ESSAYS ON BARGAINING AND DELEGATION

Björn Segendorff

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To Åsa

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Björn Segendorff

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Delegation and Threat in Bargaining

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Abstract

Two principals ("nations") appoint one agent each to bargain over the provision of a public good. Two institutional set-ups are studied, each with a different level of authority given to the agents. Here authority means the right to decide the own side's provision if negotiations break down. In equilibrium the principals choose agents with preferences differing from their own. The low-authority equilibrium Pareto dominates (with regard to the principals) the case of the principals deciding on the provisions simultaneously (autarchy). The high-authority equilibrium is Pareto dominated by the low-authority equilibrium and it may even be dominated by autarchy. *Journal of Economic Literature* Classification Numbers: C71, and C72.

1. Introduction

Strategic delegation in bargaining is an important economic issue not yet fully understood. It is important because the bargaining parties often choose to delegate the negotiation to some agent. It is not yet fully understood because the existing literature on strategic delegation seldom study bargaining. This study examines whether delegation is used strategically and, if so, its consequences within the framework of two nations bargaining over the provision of a public good.

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There are two populations of individuals/citizens called nations each of which has one unit of resources that can be used to produce two goods. One of the goods is private for the producing nation and the other is public between the nations. Every citizen prefers a particular national output combination of the two goods. This ideal allocation is determined by her taste parameter which has a continuous distribution across the population. We may think of the private good as health care and the public good as some environmental activity such as reduction of the emission of carbon dioxide into the atmosphere.

In each nation there is a particular citizen called *the principal* (she) who may delegate the task of negotiating to an especially selected citizen, *the agent* (he). Even though there is no voting in the model, it may be helpful to think of the principal as some decisive voter and the agent as some elected policymaker such as a president or a government. In reality, such an agent's payment scheme is often low-incentive powered.[1] Here, the agent is given a fixed wage, normalized to zero, and the agent therefore affected by his allocation decision in exactly the same way as any other citizen. This simplification allows for focusing on strategic delegation to an agent with preferences directly over the task he is to undertake, just as we would like to think of a politician, and to ask: what incentives does a principal have to choose an "extreme" agent and what are the welfare consequences? The basic model is used to study strategic delegation in two bargaining games differing in the amount of authority (power) given to the agents, each representing an institutional set-up. The two resulting sets of equilibria are compared to a benchmark called *autarchy*.

In autarchy the two principals simultaneously decide on their national output combinations. The unique Nash equilibrium is for each principal to implement her ideal allocation (which here is independent of the other nation's allocation) but the resulting global allocation is not Pareto efficient from the principals' point of view because not one of them internalizes the effect of her decision on the other principal and each nation therefore produces a too small amount of the public good. Hence, the principals would like to commit to some other global allocation.

The two studied delegation games/institutional set-ups are special cases of a general two-stage game in which the principals simultaneously choose agents in the first stage. In the second stage the agents meet and bargain over the global allocation, i.e., each nation's provision of the public good. The bargaining process is modelled as a Nash bargaining solution where the threat point is constituted by the (reservation) utilities the agents receive from some alternative global allocation. This can be interpreted as if there exists an exogenous probability of a breakdown in negotiations, or the agents believe such a probability to exist, and that the alternative global allocation is implemented if the negotiations break

[1] The observation of flat payment schemes is also made by Perry and Samuelson (1994).

down.

The difference between the two delegation games is the level of authority given to the agents. By authority is here meant the right to set the own nation's allocation in the case of a breakdown. If the agent is given this authority, then his preferences become important in determining the consequences of a breakdown. Hence, authority given to the agent enables the delegating principal to change, by her choice of agent, the reservation utility of the other agent and thereby gain a strategic advantage. Appointment of an agent who lowers the reservation level of utility of the other nation's agent will in the following be called *a threat*.

The first delegation game to be studied is called the *weak delegation game* because of the agents' limited authority. If there is a breakdown the agents return to the principals and announce the failure after which the principals simultaneously decide on the national allocations. The breakdown allocation is thus constituted by the principals' ideal allocations and every equilibrium to the weak delegation game Pareto dominates autarchy. The argument is straight forward. Each principal can, by choosing *self-representation* (choosing herself as agent or, equivalently, choosing not to delegate), assure herself a payoff no lower than the breakdown payoff and she can therefore not do worse in equilibrium. It is also shown that mutual self-representation cannot be a part of any equilibrium and that in equilibrium at least one of the principals is worse off compared to mutual self-representation.

In the second game, the *strong delegation game*, each principal delegates the whole decision making and commits herself not to intervene at a later stage, i.e. the agents simultaneously implement their ideal allocations in the case of a breakdown. One could think of a situation where it is publicly known that the cost of appointing a new agent is high compared to the gain from changing the policy decision. An example is when the population of a nation knows that opposing a presidential decision by, say, massive demonstrations in the streets would result in a political crisis which in turn would incur substantial costs for society.

In the strong delegation game it is always beneficial for a principal to choose an agent with stronger preferences for the private good than her own, i.e., to create a threat. Her agent would, in the case of a breakdown, choose to produce less of the public good than the principal herself. This lowers the reservation utility of the other agent and weakens his bargaining position. Therefore, as a result, the threatened nation's production of the public good increases while the production of the other nation decreases. The principal's utility is higher than under self-representation because more of her resources are allocated to production of the private good without the corresponding amount being globally withdrawn from production of the public good. Since the reasoning holds for both principals the equilibrium pair of agents has a stronger taste for the private good than the principals do. The resulting agreement will consequently state a lower production of the public good than in the case of mutual self-representation. Moreover, the negative effect of strategic delegation may more than offset the gains from reaching an agreement, i.e., the strong equilibrium may even be Pareto dominated by autarchy.

There are a few studies related to this study. Jones (1989) lets two principals choose agents to bargain over the division of two private goods. The bargaining is modelled by means of the Nash bargaining solution with the reservation utilities normalized to zero. His main finding is that there can never be a utility gain for both principals compared to mutual self-representation. The normalization of the threat point can be interpreted as if the bargaining was driven by the impatience of the agents who receive zero utility while bargaining. The delegation games studied in this paper are therefore quite different.

Crawford and Varian (1979) do not explicitly work with the delegation but recognize that the Nash and related solution concepts to the bargaining problem presume information that is unobservable in practice and that a bargainer who engages in such a bargaining situation may gain from misrepresenting her true preferences. In the context of Nash bargaining they find that the unique dominant-strategy Nash equilibrium is for both parties to report risk-neutral utility functions. Sobel (1981) extends Crawford and Varian's result to multiple goods and shows the result to be valid also for the Raiffa-Kalai-Smorodinsky solution. Burtraw (1992) extends the result of Crawford and Varian further by showing the existence of multiple ("report") equilibria in the Nash bargaining framework by allowing for a broader class of utility functions. Crawford and Varian, Sobel, and Burtraw normalize the agents' reservation utilities to zero which makes the institution they study different from the institutions studied in this paper. In the strong delegation game the principals choose to delegate to agents with more concave utility functions than their own. In the numerical example, this is true also for the weak delegation game.

Fershtman *et al.* (1991) let two principals delegate the bargaining to two agents. Each principal signs a contract (payment scheme) with her agent and they show that when allowing for a broad class of contracts, any cooperative outcome of the bargaining game without delegation can be made the unique subgame-perfect equilibrium of the delegation game. The delegation games studied in this paper are concerned with problems very different from those studied by Fershtman *et al.*, e.g., here the principals are not free to design the payment scheme but negotiations may break down.[2] [3]

[2]Schelling (1960) has an intuitive discussion of delegation and bargaining in chapters 2 and 5 of his book.

[3]Strategic delegation has been extensively studied in the field of policy implementation, especially monetary policy. The main finding is that monetary policy should be delegated to a

Section 2 contains the basic model and autarchy. The general delegation game is given in Section 3.1, the weak delegation game is given in Section 3.2, and the strong delegation game is given in Section 3.3. A numerical example is given in Section 4, and Section 5 contains summary and comments. All proofs are given in the Appendix.

2. The Basic Model

The two nations, 1 and 2, have one unit of resources each to allocate between the production of one good that is private for the producing nation and one good that is public between the two nations. Every citizen has preferences over her nation's production of the private good and the total amount of public good provided, determined by a taste parameter, θ. In both nations the citizens' taste parameters are continuously distributed over the unit interval. Let $\theta \in [0,1]$ denote an arbitrary citizen of nation $k = 1,2$ and $x_k \in [0,1]$ denote the share/amount of nation k's resources that is used to produce the public good. The preferences of θ, $\succsim$, are given by the von Neuman-Morgenstern utility function,

$$v_k(\mathbf{x},\theta) = \theta \ln(1 - x_k) + x_1 + x_2$$

where $\mathbf{x} = (x_1, x_2)$ (bold-face will in the following be used to denote vectors). From the concavity of $v_k(\mathbf{x},\theta)$ it follows that there exists, for every θ, an ideal resource allocation x^* that θ prefers to any other x_k and for the specified utility function is $x^*(\theta) = 1 - \theta$. Moreover, the two nations' allocations are strategically neutral since the cross derivatives are zero, $v_{k12} = v_{k21} = 0$.

Suppose that there in each nation k is a citizen called the *principal* (she) with taste parameter $0 < \theta_k^P < 1$. In *autarchy* the two principals simultaneously decide on the national output combination and they implement the allocation $\mathbf{x}^P = (x^*(\theta_1^P), x^*(\theta_2^P))$. The autarchy allocation $\mathbf{x}^P$ is not Pareto efficient because no principal internalizes the effect of her decision on the other principal and $\mathbf{x}^P$

central banker who puts more weight on fighting inflation than voters do, see, among others, Rogoff (1985). A two-nation setting is studied by Dolado et al. (1994) who, among other things, study delegation to a supra-national monetary authority where the two nations' central bankers coordinate their policy by maximizing the sum of their utilities. This coordination process may be viewed as a bargaining with side-payments. For an exhaustive exposition, see Persson and Tabellini (1995). Industrial Organization is another field where strategic delegation has been studied. Vickers (1985) shows that the owner of a firm would like to make the manager's wage dependent on the relative profit of the firm rather than on absolute profit. Fershtman and Judd (1987) show that in a Cournot oligopoly the owner of a firm signs a contract making the manager's payment conditional on both profits and sales. This makes each manager more aggressive and the equilibrium profits are lower than in the ordinary Cournot case. Szymanski (1994) studies owners' incentives to make managers' wages contingent on cost reductions.

supplies too little of the public good. Any such pair of principals would benefit from coordinating on some other mutually preferred allocation. Here, coordination is achieved through bargaining. By the introduction of delegation the model captures more general situations than direct bargaining between the principals; delegation seems to be an essential aspect of international negotiations.

3. Delegation in Bargaining Games

Consider the following two-stage game. In stage one the principals move simultaneously; each principal delegates the task of deciding on the national allocation to an especially selected citizen, the *agent* (he). The agent may be any citizen including the principal herself. The latter case is called *self-representation.* In the second stage, the agents meet and bargain over the provision of the public good. The resulting agreement is assumed to be binding. Let θ_1 be the delegate from country 1 and θ_2 the delegate from country 2 and let $\mathbf{x}^0 = (x_1^0, x_2^0)$ be the allocation that is implemented if no agreement is reached.

Definition 1. *For any* $\mathbf{x}^0$, $\boldsymbol{\theta} \in [0,1]^2$, *the* **contract zone**, $Z(\boldsymbol{\theta}, \mathbf{x}^0)$, *is the set of allocations that Pareto dominates the allocation* $\mathbf{x}^0$

$$Z(\boldsymbol{\theta}, \mathbf{x}^0) = \left\{ \mathbf{x} \in [0,1]^2 \mid \mathbf{x} \succsim_{\theta_1} \mathbf{x}^0 \text{ and } \mathbf{x} \succsim_{\theta_2} \mathbf{x}^0 \right\}.$$

The bargaining outcome is assumed to be efficient and the Nash bargaining solution is used to solve the bargaining problem facing the agents. It turns out that the agreement picked by the Nash bargaining solution is unique and a continuous function of $\boldsymbol{\theta}$ and $\mathbf{x}^0$.[4]

Definition 2. *The Nash bargaining solution is*

$$\mathbf{z}^{NB}(\boldsymbol{\theta}, \mathbf{x}^0) = \arg \max_{\mathbf{x} \in Z(\boldsymbol{\theta}, \mathbf{x}^0)} N(\mathbf{x}, \boldsymbol{\theta}, \mathbf{x}^0) \tag{3.1}$$

where

$$N(\mathbf{x}, \boldsymbol{\theta}, \mathbf{x}^0) = \left(v_1(\mathbf{x}, \theta_1) - v_1(\mathbf{x}^0, \theta_1)\right)\left(v_2(\mathbf{x}, \theta_2) - v_2(\mathbf{x}^0, \theta_2)\right).$$

In the following $\mathbf{z}^{NB}(\boldsymbol{\theta}, \mathbf{x}^0)$ is called the *agreement*, $\mathbf{x}^0$ is called the *breakdown allocation*, and $(v_1(\mathbf{x}^0, \theta_1), v_2(\mathbf{x}^0, \theta_2))$ is called the *threat point.*[5]

[4] This is shown in the first part of the proof of Proposition 1.

[5] An interpretation of the reservation utilities is that the agents know, or believe, an exogenous probability of a breakdown in negotiations to exist in which case $\mathbf{x}^0$ is implemented. The bargaining process is thus driven by the agents' fear of a breakdown that would prevent the exploitation of gains from coordination. For more on the subject, see Binmore *et al.* (1986). Crawford (1982) studies a bargaining model in Nash's (1953) spirit in which such a probability arises endogenously.

3.1. The General Delegation Game

The most convenient way to proceed is to define a general delegation game and then to treat the two delegation games below as special cases of this general game. In these games the principals simultaneously choose agents at the first stage. In the second stage, summarized by the Nash bargaining solution, the agents meet and bargain. The breakdown allocation $\mathbf{x}^0$ is a continuous function of the agents' preferences, $\mathbf{x}^0 = \mathbf{b}(\boldsymbol{\theta})$. The bargaining outcome $\mathbf{z}^{NB}(\boldsymbol{\theta}, \mathbf{b}(\boldsymbol{\theta}))$ is unique and the two-stage game can thus be reduced to a one-stage simultaneous-move game played by the principals. The two delegation games to be analyzed differ in their specification of the function $\mathbf{b}$.

Let $D = (N, \boldsymbol{\Theta}, \boldsymbol{\pi})$ denote the reduced general game where $N = \{1, 2\}$ is the set of players/principals. The set of pure strategies available to principal k is $\Theta_k = [0, 1]$ and the pure strategy θ is her choice of agent, $\boldsymbol{\Theta} = (\Theta_1, \Theta_2)$. Let δ_k denote a mixed strategy of principal k. The pair of payoff functions are $\boldsymbol{\pi} = (\pi_1, \pi_2)$ where π_k denotes the (expected) payoff to principal k as a function of the strategy profile $\boldsymbol{\delta} = (\delta_1, \delta_2)$, i.e.,

$$\pi_k(\boldsymbol{\delta}) = \int_0^1 v_k\left(\mathbf{z}^{NB}(\boldsymbol{\theta},\ \mathbf{b}(\boldsymbol{\theta})), \theta_k^P\right) \boldsymbol{\delta}(\boldsymbol{\theta})\, d\boldsymbol{\theta}.$$

If the breakdown allocation is continuous in $\boldsymbol{\theta}$ then also the Nash bargaining solution is continuous in $\boldsymbol{\theta}$. Hence, the payoff functions are continuous in $\boldsymbol{\theta}$ and there exists a Nash equilibrium, possibly in mixed strategies, to the reduced game.

Proposition 1. *For any pair of principals and any reduced game in which the breakdown allocation is a continuous function of the taste parameters of the chosen agents, there exists a Nash equilibrium.*

Proof. See the Appendix.

For later use, let $C(\boldsymbol{\delta}) = \{\boldsymbol{\theta} \mid \boldsymbol{\delta}(\boldsymbol{\theta}) > 0\}$ denote the support of a strategy profile $\boldsymbol{\delta}$ and let $NE(D)$ denote the non-empty set of Nash equilibria in game D.

The delegation games below differ by the rule determining the breakdown allocation. Each rule represents a different level of authority given to the agents. The two types of delegation to be considered are called *weak delegation* and *strong delegation.* Weak delegation means that the agents have no influence on the breakdown allocation and that in the case of a breakdown the principals' ideal allocations are implemented. Strong delegation, in contrast, gives each agent the authority to decide on the national breakdown allocation.

3.2. Weak Delegation

Denote the weak delegation game $D^W = \left(N, \Theta, \boldsymbol{\pi}^W\right)$. Here, the agents have no influence on the breakdown allocation and $\mathbf{b}^W(\boldsymbol{\theta}) = \mathbf{x}^P$ for all $\boldsymbol{\theta}$.

Principal k's payoff function is

$$\pi_k^W(\boldsymbol{\delta}) = \int_0^1 v_k\left(\mathbf{z}^{NB}\left(\boldsymbol{\theta}, \mathbf{x}^P\right), \theta_k^P\right) \boldsymbol{\delta}(\boldsymbol{\theta})\, d\boldsymbol{\theta}.$$

The breakdown allocation is continuous and by Proposition 1 there exists an equilibrium in game D^W. Moreover, in any equilibrium the agreement belongs to the contract zone of the two principals but mutual self-representation is not a part of the equilibrium. Formally:

Proposition 2. *Suppose* $\boldsymbol{\delta}^{NEW} = (\delta_1^{NEW}, \delta_2^{NEW}) \in NE\left(D^W\right)$. *Then:*

(i) $\mathbf{z}^{NB}(\boldsymbol{\delta}^{NEW}, \mathbf{x}^P) \in Z(\boldsymbol{\theta}^P, \mathbf{x}^P)$.

(ii) $\boldsymbol{\theta}^P \notin C(\boldsymbol{\delta}^{NEW})$.

Proof. See the Appendix.

The argument behind Proposition 2 is straight-forward. The breakdown allocation is the principals' ideal allocations. This means that each principal can, by self-representation, achieve an agreement that she prefers to $\mathbf{x}^P$ and consequently her best reply can not do worse. Both principals are therefore better off than in autarchy. The equilibrium agreement thus belongs to $Z(\boldsymbol{\theta}^P, \mathbf{x}^P)$ and each nation allocates more resources to production of the public good than in autarchy. However, mutual self-representation is not played in equilibrium. The resulting agreement turns out to be different from $\mathbf{z}^{NB}(\boldsymbol{\theta}^P, \mathbf{x}^P)$, which is a Pareto efficient allocation from the principals' point of view. Hence, at least one of them is worse off, possibly both, compared to mutual self-representation. In summary:

Corollary 1. *Suppose* $\boldsymbol{\delta}^{NEW} \in NE\left(D^W\right)$.

(i) $\mathbf{z}^{NB}(\boldsymbol{\delta}^{NEW}, \mathbf{x}^P) \succ_{\theta_k^P} \mathbf{x}^P$ *for* $k = 1, 2$.

(ii) $\mathbf{z}^{NB}(\boldsymbol{\delta}^{NEW}, \mathbf{x}^P) > \mathbf{x}^P$.

(iii) $\mathbf{z}^{NB}(\boldsymbol{\theta}^P, \mathbf{x}^P) \succ_{\theta_k^P} \mathbf{z}^{NB}(\boldsymbol{\delta}^{NEW}, \mathbf{x}^P)$ *for some* $k = 1, 2$.

Proof. See the Appendix.

The disadvantage of weak delegation is that it does not exploit all gains that strategic delegation may potentially offer. Suppose that it is possible for a principal to credibly delegate the right to decide on the national breakdown allocation to her agent. This would enable her, by appointing an agent with less taste for the public good than herself, to lower the reservation level of utility of the other agent and thereby to strengthen her own bargaining position.

3.3. Strong Delegation

In the strong delegation game each principal not only delegates the bargaining to her agent but also the right to decide on the nation's breakdown allocation. The breakdown allocation is thus constituted by the agents' ideal allocations, $\mathbf{b}^S(\boldsymbol{\theta}) = \mathbf{x}^*(\boldsymbol{\theta}) = (x^*(\theta_1), x^*(\theta_2))$ for all $\boldsymbol{\theta}$. Let $D^S = (N, \Theta, \boldsymbol{\pi}^S)$ denote the strong delegation game and let:

$$\pi_k^S(\boldsymbol{\delta}) = \int_0^1 v_k\left(\mathbf{z}^{NB}(\boldsymbol{\theta}, \mathbf{x}^*(\boldsymbol{\theta})), \theta_k^P\right)\boldsymbol{\delta}(\boldsymbol{\theta})\, d\boldsymbol{\theta}.$$

Lemma 1. *$dz_k^{NB}/d\theta_k < 0$ and $dz_l^{NB}/d\theta_k > 0$ for all $\boldsymbol{\theta} \in (0,1]^2$, $l \neq k$.*

Proof. See the Appendix.

Lemma 1 states that if principal k unilaterally revises her choice of an agent by sending an agent with marginally less taste for the public good, this will induce a decrease in the amount of public good provided by her nation and an increase in the provision of the other nation. The Nash bargaining solution, as it stands in Definition 2, provides unfortunately very little of the economic intuition behind Lemma 1. Some intuition can be found if the Nash bargaining solution is viewed as the limit of a Rubinstein-Ståhl alternating-offer game as the length of each bargaining period approaches zero. In the alternating-offer game the equilibrium strategy of each agent is to offer his opponent an agreement such that he is indifferent between accepting the offer and to continue bargaining. The decision of a principal to replace her agent with one who has a marginally stronger taste for the private good than the previous agent has three effects. First, the reservation level of utility of the other nation's agent decreases. The new agent can, to keep the other nation's agent indifferent, make a less favorable proposal than the old agent. Alternatively, the agent of the other nation must make a more favorable proposal to the new agent than to the old agent. Second, the new agent's marginal utility of the private good is higher than the old agent's marginal utility which makes the contract curve shift inward. Thus, either x_k must decrease or x_l $(l \neq k)$ must increase, or both. Third, the increased marginal utility of the private good can be thought of as the new agent's utility function being more concave than the old agent's utility function. Increased concavity is negative for the agent.[6] However, the third effect is dominated by the other two effects. In the following we say that principal k *threatens* the other country's agent θ_l by her choice of agent if θ_l's reservation level of utility is lower when bargaining against θ_k than θ_k^P, i.e., if $\theta_k > \theta_k^P$.

[6]The main finding of the studies by Crawford and Varian (1978) and Sobel (1981) is that each agent would like to misrepresent her utility function by claiming it to be risk-neutral, i.e. linear.

Principal k solves $\max_{\delta_k} \pi_k^S(\boldsymbol{\delta})$ and the first-order condition (for an interior solution) is

$$\delta_k(\theta_k) \int_0^1 \left(\frac{\partial v_k}{\partial x_k} \frac{dz_k^{NB}}{d\theta_k} + \frac{dz_l^{NB}}{d\theta_k} \right) \delta_l(\theta_l) \; d\theta_l = 0 \tag{3.2}$$

for all $\theta \in (0,\ 1)$, $l \neq k$. Recall that $\mathbf{b}^S(\boldsymbol{\theta})$ is continuous and that, by Proposition 1, there exists a Nash equilibrium to D^S for all $\boldsymbol{\theta}^P$. Using Lemma 1 to study (3.2) gives the following results. First, every agent θ_k that is a best reply against some agent θ_l has at least as strong a taste for the private good as his principal. Hence, self-representation is not played by any of the principals. Second, if an equilibrium in game D^S is in pure strategies and interior, $\boldsymbol{\theta}^{NES} = (\theta_1^{NES}, \theta_2^{NES}) < \mathbf{1} = (1,1)$, then each nation provides less public good than in the cases of mutual self-representation, weak delegation, and autarchy. Formally:

Proposition 3. *Suppose* $\boldsymbol{\delta}^{NES} = (\delta_1^{NES}, \delta_2^{NES}) \in NE(D^S)$.
(i) $\boldsymbol{\theta} > \boldsymbol{\theta}^P$ *for all* $\boldsymbol{\theta} \in C(\boldsymbol{\delta}^{NES})$.
(ii) If $\boldsymbol{\delta}^{NES}(\boldsymbol{\theta}^{NES}) = \mathbf{1}$ *and* $\boldsymbol{\theta}^{NES} < \mathbf{1}$ *then* $\mathbf{z}^{NB}(\boldsymbol{\theta}^P, \mathbf{x}^P), \mathbf{z}^{NB}(\boldsymbol{\delta}^{NEW}, \mathbf{x}^P) > \mathbf{x}^P > \mathbf{z}^{NB}(\boldsymbol{\theta}^{NES}, \mathbf{x}^*(\boldsymbol{\theta}^{NES}))$.

Proof. See the Appendix.

For every agent $\theta_l > 0$ principal l chooses, it is a best reply for θ_k^P to threaten θ_l^P's agent. This, by Lemma 1, induces nation l to provide more of the public good. This is beneficial for θ_k^P who in turn provides less compared to self-representation in which case she must provide more of the public good than what her ideal allocation prescribes. However, in equilibrium, θ_k^P chooses to delegate so that $z_k^{NB} < x^*(\theta_k^P)$ is seen by studying (3.2). If a best reply of principal k belongs to the interior of her pure strategy space then must, by Lemma 1, $\partial v_k / \partial x_k > 0$ for the first-order condition to hold which, in turn, implies $z_k^{NB} < x^*(\theta_k^P)$. Any reduction below $x^*(\theta_k^P)$ is costly for θ_k^P since her ideal allocation states a higher production, but this cost is more than outweighed by the increase in the other nation's provision. The analogous reasoning holds for principal l and the two principals are caught in a situation similar to the prisoners' dilemma. Just like "*defect*" is a dominant strategy in the prisoners' dilemma game "*delegate strategically*" is a dominant "strategy" in the strong delegation game and in equilibrium both principals exaggerate their taste for the private good. However, as a consequence, the equilibrium pair of agents has a bias toward the private good and the agreed allocation may provide less of the public good than the autarchy allocation. If an equilibrium in game D^S is in pure strategies and interior then mutual weak delegation strictly Pareto dominates autarchy, with regard to the principals, and autarchy strictly Pareto dominates mutual strong delegation.

Corollary 2. *If* $\boldsymbol{\delta}^{NES} \in NE(D^S)$, $\boldsymbol{\delta}^{NES}(\boldsymbol{\theta}^{NES}) = \mathbf{1}$, *and* $\boldsymbol{\theta}^{NES} \in (0,1)^2$ *then* $\mathbf{z}^{NB}(\boldsymbol{\theta}^P, \mathbf{x}^P), \mathbf{z}^{NB}(\boldsymbol{\delta}^{NEW}, \mathbf{x}^P) \succ_{\theta_k^P} \mathbf{x}^P \succ_{\theta_k^P} \mathbf{z}^{NB}(\boldsymbol{\theta}^{NES}, \mathbf{x}^*(\boldsymbol{\theta}^{NES}))$ *for* $k = 1, 2$.

Proof. See the Appendix.

Proposition 3(ii) states that $\mathbf{z}^{NB} < \mathbf{x}^P$ if the equilibrium is pure and interior. This may still be the case even if the equilibrium is not interior, say θ_1 or $\theta_2 = 1$, or both. For those equilibria in which $\mathbf{z}^{NB} < \mathbf{x}^P$, Corollary 2 still holds.[7]

4. An Example

A numerical example is summarized in Tables I and II. Two cases are accounted for, one in which the principals are identical, $\boldsymbol{\theta}^P = (0.2, 0.2)$, and one in which they differ, $\boldsymbol{\theta}^P = (0.3, 0.2)$. In both cases the principals strictly prefer mutual self-representation to weak delegation, weak delegation to autarchy, and autarchy to strong delegation.

TABLE I
The Two Cases of Autarchy and Mutual Self-Representation

	Autarchy		Self-Representation	
	$\theta_1^P = 0.2$	$\theta_1^P = 0.3$	$\theta_1^P = 0.2$	$\theta_1^P = 0.3$
$\mathbf{x}^*, \mathbf{z}^{NB}$	$(0.8, 0.8)$	$(0.7, 0.8)$	$(0.9, 0.9)$	$(0.834, 0.911)$
$v_1(., \theta_1^P)$	1.2781	1.1388	1.3395	1.206
$v_2(., \theta_2^P)$	1.2781	1.1781	1.3395	1.2615

All principals delegate, in both the weak and the strong delegation games, to agents with stronger taste for the private good than themselves. See also Fig. 4.1.

TABLE II
Outcomes of the Weak and Strong Delegation Games

	Weak Delegation		Strong Delegation	
	$\theta_1^P = 0.2$	$\theta_1^P = 0.3$	$\theta_1^P = 0.2$	$\theta_1^P = 0.3$
$\boldsymbol{\theta}^{NEW}, \boldsymbol{\theta}^{NES}$	$(0.28, 0.28)$	$(0.41, 0.29)$	$(0.5, 0.5)$	$(0.69, 0.58)$
$\mathbf{z}^{NEW}, \mathbf{z}^{NES}$	$(0.86, 0.86)$	$(0.778, 0.867)$	$(0.75, 0.75)$	$(0.639, 0.723)$
$\mathbf{v}_1^{NEW}, \mathbf{v}_1^{NES}$	1.3268	1.1938	1.2227	1.0567
$\mathbf{v}_2^{NEW}, \mathbf{v}_2^{NES}$	1.3268	1.2411	1.2227	1.1055

[7]Example: Let $\boldsymbol{\theta}^P = (0.45, 0.45)$. Then is $\boldsymbol{\theta}^{NES} = (1,1)$, $\mathbf{z}^{NB} = (0.5, 0.5)$ and $\mathbf{x}^P = (.55, .55)$.

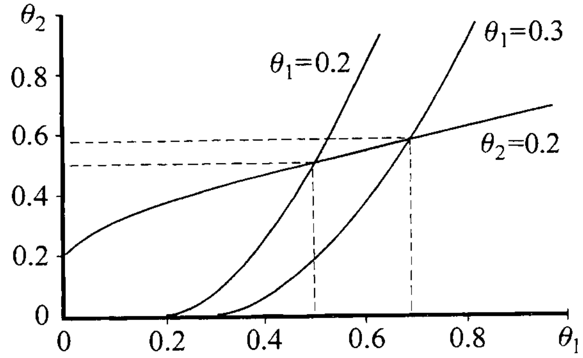

Figure 4.1: The principals' reaction curves in the strong delegation game.

5. Summary and Comments

When principals appoint agents with flat incentive schemes to bargain over the provision of a public good, delegation makes at least one of the principals worse off compared to mutual self-representation. It is also possible that all gains from reaching an agreement is more than offset from the viewpoint of the principals when delegation is strong. Here, each principal is able to create a threat to the other nation's agent by her choice of agent and this ability encourages the principals to delegate extremely, to the worse for them both. This is not the case in the weak delegation game and its equilibrium Pareto dominates both the strong delegation equilibrium and autarchy.

The results arrived to depend on the choice of utility function. Even though a more general formulation is desirable, I argue that the chosen formulation should not be considered a major drawback since it allows for a rather simple model which still provides some important insights, e.g., the incentives to vote for a politician who is more to the extreme than yourself and that even though you, for strategic reasons, have an incentive to give this politician a lot of power, it may hurt you.

The paper studies strategic delegation within two institutional set-ups differing in the level of authority given to the agents. The simplicity of the model makes it relatively easy to introduce a broad spectrum of institutions, or maybe to introduce variable institutions. Focus could thereby be switched from delegation to institutional design by letting the principals choose institutions before choosing agents. One natural question that arises is how much authority is given to the agents in equilibrium, i.e., what do the equilibrium institutions look like?[8]

[8]A related issue that has been studied by Crawford (1982) and Muthoo (1996) is how the

Appendix; Proofs

Proposition 1. The proof is carried out in two steps. First it is shown that the Nash bargaining solution in Definition 2 is a singleton set and that it is continuous over $[0,1]^4$ which is essential when constructing the reduced general delegation game. In step 2 the existence of a Nash equilibrium is shown by the use of Fan-Debreu-Glicksberg's theorem.

Step 1: The Nash bargaining solution in Definition 2 is equivalent to

$$\mathbf{z}^{NB}\left(\boldsymbol{\theta},\mathbf{y}^0\right) = \arg\max_{\mathbf{y}\in Y}\left(y_1 - y_1^0\right)\left(y_2 - y_2^0\right)$$

where $Y = \{(v_1(\mathbf{x},\theta_1), v_2(\mathbf{x},\theta_2)) \mid \mathbf{x} \in Z(\boldsymbol{\theta},\mathbf{x}^0)\}$ and $\mathbf{y}^0 = (v_1(\mathbf{x}^0,\theta_1), v_2(\mathbf{x}^0,\theta_2))$. To show the existence of a unique solution it is sufficient to show that the bargaining set Y is compact and convex. The compactness of Y follows directly from the compactness of $Z(\boldsymbol{\theta},\mathbf{x}^0)$ and the continuity of v_k. The convexity of Y follows from the concavity of v_k and the convexity of $Z(\boldsymbol{\theta},\mathbf{x}^0)$. The continuity of $\mathbf{z}^{NB}$ on $[0,1]^4$ follows from Berge's maximum theorem.

Step 2: In any considered reduced game the pure strategy spaces Θ_k are compact. Since $\mathbf{z}^{NB}(\boldsymbol{\theta},\mathbf{b}(\boldsymbol{\theta}))$ is continuous in $\boldsymbol{\theta}$ so are $v_k\left(\mathbf{z}^{NB}(\boldsymbol{\theta},\mathbf{b}(\boldsymbol{\theta})),\theta_k^P\right)$ and the payoff functions π_k. By the Debreu-Fan-Glicksberg Theorem (1952, source: Fudenberg and Tirole (1991) pp. 35) there exists a Nash equilibrium, possibly in mixed strategies. ■

Proposition 2. (i) Let θ_k^P play self-representation, $\delta_k(\theta_k^P) = 1$. By construction of D^W is

$$\pi_k^W(\boldsymbol{\delta}) \geq v_k(\mathbf{x}^P,\theta_k^P) \tag{5.1}$$

for all δ_l $(l \neq k)$ and since only best replies are played in equilibrium it follows that

$$\pi_k^W(\boldsymbol{\delta}^{NEW}) \geq v_k(\mathbf{x}^P,\theta_k^P),\ k = 1,2 \tag{5.2}$$

for all $\boldsymbol{\delta}^{NEW} \in NE(D^W)$ and for all $\boldsymbol{\theta}^P$. Hence, $\mathbf{z}^{NB}(\boldsymbol{\delta}^{NEW},\mathbf{x}^P) \in Z(\boldsymbol{\theta}^P,\mathbf{x}^P)$ by definition of Z.

(ii) Below it is shown that $\theta_k = \theta_k^P$ can never be a best reply to any $\theta_l > 0$ implying that $\boldsymbol{\theta}^P \notin C(\boldsymbol{\delta}^{NEW})$ for all $\boldsymbol{\delta}^{NEW} \in NE(D^W)$. Let

$$A_k = \left(v_k\left(\mathbf{z}^{NB}\left(\boldsymbol{\theta},\mathbf{x}^P\right),\theta_k\right) - v_k\left(\mathbf{x}^P,\theta_k\right)\right). \tag{5.3}$$

Differentiation of the system of first-order conditions of the Nash product with

cost of revoking a partial commitment affects the outcome of a Nash bargaining.

respect to x_1, x_2, and θ_k and rearranging yields

$$\begin{bmatrix} \frac{\partial^2 N}{\partial x_k^2} & \frac{\partial^2 N}{\partial x_l \partial x_k} \\ \frac{\partial^2 N}{\partial x_k \partial x_l} & \frac{\partial^2 N}{\partial x_l^2} \end{bmatrix} \begin{bmatrix} \frac{dz_k^{NB}}{d\theta_k} \\ \frac{dz_l^{NB}}{d\theta_k} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 N}{\partial \theta_k \partial x_k} \\ -\frac{\partial^2 N}{\partial \theta_k \partial x_l} \end{bmatrix}.$$

The determinant of the Hessian ($\mathbb{H}$) to N is positive since N is a concave function evaluated at its maximum. If $\mathbf{z}^{NB} > \mathbf{x}^P$ then Cramer's rule gives ($l \neq k$)

$$\frac{dz_k^{NB}}{d\theta_k} = -\frac{\frac{\partial^2 v_k}{\partial \theta_k \partial x_k} \frac{\partial^2 v_l}{\partial x_l^2} A_k A_l + 2 \frac{\partial^2 v_k}{\partial \theta_k \partial x_k} \frac{\partial v_l}{\partial x_l} A_l + \frac{\partial^2 v_l}{\partial x_l^2} \frac{\partial A_k}{\partial \theta_k} A_k}{\det(\mathbb{H})} < 0 \tag{5.4}$$

and

$$\frac{dz_l^{NB}}{d\theta_k} = \frac{2 \frac{\partial^2 v_k}{\partial \theta_k \partial x_k} A_l - \frac{\partial^2 v_k}{\partial x_k^2} \frac{\partial v_l}{\partial x_l} \frac{\partial A_k}{\partial \theta_k} A_l}{\det(\mathbb{H})} \gtreqless 0. \tag{5.5}$$

Let θ_l^P play the pure strategy $\theta_l > 0$. If $\theta_k = \theta_k^P$ is a best reply for θ_k^P then the first-order condition

$$\left(\frac{\partial v_k}{\partial x_k} \frac{dz_k^{NB}}{d\theta_k} + \frac{dz_l^{NB}}{d\theta_k} \right) \Bigg|_{\theta_k = \theta_k^P} = 0$$

to θ_k^P's maximization problem must be satisfied. Substitute (5.4) and (5.5) into the first-order condition and simplify

$$-\frac{\partial v_k}{\partial x_k} \frac{\frac{\partial^2 v_k}{\partial \theta_k \partial x_k} \frac{\partial^2 v_l}{\partial x_l^2} A_k A_l + \frac{\partial^2 v_l}{\partial x_l^2} \frac{\partial A_k}{\partial \theta_k} A_k + \left(\frac{\partial v_l}{\partial x_l} \right)^2 \frac{\partial^2 v_k}{\partial x_k^2} \frac{\partial A_k}{\partial \theta_k} A_l}{\det(\mathbb{H})} > 0.$$

Hence, $\theta_k = \theta_k^P$ is not a best reply to any $\theta_l > 0$.

Corollary 1. (i) Follows directly from proof of Proposition $2(i)$ and the definition of Z.

(ii) Follows from the proof of Proposition $2(i)$ and the properties of Z.

(iii) Because $\mathbf{z}^{NB}(\boldsymbol{\theta}^P, \mathbf{x}^P)$ is Pareto efficient it is sufficient to show that $\mathbf{z}^P = \mathbf{z}^{NB}(\boldsymbol{\theta}^P, \mathbf{x}^P)$ can only be reached by mutual self-representation in order to prove (iii). Combining the first-order conditions of the Nash product and evaluating at $(\boldsymbol{\theta}^P, \mathbf{z}^P)$ gives ($l \neq k$)

$$\frac{\partial v_k}{\partial x_k} \Bigg|_{(\boldsymbol{\theta}^P, \mathbf{z}^P)} = \frac{1}{\frac{\partial v_l}{\partial x_l}} \Bigg|_{(\boldsymbol{\theta}^P, \mathbf{z}^P)}.$$

Solving for θ_k^P we get $\theta_k^P = f(\theta_l^P)$. Using the second first-order condition

$$\left(\frac{\partial v_l}{\partial x_l} A_k + A_l\right)\bigg|_{(\boldsymbol{\theta}^P, \mathbf{z}^P)} = 0$$

θ_k^P can be written $\theta_k^P = g(\theta_l^P)$. Differentiating with respect to θ_l^P gives

$$\begin{array}{c} f' < 0 \\ g' > 0 \end{array}. \tag{5.6}$$

By construction is $f\left(\theta_l^P\right) = g\left(\theta_l^P\right) = \theta_k^P$and (5.6) implies that if $\theta_l \neq \theta_l^P$ then is $f\left(\theta_l\right) \neq g\left(\theta_l\right)$ and, hence, $\mathbf{z}^{NB}\left(\boldsymbol{\theta}, \mathbf{x}^P\right) \neq \mathbf{z}^P$.

Lemma 1. Using the same method and notation as in the proof of Proposition 2(*ii*) but replacing $\mathbf{b}^W(\boldsymbol{\theta}) = \mathbf{x}^P$ by $\mathbf{b}^S(\boldsymbol{\theta}) = \mathbf{x}^*(\boldsymbol{\theta})$ yields $(l \neq k)$

$$\frac{dz_k^{NB}}{d\theta_k} = -\frac{\frac{\partial^2 v_k}{\partial\theta_k \partial x_k}\frac{\partial^2 v_l}{\partial x_l^2} A_k A_l + 2\frac{\partial^2 v_k}{\partial\theta_k \partial x_k}\frac{\partial v_l}{\partial x_l} A_l - \frac{\partial v_k}{\partial x_k}\frac{\partial^2 v_l}{\partial x_l^2}\frac{\partial x^*}{\partial \theta_k} + \frac{\partial^2 v_l}{\partial x_l^2}\frac{\partial A_k}{\partial \theta_k} A_k}{\det\left(\mathbb{H}\right)} < 0 \tag{5.7}$$

and

$$\frac{dz_l^{NB}}{d\theta_k} = \frac{2\frac{\partial^2 v_k}{\partial\theta_k \partial x_k} A_l + \frac{\partial^2 v_k}{\partial x_k^2}\frac{\partial x^*}{\partial \theta_k} - \frac{\partial^2 v_k}{\partial x_k^2}\frac{\partial v_l}{\partial x_l}\frac{\partial A_k}{\partial \theta_k} A_l}{\det\left(\mathbb{H}\right)} > 0. \tag{5.8}$$

The sign of (5.7) is at first glance ambiguous but Taylor approximating $(\ln(1 - z_k^{NB}) - \ln(1 - x^*(\theta_k))$ at $x^*(\theta_k)$ and simplifying the nominator gives

$$\frac{-1}{1 - z_k^{NB}}\left(A_l - \frac{1 - x^*(\theta_k)}{1 - z_k^{NB}}\right).$$

Taylor approximating A_l as unfavorable as possible (at $x^*(\theta_l)$) and simplifying further yields

$$\frac{-1}{1 - z_k^{NB}}\left(\left(z_k^{NB} - x^*(\theta_k)\right) - \frac{1 - x^*(\theta_k)}{1 - z_k^{NB}}\right) > 0$$

showing (5.7) to be negative for all $\boldsymbol{\theta} \in (0, 1]^2$.

Proposition 3. (*i*) The proof is carried out in three steps. Recall that $\boldsymbol{\theta}^P \in (0, 1)^2$. First, it is shown that $\theta_k = 0$ is not a best reply against $\theta_l > 0$. Second, it is shown that $\theta_k > \theta_k^P$ for all θ_k that are best replies against $\theta_l > 0$. Third, it is shown that the used first-order condition can not define a minimum.

Step 1: Let $\beta_k^S(\delta_l) = \arg\max_{\delta_k} \pi_k^S(\boldsymbol{\delta})$. If $\delta_k(0) = 1$ then is $z_k^{NB} = 1 > x^*(\theta_k^P)$ and $z_l^{NB} = x^*(\theta_l) \leq z_l^{NB}(\boldsymbol{\theta}, \mathbf{x}^*(\boldsymbol{\theta}))$ for any $\theta_k > 0$. Hence, $0 \notin C_k\left(\beta_k^S(\delta_l)\right)$ for any δ_l where $C_k(\delta_k) = \{\theta_k \mid \delta_k(\theta_k) > 0\}$.

Step 2: The first-order condition to θ_k^P's maximization problem for an interior solution is

$$\delta_k(\theta_k)\int_0^1\left(\frac{\partial v_k}{\partial x_k}\frac{dz_k^{NB}}{d\theta_k}+\frac{dz_l^{NB}}{d\theta_k}\right)\delta_l(\theta_l)\,d\theta_l=0 \tag{5.9}$$

for all $\theta_k\in(0,1)$. Hence, for every $0<\theta_k<1$ is

$$\delta_k(\theta_k)=0 \tag{5.10}$$

or/and

$$\int_0^1\left(\frac{\partial v_k}{\partial x_k}\frac{dz_k^{NB}}{d\theta_k}+\frac{dz_l^{NB}}{d\theta_k}\right)\delta_l(\theta_l)\,d\theta_l=0. \tag{5.11}$$

Since $\int_0^1\delta_k(\theta_k)\,d\theta_k=1$ and 0 never is a best reply it must be the case that (5.11) is satisfied for some $\theta_k\in(0,1)$ or that $\delta_k(1)=1$. Let $\theta_l>0$, for any $0<\theta_k<1$ such that $\theta_k\in C_k\left(\beta_k^S(\theta_l)\right)$ must be

$$\frac{\partial v_k}{\partial x_k}\frac{dz_k^{NB}}{d\theta_k}+\frac{dz_l^{NB}}{d\theta_k}=0. \tag{5.12}$$

From the proof of Lemma 3 we have that $dz_k^{NB}/d\theta_k<0$ and $dz_l^{NB}/d\theta_k>0$ for all $\boldsymbol{\theta}\in(0,1]^2$. Hence, satisfying (5.12) requires $\partial v_k/\partial x_k>0$. By the properties of the Nash bargaining solution is $z_k^{NB}(\boldsymbol{\theta},\mathbf{x}^*(\boldsymbol{\theta}))>x^*(\theta_k)$ and by (5.12) is $z_k^{NB}(\boldsymbol{\theta},\mathbf{x}^*(\boldsymbol{\theta}))<x^*(\theta_k^P)$, hence

$$x^*(\theta_k^P)>z_k^{NB}(\boldsymbol{\theta},\mathbf{x}^*(\boldsymbol{\theta}))>x^*(\theta_k). \tag{5.13}$$

x^* is strictly decreasing in θ_k and consequently is $\theta_k>\theta_k^P$ for $\theta_k\in C_k\left(\beta_k^S(\theta_k)\right)$.

Step 3: Let $0<\theta_k\leq\theta_k^P$, then by the properties of the Nash bargaining solution is $z_k^{NB}((\boldsymbol{\theta}),\mathbf{x}^*(\boldsymbol{\theta}))>x^*(\theta_k^P)$. It follows that (5.12) is strictly positive and that $v_k(\mathbf{z}^{NB},\theta_k^P)$ increases with θ_k. Hence, (5.12) does not define a minimum. Moreover, if there does not exist a $\theta_k<1$ large enough to satisfy (5.12) then $\delta_k^{NES}(1)=1$. Furthermore, $\theta_k^P\in C_k(\beta_k^S(\delta_l))$ iff $\delta_l(0)=1$.

Conclusion: For every δ_l and $\theta_k^P>0$ must $C_k(\beta_k^S(\delta_l))\subset\left[\theta_k^P,1\right]$. Hence, if $\boldsymbol{\theta}^P\in(0,1)^2$ then $C_k(\beta_k^S(\delta_l^{NES}))\subset\left(\theta_k^P,1\right]$ for all $\boldsymbol{\delta}^{NES}\in NE\left(D^S\right)$, $k=1,2$.

(ii) Follows from Corollary $1(ii)$ and the proof of Proposition $3(i)$.■

Corollary 2. Follows immediately from Corollary $1(ii)$, (5.13), and the properties of v_k.

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Delegation of Bargaining and Power

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Abstract

Two principals simultaneously appoint one agent each and decide how much power to give to their agents. The agents' task is to bargain over the provision of a public good. Power here means the right to decide the own side's provision if negotiations break down. In equilibrium the principals delegate to agents that are relatively disinterested in the public good and give them all power. The fact that both principals have the possibility to delegate is, in equilibrium, harmful to at least one of them. The equilibrium may even be Pareto dominated by the outcome under autarchy. *Journal of Economic Literature* Classification Numbers: C71, and C72.

1. Introduction

The nature of many important decisions is such that they cannot be made by those who are most concerned. In the case of negotiations between countries it is usually impractical, if not impossible, to gather the citizens for a referendum every time a decision has to be made. Instead we delegate that kind of decision making to political institutions. The main concern of the individual citizen is then not the decision making itself but to choose a policymaker with the appropriate ideology (preferences) and to give this policymaker a well-balanced amount of power. The natural question is: what characterizes a good combination of ideology and power? Is it in the interest of the citizen to have a policymaker who is ideologically very

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different from herself and to give this policymaker limited power, or does she want the policymaker to be more powerful and ideologically close to herself? This problem of ideology and power is the problem addressed in this study. As it turns out, the citizen wants the policymaker to be ideologically different from herself and she wants him to have an extensive authority.

The framework studied is international negotiations but the model and its logic apply to a much broader spectrum of situations. A few examples are given later in the introduction. The basic model is similar to the one used in Segendorff (1998); there are two countries with one unit of resources each that can be allocated between the production of two goods. One of the goods is private for the producing country and the other good is public between the countries. The private good can be thought of as health care and the public good as reduction of the emission of carbon dioxide into the atmosphere. Every citizen in each country prefers a particular national resource allocation (output combination of the two goods). This ideal allocation is determined by her taste parameter which is continuously distributed across the population.

In each country there is a particular citizen called *the principal* (henceforth *she*) who delegates the national allocation decision to an especially selected citizen, *the agent* (henceforth *he*). She also decides on how much power, in a sense explained below, to give to her agent. There is no voting in the model but it may be helpful to think of the principal as some decisive voter and the agent as some elected policymaker such as a president or a prime minister. Alternatively, we may think of the principal as a prime minister appointing a member of the cabinet. The payment scheme of such an agent is usually low-incentive powered (see also Perry and Samuelson (1994)) and here the agent is given a fixed wage normalized to zero. There is no monetary aspect of the national allocation decision and the agent's decision is consequently based on his preferences directly over the resource allocation of his country. We like to think of a politician as driven by ideological motives and this is how we can think of the agent.[1] This basic model is the foundation of two delegation games that are used to study strategic delegation of bargaining and power. The resulting sets of Nash equilibria are compared to a benchmark called *autarchy*.

By autarchy is meant a situation where the two principals simultaneously decide on their national output combinations. The unique Nash equilibrium is for the two principals to implement their ideal allocations. This equilibrium is not

[1] High-powered incentive schemes are not ruled out but they require a different interpretation of the model. Suppose that agents only care about money and that the chosen agent is rewarded in proportion to the principal's utility. The agents are uninformed of the principal's preferences and instead of differing in interest for the public good they differ in their belief over the principal's preferences. Agents who are relatively disinterested in the private good believe the principal to care more about the public good than about the private good, and vice versa.

Pareto efficient with regard to the principals because not one of them internalizes the effect of her decision on the other principal. Therefore, the principals have an incentive to coordinate on some mutually preferred resource allocation. In this study, cooperation is achieved through bargaining between agents.

The first delegation game is a two-stage game. In the first stage the principals simultaneously choose agents and the amount of power to give to the agents. In the second stage the agents meet and bargain over the global resource allocation, i.e., the output combinations of the two countries. The bargaining is modelled by way of the Nash bargaining solution and the disagreement point is constituted by the agents' utilities of some alternative global allocation. This alternative global allocation is implemented in the case of a break down in negotiations (see Binmore *et al.* (1986) for a more extensive discussion on the subject). By power we mean the agent's influence on the resource allocation of his country in the case of a break down. In the model, the break-down allocation is a linear combination of the principal's and the agent's ideal allocations. If the agent is given no (all) power then he has no (total) influence on the break-down allocation which then becomes the principal's (agent's) ideal allocation. In the case of intermediate power the break-down allocation is somewhere between the two ideal allocations. Power can thus be thought of as a politician's ability to implement his preferred policy after a break down in negotiations. However, delegation of power is only important if it is credible, i.e., if the principal can commit to such delegation. In the model we assume that delegation is credible.

Delegation of power is just as important as the preferences of the agent because it is the combination of power and preferences that allows the principal to threat the agent of the other country. If she appoints an agent with less taste for the public good than herself then this agent would like to allocate relatively less resources to production of the public good in the case of a break down. Giving the agent power to influence the break-down allocation lowers (increases) the disagreement utility of the agent of the other (own) country and thus works as a threat. An increase in the disagreement utility of an agent induces an increase in that agent's payoff. This is called *disagreement point monotonicity* (Thomson (1987)). In the model, an increase in the payoff of an agent implies a decrease in the provision of public good of that agent's country and an increase in the other country's provision of public good. This is beneficial for both the principal and her agent since the amount of resources allocated to production of their private good increases without the corresponding amount of resources being withdrawn from production of the public good. In equilibrium the principals delegate to agents who are less interested in the public good than the principals themselves and they give their agents total power. At least one off the principals is worse of in equilibrium compared to a situation where the principals bargain themselves.

Moreover, the negative effects of delegation may more than offset the gains from coordination and both principals may be worse off than in autarchy.

The second delegation game is a three-stage game where the principals simultaneously decide on the agents' power in the first stage and thereafter, in the second stage, simultaneously choose agents after having observed the choices made in the first stage. In the third stage the agents meet and bargain. We show that in any subgame-perfect Nash equilibrium to this game the principals delegate to agents who are relatively disinterested in the public good. The main contribution of the three-stage delegation game is thus to show the robustness of the principals' incentive to delegate strategically to agents who cares little about the public good.

The model does not only apply to international negotiations but to a much broader class of situations where delegation of bargaining and power occurs. One example is two firms that can gain from cooperation in R&D. The firm owners choose managers who meet and negotiate. Managers differ in what they believe maximizes profits; investment in the sales organization (private good) or investment in R&D (public good). This example presumes the reasonable assumption of spill-over of knowledge between the firms in the case of no cooperation. Bargaining between local governments over the provision of a (locally) public good, say libraries, is another example. One more example is bargaining between interest groups who have partly coinciding objectives and who try to convince a third party about something, e.g., a union and an employers' association lobbying for subsidies to an industry or protection from international competition by trade barriers.

This study is closely related to Segendorff (1998) who lets the power of the agents be exogenously given and who studies two delegation games differing in the amount of power given to the agents. In the *weak delegation game* the agents have no power and in the *strong delegation game* they have total power. The main findings are that the equilibrium of the weak delegation game Pareto dominates autarchy while the equilibrium of the strong delegation game may be Pareto dominated by autarchy. The study presented here is different in one important respect; the amount of power given to the agents is determined endogenously. A principal will thus only give her agent power if it is in her interest to do so.

Jones (1989) studies a situation where two principals choose agents to bargain over the division of two private goods. The bargaining is modelled by way of the Nash bargaining solution and the disagreement point is normalized to zero. The main finding is that there can never be a utility gain for both principals compared to a situation where the principals bargain themselves. Fershtman *et al.* (1991) let two principals delegate a bargaining to two agents. Each principal signs a contract (payment scheme) with her agent where the payment is determined by the bargaining outcome. The principal is free to design the contract and the agent has preferences over the payment only. Their main result is that when allowing for a broad class of contracts, any cooperative outcome of the bargaining game without delegation can be made the unique subgame-perfect equilibrium of the delegation game. The delegation game studied below is concerned with a problem very different from the problem studied by Jones since the bargaining in this study is over a public good and the Nash bargaining solution is interpreted differently. It is different from the study by Fershtman *et al.* because the agents' incentives are non-monetary.

Finally, Crawford and Varian (1979), Sobel (1981), and Burtraw (1992) recognize that the Nash and related solution concepts to the bargaining problem presume information that is unobservable in practice and that a bargainer may gain from misrepresenting her true preferences. In the context of the Nash bargaining solution, the unique dominant-strategy Nash equilibrium is for both parties to report risk-neutral utility functions. These studies, even though distortion of preferences and delegation are related to each other, cannot capture some important aspects of delegation such as delegation of power.

The basic model and the autarchy benchmark are given in Section 2. Power and the Nash bargaining solution are defined in Section 3 and the two delegation games are given in Section 4. A numeric example is given in Section 5 and Section 6 contains the summary and comments. All proofs are given in the Appendix.

2. The Basic Model

The basic model is similar to the two-country model in Segendorff (1998). Each country has one unit of resources to allocate between the production of two goods of which one is private for the producing country and the other is public between the two countries. Every citizen has preferences over her country's production of the private good and the total production of the public good. These preferences are determined by a taste parameter, θ, and in both countries the taste parameters are continuously distributed over the interval $[a, 1]$ where $a \in (0, 1)$. An arbitrary citizen of country $k = 1, 2$ is denoted $\theta \in [a, 1]$ and the share of country k's resources that is allocated to production of the public good is denoted $x_k \in [0, 1]$. The preferences of θ for $x_k < 1$ are represented by the von Neuman-Morgenstern utility function

$$v_k(\mathbf{x}, \theta) = \theta \ln (1 - x_k) + x_1 + x_2$$

where $\mathbf{x} = (x_1, x_2)$ and if $x_k = 1$, then $v_k(\mathbf{x}, \theta) = -\infty$. (Bold-face will in the following be used to denote vectors.) Every θ has an ideal resource allocation $x^* (\theta) = 1 - \theta$ that θ prefers to any other x_k. The assumption $0 < a$ can thus be interpreted as every citizen receives some utility from the private good. The two countries' allocations are strategically neutral ($v_{k12} = v_{k21} = 0$) and θ's ideal allocation is the same for all resource allocations of the other country.

In each country there is a citizen called the *principal* (she) with taste parameter $a < \theta_k^P < 1$. Agents with stronger taste for the private good than the principal, $\theta_k > \theta_k^P$, will in the following be said to be *to the right* and agents with less taste for the private good than the principal, $\theta_k < \theta_k^P$, will consequently be said to be *to the left*. In *autarchy* the two principals simultaneously decide on the national output combination and they implement the allocation $\mathbf{x}^P = (x_1^P, x_2^P) = (x^*(\theta_1^P), x^*(\theta_2^P))$ which is not Pareto efficient from their point of view. Hence, any such pair of principals would benefit from coordinating on some other mutually preferred allocation. In the delegation game presented below, the countries coordinate their allocation decisions through bargaining. In reality are negotiations often carried out by delegates who represent the bargaining parties and below is delegation introduced in order to capture that important aspect of negotiations.

3. Delegation of Bargaining and Power

Consider the following two-stage game. In the first stage the principals simultaneously delegate the task of deciding on the national allocation to an especially selected citizen, the *agent* (he). At the same time each principal chooses how much power to give to her agent. Let θ_1 be the agent from country 1 and θ_2 the agent from country 2. The agent may be any citizen including the principal herself. The latter case is called *self-representation*. In the second stage the appointed agents meet and bargain over the resource allocations of the two countries, i.e., the provision of the public good. The resulting agreement is assumed to be binding. If no agreement is reached, some alternative break-down allocation is implemented.

The break-down allocation of a country is determined by the principal and her agent. Let the break-down allocation of country k be

$$b_k(\alpha_k, \theta_k) = x_k^P + \alpha_k(x^*(\theta_k) - x_k^P) = 1 - \theta_k^P + \alpha_k(\theta_k^P - \theta_k)$$

where $\alpha_k \in [0, 1]$ represents the power of the agent. If no (all) power is given to the agent, $\alpha_k = 0$ ($\alpha_k = 1$), then the ideal allocation of the principal (agent) is implemented in the case of a break-down in negotiations. In the intermediate case the break-down allocation is a linear combination of the two ideal allocations. In the following $b_k(\alpha_k, \theta_k)$ is viewed as a compromise (some bargaining outcome) between the principal and the agent where their relative bargaining strength is determined by the power of the agent. This simplification is easily justified since any Pareto efficient bargaining outcome between the principal and her agent can be described as a linear combination of the ideal allocations x_k^P and $x^*(\theta_k)$. The variable α_k can then be thought of as reflecting properties of the underlying bargaining game.[2]

Let $\mathbf{b}(\boldsymbol{\theta},\boldsymbol{\alpha}) = (b_1(\theta_1,\alpha_1), b_2(\theta_2,\alpha_2))$ where $\boldsymbol{\theta} = (\theta_1,\theta_2)$ and $\boldsymbol{\alpha} = (\alpha_1,\alpha_2)$. The bargaining between the two agents is modelled by way of the Nash bargaining solution.

Definition 1. *Let*

$$N(\mathbf{x},\boldsymbol{\alpha},\boldsymbol{\theta}) = (v_1(\mathbf{x},\theta_1) - v_1(\mathbf{b}(\boldsymbol{\alpha},\boldsymbol{\theta}),\theta_1))(v_2(\mathbf{x},\theta_2) - v_2(\mathbf{b}(\boldsymbol{\alpha},\boldsymbol{\theta}),\theta_2)).$$

Then the Nash bargaining solution is

$$\mathbf{x}^{NB}(\boldsymbol{\alpha},\boldsymbol{\theta}) = \arg\max_{\mathbf{x}\in Z(\boldsymbol{\alpha},\boldsymbol{\theta})} N(\mathbf{x},\boldsymbol{\alpha},\boldsymbol{\theta}) \tag{3.1}$$

where

$$Z(\boldsymbol{\alpha},\boldsymbol{\theta}) = \left\{\mathbf{x}\in[0,1]^2 \mid \mathbf{x} \succsim_{\theta_1} \mathbf{b}(\boldsymbol{\alpha},\boldsymbol{\theta}) \text{ and } \mathbf{x} \succsim_{\theta_2} \mathbf{b}(\boldsymbol{\alpha},\boldsymbol{\theta})\right\}$$

is the contract zone of the two agents.

The bargaining outcome $\mathbf{x}^{NB}(\boldsymbol{\alpha},\boldsymbol{\theta})$ is unique and the described two-stage delegation game can be reduced to a one-stage simultaneous-move game played by the principals.

4. The Delegation Game

In this section we formalize and analyze the two-stage delegation game. Let $D = (N, S, \boldsymbol{\pi})$ denote the delegation game where $N = \{1,2\}$ is the set of principals that play the game. The principals simultaneously choose agents and decide on how much power to give to their agents. The set from which principal k chooses her agent's power is $A_k = [0,1]$ and the set of agents available to her is $\Theta_k = [a,1]$.

[2] An alternative but not adopted view is to think of $b_k(\alpha_k,\theta_k)$ as the expected break-down allocation and α_i as the probability of the agent winning a political struggle after a break down. The winner of the struggle implements his/her ideal allocation. Thus, the power of the agent determines the probabilities of the two allocations x_k^P and $x^*(\theta_k)$. Taking the preferences of the principal and her agent as given, the break-down allocation is interpreted as a lottery induced by α_k. The principal and her agent are risk averse with respect to the own country's allocation. Their expected utilities from participating in the lottery are thus lower than their utilities from implementing the corresponding compromise. Moreover, the utility of the other country's agent is the same under both interpretations of b_k since his utility is linear in country k's provision of public good. In the model, an increase in the disagreement utility of country k's agent induces an increase in the amount of public good provided by the other country and a decrease in the amount of public good provided by the own country. This is beneficial for country k's agent.

A strategy for principal k thus a pair $s_k = (\alpha_k, \theta_k) \in S_k$ where $S_k = A_k \times \Theta_k$ is her set of strategies. Let $S = S_1 \times S_2$. The pair of payoff functions is $\boldsymbol{\pi} = (\pi_1, \pi_2)$ where π_k denotes the payoff to principal k as a function of the strategy profile $\mathbf{s} = (s_1, s_2) \in S$, i.e.,

$$\pi_k(\mathbf{s}) = v_k\left(\mathbf{x}^{NB}(\mathbf{s}), \theta_k^P\right).$$

For the moment, treat the strategy of principal $l \neq k$ and the power of agent k as given. Let $\xi_k(\alpha_k, s_l)$ be the set of agents that maximizes principal k's utility given $\alpha_k \in A_k$ and $s_l \in S_l$

$$\xi_k(\alpha_k, s_l) = \arg\max_{\theta_k \in \Theta_k} \pi_k(\mathbf{s}). \tag{4.1}$$

From the first-order condition to Equation 4.1 it can be shown that any agent that maximizes principal k's utility against $s_l \in S_l$ must be to the right of her and this is true for all levels of power given to the agent.[3]

Lemma 1. $\xi_k(\alpha_k, s_l) \subseteq \left(\theta_k^P, 1\right]$ $\forall \alpha_k \in A_k$, $\forall s_l \in S_l$.

Proof. See the Appendix.

The logic behind Lemma 1 is the following. Because the bargaining outcome is Pareto efficient it will stipulate each country to provide more public good than prescribed by the agents' ideal allocations, $\mathbf{x}^{NB}(\mathbf{s}) > (x^*(\theta_1), x^*(\theta_2))$ $\forall \mathbf{s} \in S$. In the case of self-representation, $\theta_k = \theta_k^P$, country k has to provide more of the public good than the principal wishes, $x_k^{NB}((\alpha_k, \theta_k^P), s_l) > x_k^P$, and she has an incentive to lower her country's provision of public good. If she appoints an agent to her left then she can not achieve this reduction since this agent is more interested in the public good than the principal is. Delegation aiming at reducing the own production of public good to x_k^P must therefore be to an agent who is relatively disinterested in the public good, i.e., who is to the right of the principal. The welfare of principal k does not only depend on her country's resource allocation but also on the resource allocation of the other country which in turn depends on principal k's choice of agent and the amount of power given to that agent. In Lemma 1 we learn that principal k, for every strategy played by the other principal and for any amount of power given to agent k, gains from strategic delegation to some agent to her right. The eventual utility loss from a decrease in the other country's provision of public good is outweighed by the utility gain from the reduction in the own country's provision of public good.

The Nash bargaining solution has a property called *disagreement point monotonicity* (Thomson (1987)); an increase in the disagreement utility of an agent

[3] The weak delegation game in Segendorff (1998) is the mixed extension of D where $A_k = \{0\}$ and $\Theta_k = [0, 1]$ for $k = 1, 2$. Lemma 1 can be extended to provide a strengthening of Proposition 2(ii) in Segendorff (1998) by making it possible to say that delegation is made to the right.

induces an increase in that agent's payoff. Suppose principal k delegates to an agent to her right, $\theta_k > \theta_k^P$. By increasing the power of her agent she increases the disagreement utility of the agent and lowers the disagreement utility of the other agent. Because of the disagreement point monotonicity this change induces a change in the agreement making country k provide less of the public good and country l provide more.

Lemma 2. *Let $\theta_k > \theta_k^P$. Then $\frac{dx_k^{NB}}{d\alpha_k} < 0$ and $\frac{dx_l^{NB}}{d\alpha_k} > 0$ $\forall \alpha_k \in A_k$, $\forall s_l \in S_l$.*

Proof. See the Appendix.

Changes in the power of the agent does not affect the slope of the agents' (here linear) contract curve but moves $\mathbf{x}^{NB}(\mathbf{s})$ along the contract curve. Let $\beta_k(s_l)$ denote the set of strategies that maximize π_k against $s_l \in S_l$, $l \neq k$

$$\beta_k(s_l) = \arg\max_{s_k \in S_k} \pi_k(\mathbf{s}).$$

The set $\beta_k(s_l)$ is nonempty because S_k is compact and convex and π_k is continuous in s_k. From Lemma 1 we have that $\theta_k > \theta_k^P$ for all $s_k \in \beta_k(s_l)$ and using Lemma 2 together with the properties of the Nash bargaining solution gives that $\alpha_k > 0$. In order to see this, suppose $s_k \in \beta_k(s_l)$ is such that $\alpha_k = 0$. Then $x_k^{NB}(\mathbf{s}) > x_k^P$ and increasing α_k unambiguously increases π_k by Lemma 2. Hence, any best reply for principal k includes giving her agent some power. Moreover, principal k will give her agent total power since her utility function is quasi-concave in $\mathbf{x}$. Suppose this is not true. Then $0 < \alpha_k < 1$ and the contract curve between the two agents is tangent to one of principal k's indifference curves at $\mathbf{x}^{NB}(\beta_k(s_l), s_l)$. By playing $\theta_k' = \theta_k - \varepsilon$ for some small $\varepsilon > 0$ instead of θ_k she can marginally shift the contract curve upward so that it cuts through the old indifference curve and becomes tangent to a new indifference curve representing a higher level of utility. The new tangency point $\mathbf{x}'$ is feasible since ε is arbitrary small, i.e., there exists an $\alpha_k' < 1$ such that $\mathbf{x}^{NB}(\alpha_k', \theta_k', s_l) = \mathbf{x}'$. This argument is illustrated in Figure 4.1 and it applies to every $\alpha_k \in (0,1)$. Hence, $\alpha_k = 1$. Finally, if $\alpha_k = 1$ then π_k is strictly concave in θ_k which implies that the utility maximizing strategy s_k is unique.

Lemma 3. *Let $s_l \in S_l$, $l \neq k$. Then $\beta_k(s_l)$ is a singleton set and if $\beta_k(s_l) = \{(\alpha_k, \theta_k)\}$ then $\alpha_k = 1$ and $\theta_k > \theta_k^P$.*

Proof. See the Appendix.

By using Lemma 3 we can show the existence of a Nash equilibrium to the game D. Principal k's best-reply correspondence, β_k, is a continuous function since the best reply always is unique by Lemma 3 and the payoff function π_k is

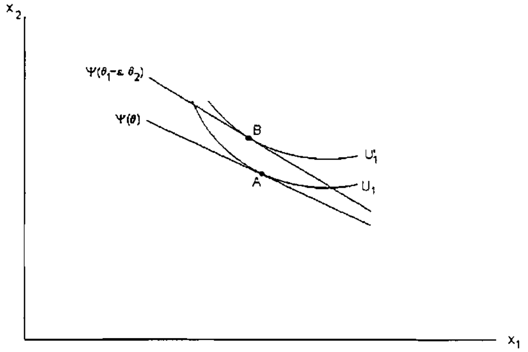

Figure 4.1: Principal 1's indifference curve U_1 tangent to the agents' contract curve $\Psi(\boldsymbol{\theta})$ at point A and agent 1 is given intermediate power, $0 < \alpha_1 < 1$. By delegating to an agent who is slightly more interested in the public good principal 1 shifts the agents' contract curve upwards. For a small enough change there exists a new intermediate level of power such that the point B is reached. At B, the new higher contract curve is tangent to an indifference curve representing a higher level of utility than U_1.

continuous in $\mathbf{s}$. Hence, the combined best-reply function $\beta = \beta_1 \times \beta_2$ has a fixed point $\mathbf{s}^{NE} = (s_1^{NE}, s_2^{NE}) \in S$ which is a Nash equilibrium to the game D. Let $NE(D)$ denote the set of Nash equilibria to the game D.

Proposition 1. $NE(D) \neq \emptyset$.

Proof. See the Appendix.

We now turn our attention to the qualitative properties of $NE(D)$. Any Nash equilibrium to the game D must, by Lemma 3, be constituted by a strategy profile such that both principals delegate to agents to their right and give them all power.[4]

Proposition 2. *Let* $\mathbf{s}^{NE} \in NE(D)$ *where* $s_k^{NE} = (\alpha_k^{NE}, \theta_k^{NE})$. *Then* $\alpha_k^{NE} = 1$ *and* $\theta_k^{NE} > \theta_k^P$ *for* $k = 1, 2$.

Proof. See the Appendix.

[4]The strong delegation game in Segendorff (1998) is the mixed extension of D where $A_k = \{1\}$ and $\Theta_k = [0, 1]$. Propositions 1 and 2 above can be extended to the strong delegation game, i.e., there exists a pure-strategy Nash equilibrium with the properties stated in Proposition 2 to the strong delegation game.

In Lemma 4 we have reformulated Lemma 1 from Segendorff (1998). It says that if principal k gives all power to her agent then country k's (country l's) provision of the public good decreases (increases) with the agent's taste for the private good. This is due to the disagreement point monotonicity and the Pareto efficiency of the Nash bargaining solution.

Lemma 4. *Let* $\alpha_k = 1$, *then* $\frac{dx_k^{NB}}{d\theta_k} < 0$ *and* $\frac{dx_l^{NB}}{d\theta_k} > 0$, $l \neq k$.

Proof. See the proof of Lemma 1 in Segendorff (1998).

The equilibrium agreement $\mathbf{x}^{NE}(\mathbf{s}^{NE})$ and the principals' welfare in equilibrium can be studied by applying Lemma 4 to the first-order conditions of principal k's maximization problem. Suppose principal k does not delegate to her right most agent in equilibrium, i.e., $\theta_k^{NE} < 1$. Then Lemma 4 tells us that principal k's marginal utility of her own provision of public good is positive, $dv_k/dx_k > 0$, which implies that the share of her country's resources that is allocated to production of the public good in equilibrium is smaller the principal's ideal allocation, $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$. This necessarily means that principal $l \neq k$ is worse of in equilibrium than in autarchy because even if she implements her ideal allocation, this can not compensate for the reduction of country k's provision of public good. Consequently, if both principals delegate such agents then they both provide less public good than in autarchy and they both worse off in equilibrium than in autarchy. Let $\boldsymbol{\theta}^P = (\theta_1^P, \theta_2^P)$ and let $A = A_1 \times A_2$.

Corollary 1. *Let* $\mathbf{s}^{NE} \in NE(D)$. *Then:*

(i) If $\theta_k^{NE} < 1$ *then* $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$ *and* $\pi_l(\mathbf{s}^{NE}) < v_l(\mathbf{x}^P, \theta_l^P)$ *for* $l \neq k$.

(ii) If $\theta_k^{NE} < 1$ *for* $k = 1, 2$, *then* $\pi_k(\mathbf{s}^{NE}) < v_k(\mathbf{x}^P, \theta_k^P)$.

(iii) $\pi_k(\mathbf{s}^{NE}) < v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P), \theta_k^P)$ *for some* $k = 1, 2$, $\forall \boldsymbol{\alpha} \in A$.

Proof. See the Appendix.

The third statement in Corollary 1 comes from Proposition 2 and the Pareto efficiency of the Nash bargaining solution. In the case of mutual self-representation the bargaining outcome lies on the two principals' contract curve. Lemma 2 states that $\theta_k^{NE} > \theta_k^P$ for $k = 1, 2$ and this implies that the agents' contract curve lies below the contract curve of the two principals. The equilibrium agreement, $x^{NB}(\mathbf{s}^{NE})$, can therefore not be on principals' contract curve. It follows that at least one of the principals is worse off in equilibrium compared to the case of mutual self-representation.

4.1. Separate Choices of Agents and Power

In international negotiations the power of a delegate is sometimes defined by the constitution of his country. Because of the often complex political process required to change a constitution it is natural to view the choice of constitution as a long-term choice and the choice of delegate as a short-term choice. It is also natural to think of the constitutions of the concerned countries as common knowledge when the delegates are chosen. This situation can be modelled by a three-stage delegation game where the principals simultaneously choose how much power to give to their agents in the first stage - this is their choice of constitutions. In the second stage the principals observe the chosen amounts of power and simultaneously choose agents. In the third stage, the agents meet and bargain over the two countries' resource allocations.

Let $D' = (N, S', \boldsymbol{\pi}')$ be the reduced three-stage delegation game. The set of principals is the same as in the definition of the game D but the strategy sets and the payoff functions are different. The strategy of principal k is a pair (α_k, φ_k) where φ_k is a function that to every $\boldsymbol{\alpha} \in A$ assigns a probability distribution $F_k(\cdot \mid \boldsymbol{\alpha})$ over principal k's set of agents, Θ_k. Hence, principal k's strategy set is $S'_k = A_k \times \Phi_k$ where Φ_k is the set of all functions φ_k from A to the set of all probability distributions over Θ_k. Let $S' = S'_1 \times S'_2$ and let $F(\cdot \mid \boldsymbol{\alpha})$ be the joint (product) probability distribution. The expected payoff for principal k from the strategy profile $\mathbf{s} \in S'$ is

$$\pi'_k(\mathbf{s}) = \int_{\boldsymbol{\theta}\in\Theta} v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}), \theta_k^P) dF(\boldsymbol{\theta} \mid \boldsymbol{\alpha})$$

and $\boldsymbol{\pi}' = (\pi'_1, \pi'_2)$.

A subgame-perfect equilibrium is a strategy profile $\mathbf{s}^{SPE} \in S'$ such that (i) $F_k^{SPE}(\cdot \mid \boldsymbol{\alpha})$ is a best reply against $F_l^{SPE}(\cdot \mid \boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in A$, and (ii) α_k^{SPE} is a best reply against α_l^{SPE} given $\boldsymbol{\varphi}^{SPE} = (\varphi_1^{SPE}, \varphi_2^{SPE})$, $l \neq k$. Let $SPE(D')$ denote the set of subgame-perfect Nash equilibria to the game D'. From Lemma 1 it follows that for all choices of power $\boldsymbol{\alpha}$, $F_k^{SPE}(\cdot \mid \boldsymbol{\alpha})$ assigns positive probability only to agents who are less interested in the public good than their principals.

Proposition 3. *If* $\mathbf{s}^{SPE} \in SPE(D')$ *then* $F^{SPE}(\boldsymbol{\theta}^P \mid \boldsymbol{\alpha}^{SPE}) = 0$.

Proof. See the Appendix.

In a subgame-perfect equilibrium we only observe bargaining between agents who are less interested in the public good than their principals. The difference in taste induces a bargaining outcome in which at least one of the principals provides less of the public good than in the case of mutual self-representation. This is true for all pair of agents in the support of $F^{SPE}(\cdot \mid \boldsymbol{\alpha})$, for all $\boldsymbol{\alpha} \in A$, and thus also true for the expected bargaining outcome. Hence, at least on of the principals is worse off in equilibrium than under mutual self-representation.

Corollary 2. *Let $\mathbf{s}^{SPE} \in SPE(D')$. Then:*

(i) $x_k^{NB}(\mathbf{s}^{SPE}) < x_k^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$ *for some* $k = 1, 2$, $\forall \boldsymbol{\alpha} \in A$.

(ii) $\pi'_k(\mathbf{s}^{SPE}) < v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P), \theta_k^P)$ *for some* $k = 1, 2$, $\forall \boldsymbol{\alpha} \in A$.

Proof. See the Appendix.

It is clear from Proposition 3 and Corollary 2 that separating the choices of power and agents in time can not eliminate the incentives for the principals to delegate strategically to agents to their right.

5. An Example

In the numerical example below we compute the Nash equilibrium $\mathbf{s}^{NE}$ to the two-stage delegation game for three pairs of principals and compare the welfare properties of each equilibrium with the cases of autarchy and mutual self-representation. For each of the three pairs of principals there exists an unique Nash equilibrium. Figure 5.1 is based on deriving the Nash equilibria for a large number of pairs of principals and in every case the Nash equilibrium is unique. The set of Nash equilibria to D is therefore likely to be a singleton set for every pair of principals.

Throughout the example, let $a = 0.05$. Let $\theta_1^P = 0.3$ and let $\theta_2^P = 0.2$. Then is $\mathbf{s}^{NE} = ((1, 0.69), (1, 0.58))$ and both principals are worse off than in the cases of autarchy and mutual self-representation. If we increase principal 2's taste for the private good, say $\theta_2^P = 0.7$, then $\mathbf{s}^{NE} = ((1, 0.7), (1, 1))$ and principal 1 prefers the equilibrium to autarchy but prefers mutual self-representation to the equilibrium. The reason to why principal 1 prefers the equilibrium to autarchy is that the restriction $\theta_2 \leq 1$ is binding. This means that principal 2, being relatively disinterested in the public good, can not lower her provision of the public good much compared to her provision in autarchy. The difference in country 2's provision of public good in equilibrium and under mutual self-representation is still to large to make principal 1 prefer the equilibrium to mutual self-representation. Again, Principal 2 is worse of than in autarchy.

If we increase principal 2's taste parameter further to $\theta_2^P = 0.9$, then the Nash equilibrium is the same as above, $\mathbf{s}^{NE} = ((1, 0.7), (1, 1))$. This is because the best reply of principal 1 does not depend on θ_2^P but only on θ_2. However, the change in θ_2^P changes the benchmarks and now principal 1 is better of in equilibrium than in the case of mutual self-representation. The difference in country 2's provision of public good in equilibrium and under mutual self-representation is smaller than before since θ_2^P has increased. Principal 2 is still worse off than in autarchy.

In Figure 5.1 the equilibrium utilities of all possible combinations of principals are compared with the cases of autarchy and mutual self-representation. In the example above, we move from region A to region B, and then to region E.

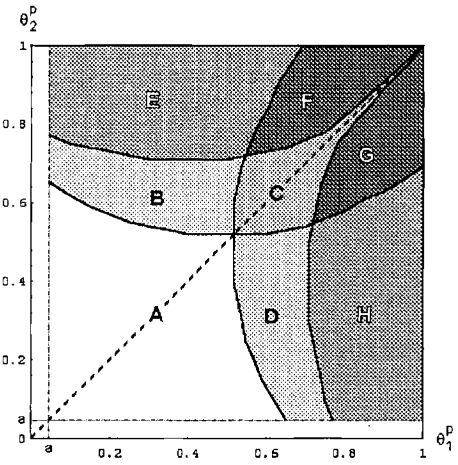

Figure 5.1: The welfare properties of the equilibrium provision of public good depends on the principals' taste. Principal 1 is prefers $\mathbf{s}^{NE}$ to mutual self-representation in regions E and F, and $\mathbf{s}^{NE}$ to autarchy in regions B, C, E, F, and G. Principal 2 analogously prefers $\mathbf{s}^{NE}$ to mutual self-representation in regions G and H, and $\mathbf{s}^{NE}$ to autarchy in regions C, D, F, G, and H. Both principals worse off in equilibrium than in autarchy in region A.

6. Summary and Comments

Any Nash equilibrium to the studied two-stage delegation game is such that each principal delegates the bargaining to an agent to her right and gives this agent all power. Each principal does so in order to reach a more favorable agreement than if she had bargained herself. However, since both principals reason in the same way and delegate as described above, they may end up in an equilibrium in which they both are worse off than under autarchy. In equilibrium, at least one of them is worse of in equilibrium compared to mutual self-representation.

In equilibrium, each principal correctly anticipates the strategy played by her opponent and plays a best reply against it. The principals therefore realize the bad nature of the equilibrium and they would benefit from coordinating on some institutional set-up in which the agents are given no or little power or/and in which mutual self-representations is played in equilibrium. One natural and important question is if such institutional set-ups can be achieved as the equilibrium outcome of some (political) delegation game and if so, what set of rules characterizes that game? Finally, we show that strategic delegation to the right is a part of every subgame-perfect Nash equilibrium of the three-stage delegation game. This suggests that the incentive to delegate strategically to the right is a robust result.

The results arrived to in this study partly depend on the chosen utility function. In order to determine the qualitative properties of the set of Nash equilibria to the delegation game one must have clear-cut results from the comparative statics carried out on the Nash bargaining solution. With a general utility function, this is not possible without imposing several restrictions on the form of the utility function. The explicit utility function used in this study was chosen for the reasons of simplicity and clearness. It keeps the model fairly simple and still provides some important insights. Even though a more general utility function is desirable we argue that this should not be considered a major drawback.

Appendix: Proofs

Lemma 1. $\xi_k(\theta_l, \boldsymbol{\alpha}) = \arg\max_{\theta_k \in \Theta_k} v_k\left(x^{NB}\left(\boldsymbol{\theta}, \boldsymbol{\alpha}\right), \theta_k^P\right)$ and the derivative of θ_k^P's maximization problem w.r.t. θ_k is

$$\frac{\partial \pi_k}{\partial \theta_k} = \frac{\partial v_k^P}{\partial x_k}\frac{dx_k^{NB}}{d\theta_k} + \frac{dx_l^{NB}}{d\theta_k} \tag{6.1}$$

where v_k^P indicates the principal's utility function. Lemma 1 is proved by showing that Equation 6.1 is strictly positive for all $\theta_k \leq \theta_k^P$ and the proof is carried out in two steps. First we derive the expressions for $\frac{dx_k^{NB}}{d\theta_k}$ and $\frac{dx_l^{NB}}{d\theta_k}$ where $l \neq k$ and then we use the derived expressions to determine the sign of Equation 6.1.

Step 1: Let

$$B_k(\boldsymbol{\theta}, \boldsymbol{\alpha}) = \left(v_k\left(\mathbf{x},\theta_k\right) - v_k\left(\mathbf{b}(\boldsymbol{\theta}, \boldsymbol{\alpha}), \theta_k\right)\right) \tag{6.2}$$

and the first-order conditions to the Nash bargaining solution are

$$\frac{\partial NB}{\partial x_k} = \frac{\partial v_k}{\partial x_k} B_l(\boldsymbol{\theta}, \boldsymbol{\alpha}) + B_k(\boldsymbol{\theta}, \boldsymbol{\alpha}) = 0. \tag{6.3}$$

The system 6.3 defines the unique bargaining outcome $\mathbf{x}^{NB}\left(\boldsymbol{\theta}, \boldsymbol{\alpha}\right)$. The outcome $\mathbf{x}^{NB}\left(\boldsymbol{\theta}, \boldsymbol{\alpha}\right)$ is, by the Implicit function theorem, locally continuous in $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$. In the following we consider principal 1's problem, i.e., $k = 1$. Differentiation of the system 6.3 w.r.t. x_1, x_2, and θ_1 and rearranging gives

$$\begin{bmatrix} \frac{\partial^2 NB}{\partial x_1^2} & \frac{\partial^2 NB}{\partial x_2 \partial x_1} \\ \frac{\partial^2 NB}{\partial x_1 \partial x_2} & \frac{\partial^2 NB}{\partial x_2^2} \end{bmatrix} \begin{bmatrix} \frac{dx_1^{NB}}{d\theta_1} \\ \frac{dx_2^{NB}}{d\theta_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 NB}{\partial \theta_1 \partial x_1} \\ -\frac{\partial^2 NB}{\partial \theta_1 \partial x_2} \end{bmatrix}.$$

The determinant of the Hessian to NB, denoted $\mathbb{H}$, is positive since NB is a concave function evaluated at its maximum. Cramer's rule gives

$$\frac{dx_1^{NB}}{d\theta_1} = -\frac{\begin{pmatrix} \frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}B_1(\boldsymbol{\theta},\boldsymbol{\alpha})B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) + 2\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial v_2}{\partial x_2}B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) \\ -\frac{\partial v_1}{\partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial b_1}{\partial\theta_1} + \frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial B_1}{\partial\theta_1}B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) \end{pmatrix}}{\det(\mathbb{H})} \tag{6.4}$$

and

$$\frac{dx_2^{NB}}{d\theta_1} = \frac{2\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) + \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial b_1}{\partial\theta_1} - \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial v_2}{\partial x_2}\frac{\partial B_1}{\partial\theta_1}B_2(\boldsymbol{\theta},\boldsymbol{\alpha})}{\det(\mathbb{H})} \tag{6.5}$$

Step 2: Let $\boldsymbol{\alpha} \in A = A_1 \times A_2$, $\theta_2 \in \Theta_2$ and $\theta_1 \leq \theta_1^P$. Then is $\partial B_1/\partial\theta_1 < 0$. Substituting Equations 6.4 and 6.5 into Equation 6.1 and gives

$$\begin{aligned} \frac{\partial\pi_1}{\partial\theta_1} &= -\frac{\frac{\partial v_1^P}{\partial x_1}}{\det(\mathbb{H})}\begin{pmatrix} \frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}B_1(\boldsymbol{\theta},\boldsymbol{\alpha})B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) + 2\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial v_2}{\partial x_2}B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) \\ -\frac{\partial v_1}{\partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial b_1}{\partial\theta_1} + \frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial B_1}{\partial\theta_1}B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) \end{pmatrix} \\ &\quad +\frac{1}{\det(\mathbb{H})}\left(\frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial b_1}{\partial\theta_1} - \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial v_2}{\partial x_2}\frac{\partial B_1}{\partial\theta_1}B_2(\boldsymbol{\theta},\boldsymbol{\alpha})\right) + \frac{2B_2(\boldsymbol{\theta},\boldsymbol{\alpha})}{\det(\mathbb{H})}\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}. \end{aligned} \tag{6.6}$$

The sign of Equation 6.6 is ambiguous because the last term is negative while the first two terms are positive. Rewriting Equation 6.6 slightly gives

$$\begin{aligned} \frac{\partial\pi_1}{\partial\theta_1} &= -\frac{\frac{\partial v_1^P}{\partial x_1}}{\det(\mathbb{H})}\begin{pmatrix} \frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}B_1(\boldsymbol{\theta},\boldsymbol{\alpha})B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) \\ -\frac{\partial v_1}{\partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial b_1}{\partial\theta_1} + \frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial B_1}{\partial\theta_1}B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) \end{pmatrix} \\ &\quad +\frac{1}{\det(\mathbb{H})}\left(\frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial b_1}{\partial\theta_1} - \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial v_2}{\partial x_2}\frac{\partial B_1}{\partial\theta_1}B_2(\boldsymbol{\theta},\boldsymbol{\alpha})\right) \\ &\quad +\frac{2B_2(\boldsymbol{\theta},\boldsymbol{\alpha})}{\det(\mathbb{H})}\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\left(-\frac{\partial v_1^P}{\partial x_1}\frac{\partial v_2}{\partial x_2} + 1\right). \end{aligned} \tag{6.7}$$

Equation 6.7 is positive if the last term is positive, i.e., if

$$-\frac{\partial v_1^P}{\partial x_1}\frac{\partial v_2}{\partial x_2} + 1 < 0. \tag{6.8}$$

From the system of first-order conditions (system 6.3) we have $\frac{\partial v_1}{\partial x_1}\frac{\partial v_2}{\partial x_2} = 1$. Because $\theta_1 < \theta_1^P$ by assumption and $x_1^{NB}(\boldsymbol{\theta},\boldsymbol{\alpha}) > x^*(\theta_1) > b_1(\theta_1,\alpha_1)$ we have $\frac{\partial v_1^P}{\partial x_1}\frac{\partial v_2}{\partial x_2} > 1$. Hence, $\frac{\partial\pi_1}{\partial\theta_1} > 0\ \forall\theta_1 \leq \theta_1^P, \forall\boldsymbol{\alpha} \in A$.

It follows that if $\theta_1 \leq \theta_1^P$ then $\theta_1 \notin \xi_1(\theta_2,\boldsymbol{\alpha})$ and it follows from the continuity of v_1 that this is true also for $\theta_1 = a$. Hence $\xi_1(\theta_2,\boldsymbol{\alpha}) \subseteq \left(\theta_1^P, 1\right]$. By analogy is $\xi_2(\theta_1,\boldsymbol{\alpha}) \subseteq \left(\theta_2^P, 1\right]$. ■

Lemma 2. Lemma 2 is proved by showing by first deriving the expressions $\frac{dx_k^{NB}}{d\alpha_k}$ and $\frac{dx_l^{NB}}{d\alpha_k}$, $l \neq k$. Thereafter we determine their signs using that $\theta_k > \theta_k^P$.

Step 1: In the following we consider country 1, i.e., $k = 1$. Differentiating the system of first-order conditions to the Nash bargaining solution (system 6.3) w.r.t. x_1, x_2, and α_1 gives

$$\begin{bmatrix} \frac{\partial^2 NB}{\partial x_1^2} & \frac{\partial^2 NB}{\partial x_2 \partial x_1} \\ \frac{\partial^2 NB}{\partial x_1 \partial x_2} & \frac{\partial^2 NB}{\partial x_2^2} \end{bmatrix} \begin{bmatrix} \frac{dx_1^{NB}}{d\alpha_1} \\ \frac{dx_2^{NB}}{d\alpha_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 NB}{\partial \alpha_1 \partial x_1} \\ -\frac{\partial^2 NB}{\partial \alpha_1 \partial x_2} \end{bmatrix}.$$

Using Cramer's rule and simplifying gives

$$\frac{dx_1^{NB}}{d\alpha_1} = \frac{-(\theta_1 - \theta_1^P)\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_1}{\partial x_1}\right)\frac{\partial^2 v_2}{\partial x_2^2} B_1(\boldsymbol{\theta}, \boldsymbol{\alpha})}{\det(\mathbb{H})}. \tag{6.9}$$

and

$$\frac{dx_2^{NB}}{d\alpha_1} = \frac{-(\theta_1 - \theta_1^P)\frac{\partial^2 v_1}{\partial x_1^2}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_1}{\partial x_1}\right)\frac{\partial v_2}{\partial x_2} B_2(\boldsymbol{\theta}, \boldsymbol{\alpha})}{\det(\mathbb{H})}. \tag{6.10}$$

Step 2: If $\theta_1 > \theta_1^P$ then (i) $(\theta_1 - \theta_1^P) > 0$ and (ii) $\frac{\partial v_1}{\partial x_1} \leq 0$ because $b_1(\theta_1, \alpha_1) \geq x^*(\theta_1)$. $\frac{\partial^2 v_2}{\partial x_2^2} < 0$ since $\theta_2 > 0$. Hence, $\frac{dx_1^{NB}}{d\alpha_1} < 0$ and $\frac{dx_2^{NB}}{d\alpha_1} > 0$. ■

Lemma 3. This proof is carried out in three steps. First we show that $\alpha_k = 1$ in all best replies. Thereafter this is used to show β_k is a singleton set.

Step 1: First it is shown that $\alpha_k > 0$ and thereafter it is shown that $\alpha_k = 1$. The first-order condition of $\pi_k(\mathbf{s})$ w.r.t. α_k is:

$$\frac{\partial \pi_k}{\partial \alpha_k} = \frac{\partial v_k^P}{\partial x_k}\frac{dx_k^{NB}}{d\alpha_k} + \frac{dx_l^{NB}}{d\alpha_k} = 0 \tag{6.11}$$

for interior solutions where v_k^P denotes principal k's utility function. Now, let $\alpha_k = 0$. Since $\theta_k \in \xi_k(0, s_l)$ we have $\mathbf{x}^{NB}((0,\theta_k), s_l) \succ_{\theta_k^P} \mathbf{x}^{NB}((0,\theta_k^P), s_l)$. Assume that $\mathbf{x}^{NB}((0,\theta_k), s_l) < \mathbf{b}((0,\theta_k), s_l)$, then $\mathbf{b}((0,\theta_k), s_l) \succ_{\theta_k^P} \mathbf{x}^{NB}((0,\theta_k), s_l)$ which contradicts $\theta_k \in \xi_k(0, s_l)$. Hence, $\mathbf{x}^{NB}((0,\theta_k), s_l) > \mathbf{b}((0,\theta_k), s_l)$. Then, by Lemma 2, can principal k increase her utility by increasing α_k which decreases x_k^{NB} and increases x_l^{NB}, $l \neq k$. Hence, $(0, \theta_k) \notin \beta_k(s_l)$ for any $\theta_k > \theta_k^p$.

The set of Pareto efficient allocations for θ_1 and θ_2 is

$$\Psi(\boldsymbol{\theta}) = \left\{\mathbf{x} \in [0,1]^2 \mid x_2 = (1-\theta_2) + \frac{\theta_2}{\theta_1}(1 - x_1)\right\}$$

and an indifference curve for principal k is

$$U_k(\mathbf{s}) = \left\{\mathbf{x} \in [0,1]^2 \mid v_k\left(\mathbf{x}, \theta_k^P\right) = \pi_k(\mathbf{s})\right\}.$$

Let $s_l \in S_l$ and let $s_k \in \beta_k(s_l)$. From Lemma 1 we know $\theta_k > \theta_k^P$ and from Step 1 above we know $\alpha_k > 0$. Suppose that $\alpha_k < 1$, then $U_k(\mathbf{s})$ is tangent to $\Psi(\boldsymbol{\theta})$ at $\mathbf{x}^{NB}(\mathbf{s})$. Let $\theta_k' = \theta_k - \varepsilon$ for some small $\varepsilon > 0$. Then $\Psi(\theta', \theta_l)$ intercepts $U_k(\mathbf{s})$ twice since v_k is quasi-concave in $\mathbf{x}$. Due to the continuity of principal k's first-order condition with respect to α_k (Equation 6.11) we can find some $\varepsilon > 0$ and some $\alpha_k' \in (0,1)$ such that $\Psi(\theta', \theta_l)$ is tangent to $U_k(\alpha_k', \theta_k', s_l)$ where $\pi_k(\alpha_k', \theta_k', s_l) > \pi_k(\mathbf{s})$. Hence, by contradiction we showed that if $(\alpha_k, \theta_k) \in \beta_k(s_l)$ then $\alpha_k = 1$.

Step 2: Let

$$graph(s_l) = \left\{\mathbf{x}^{NB}(\mathbf{s}) \in [0,1]^2 \mid s_k = (1, \theta_k),\ 0 \le \theta_k \le 1\right\}.$$

In this part of the proof we show that for any two points on $graph(s_l)$ the part of the graph between these two points lies above the line segment joining the two points. From this we show that $\beta_k(s_l)$ is singleton. Throughout the proof is $l \neq k$.

Let the strategy profile $s = (\boldsymbol{\alpha}, \boldsymbol{\theta})$ describe a pair of agents and a pair of constitutions where $\alpha_k = 1$. Let $\varepsilon > 0$ be arbitrary small and let $\mathbf{s}' = (\boldsymbol{\alpha}, (\theta_k + \varepsilon, \theta_l))$ and $\mathbf{s}'' = (\boldsymbol{\alpha}, (\theta_k + 2\varepsilon, \theta_l))$. We then have three corresponding agreements; $\mathbf{x} = \mathbf{x}^{NB}(\mathbf{s})$, $\mathbf{x}' = \mathbf{x}^{NB}(\mathbf{s}')$, and $\mathbf{x}'' = x^{NB}(\mathbf{s}'')$. From the Pareto efficiency of the Nash bargaining solution it follows that $\mathbf{x}' \in \Psi(\theta_k + \varepsilon, \theta_l)$ and that $\mathbf{x}'' \in \Psi(\theta_k + 2\varepsilon, \theta_l)$. Let $dx_k' = x_k' - x_k = \frac{\partial x_k^{NB}}{\partial \theta_k}\varepsilon < 0$ and let $dx_k'' = x_k'' - x_k = 2\frac{\partial x_k^{NB}}{\partial \theta_k}\varepsilon < 0$. Divide dx_k' into two parts $a', b' < 0$, i.e. $dx_k' = a' + b'$, where b' is such that $dx_l' = -\frac{\theta_l}{\theta_k + \varepsilon} b > 0$ and a' is such that $(1 - \theta_l) + \frac{\theta_l}{\theta_k + \varepsilon}(1 - x_k - a') = x_l$, i.e., a' is such that $(x_k + a', x_l) \in \Psi(\theta_k + \varepsilon, \theta_l)$ and b' is such that $(x_k + a' + b', x_l') \in \Psi(\theta_k + \varepsilon, \theta_l)$. Solving the latter expression for a' gives $a' = \frac{\varepsilon}{\theta_k}(1 - x_k)$ when using that $x_l = (1 - \theta_l) - \frac{\theta_l}{\theta_k}(1 - x_k)$. Analogously, letting $dx_k'' = a'' + b''$ where $a'', b'' < 0$ and where $dx_l'' = -\frac{\theta_l}{\theta_k + 2\varepsilon} b'' > 0$ and $(1 - \theta_l) + \frac{\theta_l}{\theta_k + 2\varepsilon}(1 - x_k - a'') = x_l$ gives $a'' = \frac{2\varepsilon}{\theta_k}(1 - x_k)$. Thus, $a'' = 2a'$ and since $dx_k'' = 2dx_k'$ we have $b'' = 2b'$. This implies that $dx_l'' < 2dx_l'$ and hence $graph(s_l)$ describes x_l locally as a concave function of x_k. The same reasoning applies for all $\mathbf{x} \in graph(s_l)$ and π_k is locally a concave function of θ_k around $\mathbf{x}^{NB}(\mathbf{s})$ when holding s_2 fixed and $\alpha_k = 1$.

It follows that $\beta_k(s_l)$ is a singleton set. Suppose that this is not true and that $s_k = (1, \theta_k)$ and $s_k' = (1, \theta_k')$ both belong to $\beta_k(s_l)$. Let $s_k'' = (1, (\theta_k + \theta_k')/2)$. Then $x_k^{NB}(s_k'', s_l) = \lambda x_k^{NB}(\mathbf{s}) + (1 - \lambda) x_k^{NB}(s_k', s_l)$ for some $0 < \lambda < 1$ and $x_l^{NB}(s_k'', s_l) > \lambda x_l^{NB}(\mathbf{s}) + (1 - \lambda) x_l^{NB}(s_k', s_l)$. Since $\lambda \mathbf{x}^{NB}(\mathbf{s}) + (1 - \lambda)\mathbf{x}^{NB}(s_k', s_l) \succeq_{\theta_k^P} \mathbf{x}^{NB}(\mathbf{s}),\ \mathbf{x}^{NB}(s_k', s_l)$ we have that $\mathbf{x}^{NB}(s_k'', s_l) \succeq_{\theta_k^P} \mathbf{x}^{NB}(\mathbf{s}),\ \mathbf{x}^{NB}(s_k', s_l)$ which is a contradiction. Hence, $\beta_k(s_l)$ is a singleton set. ■

Proposition 1. Let the correspondence $\gamma_k : S_l \twoheadrightarrow S_k$ be defined by $\gamma_k(s_l) = S_k$ for all $s_l \in S_l$ and notice that let $\beta_k(s_l) = \arg\max_{s_k \in \gamma(s_l)} \pi_k(\mathbf{s})$. Lemma 3 states that $\beta_k(s_l)$ is singleton and hence closed at s_l. γ_k is a continuous correspondence and $\gamma_k(s_l)$ is compact. From the properties of γ_k and β_k it follows that β_k is continuous at s_l (Border (1985), pp. 59, Proposition 11.21(b)) and hence that β_k is continuous function. Define the function $\beta : S \to S$ by $\beta(\mathbf{s}) = \beta_1(s_2) \times \beta_2(s_1)$. From above we have that S is convex and compact and that β is a continuous function. By Brouwer's fixed point theorem β has a fixed point (Border (1985), pp. 29, Corollary 6.6). Hence, the game D has a Nash equilibrium $\mathbf{s}^{NE} = (s_1^{NE}, s_2^{NE}) \in NE(D) \neq \emptyset$. ■

Proposition 2. Follows directly from Lemma 3. ■

Corollary 1. (i) Let v_k^P denote the principals utility function. If $\theta_k^{NE} < 1$ then $\frac{\partial v_k^P}{\partial x_k}\frac{dx_k^{NB}}{d\theta_k} + \frac{dx_l^{NB}}{d\theta_k} = 0$ which by Lemma 4 implies $\frac{\partial v_k^P}{\partial x_k} < 0$. Hence, $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$. If $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$ then $\pi_l(\mathbf{s}^{NE}) \leq v_l(x_k^{NB}(\mathbf{s}^{NE}), x_l^P, \theta_l^P) < v_l(\mathbf{x}^P, \theta_l^P)$ by definition of x_l^P.

(ii) If $\theta_k^{NE} < 1$ for $k = 1, 2$, then $\mathbf{x}^{NB}(\mathbf{s}^{NE}) < \mathbf{x}^P$ by (i) above. From (i) it also follows that $\mathbf{x}^P \succ_{\theta_k^P} \mathbf{x}^{NB}(\mathbf{s}^{NE})$.

(iii) Let $s_k^P = (\alpha_k, \theta_k^P)$ and let $\mathbf{s}^P = (s_1^P, s_2^P)$. By the Pareto efficiency of the Nash bargaining solution we have that $\mathbf{x}^{NB}(\mathbf{s}^P) \in \Psi(\boldsymbol{\theta}^P)$ where $\boldsymbol{\theta}^P = (\theta_1^P, \theta_2^P)$ and $\Psi(\boldsymbol{\theta})$ is defined as in the proof of Lemma 3. We also have that $\mathbf{x}^{NB}(\mathbf{s}^{NE}) \in \Psi(\boldsymbol{\theta}^{NE})$. Because $\theta_k^{NE} > \theta_k^P$ for $k = 1, 2$ we have $\Psi(\boldsymbol{\theta}^P) \cap \Psi(\boldsymbol{\theta}) = \emptyset$ by the definition of Ψ in the proof of Lemma 3. Hence, $\mathbf{x}^{NB}(\mathbf{s}^P) \neq \mathbf{x}^{NB}(\mathbf{s}^{NE})$ and one of the principals are worse off. ■

Proposition 3. Suppose $\mathbf{s}^{SPE} = (\boldsymbol{\alpha}^{SPE}, \boldsymbol{\varphi}^{SPE}) \in SPE(D')$. Then $(l \neq k)$

$$\int_{\theta_l \in \Theta_l} \frac{\partial \pi'_k}{\partial \theta_k} dF_l(\theta_l \mid \boldsymbol{\alpha}^{SPE}) = 0 \tag{6.12}$$

for all $\theta_k \in (0, 1)$. By Lemma 1, the right-hand side of Equation 6.12 is strictly positive for all $\theta_k \leq \theta_k^P$. Moreover, $F_k^{SPE}(a \mid \boldsymbol{\alpha}^{SPE}) = 0$ by the continuity of π'_k. Hence, $F_k^{SPE}(\theta^P \mid \boldsymbol{\alpha}^{SPE}) = F^{SPE}(\boldsymbol{\theta}^P \mid \boldsymbol{\alpha}^{SPE}) = 0$. ■

Corollary 2. (i) Let $C^{SPE} \subseteq (\theta_1^P, 1] \times (\theta_2^P, 1]$ be the set of pairs of agents that are assigned positive probability by $\mathbf{s}^{SPE}$. The set of possible bargaining outcomes under $\mathbf{s}^{SPE}$ is

$$X^{SPE} = \left\{\mathbf{x}^{NB}(\boldsymbol{\alpha}^{SPE}, \boldsymbol{\theta}) \in [0, 1]^2 \mid \boldsymbol{\theta} \in C^{SPE}\right\}.$$

Let $con(X^{SPE})$ be the convex hull of X^{SPE}. We have that $\mathbf{x}^{NB}(\mathbf{s}^{SPE}) \in con(X^{SPE})$. By the definition of Ψ in the proof of Lemma 3 and by Proposition 3 is $\Psi\left(\boldsymbol{\theta}^P\right) \cap$

$con(X^{SPE}) = \emptyset$. Hence, $\mathbf{x}^{NB}(\mathbf{s}^{SPE}) \neq \mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$. Moreover, $x_k^{NB}(\mathbf{s}^{SPE}) < x_k^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$ for some $k = 1, 2$ since $con(\mathbf{x}^{SPE})$ lies below $\Psi\left(\boldsymbol{\theta}^P\right)$.

(*ii*) The vN-M utility function v_k is concave in $\mathbf{x}$. Then, from $\mathbf{x}^{NB}(\mathbf{s}^{SPE}) \neq \mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$ it follows that $v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P), \theta_k^P) > v_k(\mathbf{x}^{NB}(\mathbf{s}^{SPE}), \theta_k^P) > \pi_k'(\mathbf{s}^{SPE})$. ■

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Labor- and Product-Market Structure and Excess Labor

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Abstract

This study analyzes under what labor- and product-market structures a firm may hire more labor than needed to produce its profit maximizing output. Three labor-market structures are studied: (1) *decentralized* (firm-specific unions), (2) *one-sided centralization* (central union and several firms), and (3) *centralized* (central union and employers' association). Excess labor is explained by the risk-sharing motive that in the model exists between the risk-averse workers and the risk-neutral firm owner. Labor may be excessively hired in any of the labor-market structures and under a wide range of product-market structures; duopoly, oligopoly etc. *Journal of Economic Literature* Classification Numbers: J21, J51, and L11.

1. Introduction

This paper explains why a firm may hire more labor than it needs to produce its profit maximizing quantity. It also analyzes under what labor- and product-market structures excess labor is hired. The analysis is carried out within the framework of a standard wage-bargaining model. In the model, the product-market demand is known but the size of the union will typically be larger than

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the number of available jobs. The only uncertainty is therefore faced by the individual worker who does not know in advance if he will be employed or not. It turns out that hiring of excess labor can be realized as a consequence of the risk-sharing motive that exists between the risk-averse workers and the risk-neutral firm owner (see Parsons (1986) for a more detailed discussion). If the quantity produced by a firm is less than the firm's capacity at full employment, some workers will be unemployed if the production is technically efficient. Then, if the access to public income insurance is limited, any Pareto efficient agreement over wages and employment between the firm and the union prescribes higher employment than the firm's labor-demand curve does at the same wage. This is to be viewed as additional protection for the workers compared to a situation where the firm unilaterally determines the employment since the probability of being employed is higher. The size of this additional employment and its use in production depends on the structures of the labor and product markets. It is these relationships that are investigated in this study.

The timing in the model is the following: first there is simultaneous bargaining over *labor contracts* (wages and employment) between one or several firms and firm-specific unions or the corresponding central organizations. Prior to the bargaining all firms and unions share a common belief over the future agreements. This stage will be called the *labor-market stage.* Thereafter the firms compete in a Cournot market for their products. They simultaneously make their production decisions without being informed about the labor contracts of the other firms and subject to the restrictions set out by their labor contracts and the belief discussed above. This stage will be called the *product-market stage.* Three different labor-market structures are studied; decentralized bargaining, one-sided centralized bargaining, and centralized bargaining.

Throughout the study, firms are assumed to be identical and workers to have no mobility between firms, to have identical well-behaved preferences, and to be divided into equally large firm-specific unions. The central- or firm-specific union maximizes the joint utility of its members, or alternatively, the expected utility of a representative member. The firm or employers' association maximizes profits or joint profits, respectively. The bargaining is modelled by way of the Nash bargaining solution and it is assumed to be over both wages and employment. This assumption is made because it makes the bargaining outcome Pareto efficient. We believe that it is reasonable to expect the outcome of a bargaining between two rational parties to be Pareto efficient. Moreover, it can be shown that in the context of the model studied, it is in the interest of the union to bargain over both wages and employment. For more exhaustive discussions of bargaining over wages versus bargaining over wages and employment, see Farber (1986), Flanagan *et al.* (1993), Andrews and Simmons (1995), and Nickell and Nicolitsas (1995).

Labor is assumed to be indivisible, either a worker is employed and supplies one unit of labor or he is unemployed. If the labor contract specifies a labor force smaller than the size of the union, then every worker has equal probability of being employed. Unemployed workers receive an unemployment benefit paid by the government.[1]

After the labor-market stage the firms enter the product-market stage where they compete in quantities in a market with a known inverse demand function. Every labor contract is assumed to be private information to the firm and union in question (this assumption is discussed further in Section 6). The production technology is a constant returns-to-scale technology using only labor as input.

The individual firm is assumed to have *free disposal* of its contracted labor force. The firm's expectation over its competitors' output decisions is contingent on its belief over their labor contracts. If every firm and union anticipates the labor contracts correctly, i.e., if expectations are rational, every labor contract will be a "best reply" to all the other contracts and the produced quantities will constitute an equilibrium in the product market. In the following we will call a situation where both markets are in equilibrium a *labor-market equilibrium* or just an *equilibrium*. If the hired labor force of a firm is greater than needed to produce the profit maximizing quantity, the firm will not produce on the boundary of its production-possibility set and some or all workers will be told to "shirk."

In the decentralized setting, every firm-specific union negotiates with "its" firm and the unique equilibrium vector of labor contracts is symmetric, i.e., it specifies the same wage and employment in every firm. Excess labor may be hired under any consumer-market structure depending on the degree of workers' risk aversion and the size of the unemployment benefit. The higher the degree of risk aversion and the lower the unemployment benefit, the higher is the largest number of firms that is consistent with excess employment in equilibrium.

The second case is one-sided centralization. Here, the firm-specific unions join together into one central union that negotiates separately and simultaneously with every firm. The equilibrium vector of labor contracts turns out to be the same as in the decentralized case. This is because the centralized union in every bargaining has the same objective as the firm-specific union which in turn is due to the assumption of the labor contracts being private information. The central union can treat every bargaining as a separate problem since the firms' output

[1] In the model, if a labor contract specifies less than full employment then it is not in the interest of the union to share the available jobs equally among its members, i.e., to create part-time jobs. This is because the members would then lose the unemployment benefit paid by the government, and the total amount of money received would be smaller. The elimination of the risk of being unemployed can not compensate for this loss. Moreover, in the model this is true for any income-insurance system financed by the government and in which the benefit is a non-decreasing function of the wage.

decisions are contingent on their belief and not on the actual labor contracts. Hence the identical objectives.

The last labor-market structure to be analyzed is the fully centralized structure. Here, the firm owners join together into an employers' association that bargains with the central union over a central labor contract specifying the same wage and employment in all firms. The employers' association maximizes joint profit conditional on that its members cannot collude in the product market but will compete as an one-shot oligopoly. It turns out that the central labor contract may not change continuously with the degree of risk aversion, unemployment benefit, and number of firms. This discontinuity renders difficulties in the analysis but some comparisons to the decentralized case are made. If the parameters (workers' risk aversion, number of firms etc.) are such that no excess labor is hired in equilibrium in neither the decentralized setting nor in the centralized setting, then wages are higher and employment is lower in the centralized setting than in the decentralized setting. The reason is that the employers' association internalizes the effect of increased employment on joint profits while the individual firm does not. Therefore, the employers' association experiences a stronger marginal effect of employment on profits than the individual firm and it is consequently more reluctant than the firm to insure workers against income uncertainty by means of employment. On the other hand, if labor is excessively hired, output is unaffected by an increase in employment and the marginal effect of employment on profits is the same for the firm and the employers' association. Hence, if the parameters are such that excess labor is hired in equilibrium in both the decentralized and centralized settings, then wages and employment are the same.

In the literature on implicit contracts, labor hoarding is explained by the risk-sharing motive discussed earlier. Employment and wage stability offered in implicit contracts are two ways for the firm to provide some income insurance to their workers. This explanation is first posed in some early works by Baily (1974), Gordon (1974), and Azariadis (1975). More recent studies consider more complex informational structures than the early works and some allow for risk-averse firm owners, see Hart (1983). Kahn and Reagan (1993) introduce working rules and they find that this may cause labor hoarding and inefficient use of the hired labor force. This finding has some empirical support. Especially, increases in the level of competition in the product market tend to increase productivity via less restrictive working rules and lower wages, see Nickell *et al.* (1994) and Nickell and Nicolitsas (1995).

The study presented below is related to the implicit-contract literature but there are important differences. In the implicit-contract literature firms often design the contracts unilaterally. The models are usually dynamic and the main source of uncertainty is typically a stochastic product-market demand. The study presented here is different. It focuses on employment and firms' use of hired labor. Labor contracts are negotiated and an individual firm may have product-market power. In particular, it connects employment and the use of labor in production with the structures of the labor and product markets. Finally, the source of uncertainty is not a stochastic product-market demand but the fact that each labor contract is negotiated before the jobs are distributed and thus before the individual worker knows whether he will be employed or not.

Finally, Dixit (1980) studies entry deterrence and investment in capacity. One main result is that excess capacity is never installed when capacity can be bought in an unlimited amount at a fixed price. In the model presented below, one may interpret employment as capacity. The reason why excess labor may be hired is that labor is not available in an unlimited quantity at some fixed wage. Instead, wage and employment are bargained over and consequently Dixit's result does not carry over.

The product-market stage and the decentralized labor market are presented in Section 2. Sections 3 and 4 contain the cases of one-sided centralized bargaining and centralized bargaining, respectively. The numerical example is given in Section 5 and Section 6 contains the discussion. All proofs are given in the Appendix.

2. The Model

The model consists of two stages, the labor-market stage and the product-market stage. Firms and workers are rational and thus forward looking at the labor-market stage. For this reason the properties of the product market will be essential when modelling the bargaining. The product market is therefore presented before we proceed to the labor-market stage and the analysis.

2.1. The Product Market

Consider a product market in which there are $m \geq 1$ identical firms using only one input, labor. Let $\mathbf{n} = (n_1, ..., n_m)$ and $\mathbf{w} = (w_1, ..., w_m)$ denote the vectors of employment and wages of the firms. The wage in firm i is $w_i \geq 0$ and for reasons explained below is the employment $n_i \in [0, 1/m]$. The firms compete in quantities and they make their production decisions simultaneously. The inverse demand function is $P(\mathbf{q}) = 1 - \sum_{i=1}^{m} q_i$ where $\mathbf{q} = (q_1, ..., q_m)$ is the output vector. The firms have identical CRS technologies and every firm is assumed to have *free disposal* of its labor force. Hence, $q_i \leq n_i$. The profit of firm i is

$\pi_i(\mathbf{q}, n_i, w_i) = q_i P(\mathbf{q}) - n_i w_i$ where the cost $w_i n_i$ is fixed.[2] The unique profit-maximizing output of firm i for a given level of employment n_i is given by

$$q^o(n_i, \mathbf{q}_{-i}) = \min\left\{n_i, \frac{1 - \sum_{j \neq i} q_j}{2}\right\}$$

where $\mathbf{q}_{-i} = (q_1, ..., q_{i-1}, q_{i+1}, ..., q_m)$. Given $\mathbf{n}$, the system $q_i = q^o(n_i, \mathbf{q}_{-i})$, $i = 1, 2, ..., m$ has a unique solution. Let $\mathbf{q}^e(\mathbf{n}) = (q_1^e(\mathbf{n}), ..., q_m^e(\mathbf{n}))$ be this solution.

The difference between this simple product-market model and a standard Cournot model is the firms' capacity constraints. Labor is here viewed as irrevocably hired before the making of the production decision and this induces a (capacity) constraint on the individual firm's maximization problem, $q_i \leq n_i$. Because employment is irrevocable, the marginal cost of production can be viewed as zero up to the capacity limit and infinite for quantities above the capacity limit. In the symmetric case, if the capacity constraints are not binding then the firms produce as in the corresponding Cournot model with zero marginal cost and if the constraints are binding then the firms produce at their capacity limits. A simple example will illustrate this. Let $n_i = n$ for all i. If $n \geq 1/(m+1)$ then the product-market equilibrium is as in the standard Cournot case, $q_i^e(\mathbf{n}) = 1/(m+1)$. On the other hand, if $n < 1/(m+1)$ then $q_i^e(\mathbf{n}) = n$.

Definition 1. *Firm i hires excess labor if $n_i > q_i$.*

Thus, if the capacity constraint does not bind we say that firm i hires excess labor. In the following we also say that the product market has *structure m* if there are m firms.

2.2. The Decentralized Labor Market

In the labor market, workers are identical, they have no mobility, and they are organized in m equally large firm-specific unions. The size of the labor market is normalized to 1 and the size of every union is $1/m$. At employment $n_i \in [0, 1/m]$, every worker in union i has equal probability of being employed, mn_i. The utility of income to a worker is given by the von Neumann - Morgenstern utility function $u(w_i) = 1 - e^{-\theta w_i}$ where $\theta > 0$ determines the marginal utility of income. Moreover, θ is equal to the Arrow-Pratt measure of absolute risk aversion

[2] At the labor-market stage, only labor contracts that yield a non-negative expected profit will be entered - even out of equilibrium. Every firm that is active ($n_i > 0$) at the product-market stage will thus have some positive quantity that maximizes the (non-negative) expected profit. Hence, when searching for solutions to the firm's profit-maximization problem it is not a restrictive simplification not to consider the possibility of bankruptcy etc., i.e., to view $n_i w_i$ as fixed.

which here is constant. With probability $1 - mn_i$ the worker will be unemployed and receive an unemployment benefit $0 \leq b < 1$ paid by the government. The benefit may be interpreted as also capturing the utility from leisure in case of unemployment.

Let (n_i, w_i) be the labor contract specifying employment and wage in firm i. If the firm and the union do not reach an agreement, then every worker will be unemployed and receive the unemployment benefit. The expected utility gain for a worker in union i from the labor contract is

$$v(n_i, w_i) = mn_i(u(w_i) - u(b)).$$

In the decentralized labor market, the labor contracts are determined in m pair-wise and simultaneous negotiations where each union bargains with "its" firm. The objective of union i is to maximize the expected utility of a representative member and the objective of the firm is to maximize expected profits. Prior to the bargaining all firms and unions hold the same belief over the future vector of employment, $\mathbf{n}^e = (n_1^e, ..., n_m^e) \in [0, 1/m]^m$, and this is common knowledge. Then, it is reasonable to assume every firm and union also to believe $\mathbf{q}^e(\mathbf{n}^e)$ to be the vector of future output decisions. Every labor contract (n_i, w_i) arrived to in the wage bargaining is assumed to be private information to the firm and union in question, i.e., after the bargaining firm and union i still believe the other firms to employ $\mathbf{n}_{-i}^e = (n_1^e, ..., n_{i-1}^e, n_{i+1}^e, ..., n_m^e)$ and know them to believe that firm i has employed n_i^e workers. Hence, firm and union i expect the other firms to produce $\mathbf{q}_{-i}^e(\mathbf{n}^e)$. Given this expectation, the best firm i can do is to produce $q^o(n_i, \mathbf{q}_{-i}^e(\mathbf{n}^e)$ and its expected profit is

$$\pi_i^e(\mathbf{n}^e, n_i, w_i) = \pi_i(\mathbf{q}_{-i}^e(\mathbf{n}^e), q^o(n_i, \mathbf{q}_{-i}^e(\mathbf{n}^e)), n_i, w_i).$$

The set of feasible labor contracts between firm and union i is

$$B_i^D(\mathbf{n}^e) = \{(n_i, w_i) \in [0, 1/m] \times [0, 1] \mid v(n_i, w_i), \pi_i^e(\mathbf{n}^e, n_i, w_i) \geq 0\}.$$

The Nash bargaining solution is

$$(n_i^D, w_i^D) = \arg \max_{(n_i, w_i) \in B_i^D(\mathbf{n}^e)} (v(n_i, w_i) - \underline{v})(\pi_i^e(\mathbf{n}^e, n_i, w_i) - \underline{\pi}) \tag{2.1}$$

where $\underline{v} = \underline{\pi} = 0$ because if there is no agreement then there is no utility gain to the union and the firm makes zero profit since it has no revenue and no expenses. The Nash bargaining solution implicitly defines the employment and wage as continuous functions of the parameters of the model, $\eta_i^D(b, \theta, \mathbf{n}^e)$ and $\omega_i^D(b, \theta, \mathbf{n}^e)$ respectively. The wage is decreasing in θ and increasing in b. The workers' gain from trading wage for employment increases with their risk aversion, θ. Higher employment requires lower wage by the first-order conditions of the Nash bargaining solution. Increases in b decreases the workers' gain from trading wage for employment and increases therefore the wage. Employment is non-increasing in w and therefore non-decreasing in θ and non-increasing in b. The "non"-parts of the two "employment" statements are due to the restriction $0 \leq n \leq 1/m$ which can be interpreted as the union maximizing the expected utility of its current members. Employment higher than $1/m$ lowers the wage without increasing currents members' probability of being employed and this is not in the interest of the union. A vector of labor contracts

$$(\mathbf{n}^{D*}, \mathbf{w}^{D*}) = (n_1^{D*}, ..., n_m^{D*}, w_1^{D*}, ..., w_m^{D*})$$

is a labor-market equilibrium if $(n_i^{D*}, w_i^{D*}) = (\eta_i^D(b, \theta, \mathbf{n}^e), \omega_i^D(b, \theta, \mathbf{n}^e))$, $i = 1, ..., m$, and expectations are rational, $\mathbf{n}^e = \mathbf{n}^{D*}$. Due to the symmetry of the m bargaining problems and the continuity of η_i^D and ω_i^D there exists a symmetric equilibrium, $(n_i^{D*}, w_i^{D*}) = (n^{D*}, w^{D*})$ for all i. This is also the unique equilibrium.

Proposition 1. *There exists a symmetric labor-market equilibrium* $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$ *to the decentralized labor market and, moreover, this equilibrium is unique.*

Proof. See the Appendix.

From the symmetry it follows that all firms make the same output decision and the same profit. The expected utility of an arbitrary worker is consequently the same in all unions. Using the first-order condition of the Nash bargaining solution with respect to employment and the symmetry of the equilibrium gives for every product-market structure m a critical wage such that excess labor is hired in equilibrium if and only if the equilibrium wage is lower than this critical wage.

Lemma 1. *Let the labor market be decentralized. Then labor is excessively hired in equilibrium if and only if*

$$w^{D*} < \overline{w}^D(m) = \frac{1}{2(m+1)}. \tag{2.2}$$

Proof. See the Appendix.

The equilibrium wage w^{D*} is greater than the unemployment benefit for all degrees of workers' risk aversion. This makes $\overline{w}^D(m) > b$ a necessary condition for hiring of excess labor in equilibrium at product-market structure m. The critical wage level $\overline{w}^D$ is in turn decreasing in the number of firms. An increased product-market competition in the sense of an increased number of firms induces a decrease in the critical wage level $\overline{w}^D$. This has two effects on the set of values of b and θ at which excess labor is hired in equilibrium. First, it requires a lower highest value of the unemployment benefit, i.e., the highest value of b such that $\overline{w}^D(m) > b$ goes down. Secondly, it requires a higher lowest degree of workers' risk aversion for a given level of the unemployment benefit, i.e., the lowest value of θ such that $\overline{w}^D(m) > w^{D*}$ for a given b goes up. In this sense, increased competition in the product market can be said to make equilibrium hiring of excess labor more difficult.

The critical wage $\overline{w}^D$ is decreasing in m and if $b > 0$ then there exists a product-market structure $\overline{m}(b)$ at which firms may hire excess labor in equilibrium but not if one more firm is added, i.e., a product-market structure such that $\overline{w}^D(\overline{m}(b)) > b$ and $\overline{w}^D(\overline{m}(b) + 1) < b$. By use of Equation 2.2, the critical product-market structure $\overline{m}(b)$ is derived.

Proposition 2. *The critical product-market structure is given by*

$$\overline{m}(b) = \begin{cases} +\infty & \text{if } b = 0 \\ \max\left\{x \geq 0 \mid x < \frac{1-2b}{2b} \text{ is an intiger}\right\} & \text{if } b > 0. \end{cases}$$

Proof. See the Appendix.

Thus, if the unemployment benefit is small enough then labor may be excessively hired even if the number of firms is large. For example, let $b = 0.02$, then the largest number of firms consistent with excess labor is 24. The key to Proposition 2 is that workers may be infinitely risk averse. The equilibrium wage w^{*D} is decreasing in θ and approaching b as θ goes to infinity. Hence, if $b < \overline{w}^D(m)$ then there exists a well-defined value of θ such that the equilibrium wage is lower than the critical wage only if θ exceeds this value. We then say that the product-market structure is consistent with excess labor at unemployment benefit level b and $\overline{m}(b)$ is the most competitive product-market structure that is consistent with excess labor at b.

3. One-Sided Centralization

In the case of one-sided centralization, the firm-specific unions join together into a central union that simultaneously and separately bargains with the m firms. As in the decentralized setting it is common knowledge that the union and the firms all hold the belief $\mathbf{n}^e$. Let $\mathbf{w}^e = (w_1^e, ..., w_m^e)$ be the central union's belief over future wages. The union's objective is to maximize the sum of expected utilities of its members

$$V(\mathbf{n}, \mathbf{w}) = \sum_{i=1}^{m} v(n_i, w_i).$$

Workers have no mobility and every labor contract is assumed to be private information to the union and the firm in question. A change in the centralized union's contract with firm i does therefore not induce a change in any of the $m-1$ other bargaining problems. The union consequently treats the m negotiations as separate and its objective coincides with the objective of the firm-specific union in every bargaining. The utility gain of the union from an agreement with firm i is

$$V(\mathbf{n}^e_{-i}, n_i, \mathbf{w}^e_{-i}, w_i) - V(\mathbf{n}^e_{-i}, \frac{1}{m}, \mathbf{w}^e_{-i}, b) = v(n_i, w_i)$$

where $\mathbf{w}^e_{-i} = (w^e_1, ..., w^e_{i-1}, w^e_{i+1}, ..., w^e_m)$. Hence, the set of feasible labor contracts between firm i and the central union is

$$B_i^S(\mathbf{n}^e) = \{(n_i, w_i) \in [0, 1/m] \times [0, 1] \mid v(n_i, w_i), \pi_i^e(\mathbf{n}^e, n_i, w_i) \geq 0\} = B_i^D(\mathbf{n}^e).$$

Because the sets $B_i^S(\mathbf{n}^e)$ and $B_i^D(\mathbf{n}^e)$ are identical, given $\mathbf{n}^e$, every bargaining outcome in the one-sided centralized setting is identical to the corresponding outcome in the decentralized setting. Let $(\mathbf{n}^{S*}, \mathbf{w}^{S*})$ denote the equilibrium to the one-sided centralized labor market.

Proposition 3. $(\mathbf{n}^{S*}, \mathbf{w}^{S*}) = (\mathbf{n}^{D*}, \mathbf{w}^{D*})$ *for all* θ, b, *and* m.

Proof. See the Appendix.

From Proposition 3 it follows that the results arrived to in Section 2.2 carry over to the case of one-sided centralization.

Corollary 2. *Propositions 1 and 2 and Lemma 1 apply to case of one-sided centralization.*

Proof. See the Appendix.

4. The Centralized Labor-Market

In the centralized labor market the central union negotiates over wages and employment with an employers' association formed by the m firms. The employers' association maximizes total profits conditional on subsequent non-collusion in the product market. We have required the bargaining outcome to be symmetric, i.e., the central labor contract is required to stipulate the same wage and employment in all firms, $\mathbf{n} = (n, ..., n)$ and $\mathbf{w} = (w, ..., w)$. The symmetry of the central labor contract is common knowledge and every firm thus knows the employment of its competitors. We therefore write $\mathbf{n}^e = \mathbf{n}$ and the set of feasible contracts is

$$B^C = \{(n, w) \in [0, 1/m] \times [0, 1] \mid v(n, w), \pi^e(\mathbf{n}, w) \geq 0\}$$

where $\pi^e(n,w) = \pi_i^e(\mathbf{n},w)$ for all i. The Nash bargaining solution is

$$(n^{C*}, w^{C*}) = \arg \max_{(n,w)\in B^C} (v(n,w) - \underline{v})(\pi^e(\mathbf{n},w) - \underline{\pi}). \tag{4.1}$$

The bargaining outcome (n^{C*}, w^{C*}) where $w^{C*} = \omega^C(b,\theta)$ and $n^{C*} = \eta^C(b,\theta,m)$ is automatically a labor-market equilibrium since expectations are rational by assumption. However, for some values of b and θ the functions ω^C and η^C are not continuous if $m \geq 2$. Intuitively, the discontinuities arises because the employers' association internalizes the effect of employment on total profits in combination with the assumption of the individual firm's free disposal of hired labor.

In the product market the individual firm produces until either the marginal revenue is zero or the capacity restriction is binding. The employers' association, on the other hand, aims to maximize total profits and it rationally anticipates the product-market equilibrium that follows from the central labor contract. The joint profit is maximized when the firms act as a monopolist but the firms cannot collude in the product market. Every firm maximizes its profit and when $1/2m \leq n < 1/(m+1)$ then an increase in employment induces an increase in production which in turn lowers the negative marginal joint revenue from employment. But if $n > 1/(m+1) = q_i$ then the production is unaffected by small changes in employment and the joint revenue from employment is zero. In both cases the marginal joint cost of employment is $-wm$ making the marginal effect of employment on joint profits discontinuous at $n = 1/(m+1)$.

The central labor contract is Pareto efficient and the marginal rate of substitution between wage and employment is the same for the union as for the employers' association. At some parameter values is the marginal effect of employment on joint profits discontinuous and this causes a discontinuous shift in the central labor contract. This problem does not arise in the decentralized- and one-sided centralized labor markets because the individual firm considers its environment as fixed when bargaining in a decentralized labor market and the marginal effect of employment on profits is continuous at. The first-order conditions to Equations 2.1 and 4.1 gives Proposition 4.

Proposition 4. *Let m, θ and b be given. Then:*
(i) If $n^{C*}, n^{D*} \leq 1/(m+1)$, then $w^{C*} > w^{D*}$ and $n^{C*} < n^{D*}$.
(ii) If $n^{C*}, n^{D*} > 1/(m+1)$, then $w^{C*} = w^{D*}$ and $n^{C*} = n^{D*}$.

Proof. See the Appendix.

Proposition 4(i) states that if no excess labor is hired in any of the two settings, then wages are higher and employment is lower in the centralized setting than in the decentralized. The intuition behind this result is that the marginal effect of employment on profits faced by the employers' association is higher than the

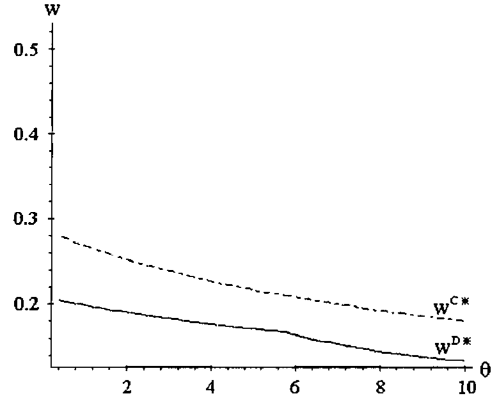

Figure 5.1: The wages w^{C*} (dashed) and w^{D*} as functions of θ for $b = 0.05$ and $m = 2$.

corresponding effect faced by the individual firm. The employers' association is therefore more reluctant than the individual firm to insure the workers against income uncertainty by means of employment. Also, the assumption of a symmetric central labor contract makes this difference even larger. If, on the other hand, excess labor is hired in both settings then the marginal effect of employment is the same in both settings. Hence the identical wage and employment.

5. An Example

Consider the case of a duopoly, $m = 2$, and let the unemployment benefit be $b = 0.05$. Suppose the wage bargaining is decentralized or one-sided centralized. Then the equilibrium wage must be less than $\overline{w}^D(2) = 1/6$ for excess labor to be hired. The wage w^{D*} as a function of θ is shown in figure 5.1.
For the chosen values of b and m excess labor is hired if $\theta > 5.79$. The highest number of firms that is consistent with excess labor is $\overline{m}(0.05) = 8$.

Consider the centralized labor market. The central labor contract specifies the same wage and employment in every firm by assumption and expectations are automatically rational, $\mathbf{n}^e = \mathbf{n}$. For the chosen parameter values, excess labor is not hired in the centralized setting. The wage w^{C*} as a function of θ is shown as the dashed line in Figure 5.1.

6. Discussion

One important assumption in the decentralized and one-sided centralized settings is that labor contracts are private information. This eliminates the strategic interaction among firms (and unions) at the labor-market stage. If labor contracts were not private information but announced before the making of the production decisions, then the labor contract between a firm and a union would no longer change continuously for all parameter values as the other firms marginally revise their contracts. Therefore it may not always exist a labor-market equilibrium. However, the qualitative results of the private-information setting carry over in those cases an equilibrium exists. The simplicity of the private-information framework is therefore an attractive property. Furthermore, hiring of excess labor is more easily obtained in the decentralized- and one-sided centralized settings if labor contracts are public information than if not. The reason is that the (negative) marginal effect of employment on the firm's profit decreases when labor contracts are made public; increased employment in one firm makes that firm more aggressive and the best response of every other firms is to produce less. The expanding firm does consequently not bear the full cost of its action and this increases its willingness to employ. A union may consequently have an incentive to make labor contracts public information and so may the individual firm and the government.

An other strong assumption is that of no labor mobility between firms. This is a simplifying assumption and allowing for labor mobility is not likely to eliminate the possibility of excess labor in equilibrium because of the symmetry of the model, but it may allow for multiple equilibria.

Perhaps the most important aspect abstracted from in the model is the union's internal democratic decision process. Two possible conflicts are thereby ruled out; the conflict in interests between the leaders and the members and the conflict in interests between different groups of members. Such extensions may substantially change the results arrived to in this study. As an example, making the model dynamic and letting the union in time t represent workers employed in $t-1$ as in the "insider-outsider theory" (Lindbeck and Snower (1989)) is likely to eliminate employment of excess labor.

Appendix: Proofs

Proposition 1. This proof is carried out in two steps. First, the existence of a symmetric equilibrium is shown and secondly it is shown that this equilibrium is unique. Throughout the proof only $\mathbf{n}^D$ is considered and this is possible because the uniqueness of the labor contracts; w_i^D can thus be viewed as a function of n_i^D.

Step 1: In equilibrium $\mathbf{n}^e = \mathbf{n}$ and each labor contract $\eta_i^D(b, \theta, \mathbf{n})$ is a "best response" by firm-union pair i to $\mathbf{n}_{-i}$. Let $\mathfrak{N}(\mathbf{n}) = \times_{i=1}^{m} \eta_i^D(b, \theta, \mathbf{n})$. From the uniqueness and continuity of the Nash bargaining solution it follows that $\mathfrak{N}$ is a continuous function mapping $[0,1/m]^m$ into itself. Consider the compact and convex subset of $[0,1/m]^m$

$$D=\{(d,...,d)\in[0,1/m]^m \mid 0\leq d\leq 1/m\}.$$

The correspondence $\mathfrak{N}$ maps D into D and hence has a fixed point $\mathbf{n}^D$ in D by Kakutani's fixed point theorem.

Step 2: Here it is first shown that if $\mathbf{n}^D, \mathbf{n}' \in [0,1/m]^m$ solves $\mathfrak{N}(\mathbf{n})=\mathbf{n}$ and $\mathbf{n}^D$ is the symmetric equilibrium from step 1, then must $n_i', n_j' \neq n^D$ for some $i\neq j$. This is then used to show a contradiction.
Let $\boldsymbol{\varepsilon}=(\varepsilon_1,...,\varepsilon_m)=\mathbf{n}'-\mathbf{n}^D$ and suppose $\mathbf{q}^e(\mathbf{n}^D)=\mathbf{q}^e(\mathbf{n}')$. Then the bargaining problem between firm and union i is the same in both equilibria. Hence, $\eta_i^D(b,\theta,\mathbf{n}')=\eta_i^D(b,\theta,\mathbf{n}^D)$ for all i and $\mathbf{n}'=\mathbf{n}^D$. It follows that if $\mathbf{n}'\neq\mathbf{n}^D$ then $q_i^e(\mathbf{n}^D)<q_i^e(\mathbf{n}')$ and $q_j^e(\mathbf{n}^D)>q_j^e(\mathbf{n}')$ for some $i\neq j$. From the relevant first-order conditions to the Nash bargaining solution (Equations 6.1 and 6.2) it follows that then is $\varepsilon_i>0$ and $\varepsilon_j<0$. Moreover, it also follows that $0>dn_i/dn_j>-1$ along η_i when $n_i\in(0,1/m)$ and $dq_j^e/dn_j\neq 0$. Then, the following must be true

$$\begin{aligned}\varepsilon_i &< -(\textstyle\sum_{k\neq i,j}\varepsilon_k+\varepsilon_j)\\ \varepsilon_j &> -(\textstyle\sum_{k\neq i,j}\varepsilon_k+\varepsilon_i).\end{aligned}$$

The system can be rewritten

$$\begin{aligned}\varepsilon_i &< -(\textstyle\sum_{k\neq i,j}\varepsilon_k+\varepsilon_j)\\ \varepsilon_i &> -(\textstyle\sum_{k\neq i,j}\varepsilon_k+\varepsilon_j).\end{aligned}$$

which is a contradiction. Hence, $\mathbf{n}^D$ is the unique equilibrium. ■

Lemma 1. If $n_i^D=q^o(n_i^D,\mathbf{q}_{-i}^e(\mathbf{n}^e))$ then (n_i^D,w_i^D) must satisfy the system of first-order conditions to the Nash bargaining solution

$$n_i^D-\frac{2(1-w_i^D-\sum_{j\neq i}q_j^e(\mathbf{n}^e))}{3} = 0 \tag{6.1}$$

$$\frac{1-w_i^D-\sum_{j\neq i}q_j^e(\mathbf{n}^e)}{3}-\frac{e^{-\theta(b-w_i^D)}-1}{\theta} = 0 \tag{6.2}$$

and if $\frac{1}{m}>n_i^D>q^o(n_i^D,\mathbf{q}_{-i}^e(\mathbf{n}^e))$ then it must satisfy the system

$$n_i^D-\frac{(1-\sum_{j\neq i}q_j^e(\mathbf{n}^e))^2}{8w} = 0 \tag{6.3}$$

$$w_i^D-\frac{e^{-\theta(b-w_i^D)}-1}{\theta} = 0. \tag{6.4}$$

If $n = \frac{1}{m}$ then w_i^D must satisfy

$$\frac{(1 - \sum_{j \neq i} q_j^e(\mathbf{n}^e))^2}{4} - \frac{w}{m} - \frac{e^{-\theta(b - w_i^D)} - 1}{\theta} = 0. \tag{6.5}$$

When $w_i^D = (1 - \sum_{j \neq i} q_j)/4$ is $n_i^D = (1 - \sum_{j \neq i} q_j)/2$ which is the highest level of employment in firm i such that the firm does not hire excessive labor. At this point Equations 6.1 and 6.2 coincide with Equations 6.3 and 6.4. If $w_i^D = m(1 - \sum_{j \neq i} q_j^e(\mathbf{n}^e))^2/8$ then is $n_i^D = 1/m$ and Equations 6.3 and 6.4 coincide with Equation 6.5. Hence, the employment and the wage are continuous functions of the parameters of the model.

Using that the unique equilibrium is symmetric gives that Equations 6.1 and 6.2 coincides with Equations 6.3 and 6.4 when $n^{D*} = 1/(m+1)$ and $w^{D*} = (1 - (m-1)/(m+1))/4 = 1/2(m+1)$. ■

Proposition 2. First $\overline{m}(b)$ is derived and then it is shown that there exists a θ such that excess labor is hired at consumer market structure $\overline{m}(b)$.

That $w_i^D > b$ for all $\theta > 0$ follows from the properties of the Nash bargaining solution. Excess labor at product-market structure m thus requires $b < \overline{w}^D(m) \Leftrightarrow m < (1 - 2b)/2b$. If $b > \overline{w}^D(1)$ then there exists no $m \geq 1$ such that $b < \overline{w}^D(m)$. Let $\overline{m}(b)$ be the largest integer m that satisfies $m < (1 - 2b)/2b$. If $b = 0$ then every integer satisfies the inequality and $\overline{m}(b) = +\infty$.

Let $b < \overline{w}^D(1)$ and let $m = \overline{m}(b)$. If there exists a θ such that $b < w^{D*} < \overline{w}^D(\overline{m}(b))$ then product-market structure $\overline{m}(b)$ is consistent with excess employment. Let $b < w^{D*} < \overline{w}^D(\overline{m}(b))$ and substitute w^{D*} into Equation 6.4. By the properties of the Equation 6.4 gives that a unique solution $\overline{\theta} > 0$ exists. Since $\overline{\theta}$ is such that $w^{D*} < \overline{w}^D(\overline{m}(b))$ there exists no solution to Equations 6.1 and 6.2 that maximizes the Nash product. Hence, $\overline{m}(b)$ is consistent with excess employment and by Lemma 1 so is every product-market structure $m < \overline{m}(b)$. ■

Proposition 3. Consider the expression $V - \underline{V}$. Simplifying gives

$$V(\mathbf{n}_{-i}^e, n, \mathbf{w}_{-i}^e, w) - V(\frac{1}{m}, \mathbf{n}_{-i}^e, b, \mathbf{w}_{-i}^e) = v(n, w) \tag{6.6}$$

which makes the bargaining problem of the firm and union identical to that of a decentralized labor market. Hence, $(n_i^S, w_i^S) = (n_i^D, w_i^D)$ for all $\theta, b, m,$ and i. ■

Corollary 1. Follows directly from the proof of Proposition 3. ■

Proposition 4. If $n^{C*} = q^o(n^{C*}, \mathbf{q}^e_{-i}(\mathbf{n}^{C*}))$ then (n^{C*}, w^{C*}) must satisfy the system of first-order conditions to the Nash bargaining solution

$$\begin{aligned} n^{C*} - \frac{2(1-w^{C*})}{3m} &= 0 && (6.7) \\ \frac{1-w^{C*}}{3} - \frac{e^{-\theta(b-w^{C*})}-1}{\theta} &= 0 && (6.8) \end{aligned}$$

and if $\frac{1}{m} > n^{C*} > q^o(n^{C*}, \mathbf{q}^e_{-i}(\mathbf{n}^{C*}))$ then it must satisfy the system

$$\begin{aligned} n^{C*} - \frac{1}{2(m+1)^2 w^{C*}} &= 0 && (6.9) \\ w^{C*} - \frac{e^{-\theta(b-w^{C*})}-1}{\theta} &= 0. && (6.10) \end{aligned}$$

When $n^{C*} = 1/m$ then w^{C*} must satisfy

$$\frac{m}{2(m+1)^2} - \frac{e^{-\theta(b-w^{C*})}-1}{\theta} = 0. \tag{6.11}$$

Suppose b, θ are such that $n_i^{D*}, n^{C*} \le 1/(m+1)$. Compare Equation 6.8 with Equation 6.2 using that $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$ is symmetric. From $\sum_{j\neq i} q_j^e > 0$ it follows that $w_i^{D*} < w^{C*}$. Using this inequality and Equations 6.7 and 6.1 gives $n_i^{D*} > n^{C*}$.

Suppose b, θ are such that $n_i^{D*}, n^{C*} > 1/(m+1)$. The Equations 6.4 and 6.10 are identical and so are Equations 6.5 and 6.11 when using the symmetry of $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$. Hence, $w_i^{D*} = w^{C*}$ and $n_i^{D*} = n^{C*}$. ■

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