On Capital Formation and the Effects of Capital Income Taxation

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On Capital Formation and the Effects of Capital Income Taxation

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Sweden is a land of taxes.

Taxes affect the lives of most citizens and not least of those who have the ambition to build up capital. Some of my friends and relatives have actively exploited the deficiencies of the tax system in order to become rich, others have reluctantly adjusted their lives to avoid excessive taxes, still others have paid and suffered. They have all been a source of inspiration for this dissertation, though few will recognize themselves in the 'representative household' with its simple utility function. It is one of the wonders of the craft of the economist that we can abstract so far from the manifold reality and still get results of practical interest.

My interest in political economy and the effects of taxes goes far back. Many teachers, superiors and colleagues have contributed to my understanding of the subject.

A starting point was an elementary course in economics taught by Tord Palander at the Royal Institute of Technology 29 years ago. Professor Palander evidently liked to teach the technology students and he spent 7 hours on an oral examination. He also advised me to pursue graduate studies in economics, but at that time I was planning a career in industrial management and when I continued my studies at the Stockholm School of Economics my main subject was business administration. I did, however, also study undergraduate economics and tax law.

At the end of our business administration studies in 1960 my friend Kaj Kjellqvist and I wrote a paper on the profitability rating of public investment projects. In this paper we developed complicated profitability measures which took account of the excess burden of tax financing and we also showed how this excess burden could be related to the distortion of labour supply.

After leaving school I spent 15 years in various positions in the steel industry (accountant, research metallurgist, financial vice-president, part-time director) and 5 years as a business consultant and private investor before returning to graduate studies. During this period a large part of my work was devoted to the evaluation of capital expenditure programmes. The effects of taxes and inflation on profitability were a constant concern. Of great help in analyzing these effects was the Ph.D. dissertation of Sven-Erik Johansson which was published in 1961. It might be regarded as a tribute both to the quality of his dissertation and to the intellectual interests of the management of Avesta Jernverk that I was sent to Stockholm on company expense to listen to the oral defence of the thesis.

When I first resumed my studies in economics it was as a hobby in my spare time and with no plans for the future. But thanks to Peter Bohm I became seriously interested and was accepted as a Ph.D. student at the University of Stockholm.

When the time came to start on the thesis work I returned to the Stockholm School of Economics and became a member of the tax policy study group under the guidance of Karl G. Jungenfelt. Professor Jungenfelt, with his fatherly interest in the welfare of his students and his great generosity in spending time and effort in helping us, has been my single most important support for this work. But I also owe many thanks to the other senior members of the tax policy group, Peter Englund and Mats Persson, who throughout the work have read many drafts, asked many questions, given me many literature references and generally helped to educate me in the scientific tradition.

Many of my colleagues at the Stockholm School of Economics have given me valuable help in scrutinizing the text of different chapters and in making suggestions which have led to major improvements. Thanks are due to Clas Bergström, Lars Hörngren, Bo Nordin, Claes Thimrén and Staffan Viotti. I also received good advice and interesting comments from Michael Brennan, Mervyn King and Jan Södersten. It goes without saying that I am solely responsible for remaining faults and questionable assertions.

Kerstin Niklasson and Monica Peijne typed the many drafts with their usual competence and helpfulness. I promise never more to use capital U and miniscule u as symbols in the same text. Siw Andersson drew the figures and Barbara Mikalson checked the English.

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Stockholm School of Economics — Ragnar Lindgren
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1. Introduction and General Assumptions

1.1 INTRODUCTION

In most developed countries a large part of the gross income is paid to the government in the form of taxes to be used for public consumption and transfer payments. These taxes affect the economy in many ways. Households adjust their labour supply, their investments in education, their savings and their portfolios. Firms adjust their financial structure, their capital expenditure pattern, their production technology and their product line. As a result of the multitude of effects on individual decisions, the country's economic development will be different from what it would have been under a different tax system.

The purpose of this dissertation is to contribute to the understanding of the effects of taxes on capital income and corporate profits. Such understanding has a scientific value in itself. I also hope that more knowledge may lead as well to improved economic policy.

Taxes on capital income and corporate profits yield far less revenue than taxes on labour income and consumption. However, in spite of their limited importance for tax revenues, they have far-reaching effects on capital formation and the structural development of trade and industry. The greater part of tax evasion efforts are also aimed at these taxes.

The subject is immense. The relevant Swedish tax law comprises about 500 pages and it is supplemented by thousands of legal cases. Millions of agents with different endowments, preferences and information are affected. The institutional environment is complex. In order to deduce any interesting results it is necessary to concentrate on a few aspects of the tax system and to model the institutional surroundings and the behaviour of agents in a very simplified way. At the same time one must never forget that the results depend upon the assumptions. In economic models as well as in data processing, rubbish in gives rubbish out.

The choice of assumptions naturally reflects the author's beliefs about the real world. We try to choose the assumptions in such a way that the results will be relevant for the society in which we live. However, the choice of assumptions is also a part of the definition of the problem. If one assumed that all households have equal endowments and preferences, one would not be interested in analyzing effects on income distribution. If one assumed that all goods are traded on the basis of prices determined on the international market, one would not be interested in short-term business cycle effects. In Section 1.2 I will state and discuss the assumptions generally made in this dissertation.

Section 1.3 contains an overview of the contents of the following chapters. In order to whet the appetite of the reader I also indicate the main results without reporting the nuances and background discussions which can be found in the respective chapters.

The dissertation is problem oriented. It grew out of a short paper on the effects of inflation in an economy with nominal interest loans and conventional tax rules and accounting rules. I had the ambition to analyze the effects of inflation in such an economy more thoroughly, but I soon found it necessary to first analyze the effects of taxes per se. This turned out to require so much effort that I never got around to the effects of inflation. I also found that most of the theoretical problems are associated with determining the effects of the effective tax rates. Calculating the impact of inflation on the effective tax rates is not very difficult theoretically, although it may require much empirical work and a detailed study of the tax system. One drawback of starting from a problem instead of from some recent scientific paper is that the age of the relevant literature and the originality of my own work has turned out to vary a good deal across chapters.

1.2 ASSUMPTIONS

1. Market economy with rational agents

The economy is assumed to consist of a private sector and a public sector. The public sector is financed by taxes on households and firms in the private sector. Public consumption, investment and savings will usually be assumed to be constant and independent of the tax system. All adjustments take place within the private sector. Households maximize utility and firms maximize expected profits or the market value of their shares. All agents are assumed to behave rationally and, generally, to have full information about the state of the economy and the probability distribution of future states. Households and firms behave competitively, that is they assume that prices and interest rates are independent of their own decisions.

2. Market equilibrium

All prices and wage rates are assumed to be flexible and to adjust immediately to equilibrium values. This assumption reflects my intention to concentrate on structural effects. I do not analyze how a tax cut can affect unemployment during a recession. One reason is that I believe that the use of capital income tax adjustments as a part of stabilization policy would cause excessive harm by increasing uncertainty.

3. Constant tax revenues

I will assume that the total tax revenue is kept constant. In Chapter 3 I assume that any decrease of the capital income tax or corporate profit tax is compensated by an increased tax on labour income or consumption. In Chapters 4, 5, 6 and 7 total tax revenue on capital income is kept constant but the tax structure is differentiated in various ways. I can then also assume that public expenditures and the budget deficit are constant and independent of the tax adjustments.

Separating decisions regarding tax policy from decisions regarding the level of government expenditure and the government deficit seems to be a reasonable representation of the political process and simplifies the analysis considerably. The alternative of assuming that an increase of the capital income tax is used for some increase of public expenditure would require specific assumptions about the nature and benefit of this expenditure and would make the analysis arbitrary. To assume that the additional tax revenue is used to decrease the budget deficit would make the dissertation an exercise in macroeconomics.

In older literature the effects of taxes are often analyzed using partial models. A tax is increased but the tax revenue is not used for anything of value. Such models can yield confusing and misleading results. Typically the tax has large income effects. Increasing a tax on labour income is said to reduce labour supply because of the substitution effect (leisure costs less relative to goods which are paid from taxed income) and to increase labour supply because of the income effect (because of lower net income the households reduce both expenditures on goods and leisure). The net effect is usually said to be ambiguous. But if the tax revenue is used to reduce other tax rates or to increase transfers there is no income effect. If the tax revenue is used to increase public consumption there is no income effect if such consumption is a perfect substitute to private consumption.

Under the stated assumptions, income effects would be expected to be small. They are restricted to two sources:

- The tax adjustment can change the income distribution. Increasing the capital income tax and reducing a consumption tax might reduce the net income of households with a high propensity of savings and increase the net income of households with a low propensity of savings. We then get a negative income effect on savings. Such income effects are eliminated if we assume that preferences are equal across households and homothetic (all income elasticities are equal to unity) or if we assume that the tax adjustment is made in such a way that the income of each household is kept constant.

- The tax adjustment can change the 'excess burden' or 'deadweight loss' of taxation. This will be a secondary effect which is a function of the primary effects of the tax adjustment. We will find that this income effect is always dominated by the substitution effect.

Keeping total tax revenue constant also simplifies the analysis by making it reasonable to assume that the labour supply will be unaffected by adjustments of household taxes. For plausible utility functions the substitution effect on the labour supply will be the same for equal yield labour income taxes, consumption taxes and capital income taxes. The main condition is that capital income is derived from capital which is originally created by savings from labour income. The conditions are formally derived in Appendix 3:1. (Readers who are not used to the models and mathematics of that appendix should first read Chapters 2 and 3.)

4. Perfect foresight regarding future taxes

In most studies of the effects of taxation it is implicitly assumed that the agents behave as if the tax system would be constant over time. This is a restrictive assumption. In the real world the tax planning of households and firms is as much concerned with expected changes in tax law as with actual taxation. In a theoretical work the distinction might not be important. The tax system studied can be interpreted to be the expected tax system which governs the planning of the agents. In empirical work, however, the distinction is critical. When measuring the effects of taxation one must know what expectations the tax-payers have regarding future tax law.

Unexpected changes in taxation might have special effects if the stock of savings cannot be adjusted immediately or if corporate equity capital is locked in by an increased tax on dividends. I will discuss such effects in Chapter 5 showing that an unexpected tax increase can have the same effect as a lump-sum tax on existing capital. Even though I believe that they are important, I do not discuss the effects of frequent unexpected tax changes which might create general uncertainty. Such uncertainty and mistrust of government intentions can damage the political system and can lead to neurotic passivity or emigration.

5. Prices of goods are constant

I will generally assume that all relative goods prices are determined on an international market and that they are independent of domestic taxation.

6. The labour market is closed

I will assume that no labour migration occurs because of taxes. Real wages are determined by the marginal productivity of labour.

7. Small open capital market or closed capital market

The effects of capital income taxes depend upon whether the capital market is open or closed. With an open capital market the rate of interest is determined internationally and is independent of domestic taxation. With a closed capital market the interest rate is determined endogenously by the balance of domestic supply and demand of capital which depend upon domestic taxation.

How can we judge whether the capital market of an economy is open or closed? For the interest rate to be perfectly exogenous the country should be small with respect to the world economy, changes in domestic tax rules should not affect tax rules in other countries, capital movements should not be restrained by effective exchange controls and foreign investors should not fear political risks. A preference for risk diversification would then cause domestic investors to hold much foreign assets and domestic firms would be financed to a large extent by foreign residents.

Treating the capital market as closed is reasonable if the economy is so large that the rest of the world is relatively insignificant, if the capital market is isolated by legal restrictions or underdevelopment, or if we want to study the effect of worldwide tax reforms.

For a small country like Sweden with a large proportion of international trade and many multinational corporations, the small open economy model would be the obvious choice in the absence of restrictions on capital movements. But such restrictions do exist and the government tries to control the net private capital flow. The portfolio choice of households is severely restricted and domestic credit restrictions exist.

It could be interesting to study the effects of different tax systems on the Swedish mixed economy of today. To do so, one should probably model the credit restrictions and the exchange controls as endogenously determined. I judged, however, that it would be far simpler to start with a model of a more liberal economy and that any study linked to the specific Swedish institutional conditions would probably become outdated during the course of the study. Thus I chose the open economy model as my principal case, while I also derive results for a perfectly closed economy. The reader may choose between interpreting my results as actual results in an open economy or as indications of the need for controls.

When comparing the results with those of earlier studies it is essential to observe whether the studies refer to closed or open economies. Most classical papers in the international literature refer to closed economies. During the 1980s capital markets have, however, become more obviously integrated and economists have taken more interest in the open economy model. This is reflected in the recent survey article of Kotlikoff

[1984] (see pp. 1582-1584). In Sweden the closed economy assumption has been used by Agell and Södersten [1982] whereas Normann [1979] used the open economy assumption.

1.3 SUMMARY OF CONTENTS AND CONCLUSIONS

In Chapter 2 I present the theory of savings and investment decisions. The aim of that chapter is not to derive new results, but to introduce the models and the concepts that I will use in the following chapters. I also try to give the non-specialist reader a comprehensive introduction to the subject.

In Chapter 3 I introduce idealized taxes on capital income and corporate profits and analyze the general effects of such taxes on savings, aggregate investment and non-financial risk taking. The main result of that chapter is that taxes on capital income and corporate profits, with few exceptions, create distortions in savings and investment and that, under plausible conditions, such taxes are socially inferior to a tax on labour income or consumption.

In Chapter 4 I examine the effects of taxing income from corporate equity capital at a higher rate than income from corporate debt. I model the choice of an optimal corporate debt ratio as determined by the balance of debt-related costs, such as bankruptcy costs, and the tax advantages of debt. I conclude that a more pronounced difference in tax rates will usually lead to a higher debt ratio, a lower capital intensity and less nonfinancial risk taking. The risk of bankruptcy will increase with the difference in tax rates. However, I cannot exclude the possibility of contrary results and I try to clarify under what assumptions and for which parameter values different results will be obtained.

In Chapter 5 I examine the effects of taxing corporate dividends at a higher rate than retained earnings. I model the choice of an optimal corporate dividend policy and an optimal corporate new issue policy and I derive the marginal cost of equity capital as a function of the dividend ratio. I also derive steady state equilibrium conditions for the return on equity capital and the financial policy as functions of the growth rate of the economy. With a high growth rate or little tax differentiation, marginal investments will be financed by new issues of equity capital, Tobin's q will be equal to 1 and the marginal cost of retained earnings will be equal to the marginal cost of new issues. With a low growth rate or more tax differentiation, all investment will be financed by retained earnings, Tobin's q is less than 1 and the capital cost depends on the growth rate. In both cases increased tax differentiation will reduce the dividend ratio. In the new issue case (case A) the cost of capital is increased, in the other case (case B) it is decreased. I discuss the stability of the two cases, the viability of increasing tax differentiation with constant tax revenue and the effects on social welfare.

In Chapter 6 I investigate the effects of differentiating corporate and household taxes depending on the portfolio choice. I derive the conditions under which a neutral tax will be optimal. The analysis is complicated by the effects on total savings and total capital intensity as well as by the ambiguity in social valuation of private risk taking. I discuss the cases of risk neutral and risk averse governments (a discussion of this problem is also found in Chapter 3, Section 7).

In Chapter 7 I analyze the effects of taxing capital gains on a realization rather than on accrual basis. I study not only the usual certainty case but also the case where the asset price is determined by a stochastic diffusion process. I find that a proportional tax on realization gains, which yields the same expected tax revenue as a tax on accrual gains, will lead to less investment in risky assets. I also discuss the effects of a realized gains tax when the investor can choose the holding period. The tax then causes asymmetric behaviour depending upon whether the asset price has increased or decreased. Finally I demonstrate that a tax on realized gains would have the same effects as a proportional tax on accrual gains if the tax rate were progressive in the size of the relative capital gain according to my equation (7:1).

REFERENCES

Agell, J., and Södersten, J. [1982], Skatteregler och realinvesteringar, in Kreditpolitiken: Fakta, teorier och erfarenheter, SOU 1982:53, Stockholm, pp. 271-320.

Kotlikoff, L.J. [1984], Taxation and Savings: A Neoclassical Perspective, Journal of Economic Literature, Vol. XXII (Dec.), pp. 1576-1629.

Normann, G. [1979], Teoretisk analys av reformerad bruttobeskattning, in Bruttoskatter, DsB 1979:3, Stockholm.

2. Elementary Theory of Savings and Investment Decisions

2.1 INTRODUCTION

In this chapter I will present the theory of savings, portfolio choice and corporate investment decisions in a world without taxes. The purpose of this chapter is not to present any new results, but only to introduce the models and the language that I will use in the following chapters and to give the non-specialist reader an introduction to the subject. I also hope that the chapter will serve as a review that is easier to comprehend than the original literature.

In Sections 2.2, 2.3, and 2.4 I assume that the future is known with certainty. I discuss how the consumption/savings and investment decisions are modelled in the three types of models which dominate the literature: that is, the simple two period model, the life cycle overlapping generations model and the continuous time model. I primarily discuss the small open economy case in which the rate of interest is determined on the international capital market and the savings and domestic investment decisions are taken independently and determine the balance in the current account. I show for comparison how the market rate of interest is determined endogenously in a closed economy model, equalizing savings and investment.

The portfolio choice problem is of no interest in certainty models. In open economy models investors choose the asset that pays the highest yield; in closed economy models all assets must be priced so that they pay the same yield.

In Section 2.5 I introduce uncertainty and expected utility maximization. I review the literature on the effects of uncertainty on the savings decision.

In Section 2.6 I discuss the portfolio choice under uncertainty. I also discuss separability of the savings decision and portfolio choice.

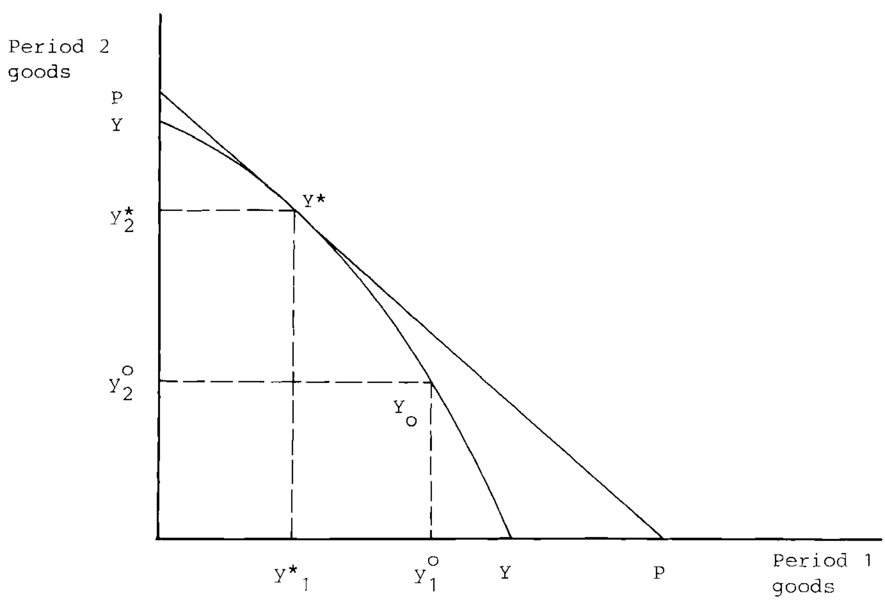
2.2 THE TWO PERIOD MODEL UNDER CERTAINTY

The simplest framework in which savings and investment decisions can be analyzed is the two period model as developed by Irving Fisher [1930] and Hirshleifer [1958] and [1970]. This model has the advantage of being simple to use and understand. It can easily be illustrated graphically. The disadvantage is that one can only see the effects on savings and investments in period 1. It is not suitable for dynamic analysis or in evaluating the development of the capital stock over time. The model implies that savings are consumed and loans are repaid in period 2. As we will see in Section 2.4, a more realistic model might show that the capital stock and foreign debt are accumulated over a long time.

Let us study a simple economy with only one type of good and one type of household. We divide time into two periods, today and tomorrow, or 1 and 2.

We assume that the production of consumption goods is y_1^o in period 1 and y_2^o in period 2 if nothing is saved and invested. If we abstain from some production in period 1 more can be produced in period 2. The maximum production possible in period 2, y_2, for any given production in period 1, y_1, is illustrated in Figure 2:1 by the production possibility curve Y. This is concave because of decreasing returns to investment. (The most profitable investments are made first. I assume that investments are perfectly divisible, so that we do not have locally increasing returns to scale.)

Figure 2:1 Allocation of production over time



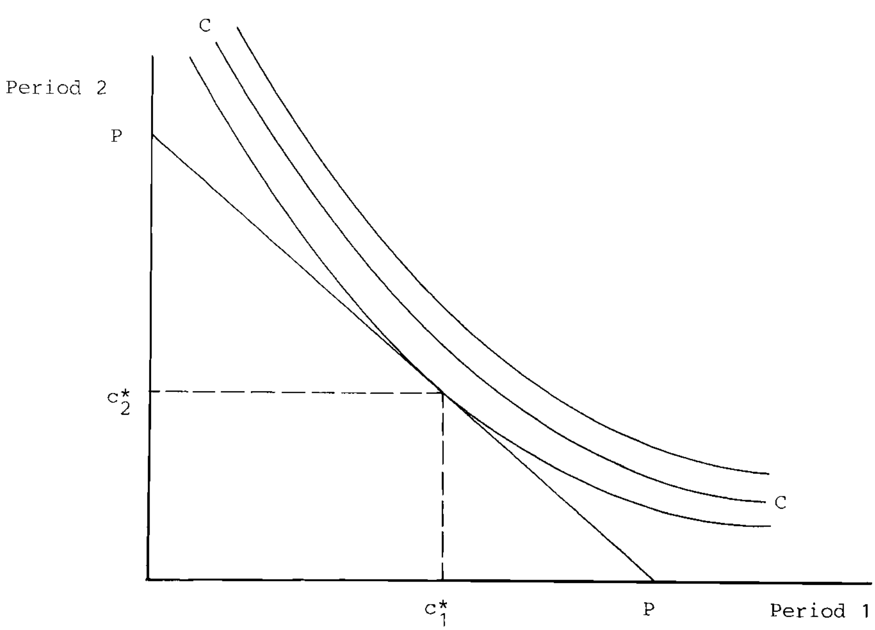
The price line P in Figure 2:1 represents the relative price of goods in periods 1 and 2. If the real interest rate is r, these prices will be 1+r and 1. The slope of the line is - (1+r). Profit maximizing firms will choose the point on Y where the present value of the total production, that is $y_1 \cdot (1+r) + y_2$, has a maximum. This is the point at which curve Y is tangent to line P. At the optimum point, Y*, the production is y_1^* and y_2^*. The firms invest $y_1^o - y_1^*$ in period 1 and raise the production in period 2 by $y_2^* - y_2^o$.

If the interest rate, r, decreases, the slope of P is less steep. We see directly that y_1^* decreases and y_2^* increases, that is the investment volume increases if the interest rate falls.

The preferences of consumers over consumption in periods 1 and 2, c_1 and c_2, can be illustrated by indifference curves as in Figure 2:2. The curves connect those combinations of c_1 and c_2 between which the consumer is indifferent. Alternatively, the preferences can be represented by an ordinal utility function

$$U = U(c_1, c_2) \qquad (2:1)$$

Figure 2:2 Allocation of consumption over time



Each value of U corresponds to one indifference curve. Consumers are always assumed to prefer more to less; that is the partial derivatives U_1 and U_2 are positive and an indifference curve further from the origin represents a higher level of utility. Consumers are also assumed to have diminishing marginal rates of substitution. This assumption implies that the indifference curves are convex. U_1/U_2 is a decreasing function of c_1/c_2.

The cost of any consumption vector (c_1, c_2) is $c_1 \cdot (1+r) + c_2$. In order to maximize the utility of any income, the consumers choose the point of tangency of an indifference curve to the budget line P. This is mathematically equivalent to maximizing U under the budget constraint.

$$y = c_1(1+r) + c_2 \tag{2:2}$$

The first order condition of an optimal solution is

$$U_1 = (1+r) \cdot U_2 \tag{2:3}$$

The interrelationship between production and consumption in an open economy with an international market rate of interest r is illustrated in Figure 2:3. With the production possibilities, the consumer preferences and the interest rate used in that figure, our country will borrow $c_1^*-y_1^*$ in period 1 and repay $y_2^*-c_2^* = (c_1^*-y_1^*) \cdot (1+r)$ in period 2. The domestic investment, $y_1^o-y_1^*$, is financed by domestic savings, $y_1^o-c_1^*$, and borrowings, $c_1^*-y_1^*$.

Figure 2:3 Intertemporal equilibrium in an open economy

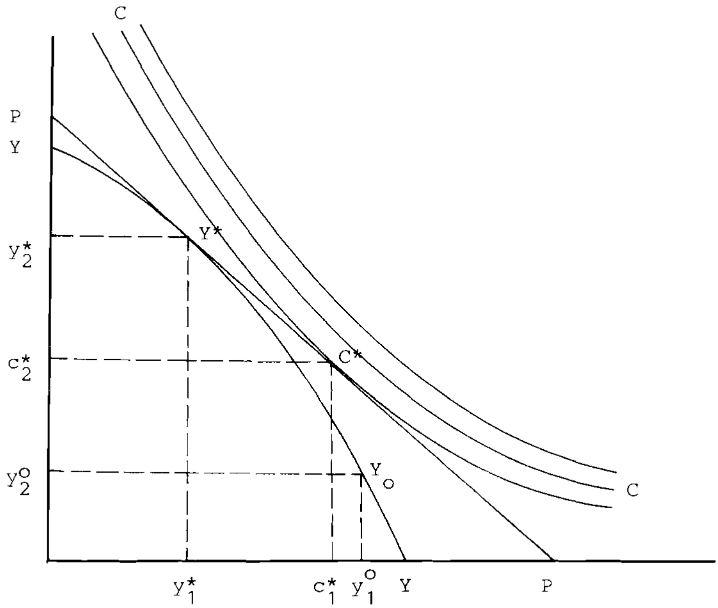
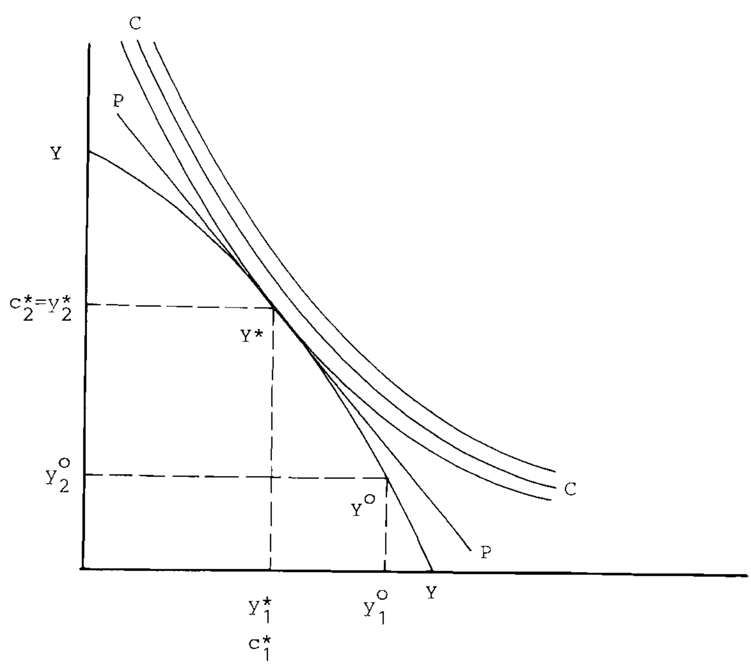


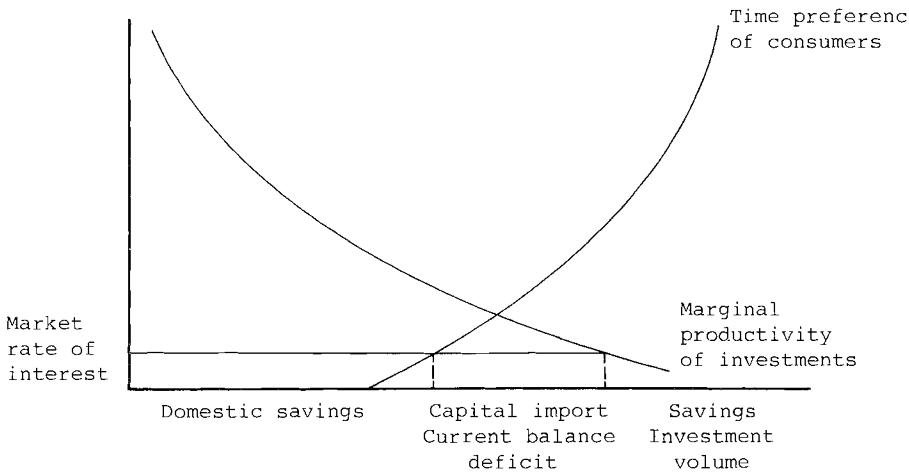
Figure 2:4 Intertemporal equilibrium in a closed economy



A usual problem is to determine whether savings increase or decrease when the market rate of interest is increased. This is equivalent to asking whether c_1^* decreases or increases. Increasing r implies a steeper slope of line P. We immediately see that c_1^* would decrease if the consumers received additional income so that they could remain on the same indifference curve: that is, the substitution effect on savings is positive. Without this hypothetical extra income the budget line through the new Y* would be tangent to a lower indifference curve if y_1^* is less than c_1^*. The utility level decreases because foreign loans become more expensive. If consumption in the two periods are normal goods (i.e. if a consumer would increase both c_1 and c_2 if income increased, which is a reasonable assumption) the "income" effect on savings is positive if y_1^* is less than c_1^*, that is, if we have foreign loans. On the other hand, the income effect will be negative if the country has net claims on foreign countries. We can thus conclude that an increased rate of interest will result in higher savings in a country with foreign net debts. For a country with foreign net claims, the result depends on the relative magnitude of the substitution and the income effects. When the net claims are small, the substitution effect will dominate; when the claims are large, the income effect might dominate. An algebraic treatment of this problem is found in Atkinson and Stiglitz [1980] pp. 69-77.

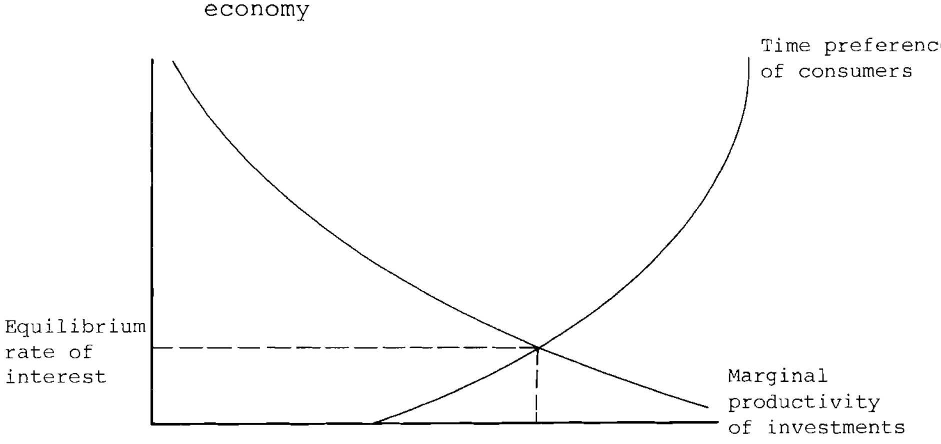
Loans to and loans from foreign countries correspond to a current balance surplus or deficit. The surplus in period 1 is $y_1^*-c_1^*$. We have found that y_1^* increases when the rate of interest increases and that c_1^* decreases if the net claims on foreign countries are not large. Thus, an increased rate of interest will result in a more positive current balance, at least if the country does not have large net claims on other countries. The relationship between interest rate, investments, savings and capital import (= current balance deficit) is illustrated in Figure 2:5.

Figure 2:5 Savings and investments as functions of r. Open economy



In a <u>closed</u> economy capital imports or exports cannot occur. c_1^* equals y_1^* and c_2^* equals y_2^*. As illustrated in Figure 2:4 we find the optimum Y* and C* where an indifference curve is a tangent to the production possibility curve. The rate of interest is determined endogenously by the slope of the curves at the optimum point. Figure 2:6 illustrates how the market rate of interest is determined by the equilibrium of savings and investments.

Figure 2:6 Savings and investments as functions of r. Closed economy



A comparison of Figures 2:3 and 2:4 illustrates that the consumers can always attain a higher utility level in an open economy than in a closed economy (if the market rate of interest does not happen to be exactly equal so that no capital movements occur). In the open economy the optimum is on an indifference curve which does not touch the production possibility curve but is further out. This corresponds to the result of foreign trade theory that trade is always beneficial when all households are equivalent.

We also note that in a closed economy savings and investment are determined simultaneously. In an open economy the investment decision is independent of the savings decision, which only affects the allocation between domestic savings and foreign financing.

2.3 THE OVERLAPPING GENERATIONS MODEL

In the overlapping generations model as developed by Samuelson [1958] and Diamond [1965] and [1970] each generation has one active period when they work and save and one period of retirement when they spend the savings and the interest income. The retirement period of one generation coincides with the active period of the next generation.

Each generation maximizes its utility of consumption in the two periods. In most applications individuals have no concern for the welfare of their children or parents and thus no private intergenerational transfers occur.[1] The utility function and the budget constraint would be

$$U = U(c_1\ c_2) \tag{2:4}$$

$$c_1^t + c_2^t = w_t + r_{t+1}\ s_t \tag{2:5}$$

$$s_t = w_t - c_1^t \tag{2:6}$$

where c_1^t and c_2^t are levels of consumption in periods t and t+1 respectively of the generation which is young in period t, w_t is wage income in period t, s_t is the savings of this generation and r_{t+1} is the anticipated interest rate in the next period. s_t is thus determined as a function of w_t and r_{t+1}

$$s_t = s(w_t, r_{t+1}) \tag{2:7}$$

In a closed economy the capital stock in period t+1 is equal to the savings of generation t. If the population is constant

$$k_{t+1} = s_t \tag{2:8}$$

The production in period t+1 and the marginal productivities of capital and labour are determined by k_{t+1} and thus by s_t.

$$w_{t+1} = w(s_t) = w[s(w_t, r_{t+1})] \tag{2:9}$$

The savings in the next period are determined by w_{t+1}. In a closed economy we thus get a dynamic adjustment process following any disturbance such as a change in tax rates. Under reasonable assumptions a steady state equilibrium with constant w, r and s is approached asymptotically.

The closed economy case has been predominant in the literature when using the overlapping generations model. Two examples of applications for open economies are Buiter [1981] and Persson [1983]. Persson shows that in a small open economy, where the interest rate is determined by the international capital market, the adjustment process is much simpler. The capital stock k_{t+1} is then independent of domestic savings and thus w_{t+1} and r_{t+1} are independent of s_t. A new equilibrium is established only one period after a parameter change. The welfare of one generation depends on the actions of the preceding generation only if intergenerational transfers or government debt exist.

The overlapping generations model has been used to analyze the effects of government debt, of government intergenerational transfers and of capital income taxes. It is well suited for the study of welfare distribution between generations. The results of such studies naturally depend on the assumptions of the model. It seems to me that great care is necessary in evaluating results from the overlapping generations models because the assumptions are often restrictive. Some points to look out for are

- The existence or non-existence of private transfers is of great importance as shown by Barro [1974].

- Diamond's well known studies refer to closed economies and the results are not applicable to small open economies.

- In the model all savings come from labour income and consumption is allocated only between the active period and the retirement period. The allocation of savings within the active period might be of equal importance.

In the overlapping generations model the direction of the effect of an increase in the interest rate on savings is equivocal and highly dependent on the form of the utility function. This can be illustrated by using the additive power utility function which is isoelastic and homothetic.

$$U = \frac{1}{\gamma} \cdot c_1^{\gamma} + \frac{1}{1+\rho} \cdot \frac{1}{\gamma} \cdot c_2^{\gamma}\ , \ \gamma < 0 \text{ or } 0 < \gamma < 1 \qquad (2:10\text{ a})$$

$$U = \ln c_1 + \frac{1}{1+\rho} \ln c_2, \ \gamma = 0 \qquad (2{:}10\ b)$$

The first order condition of optimal consumption allocation then is

$$c_1^{\gamma-1} = \frac{1+r}{1+\rho} \cdot c_2^{\gamma-1} \qquad (2{:}11)$$

and substitution in the budget constraint

$$s = w - c_1 = \frac{c_2}{1+r} \qquad (2{:}12)$$

gives

$$s = \frac{w}{1 + \left[\dfrac{1+\rho}{(1+r)^{\gamma}}\right]^{\frac{1}{1-\gamma}}} \qquad (2{:}13)$$

Thus $\partial s/\partial r$ has the same sign as γ and is zero for the logarithmic utility function.

In Section 2.2 we found that in the simple two period model $\partial s/\partial r$ could be negative if savings in the first period were positive and large compared to total consumption. This is obviously the case in the overlapping generations model where no wage income is received in the second period. In Section 2.4 I will compare these results to those of the continuous time model and I will assert that the assumptions of the overlapping generations model tend to generate extreme results.

2.4 CONTINUOUS TIME MODELS UNDER CERTAINTY

For more detailed studies of the consumer's lifetime allocation process and the development of capital stocks over time it is natural to extend the two period model to a model with an infinite number of periods or to a continuous time model.

The choice between these alternatives is just a matter of mathematical convenience if there is no uncertainty. A classical paper using the continuous time model for the analysis of the consumer's lifetime allocation process is Yaari [1964]. More recent examples are Summers [1981 b] and Chamley [1981]. The model has also been used extensively in the human capital literature. One example is Ben-Porath [1967].

Yaari assumed that an individual has a finite lifetime and that he might have a bequest motive. In order to simplify my exposition I will assume infinite life. This assumption is reasonable if individuals care for their children's welfare as for their own. We thus maximize the utility of the family over infinite time. The decision problem is to choose a consumption stream $c(t)$ which maximizes $U[c(t)]$ subject to the constraint that the discounted cost of $c(t)$ may not be higher than the sum of the endowment wealth, k_o^d, and the discounted value of the wage stream, $w(t)$, at the anticipated rate of interest, $r(t)$.

In a small <u>open</u> economy, $r(t)$ is given by the international capital market. Without loss of generality we can then assume that the interest rate is a constant, r. The invested capital $k^p(t)$ is chosen such that the marginal productivity equals the interest rate. If the interest rate is constant and the technology and the labour supply are also assumed to be constant, the optimal k^p will be constant. The wage rate, w, will then also be constant over time.

Thus the consumer will choose that consumption stream $c(t)$ which maximizes $U[c(t)]$ subject to k_o^d, w and r. The first order condition for an optimal $c(t)$ can be found by using Euler's equation

$$\dot{U}_t = - r \cdot U_t \qquad (2:14)$$

where U_t is the marginal utility at time t and $\dot{U}_t$ is the time derivative of U_t. (The discrete time equivalent of (2:14) is

$$\frac{U_{t+1} - U_t}{U_t} = - r \quad \text{or} \quad U_t = (1+r)\cdot U_{t+1} \tag{2:15}$$

which is the same condition as we derived in the two period model (equation (2:3)) and which is just the usual condition that the marginal utilities of two goods should be proportional to their prices.)

If we make the usual assumption that the utility function is additive with a subjective discount rate ρ, which is an index of impatience or the risk that the benefit of savings disappears due to some catastrophe, and if the consumption needs of the family are assumed to be constant, we have

$$U = \int_0^{\infty} u[c(t)]\cdot e^{-\rho t}\cdot dt \tag{2:16}$$

$$U_t = \frac{\partial U}{\partial c_t} = u'(c_t)\cdot e^{-\rho t} \tag{2:17}$$

$$\dot{U}_t = \frac{\partial U_t}{\partial t} = u'(c_t)\cdot e^{-\rho t} - \rho\cdot u'(c_t)\cdot e^{-\rho t} \tag{2:18}$$

Substituting in (2:14)

$$\frac{\dot{U}_t}{U_t} = \frac{\dot{u}'(c_t)}{u'(c_t)} - \rho = - r \tag{2:19}$$

$$\frac{\dot{u}'}{u'} = \rho - r \tag{2:20}$$

(2:20) indicates that if the marginal utility of consumption is positive, optimal consumption will decrease with time if $\rho > r$ and increase with time if $\rho < r$. The lower the interest rate r is, the more will consumption decrease with time.

In order to get explicit solutions of the consumption allocation and the capital stocks we must specify the form of $u(c_t)$. As in (2:10) above I assume that $u(c_t)$ is isoelastic, that is

$$u(c_t) = \frac{1}{\gamma} \cdot c_t^{\gamma} \ , \ 0 < \gamma < 1 \text{ or } \gamma < 0 \qquad (2:21\ a)$$

$$u(c_t) = \ln c_t \ , \ \gamma = 0 \qquad (2:21\ b)$$

With this specification we always get interior solutions with all $c_t > 0$ since $u'(c_t) = c_t^{\gamma-1}$ goes to infinity when c_t approaches zero. As pointed out by Yaari, the income elasticity is always 1, that is if income is doubled, consumption will be doubled in all periods.

With $u'(c_t) = c_t^{\gamma-1}$ we can rewrite (2:20)

$$\frac{\dot{u}_t}{u_t} = (\gamma-1) \cdot \frac{c_t^{\gamma-2} \cdot \dot{c}_t}{c_t^{\gamma-1}} = (\gamma-1) \cdot \frac{\dot{c}_t}{c_t} = \rho - r \qquad (2:22\ a)$$

Solving this simple differential equation

$$c_t = c_o \cdot e^{\frac{\rho-r}{\gamma-1} \cdot t} \qquad (2:22\ b)$$

where c_o is the consumption at time 0. Note that $\gamma-1$ is by definition negative. The budget constraint is

$$\int_0^{\infty} c_t \cdot e^{-rt} \cdot dt = k_o^d + \int_0^{\infty} w \cdot e^{-rt} \cdot dt \qquad (2:23)$$

Substituting (2:22) and solving we get

$$c_o = \frac{\rho - r \cdot \gamma}{1-\gamma} \cdot (k_o^d + \frac{w}{r}) \quad \text{if } \rho > r \cdot \gamma \qquad (2:24\ a)$$

$$c_o = 0 \quad \text{if } \rho \leq r \cdot \gamma \qquad (2:24\ b)$$

If γ is positive and $\rho < r$ the consumption rate might thus be infinitely low the first years. This can either be interpreted as implying that such utility function parameters are unlikely or as implying that the infinite horizon optimization is unrealistic.[2]

It can easily be seen that (2:24 a) will hold not only for t=0 but generally. The starting point is arbitrary.

$$c_t = \frac{\rho - r \cdot \gamma}{1-\gamma} \cdot (k_t^d + \frac{w}{r}) \qquad (2:25)$$

Interpreting (2:25) we observe that consumption is determined by the product of a factor containing ρ, r and γ and the present value of total resources. For a logarithmic utility function, the first factor is simply the impatience index, ρ, while the interest rate, r, influences consumption only through the wealth effect in discounting future labour income.

The savings rate s_t is the difference between income and consumption

$$s_t = w + r \cdot k_t^d - c_t \qquad (2:26)$$

Substituting (2:25)

$$s_t = \frac{r-\rho}{1-\gamma} \cdot (k_t^d + \frac{w}{r}) \qquad (2:27)$$

Thus the derivative of s_t with respect to r is

$$\frac{ds_t}{dr} = \frac{1}{1-\gamma}\,(k_t^d + \frac{\rho \cdot w}{r \cdot r}) \tag{2:28}$$

This is positive if k_t^d is not strongly negative.

We can derive k_t^d from our model

$$k_t^d = k_o^d \cdot e^{rt} + w \cdot \int_0^t e^{r \cdot (t-\tau)} \cdot d\tau - \int_0^t c_\tau \cdot e^{r(t-\tau)} \cdot d\tau =$$

$$= k_o^d \cdot e^{\frac{r-\rho}{1-\gamma} \cdot t} + \frac{w}{r}[e^{\frac{r-\rho}{1-\gamma} \cdot t} - 1] \tag{2:29}$$

Thus

$$\frac{ds_t}{dr} = \frac{1}{1-\gamma}\,[(k_o^d + \frac{w}{r}) \cdot e^{\frac{r-\rho}{1-\gamma} \cdot t} + \frac{w}{r}(\frac{\rho}{r} - 1)] \tag{2:30}$$

The first term is always positive due to the budget constraint and the second term is positive if $\rho > r$. Thus the derivative is positive if $\rho \geq r$. If $\rho < r$, k_t^d can never be negative as long as savings behaviour has always been in accordance with our model. Thus the derivative is always positive if savings have always followed our model and it seems probable that it would be so even if we started with a negative k_o^d after some shock. As noted by Summers [1981 b] we thus get a much more positive response of savings to interest rate increases in the continuous time model than in the overlapping generations model. This is due to the wealth effect of heavier discounting of future wage income, and to the existence of interest income in all periods and not only in the last period. If we divided the periods of the overlapping generations model into many subperiods, we would get a positive wealth effect on savings in most subperiods of the first period from heavier discounting of future wage income, and interest income would increase in the subperiods when we make savings decisions as well as in the last subperiod.

From (2:30) we see that if $r > \rho$ the stock of domestic savings will increase indefinitely. This would be true even if future wage income were zero.

If $r = \rho$ we find $c = r \cdot k_o^d$ and k^d is constant. Nothing is saved. (Note that we assumed constant w.)

If $r < \rho$ the stock of domestic savings will approach $-\frac{w}{r}$ asymptotically. The foreign debt would approach the sum of all invested capital, k^p, and the present value of all future labour income. Our assumption of a perfect international capital market would probably not be very realistic under such circumstances!

In a closed economy the stock of savings, k^d, and the invested capital, k^p, must be equal. The interest rate, $r(t)$, is determined endogenously. If we start with a small capital stock, the marginal productivity of capital, which determines r, might be higher than ρ. Consumers will then save and the capital stock will increase. r will fall and we approach a steady state equilibrium with $\rho = r$ and a constant capital. The consumption allocation over time will thus be more stable in a closed economy than in an open economy. There is no option to trade with countries with different time preferences.

2.5 CONSUMPTION ALLOCATION UNDER UNCERTAINTY

In this section I will introduce uncertainty. I will retain the assumption that there is only one asset in which savings can be invested. The interaction with the portfolio choice problem is discussed in Section 6.

In problems with uncertainty the preference ordering of consumers is usually represented by an expected utility function. Under certain plausible assumptions there exists a utility function U such that

$$E[U(\tilde{c})] = \sum_i [p_i \cdot U(c_i)] \tag{2:31}$$

where $\tilde{c}$ is a random variable which will be c_i with probability p_i. E[U] is expected utility.

In two period models expected utility will be defined over a certain consumption in the first period and an uncertain consumption in the second period. The consumer maximizes

$$V = E[U(c_1, \tilde{c}_2)] \tag{2:32}$$

Three main types of uncertainty problems have been studied

1. Uncertain consumption needs
2. Uncertain future labour income
3. Uncertain return on savings.

<u>Uncertainty about consumption needs</u> can either be due to uncertain lifetime or to uncertain annual needs (say hospital expenses). The effect of uncertain lifetime has been analyzed by Yaari [1965]. Since good life insurance markets exist, this type of uncertainty probably is not very important. The effect of uncertainty about annual consumption needs is analogous to that of uncertainty about future labour income which is discussed below.

The effect of <u>uncertainty about future income</u> has been analyzed in a two period model by Leland [1968] and Sandmo [1970]. A good review is found in Sandmo [1974].

Assume that a consumer has income y_1 in period 1 and a random income, $\tilde{y}_2$, in period 2. He chooses consumption c_1 and savings s so as to maximize an expected utility function

$$V = E[U(c_1, \tilde{c}_2)] \tag{2:32}$$

under the constraints

$$c_1 = y_1 - s \tag{2:33}$$

$$\tilde{c}_2 = s\cdot(1+r) + \tilde{y}_2 \tag{2:34}$$

How is s influenced by increased uncertainty of $\tilde{y}_2$?

Leland [1968] and Sandmo [1970] model increased uncertainty in different ways. The result will be the same if the risk is increased according to the definitions of Rothschild and Stiglitz [1970]. I will model risk as the probability π that the consumer is unemployed in the second period and has no non-capital income ($y_2^1 = 0$). With probability $(1-\pi)$ he is employed and receives an income $y_2^2 = \bar{y}/(1-\pi)$. We see that the expected income is $\bar{y}$ for all π and all risk averters would prefer a low π to a high π. Thus π is an index of increased risk according to the definition of Rothschild and Stiglitz.

I will simplify the analysis of Leland and Sandmo by assuming that the utility function is additive. Then

$$\begin{aligned} V &= U(c_1) + E[U(\tilde{c}_2)] = U(c_1) + \pi \cdot U(c_2^1) + \\ &\quad + (1-\pi) \cdot U(c_2^2) \end{aligned} \tag{2:35}$$

$$c_2^1 = s(1+r) \tag{2:36}$$

$$c_2^2 = s(1+r) + \bar{y}_2/(1-\pi) \tag{2:37}$$

The first order condition of an optimal s is then

$$U_1(c_1) = (1+r)\ E[U_2(\tilde{c}_2)] = (1+r)\cdot[\pi\cdot U_2(c_2^1) + $$
$$+ (1-\pi)\cdot U_2(c_2^2)] \tag{2:38}$$

Differentiating (2:38), (2:33), (2:36) and (2:37) for a constant interest rate r we find

$$\frac{ds}{d\pi} = (1+r) \cdot \frac{[U_2(c_2^1) - U_2(c_2^2)] + \frac{\bar{y}_2}{1-\pi} \cdot U_{22}(c_2^2)}{-U_{11}(c_1) - (1+r)^2 \cdot E[U_{22}(c_2)]} \tag{2:39}$$

The denominator is positive for all risk averters ($U'' < 0$). The numerator is positive if the third derivative U_{222} is positive. This is seen by noting that $c_2^1 - c_2^2 = -\bar{y}_2/(1-\pi)$ and that the sign of the numerator is thus determined by the relative size of $U_{22}(c_2^2)$ and U_{22} for consumption levels below c_2^2. That U_{222} is positive is a necessary condition for the absolute risk aversion to be decreasing in income, which is a necessary (and sufficient) condition for investments in risky assets to be a normal good. Arrow [1971] notes that the hypothesis of decreasing absolute risk aversion 'certainly seems supported by everyday observation' (p. 96).

Thus we find that increased income risk will increase savings for all households with a decreasing absolute risk aversion. This is the background of 'precautionary' savings.

The effect of <u>uncertainty about the rate of return on savings</u>, the 'capital risk', was also analyzed by Sandmo [1970] in a two period model. He finds that the effect of the 'capital risk' is more complicated than that of the 'income risk'.

Assume that the return is r+a with 50 % probability and r-a with 50 % probability.[3] a is thus our index of risk. Certain wage income is y_1 and y_2. The consumer maximizes the expected utility function

$$V = U(c_1) + .5 \cdot U(c_2^1) + .5 \cdot U(c_2^2) \tag{2:40}$$

$$c_1 = y_1 - s \tag{2:41}$$

$$c_2^1 = s \cdot (1+r-a) + y_2 \tag{2:42}$$

$$c_2^2 = s \cdot (1+r+a) + y_2 \tag{2:43}$$

The first order condition for an optimal s is

$$U_1(c_1) = .5 \cdot U_2(c_2^1) \cdot (1+r-a) + .5 \cdot U_2(c_2^2) \cdot (1+r+a) \tag{2:44}$$

Differentiating (2:41)-(2:44) we get

$$\frac{ds}{da} = \frac{[U_2(c_2^2)-U_2(c_2^1)]+s \cdot a \cdot [U_{22}(c_2^1)+U_{22}(c_2^2)]+s \cdot (1+r)[U_{22}(c_2^2-U_{22}(c_2^1)]}{-2\, U_{11}(c_1) - U_{22}(c_2^1) \cdot (1+r+a)^2 - U_{22}(c_2^2) \cdot (1+ra)^2} \tag{2:45}$$

For risk averters the denominator is always positive as in the case above.

The numerator is zero if savings are zero ($c_2^2 = c_1^2$). Increased risk has no affect on savings when savings are close to zero because the risk, a, is multiplicative to savings.

If savings are nonzero the last term of the numerator is always positive if $U_{222} > 0$. The first two terms have the opposite sign of s. Thus the effect of increased risk on savings will always be positive if savings are negative. Increased risk therefore has the same effect as an increase in expected interest which might seem counterintuitive. The reason is that a increases the interest rate in the low utility state and decreases the interest rate in the high utility state. With $U_{22} < 0$ and $U_{222} > 0$ the effect in the low utility state is more important. Borrowings decrease when interest uncertainty increases.

If savings are positive the terms in the numerator will have different signs and the net effect depends on the relative size of U_{22} and U_{222}. With a Taylor expansion around the point a=0 the numerator becomes

$$N = 4 \cdot a \cdot s \cdot U_{22} + 2\, a \cdot s^2 \cdot (1+r) \cdot U_{222} \tag{2:46}$$

Thus N and the effect on savings are zero if

$$- \frac{U_{222}}{U_{22}} = \frac{2}{s \cdot (1+r)} \tag{2:47}$$

For a logarithmic utility function

$$- \frac{U_{222}}{U_{22}} = 2 \cdot \frac{1}{c_2} \tag{2:48}$$

Thus we have the result of Sandmo [1970] that with a logarithmic utility function and no wage income in the second period ($y_2=0$) the effect of capital risk on savings is zero.[4] If y_2 is positive, which would seem to be the normal case, s(1+r) is less than c_2 and the effect on savings is negative.

2.6 SAVINGS AND PORTFOLIO CHOICE UNDER UNCERTAINTY WITH MANY ASSETS

If there is more than one asset and the assets have different risk characteristics, the consumer makes a portfolio choice decision as well as a savings decision. In the general case these cannot be separated. The portfolio choice depends on the amount saved and the savings decision depends on the expected return and risk characteristics of the chosen portfolio. The interrelationship between the two decisions can be studied in models with at least two periods. I will however first introduce the pure portfolio choice problem taking the savings, and thus the wealth, as given.

Let us start with the very simplest case studied by Arrow [1971] and others. The investor has a wealth endowment W_o and can invest in two assets, a bond paying a certain return, r_b, and equity paying a random return, $\tilde{r}_s$. He invests $a \cdot W_o$ in equity and $(1-a) \cdot W_o$ in the bond. His final wealth W_1 is then

$$W_1 = a \cdot W_o \cdot (1+\tilde{r}_s) + (1-a) \cdot W_o \cdot (1+r_b) \tag{2:49}$$

How will he choose a if he wants to maximize the expected utility $E[U(W_1)]$? We easily derive the first order condition

$$E[U'(W_1) \cdot \tilde{r}_s] = r_b \cdot E[U'(W_1)] \tag{2:50}$$

Thus the expected marginal utility of investing in the risky asset must be the same as that of investing in the safe asset. If the investor is risk averse, $U'' < 0$ and the expected value of r_s must be higher than r_b for a solution with both assets contained in the optimal portfolio.

In order to evaluate how the relative investment in the risky asset, a, depends on wealth, W_o, we differentiate (2:50) and obtain

$$\frac{da}{dW_o} = - \frac{E[U'' \cdot \frac{W_1}{W_o} \cdot (\tilde{r}_s - r_b)]}{W_o \cdot E[U'' \cdot (\tilde{r}_s - r_b)^2]} \qquad (2:51)$$

The denominator is negative for $U'' < 0$. Thus da/dW_o has the sign of $E[U'' \cdot W_1 \cdot (\tilde{r}_s - r_b)]$. Introducing the relative risk aversion

$$R_R = - \frac{U''(W) \cdot W}{U'(W)} \qquad (2:52)$$

we have

$$E[U'' \cdot W_1 \cdot (\tilde{r}_s - r_b)] = -E[R_R \cdot U'(W_1) \cdot (\tilde{r}_s - r_b)] \qquad (2:53)$$

According to (2:50) this is zero for constant R_R, that is for all isoelastic utility functions. These are the power utility and logarithmic utility functions. For this class of utility functions the relative amount invested in the risky asset is thus independent of wealth. We also see from (2:53) that if the relative risk aversion R_R is increasing in wealth, da/dW is negative.

Cass and Stiglitz [1970] generalized this model to the case of several risky assets. They showed that for investors with power utility or logarithmic utility functions the relative investment in each asset is still independent of wealth. They also showed that for a much wider set of utility functions, HARA or linear risk tolerance functions, the relative investment in each risky asset is independent of wealth. Investors will invest in the risk free asset and a portfolio of risky assets. HARA utility functions comprise

- The generalized power utility function
  $U = \frac{1}{\gamma} \cdot (c - c_o)^\gamma \ , \quad \gamma < 0 \text{ or } 0 < \gamma < 1$
- the generalized logarithmic function
  $U = \ln(c - c_o) \ , \qquad \gamma = 0$
- the generalized power function
  $U = -\frac{1}{\gamma} \cdot (c_o - c)^\gamma \ , \quad c_o \text{ large}, \ \gamma > 1$
- the exponential function
  $U = -\exp(-\delta \cdot c)$

This 'two-fund' separation result makes it natural to analyze the combined savings and portfolio choice problem with only one risky asset and one risk free asset.

Let us then assume that the consumer has certain incomes y_1 and y_2 in periods 1 and 2. In period 1 he saves s and he invests $a \cdot s$ in an asset with random return $\tilde{r}_s$ and $(1-a) \cdot s$ in a risk free asset with return r_b. He will choose s and a to maximize the expected utility

$$V = U(c_1) + E[U(c_2)] = U(y_1 - s) + E[U(y_2 + s \cdot (1+r_b) + a \cdot s(\tilde{r}_s - r_b))] \tag{2:54}$$

We get two first order conditions. One for optimal savings

$$U_1 = E\left[U_2 \cdot \left(1 + r_b + a(\tilde{r}_s - r_b)\right)\right] \tag{2:55}$$

and one for optimal portfolio choice corresponding to (2:50)

$$E[U_2 \cdot (\tilde{r}_s - r_b)] = 0 \tag{2:56}$$

Substituting (2:56) into (2:55) we get the simpler expression

$$U_1 = (1 + r_b) \cdot E(U_2) \tag{2:57}$$

The condition (2:57) is affected by $\tilde{r}_s$ only through $E(U_2)$.

Sandmo [1969] examined the comparative statics of this model. He found that the effect of yield changes on savings corresponds to that in the two period model under certainty if a is positive. An increase of r_b or a positive shift of $\tilde{r}_s$ then results in a positive substitution effect on savings. The income effect is negative if s is positive and positive if s is negative. If the investment in the risky asset $a \cdot s$ is positive, a positive shift of $\tilde{r}_s$ will always lead to an increased investment in that asset. I will discuss the effect of a general capital income tax in this model in Chapter 3 and the effect of differentiated taxes (one tax rate for r_b and another for $\tilde{r}_s$) in Chapter 6.

Multiperiod and continuous time models with consumption allocation and portfolio choice have also been developed. A good presentation is found in Ingersoll [1981] chapters 7 and 9. Classic articles include Samuelson [1969], Fama [1970], Hakansson [1970], Merton [1969] and Merton [1971]. The results of the multiperiod models are well summarized by Ingersoll [1981], p. 7-12.

1) For investors with time-additive log utility, optimal consumption is a deterministic fraction of wealth which depends solely on the investor's remaining lifetime and his rate of time preference. The optimal portfolio selected each period is identical to that chosen by a single period log utility investor.

2) For investors with power utility other than log facing a constant or deterministically changing investment opportunity set similar results obtain; however, the savings rate is a deterministic function of the opportunity set. If the opportunity set is constant over time, then the investor chooses identical portfolios each period.

3) For investors with non-power HARA utility facing a constant or deterministically changing opportunity set, the savings rate and portfolio are stochastic as they depend upon wealth. However, for a constant opportunity set the risky assets are always held in the same proportions (over time and by investors with different c_o).

4) Any non-log utility investor facing a stochastic investment opportunity set will alter his portfolio from that chosen by an otherwise identical single-period investor if it is possible to "hedge" against unfavorable changes in opportunities. Hedging of this type is generally possible unless the opportunity shifts are statistically independent of contemporaneous and earlier returns.

The results 1-3 correspond to our results in the simpler models above. Result 4 can only be derived in multiperiod models and is of importance, inter alia, when analyzing the time-structure of interest rates.

NOTES

1) One exception is Barro [1974].

2) When discussing the expected utility function Arrow [1971] argues that γ is probably close to zero (p. 98). Summers [1981 b] finds this value most plausible for the multi--period utility function.

3) The two point distribution is chosen for ease of exposition. The results hold for all symmetrical distributions.

4) Merton [1969] and others have shown that this result is also obtained in continuous-time models.

REFERENCES

Arrow, K. [1971], Essays in the Theory of Risk Bearing, Markham.

Atkinson, A.B., and Stiglitz, J.E. [1980], Lectures on Public Economics, McGraw-Hill, New York and Maidenhead.

Barro, R.J. [1974], Are Government Bonds Net Wealth?, Journal of Political Economy 82, pp. 1095-1117.

Ben-Porath, Y. [1967], The Production of Human Capital and the Life Cycle of Earnings, Journal of Political Economy 75, pp. 352-365.

Buiter, W.H. [1981], Time Preference and International Lending and Borrowing in an Overlapping-Generations Model, Journal of Political Economy 89, pp. 769-797.

Cass, D., and Stiglitz, J.E. [1970], The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Allocation; A Contribution to the Pure Theory of Mutual Funds, Journal of Economic Theory, Vol. II, pp. 122-160.

Chamley, C. [1981], The Welfare Cost of Capital Income Taxation in a Growing Economy, Journal of Political Economy, Vol. 89, pp. 468-496.

Diamond, P.A. [1965], National Debt in a Neo-classical Growth Model, American Economic Review 55, pp. 1126-1150.

Diamond, P.A. [1970], Incidence of an Interest Income Tax, Journal of Economic Theory 2, pp. 211-224.

Fama, E.F. [1970], Multiperiod Consumption - Investment Decisions, American Economic Review, pp. 163-174.

Fischer, I. [1930], The Theory of Interest, Kelloy, New York.

Hakansson, N.H. [1970], Optimal Investment and Consumption Strategies under Risk for a Class of Utility Functions, Econometrica, Vol. 38, pp. 587-607.

Hirschleifer, J. [1958], On the Theory of Optimal Investment Decisions, Journal of Political Economy, Vol. 66, pp- 329-352.

Hirschleifer, J. [1970], Investment, Interest and Capital, Englewood Cliffs, N.J.: Prentice-Hall.

Ingersoll, J.E. [1981], Notes on the Theory of Financial Decisions: Class Lecture Notes, Graduate School of Business, University of Chicago, W.P. 26.

Leland, H.E. [1968], Saving and Uncertainty: The Precautionary Demand for Saving, Quarterly Journal of Economics 82, pp. 465-473.

Merton, R.C. [1969], Lifetime Portfolio Selection under Uncertainty: the Continuous-Time Case, Review of Economics and Statistics, LI, pp. 347-357.

Merton, R.C. [1971], Optimum Consumption and Portfolio Rules in a Continuous-Time Model, Journal of Economic Theory 3, pp. 373-413.

Persson, T. [1983], Deficits and Intergenerational Welfare in Open Economies, NBER working paper.

Rothschild, M., and Stiglitz, J.E. [1970], Increasing Risk: 1. A Definition, Journal of Economic Theory 2, pp. 225-243.

Samuelson, P.A. [1958], An Exact Consumption Loan Model of Interest with or without the Social Contrievance of Money, Journal of Political Economy 66, pp. 467-482.

Samuelson, P. [1969], Lifetime Portfolio Selection by Dynamic Stochastic Programming, Review of Economics and Statistics, Vol. 51, pp. 239-246.

Sandmo, A. [1969], Capital Risk, Consumption, and Portfolio Choice, Econometrica, Vol. 137, pp. 586-599.

Sandmo, A. [1970], The Effect of Uncertainty on Saving Decisions, Review of Economic Studies, 37, pp. 353-360.

Sandmo, A. [1974], Two-Period Models of Consumption Decisions under Uncertainty: A Survey, in Allocation under Uncertainty: Equilibrium and Optimality (ed. Drèze, H.H.), The Mac-Millan Press Ltd., London.

Summers, L.H. [1981 b], Capital Taxation and Accumulation in a Life Cycle Growth Model, American Economic Review, Vol. 71, pp. 533-544.

Yaari, M.E. [1964], On the Consumer's Lifetime Allocation Process, International Economic Review 5, pp. 304-317.

Yaari, M.E. [1965], Uncertain Lifetime, Life Insurance and the Theory of the Consumer, Review of Economic Studies 32, pp. 137-150.

3. Effects of Undifferentiated Capital Income Taxation

3.1 INTRODUCTION

In this chapter I shall introduce taxes on capital income and corporate profits and discuss how such taxes affect the savings and portfolio choice decisions of households and the investment decisions of firms. The taxes introduced in this chapter will be idealized in many ways in comparison to actual tax systems.

- They are not differentiated with regard to financing or asset choice.
- Capital gains are taxed in the same way as current income, the realization criterion does not apply.
- Only real income is taxed. Inflation has no real effects.
- The taxes are perfectly symmetric. If income is negative, the tax will be negative.

What I want to discuss is thus the general effect of taxes on capital income or profits. I will concentrate on the distortion of savings, aggregate investments and risk taking.

As already discussed in Chapter 1, I will assume that the alternative to capital income taxes is increased taxes on labour income or consumption, which are equivalent at my level of abstraction. Referring to Appendix 1, I assume that the utility functions are such that an equal yield tax revision from capital income taxes on households to wage or consumption taxes does not affect the labour supply. As shown in the appendix this is, in steady state, generally true for logarithmic utility functions. Under more restrictive assumptions it holds for all isoelastic utility functions. The intuitive explanation of this result is that savings originate from labour income so that a tax on the income from savings tends to affect the labour-leisure choice in the same way as a direct tax on labour income. I also make the usual assumptions of competitive behaviour, profit maximizing firms and constant returns to scale.

The results depend upon the degree of openness of the economy. In a small open economy the market interest rate can be assumed to be determined on the international capital market and it is not influenced by domestic taxes. In that case domestic taxes on households affect household savings and portfolio choice but not corporate investments. Increased household savings result in increased capital export. Corporate taxes have no direct effects on household behaviour. In a closed economy, on the other hand, the rate of interest is determined endogenously and it does not matter much whether it is households or corporations who are taxed.

In Section 3.2 I discuss the effects of capital income taxes on households in a small open economy under certainty using two period and continuous time models. In Section 3.3 I discuss the special effects which can be obtained in an overlapping generations model. In Section 3.4 I discuss the effects of corporate taxes in a small open economy under certainty. In Section 3.5 I analyze the case of a closed economy.

I then turn to uncertainty. In Section 3.6 I analyze the effect of taxes on precautionary savings and in Section 3.7 I analyze the effect on risk taking. In Section 3.8 I discuss the effect on investments and nonfinancial risk taking of a corporate tax in a small open economy. A summary of my conclusions is contained in Section 3.9.

3.2 HOUSEHOLD TAXES IN A SMALL OPEN ECONOMY UNDER CERTAINTY

Intuitively it seems obvious that a tax on capital income should reduce the willingness to save. But many economists like to show that what is intuitively obvious is not necessarily true. Thus the well known textbook of Atkinson and Stiglitz [1980] often tries to show that the effect on savings is ambiguous. When I learnt public finance from their book I reacted against this view and I am conscious that parts of this chapter reflect that reaction. I want to show that the essential effect is to decrease savings and that the opposite effect will occur only under very special circumstances. This view of the world and much of the following analysis coincides with recent papers by Summers [1981 b], [1982] and [1984].

I will first deduce the results graphically in a two period model and algebraically in a continuous time model and then discuss what assumptions have ruled out the ambiguity of the effect on savings. The effect of intergenerational transfers is analyzed in Section 3.3.

We assume that the international market rate of interest, r, is certain and constant. We also assume that all households are equal and that the pre-tax labour income is independent of the tax rates. (As discussed in Section 3.1 I assume that the labour supply is independent of the tax rates when total tax revenue from households is constant. The real wage is determined by the capital intensity of firms which is independent of household taxes in a small open economy.)

We want to study how the amount of savings s or the stock of domestic capital k^d depends on the capital income tax rate

t_r which reduces the return on savings from $r \cdot s$ to $r \cdot (1-t_r) \cdot s$. The revenue from this tax is returned to the households in form of a reduction of other taxes by G.

In a two-period model households choose levels of consumption c_1 and c_2 and savings s so that the utility function

$$U = U(c_1, c_2) \tag{3:1}$$

is maximized subject to the budget constraints

$$c_1 = y_1 - s \tag{3:2}$$

$$c_2 = y_2 + s \cdot [1 + r(1-t_r)] + G \tag{3:3}$$

$$G = s \cdot r \cdot t_r \tag{3:4}$$

where y_1 and y_2 are labour income. We assume that G is paid in the same period as the tax is received so that we do not have to take government debt into account. G is endogenous to the model but each household believes that its individual actions cannot affect G.

From Chapter 2 we know that the solution of the maximization problem must lie where the budget line (3:3), as perceived by the household, is tangent to an indifference curve, that is the household chooses a consumption bundle $(c_1^* \; c_2^*)$ where the slope of the indifference curve is

$$- U_1/U_2 = - [1 + r \cdot (1-t_r)] \tag{3:5}$$

If r is positive, this slope is less steep the higher is t_r.

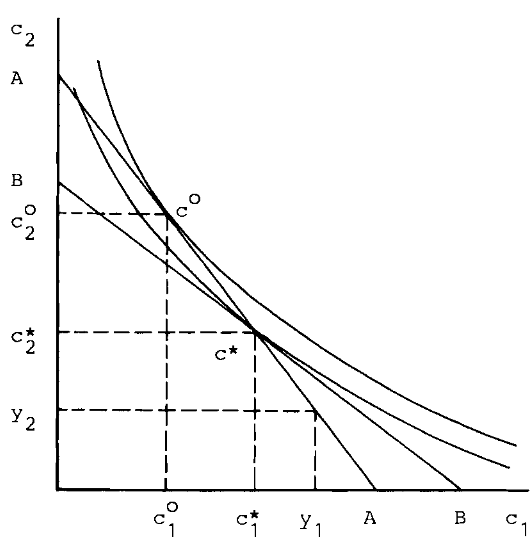
Combining (3:3) and (3:4) we also know that the solution must be on the social budget line

$$c_2 - y_2 = (y_1 - c_1) \cdot (1+r) \qquad (3:6)$$

which is also the private budget line if t_r is equal to zero.

As illustrated in Figure 3:1 all feasible solutions for a positive t_r, that is all points where the social budget line (3:6) or AA crosses an indifference curve with a slope less than that at c^o, must be to the southeast of c^o, which is the solution for $t_r = 0$. The higher is t_r, the higher must c_1^* be and the lower must c_2^* be. Thus savings, which are equal to $y_1 - c_1^*$, decrease when t_r is increased. Figure 3:1 also illustrates that the tax will cause a welfare loss (a transfer to a lower indifference curve). This is due to the distortion of the savings decision.

Figure 3:1. Consumption allocation with a capital income tax.



Line AA is the social budget constraint with slope $-(1+r)$
Line BB is the private budget constraint with slope $-[1+r(1-t_r)]$
Savings without tax $y_1 - c_1^o$.
Savings with tax $y_1 - c_1^*$.

The continuous-time model was introduced in Section 2.4. If we introduce a capital income tax $t_r \cdot r \cdot k^d$ and a corresponding lump-sum transfer $G = t_r \cdot r \cdot k^d$ in that model the optimality condition (2:20) becomes

$$\frac{\dot{u}'}{u'} = \rho - r \cdot (1-t_r) \qquad (3:7)$$

This can formally be proved using the Euler-Lagrange equation. See Dixit [1976] pp. 104 and 107.

The marginal utility of consumption will thus increase with time at the rate $\rho - r \cdot (1-t_r)$. Consumption thus decreases with time at a rate that is a monotonically increasing function of $\rho - r \cdot (1-t_r)$. The higher is t_r, the more will consumption decrease with time and the more will consumption be concentrated to the first periods.

In the special case of an additive isoelastic utility function, which was analyzed in Section 2.4, we get corresponding to (2:22)

$$c_t = c_o \cdot e^{\frac{\rho - r \cdot (1-t_r)}{\gamma - 1} \cdot t} \qquad (3:8)$$

The social budget constraint is identical to (2:23) and solving c_o we get

$$c_o = \frac{\rho - r \cdot \gamma + r \cdot t_r}{1-\gamma} \cdot (k_o^d + \frac{w}{r}) \text{ if } \rho > r \cdot \gamma - r \cdot t_r \qquad (3:9)$$

Thus c_o is an increasing function of the tax rate t_r.

For the savings rate we get corresponding to (2:26)

$$s_t = w + r \cdot (1-t_r) \cdot k_t^d + G - c_t = w + r \cdot k_t^d - c_t \qquad (3:10)$$

and thus

$$s_t = \frac{r\cdot(1-t_r) - \rho}{1-} \cdot (k_t^d + \frac{w}{r}) \qquad (3:11)$$

The savings rate is thus a decreasing function of t_r.

We also get the stock of accumulated savings corresponding to (2:30)

$$k_t^d = k_o^d \cdot e^{\frac{r\cdot(1-t_r)-\rho}{1-\gamma}\cdot t} + \frac{w}{r} \cdot \left[e^{\frac{r\cdot(1-t_r)-\rho}{1-\gamma}\cdot t} - 1\right] \qquad (3:12)$$

Evidently k_t^d is a decreasing function of t_r for all $t > 0$ and for all γ.

With our assumptions the flow and stock of savings are thus always a decreasing function of the capital income tax rate t_r. This unambiguous result is in conflict with much of the earlier literature and I will therefore discuss some of the assumptions.

1. It is easily shown that the distribution of labour income (y_t or w_t) over time does not change the results qualitatively.

2. If the transfer G is paid in an earlier period, say in period 1 instead of period 2 in the two-period model, and this prepayment is temporarily financed by a foreign loan, private income is increased by G in the first period. But the social budget restraint (3:6) is not changed, if the government has to pay the interest rate r, and the optimality condition (3:5) is not changed. Thus c_1^* and c_2^* are not affected. Private savings increase by G and public savings decrease by the same amount. Feldstein [1978] also concludes (p. 531): 'If public spending does not increase in response to the higher tax revenue [in period 1], the switch from a general income tax to a labor income tax or consumption tax will necessarily increase national saving.'

3. I have assumed that the tax revenue is returned to the taxpayers in the form of lower taxes on labour income. The income effect is then small and just due to the deadweight loss of distorting the savings decision. As shown by Figure 3:1 this income effect can never outweigh the substitution effect. It is zero around $t_r = 0$.

4. As shown in Appendix 1, my assumption of constant labour supply holds strictly only for logarithmic utility functions (for small t_r it holds for all isoelastic utility functions if the elasticity is the same for both periods). But this is a commonly used utility function and we have no reason to assume that the labour supply should normally increase or normally decrease. Besides, the effect of a change of the labour supply on savings is probably small compared to the direct substitution effect.

5. My most dubious assumption is that of equivalent households. Probably a capital income tax does result in transfers from those who save much to those who save little. But is is hard to believe that this could result in higher aggregate savings. Normally an income transfer from households with a high propensity to save to households with a low propensity should result in a negative income effect on savings which works in the same direction as the substitution effect. A special case of such transfers in an overlapping-generations model will, however, be treated in the next section.

6. In this section I assumed that there is no uncertainty. The effect of taxes on precautionary savings is discussed in Section 3.7.

3.3 THE OVERLAPPING GENERATIONS MODEL AND INTERGENERATIONAL TRANSFERS

In their section 8-4, pp. 243-248, Atkinson and Stiglitz [1980] analyze the effects of a capital income tax in an overlapping generations model such as that used by Diamond [1965]. They conclude that if the revenue of the tax is used for a transfer to the younger generation, the capital stock in a closed economy in steady state equilibrium will be an increasing function of the tax rate for, inter alia, logarithmic utility functions. I want to explain their result and to analyze whether it holds in our small open economy.

Assume that the utility function is $U = (1+\rho)\cdot\ln c_1 + \ln c_2$. In the first period each generation receives a labour income w and a transfer G_t. In the second period they receive a capital income $s_t\cdot r$ and pay a tax $s_t\cdot r\cdot t_r$.

With this utility function, optimal savings are

$$s_t = \frac{w + G_t}{2 + \rho} \tag{3:13}$$

The constant tax revenue condition is

$$G_t = s_{t-1}\cdot r\cdot t_r \tag{3:14}$$

Thus we get the difference equation

$$s_t = \frac{1}{2 + \rho}\cdot(w + s_{t-1}\cdot r\cdot t_r) \tag{3:15}$$

with the solution

$$s_t = \left(s_o - \frac{w}{2+\rho-r\cdot t_r}\right)\cdot\left(\frac{r\cdot t_r}{2+\rho}\right)^t + \frac{w}{2+\rho-r\cdot t_r} \tag{3:16}$$

The initial value s_o is the savings of a generation that does not receive any transfer G. We must have a first generation which pays the tax but does not receive any benefit. With $G_o=0$

$$s_o = \frac{w}{2+\rho} \tag{3:17}$$

and

$$s_t = \frac{w}{2+\rho-r\cdot t_r} \cdot \left[1 - \left(\frac{r\cdot t_r}{2+\rho}\right)^{t+1}\right] \tag{3:18}$$

The steady state value of the capital stock is thus

$$\lim_{t\to\infty} s_t = \frac{w}{2+\rho-r\cdot t_r} \quad \text{if} \quad r\cdot t_r < 2+\rho \tag{3:19a}$$

$$\lim_{t\to\infty} s_t = \infty \quad \text{if} \quad r\cdot t_r > 2+\rho \tag{3:19b}$$

The solution (3:19b) is very unlikely as r and the subjective discount rate ρ should be of the same order of magnitude. Denoting the solution (3:19a) s_∞, we find that the capital stock increase which is due to the tax is

$$s_\infty - s_o = \frac{w}{2+\rho-r\cdot t_r} - \frac{w}{2+\rho} = w \cdot \frac{r\cdot t_r}{(2+\rho)\cdot(2+\rho-r\cdot t_r)} \tag{3:20}$$

Of this capital stock increase

$$s_o\cdot r\cdot t_r = w \cdot \frac{r\cdot t_r}{2+\rho} \tag{3:21}$$

has been taken from the first generation and the net savings of later generations are

$$s_\infty - s_o - s_o\cdot r\cdot t_r = w\cdot\frac{r\cdot t_r}{(2+\rho)\cdot(2+\rho-r\cdot t_r)}\cdot(r\cdot t_r-\rho-1) \tag{3:22}$$

It seems reasonable to assume that $r\cdot t_r < \rho + 1$. Thus the whole capital stock increase comes from the transfer (and welfare loss) of the first generation and net savings of later generations are negative. The conclusion of Atkinson and

Stiglitz holds in this model, but the positive effect on the capital stock is financed by exploiting one generation. The effect on the capital stock would have been larger if the government had taxed only the first generation and kept the tax revenue as public savings.

In Section 2.3 we discussed the risk that the restrictive assumptions of the overlapping generations model can bias the results. In this case the observation of Barro [1974] that the existence of private transfers can upset the results is relevant.

Assume that parents can leave a private bequest B to their children in the following generation and that the utility function is

$$U = (1+\rho)\cdot \ln c_1 + \ln c_2 + a\cdot \ln(B+G+w) \qquad (3:23)$$

where B+G+w is the total income of the children.

The private budget constraint of the first generation is

$$c_2 + B = (w-c_1)\cdot[1 + r\cdot(1-t_r)] \qquad (3:24)$$

and

$$G = (w-c_1)\cdot r\cdot t_r \qquad (3:25)$$

Thus the conditions of an optimal solution are

$$B + G + w = a\cdot c_2 \qquad (3:26a)$$

$$c_1 = \frac{1+\rho}{1+r(1-t_r)} \cdot c_2 \qquad (3:26b)$$

$$c_2 + B + G = (w-c_1)\cdot(1+r) \qquad (3:26c)$$

Thus B and G are substitutes in transferring wealth and the tax does not cause any income effect. The substitution effect is of course negative on savings. Thus the result of Atkinson and Stiglitz that the tax can increase the capital stock only holds in the special case that no private intergenerational transfers exist or that parents take no interest in the welfare of their children. It is easily seen that the functional form of (3:23) is not important for this result. B+G always appears as a unit in the optimality condition and in the budget constraint.

3.4 CORPORATE TAXES IN A SMALL OPEN ECONOMY UNDER CERTAINTY

Under competitive conditions the amount of invested capital and thus the capital intensity of firms, k, is determined such that the marginal productivity of capital equals the capital cost, N.

$$f'(k) = N \tag{3:27}$$

As illustrated in Figures 2:3 and 2:5, k is a decreasing function of N.

Under certainty and with a perfect capital market and no taxes N equals the market interest rate, r. The privately optimal capital intensity is then also socially optimal.

Taxes that do not discriminate between domestic investment and investment in foreign assets do not affect N which is still equal to r. Thus the tax on capital income of households, t_r, discussed in Sections 3.2 and 3.3 has no influence on investment by firms in a perfectly open economy.

The capital cost is only affected by taxes which are specifically related to domestic investment. Such taxes are the corporate income tax, a corporate profit tax and withholding taxes on dividend and interest payments which have a net effect on the income of investors. I will assume that such taxes are equal on all domestic investment and disregard the distinction made by Harberger [1962] between corporate and non-corporate sectors of the economy.

First I will analyze the effects of an idealized corporate income tax, t_c, which is proportional to all returns on domestic production capital. In this chapter I disregard all differentiation of the tax due to the capital structure or asset structure of firms.

If investors demand a return r net of corporate taxes the capital cost, N, is $r/(1-t_c)$. The condition of an optimal k then is

$$f'(k) = \frac{r}{1-t_c} \tag{3:28}$$

and k is a decreasing function of t_c

$$\frac{dk}{dt_c} = \frac{1}{f''(k)} \cdot \frac{r}{(1-t_c)^2} \tag{3:29}$$

where $f''(k)$ is negative.

The real wage before tax w is a function of k

$$w = f(k) - k \cdot f'(k) \tag{3:30}$$

Thus

$$\frac{dw}{dt_c} = - \frac{r \cdot k}{(1-t_c)^2} < 0 \tag{3:31}$$

But an increase of t_c makes it possible to decrease the tax on wages. If the labour supply is constant,[1] the constant tax revenue condition can be written

$$R = w \cdot t_w + k \cdot \frac{r}{1-t_c} \cdot t_c \tag{3:32}$$

$$dR = 0 = w \cdot \frac{dt_w}{dt_c} + t_w \cdot \frac{dw}{dt_c} + \frac{r \cdot t_c}{1-t_c} \cdot \frac{dk}{dt_c} + \frac{k \cdot r}{(1-t_c)^2} \tag{3:33}$$

The effect on the real wage net of tax is thus

$$\frac{d[w \cdot (1-t_w)]}{dt_c} = \frac{r \cdot t_c}{1-t_c} \cdot \frac{dk}{dt_c} < 0 \tag{3:34}$$

The corporate income tax thus creates a deadweight loss due to the distortion of the capital intensity. This loss is borne by the wageearners because capital is perfectly mobile and labour is not.

If (3:34) is true and well understood, wageearners in small open economies should vote for the abolition of corporate taxes. That they do not always do so can be explained by the fact that (3:34) really holds only in the long run. In the short run, investments are irreversible and any increase of the corporate tax is to some extent a lump sum tax on those who have invested without anticipating the tax increase. If voters wish to tax existing capital but do not wish to decrease future investments, the time consistency problem arises. If the present government taxes existing capital but promises not to tax the return of new investments, firms might suspect that future governments will not be committed by this promise. Future governments might also want to tax what is then existing capital. This problem, and possible solutions, has been discussed by Hansson and Stuart [1985]. They predict that governments will subsidize investment and tax income.

The second type of tax is a corporate profit tax which only taxes profits above the normal return r. In a competitive economy under certainty such a tax yields no revenue as profits do not exist. It will be discussed in Section 3.8.

A third possible type of tax on corporate income is a withholding tax, t_k, on dividend and interest payments to foreign investors. This tax affects N for foreign capital if the investors do not get a tax credit in their own country. But the withholding tax does not affect the opportunity cost of domestic investors. We thus get a tax clientele effect so that domestic households invest in domestic firms and foreigners invest in foreign firms. If the separation in tax clientele were perfect the capital cost N would be r if the country were a net exporter of capital and $r/(1-t_k)$ if the country were a net importer of capital. In the first case the tax would have no effect and give no revenue, in the second case it would enrich domestic capital owners and reduce the capital intensity and the net wage rates.

In the real world, the tax clientele separation will not be perfect because of the preference for risk diversification. The conclusions above are then modified. If other countries give credit for the tax or treat it as a deductible cost, some part of the tax incidence may fall on foreign governments. The application of withholding taxes is therefore usually regulated in tax treaties.

3.5 CAPITAL INCOME TAXES IN A CLOSED ECONOMY

In a perfectly closed economy it does not matter much whether households or firms are taxed. With the idealized taxes and the frictionless economy, which I have modelled in this chapter, the only differences are that the taxes are based on different income measures and that they have different effects on the "market" rate of interest. The revenue of a tax on households t_r is $t_r \cdot r \cdot k$ and the revenue of a corporate tax t_c is $t_c \cdot r \cdot k/(1-t_c)$.

With a tax on the income of firms, t_c, the capital cost is increased to

$$N = \frac{r}{1-t_c} \tag{3:35}$$

The production function f(k) and the derivative f'(k) are not changed by taxes. In equilibrium f'(k) must be equal to N and

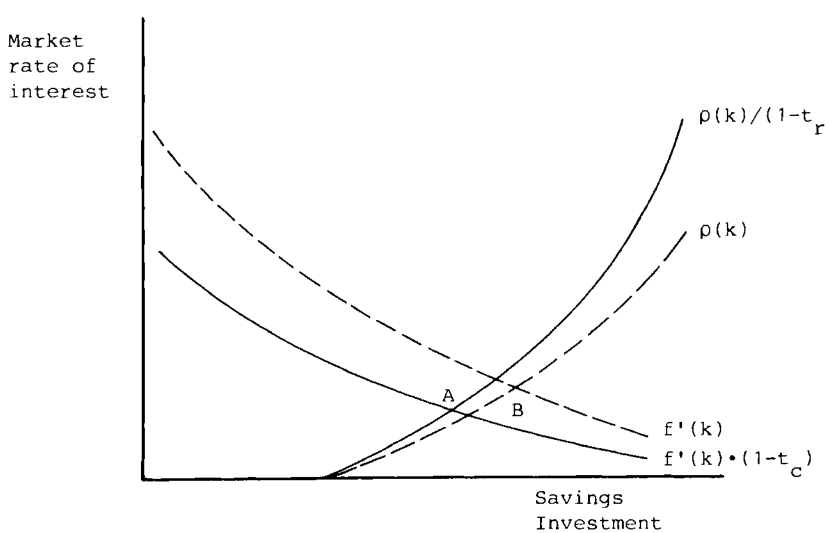
$$r = (1-t_c)\cdot f'(k) \tag{3:36}$$

The marginal time preference of households ρ is also a function of savings which are equal to k. At equilibrium $\rho(k)$ must be equal to the net of tax return on savings.

$$r = \frac{\rho(k)}{1-t_r} \tag{3:37}$$

If we disregard the income effect of tax distortions the new equilibrium can be illustrated as in Figure 3:2, point A. The no tax equilibrium is point B.

Figure 3:2. Savings and investment as functions of r. Closed economy with taxes.



We see that both forms of taxation decrease the volume of capital. A tax on households increases the market rate of interest whereas a tax on firms has the opposite effect.

In order to prove that these results are not changed by income effects, I will also give an algebraic solution for a two--period model.

Assume that the households choose consumption levels c_1 and c_2 to maximize a utility function $U = U(c_1, c_2)$ expecting non-capital disposable incomes w_1 and w_2 and an interest rate r. The budget constraint is

$$c_2 - w_2 = (w_1-c_1)\cdot[1 + r\cdot(1-t_r)] \tag{3:38}$$

and the condition of optimal savings $k = (w_1-c_1)$ is

$$U_1 = [1 + r\cdot(1-t_r)] \cdot U_2 \tag{3:39}$$

Firms will choose the capital intensity k so as to maximize profits in the second period

$$\pi = f(k) - \frac{w_2}{1-t_w} - \frac{r}{1-t_c} \cdot k \tag{3:40}$$

Thus

$$f'(k) = \frac{r}{1-t_c} \tag{3:41}$$

The zero-profit condition of competitive behaviour gives

$$\frac{w_2}{1-t_w} = f(k) - k\cdot f'(k) \tag{3:42}$$

The tax revenue R is

$$R = t_w \cdot \frac{w_2}{1-t_w} + t_r \cdot r \cdot k + t_c \cdot \frac{r}{1-t_c} \cdot k \qquad (3:43)$$

Thus from (3:42)

$$w_2 = f(k) - k \cdot r \cdot (1-t_r) + R \qquad (3:44)$$

and from (3:38)

$$c_2 - f(k) = k - R \qquad (3:45)$$

With tax revenue R constant

$$dc_2 = dk + f'(k) \cdot dk \qquad (3:46)$$

By definition $k = w_1 - c_1$. Thus

$$dc_1 = - dk \qquad (3:47)$$

Differentiating the optimality conditions (3:41) and (3:39) and substituting (3:41), (3:46) and (3:47) we have

$$f''(k) \cdot dk - \frac{1}{1-t_c} \cdot dr = \frac{r}{(1-t_c)^2} \cdot dt_c \qquad (3:48)$$

$$\{U_{11} - U_{12} \cdot [1+r\ (1-t_r)] + U_{22} \cdot [1+f'(k)]\} \cdot dk +$$

$$+ (1-t_r) \cdot U_2 \cdot dr = r \cdot dt_r \cdot U_2 \qquad (3:49)$$

The coefficient of dk in (3:49) is always negative as $U_{11} < 0$, $U_{12} \geq 0$ and $U_{22} < 0$. I will denote this coefficient -A.

If we keep t_r constant ($dt_r = 0$) we then get

$$\frac{dk}{dt_c} = \frac{r}{1-t_c} \cdot \frac{U_2 \cdot (1-t_r)}{A - f''(k) \cdot U_2 \cdot (1-t_r)(1-t_c)} < 0 \qquad (3:50a)$$

$$\frac{dr}{dt_c} = - \frac{r}{1-t_c} \cdot \frac{A}{A-f''(k)\cdot U_2\cdot(1-t_r)(1-t_c)} < 0 \qquad (3:50b)$$

With t_c constant ($dt_c=0$) we get

$$\frac{dk}{dt_r} = - r \cdot \frac{U_2}{A-f''(k)\cdot U_2\cdot(1-t_r)(1-t_c)} < 0 \qquad (3:51)$$

$$\frac{dr}{dt_r} = r \cdot \frac{-f''(k)\cdot(1-t_c)\cdot U_2}{A-f''(k)\cdot U_2\cdot(1-t_r)(1-t_c)} > 0 \qquad (3:52)$$

For our model with exogenous public consumption and constant labour supply the results are thus unequivocal.

3.6 HOUSEHOLD TAXES AND PRECAUTIONARY SAVINGS

In the preceding sections we have seen that in models with no uncertainty a capital income tax will always decrease savings. When uncertainty is introduced, the outcome is less obvious. Atkinson and Stiglitz [1980] assert on page 84 that when part of the motive for saving arises from uncertainty associated with future income or future needs, the effect of taxation on savings may be markedly different from that in the life--cycle model. As an example they take the unemployment risk model which I introduced in Section 2.5.

Analyzing the effect of taxation in that model we find that the result depends upon how the tax revenue is used. I discuss three possible cases:

1. If the tax revenue is returned to the households as a lump-sum transfer, the income effects of the tax are neutralized. The substitution effect causes decreased savings.

2. If the tax revenue is used for unemployment benefits, the income uncertainty is decreased and the income and substitution effects both reduce savings.

3. If the tax revenue is used to reduce the tax on labour income, the income uncertainty is increased, the income and substitution effects work in different directions and nothing in general can be said about the net effect on savings. In this case the warning of Atkinson and Stiglitz is motivated.

Another way to sum up the conclusions is that a capital income tax always has a negative substitution effect on savings and that any reduction of uncertainty also reduces precautionary savings. If the government wants to increase precautionary savings it must either decrease the tax or increase the income uncertainty.

I now derive the conclusions mathematically.

As in Section 2.5 I assume that the probability of unemployment in the second period is π and that the labour income in case of employment is $\bar{y}_2/(1-\pi)$. The capital income tax is t_r and there are government transfers G_1 in the unemployment state and G_2 in the employment state. The utility function and the second period consumptions are

$$V = U(c_1) + \pi\cdot U(c_2^1) + (1-\pi)\cdot U(c_2^2) \tag{3:53}$$

$$c_2^1 = s\cdot[1 + r(1-t_r)] + G_1 \tag{3:54}$$

$$c_2^2 = s\cdot[1 + r(1-t_r)] + \bar{y}_2/(1-\pi) + G_2 \tag{3:55}$$

The households will choose savings s so that

$$U_1(c_1) = [1 + r(1-t_r)]\cdot E[U_2(c_2)] =$$

$$= [1 + r(1-t_r)]\cdot[\pi\cdot U_2(c_2^1)+(1-\pi)\cdot U_2(c_2^2)] \tag{3:56}$$

Differentiating (3:56) with the employment risk, π, constant we get

$$\{U_{11}(c_1)+[1+r(1-t_r)]^2\cdot E[U_{22}(c_2)]\}\cdot ds+\pi\cdot[1+r(1-t_r)]\cdot$$

$$\cdot U_{22}(c_2^1)\cdot dG_1+(1-\pi)\cdot[1+r(1-t_r)]\cdot U_{22}(c_2^2)\cdot dG_2 =$$

$$= \{[1+r(1-t_r)]\cdot r\cdot s\cdot E[U_{22}(c_2)]+r\cdot E[U_2(c_2)]\}dt_r \qquad (3:57)$$

In my case 1 with a lump-sum transfer to all households $G_1 = G_2 = s\cdot r\cdot t_r$. Thus

$$dG_1 = dG_2 = r\cdot t_r\cdot ds + r\cdot s\cdot dt_r \qquad (3:58)$$

$$\frac{ds}{dt_r} = \frac{r \cdot E[U_2(c_2)]}{U_{11}(c_1)+[1+r(1-t_r)](1+r)\cdot E[U_{22}(c_2)]} < 0 \qquad (3:59)$$

In my case 2 with unemployment benefits $G_1 = s\cdot r\cdot t_r/\pi$ and $G_2 = 0$. Thus

$$dG_1 = \frac{1}{\pi} \cdot [r\cdot t_r\cdot ds + s\cdot r\cdot dt_r] \qquad (3:60)$$

$$dG_2 = 0 \qquad (3:61)$$

$$\frac{ds}{dt_r} = \frac{r\cdot E[U_2(c_2)]+(1-\pi)\ [1+r(1-t_r)]\cdot r\cdot s\cdot[U_{22}(c_2^2)-U_{22}(c_2^1)]}{U_{11}(c_1)+[1+r(1-t_r)]^2\cdot E[U_{22}(c_2)]+[1+r(1-t_r)]\cdot r\cdot t_r\cdot U_{22}(c_2^1)} \qquad (3:62)$$

The derivative is obviously negative if the third derivative U_{222} is positive. This is true if households have decreasing absolute risk aversion.

In my third case with decreased labour income tax $G_1 = 0$ and $G_2 = s\cdot r\cdot t_r/(1-\pi)$. Then

$$dG_1 = 0 \tag{3:63}$$

$$dG_2 = \frac{1}{1-\pi} \cdot [r \cdot t_r \cdot ds + s \cdot r \cdot dt_r] \tag{3:64}$$

$$\frac{ds}{dt_r} = \frac{r \cdot E[U_2(c_2)] - \pi \cdot [1+r(1-t_r)] \cdot r \cdot s \cdot [U_{22}(c_2^2) - U_{22}(c_2^1)]}{U_{11}(c_1) + [1+r(1-t_r)]^2 \cdot E[U_{22}(c_2)] + [1+r(1-t_r)] \cdot r \cdot t_r \cdot U_{22}(c_2^2)} \tag{3:65}$$

Thus the sign of the derivative is ambiguous if U_{222} is positive. The sign will depend on the relative size of U_2 and U_{222} and on the relative size of first period and second period labour incomes.

3.7 TAXES, UNCERTAINTY AND PORTFOLIO CHOICE

In Section 2.6 I introduced the two period, two asset model for the analysis of savings and portfolio choice. One asset has a risk free return, r_b, (usually assumed to be a bond) and one asset has a random return $\tilde{r}_s$ with the expected value $\bar{r}_s$. The returns r_b and $\tilde{r}_s$ are determined on the international capital market and are independent of domestic taxation. The return on the risky asset is assumed to be the only uncertainty in the economy so the risk cannot be diversified.

I now want to introduce capital income taxes in this model. In order to make some points more clearly I will use two tax parameters: t_r is the tax on the risk free interest rate r_b, t_e is the excess profits tax on the risk margin $\tilde{r}_s - r_b$. I will study the effects of t_r and t_e on savings, s, and investment share in the risky asset, a. We will find that t_r has the same effect as in the certainty case, whereas the effect of the excess profits tax, t_e, critically depends on the ability of the government to diversify risks. We can note that t_e gives a positive expected tax revenue $t_e \cdot (\bar{r}_s - r_b) \cdot a \cdot s$ but the utility loss to the households $t_e \cdot a \cdot s \cdot E[U_2 \cdot (\tilde{r}_s - r_b)]$ is zero at the margin if the allocation a is optimal. See also (3:70) below.

We assume that the utility function and the consumption levels are

$$V = U(c_1) + E[U(c_2)] \tag{3:66}$$

$$c_1 = y_1 - s \tag{3:67}$$

$$c_2 = y_2 + s + (1-t_r)\cdot r_b\cdot s + a\cdot(1-t_e)\cdot(\tilde{r}_s-r_b)\cdot s + G \tag{3:68}$$

y_1 and y_2 are non-capital incomes and G is the labour tax reduction or lump-sum transfer made possible by the tax revenue. I assume that each household has rational expectations about the size of G in different states but does not believe that its own actions will influence G.

In earlier literature, starting with Domar and Musgrave [1944], the tradition has been to assume that the government can diversify risk perfectly and can thus be regarded as risk neutral. This is equivalent to assuming that the transfer, G, is equal to the expected value of the tax revenues. This assumption was questioned in Stiglitz [1972] and has been asserted to be misleading by Gordon [1981] and Bulow and Summers [1984]. They assert that the government cannot diversify risk better than the households and that the government cannot use the random income in a better way than the households. This view can be modelled by assuming that the transfer, G, is equal to the actual tax revenue so that public consumption is state independent.

Which model does, then, best describe the real world? Can the government perfectly diversify risks which households cannot diversify? We can first note that it is not possible in our formal two-asset model. Nor is it possible in the Capital Asset Pricing Model (CAPM) where households are assumed to be able to diversify all risks that are diversifiable and where only the market risk remains. For the traditional view of government risk neutrality to make sense we must introduce transaction costs and heterogeneous expectations which make the households hold different portfolios of risky assets. An extreme case of high transaction costs are nonmarketable assets such as human capital. But still the 'market risk' in the form of international business cycles, oil price uncertainty, etcetera, exists. The two cases are probably extremes and the truth is somewhere in between.

The choice of model is decisive for the results. If the government can diversify risks a tax t_e will increase holdings of the risky asset and increase social welfare by transferring risk from risk averse households to the risk neutral government. If the government has no additional ability to diversify risks, t_e will have no effect (except administration costs and tax evasion costs).

From Section 2.6 we know that the first order conditions of optimal s and a are

$$U_1 = [1 + r_b(1-t_r)]\cdot E(U_2) \tag{3:69}$$

$$E[U_2\cdot(\tilde{r}_s-r_b)] = 0 \tag{3:70}$$

We see that the optimality conditions are not influenced by the excess profits tax t_e except via the budget constraints, which determine the level of c_1 and c_2.

Differentiating (3:69) and (3:70) for constant y_1, y_2, r_b and a given distribution of $\tilde{r}_s$ we get

$$U_{11}dc_1 = [1+r_b(1-t_b)]\cdot E[U_{22}\cdot dc_2]-r_b\cdot E[U_2]\cdot dt_r \tag{3:71}$$

$$E[U_{22}(\tilde{r}_s-r_b)\cdot dc_2] = 0 \tag{3:72}$$

dc_2 depends on the assumption made regarding G. I will start with the case of a risk averse government because in that case we get no income effects and the solution is simple. We then have

$$G = t_r \cdot r_b \cdot s + t_e \cdot (\tilde{r}_s - r_b) \cdot a \cdot s \tag{3:73}$$

$$c_2 = y_2 + s \cdot [1 + r_b + a(\tilde{r}_s - r_b)] \tag{3:74}$$

$$dc_2 = [1 + r_b + a(\tilde{r}_s - r_b)] \cdot ds + s(\tilde{r}_s - r_b) \cdot da \tag{3:75}$$

Substituting (3:75) in (3:72) we get an equation in only ds and da. In a single period model with s constant we would thus find da=0 and a to be independent of the tax rates. In our two period model a will be affected by the usual wealth effect

$$\frac{da}{ds} = - \frac{(1+r_b) \cdot E[U_{22} \cdot (\tilde{r}_s - r_b)] + a \cdot E[U_{22} \cdot (\tilde{r}_s - r_b)^2]}{s \cdot E[U_{22} \cdot (r_s - r_b)^2]} \tag{3:76}$$

Substituting (3:75) and (3:76) in (3:71) we get the effect on savings

$$\frac{ds}{dt_r} = \frac{r_b \cdot E[U_2]}{U_{11} + [1 + r_b(1-t_b)] \cdot (1+r_b) \cdot \left(E[U_{22}] - \frac{\{E[U_{22}(\tilde{r}_s - r_b)]\}^2}{E[U_{22}(\tilde{r}_s - r_b)^2]} \right)} \tag{3:77}$$

This is the usual negative substitution effect, modified for the adjustment of a (if the absolute risk aversion is constant, $E[U_{22} \cdot (\tilde{r}_s - r_b)] = 0$ and the existence of the risky asset does not influence s). The excess profit tax rate, t_e, has no influence on a or s.

With a <u>risk neutral government</u> we get instead

$$G = t_r \cdot r_b \cdot s + t_e(\bar{r}_s - r_b) \cdot a \cdot s \tag{3:78}$$

$$c_2 = y_2 + s \cdot [1 + r_b + a(\tilde{r}_s - r_b)] - t_e \cdot a \cdot s\ (\tilde{r}_s - \bar{r}_s) \tag{3:79}$$

$$dc_2 = [1 + r_b + a(\tilde{r}_s - r_b)]ds + s(\tilde{r}_s - r_b) \cdot da + (\bar{r}_s - \tilde{r}_s) \cdot$$

$$\cdot (t_e \cdot a \cdot ds + t_e \cdot s \cdot da + a \cdot s \cdot dt_e) \tag{3:80}$$

Substituting (3:80) in (3:72) we then find that da is a function of not only ds but also of dt_e. If s were assumed to be constant we would get

$$\left.\frac{da}{dt_e}\right|_{s\ \text{const.}} = \frac{a \cdot E[U_{22} \cdot (\tilde{r}_s - r_b)(\tilde{r}_s - \bar{r}_s)]}{E[U_{22} \cdot (\tilde{r}_s - r_b)^2] - t_e \cdot E[U_{22} \cdot (\tilde{r}_s - r_b)(\tilde{r}_s - \bar{r}_s)]} \tag{3:81}$$

This is a combination of a wealth effect of increasing welfare and the pure substitution effect. The wealth effect is zero if $E[U_{22}(\tilde{r}_s - r_b)] = 0$. For that special case

$$\frac{da}{dt_e} = \frac{a}{1 - t_e} = 0 \tag{3:82}$$

which implies that private risk taking, $a \cdot (1 - t_e)$ is constant, which is the classic result. See Stiglitz [1976].

We can also separate the wealth effect by calculating

$$\frac{d[a\cdot(1-t_e)]}{dt_e} = (1-t_e)\cdot\frac{da}{dt_e} - a =$$

$$= -a\cdot(\bar{r}_s-r_b)\cdot\frac{E[U_{22}\cdot(\tilde{r}_s-r_b)]}{E[U_{22}\cdot(\tilde{r}_s-r_b)^2]-t_e\cdot E[U_{22}\,(\tilde{r}_s-r_b)(\tilde{r}_s-\bar{r}_s)]} \tag{3:83}$$

As expected the sign of the wealth effect depends on the sign of $E[U_{22}\cdot(\tilde{r}_s-r_b)]$ which depends upon whether the absolute risk aversion is increasing or decreasing. The effect is proportional to a and to $(\bar{r}_s-r_b)$ which is the difference between the expected value of $\bar{r}_s$ and the expected value adjusted for risk (equal expected utility).

Substituting (3:80) into (3:71) we see that the derivative ds/dt_r is marginally changed in comparison to (3:77) because of the addition of new ds- and da-terms from (3:80). More interesting is that we now get a partial derivative $\partial s/\partial t_e$ with the numerator $-[1+r_b(1-t_b)]\cdot E[U_{22}\cdot(\bar{r}_s-\tilde{r}_s)]\cdot a\cdot s$ and with a negative denominator. The numerator is positive for all $U_{222} > 0$ and thus $\partial s/\partial t_e$ is negative. The tax t_e and the compensating transfer increase utility, which has a negative income effect on savings.

We can thus conclude that the tax t_r always has a negative effect on savings and an ambiguous effect via savings on the share invested in the risky asset, a. The excess profit tax, t_e, has no effect if the government is risk averse. If the government is risk neutral, t_e has a positive effect on risk taking and a, slightly, negative effect on savings. It should be stressed that the result that a increases with t_e is critically dependent on the symmetry of the tax. If only excess profits are taxed but no refunds are given for losses, the tax naturally reduces risk taking. In actual tax systems loss offsets are less than perfect and the effect of the tax is ambiguous even if the government can diversify all risks.

3.8 CORPORATE TAXES, UNCERTAINTY, CAPITAL INTENSITY AND RISK TAKING

Assume that a firm in a small open economy is free to choose its capital intensity, k, and its nonfinancial level of risk (which depends on the product program, the technology etc.). How is this choice affected by corporate taxes?

In Chapter 4 I will treat this problem in a more complicated world with debt and equity, bancruptcy costs and differentiated taxation. In this section I have only one type of capital. Risk taking is limited by the risk aversion of the investors.

For analytical purposes I will divide the ordinary corporate income tax, t_c, into two parts corresponding to t_r and t_e in the preceding section. t_{cr} is thus the tax on the riskfree interest rate, r_f, and t_{ce} is the tax on excess profits.

The expected revenue per labour unit $\bar{X}$ is assumed to be an increasing function of the capital intensity and the risk level. This function is determined by prices on the international market and is thus independent of domestic taxes.

I will denote the measure of the nonfinancial risk level σ. According to the Capital Asset Pricing Model the relevant measure of the risk level is the covariance of the return with the return on the market portfolio (this covariance might be positive or negative).

The supply price of capital to the firm depends not only on the inherent risk level σ but also on the degree to which it is reduced by taxation. If the covariance risk before tax is σ it will be $(1-t_{ce})\cdot\sigma$ after tax (for a small open economy the variance of the market portfolio is independent of domestic corporate taxes). With a 100 % excess profits tax, the investment is riskfree.

According to the Capital Asset Pricing Model the supply price of capital is a linear function of the covariance risk.

$$r = r_f + m\cdot(1-t_{ce})\cdot\sigma \qquad (3:84)$$

where $m\cdot(1-t_{ce})\cdot\sigma$ is the risk margin.

The expected profit of the firm can then be written

$$\pi = (1-t_{ce})\cdot[\bar{X}(k,\sigma) - w - \frac{r_f\cdot k}{1-t_{cr}}] - m\cdot(1-t_{ce})\cdot\sigma\cdot k \qquad (3:85)$$

The firms are assumed to choose k and σ so as to maximize π. Thus

$$\frac{\partial\pi}{\partial k} \equiv (1-t_{ce})\cdot\left[\frac{\partial\bar{X}}{\partial k} - \frac{r_f}{1-t_{cr}}\right] - m\cdot(1-t_{ce})\cdot\sigma = 0 \qquad (3:86)$$

$$\frac{\partial\bar{X}}{\partial k} = \frac{r_f}{1-t_{cr}} + m\cdot\sigma \qquad (3:87)$$

$$\frac{\partial\pi}{\partial\sigma} \equiv (1-t_{ce})\cdot\frac{\partial\bar{X}}{\partial\sigma} - m\cdot(1-t_{ce}) = 0 \qquad (3:88)$$

$$\frac{\partial\bar{X}}{\partial\sigma} = m \qquad (3:89)$$

With these assumptions we find that t_{cr} decreases the capital intensity as expected and that the excess profits tax has no effect on corporate behaviour.

If the competitive no-profit condition holds and if σ is positive (which will be true for the average product), the excess profits tax would thus give a positive expected revenue without disturbing corporate behaviour. But there are some caveats to be borne in mind:

1. The expected tax revenue is of value only if the government is risk neutral. If the government is as risk averse as the market, the expected utility of the tax revenue will be zero.

2. The conclusion holds only if the tax is perfectly symmetric.

3. The tax will reduce incentives to manage the firms well and will increase tax evasion behaviour.

Above I assumed that the CAPM assumptions hold so that the cost of risk is determined only by the covariance with the market return. If investors cannot diversify perfectly because of transaction or information costs or if the economy is not small, the variance of the return of our firm or the covariance with returns of other domestic firms might be of importance. But in that case the risk would be reduced not by $(1-t_{ce})$ but by $(1-t_{ce})^2$. Instead of (3:89) we would then get

$$\frac{\partial \bar{X}}{\partial \sigma} = m \cdot (1-t_{ce}) \qquad (3:90)$$

and risk taking should increase with the tax rate. Most empirical tests of the CAPM find, however, no significant influence of the residual variance on the risk margin. See Fama and Mac Beth [1973].

3.9 SUMMARY AND CONCLUSIONS

In this chapter I have explored the effects of capital income taxes and corporate taxes on savings and investment in a number of models. I will now try to sum up the results.

Almost generally a capital income tax reduces savings, if the tax revenue is used for anything which is of value to the households so that the tax does not create large income effects. The effect on precautionary savings for unemployment is, however, ambiguous if the tax revenue is used to decrease the tax on labour income. In that case the tax increases the income uncertainty (and reduces welfare).

A corporate income tax reduces the capital intensity of firms. In steady state equilibrium in a small open economy it has no effect on the welfare of capital owners (who can invest in foreign assets) but reduces the welfare of wage earners. In the short run the tax is partly a lump-sum tax on irreversible investments and thus decreases the welfare of the owners and might increase the welfare of wage earners.

The effect of a capital income tax or excess profits tax on the portfolio choice of households depends on the assumptions about risk diversification of the government. If the government can diversify risks better than households, a perfectly symmetric tax will tend to increase investments in risky assets.

A perfectly symmetric corporate tax will have no influence on corporate risk taking if the economy is small and open and if the assumptions of the CAPM hold. If owners cannot diversify all non-market risk, the tax might increase risk taking.

Generally the capital income taxes on households and corporations decrease social welfare more than equal revenue taxes on labour income or consumption. This is due to the fact that they, for plausible utility functions, distort labour supply in the same way and, in addition, distort savings or corporate capital intensity. The short run effects on income distribution might, however, be regarded as favourable. The excess profits tax can be socially efficient if the government can diversify risks better than the households. This positive effect applies, however, only if the tax is symmetric and if the positive effects of risk taking are not negated by the effects of tax evasion behaviour.

NOTE

1) As the net wage rate decreases there is a positive income effect and a negative substitution effect on labour supply. Any net effect is probably small in relation to the distortion of the capital intensity.

REFERENCES

Atkinson, A.B. and Stiglitz, J.E. [1980], Lectures on Public Economics, McGraw-Hill, New York and Maidenhead.

Barro, R.J. [1974], Are Government Bonds Net Wealth?, Journal of Political Economy 82, pp. 1095-1117.

Bulow, J.I. and Summers, L.H. [1984], The Taxation of Risky Assets, Journal of Political Economy, Vol. 92, pp. 20 -39.

Diamond, P.A. [1965], National Debt in a Neo-Classical Growth Model, American Economic Review, 55, pp. 1126-50.

Dixit, A.K. [1976], Optimization in Economic Theory, Oxford University Press.

Domar, E. and Musgrave, R. [1944], Effects of Proportional Taxes on Risk Taking, Quarterly Journal of Economics.

Fama, E.F. and MacBeth, J. [1973], Risk, Return and Equilibrium: Empirical Tests, Journal of Political Economy 81, pp. 607-635.

Feldstein, M. [1978], The Welfare Cost of Capital Income Taxation, Journal of Political Economy, Vol. 86, pp. 529-551.

Gordon, R.H. [1981], Taxation of Corporate Capital Income: Tax Revenues Vs. Tax Distortions, National Bureau of Economic Research, Working Paper No. 687.

Hansson, I., and Stuart, C. [1985], Investment Subsidies and the Optimality of Time-Consistent Capital Taxation, Working Paper, University of Lund.

Harberger, A.C. [1962], The Incidence of the Corporation Income Tax, Journal of Political Economy, 70, pp. 215-240.

Stiglitz, J.E. [1972], Taxation, Risk Taking, and the Allocation of Investment in a Competitive Economy. Studies in the Theory of Capital Markets (ed. Jensen, M.C.), Praeger Publishers.

Stiglitz, J. [1976], The Corporation Tax, Journal of Public Economics, Vol. 5, pp. 303-311.

Summers, L.H. [1981 b], Capital Taxation and Accumulation in a Life Cycle Growth Model, American Economic Review, Vol. 71, pp. 533-544.

Summers, L.H. [1982], Tax Policy: The Rate of Return and Savings, NBER Working Paper No. 995.

Summers, L.H. [1984], The After-Tax Rate of Return Affects Private Savings, The American Economic Review, Vol. 74, pp. 249-253.

APPENDIX 1. THE EFFECT ON LABOUR SUPPLY OF TAX ADJUSTMENTS

In the main text of the dissertation I have assumed that the labour supply is not affected by the choice between a capital income tax and a labour income tax. In this appendix I will derive the conditions that are necessary for that assumption to hold.

My analysis has much in common with that of Feldstein [1978], King [1980] and Bradford [1980]. The difference is that I primarily address the effect on labour supply whereas the other authors focus on the optimal tax issue. At the end of this appendix I will compare my solutions with the conditions derived for an optimal capital income tax rate of zero.

I make the following assumptions:

1. The capital market is open and perfect. The interest rate, r, is internationally determined.

2. The production function has constant returns to scale. The capital intensity and the real wage, w, are therefore independent of the household taxation.

3. Households have a utility function $U(c_1\ c_2\ \ell)$ where c_1 and c_2 are consumption levels in periods 1 and 2 and ℓ is the labour supply. All labour is assumed to be supplied in period 1. Households choose savings, s, and labour supply, ℓ, in order to maximize U.

4. There is a labour income tax, t_w, and a capital income tax t_r. Total tax revenue R shall be constant.

Under these assumptions the budget constraint is

$$c_2 = [1 + r(1-t_r)]\cdot[(1-t_w)w\ell - c_1] \tag{A:1}$$

The conditions defining optimal s and ℓ are

$$U_1 = U_2\cdot[1 + r(1-t_r)] \tag{A:2}$$

$$U_1\cdot(1-t_w)\cdot w = - U_3 \tag{A:3}$$

We also have the equivalent tax revenue condition

$$R = t_w\cdot w\cdot\ell\cdot(1+r) + t_r\cdot r\cdot[(1-t_w)\cdot w\cdot\ell - c_1] \tag{A:4}$$

and the social budget constraint (which can be seen as a combination of (A:1) and (A:4))

$$c_1 + \frac{c_2}{1+r} = w \cdot \ell \tag{A:5}$$

We have then four equations to determine c_1, c_2 and ℓ as functions of t_w and t_r. We can differentiate equations (A:2)-(A:5) and solve $d\ell/dt_r$. The solution is, however, complicated in the general case and in order to facilitate the reading I will start with the <u>special case of an additive utility</u> function ($U_{12} = U_{13} = U_{23} = 0$) and <u>infinitesimal tax rates t_r and t_w</u>. For that special case we get

$$U_{11}\cdot dc_1 = U_{22}(1+r)dc_2 - rU_2dt_r \tag{A:6}$$

$$U_{11}\cdot w\cdot dc_1 - U_1\cdot w\cdot dt_w = - U_{33}\cdot d\ell \tag{A:7}$$

$$w\cdot\ell\cdot(1+r)\cdot dt_w + r\cdot(w\cdot\ell-c_1)\cdot dt_r = 0 \tag{A:8}$$

$$dc_1 + dc_2/(1+r) = w\cdot d\ell \tag{A:9}$$

Substituting U_2, dc_2 and dt_w from (A:2), (A:9) and (A:8) we get

$$\begin{bmatrix} -U_{11}-U_{22}(1+r)^2 & U_{22}\cdot w\cdot(1+r)^2 \\ -U_{11}\ w & -\ U_{33} \end{bmatrix} \begin{bmatrix} dc_1 \\ d\ell \end{bmatrix} = \begin{bmatrix} 1 \\ w-c_1/\ell \end{bmatrix} \cdot \frac{r}{1+r}\, U_1 dt_r \qquad \text{(A:10)}$$

The determinant D

$$D = U_{11}U_{33} + U_{22}U_{33}(1+r)^2 + U_{11}U_{22}\cdot w^2\cdot(1+r)^2 \qquad \text{(A:11)}$$

is positive because U_{11}, U_{22} and U_{33} are negative (decreasing marginal utility).

The sign of the labour supply reaction is then determined by

$$\frac{d\ell}{dt_r} = \frac{1}{D} \cdot \frac{r}{1+r} \cdot \frac{1}{\ell} \cdot U_1\cdot[c_1\ U_{11} - c_2\cdot U_{22}\cdot(1+r)] \qquad \text{(A:12)}$$

Thus the labour supply is independent of t_r (note that we have assumed constant tax revenue R) if

$$c_1 \cdot U_{11} = c_2 \cdot U_{22}\cdot(1+r) \qquad \text{(A:13)}$$

From (A:2) we have $U_1 = (1+r)\cdot U_2$ when $t_r=0$. The expression (A:13) can then be written

$$\frac{dU_1}{dc_1} \cdot \frac{c_1}{U_1} = \frac{dU_2}{dc_2} \cdot \frac{c_2}{U_2} \qquad \text{(A:14)}$$

The labour supply is thus constant if the elasticities of U_1 and U_2 are equal. This is true for isoelastic utility functions of the types

$$U = a \cdot c_1^{\gamma} + b \cdot c_2^{\gamma} + g(\ell) \qquad \text{(A:15a)}$$

$$U = a \cdot \ln c_1 + b \cdot \ln c_2 + g(\ell) \qquad \text{(A:15b)}$$

The labour supply will increase/decrease with t_r if the elasticity of U_1 is higher/lower than the elasticity of U_2. There does not seem to be any reason to believe that the elasticities are different. The assumption of constant labour supply is thus plausible for the special case of additive utility functions.

With positive tax rates t_r and t_w the solution is somewhat more complicated and the constant labour supply result holds strictly only for logarithmic utility functions of the type (A:15b). For utility functions of the type (A:15a) labour supply will increase/decrease with increased capital income tax t_r if γ is negative/positive. Below I give the mathematical solution for this case.

The equations (A:6)-(A:9) get some additional terms

$$U_{11} \cdot dc_1 = U_{22}(1+r-rt_r)dc_2 - r \cdot U_2 \cdot dt_r \qquad \text{(A:6')}$$

$$U_{11} \cdot (1-t_w) \cdot w \cdot dc_1 - U_1 \cdot w \cdot dt_w = - U_{33} \cdot d\ell \qquad \text{(A:7')}$$

$$\begin{aligned} & dt_w \cdot w \cdot \ell \cdot (1+r) + t_w \cdot w \cdot (1+r) \cdot d\ell + \\ & \quad + r[(1-t_w) \cdot w \cdot \ell - c_1] \cdot dt_r + t_r \cdot r(1-t_w) \cdot w \cdot d\ell - \\ & \quad - r \cdot t_r \cdot dc_1 - rt_r \cdot w \cdot \ell \cdot dt_w = 0 \qquad \text{(A:8')} \end{aligned}$$

$$dc_1 + \frac{dc_2}{1+r} = w \cdot d\ell \qquad \text{(A:9')}$$

After substitution of U_2, dc_2 and dt_w we then obtain

$$\begin{bmatrix} -U_{11}-U_{22}(1+r)(1+r-rt_r) & U_{22}\cdot w(1+r-rt_r)(1+r) \\ -U_{11}(1-t_w)\cdot w+U_1 \dfrac{rt_r}{\ell(1+r-rt_r)} & -U_{33}-U_1\cdot \dfrac{w\cdot t_w}{\ell} \\ & -U_1\cdot \dfrac{rt_r\cdot w}{\ell(1+r-rt_r)} \end{bmatrix} \times \begin{bmatrix} dc_1 \\ d\ell \end{bmatrix} =$$

$$= \begin{bmatrix} 1 \\ \dfrac{c_2}{(1+r-rt_r)} \end{bmatrix} \cdot \frac{r}{1+r-rt_r} \cdot U_1 \cdot dt_r \qquad \text{(A:10')}$$

The determinant D gets some extra terms which are negative and proportional to the tax rates

$$\begin{aligned} D = {} & U_{11}U_{22} + U_{22}U_{33}(1+r)(1+r-rt_r) + \\ & + U_{11}U_{22}w^2(1+r)\cdot(1+r-rt_r)(1-t_w) - \\ & - t_w\cdot \frac{w}{\ell}\cdot U_1[-U_{11}-U_{22}(1+r)(1tr-rt_r)] - \\ & - t_r\cdot U_1 \cdot \frac{w\cdot r}{\ell(1+r-rt_r)} \cdot (-U_{11}) \qquad \text{(A:11')} \end{aligned}$$

The taxes thus reduce the value of the determinant and make the solution less stable. In note 1 I show that D becomes negative at the maximum point on the Laffer curve, that is for $dt_r/dt_w = 0$.

The expression for $d\ell/dt_r$ also gets an extra term

$$\frac{d\ell}{dt_r} = \frac{1}{D} \cdot \frac{r}{1+r-rt_r} \cdot U_1 \cdot \frac{1}{\ell}\,[c_1U_{11}-c_2U_{22}(1+r)-t_r\cdot r\cdot \frac{U_1}{1+r-rt_r}] \qquad \text{(A:12')}$$

Thus the labour supply is constant if

$$c_1U_{11} - c_2U_{22}(1+r) - t_r \cdot r \cdot \frac{U_1}{1+r-rt_r} = 0 \qquad (A:13')$$

Using (A:2) this can be written

$$\frac{dU_1}{dc_1} \cdot \frac{c_1}{U_1} - \frac{dU_2}{dc_2} \cdot \frac{c_2}{U_2} - \frac{rt_r}{1+r-rt_r} \left[1 + \frac{dU_2}{dc_2} \cdot \frac{c_2}{U_2}\right] = 0 \quad (A:14')$$

This is zero only if the elasticities of U_1 and U_2 are -1, that is if $\gamma=0$ which implies that the utility function is logarithmic.

If the utility function is not additive we get cross derivatives U_{12}, U_{13} and U_{23}. It seems probable that U_{12} is positive, reflecting a desire to maintain any given consumption standard and that U_{13} and U_{23} are negative (leisure is more valuable when the consumption standard is good).

The extra terms of equations (A:6), (A:7) and (A:10) are given in note 2. From the matrix equation in note 2 we get the determinant D

$$\begin{aligned} D = {} & U_{11}U_{33}+U_{22}U_{33}(1+r)^2+U_{11}U_{22}\cdot w^2\cdot(1+r)^2+U_{22}U_{13}\cdot 2\cdot w\cdot(1+r)^2+ \\ & + U_{11}U_{23}\cdot 2\cdot w\cdot(1+r)-U_{33}U_{12}\cdot 2(1+r)-U_{12}U_{23}\cdot 2\cdot w(1+r)^2+ \\ & + U_{13}U_{23}\cdot 2\cdot(1+r)-U_{12}^2\cdot w^2(1+r)^2-U_{13}^2-U_{23}^2\cdot(1+r)^2 \end{aligned} \qquad (A:11")$$

Most of the additional terms are positive and I assume that D is still positive. We also get

$$\begin{aligned} \frac{d\ell}{dt_r} = {} & \frac{1}{D} \cdot \frac{r}{1+r} \cdot U_1 \cdot \frac{1}{\ell}\Big\{c_1\cdot U_{11}-c_2\cdot U_{22}(1+r)-U_{12}\cdot[(1+r)c_1-c_2] + \\ & + [U_{13} - U_{23}(1+r)]\cdot\ell\Big\} \end{aligned} \qquad (A:12")$$

Looking at (A:12") it seems probable that the labour supply in this case is constant if U_1 and U_2 have the same elasticity and if c_1 and c_2 have the same relation to in the utility function. As shown in note 3 $d\ell/dt_r$ is equal to zero for utility functions of the types

$$U = c_1^a \cdot c_2^b \cdot f(\ell) + g(\ell) \qquad \text{(A:15"a)}$$

$$U = [a \cdot c_1^\gamma + b \cdot c_2^\gamma] \cdot f(\ell) + g(\ell) \qquad \text{(A:15"b)}$$

I have also made calculations for a <u>closed economy</u> with an endogenous rate of interest and an endogenous real wage w = f(k)-k•f'(k). The calculations are tedious but the result is that the labour supply is constant for logarithmic utility functions. For other isoelastic utility functions, $d\ell/dt_r$ = = 0 only for a very special case even if t_w and t_r are zero. That the set of neutral utility functions is smaller in the closed economy case than in the open economy case is due to the endogenity of the capital intensity and thus the real wage.

An important caveat is that I have assumed that there are no rents in the models analyzed above. If rents exist or if the amount of capital is larger than the amount households would hold with perfect foresight about tax rules, an increased capital income tax has partly the effect of a lump-sum tax. In that case we might get a positive effect on the labour supply.

Assume that households have a taxable rent of y each period. In that case equation (A:8) becomes

$$w \cdot \ell \cdot (1+r) \cdot dt_w + \left[\frac{r \cdot c_2}{1+r} + y + \frac{y}{1+r}\right] \cdot dt_r = 0 \qquad \text{(A:8"')}$$

and instead of (A:12) we get

$$\frac{d\ell}{dt_r} = \frac{1}{D} \cdot \frac{U_1}{1+r} \cdot \frac{1}{\ell} \cdot [rc_1U_{11} - rc_2U_{22}(1+r) - y \cdot \frac{2+r}{1+r}(U_{11}+U_{22}(1+r)^2] \quad \text{(A:12''')}$$

$d\ell/dt_r$ is then positive for most utility functions.

In industrialized societies rents are not very important. But the short run effect of excess capital could be important if I were not using models in steady state equilibrium and with perfect foresight.

Finally I will <u>compare my results with the results on optimal taxation reported in the literature</u>.

Feldstein [1978] derived a condition (p. S 33)

$$\frac{1}{c_2} \cdot \left(\frac{\partial c_2}{\partial w}\right)_U = \frac{1}{c_1} \cdot \left(\frac{\partial c_1}{\partial w}\right)_U$$

'A compensated change in the wage rate induces the same proportional changes in c_1 and c_2, that is, the ratio of c_1 to c_2 is independent of the wage rate. If this condition is satisfied, it is optimal to tax only labor income.'

Feldstein's condition corresponds to my results, (A:15) and (A:15"), indicating that the utility function should be isoelastic in c_1 and c_2 and separable between consumption and leisure. My result (A:15') is stronger than Feldstein's, which is derived under the assumption that t_r is equal to zero.

The basic model in Bradford [1980, p. 24-25] is the same as my open economy model. He gives a specific solution only for the additive logarithmic utility function where he finds that the optimal t_r is zero. Referring to Atkinson and Stiglitz [1976], he asserts that this is true for all utility functions which are separable in consumption and leisure. This would be a much more extensive result and contradicts my result that U_1 and U_2 must have the same elasticity.

The result of A-S is derived on pages 67-68 of their paper and explicitly stated on page 69. The derivation is very condensed and it is not immediately apparent what happens. They treat the utility, U, as a state variable and labour supply, L, and the consumption in one period, X_i, as control variables. Consumption in the other period X_1 is then regarded as a function of U, L and X_i. They maximize the Hamiltonian (17) with regard to X_i keeping L constant and get the optimality condition in (18). From (18) they conclude that X_1 and X_i should be taxed in the same way, which is equivalent to a zero capital income tax, if U_{L1} and U_{Li} are zero or equal. Their result is equivalent to mine if L is constant when X_i is decreased and X_1 is increased. However, I believe that they have made a mistake in assuming that L can be kept constant. There are not enough independent instruments (tax rates) to control both L and X_i. From my derivations it is obvious that any adjustment of the two tax rates t_r and t_w (which are interdependent through the equivalent tax revenue condition) will affect not only the two consumption levels but also the labour supply in the general case.

The basic model in King [1980] is a closed economy model in which the government cannot borrow. The government budget constraint is then defined for each period and not for each generation as in my model. The result therefore depends on the population growth rate n. If n=r (the golden rule) the models are equivalent. If n is not equal to r there is no simple condition for the optimal capital income tax to be zero (cf. King's equation 20).

NOTES

1. If t_r is kept constant ($dt_r=0$) and the other tax rate t_w is increased, total tax revenue R will first increase but ultimately decrease due to the distortion of labour supply and savings. At the maximum point, dR=0 and the equilibrium is described by equations (A:6')-(A:9') with $dt_r=0$. The left hand side matrix in (A:10') must then be singular and the determinant D must be zero. Thus D is positive for low values of t_r and t_w and negative for values above the Laffer curve maximum.

2. $$U_{11}dc_1 + U_{12}dc_2 + U_{13}d\ell = U_{21}(1+r)dc_1 + U_{22}(1+r)dc_2 + U_{23}(1+r)d\ell - rU_2dt_r$$

$$U_{11}\cdot w\cdot dc_1 + U_{12}\cdot w\cdot dc_2 + U_{13}\cdot w\cdot d\ell + U_{31}dc_1 + U_{32}dc_2 + U_{33}d\ell = U_1\cdot w\cdot dt_w$$

$$dt_w\cdot w\cdot \ell(1+r) + dt_r \cdot \frac{rc_2}{1+r} = 0$$

$$dc_1 + \frac{dc_2}{1+r} = w\cdot d\ell$$

$$\begin{bmatrix} U_{11}-U_{21}(1+r)-U_{12}(1+r)+U_{22}(1+r)^2 & U_{13}-U_{23}(1+r)+U_{12}(1+r)\cdot w-U_{22}(1+r)^2\cdot w \\ U_{11}\cdot w+U_{31}-U_{12}\cdot w(1+r)-U_{32}(1+r) & U_{13}\cdot w+U_{33}+U_{12}\cdot w^2(1+r)+U_{32}\cdot w(1+r) \end{bmatrix} \times$$

$$\times \begin{bmatrix} dc_1 \\ d\ell \end{bmatrix} = - \begin{bmatrix} 1 \\ \frac{c_2}{(1+r)\cdot \ell} \end{bmatrix} \cdot \frac{r}{1+r} \cdot U_1 \cdot dt_r$$

3. For the utility function

$$U = c_1^a \cdot c_2^b \cdot f(\ell) + g(\ell)$$

$$U_1 = a \cdot c_1^{a-1} \cdot c_2^b \cdot f(\ell) = (1+r)\cdot U_2$$

$$U_2 = b \cdot c_1^a \cdot c_2^{b-1} \cdot f(\ell)$$

$\therefore\ a \cdot c_2 = b \cdot c_1 \cdot (1+r)$

$c_1 \cdot U_{11} = (a-1) \cdot U_1 = (a-1) \cdot (1+r) \cdot U_2$

$(1+r) \cdot c_2 \cdot U_{22} = (b-1) \cdot (1+r) \cdot U_2$

$[(1+r)c_1 - c_2] \cdot U_{12} = [(1+r)c_1 - c_2] \cdot \frac{a}{c_1} \cdot U_2$

$\therefore\ c_1 \cdot U_{11} - c_2 \cdot U_{22}(1+r) - U_{12} \cdot [(1+r)c_1 - c_2] =$

$= [(a-1)(1+r) - (b-1)(1+r) - (1+r) \cdot a + (1+r) \cdot b] \cdot U_2 = 0$

$U_{13}/U_{23} = U_1/U_2 = 1+r$

$\therefore\ U_{13} - U_{23} \cdot (1+r) = 0$

$d\ell/dt_r = 0$

For the utility function

$U = [a \cdot c_1^{\gamma} + b \cdot c_2^{\gamma}] \cdot f(\ell) + g(\ell)$

$U_1 = a \cdot \gamma \cdot c_1^{\gamma-1} \cdot f(\ell) = (1+r) \cdot U_2$

$U_2 = b \cdot \gamma \cdot c_2^{\gamma-1} \cdot f(\ell)$

$\therefore\ a \cdot c_1^{\gamma-1} = b \cdot c_2^{\gamma-1} \cdot (1+r)$

$c_1 \cdot U_{11} = (\gamma-1) \cdot U_1 = (\gamma-1) \cdot (1+r) \cdot U_2$

$(1+r) \cdot c_2 \cdot U_{22} = (\gamma-1) \cdot (1+r) \cdot U_2$

$\therefore\ c_1 \cdot U_{11} - (1+r) \cdot c_2 \cdot U_{22} = 0$

$$U_{12} = 0$$

$$U_{13}/U_{23} = U_1/U_2 = 1+r$$

$$\therefore \; U_{13} - U_{23}(1+r) = 0$$

$$\therefore \; d\ell/dt_r = 0$$

REFERENCES

Atkinson, A.B. and Stiglitz, J.E. [1976], The Design of Tax Structure: Direct Versus Indirect Taxation, Journal of Public Economics, Vol. 6, pp. 55-75.

Bradford, D.F. [1980], The Economics of Tax Policy Towards Savings, in von Furstenberg, G.M. (ed.), The Government and Capital Formation, Cambridge, Mass.

Feldstein, M. [1978], The Welfare Cost of Capital Income Taxation, Journal of Political Economy, Vol. 86, pp. S 29 - S 51.

King, M.A. [1980], Savings and Taxation, in Hughes and Heal (eds.), Public Policy and the Tax System, London.

4. Corporate Financial Structure and the Effects of Differentiated Taxation

4.1 INTRODUCTION

In Chapter 3 I discussed the effects of undifferentiated capital income taxes on the behaviour of households and firms. The corporate financial structure is then irrelevant.

In this chapter and in Chapter 5 I will discuss the effects of differentiating taxes according to the form of financing (equity or debt), and according to the form of returns to the shareholders (dividends or capital gains). Such differentiation is in fact characteristic of most tax systems. Typically, interest payments, but not dividends, are deductible when calculating the corporate tax and the effective household tax rate is higher on dividends than on capital gains. A survey of existing tax systems is found in Chapter 3 of King [1977]. A more recent and more detailed study of the tax systems of selected countries is found in King and Fullerton [1984].

It seems intuitively obvious that higher taxes on equity than on debt will tend to increase the debt-to-capital ratio and that higher taxes on dividends than on capital gains will tend to decrease the dividend-to-income ratio. The effect on the marginal cost of capital and thus the capital intensity of firms is less obvious and I will demonstrate how the direction of this effect depends upon the circumstances. I will also analyze how the excess tax burden on equity, via an increased debt-to-capital ratio, can affect the willingness of firms to take non-financial risks.

The corporate financial structure can be analyzed independently of the dividend policy if either of the following conditions holds:

1. The amount of equity can be costlessly adjusted by new issues or the repurchase of shares.
2. The growth rate of the firm and the dividend policy are such that new shares are issued to finance the growth.

On the other hand the debt ratio will be dynamically affected by marginal changes of the dividend policy if the growth rate and the dividend policy are such that all increases in equity are financed by retained earnings (i.e. new issues do not occur) and if the repurchase of shares is legally restricted.

Most authors have treated the issue of tax discrimination of equity separately from that of tax discrimination of dividends. In order to simplify and to concentrate on the main effects I will follow this tradition. In Chapter 4 I will assume that the amounts of equity and debt can be costlessly adjusted to 'optimal' levels and that the rate of taxation on equity income can be represented by a single effective tax rate. In Chapter 5 I will study the effects of tax discrimination of dividends in two cases: A) that of costlessly adjustable equity capital and B) where the debt ratio is constrained by retained earnings. In that chapter I will also discuss under what circumstances the two cases occur.[1]

In Section 2 of this chapter I will discuss the question of modelling the corporate debt ratio as a function of the tax structure and the uncertainty of corporate returns. The mathematical model is developed in Section 3. In Section 4 I discuss how the optimal debt ratio and the optimal level of nonfinancial risk taking are determined, how they are interrelated and how they are influenced by tax rates as well as other parameters. In Section 5 I analyze how the capital intensity and the debt ratio are determined, regarding nonfinancial risk taking as an exogenous parameter, and in Section 6 I conclude with a summary of the results.

In an open economy the portfolio choice of households and the financial policy of firms are determined independently and both may be affected by tax differentials. In this chapter I will concentrate on the effects on corporate policy. The effects on household behaviour will be analogous to the general case of tax differentiation according to type of asset which I discuss in Chapter 6.

4.2 ON THE CHOICE OF ASSUMPTIONS AND ANALYTICAL MODEL

There does not exist any generally accepted model for the determination of the capital structure of firms. Modigliani and Miller [1958] and [1963] demonstrated that corporate capital structure is irrelevant and no optimal debt ratio exists if markets are perfect and complete, if there is no tax differentiation and if there are no extra costs like bankruptcy disruption costs or agency costs which depend on the debt ratio. When these conditions are fulfilled, it is of no importance whether the desired leverage is attained by corporate debt or by household debt. A generalization of the Modigliani-Miller model is found in Stiglitz [1974].

In the Modigliani-Miller world any tax discrimination of equity will lead to 100 per cent debt financing of investments. This is evidently not compatible with observed corporate behaviour. The paradox was partly resolved in a classical paper by Miller [1977]. Miller points out that the household taxation of equity income is usually lower than that on interest income. This is due to the lower effective tax rate on capital gains. Thus households with high marginal tax rates demand higher pre-tax returns for corporate bonds than for shares. In market equilibrium the corporate cost of debt financing can be equal to that of equity financing.

In order to simplify his exposition Miller assumed that there is no household tax on income from shares and that corporate bonds bear no risk. Shares must give the same certainty equivalent return as tax-exempt municipal bonds, r. If the corporate tax is t_c, corporations are then willing to supply an indefinite amount of corporate bonds yielding $r/(1-t_c)$. Investors will demand corporate bonds if the return is higher than $r/(1-t_p)$ where t_p is the marginal tax rate. The demand for corporate bonds thus depends on the investors' marginal tax rates. Those with tax rates less than t_c will invest in corporate bonds while those with higher tax rates will prefer municipal bonds. In market equilibrium the total amount of corporate bonds and thus the average debt ratio will be determined by the tax structure. For each individual firm the leverage is still irrelevant. For the model used in Miller [1977] an increase in corporate income tax (t_c) will evidently increase the average debt ratio and an increase in tax rates on households' interest income (t_p) will decrease the average debt ratio. But the effects on the debt ratio of any individual firm are indeterminate.[2]

The Miller model retains all assumptions of the Modigliani-Miller model except that of the non-existence of taxation. There are still complete and perfect markets. Firms supply bonds because tax differentiation makes their net interest cost lower than that of many investors. Senbet and Taggart [1984] extend the Miller results to other market imperfections. As an example they take the case where transaction costs for loans are smaller for firms than for households and show that all lending and borrowing will then be done by firms. Generalizing this result they find that firms will act as financial intermediaries when they can complete the markets and reduce the differences in marginal rates of substitution caused by market imperfections. As in Miller's model an optimal capital structure for the corporate sector will be obtained, but as in the case with perfect markets the capital structure of any one firm will be in equilibrium a matter of indifference.

Most economists and financial managers do not accept Miller's irrelevance results as literally true. Few managers would like to run an industrial firm with a debt ratio close to 100 per cent and most would agree that there is some relationship between the acceptable debt ratio and the variability of the income of the firm. Some authors such as Feldstein, Green and Sheshinski [1978] and [1979] simply assume that the supply prices for debt and equity capital rise as the debt-ratio rises and that the functional form and the parameters are such that an interior solution for the debt ratio is obtained. They then proceed to analyze the effect of tax changes. Other authors concentrate on discussing why the supply prices rise and how an optimal debt ratio is determined.

De Angelo and Masulis [1980] and Gordon and Malkiel [1981] start from the Miller [1977] model and show that an optimal debt ratio for each firm will be obtained if corporate results are uncertain and if the corporate tax is asymmetric so that all permissible deductions cannot be used if the debt ratio is high and the firm has a bad year, or if bankruptcy risk entails extra costs or if agency costs increase with the debt ratio (when the debt ratio is high shareholders will have an incentive to take risks that are contrary to the interests of bondholders).[3] They point out that in a Miller type model an optimal debt ratio will be obtained even if costs associated with debt are small compared to the related tax benefits. Other authors as Brennan and Schwartz [1978] and Kim [1978] have designed models with neutral household taxes in which debt costs must balance the full corporate tax benefit of debt.

There is disagreement in the literature regarding the magnitude of the debt costs and whether they are large enough to balance tax benefits. A recent discussion of the costs is found in Gordon and Malkiel [1981]. Based on my own corporate experience I believe that the marginal costs are very high when the debt ratio approaches 100 per cent because a firm cannot be run efficiently under such conditions. Haugen and Senbet [1978] argue against this position in a thought-provoking paper. Their thesis is that bankruptcy costs cannot be high because firms always have the alternative to increase the equity capital when necessary without any large excess costs. Formally, the difference seems to be that Haugen and Senbet think in terms of continuous time trading with perfect information, whereas I believe that a typical feature of the bankruptcy situation is heterogeneous information and general mistrust. I therefore believe that the debt ratio should be determined in a model with discrete time periods where no adjustments can be made within periods.

Another issue debated in the literature is whether value maximization is an appropriate goal for firms. The conditions under which shareholders will unanimously support value maximization are analyzed in Baron [1979], de Angelo [1981] and Makowski [1983]. A more intuitive presentation is found in King [1977] chapter 5. A necessary condition is competitive behaviour in the market for the firm's securities. Another necessary condition is that the securities of the firm are redundant in the sense that they can be priced forming a portfolio of other securities in the market which gives the same payoff in all states of nature. This is always possible if the market is complete and it will be approximately true if the firm is small and the market large. King [1977] asserts that the competitive assumption is untenable because monopoly is the essence of uncertainty (page 194) and he rejects the possibility of using an analytical model to derive an optimal debt ratio. King prefers to explain changes in the debt ratio using a simple empirical equation (page 222). Taggart [1980] introduces debt-related costs and incomplete markets in a Miller type model. Tax arbitrage restrictions combined with incompleteness of the capital market prevent marginal rates of substitution from being equated for all investors. Shareholder preferences are thus not unanimous and equilibrium capital structures will be those which both satisfy a majority of the current shareholders and are immune from an outside take-over. The debt-related costs dictate a tendency for more debt to be issued by firms with lower such costs as in models with complete markets. Taggart's model illustrates the effects of market incompleteness very well and it does yield reasonable results. But I believe that it is unnecessarily complicated for my purposes as I am not interested in the effects of market incompleteness per se but of the effects of taxation.

Based on the preceding review I have decided that I want a model with the following features:

1. Debt related costs will be explicitly introduced and the optimal debt ratio will be determined by the balance between these costs and tax advantages. I believe that this is a fair description of the real world and such a model permits analysis of the effects of taxation on the risk taking of firms.

2. I will use the neoclassical assumption of value maximizing firms. This is in line with the general neoclassical assumptions of this dissertation and I believe that monopoly and market incompleteness would just make the analyses more complicated but not change the main tendencies of the effects of tax differentiation. I find support for this belief in Taggart's paper.

3. I will use a single period model. All decisions must be made at the beginning of the period. This reflects my belief that the equity capital cannot be costlessly adjusted in continuous time.

Miller's model is only applicable to an economy with a closed capital market and most of the literature implicitly refers to closed economies. However, capital markets are becoming more open so I find it more interesting to model a small economy with an open capital market. The supply prices of risk free equity capital and risk free debt are then exogenous to the model and independent of domestic tax rates and domestic corporate behaviour.[4] The international capital market separates the investment and financing decisions of firms from the savings and portfolio decisions of households.

Next I will discuss modelling the effect of risk on capital costs. Uncertainty regarding corporate income will influence capital costs in two ways:

- If investors are risk averse they will demand compensation for the nondiversifiable risk in accordance with the CAPM or APT models. According to the Modigliani-Miller theorems the total magnitude of this risk compensation is independent of the financial structure. When differentiated taxes and debt related costs are introduced this does not hold exactly but any effect on the net cost of the risk compensation from increased tax differentiation must be of minor importance. In this chapter I will therefore ignore nondiversifiable risk.[5]

- Debt related costs are closely related to the uncertainty of corporate income. With perfect certainty there would not be any debt related costs and the financial structure would be determined only by tax considerations. The debt-related costs decrease the expected return and they do not depend on risk aversion. They thus cannot be diversified away by the investors. On the other hand they should, in principle, be diversifiable by the firm. A firm engaged in many industries with low correlation of risk should, ceteris paribus, have lower debt related costs than a specialized firm. We must therefore assume that diversification at the corporate level entails compensating costs for less efficient management. This assumption is formally in conflict with our assumption of constant returns to scale technology, but it is justified by the experience of many conglomerates. The existence of debt related costs, which depend on the total uncertainty of corporate income, is also in conflict with the Capital Asset Pricing Model.

It is necessary to model the size of debt related costs so that they can be expressed in terms of the decision variables, i.e. debt ratio, capital intensity and risk taking. This is complicated by the fact that these costs probably have at least three components, bankruptcy costs, tax shield losses and agency costs.

Bankruptcy costs can be expressed as the product of the probability of bankruptcy and its average cost. When analyzing the effect on the capital costs and thus the capital intensity, it is important to ascertain whether the average cost is proportional to invested capital (untimely disposal of assets etc.) or proportional to wages (low labour productivity due to production disruptions). I will allow for both possibilities.

Costs related to asymmetric taxes and unused tax shields are evidently related to the amount of capital. I will assume that the probability that such costs occur is proportional to the probability of bankruptcy. Agency costs can also be assumed to be proportional to the amount of capital and they are probably proportional to the risk of bankruptcy. I will therefore model the debt-related costs as if they were bankruptcy costs, but when judging their probable magnitude we should not forget about the other components.

A more technical, although not trivial, problem is what assumption to make regarding the tax treatment of debt related costs. After some consideration I have decided to assume that the costs are either deductible for tax purposes or that the tax burden is independent of the degree of tax differentiation and can be included in the average cost of one bankruptcy parameter. Other assumptions complicate the mathematical analysis to no benefit.

4.3 THE MODEL

I will model the corporate choice of three variables, 1) debt ratio, 2) capital intensity and 3) nonfinancial risk, in order to analyze how this choice is influenced by the tax parameters. Capital intensity, k, is a measure of the invested capital per labour unit. The debt ratio, b, is the debt to capital ratio. The amount of debt per labour unit thus is b•k and the amount of equity is (1-b)•k.

As a measure of the nonfinancial risk I will use the normalized standard deviation, σ, of the corporate revenue (really the value added) per labour unit. I will assume that the revenue can be expressed as the product of the expected revenue, $\bar{X}$, and a stochastic variable Θ with expected value unity and standard deviation σ. The 'choice' of σ thus represents the choice of products, markets and technologies which entail a distribution function $F_\sigma(\Theta)$ with the standard deviation σ. I will assume that the form of $F_\sigma(\Theta)$ is determined by the expected value which is equal to unity and by the standard deviation σ. The typical example of such distributions is the normal and most of my comments will relate to a normal distribution of Θ.

A fourth endogenous variable of great importance is expected bankruptcy cost γ which is determined by the decision variables b, k and σ. Risk of bankruptcy and thus the expected cost, is evidently an increasing function of b and σ. The relationship to capital intensity, k, is more ambiguous. Risk of bankruptcy decreases if k increases, but the expected cost for each bankruptcy might increase.

The wage rate, w, is regarded as an exogenous parameter by firms. It will be an endogenous variable in general equilibrium but the effects on w are not given much attention in this chapter. The corporate tax rates, t_d on interest costs and t_e on income from equity capital, are determined by the government and will be restricted by an equal tax revenue condition. The other parameters are determined on the international market.

The expected value added per labour unit $\bar{X}$ is evidently an increasing function of both capital intensity and risk. The size of the derivatives, $\partial\bar{X}/\partial k$ and $\partial\bar{X}/\partial\sigma$, is determined on the international market. For mathematical convenience I will assume that $\bar{X}$ is isoelastic in k and σ so that the elasticities, ε_{xk} and $\varepsilon_{x\sigma}$, are constants.

The interest rate which is determined on the international capital market is r_d and the supply price of equity capital is r_e. If the risk of bankruptcy were zero, the pretax capital cost would thus be $b \cdot k \cdot r_d/(1-t_d) + (1-b) \cdot k \cdot r_e/(1-t_e)$. The compensation for the expected bankruptcy cost, γ, will be paid partly to debt holders and partly to shareholders. I do not discuss how the compensation is distributed but merely assume that the compensation is fair and that the total cost to the firm is γ.

Under assumptions of competitive behaviour, value maximization will be equivalent to maximizing the expected profit, p

$$p = (1-t_e)\left[\bar{X}(k,\sigma) - w - \frac{r_d}{1-t_d} \cdot b \cdot k - \frac{r_e}{1-t_e}\,(1-b) \cdot k - \gamma\right] \qquad (4:1)$$

The optimal values of b, k and σ are thus found by differentiating (4:1). As γ is a function of all these variables they must be determined simultaneously. The effects of changing the tax parameters can be found by differentiating the first order conditions and solving the resulting equation system.

In order to go further we must express γ as a function of the other endogenous variables and the parameters. As discussed in Section 2, γ can be expressed as the product of an expected cost per bankruptcy, A, which is probably a function of k, and the probability of bankruptcy.

Bankruptcy will occur if Θ and the revenue $\Theta \cdot \bar{X}$ are so small that the loss exceeds the equity capital $(1-b) \cdot k$. Therefore if the corporate tax is perfectly symmetric, so that a loss is partly compensated by a tax refund, default occurs if

$$(1-t_e)\left[w + \frac{r_d}{1-t_d}\cdot b\cdot k + \gamma - \Theta\cdot\bar{X}\right] > (1-b)\cdot k \tag{4:2}$$

In order to eliminate w we must use the zero profit condition

$$\bar{X} = w + \frac{r_d}{1-t_d}\cdot b\cdot k + \frac{r_e}{1-t_e}\cdot(1-b)\cdot k + \gamma \tag{4:3}$$

Substituting (4:3) in (4:2) we see that default occurs if Θ is less than Θ_d^s

$$\Theta_d^s = 1 - \frac{(1-b)\cdot k\cdot(1+r_e)}{(1-t_e)\cdot\bar{X}(k,\sigma)} \tag{4:4}$$

The probability of default is thus equal to $F_\sigma(\Theta_d^s)$, where the form of the function F_σ depends on σ, and the expected bankruptcy cost is

$$\gamma_s = A(k)\cdot F_\sigma\left(1 - \frac{(1-b)\cdot k\ (1+r_e)}{(1-t_e)\cdot\bar{X}(k,\sigma)}\right) \tag{4:5}$$

If the tax is asymmetric, and no tax refund is paid, t_e disappears from (4:2) and the critical value of Θ becomes

$$\Theta_d^a = 1 - \frac{(1-b)\cdot k\cdot\left(1 + r_e/(1-t_e)\right)}{\bar{X}(k,\sigma)} \approx$$

$$\approx 1 - \frac{(1-b)\cdot k\cdot(1+r_e)}{\bar{X}(k,\sigma)} \tag{4:6}$$

Θ_d^a and the corresponding bankruptcy cost $_a$ are thus not directly affected by t_e. We shall see that this simplifies the analysis of the effects of adjusting t_e.

4.4 THE DEBT RATIO AND THE NONFINANCIAL RISK TAKING

In an earlier version of this chapter I solved the model with simultaneous adjustments of all three decision variables. I found that each variable is then affected by so many different mechanisms that it is very difficult to present the discussion and the results in an intelligible way. I also found that the sign of the effect on the capital intensity is ambiguous and dependent on a number of assumptions and parameters.

In this version I have therefore preferred to start with one section in which I assume that the effects on the capital intensity are unimportant and can be disregarded. I thus concentrate on determination of the debt ratio and nonfinancial risk taking, examining their interdependence. In Section 4.5 I will discuss the interdependence of the debt ratio and capital intensity, keeping the product choice and thus σ constant.

Let us start with the determination of the optimal debt ratio, b. Differentiating (4:1) and (4:5) we get the first order condition for an interior solution

$$\Delta r = \frac{r_e}{1-t_e} - \frac{r_d}{1-t_d} = \frac{1}{k} \cdot \frac{\partial \gamma_s}{\partial b} = A(k) \cdot F'_\sigma(\Theta_d) \cdot \frac{1+r_e}{\bar{X}(1-t_e)} \qquad (4:7)$$

For any given values of k and σ we can solve Θ_d as a function of the tax-adjusted interest rate differential. By (4:4) b is a linear function of Θ_d. Equation (4:7) thus implicitly defines the optimal value of b as a function of the interest rate differential, the tax rate t_e, the other endogenous variables, k and σ, and the forms of the functions $A(k)$, $F(\Theta_d)$ and $\bar{X}(k,\sigma)$.

A first question is under what circumstances we get an interior solution. In the real world the debt ratio is almost always between zero and unity but what about our model? We can first note that the right hand side of (4:7) is always positive because F' is a density function which is by definition positive (or zero). The interest rate differential must then be positive. If equity were cheaper than debt, no debt would be used. On the other hand the interest rate differential must not be larger than the maximum of the right hand side which is found at the peak of the density function $F'_\sigma(\Theta)$. The height of the peak depends on σ. With a small σ the peak is high and we get interior solutions even with large interest rate differentials; with a large σ the density function is flat and no interior solution might be obtained. According to our model the debt ratio would then be infinite which implies that the probability of default is equal to unity. Firms would thus choose the highest debt ratio that is feasible when institutional restraints are taken into account.

The second order condition of an interior optimum is

$$-\frac{p_{bb}}{1-t_e} = A(k) \cdot F''_\sigma(\Theta_d) \cdot \left[\frac{k(1+r_e)}{\bar{X}(1-t_e)}\right]^2 > 0 \qquad (4:8)$$

which implies that F''_σ must be positive. The density function F'_σ must be increasing at Θ_d. For a symmetric distribution of Θ this implies that $\Theta_d \leq 1$ and thus $b \leq 1$. The highest bankruptcy risk which can exist for an interior solution is thus 50 %.

A second question is how the optimal value of b depends upon the nonfinancial risk σ. This is a meaningful question even though σ is an endogenous variable. We will see that σ is determined by the form of the function $\bar{X}(k,\sigma)$ and discussing how b depends on σ can be interpreted as a discussion of how b depends on the form of $\bar{X}(k,\sigma)$. Alternatively we can argue that b is much more flexible than σ and that we see the effects on b before σ can adjust.

Differentiating (4:7) with respect to b and σ we get

$$\frac{\partial b}{\partial \sigma} = - \frac{p_{b\sigma}}{p_{bb}} = - \frac{\partial^2 \gamma}{\partial b \partial \sigma} \Big/ \frac{\partial^2 \gamma}{\partial b^2} \tag{4:9}$$

Differentiating $\partial\gamma/\partial b$ with respect to σ we must remember that σ affects both the form of the function $F'_\sigma(\Theta)$ and the size of $\bar{X}$ and thus of Θ_d. I will denote the change of $F'_\sigma(\Theta_d)$ which is due to the change of the functional form $\partial F'_\sigma/\partial\sigma$. We then get

$$\frac{\partial b}{\partial \sigma} = - \frac{\frac{\partial F'_\sigma}{\partial \sigma}(\Theta_d) + F''_\sigma(\Theta_d) \cdot \frac{1-\Theta_d}{\bar{X}} \cdot \frac{\partial \bar{X}}{\partial \sigma} - F'_\sigma(\Theta_d) \cdot \frac{1}{\bar{X}} \cdot \frac{\partial \bar{X}}{\partial \sigma}}{F''_\sigma(\Theta_d) \cdot \frac{1-\Theta_d}{1-b}} \tag{4:10}$$

We know from (4:8) that the denominator is positive. The first term of the numerator is the change of the density function when the standard deviation increases. This is positive for low values of Θ_d and negative for high values (the function is flattened with a constant area). For a normal distribution this term equals zero when $\Theta_d = 1-\sigma$. The sign of the sum of the last two terms is equal to the sign of $F'' \cdot (1-\Theta_d) - F'$. This expression is also positive for small Θ_d and negative for large Θ_d (that is for Θ_d approaching 1). For normal distributions it equals zero when $\Theta_d = 1-\sigma$.

The optimal value of b will thus be a decreasing function of σ for low values of Θ_d and an increasing function for high values. For a normal distribution the switching point is $\Theta_d = 1-\sigma$, which corresponds to a 16 % probability of bankruptcy. Financial (b) and nonfinancial (σ) risk will thus be substitutes if Θ_d is less than $1-\sigma$ implying a bankruptcy risk of less than 16 %. If the bankruptcy risk is higher due to a high interest rate differential and a high $\partial\bar{X}/\partial\sigma$, the two forms of risk are complements. This result is contrary to my intuition and it would seem that this case is rare in the real world.

A third question is how b depends on the expected cost of bankruptcy, A. From (4:7) it is obvious that b will be lower the higher A is.

A fourth question is how b depends on the tax variables. Differentiating (4:7) with respect to b, t_e and t_d, we get (for the case of symmetric taxes)

$$A \cdot F''(\Theta_d) \cdot \frac{1-\Theta_d}{1-b} \cdot \frac{1+r_e}{\bar{X}} \cdot \frac{db}{dt_e} = \frac{r_d}{1-t_d}\left(1 - \frac{1-t_e}{1-t_d} \cdot \frac{\partial t_d}{\partial t_e}\right) +$$

$$+ A \cdot F''(\Theta_d) \cdot \frac{1-\Theta_d}{1-t_e} \cdot \frac{1+r_e}{\bar{X}} \qquad (4:11\text{ a})$$

With asymmetric taxes t_e would not appear on the right hand side of (4:7) and we would get

$$A \cdot F''(\Theta_d) \cdot \frac{1-\Theta_d}{1-b} \cdot \frac{1+r_e}{\bar{X}} \cdot \frac{db}{dt_e} = \frac{r_e}{(1-t_e)^2} - \frac{r_d}{(1-t_d)^2} \cdot \frac{\partial t_d}{\partial t_e} \qquad (4:11\text{ b})$$

Thus db/dt_e is always positive if dt_d/dt_e is negative and if no account is taken of changes of σ and k.

Another question is whether b is largest in the case of symmetric or asymmetric taxes. From (4:7) we have

$$F'(\Theta_d) = (1-t_e) \cdot C \qquad (4:12)$$

where C is a constant. Differentiating with respect to b and t_e

$$F''(\Theta_d) \cdot \frac{1-\Theta_d}{1-b} \cdot \frac{db}{dt_e} = - C + F''(\Theta_d) \cdot \frac{1-\Theta_d}{1-t_e} =$$

$$= \frac{1}{1-t_e}\left(F''(1-\Theta_d) - F'\right) \qquad (4:13)$$

From the discussion following (4:10) we know that this is positive for low bankruptcy risk and negative for high bankruptcy risk. If the government shares the risk through a symmetric tax the debt ratio would increase if the bankruptcy risk was moderate in the initial optimal solution but decrease if the bankruptcy risk was high. That case yields many counterintuitive results!

In order to get a better feeling for the results I now assume that the distribution of Θ is normal and work through some numerical examples.

For a normal distribution

$$F'(\Theta_d) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot \exp\left[-\frac{1}{2}\left[\frac{1-\Theta_d}{\sigma}\right]^2\right] \tag{4:14}$$

Substituting (4:14) into (4:7) and taking logarithms

$$(1-b)^2 = \frac{2\,\sigma^2\cdot(1-t_e)^2\cdot\bar{X}^2}{k^2\cdot(1+r_e)^2} \cdot \ln\left[\frac{A\cdot(1+r_e)}{\sqrt{2\pi}\cdot\sigma\cdot\bar{X}\cdot(1-t_e)} \cdot \frac{1}{\dfrac{r_e}{1-t_e} - \dfrac{r_d}{1-t_d}}\right] \tag{4:15}$$

With asymmetric taxes $(1-t_e)$ disappears from its first two ocurrences in (4:15). Choosing some feasible values of σ, A and the interest rates we get:

Example	1	2	3	4	5	6
$\sigma \cdot \bar{X}/k$	.20	.20	.45	.50	.20	.20
A/k	.20	.60	.20	.20	.20	.20
r_e	.20	.05	.20	.20	.12	.105
r_d	.20	.05	.20	.20	.20	.20
t_e	.50	.50	.50	.50	.50	.50
t_d	0	0	0	0	0	0
b (symm. t_e)	.85	.73	.77	.80	.78	.73
b (asymm. t_e)	.78	.52	.87	∞	.61	.50

I first calculated the debt ratios for the case of symmetric taxes and I found the results surprisingly high. The debt ratio never got below 70 per cent even though I varied the parameter assumptions a good deal. One part of the explanation we get when we regard the results for the case of asymmetric taxes. Probably this reflects the actual tax system in Sweden better than the assumption of symmetric taxes.

Another possible explanation is that I tried only to estimate the costs for the actual bankruptcy case, while ignoring the tax shield losses and agency costs discussed in Section 2. The probability of losing half the equity capital is much greater than the probability of actual default according to our model, and such a loss may be large enough to cause agency costs and tax shield losses. Thus I might have severely underestimated total debt-related costs.

Comparing the examples illustrates some of my analytical results. From examples 1 and 2 we see that a trebling of the cost per bankruptcy and a reduction of the interest rate differential to one fourth doubles the equity capital. Examples 3 and 4 illustrate that the optimal debt ratio increases with nonfinancial risk (as measured by σ) when the parameter values are such that bankruptcy risk is high enough in the original optimal solution. The examples also illustrate that in that case asymmetric taxes lead to higher debt ratios. In example 4 there is no interior solution for asymmetric taxes. Examples 5 and 6 illustrate the effect of decreasing the interest rate differential.

We now proceed with the determination of nonfinancial risk taking σ. Differentiating (4:1) and (4:5) we get the first and second order conditions of an interior solution

$$\frac{\partial\bar{X}}{\partial\sigma} = \frac{\partial\gamma}{\partial\sigma} = A \cdot \frac{\partial F_\sigma}{\partial\sigma}(\Theta_d) + A \cdot F'_\sigma(\Theta_d) \cdot \frac{1-\Theta_d}{\bar{X}} \cdot \frac{\partial\bar{X}}{\partial\sigma} \tag{4:16}$$

$$\therefore \frac{\partial\bar{X}}{\partial\sigma} = \frac{A \cdot \frac{\partial F_\sigma}{\partial\sigma}(\Theta_d)}{1 - A \cdot F' \cdot \frac{1-\Theta_d}{\bar{X}}} \tag{4:17}$$

$$P_{\sigma\sigma} = \left(1 - t_e\right)\left(\frac{\partial^2\bar{X}}{\partial\sigma^2} - \frac{\partial^2\gamma}{\partial\sigma^2}\right) < 0 \tag{4:18}$$

According to equation (4:7), which holds exactly for an optimal debt ratio, $A \cdot F' \cdot (1-\Theta_d) = k \cdot (1-b) \cdot \Delta r$. This must be far less than the expected revenue, $\bar{X}$. The denominator of (4:17) is thus close to 1. The numerator is positive for $\Theta_d < 1$ and equal to zero for $\Theta_d = 1$ if the distribution is symmetric. Thus we know that $\partial\bar{X}/\partial\sigma$ and $\partial\gamma/\partial\sigma$ are non-negative for all interior solutions. We also know that $d\bar{X}/d$ would be equal to zero at $\Theta_d = 1$. This implies that firms at this point, which corresponds to $b = 1$, would be willing to take any risk which increases expected revenue.

It is tempting to try to use the second order condition (4:18) in order to reduce the set of feasible interior solutions and exclude the 'abnormal' solutions with $\sigma > 1 - \Theta_d$. Let us try this for a normal distribution. If G is a standardized normal distribution function with mean 1 and standard deviation 1

$$F_\sigma(\Theta_d) = G\left(1 - \frac{1-\Theta_d}{\sigma}\right) \tag{4:19a}$$

$$F'_\sigma(\Theta_d) = \frac{G'}{\sigma} \quad ; \quad F''_\sigma(\Theta_d) = \frac{G''}{\sigma^2} \tag{4:19b}$$

$$F''_\sigma = \frac{1-\Theta_d}{\sigma^2} \cdot F'_\sigma \quad ; \quad G'' = \frac{1-\Theta_d}{\sigma} \cdot G' \tag{4:19c}$$

$$\begin{aligned}\frac{\partial\gamma}{\partial\sigma} &= A \cdot \frac{\partial G}{\partial\sigma} = A \cdot G' \cdot \frac{1-\Theta_d}{\sigma^2} \cdot (1+\varepsilon_{x\sigma}) = \\ &= A \cdot F' \cdot \frac{1-\Theta_d}{\sigma} \cdot (1+\varepsilon_{x\sigma})\end{aligned} \tag{4:20}$$

$$\begin{aligned}\frac{\partial^2\gamma}{\partial\sigma^2} &= A \cdot \frac{\partial^2 G}{\partial\sigma^2} = \\ &= A \cdot G' \cdot \frac{1-\Theta_d}{\sigma^3} \cdot \left[\frac{(1-\Theta)^2}{\sigma^2} \cdot (1+\varepsilon_{x\sigma}) - 2 - \varepsilon_{x\sigma}\right] \cdot (1+\varepsilon_{x\sigma})\end{aligned} \tag{4:21}$$

When deriving (4:21) I treated $\varepsilon_{x\sigma}$ as a constant. This is unlikely as we do not expect $\bar{X}$ to be equal to zero when $\sigma = 0$, but it is of little importance since the elasticity is much smaller than 1.

We find that $\partial^2\gamma/\partial\sigma^2$ is positive for

$$\frac{1-\Theta_d}{\sigma} > \sqrt{\frac{2 + \varepsilon_{x\sigma}}{1 + \varepsilon_{x\sigma}}}$$

and negative for the abnormal case $\sigma > 1 - \Theta_d$. If $\partial^2\bar{X}/\partial\sigma^2$ were zero or positive (that is if $\bar{X}$ were linear or convex in σ) we could then exclude the case $\sigma > 1 - \Theta_d$. Investors would choose either $\sigma \leq 1 - \Theta_d$ or an infinite σ. But the latter result seems absurd. At least for large σ, $\bar{X}$ must be a concave function of σ and we cannot exclude the possibility that the second order condition may be satisfied for any σ.

From (4:16) we see that the main factors which determine σ are the function $\partial\bar{X}/\partial\sigma$, the form of the distributions F_σ and the size of Θ_d which depends on b and, in the case of symmetric taxes, t_e. The effect of b on σ is the same as the effect of σ on b; that is, they are substitutes for low bankruptcy risks and complementary for higher bankruptcy risks. This can be checked by differentiating (4:16). In the case of an asymmetric corporate tax, the tax rate has no direct effect on σ, because t_e does not occur in (4:16). t_e then affects the solution only by changing b which affects the optimal σ. In the case of symmetric taxes we do, however, get a direct effect on σ. Differentiating (4:16) we get

$$\frac{\partial\sigma}{\partial t_e} = \frac{1-t_e}{p_{\sigma\sigma}} \cdot \frac{\partial^2\gamma}{\partial\sigma\ \partial t_e} = -\frac{1-t_e}{p_{\sigma\sigma}}\left[A \cdot \frac{\partial F'_\sigma}{\partial\sigma}(\Theta_d) \cdot \frac{1-\Theta_d}{1-t_e} - \right.$$

$$\left. - A\left(F' - F''(1-\Theta_d)\right) \cdot \frac{1-\Theta_d}{1-t_e} \cdot \frac{1}{\bar{X}} \cdot \frac{\partial\bar{X}}{\partial t_e}\right] \qquad (4:22)$$

From (4:17) we know that $p_{\sigma\sigma}$ is negative. The numerator resembles that of (4:10). We thus know that, for a normal distribution, $\partial\sigma/\partial t_e$ is positive for $\Theta_d < 1-\sigma$ and negative for $\Theta_d > 1-\sigma$. At the inflection point, $\Theta_d = 1-\sigma$, σ is independent of t_e as well as of b.

Implicitly we can <u>solve b and σ simultaneously</u> from (4:7) and (4:17).

The general solution is, however, not explicit enough to be of much interest. Differentiating (4:7) and (4:17) we can study how the solution is affected by increased tax differentiation. As can be expected from (4:11) and (4:22) we find that with an asymmetric equity tax (i.e. there are no tax refunds in the case of default) the whole effect comes from the change of the tax-adjusted interest rate differential. Increased tax differentiation will make debt cheaper and thus increase b. This effect will spill over on σ according to (4:10) and σ will decrease or increase depending upon whether $\sigma - (1-\Theta_d)$ in the initial solution is negative or positive, that is depending upon whether the bankruptcy risk is small or large. With a symmetric tax we also get an effect of increased government risk sharing, which tends to increase or decrease b and σ depending upon the sign of $\sigma - (1-\Theta_d)$. The two effects on σ work in different directions and I have not been able to show that any one of them always dominates. Even more frustrating is that I have not been able to show that the net effect on b is always positive. If the direct effect on σ is large enough ($p_{\sigma\sigma}$ is small enough), it is possible that the indirect effect on b from σ dominates the direct positive effect. The critical factor is the relative size of $p_{\sigma\sigma}$, p_{bb} and $p_{\sigma b}$.

In order to get more intuitive results we can assume that Θ is normally distributed and that we have a simple revenue function $\bar{X}(\Theta)$. I had originally intended to use an isoelastic function $\bar{X} = X_o \cdot \sigma^{\varepsilon}$ but this does not work because $\bar{X}(0) = 0$ and all solutions with low σ are non-viable. So I will use the somewhat more complicated

$$\bar{X} = X_o + \alpha \cdot \sigma^{\gamma} \cdot X_o \tag{4:23a}$$

Approximating (4:17) with

$$\frac{\partial \bar{X}}{\partial \sigma} = A \cdot \frac{\partial F_{\sigma}}{\partial \sigma} = A \cdot F' \cdot \frac{1-\Theta_d}{\sigma} \tag{4:23b}$$

we then have

$$X_o \cdot \gamma \cdot \alpha \cdot \sigma^{\gamma-1} = A \cdot F' \cdot \frac{1-\Theta_d}{\sigma} = (1-b) \cdot k \cdot \Delta r \cdot \frac{1}{\sigma} \tag{4:24}$$

$$X_o \cdot \gamma \cdot \alpha \cdot \sigma^{\gamma} = (1-b) \cdot k \cdot \Delta r \tag{4:25}$$

We may note that for a linear revenue function ($\gamma = 1$) the amount of risk and the equity capital are proportional and the return to risk is equal to the excess cost of the equity capital.

We thus have

$$\sigma = \left[\frac{(1-b)\cdot k\cdot \Delta r}{\gamma\cdot\alpha\cdot X_o}\right]^{\frac{1}{\gamma}} \tag{4:26}$$

Note that (4:26) does not show how σ depends on b. It is just a relation that must hold between the optimal σ and the optimal b.

Substituting σ from (4:26) and $\bar{X}$ from (4:23) into (4:15) we get an equation in b and exogenous parameters

$$(1-b)^2 = \frac{2\cdot\left[\frac{(1-b)\cdot k\cdot\Delta r}{\gamma\cdot\alpha\cdot X_o}\right]^{\frac{2}{\gamma}}\cdot(1-t_e)^2\cdot\left[X_o+\frac{(1-b)\cdot k\cdot\Delta r}{\gamma}\right]^2}{k^2\cdot(1+r_e)^2}\cdot$$

$$\cdot\ \ln\left[\frac{A\cdot(1+r_e)}{\sqrt{2\pi}\cdot\left[\frac{(1-b)\cdot k\cdot\Delta r}{\gamma\cdot\alpha\cdot X_o}\right]^{\frac{1}{\gamma}}\cdot\left[X_o+\frac{(1-b)\cdot k\cdot\Delta r}{\gamma}\right]\cdot(1-t_e)\cdot\Delta r}\right] \tag{4:27}$$

This can be rewritten

$$\sqrt{2}\cdot(1-b)^{\frac{1}{\gamma}-1}\cdot k^{\frac{1}{\gamma}-1}\cdot\Delta r^{\frac{1}{\gamma}}\cdot\gamma^{-\frac{1}{\gamma}}\cdot\alpha^{-\frac{1}{\gamma}}\cdot X_o^{-\frac{1}{\gamma}}\cdot$$

$$\cdot\ (1-t_e)\cdot(1+r_e)^{-1}\cdot\left[X_o+\frac{(1-b)\cdot k\cdot\Delta r}{\gamma}\right]\cdot$$

$$\cdot\ \ln^{\frac{1}{2}}\left[A\cdot(1+r_e)\cdot(\sqrt{2\pi})^{-1}\cdot\left(\frac{(1-b)\cdot k\cdot\Delta r}{\gamma\cdot\alpha\cdot X_o}\right)^{-\frac{1}{\gamma}}\cdot\right.$$

$$\left.\cdot\ \left[X_o+\frac{(1-b)\cdot k\cdot\Delta r}{\gamma}\right]^{-1}\cdot(1-t_e)^{-1}\cdot\Delta r^{-1}\right] = 1 \tag{4:28}$$

If the tax is asymmetric, t_e disappears and the tax differentiation only works through Δr. The results depend upon the choice of α and λ for which I had little intuition. Trying values of $\lambda > 1$ I found that I only got saddle point solutions with $p_{bb} \cdot p_{\sigma\sigma} - p_{b\sigma}^2 < 0$ and in some cases $p_{\sigma\sigma} > 0$. The reason is that $\partial^2 x/\partial\sigma^2$ is positive so $p_{\sigma\sigma}$ becomes very small.

For the examples below I used $\lambda = 0.5$. I also assumed that $k/X_o = 2$. First, I calculate b, σ, $1-\Theta_d$ and $\varepsilon_{x\sigma}$ for the following examples with a symmetric tax on equity.

Example	1	2	3	4	5	6
α	.01	.01	.01	.03	.02	.02
A/X_o	.4	.4	.4	.4	.4	1.2
t_e	.50	.45	.75	.50	.50	.50
t_d	0	0	0	0	0	0
r_e	.05	.05	.05	.05	.05	.05
r_d	.09	.09	.09	.09	.09	.09
Δr	.01	.0009	.11	.01	.01	.01
b	.92	-15	.999	.11	.67	.71
σ	.10	16	.002	1.4	.44	.34
$1-\Theta_d$	.33	35	.008	3.7	1.39	1.21
$\varepsilon_{x\sigma}$	.002	0.02	.0005	.018	.006	.006

We see that in these examples increased tax differentiation increases the debt ratio and decreases the nonfinancial risk-taking. Increasing the market price of nonfinancial risk α increases σ and decreases b. In these examples increasing the average cost of bankruptcy, A, decreases nonfinancial risk taking and increases the debt ratio. For larger α no interior solutions exist. In all our examples $(1-\Theta_d) > \sigma$ and $\varepsilon_{x\sigma}$ is small which implies that the approximation of (4:23b) has not affected the results.

For asymmetric taxation in which losses are not offset, we obtain the following results

Example	1	2	3	4	5	6
b	.97	-4	.9997	.62	.85	.86
σ	.02	3.5	.0001	.25	.09	.07
$1-\theta_d$	.07	11	.0005	.79	.31	.29

In all our examples σ is decreased because of the tax asymmetry and the debt ratio is increased.

4.5 THE CAPITAL INTENSITY

In this section I will assume that nonfinancial risk σ is a given exogenous parameter and I will concentrate on the effects of increased tax differentiation on the capital intensity k and on the interdependence of the debt ratio, b, and the capital intensity, k.

To regard σ as an exogenous parameter might be a realistic assumption for the short to medium run. The choice of products, markets and technology which is represented by σ is probably less flexible than the debt ratio and the capital intensity.

The first order condition of an optimal k is obtained by differentiating (4:1)

$$p_k = (1-t_e)\left(\frac{\partial\bar{X}}{\partial k} - b\cdot\frac{r_d}{1-t_d} - (1-b)\cdot\frac{r_e}{1-t_e} - \frac{\partial\gamma}{\partial k}\right) = 0 \qquad (4:29)$$

In this expression $\partial\bar{X}/\partial k$ is the marginal expected revenue of increasing k and $\partial\gamma/\partial k$ is the marginal expected bankruptcy cost. By differentiating (4:5) we see that $\partial\gamma/\partial k$ is less than the average bankruptcy cost γ/k

$$\frac{\partial \gamma}{\partial k} = - A \cdot F'_{\sigma}(\Theta_d) \cdot \frac{1-\Theta_d}{k} \cdot (1-\varepsilon_{xk}) + A'(k) \cdot F_{\sigma}(\Theta_d) \quad (4:30)$$

The first term is negative which reflects the fact that a higher k and thus a larger equity capital reduces the probability of default. The second term is positive and reflects the fact that each bankruptcy is more expensive when more capital is involved. But $A'(k) < A/k$ because the cost of a bankruptcy also depends on the labour costs. Evidently the sign of $\partial\gamma/\partial k$ depends upon the elasticity ε_{xk}, the elasticity of A with regard to k (ε_{Ak}) and the size of $(1-\Theta_d)/\sigma$.

The fact that $\partial\gamma/\partial k < \gamma/k$ implies that the marginal revenue $\partial\bar{X}/\partial k$ will be smaller than the average capital cost.

We can get some further information about the size of $\partial\bar{X}/\partial k$ at optimal capital intensity if we assume that the debt ratio is always optimal. Substituting (4:7) we then have

$$\frac{\partial \bar{X}}{\partial k} = \frac{r_d}{1-t_d} + A \cdot F'_{\sigma}(\Theta_d) \cdot \frac{1-\Theta_d}{k} \cdot \varepsilon_{xk} + A'(k) \cdot F_{\sigma}(\Theta_d) \quad (4:31)$$

The marginal capital cost is thus equal to the sum of

- the tax-adjusted cost of riskless debt

- the increase of expected bankruptcy cost which is due to increased expected revenue (assuming that the standard deviation increases in proportion to the mean)

- and the increase of the expected bankruptcy cost which is due to the positive elasticity of A on k, ε_{Ak}.

If the last two terms were insignificant the capital intensity would only depend on r_d and t_d. The other parameters would influence only the debt ratio b. Increasing tax differentiation ($dt_e > 0$, $dt_d < 0$) would then always lead to an increasing capital intensity.

Normally, however, A'(k) will be positive and at least the last term will be significant. It is then impossible to see from only (4:31) how k is influenced by different parameters. We have to solve for the effects on b and k simultaneously. This can be done using any pair of (4:7), (4:29) and (4:31). In order to be able to use the second order conditions of an interior optimum ($p_{bb} < 0$, $p_{kk} < 0$ and $p_{bb} \cdot p_{kk} - p_{bk}^2 > 0$) for the determination of signs, I will differentiate the first order conditions (4:7) and (4:29).

First I will hold the tax rates constant and examine how the optimal values of k and b depend on σ. We then have

$$p_{bb}\, db + p_{bk} \cdot dk = -p_{b\sigma} \cdot d\sigma \tag{4:32a}$$

$$p_{bk}\, db + p_{kk} \cdot dk = -p_{k\sigma} \cdot d\sigma \tag{4:32b}$$

and

$$\frac{db}{d\sigma} = \frac{-p_{kk}\, p_{b\sigma} + p_{bk}\, p_{k\sigma}}{p_{bb}\, p_{kk} - p_{bk}^2} \tag{4:33a}$$

$$\frac{dk}{d\sigma} = \frac{-p_{bb}\, p_{k\sigma} + p_{bk} \cdot p_{b\sigma}}{p_{bb}\, p_{kk} - p_{bk}^2} \tag{4:33b}$$

where the denominators are positive for all interior solutions and where

$$p_{bb} = -(1-t_e) \cdot A \cdot F''_{\sigma}(\theta_d) \cdot \left(\frac{1-\theta_d}{1-b}\right)^2 < 0 \tag{4:34a}$$

$$p_{kk} = (1-t_e) \cdot \left(\frac{\partial^2 \bar{x}}{\partial k^2} - \frac{\partial^2 \gamma}{\partial k^2}\right) < 0 \tag{4:34b}$$

$$p_{bk} = (1-t_e)\left[\Delta r - \frac{\partial^2 \gamma}{\partial b\, \partial k}\right] = (1-t_e)\left[\Delta r - \right.$$

$$- \frac{A}{k} \cdot \frac{1-\Theta_d}{1-b} \cdot (1-\varepsilon_{xk})\left(F'_\sigma - F''_\sigma \cdot (1-\Theta_d)\right) -$$

$$\left. - A'(k) \cdot F'_\sigma \cdot \frac{1-\Theta_d}{1-b}\right] \tag{4:34c}$$

$$p_{b\sigma} = -(1-t_e) \quad \frac{\partial^2 \gamma}{\partial b\, \partial \sigma} = -(1-t_e) \cdot A \cdot \frac{1-\Theta_d}{1-b}\left[\frac{\partial[F'_\sigma(\Theta_d)]}{\partial \sigma} + \right.$$

$$\left. + \frac{\varepsilon_{x\sigma}}{\sigma} \cdot F'_\sigma(\Theta_d)\right] \tag{4:34d}$$

$$p_{k\sigma} = (1-t_e)\left[\frac{\partial^2 \bar{X}}{\partial k\, \partial \sigma} - \frac{\partial^2 \gamma}{\partial k\, \partial \sigma}\right] = (1-t_e)\left[\frac{\partial^2 \bar{X}}{\partial k\, \partial \sigma} + \right.$$

$$+ A \cdot \frac{1-\Theta_d}{k} \cdot (1-\varepsilon_{xk})\left(\frac{\partial[F'_\sigma(\Theta_d)]}{\partial \sigma} - \frac{\varepsilon_{x\sigma}}{\sigma} F'_\sigma(\Theta_d)\right) -$$

$$\left. - A'(k) \cdot \frac{\partial[F_\sigma(\Theta_d)]}{\partial \sigma}\right] \tag{4:34e}$$

If the distributions of Θ are normal we can simplify these expressions by using the following equations (G is the standardized normal distribution)

$$F''(\Theta_d) = \frac{1-\Theta_d}{\sigma^2} \cdot F'_\sigma(\Theta_d) \tag{4:35a}$$

$$F_\sigma(\Theta_d) = G\left(1 - \frac{1-\Theta_d}{\sigma}\right) \tag{4:35b}$$

$$F'_\sigma(\Theta_d) = \frac{1}{\sigma} \cdot G' \tag{4:35c}$$

$$\frac{\partial[F_\sigma(\Theta_d)]}{\partial \sigma} = G' \cdot \frac{1-\Theta_d}{\sigma^2} \cdot (1 + \varepsilon_{x\sigma}) \tag{4:35d}$$

$$\frac{\partial[F'_\sigma(\Theta_d)]}{\partial\sigma} = -\frac{1}{\sigma^2} \cdot G' + \frac{1-\Theta_d}{\sigma^3} \cdot G'' \cdot (1 + \varepsilon_{x\sigma}) =$$

$$= \frac{G'}{\sigma^2}\left[\left(\frac{1-\Theta_d}{\sigma}\right)^2 (1 + \varepsilon_{x\sigma}) - 1\right] \tag{4:35e}$$

$$\Delta r = \frac{A}{k} \cdot F'_\sigma \cdot \frac{1-\Theta_d}{1-b} \tag{4:35f}$$

$$A'(k) \cdot \frac{k}{A} = \varepsilon_{Ak} \tag{4:35g}$$

Thus

$$p_{bb} = -(1-t_e) \cdot \Delta r \cdot k \cdot \frac{1-\Theta_d}{1-b} < 0 \tag{4:36a}$$

$$p_{bk} = (1-t_e) \cdot \Delta r \cdot \left[\varepsilon_{xk}\left(1 - \frac{1-\Theta_d}{\sigma}\right) + \frac{1-\Theta_d}{\sigma} - \varepsilon_{Ak}\right] \tag{4:36b}$$

$$p_{b\sigma} = -(1-t_e) \cdot \Delta r \cdot k \cdot \frac{1}{\sigma} \cdot \left[\left(\frac{1-\Theta_d}{\sigma}\right)^2 - 1\right] (1 + \varepsilon_{x\sigma}) \tag{4:36c}$$

$$p_{k\sigma} = (1-t_e)\left[\frac{\partial^2\bar{X}}{\partial k\ \partial\sigma} + \Delta r \cdot (1-b) \cdot \frac{1}{\sigma} (1 + \varepsilon_{x\sigma}) \cdot \right.$$

$$\left. \cdot \left[\left(\left(\frac{1-\Theta_d}{\sigma}\right)^2 - 1\right)(1-\varepsilon_{xk}) - \varepsilon_{Ak}\right]\right] \tag{4:36d}$$

As before we find that $p_{b\sigma}$ is negative if the parameter values are such that $1-\Theta_d > \sigma$, that is if the bankruptcy risk is less than 16 %. From (4:36b) we see that the opposite is true for p_{bk} if $\varepsilon_{Ak} = 1$. But if ε_{Ak} is equal to zero, p_{bk} is always positive.

In $p_{k\sigma}$ the first term is probably positive. The second term is positive if $\varepsilon_{Ak} < 1-\varepsilon_{xk}$ and if $(1-\Theta_d) > \sigma$ that is for the case of low bankruptcy risk. In other cases the sign of $p_{k\sigma}$ is ambiguous.

What can we say about the signs of $db/d\sigma$ and $dk/d\sigma$?

We see that $db/d\sigma$ is equal to zero if $1-\Theta_d = \sigma$. Marginal changes of σ from this point do not affect the optimal debt ratio. For $1-\Theta_d > \sigma$, the first term is negative and p_{bk} is positive. $db/d\sigma$ would thus be unequivocally negative if $p_{k\sigma} \leq 0$. But $p_{k\sigma}$ will tend to be positive so the sign of $db/d\sigma$ is uncertain. If k increases much enough, b can increase with σ. For $1-\Theta_d < \sigma$ the sign of p_{bk} depends on the size of ε_{Ak} and the sign of $db/d\sigma$ is still uncertain.

We also see that if $1-\Theta_d = \sigma$, $dk/d\sigma$ will have the same sign as $p_{k\sigma}$. But this sign is ambiguous. For $1-\Theta_d > 0$, the second term is negative. But the first term gets more positive and the outcome is still uncertain.

Evidently, we cannot say very much about how the optimal values of b and k depend on σ without specifying the parameter values.

What then about the effects of increased tax differentiation ($dt_e > 0$, $dt_d < 0$)? Using the same method we find

$$p_{bb} \cdot db + p_{bk} \cdot dk = -p_{bt} \cdot dt_e \tag{4:37a}$$

$$p_{bk} \cdot db + p_{kk} \cdot dk = -p_{kt} \cdot dt_e \tag{4:37b}$$

$$\frac{db}{dt_e} = \frac{-p_{kk} \cdot p_{bt} + p_{bk} \cdot p_{kt}}{p_{bb} \cdot p_{kk} - p_{bk}^2} \tag{4:38a}$$

$$\frac{dk}{dt_e} = \frac{-p_{bb} \cdot p_{kt} + p_{bk} \cdot p_{bt}}{p_{bb} \cdot p_{kk} - p_{bk}^2} \tag{4:38b}$$

p_{bt} and p_{kt} depend upon whether the corporate tax is symmetric or asymmetric and upon how the equal tax revenue condition is formulated.

Let us start with the simplest case of asymmetric taxation and a naive (in the sense that the government does not take effects on b and k into account) adjustment of the tax rates. We then have

$$\frac{dt_d}{dt_e} = - \frac{(1-b) \cdot \frac{r_e}{(1-t_e)^2}}{b \cdot \frac{r_d}{(1-t_d)^2}} \tag{4:39}$$

$$p_{bt} = (1-t_e) \cdot k \cdot \left(\frac{r_e}{(1-t_e)^2} - \frac{r_d}{(1-t_d)^2} \cdot \frac{dt_d}{dt_e} \right) =$$

$$= (1-t_e) \cdot \frac{r_e}{(1-t_e)^2} \cdot \frac{k}{b} > 0 \tag{4:40a}$$

$$p_{kt} = -(1-t_e) \cdot \left[(1-b) \cdot \frac{r_e}{(1-t_e)^2} + b \cdot \frac{r_d}{(1-t_d)^2} \quad \frac{dt_d}{dt_e} \right] = 0 \tag{4:40b}$$

With no risk sharing by the government and no effect on the average tax rate from distortion of the tax basis, there is no direct effect on capital intensity. db/dt_e is then always positive. dk/dt_e has the same sign as p_{bk} which is positive for $1-\Theta_d > \sigma$ and might be positive or negative for $1-\Theta_d < \sigma$ depending upon the size of ε_{Ak}.

With symmetric taxes we get additional terms from the effects upon $\partial\gamma/\partial b$ and $\partial\gamma/\partial k$

$$p_{bt} = (1-t_e) \cdot \left[\frac{k}{b} \cdot \frac{r_e}{(1-t_e)^2} - A \cdot \frac{1-\Theta_d}{(1-b)(1-t_e)} \left(F' - F''(1-\Theta_d) \right) \right] =$$

$$= \frac{r_d \cdot k}{1-t_d} + \frac{1-b}{b} \cdot \frac{r_e \cdot k}{1-t_e} + A \cdot \frac{(1-\Theta_d)^2}{(1-b)} \quad F'' > 0 \tag{4:41a}$$

$$p_{kt} = -(1-t_e) \cdot \frac{\partial^2 \gamma}{\partial k\ \partial t_e} = \frac{A}{k} \cdot \frac{1-\Theta_d}{1-t_e} \cdot (1-\varepsilon_{xk})\left[F'-F''(1-\Theta_d)\right] +$$

$$+ A'(k) \cdot F' \cdot \frac{1-\Theta_d}{1-t_e} \qquad (4:41b)$$

For a normal distribution we have

$$p_{kt} = \Delta r \cdot \frac{1-b}{1-t_e}\left[(1-\varepsilon_{xk}) \cdot \left(1 - \left(\frac{1-\Theta_d}{\sigma}\right)^2\right) + \varepsilon_{Ak}\right] \qquad (4:41c)$$

We see that p_{kt} is negative for small $(1-\Theta_d)/\sigma$, that is for low bankruptcy risk, and positive for high risk. The switching point depends upon the size of ε_{Ak}. In the normal case of moderate risk of bankruptcy more symmetric taxes will thus tend lead to a lower capital intensity and a lower debt ratio.

If we take the Laffer curve effects into account the results become even more complicated. Any increase of the debt ratio reduces the tax basis and makes it necessary to increase the average tax rate and this will have a negative effect on the capital intensity. Any increase of the capital intensity makes it possible to reduce the tax rates. Effects on the capital intensity will thus be magnified. If the economy is close to the peak of the Laffer curve, k might be very sensitive to increased tax differentiation.

We might also get an effect on labour income tax revenues when the capital intensity changes but that effect is probably proportional to the effect on capital intensity and I will not take it into account below but assume that the government wants a constant corporate income tax revenue

$$R = t_e \cdot \frac{r_e}{1-t_e} \cdot (1-b) \cdot k + t_d \cdot \frac{r_d}{1-t_d} \cdot b \cdot k \qquad (4:42)$$

We then have

$$\frac{dt_d}{dt_e} = - \frac{\frac{r_e}{(1-t_e)^2}\cdot(1-b)}{\frac{r_d}{(1-t_d)^2}\cdot b} + \frac{\frac{r_e t_e}{1-t_e} - \frac{r_d t_d}{1-t_d}}{\frac{r_d}{(1-t_d)^2} \quad b} \cdot \frac{db}{dt_e} -$$

$$- \frac{\frac{(1-b)\cdot r_e \cdot t_e}{1-t_e} + \frac{b\cdot r_d \cdot t_d}{1-t_d}}{\frac{r_d}{(1-t_d)^2}\cdot b \cdot k} \cdot \frac{dk}{dt_e} \qquad (4:43)$$

In the case of asymmetric taxes we then get

$$p_{bt} = (1-t_e)\cdot\left[\frac{r_e}{(1-t_e)^2}\cdot\frac{k}{b} - \left(\frac{r_e t_e}{1-t_e} - \frac{r_d t_d}{1-t_d}\right)\cdot\frac{k}{b}\cdot\frac{db}{dt_e} + \right.$$

$$\left. + \left(\frac{1-b}{b}\cdot\frac{r_e t_e}{1-t_e} + \frac{r_d t_d}{1-t_d}\right)\cdot\frac{dk}{dt_e}\right] =$$

$$= \frac{r_e}{1-t_e}\cdot\frac{k}{b} - p_{btb}\cdot\frac{db}{dt_e} + p_{btk}\cdot\frac{dk}{dt_e} \qquad (4:44a)$$

where p_{btb} and p_{btk} are positive.

$$p_{kt} = -(1-t_e)\cdot\left[\left(\frac{r_e t_e}{1-t_e} - \frac{r_d t_d}{1-t_d}\right)\cdot\frac{db}{dt_e} - \right.$$

$$\left. - \left(\frac{(1-b)\cdot r_e t_e}{1-t_e} + \frac{b\cdot r_d t_d}{1-t_d}\right)\cdot\frac{1}{k}\cdot\frac{dk}{dt_e}\right] =$$

$$= -\frac{b}{k}\cdot p_{btb}\cdot\frac{db}{dt_e} + b\cdot p_{btk}\cdot\frac{dk}{dt_e} \qquad (4:44b)$$

Solving (4:37) we then have

$$\frac{db}{dt_e} = \frac{-(p_{kk}+b\cdot p_{btk})\cdot\frac{r_e}{1-t_e}\cdot\frac{k}{b}+(p_{bk}+p_{btk})\cdot 0}{(p_{bb}-p_{btb})(p_{kk}+b\cdot p_{btk})-(p_{bk}+p_{btk})\left(p_{bk}-\frac{b}{k}\cdot p_{btb}\right)}$$

(4:45a)

$$\frac{dk}{dt_e} = \frac{-(p_{bb}-p_{btb})\cdot 0+\left(p_{bk}-\frac{b}{k}\cdot p_{btb}\right)\cdot\frac{r_e}{1-t_e}\cdot\frac{k}{b}}{(p_{bb}-p_{btb})(p_{kk}+b\cdot p_{btk})-(p_{bk}+p_{btk})\left(p_{bk}-\frac{b}{k}\cdot p_{btb}\right)}$$

(4:45b)

We know that the denominators are positive for low tax rates (p_{btb} and p_{btk} are equal to zero when the tax rates are equal to zero). As the denominators are continuous functions of t_e and t_d and as db/dt_e and dk/dt_e go to infinity if the denominators approach zero, they cannot be negative for any interior solutions. It is also obvious that $(p_{kk} + b\cdot p_{btk})$ must be negative if the tax rates are not so high that we have passed the Laffer curve maximum. If $(p_{kk} + b\cdot p_{btk})$ were positive we could increase tax revenue by lowering both tax rates.

We can then see that with asymmetric taxes db/dt_e is still positive. The sign of dk/dt_e depends on the relative size of p_{bk} and $b/k\cdot p_{btb}$ which depends on the tax rates. If the debt ratio is increased because of increased tax differentiation, the capital intensity tends to increase in order to maintain the equity capital but it tends to decrease because of higher effective tax rates.

With symmetric taxes the zeroes in (4:45) are replaced by the p_{kt} of (4:41) which is negative for moderate bankruptcy risks. The two terms of db/dt_e then have opposite signs. Theoretically we cannot exclude the possibility that the second term dominates the first and that increased tax differentiation leads to a lower debt ratio. We cannot theoretically compare the size of p_{kk} and p_{bk} and if p_{kk} is small or if we are close to the Laffer curve maximum, the whole effect might fall on k with $db/dt_e < 0$ and $dk/dt_e \ll 0$. But I would not expect to experience this reaction.

4.6 SUMMARY AND CONCLUSIONS

I have developed a model of the determination of corporate capital structure as a function of tax differentials and debt-related costs. Firms were assumed to be value maximizing. The net-of-tax supply prices of equity capital and risk free debt were assumed to be determined by an international capital market.

I analyzed the effect of increased tax differentiation on the optimal values of the debt ratio, the capital intensity and the nonfinancial risktaking. The results turn out to be very complicated and even the signs of the effects depend on a number of parameter values. Most of the complications arise from the fact that capital intensity is influenced in many ways by increased tax differentiation.

If we at first disregard changes in capital intensity and concentrate on other effects on the debt ratio (financial risk taking) and the nonfinancial risk taking, we find that the results depend on the risk of bankruptcy in the initial solution. At an intermediate level of risk (16 per cent with a normal distribution, which corresponds to the inflection point of the density function) the effects on the debt ratio and on nonfinancial risk taking are mutually independent. For lower bankruptcy risk the two forms of risk taking are substitutes, at higher risk they are complementary. The effect on nonfinancial risk taking from the risk sharing effect of a symmetric corporate tax behaves in the same way, being positive for low risk and negative for high risk. The effects associated with low bankruptcy risk are those that we would intuitively expect and I have never experienced the high risk case myself, but I cannot exclude it theoretically. The direct effect of increased tax differentiation on the debt ratio will always be positive. The substitution effect of more expensive equity capital and less expensive debt outweighs the effect of increased risk sharing and it also outweighs the Laffer curve effect if it is at all possible to maintain tax revenues while increasing the tax on equity and reducing that on debt. If the tax is asymmetric (no loss offsets) there is no direct effect on nonfinancial risk taking and we then have the unequivocal result that the debt ratio will increase and nonfinancial risk will decrease, remain constant or increase depending on the level of bankruptcy risk. With a symmetric tax we get a countervailing effect on the nonfinancial risk taking and we cannot a priori determine the sign of the net effect. If the optimal value of the nonfinancial risk is sensitive enough to parameter values, this risk might increase so much in the low bankruptcy risk case that the net effect on the debt ratio is negative. This outcome does not seem likely, but I cannot exclude the possibility.

The direct effect on capital intensity of increased tax differentiation is derived from the risk sharing of government and it does not exist if the tax is asymmetric with no tax refunds. In that case, changes in capital intensity do not change the conclusions about the effects on the financial and nonfinancial risk taking drawn in the preceding paragraph. The indirect effect on the capital intensity of an increasing debt ratio has two components. The Laffer curve effect of an increasing debt ratio is always negative because the tax basis is decreased, the effective tax rate per unit of capital is increased and thus the marginal cost of capital is increased. The effect with respect to bankruptcy costs is always positive if the cost of bankruptcy is independent of the amount of capital. If the cost of bankruptcy is proportional to the amount of capital, the effect is positive for low bankruptcy risk and negative for high bankruptcy risk changing signs at the usual inflection point. The net effect thus depends on the relative size of the Laffer curve effect which is proportional to the tax rates. The indirect effect via a change in nonfinancial risk taking is also composed of two effects which often have opposite signs. It is thus impossible to draw any general conclusions about the direction of the change in capital intensity.

An increase of a symmetric tax on equity has a negative direct effect on capital intensity in the low risk case and a positive effect in the high risk case. This will result in a negative indirect effect on the debt ratio, which might be large if the optimal capital intensity is very sensitive to the tax rate, which might be the case if the tax rates are close to the Laffer curve maximum. We cannot exclude the possibility that the net adjustment of the optimal debt ratio is negative, even if this outcome does not seem to be very probable.

We thus have to conclude that this chapter has demonstrated more complexities and ambiguities than simple truths. But it should still be possible to use the analysis to derive unequivocal results for any specific tax system with known parameter values.

NOTES

1) An example of an integrated treatment of capital structure and dividend policy is Brennan [1970].

2) Auerbach and King [1983] have formalized the Miller model and examined the underlying assumptions. They show that restraints on the behaviour of investors and firms are essential for an equilibrium to exist. They also examine the effect of incomplete markets in this context.

3) See Jensen and Meckling [1976] and Myers [1977].

4) The supply prices reflect the rules of household taxation in other countries but these rules are assumed to be independent of decisions in the home country.

5) In Chapter 3, Section 8, I discuss the effect of a general corporate tax on risk taking behaviour taking the compensation for nondiversifiable risk into account.

REFERENCES

de Angelo, H. [1981], Competition and Unanimity, American Economic Review, March 1981, pp. 18-27.

de Angelo, H. and Masulis, R.W. [1980], Optimal Capital Structure Under Corporate and Personal Taxation, Journal of Financial Economics, Vol. 8, pp. 3-29.

Auerbach, A.J. and King, M.A. [1983], Taxation, Portfolio Choice and Debt-Equity Ratios: A General Equilibrium Model, The Quarterly Journal of Economics, Vol. 98, pp. 587-610.

Baron, D. [1979], Investment Policy, Optimality and the Mean-Variance Model, Journal of Finance, pp. 207-232.

Brennan, M.J. [1970], Taxes, Market Valuation and Corporate Financial Policy, National Tax Journal, Vol. 23, pp. 417-427.

Brennan, M. and Schwartz, E. [1978], Corporate Income Taxes, Valuation and the Problem of Optimal Capital Structure, Journal of Business, Vol. 51, pp. 103-114.

Feldstein, M., Green, J. and Sheshinski, E. [1978], Inflation and Taxes in a Growing Economy with Debt and Equity Finance, Journal of Political Economy, Vol. 86, pp. 553-569.

Feldstein, M., Green, J. and Sheshinski, E. [1979], Corporate Financial Policy and Taxation in a Growing Economy, Quarterly Journal of Economics, pp. 411-431.

Gordon, R.H. and Malkiel, B.G. [1981], Corporation Finance, in Aaron, H.J. and Pechman, J.A. (eds.), How Taxes Affect Economic Behavior, The Brookings Institution, Washington, D.C.

Haugen, R.A. and Senbet, L.W. [1978], The Insignificance of Bankruptcy Costs to the Theory of Optimal Capital Structure, Journal of Finance, Vol. 33, p. 383.

Jensen, M.C. and Meckling, W.H. [1976], Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, Journal of Financial Economics, Vol. 4, pp. 305-360.

Kim, H. [1978], A Mean-Variance Theory of Optimal Capital Structure and Corporate Debt Capacity, Journal of Finance, Vol. 33, pp. 45-64.

King, M.A. [1977], Public Policy and the Corporation, Chapman & Hall, London.

King, M.A., and Fullerton, D. [1984], The Taxation of Income from Capital. A Comparative Study of the U.S., U.K., Sweden and West Germany, University of Chicago Press.

Makowski, I. [1983], Competition and Unanimity Revisited, American Economic Review, pp. 329-339.

Miller, M.H. [1977], Debt and Taxes, Journal of Finance, Vol. 32, pp. 261-275.

Modigliani, F. and Miller, M.H. [1958], The Cost of Capital, Corporation Finance and the Theory of Investment, American Economic Review, Vol. 48, pp. 261-297.

Modigliani, F. and Miller, M.H. [1963], Corporate Income Taxes and the Cost of Capital: A Correction, American Economic Review, Vol. 63, pp. 433-443.

Myers, S.C. [1977], Determinants of Corporate Borrowing, Journal of Financial Economics, Vol. 5, pp. 147-175.

Rothschild, M. and Stiglitz, J.E. [1970], Increasing Risk: 1. A Definition, Journal of Economic Theory, Vol. 2, pp. 225-243.

Senbet, L.W. and Taggart, R.A. [1984], Capital Structure Equilibrium Under Market Imperfections and Incompleteness, Journal of Finance, Vol. 39, pp. 93-103.

Stiglitz, J.E. [1974], On the Irrelevance of Corporate Financial Policy, American Economic Review, Vol. 64, pp. 851-866.

Taggart, R.A. [1980], Taxes and Corporate Capital Structure in an Incomplete Market, Journal of Finance, Vol. 35, pp. 645-658.

5. Dividend Policy and the Effects of Differentiated Taxation

5.1 INTRODUCTION

In Chapter 4 we assumed that all returns to equity are taxed at a uniform tax rate, t_e. We also assumed that capital can be transferred costlessly between the firm and the shareholders so that the amount of equity capital always corresponds to the optimal capital intensity, k, and the optimal debt ratio, b. Concepts like dividends, retained earnings, new issues of shares and redemption of shares are then irrelevant. With a competitive economy, constant returns to scale and a perfect capital market, the market value of shares will be equal to the amount of equity. Tobin's q equals 1.

In this chapter I will discuss what happens if the tax on dividends is higher than that on retained earnings and if the equity capital can be increased by new issues of shares but cannot be decreased by repurchase of shares. A discontinuity is then created and tax adjustments will have different effects depending upon whether the marginal equity capital is financed by retained earnings (my case B) or by new share issues (my case A), that is, depending on whether the share redemption constraint is binding or not. If the constraint is binding, firms have an excess of equity capital and the capital market will value this capital at less than unity. Tobin's q is less than 1.

That the tax on dividends is higher than that on retained earnings is true for most countries. Usually the tax rate on capital gains (which result from retained earnings) is lower than the tax rate on current income and the effective tax rate on capital gains is further reduced by the realization criterion (see Chapter 7). Some years Sweden has also had a special corporate tax on dividends.

The effective corporate tax rate on dividends might also be increased by institutional rules which link the permissible dividend payment to the income on which tax is paid. Swedish firms must typically pay at least as much corporation tax as dividends. In periods of low profitability or high investment, corporate tax payments will be determined by the level of dividends and the use of accelerated depreciation and other forms of tax shield will have to be saved for later.

In order to analyze the effects of tax adjustments we must first have a consistent model of how corporations determine their dividend policy and issues of new shares. No generally accepted model exists and economists have often found it hard to explain the actual behaviour of firms. In Section 2 of this chapter I will discuss the choice of model with the ambition to find a model which is consistent with observed behaviour and which can be applied whether or not the share redemption constraint is binding.

In Section 5.3 I will develop the mathematical model and derive expressions for the optimal dividend ratio of firms in a closed economy. I will also derive expressions for the marginal cost of equity capital and discuss the steady state growth equilibrium relating the financial policy, the long term profitability and the share valuation to the growth rate of the economy.

In Section 5.4 I will discuss the dynamic stability of the two cases with and without new share issues on the margin. I will sketch the adjustment paths which result when the profitability is reduced by an increased corporation tax or the growth rate is decreased.

In Section 5.5 I will use the model of Section 5.3 to derive the effects of increased tax differentiation in a closed economy. I will show that these effects depend upon whether the share redemption constraint is binding.

In Section 5.6 I will extend the analysis to open economies with differentiated corporate taxation. Previous studies of dividend policy have used the closed economy model exclusively. An explanation might be that the majority of equity tends to be owned by domestic investors and that the board of directors is often dominated by the interests of domestic shareholders. Stock markets are, however, rapidly becoming more international and the corporations must take the reactions of foreign shareholders into account. For most countries in Western Europe the open economy model is probably of approximately the same relevance as the closed economy model.

In this chapter I will never explicitly introduce debt financing, discussing only the profitability and marginal cost of equity capital. From the model in Chapter 4 it is obvious, however, that tax adjustments which increase the marginal cost of equity capital will lead to a higher debt ratio and a lower capital intensity.

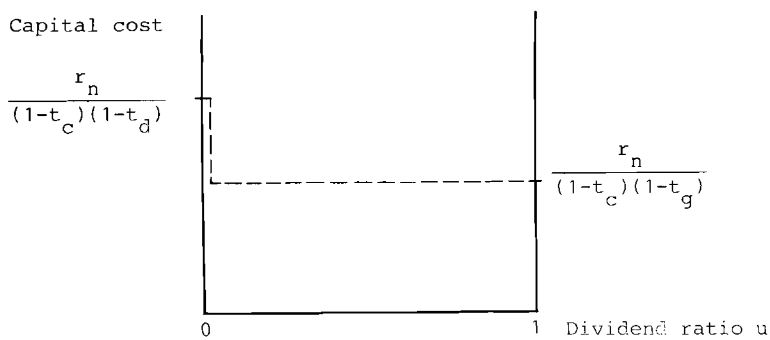
5.2 ON THE CHOICE OF ASSUMPTIONS AND ANALYTICAL MODEL

No generally accepted model for the determination of corporate dividend policy exists. Miller and Modigliani [1961] demonstrated that the dividend policy is irrelevant and no optimal dividend ratio exists if markets are perfect and complete, if no taxes or transaction costs exist and if investors are 'rational' in the sense that they have no preference for obtaining their income in the form of dividends instead of capital growth. When these conditions are fulfilled, capital can costlessly be transferred between the corporate sector and the investors. Tobin's q is then always equal to 1. If investors demand a rate of return r_n, the marginal cost of equity capital N_e is equal to r_n.

If an undifferentiated corporation tax, t_c, is introduced into the Miller and Modigliani model the marginal cost of equity capital, N_e, increases to $r_n/(1-t_c)$. But the dividend ratio is still irrelevant and Tobin's q still equals 1. As long as there are no household taxes, equity capital can be costlessly transferred to investors.

If differentiated household taxes (a tax t_d on dividend income and a tax t_g on capital gains with t_d higher than t_g) are also introduced into the Miller and Modigliani model, financing by retained earnings will always be cheaper than financing by new share issues. As shown by many authors (see Bergström and Södersten [1981], p. 174 or Poterba and Summers [1984], p. 16) the marginal cost of retained earnings is then equal to $r_n/(1-t_c)(1-t_g)$. The marginal cost of new issues is said to be $r_n/(1-t_c)(1-t_d)$ by Bergström and Södersten [1981], Atkinson and Stiglitz [1980] and others. This is, however, strictly true only for two-period models. As pointed out by Auerbach [1982, p. 31 and note 34] the cost of new issues is lower in a multiperiod model where parts of the earnings in later periods are retained. Anyway, the implication of this model is that firms would always prefer to finance investments by retained earnings and that they would never make new share issues and pay dividends at the same time. In Figure 5:1 I plot the capital cost of equity capital as a function of the dividend ratio u for this model.

Figure 5:1. Marginal cost of equity capital. Miller and Modigliani model with taxes.



The effects of increases in the tax on dividends implied by this model have been explored by Auerbach [1979] and Bradford [1981] for the case that new share issues are unimportant. They show that a tax increase will have no effect on investment or dividend ratio. It will only reduce the share price q. q will be equal to $(1-t_d)/(1-t_g)$. Poterba and Summers [1983] and [1984] name this view of the effects of dividend taxation the tax capitalization view.

It is easily seen that the effects of tax changes in the model above are drastically different for the case without dividend payments and with marginal financing by new share issues. This case is, however, hardly discussed in the literature. Evidently the implication that the dividend ratio is equal to zero is too unrealistic. Empirical observation shows that almost all publicly held corporations pay dividends and that the dividend ratio does not decline when new issues are made. Cf. textbooks in corporate finance such as Brealey and Myers [1981].

Bergström and Södersten [1981] studied a variant of the model above assuming that a fixed percentage of investment is financed by new issues. They do not, however, attempt to motivate such a financial policy. Auerbach [1982], on the other hand, assumes that there is a minimum dividend ratio constraint although he does not explain the existence of this constraint nor investigate the possibility that the constraint might depend on the tax parameters.

A very different model or 'view' was developed by Miller and Scholes [1978]. They assert that the investors do not have to pay any tax on dividends if they use all tax planning possibilities and that taxes are thus irrelevant. As I personally pay considerable amounts of dividend income tax and capital gains tax, I do not believe in this theory. Poterba and Summers [1984] also reject the theory on both theoretical and empirical grounds.

So far we have found that none of the models above can explain observed behaviour without making arbitrary assumptions regarding constraints. Let us sum up stylized facts as reported in textbooks in corporate finance and confirmed by a large amount of empirical research:

1. Dividends vary less than profits over time and corporations slowly adapt dividends to profits tending to maintain a constant long run dividend ratio.

2. The great majority of corporations pay dividends <u>and</u> retain earnings, e.g. the dividend ratio is positive but less than one.

3. The dividend ratio is not decreased when new shares are issued.

Observation 1 makes it natural to characterize dividend policy by the dividend ratio, u, which I will define as the ratio of dividends to profits (net of corporate tax).

Observations 2 and 3 imply that the net yield after tax on equity demanded by investors, r_n, is a decreasing and convex function of the dividend ratio u or, at least, that firms behave as if they believe that $r_n(u)$ is decreasing and convex. It is then possible to find an optimal dividend ratio where the tax advantage of reducing dividends is just balanced by the increased demanded after-tax return.

Models with a decreasing $r_n(u)$ have been developed by Poterba and Summers [1983] and [1984] for my case A with new share issues on the margin. They call this combination of model and case the traditional view or double tax view and compare it empirically to the tax capitalization view discussed above. It is not obvious to me why Poterba and Summers have not used this model also for my case B without new share issues. If we believe that a model with a decreasing $r_n(u)$ is correct for case A situations it should also be correct for case B situations. One reason might be that Poterba and Summers seem to find the available explanations of the decreasing $r_n(u)$ unsatisfactory even though they are forced by the facts to accept it. As this assumption is critical for the remainder of the chapter I will try to explain why the assumption is not only necessary but also reasonable under existing institutional conditions.

Anyone reading a financial magazine or a stock broker's news letter can observe that reporters and analysts usually take for granted that increased dividends, everything else equal, will lead to increased share prices. This is true at least for companies with low dividend ratios. If the dividend ratio approaches 1, reporters might discuss whether the dividends have a satisfactory earnings basis and can be maintained in the long run. This corresponds to our assumption that the function $r_n(u)$ is decreasing and convex. But why do markets react in this way? Is it merely the result of a pervasive market irrationality with self-fulfilling expectations or is it founded on some type of rational behaviour?

This question has intrigued economists for a long time and a fairly extensive literature on the topic exists. A recent review is found in Feldstein and Green [1983] and some papers which confirm my own beliefs were published in 1984. Below I will first discuss two 'explanations' which do not hold and then present three classes of arguments which I believe are sufficient to explain a decreasing $r_n(u)$. These are

- Income-principal distinction
- Signalling
- Principal-agent models

1. The simplistic explanation is that investors own shares in order to get dividends and that the share price is determined by the dividend yield. This explanation was thoroughly refuted by Miller and Modigliani [1961] who showed that the discounted value of future dividends per share is independent of the dividend ratio. The profit rate p must then equal the discount rate, r. If the corporation pays a dividend, $u \cdot p$, the number of shares corresponding to one dollar of equity capital will be $e^{-(1-u) \cdot p \cdot t}$ at time t. The dividend per share will thus be $u\ p\ e^{(1-u) \cdot p \cdot t}$. Adding the discounted values we obtain the share value

$$V = \int_0^\infty u \cdot p \cdot e^{(1-u) \cdot p \cdot t} \cdot e^{-rt} dt = \int_0^\infty u \cdot r \cdot e^{-U \cdot r \cdot t} dt =$$
$$= u \cdot r \cdot (u \cdot r)^{-1} = 1 \qquad (5:1)$$

V is thus independent of u.

2. A more sophisticated argument is that investors prefer secure present dividends to uncertain future dividends. This was also refuted by Miller and Modigliani [1961]. The proof resembles that of the preceding paragraph. In equilibrium with new issues on the margin, the expected value of p must increase as much as the certainty equivalent discount rate. Bhattacharya [1981] calls this argument the 'bird in the hand' fallacy and remarks that it is 'fallacious in a perfectly informed, competitive financial market'.

3. I now turn to explanations which I believe might be correct. The first one is the distinction between 'income' and 'principal'. Evidently many investors prefer to get that part of their total income, which they use for consumption, in the form of dividends. There are many possible motives. One is transaction costs. It is less trouble to get a regular dividend income than to sell shares. Another motive is that many shareholders regard consumption out of current income as morally right but consumption out of capital as wrong. This might be a socially rational rule of thumb. This idea is further developed by Shefrin and Statman [1984] who use the term self--control. Many charitable trusts and foundations also are legally bound to use only current income for expenditures and must not touch the 'principal'. Referring to empirical psychological studies by Kahneman and Tversky [1982], Shefrin and Statman also discuss the effect of 'regret aversion'. If dividends are low shareholders must sell shares in order to finance consumption. The decision to sell might cause more regret than using a higher dividend income for consumption without having to make any buy/sell decisions.

If the preference for dividends is explained in this way it is also reasonable that $r_n(u)$ is convex. Decreasing marginal utility of consumption is a sufficient condition.

4. A more fashionable explanation is that of signalling. Bhattacharya [1979] and Miller and Roch [1982] have developed models to show that a signalling equilibrium dividend ratio can exist if the management of a firm has better information on present or future earnings than the investors and if the deadweight costs of paying dividends are a decreasing function of present or future earnings. When the market has accepted the convention that dividends yield information about future earnings it might be very difficult for any firm to convince the market that a dividend cut is only motivated by tax savings and should have a positive effect on the share price.

It is obvious that firms believe that dividends, and especially dividend cuts, have a signalling effect. I personally believe that managers have a tendency to exaggerate the effect and to be too pessimistic about the possibilities of informing investors in other ways. A modern annual report should be more informative than any dividend policy. But I cannot deny that the market does react as if dividends actually provide some information. This is at least true with respect to the short run reaction to the announcement of a dividend change. Empirical evidence can be found in Kwan [1981] and Aharony and Swary [1980] and is reviewed in Copeland and Weston [1983], p. 509-512. Hakansson [1982] discusses under what assumptions an informative dividend policy can be welfare improving.

5. Easterbrook [1984] has developed the principal-agent argument for investors preferring dividends. He argues that the important effect of a high dividend ratio is that the firms must frequently approach the market with new share issues. This leads to better control of the investment activities of the firm. If management can finance investments with retained earnings, the screening of investment projects might be less strict. Investors will therefore distrust firms which pay low dividends and make no new issues.

There also exist a number of empirical papers which indicate that investors prefer cash dividends. Thus Long [1978] studied the experience of a U.S. company which had one type of shares paying cash dividends and another paying the same pre-tax income in the form of new shares. In spite of the tax disadvantage, investors preferred cash dividends. Gordon and Bradford [1980] studied the relative valuation of dividends and capital gains using the capital asset pricing model. They found that, over the sample period (1926-1978), the capital gain regarded by the market as equivalent to a dollar of dividends has followed a cyclical path around one. The firms have thus used the value maximizing dividend policy and the tax disadvantage of dividends for households has been balanced by non-tax advantages.

We have seen that a number of factors contribute to the form of the $r_n(u)$ function. I therefore do not try to deduce it from any special model but just assume that it exists and that it is exogenous to my model. As a simplification I assume that the level and the form of $r_n(u)$ is constant and independent of the capital intensity of firms as well as the tax structure. Formally this implies that r_n is determined by a constant yield on other assets or that savings are infinitely elastic. It actually implies that I judge the general equilibrium effects on $r_n(u)$ to be negligible. (The assumption is defended by Summers [1981].)

I will also assume that the effective tax rates are the same for all investors or that investors can be represented by a representative investor with the tax rates t_d and t_g. This might seem to be a heroic assumption. There could be a strong tax clientele effect so that investors with high marginal tax rates buy shares which pay no dividends and tax exempt investors buy shares which pay high dividends. Empirical investigations (see Lewellen, Stanley, Lease and Schlarfbaum [1978]) do find a statistically significant tax clientele effect, but the effect on the average dividend of a 10 % increase in an investor's marginal tax rate is small (about .1 %). Demand for diversification and heterogeneous expectations evidently dominate.

5.3 A MATHEMATICAL MODEL OF THE DETERMINATION OF THE DIVIDEND RATIO IN A CLOSED ECONOMY

In this section I will model the choice of an optimal dividend ratio for a value maximizing firm in a closed economy in steady state equilibrium.[1] I will make the usual assumptions about competitive behaviour, free entry and constant returns to scale. Tax rates t_d and t_g are assumed to be equal for all investors and the supply price of equity capital $r_n(u)$ is a given function.

I will first derive the market valuation of a share representing one unit of equity capital as a function of the profit rate, p, the dividend ratio, u, and the tax parameters. I then derive the value maximizing dividend ratio as a function of the profitability and the tax parameters. Inverting this function I get the marginal cost of retained earnings as a function of the dividend ratio and the tax parameters. This cost is compared to the marginal cost of new issues and it is shown that the marginal costs of these two modes of financing are equal at the dividend ratio which is optimal for a new issue policy. Finally, I derive the steady state equilibrium relationships between the profitability, the dividend ratio and the growth rate. Implicitly, the dividend ratio and profitability are determined as functions of the growth rate and the tax parameters.

We have assumed that investors demand an after-tax yield $r_n(u)$. If the share price is q, they demand an income $r_n \cdot q$ per unit of equity capital. If the firm makes a profit before taxes of p, the profit after corporate income tax is $p(1-t_c)$. It will then pay a dividend $u \cdot p \cdot (1-t_c)$ and this is worth $u \cdot p \cdot (1-t_c) \cdot (1-t_d)$ to the investor. Retained earnings are $(1-u) \cdot p\ (1-t_c)$. The market value of these is $q \cdot (1-u) \cdot p \cdot (1-t_c)$ and the investor receives $q \cdot (1-u) \cdot p \cdot (1-t_c) \cdot (1-t_g)$.[2] In steady state with a constant market valuation of equity capital, q is thus determined by the condition

$$r_n(u) \cdot q = u \cdot p \cdot (1-t_c) \cdot (1-t_d) + q \cdot (1-u) \cdot p \cdot (1-t_c)(1-t_g) \tag{5:2}$$

Solving q we get

$$q = \frac{p \cdot u \cdot (1-t_c) \cdot (1-t_d)}{r_n(u) - p \cdot (1-u) \cdot (1-t_c)(1-t_g)} \tag{5:3}$$

This can be interpreted as the usual Gordon condition. The numerator is the value of the actual dividend flow; the denominator is the difference between the discount rate and the growth rate of equity capital which, in steady state, is equal to the growth rate of dividends.

If firms are value maximizers they will choose the dividend ratio so as to maximize q.[3] Differentiating (5:3) we find the first order condition of an optimal dividend ratio $\hat{u}$

$$\hat{u} \cdot r_n'(\hat{u}) = r_n(\hat{u}) - p \cdot (1-t_c)(1-t_g) \tag{5:4}$$

Thus we get $\hat{u}$ as a function of the profit rate (p) and the tax parameters. Differentiating we get the derivative

$$\frac{d\hat{u}}{dp} = - \frac{(1-t_c)(1-t_g)}{\hat{u} \cdot r_n''(\hat{u})} \tag{5:5}$$

The second order condition for an optimal dividend ratio is

$$r_n''(\hat{u}) > 0 \tag{5:6}$$

Thus $d\hat{u}/dp$ is always negative. The higher the profitability on invested capital, the lower the optimal dividend ratio. More earnings are retained for investment.

A different way to interpret equation (5:4) is to say that it shows how high profitability must be in order to motivate a level of retained earnings, and investment in equity capital, corresponding to the optimal dividend ratio $\hat{u}$. We then find the marginal capital cost of retained earnings N_e^r as a function of the dividend ratio

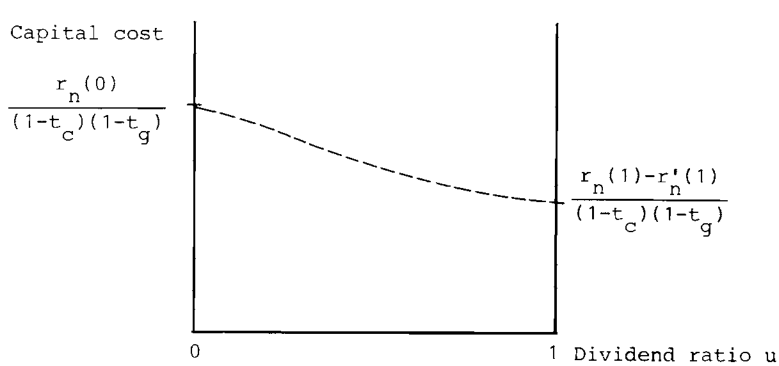
$$N_e^r = \frac{r_n(u) - u \cdot r_n'(u)}{(1-t_c)\cdot(1-t_g)} \tag{5:7}$$

We see that if $r'_n(u)$ were zero, that is if r_n were independent of u, we would get the capital cost expression of the tax capitalization view models. With r_n' negative, the marginal capital cost of retained earnings is always higher.[4] We also have

$$\frac{dN_e^r}{du} = - \frac{u \cdot r_n''(u)}{(1-t_c)(1-t_g)} < 0 \tag{5:8}$$

In other words N_e^r increases as more earnings are retained. This relationship is illustrated in Figure 5:2.

Figure 5:2. Marginal cost of retained earnings. My model.



But, if $q \geq 1$, increases in equity capital can also be financed by new issues. For a new issue to be feasible, the value of shares must be at least as high as the capital invested, that is the profitability must be so high that q is at least unity. From (5:3) we then get the marginal cost of new issues

$$N_e^n = \frac{r_n(u)}{u(1-t_c)(1-t_d)+(1-u)(1-t_c)(1-t_g)} \qquad (5:9)$$

This corresponds to equation (1.35) in Poterba and Summers [1984].

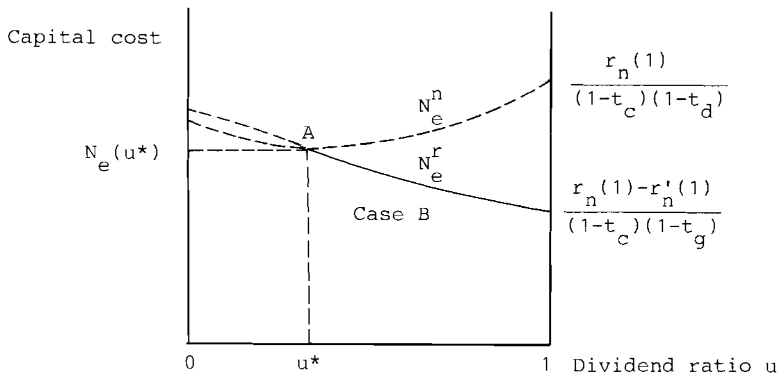
Differentiating (5:9) we find the first and second order conditions for an optimal steady state dividend ratio when making new issues $0 < u^* < 1$

$$r_n'(u^*) = - N_e^n \cdot (1-t_c)(t_d-t_g) \qquad (5:10)$$

$$r_n''(u^*) > 0 \qquad (5:11)$$

Substituting $r_n(u^*)$ and $r_n'(u^*)$ from (5:9) and (5:10) into (5:7) we can easily see that for $u = u^*$, $N_e^r = N_e^n$. Thus the marginal cost of retained earnings is equal to that of new issues at the dividend ratio which is optimal when making new issues. This is the dividend ratio which will be used by firms who finance the marginal equity capital by new issues (my case A). When the profitability is lower than $N_e(u^*)$, higher dividend ratios are chosen and the marginal cost is $N_e^r < N_e(u^*)$. This is my case B with a binding share redemption constraint and $q < 1$. Dividend ratios lower than u* will never be chosen because the marginal cost of retained earnings is then higher than the marginal cost of new issues. In Figure 5:3 I illustrate the marginal cost of retained earnings <u>and</u> new issues.

Figure 5:3. Marginal cost of equity capital. My model.



We might note that the condition for $u^* > 0$, that is for a non-zero dividend policy in case A is

$$r_n'(0) < - N_e^n(u=0)\cdot(1-t_c)(t_d-t_g) \tag{5:12}$$

$r_n'(0)$ must be negative enough for an inner solution to occur. If, as I believe, $r_n'(u)$ is close to zero for u=1, u* will always be less that unity for $t_d > t_g$. With neutral taxes, u* is equal to 1 if $r_n'(1) = 0$.

I am now ready to discuss the steady state equilibrium relations between the profitability, the dividend ratio and the growth rate.

In a competitive economy all firms regard the profitability level p as independent of their own actions. But if all firms increase their equity capital rapidly, the demand for labour will lead to increased real wages and profitability decreases.

In the long run an equilibrium must exist between the equity capital growth rate of firms and the growth rate (due to population growth or technology change) of the economy. I assume that the growth is exponential so that the equity capital of firms can grow at a rate n with a constant rate of profit, i.e., in steady state.

A firm in case B retains earnings $(1-u)\cdot p\cdot(1-t_c)$. In steady state this growth rate must be equal to n

$$(1-u)\cdot p\cdot(1-t_c) = n \qquad (5:13)$$

u is determined as a function of p and the tax parameters by equation (5:4). By (5:13) u and $p(1-t_c)$ are thus determined as functions of n and t_g. Substituting in (5:3) the share value q is determined as a function of n, t_d and t_g.[5]

Case A is stable if retained earnings at u* are smaller than n. The new share issue rate then is

$$ne = n - N_e(u=u^*)\cdot(1-t_c)\cdot(1-u^*) \qquad (5:14)$$

Equations (5:9) and (5:10) determine $N_e(u^*)$ and u* as functions of t_c, t_d and t_g.

5.4 VIABILITY AND STABILITY OF CASES A AND B

In this section I will first discuss the viability and stability of the two cases under our usual assumption of homogeneous firms and then discuss the consequences of allowing firms to be different.

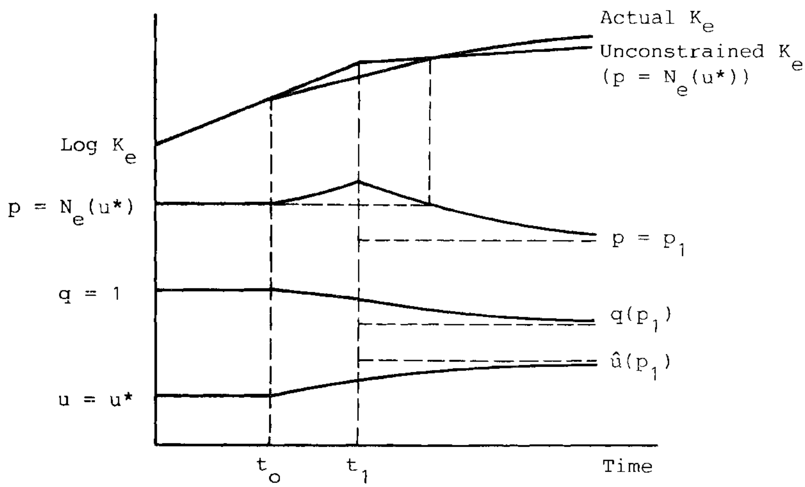
With homogeneous firms, case A will evidently be stable if

$$N_e(u^*)\cdot(1-t_c)\cdot[1-u^*] \leq n \qquad (5:15)$$

is expected to hold under the foreseeable future. If p should deviate from $N_e(u^*)$ and q from 1, the rate of new issues will change in a compensating direction forcing p and q back to the equilibrium values.

If (5:15) is expected to hold only for a few years investors will anticipate a decreasing p and a negative price change ($\dot{q}<0$). Assume that the growth rate is expected to change from n_o to n_1 at time t_1 and that (5:15) holds for n_o but not for n_1. By analogy with the theory of irreversible investment decisions (see Nickell [1978] Ch. 4), we see that new issues will end at time $t_o < t_1$. At t_o the share value q is still 1. This implies that the discounted value of all net-of-tax profits after t_o must equal 1. Loosely speaking, the weighted average value of p after t_o must equal $N_e^n(u)$. In steady state with $n = n_1$ p is less than $N_e^n(u)$. Thus p is higher than $N_e^n(u)$ for some period after t_o. This is possible because the equity capital grows slower than n_o from t_o until t_1. From t_1 it will grow faster than n_1, but the growth rate will approach n_1 as p approaches the new steady state value p_1. The adjustment paths are illustrated in Figure 5:4. Evidently the steady state solution of case B will be approached asymptotically. During some time period around t_1 the marginal capital cost will be higher than in the stable case A.

Figure 5:4 Adjustment paths when a drop of the growth rate causes a transfer from case A to case B.

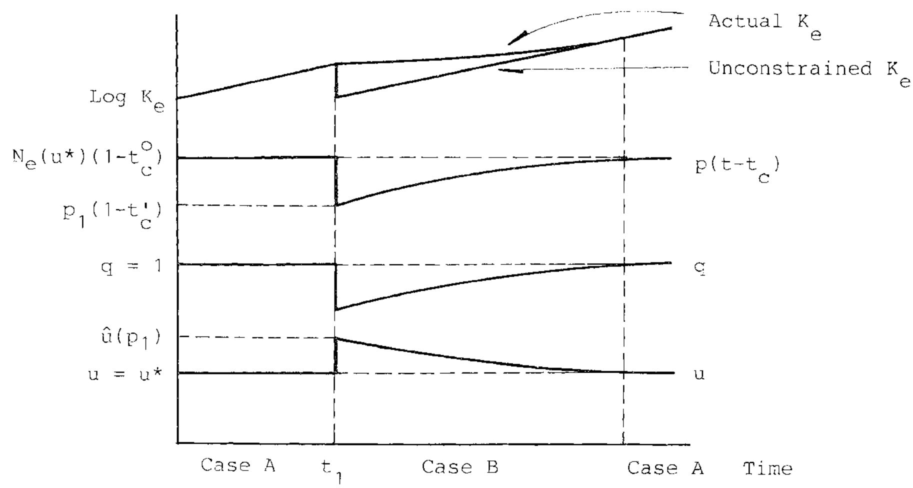


Another interesting possibility is that the net of corporate tax profit rate $p(1-t_c)$ drops instantly at t_1 because of an increase in corporate tax or new minimum wage legislation. If this change was not anticipated, the optimal value of K_e will drop momentarily, but actual K_e will adjust only over time. Thus q drops momentarily and we enter case B. But if the growth rate n is unchanged, case B is not stable. After some time the growth rate will dominate the momentary drop and we reenter case A. Thus investors can anticipate a positive $\dot{q}$ at t_1. Adjusting (5:3) we then have

$$q(t_1) = \frac{p_1(1-t_c)\cdot u\cdot(1-t_d) + \dot{q}(1-t_g)}{r_n(u) - p_1(1-t_c)(1-u)\cdot(1-t_g)} \qquad (5:16)$$

Thus the expectations of an increasing q will decrease the momentary drop of q. The adjustment process is sketched in Figure 5:5.

Figure 5:5 Adjustment paths when a temporary transfer to case B occurs because of an increased corporation tax.



If the corporation tax increase is anticipated, we evidently get a period of no new issues and compensating "excess" profits before t_1. The change back from case B to case A will then occur earlier.

In real economies with heterogeneous firms some firms will be in the case A situation and some will be in the case B situation. The case B firms will have lower marginal costs of capital than the case A firms. This difference will tend to lead to allocational inefficiencies. In the next section I will show that an increased tax differentiation increases the marginal cost gap.

The most recent empirical study of the relative importance of cases A and B is Poterba and Summers [1983]. Using British data for the period 1948 to 1980, they find that case A has been dominant and that "it appears that in making investment decisions, corporations act as if marginal investment is financed through new share issues. This suggests that the capitalization hypothesis cannot account for dividend behaviour in the United Kingdom".

A casual look at Swedish data will show that most Swedish firms with shares traded on the stock exchange were in the case B situation during the late 1970s. At the same time, we can observe a phenomenon which might make case B untenable in the long run. Firms bought shares in other firms instead of investing directly in real assets. As q was far below one, this was a cheap way to buy new capacity. The share redemption constraint might, in the long run, be ineffective because firms can buy shares in other firms and thus, at higher transaction costs, eliminate the excess equity capital. The amount of shares held by households will decrease in the same way as if firms could repurchase their own shares.

5.5 EFFECTS OF INCREASED TAX DIFFERENTIATION IN A CLOSED ECONOMY

We are now ready to study the effects of increased tax differentiation, that is of increasing t_d and decreasing t_g keeping the total tax revenue constant. These effects will turn out to be different for the two cases A and B. In case A an increased dividend tax tends to make new issues more expensive and thus has a negative effect on the capital intensity. In case B an increased dividend tax tends to lock in retained earnings and thus increase the capital intensity. We must therefore examine the two cases separately.

In case A the tax adjustment will evidently decrease the optimal dividend ratio u*. Investors then demand a higher after-tax return ($r_n'(u) < 0$). As the total tax revenue should be constant, the return before tax and thus the marginal equity capital cost, must also increase. This will tend to decrease capital intensity and to increase the debt ratio. A lower capital intensity entails a lower real wage rate. The tax adjustment would thus, in case A, leave the welfare of capital owners the same and decrease the welfare of wage earners. If firms are in case A and are expected to remain in case A, it would seem to be preferable to decrease the difference between t_d and t_g as far as possible and thus get a high dividend ratio and a large amount of new issues.

Trying to show these results mathematically we encounter some complications from Laffer curve effects.

Differentiating the optimal dividend ratio condition (5:10) we get

$$r_n'' \cdot \frac{du^*}{dt_d} = -(t_d - t_g)\cdot(1-t_c)\cdot\frac{dN_e^n}{dt_d} - N_e^n(1-t_c)\left(1 - \frac{dt_g}{dt_d}\right) \tag{5:17}$$

If the tax revenue per unit of equity capital, R, is held constant[6]

$$N_e^n = r_n(u^*) + R \tag{5:18}$$

$$\frac{dN_e^n}{dt_d} = r_n'(u^*) \cdot \frac{du^*}{dt_d} \tag{5:19}$$

The tax revenue R is equal to

$$R = t_c \cdot N_e^n + (1-t_c)\cdot N_e^n \cdot [u \cdot t_d + (1-u)\cdot t_g] \tag{5:20}$$

$$\begin{aligned}\frac{dR}{dt_d} = {} & [t_c + (1-t_c)(ut_d + (1-u)t_g]\cdot\frac{dN_e^n}{dt_d} + \\ & + (1-t_c)\cdot N_e^n\cdot(t_d - t_g)\cdot\frac{du^*}{dt_d} + u\cdot(1-t_c)\cdot N_e^n - \\ & - (1-u)(1-t_c)N_e^n \cdot \frac{dt_g}{dt_d} = 0\end{aligned} \tag{5:21}$$

Solving the equation system (5:17), (5:19) and (5:21) using (5:10) we get

$$\frac{dN^e}{dt_d} = r_n'(u^*) \cdot \frac{du^*}{dt_d} \tag{5:22}$$

$$\frac{du^*}{dt_d} = r_n' \cdot \frac{1 - \frac{dt_g}{dt_d}}{(t_d-t_g)[r_n''+(1-t_c)\cdot r_n'\cdot(t_d-t_g)]} \tag{5:23}$$

$$\frac{dt_g}{dt_d} = - \frac{1}{1-u} \cdot \frac{u^*\cdot r_n'' + (1-t_c)(1-t_g)\cdot r_n'}{r_n'' + (1-t_c)(t_d-t_g)\cdot r_n'} \tag{5:24}$$

(5:24) is negative if

$$u^*\cdot r_n'' + (1-t_c)(1-t_g)\cdot r_n' > 0 \tag{5:25}$$

(5:23) is then negative and (5:22) positive which confirms the intuitive analysis above.

If (5:25) does not hold the tax revenue can be increased by reducing t_d without increasing t_g. This case can occur if the dividend tax rate, t_d, is high and thus u* is low.

In <u>case B</u> the dividend ratio will also decrease if the capital gains tax, t_g, is decreased. This will increase the accumulation of retained earnings and thus increase the equity capital, forcing down profitability and increasing real wages. The share value, q, will be reduced both by the reduced profitability and by an increase of r_n due to the lower dividend ratio. As shown in the mathematical analysis below, Laffer curve effects can be important in case B. The revenue from the capital gains tax is reduced when q is reduced.

In case B the dividend ratio is determined by (5:4)

$$\hat{u}\cdot r_n'(\hat{u}) = r_n(\hat{u}) - p(1-t_c)(1-t_g) \qquad (5:4)$$

Differentiating (5:4) we get

$$\hat{u}\cdot r_n'' \cdot \frac{d\hat{u}}{dt_d} = -(1-t_c)(1-t_g)\cdot\frac{dp}{dt_d} + p(1-t_c)\cdot\frac{dt_g}{dt_d} \qquad (5:26)$$

In the short run p is constant and $dp/dt_d = 0$. A change of the dividend tax then has no effect on $\hat{u}$ per se. $\hat{u}$ is reduced if the capital gains tax is reduced. In steady state p is determined by (5:13)

$$(1-\hat{u})\cdot p\cdot(1-t_c) = n \qquad (5:13)$$

Differentiating we get

$$\frac{dp}{dt_d} = \frac{p}{1-\hat{u}}\cdot\frac{d\hat{u}}{dt_d} \qquad (5:27)$$

Substituting in (5:26)

$$\frac{d\hat{u}}{dt_d} = \frac{p(1-t_c)\cdot(1-\hat{u})}{\hat{u}(1-\hat{u})\cdot r_n'' + (1-t_c)(1-t_g)\cdot p}\cdot\frac{dt_g}{dt_d} \qquad (5:28)$$

In steady state equilibrium as well, the dividend ratio and profitability are thus not affected by isolated changes of the dividend tax. The effect comes from the change of the capital gains tax. This corresponds to the conclusions of the tax capitalization view models.

The tax revenue per unit of equity capital in steady state is

$$R = t_c\cdot p + (1-t_c)\cdot p\cdot[t_d\cdot\hat{u} + t_g\cdot q\cdot(1-\hat{u})] \qquad (5:29)$$

The condition for the tax revenue to increase with the dividend tax rate when the capital gains tax is kept constant, that is for $dt_g/dt_d < 0$ when R is constant, is

$$\frac{dR}{dt_d}\bigg|_{dt_g=0} = (1-t_c)\cdot p\cdot(1-\hat{u})\cdot t_g\cdot\frac{dq}{dt_d}\bigg|_{dt_g=0} + \\ + (1-t_c)\cdot p\cdot\hat{u} > 0 \tag{5:30}$$

Terms with dp/dt_d and $d\hat{u}/dt_d$ do not occur in (5:30) because they are zero when dt_g is zero.

From (5:3) we get

$$\frac{dq}{dt_d}\bigg|_{dt_g=0} = -\frac{q}{1-t_d} \tag{5:31}$$

Substituting in (5:30)

$$\frac{dR}{dt_d}\bigg|_{dt_g=0} = (1-t_c)\cdot p\cdot\left[\hat{u} - t_g\cdot\frac{1-\hat{u}}{1-t_d}\cdot q\right] \tag{5:32}$$

The effect on total tax revenues is thus positive if

$$\frac{\hat{u}}{1-\hat{u}} > \frac{q\cdot t_g}{1-t_d} \tag{5:33}$$

Substituting from (5:3) and (5:13) the condition can be written

$$r_n(\hat{u}) > p\cdot(1-t_c)\cdot(1-\hat{u}) = n \tag{5:34}$$

This condition can be given either the interpretation that the effective household tax rate $[p(1-t_c) - r_n]/p(1-t_c)$ must not be higher than $\hat{u}$, or, in steady state equilibrium, that the growth rate must be less than the demanded return.

If (5:34) holds it is possible to increase the dividend tax and reduce the capital gains tax, keeping steady state tax revenue constant (the transition will cause q to drop and thus lead to capital gains tax disbursements). It is then possible to increase the welfare of wage earners, reducing that of shareholders, if firms are in case B. But there might be negative long term dynamic effects because the increased tax differentiation makes entry of new firms and expansion of case A firms less likely.

If (5:34) does not hold, the dividend tax should be reduced until (5:32) no longer holds, that is until case A is attained. This will increase tax revenues. Shareholders gain as q is increased to 1 and wage earners gain because the tax on wages can be reduced and the capital intensity and thus the real wage does not change.

If firms are heterogeneous, some firms may experience case A with a marginal cost of equity capital N_e^n and other firms are in case B with a marginal cost of equity capital $N_e^r <$ $< N_e^n$. The difference between N_e^n and N_e^r leads to allocational inefficiency. If tax differentiation is increased, N_e^n increases for the case A firms according to (5:19) and N_e^r decreases for the case B firms. Thus allocational inefficiency is aggravated.

5.6 EFFECTS OF CORPORATE TAX DIFFERENTIATION IN A SMALL OPEN ECONOMY

In a small open economy the behaviour of households and the behaviour of firms are separated by an international capital market. Differentiation of the household taxation has no effect on the dividend policy of domestic firms. Differentiation of corporate taxation has no effect on the behaviour of households. Presented in this way the open economy model is obviously a limiting case when domestic ownership of domestic firms is unimportant.

In this section I will analyze the case of differentiated corporate taxes. The effect of household tax differentiation on the portfolio choice of investors is discussed in Chapter 6.

Assume that the effective corporate tax on distributed profits is t_{cd} and the effective tax on retained earnings is t_{cr}. If pre-tax profits are p, the tax revenue is thus[6]

$$R = p \cdot u \cdot (1-t_{cd}) + p \cdot (1-u) \cdot (1-t_{cr}) \tag{5:35}$$

If the international capital market demands a yield $r_e(u)$[7] new share issues (case A) are possible if

$$p \geq N_e^n = \frac{r_e(u)}{u(1-t_{cd}) + (1-u)(1-t_{cr})} \tag{5:36}$$

Firms in case A will choose u in order to minimize N_e^n. The optimality conditions are

$$r_e'(u^*) = - N_e(u^*) \cdot (t_{cd} - t_{cr}) \tag{5:37}$$

$$r_e''(u^*) > 0 \tag{5:38}$$

We note that (5:37) closely resembles (5:10). If $t_{cd} > t_{cr}$ firms will choose a u where $r_e'(u)$ is positive. If $t_{cd} < t_{cr}$ firms will choose (a higher) u where $r_e'(u)$ is positive. The $r_e(u)$ function is probably convex with an interior minimum because of dividend preference and differentiated household taxes in most countries.

Increasing the dividend tax and reducing the tax on retained earnings will give the same effects as in the closed economy model. The equations are almost equivalent. The only difference is that $r_e'(u^*)$ can be positive if $t_{cd} < t_{cr}$. But still the marginal capital cost will increase with the degree of tax differentiation. For given tax revenues R, N_e^n has a minimum for neutral corporate taxes $t_{cd} = t_{cr}$. Differentiating the taxation of firms leads to an inefficient adaptation to the preferences of international investors and thus results in higher capital costs, lower capital intensity and lower real wages.

Case B is less easily defined in an open economy than in a closed economy. If firms can invest freely in many countries and if profits earned in foreign countries are taxed in those countries, the case B situation can hardly exist. If it should exist for individual firms in low growth industries, the effect of domestic corporate taxes on the constrained optimum would anyway be small.

If firms are restricted to operate on the domestic market or if all profits are taxed in the domicile country, the case B situation may apply and the solution will resemble that in the closed economy model. A technical difference is, however, that retained earnings are taxed instead of capital gains. Thus the valuation of retained earnings, q, does not influence the amount of taxes. This is not a consequence of the open economy assumption but a consequence of differentiating corporate taxes instead of household taxes.

The expression (5:3) is now replaced by the equivalent

$$q = \frac{p \cdot u \cdot (1-t_{cd})}{r_e(u) - p \cdot (1-u) \cdot (1-t_{cr})} \tag{5:39}$$

The first order condition is equivalent to (5:4)

$$\hat{u} \cdot r_e'(\hat{u}) = r_e(\hat{u}) - p \cdot (1-t_{cr}) \tag{5:40}$$

The immediate effect of changing the tax on retained earnings is thus

$$\frac{d\hat{u}}{dt_{cr}} = \frac{p}{\hat{u}\cdot r_e(u)} \tag{5:41}$$

Decreasing t_{cr} thus reduces $\hat{u}$ and increases the amount of equity capital.

The growth equilibrium, which determines the steady state p and which in (5:13) is a relation between p, t_c, $\hat{u}$ and the growth rate n, now depends on the corporate tax on retained earnings t_{cr}

$$p\cdot(1-\hat{u})\cdot(1-t_{cr}) = n \tag{5:42}$$

Substituting $p\cdot(1-t_{cr})$ from (5:42) into (5:40) we find that the steady state equilibrium $\hat{u}$ is now independent of the tax rates

$$\hat{u}\cdot r_e'(\hat{u}) = r_e(\hat{u}) - \frac{n}{1-\hat{u}} \tag{5:43}$$

Thus $\hat{u}$ in this case depends only on the growth rate n ($r_e(\hat{u})$ and $r_e'(\hat{u})$ are determined on the international capital market). For any given growth rate p $(1-t_{cr})$ will be a constant. Decreasing t_{cr} will thus decrease p and increase the amount of equity capital and capital intensity in steady state equilibrium. This has a positive effect on wage earnings.

As in the closed economy model with differentiated household taxes the tax on dividends t_{cd} has no effect on $\hat{u}$ or p. As tax revenues in this case are independent of share prices q, increasing t_d will always raise the tax revenues and have the effects of a lump sum tax. It will reduce the wealth of shareholders, but most of these shareholders may well be foreigners.

In an economy where case B is predominant it might thus seem favourable to increase the corporate tax on dividends and to reduce the corporate tax on retained earnings. But it should always be kept in mind that case B is associated with low growth and no entry of new firms. We have seen above that if case A is predominant the taxes should be equalized.

5.7 SUMMARY AND CONCLUSIONS

I have examined the effects of increased tax differentiation in the two polar case models of a closed economy and a small open economy. Due to the irreversibility of equity capital increases by share issues, two cases must be distinguished; case A with marginal financing through share issues and case B with excess equity capital and a market valuation of equity capital below 1.

I have shown that in case A increased dividend taxes, and correspondingly decreased taxes on capital gains or retained earnings, will lead to a decreased dividend ratio and a decreased amount of equity capital which will entail decreased capital intensity and an increased debt ratio. Lower capital intensity leads to lower real wages. Workers lose and no one gains from increased tax differentiation. Tax neutrality ($t_d = t_g$) minimizes capital costs at given tax revenues and is thus socially optimal.

Case B can only occur if t_d is higher than t_g so that $u^* <$ < 1. The probability of case B increases with increased tax differentiation. Within case B increased tax differentiation leads to lower marginal costs of equity capital and thus to higher capital intensity and higher real wages. Share values are depressed. Thus income is transferred from capital owners to workers. This mechanism can, however, only function if increased dividend taxes increase tax revenue and thus make it possible to decrease t_g. In a closed economy with differentiated household taxes, an increased dividend tax rate will decrease the capital gains tax revenue by decreasing the share price q. The net effect of increasing t_d will be negative in steady state if the growth rate of the economy is higher than the net-of-tax interest rate. With such growth rates it is obviously preferable to decrease t_d until case A is obtained. In an open economy with differentiated corporate taxes, this effect does not arise, because taxes are then independent of the market valuation of shares.

As the effects are different in case A and case B the relative importance of the cases might be important for policy decisions. Existing empirical work by Poterba and Summers [1983] and others indicate that case A is predominant. Even though the greater part of equity capital increases are financed by retained earnings, the marginal financing comes from new issues.

NOTES

1) Dynamic adjustment paths are sketched in Section 5:4.

2) Hayashi [1982] discusses under what assumptions the average q and the marginal q are equal. t_g is regarded as an accrual tax. The translation of realization tax rates to t_g is examined in Chapter 7.

3) As shown in a review article by Baron [1979] shareholder unanimity about value maximization implies not only competitive behaviour but also marginal spanning, Pareto efficiency or a large market in the sense of Makowski [1983].

4) Personally I believe that $r_n'(u)$ is close to zero when u approaches 1. If this is true, the capital costs of the two models are equal when all profits are distributed.

5) Note that in a closed economy in case B the corporation tax, t_c, has no long run effect on the share price q. Any corporation tax increase is compensated by an increased steady state profitability.

6) The tax revenue R will be affected by adjustments in capital intensity, k, and the wage rate, w. In order to concentrate on the main effects of the dividend policy I abstract from these indirect effects.

7) Note that $r_e(u)$ is the yield gross of household taxes whereas $r_n(u)$ is the yield net of household taxes.

REFERENCES

Aharony, J., and Swary, I [1980], "Quarterly Dividends and Earnings Announcements and Stockholders' Returns: An Empirical Analysis", Journal of Finance, March, pp. 1-12.

Auerbach, A. [1979], "Share Valuation and Corporate Equity Policy", Journal of Public Economics 11, pp. 291-305.

Auerbach, A. [1982], "Taxes, Firm Financial Policy and the Cost of Capital: An Empirical Analysis", NBER Working Paper No. 955.

Baron, D. [1979], "Investment Policy, Optimality and the Mean--Variance Model", Journal of Finance, pp. 207-32.

Bergström, V., and Södersten, J. [1981], "Double Taxation and Corporate Capital Cost, in Business Taxation, Finance and Firm Behaviour, IUI, Stockholm, pp. 163-190.

Bhattacharya, S. [1979], "Imperfect Information, Dividend Policy, and 'the Bird in the Hand' Fallacy", Bell Journal of Economics 10, pp. 259-270.

Bradford, D.F. [1981], "The Incidence and Allocation Effects of a Tax on Corporate Distributions", Journal of Public Economics 15, pp. 1-22.

Brealey, R., and Myers, S. [1981], Principles of Corporate Finance, McGraw-Hill, New York.

Copeland, T.E., and Weston J.F. [1983], Financial Theory and Corporate Policy, second edition, Addison-Wesley Publishing Company, Reading, Mass.

Easterbrook, F.H. [1984], "Two Agency-Cost Explanations of Dividends". The American Economic Review, Vol. 74, pp. 650-659.

Feldstein, M., and Green, J. [1983], "Why Do Companies Pay Dividends?", The American Economic Review, Vol. 73, pp. 17-30.

Gordon, R.H., and Bradford, D.F. [1980], Taxation and the Stock Market Valuation of Capital Gains and Dividends, Journal of Public Economics 14, pp. 109-136.

Hayashi, F. [1982], "Tobin's Marginal q and Average q: A Neo-classical Interpretation", Econometrica, 40, pp. 213--225.

Hakansson, N. [1982], "To Pay or not to Pay Dividends", The Journal of Finance, Vol. 37, pp. 415-427.

Kahneman, D., and Tversky, A. [1982], "The Psychology of Preferences", Scientific American 246, pp. 167-173.

Kwan, C. [1981], "Efficient Market Tests of the Informational Content of Dividend Announcements: Critique and Extension", Journal of Financial and Quantitative Analysis, June 1981, pp. 193-206.

Lewellan, W., Stanley, K., Lease, R., and Schlarbaum, G. [1978], "Some Direct Evidence on the Dividend Clientele Phenomenon, Journal of Finance, Dec. 1978, pp. 1385-1399.

Long, J.B. [1978], "The Market Valuation of Cash Dividends: A case to Consider", Journal of Financial Economics 6, pp. 235-264.

Makowski, I. [1983], "Competition and Unanimity Revisited", American Economic Review, pp. 329-339.

Miller, M.H., and Modigliani, F. [1961], "Dividend Policy, Growth, and the Valuation of Shares", Journal of Business, Vol. 34, pp. 411-433.

Miller, M.H., and Roch, K. [1982], "Dividend Policy Under Asymmetric Information", Working Paper, Graduate School of Business, University of Chicago.

Miller, M., and Scholes, M. [1978], "Dividends and Taxes", Journal of Financial Economics 6, pp. 333-364.

Nickel, S.J. [1978], The Investment Decisions of Firms, Cambridge University Press.

Poterba, J.M., and Summers, L.H. [1983], "Dividend Taxes, Corporate Investment, and 'Q'", Journal of Public Economics 22, pp. 135-167.

Poterba, J.M., and Summers, L.H. [1984], "The Economic Effects of Dividend Taxation", NBER Working Paper No. 1353.

Ross, S. [1977], "The Determination of Financial Structures: The Incentive Signalling Approach", The Bell Journal of Economics 8, pp. 23-40.

Shefrin, H.M., and Statman, M. [1984], "Explaining Investor Preference for Cash Dividends", Journal of Financial Economics, Vol. 13, pp. 253-282.

Summers, L.H. [1981], "Taxation and Corporate Investment: A q-Theory Approach", Brookings Papers on Economic Activity 1, pp. 67-140.

6. Differentiated Taxation, Asset Structure and Portfolio Choice

6.1 INTRODUCTION

In real world tax systems the tax rates often vary dramatically depending on the type of asset from which income is derived. Thus the effective tax rates on income from bank accounts and bonds are often more than 100 % due to nominal taxation and inflation, whereas the tax on capital gains from gold or antiques is usually zero. The effective tax rates on machinery and equipment often differ from those on buildings due to different rules regarding accelerated depreciation and investment grants. Investments in research and development are usually deductible immediately and are thus favoured in relation to investments in physical equipment. Many other examples could be given. A numerical example might be illuminating. According to the calculations in The Taxation of Income from Capital [1984], the effective tax rates in the United Kingdom were -37 % for machinery and +39 % for buildings.

Some tax differentiation is a deliberate part of government policy. A typical example is the beneficial treatment of owner-occupied housing in many countries. The motive for this beneficial treatment is probably that housing consumption is regarded as a merit good. Below I will not discuss the merit good aspect but concentrate on other factors.

Most tax differentiation seems, however, to be unintended. One major reason is inflation in partly nominal tax systems. Other reasons are assessment difficulties and accounting conventions. Typically, income from capital gains is taxed less than current income. Much tax differentiation also seems to be due to ad hoc decisions on the fine points in complex tax systems.

An immediate consequence of increasing the relative effective tax rate on some asset is obviously to decrease the demand for that asset. Whether this results in price or quantity adjustments depends on the elasticity of supply. In the short run, before asset stocks have time to adjust, and in closed or large economies, price adjustment will dominate. In the long run and in small open economies quantity adjustments will dominate.

Tax differentiation will thus distort the allocation of capital between assets. Such distortion will cause welfare losses from less efficient production and less efficient portfolio selection at prevailing international prices. Tax differentiation will, however, have effects as well on aggregate capital intensity and total savings. The distortionary effects of taxes on the savings and investment decisions might in fact be reduced by differentiation. We must study the net effect.

In Chapter 3 I discussed the general effects of taxing capital income instead of taxing wages or consumption and found that such taxation is usually non-optimal. In this chapter I assume that the government wants to derive a certain amount of tax revenue from returns to capital and I discuss how the negative effects should be minimized. This can be regarded as an exercise in suboptimality.

I will discuss the effects of tax differentials on capital stock in the productive sector in Section 6.2 and the effects on savings and portfolio choice decisions of households in

Sections 6.3 and 6.4. In Section 6.3 I will discuss the effect on portfolio choice if investors have a preference for dividends. In Section 6.4 uncertainty is introduced.

6.2 CAPITAL ALLOCATION BY FIRMS

Assume that the representative firm with a constant returns to scale technology has the production function

$$x = f(k_1, k_2 \ldots k_i \ldots k_n) \tag{6:1}$$

where x is the value added per labour unit and k_1 to k_n are the capital stocks invested in different assets such as buildings, machinery, inventories, research and development and marketing organization. Assume also competitive, profit maximizing behaviour and a small open economy where the prices of goods are exogenously given. The wage rate is w and the interest rate, r, is determined by the international capital market. The tax rate on income from asset i is denoted t_i. Disregarding uncertainty, the pre-tax cost of capital invested in asset i is then $r/(1-t_i)$ and the profit of the firm net of all capital costs is

$$\pi = x - w - \sum_{1}^{n} \frac{k_i}{1 - t_i} \cdot r \tag{6:2}$$

The firm will choose each k_i so as to maximize π. Thus k_i is determined so that

$$\frac{\partial x}{\partial k_i} = \frac{r}{1 - t_i} \tag{6:3}$$

k_i will be distorted from the socially optimal, no-tax, investment in two ways.

1. All taxes on capital income decrease the optimal capital intensity. This effect has been discussed in Chapter 3.

2. Nonneutral taxation also distorts the allocation of capital between different assets. The marginal productivity of capital varies according to assets as shown by equation (6:3).

If we want a given corporate tax revenue

$$R = r \cdot \sum_i \frac{t_i \, k_i}{1 - t_i} \tag{6:4}$$

it might seem preferable to use a neutral tax (t_i=t) and thus avoid distortions of the second type. On the other hand it is well known from the theory of optimal taxation that this intuitive understanding might be misleading. If the price elasticities of demand are not equal for all assets, a non-neutral tax might distort the aggregate capital intensity less and might thus be favourable in spite of causing more type 2 distortion.

Let us study under what circumstances a marginal change from a neutral tax system (with all t_i=t) to a nonneutral system might increase social welfare. Without loss of generality we can model the change as an increase of t_1 by dt_1 and a decrease of t_2 by $-dt_2$. Firms will then adjust k_i by dk_i

$$dk_i = \frac{\partial k_i}{\partial t_1} \cdot dt_1 + \frac{\partial k_i}{\partial t_2} \cdot dt_2 \tag{6:5}$$

The condition for constant corporate tax revenue is found by differentiating (6:4)

$$dR \equiv k_1 \cdot \frac{r}{(1-t_1)^2} \cdot dt_1 + k_2 \cdot \frac{r}{(1-t_2)^2} \cdot dt_2 +$$

$$+ r \cdot \sum \frac{t_i \cdot dk_i}{1 - t_i} = 0 \tag{6:6}$$

Starting from the neutral tax all t_i = t. Substituting (6:5) in (6:6) we then have

$$dt_2 = - \frac{\frac{k_1}{1-t} + t \cdot \sum \frac{\partial k_i}{\partial t_1}}{\frac{k_2}{1-t} + t \cdot \sum \frac{\partial k_i}{\partial t_2}} \; dt_1 \tag{6:7}$$

The national income in this model is the sum of π, w, R and interest on domestic savings. With R constant and the interest rate constant we obtain the condition of increasing social welfare by differentiating (6:2). Generally,

$$\frac{\partial(\pi+w)}{\partial t_i} = \sum_j \frac{\partial x}{\partial k_j} \cdot \frac{\partial k_j}{\partial t_i} - \sum_j \frac{\partial k_j}{\partial t_i} \cdot \frac{r}{1-t_i} - r \cdot \frac{k_i}{(1-t_i)^2} =$$

$$= - r \cdot \frac{k_i}{(1-t_i)^2} \tag{6:8}$$

That the sum of the first two terms is equal to zero can be seen from (6:3).

The effect on $(\pi+w)$ of an adjustment of t_1 and t_2 which satisfies (6:6) thus is (at the point $t_1 = t_2 = t$)

$$\frac{\partial(\pi+w)}{\partial t_1} \cdot dt_1 + \frac{\partial(\pi+w)}{\partial t_2} \cdot dt_2 = - k_1 \cdot \frac{r}{(1-t_1)^2} \cdot dt_1 -$$

$$- k_2 \cdot \frac{r}{(1-t_2)^2} \cdot dt_2 = r \cdot \left(\frac{t_1}{1-t_1} \cdot dk_1 + \frac{t_2}{1-t_2} \cdot dk_2\right) =$$

$$= r \cdot \frac{t}{1-t} \cdot \Sigma dk_i \tag{6:9}$$

(6:9) confirms that a deviation from neutral taxation is beneficial if aggregate capital intensity is increased.

Using (6:9) and (6:7) we get

$$t \cdot \Sigma dk_i = - \frac{k_1}{1-t} \cdot dt_1 - \frac{k_2}{1-t} \cdot dt_2 =$$

$$\frac{\frac{k_2}{1-t} \cdot \sum \frac{\partial k_i}{\partial t_1} - \frac{k_1}{1-t} \cdot \sum \frac{\partial k_i}{\partial t_2}}{\frac{k_2}{1-t} + t \cdot \sum \frac{\partial k_1}{\partial t_2}} \cdot dt_1 \tag{6:10}$$

The neutral tax is optimal if (6:10) is non-positive for all dt_1 (positive or negative). Thus the optimality condition is

$$\frac{k_2}{1-t} \cdot \sum \frac{\partial k_i}{\partial t_1} - \frac{k_1}{1-t} \cdot \sum \frac{\partial k_i}{\partial t_2} = 0 \qquad (6:11)$$

or

$$\frac{\sum \frac{\partial k_i}{\partial t_1}}{\frac{k_1}{1-t}} = \frac{\sum \frac{\partial k_i}{\partial t_2}}{\frac{k_2}{1-t}} \qquad (6:12)$$

This condition holds if the production function is separable and isoelastic, that is where[1]

$$x = \Sigma\alpha_i \cdot \ln k_i \qquad (6:13a)$$

$$x = \frac{1}{\gamma}\Sigma\alpha_i \cdot k_i^{\gamma} \qquad (6:13b)$$

It can also be shown to hold for the symmetric functions

$$x = \frac{1}{\gamma} \cdot k_1^{\gamma} \cdot k_2^{\gamma} \cdot k_3^{\gamma} \qquad (6:13c)$$

$$x = [\alpha_o + \alpha \cdot \Sigma k_i^{\rho}]^{1/\rho} \qquad (6:13d)$$

whereas it does not hold for the general CES-function with different α_i and thus different optimal k_i.[2] The symmetric functions (6:13c) and (6:13d) are very special cases.

We can thus conclude that neutral corporate taxes are optimal only under the restrictive assumption that the production functions are separable and isoelastic. In other cases there would seem to be a potential for differentiation if the government has enough information (which it probably does not have). Going to the literature we find, however, that the assumption of Cobb-Douglas production functions and unitary elasticities for all assets is predominant.

One example is Gravelle [1981]. She estimated the deadweight loss of nonneutral taxation of corporate assets for the United States in 1978 to be $ 2.5 billion, assuming that all elasticities were 1. As she included only buildings and machinery but not inventories or investment in R&D the estimate is probably too low.

Another example is Hendershott and Hu [1980] and [1981] who assume that aggregate capital intensity is constant and that the production function is Cobb-Douglas. For a discussion of this assumption they refer to Ando, Modigliani, Rasche and Turnovsky [1974], note 29. In this note the authors suggest that total demand for producers' durable equipment has the elasticity 1 and that the Cobb-Douglas assumption is preferable. They do not, however, discuss differentiation between different types of equipment.

A conclusion might be that all estimates of the losses from nonneutral taxation of corporate assets are dubious if the authors cannot demonstrate that the production functions are such that a neutral taxation is optimal.

6.3 PORTFOLIO CHOICE OF HOUSEHOLDS UNDER CERTAINTY

In asset pricing models households are usually assumed to behave as if they maximized an expected utility function defined on the consumption in one or two or several periods. One period is enough if the total amount of savings is regarded as constant; the use of two periods permits an integrated analysis of the savings decision and the portfolio choice.

Sometimes it might be necessary to introduce more factors in the utility function. One example is the model in Chapter 5 where households were assumed to have preferences over the dividend ratio u. Another possibility is that special kinds of consumption are not marketable but can only be attained by investment in certain assets. One example might be home ownership.

Let us start with the simplest case where only aggregate consumption matters and there is no uncertainty. The utility function is then

$$U = U(c_1, c_2) \tag{6:14}$$

If the endowed wealth is W_o and amounts w_j are invested in assets with an internationally determined yield r_j and domestic tax rates t_j, we find

$$c_1 = W_o - \Sigma w_j \tag{6:15a}$$

$$c_2 = \Sigma w_j \cdot [1 + r_j(1-t_j)] \tag{6:15b}$$

The first order conditions for an optimal investment in asset j are

$$U_1 = U_2 \cdot [1 + r_j(1-t_j)] \quad \text{if} \quad w_j \neq 0 \tag{6:16a}$$

$$U_1 > U_2 \cdot [1 + r_j(1-t_j)] \quad \text{if} \quad w_j = 0 \tag{6:16b}$$

With this specification households will only invest in the asset which pays the highest net-of-tax return $r_j \cdot (1-t_j)$. It is then obvious that taxes should be neutral (all $t_j = t$) so that the same asset has the highest $r_j \cdot (1-t_j)$ and the highest r_j. Differentiated t_j might change the ranking of the investment alternatives and might thus cause distortion of the asset choice decreasing national income (and increasing the distortion of the savings decision).

If the utility function is defined not only on aggregate consumption but also on other variables, results might be more complicated. If the benefit of these other variables is not taxed in the same way as income, a neutral income tax might be distorting and a differentiated income tax might decrease distortions. The reasoning is analogous to that of optimal consumption taxes when leisure is not taxed and goods are complementary or substitutes to leisure.

One example of such utility functions is the dividend preference of investors in the model in Chapter 5. In that model I assumed that investors are not only interested in the total amount of net-of-tax income from shares but also in the distribution between dividend income and capital gains.

We write the utility function

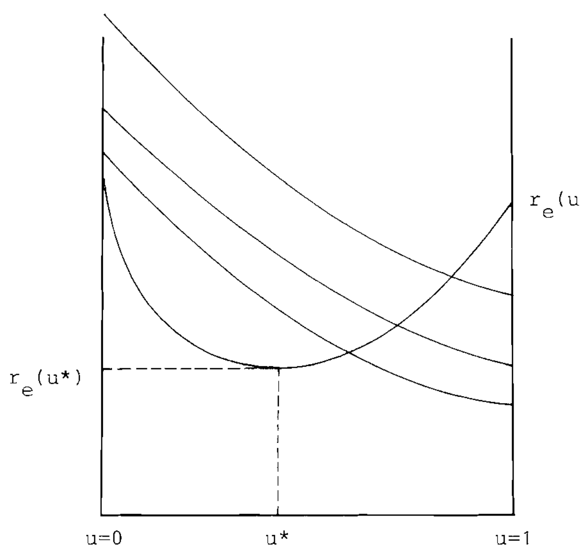
$$U = U(r_n, u) \tag{6:17}$$

where r_n is total net-of-tax yield and u is the dividend to profit ratio. Note that $r_n(u)$ was regarded as the supply price of equity capital in Chapter 5 when we focused on corporate behaviour. In this chapter it is more natural to introduce an explicit utility function in r_n and u where r_n is the after-tax yield that can be obtained on the perfect international capital market.

We found in Chapter 5 that in countries with tax differentiation in favour of retained earnings, the observed behaviour with a positive dividend ratio and new issues of shares is only possible if $r_n'(u) < 0$ and $r_n''(u) > 0$. The indifference curves of (6:17) must thus have the form depicted in Figure 6:1.

The yield that can be obtained on the international market before domestic household taxes is denoted $r_e(u)$ as in Section 5:6. Due to the combination of investor preferences for dividends and tax discrimination of dividends in most countries $r_e(u)$ will be convex with a minimum $r_e(u^*)$. In Section 5:4 we found that if the economy is small and open and if case A is predominant the corporate taxes on dividends and retained earnings should be equal in order to make firms choose the socially optimal dividend ratio u*.

Figure 6:1 Available yield $r_e(u)$ and domestic investors' indifference curves



Corporate tax differentiation would lead to an inferior adjustment to the preferences of international investors.

What is then the socially optimal choice of u for domestic investors? If the government wants a tax revenue R per unit of capital, where R is independent of u, then

$$r_n(u) = r_e(u) - R \tag{6:18}$$

The optimal portfolio choice is that for which the highest possible indifference curve is attained. From Figure 6:1 it is obvious that this will usually be at u=1 where $r_e(u)$ is high and where the preference for a high dividend ratio is well satisfied. The only viable alternative would be u=0 but this solution is only possible if the dividend preference of domestic households is less pronounced than that of the representative international investor.

The private optimum would automatically correspond to the social optimum if a wealth tax were used. The cost of the tax would then not depend upon the choice of u. A neutral income tax ($t_d=t_g$) might distort the choice because it is proportional to $r_e(u)$ and thus reduces the difference between the yields available at u=0 and u=1. But if u=1 is socially optimal it will probably also be privately optimal.

With tax differentiation in favour of capital gains the slope of the net-of-tax yield curve will approach that of the indifference curves. If the domestic tax system were equal to that of other countries and if the utility functions of domestic households were equal to those of domestic investors the yield curve and the indifference curves would have the same slope. Any interior solution might be obtained. Such solutions are evidently socially inferior and should be avoided. The neutral tax system is thus preferable.

In summary, for the special case of the model in Chapter 5 the taxes should be neutral. This was partly due to the fact that the social optimum was represented by corner solutions. If an interior solution had been socially optimal, it would have been more important to differentiate the taxes so that, close to the optimum, the slope of the net-of-tax income function were equal to the slope of the gross-of-tax income function.

This demands, however, good information. The same effect can be obtained, with less information needs, by using a wealth tax instead of the income tax.

6.4 PORTFOLIO CHOICE OF HOUSEHOLDS UNDER UNCERTAINTY

Uncertainty or risk can be regarded as a good for which the consumer/investor has preferences and which is not taxed in the same way as income. Thus it is not evident that the income of riskless and risky investments should be taxed at the same rates.

Let us start with a <u>single period model</u> with two assets, one with a riskless interest rate, r_b, and one with a random return, $\tilde{r}_s$. The expected value of $\tilde{r}_s$ is $\bar{r}_s$. The tax rates are t_b for the riskfree asset and t_s for the risky asset. If endowed wealth is W_o and if $a \cdot W_o$ is invested in the risky asset and the government pays a lump-sum transfer, G, final wealth is

$$W_1 = W_o \cdot [1 + a \cdot \tilde{r}_s \cdot (1-t_s) + (1-a) \cdot r_b \cdot (1-t_b)] + G \qquad (6:19)$$

Maximizing the expected utility of W, $E[U(W_1)]$, we get the first order condition of an optimal a

$$(1-t_s) \cdot E[U'(W_1) \cdot \tilde{r}_s] = (1-t_b) \cdot r_b \cdot E[U'(W_1)] \qquad (6:20)$$

As noted by Stiglitz [1972] it is no easy task to define an equal tax yield in this type of model. I will assume that the tax rates are adjusted in such a way that the expected tax revenue, $\bar{R}$, is constant:

$$\bar{R} = W_o \cdot [a \cdot \bar{r}_s \cdot t_s + (1-a) \cdot r_b \cdot t_b] \qquad (6:21)$$

The government then gets a random surplus or deficit

$$R - \bar{R} = W_o \cdot a \cdot t_s \cdot (\tilde{r}_s - \bar{r}_s) \tag{6:22}$$

We can now either assume (case 1) that this random surplus is the same for all households and is transferred to the households

$$G = R - \bar{R} = W_o \cdot a \cdot t_s \cdot (\tilde{r}_s - \bar{r}_s) \tag{6:23}$$

or assume, case 2, that $R - \bar{R}$ varies across households and that the aggregate government revenue is riskfree. In case 2 $G = 0$.

An alternative interpretation of case 1 is that the government uses the surplus funds for public goods which enter the utility functions in the same way as private goods. The assumption of case 2 corresponds to the usual assumption that the government is risk neutral. This assumption might be reasonable if the tax on capital income is only a small part of government revenue even though capital income is important for those who own wealth. The relevance of the two cases was discussed in Chapter 3, Section 7.

The two cases have been defined such that government expenses, transfers excepted, are constant. Changes in social welfare can then be measured by changes in private utility $E[U(W_1)]$. Using the envelope theorem

$$dU = -W_o \cdot a \cdot E[U'(W_1) \cdot \tilde{r}_s] \cdot dt_s -$$

$$- W_o \cdot (1-a) \cdot r_b \cdot E[U'(W_1) \cdot dt_b + E[U'(W_1) \cdot dG] \tag{6:24}$$

where in case 1 (see (6:23))

$$dG = W_o \cdot a \cdot (\tilde{r}_s - \bar{r}_s) \cdot dt_s + W_o \cdot t_s \cdot (\tilde{r}_s - \bar{r}_s) \cdot da \qquad (6:25)$$

and in case 2

$$dG = 0 \qquad (6:26)$$

<u>In case 1</u> (6:24) can thus be written

$$dU = -W_o \cdot [a \cdot \bar{r}_s \cdot dt_s + (1-a) \cdot r_b \cdot dt_b] \cdot E[U'(W_1)] +$$

$$+ W_o \cdot t_s \cdot E[U' \cdot (\tilde{r}_s - \bar{r}_s)] \cdot da \qquad (6:27)$$

Possible combinations of dt_s and dt_b are given by (6:21)

$$d\bar{R} = W_o \cdot [a \cdot \bar{r}_s \cdot dt_s + (1-a) \cdot r_b \cdot dt_b] +$$

$$+ W_o \cdot [\bar{r}_s \cdot t_s - r_b \cdot t_b] \cdot da = 0 \qquad (6:28)$$

Substituting (6:28) in (6:27) we have for case 1

$$dU = W_o \cdot E[U'(W_1) \cdot (\tilde{r}_s \cdot t_s - r_b t_b)] \cdot da \qquad (6:29)$$

Substituting (6:20)

$$dU = W_o \cdot r_b \cdot E[U'(W_1)] \cdot \frac{t_s - t_b}{1 - t_s} \cdot da \qquad (6:30)$$

The optimal tax system is that for which dU is zero for any admissible combination of dt_s and dt_b and thus for any da. Thus the optimality condition in case 1 is neutral taxation:

$$t_s = t_b \qquad (6:31)$$

An intuitive explanation is that the social risk aversion is equal to the private risk aversion. Therefore the existence of risk aversion in the utility function does not change our normal tax neutrality result. The same result was derived by Stiglitz [1972] under more restrictive assumptions.

For case 2 we substitute (6:26) and (6:28) into (6:24)

$$dU = -W_o \cdot E[U'(W_1) \cdot (\tilde{r}_s - \bar{r}_s)] \cdot a \cdot dt_s - W_o \cdot E[U'(W_1)] \cdot$$
$$\cdot (\bar{r}_s t_s - r_b t_b) \cdot da \tag{6:32}$$

Substituting (6:20)

$$\frac{dU}{dt_s} = -W_o \cdot E[U'(W_1)] \cdot \left[\frac{a}{1 - t_s} \cdot \left((1-t_b) \cdot r_b - (1-t_s) \cdot \bar{r}_s\right) - \right.$$
$$\left. - (\bar{r}_s t_s - r_b t_b) \cdot \frac{da}{dt_s}\right] \tag{6:33}$$

Formally we could get an explicit expression for da/dt_s by differentiating (6:20) and using the equal tax revenue condition (6:28). However, it is obvious that the solution will depend on the form of the utility function. What we can say generally is that da/dt_s must be negative.[3] For dU to be zero for all dt_s we then have

$$\frac{a}{1-t_s} = \frac{(\bar{r}_s t_s - r_b t_b) \cdot \frac{da}{dt_s}}{(1-t_b) \cdot r_b - (1-t_s) \cdot \bar{r}_s} > 0 \tag{6:34}$$

The denominator is negative if any interior solution exists and da/dt_s is negative. Thus

$$\bar{r}_s t_s > r_b t_b \tag{6:35}$$

To interpret this condition we can note that if $\bar{r}_s t_s = r_b t_b$ expected tax revenues are not affected by the choice of a. But by increasing t_s marginally we transfer risk from risk averse individuals to the risk neutral government and thus increase welfare. It seems probable that the optimal difference $\bar{r}_s t_s - r_b t_b$ will depend on the degree of risk aversion.

The result (6:35) is not equivalent to the result in Stiglitz [1972], p. 319. He finds $\bar{r}_s t_s = r_b t_b$. That his result cannot be correct is evident from the fact that it satisfies one of the optimality conditions (45 a) but not the other (45 b). The trouble seems to be that Stiglitz assumed that the marginal utility of income is the same for the government and the investors, which restricts the average tax rate, and at the same time optimized t_b and t_s independently. For case 1 this does not affect Stiglitz' result because in that case the two optimality conditions happen to become identical.

In a <u>two period model</u> we must also study the effect of tax adjustments on the volume of savings. As in Section 2, non-neutral taxation might be favourable if the total volume of savings, and thus the tax basis, is increased by deviations from neutrality.

I will denote non-capital income in the two periods y_1 and y_2, consumption c_1 and c_2 and savings s. Thus

$$c_1 = y_1 - s \tag{6:36}$$

$$\tilde{c}_2 = y_2 + s\left(1 + a\cdot\tilde{r}_s\cdot(1-t_s) + (1-a)\cdot r_b\cdot(1-t_b)\right) + G \tag{6:37}$$

Expected tax revenue is then

$$\bar{R} = s\cdot[a\cdot\bar{r}_s\cdot t_s + (1-a)\cdot r_b\cdot t_b] \tag{6:38}$$

and instead of (6:28) we have

$$d\bar{R} = s\cdot[a\cdot\bar{r}_s\cdot dt_s + (1-a)\cdot r_b\cdot dt_b + (\bar{r}_s\cdot t_s - r_b\cdot t_b)\cdot da] +$$

$$+ [a\cdot\bar{r}_s\cdot t_s + (1-a)\cdot r_b\cdot t_b]\cdot ds = 0 \tag{6:39}$$

For our simple case 1 with a riskaverse government we also have

$$dG = s\cdot a\cdot(\tilde{r}_s - \bar{r}_s)\cdot dt_s + s\cdot t_s\cdot(\tilde{r}_s - \bar{r}_s)da +$$

$$+ a\cdot t_s\cdot(\tilde{r}_s - \bar{r}_s)\cdot ds \tag{6:40}$$

$$dU = s\cdot E[U_2\cdot(\tilde{r}_s\cdot t_s - r_b\cdot t_b)]\cdot da +$$

$$+ E[U_2\cdot(a\cdot\tilde{r}_s\cdot t_s + (1-a)\cdot r_b\cdot t_b)]ds \tag{6:41}$$

Substituting (6:20)

$$dU = s\cdot r_b\cdot E[U_2]\cdot\frac{t_s-t_b}{1-t_s}\cdot da +$$

$$+ r_b\cdot E[U_2]\cdot\left[a\cdot t_s\cdot\frac{1-t_b}{1-t_s} + (1-a)\cdot t_b\right]\cdot ds \tag{6:42}$$

The first term corresponds to (6:30). A neutral tax would be optimal ($t_s=t_b$) if ds/dt_s were equal to zero at the point $t_s = t_b$.

We could now either solve da/dt_s and ds/dt_s simultaneously by differentiating the private optimality conditions and the equal tax yield condition in order to get an expression for dU/dt_s, or just study under what conditions ds/dt_s will turn out to be equal to zero. The first alternative leads to cumbersome expressions and the results depend not only on the form of the utility function but also on the amount of non-capital income y_2 and on the average tax level. I will therefore pursue the second alternative.

From Chapter 2 (2:57) we have the first order condition for optimal savings

$$E[U_1] = E[U_2\cdot(1+a\cdot\tilde{r}_s(1-t_s) + (1-a)\cdot r_b\cdot(1-t_b))] =$$

$$= [1 + r_b(1-t_b)]\cdot E[U_2] \tag{6:43}$$

From (6:36), (6:37) and (6:23) we also have

$$c_1 = y_1 - s \tag{6:44}$$

$$\tilde{c}_2 = y_2 + s\cdot[1 + a\cdot\tilde{r}_s + (1-a)\cdot r_b] - \bar{R} \tag{6:45}$$

Differentiating (6:43) keeping $\bar{R}$ constant and assuming that the utility function is additive

$$-\{U_{11} + [1 + r_b(1-t_b)]\cdot E[U_{22}\cdot(1+a\cdot\tilde{r}_s+(1-a)\cdot r_b)]\}ds =$$

$$= - r_b\cdot E[U_2]\cdot dt_b + [1 + r_b(1-t_b)]\cdot s\cdot E[U_{22}\cdot(\tilde{r}_s-r_b)]da \tag{6:46}$$

Thus ds/dt_b is negative if da/dt_b is positive and $E[U_{22}(\tilde{r}_s-$ $-r_b)]$ is not positive. With constant absolute risk aversion $E[U_{22}(\tilde{r}_s-r_b)]$ is equal to zero if $t_b=t_s$. With decreasing absolute risk aversion it is positive if $t_b=t_s$. If the absolute risk aversion were constant t_b should then be lower than t_s. But in the more likely case of decreasing absolute risk aversion, the outcome is ambiguous.

The main conclusion, then, is negative. Even in the case where we could derive a clear result for a single period model, the result is ambiguous when effects on the savings decision are taken into account. Therefore we cannot make any general recommendation about in what direction the tax rates should deviate from the neutrality condition $t_s = t_b$.

6.5 SUMMARY AND CONCLUSIONS

In this chapter I have studied the effect of tax differentiation on the choice of capital assets and on aggregate investment or aggregate savings. I have tried to deduce the optimal degree of tax differentiation.

In Section 6.2 I analyze the effect of corporate taxes on the investment behaviour of firms. I show that neutral taxes are favourable if aggregate capital intensity is not increased by tax differentiation. Neutral taxes are thus optimal if the production function is Cobb-Douglas or of the additive power type. In other cases differentiated taxation can be optimal if one knows the form of the representative production function and thus in what direction to differentiate.

In Sections 6.3 and 6.4 I analyze the effects of differential household taxes on the portfolio choice and savings of households. In the certainty case taxes should normally be neutral. A possible exception is the taxation of goods which confer benefits beside taxable income. As a special case I discuss the choice of shares with different dividend ratios by investors with a preference for dividend income. For this special case it turns out that neutral taxes are preferable.

When uncertainty is introduced, the results are less clear and results which seem clear in a single period model become ambiguous in a two period model where savings is endogenous. It is shown that the results depend upon the degree of risk diversification by the government and upon the form of the households' utility functions.

NOTES

1) From (6:3) and (6:13b) the first order condition of optimal k_i is

$$\alpha_i \cdot k_i^{\gamma_i - 1} = \frac{r}{1 - t_i}$$

Thus

$$\varepsilon_i = - \frac{\partial k_i}{\partial t_i} \cdot \frac{1 - t_i}{k_i} =$$

$$= - \frac{1}{\alpha_i} \cdot \frac{1}{\gamma_i - 1} \cdot \frac{1}{k_i^{\gamma_i - 2}} \cdot \frac{r}{(1-t_i)^2} \cdot \frac{1-t_i}{k_i} = \frac{1}{1-\gamma_i}$$

For logarithmic production function $\gamma=0$ and $\varepsilon=1$.

2) CES-function

$$x = \left[\alpha_o + \sum_i \alpha_i k_i^{\rho}\right]^{1/\rho}$$

$$\frac{\partial x}{\partial k_j} = x^{1-\rho} \cdot \alpha_j \cdot k_j^{\rho-1} = \frac{r}{1-t_j}$$

Differentiate with regard to t_j and all k_i

$$\sum_i (1-\rho) \cdot x^{-\rho} \cdot \frac{\partial x}{\partial k_i} \cdot \alpha_j \cdot k_j^{\rho-1} \cdot dk_i +$$

$$+ (\rho-1) \cdot x^{1-\rho} \cdot \alpha_j \cdot k_j^{\rho-2} \cdot dk_j = \frac{r}{(1-t_j)^2} \, dt_j$$

At the point $t_i = t_j = t$ all $\partial x/\partial k_j$ are equal to $r/(1-t)$. Thus

$$(1-\rho)\left[\frac{r}{x} \cdot \Sigma dk_i - \frac{1-t}{k_j} \cdot dk_j\right] = dt_j$$

$$\frac{\partial k_i}{\partial t_j} = \frac{x}{(1-\rho) \cdot r} \qquad \text{all } i \neq j$$

2) (cont.)

$$\frac{\partial k_j}{\partial t_j} = \frac{x}{(1-\rho)\cdot r} - \frac{k_j}{(1-\rho)(1-t)}$$

$$\frac{\sum_i \frac{\partial k_i}{\partial t_j}}{k_j} = \frac{\frac{n\cdot x}{(1-\rho)\cdot r} - \frac{k_j}{(1-\rho)(1-t)}}{k_j}$$

This is independent of j only if all k_i are equal.

3) If t_s is increased and t_b is decreased, keeping the tax revenue constant, there will be a strong substitution effect decreasing the relative investment in the risky asset and there are no strong income effects.

REFERENCES

Ando, A.K., Modigliani, F., Rasche, R., and Turnovsky, S.J., [1974], On the Role of Expectations of Price and Technological Change in an Investment Function, International Economic Review, Vol. 15, p. 412.

Gravelle, J.G., [1981], The Social Cost of Nonneutral Taxation: Estimates for Nonresidential Capital, in Depreciation, Inflation and The Taxation of Income from Capital (ed. Hulten, C.R.), The Urban Institute Press, Washington D.C.

Hendershott, P.H., and Hu, S.-C., [1980], Government-Induced Biases in the Allocation of the Stock of Fixed Capital in the United States, in Capital, Efficiency and Growth (ed. von Furstenberg, G.M.), Bollinger Publishing Co., Cambridge, Mass., pp. 323-360.

Hendershott, P.H., and Hu, S.-C., [1981], Investment in Producers' Durable Equipment, in How Taxes Affect Economic Behaviour (ed. Aaron, H.J., and Pechman, J.A.), The Brookings Institution, Washington D.C., pp. 85-126.

Stiglitz, J.E., [1972], Taxation, Risk Taking, and the Allocation of Investment in a Competitive Economy, in Studies in the Theory of Capital Markets (ed. Jensen, M.C.), Praeger Publishers.

The Taxation of Income from Capital. A Comparative Study of the U.S., U.K., Sweden and West Germany. University of Chicago Press, 1984.

7. The Realization Criterion

7.1 INTRODUCTION

In Chapter 5 I assumed that an effective capital gains tax rate t_g can be defined. All calculations were made as if capital gains were taxed each period as they occur. The technical term is taxation on accrual. An accrual tax on capital gains is directly comparable to an ordinary income tax on dividends and interest payments.

In real world tax systems a capital gain is usually taxed not on accrual but when the asset is sold and the gain is realized. The tax payment is thus deferred from the accrual date to the realization date. The investor can be said to receive a tax credit free of interest. This deferral of the tax payment reduces the present value of the tax and thus reduces the effective tax rate below the nominal rate. The reduction of the effective tax rate is larger the longer the asset is held. The realization criterion thus tends to make it more profitable to keep assets for a long time rather than trading them often (the lock-in effect).

An investor in marketable securities with uncertain returns can reduce the effective tax rate of a realization tax even further by continuously realizing all capital losses but holding on to assets with capital gains. As shown by Stiglitz [1983] such asymmetric investor behaviour can make the effective tax rate negative. In order to counteract such effects governments make the tax on realized capital gain asymmetric.

One purpose of this chapter is to analyze those factors which must be taken into account when calculating the effective tax rate corresponding to any realization tax rate t_{gr}. In Section 7.2 I first repeat the calculations made by Diamond [1975] for the case of a certain rate of price increase. The only difference is that I make the calculations in continuous time which makes it easier to present the results. I also show that the effective tax on capital gains will be still lower if only a part of the return is received in the form of capital gains. In Section 7.4 I analyze the case of uncertain returns and symmetric behaviour and in Section 7.5 I discuss the effect of asymmetric behaviour (endogenous holding period) and of asymmetric tax systems. My main conclusion is that great care is needed in calculating the effective tax rate on capital gains. In empirical work the effects of asymmetry are usually not taken into account.

Another purpose of the chapter is to demonstrate the effects of the realization criterion on portfolio choice and risk taking. In Section 7.2 I show that the realization tax rate should increase with the length of the holding period for the effective tax rate to be constant. In Section 7.3 I demonstrate the lock-in effect of a realization tax rate that does not increase with the holding period. In Section 7.4 I show that under uncertainty the net-of-tax variance will be higher with a realization tax than with an equal yield accrual tax, which will entail lower investments in risky assets. In Section 7.5, I discuss the effect of asymmetric taxation on incentives to diversify the portfolio.

The analysis will show that the existence of the realization criterion distorts the portfolio choice in many ways. An accrual tax on capital gains might seem to be preferable. But there are a number of good reasons for the existence of the realization criterion. One reason is the difficulty in measuring a capital gain before it is realized. This is especially true for those assets for which no organized markets exist. Due to this assessment problem the inclusion of unrealized capital gains in reported income is not permitted by standard accounting rules. Another reason for the existence of the realization criterion is liquidity effects in an imperfect capital market. It might be difficult to finance tax payments before the gain is realized.

Finally, an accrual tax on unrealized capital gains may be in conflict with the public sense of justice. A good example might be a farm which has been farmed by the same family for many generations. Probably most citizens would react negatively, either to the owners paying an accrual tax on the capital gain if the market value happened to increase by 10 per cent from one year to the next, or to the owners receiving a corresponding transfer if the market price happened to decline.

Real world tax systems have a number of features which I will not discuss below but which must be taken into account when evaluating the effective capital gains tax rate in a particular country. Very often the tax rate is higher for short vs. long holding periods, which increases the lock-in effect for assets with a positive capital gain and which also increases the gains of asymmetric behaviour. In some countries, the capital gains tax liability vanishes at death which increases the lock-in effect and decreases the effective tax rate. In other countries, like Sweden, the inheritance tax is based on the market value of assets without any deduction for capital gains tax liability, which can make the effective inheritance tax close to 100 per cent. It is then of great importance to foresee the date of death and to realize all gains in time.

7.2 EFFECTIVE TAX RATES UNDER CERTAINTY

If capital gains are taxed on realization instead of on accrual, the government is essentially giving the investor a tax credit free of interest. The benefit to the investor of this tax credit increases with the holding period. If the realization tax rate is independent of the holding period, the effective tax rate will be a decreasing function.

Assume that we have a tax system with a tax rate t_r on current capital income and a tax rate t_{gr} on realized capital gains. We assume that the rate of price increase of an asset, p, is known with certainty and that the asset's holding period, T, is independent of the tax rates. We want to calculate what realization tax rate is equivalent to some accrual tax t_r.

If one dollar is invested in an asset with a price which is increasing at a rate of p and yielding no current income, the value of the investment at time T is e^{pT}. If the asset were sold at T, one would have to pay a realization tax $t_{gr} \cdot (e^{pT}-1)$. The net value would be $e^{pT} - t_{gr}(T)(e^{pT}-1)$. If we paid an accrual tax t_r instead, the net value would be $e^{(1-t_r) \cdot pT}$. Thus the realization tax gives the same yield as an accrual tax if

$$t_{gr} = \frac{1 - e^{-t_r \cdot p \cdot T}}{1 - e^{-p \cdot T}} \tag{7:1}$$

We can note that as the asset yields no current income and as the accrual tax on capital gains is assumed to be equal to the tax on interest, the rate of price increase of the asset must be equal to the interest rate for an equilibrium to exist.

Evaluating (7:1) with d'Hôpital's rule we see that t_{gr} is close to t_r for small T and goes to 1 as T approaches infinity. Inverting this relationship we see that for any realization tax rate $t_{gr} < 1$, the effective tax rate goes to zero as the holding period approaches infinity.

Evaluating (7:1) numerically for reasonable values of t_r and $p \cdot T$ we obtain the following values of equivalent t_{gr}

	t_{gr} corresponding to	
$p \cdot T$	t_r=0.25	t_r= 0.50
0.1	0.26	0.51
0.5	0.30	0.56
1.0	0.35	0.62
2.0	0.45	0.73

The results correspond to those of Diamond [1975] with the difference that Diamond did not discount in continuous time and therefore did not get results exactly related to $p \cdot T$. But the form of (7:1) also makes another interpretation possible. Maybe t_{gr} should not be a function of T but of the capital gain $p \cdot T$. The difference is important if the price increase rate is uncertain and the second interpretation would coincide with one of the results of Green and Sheshinski [1978]. I will return to this issue in Section 7.4.

The results in King [1977], Section 3.3, cannot be easily compared to those reported here because King assumes that the investor sells a fixed proportion of his holdings each year instead of holding them for a fixed time T. The general tendency is, however, the same.

My subjective interpretation of the numerical results is that the effect of the realization criterion on the effective tax rate is modest if $p \cdot T$ is less than 0.5. If the tax base is adjusted for the effect of inflation, p will correspond to the real rate of interest and should usually be less than 0.05 per year. The effect of the realization criterion on the effective tax rate would then be modest for holding periods of up to 10 years. With nominal taxation and high rates of inflation, the effect can be very large.

If we make the same comparison for an asset with a rate of price increase $p < r$ and a current dividend rate $r-p$ we get a more complicated result and the realization tax rate which is equivalent to an accrual tax t_r turns out to be higher than according to (7:1). The reason is that with a realization tax the value of the investment and thus the dividends are always higher. The realization criterion thus results in not only a higher value of the investment, but also in a higher dividend income. The relative importance of the dividend effect increases if p decreases.

Assume that the price increase effect is p and that the dividend income is r-p. The dividend income is taxed at the tax rate t_r. The net-of-tax income which can be reinvested is then $(r-p)(1-t_r)$. I assume that this cash flow is invested in bonds or other assets which pay a net-of-tax yield $r(1-t_r)$.[1] The net-of-tax value of the asset and the dividends at time T is then

$$V_T = e^{pT} - t_{gr}(e^{pT}-1) + (r-p)(1-t_r) \int_0^T e^{pt} \cdot e^{r(1-t_r)(T-t)} dt =$$

$$= e^{pT} - t_{gr}(e^{pT}-1) + \frac{(r-p)(1-t_r)}{p-r(1-t_r)} (e^{pT} - e^{r(1-t_r)T})$$

For this to be equivalent to the value with an accrual tax, $e^{r(1-t_r)T}$, the tax rate t_{gr} must be equal to

$$t_{gr} = \frac{1 - e^{-(p-r(1-t_r))T}}{1 - e^{-pT}} \cdot \frac{p \cdot t_r}{p - r(1-t_r)} \qquad (7:2)$$

We see immediately that (7:2) is equivalent to (7:1) if p=r. We also find

$$\lim_{T \to 0} t_{gr} = t_r$$

$$\lim_{T \to \infty} t_{gr} = t_r \cdot \frac{p}{p - r(1-t_r)} > 1, \text{ if } p > r(1-t_r)$$

$$\lim_{T \to \infty} t_{gr} = \infty \qquad\qquad , \text{ if } p < r(1-t_r)$$

$$\lim_{p \to 0} t_{gr} = t_r \cdot \frac{e^{r(1-t_r)\cdot T} - 1}{r(1-t_r)\cdot T}$$

Evaluating (7:2) numerically for $t_r = 0.50$ and for reasonable values of $r \cdot T$ and p/r, we obtain the following equivalent values of t_{gr}

	t_{gr} corresponding to $t_r=0.50$		
$r \cdot T$	$p/r=1.0$	$p/r=0.5$	$p/r=0.1$
0.1	0.51	0.52	0.52
0.5	0.56	0.57	0.58
1.0	0.62	0.63	0.65
2.0	0.73	0.79	0.85

We can see that t_{gr} increases more rapidly with $p \cdot T$, and even with $r \cdot T$, when a part of the income is received in the form of dividends. For the effective tax rate to be 50 per cent when $p \cdot T = 1.0$, the realization tax rate must be 62 per cent if there is no dividend income ($r \cdot T = 1.0$) and 79 per cent if the dividend income is of the same magnitude as the capital gain ($r \cdot T = 2.0$).

7.3 THE LOCK-IN EFFECT

Differentiating (7:1) and (7:2) it is easily seen that the realization tax rate which is equivalent to an effective tax rate t_r is an increasing function of the holding period, T. In real world tax systems t_{gr} is constant (or a decreasing function of T). If the holding period is extended the marginal effective tax rate is then lower than the average effective tax rate. It might be profitable to hold an asset for another period even though the expected return is lower than that of alternative investments. This will result in a lower turnover on capital asset markets and in decreased market performance. If the asset is owner occupied housing, the lock-in effect will reduce adaptability to changing consumption preferences.[2]

In order to demonstrate the lock-in effect mathematically I assume that an investor owns an asset which pays no current dividends and which has increased in price at a rate r during the holding period T. He thus has the alternative to sell the asset at the current price, e^{rT}, and pay the tax, $t_{gr}(e^{rT}-1)$. I assume that he can invest the net proceeds in bonds which pay a net-of-tax return $r(1-t_r)$. His wealth at time T+dT is then

$$[e^{rT} - t_{gr}(e^{rT}-1)]\cdot e^{r(1-t_r)\cdot dT}$$

He also knows that the future price increase of the asset will be $x < r$. If he does not sell the asset, his wealth at t+dT will thus be

$$e^{rT+x\cdot dT} - t_{gr}(e^{rT+xdT}-1)$$

Holding the asset and selling it are thus equally profitable if x is such that

$$(1-t_{gr})\cdot e^{rT}\cdot[e^{r(1-t_r)\cdot dT} - e^{x\cdot dT}] = t_{gr}(1-e^{r(1-t_r)dT}) \quad (7:3)$$

For small dT, $e^{a\cdot dT} = 1 + a\cdot dT$, and we have

$$(1-t_{gr})\cdot e^{rT}\cdot[x - r(1-t_r)] = t_{gr}\cdot r(1-t_r) \quad (7:4)$$

Thus

$$x = r(1-t_r)\left[1 + \frac{t_{gr}}{1-t_{gr}}\cdot e^{-rT}\right] \quad (7:5)$$

The marginal cost of holding the asset for another period will thus decrease towards $r(1-t_r)$ as T, and thus the accumulated tax credit increases.

If we substitute condition (7:1) into (7:5) assuming that the realization tax rate is equivalent to t_r for the holding period T and the price increase rate up to T of r, we get

$$x = r \cdot (1-t_r) \cdot \left[1 + \frac{\dfrac{1-e^{-t_r \cdot r \cdot T}}{1-e^{-rT}} \cdot e^{-rT}}{1 - \dfrac{1-e^{-t_r \cdot r \cdot T}}{1-e^{-rT}}} \right]$$

$$\therefore \quad x = r \cdot (1-t_r) \cdot \frac{1 - e^{-rT}}{1 - e^{-(1-t_r) \cdot rT}} \qquad (7:6)$$

We then see that the marginal cost x is close to r when T is small (little accumulated tax credit) and falls towards $r(1-t_r)$ as the holding period is increased.

7.4 EFFECTS ON RISK TAKING WITH EXOGENOUSLY DETERMINED HOLDING PERIODS AND SYMMETRIC TAXATION

Below I will demonstrate that even if we choose a realization tax rate which is equivalent to an accrual tax t_r and increases with T according to (7:1), the net-of-tax return of any given asset will be more uncertain with a realization tax than with an accrual tax. This will make investors invest less in risky assets.

In continuous time, the price of an asset is often assumed to follow a stochastic diffusion process

$$dP = r \cdot P \cdot dt + \sigma \cdot P \cdot dz \qquad (7:7)$$

where P is the price at time t, r is the expected rate of price increase, σ is a measure of the uncertainty and dz is a Wiener process. The expected value of P and the standard deviation of P are then (see Tintner and Sengupta [1972] p. 66)

$$E[P] = P_o \cdot e^{rt} \tag{7:8}$$

$$s[P] = P_o \cdot e^{rt} \cdot \sqrt{e^{\sigma^2 t} - 1} \tag{7:9}$$

For small $\sigma^2 t$ the square root expression is approximately equal to $\sigma \cdot \sqrt{t}$.

If we invest one dollar at t=0 and pay an accrual tax t_r the value of our investment, V_a, is determined by

$$dV_a = r(1-t_r) \cdot V_a \cdot dt + \sigma(1-t_r) \cdot V_a \cdot dz \tag{7:10}$$

Thus at time T

$$E(V_a) = e^{r(1-t_r) \cdot T} \tag{7:11}$$

$$s(V_a) = e^{r(1-t_r) \cdot T} \sqrt{e^{\sigma^2 (1-t_r)^2 \cdot T} - 1} \tag{7:12}$$

With a symmetric realization tax t_{gr} we obtain instead

$$E(V_r) = e^{rT} - t_{gr}(e^{rT} - 1) \tag{7:13}$$

$$s(V_r) = (1-t_{gr}) \cdot e^{rT} \cdot \sqrt{e^{\sigma^2 \cdot T} - 1} \tag{7:14}$$

Substituting the expression (7:1) with p≡r for t_{gr}

$$E(V_r) = E(V_a) \tag{7:15}$$

$$s(V_r) = s(V_a) \cdot \frac{1-e^{-r(1-t_r) \cdot T}}{1-e^{-rT}} \cdot \sqrt{\frac{e^{\sigma^2 T} - 1}{e^{\sigma^2 (1-t_r)^2 T} - 1}} \tag{7:16}$$

Using a Taylor expansion for small T we find

$$\lim_{T\to 0} \frac{s(V_r)}{s(V_a)} = 1 \tag{7:17}$$

For large T the expression under the square root approaches $e^{(t_r \cdot \sigma \cdot T)^2}$ and

$$\lim_{T\to \infty} \frac{s(V_r)}{s(V_a)} = \infty \tag{7:18}$$

The result (7:15) confirms that with symmetric taxes and exogenously determined holding periods, relation (7:1) holds for the expected values of stochastic distributions. (7:17) and (7:18) show that the standard deviation (and thus the variance) net-of-tax will always be larger with a realization tax than with an equal yield accrual tax if the realization tax rate only depends on the length of the holding period.[3]

If t_{gr} is prescribed to be an increasing function of not only T but also of the actual rate of price increase according to the second interpretation of (7:1) the standard deviation of V_r will evidently be smaller than according to (7:16) as t_{gr} and p would be positively correlated. In fact we find

$$V_r = e^{pT} - t_{gr}(e^{pT}-1) = e^{(1-t_r)\cdot pT} \tag{7:19}$$

This is for every p equal to V_a and we thus confirm the findings of Green and Sheshinski [1978] that t_{gr} should be an increasing function of pT when the rate of return is uncertain and we do not want the realization criterion to influence the portfolio choice. The assumptions of this model are quite different from those of Green and Sheshinski which might imply that the result is robust.

7.5 THE EFFECTS OF ASYMMETRY

So far we have made the analysis of the effects of the realization criterion more complicated, and more realistic, by introducing the case where only a part of the return on an investment is received in the form of capital gains (Section 7.2) and by introducing uncertainty (Section 7.4). But the results are still misleading for most practical purposes. They ignore the possibility of asymmetric investor behaviour and the resulting asymmetric character of real world realization tax systems.

This statement can be qualified by asserting that the results would be correct if the capital gains were always positive, that is if the price increase trend r dominated the uncertainty σ. But in real life, as well as in the model described by (7:7), there is always a risk of negative outcomes. If the realization tax rate is constant or decreasing in the length of the holding period and transaction costs are small, investors will have an incentive to realize losses much earlier than they would realize positive capital gains.

Stiglitz [1983] has demonstrated that if the realization tax is symmetric, if short sales are allowed and if the capital market is perfect, households can easily avoid all taxation by buying and selling short[4] the same asset and realizing all losses as they occur. When the asset price decreases the investor sells his positive holdings and thus realizes a taxable loss. He then immediately buys back the same securities. When the asset price increases, he liquidates the short sale and thus realizes a taxable loss. In this way he would be able to realize losses for any price change without having made any net investment.

Probably most tax authorities would disallow deductions for the losses in this obvious case. The authorities can assert that the investor has not made any net investment in the security. But the investor can attain almost the same result by buying one security and short selling another security, if the returns on the two securities are highly correlated. Possible combinations would be two oil companies or two steel companies.

Tax authorities thus have reasons to dislike short sales and in many countries they are severely restricted. But the general idea can be used without short sales. With a diversified portfolio it is probable that some assets will give gains and other losses. Most popular guides for investors stress that losses should be realized. After a few days, the asset can be repurchased. The potential gain of such behaviour is even greater if the realization tax rate decreases with time. We then realize all losses when the tax rate is high and save all gains until the tax rate is low.

The effects of such asymmetric behaviour could be quantified by assuming that the asset price moves according to the stochastic process of (7:7), that transaction costs are negligible and that the tax is symmetric. The tax would then function as an accrual tax when the price decreases below any preceding low, and as a realization tax when the price increases. At the end of the total holding period T the realization tax would be paid on the difference between the value at T and the lowest preceding price. If the price never falls below the original purchase price, this case is obviously identical to the model of Section 7.4. But when any price decrease below the purchase price occurs, the effective tax rate is lower. Thus the expected tax burden is lower. If T is long or if the variance is large relative to the trend, the effective tax rate might easily be negative for any t_{gr}.

The option of using the investment policy discussed in the preceding paragraphs will cause a number of distortions:

- The effective tax rate will be lower for assets with low transaction costs (such as securities traded on an exchange) than for assets with high transaction costs such as private homes and farms, which cannot be sold in order to realize a small capital loss.

- The effective tax rate is lower for active investors than for 'widows and children'. Citizens who regard aggressive tax planning as undignified are penalized.

- Assets with high price volatility are favoured (cf. King [1977] p. 60).

The possibility of attaining a negative tax rate on capital gains is also one of the reasons for making the tax asymmetrical. This can be done in at least two different ways. One way is to make it impossible to deduct net capital losses from other types of income. In that case the effective tax rate will be positive or zero. Investors will have a strong incentive to diversify their portfolios and reduce the portfolio variance so that the risk of getting a net capital loss is small.

Another way is to disallow deductions for capital losses on one asset even if taxable gains are declared for other assets. This can increase the effective tax rate drastically and will reduce investments in risky assets. A modification of this technique is to calculate taxable gains and deductible losses in different ways. One example is the Swedish taxation of capital gains on real property. Positive gains are calculated with a mixture of nominal and real principles in order to take account of the deductibility of nominal interest payments. But only nominal losses are deductible, and such losses do not often occur in an economy with inflation.

We thus find that it is very difficult to assess the effects of the realization criterion correctly when asymmetric behaviour and asymmetric tax rules are taken into account. Asymmetric behaviour will reduce the effective tax rate for marketable assets but asymmetric tax rules will increase it. Asymmetric behaviour will favour investments in assets with a high price volatility and assymetric tax rules will penalize it.

Of at least theoretical interest is what would happen if the realization tax rate were an increasing function of $p \cdot T$ according to the second interpretation of (7:1). Would asymmetric behaviour still be favourable? No, it would not. If we realized a loss, the tax rate t_{gr} would be less than t_r and it would decrease as the loss increased. If we then repurchased the asset and the value increased, the realization tax would be calculated not only on a higher gain but also with a higher tax rate. We can show with a simple example that the net result will be the same as if the asset were held the entire time.

Let us assume that the price is e^{pT} at T and $e^{p_1T_1}$ at the earlier point of time T_1 where p_1 is negative. If we sell and repurchase at T_1 and sell again at T the final wealth will be (assuming that the tax credit at T_1 is invested in the asset)

$$W = \left[e^{p_1T_1} + t'_{gr}(1-e^{p_1T_1})\right]\left[e^{pT-p_1T_1} - t''_{gr}(e^{pT-p_1T_1}-1)\right] \quad (7:20)$$

where for (7:1)

$$t'_{gr} = \frac{1-e^{-t_r \cdot p_1T_1}}{1-e^{-p_1T_1}} \quad (7:21)$$

$$t''_{gr} = \frac{1-e^{-t_r(pT-p_1T_1)}}{1-e^{pT-p_1T_1}} \tag{7:22}$$

Substituting (7:21) and (7:22) we get

$$W = e^{(1-t_r)\cdot p_1 T_1} \cdot e^{(1-t_r)(pT-p_1T_1)} = e^{(1-t_r)\cdot p\cdot T} \tag{7:23}$$

Thus W is independent of the price path up to T and only depends on the total gain pT. W is also independent of sales and repurchases before T. A realization tax which is progressive in the relative capital gain pT according to (7:1) does not cause asymmetric behaviour. In all respects addressed by this model it is equivalent to an accrual tax t_r.

7.6 CONCLUSIONS

A realization tax on capital gains is equivalent to a proportional accrual tax t_r if the realization tax rate t_{gr} is progressive in the accumulated relative capital gain pT according to the expression (7:1)

$$t_{gr} = \frac{1-e^{-t_r\cdot p\cdot T}}{1-e^{-pT}} \tag{7:24}$$

If the realization tax is not progressive in gains but only in the length of the holding period according to the expression

$$t_{gr} = \frac{1-e^{-t_r\cdot r\cdot T}}{1-e^{-rT}} \tag{7:25}$$

where r is the expected price increase rate, the effective expected tax rate will be equal to t_r for all T if the investor does not have the possibility to sell and repurchase assets during the period T in order to realize losses. Under the same assumption the net-of-tax standard deviation of the return will be higher than it is with an accrual tax. If (7:25) holds and if it is impossible to sell and repurchase during T, the realization criterion will thus tend to decrease the investment in risky assets.

If the sale and repurchase of assets during T is possible without high transaction costs, the expected efficient tax rate will be lower than t_r and investment in risky assets is favoured.

Real world tax systems are not progressive in T and only vaguely progressive in p. The effective tax rate will then be lower the longer is the holding period and investors will experience the lock-in effect. Assets with accumulated capital gains should be held even if alternative assets are expected to give a higher pre-tax return. This effect will reduce the efficiency of capital markets.

If the tax rate is lower for long holding periods than for short holding periods, the lock-in effect and the incentive of asymmetric behaviour are intensified.

Calculating the effective tax rate corresponding to a nominal rate of realization tax is difficult when the effects of asymmetric behaviour and asymmetric tax rules are taken into account. The effective tax rate will be strongly dependent on the holding period, the asset characteristics and the characteristics of the investor.

NOTES

1) An alternative would be to assume that the cash flow is invested in the original asset and held until T. This would, however, decrease the average holding time, which makes it difficult to interpret the results.

2) See Englund [1983]. Englund also demonstrates that the conclusion does not hold, if transaction costs are large in comparison to capital gains and if the transaction costs are deductible from the capital gains.

3) That $s(V_r)/s(V_a) > 1$ for $T > 0$ can be seen by a second order Taylor expansion of (7:16).

4) 'To short sell a security, an individual borrows the security from a current owner and then immediately sells the security in the capital market at the current price. Then at a later date, the individual goes back to the capital market and repurchases the security at the then current market price and immediately returns the security to the lender. If the security price fell over the period of the short sale, the individual makes a profit; if the security price rose, he or she takes a loss. In either case the short seller's gain or loss is always the negative of the owner's gain or loss over the same period.' Copeland and Weston [1983] p. 115.

REFERENCES

Diamond, P.A. [1975], "Inflation and the Comprehensive Tax Base", Journal of Public Economics, Vol. 4, 227-244.

Englund, P. [1983], "Taxation of Capital Gains on Owner-Occupied Homes - Accrual Vs. Realization", EFI Research Paper 6262, Stockholm School of Economics.

Green, J.R., and Sheshinski, E. [1978], "Optimal Capital-Gains Taxation under Limited Information, Journal of Political Economy, Vol. 86, 1143-1158.

King, M.A. [1977], Public Policy and the Corporation, Chapman & Hall, London.

Stiglitz, J.E. [1983], "Some Aspects of the Taxation of Capital Gains", Journal of Public Economics, Vol. 21, 257-294.

Tintner, G. and Sengupta, J.K. [1972], Stochastic Economics, Academic Press, New York and London.

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