Essays in Company Valuation

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April 1998

Joakim Levin

Contents

Preface

Given that it is a complex issue, it is perhaps not very surprising that *valuation of individual companies*[1] was not at the top of the research agenda for many years. The seminal works by Modigliani and Miller[2] took an overall view of the value of an individual company and the interrelation between business performance, investments, capital structure, risk, cost of capital, and dividend policy. However, most of the subsequent research focused on a few of these issues at a time, and not on the "whole package".

From the late 1980's and on, academic interest in company valuation has increased. Several textbooks made their way into academic education and put the focus both on how value is created in companies and how companies can be valued.[3] Works by researchers in the accounting field highlighted valuation models based on accounting earnings.[4] The literature has suggested several different valuation models for valuing companies: e.g., the discounted free cash flow model (and the related adjusted present value model), the abnormal earnings (or residual income) model, and the economic value added (or economic profit) model. Together with the discounted dividend model, which is the theoretical benchmark, the models mentioned may be regarded as the most prominent examples of company valuation models. One problem is that when they are implemented under the guidance of textbooks in the area, different models may give different results. A main theme of this dissertation will be to analyse how an equivalence between different models can be achieved. Another issue of concern is the practical implementation of the theoretical results. Emphasis will be put on deriving procedures for implementing the results relating to the equivalence of models and to the links between the cost(s) of capital and the projected future development of the company, and in particular, how a synthesis of these issues can be achieved via implementable discounting procedures.

The term *company valuation* may in itself not be very well defined. In the literature, company valuation sometimes means the valuation of a company's shares, sometimes the valuation of the total company. In this dissertation, *company (firm) valuation* is defined as *valuation that aims at determining the value of a company's equity.*[5] The explicit focus in this dissertation is on *models* for company valuation. The above definition means that both valuation models directly addressed to

[1] Valuation of individual companies (firms) will throughout this dissertation be referred to as *company (firm) valuation.*
[2] Modigliani & Miller (1958, 1963) and Miller & Modigliani (1961).
[3] For example, Rappaport (1986), Copeland, Koller & Murrin (1994) (note: first edition 1990), and Stewart (1991).
[4] For example, Ohlson (1995), Feltham & Ohlson (1995), and Penman (1997).
[5] To avoid confusion, the term *company value* will henceforth be used to describe the value of the company's operations (assets), or equivalently, the sum of the values of the different financing items (debt, convertible debt, preferred stock, common stock, etc.). The value of the company's outstanding equity shares will be denoted by *equity value.*

value the company's *equity* and models where the company's *total value* (as well as the value of each of the financing items) is estimated are considered as *company valuation models* in this dissertation. The dissertation consists of four essays (see List of Essays below) focusing on models *derived from capital value theory*[6] with an aim of *determining value.*[7]

An important issue to take into consideration is a model's transparency. The more transparent a model is, the more easily its validity can be judged by those who are going to make decisions based on its results. It is definitely not true that a theoretically consistent valuation model has to be a "black box". A guideline is that the main ideas and concepts of the model should be understandable to anyone with a basic economic knowledge. In this respect, the widespread textbook by Copeland, Koller & Murrin (1994) has been an important addition to the literature. One main contribution is that it shows the usefulness of explicitly forecasting future balance sheets and income statements. Such an approach makes it possible to undertake explicit account analyses of many different types.[8] The principle of basing valuations on explicitly forecasted financial statements will be followed throughout this dissertation. This approach is suited for implementation in a spreadsheet model where one typically assigns one column to each year and one row for each accounting item.

Essay 1, *On the Fundamentals of Company Valuation*, surveys the literature relevant for company valuation. As already indicated, company valuation involves many interacting components: projections of the company's future operating performance and its relation to investment policy, the company's anticipated future financing policy including the related dividend policy question, and how these issues together are related to the costs of capital. One purpose of this essay is to clarify the different roles of these issues, and how their interaction can be captured in a company valuation framework. Moreover, the essay is devoted to identifying a framework for the author's further studies, reported in Essays 2 – 4. Consequently, this essay may serve as an introduction to the field of company valuation as well as to the studies in Essays 2 – 4.

Essay 2, *Looking Beyond the Horizon and Other Issues in Company Valuation* (jointly with Per Olsson), deals with several issues in the field of company valuation with special emphasis on discounted free cash flow analysis in general and the Copeland, Koller & Murrin (1994) model in particular. The popularity of this model in academic education will make it even more widespread among practitioners once today's students graduate and start to work. This observation is the pragmatic reason for this essay. In other words, the purpose is to undertake a detailed analysis of a

[6] As opposed to models based on statistical analysis and directed at finding relations between observable variable(s) and equity value.
[7] As opposed to models with a purpose of *explaining* relations between equity value and different phenomena.
[8] If a valuation is based on an explicit forecast of balance sheets and income statements, it is possible to translate the forecasted development of the company into well-known economic concepts such as return on owner's equity, profit margin, asset turnover, consolidation ratio, etc.

number of questions connected with the model. A large part of this essay is devoted to problems connected with the horizon value. The primary focus is on clarifying the steady state assumption that underlies the use of horizon values, and the conditions necessary to make this assumption operational. The objective is to suggest a systematic approach, making explicit use of properties of the accounting system. In particular, we note that the time series of forecasted financial statements can be seen as a system of difference equations. Seen in that light, the steady state concept can be made operational by an analysis of difference equations, where all conditions necessary for a steady state can be derived as initial value conditions on the system of equations. The main result is of a normative nature: Flows in the first year after the horizon should be decided such that corresponding stocks grow at the (assumed constant) revenue growth rate. This rule ensures that the company remains qualitatively similar throughout the post-horizon period, which is the main implication of the steady state concept. Although this result is quite simple (and perhaps obvious), the analysis can be viewed as providing a methodology which can be applied to larger and more complicated models as well.

The rest of the report covers a range of subjects that are insufficiently dealt with in Copeland, Koller & Murrin, and that are of general interest in company valuation. The next step is to take on the issue of equivalence: the purpose is to find a discounting procedure that ensures that free cash flow valuation is consistent with the principle that the equity value equals the present value of expected dividends in a setting with corporate taxes. *Complete valuations* (i.e., not just valuations at a horizon) are here considered in an accounting model based on explicit forecasts of financial statements. Further, the essay takes on the implementation related issues by empirically investigating how well the parameters in the Copeland, Koller & Murrin forecasting model of property, plant & equipment items work in three Swedish companies. Three different specifications of these items are investigated in order to assess their usefulness as predictors of the future. Also, the economic interpretations of the parameters are discussed. Other parameters in the forecasting model for some company may for some reason be correlated to industry average figures - one example being the profit margin of businesses operating in highly competitive markets. In such cases, industry average figures may be of some guidance. Such figures are empirically derived for three parameters. Moreover, an implementation of a Copeland, Koller & Murrin type of valuation model is carried out using the results from the previous analysis. The idea is to highlight many of the practical problems that arise in a valuation of this type and to suggest ways of dealing with them

Essay 3, *Company Valuation with a Periodically Adjusted Cost of Capital* (jointly with Per Olsson), takes as its starting point a result from Essay 2: It is perfectly possible to use the popular and intuitively appealing free cash flow approach by periodically adjusting the weighted average cost of capital for anticipated changes in capital structure, so that the discounted free cash flow model is consistent with the present value of expected dividends. This essay extends the analysis by showing how the *cost of equity* and the weighted average cost of capital can be simultaneously adjusted to reflect a varying capital structure. Moreover, different cost of capital settings are introduced and the underlying assumptions of the different settings are related to specific valuation situations, with particular reference to different assumptions of financing policy and the valuation of interest tax shields. Another main purpose is to develop a method for implementing the periodical cost of capital adjustment (and discounting) procedure under the different cost of capital settings.

Essay 4, *On the General Equivalence of Company Valuation Models - Free Cash Flow, Economic Value Added, Abnormal Earnings, Dividends, and the Adjusted Present Value Model in Equity Valuation,* takes its starting point in the fact that many different company valuation models have been proposed in the literature. The main purpose of this study is to analyse the factual relations between the dividend, free cash flow, economic value added, abnormal earnings and adjusted present value models, and to provide a company valuation framework where the *valuation result* is independent of the choice of valuation model. It should be recognised that equivalence results have been established by other authors, but in different settings. An important contribution of this essay is to translate the equivalence results of Feltham & Ohlson (1995) and Penman (1997) into a context 1) where all risk-adjustments are done through the discount rates (i.e., the appropriate costs of capital), 2) where corporate taxation (including tax deferrals) can be handled, 3) where the specifications of the costs of capital are explicitly considered and linked to the anticipated future development of the company (i.e., cost of equity, cost of debt, and weighted average cost of capital allowed to be non-flat), and 4) where the valuation concepts are defined in terms of explicitly forecasted financial statements of the same type as in annual reports. The equivalence is also extended to the adjusted present value model, and shown to hold on a year-to-year basis. Moreover, further objectives are 1) to discuss and analyse horizon value calculations for the different models, 2) to discuss valuation of companies with finite lives, 3) to generalise, and to discuss the implementation of, the discounting procedure derived in Essays 2 and 3. Finally, a comparison of the different valuation models is carried out.

List of Essays

Essay 1: Levin, J. (1998), *On the Fundamentals of Company Valuation.*

Essay 2: Levin, J., Olsson, P. (1995), *Looking Beyond the Horizon and Other Issues in Company Valuation.*

Essay 3: Levin, J., Olsson, P. (1998), *Company Valuation with a Periodically Adjusted Cost of Capital.*

Essay 4: Levin, J. (1998), *On the General Equivalence of Company Valuation Models – Free Cash Flow, Economic Value Added, Abnormal Earnings, Dividends, and the Adjusted Present Value Model in Equity Valuation.*

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Essay 1:

On the Fundamentals of Company Valuation

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Abstract. The purpose of this essay is to discuss some existing theories in corporate finance and how these theories are related to the complex issue of company valuation. Specific issues that are treated are i) how companies determine their capital structures, ii) how the capital structure policy together with the projected development of the company's operating performance may affect its (expected) future costs of capital, and, thus, its value, and iii) the dividend policy's (possible) effect on company value. Further, the essay highlights the issues studied in Essays 2 - 4 in this dissertation and discusses the underlying assumptions of these studies. Thus, this essay may serve as an introduction to the other essays in the dissertation, as well as to the field of company valuation in general.

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1. Introduction

Company valuation[1] is without doubt a complex issue. It involves many interacting components: e.g., projections of the company's future *operating performance* (often measured in terms of accounting concepts) and its relation to *investment policy*, the company's anticipated future *financing policy* including the related *dividend policy* question, and how this altogether is related to the different *costs of capital*. The scope of this essay is to clarify the different roles of these issues, and how their interaction can be captured in a company valuation framework. Moreover, this essay will discuss the points of departure of the other studies in this dissertation (Essays 2 - 4).

One difference between company valuation models is attributable to the actual *purpose* of the model. Some models are of course developed with a sole purpose of *determining value*, while others are devoted to *explaining relations* between different phenomena and value. Examples of the latter are the model developed in Skogsvik (1997), which in an equity valuation framework aims at explaining the contents (and importance) of voluntary disclosures (i.e., voluntary announcements of private information held by firms), and the model in Feltham & Ohlson (1994a) which examines how a company's depreciation method affects the relation between accounting figures and equity value. Furthermore, large parts of the *market microstructure* literature aim at explaining how new (or private) information affects equity values.[2] Different models have also been proposed both as *measures of managerial performance* and for their usefulness as *managerial tools*.[3]

Company valuation models can also be divided into categories on the basis of *the methodology used* for designing each model. Following Skogsvik (1994) we can here identify two categories:

1) models developed from statistical analysis of empirical observations (of, e.g., accounting measures and stock values),
2) models deduced with the support of established capital value theory.

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[1] *Company (firm) valuation* is here defined as *valuation that aims at determining the value of a company's equity*. This means that both valuation models *directly* addressed to value the company's *equity* and models where the values of each of the financing items (that together define the company's *total value*) are estimated are considered as company valuation models.

[2] Some prominent examples are Grossman & Stiglitz (1980) and Kyle (1985, 1989).

[3] Stewart (1991) advocates the economic value added (EVA) model for these purposes.

The first one of these categories of valuation models, based on statistical analysis, will not be considered in this dissertation. Admittedly, this type of models is quite (often very) simplistic in the design. The key point in this approach is to (empirically) find relations between observable variable(s) and equity value, but, as Skogsvik (1994) points out, this type of models often lacks logic relations between equity value and the measures used in the model.

The focus of this dissertation will instead be on theoretically consistent models *derived from capital value theory* that have a sole purpose of *determining value*. One of the most central points of departure in the established capital value theory is that *the value of a stock can be determined as a discounted present value of future expected net cash flows to its owner.*[4] In the literature, this principle is referred to as *the present value of future expected dividends*[5] (PVED), and the following expression will henceforth be mentioned as the PVED-principle:

$$\text{(PVED)} \qquad EV_t = \sum_{s=t+1}^{\infty} \mathrm{PV}_t\left[\mathrm{E}_t\left(DIV_s\right)\right]$$

where EV_t is the equity value at (the end of) period t, DIV_s is dividends in period s, $\mathrm{PV}_t[\cdot]$ denotes the present value operator, and $\mathrm{E}_t(\cdot)$ denotes the expectations operator. The ∞-sign in the (PVED)-expression shows that the expression applies for going concerns (i.e., companies with projected infinite life), but it should also be recognised that it applies to companies with finite life.[6]

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[4] See, e.g., Skogsvik (1994), or Ohlson (1995).

[5] *Dividends* are throughout this dissertation defined as *net* dividends, i.e., dividends minus issued new shares.

[6] T will denote the last period in the life of a company. With respect to the (PVED)-formula this of course means that $DIV_s \equiv 0$ for all $s \geq T+1$. This end of life, T, should not be confused with the (valuation) horizon concept. By a valuation horizon, H, is meant a future point in time where a finite (often detailed) forecast is ended. The future beyond H is accounted for by a horizon value (terminal value). A horizon value approach in the PVED case gives the following modified valuation expression: $EV_t = \sum_{s=t+1}^{H} \mathrm{PV}_t\left[\mathrm{E}_t\left(DIV_s\right)\right] + \mathrm{PV}_t\left[\mathrm{E}_t\left(EV_H\right)\right]$, where the last term is the horizon value (the present value of the expected equity value at the horizon). Note that this expression can be seen as a no arbitrage condition (see, e.g., Penman (1997)). See further Essay 4, section 2.4, for a more general discussion of horizon value techniques in company valuation.

Particularly, the models considered will be discounting oriented models, i.e., models where specific accounting concepts (henceforth referred to as *valuation attributes*) are discounted at the (in each case) appropriate discount rate to obtain the present value of the company's equity.[7] The aim is to develop an accounting based framework of company valuation that will ensure a theoretical consistence[8] in different, more or less complex, valuation situations. This does not mean that the valuation models need to be very complicated. It is in my view quite important to take a model's practical implementation and transparency into consideration. The more transparent a model is, the easier its validity can be judged by those who are going to make decisions based on its results. The topics of implementation, transparency and validity will be further commented upon in section 2.1.

Elton & Gruber (1991) write:

> *"A valuation model can be considered as the black box that converts forecasts of fundamental data about companies and/or the economy into forecasts or evaluations of market price."*
> (Elton & Gruber (1991), p. 475)

However, it is not necessarily so that a theoretically consistent valuation model has to be a "black box". In fact, a good valuation model ought to be understandable by anyone with basic economic knowledge. Not in every detail, perhaps, but the main ideas and concepts of the model should be intelligible. An important publication, which has received much attention and is in line with this principle, is the book *Valuation: Measuring and Managing the Value of Companies* by Copeland, Koller & Murrin (1994). The main contribution is that it conveys the usefulness of explicitly forecasting future balance sheets and income statements.[9] The principle of basing valuations on explicitly forecasted financial statements will be followed throughout this dissertation.

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[7] Option pricing techniques or real option approaches are not considered. Issues like risk-shifting policies, power, and blocking of shares will not be covered.

[8] Consistent with what, one can ask. One principle I will hold on to in this work is that the value of a stock is equal to the present value of future expected dividends (the *PVED-principle).* Other theoretical guidelines will follow from the overview of aspects relevant for company valuation modelling presented in Chapter 2 (see further Chapter 3 for a more complete discussion).

[9] However, the specification of these forecasts that is proposed by Copeland, Koller & Murrin is perhaps not the best one possible (see further Chapter 4 in Essay 2).

Brennan & Schwartz (1984) comment upon the academic interest in company valuation:

> The "*academic interest in the problem of valuing the individual firm has waned. Yet, in addition to the obvious importance of this problem for security analysts, investors, and acquirers of corporations, the issue of firm valuation is fundamental to much of the theory of corporate finance: what determines the risk of the firm; how the rate of return required by investors may be inferred from capital market data; the influence of financing policy on firm value, and other issues of central concern to the theory of financial management.*" (Brennan & Schwartz (1984), p. 593)

One explanation for the less than extensive academic interest in firm valuation in the 1980's is perhaps just that: Firm valuation is a complex task and involves many different issues, each of enough substance and complexity to generate a vast theoretical treatment of its own: the capital structure problem, the cost of capital and risk problems, the dividend puzzle, the capital budgeting issues, and so forth. When considering company valuation, all of these different issues have to be treated at the same time.

However, in recent years company valuation has been brought back to the research agenda. Pioneering work by, e.g., James Ohlson, Gerald Feltham and Stephen Penman have put valuation models based on accounting earnings in the spotlight.[10] Also, company valuation modelling has, in both business and academy, received more attention through the already mentioned textbook by Copeland, Koller & Murrin. In the first edition, they emphasised the free cash flow concept, while they in the second edition also recommend the economic value added model.[11] The economic value added concept was introduced to a larger audience, and promoted, by Stewart (1991).

In the next chapter, some fundamental aspects of company valuation modelling will be identified and further discussed. In section 2.1 the different parts of a company valuation model will be considered: *the theoretical basis* and *the estimation procedure*. Then, the interrelated issues *(choice of) capital structure*, and *cost of capital (and risk) determination* will be discussed in section 2.2, before section 2.3 is devoted to the issue of *dividend policy*. In Chapter 3, a framework for the studies in Essays 2 - 4 is presented: In section 3.1 the general assumptions underlying the studies are presented and

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[10] See, e.g, Ohlson (1990, 1995), Feltham & Ohlson (1994b, 1995) and Penman (1991, 1997).

[11] However, they use the term economic profit instead of economic value added.

discussed, while section 3.2 gives an introduction to the research issues covered in the essays. Finally, Chapter 4 will make a brief summary of the main results of the dissertation and present some implications for future research.

2. Company Valuation Modelling

2.1 Dimensions of company valuation models

I distinguish two important dimensions of a company valuation model:

1) its *theoretical basis* which describes the equity value in terms of a valuation attribute (an economic variable that is fundamentally related to the value of the company's equity), and

2) its *procedure for estimating/calculating* the valuation attribute and other necessary variables.

The first dimension can thus be regarded as the basic valuation formula which the model uses to calculate the equity value, EV_t, at time t. This formula specifies how the expected future values of the economic variable used as valuation attribute are *transformed* into the equity value, EV_t. The most prominent examples of valuation attributes are *free cash flow* (FCF)[12], *dividends* (DIV), *net profit* (NP) (i.e., accounting earnings), *economic value added* (EVA)[13], and *abnormal earnings* (AE)[14]. This theoretical dimension can be characterised further into two sub-dimensions: *valuation attribute* and *transformation.* To clarify, consider the following example: The model presented by Copeland, Koller

[12] FCF is proposed by e.g. Copeland, Koller & Murrin (1994). Note also that company valuation applications of Myers' (1974) APV model can be considered as FCF models. One example is the APV valuation of Gimo AB by Jennergren & Näslund (1996).

[13] The concept of EVA is "marketed" by Stewart (1991). It is, however, in the literature also known as *economic profit.* See further, e.g., Copeland, Koller & Murrin (1994).

[14] See, e.g., Ohlson (1995) for a discussion of this concept for company valuation purposes. It is also referred to as *residual income.*

& Murrin[15] uses the *valuation attribute* FCF and a *transformation* that discounts the expected future FCF by a constant weighted average cost of capital.[16]

The other dimension of the company valuation model, how the valuation attribute and other necessary variables should be estimated and calculated can in a first step be regarded as a "zero-one" variable: either the model explicitly specifies how the necessary variables should be estimated, or it does not. The latter is usually the case in models aimed at theoretical analyses of relations between equity values and other phenomena. However, even models aimed directly at valuation can lack a more detailed description of how the variables should be estimated. It is a valid question why this dimension is considered here at all. A valuation model does not necessarily have to be directly implementable. However, when comparing different models this is an interesting factor to take into account. Company valuation is a complex academic question, but no doubt of large impact and importance for practitioners, whether they are investors, analysts or corporate owners, so hence there is a demand for implementable models.

We are now ready to identify a number of areas that are critical for, or important in, a company valuation. From the first dimension (the theoretical basis), the choice of *valuation attribute,* as well as the specification of the *transformation* that the attribute has to be processed through in order to arrive at the equity and/or company value, appear as the most fundamental topics. From the second dimension follows the question how the valuation attribute and other necessary variables could be *estimated.*

As previously mentioned, there are several valuation models that can be employed. One obvious desire is that the choice of valuation model, given a forecast of fundamental data, should *not* affect the final value obtained from the valuation. This is, however, not automatically the case, as is commented upon later in this dissertation (in Essays 2 and 4). What is needed is that the transformations of the different valuation attributes are carried out in a consistent way. In a world without taxes, equivalence results between the most common models have been provided by Feltham & Ohlson (1995) and Penman (1997).

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[15] This specific model will henceforth be referred to as the CKM model.

[16] To make the story complete, discounting the expected FCF by the WACC does not yield the *equity* value, but the total company value (i.e., the value of the company's operations). In order to end up with the equity value, the value of debt and other non-equity securities has to be subtracted from the total company value.

If the valuation result is independent of the valuation model, and thus of the *valuation attribute*, the choice becomes a matter of taste. A vital aspect to take into consideration is then which attribute is easiest to use from a forecasting point of view. The main purpose of the (valuation) model can also affect the choice. If the model, e.g., is used internally as a management accounting tool, for the evaluation of managers, divisions, etc., then the attribute's characteristics regarding such uses have to be considered. Stewart (1991) advocates the economic value added (EVA) concept on the basis of its usefulness as a "management tool". In this dissertation, a comparison between the most prominent models is carried out in Essay 4 (section 4).

Also, as argued by, e.g., Feltham & Ohlson (1995) and Penman & Sougiannis (1997b), the different valuation attributes (and models) can be seen as consequences of particular accounting systems. In particular, the FCF model, can be seen as a result of cash accounting, whereas earnings based models typically are the result of accrual accounting systems. However, in this dissertation we will start out from one type of accounting system, namely from balance sheets and income statements linked together via clean surplus accounting in a parsimonious, but yet easily extendable, manner. The different models will thus be compared on merits other than how they are related to different types of accounting systems.

The PVED-model is uncomplicated concerning computational effort, because it only requires the use of one cost of capital, the cost of equity capital. However, since dividend pay-outs are much of a policy question, and the dividend sequence may be irrelevant for company value, direct estimation of dividends alone may not seem very logical. Conceptually, however, the PVED-principle (meaning that the value of a stock is equal to the present value of all future expected net dividends to the owner) is often argued to be the correct "benchmark" for any calculation of the equity value. The PVED-model is therefore often used as a starting point in the derivation of equity valuation models.[17] It is hence also desirable for any equity valuation model to be equivalent to the PVED-model.

The *transformation* of the valuation attribute into equity value is in most textbook models some kind of discounting procedure. As an example, consider the basic free cash flow approach, as it is proposed by CKM:[18]

[17] See, e.g., Ohlson (1995), Penman (1997) or Skogsvik (1994, 1997).

[18] For practical reasons we (of course) never do an infinitely long detailed forecast of the FCF: either the forecast period is made long enough, so that the present value of the forecasted FCF for the last particular is (almost) equal to zero, or a horizon value technique is used to calculate the value of the FCF after a certain time horizon. See, e.g., CKM, pp. 274-276, Essay 4 (section 2.4), or Penman (1997).

(1) $$EV_t = \sum_{s=t+1}^{\infty} \frac{FCF_s}{(1+k_{WACC})^{s-t}} - D_t$$

where k_{WACC} is the WACC,[19] calculated at the valuation date, t. D_t is the debt value at the same time.

In contrast, we take a preview of a result in Essay 2. There it is shown that an approach of the CKM type,[20] except for a special case, is *not* consistent with the PVED-principle. However, if the transformation is altered in the following way, the desirable equivalence is achieved:

(2) $$\begin{aligned} EV_t &= \sum_{k=1}^{\infty} \frac{FCF_{t+k}}{\prod_{j=1}^{k}(1+k_{WACC,t+j})} - D_t = \\ &= \sum_{k=1}^{\infty} \frac{FCF_{t+k}}{\prod_{j=1}^{k}\left(1+\left(\frac{D_{t+j-1}}{D_{t+j-1}+EV_{t+j-1}}\cdot(1-\tau)r_D + \frac{EV_{t+j-1}}{D_{t+j-1}+EV_{t+j-1}}\cdot k_E\right)\right)} - D_t \end{aligned}$$

The transformation is in this case a discounting procedure with a WACC, $k_{WACC,t}$, that is continuously updated when the capital structure (in market terms) changes during the forecast period (r_D denotes the borrowing rate, k_E the cost of equity capital). The updating is performed by an iterative procedure that changes the weights in the WACC formula and that simultaneously also can adjust the cost of equity capital accordingly (see Essay 3).[21]

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[19] It should be noted here that a crucial difference between company valuation models and project valuation models is that the discount rate in company valuation does not reflect a hurdle rate (as in project valuation where the project's contribution is measured vis-à-vis the company as a whole) *but the actual cost of financing for the company.* The task of discount rates in company valuation is therefore less complicated than in capital budgeting, since the riskiness of individual projects, which may differ from the overall riskiness of the company, need not be explicitly estimated. (See further section 2.2.)

[20] Essay 2: The following assumptions were made: the accounting is done in accordance with the *clean surplus relation* (see below), the market value of debt is equal to the book value (i.e., the cost of debt is equal to the coupon rate), the dividends are determined as a residual *after* the debt amount has been determined by some assumption about debt policy, and the coupon rate is constant.

[21] The procedure does not require that the model specifications of CKM are used, but just that explicit forecasts of balance sheets and income statements are made.

However, it should be noted that there exists another way of dealing with risk and time than through discounting (or closely related capitalisation): the framework related to certainty equivalent attributes. Instead of adjusting the discount rate for risk, the attribute (e.g., the free cash flows) is risk-adjusted instead. One important contribution was the Rubinstein (1976) article, which derives a valuation formula based on rational risk averse behaviour of investors and no arbitrage in perfect capital markets. This result has been further developed into a company valuation framework by Feltham & Ohlson.[22] The approach is theoretically very interesting, since the valuation expressions are of an appealing, transparent form, which have powerful implications for their usefulness for making theoretic analysis. However, the risk-adjustment of the valuation attribute tends to be a very complex issue, involving complicated contingent probability measures.[23] This apparently makes the approach very hard to implement in a real world company, and such an implementation has not yet been seen (to the author's knowledge). Feltham & Ohlson (1995) circumvent such problems by simply assuming that investors are risk-neutral.

Yet another approach should be mentioned: the *continuous time* valuation approach, closely related to option pricing theory and the seminal works by Black & Scholes (1973) and Merton (1973). One example is the model developed by Brennan & Schwartz (1984). One basic characteristic of this line of models is that firm related variables are assumed to follow different *stochastic processes.* The practical implementation will because of that be quite complicated when the parameters of these processes have to be estimated. This might be one reason why this approach has not gained very much attention in the company valuation literature. Another reason may be that the underlying assumptions concerning the stochastic processes and the continuous time model may be hard to validate in the context of a single company.

As is pointed out by, e.g., Jennergren & Näslund (1988), the most difficult part of a company valuation is the *forecasting procedure* of the value-relevant measures. Even if the model we use is theoretically consistent, it is not very useful if there is no possibility of making good forecasts of the model's parameters.

When taking an overall view on company valuation one should therefore also consider the forecasting procedures. One possible way is to make a *direct* forecast of the valuation attribute(s) and the other

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[22] See specifically Feltham & Ohlson (1994b), or previous analysis and discussion by Ohlson (1990), and Penman (1991).

[23] See, e.g., section IV of Feltham & Ohlson (1994b).

necessary parameters. The valuation attributes are however often "added-up" items[24], and because of that affected by many different factors. A direct forecast, e.g. through time-series analysis in combination with general assumptions, without more explicit consideration of the underlying economic, company specific, factors, could result in valuations that would be hard validate.[25] In many cases, it is in the forecasting procedure therefore better to "step down" to the components that together sum up to the valuation attribute. A powerful approach, already mentioned above, is to explicitly forecast the individual items of the company's balance sheet and income statement. From these forecasted financial statements it is then possible to derive practically *any* valuation attribute. Moreover, this makes it possible to make analyses of many different types. [26] This means that assessments of the reasonableness of a model's assumptions, forecasts and final result are easy to perform. The validity of the final valuation result will thus also be much easier to assess for a decision maker, two parties in a negotiation, an outside auditor, a board of arbitrators, etc., who perhaps lack the technical knowledge of valuation modelling, but possess other types of economic expert skills. Instead of just observing how the made assumptions give one specific value, it can explicitly be made clear how the assumptions made affect the company's forecasted performance over time, through the different types of possible accounting and financial analyses. Different consequences of the assumptions are thus made clear since we do *not* just see how much the equity value is affected, but also the mechanisms behind. The CKM model mentioned earlier is just one example of an application of a more general approach where the common factor is *explicit forecasts of balance sheets and income statements.* To develop and assess different forecasting tools is in my view an important area for future research.[27]

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[24] FCF is, e.g., equal to *revenues* minus *operating expenses* minus *taxes (adjusted for tax savings from depreciation and deferred taxes)* minus *gross investments.*

[25] How could one assess the assumption *'the net profit will grow by 4% a year'* if one at the same time does not make any references to components such as the company's revenue development, profit margin, depreciation policy, etc.?

[26] If the result is based on an explicit forecast of balance sheets and income statements, it is possible to translate the forecasted development of the company into well-known economic concepts such as return on stockholder's equity, profit margin, asset turnover, consolidation ratio, etc.

[27] This view is supported by findings by Rajan & Servaes (1997). They have studied analysts' assessments of initial public offerings (IPOs). Their results show that the analysts systematically overvalue these companies with respect to profits and growth. Another remarkable result is that companies that are judged by the analysts to give the best future development in the long run instead perform worse than others.

The transformation procedures involve different costs of capital that have to be estimated. CKM as well as, e.g., Dimson & Marsh (1981) & Copeland & Weston (1988) propose the traditional CAPM as the way to go when estimating the cost of equity capital. Recently, CAPM has been questioned by, e.g., Fama & French (1993, 1996), who claim that multifactor models (based on the Ross (1977b) APT model) are better.[28] As pointed out by Jagannathan & McGrattan (1995), there are other studies that contradict these findings. In a recent paper, Jagannathan & Wang (1996) assume CAPM to hold in a "conditional sense", i.e., betas and market risk premia may vary over time. Their specification manages to explain the cross-section of average returns in a satisfactory way. Further, the intuitive and simple structure of the CAPM makes it very appealing for the determination procedure of the cost of capital.[29] Nevertheless, cost of capital estimations using CAPM must be done carefully. Further research is definitely necessary before a consensus on what approach to use can be achieved.

Even if there definitely are estimation problems in determining the (relevant) cost(s) of capital, this is no excuse for not considering the cost of capital issue at a greater depth. The cost of capital component in the valuation model is important, especially in a world with corporate taxes where (as will be discussed in section 2.2) capital structure indeed is value relevant. But the cost of capital is not just depending on capital structure. In fact, properly applied the cost of capital specification provides a linkage between projected future performance of the company, the business risk, and the anticipated debt management policy (see further section 2.2 below). This linkage is often missing in the company valuation literature, but is a major concern in the works in this dissertation (see Essays 3 and 4).

However, there is yet another important issue that has to be taken into consideration: the *dividend policy* question. Is dividend policy of relevance for the company's value, or not? This issue will be discussed in section 2.3 below. First, however, the interdependence between costs of capital, capital structure, and other company characteristics is treated in section 2.2.

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[28] Fama & French (1996) find that most anomalies (the size, price/earnings, book-to-market equity anomalies, etc.) resulting from the use of CAPM *disappear* in a three factor model, consistent with APT.

[29] See, e.g., the Betlehem Steel case in Copeland & Weston (1988, p. 531) for a good example on how the WACC is calculated through a CAPM approach. Dimson & Marsh (1981) give a detailed and practical description on how the cost of capital can be estimated using CAPM techniques.

2.2 The interrelation between costs of capital, capital structure and expected performance

The discussion about the relation between different costs of capital, the capital structure and operating performance (including business risk) will start out by considering the capital structure issue. This part will also consider the relevant question for forecasting purposes of how companies determine their capital structures. Then, the different cost of capital specifications developed in the literature on project valuation will be brought to attention, including links to operating and financing risk. Finally, a synthesising framework for company valuation, following Holthausen & Zmijevski (1996), will be discussed and possible extensions will be identified.

The two main questions regarding *the capital structure issue* that appear in the literature are:
1) Is the capital structure relevant for the valuation of the company?
2) How do companies determine their capital structure?

The first of these questions is the one that has primary impact on valuation and cost of capital, while the second question is the one that has received most attention in recent years. The second question has an impact on the forecasting of future financial statements, but also relates to the cost of capital issue by its impact on how interest tax shields should be valued. First, the relevance question will be treated.

As is pointed out by Harris & Raviv (1991), the origin of modern capital structure theory was the Modigliani & Miller (1958) paper.[30] Their since long famous Proposition I states that in an ideal world financing has no impact what so ever on the company's value. This means that the cost of capital for the company, *the weighted average cost of capital* (WACC), is constant with respect to capital structure, or in other words that the required return on operations (also known as the unlevered cost of equity), k_U, is equal to WACC. In this case, by an 'ideal world' is meant that investors act rationally, that the capital market is perfectly competitive, that there are no corporate or personal income taxes or transaction costs, and that companies can be divided into homogeneous risk classes.[31] Further, the company's operations are viewed to be independent of its financing.

[30] Modigliani & Miller (or, as well, Miller & Modigliani) will henceforth be denoted by MM.

[31] This means that the shares of two companies in the same risk class are perfect substitutes (MM (1958), p. 266).

Adding corporate taxes and the possibility of tax-deductible interest payments into the analysis, MM (1963) show that, under these more real-world-like assumptions, there will be a capital structure effect on the cost of capital and thus on value. As MM stress (p. 439), this effect on value will entirely be attributable to the deductibility of interest payments.[32]

The MM results were not to remain unquestioned. One early example of empirical tests of the MM (1958) Proposition I was Barges (1963). Through three different empirical tests, he found that the proposition appeared untenable. A later example was Baxter (1967) who aimed at the MM assumption of independence between financing and the anticipated stream of net operating earnings. The MM proof starts out with two identical firms, both all-equity financed and with equal streams of net anticipated earnings. Then one of the firms swaps some of its equity for debt. By a pioneering no-arbitrage argument, MM show that the firms must have the same value. However, Baxter argues that the levered firm is more likely to be thrown into receivership, from, e.g., a succession of bad years, than the unlevered. Thus the risk of ruin (in effect different bankruptcy costs)[33] will be higher for the levered firm, and this risk can not be cancelled out in the arbitrage argument. Baxter concludes his analysis by stressing that high leverage can lead to an increased average cost of capital, and "the result is the traditional cost of capital curve, declining at low amounts of debt, but rising where leverage becomes substantial" (p. 402).

Baxter's view received support by some empirical studies and became rather widespread.[34] His view implies that there exists an optimal capital structure as a result of the U-shaped cost of capital curve (with respect to the debt ratio) that in turn stems from the trade off between the gains[35] and the associated (bankruptcy) costs of debt financing. As Miller (1977) comments, this view takes the theory of capital structure back to the traditionalist view, though for different reasons.[36]

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[32] As opposed to the so-called *traditionalist view* that viewed debt financing in itself, regardless of taxation, as lowering the cost of capital (MM (1963), p. 439). See, e.g., MM (1958), pp. 276-77 for a more thorough discussion.

[33] Baxter (p. 399) points to "financial embarrassment" as one of the most substantial costs of bankruptcy, e.g. the firm's reliability may be questioned by customers who then take their business elsewhere.

[34] Miller (1977) comments on this view and calls it "currently fashionable" (p. 262). See also Miller's paper for more references.

[35] That is, the tax-deductible interest payments.

[36] However, there were other works that supported MM Proposition I. Most prominent perhaps the work by Stiglitz. Using general equilibrium models, in his (1969) and (1974) papers he shows that the risk class

Miller challenges this bankruptcy cost versus debt tax shields view, by arguing that the bankruptcy costs "seem disproportionately small" in contrast to the tax savings they ought to balance. Further, by taking personal taxes into consideration, he shows that in market equilibrium *there is no optimal capital structure* and that *the company value will be independent of financial structure.*[37]

But, as Miller recognises, the question now turns to *how* companies determine their capital structures. As he points out no company can be expected to have "enough reliable information" to maximise the company value in accordance with the equilibrium. Instead, the actual decision making procedures in companies "are inevitably heuristic, judgmental, imitative and groping"[38]. Through "evolutionary mechanisms" the used "rule-of-thumb" procedures that lead to consistency with rational market equilibrium survive, while the "harmful heuristics" will be exterminated. The surviving heuristics, which Miller calls *neutral mutations,* will thus survive as long as they do not lose their unharmful characteristics through environmental changes.

If we return to the irrelevancy question, DeAngelo & Masulis (1980) show that Miller's irrelevance result is "extremely sensitive to realistic and simple modifications in the corporate tax code".[39] By introducing *non-debt (corporate) tax shields* (such as accounting depreciation and tax credits from investments) there will exist an optimal capital structure for each company in the market equilibrium, even in absence of bankruptcy or other leverage-related costs.

So far, the discussion of capital structure has only been concerned with tax and bankruptcy considerations. However, as we will discover when we take a deeper look into the literature dealing with the question of how firms determine their capital structure, there are many other variables that can be put into the analysis. New inputs to the explanatory models of financial policy have over the

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assumption can be substituted for an assumption that companies issue only so much debt so as to avoid a positive probability of bankruptcy, and still the firm's financial policy does not matter. (In the 1969 article this holds for the firm's choice of debt ratio, while the 1974 article extends the proposition to all aspects of financial policy, e.g. dividend payout ratio.)

[37] A recent empirical study of the implication of Miller's model is Simon (1996) which finds that the "Miller model represents a long-run equilibrium from 1970 through 1985" (p. 54). However, in the short run, deviations from the model are caused by corporate bond default risk and bank borrowing costs. See further Simon's article for more references on emprical tests of the Miller model.

[38] Miller (1977), p. 272.

[39] DeAngelo & Masulis (1980), p. 4.

last 20 years resulted in a quite extensive academic treatment of the capital structure issue. One of the seminal papers in this literature is Ross (1977a) article.

> *"One unfortunate consequence of the Modigliani-Miller insights has been the discarding (and denial) of theories for determining the financial structure."* (Ross, 1977a, p. 24)

To encourage a wider theoretical treatment of capital structure, Ross introduces the idea of asymmetric information into the analysis:[40] the managers of the firm possess inside information that is not available to the market. The MM irrelevancy proposition is however (implicitly) based on the assumption that "the market knows the (random) return stream of the firm and values this stream to set the value of the firm". Ross argues that in the actual marketplace companies are valued on basis of the "*perceived* stream of returns".[41] The basic idea is then that a change in the capital structure can change the market's perception of returns. The change in capital structure can be viewed as a *signal* to the capital market of how well the company is doing. The main mechanism in Ross' model is that an increase in the company's debt will give a positive signal to the market (*positive* in the sense that the market will perceive *higher* returns and thus set a higher value of the company) and vice versa.[42]

Ross' wish for a wider theoretical treatment of capital structure was fulfilled. During the 1980s the capital structure literature was quite extensive. This literature was mainly devoted to answering the question of how capital structure is determined, or more explicitly, to finding the factors that determine the capital structure of a firm. In a survey paper, Harris & Raviv (1991) identify four categories of determinants of capital structure: the desire to resolve/reduce conflicts of interest between different stockholders in the company, the desire to convey private information to markets, the desire to influence the result of corporate control issues, and finally, the desire to affect the nature

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[40] Another important concept, *agency costs*, was introduced into the field of capital structure studies by Jensen & Meckling (1976).

[41] Ross (1977a), p. 25.

[42] The impact of a change in capital structure on the stock price has been empirically tested. Masulis (1980) finds (in a study of exchange offers) statistically significant effects on the returns on common stock, preferred stock, and debt. The findings indicate both a "corporate debt tax shield effect and a wealth redistribution effect" (p. 175), while no evidence of a cost of bankruptcy effect is found. Moreover, the findings are possibly consistent with Ross's signalling hypotheses (see Copeland & Weston, 1988, p. 519). In a follow-up study (on exchange offers and recapatilisations), Masulis (1983) finds support for the Ross hypothesis when it is tested more explicitly.

of products or competition in the product/input market.[43] Each category consists of a number of different determinants. Harris & Raviv state that the empirical tests so far have *not* "sorted out which of these [determinants that] are important in various contexts", but evidence from these studies is in most cases consistent with theory.

Recently, an approach closely related to theories on agency problems and the issue of control rights (to assets) has gained attention in its attempt to construct a theory of the firm and firm policy, including the capital structure question. It is based on the presumption that people are unable to write contracts that are detailed enough to avoid (all possible) conflicts and that power (i.e., rights of control over assets) matters. The findings of this *incomplete contracting* approach so far are summarised by Hart (1995). Aghion & Bolton (1992) claim that "*debt financing is a natural way of implementing contingent control allocations*" (p. 490). Further, their model, which concerns a situation where an entrepreneur needs outside financing, provides a characterisation of the efficient control allocations depending on *future profitability* and the entrepreneur's *private benefits.* This results in a pecking-order: if possible issue *preferred stock* (i.e., entrepreneur control), then try *debt* (i.e., contingent control, depending on a signal of the profitability),[44] and finally issue *voting equity* (i.e., investor control). As can be seen, this type of modelling depicts partnerships in their very *beginnings,* but is not, in my opinion, very applicable to the theory of capital structure for existing or large firms, due to its very basic set-up.[45]

The "capital structure puzzle" will now be summarised by referring to the discussion in Myers (1984): Myers identifies four different frameworks for understanding why capital structures look the way they do:[46] *the managerial framework* which is based on the presumption that different types of conflicts between managers and owners can arise, calling for capital structure choice such that these conflicts are neutralised; *the neutral mutation framework* proposed by Miller (1977) (see above); *the static trade off framework,* which holds the view that a company is striving towards a *target* capital structure ("debt-to-value ratio"); and finally, *the pecking order framework.*[47] The latter framework is the one that Myers finds to be most in line with empirical evidence. He (p. 589) describes the pecking

[43] Harris & Raviv (1991), p. 299.

[44] Note that Aghion & Bolton do not model the debt contingency in a standard way; the signal is not the same as default on debt payments. Therefore, as is pointed out by Hart (1995), the debt reference is not very convincing.

[45] See the Hart (1995) book for a general treatment of the incomplete contracts literature as well as for more references.

[46] See the Myers article for more references on the different hypotheses.

[47] Not to be confused with the Aghion & Bolton (1992) pecking order mentioned previously.

order in the following way: 1) firms avoid to finance investment by issuing common stock (or other risky securities);[48] 2) dividend pay-out ratios are set in order to assure that the normal rates of equity investment are met by internally generated funds; 3) firms *may* cover part of the normal investments with new debt, but they avoid excessive borrowing in order to keep the debt reasonably close to default risk-free;[49] 4) firms will not always be able to issue safe debt;[50] then they issue securities in order of riskiness, starting with the less risky securities, such as risky debt or convertible debt before turning to equity.

Myers views the pecking order and the static trade-off frameworks as the two main combatants in the battle that is to be resolved by more empirical testing. These two combatants differ most importantly in the fact that the observed capital structures will be of a different nature. The capital structure that follows from the pecking order "will reflect the cumulative requirement for external financing"[51] over a long period, while the capital structure that results from a static trade-off point of view will just reflect its current phase of adjustment to the target structure. The pecking order framework, as defined by Myers, recognises both asymmetric information problems and bankruptcy related costs.[52]

Even if the Myers (and Majluf) pecking order framework is supported by some empirical findings and, further, that it seems theoretically appealing, it may be fruitful to pose a number of questions in the attempt to summarise the complex theory of capital structure. As Myers says, "[i]f neither [the static trade-off nor the pecking order] story explains actual behaviour, the neutral mutations story will be there faithfully waiting" (p. 577). A valid question is then: must all firms do the same thing?[53]

[48] The reason for this is that they "do not want to run the risk of falling into the dilemma of either passing by positive-NPV projects or issuing stock at a price they think is too low" (p. 589). See pp. 584-585 for a more detailed discussion.

[49] There are two reasons: "to avoid any material costs of financial distress" and to maintain "financial slack", i.e., to be able to keep the option of issuing more (safe) debt in case it is needed.

[50] Because of, e.g., fluctuating investment opportunities or the "stickiness" of target dividend pay-out ratios.

[51] Myers (1984), p. 589.

[52] The pecking order theory is developed in Myers' joint paper with Majluf (Myers & Majluf (1984)). Its implications have also been empirically supported by several studies, e.g., Eckbo (1986), Mikkelsen & Partch (1986) and Masulis & Korwar (1986). In a recent paper, somewhat related to these issues, McConnell & Servaes (1995) conclude that there is a difference between high-growth and low-growth firms when the effect of debt is measured; high-growth firms get a negative effect from debt, whereas low-growth firms get a positive effect.

[53] Or put differently, is the somewhat contradictory empirical findings a result of the fact that companies do behave in different ways, just like individuals?

Could it be so that some firms try to adjust their capital structures to target debt-to-value ratios, while others follow Myers' modified pecking order, and that both kinds of firms are still not conflicting with capital market equilibria because none of their practises are harmful (in analogy with the neutral mutations hypothesis)?[54] Perhaps, a 'some firms do, and some firms do not' hypothesis is something to take into consideration. This of course makes it of vital importance to try to get an understanding of how the particular firm one is to value carries out its capital structure management.

Anyway, let us conclude that the capital structure issue is a complex one. Under some conditions, capital structure matters, under others, it does not. As Stiglitz comments,

> "*[t]he question has not been so much whether [the MM] assumptions are 'realistic', but whether, or under what circumstances, altering these assumptions leads to situations where financial structure does indeed matter.*" (Stiglitz (1988), p. 122)

The lesson to be learnt from a *company valuation* point of view is that there is a need to identify what makes up the "realistic" assumptions in each particular case and be aware of what those assumptions imply for the theoretical foundation of the valuation model used. In particular, different debt management assumptions will not only affect the particular forecast of the capital structure, but will also affect the valuation of interest tax shields, which in turn will affect the relation between different costs of capital.

Thus, let us now turn our attention to *the cost of capital issue*, and specifically to how the (relevant) cost of capital is related to value (of operations and financing). The cost of capital issue has in the literature historically been developed in close relation to different project valuation (or capital budgeting) models. Thus, the route followed here will be to pursue how the cost of capital has been specified during the theoretic development of project valuation models. Again, we take as a point of departure the MM (1958) Proposition I that implies that the weighted average cost of capital of the firm is independent of the capital structure. When corporate taxes and tax-deductibility of interest payments are considered, the cost of capital of the firm is again dependent on the capital structure, as shown in MM (1963).

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[54] That is to say (to some extent) that all three hypotheses might hold at the same time, but for different companies (or for the same company in different periods).

The basic capital budgeting criterion that evolved in the literature (and in practice) after the work of MM was the net present value (NPV) criterion.[55] NPV is the present value of the cash flows pertaining to the project, i.e., the net contribution to the company's market value if the project is undertaken. The present value is in the standard application obtained by discounting the project's cash flows at WACC.[56,57,58,59]

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[55] See Myers (1974), especially p. 19.

[56] See any finance textbook, e.g., Brealey & Myers (1991).

[57] One slightly different approach is the *flows to equity* (FTE) method, the capital budgeting analogue to the dividend discount model in company valuation. The FTE method calculates present value of the equity owners' *residual* claim to the cash flows (i.e., after adjusting the cash flows for net borrowing) by discounting these residual cash flows at the cost of equity capital. It has not been subject to any extensive treatment in the recent literature, but has been reported to be used in the valuation of real estate projects. See, e.g., Chambers, Harris & Pringle (1982). Note that both the FTE and the standard NPV (with WACC) methods can be classified as risk adjusted discount methods, i.e., as *discounted cash flow* models. For an introduction to discounted cash flow models, see Olsson & Levin (1997).

[58] A completely different approach is the *certainty-equivalent* (CE) method. Instead of using a discount rate that reflects both time and risk, the method separates these two components. Time preference is handled by using the riskfree rate of interest as discount rate, while risk is dealt with through transformations of the expected cash flows into certainty-equivalent cash flows. It should be noted that it is possible to assume that cash flows that take place further ahead in the future are more uncertain, i.e., that the certainty-equivalent also might be smaller for a distant cash flow than for a more immediate cash flow (the ordinary discounting models handle this "automatically" by reducing the value *per period*). As Sick (1986) points out, the CE approach also is more concerned with individual flows than with the aggregated flows of a particular year. This also means that different flows can be assigned different risks (through a cash flow beta). However, the CE approach has not been very extensively covered in textbooks. One reason for this might be that in order to produce a valuation model of reasonable complexity, the cash flows must be an additive function of unsystematic risk and of current and previous market factors (Sick, 1986, pp. 24-25).

[59] Yet another class of project valuation techniques is the one referred to by Keeley & Westerfield (1972) as the *single certainty equivalent* (SCE) approach. The SCE approach first determines a probability distribution of discounted present values (by discounting cash flow sequences at the riskfree rate, i.e., a pure time preference discount rate). Then the mean of the distribution of discounted present values is multiplied by a certainty equivalence factor, before the initial investment outlay is finally subtracted to arrive at the project's value. This method thus separates the timing of the cash flows and the riskiness of these flows. But as Keeley & Westerfield (1972) point out, the result is a negligence of uncertainty beyond the first period of the valuation, and thus incorrect project valuations.

This is not unproblematic, however. Arditti (1973) concludes that WACC will be correct only if the project is financed at the company's optimal debt-ratio *and* if the project does "not alter the overall risk level of the firm's assets".[60] To overcome the problems[61] of the NPV-method, Myers (1974) introduces the concept of *adjusted present value* (APV). The first step in the APV-method is to calculate the "base case value" of the project, i.e., the project's contribution to firm value under the assumption of *all-equity financing and irrelevance of dividend policy.*[62] To this, the value from other sources of financing and/or from relevancy of dividend policy is added.[63] This means that the APV method itself does not rely on any assumptions about the relevance/irrelevance of debt financing and/or dividend policy. At the time, there was a practical problem with the formulation of the APV method: It required a tedious backward computational procedure. This procedure was however simplified by the findings of Ashton & Atkins (1978).

Miles & Ezzell (1980) repropose the standard (textbook) WACC formula approach.[64] They show that the special cases when this formula can validly be used (identified by Myers (1977) and mentioned in footnote 61 above) can in fact be replaced by a particular assumption about the company's financing policy. This critical assumption states that the company adjusts its amount of outstanding debt in each period to keep a constant leverage ratio.[65] This is referred to as *active debt management.* Thus, the

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[60] Arditti (1973), p. 1004, footnote 4. The result is further developed and summarised in Myers (1974) pp. 11-12. Myers analyses three different WACC formulas, the original MM, the generalised MM (all projects do not have to be in the same risk class), and the textbook formula.

[61] After a comment by Bar-Yosef (1977) on Myers' presentation of the necessary conditions needed for a valid use of the WACC approach, Myers (1977) is able to define the special cases when the textbook WACC formula are equivalent to the APV approach (under the assumption that the project and the firm have the same cash flow pattern) as "when 1) the firm is entirely composed of one-period assets, when 2) the firm is a perpetuity, and when 3) the firm is on a steady-state growth path ($C_t = C_1(1+g)^{t-1}$)" (p. 220).

[62] This can practically be done by using the NPV-method with the unlevered cost of equity capital as discount rate.

[63] Myers, p. 5.

[64] Henceforth, the standard (textbook) WACC formula will be referred to as the WACC formula: $k_{WACC} = \omega_D k_D (1-\tau) + (1-\omega_D) k_E$, where k denotes cost of capital, ω_D is the market debt ratio, $\frac{D}{D+EV}$, the subscripts refer to debt (D) and equity (E), and τ is the corporate tax rate.

[65] Given this assumption, the WACC can be computed as: $k_{WACC} = k_U - \tau \cdot i \cdot \omega_D \cdot \left(\frac{1+k_U}{1+i}\right)$.

question *which capital budgeting model to use* became one of the reasonableness, in the specific case, of the assumption about the company's debt management policy.

An implication of the Miles & Ezzell analysis was that active debt management (i.e., keeping a constant leverage ratio) corresponds to discounting the tax-shields of debt financing at a rate equalling the unlevered cost of equity capital, k_U, instead of discounting them at the cost of debt, k_D, as under the assumption of passive debt management.[66] To be precise, at a given point in time the tax shelter from the next period should (still) be discounted at the cost of debt, while the one-period ahead ("present") value of the tax shelters from all other future periods should be discounted at the unlevered cost of equity capital. In formal notation (note that this in practice requires a backward going procedure):

(ME)
$$\begin{cases} PVTS_{t,all} = PVTS_{t,k_D} + PVTS_{t,k_U} \\ PVTS_{t,k_D} = \dfrac{TS_{t+1}}{1+k_D} \\ PVTS_{t,k_U} = \dfrac{PVTS_{t+1,all}}{1+k_U} \end{cases}$$
for *all t* from the valuation date and forward

where $PVTS_{t,all}$ = present value at the beginning of year $t+1$ of all interest tax shields

$PVTS_{t,k_D}$ = present value at the beginning of year $t+1$ of the part of the interest tax shields that is discounted at the cost of debt, k_D, during year $t+1$

[66] k_D has often been argued to be the valid discount rate for tax-shields from debt financing. This was proposed by MM (1963), and used by, e.g., Ashton & Atkins (1978). This proposition is related to the assumption of *passive debt management,* i.e., that the firm determines its financing plan once and for all. However, Modigliani argues later in his (1988) article that "[t]his assumption seems untenable [...]. It seems much more reasonable to suppose that the leverage policy of the representative firm can be described as aiming at maintaining the debt in a stable relation to the scale of the firm as seen at any given date." (p. 152). The explanation for the difference in discount rate between the two cases is that with *passive* debt management the debt amount at any future time is certain, while with *active* debt management the expected debt amount outstanding at any future end of period becomes a direct function of the expected company value at that point in time. In the active debt management case, interest tax shields are regarded as being as risky as the operating flows of the company by the direct link between company value and the size of interest tax shields. The important lesson to be learnt from this, from a valuation point of view, is that the discount rate should reflect the (un)certainty of the tax-shields.

$PVTS_{t,k_U}$	=	present value at the beginning of year $t+1$ of the part of the interest tax shields that is discounted at the unlevered cost of equity, k_U, during year $t+1$
TS_{t+1}	=	interest tax shield in year $t+1$ (corporate tax rate times borrowing rate times beginning debt)

In a recent paper, Clubb & Doran (1995) aim at the middle of the active and passive debt management spectrum, by analysing a "partially active" debt management in a context contrasting APV versus NPV with (constant) WACC. Though APV has the ability of "model[ling] a variety of debt management assumptions", Clubb & Doran conclude that the Miles & Ezzell WACC approach's possible valuation errors are insignificant in relation to the more reasonable debt management assumptions that can be handled by APV.

An alternative capital budgeting approach was introduced by Harris & Pringle (1985). The main contribution is an explicit treatment of projects with differing risk. This is done by rearranging the usual WACC formula into two components: required return due to operating risk (i.e., the unlevered cost of equity), k_U, *minus* tax benefits of debt financing: $k_{WACC} = k_U - \tau \cdot k_D \cdot \frac{D}{D+EV}$ where k_U is specified as $k_D \cdot \frac{D}{D+EV} + k_E \cdot \frac{EV}{D+EV}$ and assumed to be constant. This formulation implies a particular assumption about the valuation of tax shields: all interest tax shields are valued at k_U. This is almost the same as in the Miles & Ezzell formulation (where, for any given future point in time, the first period interest tax shield instead is discounted at the cost of debt, k_D) and the two approaches will yield almost the same valuation (see, e.g., Essay 4 (section 3) or Taggart (1991)).

Another way of attacking the problems with risky projects (or assets) was provided by Myers & Ruback (1987). They present a model where the discount rate, r^*, for a risky asset can be calculated as a weighted average of the *after-tax return on riskless debt* and *the expected return on the market portfolio*:[67] $r^* = (1-\beta)(1-\tau)r_f + \beta r_m$. The only asset specific item that has to be forecasted is thus the

[67] β is the beta of the asset's cash flow, τ is the corporate tax rate, r_f is the risk-free rate, r_m is the expected rate of return on the market.

asset's beta. As suggested by Taggart (1991), this approach may be useful since it is independent of tax regime.[68]

Harris & Pringle (1983) study the arguments implied by the Miller (1977) article on the irrelevancy of capital structure. They are able to show the important result that the textbook WACC formula is the correct specification of the weighted average cost of capital, regardless of whether Miller's proposition is valid or not (p. 14).[69] However, the textbook WACC approach still has the same limitations that have been discussed above. Harris & Pringle state that the main contribution of Miller's (1977) work is the focus on the fact that "the market value of a project (or firm) ultimately depends upon the cash flows that can be made to investors after payment of all taxes, both corporate and personal". Therefore, there is a need to include the effect of personal taxes into the standard capital budgeting procedures.[70]

This argument was followed up by Taggart (1991), who gives valuation with *both* corporate and personal taxes a thorough treatment under the assumption of risk-free debt. He derives different systems of equations (for the determination the different costs of capital) that are valid under different sets of assumptions.[71] Moreover, he produces "a road map to choice of valuation method with corporate and personal taxes".[72] This "map" identifies the best method to use in four situations under the assumption that debt is risk-free. It may be a natural starting point for valuations that consider both corporate and personal taxes in the risk-free debt case.

One important aspect to be aware of is that many of the limitations of the WACC approach as a *project* valuation method vanish when it is used as for *company* valuation purposes. The limitations of the WACC approach for project valuation purposes follow mainly from the fact that a project may have a different risk than the company as a whole. When valuing companies we are concerned with the overall risk of the company which in fact is exactly what the WACC describes.

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[68] The usefulness of the approach is limited by the assumption that the company will maintain a very specific capital structure. As Myers & Ruback (p. 2) recognise: "If there is a different optimal [capital structure] policy, and the manager knows what that policy is, project value can exceed our guaranteed value." In any case, the r^* discount rate can be seen as a lower bound.

[69] However, WACC is here constant with respect to financial leverage, which means that the cost of equity and the cost of debt are adjusted accordingly when the financial structure changes.

[70] Harris & Pringle (1983), p. 22.

[71] The different sets of assumptions are in no way complete in the sense that all possible cases are treated.

[72] Taggart (1991), p. 17, exhibit 6.

In project valuation applications, the WACC is almost always assumed to be constant. However, in company valuation applications where the company's future operations *and* financing situation are estimated there is no rationale for assuming a constant WACC. Instead, it is definitely possible to let the WACC reflect the company's (possibly changing) risk characteristics (given some forecasted scenario) by using a non-constant updated WACC.

To summarise the cost of capital issue, capital structure definitely matters when we look at corporate taxation only. The main message from MM (1963) that the value of the firm is equal to the value of the *unlevered* firm plus the value of the interest tax shields is thus a very useful point of departure. However, the MM (1963) analysis and its resulting formulas depend on specific assumptions. Thus the MM (1963) cost of capital formulas are not valid in a general setting. Holthausen & Zmijewski (1996) have however presented a more general framework where the MM analysis arises as a special case.

One key feature of the Holthausen & Zmijewski framework is the recognition that interest tax shields at a given point in time can be discounted at the cost of capital for the unlevered firm, k_U, *and/or* at the cost of debt, k_D. That means that the market value of tax shelters can be expressed as the sum of the present value of the interest tax shield discounted at k_U, plus the present value of the interest tax shield discounted at k_D. Thus, the framework can handle any assumption about the (proper) tax shield discount rate that involves discounting at one of the two discount rates, or cases where some part of the tax shield is discounted at one rate and the rest of the tax shield is discounted at the other.

In a case where the company in year t is financed with debt and equity only, the general relation between the cost of equity (in year t), $k_{E,t}$, the cost of equity for the unlevered firm, k_U, and the cost of debt (in year t), $k_{D,t}$, is the following (Holthausen & Zmijewski, Chapter 2, p. 12):

(3) $$k_{E,t} = k_U + \left(k_U - k_{D,t}\right) \cdot \frac{\left(D_{t-1} - PVTS_{t-1,k_{D,t}}\right)}{EV_{t-1}}$$

where	D_{t-1}	=	market value of debt at the beginning of year t
	$PVTS_{t-1,k_{D,t}}$	=	present value at the beginning of year t of the part of the interest tax shields that is discounted at $k_{D,t}$ during year t
	EV_t	=	market value of equity at time t

The unlevered cost of equity, k_U, is dependent on the firm's operations only and (by Holthausen & Zmijevski) assumed to be constant over time. $k_{E,t}$ is the cost of equity that is adjusted for the beginning of period financing. Note that formula (3) is general with respect to assumptions about the tax shields in that it holds for any case where the tax shields are discounted at k_U *or* $k_{D,t}$, and for any case where some part of the tax shields is discounted at one of the rates and the rest is discounted at the other. We have already touched on the issue of choosing the appropriate discount rate for the interest tax shields above. Under passive debt management $k_{D,t}$ is the proper discount rate[73] and under active debt management in accordance with Miles & Ezzell (1980) the tax shields are valued by a combination of $k_{D,t}$ and k_U, as described in equation system (ME) above. Finally, if *all* tax shields are considered to be as risky as the unlevered cash flows all of them should be discounted at k_U, as in Harris & Pringle (1985).

The Holthausen & Zmijewski framework is attractive since it provides consistent formulations of the relations between the different costs of capital needed for the valuation of a company financed with debt and equity in a world with corporate taxes only. The framework builds on the assumption that the *required rate of return on the economic assets and on the securities financing those assets are equal for any period.*[74] By applying this intuitive principle it is straight-forward to extend the analysis to, e.g., more complex capital structures, and/or alternative tax shield valuation principles, as well as to other tax regimes.

Usually, the framework is applied in the following way: Two of the costs of capital (the cost of debt and the unlevered cost of equity) are assumed to be constant, while the third (the cost of equity) is derived from equation (3). This is a pragmatic assumption: we cannot use a single equation to derive more than one unknown; the other two must be given "exogenously". But as recognised in Essay 4 (section 3) neither the cost of debt, nor the unlevered cost of equity has to be *constant.* We will however need an additional set of equations/relations that defines at least two of the costs of capital,

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[73] This is a result of the fact that with passive debt management we assume that the interest tax shelters are independent of the value of the levered firm. That is, the debt maintenance is assumed to follow a fixed schedule so that all future interest tax shelters are known at the valuation date. If the debt amount is fixed and the company's cash flows are constant in perpetuity, equation (3) can be simplified to the well-known formula provided by MM (1963): $k_{E,t} = k_U + \left(k_U - k_{D,t}\right)\cdot\frac{(1-\tau)D_t}{EV_t}$ where in addition to (3) τ is the corporate tax rate.

[74] By economic assets are meant operating assets and tax shields.

so that equation (3) can be used to derive the third one. This may not be an easy task, but in the following cases it might prove to be useful: Firstly, the term-structure of interest rates may at times definitely imply that the expected costs of capital are different between periods. By specifying, e.g., the cost of debt and the unlevered cost of equity as functions of the (non-constant) risk-free rate, the cost of equity for a given period could be calculated via equation (3). Secondly, we may in a specific case have reason to anticipate that a firm will change the riskiness of its operations, e.g., from risky projects to fairly safe. Thus, it may be reasonable to forecast a gradually lowering unlevered cost of equity. Thirdly, as recognised by Harris & Pringle (1985), it is at extreme debt ratios possible that the required return on operations (i.e., the unlevered cost of equity) is affected by changes in leverage: "[o]ne can argue that eventually financial leverage will affect operating operating risks" (p. 239) (see also the arguments in Baxter (1967)). And, of course, bankruptcy and agency costs may in such cases affect the cost of debt. If such considerations could be quantitatively modelled, equation (3) could be used for obtaining the levered cost of equity.

To conclude, the framework (exemplified in equation (3)) provides an important link between the anticipated future performance of the company (reflected in total company value, i.e., equity value plus debt value), the riskiness of its operations (reflected by the unlevered cost of equity), the debt management policy of the firm (reflected in both the valuation of interest tax shields and the separation of the total company value into equity value and debt value), and the costs of equity and debt. This link is often missing in the literature on company valuation, but is discussed in detail for FCF valuation in Essay 3. Essay 4 is directed at creating a synthesis of 1) the explicit inclusion of this link in a world with corporate taxes, *and* of 2) the valuation model equivalence results in Feltham & Ohlson (1995) and Penman (1997) (which are derived in a setting without taxes).

2.3 Dividend policy

The capital structure and cost of capital issues are as discussed above quite complex. The dividend policy issue is in this respect very similar. Again, we refer to a work by MM as the point of departure. In their (1961) article they claim their famous *dividend irrelevance* result, i.e., that the company's current market value is independent of its dividend policy, *given its investment policy.*[75] This is in

[75] Note that changes in dividend policy will affect future dividends, future (accounting) earnings, future book equity values, and future market values (but not the *current* market value). See Feltham & Ohlson (1994b, section V) for a deeper discussion of this.

contrast to the view that dividend policy *is relevant* for security valuation.[76] The MM (1961) analysis was done without taxes[77] (and transaction costs). Including differential taxes on capital gains and (dividend) income implies that investors would favour low dividends if the tax on income was higher than on capital gains.[78] Even so, irrelevance could hold if the different investors, for some reasons, preferred different pay-out ratios, i.e., if there could exist different clienteles.[79]

Miller & Scholes (1978) do not accept the MM argument of 'clientele' effects to explain dividend irrelevancy in a world with both personal and corporate taxes, but they are able to indicate that for reasons in the tax code, many investors in effect will pay the same tax on dividends as on capital gains. Therefore, dividend policy may be irrelevant even if personal taxes are considered.

The MM proposition could not be rejected by an empirical study by Black & Scholes (1974). They argue (p. 21) that a change in dividend policy may have an immediate effect on the stock price, due to the market believing "that the change indicates something about the probable future course of earnings". However, this temporary effect ought to disappear when it is clear to the market that this is not the case. Thus, in a somewhat longer perspective, dividend policy does not affect the value of the company, and a dividend cut could be "an inexpensive way of providing [...] capital".

Bhattacharya (1979) considers the informational effect of dividend policy changes to be larger. He introduces the signalling framework (as Ross (1977) did for the capital structure issue) in the dividend policy context. The model contrasts the benefits of dividends as a signalling tool (i.e., the "dividends function as a signal of expected cash flows of firms in an imperfect-information setting", p. 259) vs. the costs arising from personal taxation, and the model may explain why companies in fact pay dividends even if dividends are disadvantageous from a tax perspective.

Hakansson (1982) analyses the information content of dividends further and identifies three different cases: "[W]hether informative or not, dividends serve no useful role when investors are substantially homogenous, have additive utility, and markets are complete. When associated with positive costs,

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[76] See, e.g., Friend & Puckett (1964) for the early references. The basic explanation given for this view is that "*investors prefer a dollar of dividends to a dollar of capital gains, because 'a bird in the hand is worth more than one in the bush'*" (Black & Scholes (1974), p. 1)

[77] Although it extends to a world with corporate taxes only. See Copeland & Weston (1988) pp. 548-550.

[78] Some authors in fact argued that it perhaps is best to pay *no dividends at all.* See Copeland & Weston (1988), p. 561 for a few references.

[79] MM (1961), section V.

dividends are under these circumstances deleterious to efficiency. On the other hand, dividends are capable of improving welfare (efficiency) when they are informative provided investors have heterogeneous beliefs, utility is non-additive, *or* markets are incomplete, even in the presence of dead-weight costs. In this context, the power of informative dividends [...] is especially significant; *dividend announcements may under certain circumstances bring an incomplete market to or even beyond the level of efficiency that would be attained if the market were complete*" (p. 416).

In empirical studies on the effect of announcements of dividend policy changes, the first results seemed to confirm that corresponding changes in stock value occur. However, as discussed by Gosnell, Keown & Pinkerton (1996), several later studies found that the informational content of dividends is small. Studies of *large* dividend policy changes have indicated that price is "materially affected" by such large changes,[80] however. Since past studies have relied on *daily* data, Gosnell, Keown & Pinkerton choose to use *intraday* data. They find an asymmetric reaction by the market to dividend increases versus decreases. "In particular, the reaction to negative announcements is stronger and lasts for a longer period than the reaction to favourable announcements" (p. 262).

As in the capital structure case, agency cost approaches have been applied. The points that have been emphasised in this literature are that dividend payments make the firms return to the markets more often to reap the benefits of external monitoring, and that the firms face a trade-off between these benefits (which lower the agency costs) and the cost of external financing.[81] In a recent paper, Noronha, Shome & Morgan (1996) combine the capital structure and dividend policy issues in an agency cost framework. Regarding dividend policy, they find that the rationale for using dividends as a tool for reducing agency costs is dependent on the presence of alternative monitoring vehicles (such as, e.g., a large shareholder) in the particular firm.

Again, we do not get an absolutely clear picture. In a world where every investor has full information and dividend payments are tax-neutral in relation to capital gains, dividend policy is irrelevant. But it is easy to construct situations where this does not hold. To answer the question *why* firms pay dividends (or how they determine what to pay), several stories have been told. Which one of them that is right (or if they all in some sense are right) is still an open question.

[80] Gosnell, Keown & Pinkerton (1996), p. 249.

[81] Rao & White (1994), p. 449.

For company valuation modelling purposes, the important thing is to make sure that the model treats dividend policy irrelevance/relevance in a way that is consistent with the underlying assumptions of the state of the world. In fact, it is desirable that the model should be able to handle both cases, since the dividend irrelevancy question may, perhaps, be most suitable to handle in the actual forecasting procedure and not in the basic formulation of the valuation model. Let us look at what dividend irrelevance really means in terms of accounting concepts: Let NP_t denote the net profit (accounting earnings) in year t, DIV_t the dividends in year t, and BV_t the (ex-dividend) book value of equity at the end of year t. Then clean surplus accounting implies the following relation between these items:

$$(4) \qquad DIV_t = BV_{t-1} + NP_t - BV_t$$

Now let us denote the invested capital at the end of year t by IC_t, and the debt outstanding at the end of year t, by D_t. In a firm with assets consisting of working capital and property, plant and equipment (PPE), the invested capital equals the net working capital plus net (book) PPE. If the firm is financed with debt and equity only (and there are no deferred taxes) then $IC_t = BV_t + D_t$. Equation (4) can now be rewritten as:

$$(5) \qquad DIV_t + (IC_t - IC_{t-1}) - (D_t - D_{t-1}) = NP_t$$

Equation (5) thus describes the relation between the forecasted (accounting) earnings, and the forecasted dividend, investment, and financing decisions of the company. Following Ohlson (1995), equations (4) and (5) should be treated consistently with the accounting rule that DIV_t reduce BV_t but leaves NP_t unaffected. Equation (5) demonstrates that the earnings can be used for paying dividends, for investments, and for reducing debt. It follows from (5) that dividends reduce the investments dollar for dollar (given a specific financing decision), and from (4) that dividends reduce book equity dollar for dollar (independently of financing).

The question of dividend irrelevance hinges on the trade off between dividends and investments. If one dollar less is paid out in dividends then one dollar more can be invested. Now, the question is whether this increase in investments will generate a sufficient increase in *subsequent* earnings (which in turn can lead to higher subsequent dividends, i.e., less dividends "now" give potential for more dividends "later on") so that the equity value remains unaffected. Penman & Sougiannis (1997a) (section IV) find empirical evidence that this in fact is the case: dividends displace subsequent US GAAP earnings, so that *cum-dividend* earnings are insensitive to the dividend pay-out ratio. This contradicts the signalling hypotheses that a high dividend pay-out signals a high performance company. Moreover, an important implication for valuations is that "earnings (or dividends) alone cannot be targeted" (p. 18). That is, a multi-year dividend (or earnings) forecast is not alone a sufficient base for providing a meaningful equity value estimate, but must be complemented with an associate earnings (or dividend) forecast (in order to provide a reliable horizon value). Approaches where financial statements are explicitly forecasted using a clean surplus accounting system can here be useful, since they explicitly link forecasts of dividends and earnings via explicitly forecasted investments (and financing decisions), as described in equation (5). In practice, for a particular company, the dividend irrelevance holds as long as there exist investment opportunities returning the cost of equity. That is, dividend irrelevance may hold for a limited range of (possible) investments.

3. Framework for the Studies

3.1 General assumptions

To set up a framework for these studies, the different issues discussed above have to be considered in the context of company valuation modelling. Firstly, an overall view on company valuation, including concern for the estimation procedures of the relevant data, will be taken. As already has been indicated, a modelling approach that utilises the structure of accounting/financial statements, will be one starting point. The following accounting system with forecasted balance sheets and income statements will be used throughout the dissertation (with some minor extensions/alterations):

Assets	**Debt and Equity**
(Net) Working Capital, NWC_t	Debt, D_t
Net Property, Plant & Equipment, N_t	Deferred Taxes, T_t
= Gross Property, Plant & Equipment, G_t - Accumulated Depr., A_t	Book Equity, BV_t

The company's forecasted balance sheet (in book value terms) in year t[82]

[82] Note that this structure of the balance sheet implies that *invested capital* equals the balance sheet total.

+Revenues, R_t
-Operating Expenses, OpX_t
-Depreciation Expense, $DepX_t$
-Interest Expense, IX_t
-Taxes, IT_t
Net profit, NP_t

The company's forecasted income statement for year t

The development of balance sheets and income statements will be linked via the *clean surplus relation.*[83] Moreover, book debt is always assumed to be equal to market debt, and interest expense is paid on beginning of year debt. In Essays 2 and 3, the borrowing (coupon) rate (which always is assumed to be equal to the cost of debt) and the corporate tax rate are assumed to be constant, while they are allowed to vary over time in Essay 4. As can be seen from the balance sheet above, it is assumed that there are no excess marketable securities in the forecast period (i.e., in the future.), which means that the company has operating *assets* only. Deferred taxes are treated as "a quasi-equity account".[84] For valuation purposes they are not really assets or liabilities; the deferred taxes "are important only to the extent that we need them to calculate income tax payments."[85] That means that the net flow to/from the deferred taxes account (i.e., the net savings on taxes to be paid) each year is included in cash flow calculations to adjust the income statement item *Taxes* to a cash basis. They can be considered as a non-interest bearing debt item, but as discussed in Essay 4, they could also be thought of as negative item on the asset side (like accumulated depreciation). Further, only corporate taxation is considered.

Secondly, considering the question of which "world" to work in, one of course would say the "real world". But assumptions have to be made: Considering taxes, the natural point of departure when considering *company valuation models* has (historically) been to include corporate taxes only (or even no taxes at all). In a world with corporate taxes only, it is clear that financial structure matters. One obvious task to look into is the use of *constant* discount rates in company valuation (a common

[83] The *clean surplus relation* (CSR) means that the change in book equity (including retained earnings) is equal to net profits minus net dividends: $BV_t = BV_{t-1} + NP_t - DIV_t$. See, e.g., Ohlson (1995). This assumption is uncontroversial; why would we *forecast* deviations from CSR?

[84] CKM, p. 162. See also Brealey & Myers (1996), p. 521.

[85] Holthausen & Zmijewski (1996), Chapter 5B, p. 6.

procedure in practical situations). Since the starting assumption concerning taxes is corporate taxes only, the starting assumption on financial structure is given: it *matters*. When it comes to dividends, the basic presumption made is that equity owners are convinced that dividends are paid some time (e.g., at least as a final lump sum dividend of the break-up value when a company of finite life ends its business), and that the value of the equity can be described as the present value of all future expected dividends. No explicit assumption about dividend irrelevance is introduced in the modelling approach. As discussed previously this topic is perhaps best treated when performing the actual forecasting of dividends, investments and subsequent earnings. One other thing to be consistent about is the treatment of inflation. It will throughout the papers be presumed that *all* variables will be described in nominal terms. With respect to cost of capital considerations, no general assumptions are made. Instead, different settings are considered throughout the dissertation, since this is one of the explicit research issues.

3.2 Research issues

The central topic of this dissertation, specially highlighted in Essay 4, regards the choice of valuation attribute and securing that the transformation of this attribute into the equity value can be done in a theoretically consistent way, under different "theoretical settings" (as, e.g., different cost of capital regimes). The theoretical settings may be different in the distinct parts of the dissertation, but the point is thoroughly to provide an extensive description of each setting so that it will be easy to understand under which conditions certain combinations of valuation attributes and transformations are consistent with each other. In particular, Essays 2 and 3 consider the consistency between free cash flow valuation and the PVED-principle, while Essay 4 considers a "general" equivalence also including the abnormal earnings (residual income) model, the economic value added model (with two alternative specifications),[86] and the adjusted present value model. Essay 4 also presents a comparison between these different models.

While the cost of capital assumption underlying Essay 2 is pretty ad hoc with constant costs of equity and debt, Essays 3 and 4 develop how an explicit linkage between different costs of capital, the valuation of interest tax shields *and* the forecasts of fundamental data (as discussed in section 2.2 above) can be achieved by the use of a backward going discounting procedure. Further, a method for implementing the backward going discounting in a spreadsheet model is developed and thoroughly

[86] Essay 4 provides an alternative specification of the EVA model, more similar to the abnormal earnings concept than the standard Stewart (1991) specification.

described. In Essay 3, the cost of debt is still assumed to be constant, while it is evident from Essay 4 that this assumption can be relaxed. Essay 4 also gives an alternative starting point for the discounting procedure, using the estimated cost of equity at the valuation date instead of the *unlevered* cost of equity as the initial forecast target. The essay provides an implementation of all the five valuation models using the backward going discounting procedure. A further issue considered here is how much different assumptions about the valuation of interest tax shields (which are related to how the company is perceived to manage its capital structure) affect the valuation result. Essay 4 also studies under which conditions the general equivalence among the leading models can be preserved in a more traditional approach with constant discount rates.

Moreover, the concept of horizon values is another important issue of this dissertation. In Essay 2, Chapter 2, the main aim is to identify the many hidden assumptions inherent in continuing value formulas and to develop a methodology for identifying conditions, in terms of the used forecasting model's parameters, that must hold when using such formulas. This is done by providing an analysis of the development of valuation attributes and other accounting items under steady state assumptions. In Essay 4, a more general discussion and analysis of horizon value techniques for the five valuation models considered is presented.

The specification of forecasting models for providing forecasted balance sheets and income statements, including estimation of parameters, are also a topic that is considered in this dissertation. An empirical, descriptive, study (initiated in Levin & Olsson (1995a)) of how well the CKM specifications of the property, plant & equipment work in three Swedish companies, as well as discussions about the meaning and the practical usefulness of different parameters, are presented in Essay 2 (Chapter 4). More comments on these issues are given in the case study of Eldon AB, presented in Chapter 5 of Essay 2.

4. Results and Implications for Further Research

With respect to the central topic of this dissertation, the equivalence between different valuation models, the main results are synthesised in Essay 4: In a world with corporate taxes, a constant discount rate approach is theoretically valid only when the company is expected to maintain a constant market debt ratio. The only case where this will be appropriate is under an assumption of *active debt management* such that the company periodically adjusts its borrowing to keep a constant market debt ratio.[87] This dissertation has however developed a valuation framework that ensures equivalence between five leading valuation models even if the projected capital structure in market terms changes over time. The equivalence holds for both finite and infinite valuations, as well as year by year. An important implication is that, *given* projected financial statements, the different company valuation models give the same result. If more direct estimation techniques are (independently) used then the equivalence may not be achieved. However, the important lesson to be learnt is that differences in valuations are due to inconsistencies between *forecasts* of valuation attributes and/or estimated discount rates, and *not* to fundamental differences between the valuation models. If the forecasts are internally consistent, then each of the five models will give the same answer. This also points out the direction for future research: Forecasting procedures for estimation of the value relevant items are definitely something to rank high on the research agenda. Such procedures could possibly also be directed at including an explicit linkage between investments, dividends and the subsequent earnings (see section 2.3 in this essay).

From an analyst's point of view, the detailed modelling of the cost of capital and value interdependence through the backward discounting procedure may seem to be an "overkill". In practice, less (often much less) detailed approaches are used. But, one important contribution is that the framework specified in this dissertation will, given the assumptions made, give a value without any approximation errors from the calculation procedure. Thus, this framework can in a comparative study be used as benchmark for evaluating more *ad hoc* approaches. This may be an interesting topic for future research.

It may however also be interesting to develop the discounting procedure further. As indicated in this essay (section 2.2) the underlying framework can both be extendable to more complex capital structures, and be generalised to settings with both non-constant required return on operations and cost of debt.

Finally, this dissertation (Essay 2 in particular) has provided a methodology for deriving conditions that ensures that valuation attributes grow at a constant rate, so that continuing value formulas are appropriate as horizon values. Moreover, it has been shown in Essay 4 that the choice of horizon value technique boils down to finding the attribute that can validly be expected to grow at a constant

[87] A constant market debt ratio can under certain conditions also be achieved in constant growth cases. Practically, this is however only valid *after some future horizon* where the company is expected to enter a steady state, and is thus not applicable to a full scale valuation (i.e., from the valuation date until the end of the life of the company).

rate (or be zero) from a specific point in time. Since recent empirical studies have indicated that the horizon value calculations are very important for the overall performance of a valuation model,[88] an important issue for future research is to find out which horizon value technique that requires the least restrictive conditions in practical implementations.

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[88] See, e.g., Francis, Olsson & Oswald (1997) and Penman & Sougiannis (1997b).

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Essay 2:

Looking Beyond the Horizon and Other Issues in Company Valuation

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Abstract. This essay deals with a number of issues concerning company valuation models with special reference to the so-called McKinsey valuation model, presented in *Valuation: Measuring and Managing the Value of Companies* by Tom Copeland, Tim Koller, and Jack Murrin. Special attention is given to horizon value problems, and we investigate the many hidden assumptions inherent in continuing value formulas. Different equity valuation approaches are also commented upon, and we develop a procedure that yields equivalence between the discounted free cash flow approach and the discounted dividend approach. The procedure is shown to hold under quite general conditions, also when the capital structure is non-constant. Finally, the traditional free cash flow approach, our modified free cash flow approach and the dividend valuation approach are all applied to a real-world company, and it is shown that the latter two yield exactly the same value.

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0. Introduction

Company valuation with discounted free cash flow models has received much attention during the last few years. One of the most prominent examples is the McKinsey valuation model, described in *Valuation: Measuring and Managing the Value of Companies* by Tom Copeland, Tim Koller, and Jack Murrin.[1] The book has sold more than 50,000 copies and is used as textbook in many leading business schools - favoured more by finance teachers than by their colleagues in accounting, however. Letting the question whether free cash flow valuation is superior to other valuation approaches remain unanswered, we make the observation that the popularity of the McKinsey model in academic education will make it even more widespread among practitioners once today's students graduate and start to work. This observation is the pragmatic reason for our work: an in-depth look at many of the questions connected with free cash flow models in general and the McKinsey model in particular.

The title of this report - Looking Beyond the Horizon - alludes to the concept of horizon value, common in valuation models, and also present in the McKinsey model. The approach is explained in Brealey & Myers (1991): "The value of a business is usually computed as the discounted value of free cash flows out to a **valuation horizon** (*H*), plus the forecasted value of the business at the horizon, also discounted back to present value. That is,

$$PV = \frac{FCF_1}{1+r} + \frac{FCF_2}{(1+r)^2} + \ldots + \frac{FCF_H}{(1+r)^H} + \frac{PV_H}{(1+r)^H}$$

Of course, the [...] business will continue after the horizon, but it's not practical to forecast free cash flow year by year to infinity."[2] The horizon value is almost always calculated as a continuing value, using for instance the Gordon formula.[3]

The fact that the horizon value is calculated using a very simplifying formula can in no way be taken as evidence that it is somehow unimportant. On the contrary, Copeland et al report typical values for some industries: for a company in the tobacco industry the horizon value accounts for 56% of the total company value, in the sporting goods industry it is 81%, for the typical skin care business the figure is 100% and for a high tech company 125% (the figures are calculated using a horizon eight years into the future).[4] Furthermore, practitioners like horizon values. A colleague told us that when he had discussed the McKinsey book with corporate analysts at Swedish investment banks, they

[1] The book and the model will be referred to alternately as the *McKinsey book/model* and *Copeland et al.*
[2] Brealey & Myers (1991), p. 64
[3] See, e.g., Brealey & Myers (1991), p. 34.
[4] Copeland et al, p. 275

typically wanted to use a horizon value after not more than five years, some even after only two or three years.

A large part of this report is devoted to problems connected with the horizon value. We particularly want to uncover the many hidden assumptions inherent in continuing value formulas. This also makes the reverse possible, namely to say what conditions must be fulfilled for a continuing value calculation to be admissible - or at least create an awareness about the problems connected with it. The rest of the report covers a range of subjects that we feel are insufficiently dealt with in Copeland et al, and that are of general interest in company valuation.

The findings in the report also lend themselves to conclusions regarding the choice of discounting method, a problem treated rather sketchily in many practically oriented textbooks.

The methodology we use in much of the report is to investigate the structure of difference equations that govern the different versions of the valuation model. This technique makes it possible to see exactly what restrictions are implied by the different proposed modelling approaches - and only when such restrictions are stated explicitly can one judge whether they are reasonable or not.

It should be noted that the McKinsey model does not include any stochastic elements. The free cash flows, which are discounted, could be interpreted as expected free cash flows. The stochastic processes that generate them are left out. In this report we adhere to this mode of presentation.

The first section in Chapter 1 contains a short description of the McKinsey model. This is meant more as an introduction for the reader not fully acquainted with the particulars of the Copeland et al approach. The second section introduces the alternative specifications of the property, plant and equipment (PPE) items in the McKinsey model. In Chapter 2, 3 and 4 the actual methods and results of this report are developed. Chapter 5 is a "How to do it"-guide, where we use the results in a step by step implementation of the model on the Swedish company Eldon AB. Chapter 6, finally, contains the concluding remarks.

1. The Modelling Framework

1.1 The model's structure

The model contains two sections: one historical and one for the future. In the historical section, balance sheets and income statements from a number of years are inserted into the model in order to calculate several financial ratios. These historical ratios can then be used as a basis for forecasting the corresponding ratios in the future, which in turn are used for calculating future balance sheets and income statements. From these, it is possible to derive the free cash flow (FCF), the net profit (NP) and the dividends (DIV) for each future year. The company value can then be calculated:[5]

Definitions:

> *The free cash flow (FCF) valuation approach* is a model where the forecasted free cash flows are discounted by the weighted average cost of capital (WACC). The sum of the discounted free cash flows is then the *total company value*. Deducting the market value of the debt yields the *equity value*.
>
> *The dividend (DIV) valuation approach* is a model where the forecasted future dividends are discounted by the cost of equity capital (k_e). The sum of the discounted dividends makes up the *equity value*, and by adding the market value of the debt, one arrives at the *total company value*.
>
> *The net profit (NP) valuation approach* is a family of models where the forecasted future net profits are calculated and capitalised, often after deducting a charge for the use of capital.

In the original version, the book by Copeland et al advocates the free cash flow valuation method. This method has also become fashionable with many financial economists, and we will also concentrate on it. Some consideration will also be given to the dividend valuation approach. Lately, many academics in the accounting field have begun advocating earnings, or net profits, as the relevant valuation measure. We do not share their enthusiasm. However, as there is a demand for explicit net profit figures, we show how these can be derived from the same modelling framework as we use to derive free cash flows and dividends. In section 3.2 we discuss the earnings issue, and explain why we are somewhat sceptical towards it.

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[5] The terminology is not standardised in the literature: "Company value" sometimes stands for the equity value, other times it refers to the total value of the company, i.e. equity value plus debt value. In this report we will use the terminology *equity value* and *total company value* where it is necessary to distinguish between the two.

Since it is very difficult to make detailed forecasts for periods in the distant future, the valuation can be divided into two periods: the explicit forecast period and a continuing value for the time after this period.[6] This so-called horizon value is often calculated with the Gordon formula,[7] but there are a number of interesting problems connected with the horizon value, which will be dealt with in coming chapters.

For a hypothetical company, the "McKay" company (adapted from a teaching note by L. Peter Jennergren (1994) and similar to the Preston company in Copeland et al), historical income statements and balance sheets are set up in order to calculate the historical values of a number of financial ratios that are later used as forecast assumptions, e.g. revenue growth, operating expenditures as a percentage of revenues, etc. An example from 1986 and 1987 can be seen in Table 1, below (note that it is necessary to present more than one year since some of the financial ratios involve figures from previous years). The whole model is in Appendix 1. For ease of understanding some simplifications from the Copeland et al version have been made, which do not affect the issues we are concerned with here.[8] The valuation is as of Jan. 1, 1993.

From the figures in Table 1 financial ratios are easily calculated, some examples are given in Table 2. Most ratios are self-explanatory, but some are accompanied by comments. Items and ratios refer to the current year unless they have a subscript that indicates otherwise (thus *gross PPE* means gross property, plant and equipment for the year in question, whereas $gross\,PPE_{t-1}$ refers to the same item the preceding year).

[6] See Copeland et al, p. 207.
[7] The Gordon model is described in Brealey & Myers (1991), p. 34.
[8] McKay has no capitalised leases or goodwill. The capital structure consists only of equity and debt.

Income statement	1986	1987
Revenues	197.6	222.3
- Operating expenses	-175.4	-206.9
- Depreciation expense	-12.8	-9.3
	------	------
Operating income	9.4	6.1
Interest income	0.0	0.0
- Interest expense	-0.1	-0.4
	------	------
Earnings before taxes	9.3	5.7
- Taxes	-5.3	-2.4
	------	------
Net profit	4.0	3.3
	======	======

Statement of retained earnings	1986	1987
Beginning retained earnings	60.6	62.5
Net profit	4.0	3.3
- Common dividends	-2.1	-2.1
	------	------
Ending retained earnings	62.5	63.7
	======	======

	1986	1987
Invested capital	70.7	88.9
	======	======

Balance sheet	1986	1987
Operating cash	4.0	4.4
Excess marketable securities	10.9	3.0
Trade receivables	17.9	24.4
Other receivables	1.5	2.0
Inventories	1.9	2.1
Prepaid expenses	4.3	5.1
	------	------
Current assets	40.5	41.0
Gross property, plant and equipment	100.0	117.7
- Accumulated depreciation	-37.7	-42.3
	------	------
Net property, plant and equipment	62.3	75.4
	------	------
Total assets	102.8	116.4
	======	======
Short-term debt	0.3	0.8
Accounts payable	7.3	11.0
Other current liabilities	13.9	13.5
	------	------
Total current liabilities	21.5	25.3
Long-term debt	5.5	11.5
Deferred income taxes	8.7	11.0
Common stock	4.6	4.9
Retained earnings	62.5	63.7
	------	------
Total common equity	67.1	68.6
	------	------
Total liabilities and equity	102.8	116.4
	======	======

Table 1 - The McKay Company in 1986 and 1987

	1986	1987	
Operations			
Revenue growth		12.5%	
Operating exp. / revenues	88.8%	93.1%	
Working capital / revenues			
Operating cash / revenues	2.0%	2.0%	
Trade receivables / rev.	9.1%	11.0%	
Other receivables / rev.	0.8%	0.9%	
Inventories / rev.	1.0%	0.9%	
Prepaid expenses / rev.	2.2%	2.3%	
Accounts payable / rev.	3.7%	4.9%	
Other current liabilities / rev.	7.0%	6.1%	
Gross Property, Plant and Equipment (Gross PPE)			Note: Specification A (see section 1.2)
Gross PPE / revenues	50.6%	52.9%	
Depreciation / gross PPE_{t-1}		9.3%	Depr. as percentage of *preceding year's* gross PPE
Retirements / gross PPE_{t-1}		4.7%	Depr. as percentage of *preceding year's* gross PPE
Taxes			
Tax rate	49.0%	49.0%	
Increase in deferred taxes / gross PPE		2.0%	

Table 2 - Some financial ratios for the McKay Company in 1986 and 1987

The historical ratios should be calculated for several years before they are used as a base for the estimation of future ratios. The expected future values of the ratios then determine the (forecasted) future performance of the company and thus the total company value. See the following tables for an example of how this works, the example is for 1994. In this case, the forecast period starts in 1993, so 1994 is the second prediction year.[9] Note the large number of items in the income statement and balance sheet that are *ratio driven*, i.e. that are decided by one of the financial ratios described in Table 2 and Table 3.

[9] The entire explicit forecast period can be found in the appendix.

Forecast assumptions	1994	
Operations		
Real growth	12.0%	Forecast
Inflation	3.0%	Forecast
Revenue growth	15.4%	[1 + Real growth] × [1 + Inflation] - 1
Operating exp. / revenues	91.0%	Forecasted ratio
Working capital / revenues		
Operating cash / revenues	2.0%	Forecasted ratio
Trade receivables / rev.	11.9%	Forecasted ratio
Other receivables / rev.	1.4%	Forecasted ratio
Inventories / rev.	2.5%	Forecasted ratio
Prepaid expenses / rev.	0.9%	Forecasted ratio
Accounts payable / rev.	3.9%	Forecasted ratio
Other current liabilities / rev.	6.1%	Forecasted ratio
Gross Property, Plant and Equipment (Gross PPE)		Note: Specification A
Gross PPE / revenues	54.6%	Forecasted ratio
Depreciation / gross PPE_{t-1}	9.7%	Forecasted ratio
Retirements / gross PPE_{t-1}	4.1%	Forecasted ratio
Taxes		
Tax rate	39.0%	Forecast (or perhaps known)
Increase in deferred taxes / gross PPE	0.8%	Forecasted ratio
Other		
Borrowing rate	9.0%	Forecast
Current year short term debt / prec. year's long term debt	20.0%	Forecasted ratio

Table 3 - Forecast assumptions for the McKay Company in 1994

Income statement	1994	
Revenues	690.6	Revenues 1993 + revenue growth
- Operating expenses	-628.4	Ratio driven: 91% of revenues.
- Depreciation expense	-31.9	Ratio driven: 9.7% of gross PPE in 1993

Operating income	30.2	
Interest income	0.0	Direct forecast
- Interest expense	-12.2	Borrowing rate × Debt (at beginning of year, i.e. $Debt_{t-1}$)

Earnings before taxes	18.0	
- Taxes	-7.0	Tax rate × Earnings before taxes

Net profit	11.0	
	======	

Statement of retained earnings	1994	
Beginning retained earnings	77.5	Preceding year's "Ending retained earnings"
Net profit	11.0	From income statement
- Common dividends	0.0	Direct forecast

Ending retained earnings	88.5	
	======	

Table 4 - Forecasted income statement and statement of retained earnings for the McKay Company in 1994

Balance sheet	1994	
Operating cash	13.8	Ratio driven: 2.0% of revenues
Excess marketable securities	0.0	Direct forecast
Trade receivables	82.2	Ratio driven: 11.9% of revenues
Other receivables	9.7	Ratio driven: 1.4% of revenues
Inventories	17.3	Ratio driven: 2.5% of revenues
Prepaid expenses	6.2	Ratio driven: 0.9% of revenues

Current assets	129.1	
Gross property, plant and equipment (Gross PPE)	377.1	Ratio driven: '54.6% of revenues
- Accumulated depreciation	-140.1	Ratio driven: Accumulated depreciation in 1993 + + This year's depr. expense - This year's retirements = = Acc. depr. 1993 + Depr. exp. 1994 - 4.1 % of gross PPE in 1993

Net property, plant and equipment (Net PPE)	237.0	

Total assets	366.2	
	======	
Short-term debt	23.0	Ratio driven: 20% of long-term debt in 1993
Accounts payable	26.9	Ratio driven: 3.9% of revenues
Other current liabilities	42.1	Ratio driven: 6.1% of revenues
	------	
Total current liabilities	92.1	
Long-term debt	136.0	Residual item: Total assets - Total current liabilities - - Deferred income taxes - Total common equity
Deferred income taxes	26.0	Ratio driven: Deferred taxes in 1993 + + 0.8% of this year's gross PPE
Common stock	23.6	Same as preceding year (no changes foreseen)
Retained earnings	88.5	Current year's "Ending retained earnings"

Total common equity	112.1	

Total liabilities and equity	366.2	
	======	

Table 5 - Forecasted balance sheet for the McKay Company in 1994

The free cash flow for 1994 can be derived from the forecasted income statement and balance sheet:

Free cash flow	1994	
Revenues	690.6	From income statement
- Operating expenses	-628.4	From income statement
- Depreciation expense	-31.9	From income statement

Adjusted EBIT	30.2	(EBIT = Earnings before interest and taxes)
- Taxes on EBIT	-11.8	Tax rate × Adjusted EBIT
Change in deferred taxes	3.0	Current year's deferred taxes - Preceding year's deferred taxes
	------	
NOPLAT	21.4	(NOPLAT = Net operating profit less adjusted taxes)
Depreciation expense	31.9	From income statement

Gross cash flow	53.4	
Change in working capital	8.0	(Between current and preceding year) [10]
Capital expenditures	61.2	Current year's net PPE - Preceding year's net PPE + + Current year's depreciation expense
	------	
Gross investment	69.2	

Free cash flow	-15.9	Gross cash flow - Gross investment
	======	

Table 6 - Forecasted free cash flow for the McKay Company in 1994

The free cash flow should correspond to the financial cash flow:

Financial cash flow	1994	
Increase(+) / Decrease(-) in excess marketable securities	0.0	Between current and preceding year
After-tax interest income (-)	0.0	[1 - Tax rate] × Interest income (from income statement)
Increase(-) / Decrease(+) in debt	-23.3	Between current and preceding year
After-tax interest expense (+)	7.5	[1 - Tax rate] × Interest expense (from income statement)
Common dividends (+)	0.0	From statement of retained earnings
Incr.(-) / Decr.(+) in common stock	0	Between current and preceding year
	------	
Financial cash flow	-15.9	
	======	

Table 7 - Forecasted financial cash flow for the McKay Company in 1994

Since the model is completely specified by the parameters (the ratios) shown in Table 3, it is of vital importance to get these right. This problem consists of two separate issues. The obvious one is, of

[10] *Increase* in operating cash, trade receivables, other receivables, inventories, and prepaid expenses; *decrease* in accounts payable and other current liabilities.

course, to have estimation routines that yield reasonable parameter values. Estimation procedures will be discussed in detail in Chapter 4. Notwithstanding the importance of estimation (obviously, the choice of profit margin has an enormous impact) we will also give considerable attention to the other aspect, namely the analytical and empirical feasibility of the ratios as such. For forecasting purposes, it is desirable to have ratios that are fairly stable over time (or at least predictable), since the whole idea of using ratios as driving parameters builds on the notion that there exist a number of relationships in a company that remain predictable over time and business cycles.

1.2 The specification of PPE

This section introduces different specifications of the *property, plant and equipment* related items. These closely related items are the following: *net* and *gross property, plant and equipment (net* and *gross PPE), accumulated depreciation, capital expenditures (CapX), depreciation expense (DepX)* and *retirements (Ret).*

The specifications under consideration in this report are all presented by Copeland et al.: Specification A in the first edition of their book, while the other two (Spec. B and C) are proposed in the second edition. The McKay example earlier in this chapter utilises Specification A.

As mentioned in the previous section, the McKinsey model is driven by ratios. The main difference between the specifications is which item is being driven by a ratio to *revenues.* The three different revenue-related ratios are referred to as the *main driving ratios.* In Specification A the main driving ratio is *gross PPE / revenues*, in Specification B the ratio is *CapX / revenues* and in Specification C it is *net PPE / revenues.* Both *depreciation expense* and *retirements* are on the other hand determined in the same way in all three specifications. The specifications will be presented in full after some matters of notation:

The PPE-items are denoted in the following way:

$A_t =$ Accumulated depreciation at the end of year t

$CapX_t =$ Capital expenditures in year t

$DepX_t =$ Depreciation expense in year t

$G_t =$ Gross PPE at the end of year t

$N_t =$ Net PPE at the end of year t

$Ret_t =$ Retirements in year t

Further, revenues in year t will be denoted R_t.

Now it remains to define the input ratios. The ratios, common to all three specifications, are defined as:

$$d_t \equiv \left[\frac{DepX_t}{G_{t-1}}\right]_t = \text{Depreciation expense as a percentage of preceding year's gross PPE.}$$

$$r_t \equiv \left[\frac{Ret_t}{G_{t-1}}\right]_t = \text{Retirements as a percentage of preceding year's gross PPE.}$$

The main driving ratios are defined as:

$$b_t \equiv \left[\frac{G_t}{R_t}\right]_t = \text{Gross PPE as a percentage of revenues. (Spec. A)}$$

$$e_t \equiv \left[\frac{CapX_t}{R_t}\right]_t = \text{Capital expenditures as a percentage of revenues. (Spec. B)}$$

$$n_t \equiv \left[\frac{N_t}{R_t}\right]_t = \text{Net PPE as a percentage of revenues. (Spec. C)}$$

The entire specifications of the PPE-items can now be written in the following way:

<u>Specification A:</u>
Items directly determined by ratios:

$$G_t = b_t \cdot R_t$$
$$DepX_t = d_t \cdot G_{t-1}$$
$$Ret_t = r_t \cdot G_{t-1}$$

Items derived indirectly:

$$A_t = A_{t-1} + DepX_t - Ret_t$$
$$N_t = G_t - A_t$$
$$CapX_t = N_t - N_{t-1} + DepX_t = G_t - G_{t-1} + Ret_t$$

<u>Specification B:</u>
Items directly determined by ratios:

$$CapX_t = e_t \cdot R_t$$
$$DepX_t = d_t \cdot G_{t-1}$$
$$Ret_t = r_t \cdot G_{t-1}$$

Items derived indirectly:

$$G_t = G_{t-1} + CapX_t - Ret_t$$
$$A_t = A_{t-1} + DepX_t - Ret_t$$
$$N_t = G_t - A_t$$

Specification C:

Items directly determined by ratios:

$N_t = n_t \cdot R_t$

$DepX_t = d_t \cdot G_{t-1}$

$Ret_t = r_t \cdot G_{t-1}$

Items derived indirectly:

$CapX_t = N_t - N_{t-1} + DepX_t$

$A_t = A_{t-1} + DepX_t - Ret_t$

$G_t = N_t + A_t$

In Chapter 4, the different specifications are analysed, both empirically and theoretically. It turns out that Specification C is very similar to Specification A and does not add any improvements. Accordingly, the horizon value study in Chapter 2 has been concentrated on Specifications A and B.

Appendix 1 - The McKay Company

Table A1:1 - Historical income statement

Income statement	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Revenues	197.6	222.3	272.3	299.5	350.0	418.9	505.4
Operating expenses	-175.4	-206.9	-249.6	-274.7	-327.5	-383.6	-467.4
Depreciation expense	-12.8	-9.3	-11.2	-13.0	-15.0	-17.7	-26.4
Operating income	9.4	6.1	11.5	11.8	7.5	17.6	11.6
Interest income	0.0	0.0	0.0	0.0	0.9	0.7	0.6
Interest expense	-0.1	-0.4	-0.8	-1.0	-3.4	-4.1	-10.1
Earnings before taxes	9.3	5.7	10.7	10.8	5.0	14.2	2.1
Income taxes	-5.3	-2.4	-5.8	-5.2	-1.0	-7.1	-0.7
Net profit	4.0	3.3	4.9	5.6	4.0	7.1	1.4

Statement of retained earnings	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Beginning retained earnings	60.6	62.5	63.7	65.8	68.6	69.8	74.0
Net profit	4.0	3.3	4.9	5.6	4.0	7.1	1.4
Common dividends	-2.1	-2.1	-2.8	-2.8	-2.8	-2.9	-2.9
Ending retained earnings	62.5	63.7	65.8	68.6	69.8	74.0	72.5

Table A1:2 - Historical balance sheets

	1986	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Operating cash	4.0	4.4	5.4	6.0	6.0	8.4	10.1
Excess marketable securities	10.9	3.0	20.5	10.3	0.0	5.8	3.2
Trade receivables	17.9	24.4	33.0	33.3	43.9	49.8	57.7
Other receivables	1.5	2.0	2.7	2.7	6.2	4.9	5.7
Inventories	1.9	2.1	2.8	2.5	9.0	10.9	11.9
Prepaid expenses	4.3	5.1	5.3	6.0	2.4	4.4	5.0
Current assets	40.5	41.0	69.7	60.8	67.5	84.2	93.6
Gross property, plant and equipment	100.0	117.7	128.2	155.6	204.7	272.5	297.6
Accumulated depreciation	-37.7	-42.3	-48.7	-56.5	-71.9	-86.9	-103.4
Net property, plant and equipment	62.3	75.4	79.5	99.1	132.8	185.6	194.2
Total assets	102.8	116.4	149.2	159.9	200.3	269.8	287.8
Short-term debt	0.3	0.8	0.9	1.5	11.5	12.5	20.7
Accounts payable	7.3	11.0	11.9	10.5	14.2	16.2	18.9
Other current liabilities	13.9	13.5	18.2	18.5	21.4	27.8	28.8
Total current liabilities	21.5	25.3	31.0	30.5	47.1	56.5	68.4
Long-term debt	5.5	11.5	16.2	21.7	40.2	90.6	103.0
Deferred income taxes	8.7	11.0	12.6	15.5	19.6	25.1	20.3
Common stock	4.6	4.9	23.6	23.6	23.6	23.6	23.6
Retained earnings	62.5	63.7	65.8	68.6	69.8	74.0	72.5
Total common equity	67.1	68.6	89.4	92.2	93.4	97.6	96.1
Total liabilities and equity	102.8	116.4	149.2	159.9	200.3	269.8	287.8
Invested capital	70.7	88.9	98.6	120.6	164.7	220.0	236.9
Debt/invested cap	8.2%	13.8%	17.3%	19.2%	31.4%	46.9%	52.2%
Debt+deferred taxes/ invested cap	20.5%	26.2%	30.1%	32.1%	43.3%	58.3%	60.8%
NOPLAT/invested cap		6.5%	7.0%	7.5%	5.7%	6.5%	0.8%

Table A1:3 - Historical free cash flow

Free cash flow	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Revenues	222.3	272.3	299.5	350.0	418.9	505.4
Operating expenses	-206.9	-249.6	-274.7	-327.5	-383.6	-467.4
Depreciation expense	-9.3	-11.2	-13.0	-15.0	-17.7	-26.4
EBIT	6.1	11.5	11.8	7.5	17.6	11.6
Taxes on EBIT	-2.6	-6.2	-5.7	-2.2	-8.8	-5.0
Change in deferred taxes	2.3	1.6	2.9	4.1	5.5	-4.8
NOPLAT	5.8	6.9	9.0	9.4	14.3	1.8
Depreciation expense	9.3	11.2	13.0	15.0	17.7	26.4
Gross cash flow	15.1	18.1	22.0	24.4	32.0	28.2
Change in working capital	5.1	5.6	2.4	10.4	2.5	8.3
Capital expenditures	22.4	15.3	32.6	48.7	70.5	35.0
Gross investment	27.5	20.9	35.0	59.1	73.0	43.3
Free cash flow	-12.4	-2.8	-13.0	-34.7	-41.0	-15.1

Financial cash flow	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Incr.(+)/Decr..(-) excess mkt. securities	-7.9	17.5	-10.2	-10.3	5.8	-2.6
After-tax interest income (-)	0.0	0.0	0.0	-0.5	-0.4	-0.3
Incr.(-)/Decr..(+) in short- and long-term debt	-6.5	-4.8	-6.1	-28.5	-51.4	-20.6
After-tax interest expense (+)	0.2	0.4	0.5	1.7	2.1	5.6
Common dividends (+)	2.1	2.8	2.8	2.8	2.9	2.9
Incr.(-)/Decr..(+) in common stock	-0.3	-18.7	0.0	0.0	0.0	0.0
Financial cash flow	-12.4	-2.8	-13.0	-34.7	-41.0	-15.1

Table A1:4 - Historical ratios for forecast assumptions

	1986	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Operations							
Real growth							
Inflation							
Revenue growth		12.5%	22.5%	10.0%	16.9%	19.7%	20.6%
Operating exp./							
revenues	88.8%	93.1%	91.7%	91.7%	93.6%	91.6%	92.5%
Working cap/revenues							
Operating cash	2.0%	2.0%	2.0%	2.0%	1.7%	2.0%	2.0%
Trade receiv.	9.1%	11.0%	12.1%	11.1%	12.5%	11.9%	11.4%
Other receiv.	0.8%	0.9%	1.0%	0.9%	1.8%	1.2%	1.1%
Inventories	1.0%	0.9%	1.0%	0.8%	2.6%	2.6%	2.4%
Prepaid expenses	2.2%	2.3%	1.9%	2.0%	0.7%	1.1%	1.0%
Accounts payable	3.7%	4.9%	4.4%	3.5%	4.1%	3.9%	3.7%
Other curr. liab's	7.0%	6.1%	6.7%	6.2%	6.1%	6.6%	5.7%
Property.Plant and							
Equipment (PPE)							
Gross PPE/rev's	50.6%	52.9%	47.1%	52.0%	58.5%	65.1%	58.9%
Depr/gross PPE *		9.3%	9.5%	10.1%	9.6%	8.6%	9.7%
Retirements/							
gross PPE *		4.7%	4.1%	4.1%	-0.3%	1.3%	3.6%
Taxes							
Tax rate	49.0%	49.0%	49.0%	49.0%	49.0%	49.0%	45.0%
Incr. in deferred							
tax/gross PPE		2.0%	1.2%	1.9%	2.0%	2.0%	-1.6%

* last year's gross PPE

Table A1:5 - Forecasted income statement

Income statement	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Revenues	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1241.7	1279.0	1317.3
Operating expenses	-550.8	-628.4	-699.1	-770.5	-841.2	-909.7	-974.5	-1033.9	-1086.2	-1130.0	-1163.8	-1198.8
Depreciation expense	-28.9	-31.9	-36.6	-40.4	-44.2	-47.9	-51.4	-54.6	-57.5	-60.0	-61.9	-63.3
Operating income	19.0	30.2	32.6	35.8	39.0	42.1	45.0	47.6	49.9	51.8	53.2	55.3
Interest income *	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest expense	-11.1	-12.2	-14.3	-15.5	-16.5	-17.3	-17.9	-18.2	-18.1	-17.6	-16.7	-17.4
Earnings before taxes	8.2	18.0	18.2	20.4	22.6	24.8	27.1	29.5	31.8	34.1	36.5	37.9
Income taxes	-3.2	-7.0	-7.1	-7.9	-8.8	-9.7	-10.6	-11.5	-12.4	-13.3	-14.2	-14.8
Net profit	5.0	11.0	11.1	12.4	13.8	15.1	16.5	18.0	19.4	20.8	22.2	23.1

Statement of retained earnings	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Beginning retained earnings	72.5	77.5	88.5	99.6	112.0	125.8	140.9	157.5	175.4	194.8	215.7	215.9
Net profit	5.0	11.0	11.1	12.4	13.8	15.1	16.5	18.0	19.4	20.8	22.2	23.1
Common dividends **	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-22.0	-16.5
Ending retained earnings	77.5	88.5	99.6	112.0	125.8	140.9	157.5	175.4	194.8	215.7	215.9	222.5

* direct forecast

** direct forecast until 2002. residual thereafter

Table A1:6 - Forecasted balance sheet

	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Operating cash	12.0	13.8	15.4	16.9	18.5	20.0	21.4	22.7	23.9	24.8	25.6	26.3
Excess marketable securities *	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Trade receivables	71.2	82.2	91.4	100.8	110.0	119.0	127.4	135.2	142.0	147.8	152.2	156.8
Other receivables	8.4	9.7	10.8	11.9	12.9	14.0	15.0	15.9	16.7	17.4	17.9	18.4
Inventories	15.0	17.3	19.2	21.2	23.1	25.0	26.8	28.4	29.8	31.0	32.0	32.9
Prepaid expenses	5.4	6.2	6.9	7.6	8.3	9.0	9.6	10.2	10.7	11.2	11.5	11.9
Current assets	111.9	129.1	143.7	158.3	172.9	186.9	200.3	212.5	223.2	232.2	239.2	246.3
Gross property, plant and equipm.	329.3	377.1	416.4	455.5	493.6	529.9	563.3	593.1	618.3	638.2	652.3	671.8
Accumulated depreciation	-121.6	-140.1	-159.4	-178.8	-197.9	-216.1	-233.2	-248.6	-262.0	-273.0	-281.3	-289.8
Net property, plant and equipment	207.7	237.0	256.9	276.7	295.7	313.7	330.1	344.4	356.2	365.2	370.9	382.0
Total assets	319.6	366.2	400.6	435.0	468.6	500.7	530.3	556.9	579.4	597.4	610.1	628.3
Short-term debt	20.6	23.0	27.2	28.9	30.8	32.2	33.2	33.7	33.5	32.5	30.7	32.4
Accounts payable	23.3	26.9	30.0	33.0	36.1	39.0	41.8	44.3	46.6	48.4	49.9	51.4
Other current liabilities	36.5	42.1	46.9	51.6	56.4	61.0	65.3	69.3	72.8	75.7	78.0	80.4
Total current liabilities	80.5	92.1	104.0	113.6	123.2	132.2	140.3	147.3	152.8	156.7	158.6	164.2
Long-term debt	115.2	136.0	144.5	154.0	161.2	166.2	168.4	167.3	162.5	153.6	162.2	166.2
Deferred income taxes	22.9	26.0	28.9	31.8	34.8	37.7	40.5	43.2	45.7	47.9	49.9	51.9
Common stock	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6
Retained earnings	77.5	88.5	99.6	112.0	125.8	140.9	157.5	175.4	194.8	215.7	215.9	222.5
Total common equity	101.1	112.1	123.2	135.6	149.4	164.5	181.1	199.0	218.4	239.3	239.5	246.1
Total liabilities and equity	319.6	366.2	400.6	435.0	468.6	500.7	530.3	556.9	579.4	597.4	610.1	628.3
Invested capital	259.8	297.1	323.8	350.3	376.2	400.7	423.3	443.3	460.1	473.2	482.2	496.6
Debt/invested cap **	52.3%	53.5%	53.0%	52.2%	51.0%	49.5%	47.6%	45.4%	42.6%	39.3%	40.0%	40.0%
Debt+deferred taxes/ invested cap	61.1%	62.3%	61.9%	61.3%	60.3%	58.9%	57.2%	55.1%	52.5%	49.4%	50.3%	50.4%
NOPLAT/invested cap	5.5%	7.2%	7.0%	7.1%	7.1%	7.1%	7.1%	7.2%	7.2%	7.1%	7.1%	7.2%

* direct forecast

** assumption from 2003

Table A1:7 - Forecasted free cash flow

Free cash flow	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Revenues	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1241.7	1279.0	1317.3
Operating expenses	-550.8	-628.4	-699.1	-770.5	-841.2	-909.7	-974.5	-1033.9	-1086.2	-1130.0	-1163.8	-1198.8
Depreciation expense	-28.9	-31.9	-36.6	-40.4	-44.2	-47.9	-51.4	-54.6	-57.5	-60.0	-61.9	-63.3
EBIT	19.0	30.2	32.6	35.8	39.0	42.1	45.0	47.6	49.9	51.8	53.2	55.3
Taxes on EBIT	-7.4	-11.8	-12.7	-14.0	-15.2	-16.4	-17.5	-18.6	-19.5	-20.2	-20.7	-21.6
Change in deferred taxes	2.6	3.0	2.9	3.0	3.0	2.9	2.8	2.7	2.5	2.2	2.0	2.0
NOPLAT	14.2	21.4	22.8	24.8	26.8	28.6	30.3	31.7	32.9	33.8	34.4	35.7
Depreciation expense	28.9	31.9	36.6	40.4	44.2	47.9	51.4	54.6	57.5	60.0	61.9	63.3
Gross cash flow	43.1	53.4	59.4	65.2	70.9	76.5	81.7	86.4	90.4	93.8	96.3	99.0
Change in working capital	9.4	8.0	6.8	6.8	6.8	6.6	6.2	5.7	5.0	4.2	3.2	3.3
Capital expenditures	42.4	61.2	56.5	60.1	63.3	65.8	67.8	69.0	69.4	68.9	67.6	74.4
Gross investment	51.8	69.2	63.3	66.9	70.0	72.4	74.0	74.6	74.4	73.1	70.9	77.7
Free cash flow	-8.6	-15.9	-3.9	-1.7	0.9	4.1	7.7	11.7	16.1	20.7	25.4	21.3

Financial cash flow	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Incr.(+)/Decr..(-) excess mkt. sec'ties	-3.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
After-tax interest income (-)	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Incr.(-)/Decr..(+) in short- and long-term debt	-12.1	-23.3	-12.6	-11.2	-9.1	-6.5	-3.2	0.6	5.0	9.9	-6.8	-5.8
After-tax interest expense (+)	6.8	7.5	8.7	9.4	10.0	10.5	10.9	11.1	11.0	10.8	10.2	10.6
Common dividends (+)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	22.0	16.5
Incr.(-)/Decr..(+) in common stock	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Financial cash flow	-8.6	-15.9	-3.9	-1.7	0.9	4.1	7.7	11.7	16.1	20.7	25.4	21.3

Table A1:8 - Forecast assumptions

	1993	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*
Operations												
Real growth	15.0%	12.0%	8.0%	7.0%	6.0%	5.0%	4.0%	3.0%	2.0%	1.0%	0.0%	0.0%
Inflation	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
Revenue growth	18.5%	15.4%	11.2%	10.2%	9.2%	8.2%	7.1%	6.1%	5.1%	4.0%	3.0%	3.0%
Operating exp./ revenues	92.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%	91.0%
Working cap/revenues												
Operating cash	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%
Trade receiv.	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%	11.9%
Other receiv.	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%
Inventories	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%
Prepaid expenses	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%
Accounts payable	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%	3.9%
Other curr. liab's	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%
Property.Plant and Equipment (PPE)												
Gross PPE/rev's	55.0%	54.6%	54.2%	53.8%	53.4%	53.0%	52.6%	52.2%	51.8%	51.4%	51.0%	51.0%
Depr/gross PPE *	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%
Retirements/ gross PPE *	3.6%	4.1%	4.6%	5.0%	5.5%	6.0%	6.5%	7.0%	7.4%	7.9%	8.4%	8.4%
Taxes												
Tax rate	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%
Incr. in deferred tax/gross PPE	0.8%	0.8%	0.7%	0.7%	0.6%	0.6%	0.5%	0.5%	0.4%	0.4%	0.3%	0.3%
Other												
Borrowing rate	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%
This year short-term/ last year long-term debt	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%

* last year's gross PPE

2. The Value Beyond the Horizon - Development in Steady State

2.0 Introduction

When performing a valuation, one can either make the explicit forecast period infinitely long or make explicit forecasts for a limited number of years only and account for the time after that with a horizon value.

An *infinitely* long explicit forecast period is only a theoretical notion. In practice, one tries to forecast free cash flows for perhaps 100 or 150 years ahead - until the *present value* of the cash flow from a certain year no longer contributes anything (almost) to the company value. This is obviously quite difficult to do: It is not an easy task to estimate how different input parameters will develop in, say, the 2050s.

Instead, the second approach is often used,[11] and the future is divided into two periods: the explicit forecast period and the infinite future after that, accounted for by the horizon value. As mentioned earlier, the horizon value can take up a very large part of the total company value, so it is obviously very important that this part of the valuation is made carefully and with correct calculation techniques. If the horizon value is calculated in an erroneous or sloppy way, it does not matter how precise and sophisticated the estimation procedures for the explicit forecast period are. The issue becomes even more problematic when taking into consideration that practitioners often ask for very short explicit forecast periods - 2 to 5 years - and an incorrect horizon value of course makes the whole valuation totally unreliable.

The use of a horizon value requires the assumption that the company has settled down to a steady state by the beginning of the period after the explicit forecast period. The meaning of steady state might seem somewhat unclear, but it will be explained in more detail below. Generally, the way steady state is achieved is by setting the input parameters constant from the first year after the explicit forecast period.[12] As we will see later this may not be sufficient, however.

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[11] Proposed in, e.g., Copeland et al.

[12] This year will in this essay henceforth be called year 0. Consequently, the last year in the explicit forecast period will be denoted year (-1).

In order to assess the reasonableness of a steady-state assumption one must know what it implies. Although this type of assumption is often recommended in textbooks, the authors seldom give any deeper answers to this valid question. In this chapter we therefore analyse the concept of steady state thoroughly.

2.0.1 Definitions and problem identification

It is useful, for theoretical and technical reasons, to distinguish between different types of steady states:

Definitions:

A **parametric steady state** (PSS) means that the parameters (of the model) that describe the company's development are constant over the coming years.

> *This is the most intuitive steady state definition. It means that one assumes, e.g., a constant revenue growth, a constant profit margin, etc. PSS is also the weakest form of steady state. No restrictions are placed on the parameters other than that they remain constant.*

A **FCF steady state** (FSS) means that the company's predicted free cash flow grows at a constant rate.

> *In other words, the free cash flow in any year t+1 will be described by* $FCF_{t+1} = (1+g)\cdot FCF_t$ *where g is the constant growth rate. This is supposed to hold for all future years.*

A **NP steady state** (NSS) means that the company's predicted net profit grows at a constant rate.

$$NP_{t+1} = (1+g)\cdot NP_t$$

A **DIV steady state** (DSS) means that the company's predicted dividends grow at a constant rate.

$$DIV_{t+1} = (1+g)\cdot DIV_t$$

A **Textbook steady state** (TSS) means that the following conditions are fulfilled (according to Copeland et al): "The company earns constant margins, maintains a constant capital turnover, and, therefore, earns a constant return on existing capital. The company grows at a constant rate and invests the same proportion of its gross cash flow in its business each year. The company earns a constant return on all new investments."[13]

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[13] Copeland et al p. 290

Copeland et al claim that *any* continuing-value approach relies on the key assumptions stated above under "Textbook steady state".[14] Less is said about how one practically goes about to ensure TSS. This will therefore be one of the main tasks in this chapter.

The Copeland et al conditions for TSS may need some clarifications to become operational:

Constant margins - this will hold if the company is in PSS, since operating expenditures will then be a constant percentage of revenues.

Constant capital turnover - this will hold if invested capital grows at the same rate as revenues.[15] By PSS, revenues grow at a constant rate (the revenue growth rate is one of the parameters); consequently invested capital will have to grow at the same rate. In all cases in our settings below, invested capital will equal the balance sheet total,[16] and the condition can be stated that the balance sheet total must grow at the same rate as revenues.

Constant return on existing capital - this will hold as a consequence of constant margins and a constant capital turnover, since the return on existing invested capital can be calculated as: *operating margin* × *capital turnover* × [1 - *tax rate*].[17]

The company grows at a constant rate - this can be interpreted in different ways. The first possibility is a constant revenue growth. That would follow immediately from PSS. The second possibility is that the balance sheet total grows at a constant rate. The third and final possibility is that the free cash flow generated by the company grows at a constant rate, i.e. FSS.

The company invests the same proportion of its gross cash flow in its business each year - this will hold if gross cash flow grows at the same rate as gross investments *or*, equivalently, if gross cash flow grows at the same rate as FCF.[18] If FSS holds the free cash flow will be proportional to preceding year's revenues (see FSS definition above and expression (7) below). In the setting of Specification A, to be discussed later, the gross cash flow also turns out to be be proportional to preceding year's revenues, i.e. *gross cash flow* = *constant* × *preceding year's revenues*. Thus a sufficient condition for a constant relation between investments and cash flows is that FSS is

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[14] Copeland et al p. 290

[15] Capital turnover is the ratio between *revenues* and *invested capital*. See Copeland et al p. 167.

[16] In our setting, balance sheet total means the sum of *net* working capital and *net PPE*.

[17] Copeland et al. p. 167

[18] Follows from the fact that FCF equals gross cash flow minus gross investments. See Copeland et al p. 169.

established. This will be shown to hold also for Specification B, since the condition for FSS in Specification B reduces it to Specification A.

Constant return on all new investments - this will hold if the balance sheet total grows at the same rate as revenues.

All of the TSS conditions may seem economically intuitive features of a steady state. The rather long list of conditions can be shortened considerably, however, since many of the requirements are only different ways of saying the same thing, and the above conditions for TSS can hence be summarised:

1. The company is in PSS.
2. It exhibits FSS.
3. The balance sheet total grows at the same rate as revenues.

Are these three conditions reasonable then?

1. PSS

For obvious practical reasons, the concept of parametric steady state will hereafter be treated as a necessary condition: the reason for using a horizon value approach in the first place is to *simplify* the valuation (this is indeed also what Copeland et al do). Without constant input parameters one would be back in the first approach with a very long explicit forecast, and the whole point of introducing a horizon is to avoid that.

2. FSS

To calculate a horizon value, a continuing value formula similar to the familiar Gordon-formula[19] is often proposed.[20] However, a first prerequisite for the use of the simple Gordon-formula is that the company is in steady state with respect to the valuation measure. When using the free cash flow approach, the free cash flow must grow at a constant rate in all future years, i.e. FSS. Otherwise one cannot calculate the sum of discounted future free cash flows by using a simple geometric series formula.

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[19] See, e.g., Brealey & Myers p. 34.
[20] See, e.g., Copeland et al pp. 274-277.

3. Balance sheet total growing at the same rate as revenues

This third condition may seem less self-evident than the former two, but it is not unrealistic. The growth rate of the balance sheet total and the sales are empirically closely related. This is commented upon by Johansson (1995).[21]

Some other questions are also of importance:

When is steady state established? The parameters are set to be constant from year 0, so PSS will by definition hold from the beginning. But the other conditions? For instance FSS - is it established already in year 0 (so one can use the free cash flow from year 0 in the continuing value calculation) or later?

What if steady state (other than the assumed PSS) is *not* implied, not in year 0 nor in any future year? One must then take the actual future behaviour of the valuation measure (FCF, DIV or NP) into consideration when calculating the horizon value. Will there be a way to do this with a formula or must one resort to the long explicit forecast, and set up financial statements for 100 years or more in the spreadsheet program?

It is also of interest to find out how one can ensure that an intuitive development of the company will be implied by the steady state assumption: what is required for the model to give intuitive results? One example is that a more efficient use of property, plant and equipment should increase the company value.[22]

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[21] Johansson (1995), p. 21

[22] In order to answer questions like this, the approach in this report is superior to the "infinitely long" forecast period approach. The latter numerical approach will of course give indications when something counter-intuitive is going on, but it does not allow any explicit analysis on the parameter level regarding what intuitive conditions are being violated.

So far, the following problems have been identified:

- Under which conditions on the input parameters does *parametric* steady state imply the other types of steady state (FSS, NSS, DSS, TSS)?
- Is it possible to perform a horizon value calculation if steady state, with respect to the valuation measure, is *not* implied?
- Under which conditions on the input parameters does one obtain intuitive results?
- If steady state is implied, *when* is it established?

2.0.2 Steady state models - framework and assumptions

It is assumed that the company has reached a point in time where the ratios driving the company's balance sheets and income statements have settled to be constant over the years, i.e. the company exhibits a parametric steady state. The company's balance sheet is defined to consist of the following items:

Assets	**Debt and Equity**
(A1) (Net) Working Capital	(D1) Debt
(A2) Net Property, Plant & Equipment	(D2) Deferred Taxes
= Gross PPE - Accumulated Depr.	(D3) Ending equity

Two different Specifications of the PPE-related items are considered in this chapter. We define the following parameters, denoting the forecasted values of the different items, which by the PSS assumption are constant over all future years:[23]

[23] To distinguish between the parameters in the PSS period and the corresponding ones in the explicit forecast period, PSS parameters are henceforth denoted **without** any time-index.

a net working capital in % of revenues (sales)

b gross PPE in % of revenues (sales) [Specification A only]

c change in deferred taxes in % of gross PPE

d depreciation in % of preceding year's gross PPE

e capital expenditures in % of revenues (sales) [Specification B only]

g nominal growth rate,[24] revenues (sales)

i interest rate on debt

p operating expenses in % of revenues (sales)

r retirements in % of preceding year's gross PPE

τ tax rate

w debt in % of balance sheet total (book value)

All parameters except a and b are assumed to be greater than 0 but smaller than 1. The parameters a and b are assumed only to be greater than 0. The revenue growth rate, g, can never exceed the discount rate in the perpetuity period since the equity value then would become infinite, which is clearly absurd.

The use of a single parameter to determine *net* working capital differs at a first glance from the way Copeland et al deal with this. However, since we here consider a PSS period, we can without loss of generality or information sum all the different *working capital* to *revenues* ratios to a single parameter. Of course, when performing the forecast of the explicit forecast period, this type of aggregation may lead to information losses.

In order to determine the development of the debt and equity side of the balance sheet it is assumed that the clean surplus relation (CSR) holds and that net book value of equity is the residual item of the balance sheet. The clean surplus relation means that the change in net book value of equity equals earnings *minus* dividends (see, e.g., Ohlson (1995)). It should be noted that CSR also holds in the McKinsey model, but we use it for the purpose of establishing dividends as the residual item of the model.[25]

---

[24] This means that the revenues of all years in the PSS period can be calculated as $(1+g)$ times the preceding year's revenues.

[25] This differs from Copeland et al who define the debt as the residual item. Our approach implies that dividends are the residual item in the whole system of equations. The reason is twofold: First, this is more intuitive when the company is in steady state, since the excess capital will then be directly distributed to equity owners (it could otherwise lead to negative debt if the company is profitable). Secondly, it also allows us to explicitly derive an expression for the dividend development which makes it possible to compare different valuation methods.

The company debt is assumed to be on market terms, i.e. that the book value of debt is equal to the market value of debt. It should also be noted that the excess marketable securities (EMS) equal zero in the forecast period, which also holds in the Eldon case in Chapter 5. Any ending excess marketable securities of the last historical year are added to the resulting equity value from the model.

Deferred taxes are "a quasi-equity account".[26] For valuation purposes we will treat them as equity. This is also the recommendation in Copeland et al (p. 163).

The first year in the parametric steady state period is denoted year 0. When the case-specific definitions are made in addition, the following state variables can be identified:

Specification A

R_t the revenues (sales) of year *t*,
A_t the accumulated depreciation at the end of year *t*,
T_t deferred taxes at the end of year *t*.

Specification B

R_t revenues (sales) of year *t*,
A_t accumulated depreciation at the end of year *t*,
G_t gross PPE at the end of year *t*,
T_t deferred taxes at the end of year *t*.

Utilising the fact that the balance sheet over the years is a set of difference equations, analytical expressions for the state variables are calculated, and other important economic variables' development over time is in turn derived from the state variables.

[26] Copeland et al, p. 162

2.0.3 A numerical example company

In order to visualise the results an example company will be used throughout this chapter. The model input items have been forecasted to be constants and the company is thus assumed to have entered into a parametric steady state, PSS. The parameters take on the following values:

a=5%

b=40%

c=0.3%

d=6%

e=3.428%

g=5%

i=10%

p=90%

r=4%

τ=30%

w=40%

The initial values of the state variables are the following:

R_0=500

A_0=125

G_0=200

T_0=5.4

2.0.4 Outline of the chapter

Each of the two specifications is treated in a section of its own (sections 2.1-2.2). In each section the case under consideration is analysed in order to find solutions to the problems identified above. Most of the derivations and proofs of each section have been brought together in the appendix at the end of the chapter. Still, a few illustrative proofs have been kept in the sections. Section A2.3 of the appendix gives an example of how the general modelling approach considered in this report can be extended to more complicated settings than the basic McKinsey model setting.

2.1 Specification A - Constant *gross PPE* to *revenues* ratio

In this section we consider Specification A of the PPE items:

$$
\begin{aligned}
G_t &= b \cdot R_t \\
DepX_t &= d \cdot G_{t-1} \\
Ret_t &= r \cdot G_{t-1}
\end{aligned}
$$

By the PSS assumption the parameters are constant. Thus the items in the balance sheet can be defined as follows:

$$
\begin{aligned}
&\text{A1}: aR_t \\
&\text{A2}: bR_t - A_t \\
&\qquad \text{where } A_t = \left[(d-r)bR_{t-1} + A_{t-1}\right] \\
&\text{D1}: w(aR_t + bR_t - A_t) \\
&\text{D2}: T_t = cbR_t + T_{t-1} \\
&\text{D3}: (1-w)(aR_t + bR_t - A_t) - T_t
\end{aligned}
$$

Note that, for t=0, we have $A_0 = (d-r)b'R_{(-1)} + A_{(-1)}$ where b' may not equal b.

The following entities for $t \geq 1$ can now be derived:

Free cash flow, FCF_t:

$$
\begin{aligned}
&(1-\tau)(R_t - pR_t - dbR_{t-1}) + dbR_{t-1} + T_t - T_{t-1} \\
(1)\qquad &-(aR_t - aR_{t-1}) - (bR_t - bR_{t-1} + rbR_{t-1}) = \\
&= R_{t-1}\left[a + (1-r)b + \tau db\right] + R_t\left[(1-\tau)(1-p) - a - b\right] \\
&+T_t - T_{t-1}
\end{aligned}
$$

Net profit, NP_t:

$$
(2)\qquad (1-\tau)\left[R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1})\right]
$$

Dividends (residual item: last year's ending equity plus net profit minus this year's ending equity), DIV_t:

$$
\begin{aligned}
&(1-w)(aR_{t-1}+bR_{t-1}-A_{t-1})-T_{t-1}\\
(3)\qquad &+(1-\tau)(R_t-pR_t-dbR_{t-1}-iw(aR_{t-1}+bR_{t-1}-A_{t-1}))\\
&-[(1-w)(aR_t+bR_t-A_t)-T_t]
\end{aligned}
$$

Three state-variables R_t, A_t and T_t. can also be identified from the balance sheet since the balance sheets over the years is a system of difference equations. The solution to this system is found to be (for $t\geq 1$):[27]

$$(4)\quad R_t=(1+g)R_{t-1}=(1+g)^t R_0$$

$$(5)\quad A_t=\frac{(1+g)^t-1}{g}\cdot(d-r)bR_0+A_0$$

$$
\begin{aligned}
(6)\quad T_t&=\frac{(1+g)^t-1}{g}\cdot c(1+g)bR_0+T_0=\\
&=\left[\frac{(1+g)^{t+1}-1}{g}-1\right]cbR_0+T_0
\end{aligned}
$$

These findings will now be used to analyse the parametric steady state behaviour of free cash flow, net profit and dividends.

2.1.1 Free cash flow

Substituting expressions (4 - 6) into the equation for the free cash flow at each point in time (equation 1) yields the following for $t\geq 1$:

$$(7)\quad FCF_t=(1+g)^t R_0\cdot(m+z_{FCF})$$

where m is the constant after - tax margin $(1-\tau)(1-p)$

and $z_{FCF}=\dfrac{b(c+\tau d-r)-g(b(1-c)+a)}{1+g}$ is a constant.

---

[27] Note again that derivations are provided in Appendix 2 at the end of this chapter.

This is a very neat expression. The *free cash flow* of any year t in the parametric steady state period is given by letting the constant term $R_0(m + z_{FCF})$ grow with $(1+g)^t$. The term $(m + z_{FCF})$ shows that the free cash flow will grow with the after-tax margin of year 0's revenues plus a term involving the ratios *gross PPE / revenues (b), working capital / revenues (a), change in deferred taxes / gross PPE (c), depreciation / preceding year's gross PPE (d)* and *retirements / preceding year's gross PPE (r)* multiplied by revenues.

Proposition 2.1

A company in PSS exhibits FSS without any restrictions on the constant input ratios.

So, by just letting the ratios being constants, a FCF steady state is achieved in the free cash flow development. The proposition follows directly from equation (7).

Properties of the FCF function

Now turning to the question whether the free cash flow develops intuitively, comparative statics can be performed, the results from which make it possible to derive conditions for the intuitive behaviour of the free cash flows. These conditions can be tested numerically when performing a valuation to ensure that the assumptions do not contradict each other or basic economic intuition. The derivations for *FCF* as well as for *NP* and *DIV* in Specification A are presented in the appendix. Since the overall methodology, of deriving properties and suitable conditions, is apparent from the treatment here (i.e. for Spec. A), the corresponding part for Specification B is left out.

In a model like this, comparative statics, i.e. changing one parameter while keeping all other constant, can for some parameters be less realistic. This is especially the case for the *depreciation* and *retirements* parameters (r and d), which are closely related provided one looks at depreciation as a real economic variable and not only something used for tax accounting purposes. These two parameters are therefore not considered here.

- FCF_t is **decreasing** in *a, net working capital / revenues,* without any conditions.

The result is intuitive: the more efficient the use of capital (i.e. the lower the a), the higher the free cash flow. This interpretation is straightforward when considering decreases in items from the asset side of the balance sheet, like, e.g., operating cash. Even if the lower a comes from an increase in one of the debt items included in net working capital, however, it should be interpreted as increased efficiency. For example, larger accounts payable means, all else equal, that the company has negotiated better terms (longer time of payment) with its suppliers.

- FCF_t is **decreasing** in *b, gross PPE / revenues,* if the following inequality is fulfilled:

$$(8) \qquad \tau d - r + (1+g)c < g$$

This should intuitively hold, since a higher b-value means a less efficient use of the company's capital and should therefore yield smaller cash flows. A first glance at the condition seems to indicate that this does not have to be the case. However, when examining the terms of the left hand side for reasonable parameter values, it can be seen that the condition will hold in almost all reasonable situations. First, the difference $\tau d - r$ will be negative in most reasonable cases or at least very small.[28] Further, since the next term c (multiplied by [1+g]) will generally be small in magnitude, the left hand side will be negative or at worst relatively small. The only critical companies are then companies with very low growth. As noted, intuition dictates that the free cash flows be decreasing in b, however, and hence the condition above can be used as a restriction on the parameter c.

- FCF_t is **increasing** in *c, change in deferred taxes / gross PPE.*

The intuition is basic tax-evasion: the more that can be hidden from taxation (by increasing deferred taxes), the better the free cash flow.

- FCF_t is **decreasing** in g, the growth rate, in the *beginning* of the PSS period, but turns to be **increasing** in g after a number of years.

The result can be attributed to two effects:

---

[28] The relationship between d and r will be discussed in Chapter 4.

1. an increase in g lowers the constant z_{FCF}, which thus lowers the constant term $R_0(m+z_{FCF})$ (i.e. the initial value of *FCF*)
2. but g **is** the growth rate, so an increase will eventually raise the free cash flow when t gets large enough.

- FCF_t is **decreasing** in *p, operating expenses / revenues,* without any conditions.

This is trivially intuitive: the higher the operating profit margin (1-p) the higher the free cash flow.

- FCF_t is **decreasing** in τ, *the tax rate*, as long as the *operating profit after depreciation* is positive.

Also this result is intuitive: the more paid out in taxes, the smaller the free cash flow for a profitable company. The explicit parameter condition for a positive operating profit after depreciation is:

(9) $$p+\frac{bd}{1+g}<1$$

2.1.2 Net profit

The net profit in year t is given by:

(2) $$NP_t=(1-\tau)\left[R_t-pR_t-dbR_{t-1}-iw\left(aR_{t-1}+bR_{t-1}-A_{t-1}\right)\right]$$

Rearranging and substituting expressions (4 - 6) into (2) yields:

(10) $$NP_t=(1+g)^t R_0(m+z_{NP})-\chi(\gamma R_0-A_0)$$

with the following constants: $m=(1-\tau)(1-p)$, $\chi=(1-\tau)iw$, $\gamma=\dfrac{(d-r)b}{g}$

$$z_{NP}=-\frac{(1-\tau)db+(1-\tau)iw(a+b)-\dfrac{(d-r)b}{g}(1-\tau)iw}{1+g}$$

The expression for each year's net profit is more complicated than the corresponding one for the free cash flow, expression (7). The major difference is that in expression (10) the time-dependent growth expression has to be reduced by a constant. This means that the net profit will not grow at a constant rate, unless one or more of the following conditions are fulfilled:

1. All equity financing (or zero-interest rate)
2. $gA_0 = (d - r)bR_0$
3. 100% tax-rate

Obviously, the third condition is totally unrealistic and can be ruled out. The remaining two are thus conditions for a NP steady state (NSS), and one of them has to be fulfilled. In reality most companies have at least some debt, and the interesting condition will be the second one.

Properties of the NP function

- NP_t is **decreasing** in *a, net working capital / revenues,* without any restrictions.

This is intuitive for the same reason as in the free cash flow case; the more efficient the use of capital (i.e. the lower the *a*), the higher the net profit.

- NP_t is **decreasing** in *b, gross PPE / revenues,* if the sufficient, but not necessary, condition that *net PPE* does not decrease year by year is fulfilled.

As in the free cash flow case, one would intuitively like to have NP_t decreasing in b. This is actually the case as long as *net PPE* does not decrease between years in the parametric steady state period. This should be a reasonable (and in reality non-binding) restriction on the parameters. If this restriction is not posed, *net PPE* can at some point in the future become negative. The condition can in terms of the input parameters be stated as:

(11) $$(d - r) \leq g$$

The condition is necessary to ensure an intuitive development of the PPE-items, and it can be used as a test in a practical valuation.

- NP_t is **independent** of *c*, *change in deferred taxes / gross PPE.*

The parameter *c* only affects the distribution of tax-payments (between now and the future) and not the tax on the income statement.

- NP_t is **increasing** in *g, the revenue growth rate,* for large *t*:s.

This case is more complicated than the corresponding one for free cash flows. Effects with bearing on the sign of the derivative come from either the growth rate or from the constant z_{NP}

1. The effect from the growth rate is positive since *g* **is** the growth rate.
2. The sign of the effect from the constant term z_{NP} is not clear and will depend on the values of other input parameters, since it in fact consists of two opposite effects: one in the numerator and one in the denominator.

In the beginning of the parametric steady state period the case is not clear-cut: net profits can be either increasing or decreasing depending on the values of the parameters included in z_{NP}. But as $t \to \infty$ the growth effect will dominate the constant term effect, which will ensure that after a certain point in time NP_t will be increasing in *g*.

- NP_t is **decreasing** in *i, the interest rate on debt,* for all relevant cases.

Intuitive and trivial.

- NP_t is **decreasing** in *p, operating expenses / revenues,* without any conditions.

This is also trivially intuitive as it was in the free cash flow case.

- NP_t is **decreasing in** τ, *the tax rate,* for all relevant cases.

As shown in the derivation in the appendix, the condition for this to hold is that the pre-tax net profit is positive in the PSS period. Since we are in a steady state period the company under consideration must always be profitable - an eternally loss-making company is hardly conceivable - and the alternative of zero tax for negative income will not have to be modelled explicitly. This implies that one should check whether the pre-tax net profits implied by the parameter assumptions are positive.

2.1.3 Dividends

Now the dividend stream will be considered. Rearranging the clean surplus definition for dividends at year *t*, expression (3), and substituting expressions (4 - 6) yields:

$$(12) \qquad DIV_t = (1+g)^t R_0\left[m + z_{DIV}\right] - \chi\left[R_0 \cdot \gamma - A_0\right]$$

with the following constants:

$$m = (1-\tau)(1-p), \quad \chi = (1-\tau)iw, \quad \gamma = \frac{(d-r)b}{g}$$

$$z_{DIV} = cb + \frac{b\left[d\tau + w(r-d+g) - r - g - (1-\tau)iw\right] - a\left[g(1-w) + (1-\tau)iw\right] + \frac{(d-r)b}{g}(1-\tau)iw}{1+g}$$

Equation (12), like expression (10) for net profit, is more complicated than the corresponding one for the free cash flow, expression (7). The difference is that here (as in the net profit case), the time-dependent growth expression has to be reduced by a constant, which is the same, independent of time.

Properties of the DIV function

- DIV_t is **decreasing** in *a, net working capital / revenues,* without any conditions.

The more efficient the use of capital (i.e. the lower the *a*), the higher the surplus that can be used for dividends.

- DIV_t is **decreasing** in *b, gross PPE / revenues* if and only if the following inequality is fulfilled:

$$(13) \qquad d\tau + c(1+g) + \frac{\chi(d-r)}{g} - r - w(d-r) - \chi < g(1-w)$$

Also regarding b, the requirement for dividends to be decreasing is more complicated and harder to interpret than in the free cash flow case. Looking at the expression, one can conclude that the negative terms in realistic cases will be large in relation to the positive terms, thus securing that the desired property will hold.

- DIV_t is **increasing** in *c, change in deferred taxes / gross PPE,* without any conditions.

That the dividend payments are increasing in *c* follows from the modelling approach where the book equity value is the residual item of the system and from the clean surplus assumption: Larger deferred taxes reduce book equity. A lower book equity means that a larger part of the earnings has been paid out as dividends. Thus, an increase in deferred taxes increases the dividends paid out since the ending equity is lowered.[29] The book equity should never be negative, and the following two boundary conditions must hold in order to satisfy the *non-negative book equity constraint*:

$$(14) \qquad (1-w)(a+b) > \left(\frac{b}{g} - \frac{b}{g(1+g)}\right)\left[(d-r)(1-w) + c(1+g)\right] + \frac{(1-w)A_0 + T_0}{R_0(1+g)}$$

$$(15) \qquad (1-w)(a+b) > \frac{b}{g}\left[(d-r)(1-w) + c(1+g)\right]$$

- DIV_t is **decreasing** in *i, the interest rate on debt,* for all relevant cases.

The result is trivially intuitive: the more of the company's free cash flow that is paid out to debt-holders, the less can be paid out as dividends.

- DIV_t is **decreasing** in *p, operating expenses / revenues,* without any conditions.

Trivial.

- DIV_t is **decreasing** in τ, *the tax rate,* for all relevant cases.

As in the net profit case, the condition for this to hold is that the pre-tax profits is positive. As argued there, this should always be true in a steady state period.

[29] Note that the TSS conditions do not necessarily imply a constant relation between book equity and deferred taxes.

2.1.4 The transition to steady state - calculational implications

It has so far been seen that FSS will be established from $t \geq 1$ if a parametric steady state is assumed from year 0. For the net profit and dividend cases, NSS and DSS are obtained only under certain conditions. If steady state with respect to the measure used for valuation prevails and in addition the discount rate is constant, a continuing value formula can be used to calculate the value from the future years. Of course, one must then know what year to use as base year for the continuing value calculation. The approach in Copeland et al suggests that the year we call year 0, the first year in the perpetuity period, should be used as base year for calculating the FCF continuing value.

Proposition 2.2

*If at least one of the ratios gross PPE / revenues (b) and net working capital / revenues (a) is changed between year (-1) and year 0 (where year 0 is the first year in the period with constant input parameters) year 0 will **not** be a FSS year: the first actual FSS will be year 1. Thus year 1 should be used as base-year in the continuing value calculation in such cases.*

This means that only if these ratios[30] remain constant between the last year of the explicit forecast period and the first parametric steady state year (year 0), the continuing value can be calculated using year 0 as basis. When using free cash flow valuation, the FCF formula then also applies to year 0. Whenever any one of these ratios is changed, however, the continuing value calculation must be moved one year ahead. The FCF equation (7) can then be used to determine the free cash flow of year 1, whereas a spreadsheet model must be used for calculating the free cash flow of year 0, which for discounting purposes can be seen as belonging to the explicit forecast period.[31] For simplicity, year 1 should always be used as base-year for calculating the continuing free cash flow value.

The way Copeland et al deal with this problem is in our opinion less transparent. They suggest one should adjust the forecast of capital expenditures in year 0, which they use as base-year, to "normalise" the free cash flow. In our approach this normalisation falls out automatically by just

[30] The ratio *net working capital / revenues* used in steady state is of course equal to the sum of the different working capital to revenues ratios used in the explicit forecast period.
[31] Conceptually, the present value of the free cash flow of year 0 plus the present value of the continuing value (for year 1 to infinity) give what is called the *horizon value.*

adding another equivalent column (year) in the spread-sheet model and by using this equivalent year (here: year 1) as base-year for the continuing value. Further, since this approach will give the same value even if the adding of year 1 is unnecessary (and the adding is a very simple operation in itself) one never has to worry about whether adjustments should be made or not. Another advantage is that one explicitly sees what is going on numerically.

The intuition behind Proposition 2.2 is that the free cash flow in year zero depends on the change in working capital and gross PPE between year (-1) and year 0. Since year (-1) does not belong to the PSS period, the change in these items can be different from what would be the case had the company already settled down to a steady state.

2.1.5 Capital structure

In the beginning of this chapter an assumption was made about a constant book value debt ratio (in the example company w=40%) in the PSS period. It is assumed that the company has found this to be the optimal capital structure and wishes it to remain at that level. There may exist a number of reasons for this, as is evident from the vast corporate finance literature on the subject. For a further discussion regarding different motives we refer to the major corporate finance books.[32] Here, we will look at the difference between the book value debt ratio and the market value debt ratio. Any differences between the two will be attributable to differences between the book asset value and the market asset value, since, as stated earlier, the book value of debt equals the market value by assumption.[33]

The commitment of the example company to a debt ratio of 40% means that the debt will each year constitute 40% of the balance sheet total. This by no means guarantees that the market value debt ratio will remain constant, however, as is evident from Figure 1 and Table 8 below:

[32] A more specific discussion on the subject can be found in, e.g., Arditti (1973).
[33] The equality between book and market value of debt is implied by assuming that the interest rate on debt, i, is the market rate of debt for the risk class of companies to which the company belongs.

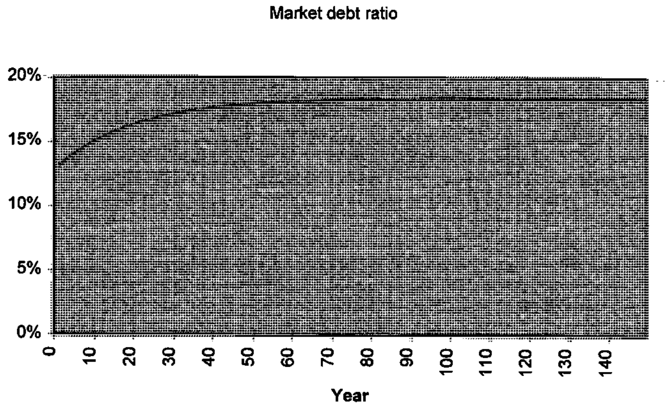

Figure 1 - Market debt ratios in the steady state period using year-to-year WACC

Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)	Year	Debt Ratio (market)
1	13,04%	26	16,85%	51	17,95%	76	18,27%	101	18,37%	126	18,39%
2	13,30%	27	16,92%	52	17,97%	77	18,28%	102	18,37%	127	18,39%
3	13,55%	28	17,00%	53	17,99%	78	18,28%	103	18,37%	128	18,39%
4	13,79%	29	17,06%	54	18,01%	79	18,29%	104	18,37%	129	18,40%
5	14,01%	30	17,13%	55	18,03%	80	18,29%	105	18,37%	130	18,40%
6	14,22%	31	17,19%	56	18,05%	81	18,30%	106	18,37%	131	18,40%
7	14,43%	32	17,25%	57	18,06%	82	18,30%	107	18,38%	132	18,40%
8	14,62%	33	17,30%	58	18,08%	83	18,31%	108	18,38%	133	18,40%
9	14,80%	34	17,36%	59	18,10%	84	18,31%	109	18,38%	134	18,40%
10	14,98%	35	17,41%	60	18,11%	85	18,32%	110	18,38%	135	18,40%
11	15,14%	36	17,45%	61	18,13%	86	18,32%	111	18,38%	136	18,40%
12	15,30%	37	17,50%	62	18,14%	87	18,33%	112	18,38%	137	18,40%
13	15,45%	38	17,54%	63	18,15%	88	18,33%	113	18,38%	138	18,40%
14	15,59%	39	17,58%	64	18,16%	89	18,33%	114	18,38%	139	18,40%
15	15,73%	40	17,62%	65	18,17%	90	18,34%	115	18,39%	140	18,40%
16	15,86%	41	17,66%	66	18,19%	91	18,34%	116	18,39%	141	18,40%
17	15,98%	42	17,70%	67	18,20%	92	18,34%	117	18,39%	142	18,40%
18	16,10%	43	17,73%	68	18,21%	93	18,35%	118	18,39%	143	18,40%
19	16,21%	44	17,76%	69	18,22%	94	18,35%	119	18,39%	144	18,40%
20	16,32%	45	17,79%	70	18,22%	95	18,35%	120	18,39%	145	18,40%
21	16,42%	46	17,82%	71	18,23%	96	18,35%	121	18,39%	146	18,40%
22	16,51%	47	17,85%	72	18,24%	97	18,36%	122	18,39%	147	18,40%
23	16,60%	48	17,88%	73	18,25%	98	18,36%	123	18,39%	148	18,40%
24	16,69%	49	17,90%	74	18,26%	99	18,36%	124	18,39%	149	18,40%
25	16,77%	50	17,93%	75	18,26%	100	18,36%	125	18,39%	150	18,40%

Table 8 - Market debt ratios in the steady state period using year-to-year WACC

The values in Table 8 are calculated using the balance sheet and income statement for each year 150 years ahead. A market value calculation is then performed each year, starting from the last year, using the particular year's balance sheet and income statement information and the particular year's weighted average cost of capital as discount rate. From the final result in Table 8, it can be seen that it takes quite a number of years for the market value debt ratio to converge towards the steady-state-level in the example company: 18.4 %.

The different behaviour of the debt ratio in book-value terms and in market-value terms is interesting in itself, but it also has implications for the choice of discounting method. With a varying market debt ratio the weighted average cost of capital will also be non-constant over time, since the market debt ratio constitutes the weights in WACC formula:

(16) $$k_{WACC,\ t} = \omega_{t-1}(1-\tau)i + (1-\omega_{t-1})k_E$$

where: $k_{WACC,\ t}$ is the weighted average cost of capital[34]
ω_{t-1} is the entering market debt ratio
τ is the tax rate
i is the borrowing rate
k_E is the cost of equity capital

This presents the analyst with practical as well as conceptual problems. To deal with the latter first, one should note that a varying discount rate is actually the rule rather than the exception. The basic DCF model[35] for valuation of an asset has a time-dependent discount rate, r_t:

(17) $$PV = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t}$$

A constant discount rate is only a special case - where all r_t are the same. That the discount rate should be varying over time when valuing such a complex asset as a company is nothing to be surprised at. The fact that in practice the discount rate is often assumed to be constant more reflects the computational problems involved when working with time-dependent rates.

Returning to the practical problems, they can be overcome by starting the discounting process sufficiently long into the future, at a time when the market value debt ratio has itself approached a

[34] The defintion in expression (16) means that the discount rate applied for a particular year is calculated using entering values for the market debt ratio.
[35] Used to determine the present value (PV) of an asset by discounting all future incremental cash flows, C_t, pertaining to the asset at the appropriate discount rate r_t. See, e.g., Brealey & Myers (1991), p. 30.

steady state. When this occurs depends on the parameter values; for normal values using about 100 years will make the approximation error negligible.

Table 8 is based upon an explicit modelling of the example company's operations 150 years ahead, where the revenues each year grow at the rate g. All other financial items are calculated via the formulas in section 2.1 (all parameters are constants by the PSS assumption). This gives explicit financial statements for each of the 150 years. The value of the operations at the end of year 149 is then calculated using the normal *FCF* valuation formula (the one proposed in Copeland et al):

$$(18) \qquad EV_{149} = \frac{FCF_{150}}{k_{WACC,\,150} - g} - D_{149}$$

(*EV* stands for equity value and *D* for debt value; the subindices mark the year.[36]) The equation is solved iteratively, since the equity value affects the WACC which in turn affects the equity value and so on. The actual market value debt ratio in Table 8, i.e. ω_{149}, is calculated by rearranging expression (16):

$$(19) \qquad \omega_{149} = \frac{k_E - k_{WACC,\ 150}}{k_E - (1-\tau)i}$$

The equity value at the end of year 148 is then calculated as (also using an iterative procedure):

$$(20) \qquad EV_{148} = \frac{FCF_{149} + EV_{149} + D_{149}}{1 + k_{WACC,\,149}} - D_{148}$$

The market value debt ratio is subsequently calculated in the same way as in expression (19).

The equity value at the end of each year is then calculated as in expression (20) yielding the market value debt ratios in Table 8 above. This may seem (and indeed is) quite complicated and time-consuming, and hence the practice of applying the WACC from year 0 as the discount rate throughout is certainly understandable - but it is an approximation only, and in many cases an approximation too crude for comfort. In Chapter 3, below, the problem is discussed further and a solution is proposed. First, however, we will look into what actually causes the market value debt ratio to change over time even though the book debt ratio (the parameter w) remains constant.

[36] End of year to be precise.

The discounting method described above is not in itself the cause of the non-constant capital structure, as can be seen in Figure 2 below, where the market value debt ratio using a constant discount rate (i.e. the WACC from year zero) is plotted:

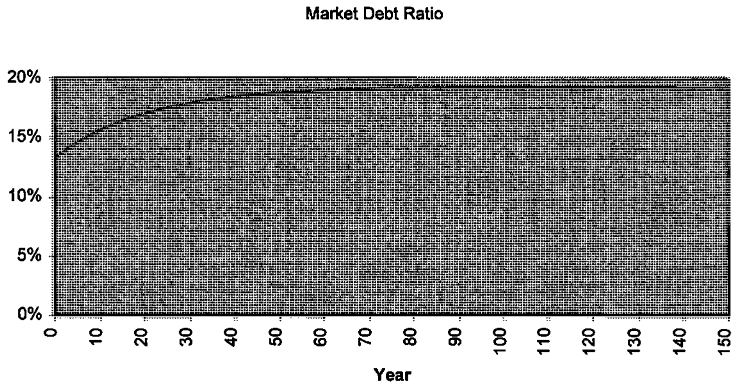

Figure 2 - Market debt ratios in the steady state period using constant (year 0) WACC

In this case the market debt ratio converges towards 19.23%. Using a constant discount rate makes it possible to analyse analytically what is happening: The market value debt ratio is defined as the market value of the debt divided by the market value of the assets. The market value of debt is by assumption equal to the book value and the market value of the assets is the sum of the discounted free cash flows:

$$\frac{D_t}{Assets_t} = \frac{w(aR_t + bR_t - A_t)}{\frac{(1+g)R_t(m+z_{FCF})}{k_{WACC}-g}} = \frac{w(aR_t + bR_t - A_t)(k_{WACC}-g)}{(1+g)R_t(m+z_{FCF})} =$$

$$= \frac{w(a+b)(k_{WACC}-g)}{(1+g)(m+z_{FCF})} - \frac{wA_t(k_{WACC}-g)}{(1+g)R_t(m+z_{FCF})} =$$

$$(21) \quad = \frac{w(a+b)(k_{WACC}-g)}{(1+g)(m+z_{FCF})} - \frac{w\left(\frac{(d-r)b}{g}R_t - \frac{(d-r)b}{g}R_0 + A_0\right)(k_{WACC}-g)}{(1+g)R_t(m+z_{FCF})} =$$

$$= \frac{w\left(a+b-\frac{(d-r)b}{g}\right)(k_{WACC}-g)}{(1+g)(m+z_{FCF})} - \frac{w\left(A_0 - \frac{(d-r)b}{g}R_0\right)(k_{WACC}-g)}{(1+g)^{t+1}R_0(m+z_{FCF})}$$

The first of the two terms in the last row of equation (21) is the steady-state debt ratio. The second term goes to zero as t becomes very large. From equation (21) it is also possible to derive the general condition that must hold if a constant book value debt ratio shall always imply a constant debt ratio also in market value terms. The technical version of this initial condition is obviously:[37]

$$(22) \qquad gA_0 = (d-r)bR_0.$$

This is perhaps more intuitively stated in Proposition 2.3:

Proposition 2.3

In order for the debt ratio to remain constant over time in book value terms as well as in market value terms, the accumulated depreciation must grow at the same rate as the revenues.

Proof of Proposition 2.3:

From equation (21) clearly gA_0 must equal $(d-r)bR_0$ if the debt ratio is to remain constant each year, and hence:

$$(22) \qquad gA_0 = (d-r)bR_0$$

The expression for the accumulated depreciation is:

$$(23) \qquad A_t = \frac{(1+g)^t - 1}{g}(d-r)bR_0 + A_0$$

Substituting equation (22) into equation (23) yields:

$$(24) \qquad A_t = (1+g)^t \frac{(d-r)b}{g} R_0 \Leftrightarrow A_t = (1+g)^t A_0$$

and since all parameters are constant by assumption, the growth rate is clearly g, the same as the revenue growth rate. Q.E.D.

Proposition 2.3 also has some interesting implications:

[37] Remember that g is always less than the discount rate.

Corollary 2.1

A constant market value debt ratio, attained by letting the initial condition $gA_0 = (d-r)bR_0$ *hold, also implies that:*

i) *the net property plant and equipment will grow at the rate g*

ii) *the balance sheet total will grow at the rate g*

iii) *the debt will grow at the rate g*

iv) *the net profit will grow at the rate g*

v) *the dividends will grow at the rate g*

Proof

i) *Gross PPE* is defined as a constant percentage of revenues (b) in Specification A. Since the revenues grow at the rate g, so will the *gross PPE*. The accumulated depreciation will grow at the rate g according to Proposition 2.3. The *net PPE*, finally, is defined as the difference between *gross PPE* and accumulated depreciation and will thus also have the growth rate g.

ii) The balance sheet total is defined as $aR_t + bR_t - A_t$, and since both revenues and accumulated depreciation grow at the rate g, so will the balance sheet total.

iii) The debt is defined as a constant percentage of the balance sheet total, so the proof for the balance sheet total also holds for the debt.

iv) The expression for the net profit is:

$$(10) \quad NP_t = (1+g)^t R_0(m + z_{NP}) - \chi\left(\frac{(d-r)b}{g} R_0 - A_0\right)$$

Since $gA_0 = (d-r)bR_0$ by assumption, the second term in the net profit expression will equal zero, and the net profits will also have the constant growth rate g.

v) The expression for the dividends is:

$$(12) \quad DIV_t = (1+g)^t R_0(m + z_{DIV}) - \chi\left(\frac{(d-r)b}{g} R_0 - A_0\right)$$

Since $gA_0 = (d-r)bR_0$ by assumption, the second term in the dividend expression will equal zero, and the dividends will also have the constant growth rate g. Q.E.D.

From Proposition 2.3 and Corollary 2.1 it is clear that a constant capital structure (in market terms) is a sufficient condition for attaining a steady state with respect to all different concepts under consideration. In particular, Corollary 2.1 shows that the conditions for TSS[38] are fulfilled. When using the model for performing valuations, the constant capital structure has the further advantage that the weighted average cost of capital will remain the same each year (since the weights in the WACC formula remain constant), and thus it is correct to use the year zero WACC as discount rate throughout.

Another interesting implication for valuations is the general irrelevancy of valuation approach implied by this "true steady state" - the constant market value debt ratio:

Proposition 2.4

A constant market value debt ratio, attained by letting the initial condition $gA_0 = (d-r)bR_0$ hold, implies that the free cash flow valuation approach using a constant WACC as discount rate will yield the same result as the dividend valuation approach.

Thus, when considering a steady-state valuation, with a constant market value debt ratio one can be certain that the neo-classical way of determining market value, as the present value of dividends, will be equal to the value obtained from a free cash flow valuation à la Copeland et al. Accordingly, in the setting under consideration here, i.e. using the constant WACC from year 0, the constant capital structure case is the only one that ensures the same value. This is also fully in line with the findings by Chambers, Harris & Pringle (1982) for project valuation. They conclude that using a constant WACC approach will give the same value as using the equity residual method (which is the project valuation case's analogue to the dividend valuation approach considered in this report) if and only if the "debt in every period equals a constant fraction of the value of the cash flows yet to be received."[39] These matters will be further discussed in Chapter 3.

[38] The conditions for TSS are, as stated earlier: 1. The company is in PSS; 2. The company exhibits FSS; 3. The balance sheet total grows at the same rate as revenues.

[39] Chambers, Harris & Pringle (1982), p. 27.

Proof

Denote the constant market-value debt ratio by ω.[40] From expression (21) and the steady-state condition, ω in the free cash flow case is given by the debt value divided by the asset value at the end of year zero:

$$(25) \qquad \omega = \frac{w(aR_0 + bR_0 - A_0)}{\dfrac{(1+g)R_0(m+z_{FCF})}{k_{WACC}-g}} = \frac{w\left(a+b-\dfrac{(d-r)b}{g}\right)(k_{WACC}-g)}{(1+g)(m+z_{FCF})}$$

The weighted average cost of capital is given by:

$$(26) \qquad k_{WACC} = \omega(1-\tau)i + (1-\omega)k_E$$

Substituting (26) into (25) and rearranging yields:

$$(27) \qquad \omega = \frac{w\left(a+b-\dfrac{(d-r)b}{g}\right)}{\dfrac{(1+g)(m+z_{FCF}) - w\left(a+b-\dfrac{(d-r)b}{g}\right)((1-\tau)i-k_E)}{k_E-g}}$$

The constant z_{FCF} is equal to:

$$(28) \qquad z_{FCF} = \frac{b(c+\tau d-r) - g(b(1-c)+a)}{1+g}$$

Thus, (27) can equivalently be stated as:

$$(29) \qquad \omega = \frac{w\left(a+b-\dfrac{(d-r)b}{g}\right)}{\dfrac{(1+g)m + (1+g)cb + b\tau d - br - g(a+b) - w\left(a+b-\dfrac{(d-r)b}{g}\right)((1-\tau)i-k_E)}{k_E-g}}$$

[40] The subindex denoting time is dropped, since the market-value debt ratio is constant over time, i.e. the same for all t.

The constant z_{DIV} is given by:

$$(30) \qquad z_{DIV} = cb + \frac{b\left(d\tau + w(r-d+g) - r - g - (1-\tau)iw\right) - a\left(g(1-w) + (1-\tau)iw\right) + \frac{(d-r)b}{g}(1-\tau)iw}{1+g}$$

Substituting (30) into (29), rearranging and multiplying numerator as well as denominator by R_0 yields:

$$(31) \qquad \omega = \frac{w(aR_0 + bR_0 - A_0)}{w(aR_0 + bR_0 - A_0) + \frac{(1+g)R_0(m + z_{DIV})}{k_E - g}}$$

The denominator in (31) is the market value of debt[41] plus the market value of all possible future dividends.

Comparing equation (25) with equation (31), the following expression is obtained:

$$(32) \qquad \frac{(1+g)R_0(m + z_{FCF})}{k_{WACC} - g} - w(aR_0 + bR_0 - A_0) = \frac{(1+g)R_0(m + z_{DIV})}{k_E - g}$$

The left-hand-side is the equity value using the free-cash-flow approach and the right-hand-side is the equity value using the dividend approach. Q.E.D.

It is now possible to summarise the findings about when the different concepts of steady state are achieved and what this implies for the calculation of the horizon value:

Proposition 2.5

If and only if the initial condition $gA_0 = (d-r)bR_0$ *is fulfilled a TSS will be established,*

and consequently,

it is only with a constant capital structure in market terms that one can use a FCF continuing value formula to calculate the horizon value without approximation errors.

[41] By assumption equal to the book value.

The intuition behind this is that a non-constant capital structure means that the weights in the *weighted average cost of capital (WACC)* [42] *formula* will change over time. A further consequence is that the different costs of capital for the company in question are not likely to remain constant if the riskiness of the company changes over time due to the varying capital structure. These issues are discussed further in Chapter 3. The more formal proof of Proposition 2.5 is given below.

One contribution in this report is the derivation of an analytical condition, in terms of forecasted parameter values, to ensure that the underlying assumptions of the continuing value approach are fulfilled. This is missing in Copeland et al; they say verbally what conditions should be fulfilled but they do not give the analyst much guidance as to how this is done in practice.

Proof of Proposition 2.5

In section 2.0.1 it was concluded that TSS is established if

1. the company is in *PSS*,
2. it exhibits *FSS* and
3. the *balance sheet total* grows at the same rate as revenues.

The first condition holds trivially by the basic PSS assumption. The same is true for the FSS condition by Proposition 2.1. Since the balance sheet total is the sum of net working capital and net PPE, and since net working capital will be growing at the same rate, *g*, as revenues (by definition of the model), the balance sheet total will grow at the same rate as revenues if and only if net PPE grows at the rate *g*. But since net PPE is equal to gross PPE minus accumulated depreciation, and since gross PPE by model definition grows at the same rate as revenues, the third condition is equivalent to the condition that accumulated depreciation grow at the rate *g*: $A_t = (1+g)^t A_0$. From equation (24) in the proof to Proposition 2.3 we know that this is true if and only if condition (22) holds. Thus, a TSS is established if and only if condition (22), i.e. $gA_0 = (d-r)bR_0$, holds. Q.E.D.

[42] The discount rate proposed by Copeland et al, p. 239.

The constant capital structure is necessary also for the dividend valuation approach.

Corollary 2.2

It is only with a constant capital structure in market terms that one can use a DIV continuing value formula to calculate the horizon value without approximation errors.

Proof

The equation for the dividends is:

$$(12) \qquad DIV_t = (1+g)^t R_0 (m + z_{DIV}) - (1-\tau) iw \left(\frac{(d-r)b}{g} R_0 - A_0 \right)$$

In order to use a continuing value, the growth rate must be constant. This is the case only when the constant term equals zero. There are four possibilities: 100% tax rate (τ=1), zero interest rate on debt (i=0), all-equity financing (w=0), and finally when the condition $gA_0 = (d-r)bR_0$ holds. 100% tax rate is absurd as is a zero interest rate. All-equity financing means that dividends will equal free cash flow and the proof of Proposition 2.5 applies. Hence, we are left with the fourth possibility, namely that $gA_0 = (d-r)bR_0$. Q.E.D.

2.2 Specification B - Constant capital expenditures to revenues ratio

If the PPE-items are modelled differently, the system of difference equations also must be modified. Here, it will be shown how the solution is affected when the *gross PPE* at the end of year *t* is derived as the preceding year's *gross PPE* plus capital expenditures made during year *t* minus retirements. The capital expenditures are forecasted as a percentage of revenues, the retirements as a percentage of the preceding year's *gross PPE*. This is what is called Specification B (presented in section 1.2):

$$(33) \qquad G_t = G_{t-1} + CapX_t - Ret_t$$

$$(34) \qquad CapX_t = e \cdot R_t$$

$$(35) \qquad Ret_t = r \cdot G_{t-1}$$

The balance sheet items are defined as follows:

A1: aR_t

A2: $G_{t-1} + eR_t - rG_{t-1} - A_t = eR_t + (1-r)G_{t-1} - A_t$

where $A_t = (d-r)G_{t-1} + A_{t-1}$

D1: $w(aR_t + G_t - A_t)$

D2: $T_t = cG_t + T_{t-1}$

D3: $(1-w)(aR_t + G_t - A_t) - T_t$

The balance sheet now contains four time-dependent state-variables: revenues (R_t), gross PPE (G_t), accumulated depreciation (A_t), and deferred taxes (T_t). Once again utilising the fact that the development of the balance sheet over the years is a system of difference equations, the expressions for the state-variables are found to be:

$$\begin{aligned}
R_t &= R_0(1+g)^t \\
G_t &= e\frac{1+g}{g+r}R_0(1+g)^t + \left(G_0 - e\frac{1+g}{g+r}R_0\right)(1-r)^t \\
A_t &= A_0 + \frac{(1+g)^t-1}{g}(d-r)e\frac{1+g}{g+r}R_0 + \frac{d-r}{r}\left(G_0 - e\frac{1+g}{g+r}R_0\right)\left(1-(1-r)^t\right) \\
T_t &= T_0 + \frac{(1+g)^t-1}{g}c(1+g)e\frac{1+g}{g+r}R_0 + \frac{c(1-r)}{r}\left(G_0 - e\frac{1+g}{g+r}R_0\right)\left(1-(1-r)^t\right)
\end{aligned} \tag{36-39}$$

Now define the constant $\beta = e\dfrac{1+g}{g+r}$ and substitute into the solution:

$$\begin{aligned}
R_t &= R_0(1+g)^t \\
G_t &= \beta R_0(1+g)^t + (G_0 - \beta R_0)(1-r)^t \\
A_t &= A_0 + \frac{(1+g)^t-1}{g}(d-r)\beta R_0 + \frac{d-r}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right) \\
T_t &= T_0 + \frac{(1+g)^t-1}{g}c(1+g)\beta R_0 + \frac{c(1-r)}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right)
\end{aligned} \tag{40-43}$$

These expressions can now be compared with the ones obtained when using the previous specification (Specification A) to describe the PPE-development:

Revenues

Specification A: $R_t = (1+g)^t R_0$

Specification B: $R_t = (1+g)^t R_0$

Gross property, plant and equipment

Specification A: $G_t = bR_0(1+g)^t$

Specification B: $G_t = \beta R_0(1+g)^t + (G_0 - \beta R_0)(1-r)^t$

Accumulated depreciation

Specification A: $A_t = A_0 + \frac{(1+g)^t - 1}{g}(d-r)bR_0$

Specification B: $A_t = A_0 + \frac{(1+g)^t - 1}{g}(d-r)\beta R_0 + \frac{d-r}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right)$

Deferred taxes

Specification A: $T_t = T_0 + \frac{(1+g)^t - 1}{g}c(1+g)bR_0$

Specification B: $T_t = T_0 + \frac{(1+g)^t - 1}{g}c(1+g)\beta R_0 + \frac{c(1-r)}{r}(G_0 - \beta R_0)\left(1-(1-r)^t\right)$

The expressions for the revenue development are the same in the two cases. The *gross PPE* differs by the term $(G_0 - \beta R_0)(1-r)^t$, the conclusion being that only in the special case where β is specified such that $G_0 - \beta R_0 = 0$ are the two specifications the same,[43] i.e. they are equal when $G_0 = \beta R_0$ - but this is exactly the definition of Specification A, when *gross PPE* was assumed to be a certain percentage of revenues in each year, also in year zero.[44] The same condition holds true also for accumulated depreciation as well as for deferred taxes, and hence:

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[43] The two expressions for the gross PPE development are also the same when $t \to \infty$. This case is never of any practical interest, however, since for large values of t, the *discounted* value is (almost) zero.

[44] Remember that the constant β is short for $e \cdot (1+g)/(r+g)$.

Observation 1

The specification of gross PPE in year t as a fixed percentage of the revenues the same year

(44) $\quad G_t = b \cdot R_t$ *("Specification A")*

is only a special case of the more general modelling of G_t *as the preceding year's gross PPE plus capital expenditures minus retirements*

(45) $\quad G_t = G_{t-1} + CapX_t - Ret_t$ *("Specification B")*

where capital expenditures are defined as a percentage of revenues and retirements as a percentage of the preceding year's gross PPE.

2.2.1 Free cash flow

The expression for the free cash flow in year t is:

(46)
$$\begin{aligned} FCF_t &= (1-\tau)\left(R_t - pR_t - dG_{t-1}\right) + dG_{t-1} + T_t - T_{t-1} - \left(aR_t - aR_{t-1}\right) - \left(G_t - G_{t-1} + rG_{t-1}\right) \\ &= aR_{t-1} + \left((1-\tau)(1-p) - a\right)R_t + \left(1 - r + \tau d\right)G_{t-1} - G_t + T_t - T_{t-1} \end{aligned}$$

Substituting the expressions for R_t, G_t, and T_t into the free cash flow equation yields eventually the following expression (see the appendix for a more detailed derivation):

(47)
$$FCF_t = (1+g)^t R_0 \left(m + \frac{\beta\left(c + \tau d - r\right) - g\left(\beta(1-c) + a\right)}{1+g} \right) + \left(c + \tau \frac{d}{1-r} \right)\left(G_0 - \beta R_0\right)\left(1-r\right)^t$$

The resulting expression for the free cash flow at year t is:

(48) $$FCF_t = (1+g)^t R_0\left(m+z_{FCF(2)}\right)+\phi(G_0-\beta R_0)(1-r)^t$$

where: $\beta = e\dfrac{1+g}{g+r}$, $\phi = \left(c+\tau\dfrac{d}{1-r}\right)$, $m=(1-\tau)(1-p)$

$$z_{FCF(2)} = \frac{\beta(c+\tau d-r)-g(\beta(1-c)+a)}{1+g}$$

This can be compared with the free cash flow expression using Specification A:

Specification A: $FCF_t = (1+g)^t R_0\left(m+z_{FCF}\right)$

Specification B: $FCF_t = (1+g)^t R_0\left(m+z_{FCF(2)}\right)+\phi(G_0-\beta R_0)(1-r)^t$

It is obvious that the expressions are equal only in the special case where $G_0 = \beta R_0$,[45] which is in line with Observation 1 above. This, however, also has implications for steady state. In Proposition 2.1 it was claimed that *a company in parametric steady state* (PSS) *exhibits FCF steady state* (FSS) *without any restrictions on the constant input ratios*. In the more general Specification B, this proposition must be modified:

Proposition 2.6

A company in parametric steady state exhibits FCF steady state with respect to the free cash flow development if and only if the initial condition

(49) $$e = \frac{G_0(g+r)}{(1+g)R_0}$$

is fulfilled.

If the initial condition in Proposition 2.6 is fulfilled Specification B reduces to Specification A, where the ratio *gross PPE / revenues* is constant, as is seen from Observation 1. Thus the only instance in this setting where steady state can be established, allowing for the use of a continuing value formula,

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[45] This can be restated as $e = \dfrac{G_0(g+r)}{(1+g)R_0}$.

is when the parameter values are such that the specification exactly equals that of Specification A! Hence, there exists no reason to use Specification B when specifying the PSS in order to yield a FSS; the simpler Specification A does the trick completely. However, as will be argued in section 4.1, Specification B does have some intuitive features which makes it useful in the explicit forecast period, but when the company is assumed to settle down to a steady state, it is easier to turn to Specification A when determining the property, plant and equipment items.

As the only steady state in Specification B exists when the FCF function of Specification B equals the one of Specification A, the analysis of the FCF function for Specification A is apparently valid also here.

2.2.2 Net profit

The net profit in year t is given by:

$$(50) \qquad NP_t = (1-\tau)\left(R_t - pR_t - dG_{t-1} - iw\left(aR_{t-1} + G_{t-1} - A_{t-1}\right)\right)$$

Substituting the expressions (40-43) into (50) and rearranging yields finally:

$$(51) \qquad NP_t = (1+g)^t R_0\left(m + z_{NP(2)}\right) - \chi\left(\frac{d-r}{g}\beta R_0 - A_0\right) - \eta\left(\beta R_0 - G_0\right) + \varphi\left(\beta R_0 - G_0\right)(1-r)^t$$

where: $\beta = e\dfrac{1+g}{g+r}$, $\chi = (1-\tau)iw$, $m = (1-\tau)(1-p)$,

$$\varphi = \chi\frac{d-r}{r(1-r)} + \frac{(1-\tau)(d+iw)}{1-r}, \qquad \eta = \chi\frac{d-r}{r},$$

$$z_{NP(2)} = -\frac{(1-\tau)d\beta + \chi\left(a + \beta\left(1 - \dfrac{d-r}{g}\right)\right)}{1+g}$$

The net profit will generally not grow at a constant rate. There is one correction term involving accumulated depreciation and another involving gross PPE. Also where net profit is concerned, a comparison between Specification A and Specification B shows that Specification A is only a special subcase of Specification B, and Observation 1 still holds:

$$(52)\qquad NP_t = (1+g)^t R_0\left(m + z_{NP(2)}\right) - \chi\left(\frac{d-r}{g}\beta R_0 - A_0\right) - \eta\left(\beta R_0 - G_0\right) + \varphi\left(\beta R_0 - G_0\right)\left(1-r\right)^t$$

$$(53)\qquad NP_t = (1+g)^t R_0\left(m + z_{NP}\right) - \chi\left(\frac{(d-r)b}{g} R_0 - A_0\right)$$

Consequently, if the initial condition (49) is fulfilled, then (52) reduces to (53), and the NP function will be exactly the same as for Specification A.

2.2.3 Dividends

Now, the dividend stream will be analysed using Specification B. The expression for dividends in year t is:

$$(54)\qquad \begin{aligned} DIV_t \;=\; & (1-w)\left(aR_{t-1} + G_{t-1} - A_{t-1}\right) - T_{t-1} \\ & +(1-\tau)\left((1-p)R_t - dG_{t-1} - iw\left(aR_{t-1} + G_{t-1} - A_{t-1}\right)\right) \\ & -\left[(1-w)\left(aR_t + G_t - A_t\right) - T_t\right] \end{aligned}$$

When substituting for revenues, *gross PPE*, accumulated depreciation and deferred taxes (expressions 40-43) the following expression is obtained:

$$(55)\qquad DIV_t = (1+g)^t R_0\left(m + z_{DIV(2)}\right) - \chi\left(\frac{d-r}{g}\beta R_0 - A_0\right) - \eta\left(\beta R_0 - G_0\right) - \kappa\left(\beta R_0 - G_0\right)\left(1-r\right)^t$$

$$\beta = e\frac{1+g}{g+r}, \qquad \chi = (1-\tau)iw, \qquad \eta = \chi\frac{d-r}{r},$$

$$\kappa = c + \frac{d}{1-r}\left(\tau - w - \frac{\chi}{r}\right), \qquad m = (1-\tau)(1-p),$$

$$z_{DIV(2)} = c\beta + \frac{\beta\left(d\tau + w\left(r-d+g\right) - r - g - \chi\right) - a\left(g(1-w) + \chi\right) + \chi\frac{(d-r)}{g}\beta}{1+g}$$

This dividend expression is quite complicated. It is, however, equal to the Specification A expression in the special case where $G_0 = \beta R_0$, which is to be expected, given Observation 1.

Appendix 2

A2.1 Specification A

A2.1.0 Solution to system of difference equations

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ A_t &= (d-r)bR_{t-1} + A_{t-1} \end{cases}$$

In matrix notation:

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1}$$

where: $\mathbf{x}_t = \begin{pmatrix} R_t \\ A_t \end{pmatrix}$ and $\mathbf{A} = \begin{pmatrix} 1+g & 0 \\ (d-r)b & 1 \end{pmatrix}$

The roots of the characteristic equation are $\lambda_1 = 1+g$ and $\lambda_2 = 1$.

A is diagonalised by **P**

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ \dfrac{(d-r)b}{g} & 1 \end{pmatrix}$$

So

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} 1+g & 0 \\ 0 & 1 \end{pmatrix}$$

Substituting $\mathbf{x}_t = \mathbf{P}\,\mathbf{u}_t$ and $\mathbf{x}_{t+1} = \mathbf{P}\,\mathbf{u}_{t+1}$ yields the system $\mathbf{u}_{t+1} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}\,\mathbf{u}_t$, the solution of which is:

$$\mathbf{u}_t = \begin{pmatrix} c_1(1+g)^t \\ c_2 \end{pmatrix}$$

Substituting back yields the solution to the original system:

$$\mathbf{x}_t = \mathbf{P}\,\mathbf{u}_t = \begin{pmatrix} 1 & 0 \\ \dfrac{(d-r)b}{g} & 1 \end{pmatrix}\begin{pmatrix} c_1(1+g)^t \\ c_2 \end{pmatrix} = \begin{pmatrix} c_1(1+g)^t \\ \dfrac{(d-r)b}{g}c_1(1+g)^t + c_2 \end{pmatrix}$$

and since the initial values are R_0 and A_0, the complete solution is:

$$\begin{cases} R_t &= (1+g)^t R_0 \\ A_t &= \dfrac{(1+g)^t - 1}{g}(d-r)bR_0 + A_0 \end{cases}$$

In the same way, the solution for T_t is derived.

A2.1.1a Free cash flow derivation - equation (7)

$$\begin{aligned} FCF_t &= R_{t-1}\left[a+(1-r)b+\tau db\right]+R_t\left[(1-\tau)(1-p)-a-b\right]+T_t-T_{t-1} = \\ &= (1+g)^{t-1}R_0\left[a+(1-r)b+\tau db\right]+(1+g)^t R_0\left[(1-\tau)(1-p)-a-b\right]+ \\ &\quad +\frac{(1+g)^t-1}{g}(1+g)cbR_0+T_0-\frac{(1+g)^{t-1}-1}{g}(1+g)cbR_0-T_0 = \\ &= (1+g)^t R_0\left[(1-\tau)(1-p)-a-b+\frac{a+(1-r)b+\tau db}{(1+g)}\right]+\frac{(1+g)^t-1}{g}\left[(1+g)cbR_0-cbR_0\right]+cbR_0 = \\ &= (1+g)^t R_0\left[\frac{(1+g)(1-\tau)(1-p)-ag+b(\tau d-r-g)}{1+g}\right]+(1+g)^t R_0 cb = \\ &= (1+g)^t R_0\left[\frac{(1+g)(1-\tau)(1-p)-ag+b(\tau d-r-g)}{1+g}+cb\right] = \\ &= (1+g)^t R_0\left[(1-\tau)(1-p)+\frac{g\left(b(c-1)-a\right)+b\left(c+\tau d-r\right)}{1+g}\right] \end{aligned}$$

So, finally for $t \geq 1$:

$$FCF_t = (1+g)^t R_0 \cdot \left(m + z_{FCF}\right)$$

where m is the constant after-tax margin $(1-\tau)(1-p)$

and $z_{FCF} = \dfrac{b\left(c+\tau d-r\right)-g\left(b\left(1-c\right)+a\right)}{1+g}$ is a constant.

A2.1.1b Derivations of the FCF function properties

- *Net working capital / revenues (a)*

Follows directly from equation (7)

- *Gross PPE / revenues (b)*

The constant b appears only in the term z_{FCF} of equation (7). The marginal effect of a change in b on FCF_t is thus equal to the first derivative of z_{FCF} with respect to b. If z_{FCF} is decreasing in b, then FCF_t is decreasing in b, since FCF_t is increasing in z_{FCF}.

$$\frac{\partial z_{FCF}}{\partial b}=\frac{gc-g+c+\tau d-r}{1+g}$$

Then z_{FCF} is decreasing in b, if and only if $\frac{\partial z_{FCF}}{\partial b}<0$:

$$\frac{gc-g+c+\tau d-r}{1+g}<0$$

After rearranging:

$$\tau d-r+(1+g)c<g$$

- *Change in deferred taxes / gross PPE (c)*

Follows directly from equation (7).

- *Growth rate (g)*

Differentiating the free cash flow expression (7) with respect to g yields:

$$\frac{\partial FCF_t}{\partial g}=(1+g)^t\frac{t}{(1+g)}R_0\left(m+z_{FCF}\right)-(1+g)^t R_0\left[\frac{(b(1-c)+a)+z_{FCF}}{1+g}\right]$$

There are two opposite effects: the left term strives towards a positive derivative, whereas the right term strives in the opposite direction. For low t:s, the negative term tends to dominate the positive, but since the positive term increases faster with respect to t, it will eventually dominate the negative.

In the original FCF formula, the positive term is attributed to the growth factor (1+g), whereas the negative term is comes from g's lowering effect on the constant term z_{FCF}

- *Tax rate, τ*

From equation (7) one can see that a change in the tax rate, τ, will affect the after-tax margin, m, and also the constant z_{FCF}. The effects of a change in τ on m and z_{FCF} go in the opposite direction, i.e. a raised τ decreases m and increases z_{FCF}. To compare these effects the partial derivatives of m and z_{FCF} are taken with respect to (-τ) and (τ) respectively:

$$\frac{\partial m}{\partial(-\tau)} = (1-p) \qquad \frac{\partial z_{FCF}}{\partial \tau} = \frac{bd}{1+g}$$

For FCF_t to be decreasing in τ, the following must hold:

$$(1-p) > \frac{bd}{1+g}$$

Rearranging then gives

$$p + \frac{bd}{1+g} < 1$$

Multiplying both sides with R_t yields:

$$R_t - pR_t - dbR_{t-1} > 0 \quad \Leftrightarrow \quad \textit{Operating income - operating expenses - depreciation} > 0$$

Thus the condition can be phrased: the *operating profit after depreciation* should be positive

A2.1.2a Derivation of net profit expression - equation (10)

$$
\begin{aligned}
NP_t &= (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right) = \\
&= (1-\tau)\left((1-p)R_t + R_{t-1}\left(-db - iw(a+b)\right) + iwA_{t-1}\right) = \\
&= R_t\left((1-\tau)(1-p) - \frac{1-\tau}{1+g}\left(db + iw(a+b)\right)\right) + (1-\tau)iwA_{t-1} = \\
&= R_t\left((1-\tau)(1-p) - \frac{1-\tau}{1+g}\left(db + iw(a+b)\right)\right) + (1-\tau)iw\left(A_0 + \frac{(d-r)b}{g}\left(R_{t-1} - R_0\right)\right) = \\
&= (1+g)^t R_0\left((1-\tau)(1-p) - \frac{(1-\tau)db + (1-\tau)iw(a+b) - \frac{(d-r)b}{g}(1-\tau)iw}{1+g}\right) - \\
&\quad -(1-\tau)iw\left(\frac{(d-r)b}{g}R_0 - A_0\right) = \\
&= (1+g)^t R_0\left(m + z_{NP}\right) - \chi\left(\gamma R_0 - A_0\right)
\end{aligned}
$$

where: $m = (1-\tau)(1-p)$, $\chi = (1-\tau)iw$, $\gamma = \dfrac{(d-r)b}{g}$

$$
z_{NP} = -\frac{(1-\tau)db + (1-\tau)iw(a+b) - \frac{(d-r)b}{g}(1-\tau)iw}{1+g}
$$

A2.1.2b Derivations of the NP function properties

- *Net working capital / revenues (a)*

Follows directly from equation (10).

- *Gross PPE / revenues (b)*

Differentiating the net profit expression (10) with respect to b yields:

(i) $$\frac{\partial NP_t}{\partial b} = R_0(1+g)^{t-1}\left(-(1-\tau)d - \chi + \frac{(d-r)}{g}\chi\right) - R_0\frac{(d-r)}{g}\chi$$

Intuitively, net profit should be decreasing in b, since a lower b-value means a more efficient use of company capital. From equation (i) it is clear that this is not necessarily the case. The critical case to examine is when t is very large. Then it must hold that:

(ii) $$-(1-\tau)d-\chi+\frac{(d-r)}{g}\chi<0$$

Then, a *sufficient* condition for (ii) to hold is that $(d\text{-}r)$ is smaller than or equal to g:

(iii) $$(d-r)\leq g$$

Multiplying both sides by bR_{t-1} yields:

(iv) $$(d-r)bR_{t-1}\leq gbR_{t-1}$$

Using the definitions of the parameters, it can be seen that this is the same thing as:

(v) $$A_t-A_{t-1}\leq G_t-G_{t-1}$$

Or, alternatively:

(vi) $$N_t-N_{t-1}\geq 0$$

Hence, it is a ***sufficient condition** that the net property plant and equipment will not be decreasing between years in the parametric steady state period*, which seems a reasonable restriction on the parameters.

- *Change in deferred taxes / gross PPE (c)*

The parameter c does not appear in the NP function.

- *Growth rate (g)*

$$\frac{\partial NP_t}{\partial g}=R_{t-1}\left[t\cdot(m+z_{NP})-\frac{1}{g}\cdot\gamma\cdot\chi-z_{NP}\right]+\frac{R_0}{g}\cdot\gamma\cdot\chi$$

This partial derivative is very hard to interpret. However as $t\rightarrow\infty$ it will definitely grow positive since the term $m+z_{NP}$ is positive for all relevant cases in a steady state.

- *Interest rate on debt (i)*

See the same section under *dividends*.

- *Operating expenses / revenues (p)*

Follows trivially from equation (10).

- *Tax rate, τ*

Differentiating equation (2):

$$\frac{\partial NP_t}{\partial \tau} = -\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} - bR_{t-1} - A_{t-1}\right)\right)$$

This is simply the pre-tax income multiplied by minus one, and the somewhat trivial conclusion is that in a profit-making company the net profit is decreasing in τ, i.e. the net profit will be smaller the higher the tax rate.

A2.1.3a Derivation of dividend expression - equation (12)

$$\begin{aligned}
DIV_t \;=\;& (1-w)\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - T_{t-1} + \\
& +(1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right) - \\
& -\left[(1-w)\left(aR_t + bR_t - A_t\right) - T_t\right] = \\
=\;& R_{t-1}\left[(1-w)(a+b) - (1-\tau)\left(db + iw(a+b)\right)\right] + \\
& +R_t\left[m - (1-w)(a+b)\right] + \\
& +A_{t-1}\left[(1-\tau)iw - (1-w)\right] + \\
& +A_t(1-w) + \\
& +T_t - T_{t-1}
\end{aligned}$$

$$\begin{aligned}
DIV_t \;=\;& (1+g)^{t-1} R_0\left[(1-w)(a+b) - (1-\tau)\left(db + iw(a+b)\right)\right] + \\
& +(1+g)^t R_0\left[m - (1-w)(a+b)\right] + \\
& +\frac{(1+g)^{t-1} - 1}{g}(d-r)bR_0\left[(1-\tau)iw - (1-w)\right] + A_0\left[(1-\tau)iw - (1-w)\right] + \\
& +\frac{(1+g)^t - 1}{g}(d-r)bR_0(1-w) + A_0(1-w) + \\
& +cbR_0(1+g)^t
\end{aligned}$$

$$
\begin{aligned}
DIV_t &= (1+g)^t R_0 \cdot [m+cb] + (1+g)^{t-1} R_0 [(d-r)b(1-w) - (a+b)(1-w)g - (1-\tau)bd - (1-\tau)iw(a+b)] + \\
&\quad + \frac{(1+g)^{t-1}-1}{g} R_0 (1-\tau)iw(d-r)b + A_0[(1-\tau)iw] = \\
&= (1+g)^t R_0 \cdot [m+cb] + (1+g)^{t-1} R_0 [b[d\tau - w(d-r) - r] - (a+b)(1-w)g - (a+b)(1-\tau)iw] + \\
&\quad + \frac{(1+g)^{t-1} R_0 (1-\tau)iw(d-r)b}{g} - R_0 \frac{(1-\tau)iw(d-r)b}{g} + A_0[(1-\tau)iw] = \\
&= (1+g)^t R_0 \cdot \left[m + cb + \frac{b[d\tau - w(d-r) - r] - (a+b)(1-w)g - (a+b)(1-\tau)iw}{1+g} + \frac{(1-\tau)iw(d-r)b}{g(1+g)} \right] - \\
&\quad -(1-\tau)iw\left[R_0 \frac{(d-r)b}{g} - A_0 \right]
\end{aligned}
$$

and following expression is obtained for the dividends payed out during year t:

$$DIV_t = (1+g)^t R_0 [m + z_{DIV}] - \chi[R_0 \cdot \gamma - A_0]$$

where:

$$
\begin{aligned}
&m = (1-\tau)(1-p) \quad \text{(the after-tax profit margin)} \\
&z_{DIV} = cb + \frac{b[d\tau + w(r-d+g) - r - g - \chi] - a[g(1-w) + \chi]}{1+g} + \frac{(d-r)b\chi}{g(1+g)} \\
&\chi = (1-\tau)iw \\
&\gamma = \frac{(d-r)b}{g}
\end{aligned}
$$

A2.1.3b Derivations of the DIV function properties

- *Net working capital / revenues (a)*

$$\frac{\partial DIV_t}{\partial a} = -(1+g)^{t-1} R_0 [g(1-w) + (1-\tau)iw] < 0$$

- *Gross PPE / revenues (b)*

$$\frac{\partial DIV_t}{\partial b} = (1+g)^t R_0 \left[c + \frac{d\tau + w(r-d+g) - r - g - \chi}{1+g} + \frac{(d-r)\chi}{g(1+g)} \right] - \chi R_0 \frac{d-r}{g}$$

Then the dividend function is decreasing in b, if and only if $\frac{\partial DIV_t}{\partial b} < 0$. We have:

$$(1+g)^t R_0 \left[c + \frac{d\tau + w(r-d+g) - r - g - \chi}{1+g} + \frac{(d-r)\chi}{g(1+g)} \right] - \chi R_0 \frac{d-r}{g} < 0$$

After rearranging:

$$c+\frac{d\tau+w(r-d+g)-r-g-\chi}{1+g}<\chi(d-r)\left[\frac{1}{g(1+g)^t}-\frac{1}{g(1+g)}\right]\Leftrightarrow$$

$$c(1+g)+d\tau+w(r-d+g)-r-g-\chi<\chi(d-r)(1+g)\frac{1-(1+g)^{t-1}}{g(1+g)^t}\Leftrightarrow$$

$$c(1+g)+d\tau+w(r-d+g)-r-g-\chi<\frac{\chi(d-r)}{g}\left[\frac{1}{(1+g)^{t-1}}-1\right]\Leftrightarrow$$

$$d\tau+wg+c(1+g)+\frac{\chi(d-r)}{g}\left[1-\frac{1}{(1+g)^{t-1}}\right]-w(d-r)-r-\chi<g$$

The most critical case will be for large t:s. Letting $t\to\infty$ yields the final condition:

$$d\tau+wg+c(1+g)+\frac{\chi(d-r)}{g}-w(d-r)-r-\chi<g$$

- *Change in deferred taxes / gross PPE (c)*

$$\frac{\partial DIV_t}{\partial c}=b\;R_0(1+g)^t>0$$

The book equity should always be non-negative:

$$\left[(1-w)(aR_t+bR_t-A_t)-T_t\right]>0\quad\forall t$$

Substituting in the solution to the system of difference equations:

$$(1-w)(a+b)(1+g)^tR_0-(1-w)\left[\frac{(1+g)^t-1}{g}bR_0(d-r)+A_0\right]-\left[\frac{(1+g)^t-1}{g}bR_0c(1+g)+T_0\right]>0$$

Rearranging gives:

$$(1-w)(a+b)>\left(\frac{b}{g}-\frac{b}{g(1+g)^t}\right)\left[(d-r)(1-w)+c(1+g)\right]+\frac{(1-w)A_0+T_0}{R_0(1+g)^t}$$

On the right hand side there are two time-dependent effects with opposite directions. Taking time into consideration, the following two boundary conditions are obtained, for $t=1$ and $t\to\infty$ respectively:

1. $(1-w)(a+b)>\left(\frac{b}{g}-\frac{b}{g(1+g)}\right)\left[(d-r)(1-w)+c(1+g)\right]+\frac{(1-w)A_0+T_0}{R_0(1+g)}$

2. $(1-w)(a+b)>\frac{b}{g}\left[(d-r)(1-w)+c(1+g)\right]$

- *The interest rate on debt (i)*

$$\frac{\partial DIV_t}{\partial i}=R_0(1+g)^t\left[\frac{-(a+b)(1-\tau)w+\frac{d-r}{g}b(1-\tau)w}{1+g}\right]-R_0\frac{d-r}{g}b(1-\tau)w+(1-\tau)wA_0\left(=\frac{\partial NP_t}{\partial i}\right)$$

Rearranging:

$$R_0(1+g)^{t-1}(1-\tau)w\left[-(a+b)+\frac{d-r}{g}b\right]-R_0\frac{d-r}{g}b(1-\tau)w+(1-\tau)wA_0$$

Looking at the two boundary cases in order to find the expression for determining the sign:

$(t=1)$:

$$(1-\tau)wR_0\left[-(a+b)+\frac{d-r}{g}b\right]-R_0\frac{d-r}{g}b(1-\tau)w+(1-\tau)wA_0$$

which, in order to determine the sign, can be simplified to

$$-(a+b)R_0+A_0=-[\text{balance sheet total}]<0$$

$(t\to\infty)$:

Searching for the condition for a negative derivative:

$$(a+b)>\frac{(d-r)b}{g}\Leftrightarrow$$
$$(a+b)R_t>\frac{(d-r)b}{g}R_t\Leftrightarrow$$
$$\text{net working capital}_t+G_t>\frac{d-r}{g}G_t$$

Applying the condition for *net PPE* to be non - decreasing over time, $(d-r)\le g$:

$$\text{net working capital}_t+\{\text{something non - negative}\}>0$$

Thus a sufficient condition for dividends to be decreasing in i is that $a>0$, i.e. that the net working capital in parametric steady state is positive. In the PSS period the balance sheet total must be positive. Hence, in addition to the non-decreasing *net PPE* condition, one must assume that a, the *net working capital* to *revenues* ratio is larger than zero, and as a consequence DIV_t will always be decreasing in i.

- *The tax rate (τ)*

$$\frac{\partial DIV_t}{\partial \tau} = \left(-1 + p + \frac{db + aiw + biw}{1+g} - \frac{(d-r)b}{g(1+g)} iw \right) (1+g)^t R_0 + iw \left(\frac{(d-r)b}{g} R_0 - A_0 \right)$$

This is exactly the same expression as for net profits with respect to τ. See that section above for the further proof.

A2.1.4 Proof of proposition 2.2

Expression (1) for the free cash flow in year 0 can be rewritten as:

$$\begin{aligned} FCF_0 &= (1-\tau)(R_0 - pR_0 - db'R_{(-1)}) + db'R_{(-1)} + T_0 - T_{(-1)} - (aR_0 - a'R_{(-1)}) - (bR_0 - b'R_{(-1)} + rb'R_{(-1)}) = \\ &= (1-\tau)(R_0 - pR_0 - db'R_{(-1)}) + db'R_{(-1)} + cbR_0 - (aR_0 - a'R_{(-1)}) - (bR_0 - b'R_{(-1)} + rb'R_{(-1)}) \end{aligned}$$

where a and b are constants for $t \geq 0$ and a' and b' are the corresponding items for $t = (-1)$.

Note that $R_0 = (1+g)R_{(-1)}$

It is easy to see that FCF_0 will generally not be the same as $FCF_1/(1+g)$, if $a \neq a'$ and / or $b \neq b'$.[46] Thus FCF_0 is not the free cash flow that will grow with growth rate g for the following years and year 0 therefore is not the first FCF steady state year, but as equation (7) above shows, year 1 will be the first FCF steady state year. Q.E.D.

[46] Unless for very specific parameter combinations

A2.2 Specification B

A.2.2.0 Solution to system of difference equations

Revenues, gross PPE and taxes:

The definitions are given in the text and can be formalised as follows:

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ G_t &= G_{t-1} + eR_t - rG_{t-1} \\ T_t &= cG_t + T_{t-1} \end{cases}$$

or equivalently:

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ G_t &= e(1+g)R_{t-1} + (1-r)G_{t-1} \\ T_t &= ce(1+g)R_{t-1} + c(1-r)G_{t-1} + T_{t-1} \end{cases}$$

In matrix notation:

$$\mathbf{x_t} = \mathbf{A}\mathbf{x_{t-1}}$$

where: $\mathbf{x_t} = \begin{pmatrix} R_t \\ G_t \\ T_t \end{pmatrix}$ and $\mathbf{A} = \begin{pmatrix} 1+g & 0 & 0 \\ e(1+g) & 1-r & 0 \\ ce(1+g) & c(1-r) & 1 \end{pmatrix}$

A is diagonalised by **P**:

$$\mathbf{P} = \begin{pmatrix} \dfrac{g+r}{e(1+g)} & 0 & 0 \\ 1 & 1 & 0 \\ \dfrac{c(1+g)}{g} & -c\dfrac{1-r}{r} & 1 \end{pmatrix}$$

and thus:

$$\mathbf{P^{-1}AP} = \begin{pmatrix} 1+g & 0 & 0 \\ 0 & 1-r & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Substituting $\mathbf{x_t} = \mathbf{P}\,\mathbf{y_t}$ and $\mathbf{x_{t+1}} = \mathbf{P}\,\mathbf{y_{t+1}}$ yields the system $\mathbf{y_{t+1}} = \mathbf{P^{-1}AP}\,\mathbf{y_t}$, the solution to which is:

$$\mathbf{y}_t = \begin{pmatrix} k_1(1+g)^t \\ k_2(1-r)^t \\ k_3 \end{pmatrix}$$

Substituting back yields:

$$\mathbf{x}_t = \mathbf{P}\,\mathbf{y}_t = \begin{pmatrix} \frac{g+r}{e(1+g)} & 0 & 0 \\ 1 & 1 & 0 \\ \frac{c(1+g)}{g} & -c\frac{1-r}{r} & 1 \end{pmatrix} \begin{pmatrix} k_1(1+g)^t \\ k_2(1-r)^t \\ k_3 \end{pmatrix} = \begin{pmatrix} k_1(1+g)^t \frac{g+r}{e(1+g)} \\ k_1(1+g)^t + k_2(1-r)^t \\ k_1(1+g)^t \frac{c(1+g)}{g} - k_2(1-r)^t c\frac{1-r}{r} + k_3 \end{pmatrix}$$

and since the initial values are R_0, G_0 and T_0, the constants k_1, k_2 and k_3 are found to be:

$$\begin{cases} k_1 &= \frac{e(1+g)}{g+r} R_0 \\ k_2 &= G_0 - \frac{e(1+g)}{g+r} R_0 \\ k_3 &= T_0 - \frac{c(1+g)}{g}\frac{e(1+g)}{g+r} R_0 + \frac{c(1-r)}{r}\left(G_0 - \frac{e(1+g)}{g+r} R_0\right) \end{cases}$$

The complete solution is:

$$\begin{cases} R_t &= R_0(1+g)^t \\ G_t &= \frac{e(1+g)}{g+r} R_0(1+g)^t + \left(G_0 - \frac{e(1+g)}{g+r} R_0\right)(1-r)^t \\ T_t &= T_0 + \frac{(1+g)^t - 1}{g} c(1+g) e \frac{(1+g)}{g+r} R_0 + \frac{c(1-r)}{r}\left(G_0 - \frac{e(1+g)}{g+r} R_0\right)\left(1-(1-r)^t\right) \end{cases}$$

The expression for the accumulated depreciation can be derived in a similar way.

A2.2.1 Derivation of FCF expression - equation (48)

The definition of free cash flow in any year t is:

$$\begin{aligned} FCF_t &= (1-\tau)(R_t - pR_t - dG_{t-1}) + dG_{t-1} + T_t - T_{t-1} - (aR_t - aR_{t-1}) - (G_t - G_{t-1} + rG_{t-1}) = \\ &= aR_{t-1} + ((1-\tau)(1-p) - a)R_t + (1 - r + \tau d)G_{t-1} - G_t + T_t - T_{t-1} \end{aligned}$$

Substituting the solutions for the state-variables yields:

$$
\begin{aligned}
FCF_t \quad = \quad & a(1+g)^{t-1}R_0 + ((1-\tau)(1-p)-a)(1+g)^t R_0 + \\
& +(1-r+\tau d)\left[e\frac{1+g}{g+r}(1+g)^{t-1}R_0 + \left(G_0 - e\frac{1+g}{g+r}R_0\right)(1-r)^{t-1}\right] - \\
& -\left[\frac{e(1+g)}{g+r}R_0(1+g)^t + \left(G_0 - \frac{e(1+g)}{g+r}R_0\right)(1-r)^t\right] + \\
& +\frac{(1+g)^t-1}{g}c(1+g)e\frac{(1+g)}{g+r}R_0 + \frac{c(1-r)}{r}\left(G_0 - \frac{e(1+g)}{g+r}R_0\right)\left(1-(1-r)^t\right) - \\
& -\left[\frac{(1+g)^{t-1}-1}{g}c(1+g)e\frac{(1+g)}{g+r}R_0 + \frac{c(1-r)}{r}\left(G_0 - \frac{e(1+g)}{g+r}R_0\right)\left(1-(1-r)^{t-1}\right)\right]
\end{aligned}
$$

Substituting $\beta = e\dfrac{1+g}{g+r}$ and $m = (1-\tau)(1-p)$, and rearranging gives:

$$
\begin{aligned}
FCF_t \quad = \quad & (1+g)^t R_0\left(m - \frac{ag}{1+g}\right) + \beta(1+g)^t R_0 \cdot \left(\frac{1-r+\tau d}{1+g} - 1\right) + \\
& +(1-r)^{t-1}(G_0 - \beta R_0)\cdot\left((1-r+\tau d)-(1-r)\right) + (1+g)^t R_0 \cdot c\beta + \\
& +\frac{c(1-r)}{r}(G_0 - \beta R_0)\cdot\left[(1-r)^{t-1} - (1-r)^t\right]
\end{aligned}
$$

Simplifying further:

$$
FCF_t = (1+g)^t R_0\left(m - \frac{ag - \beta(1-r+\tau d)}{1+g} - \beta + c\beta\right) + \tau d(G_0 - \beta R_0)(1-r)^{t-1} + (G_0 - \beta R_0)\cdot c\cdot(1-r)^t
$$

and finally:

$$
FCF_t = (1+g)^t R_0\left(m + \frac{\beta(c+\tau d - r) - g(\beta(1-c)+a)}{1+g}\right) + \left(c + \tau\frac{d}{1-r}\right)(G_0 - \beta R_0)(1-r)^t
$$

This expression is equation (47) in section 2.2.1. The final expression (48) is obtained by introducing a couple of new constants.

A2.2.2 Derivation of NP expression - equation (51)

The net profit in year t is given by:

$$NP_t = (1-\tau)\left(R_t - pR_t - dG_{t-1} - iw\left(aR_{t-1} + G_{t-1} - A_{t-1}\right)\right)$$

Rearranging and introducing the constants $m = (1-\tau)(1-p)$ and $\chi = (1-\tau)iw$:

$$NP_t = R_{t-1} \cdot \chi \cdot (-a) + R_t \cdot m + G_{t-1}(1-\tau) \cdot (-d - iw) + A_{t-1} \cdot \chi$$

Substituting the expressions for *revenues, gross PPE* and *accumulated depreciation*, and letting $\beta = e\dfrac{1+g}{g+r}$ yields:

$$\begin{aligned}
NP_t \;&=\; (1+g)^{t-1} R_0 \cdot \chi \cdot (-a) + (1+g)^t R_0 \cdot m + \\
&+\left[\beta R_0 (1+g)^{t-1} + (G_0 - \beta R_0)(1-r)^{t-1}\right](1-\tau) \cdot (-d - iw) + \\
&+\left[A_0 + \frac{(1+g)^{t-1} - 1}{g}(d-r)\beta R_0 + \frac{d-r}{r}(G_0 - \beta R_0)\left(1 - (1-r)^{t-1}\right)\right] \cdot \chi
\end{aligned}$$

Factoring out:

$$\begin{aligned}
NP_t \;&=\; (1+g)^t R_0 \left(m + \frac{\chi \cdot (-a) + (1-\tau)(-d-iw)\beta + \chi \dfrac{(d-r)}{g}\beta}{1+g} \right) + \\
&+(1-\tau)(-d-iw) \cdot (G_0 - \beta R_0)\ (1-r)^{t-1} - \chi \cdot \frac{(d-r)}{g}\beta R_0 + \chi\ A_0 + \\
&+\chi\ \left[\frac{d-r}{r} \cdot (G_0 - \beta R_0) \cdot \left(1 - (1-r)^{t-1}\right)\right]
\end{aligned}$$

Rearranging finally gives:

$$\begin{aligned}
NP_t \;&=\; (1+g)^t R_0 \left(m + \frac{\chi \cdot (-a) + (1-\tau)(-d-iw)\beta + \chi \dfrac{(d-r)}{g}\beta}{1+g} \right) - \\
&-\chi\ \left[\frac{(d-r)}{g}\beta R_0 - A_0\right] - \chi \cdot \frac{(d-r)}{r}(\beta R_0 - G_0) + \\
&+\left[\chi \frac{(d-r)}{r(1-r)} + (1-\tau)\frac{d+iw}{1-r}\right] \cdot (\beta R_0 - G_0) \cdot (1-r)^t
\end{aligned}$$

Introducing three more constants yields equation (51).

A2.2.3 Derivation of DIV expression - equation (55)

The expression for dividends in year *t* is:

$$
\begin{aligned}
DIV_t = \; & (1-w)\left(aR_{t-1} + G_{t-1} - A_{t-1}\right) - T_{t-1} + \\
& +(1-\tau)\left((1-p)R_t - dG_{t-1} - iw\left(aR_{t-1} + G_{t-1} - A_{t-1}\right)\right) - \\
& -\left[(1-w)\left(aR_t + G_t - A_t\right) - T_t\right]
\end{aligned}
$$

Rearranging and recognising that $T_t - T_{t-1} = cG_t$:

$$
\begin{aligned}
DIV_t = \; & R_t\left((1-\tau)(1-p) - a(1-w)\right) + R_{t-1}\left(a\left((1-w) - (1-\tau)iw\right)\right) + \\
& +G_t\left(c-1+w\right) + G_{t-1}\left(1-w-d(1-\tau)-(1-\tau)iw\right) + A_t\left(1-w\right) + A_{t-1}\left(w-1+(1-\tau)iw\right)
\end{aligned}
$$

Substituting in the expressions for the state variables and simplifying:

$$
\begin{aligned}
DIV_t = \; & R_0(1+g)^t \cdot \left((1-\tau)(1-p) - a(1-w)\right) + R_0(1+g)^{t-1} \cdot \left(a\left((1-w) - (1-\tau)iw\right)\right) - \\
& -\left(\beta R_0(1+g)^t + \left(G_0 - \beta R_0\right)(1-r)^t\right) \; (1-c-w) + \\
& +\left(\beta R_0(1+g)^{t-1} + \left(G_0 - \beta R_0\right)(1-r)^{t-1}\right) \cdot \left(1 + w\left(r-d\right) - \left(1-\tau\right)iw + \tau d - w - r\right) + \\
& +\left(A_0 + \frac{(1+g)^{t-1}-1}{g}\left(d-r\right)\beta R_0 + \frac{d-r}{r}\left(G_0 - \beta R_0\right)\left(1-(1-r)^{t-1}\right)\right) \cdot \left(1-\tau\right)iw
\end{aligned}
$$

Collecting terms and using the constants $m = (1-\tau)(1-p)$ and $\chi = \left(1-\tau\right)iw$:

$$
\begin{aligned}
DIV_t = \; & R_0(1+g)^t \left(m + c\beta + \frac{\beta\left(d\tau + w\left(r-d+g\right) - r - g - \chi\right) - a\left(g(1-w) + \chi\right) + \chi \frac{(d-r)}{g}\beta}{1+g} \right) - \\
& -\chi\left(\frac{d-r}{g}\beta R_0 - A_0\right) - \left(\chi \frac{d-r}{r}\right)\left(\beta R_0 - G_0\right) - \\
& -\left(c + \frac{d}{1-r}\left(\tau - w - \frac{\chi}{r}\right)\right)\left(\beta R_0 - G_0\right)\left(1-r\right)^t
\end{aligned}
$$

Introducing two more constants finally gives equation (55).

A2.3 Specification A with capital-based reserve

A2.3.1 A more complex tax-system

In order to show the general usefulness of the modelling approach considered in this report, we will in this appendix apply the methods to a very complicated and specific case.[47] Thereby, we wish to visualise the fact that this modelling approach can be extended and specialised to capture virtually any specific legal system and/or relations between parameters.

In the report, the specification of taxes has been rather rudimentary. The possibility of tax deferrals has been modelled as the difference equation:

$$T_t = cbR_t + T_{t-1}$$

with:	T_t	deferred taxes, year t
	c	increase in deferred taxes in % of gross PPE
	bR_t	gross PPE, year t (using Spec. A)

In a specific case, it is possible to explicitly model the company's tax operations in accordance with the tax system under consideration. A modified system of difference equations is then obtained. An interesting example from Sweden is the *capital-based reserve* (*K-SURV*), which was proposed in a government committee report in 1989 and a variation of it has also been in operation for a few years but is now being gradually abolished. The technical details are taken from the government committee report SOU 1989:34.

The *capital-based reserve* is an untaxed reserve, which can maximally be 30% of the capital base, defined as the firm's ending equity plus the capital-based reserve itself. Hence, the expression for the untaxed capital-based reserve, U_t, becomes:[48]

$$\begin{aligned} U_t \;\; = \;\; & s\big[aR_{t-1} + bR_{t-1} - A_{t-1} + \\ & + R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - \\ & - w\left(aR_t + bR_t - A_t\right) - \\ & - \tau\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - \left(U_t - U_{t-1}\right)\right)\big] \end{aligned}$$

where s denotes the (maximal) percentage of the capital base that may be considered an untaxed reserve (in Sweden 30%) and U_t is the untaxed reserve in year t.

---

[47] All derivations are presented in section A2.3.6.

[48] In words, the expression states the following:

$$\begin{aligned} U_t \;\; = \;\; & s\cdot\big[\text{entering assets} \\ & + \text{this yr's profits before taxes} \\ & - \text{debts} \\ & - \text{this yr's taxes}\big] \end{aligned}$$

Solving the system of difference equations now yields the following result:

$$\begin{aligned}
R_t &= R_0(1+g)^t \\
A_t &= A_0 + \frac{(1+g)^t - 1}{g}(d-r)bR_0 \\
U_t &= \theta R_0(1+g)^t + (U_0 - \theta R_0 - \xi)(-\psi)^t + \xi
\end{aligned}$$

where θ, ξ and ψ are constants:

$$\theta = \frac{s\left((a+b)\left(1-\left(1+(1-\tau)i\right)w - gw\right) - (1-\tau)db + (d-r)bw + (1+g)(1-p)(1-\tau) - \left(1-\left(1+(1-\tau)i\right)w\right)\frac{d-r}{g}b\right)}{1+g(1-s\tau)}$$

$$\xi = s\left(1-\left(1+(1-\tau)i\right)w\right)\left(\frac{d-r}{g}b\,R_0 - A_0\right)$$

$$\psi = \frac{s\tau}{1-s\tau}$$

The equation for the untaxed reserve may look quite complex, and there is indeed not much insight to be gained from trying to interpret the new parameters θ, ξ and ψ Instead, attention will be focused on the development over time of the untaxed-reserves-variable. The constant ψ is different from zero[49] so the untaxed reserves will actually exhibit an oscillating behaviour over time. Since (the absolute value of) ψ is also strictly smaller than one in all reasonable cases, the untaxed reserves will eventually approach $\theta R_0(1+g)^t + \xi$. This will have implications for the free cash flow, the net profit and the dividends.

A2.3.2 Free cash flow

The expression for the free cash flow in year t is:

$$\begin{aligned}
FCF_t &= R_t - pR_t - \tau\left(R_t - pR_t - dbR_{t-1} - (U_t - U_{t-1})\right) - (aR_t - aR_{t-1}) - (bR_t - bR_{t-1} + rbR_{t-1}) = \\
&= R_{t-1}\left(a + (1-r)b + \tau db\right) + R_t\left((1-\tau)(1-p) - a - b\right) + \tau(U_t - U_{t-1})
\end{aligned}$$

Substituting the expressions for R_t and U_t into the free cash flow equation yields the following equation:

$$FCF_t = (1+g)^t R_0\left(m + z_{FCF(UR)}\right) + \tau\varsigma(-\psi)^t$$

where the constants are defined as before, but adding:

[49] In the system under consideration s=0.3 and τ=0 3, and thus ψ=0 0989

$$z_{FCF(UR)} = \frac{b(\tau d - r) - g(a + b - \tau\theta)}{1+g} \quad \text{and} \quad \varsigma = \frac{\psi+1}{\psi}(U_0 - \theta R_0 - \xi)$$

The original free cash flow expression was:

$$(7) \quad FCF_t \quad = (1+g)^t R_0 (m + z_{FCF})$$

where $z_{FCF} = \frac{b(c + \tau d - r) - g(b(1-c) + a)}{1+g}$

There is a slight difference in the two z-constants, but this is due to the different technical specifications of tax deferrals. The other - and more interesting - difference is of a qualitative nature: The free cash flow in the capital-based reserve case will have an oscillating trajectory *around* the straight line given by $(1+g)^t R_0 (m + z_{FCF(UR)})$, whereas in the original case the free cash flow grows at the constant rate g. The growth rate in the capital-based reserve case will, however, approach g as t becomes larger; with reasonable parameters after only a few years. In the following table and graph the development of free cash flow in the example company is shown (g is 5%):[50]

Year	FCF	Growth rate
1	23,34011	
2	22,46945	-3,7303%
3	23,79445	5,8969%
4	24,96424	4,9162%
5	26,21442	5,0079%
6	27,52495	4,9993%
7	28,90121	5,0001%
8	30,34627	5,0000%
9	31,86358	5,0000%
10	33,45676	5,0000%

Table A2 3:1 - Free cash flows and growth rates with example company characteristics using untaxed capital-based reserve

[50] a=5%, b=40%, d=6%, r=4%, w=40%, g=5%, i=10%, τ=30%, p=90%, R_0=500, A_0=125 as before and adding: s=30%, U_0=18

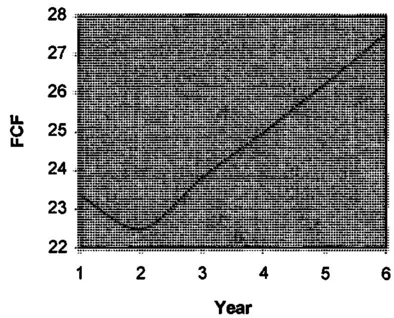

Figure A2 3:1 - Free cash flow with untaxed capital-based reserve

With a tax system of this kind, there is thus a time lag between the *parametric steady state* and the *free cash flow steady state*, i.e. it takes a few years with constant parameters for the firm's cash flow development to settle down to a constant growth (note that mathematically this is strictly true only when $t \to \infty$). In order to calculate the horizon value one must take the discount rate into consideration. In this case the initial condition of Proposition 2.3 and Corollary 2.1 will not be sufficient to ensure a constant capital structure over the PSS period, due to the oscillating behaviour.

In the final value calculation, which in this case only will be an approximation (since the WACC will not be constant), it is necessary to calculate a correction term in addition to the Gordon formula:

$$Equityvalue(FCF) \approx \frac{(1+g)R_0(1+z_{UR})}{k_{WACC}-g} - \tau(U_0 - \theta R_0 - \xi)\frac{1+\psi}{1+\psi+k_{WACC}} - D_0$$

The correction term is quite small for normal parameter values. This is due to the fact that the oscillating behaviour of the free cash flow will be dampened and almost fade after only a few years.

A2.3.3 Net profit

The net profit expression is:

$$NP_t = (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}) - (U_t - U_{t-1})\right)$$

Substituting the expressions for revenues, accumulated depreciation and untaxed reserves, equations and rearranging yields:

$$NP_t = (1+g)^t R_0 (m + z_{NP(UR)}) - \chi\left(R_0 \frac{(d-r)b}{g} - A_0\right) - (1-\tau)\varsigma(-\psi)^t$$

with: $\quad m = (1-\tau)(1-p)\,,\quad \chi = (1-\tau)iw\,,\quad \varsigma = \frac{\psi+1}{\psi}(U_0 - \theta R_0 - \xi)$

$$z_{NP(UR)} = -\frac{(1-\tau)\left(db + g\theta + iw\left(a + b - \frac{(d-r)b}{g}\right)\right)}{1+g}$$

The net profit will thus also exhibit an oscillating behaviour, albeit the oscillation quickly becomes negligible using reasonable parameter values. Figure A2.3:2 below is obtained using the example company data:

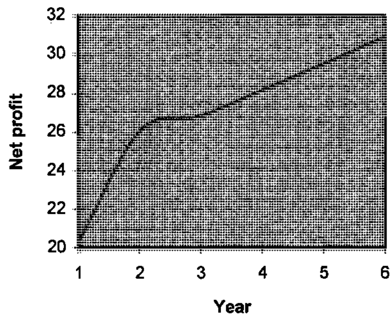

Figure A2 3:2 - Net profit with untaxed capital-based reserve

The oscillation in the net profit is somewhat more pronounced than in the free cash flow, since the tax rate is set at 30% and the net profit oscillation therefore is multiplied by 0.7 whereas the free cash flow oscillation is multiplied by 0.3. The presence of an oscillating term is, however, further obscured in the net profit case by the constant $\chi\left(R_0(d-r)b/g - A_0\right)$, which has to be deducted each year, thus causing the net profit not to grow constantly even if there were no untaxed reserves.

A2.3.4 Dividends

The dividend expression is similar to the original case (equation 12), but also here with an additional term causing an oscillating behaviour:

$$DIV_t = (1+g)^t R_0\left(m + z_{DIV(UR)}\right) - \chi\left(R_0\gamma - A_0\right) + \tau\varsigma(-\psi)^t$$

with the following constants:

$$m = (1-\tau)(1-p),\quad \chi = (1-\tau)iw,\quad \gamma = \frac{(d-r)b}{g},\quad \varsigma = \frac{\psi+1}{\psi}\left(U_0 - \theta R_0 - \xi\right)$$

$$z_{DIV(UR)} = \frac{b\left[d\tau + w(r-d+g) - r - g - (1-\tau)iw\right] - a\left[g(1-w) + (1-\tau)iw\right] + \frac{(d-r)b}{g}(1-\tau)iw + \tau\theta g}{1+g},$$

Qualitatively the dividends exhibit the same behaviour as the free cash flow: an oscillating term, $(-\psi)^t$, multiplied by the constant $\tau\cdot\varsigma$. Using once again the example company to visualise:

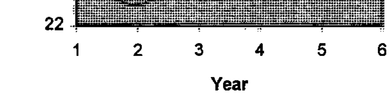

Figure A2 3:3 - Dividends with untaxed capital-based reserve

A2.3.5 Concluding remarks

When introducing capital-based reserves, there will not exist any FCF, DIV or NP steady state, at least not in the strict mathematical sense. The reason for this is that the *FCF, NP,* and *DIV* sequences have an oscillating behaviour. Only as $t\rightarrow\infty$ will the functions grow exactly by a constant growth rate. Numerically, however, the different functions will tend towards a steady state in a few years.

A2.3.6 Derivations for A2.3.5

Solution to system of difference equations

The expression for the untaxed reserves is:

$$
\begin{aligned}
U_t \;=\;& s\big[aR_{t-1} + bR_{t-1} - A_{t-1} + \\
+\;& R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - \\
-\;& w\left(aR_t + bR_t - A_t\right) - \\
-\;& \tau\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - \left(U_t - U_{t-1}\right)\right)\big]
\end{aligned}
$$

After rearranging:

$$
\begin{aligned}
U_t \;=\;& \frac{s}{1-s\tau}\big[aR_{t-1} + bR_{t-1} - A_{t-1} + \\
+\;& R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - \\
-\;& w\left(aR_t + bR_t - A_t\right) - \\
-\;& \tau\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) + U_{t-1}\right)\big]
\end{aligned}
$$

Keeping in mind that $A_t = (d-r)bR_{t-1} + A_{t-1}$, U_t can equivalently be written:

$$
\begin{aligned}
U_t \;=\;& \frac{s}{1-s\tau}\big[aR_{t-1} + bR_{t-1} - A_{t-1} + \\
+\;& R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) - \\
-\;& w\left(aR_t + bR_t - (d-r)bR_{t-1} - A_{t-1}\right) - \\
-\;& \tau\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right) + U_{t-1}\right)\big]
\end{aligned}
$$

Now define the following constants:

$$q_1 = a + b - (1-\tau)db - (1-\tau)iw(a+b) + w(d-r)b + (1+g)\left((1-\tau)(1-p) - w(a+b)\right)$$

$$q_2 = 1 - \left(1 + (1-\tau)i\right)w$$

The untaxed reserves will be:

$$U_t = \frac{s}{1-s\tau}\left(q_1 R_{t-1} - q_2 A_{t-1} - \tau U_{t-1}\right)$$

The expression for A_{t-1} is known (from section A2.1):

$$A_{t-1} = \frac{(1+g)^{t-1} - 1}{g}(d-r)bR_0 + A_0$$

Define the following constants:

$$q_3 = \frac{s}{1-s\tau}\left(q_1 - q_2\frac{(d-r)b}{g}\right)$$

$$q_4 = \frac{s}{1-s\tau}q_2\left(\frac{(d-r)b}{g}R_0 - A_0\right)$$

$$q_5 = \frac{s\tau}{1-s\tau}$$

The untaxed reserves can now be expressed as:

$$U_t = q_3 R_{t-1} + q_4 - q_5 U_{t-1}$$

The following system of difference equations can now be solved:

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ U_t &= q_3 R_{t-1} - q_5 U_{t-1} + q_4 \end{cases}$$

In matrix notation:

$$\mathbf{x_t} = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{Q}$$

where: $\mathbf{x_t} = \begin{pmatrix} R_t \\ U_t \end{pmatrix}$; $\mathbf{A} = \begin{pmatrix} 1+g & 0 \\ q_3 & -q_5 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 0 \\ q_4 \end{pmatrix}$

The homogenous equation is:

$$\mathbf{x_t} = \mathbf{A}\mathbf{x}_{t-1}$$

The roots of the characteristic equation are $\lambda_1 = 1+g$ and $\lambda_2 = -q_5$.

A is diagonalised by **P**:

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ \dfrac{q_3}{q_5+1+g} & 1 \end{pmatrix}$$

And:

$$\mathbf{P}^{-1}\mathbf{AP} = \begin{pmatrix} 1+g & 0 \\ 0 & -q_5 \end{pmatrix}$$

Substituting $\mathbf{x}_t = \mathbf{P}\,\mathbf{y}_t$ and $\mathbf{x}_{t+1} = \mathbf{P}\,\mathbf{y}_{t+1}$ yields the system $\mathbf{y}_{t+1} = \mathbf{P}^{-1}\mathbf{AP}\,\mathbf{y}_t$, the solution to which is:

$$\mathbf{y}_t = \begin{pmatrix} c_1(1+g)^t \\ c_2(-q_5)^t \end{pmatrix}$$

Substituting back yields:

$$\mathbf{x}_t = \mathbf{P}\,\mathbf{y}_t = \begin{pmatrix} 1 & 0 \\ \dfrac{q_3}{q_5+1+g} & 1 \end{pmatrix} \begin{pmatrix} c_1(1+g)^t \\ c_2(-q_5)^t \end{pmatrix} = \begin{pmatrix} c_1(1+g)^t \\ c_1(1+g)^t \dfrac{q_3}{q_5+1+g} + c_2(-q_5)^t \end{pmatrix}$$

A particular solution to the system of equations has to be added. The steady state will serve as particular solution:

$$\mathbf{x}^{SS} = \mathbf{A}\mathbf{x}^{SS} + \mathbf{Q}$$

$$\mathbf{x}^{SS} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{Q} = \begin{pmatrix} -g & 0 \\ -q_3 & 1+q_5 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ q_4 \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{g} & 0 \\ -\dfrac{q_3}{g(q_5+1)} & \dfrac{1}{q_5+1} \end{pmatrix} \begin{pmatrix} 0 \\ q_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \dfrac{q_4}{q_5+1} \end{pmatrix}$$

and since the initial values are R_0 and U_0, the complete solution is:

$$\begin{cases} R_t &= (1+g)^t R_0 \\ U_t &= \dfrac{q_3}{q_5+1+g}(1+g)^t R_0 + \left(U_0 - \dfrac{q_3}{q_5+1+g} R_0 - \dfrac{q_4}{q_5+1} \right)(-q_5)^t + \dfrac{q_4}{q_5+1} \end{cases}$$

Define the following constants:

$$\theta = \frac{q_3}{q_5+1+g} = \frac{s\left(q_1 - q_2\frac{(d-r)b}{g}\right)}{s\tau+(1+g)(1-s\tau)} =$$

$$= \frac{s\left((a+b)\left(1-\left(1+(1-\tau)i\right)w-gw\right)-(1-\tau)db+(d-r)bw+(1+g)(1-p)(1-\tau)-\left(1-\left(1+(1-\tau)i\right)w\right)\frac{(d-r)b}{g}\right)}{1+g(1-s\tau)}$$

$$\xi = \frac{q_4}{q_5+1} = sq_2\left(\frac{(d-r)b}{g}R_0 - A_0\right) = s\left(1-\left(1+(1-\tau)i\right)w\right)\left(\frac{(d-r)b}{g}R_0 - A_0\right)$$

$$\psi = q_5 = \frac{s\tau}{1-s\tau}$$

and the expression for the untaxed reserves will be:

$$U_t = \theta R_0(1+g)^t + (U_0 - \theta R_0 - \xi)(-\psi)^t + \xi$$

Free cash flow

$$\begin{aligned}
FCF_t &= R_t - pR_t - \tau\left(R_t - pR_t - dbR_{t-1} - (U_t - U_{t-1})\right) - (aR_t - aR_{t-1}) - (bR_t - bR_{t-1} + rbR_{t-1}) = \\
&= R_{t-1}\left(a + (1-r)b + \tau db\right) + R_t\left((1-\tau)(1-p) - a - b\right) + \tau(U_t - U_{t-1}) = \\
&= R_t\left((1-\tau)(1-p) + \frac{b(\tau d - r) - g(a+b)}{1+g}\right) + \tau(U_t - U_{t-1}) = \\
&= R_t\left((1-\tau)(1-p) + \frac{b(\tau d - r) - g(a+b)}{1+g}\right) + \tau\left(g\theta R_{t-1} + \frac{\psi+1}{\psi}(U_0 - \theta R_0 - \xi)(-\psi)^t\right) = \\
&= R_t\left((1-\tau)(1-p) + \frac{b(\tau d - r) - g(a+b-\tau\theta)}{1+g}\right) + \tau\frac{\psi+1}{\psi}(U_0 - \theta R_0 - \xi)(-\psi)^t
\end{aligned}$$

Define the following constants:

$$m = (1-\tau)(1-p)$$

$$z_{FCF(UR)} = \frac{b(\tau d - r) - g(a + b - \tau\theta)}{1+g}$$

$$\varsigma = \frac{\psi + 1}{\psi}(U_0 - \theta R_0 - \xi)$$

and the free cash flow expression will be:

$$FCF_t = (1+g)^t R_0 \left(m + z_{FCF(UR)}\right) + \tau\varsigma(-\psi)^t$$

Net Profit

$$NP_t = (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}) - (U_t - U_{t-1})\right) =$$

$$= (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}) - \left(\frac{g\theta}{1+g}R_t + (U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t\right)\right) =$$

$$= (1-\tau)\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - \left(A_0 + \frac{(d-r)b}{g}(R_{t-1} - R_0)\right)\right) -\right.$$

$$\left. - \left(\frac{g\theta}{1+g}R_t + (U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t\right)\right) =$$

$$= R_t\left((1-\tau)(1-p) - \frac{(1-\tau)db + (1-\tau)iw(a+b) - (1-\tau)iw\dfrac{(d-r)b}{g}}{1+g}\right) -$$

$$-(1-\tau)iw\left(\frac{(d-r)b}{g}R_0 - A_0\right) - (1-\tau)\left(\frac{g\theta}{1+g}R_t + (U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t\right) =$$

$$= R_t\left((1-\tau)(1-p) - \frac{(1-\tau)db + (1-\tau)g\theta + (1-\tau)iw(a+b) - (1-\tau)iw\dfrac{(d-r)b}{g}}{1+g}\right) -$$

$$-(1-\tau)iw\left(\frac{(d-r)b}{g}R_0 - A_0\right) - (1-\tau)(U_0 - \theta R_0 - \xi)\frac{\psi+1}{\psi}(-\psi)^t$$

Define the following constants:

$$m = (1-\tau)(1-p)$$

$$z_{NP(UR)} = -\frac{(1-\tau)\left(db + g\theta + iw\left(a + b - \frac{(d-r)b}{g}\right)\right)}{1+g}$$

$$\chi = (1-\tau)iw$$

$$\varsigma = \frac{\psi+1}{\psi}(U_0 - \theta R_0 - \xi)$$

and the expression for the net profit will be:

$$NP_t = (1+g)^t R_0(m + z_{NP(UR)}) - \chi\left(R_0\frac{(d-r)b}{g} - A_0\right) - (1-\tau)\varsigma(-\psi)^t$$

Dividends

$$\begin{aligned} DIV_t = \ & (1-w)(aR_{t-1} + bR_{t-1} - A_{t-1}) - U_{t-1} + \\ & +(1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}) - (U_t - U_{t-1})) - \\ & -((1-w)(aR_t + bR_t - A_t) - U_t) = \end{aligned}$$

$$\begin{aligned} = \ & (1-w)(aR_{t-1} + bR_{t-1} - A_{t-1}) + \\ & +(1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1})) - \\ & -((1-w)(aR_t + bR_t - A_t)) + \\ & +\tau(U_t - U_{t-1}) = \end{aligned}$$

$$\begin{aligned} = \ & R_{t-1}((1-w)(a+b) - (1-\tau)(db + iw(a+b))) + \\ & +R_t((1-\tau)(1-p) - (1-w)(a+b)) + \\ & +A_{t-1}((1-\tau)iw - (1-w)) + \\ & +A_t(1-w) + \\ & +\tau(U_t - U_{t-1}) = \end{aligned}$$

$$
\begin{aligned}
= \ & (1+g)^{t-1} R_0\Big(\big(1-w\big)\big(a+b\big)-\big(1-\tau\big)\big(db+iw\big(a+b\big)\big)\Big)+ \\
& +(1+g)^{t} R_0\Big(\big(1-\tau\big)\big(1-p\big)-\big(1-w\big)\big(a+b\big)\Big)+ \\
& +\frac{(1+g)^{t-1}-1}{g}(d-r)bR_0\Big(\big(1-\tau\big)iw-\big(1-w\big)\Big)+A_0\Big(\big(1-\tau\big)iw-\big(1-w\big)\Big)+ \\
& +\frac{(1+g)^{t}-1}{g}(d-r)bR_0\big(1-w\big)+A_0\big(1-w\big)+ \\
& +\tau\big(U_t-U_{t-1}\big)=
\end{aligned}
$$

$$
\begin{aligned}
= \ & (1+g)^{t} R_0(1-\tau)(1-p)+ \\
& +(1+g)^{t-1} R_0\Big(\big(d-r\big)b\big(1-w\big)-\big(a+b\big)\big(1-w\big)g-\big(1-\tau\big)bd-\big(1-\tau\big)iw\big(a+b\big)\Big)+ \\
& +\frac{(1+g)^{t-1}-1}{g}R_0\big(1-\tau\big)iw\big(d-r\big)b+A_0\big(1-\tau\big)iw+ \\
& +\tau\left(\frac{g\theta}{1+g}(1+g)^{t}R_0+\big(U_0-\theta R_0-\xi\big)\frac{\psi+1}{\psi}\big(-\psi\big)^{t}\right)
\end{aligned}
$$

Define the following constants:

$$
z_{DIV(UR)}=\frac{b\big[d\tau+w\big(r-d+g\big)-r-g-\big(1-\tau\big)iw\big]-a\big[g\big(1-w\big)+\big(1-\tau\big)iw\big]+\frac{(d-r)b}{g}\big(1-\tau\big)iw+\tau\theta g}{1+g}
$$

$$
m=(1-\tau)(1-p), \qquad \chi=\big(1-\tau\big)iw, \qquad \gamma=\frac{(d-r)b}{g}, \qquad \varsigma=\frac{\psi+1}{\psi}\big(U_0-\theta R_0-\xi\big)
$$

and the dividend expression will be:

$$
DIV_t=\big(1+g\big)^{t}R_0\big(m+z_{DIV(UR)}\big)-\chi\big(R_0\gamma-A_0\big)+\tau\varsigma\big(-\psi\big)^{t}
$$

3. Value Calculation Methods

3.1 Discounting methods

What if the market value debt ratio is not constant according to the predictions and consequently there exists no textbook steady state? Surely, this will very often be the case in practice. A constant capital structure is even more unlikely in the explicit forecast period. As has already been indicated a non-constant market debt ratio has implications for the discounting procedure.

It was established in Proposition 2.5 that it is *only with a constant capital structure in market terms that one can use a continuing value formula to calculate the FCF horizon value without approximation errors.* Such approximation errors normally arise from two causes: 1) the weights in the WACC formula changes over time when the capital structure changes, and 2) the costs of different kinds of capital are likely to change when the capital structure changes, since capital structure influences the risk. This will be formalised in the following way:

Definitions:

Type 1 approximation error is the error introduced into the valuation by neglecting that the weights in the WACC formula change over time when the market debt ratio changes.

Type 2 approximation error is the error introduced into the valuation by neglecting that the risk of the company may very well change over time as the market debt ratio changes. Hence, it is often not correct to assume a constant cost of equity capital, k_e (or for that matter a constant cost of debt).

The task for the practically oriented analyst must then be to minimise such approximation errors. We will here show how one can avoid type 1 errors completely. (In Chapter 5 we show how one can get at least a rough estimate of the severity of type 2 approximation errors.)

One way of removing type 1 errors for the FCF valuation approach was suggested in section 2.1.5: to start from a year long into the future (when the market value debt ratio has converged towards its steady-state level) and work oneself backwards, continuously updating the WACC. This is a rather cumbersome procedure, however, and we have never seen it suggested in the literature.

One obvious way to avoid type 1 errors altogether is to use the dividend valuation approach, i.e. to discount the future dividends by the equity cost of capital, k_E. Many financial economists prefer the free cash flow approach, however, claiming that it is conceptually more valid since it explicitly values the asset side operations of the company, i.e. the actual value-creating process. This is also the view taken by Copeland et al. The discrepancy between practical usefulness and theoretical viability may seem discouraging, but one should then remember that *the two valuation approaches should yield exactly the same value*. This equality has been commented upon by several economists, most notably Miller and Modigliani,[51] and it is also mentioned in Copeland et al.[52]

Discussions regarding the equality of valuation approaches tend to be normative in nature and on a rather high level of abstraction. Copeland et al write that the valuation approaches are equal "as long as the discount rates are selected properly" [53] - but without stating what this "proper" method should look like. The discounting method they employ[54] certainly does not yield equivalence. Financial economists often assume that the capital structure remains constant. This is a rather restrictive assumption, as has been seen in Chapter 2, but if fulfilled the equality holds. The method we suggested in section 2.1.5, to continuously update the WACC, turns out to yield the equivalence without any restricting assumptions. This is formalised in Proposition 3.1, which can be viewed as a fairly substantial extension of Proposition 2.4:

Proposition 3.1

Valuation by discounting the free cash flows at a continuously updated weighted average cost of capital ("year-to-year-WACC") will yield the same value as valuation by discounting the future dividends at the cost of equity capital.

[51] Miller and Modigliani (1961), p. 416 and p. 419. The Miller-Modigliani model is more stylised than the one discussed in this chapter, but the general discussion concerning these issues is clearly of interest.
[52] Copeland et al, p. 132.
[53] Copeland et al, p. 132.
[54] The weighted average cost of capital at the moment of valuation ("year 0") used throughout.

Proof

The free cash flow is - in the general case - defined as follows:[55]

$$(56) \quad FCF_t = \textit{Net profit, } NP_t$$
$$+ \ \textit{Net interest payments after taxes, } (1-\tau)iD_{t-1}$$
$$+ \ \textit{Increase in deferred taxes, } (T_t - T_{t-1})$$
$$- \ \textit{Increase in asset side } (AS_t - AS_{t-1})$$

where: τ is the tax rate
i is the interest rate
D_{t-1} is the (net) debt at the end of year *t-1* on which interest is paid

The dividends are by the clean surplus relationship defined as:[56]

$$(57) \quad DIV_t = \textit{Net profit, } NP_t$$
$$+ \ \textit{Increase in debt, } (D_t - D_{t-1})$$
$$+ \ \textit{Increase in deferred taxes, } (T_t - T_{t-1})$$
$$- \ \textit{Increase in asset side, } (AS_t - AS_{t-1})$$

Assume there exists an equity value at time *T*, called EV_T. This value is the same for both the *FCF* and the *DIV* approach and it is calculated after a possible dividend at time *T*. This means that the total company value at time *T*, called TCV_T, will be $EV_T + D_T$ (i.e. equity value plus debt value). Valuation by the *FCF* approach will then yield the following total company value at the end of year *T-1*:[57]

[55] Using the McKinsey definition in full: *FCF = Net profit + Net interest payments after taxes + Depreciation expense + Increase in deferred taxes - Increase in working capital - Capital expenditures = Net profit + Net interest payments after taxes + Depreciation expense + Increase in deferred taxes - Increase in working capital - (Current year's net PPE - Preceding year's net PPE + Depreciation expense) = Net profit + Net interest payments after taxes + Increase in deferred taxes - Increase in working capital - Increase in net PPE = Net profit + Net interest payments after taxes + Increase in deferred taxes - Increase in asset side.* Note that excess marketable securities are not present in the forecast period.

[56] As in expression (3)

[57] Note again that the WACC formula, expression (16), implies that the WACC used for discounting during year *t* is based on the entering market values of debt and equity, i.e. D_{t-1} and EV_{t-1}, and hence also the total company value, TCV_{t-1}.

$$(58) \qquad TCV_{T-1} = \frac{FCF_T + EV_T + D_T}{1 + k_{WACC,\,T}}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T}{1 + k_{WACC,\,T}}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T}{1 + \frac{D_{T-1}}{TCV_{T-1}}(1-\tau)i + \left(1 - \frac{D_{T-1}}{TCV_{T-1}}\right)k_E}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T}{\frac{TCV_{T-1} + (1-\tau)iD_{T-1} + TCV_{T-1}k_E - D_{T-1}k_E}{TCV_{T-1}}}$$

Rearranging yields:

$$(59) \qquad TCV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T + D_{T-1}k_E}{1 + k_E}$$

The equity value is then obtained by deducting the debt, i.e. by deducting D_{T-1}:

$$(60) \qquad EV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + D_T + D_{T-1}k_E}{1 + k_E} - D_{T-1}$$

$$= \frac{NP_T + (T_T - T_{T-1}) - (AS_T - AS_{T-1}) + EV_T + (D_T - D_{T-1})}{1 + k_E}$$

By inserting (57) into (60) one obtains:

$$(61) \qquad EV_{T-1} = \frac{DIV_T + EV_T}{1 + k_E}$$

which is exactly the valuation formula used when discounting the dividends by the equity cost of capital. Having thus established that the value at *T-1* will be the same when employing the different methods, one can go on to time *T-2* and so on. Q.E.D.

The proof hinges on the assumption that there exists an equity value at some future date (the same for both the *FCF* and the *DIV* approach) from which the discounting process can start. In the example company of Chapter 2 there is no TSS, and hence such a value does not exist. However, the approximation error can be made arbitrarily small by choosing a starting point sufficiently long into the future. Since the market value debt ratio strives towards its theoretical steady state value as t increases (visualised in Figure 1 in section 2.1.5), the approximation will become better and better with time, and the approximation error becomes zero as $t \rightarrow \infty$. Furthermore, since any approximation error will occur at a date far into the future, the discounting process itself will also make the impact of an approximation error on the company value smaller and smaller as t increases.

Having thus established that the *FCF* approach, using the correct (updated) WACC discounting method, is equivalent to the *DIV* approach (using k_E), one can use the dividend valuation as the technical way of calculating the equity value, regardless of approach. If one for theoretical or other reasons prefers the *FCF* approach, then the argument is that one actually uses this approach when modelling (*FCF* statements are set up, etc.), but the actual computations are programmed in a way that makes them equal to the dividend valuation approach. This simplifies the computational work considerably: In the example company instead of performing 150 different iterative calculations one can simply use the dividend valuation formula:[58]

(62) $$EV_0 = \frac{(1+g)R_0(m + z_{DIV})}{k_E - g} - \frac{\chi(\gamma R_0 - A_0)}{k_E}$$

The second term in (62) is a correction term to account for the fact that the company never reaches a (textbook) steady state (see expression (12) for parameter explanations).

So far the discussion has been exemplified by a company in parametric steady state, i.e. the part of the valuation that concerns the horizon value. Proposition 3.1 is quite general, however, and is clearly valid also for the explicit forecast period. For the calculations in the explicit forecast period, one does not have to worry about any asymptotic proofs, since there clearly exists an equity value at time T - the horizon value - and the existence of such a value guarantees that Proposition 3.1 holds. Another suggestion often made in practically oriented textbooks is to limit the life of the company to, say, 100

[58] Expression (62) is the simply the sum of the discounted dividend expression (12) taken from now to eternity.

years and assume zero value thereafter. In this case Proposition 3.1 also holds, since there will exist an exact value at some future date, namely the value zero in year 100.

To conclude this discussion: There is never any reason to resort to using a *constant* weighted average cost of capital when the capital structure, and consequently the weights in the WACC formula, changes over time. It would inevitably introduce type 1 approximation errors into the valuation. The reason often given for using a constant WACC is that it simplifies the computational work considerably, but as we have shown above, any practical problems can be overcome by *calculating* the value by the dividend approach instead. Furthermore, it is often argued that the present value of future dividends is the correct benchmark for any calculation of the equity value.[59]

There remains, however, the other approximation problem: that the cost of equity capital could change when the capital structure (and hence the riskiness) changes. It should be noted that Proposition 3.1 holds even if the cost of equity capital (k_E) is allowed to vary over time. This is easily seen by just adding the subindex *t* to the cost of equity capital (i.e. $k_{E,t}$) in the proof. If k_E varies, no easy summation formula can be used, but it is still easier to perform dividend discounting than *FCF* discounting, since the latter involves tedious and time-consuming iterative calculations. How to address these issues in practice is shown in Chapter 5.

3.2 Earnings capitalisation

During the last few years, a number of papers and articles have begun advocating earnings as valuation measure. Such notions originate in the accounting field, and finance academics are on the whole less enthusiastic about earnings. We also lean more towards free cash flows and dividends as the relevant measures to be used in valuations, but the ardent support for earnings from some accounting researchers has made us include net profit calculations in earlier chapters. The reader will have noticed, however, that we have not used net profits for valuation purposes, and we will here give some indications as to why we are somewhat more sceptical.

[59] This is for example emphasized by Ohlson (1995), who starts out by the assumption that "value equals the present value of expected dividends" (p. 1).

In his article *Return to Fundamentals*[60] Stephen Penman discusses the development of the theory behind earnings as a valuation measure. He points out that the present value of expected dividends, P_t, is a "non-controversial description of price" (p. 23). Thus having argued that P_t is a relevant bench-mark, he continues: "Ohlson has shown that

$$[(63)] \quad V_t^T = \left(\rho^T - 1\right)^{-1} E\left[\left(\sum_{\tau=1}^{T} \tilde{X}_{t+\tau} + \sum_{\tau=1}^{T} \left(\rho^{T-\tau} - 1\right)\tilde{d}_{t+\tau}\right) \middle| Z_t\right]$$

(where $\tilde{X}_{t+\tau}$ is earnings in period $t+\tau$) approaches P_t as T, the number of periods ahead, gets large."

[V_t^T is the equity value using formula (63). P_t is the present value of expected dividends; ρ is $1+k$, i.e. 1+ the discount rate; $\tilde{d}_{t+\tau}$ stands for dividends in period $t+\tau$; Z_t is the information set at time t.]

Penman continues: "One might be thrown off by the inclusion of future dividends in [(63)]. Does this mean one has to predict future dividends also? No. These appear because, if dividends are paid out in the future, expected subsequent earnings are also reduced. The formula simply puts the dividends into an earning escrow account. Indeed, their presence in the formula is because they are irrelevant for value (but affect future earnings). Again, earnings-based valuation gets rid of the discretionary dividends. One can legitimately forget them and consider expected future earnings as if all dividends are reinvested in the firm."

We find such arguments somewhat hard to swallow. On intuitive grounds, one would suspect that dividends *do* matter (in the sense that they cannot always be zero), and that the reason that V_t^T approaches P_t is that the capitalised earnings sequence actually approaches zero as T gets large.

This supposition can be checked by separately assessing the earnings contribution to value, VX_t^T:

$$(64) \quad VX_t^T = \left(\rho^T - 1\right)^{-1} E\left[\sum_{\tau=1}^{T} \tilde{X}_{t+\tau} \middle| Z_t\right]$$

[60] Penman (1991), pp. 23-25

The conditional expected earnings sequence has to be specified. In Corollary 2.1, it was shown that with a constant capital structure, the earnings (i.e. net profit) would grow by a constant each year. It turned out that this constant was equal to the revenue growth rate, *g*. This is a very commonly used assumption, and any valuation formula would have to be able to handle this case.

Hence, given the information set at time *t* (i.e. the conditioning Z_t), where time *t* corresponds to year zero in our Chapter 2 model, we would expect:

$$(65)\qquad E\left[\sum_{\tau=1}^{T}\widetilde{X}_{t+\tau}|Z_t\right]=E\left[\widetilde{X}_{t+1}|Z_t\right]+E\left[\widetilde{X}_{t+2}|Z_t\right]+\ldots+E\left[\widetilde{X}_{t+T}|Z_t\right]=$$

$$=(1+g)x_0+(1+g)^2x_0+\ldots+(1+g)^Tx_0=\sum_{\tau=1}^{T}x_0(1+g)^\tau=x_0\sum_{\tau=1}^{T}(1+g)^\tau$$

where x_0 stands for the earnings (net profit) in year zero.

The test will now be to evaluate expression (64) as $T\to\infty$, using expression (65) as the expected earnings sequence.

$$(66)\qquad \lim_{T\to\infty}VX_t^T=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}E\left[\sum_{\tau=1}^{T}\widetilde{X}_{t+\tau}|Z_t\right]=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}x_0\sum_{\tau=1}^{T}(1+g)^\tau=$$

$$=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}x_0\left(\frac{(1+g)^{T+1}}{g}-\frac{(1+g)}{g}\right)=\lim_{T\to\infty}\left(\rho^T-1\right)^{-1}x_0\frac{(1+g)}{g}\left((1+g)^T-1\right)=$$

$$=x_0\frac{(1+g)}{g}\lim_{T\to\infty}\frac{\left((1+g)^T-1\right)}{\left(\rho^T-1\right)}=x_0\frac{(1+g)}{g}\lim_{T\to\infty}\frac{\left((1+g)^T-1\right)}{\left((1+k)^T-1\right)}=$$

$$=\begin{cases}0 & if\quad g<k\\ x_0\dfrac{(1+g)}{g} & if\quad g=k\\ \to\infty & if\quad g>k\end{cases}$$

(*k* stands for the discount rate; in our earlier models this would be the cost of equity capital, k_E.)

The magnitude of the revenue growth rate, *g*, is supposed to be less than the discount rate (see Chapter 2). This is surely not a very restricting assumption: The discount rate (in Sweden) is typically somewhere around 13%, and a revenue growth rate as high as that in every year from now to eternity is not realistic when inflation is assumed to be around 3%. This means that we can discard the possibility that $g \geq k$.[61] Miller & Modigliani (1961) also argue that the growth rate in a constant growth scenario must be strictly smaller than the discount rate. [62]

Thus the first possibility in the solution to expression (66) remains: that the growth rate is smaller than the discount rate. This means, however, that the value of the earnings are zero, and hence the entire value stems from the dividend stream. (This can be verified the other way around as well. Specify, e.g., a continuous dividend growth, i.e. $DIV_{t+1} = (1+g)DIV_t$. and the limit of the "dividends part" of formula (63) will be exactly equal to what we get from the Gordon formula, which is the standard "finance" way of calculating the sum of discounted dividends. See Appendix 3.)

If the earnings sequence has no value, then the informational content of the aggregate earnings figure *in itself* should also be zero. This does not mean that earnings are somehow unimportant. The earnings, or net profit, figure is a necessary link between the income statement and the balance sheet, and it is thus necessary to calculate earnings as a way of deriving the development over time of the financial statements.

Two objections to our reasoning immediately present themselves:

- Is it reasonable to use asymptotic proofs, thereby assuming an infinite life of the company?
- Many practitioners seem to use the earnings figures when contemplating an investment, is this not an indication that earnings valuation is reasonable?

The answer to the first objection is short and straightforward: When modelling a finite life, there will generally be a liquidating dividend and retained earnings will serve as one component in the determination of the size of the liquidating dividend. This is, however, a dividend valuation approach, albeit a very special case. The earnings figure serves as a link in determining the size of the dividend, not as a valuation measure itself. Hence any finite life assumption implies a dividend valuation approach.

---

[61] A growth rate larger than the discount rate is never the case in reality. It might be in a single year, but not on average in every year from now to infinity. Since there is a finite value for every company, the market clearly shares this view.

[62] Miller and Modigliani (1961), p. 421. They consider what they call "*the convenient case of constant growth rates*", i.e. the same case that we have investigated in expression (66).

The second objection: Any observer of stock market reporting in the news media will notice the prominence of the earnings figure. The standard answer to this is that while earnings have no direct value to the stock holder, it will give an indication as to future dividends and future capital gains. Quite possibly the earnings figure works well enough as a proxy variable for value, and hence some investors stop there. We do not disagree with this, we are merely pointing out that it is in reality nothing but a proxy variable, giving guidance as to the level of future dividends. As was seen in Chapter 2, the net profit figure combined with *additional* balance sheet information can be translated into free cash flows or dividends. This should be nothing controversial. Accounting academics will only phrase this somewhat differently: that the earnings figure combined with the book value of the company provides value relevant information. And yet it is obviously controversial: Penman spends four pages (pp. 28-31) telling us why the free cash flow approach is inferior, with special reference to Copeland et al.

Ohlson (1995) claims that the dividend approach is equal to the earnings approach if one introduces the concept of "abnormal earnings".[63] We have shown earlier in this report that the dividend approach is equal to the free cash flow approach provided the discounting is done properly. So it seems that one has a choice: abnormal earnings, dividends, or free cash flows. Personally, we remain unconvinced that "abnormal earnings" is a clearer concept than "free cash flows", and, as was demonstrated earlier in this section, "earnings" as such do not work as a valuation measure.

[63] Earnings in excess of the discount rate multiplied by the entering book value.

Appendix 3 - Constant dividend growth

Assume a scenario with constant dividend growth, i.e.

$$\text{(A3:1)} \quad DIV_\tau = (1+g)^\tau DIV_0$$

where g is the constant dividend growth. The value of the equity is the sum of the discounted future dividends. Assuming an infinite life of the company:

$$\text{(A3:2)} \quad V_{DIV} = \sum_{\tau=1}^{\infty} \frac{DIV_\tau}{(1+k_E)^\tau} = \sum_{\tau=1}^{\infty} \frac{(1+g)^\tau DIV_0}{(1+k_E)^\tau}$$

Evaluating the infinite sum and simplifying, the Gordon formula is obtained (provided $g<k_E$):

$$\text{(A3:3)} \quad V_{DIV} = \frac{(1+g)DIV_0}{k_E - g} = \frac{DIV_1}{k_E - g}$$

This definition of equity value is non-controversial.

We now want to show that the capitalised dividend sequence in expression (63) equals (A3:3) when using an infinite horizon, i.e. we want to show that:

$$\text{(A3:4)} \quad \lim_{T\to\infty}(\rho^T - 1)^{-1} E\left[\left(\sum_{\tau=1}^{T}(\rho^{T-\tau}-1)\tilde{d}_{t+\tau}\right)|Z_t\right] = \frac{(1+g)DIV_0}{k_E - g}.$$

We denote dividends with DIV, Penman uses d. ρ is equal to $(1+k_E)$, so the left-hand-side of (A3:4) can equivalently be written:

$$\text{(A3:5)} \quad LHS = \lim_{T\to\infty}\left((1+k_E)^T - 1\right)^{-1} E\left[\left(\sum_{\tau=1}^{T}\left((1+k_E)^{T-\tau}-1\right)\tilde{d}_{t+\tau}\right)|Z_t\right]$$

The scenario under consideration is constant dividend growth. This means that the conditional expectations are:

$$\text{(A3:6)} \quad E\left[\left((1+k_E)^{T-\tau}-1\right)\tilde{d}_{t+\tau}|Z_t\right] = \left((1+k_E)^{T-\tau}-1\right)(1+g)^\tau d_0$$

Hence, expression (A3:5) can be rewritten:

$$\text{(A3:7)} \quad LHS = \lim_{T\to\infty}\left(\left(1+k_E\right)^T - 1\right)^{-1}\sum_{\tau=1}^{T}\left(\left(1+k_E\right)^{T-\tau} - 1\right)\left(1+g\right)^{\tau} d_0$$

Rearranging and noting that the limit of a sum is equal to the sum of the limits:

$$\text{(A3:8)} \quad LHS = \lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left(\left(1+k_E\right)^{T-\tau}\right)\left(1+g\right)^{\tau} d_0}{\left(1+k_E\right)^T - 1} - \lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left(1+g\right)^{\tau} d_0}{\left(1+k_E\right)^T - 1}$$

The two terms in (A3:8) can be evaluated separately. Define:

$$\text{(A3:9)} \quad A \equiv \lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left(\left(1+k_E\right)^{T-\tau}\right)\left(1+g\right)^{\tau} d_0}{\left(1+k_E\right)^T - 1}$$

$$\text{(A3:10)} \quad B \equiv \lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left(1+g\right)^{\tau} d_0}{\left(1+k_E\right)^T - 1}$$

Evaluating (A3:9):

$$\text{(A3:11)} \quad A \equiv \lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{\left(\left(1+k_E\right)^{T-\tau}\right)\left(1+g\right)^{\tau} d_0}{\left(1+k_E\right)^T - 1} =$$

$$= d_0 \lim_{T\to\infty}\left(-\frac{\left(1+g\right)^{T+1}}{\left(\left(1+k_E\right)^T - 1\right)\left(k_E - g\right)} + \frac{\left(1+g\right)\left(1+k_E\right)^T}{\left(\left(1+k_E\right)^T - 1\right)\left(k_E - g\right)}\right) =$$

$$= \frac{\left(1+g\right)d_0}{\left(k_E - g\right)}\lim_{T\to\infty}\left(-\frac{\left(1+g\right)^T}{\left(\left(1+k_E\right)^T - 1\right)} + \frac{\left(1+k_E\right)^T}{\left(\left(1+k_E\right)^T - 1\right)}\right) =$$

$$= \frac{\left(1+g\right)d_0}{\left(k_E - g\right)}\left(\lim_{T\to\infty}\left(-\frac{\left(1+g\right)^T}{\left(\left(1+k_E\right)^T - 1\right)}\right) + \lim_{T\to\infty}\left(\frac{\left(1+k_E\right)^T}{\left(\left(1+k_E\right)^T - 1\right)}\right)\right) =$$

/ $g < k_E$ by assumption /

$$= \frac{\left(1+g\right)d_0}{\left(k_E - g\right)}\left(-0+1\right) = \frac{\left(1+g\right)d_0}{\left(k_E - g\right)}$$

Evaluating (A3:10):

(A3:12) $$B \equiv \lim_{T\to\infty}\sum_{\tau=1}^{T}\frac{(1+g)^{\tau}d_0}{(1+k_E)^T-1}=d_0\lim_{T\to\infty}\left(\frac{(1+g)^{T+1}}{\left((1+k_E)^T-1\right)g}-\frac{(1+g)}{\left((1+k_E)^T-1\right)g}\right)=$$

$$=d_0\frac{1+g}{g}\lim_{T\to\infty}\frac{(1+g)^T-1}{(1+k_E)^T-1}=0 \qquad \text{/ since } g<k_E \text{ by assumption /}$$

And the entire LHS expression (A3:8) becomes:

(A3:13) $$LHS=A+B=\frac{(1+g)d_0}{(k_E-g)}+0=\frac{(1+g)d_0}{(k_E-g)}$$

The LHS of expression (A3:4) equals the RHS. QED.

4. Model Implementation - Empirical and Theoretical Analysis

In this chapter topics related to the practical implementation of the modelling approach will be discussed. An analysis of the different input parameters is performed, both from an empirical and a theoretical point of view: we look at the parameters themselves and also at their interaction in determining the financial statements from which valuation measures are derived.

In section 4.1 the different specifications of the property, plant and equipment (PPE) items presented by Copeland et al are analysed. Section 4.2 contains the determination of an analytical expression for the steady-state parameter *r, retirements / preceding year's gross PPE,* based on steady state considerations. In section 4.3 a solution is derived for determining the parameter *c, change in deferred taxes / gross PPE,* in steady state, by using assumptions about the *effective* tax rate. In section 4.4 we look at the parameter *p, operating expenses in % of revenues,* before turning to the *working capital* to *revenues* relation in section 4.5.

Some of the model's parameters are very company specific, and there is not much help to be drawn from an analytical treatment or from empirical industry data. In those cases the company's own historical figures together with (the often treacherous) intuition will have to suffice. For other parameters, the company's parameter value may for some reason be correlated to an industry average figure - one example being the profit margin of businesses operating in highly competitive markets. In those cases, industry average figures may be of some guidance. Such figures are presented in tables for the parameters *a, e* and *p.* They all originate from Statistics Sweden (SCB).[64] Since, generally, the information content of accounting data is rapidly decreasing with time we only include figures from 1988 and onwards (the latest available year is 1993). This still enables us to include both ends of the business cycle (such as the top years at the end of the 1980's and the in many businesses almost depression-like year 1992).

[64] See the appendix at the end of the chapter for a brief discussion about the data quality.

4.1 The determination of PPE

4.1.1 Introduction

This section contains a discussion about *property, plant and equipment* related items. These closely related items are the following: *net* and *gross property, plant and equipment (net* and *gross PPE), accumulated depreciation (A), capital expenditures (CapX), depreciation expense (DepX)* and *retirements (Ret).* Copeland et al give only limited attention to the determination of them.

The three different specifications of these items (already presented in section 1.2) will be investigated here in order to assess their usefulness as predictors of the future. This is done by examining the behaviour over time of the different driving ratios (*gross PPE / revenues*, *CapX / revenues*, and *net PPE / revenues*) in three large Swedish companies: Astra, Volvo and MoDo. Although an investigation of only three firms yields more an indication than substantive proof in the statistical sense, we believe the study to be of interest. The companies have been chosen because they represent different kinds of firms and businesses. The pharmaceutical company Astra has experienced tremendous growth in recent years and it is interesting to see how this affects the percentages. Volvo, the largest Swedish car manufacturer, has a much more fluctuating revenue development. MoDo is in the pulp and paper business, which traditionally is a cyclical business. Also, the economic interpretation of the ratios will be discussed.

4.1.2 The main driving ratios - empirical findings and interpretations

For forecasting purposes one would like to have input ratios with a fairly stable development over time. Moreover, the ratio should ideally be some kind of "identity" with a development of its own, so that the forecast of the ratio is unconditional of the revenue forecast. At the very least, it should be easier to predict the ratio than to make a direct forecast of the item determined by the ratio. Otherwise, the whole approach with revenue-related ratios would be meaningless.

In Figure 3 the three different ratios are plotted for Astra, MoDo and Volvo. Note that *net* and *gross PPE* are denoted by *NPPE* and *GPPE* respectively in the table legends throughout this chapter.

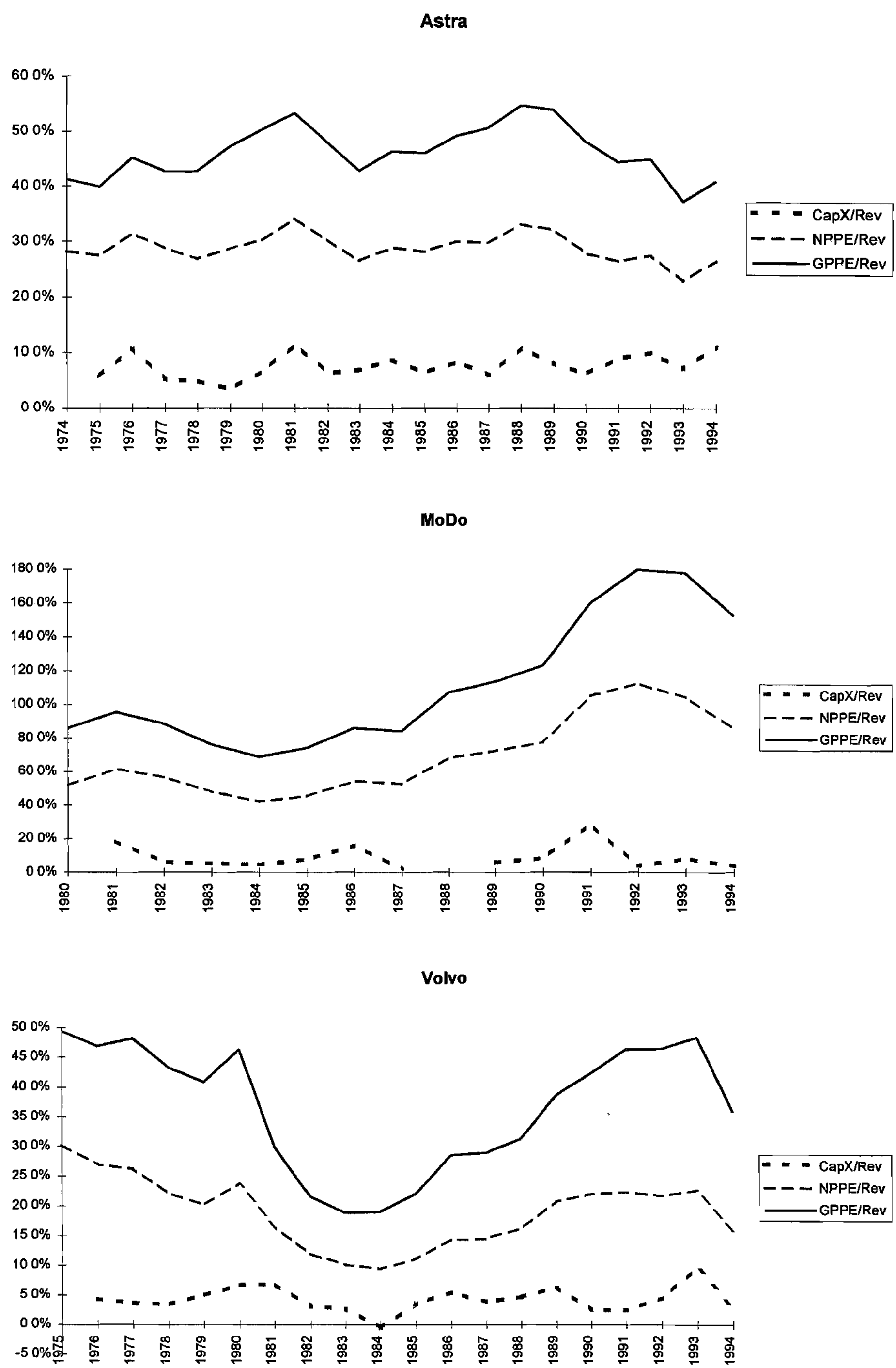

Figure 3 - The historical behaviour of the main driving input ratios in Astra, MoDo and Volvo

Gross PPE as a percentage of revenues, b_t (Specification A)

In Specification A the main driving ratio is *gross PPE / revenues*. The thought behind this is that a certain amount of property, plant and equipment is required to generate a certain amount of revenues.[65] The assumption conflicts, one suspects, with reality since revenues often tend to be fluctuating whereas *gross PPE* should be more stable. Astra, Volvo and MoDo will provide an indication on whether this is true or not:

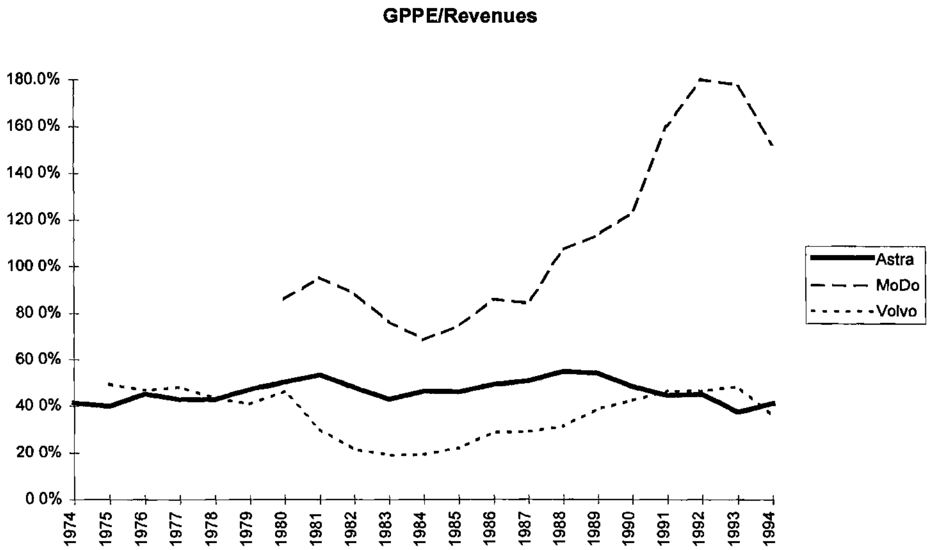

Figure 4 - Ratio between gross PPE and revenues in Astra, MoDo and Volvo

As can be seen from Figure 4, the *gross PPE* to *revenues* ratio works well in Astra, the company with continuous revenue growth. In Volvo and MoDo, the ratio fluctuates much more. This effect is mainly attributable to the difference in revenue development as is obvious from Figure 5.

[65] This means that a decrease in this percentage can be viewed as a more efficient use of *gross PPE*; less *gross PPE* being required to yield a certain amount of revenues. The reverse, of course, holds for an increase.

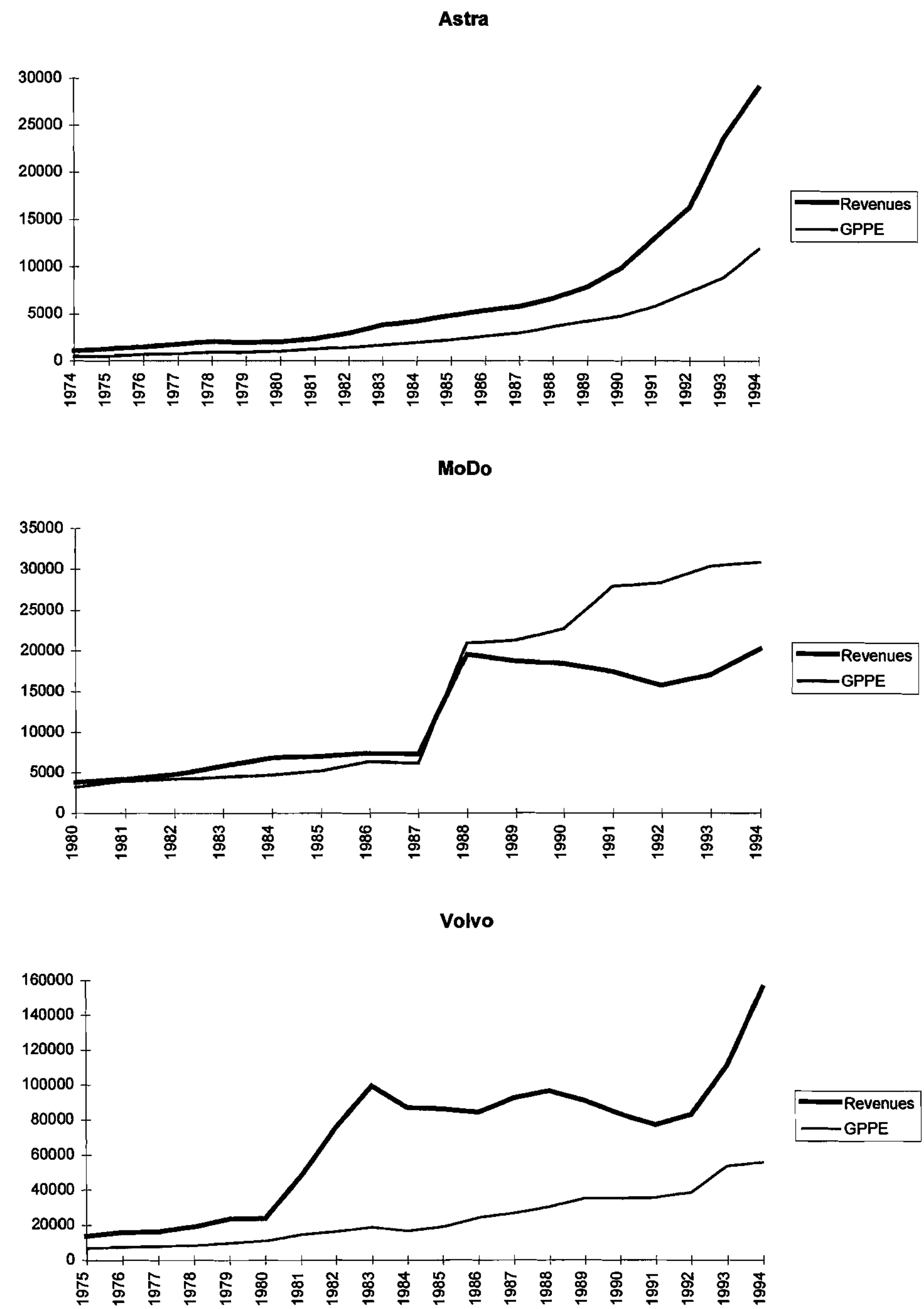

Figure 5 - Revenues and gross PPE in Astra, MoDo and Volvo

In all three companies, *gross PPE* grows every year (with one exception). This is the case irrespective of revenue development: the revenues are increasing in Astra, fluctuating in Volvo and cyclical in MoDo.[66] The historical behaviour of *gross PPE / revenues* is thus very company specific. In Astra, with a long history of growth, the ratio tends to lie within a specific range. In the other two companies, more fluctuating revenue developments result in fluctuating ratios.

Fluctuating ratios in themselves are not necessarily bad. If the ratio really is an independent economic entity, it could of course have its own fluctuating development, which could be used for forecasting the future development of the ratio, unconditional on the forecast of the denominator. In the case of the *gross PPE* to *revenues* ratio for MoDo and Volvo this ideal does not hold, however, mainly because *gross PPE* tends to increase steadily with time, whereas revenues go up and down. This implies that the ratio's development will be very much determined by the revenue development: the ratio increases whenever revenues are decreasing or at a constant level, and vice versa. Therefore, forecasts of the ratio must be made *conditional* on the forecast of the future revenue growth for Volvo and MoDo. The choice, in such cases, stands between assuming a non-fluctuating revenue development or adjusting the *gross PPE* to *revenues* ratio in order to compensate for the fluctuations in revenues, so that the *gross PPE* obtains a stable development. To conclude: for this type of companies, with fluctuating/cyclic revenue development, the main driving ratio of Specification A clearly works poorly.

Turning to the economic interpretation of the ratio, a first-hand interpretation is: *a certain amount of property, plant and equipment is required to generate a certain amount of revenues*. This implies that the ratio is a kind of efficiency measure. A problem is that the components (i.e. machines, buildings, etc.) of *gross PPE* are valued at their purchase value, while the revenues are "inflated", i.e. the revenues in each calculated ratio come from a specific year, whereas the components of *gross PPE* come from a wide dispersion of years, with different monetary value. If one uses inflation-adjusted accounting the interpretation would indeed be what was suggested above: a certain amount of property, plant and equipment required to generate a certain amount of revenues. When not using inflation-adjusted accounting, the validity of this interpretation is not that clear anymore. This is due to the fact that the denominator, *gross PPE,* is then not the actual amount of the property, plant and equipment, but merely some kind of weighted average of PPE-values, with much larger weights on *recently bought* property, plant and equipment. It is desirable to have a measure that takes into account how *many/much* (in physical terms) PPE used, or alternatively a measure of how much

[66] The peak in both curves in 1987 for MoDo, is attributed to the fact that two other companies were absorbed by MoDo.

money (at today's prices) that has been invested in PPE, neither of which is the case with *gross PPE.* Therefore the interpretation is somewhat unclear.

In conclusion, the main interpretation problem with inflation concerns *gross PPE* and how it develops over time. In times of high inflation the PPE bought recently makes up a larger part of the total PPE, than in times with low inflation. This means that if a company purchases a great deal of new equipment during a high inflation period, *gross PPE* will increase at a faster rate than in years of low inflation, even if *gross PPE* in terms of plants, numbers of machines, etc., is the same.

Capital expenditures as a percentage of revenues, e_t (Specification B)

The parameter e_t, *CapX / revenues,* is used as main driving input ratio in Specification B. The interpretation is straightforward: a certain amount of the revenues is used for buying new property, plant and equipment.

Specification B seems to be more intuitive than Specification A, mainly because the *gross PPE*-development is modelled in a much more stable way. The *gross PPE* is determined by adding this year's *capital expenditures* (minus *retirements*) to the preceding year's *gross PPE.* This ensures that *gross PPE* will have a smooth development.

Empirically the ratio is very unstable for the pulp and paper company MoDo, as can be seen in Figure 6. This is not surprising taking into account the characteristics of machinery and plants in the pulp and paper business, where new investments are large and occur at discrete, distant points in time. This is also obvious from Figure 7.

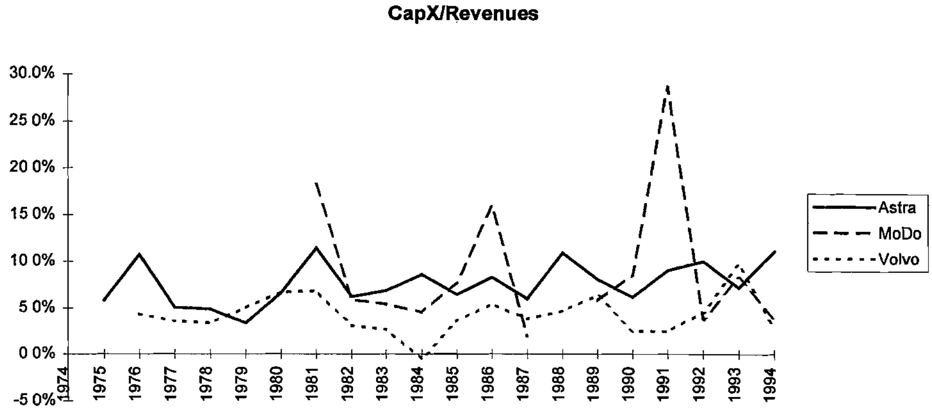

Figure 6 - Ratio between capital expenditures and revenues in Astra, MoDo and Volvo

On the other hand, it is obvious from Figure 7 that Astra is a company where the interpretation of the *CapX* to *revenues* ratio is applicable; the capital expenditure development closely follows the revenue development. The ratio itself is not *exceptionally* stable, but it tends to fluctuate around 8%. For Volvo, the relation between capital expenditures and revenues are not as obvious as for Astra, but working better than for MoDo.

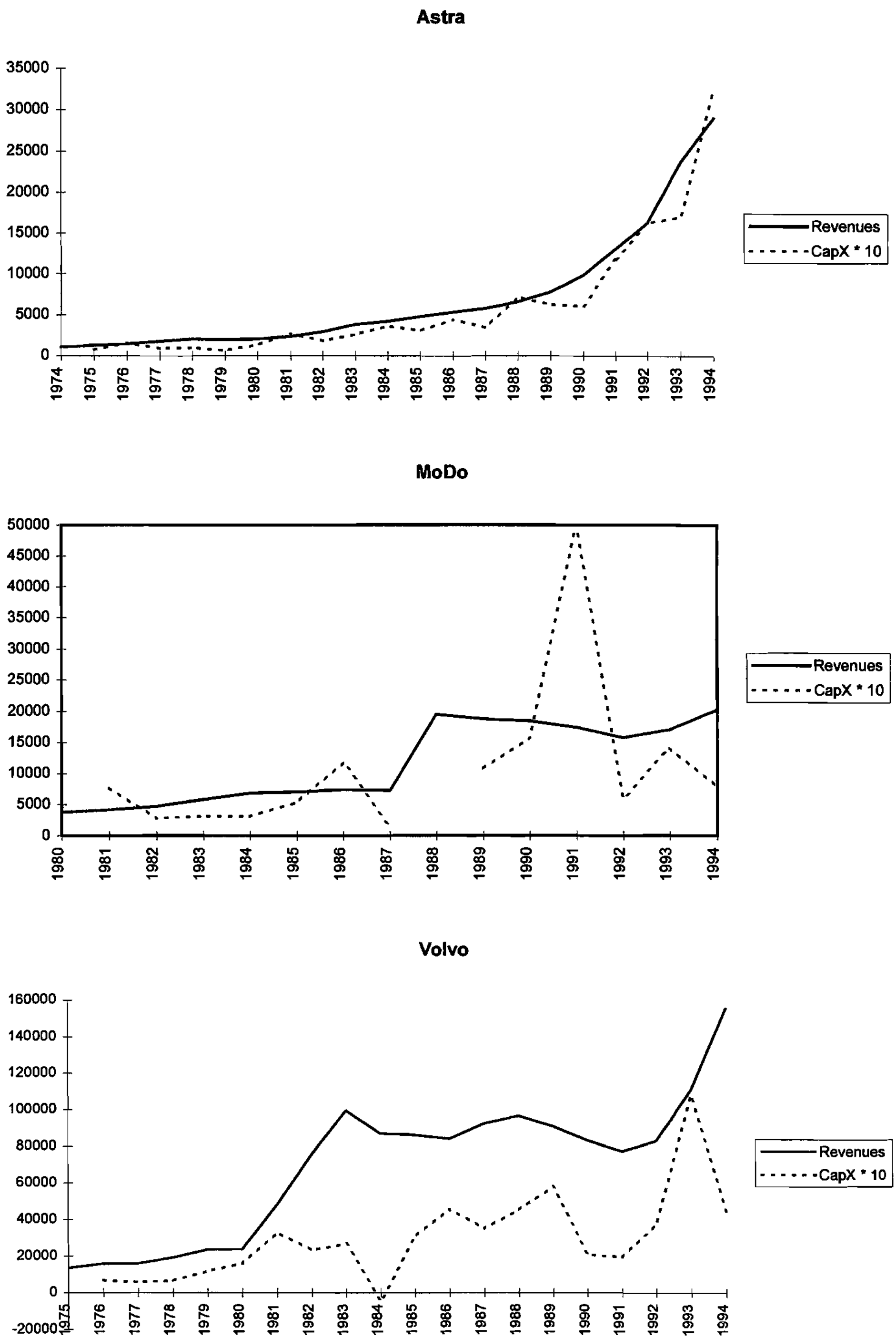

Figure 7 - Capital expenditures (scaled) and revenues in Astra, MoDo and Volvo

It is important to note that Specification B differs quite a lot from Specification A when it comes to the determination of *gross PPE*. In Specification B it is just the yearly *change* in *gross PPE* that is determined through the ratio, and not the whole stock of *gross PPE* as was the case with Specification A. This means that Specification B provides a way of modelling property, plant and equipment that is more consistent with the empirical finding that *gross PPE* is growing steadily over time.

It is reasonable to believe that the parameter e_t, the *capital expenditures* to *revenues* ratio, can differ between industries. Table 9 shows that this in fact is the case: it is low in trade industries, whereas it is quite high in heavy industries like *mining and quarrying, electricity, gas and water* and *manufacture of pulp and paper products.* A discussion about the data in Tables 9, 10 and 11 can be found in Appendix 4.

Industry	median	min	max	latest value	
Agriculture, hunting, forestry and fishing	4,1%	3,5%	5,0%	3,7%	(1992)
Mining and quarrying	10,8%	8,0%	14,3%	14,3%	
Manufacture of food, beverages and tobacco	3,1%	1,8%	3,5%	3,3%	
Textile, wearing apparel and leather industries	3,4%	2,9%	9,3%	2,9%	
Manufacture of wood and wood products	4,8%	4,0%	6,2%	4,8%	(1992)
Manufacture of pulp, paper products	6,9%	4,0%	9,0%	4,7%	
Printing and publishing	4,4%	-2,0%	6,1%	5,2%	
Manufacture of chemicals, petroleum, coal rubber and plastic products	5,2%	3,5%	6,2%	5,4%	
Manufacture of non-metallic mineral products, except products of petroleum and coal	5,4%	1,7%	7,0%	1,7%	
Basic metal industries	3,6%	2,8%	4,4%	2,8%	
Manufacture of fabricated metal products	4,7%	4,5%	5,4%	4,7%	(1992)
Manufacture of fabricated machinery and equipment	2,9%	1,8%	3,5%	1,8%	(1992)
Manufacture of electromechanical products	2,9%	2,4%	3,9%	2,9%	(1992)
Electricity, gas and water	11,6%	9,1%	15,3%	9,3%	
Construction	2,5%	1,7%	2,8%	1,7%	
Wholesale trade	1,2%	1,1%	1,8%	1,1%	
Retail trade	1,7%	1,5%	2,3%	1,5%	(1992)
Restaurants and hotels	4,2%	2,3%	5,6%	2,7%	
Land transportation	8,4%	6,0%	12,8%	6,4%	
Sea transportation	2,5%	0,2%	6,6%	5,4%	(1992)
Air transportation	11,6%	8,0%	12,9%	8,0%	(1992)
Consulting business	4,1%	3,0%	5,8%	3,0%	(1992)
Social and personal services	4,7%	3,4%	5,8%	3,4%	(1992)

Table 9 - capital expenditures in % of revenues (sales) in Swedish businesses 1988-1993

The observation made earlier that e_t is rather unstable over time is also obvious from these figures.

Net PPE as a percentage of revenues, n_t (Specification C)

Specification C is very much the same as Specification A. The difference is that instead of relating *gross* PPE to revenues, one here relates *net* PPE to revenues. The analogous (to Specification A) interpretation of the ratio *net PPE / revenues* is thus that a certain amount of *net* property, plant and equipment (i.e. the book value of PPE) is required to generate a certain amount of revenues.

Empirically, it can be seen from Figure 3 that the ratio *net PPE / revenues* behaves in almost exactly the same way as *gross PPE / revenues.* In fact, both net *and* gross PPE exhibit much more stable developments than revenues. This means that there will be the same problems with Specification C as was noticed for Specification A above, namely that forecasts of the ratio often must be made *conditional* on the forecast of the future revenue growth, which makes the approach with revenue-related ratios not very practical. Further, since the ratio *net PPE / revenues* is not easier to interpret than the corresponding ratio in Specification A, the modelling will not be enhanced by using Specification C instead of Specification A.

Other revenue-related ratios

In the McKinsey model, there are a lot of ratios that have revenues as denominator. Most of these other ratios tend, theoretically, to be more natural than *gross PPE / revenues,* e.g. *trade receivables / revenues.*[67] But, they can also fluctuate quite a lot in some cases. The reason for this can be that the items driven by these ratios are influenced by changes in the management policy concerning, for example, cash or inventory levels. By assuming constant management policy, these items should be closely tied to the revenue development in the future, however, and are therefore useful as forecasting tools.

[67] "Natural" meaning "more naturally related to the revenue development", i.e. for items, the amount of which depend on the amount of revenues in a specific year.

4.1.3 The retirements and depreciation ratios

Attention will now be turned to the two parameters that are the same in all three specifications: *retirements / preceding year's gross PPE* and *depreciation expense / preceding year's gross PPE*. The problems with the denominator, *gross PPE*, have already been discussed, and the analysis here will focus on the respective numerator.

The ratio *retirements / preceding year's gross PPE* is difficult to interpret since the item *historical retirements* in the McKinsey model, in excess of "real" retirements may contain accounting transactions, common in real world Swedish companies. If "retirements" were indeed real retirements, then the interpretation would be "the fraction of preceding year's *gross PPE* retired the next year." With the Copeland et al definition of retirements, however, the interpretation will be something like "the fraction of preceding year's *gross PPE* that will be the difference between the next year's depreciation expense and the next year's actual[68] change in accumulated depreciation." This is due to the fact that the Copeland et al definition is based on a clean surplus relation (CSR),[69] which empirically seems to be violated. Thus, when retirements are calculated according to Copeland et al's definition, a noise term stemming from the violation of the CSR may be included in the retirement figure. Obviously, *retirements as a percentage of preceding year's gross PPE* is not a very meaningful entity, and is even less meaningful if one also applies the interpretation of *gross PPE* stated above.

[68] After accounting transactions.

[69] Restating the CSR definition of Feltham & Ohlson (1994) in the terminology used here, one obtains: $N_{t+1} = N_t + CapX_{t+1} - DepX_{t+1}$. Using the Copeland et al definition of *Retirements,* $Ret_{t+1} = DepX_{t+1} - (A_{t+1} - A_t)$, to substitute for depreciation yields: $G_{t+1} = G_t + CapX_{t+1} - Ret_{t+1}$. Since this expression intuitively is consistent with clean surplus accounting, the same must be true for the Copeland et al definition of retirements.

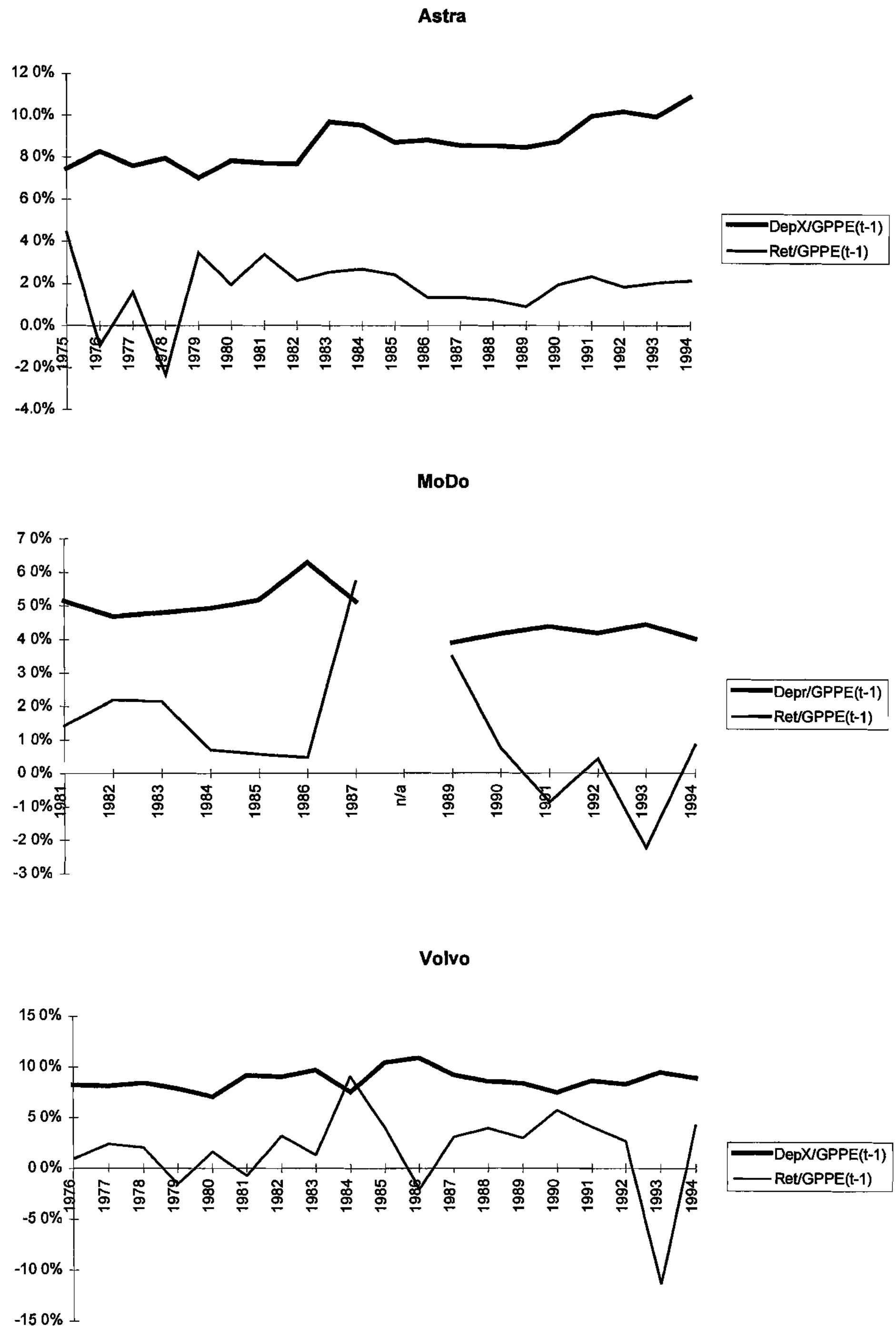

Figure 8 - Depreciation expense and retirements as percentages of preceding year's gross PPE in Astra, MoDo & Volvo

The real retirements should intuitively be correlated with the size of the *gross PPE*. However, when the determination of historical retirements is done according to a clean surplus relation that historically will not hold, and consequently other things than pure retirements are being measured, the correlation between "*retirements*" and *gross PPE* seems dubious at best. The empirical study of the three companies supports this suspicion.

As can be seen from Figure 8 the *retirements / preceding year's gross PPE* ratio, *r*, is not at all stable in Volvo and MoDo, the two companies with fluctuating and cyclical revenue development. The ratio is more stable in Astra, the growth company. This indicates that *r* might not be a robust predictor. Thus there are three major weaknesses of the *retirements* to *gross PPE* ratio: that it is difficult to interpret, that it indeed can be unstable over time, and that it is not a precise measure of true retirements. Later in this report we point at ways to overcome the forecasting problems by using the steady state concept to determine *r*.

The parameter d_t, *DepX / preceding year's gross PPE,* is intuitively easy to interpret: a fraction of the preceding year's gross PPE will be written off each year. In a normal going concern, this fraction should be fairly constant over time. This is also the case for Astra, MoDo and Volvo as can be seen from Figure 8. Empirically the ratio tends to remain at a specific level for each company, with relatively small variations each year. This makes the ratio quite useful for forecasting.

4.1.4 Concluding remarks on PPE-determination

Specification A of the property, plant and equipment items has one major drawback: for companies with fluctuating revenue development the main driving ratio *gross PPE / revenues* is not working well for forecasting purposes. This problem does not appear when there is stable revenue growth, as in the continuous growth firm Astra. This means that Specification A will work well when modelling a steady state development where revenues grow at a constant rate.

Specification B automatically gives a smoother, and thus more intuitive, development of the property, plant and equipment. Unfortunately, it can be difficult to make forecasts of the main driving ratio based on historical data, especially for companies where new investments in property, plant and equipment take place with large portions at distant points in time. However, since the main driving ratio of Specification B just determines the yearly *change* in *gross PPE*, and not the whole *gross*

PPE-stock, the overall sensitivity of the *gross PPE*-determination, with regard to the forecasting problem, will be smaller than for Specification A. In order to yield any steady state (other than PSS), it was shown in Chapter 2 that the parameters have to fulfil a condition that actually reduces Specification B to Specification A.

Specification C is very similar to Specification A and does not add any improvements to Specification A: the forecasting problems are the same. Accordingly, the analysis in previous chapters has been concentrated on Specifications A and B.

A common problem for all three specifications is the determination of *retirements*. The ratio *retirements / preceding year's gross PPE* empirically has a very unstable historical development, which may make forecasts based on historical data impossible.

The determination of *depreciation expense* through the ratio *DepX / preceding year's gross PPE* on the other hand works well for forecasting purposes in all three companies in the study.

4.2 An analytical determination of the *retirements* to *preceding year's gross PPE* ratio

It was noted above that forecasts of the ratio *retirements / preceding year's gross PPE* are almost impossible to infer from the data provided by annual reports, and we will here instead use the steady state concept to determine the retirements by its relation to growth and depreciation.

If the company's new investments in PPE, its depreciation and its retirements each year together have settled to a steady state,[70] a constant fraction of the old PPE will be replaced by new PPE each year. Using this relation, the following can be concluded:

[70] The exact meaning of this is obvious from the proof below.

Proposition 4.1

If a company's new investments in PPE, retirements and depreciation together have settled to a steady state, the gross PPE grows with growth rate g, and the economic life equals the depreciation period, then the fraction, r, of preceding year's gross property, plant & equipment that is retired is given by the following relation:

$$(67) \qquad \left[{}^{Retirements_t}\!\Big/_{G_{t-1}}\right]_t = r = \frac{g}{(1+g)^{1/d}-1}$$

where d is the fraction of preceding year's gross PPE that is depreciated.

Proposition 4.1 may seem very convenient when determining the *retirements / gross PPE* ratio in the steady state period. By the steady state assumption this ratio-value automatically falls out once the depreciation rate d and the growth rate g have been determined. Since the company is in steady state these two parameters are constants. The usefulness of Proposition 4.1 is somewhat limited, however, because of the assumptions underlying it. The derivation rests on the assumption that the economic life equals the depreciation period, and that this has been the case for quite some time. This is seldom the case in real world companies. Hence, the r-figure from expression (67) will generally be only an approximate bench-mark figure. In the Eldon valuation in Chapter 5 we use the r-value derived using expression (67) only as a starting point for a more complicated procedure.

Proof of proposition 4.1

Consider a model in continuous time. The assumptions are the following: The company is in steady state where the relations between the company's new investments in PPE, depreciation and retirements are constant. Furthermore, a constant fraction, d, of the *gross PPE* one year ago is depreciated. The *gross PPE* grows at the rate g. All this means that at each point in time, the PPE bought $1/d$ years ago will be retired and replaced by the amount $x \cdot (1+g)^{(1/d)}$ of new PPE. Let r denote the constant fraction of the PPE one year ago that is retired. The *gross PPE* at each point in time only consists of the PPE bought during the last $1/d$ years. All PPE bought earlier has been retired.

Now, suppose that *gross PPE* at time $t + 1/d$ in steady state is $G_{t+(1/d)}$. This can be written as (where x denotes the amount spent on PPE at time t):

$$G_{t+(1/d)} = x \int_{t}^{t+(1/d)} (1+g)^{s-t}\, ds = x \cdot \frac{1}{(1+g)^{t} \ln(1+g)} \left[(1+g)^{t+(1/d)} - (1+g)^{t}\right] =$$

$$x \cdot \frac{1}{\ln(1+g)} \left[(1+g)^{(1/d)} - 1\right] \quad .$$

At time $t + (1/d) + 1$ the depreciation will be:

$$d \cdot G_{t+(1/d)} = x \frac{d}{\ln(1+g)} \left[(1+g)^{1/d} - 1\right]$$

At time $t + 1/d$ the PPE bought $1/d$ years ago cost the amount x. To stay in steady state, the amount x of the PPE has to be retired. Also by the steady state assumption the amount retired will grow at the rate g, simply because retirements are constantly related to *gross PPE*, which in turn grows at this rate g. The retirements the coming year will then be:

$$x \int_{t+(1/d)}^{t+(1/d)+1} (1+g)^{s-(t+(1/d))}\, ds = x \frac{1}{(1+g)^{t+(1/d)} \ln(1+g)} = \left[(1+g)^{t+(1/d)+1} - (1+g)^{t+(1/d)}\right] =$$

$$= x \frac{1}{\ln(1+g)} \left[(1+g)^{1} - 1\right] = x \frac{g}{\ln(1+g)}$$

This is by definition equal to $r \cdot G_{t+(1/d)}$. The relation between the retirement and depreciation ratios, r and d, then becomes:

$$\frac{r}{d} = \frac{g}{d\left[(1+g)^{1/d} - 1\right]}$$

This relation between retirements and depreciation must hold in order for the company to be in steady state with respect to its replacement and growth of PPE. From this relation an explicit expression for r can be found:

(67) $$\left[{}^{Ret_t}\!/\!_{G_{t-1}}\right]_t = r = \frac{g}{(1+g)^{1/d} - 1}$$ Q.E.D.

4.3 Determining the development of deferred taxes

The parameter c, the change in deferred taxes as a percentage of *gross PPE*, is closely related to τ, the corporate tax rate. The nominal corporate tax rate is notoriously unstable over time (during the last ten years in Sweden it has been 52%, 40%, 30%, and 28%), and it is thus hazardous to assume that the present tax rate will remain constant in the future. More stable is the *effective* tax rate, i.e. the tax rate that firms actually face (i.e. pay) when taking all sorts of tax deferrals and tax exemptions into consideration. It can be argued that the government wants to keep the effective tax rate relatively constant, whereas the nominal tax rate is subject to all sorts of political considerations. This is one possible starting-point for the determination of the steady-state value c. In the steady-state period this would mean (using Specification A and denoting the effective tax rate by α):

$$(68) \qquad \alpha = \frac{\tau\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right) - \left(T_t - T_{t-1}\right)}{\left(R_t - pR_t - dbR_{t-1} - iw\left(aR_{t-1} + bR_{t-1} - A_{t-1}\right)\right)}$$

Simplifying and substituting for expressions (4 - 6) yields

$$(69) \qquad \alpha = \tau - \frac{cb}{1 - p - \frac{db}{1+g} - \frac{iw}{1+g}\left(a + b - \frac{(d-r)b}{g}\right) - \frac{iw}{(1+g)^t R_0}\left(\frac{(d-r)b}{g}R_0 - A_0\right)}$$

It should be noticed that α is time-dependent in the general case, but in the special case of a constant capital structure, $(d-r) \cdot b / g \cdot R_0$ equals A_0 (see Proposition 2.3 and Corollary 2.1) and the time-dependent part of (69) disappears.

Manipulating (69) yields the expression for c:

$$(70) \qquad c = \frac{\tau - \alpha}{b}\left(1 - p - \frac{db}{1+g} - \frac{iw}{1+g}\left(a + b - \frac{(d-r)b}{g}\right) - \frac{iw}{(1+g)^t R_0}\left(\frac{(d-r)b}{g}R_0 - A_0\right)\right)$$

If α, the *effective* tax rate, is kept constant, c obviously becomes time-dependent except in the constant capital structure case.

4.4 The parameter *p* - operating expenses in % of revenues (sales)

This is the most important parameter in a valuation since it completely determines the profit margin (1-*p*), which is the basic foundation for any corporate value. Effort should thus not be spared on the estimation of the steady-state value for this parameter. Obviously, a thorough analysis of the industry is of the essence. For example, the profit margin is higher (and consequently the value of *p* lower) in monopolistic industries with high barriers of entry than in industries where free competition prevails. Notwithstanding the importance of this subject, we will refrain from pursuing it, as it is a science in itself and lies far beyond the scope of this report. For an excellent introduction we refer to Porter (1980). As a starting-point for an analysis, the averages for a number of Swedish industries are presented in Table 10, below:

Industry	median	min	max	latest value	
Agriculture, hunting, forestry and fishing	92,3%	92,1%	93,5%	92,1%	(1992)
Mining and quarrying	87,7%	81,5%	90,3%	88,9%	
Manufacture of food, beverages and tobacco	93,8%	92,3%	94,6%	92,3%	
Textile, wearing apparel and leather industries	93,6%	91,9%	95,4%	91,9%	
Manufacture of wood and wood products	92,0%	90,4%	95,0%	95,0%	(1992)
Manufacture of pulp, paper products	88,5%	84,2%	92,9%	88,1%	
Printing and publishing	93,5%	92,4%	94,7%	92,4%	
Manufacture of chemicals, petroleum, coal rubber and plastic products	88,0%	83,6%	89,4%	83,6%	
Manufacture of non-metallic mineral products, except products of petroleum and coal	89,3%	86,7%	92,2%	90,8%	
Basic metal industries	93,5%	88,9%	98,5%	91,1%	
Manufacture of fabricated metal products	92,0%	91,4%	93,7%	92,9%	(1992)
Manufacture of fabricated machinery and equipment	94,1%	93,6%	94,8%	94,6%	(1992)
Manufacture of electromechanical products	94,5%	93,2%	96,9%	94,5%	(1992)
Electricity, gas and water	75,6%	72,9%	77,5%	72,9%	
Construction	94,9%	94,5%	95,1%	95,0%	
Wholesale trade	97,0%	96,5%	97,7%	96,5%	
Retail trade	96,6%	95,9%	96,8%	96,8%	(1992)
Restaurants and hotels	94,4%	92,6%	95,5%	93,0%	
Land transportation	89,6%	88,1%	95,3%	88,1%	
Sea transportation	92,0%	89,7%	95,3%	94,6%	
Air transportation	94,2%	90,3%	99,3%	99,3%	
Consulting business	93,5%	92,8%	94,1%	94,1%	(1992)
Social and personal services	91,5%	90,5%	92,7%	91,5%	(1992)

Table 10 - Operating expenditures in % of revenues in Swedish businesses 1988-1993

The percentage is rather stable in most industries, a fact that may be taken as justification for the use of industry averages as benchmarks.

4.5 The parameter *a* - net working capital in % of revenues (sales)

The definition is *(Current assets-Current non-interest bearing liabilities)/Revenues*. This is the empirically most troublesome parameter, since it varies a lot both over time and across businesses. The management policy on different working capital items may be quite company specific, customs regarding the length of payment periods may differ substantially over time, etc. The unstable nature of the parameter is evident from Table 11, below, where industry averages vary substantially over time.

It should be noted once again that in order not to lose valuable information, the aggregate measure should not be used in the explicit forecast period. Instead the different balance sheet items of the working capital should be related to revenues, one by one.

Industry	median	min	max	latest value	
Agriculture, hunting, forestry and fishing	10,4%	9,7%	12,8%	12,8%	(1992)
Mining and quarrying	40,6%	26,7%	45,2%	45,2%	
Manufacture of food, beverages and tobacco	8,0%	4,8%	9,3%	5,1%	
Textile, wearing apparel and leather industries	24,0%	19,3%	25,6%	19,3%	
Manufacture of wood and wood products	15,5%	13,3%	16,3%	16,3%	(1992)
Manufacture of pulp, paper products	11,6%	-10,1%	21,5%	14,1%	
Printing and publishing	6,7%	2,0%	12,1%	12,1%	
Manufacture of chemicals, petroleum, coal rubber and plastic products	12,2%	4,0%	18,1%	15,7%	
Manufacture of non-metallic mineral products, except products of petroleum and coal	11,7%	4,1%	14,0%	13,0%	
Basic metal industries	14,0%	6,7%	25,9%	14,7%	
Manufacture of fabricated metal products	15,6%	13,8%	20,2%	20,2%	(1992)
Manufacture of fabricated machinery and equipment	16,9%	14,1%	18,4%	17,8%	(1992)
Manufacture of electromechanical products	20,9%	16,0%	22,5%	16,0%	(1992)
Electricity, gas and water	-1,4%	-11,3%	10,7%	1,2%	
Construction	7,5%	1,8%	11,3%	7,8%	
Wholesale trade	9,5%	8,8%	10,1%	8,8%	
Retail trade	9,0%	8,5%	10,0%	8,5%	(1992)
Restaurants and hotels	-1,5%	-6,1%	0,3%	-4,5%	(1992)
Land transportation	-1,2%	-5,1%	0,9%	0,9%	
Sea transportation	11,0%	-7,2%	31,2%	4,6%	
Air transportation	20,6%	15,7%	28,3%	28,3%	
Consulting business	8,0%	5,9%	12,4%	8,4%	(1992)
Social and personal services	7,9%	7,3%	9,1%	7,6%	(1992)

Table 11 - Net working capital in % of revenues in Swedish businesses 1988-1993

Appendix 4 - Empirical industry figures.

The *industry average* is a concept open for different interpretations. The Swedish empirical figures used in this chapter are derived from treating the entire industry as one company, calculating the different financial ratios from its income statement and balance sheet. Conceptually, this is the same as a weighted mean, where the weights are given by the size of the companies. Another possibility would of course be an unweighted mean, a third would be the median.

There is an inherent difficulty in making empirical observations regarding the stability over time of financial ratios. On the one hand one wants a long time period to be able to draw statistically valid conclusions, on the other hand the underlying relationships may change over a long period of time and the principles of financial reporting may also change, causing additional "definition bias". We do not go further back than 1988 in the tables since the informational content in earlier figures is rather questionable.

The data in the empirical tables are taken from the industry-specific income statements and balance sheets in *Official Statistics of Sweden, Statistics Sweden: Enterprises 1987, Enterprises 1988, ..., Enterprises 1993.* The official industry classification (SNI) has changed from 1993. In some cases this makes the figures up until 1992 incompatible with the 1993 figures, so sometimes the 1993 figure has had to be left out. This is marked in the tables.

The many problems and reservations connected with the subject of average industry data means they should be used with caution. We present them merely as a possibility for situations where one can find no other guidance as to reasonable parameter values.

5. Eldon AB

In this chapter it will be shown how one in practice can implement a model of the Copeland et al type on a Swedish company using the results from previous chapters. The idea is to highlight many of the practical problems that arise in a valuation of this type and to suggest ways of dealing with them. The Swedish company Eldon AB will be used as example. In many cases it makes more sense to value the (consolidated) company group rather than the parent company; this is also what will be done here: what we refer to as Eldon is henceforth the consolidated group.

The actual procedure will be dealt with step by step:

5.1 Setting up historical financial statements
5.2 Calculating historical financial ratios, i.e. historical parameter values
5.3 Forecasting future parameter values
5.4 Setting up future (forecasted) financial statements
5.5 Calculating future (forecasted) free cash flows
5.6 Calculating the equity value

It should be noted that the aim here is to construct a working model that is internally consistent and that the numbers plugged into it are for demonstration only. We have chosen to use as a sort of base-case a rather gloomy picture of the company's future with virtually no real growth. This is in no way meant to be a realistic view of this specific company - in reality there are growth opportunities (indicated by the fact that the market value is considerably higher than our "base-case value"). Questions of future growth, operating margin, etc., are more for the corporate analyst, however, and we will concentrate on the actual model building.

5.1 Historical financial statements

The first step in the valuation procedure is to insert historical accounting data into a spreadsheet in order to calculate the historical values of the parameters.

5.1.1 Looking for changes in the structure of financial statements

A natural starting point is to look at the structure of the financial statements (balance sheets and income statements) over the time period of interest in order to find out whether there have been any major structural changes in the company's financial statements and/or accounting principles. In the Eldon case, one important change has occurred: in 1990 the group accounting changed from untaxed-reserves accounting to deferred-taxes accounting. This will have as a consequence that some of the accounting figures from 1989 and 1990 will not be consistent with later years, and hence figures from 1989 and 1990 should be used with caution.

5.1.2 Creating a spreadsheet model for the historical financial statements

Let the first column be reserved for text description of the items and the following columns be reserved for each one of the historical years. Then add the different items of the income statement and the balance sheet in a row each, in column 1. The summarising items (like *operating income*) are indicated in the tables below by a straight line *above* them. Other figures that are calculated (and *not* directly inserted) are indicated by *italics.*

The next step is then to insert the figures of the historical financial statements. All summarising items can be calculated in the spreadsheet instead of inserting them directly from the annual report. They can then be used as checkpoints against the corresponding items in the annual reports in order to ensure that all inserted figures are correct. The historical income statements are presented in Table 12:

INCOME STATEMENT:	1989	1990	1991	1992	1993	1994
Revenues	1,136.6	1,169.4	1,289.4	1,271.8	1,407.2	1,663.9
Operating expenses	-1,027.7	-1,057.9	-1,205.9	-1,182.1	-1,277.0	-1,478.4
Depreciation expense	-35.5	-35.4	-41.6	-44.9	-47.3	-46.5
Operating income	73.4	76.1	41.9	44.8	82.9	139.0
Net financial income	-21.1	-17.8	-29.2	-26.8	-37.4	-26.3
Extraordinary items	0.0	25.8	-10.1	-40.4	10.4	12.6
Earnings before taxes	52.3	84.1	2.6	-22.4	55.9	125.3
Appropriations	32.4	32.1				
Taxes	-32.6	-52.6	-8.1	-6.6	-19.0	-56.6
Net profit	**52.0**	**63.5**	**-5.5**	**-29.0**	**36.9**	**68.7**

Table 12 - Historical income statements, Eldon

The insertion of the balance sheet figures requires some extra considerations, especially when it comes to the property, plant and equipment items, since these are generally presented as net values in Swedish annual reports, whereas most of the models in the Copeland et al book and in this report require that the net PPE is explicitly calculated as gross PPE minus accumulated depreciation. We also need a measure of retirements.

Practically, the insertion of the respective PPE figures is done in the following way: gross PPE in the balance sheet is calculated as the sum of the gross property, gross plant, gross equipment and installations in progress from the PPE calculations table (Table 13); accumulated depreciation and net PPE are calculated in the same manner.[71] The figures in Table 13 come from the depreciation expense note of the balance sheet with the exception for retirements, which are derived by calculating the difference between the depreciation expense (from Table 12) and the change in accumulated depreciation.

CALCULATION OF PROPERTY, PLANT & EQUIPMENT ITEMS:	1989	1990	1991	1992	1993	1994
Installation in progress	0.0	4.3	0.3	12.4	0.1	5.0
Property:						
Gross	172.6	197.8	220.5	248.1	257.8	257.4
Acc. depr	-42.0	-37.8	-47.1	-53.9	-58.2	-64.6
Appreciation	4.6	4.4	4.0	3.8	3.0	3.0
Net Property	135.2	164.4	177.4	198.0	202.6	195.8
Equipment:						
Gross	352.3	360.9	420.3	413.7	439.0	459.3
Acc. depr	-187.1	-178.2	-207.6	-215.8	-236.3	-260.7
Net Equipment	165.2	182.8	212.7	197.9	202.7	198.6
Plant:						
Gross	18.9	29.7	31.4	36.4	38.1	38.0
Acc. depr	-2.1	-2.0	-2.3	-2.4	-2.4	-2.6
Net Plant	16.8	27.6	29.1	34.0	35.7	35.4
Depreciation	35.5	35.4	41.6	44.9	47.3	46.5
Property	4.5	4.0	4.8	5.8	6.6	6.8
Plant	0.2	0.2	0.2	0.2	0.2	0.2
Equipment	30.8	31.3	36.6	38.9	40.5	39.5
Retirements	*n/a*	*48.5*	*2.2*	*29.6*	*21.7*	*15.5*
Property		*8.0*	*-4.9*	*-1.2*	*1.5*	*0.4*
Plant		*0.2*	*-0.1*	*0.1*	*0.2*	*0.0*
Equipment		*40.2*	*7.2*	*30.7*	*20.0*	*15.1*

Table 13 - Calculation of PPE, depreciation and retirements, historically, Eldon

With the additional information from Table 13, the balance sheets from the annual reports can be inserted as in Table 14:

[71] Appreciation will be netted against accumulated depreciation in the balance sheet (Table 14).

BALANCE SHEET:						
	1989	1990	1991	1992	1993	1994
ASSET SIDE						
Operating cash	38.4	78.8	82.5	95.9	122.1	99.7
Trade receivables	166.9	190.0	175.7	175.2	200.8	262.2
Prepaid expenses	10.3	9.3	12.4	14.9	17.3	17.5
Other receivables	15.6	25.0	35.8	47.0	21.5	14.6
Inventories	331.2	344.6	350.1	320.9	336.8	349.0
Current assets	562.5	647.8	656.5	653.9	698.5	743.0
Excess marketable securities	8.8	7.5	7.5	7.8	7.7	0.9
Investment fund	47.4	39.3	9.6	1.2		
Gross property, plant and equipm	*543.8*	*592.6*	*672.5*	*710.6*	*735.0*	*759.7*
Accumulated depreciation	*-226.6*	*-213.6*	*-253.0*	*-268.3*	*-293.9*	*-324.9*
Net property, plant and equipment	317.2	379.1	419.5	442.3	441.1	434.8
Total assets	**935.9**	**1,073.6**	**1,093.1**	**1,105.2**	**1,147.3**	**1,178.7**
Net Total Assets*	739.1	797.6	850.5	874.3	888.6	863.7
* Total assets - Working capital liabilities						
	1989	1990	1991	1992	1993	1994
DEBT & EQUITY SIDE						
Short-term debt	83.1	116.6	104.0	163.4	139.7	91.7
Accounts payable	82.4	130.8	103.9	92.2	118.3	140.0
Accrued expenses	75.7	75.7	89.5	98.8	78.1	91.3
Taxes Payable	17.3	19.9	7.0	3.4	33.0	47.2
Other current liabilities	21.3	49.7	42.2	36.5	29.3	36.5
Total current liabilities	279.8	392.6	346.6	394.3	398.4	406.7
Long-term debt	80.8	92.0	170.0	158.3	172.1	152.6
Used check-credit	30.8	38.6	60.5	66.9	73.3	56.3
Pension funds	51.6	53.2	59.5	61.8	62.4	63.5
Deferred taxes	8.2	5.4	92.6	90.0	65.0	70.5
Total longterm liabilities	171.4	189.3	382.6	377.0	372.8	342.9
Untaxed Reserves	300.3	268.6				
Common stock	43.3	51.9	51.9	51.9	51.9	51.9
Restricted reserves	19.6	13.9	194.8	160.0	151.6	139.6
Non-restr. equity	121.5	157.3	117.2	122.0	172.6	237.6
Total common equity	184.4	223.1	363.9	333.9	376.1	429.1
Total liabilities and equity	**935.9**	**1,073.6**	**1,093.1**	**1,105.2**	**1,147.3**	**1,178.7**

Table 14 - Historical balance sheets, Eldon

In Eldon, equity consists of three categories: common stock, restricted reserves (which together make up restricted equity) and non-restricted equity (basically retained earnings). When deferred-taxes accounting was introduced at Eldon (from 1991) it meant that the restricted reserves were raised dramatically. The untaxed reserves were then divided between deferred taxes and restricted reserves.

5.2 Calculating historical financial ratios (parameters)

We are now ready to calculate the historical performance measures, i.e. the historical values of the model parameters. We also insert the historical inflation in order to make it possible to calculate the real growth in revenues.[72]

Operations:

Operations:	1989	1990	1991	1992	1993	1994
Real growth	1.3%	-6.8%	0.8%	-3.6%	5.7%	15.7%
Inflation	6.4%	10.4%	9.4%	2.3%	4.7%	2.2%
Revenue growth (g)	7.8%	2.9%	10.3%	-1.4%	10.6%	18.2%
Operating exp/revenues (p)	90.4%	90.5%	93.5%	92.9%	90.7%	88.9%

Table 15 - Historical parameter values, operations

Working capital:

Working cap./revenues:	1989	1990	1991	1992	1993	1994
+Operating cash/rev's	3.4%	6.7%	6.4%	7.5%	8.7%	6.0%
+Trade receivables/rev's	14.7%	16.3%	13.6%	13.8%	14.3%	15.8%
+Other receivables/rev's	1.4%	2.1%	2.8%	3.7%	1.5%	0.9%
+Inventories/rev's	29.1%	29.5%	27.2%	25.2%	23.9%	21.0%
+Prepaid expenses/rev's	0.9%	0.8%	1.0%	1.2%	1.2%	1.1%
-Accounts payable/rev's	7.2%	11.2%	8.1%	7.2%	8.4%	8.4%
-Other curr liab's/rev's	1.9%	4.2%	3.3%	2.9%	2.1%	2.2%
-Accrued expenses/rev's	6.7%	6.5%	6.9%	7.8%	5.6%	5.5%
-Taxes payable/rev's	1.5%	1.7%	0.5%	0.3%	2.3%	2.8%
Net working cap./revenues (a)	32.2%	31.8%	32.1%	33.3%	31.3%	25.7%

Table 16 - Historical parameter values, working capital

[72] Real growth = ((1 + revenue growth) / (1 + inflation)) - 1

Property, plant and equipment:

Property,Plant and Equipment (PPE):	1989	1990	1991	1992	1993	1994
CapX/rev's (e)		8.3%	6.7%	5.3%	3.3%	2.4%
Gross PPE/rev's (b)	47.8%	50.7%	52.2%	55.9%	52.2%	45.7%
DepX/prec. year's gross PPE (d)		6.5%	7.0%	6.7%	6.7%	6.3%
Ret/prec. year's gross PPE (r)		8.9%	0.4%	4.4%	3.1%	2.1%

Table 17 - Historical parameter values, PPE

Deferred taxes and debt:

Deferred taxes and debt:	1989	1990	1991	1992	1993	1994
Incr in def. taxes/gross PPE (c)		-0.5%	0.9%	-0.4%	-3.4%	0.7%
Debt/net total assets (w)	33.3%	37.7%	46.3%	51.5%	50.4%	42.2%
Short-term/net total assets	11.2%	14.6%	12.2%	18.7%	15.7%	10.6%
Long-term/net total assets	10.9%	11.5%	20.0%	18.1%	19.4%	17.7%
Used check-cr./net total asset	4.2%	4.8%	7.1%	7.7%	8.2%	6.5%
Pension funds/net total asset	7.0%	6.7%	7.0%	7.1%	7.0%	7.4%

Table 18 - Historical parameter values, deferred taxes and debt

The figure for *increase in deferred taxes / gross PPE* in 1991 is calculated using the adjusted figure for deferred taxes in 1990, given in the annual report of 1991 (adjusted for change in accounting principles).[73]

5.3 Forecasting future parameter values

The next step in the valuation process is to predict the parameter values for future years, i.e. to specify the forecast assumptions. The future has been divided into a ten-year explicit forecast period and the time after that has been set as a steady state, where all input parameters remain constant and, as will be seen further on, we will also have achieved the different steady states discussed in earlier chapters.

The first year in the constant parameter period is 2005. This is *year zero* in the parametric steady state period, as discussed previously. This year is marked by italics in the tables below. Year 1 in the

[73] As mentioned earlier, the Eldon group accounting changed from untaxed-reserves accounting to deferred-taxes accounting in 1991.

perpetuity period is thus 2006, and in accordance with earlier results 2006 will be the base year for any continuing value calculations.

The revenue growth and the operating margin related parameters are without doubt the most important ones and no effort should be spared in trying to make the forecasts as accurate as possible. As was indicated in previous chapters, this is more a problem for the corporate analyst, however, and our treatment here will be a simple *base-case scenario*, which of course can be altered to incorporate any features deemed reasonable by the analyst.

5.3.1 Operations

Operations:	1995	1996	1997	1998	1999	2000
Real growth	7.0%	5.0%	3.0%	1.5%	0.6%	0.0%
Inflation	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
Revenue growth (g)	10.2%	8.2%	6.1%	4.5%	3.6%	3.0%
Operating exp/revenues (p)	90.0%	90.0%	90.0%	90.0%	90.0%	90.0%
Operations:	2001	2002	2003	2004	*2005*	2006 (perpetuity)
Real growth	0.0%	0.0%	0.0%	0.0%	*0.0%*	0.0%
Inflation	3.0%	3.0%	3.0%	3.0%	*3.0%*	3.0%
Revenue growth (g)	3.0%	3.0%	3.0%	3.0%	*3.0%*	3.0%
Operating exp/revenues (p)	90.0%	90.0%	90.0%	90.0%	*90.0%*	90.0%

Table 19 - Forecasted parameter values, operations

The growth was exceptionally high in 1994, due to the very large depreciation of the Swedish currency in 1993 and 1994. This effect may fade in the coming years and we have set a zero real growth from the year 2000. The official inflation goal, set by the Swedish central bank, is 2 ±1%, and since the upper limit tends to be the rule rather than the exception in these matters, our forecast is 3% throughout.[74] The *operating expenditures to revenues* ratio has revolved around 90% in the historical period, hence the 90%-figure also in the forecast period.

[74] Revenue growth (g) = ((1 + real growth) × (1 + inflation)) - 1

5.3.2 Working capital

The different working capital figures are set so as to achieve a (parametric) steady state from 2005 and onwards. Other than that we make no attempt to justify the figures in Table 20, except that they are reasonable from a historical perspective.

Working cap./revenues:	1995	1996	1997	1998	1999	2000
+Operating cash/rev's	6.0%	5.9%	5.7%	5.6%	5.5%	5.4%
+Trade receivables/rev's	15.5%	15.4%	15.3%	15.3%	15.2%	15.2%
+Other receivables/rev's	1.0%	1.2%	1.3%	1.4%	1.5%	1.6%
+Inventories/rev's	21.5%	22.0%	22.5%	23.0%	23.5%	24.0%
+Prepaid expenses/rev's	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%
-Accounts payable/rev's	8.4%	8.4%	8.4%	8.4%	8.4%	8.4%
-Other curr liab's/rev's	2.3%	2.3%	2.4%	2.4%	2.4%	2.4%
-Accrued expenses/rev's	5.4%	5.6%	5.8%	5.9%	6.0%	6.1%
-Taxes payable/rev's	2.5%	2.4%	2.3%	2.3%	2.2%	2.2%
Net working cap./revenues (a)	26.4%	26.8%	26.9%	27.3%	27.7%	28.1%
Working cap./revenues:	2001	2002	2003	2004	*2005*	2006 (perpetuity)
+Operating cash/rev's	5.3%	5.2%	5.1%	5.0%	*5.0%*	5.0%
+Trade receivables/rev's	15.1%	15.1%	15.0%	15.0%	*15.0%*	15.0%
+Other receivables/rev's	1.7%	1.8%	1.9%	2.0%	*2.0%*	2.0%
+Inventories/rev's	24.5%	24.7%	24.9%	25.0%	*25.0%*	25.0%
+Prepaid expenses/rev's	1.0%	1.0%	1.0%	1.0%	*1.0%*	1.0%
-Accounts payable/rev's	8.4%	8.4%	8.4%	8.4%	*8.4%*	8.4%
-Other curr liab's/rev's	2.5%	2.5%	2.5%	2.5%	*2.5%*	2.5%
-Accrued expenses/rev's	6.2%	6.3%	6.4%	6.5%	*6.5%*	6.5%
-Taxes payable/rev's	2.2%	2.1%	2.0%	2.0%	*2.0%*	2.0%
Net working cap./revenues (a)	28.3%	28.5%	28.6%	28.6%	*28.6%*	28.6%

Table 20 - Forecasted parameter values, working capital

5.3.3 *Taxes and debt ratio*

The forecasts look as follows:

Taxes and debt:	1995	1996	1997	1998	1999	2000
Tax rate (tau)	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Effective Tax rate						
Incr in def. taxes/gross PPE (c)	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%
Borrowing rate (i)	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
Debt/net total assets (w)	41.8%	41.4%	41.2%	41.0%	40.8%	40.6%
Short-term/net total assets	10.6%	10.5%	10.5%	10.4%	10.4%	10.3%
Long-term/net total assets	17.4%	17.2%	17.1%	17.0%	17.0%	16.9%
Used check-cr./net total assets	6.2%	5.9%	5.6%	5.4%	5.1%	4.9%
Pension funds/net total assets	7.6%	7.8%	8.0%	8.2%	8.3%	8.5%

Taxes and debt:	2001	2002	2003	2004	*2005*	2006 (perpetuity)
Tax rate (tau)	30.0%	30.0%	30.0%	30.0%	*30.0%*	30.0%
Effective Tax rate					*27.5%*	27.5%
Incr in def. taxes/gross PPE (c)	0.3%	0.3%	0.3%	0.3%	*0.318%*	0.318%
Borrowing rate (i)	11.0%	11.0%	11.0%	11.0%	*11.0%*	11.0%
Debt/net total assets (w)	40.4%	40.2%	40.1%	40.0%	*40.0%*	40.0%
Short-term/net total assets	10.3%	10.3%	10.2%	10.2%	*10.2%*	10.2%
Long-term/net total assets	16.9%	16.8%	16.8%	16.7%	*16.7%*	16.7%
Used check-cr./net total assets	4.6%	4.4%	4.3%	4.2%	*4.2%*	4.2%
Pension funds/net total assets	8.6%	8.7%	8.8%	8.9%	*8.9%*	8.9%

Table 21 - Forecasted parameter values, taxes and debt

Corporate tax rate, effective tax rate and increase in deferred taxes

The corporate tax rate is assumed to be 30%. The increase in deferred taxes is not easy to predict by itself, but in section 4.3 it was shown how the parameter *c*, the increase in deferred taxes, should be specified in steady state, once the *effective* tax rate had been set - and the effective tax rate is much easier to have an opinion about. In this example we have set it at 27.5% (in the steady state period from 2005 and onwards) which yields an increase in deferred taxes of 0.318% when applying expression (70). It should be stressed once again that this modelling approach is insensitive to the beliefs about the nominal corporate tax rate. Should one believe that the government will tax at 28% instead, the value of *c* will be adjusted accordingly.

Book value debt ratio (w)

The *debt / net assets* ratio, i.e. the parameter w, is set at 40% in the steady state period, assuming that this is the optimal capital structure as the management sees it. It should be noted that this is the book debt ratio and *not* the market debt ratio. The ratio is also decomposed in four different debt item ratios (which are later used for calculating the different debt items of the balance sheet).

5.3.4 Property, plant and equipment (PPE)

In accordance with the findings in Chapter 4, the specification of PPE will be best made using *Specification B*, i.e.:

Specification B:

Items directly determined by ratios:

$$CapX_t = e_t \cdot R_t$$
$$DepX_t = d_t \cdot G_{t-1}$$
$$Ret_t = r_t \cdot G_{t-1}$$

Items derived indirectly:

$$G_t = G_{t-1} + CapX_t - Ret_t$$
$$A_t = A_{t-1} + DepX_t - Ret_t$$
$$N_t = G_t - A_t$$

$CapX$ = Capital expenditures
$DepX$ = Depreciation expense
R = Revenues
Ret = Retirements
G = Gross property, plant and equipment
A = Accumulated depreciation
N = Net property, plant and equipment
e = Capital expenditures as a percentage of revenues
d = Depreciation exp. as a percentage of preceding year's gross PPE
r = Retirements as a percentage of preceding year's gross PPE
t = time-index denoting year

The forecasted parameter values are presented in Table 22. Note that the parameter b (gross PPE as a percentage of revenues) is not meaningful when working with Specification B in the explicit forecast period (until 2004). In the steady state period (from 2005), there are values of b presented in Table 22. The rationale for this will be discussed below.

Property,Plant and Equipment (PPE):	1995	1996	1997	1998	1999	2000
CapX/rev's (e)	2.9%	2.9%	3.0%	3.0%	3.0%	3.0%
Gross PPE/rev's (b)	n/m	n/m	n/m	n/m	n/m	n/m
DepX/prec. year's gross PPE (d)	6.5%	6.5%	6.5%	6.5%	6.5%	6.5%
Ret/prec. year's gross PPE (r)	3.2%	3.4%	3.6%	3.7%	3.9%	4.1%
Property,Plant and Equipment (PPE)	**2001**	**2002**	**2003**	**2004**	***2005***	**2006 (perpetuity)**
CapX/rev's (e)	3.1%	3.1%	3.1%	3.2%	*3.195%*	3.195%
Gross PPE/rev's (b)	n/m	n/m	n/m	n/m	*41.162%*	41.162%
DepX/prec. year's gross PPE (d)	6.5%	6.5%	6.5%	6.5%	*6.5%*	6.5%
Ret/prec. year's gross PPE (r)	4.3%	4.5%	4.6%	4.8%	*4.995%*	4.995%

Table 22 - Forecasted parameter values, PPE

Depreciation / preceding year's gross PPE (d)

The parameter d generally tends to lie on a company specific level as was indicated in section 4.1.4. In Eldon it has historically been around 6.5%. There is no reason to believe that this will change in the future.

Capital expenditures / revenues (e) **and** ***retirements / preceding year's gross PPE (r)***

The two remaining PPE-related parameters e and r are important for the fulfilment of the initial conditions necessary to ensure TSS:

(22) $$gA_0 = (d-r)bR_0$$

(49) $$e = \frac{G_0(g+r)}{(1+g)R_0}$$

Condition (22) is derived using Specification A. However, it was shown in section 2.2 that if expression (49) holds, Specification B reduces to Specification A, where the ratio *gross PPE / revenues* (b) is constant. This means that in the perpetuity period, i.e. from 2005 and onwards, the only alternative is to use Specification A in order to be able to use a continuing value. One can of course work with the *terminology* of Specification B as long as e is determined by expression (49). The "translation" between Specification A and Specification B will then be:

(71) $$b = \beta = \frac{e(1+g)}{(g+r)}$$

and condition (22) can, using Specification B terminology, be stated as:

(72) $$gA_0 = (d-r)e\frac{(1+g)}{(g+r)}R_0$$

Expression (72) can be rearranged as:

(73) $$r = \frac{de(1+g)R_0 - g^2 A_0}{e(1+g)R_0 + gA_0}$$

As seen in Chapter 4, the historical parameter values of e and r give little guidance in forecasting, hence other procedures have to be employed. As before, the forecast will be divided into two parts: the explicit forecast period and the perpetuity period.

We will make a direct forecast for 1995, the first year in the explicit forecast period. For 2005 the e- and r-values will be given by the initial value conditions (49) and (73) to guarantee textbook steady state in the perpetuity period. The parameter values between 1995 and 2005 are then determined by a linear interpolation.

A problem is that in order to determine the e- and r-values in 2005, we need A_0 and G_0. This is obvious from the initial conditions (49) and (73). A_0 and G_0 will be given from the spreadsheet model only after all parameters up to 2005 have been specified, including the e- and r-values. Thus, a more complex and partly iterative procedure is needed:

1. Make direct forecasts of the e- and r-values in 1995.
2. Set trial values for e and r in 2005.
3. Model values for the years in-between by way of linear interpolation.
4. This yields complete financial statements for the years in question including values in 2005 for accumulated depreciation and gross PPE, i.e. A_0 and G_0. The initial value conditions (49) and (73) will generally not hold for these values of A_0 and G_0.
5. Calculate r in 2005 according to condition (73). Since the 2005 value of r is changed, the linearly interpolated r-values of earlier years will also be changed. This will give new financial statements and new, updated values of A_0 and G_0.
6. Use the updated G_0 to calculate e in 2005 according to condition (49). Since the e-value of 2005 is changed, the linearly interpolated e-values of earlier years will also be changed. This will give new financial statements and new, updated values of A_0 and G_0.
7. Repeat procedures 5 and 6 until there is convergence.

The direct forecasts of e and r for 1995 are apparent from Table 22. The first trial values for 2005 were e=2.9% and r=5.2%.[75] Using these figures the program converged very quickly.

[75] The first trial value of r in 2005 is given by expression (67) in Proposition 4.1. This formula is valid under the assumption that the economic life equals the depreciation period, and although these ideal conditions do not quite hold here, the r-value arrived at with this method should be a reasonable first guess. The first trial value of e in 2005 is simply set equal to the 1995 value.

5.3.5 Checks for the steady state period

Having specified all parameter values, their reasonableness should be checked. By inspecting the resulting balance sheets and income statements, it can be verified that the development of the company in the explicit forecast period is reasonable (Tables 23, 24 and 25, below). For the steady state period, the checks derived in section 2.1 can be used, since we are back in Specification A.[76] These checks were the following:

- *A higher gross PPE / revenues ratio should lead to lower FCF and vice versa,*
- *FCF decreasing in the tax rate,*
- *Net PPE non-decreasing over time,*
- *Positive pre-tax profits,*
- *Dividends decreasing in the gross PPE / revenues ratio,*
- *Non-negative book equity.*

1. A higher gross PPE / revenues ratio should lead to lower FCF and vice versa:

The condition to check is the following:

(8) $\tau d - r + (1+g)c < g$

Inserting the predicted values: $0.3 \cdot 0.065 - 0.05 + (1.03) \cdot 0.003 \approx -2.7\% < g = 3\%$

Obviously, the condition is fulfilled by a wide margin.

2. FCF decreasing in the tax rate:

The condition for this property is the following:

(9) $p + \dfrac{bd}{1+g} < 1$

Inserting the parameter values yields:

[76] Expression (71) should be applied whenever the parameter b is appearing: $b = \beta = e(1+g)/(g+r)$.

$$0.90+\frac{0.416\cdot 0.065}{1.03}\approx 92.6\%<1$$

Also this condition holds without any problems.

3. Net PPE non-decreasing over time:

This condition, which is sufficient for NP to be decreasing in the ratio *gross PPE / revenues*, is:

(11) $(d-r)\leq g$

Inserting the parameter values:

(0.065-0.05) < 0.03

and consequently, the condition is fulfilled.

4. Positive pre-tax profits:

From the forecasted income statement of year 2006 one can see directly that this holds.

5. Dividends decreasing in the gross PPE / revenues ratio:

The parameter condition to check here is:

(13) $$d\tau+wg+c(1+g)+\frac{\chi(d-r)}{g}-w(d-r)-r-\chi<g$$

Inserting the predicted parameter values for 2005 and onwards yields a LHS value of -3.7%, which is clearly less than +0.03.

6. Non-negative book equity:

The following two boundary conditions must be fulfilled:

$$(14) \qquad (1-w)(a+b) > \left(\frac{b}{g} - \frac{b}{g(1+g)}\right)\left[(d-r)(1-w)+c(1+g)\right] + \frac{(1-w)A_0 + T_0}{R_0(1+g)}$$

$$(15) \qquad (1-w)(a+b) > \frac{b}{g}\left[(d-r)(1-w)+c(1+g)\right]$$

The left hand side takes on the value 41.9%. The right hand side (RHS) of condition (14) is equal to 16.2% and for condition (15) the RHS equals 16.9%, so the conditions hold.

5.4 The forecasted financial statements

The forecasted financial statements, finally, are presented below. Most items follow directly from the forecast assumptions (i.e. the parameter values) described in the previous sections. Some additional comments will be made, however.

5.4.1 Income statements

The setting up of income statements is straightforward:

FORECASTED INCOME STATEMENT:	1995	1996	1997	1998	1999	2000
Revenues	1,833.8	1,983.2	2,104.0	2,199.6	2,279.2	2,347.6
Operating expenses	-1,650.4	-1,784.9	-1,893.6	-1,979.7	-2,051.3	-2,112.8
Depreciation expense	-49.4	-51.3	-53.3	-55.5	-57.7	-59.9
Operating income	134.0	147.1	157.1	164.5	170.3	174.9
Net financial income	-40.1	-42.4	-44.5	-46.2	-48.0	-49.7
Extraordinary items	0.0	0.0	0.0	0.0	0.0	0.0
Earnings before taxes	93.9	104.6	112.6	118.3	122.2	125.2
Taxes	-28.2	-31.4	-33.8	-35.5	-36.7	-37.6
Net profit	**65.8**	**73.2**	**78.8**	**82.8**	**85.6**	**87.6**

FORECASTED INCOME STATEMENT:	2001	2002	2003	2004	*2005*	2006
Revenues	2,418.0	2,490.6	2,565.3	2,642.2	*2,721.5*	2,803.2
Operating expenses	-2,176.2	-2,241.5	-2,308.8	-2,378.0	*-2,449.4*	-2,522.8
Depreciation expense	-62.1	-64.2	-66.4	-68.6	*-70.7*	-72.8
Operating income	179.7	184.8	190.1	195.7	*201.5*	207.5
Net financial income	-51.2	-52.6	-54.1	-55.6	*-57.1*	-58.8
Extraordinary items	0.0	0.0	0.0	0.0	*0.0*	0.0
Earnings before taxes	128.5	132.2	136.0	140.1	*144.4*	148.7
Taxes	-38.6	-39.7	-40.8	-42.0	*-43.3*	-44.6
Net profit	**90.0**	**92.5**	**95.2**	**98.0**	***101.0***	**104.1**

Table 23 - Forecasted income statements

5.4.2 Balance sheets

Now consider the balance sheets:

FORECASTED BALANCE SHEET:						
	1995	1996	1997	1998	1999	2000
ASSET SIDE						
Operating cash	110.0	117.0	119.9	123.2	125.4	126.8
Trade receivables	284.2	305.4	321.9	336.5	346.4	356.8
Prepaid expenses	18.3	19.8	21.0	22.0	22.8	23.5
Other receivables	18.3	23.8	27.4	30.8	34.2	37.6
Inventories	394.3	436.3	473.4	505.9	535.6	563.4
Current assets	825.2	902.4	963.6	1,018.4	1,064.4	1,108.1
Excess marketable securities	0.0	0.0	0.0	0.0	0.0	0.0
Investment fund	0.0	0.0	0.0	0.0	0.0	0.0
Gross property,plant and equip	788.6	820.0	853.1	886.9	921.0	954.8
Accumulated depreciation	-350.0	-374.6	-398.7	-422.3	-445.2	-467.3
Net property, plant and equipment	438.6	445.4	454.4	464.7	475.8	487.5
Total assets	**1,263.8**	**1,347.8**	**1,418.0**	**1,483.1**	**1,540.2**	**1,595.6**
Net Total Assets*	922.7	976.9	1,020.4	1,065.2	1,107.2	1,147.2
* Total assets - Working capital liabilities						
FORECASTED BALANCE SHEET:						
	2001	2002	2003	2004	*2005*	2006
ASSET SIDE						
Operating cash	128.2	129.5	130.8	132.1	*136.1*	140.2
Trade receivables	365.1	376.1	384.8	396.3	*408.2*	420.5
Prepaid expenses	24.2	24.9	25.7	26.4	*27.2*	28.0
Other receivables	41.1	44.8	48.7	52.8	*54.4*	56.1
Inventories	592.4	615.2	638.8	660.6	*680.4*	700.8
Current assets	1,151.0	1,190.5	1,228.8	1,268.3	*1,306.3*	1,345.5
Excess marketable securities	0.0	0.0	0.0	0.0	*0.0*	0.0
Investment fund	0.0	0.0	0.0	0.0	*0.0*	0.0
Gross property,plant and equip	988.3	1021.7	1054.8	1087.6	*1120.2*	1153.8
Accumulated depreciation	-488.5	-508.7	-527.7	-545.5	*-561.9*	-578.7
Net property, plant and equipment	499.9	513.0	527.0	542.1	*558.4*	575.1
Total assets	**1,650.8**	**1,703.5**	**1,755.8**	**1,810.4**	***1,864.7***	**1,920.6**
Net Total Assets*	1,184.2	1,222.8	1,260.7	1,297.8	*1,336.7*	1,376.8
* Total assets - Working capital liabilities						

Table 24 - Forecasted balance sheets, asset side

	1995	1996	1997	1998	1999	2000
DEBT & EQUITY SIDE						
Short-term debt	97.8	102.6	107.1	110.8	115.1	118.2
Accounts payable	154.0	166.6	176.7	184.8	191.5	197.2
Accrued expenses	99.0	111.1	122.0	129.8	136.8	143.2
Taxes Payable	45.8	47.6	48.4	50.6	50.1	51.6
Other current liabilities	42.2	45.6	50.5	52.8	54.7	56.3
Total current liabilities	438.9	473.4	504.8	528.7	548.2	566.6
Long-term debt	160.6	168.0	174.5	181.1	188.2	193.9
Used check-credit	57.2	57.6	57.1	57.5	56.5	56.2
Pension funds	70.1	76.2	81.6	87.3	91.9	97.5
Deferred taxes	72.9	75.3	77.9	80.5	83.3	86.2
Total longterm liabilities	360.8	377.2	391.1	406.5	419.9	433.8
Common stock	51.9	51.9	51.9	51.9	51.9	51.9
Other equity	412.3	445.3	470.2	496.0	520.2	543.4
Total common equity	464.2	497.2	522.1	547.9	572.1	595.3
Total liabilities and equity	**1,263.8**	**1,347.8**	**1,418.0**	**1,483.1**	**1,540.2**	**1,595.6**

	2001	2002	2003	2004	*2005*	2006
DEBT & EQUITY SIDE						
Short-term debt	122.0	125.9	128.6	132.4	*136.3*	140.4
Accounts payable	203.1	209.2	215.5	221.9	*228.6*	235.5
Accrued expenses	149.9	156.9	164.2	171.7	*176.9*	182.2
Taxes Payable	53.2	52.3	51.3	52.8	*54.4*	56.1
Other current liabilities	60.5	62.3	64.1	66.1	*68.0*	70.1
Total current liabilities	588.6	606.6	623.7	645.0	*664.3*	684.3
Long-term debt	200.1	205.4	211.8	216.7	*223.2*	229.9
Used check-credit	54.5	53.8	54.2	54.5	*56.1*	57.8
Pension funds	101.8	106.4	110.9	115.5	*119.0*	122.5
Deferred taxes	89.1	92.2	95.4	98.6	*102.2*	105.9
Total longterm liabilities	445.6	457.8	472.3	485.4	*500.5*	516.2
Common stock	51.9	51.9	51.9	51.9	*51.9*	51.9
Other equity	564.7	587.1	607.9	628.1	*647.9*	668.3
Total common equity	616.6	639.0	659.8	680.0	*699.8*	720.2
Total liabilities and equity	**1,650.8**	**1,703.5**	**1,755.8**	**1,810.4**	***1,864.7***	**1,920.6**

Table 25 - Forecasted balance sheets, debt and equity side

Excess marketable securities

Excess marketable securities are assumed to be zero. They were very low during the last historical year (1994). The cash required to run the operations is modelled as operating cash. Any excess capital generated by the company will be distributed to the shareholders as dividends (same assumption as in earlier chapters) unless it is needed for investments or for maintaining the desired capital structure.

Equity

No stock issues are foreseen. Hence the *common stock* remains unchanged. The item *other equity* corresponds to retained earnings plus other non-restricted and restricted reserves. *Other equity* is the residual item of the balance sheet (i.e. decided so as to make the asset side equal to the debt and equity side).

5.4.3 Statements of equity

The statement of equity is used to forecast the dividends.

STATEMENT OF EQUITY:						
	1995	1996	1997	1998	1999	2000
COMMON STOCK:						
Beginning common stock	51.9	51.9	51.9	51.9	51.9	51.9
Issues/Repurchases	0.0	0.0	0.0	0.0	0.0	0.0
Ending common stock	51.9	51.9	51.9	51.9	51.9	51.9
OTHER EQUITY:						
Beginning other equity	376.3	412.3	445.3	470.2	496.0	520.2
Net profit	65.8	73.2	78.8	82.8	85.6	87.6
Common dividends	-29.8	-40.2	-53.9	-57.0	-61.3	-64.5
Ending other equity	412.3	445.3	470.2	496.0	520.2	543.4

STATEMENT OF EQUITY:						
	2001	2002	2003	2004	2005	2006
COMMON STOCK:						
Beginning common stock	51.9	51.9	51.9	51.9	51.9	51.9
Issues/Repurchases	0.0	0.0	0.0	0.0	0.0	0.0
Ending common stock	51.9	51.9	51.9	51.9	51.9	51.9
OTHER EQUITY:						
Beginning other equity	543.4	564.7	587.1	607.9	628.1	647.9
Net profit	90.0	92.5	95.2	98.0	101.0	104.1
Common dividends	-68.6	-70.1	-74.5	-77.8	-81.3	-83.7
Ending other equity	564.7	587.1	607.9	628.1	647.9	668.3

Table 26 - Forecasted statements of equity

Beginning other equity is the same as *ending other equity* from the preceding year. *Net profit* comes from the income statement. *Ending other equity* is taken directly from the balance sheet. To balance the entire system, this leaves us with *common dividends* as the residual item. This follows immediately from the clean surplus relationship (the change in net book value of equity equals net profit minus dividends).

The equity/dividend forecasting may seem technically complicated, but conceptually it can be summarised in a very simple rule, namely that the forecasted excess capital not required for internal company use (after rebalancing the debt) is distributed to the share-holders as dividends.

We have not touched on the subject of restricted vs. non-restricted reserves. The subject is somewhat peripheral in this context and is addressed in Appendix 5.

5.5 Free cash flow

All items in the free cash flow calculations can be derived directly from the earlier tables:

FORECASTED FREE CASH FLOW:	1995	1996	1997	1998	1999	2000
Revenues	1833.8	1983.2	2104.0	2199.6	2279.2	2347.6
Operating expenses	-1650.4	-1784.9	-1893.6	-1979.7	-2051.3	-2112.8
Depreciation expense	-49.4	-51.3	-53.3	-55.5	-57.7	-59.9
Adjusted EBIT	134.0	147.1	157.1	164.5	170.3	174.9
Taxes on EBIT	-40.2	-44.1	-47.1	-49.4	-51.1	-52.5
Change in deferred taxes	2.4	2.5	2.6	2.7	2.8	2.9
NOPLAT	96.2	105.4	112.5	117.8	122.0	125.3
Depreciation expense	49.4	51.3	53.3	55.5	57.7	59.9
Gross cash flow	145.5	156.7	165.8	173.3	179.6	185.2
Change in working capital	56.1	47.4	34.5	34.5	30.8	28.3
Capital expenditures	53.2	58.1	62.3	65.7	68.8	71.5
Gross investment (-)	109.3	105.5	96.7	100.3	99.6	99.9
Operating FCF	**36.2**	**51.2**	**69.1**	**73.0**	**80.0**	**85.3**
Non-operating cash flow	0.0	0.0	0.0	0.0	0.0	0.0
Total FCF	**36.2**	**51.2**	**69.1**	**73.0**	**80.0**	**85.3**

Table 27a - Forecasted statements of free cash flow 1995 - 2000

FORECASTED FREE CASH FLOW:	2001	2002	2003	2004	2005	2006
Revenues	2418.0	2490.6	2565.3	2642.2	2721.5	2803.2
Operating expenses	-2176.2	-2241.5	-2308.8	-2378.0	-2449.4	-2522.8
Depreciation expense	-62.1	-64.2	-66.4	-68.6	-70.7	-72.8
Adjusted EBIT	179.7	184.8	190.1	195.7	201.5	207.5
Taxes on EBIT	-53.9	-55.4	-57.0	-58.7	-60.4	-62.3
Change in deferred taxes	3.0	3.1	3.2	3.3	3.6	3.7
NOPLAT	128.8	132.4	136.2	140.2	144.6	148.9
Depreciation expense	62.1	64.2	66.4	68.6	70.7	72.8
Gross cash flow	190.8	196.7	202.7	208.8	215.3	221.7
Change in working capital	24.6	25.5	23.9	22.0	22.7	23.4
Capital expenditures	74.4	77.4	80.5	83.6	87.0	89.6
Gross investment (-)	99.0	102.9	104.3	105.7	109.6	112.9
Operating FCF	**91.8**	**93.8**	**98.3**	**103.1**	**105.7**	**108.8**
Non-operating cash flow	0.0	0.0	0.0	0.0	0.0	0.0
Total FCF	**91.8**	**93.8**	**98.3**	**103.1**	**105.7**	**108.8**

Table 27b - Forecasted statements of free cash flow 2001 - 2006

The free cash flow should correspond to the financial cash flow:

FORECASTED FINANCIAL CASH FLOW:	1995	1996	1997	1998	1999	2000
Incr(+)/Decr(-) exc. mkt sec'ties	0.0	0.0	0.0	0.0	0.0	0.0
Incr(-)/Decr(+) debt, check-cr.	-15.0	-12.7	-10.5	-10.6	-10.4	-8.4
Incr(-)/Decr(+) in pension funds	-6.6	-6.1	-5.4	-5.7	-4.5	-5.6
After-tax net interest exp. (+)	28.0	29.7	31.1	32.4	33.6	34.8
Common dividends (+)	29.8	40.2	53.9	57.0	61.3	64.5
Incr(-)/Decr(+) in common stock	0.0	0.0	0.0	0.0	0.0	0.0
Financial cash flow	**36.2**	**51.2**	**69.1**	**73.0**	**80.0**	**85.3**
FORECASTED FINANCIAL CASH FLOW:	2001	2002	2003	2004	2005	2006
Incr(+)/Decr(-) exc. mkt sec'ties	0.0	0.0	0.0	0.0	0.0	0.0
Incr(-)/Decr(+) debt, check-cr.	-8.3	-8.6	-9.4	-9.0	-12.1	-12.5
Incr(-)/Decr(+) in pension funds	-4.3	-4.5	-4.6	-4.6	-3.5	-3.6
After-tax net interest exp. (+)	35.9	36.8	37.9	38.9	40.0	41.2
Common dividends (+)	68.6	70.1	74.5	77.8	81.3	83.7
Incr(-)/Decr(+) in common stock	0.0	0.0	0.0	0.0	0.0	0.0
Financial cash flow	**91.8**	**93.8**	**98.3**	**103.1**	**105.7**	**108.8**

Table 28 - Forecasted statements of financial cash flow

5.6 Value calculation

We now have access to forecasts of free cash flows and dividends and can hence proceed to the actual equity value calculation. Three different ways have been discussed earlier in this report:

1. Discounting the free cash flows at a constant WACC. Calculating the equity value as the sum of the discounted free cash flows minus the debt value.

2. Discounting the free cash flows at a year-to-year updated WACC. Calculating the equity value as the sum of the discounted free cash flows minus the debt value.

3. Discounting the dividends at the equity cost of capital. Calculating the equity value as the sum of the discounted dividends.

As discussed earlier, methods 2 and 3 yield the same value. For completeness we will present all three valuations.

Cost of equity capital

First, the cost of equity capital must be estimated. This is necessary for all three valuation methods. As suggested by Copeland et al, the capital asset pricing model[77] is used for determining the cost of equity capital, k_E:

(74) $$k_E = r_f + \beta\left[\mathrm{E}(r_m) - r_f\right]$$

where: r_f is the riskfree rate
β is the beta value
$E(r_m) - r_f$ is the market risk premium

[77] The capital asset pricing model (CAPM) by Sharpe (1964) has been very popular as a way of determining the cost of capital. The discount rate is determined by adding a risk premium to the risk-free interest rate. The risk premium is calculated by multiplying the asset's sensitivity to general market movements (its *beta*) by the market risk premium. A useful practical article on the subject is Dimson & Marsh (1982). However, the empirical validity of CAPM is a matter of debate. Proposed alternatives to the single-factor CAPM include different multi-factor approaches based on Ross' arbitrage pricing model (Ross (1977)). Fama & French (1993) identifies five common risk factors in returns on stocks and bonds that can be used for estimating the cost of capital.

For Eldon, the beta value has been estimated to be 1.08,[78] the market risk premium has been estimated to be 5.7% in Sweden,[79] and consequently the following cost of equity capital is estimated for Eldon:

$$k_E = r_f + \beta\left[\mathrm{E}(r_m) - r_f\right] = 7.0\% + 1.08 \cdot 5.7\% = 13.156\%$$

As was mentioned in Chapter 3, a constant cost of equity capital may not be theoretically viable if the capital structure is not constant (referred to as "type 2 approximation error"). In the Eldon case the capital structure is rather stable, as can be seen from Figure 9, and hence there are no substantial indications that the risk level, and with it k_E, should change.[80]

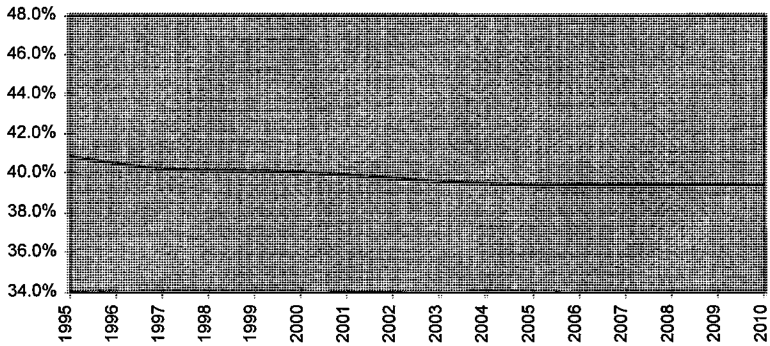

Figure 9 - Forecasted market debt ratio in Eldon

Thus, we in this example use a constant k_E. Note, however, that it is perfectly possible to use a time-varying cost of equity capital whenever the analyst may find this reasonable (if using either method 2 or method 3).

5.6.1 FCF valuation with constant WACC

This is the model proposed in Copeland et al. As has been shown in Chapter 3, it will suffer from approximation errors if the market debt ratio varies over time.

[78] Öhmans Börsguide 1995.
[79] Nyman & Smith (1994).
[80] The market value debt ratio turns out to be very close to the book value debt ratio in the steady state period. As was seen in Chapter 2, this is not always the case.

The state of the company at 1 January 1995, is the basis for the calculation of the WACC. The formula looks as follows:

$$(75) \qquad k_{\text{WACC}} = \frac{D}{D+EV} \cdot (1-\tau)i + \left(1 - \frac{D}{D+EV}\right) \cdot k_E$$

D is the market debt value (Jan 1, 1995), *EV* is the market equity value (Jan 1, 1995). The market interest rate on debt, *i*, is predicted to be 11%. The outstanding debt is assumed to be on market terms.[81] Since k_{WACC} depends on the market value of equity, which in turn depends on k_{WACC}, the value calculation and the calculation of k_{WACC} have to be made simultaneously. This requires a recursive procedure:

a) Use a trial input WACC.
b) Calculate the equity value.
c) Calculate the WACC implied by the equity value from b) by using formula (75). If this resulting WACC value differs from a), the trial input WACC in a) has to be adjusted. The procedure has to be redone until the resulting WACC value (c) equals the trial input value (a). (The procedure can be automated in a modern spreadsheet program.)

The final value calculation from the spreadsheet is presented in Table 29:

Value Calculation:						
FCF with constant WACC						
Trial Input WACC	10.943%					
Resulting WACC	10.943%					
	1995	1996	1997	1998	1999	2000
Forecasted Free Cash Flow	36.2	51.2	69.1	73.0	80.0	85.3
Present Value as of Jan. 1, 1995	32.7	41.6	50.6	48.2	47.6	45.7
						Perpetuity
	2001	2002	2003	2004	2005	2006
Forecasted Free Cash Flow	91.8	93.8	98.3	103.1	105.7	108.8
Present Value as of Jan. 1, 1995	44.4	40.9	38.6	36.5	33.7	437.2
Present Value of Forecasted FCF	897.6					
Excess Marketable Securities	0.9					
-Debt Value	-364.1					
Final Equity Value	534.4					

Table 29 - Value calculation 1 using a constant WACC

[81] That means that the market interest rate on debt equals the borrowing rate.

The value of the perpetuity period is calculated using the Gordon formula. The market value capital structure is non-constant in the explicit forecast period (evident from Figure 9) and hence using a constant WACC will yield a small approximation error of type 1.

5.6.2 FCF valuation with updated WACC

As argued in Chapter 3, this is in fact the correct discounting method to use when discounting free cash flows since it explicitly incorporates possible variations in the capital structure. The procedure was described in detail in section 2.1.5. The market interest rate on debt, i, is predicted to be 11%. The outstanding debt is assumed to be on market terms:

Value Calculation: FCF with updated WACC						
	1995	1996	1997	1998	1999	2000
Forecasted Free Cash Flow	36.2	51.2	69.1	73.0	80.0	85.3
Present Value of Forecasted FCF at beginning of each year	892.1	953.4	1006.6	1047.8	1089.8	1129.3
WACC	10.929%	10.949%	10.964%	10.967%	10.969%	10.974%
						Perpetuity
	2001	2002	2003	2004	*2005*	2006
Forecasted Free Cash Flow	91.8	93.8	98.3	103.1	*105.7*	108.8
Present Value of Forecasted FCF at beginning of each year	1168.0	1204.4	1243.0	1281.3	*1319.2*	1358.7
WACC	10.980%	10.989%	10.998%	11.003%	*11.009%*	11.009%
Present Value of Forecasted FCF	892.1					
Excess Marketable Securities	0.9					
-Debt Value	-364.1					
Final Equity Value	528.9					

Table 30 - Value calculation 2 using updated WACC

The equity value arrived at by using the updated WACC approach does not differ very much from the value given by the constant WACC approach (529 million instead of 534 million). Thus the approximation error introduced by wrongly assuming a constant capital structure is only 1%. The size of the error will depend on how much the capital structure (debt vs. value of future operations) changes over the forecast period. Eldon is an exceptionally stable company, but for many other companies the magnitude of the approximation error would be much larger.

5.6.3 Dividend valuation

Now we can calculate the equity value by discounting the estimated future dividends at the cost of equity capital. As was argued in Chapter 3, this should yield exactly the same value as the updated WACC approach - a proposition that can now be checked:

Value Calculation: Dividend Valuation						
	1995	1996	1997	1998	1999	2000
Forecasted Dividends	29.8	40.2	53.9	57.0	61.3	64.5
Present Value as of Jan. 1, 1995	26.3	31.4	37.2	34.7	33.1	30.7
						Perpetuity
	2001	2002	2003	2004	*2005*	2006
Forecasted Dividends	68.6	70.1	74.5	77.8	*81.3*	83.7
Present Value as of Jan. 1, 1995	28.9	26.1	24.5	22.6	*20.9*	211.6
Excess Marketable Securities	0.9					
Final Equity Value	528.9					

Table 31 - Value calculation 3 using dividends

As can be seen the dividend approach yields the same value as the FCF approach using updated WACC.

5.7 Concluding remarks on Eldon

The base-case scenario, with no real growth after year 2000, yields an equity value of SEK 529 million (using method 2 or 3). This is, as predicted, lower than the observed market value of SEK 722 million. There are clearly growth opportunities, which we have not included here. Neither have we made any thorough analysis of future profit margins, which of course are crucial to any value calculation. The purpose of this chapter has instead been to show how some practical problems can be overcome, also how some of the results derived in earlier chapters can be incorporated into a "real-world" company valuation. We hope also to have conveyed a sense of how flexible these kinds of valuation approaches are. Some items have been modelled/forecasted very carefully whereas others have been treated more sketchily. This shows that one can basically be as explicit and thorough as one deems reasonable when setting up a valuation model of this kind.

Appendix 5 - Restricted reserves

Swedish companies generally have restricted reserves as part of their equity, as well as non-restricted reserves. We have not modelled this explicitly in the Eldon case. However, one has to remember that only non-restricted reserves can be used for paying dividends. Thus, it may be a good idea to check that the dividend forecasts are not violating this requirement, i.e. that dividends only are paid out of non-restricted equity. We suggest the following model:

1. Retrieve the item *other equity* from the balance sheet (i.e. book equity other than common stock).
2. Calculate the *restricted reserves.*
3. Calculate the *non-restricted equity* as the difference between *other equity* and *restricted reserves.*
4. Check that *non-restricted equity* is not negative. Since *non-restricted equity* is after dividends, a negative value would indicate unrealistically large dividends.

We will go through the procedure, and use the year 2005 to visualise:

1 Other equity

Other equity is 647.9 in 2005.

2. Restricted reserves

We propose the following model for calculating the *restricted reserves*:

$$\text{(A5:1)} \quad \text{Restricted reserves} = 0.20 \times \text{common stock} + \frac{1 - \text{tax rate}}{\text{tax rate}} \times \text{deferred taxes}$$

Firms are required by law to keep restricted reserves at least as high as 20% of the common stock[82] (unless losses make this impossible), hence the first term in the right-hand-side of expression (A5:1).

The second term has to do with untaxed reserves Although deferred-taxes accounting is used in the group accounting, the deferred taxes are *derived* from the untaxed reserves of the different companies that belong to the group. This is a complicated issue, and it is by no means clear how one should address it when valuing the consolidated group. Our suggestion here builds on the following facts:

[82] Swedish: "reservfond"

- Untaxed reserves can be divided into deferred taxes and equity.
- Parent company and subsidiaries have untaxed reserves (which cannot be used for dividends).
- The group is basically a sort of "sum" of the parent company and its subsidiaries.
- The group accounting is done using deferred taxes accounting.
- When adding the untaxed reserves of the different group companies, part of it will be counted as deferred taxes, part of it will be counted as equity.
- The equity part will in reality be part of the untaxed reserves of the different companies belonging to the group, hence it should by definition be labelled *restricted* equity since it cannot be used for paying dividends.

This "split-up" of the untaxed reserves is done according to the following formula:[83]

- To *deferred taxes*: tax rate × untaxed reserves
- To *restricted reserves*: (1 - tax rate) × untaxed reserves

Since the deferred taxes are known (modelled via the parameter c as described in section 5.3.4), it is possible to calculate also the "restricted reserves part" of the untaxed reserves, i e. the second term in (A5:1). In the year 2005:

Common stock	51.9
Deferred taxes	102.2

$$\textit{Restricted reserves} = 0.20 \times 51.9 + \frac{1-0.30}{0.30} \times 102.2 \approx 248\,8$$

This is obviously a rather heuristic way of estimating the restricted reserves.

3 Calculation of non-restricted equity

In year 2005:

Other equity	647.9
-Restricted reserves	-248.8
Non-restricted equity	399.1

[83] It can be noted (see Table 14) that the Eldon group changed from untaxed reserves to deferred taxes accounting in 1990-91, and the split-up of untaxed reserves seems to have been made according to this division formula. This was also the recommended accounting standard (see FAR (1991) pp. 286).

4. Non-restricted equity greater than zero

The non-restricted equity is positive in year 2005. Remember that non-restricted equity (as well as the other equity items) is calculated after dividends. Since the remaining non-restricted equity is positive, the forecasted dividends in 2005 have been fully paid out of non-restricted reserves. Had the *non-restricted equity* item been negative, the forecasted dividends would have been unrealistically high.

6. Concluding Remarks

We have in Chapter 5 shown how a McKinsey-style valuation of Eldon works in practice. We have, however, also made use of some of the improvements derived in earlier chapters of this report. The purpose has not been to show a "recommended" model, rather we have shown the consequences implied when using different versions of the valuation model.

Some more normative remarks may be warranted. Eldon turned out to be quite a stable company in all respects, and approximation errors are of fairly limited magnitude. In more unstable circumstances the problems with approximation errors would be much larger. Small or large - the point is that it is *unnecessary* to introduce approximation errors when there is absolutely nothing to be gained from the approximation in the first place:

In Eldon (and indeed in almost any real-world company), using the original Copeland et al model[84] would introduce approximation errors into the valuation. Both type 1 errors[85] and type 2 errors[86] would be present. Type 1 errors can be avoided by updating the weighted average cost of capital, thus taking into consideration the impact of the changing weights in the WACC formula, implied by the actual forecast. This is a rather cumbersome procedure. There exists a computationally simpler alternative that yields exactly the same result: to discount the forecasted dividends at the equity cost of capital. This in no way alters the approach (free cash flow valuation), it should merely be thought of as a calculational tool.

Type 2 errors may be present, even if the actual value calculation is done by discounting the dividends. The importance of such errors is a qualitative question. A useful diagnostic tool is a graph of the development of the capital structure over time (Figure 9 in Chapter 5). In the Eldon case, the capital structure remained fairly constant over time, and the conclusion was that any type 2 errors are small in magnitude.

Continuing value calculations are popular among practitioners because of their computational simplicity. What is typically done with revenue-driven models, such as the McKinsey model, is to assume a constant revenue growth after, say, ten years. Continuing value calculations do, however, require assumptions much stronger than "continuous revenue growth" in order to yield correct results - assumptions rarely fulfilled in practice, one would suspect. The McKinsey model requires a

[84] Discounting the free cash flows at a constant WACC.
[85] The calculational errors stemming from the use of a constant weighted average cost of capital although the weights are not constant over time.
[86] The errors stemming from ignoring that the risk of the company probably changes when the capital structure changes which should have effects on the cost of equity capital.

"Textbook steady state," which means that the company earns constant margins, grows at a constant rate, invest a constant proportion of its gross cash flow each year, and that it earns a constant return on existing capital as well as on all new investments. The detailed analysis in Chapter 2 showed that this boils down to a one or two (depending on specification) initial value conditions: For Specification A it is concluded that a "true" or textbook steady state (TSS) is implied by the general assumption that the model's parameters are constants (PSS) and the initial parameter condition (22) that ensures a constant capital structure. For Specification B, the only possibility to establish a steady state is where the parameters fulfil condition (49) that reduces the more general Specification B setting to the more specific Specification A.

Trying to fulfil these conditions will almost inevitably lead the analyst to assign values to the parameters that are dictated by his spreadsheet model rather than by his expert analysis and knowledge of the company and the industry. Thus, even if the calculations are formally correct, there might be errors of a more qualitative nature stemming from the analyst's wish to make the input numbers fit the modelling requirements. This is evident in the Eldon valuation - many degrees of freedom are lost because of the TSS requirement. Is TSS really necessary then?

TSS is necessary only if one requires that there be no approximation errors whatsoever. But it is generally sufficient that the approximation errors are small in magnitude and that there are no counter-intuitive effects in the valuation. To achieve this, much less stringent assumptions are needed, making the valuation more realistic.

A *valuation schedule* might, with this in mind, look something like this:

1. Set up the historical financial statements.
2. Calculate the historical financial ratios.
3. Forecast the financial ratios (the parameters) year by year for, say, the next ten years (i.e. the explicit forecast period).
4. Calculate the explicit forecast period's income statements, statements of equity, balance sheets and *FCF* statements, implied by the forecasted parameter values. Check that everything looks reasonable, otherwise correct the parameter values.
5. Set the parameter values in the perpetuity period. These must be set such that no counter-intuitive effects occur. Such parameter restrictions were derived in Chapter 2. For example, the condition $(d\text{-}r) \leq g$ guarantees that the net PPE will be non-decreasing over time. The restrictions derived in Chapter 2 are appropriate for this model. With other models, other restrictions may apply, but the methodology used in Chapter 2 can be applied to any model. Note that these conditions are in the form of inequalities and hence are much less restrictive than many of the TSS conditions, which completely specify the values of some parameters.

6. Calculate year zero and year one in the perpetuity period (income statement, balance sheet, equity statement and FCF statement).
7. Calculate the horizon value, using the dividends from year one as valuation measure:

$$Horizon\ value = \frac{(1+g)R_0(m+z_{DIV})}{k_E - g} - \frac{(1-\tau)iw\left(\frac{d-r}{g}bR_0 - A_0\right)}{k_E}$$

   Equivalently, the horizon value can be calculated using a long (100-150 years) explicitly modelled forecast, which is really only a copy-routine in the spreadsheet program since the parameter values are constant. The latter approach makes the valuation more flexible (see step 9 below).
8. Discount the horizon value and the forecasted dividends from the years modelled explicitly. The best way to do this is to start with the last value and "discount backwards" one period at a time. This way of modelling makes it possible to change the discount rate, should the riskiness of the company change (see step 9 below).
9. Check for type 2 approximation errors by constructing a graph of the capital structure over time.[87] If the graph shows substantial deviations from today's capital structure, the analyst may consider changing the equity cost of capital over time to make the discount rate better reflect the changing risk of the company, and then go back to the previous step in the schedule. If one does not want to go that far and make actual adjustments, a look at the capital structure over time will still give the analyst an indication as to whether the valuation is biased upwards or downwards. An increasing debt ratio may indicate an increasing risk which would tell the analyst that the estimated equity value is probably a bit too high.

Using this approach there will be no type 1 approximation errors, possible type 2 approximation errors are explicitly searched for, and there will be a large amount of freedom in specifying parameter values and still no counter-intuitive implicit effects.

This report has in part been very detailed. We will end it on a more general note. The value of an asset is in its most general sense equal to the present value of all future cash flows pertaining to the asset. Both the *free cash flow approach* and the *dividend approach* utilise this basic valuation concept. In the free cash flow approach the "asset" is the company's operations. In the dividend approach the asset is the shares. Intuitively, two valuation approaches stemming from the same basic concept should result in the same value. We have established a procedure that ensures that the

[87] The graph of the perpetuity period will be much easier to construct if this period is modelled explicitly, as suggested in step 7.

equivalence of approaches holds under quite general conditions, even if the capital structure is non-constant. This has also been shown to hold in a real-world case.

What we originally set out to do was to value the equity of a company, i.e. to derive a price for its shares. Subscribing to the wide-held view that the value of a share equals the present value of expected dividends, it is reassuring that the FCF valuation approach can be applied in such a way that the value derived equals the value from the dividend approach.

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Essay 3:

Company Valuation
with a Periodically Adjusted Cost of Capital

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Abstract. This essay points out procedures and relations that are useful in applications of company valuation methods using discounted free cash flow valuation models. The essay, which can be viewed as an extension of Levin & Olsson (1995) (i.e., Essay 2 in this dissertation), develops a general discounting procedure where the equity cost of capital and the weighted average cost of capital can be simultaneously adjusted to reflect a varying capital structure under different assumptions about the value of interest tax shields. Implementation issues including programming are also discussed.

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1. Introduction

One of the most popular company valuation techniques is discounted free cash flow valuation. The free cash flows of the unlevered firm (i.e., independent of financing) are calculated and discounted at the weighted average cost of capital. The McKinsey book *Valuation: Measuring and Managing the Value of Companies* (Copeland et al. (1994)) has substantially added to the popularity of such models – indeed, the valuation technique itself is often referred to as "the McKinsey model". The latter fact tends to anger some academics, who recognise that free cash flow valuation is not an invention of McKinsey and Company. While this is certainly true, the point highlights the fact that much of the importance in business related academia lies in the applications and in the possibility of implementing results. In this respect the McKinsey book is an important addition to the literature.

There are, however, some issues left uncommented on in Copeland et al. For example, a number of financial ratios are chosen as parameters in their forecasting model, but the rationale for these choices is not always obvious. Another issue on which the book is silent is the interdependence between the costs of capital and the development of the company. We discuss these problems in Essay 2. The particular aspects pursued further here have to do with the latter issue – the relations between the cost(s) of capital and the forecasted development of the company.

The common use of a constant weighted average cost of capital (WACC) as discount rate assumes a constant capital structure (in market value terms). If the projected capital structure varies, then the use of a constant WACC is inappropriate. This "conventional wisdom" seems to have different implications for different people. The prudent university lecturer is quick to point out that with a varying capital structure one should choose a valuation technique that can accommodate such variation. The adjusted present value (APV) method suggests itself as the prime candidate for this role.[1] Another approach is to ignore that the assumption of constant capital structure is being violated, and still use a constant WACC. Copeland et al. (1994) follow the latter path. It makes the exposition clear, but comes at a price, namely approximation errors that may or may not be serious.

Essay 2 shows that it is perfectly possible to use the popular and intuitively appealing WACC approach, but continuously adjust the weighted average cost of capital for anticipated changes in capital structure, thereby alleviating the approximation error problems, in particular those stemming

[1] See, e.g., Brealey & Myers (1991), p. 458.

from changing weights in the WACC formula.[2] This essay extends the analysis by showing how the *equity cost of capital* and the weighted average cost of capital can be simultaneously adjusted to reflect a varying capital structure. The cost of debt is still assumed to be constant (as discussed further in the next section).

1.1 The modelling framework

The company's forecasted future financial statements can be expressed as:

Assets	**Debt and Equity**
(Net) Working Capital, NWC_t	Debt, D_t
Net Property, Plant & Equipment, N_t	Deferred Taxes, T_t
= Gross Property, Plant & Equipment, G_t - Accumulated Depr., A_t	Book Equity, BV_t

Forecasted balance sheet for period t

+Revenues, R_t
-Operating Expenses, OpX_t
-Depreciation Expense, $DepX_t$
-Interest Expense, $IX_t = i \cdot D_t$
-Taxes, IT_t
=Net profit, NP_t

Forecasted income statement for period t

Free cash flow equals the gross cash flow minus the gross investments. Using the notation from the financial statements above:

$$\begin{aligned} FCF_t = {} & (1-\tau)(R_t - OpX_t - DepX_t) + (T_t - T_{t-1}) + DepX_t \\ & - (NWC_t - NWC_{t-1}) - (N_t - N_{t-1} + DepX_t) \end{aligned}$$

[2] Specifically, the equity cost of capital and the cost of debt are assumed to be constant.

Invested capital, IC_t, is defined as net working capital plus net property, plant and equipment:

$$IC_t = NWC_t + N_t$$

An alternative way of calculating free cash flow is to start from net profit:

$$FCF_t = NP_t + (1-\tau)IX_t + (T_t - T_{t-1}) - (IC_t - IC_{t-1})$$

Further, the clean surplus relation is supposed to hold (i.e., the change in book equity equals net profit minus dividends):[3]

$$BV_t - BV_{t-1} = NP_t - DIV_t$$

The company's debt is assumed to be on market terms, i.e., the book value of debt is equal to the market value; equivalently, the coupon rate on debt, is assumed to equal the (market) cost of debt for all future periods. Moreover, the coupon rate (and thus the cost of debt) is assumed to be constant over time, and both (equal) rates are denoted by i. These simplifying assumptions are made to avoid cumbersome modelling of the detailed debt structure at each future period. The reader will note that we explicitly abstract from all non-quantifiable costs of debt, since the extreme cases where they may be important are not the primary concern of this essay. For example, expected costs of financial distress and agency costs are not addressed here. Also, we disregard the issue of personal taxes and consider only taxes on the corporate level. Our framework is thus similar to Modigliani & Miller (1963), and consequently, an increase in leverage leads to a lower cost of capital, because of tax consequences. This is obviously a valid approximation only within a limited range of debt ratios. At extreme debt ratios, financial distress and agency costs cannot be disregarded. The reader is referred to the vast corporate finance literature on the subject.

There are no excess marketable securities in the forecast period. Deferred taxes are treated as a quasi-equity account.[4] Conceptually, deferred taxes are really neither assets nor liabilities – they are linked to the firm's operations and one may even think about them as a negative item on the asset side. For valuation purposes they are "important only to the extent that we need them to calculate income tax

[3] Dividends, DIV_t, are defined net of capital contributions/withdrawals.
[4] See Copeland et al. (1994), p. 162.

payments."[5] That means that the net flow to/from the deferred taxes account each period is included in the FCF calculation as an adjustment to the income statement item *Taxes*.

2. A General Discounting Procedure

2.1 The basic discounting procedure

In many ways, this essay can be viewed as an extension of Essay 2. The particular result of interest here is Proposition 3.1:

Valuation by discounting the free cash flows at a continuously updated weighted average cost of capital will yield the same value as valuation by discounting the future dividends at the cost of equity capital.

The key point is that Proposition 3.1 makes no statement about the cost of equity capital (hereinafter denoted $k_{E,t}$) being constant – indeed, the proposition will hold regardless of how $k_{E,t}$ is defined.[6] We use the t-subscript to highlight the fact that the cost of equity capital may vary over time.

The basic idea behind the valuation procedure is the following: Starting at a future point in time where the equity value is either zero (a company with finite life) or can be expressed as a terminal value, one then goes backwards, one period at a time, and calculates the equity value at the beginning of each period. One should use an 'updated' discount rate that reflects the anticipated capital structure at each point. This is repeated until the valuation date is reached. Traditionally, the period length in this type of company valuation models is one year. We will adhere to this convention and refer to periods as years. Note, however, that it is perfectly possible to use any period length in the modelling framework developed in this essay.

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[5] Holthausen & Zmijewski (1996), Chapter 5B, p. 6.

[6] See Appendix 2 for a proof.

The discounting procedure is based on the following difference equation (1):

$$(1) \qquad EV_t = \frac{FCF_{t+1} + EV_{t+1} + D_{t+1}}{1 + k_{WACC,\,t+1}} - D_t$$

where EV_t is the market equity value at the end of year t,
FCF_t is the free cash flow in year t,
D_t is the debt at the end of year t,
$k_{WACC,t}$ is the weighted average cost of capital during year t, defined as

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \left(1 - \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}\right)k_{E,t}$$

where i is the market rate on debt,
$k_{E,t}$ is the cost of equity capital during year t (computed at beginning of year t)
τ is the (company) tax rate.

Formula (1) thus expresses the equity value at the end of year t-1 as the present value of the next year's free cash flow and the (ex-dividend) total company value at the end of year t minus the value of the debt at t-1.

In practice, one can start the valuation procedure either at the horizon H, and use some terminal value technique to calculate the equity value at that point in time, EV_H, or, if the company has finite life, at the end of its lifetime. One then goes back one year in time, and uses (1) to calculate the equity value at H-1, and then H-2, and so on, until the valuation date is reached.

2.2 Updating the cost of equity

Arguably, the best known way of updating the cost of equity with respect to changes in the capital structure starts from the approach formulated in Modigliani & Miller (1963)[7] and is discussed in popular corporate finance textbooks. The approach is quite straight-forward to implement.[8] It has an important drawback, however, in that it is limited to the following special case:

1) the debt is a fixed dollar amount, i.e., constant for all future years
2) the company is expected to generate the same amount of cash flow in perpetuity

[7] Modigliani & Miller (1963) will henceforth be referred to as MM.
[8] See, e.g., Copeland & Weston (1988), p. 451.

These underlying assumptions make the MM approach less useful in many real world applications,[9] and a more general approach is often needed. Holthausen & Zmijewski (1996) develop a more general analysis for the relations between different cost of capital items. The MM approach arises as a special case in this framework.

In a case where the company at the beginning of year t is financed with debt and equity only, the general relation between the cost of equity, $k_{E,t}$, the cost of equity for the unlevered firm, k_U, and the cost of debt, i, is (Holthausen & Zmijewski, Chapter 2, pp. 12-14):[10]

$$(2) \qquad k_{E,t} = k_U + (k_U - i) \cdot \frac{(D_{t-1} - PVTS_{t-1,i})}{EV_{t-1}}$$

where D_{t-1} = market value of debt at the beginning of year t
$PVTS_{t-1,i}$ = present value at the beginning of year t of the part of the interest tax shields that is discounted at i during year t
EV_{t-1} = market value of equity at the beginning of year t

The unlevered cost of equity, k_U, depends on the company's operations only and is assumed to be constant over time; $k_{E,t}$ is the cost of equity adjusted for current financing. Note that interest tax shields are either discounted by i or k_U. The present value of all tax shields, $PVTS_{t,all}$, can thus be expressed as $PVTS_{t,i} + PVTS_{t,k_U}$, where the second index refers to the discount rate (i or k_U). Formula (2) is quite general with respect to assumptions about the valuation of interest tax shields: it holds for any case where the tax shields are discounted at k_U or i, and for any case where some part of the tax shields is discounted at one of the rates and the rest is discounted at the other. This also means that one must make an explicit assumption about how interest tax shields should be valued. This assumption may depend on the specifics of the company under consideration and is closely tied to how the company determines its capital structure, since the riskiness of the tax shields (and thus their value) depends on how the company carries out its financing.

If the company has decided its financing plan once and for all, independent of the value of the company, then it is reasonable to regard the interest tax shields as being just as risky as the debt, since the size of the tax shields is completely determined by the pre-specified debt schedule. The proper discount rate for the interest tax shields in this case is the cost of debt, i. Given such *passive debt management* it is straight-forward to use the general formula (2) directly if one has a forecast of the

[9] Basically, since the original MM setting implies a non-changing financial structure, there is no *need* to update any costs of capital, once the financial structure has been set.
[10] See Appendix 4 for a derivation.

debt development from year to year. Note, however, that we explicitly have to forecast the value of tax shields at the end of each future year.

This passive debt management view also corresponds to the MM article. However, MM further assume that debt is a fixed dollar amount (determined once and for all). Their assumptions result in the following well-known cost of equity formula for the MM case:

$$(3) \qquad k_{E,t} = k_U + (k_U - i) \cdot \frac{D_{t-1}}{EV_{t-1}} (1 - \tau)$$

It is sometimes argued that an assumption about passive debt management is unrealistic in many cases, since borrowing decisions may very well be made to approach a target capital structure.[11] Pursuing this line of reasoning, a different picture of the valuation of interest tax shields emerges. With perfect *active debt management*, i.e., if the size of the company's debt is *continuously* adjusted in order to maintain a constant market debt-to-value ratio, the size of the interest tax shields will be directly related to the development of the company's operations. Consequently, the proper discount rate for the interest tax shields in this case would be the unlevered cost of equity, k_U. This seems to be the underlying assumption in the so-called *compressed APV technique* (Kaplan & Ruback (1995)) and results in the following formula for the calculation of the cost of equity:[12]

$$(4) \qquad k_{E,t} = k_U + (k_U - i) \cdot \frac{D_{t-1}}{EV_{t-1}}$$

In practice, it will be quite problematic to *continuously* adjust the borrowing to maintain a constant capital structure. A common way of handling this problem, introduced by Miles & Ezzell (1980), is that the debt amount is determined at the end of each year and then kept constant until the end of next year. Thus, at the end of each year the company's capital structure equals its target capital structure. This means that the interest tax shield for the coming year is as risky as the outstanding debt, whereas the interest tax shields from all subsequent years will be as risky as the unlevered equity. The tax shield valuation at the beginning of each year will be performed using the cost of debt as discount rate for the interest tax shield from the immediately following year, whereas the unlevered cost of equity is used as discount rate for the interest tax shield from the years thereafter.

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[11] See Fama & French (1997) for empirical evidence on this.

[12] See Appendix 4 for the derivation.

Inserting this valuation principle in the general equation (2) gives the Miles & Ezzell (1980) formula for the cost of equity (Holthausen & Zmijewski (1996), Chapter 2, pp. 15-16):[13]

(5) $$k_{E,t} = k_U + (k_U - i) \cdot \frac{D_{t-1}}{EV_{t-1}} \left(1 - \frac{\tau \cdot i}{1+i}\right)$$

This formula is even more general than it may appear at first sight. Assume that the yearly debt adjustment is contingent on the company's market value, but only approximately (the target debt ratio is not reached exactly). The debt is held constant for one year until the next debt adjustment at the end of the following year. The debt being contingent on the company's market value makes it appropriate to assume that subsequent (i.e., *beyond* the next year) interest tax shields be discounted at k_U. The tax shield from the *first* year should be discounted at the cost of debt, *i*, since debt is assumed to be kept constant for one year at a time. Consequently, the crucial assumption for (5) to be the appropriate formula is not that the debt adjustment is made to reach the exact target debt ratio at the end of each year, but that interest tax shields beyond the end of the following year are assumed to have the same risk as the firm's operations and hence should be discounted at k_U.

Finally, the underlying assumptions about debt policy have an impact not only on the perceived riskiness of tax shields but also on the actual forecasts:

No debt adjustment (i.e., *passive debt management*) implies that:

- interest tax shields are discounted at the cost of debt:

  $$PVTS_{t,all} = PVTS_{t,i} = \sum_{s=t+1}^{\infty} \frac{\tau \cdot i \cdot D_{s-1}}{(1+i)^{s-t}}$$

  in the MM case this reduces to:

  $$PVTS_{t,all} = PVTS_{t,i} = \tau \cdot D_t$$

- the debt schedule is predetermined, i.e., fixed once and for all.

[13] A derivation is provided in Appendix 4.

Yearly adjusted debt (i.e., *active debt management*) implies that:

- interest tax shields are valued using the Miles & Ezzell (1980) procedure:

$$PVTS_{t,all} = PVTS_{t,i} + PVTS_{t,k_U} \text{ and } \begin{cases} PVTS_{t,i} = \dfrac{\tau \cdot i \cdot D_t}{1+i} \\ PVTS_{t,k_U} = \dfrac{PVTS_{t+1,all}}{1+k_U} \end{cases}$$

- the debt schedule is contingent on the expected development of the company value.

Continuously adjusted debt (i.e., *perfect active debt management)* implies that:

- interest tax shields are discounted at the unlevered cost of equity, k_U:

$$PVTS_{t,all} = PVTS_{t,k_U} = \sum_{s=t+1}^{\infty} \frac{\tau \cdot i \cdot D_{s-1}}{(1+k_U)^{s-t}}$$

- the debt schedule is contingent on the expected development of the company value.

In the passive debt management case the debt schedule is fixed. In the two active cases, the debt amount in each period is variable, it is tied to the expected company value through the target capital structure. The passive and active methods will generally not yield the same forecasted debt schedule.

2.3 Simultaneous updating of the WACC and the cost of equity

Before turning to the issue of how to combine the cost of equity updating with the basic discounting procedure, some problems of a technical nature must be addressed.

The situation is the following: the equity value at the end of (the horizon) year H, EV_H, has been computed and forecasts of FCF_H, D_H, and D_{H-1} have been obtained. Equation (1) can in principle be used to calculate EV_{H-1}, the equity value at the end of year H-1.

In order to compute $EV_{H\text{-}1}$, however, one must know WACC during year H, but to calculate WACC both $EV_{H\text{-}1}$ and the cost of equity, $k_{E,H}$, must be known (and the latter is itself a function of $EV_{H\text{-}1}$). An iterative numerical procedure must be used, one that eventually will make $EV_{H\text{-}1}$, $k_{E,H}$, and $k_{WACC,H}$ converge to their mutually consistent values.

The simultaneity problem is solved in part by inserting the general formula (2) for the cost of equity updating into the standard WACC definition, and the general WACC formula (6) is obtained:[14]

$$(6) \qquad k_{WACC,t} = k_U \cdot \left(1 - \frac{PVTS_{t-1,i}}{D_{t-1} + EV_{t-1}}\right) + i \cdot \left(\frac{PVTS_{t-1,i} - \tau \cdot D_{t-1}}{D_{t-1} + EV_{t-1}}\right)$$

Formula (6) can be simplified in some of the cases discussed above:

i) for the MM case:

$$(7) \qquad k_{WACC,t} = k_U \cdot \left(1 - \frac{\tau \cdot D_{t-1}}{D_{t-1} + EV_{t-1}}\right)$$

ii) for the active debt management case (with yearly debt adjustments, including the Miles & Ezzell (1980) approach):

$$(8) \qquad k_{WACC,t} = k_U - \tau \cdot i \cdot \frac{D_{t-1}}{D_{t-1} + EV_{t-1}} \cdot \left(\frac{1 + k_U}{1 + i}\right)$$

iii) for the perfect active debt management case (with continuously adjusted debt):

$$(9) \qquad k_{WACC,t} = k_U - \tau \cdot i \cdot \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}$$

Thus, the yearly iterative procedure will only have to consider the interdependence between the equity value and the weighted average cost of capital, since the appropriate WACC formula already includes the correct specification of the updated cost of equity. To complete the valuation the iterative procedure must be carried out for years H, H-1, H-2, etc., until the valuation date is reached.

[14] The derivation of equations (6), (8) and (9) is in Appendix 4. Equation (6) is taken from Holthausen & Zmijewski (1996, Ch. 2, p. 20). Equation (8) is from Holthausen & Zmijewski (1996, Ch. 2, p. 21).

The general WACC formula (6) as well as the three special cases ((7), (8) and (9)) also contain the unlevered cost of equity k_U, which is a measure of the riskiness of the company's operations. We assume that this parameter is constant through time. A problem is that k_U may be unobservable. Intuitively, the value of k_U may be regarded as an industry specific number – firms in a certain industry can be assumed to have equally risky operations. If this assumption is made, then k_U can be inferred from industry data using some quantitative method.[15] Once an estimate of k_U has been obtained the yearly iterative procedure, *Procedure Y*, is uncomplicated:

<u>*Procedure Y*</u>

Y1 Assign a trial value to the WACC, $k_{WACC,t+1}$.

Y2 Calculate the equity value at t through equation (1) (or by using a terminal value formula if at the horizon, H).

Y3 Compute the implied resulting WACC by inserting the equity value from Y2 into the appropriate WACC formula ((6), (7), (8) or (9)).

Y4 Compare the implied resulting $k_{WACC,t+1}$ with the trial value. If equal, the equity value is correct, and one can go on to the preceding year. If the resulting WACC differs from the trial value, go to Y1 again, where the resulting WACC from Y3 can be used as a new trial value.

One starts at the forecast horizon (year H) with *Procedure Y.* The procedure is then repeated every forecasted year until the valuation date is reached. More detailed implementation issues will be discussed in the next section.

3. Implementing the General Discounting Procedure

To visualise the implementation of the general discounting procedure we will use the stylised (fictitious) company XMPL as illustration. XMPL is the same company that was referred to as *the example company* in Chapter 2 of Essay 2. Only the years in the *parametric steady state* (PSS) – the period after the explicit forecast period when all parameters are assumed constant – were considered

[15] See, e.g., Jennergren & Näslund (1996) pp. 68-69 for an implementation of a CAPM related approach.

there, however. Here, we make a full-scale valuation including nine explicitly forecasted years. This will further highlight the problems introduced by a non-constant capital structure.

The XMPL company follows a pre-set financing plan in the explicit forecast period, i.e., in the first nine years. This will exemplify the passive debt management case discussed in section 2.2. The company will enter into a parametric steady state in year 10. The expected market debt ratio will remain non-constant, however, until it asymptotically approaches its steady state value. The PPE items are forecasted as in the first edition of Copeland et al. (1990) and *Specification A* in Essay 2.

It may well be argued that following MM and viewing the debt level as pre-determined is unrealistic, especially over longer time periods. Indeed, Modigliani (1988) acknowledges this point: "It seems much more reasonable to suppose that the leverage policy of the representative firm can be described as aiming at maintaining the debt in a stable relation to the scale of the firm as seen at any given date" (Modigliani (1988), p. 152). Even so, the influence of MM on mainstream corporate finance makes it interesting to see what is involved when making the passive debt management assumption operational in a full-scale company valuation.

In the period after the explicit forecast period we assume that the debt management will be related to the development of the company. In particular, the (assumed) debt policy is to keep debt as a constant fraction of the balance sheet. So whereas the first nine years exemplify *passive debt management*, the period after that will visualise *active debt management.*

Interest tax shields in the explicit forecast period will be discounted at the cost of debt, *i.* As argued in section 2.2, this is appropriate when the debt levels are pre-set and unrelated to the company's development. In this case the debt is not a fixed dollar amount, however, so we cannot use the MM formulas. Instead the cost of equity can be updated using formula (2) and the WACC can be updated using formula (6). In the steady state period, the debt level is explicitly tied to the company's operations in that it is forecasted as a percentage of the balance sheet, and the cost of equity will be updated using formula (5) and the weighted average cost of capital using formula (8). The unlevered cost of equity, k_U, is assumed to be 12%. Year 0 will denote the last historical year, years 1 to 9 are the explicitly forecasted years, and the parametric steady state period starts in year 10. Parametric steady state (PSS) means that the parameters in the forecasting model are expected to be constant. In our framework, PSS also means that the forecasted accounting items for any year can be derived directly through the formulas in Essay 2, Chapter 2. For a description of how the corresponding accounting data can be computed for any year in the PSS period, see Appendix 1.

XMPL, Forecasted Balance Sheets, Years 1-10											
Year	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
ASSET SIDE:											
+Net Working Capital	15.51	15.36	15.72	16.39	17.77	19.41	20.57	21.60	22.68	23.81	25.00
+Gross PPE	111.30	124.60	134.92	142.14	149.24	156.71	164.54	172.77	181.41	190.48	200.00
-Acc. Depr.	94.44	96.66	99.16	101.85	104.70	107.68	110.82	114.11	117.56	121.19	125.00
+Net PPE	16.86	27.94	35.76	40.28	44.55	49.02	53.72	58.66	63.84	69.29	75.00
Total Assets	32.37	43.30	51.48	56.67	62.31	68.43	74.29	80.26	86.52	93.10	100.00
DEBT & EQUITY SIDE:											
+Debt	12.95	22.50	24.50	27.00	27.00	27.37	29.72	32.10	34.61	37.24	40.00
+Deferred Taxes	0.55	0.92	1.33	1.75	2.20	2.67	3.17	3.68	4.23	4.80	5.40
+Book Equity	18.87	19.88	25.65	27.92	33.11	38.39	41.41	44.47	47.68	51.06	54.60
Total Debt & Equity	32.37	43.30	51.48	56.67	62.31	68.43	74.29	80.26	86.52	93.10	100.00
Note:	Year 0 = Last historical year Year 10 = First year forecasted to be in Parametric Steady State										

Table 1 - XMPL forecasted balance sheets, years 0 - 10.

XMPL, Forecasted Income Statements, Years 1-10											
Year	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
+Revenues	310.11	307.26	314.30	327.80	355.34	388.22	411.35	431.92	453.51	476.19	500.00
-Operating exp.	-279.10	-286.44	-282.87	-301.23	-322.66	-352.23	-368.32	-390.22	-410.00	-428.57	-450.00
-Depr. exp.	-7.00	-6.68	-7.48	-8.10	-8.53	-8.95	-9.40	-9.87	-10.37	-10.88	-11.43
+Operating income	24.01	14.14	23.95	18.47	24.15	27.04	33.63	31.83	33.15	36.73	38.57
-Interest exp.	-1.00	-1.29	-2.25	-2.45	-2.70	-2.70	-2.74	-2.97	-3.21	-3.46	-3.72
+Earnings bef. taxes	23.01	12.85	21.70	16.02	21.45	24.34	30.89	28.85	29.94	33.27	34.85
-Taxes	-6.90	-3.85	-6.51	-4.81	-6.44	-7.30	-9.27	-8.66	-8.98	-9.98	-10.45
Net profit	16.11	8.99	15.19	11.22	15.02	17.03	21.62	20.20	20.96	23.29	24.39
Note:	Year 0 = Last historical year Year 10 = First year forecasted to be in Parametric Steady State										

Table 2 - XMPL forecasted income statements, years 0 - 10.[16]

From the forecasted balance sheets and income statements (Tables 1 and 2) one can then obtain XMPL's forecasted free cash flow (Table 3). The forecasted statements of equity are presented in Table 4.

XMPL, Forecasted Free Cash Flow Calculation, Years 1-10										
Year	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
+Net profit	8.99	15.19	11.22	15.02	17.03	21.62	20.20	20.96	23.29	24.39
+Interest exp. aft. taxes	0.91	1.58	1.72	1.89	1.89	1.92	2.08	2.25	2.42	2.61
+Inc. deferred taxes	0.37	0.40	0.43	0.45	0.47	0.49	0.52	0.54	0.57	0.60
-Increase in invested capital	-10.93	-8.18	-5.19	-5.64	-6.12	-5.86	-5.96	-6.26	-6.58	-6.90
Free cash flow	-0.66	8.99	8.17	11.71	13.27	18.18	16.83	17.49	19.71	20.70
Note:	Year 10 = First year forecasted to be in Parametric Steady State									

Table 3 - XMPL forecasted free cash flow calculations, year 1-10

[16] Note that the printed tables do not necessarily sum correctly at the last decimal because of rounding.

XMPL, Forecasted Statements of Equity, Years 1-10										
Year	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Beginning Book Equity	18,87	19,88	25,65	27,92	33,11	38,39	41,41	44,47	47,68	51,06
+Net profit	8,99	15,19	11,22	15,02	17,03	21,62	20,20	20,96	23,29	24,39
-Dividends	-7,99	-9,42	-8,95	-9,82	-11,76	-18,60	-17,14	-17,74	-19,92	-20,85
=Ending Book Equity	19,88	25,65	27,92	33,11	38,39	41,41	44,47	47,68	51,06	54,60
Note:	Year 10 = First year forecasted to be in Parametric Steady State									

Table 4 - XMPL forecasted statements of equity, year 1-10

In the parametric steady state period FCF will grow at a constant annual rate of 5%. The initial condition for the capital structure in market terms to remain constant in the PSS period is not fulfilled in the XMPL case.[17] Hence, one cannot use a simple continuing value formula at the end of the explicit forecast period to get a correct horizon value. The market debt ratio will, however, approach a steady state value as t gets large,[18] and this means that one can use a continuing value at some point in the distant future where the market debt ratio is close enough to the steady state value, as a very good approximation of the company value at that moment. If this continuing value calculation is done sufficiently long into the future, the approximation error will have no impact on the company value at valuation date. To find a reasonable cut-off-year where we can compute the company value with a continuing value, we simply calculate the implied market debt ratio at a number of points of time in the distant future (Table 5). [19]

H	25	100	150	200	500	1000	10000
Implied market debt ratio	16.337%	18.870%	18.931%	18.936%	18.936%	18.936%	18.936%

Table 5 - XMPL market debt ratios implied by continuing value formula at different horizon years (H)

Thus, after about 200 years we arrive at the steady state market debt ratio level. We choose year 210 as this 'final horizon', and start the discounting procedure here by computing the company value via the FCF continuing value formula.

[17] Basically, this condition states that the depreciation and retirements parameters in the parametric steady state period must be decided such that accumulated depreciation grows at the revenue growth rate, g, or – in other words – the depreciation related flows must be such that the stock grows by g.

[18] See Essay 2, Section 2.1.5.

[19] Implied market debt ratio = $D_t / \left(\frac{FCF_{t+1}}{(k_{WACC,t+1} - g)} \right)$

In the explicit forecast period with passive debt management we use the general case formula (6) to calculate WACC. Accordingly, we have to forecast the present value of interest tax shields *discounted at i* at the beginning of each year in the explicit forecast period, i.e., $PVTS_{t,i}$ for t = 9 to 0. The interest tax shield in any year t is simply $\tau \cdot i \cdot D_{t-1}$, but since *all* tax shields here are discounted at i, we must also take the present value of all tax shields at the end of year 10 into consideration. The calculation of $PVTS_{9,i}$ will thus represent a link between the two periods with different debt management policies, and thus, different tax shields valuation principles:

$$(10) \qquad PVTS_{9,i} = \frac{\tau \cdot i \cdot D_9 + PVTS_{10,all}}{1+i}$$

However, we must first calculate $PVTS_{10,all}$, the value (at the end of year 10) of *all* tax shields from the years after the explicit forecast period, which is obtained using the Miles & Ezzell (1980) backward going procedure:[20]

$$(11) \qquad \begin{cases} PVTS_{t,i} = \dfrac{\tau \cdot i \cdot D_t}{1+i} \\ PVTS_{t,k_U} = \dfrac{PVTS_{t+1,all}}{1+k_U} \\ PVTS_{t,all} = PVTS_{t,i} + PVTS_{t,k_U} \end{cases} \qquad (t = 209 \text{ to } 10).$$

Then, we can use equation (10) to calculate $PVTS_{9,i}$. Finally, a similar calculation is repeated for t = 8, for t = 7 and so on, until the valuation date (t = 0) is reached:

$$(12) \qquad PVTS_{t,i} = \frac{\tau \cdot i \cdot D_t + PVTS_{t+1,i}}{1+i} \; (= PVTS_{t,all}) \qquad (t = 8 \text{ to } 0).$$

This altogether produces a forecasted sequence of $PVTS_{t,i}$ for the relevant years (0 to 9). We can then go to the actual valuation by using *procedure Y* and start at the horizon:

[20] We start at the horizon by using the Miles & Ezzell continuing value formula provided by Holthausen & Zmijevski (1996), Chapter 2, p. 17: $PVTS_{210,all} = \dfrac{\tau \cdot i \cdot D_{210}}{k_U - g_D}\left[\dfrac{(1+k_U)}{(1+i)}\right]$ where g_D is the debt growth rate (which has asymptotically approached 5% in year 210).

Y1 We guess that the WACC at the horizon is 11.5%.

Y2 The 'first guess' equity value at the horizon is computed using the continuing value formula:

$$EV_{210} = TCV_{210} - D_{210} = \frac{FCF_{211}}{k_{WACC,211\to\infty} - g} - D_{210} \text{ where } g \text{ is 5\%.}$$

The result is 4778091.10.

Y3 The appropriate WACC formula in this setting for the years in the PSS period is formula (8):

$$k_{WACC,t} = k_U - \tau \cdot i \cdot \frac{D_{t-1}}{D_{t-1} + EV_{t-1}} \cdot \left(\frac{1+k_U}{1+i}\right).$$

Inserting the 'first guess' equity value from step Y2 and the D_{210} value implies a WACC of 11.470%.

End of year 210	
FCF_{211}	375767.78
EV_{210}	4778091.10
Trial $k_{WACC,211\to\infty}$	11.50000%
Resulting $k_{WACC,211\to\infty}$	11.47007%
$k_{WACC,211\to\infty}$ difference	0.02993%

Y4 This is clearly not identical to the trial value of 11.5%, and the procedure must be restarted from Y1 using 11.470% as a new guess of WACC. After a number of loops we arrive at the following:[21]

End of year 210	
FCF_{211}	375767.78
EV_{210}	4802811.12
Trial $k_{WACC,211\to\infty}$	11.47232%
Resulting $k_{WACC,211\to\infty}$	11.47232%
$k_{WACC,211\to\infty}$ difference	0.00000%

[21] In practice, this is done through the goal-seek function of Excel, where the 'WACC difference' (Trial WACC *minus* Resulting WACC) should be set to 0 (zero) by changing the Trial WACC cell in the spreadsheet model. See appendix 3 for exact programming.

Procedure Y is then repeated for all preceding years, using equation (1) instead of the continuing value formula in step Y2, and using equation (6) instead of (8) in step Y3 for the years in the explicit forecast period. When we reach the valuation date we get:

End of year 0	
FCF_1	-0.66
EV_0	164.78
Trial $k_{WACC,1}$	11.63796%
Resulting $k_{WACC,1}$	11.63796%
$k_{WACC,1}$ difference	0.00000%

This whole procedure (i.e., *procedure Y* for all years) took 24 seconds on a PC with 90 MHz Pentium processor. The macro program (for use in Excel) for the whole procedure is in Appendix 3. The market debt ratio of the company is shown in Figure 1.

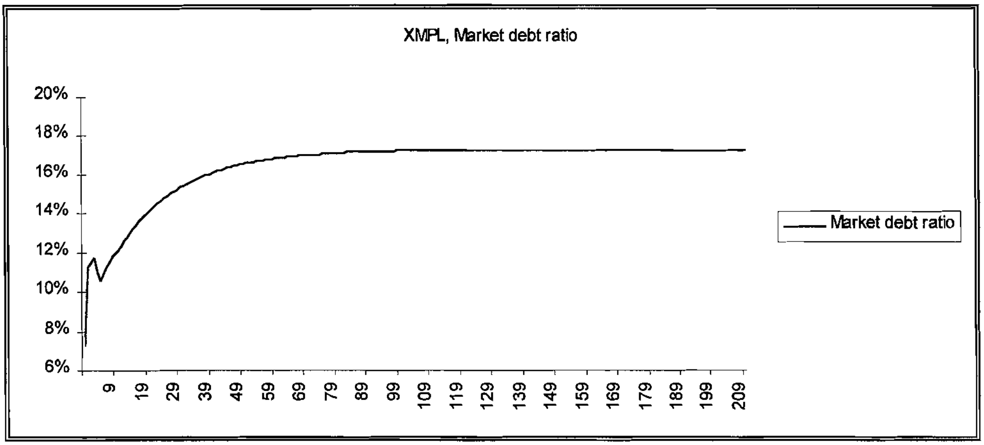

Figure 1 - XMPL market debt ratio, with updated WACC and simultaneously updated cost of equity capital.[22]

Our valuation of XMPL with both non-constant WACC and non-constant cost of equity capital results in an equity value of 164.8. As a comparison, a more naïve approach using a constant WACC of 11.63% (calculated at the valuation date through equation (6)) gives an equity value of 162.4. Copeland, Koller & Murrin recommends that one should use some target capital structure as basis for the (constant) WACC calculation. As can be seen from Figure 1, the market debt ratio will eventually

[22] The kink at the end of the explicit forecast period reflects the adjustment that takes place when the parameters are set to their steady state values.

approach a steady state level slightly above 17%. If this is used as a 'target' when calculating a constant WACC (=11.47%), the resulting equity value would be 167.3. If instead the projected capital structure at the end of the explicit forecast period is used to calculate a constant WACC (=11.52%), the equity value will be 165.8. The constant WACC approaches tend to be somewhat 'ad hoc', and as a consequence drive the valuation procedure towards guessing and approximations. The methodology developed in this essay, as a contrast, results in a equity value that is exactly correct given the assumptions made. However, as our example indicates, the differences in estimated equity value do not seem to be overly severe.

4. Concluding Remarks

The aim of this essay is to point out procedures and relations that are useful in applications of company valuation methods using discounted free cash flow valuation models, first and foremost the so-called McKinsey model.

One of the main problems with such models is the discount rate. This is often assumed to be constant, although when one looks at the company's forecasted future capital structure, this almost always varies over time. Hence, the use of a constant average weighted cost of capital is inappropriate. In Essay 2, we developed procedures for continuously adjusting the weights to reflect a changing capital structure. In this essay, the focus has been on the cost of equity capital and how one simultaneously can adjust both the WACC weights and the cost of equity capital that also are parts of the WACC formula. One might conceivably go on and consider adjustments of the cost of debt capital, using some quantitative model. An interesting extension would be to include the term structure of interest rates. Also, other opinions about the riskiness of tax shields will lead to models other than the ones described and used here (but similar) for determining the equity cost of capital. Such additions or alterations do not change the operational principles, however, and those are our prime interest in this essay.

One sometimes hears comments to the effect that it is not worth the extra effort to use correct and precise calculation techniques when valuing companies, since there is so much uncertainty anyhow in the data that must ultimately be fed into the model. We object to such statements on two accounts: First, it is not really much of an extra effort. The Excel model can be constructed once, and then used for different companies. Second, uncertainty is additive. The fact that there is a lot of uncertainty in the data should really spur the analyst even more to do what he or she can to reduce the over-all uncertainty. One way of doing this is obviously to use calculation techniques that do not by themselves introduce approximation errors. At the same time it must be noted that the consequences of basing the discount rate calculation on an assumed target capital structure are often not that severe. In our example the approximation error in estimated equity value was within a few percent. The balance between computational complexity and exactness is in the end a matter of judgement.

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Appendix 1 - Parametric Steady State Formulas

The parameters in the Copeland et al. type of forecasting model are (Essay 2, p. 73):[23]

a	net working capital in % of revenues (sales)
b	gross PPE in % of revenues (sales)
c	change in deferred taxes in % of gross PPE
d	depreciation in % of preceding year's gross PPE
g	nominal growth rate, revenues (sales)[24]
i	interest rate on debt
p	operating expenses in % of revenues (sales)
r	retirements in % of preceding year's gross PPE
τ	tax rate
w	debt in % of balance sheet (book value)

The first year in the parametric steady state period for XMPL is year 10. This means that the parameters defined above are constant from this point in time. The following state variables are also identified:

R_t	revenues (sales) of year *t*,
A_t	accumulated depreciation at the end of year *t*,
T_t	deferred taxes at the end of year *t*.

The state-variables in the parametric steady state period (for $t \geq 11$) are given through the following set of equations (Essay 2, p. 77):

$$R_t = (1+g)R_{t-1} = (1+g)^{t-10} R_{10}$$

$$A_t = \frac{(1+g)^{t-10} - 1}{g} \cdot (d-r) b R_{10} + A_{10}$$

$$T_t = \frac{(1+g)^{t-10} - 1}{g} \cdot c(1+g) b R_{10} + T_{10} =$$

$$= \left[\frac{(1+g)^{t+1-10} - 1}{g} - 1 \right] cbR_{10} + T_{10}$$

where R_{10}, A_{10} and T_{10} are the initial values of the state-variables in the PSS period (see tables 1 and 2).

---

[23] In the XMPL case in the parametric steady state period: a=5%; b=40%; c=0.3%; d=6%; g=5%; i=10%; p=90%; r=4%; τ=30%; w=40%.

[24] This means that the revenues of all years in the PSS period can be calculated as (1+g) times the preceding year's revenues.

The forecasting model defines the balance sheet items as follows (for ($t \geq 11$)):

Net working capital:	aR_t
Net PPE	$bR_t - A_t$
	$where\ A_t = \left[(d-r)bR_{t-1} + A_{t-1}\right]$
Debt:	$w(aR_t + bR_t - A_t)$
Deferred taxes:	$T_t = cbR_t + T_{t-1}$
Book equity:	$(1-w)(aR_t + bR_t - A_t) - T_t$

The income statement is also defined (for ($t \geq 11$)):

Revenues:	$R_t = (1+g)^{t-10} R_{10}$
Operating expenses:	pR_t
Depreciation expense:	$dG_{t-1} = dbR_{t-1}$
Interest expense:	$iw(aR_{t-1} + bR_{t-1} - A_{t-1})$
Taxes:	$\tau((1-p)R_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))$
Net profit:	$(1-\tau)((1-p)R_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))$

Appendix 2 - Modified Proof of Proposition 3.1 in Essay 2

The free cash flow can be written as follows:

$$
\begin{array}{llll}
\text{(A2:1)} & FCF_t & = & \textit{Net profit, } NP_t \\
& & + & \textit{Net interest payments after taxes, } (1-\tau)iD_{t-1} \\
& & + & \textit{Increase in deferred taxes, } (T_t - T_{t-1}) \\
& & - & \textit{Increase in invested capital } (IC_t - IC_{t-1})
\end{array}
$$

where: τ is the tax rate

i is the interest rate

D_{t-1} is the (net) debt at the end of year *t-1* on which interest is paid

The dividends are by the clean surplus relationship calculated as:

$$
\begin{array}{llll}
\text{(A2:2)} & DIV_t & = & \textit{Net profit, } NP_t \\
& & + & \textit{Increase in debt, } (D_t - D_{t-1}) \\
& & + & \textit{Increase in deferred taxes, } (T_t - T_{t-1}) \\
& & - & \textit{Increase in invested capital } (IC_t - IC_{t-1})
\end{array}
$$

Assume there exists an equity value at some future time *T*, called EV_T, which is calculated after a possible dividend at time *T*. This means that the total company value at time *T*, called TCV_T, will be $EV_T + D_T$ (i.e. equity value plus debt value). Valuation by the *FCF* approach will then yield the following total company value at the end of year *T-1*:[25]

$$\text{(A2:3)} \quad TCV_{T-1} = \frac{FCF_T + EV_T + D_T}{1 + k_{WACC,\,T-1}}$$

which can be rearranged:

[25] Note that the WACC formula implies that the WACC used for discounting during year *T* is based on the entering market values of debt and equity, i.e. D_{T-1} and EV_{T-1}, and hence also the total company value, TCV_{T-1}.

$$(A2:4)\quad TCV_{T-1} = \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (IC_T - IC_{T-1}) + EV_T + D_T}{1 + k_{WACC,T-1}}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (IC_T - IC_{T-1}) + EV_T + D_T}{1 + \frac{D_{T-1}}{TCV_{T-1}}(1-\tau)i + \left(1 - \frac{D_{T-1}}{TCV_{T-1}}\right)k_{E,T}}$$

$$= \frac{NP_T + (1-\tau)iD_{T-1} + (T_T - T_{T-1}) - (IC_T - IC_{T-1}) + EV_T + D_T}{\frac{TCV_{T-1} + (1-\tau)iD_{T-1} + TCV_{T-1}k_{E,T} - D_{T-1}k_{E,T}}{TCV_{T-1}}}$$

Dividing through by TCV_{T-1} and rearranging further yields:

$$(A2:5)\quad TCV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (IC_T - IC_{T-1}) + EV_T + D_T + D_{T-1}k_{E,T}}{1 + k_{E,T}}$$

The equity value is then obtained by deducting the debt, i.e. by deducting D_{T-1}:

$$(A2:6)\quad EV_{T-1} = \frac{NP_T + (T_T - T_{T-1}) - (IC_T - IC_{T-1}) + EV_T + D_T + D_{T-1}k_{E,T}}{1 + k_{E,T}} - D_{T-1}$$

$$= \frac{NP_T + (T_T - T_{T-1}) - (IC_T - IC_{T-1}) + EV_T + (D_T - D_{T-1})}{1 + k_{E,T}}$$

By inserting (A2:2) into (A2:6) one obtains:

$$(A2:7)\quad EV_{T-1} = \frac{DIV_T + EV_T}{1 + k_{E,T}}$$

which is exactly the valuation formula used when discounting the dividends by the equity cost of capital. Having thus established that the value at *T-1* will be the same when employing the different methods, one can go on to time *T-2* and so on.

Appendix 3 - Macro Programming

Here is an example of how *Procedure Y* can be implemented in Excel 5.0 using Visual Basic macro programming. Notation terms in brackets, e.g., [example], refer to physical cells in the spreadsheet model. For example, [kU difference] means the spreadsheet cell in which the difference between the Trial k_U and Implied k_U is computed. Moreover, year *H* denotes the horizon, which in XMPL was at the end of year 210, and year *0* denotes the valuation date (i.e., the valuation date is at the end of year 0).

Procedure Y (for one particular year *t*):

1) Manually set [Trial kWACC,*t*] to the initial trial value of $k_{WACC,t}$

2) Run the following macro program:

```
Sub procedure_Y()
  Range("[WACC difference year t]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC year t]")
End Sub
```

Procedure Y (for all years from year *H* down to the valuation date, *0*):

1) Manually set all [Trial kWACC,*t*] cells (*t*=1, ..., H+1) to their initial trial values.

2) Run the following macro program:

```
Sub procedure_Y_all_years()
  Range("[WACC difference year H+1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial
      WACC year H+1]")
  Range("[WACC difference year H]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial   WACC year
H]")
  Range("[WACC difference year H-1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial
      WACC year H-1]")
...
  Range("[WACC difference year 2]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC      year
2]")
  Range("[WACC difference year 1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC      year
1]")
End Sub
```

Appendix 4 – Derivations

Equation (2)

As derived by Holthausen & Zmijewski, Chapter 2, pp. 12-14:

Since the required rate of return on the economic assets of the firm must equal the required of return on the securities financing the same assets we have:[26]

$$\left(EV_{t-1} + D_{t-1} - PVTS_{t-1,i} - PVTS_{t-1,k_U}\right)\cdot k_U + PVTS_{t-1,i}\cdot i + PVTS_{t-1,k_U}\cdot k_U = D_{t-1}\cdot i + EV_{t-1}\cdot k_{E,t}$$

Solving for $k_{E,t}$:

$$k_{E,t} = \frac{\left(EV_{t-1} + D_{t-1} - PVTS_{t-1,i} - PVTS_{t-1,k_U}\right)}{EV_{t-1}}\cdot k_U + \frac{PVTS_{t-1,i}}{EV_{t-1}}\cdot i + \frac{PVTS_{t-1,k_U}}{EV_{t-1}}\cdot k_U - \frac{D_{t-1}}{EV_{t-1}}\cdot i$$

Simplifying now yields:

$$k_{E,t} = k_U + \frac{\left(D_{t-1} - PVTS_{t-1,i}\right)}{EV_{t-1}}\cdot k_U - \frac{\left(D_{t-1} - PVTS_{t-1,i}\right)}{EV_{t-1}}\cdot i$$

Rearranging finally yields equation (2):

$$k_{E,t} = k_U + \left(k_U - i\right)\cdot\frac{\left(D_{t-1} - PVTS_{t-1,i}\right)}{EV_{t-1}}$$

[26] Note that the first parenthesis equals the unlevered value of the company.

Equation (4)

In the same way as for equation (2), the required rate of return on the economic assets and on the financing securities are equal. All tax-shields are discounted at k_U, however. Thus:

$$\left(EV_{t-1} + D_{t-1} - PVTS_{t-1,all}\right)\cdot k_U + PVTS_{t-1,all}\cdot k_U = D_{t-1}\cdot i + EV_{t-1}\cdot k_{E,t}$$

Solving for $k_{E,t}$ and simplifying yields equation (4):

$$k_{E,t} = \frac{\left(EV_{t-1} + D_{t-1}\right)}{EV_{t-1}}\cdot k_U - \frac{D_{t-1}}{EV_{t-1}}\cdot i = k_U + \left(k_U - i\right)\cdot\frac{D_{t-1}}{EV_{t-1}}$$

Equation (5)

In the general case, the cost of equity function is given by equation (2):

$$k_{E,t} = k_U + \left(k_U - i\right)\cdot\frac{\left(D_{t-1} - PVTS_{t-1,i}\right)}{EV_{t-1}}$$

In the Miles & Ezzell (1980) setting, at any valuation date t-1 only the interest tax shield from year t is to be discounted at i, the cost of debt. The tax shields from years beyond t is at time t-1 discounted at k_U. Thus:

$$\begin{cases} PVTS_{t-1,i} = \dfrac{taxshield_t}{1+i} = \dfrac{\tau\cdot i\cdot D_{t-1}}{1+i} \\ PVTS_{t-1,k_U} = \dfrac{PVTS_{t,all}}{1+k_U} \end{cases}$$

Now inserting the expression for $PVTS_{t-1,i}$ into the general cost of equity formula yields equation (5):

$$k_{E,t} = k_U + \left(k_U - i\right)\cdot\frac{\left(D_{t-1} - \dfrac{\tau\cdot i\cdot D_{t-1}}{1+i}\right)}{EV_{t-1}} = k_U + \left(k_U - i\right)\cdot\frac{D_{t-1}}{EV_{t-1}}\left(1 - \frac{\tau\cdot i}{1+i}\right)$$

Equation (6)

The standard WACC definition is:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1} + EV_{t-1}} k_{E,t}$$

The cost of equity is in the general case given by equation (2):

$$k_{E,t} = k_U + (k_U - i)\cdot\frac{(D_{t-1} - PVTS_{t-1,i})}{EV_{t-1}}$$

Inserting the cost of equity equation into the WACC definition yields:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1} + EV_{t-1}}\left(k_U + (k_U - i)\cdot\frac{(D_{t-1} - PVTS_{t-1,i})}{EV_{t-1}}\right)$$

Rearranging gives:

$$k_{WACC,t} = \frac{k_U \cdot EV_{t-1} + k_U \cdot (D_{t-1} - PVTS_{t-1,i})}{D_{t-1} + EV_{t-1}} + \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i - \frac{(D_{t-1} - PVTS_{t-1,i})}{D_{t-1} + EV_{t-1}}i$$

Finally, equation (6) is obtained:

$$\begin{aligned} k_{WACC,t} &= \frac{k_U \cdot (D_{t-1} + EV_{t-1}) - k_U \cdot PVTS_{t-1,i}}{D_{t-1} + EV_{t-1}} + \frac{i \cdot PVTS_{t-1,i} + [(1-\tau)i - i]D_{t-1}}{D_{t-1} + EV_{t-1}} = \\ &= k_U \cdot \left(1 - \frac{PVTS_{t-1,i}}{D_{t-1} + EV_{t-1}}\right) + i\frac{PVTS_{t-1,i} - \tau \cdot D_{t-1}}{D_{t-1} + EV_{t-1}} \end{aligned}$$

Equation (8)

The standard WACC definition can be written $k_{WACC,t} = \omega_{D,t-1}(1-\tau)i + (1-\omega_{D,t-1})k_{E,t}$,

where $\omega_{D,t-1} = \dfrac{D_{t-1}}{D_{t-1} + EV_{t-1}}$ (the debt to value ratio).

The cost of equity is given by $k_{E,t} = k_U + (k_U - i)\cdot \dfrac{D_{t-1}}{EV_{t-1}}\left(1 - \dfrac{\tau \cdot i}{1+i}\right)$.

Substituting into the WACC definition

$$k_{WACC,t} = \omega_{D,t-1}(1-\tau)i + (1-\omega_{D,t-1})\left(k_U + (k_U - i)\cdot \frac{D_{t-1}}{EV_{t-1}}\left(1 - \frac{\tau \cdot i}{1+i}\right)\right)$$

Recognising that $D_{t-1}/EV_{t-1} = \omega_{D,t-1}/(1-\omega_{D,t-1})$ yields

$$k_{WACC,t} = \omega_{D,t-1}(1-\tau)i + (1-\omega_{D,t-1})k_U + (k_U - i)\omega_{D,t-1}\left(1 - \frac{\tau \cdot i}{1+i}\right)$$

Simplifying:

$$k_{WACC,t} = k_U - \omega_{D,t-1}\cdot \tau \cdot i - (k_U - i)\omega_{D,t-1}\frac{\tau \cdot i}{1+i}$$

Rearranging further:

$$k_{WACC,t} = k_U - \omega_{D,t-1}\cdot \tau \cdot i\left(1 + \frac{k_U - i}{1+i}\right) =$$
$$= k_U - \omega_{D,t-1}\cdot \tau \cdot i\left(\frac{(1+i) + k_U - i}{1+i}\right)$$

This yields equation (8):

$$k_{WACC,t} = k_U - \omega_{D,t-1}\cdot \tau \cdot i\left(\frac{1+k_U}{1+i}\right) =$$
$$= k_U - \tau \cdot i \cdot \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}\left(\frac{1+k_U}{1+i}\right)$$

Equation (9)

The standard WACC definition is:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1} + EV_{t-1}} k_{E,t}$$

The cost of equity is given by:

$$k_{E,t} = k_U + (k_U - i) \cdot \frac{D_{t-1}}{EV_{t-1}}$$

Substituting into the WACC definition:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1}}{D_{t-1} + EV_{t-1}}\left(k_U + (k_U - i) \cdot \frac{D_{t-1}}{EV_{t-1}}\right)$$

Rearranging:

$$k_{WACC,t} = \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}(1-\tau)i + \frac{EV_{t-1} \cdot k_U + (k_U - i)D_{t-1}}{D_{t-1} + EV_{t-1}}$$

$$k_{WACC,t} = \frac{D_{t-1} \cdot i - D_{t-1} \cdot \tau \cdot i + EV_{t-1} \cdot k_U + (k_U - i)D_{t-1}}{D_{t-1} + EV_{t-1}}$$

$$k_{WACC,t} = \frac{(D_{t-1} + EV_{t-1})k_U - D_{t-1} \cdot \tau \cdot i}{D_{t-1} + EV_{t-1}}$$

And equation (9) is obtained:

$$k_{WACC,t} = k_U - \tau \cdot i \cdot \frac{D_{t-1}}{D_{t-1} + EV_{t-1}}$$

Essay 4:

On the General Equivalence of Company Valuation Models

- Free Cash Flow, Economic Value Added, Abnormal Earnings, Dividends, and the Adjusted Present Value Model in Equity Valuation

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Abstract. This essay analyses the relations between prominent valuation concepts and models for company valuation in the traditional constant discount rate setting as well as in settings where the cost of capital is adjusted according to anticipated changes in the capital structure. The essay provides a company valuation framework with corporate taxes where the valuation result is independent of the choice of valuation model, and discusses the usefulness of the different concepts and models. The implementation of the valuation framework is described in the Eldon AB case study.

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1. Introduction

Several different discounting based models have been suggested as the way to proceed when performing a company valuation. The most prominent examples are perhaps

- the *dividend* (DIV) model[1], where equity value is calculated as the present value of all future dividends,
- the *free cash flow* (FCF) model, where total company value (i.e., debt plus equity) is calculated as the present value of all future free cash flows,
- the *economic value added* (EVA) model[2], where total company value is calculated as the present value of all future economic value added plus the existing capital,

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[1] This model is often referred to as *the present value of future expected dividends* (PVED) model. For simplicity we will use the term *the DIV model* throughout the essay. Note also that in the framework used in this essay, the DIV model is equivalent to *the flow to equity (FTE) model* (also referred to as the *FCF to common equity* model). This depends on the assumption that there are only non-financial assets in the future. That is, all generated cash flows *not* used for investments and debt servicing will be paid out as dividends. The FTE for a given year would exceed the dividends paid out if instead some of the cash flow was retained in the company as (excess) cash and/or securities. But since these retained resources 1) by definition are *not* investments, and 2) can not be expected to earn the cost of equity (if they do, this means that we have a "bank", i.e., the (excess) cash and securities can be regarded as being part of the company's normal operations and they should thus be treated as investments, and not something excess, and the FTE and DIV would once again be equal), allowing for a difference between FTE and DIV in the tax regime considered in this essay would be to explicitly model a breakdown of the dividend policy irrelevance. The issue of dividend policy irrelevance should instead be viewed as the trade-off between dividends and investments. That is, dividend irrelevance implies that if dividends are reduced in favour of investments then the additional investments should return the cost of equity (i.e., increase subsequent earnings by a sufficient amount), and vice versa. See, e.g., the discussion in Penman & Sougiannis (1997a) about the "dividend displacement property" that links dividends to subsequent years' earnings (net profits), or Feltham & Ohlson (1994), section V.

[2] The economic value added (EVA) concept (a trademark in USA owned by Stern & Stewart) is also known under the name *economic profit*. Economic profit is the term used by Copeland, Koller & Murrin (1994). Further, Frankel & Lee (1996) refer to this concept as the *economic value added to long-term investors*. In this essay, to avoid any confusion, this concept will exclusively be referred to as EVA, economic value added.

- the *abnormal earnings* (AE) model[3], where equity value is calculated as the present value of all future abnormal earnings plus the book value of equity, and

- the *adjusted present value* (APV) model,[4] where total company value is calculated as the (unlevered) value of the company's operations plus the value of financing (interest tax shields in particular).

One problem is that when these models are implemented for company valuation under guidance of textbooks in the area (e.g., Copeland, Koller & Murrin (1994), Stewart (1991), and Copeland & Weston (1988)), the different models may give different results, even though they are theoretically equivalent. An implication of the findings in Essay 2 is that a valuation using the Copeland, Koller & Murrin (1994) (CKM)[5] version of the FCF model will not generally be equal to a valuation using the DIV model (in a world with corporate taxes). But Essay 2 shows that the equivalence can be achieved if the changing capital structure in market terms is taken into consideration by the discounting procedure. This result is further elaborated upon in Essay 3 where it is shown how the discounting procedure can also handle a non-constant cost of equity capital with respect to varying leverage over time in accordance with several different cost of capital models, e.g., the Miles & Ezzell (1980) model.

The main purpose of this essay is to analyse the factual relations between the different valuation concepts and models, and to provide an implementable company valuation framework (with corporate taxes) where the valuation result is independent of the choice of valuation model. Further, the essay will discuss the link between the expected future characteristics of the company and different cost of capital concepts, and show how this link can be incorporated in the valuation models through the discounting procedure.

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[3] The abnormal earnings concept is also referred to as *residual earnings* and *residual income*, and *economic value added to equityholders* (Frankel & Lee (1996)). In this essay the term abnormal earnings (AE) will be used.

[4] For company valuation purposes, APV may be regarded more as an alternative calculation technique than an independent valuation model (see further section 2.2.5). It should also be noted that the APV model differs from the other models in one main aspect: APV usually involves more than one discount rate function since the operating cash flows are explicitly separated from financing effects.

[5] Copeland, Koller & Murrin (1994) will henceforth be referred to as CKM.

This essay is developed in a discounting framework where all risk- and time-preferences are reflected in the discount rate function. This approach is supported by, e.g., CKM for FCF valuation and by Stewart (1991) for EVA valuation. Alternative approaches are capitalisation (as opposed to discounting) techniques,[6] approaches using an assumption of risk neutrality, or approaches where risk-adjustments are carried out in the numerator (as opposed to risk-adjusted discount rates), i.e., where the expected values are risk-adjusted following Rubinstein (1976).[7] The approach following Rubinstein is theoretically very interesting, since the valuation expressions are of an appealing, transparent form. However, the risk-adjustment of the valuation attribute tends to be a very complex issue in practice, involving complicated contingent probability measures,[8] and this makes this approach hard to implement.

It should be recognised that equivalence results have been established by other authors, but in different settings. Feltham & Ohlson (1995) show that the DIV, FCF, EVA and AE models are equivalent in infinite valuations of going concerns. In their framework there is no taxes and investors are risk-neutral, which together means that all discounting operations are carried out at the risk-free rate. Their model can be extended to the (more general) approach with risk-adjusted expectations following Rubinstein (1976), but as mentioned above this approach may be complicated to implement. Penman (1997) shows that the AE, FCF and EVA models (in a finite valuation context)[9] can be "recast" as the DIV model, given appropriate terminal value calculations. But the Penman (1997) analysis is explicitly demarcated from the specifications of costs of capital: The cost of equity is assumed to be non-stochastic and flat.[10] The free cash flows are discounted at the weighted average cost of capital such that the value for operations "consistent with [Modigliani & Miller] (1958) is

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[6] For an overview see, e.g., the Penman (1997) paper, which derives both discounting and capitalisation based versions of company valuation models.

[7] For example, Feltham & Ohlson (1995) assume risk-neutrality and apply the risk-free rate as discount rate for dividends and abnormal earnings. Ohlson (1990) and Feltham & Ohlson (1994) specify how risk-adjusted expectations along with risk-free discount factors can be used in order to deal with systematic risk and risk-averse investors.

[8] See, e.g., section IV of Feltham & Ohlson (1994).

[9] See section 2.4 below for a discussion of the term "finite valuation".

[10] As in Feltham & Ohlson (1995), it is claimed that it is possible to extend the results to the Rubinstein (1976) setting with risk-adjusted expectations of cash flows and earnings along with (varying) stochastic discount rates. But again, if we are considering implementations, this is not a very useful approach.

independent of the level of financial assets (or debt)"(p. 10).[11] But this implies that the cost of equity is a function of leverage, and for the cost of equity to be flat the leverage ratio must be (expected to be) constant over time.

An important contribution of this essay is to translate the equivalence results of Feltham & Ohlson (1995) and Penman (1997) into an implementable context 1) where all risk-adjustments are done through the discount rates (i.e., the appropriate costs of capital), 2) where corporate taxation (including tax deferrals) can be handled, 3) where the specifications of the costs of capital are explicitly considered and linked to the anticipated future development of the company (i.e., cost of equity, cost of debt, and weighted average cost of capital allowed to be non-flat), and 4) where the valuation concepts are defined in terms of explicitly forecasted financial statements of the same type as in annual reports. The equivalence is also extended to the APV model. Moreover, the equivalence is shown to hold on a year-to-year basis (and not only when considering infinite valuations or finite valuations over specific horizons), thus revealing that the models can be combined in any way. Further, advantages and disadvantages of the models as well as implementation related issues are discussed. The implementations of the different valuation models are visualised through valuations of the Swedish firm Eldon AB.

The basic foundation of the framework in this essay is that explicit forecasts of future financial statements are made, as proposed by CKM. The question of how these forecasts can be made is discussed in CKM and Essay 2. Here, only the structure of the forecasted financial statements and the relations between the items in these statements are modelled. The whole approach is suited for implementation in a spreadsheet model where one typically assigns a column to each year and a row for each accounting item. An implementation is exemplified by the Eldon AB case (see Appendix 4 for the forecasted financial statements). The usefulness of explicitly forecasting the future development of the company's balance sheets and income statements is that it makes it possible to undertake explicit account analyses of many different types.[12]

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[11] This implies that the weighted average of cost capital is constant and equal to the unlevered cost of equity, which is the case only if there are no taxes or taxes are irrelevant.

[12] If the result is based on an explicit forecast of balance sheets and income statements, it is possible to translate the forecasted development of the company into well-known economic concepts as return on owner's equity, profit margin, asset turnover, consolidation ratio, etc. See further the discussion in Essay 1, section 2.1.

1.1 Organisation of the essay

The essay has the following organisation: In section 2 the modelling framework of the essay is presented, the underlying assumptions are stated, and the different valuation concepts and models defined (see Appendix 1 for a summary of the notation used). In section 3 the analysis of the different concepts and models is carried out and the results are presented. Moreover, the five different valuation models are implemented using the Eldon AB case as example (see further Appendices 4 and 5 for details). In section 4, the differences between the models are discussed. Finally, in section 5, the results of the essay are summarised and further discussed.

2. A framework for Company Valuation

In section 2.1 the notation and the basic accounting assumptions are presented, before section 2.2 defines the different valuation concepts in terms of the accounting items, and how the valuation concepts are related to the equity value of the company according to the five models considered in this essay. Section 2.3 introduces different discounting procedures, and section 2.4 discusses the horizon value concept. Finally, section 2.5 considers valuation of companies with finite lives.

2.1 Notation and accounting related assumptions

The valuation framework in this essay is based on forecasts of future financial accounting statements of the type that is officially published in annual reports. Consider a company whose forecasted future balance sheets can be expressed as:[13]

[13] See Appendix 1 for a summary of used notation, relations and definitions.

Assets	Debt and Equity
Working Capital Assets, WA_t	Working Capital Liabilities, WL_t
Net Property, Plant & Equipment, N_t	Debt, D_t
= Gross Property, Plant & Equipment, G_t	Deferred Taxes, T_t
- Accumulated Depr., A_t	Book Equity, BV_t

Forecasted balance sheet (year t)

The invested capital, IC_t, is defined according to the following balance sheet relation:

(BSR1) $$IC_t \equiv (WA_t - WL_t) + N_t$$

Equivalently, the following must hold:[14]

(BSR2) $$D_t + T_t + BV_t = IC_t$$

However, one can view the relation between invested capital and deferred taxes differently. It is thus useful to define an alternative specification of invested capital, IC_t^* :[15]

(IC) $$IC_t^* \equiv IC_t - T_t = D_t + BV_t = (WA_t - WL_t) + N_t - T_t$$

[14] The inclusion of deferred taxes in invested capital in relation (BSR2) follows the standard exposition in valuation textbooks as CKM and Stewart (1991). The argument behind this is that "investors expect the company to continue to earn a return on the capital saved as a result of tax deferrals" (CKM, p. 159). Deferred taxes are however non-interest bearing, and arise as a consequence of temporary differences between the *tax* and the *financial* records of the company's assets and liabilities (see, e.g., Holthausen & Zmijevski (1996), Chapter 5B). Note that deferred taxes in financial accounting may be divided into *long*- and *short*-term deferred taxes. T_t is here regarded as the sum of these two categories.

[15] The general notation for invested capital (i.e., for IC_t or IC_t^*) will be Cap_t.

The company's projected income statement for any future year is defined as:

+Revenues, R_t
-Operating Expenses, OpX_t
-Depreciation Expense, $DepX_t$
=Operating Profit, OP_t
-Interest Expense, $IX_t = i_t \cdot D_{t-1}$
-Taxes, $IT_t = \tau_t (OP_t - IX_t)$
=Net profit, NP_t

Forecasted income statement (year t)[16]

In addition to operating profit (before taxes), OP_t, and net profit, NP_t, it may be useful to define a few other profit concepts. First, operating profit after taxes, $OPAT_t$, is defined as:

(OP) $$OPAT_t \equiv (1-\tau_t) \cdot OP_t$$

There is another important profit concept often considered in valuation textbooks: *net operating profit less adjusted taxes*, $NOPLAT_t$. In terms of the used notation it is defined in the following way:[17],[18]

(N1) $$\begin{aligned} NOPLAT_t &\equiv OP_t - \left(IT_t - (T_t - T_{t-1}) + \tau_t \cdot IX_t\right) \\ &= OPAT_t + (T_t - T_{t-1}) \\ &= NP_t + (1-\tau_t)IX_t + (T_t - T_{t-1}) \end{aligned}$$

The reverse relation between NP_t, OP_t, and $NOPLAT_t$ is consequently:

(N2) $$\begin{aligned} NP_t &= (1-\tau_t)(OP_t - IX_t) \\ &= NOPLAT_t - (1-\tau_t)IX_t - (T_t - T_{t-1}) \end{aligned}$$

Further, let us denote the yearly growth in deferred taxes as:

(DT) $$g_{T,t} \equiv \frac{(T_t - T_{t-1})}{T_{t-1}}$$

[16] The tax rate, τ_t, is the expected corporate tax rate that determines the income statement item Taxes, IT_t.

[17] This concept can be thought of as the net profit if 1) the company is financed with equity only, and 2) if only cash operating taxes are considered. See further, e.g., CKM, pp. 155-159, or Stewart (1991), p. 742.

[18] The general notation for (after-tax) operating profit (i.e., $NOPLAT_t$ or $OPAT_t$) will be Π_t.

In order to link the development of the debt and equity side of the balance sheet with the income statement, it is assumed that the clean surplus relation (CSR) holds. The clean surplus relation means that the change in net book value of equity equals net profit *minus* net dividends[19] (see, e.g., Ohlson (1995)):

(CSR) $BV_t = BV_{t-1} + NP_t - DIV_t$

Similarly, the property, plant and equipment (PPE) items are assumed to develop according to the following relations:

$$\text{(PPE)} \quad \begin{aligned} A_t &= A_{t-1} + DepX_t - Ret_t \\ G_t &= G_{t-1} + CapX_t - Ret_t \end{aligned}$$

where Ret_t and $CapX_t$ respectively denote the retirements[20] and capital expenditures[21] in year t.

The company debt is assumed to be on market terms, i.e., the book value of debt is equal to the market value of debt. Thus, the (effective) coupon rate in year t, i_t, is assumed to equal the (market) cost of debt for the same year (thus, i_t will denote both the coupon rate and the cost of debt).[22] It is assumed that the company has only operating assets, which means that there are no excess marketable securities (EMS) in the forecast period.[23]

[19] That is, *net* of capital contributions.

[20] The retirements are measured in gross book value of the retired assets. If an asset is retired before it has been fully depreciated, the remaining (i.e., non-depreciated) net book value is depreciated. Consider an example: a machine was initially purchased for SEK 1000. It is written off linearly in 5 years:

	G_t	$DepX_t$	A_t	N_t
$t = 1$:	1000	200	200	800
$t = 2$:	1000	200	400	600

Retiring the machine in year 3 means that for this particular machine: Ret_3=1000 and $DepX_3$=600, so that $A_3 = G_3 = N_3 = 0$.

[21] The capital expenditures can thus be expressed as: $CapX_t = G_t - G_{t-1} + Ret_t = N_t - N_{t-1} + DepX_t$.

[22] In practical applications the cost of debt is typically assumed to be constant. The subscript t is used here in order to emphasise that it (from a modelling point of view) is fully possible to use a non-flat cost of debt, as long as the cost of debt for each year can be assumed to be equal to the effective borrowing rate in the same year.

[23] See Appendix 7 for a discussion about how excess marketable securities existing at the valuation date can be treated.

Further, there is no preferred stock: book equity consists of common stock and retained earnings. Deferred taxes are treated as "a quasi-equity account".[24] For valuation purposes they are not really assets or liabilities; the deferred taxes "are important only to the extent that we need them to calculate income tax payments."[25] That means that the net flow to/from the deferred taxes account (i.e., the net savings on taxes to be paid) each year is included in the FCF calculation to adjust the income statement item *Taxes* to a cash basis.[26] Only corporate taxes are considered.

Dividends are assumed to be paid out at the end of each year. However, in Sweden the actual dividend pay out is delayed a few months. It is recognised that the equity value obtained from the models can be parallel shifted to reflect the 'delay'.[27]

Note that time index t on balance sheet or other stock items means "at the end of year t", or equivalently, "the beginning of year $t+1$". That is, if we are valuing the equity of a company as of January 1, 1996, this will be denoted as EV_{1995}. Further, T (without index) denotes the expected end of life of the company. Typically, $T=\infty$ for going concerns, but can in specific cases be a finite year.[28] Note that T has nothing to do with the concept of valuation horizons. Instead, H will denote a valuation horizon, at which the future beyond the horizon is accounted for by some horizon value (terminal or continuing value) calculation, if such a calculation is appropriate (see further section 2.4 below). The period up to the horizon, H, will be referred to as *the explicit forecast period* under which the valuation attributes are forecasted year by year (from projected financial statements).

The present value operator is applied in the following way:

> $\mathrm{PV}_t^{k_X}[X]$ = the present value at time t of X, where k_X is the appropriate discount rate function for X.

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[24] CKM, p. 162.

[25] Holthausen & Zmijewski (1996), Chapter 5B, p. 6.

[26] That is, the tax cost is adjusted for changes in deferred taxes. Note also that actual tax *payments* of the tax pertaining to year s often occur in year $s+1$, and that this amount is thus included in working capital liabilities (at the end of year s).

[27] For example, if dividends are paid out at, say, April 30 in all future years, the equity value obtained from the models, EV_0, can be parallel shifted a third of a year, such that $EV_0^* = \frac{EV_0}{(1+k_E)^{1/3}}$ (where k_E is the cost of equity capital).

[28] For example, think of a Swedish nuclear plant which clearly will have a finite life.

Note also that the expectations operator is suppressed for all forecasted items. For example, R_{2000} denotes the *expected value* of revenues in year 2000 given the information at the valuation date t, I_t, and *not* the realisation. That is, $\mathrm{E}_t\left(\tilde{R}_{2000} \mid I_t\right)$ is here for simplicity denoted R_{2000}.

Finally, the *market* values of the company's assets and liabilities are denoted in the following way:

Assets (market values)	**Debt and Equity (market values)**
Operating Assets, OA_t	Debt, D_t
Present Value of Interest Tax Shields, $PVTS_{t,all}$	Equity, EV_t
=Company Value, TCV_t	=Company Value, TCV_t

Forecasted balance sheet in market terms at the end of period t

In this formulation, OA_t is the *unlevered* market value of operating assets (i.e., independent of financing). Since the company is assumed to have only operating assets (i.e., does not have any financial *assets*), the term *company value*, TCV_t, thus refers to the *levered* market value of operating assets, which is dependent on (the tax shields, $PVTS_{t,all}$, from) financing.

2.2 Valuation models

Here, different valuation models that have been proposed for valuing companies will be introduced. Each model relates an accounting concept (valuation attribute) to the market value of equity, EV_t. Here, the relations between the different valuation attributes and the equity value are defined. Further, the valuation attributes are defined in terms of the accounting variables defined in the previous section.

Note again that T denotes the *end of life of the company* which may be a finite year *or* infinity (i.e., $T=\infty$). This T should not be confused with the *horizon* concept (denoted H, and discussed in section 2.4 below). In most real-world cases, $T=\infty$, and in the valuation formulas in this essay, T can thus (almost always) be replaced by ∞, except for those specific cases where the company is expected to have a finite life. How the final year T for a company with finite life should be treated for valuation purposes is discussed in section 2.5.

Also, note that the FCF and EVA models involve a two-step procedure: first TCV_t is calculated, then D_t is subtracted to arrive at EV_t. In other words, the valuation attributes FCF and EVA are directly related to the *total* market value of the company, which means that the present value calculations should be carried out with a discount rate that takes the company's entire financing and risk structure into account. This is traditionally done using the *weighted average cost of capital* (WACC). In contrast, the DIV and AE valuation attributes are directly related to the equity of the company. Consequently, these attributes are to be discounted at the cost of equity capital.

2.2.1 The dividend model

In the DIV model, the market value of equity, EV_t, is simply equal to the *present value of all future net payments (dividends) to the equity owners*:

(EV:DIV) $$EV_t(DIV) \equiv \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_E}\left[DIV_s\right]$$

This is the method often used as the correct benchmark in company valuation.[29] As a theoretic starting point this is non-controversial, although it may be less useful in practice, since the dividend pay-out sequence can be regarded as a policy question, and as a such may be 'irrelevant' for the valuation result (i.e., the timing of the dividend pay-outs may be irrelevant for value).[30] A fact that sometimes is overlooked in the literature, but recognised by, e.g., Penman (1997), is that *any* valuation approach that satisfies CSR accounting gives an implicit forecasted dividend sequence.

The expected dividends for a particular year *t* in the forecast period are calculated using (CSR):

(D1) $$DIV_t = BV_{t-1} + NP_t - BV_t$$

A useful equivalent statement (I utilise (BSR2)) is:

(D2) $$DIV_t = NP_t + (T_t - T_{t-1}) + (D_t - D_{t-1}) - (IC_t - IC_{t-1})$$

(D2) is the dividend analogue to the FCF formula (F2) below, and describes the relation between the anticipated financing, dividend, and investment decisions of the company. The dividend relations

[29] For example by Ohlson (1995), Skogsvik (1994), and Feltham & Ohlson (1995).

[30] See, e.g., section V of Feltham & Ohlson (1994).

(D1) and (D2) should be treated consistently with the accounting rule that DIV_t reduce BV_t but leaves NP_t unaffected.[31] Equation (D2) demonstrates that the (cash-tax adjusted) net profit can be used for paying dividends, for investments, and for reducing debt. It follows that dividends, 1) reduce the investments dollar for dollar (given a specific financing decision), and 2) reduce book equity dollar for dollar (independently of financing). See further Essay 1, section 2.3, for a discussion about how these issues relate to the topic of *dividend irrelevance.*

2.2.2 The free cash flow model

The FCF model suggests that the *total company value,* TCV_t, is equal to the present value of all future free cash flows generated by the company. To arrive at the *equity value,* EV_t, the debt value at the valuation date has to be subtracted from TCV_t:

(EV:FCF) $$EV_t(FCF) \equiv \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_{WACC}}[FCF_s] - D_t$$

Essay 2 as well as CKM describe the free cash flow concept and I refer to them for more detailed discussions. Following CKM, the free cash flow in year *t*, FCF_t, is defined in the following way:

(F1) $$\begin{aligned} &FCF_t \equiv \\ &\equiv NOPLAT_t + DepX_t && \text{(Gross FCF)} \\ &-((WA_t - WL_t) - (WA_{t-1} - WL_{t-1})) - CapX_t && \text{(-Gross investment)} \end{aligned}$$

This definition can be reformulated by first using (PPE) and (BSR1), and then applying (N1) and (IC):

(F2) $$\begin{aligned} &FCF_t = \\ &= NOPLAT_t + DepX_t \\ &-((WA_t - WL_t) - (WA_{t-1} - WL_{t-1})) - (N_t - N_{t-1} + DepX_t) = \\ &= NOPLAT_t - (IC_t - IC_{t-1}) = \\ &= OPAT_t - (IC_t^{*} - IC_{t-1}^{*}) \end{aligned}$$

Equation (F2) tells us that the FCF concept measures what is left (for distribution to the investors) from the (after-tax) operating profit *after* investments have been done. Note that FCF is independent

[31] The importance of this rule in (CSR)-based accounting models was recognised by Ohlson (1995).

of financing and that it sometimes is referred to as *the free cash flow of the unlevered firm.* Note also that the FCF concept is used both in the FCF model and in the APV model.

2.2.3 The economic value added model

The third valuation approach here is the concept of *economic value added* (EVA). EVA is most strongly advocated by Stewart (1991). Stewart favours EVA before free cash flow on the basis of its usefulness as a managerial tool. He claims that the FCF method "fails to provide any meaningful measures to assess progress in creating value or useful benchmarks to judge performance",[32] but that the EVA concept is good for these tasks.

What is EVA then? - EVA, the economic value added, is a measure of how much value the operations/management of the company in a particular period has added to the existing invested capital. Let us first make a general definition (i.e., without direct linkage to the previously defined accounting concepts):

(EVA) $$EVA_t \equiv \Pi_t - k_{WACC,t} \cdot Cap_{t-1}$$

where Π_t is an (after-tax) operating profit concept, and
Cap_{t-1} is year t's beginning (invested) capital.

To arrive at the EVA one thus deducts a capital charge for the capital in use from the after-tax profit from operations. The EVA model defines equity value as:

(EV:EVA) $$EV_t(EVA) \equiv Cap_t + \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_{WACC}}[EVA_s] - D_t$$

Thus, the total company value is obtained by adding the present value of all future years' expected EVA to the existing capital, and then the market value of debt is deducted to arrive at the equity value. The present value of all future years' EVA is commonly referred to as the *market value added* (MVA).

The purpose for introducing the (general) notation Π_t and Cap_{t-1} is that it may be useful to analyse two alternative (joint) specifications of the after-tax net operating profit and the invested capital: As EVA is defined by Stewart (1991), $\Pi_t = NOPLAT_t$, and $Cap_{t-1} = IC_{t-1}$. This definition of EVA thus uses operating profit after *cash* taxes and includes deferred taxes in the invested capital. From an

[32] Stewart (1991), p. 350

accounting point of view, it may however be more straight-forward to consider accounted taxes when calculating the (after-tax) operating profit, and to treat deferred taxes as a negative item on the asset side. In this alternative way of defining EVA, $\Pi_t = OPAT_t$, and $Cap_{t-1} = IC^{*}_{t-1}$.

In the numerical example of Eldon AB, EVA will be exemplified by the Stewart definition. However, in the theoretical analysis both specifications will be explicitly considered.

2.2.4 The abnormal earnings model

The AE method calculates (market) equity value as the book value of equity plus a premium (often called *goodwill*) which is calculated as the present value of all future years' abnormal earnings:

$$\text{(EV:AE)} \qquad EV_t(AE) \equiv BV_t + \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_E}\left[AE_s\right]$$

Abnormal earnings simply mean earnings above (or below) the 'normal' level of earnings.[33] This concept is advocated by, e.g., Feltham & Ohlson (1994) and Ohlson (1995). Brief & Lawson (1992) and Skogsvik (1994, 1997) use a modified version, where the abnormal earnings are measured as the above cost of capital return on book equity.[34] AE is, in fact, very similar to the EVA concept, but is more directly related to the equity of the company, since the capital charge here is calculated using book *equity* (instead of invested capital as in EVA) and the required rate is the cost of *equity* capital:

$$\text{(AE)} \qquad AE_t \equiv NP_t - k_{E,t} BV_{t-1}$$

Note that some authors (e.g., Frankel & Lee (1996)) refer to the abnormal earnings concept as *EVA to equityholders*. EVA as defined in this essay is by Frankel & Lee referred to as *EVA to long-term investors*. As we will see in Proposition 2 these two concepts are not the same.

[33] 'Normal' here means earnings on a level such that the return on book equity equals the cost of equity capital.

[34] $AE_t = BV_{t-1} \cdot \left(r_t - k_{E,t}\right)$ where $r_t = \dfrac{NP_t}{BV_{t-1}}$ is the expected book return on owners' equity (Skogsvik (1997)).

2.2.5 The adjusted present value model

The APV model, originally derived by Myers (1974) as a general approach for project valuation, calculates the value of a project by adjusting "the direct contribution" of the project to company value for "side effects" that "occur because of the project's effects on debt capacity and sources uses constraints" (p. 4). By *direct contribution* is meant the project's contribution to company value under the assumption of all equity financing, and is calculated by discounting the expected net after-tax cash flows from each future period at the unlevered cost of (equity) capital. Considering a company as the sum of the company's projects (which of course is uncontroversial in the APV context since APV measures each project's contribution to company value) simply means that summing the direct contributions of the company's projects yields the (unlevered) market value of the company's operations, to which the present value of future tax shields is added to yield the total company value. Note also that aggregating the expected net after-tax cash flows from *all* projects in a particular year yields the FCF in that year. Hence, FCF is the appropriate valuation attribute when considering the (original) APV model for company valuation purposes.

The APV model for company valuation[35] is thus similar to the FCF model in that it also uses the future free cash flows as valuation attribute. However, the *present value* of the forecasted FCF is in the APV model computed as if the company were all-equity financed. To this unlevered present value (of the company's operating assets) the value of financing, $PVTS_{t,all}$,[36] is added to yield the total company value:

(EV:APV)
$$EV_t(APV) \equiv OA_t + PVTS_{t,all} - D_t = \\ = \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_U}[FCF_s] + PVTS_{t,all} - D_t$$

where $PVTS_{t,all} = \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_{TS}}[TS_s]$ and

(TS) $TS_t = \tau_t \cdot i_t \cdot D_{t-1}$

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[35] The APV model has been used for company valuation by, e.g., Jennergren & Näslund (1996). A modified version is used by Kaplan & Ruback (1995).

[36] $PVTS_{t,all}$ is the present value at the beginning of year t+1 of *all* future tax shields from financing. This present value, $PVTS_{t,all}$, may be obtained using more than one discount rate (the discount rate *function* is denoted by k_{TS}). In the case of two discount rates (say, i_t and k_U) for the tax shields, we have: $PVTS_{t,all} = PVTS_{t,i} + PVTS_{t,k_U}$ where $PVTS_{t,i}$ is the part of the tax shield value that, *during year* t+1, has been discounted at the cost of debt, i_{t+1}, and $PVTS_{t,k_U}$ is the part of the tax shield value that, *during year* t+1, has been discounted at the unlevered cost of equity, k_U.

The basic implication of the APV model is the separation of the value of operations from the value of financing. This *idea* does not necessarily imply that FCF has to be used as valuation attribute; Indeed, the APV model can be implemented using valuation attributes that are independent of financing. For example, as the reader can verify, it is fully possible to implement the separation idea on the EVA model. Thus, APV may be viewed more as a calculation *technique* (that targets unlevered flows and financing flows at (possibly) different discount rates) than an independent valuation model. But as originally defined by Myers (1974) the intention was clearly to provide a new (and more general) model for evaluating investment opportunities, and APV is in light of this referred to as a model in this essay.

Since the APV model is similar to the FCF model it is directly related to the total company value. However, it involves at least two different discounting operations: the free cash flows are discounted at the *unlevered cost of equity*, k_U, while the tax shields are discounted at some appropriate rate(s), often the cost of debt or the unlevered cost of equity (or a combination of the two). One advantage of APV is that k_U generally is considered as being constant, making the FCF discounting operation easy to perform. The simplicity of the tax shield discounting operation depends, however, on the assumptions made about the valuation of the tax shields. If *one* constant rate (e.g., the unlevered cost equity, k_U) is the appropriate discount rate, the TS discounting operation is easy to perform. However, given other assumptions, e.g., the Miles & Ezzell (1980) framework, the TS discounting operation will be more complicated.

As discussed in Essay 3 there are three main cases of TS valuation: the tax shields are discounted 1) at the cost of debt, i_t, 2) at the unlevered cost of equity, k_U, or 3) by a combination of i_t and k_U. In the first case, the underlying assumption is that the company follows a pre-specified borrowing plan, so that the debt (as well as the tax shield) is known at each future date. Thus, in this *passive debt management* context, the interest tax shields are regarded as being as risky as the debt, and consequently, the appropriate discount rate is the cost of debt. The cost of capital model derived by Modigliani & Miller (1963) is a special case of this principle.[37] Secondly, as suggested by Myers (1974), the debt can be adjusted to maintain a constant leverage (i.e., *active debt management*) implying that k_U is the appropriate discount rate, since the debt outstanding will be "a random variable perfectly correlated with APV and *thus has the same risk characteristics*" (p. 22). The higher discount rate can also be motivated by another argument: Since there exist a possibility that the

[37] Even though the cost of debt in their original setting equals the risk-free rate.

company may not be in tax paying position in *all* future years, all tax shields may not be exploited, and thus the tax shields bear a higher risk than the debt.[38]

Based on the assumption of a periodically adjusted debt to maintain a target capital structure, the Miles & Ezzell (1980) analysis makes one distinction from the Myers proposal: Using a backward valuation procedure, they assume that the appropriate rate at *any* (future) valuation date s for the tax shield from the interest payment in year s+1 is the cost of debt, whereas the appropriate rate for discounting the (already calculated) value of future tax shields at date s+1 (i.e., $PVTS_{s+1,all}$) is the unlevered cost of equity. The argument behind this valuation scheme is that (at any valuation date) the interest tax shield from the interest payment the immediately following year is as risky as the debt, while the rest of the tax shields at that particular moment is as risky as the operations of the company. In light of this, using the unlevered cost of equity as the one and only discount rate for interest tax shields, as proposed by Myers (1974) and Harris & Pringle (1985), can be interpreted as *"perfect" active debt management*, implying that the tax shields always are as risky as the operations of the company, which could be the consequence of *continuously* adjusting the leverage to a specific target. See further Essay 3 (section 2) for a discussion of how these different tax shield valuation principles relate to the calculation of the (levered) cost of equity and the WACC.

2.3. Discounting procedures

The next issue to consider is how to perform the present value calculations, i.e., what discounting procedure to use. The use of a *constant* WACC is what has been proposed in the literature (e.g., by CKM and Copeland & Weston, 1988) for the FCF model, and (by Stewart (1991)) for EVA. Using a constant WACC in FCF valuation in the tax regime considered here will *not* give the same valuation result as the traditional benchmark, the DIV method, unless the forecasted capital structure is constant in market terms, as has been shown in Essay 2. The constant WACC will be denoted by $\bar{k}_{WACC}$.

[38] See, e.g., Harris & Pringle (1985), section IV.

(PV_{const}:DIV)	$\sum_{s=t+1}^{T} PV_t^{k_E}[DIV_s] = \sum_{s=t+1}^{T} \frac{DIV_s}{(1+\bar{k}_E)^{s-t}}$
(PV_{const}:FCF)	$\sum_{s=t+1}^{T} PV_t^{k_{WACC}}[FCF_s] = \sum_{s=t+1}^{T} \frac{FCF_s}{(1+\bar{k}_{WACC})^{s-t}}$
(PV_{const}:EVA)	$\sum_{s=t+1}^{T} PV_t^{k_{WACC}}[EVA_s] = \sum_{s=t+1}^{T} \frac{EVA_s}{(1+\bar{k}_{WACC})^{s-t}}$
(PV_{const}:AE)	$\sum_{s=t+1}^{T} PV_t^{k_E}[AE_s] = \sum_{s=t+1}^{T} \frac{AE_s}{(1+\bar{k}_E)^{s-t}}$
(PV:OA/APV)	$\sum_{s=t+1}^{T} PV_t^{k_U}[FCF_s] = \sum_{s=t+1}^{T} \frac{FCF_s}{(1+k_U)^{s-t}}$

Table 1 - Present value calculations with constant discount rates

Analogously, the constant cost of equity will be denoted by $\bar{k}_E$ and the constant cost of debt as $\bar{i}$. The constant WACC, $\bar{k}_{WACC}$ is defined as:

$$(\text{WACC}_{const}) \quad \bar{k}_{WACC} = \frac{D_t}{TCV_t}(1-\bar{\tau})\bar{i} + \left(1 - \frac{D_t}{TCV_t}\right)\bar{k}_E$$

where t typically refers to the valuation date (or to a normalised year, where some target capital structure is reached), and $\bar{\tau}$ denotes the (constant) corporate tax rate.

The present value calculations, using the concept of *constant* discount rates, are presented in Table 1. Next, the discounting procedure developed in Essays 2 and 3 will be considered.[39] The procedure's main difference from the previous one is that the discount rate is allowed to vary over time according to the anticipated capital structure. The backward discounting procedure works in the following way: One starts at a future point in time where the equity value either is 0 (zero)[40] or can be expressed as a horizon value.[41] One then goes backwards, one year at a time, and computes the equity value at the beginning of each year, according to a yearly dynamics equation (see below), using a yearly adjusted discount rate that reflects the anticipated capital structure at each point, until one finally reaches the

[39] This procedure will henceforth be referred to as the *backward discounting procedure.*

[40] Typically at the end of the life of a company with projected finite life, i.e., at the end of year T.

[41] The use of horizon values is commented upon in the next section.

valuation date. For a detailed description of how the procedure is implemented for the FCF case: see Essay 3.[42] There, it is also shown how this procedure is implemented in a spreadsheet program like Microsoft Excel.

The cost of *equity* capital for year t is denoted by $k_{E,t}$. Assume that $k_{E,t}$ is some function that defines the cost of equity capital in terms of the state of nature at the beginning of year t.[43] Consequently, the weighted average cost of capital can be defined as:

$$\text{(WACC)} \qquad k_{WACC,t} = \frac{D_{t-1}}{TCV_{t-1}}(1-\tau_t)i_t + \left(1 - \frac{D_{t-1}}{TCV_{t-1}}\right)k_{E,t}$$

In order to use the backward discounting procedure we must establish the yearly dynamics of the equity value in terms of the different valuation models. The yearly dynamics equations explain how the equity value can be written as functions of the valuation attributes and the equity value one year ahead. The yearly dynamics[44] for the four valuation models considered here are presented in Table 2.

To see how the yearly dynamics equations have been derived, let us look at the most complicated case, the EVA concept. According to the EVA model, market value of the company at time t, TCV_t, can be written:

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[42] When using the backward discounting procedure for the other valuation models one can follow the description in Essay 3 with the following adjustments:

- EVA model: Note that EVA itself depends on the WACC. The 'trial' WACC should thus be used as basis for the EVA calculations.
- DIV model: Since the discount rate in the DIV model is the cost of equity capital, one uses a 'trial' and 'resulting' $k_{E,t}$ instead of WACC.
- AE model: As in the DIV model one uses a 'trial' and 'resulting' $k_{E,t}$. Moreover, the 'trial' $k_{E,t}$ should be used in the calculations of the yearly AE.

[43] This function could, e.g., be the formula initially provided by Modigliani & Miller (1963):

$k_{E,t} = k_U + (1-\tau_t)(k_U - i_t)\frac{D_{t-1}}{EV_{t-1}}$ where k_U is the cost of equity for an unlevered company. See Essay 3 for a discussion about different cost of capital settings, and the implications of different cost of capital formulas. Typically, the cost of equity is a function of the company's leverage.

[44] The DIV and FCF dynamics were provided in Essay 2 (Chapter 3).

(EV_{dyn}:DIV)	$EV_{t-1} = \frac{DIV_t + EV_t}{1 + k_{E,t}}$
(EV_{dyn}:FCF)	$EV_{t-1} = \frac{FCF_t + EV_t + D_t}{1 + k_{WACC,\,t}} - D_{t-1}$
(EV_{dyn}:EVA)	$EV_{t-1} = Cap_{t-1} + \frac{EVA_t + EV_t + D_t - Cap_t}{1 + k_{WACC,t}} - D_{t-1}$
(EV_{dyn}:AE)	$EV_{t-1} = BV_{t-1} + \frac{AE_t + EV_t - BV_t}{1 + k_{E,t}}$

Table 2 - The yearly equity value dynamics for the models with one discount rate function

(TCV:EVA) $$TCV_t = Cap_t + \sum_{\tau=t+1}^{T} \mathrm{PV}_t^{k_{WACC}}\left[EVA_\tau\right]$$

(TCV:EVA) can be rewritten as:

(2.3:1) $$TCV_t = Cap_t + \frac{EVA_{t+1}}{1 + k_{WACC,t+1}} + \frac{\sum_{\tau=t+2}^{T} \mathrm{PV}_{t+1}^{k_{WACC}}\left[EVA_\tau\right]}{1 + k_{WACC,t+1}}$$

Analogously, the value at time $t+1$ can be written:

(2.3:2) $$TCV_{t+1} = Cap_{t+1} + \sum_{\tau=t+2}^{T} \mathrm{PV}_{t+1}^{k_{WACC}}\left[EVA_\tau\right]$$

Rearranging (2.3:2):

(2.3:3) $$\sum_{\tau=t+2}^{T} \mathrm{PV}_{t+1}^{k_{WACC}}\left[EVA_\tau\right] = TCV_{t+1} - Cap_{t+1}$$

Substituting (2.3:3) into (2.3:1) yields:

(2.3:4) $$TCV_t = Cap_t + \frac{EVA_{t+1}}{1 + k_{WACC,t+1}} + \frac{TCV_{t+1} - Cap_{t+1}}{1 + k_{WACC,t+1}}$$

Shifting the time index finally yields:

$$(2.3{:}5) \qquad TCV_{t-1} = Cap_{t-1} + \frac{EVA_t + TCV_t - Cap_t}{1 + k_{WACC,t}}$$

By deducting the value of debt at the valuation date t-1, and using the TCV definition, we arrive at the equity value:

$$(\mathrm{EV}_{dyn}{:}\mathrm{EVA}) \quad EV_{t-1} = Cap_{t-1} + \frac{EVA_t + EV_t + D_t - Cap_t}{1 + k_{WACC,t}} - D_{t-1}$$

The three other models' yearly dynamics are derived through the same line of reasoning.

One starting point of the backward discounting procedure is that the unlevered cost of equity (i.e., the cost of equity for the company given all-equity financing), k_U, has been estimated. However, in some cases, it may be more practical to start out with an estimate of the first year cost of equity capital, $k_{E,t}$ (see further the discussion in section 3 below). However, this has implications for the valuation procedure: Specifically, an additional iterative procedure will be needed at the valuation date to find the unlevered cost of equity that is implied by the known cost of equity capital, $k_{E,t}$, and the resulting market capital structure at the valuation date t-1. In short, this means that the backward discounting procedure is repeated a number of times, using trial values on k_U, until the true k_U is found (i.e., the k_U that is consistent with the known $k_{E,t}$ and the resulting market capital structure). For more detailed descriptions of the proceedings in this case, see Appendix 3 as well as the Eldon AB implementation in Appendix 5.

2.4 Horizon values

When performing a valuation, one often makes explicit forecasts for a limited number of years only and accounts for the time after that with a horizon value.[45] The future is thus divided into two periods: the *explicit forecast period* and the *infinite future* after that, accounted for by the horizon value, and here called the *perpetuity period.* This section will discuss some of the most commonly used techniques for horizon value calculations.

The horizon value concept can be thought of in different ways. The typical textbook approach links the horizon value to the expected performance of the firm in perpetuity, and recommends the use of *continuing value formulas*. Alternatively, the horizon value can be based on the measurement error in the accounting, i.e., how stocks of value are recognised and measured by the accounting system in the valuation model (Penman (1997), p. 11). In this approach the explicit forecast period can be regarded as a *finite valuation,* while the horizon value can be thought of as *terminal value correction.*

The use of continuing value formulas to calculate the horizon value requires that the company is expected to have settled down to a steady state by the beginning of the period after the explicit forecast period, so that the valuation attribute(s) can be expected to grow at a constant rate. The steady state requirements involve, e.g., that the company is foreseen to grow at a constant rate, to earn constant margins, to maintain a constant capital turnover, and, therefore, is projected to earn a constant return on existing invested capital as well as on new investments.[46] Essay 2 shows that these requirements taken together imply that the forecasted debt ratio in market terms is constant in the steady state period.

Continuing value formulas thus involve very strong requirements about the company's forecasted performance, and the suggestion is that before using a continuing value in an actual valuation, one should thoroughly examine if these requirements are reasonable in the case in question. If not, a continuing value should *not* be used (at least not at that point in time). In this section it is assumed that the steady state requirements *are fulfilled* from the horizon H, which also is the case in the Eldon AB valuation (H=2005 in Eldon). When these requirements are not met, other means have to be used,

[45] The term *terminal value* is sometimes used instead of *horizon value* in the literature.

[46] See CKM p. 290 for the complete listing of required assumptions for use of continuing values, or Essay 2, Chapter 2, for an analysis of these requirements.

e.g., making a long explicit forecast of the necessary accounting items. For an example, see the abnormal earnings valuation of Eldon AB in Appendix 5, section A5.1. Another situation in which horizon values are inappropriate is for companies with finite *lives*, where explicit forecasts should be done over the lifetime of each company (see section 2.5 below).

The alternative approach specified in Penman (1997) makes use of the characteristics of the accounting system behind the particular valuation model, i.e., how the accounting in each model measures flows and book value of (the relevant) stock. Let us denote the expected values of 1) the flow item (in year s) by F_s and 2) the book value of the stock item (at the end of year s) by B_s. The DIV model is in this respect a pure flow model, i.e., the flow item is the dividends themselves, and there is no stock item. In the AE model the flow item is *net profit*, and the stock item is simply *book equity.* Considering the accounting system behind the FCF model, as specified by Penman, the relevant flow item is *not* free cash flow,but *free cash flow minus (after-tax) interest expense*, and the stock item is financial assets, which in this essay is represented by *(negative) debt.* Moreover, for the EVA model, the relevant flow item is the *(after-tax) operating profit minus (after-tax) interest expense*, and the stock item is (net) operating assets plus financial assets, which in this essay is represented by *invested capital minus debt.*[47] The horizon H is in the Penman approach chosen such that for S periods subsequent to H,

(2.4:1) $$EV_{H+S}^{CS} - B_{H+S}^{CS} = K_S\left(EV_H - B_H\right),$$

where CS denotes expected values *cum-dividend* from H to H+S, and K_S can be interpreted as the expected growth[48] (over S years) of the measurement error in the accounting (for book value). Then the generalised representation of the equity value at the horizon is (Penman (1997), equation (9)):

(2.4:2) $$EV_H = \frac{\sum_{s=1}^{S} F_{H+s}^{CS} - \left(K_S - 1\right)\cdot B_H}{\left(1+\bar{k}_E^H\right)^S - K_S}$$

[47] In formal notation:

- for the FCF model $\left[F_s; B_s\right] = \left[\left(FCF_s - (1-\tau_s)IX_s\right); \left(-D_s\right)\right]$,
- for the EVA model $\left[F_s; B_s\right] = \left[\left(\Pi_s - (1-\tau_s)IX_s\right); \left(Cap_s - D_s\right)\right]$,
- for the AE model $\left[F_s; B_s\right] = \left[NP_s; BV_s\right]$, and
- for the DIV model $\left[F_s; B_s\right] = \left[DIV_s; 0\right]$.

[48] That is, one plus the growth rate over S years.

In practice, the difficulty lies in calculating K_S. As is recognised by Penman (pp.16-17), a *constant* K_S between any subsequent set of S years (beyond H) is typically required, but this requirement implies that the (S-year) forecasted abnormal earnings must grow at a constant rate fully determined by K_S. Consequently, K_S can be inferred from the projected growth rate of abnormal earnings beyond the horizon.[49] Thus, the finite valuation approach with a terminal value calculation is finite only to the extent that one explicitly forecasts the valuation attributes for a finite number of years: The terminal value calculation inevitably introduces (implicit) assumptions regarding the future development of the company (into infinity). In particular, the constant growth of the measurement error of the accounting implies specific conditions on the projected performance of the company beyond the horizon, just in the same way as a simple continuing value calculation does. The important thing in applying terminal values is thus to analyse what the specific conditions required for a particular terminal value calculation really mean in terms of the forecasted operations of the company and specifically if the fulfilment of the conditions can be justified (in each particular case). (See, e.g., Essay 2, for a methodology of deriving conditions for horizon value calculations - parameter conditions are supplied for the DIV and FCF models in terms of the underlying forecasting model of accounting data.)

Generally, the simplest continuing value formulas are of the same type as Gordon's growth formula; the common factor is that the valuation attribute in each case is forecasted to grow at a constant rate, while the discount rate is constant and larger than the growth rate. Let us denote the attribute by X_t , the growth rate by g_X, the appropriate discount rate for attribute X after the horizon by $\bar{k}_X^H$ (here either the cost of equity capital or the weighted average cost of capital, in the perpetuity period denoted by $\bar{k}_E^H$ and $\bar{k}_{WACC}^H$ respectively),[50] and the continuing value at the horizon by CV_H. Further, consider the following sequence of the valuation attribute:

$$\mathbf{X}_{H+1}^{\infty} = X_{H+1}, X_{H+1}(1+g_X), X_{H+1}(1+g_X)^2, X_{H+1}(1+g_X)^3, \ldots$$

The continuing value for the attribute is then given by:

$$CV_H\left[\mathbf{X}_{H+1}^{\infty}\right] \equiv \mathrm{PV}_H^{\bar{k}_X^H}\left[\mathbf{X}_{H+1}^{\infty}\right] = \sum_{s=H+1}^{\infty} \frac{X_{H+1}(1+g_X)^{s-(H+1)}}{\left(1+\bar{k}_X^H\right)^{s-H}} = \frac{X_{H+1}}{\bar{k}_X^H - g_X}$$

[49] Equivalently, it is also possible to derive K_S from how the *difference* capitalised net profit (over S years) *minus* the book value of equity (at the beginning of each S-year period) is projected to grow. (See further Penman, p. 17.)

[50] $\bar{k}_{WACC}^H$ is determined according to (WACC$_{const}$) with H as date t.

Note that the general formulation (2.4:2) provided by Penman (1997) specifies the horizon value as the *equity* value at the horizon. This differs from the formulations below where the continuing value formulas do not necessarily represent the equity value (e.g., the EVA continuing value is the *market value added* at the horizon, i.e., the present value at the horizon of all economic value added beyond the horizon). However, all the formulas below are special cases of Penman's formula (2.4:2), for S = 1 and $K_1 = (1 + g_X)$ (except when the measurement error is zero as in (2.4:8)). For a discussion regarding the implementation of the Penman terminal value approach for cases where $S > 1$ the reader is referred to, e.g., Sougiannis & Yaekura (1997).

2.4.1 Dividends

In the dividend case, the continuing value is very straightforward. It is calculated through the Gordon formula:

(CV:DIV) $$CV_H\left[\mathbf{DIV}_{H+1}^{\infty}\right] = \frac{DIV_{H+1}}{\bar{k}_E^H - g_{DIV}}$$

In order to use this continuing value, the forecasted dividends must be expected to grow at a constant rate and the cost of equity capital must be constant. But as noted earlier, this may not be unproblematic in terms of the forecasting model used for forecasting the necessary accounting items. The valuation equation (EV:DIV) can with a continuing value approach thus be rewritten as:

(2.4:3) $$EV_t(DIV) \equiv \sum_{s=t+1}^{H} \mathrm{PV}_t^{k_E}\left[DIV_s\right] + \mathrm{PV}_t^{k_E}\left[CV_H\left[\mathbf{DIV}_{H+1}^{\infty}\right]\right]$$

2.4.2 Free cash flow

The free cash flow continuing value formula is analogously:

(CV:FCF) $$CV_H\left[\mathbf{FCF}_{H+1}^{\infty}\right] = \frac{FCF_{H+1}}{\bar{k}_{WACC}^H - g_{FCF}}$$

Consequently, the valuation equation (EV:FCF) for the continuing value approach can be rewritten:

(2.4:4) $$EV_t(FCF) \equiv \sum_{s=t+1}^{H} \mathrm{PV}_t^{k_{WACC}}\left[FCF_s\right] + \mathrm{PV}_t^{k_{WACC}}\left[CV_H\left[\mathbf{FCF}_{H+1}^{\infty}\right]\right] - D_t$$

2.4.3 Economic value added

The continuing value expression for economic value added is:

$$\text{(CV:EVA)} \qquad CV_H\left[\mathbf{EVA}_{H+1}^{\infty}\right] = \frac{EVA_{H+1}}{\bar{k}_{WACC}^{H} - g_{EVA}}$$

which can be inserted in the valuation formula (EV:EVA) to yield the continuing value version:

$$(2.4{:}5) \qquad EV_t(EVA) = Cap_t + \sum_{s=t+1}^{H} \mathrm{PV}_t^{k_{WACC}}\left[EVA_s\right] + \mathrm{PV}_t^{k_{WACC}}\left[CV_H\left[\mathbf{EVA}_{H+1}^{\infty}\right]\right] - D_t$$

CKM have provided another continuing value formula for the EVA model which is equivalent to (CV:EVA) but more complicated and hard to interpret. See Appendix 6 for an analysis of the CKM continuing value.

Moreover, Stewart (1991) suggests another approach: Find the horizon, H^*, from where on the expected EVA will be zero ($\mathbf{EVA}_{H^*+1}^{\infty} = 0, 0, 0, ..., 0, ... = \mathbf{0}$), and consequently $\mathrm{PV}_{H^*}^{k_{WACC}}\left[\mathbf{EVA}_{H^*+1}^{\infty}\right] = 0$. If such a horizon H^* can be found, the valuation has only to consider the time up to H^*:

$$(2.4{:}6) \qquad EV_t = Cap_t + \sum_{s=t+1}^{H^*} \mathrm{PV}_t^{k_{WACC}}\left[EVA_s\right] - D_t$$

Given that H^* can be found, this approach is certainly very appealing. See further section 3 for conditions for the existence of H^*.

2.4.4 Abnormal earnings

Provided that the forecasted abnormal earnings in the perpetuity period is growing at a constant rate,[51] and that the cost of equity capital is projected to be constant, we can as in the previous cases use a

[51] This is *not* the case however in the Eldon AB valuation. Even though all the economic requirements for using a continuing value are fulfilled, the abnormal earnings will *not* grow at a constant rate in the perpetuity period! Thus (CV:AE) cannot be used in the Eldon case. Instead of using a continuing value formula, a long explicit forecast is done. The forecasted AE do not grow at a constant rate since the book equity does not. See further Appendix 5, section A5.1.

continuing value of this form:

$$\text{(CV:AE)} \qquad CV_H\left[\mathbf{AE}_{H+1}^{\infty}\right] = \frac{AE_{H+1}}{\bar{k}_E^H - g_{AE}}$$

which can be inserted in the valuation formula in the following way:

$$\text{(2.4:7)} \qquad EV_t(AE) \equiv BV_t + \sum_{s=t+1}^{H} \mathrm{PV}_t^{k_E}[AE_s] + \mathrm{PV}_t^{k_E}\left[CV_H\left[\mathbf{AE}_{H+1}^{\infty}\right]\right]$$

Note that the AE concept is similar to the EVA concept in that it measures the value added to some capital stock (book equity and invested capital, respectively) by the company's expected future performance. Thus, it may be useful to search for a horizon H^{**} such that the future expected AE from that point will equal zero (implying that $EV_H = BV_H$, i.e., that the measurement error of the accounting is zero, or equivalently, that the expected *goodwill* beyond H is zero), and resulting in the following valuation expression:[52]

$$\text{(2.4:8)} \qquad EV_t = BV_t + \sum_{s=t+1}^{H^{**}} \mathrm{PV}_t[AE_s]$$

2.4.5 The adjusted present value model

In the APV model, the usual constant growth rate assumptions yield a simple continuing value formula for the unlevered free cash flow part of the horizon value, while the continuing value formula for the tax shield part will be different for different assumptions about the valuation of tax shields.

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[52] Skogsvik (1997) extends this horizon application of the abnormal earnings model by recognising that goodwill can be specified as the sum of the *business goodwill* and the *cost matching bias* (in the accounting of owners' equity). Even if the *business goodwill* is expected to be zero at the horizon, there still may be a measurement error in the accounting from the cost matching bias that must be considered, and (2.4:8) will not be applicable.

Inserting Skogsvik's equation (13) into (CV:AE) yields: $CV_H\left[\mathbf{AE}_{H+1}^{\infty}\right] = \frac{AE_{H+1}}{\bar{k}_E^H - g_{AE}} =$

$\frac{BV_H\left[\left(\bar{k}_E^H + q_{CM}\cdot\left(\bar{k}_E^H - g_{AE}\right)\right) - \bar{k}_E^H\right]}{\bar{k}_E^H - g_{AE}} = BV_H \cdot q_{CM}$ where q_{CM} is the ratio [*cost matching bias of owners' equity*] / [*(book) owners' equity*]. The continuing value in this case will thus equal the cost matching bias of owners' equity at the horizon (in absolute terms), but can easily be extended to include expected business goodwill, so that the continuing value equals the expected (total) measurement error of the accounting at the horizon (in absolute terms).

However, in general we have:

(CV:APV) $$CV_{H|unlevered}\left[\mathbf{FCF}_{H+1}^{\infty}\right]+CV_{H}\left[\mathbf{TS}_{H+1}^{\infty}\right]=\frac{FCF_{H+1}}{k_U-g_{FCF}}+CV_{H}\left[\mathbf{TS}_{H+1}^{\infty}\right]$$

The basic valuation expression can here be reformulated as:

$$EV_t \equiv$$

(2.4:9) $$\equiv \sum_{s=t+1}^{H}\mathrm{PV}_t^{k_U}\left[FCF_s\right]+\sum_{s=t+1}^{H}\mathrm{PV}_t^{k_{TS}}\left[TS_s\right]+\mathrm{PV}_t^{k_U}\left[CV_H\left[\mathbf{FCF}_{H+1}^{\infty}\right]\right]+\mathrm{PV}_t^{k_{TS}}\left[CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right]\right]-D_t$$

In the most important cases, introduced and discussed in section 2.2.5, we get the following continuing value formulas for the interest tax shields:

- Interest tax shields discounted at the (constant) cost of debt, $\bar{i}$:

  $CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right]=\dfrac{TS_{H+1}}{\bar{i}-g_{TS}}=\dfrac{\bar{\tau}\cdot\bar{i}\cdot D_H}{\bar{i}-g_D}$ where g_{TS}, the growth rate of interest tax shields, equals g_D, the growth rate of debt.

- Interest tax shields discounted at the unlevered cost of equity, k_U:

  $CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right]=\dfrac{TS_{H+1}}{k_U-g_{TS}}=\dfrac{\bar{\tau}\cdot\bar{i}\cdot D_H}{k_U-g_D}$ where $g_{TS}=g_D$.

- Interest tax shields valued by a combination of $\bar{i}$ and k_U, according to the Miles & Ezzell (1980) procedure: $CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right]=\dfrac{TS_{H+1}}{(k_U-g_{TS})}\cdot\left[\dfrac{(1+k_U)}{(1+\bar{i})}\right]=\dfrac{\bar{\tau}\cdot\bar{i}\cdot D_H}{(k_U-g_D)}\cdot\left[\dfrac{(1+k_U)}{(1+\bar{i})}\right]$ where $g_{TS}=g_D$ (Holthausen & Zmijewski (1996), Chapter 2, p. 17).

See Appendix 8 for further discussion about the intuition behind the Miles & Ezzell tax shield valuation procedure.

2.5 Companies with finite lives

As noted above, some companies may be expected to have finite lives. If that is the case, the following is to be expected in the final year:

1) at the end of year T, the company's assets are realised (i.e., simply sold off),

2) the proceeds are distributed among the investors (debt- and equity-holders), and

3) at the very end of year T the debt and equity values (as well as all balance sheet items) will be zero (i.e., $EV_T = BV_T = D_T = T_T = N_T = WA_T = WL_T = 0$).

The debt-holders' final claim equals the outstanding debt at the beginning of the year plus interest, i.e., $D_{T-1} + i_T \cdot D_{T-1}$. Since interest payments are tax-deductible, the *net* debt payment from the company's standpoint is $D_{T-1} + (1 - \tau_T) \cdot i_T \cdot D_{T-1}$. The net (after-tax) proceeds from the 'disinvestment' of assets plus the cash generated from operations in year *T*, will together equal the company's (final) cash flow, CF_T. CF_T is *expected* to be positive and at least as large as the final claim of the debt-holders (otherwise the expected life of the company should be shorter). As residual claimants, the equity-holders will receive a final dividend, $DIV_T = CF_T - (D_{T-1} + (1 - \tau_T) \cdot i_T \cdot D_{T-1})$. The question is then - how is the final cash flow calculated?

Two different cases will emerge: First, let us assume that the disinvestment of assets will generate exactly their (beginning of year *T*) net value minus the deferred taxes, $IC_{T-1} - T_{T-1}$, i.e., that they can be sold for exactly their beginning book value. But this just means that the standard FCF calculations ((F1) or (F2)) apply and $CF_T = FCF_T$. Since the accounting system in this case correctly links the change in stock (from $IC_{T-1} - T_{T-1}$, to $IC_T - T_T = 0$) to the generated flows all formulas will apply as usual: dividends, abnormal earnings, free cash flow and economic value added can as be calculated precisely as for all other years. For example, consider the standard dividend relation (D1):

(D1) $$DIV_T = BV_{T-1} + NP_T - BV_T$$

Using (N2) and (BSR2) and the fact that all balance sheet items at *T* equals zero, this can be rewritten as:

(2.5:2) $$\begin{aligned} DIV_T &= BV_{T-1} + NOPLAT_T - (1-\tau_T)IX_T + T_{T-1} = \\ &IC_{T-1} - D_{T-1} - T_{T-1} + NOPLAT_T - (1-\tau_T)IX_T + T_{T-1} = \\ &IC_{T-1} + NOPLAT_T - D_{T-1} - (1-\tau_T)i_T D_{T-1} \end{aligned}$$

Now, since $IC_T = 0$, the free cash flow equation (F2) gives:

(2.5:3) $$DIV_T = FCF_T - \left(D_{T-1} + (1-\tau_T)i_T D_{T-1}\right)$$

The standard dividend calculation thus gives the residual claim of the company's final cash flow. However, if the (after-tax) proceeds from the realisation of assets are expected to differ from the net book value ($IC_{T-1} - T_{T-1}$), then the *T*-year calculations must be different from the standard ones. Let RP_T denote the total realised (after-tax) proceeds from the liquidation of assets. Then define the (non-accounted) expected premium, $Prem_T$, as $Prem_T = RP_T - (IC_{T-1} - T_{T-1})$. Consequently, $Prem_T$ shows how much the liquidation of assets is expected to generate above (or under) the assets' net book value. Since this value will not be evident from the accounting system it must be estimated independently, and then added to the valuation attributes (derived from the accounting system in the standard way). In this case the final cash flow, CF_T, will equal $FCF_T + Prem_T$. The correctly adjusted valuation attributes (denoted with superscript *) can be calculated as:

$$DIV_T^* = DIV_T + Prem_T = BV_{T-1} + NP_T + Prem_T$$

$$FCF_T^* = FCF_T + Prem_T = OPAT_T + (IC_{T-1} - T_{T-1}) + Prem_T$$

$$EVA_T^* = EVA_T + Prem_T = \Pi_T + Prem_T - k_{WACC,T}Cap_{T-1}$$

$$AE_T^* = AE_T + Prem_T = NP_T + Prem_T - k_{E,T}BV_{T-1}$$

3. Analysis and Results

The first part of the analysis considers the continuing value formulas and the alternative horizon value techniques defined in the previous section, before the complete valuation models are analysed. All proofs are given in Appendix 2.

Let us first, by following the CKM requirement for continuing values that the company should grow at a constant rate, consider the case where the annual growth rates in the perpetuity period (i.e., from the horizon *H*) for the free cash flow, the dividends and the economic value added are all equal:[53]

(GR) $g = g_{DIV} = g_{FCF} = g_{EVA} = g_{AE}$

where *g* equals the growth rate in perpetuity (from the horizon *H*) for debt, book equity and invested capital.

[53] One of the results of the steady state analysis in Essay 2 (Chapter 2) is that the necessary conditions (for use of continuing values in a CKM framework) imply that both FCF, NP, and DIV (as well as Revenues, NOPLAT, debt, and invested capital) grow at the same rate. This is not implied for the book value of equity as was noted above. So, the (GR) assumption is non-controversial for DIV, FCF and EVA when we have clean surplus accounting along with the CKM requirements for continuing values, but may not generally hold in this setting for AE due to the fact that book equity may not constantly grow with the same rate as the other items in the perpetuity period. This is, in the Eldon case, because the CKM-type forecasting model models the development of the deferred taxes in such a way that a constant growth may not be feasible and the fact that book equity is the residual item of the balance sheet.

Then:

Proposition 1

Given condition (GR) *the equity value at the horizon H is equal to:*

$$
\begin{aligned}
& EV_H = \\
& = CV_H\left[\mathbf{DIV}_{H+1}^{\infty}\right] = \frac{DIV_{H+1}}{\bar{k}_E^H - g} \\
\text{(P1)} \quad & = CV_H\left[\mathbf{FCF}_{H+1}^{\infty}\right] - D_H = \frac{FCF_{H+1}}{\bar{k}_{WACC}^H - g} - D_H \\
& = IC_H + CV_H\left[\mathbf{EVA}_{H+1}^{\infty}\right] - D_H = IC_H + \frac{EVA_{H+1}}{\bar{k}_{WACC}^H - g} - D_H \\
& = BV_H + CV_H\left[\mathbf{AE}_{H+1}^{\infty}\right] = BV_H + \frac{AE_{H+1}}{\bar{k}_E^H - g}
\end{aligned}
$$

◊

Proposition 1 implies that if one of the valuation attributes grows at a constant rate from *H*, then if any of the other attributes grows at a constant rate, the growth rate must be the same. More implications of Proposition 1 (in combination with Proposition 3) will be discussed below.

Let us next consider the horizon approach suggested by, e.g., Stewart (1991), which is directed at finding a horizon at which the EVA and/or AE horizon values will be zero. For the EVA case we have that: *the economic value added equals zero* ($EVA_t = 0$) *if and only if the expected return on invested capital equals the weighted average cost of capital, i.e.,* $\frac{\Pi_t}{Cap_{t-1}} = k_{WACC,t}$. For the "Stewart" specification this means simply that $NOPLAT_t$ should equal *the required after-tax return on capital (i.e., the WACC) times the beginning invested capital,* IC_{t-1}. Similarly, for the "alternative" EVA specification, $OPAT_t$ should equal *the required after-tax return on capital (i.e., the WACC) times the beginning invested capital (excluding deferred taxes),* IC_{t-1}-T_{t-1}.

This means that if we can find a horizon H^* where this will hold (in expectation) for all future years, the valuation can be cut off at this horizon since the company is just expected to earn its required return on invested capital.

Similarly, for the AE case we have that *the abnormal earnings are zero* ($AE_t = 0$) *if and only if the expected return on book equity equals the cost of equity capital, i.e.,* $\frac{NP_t}{BV_{t-1}} = k_{E,t}$. The straightforward implication of this is that the net profit, NP_t, should equal the *required rate of return on equity (i.e., the cost of equity,* $k_{E,t}$*) times the beginning book value of equity.* Analogous to the EVA case, if this condition is expected to hold for all future years beyond a horizon H^{**} we can cut off the valuation calculations at H^{**}. One may think that H^* and H^{**} should be the same point in time. This *may* not be the case in general, however (see Proposition 2, below).

Next issue to be considered is the relation between the EVA and AE concepts. There appears to exist some conflicting interpretations of the AE and EVA attributes,[54] and given their similarities one may easily believe that they are the same. Remember also that we have specified two different versions of the EVA attribute (which in general notation is defined as $EVA_t \equiv \Pi_t - k_{WACC,t} \cdot Cap_{t-1}$):

1) the Stewart (1991) specification, where $\Pi_t = NOPLAT_t$ and $Cap_{t-1} = IC_{t-1}$

2) the alternative specification, where $\Pi_t = OPAT_t$ and $Cap_{t-1} = IC^*_{t-1} = IC_{t-1} - T_{t-1}$.

The relation between the AE concept and the EVA specifications can be described in the following terms:

Proposition 2

(a) *For the "Stewart" EVA specification:*

(P2a) $$\left(g_{T,t} - k_{WACC,t}\right)T_{t-1} = \left(k_{E,t} - k_{WACC,t}\right)\left(EV_{t-1} - BV_{t-1}\right) \Leftrightarrow AE_t = EVA_t$$

(b) *For the "alternative" EVA specification:*

(P2b) $$\left(1 + k_{E,t}\right)\left(EV_{t-1} - BV_{t-1}\right) = \left(1 + k_{WACC,t}\right)\left(EV_{t-1} - BV_{t-1}\right) \Leftrightarrow AE_t = EVA_t$$

◊

As can be seen from Proposition 2 the EVA and AE attributes (regardless of EVA specification) are not generally equivalent: For a particular year, the valuation attribute AE will in general not take the

[54] See, e.g., Frankel & Lee (1996), pp. 5-7. They use the notation EVA_t for both the EVA to long-term investors (what here is referred to as EVA) and the EVA to equityholders (what here is referred to as AE).

same numeric value as the EVA attribute unless a specific condition is fulfilled. Note that this does *not* mean that an AE valuation will give a result different from an EVA valuation. It only means that the attributes' values in a particular year *t*, AE_t and EVA_t, are different. Now, let us consider some interesting cases where in fact the attributes will be the same for a particular year: from (P2b) it is evident that if the expected measurement error of the book equity accounting $(EV_{t-1} - BV_{t-1})$ is zero in the beginning of a particular year *t*, then the "alternative" EVA_t will equal AE_t, but this is not a sufficient condition for the "Stewart" EVA_t to equal (the same) AE_t. So, if the measurement error of the book equity accounting is zero at some point in time, then this point in time *is* H^* (for the "alternative" EVA specification) *and* H^{**}.[55] From (P2b) it is also clear that if WACC equals the cost of equity (i.e., that the company is all-equity financed) the "alternative" EVA is equivalent to AE (but again, this is not a sufficient condition for the "Stewart" EVA to be equivalent to AE). For the "Stewart" EVA to be equal to AE (and to "alternative" EVA) some additional condition regarding the deferred taxes must be fulfilled. The conclusion is that the AE attribute and the "alternative" EVA attribute are more similar than the two EVA specifications, and that it is hardly ever the case that H^* for "Stewart" EVA could be equal to H^{**}, unless there are no deferred taxes (which means that the two EVA specifications would be equivalent).

Now, let us finally consider complete valuations, from the valuation date to the end of the life of the company (possibly assumed to be infinite). In the infinite case, some type of horizon value as discussed previously could possibly be used, if the underlying assumptions are appropriate.

The case of Eldon AB, described in Essay 2 (Chapter 5), will be used as an example. The valuation date is December 31, 1994. This example provides a forecast of all necessary accounting items for the valuation concepts discussed in this essay. The forecasted financial statements can be found in Appendix 4.[56] This forecast of the future performance of Eldon is here taken as the 'given set of fundamental data'. The borrowing rate (equal to the cost of debt) is 11% and assumed to be constant. In addition to the assumptions stated in Essay 2 the following are here made:

[55] By definition $(EV_H - BV_H) = 0$ typically implies that $AE_{H+1} = 0$, but then (P2b) implies that "alternative" $EVA_{H+1} = 0$.

[56] The Property, Plant and Equipment (PPE) items for Eldon AB are forecasted using Specification B in Essay 2. This is taken from the second edition of Copeland et al. (1994). As discussed in Essay 2, p. 137, this specification reduces to Specification A in the post-horizon period, if a steady state is to be achieved. See Appendix 4 for a description of how the accounting items can be computed for any year in the post-horizon period.

1) The existing excess marketable securities (EMS) at the end of 1994 (0.9 MSEK) are assumed to be immediately 'netted' against the outstanding long-term debt, so that the market value of debt at the valuation date is 363.2 MSEK.[57]
2) The cost of equity is allowed to be non-constant.
3) The yearly debt adjustment is assumed to be contingent on the company's market value. However, the debt is not expected to be exactly adjusted to a target market debt ratio during the first 10 years. The foreseen adjustments will make the *book* debt ratio to (gradually) approach a steady state level of 40% (the last historical observation in 1994 is 42.2%).

The first 10 years of the forecast constitute a phase where Eldon AB is expected to 'adjust' its operations and financing decisions so that the company, in expectation, enters into a steady state from year 2005 (among other things, it will then have a constant *market* debt ratio). Moreover, all steady state conditions for a simple continuing value formula are fulfilled at the end of year 2005.[58]

One of the first questions when considering different valuation models is:
Given the same forecast of fundamental data, do the different models give the same valuation result?

Intuitively this is of course what we would like them to do. However, this may not be the case: as already mentioned, we know that the DIV and FCF models in the traditional setting with constant discount rates are generally not equivalent. The valuation concepts of EVA and AE as well as the APV model will be taken into consideration. It will be shown how all these concepts relate to each other (see further the proofs in Appendix 2), and if it is possible to establish a general equivalence of company valuation methods.

Let us start out by considering the traditional textbook approach with constant discount rates in the DIV, FCF, EVA and AE models.[59] The equity values for Eldon AB according to the different models[60]

[57] This is slightly different from how the EMS were treated in Essay 2. There it was implicitly assumed that the EMS were sold off at the valuation date as an immediate (i.e., pre-forecast period) 0.9 MSEK 'dividend' to the equity owners, and that this dividend did not influence the capital structure at the valuation date. See Appendix 7 for more on excess marketable securities.

[58] See Chapter 5 of Essay 2 for the details regarding Eldon AB, and section 2.1.5 for a general analysis (especially Proposition 2.5).

[59] Note, again, that Eldon AB is not foreseen to maintain a constant market debt ratio in the first years.

Model:	Equity value, EV_{1994}:	Discount rate:
The DIV Model	528.88 MSEK	$\bar{k}_E = 13.156\%$
The FCF Model	533.91 MSEK	$\bar{k}_{WACC} = 10.947\%$ ($\bar{k}_E = 13.156\%$)
The EVA Model	533.91 MSEK	$\bar{k}_{WACC} = 10.947\%$ ($\bar{k}_E = 13.156\%$)
The AE Model	528.88 MSEK	$\bar{k}_E = 13.156\%$

Table 3 - Valuations of Eldon AB with constant initial discount rates

with constant *initial* discount rates (i.e., computed at the valuation date)[61] are presented in Table 3. The DIV and AE valuations yield an estimated market equity value of 528.88 MSEK while the FCF and EVA model results in a slightly higher value, 533.91 MSEK.[62]

Observation 1

At time *t*, $EV_t[\text{DIV}] = EV_t[\text{AE}] \equiv EV1_t$ and $EV_t[\text{FCF}] = EV_t[\text{EVA}] \equiv EV2_t$ but $EV1_t$ is generally *not* equal to $EV2_t$ when traditional (i.e., constant discount rate) present value calculations are used.
◊

The equivalence between the DIV and the AE methods, as well as the equivalence between the FCF and EVA methods, can be formally proved (see Appendix 2). These equivalencies are also of a general form since they apply to both constant and non-constant discount rate cases.

But the WACC-oriented FCF and EVA approaches do not give the same value as the cost of equity-related DIV and AE approaches. The exception is when the fundamental data underlying the valuation implies that the forecasted capital structure in market terms is constant over time (of course suggesting that a constant discount rate is non-controversial). But:

---

[60] That is, as described in the valuation formulas (EV:DIV, EV:FCF, EV:EVA, EV:AE). The value calculations for each case are provided in Appendix 5.

[61] This is for the FCF and EVA cases done by an iterative procedure that simultaneously solves for the equity value and the WACC, so that the WACC weights at the valuation date are mutually consistent with the calculated equity value and the observed debt value at the valuation date.

[62] The latter result differs from the FCF valuation with constant discount rate in Essay 2: 534.4 MSEK. The difference can be fully attributed to the different treatment of EMS in the two cases.

Observation 2

The forecasted capital structure in market terms is constant over time (and a general equivalence between the constant discount rate versions of the models exists) only

1) under some assumption of active debt management which by specification requires the company to maintain a constant market leverage ratio in expectation,[63]

or

2) in some constant growth case where the expected total market value of the company, TCV_t, will grow at the same rate as the expected debt.[64]

◊

Note, however, that different assumptions regarding debt management in practice may have an impact on the actual forecast,[65] which in turn may influence the usefulness of the different valuation models. For instance, if we assume *active debt management* the debt forecast should be contingent on the forecasted development of the company value, and thus the debt forecast and the value calculation will be interrelated in the overall valuation proceedings. However, this will require a different forecasting approach than the one suggested by CKM (and followed in the Eldon AB valuation in this essay), since one will need the company value at the end of each year in order to compute the expected debt at the same moment. This modified forecasting approach implies, as we will see, that valuation attributes that are independent of financing are easier to use, i.e., FCF or EVA. First, one must obtain forecasts of the accounting items necessary for calculating the development of FCF or EVA, specifically the items that add up to the invested capital and NOPLAT,[66] and of what the company's expected (constant) market debt ratio should be in order to calculate the (constant) discount rate. At this stage we *cannot* obtain forecasts of the valuation attributes AE and DIV, since

[63] Note that *active debt management* here is referred to as meaning that the company adjusts its debt periodically to maintain a constant leverage in *market terms.* Alternatively, one could by *active debt management* of course mean that the company adjusts its borrowing to maintain a constant *book* debt ratio. This case is *not* treated in this essay. Consequently, *active debt management* will henceforth mean that the company is assumed to maintain a constant *market* debt ratio in expectation.

[64] See, e.g., Essay 2, Proposition 2.3 (p. 51).

[65] See further Essay 3, section 2.2.

[66] The invested capital is calculated through (BSR1): $IC_t = (WA_t - WL_t) + N_t$ and NOPLAT through (N1): $NOPLAT_t = (1-\tau_t)OP_t + (T_t - T_{t-1}) = (1-\tau_t)(R_t - OpX_t - DepX_t) + (T_t - T_{t-1})$. All items needed, including the deferred taxes, are asset related.

the calculation of these items depends on the interest expense which in turn depends on the outstanding debt amount at the beginning of each year (which by the active debt management assumption cannot be found until we have computed the total company value at each corresponding point in time).[67] Then the FCF or EVA model can be used to calculate the equity value (at the valuation date). Moreover, to get complete forecasts of the financial statements one must calculate the total company value at the end of each explicitly forecasted future year before we can calculate the debt outstanding at the corresponding points in time. Only after that, one can compute the debt and equity items in the forecasted balance sheets, the remaining income statement items interest expense and net profit, and finally, the dividends and abnormal earnings.

So, we have seen that the traditional valuation approach fails in achieving a general equivalence among all four valuation models, which is not very surprising since it neglects to take anticipated future changes in the company's capital structure into consideration. Now the question is: *Is it possible to establish a general equivalence even if the capital structure is non-constant?* - Yes. As indicated in Essay 3, the equivalence of the FCF and DIV models can be established by updating discount rates to reflect the changing capital structure (and thus risk) over time. Thus, let us consider the backward discounting procedure, which in combination with the framework derived in section 2, gives the following result (if T is finite then the adjusted valuation attributes as specified in section 2.5 should be used for year T):

Proposition 3

Valuation by discounting the free cash flows at a periodically adjusted weighted average cost of capital and subtracting the debt value will yield the same equity value as

(1) *valuation by discounting the economic value added at a periodically adjusted weighted average cost of capital, adding the invested capital, and subtracting the debt value*

(2) *valuation by discounting the abnormal earnings at the cost of equity capital (allowed to be non-constant), and adding the book value of equity*

(3) *valuation by discounting the future dividends at the cost of equity capital (allowed to be non-constant):*

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[67] Thus, to calculate the valuation attributes abnormal earnings and dividends, as well as the tax shields in the APV model, an iterative procedure would be needed in the constant (market) leverage case. If the constant leverage assumption is introduced in order to avoid iterative valuation procedures, then the AE, DIV and APV models should not be used in this case.

$$
\text{(P3)} \quad \begin{aligned}
EV_t &= \sum_{k=1}^{T} \frac{FCF_{t+k}}{\prod_{j=1}^{k}(1+k_{WACC,t+j})} - D_t = \\
&= IC_t + \sum_{k=1}^{T} \frac{EVA_{t+k}}{\prod_{j=1}^{k}(1+k_{WACC,t+j})} - D_t = \\
&= BV_t + \sum_{k=1}^{T} \frac{AE_{t+k}}{\prod_{j=1}^{k}(1+k_{E,t+j})} = \\
&= \sum_{k=1}^{T} \frac{DIV_{t+k}}{\prod_{j=1}^{k}(1+k_{E,t+j})}
\end{aligned}
$$

where EV_t = Market value of equity value at the valuation date t
FCF_s = Forecasted free cash flow in year s
D_t = Debt at the valuation date t
EVA_s = Forecasted economic value added in year s
IC_t = Invested capital at the valuation date t
AE_s = Forecasted abnormal earnings in year s
BV_t = Book value of equity at the valuation date t
DIV_s = Forecasted dividends in year s
T = end of life of the company (T may be finite or infinite, i.e. $T=\infty$)
$k_{E,s}$ = cost of equity capital in year s
$k_{WACC,s}$ = weighted average cost of capital[68] in year s

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It is thus possible to not just update the weights in the WACC formula (as in Essay 2), but also to update the cost of equity capital according to the changing capital structure (as in Essay 3) *and* to a (possibly) changing cost of debt. Thus, the modelling framework proposed in this essay is consistent with the general formulation (GEN) (see below) provided by Holthausen & Zmijewski (1996), in which the well-known cost of capital models developed by Modigliani & Miller (1963), Miles and Ezzell (1980), and Harris & Pringle (1985) arise as special cases.[69] However, it should be recognised that the equivalence of the four methods in Proposition 3 is achieved by the updating of the weights in the WACC formula; thus, the equivalence is preserved independently of how, or if, the cost of equity capital is updated. *The important point is that the backward discounting procedure in fact always ensures that the different valuation models are perfectly equivalent.* An important implication (that is evident from the proof) is that the models are not equivalent just in infinite valuations, but they are

[68] Calculated according to the WACC definition: $k_{WACC,s} = \frac{D_{s-1}}{TCV_{s-1}}(1-\tau_s)i_s + \left(1 - \frac{D_{s-1}}{TCV_{s-1}}\right)k_{E,s}$

[69] See further the discussion in Essay 3.

also equivalent *year by year*. Hence, it is fully possible to combine the different models in any way: For example, one can use FCF for the first three years, then AE for one year, then use EVA for a number of years, and so on.

Further, considering Propositions 1 and 3 together gives more implications for the use of horizon values:

1) It is fully possible to combine any of the horizon values in Proposition 1, with any of the models in the explicit forecast period. That is, we can, e.g., forecast FCF (or any combination of the models) in the explicit forecast period and use the most suitable model for the horizon value calculation.

2) If one of the attributes in expectation starts to grow at a constant rate from year H, and the others do *not*,[70] one can correctly use the simple continuing value formula for the attribute that grows at the constant rate to obtain the horizon value (which will be equivalent to one obtained from a long explicit forecast beyond the horizon for each of the other attributes). That is, *the horizon value problem boils down to finding the attribute that validly can be expected to grow at a constant rate (or be zero) from a specific point in time.*

The analysis is now taken one step further, by the introduction of the APV model into the analysis, and the explicit inclusion of cost(s) of capital specifications. Let us first assume that interest tax shields are valued at the cost of debt and/or the unlevered cost of equity capital, so that:

(PVTS) $$PVTS_{t,all} = PVTS_{t,i} + PVTS_{t,k_U}$$ for all t (from the valuation date and forward)

where $PVTS_{t,i}$ is the part of the interest tax shields that is discounted at the cost of debt during year $t+1$, and

$PVTS_{t,k_U}$ is the part of the interest tax shields that is discounted at the unlevered cost of equity during year $t+1$

[70] If there is a steady state such that one of the attributes grow at a constant rate, and the others do not, then the yearly growth of the other attributes will asymptotically approach the steady state growth rate (which will equal the growth rate of revenues). See, e.g., the steady state analysis in Essay 2.

Following Holthausen & Zmijewski (1996), the general[71] relation between a company's cost of equity, cost of debt and *unlevered* cost of equity is now (see Essay 3, Appendix 4 for a derivation):

(GEN) $$k_{E,t} = k_U + (k_U - i_t) \cdot \frac{(D_{t-1} - PVTS_{t-1,i})}{EV_{t-1}}$$

Then (again, if T is finite then the adjusted valuation attributes as specified in section 2.5 should be used for year T):

Proposition 4

Given (PVTS) *and* (GEN), *valuation by using the adjusted present value model and subtracting the debt value will yield the same equity value as*

(1) *valuation by discounting the free cash flows at a periodically adjusted weighted average cost of capital, and subtracting the debt value*

(2) *valuation by discounting the economic value added at a periodically adjusted weighted average cost of capital, adding the invested capital, and subtracting the debt value*

(3) *valuation by discounting the abnormal earnings at the cost of equity capital (allowed to be non-constant), and adding the book value of equity*

(4) *valuation by discounting the future dividends at the cost of equity capital capital (allowed to be non-constant):*

[71] General in the sense *'independent of tax shield valuation assumptions'* given (PVTS). By adding assumptions about the valuation of tax shields and/or the development of the valuation relevant characteristics of the company the relation can be specialised for certain cases.

$$EV_t = \sum_{k=1}^{T} \frac{FCF_{t+k}}{(1+k_U)^k} + PVTS_{t,all} - D_t =$$

$$= \sum_{k=1}^{T} \frac{FCF_{t+k}}{\prod_{j=1}^{k}(1+k_{WACC,t+j})} - D_t =$$

$$\text{(P4)} \qquad = IC_t + \sum_{k=1}^{T} \frac{EVA_{t+k}}{\prod_{j=1}^{k}(1+k_{WACC,t+j})} - D_t =$$

$$= BV_t + \sum_{k=1}^{T} \frac{AE_{t+k}}{\prod_{j=1}^{k}(1+k_{E,t+j})} =$$

$$= \sum_{k=1}^{T} \frac{DIV_{t+k}}{\prod_{j=1}^{k}(1+k_{E,t+j})}$$

where EV_t = Market value of equity value at the valuation date t
FCF_s = Forecasted free cash flow in year s
$PVTS_{t,all}$ = Value of all future interest tax shields at the valuation date t
D_t = Debt at the valuation date t
EVA_s = Forecasted economic value added in year s
IC_t = Invested capital at the valuation date t
AE_s = Forecasted abnormal earnings in year s
BV_t = Book value of equity at the valuation date t
DIV_s = Forecasted dividends in year s
T = end of life of the company (T may be finite or infinite, i.e. T=∞)
k_U = the unlevered cost of equity capital
$k_{E,s}$ = cost of equity capital in year s
$k_{WACC,s}$ = weighted average cost of capital in year s

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This effectively means that the modelling framework set up in this essay ensures a general equivalence between prominent company valuation models: the adjusted present value model, the free cash flow model, the economic value added model, the abnormal earnings model *and* the dividend model. Thus, if the definitions and procedures proposed in this essay are followed, the choice of valuation model has no impact whatsoever on the final valuation result. It should also be noted that Proposition 4 also holds if the unlevered cost of equity is allowed to be non-constant.[72]

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[72] That is, equation (GEN) can be generalised to (GEN*) $k_{E,t} = k_{U,t} + (k_{U,t} - i_t) \cdot \frac{(D_{t-1} - PVTS_{t-1,i})}{EV_{t-1}}$. In most cases, $k_{U,t}$ as well as i_t are assumed to be constant. However, the generalised formulation allows for including projected changes in operating risk of the company into the valuation, and/or for the inclusion of the term structure of interest rates. For example, if two of the required rates of return can be modelled as a function of the

In order to implement the framework we need to define the assumptions regarding the valuation of tax shields. These assumptions are partly related to the financial management procedures of the particular company in question and I here refer to Essay 3 for a detailed discussion of this topic. Even if the original setting in Miles & Ezzell (ME) does not hold for the Eldon AB case, Essay 3 (section 2.2) finds that the resulting ME formula is applicable under more general conditions. The bottom line is that the ME formula may be used provided that the debt forecast is based on an assumption that the management's debt decisions are related to company value (even if the result is a non-constant debt ratio). Consequently, the ME formula for adjusting the cost of equity is appropriate for Eldon AB (see Essay 3, Appendix 4, for a derivation):

$$k_{E,t} = k_U + (k_U - i_t) \cdot \frac{D_{t-1}}{EV_{t-1}} \left(1 - \frac{\tau_t \cdot i_t}{1 + i_t}\right) \tag{3:1a}$$

which is consistent with the following valuation scheme of the tax shields (see further Appendix 8):

$$\begin{cases} PVTS_{t,i} = \dfrac{\tau_{t+1} \cdot i_{t+1} \cdot D_t}{1 + i_{t+1}} \\ PVTS_{t,k_U} = \dfrac{PVTS_{t+1,all}}{1 + k_U} \end{cases} \quad \text{for all } t \text{ (from the valuation date and forward)} \tag{3:1b}$$

This basically means that Eldon AB is assumed to *aim* for a specific market debt ratio target, but that the company is not foreseen to be able to maintain an exactly constant market debt ratio until Eldon AB is projected to enter into a steady state from year 2006. This finally results in an equity value for Eldon AB of 530.76 MSEK (the value calculations for all the different models are shown in Appendix 5).[73] This will henceforth be referred to as the *base-case valuation* of Eldon AB.

(periodical) risk-free rate (derived from a term structure model), the third will follow from (GEN*). In this essay, the unlevered cost of equity is treated as a constant.

[73] This differs from the result obtained in Essay 2: 528.9 MSEK. The difference is due to the facts that 1) the existing EMS are treated slightly different, and that 2) the cost of equity capital is here updated according to the ME formula (in Essay 2 the cost of equity capital was assumed to be constant, 13.156%). The first fact is the one with least effect on the result. If the EMS had been treated as in Essay 2, the result with ME updating of the cost of equity would have been 531.0 MSEK due to the preservation of a larger tax shield (see further Appendix 7).

However, it may be of interest to study how much the value will deviate if different assumptions are made about the value of the tax shields.[74] Two alternative cases are thus also considered: all interest tax shields are *valued at the cost of debt* (which is related to an assumption about passive debt management), and all interest tax shields are *valued at the unlevered cost of equity* (which is related to an assumption about continuously adjusted debt). In the first case the cost of equity updating is carried out via equation (GEN), which requires that the tax shields explicitly have to be discounted in order to provide the value of $PVTS_{t,i}$ for all years up to the horizon.[75] In the second case the cost of equity adjustment is done through the following formula (see Essay 3, Appendix 4 for a derivation):

(3:2) $$k_{E,t} = k_U + (k_U - i_t) \cdot \frac{D_{t-1}}{EV_{t-1}}$$

Moreover, there are two alternative starting points for the valuation procedure (see further Appendix 3): Either the unlevered cost of equity is estimated (and the cost of equity at the valuation date is inferred from (GEN)[76]), or the cost of equity at the valuation date is estimated (and the unlevered cost of equity is inferred from (GEN)[77] after the use of an iterative procedure). In the base-case valuation of Eldon AB the starting point was an estimated cost of equity capital of 13.156% (which resulted in an unlevered cost of equity of 12.296%).

As is seen from the sensitivity analysis[78] (Tables 4 - 9), the valuation results, given an estimated cost of equity, do not differ substantially between different assumptions about the valuation of interest tax shields (the maximum difference between equity values in each table of Tables 4 - 6 is 0.38%). Note, however, that the valuation results in Tables 4 - 6 are counter-intuitive in the following sense: in each table the valuation method with highest discount rate for interest tax shields give the highest equity value. This is due to the fact that a given cost of equity will produce quite different *unlevered* costs of equity for the different methods. That is, the effect from a lower discount rate for tax shields will,

[74] That is, *given* the estimated debt schedule in Appendix 4. Note, again, that different assumptions regarding debt management, and thus also regarding the valuation of interest tax shields, in practice may have an impact on the actual forecast.

[75] See the XMPL case in Essay 3, Chapter 3, for a more detailed description of the procedure.

[76] Or from the special cases (3:1a) or (3:2) if the corresponding assumptions are fulfilled.

[77] Or, again, from the special cases (3:1a) or (3:2) if the corresponding assumptions are fulfilled.

[78] To explicitly study the sensitivity of the valuation result 13.156% ± 0.2% have been used as cost of equity starting points, and 12.3% ± 0.2% as *unlevered* cost of equity starting points, for three different cases of assumptions about valuation of tax shields.

Tax shield valuation	Equity value, EV_{1994} MSEK	Cost of equity, $k_{E,1995}$	Unlevered cost of equity, k_U	WACC, $k_{WACC,1995}$
All at cost of debt, i	528.96	**13.156%**	12.554%	10.935%
Miles-Ezzell (i and k_U)	530.76	**13.156%**	12.296%	10.939%
All at unlevered cost of equity, k_U	530.79	**13.156%**	12.281%	10.939%

Table 4 - Eldon AB valuations, given $k_{E,1995} = 13.156\%$

(The Miles-Ezzell case here is the base-case valuation)

Tax shield valuation	Equity value, EV_{1994} MSEK	Cost of equity, $k_{E,1995}$	Unlevered cost of equity, k_U	WACC, $k_{WACC,1995}$
All at cost of debt, i	540.56	**12.956%**	12.419%	10.844%
Miles-Ezzell (i and k_U)	542.22	**12.956%**	12.186%	10.848%
All at unlevered cost of equity, k_U	542.26	**12.956%**	12.171%	10.848%

Table 5 - Eldon AB valuations, given $k_{E,1995} = 12.956\%$

Tax shield valuation	Equity value, EV_{1994} MSEK	Cost of equity, $k_{E,1995}$	Unlevered cost of equity, k_U	WACC, $k_{WACC,1995}$
All at cost of debt, i	517.82	**13.356%**	12.688%	11.024%
Miles-Ezzell (i and k_U)	519.75	**13.356%**	12.404%	11.029%
All at unlevered cost of equity, k_U	519.78	**13.356%**	12.387%	11.030%

Table 6 - Eldon AB valuations, given $k_{E,1995} = 13\ 356\%$

given a specific cost of equity, be counteracted by a higher discount rate for operations. That is, if we start out by an estimate of the cost of equity, the choice of interest tax shield valuation method will not only affect the valuation of tax shields, but also the valuation of the unlevered operating assets (in opposite direction). Consequently, the stableness of the valuation result in this case depends on two counteracting effects.

However, for a given *unlevered* cost of equity capital, the different assumptions have larger impact on the valuation result (the maximum difference between equity values in each table of Tables 7 - 9 is

Tax shield valuation	Equity value, EV_{1994} MSEK	Cost of equity, $k_{E,1995}$	Unlevered cost of equity, k_U	WACC, $k_{WACC,1995}$
All at cost of debt, i	550.99	12.783%	**12.3%**	10.764%
Miles-Ezzell (i and k_U)	530.31	13.164%	**12.3%**	10.943%
All at unlevered cost of equity, k_U	528.72	13.193%	**12.3%**	10.956%

Table 7 - Eldon AB valuations, given $k_U = 12.3\%$

Tax shield valuation	Equity value, EV_{1994} MSEK	Cost of equity, $k_{E,1995}$	Unlevered cost of equity, k_U	WACC, $k_{WACC,1995}$
All at cost of debt, i	569.20	12.496%	**12.1%**	10.764%
Miles-Ezzell (i and k_U)	551.32	12.803%	**12.1%**	10.943%
All at unlevered cost of equity, k_U	549.94	12.826%	**12.1%**	10.956%

Table 8 - Eldon AB valuations, given $k_U = 12.1\%$

Tax shield valuation	Equity value, EV_{1994} MSEK	Cost of equity, $k_{E,1995}$	Unlevered cost of equity, k_U	WACC, $k_{WACC,1995}$
All at cost of debt, i	533.56	13.076%	**12.5%**	10.898%
Miles-Ezzell (i and k_U)	510.21	13.536%	**12.5%**	11.109%
All at unlevered cost of equity, k_U	508.40	13.572%	**12.5%**	11.125%

Table 9 - Eldon AB valuations, given $k_U = 12.5\%$

4.95%). This may, at a first glance, suggest that it may be wise to concentrate on direct estimation of the cost of equity capital at the valuation date, rather than estimating the unlevered cost of equity. A drawback is that the discounting procedure will require additional iterations, but this problem can be overcome through the procedure provided in Appendix 3. More important is that the unlevered cost of equity implied by an estimated cost of equity will depend on the chosen method for valuing interest tax shields.

This implies that it may be hazardous to estimate the unlevered cost of equity from just one observation. The unlevered cost of equity could be viewed as being the same within an industry and could thus be estimated using industry data. If this is the case, it may be better to start out with an estimated unlevered cost of equity even though this makes the valuation more sensitive to the assumptions made about the valuation of tax shields. On the other hand, if the unlevered cost of equity is believed to be company specific,[79] and one has a good idea about the riskiness of this particular company's interest tax shields, it may be easier to start out with an estimate of the cost of equity capital. In either situation, it is definitely important to carefully consider which set of assumptions is reasonable and appropriate for a particular company.

4. A Comparison of Equity Valuation Models

As have been demonstrated, correctly implemented the five different valuation models are equivalent. So, which one is the best?

Figure 1 visualises how the different valuation attributes[80] considered in this essay develop over time in the Eldon case. Figure 1 indicates that the EVA concept says more about the qualitative development of the company than the other items. During the first years, which are forecasted to be good performance years with high revenue growth, the forecasted EVA is high, while it later on settles down to a lower steady state level. Looking at the FCF or DIV for these years we can see that the growth rate is slightly higher than in later years, but the graph is not as illustrative as the one for EVA. The AE develops in the same pattern as EVA, but not with the same indicative magnitude. This (single case) indicates that the Stewart (1991) proposal about EVA's superiority as a managerial tool, i.e., how it supports managerial decision making (by focusing on what creates value for shareholders) at least can not be rejected. However, it may very well prove that the similar AE concept may be as useful. Anyway, this feature is fairly unimportant for an outside analyst.

[79] That is, different companies in an industry may have different *operating* risk characteristics, due to, e.g., different marketing strategies.

[80] Note that by valuation attributes are meant the accounting concepts defined in section 2.2, i.e., free cash flow (FCF), economic value added (EVA), abnormal earnings (AE), and dividends (DIV). Note also that in the Eldon AB case EVA is calculated using the Stewart (1991) specification.

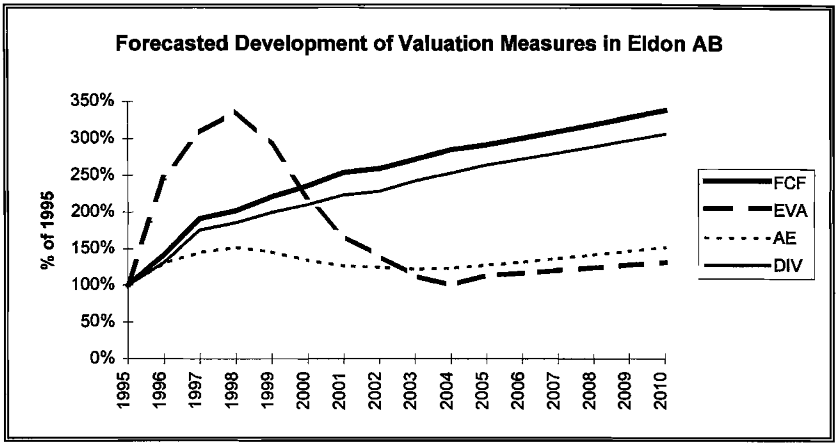

Figure 1 - Development over time of the forecasted valuation attributes for Eldon AB, Index: 100%=1995

A conceptual advantage of the AE and DIV approaches in *equity* valuation is that by definition they are more directly related to the market value of equity than FCF and EVA. Even if it is the (possible) dividends that ultimately *define* the equity value, these dividends cannot be projected without estimating the operating performance of the company. Thus, company valuation is very much about projecting the operating performance, and as measures of this FCF and EVA are superior. In particular, the FCF (and to some extent, EVA[81]) attribute is *independent of financing*, whereas either the AE or the DIV attribute cannot be calculated without explicitly projecting the company's financing policy. In general, a FCF valuation can be performed by first estimating the operating performance of the company (given the investments), and then separately bring the financing issues into consideration through the discount rate. Also, the (FCF based) APV model has the explicit advantage of separating the total company value into the value of the company's operations and the value of financing. With the AE and DIV models both operating and financing issues must (jointly) be considered in the first stage (i.e., when estimating the valuation attribute).

Moreover, the present value of the forecasted EVA a particular year is the *value added* to the estimated market value of the total company, stemming from that particular year's performance. In the same way, the present value of the forecasted abnormal earnings a particular year is the *value added* to the estimated market value of equity, stemming from that particular year's performance.

[81] Note that EVA calculation includes a capital charge based on WACC, which may or may not be independent of financing (e.g., in a Modigliani & Miller (1958) setting, EVA clearly is independent of financing).

Thus EVA is perhaps favoured to FCF (and the APV model) in this 'additive' sense, while AE might be favoured to DIV.

Regarding computational ease, the FCF, EVA, AE, and DIV models are similar in the framework of this essay, requiring the use of a backward discounting procedure (unless the company is expected to maintain a constant leverage in market terms, when FCF and EVA are much easier to implement). The APV model is perhaps the winner in this sense: The FCF part of the valuation is very easy since it (in most applications) uses a constant discount rate, the unlevered cost of equity; The financing/tax shield part may be more complicated, e.g., in the Miles & Ezzell setting, but may under other assumptions as well involve valuation with a single constant discount rate.

Another important aspect is of course the possibility of getting good forecasts of the valuation relevant attributes and parameters. In a framework where complete financial statements are forecasted the different attributes are equally easy to calculate. However, the models differ in one important aspect: They employ different discount rates. FCF and EVA use the WACC, AE and DIV the cost of equity capital, and APV the unlevered cost of equity capital and in many cases (in some way) also the cost of debt for the interest tax shields. If one of the rates is better for predictability reasons this will of course favour the model(s) using this discount rate. This is apparently an important question for future research.

Now, if we discard the case where complete financial statements are projected, the possibility of getting good forecasts of the valuation relevant attributes is certainly a matter to consider. Moreover, it is important to also consider how large part of the value from each model that is subject to forecasts (and possible forecast errors), and how large part that is (more or less) directly observable at the valuation date. Consider the Eldon AB case as an example: In the DIV model the forecast attribute is future dividends, amounting to an estimated present value of 530.8 MSEK. In the FCF model the forecast attribute is future free cash flows, amounting to an estimated present value of 894.0 MSEK. In the AE model the target is the future abnormal earnings, which amount to an estimated present value of 101.7 MSEK. Finally, in the EVA model the forecast target is the future economic value added each year, amounting to an estimated present value of 31.2 MSEK. Thus, it stands pretty clear that the EVA and AE models are more insensitive to forecast errors than the flow oriented DIV and FCF models.

However, despite this, Kaplan & Ruback (1995) find that discounted cash flow analysis is reliable.[82] Penman & Sougiannis (1997b) (PS) as well as Francis, Olsson & Oswald (1997) (FOO) find that the AE model performs relatively well, and outperforms both dividend and free cash flow valuations. In contrary to Kaplan & Ruback (1995), PS find that free cash flow valuations perform devastatingly bad, which however not is the case in FOO (even if FCF is not as good as AE).[83] DIV valuations have not been found to perform particularly well in any of the recent studies. An implication of both PS and FOO, as well as of Sougiannis & Yaekura (1997) (which tests different horizon value calculations for the AE model) that the horizon value calculation is a very important task. To conclude, the AE model seems to be a robust and reliable valuation tool. The characteristic that the AE valuation starts out by (the "known") book value and then targets an attribute that only should explain the value added (goodwill) to this book value is certainly appealing.[84] The EVA model is in this respect very similar and could thus be expected to perform as well as the AE model: This is an interesting question for future empirical research.

Taking all these issues in consideration there is no clear winner in the company valuation model contest, but the AE or EVA models are the first hand candidates. Given the general equivalence of the models when forecasts are produced consistently, the bottom line is that one should use the model one finds most comfortable in each specific valuation situation.

5. Summary and Concluding Remarks

The traditional textbook approach to company valuation with constant discount rates fails to achieve equivalence between the different valuation models. On the one hand, we have the models whose valuation attributes relate to total company value: the *free cash flow* and *economic value added* models. These two models are always equivalent to each other, but in the constant discount rate case they are not generally equivalent to the *dividend* and *abnormal earnings* models. The latter two

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[82] Kaplan & Ruback (1995) do not exactly use free cash flow forecasts according to the definition in this essay. Instead they consider "capital cash flows", i.e., they explicitly include the interest tax shields in the valuation attribute, and discount this at the unlevered cost of equity, which can be seen as a "compressed APV" technique.

[83] The main difference between the PS and the FOO studies is that PS use realisations, while FOO use analysts' forecasts.

[84] The FCF valuation targets an attribute that should explain *total* company value (and not just the value added to a known book value), and thus a relatively small forecasting error can have a large impact on the valuation.

models, whose valuation attributes relate more directly to the *equity value,* are also equivalent to each other.

Further, in a world with corporate taxes, the traditional approach is theoretically valid only when the company is expected to maintain a constant market debt ratio. The only case where this will be appropriate is under an assumption of *active debt management* such that the company periodically adjusts its borrowing to keep a constant market debt ratio.[85] But this leads to estimation problems for the abnormal earnings and dividend models: One cannot calculate the abnormal earnings or the dividends until one has estimated the future debt. But one cannot estimate the future debt until one has estimated the expected company value at each future debt restructuring date, which in turn cannot be calculated until we have computed the abnormal earnings or dividends. Thus, an iterative procedure is required to estimate the *valuation attributes* abnormal earnings and dividends if the AE and DIV models are used. In this case, in practice, it is more straightforward to use a valuation model where the valuation attribute is independent of financing: either the free cash flow or the economic value added model.

This essay has developed a valuation framework that ensures equivalence between all the different valuation models even if the projected capital structure in market terms changes over time. By using the backward discounting procedure suggested in Essays 2 and 3, together with the modelling framework presented here (including the possibility of using a non-constant corporate tax rate and a non-constant cost of debt), the general equivalence of these four valuation models is achieved. Moreover, when the cost of capital setting is specified, including assumptions about the valuation of interest tax shields, the equivalence is further extended to include the adjusted present value model. Thus, the framework developed in this essay *ensures the equivalence among the five leading discounting-based company valuation models*. The equivalence holds for both finite and infinite valuations, as well as year by year.

The main theme of this essay is that, given projected financial statements, the different company valuation models give the same result. There exist other methods that more directly forecast valuation

[85] As commented earlier, a constant market debt ratio can under certain conditions also be achieved in constant growth cases. Practically, this is however only valid *after some future horizon* where the company is expected to enter a steady state, and is thus not applicable to a full scale valuation (i.e., from the valuation date until the end of the life of the company).

attributes, e.g., through time-series analysis.[86] If such direct forecasts of the different valuation attributes are (independently) used rather than the financial statements approach used in this essay we may very well observe different values from the models. Then, the abnormal earnings or the economic value added model may be the best choice, given their relative insensitivity to forecast errors. However, the important lesson to be learnt from this essay is that differences in valuations are due to inconsistencies between *forecasts* of valuation attributes and/or estimated discount rates, and *not* to fundamental differences between the valuation models. If the forecasts are internally consistent, then each of the five models considered here will give the same answer.

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[86] Lorek & Willinger (1996) investigate the time-series properties of cash flows. Brown & Rozeff (1979) do the same for earnings. Ohlson (1995) develops an approach where the time-series behaviour of earnings and book equity (i.e., the main components of the abnormal earnings valuation attribute), as well as cash flows are explicitly modelled. For a recent empirical test of the Ohlson model, see Dechow, Hutton & Sloan (1997).

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Appendix 1 - Notation

The notation used in this paper is as follows: (Note that the statement "the end of year *t*" should be regarded as being equivalent to "the beginning of year $t+1$".)

Items directly related to valuation:

EV_t = Equity value, end of year t (ex-dividend year t

D_t = Debt value, end of year t (ex-interest year t

TCV_t = (Total) Company value, end of year t (i.e., EV_t plus D_t)

AE_t = Abnormal earnings, year t

DIV_t = (Net) Dividends, year t

EVA_t = Economic value added, year t

FCF_t = Free cash flow, year t

TS_t = Interest tax shield, year t

$PVTS_{t,arg}$ = Present value at the end of year t of

[*arg* = all]: all future tax shields

[*arg* = *k*]: the part of future tax shields that is *discounted at* rate k (in this essay i (time-index suppressed) or k_U) *during* year $t+1$

H = Horizon (at which some horizon value technique is used)

I_t = Information available, end of year t

K_S = Growth factor of the measurement error in the accounting (of a particular model) over S years

T = The expected end of the company's life (a finite year or ∞)

g_{item} = (constant) annual growth rate after horizon of valuation item, where *item* can be AE (abnormal earnings), D (debt), EVA (economic value added), FCF (free cash flow), or TS (interest tax shields)

Cost of capital items:

$k_{WACC,t}$ = Weighted average cost of capital,[87] year t

$k_{E,t}$ = Cost of equity capital, year t

k_U = The unlevered cost of equity (constant)

i_t = Cost of debt = (Effective) Borrowing rate, year t

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[87] $k_{WACC,t} = \frac{D_{t-1}}{TCV_{t-1}}(1-\tau_t)i_t + \left(1 - \frac{D_{t-1}}{TCV_{t-1}}\right)k_{E,t}$

τ_t = Corporate tax rate, year t

$\bar{k}_{WACC}$ = Constant weighted average cost of capital

$\bar{k}_E$ = Constant cost of equity capital

$\bar{i}$ = Constant cost of debt

$\bar{\tau}$ = Constant corporate tax rate

$\bar{k}_{WACC}^H$ = Constant weighted average cost of capital from horizon H (to infinity)

$\bar{k}_E^H$ = Constant cost of equity capital from horizon H (to infinity)

Accounting items:

A_t = Accumulated depreciation, end of year t

B_t = Book value of stock item (general notation), end of year t

BV_t = Book value of (owners') equity, end of year t (ex-dividend year t)

Cap_t = (Invested) Capital, end of year t (general notation for two alternative specifications, IC_t and IC_t^*)

$CapX_t$ = Capital expenditures, year t

$DepX_t$ = Depreciation expense, year t

F_t = Flow variable (general notation), year t

G_t = Gross property, plant and equipment, end of year t

IC_t = Invested capital, end of year t

IC_t^* = Invested capital, end of year t (alternative specification)

IT_t = (Income) Taxes, year t

IX_t = Interest expense, year t (equals $i_t \cdot D_{t-1}$)

N_t =Net property, plant and equipment, end of year t

$NOPLAT_t$ = Net operating profit less adjusted taxes, year t

NP_t = Net profit, year t

OA_t = Market value of operating assets, end of year t

OP_t = Operating profit (before taxes), year t

$OPAT_t$ = Operating profit after taxes, year t

OpX_t = Operating expenses, year t

R_t = Revenues, in year t

Ret_t = Retirements, year t

T_t = Deferred taxes, end of year t

WA_t = Working capital assets, end of year t

WL_t = Working capital liabilities, end of year t

$g_{T,t}$ = annual growth (or decline) rate during year t of deferred taxes

q_{CM} = relative cost matching bias of owners' equity

r_t = expected book return on owners' equity, year t

Π_t = general notation for after-tax operating profit (either $NOPLAT_t$ or $OPAT_t$)

<u>Accounting relations and definitions:</u>

Balance sheet relation, asset side (definition of invested capital, IC_t):

(BSR1) $$IC_t \equiv (WA_t - WL_t) + N_t$$

Balance sheet relation, debt and equity side:

(BSR2) $$D_t + T_t + BV_t = IC_t$$

Clean surplus relation:

(CSR) $$BV_t = BV_{t-1} + NP_t - DIV_t$$

Definition of growth in deferred taxes:

(DT) $$g_{T,t} \equiv \frac{(T_t - T_{t-1})}{T_{t-1}}$$

Alternative specification of invested capital, IC_t^*:

(IC) $$IC_t^* \equiv IC_t - T_t = D_t + BV_t = (WA_t - WL_t) + N_t - T_t$$

Definition of net operating profit less adjusted taxes:

(N1) $$\begin{aligned} NOPLAT_t &\equiv OP_t - (IT_t - (T_t - T_{t-1}) + \tau_t \cdot IX_t) \\ &= OPAT_t + (T_t - T_{t-1}) \\ &= NP_t + (1-\tau_t)IX_t + (T_t - T_{t-1}) \end{aligned}$$

Reverse relation between profit concepts:

(N2) $$\begin{aligned} NP_t &= (1-\tau_t)(OP_t - IX_t) \\ &= NOPLAT_t - (1-\tau_t)IX_t - (T_t - T_{t-1}) \end{aligned}$$

Definition of operating profit after taxes:

(OP) $$OPAT_t \equiv (1-\tau_t) \cdot OP_t$$

Property, plant and equipment relations:

(PPE) $$\begin{aligned} A_t &= A_{t-1} + DepX_t - Ret_t \\ G_t &= G_{t-1} + CapX_t - Ret_t \end{aligned}$$

<u>Valuation related definitions and relations:</u>

Definition of abnormal earnings:

(AE) $$AE_t \equiv NP_t - k_{E,t} BV_{t-1}$$

Dividend relation:

(D1) $$DIV_t = BV_{t-1} + NP_t - BV_t$$

Alternative (equivalent) dividend formula:

(D2) $$DIV_t = NP_t + (D_t - D_{t-1}) + (T_t - T_{t-1}) - (IC_t - IC_{t-1})$$

Definition of economic value added:

(EVA) $$EVA_t \equiv \Pi_t - k_{WACC,t} Cap_{t-1}$$

Definition of free cash flow:

(F1)
$$\begin{aligned} FCF_t &\equiv \\ &\equiv NOPLAT_t + DepX_t && \text{(Gross FCF)} \\ &-((WA_t - WL_t) - (WA_{t-1} - WL_{t-1})) - CapX_t && \text{(-Gross investment)} \end{aligned}$$

Alternative (equivalent) free cash flow formula:

(F2)
$$\begin{aligned} FCF_t &= \\ &= NOPLAT_t + DepX_t \\ &-((WA_t - WL_t) - (WA_{t-1} - WL_{t-1})) - (N_t - N_{t-1} + DepX_t) \\ &= NOPLAT_t - (IC_t - IC_{t-1}) = OPAT_t - (IC_t^* - IC_{t-1}^*) \end{aligned}$$

General relation between cost of equity, cost of debt and unlevered cost of equity:

(GEN) $$k_{E,t} = k_U + (k_U - i_t)\frac{D_{t-1} - PVTS_{t-1,i}}{EV_{t-1}}$$

Definition of interest tax shields:

(TS) $$TS_t \equiv \tau_t \cdot i_t \cdot D_{t-1}$$

<u>Valuation model definitions:</u>

Equity value according to the AE model:

(EV:AE) $$EV_t(AE) \equiv BV_t + \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_E}[AE_s]$$

Equity value according to the APV model:

(EV:APV)
$$\begin{aligned} EV_t(APV) &\equiv OA_t + PVTS_{t,all} - D_t = \\ &= \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_U}[FCF_s] + PVTS_{t,all} - D_t \end{aligned}$$

Equity value according to the DIV model:

(EV:DIV) $$EV_t(DIV) \equiv \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_E}[DIV_s]$$

Equity value according to the EVA model:

(EV:EVA) $$EV_t(EVA) \equiv Cap_t + \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_{WACC}}[EVA_s] - D_t$$

Equity value according to the FCF model:

(EV:FCF) $$EV_t(FCF) \equiv \sum_{s=t+1}^{T} \mathrm{PV}_t^{k_{WACC}}[FCF_s] - D_t$$

Yearly equity value dynamics according to the AE model:

(EV$_{dyn}$:AE) $$EV_{t-1} = BV_{t-1} + \frac{AE_t + EV_t - BV_t}{1 + k_{E,t}}$$

Yearly equity value dynamics according to the APV model (see Proof of Proposition 4 in Appendix 2):

(EV$_{dyn}$:APV) $$EV_{t-1} = PVTS_{t-1,all} + \frac{FCF_t + EV_t + D_t - PVTS_{t,all}}{1 + k_U} - D_{t-1}$$

Yearly equity value dynamics according to the DIV model:

(EV$_{dyn}$:DIV) $$EV_{t-1} = \frac{DIV_t + EV_t}{1 + k_{E,t}}$$

Yearly equity value dynamics according to the EVA model:

(EV$_{dyn}$:EVA) $$EV_{t-1} = Cap_{t-1} + \frac{EVA_t + EV_t + D_t - Cap_t}{1 + k_{WACC,t}} - D_{t-1}$$

Yearly equity value dynamics according to the FCF model:

(EV$_{dyn}$:FCF) $$EV_{t-1} = \frac{FCF_t + EV_t + D_t}{1 + k_{WACC,t}} - D_{t-1}$$

<u>Continuing value formulas:</u>

AE continuing value formula:

(CV:AE) $$CV_H\left[\mathbf{AE}_{H+1}^{\infty}\right] = \frac{AE_{H+1}}{\bar{k}_E^H - g_{AE}}$$

APV continuing value formula:

(CV:APV) $$CV_{H|unlevered}\left[\mathbf{FCF}_{H+1}^{\infty}\right] + CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right] = \frac{FCF_{H+1}}{k_U - g_{FCF}} + CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right]$$

DIV continuing value formula:

(CV:DIV) $$CV_H\left[\mathbf{DIV}_{H+1}^{\infty}\right] = \frac{DIV_{H+1}}{\bar{k}_E^H - g_{DIV}}$$

EVA continuing value formula:

(CV:EVA) $$CV_H\left[\mathbf{EVA}_{H+1}^{\infty}\right] = \frac{EVA_{H+1}}{\bar{k}_{WACC}^H - g_{EVA}}$$

FCF continuing value formula:

(CV:FCF) $$CV_H\left[\mathbf{FCF}_{H+1}^{\infty}\right] = \frac{FCF_{H+1}}{\bar{k}_{WACC}^H - g_{FCF}}$$

Appendix 2 - Proofs

Proof of Proposition 1

The equity value at time H obtained from the DIV model is, by (EV:DIV), (CV:DIV) and (GR) given by:

(A2:1) $$EV_H = CV_H\left[\mathbf{DIV}_{H+1}^{\infty}\right] = \frac{DIV_{H+1}}{\bar{k}_E^H - g}$$

Analogously, the equity value at time H from the AE model is, by (EV:AE), (CV:AE) and (GR), given by:

(A2:2) $$EV_H = BV_H + CV_H\left[\mathbf{AE}_{H+1}^{\infty}\right] = BV_H + \frac{AE_{H+1}}{\bar{k}_E^H - g}$$

As the AE model is derived from the DIV model by applying (CSR),[88] it should not come as a surprise that the AE and DIV continuing value approaches are equivalent. Using the definition (AE) and setting the RHS terms in (A2:2) on a common denominator yields:

(A2:3) $$EV_H = \frac{BV_H\left(\bar{k}_E^H - g\right) + NP_{H+1} - \bar{k}_E^H \cdot BV_H}{\bar{k}_E^H - g}$$

This can be simplified to:

(A2:4) $$EV_H = \frac{NP_{H+1} - BV_H \cdot g}{\bar{k}_E^H - g}$$

Note that, by (GR), $BV_H \cdot g$ is just another way of writing BV_{H+1} - BV_H. By this fact and (CSR) or (D1), expression (A2:4) can be reformulated as:

(A2:5) $$EV_H = \frac{DIV_{H+1}}{\bar{k}_E^H - g}$$

But this is just the equity value at time H from the DIV model. The DIV and AE continuing value approaches are equivalent.

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[88] See, e.g., Ohlson (1995).

Now, consider the FCF model. By (EV:FCF), (CV:FCF) and (GR) we have:

(A2:6) $$EV_H = CV_H\left[\mathbf{FCF}_{H+1}^{\infty}\right] - D_H = \frac{FCF_{H+1}}{\bar{k}_{WACC}^{H} - g} - D_H$$

This can by using the FCF definition (F2) be rearranged into the following equation:

(A2:7) $$EV_H + DV_H = \frac{NOPLAT_{H+1} - \left(IC_{H+1} - IC_H\right)}{\bar{k}_{WACC}^{H} - g}$$

Substituting (N1) and simplifying yields:

(A2:8) $$EV_H + D_H = \frac{NP_{H+1} + (1-\bar{\tau})\bar{i}D_H + \left(T_{H+1} - T_H\right) - \left(IC_{H+1} - IC_H\right)}{\bar{k}_{WACC}^{H} - g}$$

By definition $\bar{k}_{WACC}^{H} = \frac{D_H}{TCV_H}(1-\bar{\tau})\bar{i} + \frac{EV_H}{TCV_H}\bar{k}_E^H = \frac{D_H}{TCV_H}(1-\bar{\tau})\bar{i} + \left(1 - \frac{D_H}{TCV_H}\right)\bar{k}_E^H$ where $TCV_H = EV_H + D_H$.

Inserting this into (A2:8) yields:

(A2:9) $$TCV_H = \frac{NP_{H+1} + (1-\bar{\tau})\bar{i}D_H + \left(T_{H+1} - T_H\right) - \left(IC_{H+1} - IC_H\right)}{\frac{D_H}{TCV_H}(1-\bar{\tau})\bar{i} + \left(1 - \frac{D_H}{TCV_H}\right)\bar{k}_E^H - g}$$

Rearranging the right hand side (RHS) denominator yields:

(A2:10) $$TCV_H = \frac{NP_{H+1} + (1-\bar{\tau})\bar{i}D_H + \left(T_{H+1} - T_H\right) - \left(IC_{H+1} - IC_H\right)}{\frac{D_H(1-\bar{\tau})\bar{i} - D_H \cdot \bar{k}_E^H + TCV_H\left(\bar{k}_E^H - g\right)}{TCV_H}}$$

Multiplying both sides with the RHS denominator gives:

(A2:11) $$\begin{aligned} &D_H(1-\bar{\tau})\bar{i} - D_H \cdot \bar{k}_E^H + TCV_H\left(\bar{k}_E^H - g\right) = \\ &= NP_{H+1} + (1-\bar{\tau})\bar{i}D_H + \left(T_{H+1} - T_H\right) - \left(IC_{H+1} - IC_H\right) \end{aligned}$$

Rearranging yields:

(A2:12) $$TCV_H = \frac{NP_{H+1} + \left(T_{H+1} - T_H\right) - \left(IC_{H+1} - IC_H\right) + D_H \cdot \bar{k}_E^H}{\bar{k}_E^H - g}$$

The equity value is then obtained by deducting the debt, i.e., by deducting D_H from both sides of (A2:12):

(A2:13) $$EV_H = \frac{NP_{H+1} + (T_{H+1} - T_H) - (IC_{H+1} - IC_H) + D_H \cdot \bar{k}_E^H}{\bar{k}_E^H - g} - D_H$$

The RHS can be rearranged:

(A2:14) $$\begin{aligned} EV_H &= \frac{NP_{H+1} + (T_{H+1} - T_H) - (IC_{H+1} - IC_H) + D_H \cdot \bar{k}_E^H - D_H \cdot (\bar{k}_E^H - g)}{\bar{k}_E^H - g} = \\ &= \frac{NP_{H+1} + (T_{H+1} - T_H) - (IC_{H+1} - IC_H) + D_H \cdot g}{\bar{k}_E^H - g} \end{aligned}$$

By (GR) and substitution of (BSR2) we get the following new equation:

(A2:15) $$EV_H = \frac{NP_{H+1} - BV_H \cdot g}{\bar{k}_E^H - g}$$

But this is the same as (A2:4) which is equivalent to the equity value obtained from the AE and DIV models. Thus we have proved that the continuing value approaches of the FCF, AE and DIV models are equivalent, subject to (GR).

Remains to prove that this also holds for EVA. The equity value at time H obtained from the EVA model is, by (EV:EVA), (CV:EVA) and (GR) given by:

(A2:16) $$EV_H = CV_H\left[\mathbf{EVA}_{H+1}^{\infty}\right] = Cap_H + \frac{EVA_{H+1}}{\bar{k}_{WACC}^H - g} - D_H$$

Using the definition (EVA) and rearranging yields:

(A2:17) $$EV_H = \frac{\Pi_{H+1} - \bar{k}_{WACC}^H \cdot Cap_H + Cap_H \cdot (\bar{k}_{WACC}^H - g)}{\bar{k}_{WACC}^H - g} - D_H$$

This can be simplified to:

(A2:18) $$EV_H = \frac{\Pi_{H+1} - Cap_H \cdot g}{\bar{k}_{WACC}^H - g} - D_H$$

Note that, by (GR), $Cap_H \cdot g$ is just another way of writing $Cap_{H+1} - Cap_H$. By this fact and the FCF (F2) formula, expression (A2:18) can (for both specifications of EVA)[89] be reformulated as:

(A2:19) $$EV_H = \frac{FCF_{H+1}}{\bar{k}_{WACC}^{H} - g} - D_H$$

But this is just the equity value at time H from the FCF model. Since we already have proved that the FCF model's continuing value is equivalent to those of the AE and DIV models, the same holds for EVA.

Q.E.D.

Proof of Proposition 2

The proposition will be proved by using the yearly equity dynamics of the AE and EVA models:

(EV_{dyn}:AE) $$EV_{t-1} = BV_{t-1} + \frac{AE_t + EV_t - BV_t}{1 + k_{E,t}}$$

(EV_{dyn}:EVA) $$EV_{t-1} = Cap_{t-1} + \frac{EVA_t + EV_t + D_t - Cap_t}{1 + k_{WACC,t}} - D_{t-1}$$

First, assume that the dynamics are equivalent (which they *are*, see Proof of Proposition 3, below), such that equity value is transferred between years in exactly the same way in both models. Now, solve for AE_t and EVA_t, respetively:

(A2:21) $$AE_t = (1 + k_{E,t})(EV_{t-1} - BV_{t-1}) - (EV_t - BV_t)$$

(A2:22) $$EVA_t = (1 + k_{WACC,t})(EV_{t-1} + D_{t-1} - Cap_{t-1}) - (EV_t + D_t - Cap_t)$$

Assume that AE_t and EVA_t are equal. Thus:

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[89] Recall that we consider two different specifications:

1) "Stewart" specification: $\Pi_t = NOPLAT_t; Cap_t = IC_t$ for all t

2) "alternative" specification: $\Pi_t = OPAT_t; Cap_t = IC_t^*$ for all t

(A2:23)
$$\begin{aligned}&(1+k_{E,t})(EV_{t-1}-BV_{t-1})-(EV_t-BV_t)=\\&=(1+k_{WACC,t})(EV_{t-1}+D_{t-1}-Cap_{t-1})-(EV_t+D_t-Cap_t)\end{aligned}$$

Remember that there exist two specifications of EVA. First, consider the "Stewart" specification, where $Cap_t = IC_t$. This fact and the IC_t relation (BSR2) yields:

(A2:24)
$$\begin{aligned}&(1+k_{E,t})(EV_{t-1}-BV_{t-1})-(EV_t-BV_t)=\\&=(1+k_{WACC,t})(EV_{t-1}+D_{t-1}-(BV_{t-1}+D_{t-1}+T_{t-1}))-(EV_t+D_t-(BV_t+D_t+T_t))\end{aligned}$$

Simplifying gives:

(A2:25)
$$(1+k_{E,t})(EV_{t-1}-BV_{t-1})=(1+k_{WACC,t})(EV_{t-1}-BV_{t-1}-T_{t-1})+T_t$$

Rearranging and using the (GT) notation for growth in deferred taxes finally yields:

(A2:26)
$$(g_{T,t}-k_{WACC,t})T_{t-1}=(k_{E,t}-k_{WACC,t})(EV_{t-1}-BV_{t-1})\Leftrightarrow AE_t=EVA_t$$

Next, consider the "alternative" EVA specification, where $Cap_t = IC_t^*$. This fact and relation (IC) yields from (A2:23):

(A2:27)
$$\begin{aligned}&(1+k_{E,t})(EV_{t-1}-BV_{t-1})-(EV_t-BV_t)=\\&=(1+k_{WACC,t})(EV_{t-1}+D_{t-1}-(BV_{t-1}+D_{t-1}))-(EV_t+D_t-(BV_t+D_t))\end{aligned}$$

Now, simplifying yields:

(A2:28)
$$(1+k_{E,t})(EV_{t-1}-BV_{t-1})=(1+k_{WACC,t})(EV_{t-1}-BV_{t-1})\Leftrightarrow AE_t=EVA_t$$

Q.E.D.

Proof of Proposition 3

Proposition 3 will be proved by first considering an arbitrary year in the forecast period. We will show that the yearly dynamics[90] of the different valuation models with the proposed backward discounting procedure are *equivalent* for an arbitrary year. Then this must hold for any and *all* years in the forecast period, since the models and the discounting procedure are applied in the same way for any year. By this argument Proposition 3 is finally proved.

[90] That is, the way the market value of equity value at the beginning of one year is related to the market value of equity at the end of the year through the valuation attribute in question.

The yearly dynamics for the different models are (as described in section 2.4):

(EV$_{dyn}$:DIV) $$EV_{t-1} = \frac{DIV_t + EV_t}{1 + k_{E,t}}$$

(EV$_{dyn}$:FCF) $$EV_{t-1} = \frac{FCF_t + EV_t + D_t}{1 + k_{WACC,t}} - D_{t-1}$$

(EV$_{dyn}$:EVA) $$EV_{t-1} = IC_{t-1} + \frac{EVA_t + EV_t + D_t - IC_t}{1 + k_{WACC,t}} - D_{t-1}$$

(EV$_{dyn}$:AE) $$EV_{t-1} = BV_{t-1} + \frac{AE_t + EV_t - BV_t}{1 + k_{E,t}}$$

We start out with FCF and consider year s. The valuation date is December 31 in year s-1. By (EV$_{dyn}$:FCF) and (F2) we have:

(A2:29) $$EV_{s-1} + D_{s-1} = \frac{NOPLAT_s - (IC_s - IC_{s-1}) + EV_s + D_s}{1 + k_{WACC,s}}$$

Substituting (N1) and simplifying yields:

(A2:30) $$EV_{s-1} + D_{s-1} = \frac{NP_s + (1-\tau_s)i_s D_{s-1} + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + D_s}{1 + k_{WACC,s}}$$

By the definition of the backward discounting procedure we have:

$$k_{WACC,s} = \frac{D_{s-1}}{TCV_{s-1}}(1-\tau_s)i_s + \frac{EV_{s-1}}{TCV_{s-1}}k_{E,s} = \frac{D_{s-1}}{TCV_{s-1}}(1-\tau_s)i_s + \left(1 - \frac{D_{s-1}}{TCV_{s-1}}\right)k_{E,s}$$ where $TCV_{s-1} = EV_{s-1} + D_{s-1}$.

Substituting this into (A2:30) yields:

(A2:31) $$TCV_{s-1} = \frac{NP_s + (1-\tau_s)i_s D_{s-1} + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + D_s}{1 + \frac{D_{s-1}}{TCV_{s-1}}(1-\tau_s)i_s + \left(1 - \frac{D_{s-1}}{TCV_{s-1}}\right)k_{E,s}}$$

The right hand side (RHS) denominator can be rearranged, which yields:

$$\text{(A2:32)} \qquad TCV_{s-1} = \frac{NP_s + (1-\tau_s)i_s D_{s-1} + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + D_s}{\frac{D_{s-1}(1-\tau_s)i_s - D_{s-1}k_{E,s} + TCV_{s-1}(1+k_{E,s})}{TCV_{s-1}}}$$

Now, we multiply both sides with the RHS denominator:

$$\text{(A2:33)} \qquad \begin{aligned} &D_{s-1}(1-\tau_s)i_s - D_{s-1}k_{E,s} + TCV_{s-1}(1+k_{E,s}) = \\ &= NP_s + (1-\tau_s)i_s D_{s-1} + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + D_s \end{aligned}$$

Rearranging yields:

$$\text{(A2:34)} \qquad TCV_{s-1} = \frac{NP_s + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + D_s + D_{s-1}k_{E,s}}{(1+k_{E,s})}$$

Then we can obtain the equity value by deducting the debt, D_{s-1}, from both sides of (A2:34):

$$\text{(A2:35)} \qquad EV_{s-1} = \frac{NP_s + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + D_s + D_{s-1}k_{E,s}}{(1+k_{E,s})} - D_{s-1}$$

The RHS can be rearranged in the following way:

$$\text{(A2:36)} \qquad \begin{aligned} &\frac{NP_s + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + D_s + D_{s-1}k_{E,s} - D_{s-1}(1+k_{E,s})}{(1+k_{E,s})} = \\ &= \frac{NP_s + (T_s - T_{s-1}) - (IC_s - IC_{s-1}) + EV_s + (D_s - D_{s-1})}{(1+k_{E,s})} \end{aligned}$$

By (BSR2) we now get the following equation:

$$\text{(A2:37)} \qquad EV_{s-1} = \frac{NP_s - (BV_s - BV_{s-1}) + EV_s}{(1+k_{E,s})}$$

Now, we add $k_{E,s}BV_{s-1} - k_{E,s}BV_{s-1}$ to the RHS numerator:

$$\text{(A2:38)} \qquad EV_{s-1} = \frac{NP_s - (BV_s - BV_{s-1}) + k_{E,s}BV_{s-1} - k_{E,s}BV_{s-1} + EV_s}{(1+k_{E,s})}$$

Then rearranging yields:

$$\text{(A2:39)} \qquad EV_{s-1} = \frac{NP_s + BV_{s-1}(1+k_{E,s}) - k_{E,s}BV_{s-1} + EV_s - BV_s}{(1+k_{E,s})}$$

Simplifying and using the (AE) definition yields:

(A2:40) $$EV_{s-1} = BV_{s-1} + \frac{AE_s + EV_s - BV_s}{1 + k_{E,s}}$$

But (A2:40) is just the yearly dynamics according to the AE model. Thus we have now proved that the yearly dynamics of the FCF models equal the dynamics of the AE model. Now, since the dynamics apply to any year in the forecast period, the FCF and AE models are equivalent when the proposed backward discounting procedure is used.

Now, the next step will be to show that the FCF model equals the EVA model. Consider the EVA model. (EV_{dyn} :EVA) and the (EVA) definition yields

(A2:41) $$EV_{s-1} = Cap_{s-1} + \frac{\Pi_s - k_{WACC,s}Cap_{s-1} + EV_s + D_s - Cap_s}{1 + k_{WACC,s}} - D_{s-1}$$

Rearranging yields:

(A2:42) $$EV_{s-1} = \\ = \frac{\Pi_s - k_{WACC,s}Cap_{s-1} + Cap_{s-1}(1 + k_{WACC,s}) + EV_s + D_s - Cap_s}{1 + k_{WACC,s}} - D_{s-1}$$

Simplifying now yields:

(A2:43) $$EV_{s-1} = \frac{\Pi_s - (Cap_s - Cap_{s-1}) + EV_s + D_s}{1 + k_{WACC,s}} - D_{s-1}$$

Since $\Pi_s - (Cap_s - Cap_{s-1}) = NOPLAT_s - (IC_s - IC_{s-1}) = OPAT_s - (IC_s^* - IC_{s-1}^*)$, we by (F2) finally get:

(A2:44) $$EV_{s-1} = \frac{FCF_s + EV_s + D_s}{1 + k_{WACC,s}} - D_{s-1}$$

(A2:44) equals the yearly dynamics of the FCF model. By the same argument as above the EVA-equalling-FCF part of Proposition 3 is finally proved. Note that the equivalence between the FCF and EVA models holds without any restrictions on how WACC is calculated (except that the WACC each year must be the same when applied to the two models). Thus the FCF and EVA equivalence is preserved with the initial constant WACC approach.[91]

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[91] This equivalence is well known. See, e.g., Stewart (1991) or CKM.

Finally, it remains to show the equivalence between the AE and DIV models. This is very straightforward since, as mentioned above, the AE model in the literature is directly derived from the DIV model, but commonly only with a constant discount rate and/or risk-adjustments in the numerator.[92]

Consider the DIV model. At January 1, year *s*, the equity value can according to (EV_{dyn} :DIV) be expressed as:

(A2:45) $$EV_{s-1} = \frac{DIV_s + EV_s}{1 + k_{E,s}}$$

By using the dividend definition (D1) we get:

(A2:46) $$EV_{s-1} = \frac{NP_s - (BV_s - BV_{s-1}) + EV_s}{(1 + k_{E,s})}$$

But (A2:46) is exactly equal to equation (A2:37) above, which through equations (A2:38-40) was proved to be the AE model's yearly dynamics. Thus, by the fact that the dynamics apply to any year, the DIV model and the AE model are as could be expected fully equivalent. The DIV and AE models are obviously also equivalent with a constant cost of equity as discount rate.

Q.E.D.

Proof of Proposition 4

I will follow the same line of argumentation as in the proof of Proposition 3 and show that, under (PVTS) and (GEN), the yearly dynamics of the APV and FCF models are equivalent. The equivalence with the rest of the models follows then from Proposition 3, since Proposition 3 holds independently of how the cost of equity is specified.

First, note that (PVTS) implies the following yearly dynamics for the present value of all future tax shields:[93]

[92] See, e.g, Ohlson (1995), Feltham & Ohlson (1994), and Skogsvik (1994).

[93] The following are the [Δ_t^i ; δ_t^i] values in the three cases discussed in this paper:

$$(\mathrm{TS}_{dyn})\quad \begin{aligned} PVTS_{t,all} &= PVTS_{t+1,all}\cdot\left[\frac{\Delta_t^i}{1+i_{t+1}}+\frac{(1-\Delta_t^i)}{1+k_U}\right]+TS_{t+1}\left[\frac{\delta_t^i}{1+i_{t+1}}+\frac{(1-\delta_t^i)}{1+k_U}\right]= \\ &=\left[\frac{\Delta_t^i PVTS_{t+1,all}+\delta_t^i TS_{t+1}}{1+i_{t+1}}\right]+\left[\frac{(1-\Delta_t^i)PVTS_{t+1,all}+(1-\delta_t^i)TS_{t+1}}{1+k_U}\right]\end{aligned}$$

where Δ_t^i = the fraction of $PVTS_{t+1,all}$ that is discounted at the cost of debt

δ_t^i = the fraction of TS_{t+1} that is discounted at the cost of debt

Moreover, by (TS_{dyn}) and (PVTS) we also have:

$$(\mathrm{TS}_{dyn}:i)\ PVTS_{t,i}=\left[\frac{\Delta_t^i PVTS_{t+1,all}+\delta_t^i TS_{t+1}}{1+i_{t+1}}\right]$$

$$(\mathrm{TS}_{dyn}:k_U)\ PVTS_{t,k_U}=\left[\frac{(1-\Delta_t^i)PVTS_{t+1,all}+(1-\delta_t^i)TS_{t+1}}{1+k_U}\right]$$

Let us now derive the yearly dynamics of the APV model. We start out by rearranging (EV:APV):

$$(\mathrm{TCV:APV})\qquad TCV_t=\sum_{s=t+1}^{T}\mathrm{PV}_t^{k_U}[FCF_s]+PVTS_{t,all}$$

This can be rewritten:

$$(\mathrm{A2:47})\qquad TCV_t=\frac{FCF_{t+1}}{1+k_U}+\frac{\sum_{s=t+2}^{T}\mathrm{PV}_{t+1}^{k_U}[FCF_s]}{1+k_U}+PVTS_{t,all}$$

Analogously, the value at time $t+1$ can be written:

$$(\mathrm{A2:48})\qquad TCV_{t+1}=\sum_{s=t+2}^{T}\mathrm{PV}_{t+1}^{k_U}[FCF_s]+PVTS_{t+1,all}$$

Rearranging (A2:48):

$$(\mathrm{A2:49})\qquad \sum_{s=t+2}^{T}\mathrm{PV}_{t+1}^{k_U}[FCF_s]=TCV_{t+1}-PVTS_{t+1,all}$$

- [1; 1] in the case where all interest tax shields are discounted at i
- [0; 1] in the ME case
- [0; 0] in the case where all interest tax shields are discounted at k_U

Substituting (A2:49) into (A2:47) yields:

(A2:50) $$TCV_t = \frac{FCF_{t+1}}{1+k_U} + \frac{TCV_{t+1} - PVTS_{t+1,all}}{1+k_U} + PVTS_{t,all}$$

Shifting the time index finally yields:

(A2:51) $$TCV_{t-1} = PVTS_{t-1,all} + \frac{FCF_t + TCV_t - PVTS_{t,all}}{1+k_U}$$

By deducting the value of debt at the valuation date t-1, and using the TCV definition, we arrive at the equity value and have thus derived the yearly equity value dynamics of the APV model:

(EV_{dyn}:APV) $$EV_{t-1} = PVTS_{t-1,all} + \frac{FCF_t + EV_t + D_t - PVTS_{t,all}}{1+k_U} - D_{t-1}$$

The proof is completed by showing that (EV_{dyn}:APV) is equal to (EV_{dyn}:FCF). Rearranging (EV_{dyn}:APV) gives:

(A2:52) $$TCV_{t-1} = \frac{FCF_t + EV_t + D_t + (1+k_U)PVTS_{t-1,all} - PVTS_{t,all}}{1+k_U}$$

By (TS_{dyn}) we have:

(A2:53) $$TCV_{t-1} = \frac{FCF_t + EV_t + D_t + (1+k_U)\left[PVTS_{t,all}\left(\frac{\Delta^i_{t-1}}{1+i_t} + \frac{1-\Delta^i_{t-1}}{1+k_U}\right) + TS_t\left(\frac{\delta^i_{t-1}}{1+i_t} + \frac{1-\delta^i_{t-1}}{1+k_U}\right)\right]}{1+k_U} - \frac{PVTS_{t,all}}{1+k_U}$$

Rearranging now yields:

(A2:54) $$TCV_{t-1} = \frac{FCF_t + EV_t + D_t + (1+k_U)\frac{\Delta^i_{t-1}PVTS_{t,all} + \delta^i_{t-1}TS_t}{1+i_t} + (1-\Delta^i_{t-1})PVTS_{t,all} + (1-\delta^i_{t-1})TS_t}{1+k_U} - \frac{PVTS_{t,all}}{1+k_U}$$

This can be simplified to:

(A2:55)
$$TCV_{t-1} = \\ = \frac{FCF_t + EV_t + D_t + (1+k_U)\frac{\Delta^i_{t-1}PVTS_{t,all} + \delta^i_{t-1}TS_t}{1+i_t} - \Delta^i_{t-1}PVTS_{t,all} - \delta^i_{t-1}TS_t + TS_t}{1+k_U}$$

Further, (A2:55) can be rewritten as:

(A2:56)
$$TCV_{t-1} = \\ = \frac{FCF_t + EV_t + D_t + (1+k_U)\frac{\Delta^i_{t-1}PVTS_{t,all} + \delta^i_{t-1}TS_t}{1+i_t} - (1+i_t)\frac{\Delta^i_{t-1}PVTS_{t,all} + \delta^i_{t-1}TS_t}{1+i_t} + TS_t}{1+k_U}$$

Simplifying (A2:56) yields:

(A2:57)
$$TCV_{t-1} = \frac{FCF_t + EV_t + D_t + (k_U - i_t)\frac{\Delta^i_{t-1}PVTS_{t,all} + \delta^i_{t-1}TS_t}{1+i_t} + TS_t}{1+k_U}$$

By (TS_{dyn} :i) (A2:57) can be written as:

(A2:58)
$$TCV_{t-1} = \frac{FCF_t + EV_t + D_t + (k_U - i_t)PVTS_{t-1,i} + TS_t}{1+k_U}$$

Now multiplying both sides with $(1+k_U)$ and using the TCV definition gives:

(A2:59)
$$TCV_{t-1} + (EV_{t-1} + D_{t-1})k_U = FCF_t + EV_t + D_t + (k_U - i_t)PVTS_{t-1,i} + TS_t$$

Moving terms and adding $i_t D_{t-1} - i_t D_{t-1}$ to the LHS now gives:

(A2:60)
$$TCV_{t-1} + EV_{t-1}k_U + \left[(k_U - i_t)(D_{t-1} - PVTS_{t-1,i})\right] + i_t D_{t-1} - TS_t = \\ = FCF_t + EV_t + D_t$$

By using the definition of interest tax shields (TS), equation (A2:60) transforms into

(A2:61)
$$TCV_{t-1} + EV_{t-1} \cdot k_U + \left[(k_U - i_t)(D_{t-1} - PVTS_{t-1,i})\right] + D_{t-1}(1-\tau_t)i_t = \\ = FCF_t + EV_t + D_t$$

Now using the TCV definition and rearranging yields:

(A2:62)
$$TCV_{t-1}\left[1 + \frac{TCV_{t-1} - D_{t-1}}{TCV_{t-1}}\left(k_U + (k_U - i_t)\frac{(D_{t-1} - PVTS_{t-1,i})}{EV_{t-1}}\right) + \frac{D_{t-1}}{TCV_{t-1}}(1-\tau_t)i_t\right] = \\ = FCF_t + EV_t + D_t$$

Dividing both sides of (A2:62) with the term

$$\left[1+\left(1-\frac{D_{t-1}}{TCV_{t-1}}\right)\left(k_U+\left(k_U-i_t\right)\frac{\left(D_{t-1}-PVTS_{t-1,i}\right)}{EV_{t-1}}\right)+\frac{D_{t-1}}{TCV_{t-1}}\left(1-\tau_t\right)i_t\right]$$ and rearranging gives:

$$TCV_{t-1}=$$

(A2:63) $$=\frac{FCF_t+EV_t+D_t}{1+\frac{D_{t-1}}{TCV_{t-1}}\left(1-\tau_t\right)i_t+\left(1-\frac{D_{t-1}}{TCV_{t-1}}\right)\left[k_U+\left(k_U-i_t\right)\frac{\left(D_{t-1}-PVTS_{t-1,i}\right)}{EV_{t-1}}\right]}$$

Using (GEN) and rearranging gives:

(A2:64) $$EV_{t-1}=\frac{FCF_t+EV_t+D_t}{1+\frac{D_{t-1}}{TCV_{t-1}}\left(1-\tau_t\right)i_t+\left(1-\frac{D_{t-1}}{TCV_{t-1}}\right)k_{E,t}}-D_{t-1}$$

Finally, substituting the (WACC) definition into (A2:64) gives:

(A2:65) $$EV_{t-1}=\frac{FCF_t+EV_t+D_t}{1+k_{WACC,t}}-D_{t-1}$$

(A2:65) is the equity value dynamics of the FCF model (EV_{dyn} :FCF).

Q.E.D.

Appendix 3 - Valuation Procedures and Macro Programming

A starting point of the backward discounting procedure in Essay 3 is that an estimate of the unlevered cost of equity, k_U, has been obtained. This can, e.g., be done using industry data.[94] However, as been discussed previously, this is not the only possible starting point: In some cases, it may be more practical to start out with an estimate of the cost of equity in the first year (i.e., computed at the valuation date t-1), $k_{E,t}$. An additional simultaneity problem then arises: To calculate k_U one needs the equity value at t-1, but this equity value is a function of the discount rate, which in turn depends on k_U. Thus, an initial iteration at the valuation date, *procedure I*, is necessary to determine the unlevered cost of equity (given the determined cost of equity, $k_{E,t}$), in addition to the backward discounting procedure developed in Essay 3. The backward discounting procedure is carried out by repeating the yearly iterative procedure, *procedure Y*, for all years from the horizon, *H*, to the valuation date (see Essay 3, section 2.3, for a description of procedure Y).

The *initial* iterative procedure at the valuation date, *procedure I*, is carried out as follows (the valuation date is t-1):

<u>Procedure I</u>

I0. The procedure is initialised by determining the cost of equity capital for the first period, $k_{E,t}$.

I1. Assign a trial value to k_U.

I2. Perform the backward discounting procedure: The *yearly* iterative procedure, *procedure Y*, is carried out for all years, from the horizon *H* until the valuation date is reached, using the trial k_U as input for the discount rate (i.e., WACC or cost of equity) computations. (This will yield a first guess equity value at the valuation date as well as sequences of WACC and/or cost of equity estimates for all years, given the trial k_U and the estimated cost of debt-sequence)

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[94] See, e.g., Jennergren & Näslund (1996) for a practical implementation of such an approach.

I3. To test if the trial k_U is correct, compute the implied k_U by inserting the equity value from step I2 and the already determined first year cost of equity, $k_{E,t}$, into the appropriate k_U-formula (see Essay 3, section 2.2, for a discussion of the different cases. The first three equations are taken from Holthausen & Zmijewski, Chapter 2, pp. 24-25):

i) for the passive debt management case:

$$\text{(A3:1)} \quad k_U = k_{E,t}\frac{EV_{t-1}}{EV_{t-1}+D_{t-1}-PVTS_{t-1,i}}+i_t\frac{D_{t-1}-PVTS_{t-1,i}}{EV_{t-1}+D_{t-1}-PVTS_{t-1,i}}$$

ii) for the Modigliani & Miller case:

$$\text{(A3:2)} \quad k_U = k_{E,t}\frac{EV_{t-1}}{EV_{t-1}+D_{t-1}(1-\tau_t)}+i_t\frac{D_{t-1}(1-\tau_t)}{EV_{t-1}+D_{t-1}(1-\tau_t)}$$

iii) for the Miles & Ezzell case:

$$\text{(A3:3)} \quad k_U = k_{E,t}\frac{EV_{t-1}}{EV_{t-1}+D_{t-1}\left(1-\frac{\tau_t\cdot i_t}{1+i_t}\right)}+i_t\frac{D_{t-1}\left(1-\frac{\tau_t\cdot i_t}{1+i_t}\right)}{EV_{t-1}+D_{t-1}\left(1-\frac{\tau_t\cdot i_t}{1+i_t}\right)}$$

iv) for the perfect active debt management (Harris & Pringle) case:[95]

$$\text{(A3:4)} \quad k_U = k_{E,t}\frac{EV_{t-1}}{EV_{t-1}+D_{t-1}}+i_t\frac{D_{t-1}}{EV_{t-1}+D_{t-1}}$$

I4. Compare the implied k_U with the trial value. If equal, the equity value is correct, as are k_U as well as the $k_{WACC,t}$- and/or $k_{E,t}$-sequences.[96] If *not* equal, go back to step I1, where the implied k_U from step I3 can be used as a new trial value.

The procedure has been tested on numerous cases and has been found to converge in each case. The approach may seem cumbersome. Spreadsheet programs like Excel, however, have powerful features that make the procedures easy to implement (e.g., Macro programming and the goal-seek function). Below is an example of how *procedure I* and *procedure Y* jointly can be implemented in Excel 5.0 using Visual Basic macro programming. See further the implementation in the Eldon AB case in Appendix 5.

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[95] See Harris & Pringle (1985), Table 1, p. 241.

[96] This now holds for any year in the forecast period. This is because the k_U (that has been found to be correct at the valuation date) is the same as the (trial) k_U that already has been used for calculating the discount rates for each and every year in step I2 in the last iteration.

Notation terms in brackets, e.g., [example], refer to physical cells in the spreadsheet model. For example, [kU difference] means the spreadsheet cell in which the difference between the Trial k_U and Implied k_U is computed. Moreover, year *H* denotes the horizon, which in Eldon AB was at the end of year 2005, and year *t*-1 denotes the valuation date (corresponding to the end of year 1994 in Eldon AB).

Procedure Y (for one particular year *s*) (Essay 3, Appendix 3):

1) Manually set [Trial kWACC,s] to the initial trial value of $k_{WACC,s}$.
2) Run the following macro program:

```
Sub procedure_Y()
  Range("[WACC difference year s]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC year s]")
End Sub
```

Procedure Y (for all years from year *H* down to the valuation date, *t*-1) (Essay 3, Appendix 3):

1) Manually set all [Trial kWACC,s] cells (*s*=*t*, ..., *H*+1) to their initial trial values.
2) Run the following macro program:

```
Sub procedure_Y_all_years()
  Range("[WACC difference year H+1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial
        WACC year H+1]")
  Range("[WACC difference year H]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial   WACC year
H]")
  Range("[WACC difference year H-1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial
        WACC year H-1]")
...
  Range("[WACC difference year t+1]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC   year
t+1]")
  Range("[WACC difference year t]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial WACC      year
t]")
End Sub
```

Procedure I:

1) Manually set [Trial kU] to the initial trial value of k_U. and set all [Trial kWACC,s] cells (s=t, ..., H+1) to their initial trial values.

2) Run the following macro program:

```
Sub procedure_I()
  While Abs(Range("[kU difference]")) > 0.00000000000001
     Range("[kU difference]").GoalSeek Goal:=0, ChangingCell:=Range("[Trial kU]")
     Application.Run Macro:="procedure_Y_all_years"
  Wend
End Sub
```

Appendix 4 - The Eldon AB Case

These are the forecasted financial statements of Eldon AB used as the basis for all valuation examples in this paper. These forecasted financial statements are based on the forecasts in Essay 2. Here, the statements have been reorganised slightly to match the stylised balance sheets defined in section 2.1. For a detailed description of how the forecasts were made, using the CKM forecasting framework, see Essay 2, Chapter 5.

FORECASTED INCOME STATEMENTS:	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
Revenues	1,833.8	1,983.2	2,104.0	2,199.6	2,279.2	2,347.6	2,418.0	2,490.6	2,565.3	2,642.2	*2,721.5*	2,803.2
Operating expenses	-1,650.4	-1,784.9	-1,893.6	-1,979.7	-2,051.3	-2,112.8	-2,176.2	-2,241.5	-2,308.8	-2,378.0	*-2,449.4*	-2,522.8
Depreciation expense	-49.4	-51.3	-53.3	-55.5	-57.7	-59.9	-62.1	-64.2	-66.4	-68.6	*-70.7*	-72.8
Operating profit	134.0	147.1	157.1	164.5	170.3	174.9	179.7	184.8	190.1	195.7	*201.5*	207.5
Interest expense	-40.0	-42.4	-44.5	-46.2	-48.0	-49.7	-51.2	-52.6	-54.1	-55.6	-57.1	-58.8
Extraordinary items	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	0.0
Earnings before taxes	94.0	104.6	112.6	118.3	122.2	125.2	128.5	132.2	136.0	140.1	*144.4*	148.7
Taxes	-28.2	-31.4	-33.8	-35.5	-36.7	-37.6	-38.6	-39.7	-40.8	-42.0	*-43.3*	-44.6
Net profit	**65.8**	**73.2**	**78.8**	**82.8**	**85.6**	**87.6**	**90.0**	**92.5**	**95.2**	**98.0**	***101.0***	**104.1**

Table A1 - Forecasted income statements, Eldon AB

FORECASTED BALANCE SHEETS:	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
ASSET SIDE												
Operating cash	110.0	117.0	119.9	123.2	125.4	126.8	128.2	129.5	130.8	132.1	*136.1*	140.2
Trade receivables	284.2	305.4	321.9	336.5	346.4	356.8	365.1	376.1	384.8	396.3	*408.2*	420.5
Prepaid expenses	18.3	19.6	21.0	22.0	22.8	23.5	24.2	24.9	25.7	26.4	*27.2*	28.0
Other receivables	18.3	23.8	27.4	30.8	34.2	37.6	41.1	44.8	48.7	52.8	*54.4*	56.1
Inventories	394.3	436.3	473.4	505.9	535.6	563.4	592.4	615.2	638.8	660.6	*680.4*	700.8
Work. cap. assets	825.2	902.4	963.6	1,018.4	1,064.4	1,108.1	1,151.0	1,190.5	1,228.8	1,268.3	*1,306.3*	1,345.5
Excess mark. sec.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	0.0
Investment fund	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	0.0
Gross PPE	788.6	820.0	853.1	886.9	921.0	954.8	988.3	1021.7	1054.8	1087.6	*1120.2*	1153.8
Acc. depreciation	-350.0	-374.6	-398.7	-422.3	-445.2	-467.3	-488.5	-508.7	-527.7	-545.5	*-561.9*	-578.7
Net property, plant and equipment	438.6	445.4	454.4	464.7	475.8	487.5	499.9	513.0	527.0	542.1	*558.4*	575.1
Total assets	**1,263.8**	**1,347.8**	**1,418.0**	**1,483.1**	**1,540.2**	**1,595.6**	**1,650.8**	**1,703.5**	**1,755.8**	**1,810.4**	***1,864.7***	**1,920.6**
Invested capital*	922.7	976.9	1,020.4	1,065.2	1,107.2	1,147.2	1,184.2	1,222.8	1,260.7	1,297.8	*1,336.7*	1,376.8
* Total assets - Working capital liabilities												

Table A2a - Forecasted balance sheets, asset side, Eldon AB

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
DEBT & EQUITY SIDE												
Accounts payable	154.0	166.6	176.7	184.8	191.5	197.2	203.1	209.2	215.5	221.9	*228.6*	235.5
Accrued expenses	99.0	111.1	122.0	129.8	136.8	143.2	149.9	156.9	164.2	171.7	*176.9*	182.2
Taxes Payable	45.8	47.6	48.4	50.6	50.1	51.6	53.2	52.3	51.3	52.8	*54.4*	56.1
Other current liabilities	42.2	45.6	50.5	52.8	54.7	56.3	60.5	62.3	64.1	66.1	*68.0*	70.1
Work. cap. liabilities	341.1	370.9	397.7	417.9	433.1	448.4	466.7	480.7	495.1	512.6	528.0	543.8
Short-term debt	97.8	102.6	107.1	110.8	115.1	118.2	122.0	125.9	128.6	132.4	*136.3*	140.4
Long-term debt	160.6	168.0	174.5	181.1	188.2	193.9	200.1	205.4	211.8	216.7	*223.2*	229.9
Used check-credit	57.2	57.6	57.1	57.5	56.5	56.2	54.5	53.8	54.2	54.5	*56.1*	57.8
Pension funds	70.1	76.2	81.6	87.3	91.9	97.5	101.8	106.4	110.9	115.5	*119.0*	122.5
Debt	385.7	404.5	420.4	436.7	451.7	465.8	478.4	491.6	505.5	519.1	534.7	550.7
Deferred taxes	72.9	75.3	77.9	80.5	83.3	86.2	89.1	92.2	95.4	98.6	*102.2*	105.9
Common stock	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	*51.9*	51.9
Other equity	412.3	445.3	470.2	496.0	520.2	543.4	564.7	587.1	607.9	628.1	647.9	668.3
Total common equity	464.2	497.2	522.1	547.9	572.1	595.3	616.6	639.0	659.8	680.0	*699.8*	720.2
Tot. liab. & equity	**1,263.8**	**1,347.8**	**1,418.0**	**1,483.1**	**1,540.2**	**1,595.6**	**1,650.8**	**1,703.5**	**1,755.8**	**1,810.4**	**1,864.7**	**1,920.6**

Table A2b - Forecasted balance sheets, debt and equity side, Eldon AB

FORECASTED STATEMENT OF EQUITY:	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
COMMON STOCK:												
Beginning common stoc	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	*51.9*	51.9
Issues/Repurchases	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	0.0
Ending common stock	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	51.9	*51.9*	51.9
OTHER EQUITY:												
Beginning other equity	377.2	412.3	445.3	470.2	496.0	520.2	543.4	564.7	587.1	607.9	*628.1*	647.9
Net profit	65.8	73.2	78.8	82.8	85.6	87.6	90.0	92.5	95.2	98.0	*101.0*	104.1
Common dividends	-30.8	-40.2	-53.9	-57.0	-61.3	-64.5	-68.6	-70.1	-74.5	-77.8	*-81.3*	-83.7
Ending other equity	412.3	445.3	470.2	496.0	520.2	543.4	564.7	587.1	607.9	628.1	*647.9*	668.3

Table A3 - Forecasted statements of equity, Eldon AB

FORECASTED STATEMENTS OF FREE CASH FLOW:	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
Revenues	1833.8	1983.2	2104.0	2199.6	2279.2	2347.6	2418.0	2490.6	2565.3	2642.2	*2721.5*	2803.2
Operating expenses	-1650.4	-1784.9	-1893.6	-1979.7	-2051.3	-2112.8	-2176.2	-2241.5	-2308.8	-2378.0	*-2449.4*	-2522.8
Depreciation expense	-49.4	-51.3	-53.3	-55.5	-57.7	-59.9	-62.1	-64.2	-66.4	-68.6	*-70.7*	-72.8
Operating profit	134.0	147.1	157.1	164.5	170.3	174.9	179.7	184.8	190.1	195.7	*201.5*	207.5
Taxes on operations	-40.2	-44.1	-47.1	-49.4	-51.1	-52.5	-53.9	-55.4	-57.0	-58.7	*-60.4*	-62.3
Change in def. taxes	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	*3.6*	3.7
NOPLAT	96.2	105.4	112.5	117.8	122.0	125.3	128.8	132.4	136.2	140.2	*144.6*	148.9
Depreciation expense	49.4	51.3	53.3	55.5	57.7	59.9	62.1	64.2	66.4	68.6	*70.7*	72.8
Gross cash flow	145.5	156.7	165.8	173.3	179.6	185.2	190.8	198.7	202.7	208.8	*215.3*	221.7
Change in working capit	56.1	47.4	34.5	34.5	30.8	28.3	24.6	25.5	23.9	22.0	*22.7*	23.4
Capital expenditures	53.2	58.1	62.3	65.7	88.8	71.5	74.4	77.4	80.5	83.6	*87.0*	89.6
Gross investment (-)	109.3	105.5	96.7	100.3	99.6	99.9	99.0	102.9	104.3	105.7	*109.6*	112.9
Free cash flow	**36.2**	**51.2**	**69.1**	**73.0**	**80.0**	**85.3**	**91.8**	**93.8**	**98.3**	**103.1**	***105.7***	**108.8**

Table A4 - Forecasted statements of free cash flow, Eldon AB

The forecasts of the Property, Plant and Equipment items were obtained by using Specification B in Essay 2. This is taken from the second edition of CKM (1994). As discussed in Essay 2, p. 177, this specification reduces to (the somewhat simpler) Specification A in the parametric steady state period if the conditions for a steady state is to be fulfilled. This is the case for Eldon AB, and Specification A is most suitable for computing the accounting items for any year after the horizon.

The (constant) parameters are:[97]

a	net working capital in % of revenues (sales)	
b	gross PPE in % of revenues (sales)	**(Specification A only)**
c	change in deferred taxes in % of gross PPE	
d	depreciation in % of preceding year's gross PPE	
e	capital expenditures in % of revenues (sales)	**(Specification B only)**
g	nominal growth rate, revenues (sales)[98]	
i	interest rate on debt	
p	operating expenses in % of revenues (sales)	
r	retirements in % of preceding year's gross PPE	
τ	tax rate	
w	debt in % of balance sheet total (book value)	

[97] For Eldon AB in the parametric steady state period, using Specification A: a=28.6%; b=41.162%; c=0.318%; d=6.5%; g=3%; i=11%; p=90%; r=4.995%; τ=30%; w=40%.

[98] This means that the revenues of all years in the PSS period can be calculated as (1+g) times the preceding year's revenues.

The first year in the parametric steady state period for Eldon is year 2005. This means that the parameters defined above are constant from this point in time. Moreover, the following state variables can be identified:

R_t the revenues (sales) of year t,
A_t the accumulated depreciation at the end of year t,
T_t deferred taxes at the end of year t.

The state-variables in the parametric steady state period (for $t \geq 2006$) are, for Specification A, given by the following set of equations (Essay 2, p. 77):

$$R_t = (1+g)R_{t-1} = (1+g)^{t-2005} R_{2005}$$
$$A_t = \frac{(1+g)^{t-2005} - 1}{g} \cdot (d-r)bR_{2005} + A_{2005}$$
$$T_t = \frac{(1+g)^{t-2005} - 1}{g} \cdot c(1+g)bR_{2005} + T_{2005} =$$
$$= \left[\frac{(1+g)^{t+1-2005} - 1}{g} - 1 \right] cbR_{2005} + T_{2005}$$

where R_{2005}, A_{2005}, T_{2005} are the initial values of the state-variables in the PSS period (2,721.5 MSEK, 561.9 MSEK, and 102.2 MSEK respectively).

The forecasting model now defines the balance sheet items in terms of the parameters in the following way (for ($t \geq 2006$)):

Net working capital: aR_t

Net property, plant & equipment: $bR_t - A_t$

where $A_t = [(d-r)bR_{t-1} + A_{t-1}]$

Debt: $w(aR_t + bR_t - A_t)$

Deferred taxes: $T_t = cbR_t + T_{t-1}$

Book equity: $(1-w)(aR_t + bR_t - A_t) - T_t$

Likewise, the items in the income statement are expressed in terms of parameters (for ($t \geq 2006$)):

Revenues:	$R_t = (1+g)R_{t-1}$
Operating expenses:	$p \cdot R_t$
Depreciation expense:	$d \cdot G_{t-1} = d \cdot b \cdot R_{t-1}$
Interest expense:	$i \cdot w(aR_{t-1} + bR_{t-1} - A_{t-1})$
Taxes:	$\tau \cdot ((1-p)R_t - d \cdot b \cdot R_{t-1} - i \cdot w(aR_{t-1} + bR_{t-1} - A_{t-1}))$
Net profit:	$(1-\tau) \cdot ((1-p)R_t - d \cdot b \cdot R_{t-1} - i \cdot w(aR_{t-1} + bR_{t-1} - A_{t-1}))$

Appendix 5 - Value Calculations (Eldon AB)

A5.1 - The case of constant initial discount rates

Here are the value calculations of the valuations presented in Table 5 (i.e., valuations using the method of constant initial discount rates).

Dividend value calculation

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
DIV(t)	30.8	40.2	53.9	57.0	61.3	64.5	68.6	70.1	74.5	77.8	81.3	83.7
PV(1994)[DIV(t)]	27.2	31.4	37.2	34.7	33.1	30.7	28.9	26.1	24.5	22.6	20.9	*211.6*
k(E)	13.156%											*Cont. value*

EV(1994) =	
= Sum of PV(1994)[DIV(t)]	**528.88**

The value of all future expected dividends at the horizon, $PV_{1994}[DIV_{H+1}^{\infty}]$, is calculated using (CV:DIV).

Free cash flow value calculation

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
FCF(t)	36.2	51.2	69.1	73.0	80.0	85.3	91.8	93.8	98.3	103.1	*105.7*	108.8
PV(1994)[FCF(t)]	32.7	41.6	50.6	48.2	47.6	45.7	44.4	40.9	38.6	36.5	33.7	*436.7*
k(WACC)	10.947%											*Cont. value*

TCV(1994) = Sum of PV(1994)[FCF(t)]	897.11
-D(1994)	-363.20
EV(1994)	**533.91**

The value of all future expected FCF at the horizon, $PV_{1994}[FCF_{H+1}^{\infty}]$, is computed by (CV:FCF).

Economic value added value calculation

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
IC(t-1)	862.8	922.7	976.9	1,020.4	1,065.2	1,107.2	1,147.2	1,184.2	1,222.8	1,260.7	*1,297.8*	1,336.7
* k(WACC)	10.947%	10.947%	10.947%	10.947%	10.947%	10.947%	10.947%	10.947%	10.947%	10.947%	*10.947%*	10.947%
=Cap.ch.(t)	94.5	101.0	106.9	111.7	116.6	121.2	125.6	129.6	133.9	138.0	*142.1*	146.3
NOPLAT(t)	96.2	105.4	112.5	117.8	122.0	125.3	128.8	132.4	136.2	140.2	*144.6*	148.9
-Cap.ch.(t)	-94.5	-101.0	-106.9	-111.7	-116.6	-121.2	-125.6	-129.6	-133.9	-138.0	*-142.1*	-146.3
=EVA(t)	1.7	4.4	5.6	6.1	5.3	4.1	3.2	2.8	2.4	2.2	2.5	2.6
PV(1994)[EVA(t)]	1.5	3.6	4.1	4.0	3.2	2.2	1.5	1.2	0.9	0.8	*0.8*	*10.4*
k(WACC)	10.947%											*Cont. value*

Sum of PV(1994)[EVA(t)]	34.31
IC(1994)	862.80
TCV(1994)	897.11
-D(1994)	-363.20
EV(1994)	**533.91**

The value of all future expected EVA at the horizon, $PV_{1994}[\text{EVA}_{H+1}^{\infty}]$, is calculated using (CV:EVA).

Abnormal earnings value calculation

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
BV(t-1)	429.1	464.2	497.2	522.1	547.9	572.1	595.3	616.6	639.0	659.8	*680.0*	699.8
* k(E)	13.156%	13.156%	13.156%	13.156%	13.156%	13.156%	13.156%	13.156%	13.156%	13.156%	*13.156%*	13.156%
=Cap.ch.(t)	56.5	61.1	65.4	68.7	72.1	75.3	78.3	81.1	84.1	86.8	*89.5*	92.1
NP(t)	65.8	73.2	78.8	82.8	85.6	87.6	90.0	92.5	95.2	98.0	*101.0*	104.1
-Cap.ch.(t)	-56.5	-61.1	-65.4	-68.7	-72.1	-75.3	-78.3	-81.1	-84.1	-86.8	*-89.5*	-92.1
AE(t)	9.4	12.2	13.4	14.1	13.5	12.4	11.6	11.4	11.2	11.2	*11.6*	12.0
PV(1994)[AE(t)]	8.3	9.5	9.3	8.6	7.3	5.9	4.9	4.2	3.7	3.3	*3.0*	*31.9*
												Horizon value based on explicit forecast

Sum of PV(1994)[AE(t)]	99.78
BV(1994)	429.10
EV(1994)	**528.88**

Note that I do not use a continuing value to calculate the equity value at the horizon, since the abnormal earnings do not grow exactly at a constant rate in the perpetuity (the yearly growth rate asymptotically approaches the revenue growth rate as $t \to \infty$.) Instead a long explicit forecast of the abnormal earnings measure is used. The forecasted AE does not grow at a constant rate since the book equity does not. Even if the forecasted book equity does not grow at a constant rate, it has of course a stable development since we have projected a steady state development of the firm. According to the used forecasting model the book equity of any year s in the perpetuity period (that starts at the horizon H, 2005) is computed as:

$BV_s = IC_H(1+g)^{s-H} - D_H(1+g)^{s-H} - \left(T_{s-1} + 0.318\% \cdot G_H(1+g)^{s-H}\right)$ (Essay 2, p. 175). The growth rate, g, here equals 3%.

A5.2 - The case of periodically adjusted cost of capital

The Eldon AB value calculations with periodically adjusted discount rates for the base-case valuation are presented below (i.e., for the case where the cost of equity is updated according to the Miles & Ezzell (1980) formula (equation (3:1a)). But first, the implementation of the backward discounting procedure with the first period cost of equity, $k_{E,1995}$, as starting point, is described (see Appendix 3 above for the general definition of the procedure), using the FCF approach as an example.

As is recognised in Essay 3, the following expression can be used to directly update the WACC if the Miles & Ezzell formula (3:1a) is used to update the cost of equity:

(A5:1) $$k_{WACC,t} = k_U - \tau_t \cdot i_t \cdot \frac{D_{t-1}}{D_{t-1} + EV_{t-1}} \cdot \left(\frac{1+k_U}{1+i_t}\right)$$

Correspondingly, the implied k_U can be calculated by formula (A3:3).

The valuation procedure starts with the first loop of *procedure I*, since there is no estimate of k_U.

I0. Eldon AB's cost of equity at the valuation date, $k_{E,1995}$, has been estimated to be 13.156%.

I1. Guess that the unlevered cost of equity will be slightly lower than the levered cost of equity (13.156%), say 12.5%.

I2. Start *procedure Y* at the horizon, i.e., at the end of year 2005:

Y1. Guess that the WACC at the horizon, $k_{WACC,2006}$, is 11.0%.

Y2. The (first guess) equity value at the horizon is calculated using a continuing value formula and the trial WACC:

$$EV_{2005} = TCV_{2005} - D_{2005} = \frac{FCF_{2006}}{k_{WACC,2006\to\infty} - g} - D_{2005}$$

The growth rate, g, is 3%. The 'first guess' equity value at the horizon equals 825.56 MSEK.

Y3. Inserting this equity value and the trial k_U into the WACC formula (A5:1) yields an implied WACC of 11.185%:

End of year:	2005
FCF_{2006}	108.82
EV_{2006}	825.58
Trial $k_{WACC,2006}$	11.00000%
Resulting $k_{WACC,2006}$	11.18532%
$k_{WACC,2006}$ difference	-0.18532%

Y4. This WACC value is clearly not identical to the trial value of 11.0%. One has to go back to step Y1 with a new guess of the WACC (use 11.185% from step Y3).

After a number of iterations, the following is obtained:[99]

End of year:	2005
FCF_{2006}	108.82
EV_{2006}	799.05
Trial $k_{WACC,2006}$	11.15916%
Resulting $k_{WACC,2006}$	11.15916%
$k_{WACC,2006}$ difference	0.00000%

Procedure Y is then repeated for all preceding years, one year at a time (but now using yearly equity dynamics for the FCF model (EV_{dyn}:FCF) instead of the continuing value formula), and finally at the valuation date:

[99] In practice, we use the goal-seek function of Excel, specifying that the WACC difference (Trial WACC *minus* Resulting WACC) should be zero by changing the Trial WACC cell in the spreadsheet model.

End of year:	1994
FCF_{1995}	36.25
EV_{1994}	510.21
Trial $k_{WACC,1995}$	11.10917%
Resulting $k_{\hat{W}ACC,1995}$	11.10917%
$k_{WACC,1995}$ difference	0.00000%

I3. The implied unlevered cost of equity, k_U, is computed via equation (A3:3). The resulting equity value from step I2 and the known cost of equity (13.156%) are used as input:

End of year:	1994
Known $k_{E,1995}$	13.15600%
Computed $k_{E,1995}$	13.53606%
Trial k_U	12.50000%
Implied k_U	12.27521%
k_U difference	0.22479%

I4. The trial value of the unlevered cost of equity was apparently not a perfect guess. Hence, *procedure I* is performed again from step I1 using the resulting value from the previous iteration, 12.27521%, as the new trial k_U. This must go on until the implied unlevered cost of equity is equal to the trial value.

After eight additional iterations of *procedure I* the difference between the trial and implied unlevered cost of equity is zero:[100]

End of year:	1994
Known $k_{E,1995}$	13.15600%
Computed $k_{E,1995}$	13.15600%
Trial k_U	12.29570%
Implied k_U	12.29570%
k_U difference	0.00000%

Finally, we arrive at an equity value of Eldon AB of 530.76 MSEK.

End of year:	1994
FCF_{1995}	36.25
EV_{1994}	530.76
Trial $k_{WACC,1995}$	10.93931%
Resulting $k_{WACC,1995}$	10.93931%
$k_{WACC,1995}$ difference	0.00000%

Dividend value calculation

The equity value is calculated via (EVdyn:DIV), i.e., EV(t-1) = (DIV(t) + EV(t)) / (1 + k(E,t)):

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
DIV(t)	30.8	40.2	53.9	57.0	61.3	64.5	68.6	70.1	74.5	77.8	*81.3*	83.7
k(E,t)	13.156%	13.147%	13.137%	13.135%	13.133%	13.130%	13.126%	13.120%	13.114%	13.111%	*13.108%*	13.108%
EV(t-1)	**530.76**	**569.8**	**604.5**	**630.0**	**655.8**	**680.5**	**705.4**	**729.4**	**754.9**	**779.5**	***803.9***	**828.0**

At the horizon H (end of year 2005), the equity value is calculated by inserting (CV:DIV) into the model definition (EV:DIV), and valuing this expression at the horizon:

$$EV_H = \sum_{s=H+1}^{\infty} \mathrm{PV}_H\left[DIV_s\right] = \frac{DIV_{H+1}}{\bar{k}_E^H - g_{DIV}}$$

[100] Each iteration takes about 1 second on a PC with a 90 MHz Pentium processor.

Free cash flow value calculation

In the FCF case, the equity value is calculated via (EV_{dyn}:FCF), i.e.,

EV(t-1) = [(FCF(t) + TCV(t)) / (1 + k(WACC,t))] - D(t-1):

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
FCF(t)	36.2	51.2	69.1	73.0	80.0	85.3	91.8	93.8	98.3	103.1	*105.7*	108.8
k(WACC,t)	10.939%	10.948%	10.957%	10.960%	10.961%	10.964%	10.968%	10.973%	10.979%	10.982%	*10.986%*	10.986%
TCV(t-1)	894.0	955.5	1008.9	1050.4	1092.5	1132.3	1171.1	1207.8	1246.5	1285.0	*1323.0*	1362.7
D(t-1)	363.2	385.7	404.5	420.4	436.7	451.7	465.8	478.4	491.6	505.5	*519.1*	534.7
EV(t-1)	**530.76**	**569.8**	**604.5**	**630.0**	**655.8**	**660.5**	**705.4**	**729.4**	**754.9**	**779.5**	***803.9***	**828.0**

The equity value at the horizon is computed by first inserting (CV:FCF) into the model definition (EV:FCF), and then valuing this new expression at the horizon:

$$EV_H = \sum_{s=H+1}^{\infty} \mathrm{PV}_H\left[FCF_s\right] - D_H = \frac{FCF_{H+1}}{\bar{k}_{WACC}^{H} - g_{FCF}} - D_H$$

Economic value added value calculation

t	1995	1998	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
IC(t-1)	862.8	922.7	976.9	1,020.4	1,065.2	1,107.2	1,147.2	1,184.2	1,222.8	1,260.7	*1,297.8*	1,336.7
* k(WACC,t)	10.939%	10.948%	10.957%	10.960%	10.961%	10.964%	10.968%	10.973%	10.979%	10.982%	10.988%	10.986%
=Cap.ch.(t)	94.4	101.0	107.0	111.8	116.8	121.4	125.8	129.9	134.3	138.5	142.6	146.8
NOPLAT(t)	96.2	105.4	112.5	117.8	122.0	125.3	128.8	132.4	138.2	140.2	144.6	148.9
-Cap.ch.(t)	-94.4	-101.0	-107.0	-111.6	-116.8	-121.4	-125.6	-129.9	-134.3	-138.5	-142.6	-146.8
=EVA(t)	1.8	4.4	5.5	6.0	5.2	3.9	3.0	2.5	2.0	1.8	2.0	2.1

Value calculation using (EVdyn:EVA), i.e., EV(t-1) = IC(t-1) + [(EVA(t) + TCV(t) - IC(t)) / (1 + k(WACC,t))] - D(t-1):

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
IC(t-1)	862.8	922.7	976.9	1,020.4	1,065.2	1,107.2	1,147.2	1,184.2	1,222.8	1,260.7	*1,297.8*	1,336.7
EVA(t)	1.8	4.4	5.5	6.0	5.2	3.9	3.0	2.5	2.0	1.8	*2.0*	2.1
k(WACC,t)	10.939%	10.948%	10.957%	10.960%	10.961%	10.964%	10.968%	10.973%	10.979%	10.982%	*10.986%*	10.986%
TCV(t-1)	894.0	955.5	1,008.9	1,050.4	1,092.5	1,132.3	1,171.1	1,207.8	1,246.5	1,285.0	*1,323.0*	1,362.7
D(t-1)	363.2	385.7	404.5	420.4	436.7	451.7	465.8	478.4	491.6	505.5	*519.1*	534.7
EV(t-1)	**530.76**	**569.8**	**604.5**	**630.0**	**655.8**	**680.5**	**705.4**	**729.4**	**754.9**	**779.5**	***803.9***	**828.0**

The equity value at the horizon is calculated in the same fashion as in the DIV and FCF models: first, (CV:EVA) is inserted into the model definition (EV:EVA), then this expression is valued at the horizon, the end of year 2005:

$$EV_H = IC_H + \sum_{s=H+1}^{T} \mathrm{PV}_H\left[EVA_s\right] - D_H = IC_H + \frac{EVA_{H+1}}{\bar{k}_{WACC}^{H} - g_{EVA}} - D_H$$

Abnormal earnings value calculation

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
BV(t-1)	429.1	464.2	497.2	522.1	547.9	572.1	595.3	616.6	639.0	659.8	*680.0*	699.8
* k(E,t)	13.156%	13.147%	13.137%	13.135%	13.133%	13.130%	13.126%	13.120%	13.114%	13.111%	*13.108%*	13.108%
=Cap.ch.(t)	56.5	61.0	65.3	68.6	72.0	75.1	78.1	80.9	83.8	86.5	*89.1*	91.7
NP(t)	65.8	73.2	78.8	82.8	85.8	87.6	90.0	92.5	95.2	98.0	*101.0*	104.1
-Cap.ch.(t)	-56.5	-61.0	-65.3	-68.6	-72.0	-75.1	-78.1	-80.9	-83.8	-86.5	*-89.1*	-91.7
AE(t)	9.4	12.2	13.5	14.2	13.6	12.5	11.8	11.6	11.4	11.5	*11.9*	12.3

Value calculation using (EVdyn:AE), i.e., EV(t-1) = BV(t-1) + [(AE(t) + EV(t) - BV(t)) / (1 + k(E,t))]:

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	*2005*	2006
BV(t-1)	429.1	464.2	497.2	522.1	547.9	572.1	595.3	616.6	639.0	659.8	*680.0*	699.8
AE(t)	9.4	12.2	13.5	14.2	13.6	12.5	11.8	11.6	11.4	11.5	*11.9*	12.3
k(E,t)	13.156%	13.147%	13.137%	13.135%	13.133%	13.130%	13.126%	13.120%	13.114%	13.111%	*13.108%*	13.108%
EV(t-1)	**530.76**	**569.8**	**604.5**	**630.0**	**655.8**	**680.5**	**705.4**	**729.4**	**754.9**	**779.5**	***803.9***	**828.0**

Note also here that I do not use an AE continuing value to calculate the equity value at the horizon, since the abnormal earnings do not grow exactly at a constant rate in perpetuity. I have again used a long explicit forecast of the abnormal earnings measure as described in section A5.1, and repeatedly used (EV_{dyn}:AE) to finally get the equity value at the horizon.

A5.3 - The adjusted present value model

The Eldon AB value calculation for the APV model is presented below for the case where the tax shields are valued according to (3:1b), i.e., the active debt management case consistent with the Miles & Ezzell (1980) framework.

APV value calculation

t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
FCF(t)	36.2	51.2	69.1	73.0	80.0	85.3	91.8	93.8	98.3	103.1	105.7	108.8
OA(t-1)	756.1	812.9	861.6	898.5	935.9	971.1	1005.2	1037.0	1070.7	1104.0	1136.6	1170.7
discounted at k(U) = 12.2957%												
TS(t)	12.0	12.7	13.3	13.9	14.4	14.9	15.4	15.8	16.2	16.7	17.1	17.6
PV(t-1)[TS,i]	10.8	11.5	12.0	12.5	13.0	13.4	13.8	14.2	14.6	15.0	15.4	
PV(t-1)[TS,k(U)]	127.0	131.2	135.3	139.4	143.6	147.8	152.1	156.6	161.2	166.0	171.0	
PV(t-1)[TS,all]	137.8	142.6	147.3	151.9	156.5	161.2	166.0	170.8	175.8	181.1	186.4	192.0
OA(t-1)	756.1	812.9	861.6	898.5	935.9	971.1	1005.2	1037.0	1070.7	1104.0	1136.6	1170.7
PV(t-1)[TS,all]	137.8	142.6	147.3	151.9	156.5	161.2	166.0	170.8	175.8	181.1	186.4	192.0
TCV(t-1)	894.0	955.5	1008.9	1050.4	1092.5	1132.3	1171.1	1207.8	1246.5	1285.0	1323.0	1362.7
D(t-1)	363.2	385.7	404.5	420.4	436.7	451.7	465.8	478.4	491.6	505.5	519.1	534.7
EV(t-1)	**530.76**	**569.8**	**604.5**	**630.0**	**655.8**	**680.5**	**705.4**	**729.4**	**754.9**	**779.5**	**803.9**	**828.0**

At the horizon, the (unlevered) market value of the operating assets are calculated using a FCF continuing value with k_U as discount rate. The horizon value of the tax shields (192.0 MSEK) is, following section 2.4.5, calculated using the following continuing value formula:

$$CV_{2005}\left[\mathbf{TS}_{2006}^{\infty}\right]=\frac{\bar{\tau}\cdot\bar{i}\cdot D_{2005}}{\left(k_U-g_D\right)}\cdot\left[\frac{\left(1+k_U\right)}{\left(1+\bar{i}\right)}\right].$$

Appendix 6 - A Note on the Copeland, Koller & Murrin Continuing Value Formula for EVA

Copeland, Koller & Murrin (CKM) suggest that the EVA continuing value should be be calculated by the following formula:[101]

$$\text{(CV:EVA}_{CKM}\text{)} \quad CV_H^{CKM}\left[\mathbf{EVA}_{H+1}^{\infty}\right]=\frac{EVA_{H+1}}{\bar{k}_{WACC}^{H}}+\frac{NOPLAT_{H+1}(g_N\,/\,ROIC)(ROIC-\bar{k}_{WACC}^{H})}{\bar{k}_{WACC}^{H}(\bar{k}_{WACC}^{H}-g_N)}$$

where *ROIC* is return on new invested capital, g_N is the growth rate of NOPLAT, and $\bar{k}_{WACC}^{H}$ is the weighted average cost of capital in the perpetuity period.

This formula can at a first glance be rather hard to interpret, and CKM do not tell how this formula has been derived. To shed some more light on this formula, we will take a closer look at it.

First a few definitions, following CKM (p. 514):

$$\text{(ROIC)} \qquad ROIC_t=\frac{NOPLAT_t}{IC_{t-1}}$$

$$\text{(INV)} \qquad Inv_t=IC_t-IC_{t-1}$$

Note that *ROIC* is constant for all years in the perpetuity period, since a constant *ROIC* is one of the steady-state requirements for using a continuing value in the first place. Thus we have

$$(\text{ROIC}_{H\to\infty}) \qquad ROIC^H\equiv\frac{NOPLAT_{t+1}}{IC_t}\{=\text{constant}\}\ \text{for}\ t\geq H$$

[101] Note that this continuing value (as opposed to the FCF continuing value) does *not* represent the company value at the end of the explicit forecast period, but the *incremental value* over the company's invested capital at this point in time (here: end of 2005), often referred to as the *market value added*. This means that adding the invested capital at the horizon to this continuing value will yield the company value at the horizon (which of course then is equal to the FCF continuing value).

The ratio $\left(g_N / ROIC^H\right)$ is the *net investment rate*. Since we are in the perpetuity (steady state) period, the company among other things has to earn a constant return on existing capital. This means that the change in NOPLAT between two years will be attributable to the return on last year's net investment in new capital (CKM, p. 514). Thus the growth rate of *NOPLAT*, g_N, equals *the return on last year's net investment in new capital* divided by last year's *NOPLAT*. In formula notation this by definition is:

$$\text{(GN)} \quad g_N = \frac{\left(ROIC^H \cdot Inv_t\right)}{NOPLAT_t}.$$

Now we rearrange (CV:EVA$_{CKM}$):

$$\text{(A6:1)} \quad CV_H^{CKM}\left[\mathbf{EVA}_{H+1}^{\infty}\right] = \frac{EVA_{H+1}}{\bar{k}_{WACC}^H} + \frac{ROIC^H - \bar{k}_{WACC}^H}{\bar{k}_{WACC}^H} \cdot \frac{NOPLAT_{H+1}(g_N / ROIC^H)}{\left(\bar{k}_{WACC}^H - g_N\right)}$$

Now, substituting (GN) into (A6:1) yields:

$$\text{(A6:2)} \quad \begin{aligned} CV_H^{CKM}\left[\mathbf{EVA}_{H+1}^{\infty}\right] &= \frac{EVA_{H+1}}{\bar{k}_{WACC}^H} + \frac{ROIC^H - \bar{k}_{WACC}^H}{\bar{k}_{WACC}^H} \cdot \frac{Inv_{H+1}}{\bar{k}_{WACC}^H - g_N} = \\ &= \frac{EVA_{H+1}}{\bar{k}_{WACC}^H} + \frac{\dfrac{\left(ROIC^H - \bar{k}_{WACC}^H\right) \cdot Inv_{H+1}}{\bar{k}_{WACC}^H}}{\bar{k}_{WACC}^H - g_N} \end{aligned}$$

By substituting (ROIC$_{H \to \infty}$) and (INV) formula (A6:2) transforms into:

$$\text{(A6:3)} \quad CV_H^{CKM}\left[\mathbf{EVA}_{H+1}^{\infty}\right] = \frac{EVA_{H+1}}{\bar{k}_{WACC}^H} + \frac{\dfrac{\left(\dfrac{NOPLAT_{H+1}}{IC_H} - \bar{k}_{WACC}^H\right) \cdot \left(IC_{H+1} - IC_H\right)}{\bar{k}_{WACC}^H}}{\bar{k}_{WACC}^H - g_N}$$

Rearranging (A6:3) further:

$$\text{(A6:4)} \quad CV_H^{CKM}\left[\mathbf{EVA}_{H+1}^{\infty}\right] = \frac{EVA_{H+1}}{\bar{k}_{WACC}^H} + \frac{\left(\dfrac{NOPLAT_{H+1}}{\bar{k}_{WACC}^H \cdot IC_H} - 1\right) \cdot \left(IC_{H+1} - IC_H\right)}{\bar{k}_{WACC}^H - g_N}$$

Since the return on invested capital is constant, the invested capital must grow at rate g_N:

$$CV_H^{CKM}\left[\mathbf{EVA}_{H+1}^{\infty}\right]=$$

(A6:5)
$$=\frac{EVA_{H+1}}{\bar{k}_{WACC}^{H}}+\frac{\left(\dfrac{NOPLAT_t-\bar{k}_{WACC}^{H}\cdot IC_H}{\bar{k}_{WACC}^{H}\cdot IC_H}\right)\cdot g_N\cdot IC_H}{\bar{k}_{WACC}^{H}-g_N}=\frac{EVA_{H+1}}{\bar{k}_{WACC}^{H}}+\frac{\dfrac{EVA_{H+1}\cdot g_N}{\bar{k}_{WACC}^{H}}}{\bar{k}_{WACC}^{H}-g_N}$$

Now, using a common denominator and then rearranging, (A6:5) becomes:

$$CV_H^{CKM}\left[\mathbf{EVA}_{H+1}^{\infty}\right]=\frac{EVA_{H+1}}{\bar{k}_{WACC}^{H}}\cdot\frac{\bar{k}_{WACC}^{H}-g_N}{\bar{k}_{WACC}^{H}-g_N}+\frac{\dfrac{EVA_{H+1}\cdot g_N}{\bar{k}_{WACC}^{H}}}{\bar{k}_{WACC}^{H}-g_N}\cdot\frac{\bar{k}_{WACC}^{H}}{\bar{k}_{WACC}^{H}}=$$

(A6:6)
$$=\frac{EVA_{H+1}\left(\bar{k}_{WACC}^{H}-g_N\right)+EVA_{H+1}\cdot g_N}{\bar{k}_{WACC}^{H}\left(\bar{k}_{WACC}^{H}-g_N\right)}=\frac{EVA_{H+1}\cdot\bar{k}_{WACC}^{H}}{\bar{k}_{WACC}^{H}\left(\bar{k}_{WACC}^{H}-g_N\right)}=$$
$$=\frac{EVA_{H+1}}{\bar{k}_{WACC}^{H}-g_N}$$

Since the return on invested capital is constant in the perpetuity period we have already noted that both *NOPLAT* and *invested capital* grow at rate g_N. By the definition (EVA) it is then clear that *EVA* grows at rate g_N as well, i.e. $g_{EVA}=g_N$. Thus we have shown:

The CKM continuing value EVA formula (CV:EVA$_{CKM}$) is equivalent to the ordinary Gordon-type formula (CV:EVA).

The CKM formula only represents the growth of future EVA in a different way using their proposed "value driver" *ROIC* as a parameter. But by doing so they get a much more complicated and harder-to-interpret formula, when a simple Gordon-type of formula does the work completely.[102]

As we have seen the CKM formula is divided into two terms. To see what these terms really mean consider the following:

[102] CKM holds forth one feature of their proposed continuing value formula: when the return on invested capital equals the WACC the second term of their formula vanishes. But this is nothing particular for their formulation: by the (EVA) definition, the (ROIC$_{H\to\infty}$) relation, and the fact that the return on invested capital equals WACC, the EVA growth rate g_{EVA} equals 0 (zero) in (CV:EVA) and both continuing value formulas become the present value of EVA_{H+1} in perpetuity.

We know that NOPLAT and invested capital grow at rate g_N. Accordingly EVA also grows at g_N, which we henceforth denote with g_{EVA}. Instead of using the continuing value formula (CV:EVA) the present value at H can of course be written:

$$\text{(A6:7)}\quad \mathrm{PV}_H^{k_{WACC}}\left[\mathbf{EVA}_{H+1}^{\infty}\right]=\frac{EVA_{H+1}}{1+\bar{k}_{WACC}^{H}}+\frac{\left(1+g_{EVA}\right)\cdot EVA_{H+1}}{\left(1+\bar{k}_{WACC}^{H}\right)^{2}}+\frac{\left(1+g_{EVA}\right)^{2}\cdot EVA_{H+1}}{\left(1+\bar{k}_{WACC}^{H}\right)^{3}}+\ldots$$

This can be rewritten as:

$$\begin{aligned}
&\mathrm{PV}_H^{k_{WACC}}\left[\mathbf{EVA}_{H+1}^{\infty}\right]=\frac{EVA_{H+1}}{1+\bar{k}_{WACC}^{H}}+\frac{EVA_{H+1}}{\left(1+\bar{k}_{WACC}^{H}\right)^{2}}+\frac{g_{EVA}\cdot EVA_{H+1}}{\left(1+\bar{k}_{WACC}^{H}\right)^{2}}+\\
\text{(A6:8)}\quad &+\frac{EVA_{H+1}}{\left(1+\bar{k}_{WACC}^{H}\right)^{3}}+\frac{\left[\left(1+g_{EVA}\right)^{2}-1\right]\cdot EVA_{H+1}}{\left(1+\bar{k}_{WACC}^{H}\right)^{3}}+\ldots=\\
&=\frac{EVA_{H+1}}{\bar{k}_{WACC}^{H}}+\sum_{s=2}^{\infty}\frac{\left[\left(1+g_{EVA}\right)^{s-1}-1\right]\cdot EVA_{H+1}}{\left(1+\bar{k}_{WACC}^{H}\right)^{s}}
\end{aligned}$$

From the last row in (A6:8) we can now interpret the (CV:EVA$_{CKM}$) expression, since the left term is the same in both expressions; the perpetuity of *EVA* in year *H*+1. The second term thus must be equal in the two cases. We can from (A6:8) conclude that the second, hard-to-interpret, term of the CKM continuing value just equals the present value of the *EVA* in each subsequent year *minus* the "starting" value of EVA from the first year in the perpetuity period.

Thus the first term of (CV:EVA$_{CKM}$), the perpetuity of EVA in year *H*+1, captures the "normal" or no-growth expected EVA (i.e. normal in the sense of a return equalling the discount rate, the weighted average cost of capital), while the second term is an adjustment of this no-growth EVA that takes into account the value added from the fact that EVA grows.

Appendix 7 - Excess Marketable Securities

Excess marketable securities, EMS, (or 'excess cash and marketable securities') are the short-term cash and investments that the company holds over and above its target cash balances to support operations (Copeland et al., 1994, p.161). The very fact that EMS are defined as 'excess' means that they are not part of the company's operations, but 'represent temporary imbalances in the company's cash flow' (Copeland et al., 1994, p.160). Consequently, it is reasonable to assume that the expected value of EMS in future periods is zero. The question then arises how to handle EMS (if any) existing at the valuation date.

In a valuation context, we are interested in how EMS affect the valuation through financing consequences. In particular, in what way it is assumed to affect capital structure and hence discount rates. One might think that the issue is trivial from a practical perspective, but Copeland et al. (1994, p.161) report that they have seen cash balances not needed for operations as high as USD 5 billion.

One obvious way to handle the problem is to explicitly recognise that EMS really are "excess" and hence not part of operations. To make this operational in the valuation model one can *exclude* EMS from the valuation of the firm's operations and then add them to the company value at market value. This could be thought of, e.g., as an immediate dividend to shareholders. In a consistent scenario this would apply both to a dividend valuation model and to a free cash flow valuation model (and indeed to other valuation models as well, e.g., EVA and APV). Hence, the equivalence of valuation models would not be affected. Practically, this is implemented by deleting the EMS item and subtracting the same amount from the book equity account (by reducing retained earnings) when transferring the last historical balance sheet to the entering balance sheet of the forecasting model. Finally, to obtain the equity value of the company, the value of EMS is added to the value obtained from the valuation models.[103]

Now, it can be argued that although the EMS are labelled "excess", they may in reality often have some real function. For instance, they can serve as collateral for some of the debt, possibly lowering the cost of debt. Intuitively (and possibly formally) it can hence be better to net EMS against debt. This can be thought of as immediately retiring the corresponding part of debt. Also in this case the equivalence of valuation models is ensured, since technically this is only a re-definition of entering

[103] This approach is described in Essay 2.

debt, and hence the consistency proofs in the main text still apply. Practically, this approach (used in this paper) is implemented by deleting the EMS item and subtracting the same amount from the debt account when transferring the last historical balance sheet to the entering balance sheet of the forecasting model. The equity value is in this case given directly by the valuation models since the debt value at the point of valuation already has been adjusted.

Another approach could be to keep existing EMS in the entering balance sheet of the model, and then set EMS at the end of the first year to zero. The equivalence between valuation models is preserved, if one poses the additional assumption that the EMS will earn an expected return equal to the borrowing rate. This line of reasoning easily extends to multiple periods. The addition of EMS in future periods do not alter the valuation result if it on the other side of the balance sheet is followed by an equal addition to debt, and if the expected return on EMS is equal to the borrowing rate. But would we really gain anything by introducing EMS in the forecasting model? – No! We would still get the same value as had we assumed zero EMS in the future. Including EMS in the forecasting model will thus only make the valuation more complicated.

It is however important to note that the choice of method for treating EMS have an impact on the final valuation result. The method of netting EMS against debt will (in the tax regime considered in this paper) give a lower value than if they are used for a direct dividend payment. The reason is that lowering the debt of course also lowers the interest tax shield (while the (unlevered) value of the company will be the same regardless which method is used). It can thus be concluded that the choice of approach for dealing with EMS affects the valuation, whereas the choice of valuation model does not (*given* a certain way of treating EMS).

Appendix 8 - Interest Tax Shield Valuation Procedures

As noted in section 2.4.5, the following continuing value can be used in a Miles & Ezzell (1980) consistent case:

$$\text{(A8:1)}\quad CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right]=\frac{\bar{\tau}\cdot\bar{i}\cdot D_H}{\left(k_U-g_D\right)}\cdot\left[\frac{\left(1+k_U\right)}{\left(1+\bar{i}\right)}\right]$$

This continuing value can be rewritten as (the derivation is provided at the end of this section):

$$\text{(A8:2)}\quad CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right]=\frac{\bar{\tau}\cdot\bar{i}\cdot D_H+\dfrac{\bar{\tau}\cdot\bar{i}\cdot D_H\left(1+g_D\right)}{k_U-g_D}}{\left(1+\bar{i}\right)}$$

Apparently, (A8:2) implies the following tax shield valuation scheme at the horizon:

$$\text{(A8:3)}\quad \begin{cases} PVTS_{H,all}=PVTS_{H,i}=\dfrac{\bar{\tau}\cdot\bar{i}\cdot D_H+PVTS_{H+1,all}}{1+\bar{i}} \\ PVTS_{H+1,all}=PVTS_{H+1,k_U}=\dfrac{\bar{\tau}\cdot\bar{i}\cdot D_{H+1}}{k_U-g_D}\quad\left(=\dfrac{\bar{\tau}\cdot\bar{i}\cdot D_H\left(1+g_D\right)}{k_U-g_D}\right) \end{cases}$$

Formulation (A8:3) gives a different interpretation of the tax shield valuation procedure underlying the Miles & Ezzell framework than as described in formulation (3:1b) in section 3, which is:

$$\text{(3:1b)}\quad \begin{cases} PVTS_{t,i}=\dfrac{\tau_{t+1}\cdot i_{t+1}\cdot D_t}{1+i_{t+1}} \\ PVTS_{t,k_U}=\dfrac{PVTS_{t+1,all}}{1+k_U} \end{cases}\quad\text{for all } t$$

Note, however, that it is *only under an assumption of constant (or zero) debt growth* that (A8:3) is valid. (3:1b) is on the other hand valid in *all* cases, including the constant growth case (see, e.g., the derivation of (A8:1) in Holthausen & Zmijewski (1996), Chapter 2, pp. 16-17). Thus, formulations (A8:3) and (3:1b) are equivalent interpretations in the constant growth case, but (3:1b) gives the general interpretation.

But regardless of which one of these interpretations that is considered, there is something counter-intuitive in the Miles & Ezzell tax shield valuation procedure. We can think of it in the following terms: We are valuing the company at one specific valuation date (say, at the end of year 0). This means that the interest tax shield in year 1 is known. The interest tax shields for all future years (i.e., for years 2 and beyond) will be a function of the company value at the beginning of each year through the yearly debt adjustment to the target capital structure, and thus at t=0, not certain. But in the Miles & Ezzell procedure, at the end of each future year, the interest tax shield stemming from the immediately following year is always discounted at the cost of debt, thus basically assuming that if we stand at the end of a year, say year 8, the tax shield from year 9 is known. But, since we are valuing the company given our knowledge at year 0, it can not be known. Thus, the Miles & Ezzell procedure partly implies that the (at year 0) expected company value at the end of year 8 will be realised (even if it on the other hand implies that it may *not* be realised, since during year 8, the computed total value of tax shields at the end of year 8 is discounted back at the unlevered cost of equity.) Given the original arguments behind the Miles & Ezzell framework, it would perhaps be more straightforward to use the following tax shield valuation scheme, where t=0 denotes the valuation date:

$$\begin{cases} PVTS_{0,all} = \dfrac{\tau_1 \cdot i_1 \cdot D_0 + PVTS_{1,all}}{1+i_1} \\ PVTS_{t,all} = \dfrac{\tau_{t+1} \cdot i_{t+1} \cdot D_t + PVTS_{t+1,all}}{1+k_U} \quad \text{for } t \geq 1 \end{cases}$$

Derivation of (A8:2)

Equation (A8:1) can be rearranged in the following way:

$$\text{(A8:4)} \quad CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right] = \frac{\bar{\tau} \cdot \bar{i} \cdot D_H}{(k_U - g_D)} \cdot \left[\frac{(1+k_U)}{(1+\bar{i})}\right] = \bar{\tau} \cdot \bar{i} \cdot D_H \left[\frac{1+k_U}{(1+\bar{i})(k_U - g_D)}\right]$$

Now, adding g_D - g_D to the numerator gives:

$$\text{(A8:5)} \quad CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right] = \bar{\tau} \cdot \bar{i} \cdot D_H \left[\frac{k_U - g_D + 1 + g_D}{(1+\bar{i})(k_U - g_D)}\right]$$

Rearranging finally yields:

$$CV_H\left[\mathbf{TS}_{H+1}^{\infty}\right] = \bar{\tau} \cdot \bar{i} \cdot D_H \left[\frac{1}{(1+\bar{i})} + \frac{1+g_D}{(1+\bar{i})(k_U - g_D)}\right] = \frac{\bar{\tau} \cdot \bar{i} \cdot D_H + \dfrac{\bar{\tau} \cdot \bar{i} \cdot D_H (1+g_D)}{k_U - g_D}}{(1+\bar{i})}$$

Q.E.D.

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