

*On monetary policy and interest rate  
determination in an open economy*



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On  
Monetary Policy  
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Determination

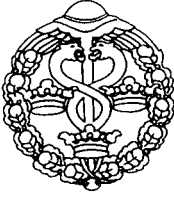
in an open economy

by Lars Hörngren



STOCKHOLM SCHOOL OF ECONOMICS  
THE ECONOMIC RESEARCH INSTITUTE

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# Preface

Once upon a time there was (and in some sense there maybe still is) a small-scale macroeconomic model of Sweden that had a very rudimentary financial sector. Not that the economy itself was characterized by a rich financial structure, quite the contrary, but there was a number of aspects of monetary policy as implemented in practice about which the model had very little to say. It was decided that a special study of this sector was necessary, in particular, to clarify the effects of the many regulatory monetary policy instruments. I set out to look into this problem.

A number of years later, that study has materialized in the form of this dissertation. However, the initial interest in the regulatory system has been pushed aside. During the time that this project has been under way, Swedish monetary policy and financial markets have undergone fundamental changes, which have brought new problems to the fore. In just a few years, virtually all the old regulations have been done away with and a rich structure of financial markets has developed. At times, the pace of reform and innovation has been almost frustrating for someone trying to understand and analyze the system. However, on the positive side, it must be noted that this development also has made the subject of this dissertation all the more interesting and topical. In addition, several of the policy changes conform to conclusions reached in the current project. On balance, it is therefore rather difficult to maintain a negative attitude to what has happened.

It is of course a cliché to point out in the preface to a dissertation that it is the result of a collective effort. In this case I see no way of avoiding it, however. Although only my name appears on the title page of this book, it would be equally appropriate to describe it as a presentation of material developed within the monetary policy research group at the Stockholm School of Economics, to which I have the privilege of belonging. Several of the chapters in this book are thus the results of joint work with colleagues in this group. I owe a special debt to Staffan Viotti, my thesis advisor, and Anders Vredin, equally important as unofficial advisor. It was Staffan who started me on this project and he has kept me going ever since with the help of his special blend of encouraging advice and caustic comments. I have learned to appreciate both. He and Anders have always been available and willing to discuss any problem, no matter how big or how small. The help and support I have received from them and the other members in the group, Peter Englund, Per Krusell, and Ingrid Werner, have been indispensable, both for the completion of this book and for me personally.

Discussions with Johan Myhrman have also been very helpful, as well as comments from other colleagues, past and present, at the Stockholm School of Economics. Thanks are also due to Thomas Franzén, who did a very thorough job as discussant when parts of this material were presented as a "licentiat" thesis.

It would all have been in vain, however, without the help of Kerstin Niklasson and Monica Peijne, who have typed the many drafts of these chapters with their usual excellence, helpful and patient also when time was running short. Had I been equally competent in my work, there would have been no mistakes of any kind in this book, but being realistic, I must add that I take full responsibility for all remaining errors and misunderstandings.

I have received generous financial support for this project from Finanspolitiska Forskningsinstitutet, PKbanken and the Swedish Post Office, and Jan Wallander's Research Foundation. Also, I had the opportunity to spend the academic year 1982-83 at the Center for Research in Government Policy and Business, Graduate School of Management, University of Rochester, and I thank the Center and its Director, Professor Karl Brunner, for their generous hospitality and Johan Myhrman for his help in making this visit possible.

Stockholm in September, 1986

Lars Hörngren





# Contents

1.	BACKGROUND AND SUMMARY	1
1.1	Introduction	1
1.2	Swedish monetary policy 1970-1986: A brief review	5
1.3	Summary and main conclusions	18
	Notes	29
	References	31
2.	REGULATORY MONETARY POLICY AND UNCONTROLLED FINANCIAL INTERMEDIARIES	33
2.1	Introduction	33
2.2	The liquidity ratio as a policy instrument	35
2.3	The portfolio model	36
2.4	The effects of changing the liquidity ratio	44
2.5	Concluding remarks	52
	Appendix	55
	Notes	57
	References	59

3.	DISCOUNT WINDOW POLICIES AND THE DETERMINATION OF THE MONEY MARKET RATE	61
3.1	Introduction	61
3.2	A money market model with discount window borrowing	63
3.3	The role of the discount window: A closer look	77
3.4	Discount window policies in Sweden	86
3.5	Concluding comments	95
	Appendix	98
	Notes	102
	References	105
4.	OPTIMAL MONETARY POLICY RULES: THE ROLE OF THE DISCOUNT WINDOW	107
4.1	Introduction	107
4.2	Analytical framework	109
4.3	The model	111
4.4	A comment on Poole	116
4.5	Comparative statics	118
4.6	Optimal policy rules	121
4.7	Concluding comments	126
	Appendix	128
	Notes	134
	References	136

5.	INTERNATIONAL INTEREST RATE DEPENDENCE AND THE FOREIGN EXCHANGE RISK PREMIUM	139
5.1	Introduction	139
5.2	Monetary policy and the foreign exchange risk premium	140
5.3	Concluding comments	148
	Notes	150
	References	151
6.	THE FOREIGN EXCHANGE RISK PREMIUM: A REVIEW OF THEORY AND EVIDENCE	153
6.1	Introduction	153
6.2	Theoretical models of the foreign exchange risk premium	154
6.2.1	The mean-variance optimization approach	155
6.2.2	Models without purchasing power parity	158
6.2.3	Models with stochastic investment and consumption opportunities	164
6.2.4	General equilibrium asset pricing models	169
6.2.5	Summary of the theoretical work	174
6.3	Empirical work on the foreign exchange risk premium	175
6.3.1	The atheoretical approach	176
6.3.2	Portfolio balance models	179
6.3.3	Tests related to intertemporal asset pricing models	184
6.4	Concluding comments	189
	Notes	192
	References	195

7.	THE FOREIGN EXCHANGE RISK PREMIUM IN A CURRENCY BASKET SYSTEM	199
7.1	Introduction	199
7.2	Risk premia in a currency basket system	201
7.3	Empirical tests	210
7.3.1	The data and the formulation of the tests	211
7.3.2	Regression results	215
7.4	Concluding comments	220
	Notes	221
	References	224
8.	THE TERM STRUCTURE OF INTEREST RATES IN THE SWEDISH MONEY MARKET	227
8.1	Introduction	227
8.2	Expectations and risk premia in the term structure of interest rates	229
8.3	The data	236
8.4	The predictive power of forward rates: A comparison to the martingale model	239
8.5	The predictive power of forward rates: Regression tests	248
8.5.1	The regression equations	249
8.5.2	Regression results	253
8.6	Concluding comments	261
	Notes	264
	References	268

# 1. Background and Summary

## 1.1 INTRODUCTION

Money has always attracted a lot of attention, from laymen and economists alike. Although economists are interested in money for the same reasons as everybody else, they are (usually) not only concerned with having more of it, but also with understanding in a more fundamental sense what money is and what money does.

Despite continuing efforts in this field, the role of money is not particularly well understood. In particular, the aggregate effects of monetary variables, i.e., their influence on output, inflation, employment, etc., are still controversial issues. At a general level, the long run connection between money and prices is widely accepted, but the short run mechanisms and effects have yet to be adequately modeled and explained, i.e., we know relatively little about the transmission mechanism.

Nevertheless, in virtually all countries attempts are made to control monetary variables in order to improve (or at least influence) the macroeconomic development. In other words, monetary policy is considered important, despite the difficulties involved in knowing what its ultimate effects are.

This state of affairs is also reflected in how monetary policy is implemented in practice.<sup>1</sup> The policy makers can be thought of as having a set of ultimate targets, i.e., variables whose developments are important for economic and social welfare. Standard examples would be the growth of aggregate output, the rate of inflation, and the rate of unemployment. However, these variables are typically not subject to direct policy control. What the policy makers have at their disposal is a set of policy instruments, defined as variables that are directly controllable. The instruments of monetary policy include, for example, the central bank's holdings of government bonds and the interest rate charged for borrowed reserves.

Ideally, the policy makers should be able to choose values of the policy instruments that are consistent with the desired time paths for the ultimate target variables. However, given the uncertainty surrounding the effects on the ultimate targets of monetary events, it is in general not possible to derive an "optimal" policy in this way. As implemented in practice, the policy makers instead use intermediate targets. These variables are, in general, not considered important in their own right, but they are believed to be (somehow) connected to the ultimate targets and, in addition, at least to some extent controllable with the help of the policy instruments. The intermediate targets are typically monetary and financial variables, for example, the money supply, the foreign exchange reserves, or some market interest rate(s).

Under these procedures, the explicit policy targets are formulated in terms of the intermediate targets and the policy instruments are adjusted so as to keep these at their desired paths. To the extent that the target paths of the intermediate variables are determined with reference to the ultimate targets, the underlying reasoning is based on relatively loosely formulated ideas about, for example, how the rate of inflation is affected by an increase in the growth rate of the money

supply or how aggregate demand responds to a change in interest rates. Although formal econometric models (of the money market) may be used in the first stage when the setting of the policy instrument is determined, the second stage of policy formation usually relies almost exclusively on informal and judgmental methods.

Given the limits of our current knowledge, a purist would maybe recommend a moratorium on monetary policy until our understanding of its effects has been substantially improved. Policy makers cannot always afford to be purists, however, and even if the prevailing uncertainty would seem to call for a cautious policy, it must be considered highly unwise to let the monetary development go out of hand just because we do not know exactly why we should be concerned about it. After all, the general notion that monetary variables are important for the aggregate economic development is supported by considerable amounts of empirical evidence and hard-earned practical experience.

This book is also based on the premise that it is meaningful to analyze monetary policy in terms of its effects on (potential) intermediate targets, for example, financial stock variables and interest rates. Hence, the models presented in the following chapters study the connection between policy instruments (open market operations, discount window policies, etc.) and endogenously determined financial variables. No attempt is made to incorporate the channels through which the financial and real sectors of the economy interact.

This also implies that no specific stand is taken on issues related to the choice of intermediate target variables or how the optimal path for the chosen target, for example, the money supply, is to be determined. The arguments for and against a constant growth rate of the money supply are thus not addressed as this requires a model of how money, somehow defined, affects, for example, the rate of inflation.

The assumption underlying the approach taken in this book is that monetary events are important and that policy makers therefore should be concerned with them. To ignore the transmission mechanism is undeniably a limitation, but it is a limitation that this study shares with most forms of monetary policy applied in practice. Moreover, for a monetary policy implemented using intermediate targets of the type discussed above, the problems of monetary control, i.e., how the target variables (money supply, interests rates, or whatever) are to be influenced and kept at their desired levels, are of fundamental importance.

Examples of problems of monetary control addressed in this book are:<sup>2</sup>

- What are the effects of certain policy instruments on possible intermediate target variables?
- How do the institutional rules and regulations affect the usefulness of the policy instruments?
- How should policy rules be formulated so as to improve the control of intermediate targets?
- To what extent is the possibility of controlling intermediate targets in an open economy limited by international influences?

Most of the studies presented below are the results of attempts to analyze aspects that are important for understanding Swedish monetary policy. In most respects, there is nothing uniquely Swedish about these problems, however. It is nevertheless useful to have a concrete example as point of departure. The next section therefore gives a brief review of recent developments of monetary policy and financial markets in Sweden. With the help of a picture of the wider setting, the connection between the different parts of the book will also be seen more clearly. The summary of the book in Section 1.3 also serves this purpose.



## 1.2 SWEDISH MONETARY POLICY 1970-86: A BRIEF REVIEW

The Swedish financial system has changed dramatically during the 1980s. In just a few years, it has been transformed from a highly regulated and in many respects underdeveloped state into a relatively sophisticated financial network where the role of regulations is limited. Concurrently, an equally drastic change has been made in the types of policies pursued by the Swedish central bank, the Riksbank.

In this book various aspects of this transformation of policy and institutions are discussed in some detail. The purpose of this section is to give a background to these analyses by reviewing the development during the last two decades of both the institutional system, and the targets and instruments of monetary policy. The review is sketchy and emphasizes the general development rather than the specific policies pursued at different points in time.<sup>3</sup>

The main characteristic of Swedish domestic monetary policy in the 1970s was the steady increase in the reliance on regulatory policy measures, i.e., instruments that were intended to set interest rates or quantitative variables (or both) in accordance with central bank directives. With the exception of changes in the discount rate, very little use was made of open market operations and other "traditional" instruments that work by influencing the market outcome rather than by putting the market forces out of action. The roots of this regulatory system can be traced back to the period after World War II when Sweden like a number of other countries used policies intended to keep interest rates at a low level.

The emphasis on low interest rates was partly motivated by a desire to use monetary policy to achieve specific allocative and distributive goals. In particular, the purpose was to

guarantee low cost credit to the housing and government sectors. In principle, a policy with low interest rates is possible without the use of regulations, but only at the price of giving up control of quantitative variables. For example, an increase in the demand for credit has to be accommodated by the central bank, if a rise in the interest rate is to be prevented. When problems of controlling the rate of credit expansion or the currency flows emerged in Sweden, the reaction was typically to try to get control over the target variables with the help of direct regulations. The extreme emphasis on low interest rates had been gradually abandoned in the second half of the 1950s, but the reliance on regulatory policies continued throughout the 1970s.

The most important instruments were interest rate regulations of both loan and deposit rates, direct ceilings on bank lending, strict control of corporate bond issues, and so called liquidity ratios<sup>4</sup>, rules that forced banks and insurance companies to hold a substantial fraction of their assets in the form of government and housing bonds with below market interest rates. The liquidity ratio in particular reflects the strong selective element in the regulatory system. Priority was also given to the corporate sector in the allocation of bank credit, whereas the households often had to bear the main burden of credit contractions. One reason for this was that the tax system with full deductibility of interest expenses and high marginal tax rates was believed to make the household sector's credit demand highly interest inelastic so that quantitative regulations were necessary to limit, for example, consumer credit.<sup>5</sup>

Although this was hardly intentional, these regulations also had a highly selective effect on the financial intermediaries. Most of the rules were directed towards controlling bank credit, but there were of course other institutions that could offer

loans to households if the banks had to turn down loan applications. In the wake of the strong regulations enforced on banks, new and unregulated financial intermediaries emerged. In particular, finance companies that could work independently of central bank influence flourished. In other words, an unregulated financial sector developed alongside the conventional and heavily regulated banking sector.

It should be noted that most of the restrictions imposed on banks had no formal legal basis at the start of the 1970s. Instead, the central bank entered into "voluntary" agreements with the banks where the latter promised to meet the prescribed liquidity ratios, keep the rate of credit expansion below the limits set by the loan ceiling, etc. The central bank could secure compliance with these agreements by penalizing banks, for example, by limiting their access to discount window borrowing and by other more subtle measures. The banks were highly aware of the fact that the central bank, especially in a banking system of the Swedish type with a high degree of concentration and thus very few banks to monitor, has many (informal) ways of making times hard for a bank that fails to comply with its intentions.

In addition to these regulations of domestic transactions, there was a system of strict foreign exchange controls. In principle, it outlawed all foreign exchange transactions, but a general exception had been made for payments and credits directly related to trade in goods and services. The restrictions on financial transactions were very strict, however, one purpose being to insulate as much as possible Swedish financial markets from international developments.

One reason for the increased reliance on regulations from the early 1970s and onwards was that the change in the international monetary system put new demands on stabilization policy. In the past fiscal policy had been used to attain

the goals of stabilization policy and monetary policy could be geared towards keeping interest rates down, as discussed above, partly for distributive and allocative purposes. However, it soon became clear that a passive monetary policy was no longer possible in the face of disturbances that led to rapid outflows of foreign currency. The first examples of this were seen in the late 1960s, but as the degree of integration and the magnitudes of the international disturbances increased during the 1970s, monetary policy interventions motivated by concern for the external balance were to become recurring events. Monetary policy was thus faced with new policy problems and Sweden's international dependence became a more important concern. However, the authorities were not willing to give up the goal of keeping interest rates at levels consistent with domestic targets, which meant that stronger and stronger regulations were required.

At the start of the 1970s the regulatory system was more or less fully developed and in 1974 it was formalized with the enactment of "Lagen om kreditpolitiska medel, LKM" (The law on monetary policy measures). This law did not really change anything in practice, but only codified the informal rules that the central bank already was applying. The law specifies a set of regulatory instruments that the central bank can use. It is noteworthy that the law does not give the central bank unconditional right to impose these restrictions. It states that they can be applied only after government approval and if they are "needed for fulfillment of the goals specified for the monetary policy activities of the Riksbank". For some instruments, for example, interest rate regulation and loan ceilings, stronger rules apply in that they can be used only if "special reasons" are present. It is not specified, however, what is meant by "special reasons", i.e., this is left to the government and the central bank to interpret and, in practice, these limitations have been of no importance. Many of

the regulations were applied continuously and, on occasion, the central bank imposed loan ceilings despite the fact that use of this instrument had not been officially approved. The banks complied, knowing that if they did not the central bank could get formal government approval. In addition, the informal methods for penalizing a "disobedient" bank remained an effective threat. This points to the important fact that the central bank's general attitude and intentions are far more important for how monetary policy is conducted than the formal rules set down in law. The discussion of the recent change in policy below will give additional support to this assertion.

Toward the middle of the 1970s problems with the regulatory system started to emerge. First, the growth of the unregulated sector meant that the control of bank lending became a less effective way of limiting the growth of total credit in the economy. The response to this development was to extend the monetary policy regulations to include also the finance companies.

Second, the macro development both domestically and internationally put additional strains on the system. The world recession that followed on the first oil price increases was met by a highly expansionary fiscal policy 1974-76. The domestic rate of inflation rose above the international level and growing current account problems emerged. With the breakdown of the international system of fixed exchange rates, the value of the currency had ceased to be a natural restriction on economic policy. The official ambition in Sweden has consistently been to keep a fixed exchange rate, first relative to the European currency "snake", later to a currency basket index defined unilaterally. However, rapid inflation soon made this policy untenable and in 1976 the first of a series of devaluations was made.

To help defend the foreign exchange reserves, a policy that encouraged private sector borrowing from abroad was also introduced. This meant a partial liberalization of the foreign exchange controls, which up to that point had severely restricted the access to international financial markets. This deregulation was thus in contrast to the steady increase in the reliance on regulatory instruments in the domestic system. As the purpose was to create a currency inflow, lending to foreign agents was still not permitted. In addition, foreign investors were denied access to Swedish financial markets. The main reason for this was presumably that they could not be prevented from withdrawing from the Swedish markets, for example, in connection with expectations of a devaluation of the krona. Domestic agents, on the other hand, were prevented from making premature payments on their outstanding foreign currency loans and could therefore be expected to generate a more "permanent" inflow of foreign currency.

Third, the highly expansionary stance of fiscal policy also meant a very rapid increase in the government budget deficit, putting additional strains on the regulatory system. The liquidity ratios for banks and insurance companies were gradually raised, forcing them to absorb more and more government bonds. The government also started to borrow from abroad, thereby assuming the responsibility for creating the inflows of foreign currency that were necessary to offset the outflows over the current account. This policy was also partly motivated by an ambition to keep domestic interest rates down, as it was believed that in order to encourage the same volume of borrowing by the private sector, it would have been necessary to maintain a bigger interest rate differential.

The stylized picture of Sweden in the early 1980s is thus of an economy with severe problems with both the internal and external balance. As noted above, the standard reaction when problems emerged in the regulatory system was to tighten

the regulations and introduce new ones. It was now becoming increasingly clear that this approach had its limitations. For example, the need to have banks and insurance companies absorb the growing stock of government bonds had raised the liquidity ratios close to their legal maximum, 50 percent. The authorities were faced with the choice of either raising this limit or finding new ways of financing the budget deficit. Obviously, the former alternative would, sooner or later, lead to the final transformation of the banks from important financial market institutions to semi-public agencies for the storing of government bonds.

Similarly, the inclusion of the finance companies under the regulatory policies had not eliminated the unregulated financial sector. A new phenomenon was that many major non-financial companies put more and more resources into their financial activities, forming departments or subsidiaries that highly resembled regular finance companies although they were exempt from the regulations. The strategic choice for the central bank was between extending the regulations even further, i.e., to include also non-financial companies in the system of monetary policy regulations, or finding new methods for controlling this development as well as the regular financial system.

Concurrently with the growing tensions in the regulatory system, a development of new financial markets and institutions had begun. In March 1980 Swedish commercial banks started issuing "bankcertifikat", certificates of deposit (CDs). The important innovation was that, after some initial problems, a secondary market for these deposit instruments developed.<sup>6</sup> Prior to this, so-called special deposits had been used for wholesale deposit transactions, but they were based on bilateral agreements between the depositor, usually a major corporation, and the bank and therefore not marketable instruments. With the introduction of the CDs, however, the foundation for a functioning Swedish money market was laid.

This development had several important implications, both for monetary policy and government debt policy. The introduction of CDs opened up new possibilities for active bank liability management, which, for example, made it easier for banks to circumvent the liquidity ratios. Hence, there is reason to believe that the development of a largely unregulated money market caused additional problems for a monetary policy based on regulatory instruments.

On the other hand, this development also created new opportunities for the authorities. It was realized that this new market could be used also for financing the government budget deficit. In July 1982, the National Debt Office - "Riksgäldskontoret" - started issuing a new type of short term discount bonds - "statsskuldväxlar" - mainly with maturities up to one year. This was certainly also an innovation as it meant that part of the deficit was being financed on a regular asset market and with market determined interest rates. Prior to this, all government bonds had been "priority bonds" that had to be bought and held by banks and insurance companies under the rules of the liquidity ratio. They had no secondary market and were issued at regulated interest rates. The use of marketable government debt instruments was later continued with the introduction of coupon bonds - "riksobligationer" - with maturities up to seven years. An active secondary market was established also for these long term bonds, which means that a whole term structure of market determined interest rates is developing.

With the introduction of the "statsskuldväxel", the development of the Swedish money market really took off as, due to the size of the budget deficit, these instruments were issued in large quantities. This meant that the trading volume in the market became large enough to allow open market operations of conventional type, i.e., it was suddenly possible for the central bank to influence the money market interest rates



by buying and selling bonds. The institutional development thus both undermined the old regulations and created opportunities for a new type of monetary policy that used the market mechanism rather than tried to prevent it from working.

As indicated above, a policy of continued regulations would have had far-reaching consequences as this most probably would have made it necessary to include the entire Swedish economy in a web of financial regulations. The choice of the central bank was also to move towards deregulation. It was not quite a "clean break" with the regulatory policies, but starting with the abolishment of the liquidity ratios for the banks in 1983, a consistent policy of gradual deregulation has been followed.

The second step was to ease the very strict control on corporate bond issues. This led to a marked revival of the bond market and many companies also started using the money market for short term financing. In the wake of the deregulations, the financial markets have developed very rapidly, not least in terms of the types of financial instruments that are traded. For example, regular markets for interest rate options and forward contracts for various maturities have been established. The degree of sophistication of the Swedish financial system has thus increased substantially, not least considering that in 1980 the only financial market was the regular stock market.

The central bank also started using open market operations although only at a small scale at first. This had to do partly with the fact that the discount window policies followed by the central bank had the effect of often making open market interventions inoperative for influencing the money market. Banks were given unlimited access to reserve borrowing, which meant that they could offset the effect of an open market intervention by adjusting the level of borrowed reserves.<sup>7</sup>

In 1985, the final steps in the deregulation of the domestic system were taken. They involved the lifting of the regulation of interest rates and, in November, the abolishment of the strict guidelines (loan ceilings) for the growth rates of lending from banks and finance companies. This meant that financial intermediaries all of a sudden were free to determine both the volume and the interest rates on their lending without direct central bank intervention.

At the same time, the central bank reformulated its rules for discount window borrowing. The system in which banks were given in practice unlimited access to reserve borrowing was replaced by rules that made the central bank lending rate increase with the level of borrowing. The central bank lending rate ranges (as of May 1986) between 6 and 16 percent and increases in steps of two percentage points. One important effect of this reform is that open market operations have been made a useful instrument for affecting the money market interest rate. By buying and selling bonds in the money market, the central bank can influence the supply of non-borrowed reserves and, hence, the banks' demand for borrowed reserves. As the banks' marginal funding cost is determined by the cost of reserve borrowing, an intervention that moves the bank to a higher step on the reserve borrowing function, and thus raises the central bank lending rate, has repercussions also for other market interest rates.

While there are thus virtually no monetary policy motivated regulations of the domestic money market left, one important set of regulations remain (in the summer of 1986), namely, the foreign exchange controls.<sup>8</sup> Also in this area, several steps in the direction of a deregulation have been taken, however. It started in the mid 1970s when Swedish companies were actively encouraged to go out and borrow abroad. This has come to mean in practice that the major export oriented companies have unlimited rights to take loans with maturities

up to six months. Also longer term loans have been permitted, but there restrictions against early repayment have made the positions less flexible. These rules are intended to limit the risk for drastic currency outflows when borrowers decide that a debt in foreign currency is less desirable, for example, in connection with fears for a Swedish devaluation.

In general, there is also a clear "bias" in the foreign exchange regulations in that they permit transactions leading to currency inflows, but try to prevent outflows. One example of this is the strict rules against portfolio investments in foreign assets, for example, foreign government bonds. However, this rule probably has less significance for maturities of less than six months. With no restrictions on foreign loans, this rule becomes binding in aggregate terms only when the stock of outstanding loans is zero as otherwise a repayment is equivalent to giving a loan. It is therefore likely that the more liberal rules concerning foreign borrowing have contributed to making the exchange controls in general less effective.

Despite this "currency mercantilism" of the foreign exchange regulations, there is one type of inflow that has not been permitted, namely, investments in Swedish bonds and money market instruments by foreign investors. The motivation for this is that if a major stock of Swedish assets in the hands of foreign investors is allowed to accumulate, it would be a highly mobile stock. If any doubts concerning the stability of the krona were to appear, massive return inflows of these assets would occur and there would be heavy losses of foreign exchange reserves. This aspect of the regulations thus indicates a considerable degree of risk aversion on the part of the authorities.

In the spring of 1986, further deregulations were heralded by the central bank, including more liberal rules concerning early repayment of foreign loans. However, foreigners will

still be denied access to the Swedish money and bond markets. This indicates that the overriding concern is with limiting the size of the flows that can occur in a critical situation for the krona.

This rule also has another dimension, however. In addition to defending the foreign exchange reserves and, ultimately, the pegged exchange rate, the Swedish central bank puts considerable weight on trying to maintain a degree of independence for monetary policy. The position taken by the central bank has been that if the domestic markets are opened up to foreign investors, they would be more effective in pushing Swedish interest rates in line with the international level as the risky foreign exchange positions would be spread over a much greater number of individual portfolios.<sup>9</sup>

Another example of the ambition to maintain some degree of interest rate independence can be seen in the central bank's treatment of the krona exchange rate. As mentioned above, the Swedish krona is tied to an index of foreign currencies, a currency basket. This is a variant of a pegged exchange rate. There is also a benchmark value for this index, but for a long time the central bank did not specify the range of deviations from this benchmark that it would permit and the actual variability was substantial.<sup>10</sup> In June 1985 a band within which the index would be kept was announced, but it was still relatively wide (+/- 1.5 percent) which means that a considerable degree of uncertainty concerning the future value of the krona remains. Other things equal, this can be expected to decrease agents' willingness to take positions in the Swedish krona and, hence, to increase the possibility for making domestic interest rates deviate from the international level.<sup>11</sup>

There can thus hardly be any doubt that the Swedish authorities want to maintain some latitude for an autonomous monetary policy. A highly interesting question is whether it is desirable to have the opportunity to pursue an independent policy. Here no very clear statements have been issued by the policy authorities which means that it is difficult to tell what they want to use the autonomy for. This may of course depend on the state of affairs discussed in the introduction, namely, that it is hard to say anything very specific about what the effects of monetary policy are.

The purpose of this brief review has been to sketch major traits in the development of Swedish monetary policy in recent years, in particular, the shift from a system based almost entirely on regulatory instruments to one that uses the financial markets to attain the desired policy targets. The speed with which this almost complete turn-around has been made is undisputably quite impressive. Suggestions about a deregulation had been made repeatedly over the years, most emphatically perhaps in the government committee report SOU 1982:52. In its proposals, the committee was rather modest, however, emphasizing that the road to deregulation would be very long. Formally, the committee report was also ignored, for example, its proposal for a revision of the law on monetary policy measures. In practice, however, the deregulation has already gone much further than the committee at that time thought it advisable to suggest.

It bears emphasizing that the whole process of deregulation has taken place within an unchanged legal framework, i.e., "LKM" is still in force. The important changes have thus been in the rest of the institutional framework and, not least, in the policy makers' views about what constitutes an effective monetary policy. For example, this has meant that the constraints previously imposed on monetary policy by the ambition

to use it for allocative and distributive purposes have been lifted and other (and more direct) instruments are used to achieve such goals.

There is no way for an outsider to determine whether the authorities were aware of the forces that were set free when the money market was allowed to develop. (As a matter of fact, it is not obvious that anyone was.) However, they have been willing to adapt and, in a sense, take advantage of it, because there is reason to believe that the new system has made it possible to use monetary policy more effectively in macro-economic policy and with less harmful consequences for the allocative efficiency of the financial system.

While it is difficult to object to this development, it should be noted that it is not obvious that the authorities had a real choice in this matter. Given the tensions that had appeared in the regulatory system as a result of the financial imbalances in the economy, an attempt to continue to patch up the regulations where a leak appeared would have made it necessary to impose very rigorous restrictions on virtually all forms of economic activity. In particular, also non-financial companies would have had to be subjected to regulatory policy measures. Whether this would have been sufficient to stop the leaks is unclear, but the costs in terms of resource misallocation would have been substantial.

### 1.3 SUMMARY AND MAIN CONCLUSIONS

The following chapters present analyses of various aspects of monetary policy and interest rate determination in both regulated and deregulated systems, issues that have been brought to the fore by the development discussed in Section 1.2.

Chapter 2 is devoted to a theoretical analysis of one of the most important regulatory instruments, the so-called liquidity ratio. It specifies the fraction of total assets that banks must hold in the form of bonds and reserves and can thus be interpreted as an extended reserve requirement. The analytical framework is a conventional portfolio model. It is used to study the effects of changes in the liquidity ratio on a set of possible intermediate target variables, namely, interest rates, money supply, and the volume of credit. An important feature of the model is that it acknowledges the possibility for banks to engage in active liability management, i.e., meet an increase in the liquidity ratio by attracting new deposits rather than cutting back their lending to the public. They can do this by issuing marketable deposit instruments, CDs. Interpreted in terms of the development of the Swedish system, the model can be said to give a stylized representation of a situation when a money market is developing, making active liability management possible, but monetary policy relies on regulatory instruments. The model also considers the effects that bank liability management has on the unregulated intermediaries.

The analysis shows that the liquidity ratio has the intended effect on market interest rates in the sense that an increase in the required ratio raises the general level of interest rates. The effect on the money supply, on the other hand, is ambiguous and quite possibly perverse. In particular, if banks primarily react by issuing CDs, thereby protecting their customers from credit contractions, a higher liquidity ratio may result in an increase in the money supply. The effect on total credit, including lending from non-bank intermediaries, is also ambiguous. These ambiguities mean that the efficacy of the liquidity ratio depends to some extent on the choice of intermediate target, but the analysis indicates that the standard argument that a credit reduction is achieved is not necessarily correct in a framework where liability management is possible.

In Chapter 3 attention is turned to problems of monetary policy in a deregulated system. It analyzes the role of discount window borrowing, in particular, the effects of the conditions under which banks are given access to reserve borrowing from the central bank. One of the classical functions of a central bank is to act as "lender of last resort" in order to prevent massive bank failures. In many countries, central bank lending has become an important part of monetary policy, however, and the central bank lending rate - the discount rate - is used as a policy instrument.

The role of the discount window is analyzed in this chapter using a two interest rate money market model, where an explicit supply function for borrowed reserves is introduced. This makes it possible to study the effects of varying degrees of openness of the discount window on the usefulness of various policy instruments and on how the system is affected by non-policy shocks. The model determines simultaneously the money market rate and the interest rate in the overnight (interbank) market for borrowed reserves. An important feature of the model is that interbank loans and borrowed reserves from the central bank are treated as perfect substitutes. This follows from the fact that there are no intertemporal constraints on reserve borrowing, in the sense that borrowing today does not limit access to borrowing tomorrow. It is argued that this assumption is representative of central bank procedures in most European countries.

After a general analysis, the model is used to study the special case of a completely open discount window, i.e., unlimited access to borrowed reserves at a constant lending rate, a system that was applied in Sweden until December 1985. It is shown that this system, especially if combined with a lending rate that is kept constant for extended periods of time, reduces the efficacy of open market policies. Changes in the supply of non-borrowed reserves will be offset by adjustments of



reserve borrowing and an effective interest rate peg is achieved. On the other hand, changes in the pegged discount rate, when they do occur, have a direct impact on market interest rates and financial stock variables. It is also shown that this type of policy may tend to make the foreign exchange reserves more sensitive to shocks originating from abroad.

These results are then applied in a discussion of discount window policies in Sweden. It is concluded that given the central bank's explicit ambition to stabilize foreign currency flows, the choice of permitting unlimited access to borrowed reserves is relatively inefficient. The operating procedures introduced in December 1985 is seen to correspond to the general version of the model and has several advantages. They make it possible to use open market interventions, which provides more flexibility than the system with a pegged lending rate as the only useful instrument. Policy can therefore be more quickly adjusted to offset disturbances to the financial markets and should therefore improve monetary control, i.e., make it possible to increase the stability of quantitative intermediate targets.

In Chapter 4, the analysis of discount window policy is taken one step further. The model in Chapter 3 is deterministic and the discussion of random shocks is therefore based on ad hoc reasoning. Chapter 4 addresses the question of whether there is an optimal discount window policy within an explicitly stochastic framework. The model, which is otherwise similar to that used in Chapter 3, extends previous studies of optimal monetary policy by considering the simultaneous choice of open market strategy and discount window policy, i.e., the central bank is assumed to operate both in the money market and the interbank reserve market.

The (intermediate) target that the central bank is trying to stabilize is the money supply. It is assumed that the money

stock is observable only with a lag, which means that the central bank must extract information about underlying disturbances by monitoring the endogenous interest rates, both of which are instantaneously observable. The optimal policy is derived by finding values of the parameters in the two reaction functions, for open market interventions and discount window policy, that minimize the variance of the money supply (around its target path).

It turns out that no unique optimal strategy exists. There are (at least) three combinations of policy parameters that each gives the same (conditional) target variability. The first strategy involves allowing unlimited reserve borrowing. Open market policies are adjusted to information in the money market rate in a certain proportion determined, *inter alia*, by the stochastic structure of the system. In the second optimal strategy, the choice of discount window policy is immaterial, i.e., only open market operations need respond to interest rate changes. In the third case, on the other hand, the open market strategy is arbitrary and policy is formulated only in terms of the rules for discount window borrowing.

The equivalence between these three cases in terms of their effect on monetary stability means that one cannot say anything in general about the role of the discount window. However, the analysis shows that, in general, it is inefficient for the central bank not to react to information in both interest rates.

Chapter 5 is the first in a sequence of three chapters studying the determinants of international interest rate dependence in an open economy. In particular, the analysis focuses on the role of the foreign exchange risk premium. The basic issues and the motivation for studying this risk premium are outlined in Chapter 5. The starting point is a set of international interest rate parity conditions. The most important of these

is the condition for uncovered interest parity (UIP), which says that the difference between the domestic and foreign interest rates must correspond to the expected rate of depreciation of the domestic currency. This means that if UIP holds, the only way to affect the domestic interest rate is by changing the expectations about the future course of the exchange rate. For a country with a basically fixed exchange rate, fulfillment of the UIP condition would imply a severe restriction on monetary policy.

In principle, UIP requires that investors are risk neutral, i.e., indifferent to the variability in asset returns, and it is noted that this must be considered a highly extreme assumption. In general, the interest rate relation will therefore also include a risk premium and provided that this can be systematically affected, the domestic interest rate can be controlled even under a basically fixed exchange rate.

It is shown that the results from the model in Chapter 3 can be reinterpreted in terms of the effects of policy interventions on the foreign exchange risk premium. In particular, it is seen that the standard assumption of imperfect substitutability creates a direct link between open market policies (changes in asset stocks) and the (implicit) risk premium.

The portfolio model gives one simple and quite plausible story about the determination of the risk premium. However, it is based more on assertion than a careful analysis of when and why a risk premium of this type will emerge. Such an analysis requires a different class of models in which it is possible to determine, for example, the conditions under which foreign and domestic assets are treated as perfect substitutes, i.e., when the risk premium is zero.

This problem is addressed in Chapter 6 which starts with a review of the theoretical literature on the nature of the

risk premium. The natural framework for this analysis is a model of international asset pricing. The chapter discusses the determination of the risk premium in different types of assets pricing models ranging from a mean-variance optimization model to a complete general equilibrium formulation. It is concluded that in models in the conventional CAPM tradition (with non-stochastic investment and consumption opportunities) a connection between asset stocks and the risk premium emerges that is similar to the mechanism postulated in conventional portfolio models. This thus implies that an open market operation, by changing the stock of domestic assets (outside the central bank) can affect the risk premium and therefore the domestic interest rate. In principle, this is also a testable hypothesis.

In more general asset pricing models, such as the consumption CAPM and complete general equilibrium models, the results are less clearcut. It is shown that the risk premium depends in a fairly complicated way on the stochastic structure of the model, but this does not allow any conclusions as to the effects of monetary policy. In fact, the generality of these models means that it is difficult even to define what is to be meant by monetary policy.

Their generality also means that they do not give rise to hypotheses that are directly amenable to empirical testing. This fact is reflected in the empirical studies on the nature of the foreign exchange risk premium that are reviewed in the second part of Chapter 6. There are two main approaches to testing for a risk premium. One can be classified as atheoretical in that it is based on pure time-series tests, either of UIP or of the unbiasedness of the forward exchange rate as a predictor of the future spot rate. The purpose is thus limited to finding out whether there is a risk premium in these relations. In the other approach, attempts are made to explain the implied premia in terms of specific asset pricing

models. These tests are either based on CAPM results or on hypotheses that are consistent with rather than explicitly derived from more general asset pricing models.

The review indicates that many authors have found evidence of risk premia, i.e., deviations from UIP or from unbiasedness. However, it has proved considerably more difficult to explain the implied premia in terms of the variables on which they are supposed to depend according to asset pricing theory.

Chapter 7 contains an analysis of the foreign exchange risk premium in a currency basket system, i.e., a system where the exchange rate is related to an index of other currencies. As usually applied, this system must be interpreted as a variant of a basically fixed exchange rate. At least, it must be said to imply that the authorities are unwilling to use exchange rate changes as part of the transmission mechanism.

It is argued that if the currency index were kept completely stable, there would be no more (or less) policy independence than under a regular fixed exchange rate. However, as applied in practice, there is usually some degree of flexibility in that the currency index is allowed to vary around some benchmark value. This makes it meaningful to discuss exchange rate risk and, consequently, to study the determination of a risk premium in such a system.

The first step in the analysis is to study the determination of the risk premium in an explicit asset pricing model, based on mean-variance optimization, where one country pursues a currency basket policy. As the ultimate purpose is to apply these results to Sweden, the pegged currency is referred to as the krona. It is shown that bilateral interest rate relations, e.g., between krona and dollar interest rates, will contain a risk premium even if the currency index is perfectly pegged as long as international exchange rates are variable, i.e., the dollar fluctuates relative to non-krona currencies.

However, these international premia are outside the control of the policy makers in a small country. The existence of such premia therefore has no implications for the degree of independence of monetary policy.

For empirical purposes the important conclusion from the theoretical model is that if a risk premium that (potentially) is controllable by means available to the domestic authorities is to be identified, one must study the relation between the krona interest rate and a currency weighted portfolio. In tests of bilateral relations, one may find premia that are due entirely to international exchange rate variability and, hence, irrelevant from the point of view of interest rate dependence. This insight is applied in a test for the existence of a risk premium on the Swedish krona. The test can be characterized as atheoretical in terms of the classification used in Chapter 6, as it involves a time-series test of the unbiasedness of a basket weighted index of forward exchange rates as a predictor of the future value of the currency index. For comparison, results from bilateral tests are also reported.

The tests indicate significant deviations from unbiasedness for two out of three bilateral relations and for the currency index. The latter finding in particular is thus consistent with the existence of a risk premium on the currency basket. This finding also appears plausible given the actual variability in the Swedish currency index. In the conclusions it is emphasized that this result is far from sufficient to establish that Swedish monetary policy can be effectively used to control the domestic interest rate. To introduce a risk premium, all one has to do is let the index vary within wide limits, but it is far more difficult to determine whether the size of the premium can be influenced in any systematic way. That question is left for future research, however.

In Chapter 8 attention is turned to the term structure of interest rates, i.e., the relation between the interest rates on assets with varying times to maturity. In the preceding chapters, no explicit attention is paid to the relation between short and long term interest rates. This chapter is a first step in a richer analysis of these issues, which are important also for the discussion of monetary policy. For example, the behavior of the term structure relation determines how policies directed at the short end of the market, such as discount window rules, affect longer term interest rates that may be more important for both intermediate and ultimate target variables.

One common starting point for the discussion of the term structure is the pure expectations hypothesis, which says that the sole determinants are the market's expectations about future spot interest rates. This implies in particular that the forward rates that are implicit in the term structure are unbiased predictors of future spot rates. In general, however, forward rates will also contain risk premia, i.e., expected excess returns to investors who take on reinvestment risk, for example. The emphasis in this chapter is on studying the relative importance of expectations of future spot interest rates and risk premia for the behavior of the term structure.

The first step is to study directly the predictive power of the implicit forward rates. Forward rate forecasts are compared to the predictions from a simple martingale model. It is shown that the forward rates perform no worse and in some cases even better than the martingale model, which is in contrast to findings in similar studies on U.S. data. Also in contrast to previous results is that ex post excess returns (risk premia) are consistently (although only rarely significantly) negative. This indicates that the market has paid a premium to investors who have chosen to hold one-period assets.

The simple forecast comparisons are complemented by regression tests on forward rates as predictors of the development of future spot rates. These tests give a direct indication of the relative importance of expected future spot rates and expected premia as determinants of the forward rates. The first set of tests shows that the forward rates contain information about the levels of future spot interest rates even though they are not unbiased predictors, i.e., the pure expectations hypothesis can be rejected. The fraction of the variation in the forward rate that is attributable to variations in expected future spot rates is highest in the one-month ahead forecasts. The one-month forward rates also contain information about future excess returns and the importance of future premia increases as the forecast horizon is lengthened.

In a second set of tests, the forward-spot rate differentials are shown to have low power as predictors of both the changes in the monthly spot rates and the future premia. However, as something of an anomaly, it turns out that it is not possible to reject the hypothesis that the three-month forward rate (the longest horizon studied) gives unbiased predictions of the change in the quarterly spot rate. In this case, the forward rate thus seems to be completely dominated by the expectations component. Nevertheless, the overall impression from the regression tests is that the relative importance of the expected future spot rates decreases as the forecast horizon is lengthened.

The concluding comments point to the need for further studies of the term structure relation. In particular, an integrated analysis of the term structure and the problems of international interest rate dependence discussed in Chapter 7 is seen as an important next step in the future work on monetary policy and interest rate determination.



## NOTES

1. For a detailed discussion (and critique) of the policy procedures outlined her, see Bryant (1980), who also gives a broad presentation of the literature. For a more recent (and more positive) review of monetary targeting, see McCallum (1985).
2. A more detailed summary of the contents of the book is given in Section 1.3.
3. More details on these aspects can be found (in Swedish) in SOU 1982:52, Myhrman (1979), and Jonung (1982). A review of recent problems and issues in Swedish monetary policy is presented by Hörngren (1985). Note also that the institutions and regulations discussed in this section are those that have been motivated and used as instruments of monetary policy. Additional rules, for example, the bank law that specifies capital adequacy requirements, etc., are best seen as structural regulations. This type of legislation undoubtedly raises a number of interesting questions, but these are preferably treated separately from the monetary policy aspects and will therefore not be discussed in the current context.
4. This instrument is sometimes also referred to as a secondary reserve requirement. Its effects will be analyzed in Chapter 2.
5. For a discussion of how to model the Swedish system in the 1970s and whether the regulated interest rates meant that there was "disequilibrium" credit rationing, see Myhrman (1973, 1975), and Ettlin and Lybeck (1975).
6. For descriptions of the development of the Swedish money market, see Nyberg (1982), or Kvist, Nyberg and Wissén (1985).

7. The role of the discount window policy, with special emphasis on the Swedish system, is analyzed in Chapter 3.
8. For a detailed presentation (in Swedish) of the system of foreign exchange controls, see the government committee report SOU 1985:52. However, note that changes in the rules and in their implementation have already been made and this deregulation can be expected to continue; cf. the discussion below.
9. Cf. Franzén (1985). Much more on this issue and on Sweden's interest rate dependence will be said in Chapters 5-7.
10. This policy was criticized by Hörngren and Viotti (1985). See also Franzén (1985).
11. This assertion is discussed and motivated in Chapters 5-7.

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## 2. Regulatory Monetary Policy and Uncontrolled Financial Intermediaries\*

### 2.1 INTRODUCTION

In textbooks and in much of the theoretical literature, discussion of monetary policy focuses on open market operations as the most important policy instrument. However, in many small countries bond markets are too thin to allow any sizable open market operations. Instead, the central banks frequently use various regulatory measures. Despite their widespread use, these instruments have received very limited attention. It is this gap between theory and practice that this chapter tries to narrow.

The regulatory measures are typically selective and limited to the commercial banks. Often supplemented by constraints on interest rates, they give the central bank considerable control over bank behavior. However, control of the banks is not equivalent to controlling the financial sector, since there are also other intermediaries. They may be expected to offset the direct effects of the regulation. This line of reasoning has led some observers to question the efficacy of regulatory monetary policy (see, e.g., Hodgman 1972).

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\* This chapter is a modified version of Hörngren (1985).

In this chapter I introduce an additional factor that might mitigate the effects of regulatory instruments. This is the opportunity for banks to use active liability management, that is, in the face of tighter regulations attract loanable funds by issuing money market liabilities. This option allows banks to circumvent the direct effects of some of the most common regulations. Since the nonbanks raise their funds on the money market, the introduction of bank liability management also affects the role of the unregulated intermediaries.

The purpose is to investigate the effects of regulatory instruments in a framework with unregulated intermediaries and bank liability management. As a prerequisite for bank liability management is that there are large-scale deposits with market determined interest rates, the analysis can be interpreted as showing the effects of regulatory monetary policy in a system where at least a rudimentary money market exists. In this sense, the model attempts to capture one stylized feature of the Swedish system in the early 1980s when a money market was developing, but the central bank still relied almost exclusively on regulatory instruments. The analysis focuses on one of these, the liquidity ratio<sup>1</sup>, which forces banks to hold a certain fraction of their assets in the form of bonds and reserves.

The outline of the chapter is as follows: Section 2.2 contains a discussion of the difference between the liquidity ratio and a regular reserve ratio requirement and a summary of previous work; in Section 2.3 the model is presented; the results are analyzed in Section 2.4. Finally, some tentative conclusions are attempted.

## 2.2 THE LIQUIDITY RATIO AS A POLICY INSTRUMENT

The liquidity ratio is one example of the regulatory instruments used in various countries (see Hodgman (1974) and Myhrman (1973) for more examples). Like many of these, it has been rationalized in terms of the availability theory of credit (see Lindbeck 1963). By forcing banks to hold more government bonds, it is argued that credit to the public can be limited. One counterargument emphasized by Lindbeck (1963) is that the requirement typically is imposed only on commercial banks and has no direct effects on other intermediaries. However, he does not discuss the indirect effects that are important in the present model.

In a critical analysis of the use of the liquidity ratio, Myhrman (1973) stresses the importance of the effect on the money supply. He uses a modified Brunner and Meltzer (1964) model with banks as the only intermediaries. Liability management is not possible, which means that the direct effect of a higher ratio is a shift from loans to bonds in bank portfolios. The public's borrowing is reduced but this is matched exactly by the sale of bonds to the banks, leaving the money stock unchanged. Hence, only the substitutions caused by interest changes matter. Myhrman shows that these elasticities are of opposite signs and argues that both are small in absolute value. His conclusion is that the liquidity ratio does not have any significant effect on the money stock and therefore is ineffective.

Myhrman's result shows that a liquidity ratio is quite different from a regular reserve ratio requirement, which, although this is not discussed, clearly would affect the money supply. Essentially, this stems from the different nature of the assets

included in the numerators of these ratios. Under a reserve ratio the banks have to attract cash from the public if reductions of loans and deposits outstanding are to be avoided. Since the own rate on cash is institutionally fixed at zero and the cross-rate elasticities of the public's cash ratio presumably are quite small - Myhrman assumes that they are zero - this would require substantial interest rate changes, invalidating such an attempt to keep lending fixed.

Under a liquidity ratio, banks have to attract bonds, which is much easier since the own rate can be affected. With smaller changes in interest rates, the effect on the money stock (deposits) is also smaller. The crucial difference here is that bonds have a variable interest rate. Therefore the liquidity ratio is not just an extended reserve ratio requirement but can be expected to exert a very different influence on the financial system. The purpose of this chapter is to analyze these effects in a setup that acknowledges the existence of unregulated intermediaries and liability management.

### 2.3 THE PORTFOLIO MODEL

The effects of the liquidity ratio will be analyzed using a closed economy, general equilibrium portfolio model of conventional type (see, e.g., Tobin and Brainard 1963). The model is static and it is partial in the sense that the interdependencies with nonfinancial variables are disregarded. This implies that it must be interpreted as a short-run equilibrium model - it covers a time period so short that dynamic phenomena, such as asset accumulation, and the feedbacks from nonfinancial developments can be ignored. In this type of model it is also important to distinguish between markets that are fully flexible and others where interest rates and/or quantities are fixed in the short run. Okun's (1981) distinction between auction and customer markets is also relevant for financial assets since some markets, notably for bank loans and demand



deposits, are characterized by long-term commitments between banks and their customers. On the other hand, the impersonal trade in bonds and certificates of deposit takes place in typical auction markets. The implications of this distinction will be made clear in the following presentation of the behavioral assumptions for the agents identified, namely, banks, nonbank intermediaries, the public, the central bank, and the Treasury.

### The Banks

The assets of the banks are government bonds ( $B^b$ ), loans to the public ( $L^b$ ), and reserves ( $R$ ). They finance these holdings by accepting demand deposits ( $D$ ) and issuing certificates of deposit ( $C^b$ ). Bank equity is disregarded. This gives the following balance sheet constraint:

$$B^b + L^b + R = D + C^b \quad (2.1)$$

Bank portfolios are directly influenced by the central bank through a liquidity ratio requirement ( $q$ ). It regulates the fraction of total deposits banks must allocate to bonds and reserves, that is,

$$B^b + R = q(D + C^b); \quad 0 \leq q < 1 \quad (2.2)$$

The liquidity ratio is assumed to be strictly binding. This constraint also puts a limit on bank lending. Solving for  $L^b$  from (1) and (2) gives

$$L^b = (1 - q)(D + C^b) \quad (2.3)$$

An increase in  $q$  forces banks to hold more eligible assets, that is, reserves or bonds. There are two ways to achieve this. (i) They can reduce  $L^b$ . The termination of loans forces the public to sell bonds, which are then absorbed by the banks.<sup>2</sup>

This is just a shift on the asset side of the banks' balance sheet between  $L^b$  and  $B^b$ . This is the case considered by Myhrman (1973). (ii) Banks can issue certificates of deposits that are more attractive than bonds. The public switches from bonds to certificates, again giving the banks the bonds they are required to hold. A combination of these two alternatives is also possible.

By using the second alternative, active liability management, banks can circumvent the direct effects of the liquidity ratio and no reduction of loans outstanding is necessary. One rationale for such a behavior is that it is costly for a bank to cut back credit. As discussed by Wood (1974), premature termination of loans or refusal to fulfill agreements on loans not yet paid out may upset established customer relations. If dissatisfied customers decide to take their business elsewhere, this will lead to future losses of deposits. Wood's model illustrates the customer market characteristics of bank loans and deposits. This implies that it may be sensible for a bank to accommodate the need for more "liquid" assets by increasing the supply of certificates. Presumably this is primarily a short-run effect since certificates are an expensive source of funds, but as discussed above the short run is the relevant time horizon for this model.

Another important consideration in this context is whether the change in  $q$  is expected to be temporary or permanent. If the liquidity ratio is used as a policy tool and is varied over the business cycle, banks will perceive the changes as temporary. This would be an additional reason to keep lending fixed and to issue certificates that can be recalled when  $q$  is lowered again. On the other hand, with a permanent rise in  $q$  banks will immediately start to readjust their portfolios and, as loans mature, reduce their certificate liabilities. The implication is that frequent changes in the liquidity ratio would reduce the impact on bank lending as banks learn

to bridge periods with high ratios by liability management. Of course the distinction between temporary and permanent events and this learning process are dynamic aspects and therefore impossible to treat explicitly in this model. They are introduced only as explanations of the kind of behavior modeled. Here the banks' liability management policy is captured by constant parameters.

These considerations lead to the following specification of the bank's asset demand functions:

$$B^b = B^b(r_B^+, r_L^+ - r_C^+, q) \quad (2.4)$$

$$R = R(r_B^-, r_L^+ - r_C^+, q) \quad (2.5)$$

$$L^b = L^b(r_L^+ - r_C^-, q) \quad (2.6)$$

$$C^b = C^b(r_L^+ - r_C^+, q) \quad (2.7)$$

where  $(r_B, r_L, r_C)$  are the interest rates on bonds, loans, and certificates, respectively. For convenience, it is assumed that only the interest rate spread  $(r_L - r_C)$  affects the banks' choice of liability management policy, which means that  $L_L^b = -L_C^b$ ,  $C_L^b = -C_C^b$ , etc.<sup>3</sup> (Subscripts will be used to denote partial derivatives, i.e.,  $L_L^b = \partial L^b / \partial r_L$ ,  $C_C^b = \partial C^b / \partial r_C$ , etc.)

There is no function describing desired demand deposit liabilities, which implies that banks accept whatever amounts of deposits the public chooses to make. The interest rate on demand deposits is fixed (at zero, for simplicity).

The inclusion of the liquidity ratio as an argument in (2.4)-(2.7) reflects the fact that an increase in  $q$  forces banks to readjust their portfolios. Their liability management policy, implicitly determined by the costs of reducing lending and

the perceived nature of the change in  $q$  as discussed above, is captured by the partials  $C_q^b \geq 0$  and  $L_q^b \leq 0$ .<sup>4</sup> However, interest rates also influence this decision. For example, the total derivative of (2.7) is

$$\frac{dC^b}{dq} = C_q^b + C_L^b \left( \frac{dr_L}{dq} - \frac{dr_C}{dq} \right) \quad (2.8)$$

which shows that the banks' willingness to issue certificates is reduced if  $r_C$  rises relative to  $r_L$ . This implies a correspondingly larger reduction in  $L^b$ .

The bond rate does not appear in (2.6) and (2.7) because of the constraints imposed by the liquidity ratio. Since the requirement is assumed to be strictly binding, the banks can only shift between bonds and reserves on the one hand and between loans and certificates on the other.<sup>5</sup> Nevertheless,  $r_L$  and  $r_C$  affect  $B^b$  and  $R$  indirectly via  $C^b$ . Because of the liquidity ratio, any change in  $C^b$  makes corresponding adjustments in bonds and/or reserve holdings necessary, since from (2.2)

$$qC_i^b = R_i + B_i^b, \quad i = C, L \quad (2.9)$$

It turns out that it is useful to identify some special cases of liability management policy. First, consider a policy of fixed bank lending. This implies  $L_q^b = 0$ . If banks have this accommodative policy, it is natural to assume that it is also unaffected by interest changes, that is, that  $R_L = C_L^b = L_L^b = 0$ . In the opposite extreme case, where banks are unwilling to issue certificates ( $C_q^b=0$ ), they can also be expected to be sensitive to interest changes. Given the complexity of the model, these two special cases are useful for interpreting the results.

### Nonbank Intermediaries

The most important characteristic of the nonbanks in this context is that their activities are not regulated by the central bank. In order to focus on this aspect their portfolios are highly simplified, including only one asset, loans to the public, and one liability. Since nonbank debt is assumed to be a perfect substitute for banks' certificates, it is denoted by  $C^f$ .<sup>6</sup> Hence, the balance sheet constraint is  $C^f - L^f = 0$ . The portfolio decisions of the nonbanks are governed by the following functions:

$$C^f = C^f(r_L^+ - r_C) \quad (2.10)$$

$$L^f = L^f(r_L^+ - r_C) \quad (2.11)$$

### The Public

The public includes the entire nonfinancial private sector, that is, both households and firms. Their assets are bonds, demand deposits, and certificates of deposit. Certificates issued by banks and nonbanks are perfect substitutes. For simplicity, cash holdings are disregarded. The liabilities are loans from banks and nonbanks, also treated as perfect substitutes. Holdings of real capital are ignored. Consequently, the balance sheet constraint is

$$W = B^p + C + D - L \quad (2.12)$$

where  $W$  is net public wealth.

The asset supply and demand functions are

$$B^p = B^p(r_B^+, r_L^-, r_C^-, Y, W) \quad (2.13)$$

$$C = C(r_B^-, r_L^-, r_C^+, Y, W) \quad (2.14)$$

$$L = L(r_B^+, r_L^-, r_C^+, Y^+, W^+) \quad (2.15)$$

$$D = D(Y^+) \quad (2.16)$$

where  $Y$  is a transactions variable, for example, income. The signs of the partial derivatives in (2.13)-(2.16) reflect an assumption of gross substitutability. In addition, it is assumed that own rate effects dominate cross effects. That implies, for example, that  $B_B^D > |B_L^D|$  and  $B_B^D > |B_C^D|$ .

The assumption in (2.16) that  $D$  depends only on income might seem restrictive. In effect it implies that  $D$  is exogenous and that the analysis could be carried through without ever introducing demand deposits. The distinction between two types of deposits is made for expositional reasons. The point is that banks typically have part of their funds from established customers. These demand deposits bear a low interest rate and losing them because of loan terminations would be costly. The banks would have to rely more on the expensive source of funds, certificates, which are traded on markets where established relations do not matter.<sup>7</sup>

### The Central Bank and the Treasury

The central bank sets the liquidity ratio and controls the supply of bank reserves subject to the balance sheet constraint  $B^C - R = 0$ , where  $B^C$  is the central bank's holdings of bonds.

The Treasury issues bonds to finance any government budget deficits. Accumulated deficits have given rise to a supply of bonds equal to the government debt,  $G = B^D + B^b + B^C + B^f$ . In the absence of real capital, the government debt is also identical to the public's net wealth.

### Equilibrium Conditions

The model consists of markets for bonds, loans, certificates of deposit, and base money (bank reserves). On these four markets three interest rates are determined ( $r_B$ ,  $r_L$ ,  $r_C$ ) and given these the equilibrium stocks of all assets can be computed. The exogenous variables are the central bank's policy parameters ( $q$ ,  $B^C$ ), income ( $Y$ ), and the government debt ( $G$ ).

It is assumed that all markets are functioning well and that interest rates are market determined. Hence, I abstract from both the thinness typical of secondary markets in countries using regulatory policy and the possibility that interest rates are also regulated. This is done at this stage to avoid the complexities involved in interest determination on inactive markets and disequilibrium analysis. Therefore, the equilibrium conditions are

$$B^C - R = 0 \quad (2.17)$$

$$L^b + L^f - L = 0 \quad (2.18)$$

$$C - C^b - C^f = 0 \quad (2.19)$$

$$B^b + B^p + B^C - G = 0 \quad (2.20)$$

A four-market general equilibrium model does not have more than three independent equilibrium conditions. Henceforth I will ignore the bond market relation (2.20). Using standard methods in comparative static analysis, it is now possible to derive the effects of changes in the exogenous variables. The differentiated system is shown in the Appendix.

## 2.4 THE EFFECTS OF CHANGING THE LIQUIDITY RATIO

In and of themselves, monetary events are rarely of prime concern in economic policy. They are interesting only to the extent that they influence the ultimate policy targets (GNP, inflation, etc.). Without a complete model of the interaction between the financial sector and the rest of the economy, the effects of monetary policy have to be gauged by an indicator variable. The problem of the choice of optimal indicator(s) is complex, however, and beyond the scope of this paper (see, e.g., Friedman (1975) and the references cited there). Here I will simply derive the effects on the possible indicator variables. In the present model there are three such variables: (i) the interest rates, (ii) the money stock, and (iii) total credit. The stated purpose of an increase in the liquidity ratio is to create a tightening of monetary policy, which in conventional macro analysis means raise interest rates and/or reduce the stocks of money and credit. In the remainder of this section the conditions for these effects to materialize will be analyzed.

### Interest Rate Effects

The effect on the bond rate of a change in the required liquidity ratio is

$$\frac{dr_B}{dq} = \frac{1}{\Delta} \left\{ R_q \begin{matrix} + & + & + & + \\ A_{22} & A_{33} & - & - \end{matrix} A_{32} \right] - R_L [L_q^b \begin{matrix} + & - & - & + \\ A_{32} & + & A_{33} & \end{matrix} + C_q^b \begin{matrix} + & + & - \\ A_{22} & + & A_{23} \end{matrix}] \right\} \quad (2.21)$$

where  $\Delta > 0$  and the  $A_{ij}$ 's are defined in the Appendix. The effect on  $r_B$  depends essentially on what happens to the demand for reserves. The first term, which is positive<sup>8</sup>, captures the banks' desire to hold a certain fraction of their liquid assets in the form of reserves. If  $q$  is raised, they would



want to increase  $R$  but since the supply is fixed, this is impossible. Instead,  $r_B$  has to rise to make them satisfied with a lower reserves/bond ratio. This effect is modified by the change in  $r_L - r_C$ , captured by the second term in (2.21). Its sign depends on the relative size of  $L_q^b$  and  $C_q^b$ . Specifically, it is positive if

$$\frac{L_q^b}{C_q^b} > \frac{-(L_C + L_L)}{C_C + C_L} \quad (2.22)$$

If  $L_q^b$  is large (in absolute value) relative to  $C_q^b$ ,  $r_L - r_C$  tends to rise, which, other things being equal, increases the demand for  $R$  and makes an additional rise in  $r_B$  necessary. If (2.22) does not hold, the sign of  $dr_B/dq$  is ambiguous. However, note that a high value of  $C_q^b$  (i.e., if banks primarily react by issuing certificates) implies a high value of  $R_q$ , since

$$R_q = C_q^b - L_q^b - B_q^b = qC_q^b + (D + C^b) - B_q^b$$

from the adding up constraint. Hence, as the second term drops, the weight of the positive first term rises. Also, the second term is weighted by  $R_L$ , which is likely to be small, particularly for low values of  $q$ , since from (2.9)

$$R_L = qC_L^b - B_L^b \quad (2.9')$$

For these reasons one can conjecture that  $dr_B/dq > 0$  holds.<sup>9</sup> Intuitively this is also the most plausible outcome since there will be a net increase in liquid asset holdings, unless the banks' desire to issue new certificates, as captured by  $C_q^b$ , drives up  $r_C$  to such a level that the countervailing interest effect more than offsets this and  $C^b$  actually falls. This seems highly unlikely, which means that  $r_B$  can be expected to rise.

The effects on the loan and certificate rates are

$$\frac{dr_L}{dq} = \frac{1}{\Delta} [R_q(A_{23}A_{31} - A_{21}A_{33}) + L_q^b(R_L A_{31} + R_B A_{33}) + C_q^b(R_L A_{21} - R_B A_{23})]$$

$\begin{array}{ccccccccccc} + & + & - & - & - & + & - & + & - & - & + \\ & & & & & & & & + & + & - & - & - \end{array}$

(2.23)

$$\frac{dr_C}{dq} = \frac{1}{\Delta} [R_q(A_{21}A_{32} - A_{22}A_{31}) + L_q^b(R_L A_{31} - R_B A_{32}) + C_q^b(R_L A_{21} - R_B A_{22})]$$

$\begin{array}{ccccccccccc} + & + & - & - & + & - & - & + & - & - & + \\ & & & & & & & & + & + & - & - & + \end{array}$

(2.24)

As is clear from the signs indicated in (2.23) and (2.24), the terms containing  $R_q$  and  $L_q^b$  are all positive. The only ambiguity is related to the  $C_q^b$  terms. A sufficient condition for  $dr_L/dq > 0$  is

$$\frac{-R_B}{R_L} > - \left[ \frac{L_B}{L_C^b + L_C^f - L_C} \right] \quad (2.25)$$

and  $dr_C/dq > 0$  if

$$\frac{-R_B}{R_L} > \frac{L_B}{L_L^b + L_L^f - L_L} \quad (2.26)$$

Of these conditions (2.26) is most likely to hold since the denominator on the right is the sum of three own rate effects. Provided  $R_L$  is reasonably small, that is, that the liability management policy is not extremely sensitive to interest rate changes,  $dr_C/dq > 0$  is guaranteed.

The conditions ensuring that (2.25) holds are more restrictive since it involves relative cross-derivatives. It is a sufficient condition, however, and even if  $L_q^b = 0$ , which is expected to minimize the change in  $r_L$ ,  $dr_L/dq > 0$ , since this implies  $R_L = 0$ .

The result that interest rates rise is also plausible. If the banks try to meet the higher ratio by issuing certificates, they will have to offer a higher interest rate. Specifically,  $r_C$  has to rise relative to  $r_B$  to make the public replace bonds with certificates. The rise in  $r_C$  increases loan demand, which is expected to push  $r_L$  upwards, particularly if banks simultaneously reduce their lending.

Note also that the parameter  $R_L = -R_C$ , the interest elasticity of reserve demand, is crucial for the interest effects. If  $R_L = 0$ , all interest rates rise unambiguously. The explanation is that  $R_L$  captures the interest elasticity of the banks' liability management policy.  $R_L$  is high only if  $C_L^b$  is high (cf. (2.9')). In this case, the direct effect on  $C^b$  is offset, which mitigates the increase in  $r_C$  and consequently the other interest rates also. From (2.9') it is clear that  $R_L$  rises with the level of the liquidity ratio. Hence, the conclusion that an increase in the liquidity ratio raises the general level of interest rates is stronger the lower the initial level of the ratio. The results therefore indicate that at least as long as  $q$  is kept at a reasonable level, changing it is an effective way to influence interest rates.

#### The Effect on the Money Stock

A definition of the relevant monetary aggregate is a prerequisite for an analysis of the money stock effect. By assumption, demand deposits cannot be influenced by monetary policy, which means that a very narrow definition is void of interest. Therefore money is defined to include certificates of deposit issued by banks:

$$M = D + C^b \quad (2.27)$$

This definition is undoubtedly somewhat arbitrary. It could be argued that  $C^f$  should also be included given that  $C^b$  and

$C^f$  are treated as perfect substitutes. Under current assumptions, the conclusions are not materially affected by the exclusion of  $C^f$ , but this is hardly a general result. Most money supply definitions are arbitrary, however, and the question whether this aggregate is also the "best" policy indicator cannot be addressed in this framework.

Differentiation of (2.27) with respect to  $q$  yields

$$\frac{dM}{dq} = \underset{+}{C_q^b} + \underset{+}{C_L^b} \left( \frac{dr_L}{dq} - \frac{dr_C}{dq} \right) \quad (2.28)$$

Since  $C_q^b \geq 0$ , a necessary condition for  $dM/dq < 0$  is that  $r_C$  rises relative to  $r_L$ . Such a change in relative interest rates makes banks more reluctant to issue new certificates. This tends to offset the direct effect,  $C_q^b$ , which measures the desired change in  $C^b$  at constant interest rates. Since  $dM/dq = dC^b/dq$ , this interest effect has to be so strong that the supply of  $C^b$  actually is reduced for  $dM/dq < 0$  to obtain. This requires a fall in  $L^b$  over and above the direct effect of the higher liquidity ratio. From (2.23) and (2.24) the change in  $r_L - r_C$  is

$$\frac{dr_L}{dq} - \frac{dr_C}{dq} = \frac{1}{\Delta} \left[ \underset{+}{(R_q A_{31} + C_q^b R_B)} \underset{+}{(A_{22} + A_{23})} - \underset{+}{(R_q A_{21} - L_q^b R_B)} \underset{+}{(A_{32} + A_{33})} \right] \quad (2.29)$$

In (2.29) the first term is negative and the second positive making the sign ambiguous.<sup>10</sup> It is clear, however, that a high willingness to issue certificates tends to raise  $r_C$  relative to  $r_L$  and vice versa if  $L_q^b$  is large (in absolute value). The ambiguity of (2.29) means that no general results for the money stock effect can be derived; the analysis has to be limited to some special cases.

First, assume that banks completely accommodate the change in  $q$  by issuing certificates, that is,  $L_q^b = 0$ . As discussed

in Section 2.3, this also implies  $C_L^b = 0$ , which means that (2.28) simplifies to

$$\frac{dM}{dq} = C_q^b \quad (2.30)$$

If banks choose to keep lending constant, a higher liquidity ratio clearly increases the money stock. The explanation is that the public shifts from bonds to certificates, and since only the latter is defined as money, this is tantamount to a monetary expansion.

The opposite extreme case is when banks choose not to issue certificates,  $C_q^b = 0$ . This corresponds to the traditional view of how banks react to changes in the liquidity ratio. This eliminates the direct effect in (2.28) but the change in  $(r_L - r_C)$  must be taken into account. The money stock effect is

$$\frac{dM}{dq} = \frac{C_L^b}{\Delta} \left\{ \begin{array}{ccccccc} L_q^b R_B (A_{32} + A_{33}) & + & R_q [A_{31} (A_{22} + A_{23}) & - & A_{21} (A_{32} + A_{33})] \end{array} \right\} \quad (2.31)$$

Since the first term is positive, a necessary condition for  $dM/dq < 0$  is that the term in square brackets is negative. Its sign is indeterminate,<sup>11</sup> but since  $L_q^b > R_q$ , it is doubtful whether the second term can outweigh the first. In any case, even if the banks react as the authorities want them to, that is, by reducing outstanding loans, there is definitely no clear presumption that a higher liquidity ratio is associated with a smaller money stock. On the contrary, a monetary expansion can be considered likely, particularly if banks are relatively accommodative.

Other special cases can be analyzed, but the ones already discussed suffice to show the difficulties of using an increase

in the liquidity ratio to reduce the stock of money. Intuitively, the case where the banks' basic policy is not to issue certificates is most likely to give the desired result, but not even then is the net effect clearly negative. If banks choose to fulfill the higher ratio by attracting deposits, an expansion of the equilibrium money stock is more than likely. An action intended to reduce  $M$  might thus turn out to be expansionary.

### The Effect on Total Credit

The liquidity ratio is an instrument developed essentially within an availability theory framework. Given both its origin and the possibility of using total credit as an indicator, it is interesting to derive the effect of  $q$  on the availability of credit. Once more a definition is needed, and even if bank credit is what matters in traditional availability models, total credit,  $L = L^b + L^f$ , is assumed to be the relevant aggregate in the context of the present model. Differentiation of  $L$  with regard to  $q$ , again noting that only the interest spreads matter, yields

$$\frac{dL}{dq} = L_q^b + (L_L^b + L_L^f) \left( \frac{dr_L}{dq} - \frac{dr_C}{dq} \right) \quad (2.32)$$

The change in  $r_L - r_C$  is crucial once more. If  $r_C$  rises relative to  $r_L$ , a reduction in  $L$  is certain, since  $L_q^b \leq 0$ . In the opposite case, the effect is ambiguous. As noted above, the change in the interest differential cannot be signed which means that the analysis must be confined to special cases.

First, assume that there is no direct effect on bank lending, i.e.,  $L_q^b = 0$ .<sup>12</sup> A priori this can be expected to minimize the reduction in  $L$ . Then (2.32) can be written as

$$\frac{dL}{dq} = \frac{1}{\Delta} \left\{ \begin{array}{ccccccc} L_L^f [R_q (A_{31} (A_{22} + A_{23}) - A_{21} (A_{32} + A_{33})) + C_q^b R_B (A_{22} + A_{23})] \end{array} \right\} \quad (2.33)$$

The effect on  $L$  depends only on how nonbanks respond to the change in  $(r_L - r_C)$ . The banks' issues of certificates are expected to raise  $r_C$  relative to  $r_L$  inducing nonbanks to reduce both  $C^f$  and  $L^f$ . A sufficient condition for this is that condition (A2.2) in the Appendix holds, but  $dL/dq < 0$  must be considered likely in any case as  $C_q^b > R_q$ .

Second, consider the opposite extreme case,  $C_q^b = 0$ , when (2.32) becomes

$$\begin{aligned} \frac{dL}{dq} = & L_q^b \left\{ 1 + \frac{1}{\Delta} [R_B (L_L^b + L_L^f) (A_{32} + A_{33})] \right\} + \\ & + \frac{R_q}{\Delta} \left\{ (L_L^b + L_L^f) [A_{31} (A_{22} + A_{23}) - A_{21} (A_{32} + A_{33})] \right\} \end{aligned} \quad (2.34)$$

As it can be shown that the first term in (2.34) is negative, the sufficient condition for  $dL/dq < 0$  is the same as in the previous case, i.e., that (A2.2) holds making also the second term negative. Hence, it seems that the liquidity ratio can be used to reduce the supply of credit even in the presence of unregulated intermediaries.

The analysis has shown that nonbank intermediaries play an interesting role in the impact on total credit. First, it is clear that the argument that nonbanks automatically fill gaps left by regulated banks is not generally valid. It ignores the fact that the cost of funds for nonbanks is affected by how banks react. If they issue certificates,  $r_C$  tends to rise relative to  $r_L$  and nonbanks will contribute to a reduction of total credit. On the other hand, if banks choose to reduce  $L^f$ ,  $r_L$  will rise and nonbanks will expand their lending. This shows that unregulated intermediaries can be expected to act as a buffer, moderating the impact of policy changes and bank behavior on the financial markets. Moreover, from the central bank's point of view their activities are not necessarily detrimental. This is a somewhat different role than the one typically assigned to unregulated intermediaries by central banks arguing for extended regulations.

## 2.5 CONCLUDING REMARKS

In this chapter I have analyzed a financial general equilibrium model with regulated and unregulated intermediaries. The purpose has been to highlight two possible impediments to the use of bank regulations as instruments of monetary policy, namely, the existence of nonbank intermediaries and the opportunity for banks to circumvent the regulations by active liability management. To capture these phenomena a number of simplifications of other aspects have been made. Still, it has not been possible to derive unambiguous qualitative results in some cases. However, while not giving the final answers to all questions concerning the efficacy of regulatory monetary policy, the analysis allows some tentative conclusions.

The results on the effects of the liquidity ratio can be summarized as follows. First, it was shown that an increase in the ratio leads to higher interest rates. Hence, there is no apparent need for a central bank that uses  $q$  to influence interest rates to reconsider its choice of policy instrument. Second, the change in total credit cannot be unambiguously determined, but there is a presumption that the net effect is negative. Third, the analysis indicates that the effect on the money stock is at best uncertain and quite possibly perverse in that an increase in the ratio might lead to a monetary expansion.

There are clear ambiguities in the signals given by the different policy indicators in this model. Hence, the choice of indicator variable is crucial in the evaluation of the liquidity ratio as a tool of monetary policy. This paper has explored the effects, but pending further work on the choice of indicator and on the nature of the transmission mechanism, it gives no firm basis for a definite conclusion about the efficacy of the liquidity ratio. Nevertheless, the analysis has revealed some fundamental problems. The traditional argu-



ment for using the liquidity ratio is that it forces banks to reduce their lending to the public. If they behave in this way, there will be a reduction in bank lending and the possible ensuing expansion of nonbank credit is unlikely to offset this effect completely. However, the central bank cannot control the way in which banks meet a change in the required ratio. They have the option to maintain their lending, instead fulfilling the higher ratio by obtaining bonds from the public in exchange for certificates. Unfortunately, there is no empirical evidence that bears on this issue - we do not know how banks typically respond. However, there are reasons to believe that it is costly for banks to curtail credit, which gives a theoretical motivation for studying this case carefully. Moreover, if the liquidity ratio is used as a policy variable and is changed over the business cycle, banks can be expected to learn to bridge the periods with high ratios by issuing certificates that can be recalled when the ratio is lowered again. In terms of the model, this means that  $C_q^b$  is likely to rise if the ratio is changed frequently, making a perverse effect on the money stock more likely.

Even if interest rates were shown to be the relevant indicator so that the liquidity ratio would have the intended effect, there is another fundamental problem inherent in the use of any regulatory measure. This is the effect on the allocative efficiency of the financial system. Since only some intermediaries are regulated (and, by necessity, some are always exempt) and only certain assets qualify as "liquid", this system unavoidably favors some sectors among both debtors and creditors. Hence, for the liquidity ratio to be a really useful policy tool, it must be shown that these results cannot be achieved by other means with less harmful consequences for efficiency. This may be a difficult test to pass. For example, it can be argued that the regulations contribute to the thinness of the secondary bond market by forcing banks to buy and hold a substantial fraction of the outstanding

stock. If a deregulation should contribute to a revival of the bond market, one of the most frequently used arguments for the liquidity ratio would be invalid. A more conventional monetary policy, for example, using open market operations, would then be feasible.

Seen from another perspective, the analysis in this chapter gives an example of the problems of effectively using regulatory policy instruments in a system where secondary asset markets have emerged. In a completely regulated system, bank liability management is impossible and a liquidity ratio is very difficult to circumvent. The development in Sweden discussed in Chapter 1 illustrates the process through which a gradual liberalization of the financial system can reduce the efficacy of the regulatory policy instruments. The start of the money market in 1980 led the way for a major shift in monetary policy and, as pointed out in Chapter 1, neither liquidity ratios nor loan ceilings are currently in use. The legislation authorizing the central bank to impose these regulations is still in force, but the efficacy of these instruments, should the central bank try to use them within the current institutional framework, is very much in doubt.

## APPENDIX

In this appendix, the comparative statics framework underlying the results presented in Section 2.4 is discussed briefly. By totally differentiating the three equilibrium conditions (2.17)-(2.19) and making various substitutions to ensure consistency with the sectoral balance sheet constraints, the following system of equations is derived:

$$\begin{bmatrix} -R_B & -R_L & -R_C \\ -(B_B^p + C_B) & L_L^b + L_L^f - B_L^p - C_L & L_C^b + L_C^f - B_C^p - C_C \\ C_B & C_L - C_L^b - L_L^f & C_C - C_C^b - L_C^f \end{bmatrix} \cdot \begin{bmatrix} dr_B \\ dr_L \\ dr_C \end{bmatrix} = \begin{bmatrix} R_q \\ -L_q^b \\ C_q^b \end{bmatrix} dq$$

The sign pattern in the matrix is

$$\begin{bmatrix} + & - & + \\ - & + & - \\ - & - & + \end{bmatrix}$$

Denoting the elements by  $A_{ii}$  and noting that  $-R_L = R_C$ , that is,  $A_{12} = -A_{13}$ , the determinant is

$$\Delta = A_{11} \begin{bmatrix} A_{22}A_{33} & - & A_{23}A_{32} \\ + & + & - \end{bmatrix} - A_{12} \begin{bmatrix} A_{21}(A_{32}+A_{33}) & - & A_{31}(A_{22}+A_{23}) \\ - & - & + \end{bmatrix}$$

It is clear that the determinant cannot be signed directly. However, it can be shown that  $\Delta > 0$  is a necessary condition for stability in the Walrasian sense. Sufficient conditions for this are

$$A_{22}A_{33} - A_{23}A_{32} > 0 \quad (A2.1)$$

$$A_{21}(A_{32}+A_{33}) - A_{31}(A_{22}+A_{23}) > 0 \quad (A2.2)$$

Since the positive term in (A2.1) consists of own rate effects, (A2.1) cannot be considered restrictive. Conditions (A2.2) is less obvious. It depends on the relative interest elasticities of the public's portfolio, but the magnitudes involved cannot be determined a priori. Even if negative, (A2.2) is not likely to change the sign of  $\Delta$  since it is weighted by  $-A_{12} = R_L = qC_L^b - B_L^b$ , which is expected to be small, particularly for low values of  $q$ . Therefore I assume that (A2.1) holds and that  $\Delta$  is positive, that is, that (A2.2) is not a necessary condition. As discussed in Section 2.4, if (A2.2) does not hold, the likelihood of a perverse money stock effect is greater and the presumed negative impact on total credit is reduced.

## NOTES

1. The term liquidity ratio is a misnomer, since the assets included usually are quite illiquid. Similar arrangements have been referred to as placement ratios or secondary reserve requirements.
2. Note that it is crucial that bonds be held by agents other than banks, since otherwise a liquidity ratio would be equivalent to a regular reserve ratio requirement.
3. Because of the budget constraint, there are various adding-up restrictions on the partial derivatives in (2.4)-(2.7) (cf. Brainard and Tobin 1968), but to economize on notation they will mostly be kept implicit.
4. Note that  $C_q^b$  and  $L_q^b$  are not independent. From (2.3) it follows that  $L_q^b = (1-q)C_q^b - (D+C^b)$ .
5. This is an implicit restriction on  $r_B$ ; bonds are not allowed to become so attractive that banks want to hold excess liquid assets. Analyzing changes in  $q$  is meaningful only if it is assumed to be binding since otherwise the effects are trivially zero.
6. Disregarding reserves is not necessarily an innocuous assumption. Guttentag and Lindsay (1968) show that the form in which nonbank reserves are held is crucial for their ability to expand credit. This problem is not addressed here, however. Coghlan (1977) reviews the debate about the relative importance of banks and nonbanks.
7. Given the treatment of  $C^b$ , banks' total deposit liabilities and their yield are endogenous and in this respect the model is less restrictive than under the standard assumption of fixed deposit rates in, for example, Myhrman (1973).

8. See the stability conditions stated in the Appendix.
9. Note that in the special case  $L_q^b = 0$ , when (2.22) is violated  $R_L = 0$ , implying that the second term drops out, that is, that  $dr_B/dq > 0$  still holds.
10. The terms  $(A_{22}+A_{23}) = -(L_C+L_L)$  and  $(A_{32}+A_{33}) = C_C + C_L$  are both positive because of dominating own rate effects.
11. The term is the negative of the second sufficient stability condition (A2.2) in the Appendix. Hence, if (A2.2) is assumed to be positive, the  $R_q$  term in (2.31) will mitigate the increase in  $M$ .
12. Note that the assumption of perfect substitutability between  $L^b$  and  $L^f$  becomes somewhat tenuous if banks have this very accommodative policy. The risk of premature termination is then much greater for a loan from a nonbank, whereas in the case where  $C_q^b = 0$  this risk is about the same. The problem of variable substitutability cannot be treated in this model, however.

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### 3. Discount Window Policies and the Determination of the Money Market Rate\*

#### 3.1 INTRODUCTION

The role as "lender of last resort" is one of the classical functions of a central bank. The standard description of this role is a situation where the central bank intervenes by giving credit (reserves) to a bank that is on the verge of a collapse. The purpose of such an action is to protect the depositors in this bank and, ultimately, the confidence in and the stability of the banking system in general.

In practice, however, bank borrowing from the central bank in many countries has come to play a role that is very different from that envisaged in the description of the central bank as lender of last resort. In Sweden, for example, where no bank has been even close to failure for at least 50 years, banks have borrowed 20-30 billion kronor (5-7 percent of the money stock) for extended periods during the 1980s. This indicates that the central bank in some cases has become a lender of first rather than last resort. The explanation is that discount window borrowing and, in particular, the interest rate charged for such loans have become important instruments

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\* This chapter draws partly on joint work with Peter Englund and Staffan Viotti, e.g., Englund, Hörngren, and Viotti (1985a, 1985b, 1986), and Hörngren and Viotti (1985).

of monetary policy. The strategy has been to make the supply of non-borrowed reserves small enough to force banks into central bank borrowing and then use the central bank lending rate - the discount rate - to control money market interest rates.

One important motive for this type of system has been that financial markets in many countries have been so thin that open market operations have not provided a sufficiently flexible instrument of monetary policy. In recent years, financial markets have developed rapidly in many European countries, but Bingham's (1985) review of the institutional systems in the OECD area indicates that discount window borrowing continues to play an important role (see especially pp. 134-144). Bingham also shows that the conditions under which the banks are given access to discount window borrowing differ greatly between countries. In many European countries commercial banks have virtually unlimited access to borrowed reserves and, frequently, at a fixed discount rate. In the U.S., in contrast, there are formal and informal restrictions on both the amounts and the periods of borrowing that are permitted. Not only do these rules differ between countries, they also change over time. As discussed in Chapter 1, the Swedish central bank used to work with a completely open discount window and a constant lending rate, the penalty rate. However, in December 1985 a system with a lending rate that increases stepwise as reserve borrowing rises was introduced.

It is thus clear that the rules for discount window borrowing have been a matter of concern for the policy makers. This is as it should be, of course, since these rules are of fundamental importance both for how the money market in general works and for the effects of various monetary policy measures.

The purpose of this chapter is to analyze the effects of varying degrees of "openness" of the discount window - a concept

that will be made precise below - within a simple money market model. This analytical framework will then be used to discuss the policies pursued in Sweden, in particular, how they have affected the usefulness of the different policy instruments and the effects of various shocks to the economy. One important question is whether the discount window policies increase or decrease the variability of the targets (indicators) that the central bank might want to stabilize. This analysis is taken one step further in Chapter 4, where the question is whether optimal discount window rules can be devised within an explicitly stochastic framework.

The chapter is organized as follows. In Section 3.2, the model is presented and its general properties analyzed. Section 3.3 discusses in greater detail the role of the discount window policies. Special attention is given to the case with unlimited access to borrowed reserves as this is especially relevant for the policies in Sweden. In Section 3.4, reserve borrowing in Sweden is discussed and a comparison is made between the penalty rate system with a fixed discount rate and the rules introduced in December 1985 with a positively sloped supply curve for borrowed reserves. Finally, Section 3.5 gives some concluding comments.

### 3.2 A MONEY MARKET MODEL WITH DISCOUNT WINDOW BORROWING

In this section, a simple two interest rate money market model for an open economy is presented. In its basic structure it is a conventional portfolio balance model, but special emphasis is given to the analysis of the role of the discount window policies, an issue that has received relatively little attention in the literature. Among the exceptions are Aftalion and White (1977) who analyze a model with a pegged discount rate, a case that is given special attention also in this chapter. In contrast to the current model, however, they only consider the case when banks treat discount window borrowing

and loans to the public as perfect substitutes, implying that there is only one market determined interest rate. Langohr and Santomero (1985) analyze the implications for economic stability of different methods for providing banks with borrowed reserves in a model with features typical of the European system for central bank borrowing.

Goodfriend (1982, 1983) also pays explicit attention to discount window borrowing. He is concerned with the U.S. system in which there are various administrative and non-pecuniary costs associated with reserve borrowing. There is also an inter-temporal aspect in that borrowing today may reduce the access to borrowed reserves tomorrow. In the typical European system, on which I focus in this chapter, neither of these aspects is important, however. This has several interesting consequences, for example, for the appropriate aggregation of bank assets and liabilities in the current model.

The model identifies three domestic sectors: the central bank, the public, and the banking sector.<sup>1</sup> Each sector is subject to a balance sheet constraint:

$$T^C + FX + RB^C = R^C \quad (\text{central bank}) \quad (3.1)$$

$$T^b + R^b = M + RB^b \quad (\text{banks}) \quad (3.2)$$

$$T^D + F^D + M = W \quad (\text{public}) \quad (3.3)$$

where  $T^i$  = sector  $i$ 's holdings of t-bills

$M$  = money stock (= deposits)

$R$  = total reserves (the monetary base)

$RB$  = borrowed reserves

$FX$  = foreign exchange reserves

$F^D$  = net holdings of foreign currency denominated assets

$W$  = private net financial (short-term) wealth.

The balance sheets indicate a highly aggregated picture of the financial system. The model only considers the short end of the system, i.e., long-term bonds and equity are disregarded. It is also assumed that since the focus of the analysis is on the short run, bank loans can be considered fixed (at zero), the idea being that the market for bank loans can be characterized as a customer market making it difficult or impossible for banks to vary their stocks of loans at short notice. T-bills, on the other hand, is an aggregate containing all domestic (short-term) assets with well functioning secondary markets, primarily treasury bills ("statsskuldväxlar") and certificates of deposit (CDs). Sudden deposit changes will induce banks to adjust their holdings of marketable assets instead of their loan portfolio.

The central bank's reserves supply is the sum of its holdings of t-bills, the foreign exchange reserves, and borrowed reserves. Cash holdings are disregarded as, in particular in the short run, these can be assumed to be insensitive to interest rate variations. This means that only banks demand reserves.

Banks hold t-bills and reserves and finance these by accepting deposits and by borrowing from the central bank. The public's portfolio choice is to divide its short term assets between deposits, t-bills, and foreign assets. It is assumed that the exchange rate is fixed and that the country is small so that foreign assets are available to the domestic public at a fixed interest rate.

Implicit in the banks' balance sheet constraint is also their position in the overnight (interbank) market. As this must be zero in the aggregate it cancels out from the sector balance sheet.

### Behavioral assumptions

The public allocates its given financial wealth between deposits, t-bills, and foreign assets according to the following asset demand functions:

$$M = m(r_T, r_F, W) \quad m_T < 0, m_F < 0 \quad (3.4)$$

$$T^D = t^D(r_T, r_F, W) \quad t_T^D > 0, t_F^D < 0 \quad (3.5)$$

$$F^D = f(r_T, r_F, W) \quad f_T < 0, f_F > 0 \quad (3.6)$$

where  $x_T$  and  $x_F$  ( $x = m, t, f$ ) denote the partial derivatives with respect to  $r_T$  and  $r_F$ , the domestic t-bill rate and the foreign interest rate, respectively. Equations (3.4)-(3.6) are subject to the balance sheet constraint (3.3) and, hence, to standard adding up constraints on the partial derivatives.

The banks passively accept deposits from the public at a given interest rate, implying that the volume of deposits is completely demand determined. The short-term decision problem for the banks is therefore how to invest the funds put at their disposal by the public. However, they must allocate part of their assets to meet a reserve requirement imposed by the central bank. Reserve demand is given by

$$R^D = a\bar{M} \quad 0 < a \leq 1 \quad (3.7)$$

where  $a$  is the required reserve ratio and  $\bar{M}$  is the (pre-determined) level of deposits used to compute the reserve requirement, i.e., the central bank uses lagged reserve accounting. This has the effect of making reserve holdings independent of the (current) interest rate, but an analysis using contemporaneous reserve accounting would be straightforward and would not lead to fundamentally different conclusions.<sup>2</sup>

What remains for the banks is thus to determine their holdings of t-bills,  $T^b$ , and the amount of reserve borrowing,  $RB^b$ . T-bills demand is given by

$$T^b = t^b(r_T, r_R) \quad t_T^b > 0, \quad t_R^b < 0 \quad (3.8)$$

which from the balance sheet constraint implies that the banks' demand for borrowed reserves is

$$RB^b = t^b(r_T, r_R) - M + a\bar{M} \quad (3.9)$$

Equation (3.9) says that the demand for borrowed reserves is an increasing function of the money market rate  $r_T$  and a decreasing function of the rate on borrowed reserves  $r_R$ .

It is important to note that the banks are assumed to treat central bank borrowing and overnight interbank loans as perfect substitutes. The motivation for this is that in this model, as in the typical European system, there are no (formal or informal) rules constraining the access to discount window borrowing based on past borrowings, for example. This means that provided  $RB$  is non-zero, the overnight rate and the central bank's lending rate will be equalized by arbitrage<sup>3</sup> and the market for overnight interbank loans need not be introduced explicitly.

The aggregation of interbank loans and discount window borrowing is important and one aspect in which this model deviates from models where the U.S. system for discount window borrowing is taken as point of departure. For example, Goodfriend (1983) demonstrates that it is optimal for a U.S. bank to choose borrowed reserves so that the interbank rate (the Federal funds rate) equals the official discount rate plus the marginal administrative costs - "the costs of harassment" in the terminology of Brunner (1974) - imposed by the central bank.<sup>4</sup> When there are such costs, it would be inappropriate to treat inter-

bank loans and discount window borrowing as perfect substitutes. The importance of this rationing mechanism is also illustrated by the discussion in Goodfriend and Hargraves (1983) of the U.S. discount window policies in the 1920s. Then the discount rate was the Fed's main instrument and the administrative costs were very low. This led to a system that was highly similar to the open discount window case in the current model.

The central bank supplies the system with reserves (base money). There are three sources of reserves. First, the central bank has a stock of government bonds ( $T^C$ ) and by performing open market operations, i.e., selling and buying t-bills, it can change the stock of reserves. Second, because of its commitment to a fixed exchange rate, the central bank must maintain a stock of foreign exchange reserves (FX) and buy and sell foreign currency to keep the exchange rate at the target level. The foreign exchange reserves are thus the sum of cumulated current account (CA) and capital account surpluses (deficits) ( $F^P$ ) so, by definition,

$$FX = CA - F^P \quad (3.10)$$

where CA is exogenous. For simplicity, it is assumed that only the public holds foreign assets. Third, the central bank offers to lend reserves to the banks. Its reserve supply function is

$$RB^C = \rho(r_R) \quad \rho_R \geq 0 \quad (3.11)$$

where  $r_R$  is the central bank lending rate. The interpretation of (3.11) is clarified if it is linearized around some value  $\bar{r}_R$  and solved for the lending rate:

$$r_R = \frac{1}{\rho_R} RB^C + \bar{r}_R \quad (3.12)$$



Written on this form, it is seen that  $\rho_R$ , the derivative of the  $RB^C$  function with respect to  $r_R$ , measures the sensitivity of the lending rate to the level of borrowing. If  $\rho_R = \infty$ , the lending rate is constant and equal to  $\bar{r}_R$  which is the case of the Swedish penalty rate system, i.e., a completely open discount window. Conversely, if  $\rho_R = 0$ , the lending rate becomes infinite which is equivalent to a completely closed discount window. Values of  $\rho_R$  less than infinity thus gives rise to an upward sloping supply function, i.e., the cost of funds for the banks is higher the more they borrow.<sup>5</sup>

Equation (3.11) is thus a convenient way of formalizing the concept of varying degrees of openness of the discount window. It bears emphasizing that it should not be confused with a standard demand function for borrowed reserves in a model of the U.S. system. In Poole (1982), for example,  $RB^b$  is written as a function of the difference between  $r_R$  and the discount rate, but as discussed above, these are equalized by arbitrage in the current model. This is another reflection of the differences in aggregation between models of the U.S. and the European type systems.

In the general case, the supply function for borrowed reserves will have the shape illustrated in Figure 3.1.

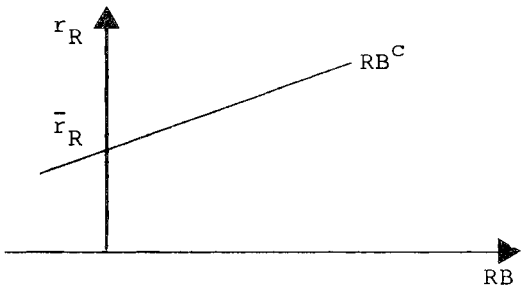


Figure 3.1. The supply of borrowed reserves.

The fact that the function extends into the fourth quadrant indicates that it can also be thought of as applying to excess reserves. Then  $r_R$  measures the interest rate paid to banks for excess reserves deposited with the central bank. As an alternative to assuming interest payments on excess reserves according to the same scheme as for borrowed reserves, the analysis could be motivated by saying that the central bank keeps the supply of non-borrowed reserves so tight that  $RB$  always is positive. The consequences of a discontinuous  $RB^C$  schedule will be discussed in Section 3.4 in connection with the analysis of the Swedish system.

#### Equilibrium conditions

Formally, there are six markets in this model: the markets for t-bills, non-borrowed reserves, borrowed reserves, interbank loans, deposits, and foreign assets. However, the markets for borrowed reserves and interbank loans can be aggregated, and equilibria in the deposit and foreign asset markets are completely demand determined and can be solved for recursively. This leaves three markets which determine two interest rates, the money market (t-bills) rate ( $r_T$ ) and the central bank lending rate ( $r_R$ ).

Eliminating the bond market by Walras' law, the remaining equilibrium conditions are

$$R^b - RB^b = R^C - RB^C \quad (\text{Non-borrowed reserves}) \quad (3.13)$$

$$RB^b = RB^C \quad (\text{Borrowed reserves}) \quad (3.14)$$

It turns out to be convenient, however, to use the equilibrium condition for total reserves, i.e., the sum of (3.13) and (3.14), rather than (3.14) directly. Using (3.1) to ensure consistency with the central bank's balance sheet constraint, the equilibrium system can then be written as

$$R^b - RB^b = T^C + FX \quad (\text{Non-borrowed reserves}) \quad (3.13')$$

$$R^b = T^C + FX + RB^C \quad (\text{Total reserves}) \quad (3.14')$$

Note that the demand for total reserves,  $R^b$  on the right hand side of (3.14'), is given by (3.7) and is independent of current interest rates. This implies that the demand for borrowed reserves is simply the difference between the required reserves and the stock of non-borrowed reserves and no (non-interest bearing) excess reserves will ever be held. This is another implication of the assumption of perfect substitutability between discount window borrowing and interbank loans. As there are no intertemporal links in the system, it can never pay to hoard reserves today. With unconditional access to discount window borrowing, reserves will always be available tomorrow at a market determined rate.

This property of the model can be used to throw some light on the discussion in Langohr (1981) of competing hypotheses concerning the determination of bank borrowing from the central bank.

His main purpose is to analyze the relative merits of targeting the monetary base (total reserves) and the banks' free reserve position, respectively. He distinguishes between two main theories, referred to as "the reserve position doctrine" and "the profit theory of borrowing". The former "argues that central banks cannot have effective control of total bank reserves because bank borrowing would automatically offset open market operations intended to change total reserves" (Langohr (1981), p. 109). Banks will borrow whatever amount of reserves they "need" to fulfill the reserve requirement. The profit theory, on the other hand, asserts that the amount of borrowing depends on the relation between the discount rate and the relevant market interest rate.

The issue of optimal monetary targeting is far beyond the scope of this chapter and here Langohr's critique of the arguments leading to a free reserves target seems motivated. However, the treatment of discount window borrowing in the theories he discusses merits some comments. With the starting point taken in this chapter, it is clear that a distinction between these two hypotheses cannot be made without a specification of the discount window policies and the reserve accounting system. First, in a system with lagged reserve accounting and binding reserve requirements the reserve position doctrine would be trivially true, because banks are forced to borrow exactly the difference between required reserves and stock of non-borrowed reserves which in the closed economy case considered by Langohr is set by the central bank. Second, it will be shown that whether there is complete offset, as asserted in the reserve position doctrine, depends on the degree of substitutability between central bank borrowing and money market investments. If the discount window is completely open, the reserve position doctrine will provide accurate predictions, despite the fact that it is "profit motives" that drive the arbitrage mechanism giving this result. One element that is missing in the theories tested by Langohr is thus the supply of borrowed reserves.<sup>6</sup> The current model demonstrates that the determinants of reserve borrowing cannot be analyzed only in terms of the demand side.

Before proceeding with the analysis, a brief comment on the relation between the two endogenous interest rates in the model is in order. The central bank lending rate (the overnight rate) is the very shortest interest rate in the maturity structure in the financial system. The t-bills rate, on the other hand, should be interpreted as some representative market rate on an asset with, say, 30 or 90 days to maturity. In a strict sense, these two interest rates are not independent. Under the (extreme) expectations theory of the term structure

of interest rates, the 30 day rate is determined exclusively by the expected overnight rates over the 30 day period. If these are expected to be constant, the two interest rates would actually coincide and there would in effect be only one interest rate in the model. Although this is an interesting case, in particular in situations with a completely open discount window, it is certainly not the general case.

The problem in this context is that it is impossible to give a fully adequate treatment of the term structure relation in a static framework. However, it can be argued that considering a model with two assets that, in the general case, are imperfect substitutes is at least no more restrictive than the standard assumption in macro analysis of having only one interest rate that is used to represent the entire term structure, as in, for example, Goodfriend (1982) or Poole (1982).

In a sense, this is a static analog to the expectations hypothesis which also implies that the (expected) return on all assets are equalized. Given that this is not an entirely satisfactory (dynamic) model of the term structure, it would seem worthwhile to study models with more general interest rate relations also in a static setting.

The approach used here is analogous to the macro portfolio model analysis pioneered by Tobin and employed also in Chapter 2. There it is standard practice to distinguish between several interest rates (loan rate, bond rate, etc.) without explicitly modeling their different characteristics in terms of risk or time to maturity. The absence of a term structure relation becomes more apparent in the current model where the interest rates are referred to as overnight and 30 day rates, but the basic assumptions are the same as in any macro portfolio model.

Differentiation of the equilibrium conditions (3.13') and (3.14') yields the following system:

$$\begin{bmatrix} m_T + f_T - t_T^b & -t_R^b \\ f_T & -\rho_R \end{bmatrix} \begin{bmatrix} dr_T \\ dr_R \end{bmatrix} = \begin{bmatrix} 1 & -(f_F + m_F) & 0 \\ 1 & -f_F & -\rho_R \end{bmatrix} \begin{bmatrix} dT^C \\ dr_F \\ d\bar{r}_R \end{bmatrix} \quad (3.15)$$

The determinant of the coefficient matrix is

$$\Delta = \rho_R (t_T^b - m_T - f_T) + f_T t_R^b > 0 \quad (3.16)$$

It will prove useful to have a graphical illustration of the equilibrium solution. First, consider the locus of interest rate combinations giving equilibrium in the market for non-borrowed reserves. Denoting this locus by NBR, its slope in  $r_R/r_T$ -space can be expressed as

$$\left. \frac{dr_R}{dr_T} \right|_{\text{NBR}} = \frac{m_T + f_T - t_T^b}{t_R^b} > 0 \quad (3.17)$$

Similarly, for the total reserves market the equilibrium locus, denoted TR, has the slope

$$\left. \frac{dr_R}{dr_T} \right|_{\text{TR}} = \frac{f_T}{\rho_R} < 0 \quad (3.18)$$

The positive slope of the NBR locus can be intuitively understood as follows. Suppose that, for some reason,  $r_T$  increases. This decreases the public's demand for foreign assets and the ensuing capital inflow increases the supply of non-borrowed reserves as FX rises. To maintain equilibrium in the NBR market, demand must increase which happens if  $r_R$  rises as this makes banks less willing to borrow reserves, other things equal.

The same disturbance creates an excess supply in the market for total reserves as FX increases due to an inflow of capital. To maintain equilibrium reserve borrowing must fall, i.e.,  $r_R$  decreases, which explains the negative slope of the TR locus.

The equilibrium solution can be illustrated as in Figure 3.2.

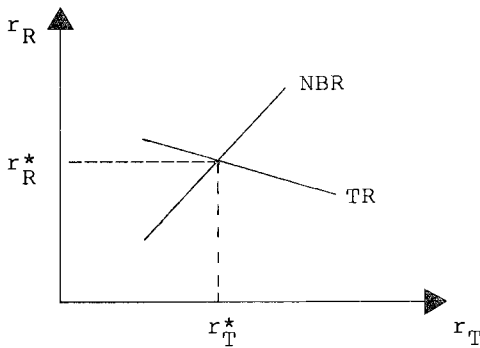


Figure 3.2. Market equilibrium.

In this figure, the TR locus is relatively flat which indicates that  $\rho_R$  is large, i.e., that the discount window is relatively open.

The slope of the NBR locus depends on the partial derivatives of the banks' and the public's asset demand functions. An interesting special case is when banks regard t-bills and overnight loans as perfect substitutes. Then  $t_T^b = |t_R^b| = \infty$  and the slope of NBR is unity, which means that  $r_R = r_T$  holds as an arbitrage condition. In this case, the markets for borrowed reserves and t-bills can be aggregated so that there are only two equilibrium conditions determining one interest rate.

It should be noted that if  $f_T$  is large in absolute value, i.e., if capital flows are highly sensitive to interest rate changes, both loci become very steep. In the special case when domestic and foreign assets are treated as perfect substitutes, they become vertical which corresponds to the standard result that a fixed exchange economy with perfectly integrated capital markets has no latitude for an independent interest rate policy. In this chapter, it is assumed that there is less than perfect substitutability, but much more on this issue will be said in later chapters.

In its general version, this model has conventional properties. As an example, consider an open market operation involving an increase in  $T^C$ . The vertical shifts of the equilibrium loci are

$$\left. \frac{dr_R}{dT^C} \right|_{NBR} = -\frac{1}{t_R^b} > 0; \quad \left. \frac{dr_R}{dT^C} \right|_{TR} = -\frac{1}{\rho_R} < 0$$

This experiment is illustrated graphically in Figure 3.3.

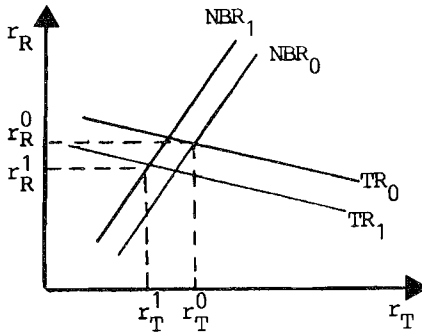


Figure 3.3. The effects of an open market purchase,  $dT^C > 0$ .

Both interest rates fall unambiguously. The increase in the supply of non-borrowed reserves means that reserve borrowing



falls and thereby also the central bank's lending rate. With a larger portion of t-bills held by the central bank, the public finds itself with more deposits and less t-bills. To be satisfied with this change in their portfolio composition  $r_T$  must fall, making t-bills less and deposits more attractive. This effect is mitigated by an increase in the demand for foreign assets which counteracts the initial increase in the supply of non-borrowed reserves as FX is reduced. In the general case of less than perfect substitutability between foreign and domestic assets, this offset will not be complete, however, and  $r_T$  will fall. The Appendix gives a formal and more complete presentation of the comparative statics of this model.

### 3.3 THE ROLE OF THE DISCOUNT WINDOW: A CLOSER LOOK

In the model presented in the previous section, the degree of openness of the discount window is captured by the parameter  $\rho_R$ , which essentially measures the slope of the central bank's supply function for borrowed reserves. In this section, the focus is on how the setting of this parameter affects the money market.

As can be seen from the analysis above, both the slope and the magnitudes of the shifts of the TR equilibrium locus depend crucially on the degree of openness of the discount window. An interesting situation, not least with reference to Sweden, is when  $\rho_R = \infty$ , i.e., when unlimited borrowing is permitted. As discussed in Section 3.2, this is the stylized representation of the Swedish penalty rate system that was in force prior to December 1985.

Considering first how this affects the graphical description of the system, it is immediately seen that the TR locus becomes horizontal at the level given by  $\bar{r}_R$ , as shown in Figure 3.4. This parameter can thus be interpreted as the penalty rate

in a system with unlimited borrowing. A penalty rate system as the one practiced in Sweden has another important characteristic, however. This is the fact that the penalty rate is kept fixed for extended periods of time. Combined with the open discount window, this has important consequences for how the money market can be expected to work.

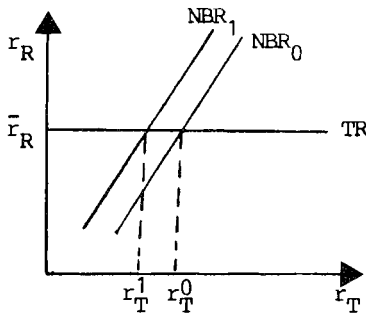


Figure 3.4. An open market purchase ( $dt^C > 0$ ) under a completely open discount window.

If the central bank attempts an open market purchase similar to that discussed above, only the NBR locus will shift as shown in Figure 3.4. Hence, the t-bills rate still falls, but the interbank rate is unaffected. It is interesting to note that the magnitude of the change in  $r_T$  is inversely proportional to how sensitive banks' t-bills demand is to changes in the interbank rate. The higher the substitutability between t-bills and central bank borrowing, the smaller the deviation between  $r_R$  and  $r_T$  that a given disturbance can give rise to.

There are also strong reasons to believe that this substitutability is high under a penalty rate system as defined here. In particular, as the lending rate is pegged and only rarely changed, the current overnight rate is a reasonably accurate predictor of the overnight rates for the future 30 day period,

say. This means that if, for example, the t-bills rate were to rise above the central bank lending rate, banks would be willing to borrow at the discount window to buy t-bills. Such activities would tend to decrease  $r_T$  until the spread is considered too small to compensate for the interest rate risk that the banks are taking when financing a 30 day position with overnight loans. Obviously, the more stable the central bank rate, the smaller is this risk and the closer the correspondence between  $r_T$  and  $\bar{r}_R$ .

This line of reasoning implies that the degree of substitutability between t-bills and overnight loans is not independent of the discount window policies, i.e.,  $t_T^b$  and  $t_R^b$  need not, as was assumed in Section 3.2, be constant parameters. In particular, high values of  $\rho_R$  is expected to be associated with high (absolute) values of  $t_T^b$  and  $t_R^b$ . Conversely, a closed discount window will make  $r_R$  more variable, other things equal, and the interest risk involved in exploiting differences between  $r_T$  and  $r_R$  may be substantial, implying low values of  $t_T^b$  and  $t_R^b$ . These aspects cannot be adequately captured within the current model, but neither can they be ignored in the (informal) discussion of the choice of discount window policy.

Note that open market operations are of little use as an instrument of monetary policy in a penalty rate system. If the central bank increases the supply of non-borrowed reserves via an open market purchase, this is more or less completely offset by the banks reducing the amount of borrowed reserves. Total reserves are unaffected and, consequently, all the interest rates and quantitative aggregates that the central bank may have wanted to influence, e.g., the money supply or the foreign exchange reserves.

It is important to note that the situation described here is characterized not only by a completely open discount window

but also by a constant lending rate. If  $\rho_R = \infty$ , but  $\bar{r}_R$  is changed weekly or even daily, there is no reason to expect equality between  $r_R$  and  $r_T$ . A completely open discount window does therefore not imply that the central bank has to give up the control of the money market. In particular, changes in  $\bar{r}_R$  can be highly effective. An increase in  $\bar{r}_R$  shifts the horizontal TR locus by exactly the same amount since

$$\left. \frac{dr_R}{d\bar{r}_R} \right|_{TR} = 1$$

This experiment is illustrated in Figure 3.5.

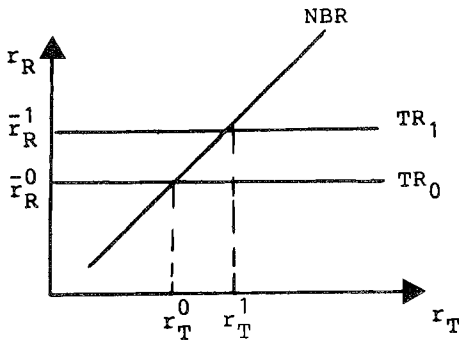


Figure 3.5. An increase in the penalty rate.

The penalty rate is raised from  $\bar{r}_R^0$  to  $\bar{r}_R^1$  shifting the TR locus upward. In general, this will also raise  $r_T$ . By how much depends on the slope of the NBR locus, i.e., on the substitutability between foreign and domestic assets and t-bills and overnight loans, respectively, as can be seen in (3.17).

The conclusion is thus that a system with unlimited access to discount window borrowing creates a situation where open market operations are, in general, less useful for controlling money market interest rates. On the other hand, changes in

the pegged lending rate become relatively more effective. Hence, an open discount window does not necessarily imply loss of control over the financial system. The relative merits of the two instruments depend instead primarily on how efficiently the control can be exercised. This issue will be discussed in Section 3.4 below.

The choice to keep a fully open discount window also has important implications for how non-policy shocks affect the financial system. As an example, consider an exogenous increase in the foreign interest rate, making foreign assets relatively more attractive, other things equal. Returning to the system (3.15), the effects on the two equilibrium loci are seen to be

$$\left. \frac{dr_R}{dr_F} \right|_{NBR} = \frac{f_F + m_F}{t_R^b} < 0$$

and

$$\left. \frac{dr_R}{dr_F} \right|_{TR} = \frac{f_F}{\rho_R} > 0$$

To sign the NBR shift, it is necessary to assume that there are dominating own rate effects in the public's portfolio so that  $f_F > |m_F|$ . As shown in the Appendix, this is also sufficient (but not necessary) to guarantee that both interest rates rise. The increase in the domestic money market rate thus tends to reduce the relative attractiveness of foreign assets and mitigate the capital outflow. However, in the penalty rate system where  $t_R^b = \rho_R = \infty$ , neither locus is shifted and the interest rates are unchanged.

Consequently, the entire adjustment to this disturbance is made through a loss of foreign exchange reserves in the form

of a capital outflow. This reduction in the supply of non-borrowed reserves is compensated by an equally large increase in borrowed reserves, i.e., there is automatic and complete sterilization of all currency flows. This is, of course, the same arbitrage mechanism that makes open market interventions inoperative in a penalty rate system.

The central bank can counteract the capital outflow by raising the penalty rate as illustrated in Figure 3.5. However, the automatic stabilization of the foreign exchange reserves that occurs when the supply of borrowed reserves is not infinitely elastic is absent. Instead, a discrete policy intervention is required. Working with a completely open discount window may thus be inefficient if stabilization of the foreign exchange reserves is considered desirable.

On the other hand, if the central bank's aim is to stabilize interest rates, then a penalty rate system is obviously highly effective, as these become insulated from all non-policy disturbances.<sup>7</sup>

The opposite extreme case is when banks are denied access to discount window borrowing, i.e., when  $\rho_R = 0$ . It should be noted that  $\rho_R = 0$  need not imply that the central bank refuses to act as lender of last resort. Such loans can still be given, but only under special conditions motivating such an intervention, e.g., an acute financial crisis. The point here is that unconditional reserve borrowing is impossible. In this case, the market equilibrium can be illustrated as in Figure 3.6.

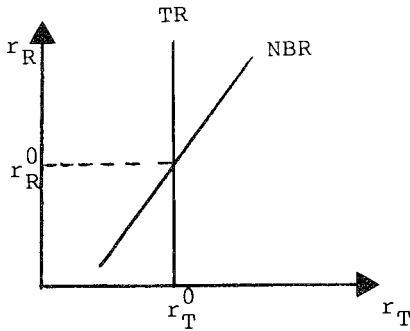


Figure 3.6. Market equilibrium with a closed discount window.

The TR locus becomes vertical. The NBR locus is drawn relatively steep as the degree of substitutability between t-bills and interbank loans is expected to be low ( $t_R^b$  low) in this case. The reason is that  $r_R$  will tend to be variable and, especially compared to the penalty rate system, it will be relatively more risky to finance a money market investment with a series of overnight loans. Note that as the central bank's lending rate is infinite in this case,  $r_R$  is the interest rate in interbank market for overnight loans. Equilibrium in this market occurs when the aggregate demand for borrowed reserves is zero.

Consider once again the effects of an open market purchase of t-bills,  $dT^C > 0$ . The (horizontal) shifts of the equilibrium loci are

$$\left. \frac{dr_T}{dT^C} \right|_{\text{NBR}} = \frac{1}{m_T + f_T - t_T^b} < 0$$

$$\left. \frac{dr_T}{dT^C} \right|_{\text{TR}} = \frac{1}{f_T} < 0$$

The experiment is illustrated in Figure 3.7.

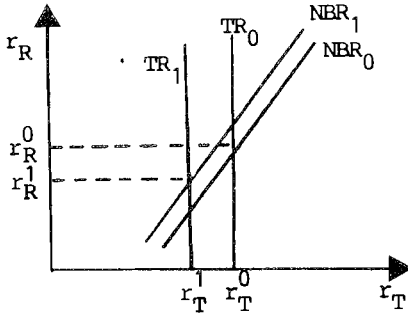


Figure 3.7. Open market purchase ( $dT^C > 0$ ) under a closed discount window.

Both interest rates thus fall just as in the general model.<sup>8</sup> With no access to discount window borrowing, open market operations have a relatively strong effect on interest rates. In this case, the only offsetting mechanism is the currency outflow that the fall in domestic interest rates will induce. Hence, the greater the value of  $f_T$ , the smaller the change in interest rates for any given open market intervention.

It is also interesting to study the effects of an exogenous foreign disturbance in this case. Assume as before that the foreign interest rate rises. From (3.15) the expressions for the (horizontal) shifts are

$$\left. \frac{dr_T}{dr_F} \right|_{NBR} = \frac{f_F + m_F}{t_T^b - m_T - f_T} > 0$$

$$\left. \frac{dr_T}{dr_F} \right|_{TR} = - \frac{f_F}{f_T} > 0$$

and the graphical description is shown in Figure 3.8.



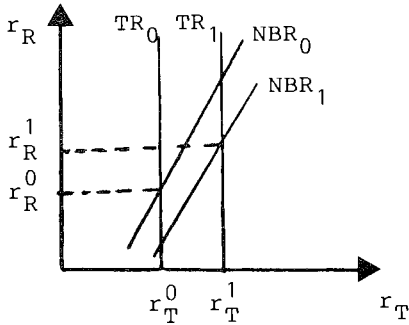


Figure 3.8. An increase in the foreign interest rate with a closed discount window.

As expected, there are strong interest rate effects in this case and, hence, also a substantial automatic stabilization of the capital flows. In effect, closing the discount window minimizes the impact of foreign disturbances on quantitative aggregates and gives maximum effects on domestic interest rates. Also in this respect it is therefore the mirror image of the system with unlimited access to central bank borrowing. Similarly, if that regime is preferable for a central bank that puts emphasis on stabilizing interest rates, closing the discount window seems to be a superior strategy if stabilization of capital flows (the foreign exchange reserves) has high priority. In addition, it is necessary to consider the source of the destabilizing disturbances, for example, whether they come from the international markets or are generated domestically. More will be said on this issue in Chapter 4.

### 3.4 DISCOUNT WINDOW POLICIES IN SWEDEN

As mentioned in Section 3.1, the Swedish central bank reformulated its discount window policies in December 1985. The abandoned system in effect combined the two extremes discussed in the previous section in that it gave banks unlimited access to discount window borrowing, but in such a way that there were market situations when this option was uninteresting for the banks. When this occurred, the system worked much like in a model where no discount borrowing is permitted.

The  $RB^C$  schedule in this system can be illustrated as in Figure 3.9.

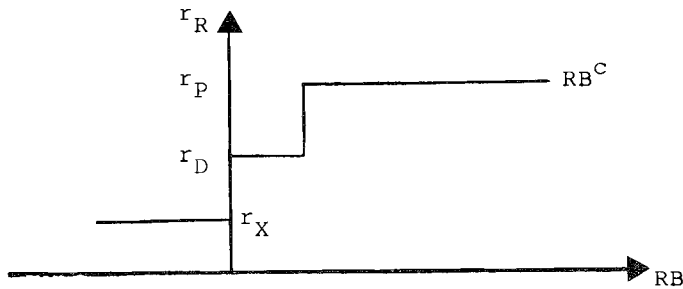


Figure 3.9. The Swedish penalty rate system.

Banks had the right to borrow an amount corresponding to 25 percent of their equity at the discount rate,  $r_D$  in the figure, and at the penalty rate, denoted  $r_P$ , unlimited borrowing was permitted. Banks could also deposit excess reserves, i.e., have negative borrowing, and on these deposits they earned the fixed interest rate  $r_X$ .

When reserve borrowing exceeded the limit where the penalty rate became effective (approximately 2.5 billion kronor), the system worked exactly as in the case with  $\rho_R = \infty$  described above. On the other hand, when the supply of non-borrowed reserves was high, the money market rate tended to fall below  $r_p$  and central bank borrowing was uninteresting for the banks. However,  $r_T$  was still higher than  $r_X$ , the deposit rate, so that no bank would (voluntarily) hold excess reserves. There was thus an interest rate interval where  $RB = 0$  and in this range the system worked as if the discount window had been closed.<sup>9</sup>

Note that it is of no particular significance whether the discount window policy is specified in terms of the central bank borrowing rate or the central bank deposit rate. This chapter considers a system with positive borrowing because the focus is on the Swedish system. However, a model with analytically exactly the same properties could be expressed in terms of a central bank deposit demand function.

A concrete example of this is the system that was introduced in Denmark in August 1985 (cf. Finanstidene, July 26, 1985, pp. 793-795). There the central bank sells certificates of deposit to the banks, which they in turn can use as collateral for overnight loans from the central bank at a fixed interest rate. It is easily seen that this corresponds to a horizontal  $RB^C$  function defined only over negative values of  $RB$ , i.e., for excess reserves, in Figure 3.9. The properties of this system are the same as with a pegged lending rate and  $\rho_R = \infty$  as in the Swedish penalty rate zone.<sup>10</sup>

A stepwise function as the one shown in Figure 3.9 is not easily incorporated into the analysis of the preceding section. However, by far the most interesting segment of the function is the penalty rate zone as the central bank kept the banks at such levels of borrowing that the penalty rate became binding in most periods. This implies that the case with  $\rho_R = \infty$  analyzed above is the most interesting when discussing the Swedish system prior to December 1985. Many of the implications of that analysis are also borne out by the Swedish experience.<sup>11</sup>

In particular, one distinguishing feature of the system was that a change in the penalty rate led to an equiproportional change in the money market rate. The fact that the penalty rate was changed only two or three times per year meant that the new level could reasonably be assumed to be stable for at least the next month, which as discussed in Section 3.3 creates arbitrage opportunities for banks, should  $r_T$  exceed  $r_P$ . The assumption of perfect arbitrage between 30 day t-bills and overnight loans thus provides a reasonably good approximation of the behavior of Swedish interest rates in periods when banks were borrowing heavily at the penalty rate.

It is also clear from the practical experience with this system that open market operations were futile when the banks were deep into the penalty rate zone. If the initial situation was such that reserve borrowing was so small that the penalty rate did not matter, open market sales could be used to reduce the supply of non-borrowed reserves and thus force the banks into the penalty rate zone. This is illustrated schematically in Figure 3.10 which shows the (partial) equilibrium in the market for total reserves.

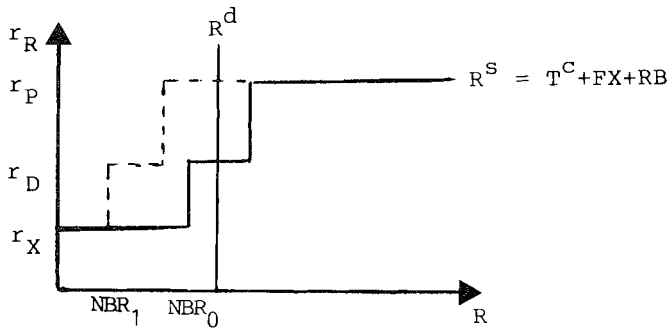


Figure 3.10. An open market sale in the penalty rate system.

The supply function is the sum of the stock of non-borrowed reserves ( $T^C + FX$ ) and the supply of borrowed reserves ( $RB$ ) as shown in Figure 3.9. Reserve demand ( $R^d$ ) is a fixed quantity because of lagged reserve accounting. An open market sale reduces  $T^C$  and shifts the  $R^S$  function to the left to the position illustrated by the dashed curve. This raises the overnight rate from  $r_D$  to  $r_P$  as banks are forced to borrow at the penalty rate. The figure also makes it clear that additional reductions in  $T^C$  will not have any effect on  $r_R$  as the banks will offset the change in non-borrowed reserves by increasing their borrowing at the discount window. If interest rates higher than  $r_P$  are required, it is necessary to raise the penalty rate which shifts the horizontal segment of  $R^S$  upward and, as indicated in Section 3.3, gives an equiproportional increase in the overnight rate.

This illustration ignores the interaction with the t-bills market, but it gives a description of a typical chain of events when a monetary contraction was made in the penalty rate system. One example is the development during the first half of 1985 when persistent currency outflows made the central bank intervene to raise the domestic interest rates. In the first step

in January, open market sales were used to move the banking system deep into the penalty rate zone. When the outflows continued, the next step in May was to raise the penalty rate.<sup>12</sup>

The development in 1985 also illustrates one of the greatest disadvantages of the penalty rate system from the point of view of the Swedish central bank, namely, the absence of a mechanism for automatic stabilization of currency flows. Most policy statements from the central bank in recent years have emphasized the need to stabilize currency flows, in particular, to avoid losses of foreign exchange reserves. This goal has been motivated primarily as part of the policy to maintain the credibility of the fixed exchange rate.

Given this ambition, the choice of giving banks unlimited access to reserve borrowing seems ill-advised since, as shown in Section 3.3, this policy tends to destabilize currency flows, in particular, following international disturbances. The penalty rate system applied in Sweden was in effect a mechanism for automatic sterilization of currency flows. For example, an outflow leading to a reduction in the supply of non-borrowed reserves did not affect the stock of total reserves. Instead, banks offset the loss of reserves by increasing their borrowing and the domestic interest rate could remain unchanged (at the level determined by the current penalty rate). If interest rates had been permitted to rise, this would have limited the outflow which would have been far more conducive for the achievement of the monetary policy targets.

An alternative instrument was of course the penalty rate. However, it was considered undesirable to have too frequent changes in the penalty rate. It was set by the board of the central bank which meets once a week, but as pointed out above, more than two or three changes per year were not made. This (self-imposed) constraint obviously limited the flexibility of monetary policy.

It is therefore not difficult to find sufficient motives for the reform of the rules for discount window borrowing that came into effect on December 9, 1985. The official announcement of the new policies in Sveriges Riksbank Quarterly Review 1985:4 also emphasizes the need for automatic interest rate adjustment and for making open market operations an effective instrument for monetary policy.

Technically, the new system involves only minor modifications relative to the old. The supply function for borrowed reserves with three steps, shown in Figure 3.9, is replaced by a function with six steps illustrated in Figure 3.11.

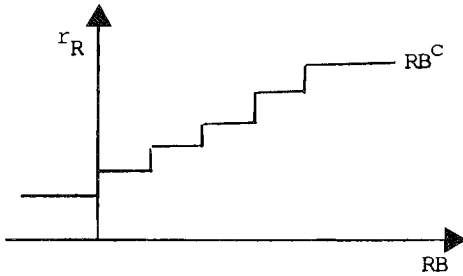


Figure 3.11. The supply of borrowed reserves in the post December 9, 1985 system.

The interest rate increases by two percentage points for each step and ranges from the deposit rate at 6 percent to the highest lending rate at 16 percent. The length of each step corresponds to approximately 6 billion kronor which means that the 16 percent level is reached when borrowing exceeds 24 billion kronor.<sup>13</sup> The borrowing rights for each individual bank are defined in terms of (an extended definition of) its equity, but the important thing for the analysis of the money market is of course the aggregate supply function shown in the figure.

While the technical aspects of the two systems may be similar, the implications for monetary policy and interest rate determination are radically different. If the central bank were to reduce the supply of non-borrowed reserves so that lending exceeded 24 billion kronor, it would of course be back in the old system as there are no limits on the amount of borrowing permitted at the highest interest rate level. However, under current (the summer of 1986) conditions 16 percent is an extreme interest rate level. Instead, the central bank tries to control the supply of non-borrowed reserves in order to create a level of borrowing that is consistent with its interest rate target. For example, if 10 percent is considered a suitable level for the interbank rate, the central bank has to ensure that the need for borrowed reserves (required reserves minus non-borrowed reserves) is in the range between 6 and 12 billion.

Provided the system is neither on the lowest, nor the highest step, the central bank can move the interest rate up or down by changing the supply of non-borrowed reserves. This system thus gives the central bank a "menu" of penalty rates to choose from. In effect, the right to determine the penalty rate in the short run has been given to the central bank officials responsible for open market operations, which makes it possible to change the rate daily. The role of the board of the central bank is to decide on the basic rules, in particular, the shape of the  $RB^C$  function, and the guidelines for the daily market interventions.<sup>14</sup>

However, the central bank's control of the system is not perfect and, for example, when mistakes are made in the forecasts of the autonomous components in the supply of non-borrowed reserves, "too high" or "too low" overnight rates may be recorded. However, the overnight rate itself is not of primary concern for the central bank. For currency flows the most important interest rates are presumably the money market rates for assets with 30 to 180 days to maturity. Experience has



shown that these interest rates are quite insensitive to movements in the overnight rate that can reasonably be thought of as temporary, for example, due to an erroneous forecast of the supply of non-borrowed reserves by the central bank. These rates adjust instead to what is perceived as the "normal" overnight rate in the foreseeable future. This is another difference compared to the previous system: the money market rates and the penalty rate do no longer coincide on a daily basis. The degree of substitutability between t-bills and overnight loans has thus fallen.

As with the old penalty rate system, it is not possible to incorporate the step function directly into the analytical model. However, it can be argued that it can be fairly well approximated by a continuous  $RB^C$  function with  $\rho_R > 0$ , but less than infinity, i.e., what was considered the general case in Section 3.2.

To see this, it is necessary to look a bit more closely on how the trades in the interbank market for overnight funds are made. The ultimate restriction on the banks' daily liquidity management is that at the end of the market day, they have to have an amount corresponding to the required reserves in their accounts with the central bank. If they have excess funds they earn the interest corresponding to the lowest step in the function (6 percent) and if they do not have enough they have to borrow according to their individual step functions.

The purpose of the interbank trade that goes on during any market day is to see to it that mutually profitable arbitrage transactions are carried out. Ideally, funds should be re-distributed among the banks so that the marginal funding cost for each bank is the same, i.e., that they are all on the same interest rate step at the end of the day. Hence, a bank

with a strong reserve position should use its borrowing rights and relend the funds to a bank with deficient reserves via the interbank market.<sup>15</sup>

Note, however, that all the trades that are made during the day are made under uncertainty. No one knows what the aggregate liquidity position of the banking system is going to be at the end of the day and therefore no one can tell with certainty what the appropriate marginal funding cost is for the system as a whole. This means that the steps in the central bank's supply function for borrowed reserves are not binding for the interbank trades that are made during the market day. For example, a bank that expects the system to end up on the 12 percent level may make an (ex ante) mutually attractive deal at 11 percent with another bank that believes that 10 percent will be the level reached in the final reckoning. Hence, in the interbank trade, closing rates at in principle any level may be recorded. The general version of the model discussed in Section 3.2 would therefore seem to give a reasonable description of the Swedish system even under the stepshaped supply function.<sup>16</sup>

One difference between the two systems should be noted, however. In the actual lending system applied by the central bank, it is the marginal lending rate that is determined by the amount of borrowing. As there is an individual supply function for each bank, it is possible to make an individual allocation of the total costs for the banking system's borrowing. For example, consider a bank that can borrow a maximum of 1 billion kronor at 8 percent corresponding to the lowest step. If it on a particular day borrows 1.2 billion, it is charged 10 percent only for the final 0.2 billion.

In the theoretical system with a continuously increasing lending rate defined in terms of aggregate reserve borrowing, it is difficult to achieve such an allocation. Hence, each bank

would be charged the same interest rate on its entire volume of borrowed reserves and the average and marginal funding costs would therefore coincide. It is not obvious that this difference would be of any significance from the point of view of monetary policy, however. Other things equal, it would affect bank profits, but it is difficult to see any reason why it would matter for the interest rate determination. Interpreted in this way, it would seem that the central bank has found a practical way of implementing a system with a positively sloped supply function for borrowed reserves.

### 3.5 CONCLUDING COMMENTS

This chapter has analyzed the role of the discount window policy for the determination of money market interest rates and the effects of monetary policy. The purpose has not been to provide a fully comprehensive analysis of all the issues involved, but primarily to present a framework within which some important aspects can be examined.

At the most general level, the analysis has shown that the rules for banks' access to discount window borrowing have important consequences for the effects of both monetary policy interventions and non-policy shocks to the money market. The choice of discount window policy therefore affects the usefulness of the various monetary policy instruments and the possibilities to influence and stabilize the (intermediate) target variables. In particular, it was concluded that the instrument-target combination in the Swedish penalty rate system was relatively inefficient. There the penalty rate was the important instrument, but especially given the administrative problems involved in changing its value, it was not suitable for stabilizing foreign currency flows which have been nominated as a primary (intermediate) target by the central bank.

It was also possible to show with the help of the model developed in this chapter that the operating procedure introduced in December 1985 has several desirable properties compared to the old penalty rate system. With the positively sloped supply function for borrowed reserves, the central bank can use open market operations to control the demand for borrowed reserves. Provided the forecasts of the autonomous determinants of the stock of non-borrowed reserves are sufficiently accurate, this enables the central bank to choose the overnight rate that is considered consistent with the target for the currency flows. The direct instrument in this system is thus the open market interventions.

This system certainly provides more flexibility and uses an instrument that can be more quickly adjusted to offset new disturbances to the financial markets. Handled properly, it should therefore improve the central bank's control over its main target variable, flows of foreign currency. The shift from a completely open discount window to one that is "half-open" thus means that the rules are better adapted to the policy goals that the Swedish central bank has set up for itself.

It should be noted, however, that if the central bank had been concerned only with currency flows and disturbances coming from abroad, it would have been preferable to close the discount window completely. Then by fixing the value of  $T^C$ , the domestic interest rate would always adjust so that FX remained constant. This policy could lead to very wide interest rate fluctuations, however, and would not be consistent with the desire, discussed in Chapter 1, to maintain some degree of stability and independent control of domestic interest rates.

Nevertheless, the revision of the rules for discount window borrowing indicates that a higher degree of flexibility and, possibly, variability in interest rates have become necessary

in order to attain the desired stability in the currency flows. The growing difficulties with controlling FX in the previous system would therefore seem to imply that the latitude for an independent interest rate policy has decreased during the 1980s.

This raises the fundamental question concerning Sweden's international financial dependence, which has been given a very cursory treatment in this chapter. It was simply asserted, without any specific motivation, that some autonomy exists. The accuracy of this assertion and the conditions giving a country the opportunity to pursue an independent monetary policy will be discussed in Chapters 5, 6, and 7. Before that, Chapter 4 is devoted to a more rigorous analysis of the choice of discount window policy.

## APPENDIX

In this appendix a formal presentation of the comparative statics results discussed in Chapter 3 is given. The experiments analyzed are changes in  $T^C$ ,  $r_F$ , and  $\bar{r}_R$ . An additional policy instrument is the reserve ratio,  $a$ , but under lagged reserve accounting its effects are identical to those obtained from an open market operation ( $dT^C$ ).

(i) Open market operations ( $dT^C$ )

The interest rate effects of an open market purchase are, using the system (3.15),

$$\frac{dr_T}{dT^C} = \Delta^{-1}(t_R^b - \rho_R) < 0$$

$$\frac{dr_R}{dT^C} = \Delta^{-1}(m_T - t_T^b) < 0$$

where  $\Delta$  is defined in (3.16).

The effects on the recursively determined stock variables are easily seen to be

$$\frac{dM}{dT^C} = m_T \frac{dr_T}{dT^C} > 0$$

$$\frac{dFX}{dT^C} = -f_T \frac{dr_T}{dT^C} < 0$$

where  $dFX/dT^C = -1$  under perfect substitutability, i.e., if  $|f_T| = \infty$  there is complete offsetting, but, in general,  $dFX/dT^C > -1$ . Concerning the money stock effect, it can be noted that no conventional money multiplier can be defined in the current model. A priori, one may be inclined to assume that  $M$  would be given by the product of the inverse of the

reserve ratio,  $a$ , and the monetary base (i.e., the supply of total reserves). The standard model giving such a relationship is based on the assumption of contemporaneous reserve accounting, however. With lagged reserve requirements, the current money stock is not constrained by the supply of reserves, as reserve holdings depend only on the volume of deposits in a previous period. Hence, the multiplier is irrelevant for the determination of the money stock.

In the case with a completely open discount window, there is no effect on  $r_R$  and the change in  $r_T$  can be written as

$$\left. \frac{dr_T}{dT^C} \right|_{\rho_R=\infty} = - \frac{1}{t_T^b - m_T - f_T} < 0$$

While negative, this effect is smaller (in absolute value) than in the general case and also smaller the higher is  $t_T^b$ , i.e., the higher the substitutability between  $t$ -bills and overnight loans.

With  $\rho_R = 0$  the interest rate effects are

$$\left. \frac{dr_T}{dT^C} \right|_{\rho_R=0} = \frac{1}{f_T} < 0$$

$$\left. \frac{dr_R}{dT^C} \right|_{\rho_R=0} = \frac{m_T - t_T^b}{f_T t_R^b} < 0$$

Note that also in this case will the increase in  $T^C$  be fully offset by a decrease in  $FX$  since

$$\frac{dFX}{dT^C} = - f_T \cdot \frac{1}{f_T} = - 1$$

The reason is that with lagged reserve accounting the demand for reserves is fixed in kronor. Therefore, when  $T^C$  increases

there will be an excess supply of reserves and, as  $RB = 0$  in this case, this can only be eliminated by an outflow of foreign currency, i.e., a decrease in FX of equal magnitude. The fall in  $r_T$  is sufficient to make the public want to hold more foreign assets, i.e.,  $F^D$  rises.

(ii) A change in the foreign interest rate ( $dr_F$ )

The interest rate effects are

$$\frac{dr_T}{dr_F} = \Delta^{-1} [\rho_R(f_F + m_F) - f_F t_R^b] > 0$$

$$\frac{dr_R}{dr_F} = \Delta^{-1} [f_T m_F - f_F (m_T - t_T^b)] > 0$$

In order to guarantee  $dr_T/dr_F > 0$ , it must be assumed that  $f_F + m_F > 0$ , which is equivalent to assuming dominating own rate effects in the public's asset demand functions.

The implied effect on the foreign exchange reserves is

$$\begin{aligned} \frac{dFX}{dr_F} &= -f_T \frac{dr_T}{dr_F} - f_F \\ &= -\Delta^{-1} \rho_R [(f_T m_F + f_F (t_T^b - m_T))] < 0 \end{aligned}$$

Note that setting  $\rho_R = \infty$  reduces the increase in  $r_T$  and, hence, gives a larger outflow of foreign exchange reserves, other things equal. If in addition  $t_T^b$  is very large, which as argued in Section 3.2 is likely in a penalty rate system,  $dr_T/dr_F$  becomes very small and the sensitivity of FX to foreign disturbances even greater. Conversely,  $\rho_R = 0$ , i.e., a closed discount window, effectively insulates FX as this implies  $dFX/dr_F = 0$ .



(iii) A change in the benchmark discount rate ( $d\bar{r}_R$ )

This experiment is most easily interpreted in a situation with a completely open discount window when  $\bar{r}_R$  is the pegged central bank lending rate - the penalty rate. In the general case, the interest rate effects are

$$\frac{dr_T}{d\bar{r}_R} = -\Delta^{-1}\rho_R t_R^b > 0$$

$$\frac{dr_R}{d\bar{r}_R} = \Delta^{-1}\rho_R(t_T^b - m_T - f_T) > 0$$

It is immediately seen that both these expressions are zero when  $\rho_R = 0$ , since obviously the benchmark rate does not matter in a situation when the discount window is effectively closed. On the other hand, when  $\rho_R = \infty$ ,  $dr_R/d\bar{r}_R = 1$  and

$$\left. \frac{dr_T}{d\bar{r}_R} \right|_{\rho_R=\infty} = - \frac{t_R^b}{t_T^b - m_T - f_T}$$

which also goes to unity in the extreme case of perfect substitutability between overnight loans and t-bills,  $t_T^b = |t_R^b| = \infty$ .

## NOTES

1. This chapter generalizes the model discussed by Englund, Hörngren, and Viotti (1985a, b). There we only analyze the two extreme cases of completely closed and completely open discount window policies, and assume that central bank lending and money market instruments are treated as perfect substitutes.
2. Cf. Englund, Hörngren, and Viotti (1985a).
3. As discussed below this is also a good approximation of the actual Swedish system.
4. For a micro analysis of the Federal funds market, see Ho and Saunders (1985).
5. As the analysis in Chapter 4 will show, values of  $\rho_R < 0$ , i.e., a downward sloping supply curve may be desirable in some situations, but in this chapter only non-negative values of  $\rho_R$  will be considered.
6. Another example of this is Poole (1982) which is discussed in Chapter 4. For an analysis similar to Langohr's (1981), but with an explicit supply side, see Wessels (1982).
7. These conclusions are motivated more formally in the Appendix.
8. Although this is not directly obvious from the graph,  $r_R$  must fall. The horizontal shift of TR is always numerically greater than that of NBR as  $|f_T| < |m_T + f_T - t_T^b|$ . The Appendix gives the exact expressions. Note also that in a closed economy where  $f_T = 0$ , the model would collapse in this case. The reason is that the banks' demand for reserves is fixed and independent of interest rates, but

8. (continued)  
with no capital flows so is the supply of (non-borrowed) reserves. The model would therefore not be able to determine an equilibrium value of  $r_R$ . This indeterminacy would not occur under contemporaneous reserve accounting, however; cf. Englund, Hörngren, and Viotti (1985a).
9. Note that the assumption of perfect substitutability between discount loans and t-bills does not hold in this market situation. This is the reason why  $r_T$  did not momentarily fall to  $r_X$  when RB was reduced to zero. For a more detailed discussion and a formal analysis of the interest rate determination in this two-regime model, see Englund, Hörngren, and Viotti (1985a).
10. As the system that was abandoned in Denmark in August 1985 involved a stepshaped supply function for borrowed reserves highly similar to that formulated by the Swedish central bank, one can say that Sweden and Denmark both felt the need for reform but in opposite directions. One explanation of this is that the opportunities for making effective open market operations appear to be better in Sweden than in Denmark, where the money market is less developed.
11. Cf. the discussion in Englund, Hörngren, and Viotti (1985b).
12. See Hörngren and Viotti (1985) for a detailed discussion of these events.
13. This is the form of the supply function introduced in May 1986. The modifications compared to the original schedule are purely technical, however.
14. Englund, Hörngren, and Viotti (1986) gives a more detailed description of the workings of the new system.

15. As the market is organized without a central auctioneer, the arbitrage mechanism is not perfect, but normally the banks manage to even out most of the imbalances in the reserve holdings among the individual banks.
16. It can be noted that one effect of the new system is also that the interbank market has grown in importance and scope. Trading is no longer limited to overnight funds, but rates for longer maturities and various forward agreements are also quoted. This reflects the increased importance for banks of short run liquidity management under a system with a variable overnight interest rate.

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## 4. Optimal Monetary Policy Rules: the Role of the Discount Window\*

### 4.1 INTRODUCTION

Central bank lending policies differ between countries. This was one of the points of departure for the analysis in the previous chapter. As noted there, many European central banks give commercial banks more or less unlimited access to reserve borrowing at a fixed discount rate. In the U.S., in contrast, there are formal and informal restrictions on the amount of borrowing allowed. Indeed the standard textbook model, which is based on the U.S. system, presumes that in practice central bank borrowing is so small as to be inconsequential for money stock control. This is a system in which the central bank acts as lender of last resort but where the discount window is not considered an instrument of monetary policy.

In this chapter we address the question whether there is an optimum discount window policy in the sense of a supply curve relating the cost (implicit or explicit) of central bank borrowing to the amount borrowed. This is done within an optimal

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\* This chapter was written together with Peter Englund and Staffan Viotti.

policy framework of the type pioneered by Poole (1970). His analysis was conducted in a conventional IS/LM framework. Subsequent developments have looked at instrument choice in partial money market models - LeRoy and Waud (1977), LeRoy and Lindsey (1978) - and aggregate demand and supply models with and without rational expectations - Turnovsky (1980), Craine and Havenner (1981), Santomero and Siegel (1981), Canzoneri, Henderson and Rogoff (1983) and others. Our model belongs to the former category. It is a short term money market model, where the central bank is concerned with minimizing the variance of the money supply around its target value. This appears to be the framework within which operating procedures are discussed in practice; see e.g. Axilrod and Lindsey (1981), *Current Issues in the Conduct of U.S. Monetary Policy* (1982), Bryant (1983), and McCallum (1985).

Compared with previous studies we extend the analysis by letting the central bank consider not only its open market operations strategy but also its discount window policy. There are two markets where the central bank can have a direct impact: the t-bills market and the interbank market. As in Chapter 3 interbank and central bank overnight loans are assumed to be perfect substitutes. The problem for the central bank is how to use the interest rates in these two markets to extract information about underlying disturbances affecting the stock of money which is not instantaneously observable.

Our primary question is: What, if any, is the role of the discount window? This question turns out not to have a clear answer. We derive three "optimal" policies that give exactly the same value of the variance of the money stock. One of these specifies a discount window rule in which case the choice of open market strategy is inconsequential. But there is another policy where the roles are reversed; only the open market strategy is specified and the discount window does not matter. There is, however, also a case with a role for both, and we



argue in the chapter that this may be the administratively most appealing policy. This uses the discount window to peg the interbank rate, whereas open market operations are only conditioned on the t-bills rate.

## 4.2 ANALYTICAL FRAMEWORK

It is customary to distinguish between ultimate goals, intermediary targets and instruments of economic policy. In this paper we adopt a two-stage view of policy making where a value of the intermediary target is assumed to have been derived from some ultimate goal(s). Given this the central bank attempts to handle its instruments so as to minimize the expected deviation of the money stock from the desired value.

Such a strategy is certainly open to critique for unnecessarily focusing on a target of doubtful significance instead of deriving the optimal instrument policy rules directly from the ultimate goals. For forceful examples of such criticism see Friedman (1975) and Bryant (1980), part 4. Thinking in terms of the hierarchy goal-target-instrument is convenient, however, since it allows us to use a more detailed model of the money market without being explicit about general equilibrium repercussions.

As a background to our analysis we shall briefly review the money market model of LeRoy and Waud (1977). Demand for money,  $M$ , depends on the short term interest rate,  $r$ , and on a stochastic factor,  $u_1$ , assumed to be normally distributed and unobservable to the central bank;

$$M = a_0 - a_1 r + u_1 \quad (4.1)$$

Banks are assumed to accommodate the public's money demand and to demand reserves,  $R$ , as a function of  $M$ ,  $r$ , and a stochastic term,  $u_2$ , also assumed normally distributed and unobservable to the central bank;

$$R = b_0 + b_1 M - b_2 r + u_2 \quad (4.2)$$

The information structure is central to the problem. It is assumed that the central bank knows the parameters of the model, but it cannot observe  $M$  and, hence, not infer the realizations of the stochastic terms  $u_1$  and  $u_2$ . If it could, there would not be any policy problem; the central bank could simply set  $R$  so as to get the desired value of  $M$ .

The problem is now for the central bank to design its policy while exploiting all available information in the best possible way, where "best" is interpreted in terms of a quadratic loss function. There are basically two equivalent ways to solve this problem. The first one uses properties of conditional expectations for jointly normal variables. Assuming that the central bank can observe  $r$  and  $R$ , it is easily seen from the reduced forms of (4.1) and (4.2) that the central bank can infer the value of the composite stochastic term  $u = u_2 + b_1 u_1$ .<sup>1</sup> The expected value of  $u_1$  conditional on a given value of  $u$  can then be calculated. By certainty equivalence, this is the value to plug into (4.1) and then solve for the  $r$  that would lead to target fulfillment for  $M$ .

Substituting away  $u$  this may be expressed in terms of a policy reaction function;

$$R = A_0 + A_1 r \quad (4.3)$$

where the  $A$ 's are functions of the parameters of the model including the variance-covariance matrix of the stochastic terms.

Alternatively, the optimal values of  $A_0$  and  $A_1$  can be calculated by adding (4.3), which we know is the structure of the optimal policy reaction function, to (4.1) and (4.2) and then solving

the system. This gives  $M$ , and the unconditional loss function in  $M$ , as functions of the parameters of the model. Minimizing the loss function with respect to  $A_1$  gives the same result as above.

The purpose of this chapter is to analyze the simultaneous choice of discount window policy and open market strategy using this approach. This means that we recognize that there are at least two markets where the central bank may have a direct impact: the t-bills market and the interbank market for overnight loans. While these two assets may be rather close substitutes from the point of view of commercial banks, they are certainly not, as discussed in Chapter 3, perfect substitutes.

In reality the central bank operates with several instruments of varying maturities. Since our focus in this paper is on the distinction between overnight loans and assets with longer terms to maturity we will ignore this complication. The t-bills market should be considered a catch-all for all markets except the overnight market on which the central bank operates.

#### 4.3 THE MODEL

We will consider a simplified short term money market model, which is highly similar to that used in Chapter 3. The main differences are that we are considering a closed economy and that the current model is explicitly stochastic. This allows a formal analysis of the effects of random shocks to the model. For convenience and clarity, the current version of the model is presented in some detail despite its similarities to the model in Chapter 3.

There are three sectors, the central bank, banks, and the public, with the following balance sheets:

$$\begin{aligned} T^C + RB^C &= R^C && \text{(Central Bank)} \\ T^b + R^b &= M + RB^b && \text{(Banks)} \\ T^p + M &= W && \text{(Public)} \end{aligned}$$

where

$$\begin{aligned} T^i &= \text{Sector } i\text{'s holdings of t-bills} \\ M &= \text{Bank deposits (= Money stock)} \\ R &= \text{Bank reserves} \\ RB &= \text{Borrowed reserves (= Overnight money)} \\ W &= \text{Private short-term financial net wealth} \end{aligned}$$

The central bank's reserves supply is the sum of its holdings of t-bills and borrowed reserves. We disregard cash, which in the short run can be assumed to be insensitive to interest rate variations, so all reserves are held by banks. Banks hold t-bills and reserves, and accept deposits and borrow reserves in the central bank. The public has a portfolio choice to divide its short term assets between demand deposits, which equal the money stock, and t-bills.

The reason for limiting the analysis in this chapter to a closed economy model is primarily analytical tractability. The main problem in an open economy model is that the number of potential shocks increases and the stochastic structure therefore becomes more complicated. There is also an additional signal extraction problem for the central bank to handle. Assume that it is deemed desirable to stabilize the foreign exchange reserves. This is of course a continuously observable variable, but it is difficult to determine whether a given change is due to current account or capital account transactions. As the appropriate policy reactions to these disturbances may differ, this is a real signal extraction problem, which also has considerable practical significance. The analytical difficulty encountered in this case is that current

account shocks involve changes in wealth which have repercussions on all markets. We intend to look more carefully at this issue in future work.

Under current assumptions we have four markets: the markets for borrowed reserves, non-borrowed reserves, t-bills and deposits. Assuming all four always to clear it suffices to look at three of them. Furthermore, equilibrium in the deposit market is completely demand determined and thus can be solved recursively. The equilibrium system becomes

$$R^b - RB^b = T^C \quad (\text{Non-borrowed Reserves}) \quad (4.4)$$

$$R^b = T^C + RB^C \quad (\text{Total Reserves}) \quad (4.5)$$

$$M = M^D \quad (\text{Money}) \quad (4.6)$$

$R^b$  is the banks' reserve demand and is given by

$$R^b = a \bar{M} + u_R \quad (4.7)$$

where the coefficient  $a$  is a reserve requirement imposed by the central bank, and the stochastic variable  $u_R$  represents excess reserves.  $\bar{M}$  is bank deposits at some earlier point in time, which is left unspecified, because we never deal with the dynamics inherent in lagged reserve accounting.<sup>2</sup>

To derive banks' demand for borrowed reserves (and hence for non-borrowed reserves) we first specify the banks' demand for t-bills<sup>3</sup>

$$T^b = t_0 + t_T r_T - t_R r_R + u_T \quad (4.8)$$

T-bill demand is thus a function of the t-bills rate, the interest rate on borrowed reserves (overnight loans) and a

stochastic term,  $u_T$ . We note that with perfect substitutability between t-bills and overnight money, i.e. with  $t_T = t_R = \infty$  and  $t_T/t_R = 1$ , (4.8) implies  $r_T = r_R$ .

Banks' demand for borrowed reserves is now obtained from the balance sheet constraint

$$RB^b = t_0 + t_T r_T - t_R r_R - M + a\bar{M} + u_R + u_T \quad (4.9)$$

Further, the public's demand for money (bank deposits) is determined by

$$M^D = m_0 - m_T r_T + u_M \quad (4.10)$$

The stochastic terms  $u_i$  ( $i=M,R,T$ ) are taken to be independently normally distributed with zero mean and variance  $\sigma_i^2$ . Note that the assumption of zero covariances implies from the balance sheet constraint that the stochastic term of (4.9) is correlated with those of (4.7) and (4.8). We also observe that in principle it is not appropriate to assume  $u_R$  to be normally distributed with mean zero, since it will not except by mistake be negative. This complexity is ignored here, however.

The supply of reserves (borrowed as well as non-borrowed) is of course determined by the central bank. In analogy with the policy reaction function (4.3) in the preceding section we assume that the central bank supplies reserves according to the following two policy rules:

$$T^C = \eta_0 + \eta_T r_T + \eta_R r_R \quad (4.11)$$

$$RB^C = \rho_0 + \rho_T r_T + \rho_R r_R \quad (4.12)$$

These equations can be said to represent, respectively, the open market operations and discount window policies of the central bank. Since both  $r_T$  and  $r_R$  provide information about

the realizations of the unobservable stochastic variables in the system and both are (contemporaneously) observable to the central bank, they both appear in the policy reaction functions. With  $\eta_R = 0$  (4.11) resembles (4.3), the ordinary Poole-inspired reaction function for open market operations, where  $\eta_T = 0$  and  $\eta_T = \infty$  correspond to the extreme cases of complete inaction in the open market and complete interest pegging, respectively. In (4.12)  $\rho_R = 0$  and  $\rho_R = \infty$  represent the cases of a completely closed and completely open discount window policy, respectively, provided that  $\rho_T = 0$ . In the former case no borrowing is allowed in the central bank, in the latter case unlimited amounts could be borrowed at the rate set by the central bank. With  $\rho_T = 0$ , (4.12) is equivalent to the supply function for borrowed reserves used in Chapter 3, equation (3.8), with  $\bar{r}_R$  included in the constant term  $\rho_0$ . The difference is that here we are interested in finding an optimal value for  $\rho_R$  (and the other policy parameters) rather than in the effects of any particular choice of discount window policy.

Policies according to (4.11) and (4.12) are both implemented along the lines suggested by LeRoy and Waud (1977) (cf. Section 4.2 above), only in this case the central bank lets its broker act in both the overnight loan and t-bills markets.

The discussion in Chapter 3 concerning the relation between the interest rates in these two markets is relevant also in this context, of course. If overnight loans and t-bills are considered perfect substitutes, which in a sense is the static analog to the expectations hypothesis of the term structure of interest rates,  $r_R$  and  $r_T$  are invariably equal. In this case, the central bank can obviously not gain anything by looking at more than one interest rate. As the previous literature has studied models with only one interest rate, perfect substitutability is the (implicit) assumption on which these analyses have been based. As argued in Chapter 3, it seems

worthwhile to consider a more general case even though we cannot provide a fully satisfactory account of the term structure relation. To study this problem in the context of a dynamic model with an explicit term structure relation is another interesting topic for future work.

#### 4.4 A COMMENT ON POOLE

A previous study incorporating reserve borrowing in a simple money market model of this type is Poole (1982) and it is useful to compare his approach to ours. Poole specifies a borrowed reserves function which in our notation reads:

$$RB = t_0 + t_T(r_T - r_R) + u \quad (4.13)$$

where  $t_T$  is specified to be positive. This function is then plugged into the equilibrium condition for total reserves which is identical to (4.5) above. Solving this equation for  $M$  yields "the deposit supply function". This is inserted into a money market equilibrium condition which is solved for the market interest rate, identified as the federal funds rate. By computing the variance of deposits under the assumptions of an exogenous interest rate and exogenous supply of non-borrowed reserves Poole discusses the relative usefulness of "interest rate control" and "reserve control", respectively.

The crucial difference between our approach and Poole's lies in the treatment of borrowed reserves. We specify an explicit supply function (4.12) that captures the central bank's discount window policy. This makes it clear that  $r_R$  is an endogenous variable in the system determined by equating (4.12) and the demand function (4.9). A constant borrowing rate is just the special case of an infinitely elastic supply of borrowed reserves.



Poole, however, treats  $r_R$ , the discount rate in his terminology, as a constant. This makes borrowed reserves demand determined and (4.13) should therefore be interpreted as a demand function, which is also consistent with the assumption that  $t_T$  is positive and with the introduction of a stochastic term. The absence of a supply function for borrowed reserves means that there is no clear definition of "discount window policy". Poole makes several references to varying degrees of openness of the discount window, but he is never explicit about how this is captured in the model.

In effect, he treats the case with a fully open discount window. This means, for example, that a reserve control policy is not necessarily feasible since, as discussed in detail in Chapter 3, open market operations that change the supply of non-borrowed reserves will be offset by adjustments in the banks' borrowing position. If discount borrowing and federal funds are considered perfect substitutes,  $t_T$  in (4.13) is infinite and it is easily seen that  $dM/dT^C = 0$  in Poole's model.

Under unlimited borrowing, it is difficult to see any reason why they should not be perfect substitutes. It could be argued that in the U.S. the Fed does not permit unlimited borrowing, but then the discount rate is in effect not constant, at least in the sense that increased borrowing today will lead to higher borrowing costs in the future. Either formally or informally, e.g., through a "cost of harassment" term of the type discussed by Brunner (1974), the banks face an upward-sloping supply curve for borrowed reserves.<sup>4</sup> This element is missing in Poole's framework. In our model, we take these considerations into account and can therefore provide an explicit analysis of the optimal operating procedures both for discount window and open market policies.

## 4.5 COMPARATIVE STATICS

Before explicitly analyzing the policy problem of how to find the optimal values for  $\eta_i$  and  $\rho_i$  ( $i = T, R$ ) in (4.11) and (4.12),<sup>5</sup> some insights can be gained by looking at the comparative-static properties of the model. Substituting the behavioral relations into the market equilibrium conditions (4.4), (4.5) and (4.6) and differentiating yields

$$\begin{bmatrix} -1 & \eta_T + t_T & \eta_R - t_R \\ 0 & \eta_T + \rho_T & \eta_R + \rho_R \\ 1 & m_T & 0 \end{bmatrix} \begin{bmatrix} dM \\ dr_T \\ dr_R \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} du_T \\ du_R \\ du_M \end{bmatrix}$$

Solving this system for  $dM$  we can study how the disturbance variables  $u_i$  affect the (intermediate) target variable  $M$  for given parameter values in the policy reaction functions. The effects are

$$dM/du_T = m_T(\eta_R + \rho_R)/\Delta$$

$$dM/du_R = m_T(\eta_R - t_R)/\Delta$$

$$dM/du_M = [(t_T + \eta_T)(\eta_R + \rho_R) - (\rho_T + \eta_T)(\eta_R - t_R)]/\Delta$$

where the determinant  $\Delta = (m_T + t_T + \eta_T)(\eta_R + \rho_R) - (\rho_T + \eta_T)(\eta_R - t_R)$

We can now ask what values of the policy function parameters ( $\eta_R$ ,  $\eta_T$ ,  $\rho_R$  and  $\rho_T$ ) will fully neutralize one or more of the disturbances, i.e., will give  $dM/du_i = 0$  for at least one  $i$  ( $i = M, R, T$ ).<sup>6</sup>

We start with disturbances emanating from the banking sector, and note that a change in  $u_R$  and/or  $u_T$  can only affect  $M$  indirectly through  $r_T$ . It then follows that if the supply of

reserves, borrowed or non-borrowed, is perfectly elastic with respect to  $r_T$  - i.e., the t-bills rate is pegged - then the money stock is perfectly controlled. In that case disturbances will only affect the banks reserve holdings - borrowed or non-borrowed depending on the disturbance - and the overnight interest rate. On the other hand, a money demand disturbance will have full impact on  $M$ . Formally

$$\eta_T = \infty \text{ and/or } \rho_T = \infty \Rightarrow dM/du_R = dM/du_T = 0; dM/du_M = 1 \quad (4.14)$$

We also note from inspecting the comparative static expressions that another way to insulate the money stock from either of the disturbances originating in the banking sector is

$$\eta_R = t_R \Rightarrow dM/du_R = 0 \quad (4.15)$$

and

$$\eta_R = -\rho_R \Rightarrow dM/du_T = 0 \quad (4.16)$$

An increase in  $u_R$  will, *ceteris paribus*, increase the demand for reserves. To meet the increased demand, supply has to change. According to the reaction functions (4.11) and (4.12) this can happen only if  $r_T$ ,  $r_R$  or both change. To avoid changes in  $r_T$  and thereby in the money stock the policy functions must be designed so that  $r_T$  is left unaffected. This can be achieved if changes in the demand for non-borrowed reserves caused by changes in  $r_R$  are always exactly matched by corresponding changes in the supply, i.e., if  $\eta_R = t_R$ . In this case, although  $r_R$  changes to clear the market for total reserves, this will have no repercussions on  $r_T$  and  $M$ . An increase in  $u_T$ , having its direct effect on the market for non-borrowed reserves, would by similar reasoning have no effect on  $r_T$  and  $M$  if equilibrium in the market for total reserves were independent of  $r_R$ , i.e., if (4.16) were to hold. Conditions (4.15) and (4.16) cannot, however, be applied simultaneously

as in this case the equilibrium equations (4.4) and (4.5) become linearly dependent and only one interest rate can be determined.

We also note that a simultaneous shock of equal magnitude in  $u_R$  and  $u_T$  but with opposite signs, i.e., a ceteris paribus switch between t-bills and reserves in bank portfolios, can be neutralized by having  $\rho_R = -t_R$ , i.e.,

$$\rho_R = -t_R \Rightarrow dM/du_R - dM/du_T = 0 \quad (4.17)$$

Let us now look at disturbances in money demand,  $u_M$ . The general expression for  $dM/du_M$  involves all of the policy parameters. It turns out that

$$\rho_T = -\eta_T = t_T \Rightarrow dM/du_M = 0 \quad (4.18)$$

In this case both (4.4) and (4.5) are independent of  $r_T$ . This means that an increase in  $u_M$  will only affect  $r_T$  so as to leave the quantity of money demanded unaffected. It will have no repercussions on the reserve markets.

Conditions (4.14) - (4.18) insulate the money stock from one or two types of shocks. It is not possible, however, with the type of policy rules that we have specified to insulate our model economy from all kinds of disturbances simultaneously. Condition (4.18) may be combined with either of (4.15), (4.16) or (4.17) but not with more than one.<sup>7</sup>

The conclusion that can be drawn from the comparative static analysis is that if one of the three stochastic terms has a very low variance relative to the others, then there is a policy rule that is (almost) perfect in the sense of making the money stock insensitive to shocks in these two terms. In general, however, the task of stabilizing the money stock causes a real trade-off problem. That trade-off will be analyzed in the next section.

## 4.6 OPTIMAL POLICY RULES

To analyze the general policy problem the solution of the model is used to derive an expression for the variance of  $M$ . Since we have assumed that the random shocks are uncorrelated the variance of  $M$  is a weighted average of  $\sigma_i^2$  ( $i = T, R, M$ ), with the weights given by the squares of  $dM/du_i$ , the comparative static effects of changes in either of the disturbance factors. Thus we can write

$$\begin{aligned} \text{Var } M = & \Delta^{-2} \left\{ m_T^2 (\eta_R + \rho_R)^2 \sigma_T^2 + m_T^2 (\eta_R - \tau_R)^2 \sigma_R^2 + \right. \\ & \left. + [(\tau_T + \eta_T)(\eta_R + \rho_R) - (\rho_T + \eta_T)(\eta_R - \tau_R)]^2 \sigma_M^2 \right\} \end{aligned} \quad (4.19)$$

The optimal policy is derived from minimizing the variance of  $M$ .<sup>8</sup> Differentiating (4.19) with respect to the parameters of the policy rule equations gives, after a considerable amount of algebra, the following set of first order conditions:

$$\begin{aligned} d \text{ Var } M / d \eta_R &= 2 \Delta^{-3} (\tau_R + \rho_R) \Omega = 0 \\ d \text{ Var } M / d \rho_R &= 2 \Delta^{-3} (\tau_R - \eta_R) \Omega = 0 \\ d \text{ Var } M / d \eta_T &= 2 \Delta^{-3} (\tau_R + \rho_R) \Gamma = 0 \\ d \text{ Var } M / d \rho_T &= 2 \Delta^{-3} (\tau_R - \eta_R) \Gamma = 0 \end{aligned} \quad (4.20)$$

where

$$\begin{aligned} \Omega = & [\eta_R (\tau_T - \rho_T) + \tau_R (\eta_T + \rho_T) + \rho_R (\tau_T + \eta_T)] (\rho_T + \eta_T) \sigma_M^2 + \\ & + m_T (\eta_R + \rho_R) (\eta_T + \rho_T) \sigma_T^2 + m_T (\eta_R - \tau_R) (\eta_T + m_T + \tau_T) \sigma_R^2 \\ \Gamma = & [\eta_R (\tau_T - \rho_T) + \tau_R (\eta_T + \rho_T) + \rho_R (\tau_T + \eta_T)] (\rho_R + \eta_R) \sigma_M^2 - \\ & - m_T (\eta_R + \rho_R)^2 \sigma_T^2 - m_T (\eta_R - \tau_R)^2 \sigma_R^2 \end{aligned}$$

The system of first order conditions (4.20) is highly non-linear. One can easily find several combinations of the policy parameters which imply that all conditions are fulfilled simultaneously. In the Appendix we give a derivation of the 8 cases that we have found, but due to the non-linearity we cannot guarantee that the list is exhaustive. Out of these there are three which unambiguously dominate all the others in the sense of giving a lower variance of  $M$  irrespective of the relative variances. These cases are:

$$A. \quad \rho_R \rightarrow \infty; \quad \eta_T = \eta_T^* \equiv m_T \frac{\sigma_T^2}{\sigma_M^2} - t_T; \quad \eta_R \text{ and } \rho_T \text{ arbitrary.}$$

This policy amounts to pegging the interbank interest rate by allowing unlimited discount window borrowing at a certain rate. The rule for open market operations uses only the  $t$ -bills rate as information variable. As the interbank rate is constant, it is obvious that it cannot contain any information about the shocks to the economy.

$$B. \quad \eta_R = t_R; \quad \eta_T = \eta_T^*; \quad \rho_R \text{ and } \rho_T \text{ arbitrary.}$$

In this case the choice of discount window parameters is immaterial, and the rule for open market operations uses both interest rates as information variables.

$$C. \quad \rho_T, \quad \rho_R \rightarrow \infty \text{ and } \rho^* \equiv \frac{\rho_T}{\rho_R} = \frac{t_T + \eta_T - m_T \frac{\sigma_T^2}{\sigma_M^2}}{\eta_R - t_R};$$

$$\eta_R \text{ and } \eta_T \text{ arbitrary.}$$

This policy amounts to letting both  $\rho_T$  and  $\rho_R$  go to infinity, but in a fixed proportion given by  $\rho^*$ .<sup>9</sup> In this case the open market operations parameters become indeterminate. The discount window policy is equivalent to pegging a weighted sum of the interest rates,  $r_R + \rho^* r_T$ .<sup>10</sup>

All these cases give

$$\text{Var } M^* = \frac{\sigma_T^2 \sigma_M^2}{\sigma_T^2 + \sigma_M^2} \quad (4.21)$$

Taking the difference between the objective function (4.19) and the minimum variance (4.21), it can be shown, after some manipulations, that

$$\begin{aligned} \text{sgn}[\text{Var } M - \text{Var } M^*] = \\ \text{sgn}[\{m_T(\eta_R + \rho_T)\sigma_T^2 - [\eta_R(t_T - \rho_T) + t_R(\eta_T + \rho_T) + \rho_R(t_T + \eta_T)]\sigma_M^2\}^2 \\ + m_T^2(\eta_R - t_R)^2\sigma_R^2(\sigma_T^2 + \sigma_M^2)]. \end{aligned}$$

This expression is clearly non-negative independently of the values of the policy parameters, which shows that a smaller target variance cannot be attained. However, while  $\text{Var } M^*$  is the global minimum, it is obviously not unique, and it cannot be excluded a priori that there are combinations of policy parameters other than those discussed above that would give the same target variance.

Looking at the three optimal policies it is clear that one cannot say anything in general about what constitutes the role for the discount window. According to A there is a specific division of labor between the discount window and the open market trading desk, but according to B and C either of these is superfluous. The important general conclusion is that it is inefficient for the policy maker not to consider both interest rates. This is in contrast to the previous literature where the interbank rate typically has been ignored. However, note that it is not necessary to use both interest rates as information variables. This is illustrated by case A, where it is optimal to peg the interbank rate, a policy that makes it lose all of its potential information content.

Given that the three policy rules A, B, and C are equivalent in terms of their effects on the central bank's policy target, other criteria must be used to distinguish between them. From an administrative point of view rule A appears to be the simplest. Pegging the interbank rate by allowing unlimited borrowing at the discount window has been practiced by many European central banks for decades, and using the t-bills rate as an information variable is explicitly or implicitly done in practice by those central banks that pursue an active policy of open market operations. Also rule B seems rather straightforward to handle. It would, however, require that the central bank's trading desk collect information not only on the interest rate in the market they are trading in, but also on the interest rate in a quite separate market, the interbank market. Case C appears more complicated than the others, since it involves use of the discount window to peg an index of the two interest rates. In a sense the discount window is fully open, but the discount rate is continually adjusted in response to changes in the t-bills rate. This is a technically complicated rule that may be difficult to implement.

In trying to get an intuitive understanding of the optimal policies we will concentrate on policies A and B, and refer the reader to the Appendix for some comments on the relation between these and C. We first note that it is optimal to insulate the money stock from disturbances coming from the bank demand for reserves. This result is related to our assumption that an increase in reserve demand is matched by an equal increase in the demand for borrowed reserves (the covariance between  $u_R$  and  $u_T$  is zero). Hence,  $u_R$  does not affect the supply of money and only has an indirect effect on  $M$  via interest rate effects. This is accomplished by  $\rho_R \rightarrow \infty$  or  $\eta_R = t_R$ . In either of these cases we have  $dM/du_R = 0$  and



$$dM/du_T = \frac{t_T + \eta_T}{m_T + t_T + \eta_T}$$

$$dM/du_M = \frac{m_T}{m_T + t_T + \eta_T}$$

implying

$$\text{Var } M = \frac{(t_T + \eta_T)^2 \sigma_M^2 + m_T^2 \sigma_T^2}{(m_T + t_T + \eta_T)^2}$$

which shows that the choice is in effect reduced to a trade off between  $\sigma_T^2$  and  $\sigma_M^2$ . This expression motivates that the optimal value  $\eta_T^*$  goes from  $-t_T$  to  $+\infty$  as  $\sigma_T^2/\sigma_M^2$  goes from 0 to  $+\infty$ . It also shows that it is not a priori clear whether the central bank should react to an observed increase in  $r_T$  by buying or selling t-bills. If the demand for money is relatively more variable, it is likely that an observed interest increase is due to an increase in the demand for money in which case it should be counteracted by a decrease in the supply of reserves, i.e.,  $\eta_T^* < 0$ . On the other hand, if bank demand for t-bills, i.e., the supply of money, is relatively more variable, then it is likely that an observed interest increase is the result of a reduced t-bills demand on part of the banks, which in turn will lead the public to switch from money to t-bills resulting in a decrease in the money stock. In this situation the central bank should react by increasing the supply of reserves, i.e.,  $\eta_T^* > 0$ .

#### 4.7 CONCLUDING COMMENTS

Some authors (e.g., Brunner (1974)) attach a great import to the choice of discount window policy. In particular, it is often argued that a completely open discount window means that the central bank gives up control over the money stock. The analysis of the present paper indicates that this assertion is not necessarily valid. By widening the perspective and simultaneously considering the policy for open market operations, we have shown that a completely open discount window may indeed be a part of a policy aimed at minimizing the variance of the money stock.

This finding may seem to be at odds with the (tentative) conclusion in Chapter 3 that a completely open discount window as in the Swedish penalty rate regime has several undesirable properties. It should be noted, however, that this statement was made in a somewhat different context. In particular, the concern in Chapter 3 was with a central bank that primarily wanted to stabilize the foreign exchange reserves. In addition, disturbances originated from abroad, for example, in the form of unexpected changes in the foreign interest rate. The closed economy model used in the more rigorous analysis in this chapter is of course not able to handle either this policy target or any foreign disturbances. A priori, it is also highly plausible that the optimal policy rules depend on the choice of policy target, which means that the optimality of rule A in no way contradicts the more intuitive conclusions stated in Chapter 3.

Nevertheless, this points to the need for a more formal analysis along the lines of this chapter of an open economy model. This would also make the results more directly relevant for a discussion of Swedish monetary policy. In particular, it would be possible to address the problem of how to extract information about the sources of variations in the foreign

exchange reserves, i.e., whether they emanate from the current account or the capital account. Given the central bank's concern with stabilizing currency flows, this is a highly interesting policy problem seen from a Swedish perspective. As mentioned in Section 4.3, this is a topic for future research.

The conclusions concerning the rules for the central bank's open market policies in a closed economy are less specific. In particular, we see that one cannot determine the sign of  $\eta_T^*$ , which is important in both cases A and B, on a priori grounds, i.e., whether the central bank should react to a high t-bills rate by buying or selling t-bills. This depends both on the relative variability of the shocks involved and on the properties of household and bank demand functions. Despite this indeterminacy the analysis in this chapter indicates, in contrast to the previous literature where the standard practice has been to study models where the central bank only observes one interest rate, that the policy performance may be improved by considering additional information variables. Future work will have to show whether this finding is robust to more complete specifications of, in particular, the relation between the different interest rates.

## APPENDIX

In this appendix we derive stationary points of Var M. The first three cases fulfill  $dM/du_i=0$  ( $i=R,T,M$ ).

Case 1:  $dM/du_R = 0$ . From (4.19) we see that setting  $\eta_R = t_R$  completely insulates the system from variations in  $u_R$ . It is immediately seen from (4.20) that the second and fourth first-order conditions are then satisfied. Then insert  $\eta_R = t_R$  into  $\Omega$  and  $\Gamma$  and set them equal to zero.<sup>1</sup> It turns out that

$$\eta_T = m_T \frac{\sigma_T^2}{\sigma_M^2} - t_T \equiv \eta_T^*$$

satisfies both  $\Omega = 0$  and  $\Gamma = 0$ . Thus  $\eta_R = t_R$  and  $\eta_T = \eta_T^*$  satisfy (4.20) for arbitrary values of  $\rho_i$ . Substituting these parameter values into the objective function (4.19)  $\text{Var } M_1 = \sigma_T^2 \cdot \sigma_M^2 / (\sigma_T^2 + \sigma_M^2)$  is easily derived.

Case 2:  $dM/du_T = 0$ . From (4.19) we know that  $\eta_R = -\rho_R$  eliminates all effects on M from variations in  $u_T$ . With this condition inserted it turns out that setting  $\eta_T$  or  $\rho_T$  equal to infinity makes  $\Omega/B^3$  and  $\Gamma/B^3$  go to zero. Thus  $\eta_R = -\rho_R$  and  $\eta_T = \infty$  or  $\rho_T = \infty$  satisfies (4.20). The variance in case 2

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<sup>1</sup> Note that setting  $\eta_R = t_R = -\rho_R$  is no solution since it implies  $\Delta = 0$ , i.e., the system becomes undetermined.

is  $\text{Var } M_2 = \sigma_M^2$  which is obviously larger than  $\text{Var } M_1 = \sigma_T^2 \cdot \sigma_M^2 / (\sigma_T^2 + \sigma_M^2)$  provided that neither  $\sigma_T^2$  nor  $\sigma_M^2$  is zero. We also note that in case 2 the conditions on the policy parameters do not contain any of the structural parameters. The absolute values of  $\eta_R$  and  $\rho_R$  do not matter as long as they sum to zero.

Case 3:  $dM/du_M = 0$ . Again from (4.19) we see that setting  $\rho_T = -\eta_T = t_T$  eliminates all money demand disturbances. If in addition  $\rho_R$  is set equal to infinity (4.20) is fulfilled. The variance in case 3 is  $\text{Var } M_3 = \sigma_T^2$  which is again, in general, larger than  $\text{Var } M_1$ .

Case 4. After having studied the three cases where the effects on  $M$  of at least one of the disturbances is completely eliminated, a fourth case is suggested from inspection of (4.20). Setting  $\rho_R = -t_R$  makes the first and third conditions equal to zero. Insertion into the fourth shows that  $\Gamma = 0$  for  $\rho_T = t_T - m_T \left( \sigma_T^2 + \sigma_R^2 / \sigma_M^2 \right) \equiv \rho_T^*$ . Finally it turns out that setting  $\eta_R = \infty$  will make  $\Omega = 0$  and thus (4.20) is fulfilled (for arbitrary values of  $\eta_T$ ).  $\text{Var } M_4 = \sigma_M^2 \left( \sigma_T^2 + \sigma_R^2 \right) / \left( \sigma_M^2 + \sigma_T^2 + \sigma_R^2 \right)$  can be shown to be larger than  $\text{Var } M_1$ .

Case 1 is thus superior to the other cases. Note also the strong parallels between cases 1 and 4. In case 1 the rules refer to the values of the parameters of the open market ope-

ration function. Case 4 is characterized by highly similar expressions for  $\rho_R$  and  $\rho_T$ , the parameters defining the discount window policies. The implied superiority of case 1 derives from the fact that it eliminates  $\sigma_R^2$  completely. It is easily seen that if  $\sigma_R^2 = 0$ , the cases are exactly equivalent. Moreover, the condition  $\eta_R = \infty$  becomes superfluous. We can therefore conclude that in the case with  $\sigma_R^2 = 0$  it is immaterial whether policy is derived in terms of open market operations or discount window policies. The two instruments are perfect substitutes.

There is an alternative way to obtain the conditions in the four cases above, which is instructive. Consider the extreme cases in which each of the policy parameters is set to infinity. For arbitrary values (less than infinity) of the other three parameters, we find the variances of M:

$$\lim_{\rho_T \rightarrow \infty} \text{Var } M = \sigma_M^2 \quad (\text{A4.1})$$

$$\lim_{\rho_R \rightarrow \infty} \text{Var } M = \frac{m_T^2 \sigma_T^2 + (t_T + \eta_T)^2 \sigma_M^2}{(t_T + m_T + \eta_T)^2} \quad (\text{A4.2})$$

$$\lim_{\eta_T \rightarrow \infty} \text{Var } M = \sigma_M^2 \quad (\text{A4.3})$$

$$\lim_{\eta_R \rightarrow \infty} \text{Var } M = \frac{m_T^2 (\sigma_T^2 + \sigma_R^2) + (t_T - \rho_T)^2 \sigma_M^2}{(t_T + m_T - \rho_T)^2} \quad (\text{A4.4})$$

Apparently, (A4.1) and (A4.3) correspond to case 2 in which either  $\rho_T$  or  $\eta_T$  is set to infinity. This shows that the other condition in case 2,  $\eta_R = -\rho_R$ , is not really necessary for the first-order conditions to be satisfied. Both  $\rho_T$  and  $\eta_T$  appear in higher power in the denominator ( $\Delta^3$ ) than in the numerators in (4.19). Hence, additional restrictions are in fact superfluous.

In the other two cases, (A4.2) and (A4.4), the variance expressions contain one policy parameter,  $\eta_T$  and  $\rho_T$ , respectively. The reason is that  $\eta_R$  and  $\rho_R$  each appear in the third power in both the numerators and the denominators in the differentials with respect to  $\eta_T$  and  $\rho_T$ . Hence, letting  $\eta_R$  go to infinity is not sufficient for  $d\text{VarM}/d\rho_T = 0$  and  $\rho_R = \infty$  is not sufficient for  $d\text{VarM}/d\eta_T = 0$ .

The obvious thing to do then is to minimize (A4.2) and (A4.4) with respect to  $\eta_T$  and  $\rho_T$ , respectively. This gives

$$\eta_T^* = m_T \frac{\sigma_T^2}{\sigma_M^2} - t_T$$

$$\rho_T^* = t_T - m_T \frac{\sigma_T^2 + \sigma_R^2}{\sigma_M^2}$$

These solutions are identical to those derived in cases 1 and 4, respectively and we can define

Case 5:  $\rho_R \rightarrow \infty$  ;  $\eta_T = \eta_T^*$

$$\text{Var } M_5 = \frac{\sigma_T^2 \sigma_M^2}{\sigma_T^2 + \sigma_M^2}$$

and

Case 6:  $\eta_R \rightarrow \infty$ ;  $\rho_T = \rho_T^*$

$$\text{Var } M_6 = \frac{\sigma_M^2(\sigma_T^2 + \sigma_R^2)}{\sigma_M^2 + \sigma_T^2 + \sigma_R^2}$$

Finally, we may investigate the cases where either the discount window or the open market operations policy is used to peg an index of the two interest rates.

Case 7: Defining  $\eta^* = \eta_T/\eta_R$ , we have

$$\lim_{\eta_T, \eta_R \rightarrow \infty} \text{Var } M = \frac{m_T^2(\sigma_T^2 + \sigma_R^2) + [t_T - \rho_T + \eta^*(\rho_R + t_R)]^2 \sigma_M^2}{[m_T + t_T - \rho_T + \eta^*(\rho_R + t_R)]^2}$$



Minimizing this expression with respect to  $\eta^*$  yields

$$\eta^* = \frac{m_T \frac{\sigma_T^2 + \sigma_R^2}{2} - t_T + \rho_T}{\rho_R + t_R}$$

and

$$\text{Var } M_7 = \frac{(\sigma_T^2 + \sigma_R^2) \sigma_M^2}{\sigma_T^2 + \sigma_R^2 + \sigma_M^2}$$

We note that  $\text{Var } M_7 = \text{Var } M_4$ .

Case 8: Defining  $\rho^* = \rho_T / \rho_R$ , we have

$$\lim_{\rho_T, \rho_R \rightarrow \infty} \text{Var } M = \frac{m_T^2 \sigma_T^2 + [t_T + \eta_T - \rho^* (\eta_R - t_R)]^2 \sigma_M^2}{[m_T + t_T + \eta_T - \rho^* (\eta_R - t_R)]^2}$$

Minimization with respect to  $\rho^*$  gives

$$\rho^* = \frac{t_T + \eta_T - \frac{\sigma_T^2}{2} m_T}{\eta_R - t_R}$$

and

$$\text{Var } M_8 = \frac{\sigma_T^2 \sigma_M^2}{\sigma_T^2 + \sigma_M^2}$$

To summarize we have three policies yielding the same minimum variance;  $\text{Var } M^* = \text{Var } M_1 = \text{Var } M_5 = \text{Var } M_8$ . These correspond to cases B, A, and C in the main text.

## NOTES

1. This statement needs some clarification. It is required that the central bank knows the demand for reserves conditional on any given interest rate prior to determining the supply. This means that they have to gather this information through their dealers:

"The banks are assumed to submit their demands for reserves contingent on the interest rate to the Federal Reserve System (Equation (4.2)) which then, acting as the Walrasian auctioneer or market maker, can determine the quantity of reserves which are desired at any interest rate. The government bond dealers are the transmission vehicle conveying this information to the open-market desk" (LeRoy and Waud (1977), p. 201).

Note that, given the assumed linearity of the model, the central bank only has to ask for quantity demanded given one particular interest rate. The informational requirements in this case may hence be rather limited.

2. Contemporaneous reserve accounting would lead to a somewhat more complicated expression for the variance of the money stock. It would not, however, change anything fundamental with regard to the optimal policy rules. For an analysis of a money market model comparing lagged and contemporaneous reserve accounting, see Goodfriend (1982).
3. The behavioral equations are here given in linearized form and with all parameters defined to be positive.
4. Goodfriend (1982) uses this approach in an analysis of the U.S. system.
5. Note that we will not determine the intercepts  $\eta_0$  and  $\rho_0$  in (4.11) and (4.12) in the policy optimization process. These constants should be interpreted as containing the values of  $r_T$  and  $r_R$  towards which pure pegging policies

could be directed. To illustrate, take (4.12) and assume  $\rho_T = 0$ . A policy with a completely open discount window is given by letting  $\rho_R \rightarrow \infty$ . Rewrite (4.12) as

$$RB^C = \rho_0' + \rho_R(r_R - \bar{r}_R) \quad (4.12')$$

where  $\rho_0' - \rho_R \bar{r}_R = \rho_0$ .  $\bar{r}_R$  is the arbitrarily chosen discount rate that would be obtained with a completely open discount window, i.e., the penalty rate in a system of the type discussed in Section 3.4.

6. Note that a completely passive policy with  $\rho_i = \eta_i = 0$  ( $i=T, R$ ) is not feasible in this model as it implies  $\Delta=0$ . The explanation is that this makes the supply curve for reserves vertical and since reserve demand also is interest independent (due to lagged reserve accounting) no equilibrium interest rate can be determined.
7. If either (4.15), (4.16) or (4.17) holds, then only one of the equalities in (4.18) is needed to give  $dM/du_M = 0$ ;
 
$$\begin{array}{lll} \eta_R = t_R & \text{and} & \eta_T = -t_T \\ \eta_R = -\rho_R & \text{and} & \eta_T = -\rho_T, \text{ or} \\ \rho_R = -t_R & \text{and} & \rho_T = t_T. \end{array}$$
8. To be exact, we want to minimize the mathematical expectation of the squared deviation from some target value  $M^*$ . With our framework these approaches are equivalent.
9. Note that  $\rho^* = 0$  if  $\eta_T = \eta_T^*$ . For this particular choice of  $\eta_T$ , case C is therefore identical to case A, as  $\rho^* = 0$  implies  $\rho_R \rightarrow \infty$ .
10. Some authors, e.g. Friedman (1982), have suggested that the borrowing rate should be kept consistently somewhat above the money market rate. In our notation this may be interpreted as
 
$$r_R = r_T + \rho.$$
 This is a variant of case C, where  $\rho^* = -1$  and  $\rho$  is the value at which the index of  $r_R$  and  $r_T$  is pegged. It is obvious from the expression for  $\rho^*$  that setting it equal to -1 is not generally optimal in the current model.

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## 5. International Interest Rate Dependence and the Foreign Exchange Risk Premium

### 5.1 INTRODUCTION

It is a standard result, going back to Fleming (1962) and Mundell (1963), that in an economy with a fixed exchange rate the efficacy of monetary policy depends crucially on the degree of substitutability between domestic and foreign assets. In particular, if investors treat them as perfect substitutes, any attempt to influence domestic interest rates, for example, via open market operations, will be completely offset by international capital movements.

If indeed exchange rates were permanently and credibly fixed, there would be little reason to expect assets differing only in currency denomination not to be perfect substitutes.<sup>1</sup> However, in practice exchange rates are not rigidly fixed even in systems that contain strong elements of pegging as, for example, the Swedish currency basket. Even ignoring the fact that the benchmark value for the currency index has been changed twice in connection with substantial devaluations, there are daily fluctuations around the announced benchmark value. Elements of uncertainty are thus present also in the "basically" fixed currency basket system. This implies that assets denominated in different currencies will not in general be treated as perfect substitutes by investors who are concerned with risk. The model in Chapter 3, which was based on the assumption of imperfect substitutability between domestic and foreign

assets, may therefore be acceptable in this respect. Nevertheless, that model introduces capital flows in a rather superficial manner and no attempt is made to discuss the determinants of the degree of asset substitutability.

The purpose of this and the next two chapters is to try to remedy these deficiencies by providing a more thorough analysis of issues related to the degree of international interest rate dependence. In particular, the analysis focuses on a concept known as the foreign exchange risk premium which will be shown to play a crucial role in this context.

The current chapter outlines the basic principles and motivates explicitly why the foreign exchange risk premium is so important for the discussion of monetary policy in an economy of the Swedish type. This is done with the help of some standard international parity condition and a reinterpretation of the portfolio model of Chapter 3 to account explicitly for one simple story about the determination of the risk premium. This provides an introduction to Chapter 6 which reviews more rigorous attempts to model the risk premium theoretically and empirical attempts to capture it. Chapter 7 contains a theoretical and empirical analysis of the risk premium in a currency basket system.

## 5.2 MONETARY POLICY AND THE FOREIGN EXCHANGE RISK PREMIUM

As noted above, the model discussed in Chapter 3 treats international capital flows in the simplest possible way. The interest rate elasticities of the public's demand for foreign assets determine to what extent the domestic interest rate can deviate from the international level. In the extreme case of infinite elasticities, the model implies that the domestic interest rate invariably equals  $r_f$  and this is thus the condition for complete interest rate dependence in a small open economy.



The purpose here is to study more closely the relationship between domestic and foreign nominal interest rates. The starting point for this discussion is a set of international parity conditions. The first of these is covered interest parity (CIP) which holds if

$$r_t = r_t^f + f_t - s_t \quad (5.1)$$

where  $r_t$  and  $r_t^f$  are the domestic and foreign nominal interest rates, respectively, on assets maturing at  $t+1$  that are identical in all respects except currency denomination, and  $s_t$  and  $f_t$  are (the logs of) the spot and forward exchange rates. The CIP condition thus states that the interest rate differential should equal the forward premium ( $f_t - s_t$ ). In other words, the return on the domestic asset is equated to the return on the foreign asset adjusted for the cost of forward coverage. As these two alternatives are equivalent ways of obtaining a given amount of domestic currency at some time in the future there is no uncertainty involved, i.e., CIP is a pure arbitrage condition. This is also clear from the fact that all the variables in (5.1) are known at time  $t$ . Given that markets are unregulated, in particular, that there are no foreign exchange controls limiting covered interest arbitrage, CIP can therefore be expected to hold (with due allowance for transactions costs).<sup>2</sup>

What then does CIP, which in the terminology of Frankel (1983) is the condition for perfect capital mobility, mean for interest rate dependence? The answer is that CIP in and of itself implies no constraints on domestic monetary policy. All it says is that the interest rate differential will be (approximately) equal to the forward premium. CIP is therefore, in principle, consistent with a unidirectional causality from  $r_t - r_t^f$  to the forward premium and the central bank is not constrained in its choice of domestic interest rate level

by the fact that CIP holds. Being an arbitrage condition, CIP cannot help determine the absolute levels of  $r_t - r_t^f$  and  $f_t - s_t$ , but only demonstrate that they must be equal.

The implication of the reasoning above that the forward premium passively adjusts to any interest rate differential chosen by the domestic central bank does seem rather implausible, however. In particular, one would expect the forward exchange rate, which is the price at time  $t$  of one unit of foreign currency to be delivered at  $t+1$ , to stand in some relation to the spot rate expected to prevail at time  $t+1$ . If investors observe that  $f_t$  is substantially higher than the expected future spot rate, they are likely to sell currency forward and then buy spot at  $t+1$  when delivery is due. This leads to an open position, i.e., the investor is exposed to foreign exchange risk, but if  $f_t$  is sufficiently high relative to the expected future spot rate he might consider this a risk worth taking.

This line of reasoning implies that if a central bank decides to try to raise the domestic interest rate, thereby raising  $f_t$  in accordance with the CIP condition, private investors can be expected to engage in offsetting transactions. For example, they can find it desirable to borrow abroad without making offsetting forward transactions and/or take open forward positions. Such transactions tend to reduce the domestic interest rate and, given CIP, lower the forward rate. In terms of the model in Chapter 3, this is explained by an inflow of foreign currency, which adds to the foreign exchange reserves and therefore increases the supply of reserves.

It should be noted that the covered foreign exchange transactions underlying CIP do not in general lead to capital inflows. To see this consider an example with a company taking a loan in foreign currency. When the loan is converted to domestic currency there is an increase in the foreign exchange

reserves. Simultaneously, the company buys foreign currency in the forward market to close its position which, however, means that the counterpart in the forward transaction (a bank, say) has an open position. Assuming that nothing has happened that makes the bank want to change its position, it will cover its forward contract by buying the currency spot and, for example, deposit it with a foreign bank. Hence, the increase in the foreign exchange reserves is immediately offset by the bank taking a spot position and the net effect on the domestic reserve market is zero. This simple example illustrates the fact that for there to be a capital inflow some agent must be willing to take an open position in the foreign currency, i.e., actively try to exploit interest rate differentials.

If investors are willing to fully offset any deviations between the forward rate and the expected future spot rate, the CIP condition becomes

$$r_t = r_t^f + E(s_{t+1}) - s_t \quad (5.2)$$

where  $E(s_{t+1})$  is the expectation at time  $t$  of the spot exchange rate at time  $t+1$ . This is known as the uncovered interest parity (UIP) condition, which thus says that the interest rate differential should equal the expected rate of depreciation of the domestic currency. Equations (5.1) and (5.2) together imply that  $f_t = E(s_{t+1})$ , i.e., that the forward rate observed at time  $t$  is an unbiased predictor of the spot rate at  $t+1$ .

The UIP condition (5.2) thus says that the expected returns on domestic and foreign assets are equalized. Frankel (1983) refers to UIP as the condition for perfect asset substitutability. It is easily seen that (5.2) is a generalization of the case in which monetary policy becomes inoperative in the model of Chapter 3 to a situation where the exchange rate

change is not necessarily zero. What UIP implies for the efficacy of monetary policy therefore depends crucially on the ruling exchange rate system, an issue that will be discussed further below. However, before proceeding it is illuminating to consider also the consequences of imposing a third international parity condition that is frequently applied not least in theoretical open economy models, namely, purchasing power parity (PPP). The condition for (relative) ex ante PPP is

$$E(s_{t+1} - s_t) = E(\pi_{t+1} - \pi_{t+1}^f) \quad (5.3)$$

where  $\pi_{t+1}$  and  $\pi_{t+1}^f$  are the domestic and foreign rates of inflation between  $t$  and  $t+1$ , respectively. Condition (5.3), which implies perfect goods market integration, postulates that the expected rate of depreciation should correspond to the expected differential in inflation rates. Combining PPP with the UIP condition (5.2) gives

$$r_t = r_t^f + E(\pi_{t+1} - \pi_{t+1}^f) \quad (5.4)$$

From the standard Fisher relation the (expected) real rate of interest ( $i_{t+1}$ ) is defined as

$$E(i_{t+1}) = r_t - E(\pi_{t+1}) \quad (5.5)$$

which means that (5.4) reduces to

$$E(i_{t+1}) = E(i_{t+1}^f) \quad (5.6)$$

Hence, the joint assumption of UIP and PPP implies that ex ante real interest rates are equalized or real interest parity (RRP). Perfect international goods and asset market integration thus seem to leave very little scope for monetary policy to the extent that interest rates are important in the transmission mechanism.

However, it is not difficult to think of reasons why both PPP and UIP are likely to be violated in the real world. For example, the existence of nontraded goods is sufficient to cause deviations from PPP without there being any implication of imperfect integration of the markets for tradable goods. Similarly, note that the UIP condition is expressed in terms of expected magnitudes. As it is not an arbitrage condition but risk is involved, this implicitly assumes that agents are risk neutral, i.e., that they ignore the variability in the returns on their investments. However, risk neutrality is not a requirement for financial market efficiency which means that UIP may be violated even if markets are fully efficient. In general, risk averse investors will treat assets denominated in different currencies as imperfect substitutes if exchange rates are not fixed.<sup>3</sup>

The emphasis here is on financial market integration and in the following attention will be focused on the relation between nominal interest rates, i.e., the UIP condition. If UIP is violated, condition (5.2) is modified to read

$$r_t = r_t^f + E(s_{t+1}) - s_t + p_t \quad (5.7)$$

where  $p_t$  can be interpreted as a (possibly time varying) risk premium. Such a premium causes deviations also from RRP, i.e., the same term would appear in equation (5.6). Intuitively, it measures the compensation required by investors in order to take on foreign exchange risk, i.e., engage in uncovered borrowing and lending transactions involving assets denominated in different currencies.

Combining (5.7) and the CIP condition (5.1), it is easily seen that

$$f_t = E(s_{t+1}) + p_t. \quad (5.8)$$

Hence,  $p_t$  is (by definition) the bias in the forward rate as a predictor of the future spot rate. Under the assumption that the CIP arbitrage condition holds, it does not matter therefore whether the premium is measured in terms of interest rates as in (5.7) or as the bias in the forward rate as in (5.8).

The question now is whether it matters for the effects of monetary policy if the interest rate relation is described by the UIP condition (5.2) or by (5.7) containing also a risk premium. First, this depends on the ruling exchange rate system. In the flexible exchange rate regime, perfect asset market integration does not hamper monetary policy since in this case, for example, an increase in the money supply in a conventional analysis leads to a depreciation of the domestic currency, presumably with real effects on the economy.

However, in a country with a fixed (or at least pegged) exchange rate the authorities are unwilling to use exchange rate changes as part of the transmission mechanism, which means that this channel is cut off, i.e., systematic changes in the expected rate of depreciation are ruled out. In this case, the UIP condition (5.2) implies a severe restriction on domestic monetary policy. With a permanently and credibly fixed exchange rate, (5.2) reduces to the parity condition that was implicit in the model in Chapter 3 under perfect substitutability, namely,  $r_t = r_t^f$ , i.e., there is complete nominal interest rate equalization.

On the other hand, if the exchange rate is not perfectly rigid, some degree of exchange rate uncertainty is present and there may be a risk premium in the interest rate relation, i.e., (5.7) would be valid. However, (5.7) says that for a given expected rate of depreciation (possibly zero), the central bank can affect the domestic interest rate if and only if it can influence the size of the risk premium.

In a country (like Sweden) where the exchange rate is basically fixed the risk premium thus becomes a variable of considerable importance. The possibility for an independent interest rate policy hinges on some knowledge of the determinants of the risk premium and how they can be influenced by systematic policy actions. Otherwise, the policy independence is no greater than if the UIP condition (5.2) held exactly.

This conclusion obviously raises questions concerning the more exact properties of the risk premium which so far has been given a very loose motivation. It is possible to rephrase the results derived in Chapter 3 concerning the effects of monetary policy on the domestic interest rate in terms of changes in the risk premium.<sup>4</sup> For example, consider an open market purchase of t-bills which, in the general case, leads to an increase in the money market rate. The intuitive explanation of this result, reinterpreted as an increase in the foreign exchange risk premium, would be that this intervention leads to an increase in the proportion of t-bills in the portfolios of domestic investors. They must therefore reduce their holdings of (increase their debts in) foreign currency denominated assets, which makes them more exposed in case the domestic currency should depreciate. To accept this situation, they require a (higher) premium on domestic relative to foreign assets.

In this example, the introduction of a risk premium is clearly just a matter of exposition, but it serves to highlight the implicit connection between asset stocks and the risk premium that exists in macro portfolio models. For contrast, assume that there is uncovered interest parity. Under UIP the premium is zero and the interest rate differential uniquely determined by the expected rate of depreciation. Provided that the latter is independent of the central bank's open market policies, conventional monetary policy instruments cannot be used to

influence the domestic interest rate. This is recognized from Chapter 3 as the case when foreign and domestic assets are treated as perfect substitutes.

This shows that the conditions for UIP to obtain are quite stringent. In particular as it is an atemporal relation also in its general version (5.2), it requires instantaneous portfolio adjustment to any deviation between the interest rate differential and the expected rate of depreciation. Hence, even if it takes no more than one week, say, for private investors to rebalance their portfolios, deviations from UIP could be observed. This implies that violations of UIP are necessary but not sufficient for interest rates to be independent in any meaningful sense in a small open economy with a (more or less) fixed exchange rate. In addition, it must be shown that these deviations are persistent enough to really matter, i.e., the speed of adjustment in international capital flows must be analyzed in order to determine the duration of any observed deviations from UIP.<sup>5</sup>

### 5.3 CONCLUDING COMMENTS

The purpose of the following chapter is to analyze more thoroughly when and why a risk premium of the type discussed here will emerge. This makes it necessary to consider more rigorous asset pricing models that explicitly determine, for example, the conditions under which domestic and foreign assets are treated as perfect substitutes, i.e., when the risk premium is zero. In addition, from the point of view of monetary policy it is of fundamental importance whether a non-zero premium can be systematically influenced using standard policy instruments such as open market interventions. As emphasized above, a non-zero but uncontrollable risk premium gives no more room for independent policy than if uncovered interest parity holds exactly.



Before proceeding, it should be noted that although the term  $p_t$  in (5.7) and (5.8) is referred to throughout as a risk premium there are numerous reasons other than risk aversion why UIP may be violated or, equivalently, why the forward rate may not be an unbiased predictor of the future spot rate. For example, imperfect market integration due to foreign exchange controls or general market inefficiencies will also introduce a bias in these relations. Moreover, in an explicitly stochastic setting it is well known that the UIP condition will not be well defined even in the absence of market imperfections and risk aversion because of Jensen's inequality.<sup>6</sup>

## NOTES

1. This statement ignores considerations relating to "political risk" or "country risk", emphasized by, e.g., Aliber (1973) and Dooley and Isard (1980). This is the possibility that not even government securities are in effect default free, which of course is a relevant problem in many cases. In this context it will be assumed that all assets are default free, however, as the focus is on interest rates on short term securities issued by politically and otherwise stable industrialized countries. For example, the default risk on a Swedish government bond with less than one year to maturity can undoubtedly be set to zero. For a discussion of these concepts in a world with nominally risky assets, see Stulz (1986).
2. For empirical evidence on CIP based on Swedish data, see McPhee (1984) and Englund, McPhee, and Viotti (1985).
3. This assertion will be motivated in detail in Chapter 6.
4. Dooley and Isard (1983) analyze in greater detail a simple two-country portfolio model with an endogenous risk premium. See also Hörngren and Viotti (1985).
5. This is discussed in detail by Vredin (1986), who also reports empirical results for Sweden.
6. See, e.g. Obstfeld (1986) for a demonstration of this fact which in the literature often is referred to as "Siegel's paradox"; cf. Siegel (1972).

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## 6. The Foreign Exchange Risk Premium: a Review of Theory and Evidence\*

### 6.1 INTRODUCTION

The conclusion from the discussion in Chapter 5 was that the foreign exchange risk premium is an important concept in the analysis of the effects of monetary policy in a country with a basically fixed exchange rate. It was introduced in a very cursory manner, however, and very little was said about its determinants. A more thorough analysis of this problem requires models of a different type than the macro portfolio balance models considered so far, where the existence of a risk premium is simply (and implicitly) postulated.

The appropriate framework for this analysis is a model of international asset pricing in which the risk premium can be endogenously determined. In recent years there has been a surge of work where the methods of the theory of finance are applied to problems of international finance. This line of research has resulted in a number of open economy asset pricing models that have a strong bearing on the question of the determination of the foreign exchange risk premium.

The purpose of this chapter is to review the literature on international asset pricing (with no pretensions to completeness). In particular, we discuss the implications for the

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\* This chapter was written together with Anders Vredin.

behavior of the foreign exchange risk premium and, ultimately, whether it can be systematically controlled with the help of conventional instruments of monetary policy.

The second part of this chapter gives a similar review of empirical studies of the risk premium. There are two main strands in this literature. One is primarily atheoretical and concerned with trying to find evidence on whether risk premia exist or, equivalently, whether there are deviations from uncovered interest parity (UIP) or not. In the second line of research, attempts are made to explain observed premia (deviations from UIP) by testing specific asset pricing models. This review of the literature forms the basis for the theoretical and empirical analysis of the Swedish currency basket system and its implication for the foreign exchange risk premium that is presented in Chapter 7.

## 6.2 THEORETICAL MODELS OF THE FOREIGN EXCHANGE RISK PREMIUM

There is a rapidly growing literature in which the methods of the theory of finance are applied to problems in international finance. This has resulted in a number of open economy extensions of the traditional capital asset pricing models, ranging from the original Sharpe-Lintner version to the more general consumption CAPM developed by Breeden (1979). Recently, complete general equilibrium models have also been presented, building on, in particular, Lucas (1982).

The purpose here is not to provide a complete and general survey of the vast literature on international asset pricing.<sup>1</sup> The focus is limited by our concern for the foreign exchange risk premium and the review will concentrate on the implications that these models have for the determination of this variable and for the question of international interest rate dependence.

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### 6.2.1 The mean-variance optimization approach

Frankel (1982) develops an expression for the risk premium using an international portfolio balance model derived from mean-variance optimization. The representative investor is assumed to maximize a function of the mean and the variance of his end-of-period real wealth (evaluated in U.S. dollars). With the dollar as the reference currency, the investor chooses a vector  $x_t$  of portfolio shares allocated to  $n$  other assets denominated in different currencies. The only uncertainty in the model relates to exchange rate changes. However, the investor is assumed to pay for goods from a given country with the currency of that country, which means that the real value of his portfolio also depends on the composition of his consumption basket, summarized by the vector  $\alpha$  of consumption shares allocated to goods produced in the  $n$  foreign countries. The consumption pattern is exogenous and assumed to be constant over time.

Solving the optimization problem gives the following asset demand equation:<sup>2</sup>

$$x_t = \alpha + (\rho\Omega)^{-1}[r_t - \iota r_t^{\$} - E(s_{t+1} - s_t)] \quad (6.1)$$

Here  $x_t$  is a  $(n \times 1)$  vector of portfolio shares;  $s_t$  is a  $(n \times 1)$  vector of (logs of) dollar exchange rates (DM/\$, etc.);  $r_t$  is a  $(n \times 1)$  vector of nominal non-dollar interest rates, while the dollar interest rate is denoted  $r_t^{\$}$ ;  $\Omega$  is the variance-covariance matrix of exchange rate changes;  $\iota$  is a vector of  $n$  ones; and  $\rho$  is the coefficient of relative risk aversion.  $\rho$  is assumed to be constant, which implies that the underlying utility function is isoelastic.  $\alpha$ , the  $(n \times 1)$  vector of consumption shares, also represents the minimum-variance portfolio that an extremely risk averse investor would hold. The second term in (6.1) can be referred to as the "speculative portfolio".

Assuming asset supplies to be exogenously fixed, (6.1) can be interpreted as an equilibrium condition which we can invert to get an expression for the deviation from UIP:

$$r_t - r_t^{\$} - E(s_{t+1} - s_t) = \rho \Omega(x_t - \alpha) \quad (6.2)$$

The interest rate relation thus reduces to UIP if the representative investor is risk neutral ( $\rho=0$ ) or if there is no randomness in exchange rate changes ( $\Omega=0$ ). Otherwise, there will be risk premia in foreign exchange markets; assets denominated in different currencies will be imperfect substitutes, and the forward foreign exchange rate will be a biased predictor of the future spot rate. In addition, the mean-variance optimization approach tells us that as long as investors are risk averse, relative returns of foreign and domestic assets will be affected by changes in assets supplies ( $x$ ). In this sense, the mean-variance optimization approach offers some micro foundations to the macroeconomic portfolio-balance models as this implies that an open market sale increasing the outstanding stock of a given asset would raise the required premium just as in the model discussed in Chapter 5.

Another way of looking at the risk premium is to express (6.2) as a CAPM relation. To do this, assume first that  $\alpha$  is a null vector, so that a portfolio of only dollar assets will be a riskless portfolio. Next, note that the return on the (world) market portfolio will be

$$r^m = x'(r - \Delta s) + (1 - x'1)r^{\$} \quad (6.3)$$

where time subscripts are dropped for convenience and  $\Delta s = s_{t+1} - s_t$ .

$$r^m - E(r^m) = -x'[\Delta s - E(\Delta s)]$$

the variance of the market portfolio will be



$$\begin{aligned}
\text{Var}(r^m) &= x' \Omega x = \\
&= \rho^{-1} x' [r - r^{\$} - E(\Delta s)] = \\
&= \rho^{-1} [E(r^m) - r^{\$}]
\end{aligned}$$

where the second equality follows from (6.1) (under the assumption that  $\alpha$  is the null vector) and the third equality follows from (6.3). Similarly, the covariance of the market portfolio with an asset denominated in currency  $i$  can be written as

$$\begin{aligned}
\text{Cov}(r^m, r^i - \Delta s^i) &= E \left\{ \left[ r^m - E(r^m) \right] \left[ r^i - \Delta s^i - \left( r^i - E(\Delta s^i) \right) \right] \right\} = \\
&= E \left\{ x' [\Delta s - E(\Delta s)] [\Delta s^i - E(\Delta s^i)] \right\} = \\
&= \rho^{-1} [r^i - E(\Delta s^i) - r^{\$}]
\end{aligned}$$

where  $r^i$  and  $\Delta s^i$  are the  $i$ :th elements of  $r$  and  $\Delta s$ , respectively, and where the last equality follows from (6.2). Substituting the above into the definition

$$\beta^i = \frac{\text{Cov}(r^m, r^i - \Delta s^i)}{\text{Var}(r^m)}$$

we get the CAPM relation

$$r^i - r^{\$} - E(\Delta s^i) = \beta^i [E(r^m) - r^{\$}] \quad (6.4)$$

which implies that the risk premium on an asset denominated in currency  $i$  will be higher, the higher the covariance between its return and that of the (world) market portfolio. Thus, the less correlated is the krona with other currencies, the less risky will it be.

The conclusions that follow from the mean-variance optimization approach have been derived within similar frameworks by, e.g., Kouri (1977) and Fama and Farber (1979). In contrast to Frankel

(1982) they explicitly analyze the demand for different kinds of financial assets - money, bonds, and equity. (A demand for money is secured by assuming that utility depends on the size of money holdings.)

Kouri and Fama and Farber also discuss the risk premia in terms of CAPM relations, but they prefer to interpret the "exchange risk" as an "inflation risk", i.e., relate it to the uncertainty about the future purchasing power of different currencies. This is not a very meaningful distinction, however, as they model one-good economies where PPP holds as long as the law-of-one-price holds. In this type of model exchange rates and prices do not change independently, and whether the risk involved should be labeled exchange risk or inflation risk is purely a matter of semantics.<sup>3</sup>

In general equilibrium models of international asset pricing, to be discussed in greater detail in Section 6.2.4, monetary and real uncertainty interact in the determination of the risk premium, as in Hodrick's and Srivastava's (1984) analysis of Lucas' (1982) model or in Olofsson's (1985) treatment of the model developed by Svensson (1985). Hodrick and Srivastava argue that the CAPM interpretation of the risk premium is "perfectly consistent" with the representation that can be derived from Lucas' model. Nevertheless, the interpretation of the risk premium given by the mean-variance optimization/CAPM approach will be altered once the restrictive assumptions of a representative investor, a fixed consumption pattern, and non-stochastic prices are relaxed.

### 6.2.2 Models without purchasing power parity

The assumptions of identical investors (or, in effect, one investor) and non-stochastic goods prices mean that purchasing power parity is upheld in Frankel's (1982) model (irrespective

of the values in the consumption vector  $\alpha$ ). The importance of PPP for international asset pricing stems from the fact that in the absence of PPP investors in different countries will not use the same price index when computing the real rates of return. This heterogeneity among investors is in contrast to the traditional (closed economy) CAPM where it is implicitly assumed that all investors use the same price index to deflate the returns.

This aspect is emphasized by Adler and Dumas (1983), who develop an asset pricing model where PPP does not hold. Technically, their model also differs from Frankel's in that they formulate it in continuous time and use stochastic calculus. Our purpose here is to discuss the economic differences, however, and the technical issues will be dealt with only parenthetically.

Adler and Dumas analyze the portfolio choice problem of an investor who maximizes a standard time-additive von Neumann-Morgenstern expected utility of life-time consumption function. All assets are freely traded, but national differences causing deviations from PPP imply that the real returns from these assets may differ between investors. This assumption is implemented by specifying separate stochastic processes for the national consumer price indices,

$$\frac{dP_j}{P_j} = \pi_j dt + \sigma_{\pi}^j dz_{\pi}^j \quad j = 1, \dots, J+1 \quad (6.5)$$

These are so-called Ito processes, where  $\pi^j$  is the (instantaneous) expected rate of inflation in country  $j$ ,  $\sigma_{\pi}^j$  is the (instantaneous) standard deviation of inflation, and  $dz_{\pi}^j$  is a Wiener process.<sup>4</sup> There are  $J+1$  countries and the  $J+1$ st currency (say, dollar) is used to measure all nominal returns. There are  $N$  nominally risky assets whose prices also follow Ito processes,

$$\frac{dy_i}{y_i} = \mu_i dt + \sigma_i dz_i \quad i = 1, \dots, N \quad (6.6)$$

where  $\mu_i$  and  $\sigma_i$  are the (instantaneous) expected nominal rate of return and the standard deviation of this return, respectively. There is also a nominally riskless bond, the  $N+1$ st asset, with an (instantaneous) yield denoted by  $r$ .<sup>5</sup>

Solving the investor's optimization problem yields the following expression for investor  $j$ 's required nominal yield on security  $i$ :

$$\mu_i = r + (1-\rho)\sigma_{i,\pi} + \rho \sum_{k=1}^N x_k \sigma_{i,k} \quad (6.7)$$

where  $\rho$  is the degree of risk aversion,  $\sigma_{i,\pi}$  is the covariance of the return on asset  $i$  with the rate of inflation,  $\sigma_{i,k}$  is the covariance between the returns on assets  $i$  and  $k$ , and  $x_k$  is the portfolio share of asset  $k$ . (The  $j$  superscripts are left out for notational simplicity.)

We can rewrite (6.7) so as to make the connection with eq. (6.2) in Frankel's model explicit. Note that  $r = \mu_{N+1} = r^{\$}$  and that  $\mu_i = r^i - \theta^i + \sigma_{i,i}$ ,<sup>6</sup> where  $\theta^i$  is the (instantaneous) expected rate of depreciation of currency  $i$  against the dollar (the  $J+1$ st currency), and  $\sigma_{i,i}$  is the variance of the return on the nominal bond denominated in currency  $i$ , i.e., the variance of the exchange rate in question, so that (6.7) reads

$$r^i - \theta^i + \sigma_{i,i} - r^{\$} - \sigma_{i,\pi} = \rho \left[ \left( \sum_{k=1}^N x_k \sigma_{i,k} \right) - \sigma_{i,\pi} \right] \quad (6.8)$$

The first (and trivial) difference is that the consumption pattern is missing in (6.8), the explanation being that Adler and Dumas define their price indices directly, i.e., the consumption shares do not enter the model. Second, the term  $\sigma_{i,\pi}$  is absent in Frankel's model.

As can be seen from (6.8)  $\sigma_{i,\pi}$  will affect the nominal interest rate differential irrespective of the degree of risk aversion. To see why, note that the risk premium is a difference between expected excess real returns. The expected real return, in turn, depends not only on the expected nominal return and expected inflation, but also on the variance of inflation and on the covariance between the nominal return and the rate of inflation.<sup>7</sup>

This means that the presence of  $\sigma_{i,\pi}$  on the L.H.S. of (6.8) has nothing to do with risk or whether PPP holds or not; for example, it appears also in an exact formulation of Frankel's asset pricing equation (6.2). Frankel shows in an appendix that in his model,

$$E(i^i - i^{\$}) \equiv r^i - r^{\$} - \theta^i + (1-\alpha^i)\sigma_{i,i} - \sum_{j \neq i} \alpha^j \sigma_{i,j}$$

where  $\sigma_{i,j}$  is the  $i,j$ :th argument of  $\Omega$  and  $i^i$  is the real interest rate on the asset which is denominated in currency  $i$ . The nature of  $\sigma_{i,\pi}$  thus depends on the stochastic specification of the nominal returns and the price index (cf. Branson and Henderson (1985)).

However,  $\sigma_{i,\pi}$  obviously also affects the risk premium, i.e., the R.H.S. of (6.8). This will be the case when goods prices (or the price of the single good in a one-good economy) are stochastic, since the risk of a certain asset then does not only depend on its contribution to total asset market risk, but also on its covariance with goods prices. For example, a high covariance between a specific asset and the average price level may make that asset attractive even if it has a positive covariance with the market portfolio.

The presence of  $\sigma_{i,\pi}$  in (6.8) implies that an increase in the covariance between the (dollar) value of assets denominated in currency  $i$  and the price index, will lower the required

excess nominal return on such assets, provided that the risk aversion coefficient  $\rho$  is greater than unity. Another implication is that the risk premium can be interpreted as consisting of both an "inflation risk" and an "exchange risk". In Frankel's (1982) model it is only exchange risk that gives rise to a time varying risk premium, and he uses this fact to justify his neglect of  $\sigma_{i,\pi}$ .

For the solution to the individual optimization problem the PPP assumption is not really crucial. The conceptual differences appear when (6.8) is to be aggregated to give a meaningful asset pricing relationship. For Frankel this is trivial since he considers a representative investor, i.e., (6.2) is the aggregate asset pricing model. In Adler's and Dumas's model, on the other hand, investors are not identical and a nontrivial aggregation problem emerges.

The procedure followed by Adler and Dumas is as follows: Multiply (6.8) by  $\frac{1}{\rho^j}$ , the risk tolerance of the  $j$ th investor, and aggregate across investors by forming a weighted average where the weights are their relative shares of aggregate (world) wealth. This gives

$$\begin{aligned} \mu_i = r + (1 - \rho^m) & \frac{\sum_j \left(1 - \frac{1}{\rho^j}\right) x^j \sigma_{i,\pi}^j}{\sum_j \left(1 - \frac{1}{\rho^j}\right) x^j} + \\ & + \rho^m \left( \sum_{k=1}^N x_k^m \sigma_{i,k} \right) \end{aligned} \quad (6.9)$$

where  $\frac{1}{\rho^m}$  is a wealth weighted average of individual risk tolerances,

$$\frac{1}{\rho^m} = \frac{\sum_j x^j \frac{1}{\rho^j}}{\sum_j x^j}$$

i.e., a measure of "world" risk tolerance, and  $x_k^m$  is the share of the  $k$ th asset in the world market portfolio.

The interesting difference compared to Frankel's model lies in the second term, which shows that the equilibrium rate of return depends both on individual risk aversion coefficients and the covariances between the return and the individual price indices,  $\sigma_{i,\pi}^j$ . This complicated sum is directly attributable to the fact that PPP does not hold in the Adler-Dumas model.

The very real possibility of deviations from PPP thus imposes a tremendous aggregation problem on empirical studies of international asset pricing. This aggregation problem, however, is not unique to asset pricing models but hampers empirical work on macroeconomic models of any kind. Furthermore, the theoretical properties of risk premia are not much altered by relaxation of the PPP assumption. The expression in (6.9) still suggests that the size of the risk premia depends (although in a complicated manner) on the composition of the world market portfolio, and, hence, that the premia can be directly affected by open market operations. This result has been used as a starting point for empirical work on risk premia, e.g., by Frankel (1982) and Park (1984), studies which will be discussed in detail in Section 6.3 below. Unfortunately, the dependence of risk premia on asset supplies become less clear when the conventional CAPM assumption of non-stochastic investment and consumption opportunities is abandoned. The following section will be devoted to a discussion of this issue.

### 6.2.3 Models with stochastic investment and consumption opportunities

The models we have considered so far are of the conventional CAPM type. This means, for example, that the investment opportunity set is constant through time (state independent). Merton (1973) points out that in this case it does not matter whether asset prices are derived from a model in which investors maximize expected utility of lifetime consumption or solve a one-period mean-variance optimization problem. This equivalence is also evident from the strong similarities between the results derived by Frankel (1982) and Adler and Dumas (1983).

To make this equivalence break down it is sufficient to introduce a stochastic risk-free (default free) interest rate. This exposes the investor to an additional form of risk. For example, an increase in the risk-free (one-period) interest rate leads to capital losses for holders of fixed income securities such as (multi-period) bonds. Hence, end-of-period wealth is imperfectly correlated with future resources for consumption and the investor is faced with a term-structure problem. As the real-world risk-free rate appears to be highly variable, the conventional CAPM formulation must be considered quite restrictive, both empirically and theoretically, and it is reasonable to assume that a risk-averse investor will want to hedge against this type of risk as well. This means that the required excess return on a given asset no longer depends only on its relation to the market portfolio but also on how it covaries with the return on a portfolio that is a perfect hedge against changes in the riskfree rate. This is referred to as three-fund separation. The investor chooses his optimal portfolio by obtaining a mix of three "mutual funds", namely, (i) the riskless asset, (ii) the market portfolio, and (iii) the portfolio that hedges against interest rate variability.<sup>8</sup> The conclusion is that the investor has



to be compensated both for bearing market risk and for bearing the risk of changes in the investment opportunity set (the risk-free rate).

For each new state variable that is introduced, a new element of risk is added and an additional "mutual fund" must be included in the asset pricing equation. This is Merton's (1973) "multi-beta" model, which has one beta coefficient for each state variable.

Breeden (1979) shows that Merton's multi-beta model can be converted into an equivalent single-beta model by exploiting the fact that in optimum the marginal utility of wealth equals the marginal utility of consumption. This enables him to express the asset pricing relation in terms of just one parameter, namely, a "beta" that measures the covariance between the return on the asset and the change in the rate of consumption. Formally, assuming that there is just one good,

$$\mu_i - r = \beta_{iC} \left( \frac{\mu_M - r}{\beta_{MC}} \right) \quad (6.10)$$

where

$$\beta_{iC} = \frac{\text{cov}(\mu_i, d \ln C)}{\text{var}(d \ln C)},$$

$C$  is the aggregate rate of consumption,  $\mu_M$  measures the expected return on any portfolio (i.e., not necessarily the market portfolio), and  $\beta_{MC}$  is defined analogously to  $\beta_{iC}$ .

Breeden's model has a highly intuitive interpretation. From (6.10) we see that a risky asset in this framework is one that has a rate of return that is highly correlated with (the rate of) consumption. Under standard assumptions, the marginal utility of consumption is low in states where the level (or rate) of consumption is high. The mirror image of high consumption is that investment opportunities are unfavorable,

or as expressed by Breeden (1979): "Always, when the value of an additional dollar payoff in a state is high, consumption is low in that state, and when the value of an additional investment is low, optimal consumption is high" (p. 278). Therefore investors/consumers are willing to pay a premium (require a lower rate of return) for an asset that has high payoffs in the low consumption (favorable investment) states.

This basic intuition of the consumption CAPM model carries over to Stulz' (1981) extension to international asset pricing. Stulz considers a two-country multi-commodity world where deviations from PPP occur.<sup>9</sup> He derives the following international asset pricing relation:<sup>10</sup>

$$\mu_i - r - \sigma_{i,\pi} = \rho^W C(\sigma_{i,C} - \sigma_{i,\pi}) \quad (6.11)$$

where  $\rho^W$  is a measure of world (absolute) risk aversion,  $C$  is now world consumption expenditure, and  $\sigma_{i,C} = \text{cov}(\mu_i, d \ln C)$ . By introducing an arbitrary reference portfolio,  $M$ , we can write (6.11) in terms of the consumption beta as

$$\mu_i - r - \sigma_{i,\pi} = \beta_{iC} \left( \frac{\mu_M - r - \sigma_{M,\pi}}{\beta_{iM}} \right) \quad (6.12)$$

The L.H.S. is, as demonstrated above, the expected real excess return on asset  $i$ , and we conclude that it is proportional to the covariance of the return on that asset (in home currency) with changes in the real rate of world consumption. The economic intuition behind this result is easily seen to be the same as in the original Breeden model.<sup>11</sup>

Given the general asset pricing equation in (6.11) it is straight forward to derive an expression for the foreign exchange risk premium. To facilitate the comparison with the previous results we assume that the measurement (home) currency is the dollar, implying  $r = r^{\$}$  in (6.11). Then if asset  $i$  is a nominally riskless bond in the foreign currency, we can split  $\mu_i$  into

two components, namely, the nominal interest rate and the expected rate of depreciation relative to the dollar, and write

$$r^i - \theta^i + \sigma_s^2 - r^{\$} - \sigma_{s,\pi} = \rho^W_C(\sigma_{s,C} - \sigma_{s,\pi}) \quad (6.13)$$

In (6.13),  $\sigma_s^2$ ,  $\sigma_{s,\pi}$  and  $\sigma_{s,C}$  refer to the variance of the spot rate, and the covariances between the spot exchange rate and  $\pi$  and  $C$ , respectively, as exchange rate variability is the only source of uncertainty for a nominally riskless asset. Equation (6.13) gives the risk premium on the foreign (non-dollar) asset as an increasing function of the covariance between the spot exchange rate and the real rate of consumption.

Equation (6.13) implies that the forward premium on the dollar, equal to  $r^{\$} - r^i$  under CIP, is a decreasing function of  $\sigma_{s,C}$ , the covariance of the rate of change in spot exchange rate and the rate of real consumption. This result can be interpreted in several ways. For example, consider a holder of a short forward position, i.e., a seller of forward (foreign) currency. His expected gain equals the difference between the forward rate and the expected future spot rate. The forward rate (or premium) and the return are low when the covariance between the spot rate and the rate of consumption is high, as this means that the return tends to be high in states when  $C$  is low, i.e., when the additional income gives more in terms of added utility. This implies that a short forward position acts as a hedge against consumption risk and that therefore a lower forward rate is required in order to make the expected gain high enough to induce investors to sell foreign currency forward (for any given expected rate of depreciation).

Equation (6.13) is easily seen to be analogous to (6.8) and (6.9) which were derived in Adler and Dumas's (1983) model. Under Stulz' assumption that aggregation is possible, the important difference lies in the definition of the reference

portfolio. In (6.8) this is a financial market portfolio, whereas in (6.13) it is acknowledged that utility is ultimately derived from consumption. Hence, risk is more appropriately defined in terms of "consumption risk".

Despite the strong formal similarities between (6.12)-(6.13) and (6.8), there are important differences when it comes to the determinants of the risk premium. Stulz (1981) points out that in his model "a change in the supply of government bonds does not necessarily affect the risk premium incorporated in the forward exchange rate" (p. 400). On the other hand, there is nothing in the model that precludes open market operations from having effects on the risk premium. The stock of government bonds must be interpreted as one of the state variables in the model.

It should be noted that, conceptually, it is not obvious how an open market operation is to be defined in these multiple-state models. One might conjecture that an open market operation that is the outcome of a known state-dependent process - "if state  $s$  obtains next period, the stock of bonds will change by  $\Delta x_s$ " - will not affect asset prices or premia at all. Only by changing the process can the central bank expect its actions to be effective, if this interpretation is correct. While highly interesting, this problem opens up a wide range of issues in the theory of economic policy that are far beyond the scope of this chapter. They will therefore not be pursued in this context.

Despite its readily apparent connection to the fundamental optimization problem, Stulz' (1981) model is far from being a complete general equilibrium model. In particular, a connection between the processes that govern prices and the exchange rate, and the more fundamental stochastic forces impinging on the economy is missing.

One step in this direction is taken by Stulz (1984). He develops a consumption CAPM model where the exchange rate is endogenously determined. In order to handle this extension, he assumes that there is only one good, which of course reintroduces PPP. Stulz shows that the exchange rate depends positively on the domestic money stock and interest rate and negatively on the corresponding foreign variables. He therefore interprets his model as a monetary approach model derived in an optimizing framework.<sup>12</sup>

Just as in Stulz (1981), the forward premium is a decreasing function of the covariance between the exchange rate and consumption. The endogeneity of the spot rate means that it is possible to go one step further and express this covariance in terms of the more fundamental variables, in particular, the money supply processes. For example, an increase in the covariance between the domestic money supply and consumption raises  $\sigma_{s,C}$  and, hence, lowers the forward premium by making a short forward position less risky for the reasons discussed in the example above.

#### 6.2.4 General equilibrium asset pricing models

Hodrick and Srivastava (1984, 1986) analyze the determination of the foreign exchange risk premium in a general equilibrium setting. They use Lucas' (1982) dynamic general equilibrium model of international asset pricing, where the exogenous stochastic processes determine the paths for output and money supplies. All goods and asset prices are therefore endogenous functions of these "fundamental" stochastic variables. Money is introduced in this model by imposing a cash-in-advance constraint, i.e. by requiring that a means-of-exchange is used in all goods transactions.

The assumptions concerning the sequence of transactions in each period are important for the properties of models of this type. In Lucas (1982), no uncertainty remains when the

financial markets open which means that the cash constraint is always exactly binding. Svensson (1985) modifies the Lucas model by assuming that agents have to decide on their money holdings before the state of the world is revealed which leads to a more general money demand function. Olofsson (1985) uses Svensson's version of the model to study the foreign exchange risk premium. As the results in this respect are highly similar, we will only discuss the Lucas model as presented by Hodrick and Srivastava (1984).

The world consists of two countries with agents that have identical preferences but are endowed with two different consumption goods in period  $t$ , agents in the home country are endowed with  $x_t$  units of good  $x$  and nothing of good  $y$ , whereas in the foreign country agents receive  $y_t$  units of good  $y$ . These endowments follow a known (stochastic) Markov process.

The nominal uncertainty in the model derives from changes in the money supplies in the two countries. They are also assumed to follow known exogenous Markov processes, where the growth rates may also depend on the real state of the world, i.e., the realization of the output process.

The representative agent in each country wishes to maximize a standard expected utility function

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t U(x_{it}, y_{it}) \right\} \quad 0 < \beta < 1 \quad (6.14)$$

where  $x_{it}$  and  $y_{it}$  are the quantities consumed by agent  $i$ , and  $\beta$  is a constant discount factor.

It is assumed that the home good  $x$  can only be bought with home currency and since no uncertainty remains when goods and asset markets open, the nominal price of good  $x$  is simply

$$p_{xt} = \frac{M_t}{x_t} \quad (6.15)$$

where  $M_t$  is the supply of domestic money. Similarly, if  $N_t$  is the supply of foreign money, the nominal price of good  $y$  is

$$p_{yt} = \frac{N_t}{Y_t} \quad (6.16)$$

Under these assumptions, the spot exchange rate can be determined by the arbitrage condition that  $S_t$  must equal the ratio between the two nominal prices,  $p_{x,t}$  and  $p_{y,t}$  multiplied by the relative price of good  $y$  in terms of good  $x$ , denoted by  $q_t$ , i.e.,

$$S_t = \frac{p_{x,t}}{p_{y,t}} \cdot q_t = \frac{M_t Y_t}{N_t X_t} \cdot q_t \quad (6.17)$$

Equation (6.17) thus implies (as expected) that an increase in the domestic money supply  $M_t$  raises  $S_t$ , i.e., leads to a depreciation of the home currency.

Next, we must determine the forward exchange rate. Denoting the home and foreign one-period interest rates by  $r_{x,t}$  and  $r_{y,t}$ , respectively, covered interest parity implies<sup>13</sup>

$$1 + r_{x,t} = (1 + r_{y,t}) \frac{F_t}{S_t} \quad (6.18)$$

By definition, the return on an asset paying one unit of home currency at  $t+1$  is the inverse of the price at time  $t$  of that asset, or  $1/b_{x,t}$ . Hence, (6.18) becomes

$$F_t = S_t \cdot \frac{b_{y,t}}{b_{x,t}} \quad (6.19)$$

However, the asset prices are endogenously determined. In general, the equilibrium price of any asset in this framework is "such that the marginal utility foregone by purchasing the asset is equal to the conditional expectation of the marginal utility of the return from holding the asset" (Hodrick and Srivastava (1984, p. 8)).

Let us consider a discount bond paying one unit of home currency at  $t+1$ . The expected (discounted) marginal utility of one unit of  $M$  is given by the amount of good  $x$  it is expected to buy, evaluated by the agents at the marginal utility of good  $x$  at  $t+1$  (since  $M$  cannot be used to buy good  $y$ ), or, formally,  $E_t[\beta U_{x,t+1} \cdot \frac{1}{p_{x,t+1}}]$ . In order to get this expected payoff, the agent has to pay  $b_{x,t}$  which in terms of marginal utility foregone is worth  $b_{x,t} U_{x,t} \cdot \frac{1}{p_{x,t}}$ . Equality between these two expressions implies that the asset price must be

$$b_{x,t} = E_t \left[ \frac{\beta U_{x,t+1}}{p_{x,t+1}} \bigg/ \frac{U_{x,t}}{p_{x,t}} \right] \equiv E(Q_{t+1}^M) \quad (6.20)$$

The price of a one-period security in home currency is thus equal to the conditional expectation of the intertemporal marginal rate of substitution of the home currency which Hodrick and Srivastava (1984) denote  $E_t(Q_{t+1}^M)$ , for brevity.  $Q_{t+1}^M$  can be interpreted as a measure in utility terms of the change in the purchasing power of the home currency,  $p_{x,t}/p_{x,t+1}$ , and this variable will prove to be crucial for the determination of the risk premium.

Obviously, a similar expression holds for the foreign currency denominated asset and (6.19) can be rewritten as

$$F_t = S_t \cdot \frac{E_t(Q_{t+1}^N)}{E_t(Q_{t+1}^M)} \quad (6.21)$$

Computing  $E_t[S_{t+1}]$  from (6.17) and subtracting it from (6.21) gives

$$\begin{aligned} F_t - E(S_{t+1}) &= q_t \frac{M_t}{N_t} \frac{Y_t}{x_t} \cdot \frac{E_t(Q_{t+1}^N)}{E_t(Q_{t+1}^M)} - \\ &- E_t \left[ q_{t+1} \cdot \frac{M_{t+1}}{N_{t+1}} \frac{Y_{t+1}}{x_{t+1}} \right] \end{aligned} \quad (6.22)$$



This is the foreign exchange risk premium in absolute terms, but by dividing through by  $S_t$  we get an expression for the risk premium in relative terms, analogous to the definition in previous sections:

$$\begin{aligned} \frac{F_t - E(S_{t+1})}{S_t} &= \frac{E_t(Q_{t+1}^N)}{E_t(Q_{t+1}^M)} - E_t \left[ \frac{Q_{t+1}^N}{Q_{t+1}^M} \right] \\ &= \frac{E_t(Q_{t+1}^N)}{E_t(Q_{t+1}^M)} - E_t(Q_{t+1}^N) \cdot E_t \left( \frac{1}{Q_{t+1}^M} \right) - C \left( Q_{t+1}^N, \frac{1}{Q_{t+1}^M} \right) \end{aligned} \quad (6.23)$$

where  $C(\cdot, \cdot)$  denotes the covariance. While demonstrating that the risk premium is determined by both real and monetary uncertainty interacting with the preferences of the representative agents, (6.23) has no readily apparent intuitive interpretation. It is easily seen that in the absence of uncertainty, the risk premium will be zero.<sup>14</sup> We can also note that a positive covariance between  $Q_{t+1}^N$  and  $Q_{t+1}^M$ , implying a negative sign on the covariance term in (6.23), tends to increase the size of the premium, other things equal. Such a covariance pattern can occur, for example, if the money supply processes in the two countries are positively correlated.

In general, however, the level of abstraction in this model is too high to permit any conclusions concerning the empirical determinants of the risk premium. It is also impossible to use a model where money supplies follow exogenous Markov processes to analyze "the role of monetary policy" as it is difficult in this context even to specify what is to be meant by monetary policy.

### 6.2.5 Summary of the theoretical work

In this section we have reviewed some models of international asset pricing, emphasizing the determination of the foreign exchange risk premium. While the formal similarities between the models are strong they tell somewhat different stories about the risk premium. In the models based on the conventional CAPM formulation, there is a direct connection between asset stocks and the size of the risk premium via the composition of the market portfolio. This is also, at least in principle, a testable hypothesis.

This result turns out not to be robust to more general formulations of the asset pricing model such as the consumption CAPM. It is obvious that this formulation is a substantial step in the direction of providing asset pricing models with a stronger general equilibrium foundation. The other side of the coin is that, e.g., the Stulz (1981) model does not give rise to any hypotheses that are directly amenable to empirical testing. This is of course the case also for the complete general equilibrium models of Lucas (1982) and Svensson (1985).

Recently, estimates of the closed economy consumption CAPM have been published by Grossman, Melino, and Shiller (1985), but the open economy version has so far (to our knowledge) not been explicitly tested. The empirical literature to be reviewed in the next section is based either on the standard CAPM results or on hypotheses that are consistent with rather than derived from more general asset pricing models.

### 6.3 EMPIRICAL WORK ON THE FOREIGN EXCHANGE RISK PREMIUM

One conclusion from the review of the theoretical literature in the previous section was that the more general asset pricing models are not easily adaptable to direct empirical testing. This is also reflected in the empirical work, where considerable attention has been given to basically atheoretical tests of the implied parity conditions and the relation between forward rates and future spot rates. The purpose in these tests is thus limited to finding out whether indications of risk premia can be found.

More recently, a number of studies attempting to explain observed deviations from UIP or unbiasedness in terms of specific models of the risk premium have appeared. However, these are, with a few exceptions, not structural tests of asset pricing models. In particular, when the starting point is a more general model of international asset pricing, it is usually only possible to test constraints that are consistent with that particular model.

In this section, we will, first, present the atheoretical approach and review some of the results obtained in this type of studies. Second, we will see how some of the theoretical risk premium models discussed in Section 6.2 have been applied in empirical work. As it turns out, the evidence is quite mixed. There is strong evidence indicating that deviations from UIP occur, but it has proved considerably more difficult to connect them with specific models of the risk premium.

### 6.3.1 The atheoretical approach

In Chapter 5, we saw that the general interest rate relation is

$$r_t = r_t^f + E(s_{t+1} - s_t) + p_t \quad (6.24)$$

and UIP obtains if  $p_t = 0$ . Combining (6.24) with the CIP condition

$$r_t = r_t^f + f_t - s_t \quad (6.25)$$

we note that

$$f_t = E(s_{t+1}) + p_t \quad (6.26)$$

This implies that under the maintained hypothesis of CIP a test of UIP is equivalent to a test of the unbiasedness of the forward exchange rate as a predictor of the future spot rate. This observation suggests the following regression model:

$$s_{t+1} = \alpha_1 + \beta_1 f_t + \varepsilon_{t+1} \quad (6.27)$$

which is used, for example, by Frenkel (1981), who cannot reject UIP, i.e., the hypothesis  $[\alpha_1, \beta_1] = [0, 1]$ .

There is a potential statistical problem in this formulation, however. As pointed out by Hansen and Hodrick (1980) and confirmed by Meese and Singleton (1982), the stochastic processes that generate the logarithms of spot and forward exchange rates tend to be non-stationary. Under these circumstances OLS estimation still yields consistent parameter estimates, but usual (asymptotic) hypothesis tests may be invalid.

This so-called unit-root problem can be circumvented, however, by adopting a first difference formulation:

$$s_{t+1} - s_t = \alpha_1 + \beta_1(f_t - s_t) + \epsilon_{t+1} \quad (6.28)$$

which is equivalent to (6.27) under the null hypothesis of UIP. As  $s_{t+1} - s_t$  is likely to be stationary, the inference based on (6.28) can be expected to be more reliable than when (6.27) is used. In general, studies based on (6.28) have rejected UIP; cf. Cumby and Obstfeld (1984) and the references cited there, and Fama (1984).

It is interesting to go beyond this finding and inquire into the properties of the implied risk premium. Fama (1984) notes that the deviation of  $\beta_1$  from unity is "a direct measure of the variation in the premium in the forward rate" (p. 321). Using (6.26) we can write the forward premium,  $f_t - s_t$ , as the sum of two components, namely, the expected rate of depreciation and the risk premium:

$$f_t - s_t = E(s_{t+1} - s_t) + p_t$$

As pointed out by Fama (1984), this implies that, under rational expectations, the regression coefficient  $\beta_1$  can be expressed as

$$\begin{aligned} \beta_1 &= \frac{\text{cov}(s_{t+1} - s_t, f_t - s_t)}{\sigma^2(f_t - s_t)} = \\ &= \frac{\sigma^2[E(s_{t+1} - s_t)] + \text{cov}[p_t, E(s_{t+1} - s_t)]}{\sigma^2(p_t) + \sigma^2[E(s_{t+1} - s_t)] + 2 \text{cov}[p_t, E(s_{t+1} - s_t)]} \quad (6.29) \end{aligned}$$

If there is zero covariance between the risk premium and the expected rate of depreciation,  $\beta_1$  measures the proportion of the variance that can be attributed to  $\sigma^2[E(s_{t+1} - s_t)]$ . Hence,  $1 - \beta_1$  is the fraction that is due to the variance in the risk premium.

However, it cannot be ruled out that the risk premium and the expected rate of depreciation are correlated and in Fama's study there is actually evidence of such a correlation. Using spot and forward dollar exchange rates for nine currencies, Fama consistently gets negative values for  $\beta_1$ . From (6.29) this is seen to imply that

$$\text{cov}[p_t, E(s_{t+1}-s_t)] < 0$$

and

$$\sigma^2[E(s_{t+1}-s_t)] < |\text{cov}[p_t, E(s_{t+1}-s_t)]|$$

Moreover,  $\beta_1 < 0$  implies that

$$1 - \beta_1 = \frac{\sigma^2(p_t) + \text{cov}[p_t, E(s_{t+1}-s_t)]}{\sigma^2(f_t-s_t)} > 1$$

and, the denominator being positive, we see that

$$\sigma^2(p_t) > |\text{cov}[p_t, E(s_{t+1}-s_t)]|$$

Hence, it must hold that

$$\sigma^2(p_t) > \sigma^2[E(s_{t+1}-s_t)]$$

i.e., the variance in the risk premium exceeds the variance in the expected rate of depreciation.<sup>15</sup>

Fama's (1984) results thus indicate that time varying risk premia exist and he can also say something about their properties. However, his approach does not permit any conclusions about either the absolute magnitudes or the determinants of the premia. The former would require observations of the expected rate of depreciation. In principle, this problem could be approached along the lines suggested by Mishkin (1983) in the context of obtaining indirect measures of expectations in market efficiency tests. We will not pursue this approach here, however, and the reasons for this are twofold.

First, market efficiency is not our concern in this chapter. Second, and more importantly, a Mishkin test is always a joint test of market efficiency and a model of market equilibrium. To formulate the latter one must have a theory of the determination of the risk premium. (Although there has been some confusion in the literature on this point, when UIP tests were interpreted as tests of forward market efficiency, e.g., by Levich (1978), risk neutrality is clearly not a prerequisite for efficiency.) A thorough examination of the issues raised here is therefore an important first step in an efficiency test. (For an application of Mishkin's approach to the field of exchange rate determination, see Hoffman and Schlagenhauf (1985).) Here, we will proceed by discussing the attempts to test specific models of the risk premium.

### 6.3.2 Portfolio balance models

Frankel (1982) bases his test on the asset pricing relation derived in the mean-variance optimization model discussed in Section 6.2.1. Repeated here for convenience it is

$$r_t - r_t^{\$} - E(s_{t+1} - s_t) = \rho\Omega(x_t - \alpha) \quad (6.30)$$

To adapt the model to econometric estimation Frankel invokes rational expectations, i.e.,

$$E(s_{t+1}) = s_{t+1} + \epsilon_{t+1} \quad (6.31)$$

where  $\epsilon_{t+1}$  is the error in the exchange rate forecast, which (by rational expectations) is independent of all information available at time  $t$ . Combining (6.30) and (6.31) gives the regression model

$$r_t - r_t^{\$} - (s_{t+1} - s_t) = \rho\Omega(x_t - \alpha) + \epsilon_{t+1} \quad (6.32)$$

Frankel emphasizes that " $\Omega$  is precisely the variance-covariance matrix of the error term, and the system should be estimated subject to this constraint" (p. 260). Applying maximum likelihood techniques, he estimates the crucial parameter  $\rho$  simultaneously with  $\Omega$ .

This test requires data on interest rates and exchange rates for the L.H.S. variables and observations on  $x_t$  and  $\alpha$ . Recalling that these are the vector of asset shares in the world market portfolio and the vector of consumption shares, respectively, it is obvious that Frankel encounters considerable measurement problems.<sup>16</sup> The definitions used are presented in a lengthy appendix to Frankel (1982) and although the computations are done with great care, there is certainly a considerable degree of arbitrariness involved. The well-known difficulties of defining "the market portfolio" in regular CAPM formulations (cf. Roll (1977)) are inevitably aggravated when international aspects are introduced. The data problem is thus an important caveat when interpreting Frankel's results.

When (6.32) is estimated using a maximum likelihood procedure, it turns out that the likelihood function is extremely flat, but that its maximum is at  $\rho=0$ . The hypothesis of risk neutrality can therefore not be rejected. However, the evidence is not very strong and the test gives no basis for asserting the null hypothesis  $\rho=0$  as Frankel, e.g., cannot reject  $\rho=1$  or  $\rho=2$  either. The power of the test is therefore quite low.

Frankel concludes that while the power of the test may be low, he has found no evidence of a systematic relationship between the deviation from uncovered interest parity and the variables on which theory tells us that the risk premium should depend.

As Frankel himself points out, there are a number of reasons to remain skeptical about these results. The model is based on a number of restrictive assumptions. These include constant



degree of relative risk aversion, nonstochastic prices and homogeneous investors. Also, there may be an identification problem in that Frankel tries to estimate the parameters in the asset demand function from what is in effect an equilibrium condition. Unless outside asset supplies have been exogenously and sufficiently varied, the demand function is poorly identified.

In the beginning of this section we noted that, under the hypothesis of CIP, a test of UIP is equivalent to a test of whether the forward exchange rate is an unbiased predictor of the future spot rate. We can elaborate on this point by comparing Frankel's test of UIP with Park's (1984) test of unbiasedness; Park claims that his test is a two-currency version of Frankel's.

In a world of only two currencies, (6.30) will read

$$r_t - r_t^{\$} - E(s_{t+1} - s_t) = \rho\sigma^2(x_t - \alpha) \quad (6.33)$$

Here  $\sigma^2$  is the variance of the exchange rate and all variables and parameters are now scalars. Under the assumptions of CIP and rational expectations (6.33) can be transformed into the following regression equation:

$$s_{t+1} - f_t = -\rho\sigma^2(x_t - \alpha) + \varepsilon_{t+1} \quad (6.34)$$

where

$$\sigma^2 = \sigma_{\varepsilon}^2$$

Now, Park (1984) specifies his model as

$$s_{t+1} = c_0 + c_1 f_t + c_2 (x_t - \alpha) + \eta_{t+1} \quad (6.35)$$

and tests the null hypothesis  $c_2=0$  against  $c_2<0$ . There are several differences between this test and Frankel's. First,

Park imposes less structure on his model by not acknowledging the theoretical constraint that  $c_2 = -\rho\sigma^2$ . Second, Park allows  $[c_0, c_1]$  to differ from  $[0, 1]$ . An interesting question then is: what conclusion should be drawn if it turns out that one can reject neither that  $c_2=0$  (which is consistent with UIP) nor that  $c_1 \neq 1$  (which is an indication of biasedness in the forward exchange rate)?

Given an estimate of  $c_0$  not significantly different from zero, the conclusions from different estimates of  $[c_1, c_2]$  are summarized in the table below.

	$c_1 = 1$ Not rejected	$c_1 \neq 1$ Rejected
$c_2 = 0$ Not rejected	<ul style="list-style-type: none"> <li>. Consistent with UIP</li> </ul>	<ul style="list-style-type: none"> <li>. Not consistent with UIP</li> <li>. Wrong model of the risk premium</li> </ul>
$c_2 < 0$ Not rejected	<ul style="list-style-type: none"> <li>. Not consistent with UIP</li> <li>. Correct model of the risk premium</li> </ul>	<ul style="list-style-type: none"> <li>. Not consistent with UIP</li> <li>. Insufficient model of the risk premium</li> </ul>

The only result that is consistent with UIP is if it cannot be rejected that  $[c_1, c_2] = [1, 0]$ . While Frankel (1982) is unable to reject that  $c_2 = 0$ , he cannot say anything about the value of  $c_1$ . Therefore it cannot be ruled out that his failure to reject UIP is due to use of an incorrect model of the risk premium.

Park (1984) reaches a conclusion different from that of Frankel. Using U.S. dollar/D-mark rates, Park obtains estimates of  $c_2$  that are, in most cases, significantly negative. In quarterly exchange rates, the adjusted market portfolio  $(x-\alpha)_t$  explains 10-20 percent of the total variance. The proportion is somewhat smaller for monthly data. Park concludes that "[t]he tests provide firm evidence for a risk premium in the foreign exchange

market" (p. 175). Park makes no joint test of  $[c_1, c_2]$ , however. In fact, in all three regressions on quarterly data,  $c_1$  is significantly less than unity. This is true also for one of the regressions on monthly data. It can therefore be questioned whether Park offers a sufficient explanation of the risk premium.

It is illuminating to relate Park's model to the time-series test used by, e.g., Fama (1984). To do this, we must first rewrite Park's model to take the aforementioned unit-root problem into account. Park estimates a model where the dependent variable is likely to be non-stationary and his hypothesis tests should therefore be interpreted with caution. A more appropriate formulation than (6.35) is therefore

$$s_{t+1} - s_t = c_0 + c_1(f_t - s_t) + c_2(x - \alpha)_t + \eta_{t+1} \quad (6.36)$$

which is equivalent to (6.35) under the null hypothesis of UIP,  $[c_0, c_1, c_2] = [0, 1, 0]$ .

Comparing (6.36) to Fama's regression model, repeated here for convenience,

$$s_{t+1} - s_t = \alpha_1 + \beta_1(f_t - s_t) + \varepsilon_{t+1} \quad (6.28)$$

we see immediately that

$$\varepsilon_{t+1} \equiv c_2(x - \alpha)_t + \eta_{t+1}$$

The essence of Park's test is therefore to try to decompose Fama's residual term into a systematic risk premium and a random component. Analogously, Frankel, by assuming  $c_1=1$ , tries to find a systematic component in the deviations between the realized spot rate and the forward rate.

Seen from this point of view, Frankel's formulation appeals stronger to intuition by going straight at the hypothesis that is to be tested. Park's free estimate of  $c_1$  may give indications of deficiencies in the model of the risk premium, but no guidance on what the nature of these problems may be. In this sense, Park's formulation is a combination of the atheoretical time-series test and a test of a structural model. It seems preferable to keep these approaches apart and use the first to see if there are deviations from UIP and, if such evidence is found, use the second to investigate the nature and cause of these deviations.

### 6.3.3 Tests related to intertemporal asset pricing models

Empirical research on the foreign exchange market risk premium is sometimes related to intertemporal asset pricing models such as those of Lucas (1982) and Stulz (1981). No attempts to test structural equations derived from these models have been made, however. The usual approach is instead to test hypotheses consistent with or restrictions implied by these asset pricing models.

One example of this approach is a paper by Hodrick and Srivastava (1984). Drawing on previous work by Hansen and Hodrick (1983), they derive testable cross-equation constraints from a conventional capital asset pricing model. Their starting point is the following relation,

$$E \left( \frac{S_{t+1}^i - F_t^i}{S_t^i} \right) = \beta_t^i \cdot y_t \quad i=1, \dots, n \quad (6.37)$$

$(S_{t+1}^i - F_t^i)/S_t^i$  is the excess of the relative return of an uncovered position in currency  $i$  over that of a covered position ( $s = \ln S$ ,  $f = \ln F$ ),  $y_t$  is the expected difference between the return of a benchmark asset and a nominal risk-free return, and  $\beta_t^i$  is the standard CAPM- $\beta$ , i.e., the covariance between

$(S_{t+1}^i - F_t^i)/S_t^i$  and the return on the benchmark asset, divided by the variance of the latter. It is assumed that the  $\beta^i$ 's are constant.

As demonstrated in Section 6.2.1, such a CAPM relation can be derived in Frankel's model. Hodrick and Srivastava (1984) choose to interpret it as "perfectly consistent" with Lucas' (1982) asset pricing model.

This is a somewhat misleading interpretation, since, as shown in Section 6.2.3, the simple CAPM relations break down when one goes from the mean-variance optimization approach to more general equilibrium treatments. The difference between Frankel's (1982) study and that of Hodrick and Srivastava (1984) therefore lies more in the empirical work than in the theoretical underpinnings.

Instead of defining a benchmark asset (or portfolio) and compute its rate of return relative to the riskfree rate in order to obtain a direct measure of  $y_t$ , which would be analogous to Frankel's approach, Hodrick and Srivastava treat  $y_t$  as an unobservable variable. Being unknown to the econometrician,  $y_t$  must be replaced by the best linear predictor based on a subset of available information, using instrumental variables. Specifically, Hodrick and Srivastava assume that

$$y_t = \gamma_0 + \sum_i \gamma_1^i \left( \frac{F_t^i - S_t^i}{S_t^i} \right) + \eta_t \quad (6.38)$$

i.e., the forward premia on all the currencies in the sample are used as instruments. Using (6.38), the following system of  $n$  regression equations is obtained:

$$\frac{S_{t+1}^j - F_t^j}{S_t^j} = \beta^j \gamma_0 + \beta^j \sum_i \gamma_1^i \left( \frac{F_t^i - S_t^i}{S_t^i} \right) + v_{t+1}^j \quad j=1, \dots, n \quad (6.39)$$

where

$$v_{t+1}^j = \beta^j \eta_t + \phi_{t+1}^j$$

$$\phi_{t+1}^j = E \left[ \frac{S_{t+1}^j - F_t^j}{S_t^j} \right] - \frac{S_{t+1}^j - F_t^j}{S_t^j}$$

The cross-equation constraints in (6.39) enable them to recover separate estimates of the  $\beta$ 's and  $\gamma$ 's.

Hodrick and Srivastava (1984) get significant estimates of the  $\beta^i$ 's and for two out of five currencies they can reject the hypothesis that the composite parameters  $\hat{\theta}_{ij} = \hat{\beta}^j \hat{\gamma}_1^i$  are all zero, which would seem to give some support to their particular CAPM model of a time-varying risk premium. However, the cross-equations constraints imposed by the model are rejected at the five percent level. Hodrick and Srivastava (1984) conclude: "If the source of the rejection of the unbiasedness hypothesis is a time-varying risk premium, it appears that the assumptions of the [Hansen and Hodrick (1983)] model are too strong. Either the betas ... are not constant, or some other model of risk and return is necessary to describe the risk premium" (p. 15).

Note the formal similarity between (6.39) and the test equation used by Fama (1984). Ignoring the cross-equation constraints, (6.39) is a multi-currency version of (6.28) where the forward premia for all the currencies are included in each equation and the regression coefficients are defined as  $\hat{\theta}_{ii} = \beta_1 - 1$ .<sup>17</sup>

Under unbiasedness ( $\beta_1 = 1$ ) we would expect  $\hat{\theta}_{ii} = 0$ . Examining the results from Hodrick and Srivastava's (1984) estimates of the unconstrained model, we find that the  $\hat{\theta}_{ii}$ 's are negative in four out of five cases and significantly so in three. The pattern is thus the same as in Fama (1984) although less uniform.

As we noted above, Hodrick and Srivastava (1984) and Frankel (1982) have similar starting points, but follow quite different

econometric procedures. Frankel undertakes the arduous task of measuring the composition of the world market portfolio. Hodrick and Srivastava circumvent this problem by using an instrumental variables technique. The other side of the coin is that this forces them to choose, more or less arbitrarily, a set of instrumental variables. There is no denying that there is a considerable degree of arbitrariness also in this procedure although of a different nature than in Frankel's approach. In any case, the two procedures give similar results.

Korajczyk (1985) also derives a testable hypothesis which is consistent with intertemporal asset pricing models. He points out that these "models imply that the risk premia in forward prices should be identical to the risk premia differential in the real returns on default-free nominal bonds denominated in the respective currencies" (p. 347).

Korajczyk starts with the covered interest parity condition (in logarithms),

$$f_t - s_t = r_t - r_t^f \quad (6.40)$$

and an expression for the spot exchange rate that allows for deviations from PPP,

$$S_t = k \frac{P_t}{P_t^f} \cdot D_t \quad (6.41)$$

where  $P_t$  and  $P_t^f$  are the domestic and foreign price levels,  $D_t$  measures the deviation from PPP, and  $k$  is some constant. Taking logarithms in (6.41) and combining it with (6.40) gives an expression for the difference between realized spot rate and the forward rate:

$$s_{t+1} - f_t = \pi_{t+1} - \pi_{t+1}^f + d_{t+1} - d_t - (r_t - r_t^f) \quad (6.42)$$

where  $\pi_{t+1}$  and  $\pi_{t+1}^f$  are the domestic and foreign rates of inflation and  $d_t = \ln D_t$ . The standard Fisher equation implies that

$$r_t = E(\pi_{t+1}) + E(i_{t+1}) \quad (6.43)$$

where  $i_{t+1}$  is the real return from  $t$  to  $t+1$ . Combining (6.43) and the corresponding expression for  $r_t^f$  with (6.42) and taking expectations we get

$$E(s_{t+1} - f_t) = E(i_{t+1}^f - i_{t+1}) + E(d_{t+1} - d_t) \quad (6.44)$$

From (6.44) we see that there are two elements that may introduce a bias in the forward rate as a predictor of the spot rate, namely, expected differences in real interest rates and expected changes in the relative deviation from PPP.

Korajczyk points out that the condition

$$E(d_{t+1}) = d_t \quad (6.45)$$

is equivalent to Roll's (1979) "efficient market version of PPP", i.e., that the deviations from PPP follow a martingale. Korajczyk cites results from Roll (1979) and Adler and Lehmann (1983) that support the martingale hypothesis and his tests are based on the assumption that condition (6.45) holds. This leads to the following regression model:

$$s_{t+1} - f_t = \beta_0 + \beta_1 E(i_{t+1}^f - i_{t+1}) + \eta_{t+1} \quad (6.46)$$

The null hypothesis is that  $\beta_0 = 0$  and  $\beta_1 = 1$ . Korajczyk argues that "rejection of the restrictions is a rejection of the joint hypothesis that (i) risk premia are the cause of deviations from [the unbiased expectations hypothesis] and (ii)  $E(d_{t+1} - d_t) = 0$ " (p. 351).



When interpreting Korajczyk's tests, it should be noted that (6.46) was derived above without reference to any intertemporal asset pricing model. In Chapter 5, it was shown that UIP and PPP jointly imply real interest rate parity. Korajczyk assumes (a version of) PPP and proceeds to attribute deviations from UIP to deviations from real interest rate parity. These expected real interest rate differentials are not explained, however. As it is possible that they are caused by, for example, imperfect asset market integration, Korajczyk's tests do not really give any firm evidence on the role of risk premia and no evidence whatsoever on their determinants. In this sense, Korajczyk's study is more in line with the atheoretical approach to testing parity conditions adopted by Cumby and Obstfeld (1984) and Fama (1984), who do not attempt to motivate their tests with reference to any asset pricing model.

Using a 3SLS procedure where the first step is used to find estimates of the unobservable expected real interest rate differential, Korajczyk cannot reject the null hypothesis, in general. Hence, all we can say is that the deviations between the realized spot rates and the forward rates, and expected real interest rate differentials tend to move together. The common determinants of these movements remain to be identified.

#### 6.4 CONCLUDING COMMENTS

When trying to summarize the empirical evidence on the risk premium in the foreign exchange market we have to conclude that the results are inconclusive. There are some studies, e.g., by Fama (1984), Cumby and Obstfeld (1984), and Korajczyk (1985), that show that the forward rate is not an unbiased predictor of the future spot rate. This is consistent with the hypothesis that a risk premium exists, but can also be attributable to, e.g., imperfect market integration or simple market inefficiency.

When attempts have been made to relate these deviations from UIP (or unbiasedness) to economic variables on which the risk premium according to standard models of asset pricing theory should depend the results have been negative or, at least, inconclusive. As pointed out by Huang (1984), it is relatively easy to reject unbiasedness against a very general alternative hypothesis, e.g.,  $\beta_1 \neq 1$  in (6.28), but when a more specific alternative is introduced, the tests seem to fail to reject unbiasedness.

These results are open to several interpretations. First, it can be argued that risk premia are unimportant and that other explanations to the observed bias should be tested. On the other hand, it is clear that the risk premium models tested are very crude and that empirical work in this field is plagued by difficult measurement problems. The inherent unobservability of many important variables means that the researcher has to resort to heroic assumptions (as, e.g., Frankel (1982)) or basically ad hoc choices of instrumental variables (as, e.g., Hodrick and Srivastava (1984)). Explicit tests of the more complete asset pricing models, such as Stulz' (1981), have so far not been carried out.

The inconclusiveness of the empirical results can also be due to tests with low statistical power. As noted by many authors, there are a priori reasons to suspect that tests of unbiasedness are subject to such problems. To see this, let us consider the residual in the time-series tests in equation (6.28). Under the null hypothesis, it can be written as

$$\epsilon_{t+1} = s_{t+1} - E(s_{t+1})$$

i.e., as (minus) the error in the forecast of the future spot rate. As spot rates are generally extremely difficult to predict, a high residual variance is to be expected in a regression of (6.28), which gives hypothesis tests with low power.<sup>18</sup>

Hence, while the evidence in favor of the risk premium hypothesis is far from overwhelming neither is the evidence against it. The tests have so far been unable to discriminate between this and alternative explanations for the deviations from UIP, mainly because of fundamental empirical problems. Of course, most of these are present also in the Swedish system that will be analyzed in Chapter 7. However, the currency basket to which the Swedish krona is attached will be shown to have some features that at least may reduce the forecast error variance problem.

## NOTES

1. For a more general survey of international asset pricing, see Adler and Dumas (1983). Branson and Henderson (1985) discuss what implications asset pricing models have for the micro foundations of portfolio balance macro models.
2. A more detailed analysis of a model that is similar to Frankel (1982) is presented in Chapter 7.
3. This has also been stressed by Branson and Henderson (1985).
4. For details about Ito processes and other elements of stochastic calculus, see Merton (1971) or Malliaris and Brock (1982).
5. In Frankel's (1982) model there is only one, nominally riskless, asset in each currency, so  $J = N$ . With the dollar as the reference currency,  $\mu_{N+1} = r^{\$} = r$ .
6. The difference between  $\mu_i$  and  $r^i - \theta^i$  is due to Jensen's inequality.
7. Note that

$$\frac{d(Y_j/P)}{(Y_j/P)} = \frac{dY_j}{Y_j} - \frac{dP}{P} + \left(\frac{dP}{P}\right)^2 - \left(\frac{dY_j}{Y_j}\right) \cdot \left(\frac{dP}{P}\right)$$

If

$$\frac{dY_i}{Y_i} = (r^i - \theta^i)dt + \sigma_i dz_i$$

and

$$\frac{dP}{P} = \pi dt + \sigma_{\pi} dz_{\pi}$$

then, by Ito's lemma, the expected real rate of return is

$$E \left[ \frac{d(Y_i/P)}{(Y_i/P)} \right] = (r^i - \theta^i - \pi + \sigma_\pi^2 - \sigma_{i,\pi})$$

Similarly, if

$$\frac{dY_{N+1}}{Y_{N+1}} = r^\$ dt$$

then

$$E \left[ \frac{d(Y_{N+1}/P)}{(Y_{N+1}/P)} \right] = r^\$ - \pi + \sigma_\pi^2$$

since  $\sigma_{N+1,\pi} = 0$  as  $Y_{N+1}$  is non-stochastic.

Hence,

$$E \left[ \frac{d(Y_i/P)}{(Y_i/P)} - \frac{d(Y_{N+1}/P)}{(Y_{N+1}/P)} \right] = r^i - \theta^i - r^\$ - \sigma_{i,\pi}$$

8. For details and generalizations, see Merton (1971, 1973) and Breeden (1979).
9. For a multi-good version of the closed economy model, see Breeden (1979), section 7.
10. We simplify Stulz' model by assuming homothetic preferences. Under non-homotheticity, two separate price indices must be defined, but such index problems are not crucial for our purposes.
11. The reason that (6.12) is expressed in real terms is the multi-good assumption and, hence, has nothing to do with the international aspects as such.
12. Contrary to the earlier model, Stulz (1984) gives an explicit role to money via the money-in-the-utility-function approach. This assumption is used also by Kouri (1977) in a conventional CAPM.

13. Recall that the CIP condition (5.1) was expressed in terms of the logs of the spot and forward exchange rates. It is therefore an approximation of (6.18).
14. However, note that risk neutrality does not imply a zero premium because of Jensen's inequality.
15. Hodrick and Srivastava (1986) show that such a pattern can be explained in terms of Lucas' (1982) general equilibrium asset pricing model.
16. Frankel (1982) also tests a model where the consumption pattern is allowed to vary. The results are highly similar to those discussed below.
17. The dependent variables in (6.28) and (6.39) are different, but this is purely a matter of convenience. As shown by Fama (1984) the regression

$$f_t - s_{t+1} = \alpha_2 + \beta_2(f_t - s_t) + \epsilon'_{t+1}$$

is equivalent to (6.28) with  $\beta_2 = 1 - \beta_1$ .

18. This is emphasized by Mishkin (1984). He attributes his failure to reject UIP and PPP separately when the joint hypothesis of real interest rate parity is strongly rejected to the errors in exchange rate forecasts. The nature of the problem is seen by recalling that the standard error of  $\beta_1$  in (6.28) is  $[\text{Var}(\epsilon_{t+1})/n \cdot \text{Var}(f_t - s_t)]^{1/2}$ , where  $\text{Var}(\epsilon_{t+1})$  is the forecast error variance.

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## 7. The Foreign Exchange Risk Premium in a Currency Basket System\*

### 7.1 INTRODUCTION

The discussion of theoretical and empirical models of the foreign exchange risk premium in the previous chapter was based on the assumption of more or less flexible exchange rates. When applying the theoretical concepts "fixed" and "flexible" exchange rates to the real world, one must give careful consideration to the intricacies of actual exchange rate systems. After the breakdown of the fixed exchange regime under the Bretton Woods system a variety of exchange rate arrangements have emerged. While some of the major currencies are floating more or less freely relative to one another, a number of countries have tried to establish more stable currency arrangements. These attempts have been especially common in Western Europe, where, for example, the European Monetary System (EMS) is intended to maintain stable exchange rates within the European Community. Several of the non-member countries, including Sweden, have chosen unilateral systems in which they tie their exchange rates to indices of foreign currencies, so-called currency basket systems.

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The policy of tying the exchange rate to a currency basket and keeping the value of the currency index stable can be interpreted as a basically fixed exchange rate system. As emphasized in Chapter 5, this policy must be said to imply that the authorities are unwilling to use exchange rate changes as part of the transmission mechanism of monetary policy.

If the currency had been perfectly pegged to a basket index, there would presumably be no more monetary independence in a currency basket system than in a system of rigidly fixed exchange rates. However, as implemented in practice there is usually some flexibility in the value of the currency index, which makes it meaningful to study the role of risk premia in this context. If such premia can be shown to exist, one would eventually want to know whether their determinants include variables that the central bank can control in a systematic way.

We should note at the outset that even if there is such an exploitable relationship, it is not at all self-evident that it should be exploited. "Independence" in itself is hardly a policy goal and to the extent that it is achieved by increasing the degree of uncertainty in the economy, it can prove to be quite costly. Although this is not an issue in this chapter, the distinction between what possibly can be done and what should be done is important.

The purposes of this chapter are, first, to analyze theoretically how the foreign exchange risk premium is determined in a currency basket system. Second, on the basis of the theoretical analysis we will test for the existence of a risk premium on the Swedish krona.

The chapter is organized as follows. Section 7.2 contains a theoretical analysis of the determination of the foreign

exchange risk premium in a currency basket system. In Chapter 6, we demonstrated that the appropriate framework for this analysis is a model of international asset pricing. We modify the portfolio optimization model used by Frankel (1982), that was briefly presented in Section 6.2, by letting one country pursue a currency basket policy. This analysis turns out to have important implications for how the relevant risk premium on the currency that is tied to an index should be measured. This insight is the basis for the empirical tests in Section 7.3, where we use the method applied by Fama (1984), also discussed in Chapter 6, to investigate whether there is a risk premium on the Swedish krona. We find evidence consistent with this hypothesis, but given the atheoretical nature of the test, we cannot study the determinants of the implied premium. Nevertheless, our results indicate that what could be considered a necessary (but not sufficient) condition for a country with a basically fixed exchange rate to have some monetary independence is met in the case of Sweden.

## 7.2 RISK PREMIA IN A CURRENCY BASKET SYSTEM

In this section, we analyze the determination of the risk premium in a simple optimization model of international asset pricing. As a background, we will first discuss a concrete example of the functioning of a currency basket system. This is the Swedish system which is also the focus of the empirical tests in Section 7.3.

The Swedish krona is tied to an index of foreign currencies, a so-called foreign currency basket. In principle, the value of the krona is fixed to this basket, at the same time floating bilaterally against all the individual currencies in the basket. In practice, this system is complicated by the fact that the central bank allows the krona to deviate from a benchmark

value of the currency index. This is analogous to the fluctuations of  $\pm 1$  percent that were permitted against the dollar in the Bretton Woods system. However, the Swedish central bank has, until very recently, declined to specify any exact bounds on the deviations from the benchmark value that it will permit. Since 1977 when the currency basket system was instituted deviations up to two percent have been observed, which gives an indication of what the phrase "minor fluctuations" used in central bank policy statements has meant in practice. The official bounds announced in June 1985 are (approximately)  $\pm 1.5$  percent. To the "flexibility" of the system adds the fact that there have been two changes in the benchmark value - devaluations of 10 % in September 1981 and 16 % in October 1982.<sup>1</sup>

If we let  $s_t^j$  denote the log of the currency  $j$ /dollar exchange rate in period  $t$ , we can define the index of the Swedish currency basket ( $I_t$ ) as follows:

$$\ln I_t = \omega'[(\iota s_t^{Kr} - s_t) - (\iota s_0^{Kr} - s_0)] + (1 - \omega' \iota)(s_t^{Kr} - s_0^{Kr}) \quad (7.1)$$

Here,  $s_t' \equiv [s_t^1, \dots, s_t^m]$  contains all dollar exchange rates, except the krona/dollar rate, in period  $t$ ; the  $(m \times 1)$  vector  $s_0$  contains the same exchange rates in the base period;  $s_t^{Kr}$  is the krona/dollar exchange rate in period  $t$  (consequently,  $s_t^{Kr} - s_t^j$  is the krona/currency  $j$  exchange rate in period  $t$ );  $\iota$  is a  $(m \times 1)$  vector of ones; and  $\omega' \equiv [\omega^1, \dots, \omega^m]$  contains the currency basket's weights of all currencies except the dollar. The weight of the dollar is thus equal to  $(1 - \omega' \iota)$ .

From the definition of the index we can derive an expression for the depreciation of the krona against the dollar:

$$s_{t+1}^{Kr} - s_t^{Kr} = \omega'(s_{t+1} - s_t) + \ln I_{t+1} - \ln I_t \quad (7.2)$$

Hence, an observed depreciation of the krona against the dollar can be due to an overall depreciation of the krona (an increase in the currency basket's index) and/or to an overall appreciation of the dollar against an average of the non-krona currencies.<sup>2</sup>

What implications does the Swedish Riksbank's exchange rate policy have for the question about monetary autonomy? If the average value of the krona (as expressed in the currency basket's index) were rigidly and credibly fixed, a nominally safe Swedish asset would be a perfect substitute to a certain portfolio of nominally safe foreign currency assets, namely, a portfolio with the same composition as the official currency basket. Hence, the Swedish interest rate would have to equal a weighted average of foreign interest rates, and there would be little scope for an independent monetary policy. It would still be possible for UIP to be violated bilaterally, since there would be no reason to expect assets denominated in kronor to be perfect substitutes to investments in any other single currency. Therefore, it is the variability in the currency index that (possibly) gives the Riksbank some independence.

The most important implication of the exchange rate policy is that if we want to test whether there is a risk premium that drives a wedge between Swedish and foreign interest rates, we cannot rely on bilateral tests. We will elaborate on this point below and formalize our argument by analyzing a model of international asset pricing in which one country pursues a currency basket policy.

The starting point is Frankel's (1982) international portfolio balance model. Like Frankel, we study a representative investor who is assumed to maximize a utility function of the mean and variance of his end-of-period real wealth (evaluated in U.S. dollars which is the measurement currency). The investor chooses a vector  $x_t$  of portfolio shares. The only uncertainty in the model is related to exchange rate changes, i.e., goods prices are non-stochastic. However, the investor consumes goods from different countries which means that the real value of his portfolio, via the exchange rates, also depends on the composition of his consumption basket, summarized by the vector  $\alpha$  of consumption shares. The consumption pattern is exogenous and assumed to be constant over time.

The investor's problem is to allocate his real wealth  $W_t$  among assets denominated in  $m+2$  different currencies. There is one asset in each currency. His real wealth in period  $t+1$  will be

$$W_{t+1} = W_t + W_t x_t' i_{t+1} + W_t x_t^{Kr:Kr} i_{t+1}^{Kr:Kr} + W_t (1 - x_t' - x_t^{Kr:Kr}) i_{t+1}^{\$} \quad (7.3)$$

Here  $x_t^{Kr}$  is the portfolio share allocated to assets denominated in Swedish kronor, and  $x_t$  is a  $(m \times 1)$  vector of portfolio shares for non-dollar assets. Hence,  $(1 - x_t' - x_t^{Kr})$  is the portfolio share of dollar assets. In analogy,  $i_{t+1}^{Kr}$ ,  $i_{t+1}^{\$}$  and the  $(m \times 1)$  vector  $i_{t+1}$  give the real interest rates of the  $m+2$  different assets. These are, in turn, given by

$$\begin{aligned} i_{t+1}' &\equiv [i_{t+1}^1, \dots, i_{t+1}^m] \\ i_{t+1}^j &\approx r_t^j - \pi_{t+1}^* - (s_{t+1}^j - s_t^j) \quad j = 1, \dots, m \\ i_{t+1}^{Kr} &\approx r_t^{Kr} - \pi_{t+1}^* - (s_{t+1}^{Kr} - s_t^{Kr}) \\ i_{t+1}^{\$} &\approx r_t^{\$} - \pi_{t+1}^* \end{aligned} \quad (7.4)$$



where  $r_t^j$  is the (safe) one-period nominal interest rate on currency  $j$  assets; and  $\pi_{t+1}^*$  is the rate of inflation for "the appropriate basket of goods" (measured in dollars). This inflation index is a weighted average of the  $m+2$  countries' inflation rates, where each rate is adjusted for the exchange rate change, so as to give a dollar inflation index. The weights are equal to the shares of the investor's consumption allocated to products from the different countries:

$$\begin{aligned}\pi_{t+1}^* &= \alpha'(\pi_{t+1} - (s_{t+1} - s_t)) + \alpha^{Kr}(\pi_{t+1}^{Kr} - (s_{t+1}^{Kr} - s_t^{Kr})) \\ &\quad + (1 - \alpha' - \alpha^{Kr})\pi_{t+1}^{\$}\end{aligned}\quad (7.5)$$

where

$$\alpha' \equiv [\alpha^1, \dots, \alpha^m]$$

$$\pi_{t+1}' \equiv [\pi_{t+1}^1, \dots, \pi_{t+1}^m]$$

Next we rewrite (7.3) as

$$W_{t+1} = W_t [1 + x_t'(i_{t+1} - i_{t+1}^{\$}) + x_t^{Kr}(i_{t+1}^{Kr} - i_{t+1}^{\$}) + i_{t+1}^{\$}] \quad (7.6)$$

and we use (7.4) to get

$$\begin{aligned}W_{t+1} &= W_t [1 + x_t'(r_t - (s_{t+1} - s_t) - r_t^{\$}) + x_t^{Kr}(r_t^{Kr} - (s_{t+1}^{Kr} - s_t^{Kr}) - r_t^{\$}) \\ &\quad + r_t^{\$} - \pi_{t+1}^*]\end{aligned}\quad (7.7)$$

Further substitutions from (7.5) and (7.2) yield

$$\begin{aligned}
 W_{t+1} = & W_t [1 + x'_t(r_t - (s_{t+1} - s_t) - r_t^{\$}) + \\
 & + x_t^{Kr}(r_t^{Kr} - \omega'(s_{t+1} - s_t) + \ln I_t - \ln I_{t+1} - r_t^{\$}) + \\
 & + r_t^{\$} - \alpha'(\pi_{t+1} - (s_{t+1} - s_t)) \\
 & - \alpha^{Kr}(\pi_{t+1}^{Kr} - \omega'(s_{t+1} - s_t) + \ln I_t - \ln I_{t+1}) \\
 & - (1 - \alpha' - \alpha^{Kr})\pi_t^{\$}] \quad (7.8)
 \end{aligned}$$

Following Frankel (1982) we then assume that the representative investor chooses portfolio shares in order to maximize a function of the mean and variance of next period's wealth. The variance of  $W_{t+1}$  is

$$\begin{aligned}
 V(W_{t+1}) = & W_t^2 \{ [-x'_t + \alpha' - (x_t^{Kr} - \alpha^{Kr})\omega'] \Omega [-x_t + \alpha - (x_t^{Kr} - \alpha^{Kr})\omega] \\
 & + (x_t^{Kr} - \alpha^{Kr})^2 \sigma_I^2 \} \quad (7.9)
 \end{aligned}$$

where  $\Omega$  is a  $(m \times m)$  variance-covariance matrix of currency depreciation rates for all dollar exchange rates except the krona/dollar rate, and  $\sigma_I^2$  is the variance of  $\ln I_{t+1} - \ln I_t$ . Behind (7.9) is the assumption that

$$\text{cov}[(s_{t+1}^j - s_t^j), \ln I_{t+1} - \ln I_t] = 0 \quad \forall j \quad (7.10)$$

In other words, we assume that there is no covariance between international exchange rate changes and the shocks affecting the value of the currency index. This assumption can be justified on both theoretical and empirical grounds. Theoretically, an assumption of a (negative or positive) covariance between the index and the dollar value of other currencies would make a distinction between a currency basket exchange rate policy and a flexible exchange rate policy rather meaningless. In

effect, it could be argued that this covariance condition comes close to a definition of a currency basket policy. Empirically, it seems as if there is enough strength (and confidence) in the Riksbank's day-to-day actions to keep the index stable in spite of large fluctuations in important bilateral exchange rates.

Assuming asset supplies to be exogenously fixed, interest rate differences must in equilibrium satisfy:

$$r_t - r_t^{\$} - E(s_{t+1} - s_t) = \rho \Omega (x_t - \alpha + (x_t^{Kr} - \alpha^{Kr}) \omega) \quad (7.11)$$

$$\begin{aligned} r_t^{Kr} - r_t^{\$} - E(s_{t+1}^{Kr} - s_t^{Kr}) &= \rho \omega' \Omega (x_t - \alpha + (x_t^{Kr} - \alpha^{Kr}) \omega) \\ &+ \rho (x_t^{Kr} - \alpha^{Kr}) \sigma_I^2 \end{aligned} \quad (7.12)$$

where  $\rho$  is the coefficient of relative risk aversion and

$$E(s_{t+1}^{Kr} - s_t^{Kr}) = \omega' E(s_{t+1} - s_t) + E(\ln I_{t+1}) - \ln I_t \quad (7.13)$$

Condition (7.11) is the equilibrium relation derived by Frankel (1982) except that in his model  $\omega$  is a null-vector as there is no country pursuing a currency basket policy. It gives the relation between all the non-krona interest rates and the dollar interest rate. As the right-hand side (by definition) is the risk premium, we see that UIP will obtain either if the representative investor is risk neutral ( $\rho=0$ ) or if there is no randomness in exchange rate changes ( $\Omega=0$ ). Otherwise, there will be risk premia and the signs and magnitudes of these will depend, in addition to  $\rho$  and  $\Omega$ , on  $(x_t - \alpha)$ , the difference between the outstanding supply of assets in a given currency and the consumption share of that currency. This dependence is due to the fact that the greater the difference between portfolio and consumption shares, the greater is the uncertainty about the real value of the currency, other things equal. Whether there will be a premium or a discount depends not only on  $(x_t - \alpha)$ , however, but also on the covariance structure in  $\Omega$ .

One implication of this mean-variance optimization model is that, as long as investors are risk averse, changes in relative asset supplies ( $x$ ), e.g., via open market operations, affect the risk premia. There would thus be a direct channel through which monetary policy could influence the relative returns on domestic and foreign assets.<sup>3</sup>

Substituting (7.11) into (7.12) we get the risk premium on assets in kronor vs. dollar assets as

$$r_t^{Kr} - r_t^{\$} - E(s_{t+1}^{Kr} - s_t) = \omega'(r_t - r_t^{\$} - E(s_{t+1} - s_t)) + \rho(x_t^{Kr} - \alpha^{Kr})\sigma_I^2 \quad (7.14)$$

We see that this risk premium is determined by two factors: the risk premia between all other currencies and the dollar and the uncertainty about the central bank's exchange rate policy. Thus, even if the krona were rigidly fixed to an index of all other currencies, there would still be a risk premium between the krona and the dollar. This latter proposition is a general result which does not depend on the features of the specific asset pricing model used here, as it can be derived from the definitions of the currency index and the risk premium.

Another exercise will be illuminating. Rewrite (7.14), using (7.13), to obtain the risk premium on the krona relative to the currency basket weighted portfolio:

$$r_t^{Kr} - \omega'r_t - (1-\omega')r_t^{\$} - E(\ln I_{t+1} - \ln I_t) = \rho\sigma_I^2(x_t^{Kr} - \alpha^{Kr}) \quad (7.15)$$

This expression confirms our previous assertion that the variability of the index is crucial for the interest rate relation. If the currency index is fixed ( $E(\ln I_{t+1} - \ln I_t) = \sigma_I^2 = 0$ ), a perfect hedge against exchange rate fluctuations can be created.<sup>4</sup> Since there is no risk on a basket weighted

position over and above that of a Swedish asset (remember that the consumption pattern and commodity prices are non-stochastic in this model), (7.15) is turned into an arbitrage relation. Hence, there will be no excess return on Swedish assets in comparison to a currency basket portfolio and complete interest rate dependence. The only difference then between a basket system and a regular fixed exchange rate is that the domestic interest rate has to equal a weighted average of interest rates rather than a specific foreign rate.

The conclusion that a basket weighted foreign currency position is insulated from exchange rate risk (assuming  $E(\ln I_{t+1} - \ln I_t) = \sigma_I^2 = 0$ ) may seem inconsistent with the result that there are risk premia in bilateral relations. This seeming paradox can be easily resolved, however. To take advantage of the protection offered by the currency basket, the investor has to borrow (or lend) exactly in the proportions given by the index weights. If an individual foreign interest rate is lower than the domestic rate but the condition that

$$r_t^{Kr} - \omega' r_t - (1 - \omega' \iota) r_t^S = 0 \quad (7.16)$$

holds, some other interest rate must be higher than  $r_t^{Kr}$ . As the investor would have to borrow in both these currencies in order to be protected he cannot exploit the bilateral interest rate differential without taking on some risk. Hence, he cannot use hedged positions to eliminate bilateral interest rate differences as long as (7.16) holds. There is therefore no presumption that all risk premia will be eliminated and international interest rate equalization obtain just because one country ties its exchange rate to a currency basket.

In this model, it is the variability in the currency index that (possibly) gives the central bank some independence. The choice to let the currency index fluctuate within a rather

wide range around its benchmark value can therefore be interpreted as an attempt to maintain some latitude for an independent monetary policy.

Given the opportunity for diversification of exchange risks that a currency basket creates, a test for a risk premium that is specific to the Swedish krona should not be performed on bilateral parity relations. Instead, the parity condition should be expressed in terms of an index of foreign interest and exchange rates, i.e., UIP implies

$$r_t^{Kr} = \omega' r_t + (1-\omega') r_t^{\$} + E(\ln I_{t+1}) - \ln I_t \quad (7.17)$$

An advantage is that a test based on the currency index can be expected to have considerably higher power than a bilateral test as the variability of the currency index is likely to be substantially lower than in the individual exchange rates and the forecast error variance is therefore expected to be smaller (cf. the discussion in Section 6.4).

### 7.3 EMPIRICAL TESTS

In recent years there has been a surge of empirical work on international interest rate relations, both uncovered interest parity and real rate parity. The studies include both basically atheoretical tests of whether these parity conditions hold (e.g., Cumby and Obstfeld (1984), Fama (1984), and Mishkin (1984)) and tests of structural models to explain parity deviations (e.g., Frankel (1982), Hansen and Hodrick (1983), and Hodrick and Srivastava (1984)). In general, as concluded in Chapter 6, recent studies have tended to reject the UIP condition, but it has proved more difficult to explain the implied risk premia in terms of asset pricing models.

In this section, we apply the atheoretical approach used by Fama (1984) to test whether there is a risk premium on the

Swedish krona. This test exploits the equivalence of UIP and unbiasedness which obtains under the assumption of CIP. The unbiasedness test has the advantage of requiring data only on spot and forward exchange rates.

We argued in Section 7.2 that the relevant risk premium can be identified only by comparing the return on an asset denominated in kronor with that on a particular portfolio of foreign assets. This is a portfolio that replicates the composition of the official currency basket. Consequently, we are not primarily interested in whether a specific spot exchange rate can be predicted without bias from its forward rate. The premium in this relation will, as demonstrated in Section 7.2, largely be determined by the conditions in the international foreign exchange markets.

### 7.3.1 The data and the formulation of the tests

The official Swedish currency basket consists of fifteen currencies. Their index weights correspond to the relative shares in total Swedish foreign trade, i.e., exports and imports, of the respective countries. The countries included are those whose trade shares exceed one percent and also have convertible currencies. The weights are changed on April 1 each year to reflect changes in the trade pattern.<sup>5</sup> When the currency basket was instituted in August 1977, the benchmark value of the index was set at 100. After the two devaluations in September 1981 and October 1982, the Riksbank has set the benchmark at 132.<sup>6</sup> In June 1985, it also announced bounds of  $\pm 2$  units ( $\pm 1.5$  percent) around the benchmark outside of which the currency index would not be permitted to move.

Table 7.1 shows some descriptive statistics for three krona exchange rates and for the official currency basket. The relative change in "Index" is the percentage change in the value of the currency basket, i.e.,

$$s_{t+1}^I - s_t^I \equiv \ln I_{t+1} - \ln I_t \quad (7.18)$$

Table 7.1. Krona exchange rates, January 1980 - December 1985

	Autocorrelations						Jan 80 - Aug 81		Oct 81 - Sep 82		Nov 82 - Dec 85	
Country	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.
<u><math>s_{t+1} - s_t</math></u>												
West Germany	0.18	0.02	-0.01	-0.06	0.35	2.67	-0.60	1.52	0.26	1.04	0.14	1.29
U.S.A.	0.04	-0.01	0.14	0.13	0.86	3.08	1.17	2.51	0.98	2.05	0.08	2.26
United Kingdom	-0.07	-0.04	-0.06	-0.02	0.21	2.96	-0.11	2.48	0.49	1.42	-0.34	2.40
Index	0.11	-0.09	-0.06	-0.04	0.34	2.08	-0.04	0.35	0.11	0.50	-0.00	0.49
<u><math>f_t - s_{t+1}</math></u>												
West Germany	0.19	0.03	0.00	-0.04	0.86	3.08	1.04	1.63	0.07	1.09	0.46	1.25
U.S.A.	0.08	0.02	0.17	0.16	-0.70	3.14	-1.14	2.50	-1.01	1.97	0.20	2.38
United Kingdom	-0.06	-0.03	-0.04	-0.01	-0.11	3.02	-0.09	2.68	-0.51	1.49	0.51	2.44
Index	0.14	0.00	-0.03	-0.01	-0.18	2.12	0.20	0.60	-0.14	0.57	0.23	0.52
<u><math>f_t - s_t</math></u>												
West Germany	0.57	0.43	0.44	0.36	0.51	0.24	0.46	0.31	0.32	0.19	0.60	0.16
U.S.A.	0.70	0.47	0.38	0.27	0.15	0.29	0.03	0.35	-0.03	0.23	0.28	0.21
United Kingdom	0.65	0.49	0.34	0.23	0.10	0.25	0.02	0.39	-0.02	0.23	0.17	0.12
Index	0.66	0.45	0.39	0.28	0.16	0.25	0.15	0.32	-0.03	0.23	0.22	0.18



The data are monthly observations from January 1980 to December 1985, i.e., a total of 72 observations. The exchange rate quotations are end-of-day buying rates, spot and thirty-day forward, on the last trading day of the month, obtained from SEB International and Götabanken. Data concerning the official currency basket were obtained from the Riksbank.

These figures can be compared to those reported by Fama (1984, Table 5) on nine dollar exchange rates (not including the Swedish krona). He notes that the standard deviations of  $f_t - s_{t+1}$  are larger than those of  $s_{t+1} - s_t$  in his data. This indicates that "the current spot rate is a better predictor of the future spot rate than the forward rate" (p. 323). As can be seen from Table 7.1, this pattern is present also in the Swedish data. This is also consistent with the findings of Meese and Rogoff (1983), who show that the current spot rate outperforms a number of standard models of exchange rate determination as predictors of the future spot rate, including the forward rate.

When studying the Swedish data, one should look at the time periods before, between, and after the two devaluations in September 1981 and October 1982 separately. For example, from the whole period since January 1980 it does not seem as if the currency basket's index has been much more stable than the bilateral exchange rates. On the other hand, once the effects of the devaluations are eliminated, it can be seen that the standard deviation of the change in the index is much less than that of either bilateral exchange rate.

Evidently, there is autocorrelation in the forward premia ( $f_t - s_t$ ) in Table 7.1. Exchange rate changes do not seem to be autocorrelated, however, nor do the differences between forward rates and future spot rates. This may seem surprising, since

$$f_t - s_t = (s_{t+1} - s_t) + (f_t - s_{t+1}) \quad (7.19)$$

i.e., the sum of two variables that show no sign of autocorrelation is highly correlated over time. The pattern is the same in Fama's data. Noting that from definitions of the forecast error,

$$\varepsilon_{t+1} = s_{t+1} - E(s_{t+1}) \quad (7.20)$$

and the risk premium,

$$p_t = f_t - E(s_{t+1}) \quad (7.21)$$

(7.19) can be written as

$$f_t - s_t = [E(s_{t+1} - s_t) + \varepsilon_{t+1}] + (p_t - \varepsilon_{t+1}) \quad (7.22)$$

Fama suggests that the autocorrelation of the risk premium and/or the expected changes in the exchange rate, which show up in the forward premium, is dominated in the time series of  $(s_{t+1} - s_t)$  and  $(f_t - s_{t+1})$  by the high variability of the forecast error  $(-\varepsilon_{t+1})$ . This means that the observation that the difference between the forward rate and the realized spot rate is white noise does not warrant the conclusion that there is no risk premium.

We will make a regression test for the existence of a (possibly time-varying and autocorrelated) risk premium on the currency basket. Specifically, we test the hypothesis of unbiasedness of the forward rate as a predictor of the future spot rate. As shown in Chapter 6, this is equivalent to a test of UIP under the maintained hypothesis of CIP.

Following Fama (1984), we test this hypothesis by adopting a first difference formulation:<sup>7</sup>

$$s_{t+1} - s_t = \alpha_1 + \beta_1(f_t - s_t) + \epsilon_{t+1} \quad (7.23)$$

where UIP implies the null hypothesis  $[\alpha_1, \beta_1] = [0, 1]$ . Under rational expectations  $E(\epsilon_{t+1}) = 0$  and (7.23) can be consistently estimated by OLS. This is thus a joint test of unbiasedness and rational expectations.

Hodrick and Srivastava (1986) point out that under the alternative hypothesis that there is a time-varying risk premium, there may be a bias in the standard errors obtained from OLS estimates of this equation. However, they also argue that this problem is unlikely to be severe, a conjecture that is verified as results from techniques that are not subject to this bias confirm Fama's findings. We will thus use the simpler OLS technique.

We are primarily interested in the risk premium on the currency basket, i.e., when  $s_t$  and  $f_t$  refer to indices of spot and forward rates, but for comparison results from bilateral tests will also be reported.<sup>8</sup> As the variance of the dependent variable ( $s_{t+1} - s_t$ ) is much higher than the variance of the regressor ( $f_t - s_t$ ), the tests can be expected to have low power. However, we expect the test in terms of the basket to have higher power as the variability in the index is much lower than in any of the individual currencies.

### 7.3.2 Regression results

Our regression results are summarized in Table 7.2, where we report the estimates from equation (7.23) for the German Mark, the U.S. dollar, the British pound, and the Swedish currency basket index. The estimation period is November 1982 to December 1985. The reasons for studying only this period are twofold. First, evidence presented by McPhee (1984) indicates that the CIP arbitrage condition was violated prior

to 1982 even on major currencies. Consequently, the unbiasedness tests for that period cannot be interpreted as tests of UIP which is our main concern.

Second, and this is the reason we start in November 1982, we want to avoid the effects of the devaluation in October that year. A discrete change in the benchmark value of the currency index is an extraordinary event that may obscure the results on the "normal" relation between forward and spot rates. The occurrence of devaluations within the currency basket system may nevertheless affect our tests, an issue that we will return to below.

Table 7.2. OLS regressions, November 1982 - December 1985

$$s_{t+1} - s_t = \hat{\alpha}_1 + \hat{\beta}_1(f_t - s_t) + \hat{\epsilon}_{t+1}$$

Country	$\hat{\alpha}_1$	$\hat{\beta}_1$	$s(\hat{\alpha}_1)$	$s(\hat{\beta}_1)$	$R^2$	$s(\hat{\epsilon})$	DW
West Germany	-0.012	2.219	0.008	1.260	0.08	0.013	1.76
U.S.A.	0.017*	-5.999*	0.005	1.511	0.30	0.019	2.48
United Kingdom	0.008	-6.432*	0.007	3.189	0.10	0.023	1.91
Index	0.026	-0.139*	0.128	0.454	0.03	0.491	1.51

\* indicates that the coefficient is significantly different from its value under the null hypothesis ( $\alpha_1 = 0$  and  $\beta_1 = 1$ ) at the 5 per cent level.  
 $s(\ )$  denotes standard error of estimate.

Looking first at the estimates for the bilateral exchange rates, which are most easily compared to previous results, we find significant deviations from unbiasedness for the dollar and the pound. The  $\beta_1$  coefficients in these equations are also markedly negative, which, as demonstrated in Chapter 6, indicates that the variance in the risk premium exceeds the variance in the expected rate of depreciation of the exchange rate in question. This is thus consistent with there being a time varying risk premium. In these respects our results

are similar to Fama's (1984), although our estimates are far greater in absolute values than his. He does not, incidentally, include the dollar/krona exchange rate which means that our result complements his study by adding a tenth currency, albeit for a different time period.

The results for the German Mark are quite different and although the  $\beta_1$  estimate is not numerically close to unity, we cannot reject unbiasedness. However, neither can we show that it is significantly different from zero, which illustrates the imprecision and power problem inherent in these tests.

These results can also be compared to those presented for krona exchange rates by Oxelheim (1985). He analyzes nonoverlapping three month forward rates, 1974-84, for five currencies, namely, those in Table 7.2 plus the Swiss franc and the Japanese yen. In regressions comparable to ours (Table 4.27, p. 186)<sup>9</sup>, he can in no case reject unbiasedness. The point estimates of  $\beta_1$  are all positive, but only for the pound is it significantly different from zero. This gives an additional illustration of the power problem in bilateral tests, but the reasons for the marked differences between his estimates and ours are difficult to determine. Apart from using a different observation interval, it should be noted that Oxelheim has five devaluations and the shift to the currency basket system in 1977 in his sample period. A priori it seems likely that this should increase the errors and tend to reduce the forecast power. What we find, however, is point estimates that are closer to unity and standard errors of estimates that are considerably smaller than ours.<sup>10</sup> It is thus possible that there has been a change in the relations between the krona and, in particular, the dollar and the pound in recent years. Our analysis of the currency basket system demonstrates, however, that this need not be due to domestic events as the bilateral relation is dominated by the internationally determined premium (or discount) on the currency in question.

This brings us to the more interesting test of the ability of a basket weighted index of forward premia to predict the future relative change in the currency index. The results are reported in the bottom line of Table 7.2. The  $\beta_1$  estimate is significantly less than unity (although not significantly different from zero), which indicates that the forward premia do not provide an unbiased prediction of the index. The coefficient of determination is far from impressive and considerably smaller than in the bilateral regressions. This reflects the fact that whereas there is a relatively stable relationship between bilateral forward rates and future spot rates<sup>11</sup>, albeit for the dollar and the pound not at all the relation predicted by the unbiased expectations hypothesis, the index changes are more or less independent of observed forward premia,  $\beta_1$  being close to zero both numerically and statistically.

The fact that the point estimate of  $\beta_1$  is so small in absolute value means that we cannot make the type of inference concerning the relative variances of the risk premium and the expected change in the spot rate that Fama (1984) emphasizes. There is some indication that the former is greater as in Fama's tests, but the evidence is weak.<sup>12</sup>

The result is consistent with the existence of a risk premium on the krona. This finding is not surprising, of course, given the actual variability in the index. A necessary condition for Sweden to have some latitude for an independent monetary policy would thus be fulfilled. The fact that we obtained mixed results in the bilateral tests illustrates our point that these may be of limited interest for the question of monetary independence in a country pursuing a currency basket policy.

It should be stressed that the fact that forward exchange rates do not predict future spot rates without bias, need not be interpreted as evidence in favor of the existence of risk premia. Such an interpretation seems reasonable when CIP is upheld, but if CIP is violated the unbiasedness tests have no direct implications for UIP. The question arises what unbiasedness tests (on bilateral exchange rates and on the currency basket) tell about the scope for monetary policy if CIP holds between the krona and some (important) currencies but not between the krona and other (minor) currencies.

Finally, some statistical caveats relating to our tests must be pointed out. In particular, one may have some doubts about whether the innovations in the currency index are homoskedastic and normally distributed as implicitly assumed in our OLS regression.<sup>13</sup> As the Riksbank's policy is to keep the index within a certain band around the benchmark value, the normality assumption may not be justified when the index is close to one of the limits (and when the limits are credible). Experience should also have taught market participants that there is a non-negligible risk of a discrete change in the benchmark value in the form of a devaluation.

However, it can also be argued that these problems are not so severe in the period under study. First, the bounds for the currency index were not publicly announced until June 27, 1985. Prior to that, the central bank had been working with a secret band which means that market participants could never know for sure that the index could go only in one direction. Moreover, the index value has never been on or even very close to either of the limits in the more recent part of the observation period.

Second, the devaluation in October 1982 was announced as the definitely last attempt to cure the imbalances in the Swedish

economy by means of exchange rate changes. Although such an announcement in itself is of little informative value, it seems to have gained considerable credibility, at least for the near term future. As we are studying one month ahead forward rates, it can thus be hoped that our estimates are not affected by these factors.

#### 7.4 CONCLUDING COMMENTS

Our study shows that there are theoretical reasons to believe that investors' risk aversion should drive a wedge between domestic and foreign exchange rates in a currency basket system of the Swedish type. In the empirical test we find evidence consistent with this hypothesis.

This is, of course, far from sufficient to show that Swedish monetary policy can be effectively used to control the domestic interest rate level. To introduce a risk premium, all one has to do is let the currency index vary sufficiently, but it is an entirely different matter whether one can also influence the size of that premium in any useful sense. The questions of whether this premium can and should be exploited for stabilization policy purposes and the degree of monetary independence under imperfect capital mobility deserve further attention, but require more elaborate models and tests than those considered here and must therefore be left for future work.



## NOTES

1. The currencies' weights in the basket are basically foreign trade shares, except that U.S. dollar has been given a weight which is twice the share of U.S.-Sweden trade in Sweden's total exports and imports. The weights are changed once a year and reflect the trade pattern of the preceding five-year period. Since the annual changes in weights can be fairly well predicted, they add no uncertainty of theoretical interest, although they have to be taken into account in empirical work.
2. Our definition of the index does not give a completely accurate description of the Swedish currency basket, for two reasons. First, the official index is not defined as a geometric average but as an arithmetic average. Second, the weights are not constant. To take account of the changes in weights we should write

$$\ln I_t \equiv \left( s_t^{Kr} - s_{o,t}^{Kr} \right) - \omega'_t \left( s_t - s_{o,t} \right) \quad (i)$$

$$\ln I_{t+1} \equiv \left( s_{t+1}^{Kr} - s_{o,t+1}^{Kr} \right) - \omega'_{t+1} \left( s_{t+1} - s_{o,t+1} \right) \quad (ii)$$

where it can be seen that the base period is changed every time the weights change. This is to guarantee that the changes in weights do not affect the index as long as spot rates are constant, i.e., that

$$s_t^{Kr} - s_{o,t+1}^{Kr} - \omega'_{t+1} \left( s_t - s_{o,t+1} \right) = \ln I_t \quad (iii)$$

Using (i)-(iii) we can rewrite (7.9) as

$$s_{t+1}^{Kr} - s_t^{Kr} = \omega'_{t+1} \left( s_{t+1} - s_t \right) + \ln I_{t+1} - \ln I_t$$

3. Frankel (1982) makes a structural test of this model by estimating  $\rho$ . He cannot reject the hypothesis  $\rho=0$ , but the power of his test is quite low. The empirical importance of this connection between asset supplies and premia is thus an open question.
4. A portfolio with the same composition as the currency basket is a "perfect hedge" in the sense that it is no more risky than an asset denominated in the Swedish krona. It will not be optimal for an investor to put all his wealth in such a portfolio unless he (i) consumes only Swedish goods, and (ii) is extremely risk averse.
5. Cf. footnotes 1 and 2. For further discussions about the Swedish currency basket, see Franzén, Markowski, and Rosenberg (1980) and Franzén and Rosenberg (1983).
6. In 1981 and 1982 the krona was devalued relative to the index by 10 and 16 percent, respectively. Hence, the average foreign currency price of the krona should have dropped to 75.6 percent ( $0.9 \cdot 0.84 \cdot 100$ ) of the price in August 1977. The average value of foreign currency in terms of kronor, and the benchmark value of the currency basket, is therefore  $100/0.756 = 132.2$ .
7. As pointed out in Chapter 6, a first difference formulation is preferable to a test in level form because of possible stationarity problems in exchange rate levels; cf. Hansen and Hodrick (1980).
8. Previous studies of basket related currencies, e.g., Oxelheim (1985) of the Swedish krona and Haaparanta and Kähkönen (1985) of the Finnish mark, have only considered bilateral exchange rate relations.

9. Oxelheim (1985) also reports tests of the ability of the forward rate to predict the level of the spot rate. As we pointed out above (cf. footnote 7), there may be problems in interpreting results from this formulation.
10. Oxelheim's (1985)  $\beta_1$  estimate of 0.78 in the krona/dollar regression is also in contrast to the generally negative coefficients obtained by Fama (1984) for other dollar exchange rates in a sample period similar to Oxelheim's.
11. In particular, the slope coefficients are significantly different from zero in all the bilateral regressions, although only at the 10 per cent level for the German Mark. In this sense, the forward premia contain information about future spot rate changes even though they are not unbiased predictors.
12. For a theoretical analysis of this finding, see Hodrick and Srivastava (1986).
13. The assumption of variance stationarity is important not only for our test on the currency basket but for all tests of this type. This is emphasized by Krasker (1980), who argues that this assumption may be incorrect. See also Hodrick and Srivastava (1986).

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## 8. The Term Structure of Interest Rates in the Swedish Money Market\*

### 8.1 INTRODUCTION

In the theoretical analysis of monetary policy and financial markets, it is often useful to study the determination of some specific interest rate(s) or "the level of interest rates" in some generic sense. The previous chapters in this book give several examples of this approach.

In practice, however, there is a whole spectrum of interest rates on securities with varying times to maturity, i.e., a term structure of interest rates. This is acknowledged in Chapters 3 and 4 where a distinction is made between the overnight rate in the interbank market and some representative money market rate on assets with, say, 30 or 90 days to maturity. However, as emphasized there, the static nature of the models means that it is impossible to explicitly analyze the intertemporal links between these two interest rates. While a model with two assets is no more limited than the standard practice of letting one interest rate represent the whole term structure, which implicitly assumes that the yield curve always shifts uniformly, a more careful analysis of these issues is certainly called for.

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\* This chapter is based on results from a joint project with Ingrid Werner.

The fact that a given disturbance or policy intervention may have different effects on short and long term rates means that the term structure relation is important for the effects of monetary policy. For example, it is often the case that the central bank can exert a strong influence on the very short rates, in particular the overnight rate with the help of discount window policies, as discussed in Chapter 3. However, these rates are often of limited importance for both intermediate and ultimate targets of monetary policy. Financial stocks and flows depend primarily on one to six month rates and aggregate demand on even longer rates, at least in a conventional Keynesian analysis (cf. Clarida and Friedman (1983)). The behavior of the term structure is therefore relevant both for monetary control and (possibly) for the effects on ultimate targets of monetary policy.

An integration of the term structure relation into a money market model would therefore be an important step in a richer analysis of monetary control and monetary policy. Unfortunately, this is also a highly complex problem and this step will not be attempted here as our purposes are more modest. The emphasis in this chapter is on a partly exploratory empirical study of the properties of the term structure of interest rates in the Swedish money market. This relation has not previously been subjected to any systematic investigations. Our prime concern is the information content in the term structure, in particular, the extent to which the forward interest rates that are implicit in the term structure can be used to forecast future interest rates.

Conceptually, the implicit forward rate can be decomposed into two components, one measuring the expected future spot interest rate and the other the expected premium (excess return) on a multi-period security. Being expectations, neither of these is directly observable, but by studying the predictive



power of forward rates we can learn something about their relative importance. The better the predictions produced by forward rates, the smaller the role of the premium component, and vice versa. Our study is not motivated by a concern for interest rate forecasting, per se, but by the belief that information about the empirical roles of premia and spot rate expectations is important for understanding the determination of the term structure and, thus, as a guideline for future work on these issues.

The chapter is organized as follows. Section 8.2 gives some background material on the theory of the term structure of interest rates and common hypotheses concerning the role of risk premia in this relation. Section 8.3 contains a brief presentation of the data, and in Section 8.4 the predictive power of forecast rates is compared to that of another naive forecasting technique, the simple martingale model. Regression tests of the information content in forward interest rates, inspired by Fama (1984), are presented in Section 8.5 and the final section gives some concluding comments and directions for future work.

## 8.2 EXPECTATIONS AND RISK PREMIA IN THE TERM STRUCTURE OF INTEREST RATES

Most models of the term structure of interest rates agree that expectations of future short term rates are an important ingredient. In its extreme version, this assertion is formulated as the pure expectations hypothesis which states that expected future spot rates uniquely determine the term structure.<sup>1</sup>

This general hypothesis can be expressed in several different ways and it is useful to start by introducing some notational conventions. Assume continuously compounded interest<sup>2</sup> and let

- $Q_{\tau_t}$  = the price at time  $t$  of a zero coupon bond that pays 1 krona at time  $t+\tau$ ,
- $R_{\tau_t}$  = the rate on a  $\tau$ -period bond observed at time  $t$ ,  
i.e.,  $R_{\tau_t} \equiv -\frac{\ln Q_{\tau_t}}{\tau}$ ,
- $R1_{t+\tau-1}$  = the spot (one-period) rate, observed at  $t+\tau-1$ ,  
of a bond that matures at  $t+\tau$ ,
- $H_{\tau_{t+1}}$  = the holding period yield of a  $\tau$ -period bond  
purchased at  $t$  and sold at  $t+1$ , i.e.,  
 $H_{\tau_{t+1}} \equiv \ln \frac{Q_{(\tau-1)_{t+1}}}{Q_{\tau_t}}$ . By definition,  
 $H1_{t+1} \equiv R1_t$ , the spot rate observed at time  $t$ ,
- $F_{\tau_t}$  = the (implicit) forward (one-period) rate, observed  
at time  $t$ , i.e.,  $F_{\tau_t} \equiv \ln \left( \frac{Q_{(\tau-1)_t}}{Q_{\tau_t}} \right)$ ,
- $P_{\tau_{t+1}}$  = the ex post premium (excess return) on a  $\tau$ -period  
bond held from  $t$  to  $t+1$  relative to a one-period  
bond held over the same interval, i.e.,  
 $P_{\tau_{t+1}} \equiv H_{\tau_{t+1}} - R1_t$ .

In a non-stochastic setting, the term structure problem is trivial, but it is nevertheless useful to take this case as a starting point. The various forms of the expectations hypothesis can then be seen as reinterpretations of results derived under certainty.

The assumption of no uncertainty implies in particular that future bond prices are known. Hence, the holding period yield of any bond can be computed at time  $t$ . Absence of arbitrage opportunities therefore requires that

$$H\tau_{t+1} = R1_t \quad \forall \tau > 1 \quad (8.1)$$

i.e., the holding period yield on any multi-period bond must equal the yield on a one-period bond.

A second arbitrage relation is between the return on a multi-period bond and the return on rolling over several one-period bonds. For example, in the case of a two-period bond

$$\frac{1}{Q2_t} = \frac{1}{Q1_t \cdot Q1_{t+1}} \quad (8.2)$$

or, in terms of interest rates,

$$2R2_t = R1_t + R1_{t+1} \quad (8.3)$$

which shows that the yield on a multi-period bond is a simple average of the one-period rates under its time to maturity.

Finally, consider the relation between (implicit) forward rates and future spot (one-period) rates. A forward transaction at time  $t$  can be made by buying a two-period bond and, to finance this, sell short (issue) one-period bonds worth  $Q2_t$ . The net investment in the first period is thus zero. At  $t+1$ , the short position is covered which costs  $(Q1_t/Q2_t)$  and at  $t+2$  one krona is obtained for the maturing two-period bond. This is a perfect alternative to a one-period investment between  $t+1$  and  $t+2$ , which implies that zero arbitrage requires

$$R1_{t+1} = \ln \left[ \frac{Q1_t}{Q2_t} \right] \equiv F2_t \quad (8.4)$$

The example above considered a two-period case, but as this must hold for any  $\tau$ ,<sup>3</sup>

$$F\tau_t = R1_{t+\tau-1} \quad (8.5)$$

In words, (8.5) says that under certainty the (one-period) forward rates that are implicit in the term structure at time  $t$  must equal the future spot rates.<sup>4</sup>

In the certainty case, we can therefore conclude that the term structure of interest rates is uniquely determined by the future spot rates. Hence, an upward sloping yield curve would indicate that interest rates in future periods will be higher, and vice versa.

While illuminating in some ways, the analysis of the term structure of interest rates under perfect foresight is of little practical relevance. Future spot interest rates are not known and, hence, uncertainty is at the very heart of the problem.

The procedure leading to the expectations hypothesis in its various forms is to replace the future interest rates and prices with their expectations. For example, (8.5) combined with rational expectations leads to the unbiased expectations hypothesis:

$$F\tau_t = E_t[R1_{t+\tau-1}] \quad (8.6)$$

i.e., that the forward rate is equal to the mathematical expectation of the future spot rate. This says that the term structure is uniquely determined by the market's expectations of the future spot rates.

Similarly, (8.1) is reinterpreted as

$$E_t[H\tau_{t+1}] = R1_t \quad (8.7)$$

which says the expected excess return on holding a multi-period bond is zero, or equivalently,

$$E_t[P\tau_{t+1}] = 0 \quad (8.8)$$

This may be referred to as the holding period yield hypothesis.

The replacement of a known future interest rate with its expectations may seem like a straightforward operation, but it is not without problems. In fact, it can be shown that Jensen's inequality means that the seemingly equivalent formulations of the pure expectations hypothesis are mutually exclusive.<sup>5</sup> The traditional critique of the pure expectations hypothesis, however, is based on the more obvious fact that it ignores the risk that an agent must take on if, for example, the expected holding period yield of long bonds is to be driven to equality with the certain yield of a one-period bond. Given that investors can be expected to require some compensation for bearing risk, (8.6) is modified to read<sup>6</sup>

$$F\tau_t = E_t[R]_{t+\tau-1} + E_t[P\tau_{t+1}] \quad (8.9)$$

where  $E_t[P\tau_{t+1}]$  is the expected excess return on a  $\tau$ -period bond from  $t$  to  $t+1$ , i.e.,  $E_t[P\tau_{t+1}] = E_t[H\tau_{t+1}] - R]_t$ .<sup>7</sup>

There are several hypotheses in the literature that postulates the existence of such a risk (or term) premium, although they differ in their predictions of the sign of the premium. One of these is the liquidity preference hypothesis, originated by Hicks (1946). Under this hypothesis, the premium is expected to be positive, i.e., the forward rate would systematically tend to overpredict the future spot rates. This view is based on the following reasoning.

Consider an agent with a one-period investment horizon. He can obtain a predetermined rate of return by buying a one-period bond. If he chooses a multi-period debt instrument, he will have to sell it at the end of the period and faces the risk of making a capital loss. In order to bear this risk,

he will require a premium. To turn this story into a theory of the term structure, the simple portfolio allocation model is combined with the assumptions that all investors are of this type, i.e., they require a premium to invest long, and that all borrowers in the market prefer to issue long-term debt. This implies that if  $P\tau_{t+1} = 0$  for all  $\tau$ , there would be an excess supply of multi-period debt. Therefore, in equilibrium there must be positive risk premia in the term structure to induce the investors to hold the outstanding stock of debt. Moreover, the premia are expected to be higher the longer the maturity of the bond.

The implication of the liquidity preference hypothesis is thus that the yield curve is upward sloping even if the expected future spot rates are constant. Only if the expected fall in the interest rates is large enough to offset the premia would the yield curve be negatively sloped.

A somewhat peculiar feature of the liquidity preference theory is the very strong assumptions about lender and borrower behavior. For example, no real motivation is given for why investors should all prefer to invest short. This type of objections led to the formulation of a third hypothesis of which the liquidity preference theory is a special case. It is the preferred habitat hypothesis of Modigliani and Sutch (1966). They argue that different types of investors have different investment horizons and that a premium is required to induce them to invest in assets with maturities that deviate from their "habitats".

Whether a positive or negative premium will arise in equilibrium depends on the relative distributions of investors and outstanding debt along the time axis. The premia will tend to be positive if there is a surplus of long-term assets relative to the number of investors who have long planning horizons and negative if the short-term assets dominate. An interesting

implication of this is that changes in the composition of the outstanding debt, for example, via open market operations, may influence the form of the yield curve. Contrary to the liquidity preference hypothesis, the preferred habitat hypothesis gives no prediction as to the sign of the risk premium in the term structure.

These comments on the theory of the term structure are necessarily brief and superficial and are only intended to give a hint of what Cox, Ingersoll, and Ross (1981) call the "traditional" hypotheses (the pure expectations, liquidity premium, and preferred habitat theories) and the intuitive motivation for considering expectations and risk premia as determinants of the term structure.<sup>8</sup>

It should be noted that, intuitively plausible as they may be, these hypotheses have been subject to strong critique on theoretical grounds by, in particular, Cox, Ingersoll, and Ross (1981).<sup>9</sup> They show that the expectations hypothesis is, by and large, incompatible with modern continuous time asset pricing theory and that major reformulations and re-interpretations of the other two are necessary. This critique notwithstanding, the "traditional" hypotheses continue to play important roles in the analyses of interest rate relations, not least in the empirical literature.

A recent example is Flavin (1984a), who studies the expectations hypothesis. In a reply to the critique that she has ignored the results of Cox, Ingersoll, and Ross (1981), Flavin (1984b) points out that although various representations of the expectations hypothesis are mathematically inconsistent they all have the same first-order Taylor series approximation.<sup>10</sup> Given the very widespread use of linear approximations in all areas of economics, not least in applied work, the empirical relevance of the problems expounded by Cox, Ingersoll, and Ross (1981) is not obvious.

In addition, the purpose of the current study is not to test the expectations hypothesis as such, but rather to investigate the roles of expectations and risk premia in the determination of interest rates in the Swedish money market. The empirical evidence against the pure expectations hypothesis is overwhelming,<sup>11</sup> but it is nevertheless interesting to study how well (or how poorly) forward rates predict future spot rates. This can give an indication of the relative importance of expectations and risk premia in this relation and these are bound to be crucial variables in any model of the term structure of interest rates.

### 8.3 THE DATA

The data used in this study are monthly end of period observations of the interest rates on bank certificates of deposit from the advent of the Swedish money market in March 1980 up until December 1985. Although not issued by the government, the structure of the Swedish banking system means that CDs are effectively default free. Quotations on 30, 60, 90, and 180 day rates are available for the whole period. Due to the fact that securities with longer maturities were not traded in a regular secondary market prior to 1984, we are restricted to the short end of the term structure. Specifically, we will study one and two months ahead 30 day forward rates and three months ahead 90 day forward rates (denoted by lower case letters). These are the only implicit forward rates that it is meaningful to compute on the basis of available data.

Table 8.1 contains some summary statistics - autocorrelations, means, and standard deviations - of the spot and forward interest rates and some additional relevant variables. These figures will be discussed in greater detail below, but we can note that both spot and forward rates follow mean stationary first-order autocorrelated processes. Autocorrelation also appears in the series that contain overlapping observations, for example,



$R1_{t+2} - R1_t$ , the two month change in the spot rate. A series of monthly observations of two month changes in an autocorrelated variable will tend to have some moving average characteristics, since a given innovation in the interest rate level will affect two consecutive observations.<sup>12</sup> A similar pattern occurs in the monthly observations of the quarterly rates in the bottom panel, where there is a double overlap. This phenomenon causes residual autocorrelation in some of the regressions reported below and we will therefore have reason to return to it.

It should be noted that Table 8.1 is based on observations of interest rates as they are quoted in the market, i.e., using discrete compounding. This is in contrast to most studies based on U.S. data, for example, Fama (1984), where the primary data are observations of bond prices, and interest rates are computed under the assumption of continuous compounding. As noted above (footnote 7), this assumption matters for the interpretation of the risk premium and in the tests that follow Fama (1984), reported in Section 8.5, we have recomputed the data to express interest rates in terms of continuously compounded rates.<sup>13</sup> This turns out to have marginal effects on the results, however. The summary statistics are virtually unchanged and Table 8.1 is therefore sufficient to describe both data sets.

It should also be pointed out that this data base is far from ideal. For example, it is not possible to make sure that the comparisons always involve future and spot rates with exactly the same maturity date. This implies that we are dealing with approximations and all results should be interpreted with this caveat in mind. The current data base is the best there is, however.

Table 8.1 Autocorrelations, means, and standard deviations: March 1980 to October 1985, N=68

Variable	Autocorrelations												Mean	Std. dev.
	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\rho_5$	$\rho_6$	$\rho_7$	$\rho_8$	$\rho_9$	$\rho_{10}$	$\rho_{11}$	$\rho_{12}$		
$R1_t$	0.76	0.63	0.44	0.28	0.14	0.08	0.07	0.02	0.04	0.05	0.06	0.06	13.134	1.6416
$R1_{t+1} - R1_t$	-0.24	0.14	-0.06	-0.04	-0.18	-0.10	0.10	-0.17	0.02	0.01	0.00	-0.00	- 0.004	1.1484
$R1_{t+2} - R1_t$	0.43	-0.01	-0.00	-0.20	-0.33	-0.19	-0.05	-0.14	-0.06	0.04	-0.00	0.06	- 0.002	1.4124
$F2_t$	0.78	0.60	0.46	0.30	0.15	0.05	0.04	0.00	-0.01	0.01	0.01	0.04	13.092	1.5600
$F3_t$	0.74	0.59	0.42	0.20	0.05	-0.04	-0.05	-0.08	-0.01	-0.000	0.03	-0.01	12.846	1.5230
$P2_{t+1}$	-0.07	0.02	0.05	-0.01	-0.15	-0.15	0.08	-0.15	-0.09	0.01	-0.04	0.06	- 0.048	1.1520
$P3_{t+1}$	-0.04	0.01	0.10	-0.02	-0.15	-0.16	0.10	-0.14	0.02	-0.02	-0.04	0.01	- 0.240	2.1240
$F2_t - R1_t$	0.02	0.14	0.07	0.04	0.07	0.03	0.02	0.03	0.06	-0.18	0.12	-0.14	- 0.048	0.4560
$F3_t - R1_t$	0.39	0.40	0.16	0.07	0.04	0.08	0.12	0.08	-0.01	-0.05	-0.11	-0.10	- 0.264	0.7680
$r3_t$	0.78	0.62	0.46	0.26	0.11	0.04	0.02	-0.03	0.01	0.04	0.04	0.04	13.162	1.5800
$r3_{t+3} - r3_t$	0.62	0.32	-0.09	-0.24	-0.34	-0.38	-0.24	-0.18	-0.04	-0.00	0.07	0.06	- 0.006	1.6564
$f6_t$	0.73	0.55	0.40	0.21	0.06	-0.04	-0.02	-0.06	-0.05	-0.000	-0.03	-0.01	12.704	1.1873
$p6_{t+3}$	0.62	0.33	0.01	-0.14	-0.25	-0.34	-0.23	-0.16	-0.08	0.01	0.04	0.12	- 0.431	1.5456
$f6_t - r3_t$	0.64	0.44	0.20	0.06	0.08	0.05	0.10	0.10	0.16	0.11	0.05	-0.04	- 0.443	0.5780

#### 8.4 THE PREDICTIVE POWER OF FORWARD RATES: A COMPARISON TO THE MARTINGALE MODEL

It was concluded in Section 8.2 that a basic ingredient in the theories of the term structure is that the implicit forward rates contain assessments of future spot rates. However, in general, they will also include other elements, in particular, the expected future premium in the return on multiperiod bonds, i.e., in the two-period case,

$$F2_t = E_t(R1_{t+1}) + E_t(P2_{t+1}) \quad (8.10)$$

Both the magnitude and sign of the premium term are uncertain, but it is clear that unless it is constant over time, this term will tend to reduce the predictive power of the forward rate. How serious a problem this is, is ultimately an empirical question and in this section we will study the accuracy of forward rates as predictors of future spot rates.

The forecasts obtained from the term structure relation will be compared to another simple forecast, the naive martingale model, i.e., that the interest rate next period is expected to equal the current one-period interest rate,

$$E_t(R_{t+1}) = R1_t \quad (8.11)$$

Another interesting comparison forecast would be to estimate a univariate time-series model. Since the spot rate in Table 8.1 is seen to follow a stationary AR(1) process, such a model could be expected to perform better than the martingale assumption, at least within sample. However, given the short observation period, a full scale comparison using an ARIMA model is impossible as we do not want to use within sample forecasts. Only results from the martingale comparisons will therefore be reported here, although an AR(1) model could be a relevant alternative for someone interested in practical interest rate forecasting.

If the martingale forecasts can be shown to outperform the forward rates as predictors of future spot rates which, e.g., is what Fama (1976) finds, this indicates either that the premium term is important or that the market does not make efficient use of (readily available) current information.

It should be noted that the assumption of complete rationality, which is necessary for the interpretation in terms of risk premia, may be regarded as rather strong given the very short history of the Swedish money market. It is not unreasonable to think that it takes time both for the market to "settle down" and for market participants to learn how the system works well enough to avoid systematic errors. This must be kept in mind when interpreting the results discussed below, but problems of this kind are inevitable in a study of a market that is still in its infancy.

The first comparison concerns the forecasts of the one-month rate one month in the future using the implicit forward rates and the martingale model, respectively. The forecast errors (predicted minus actual) are plotted in Figure 8.1, with the solid curve representing the forward rate forecasts.

The most striking feature of the graph is the very high correlation between the two forecast error series. This indicates that the implicit one month forward rate in practice differs very little from the prevailing spot rate, i.e., the term structure is relatively flat over this interval. One possible interpretation of this is that agents form their expectations using a martingale model, i.e., normally predict that the spot interest rate will remain unchanged. In periods when the banks were borrowing heavily at the penalty rate, this might well have been a rational forecast unless there was reason to expect that the central bank would change the penalty rate. However, this explanation is made less plausible by the fact that the correlation between the forward and the current spot rates is equally high in periods when there was little or no borrowing at the penalty rate as, for example, in the end of 1984.

Figure 8.1

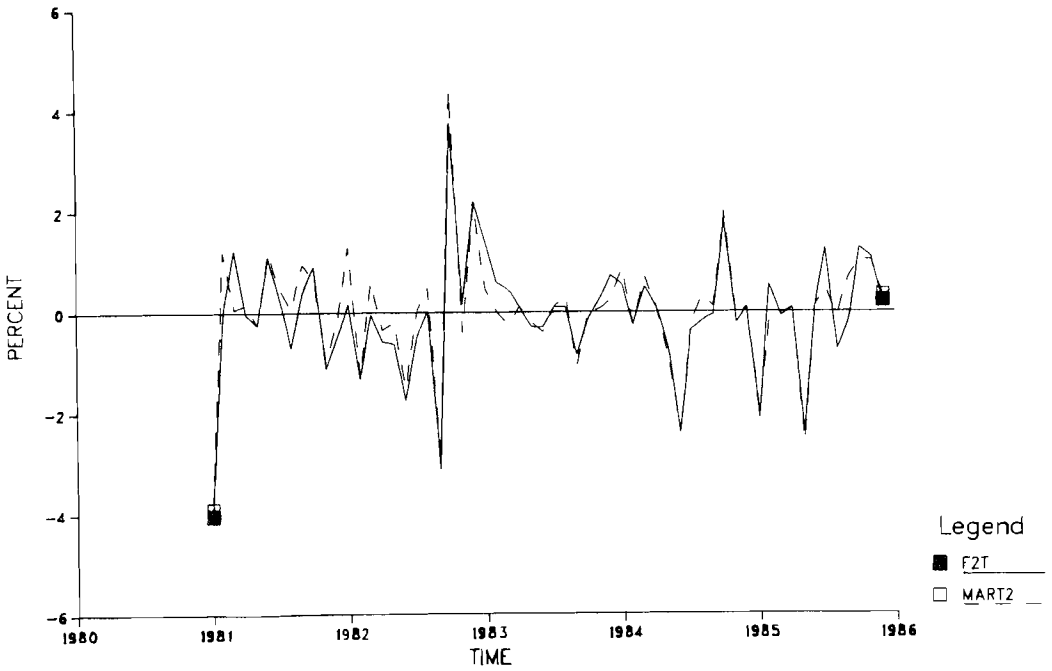


Table 8.2. Forward rate (FR) and martingale model (MM) forecasts of the one-month rate one month ahead

Period	Mean error		Standard deviation		RMSE*		MAE*	
	FR	MM	FR	MM	FR	MM	FR	MM
1981 - 85	-0.0713	0.0033	1.2108	1.2175	<u>1.2028</u>	1.2073	0.7888	<u>0.7560</u>
1981	-0.2284	-0.0500	1.3984	1.3718	1.3584	<u>1.3143</u>	0.8817	<u>0.8333</u>
1982	-0.1357	0.1875	1.7664	1.8787	<u>1.6966</u>	1.8085	1.1861	1.3042
1983	0.1789	-0.0542	0.5853	0.3940	0.5882	<u>0.3811</u>	0.4412	<u>0.2625</u>
1984	-0.0866	0.0250	0.9730	1.0448	<u>0.9356</u>	1.0006	<u>0.6000</u>	0.6667
1985	-0.0845	-0.0917	1.1824	1.1130	1.1352	<u>1.0695</u>	0.8348	<u>0.7133</u>

\*) The root mean square error (RMSE) is computed as

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{x}_i - x_i)^2},$$

where  $\hat{x}_i$  is the predicted value,  $x_i$  is the actual, and  $N$  is the number of periods. Similarly, the mean absolute error (MAE) is defined as

$$MAE = \frac{1}{N} \sum_{i=1}^N |\hat{x}_i - x_i|.$$

The underlined figure indicates the smallest prediction error in each case.

The quantitative forecasting results are summarized in Table 8.2. Looking first at the whole five year period, we note that neither method gives a mean error that is significantly different from zero.<sup>14</sup> The martingale model also has a mean error that is numerically very close to zero. The overall forecast performance is not impressive under either of the criteria reported in Table 8.2, the root mean square error (RMSE) and the mean absolute error (MAE). The two forecasts are approximately equally imprecise, however, with only a marginal advantage for the martingale model in terms of the MAE. When the sample is split into subperiods, a very mixed pattern appears. In effect, the forward rate forecasts dominate in two out of five years. Hence, for someone trying to find the best (simple) method for practical interest rate forecasting, these results give little guidance.

The corresponding results from the two months ahead forecasts of the one-month interest rate are presented in Figure 8.2 and Table 8.3. Although still highly correlated, the errors for the two-months ahead predictions deviate more than in the one-month case shown in Figure 8.1. The pattern that the martingale model predicts higher interest rates, although present in Figure 8.1, is more pronounced in Figure 8.2. We note that the error variance seems to increase. This is to be expected as the forecasts look one month further into the future. The forecast errors also become autocorrelated in that one overprediction tends to be followed by another.

The impression of increased forecast errors and error variances is confirmed in Table 8.3. However, the mean errors are still not significantly different from zero. The mean absolute error increases by 50 percent compared to the one month ahead forecasts to 1.13 percentage points. The smallest errors are recorded using the martingale forecasts, but the margin is again quite narrow.

Figure 8.2

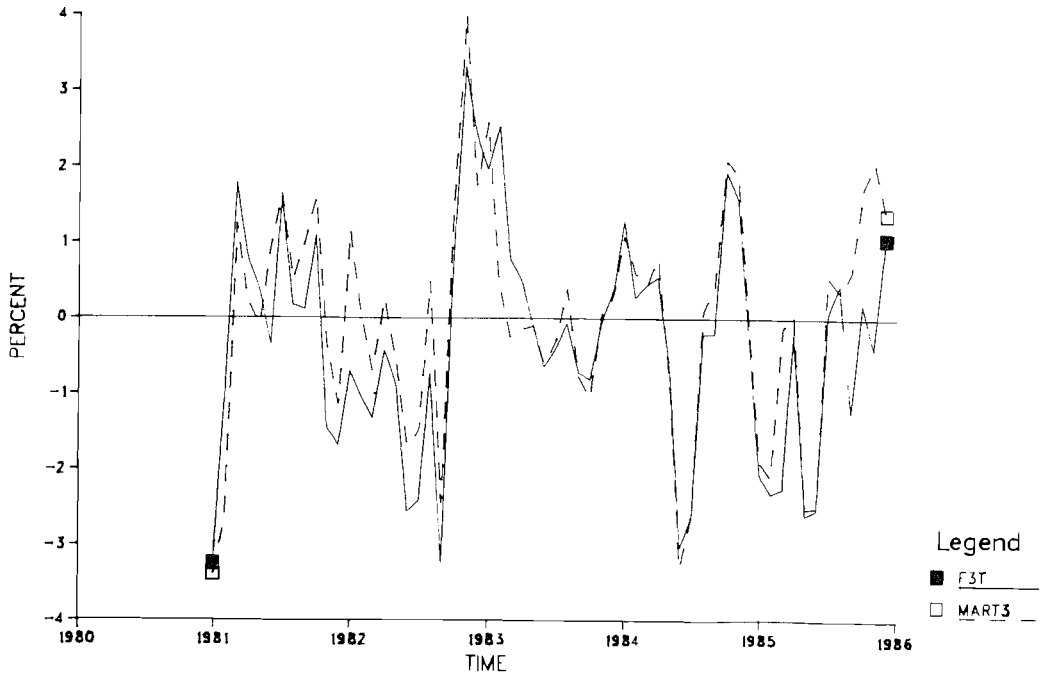


Table 8.3. Forward rate (FR) and martingale model (MM) forecasts of one-month rate two months ahead

Period	Mean error		Standard deviation		RMSE*		MAE*	
	FR	MM	FR	MM	FR	MM	FR	MM
1981 - 85	-0.3005	0.0093	1.5056	1.5010	1.5229	<u>1.4885</u>	1.1826	<u>1.1310</u>
1981	-0.1593	-0.0458	1.4835	1.6325	<u>1.4292</u>	1.5637	<u>1.1433</u>	1.2375
1982	-0.5677	0.1833	1.9423	1.7667	1.9444	<u>1.7013</u>	1.6591	<u>1.3333</u>
1983	0.2798	0.0500	1.0502	0.9242	1.0685	<u>0.8862</u>	0.7427	<u>0.5833</u>
1984	-0.0664	0.0625	1.5123	1.6194	<u>1.4494</u>	1.5517	<u>1.0861</u>	1.1792
1985	-0.9890	-0.2033	1.3233	1.6524	1.6073	<u>1.5951</u>	<u>1.2818</u>	1.3217

\*) See footnote to Table 2.

Looking at the results from individual years, the same mixed pattern as in Table 8.2. is found. In the most recent period, for 1984 and 1985, the forward rate forecasts turn out to be superior. The overall result derives primarily from the relatively high accuracy of the martingale forecasts in 1982 and 1983. We note again that the mean errors from the forward rate forecasts tend to be negative, in 1985 even significantly less than zero. No such signs of systematic underprediction (or overprediction) can be found in the martingale forecasts.

In order to study a somewhat longer forecast horizon, we will also look at the behavior of the three-months ahead forecasts of the three-month rate using the forward rates and the martingale assumption. The results are depicted in Figure 8.3. The pattern that the martingale model errors lie above the forward rate errors is even more pronounced than in Figures 8.1 and 8.2. Again not surprisingly, the lengthening of the forecast horizon leads to a higher error variance and the fact that there are two overlapping observations induces strong autocorrelation in the forecast errors.

We can also note that the major errors occur in the same time periods in all three figures. The important difference is that in the three-months ahead forecasts in Figure 8.3 these errors persist, leading to worse overall forecasts than in Figure 8.1 where the forecast can be revised each period.

The quantitative results are summarized in Table 8.4. We note that in this case the mean error from the forward rate forecasts is significantly negative, whereas in the martingale model it is again very close to zero. The lengthening of the forecast horizon means that the accuracy of the forecasts deteriorates further. In contrast to the previous tables, we find that the forward rate forecasts in the case of the 90-day rate are superior. From the subperiod results it is seen that they are relatively more successful in the last two years.



Figure 8.3

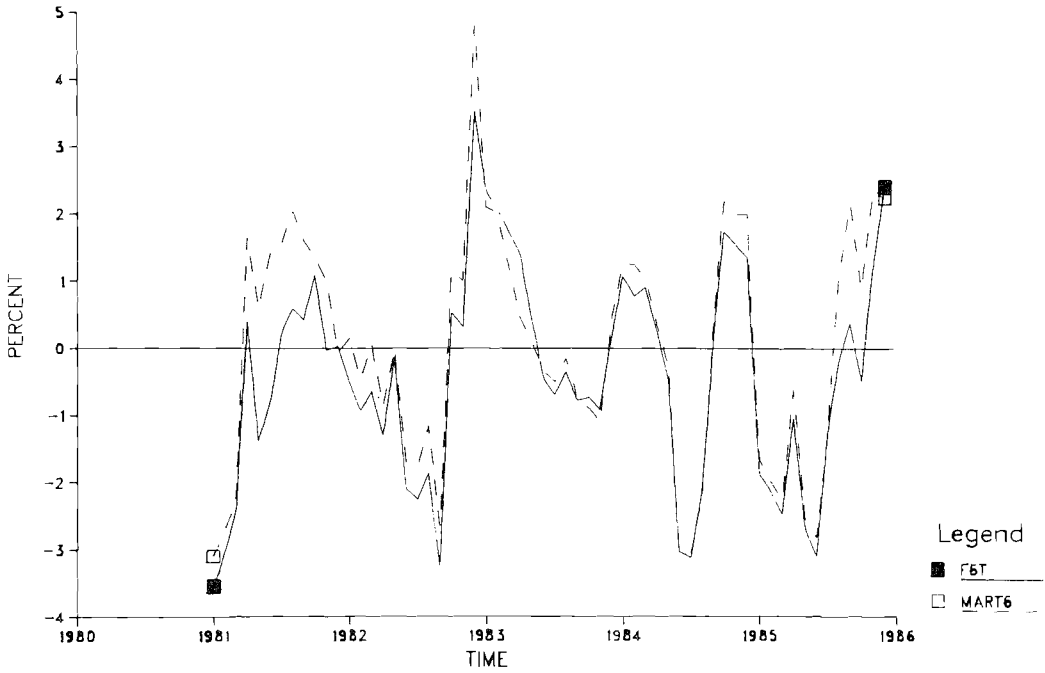


Table 8.4. Forward rate (FR) and martingale model (MM) forecasts of the three-month rate three months ahead

Period	Mean error		Standard deviation		RMSE*		MAE*	
	FR	MM	FR	MM	FR	MM	FR	MM
1981 - 85	-0.4184	0.0267	1.6187	1.7439	<u>1.6588</u>	1.7295	<u>1.3213</u>	1.4320
1981	-0.7115	0.2500	1.5569	1.8768	<u>1.6518</u>	1.8142	<u>1.1782</u>	1.6167
1982	-0.7134	-0.1167	1.7417	1.9175	<u>1.8137</u>	1.8396	1.4433	<u>1.3083</u>
1983	0.3575	0.2167	1.2166	1.0788	1.2185	<u>1.0553</u>	1.0121	<u>0.8417</u>
1984	-0.0861	0.1458	1.7548	1.9245	<u>1.6823</u>	1.8484	<u>1.3802</u>	1.5792
1985	-0.9386	-0.3625	1.6657	1.9872	<u>1.8505</u>	1.9368	<u>1.5929</u>	1.8142

\*) See footnote to Table 2.

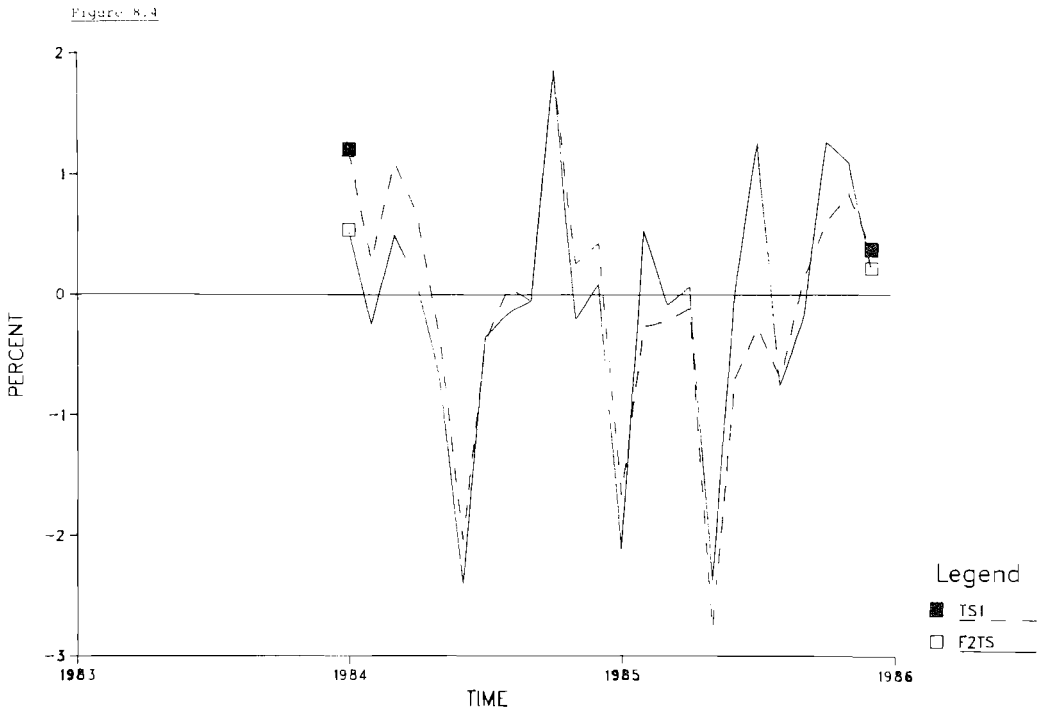


Table 8.5. Forward rate (FR) and time-series (TS) forecasts

Forecast horizon	Period	Mean error		RMSE*		MAE*	
		FR	TS	FR	TS	FR	TS
One month	1984 - 85	-0.085	-0.070	1.040	<u>1.002</u>	<u>0.717</u>	0.721
	1984	-0.087	0.251	<u>0.936</u>	0.972	<u>0.600</u>	0.726
	1985	-0.084	-0.390	1.135	<u>1.032</u>	0.835	<u>0.717</u>
Two months	1984 - 85	-0.528	-0.143	1.530	<u>1.409</u>	1.184	<u>1.182</u>
	1984	-0.066	0.404	1.449	<u>1.366</u>	<u>1.086</u>	1.164
	1985	-0.989	-0.69	1.607	<u>1.451</u>	1.282	<u>1.200</u>
Three months (three month rate)	1984 - 85	-0.512	0.120	1.768	<u>1.600</u>	1.487	<u>1.399</u>
	1984	-0.086	0.483	1.682	<u>1.574</u>	<u>1.381</u>	1.442
	1985	-0.939	-0.242	1.850	<u>1.625</u>	1.593	<u>1.356</u>

\*) See footnote to Table 2.

To summarize the results so far we are forced to conclude that no very clear pattern emerges. There are indications that the forward rate forecasts perform somewhat better when the forecast horizon is lengthened and, possibly, also that they do better in the latter part of the (short) period studied.

These results are different in several respects from those reported for the U.S. by Fama (1976, 1984). First, he finds that the martingale model is superior for practically all time periods and all forecast horizons ranging from 2 to 6 months ahead. As the spot rate in Fama's (1984) data behaves roughly as a martingale process, that model is obviously a more serious "contestant" than in the current comparisons. However, this fact only implies that forward rates could not do much better as predictors of the future spot rates, not that they should do significantly worse. This finding is thus consistent with there being risk premia in the U.S. forward rates. The fact that the martingale model is not as good a representation of Swedish interest rates means that the comparisons made here are somewhat less informative. If it had turned out that the martingale forecasts were superior also in our data, an interpretation in terms of risk premia would have been possible. The actual results, which come close to a draw, do not permit very definite conclusions in this respect, although it might possibly indicate that risk premia are less prominent in Swedish forward rates.

Second, Fama finds that the mean errors from the forward rate model are consistently and significantly positive, indicating systematic overpredictions. Note that the realized error can be written (in the one-step ahead case)

$$F2_t - R1_{t+1} = E_t(R1_{t+1}) - R1_{t+1} + E_t(P2_{t+1}), \quad (8.12)$$

i.e., as the sum of the forecast error and the expected premium. Hence, assuming that there are no systematic forecast errors,

$E[E(R1_{t+1}) - R1_{t+1}] = 0$ , a positive mean error indicates that the expected premium is positive and vice versa. In terms of "the traditional hypotheses", Fama's results are thus consistent with the liquidity preference theory. Although our results contain few significant mean errors, they are with the exception of 1983, all negative, and therefore an indication that expected premia are negative. This is not a very strong result, however, and the low significance levels indicate above all that whatever premia there may be are drowned by the large and highly variable forecast errors.

In the following section, we discuss complementary evidence from applying an alternative method for studying the information content in the term structure of interest rate to Swedish money market data.

#### 8.5 THE PREDICTIVE POWER OF FORWARD RATES: REGRESSION TESTS

As noted in Section 8.2, the forward rate can be decomposed into two elements, a forecast of the future spot rate and an expected premium term, i.e.,

$$F^{\tau}_t = E_t(R1_{t+\tau-1}) + E_t(P^{\tau}_{t+1}) \quad (8.13)$$

Neither the forecast nor the premium is observable, however, and indirect methods must be used to extract information about their relative importance. One such approach is suggested by Fama (1984). Its main purpose is to investigate whether observed forward rates have the power to predict future spot rates and/or future premia and it is analogous to the unbiasedness tests reported in Chapter 7. In this section, we will apply this approach to data from the Swedish money market. Due to differences in the time-series characteristics of our data compared to Fama's, we are also able to use an auxiliary test that gives additional insights about the information content in forward interest rates.

### 8.5.1 The regression equations

Consider the case when the forward rate contains only a forecast of the future spot rate, i.e., when  $E_t(P\tau_{t+1}) = 0$  so that (8.13) reduces to

$$F\tau_t = E_t(R1_{t+\tau-1}) \quad (8.14)$$

This hypothesis, which basically is identical to the unbiased expectations hypothesis can be tested by running the following time-series regression:

$$R1_{t+\tau-1} = \alpha_1 + \beta_1 F\tau_t + \epsilon_{t+\tau-1} \quad (8.15)$$

where  $\epsilon_{t+\tau-1} = R1_{t+\tau-1} - E_t(R1_{t+\tau-1})$  under the null hypothesis  $(\alpha_1, \beta_1) = (0, 1)$ .

Under rational expectations, the mean of the forecast error is zero and it is therefore legitimate to estimate this equation using OLS.<sup>15</sup> Note, however, that this test, as well as the others discussed in this section, is a joint test of the null hypothesis and the rationality postulate.

The opposite extreme hypothesis is that variations in the forward rate reflect only variations in the premium component, i.e., is independent of the expected future spot rate. This suggests the regression equation

$$P\tau_{t+1} = \alpha_2 + \beta_2 F\tau_t + \eta_{t+1} \quad (8.16)$$

where  $P\tau_{t+1} \equiv H\tau_{t+1} - R1_t$ , the ex post premium, and  $\eta_{t+1} = P\tau_{t+1} - E_t(P\tau_{t+1})$  under the null hypothesis  $(\alpha_2, \beta_2) = (0, 1)$ .

The regressions (8.15) and (8.16) are clearly complementary in that if  $\beta_1 = 1$ , we would expect  $\beta_2 \approx 0$  and vice versa if  $\beta_2 = 1$ .

It should be emphasized that even if it is easiest to interpret (8.15) in terms of the expectations hypothesis, our main concern here is not to test this hypothesis per se. As pointed out in Section 8.2, there are strong a priori reasons (both theoretical and empirical) to doubt the validity of the pure expectations hypothesis. However, evidence that  $\beta_1$  is reliably non-zero indicates that the forward rate contains information about the level of the future spot interest rate even though it is not an unbiased predictor.

Similarly, we do not seriously entertain the extreme pure premium hypothesis in (8.16), but are primarily interested in knowing whether  $\beta_2$  is positive. This would indicate that the forward rates contain information about next period's excess returns on multiperiod bonds.

To see the implications of this more clearly, let us consider the expressions for  $\beta_1$  and  $\beta_2$ . Under rational expectations they are (cf. Fama (1984))

$$\begin{aligned}\beta_1 &= \frac{\text{cov}(R_{t+\tau-1}, F_{\tau_t})}{\sigma^2(F_{\tau_t})} \\ &= \frac{\sigma^2(E_t(R_{t+\tau-1})) + \text{cov}(E_t(R_{t+\tau-1}), E_t(P_{\tau_{t+1}}))}{\sigma^2(F_{\tau_t})}\end{aligned}\quad (8.17)$$

$$\begin{aligned}\beta_2 &= \frac{\text{cov}(P_{\tau_{t+1}}, F_{\tau_t})}{\sigma^2(F_{\tau_t})} \\ &= \frac{\sigma^2(E_t(P_{\tau_{t+1}})) + \text{cov}(E_t(R_{t+\tau-1}), E_t(P_{\tau_{t+1}}))}{\sigma^2(F_{\tau_t})}\end{aligned}\quad (8.18)$$

For simplicity, consider the case  $\tau=2$  when both the interest rate expectation and the premium are looking one step ahead.<sup>16</sup> If the future spot rate and the excess return were uncorrelated, the regression coefficient would have a very straightforward interpretation. As this implies that

$$\sigma^2(F2_t) = \sigma^2\left(E_t(R1_{t+1})\right) + \sigma^2\left(E_t(P2_{t+1})\right) \quad (8.19)$$

$\beta_1$  and  $\beta_2$  would measure the fractions of the total variance in the forward rate that are due to variance in the expected spot rate and the expected premium, respectively.

The values of  $\beta_1$  and  $\beta_2$  would then give a direct indication of the relative variability of the two components. However, we have no a priori grounds for assuming that spot rates and premia are in fact uncorrelated, which means that this simple interpretation may be inaccurate.

Nevertheless, we can say something about their relative magnitudes. Suppose that  $\beta_1$  and  $\beta_2$  both are positive but less than unity (as will turn out to be the case; see below). Then we know, first, that if the covariance term is negative, it is at least smaller in absolute value than both  $\sigma^2\left(E_t(R1_{t+\tau-1})\right)$  and  $\sigma^2\left(E_t(P\tau_{t+1})\right)$ . Second, both variances must be positive, since a zero variance would imply also zero covariance and one of the  $\beta$ s would vanish. Third, if  $\beta_1 > \beta_2$ , it must be the case that  $\sigma^2\left(E_t(R1_{t+\tau-1})\right) > \sigma^2\left(E_t(P\tau_{t+1})\right)^2$  and vice versa if  $\beta_2 > \beta_1$ .

Note that equation (8.15) concerns the relation between the levels of the forward rate and the future spot rate. This is a version of the test that is not used by Fama (1984). The reason is that the spot interest rate in his data shows signs of mean non-stationarity, i.e., the autocorrelation function for  $R1_t$  (corresponding to the  $\rho_i$ 's in Table 8.1)

gives high values all the way to lag 12, i.e., a regression on level form would be subject to the "unit-root problem" discussed briefly in Section 6.3.1 with reference to non-stationary exchange rate levels. As the spot rate in our data follows a well-behaved stationary AR(1) process, (8.15) is a meaningful test of the ability of the forward rate to predict the level of the future spot rate.

The reasons for this striking difference in the time-series behavior of Swedish and U.S. interest rates are not immediately apparent, but a conjectural explanation can be made. On theoretical grounds, it is not obvious that the nominal interest rate is expected to be mean non-stationary. Using the standard Fisher equation, it can be decomposed into a real interest rate and an inflation component. Even though real interest rates are no longer believed to be constant (see, e.g., Mishkin (1981)), there is little reason to assume that they are mean non-stationary. Theoretically, there is also no compelling reason to believe that the rate of inflation follows a non-stationary process. However, in practice there are clearly long cycles in the inflation process which means that signs of non-stationarity may appear in a given sample. Fama's sample period is 1959 to 1982 which is precisely such a period in which there was a secular increase in the rate of inflation. This led to a gradual increase in the nominal interest rate which shows up in the high degree of autocorrelation.

In contrast to Fama, we have a sample from a period (1980-1985) when the inflation rate was high, but relatively stable. Hence, it is possible that differences in the behavior of the rate of inflation in the two samples account for much of the difference in the time-series patterns in the nominal spot interest rates.



The usual procedure when a non-stationary time-series is encountered is differencing. Fama (1984) therefore rewrites (8.13) as

$$F\tau_t - R1_t = E(R1_{t+\tau-1} - R1_t) + E(P\tau_{t+1}) \quad (8.20)$$

and specifies the regressions

$$R1_{t+\tau-1} - R1_t = \alpha_3 + \beta_3(F\tau_t - R1_t) + \varepsilon'_{t+\tau-1}, \quad (8.21)$$

and

$$P\tau_{t+1} = \alpha_4 + \beta_4(F\tau_t - R1_t) + \eta'_{t+1} \quad (8.22)$$

Under the null hypothesis  $(\alpha_3, \beta_3) = (0, 1)$ , (8.21) is seen to be equivalent to (8.21) under the null hypothesis  $(\alpha_1, \beta_1) = (0, 1)$  and a similar equivalence holds between (8.22) and (8.16).

While these tests are identical under the (extreme) null hypotheses, they pose somewhat different questions concerning the information in the term structure. In particular, an estimate of  $\beta_3 > 0$  indicates that the forward-spot differential has power to predict the future change in the spot interest rate and  $\beta_4 > 0$  indicates that this differential contains information about the future premium. Decompositions of  $\beta_3$  and  $\beta_4$  similar to those of  $\beta_1$  and  $\beta_2$  made above are also possible, of course.

### 8.5.2 Regression results

On the basis of available data, we can estimate regressions for  $P2_{t+1}$  and  $P3_{t+1}$ , i.e., the excess return from  $t$  to  $t+1$  on two-month and three-month instruments. Similarly, we can test the power of forward rates to predict the spot rate one

month and two months in the future. In order to lengthen the time perspective, we will also consider a holding period of three months. These variables will be denoted by lower case letters. We will study  $p6_{t+3}$ , the excess return on a six-month bonds relative to a three-month bond, and the relation between  $r3_{t+3}$ , the quarterly spot rate three months in the future, and  $f6_t$ , the relevant forward rate.

Before discussing the estimation results it is useful to return to Table 8.1 and the descriptive statistics of the variables involved in these tests.

As previously noted, spot and forward rates follow typical AR(1) processes. The changes in the one-month spot rate show a quite weak AR(1) pattern whereas the differences between the one-month forward rate and the spot rate,  $F2_t - R1_t$ , are uncorrelated. For  $F3_t - R1_t$  a slowly damped AR(2) process is suggested, in particular, from the partial autocorrelation function (not shown). The premia,  $P\tau_{t+1}$ , show no time series pattern, i.e., appear to be white noise.

The interpretation of the time series pattern of the three-month rate is complicated by the presence of overlapping observations. This probably accounts for the pattern in the changes in the spot rate, as well as in  $p6_{t+3}$ . One might therefore conjecture that there would be little autocorrelation in the quarterly changes in the quarterly rates. The short sample period prohibits an examination of this proposition.

We can also note that both the ex ante premia,  $F\tau_t - R1_t$ , and the actual, ex post, premia,  $P\tau_{t+1}$ , are negative on average and almost identical in magnitude. The variability of the ex post premium is almost three times as high, however. Note therefore that only the mean of  $F3_t - R1_t$  is significantly negative. The same correspondence is found for the quarterly rates where the means are significantly negative both ex ante and

ex post. We conclude therefore that there have been (systematic?) negative excess returns on securities with longer maturities.

It might be useful to compare these findings to those of Fama (1984) despite the fact that his data partly cover a different and much longer time period, February 1959 to July 1982. A similarity is that Fama finds no marked autocorrelation in the changes in the spot rate. However, in the U.S. data, both  $F\tau_t - R1_t$  and  $P\tau_{t+1}$  show significant autocorrelation for all  $\tau$ . Fama concludes: "Given that the changes in the one-month spot rate do not show much autocorrelation, the autocorrelations of  $F\tau_t - R1_t$  seem primarily to reflect its  $E_t(P\tau_{t+1})$  component rather than its  $E_t(R1_{t+\tau-1} - R1_t)$  component" (Fama (1984), p. 515). For our data, no similar statement can be made as  $P2_{t+1}$  and  $P3_{t+1}$  both are white noise and only  $F3_t - R1_t$  shows autocorrelation.

The estimation results for equations (8.15) and (8.16) are summarized in Table 8.5. We have run the regression both for the whole sample period and for the most recent subperiod. The reason for focusing on the more recent observations is the widely held view that the functioning of the Swedish money market has improved (see, e.g., McPhee (1984)). It is therefore interesting to see whether exclusion of data from the more "primitive" phase in the development of the market affects the behavior of the forward rates. The table gives estimated coefficients, standard errors (in parentheses),  $R^2$ , and the standard Durbin-Watson coefficient for first-order residual autocorrelation.

Table 8.5  $R^1_{t+\tau-1} = \alpha_1 + \beta_1 F\tau_t + \epsilon_{t+\tau-1}$  ;  $P\tau_{t+1} = \alpha_2 + \beta_2 F\tau_t + \eta_{t+1}$

Dependent	3/80 - 10/85; N = 68				1/82 - 10/85; N = 46			
	$\hat{\alpha}$	$\hat{\beta}$	$R^2$	DW	$\hat{\alpha}$	$\hat{\beta}$	$R^2$	DW
$R^1_{t+1}$	0.0024 (0.0010)	0.7778 (0.0866)	0.55	1.89	0.0022 (0.0011)	0.7997 (0.1056)	0.57	1.84
$R^1_{t+2}$	0.0042 (0.0011)	0.6291 (0.1056)	0.35	0.88	0.0040 (0.0016)	0.6384 (0.1518)	0.29	0.71
$r^3_{t+3}$	0.0147 (0.0047)	0.5646 (0.1491)	0.18	0.59	0.0172 (0.0056)	0.4680 (0.1808)	0.13	0.55
$P^2_{t+1}$	-0.0024 (0.0009)	0.2222 (0.0866)	0.09	1.89	-0.0022 (0.0011)	0.2003 (0.1056)	0.08	1.84
$P^3_{t+1}$	-0.0046 (0.0017)	0.4158 (0.1618)	0.09	1.85	-0.0050 (0.0024)	0.4595 (0.2278)	0.08	1.73
$p^6_{t+1}$	-0.0147 (0.0047)	0.4354 (0.1491)	0.12	0.59	-0.0172 (0.0056)	0.5320 (0.1808)	0.17	0.55

As a general comment, we can first note that all the coefficients in Table 8.5 are significantly different from zero. Moreover, with one exception,<sup>17</sup> they are also significantly different at the 5 percent level from their values under the extreme null hypotheses discussed in Section 8.5.1. In particular, the  $\beta$ s are consistently less than unity. Hence, we can (not surprisingly) clearly reject the unbiased expectations hypothesis.

However, it is equally clear that the forward rates contain information about the level of the future spot rates. For the spot rate one-month ahead, the forward rate accounts for almost 60 percent of the total variation in  $R^1_{t+1}$ , and  $\beta_1$  is close to 0.8 and almost nine standard errors away from zero. For the two step ahead spot rate both the fraction of explained variation and the  $\beta_1$  coefficient drop. The very marked residual autocorrelation in this equation is caused

by the presence of overlapping observations. This means that an error at  $t+1$  will persist to  $t+2$  and a first-order AR process is introduced in the residuals. While this does not bias the coefficient estimates, the standard errors tend to be underestimated. Hence, the significance levels should be interpreted with some caution.

In the quarterly interest rate regressions the autocorrelations are even stronger. The results are otherwise similar and indicate that the fraction of explained spot rate variation,  $R^2$ , as well as the  $\beta_1$  coefficient, drop when the time horizon is lengthened.

The mirror image of this is seen in the premium regressions where  $\beta_2$  and, to some extent,  $R^2$  rise as  $\tau$  increases. Interpreted in terms of relative variances, we see from the expression for  $\beta_1$  and  $\beta_2$  in (8.17) and (8.18) that the variance of the expected spot rate accounts for a smaller fraction of the total variance the longer the horizon over which the expectation is taken. Conversely, this implies that the variance of the expected premium becomes more and more important as  $\tau$  increases. This finding is thus consistent with the simple notion that premia are more important for longer term maturities than in the very short run. It is interesting to note that this effect seems to be present even though we are looking only at the very short end of the term structure.

Finally, we can also note that exclusion of the first years of data has relatively limited effects on the regression results. The most marked differences occur in the quarterly regressions where it appears that the premium component has become more important in the most recent subperiod.

The conclusion from the results presented in Table 8.5 is thus that the forward rates contain information about both

future spot rate levels and future return premia and that the importance of the expected spot rate falls as the forecast horizon is lengthened.

The result that forward rate levels are related to future spot rate levels is fairly obvious already from inspection of their autocorrelation functions. We know that current spot rates have power to predict future spot rates from the strong AR(1) pattern in the  $R1_t$  series. Moreover, it is clear that the forward rate,  $F\tau_t$ , is highly correlated with the currently observable spot rate,  $R1_t$ , which is to say that the yield curve tends to be very flat. (Cf. the forecast errors plotted in Figure 8.1, for example.) In a sense, this result thus just reconfirms that the spot rate follows an AR(1) process and we could have got almost identical results by replacing  $F\tau_t$  with  $R1_t$ .

On a more positive note, we can still conclude that the premium does not dominate the expectation component in the forward rate. This reconfirms the conclusion from Section 8.4 that forward rates on the Swedish money market provide forecasts that are at least as good as those from a martingale model.

The second set of tests, equations (8.21) and (8.22), relates the current forward-spot differential to the change in the spot rate and the premium. The regression results are summarized in Table 8.6.

Table 8.6  $R1_{t+T-1} - R1_t = \alpha_3 + \beta_3(F\tau_t - R1_t) + \varepsilon'_{t+1}$ 

$$P\tau_{t+1} = \alpha_4 + \beta_4(F\tau_t - R1_t) + \eta'_{t+1}$$

Dependent	3/80 - 10/85; N = 68				1/82 - 10/85; N = 46			
	$\hat{\alpha}$	$\hat{\beta}$	$R^2$	DW	$\hat{\alpha}$	$\hat{\beta}$	$R^2$	DW
$R1_{t+1} - R1_t$	0.00001 (0.00011)	0.4545 (0.3080)	0.04	2.32	0.00003 (0.00015)	0.5275 (0.4520)	0.03	2.22
$R1_{t+2} - R1_t$	0.00008 (0.00015)	0.4188 (0.2339)	0.05	1.07	0.00007 (0.00020)	0.3115 (0.3247)	0.02	0.92
$r3_{t+3} - r3_t$	0.00121 (0.00058)	1.1109 (0.3313)	0.15	0.70	0.00103 (0.00072)	1.2542 (0.5044)	0.13	0.61
$P2_{t+1}$	-0.00001 (0.00011)	0.5455 (0.3080)	0.05	2.32	-0.00003 (0.00015)	0.4725 (0.4520)	0.02	2.22
$P3_{t+1}$	-0.00007 (0.00022)	0.5804 (0.3536)	0.04	2.18	-0.00007 (0.00030)	0.8269 (0.4874)	0.06	2.20
$p6_{t+3}$	-0.00121 (0.00058)	-0.1109 (0.3313)	0.00	0.70	-0.00103 (0.00072)	-0.2542 (0.5044)	0.01	0.61

In the regressions of the changes in the monthly spot rates, the slope coefficients are all positive, but the significance levels are quite low.<sup>18</sup> The highest value is found in the  $R1_{t+2} - R1_t$  equation where the  $\beta$  is significantly greater than zero at the 10 percent level. We can also note that the coefficients in the shorter periods are similar in magnitude, but that increases in the standard errors lead to lower significance levels. The constant terms are insignificant in all the equations.

In the monthly premium regressions in the bottom panel of Table 8.6, the slope coefficients are relatively large, but significant only at the 10 percent level and only when the total sample is used. In the 1982-85 subperiod, the smaller sample gives higher standard errors but the coefficient in the  $P3_{t+1}$  regression increases from 0.58 to 0.83 and remains (weakly) significant. The constant terms are again very small and clearly insignificant.

To summarize the results for the monthly interest rates, we find some evidence that the forward-spot rate differential contains information about future premia. The slope coefficient is weakly significant also in the regression of the spot rate two months ahead, which indicates that the forward-spot rate differential also may contain information about future spot rates. The reliability of this result is open to question, however, because of residual autocorrelation.<sup>19</sup> These doubts are to some extent strengthened by the fact that no significant predictive power can be found in the one-month forward rate where no autocorrelation is present. A priori, we are inclined to believe that the predictive power should be greater the shorter the time horizon. This is also what we found in the previous tests reported in Table 8.5. The evidence in favor of the hypothesis that the forward-spot rate differential contains information about the future change in the monthly spot rate is thus relatively weak.

However, as the significance levels in the premium regressions are similar, we cannot conclude that the premium component in the forward rate is more important. Instead, the impression is that the variability in both spot rate changes and premia is so high that the forecast error variance dominates.

The results for the quarterly rates are markedly different. Due to overlapping observations, these regressions are marred by residual autocorrelation of the same type as in the  $R1_{t+2}-R1_t$  regression. With the same type of reservations discussed there, we see that the slope coefficient in the spot rate equation is remarkably close to unity. Given that  $\beta_4 = 1-\beta_3$ , this implies that  $\beta_4$  is close to zero. Hence, the three-month forward-spot differential contains no information about future excess returns, but is an (almost) unbiased predictor of the future change in the quarterly spot rate. These results hold also in the shorter sample period although with higher standard errors.



The results in Table 8.6 thus suggest that three month forward-spot differentials contain unbiased forecasts of future spot rate changes, whereas monthly forward-spot differentials are dominated by the influence of expected future premia. The reason for this somewhat anomalous result is difficult to determine. It also contrasts against the finding in Table 8.5 that the monthly forward rates give much better forecasts of the level of the future spot rates than the quarterly. It is clear that the residual autocorrelation might lead to an overestimate of the significance level, but the estimates are at least unbiased. It seems necessary to follow the future development of this relation to determine whether this is a reliable result or just a spurious regression coefficient.

## 8.6 CONCLUDING COMMENTS

The purpose of this chapter has been to explore the properties of the term structure relation in the Swedish money market. In particular, we have studied the relative roles of expectations of future spot interest rates and risk premia by testing the forecasting power of forward interest rates. The short history of the money and bond markets means that only the short end of the maturity structure can be studied and even then the observation period is relatively short.

It is interesting to note that the results differ in a number of respects from those obtained in similar studies of U.S. data. For example, it turns out that the forward rates provide forecasts of future spot rates that are no worse and in some cases even better than those derived from the naive martingale model. This is in contrast to Fama's (1976, 1984) findings for the U.S. money market, where martingale forecasts dominate completely.

Also in contrast to Fama's results is that ex post excess returns, i.e., realized risk premia, are consistently (although only rarely significantly) negative. This indicates that the market has paid a premium to investors who have chosen to hold one-period assets.

The regression tests showed that the forward rates contain reliable information about the level of the future spot rates. However, the pure expectations hypothesis could be rejected which means that there is also information about future excess returns. The forward-spot rate differential was showed to have weak power to predict the future change in the monthly spot rate. However, as something of an anomaly, it was found that unbiasedness could not be rejected for the change in the quarterly interest rate. With the exception of this result, the overall impression from the regression tests is that the relative importance of spot rate expectations decreases as the forecast horizon is lengthened.

The tests reported here are all atheoretical and one important purpose has been to explore some empirical characteristics of the term structure in the Swedish money market. Much work remains before we can get a coherent picture of the important underlying determinants of this relation and the implied risk premia.<sup>20</sup>

A particularly interesting aspect in an open economy is the influence of international interest rates on the returns of assets with different times to maturity. In Chapter 7, the analysis was focused on 30 day forward exchange rates, but obviously there are forward contracts for 60, 90, 180 days, etc., i.e., a term structure of forward exchange rates. The implied foreign exchange risk premia may vary and thus the conclusions concerning interest rate dependence may be different at different points along the yield curve.

This points to the need for an integrated analysis of the term structure of interest rates and the analogous term structure of forward rates.<sup>21</sup> It is in the interaction between these two relations that the effects of various monetary policy interventions on interest rates on assets with different maturities are determined. In this book, evidence consistent with there being risk premia in both these relations has been found, but the important questions of their determinants and the extent to which they can be affected by systematic monetary policy must be left unanswered. Hopefully, future work on monetary policy and interest rate determination will bring us closer to an answer to this and many other unresolved questions.

## NOTES

1. This hypothesis can be traced back to Fisher (1896). See also Meiselman (1962).
2. The use of continuous compounding here is primarily for notational convenience. However, for the tests reported in Section 8.5, this assumption is of some importance for the interpretation, which means that we will have reason to return to this problem below.
3. In terms of interest rates, the forward rate can be computed as
 
$$F\tau_t = \tau R\tau_t - (\tau-1)R(\tau-1)_t$$
4. It is straightforward to show that this also must hold for multi-period forward rates and future multi-period interest rates.
5. This is emphasized by Cox, Ingersoll, and Ross (1981). The unbiased expectations hypothesis (8.6) is easily seen to be analogous to the hypothesis of unbiasedness of forward exchange rates as predictors of future spot rates discussed in Chapters 5-7. It is therefore not surprising that the same type of problems due to Jensen's inequality appear in both relations.
6. This formulation assumes that the future premia are expected to remain constant for  $\tau > 2$ . Fama (1984) gives the more general expression.
7. Note that this is where the assumption of continuously compounded interest matters. Under discrete compounding, the two expressions for the risk premium, as the bias in the forward rate and as the excess return on  $\tau$ -period bond, will not coincide. To maintain consistency, the tests where both these relations are used employ interest rates that have been converted from discrete to continuously compounded rates. The details are discussed in Section 8.3.

8. For a good introduction to the theory of the term structure, see Van Horne (1978), who also gives useful references. This literature is huge, but some standard sources are Roll (1970), Fama (1976), Shiller (1979), and Shiller, Campbell, and Schoenholtz (1983).
9. See also LeRoy (1982).
10. This point is also made by Shiller, Campbell, and Schoenholtz (1983), and by Mankiw and Summers (1984) to motivate their interest in the expectations hypothesis.
11. For a few examples of statistical rejections of the expectations hypothesis, see the references cited in footnote 8. Note that these tests are invariably joint tests of unbiasedness and rationality and that the rejection therefore, in principle, may be due to the failure of the assumption that market forecasts are rational, i.e., without systematic error. More will be said on this issue below in the discussion of the tests employed here.

12. From the autocorrelation function, it is apparent that  $R1_t$  follows an AR(1) process, i.e.,

$$R1_{t+1} = \phi_1 R1_t + a_{t+1}$$

This implies that the two month change in the spot rate can be written as

$$\begin{aligned} R1_{t+2} - R1_t &= \phi_1 R1_{t+1} + a_{t+2} - R1_t \\ &= \phi_1 (\phi_1 R1_t + a_{t+1}) + a_{t+2} - R1_t \\ &= (\phi_1^2 - 1) R1_t + \phi_1 a_{t+1} + a_{t+2} \end{aligned}$$

Further substitutions for  $R1_t$ , etc. give

$$\begin{aligned}
 R1_{t+2} - R1_t &= (\phi_1^2 - 1)\phi_1 R1_{t-1} + (\phi_1^2 - 1)a_t + \phi_1 a_{t+1} + a_{t+2} \\
 &= (\phi_1^2 - 1) \sum_{i=0}^{\infty} \phi_1^i a_{t-i} + \phi_1 a_{t+1} + a_{t+2}
 \end{aligned}$$

This shows that although all innovations in the interest rate process persist,  $a_{t+1}$  has a relatively stronger effect (given that  $\phi_1$  is large). This tends to induce a moving average component in the time-series process. This is consistent with the autocorrelation function for  $R1_{t+2} - R1_t$  shown in Table 8.1, which (roughly) has the characteristics of an MA(1) process. Similarly,  $r3_{t+3} - r3_t$ , which contains two overlapping observations, shows signs of following an MA(2) process.

13. The continuously compounded rate,  $R_c$ , corresponding to any discrete interest rate,  $r_D$ , is computed simply as  $R_c = \ln(1+r_D)$ .
14. The test statistic here, as in all other statements about the significance of mean values in the rest of the chapter, is  $\bar{x}/(\sigma/\sqrt{N})$ , which has a standard t distribution.
15. However, note the problem pointed out by Hodrick and Srivastava (1986) that the OLS estimates may be biased under the alternative hypothesis of a time-varying risk premium. They show that this problem is unlikely to be important in a test of the unbiasedness of forward exchange rates (cf. Chapter 7), but this general caveat should be kept in mind.
16. This implies also that  $\beta_1 + \beta_2 = 1$ .
17. The exception is  $\beta_1$  in the  $R1_{t+1}$  regression in the shorter subperiod which has a t-value of 1.9.

18. Note that for  $\tau=2$ , the regressions are equivalent, i.e.,  
 $\alpha_3 = -\alpha_4$  and  $\beta_3 = 1-\beta_4$ .
19. It is somewhat surprising that Fama (1984), who, of course, notices the same pattern, does not comment on its effect on the estimated standard errors.
20. For an example of a test of possible determinants of the risk premia, see Jones and Roley (1983).
21. Studies of this problem include Campbell and Clarida (1986) and Hakkio and Leiderman (1986).

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