data can be computed for any year in the PSS period, see Appendix 1.

XMPL, Forecasted Balance Sheets, Years 1-10											
Year	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
ASSET SIDE:											
+Net Working Capital	15.51	15.36	15.72	16.39	17.77	19.41	20.57	21.60	22.68	23.81	25.00
+Gross PPE	111.30	124.60	134.92	142.14	149.24	156.71	164.54	172.77	181.41	190.48	200.00
-Acc. Depr.	94.44	96.66	99.16	101.85	104.70	107.68	110.82	114.11	117.56	121.19	125.00
+Net PPE	16.86	27.94	35.76	40.28	44.55	49.02	53.72	58.66	63.84	69.29	75.00
Total Assets	32.37	43.30	51.48	56.67	62.31	68.43	74.29	80.26	86.52	93.10	100.00
DEBT & EQUITY SIDE:											
+Debt	12.95	22.50	24.50	27.00	27.00	27.37	29.72	32.10	34.61	37.24	40.00
+Deferred Taxes	0.55	0.92	1.33	1.75	2.20	2.67	3.17	3.68	4.23	4.80	5.40
+Book Equity	18.87	19.88	25.65	27.92	33.11	38.39	41.41	44.47	47.68	51.06	54.60
Total Debt & Equity	32.37	43.30	51.48	56.67	62.31	68.43	74.29	80.26	86.52	93.10	100.00
Note:	Year 0 = Last historical year										
	Year 10 = First year forecasted to be in Parametric Steady State										

Table 1 - XMPL forecasted balance sheets, years 0 - 10.

XMPL, Forecasted Income Statements, Years 1-10											
Year	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
+Revenues	310.11	307.26	314.30	327.80	355.34	388.22	411.35	431.92	453.51	476.19	500.00
-Operating exp.	-279.10	-286.44	-282.87	-301.23	-322.66	-352.23	-368.32	-390.22	-410.00	-428.57	-450.00
-Depr. exp.	-7.00	-6.68	-7.48	-8.10	-8.53	-8.95	-9.40	-9.87	-10.37	-10.88	-11.43
+Operating income	24.01	14.14	23.95	18.47	24.15	27.04	33.63	31.83	33.15	36.73	38.57
-Interest exp.	-1.00	-1.29	-2.25	-2.45	-2.70	-2.70	-2.74	-2.97	-3.21	-3.46	-3.72
+Earnings bef. taxes	23.01	12.85	21.70	16.02	21.45	24.34	30.89	28.85	29.94	33.27	34.85
-Taxes	-6.90	-3.85	-6.51	-4.81	-6.44	-7.30	-9.27	-8.66	-8.98	-9.98	-10.45
Net profit	16.11	8.99	15.19	11.22	15.02	17.03	21.62	20.20	20.96	23.29	24.39
Note:	Year 0 = Last historical year										
	Year 10 = First year forecasted to be in Parametric Steady State										

Table 2 - XMPL forecasted income statements, years 0 - 10.[16]

From the forecasted balance sheets and income statements (Tables 1 and 2) one can then obtain XMPL's forecasted free cash flow (Table 3). The forecasted statements of equity are presented in Table 4.

[16] Note that the printed tables do not necessarily sum correctly at the last decimal because of rounding.

XMPL, Forecasted Free Cash Flow Calculation, Years 1-10										
Year	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
+Net profit	8.99	15.19	11.22	15.02	17.03	21.62	20.20	20.96	23.29	24.39
+Interest exp. aft. taxes	0.91	1.58	1.72	1.89	1.89	1.92	2.08	2.25	2.42	2.61
+Inc. deferred taxes	0.37	0.40	0.43	0.45	0.47	0.49	0.52	0.54	0.57	0.60
-Increase in invested capital	-10.93	-8.18	-5.19	-5.64	-6.12	-5.86	-5.96	-6.26	-6.58	-6.90
Free cash flow	-0.66	8.99	8.17	11.71	13.27	18.18	16.83	17.49	19.71	20.70
Note:	Year 10 = First year forecasted to be in Parametric Steady State									

Table 3 - XMPL forecasted free cash flow calculations, year 1-10

XMPL, Forecasted Statements of Equity, Years 1-10										
Year	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Beginning Book Equity	18,87	19,88	25,65	27,92	33,11	38,39	41,41	44,47	47,68	51,06
+Net profit	8,99	15,19	11,22	15,02	17,03	21,62	20,20	20,96	23,29	24,39
-Dividends	-7,99	-9,42	-8,95	-9,82	-11,76	-18,60	-17,14	-17,74	-19,92	-20,85
=Ending Book Equity	19,88	25,65	27,92	33,11	38,39	41,41	44,47	47,68	51,06	54,60
Note:	Year 10 = First year forecasted to be in Parametric Steady State									

Table 4 - XMPL forecasted statements of equity, year 1-10

In the parametric steady state period FCF will grow at a constant annual rate of 5%. The initial condition for the capital structure in market terms to remain constant in the PSS period is not fulfilled in the XMPL case.[17] Hence, one cannot use a simple continuing value formula at the end of the explicit forecast period to get a correct horizon value. The market debt ratio will, however, approach a steady state value as t gets large,[18] and this means that one can use a continuing value at some point in the distant future where the market debt ratio is close enough to the steady state value, as a very good approximation of the company value at that moment. If this continuing value calculation is done sufficiently long into the future, the approximation error will have no impact on the company value at valuation date. To find a reasonable cut-off-year where we can compute the company value with a continuing value, we simply calculate the implied market debt ratio at a number of points of time in the distant future (Table 5). [19]

[17] Basically, this condition states that the depreciation and retirements parameters in the parametric steady state period must be decided such that accumulated depreciation grows at the revenue growth rate, g, or – in other words – the depreciation related flows must be such that the stock grows by g.
[18] See Study 2, Section 2.1.5.
[19] Implied market debt ratio = $D_t \Big/ \left(\dfrac{FCF_{t+1}}{(k_{WACC,t+1} - g)} \right)$

H	25	100	150	200	500	1000	10000
Implied market debt ratio	16.337%	18.870%	18.931%	18.936%	18.936%	18.936%	18.936%

Table 5 - XMPL market debt ratios implied by continuing value formula at different horizon years (H)

Thus, after about 200 years we arrive at the steady state market debt ratio level. We choose year 210 as this 'final horizon', and start the discounting procedure here by computing the company value via the FCF continuing value formula.

In the explicit forecast period with passive debt management we use the general case formula (6) to calculate WACC. Accordingly, we have to forecast the present value of interest tax shields *discounted at i* at the beginning of each year in the explicit forecast period, i.e., $PVTS_{t,i}$ for t=9 to 0. The interest tax shield in any year t is simply $\tau i D_{t-1}$, but since *all* tax shields here are discounted at i, we must also take the present value of all tax shields at the end of year 10 into consideration. The calculation of $PVTS_{9,i}$ will thus represent a link between the two periods with different debt management policies, and thus, different tax shield valuation principles:

$$(10) \qquad PVTS_{9,i} = \frac{\tau i D_9 + PVTS_{10,all}}{1+i}$$

However, we must first calculate $PVTS_{10,all}$, the value (at the end of year 10) of *all* tax shields from the years after the explicit forecast period, which is obtained using the Miles and Ezzell (1980) backward going procedure:[20]

[20] We start at the horizon by using the Miles and Ezzell continuing value formula provided by Holthausen and Zmijevski (1996), Chapter 2, p. 17: $PVTS_{210,all} = \frac{\tau \cdot i \cdot D_{210}}{k_U - g_D}\left[\frac{(1+k_U)}{(1+i)}\right]$ where g_D is the debt growth rate (which has asymptotically approached 5% in year 210).

$$(11)\qquad \begin{cases} PVTS_{t,i} = \dfrac{\tau i D_t}{1+i} \\ PVTS_{t,k_U} = \dfrac{PVTS_{t+1,all}}{1+k_U} \\ PVTS_{t,all} = PVTS_{t,i} + PVTS_{t,k_U} \end{cases} \qquad (t = 209 \text{ to } 10).$$

Then, we can use equation (10) to calculate $PVTS_{9,i}$. Finally, a similar calculation is repeated for t = 8, for t = 7 and so on, until the valuation date (t = 0) is reached:

$$(12)\qquad PVTS_{t,i} = \frac{\tau i D_t + PVTS_{t+1,i}}{1+i} \left(= PVTS_{t,all}\right) \quad (t = 8 \text{ to } 0).$$

This altogether produces a forecasted sequence of $PVTS_{t,i}$ for the relevant years (0 to 9). We can then go to the actual valuation by using *procedure Y* and start at the horizon:

Y1 We guess that the WACC at the horizon is 11.5%.

Y2 The 'first guess' equity value at the horizon is computed using the continuing value formula:

$$EV_{210} = TCV_{210} - D_{210} = \frac{FCF_{211}}{k_{WACC,211\to\infty} - g} - D_{210}, \text{ where } g \text{ is } 5\%.$$

The result is 4778091.10.

Y3 The appropriate WACC formula in this setting for the years in the PSS period is formula (8):

$$k_{WACC,t} = k_U - \tau i \frac{D_{t-1}}{D_{t-1} + EV_{t-1}} \left(\frac{1+k_U}{1+i} \right).$$

Inserting the 'first guess' equity value from step Y2 and the D_{210} value implies a WACC of 11.470%.

End of year 210	
FCF_{211}	375767.78
EV_{210}	4778091.10
Trial $k_{WACC,211\to\infty}$	11.50000%
Resulting $k_{WACC,211\to\infty}$	11.47007%
$k_{WACC,211\to\infty}$ difference	0.02993%

Y4 This is clearly not identical to the trial value of 11.5%, and the procedure must be restarted from Y1 using 11.470% as a new guess of WACC. After a number of loops we arrive at the following:[21]

End of year 210	
FCF_{211}	375767.78
EV_{210}	4802811.12
Trial $k_{WACC,211\to\infty}$	11.47232%
Resulting $k_{WACC,211\to\infty}$	11.47232%
$k_{WACC,211\to\infty}$ difference	0.00000%

Procedure Y is then repeated for all preceding years, using equation (1) instead of the continuing value formula in step Y2, and using equation (6) instead of (8) in step Y3 for the years in the explicit forecast period. When we reach the valuation date we get:

End of year 0	
FCF_1	-0.66
EV_0	164.78
Trial $k_{WACC,1}$	11.63796%
Resulting $k_{WACC,1}$	11.63796%
$k_{WACC,1}$ difference	0.00000%

[21] In practice, this is done through the goal-seek function of Excel, where the 'WACC difference' (Trial WACC *minus* Resulting WACC) should be set to 0 (zero) by changing the Trial WACC cell in the spreadsheet model. See appendix 3 for exact programming.

This whole procedure (i.e., *procedure Y* for all years) took 24 seconds on a PC with 90 MHz Pentium processor. The macro program (for use in Excel) for the whole procedure is in Appendix 3. The market debt ratio of the company is shown in Figure 1.

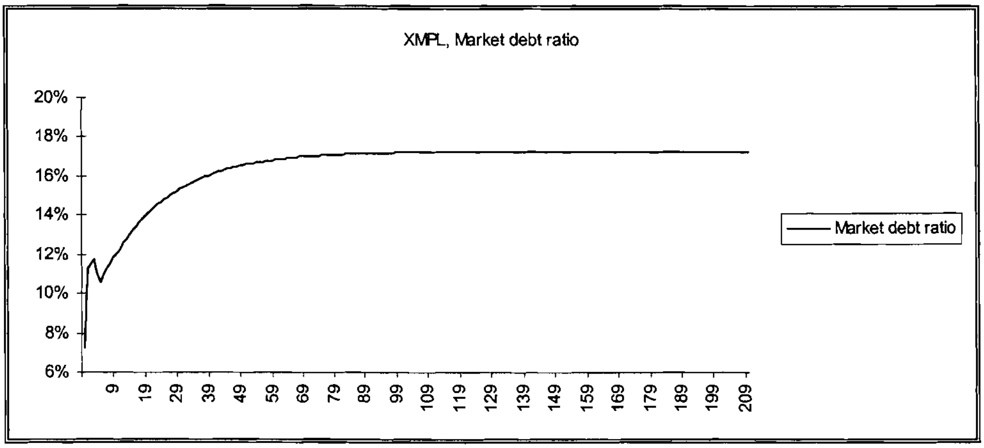

Figure 1 - XMPL market debt ratio, with updated WACC and simultaneously updated cost of equity capital.[22]

Our valuation of XMPL with both non-constant WACC and non-constant cost of equity capital results in an equity value of 164.8. As a comparison, a more naïve approach using a constant WACC of 11.63% (calculated at the valuation date through equation (6)) gives an equity value of 162.4. Copeland, Koller and Murrin recommends that one should use some target capital structure as basis for the (constant) WACC calculation. As can be seen from Figure 1, the market debt ratio will eventually approach a steady state level slightly above 17%. If this is used as a 'target' when calculating a constant WACC (=11.47%), the resulting equity value would be 167.3. If instead the projected capital structure at the end of the explicit forecast period is used to calculate a constant WACC (=11.52%), the equity value will be 165.8. The constant WACC approaches tend to be somewhat 'ad hoc', and as a consequence drive the valuation procedure towards guessing and approximations. The methodology developed in this study, as a contrast, results in a equity value that is exactly correct given the assumptions made. However, as our example indicates, the differences in estimated equity value do not seem to be overly severe.

4. Concluding Remarks

The aim of this study is to point out procedures and relations that are useful in applications of company valuation methods using discounted free cash flow valuation models, first and foremost the so-called McKinsey model.

One of the main problems with such models is the discount rate. This is often assumed to be constant, although when one looks at the company's forecasted future capital structure, this almost always varies over time. Hence, the use of a constant average weighted cost of capital is inappropriate. In Study 1, we developed procedures for continuously adjusting the weights to reflect a changing capital structure. In this study, the focus has been on the cost of equity capital and how one simultaneously can adjust both the WACC weights and the cost of equity capital that also are parts of the WACC formula. One might conceivably go on and consider adjustments of the cost of debt capital, using some quantitative model. An interesting extension would be to include the term structure of interest rates. Also, other opinions about the riskiness of tax shields will lead to models other than the ones described and used here (but similar) for determining the equity cost of capital. Such additions or alterations do not change the operational principles, however, and those are our prime interest in this study.

One sometimes hears comments to the effect that it is not worth the extra effort to use correct and precise calculation techniques when valuing companies, since there is so much uncertainty anyhow in the data that must ultimately be fed into the model. We object to such statements on two accounts: First, it is not really much of an extra effort. The Excel model can be constructed once, and then used for different companies. Second, uncertainty is additive. The fact that there is a lot of uncertainty in the data should really spur the analyst even more to do what he or she can to reduce the over-all uncertainty. One way of doing this is obviously to use calculation techniques that do not by themselves introduce approximation errors. At the same time it must be noted that the consequences of basing the discount rate calculation on an assumed target capital structure are often not that severe. In our example the

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[22] The kink at the end of the explicit forecast period reflects the adjustment that takes place when the parameters are set to their steady state values.

approximation error in estimated equity value was within a few percent. The balance between computational complexity and exactness is in the end a matter of judgement.

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Appendix 1 - Parametric Steady State Formulas

The parameters in the Copeland et al. type of forecasting model are (Study 1, p. 73):[23]

a net working capital in % of revenues (sales)
b gross PPE in % of revenues (sales)
c change in deferred taxes in % of gross PPE
d depreciation in % of preceding year's gross PPE
g nominal growth rate, revenues (sales)[24]
i interest rate on debt
p operating expenses in % of revenues (sales)
r retirements in % of preceding year's gross PPE
τ tax rate
w debt in % of balance sheet (book value)

The first year in the parametric steady state period for XMPL is year 10. This means that the parameters defined above are constant from this point in time. The following state variables are also identified:

R_t revenues (sales) of year *t*,
A_t accumulated depreciation at the end of year *t*,
T_t deferred taxes at the end of year *t*.

The state-variables in the parametric steady state period (for $t \geq 11$) are given through the following set of equations (Study 1, p. 77):

$$R_t = (1+g)R_{t-1} = (1+g)^{t-10} R_{10}$$

$$A_t = \frac{(1+g)^{t-10} - 1}{g}(d-r)bR_{10} + A_{10}$$

$$T_t = \frac{(1+g)^{t-10} - 1}{g}c(1+g)bR_{10} + T_{10} = \left[\frac{(1+g)^{t+1-10} - 1}{g} - 1\right]cbR_{10} + T_{10}$$

where R_{10}, A_{10} and T_{10} are the initial values of the state-variables in the PSS period (see tables 1 and 2).

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[23] In the XMPL case in the parametric steady state period: a=5%; b=40%; c=0.3%; d=6%; g=5%; i=10%; p=90%; r=4%; τ=30%; w=40%.

The forecasting model defines the balance sheet items as follows (for ($t \geq 11$)):

Net working capital:	aR_t
Net PPE	$bR_t - A_t$
	where $A_t = [(d - r)bR_{t-1} + A_{t-1}]$
Debt:	$w(aR_t + bR_t - A_t)$
Deferred taxes:	$T_t = cbR_t + T_{t-1}$
Book equity:	$(1 - w)(aR_t + bR_t - A_t) - T_t$

The income statement is also defined (for ($t \geq 11$)):

Revenues:	$R_t = (1+g)^{t-10} R_{10}$
Operating expenses:	pR_t
Depreciation exp.:	$dG_{t-1} = dbR_{t-1}$
Interest expense:	$iw(aR_{t-1} + bR_{t-1} - A_{t-1})$
Taxes:	$\tau((1-p)R_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))$
Net profit:	$(1-\tau)((1-p)R_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))$

Appendix 2 - Modified Proof of Proposition 3.1 in Study 1

The free cash flow can be written as follows:

(A2:1) FCF_t = *Net profit,* NP_t
+ *Net interest payments after taxes,* $(1-\tau)iD_{t-1}$
+ *Increase in deferred taxes,* $(T_t - T_{t-1})$
- *Increase in invested capital* $(AS_t - AS_{t-1})$

---

[24] This means that the revenues of all years in the PSS period can be calculated as (1+g) times the preceding year's revenues.

where: τ is the tax rate
i is the interest rate
D_{t-1} is the (net) debt at the end of year t-1 on which interest is paid

The dividends are by the clean surplus relation calculated as:

$$\text{(A2:2)}\quad DIV_t = \begin{aligned}&\textit{Net profit, } NP_t\\ &+\ \textit{Increase in debt, } \left(D_t - D_{t-1}\right)\\ &+\ \textit{Increase in deferred taxes, } \left(T_t - T_{t-1}\right)\\ &-\ \textit{Increase in invested capital } \left(IC_t - IC_{t-1}\right)\end{aligned}$$

Assume there exists an equity value at some future time T, called EV_T, which is calculated after a possible dividend at time T. This means that the total company value at time T, called TCV_T, will be $EV_T + D_T$ (i.e. equity value plus debt value). Valuation by the FCF approach will then yield the following total company value at the end of year T-1:[25]

$$\text{(A2:3)}\qquad TCV_{T-1} = \frac{FCF_T + EV_T + D_T}{1 + k_{WACC,T-1}}$$

which can be rearranged:

$$\text{(A2:4)}\qquad TCV_{T-1} = \frac{NP_T + (1-\tau)iD_{T-1} + \left(T_T - T_{T-1}\right) - \left(IC_T - IC_{T-1}\right) + EV_T + D_T}{1 + k_{WACC,T-1}}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + \left(T_T - T_{T-1}\right) - \left(AS_T - AS_{T-1}\right) + EV_T + D_T}{1 + \dfrac{D_{T-1}}{CV_{T-1}}(1-\tau)i + \left(1 - \dfrac{D_{T-1}}{CV_{T-1}}\right)k_E}$$

[25] Note that the WACC formula implies that the WACC used for discounting during year T is based on the entering market values of debt and equity, i.e. D_{T-1} and EV_{T-1}, and hence also the total company value, TCV_{T-1}.

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T}{\frac{CV_{T-1} + (1-\tau)iD_{T-1} + CV_{T-1}k_E - D_{T-1}k_E}{CV_{T-1}}}$$

Dividing through by TCV_{T-1} and rearranging further yields:

$$\text{(A2:5)} \qquad TCV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T + D_{T-1}k_E}{1+k_E}$$

The equity value is then obtained by deducting the debt, i.e. by deducting D_{T-1}:

$$\text{(A2:6)} \qquad EV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T + D_{T-1}k_E}{1+k_E} - D_{T-1}$$

$$= \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + (D_T - D_{T-1})}{1+k_E}$$

By inserting (A2:2) into (A2:6) one obtains:

$$\text{(A2:7)} \qquad EV_{T-1} = \frac{DIV_T + EV_T}{1+k_E}$$

which is exactly the valuation formula used when discounting the dividends by the equity cost of capital. Having thus established that the value at *T-1* will be the same when employing the different methods, one can go on to time *T-2* and so on.

Appendix 3 - Macro Programming

Here is an example of how *Procedure Y* can be implemented in Excel 5.0 using Visual Basic macro programming. Notation terms in brackets, e.g., [example], refer to physical cells in the spreadsheet model. For example, [kU difference] means the spreadsheet cell in which the difference between the *Trial* k_U and *Implied* k_U is computed. Year *H* denotes the horizon, which in XMPL was at the end of year 210, and year 0 denotes the valuation date (i.e., the valuation date is at the end of year 0).

Procedure Y (for one particular year *t*):

1) Manually set [Trial kWACC,*t*] to the initial trial value of $k_{WACC,t.}$

2) Run the following macro program:

```
Sub procedure_Y()
  Range("[WACC difference year t]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC year t]")
End Sub
```

Procedure Y (for all years from year *H* down to the valuation date, *0*):

1) Manually set all [Trial kWACC,*t*] cells (*t*=1, ..., H+1) to their initial trial values.

2) Run the following macro program:

```
Sub procedure_Y_all_years()
  Range("[WACC difference year H+1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial   WACC
year H+1]")
  Range("[WACC difference year H]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial     WACC
year H]")
  Range("[WACC difference year H-1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial   WACC
year H-1]")
...
  Range("[WACC difference year 2]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC   year
2]")
  Range("[WACC difference year 1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC   year
1]")
End Sub
```

Appendix 4 – Derivations

Equation (2)

As derived by Holthausen and Zmijewski, Chapter 2, pp. 12-14:

Since the required rate of return on the economic assets of the firm must equal the required of return on the securities financing the same assets we have:[26]

$$\left(EV_{t-1} + D_{t-1} - PVTS_{t-1,i} - PVTS_{t-1,k_U}\right)k_U + PVTS_{t-1,i}\,i + PVTS_{t-1,k_U}\,k_U =$$
$$= D_{t-1}i + EV_{t-1}k_{E,t}$$

Solving for $k_{E,t}$:

$$k_{E,t} = \frac{EV_{t-1} + D_{t-1} - PVTS_{t-1,i} - PVTS_{t-1,k_U}}{EV_{t-1}}k_U +$$
$$+\frac{PVTS_{t-1,i}}{EV_{t-1}}i + \frac{PVTS_{t-1,k_U}}{EV_{t-1}}k_U - \frac{D_{t-1}}{EV_{t-1}}i$$

Simplifying now yields:

$$k_{E,t} = k_U + \frac{D_{t-1} - PVTS_{t-1,i}}{EV_{t-1}}k_U - \frac{D_{t-1} - PVTS_{t-1,i}}{EV_{t-1}}i$$

Rearranging yields equation (2):

$$k_{E,t} = k_U + \left(k_U - i\right)\cdot\frac{D_{t-1} - PVTS_{t-1,i}}{EV_{t-1}}$$

[26] Note that the first parenthesis equals the unlevered value of the company.

Equation (4)

In the same way as for equation (2), the required rate of return on the economic assets and on the financing securities are equal. All tax-shields are discounted at k_U, however. Thus:

$$(EV_{t-1} + D_{t-1} - PVTS_{t-1,all})k_U + PVTS_{t-1,all}k_U = D_{t-1}i + EV_{t-1}k_{E,t}$$

Solving for $k_{E,t}$ and simplifying yields equation (4):

$$k_{E,t} = \frac{EV_{t-1} + D_{t-1}}{EV_{t-1}}k_U - \frac{D_{t-1}}{EV_{t-1}}i = k_U + (k_U - i)\frac{D_{t-1}}{EV_{t-1}}$$

Equation (5)

In the general case, the cost of equity function is given by equation (2):

$$k_{E,t} = k_U + (k_U - i)\frac{D_{t-1} - PVTS_{t-1,i}}{EV_{t-1}}$$

In the Miles and Ezzell (1980) setting, at any valuation date *t*-1 only the interest tax shield from year *t* is to be discounted at *i*, the cost of debt. The tax shields from years beyond *t* is at time *t*-1 discounted at k_U. Thus:

$$\begin{cases} PVTS_{t-1,i} = \dfrac{taxshield_t}{1+i} = \dfrac{\tau i D_{t-1}}{1+i} \\ PVTS_{t-1,k_U} = \dfrac{PVTS_{t,all}}{1+k_U} \end{cases}$$

Now inserting the expression for $PVTS_{t-1}$ into the general cost of equity formula yields equation (5):

$$k_{E,t} = k_U + (k_U - i)\frac{D_{t-1} - \frac{\tau i D_{t-1}}{1+i}}{EV_{t-1}} = k_U + (k_U - i)\frac{D_{t-1}}{EV_{t-1}}\left(1 - \frac{\tau i}{1+i}\right)$$

Equation (6)

The standard WACC definition is:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1} + EV_{t-1}}k_{E,t}$$

The cost of equity is in the general case given by equation (2):

$$k_{E,t} = k_U + (k_U - i)\frac{D_{t-1} - PVTS_{t-1,i}}{EV_{t-1}}$$

Inserting the cost of equity equation into the WACC definition yields:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1} + EV_{t-1}}\left(k_U + (k_U - i)\frac{D_{t-1} - PVTS_{t-1,i}}{EV_{t-1}}\right)$$

Rearranging gives:

$$\begin{aligned} k_{WACC,t} &= \frac{k_U EV_{t-1} + k_U(D_{t-1} - PVTS_{t-1,i})}{D_{t-1} + EV_{t-1}} + \\ &\quad + \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i - \frac{D_{t-1} - PVTS_{t-1,i}}{D_{t-1} + EV_{t-1}}i \\ &= \frac{k_U(D_{t-1} + EV_{t-1}) - k_U PVTS_{t-1,i}}{D_{t-1} + EV_{t-1}} + \frac{iPVTS_{t-1,i} + [(1-\tau)i - i]D_{t-1}}{D_{t-1} + EV_{t-1}} \end{aligned}$$

And, finally:

$$k_{WACC,t} = k_U\left(1-\frac{PVTS_{t-1,i}}{D_{t-1}+EV_{t-1}}\right)+i\frac{PVTS_{t-1,i}-\tau D_{t-1}}{D_{t-1}+EV_{t-1}}$$

Equation (8)

The standard WACC definition can be written:

$$k_{WACC,t} = \omega_{D,t-1}(1-\tau)i + (1-\omega_{D,t-1})k_{E,t}$$

where $\omega_{D,t-1} = \dfrac{D_{t-1}}{D_{t-1}+EV_{t-1}}$ (the debt to value ratio).

The cost of equity is given by:

$$k_{E,t} = k_U + (k_U - i)\frac{D_{t-1}}{EV_{t-1}}\left(1-\frac{\tau i}{1+i}\right).$$

Substituting into the WACC definition:

$$k_{WACC,t} = \omega_{D,t-1}(1-\tau)i + (1-\omega_{D,t-1})\left(k_U + (k_U - i)\cdot\frac{D_{t-1}}{EV_{t-1}}\left(1-\frac{\tau i}{1+i}\right)\right).$$

Recognising that $D_{t-1}/EV_{t-1} = \omega_{D,t-1}/(1-\omega_{D,t-1})$ yields:

$$k_{WACC,t} = \omega_{D,t-1}(1-\tau)i + (1-\omega_{D,t-1})k_U + (k_U - i)\omega_{D,t-1}\left(1-\frac{\tau i}{1+i}\right)$$

Simplifying:

$$k_{WACC,t} = k_U - \omega_{D,t-1}\,\tau i - (k_U - i)\omega_{D,t-1}\frac{\tau i}{1+i}$$

Rearranging further:

$$k_{WACC,t} = k_U - \omega_{D,t-1}\,\tau i\left(1+\frac{k_U - i}{1+i}\right) = k_U - \omega_{D,t-1}\,\tau i\left(\frac{(1+i)+k_U - i}{1+i}\right)$$

This yields equation (8):

$$k_{WACC,t} = k_U - \omega_{D,t-1}\,\tau i\left(\frac{1+k_U}{1+i}\right) = k_U - \tau i\frac{D_{t-1}}{D_{t-1}+EV_{t-1}}\left(\frac{1+k_U}{1+i}\right)$$

Equation (9)

The standard WACC definition is:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1}+EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1}+EV_{t-1}}k_{E,t}$$

The cost of equity is given by:

$$k_{E,t} = k_U + (k_U - i)\frac{D_{t-1}}{EV_{t-1}}$$

Substituting into the WACC definition:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1}+EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1}+EV_{t-1}}\left(k_U + (k_U - i)\frac{D_{t-1}}{EV_{t-1}}\right)$$

Rearranging:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1} \cdot k_U + (k_U - i)D_{t-1}}{D_{t-1} + EV_{t-1}}$$

$$k_{WACC,t} = \frac{D_{t-1}\, i - D_{t-1}\, \tau i + EV_{t-1}\, k_U + (k_U - i)D_{t-1}}{D_{t-1} + EV_{t-1}}$$

$$k_{WACC,t} = \frac{(D_{t-1} + EV_{t-1})k_U - D_{t-1}\, \tau i}{D_{t-1} + EV_{t-1}}$$

And equation (9) is obtained:

$$k_{WACC,t} = k_U - \tau i \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}$$

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