ESSAYS ON BANKING, CREDIT AND INTEREST RATES

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STOCKHOLM SCHOOL OF ECONOMICS
EFI, THE ECONOMIC RESEARCH INSTITUTE

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Abstract:

Essays on Banking, Credit and Interest Rates consists of four papers, each with an application of a discrete dependent variable model, censored regression or duration model to a credit market phenomenon or monetary policy question. The first three essays of this doctoral dissertation deal with bank lending policy, while the last one studies interest rate policy by Central Banks.

In the first essay, a bivariate probit model is estimated to contrast the factors that influence banks´ loan granting decision and individuals' risk of default. This model is used as a tool to construct a Value at Risk measure of the credit risk involved in a portfolio of consumer loans and to investigate the efficiency of bank lending policy.

The second essay takes the conclusions from the first paper as a starting point. It investigates if the fact that banks do not minimize default risk can be explained by the existence of a return maximization policy. For this purpose, a Tobit model with sample selection effects and variable censoring limits is developed and estimated on the survival times of consumer loans.

The third paper focuses on dormancy, instead of default risk or survival time, as the most important factor affecting risk and return in bank lending. By means of a duration model the factors determining the transition from an active status to dormancy are studied. The estimated model is used to predict the expected durations to dormancy and to analyze the expected profitability for a sample loan applicants.

In the fourth paper, the discrete nature of Central Bank interest rate policy is studied. A grouped data model, that can take the long periods of time without changes in the repo rate by the Central Bank into account, is estimated on weekly Swedish data. The model is found to be reasonably good at predicting interest rate changes.

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Preface

A famous Dutch football coach used to whisper every young new player in the ear that he had a good chance of making it to the squad, but also that he had selected him because of his running abilities. To stay in the team it was therefore essential to run more and faster than all the other players. "Don't mind that 49 out of the 50 balls you run after are out of reach or taken by another player before you can even get your foot on it. Just keep running and concentrate on that single chance." Of course, all the young kids hated the running. But by the time they found out that all their teammates had also been selected because of their running capacity, chasing unlikely balls had already become a habit.

At that time, I could never have thought that this instruction would be of use when writing a doctoral thesis. After training with coursework, concentrating on that single chance (an idea) seemed to take ages. The odds appeared even worse than in football. Fortunately, my coach at that time, Anders Vredin, was patient and kept me in the game. Anders, I am very grateful for your patience with me, for your help with my first paper and for the encouragement during work on the last three papers. Last but not least, I am still enjoying the fruits from the intensive course in Swedish customs and traditions you gave me.

After paper number one, Anders turned technical director and engaged Tor Jacobson to become my advisor for the remaining papers. Tor, without your encouragement, guidance, good mood, sailing stories and co-authorship, producing this dissertation would have been an impossible task. Thank you for making working on it such a pleasant experience. Especially at those moments when the data set seemed one size too big for me, I was very thankful for your unshakable (?) optimism.

Many thanks also to Kenneth Carling, who is co-author on one of the papers. I'm happy that my learning curve when playing tennis together is not as steep anymore as when working with you.

I also want to expres my gratitude to Sune Karlsson for always being there to help when Gauss did not do what I wanted it to, and when I could

not do what Gauss wanted me to do. It is a comforting thought to know that the door is always open. Thanks also to Mårten Palme and Paul Söderlind for comments and advice. Britt-Marie Eisler, Pirjo Furtenbach, Kerstin Lindskog and Kerstin Niklasson have supported me through the years in the annual struggle for (pleasant) office space, with all kinds of administrative matters (like confirming my student status to the travel agency that couldn't book my student ticket to Amsterdam) and in general with making my stay at the school enjoyable.

My stay at the Department of Economics of the University of Rochester was a valuable experience. I am grateful to Karl Jungenfelt and Torsten Persson for getting me there, and to the members of BMF Inc. for livening up the city of Rochacha. Torsten, thanks also for getting me back into the football game.

Arvid (thanks for the good mood when times were hard, and for making me stop complain ...... for a while), Ingela (knock, knock ...), Jesper (the safest password in the world), Marcus (Sabbath Bloody Sabbath), Micke (have you heard this .....), Richard and Sara (there is a small problem with the delivery of the computer .....): thanks for all the lunches, doktorandöl and dinners. Rickard, without your friendship, it would have been a boring time. We have spent quite some time discussing Swedish society, but even more time enjoying it. Your introduction to Sweden has been so thorough that I now wonder if I'm a stranger when I walk around in Amsterdam. Thanks a lot!

Everybody in Holland, Alexander, Bart, Bien, Ciel, Harm, As, Jaco, Jeroen, Pauline, Lau, Chris, Mike, Joke, Stephan, Farida, Zuur and Iloon: the great thing about this was to always be the guest. Can´t we just continue like that?

All other people that I haven´t mentioned here but should have: thanks for your friendship and support!

My parents and grandparents, Bob, Germien, Jaap en Riek, it was not always so easy, but despite the physical distance I have always felt your love and presence near me!

Finally, I am much obliged to the Dr. Hendrik Muller's Vaderlandsch Fonds foundation, the Swedish Institute/NUFFIC, the Royal Swedish Academy of Sciences, Stiftelsen Siamon and the Jan Wallander and Tom Hedelius Foundation for financial support in various stages.

Stockholm, July 1998

Part I

Thesis summary

Chapter 1

Introduction

This dissertation consists of four essays in the field of applied econometrics. The first three essays, presented in Part 2, are applications of a discrete dependent variable model, a censored regression, and a duration model to credit market data. The fourth paper applies a censored regression model to interest rate data.

Part 1 consists of two chapters. Chapter 1 provides a description of consumer credit markets in Sweden and introductions to the literature on credit scoring and the econometric models used in Part 2. Chapter 2 contains a summary of each of the four essays.

1.1 Consumer credit markets

Consumer credit has come to play an increasingly important role as an instrument in the financial planning of households. When current earnings fall below a household's permanent income and assets are either not available or not acccessible for dissaving, credit is a means to maintain consumption at a level that is consistent with permanent income. People expecting a lasting increase in their income but lacking any assets have a desire to maintain consumption at a higher level than their current earnings allow for. Borrowing can assist them in doing that. Others who have accumulated funds, for example in a pension scheme, but are unable to get access to them, can also increase their welfare by bridging the temporary fall in income with a loan.

The quantitative importance of consumer credit may be illustrated by the fact that total lending, excluding residential loans, by banks and financial companies to Swedish households amounted to SEK 207 bn., or SEK 22,698 per capita, by the end of 1996. This is the equivalent of 12 percent of Swedish GDP or 22.7 percent of total private consumption. These numbers demonstrate that the decision by a credit institution to grant or refuse a loan and the process by which this decision is reached, can greatly influence households' ability to smooth consumption. Thereby it can also affect their welfare.

At a more aggregate level, consumer credit constitutes a significant part of financial institutions' assets and activities. By the end of 1996, consumer credit made up 11 percent of total lending to the public. If one excludes residential loans from total lending, this figure rises to 25 percent. When looking at the risk involved in these loans, their importance is even greater. Rules by the Basle Committee on Banking Supervision, that works under the umbrella of the Bank for International Settlements, stipulate an eight percent capital requirement on consumer credit compared to, for example, four percent on residential loans. Any loan losses that banks incur affect lending capacity directly and can thereby influence other sectors of the economy that rely on borrowing from the financial sector. Investigating the properties of banks' lending policies is thus not merely of interest because it allows us to examine how households' ability to smooth consumption is affected. Through the credit channel, these policies *also* have implications for welfare.

Despite the importance of credit, it is common to see households being rationed in credit markets. When rationing is the mechanism that allocates resources in credit markets, some applicants will be excluded from credit due to an asymmetry in information sets, despite being ex-ante equally creditworthy as those granted a loan. This makes the resulting equilibrium inefficient.[1] Under such circumstances, for example when a lender cannot observe borrowers' probabilities of default, ranking applicants according to their default risk can improve the allocation of resources.

1.2 Credit scoring

Anyone applying for a loan at a bank can count on being subjected to one of the many procedures that exist to assess one´s creditworthiness. Some credit institutions continue to analyze applicants' personal characteristics in subjective, non-formalized ways while others use so called credit scoring models. Their varying degrees of sophistication in spite, credit scoring models share a common objective: to separate loan applicants that are expected to pay back their debts from those who are likely to fall into arrears or go bankrupt. Typically, lending institutions analyze the outcomes a sample of

[1] Stiglitz and Weiss [24] and Williamson [26] contain two different explanations of this phenomenon.

granted loans and the data that have been obtained from the applicants and some credit agency's register.

By far the most commonly used methods are discriminant analysis and logistic regression. Both models have been used to predict whether an applicant with a set of characteristics x_i, who has been granted a loan, is most likely to belong to the group of good loans or to the group of bad loans. As all other models described in this section, discriminant analysis and logistic regression suffer from two important weaknesses. Firstly, the estimated parameters are subject to a sample selection bias when the model is used to predict the behavior of *all* applicants. Secondly, the models minimize the number of accepted bad loans given an exogenous acceptance rate, without any rule for picking this rate optimally. Among practitioners, discriminant analysis has lost ground to logistic regression. In studies using discriminant analysis or logistic regression, the forecasting accuracy ranges from 54 to 90 percent. Altman, Avery, Eisenbeis and Sinkey [1] offers a good review of the literature up to 1980.

More recent studies have employed k-nearest-neighborhood models, classification trees, neural networks, and count data models. In addition to the weaknesses mentioned above, these methods tend to suffer from problems with either the calibration, estimation or interpretation of their parameters. A k-NN scoring rule classifies a new case on the basis of a majority vote amongst the k nearest sample elements, as measured by some metric. Its strong side is that it is a non-parametric method and thus not liable to any of the specification biases we mentioned before. A disadvantage is that the selection of some of the model's parameters involves a large degree of arbitrariness. In addition to this, the parameter estimates lack a clear interpretation. Henley and Hand [17] apply the k-nearest-neighbor method to credit scoring.

Dionne, Artis and Guillen [12] study the costs of defaults by estimating a count data model with sample selection effects for the number of nonpayments under a predetermined repayment scheme. Arminger, Enache and Bonne [4] compare a classification tree model, a feedforward neural network and a logistic regression. They find that all three are almost equally good at predicting loan defaults. A shortcoming of both the tree model and the neural network, is that one cannot quantify the importance of the explanatory variables. In the first model because there are no parameters, in the second because the parameters have no interpretation. Moreover, the neural network was found to be much worse at correctly predicting bad loans than good loans compared to logistic regression and the tree model.

Unfortunately, all the models described above fail to account for the multiperiod character of an optimal debt contract and the implications this has for the credit granting decision. In financial markets with perfect information, any optimal multiperiod financial contract can be obtained by a sequence of one-period loan agreements.[2] Loan applicants will be willing to pay the competitive interest rate that corresponds to their idiosyncratic risk and choose a first-best loan size. In a more general context, for example under asymmetric information, things become more intricate, however. Under these circumstances traditional credit scoring models - by enabling a lending institution to rank potential customers according to their default risk - can improve the allocation of resources, from a second best towards the first best equilibrium.

1.3 Econometric models

All four essays included in this dissertation are applications of an econometric model on consumer loan data or interest rate data. The three following sections contain an introduction to the main concepts used in these models.[3]

1.3.1 Discrete regression models

There are many settings in which the phenomenon that one seeks to represent is discrete rather than continuous. This class of models goes under the common name of *discrete regression* or *qualitative response models.* Because the exact nature of the discrete process that is to be modelled can differ a lot, there exists a large variety of models. A discrete dependent variable can be qualitative or quantitative, ordinal or cardinal, dichotomous or polychotomous. It can represent a participation decision (grant a loan or not), describe the choice between several alternative means of transportation, measure the number of loans provided, or embody a ranking of a number of alternative credit cards according to preference.

Processes where the dependent variables are not continuous do not fit very well into the standard linear regression framework. A more natural approach is to model them such that we can estimate the probability of the discrete event taking place:

$$\Pr(\text{event j occurs}) = \Pr(y_i = j) = F_j(\boldsymbol{\theta},\ \mathbf{x}_i) \qquad j = 0, 1, \ldots.., J \tag{1.1}$$

Here, $\boldsymbol{\theta}$ is a vector of parameters to be estimated, $\mathbf{x}_i$ a vector of explanatory

[2] See Townsend [25] for details.

[3] This section borrows heavily from Amemiya [3], Greene [14] and Maddala [20].

variables, J the number of possible events and $i = 1, 2, ...N$ the observation number.

In this introduction, only models where the dependent variable is binary, that is: $J = 1$, are treated. In this special case, the structure by which the factors $\mathbf{x}_i$ explain the binary decision takes the form of:

$$\begin{aligned} \Pr(y_i = 1) &= F(\boldsymbol{\theta},\ \mathbf{x}_i) \\ \Pr(y_i = 0) &= 1 - F(\boldsymbol{\theta},\ \mathbf{x}_i) \end{aligned} \tag{1.2}$$

In applications, it is generally the case that $F(\boldsymbol{\theta},\ \mathbf{x}_i) = F(\boldsymbol{\theta}'\mathbf{x}_i)$ where $F(.)$ can be chosen in a way that best accomodates the distribution of, and relation between, $\mathbf{x}$ and y. Although in principle any continuous probability distribution will do, almost only the normal and logistic distribution functions have been used in applications.

One way to better understand binary choice models, and other discrete dependent variable models, is to interpret the discrete outcome as the result of some underlying continuous process. In the case of a loan granting decision, one can think of a bank trying to estimate the benefit on a potential loan. Since the benefit is not observed, this would have to be modelled as an unobserved variable y_i^*, that follows

$$y_i^* = \boldsymbol{\theta}'\mathbf{x}_i + \varepsilon_i \tag{1.3}$$

where ε_i follows a standard Normal or Logistic distribution. Although one cannot observe the actual return on a loan, one can see if the loan has been granted or not. One can then define the observed variable y_i in the following way:

$$y_i \begin{cases} 1 & \text{(granted loan)} \\ 0 & \text{(rejected application)} \end{cases} \text{if } \begin{matrix} y_i^* > 0 \\ y_i^* \leq 0 \end{matrix} \tag{1.4}$$

If the distribution of ε_i is symmetric, as is the case with the normal and logistic distribution, then (1.3)−(1.4) imply (1.2). The loglikelihood function for this problem is

$$\ln \ell = \sum_{y_i=1} \ln F(\boldsymbol{\theta}'\mathbf{x}_i) + \sum_{y_i=0} \ln [1 - F(\boldsymbol{\theta}'\mathbf{x}_i)] \tag{1.5}$$

More recently, the single equation binary choice model has been extended to encompass multiple event situations. The general structure of these models is that the unobserved variable y_{ki}^* follows

$$y_{ki}^* = \mathbf{x}_{ki}.\boldsymbol{\alpha}_k + \varepsilon_{ki} \quad \text{for } i = 1, 2, ..., N \text{ and } k = 1, 2, ..., K \tag{1.6}$$

while the observable variable y_{ki} is defined as

$$y_{ki} = \begin{cases} 1 & \text{if } \begin{matrix} y_{1i}^{*} > 0 \\ y_{1i}^{*} \leq 0 \end{matrix} \\ 0 & \end{cases} \quad \text{for } i = 1, 2, ..., N \text{ and } k = 1, 2, ..., K \tag{1.7}$$

Here, k is the decision number and i is the observation number. Typically, the errors ε_{ki} are assumed to be Normally distributed because multivariate Normal distributions allow for an easy decomposition. Also, K is generally not bigger than two or three in applications for reasons of computational intensity. A suitable situation to apply the multivariate probit model on is a sequential decision problem. Suppose, for example, that is investigating which personal characteristics determine whether somebody will spend more than \$ 50 at a supermarket. This binary decision is preceded, however, by the decision to go and do one's shoppings at a supermarket or not. No expenditure data are available on people that do not shop at supermarkets. Because the disturbance terms affecting both decisions are likely to be correlated, the expenditure decision can only be modelled consistently in combination with the shopping decision.

The first of these four essays, *Bank lending policy, credit scoring and Value at Risk*, contains an application of a bivariate probit model to a sequential decision problem. The model is used to investigate the factors that determine whether somebody will be granted a loan, and which of the granted loans will default.

1.3.2 Censored regression models

Another common complication in the analysis of microeconomic data is censoring of the dependent variable. When the dependent variable is censored, values in a *specific* range are transformed into or reported as one single value. A typical example is the number of tickets demanded for a stadium event. If the event sells out, the number of sold tickets falls short of demand and equals the capacity of the stadium. The number of tickets demanded is censored, when measured by the number of tickets sold. A somewhat more complicated situation would arise if one studies demand when the capacity of the stadium shifts over time (e.g. per event). The censoring point of the dependent variable then varies, instead of being constant.

The oldest censored regression model, generally referred to as the *standard Tobit model*, can be formulated as follows:

$$y_i^* = \beta' x_i + \varepsilon_i \tag{1.8}$$

where y_i^* is a latent, unobservable, variable. Only values of y_i^* smaller than c can be observed:

$$\begin{array}{ll} y_i = c & \text{if} \quad y_i^* \geq c \\ y_i = y_i^* & \text{if} \quad y_i^* < c \end{array} \tag{1.9}$$

The cause of the censoring of the dependent variable over a range of values can lie in either the data collection or the nature of the process by which the outcomes of the dependent variable are brought forth. In the above mentioned example of tickets for events, c would be the capacity of the stadium. If the stadium has a varying capacity, the fixed censoring limit c is replaced by a variable c_i.

Because there are censored and continuous observations, the distribution of the sample data is a mixture of discrete and continuous distributions. Conventional regression methods fail to account for this qualitative difference between limit (censored) and non-limit (continuous) observations of the dependent variable. As a consequence, a large number of standard statistical results is distorted.

The loglikelihood function for the Tobit model is given by:

$$\ln \ell \quad = \sum_{y_i \; < \; c} \ln \phi \left(\tfrac{y_i - \beta x_i}{\sigma} \right) + \sum_{y_i \; = \; c} \ln \left[1 - \Phi \left(\tfrac{c - \beta x_i}{\sigma} \right) \right] \tag{1.10}$$

where ϕ and Φ are the Normal density and distribution function respectively. The first part of the likelihood function corresponds to the classical regression model for the non-limit observations. The second part is similar to the the second term in (1.5) and consists of normal probabilities for the limit observations. A censored regression model thus takes the difference between limit and non-limit observations into account.

In the second of these essays, a Tobit model is used to study the survival time of bank loans. Because lending behavior can only be observed with people who have been granted a loan, the standard Tobit model is modified to allow for the presence of sample selection effects and a varying censoring limit.

A large number of extensions of the Tobit model have been suggested to adapt the standard model to varying processes and sampling methods. One variation on the Tobit model, the *grouped data model*, allows the dependent variable to be censored over the full range of possible values. This model has a formulation that differs only slightly from the standard Tobit model.[4] Let y_i^* again follow

$$y_i^* = \beta' x_i + \varepsilon_i \tag{1.11}$$

with y_i^* a latent, unobservable, variable. Instead of y_i^* being censored over a limited range, it is now only possible to observe the interval to which observation i belongs:

[4] See Greene [15] and Stewart [23] for an extensive description of the model.

$$
\begin{array}{lll}
y_i = n+1 & \text{if} & y_i^* \geq c_{n+1} \\
y_i = n & \text{if} & c_{n-1} \leq y_i^* < c_n \\
. & . & . \\
y_i = 1 & & c_0 \leq y_i^* < c_1 \\
y_i = 0 & \text{if} & y_i^* < c_0
\end{array}
\tag{1.12}
$$

This kind of censoring typically occurs when the data are only available by category. In questionnaires, for example, people can be asked to report their level of consumption of a certain good by selecting one of a number of prespecified ranges instead of by writing down their exact consumption. Despite this loss of information, one can still estimate (1.11) consistently if one takes the censoring of y_i^* into account in a way that is similar to that in the Tobit model.

In the fourth essay of this dissertation, *Central Banks, adjustment costs and interest rate policy*, a grouped data model is used to model the interest rate policy of the Bank of Sweden.

1.3.3 Duration models

A third type of model for the analysis of censored data belongs to the class of *survival* or *duration models.* Duration models belong to the same general class of models as Tobit models, but also possess some special features that distinguishes them from Tobit models. If one broadly defines time series models as statistical models that specify how the distribution of random variables, that are observed over time, depends on lagged values of these variables, then survival models can be seen as a special type of time series model. In traditional time series models, the random variables take on continuous values (states) and are observed at discrete times; duration models have been developed to model discrete state, continuous time data. The most common application of survival models on an economic problem has been in the analysis of the factors that influence people's time path from unemployment to employment. See for example Carling, Edin, Harkman and Holmlund [8].

The most fundamental concepts in duration modelling are the survival function and the hazard function.[5] The distribution of survival times, conditional on a vector of characteristics x can be described by both the probability distribution function of duration, $F(t \mid x) = Pr(T < t \mid x)$and the density function $f(t \mid x) = dF(t \mid x)/dt$. The survivor function is another equivalent concept, but has greater intuitive attraction in the context of duration

[5] See Kiefer [18] and Lancaster [19] for an extensive treatment of duration models and the analysis of transition data.

analysis:

$$\begin{aligned} S(t \mid x) &\equiv 1 - F(t \mid x) \\ &= Pr(T \geq t \mid x) \end{aligned} \tag{1.13}$$

Duration data are easiest interpreted, though, by means of the hazard function. Using a hazard function specification enables one to decompose an unconditional probability into a series of conditional probabilities. Formally the hazard function is defined as $\lambda(t) = \lim_{h \to 0} Pr(t \leq T < t + h \mid T \geq t) / h$. It is thus the probability of the immediate occurence of an event at a specific point in time given that the event did not take place earlier. From the definition of the survivor function, it then transpires that

$$\lambda(t \mid x) = f(t \mid x) \ / \ S(t \mid x). \tag{1.14}$$

Suppose that our sample consists of n individuals that have completed an unemployment spell t_i, while the remaining $N - n$ people have not yet left unemployment and thus have spells that are censored at time t_i^*. The loglikelihood function for such a problem takes the following form:

$$\ln \ell = \sum_{i=1}^{n} \ln f^i(t_i) + \sum_{i=n+1}^{N} \ln\left[1 - F^i(t_i^*)\right] \tag{1.15}$$

As with the likelihood function of the standard Tobit model, this is a mixture of densities and probabilities. The first part is related to the non-limit observations (completed spells), while the second part refers to the censored observations (incomplete spells). Unlike the density and distribution functions in the Tobit model, however, those of the duration model generally are a function of the hazard rate. They can therefore differ from observation to observation. It is also usual to use other distributions than the Normal for $f^i(.)$ and $F^i(.)$.

In the third essay of this dissertation, a duration model is employed to analyze what factors determine consumer loans' transition to dormancy. The estimated model is used to make predictions of the loans′ expected time to dormancy and to investigate the implications for bank lending policy.

Chapter 2

Summary of the essays

As mentioned in Section 1.2, credit scoring models leave much room for subjective factors in the loan approval process. By ranking customers according to predicted default probabilities, banks can minimize the expected default or misclassification rate subject to some exogenous acceptance rule. However, this does not enable them to solve their ultimate problem of maximizing profit or utility. Scoring models thus make it possible for banks to predict bankruptcy, but require them to resort to ad-hoc methods to forecast profitability. Boyes, Hoffman and Low [6] provide evidence that confirms the authenticity and importance of this dilemma. In a study of the bank credit scoring problem, they find that loan granting policies are not consistent with default risk minimization.

Lenders may, nevertheless, prefer such a policy of supplying loans with a higher default risk, because they have a higher expected rate of return: the interest rate can be higher or the default is expected to occur after a long period with regular installments and interest payments. In general, the expected return on a loan depends on several factors, such as the interest rate, the discount rate, the amortization scheme, fees paid by the customer, fixed and variable costs incurred by the lender, the likelihood and time of a default, and the expected loss on the principal in case of a default. Except for fluctuations in the rate of interest, that are abstracted from in this paper, a bank thus faces two main types of risks when lending money to a household: the risk of a default and the risk of dormancy, that is: an early redemption of the loan.

A loan, being a multi-period contract, generates a flow of income until it becomes dormant or defaults. In the former case, the return on a loan depends mainly on the duration to dormancy and the amortization scheme. In the latter case, part of the principal may still be recovered. The net present value of a bad loan is thus not merely determined by whether it's paid off in full or not, but also by the duration of the repayments, the amortization scheme, the collection costs and the collateral value. Since the goal of financial institutions is to maximize profit (or utility), not to rank potential customers according to default risk, it may, for example, still be profitable to provide a loan, even if the lender is certain that it will default. Similarly, a loan that becomes dormant at a very early stage may imply a loss due to the dominating role of fixed costs in providing a loan.

The essays that make up this dissertation deal to a large extent with the above mentioned matters. The remainder of this section is devoted to a summary of the papers.

In *Bank lending policy, credit scoring and Value at Risk*[1], the bivariate probit model developed by Boyes et al. [6] is estimated on a bigger data set with more reliable and more extensive financial and personal information on the loan applicants. The results confirm that banks provide loans in a way that is not consistent with default risk minimization. Boyes et al. suggest that banks prefer *bigger* loans because they offer higher expected earnings. Since bigger loans are generally thought to be riskier, maximizing expected earnings would then imply deviating from risk minimization. However, with the data on the size of all loans that we have at our disposal, size is shown not to affect the default risk associated with a loan. Banks, even if they are risk averse, are thus not faced with a trade-off between risk and return. The fact that banks do not minimize default risk can thus not be ascribed to some relation between loan size and return, that earlier models had not accounted for. The banking behavior must thus either be a symptom of an inefficient lending policy or be the result of some other type of optimizing behavior.

The estimated bivariate probit model is used to construct an alternative measure of the risk involved in a portfolio of loans. We take Value at Risk (VaR) as the relevant risk measure and study how marginal changes in a default risk based acceptance rule would shift the the bank's VaR exposure. A comparison of the sample portfolio with an efficiently provided portfolio shows that the bank can reduce its VaR significantly by adopting a default risk based acceptance rule.

Bank lending policy, credit scoring and the survival of loans deals with two issues. Firstly, in order to improve upon the currently available methods for evaluating loan applications, a Tobit model with sample selection and variable censoring thresholds is constructed and estimated. Such a model can be used to predict the survival time on new loans. This permits a more realistic evaluation of the return on a loan than a traditional credit scoring

[1] This paper is co-authored with Tor Jacobson.

model does. Secondly, it takes up the questions that are raised about the efficiency of banks' lending policy in the first essay and investigates if banks face a trade off between default risk and returns.

The empirical results conflict with the notion that the financial institution would be trading off higher default risk against higher returns. The institution does not favor loans that survive longer - and thus have a higher rate of return - and is indifferent between loans of different sizes, given its expected survival time. This is despite the fact that longer survival times are found to imply lower default risk, and vice versa. There is thus no evidence of banks' behaving in a way that is consistent with profit maximization. This impression is strengthened by an experiment in which expected survival times are calculated for all loan applicants, including those who were rejected. In that experiment, only 49 percent of the actually granted loans would have been granted if a survival time criterion had been handled. Moreover, lending to 349 of the 388 defaulted applicants in the sample would have been avoided. These results strengthen the impression that lending institutions have attempted to minimize default or credit risk, or tried to maximize a simple return function, without any success.

In *Duration of consumer loans and bank lending policy: dormancy versus default risk*[2], credit risk is looked at from another perspective than in the first two essays. This paper takes dormancy risk, not default risk, as the economically relevant phenomenon to study. The status of dormancy, unlike that of defaultness, is a condition that every credit account sooner or later will arive at. It is therefore natural to study this phenomenon using survival, or duration, models. A semi-parametric duration model is selected to describe the transition by consumer loans from an active to a dormant state. We investigate the ability of the estimated model to describe the hazard rates observed in the data. The model captures a number of interesting relationships between applicants' characteristics and their expected durations to dormancy. In order to examine bank lending policy, the implied distribution of conditional expected durations for the sample loans is derived. Using these predicted durations to dormancy, we calculate the distribution of expected profits for the sample of loans. This enables us to make a number of inferences about the efficiency of bank lending policy.

The fourth essay, *Central Banks, adjustment costs and interest rate policy,* is an empirical investigation into Central Banks' interest rate policy. Currently, the main instrument of monetary policy in industrialized countries is currently a very short interest rate. Such an interest rate, like the

[2]This paper is co-authored with Kenneth Carling.

repo rate in Sweden or the federal funds rate target in the U.S., forms the building block for all term structure theory and is therefore of importance from both a monetary economic and a financial economic perspective. A typical feature of these interest rates, that has consequently been ignored by monetary theory, is their constancy over long periods of time despite changes in the economic environment.

No attempt has yet been made to build a theoretical framework that explains why Central Banks follow such discrete decision rules for their interest rate instrument. Modelling Central Banks' policy rules, or reaction functions, has traditionally been done by means of VAR and single equation OLS models. In such models, the discrete behavior of the interest rate has generally been removed by using monthly data - often of a market rate with the same maturity as the targeted rate. Therefore, they fail to explain the discrete jumps in the targeted interest rate. The models can be employed to produce consistent forecasts of the targeted interest rate one or more months ahead. However, doing so does not add very much to our comprehension of when and why a Central Bank will change the interest rate *within* any monthly period. The first to recognize the empirical relevance of this feature and treat the interest rate targeted by the Central Bank as a discrete dependent variable was Rudebusch [21]. He applied a non-parametric hazard rate estimator on the spells between changes in the daily U.S. federal funds rate target.

The paper addresses two issues. First, it asks why Central Banks change interest rates so seldomly and what the driving forces are behind a change? For this purpose, a simple model of interest rate policy is developed, where the monetary authority is assumed to incur a fixed cost of changing the interest rate. Next, an econometric model is formulated that can quantitatively capture the discrete behavior of the Swedish repo rate.

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Part II

Included essays

Chapter 3

Essay 1

Bank lending policy, credit scoring and Value at Risk

Bank lending policy, credit scoring and Value at Risk*

Tor Jacobson† Kasper Roszbach‡

30 July 1998

Abstract

In this paper we apply a bivariate censored probit model to investigate the implications of bank lending policy. In the first equation we model the bank's decision to grant a loan, in the second the probability of default. We confirm that banks provide loans in a way that is not consistent with default risk minimization. Loan size is shown not to affect the default risk associated with it. The lending policy must thus either be inefficient or be the result of some other type of optimizing behavior than maximization of expected profit. Value at Risk, being derived from a value weighted sum of all individual risks, provides a more adequate measure of monetary losses on a portfolio of loans than default risk. We derive a Value at Risk measure for the sample portfolio of loans and show how a VaR analysis can enable financial institutions to evaluate alternative lending policies on the basis of their implied credit risk and loss rate and make lending rates consistent with the implied Value at Risk.

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1. Introduction

Consumer credit has come to play an increasingly important role as an instrument in the financial planning of households. When current income falls below a household's permanent level and assets are either not available or not acccessible for dissaving, credit is a means to maintain consumption at a level that is consistent with permanent income. People expecting a permanent increase in their income but lacking any assets, like students, have a desire to maintain consumption at a higher level than their current income allows. Borrowing can assist them in doing that. Those who accumulate funds in a pension scheme but are unable to get access to them when they experience a temporary drop in current income can also increase their welfare by bridging the temporary fall in income with a loan.

The quantitative importance of consumer credit may be illustrated by the fact that total lending, excluding residential loans, by banks and financial companies to Swedish households amounted to SEK 207 bn., or SEK 22.698 per capita, by the end of 1996. That is the equivalent of 12 percent of Swedish GDP or 22.7 percent of total private consumption. Viewed from the perspective of financial institutions, consumer credit also constitutes a significant part of their activities, making up 25 percent of total lending to the public. If one includes residential loans in total lending, this figure drops to 11 percent. When looking at the risk involved in these loans instead of their volume, their importance is even greater, however. Rules by the Basle Committee on Banking Supervision, that works under the umbrella of the Bank for International Settlements, stipulate an 8 percent capital requirement on consumer credit compared to, for example, 4 percent on residential loans. The above numbers make it clear that lending institutions' decision to grant a loan or not and their choice for a specific loan size can thus greatly affect a great many households' ability to smooth consumption over time, and thereby their welfare.

At a more aggregate level, consumer credit makes up a significant part of financial institutions' assets and the effects of any loan losses on lending capacity will be passed through to other sectors of the economy that rely on borrowing from the financial sector. Consequently, investigating the properties of banks' lending policies is not merely of interest because it enables us to examine how households' ability to smooth consumption is affected; these policies also have indirect implications for welfare, through financial markets. We will restricts ourselves to the first channel, however.

Despite credit's importance, it is common to see households being rationed in financial markets. Stiglitz and Weiss [13] and Williamson [14] contain two different explanations of this phenomenon. When rationing is the mechanism that allocates resources in credit markets, some applicants will be excluded from credit despite being equally creditworthy as those granted a loan, making the equilibrium that results inefficient.[1] When a lender cannot observe borrowers' probabilities of default, credit scoring models - by enabling a lending institution to rank potential customers according to their default risk - can improve the allocation of resources, from a second best towards the first best equilibrium. Boyes et al. [6] investigate if the provision of credit takes place in an efficient way. For this purpose they estimate a bivariate probit model with two sequential events as the dependent variables: the lender's decision to grant the loan or not, and - conditional on the loan having been provided - the borrower's ability to pay it off or not. The parameters on variables like duration of job tenure, education and credit card ownership are, however, found to carry equal signs in both equations. Variables that increase (decrease) the probability of positive granting decision thus reduce (raise) the likelihood of a default. In addition, *unexplained* tendencies to extend credit, as measured by the regression error, were found to be positively correlated with default frequencies. Both these observations are inconsistent with a policy of default risk minimization.

In this paper, we construct an alternative risk measure for loans and present two problems to which it can be applied. Instead of an unweighted sum, a value weighted sum of all individual default risks is a more suitable measure of the risk on a portfolio of loans for a financial institution to consider when it needs to balance risk and return. Risk, after all, is only of interest because of the expected monetary losses that are associated with it. The focus of this paper will therefore be on Value at Risk rather than default risk.

First, we re-estimate the model of Boyes et al. on a bigger data set that contains both more reliable and more extensive financial and personal information on the loan applicants. This allows us to investigate the robustness of the finding that banks' lending policies are not consistent with default risk minimization. The re-estimated model will become the workhorse for the remainder of the paper. Next, we take Value at Risk as the relevant risk measure and study how marginal changes in a default risk based acceptance rule would shift the size of the bank's loan portfolio, its VaR exposure and average credit losses. Finally, we compare

[1] Here, we have in mind the unequal treatment of ex-ante equal people due to an asymmetry in information sets. Several different definitions of credit rationing exist, however. One of them is the so called 'redlining' where less creditworthy people are willing to pay a higher interest rate but do not get a loan.

the risk on the sample portfolio with that on an efficiently provided portfolio of equal size.

The rest of the paper is organized as follows. Section 2 surveys the more recent research on statistical methods for credit scoring. Those who are already familiar with credit scoring or merely interested in the empirical results of the paper can skip this section. The data set and its sources are described in section 3. In section 4, we present the parameter estimates of the econometric workhorse model. In section 5, these estimates are used in the Value at Risk experiments we described earlier. Section 6 summarizes the results and discusses roads for future research.

2. Traditional credit scoring

The starting point of each loan is an application. When lending institutions receive an application for a loan, the process by which it is evaluated and its degree of sophistication can vary greatly. Most continue to use rather naïve, subjective evaluation procedures. This could be a non-formalized analysis of an applicant's personal characteristics or 'scoring with integer numbers' on these characteristics. Some banks, however, have started to use statistical credit scoring models. The objective of most credit scoring models is to minimize the misclassification rate or the expected default rate. To achieve this, various statistical methods are used to separate loan applicants that are expected to pay back their debts from those who are likely to fall into arrears or go bankrupt. Typically, a lending institution will analyse a sample of *granted* loans, their outcomes and the data that has been extracted from the applications and a credit agency's register.

The most commonly used statistical methods have been some form of discriminant analysis (DA) and logistic regression (LR). The DA model assumes that the exogenous variables, x_i, are normally distributed but with different means (and in the case of the 'quadratic discriminant model' even a different variance-covariance matrix) conditional on the group to which the dependent variable belongs. The objective is then to estimate these means (and, in the quadratic model, the covariance matrix) and then predict which of the groups an observation with characteristics x_i is most likely to come from.[2] DA thus differs from probit and logit

[2] Depending on the group to which to which the observation $(y_i, \mathbf{x}_i)$ belongs, the vector of exogenous variables x_i follows either f_0 or f_1, where f_k are normal densities with mean $\boldsymbol{\mu}_k$ and variance-covriance $\boldsymbol{\Sigma}_k$, $k = 0, 1$. We define $y_i = 0$ and $y_i = 1$ if $\mathbf{x}_i$ is generated by f_0 and f_1 respectively. To be able to classify y_i we will need $Pr\ (y_i = 0 \mid \mathbf{x}_i)$ and $Pr\ (y_i = 1 \mid \mathbf{x}_i)$. Define

analysis in that the exogenous variables explicitly determine group membership in the latter two models whereas it is taken as given in the former. In other words, the 'causal' relationship runs from the dependent variable to the explanatory variables in DA, not the other way around. One potential weakness of the DA model is that the underlying assumptions are easy to violate. This occurs, for example, when the x_i are categorical instead of continuous and thus not normally distributed. In addition, discriminant analysis suffers from two other weaknesses that it shares with the other methods reviewed in this section. All of them merely minimize the number of accepted bad loans given an exogenous acceptance rate, without any rule for picking this rate optimally. Beside that, the models can only be estimated on samples of granted loans, which causes a sample selection bias in the parameters estimates.

Logistic regression's advantage over DA is that it does not suffer from the strict distributional assumption for $\mathbf{x}_i$.[3] Among practitioners, DA appears to have lost ground to logistic regression. Steenackers and Goovaerts [12] is the latest in a long series of applications of the logistic regression to a sample of personal loans. Their model correctly classifies 62.6 percent of the good loans and 76.6 percent of all bad loans in a holdout sample. In other studies using DA or LR studies, the forecasting accuracy ranges from 54 to 90 percent. Altman, Avery, Eisenbeis and

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the marginal probabilities by $p_0 = Pr(y_i = 0)$ and $p_1 = Pr(y_i = 1)$. Then Bayes' rule gives us that $Pr(y_i = 0 \mid \mathbf{x}_i) = \frac{f_0(\mathbf{x}_i)\cdot p_0}{f_0(\mathbf{x}_i)\cdot p_0 + f_1(\mathbf{x}_i)\cdot p_1}$, which under the normal distribution assumption equals $\boldsymbol{\Lambda}\left(\beta_0 + \boldsymbol{\beta}_1^{'}\mathbf{x}_i + \mathbf{x}_i^{'}\mathbf{A}\mathbf{x}_i\right)$, where $\boldsymbol{\Lambda}(z) = \frac{e^z}{1+e^z}$ is the logistic cdf and β_0, $\boldsymbol{\beta}_1$ are loglinear functions of p_0, p_1, $\boldsymbol{\mu}_0$, $\boldsymbol{\mu}_1$, $\boldsymbol{\Sigma}_0$, $\boldsymbol{\Sigma}_1$. If $\boldsymbol{\Sigma}_0 = \boldsymbol{\Sigma}_1$ is assumed, as common, then a linear logit model results The likelihood function needed to estimate the 6 (4 - if one uses priors for p_0 and p_1) parameters is $\ell = \prod_{i=1}^{N} [f_0(\mathbf{x}_i)\cdot p_0]^{1-y_i} [f_1(\mathbf{x}_i)\cdot p_1]^{y_i}$. With the resulting parameter estimates one can then compute $Pr(y_i = k \mid \mathbf{x}_i)$, $k = 0, 1$. Altman et al. [1] p.40 show that one can modify this likelihood function to take the perceived cost of the two possible types of misclassification into acount.

[3]The logit model has incommon with the linear DA model that $Pr(y_i = 0 \mid \mathbf{x}_i) = \boldsymbol{\Lambda}\left(\beta_0 + \boldsymbol{\beta}_1^{'}\mathbf{x}_i\right)$. Instead of arriving at this point by using Bayes' rule and assuming a normal distribution for the $\mathbf{x}_i$, it drops the assumptions of stochastic $\mathbf{x}_i$ and different group means and variance-covariances and takes the model $y_i^* = \boldsymbol{\Lambda}\left(\beta_0 + \boldsymbol{\beta}_1^{'}\mathbf{x}_i\right) + \varepsilon_i$, $y_i = \begin{cases} 0 & \text{if} \quad y_i^* \leq 0 \\ 1 & \text{if} \quad y_i^* > 0 \end{cases}$ as a starting point. As a consequence $f_0(\mathbf{x}_i) = f_1(\mathbf{x}_i)$ and $P(y_i = k \mid \mathbf{x}_i) = p_k$, causing the likelihood function to take a different form than in the DA model : $\ell = \prod_{i=1}^{N} \left[\boldsymbol{\Lambda}\left(\beta_0 + \boldsymbol{\beta}_1^{'}\mathbf{x}_i\right)\right]^{1-y_i} \left[1 - \boldsymbol{\Lambda}\left(\beta_0 + \boldsymbol{\beta}_1^{'}\mathbf{x}_i\right)\right]^{y_i}$

Sinkey [1] review the literature up to 1980.

In a couple of studies k-nearest-neighborhood, count data or neural network methods have been employed. Henley and Hand [10] apply the k-nearest-neighbor method to credit scoring. A k-NN scoring rule classifies a new case on the basis of a majority vote amongst the k nearest sample elements, as measured by some metric that is defined over the space of explanatory variables. Its strong side is that it is a non-parametric method and thus not liable to any of the specification biases we mentioned before. One of the disadvantages is that the selection of some of the model's parameters involves quite a large degree of arbitrariness.[4] In addition, the parameter estimates lack a clear interpretation.

Dionne et al. [7] study the costs of defaults by means of a count data model with sample selection effects. Their dependent variable is the number of non-payments under a predetermined repayment scheme. Their sample, however, consists merely of approved loan applications. Because of the sample selection bias in the parameter estimates that this leads to, the methodology in this study is thus of limited interest for the purpose of forecasting the profitability of future applicants.

Arminger et al. [3] compare a classification tree model[5] and a feedforward

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[4] The most popular measure of distance beween two points $\mathbf{x}$ and $\mathbf{y}$ is the Euclidean metric $d_E(\mathbf{x},\mathbf{y}) = \left\{(\mathbf{x}-\mathbf{y})'(\mathbf{x}-\mathbf{y})\right\}^{1/2}$. By using a weighting matrix $\mathbf{A}$, we can assign unequal importance to distances in different dimensions in the variable space. For example, $d_P(\mathbf{x},\mathbf{y}) = \left\{(\mathbf{x}-\mathbf{y})'\mathbf{A}(\mathbf{x}-\mathbf{y})\right\}^{1/2}$ allows for larger weights for distances in the direction of variables that are of great importance and smaller weights for variables that affect the probability of belonging to a certain group only marginally. In practice, $\mathbf{A}$ is set equal to $\mathbf{I}+\mathbf{ww}'$, where $\mathbf{w}$ is the gradient of the iso-probability curves, which is estimated by linear regression from the sample data. $\mathbf{A} = \mathbf{I}+\mathbf{ww}'$ implies a metric that weighs the Euclidean distance and the distance between the points in the direction orthogonal to the iso-probability curves. One chooses k by trading off a bias against variance, because the former increases with k whereas the latter decreases.

[5] A classification tree splits up a sample into two subsamples, each of which contains only cases from a specific range of values of the dependent variable. The split should have the lowest degree of impurity as measured by the Gini coefficient. Building up an extra branch on the decision tree is done by means of a 2-step routine. First, at an already created node one finds (if the variable is qualitative) the categorization (or the cut-off point, if the variable is metrical) that minimizes the additional impurity for each explanatory variable. The impurity at this node is measured as $p_L \cdot i(t_L) + p_R \cdot i(t_R)$, where p_L is the probability of an individual being classified as a member of the left branch and $i(t_L)$ is the impurity on the left side branch under node t. The in the second step the actual split is made by choosing the explanatory variable that minimizes the impurity over all exogenous variables at node t. How long the branching continues depends on a misclassification cost function that decreases with the number correct predictions but increases with the tree size. See Arminger et al. [3] p. 297-301.

neural network [6] with a logistic regression and find that all three are approximately equally good at predicting loan defaults. A shortcoming that the first two models share, however, is that one cannot quantify the importance of the explanatory variables; in the tree analysis because there are no parameters, and in the neural network because the parameters have no interpretation. Moreover, the neural network was found to be much worse at correctly predicting bad loans than good loans compared to the LR and the tree model.

3. Data

The data set consists of 13,338 applications for a loan at a major Swedish lending institution between September 1994 and August 1995. All loans were granted in stores where potential customers applied for instant credit to finance the purchase of a consumer good. The evaluation of each application took place in the following way. First, the store phoned to the lending institution to get an approval or a rejection. The lending institution then analysed the applicant with the help of a database with personal characteristics and credit variables to which it has on-line access. The database is maintained by Upplysningscentralen AB, the leading Swedish credit bureau which is jointly owned by all Swedish banks and lending institutions. If approval was granted, the store's salesman filled out a loan contract and submitted it to the lending institution. The loan is revolving and administered by the lending institution as any other credit facility. It is provided in the form of a credit card that can only be used in a specific store. Some fixed amount minimum payment by the borrower is required during each month. However,

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[6] An artificial neural network is a non-linear regression model $\mathbf{y} = \nu(\mathbf{x}) + \varepsilon$, where the conditional expectation $\nu(\mathbf{x})$ is approximated by a function $\mu(\mathbf{x};\boldsymbol{\theta})$. The network $\mu(\mathbf{x};\boldsymbol{\theta})$ is composed of a number of interconnected processing units. These units - called 'perceptrons' when they are functions of linear combinations of their inputs - produce an output that is passed on to another processing unit. Many ANN's are composed of a number of perceptron layers, where the output of each unit of one layer can only be passed on to units of a higher one. Such ANN's are called feedforward networks.

For example, a FfN with one hidden layer (a two-layer-perceptron) would look like $\mu(\mathbf{x};\boldsymbol{\beta},\boldsymbol{\gamma}) = \phi(\boldsymbol{\beta}'\boldsymbol{\eta})$, $\boldsymbol{\eta} = (\eta_1, \ldots\ldots, \eta_K)'$ where $\boldsymbol{\beta}$ is the parameter vector of the output unit, $\eta_k = \psi(\boldsymbol{\gamma}_k'\mathbf{x})$, $k = 1, \ldots\ldots, K$, and the $\boldsymbol{\gamma}_k$ are the parameter vectors of the K hidden units. The outputs η_k of the hidden units are not observed and can be seen as realizations of latent variables. A common parameterization is to let $\phi(\cdot)$ be a linear model and $\varphi(\cdot)$ the standardized logistic distribution. Finally, predicted values $\widehat{y}_i$ are obtained by using the threshold relation

$$\widehat{y}_i = \begin{cases} 0 & \text{if } \mu(\mathbf{x}_i;\boldsymbol{\beta},\boldsymbol{\gamma}) < 0 \\ 1 & \text{if } \mu(\mathbf{x}_i;\boldsymbol{\beta},\boldsymbol{\gamma}) \geq 0 \end{cases}$$

Table 1: Definition of variables.

Variable	Definition
AGE	age of applicant
MALE	dummy, takes value 1 if applicant is male
MARRIED	dummy, takes value 1 if applicant is married
DIVORCE	dummy, takes value 1 if applicant is divorced
HOUSE	dummy, takes value 1 if applicant owns a house
BIGCITY	dummy, takes value 1 if applicant lives in one of the three greater metropolitan areas around Göteborg, Malmö and Stockholm.
NRQUEST	number of requests for information on the applicant that the credit agency received during the last 36 months
ENTREPR	dummy, takes value 1 if applicant has taxable income from a registered business
INCOME	annual income from wages as reported to Swedish tax authorities (in 1000 SEK)
DIFINC	change in annual income from wages, relative to preceding year, as reported to Swedish tax authorities (in 1000 SEK)
CAPINC	dummy, takes value 1 if applicant has taxable income from capital
BALINC[7]	ratio of total collateral free credit facilities actually utilized and $INCOME$, expressed as percentage.
ZEROLIM	dummy, takes value 1 if applicant has no collateral-free loans outstanding
LIMIT	total amount of collateral free credit facilities already outstanding (in 1000 SEK)
NRLOANS	number of collateral free loans already outstanding
LIMUTIL	percentage of $LIMIT$ that is actually being utilized
LOANSIZE	amount of credit granted (in 1000 SEK)
COAPPLIC	dummy, takes value 1 if applicant has a guarantor

[7] This variable takes value zero when *INCOME* = 0 and is thus actually defined as $DUMMY_{\{income\,>\,0\}}$ * (*BALANCE* / *INCOME*).

since the loan is revolving, there is no predetermined maturity of the loan. Earnings on the loan come from three sources: a one-time fee paid by the customer; a payment by the store that is related to total amount of loans granted through it; and interest on the balance outstanding on the card.

For this study, the lending instutution provided us with a data file with the personal number of each applicant, the date on which the application was submitted, the size of the loan that was granted, the status of each loan (good or bad) on October 9, 1996, and the date on which bad loans gained this status.

Although one can think of several different definitions of a 'bad' loan, we classify a loan as bad once it is forwarded to a debt-collecting agency. We do not study what factors determine the differences in loss rates, if any, among bad loans. An alternative definition of the set of bad loans could have been 'all customers who have received one, two or three reminders because of delayed payment'. However, unlike 'forwarded to debt-collecting agency", one, two or three reminders were all transient states in the register of the financial institititution. Once customers returned to the agreed-upon repayment scheme, the number of reminders was reset to zero. Such a property is rather undesirable if one needs to determine unambiguously which loans have defaulted and which have not.

Upplysningscentralen provided the information that was available on each applicant at the time of application and which the financial institution accessed for *its* evaluation. By exploiting the unique personal number that each resident of Sweden has, the credit bureau was able to merge these two data sets. Before handing over the combined data for analysis, the personal numbers were removed. The database included publicly available, governmentally supplied information, such as sex, citizenship, marital status, postal code, taxable income, taxable wealth, house ownership, and variables reported by Swedish banks like the total number of inquiries made about an individual, the number of unsecured loans and the total amount of unsecured loans. In total there we disposed of some 60 different variables.

A number of the variables in the dataset were not used in the final estimation of the model described in Sections 3 and 4. Among these are the number of months since the most recent change in marital status, citizenship (Swedish, nordic, non-nordic), number of months since immigration, number of houses a person owns (partially), assessed value of all real estate a person has (partial) ownership in, the combined value of all real estate ownership shares, several measures of income, taxable wealth, a large number of entries on the two most recently submitted income-tax return forms, like total taxes due, back tax etc, and a number of

Table 2: Descriptive statistics for all loan applicants ($N = 13338$)

Variable	Rejections ($N = 6899$) mean	stdev	min	max	Granted loans ($N = 6439$) mean	stdev	min	max
AGE	38.65	12.76	18	84	41.02	12.08	20	83
MALE	.62	.48	0	1	.65	.48	0	1
DIVORCE	.13	.34	0	1	.14	.35	0	1
HOUSE	.34	.47	0	1	.47	.50	0	1
BIGCITY	.41	.49	0	1	.37	.48	0	1
NRQUEST	4.69	2.60	1	10	4.81	2.68	1	19
ENTREPR	.04	.21	0	1	.02	.16	0	1
INCOME	129.93	70.38	0	737.9	189.47	75.70	0	1093.0
DIFINC	5.37	34.06	-438.5	252.6	9.03	34.63	-6226.0	5006.0
CAPINC	.12	.32	0	1	.07	.25	0	1
BALINC [8]	91.04	894.53	0	41533	31.01	386.15	0	22387
BALINC [9]	114.01	999.73	1	41533	35.85	431.87	1	22387
ZEROLIM	.15	.36	0	1	$<.01$	.05	0	1
LIMIT	79.89	93.69	0	1703.0	50.47	51.07	.0	949.2
NRLOANS	2.99	2.42	0	18	3.65	2.04	0	16
LIMUTIL	64.34	38.88	0	278.0	53.22	33.94	0	124.0
COAPPLIC	.16	.36	0	1	.14	.35	0	1

transformations of these variables.

Most of these were disregarded because they lacked correlation with the variables of interest - the loan granting decision and the payment behavior. Examples are back tax and real estate value. Others were disregarded because they displayed extremely high correlation with variables that measured approximately the same thing but had greater explanatory power. The numerous income measures in the dataset and *BALANCE* were eliminated in this way. A number of variables was selected for the statistical analysis because of an actual or supposed covariation with the dependent variables, but omitted from the final model because they did

[8] Only computed for the 6508 rejected and 6372 approved applications with *INCOME* > 0.
[9] Only computed for the 5197 rejected and 5086 approved applications with *BALINC* > 0.

Table 3: Descriptive statistics for granted loans.

	Defaulted loans ($N = 388$)				Good loans ($N = 6051$)			
Variable	mean	stdev	min	max	mean	stdev	min	max
AGE	36.11	11.03	21	75	41.33	12.07	20	83
MALE	.67	.47	0	1	.65	.48	0	1
DIVORCE	.20	.40	0	1	.14	.35	0	1
HOUSE	.28	.45	0	1	.48	.50	0	1
BIGCITY	.41	.49	0	1	.36	.48	0	1
NRQUEST	6.15	2.85	1	14	4.72	2.64	1	19
ENTREPR	.02	.13	0	1	.03	.16	0	1
INCOME	165.36	82.35	0	1093.0	191.01	75.00	0	1031.7
DIFINC	3.52	39.01	-135.0	439.7	9.38	34.30	-622.6	500.6
CAPINC	.04	.20	0	1	.07	.26	0	1
BALINC [10]	39.92	313.51	0	6041	30.44	390.36	0	22387
BALINC [11]	46.45	337.81	1	6041	38.33	437.68	1	22387
ZEROLIM	.04	.20	0	1	$<.01$	.02	0	1
LIMIT	41.44	57.98	0	511.5	51.05	50.54	0	949.21
NRLOANS	2.34	1.64	0	11	3.74	2.04	0	16
LIMUTIL	75.69	33.37	0	124.0	51.78	33.47	0	112.0
LOANSIZE	7.08	3.95	3.0	24.5	7.12	3.83	3.0	30.0
COAPPLIC	.07	.26	0	1	.14	.35	0	1

not gain significance in any of the estimations. Citizenship, immigration related variables and real estate value were among these. Finally, wealth could not be used as an explanatory variable because not a single bad loan concerned a person with positive *taxable* wealth, thereby creating a numerical problem in the gradient of the likelihood function. Wealth up to SEK 900,000 is tax-exempted, making the group of people with *taxable* wealth extremely small in Sweden. Instead we

[10] Only computed for the 5988 good and 384 bad loans with *INCOME* > 0.

[11] Only computed for the 4756 good and 330 bad loans with *BALINC* > 0.

used *taxable* income from capital - which is taxed from the first krona - to create a dummy explanatory variable. Tables 1 and 2 contain definitions and descriptive statistics for the variables that have been selected for the estmation of the final model in Section 4.

Of all applicants, 6,899, or 51.7 percent, were refused credit. The remaining 6,439 obtained a loan ranging from 3,000 to 30,000 Swedish kronor (approximately US$ 375 - 3750). The lending institution's policy was that no loans exceeding 30,000 kronor were supplied. Although there is an indicated amortization scheme, the loans have no fixed maturity - they are revolving.

On 9 October 1996, the people in the sample were monitored by the lending institutution. On that day 388 (6.0 %) of those who obtained a loan had defaulted and been forwarded to a debt collection agency. All other borrowers still fulfilled their minimum repayment obligations at that time. Some descriptive statistics are provided in Tables 2 and 3.

4. Econometric model

In this section we present the econometric model, that will be used as a workhorse in the experiments of Section 5. The model consists of two simultaneous equations, one for the binary decision to provide a loan or not, y_{1i} , and another for the binary outcome, 'default' or 'proper repayment', of each loan, y_{2i}. We let the superscript $*$ indicate an unobserved variable and assume that y_{1i}^* and y_{2i}^* follow

$$\begin{aligned} y_{1i}^* &= \mathbf{x}_{1i}.\boldsymbol{\alpha}_1 + \varepsilon_{1i}, \\ y_{2i}^* &= \mathbf{x}_{2i}.\boldsymbol{\alpha}_2 + \varepsilon_{2i} \quad \text{for } i = 1, 2, ..., N \end{aligned} \tag{4.1}$$

where the $\mathbf{x}_{ji}$, $j = 1, 2$, are $1 \times k_j$ vectors of explanatory variables.

The disturbances are assumed to be bivariate normally distributed.

$$\begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \end{pmatrix} \sim N \begin{pmatrix} 0 & 1 & \rho \\ 0 & \rho & 1 \end{pmatrix}$$

The binary choice variable y_{1i} takes value 1 if the loan was granted and 0 if the application was rejected:

$$y_{1i} = \begin{cases} 0 & \text{if} \quad \text{loan not granted} \quad (y_{1i}^* < 0) \\ 1 & \text{if} \quad \text{loan granted} \quad (y_{1i}^* \geq 0) \end{cases} \tag{4.2}$$

The second binary variable, y_{2i}, takes the value 0 if the loan defaults and 1 if not:

$$y_{2i} = \begin{cases} 0 & \text{if} \quad \text{loan defaults} \quad (y_{2i}^* < 0) \\ 1 & \text{if} \quad \text{loan does not default} \quad (y_{2i}^* \geq 0) \end{cases} \tag{4.3}$$

Due to the fact that one only observes if a loan is good or bad if it was granted, there is not only a censoring rule for $(y_{1i},\ y_{2i})$ but even an *observation* rule. The observation rule is shown in Figure 1.

Figure 1: Observation rule for y_{1i} and y_{2i}. Entries in the 2×2 table show pairs $(y_{1i},\ y_{2i})$ that are observed for all ranges of y_{1i}^* and y_{2i}^*.

	$y_{2i}^* < 0$	$y_{2i}^* \geq 0$
$y_{1i}^* < 0$	$(0, .)$	$(0, .)$
$y_{1i}^* \geq 0$	$(1, 0)$	$(1, 1)$

Because we have three types of observations: no loans, bad loans and good loans, the likelihood function will take the following form:

$$\ell = \prod_{no\ loans} \text{pr}\,(no\ loan) \cdot \prod_{bad\ loans} \text{pr}\,(bad\ loan) \prod_{good\ loans} \text{pr}\,(good\ loan) \tag{4.4}$$

In appendix **A.1.** it is shown that (4.4) implies the following loglikelihood:

$$\begin{aligned} \ln\ell = & \textstyle\sum_{i=1}^{N} (1 - y_{1i}) \cdot \ln\left[1 - \Phi\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_1\right)\right] + \\ & \textstyle\sum_{i=1}^{N} y_{1i} \cdot (1 - y_{2i}) \left\{\Phi\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_1\right) - \Phi_2\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_1,\ \mathbf{x}_{2i}\boldsymbol{\alpha}_2;\ \rho\right)\right\} \\ & \textstyle\sum_{i=1}^{N} y_{1i} \cdot y_{2i}\ \ln\Phi_2\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_1,\ \mathbf{x}_{2i}\boldsymbol{\alpha}_2;\ \rho\right) \end{aligned} \tag{4.5}$$

where $\Phi(\cdot)$ and $\Phi_2(\cdot, \cdot, \rho)$ represent the univariate and bivariate standard normal c.d.f., the latter with correlation coefficient ρ.

The estimated parameters, their standard errors and t-statistics are presented in Table 4. Notice that *LOANSIZE* cannot be used as an explanatory variable in the first equation because no data on this variable is available for rejected applicants. The effect of many variables on the probability of obtaining a loan seems in

Table 4: Bivariate probit MLE of $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$.

	P(obtain a loan)			P(loan does not default)		
Variable	$\widehat{\alpha}_1$	std. error	t-stat.	$\widehat{\alpha}_2$	std. error	t-stat.
CONSTANT	-.2374	.06652	-3.57	2.2900	.1463	15.65
AGE	-.004303	.001166	-3.69	.006892	.002624	2.63
MALE	-.2003	.02823	-7.10	-.02456	.05812	-.43
DIVORCE	-.02588	.03696	-.70	-.2380	.07125	-3.34
HOUSE	.06391	.02759	2.32	-.02019	.05830	.35
BIGCITY	-.2382	.02659	-8.96	-.03724	.05397	-.69
NRQUEST	-.008123	.005153	-1.58	-.1000	.01017	-9.84
ENTREPR	.5223	.06294	8.30	.2065	.1613	1.28
INCOME	.008928	.0001816	49.17	-.002392	.0004968	-4.81
DIFINC	-.002336	.0003445	-6.78	.002233	.0007352	3.04
CAPINC	-.2776	.05066	-5.48	.1189	.1232	.97
BALINC	.00006548	.00001811	3.48	-.00009135	.00005922	-1.54
ZEROLIM	-2.2440	.1049	-21.40	-.6590	.2890	-2.23
LIMIT	-.008381	.0001526	-54.94	.005064	.0004928	10.28
NRLOANS	.08420	.006882	12.23	.2698	.01873	14.41
LIMUTIL	-.007746	.0004370	-17.72	-.01197	.0009266	-12.91
COAPPLIC	.1300	.03395	3.83	.4374	.09715	4.50
LOANSIZE	-	-	-	-.006637	.006794	-.98
ρ	-	-	-	-.9234	.05326	-17.34

Critical values are 1.645, 1.96,and 2.575 for the 10, 5, and 1 percent significance levels.

accordance with the behavior banks commonly display. *INCOME, HOUSE, ENTREPR, NRLOANS* and having a *COAPPLIC*ant confirm their role as important factors that contribute positively while *ZEROLIM, LIMIT,* and *LIMUTIL* weigh negatively in the bank's decision. Somewhat surprising are the coefficients on *MALE, BIGCITY, DIFINC* and *CAPINC.* Men have a significantly smaller chance of being granted a loan as do people living in one of the three metropolitan areas. The same holds for people who have capital income and those who experienced a rise in income during the last year. The latter effect deserves some further attention, though. Another way to interpret the sign of this parameter would be that people who experience large increases in wage income had quite a low income the preceeding year. Rather than reasoning that a rise in income worsens your chances of getting a loan, one could argue that income uncertainty - embodied in a low income in the year before - does so. If this were the case, then we should expect a similar effect to exist for people experiencing a fall in income. We tested for the presence of such an effect by transforming DIFINC into a variable with absolute values of income changes. We also tried with the standard deviation of income. Neither of these variables gained significance. We therefore interpret the coefficients in INCOME and DIFINC in the following way. We rewrite $\alpha_y y_t + \alpha_{dy} \bigtriangleup y_t$ as $(\alpha_y + \alpha_{dy})\, y_t - \alpha_{dy} y_{t-1}$. Current and past income then both have positive and significant coefficients in the equation, with the former carrying the largest weight - as in a calculation of permanent income.

More striking is the fact that only four variables have the equal signs in both equations that one would expect when banks are minimizing default risk. Exploiting the credit facilities one disposes of to a greater extent (a higher *LIMUTIL)* or lacking experience with servicing debt (*ZEROLIM*) reduces an applicant's odds of obtaining a loan from the bank and increases the likelihood of a default. Having more experience in borrowing money and servicing a debt, as reflected by a higher *NRLOANS*, or applying together with a *COAPPLIC*ant makes it more likely that somebody will receive a loan and also add to the chances that the loan will be paid back.

Four variables have opposite coefficients in the loan granting and default equations. *INCOME*, notwithstanding a large positive weight in the decision to grant a loan by the bank, actually increases a loan's probability of default. Although one should be careful not to rationalize each counter-intuitive finding, we can look for a tentative explanation. Table 3 clearly shows that people who default on their loans have a lower average income than those who do not. This may well lead us to infer - if we disregard the rejected applicants, who have lower incomes than those who were granted a loan - that higher income reduces default risk. Suppose, however, that it is actually the case that other factors than *INCOME* determine a loan's survival. Then the selection of applicants may be taking place on the basis of a negative bivariate correlation between *INCOME* and defaults rather than on grounds of a negative partial correlation, which controls for both the sample selection effect and the correlation with other variables. It may, for example, be the case that people with higher income have other characteristics that are associated with greater default risk. Similar arguments can be applied to *AGE, DIFINC* and *LIMIT*, that have negative weights in the first equation but positive weights in the second. Although one might, for example, expect *LIMIT* to have a negative impact on debt service, one should keep in mind that it is merely the ceiling of the credit facility that a person disposes of. *LIMUTIL* captures the extent to which he or she actually uses it, while *LIMIT* proxies for experience with servicing debt in the same way as *NRLOANS* does.[12] The positive coefficient on *DIFINC* in combination with the negative coefficient on *INCOME* illustrate how popularly assumed relations can lack factual support.

Furthermore, it is wortwhile to take notice of the large number of variables that are significant in only one equation and thus witness of inefficient use of information in the evaluation of applicants. *NRQUEST* is a proxy for people's eagerness to obtain additional credit and as such adds to the probability of a default. In the decision to grant a loan it has no role of importance, however. Being *DIVORCE*d, which can bring about a mismatch between financial obligations and income, has an effect similar to that of *NRQUEST*. Five variables, *MALE, HOUSE, BIGCITY, CAPINC* and *BALINC* carry either positive or negative weight in the bank's decision but do not affect a loan's risk of default. Finally, we point out that *LOANSIZE* has no influence whatsoever on default risk. On the margin, an extra credit with a maximum of SEK 30,000 apparently does not affect default probability. Because the average *LIMIT* is between six and seven times the the average *LOANSIZE*, the relevant variable to study in this context is *LIMIT*, the total amount of credit facilities.

The only parameter we have not yet reviewed is the correlation coefficient. The value of $-.9234$ implies that non-systematic tendencies to grant loans are almost perfectly correlated with non-systematic increases in default risk.[13] In other words: the subjective elements - that conflict with the systematic policy described by the first equation in (4.1) - in the bank's lending policy that increase individuals' odds of being granted a loan, are positively related to increases in default

[12] Strong correlation between the variables *BALANCE* and *LIMIT* tended to create numerical problems when trying to use both as explanatory variables. Some test regressions indicated that *LIMIT* and *BALANCE* have opposite effects on *SURVIVAL*, the former a positive and the latter a negative. The coefficient on *LIMIT* in Table 4 is approximately equal to the net effect of *LIMIT* minus *BALANCE*.

[13] Although the value of $-.92324$ for ρ is quite close to -1, and more than twice as large as what Boyes et al. [6] found it to be, this is no symptom of problems with convergence for the algorithm. The correlation coefficient varied between $-.51$ and $-.97$ depending on the number and the type of variables that we let $\mathbf{x}_{ji}$ consist of.

risk that cannot be unexplained by a systematic relation with the covariates $\mathbf{x}_{2i}$.

If we compare the above results with those in Boyes et al. [6] we can make three observations. Firstly, our results confirm the conclusion in Boyes et al. that banks do not appear to be minimizing default risk. Many of the variables that make the bank approve loan applications are not among those that reduce the probability of default. Secondly, non-systematic tendencies to grant loans are indeed associated with greater default risk. Thirdly, we find that the *size* of a loan does *not* affect default risk. This contradicts the interpretation of Boyes et al. that banks pick out loans with higher default risk because they have higher returns. They suggest that banks actually prefer *bigger*, not riskier, loans for the one reason that they offer higher expected earnings. Because they think of bigger loans as generally also being riskier, maximizing earnings would imply deviating from risk minimization. In this paper we *control* for *LOANSIZE* by including it in the set of explanatory variables and find that it has no significant impact on default risk. Bigger loans are thus not riskier.

As a consequence, the fact that bank behavior is not consistent with risk minimization cannot be ascribed to a disregarded relation between loan size and return. Because all loans in this sample pay the same rate of interest, there remain only two sources of differences in the expected rate of return between loans: survival time - and the amortizations and interest payments that result from it - and the loss rate on bad loans. To get a good forecast of profitability, banks may be evaluating survival and the loss rate simultaneously. In a study of the survival of bank loans Roszbach [11] finds, however, that loans are not provided in a way that is consistent with survival time maximization. As an alternative, banks have been maximizing some other objective than the rate of return on their loan portfolio; for example, the number of customers or lending volume subject to a minimum return constraint, or total profits from a range of financial products. The current organization of information flows in banks and the degree of co-ordination between different departments does not allow for the pursuit of a composite objective such as the return on a range of products. Most important of all, the alternative objectives suggested above are not in agreement with what employees from the lending institution reported to us in a series of interviews on the matter. Rather, the results bear the evidence of a lending institution that has attempted to minimize risk or maximize a simple return function without success.

5. Lending policies and Value at Risk

Risk is of importance to financial institutions only to the extent that it involves expected monetary losses. Estimating individual default risks is merely of limited help, because their linkage with credit losses is unclear. A better way to measure risk is to weigh individual default risks by value, as one does, for example, in the calculation of Value at Risk. Studying Value at Risk not only enables the financial institution to get a measure of the credit risk present in currently administered loans. It also allows an evaluation of the impact of different lending policies on (a specific measure of) risk exposure and creates a better basis for an explicit decision on the implied loss rate. For these reasons, we will shift our attention in the remainder of the paper from the estimation of default risk to the construction of a Value at Risk (VaR) measure. First, we derive a VaR measure using a Monte-Carlo simulation of the bivariate probit model of Section 4. After that, we show how it can be applied in a typical problem that a lending institution may be confronted with when supplying loans.

We define Value at Risk as "the loss that is expected to be exceeded with a probability of only x % during the total holding period of the loan portfolio", where the relevant risk measure x needs to be chosen in advance. If one sets x equal to 5%, for example, a Value at Risk of SEK 10 mn. means that total credit losses on the loan portfolio will be greater SEK 10 mn. with a probability of 5%. Observe that our VaR concept differs from more conventional types in two respects. Firstly, the computed losses concern credit risk rather than market risk. Secondly, Value at Risk is calculated for a specific time horizon. VaR measures over other time horizons require re-estimation of the bivariate probit model.

One of the purposes of this section is to illustrate how using Value at Risk instead of default risk can be auxiliary in optimizing bank lending policy. We will therefore carry out two experiments, in which the bivariate probit model from section 4 will serve as a workhorse. First, we analyse how the Value at Risk is affected by marginal changes in the bank's acceptance rule. Second, we construct a hypothetical portfolio of loans that would be granted if the bank had a default risk based decision rule instead of its current policy. Comparing the distribution of credit losses on this hypothetical portfolio with those on the actual portfolio may supply us with a crude estimate of the efficiency losses that the bank's lending gives rise to.

In the first experiment we study how the bank can affect its Value at Risk exposure by making its acceptance criterion more or less restrictive. Here, we abandon the bank's current lending policy, as described by the first equation in (4.1) and (4.2). In section 4 we showed that this policy is not consistent with risk minimization. Instead, we construct a default risk based acceptance rule of the form:

$$\left.\begin{array}{l}\text{loan not granted}\\ \text{loan granted}\end{array}\right\} \text{if} \left\{\begin{array}{l} pr\left(y_{2i}=0\right) \geq \delta' \\ pr\left(y_{2i}=0\right) < \delta' \end{array}\right. \tag{5.1}$$

By means of a Monte-Carlo simulation similar to the one described above, we can derive the probability distribution of bank credit losses associated with the acceptance/rejection rule (5.1) for any value of the threshold parameter δ'. The Monte-Carlo simulation consists of the following 5 steps:

1. Pick a value for δ'.

2. Draw one observation $\mathbf{x}_{2i}\widetilde{\boldsymbol{\alpha}_2}$ from $N\left(\mathbf{x}_{2i}\widehat{\boldsymbol{\alpha}_2}, \sigma_{\mathbf{x}_{2i}\widehat{\boldsymbol{\alpha}_2}}\right)$ for $i = 1, 2, .., 13338$,where $\sigma^2_{\mathbf{x}_{2i}.\widetilde{\boldsymbol{\alpha}_{2i}}} = \mathbf{x}_{2i} \cdot \boldsymbol{\Sigma}_{\widehat{\boldsymbol{\alpha}_2}} \cdot \mathbf{x}'_{2i}$. Number them $i = 1, 2, .., 13338$;

3. To determine which applicants will be granted a loan, calculate the expected default probabilities $E\left[pr\left(y_{2i}=0\right)\right]$ as

   $$E\left[\widetilde{p}_i\right] \;=\; 1 - \Phi\left(\mathbf{x}_{2i}.\widetilde{\boldsymbol{\alpha}_{2i}} \;/\; \left(1+\sigma^2_{\mathbf{x}_{2i}.\widetilde{\boldsymbol{\alpha}_{2i}}}\right)^{1/2}\right)$$

   and then apply (5.1). Number the approved applications $i = 1, 2, ..., N_A$;

4. For the N_A approved applications, compute the total credit losses λ on this portfolio as

   $$\lambda \;=\; \sum_{i=1}^{N_A} E\left[\widetilde{p}_i\right] \cdot q_i$$

   where q_i is the size of the loan individual i applied for. Because q_i is not available for the rejected applicants, we impute $\overline{q} = \frac{1}{N_{A,\ true}} \sum_{i=1}^{N_{A,\ true}} q_i$ in steps 1-4. Here, $N_{A,\ true}$ is the number of accepted applicants in the original sample.

5. Repeat steps 1-4 M times and compute the approximate probability distribution over losses from the M values one obtains for λ. M should be chosen such that the distribution is invariable for $M' \geq M$.

For our purpose we have picked a series of values δ' in the interval $[.01,\ .20]$. The results from these simulations are displayed in Table 5. The second column of Table 5 shows how expected loan losses increase as the bank relaxes its lending policy. The most restrictive policy, $\delta' = .01$, results in lending between 36.6 mn. and 37.2 mn. kronor, whereas the most generous policy, $\delta' = .20$, leads to approximately two and a half times as much lending. As the lending volume grows, losses increase at an accelerating rate. For the most risk averse decision rule loan losses range from .3 to .6 mn. kronor, compared to SEK 3 mn. - up to 9 times as much - on the riskiest loan portfolio. As expected the loss rate, loan losses divided by total lending, rises from .36 percent to 3.35 percent as the acceptance criterion δ is successively relaxed from .01 to .20. Total losses and the loss rate both monotonically increase with δ, at an ever decreasing rate however.

Applying a Value at Risk analysis before selecting a lending policy thus allows the lending institution to decide explicitly on either its aggregate credit risk exposure or its loss rate. Alternatively, it could choose to pick a desirable loss rate conditional on the Value at Risk not exceeding some maximum allowable amount of money. Doing so has several advantages. First, compared to current practice, the risk involved in lending becomes more transparent. Instead of registering loans that have already become non-performing, the financial institution will be able to create provisions for expected losses. This offers gains from both a private and a social perspective. From a private perspective because provisions for loan losses on banks' balance sheets will be forward-looking and only lag *un*expected events. This should facilitate a correct valuation of the firm. At an aggregate level, there would be less risk for bankruptcy of financial institutions and therefore less risk for financial disturbances to the economy. See for example Bernanke and Gertler [5]. Secondly, unless the bank sets interest rates individually, this methodology also enables a bank to pick a risk-premium on top of the risk free rate of interest that is consistent with average credit risk over the maturity in question. If the loss rate is 2.5 percent, for example, and the average duration of a loan is 3 years, then the bank could charge a risk-premium of approximately .8 percent per annum.

In the second experiment our aim is to produce an estimate of the monetary losses that the inefficiency in the current lending policy gives rise to. Table 5 has given us an impression of how lending volume, loan losses and the loss rate covary, and can help a bank choose one specific efficient lending policy from a larger set. However, before switching to a new policy, a financial institution will first want to quantify the potential gains from doing so. For this purpose, we construct the 'efficient' portfolio of loans that would be granted if the bank used a default risk

Table 5: 95 percent confidence intervals for total loan losses, total lending (both in 1000 SEK) and the loss rate (total credit losses/total lending), all for given rejection threshold δ'.

δ'	loan losses	total lending	loss rate
.01	$131 - 137$	$36,583 - 37,226$	$.36 - .37$
.02	$339 - 551$	$51,124 - 51,763$	$.66 - .68$
.03	$548 - 564$	$59,705 - 60,318$	$.92 - .94$
.04	$751 - 773$	$65,641 - 66,248$	$1.14 - 1.17$
.05	$948 - 947$	$70,083 - 70,683$	$1.35 - 1.38$
.06	$1,143 - 1,174$	$73,678 - 74,276$	$1.55 - 1.58$
.07	$1,334 - 1,370$	$76,672 - 77,225$	$1.74 - 1.77$
.08	$1,516 - 1,555$	$79,138 - 79,703$	$1.91 - 1.95$
.09	$1,687 - 1,730$	$81,181 - 81,730$	$2.08 - 2.12$
.10	$1,849 - 1,895$	$82,916 - 83,449$	$2.23 - 2.27$
.11	$2,003 - 2,053$	$84,411 - 84,923$	$2.37 - 2.42$
.12	$2,149 - 2,201$	$85,699 - 86,190$	$2.51 - 2.55$
.13	$2,285 - 2,339$	$86,812 - 87,279$	$2.63 - 2.68$
.14	$2,414 - 2,470$	$87,785 - 88,230$	$2.75 - 2.80$
.15	$2,285 - 2,339$	$88,641 - 89,066$	$2.85 - 2.91$
.16	$2,648 - 2,709$	$89,389 - 89,793$	$2.96 - 3.02$
.17	$2,752 - 2,814$	$90,034 - 90,416$	$3.06 - 3.11$
.18	$2,847 - 2,909$	$90,586 - 90,945$	$3.14 - 3.20$
.19	$2,932 - 2,995$	$91,057 - 91,398$	$3.22 - 3.28$
.20	$3,009 - 3,073$	$91,467 - 91,789$	$3.29 - 3.35$

based decision rule, instead of its current policy, but preferred a lending volume (approximately) equal to that of the actual portfolio. Executing steps 1-3 in the above Monte-Carlo experiment and picking δ' such that the simulated lending volume equals actual lending gives us the desired portfolio.

By inspecting Table 5 one can already infer that the implied value of δ' will lie between .01 and .02. We find that δ' equals .012. We then repeat steps 2-5 of the Monte-Carlo experiment for both the actual and the 'efficient' portfolio - but do not apply (5.1) in step 3 since we already know which individuals make up our

Table 6: Value at Risk at different risk levels computed for the sample portfolio and an efficiently provided portfolio of equal size (amounts × thousand SEK).

Portfolio	Risk level		
	1%	5%	10%
Sample	1,513	1,506	1,503
Efficient	263	262	261

sample. From the credit loss distributions that we obtain along these lines, we extract three different Value at Risk measures for each portfolio. These are displayed in Table 6. Credit losses on the two portfolios clearly differ greatly. At the 10 percent risk level, the value at risk amounts to SEK 1,503 thousand for the actual portfolio compared to 261 thousand for the efficient portfolio. At the 1 percent risk level these amounts are 1,513 and 263 respectively. By shifting to a default risk based decision rule and abandoning its current lending policy, the bank can can reduce its expected credit losses significantly. Continuing providing loans in the same way as has been done leads to a VaR exposure that is six times higher than with a policy consistent with default risk minimization. Switching to one of the 'efficient' lending policies displayed in Table 5 thus involves large potential benefits for the financial institution.

6. Discussion

In this paper we have applied the bivariate probit model from Boyes et al. [6] to investigate the implications of bank lending policy. With a larger and more extensive data set we confirm earlier evidence that banks provide loans in a way that is not consistent with default risk minimization. It had been suggested that banks prefer *bigger* loans because they offer higher expected earnings. Since bigger loans are generally thought to be riskier, maximizing expected earnings would then imply deviating from risk minimization. However, with the data on the size of all loans that we have at our disposal, size has been shown not to affect the default risk associated with a loan. Banks, even if they are risk averse, are thus not faced with a trade-off between risk and return. The inconsistency in banking behavior can thus not be ascribed to some relation between loan size and return, that earlier models had not accounted for. The banking behavior must thus be either a symptom of an inefficient lending policy or the result of some other type of optimizing behavior. Banks may, for example, be forecasting survival time, loss rates or both. Another alternative is that they are maximizing another objective than the rate of return on their loan portfolio, e.g. the number of customers, lending volume subject to a minimum return constraint, or total profits from a range of financial products. Current banking technology does not yet allow for the pursuit of a composite objective such as the return on a range of products, however. In addition, the above suggestions are not in agreement with the practices reported to us by the lending institution that provided our data. Rather, the results bear the evidence of a lending institution that has attempted to minimize risk or maximize a simple return function without success.

Value at Risk, being derived from a value weighted sum of all individual risks, provides a more adequate measure of monetary losses on a portfolio of loans than default risk. By means of Monte-Carlo simulation with the bivariate probit model, we have obtained a Value at Risk measure for the sample portfolio of loans. We have also shown how calculating Value at Risk can enable financial institutions to evaluate alternative lending policies on the basis of their implied credit risks and loss rates. An analysis of the VaR involved in lending policies offers both private and social gains. Provisions for loan losses on banks' balance sheets will become more forward-looking. This should facilitate a correct evaluation of the firm. At an aggregate level, the risk of bankruptcy for financial institutions and the likelihood of financial disturbances to the economy would be reduced. Banks would also be able to choose a risk-premium on top of the risk free rate of interest that is *consistent with* average credit risk over the maturity in question.

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A. Likelihood function

Combining (4.2) – (4.3) and table 1, the likelihood function in equation (4.4) becomes

$$\begin{aligned}\ell \;=\; & \prod_{i=1}^{N} \mathrm{pr}\left(y_{1i}^{*}<0\right)^{(1-y_{1i})} \cdot \prod_{i=1}^{N} \mathrm{pr}\left(y_{1i}^{*}\geq 0,\ y_{2i}^{*}\leq 0\right)^{y_{1i}\cdot y_{2i}} \times \\ & \prod_{i=1}^{N} \mathrm{pr}\left(y_{1i}^{*}\geq 0,\ y_{2i}^{*}\geq 0\right)^{y_{1i}\cdot y_{2i}}\end{aligned} \tag{A.1}$$

Substituting for (4.1), $(A.1)$implies the following loglikelihood function:

$$\begin{aligned}\ln\ell \;=\; & \textstyle\sum_{i=1}^{N}\left(1-y_{1i}\right)\cdot\ln\left[\mathrm{pr}\left(\varepsilon_{1i}<-\mathbf{x}_{1i}\boldsymbol{\alpha}_{1}\right)\right]+ \\ & \textstyle\sum_{i=1}^{N} y_{1i}\cdot\left(1-y_{2i}\right)\ \ln\ \left[\mathrm{pr}\left(\varepsilon_{1i}\geq-\mathbf{x}_{1i}\boldsymbol{\alpha}_{1}\cap\varepsilon_{2i}\leq-\mathbf{x}_{2i}\boldsymbol{\alpha}_{2}\right)\right]+ \\ & \textstyle\sum_{i=1}^{N} y_{1i}\cdot y_{2i}\ \ln\left[\mathrm{pr}\left(\varepsilon_{1i}\geq-\mathbf{x}_{1i}\boldsymbol{\alpha}_{1}\cap\varepsilon_{2i}\geq-\mathbf{x}_{2i}\boldsymbol{\alpha}_{2}\right)\right]\end{aligned} \tag{A.2}$$

Because of the symmetry property of the bivariate normal distribution, the last line in $(A.2)$ can be rewritten as:

$$\begin{aligned}& \mathrm{pr}\left(\varepsilon_{1i}\geq-\mathbf{x}_{1i}\boldsymbol{\alpha}_{1}\cap\varepsilon_{2i}\geq-\mathbf{x}_{2i}\boldsymbol{\beta}_{2}\right)\Leftrightarrow{}^{14} \\ & \Phi_{2}\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_{1},\ \mathbf{x}_{2i}\boldsymbol{\alpha}_{2};\ \rho\right)\end{aligned} \tag{A.3}$$

Since

$$\mathrm{pr}\left(y_{1i}^{*}\geq 0,\ y_{2i}^{*}\leq 0\right) \;=\; 1-\mathrm{pr}\left(y_{1i}^{*}<0\right)-\mathrm{pr}\left(y_{1i}^{*}\geq 0,\ y_{2i}^{*}\geq 0\right)$$

$\forall i$, the loglikelihood function can be written as

$$\begin{aligned}\ln\ell \;=\; & \textstyle\sum_{i=1}^{N}\left(1-y_{1i}\right)\cdot\ln\left[1-\Phi\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_{1}\right)\right]+ \\ & \textstyle\sum_{i=1}^{N} y_{1i}\cdot\left(1-y_{2i}\right)\left\{\Phi\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_{1}\right)-\Phi_{2}\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_{1},\ \mathbf{x}_{2i}\boldsymbol{\alpha}_{2};\ \rho\right)\right\} \\ & \textstyle\sum_{i=1}^{N} y_{1i}\cdot y_{2i}\ \ln\Phi_{2}\left(\mathbf{x}_{1i}\boldsymbol{\alpha}_{1},\ \mathbf{x}_{2i}\boldsymbol{\alpha}_{2};\ \rho\right)\end{aligned} \tag{A.4}$$

[14] See Greene (1993) p.661 for a summary of results on the bivariate normal cdf.

Chapter 4

Essay 2

Bank lending policy, credit scoring and the survival of loans

Bank lending policy, credit scoring and the survival of loans.*

Kasper Roszbach[†]

30 July 1998

Abstract

To evaluate loan applicants, banks use a large variety of systems, of which the more sophisticated ones go under the name of credit scoring. These models' objective typically is to minimize default rates or the number of incorrectly classified loans. Thereby they fail to take into account that loans are multiperiod contracts. From a utility maximizing perspective it is not only important to know *if* but also *when* a loan will default. In this paper a Tobit model with a variable censoring threshold and sample selection effects is estimated for (1) the decision to provide a loan or not and (2) the survival of granted loans. The model is shown to be an affective tool to separate applicants with short survival times from those with long survivals. The bank's loan provision process is shown to be inefficient. Loans are granted in a way that is not only in conflict with default risk minimization but even incompatible with survival maximization, implying there is no trade-off between higher default risk and higher return.

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1. Introduction

Consumer credit has come to play an increasingly important role as an instrument in the financial planning of households. When current income falls below a household's permanent level and assets are either not available or not acccessible for dissaving, credit is a means to maintain consumption at a level that is consistent with permanent income. People expecting a permanent increase in their income but lacking any assets, like students, have a desire to maintain consumption at a higher level than their current income allows. Borrowing can assist them in doing that. Those who accumulate funds in a pension scheme but are unable to get access to them when they experience a temporary drop in current income can also increase their welfare by bridging the temporary fall in income with a loan.

The quantitative importance of consumer credit may be illustrated by the fact that total lending, excluding residential loans, by banks and finance companies to Swedish households amounted to SEK 207 bn., or SEK 22.698 per capita, by the end of 1996. That is equivalent to 12% of Swedish GDP or 22.7% of total private consumption. Viewed from the perspective of financial institutions, consumer credit also constitutes a significant part of their activities, making up 25 percent of total lending to the public. If one includes residential loans in total lending, this figure drops to 11 percent. When looking at the risk involved in these loans instead of their volume, their importance is even greater, however. BIS rules stipulate an 8 percent capital requirement on consumer credit compared to, for example, 4 percent on residential loans.

From these numbers, it may be clear that a lending institution's decision to grant a loan or not and its choice for a specific loan size can greatly affect households' ability to smooth consumption over time, and thereby even households' welfare. At a more aggregate level, consumer credit makes up a significant part of financial institutions' assets and the effects of any loan losses on lending capacity will be passed through to other sectors of the economy that rely on borrowing from the financial sector. For this reason, the properties and efficiency of banks' credit granting process are of interest not merely because the factors determining the optimal size of financial contracts can be examined. At least as important are the implications these contracts have for the welfare of households and the stability of financial markets.

The starting point of every loan is the application. When lending institutions receive an application for a loan, the process by which it is evaluated and its degree of sophistication can vary greatly. Most continue to use rather naïve, subjective evaluation procedures. This could be a non-formalized analysis of an applicant's personal characteristics or 'scoring with integer numbers' on these characteristics. Some banks, however, have started to use a statistical 'credit scoring' model to separate loan applicants that are expected to pay back their debts from those who are likely to fall into arrears or go bankrupt.

By far the most commonly used methods are discriminant analysis and logistic regression. Altman, Avery, Eisenbeis and Sinkey [1] contains a good review of this literature. Both models have been fit to separate good loans from bad ones among *approved* applications. The estimated parameters are thus subject to sample selection bias when these models are applied to *all* applicants. More recent studies have employed k-nearest-neighborhood and [10] count data models [7], classification trees and neural networks [3]. These methods tend to suffer from problems with either the calibration, estimation or interpretation of their parameters *in addition to* the sample selection bias I mentioned earlier. All the above mentioned models, however, fail to account for the multiperiod character of an optimal debt contract and the implications this has for the credit-granting decision. In financial markets with perfect information, any optimal multiperiod financial contract can be obtained by a sequence of one-period loan agreements [12]. Loan applicants will be willing to pay the competitive interest rate that corresponds to their idiosyncratic risk and choose a first-best loan size. Under asymmetric information things become more intricate. In the literature that studies credit markets and the form of optimal financial contracts in the presence of adverse selection or costly state verification, credit rationing - the unequal treatment of ex-ante equal people - is a recurring phenomenon. See for example Stiglitz and Weiss [11] or Williamson [13]. When rationing is the mechanism that equilibrates credit markets, some applicants will be excluded from credit despite being equally creditworthy as those granted a loan. The allocation of resources will thus be inefficient.

Let us assume that *single-period* agreements are optimal[1] and that only the probability of default is unobservable to the lender. Gale and Hellwig [8] show that the optimal one-period debt contract consists of a pair (l, θ) where l is the size of the loan and θ the level of the endowment shock below which the debtor will be declared bankrupt. Under these circumstances traditional credit scoring models - by enabling a lending institution to rank potential customers according to their default risk - could improve the allocation of resources, from a second best towards the first best equilibrium. In a more general context, however, this

[1] Since the default probabilities are not observed, this would be in a second best sense.

does not solve a lender's profit maximization problem because financial contracts typically stretch out over several time periods. Townsend [12] proves that, in the presence of asymmetric information between the borrower and lender, ex-ante optimal contracts can only be created by multiperiod debt contracts because they allow payoffs to be dependent on past and present behavior of the borrower.

A loan, being a multi-period contract, generates a flow of funds until it either is paid off or defaults, in which case a part of the principal may still be recovered. The net present value of a loan is thus not determined by whether it's paid off in full or not, but - if it is not - by the duration of the repayments, amortization scheme, collection costs and possible collateral value. It may, for example, still be profitable to provide a loan, even if the lender is certain that it will default. Since the goal of financial institutions is to maximize profit (or utility), not to rank potential customers according to default risk, credit scoring models leave much room for subjective factors in the loan approval process. In a sense, banks use statistical models to forecast bankruptcy, but - conditional on this forecast - resort to ad-hoc methods to predict profitability.

Boyes, Hoffman and Low [5] address this deficiency and investigate if the provision of credit currently takes place in an efficient way. For this purpose they estimate a bivariate probit model with two sequential events as the dependent variables: the lender's decision to grant the loan or not, and - conditional on the loan having been provided - the borrower's ability to pay it off or not. If the lending institution is minimizing credit risk, we ought to find opposite signs for the parameter of one particular explanatory variable in the two different equations. This would imply that variables that increase the probability of positive granting decision also decrease the likelihood of a default, or vice versa. They find, however, that variables like duration of job tenure, education and credit card ownership carried equal signs, indicative of a policy that conflicts with default risk minimization. As we noted earlier, lenders may nevertheless prefer such a policy of supplying loans with a higher default risk because they have a higher expected rate of return (either the interest rate is higher or the default is expected to occur after a long period with regular installments and interest payments). Moreover, Boyes et al. show that unexplained tendencies to extend credit are positively correlated with default frequencies - another fact consistent with a policy that trades off default risk against profitability.

This paper deals with two issues. First, in order to improve upon the currently available methods for evaluating loan applications, I construct and estimate a Tobit model with sample selection and variable censoring thresholds. The model can be used to predict the expected survival time on a loan to any potential applicant. This allows for a more realistic evaluation of the return on a loan than an estimate of the default risk associated with an individual with a traditional credit scoring model does.

Secondly, I take up the question about the efficiency of banks' loan provision process that is raised by the results in Boyes et al. [5]. Those suggest that the fact that some variables increase the probability of a positive granting decision while at the same time increasing the likelihood of a default is a consequence of profit maximizing behavior by the lender. Here, it will be investigated if a similar relationship continues to exist when one models the survival time of a loan instead of the probability of its default. If variables that increase the likelihood of an applicant obtaining a loan also increase the expected survival and vice versa, then this would constitute further evidence of banks' behaving in a way that is consistent with profit-maximization.

The rest of this paper is organized as follows. Section 2 describes the data set and its sources. In Section 3, I derive the econometric model. Section 4 contains the empirical results and section 5 concludes the paper with a discussion of the results and possibilities for future research.

2. Data

The data set consists of 13,337 applications for a loan that were processed by a major Swedish lending institution between September 1994 and August 1995. All applications were submitted in stores where potential customers applied for instant credit to finance the purchase of a consumer good. Out of 13,337 applications, 6,899 were rejected and 6,438 were approved. The dataset includes 127 second attempts by individuals that had applied once before.

The evaluation of each application took place in the following way. First, the store phoned to the lending institution to get an approval or a rejection. The lending institution then analysed the applicant with the help of a database with personal characteristics and credit variables to which it has on-line access. The database is maintained by Upplysningscentralen AB, the leading Swedish credit bureau which is jointly owned by all Swedish banks and lending institutions. If approval was given, the store's salesman filled out a loan contract and submitted it to the lending institution. The loan is revolving and administered by the lending institution as any other credit facility. It is provided in the form of a credit card

Table 1: Definition of variables.

Variable	Definition
SURVIVAL	days between granting of loan and its default
MALE	dummy, takes value 1 if applicant is male
MARRIED	dummy, takes value 1 if applicant is married
DIVORCE	dummy, takes value 1 if applicant is divorced
HOUSE	dummy, takes value 1 if applicant owns a house
BIGCITY	dummy, takes value 1 if applicant lives in one of the three greater metropolitan areas around Göteborg, Malmö and Stockholm.
NRQUEST	number of requests for information on the applicant that the credit agency received during the last 36 months
ENTREPR	dummy, takes value 1 if applicant has taxable income from a registered business
INCOME	annual income from wages as reported to Swedish tax authorities (in 1000 SEK)
DIFINC	change in annual income from wages, relative to preceding year, as reported to Swedish tax authorities (in 1000 SEK)
CAPINC	dummy, takes value 1 if applicant has taxable income from capital
ZEROLIM	dummy, takes value 1 if applicant has no collateral-free loans outstanding
LIMIT	total amount of collateral free credit facilities already outstanding (in 1000 SEK)
NRLOANS	number of collateral free loans already outstanding
LIMUTIL	percentage of *LIMIT* that is actually being utilized
LOANSIZE	amount of credit granted (in 1000 SEK)
COAPPLIC	dummy, takes value 1 if applicant has a guarantor

that can only be used in a specific store. Some fixed amount minimum payment by the borrower is required during each month. However, since the loan is revolving, there is no predetermined maturity of the loan. Earnings on the loan come from three sources: a one-time fee paid by the customer; a payment by the store that is related to total amount of loans granted through it; and interest on the balance outstanding on the card.

Table 2: Descriptive statistics for all loan applicants ($N = 13337$).
The table splits up the sample into rejected and approved applications.

	Rejected ($N = 6899$)				Granted ($N = 6438$)			
Variable	mean	stdev	min	max	mean	stdev	min	max
MALE	.62	.48	0	1	.65	.48	0	1
MARRIED	.47	.50	0	1	.47	.50	0	1
DIVORCE	.13	.34	0	1	.14	.35	0	1
HOUSE	.34	.47	0	1	.47	.50	0	1
BIGCITY	.41	.49	0	1	.37	.48	0	1
NRQUEST	4.69	2.60	1	10	4.81	2.68	1	19
ENTREPR	.04	.21	0	1	.02	.16	0	1
INCOME	129.93	70.38	0	737.9	189.47	75.70	0	1093.0
DIFINC	5.37	34.06	-438.5	252.6	9.03	34.63	-6226.0	5006.0
CAPINC	.12	.32	0	1	.07	.25	0	1
ZEROLIM	.15	.36	0	1	<.01	.05	0	1
LIMIT	79.89	93.69	0	1703.0	50.33	49.83	.0	627.0
NRLOANS	2.99	2.42	0	18	3.65	2.04	0	16
LIMUTIL	64.34	38.88	0	278.0	53.22	33.93	0	124.0
COAPPLIC	.16	.36	0	1	.14	.35	0	1

For this study, the lending instutution provided a data file with the personal number of each applicant, the date on which the application was submitted, the size of the loan that was granted, the status of each loan (good or bad) on October 9, 1996, and the date on which bad loans gained this status.

Although one can think of several different definitions of a 'bad' loan, I classify a loan as bad once it is forwarded to a debt-collecting agency. I do not study what factors determine the differences in loss rates, if any, among bad loans. An alternative definition of the set of bad loans could have been 'all customers who have received one, two or three reminders because of delayed payment'. However, unlike 'forwarded to debt-collecting agency", one, two or three reminders were all transient states in the register of the financial instititution. Once customers

Table 3: Descriptive statistics for granted loans.
The table splits up the subsample of granted applications into defaulted and non-defaulted loans.

	Defaulted loans ($N = 388$)				Good loans ($N = 6050$)			
Variable	mean	stdev	min	max	mean	stdev	min	max
*SURVIVAL**	400.09	151.07	130	789	632.79	93.99	34	795
MALE	.67	.47	0	1	.65	.48	0	1
MARRIED	.24	.43	0	1	.48	.50	0	1
DIVORCE	.20	.40	0	1	.14	.35	0	1
HOUSE	.28	.45	0	1	.48	.50	0	1
BIGCITY	.41	.49	0	1	.36	.48	0	1
NRQUEST	6.15	2.85	1	14	4.72	2.64	1	19
ENTREPR	.02	.13	0	1	.03	.16	0	1
INCOME	165.36	82.35	0	1093.0	191.02	75.00	0	1031.7
DIFINC	3.52	39.01	-135.0	439.7	9.38	34.30	-622.6	500.6
CAPINC	.04	.20	0	1	.07	.26	0	1
ZEROLIM	.04	.20	0	1	<.01	.02	0	1
LIMIT	41.44	57.98	0	511.5	50.90	49.21	0	627.0
NRLOANS	2.34	1.64	0	11	3.74	2.04	0	16
LIMUTIL	75.69	33.37	0	124.0	51.78	33.47	0	112.0
LOANSIZE	7.08	3.95	3.0	24.5	7.12	3.83	3	30.0
COAPPLIC	.07	.26	0	1	.14	.35	0	1

* For good loans these are censored survival times.

returned to the agreed-upon repayment scheme, the number of reminders was reset to zero. Such a property is rather undesirable if one needs to determine unambiguously which observations are censored and which are not.

Upplysningscentralen provided the information that was available on each applicant at the time of application and which the financial institution accessed for *its* evaluation. By exploiting the unique personal number that each resident

Table 4: Descriptive statistics for survival time.
Percentiles for survival time and the natural logarithm of survival time.
The sample has been split up into defaulted and non-defaulted loans.

		Percentiles							
Sample	min	5	10	25	50	75	90	95	max
t, bad loans	130	156	192	278	403	514	606	648	789
t, good loans	34	470	497	564	652	704	746	767	795
ln(t), bad loans	4.87	5.05	5.26	5.63	6.00	6.24	6.41	6.47	6.67
ln(t), good loans	3.53	6.15	6.21	6.34	6.48	6.56	6.61	6.64	6.68

of Sweden has, the credit bureau was able to merge these two data sets. Before handing over the combined data for analysis, the personal numbers were removed. Overall, the database includes a total 60-70 variables. The major part consists of publicly available, governmentally supplied information such as sex, citizenship, marital status, postal code, taxable income, taxable wealth, house ownership. The remaining variables, like the total number of inquiries made about an individual, the number of unsecured loans and the total amount of unsecured loans, are reported to Upplysningscentralen by the Swedish banks. Table 1 contains definitions of all variables that are used in the analysis in Section 4. Some descriptive statistics on the explanatory variables are provided in Tables 2 and 3.

Of the applicants, 6,899, or 51.7 percent, were refused credit. The remaining 6,438 obtained a loan ranging from 3,000 to 30,000 Swedish kronor (approximately US$ 375 - 3750) . The lending institution's policy was that no loans exceeding 30,000 kronor were supplied. Although there is an indicated amortization scheme, the loans have no fixed maturity - they are revolving.

On 9 October 1996, all people in the sample were monitored by the lending institutution. At that moment 388 (6.0 %) of those who had obtained a loan had defaulted and been forwarded to a debt collection agency. All other borrowers still fulfilled their minimum repayment obligations at that time. The survival time in the sample, calculated as the number of calendar days between the date of application and the date of default, ranged from 130 days (a defaulted loan) to 795 days (a censored observation). Descriptive statistics for survival time are provided in Tables 4. Because the statistical model that will be presented in Section 3 will be estimated with the natural logarithm of survival time as a dependent variable, table 4 also contains descriptive statistics on logaritmized survival time.

3. Econometric model

Under ideal conditions evaluating loan applicants or studying efficiency in the provision of bank loans would entail modelling the revenue on each loan as a function of a set of personal characteristics and macro-economic indicators. However, since few banks store complete time series of interest payments and amortizations on loans, the information presently available and useful for such a study is limited to the *current balance and status* (good or bad) of each loan. Therefore, we will instead model the survival time of each loan. With some simplifying assumptions imposed on the amortization scheme and cost structure, one can then in principle calculate an estimate of the return on each loan as a function of survival time.

The econometric model consists of two simultaneous equations, the first one for the binary decision to provide a loan or not, y_i, and the second one for the *natural logarithm* of survival time of a loan (in days), for notational simplicity denoted by t_i. Because the bank from which we obtained our data merely considered whether it would accept an application or not, all people who were granted a loan received the amount of credit they applied for at the going rate of interest. The first equation therefore models a binary decision. I do not model how individuals determine the amount of credit they apply for.

I use the superscript $*$ to indicate an unobserved variable and let y_i^* and t_i^* follow

$$\begin{aligned} y_i^* &= \mathbf{x}_{1i}.\boldsymbol{\beta}_1 + \varepsilon_{1i} \\ t_i^* &= \mathbf{x}_{2i}.\boldsymbol{\beta}_2 + \varepsilon_{2i} \quad \text{for } i = 1, 2, \ldots.., N \end{aligned} \tag{3.1}$$

where the disturbances are assumed to be bivariate normal distributed.

$$\begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \end{pmatrix} \sim N \begin{pmatrix} 0 & 1 & \sigma_{12} \\ 0 & \sigma_{12} & \sigma_2^2 \end{pmatrix}$$

As mentioned, y_i, is a binary choice variable that takes value 1 if the loan was granted and 0 if the application was rejected:

$$y_i = \begin{cases} 0 & if \quad y_i^* < 0 \\ 1 & if \quad y_i^* \geq 0 \end{cases} \tag{3.2}$$

For loans that turn bad, one can observe the exact survival time. For loans that are still performing on the day of monitoring, survival is censored because we do not know if and when they will turn bad. Because all loans are monitored on October 9, 1996, but are granted anywhere between September 1994 and August 1995, the good loans' survival times will be censored at varying thresholds. For example, a loan granted on September 1, 1994, has a censoring threshold of 768 days. For a loan granted on August 31, 1995, this is 434 days. A loan's censoring threshold for survival time will be denoted as $\overline{t_i}$. The above can be summarized in the following censoring rule:

$$t_i = \begin{cases} t_i^* & if \quad t_i^* < \overline{t_i} \\ \overline{t_i} & if \quad t_i^* \geq \overline{t_i} \end{cases} \tag{3.3}$$

Due to the fact that one only observes survivals for loans that are actually granted, there is not only a censoring rule for t_i but even an *observation* rule:

Figure 1: Observation rule for y_i and t_i.
Entries in the 2×2 table show pairs $(y_i,\ t_i)$ that are observed for all ranges of y_i^* and t_i^*.

	$t_i^* \leq \overline{t_i}$	$t_i^* > \overline{t_i}$
$y_i^* < 0$	$(0, .)$	$(0, .)$
$y_i^* \geq 0$	$(1, t_i^*)$	$(1, \overline{t_i})$

A dummy variable d_i splits up the sample of granted loans into good ones and bad ones. If a loan's survival is uncensored, $t_i^* \leq \overline{t_i}$, it must be a defaulted one. If survival is censored, it must be a good loan.

$$d_i = \begin{cases} 0 & if \quad t_i^* \leq \overline{t_i} \\ 1 & if \quad t_i^* > \overline{t_i} \end{cases}$$

Because we have three types of observations: no loans, bad loans with survival t_i, and good loans with survival $\overline{t_i}$, the likelihood function will take the following form:

$$\begin{aligned} \ell \;=\; & \prod_{no\ loans} \mathrm{pr}\,(no\ loan) \cdot \prod_{bad\ loans} \mathrm{pr}\,(t_i\ \cap\ bad\ loan) \times \\ & \prod_{good\ loans} \mathrm{pr}\,(\overline{t_i}\ \cap\ good\ loan) \end{aligned} \tag{3.4}$$

Combining (3.2), (3.3) and Table 1, equation (3.4) becomes

$$\begin{aligned}\ell &= \prod_{i=1}^{N} \mathrm{pr}\left(y_i^* < 0\right)^{(1-y_i)} \cdot \prod_{i=1}^{N} \mathrm{pr}\left(y_i^* \geq 0,\ t_i\right)^{y_i \cdot (1-d_i)} \times \\ & \prod_{i=1}^{N} \mathrm{pr}\left(y_i^* \geq 0,\ t_i^* \geq \overline{t_i}\right)^{y_i \cdot d_i}\end{aligned} \tag{3.5}$$

In appendix **A.1.** it is shown that (3.5) implies the following loglikelihood:

$$\begin{aligned}\ln\ell &= \textstyle\sum_{i=1}^{N} (1-y_i) \cdot \ln\left[1-\Phi\left(\mathbf{x}_{1i}\boldsymbol{\beta}_1\right)\right] + \\ & \textstyle\sum_{i=1}^{N} y_i \cdot (1-d_i) \ \left\{\ln\Phi\left(\frac{\mathbf{x}_{1i}\boldsymbol{\beta}_1 - \frac{\sigma_{12}}{\sigma_2^2}(t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2)}{\sqrt{(1-\rho^2)}}\right) + \right. \\ & \qquad \left. -\tfrac{1}{2}\ln 2\pi + \ln\left(\tfrac{1}{\sigma_2}\right) - \tfrac{1}{2}\left(\tfrac{t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2}{\sigma_2}\right)^2\right\} + \\ & \textstyle\sum_{i=1}^{N} y_i \cdot d_i \ \ln\Phi_2\left(\mathbf{x}_{1i}\boldsymbol{\beta}_1,\ \frac{\mathbf{x}_{2i}\boldsymbol{\beta}_2 - \overline{t_i}}{\sigma_2};\ \rho\right)\end{aligned} \tag{3.6}$$

where $\Phi(\cdot)$ and $\Phi_2(\cdot,\cdot,\rho)$ represent the univariate and bivariate standard normal c.d.f., the latter with correlation coefficient ρ.

4. Empirical results

To find out which of the 60-70 variables in the dataset have sufficient explanatory power to become significant in the estimation of the final model, I went through several steps. First, I picked out the defaulted loans, that have completed spells of survival time, from the dataset. For this subsample I calculated simple correlation coefficients for the candidate explanatory variables, both with each other and with survival time. For categoric variables I also compared mean survival times between categories. These steps gave me a preliminary impression of what variables were close substitutes. Variables that were strongly correlated were added to the set of potential covariates. For the variables with significant correlation coefficients, I estimated linear regressions and inspected the parameter estimates and coefficients of determination. For the remaining variables, I proceeded by estimating Lowess regressions.

Next, I split up the dataset into rejected and approved applications and contrasted the mean values of all candidate explanatory variables for these two subsamples. I also compared the distribution of some of the explanatory variables between subsamples, both to check if the failure to discover any difference in means could be ascribed to the presence of outliers and to see if there was any apparent gain from transforming the variable.

If no relationship between a variable and either survival time or the loan granting decision could be discovered in any of the above steps, then the variable was deleted from the set of candidate explanatory variables. This procedure led to the selection of just over 20 variables from the dataset. Among the variables that were scrapped are the number of months since the most recent change in marital status, number of houses a person owns (partially), ownership of a registered company, a large number of entries on the two most recently submitted income-tax return forms, like total taxes due, back tax, and a number of transformations of these variables. *Taxable* wealth, although likely to be positively correlated with survival time, was excluded as an explanatory variable because assets up to a value of SEK 900,000 are exempted from taxes. This makes the group of people with *taxable* wealth too small to be useful. Instead I have exploited that income from capital is taxed and therefore registered from the first krona and have created a dummy explanatory variable 'income from capital'.

A number of variables, although they *were* selected in the above procedure, are not part of the final model because they are too strongly correlated with others that measure approximately the same thing but have greater explanatory power. The numerous income measures in the dataset and *BALANCE* were eliminated in this way in favor of *INCOME* and *LIMIT*. Some other variables were omitted from the final model because the bivariate relation with the dependent variable(s) turned insignificant when estimating the model with multiple explanatory variables. Age, citizenship (Swedish, nordic, non-nordic), the number of months since immigration, the combined value of all real estate a person has (partial) ownership in, and *BALANCE/INCOME* were removed in this way.

Finally, it is worthwhile to make some remarks on the distribution of survival time. Table 4 may have created the impression that the distribution of logarithmized survival time for bad loans is more skewed than untransformed survival. A QQ normality graph (not shown here), that compares a variable's sample distribution with a normal distribution with equal mean and variance, shows, however, that the transformation reduces the skewedness of survival time and improves the match with the normal distribution slightly.

After selecting the explanatory variables, the parameters of (3.6) are estimated with the following procedure. First, I calculate starting values for $\boldsymbol{\beta}_1$ from a univariate probit on the first equation in (3.1). These are consistent although not efficient, because the covariation between ε_1 and ε_2 is not taken into account. The starting values for $\boldsymbol{\beta}_2$ and σ_2 come from a Tobit model with variable censoring bound on the survival time of the granted loans. This model implicitly assumes that $\rho = 0$. Under the restriction that $\rho = 0$, one can estimate the second equation in (3.1) separately. Because one ignores the rejected loan applications, these parameter estimates suffer from a sample selection bias and are inconsistent if $\rho \neq 0$ - which is the case here, as we will see below. In all tests of the the model with simulated data, however, these estimates were found to be close (plus minus a decimal) to the true parameter values. The iterative procedure on the full model *with* sample selection converged rather easily when using these estimates as starting values. By comparison, when I let either an OLS or a Heckman's two-step procedure generate the starting values for $\boldsymbol{\beta}_2$ and σ_2 - thus taking the sample selection effect into account while ignoring the censoring in t_i - it was more time-consuming or even impossible to find a maximum for the loglikelihood function (3.6).

With these starting values and letting $\rho^{start} = 0$, I then estimate $\boldsymbol{\beta}_2$, σ_2 and ρ simultaneously by maximizing (3.6) under the restriction that $\boldsymbol{\beta}_1 = \widehat{\boldsymbol{\beta}}_1^{probit}$. These estimates of $\boldsymbol{\beta}_2$, σ_2 and ρ are consistent and are in their turn used as starting values in the last step. Estimating $\boldsymbol{\beta}_2$, σ_2 and ρ first and then estimating $\boldsymbol{\beta}_2$, σ_2, ρ *and* $\boldsymbol{\beta}_1$ by FIML saves a lot of time compared to doing FIML directly. The FIML iterations provide consistent and efficient estimators of $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, σ_2 and ρ and a consistent estimator of the variance-covariance matrix. The FIML parameter estimates, their standard errors and t-statistics are presented in Tables 5, 6 and 7.

Table 5 contains two sets of parameter estimates for the loan granting decision: the first one from estimation as a single equation and the second from estimation together with the the survival equation. There appears to be no clear gain in efficiency in the estimate of β_1 from estimating the two equations in (3.1) simultaneously. Remember that *LOANSIZE* could not be used as an explanatory variable because no data on this variable were available for rejected applications.

Table 5: Univariate probit and full information probit MLE of β_1.
The univariate estimators come from separate estimation of the first equation in (3.1) ; the bivariate estimators come from estimation of the complete model (3.1) - (3.4).

	Univariate			Bivariate		
Variable	$\hat{\beta}_1$	std. error	t-stat.	$\hat{\beta}_1$	std. error	t-stat.
CONSTANT	-.3361	.05125	-6.56	-.3277	.05122	-6.40
MALE	-.2068	.02812	-7.36	-.1955	.02794	-7.00
MARRIED	-.2416	.02969	-8.13	-.2328	.02951	-7.89
DIVORCE	-.1859	.03953	-4.70	-.1792	.03946	-4.54
HOUSE	.1103	.02802	3.93	.1025	.02820	3.63
BIGCITY	-.2321	.02676	-8.67	-.2223	.02655	-8.37
NRQUEST	-.007228	.005114	-1.41	-.004293	.005058	-.85
ENTREPR	.5697	.06386	8.92	.5703	.06423	8.88
INCOME	.009098	.0001817	50.06	.008863	.0001823	48.63
DIFINC	-.002429	.0003505	-6.93	-.002366	.0003480	-6.80
CAPINC	-.2837	.05098	-5.56	-.2717	.04995	-5.44
ZEROLIM	-2.2529	.1062	-21.22	-2.2180	.1135	-19.54
LIMIT	-.008609	.0001870	-46.04	-.008476	.0002082	-40.71
NRLOANS	.08621	.006834	12.62	.08641	.006961	12.41
LIMUTIL	-.007465	.0004487	-16.67	-.007587	.0004491	-16.89
COAPPLIC	.1559	.03413	4.57	.1463	.03432	4.26

Critical values are 1.645, 1.96,and 2.575 for the 10, 5, and 1 percent significance levels.

The effect of most variables on the probability of obtaining a loan is as one might have expected. *INCOME* and *HOUSE* confirm their role as important factors that contribute positively, while *LIMIT, LIMUTIL* and *DIVORCE* have the traditional negative effects. More surprising are the coefficients on *MARRIED, DIFINC* and *CAPINC.* The parameter on *MARRIED* may be capturing the positive correlation between age and marriage. In preliminary regressions where age was one of the explanatory variables, it consistently had a negative effect on the probability of being granted a loan. Its parameter estimate failed to gain significance, though.

Table 6: Univariate and bivariate Tobit MLE of β_2.
The univariate estimates are computed under the hypothesis that $\rho = 0$; the bivariate estimation takes the sample selection effect into account and estimates ρ.

	Univariate			Bivariate		
Variable	$\widehat{\beta_2}$	std. error	t-stat.	$\widehat{\beta_2}$	std. error	t-stat.
CONSTANT	8.2464	.1555	53.05	9.0647	.1925	47.08
MALE	-.1060	.06085	-1.74	.02395	.05844	.41
MARRIED	.1869	.06823	2.74	.3449	.06611	5.21
DIVORCE	-.1237	.07820	-1.58	-.008730	.06936	-.13
HOUSE	.06070	.06114	.99	-.02330	.06199	-.38
BIGCITY	-.1284	.05808	-2.21	.2325	.05250	.44
NRQUEST	-1.1547	.1218	-9.48	-.9673	.1090	-8.87
ENTREPR	.1355	.1809	.75	.1623	.1595	1.02
INCOME	.03790	.04189	.90	-.2862	.05010	-5.71
DIFINC	.1469	.07490	1.96	.1846	.07856	2.35
CAPINC	-.05713	.1233	-.46	.1948	.09965	1.95
ZEROLIM	-2.2441	.4006	-5.60	-.3277	.1403	-2.34
LIMIT	.005818	.05886	.10	.5614	.04979	11.27
NRLOANS	3.2884	.2560	12.85	2.5869	.2249	11.50
LIMUTIL	-.1295	.01203	-10.76	-.1223	.01156	-10.58
LOANSIZE	-.06863	.07300	-.94	-.06995	.06974	-1.00
COAPPLIC	.5091	.1087	4.68	.3736	.1037	3.60
σ_2	.9187	.04482	20.50	1.0961	.05684	19.28
ρ	$\equiv$0.00	-	-	-0.9855	.02137	-46.11

Critical values are 1.65, 1.96,and 2.58 for the 10, 5, and 1 percent significance levels.

Table 6 compares two different estimators of $\boldsymbol{\beta}_2$ and σ_2. The parameter estimates in the first column are obtained from a Tobit model with a variable censoring threshold that ignores the sample selection effect one generates when disregarding the rejected loan applications. This is equivalent to estimating $\boldsymbol{\beta}_2$ and σ_2 in (3.1) under the hypothesis that $\rho = 0$. One is, in other words, assuming that the likelihood of a survival of a certain length is not affected in any systematic way by the inferences one can make from observing y_i and x_{1i}. If the hypothesis is true, then the parameters in the first and second equation in (3.1) can be estimated separately from each other. However, if the disturbances ε_1 and ε_2 *are* correlated, these estimators of $\boldsymbol{\beta}_2$ and σ_2 will be biased.

The second set of coefficients in Table 6 are the consistent parameters estimates of $\boldsymbol{\beta}_2$ and σ_2 obtained by estimating the complete model (3.1) – (3.3).

The purpose of comparing these two estimators is to investigate to what extent any misunderstandings about the relation between people's characteristics and financial discipline may have originated in an incorrect way of sampling data for profitability analyses by financial institutions. A comparison of the two estimators will help us determining if inconsistencies in bank lending policy may find their origin in a sample selection bias. If there is any such sample selection bias, we will also want find out whether it is also quantitatively importantant.

In the final model, all explanatory variables enter the model linearly. I have checked for the presence of non-linear effects by adding quadratic terms of all continuous variables. Their coefficients were never significant, however. Out of 16 explanatory variables, four lose or reduce their significance and three turn significant or increase their level of significance when disregarding the sample selection effect. Of the parameters for the remaining 9 variables, 4 are insignificant while the remaining 5 are significant and have identical signs in both models. So although accounting for the sample selection effect never reverses the sign of any of the coefficients, it does clearly affect their magnitude. The influence of the variable *ZEROLIM*, for example, would be badly overestimated if one did not account for the sample selection effect. A look at Tables 2 and 3 may help us understand this phenomenon. Although having no loans outstanding is rather uncommon among the granted loans, it is stronger associated with defaulting than with proper repayment behavior. However, this overlooks the fact that 15% of all rejected applicants did not have any loan yet. If rejected applications are not so much different from approved ones, then the actual impact of having a zero limit may well be much smaller than one would expect by merely looking at granted loans.[2] Similarly, *INCOME* is not significant in the column with biased estimators, whereas the consistent parameter estimate has a significantly negative coefficient. Although one should be careful not to rationalize each counter-intuitive finding, we can look for a tentative explanation. Tables 2 and 5 clearly showed that people with higher incomes are more likely to be granted a loan. This may well lead us to infer - if we disregard the rejected applicants, who have low incomes, and consider only approved ones - that income does not influence a loan's default risk. Suppose,

[2]Rejected applications will differ very little from approved ones if the lending institution grants loans to applicants on the basis of characteristics that have little impact on survival.

however, that it is actually the case that other factors than *INCOME* determine a loan's survival. Then the selection of applicants may be taking place on the basis of a negative bivariate relation between *INCOME* and defaults (see Table 3) that disappears when one controls for both the sample selection effect and the correlation with other variables. It may, for example, be the case that people with higher income also take greater risks.

It is also wortwhile to take notice of the sign of some other parameter estimates in the fourth column of Table 6. *NRQUEST* is considered to be quite good an indicator of a person's efforts to obtain additional credit and as such expected to contribute negatively to survival. Not having any loan at all, as indicated by *ZEROLIM*, is a sign of inexperience with servicing debt and has a negative effect on survival. The reverse holds for *NRLOANS* and *LIMIT*. The positive effect on survival of two granted loan evens out the negative effect of five questions. Although one might expect *LIMIT* to have a negative influence, one should keep in mind that it is merely the ceiling of the credit facility that a person disposes of. *LIMUTIL* captures the extent to which he or she actually uses it, while *LIMIT* proxies for experience with servicing debt in the same way as *NRLOANS* does.[3] A rise in income between years increases expected survival while a higher utilization degree of the available credit facility by an applicant decreases survival. Finally, it is worth commenting the value of the correlation coefficient. Its value of -.98 may create the impression that the algorithm had problems converging. In extensive tests of the model with different sets of explanatory variables and varying sample sizes, rho took values between -.55 and -.98. In tests with the bivariate probit model, the final parameter estimates of which are reported in tables 5 and 7, ρ ranged from approximately -.65 to -.93. Boyes et al. report -.35. As is the case with most models with limited dependent variables (see Bermann [4]), the computations for the tobit and probit models did not converge for some configurations of explanatory variables. When the computations broke down, divergence always took place after relatively few iterations, however, with rho breaking its constraint before any of the other parameters had stabilized around a final value. In the estimation of the final model, all parameters settled down around their final values rather quickly.

[3] Strong correlation between the variables *BALANCE* and *LIMIT* tended to create numerical problems when trying to use both as explanatory variables. Some test regressions indicated that *LIMIT* and *BALANCE* have opposite effects on *SURVIVAL*, the former a positive and the latter a negative. The coefficient on *LIMIT* in tables 5 and 6 is approximately equal to the net effect of *LIMIT* minus *BALANCE*.

Table 7: Bivariate probit and Tobit MLE with sample selection.
The probit estimator α_2 is the parameter in the equation that models the probability of a default; the tobit estimator β_2 comes from (3.1) - (3.4). Both estimators take the sample selection effect into account.

	Bivariate probit			Tobit with sample selection		
Variable	$\widehat{\alpha}_2$	std. error	t-stat.	$\widehat{\beta}_2$	std. error	t-stat.
CONSTANT	2.4546	.1132	21.69	9.0647	.1925	47.08
MALE	-.02338	.05916	-.40	.02395	.05844	.41
MARRIED	.2527	.06563	3.85	.3449	.06611	5.22
DIVORCE	-.07018	.07427	-.95	-.008730	.06936	-.13
HOUSE	-.01004	.06001	-.17	-.02330	.06199	-.38
BIGCITY	-.04068	.05480	-.74	.02325	.05250	.44
NRQUEST	-.1036	.01026	-10.10	-.9673	.1090	-8.87
ENTREPR	.1347	.1565	.86	.1623	.1595	1.02
INCOME	-.002266	.0005015	-4.52	-.2862	.05010	-5.71
DIFINC	.002049	.0007326	2.80	.1846	.07856	2.35
CAPINC	.1477	.1265	1.17	.1948	.09965	1.95
ZEROLIM	-.6796	.2982	-2.28	-.3277	.1403	-2.34
LIMIT	.004822	.0005693	8.47	.5614	.04979	11.27
NRLOANS	.2704	.01947	13.89	2.5869	.2249	11.50
LIMUTIL	-.01208	.0009290	-13.00	-.1223	.01156	-10.58
LOANSIZE	-.006581	.006850	-.96	-.06995	.06973	-1.00
COAPPLIC	.4189	.09789	4.28	.3736	.1037	3.60
σ_2	-	-	-	1.0961	.05684	15.68
ρ	-.9110	.05624	−16.20	-0.9855	.02137	-41.80

Critical values are 1.65, 1.96,and 2.58 for the 10, 5, and 1 percent significance levels.

Overall, the conclusion one can draw from the results in Table 6 is that ignoring rejected applicants in an analysis of the duration of loans leads to large biases in the parameter estimates. Although the signs of parameters are never reversed, some of the variables that are generally thought to be among the most important determinants of creditworthiness, like income, outstanding loans, and income from assets, appear to have no relationship whatsoever with survival time when disregarding the sample selection effect. Such misunderstandings may well be the origin of inefficient lending policies at financial institutions.

Finally, in Table 7, I present parameters of the bivariate probit model, as presented in Boyes et al. [5] but re-estimated with the data used in Table 6. The first observation one can make when comparing the probit parameters that determine the probability of a loan *not* defaulting ($\boldsymbol{\alpha}_2$) with those that determine logged survival time ($\boldsymbol{\beta}_2$) is that each variable has coefficients with identical signs in both models.[4] [5] Variables that increase (decrease) the probability of a default thus also decrease (increase) the expected survival time of a loan and thus - since survival time proxies for return - reduce (raise) its expected return.

Moreover, variables like *MALE, DIVORCE, HOUSE, BIGCITY* and *ENTREPR* that are given significant weights in the loan granting decision actually do not affect default risk and survival. *NRQUEST* on the other hand does have a significant effect on survival but is not given any weight in the decision process. For variables like *MARRIED, INCOME, DIFINC, CAPINC* and *LIMIT*, the parameter estimates for the loan granting decision and the probability of a loan not defaulting have opposite signs. These variables are thus used in such a way by the bank in the loan granting process that they increase (decrease) the likelihood of a loan being granted although they in fact increase (decrease) the risk of default. Because the parameters in the survival equation have the same sign as the bivariate probit parameters, these variables also reduce (raise) expected survival and return on the loan. In other words: if the bank is not minimizing default risk in its loan granting policy, it is not doing so because loans with higher default risk have higher expected returns. Moreover, the negative values of ρ in Table 7 indicate that any non-systematic propensity to grant loans is associated with shorter survival times and higher default risk. This is consistent with the above observation that the bank does not appear to trade off risk against return. Rather, the loan

[4]As a matter of fact, all variables with significant parameters in the survival equation of the Tobit model with sample selection also have significant coeffficients in the 'probability that loan doesn't default' equation of the bivariate probit model. The reverse, however, does not hold! Variables that would have been sigificant in a bivariate probit model but are not in the Tobit model like (3.1) have therefore been omitted.

[5]The bivariate probit model implicitly assumes that loans which are still good, will not turn bad later on.

granting policy appears to be inefficient and contain non-systematic components that are strongly negatively correlated with survival.

In the estimation of $\boldsymbol{\alpha}_2$ and $\boldsymbol{\beta}_2$, we controlled for the size of the loan. Table 7 shows that neither default risk nor survival is affected by *LOANSIZE*. This has two implications. First, bigger loans do not carry greater default risk nor do they imply either shorter or longer survivals. Secondly, greater default risk is associated with shorter survival, not with *longer* survival as Boyes et al. suggest. Riskier loans thus have *lower* expected returns. The lending institution that we study, however, always extended loans with size equal to the amount applied for - independent of the risk associated with the applicant - and was thus indifferent between alternative loan sizes. Such behavior is not consistent with the hypothesis that the financial institution is trading off higher default risk against higher expected earnings (that supposedly come with bigger loans). The lending institution's behavior is neither compatible with return maximization due to a (previously assumed) positive relation between loan size and rate of return nor is it in agreement with the maximization of survival time in general. If the lending institution is minimizing default risk, it would be strictly better off granting either nothing at all or the maximum amount possible for the type of loan in question. After all, granting a bigger loan does not increase risk but it raises the revenue. For the same reason, the lending institution would also be better off with this corner solution policy if it is maximizing survival time. It raises revenues without changing the riskiness.

Model (3.1) – (3.3) can now be used to examine the lending policy of the bank. Loans with the longest expected survival time also have the greatest gross returns. The estimated model from Tables 6 and 7 can be used to calculate the expected survival time for all loan applicants. In Table 8, we show the outcome from an experiment where all loan applications are ranked and approved according to their predicted survival time $E\left[t_i^* | \mathbf{x}_{2i}\right]$. The first column in the table shows that only 3,156 out of the 6,438 granted loans (49 percent) would have been approved if selection had taken place according to expected survival time. This strongly suggests that the current lending policy is not efficient, because it selects loans with shorter survivals. These results could, however, also be indicative of an inability by the empirical model to separate good from bad loans. If we look at some other ways to evaluate the reliability of the results in Table 8, then this seems to confirm that the model is an effective tool to evaluate loan applicants. The loans that would be granted with a survival time selection criterion contain

Figure 8: Selecting applications by predicted survival times.
Entries in the 2×2 table show how many applicants that were granted a loan would even be so if applicants were ranked according to predicted survival time $E[t_i^* | \mathbf{x}_{2i}]$ and the same number of loans were granted as in the data set.

		actual			# failed loans
		granted	rejected	sum	among predicted
predict	granted	3,156	3,282	6,438	39
	rejected	3,282	3,617	6,899	349
	sum	6,438	6,899	13,337	388

merely 39 of the 338 bad loans that are present in the data set. The predicted (logarithm of) survival time for defaulted loans was 10 percent shorter than for all granted loans and all other applicants.

5. Discussion

Traditionally, the objective of credit scoring models used by financial institutions is to minimize default rates or the number of loans that is incorrectly classified as defaulted or non-defaulted. From a profit or utility maximizing perspective, however, it is not only important to know *if* but also *when* a loan will default. Traditional credit scoring models predict default risk and therefore fail to take into account this multiperiod nature of loans contracts. To allow for a more realistic evaluation of the return on a loan, a Tobit model with sample selection and variable censoring thresholds has been constructed and estimated in this paper. This model is shown to be a useful tool to predict the expected survival time on a loan to any kind of applicant. A comparison with a nested model that disregards rejected applications - as has been common in studies of creditworthiness - shows that ignoring the sample selection effect leads to a large bias in the parameters estimates.

From the empirical results we gain several insights. They confirm the findings in Boyes et al. that financial institutions' lending policies are not compatible with default risk minimization. At the same time, though, the results also conflict with the notion that the financial institution would be trading off higher default risk against higher returns. The lending policy does not favor people that survive longer and thus have a higher rate of return. Firstly, some of the variables that increase (decrease) applicants' odds of obtaining a loan reduce (raise) the expected survival time (and thus return) on a loan and raise (reduce) the likelihood of a default. Secondly, the financial institution is found to be indifferent between loans of different sizes, given its expected survival time. There is thus no evidence of banks' behaving in a way that is consistent with profit-maximization. This impression is strengthened by an experiment in which expected survival times are calculated for all loan applicants, including those who were rejected. In that experiment, only 49 percent of the actually granted loans would have been granted if a survival time criterion had been handled. Moreover, lending to 349 of the 388 defaulted applicants in the sample would have been avoided.

Lending behavior by banks must thus be either a symptom of an inefficient lending policy or the result of some other type of optimizing behavior. The current level of technology in the banking industry generally does not yet allow for the pursuit of composite objectives such as the return on a range of products or revenues from several sources of income. But banks may, for example, be maximizing some other objective like provision income from the turnover on credit cards, the number of customers or lending volume subject to a minimum return constraint. None of these suggestions agree, however, with the practices reported to us by the lending institution who provided our data. Rather, the results bear strong evidence of a lending institution that has attempted to minimize risk or maximize a simple return function without success.

Censoring of data, as is the case with the non-defaulted loans in the sample, increases the uncertainty in the parameter estimates of the survival function. Appropriate changes in sampling methods can improve their accuracy. A longer period of observation of the loans would reduce the regression error. Even better would be to set up an experiment where a predetermined number of applicants is granted a loan without consideration of their personal characteristics. If each loan is monitored at least at termination of the contract then separate survival time functions for good and bad loans can be estimated. An ideal model of bank profitability or bank efficiency will have to be built on time series data for fees, interest payments and amortizations on loans, personal characteristics, macroeconomic indicators and all costs involved.

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A. Likelihood function and gradient

A.1. Likelihood function

The likelihood function

$$\begin{aligned}\ell \;=\; & \prod_{i=1}^{N} \operatorname{pr}\left(y_i^* < 0\right)^{(1-y_i)} \cdot \prod_{i=1}^{N} \operatorname{pr}\left(y_i^* \geq 0,\ t_i\right)^{y_i \cdot (1-d_i)} \times \\ & \prod_{i=1}^{N} \operatorname{pr}\left(y_i^* \geq 0,\ t_i^* \geq \overline{t_i}\right)^{y_i \cdot d_i}\end{aligned} \tag{A.1}$$

implies that

$$\begin{aligned}\ln \ell \;=\; & \textstyle\sum_{i=1}^{N} (1-y_i) \cdot \ln\left[\operatorname{pr}\left(\varepsilon_{1i} < -\mathbf{x}_{1i}\boldsymbol{\beta}_1\right)\right] + \\ & \textstyle\sum_{i=1}^{N} y_i \cdot (1-d_i)\ \ln\ \left[\operatorname{pr}\left(\varepsilon_{1i} \geq -\mathbf{x}_{1i}\boldsymbol{\beta}_1 \cap \varepsilon_{2i} = t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2\right)\right] + \\ & \textstyle\sum_{i=1}^{N} y_i \cdot d_i \ln\left[\operatorname{pr}\left(\varepsilon_{1i} \geq -\mathbf{x}_{1i}\boldsymbol{\beta}_1 \cap \varepsilon_{2i} \geq \overline{t_i} - \mathbf{x}_{2i}\boldsymbol{\beta}_2\right)\right]\end{aligned} \tag{A.2}$$

If we use that $\varepsilon_{1i}|\varepsilon_{2i} \sim N\left(\frac{\sigma_{12}}{\sigma_2^2}\varepsilon_{2i},\ (1-\rho^2)\right)$ for $\begin{pmatrix}\varepsilon_{1i}\\ \varepsilon_{2i}\end{pmatrix} \sim \mathbf{N}(\mathbf{0},\ \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma} = \begin{pmatrix}1 & \sigma_{12}\\ \sigma_{12} & \sigma_2^2\end{pmatrix}$, then we can simplify by expressing the second line in terms of a univariate normal cdf and - pdf. As a result

$$\begin{aligned}& \operatorname{pr}\left(\varepsilon_{1i} \geq -\mathbf{x}_{1i}\boldsymbol{\beta}_1 \cap \varepsilon_{2i} = t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2\right) \Leftrightarrow \\ & \operatorname{pr}\left(\varepsilon_{1i} \geq -\mathbf{x}_{1i}\boldsymbol{\beta}_1 \middle|\ \varepsilon_{2i} = t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2\right) \operatorname{pr}\left(\ \varepsilon_{2i} = t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2\right) \Leftrightarrow \\ & \operatorname{pr}\left(\varepsilon_{1i} < \mathbf{x}_{1i}\boldsymbol{\beta}_1 \middle|\ \varepsilon_{2i} = t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2\right) \operatorname{pr}\left(\ \varepsilon_{2i} = t_i - \mathbf{x}_{2i}\boldsymbol{\beta}_2\right) \Leftrightarrow \\ & \Phi\left(\frac{\mathbf{x}_{1i}\beta_1 - \frac{\sigma_{12}}{\sigma_2^2}(t_i - \mathbf{x}_{2i}\beta_2)}{\sqrt{(1-\rho^2)}}\right) \frac{1}{\sigma_2}\phi\left(\ \frac{t_i - \mathbf{x}_{2i}\beta_2}{\sigma_2}\right)\end{aligned} \tag{A.3}$$

Taking natural logarithms we get

$$\ln \Phi\left(\frac{\mathbf{x}_{1i}\beta_1 - \frac{\sigma_{12}}{\sigma_2^2}(t_i - \mathbf{x}_{2i}\beta_2)}{\sqrt{(1-\rho^2)}}\right) - \frac{1}{2}\ln 2\pi + \ln\left(\frac{1}{\sigma_2}\right) - \frac{1}{2}\left(\ \frac{t_i - \mathbf{x}_{2i}\beta_2}{\sigma_2}\right)^2 \tag{A.4}$$

The last line in $(A.2)$ can be rewritten in terms of a bivariate normal cdf:

$$\begin{aligned}& \operatorname{pr}\left(\varepsilon_{1i} \geq -\mathbf{x}_{1i}\boldsymbol{\beta}_1 \cap \frac{\varepsilon_{2i}}{\sigma_2} \geq \frac{\overline{t_i} - \mathbf{x}_{2i}\beta_2}{\sigma_2}\right) \Leftrightarrow {}^{6} \\ & \Phi_2\left(\mathbf{x}_{1i}\boldsymbol{\beta}_1,\ \frac{\mathbf{x}_{2i}\beta_2 - \overline{t_i}}{\sigma_2};\ \rho\right)\end{aligned} \tag{A.5}$$

[6] See Greene (1993) p.661 for a summary of results on the bivariate normal cdf.

Consequently, the loglikelihood function can be written as

$$
\begin{aligned}
\ln\ell \;=\; & \sum_{i=1}^{N}(1-y_i)\cdot\ln\left[1-\Phi\left(\mathbf{x}_{1i}\boldsymbol{\beta}_1\right)\right]+ \\
& \sum_{i=1}^{N} y_i\cdot(1-d_i)\;\left\{\ln\Phi\left(\frac{\mathbf{x}_{1i}\boldsymbol{\beta}_1-\frac{\sigma_{12}}{\sigma_2^2}(t_i-\mathbf{x}_{2i}\boldsymbol{\beta}_2)}{\sqrt{(1-\rho^2)}}\right)+\right. \\
& \qquad\qquad \left.-\tfrac{1}{2}\ln 2\pi+\ln\left(\tfrac{1}{\sigma_2}\right)-\tfrac{1}{2}\left(\tfrac{t_i-\mathbf{x}_{2i}\boldsymbol{\beta}_2}{\sigma_2}\right)^2\right\}+ \\
& \sum_{i=1}^{N} y_i\cdot d_i\;\ln\Phi_2\left(\mathbf{x}_{1i}\boldsymbol{\beta}_1,\;\tfrac{\mathbf{x}_{2i}\boldsymbol{\beta}_2-\overline{t_i}}{\sigma_2};\;\rho\right)
\end{aligned}
\tag{A.6}
$$

After further simplification, by setting

$$
\begin{aligned}
\boldsymbol{\alpha}_2 &= \boldsymbol{\beta}_2/\sigma_2 \\
lng2 &= \ln\left(\tfrac{1}{\sigma_2}\right) \\
e_{2i} &= -\left(\tfrac{t_i-\mathbf{x}_{2i}\boldsymbol{\beta}_2}{\sigma_2}\right) \\
&= \mathbf{x}_{2i}\boldsymbol{\alpha}_2-t_i\cdot\exp\left(lng2\right) \\
\overline{e_{2i}} &= \mathbf{x}_{2i}\boldsymbol{\alpha}_2-\overline{t_i}\cdot\exp\left(lng2\right) \\
\delta &= \tfrac{1}{(1-\rho^2)^{1/2}}
\end{aligned}
$$

we get that

$$
\begin{aligned}
\ln\ell \;=\; & \sum_{i=1}^{N}(1-y_i)\cdot\ln\left[1-\Phi\left(\mathbf{x}_{1i}\boldsymbol{\beta}_1\right)\right]+ \\
& \sum_{i=1}^{N} y_i\cdot(1-d_i)\;\left\{\ln\Phi\left(\delta\left[\mathbf{x}_{1i}\boldsymbol{\beta}_1-\;\rho\cdot e_{2i}\right]\right)+\right. \\
& \left.-\tfrac{1}{2}\ln 2\pi+lng2-\tfrac{1}{2}e_{2i}^2\right\}+ \\
& \sum_{i=1}^{N} y_i\cdot d_i\;\ln\Phi_2\left(\mathbf{x}_{1i}\boldsymbol{\beta}_1,\;\overline{e_{2i}};\;\rho\right)
\end{aligned}
\tag{A.7}
$$

The parameters with respect to which we maximize $\ln\ell$ are $\boldsymbol{\beta}_1, \boldsymbol{\alpha}_2$, $lng2$, and ρ.

A.2. Gradients

The gradients corresponding to each observation in the above loglikelihood function are:

$$
\begin{aligned}
\frac{\partial \ln \ell_i}{\partial \beta_1} &= (1-y_i)\frac{-\phi(\mathbf{x}_{1i}\beta_1)}{1-\Phi(\mathbf{x}_{1i}\beta_1)}\cdot \mathbf{x}_{1i}+ \\
& y_i\,(1-d_i)\cdot\frac{\phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}{\Phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}\cdot \delta\mathbf{x}_{1i}+ \\
& y_i\cdot d_i\,\frac{\phi(\mathbf{x}_{1i}\beta_1)\Phi\left(\frac{\overline{e_{2i}}-\rho\mathbf{x}_{1i}\beta_1}{(1-\rho^2)^{1/2}}\right)}{\Phi_2(\mathbf{x}_{1i}\beta_1,\ \overline{e_{2i}};\ \rho)}\cdot\mathbf{x}_{1i} \\
\frac{\partial \ln \ell_i}{\partial \alpha_2} &= y_i\cdot(1-d_i)\ \left\{\frac{\phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}{\Phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}\delta\,(-\rho)\,\mathbf{x}_{2i}-e_{2i}\mathbf{x}_{2i}\right\}+ \\
& y_i\cdot d_i\frac{\phi(\overline{e_{2i}})\Phi\left(\frac{\mathbf{x}_{1i}\beta_1-\rho\overline{e_{2i}}}{(1-\rho^2)^{1/2}}\right)}{\Phi_2(\mathbf{x}_{1i}\beta_1,\ \overline{e_{2i}};\ \rho)}\cdot\mathbf{x}_{2i} \\
\frac{\partial \ln \ell_i}{\partial lng2} &= y_i\cdot(1-d_i)\left\{\frac{\phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}{\Phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}\delta\,(-\rho)\,(-t_i)\exp(lng2)+\right. \\
& \qquad\qquad \left. +1+e_{2i}\cdot t_i\cdot\exp(lng2)\right\}+ \\
& y_i\cdot d_i\left\{\frac{\phi(\overline{e_{2i}})\Phi\left(\frac{\mathbf{x}_{1i}\beta_1-\rho\overline{e_{2i}}}{(1-\rho^2)^{1/2}}\right)}{\Phi_2(\mathbf{x}_{1i}\beta_1,\ \overline{e_{2i}};\ \rho)}\cdot\left(-\overline{t_i}\cdot\exp(lng2)\right)\right\} \\
\frac{\partial \ln \ell_i}{\partial \rho} &= y_i\cdot(1-d_i)\ \frac{\phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}{\Phi(\delta[\mathbf{x}_{1i}\beta_1-\rho e_{2i}])}\left(-\delta e_{2i}+\rho\delta^3\left[\mathbf{x}_{1i}\boldsymbol{\beta}_1-\ \rho e_{2i}\right]\right)+ \\
& y_i\cdot d_i\cdot\frac{\phi_2(\mathbf{x}_{1i}\beta_1,\overline{e_{2i}};\ \rho)}{\Phi_2(\mathbf{x}_{1i}\beta_1,\ \overline{e_{2i}};\ \rho)}
\end{aligned}
\tag{A.8}
$$

After convergence of the iterative procedure, a consistent estimator of the variance-covariance matrix is obtained by applying the delta method.[7] If we define $\begin{pmatrix}\boldsymbol{\beta}_1\\ \boldsymbol{\beta}_2\\ \sigma_2\\ \rho\end{pmatrix}=\begin{pmatrix}\boldsymbol{\beta}_1\\ \boldsymbol{\alpha}_2\left[\exp(lng2)\right]^{-1}\\ \left[\exp(lng2)\right]^{-1}\\ \rho\end{pmatrix}\equiv \mathrm{f}(\boldsymbol{\theta})$, where $\boldsymbol{\theta}=\begin{pmatrix}\boldsymbol{\beta}_1\\ \boldsymbol{\alpha}_2\\ lng2\\ \rho\end{pmatrix}$, then

$$
\boldsymbol{\Gamma}\equiv\frac{\partial \mathrm{f}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}'}=\begin{bmatrix}
\mathbf{I}_{K_1} & \mathbf{0}_{\mathbf{K}_1\times\mathbf{K}_2} & \mathbf{0}_{\mathbf{K}_1\times 1} & \mathbf{0}_{\mathbf{K}_1\times 1}\\
\mathbf{0}_{\mathbf{K}_2\times\mathbf{K}_1} & \frac{\mathbf{I}_{K_2}}{[\exp(\mathrm{lng2})]} & \frac{-\boldsymbol{\alpha}_2}{[\exp(\mathrm{lng2})]} & \mathbf{0}_{\mathbf{K}_2\times 1}\\
\mathbf{0}_{1\times\mathbf{K}_1} & \mathbf{0}_{1\times\mathbf{K}_2} & \frac{-1}{[\exp(\mathrm{lng2})]} & 0\\
\mathbf{0}_{1\times\mathbf{K}_1} & \mathbf{0}_{1\times\mathbf{K}_2} & 0 & 1
\end{bmatrix}
\tag{A.9}
$$

In some of the iterations, I also made the additional transformation $\rho=\frac{\exp x-1}{\exp x+1}$ to assure that $0\leq\rho\leq 1$. In those cases the gradient $\partial\ln\ell/\partial\rho$ needs to be

[7] A description of the delta method is provided in Greene [9], pp. 297.

multiplied by the term $\partial\rho/\partial x = \frac{2\exp x}{(\exp x\ +1)^2}$ while the 4th diagonal element in $\mathbf{\Gamma}$ needs to be set equal to $\partial\rho/\partial x$.

Chapter 5

Essay 3

Duration of consumer loans and bank lending policy: dormancy versus default risk

Duration of consumer loans and bank lending policy: dormancy versus default risk*

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Abstract

A bank that lends money to a household faces two types of risk. Most commonly mentioned is the risk of a default. Hardly ever referred to is the risk of an early redemption of the loan - leading to dormancy. In this paper, we model consumer loans' transition from an active to a dormant state. For this purpose, we use data on 4,733 individuals who were granted credit by a Swedish lending institution between September 1993 and August 1995 and estimate three duration models. We analyze the factors that determine the time to maturity on consumer loans and investigate the ability of each model to match the maturities observed in the data. For the preferred model, we derive the distribution of conditional expected durations of loans. The analysis enables us to draw some conclusions about the efficiency of bank lending policy. Using a semi-parametric model we show how a duration model can be used to evaluate loan applicants by calculating their expected profits.

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1. Introduction

Anyone applying for a loan at a bank can count on being subjected to one of the many procedures that exist to assess creditworthiness. Some credit institutions continue to analyze applicants' personal characteristics in subjective, non-formalized ways while others use so called credit scoring models. Their varying degrees of sophistication in spite, credit scoring models share a common objective: to separate loan applicants that are expected to pay back their debts from those who are likely to fall into arrears or go bankrupt. Altman, Avery, Eisenbeis and Sinkey [1] offers a good review of this literature.[1]

Unfortunately, credit scoring models leave much room for subjective factors in the loan approval process. By ranking customers according to predicted default probabilities, banks can minimize the expected default or misclassification rate subject to some exogenous acceptance rule. However, this is a far cry from solving the ultimate problem of profit or utility maximization. Scoring models make it possible for banks to predict bankruptcy, but require them to resort to ad-hoc methods to forecast profitability. Boyes, Hoffman and Low [8] and Jacobson and Roszbach [20] provide empirical evidence that confirms the authenticity and importance of this dilemma. In two independent studies of the bank credit scoring problem, they find that loan granting policies are consistent with neither default risk - nor loss rate minimization.

The quantitative importance of consumer credit for financial institutions in Sweden may be illustrated by the fact that it makes up 25 percent of total lending, excluding residential loans, to the public.[2] Even from the household perspective, consumer credit has come to play an increasingly important role as a means to maintain consumption at a level consistent with permanent income. Total lending, excluding residential loans, by banks and finance companies to Swedish households amounted to SEK 207 bn., or SEK 22.698 per capita, by the end of 1996. This is the equivalent of 12 percent of Swedish GDP or 22.7 percent of total private consumption. These numbers demonstrate that the process by which a credit institution decides to grant or refuse a loan can greatly affect households' ability to smooth consumption and thereby their welfare. Lending policy is thus of interest not merely because of its implications for bank profitability, but also

[1] The most commonly used methods are discriminant analysis and logistic regression. Recent studies have even applied k-nearest-neighborhood [17] and count data models [16], classification trees and neural networks [3].

[2] If one includes residential loans in total lending, this figure drops to 11 percent.

because of its implications for households' welfare. Nevertheless, credit rationing remains a common phenomenon in financial markets.

The expected return on a loan depends on several factors such as the interest rate, the amortization scheme, fees paid by the customer, fixed and variable costs incurred by the lender, the probability of a default, and the expected loss on the principal in case of a default. Except for fluctuations in the rate of interest, that we abstract from in this paper, a bank thus faces two types of risk when lending money to a household. Most commonly mentioned is the risk of a default. Hardly ever referred to is the risk of an early redemption of the loan, leading to dormancy. Since the provision and administration of credit involves mostly fixed costs while revenues are variable and spread out over time, loans with short maturities can result in losses for financial institutions, despite the fact that they never default. Moreover, because defaults generally occur after a period of regular installments and interest payments, the discounted loss on the average bad loan exceeds the profit on the average good loan by only little. In our dataset, the discounted loss on an averagely sized bad loan with the average duration to default is -36.7 percent compared to a return of 8.1 percent on a good loan. Combining these figures with the fact that defaulted loans generally make up only one percent of the portfolio for this type of loan, it appears a matter of course that dormancy, rather than default risk, is the economically important phenomenon to study.

The status of dormancy is a condition that every credit account sooner or later will arive at. It is therefore natural to study this phenomenon using survival, or duration, models. The classical application of duration models in economics is the analysis of unemployment survival. Kiefer [21] provides an accessible introduction to this class of models. Carling, Edin, Harkman and Holmlund [10] is a recent contribution in the area of labor market economics. Other examples of economic applications of duration analysis are Diebold and Rudebusch [15], who study duration dependence of expansions and contractions in the business cycle, and Baek and Bandopadhyaya [4], who analyze the factors that affect the length of commercial bank debt rescheduling negotiations. Bandopadhyaya [5] and Lane et al. [23] study bank failure.

In this paper, we model consumer loans' transition from an active to a dormant status and analyze the factors that determine the time to maturity. We also derive the distribution of conditional expected durations of loans in order to examine bank lending policy. In Section 2, we present the data set and its sources. The data consists of 4,733 individuals who were granted a consumer loan by a Swedish lending institution between September 1993 and August 1995. When monitored in October 1996, the loans had either been amortized in full or continued to pay regular installments and interest. Except for the duration of each loan, the data set contains detailed information on a range of demographic, financial and credit history variables for each borrower. This information has been extracted from a number of registers by using the personal number that every resident of Sweden has. In Section 3, we start by analyzing the duration of loans until failure. Default risk has traditionally been modelled by discrete choice models, letting the outcome variable be failure or not. To create a uniform framework for analyzing bad and good loans and to contrast the implications of the dormancy models, we also investigate the determinants of the time to failure. Next, we model the transition from activity to dormancy and relate this transition to the characteristics of the loan applicant. At the end of the section we address a number of conceptual problems that arise in modelling consumer credits by econometric duration models, as well as practical problems in revealing the functional relationship between the transition and the covariates. Finally, using the predicted time to dormancy we calculate the expected return on each loan and compare it with the revenue on a benchmark (break-even) loan. Section 4 concludes the paper with a discussion of the results.

2. Data

The data set consists of all 4,733 individuals who were granted a loan at a major Swedish lending institution between September 1993 and August 1995. When monitored on October 6, 1996, the loans were either still paying regular installments and interest or had been amortized completely. All loans were granted in stores where potential customers applied for instant credit to finance the purchase of a consumer good. They range in value between 3,000 and 30,000 Swedish kronor (approximately US$ 375 - 3750). The lending institution's policy was that no loans exceeding 30,000 kronor were supplied.

The evaluation of each application took place in the following way. First, the store phoned to the lending institution to get an approval or a rejection. The lending institution then analyzed the applicant with the help of a database with personal characteristics and credit history variables to which it has on-line access. The database is maintained by Upplysningscentralen AB, the leading Swedish credit bureau which is jointly owned by all Swedish banks and lending institutions. When approval was given, the store's salesman filled out a loan contract and submitted it to the lending institution. The loans are revolving and administered by the lending institution as any other credit facility. They are provided in the form of a credit card that can only be used in a specific store. The loan is free of interest during the month of application plus the following 2 months. After the interest free period, a payment of at least 5 percent of the outstanding debt is required each month. However, since the loan is revolving, there is no predetermined maturity of the loan. Earnings on the loan come from four sources: a payment by the store that is related to total amount of loans granted to its customers during some time-period, a monthly invoice fee of 17 kronor, and interest on the balance outstanding on the card.

We classify a loan as 'bad' once it has been forwarded to a debt-collecting agency. One can think of several different definitions. One alternative is, for example, 'all customers who have received one, two or three reminders because of delayed payment'. However, unlike 'being forwarded to debt-collecting agency", one, two or three reminders were all transient states in the register of the financial instititution. Once customers returned to the agreed-upon repayment scheme, the number of reminders was reset to zero. Such a property is rather undesirable if one needs to determine unambiguously which loans have defaulted and which have not.

Dormancy is defined as the state into which loans move when they are terminated or reach a debt balance below SEK 200 ($\approx$US$ 25). This definition may not appear completely appropriate for loans that are revolving. Due to the way in which the loans are administered, customers hardly ever reach a zero balance in the bank's computer system. A customer that makes his final payment on a loan, but sends away his money transfer a few days too late is likely to end up with a non-zero balance on his account. If the supposedly repaid loan is neglected and not formally terminated, accrued interest and automatically added late payment fees can quickly add up to 100-200 kronor. By classifying loans with a balance under SEK 200 as dormant, we thus make sure to assign the loans that are not active anymore but were never formally terminated to their correct group. Moreover, because it is exceptional for the customers in this sample to utilize the credit facility more than once, there is only minimal risk of incorrectly classifying active loans as dormant. This can be explained by the continuous supply of loans with an interest rate free period up front. In particular, because the loans in our sample do not involve any fixed cost for the customer, terminating the contract after amortization of the loan is obviously a cost minimizing strategy. Note that people cannot take a new interest free loan elsewhere after three months to amortize the old loan that is about to become interest bearing. To be eligible for a loan with an interest free period one must purchase a good at a store that cooperates with a bank.

Because the default rate on this type of loans is close to 1 percent (53 out of 4,786 observations), the analysis of the time to default requires a larger sample. For this purpose, we obtained a sample containing 1,103 defaulted loans with somewhat different payment conditions than the 'good' loans. These loans have up to 12 months of interest exemption, but pay a contract fee of approximately 250 kronor. Intuitively, the most natural thing to do would be to analyze dormancy with loans from the same sample. Unfortunately, because dormancy generally comes about after completion of the interest free period, a long lasting interest exemption makes it difficult to separate the economically induced duration process for good loans from the contractually determined minimum survival. The effect of interest exemptions on the behavior of people that renege on their committments is negligable, however. The time to default *can* therefore be studied with this larger sample. Using a different sample for the estimation of a model of the time-to-default is not ideal. The only purpose of the model for the time to default is, however, to contrast its properties with those of the process that governs dormancy, not to forecast. The main focus of our study is on modelling dormancy.

For this study, the lending institution provided us with a data file with the personal number of each applicant, the date on which the application was submitted, the size of the loan that was granted, the status of each loan (good, bad or active) on October 9, 1996, and the date on which defaulted and dormant loans ended their 'active' status. Upplysningscentralen provided the information that was available on each applicant at the time of application and which the financial institution accessed for *its* evaluation. By exploiting the unique personal number that each resident of Sweden has, the credit bureau was able to merge these two data sets. Before handing over the combined data for analysis, the personal numbers were removed. The database included publicly available, governmentally supplied information such as sex, citizenship, marital status, postal code, taxable income, taxable wealth, house ownership, and variables reported by Swedish banks like the total number of inquiries made about an individual, the number of unsecured loans and the total amount of unsecured loans. Table 1 contains definitions of all variables that are used in the analysis described in Sections 3. Table 2 provides some descriptive statistics.

A number of the variables in the dataset have not been used in the final estimation of the model described in Section 3. Among these are the number of

Table 1: Definition of variables.

Variable	Definition
PERSONAL	
AGE	age of applicant (in years)
MALE	dummy, takes value 1 if applicant is male
MARRIED	dummy, takes value 1 if appl. is married but not a separated woman
SEPARATE	dummy, takes value 1 if appl. is a widow(er) or a separated woman
NORDIC	dummy, takes value 1 if appl. is Nordic but not Swedish
FOREIGN	dummy, takes value 1 if appl. is non-Nordic
FINANCIAL	
HOUSE	dummy, takes value 1 if appl. owns (part of) a house
HOUSEVAL	value of all real estate ($\times$10k SEK), weighted by appl. owned share (log.)
ENTREPR	dummy, equals 1 if appl. has taxable income from a business
INCOME	annual taxable income from wages ($\times$100 SEK, logarithmized)
DUMMY180	dummy, takes value 1 if appl. has INCOME under 180k SEK
INCOM180	DUMMY180 * INCOME
DIFINC	INCOME_t - INCOME_{t-1} ($\times$100 SEK, not logarithmized)
CAPINC	dummy, takes value 1 if appl. has taxable income from capital
CREDIT	
NRQUEST	ln (1 + number of requests for information on the appl. that the credit agency received during the last 36 months)
LIMIT	collateral-free credit facilities already outstanding ($\times$100 SEK, log.)
ZEROLIM	dummy, equals 1 if appl. has no collateral-free loans outstanding
NRLOANS	ln (1 + number of collateral-free loans registered)
LIMUTIL	percentage of limit that is actually being utilized
BALINC	dummy {INCOME > 0} * ln(100 * BALANCE / INCOME)
BALINCSQ	BALINC^2
BALANCE	total collateral free credit facilities actually utilized
COAPPLIC	dummy, takes value 1 if appl. has a guarantor
LOANSIZE	amount of credit granted ($\times$ 1000 SEK)
LIMBAL	LIMIT-BALANCE
DUMMY20	dummy, takes value 1 if LIMBAL > 20k SEK
LIMBAL20	LIMBAL * DUM20

Table 2: Descriptive statistics for granted loans.

Variable	Mean	Min	Q1	Median	Q3	Max
PERSONAL						
AGE	44.00	20	35	44	52	81
MALE	.70	0				1
MARRIED	.57	0				1
DIVORCE	.03	0				1
NORDIC	.02	0				1
FOREIGN	.01	0				1
FINANCIAL						
HOUSE	.45	0				1
HOUSEVAL	3.47	0	0	5.24	6.01	11.21
ENTREPR	.04	0				1
INCOME	7.42	0	7.35	7.56	7.77	9.78
DUMMY180	.41	0				1
INCOM180	2.85	0	0	0	7.22	7.50
DIFINC	.62	-44.61	-.31	.53	1.81	68.16
CAPINC	.18	0				1
CREDIT						
NRQUEST	1.51	0	1.10	1.61	1.95	2.64
LIMIT	10.00	0	9.39	10.13	10.87	13.56
ZEROLIM	.01	0				1
NRLOANS	1.32	0	1.10	1.39	1.61	2.71
LIMUTIL	32.12	0	0	16	66	115
BALINC	1.38	0	0	.69	2.71	9.15
BALINCSQ	4.27	0	0	.48	7.34	83.70
BALANCE	6.08	0	0	8.21	10.28	13.55
COAPPLIC	.08	0				1
LOANSIZE	6.53	3.00	4.00	6.00	8.00	30.00
LIMBAL	9.28	0	8.92	9.55	10.05	12.21
DUMMY20	.27	0				1
LIMBAL20	2.84	0	0	0	10.05	12.21

months since the most recent change in marital status, number of months since immigration, number of houses a person (partially) owns, several measures of income, taxable wealth, a large number of entries on the two most recently submitted income-tax return forms, like total taxes due, back tax etc, and a number of transformations of these variables. Most of these were disregarded because they lack a relation with the dependent variable. Examples are back tax and real estate value. Section 3 contains a more detailed account of the methods we used in this selection process. Other variables were disregarded because they displayed extremely high correlation with covariates that measured approximately the same thing but had greater explanatory power. The numerous income measures in the dataset were eliminated in this way. Finally, taxable wealth could not be used as an explanatory variable. Wealth up to SEK 900,000 is tax-exempted, making the group of people with *taxable* wealth extremely small. Instead we used *taxable* income from capital - which is taxed from the first krona - to create a dummy explanatory variable.

3. Empirical analysis

Ideally, evaluating loan applicants or studying efficiency in the provision of bank loans would entail modelling the revenue on each loan as a function of a set of personal characteristics and macro-economic indicators. However, since few banks store complete time series of interest payments and amortizations on loans, the information presently available and useful for such a study is limited to the current balance and status (good or bad) of each loan. Therefore, we will instead model the survival time of each loan. With some simplifying assumptions imposed on the amortization scheme, and the rates of interest and discounting, one is then in principle able to calculate the return on each loan.

Because we have both censored and uncensored observations in our sample data, estimating a model of dormancy by means of OLS will lead to inconsistency in the parameter estimates. Duration models can take the censoring of data into account and will produce consistent and efficient parameter estimates. To make the exposition in the rest of this section more accessible for the reader, we start by briefly introducing some basic concepts in duration modelling.[3]

The probability distribution function of duration, denoted by $F(t \mid x) = Pr(T < t \mid x)$ and the corresponding density function $f(t \mid x) = dF(t \mid x)/dt$ are

[3] Kiefer [21] and Lancaster [22] discuss duration models and the analysis of transition data in greater detail.

two equivalent ways of describing the distribution of survival times conditional on a vector of characteristics x. Another equivalent concept, but with greater intuitive attraction in the context of duration analysis is the survivor function

$$\begin{aligned} S(t \mid x) &\equiv 1 - F(t \mid x) \\ &= Pr(T \geq t \mid x) \end{aligned}$$

Duration data are easiest interpreted, though, by means of the hazard function

$$\lambda(t \mid x) \equiv f(t \mid x) \ / \ S(t \mid x).$$

Roughly, the hazard rate is the probability of the immediate occurence of an event at a specific point in time, conditional on this event not having taken place earlier.[4] From the definition of $\lambda(t \mid x)$, it is easy to derive that $\lambda(t \mid x) = -d$ $\ln S(t \mid x)/dt$ and

$$S(t \mid x) = exp[-\Lambda(t \mid x)]$$

where $\Lambda(t \mid x) = \int_0^t \lambda(v \mid x)\,dv$. The hazard function specification can been seen as a way to decompose an unconditional probability into a series of conditional probabilities. The hazard function also offers a practical interpretation of the meaning of duration dependence. When $d\ \lambda(t \mid x)/dt > (<)\ 0$, there exists positive (negative) duration dependence and the probability that a spell will end instantaneously increases (decreases) with the length of the spell. It makes sense to choose the parametrization in such a way that it allows the *hazard* function to behave as preconceived. Using the *distribution* function instead may fail to reveal any such properties that are undesirable for the analysis of durations - despite the distribution's appeal in other applications. For example, the Normal and Lognormal distributions are unable to comprise the constant hazard as a special case; some other distributions have monotonically increasing or decreasing hazard rates.

The rest of this section is divided into three subsections. We start by analyzing the duration of loans until failure in Section 3.1. In Section 3.2 we model the transition from active loans to dormancy and relate this transition to the characteristics of the applicant. We conclude the empirical analysis with a subsection that addresses some conceptual problems that arise when modelling consumer credits by econometric duration models, as well as practical problems in revealing the functional relationship between the transition and the covariates.

[4] A formal definition is $\lambda(t) = \lim_{h \to 0} Pr(t \leq T < t + h \mid T \geq t) \ / \ h$.

3.1. Bad loans

Traditionally, default risk has been modelled by means of discrete choice models, letting the outcome variable be failure or not. For an adequate analysis of the risk involved in bank lending one needs to consider not only the probabilities of default but even the durations of loans. Failures may occur after a long period of regular installments and interest payments. These revenues will partly or completely make up for the losses on bad loans, particularly because these will be heavily discounted in addition. By starting with an investigation of the time to failure we create a unified framework for the analysis of both risk and return in bank lending and make it possible to contrast the behavior of the defaultness and dormancy models later on.

We denote the duration of the i:th bad loan by $t_{d,i}$, the elapsed time in weeks from the approval to the time point it fails. We attempt to fit a regression model which can be expressed as,

$$\ln t_{d,i} = m(x_i, \beta) + \epsilon_i, \tag{3.1}$$

where $m(\cdot)$ is the function which links the duration variable to the applicant specific covariate vector x_i and β is a parameter vector assumed common to all loans. The functional form of $m(\cdot)$ is selected on basis of the results from non-parametric regressions as outlined in Section 3.3.

Table 3 shows the least squares estimates and the standard errors of the unknown parameters in the vector β. The purpose of fitting the regression model is purely descriptive, and our focus is on the sign, magnitude and standard errors of the parameter estimates. In obtaining the estimates, a considerabe effort has been devoted to the elimination of variation that arises from regional factors and the blend of credit products that the data set is composed of. We have, however, excluded the parameters pertaining to these factors from Table 3 because they are of limited relevance for the present investigation. Except for the regional and product related variables, having a *COAPPLIC*ant or a large number of registered loans are found to significantly increase the time to default. Having capital income or lacking experience in borrowing money reduce duration.

As a minor point of succes, we further note that the residuals closely follow the Normal distribution. Overall, however, we judge that the regression model fails to capture the relationship between survival time and applicants characteristics. We attribute this to two different causes. Firstly, the variety in loan types present in this sample blurs many behavioral relations. Although all loan types have similar

Table 3: LS regression on survival time of bad loans.
Asymptotic standard errors are presented within parentheses. Parameters that are significant at the 10 percent level or higher are printed in bold style.

Variable	Parameter	Standard error
CONSTANT	**2.615**	(.375)
AGE	−.000	(.001)
MALE	−.048	(.033)
MARRIED	.047	(.036)
DIVORCE	−.118	(.094)
SCAND	.009	(.091)
FOREIGN	−.050	(.074)
HOUSE	.050	(.037)
ENTREPR	.099	(.101)
INCOME	−.009	(.036)
ZEROINC	−.247	(.256)
DIFINC	−.000	(.001)
CAPINC	**−.172**	(.078)
NRQUEST	−.023	(.034)
LIMIT	−.013	(.020)
ZEROLIM	**−.268**	(.150)
NRLOANS	**.132**	(.049)
LIMUTIL	−.000	(.001)
BALANCE	.014	(.013)
COAPPLIC	**.210**	(.054)
LOANSIZE	.033	(.033)

$N = 1103$, $R^2 = .185$, $\sigma^2 = .230$. Dummy variables pertaining to the regional belonging of applicants or the type of credit are included in the model but suppressed in this table. They contribute with 7 percentage points to the R^2 measure of fit.

terms, small changes in terms are regularly associated with large shifts in the clientele, because different kinds of loans generally are sold by different types of stores. Any differences in clientele behavior are unlikely to be captured by a single product related dummy variable, as we have attempted here. Secondly, and more importantly, the transition to default cannot be modelled as a univariate process in the way dormancy can, because of the large share of incomplete observations in these data (around 99 percent of the loans have not completed a default spell). In principle, failure and dormancy are competing risks and should thus be modeled simultaneously, unless their error processes are independent. Ignoring this relationship when modelling dormancy will turn out to be a rather harmless act, because the defaulted loans that are disregarded constitute only one percent of the data. Even if the error processes would be strongly correlated, the dormancy model's parameter estimates will hardly be affected because of the disregarded observations negligible weight in the likelihood function. When modelling failure, however, the same strategy will imply overlooking 99 percent of the data set. Obviously, this is likely to have an important influence on the parameters that are estimated. As a consequence, the resulting model will not lend itself for prediction of the time to default. In theory, this problem can be solved by estimating a bivariate duration model with two competing risks: default and dormancy.[5] Because of the extreme imbalance between the shares of dormant and defaulted loans in the population this turned out to be practically unfeasible.

3.2. Dormant loans

The aim of this section is to disclose the transition process by which a loan moves from an active to a dormant status and to relate this process to the applicants characteristics. Unlike the model of the time to default, however, the purpose of the models in this section is not only descriptive but even to predict, i.e. to calculate the expected durations for all granted loans and compare these with the survival time required for a break-even.

We denote the elapsed time (in weeks) from approval of the i:th loan to the time point the loan becomes dormant by T_i. Due to the sampling scheme, the actual duration t_i can be either entirely or partially observed. In the former case we refer to the observation as complete, in the latter case it is called incomplete.

[5] See Carling and Jacobson [11] for an explanation of competing risk models.

The data set consists of the following complete and incomplete observations:

T_i	$= t_i$	$< t_i$	$> t_i$
no.	751	3605	377
c_i	-1	0	1

The incomplete observations with the coding variable $c = 1$ are right-hand censored observations. The observations with coding $c = 0$ are less common in duration analysis: they are a consequence of the bank's partial failure to record dormancy[6]. Table 2 contains descriptive statistics on the rich background information on the applicants, x_i, which will be used in the duration analyses. The covariates are exclusively time-invariant. Detailed discussions on how to include time-varying covariates in duration models are provided by Lancaster [22] and Narendranathan and Stewart [24].

Although duration analysis is better suited to model the duration of the active loans, we will begin with fitting a regression model for the complete observations, i.e. those for which $c = -1$. Table 4 presents as Model I the results from a least squares regression of the natural logarithms of the duration on a set of covariates (see the model in (3.1)). There are several reasons for studying the complete observations first. Firstly, least squares regression is a robust estimation method, thus eliminating the need of guessing the error distribution. Secondly, advanced univariate and multivariate statistical and graphical techniques exist for investigating the functional form of $m(\cdot)$ (see Section 3.3, where the principles and the techniques for selecting $m(\cdot)$ is discussed). Finally, estimation is fast and simple.

Model I has a reasonably good fit, the R^2 is 25.1 percent, and the distribution of the residuals follows closely the Normal distribution. However, the omission of the incomplete observations is clearly inappropriate. The censored observations with $c_i = 1$ are the loans with the longest durations and excluding them leads to biased parameter estimates and underestimation of the expected duration. We therefore turn to consider duration models.

As we mentioned above, the hazard function is the most natural entity to model duration variables since our attention is on the transition rate from one state to another. We consider models derived from the two most common clas-

[6] The presence of incomplete observations does not invalidate a consistent estimation of the unknown parameters. The drawback is a reduced precision in the estimates, manifested by larger standard errors.

Table 4: Estimated models of duration to dormancy.
Model I is an OLS regression of the complete observations (c = -1). The parametrization of models II-III conforms to that of model I in that the parameter estimates can be interpreted as the relative contribution to the duration of a loan. Asymptotic standard errors (within parentheses) are obtained from the cross-product of the gradient matrix.

Variable	OLS regression model I	Lognormal duration model II	Log-logistic duration model III	Semi-parametric dur. model IV
CONSTANT	**5.189**	**5.584**	**5.489**	–
	(1.546)	(2.438)	(1.546)	
AGE	.003	.003	.002	.001
	(.003)	(.005)	(.002)	(.002)
MALE	−.107	−.053	−.081	−.023
	(.073)	(.128)	(.052)	(.046)
MARRIED	−.092	−.184	**−.127**	**−.101**
	(.068)	(.127)	(.050)	(.046)
SEPARATE	**.422**	.180	.234	.144
	(.211)	(.358)	(.152)	(.138)
NORDIC	.128	.147	.098	.054
	(.215)	(.391)	(.154)	(.155)
FOREIGN	**−.543**	−.528	**−.407**	−.109
	(.294)	(.488)	(.206)	(.161)
HOUSE	.093	.004	.163	.022
	(.337)	(.576)	(.233)	(.216)
HOUSEVAL	−.001	−.014	−.035	−.005
	(.057)	(.099)	(.040)	(.037)
ENTREPRE	−.235	−.417	−.246	−.179
	(.209)	(.383)	(.164)	(.123)
INCOME	**−.423**	−.445	**−.402**	**−.282**
	(.197)	(.311)	(.132)	(.104)
DUMMY180	**−3.161**	−3.022	**−2.837**	**−1.942**
	(1.585)	(2.493)	(1.063)	(.835)
INCOM180	**.409**	.410	**.379**	**.262**
	(.203)	(.326)	(.139)	(.109)
DIFINC	**−.015**	−.014	−.063	−.015
	(.009)	(.137)	(.057)	(.045)
CAPINC	−.082	−.237	**−.125**	**−.090**
	(.094)	(.176)	(.075)	(.058)
NRQUEST	**.319**	**.521**	**.192**	**.110**
	(.071)	(.145)	(.052)	(.047)
LIMIT	**.116**	−.075	.021	.077
	(.055)	(.155)	(.051)	(.053)

Continued on next page.

Table 4 - continued.
Coefficients significant at the 10 percent level or higher are printed in bold style.

Variable	OLS regression model I	Lognormal duration model II	Log-logistic duration model III	Semi-parametric dur. model IV
ZEROLIM	.661	−.051	.208	**1.036**
	(.466)	(1.355)	(.450)	(.432)
NRLOANS	.126	.255	**.354**	**.304**
	(.106)	(.234)	(.088)	(.086)
LIMUTIL	−.002	.009	**.004**	**.006**
	(.002)	(.006)	(.002)	(.002)
BALINC	**.201**	−.007	.110	.042
	(.103)	(.222	(.087)	(.081)
BALINCSQ	**−.043**	.020	**−.032**	−.017
	(.017)	(.039)	(.015)	(.015)
BALANCE	−.005	.041	−.002	−.002
	(.016)	(.032)	(.013)	(.011)
COAPPL	**.053**	.285	**.184**	**.157**
	(.010)	(.186)	(.070)	(.073)
LOANSIZE	−.005	**.046**	**.022**	**.020**
	(.009)	(.019)	(.007)	(.007)
LIMBAL	−.023	−.053	−.015	.036
	(.021)	(.094)	(.020)	(.030)
DUMMY20	−2.757	**−.565**	−.133	−.077
	(1.870)	(.193)	(.082)	(.070)
LIMBAL20	.260	**.100**	−.010	**−.022**
	(.180)	(.033)	(.009)	(.011)
REGION1	**.205**	**−.014**	−.044	−.008
	(.071)	(.003)	(.052)	(.048)
REGION2	.048	−.162	**−.074**	**−.086**
	(.057)	(.109)	(.042)	(.040)
REGION3	.079	.105	.052	.029
	(.094)	(.180)	(.071)	(.064)
REGION4	.141	**.428**	**.282**	**.249**
	(.096)	(.185)	(.073)	(.068)
σ^2	.620	3.820	.850	
R^2	.251			

ses of conditional hazard models.[7] The first is the location-scale model (also known as the Accelerated Failure Time Model) which can be written in regression form as

$$\ln t_i = m(x_i, \beta) + \sigma\epsilon_i. \tag{3.2}$$

Taking $m(x_i, \beta) = h(x_i) \cdot \beta$, the parameters can be interpreted as the absolute change in the expected value of the logarithm of the duration or, approximately, the relative change in the expected duration.

An extension of Model I yields the Lognormal model with hazard function,

$$\lambda(t|x) = \frac{\phi((\ln t - x'\beta)/\sigma)}{t\sigma\overline{\Phi}((\ln t - x'\beta)/\sigma)} \tag{3.3}$$

and survival function

$$S(t|x) = \overline{\Phi}((\ln t - x'\beta)/\sigma). \tag{3.4}$$

We hereafter refer to this as Model II.

The Lognormal model belongs to the class of location-scale models and can thus be specified as regression model, we estimate $\ln t_i = x_i'\beta + \sigma\epsilon_i$, where the residuals are assumed to follow the standard Normal distribution. Thus, the maximum likelihood estimates of Model II in Table 4 can be contrasted to the estimates of Model I. The Maximum Likelihood estimates are obtained by maximizing the log likelihood function[8],

$$\begin{aligned}\ln L(\beta, \sigma) &= \textstyle\sum_{c=-1} \ln \frac{\phi((\ln t - x'\beta)/\sigma)}{t\sigma} + \\ &\textstyle\sum_{c=0} \ln\left(1 - \overline{\Phi}((\ln t - x'\beta)/\sigma)\right) + \sum_{|c|=1} \ln \overline{\Phi}((\ln t - x'\beta)/\sigma),\end{aligned}$$

by an iterative gradient method.

The Log-logistic model, Model III, is an alternative to the Lognormal model. The hazard function is given as

$$\lambda(t|x) = \frac{\exp(x\gamma)\,\alpha t^{\alpha-1}}{(1 + \exp(x\gamma)\,t^{\alpha})}. \tag{3.5}$$

[7] See Lancaster [22] and Kiefer [21].

[8] Starting values were obtained from the Approximate Maximum Likelihood method (Carling [9]), as well as from Model I, and used in conjunction with the BHHH algorithm (see Carling and Söderberg [12]). The same procedure was adopted for Model III and IV as well.

and the log likelihood function to be maximized is

$$\begin{aligned}\ln L(\beta,\sigma) &= \sum_{c=-1} \ln \frac{\exp(x\gamma)\,\alpha t^{\alpha-1}}{(1+\exp(x\gamma)\,t^{\alpha})} + \\ &\sum_{c=0} \ln \frac{\exp(x\gamma)\,t^{\alpha}}{(1+\exp(x\gamma)\,t^{\alpha})} - \sum_{|c|=1} \ln(1+\exp(x\gamma)\,t^{\alpha}).\end{aligned}$$

The Log-logistic model also belongs to the class of location-scale models described in (3.2) where the residuals follow the logistic distribution, and hence there is a simple relation between the parameters of Model II and III: $\beta = -\gamma/\alpha$. Table 4 presents the resulting parameter estimates after reparametrization in order to simplify a comparison of the estimates.

However, Model II and III are not nested and there is no obvious way to discriminate between them. Yet, they impose rather strong restrictions on the error distribution and thus an implicit assumptions on the hazard function. We therefore, by considering a fourth model, relax the assumption on the form of the hazard function. Model IV is a semi-parametric Proportional Hazard (PH) model of the form [9]

$$\lambda(t|x) = \exp(-x'\beta)\,\lambda(t)$$

where $\lambda(t)$ is the base-line hazard function stipulated to be equal for all loans. The Proportional Hazard model can also be formulated as a regression model

$$\ln \int_0^{t_i} \lambda(u)\,du = x_i'\beta + \zeta_i$$

where ζ follows the Extreme Value distribution with expectation $E[\zeta] = -0.5772$ and variance $V[\zeta] = \pi^2/6$. The interpretation of the parameters in the PH-model is different from the location-scale model: a parameter shows, approximately, the relative change in the hazard function. The non-parametric baseline hazard $\lambda(t)$ and the linking function $\exp(-x'\beta)$ are estimated simultaneously by Maximum Likelihood.[10,11] The estimates of Model IV have been reparameterized so that

[9]The somewhat unusual specification of the linking function, $\exp(-x'\beta)$, is given in the interest of making the estimates between Model I-III and Model IV directly comparable.

[10]See Carling et al [10].

[11]We considered four weeks intervals to restrict the number of parameters in the baseline hazard. As a consequence, we assume that the hazard is constant within a four weeks interval, an assumption justified by preceding analysis which showed a minuscule change in the baseline hazard by extending the time unit. Furthermore, there are 40 parameters in the baseline hazard, hence there were no attempt, due to sample restrictions, to estimate the hazard function beyond the 160:th week.

they can be compared with those of models II and III and are shown in Table 4.

The attempt to fit the models to the data has been met by a reasonable success; the question is which model provides the best description of the data. Table 5 shows the fitted probability distributions. Figure 1 shows the hazard function produced by Model II-IV for an average loan-applicant, i.e. an individual with covariate vector equal to the sample average. Table 5 shows that the tail of the lognormal hazard function is obviously too fat on the right hand side. It is less apparent whether one should prefer the log-logistic or the semi-parametric hazard function. In Figure 1, however, we immediately see that both parametric models provide a poor description of the hazard function. The lognormal hazard function is monotonically decreasing with time and fails to register any of the peaks that the semi-parametric model captures at three and twelve months. The loglogistic hazard function does a somewhat better job. It is unimodal, however, and in its attempt to capture the double peakedness it reaches its maximum in between them, at 30 to 40 weeks. It would have been possible to augment the class of location-scale models, as described in (3.2), by even more flexible distributions.[12] Because of the larger number of free parameters, the semi-parametric model will, however, always produce the most accurate description of the data. Since the sample size of our data set permits us to estimate a larger number of free parameters at a relatively low cost (in terms of the relative change in the degrees of freedom that are lost), we prefer the semi-parametric representation as the best way to model the duration to dormancy.

Unlike the model in Section 3.1, the semi-parametric model captures a number of interesting relationships between borrowers characteristics and duration to dormancy. When controlling for income and credit history, age, gender and citizenship no longer have any bearing on duration. Married applicants tend to pay back their loans faster. This may be a result of Swedish couples generally having two wage earners in the family which leads to a stable flow of income. Alternatively, it could reflect the fact that married couples are simply more diligent. Higher income also reduces the duration of loans until dormancy. For incomes over SEK 180k. this effect flattens out. Intuitively this makes sense, considering that the median gross monthly income is about SEK 15k. People with below-median income take a longer time to pay their debts; all others behave alike. Owning assets further reduces the duration of any loans people hold. Such people will

[12] See Bergstöm and Edin [6].

Table 5: The fitted probability distribution for the time to dormancy (in weeks). For models II-IV, the distribution has been calculated for an average applicant, i.e. all covariates equal their sample means. To derive the expected duration by model IV, the hazard rate has been assumed constant beyond the 160th week.

Model	mean	percentiles						
		5	10	25	50	75	90	95
Lognormal (II)	89	<1	<1	3	13	50	163	331
Log-logistic (III)	61	8	12	21	38	66	117	175
Semi-parametric (IV)	58	8	12	15	44	80	125	160

likely use liquid assets for repayment of a debt with a 30 percent interest rate. Somewhat surprising is the fact that the size of *LIMIT* does not affect duration, whereas increasing the loan size or the number of loans delays repayment. It is interesting to contrast this with the results in Jacobson and Roszbach [20] and Roszbach [25]. In two independent studies of the determinants of default, they find that *LIMIT* has a significant effect on both the risk of default and the duration to default, but *LOANSIZE* does not. Inexperience with servicing debt, as indicated by *ZEROLIM*, slows down the amortization. Similarly, *NRQUEST* is considered a good measure of a persons eagerness to obtain additional credit and as such expected to lead to a longer payback time. *LIMUTIL* has a comparable effect. Finally, having a guarantor also makes it more likely that the loan will be paid back over a longer period of time. This is rather interesting in view of the fact that a guarantor is generally asked for to reduce the risk of what is considered a likely failure.

Model IV can now be used to examine the lending policy of the bank. An averagely sized (SEK 6,300) loan needs to stay active for at least 5 months (of which 3 are free of interest) for the bank to break even. Hence, one would expect the bank to approve loans which are likely to remain active and be restrictive towards loan applications that are likely to turn dormant shortly after the free 13 weeks. For each loan in the sample, we can derive the the conditional expected duration from Model IV. Figure 2 shows by a histogram the distribution of the expected duration of the active loans for all 4,733 sample applicants. If we compare the expected durations with the break-even duration, 4,671 (98.7 %) out of the

4,733 granted loans are found to have positive expected profits. With the help of the expected durations that have been computed to construct Figure 2, we can even calculate the expected profit or return on each loan. Appendix A describes how these profits are derived. Figure 3 contains a plot of the resulting distribution of expected profits for the loans in the sample. The expected profits range from -105 to 7924 kronor, with a mean of 786 and a standard deviation of 616.

Ideally, we would have wanted to carry out the same analysis on the rejected loan applications. Due to way in which the data were sampled, we were however unable to do so. The institution that provided the data extracted loans that were granted between September 1993 and August 1995 from its customer base but only collected applications during the last of these two years. The type of loan that we study was hardly offered during 1994-95 and we thus lack information on the rejected applicants. There are no practical obstacles, however, that prevent banks from doing so for future loan applications. The semi-parametric model that we have estimated for our sample data illustrates how a duration model can be used to evaluate loan applicants by calculating their expected returns.

3.3. Econometric issues

During the completion of this analysis, we have had reason to consider some conceptual and practical problems which we address here. First, in duration analysis it is assumed that eventually the event will occur, e.g. that an active loan eventually turns dormant. Formally, this assumption arises since it is required $\lim_{t\to\infty} P[T>t]=0$ for the survival function to be well-defined. In practice, an active loan can either become dormant or it can fail. The existence of the two mutually exclusive events seems to suggest that the condition is violated. As we already mentioned in Section 3.1, we have taken a pragmatic point of view in this matter, by claiming that the bad loans are too few to invalidate the analysis of the transition to dormancy. We found support for this claim in an additional estimation of models II-IV on an expanded dataset that includes the bad loans as independent observations that are right-hand censored at the time point of default. The results confirmed the parameter estimates in Table 4. Nevertheless, this condition did prevent us from carrying out a duration analysis of the bad loans. As a consequence, we failed to give a meaning to the expected duration of a bad loan in an environment with a high transition rate to dormancy.

Other conceptual issues in duration analysis are unobserved heterogeneity and multiple spells.[13] We refer to Narendranathan and Stewart [24] and to their arguments for neglecting unobserved heterogeneity in semi-parametric proportional hazards models. Multiple spells arise when an individual has several loan spells. In the sample, there were hardly any such individuals. However, it is likely that the lending history of an individual can be very helpful in explaining the future behaviour. Whenever multiple spells are present, it seems wise to augment the duration model accordingly.

An important practical problem is how to disclose the function $m(x, \beta)$, i.e. how to perform preliminary data analysis. Duration data and duration models are quite difficult in this sense.[14] We prefer to perform the preliminary data analysis on the complete observations. Selecting the covariates and linking these to the duration variable in Table 4 has been done with the help of exploratory tools for ordinal and categorical variables and a number of non-parametric regressions for the continuous variables.[15] This decision hase been a compromise between the desire to reach effective linking and to restrict the number of parameters. We can take the variable *NRLOANS* as an example. In the sample there were applicants with 0 to 14 registered loans. A dummy-variable for each number of questions would imply 14 parameters. The transformation $y = \ln(1 + x)$ implied that the number of parameters could be reduced by 13, at a negligible reduction of the goodness of fit.

4. Discussion

A bank that lends money to a household faces two types of risk. Most commonly mentioned is the risk of default. Hardly ever referred to is the risk of an early redemption of the loan. As a result of the period of regular installments and interest payments that generally preceed a default, loan losses may be partially or completely made up for by revenues in addition to being heavily discounted. Moreover, defaulted loans generally make up only a few percent of the loan portfolio. Together, these facts suggests that dormancy is the economically relevant phenomenon to study - despite widely spread views that banks need to focus on default risk.

[13] See Blossfeld and Hamerle [7] for a clear exposition of the unobserved heterogeneity problem.

[14] Altman and de Stavola [2] provide a careful discussion on available techniques.

[15] Exploratory tools for ordinal and categorical variables are treated by Hoaglin, Mosteller, and Tukey [18]. For literature on non-parametric regressions, see Cleveland [13], Cleveland, Devlin, and Grosse [14], and Härdle [19].

In principle, failure and dormancy are competing risks and should therefore be modelled simultaneously unless their error processes are independent. If default rates had been somewhat higher than the one percent in our data set, estimation of a bivariate duration model with default and dormancy as competing risks would be required. This would be a straightforward extension of the analysis in this paper.[16] Because the defaulted loans make up only one percent of our sample data, we can ignore defaultness as a competing risk when modelling dormancy.

We model the transition by consumer loans' from an active to a dormant state by means of four different univariate representations. For each duration model, we investigate its ability to describe the hazard rates observed in the data. A semi-parametric model is found to best capture the relationship between applicants' characteristics and duration to dormancy. Among other things, the model shows that people tend to pay back their loans faster as their income increases. This effect disappears, however, when individual income reaches the median level in the sample. Bigger loans mean longer pay-back periods but the amount of already outstanding loans does not affect duration.

For the preferred model, we derive the distribution of conditional expected durations of loans. In combination with information provided by the lending institution, these durations enable us to calculate a distribution of expected profits for the sample loans. The duration model of dormancy is thus well suited for use by banks as a tool to evaluate new loan applications. However, unlike credit scoring models, which merely predict default probabilities, it is based on an evaluation of expected profitability. An interesting application of the model would be to examine the expected durations for rejected applications and compare these with the durations of granted loans. This would give us better insight into the efficiency of current bank lending and could help increasing the future profitability and efficiency of bank lending.

[16] See Carling and Jacobson [11].

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A. Return on loans

In this appendix we show how the returns on defaulted and non-defaulted loans were calculated. The average survival time for non-defaulted loans is 11 months and has been computed from the 751 observations that we have full information on. This means that we excluded the remaining 3982 censored observations from this calculation. Considering that the expected survival time implied by the non-parametric model (Table 5) is 58 weeks, this estimate appears on the conservative side. The average survival time on bad loans is 13 months and has been computed from the 53 loans that had defaulted in the sample (of 4,786 observations) from which the data for the dormancy model was drawn. Between September 1993 and October 1996, people on average borrowed SEK 6,300 (median 5,400) and paid an interest rate of 31.36 percent per annum, or 2.3 percent on a monthly basis. The maximum yield on 6 months government bonds during this period was 9.63 percent per annum. We therefore apply a discounting factor for the bank's costs and revenue of 10.63 percent.

The loans are free of interest charges during the first three months. If the loan is not paid back after three months, interest starts accumulating. Borrowers receive an invoice every month for which they are charged a fee of 17 kronor. We assume that the loan is paid back as a annuity, the amount being determined by the average survival time and the interest rate. For defaulted loans the annuity will even be a function of the average nominal loss on the principal. Because amortization is slow and only partial, bad debtors pay the bank twice as much interest as good ones do. Moreover, because loan losses are realized in the future, discounting reduces their net present value.

On the cost side, the bank incurs a fixed cost for evaluating an applicant, printing a credit card and entering the new loan into its computer system. Variable costs consist of maintenance of the computer system and sending a monthly invoice to the customer.

The above can be summarized in the following expressions. The return on good loan can thus be expressed as $R_G = \left(\sum_{t=4}^{11} \frac{x_t+17-vc}{(1+r)^t} - fc\right)/s$. For a defaulted loan, the expression is only slightly different: $R_D = \left(\sum_{t=4}^{13} \frac{z_t+17-vc}{(1+r)^t} - fc - \frac{ll}{(1+r)^{13}}\right)/s$. Here r_t is the discount rate, x_t and z_t are the interest payments at time t on a good and bad loans respectively, fc and vc are the fixed and variable cost the bank incurs for each loan, ll is the average loss of principal on a defaulted loan and s is the average loan size. Entering all parameters into the above two expressions gives that $R_G = 8.1$ and $R_D = -36.7$ percent. The break-even point for both the mean and the median loan lies at 5 months (of which 3 are exempt of interest charges).

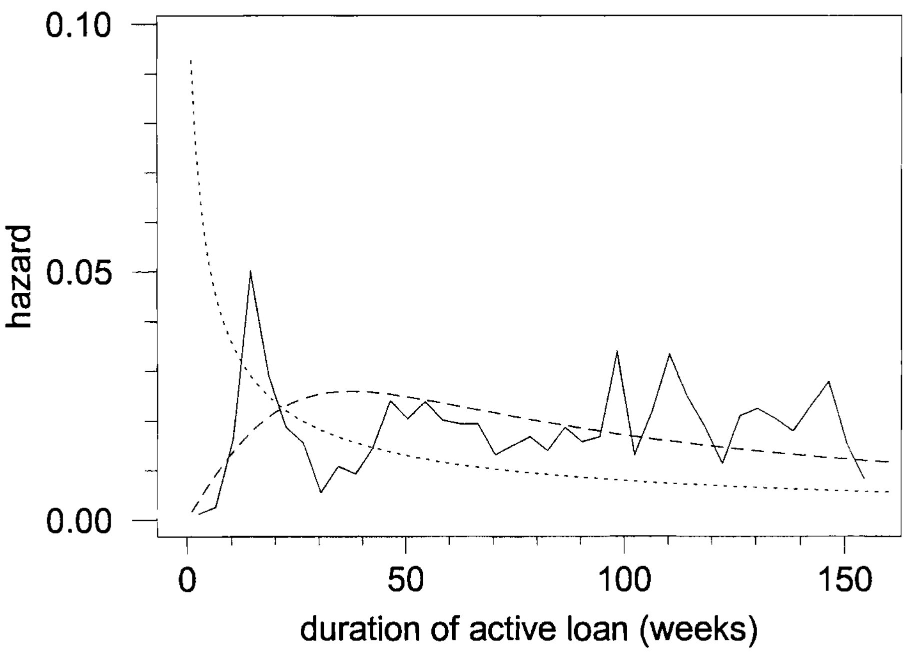

Figure 1. The hazard function for the duration of active loan until passivity. The hazard function is derived from Model II-IV assuming an average sample applicant. The solid line refers to the non-parametrically estimated hazard (Model IV), the dashed line to the Log-logistic (Model III), and the dotted line to the Lognormal hazard (Model II).

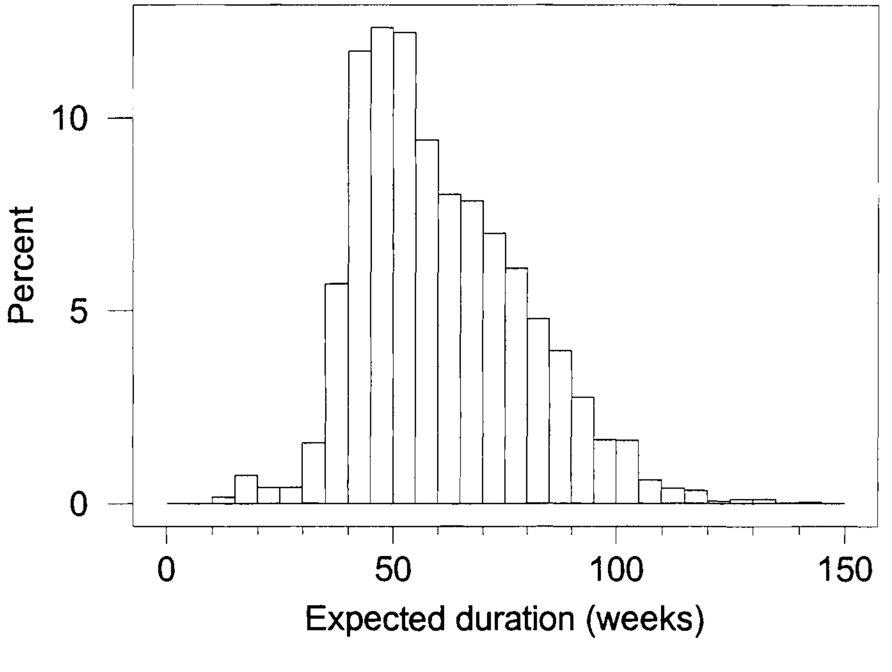

Figure 2. The distribution of the conditional expected duration of the active loans for the sample applicants. The expected duration for each applicant is derived from Model IV using the individual's values on the covariates.

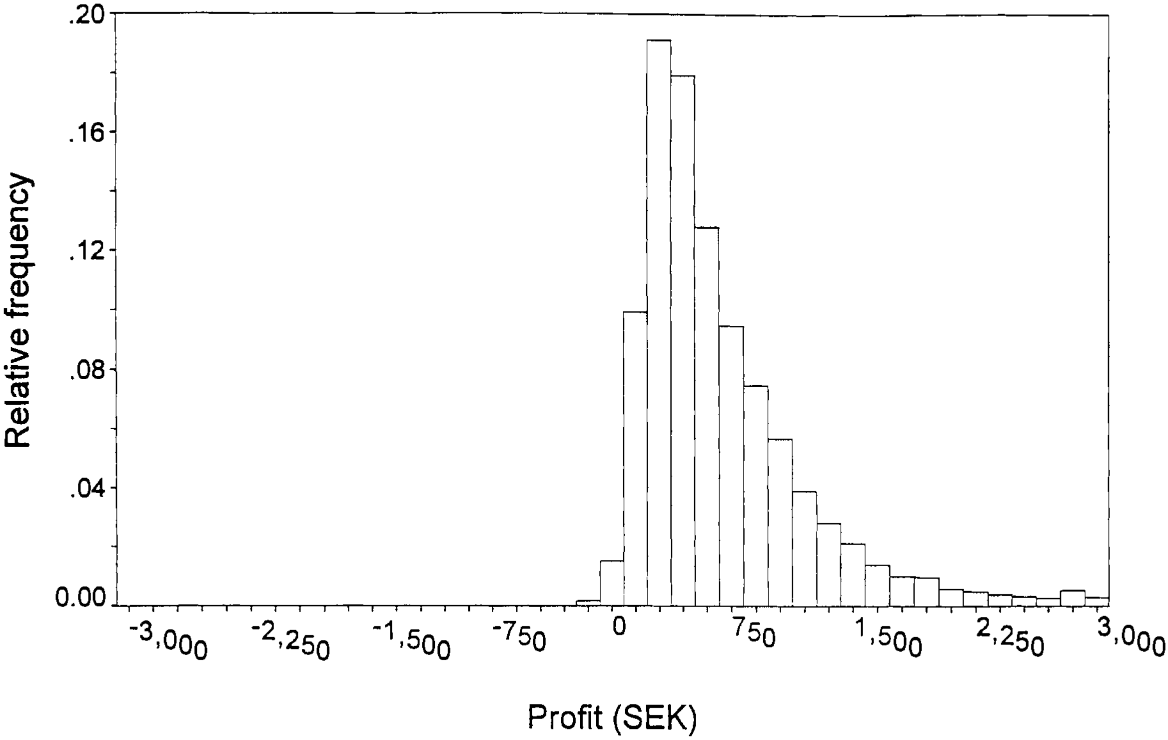

Figure 3: The distribution of expected profits for the loans in the sample.
The range of the histogram is -[3125, 3125]. Thirtythree observations fall outside this range. $N = 4{,}733$; $\sigma = 616.2$; $\mu = 786.3$; min = -105.2; max =7594.6.

Chapter 6

Essay 4

Central Banks, adjustment costs and interest rate policy

Central Banks, adjustment costs and interest rate policy *

Kasper Roszbach†

30 July 1998

Abstract

This paper addresses three questions. Why do Central Banks change targeted interest rates so seldomly? Can we model the weekly behavior of a targeted interest rate? What are the driving forces behind rate changes? This paper takes the point of view that Central Banks face a fixed cost when adjusting a targeted interest rate and therefore smoothe it by using a discrete policy rule. When modeling interest rate behavior this discrete nature is taken into account by applying a grouped data model to a Swedish data set. Changes in unemployment, inflation, retail sales, industrial production, money growth and U.S. and German interest - and exchange rates are shown to determine Swedish repo rate changes. Probabilities of the target rate being raised, lowered or kept constant are computed. The model has a prediction rate of 88% versus 78% for the best naïve estimator.

*The idea for this paper originated in a conversation with Marcus Asplund. I thank him, Torbjörn Becker, Sune Karlsson, Rickard Sandin, Peter Sellin, Paul Söderlind, Anders Vredin and seminar participants at the Stockholm School of Economics and the Bank of Sweden for their helpful comments. Financial support from the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged.

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A central banker walks into an Italian restaurant to order a pizza. When the pizza is done, he goes up to the counter to get it. There a clerk asks him :
– Should I cut it into six or eight pieces?
The central banker replies :
–I'm feeling rather hungry right now. You'd better cut it into eight."
HTTP : //WWW.ETLA.FI/PKM/JOKE.HTML

1. Introduction

In recent years, the main instrument of monetary policy in most industrial countries has been a very short interest rate controlled by the Central Bank. Despite the often explicitly stated final goal of price stability, monetary policy in many countries has in practice also been employed for the purpose of reducing fluctuations in short-term interest rates. For the U.S., for example, a number of studies has presented evidence for the thesis that interest rates became substantially more persistent and less volatile and ceased to exhibit the pattern of wide irregular and seasonal fluctuations after the foundation of the Federal Reserve System in 1914.[1] Recent empirical work on monetary policy, like Bernanke and Blinder [5], Brunner [6], Cecchetti [7], Christiano and Eichenbaum [8] and Hamilton [15] has also taken the view that the federal funds rate, an overnight rate targeted by the Federal Reserve Bank, is the Fed's foremost policy instrument.

Since very short Central Bank rates like the repo rate in Sweden and the federal funds rate in the U.S. form an important building block for term structure theory, they are of interest to us not only from a monetary economic but also from a financial economic perspective. Rudebusch [22] is one of those who linked together these two different perspectives.

Maybe the most salient feature of a policy instrument like the federal funds rate target in the U.S. and the repo rate in Sweden is their constancy over long lapses of time despite changes in the economic environment.[2] Surprisingly, except for

[1]Cukierman [9] pp. 111-115 summarizes some of the evidence.

[2]For an in-depth description of U.S. monetary policy, see Goodfriend [13]. Hamilton [15] gives a detailed description of the workings of the federal funds market. For Sweden, see [19] and [20].

Rudebusch [22], all authors modeling them have consistently ignored this property. Khoury [16] and Brunner [6] offer good overviews of the single equation OLS and VAR based reaction functions that have been estimated.

Until today, and to my knowledge, no attempt has been made to build a theoretical framework that explains why Central Banks follow discrete decision rules for their interest rate policy instrument. The first to recognize the empirical relevance of this feature and treat the interest rate targeted by the Central Bank as a discrete dependent variable was Rudebusch [22]. He applied a non-parametric hazard rate estimator on the spells between changes in the daily U.S. federal funds rate target and found that "after a target [rate] change, there is a greater likelihood of another target change in the same direction." [3] The hazard functions were also found to display duration dependence. During the first 25 business days after a change, the target rate is more likely to move once more in the same direction than in the other direction. After the fifth week the duration dependence has ebbed away. Thereafter, the target rate displays the behavior of a random walk.

In the earlier mentioned work with reaction functions estimated with VARs or single equation OLS the above described censored behavior of the interest rate has been removed by using monthly data. Those models can be employed to produce consistent forecasts of the targeted interest rate one or more months ahead. However, doing so does not add very much to our comprehension of when and why the Central Bank will move to change interest rates *within* any monthly period; they cannot explain the factors underlying the behavior found by Rudebusch at weekly frequencies.

Rudebusch's conclusions are liable to some qualifications, however. Firstly, a quick glance at figure 1.1 suggests that the size of a target rate change does depend on the sign of the change: decreases tend to be bigger than increases. A test on the means, for example, cannot reject the null that cuts in the repo rate

[3] A random-walk characterization of the federal funds rate, for which Cukierman [9] argued, would thus be incorrect.

Durations separating two target rate increases were found to have the same distribution as those separating two target rate decreases (see: Rudebusch [22]). Likewise, spells between a negative and a succeeding positive rate change have the same distribution as spells between a positive and succeeding negative target rate change. However, durations between changes of the same sign are drawn from a different distribution than the ones between changes of different sign. Because a one-to-one relationship exists between hazard ratios and probabilities, this implies that $P^{--} = P^{++} \equiv P^s$ and $P^{-+} = P^{+-} \equiv P^d$, but $P^d \neq P^s$ where P^{-+} is the probability of successive changes being first downward and then upward, etc.

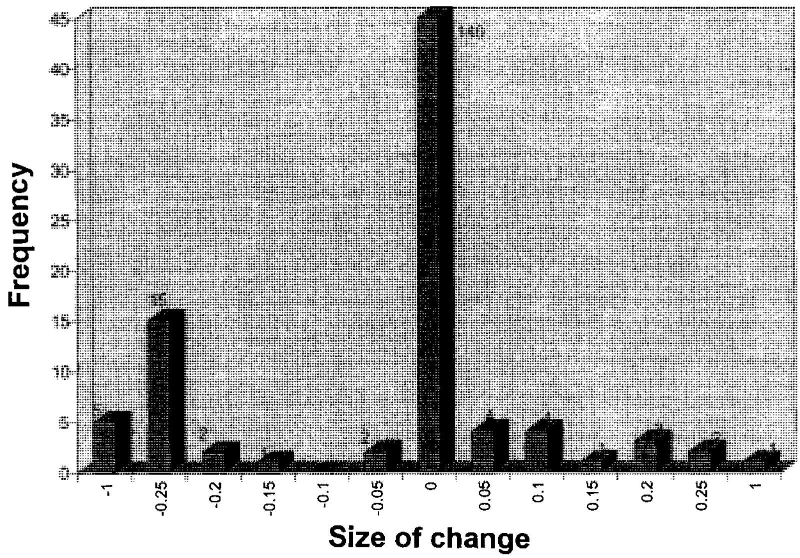

Figure 1.1: Changes (in percentage points) in the Swedish repo rate, weekly data, 24 November 1992 - 30 April 1996

target are at least 1/4 percent larger than rises.[4]

Secondly, due to the nature of duration models, Rudebusch was unable to compose one model that explains both increases and decreases in the funds rate. Instead, he estimated two separate models, one for each type of change.

Apart from that, he assumed that target rate changes can be well described by a hazard rate that doesn't depend on any covariates - thus making the funds rate behavior independent of the macro-economic environment. Thereby he conveniently circumvented the data problem that economists faced in earlier work - namely that statistics on financial, and sometimes monetary data are abundantly available at weekly or even daily frequencies, while information on most real economic aggregates is so only at monthly frequency.

The above qualifications combined with the stylized facts on the distribution of target rate changes provided in figure 1.1, raise a number of questions about the way in which monetary policy is conducted and how we can estimate a Central Bank's policy function.

Firstly, why do Central Banks change targeted interest rates so seldomly?

[4] An identical test on U.S. data used by Rudebusch cannot reject the null that decreases are 1/16 percent bigger than increases.

Secondly, when changes occur, should we expect an asymmetry in the reaction function, that is: are increases and decreases differently sized? Thirdly, what are the driving forces behind changes in the interest rate? Finally, if one could describe the hazard rates in terms of covariates, would we then still be able to find any duration dependence or asymmetries?

This paper is made up of two parts, a theoretical and an empirical. In Section 2, I formulate a simple model of interest rate policy, that casts some light on the underlying properties of Central Banks' discrete, and asymmetric, policy rules. There, a monetary authority with quadratic preferences incurs a fixed cost of changing the interest rate. With the size of the cost depending on the direction of the change, the optimal interest rate rule is both discrete and asymmetric. That section also contains a sketch of the conditions under which the model and its implications would generalize to a dynamic environment. The main objective of Section 3 is to formulate econometric models that can accurately describe the discrete behavior of the Swedish repo rate. These models form the basis for the empirical analysis in Section 4.

2. Interest rate targeting with costly adjustment

In this section, I first review a selection of studies that have offered either evidence that Central Banks face an adjustment cost when changing interest rates or an explanation for their policy of smoothing interest rates. After that, I formulate a simple model that provides us with some basic insights into the mechanisms that cause monetary authorities to change interest rates so seldomly, and to do so asymmetrically.

Although I am not aware of any paper offering a rationale for Central Banks' following discrete decision rules, a few papers have attempted to either explain the persistence in interest rates or discuss the consequences of interest rate smoothing policies. Goodfriend [12] and Barro [2] assumed some form of interest rate smoothing as an objective of monetary policy, and then pursued this goal with money as the instrument. Goodfriend started from the premiss that Central Banks seek to minimize price level forecast errors and the variation in inflation expectations. As a direct result smoothing nominal interest rates becomes the optimal means "to maintain 'orderly money markets"'. In Barro's paper, the starting point is a monetary authority that dislikes deviations of both the nominal interest rate from its target value and the price level from people's expectations. In such a model, both monetary growth and inflation are more volatile than the nominal interest rate.

Cukierman [9] has been a proponent of the view that the Fed engages in interest rate smoothing to maximize the stability of the financial sector - one of its explicitly stated tasks. Big profits generally provide financial companies with a buffer against losses and therefore further stability. To capture this mechanism, Cukierman added a banking sector to a stripped version of Goodfriend's [12] model. Under the presumption that interest rates on loan contracts are set prior to the determination of the cost of funds to banks, the best the Central Bank then can do is to supply money in such a way that shocks to the market rate of interest are smoothed.

Finally, an explanation that has regularly been brought forward but has not been explored in any formal theoretical framework, is that Central Banks loose credibility when they alter the stance of monetary policy. In this view, frequent rate changes are an expression of the Central Bank's lacking understanding of, or control over, the economy. One way to think of this loss of credibility goes as follows: when commercial banks match their borrowing and lending, they assume a certain amount of uncertainty in future rates of interest. The perceived uncertainty is a function of the Central Bank's force of conviction, which the financial sector measures, among other things, by the frequency of interest rate changes. When a Central Bank changes the centrally determined rate of interest after a longer period with a constant rate, financial institutions experience an increase in uncertainty. Due to the greater uncertainty, commercial banks require a bigger risk premium and thus widen the margin between lending and borrowing rates, thereby reducing equilibrium lending to private businesses. Central Banks, although aware of this mechanism, cannot perfectly infer the cost of their actions. Instead, they use a rule of thumb, a simpler function (possibly duration - and state-dependent), to infer the cost of adjusting interest rates. This mechanism could also account for the duration dependence found by Rudebusch if the adjustment cost increases with the number of weeks since the latest rate change and reaches its maximum after some time.

The most concrete empirical evidence for the hypothesis that Central Banks incur a fixed cost when adjusting short term interest rates has been reported by Cecchetti [7]. After comparing the simulated interest rate patterns implied by two different objective functions with actual interest rates, he concluded that [U.S.] interest rates have been so smooth over the last 10 years that policymakers must have been "attaching a cost to actually moving the federal funds rate".

The main interest of this paper is to investigate if it is possible to incorporate the mechanism through which this adjustment cost affects the implementation of monetary policy in an econometric model and match the actual interest rate behavior with the latter. For that reason, I will set up a model of Central Bank behavior in a very simple environment with adjustment costs. This closely follows Obstfeld [21], who analyzed the adjustment of exchange rate pegs in the presence of fixed adjustment costs in a Barro-Gordon [4] economy, and Alexius [1], who used the same structure to model escape clauses for inflation rules. I let the adjustment cost that the Central Bank incurs be a fixed one, as a short-cut for the more complicated, possibly state-dependent, adjustment cost the Bank would face in a full scale general equilibrium model. Thereby, I abstract from any possible interaction between the adjustment cost and the state variables. Since the main objective of this paper is to model weekly interest rate behavior with a reduced form equation, the absence of a theory-based parametrization of the cost function will not constrain the analysis.

Let y be the natural logarithm of GNP and $\overline{y}$ its steady state level. The increment in the interest rate is denoted by $\triangle\ i$ and its expectation by $\triangle\ i^e$, while ϵ represents a random disturbance with mean zero and variance σ_ϵ^2. I^+ is an indicator function that takes the value 1 if $\triangle\ i > 0$ and zero otherwise, while I^- takes the value 1 if $\triangle\ i < 0$ and zero else. Suppose that the adjustment costs are independent of the state of the economy, but contingent on the sign of the change. So $\overline{c}$ and $\underline{c}$ are the costs associated with an increase respectively a decrease in the interest rate. GDP follows a Lucas inspired supply curve. The only difference is that income deviates from its steady state level due to unexpected interest changes rather than unexpected inflation.

$$y = \overline{y} + \alpha\left(\triangle\ i - \triangle\ i^e\right) - \epsilon \tag{2.1}$$

To change an inflation based Phillips curve into (2.1) one would need a link between the interest rate and inflation. Although it is conceptually very easy, I do not formalize that link here for reasons of algebraical simplicity. The Central Bank dislikes both deviations of income from its preferred level $\overline{y}$ and changes in the interest rate, with the cost having both a fixed and a variable component. Consequently, it faces the following optimization problem:

$$\min_{\{\triangle\ i\}} \frac{1}{2}\left\{\left(y-\overline{y}\right)^2 + \beta\left(\triangle\ i\right)^2 + \overline{c}\cdot I^+ + \underline{c}\cdot I^-\right\} \tag{2.2}$$

which it solves by setting the interest rate given interest rate expectations $\triangle i^e$.[5,6]

This very simple model implies the discrete type of optimal interest rate policy that we actually observe. The Central Bank's decision rule will be discrete and non-symmetric around zero if adjustment costs are asymmetric:[7]

$$\triangle i \;=\; \begin{cases} 0 & if \quad \underline{\epsilon} \leq \epsilon \leq \overline{\epsilon} \\ \frac{\epsilon}{\alpha^2+\beta} & if \quad not \end{cases} \tag{2.3}$$

where

$$\underline{\epsilon} = -\frac{1}{\alpha}\sqrt{\underline{c}\left(\alpha^2+\beta\right)} \qquad \overline{\epsilon} = \frac{1}{\alpha}\sqrt{\overline{c}\left(\alpha^2+\beta\right)} \tag{2.4}$$

Only when the economy is exposed to big, unexpected shocks and the steady state interest rate comes to differ too much from the optimal rate, will the Central Bank adjust the interest rate. The circumstances under which this happens depend on the parameters that describe the economy, the objective of the Central Bank and the size of the adjustment costs.

Although this model is a crude abstraction of reality, its quintessential property will generalize to more complicated, dynamic settings. If the Central Bank forms its optimal decision rule in a dynamic economy then its interest rate policy could well be a function of both simultaneous and lagged values of macroeconomic and financial variables. However, even then, if one accepts the existence of some adjustment cost, will the monetary authorities' optimal policy follow a discrete, possibly non-symmetric, decision rule. In the presence of adjustment costs, the interest rate will not change when the optimal frictionless rate change is small. First when the gain of the optimal interest rate change is bigger than the cost of adjustment will the Central Bank decide to increase or decrease the interest rate. This mechanism is independent of the variables determining the optimal frictionless rate change.

Suppose $\mathbf{X}_t$ is a vector containing all relevant variables *observed* by the Central Bank, $\boldsymbol{\beta}$ a vector of structural parameters and ε_t a random disturbance, that is unrelated to ϵ above, with zero mean and σ^2 variance. Let $i_t^* = \widetilde{h}\left(\boldsymbol{\beta}, \mathbf{X}_t, \varepsilon_t\right)$ be the

[5]If we let i^{ss} be the steady state level of interest, then $\Delta i = i - i^{ss}$ in this static model, just as in Obstfeld [21]. For the same reason, we could also write $\Delta i - \Delta i^e = (i - i^{ss}) - (i^e - i^{ss}) = i - i^e$.

[6]It is tempting but confusing to talk about discretion versus rules instead of adjustment versus non-adjustment. Since both full discretion and following a rule can imply that adjustment is optimal, I prefer the latter terminology. The adjustment regime I use here is a full discretion policy.

[7]Derivations are provided in Appendix B.

optimal interest rate in the absence of adjustment costs, where the * indicates that the variable is unobserved. I will restrict myself to functions $\widetilde{h}(\cdot)$ with an additive disturbance, so that $i_t^* = h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t$. In this dynamic macroeconomic context the Central Bank's interest rate policy rule, in the presence of an adjustment cost like the one in the model (2.1) – (2.2), would be of the form:

$$\triangle\, i_t = \begin{cases} i_t^* - i_{t-1} & if \quad i_t^* < \underline{a} + i_{t-1} \\ 0 & if \quad \underline{a} + i_{t-1} \leq i_t^* \leq \overline{a} + i_{t-1} \\ i_t^* - i_{t-1} & if \quad \overline{a} + i_{t-1} < i_t^* \end{cases} \tag{2.5}$$

or equivalently, in a notation more similar to that in (2.3):

$$\triangle\, i_t = \begin{cases} h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t - i_{t-1} & if \quad h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t < \underline{a} + i_{t-1} \\ 0 & if \quad \underline{a} + i_{t-1} \leq h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t \leq \overline{a} + i_{t-1} \\ h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t - i_{t-1} & if \quad \overline{a} + i_{t-1} < h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t \end{cases} \tag{2.6}$$

In other words, in the presence of adjustment costs $\triangle\, i_t$ will be non-zero only for values of i_t^* outside the range $[\underline{a} + i_{t-1}\ ,\ \overline{a} + i_{t-1}]$. The parameters $\underline{a}$, $\overline{a}$, analogous to $\underline{\epsilon}$ and $\overline{\epsilon}$ in (2.4), are a function of the economy's parameters, the Central Bank's preferences and the adjustment cost. A difference from the static model is that the limits that i_t^* has to exceed, in order for a *change* in i_t to be optimal, now vary over time.

It is not the notion of Central Banks incurring some kind of adjustment cost that I consider being controversial, given our observation of their following discrete policy rules. Rather, as it has been in all earlier work in the adjustment cost literature, the issue really at stake is what the adjustment cost originates in. Moreover, are $\underline{a}$, $\overline{a}$ constants or are they a function of the state of the economy, $\underline{a}(\mathbf{X}_t)$, $\overline{a}(\mathbf{X}_t)$? Here, I will not go into this subject any deeper and suffice with a reference to the literature I reviewed in the beginning of this section.

The next step will be to test if a decision rule like (2.5) can accurately describe monetary policy. For this purpose, I will use a grouped data model. In addition, I will assume a specific functional form for $h(\boldsymbol{\beta}, \mathbf{X}_t)$. Section 3 reviews previous econometric work on reaction function estimation and discusses the underlying assumptions and the workings of the methods proposed here. Section 4 contains the empirical results.

3. Econometric modeling

There exists an extensive literature on the estimation of reaction functions, that broadly breaks up into two categories. Papers of the first type generally consist of ordinary least squares regressions of single equation reaction functions. Khoury [16] offers the most complete survey of this line of research. Studies in the second group usually employ a VAR approach. Brunner [6] is a recent, representative and interesting paper in this category, that contains some new insights about U.S. monetary policy between 1959 and 1993. Most importantly, he found in his study on American monetary policy that the federal funds rate is the best *indicator* of the stance of monetary policy - even in periods when the Fed was reportedly targeting reserves.[8] In the literature, this view has also been supported by Bernanke and Blinder [5]. Christiano and Eichenbaum [8] on the other hand preferred a reserve measure. Brunner also demonstrated that the federal funds rate is predetermined with respect to other reserves market variables and general macroeconomic variables. Moreover, the monetary authority does not have direct information about economic activity in the current week or month. Hence, it is permitted to estimate the reaction function by OLS regression of the target rate on the appropriate lagged variables in the Central Bank's information set.[9]

Both approaches suffer from the same weakness, namely that they ignore the high frequency behavior of interest rates. Figure 1.1 in Section 1 clearly displayed how the Swedish repo rate typically is constant during long periods of time. An econometric model estimated on monthly data is not fit to explain Central Bank behavior at higher frequencies. Interest rates, especially rates for very short maturities, can make jumps in response to changes in the Central Bank's interest rate instrument, that are especially important directly after their occurrence. Since the OLS estimator is inconsistent when the dependent variable is censored, the size

[8]The argument consists of two steps. First, open market operations are shown to be predominantly determined by deviations in the federal funds rate from its target and much less by deviations in non-borrowed reserves. Thus, the federal funds rate has been the Fed's *operational* target - even in periods when reserves were reported to play that role. Next, variance decomposition reveals that more than 86 percent of the variance of the federal funds rate is due to monetary policy shocks , while the portion of the variance of nonborrowed reserves due to policy shocks amounts to a mere 7 percent. Conditional on the identifying assumptions made by Brunner, this confirms that even to the extent that non-borrowed reserves *were* targeted, they weren't quantitatively important.

[9]Combined with the results mentioned in the preceding footnote, this also means that such a regression is the best means to uncover monetary policy shocks.

and timing of the jumps in daily or weekly series of the repo rate can't possibly be explained by the OLS or VAR based models that have been used. Some authors conveniently circumvented this problem, for example by utilizing the daily federal funds rate - which fluctuates around its target - rather than the target itself. Typically, such a daily rate is a market rate whereas the target is a pure policy instrument. The variables that cause fluctuations in the daily rate around its target, and the way in which they do this, are likely to be very different from those determining movements in the target itself.

The objective of this paper is to explain fluctuations in the targeted interest rate at weekly frequencies. Therefore one needs to take the censored nature of the interest rate data explicitly into account in order to obtain consistent estimates of the parameters in the reaction function. I estimate the Swedish Riksbank's reaction function with a grouped data model.[10]

The data might have allowed us to estimate the model with a Tobit model with both an upper and lower censoring threshold. This is less attractive, however, for at least three reasons. First, there are merely 40 uncensored obervations in the data set, making it unrealistic to expect great predictive power *within* the uncensored range. Second, many of the uncensored observations take round values like 0.05, 0.10, 0.15, etc., and there is very little reason to believe that the remaining uncensored observations (only 8) differ in any systematic way from the others. Third, as will be illustrated in Section 4, efficiency considerations require the model choice to be determined by the purpose it has to serve. As I mentioned above, the scarcity of uncensored observations means we should expect little from the model's ability to make point forecasts for the interest rate. However, the model may have a good potential to predict increases and decreases, possibly even for different intervals. On these grounds, grouping the observations seems to be the best way to handle the data. This is illustrated in figure 3.1. The 45 degree line represents all changes in the repo rate that are actually observed - without indicating relative frequencies of the various changes. Only values very close to zero are not observed. Instead of using the actual values, I group the changes according to size and maximize the likelihood that the observations belong to their respective intervals. Cuts bigger than a_1 are assigned to interval $(-\infty\ ,\ a_1)$, decreases smaller than a_1 but bigger than a_2 are assigned to the interval $[a_1\ ,\ a_2)$ etcetera - as is illustrated in the third quadrant. I will let the models' performance

[10]Greene [14] contains a compact description of both the ordered probit and the grouped data model. Stewart [23] describes the latter model more extensively.

determine the number of groups that we will allocate the observations to.[11]

Let us assume that the underlying, unobserved, interest rate is linear in the explanatory variables and has an additive normally distributed disturbance. Then the model from Section 2 where $i_t^* = h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t$ will simplify into

$$i_t^* = \boldsymbol{\beta}'\mathbf{X}_t + \varepsilon_t\ ,\ \ \varepsilon_t \sim N\left(0, \sigma^2\right) \tag{3.1}$$

where the superscript $*$ indicates an unobserved variable. The matrix $\mathbf{X}_t$ could in principle even contain simultaneous values of the the explanatory variables. Since monetary authorities do not have direct information about economic activity in the current period, $\mathbf{X}_t$ will exclusively consist of lags of the regressors.

Although i_t^* is not observed, we can interpret (3.1) as the 'first best' or 'frictionless' interest rate: if there were no adjustment costs, the interest rate would adjust continuously and such a frictionless interest rate would follow a decision rule that is continuous in the explanatory variables.

Despite i_t^* being censored, we can observe the discrete variables i_t and $\triangle i_t$. These even inform us about the range in which $\triangle i_t^*$ falls when $\triangle i_t = 0$. We collect all changes and divide them into groups according to their size. The groups, or intervals, are ordered such that an observation in the second interval can be said to be greater than one in the first, those in the third interval are greater than the ones in the second, etcetera.

Consequently, we will be replacing the policy function (2.6)

$$\triangle\, i_t \ = \begin{cases} h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t - i_{t-1} & if \ \ h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t < \underline{a} + i_{t-1} \\ 0 & if \ \ \ \underline{a} + i_{t-1} \leq h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t \leq \overline{a} + i_{t-1} \\ h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t - i_{t-1} & if \ \ \overline{a} + i_{t-1} < h(\boldsymbol{\beta}, \mathbf{X}_t) + \varepsilon_t \end{cases}$$

with the following structure, that we will find easier to estimate:

$$\triangle\, i_t \in \begin{cases} (-\infty,\ a_1) & if \quad i_t^* < i_{t-1} + a_1 \\ [a_1,\ a_2) & if \ \ i_{t-1} + a_1 \leqslant\ i_t^* < i_{t-1} + a_2 \\ \cdot & \quad\ \cdot\ \ \cdot \\ \cdot & \quad\ \cdot\ \ \cdot \\ [a_{K-2},\ a_{K-1}) & if \ \ i_{t-1} + a_{K-2} \leqslant\ i_t^* < i_{t-1} + a_{K-1} \\ [a_{K-1},\ \infty) & if \ \ i_{t-1} + a_{K-1} \leqslant\ i_t^* \end{cases} \tag{3.2}$$

[11] Group probabilities obtained from a model with for example 7 groups can simply be summed to calculate the probabilities for bigger groups, that contain one or more of these 7 groups. However, a model that has been computed with observations split up into a smaller number of groups will make better predictions for that purpose.

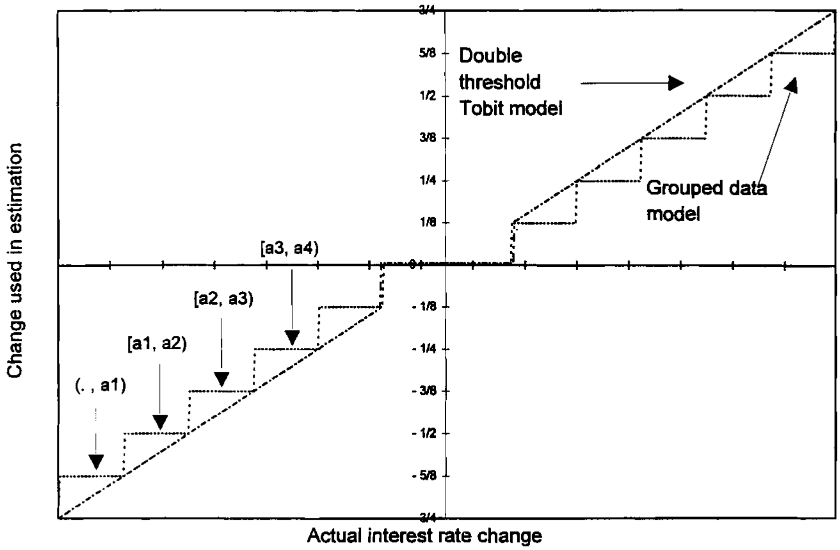

Figure 3.1: Grouping of observed interest rate changes in the grouped data model and a comparison with a double threshold Tobit model.

The corresponding likelihood function is

$$
\begin{aligned}
ln\ L = \quad & \sum_t d_t^1\ ln\ \Phi\left[\left(a_1 + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t\right)/\sigma\right] + \\
\sum_{k=2}^{K-1} & \sum_t d_t^k\ ln\left\{\Phi\left[\left(a_k + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t\right)/\sigma\right] - \Phi\left[\left(a_{k-1} + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t\right)/\sigma\right]\right\} + \\
& \sum_t d_t^K\ ln\left\{1 - \Phi\left[\left(a_{K-1} + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t\right)/\sigma\right]\right\}
\end{aligned}
\tag{3.3}
$$

where $\Phi(\cdot)$ is the standard normal cdf and d_t^k a dummy that takes value 1 if $\triangle\ i_t$ belongs to the k-th interval and zero otherwise. The number of groups K is exogenous in the estimation as are the the threshold values a_z. Note that one implicitly imposes an identifying restriction when assuming that the thresholds are exogenously given. In the estimation of (2.5) and (2.6) when i_t^* follows (3.1), $\underline{a}(\mathbf{X}_t)$, $\overline{a}(\mathbf{X}_t)$ are estimable in theory. However to be able to identify them, one would have had to parameterize these functions. Instead, what I have done now is to assume that $\underline{a}$ and $\overline{a}$, or a_1, a_2, ..., a_K if one so prefers, are exogenously given constants.

We can compute a conditional mean function in analogy to that in the Tobit model[12]:

$$E[\ i_t^* \mid \mathbf{X}_t,\ a_{k-1} + i_{t-1} \leqslant\ i_t^* < a_k + i_{t-1}] = \boldsymbol{\beta}'\mathbf{X}_t + \sigma \cdot \frac{\phi\left(z_{t,k-1}\right) - \phi\left(z_{t,k}\right)}{\Phi\left(z_{t,k}\right) - \Phi\left(z_{t,k-1}\right)} \quad (3.4)$$

where $z_{t,k} = \left(a_k + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t\right)/\sigma$ and $\phi(\cdot)$ is the standard normal pdf. Unfortunately, this is a prediction of the *un*observed variable conditional on both $\mathbf{X}_t$ *and* its group membership, and therefore a less interesting quantity[13]. Instead, as an alternative conditional forecast, one can calculate to which interval i_t belongs with the highest probability, conditional on observing $\mathbf{X}_t$. This can be used as a complement to the point estimator provided by (3.1).

Analogous to the calculation of the conditional forecast described above, it has been common in so called ordered probit models to use the marginal effect of changes in the explanatory variables on the *cell probabilities* - instead of on the point estimate (3.1) - as marginal effects:[14]

$$\partial \text{Prob}\left[cell\ k\right]/\partial \mathbf{X}_t = \\ \begin{cases} -\phi\left(\frac{a_1 + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t}{\sigma}\right) \times \boldsymbol{\beta}/\sigma & k=1 \\ \left[\phi\left(\frac{a_{k-1} + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t}{\sigma}\right) - \phi\left(\frac{a_k + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t}{\sigma}\right)\right] \times \boldsymbol{\beta}/\sigma & 2 \leq k \leq K-1 \\ \phi\left(\frac{a_{K-1} + i_{t-1} - \boldsymbol{\beta}'\mathbf{X}_t}{\sigma}\right) \times \boldsymbol{\beta}/\sigma & k=K \end{cases} \quad (3.5)$$

One typically evaluates these marginal effects at the means of the explanatory variables. Unlike the marginal effects in the linear model, (3.5) can take very different values, even opposite signs, across both observations and cells. It is thus important to be careful when interpreting the parameter estimates.

[12]The analogy derives its origin from the grouped data model's being a special case of the Tobit model - namely where the range of the dependent variable is completely censored.

[13]This can be compared to the case of the standard Tobit where the uncensored range of the dependent variable is the object of the conditional mean function. Also, observe that in the grouped data model $E\left[\Delta\ i_t^* \mid \mathbf{X}_t\right]$ once again simply equals $\boldsymbol{\beta}'\mathbf{X}_t$.

[14]The only difference between the ordered probit and the grouped data model is that the threshold values a_z are endogenous nuisance parameters that have to be estimated in the first model while they are exogenous in the second. This creates a difference in the efficiency of the parameter estimates when the dependent variable is not only ordered but even measurable on a cardinal scale.

Finally, it is important to stay aware of the conditions under which one can estimate the reaction function (3.1)-(3.2) consistently. For the U.S., both Bernanke and Blinder [5] and Brunner [6] have advanced ample evidence in favor of the thesis that the target rate is predetermined with respect to other reserves market variables and macroeconomic variables. They did *not* find any compelling reasons for macroeconomic variables not to depend simultaneously on the target rate or each other. This, however, combined with the fact that monetary authorities do not have direct information about economic activity in the current period, is enough to be able to consistently, although not efficiently, estimate the reaction function (3.1)-(3.2) while disregarding the equations that rule the evolution of the macro variables included in $\mathbf{X}_t$. Since I am primarily interested in the monetary authority's reaction function, I can and will hereafter disregard the equations that govern the motion of the macro variables in $\mathbf{X}_t$.

4. Empirical results

In this Section, I will address two questions. Firstly, is it possible to accurately represent and explain monetary policy in Sweden by means of a discrete decision rule like (2.6)? Secondly, if one can, then what are the driving forces behind changes in the policy interest rate? And thirdly, is there any evidence for pure duration dependence? In other words: is time itself an explanatory variable once one describes the hazard rates for increases and decreases in the repo rate in terms of exogenous variables?

To make the step from the static, theoretical model (2.3) to a dynamic form that we can estimate, like (2.6) or (3.1) – (3.2), one needs to identify the matrix of explanatory variables that compose the information set of the Swedish Riksbank in any week in the period 1992/50-1996/16. I have chosen weekly frequencies because changes in the repo rate have generally been announced on Tuesday morning by the Riksbank. Here, I will take $\mathbf{X}_t$ to consist of a set of commonly used financial and macro-economic variables. A proper benchmark for $\mathbf{X}_t$ to start with is the set of "factors important for future inflation" mentioned in the Bank of Sweden's *Inflationsrapport* ([*18*]). Because of the strong correlation between variables like the consumer prices and producer prices, or broad money and lending to households, some of these had to be eliminated as exogenous variables to avoid multicollinearity problems. Other variables like capacity utilization, the number of available jobs, the number of newly registered cars or the spread between long and short term interest rates were not significant and therefore not used in the final regressions. The list of variables that I employ is shown in Tables 3 and 4 and described in greater detail in appendix A. In addition to the real, monetary and financial variables, two censored time variables have been included into $\mathbf{X}_t$ as an ad-hoc way of testing if Rudebusch's [22] finding of duration dependence in the hazard rate is a robust finding or just a by-product of his exclusion of covariates from the model.[15]

Real and monetary variables including the CPI are expressed in annual growth rates, exchange and interest rates in levels. The motive for choosing this specific transformation of the macro aggregates - rather than taking their levels, which are more common in the VAR literature - is threefold. First, a widely used workhorse model in macro theory like the neoclassical growth model implies an equilibrium relationship between the real interest rate and the growth rate of consumption or GDP.[16] Variations on the basic model include the cash-in-advance economy of Stockman [24], in which equilibrium implies a relationship between the interest rate and the growth rate of money. Next, the treatment of economic news flows in the media strongly suggests that the public has little or no interest for the levels of real and monetary variables. Although the logic behind this behavior may be questioned, economic decision making appears to based on an information set consisting of growth rates of real and monetary variables plus levels of exchange and interest rates. Growth rates do have the advantage over levels of being scale-free. Last of all, in arguing against the use of levels in the regression, one ought to keep in mind that the VAR literature, being a-theoretical by nature, doesn't offer any convincing theoretical justification for its choice of variable transformation.

In order to exploit all relevant data in the Swedish Riksbank's information set, we have used both weekly and monthly data in the estimation of the reaction function. Using the release dates for all relevant macro-economic and financial series, we have composed the Swedish Riksbank's information set for every week in the period December 1992 - May 1996. The data are described in greater detail in Appendix A.

The results from the grouped data estimation of (3.1)-(3.2) for K=3 are summarized in Tables 1 and 2. With the smallest increases and decreases being 0.03 percent, the thresholds were somewhat arbitrarily set at $a_1 = -0.025$ and $a_2 = 0.025$. Moving them closer towards 0.03 did not affect the results, however.

[15] Of course, this tests for a linear type of duration dependence.

[16] For some examples, see Barro and Sala-i-Martin [3].

Although each explanatory variable shows up significantly in the regression at some lag, they do so for only one or two lags, and most of them merely at the 10 % level. I also estimated the model for K = 5, 7, and 13 but found that the model's explanatory power decreased with increasing K. This is probably a consequence of the small number of rate changes in the sample, which makes it difficult for the exogenous variables to predict interest rate changes more exactly than just "up", "unchanged" or "down".

There are two potential explanations for this result. Firstly, the sample used here is rather short, maybe too short to do any inference. Below, however, we will see that there exists another specification of the model with much higher significance levels for the estimated parameters. Secondly, the model described above, although adequately embodying the discrete character of the monetary policy rule, may have failed to capture some other aspect Central Bank behavior. To investigate in what direction we should look for modifications in the specification, I estimated an ad-hoc variant of the model (3.1)- (3.2) in which I regress the deviation of the frictionless rate from the actual federal funds rate on *changes* in the exogenous variables $\mathbf{X}_t$. In other words, I replaced (3.1) by

$$i_t^* - i_{t-1} = \boldsymbol{\beta}' \triangle \mathbf{X}_t + \xi_t \ , \ \xi_t \sim N\left(0, \widetilde{\sigma}^2\right) \tag{4.1}$$

while maintaining (3.2) as the observation rule[17].

A similar hypothesis was brought forward and tested for the U.K. by Eichengreen et al. [11], who estimated the Bank of England's policy function during the interbellum with an ordered probit model. The simple rule of thumb assumed here is that unobserved deviations of the Central Bank's target rate of interest from its current value are determined by changes in the growth rates of some macroeconomic variables and changes in interest and exchange rates.

Table 2 shows that this model performs rather well, especially compared to the model (3.1)-(3.2) above. Except for Tup and the krona-dollar exchange rate, all explanatory variables are significant at the 1 or 5 percent level, some of them at two or three different lags. The Riksbank is responsive to changes in the German

[17] I actually also estimated another modified version of (3.1)-(3.2) where I replaced (3.1) with

$$i_t^* - i_{t-1} = \boldsymbol{\alpha}' \left(\mathbf{X}_t - E\left[\mathbf{X}_t\right]\right) + \boldsymbol{\beta}' \mathbf{X}_t + \varepsilon_t \ , \ \varepsilon_t \sim N\left(0, \sigma^2\right) \tag{4.2}$$

to check if only *unexpected* changes in the exogenous variables might be of importance. The estimate of $\boldsymbol{\alpha}$ was far from significant, however. To save space, I leave out the exact results at this place.

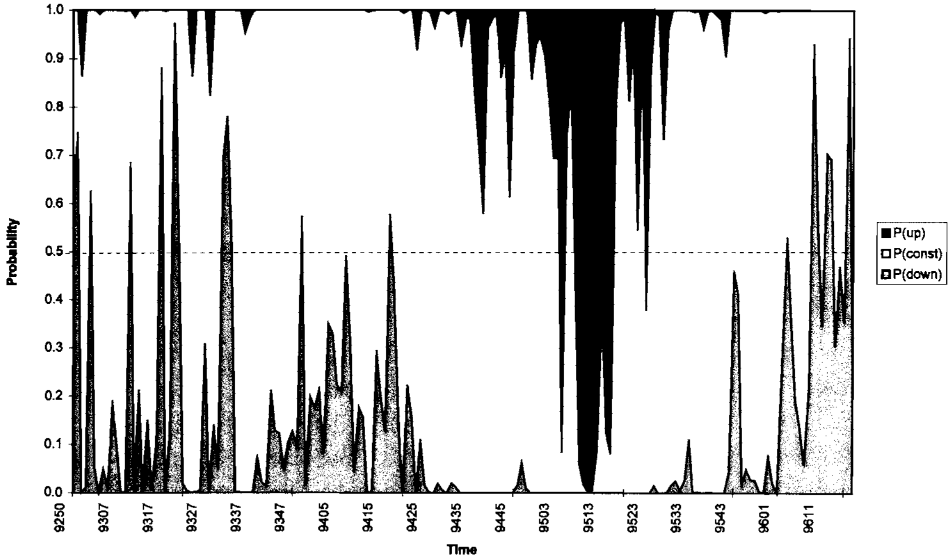

Figure 4.1: Implied probabilities of the Swedish repo rate decreasing, remaining unchanged and increasing.

repo rate two weeks earlier and reacts both simultaneously and with a one week lag to changes in the US federal funds rate. The krona-deutschemark exchange rate influences the equilibrium repo rate simultaneous as well as with a 3-week lag. Unemployment is significant at lag 1 only, while the CPI, retail sales, industrial production and credit growth are significant at two different lags each. There also appears to be a significant duration dependence effect, since Tdown is significant at the 1% level.

On the basis of these results, I proceed to evaluate the model's predictive power. For that purpose, I calculate the probabilities of the repo rate being raised, lowered or kept constant. The model's forecast is then the outcome with the highest probability.

The most straightforward means of assessing the model's forecasting ability is to compare it with the predictive power of a naive predictor. An intuitively attractive predictor would be to 'expect the same as last week to happen'. As is shown in Table 3, this predictor correctly forecasts 125 out of 177 observations (70%). Table 4 demonstrates that the most efficient naïve predictor for our sample is to 'always predict that rates will stay constant'. This has a success rate of 139 out of 178 (78%).

The grouped data model does a much better job and correctly predicts 156

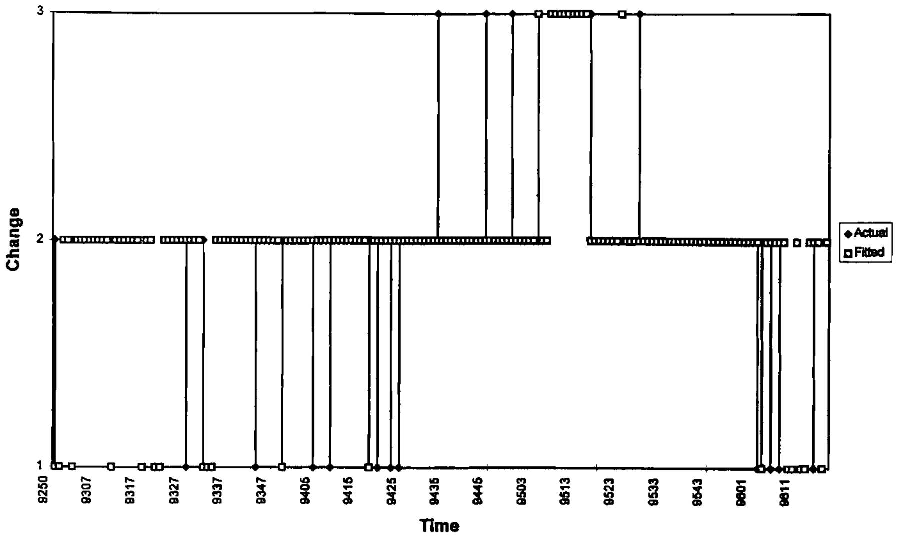

Figure 4.2: Conditional forecasts from the 3 cell grouped data model;1=down, 2=constant, 3=up. Vertical lines indicate an incorrect forecast

out of 178 observations, giving it a success rate of 88%. As one can see in figure 4.2, among the remaining 22 misses, there are another two pairs of observations where an unforecasted 'down' (constant) is immediately followed by an incorrectly predicted 'down' (constant) in the first or second period after.

Of course, as holds for all models with discrete dependent variables, an ideal measure of fit would weigh the errors by their importance. Since the model correctly predicts a rare event, its gain over the naive estimator is likely to be larger than 10 percentage points. At the margin, the cost of failing to correctly forecast any change is likely to be very high.

Figure 4.1 shows the probabilities of the Swedish repo rate increasing, staying constant or decreasing during the sample period December 1992 - April 1996. By comparing figures 4.2 and 4.1 one can easily see that the model, for part of the incorrectly forecasted observations, implies probabilities of the correct change taking place that are very high and sometimes only marginally under 0.5. A non-binary measure of fit would also take such predictive power into account in its evaluation of the model.

In view of the caution that is required when interpreting the marginal effects in the grouped data model, it is rather difficult to impute the shift from high odds of a rate cut between late 1992 and early 1994 to a high probability rate increase in early 1995, and then back again in 1996, to any specific variables. It may be helpful in understanding these shifts to inspect the broad pattern of economic changes that took place between early 1994 and early 1995.

Inflation, generally considered to be one of the most important determinants of interest rate policy, dropped below two percent in Sweden in 1994, then rose by about a percent in 1995 to fall back under the two percent level in 1996. Sales in the retail industry, one of the two indicators of economic activity in the model, lost ground uninterruptedly during the 1992-94 period. In 1996, after a period of growth in the preceding year, retail sales fell sharply. Industrial production grew modestly from the the third quarter of 1993 and quite rapidly from the first quarter of 1994 until the last quarter of 1995. It then lost speed and stagnated in the beginning of 1996. The monetary indicator used here, lending to households, shrank during the complete sample period, but experienced half a year of relative stability in early 1995. Foreign interest rates displayed quite different patterns of behavior. The German repo rate dropped from about nine percent in late 1992 to five percent in mid 1994 and continued its decline a little less than a year later. In the U.S. the federal funds rate rose most of the time, but started falling and kept pace with the German repo rate from the second half of 1995.

Together the above events give us an impression of an economy where many indicators pointed upward at the end of 1994 and in the beginning of 1995: industrial production and inflation went up, the repo rate in Germany had stopped falling whereas the federal funds rate in the U.S. continued rising. In the first half of 1995 many of these variables reversed sign: production growth and inflation flattened, retail sales continued their plunge while interest rates were lowered abroad and this deterioration in the economic climate was accompanied by a shift from a high probability rise to a high probability cut in the repo rate in Sweden.

Finally, the discussion in Section 3 about the interpretation of i_t^* created some hope about our ability to obtain implicit calculations of the 'frictionless' rate of interest. Figure 4.3 displays the outcome of these calculations for a grouped data model with three cells. [18] The general impression we get from figure 4.3 is that the model is not able to produce the same size of changes in the repo rate as we observe in reality. The conditional forecast of $i_t^* - i_{t-1}$ is *always* smaller in absolute value than $i_t - i_{t-1}$ in periods when the repo rate is actually changed. In other words, actual changes in the repo rate are always bigger than the ones

[18] I computed the implicit frictionless rate for models with 3, 5, 7, 9 and 13 groups. The qualitative results were not affected by the number of cells.

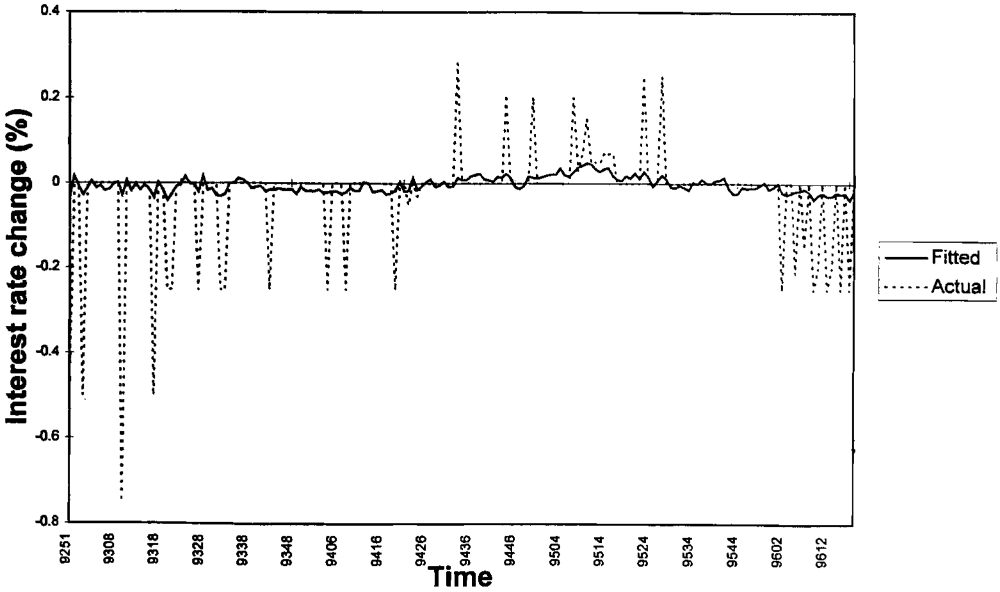

Figure 4.3: Fitted frictionless changes and actual changes in the repo rate.

implied by the 'frictionless' rate.

This should not surprise us, however. The grouped data model, as other likelihood based discrete choice models, does not maximize the model's fit in the sense of minimizing the sum of squared residuals. It maximizes the likelihood of observing the censored variable in different intervals. As a consequence, the model is less likely to produce good conditional (point) forecasts of the dependent variable than any least squares based model. Moreover the model was estimated with observations split up into three groups - up, constant and down. Its ability to make quantitatively accurate forecasts of rate changes should therefore be expected to be small. The real bottleneck preventing the production of accurate conditional forecasts is most likely not the model's intrinsic inability to do so, however, but rather the small number of rate changes in the available sample.

5. Conclusions

The main instrument of monetary policy in industrialized countries is currently a very short interest rate. Such an interest rate, like the repo rate in Sweden, forms the building-stone for all term structure theory and is therefore of interest from both a monetary economic and a financial economic perspective. A typical feature of this interest rate, that has consequently been ignored by monetary economic theory, is that it remains unchanged during long spans of time.

This paper has tried to answer three questions. Why do Central Banks change targeted interest rates so seldomly. Moreover, what are the driving forces behind a change? And finally, is there any sign of duration dependence? For this purpose a simple theory has been developed that can explain why Central Banks smooth this short interest rate through discrete decision rules. It argues that Central Banks face some cost when adjusting the interest rate. One possible interpretation of the adjustment cost is that financial institutions perceive that uncertainty is related to the frequency of interest rate changes. In face of an adjustment cost, the Central Bank's optimal policy is to follow a discrete interest rate rule.

The estimation of a Central Bank's policy rule, or reaction function, has traditionally been done with monthly data - often of a market rate with the same maturity as the targeted rate - and has therefore failed to explain the discrete jumps in the targeted interest rate. Moreover, the factors causing fluctuations in such an interest rate are likely to be very different from those determining movements in the target rate.

In this paper this discrete behavior has been taken into account in the estimation of the reaction function. A grouped data model has been used to estimate Swedish Riksbank's reaction function. It turns out that the Bank's decision rule is better represented in terms of *changes in* growth rates of macroeconomic variables and *changes in* exchange rates and interest rates than in their respective growth rates and levels - as implied by the theory. Probabilities of the repo rate being raised, lowered or kept constant have been computed. These show that repo rate cuts had likelihoods close to one in mid 1993 and early 1996, while increases had such likelihoods in early 1995. The model correctly predicts 88 percent of the observations and 23 out 39 interest rate changes. It thereby clearly outperforms the best 'naïve' estimator, which has a success rate of 78 percent and predicts 'no change' for all observations. The model has no strong ability, however, to make good quantitative predictions of the changes. This is likely to be much better in a bigger sample, where the model can be estimated with more than three cells - as was done here.

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A. Data appendix

Data were obtained from Statistics Sweden (SCB), the Bank of Sweden (Riksbanken, RB), the annual report of Federal Reserve Bank of New York (NY Fed), and Dextel Findata AB

Variable	**Definition** W = weekly data, M = monthly data	**Source**
Srepo	Swedish repo rate (W)	RB
Tdown	Weeks since last rate reduction (zero if last change was up)	-
Tup	Weeks since last rate increase(zero if last change was down)	-
CPI	Annual growth rate consumer price index (M)	SCB
IP	Annual growth rate of indust. production (M)	SCB
Retail	Annual growth rate of retail sales (M)	SCB
U	Unemployment rate (M)	SCB
L2hh	Annual growth rate of bank lending to households (M)	RB
USffr	US federal funds rate (W)	NY Fed
Grepo	German repo rate (W)	Findata
SKDM	SEK/DEM exchange rate (W)	Findata
SKUS$	SEK/USD exchange rate (W)	Findata

While composing the data set, the following has been taken into account. Changes in the repo rate are announced by the Swedish Riksbank on Tuesdays at 10 A.M. Any economic data released before that time is assumed to be in the information set that is taken into consideration in the decision to raise or lower the repo rate. Figures released after that time are assumed to be new information next week. List with release dates and times for all monthly data series were provided by Statistics Sweden and the Bank of Sweden. Foreign interest rates and exchange rates were directly available at daily and weekly frequencies.

B. Mathematical appendix

In the absence of any fixed cost the FOC's of problem (2.1) - (2.2) imply that

$$\triangle\, i = \frac{\alpha\,(\epsilon + \alpha \ \triangle\, i^e)}{\alpha^2 + \beta} \tag{B.1}$$

Given interest rate expectations $\triangle i^e$, this would imply equilibrium income of

$$y = \overline{y} - \frac{\beta.\epsilon + \alpha\beta \triangle\, i^e}{\alpha^2 + \beta} \tag{B.2}$$

We can then evaluate the loss function both when the interest rate is adjusted and when not. Using $(B.2)$ and (2.1) we find that

$$L^{adjust}\left(\triangle i^e\right) = \frac{\alpha}{\alpha^2 + \beta}\left(\epsilon + \alpha \triangle\, i^e\right)^2 \tag{B.3}$$

and

$$L^{const}\left(\triangle i^e\right) = \left(\epsilon + \alpha \triangle\, i^e\right)^2 \tag{B.4}$$

Interest rate changes are optimal only if

$$\begin{array}{l} L^{adjust}\left(\triangle i^e\right) + \overline{c} < L^{const}\left(\triangle i^e\right) \quad \Leftrightarrow \quad \epsilon > \overline{\epsilon} \\ \text{or} \\ L^{adjust}\left(\triangle i^e\right) + \underline{c} < L^{const}\left(\triangle i^e\right) \quad \Leftrightarrow \quad \epsilon < \underline{\epsilon} \end{array} \tag{B.5}$$

Define

$$\begin{aligned} \Xi\left(\triangle i^e\right) &= \{\epsilon \in R \mid \ L^{adjust}\left(\triangle i^e\right) + \overline{c} \geq L^{const}\left(\triangle i^e\right) \ \ and \\ &\qquad\qquad\quad L^{adjust}\left(\triangle i^e\right) + \underline{c} \geq L^{const}\left(\triangle i^e\right)\} \\ &= [\underline{\epsilon}, \overline{\epsilon}] \end{aligned} \tag{B.6}$$

Thus we find that

$$\begin{aligned} \underline{\epsilon}\left(\triangle i^e\right) &= -\tfrac{1}{\alpha}\sqrt{\underline{c}\left(\alpha^2 + \beta\right)} - \alpha \triangle\, i^e \quad \text{and} \\ \overline{\epsilon}\left(\triangle i^e\right) &= \tfrac{1}{\alpha}\sqrt{\underline{c}\left(\alpha^2 + \beta\right)} - \alpha \triangle\, i^e \end{aligned} \tag{B.7}$$

For simplicity we assume that ε is uniformly distributed over the interval $(-\omega, \omega)$ Using B.1 and assuming that $\underline{\varepsilon} > -\omega$ and $\overline{\varepsilon} < \omega$ it follows that

$$\begin{aligned} E\left[\triangle\, i\right] &= E\left[\triangle\, i \mid \epsilon > \overline{\epsilon}\right].\Pr\left(\epsilon > \overline{\epsilon}\right) + E\left[\triangle\, i \mid \epsilon < \underline{\epsilon}\right].\Pr\left(\epsilon < \underline{\epsilon}\right) \\ &= \tfrac{\alpha^2}{\alpha^2 + \beta} \triangle\, i^e \end{aligned} \tag{B.8}$$

The unique fixed point of this equation leads us to the equilibrium interest rate expectations: $\triangle i^e = 0$. Any other solution would imply an ever increasing or decreasing interest rate. Substituting this into $(B.2)$ and $(B.7)$ gives that

$$y = \begin{cases} \overline{y} - \epsilon & \text{if } \underline{\epsilon} \leq \epsilon \leq \overline{\epsilon} \\ \overline{y} - \frac{\beta}{\alpha^2+\beta}\epsilon & \text{otherwise} \end{cases} \tag{B.9}$$

and

$$\triangle i = \begin{cases} 0 & \text{if } \underline{\epsilon} \leq \epsilon \leq \overline{\epsilon} \\ \frac{\epsilon}{\alpha^2+\beta} & \text{otherwise} \end{cases} \tag{B.10}$$

where

$$\underline{\epsilon} = -\frac{1}{\alpha}\sqrt{\underline{c}\,(\alpha^2+\beta)} \text{and } \overline{\epsilon} = \frac{1}{\alpha}\sqrt{\underline{c}\,(\alpha^2+\beta)} \tag{B.11}$$

Table 1 : Maximum likelihood estimators from grouped data regression (3 groups) of $istar_i$ on $\mathbf{X}_t$ and lags if i_t. Standard errors in parentheses. Underlined style for coefficients significant at the 10% level, bold for the 5% level and both underlined and bold for the 1% level.

	Lag 0	Lag 1	Lag 2	Lag 3	Lag 4
Constant	0.93384E-01				
	(0.18255)				
Tup	0.18970E-03				
	(0.39532E-03)				
Tdown	0.20362E-03				
	(0.81061E-03)				
Grepo	0.12083E-01	-0.20928E-01	<u>0.44884E-01</u>	0.13033E-01	0.20978E-01
	(0.21582E-01)	(0.28611E-01)	<u>(0.27860E-01)</u>	(0.28176E-01)	(0.22569E-01)
USffr	**0.31051E-01**	-0.21381E-01	<u>0.31599E-01</u>	0.36896E-02	-0.45899E-01
	(0.16171E-01)	(0.18532E-01)	<u>(0.18787E-01)</u>	(0.16090E-01)	(0.16676E-01)
SKDM	**0.56353E-03**	0.890804E-04	-0.30121E-03	0.48419E-03	0.21897E-03
	(0.28868E-03)	(0.29191E-03)	(0.27934E-03)	(0.31096E-03)	(0.24986E-03)
SKUS$	0.53429E-02	<u>-0.31535E-01</u>	-0.58964E-03	**0.37361E-01**	-0.61547E-02
	(0.16294E-01)	<u>(0.17934E-01)</u>	(0.15979E-01)	**(0.17399E-01)**	(0.14588E-01)
Srepo		<u>**0.52735E-01**</u>	0.51648E-02	0.14230E-01	-0.36915E-04
		<u>**(0.18048E-01)**</u>	(0.18229E-01)	(0.15843E-01)	(0.57912E-04)
U	0.54499E-02	**0.86308E-02**	-0.80819E-02	**0.78225E-02**	
	(0.42323E-02)	**(0.39348E-02)**	(0.43528E-02)	**(0.40684E-02)**	
CPI	**0.78021**	-0.67875	-0.20893	-0.42392	
	(0.37411)	(0.50894)	(0.56787)	(0.52330)	
Retail	0.20032	0.10126	0.88143E-01	0.68749E-01	
	(0.12471)	(0.10522)	(0.11784)	(0.10653)	
IP	0.89463E-01	-0.23582E-01	0.15292E-01	<u>0.14597</u>	
	(0.10222)	(0.94516)	(0.90984E-01)	<u>(0.88678E-01)</u>	
L2hh	-0.21105	<u>0.70695</u>	0.38889	<u>0.45853</u>	
	(0.34605)	<u>(0.40368)</u>	(0.44781)	<u>(0.27076)</u>	

Number of observations 178
Iterations completed 6
Log likelihood function -49.62734
Standard error of regression 0.82220E-02

Table 2 : Maximum likelihood estimators from grouped data regression (3 groups) of $istar_j - i_{t-1}$ on $\Delta \mathbf{X}_t$. Standard errors in parentheses. Underlined style for coefficients significant at the 10% level, bold for the 5% level and both underlined and bold for the 1% level.

	Lag 0	Lag 1	Lag 2	Lag 3	Lag 4
Constant	0.36180E-02				
	(0.32689E-02)				
Tup	-0.46104E-04				
	(0.28891E-03)				
Tdown	**-0.17440E-02**				
	(0.55181E-03)				
Grepo	0.20159E-01	-0.10977E-01	**0.39895E-01**	0.31052E-01	0.34048E-01
	(0.18975E-01)	(0.21486E-01)	**(0.17283E-01)**	(0.23848E-01)	(0.21269E-01)
USffr	**-0.47090E-01**	**-0.42342E-01**	-0.12637E-01	-0.47222E-02	-0.13104E-01
	(0.13181E-01)	**(0.14604E-01)**	(0.13105E-01)	(0.11572E-01)	(0.15160E-01)
SKDM	**0.23810E-02**	0.12474E-02	-0.12117E-03	**-0.31943E-02**	-0.63020E-03
	(0.12369E-02)	(0.12419E-02)	(0.10422E-02)	**(0.10528E-02)**	(0.98794E-03)
SKUS$	0.55878E-03	-0.17393E-02	-0.17490E-02	0.17861E-02	0.12235E-02
	(0.10633E-02)	(0.11005E-02)	(0.97284E-03)	(0.98278E-03)	(0.93527E-03)
U	-0.10979E-02	**0.80244E-02**	0.39249E-02	0.19024E-02	
	(0.25883E-02)	**(0.28512E-02)**	(0.31095E-02)	(0.21858E-02)	
CPI	**0.10911E-01**	-0.39718E-02	-0.11086E-02	**-0.76154E-02**	
	(0.30579E-02)	(0.29819E-02)	(0.31416E-02)	**(0.32375E-02)**	
Retail	0.65910E-03	0.13269E-02	**0.19694E-02**	**0.15100E-02**	
	(0.82445E-03)	(0.10984E-02)	**(0.95057E-03)**	**(0.65623E-03)**	
IP	**0.18190E-02**	**0.14991E-02**	0.10722E-02	**0.17766E-02**	
	(0.69626E-03)	**(0.76423E-03)**	(0.68925E-03)	**(0.63219E-03)**	
L2hh	0.26055E-03	**0.46153E-02**	**0.52282E-02**	0.15559E-02	
	(0.21668E-02)	**(0.23114E-02)**	**(0.23838E-02)**	(0.24482E-02)	

Number of observations 178
Iterations completed 6
Log likelihood function -56.06951
Standard error of regression 0.80632E-02

Table 3

Actual and predicted interest rate changes,
calculated from the probabilities displayed in Figure 4.1.

A			**Predicted**		
c		up	constant	down	
t	up	10	5	0	15
u	constant	3	133	3	139
a	down	0	11	13	24
l		13	149	16	178

Table 4

Actual and predicted interest rate changes, calculated
by taking today´s outcome as the prediction for tomorrow

A			**Predicted**		
c		up	constant	down	
t	up	9	6	0	15
u	constant	6	112	20	138
a	down	0	20	4	24
l		15	138	24	177

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