Supply disruptions and the allocation of emergency reserves

by Clas Bergström

The study considers a country that faces uncertain and intermittent disruptions of a single import. It focuses on the possibility of stockpiling the good to hedge against the adverse effects of interrupted supplies. The following normative questions are addressed: How much of the good should be stockpiled in light of the perceived threat of supply disruptions? How should inventories be used (priced) during a supply disruption? How much worse off is the country when it is subject to interrupted supplies?

The optimal allocation of emergency reserves is contrasted with the market allocation. Certain specific conditions have to be specified if the market economy is to be able to replicate the optimal stockpiling and stock withdrawal policies. We investigate the bias that arises in an actual market economy lacking a complete set of risk and future markets.

Obviously, an omniscient planner can do better than an actual market economy. But what about the ''real world'' planner? To make a fair test of the welfare properties of private stockpiling we restrict the planner to operate only on the markets that do exist. That is, we place the planner and the market in a symmetric position with respect to the transaction technology. When we discuss practical planning problems an alternative restriction is imposed on the planner. In dealing with the problem of allocating a government controlled strategic reserve we consider the limitations on the information available to the planner about the demand conditions for the good.

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The Economic Research Institute at the Stockholm School of Economics
Addres: Sveavägen 65, Box 6501, S-113 83 Stockholm, tel 08-736 01 20

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Stockholm in March, 1985

Clas Bergström

Contents

1. Introduction

1.1 THE PROBLEM

Consider a small country that faces uncertain and intermittent disruptions in the supply of a single import. For brevity, we will call the commodity oil. If the oil import is disrupted the demand for oil is reduced through changes in the production technology and by shifts in the mix of goods and services consumed. In a short-term perspective the possibilities for replacing oil with other types of energy, intermediate goods, capital and labor are quite limited. Also, changes in the composition of production and consumption towards products that require less of the imported oil to produce can only be expected to take place gradually.

There are several ways to hedge against the adjustment costs of unexpected oil supply disruptions. One way is to move away from the trade pattern dictated by short-run comparative advantage, towards a domestic production and consumption pattern that is optimal in the absence of trade. Of course, the justification for such a move towards self-sufficiency presupposes that there are impediments to adjustments when the oil supply is disrupted. The optimal degree of departure from the trade pattern that is optimal in the absence of supply uncertainty is determined by the trade-off between present gains from trade and expected future adjustment costs. Unless there is some kind of market failure, the optimal trade pattern can be arrived at in a decentralized manner.[1)]

In discussions of policies of self-sufficiency it is a standard practice to treat the presence of short-term inflexibilities in production and consumption as given. But the economy's ability to adjust to a disruption in the supply of oil is rather a choice variable. Thus, an alternative way to mitigate the costs of a supply disruption would, e.g., be to design production plants and consumption patterns in which there are some short-run substitution possibilities. The idea is to sacrifice some static efficiency to increase the flexibility and thereby reduce potential losses in the future.[2)]

We shall abstract in this study from these two ways of safeguarding the economy against supply disruptions. Instead, we shall focus on the possibility of stockpiling oil to hedge against the adverse effects of supply disruptions. If there is a disruption in the imports of oil, the country can run down its stock of emergency reserves to replace interrupted supplies.

From a normative standpoint we will address the following questions:

- How much oil should be stockpiled in light of the perceived threat of supply disruptions?

- How should the inventories be used (priced) during a supply disruption?

- How much worse off is the country by being subject to interrupted supplies?

These questions aim at ideal standards of the allocation of inventories against which the market economy can be judged. Once the optimal stockpiling and stock withdrawal policies have been derived, it is interesting to inquire whether they can be replicated in a market economy. Now, it is well known that the equivalence theorems of welfare economics can be extended to an uncertain intertemporal setting. Relying on one of the equivalence theorems, we can state that any Pareto-optimal allocation of the emergency reserves can be sustained as a perfectly competitive general equilibrium for some distribution of endowments. Unfortunately, some fairly specific conditions have to be fulfilled in order to ensure that the equivalence theorems hold.

One such condition is the existence of a complete set of risk and future markets. Now, the risk and future markets that do exist are not of the type or number required for Pareto optimality. Of course one can construct model economies with a set of markets less extensive than the one in the Arrow-Debreu economy, that nevertheless result in a Pareto-optimal allocation of resources. For instance, one can establish the optimality of rational expectations equilibria and the supportability of a Pareto optimum by a rational expectations equilibrium in an economy with homogeneous and immortal agents.[3)] With less restrictive assumptions there is no presumption that actual economies will achieve Pareto optimality. Mortal agents in an economy lacking future markets will probably not look far enough into the future. Also, in an economy with heterogeneous agents lacking risk markets, there is no presumption that the marginal rate of substitution between incomes in different states of nature is equal across individuals. With the optimal stockpiling and stock withdrawals as benchmarks we shall analyze the efficiency implication of the absence of a complete set of markets in economies facing uncertain and intermittent disruptions of oil imports. What is the direction of the bias in stockpiling and stock withdrawals in an actual market economy?

The problem of an intertemporal inefficient allocation of oil inventories can, in principle, be circumvented by a planner who calculates and implements an optimal allocation. However, one may doubt that a "real world" planner can do any better than the market. In the real world there is, of course, no such thing as a planner with unlimited knowledge. So, assuming an omniscient planner might be just as unappealing as postulating the existence of a complete set of markets. Also, the cost of and obstacles to market institutions that prevent the set of markets from being complete might also create difficulties for the planner. Hence, it will perhaps not be appropriate to regard the actual incomplete set of markets as prima facie evidence of inefficiency.

In order to investigate the welfare properties of private stockpiling, probably the only sensible approach is to introduce a weaker definition of optimality. We construct such a criterion of optimality by placing the planner and the market in a symmetric position with respect to the "transaction technology". This means that the planner is restricted to operate only on the markets that do exist. A "fair" test of the market economy can then be formulated: given the set of markets that do exist, will the market economy keep an optimal stockpile under free-trade periods and, will competitive depletion of the oil inventories during the embargoes be efficient?

In principle, a fair comparison between the market and the planner can also be made by placing them in a symmetric position with respect to "the information technology". However, there seems to be no obvious way to implement such a comparison.

In dealing with practical planning problems it is, of course, more useful to consider the limitations of the information available than to impose a restriction on the planner's transaction technology. Take for instance the problem of depleting a government controlled strategic petroleum reserve during a period when supply is disrupted. How much should be withdrawn from the reserves in any given time period? How should that quantity be allocated among the various consumers? In our discussion of these allocation problems we relax the unrealistic assumption of an omniscient planner. The additional source of uncertainty owing to the stockpile manager's imperfect information about the demand conditions is taken into account.

A proper analysis of the questions outlined above necessitates a well defined intertemporal stochastic statement of the oil-importing country's problem. To keep the analysis sufficiently tractable we shall reduce the problem to its minimally necessary features. In this way it will be possible to focus more sharply on some basic allocation problems.

The environment of the oil-importing country is characterized throughout the study by one of two regimes: "free trade" or "embargo". In a free-trade regime the price of oil is an exogenously determined world price. It is also assumed that the free-trade price is constant over time and not subject to uncertainty. When an embargo is imposed we assume that the oil imports are totally cut off. The price of oil during an embargo is determined by the country's demand curve for oil and the supply from oil inventories.

In the long-run perspective, we will assume that periods of free trade come and go, interrupted by embargoes, in an infinite sequence. In this perspective we assume that the duration of an embargo is either exponentially or geometrically distributed depending on whether time enters as a continuous or discrete variable. Likewise, the duration of a free-trade regime is also assumed to be exponentially or geometrically distributed. These assumptions imply that the stochastic process generating the sequence of regimes is a two-state stationary Markov process. We will also consider models that can be characterized by a 2-period setting: In the first period the country is in a free-trade regime (an embargo regime) while there might be an embargo (a free-trade regime) with a certain probability in the second period.

Throughout the study we will regard the problem solely from the point of view of a single oil-importing country. The probability law generating shifts in regimes is regarded as given.[4)] The interaction between the oil-exporting and the oil-importing countries and the possibility of joint stockpiling and stock withdrawal policies among the oil-importing countries is not considered.

A complete study of an oil supply disruption would follow the reactions of the economy over the period of time stretching from the commencement of the supply disruption to the time a new equilibrium is reached. Such a study would include a picture of the mutual dependencies among the various sectors of the economy. The inventories of oil as well as of other inputs and final goods would also be specified. In addition, the study would include a description of the possibilities for substitution in the production system and in consumption for the entire period of supply disruption. For obvious reasons, such requirements of completeness would lead to an extremely complex model.

To be able to focus more sharply on the stochastic and dynamic aspects of supply disruptions we will neglect a detailed modelling of the production side of the economy and the interdependence among its various sectors. To economize on concepts and notation we will, except in the final chapter, assume that the country consumes only two commodities, the imported oil and a domestically produced perishable background good. We make the simplest possible assumption about the production of the background commodity: a constant flow is made available at each instant of time. A further simplification is that we do not distinguish between a short-run and a long-run demand curve for oil. It would certainly have been more realistic to treat oil as an input in the production of a consumption good instead of treating oil as a consumption good. An obvious extension would be to introduce a production side with substitution possibilities between oil and other factors of production.

A further simplification is that the models we shall formulate can in no way capture the secondary effects of an adverse supply shock on the rate of inflation and employment due to wage and price rigidities. These effects of an oil supply disruption might also provide motives for policy interventions. One might argue that the macroeconomic implications of supply shocks bring about a divergence between the expected private returns to stockpiling and social returns to holding emergency reserves. This provides an argument for government stockpiling or, alternatively, for subsidizing private stockpiling.[5)]

Another argument for intervention that we shall also abstract from can be based on the possibility that private investors' incentives to stockpile oil are somehow distorted. For example, one might fear that private speculators will not store enough oil since they anticipate that the government would impose profit taxes of price controls, should the supply be disrupted. If it is impossible for the government to make credible a pledge that it will not intervene in this way, an argument (political failure) can be made for public storage or for subsidizing private storage.[6)]

Objections to our characterization of the economy and its environment can be made. The abstractions and simplifications we introduce can, of course, be justified only by the simplifications they offer in answering the questions we have posed. It should be emphasized that the present study should be considered primarily as an analysis of methods. The study provides a starting point for an analysis of the problems we have posed. Its main purpose is to discuss some aspects of the allocation of inventories in a stochastic world.

1.2 SUMMARY

Chapter 2: Embargo Threats and the Management of Emergency Reserves

The model in Chapter 2 describes the problem of optimal reserve management for an oil importing country. Periods of free trade are assumed to come and go, interrupted by embargoes, in an infinite sequence. Using arguments based on dynamic programming we analyze optimal stockpiling in free trade and optimal depletion of reserves during embargoes. We also derive expressions for the cost to the country of being subject to embargo threats.

In Section 2.2 we set up the model. The country consumes the imported oil and a domestically produced, perishable background good. We take it that the country's utility is additively separable in time and that utility is linear in the background commodity but a concave function of oil. We also assume that the marginal utility of oil approaches infinity as the consumption of oil goes to zero. The duration of the two regimes - embargo and free trade - are assumed to be exponentially distributed, possibly with different parameters.

We show that the optimal policy under each of the two regimes must satisfy a particular functional equation and the overall problem of optimal reserve management involves the simultaneous solution of the two equations of this system. We find it instructive to analyze one equation at a time before we pull them together.

In Section 2.3 we analyze optimal depletion of an oil reserve during an embargo. The functional equation for the embargo problem defines the maximum expected present value to be derived from an optimal depletion policy with any given reserve stockpiled during the preceding free-trade regime. Here, it is taken that an optimal policy will be pursued in the following free-trade period. The analysis of the optimal depletion of emergency reserves during the embargo reveals that the reserve should be priced so that the expected rate of price increase equals the rate of interest. This is, of course, a special case of the Hotelling (1931) principle. Our modified Hotelling formula is, with proper reinterpretations, identical to the one obtained by Dasgupta and Stiglitz (1981).[7)]

As soon as the embargo regime has expired and a regime of free trade has begun the country will start to build up its emergency reserves again. We turn to the second dynamic programming equation describing the problem of optimal stockpiling in free trade in Section 2.4. This equation defines the maximum expected welfare of optimal behavior given the emergency reserves left over from the preceding embargo. We assume that an optimal depletion policy is followed in the subsequent embargo regime. The condition for optimal storage derived is simple. It says that the cost of storing one additional unit in free trade should equal the expected discounted value of that unit, should an embargo be imposed. It may seem intuitively reasonable to set the cost of storing one additional unit in free trade equal to the world market price. However, unless the capital market is perfect, this is not so. Without a capital market the stock cannot be adjusted instantaneously, and it will take some time to build up the stockpile to the desired level. During this interval the marginal valuation of oil will be greater than the world market price. We use phase diagrams to analyze the optimal paths in free trade when there is no capital market.

The two functional equations represent the planner's problem in the two regimes, respectively. Their simultaneous solution for the two maximum value functions yields the overall solution to the optimal reserve management problem. In general this problem of dynamic programming with a switching of regimes is difficult to solve.

If the country has access to a perfect capital market, a full solution (i.e., a simultaneous solution of the system of functional equations) can be obtained. This case is discussed in Section 2.5. Access to a perfect capital market together with our assumptions of a constant marginal utility of the background good and the assumption of no monopsony power makes it optimal to adjust the stock instantaneously at the beginning of the period of free trade. Then we will be in a steady state during the whole free-trade regime and the marginal valuation of oil in stock during the free-trade regime will be equal to the world market price.

The assumption of a perfect capital market implies that the system of functional equations becomes recursive and a closed form solution can be obtained. We can now express the modified Hotelling formula explicitly. With a proper interpretation of the parameters in our model, the modified Hotelling formula looks like the equation describing the optimal depletion of a natural resource with a constant marginal extraction cost. This simplifies the storage condition and permits us to derive an explicit expression for the optimal storage in free trade.

The assumption of a perfect capital market also allows us to derive explicit formulae for the two maximum value functions. In Section 2.6 we make use of these expressions to evaluate the welfare loss to an oil-importing country of being subject to embargo threats. We also gain some insight into the insurance-theoretic properties of trade disruptions. We show that the model can be solved for the maximum premium above the world market price that a country would be willing to pay for guaranteed safe deliveries of oil or of a close substitute for oil.

Chapter 3. A Comparative-Static Analysis of the Embargo Model, Some Numerical Examples and the Effects of Uncertainty on Stock Withdrawals and Stockpiling

To gain further insight into the economic interpretation of the embargo model developed in Chapter 2, we not only look at the comparative statics qualitatively, but also analyze the model numerically to illustrate the orders of magnitude that emerge. The purpose of the numerical simulations is modest. They should only be regarded as a means to provide some intuitive feeling for the dynamics and stochastics of supply disruptions. The numerical implementation of the model also makes it possible to quantify the bias in stockpiling and stock withdrawals that arises when the random date at which an embargo will end is replaced by its expected value. This enables us to contrast the embargo model with some simpler 2-period stockpiling models.

In Section 3.2 we make use of the assumption of a perfect capital market and adapt the model for numerical analysis.

In Section 3.3 we analyze optimal depletion of the emergency reserves when the size of the reserves (not necessarily at the optimal level) is given. The qualitative comparative static results can be summarized as follows: The jump in the price that occurs when the economy enters the embargo regime is smaller:

- the larger the initial stock of reserves;
- the higher the probability that the embargo will end in the next period;
- the higher the (absolute) elasticity of demand;
- the higher the rate of interest;
- the lower the world market price in a free-trade regime.

Although the above comparative static results (and those for the optimal stock to be presented below) are in accordance with what would be expected, in some cases they are the net result of several opposing forces. The Appendix to Chapter 3 gives a complete derivation of the qualitative comparative statics.

To illustrate the orders of magnitude that emerge from the comparative statics we analyze numerically the problem during the embargo regime. A time period is assumed to be one week long. We form a reference case based on the following parameter values:

the probability that the embargo will expire next week	= 0.042*)
the absolute value of the elasticity of demand	= 0.3
the annual rate of interest	= 6 %
the relative price of oil in free trade	= 1
the initial emergency reserves (in terms of number of weeks of free-trade consumption)	= 75 weeks

*) Since we assume that the duration of the embargo is exponentially distributed, this particular parameter value implies that the expected length of the embargo is $(0.042)^{-1} \approx 24$ weeks.

With these values of the parameters there is a rise in the price of oil by 90 per cent when the economy enters an embargo regime. The price jump reduces the initial rate of consumption by 18 per cent. The price follows a modified Hotelling path. The price rises in such a way that the expected rate of price increase equals the risk-free rate of interest. As the embargo unfolds, the capital loss, should the embargo terminate, increases. The implication of this is that the price rises during the embargo regime at a variable rate, initially at a weekly rate of around 2 per cent. As the spot price increases, the rate of price increase approaches a maximum weekly rate of 4.3 per cent.

This price path implies a rather slow depletion of the emergency reserves. On reaching the expected duration of the embargo (that is, after 6 months) the cumulative resource use amounts to only 24 per cent of the initial stock. The fraction of reserves remaining after 12 months (i.e. twice the expected length of the embargo) is as high as 0.53. Since there is, with an exponentially distributed length of embargoes, no upper bound on the possible duration of the embargo, and since the marginal utility of oil approaches infinity as the consumption of oil goes to zero, this conservative depletion policy makes good intuitive sense.

The initial price is quite sensitive to expectations about the duration of the embargo. The calculations show that the initial price increases at an increasing rate with respect to the expected duration of the embargo. The numerical examples also reveal that the initial price is very sensitive to changes in the elasticity of demand. Also, the size of the emergency reserves has a substantial bearing on the jump in the spot price that occurs in the transition from a free-trade regime to an embargo. The initial price in the embargo regime is decreasing in the size of reserves at a diminishing rate.

In Section 3.4 we analyze the stockpiling problem. The result of the comparative static analysis can be summarized in the following way: The optimal size of the emergency reserve is larger:

- the larger the probability that an embargo will be imposed in the next period;
- the smaller the probability that the embargo will end, given that it has been imposed;
- the lower the absolute value of the elasticity of demand;

- the lower the rate of interest;
- the lower the world market price of oil.

Our numerical analysis here is based on the values of parameters used above, except for the initial emergency reserves. The optimal initial level of the emergency reserves is now to be found. In addition to the above parameters we also assume that the probability that the economy will be in an embargo regime, in the next week, given that there is free trade now, is equal to 0.004. This probability implies that the expected duration of a free-trade regime is approximately 5 years.

With these parameter values, the optimal size of the emergency reserve is 97, that is, the optimal stock is 97 weeks of normal consumption. The impact on the optimal reserves of changes in the transition probability of leaving free trade are small in comparison with the impact of changes in the probability of leaving the embargo regime. For example, if the probability that the embargo will expire next week, given that it has not already expired is halved (that is, if the expected duration of the embargo has doubled), the optimal reserve is almost twice as large. If, on the other hand, the probability that the economy will be in an embargo regime next week, given that there is no embargo going on now is halved, the decrease in the optimal reserves is quite moderate. It is reduced from 96 to 80.

The consequence of a decrease in the absolute elasticity of demand is quite a substantial increase in the optimal size of the stockpile. For instance, if the elasticity is as low as 0.1, the optimal reserves are between 3.7 and 6.7 times larger (in the interval of an expected duration of the free-trade ranging from 1 year to 15 years) than what optimal inventory behavior would require if the elasticity were as high as 0.6.

In Section 3.5 we inquire whether the expectation of the random duration of an embargo is a certainty equivalent for that variable. The biases in stock withdrawals and stockpiling that result from acting as if the uncertain duration is equal to its expectation are large. Acting during the embargo regime as if there exists a certainty equivalence involves quite a substantial shift downward of the initial price for any given size of the emergency reserves and thereby a faster initial rate of stock withdrawals. In the free-trade regime, the bias in stockpiling that arises when the random date at which an embargo will end is replaced by its expected value may be large. For the parameter values of the reference case, the amount stockpiled is between one-fourth and one-fifth of the optimal amount.

Chapter 4. Stockpiling and Stock Withdrawals in a Market Economy

The results on optimal stockpiling and depletion of emergency reserves in Chapters 2 and 3 are derived in the context of a planning model. In Chapter 4 we ask whether a Pareto-optimal allocation of emergency reserves can be achieved through a decentralized market system. We focus the discussion on one particular assumption that is critical to the validity of the welfare theorems. The assumption is that there exists a complete set of risk and future markets.

In Section 4.2 we study stockpiling and stock withdrawals in a competitive sequence economy. We take it that the inventory holders have myopic conditional perfect foresight so that expectations about the price next period, given the regime that prevails at that time, are realized. We also assume that the inventory holders are risk neutral. Accordingly, risk markets are redundant and we can focus on the implications of an incomplete set of future markets. We find that a market economy with such a formation of expectations facing embargo threats exhibits a kind of long-run inefficiency similar to that observed in capital theory and in the theory of natural resources.[8)]

Although the market economy behaves in a locally efficient manner during both embargo regimes and free-trade periods, the allocation of inventories is intertemporally inefficient. In our framework with intermittent interruptions of oil imports it is not only the indeterminacy of the initial price during embargo regimes that leads to intertemporal inefficiency. Also, there is no mechanism endogenous to the model to ensure that the competitive economy will keep an optimal stockpile during free-trade periods.

In Section 4.3 we abstract from the problems associated with the inability of market economies to see infinitely far into the future. Instead, we focus on the implications of an incomplete set of risk markets. In a 2-period setting we pose the following questions: What are the implications of inventory holders having aversion towards risk? Should the government have a different attitude towards risk?

If oil in storage is the only risky asset in the economy, we find, in accordance with what would be expected, that risk-averse inventory holders are more cautious than those who are risk neutral. In order to reduce the likelihood of large capital losses from holding inventories, should the embargo expire, they give up some expected profit during embargo regimes by increasing the rate of stock withdrawals. Similarly, they demand less (stockpile smaller amounts) of the risky asset in free trade. The inventory holders raise the risk-free discount rate in their adjustment to risk.

Using a welfare criterion that is identical to the one employed in the previous chapters (i.e., one in which utility is linear in the background good but a concave function of oil) we contrast the competitive allocation with the optimal allocation. We find that if stockpiling and stock withdrawals are undertaken by agents who are risk averse, the market equilibrium implies too fast a depletion of the reserves during embargoes and too small an emergency reserve in free trade. However, though frequently seen, this kind of comparison is misleading. The particular welfare function employed implies price risk neutrality for the consumers. Moreover, to justify it we have to assume that all consumers are alike. So, if the inventories are held by firms owned by the identical individuals, the owners would impose on these firms their own (price-risk-neutral) utility functions. Or, if the inventories are held directly by the identical individuals, they would, of course, make stockholding decisions consistent with their preferences. So we find that risk-averse inventory holders are incompatible with the particular welfare function employed for the analysis. Hence, the argument for competitive inefficiency derived is inconclusive.

We next consider a theorem in Arrow and Lind (1970). A straight-forward application of the theorem to our problem results in the following conclusion: the government should either undertake the stockpiling itself, or it should subsidize private stockpiling. In this way the risk is transferred to the public at large, and the total cost of risk-bearing becomes negligible. Or, to put it differently, the government has a superior position with respect to risk and it can use the risk-free interest rate in evaluating investments in oil inventories. Two assumptions are essential for this result. First, the risk markets that do exist are incomplete to a substantial degree. Second, the returns to holding inventories are uncorrelated with other sources of income. Within the framework of the Capital Asset Pricing Model (CAPM) we discuss these two aspects.

Take for instance the consumption CAPM. Its basic result tells us that the risk premium on oil in storage, that is, the difference between the expected rate of appreciation of oil inventories and the risk-free rate, has a sign that is the opposite of the covariance between the rate of capital gain on oil and the marginal utility of income. If the covariance is zero, private agents use the risk-free rate of interest, just as Arrow and Lind suggest that the government should do, in evaluating investments in oil inventories. Now, the rate of return on oil in storage is presumably highly negatively correlated with the return to investments in general and the national income. Consequently, the risk premium on oil in storage is negative. This makes sense, since oil in storage is very attractive to risk-averse individuals because it provides not only diversification opportunities, but also insurance against random fluctuations of income. So the conclusion that risk-averse inventory holders raise the discount rate to adjust for risk taken on when oil in storage is the single risky asset must be recast.

In the hypothetical world of CAPM one can conclude that risk should not be treated differently for public as opposed to private investments. Any stockpiling project (private or public) should be evaluated using the risk-free discount rate adjusted for the (negative) risk allowance. The adjustment for risk in evaluating a public project is not in contrast to the Arrow-Lind theorem which is based on uncorrelated returns. What is not in the spirit of the Arrow-Lind theorem is the possibility on actual risk markets to diversify away that portion of risk which is uncorrelated with the rest of the economy.

The CAPM belongs to a set of alternative structures of the capital market that are less extensive than the Arrow-Debreu market, but that result in a Pareto-optimal allocation of risk among investors. However their common assumptions, perhaps especially those on preferences, are quite restrictive. Moreover, one essential feature of the CAPM is that it is only a partial equilibrium model of the capital markets. The quantities of the assets are given and neither saving decisions of consumers nor investment decisions of firms are considered in the model. So, in the hypothetical world of CAPM, the original problems of incomplete markets, of suboptimal allocation of risk and of potential misallocation in the form of underinvestment dealt with in Arrow and Lind are assumed away.

In fact, some strong assumptions have to be satisfied to ensure that a market economy lacking a complete set of risk markets will allocate, e.g., oil inventories Pareto optimally. The natural question to ask now is whether governmental intervention might improve welfare. To make a fair comparison between the planner and the market, we use in Section 4.4 the notion of constrained Pareto efficiency. The question we raise is the following: given the set of markets that does exist, will the market economy be efficient?

As a vehicle for discussion we introduce storage into some existing models.[9)] Restricting the planner to operating only on the markets that do exist, we discuss the welfare properties of private stockpiling in two different types of market economies, one without any risk markets whatsoever and one with a stock market.

The results are quite negative. There is no presumption that even a rational expectations market equilibrium will lead to a constrained Pareto-optimal amount of storage. Competitive storage will lead to constrained Pareto optimality only under some fairly restrictive assumptions on the preferences. These conditions are in fact the same as those required for risk market redundancy. Consequently, government intervention might improve welfare. But in order to be able to proceed with that discussion one would like to know the direction of the bias and orders of magnitude involved in the competitive misallocation. Also, the information requirement has to be considered. This requirement seems to be very demanding. Further research is necessary before we will be able to recommend the proper, if any, governmental intervention.

Chapter 5. Demand Uncertainty and the Allocation of Emergency Reserves - - A Cake-Eating Problem With Unknown Appetite and Horizon

The basic approach in Section 4.4 to a fair comparison between the "real world" planner and an actual market economy is to constrain the planner to operate only on the markets that do exist. In Chapter 5 we put an alternative constraint on the planner. The constraint is a restriction on the information available at the planning board. However, we do not make any comparison of the kind we do in Section 4.4. Instead our aim is to extend the planning models of Chapters 2 and 3 to include imperfect knowledge at the planning board of the demand conditions for oil. In this way we adapt the "theoretical" planning models to a more realistic planning environment.

Throughout the analysis we focus solely on the situation in which the imports of oil have been completely cut off. Given the reserves available, which we assume are completely owned or controlled by the government, the task of the stockpile manager is to deplete this stock in an optimal fashion. The problem dealt with here differs from the previous planning models in two fundamental ways. First, there is a difference between controlling the rate of stock withdrawals by quantitative restrictions and through price incentives. Second, the open-loop control employed in the previous planning models is inapplicable. Instead, we must rely on a model for sequential decision making in which the depletion policy is selected sequentially after observing the result of the previous policy.

In Section 5.2 we set up the formal structure of the model. Incomplete knowledge of the demand conditions is introduced by adding a stochastic term to the demand function. For simplicity we assume that the error is distributed independently and identically over time. As in the previous planning problems we assume that the duration of an embargo is unknown. As we let time enter as a discrete variable we take it that the length of an embargo is geometrically distributed. In all other respects the structure of the models of Chapters 2 and 3 remains intact.

Using dynamic programming arguments we convert the stochastic dynamic problem into the problem of determining the solution to a particular stochastic functional equation. Manipulating this equation yields a one-period maximization problem. This problem represents the planner's problem in any given subperiod of the embargo. It can be stated as the problem of choosing a rate of stock withdrawal so as to maximize the discounted expected difference between the utility in the current period and the opportunity cost. The opportunity cost (or the intertemporal cost) is equal to the expected benefits forgone in later periods (given that an optimal policy is pursued from the next period onward) owing to a stock release in the current period.

In Section 5.3 we characterize the solution of this problem. This provides some insights into the characteristics of an optimal decision within a given time period of the embargo. In particular, we find that when the demand conditions are not perfectly known by the planner there will be a difference between using quantitative controls and price incentives to regulate the rate of depletion of the emergency reserves.[10)] This is in contrast to a basic theme of microeconomics under certainty, viz. the equivalence of planning by quantities and planning through prices. It is also quite distinct from what we can infer from the analysis in Chapters 2 and 3. In implementing the optimal stock release the stockpile manager in Chapters 2 and 3 can equally well adhere to a quantity scheme for direct control as to an allocation attained by announcing an appropriate price path.

However, we find the following question crucial when the stockpile manager has only imperfect knowledge of the demand conditions: Should he rely on market allocation by announcing an appropriate price and accommodating the ex ante unknown demand from the reserves, or should he decide in advance how much of the reserves is to be released and allow the price (marginal valuation) of oil to be (ex ante) unknown? In answering this question, we make two different assumptions about the quantity control: either the stock withdrawal is efficiently allocated through a competitive market for rationing coupons, or inefficiently allocated by means of an allotment scheme without transferable rations.

In the former case, we find that the difference between the total benefits of consumption and the intertemporal cost of stock withdrawals is largest on average if one chooses the quantity control when the demand curve is relatively flat and the slope of the marginal opportunity cost is large. Naturally, the conclusion is the opposite when the marginal opportunity cost has a smaller slope than the absolute slope of the demand curve. In this case, it will be more efficient to set a price. When the absolute slopes of the two curves are equal, both methods for controlling the stock withdrawal are equally good. To interpret the above results, it is instructive to look at the benefit function and the intertemporal cost function. Due to the strict convexity of the cost function, the opportunity cost of releasing a quantity with certainty is less than the expected opportunity cost achieved when a price is announced. Also, the more convex the opportunity cost function, that is the steeper the marginal cost curve, the larger is the ceteris paribus welfare forgone by using the price option. Now, since the gross benefit function is strictly concave, one might suspect that a similar bias against the price option is felt on the benefit side of the comparison. However, we find that the negative effect of price induced variations in demand (due to the concavity of the gross benefit function) is counterbalanced by the efficiency gain of letting demand move in the appropriate direction relative to the actual valuation of oil.

While Section 5.3 focuses on a welfare comparison of a price allocation and a rationing scheme with transferable ration coupons, in Section 5.4 we investigate the relative desirability of the two control modes when the stock withdrawal decided in advance under the quantity mode is inefficiently allocated. One difference between this case and the former is found on the cost side of the comparison. Whenever the covariance between the demand curves of (any pair of) consumers i and j is negative (positive) the variations in individual demands induced by prices are in the opposite (same) direction, decreasing (increasing) the variance of total demand as compared with the situation when the covariance is zero. This means that an increase (reduction) in the competitiveness of the price option is recorded on the cost side of the comparison. On the benefit side, we have a similar effect, that is, a decrease (increase) in the variance of total demand will increase (decrease) the net advantage of charging a price over the option of regulating the stock withdrawals quantitatively. In the case where consumer's demand functions are uncorrelated we find that an increase in the number of consumers leads to an increase in the advantage of the price option. The reason behind this result is, of course, that the larger the number of consumers, the larger are the losses from inconvertible rationing. However, the trouble with the price option we found in Section 5.3 remains. That is, although efficient allocation among the consumers is guaranteed, the total demand (stock withdrawal) might settle down at an undesirable level. So, at least in principle, even if the stock withdrawal decided in advance under the quantity mode is inefficiently allocated, the cost of a too large (or too small) stock withdrawal induced by the price might be even larger.

Chapter 6. Centralized Quantitative Allocation

In Chapter 6 we discuss the losses of efficiency in a rationing program with nontransferable rations. The centralized quantitative allocation that is presently planned for the allocation of oil within the Swedish production system during an oil supply disruption is a typical example of such a rationing system. In Section 6.2 we outline the planning model that is to be used by the rationing authority to determine a program of rationing quotas for the allocation of oil among the various production units. The model is basically an input-output model of the conventional type, and the objective is to maximize private consumption with given restrictions on, e.g., other demand components, on the composition of private consumption and resource availabilities. Special treatment is given to short-term adaptation of oil coefficients and possibilities of using inventories of semimanufactures and final goods.

In Section 6.3 we apply the planning model. Using the available information about production technology, patterns of interindustry deliveries, final demand conditions and resource availabilities, we approximate an efficient allocation of oil with an aggregated (at a sector level of a subdivision into 88 sectors) rationing program for various shortfalls of oil.

The efficient allocation program for oil implies large differences in the quotas across sectors. When the cutback in oil supplies is 15%, most of the sectors involved in the production of consumer goods receive quotas that are higher than average. The oil-intensive process industries are hit relatively hard by the rationing. However, there are many exceptions to the simple rule that cutbacks be directed to where they - per cubic meter of oil - have least effect on the production in a particular plant. Consideration must also be taken of the differences among branches with regard to short-term possibilities for adaptation and the inventory situation as well as of the mutual dependencies among the various sectors. For instance, oil-intensive sectors whose products are in stock only in small quantities receive higher quotas than those that can meet demand by running down their inventories.

This program of differentiated quotas is, of course, a coarse approximation to an efficient allocation. For instance, the specification of various types of adjustments is very much simplified. The computations indicate that the consequences of an oil supply disruption are sensitive to the composition of domestic demand and foreign trade. Also, the possibilities for firms to adjust their oil coefficients in the short term and the possibilities of reducing inventories of finished goods and semimanufactures are decisive for the effects of a disruption. Given the margin of error that exists in the estimates of these data, the risk of efficiency losses is high in systems with nontransferable rations.

In addition, even without any misspecification whatsoever in the aggregated data, there remain efficiency losses due to the aggregation of goods and production processes. The problem arises when there are differences among the individual processes that make up the aggregated process with regard to production technology and product assortment. The program of differentiated quotas described above only includes the consequences of a uniform cutback in the various plants that make up a branch of industry. This means that there are possibilities for substitutions in the production function for the aggregated process that are not taken into account. In some cases a more efficient adjustment would be to close down those plants with the greatest oil requirement per unit of production.

It is, of course, very difficult to quantify the losses of efficiency that arise in a simplified, aggregated rationing program. For instance, we cannot, owing to the lack of disaggregated data, measure the "transfer gains" that would be achieved in an efficient allocation attained by reallocating oil within the branches. In <u>Section 6.4</u> we try to estimate the orders of magnitude of these losses by looking at programs that are not as differentiated as the planning model permits. The idea is that these losses are of the same orders of magnitude as the gains that would be accomplished in a rationing system that is allowed to use more (decentralized) information (e.g., a system of transferable ration coupons). Our estimates show that the losses of efficiency increase considerably if one does not take advantage of the possibilities for substitution within the branches of industry.

A rationing system with purely proportional quotas turns out to cause a total collapse in the pattern of interindustry deliveries. An exception must be made for the plastics and synthetic fiber industry which has very special conditions both regarding oil requirement and pattern of deliveries. The simulations indicate that the effect on the production possibilities of the economy will be about three times as great if undifferentiated quotas (except for plastics and synthetic fibers) are used, than if fully differentiated quotas are used. So, the advantages of an undifferentiated system of quotas - less complex administration and small requirements for information - hardly outweigh the losses that are indicated here. A system of dispensations - in which exceptions from a system of otherwise undifferentiated rations are made for those sectors for which the shadow price is greatest - leads to a considerably more efficient allocation of resources. In this way, priority is given to branches in which an additional quantity of oil leads to the greatest increases in production for the system as a whole. However, even so, there still remain considerable losses in efficiency owing to the fact that full consideration has not been taken of the differences among the branches with regard to production technology. In terms of loss of production, the effect of the embargo will be almost twice as great in a case of 10 dispensations than in the rationing program that is "fully" differentiated at the 88-sector level. However, even in the "fully" differentiated rationing program there remain efficiency losses since oil is allocated proportionally within the 88 sectors.

To sum up: The misallocation of supplies during an embargo can be very costly. Once it has been decided that rationing will be used during an embargo it is important to decentralize the decisions about the use of oil. This can be accomplished through some form of market system (e.g., transferable rations).

FOOTNOTES

1) A common argument for policy intervention in this context has its origin in the assumption that a country subject to disturbances in the supply of importables faces a situation of endogenous uncertainty. By this, it is meant that the probability of, e.g., an embargo being imposed is assumed to be an increasing function of the amount imported by the country. The optimal policy in case there are no adjustment costs is a trade tariff. If there are impediments to adjustment on the production side, Bhagwati and Srinivasan (1976) find that the appropriate policy will involve a tariff plus a production tax-cum-subsidy. A related analysis is that of Mayer (1977). He assumes a small country that is subject to *exogenous* interruptions in the supply of importables. In case adjustment impediments occur on the production side he concludes that free trade is suboptimal if individual producers disregard the implications of an embargo threat. More discussions of self-sufficiency policies are to be found in Tolley and Wilman (1977) and Bohi and Montgomery (1982). The above studies are either 2-period models or continuous time models in which an embargo can be imposed at any time, but once imposed, will last forever. Recently, Cheng (1985) has incorporated recurrent embargoes with uncertain timing and duration into a model which focuses on the pattern of production as a response to embargo threats. His model has a formal structure similar to the model in Chapter 2 in this book. Cheng's model is complementary to our model which considers stockpiling but assumes a fixed pattern of production.

2) For a discussion of the trade-off between static efficiency and *ex post* flexibility of input proportions, see Bergman and Mäler (1981). In this paper there are also numerical estimates of the economic significance of having plants in which there are *ex post* substitution possibilities. For a portfolio approach to the problem of *ex ante* plant design, see Cheng (1983).

3) See, e.g., Prescott and Mehra's (1980) introduction of capital accumulation into the asset pricing model of Lucas (1978).

4) It is conceivable that the parameters in the stochastic process that generates shifts in the regimes are decision variables of the oil-exporting countries. By a suitable choice of these parameters the oil-exporting countries, acting under collusion, could induce the importers to store oil and thereby purchase more oil during free-trade periods than they would otherwise do. On the other hand the exporters would lose revenues during embargo periods. Whether a threat strategy, i.e., a policy with trade disruptions according to a particular probability law can result in higher profits for the oil-exporters than an ordinary policy with monopoly pricing with certain deliveries is an - as yet - unresolved problem. Hence, whether a threat strategy is a credible threat, i.e., whether that strategy can be a profitable activity for the oil-exporters, remains to be shown.

5) There has been a lot of concern in the literature with the macroeconomic implications for the welfare and policy of a country subject to oil supply disruptions. See, e.g., Nordhaus (1980), Solow (1980) and Gilbert and Mork (1982). Several writers have suggested stockpiling to mitigate the "macroeconomic cost of oil". See, for instance, Tolley and Wilman (1977), Hogan (1981), Bohi and Montgomery (1982) and Mork (1982).

6) This issue has been taken up by Nichols and Zeckhauser (1977) and more recently by Bohi and Montgomery (1982) and Wright and Williams (1982).

7) In fact, the embargo model belongs to the wider class of expectations models of asset prices. Leroy (1982) surveys various expectation models of asset prices. Besides the one related to natural resources, he discusses the expectation hypothesis of the term structure of interest rates, martingale models of stock prices and expectation theories of commodity markets. Leroy identifies restriction on preferences under which various expectations models of asset prices are valid. In each case, the required restriction was shown to be related to the assumption of risk neutrality.

8) See, e.g., Malinvaud (1953) and Dasgupta and Heal (1979).

9) The models we apply to our problem are those in Newbery and Stiglitz (1982) and Stiglitz (1982).

10) The basic analysis here is inspired by the model devised by Weitzman (1974).

REFERENCES

Arrow, Kenneth J., and Lind, Robert C. "Uncertainty and the Evaluation of Public Investment Decisions." American Economic Review 60 (June 1970): 364-378.

Bergman, Lars, and Mäler, Karl-Göran. "The Efficiency-Flexibility Trade-Off and the Cost of Unexpected Oil Price Increases." Scandinavian Journal of Economics 83 (April 1981):113-128.

Bhagwati, Jagdish N., and Srinavasan, T.N. "Optimal Trade Policy and Compensation Under Endogenous Uncertainty: The Phenomenon of Market Disruption." Journal of International Economics 6 (November 1976):317-336.

Bohi, Douglas R., and Montgomery, David W. Oil Prices, Energy Security and Import Policy. Baltimore: Johns Hopkins University Press for Resources for the Future, 1982.

Cheng, Leonard. "Ex Ante Plant Design, Portfolio Theory, and Uncertain Terms of Trade." Journal of International Economics 14 (February 1983):25-51.

Cheng, Leonard. "Intermittent Trade Disruptions and Optimal Production." Research Paper. Gainesville: University of Florida, January 1985.

Dasgupta, Partha S., and Heal, Geoffrey M. Economic Theory and Exhaustible Resources. Cambridge: Cambridge University Press, 1979.

Dasgupta, Partha S., and Stiglitz, Joseph E. "Resource Depletion under Technological Uncertainty." Econometrica 49 (January 1981):85-104.

Gilbert, Richard J., and Mork, Knut A. "Coping with Oil Supply Disruptions. In: Energy and Vulnerability, Ed.; J. Plummer. Cambridge, Massachusetts: Ballinger, 1982.

Hogan, William W. Import Management and Oil Emergencies. In: Energy and Security. Eds.; D.A. Desse and I.S. Nye. Cambridge, Massachusetts: Ballinger, 1981.

Hotelling, Harold. "The Economics of Exhaustible Resources. "Journal of Political Economy 39 (April 1931):137-175.

Leroy, Stephen F. "Expectations Model of Asset Prices: A Survey of Theory." Journal of Finance 37 (March 1982):185-217.

Lucas, Robert E., Jr. "Asset Prices in an Exchange Economy." Econometrica 46 (November 1978):1429-1445.

Malinvaud, Edmund. "Capital Accumulation and Efficient Allocation of Resources." Econometrica 21 (1953):233-268.

Mayer, Wofgang. "The National Defense Tariff Argument Reconsidered." Journal of International Economics 7 (November 1977):363-377.

Mork, Knut A. The Economic Cost of Oil Supply Disruptions. In: Energy and Vulnerability. Ed.; J. Plummer. Cambridge, Massachusetts: Ballinger, 1982.

Newbery, David M.G., and Stiglitz, Joseph E. "The Choice of Techniques and the Optimality of Market Equilibrium with Rational Expectations." Journal of Political Economy 90 (April 1982):223-246.

Nichols, Albert L., and Zeckhauser, Richard J. "Stockpiling Strategies and Cartel Prices." Bell Journal of Economics 8 (Spring 1977):66-96.

Nordhaus, William D. "Oil and Economic Performance in Industrial Countries." Brookings Papers on Economic Activity 2 (1980):341-400.

Prescott, Edward C., and Mehra, Rajnish. "Recursive Competitive Equilibrium: The Case of Homogeneous Households." Econometrica 48 (September 1980):1365--1379.

Solow, Robert M. What to Do (Macroeconomically) When the OPEC Comes? In: Rational Expectations and Economic Policy. Ed.; S. Fisher. Chicago: University of Chicago Press, 1980.

Stiglitz, Joseph E. "The Inefficiency of the Stockmarket Equilibrium." Review of Economic Studies 49 (April 1982):241-261.

Tolley, George S., and Wilman, John D. "The Foreign Dependence Question." Journal of Political Economics 82 (April 1977):323-347.

Weitzman, Martin L. "Prices vs. Quantities." Review of Economic Studies 41 (October 1974):477-491.

Wright, Brian D., and Williams, Jeffrey C. "The Roles of Public and Private Storage in Managing Oil Import Disruptions." Bell Journal of Economics (Autumn 1982):341-353.

2. Embargo Threats and the Management of Emergency Reserves*

2.1 INTRODUCTION

We consider a country that in normal times is able to import a steady flow of a commodity at a given world market price. For brevity we will call this commodity "oil". Now, the imports of oil are subject to possible curtailments of deliveries from foreign countries. To hedge against the costs of such unexpected interruptions, some of the imported oil is stockpiled. In case of an embargo, the country can run down its stocks of emergency reserves and thereby mitigate the impact of the supply disruption. From the point of view of the importing country, there are two kinds of uncertainty making the problem of optimal management of emergency reserves nontrivial. First, the date at which an embargo will be imposed is not known in advance but has to be treated as a random variable. Second, given that an embargo has been imposed, the date at which it will end is uncertain.

Parts of the problem of trade disruptions have been analyzed by, for example, Nordhaus (1974); Kuenne, Blankenship, and McCoy (1979); and in a different context by Arad and Hillman (1979). These papers are characterized by a 2-period setting: in the first period the country can import freely, while there might be a trade disruption (with probability π) in the second period. A more realistic, multiperiod setup is that of Tolley and Wilman (1977), where the duration of the embargo is uncertain.

* This chapter is an extended version of Bergström, Loury and Persson (1985).

Our problem is somewhat similar to that of Tolley and Wilman, and in Section 2.3 we will discuss how our analysis differs from theirs.

Formally, our problem resembles the one in the literature dealing with the optimal depletion of a natural resource in the face of the possible future introduction of a new technology. This problem has been studied by, for example, Dasgupta and Heal (1974) and by Dasgupta and Stiglitz (1981), and we will show below that some of our results bear a close resemblance to theirs. The two sources of uncertainty in the embargo problem, however, make it somewhat different.

The chapter is organized as follows. In Section 2.2 we set up the formal structure of the model. The environment of the oil-importing country can be characterized by one of two regimes: "free trade" or "embargo". Optimal behavior can be analyzed by means of dynamic programming arguments. Under each of the two regimes the optimal policy must satisfy a particular functional equation, and the overall problem involves the solution of the simultaneous system of these two equations. It is convenient to start by analyzing the equations one at a time: in Section 2.3 we analyze the optimal depletion of an oil reserve during an embargo, when the size of the reserve is given and when the duration of the embargo is uncertain, and in Section 2.4 we analyze the optimal stockpiling policy during free trade, when the duration of the free-trade regime is uncertain. This can be done under either of two assumptions: the country does not have access to any capital markets when building up its oil reserves, or else perfect borrowing is admitted. In Section 2.4 we stick to the former assumption. In Section 2.5 we solve the general problem (i.e., the system of functional equations) under the simplifying assumption that a perfect capital market exists. This makes it possible for us to obtain an explicit expression for the optimal size of the emergency reserves. In Section 2.6 we derive expressions for the welfare of the country that is subject to embargo threats. In Section 2.7, finally, we conclude the chapter by discussing some possible extensions of the model.

2.2 THE MODEL

Assume that a country consumes two commodities - one imported, which we will call "oil", and one domestically produced, perishable commodity, which we will call "potatoes". We confine our analysis to a small country; thus the relative price of oil in terms of potatoes, denoted by p, is assumed to be exogenously given by a world market.[1)] We will make the simplest possible assumptions about the production technology and just assume that a constant flow of potatoes, Z, will be made available to the country at each instant of time. Denoting the country's consumption of oil at time t by q_t and its consumption of potatoes by z_t, we can write the budget constraint as

$$p\dot{S}_t + pq_t = Z - z_t + \xi_t,$$

where S_t is the country's stock of oil reserves, and where the dot indicates the time derivative. The variable ξ_t is the deficit in the balance of trade. While S_t, q_t and z_t are nonnegative, ξ_t is not restricted in sign. A positive value of ξ_t indicates that the country is borrowing on the international capital market (or, alternatively, is a net creditor and receives interest income and amortizations). A negative value implies that the country pays interest and amortizes loans (or, alternatively, lends money to foreign borrowers).

If no international capital market exists, we impose the restriction that

$$\xi_t = 0 \quad \text{for all t.} \tag{2.1}$$

In this case the country cannot borrow money to build up its oil reserves but has to rely solely on its potato crop to finance the stockpile. If, on the other hand, the country has access to a perfect international capital market, ξ_t has to satisfy the milder constraint

$$E\left[\int_0^\infty \xi_t \, e^{-rt} \, dt\right] = 0 \tag{2.2}$$

instead.[2] [3] Since reality is somewhere between the two extreme assumptions of "no capital market" and "a perfect capital market", we will analyze both. In this section, and in Section 2.3 (which deals with optimal depletion under an embargo regime), there is no need to distinguish between the two assumptions; the notation is sufficiently general to cover either. Under free trade, however, the distinction becomes crucial. We will therefore analyze the optimal stockpiling policy with no credit market in Section 2.4 and the optimal stockpiling policy with a perfect credit market in Section 2.5.

Assume further that the country's utility is additively separable in time and that the marginal utility of potato consumption is constant. The discounted instantaneous utility can then be written as

$$U(q_t, z_t, t) \equiv e^{-rt}\left[u(q_t) + z_t\right],$$

where r is the discount rate and where we assume that

$$u'(q) > 0, \qquad u''(q) < 0, \qquad \text{and} \quad \lim_{q \to 0} u'(q) = \infty.$$

The assumption of a constant marginal utility of the background commodity is made to facilitate the analysis and is a standard one in the literature on natural resources.[4]

In the long-run perspective, we will assume that periods of free trade come and go, interrupted by embargoes, in an infinite sequence.[5] We assume the stochastic process $\{x_t\}$ generating the sequence of regimes to be a two-state stationary Markov process that takes the values zero and one. We identify the state $x_t = 0$ with an embargo and the state $x_t = 1$ with a free-trade regime. The assumption of a stationary Markov process implies that the duration τ_0 of an embargo is exponentially distributed with parameter θ_0. This means that the density function of the duration τ_0 can be written as

$$f_0(\tau_0) = \theta_0 \cdot e^{-\theta_0 \tau_0}.$$

Likewise, the duration τ_1 of a free trade regime is exponentially distributed with parameter θ_1 according to

$$f_1(\tau_1) = \theta_1 \cdot e^{-\theta_1 \tau_1}.$$

The parameter θ_0 is the probability rate that the country will be in a free-trade regime in the next instant, given that there is an embargo going on now. Likewise, if no embargo is now in effect, θ_1 denotes the probability rate that the economy will be in an embargo regime in the next instant. Hence the stochastic process $\{x_t\}$ is completely characterized by the two transition probability rates θ_0 and θ_1. This follows because the Markovian assumption implies that the likelihood of leaving a given state in the next instant is independent of the history of the process, while the assumption that the process is stationary implies that the likelihood, given the history, is independent of the date t.

We can now formulate the problem of the oil-importing country. Assume that an optimal policy exists. Then, if <u>no embargo is now in effect</u> (i.e., if $x_0 = 1$), let V(1, S) denote maximum expected net utility to be derived from an optimal behavior, with an initial stock S in storage. Assume that physical storage per se is costless, though, of course, financial capital tied up in the storage forgoes the opportunity to earn interest. On the other hand, if <u>an embargo is now in effect</u> (i.e., if $x_0 = 0$), let V(0, S) denote the expected present value of welfare when optimal behavior is pursued from the initial inventory of S in storage. When there is an embargo going on, optimal consumption implies the following:

$$V(0, S) \equiv \underset{\{q_t\}}{\text{Max}} \int_0^\infty \theta_0 \, e^{-\theta_0 \tau_0} \left[\int_0^{\tau_0} e^{-rt} [u(q_t) + Z + \xi^*] dt \right. \tag{2.3}$$

$$\left. + \, e^{-r\tau_0} \, V(1, S_{\tau_0}) \right] d\tau_0$$

subject to $\dot{S}_t = - q_t$;

$S_0 = S$;

$S_t,\ q_t \geq 0.$

Two things should be noted here. First, we see that since there is an embargo going on, the budget constraint has been split into two: $\dot{S}_t = -q_t$ and $z_t = Z + \xi_t$. Second, we have set $\xi_t = \xi^*$ (a constant) in the functional equation; thus $z_t = Z + \xi^*$ and the maximization is made over the control variable q_t only. If no credit market exists, $\xi^* \equiv 0$ by (2.1) and $z_t = Z$. If a perfect credit market exists, however, things are somewhat different. Then ξ^* is a negative constant that can be interpreted as the steady-state rate of debt service and justified in the following way. During an embargo, there are two decisions to be made: how to deplete the stock S_t and how to divide the potato crop Z between consumption z_t and debt service ξ_t. Since the utility function is additively separable in q_t and z_t, these two decisions are independent of each other, and therefore, the optimal policies $\{z_t\}$ and $\{\xi_t\}$ do not affect the optimal depletion policy $\{q_t\}$. Now, since the utility function is linear in z_t, the country is indifferent between different time profiles of debt service $\{\xi_t\}$ satisfying (2.2). Further, lenders are also indifferent between different paths $\{\xi_t\}$ as long as (2.2) is satisfied. We therefore choose the most convenient of these paths, namely the steady-state one with $\xi_t = \xi^*$. For the time being we regard ξ^* as an exogenous constant; later in the chapter we will derive an explicit formula for it.

Integrating by parts, (2.3) can be equivalently written as

$$V(0, S) \equiv \underset{\{q_t\}}{\mathrm{Max}} \int_0^\infty [u(q_t) + Z + \xi^* + \theta_0\, V(1, S_t)]\, e^{-(r+\theta_0)t}\, dt \qquad (2.4)$$

$$\text{subject to } \dot{S}_t = -q_t;$$

$$S_o = S;$$

$$S_t,\ q_t \geq 0.$$

The functional equation (2.4) thus describes the planning problem when an embargo is in effect.[6] When there is no embargo, the problem looks different. Then optimal behavior implies

$$V(1, S) \equiv \underset{\{q_t, z_t, \xi_t\}}{\text{Max}} \int_0^\infty \theta_1\, e^{-\theta_1 \tau_1} \left[\int_0^{\tau_1} e^{-rt} [u(q_t) + z_t] dt \right. \tag{2.5}$$

$$\left. + e^{-r\tau_1}\, V(0, S_{\tau_1}) \right] d\tau_1$$

$$\text{subject to } p\dot{S}_t + pq_t = Z - z_t + \xi_t;$$

$$\text{either (2.1) or (2.2);}$$

$$S_o = S;$$

$$S_t,\ q_t,\ z_t \geq 0;$$

or equivalently (again integrating by parts):

$$V(1, S) \equiv \underset{\{q_t, z_t, \xi_t\}}{\text{Max}} \int_0^\infty [u(q_t) + z_t + \theta_1\, V(0, S_t)]\ e^{-(r+\theta_1)t}\ dt \tag{2.6}$$

subject to $p\dot{S}_t + pq_t = Z - z_t + \xi_t$;

either (2.1) or (2.2);

$S_o = S$;

$S_t, q_t, z_t \geq 0$.

We do not have to specify the borrowing constraint yet; depending on whether one assumes a perfect capital market to exist or not, (2.2) or (2.1) is applicable.

Equations (2.4) and (2.6) may be taken to represent the problem of optimal reserve management, and their simultaneous solution for the functions V(0, S) and V(1, S) yields the optimal inventory policy. In general this problem of dynamic programming with a switching of regimes is very difficult. Application of Bellman's principle of optimality would lead to a pair of nonlinear differential equations to be solved for V(0, S) and V(1, S). As will be demonstrated below, however, this problem is not insurmountable. It will be instructive to concentrate on one equation at a time; therefore, in the next section we will study the function V(0, S) taking V(1, S) as given. In Section 2.4 we will then study V(1, S) while taking V(0, S) as given, under the assumption that (2.1) is applicable. In Section 2.5 we solve the system of functional equations (i.e., the general problem) under the assumption that (2.2) is applicable.

2.3. THE OPTIMAL DEPLETION OF EMERGENCY RESERVES

The Pontryagin necessary conditions for maximization of (2.4), taking the function $V(1, S_t)$ as given, are

$$\lambda_t^o = u'(q_t) \tag{2.7}$$

$$\dot{\lambda}_t^o = (r + \theta_o)\lambda_t^o - \theta_o V_s(1, S_t), \tag{2.8}$$

where the superscript "o" indicates that the costate variable is the one associated with the maximum value function V(0, S) - that is, an embargo regime - and where the dot indicates the time derivative. Further, $V_s(1, S_t) \equiv \partial V(1, S_t)/\partial S_t$. Note that we have disregarded the shadow prices associated with the non-negativity constraints on S_t and q_t since the infinite slope of u(q) at q = 0 will always ensure strictly positive values for S_t and q_t.

The spot price λ_t^o is defined as the marginal value of oil in stock as long as the embargo is going on. An intuitive interpretation of (2.8) can be obtained if we write the price path in the following form:

$$\frac{\dot{\lambda}_t^o}{\lambda_t^o} + \theta_o\left[\frac{V_s(1, S_t) - \lambda_t^o}{\lambda_t^o}\right] = r. \tag{2.9}$$

This formula is, in fact, a special case of the Hotelling (1931) principle of resource depletion. It simply says that if the embargo is still going on at date t, then the expected rate of price increase of oil in stock should be equal to the rate of interest. That the left-hand side of (2.9) can be interpreted as the expected rate of price increase is evident from the fact that λ_t^o is the spot shadow price of oil in stock during an embargo while $V_s(1, S_t)$ is the marginal valuation of oil in stock in case the embargo should end. Thus, $V_s(1, S_t) - \lambda_t^o$ is the marginal capital gain (or rather capital loss) from holding oil, should the embargo expire, and $\theta_o[V_s(1, S_t) - \lambda_t^o]$ is the expected marginal capital gain.[7)]

Equation (2.9) is identical to the one obtained by Dasgupta and Stiglitz (1981) in their study of resource depletion under technological uncertainty. In their terminology, $V_s(1, S_t)$ is the fall back price of the resource after the new technology has been introduced. The formula differs from the one derived by Dasgupta and Heal (1974). In that paper the authors assume that $V(1, S_t) = V_s(1, S_t) = 0$, which means that after the breeder reactor (or solar energy) has been introduced, all remaining oil will be worthless.

Tolley and Wilman's (1977) paper on oil embargoes uses a formal setup which is quite different from that of our model. It is thus hard to relate their results to ours, but some comparisons can be made. They make two special assumptions that together result in a policy different from the one implied by (2.9). First, they assume explicitly that the embargo period is so short that the discount rate can be disregarded, i.e., that $r = 0$. Second, they treat the utility of oil remaining in storage at the end of an embargo in a quite different manner. In fact, they assume that the oil stock should be completely exhausted during the embargo, and they calculate the optimal date $\tilde{L}$ (which is less than or equal to the duration of the embargo) at which the stock should be depleted. This results in an optimal policy saying that the rate of price increase ($\dot{\lambda}_t^o/\lambda_t^o$ in our notation) should be equal to the probability rate that the embargo will end in the next moment, given that it has not already ended. With an exponential probability distribution for the duration of the embargo, this conditional probability rate is equal to our parameter θ_o. Thus, (2.9) would imply the same policy as that of Tolley and Wilman if we set $r = 0$ and $V_s(1, S_t) = 0$. This seems, however, rather restrictive. In general, the embargo period might last for a long time, the average length being $1/\theta_o$, and the interest costs could be rather high. Further, with our assumption of an infinite marginal utility $u'(q)$ at $q = 0$, it can never be optimal to deplete the stock before the embargo has expired, so some oil will always remain at the terminal date. This oil could in general be used later (in the free-trade regime that follows) for some purpose, for example, for consumption or to refill the emergency reserves in case a new embargo should occur, or it could be sold on the world market.

To obtain a solution to the problem of optimal depletion when the length of the embargo is uncertain, we thus have to solve the system of differential equations

$$\dot{\lambda}_t^o = (r + \theta_o)\lambda_t^o - \theta_o V_s(1, S_t) \qquad (2.10)$$

$$\dot{S}_t = - u'^{-1}(\lambda_t^o). \qquad (2.11)$$

For each function $V_s(1, S_t)$ this system has infinitely many solutions $\{\lambda_t^o\}$, $\{S_t\}$. We must therefore impose some initial conditions on the paths. For $\{S_t\}$ this is self-evident; it must hold that

$$S_0 = S. \tag{2.12}$$

To obtain the initial price λ_0^0, we impose the resource constraint

$$\int_0^\infty u'^{-1}(\lambda_t^0)dt = S. \tag{2.13}$$

Of course, the probability that the embargo will come to an end is arbitrarily close to unity under our assumptions, so the proper interpretation of (2.13) is that the inventories should be allocated during the embargo so as to ensure consumption at each date ($u'(0) = \infty$), but not to leave redundant stocks asymptotically.

Let us denote the initial shadow price λ_0^0, given by (2.12) and (2.13) by

$$\lambda_0^0 = \lambda_0^0[S, V_S(1, \cdot)], \tag{2.14}$$

where the notation $V_S(1, .)$ is used to indicate that λ_0^0 does not depend simply on a particular value of the $V_S(1, .)$ function, but on the entire functional form. The system (2.10), (2.11), together with the endpoint restrictions (2.12) and (2.13), thus gives us the optimal paths $\{\lambda_t^0\}$, $\{S_t\}$ for each initial stock S and each given function V(1, .). For a formal proof of the existence and uniqueness of a solution, the same method of proof as the one of Dasgupta and Stiglitz (1981, p. 103) can be applied. In the next section, we will deal with the V(1, .) function.

2.4 STOCKPILING UNDER FREE TRADE WITH NO CAPITAL MARKET

As soon as the embargo has expired and a regime of free trade has begun, the country will start to build up its emergency reserves again. V(1, S), as defined by the functional equation (2.6), is the value of having a stock S at the beginning of a free-trade regime. It may seem intuitively reasonable to set $V_S(1, S_t)$ equal to the world market price p; in a free market equilibrium the marginal valuation of a unit of oil should be equal to the relative price. However, that is not true in the absence of a capital market. During the embargo, the country has been depleting its oil reserves in the fashion analyzed in Section 2.3 above, and when the free-trade regime begins, the size of the remaining stock will in general be below the optimal size S^*. An instantaneous stock adjustment $(S^* - S_t)$ is possible only if a perfect international credit market exists. Otherwise the country's purchases of oil in the world market are constrained by the borrowing restriction (2.1). Thus to build up the stockpile will take some time, and during this interval (which will be shorter, the shorter was the preceding embargo) the marginal valuation of oil will be greater than the world market price.[8] Immediately after an embargo, the country will thus find itself in a corner solution; the oil reserves are so small that it must use its entire potato harvest to purchase oil.[9] In that case, $V_s(1, S_t) > p$.

To find the correct value of $V_s(1, S_t)$, we derive the Pontryagin first-order conditions for a maximum of (2.6):

$$\lambda_t^1 = u'(q_t) \tag{2.15}$$

$$\dot{\lambda}_t^1 = (r + \theta_1)\lambda_t^1 - \theta_1 V_s(0, S_t) \tag{2.16}$$

$$\lambda_t^1 \geq p, \tag{2.17}$$

where the superscript "1" indicates that the shadow price λ_t^1 is the one corresponding to (2.6), that is, to a free-trade regime. Since $u'(q)$ goes to infinity as $q \to 0$, we know that we will always have an interior solution with respect to q_t, that is, (2.15) will always be satisfied as an equality. Expression (2.17), however, corresponds to the constraint $z_t \geq 0$, and since the marginal utility of potatoes is constant and finite there is no guarantee for an interior solution.[10] Thus z_t may well be equal to zero, and (2.17) will then be satisfied as an inequality. In order to find the entire path of λ_t^1, which is what we want since λ_0^1 is identically equal to the $V_s(1, S_t)$ we needed for the solution of (2.10) and (2.11) above, we therefore have to investigate all possible solutions of z_t.

The solution to the problem of optimal stockpiling under free trade (2.6) is given by the system of differential equations

$$\dot{\lambda}_t^1 = (r + \theta_1)\lambda_t^1 - \theta_1 \, V_s(0, S_t) \tag{2.18}$$

$$\dot{S}_t = \frac{Z - z_t}{p} - u'^{-1}(\lambda_t^1). \tag{2.19}$$

We note that the entity $V_s(0, S_t)$ appearing in (2.18) is the marginal valuation of an oil stock S_t if the free-trade regime should end through an embargo being imposed at time t. That is, it is identical to the $\lambda_0^0(S, V_s(1, \cdot))$ of (2.14) in the previous section.

The equation system (2.18), (2.19) gives us the steady state solution (λ^{1*}, S^*) by setting $\dot{\lambda}_t^1 = \dot{S}_t = 0$. Thus,

$$(r + \theta_1)\lambda^{1*} = \theta_1 \, V_s(0, S^*) \tag{2.20}$$

$$u'^{-1}(\lambda^{1*}) = \frac{Z - z^*}{p}. \tag{2.21}$$

It is easily shown that two paths $\{\lambda_t^1\}$, $\{S_t\}$ that satisfy the equation system (2.18), (2.19) and lead to the steady state (λ^{1*}, S^*) also satisfy the transversality conditions. Thus, these paths are optimal.[11)]

The optimal paths can easily be analyzed by means of a phase diagram. To do this, we will study all possible paths of z_t, that is, we will study the two possible cases of inequality (2.17). This inequality can be written either as

$$\lambda_t^1 \geq p \qquad \text{and} \qquad z_t = 0, \tag{2.22}$$

or,

$$\lambda_t^1 = p \qquad \text{and} \qquad z_t > 0. \tag{2.23}$$

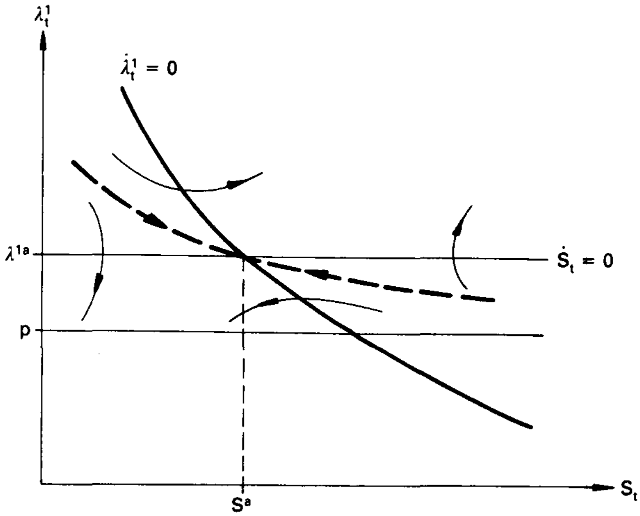
Let us study these two cases one at a time.

Case (2.22) means that the shadow price of oil is higher than the market price; the country's oil reserve is so small that the entire potato harvest has to be exported to finance the purchase of oil. Thus the state equation (2.19) becomes

$$\dot{S}_t = \frac{Z}{p} - u'^{-1}(\lambda_t^1). \tag{2.24}$$

The locus of $\dot{S}_t = 0$ is then a horizontal line in the S_t, λ_t^1 plane. Assume that the V(0, S) function is concave.[12)] Then the locus of $\dot{\lambda}_t^1 = 0$ must be a downward-sloping curve. Assuming there exists a steady state (λ^{1a}, S^a) which is a corner solution with $\lambda^{1a} \geq p$ and $z^a = 0$, the phase diagram must look like the one depicted in Figure 2.1.

Figure 2.1 Phase diagram for the case of a steady state with a corner solution



There is obviously a saddle-point configuration, corresponding to the steady-state level of oil reserves, S^a. At that point, we have by (2.24) that

$$\lambda^{1a} = u'\left[\frac{Z}{p}\right]$$

and thus the value of S^a can be obtained by (2.18):

$$V_S(0, S^a) = \frac{r + \theta_1}{\theta_1} \cdot u'\left[\frac{Z}{p}\right].$$

There is no reason to dismiss such a corner solution as impossible per se; it applies to a country with such a low level of real income that the whole potato crop will be sold to finance the purchase of oil. This kind of heavy specialization may not be entirely unrealistic. However, one might argue that it is not very interesting from an economic point of view: there is no trade-off between the consumption of oil and the consumption of potatoes, and the only decision that has to be made is that of the optimal size of the emergency reserves.

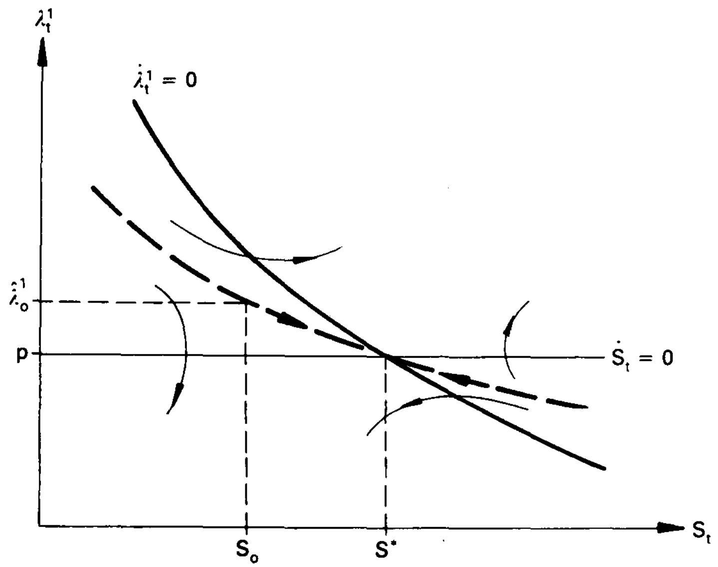
Let us therefore consider a somewhat richer and more general situation, namely that of a steady state with an inner solution $0 < z^* < Z$. We thus assume that the level of real income is high enough to support such a case. The locus of $\dot{\lambda}_t^1 = 0$ is the same as for the case of a corner solution above. The locus of $\dot{S}_t = 0$ is given by

$$\dot{S}_t = - u'^{-1}(\lambda_t^1) + \frac{Z - z_t}{p} = 0$$

for $z_t = z^* \in (0, Z)$. Hence it must be a horizontal line in the S_t, λ_t^1 plane, as shown in Figure 2.2 below. Thus we have a saddle point (λ^{1*}, S^*) given by

$$\lambda^{1*} = u'\left[\frac{Z - z^*}{p}\right]. \tag{2.25}$$

Figure 2.2 Phase diagram for the case of a steady state with an inner solution



Further we know that, at the steady state, the marginal valuation of oil in stock λ^{1*} must be equal to the world market price p. Thus (2.25) gives us the value of z^* by

$$u'\left[\frac{Z - z^*}{p}\right] = p.$$

The optimal level of inventories S^* is then given implicitly by (2.20):

$$V_S(0, S^*) = \frac{r + \theta_1}{\theta_1} p. \tag{2.26}$$

This formula is essential for the analysis, and it will be used later in Section 2.5. It lends itself to a simple intuitive interpretation. In free trade, the cost of storing one unit of oil is p. The marginal benefit is the expected, discounted value of that unit, should an embargo be imposed. The marginal value at the beginning of this future embargo is $V_S(0, S^*)$ and since it occurs at the random date τ_1, the discounted value is $e^{-r\tau_1} V_S(0, S^*)$. The date τ_1 is exponentially distributed; thus

$$E_1\left[e^{-r\tau_1} V_S(0, S^*)\right] = \int_0^\infty \theta_1\, e^{-(r+\theta_1)\tau_1} d\tau_1\, V_S(0, S^*) = \frac{\theta_1}{r + \theta_1} V_S(0, S^*).$$

Formula (2.26) therefore says that at the optimal stock, the cost p of storing one additional unit of oil should equal the expected discounted marginal benefit $V_S(0, S^*)\, \theta_1/(r + \theta_1)$.

From now on, we will only analyze the case of an interior steady-state solution $0 < z^* < Z$. Thus we will assume that, for any initial stock S_0, the initial price during a free-trade regime should be set according to the stable trajectory depicted in Figure 2.2:

$$\lambda_0^1 = \hat{\lambda}_0^1.$$

This leads eventually to the steady-state level of oil reserves S^*. The optimal policy is thus to start with an initial $\hat{\lambda}_0^1 > p$. Given the initial stock S_0, we thus follow the stable path of Figure 2.2 while consuming no potatoes but exporting our entire harvest to purchase oil. This policy continues until the stock has reached the level S^* or until we are hit by a new embargo, whichever event occurs first. If we are lucky enough, the new embargo will not occur until we have filled our reserves. This will then happen at a finite date t^*, and thereafter we have an interior solution with oil consumption given by

$$q_t^* = u'^{-1}(p), \qquad t \geq t^*.$$

It remains to show that t^* is finite. We know that an optimal policy implies, for $t < t^*$,

$$q_t = u'^{-1}(\lambda_t) < u'^{-1}(p),$$

where the inequality follows from the fact that $\lambda_t^1 > p$. But then

$$\dot{S}_t = \frac{Z}{p} - q_t > \frac{Z}{p} - u'^{-1}(p),$$

which is positive by our assumption that $z^* > 0$. Integrating both sides of the inequality from 0 to t^* yields an upper bound for t^*:

$$t^* < \frac{p(S^* - S_o)}{Z - p\, u'^{-1}(p)} < \infty.$$

We thus have characterized the optimal price path $\{\lambda_t^1\}$ for a free-trade regime with no capital market. Using the same notation as in (2.14) above, we denote the initial value of the $\{\lambda_t^1\}$ path by

$$\lambda_o^1 = \lambda_o^1\left[S,\ V_s(0,\ \cdot)\right]. \tag{2.27}$$

Identifying $V_s(0, \cdot)$ with $\lambda_o^0(\cdot)$ and $V_s(1, \cdot)$ with $\lambda_o^1(\cdot)$, (2.14) and (2.27) thus forms a nonlinear equation system in the unknown functions $\lambda_o^0(\cdot)$ and $\lambda_o^1(\cdot)$. This system is very difficult to solve in general, and the full solution to the reserve management problem with no capital market can only be achieved by numerical methods. However, if one can assume that the adjustment to the optimal stock can be accomplished instantaneously, once the free-trade regime has started, things are easier and explicit analytical solutions can be obtained. This will be the topic of the next section.

2.5 THE FULL SOLUTION WITH A PERFECT CAPITAL MARKET

Assume that the country is not constrained by (2.1), but by (2.2). Then our assumption of a constant marginal utility of income, together with the assumption of a small country with no monopsony power, makes it optimal to adjust the stock at the beginning of the free-trade regime instantaneously to the desired level S^*. This means that $\xi_t = p(S^* - S_t)$ for $t = \tau_0$, while $\xi_t = \xi^* < 0$ for all other t. The constant ξ^* can be interpreted as the steady-state rate of debt service and is derived in the following way. Throughout a cycle consisting of one free-trade period and one embargo period (with a total duration of $\tau_1 + \tau_0$) the loan that financed the stock adjustment is serviced. Due to the constant marginal utility of income, we can assume that this repayment will be in the form of a constant debt service rate ξ^*. The capital market constraint (2.2) says that during such a cycle, the loan has to be fully repaid, on the average. This repayment condition can be written

$$E_0\left[p(S^* - S_{\tau_0})\right] = \xi^* \frac{1}{r} E_1\left[E_0\left[1 - e^{-r(\tau_1+\tau_0)}\right]\right], \tag{2.28}$$

where the expectations $E_0[\cdot]$ and $E_1[\cdot]$ are defined by the exponential density functions $f_0(\tau_0)$ and $f_1(\tau_1)$ of Section 2.2 above. This therefore gives us an explicit formula for the constant ξ^*.[13)]

With instantaneous stock adjustment, we are in a steady-state (q^*, z^*, S^*, ξ^*, λ^{1*}) during the whole free-trade regime. This also means that the marginal valuation of oil in stock at the beginning of a free-trade regime is equal to the world market price:

$$V_S(1, S_t) \equiv \lambda_0^1 = \lambda_t^1 = \lambda^{1*} = p.$$

This implies that the problem can be drastically simplified. In fact, the system of functional equations (2.4) and (2.6) - or, equivalently, the system of equations (2.14) and (2.27) - becomes recursive, and a closed-form solution can be obtained.[14)]

We recall that the equation system (2.10)-(2.13) gives the optimal depletion of an oil stock S during an embargo. The Hotelling interpretation

(2.9) of the differential equation (2.10) - identifying $\theta_0[V_S(1, S_t) - \lambda_t^0]$ with the expected marginal capital gain, should the embargo expire - still holds with $V_S(1, S_t) = p$. But we can gain some further insight into the economic interpretation of the model by rewriting (2.10) as

$$\frac{\dot{\lambda}_t^0}{\lambda_t^0 - [\theta_0/(r + \theta_0)]p} = r + \theta_0. \tag{2.29}$$

Equation (2.29) looks exactly like the equation describing the optimal depletion of a natural resource with a constant marginal extraction cost[15]) equal to $p\theta_0/(r + \theta_0)$ and with an interest rate $r + \theta_0$. The reason why $p\theta_0/(r + \theta_0)$ can be interpreted as an extraction cost in our model is the following. If we remove one unit of oil from the stock, it has to be replenished when the embargo ends. The expected, discounted replenishment cost is

$$E_0\left[e^{-r\tau_0} p\right] = \frac{\theta_0}{r + \theta_0} p,$$

where the expectations operator $E_0[\cdot]$ is defined by the density function $f_0(\tau_0)$ of Section 2.2. The "discount rate" $r + \theta_0$ applies since future consumption is discounted both for time preference reasons and because the return of free trade permits the replenishment of inventories.

To derive the size of the optimal stock S^*, we now solve the differential equation (2.29):

$$\lambda_t^0 = \left[\lambda_0^0 - \frac{\theta_0}{r + \theta_0} p\right] e^{(r+\theta_0)t} + \frac{\theta_0}{r + \theta_0} p, \tag{2.30}$$

where λ_0^0 is given by the resource constraint (2.13). We can, however, obtain a simple expression for λ_0^0 when the initial stock $S = S^*$, which is the case with a perfect capital market. Instantaneous stock adjustment to S^* implies that we are in a steady state during the whole free-trade regime. Therefore,

$$\lambda_0^1 = \lambda_t^1 = \lambda^{1*} = p.$$

Equation (2.20), which must hold during any free-trade regime regardless of which assumption we make about the existence of a capital market, says that

$$(r + \theta_1)\, p = \theta_1\, V_s(0, S^*).$$

Since $V_s(0, S) \equiv \lambda_0^0$, this implies that for a stock $S = S^*$,

$$\lambda_0^0 = \frac{r + \theta_1}{\theta_1}\, p. \tag{2.31}$$

This means that when stocks are optimal, the marginal valuation of oil in storage at the beginning of an embargo should exceed the free-trade price by a factor $(r + \theta_1)/\theta_1$. Inserting this into (2.30), and substituting the solution for λ_t^0 into the resource constraint (2.13) at $S = S^*$ gives us an explicit expression for the optimal stock:

$$S^* = \int_0^\infty u'^{-1}\left[\left[\frac{r + \theta_1}{\theta_1} - \frac{\theta_0}{r + \theta_0}\right] p\, e^{(r+\theta_0)t} + \frac{\theta_0}{r + \theta_0}\, p\right] dt. \tag{2.32}$$

2.6 SOME RESULTS ON WELFARE

The model above describes the optimal reserve management of an oil-importing country. A question that naturally arises in this context is whether one can find an expression for the cost to the country, in terms of expected utility, of being subject to embargo threats. In fact, the assumption of a perfect capital market allows us to derive explicit formulae for $V(0, S^*)$ and $V(1, S^*)$. These expressions make it possible to evaluate the welfare loss to an oil-importing country of being subject to embargo threats.[16)]

Assume that the country is in a free-trade regime with an optimal stock S^*. We can obtain an expression for $V(1, S^*)$ by evaluating the integral in (2.6) at $q_t = q^* = u'^{-1}(p)$, $z_t = z^* = Z - pq^* + \xi^*$ and $S_t = S^*$:

$$V(1, S^*) = \frac{1}{r + \theta_1}\left[v(p) + Z + \xi^*\right] + \frac{\theta_1}{r + \theta_1} V(0, S^*), \tag{2.33}$$

where $v(p) + Z + \xi^*$ is the indirect instantaneous utility in steady state:

$$v(p) + Z + \xi^* \equiv \underset{q^*, z^*}{\text{Max}}\; u(q^*) + z^*$$

$$\text{subject to } pq^* + z^* = Z + \xi^*.$$

Working out the optimization, we see that the $v(p)$ function is

$$v(p) \equiv u\left[u'^{-1}(p)\right] - pu'^{-1}(p).$$

Let us now consider the maximum value function for an embargo (2.4). The basic differential equation for this dynamic programming problem is the following:[17)]

$$V(0, S) = \frac{\theta_0}{r + \theta_0} V(1, S) \tag{2.34}$$

$$+ \frac{1}{r + \theta_0} \underset{q}{\text{Max}} \left[u(q) + Z + \xi^* - V_s(0, S)q\right]$$

Assume now that we start the embargo with an optimal stock S^*, as we in fact do with a perfect capital market. Then by (2.31), $V_s(0, S) \equiv \lambda_0^0 =$ $= p(r + \theta_1)/\theta_1$ and (2.34) can be written

$$V(0, S^*) = \frac{1}{r + \theta_0}\left[v\left[p\frac{r + \theta_1}{\theta_1}\right] + Z + \xi^*\right] + \frac{\theta_0}{r + \theta_0} V(1, S^*). \tag{2.35}$$

Equations (2.33) and (2.35) form a linear system in the two unknowns $V(0, S^*)$ and $V(1, S^*)$ with the solution

$$V(1, S^*) = \frac{r + \theta_0}{r + \theta_0 + \theta_1} \frac{1}{r} [v(p) + Z + \xi^*] \tag{2.36}$$

$$+ \frac{\theta_1}{r + \theta_0 + \theta_1} \frac{1}{r}\Big[v[p(r + \theta_1)/\theta_1] + Z + \xi^*\Big]$$

$$V(0, S^*) = \frac{\theta_0}{r + \theta_0 + \theta_1} \frac{1}{r} [v(p) + Z + \xi^*] \tag{2.37}$$

$$+ \frac{r + \theta_1}{r + \theta_0 + \theta_1} \frac{1}{r}\Big[v[p(r + \theta_1)/\theta_1] + Z + \xi^*\Big].$$

Equation (2.36) says that in a free-trade regime, expected discounted utility under optimal inventory behavior is a weighted average of the welfare we would have if free trade always ruled, the price of oil were p, and our income were $Z + \xi^*$ and the welfare we would have if free trade always ruled, the price of oil were $p(r + \theta_1)/\theta_1$, and our income were $Z + \xi^*$. Similarly, equation (2.37) says that, given that an embargo has been imposed, expected discounted utility is also a weighted average of the utilities of free-trade regimes with these prices, but the weights are somewhat different. Interpreting the weights as probabilities, one can say that if free trade currently prevails, the welfare under an embargo threat characterized by the parameters θ_0 and θ_1 is equal to the welfare of a perpetual free-trade regime with prices randomly fluctuating between p and $p(r + \theta_1)/\theta_1$ with probabilities $(r + \theta_0)/(r + \theta_0 + \theta_1)$ and $\theta_1/(r + \theta_0 + \theta_1)$, respectively.

Applying Jensen's inequality to (2.36) and (2.37), we gain some insight into the insurance-theoretic properties of trade disruptions:

$$V(1, S^*) > \frac{1}{r} v\Big[(1 + \frac{r}{r + \theta_0 + \theta_1}) p\Big] + \frac{1}{r} (Z + \xi^*)$$

$$V(0, S^*) > \frac{1}{r} v\Big[(1 + \frac{r}{r + \theta_0 + \theta_1} \cdot \frac{r + \theta_1}{\theta_1}) p\Big] + \frac{1}{r} (Z + \xi^*).$$

This means that, under free trade, a country that runs an optimal stockpile and is subject to an embargo threat (θ_0, θ_1) will never be worse off than a country that has entered a long-term contract guaranteeing perpetual deliveries at a price $(1+r/(r + \theta_0 + \theta_1))p$ and that has a flow of income equal to $Z + \xi^*$. Similarly, a country subject to an embargo that has a stock S^* is never worse off than a country with a long-term contract of oil at the price $(1 + r(r + \theta_1)/(r + \theta_0 + \theta_1)\theta_1)p$ and a perpetual flow of income $Z + \xi^*$.

Thanks to the assumption of a constant marginal utility of income, we can solve the model for the maximum premium α above the price p that a country would be willing to pay for guaranteed safe deliveries of oil forever. For a country that is presently in a free-trade regime, this premium α_1 is given by

$$V(1, S^*) = \frac{1}{r} v[(1 + \alpha_1)p] + \frac{Z}{r},$$

while for a country with a stock S^* that is presently at the beginning of an embargo, the premium α_0 is given by

$$V(0, S^*) = \frac{1}{r} v[(1 + \alpha_0)p] + \frac{Z}{r}.$$

The $V(1, S^*)$ and $V(0, S^*)$ are just numbers, given by (2.36) and (2.37). The last term on the right hand side is due to the fact that if the country enters such a contract, it can sell its emergency stock S^* and repay its debts, and thus it no longer has to service its debts with a perpetual flow ξ^*.

2.7 CONCLUDING COMMENTS

In this chapter we have made one basic assumption that serves to facilitate the analysis, namely, that the price of oil is constant over time. As pointed out above, this assumption is made mainly for expositional reasons and could easily be dispensed with. Dropping it would, however, raise a few problems that have to be considered before it is possible to proceed.

Our first intuitive idea is that if we want a more realistic description of the oil market, the price of oil should be assumed to increase over time according to the Hotelling rule. This is, however, not self-evident. First, oil reserves would be depleted according to the Hotelling rule (i.e., in a fashion such that the price increases at the rate of interest) only if the oil producers work under competitive conditions.[18)] But this is not consistent with our assumption of embargo threats; if it is possible for a producer to impose embargoes, some kind of monopoly power must exist in the oil market. Second, assuming a well-functioning cartel among the producers together with a constant price elasticity of demand among the consumers, a price path $p_t = p_o e^{rt}$ means that the embargo problem would degenerate into a trivial one: the optimal stock S^* becomes infinite. And then our assumption that the oil-consuming nation is a small, price-taking country would no longer hold. That S^* is infinite is easily seen from the fact that with the world price of oil increasing (deterministically) at the rate r, which is also the interest rate, oil would dominate other assets in the country's portfolio. It would be costless to borrow huge quantities of money and hold it in the form of oil; if no embargo occurs, such a policy breaks even since the cost of borrowing is exactly matched by the capital gain of holding the asset, and if an embargo is imposed the holding of oil becomes more profitable. To cope with this, we must introduce some imperfections in the international credit markets, or we must introduce storage costs or uncertainty about future oil prices into the model to make oil a less dominating asset in the country's portfolio.

Third, assume that the price of oil increases at some rate ρ which may or may not be equal to the rate of interest. We still have to decide what happens to the world market price during the embargo. If the oil-importing country is a small one and if no other country is affected by the embargo, it is reasonable to assume that the world market price will continue to increase at the rate ρ throughout the embargo. If the importing country is a large one, however, or if a large part of the world is subject to the embargo, one could imagine that the world market price of oil would rise at a lower rate - or even remain constant - and not start to increase at the rate ρ again until the embargo has been called off. The solution to our country's decision problem is of course affected by which of these two scenarios is considered to be the most realistic one.

FOOTNOTES

1) Strictly speaking, the assumption of a constant price is unacceptable in an analysis of the oil market. Instead, world oil reserves should be depleted according to some Hotelling formula. The assumption of $p_t = p$ is made to facilitate the exposition; those who do not like it can substitute the word "oil" by, e.g., "wheat" throughout the chapter.

2) The expectation is taken since there is uncertainty involved in the problem, which means that we cannot automatically guarantee that all debts are repaid with certainty. Constraint (2.2) thus says that all debts are repaid on the average, which means that lenders are assumed to be risk neutral. The probability laws defining the expectations operator $E[\cdot]$ will be presented later in this section.

3) As will be shown later, it can sometimes be advantageous to adjust the stock of oil instantaneously. Thus the stock of oil would make a discrete jump ΔS_t at some $t = \tau_0$. This means that if the control variable ξ_t were a function of time, it would be infinite for $t = \tau_0$. In such a case the integral in (2.2) is not well defined. We can, however, get around this problem by regarding ξ_t not as a function of time but as a measure, and by regarding the integral in (2.2) as a Lebesgue integral. The intuitive meaning of this rather abstract mathematics will be clear from the discussion of the steady-state variable ξ^* at the beginning of Section 2.5 below.

4) Our assumption that oil is a consumption good is not critical for the analysis. It is quite possible to assume instead that oil is used as an input in the production of a consumption good (as is the case with, e.g., fuel or fertilizer). However, since this means that we would have to introduce a production technology into the model we have decided to economize on concepts and notation by considering only the simplest case, i.e., that in which oil is used as a consumption good.

5) In fact, the concept of an infinite sequence of shifting regimes is what makes our analysis different from that of, e.g., Dasgupta and

Heal (1974) and Dasgupta and Stiglitz (1981). In these papers it is assumed that the new technology is a once-and-forever ~~event;~~ as soon as it is introduced, it will last forever - and nothing new will happen. Hillman and Long (1983), in their discussion of how embargo threats affect the optimal depletion of a natural resource, also make this once-and-for-all assumption: the embargo begins at (the stochastic) date T and lasts forever.

6) Here we see that the assumption of an exponential probability distribution of the length of embargoes gives rise to a "discount factor" of the form $e^{-(r+\theta_0)t}$. Thus the solution to our planning problem (2.4) is dynamically consistent in the Strotz (1955-56) sense, while no other probability distribution will yield consistent depletion paths.

7) For the particular case of instantaneous stock adjustment analyzed in Section 2.5 below, the necessary condition (2.9) will be of the same functional form as the necessary condition for the deterministic problem of optimal depletion of a natural resource with a constant marginal extraction cost (see further Section 2.5). This parallel does not hold, however, for the general case (2.9).

8) Note that this does not depend on our assumption of a constant marginal utility of potato consumption. If the utility function were written as $U(q_t, z_t) \equiv u(q_t) + v(z_t)$ instead, with all the usual assumptions on $v(z_t)$ being satisfied, we could still be in a situation where our oil reserves are so small that $V_s(1, S_t) > p$. In the absence of a perfect capital market, the borrowing constraint (2.1) imposes such a restriction on the country that the marginal value of potatoes is greater than unity and the marginal value of oil is greater than p. Even for a case with a perfect capital market, $V_s(1, S_t)$ could be greater than the world market price, namely, if the country is so large that (by its monopsony power) it bids up the world market price when refilling its reserves. While this case is certainly relevant to, e.g., the strategic reserves of the United States, it requires quite

a different model structure and is therefore disregarded in the present chapter.

9) This can be seen immediately by inspection of (2.6). The functional equation is linear in the control z_t, which implies a "bang-bang" solution.

10) Strictly speaking, there should also be an upper bound on z_t when there is no capital market. We have that $z_t \leq Z + pS_t$. If this restriction were binding, it could be interpreted as a case where the oil stock is too large, and thus the country sells oil to purchase potatoes in the world market. Any country could inherit such a large stock S_0 from the past, but it would diminish quickly and after just a few embargoes be so small that the upper bound on z_t will not be binding any more. We will therefore disregard this case and assume in the following that S_t never exceeds the optimal stock S^*.

11) Proposition 10 in Arrow and Kurz (1970, p. 51) is applicable to our problem.

12) We have not been able to prove that the V(0, S) function is concave in the case with no capital markets. The trouble is that the overall problem involves the solution of the simultaneous system of equations (2.4) and (2.6). Consider (2.4). If we could take it that the V(1, S) function is concave, then the V(0, S) function would be concave, too (since the constraint is linear and since in this case the objective function in (2.4) is concave jointly as a function of q and S). However, from (2.6) we see that the functional form of V(1, S) depends on the functional form of V(0, S). In the case of perfect capital markets we know that $\partial V(1, S)/\partial S = p$. This implies that the system of functional equations (2.4) and (2.6) is recursive and that the V(0, S) function is concave (cf. Section 2.5).

13) Integrating $E\left[\int_0^{\tau_1+\tau_0} \xi_t e^{-rt} dt\right]$ in (2.2) and rearranging the terms yields the RHS of (2.28) where we have substituted ξ^* for ξ_t.

14) Even without the assumption of a perfect capital market, the parameter values θ_0, θ_1, ρ and/or Z could be such that the reserves can be fairly quickly restored to the level S^* after an embargo. Thus the assumption of instantaneous stock adjustment could be a reasonable approximation to reality, and thereby a permissible simplification, even if no capital market exists.

15) Cf. Hotelling (1931) and Solow (1974).

16) Cf. Loury (1983).

17) See e.g., Kamien and Schwartz (1981, pp. 241-242).

18) Or, under monopoly, if demand has a constant price elasticity.

REFERENCES

Arrow, Kenneth J., and Kurz, Mordecai. Public Investment, the Rate of Return, and Optimal Fiscal Policy. Baltimore: Johns Hopkins Press for the Resources for the Future, Inc., 1970.

Arad, Ruth W., and Hillman, Arye L. "Embargo Threat, Learning and Departure from Comparative Advantage." Journal of International Economics 9 (May 1979):265-275.

Bergström, Clas; Loury, Glenn C.; and Persson, Mats. "Embargo Threats and the Management of Emergency Reserves." Journal of Political Economy 93 (February 1985):26-42.

Dasgupta, Partha S., and Heal, Geoffrey M. "The Optimal Depletion of Exhaustible Resources." Review of Economic Studies (Symposium on the Economics of Exhaustible Resources 1974):3-28.

Dasgupta, Partha S., and Stiglitz, Joseph E. "Resource Depletion Under Technological Uncertainty." Econometrica 49 (January 1981):85-104.

Hillman, Arye L., and Long, Ngo Van. "Pricing and Depletion of an Exhaustible Resource When There Is Anticipation of Trade Disruption." Quarterly Journal of Economics 98 (May 1983):215-233.

Hotelling, Harold. "The Economics of Exhaustible Resources." Journal of Political Economy 39 (April 1931):137-175.

Kamien, Morton I., and Schwartz, Nancy L. Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management. New York: North-Holland, 1981.

Kuenne, Robert E.; Blankenship, Jerry W.; and McCoy, Paul F. "Optimal Drawdown Patterns for Strategic Petroleum Reserves." Energy Economics 2 (January 1979):3-13.

Loury, Glenn C. "The Welfare Effects of Intermittent Interruptions of Trade." American Economic Review. Papers and Proceedings 73 (May 1983): 272-277.

Nordhaus, William D. "The 1974 Report of the President's Council of Economic Advisors: Energy in the Economic Report." American Economic Review 64 (September 1974):558-565.

Solow, Robert M. "The Economics of Resources or Resources of Economics." American Economic Review 64 (May 1974):1-14.

Strotz, Robert H. "Myopia and Inconsistency in Dynamic Utility Maximization." Review of Economic Studies 23, no. 3 (1955-1956):165-180.

Tolley, Georg S., and Wilman, John D. "The Foreign Dependence Question." Journal of Political Economy 82 (April 1977):323-347.

3. A Comparative-Static Analysis of the Embargo Model, Some Numerical Examples and the Effects of Uncertainty on Stock Withdrawals and Stockpiling

3.1 INTRODUCTION

In this chapter we will use the embargo model developed in the preceding chapter. To gain further insight into the economic interpretation of the model and to deepen our understanding of the relationship between its key variables we shall not only look at the comparative statics qualitatively, but also analyze the model numerically to illustrate the orders of magnitude that emerge. Despite the many numerical illustrations the present chapter should be considered solely as an analysis of methods. Some further model development, such as, e.g., a description of the possibilities for adaptation in production and consumption, seems advisable before an empirical investigation can be carried out. So the numerical examples presented here are only intended to provide some feeling for the dynamics and stochastics of trade embargoes.

The organization of the chapter is as follows. In Section 3.2 we make use of some simplifying assumptions to adapt the model for numerical analysis. In Sections 3.3 and 3.4 we analyze numerically the depletion of emergency reserves during embargoes and stockpiling during free trade. In Section 3.5, finally, we contrast the embargo model with some simpler models that can be characterized by a 2-period setting. In particular, we quantify the bias in stock withdrawal and stockpiling that arises when the random date at which an embargo will end is replaced by its expected value.

3.2 THE OPERATIONAL EMBARGO MODEL

In order to design the embargo model for empirical implementation we make use of a couple of the simplifying assumptions that were employed in the previous chapter. The assumption of a perfect capital market together with the assumption of a constant marginal utility of income makes it optimal to adjust the stockpile to the desired level S^* as soon as the embargo has expired. Thanks to the assumption of a constant marginal utility of income it is a straightforward matter to define an empirically based criterion by which the effects on the economy are to be measured and evaluated. As long as we confine ourselves to the issues of optimal depletion and stockpiling, the above assumptions also make it possible for us to disregard the consumption of the background commodity and debt servicing. It is, of course, important to include these latter variables if it is desired to compute the effects on the welfare of a country that is subject to embargo threats (cf. Section 2.6).

Let the market demand for the embargoed good have a constant elasticity. Thus we can write it as

$$q_t = \lambda_t^{-\eta}, \tag{3.1}$$

where η is the absolute value of the elasticity of demand and q_t is the quantity demanded at price λ_t, both at time t.[1)] Since utility is linear in the background commodity (cf. Section 2.2) the social benefit derived from the consumption of oil can be measured by consumer's surplus. So we can take it that the gross instantaneous utility enjoyed by the country if q_t is the rate of consumption is given by[2)]

$$u(q_t) = \frac{\eta}{\eta - 1} q_t^{\frac{\eta-1}{\eta}} \qquad (\eta > 0). \tag{3.2}$$

When an embargo expires at the unknown date τ_0 the stockpile has to be replenished so that reserves will be available should there be a new embargo. Let p denote the constant acquisition price. Then the total discounted replenishment cost at the date τ_0 at which the embargo expires is equal to

$$e^{-r\tau_0}\, p(S^* - S_{\tau_0}), \tag{3.3}$$

where r is the discount rate and where the probability of the date τ_0 is distributed according to the exponential density function of Section 2.2:

$$f_0(\tau_0) = \theta_0\, e^{-\theta_0 \tau_0},$$

where the parameter θ_0 is the probability rate (hazard rate) that the country will be in a free-trade regime in the next instant, given that there is an embargo going on now.

The problem of the oil-importing country can now be formulated. Optimal depletion of emergency reserves during an embargo must satisfy a functional equation that is similar to (2.3). Since later in the chapter we want to relate the reformulated embargo model to models with a simpler stochastic specification, it is worth setting out precisely what it looks like. The new functional equation is given by

$$\bar{V}(0, S) = \underset{\{q_t\}}{\text{Max}} \int_0^\infty \theta_0\, e^{-\theta_0 \tau_0} \left[\int_0^{\tau_0} e^{-rt} \frac{\eta}{\eta - 1} q_t^{\frac{\eta-1}{\eta}}\, dt - e^{-r\tau_0}\, p(S^* - S_{\tau_0}) \right] d\tau_0 \tag{3.4}$$

subject to $\dot{S}_t = -q_t$;

$S_0 = S$;

$S_t,\ q_t \geq 0.$

The only differences between (3.4) and (2.3) are that the maximum value function V(1, S) in (2.3) has been replaced by (3.3) for the total replenishment cost and that the utility function (which is now parametrized) does not include consumption of the background good and debt services.[3] Hence the maximum value function $\bar{V}(0, S)$ does not represent the total welfare of the country as the V(0, S) function does in problem (2.3). Expression (3.4) will be the point of departure for the investigations of Section 3.5. For the analysis of Sections 3.3 and 3.4 it is more convenient to proceed by integrating (3.4) by parts. We then have:

$$\bar{V}(0, S) = \underset{\{q_t\}}{\text{Max}} \int_0^\infty \left[\frac{\eta}{\eta - 1} q_t^{\frac{\eta-1}{\eta}} - \theta_0 \, p(S^* - S_t) \right] e^{-(r+\theta_0)t} dt \tag{3.5}$$

subject to $\dot{S}_t = -q_t$;

$S_0 = S$;

$S_t, \; q_t \geq 0$.

To obtain a solution to the problem (3.5) we have to solve the system of differential equations:[4)]

$$\dot{\lambda}_t = (r + \theta_0) \lambda_t - \theta_0 \, p \tag{3.6}$$

$$\dot{S}_t = -q_t = -\lambda_t^{-\eta} \tag{3.7}$$

Since (3.6) is a linear differential equation with a constant coefficient and a constant term, its solution is

$$\lambda_t = \left[\lambda_0 - \frac{\theta_0}{r + \theta_0} p\right] e^{(r+\theta_0)t} + \frac{\theta_0}{r + \theta_0} p, \tag{3.8}$$

where the initial shadow price during the embargo λ_0 is given by the resource constraint

$$\int_0^\infty \lambda_t^{-\eta} \; dt = \int_0^\infty \left[\left[\lambda_0 - \frac{\theta_0}{r + \theta_0} p\right] e^{(r+\theta_0)t} + \frac{\theta_0}{r + \theta_0} p \right]^{-\eta} dt = S. \tag{3.9}$$

From (3.9) we see that the initial price λ_0 is an implicit function of S, θ_0, η, p and r, that is, $\lambda_0 = \lambda_0(S, \theta_0, \eta, p, r)$. By way of illustration, for $\eta = 1$ the initial shadow price λ_0 can be written as[5)]

$$\lambda_0 = \frac{\theta_0}{r + \theta_0} p\left[(e^{S\theta_0 p} - 1)^{-1} + 1\right],$$

and in case $\eta = \frac{1}{2}$, the solution of equation (3.9) is given by

$$\lambda_0 = \frac{\theta_0}{r + \theta_0} p\left[4e^{S(r+\theta_0)(\theta_0 \, p/(r+\theta_0))^{\frac{1}{2}}}\left[e^{S(r+\theta_0)(\theta_0 p/(r+\theta_0))^{\frac{1}{2}}} - 1\right]^{-2} + 1\right].$$

In Section 3.3, where we discuss how any given stock S should be used (priced) during periods of supply disruptions, we shall look at the comparative statics of (3.9). When our comparative-static analysis is quantitative in nature, we have, for other values of η than $\eta = 1$ and $\eta = \frac{1}{2}$, to solve (3.9) numerically in order to obtain the initial price λ_0.

In order to analyze optimal stockpiling during a free-trade regime, we have to take account of one additional kind of uncertainty. The date at which an embargo will be imposed - as well as the date, once it is imposed, it will end - has to be treated as a random variable. In Chapter 2 we derived a condition for optimal storage for the particular case in which the duration τ_1 of a free-trade regime is exponentially distributed. For this case the optimal level of inventories S* is given implicitly by

$$p = \frac{\theta_1}{r + \theta_1} \lambda_0(S^*, \theta_0, \eta, p, r), \tag{3.10}$$

where θ_1 is the probability rate that the country will be in an embargo regime in the next instant, given that there is presently no embargo. Expression (3.10) says that the cost of purchasing an additional unit of oil for storage should just equal the expected discounted marginal value of having slightly larger emergency reserves when the embargo comes (cf. Section 2.5). So in case the country carries an optimal stockpile S^*, the initial price λ_0 is known from equation (3.10). Inserting this into (3.9) gives us the following expression for the optimal stock S^*:

$$S^* = \int_0^\infty \left[\left[\frac{r + \theta_1}{\theta_1} - \frac{\theta_0}{r + \theta_0}\right] p \, e^{(r+\theta_0)t} + \frac{\theta_0}{r + \theta_0} p\right]^{-\eta} dt. \tag{3.11}$$

Expression (3.11) gives us the optimal stock S^* as a function of the parameters θ_0, θ_1, η, p and r, that is, $S^* = S^*(\theta_0, \theta_1, \eta, p, r)$. For $\eta = 1$ and $\eta = \frac{1}{2}$ the optimal stock can (by integration) be expressed as

$$S^* = \frac{1}{\theta_0 p} \ln\left[1 - \frac{\theta_0\theta_1 p}{(r + \theta_0)(r + \theta_1)}\right]^{-1},$$

and

$$S^* = \frac{2}{(r + \theta_0)(\theta_0 p/(r + \theta_0))^{\frac{1}{2}}} \ln \frac{1 + \left[(r + \theta_1)(r + \theta_1)/\theta_1\theta_0\, p\right]^{\frac{1}{2}}}{\left[(r + \theta_1)(r + \theta_0)/\theta_1\theta_0\, p - 1\right]^{\frac{1}{2}}},$$

respectively.[6] In Section 3.4 we will investigate how the value of S^* changes when there is a change in the parameters that determine the optimal stock.

3.3 OPTIMAL DEPLETION OF EMERGENCY RESERVES DURING AN EMBARGO

In this section we will analyze optimal depletion of the emergency reserves when the size of the reserves (not necessarily at the optimal level) is given. The important question is how the reserves should be priced so as to ensure an efficient utilization. The efficient price of the oil stock and its relation to the initial stock, the random length of the embargo, the elasticity of demand and so on, is given by the $\lambda_0(S, \theta_0, \eta, p, r)$ function and more specifically by (3.9).

Using the implicit function theorem we can derive the following comparative-static results (See Appendix 3:1):

$$\frac{\partial\lambda_0(S, \theta_0, \eta, p, r)}{\partial S} < 0, \qquad \frac{\partial\lambda_0(S, \theta_0, \eta, p, r)}{\partial \theta_0} < 0,$$

$$\frac{\partial\lambda_0(S, \theta_0, \eta, p, r)}{\partial \eta} < 0, \qquad \frac{\partial\lambda_0(S, \theta_0, \eta, p, r)}{\partial p} > 0\ ,$$

and

$$\frac{\partial\lambda_0(S, \theta_0, \eta, p, r)}{\partial r} < 0.$$

These results are of course in accordance with what would be expected. The jump in the price from p to λ_0 that occurs when the economy enters an embargo regime is smaller: the larger the initial stock of reserves S; the higher the probability θ_0 that the economy will be in a free trade period in the next period; the higher the (absolute) elasticity of demand η; the higher the rate of interest r; and the lower the world market price in a free-trade regime p.

To illustrate the orders of magnitude that emerge from the comparative statics, let us analyze the problem numerically. Consider the following parameter values as our reference case: let the (absolute) value of the elasticity of demand η be equal to 0.3, the annual rate of interest be 6 per cent and the relative price of oil p, in terms of the background commodity in a free (nondisrupted) market, be equal to unity. This means that the normal consumption during one time period (which we take to be one week) is given by $q = u'^{-1}(p) = 1$. Furthermore, our reference case is based on the assumption that the hazard rate θ_0 is 0.042. We can interpret the parameter θ_0 as the probability that the embargo will expire next week, given that it has not already expired.[7)] Since the expected duration of the embargo $E[\tau_0]$ is equal to $1/\theta_0$, the numerical value of the parameter θ_0 corresponds to an expected duration of the embargo of 24 weeks. Suppose also that the initial stock S_0 is 75, that is, that the initial stock is 75 times the rate of consumption in a free-trade period.[8)] So we assume:

η	the absolute value of the elasticity of demand	= 0.3
r	the annual rate of interest	= 6 %
p	the relative price of oil in free trade	= 1
$1/\theta_0$	the average duration of an embargo	= 24 weeks
S_0	the initial stock of emergency reserves	= 75 weeks

Under these assumptions the solution of equation (3.9) yields an initial price of 1.9.[9)] This means that when the economy enters the embargo regime there is an instantaneous rise in the price of oil by 90 per cent. The price jump reduces the initial rate of consumption by 18 per cent.

Should the initial stock only be of size 60, the price jumps up to 3.3 just when the embargo starts which in turn implies an immediate drop in the rate of consumption by around 30 per cent. Equation (3.6) does not yield a constant rate of increase in λ_t. The price rises during the embargo (in the reference case) initially at a weekly rate of around 2 per cent (or a yearly rate of around 110 per cent). As the spot price λ_t becomes larger, the rate of price increase approaches $r + \theta_0$ (i.e., a weekly rate of around 4.3 per cent). This means that the spot price of oil given that the embargo is still going on rises at a rate exceeding the risk-free return by a factor varying between 18 and 37. However, note that the expected rate of price increase is equal to the risk-free rate of interest. This can be seen if we rewrite equation (3.6) as

$$\frac{\dot{\lambda}_t}{\lambda_t} + \theta_0 \left[\frac{p - \lambda_t}{\lambda_t} \right] = r,$$

where $p - \lambda_t$ is the capital gain from holding oil should the embargo expire. In the first week of the embargo, for the reference case parameters, this capital gain (or rather loss) is -0.92 and its expected value $\theta_0(p - \lambda_t)$ is -0.038.

Figure 3.1 displays, for the reference case parameters, the ratio of emergency reserves remaining to initial stock S_0. We find that the fraction of reserves remaining after 6 months (i.e., on reaching the expected duration) is as high as 0.76. After twice the expected length of the embargo (i.e., 12 months) the cumulative resource use amounts to only 43 per cent of the initial stock, that is, the reserve level has sunk to 57 per cent of the stock that was in storage when the embargo started. In fact, it can be computed that the first and second half-lives of the emergency reserves are about 15 and 13 months, respectively (provided, of course, that the embargo is still going on). It can also be shown that as t goes to infinity, the remaining stock will approach zero asymptotically from above. The orders of magnitude involved are quite sensible under our assumptions. Since the exponential distribution implies that there is no upper bound on the duration of an embargo and since $u'(q_t) = q_t^{-1/\eta}$ goes to infinity as q_t goes to zero, it makes sense to allocate inventories so as to ensure consumption at each date, but so as not to leave redundant stocks asymptotically.

Figure 3.1 Fraction of reserves remaining

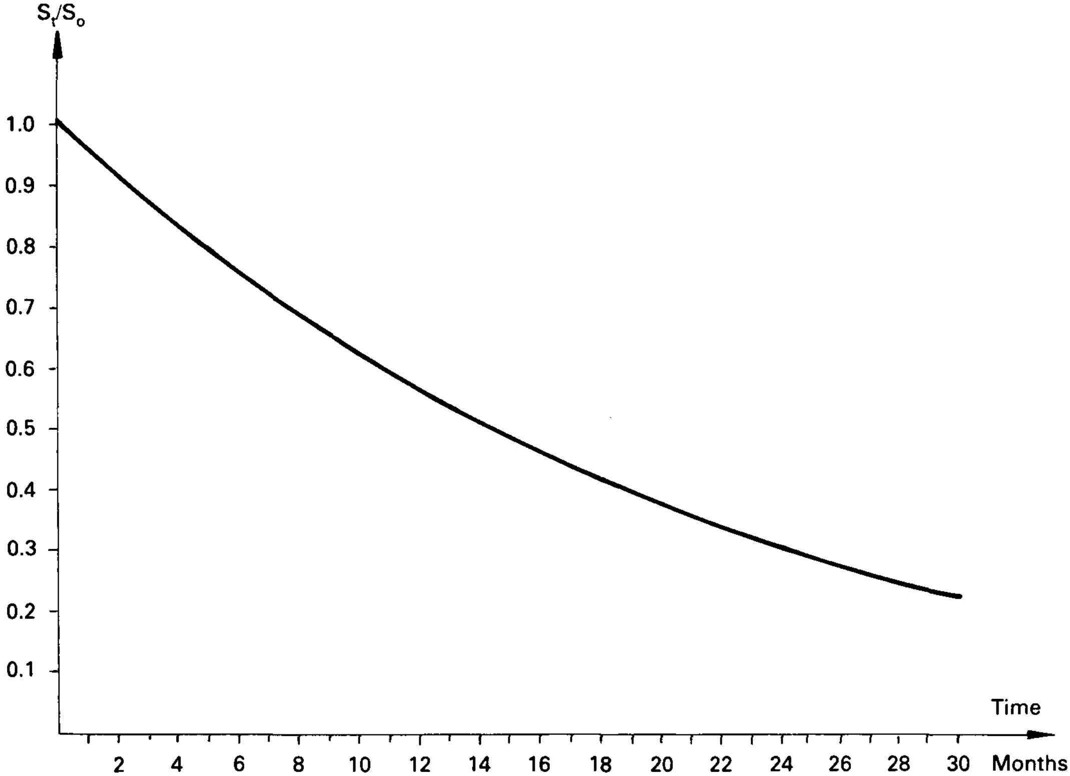
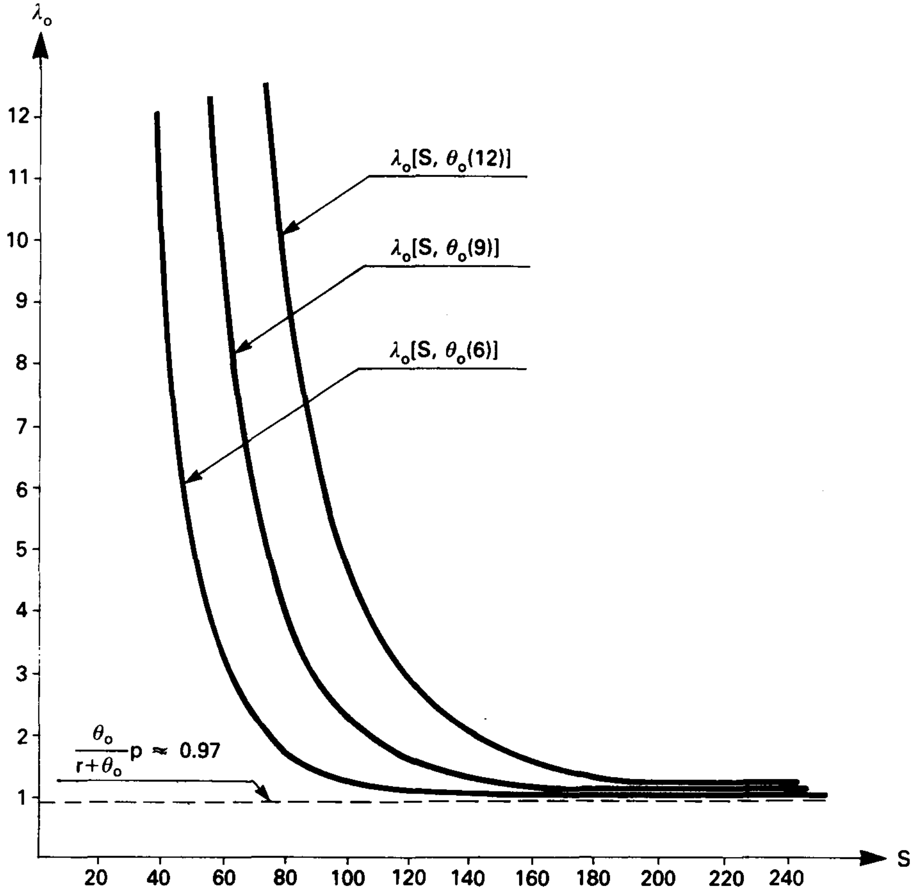


Figure 3.2 illustrates the initial price λ_0 as a function of the emergency reserves on hand when the embargo is imposed, for three different values of the parameter θ_0. The one denoted by $\lambda_0(S, \theta_0(6))$ corresponds to an expected duration of the embargo of 6 months, that is, to the reference case. The other two illustrate the dependence of λ_0 on S when the expected length of the embargo is 9 and 12 months, respectively.[10)]

The curves of Figure 3.2 are strictly convex throughout, confirming the results of the Appendix (see (A.3.2) and (A.3.3)). In fact, they look exactly like the curve describing the relationship between the price of an extracted exhaustible resource in the presence of a constant marginal extraction cost and the stock of the natural resource. So, to borrow results from the natural resource literature, let π_t denote the competitive spot price of an unextracted resource, that is, the royalty price, and let μ_t

Figure 3.2 The initial price as a function of the initial stock for three different transition probabilities θ_o *)



*) The transition probabilities (i.e., the probabilities that the embargo will end in the next week) are 0.042, 0.027 and 0.021 for the curves $\lambda_o(S, \theta_o(6))$, $\lambda_o(S, \theta_o(9))$ and $\lambda_o(S, \theta_o(12))$, respectively.

denote the price of the extracted resource. If C is the constant marginal extraction cost, we have that $\mu_t = \pi_t + C$. Stock equilibrium in the market for assets requires that $\dot{\pi}_t/\pi_t = r$. Hence we can write

$$\frac{\dot{\mu}_t}{\mu_t - C} = r, \tag{3.12}$$

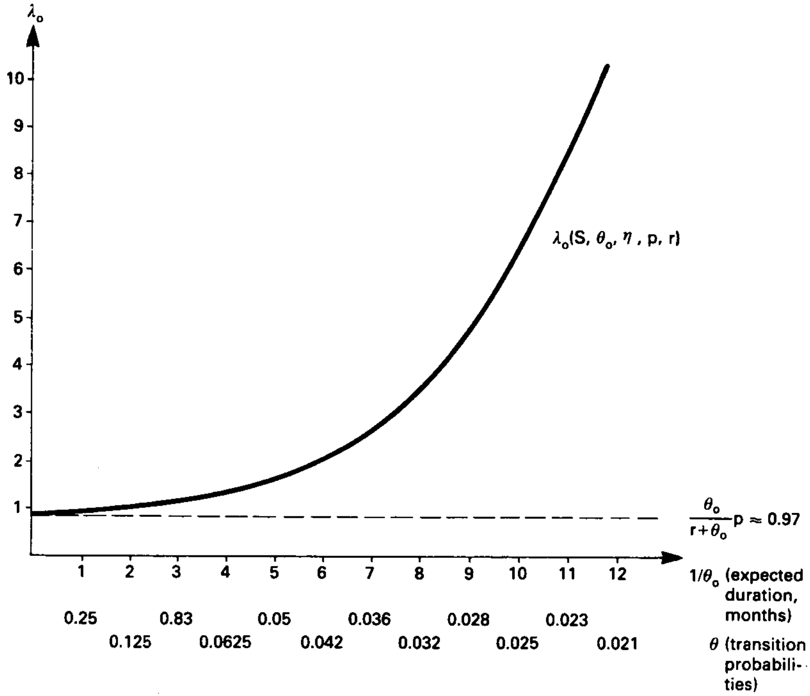
which says that the resource should be depleted in such a way that its price, net of extraction cost, increases at the rate r. Rewriting the differential equation (3.6) as

$$\frac{\dot{\lambda}_t}{\lambda_t - \frac{\theta_0}{r + \theta_0} p} = r + \theta_0, \tag{3.13}$$

and identifying the constant marginal extraction cost C in (3.12) with the constant $\theta_0 p/(r + \theta_0)$ in (3.13) and the rate of interest r in (3.12) with $r + \theta_0$ in (3.13), we see that the price paths become formally identical.[11)] From the literature of natural resources we have that the royalty $\pi_t = \mu_t - C$ of the resource owner will approach zero as the initial stock goes to infinity, and that the price μ_t will approach the extraction cost C. We also know that the royalty goes to infinity as S approaches 0 provided, of course, that the demand for the resource flow is positive irrespective of the price.[12)] Owing to our identification of the embargo problem with the natural resource problem it comes as no surprise that the curve $\lambda_0 = \lambda_0(S, \theta_0, \eta, p, r)$ approaches the value $p\theta_0/(r + \theta_0)$ asymptotically from above and that λ_0 approaches infinity as $S \to 0$, as is shown in Figure 3.2.

The effect of a change in θ_0 is also displayed in Figure 3.2. A decrease in θ_0 means that the probability that the country will be in a free-trade regime next week, given that there is an embargo going on now, decreases. For any given level of the emergency reserves the figure reveals that the response to an increase in the expected duration of embargoes $(1/\theta_0)$ is an increase in the initial price. Moreover, the initial price increases at an increasing rate with respect to the average duration of the embargo. This is more clearly brought out in Figure 3.3 for the particular stock level of 75. We see here that the curve in question displays a steeper and steeper slope as the expected length of the embargo increases (i.e., as the probability that the country will be in a free-trade regime next week decreases).

Figure 3.3 The initial price as a function of the hazard rate θ_0



These results are what intuition suggests. However, they are in fact the outcome of two forces that work in opposite directions. The following expression for the spot price path makes this apparent:

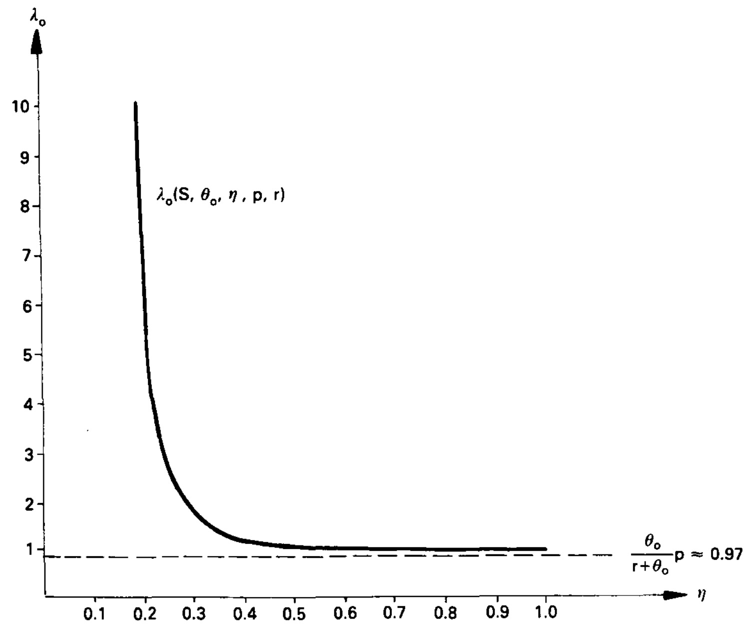
$$\frac{\dot{\lambda}_t}{\lambda_t} = (r + \theta_0)\left[\frac{\lambda_t - \frac{\theta_0}{r + \theta_0} p}{\lambda_t}\right] \tag{3.14}$$

A decrease in θ_0 (i.e., an increase in the expected duration of the embargo) can be interpreted as a decrease in the discount rate $(r + \theta_0)$ applied to future consumption during the embargo. This in turn tends to reduce the rate of increase in the component $(\lambda_0 - \theta_0 p/(r + \theta_0))$ of the spot price λ_t, i.e., the royalty price. However, a decrease in θ_0 also means that the expected discounted value of a unit in storage at the end of the embargo $(\theta_0 p/(r + \theta_0))$ is reduced (Remember that a lower value of θ_0 means that the embargo becomes longer, i.e., that the end of the embargo is more distant).[13] This latter effect tends to increase the rate of price increase. As is shown in the appendix (A.3.4), the former effect (the "interest-rate" effect) outweights the latter effect (the "extraction-cost" effect). So, the net effect of a decrease in θ_0 is to reduce the rate of price increase. And this in turn implies (since the same resource constraint must hold for any parameter value of θ_0) that the price is higher for an initial interval of time.

Equation (3.14) makes the effect of a change in the rate of interest immediately evident. For a given level of S, an increase in r makes the price path steeper which in turn implies a lower price initially. So an increase in r will cause the λ_0 curves of Figure 3.2 to shift downward. However, the initial price is not very sensitive to changes in the interest rate. In the reference case, which is based on an annual rate of interest of 6 per cent, the initial price is 1.92. If the interest rate were higher, say 10 per cent, the initial price would be reduced to 1.84.

The responsiveness of the initial price λ_0 to elasticities of demand ranging from 0.2 to 1.0 is illustrated in Figure 3.4. As the figure shows, the elasticity of demand is of critical importance for our analysis. For example, in the reference case with a demand elasticity of 0.3 the arrival of the embargo leads to a discontinuous jump in the price by 90 per cent. If the elasticity were 0.2 instead, the same event would lead to a price which is ten times higher than the price in a nondisrupted market. These results have important implications for the optimal storage question which is taken up in the next section.

Figure 3.4 The initial price as a function of the (absolute) elasticity of demand



3.4 THE OPTIMAL SIZE OF EMERGENCY RESERVES

As we concluded in the previous section, the size of the emergency reserves has a substantial bearing on the jump in the spot price that occurs when the economy enters the embargo regime. The question we raise in the present section is the following: How much of the good in question should be carried in the face of the embargo threat?

On the basis of equation (3.11) we will perform a quantitative comparative-static analysis.[14] Let us retain the parameter values used in the reference case of the previous section and suppose that the probability that the economy will be in an embargo regime in the next week, given that there is no embargo in effect now, is 0.004.[15] This probability implies that the average duration of a free-trade regime is 5 years. So we assume:

η	the absolute value of the elasticity of demand	= 0.3
r	the annual rate of interest	= 6 %
p	the relative price of oil in free trade	= 1
$1/\theta_0$	the average duration of an embargo	= 24 weeks
$1/\theta_1$	the average duration of a free-trade period	= 5 years

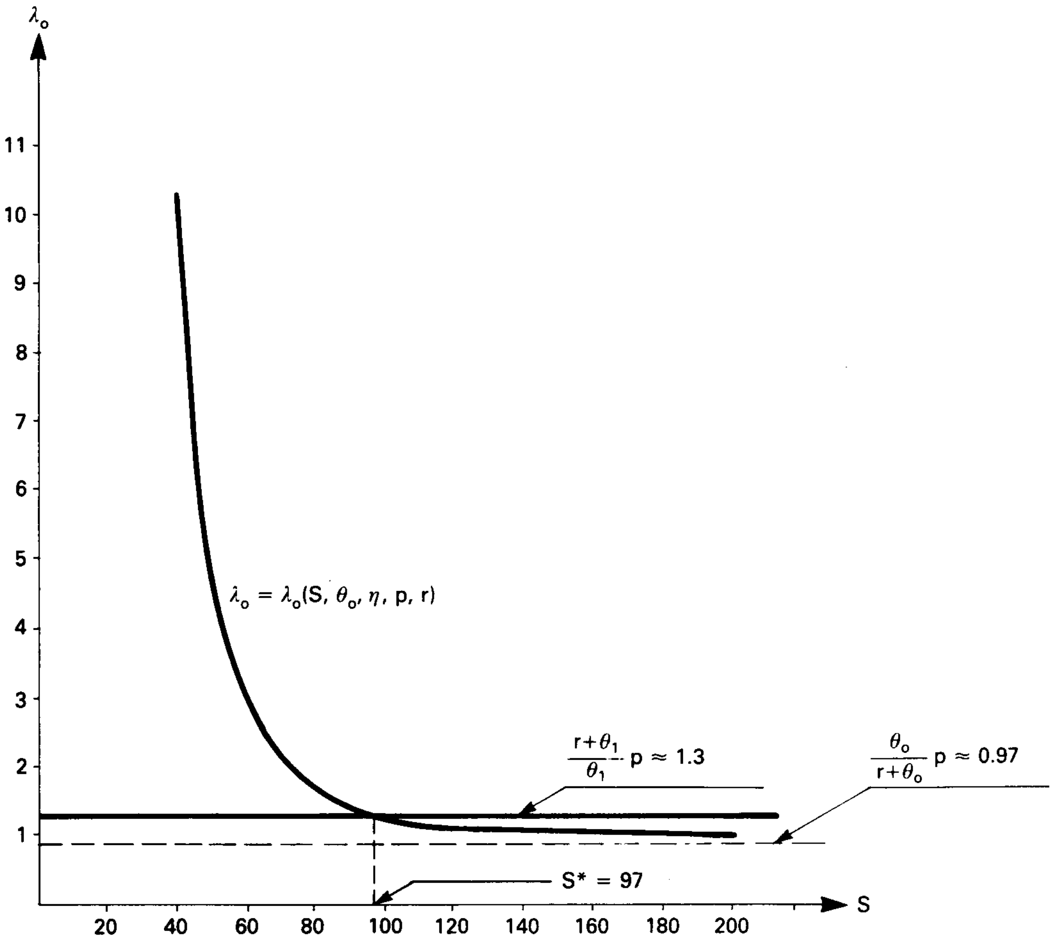
With these parameter values, the optimal size of the emergency reserves is 97, that is, the optimal stock is 97 weeks of normal consumption. For a stock $S = S^* = 97$, the initial price during an embargo is given by $(r + \theta_1)p/\theta_1 = 1.3$ (cf. equation 3.10). This in turn implies that there is an instantaneous drop in consumption by 8 per cent as the economy enters the embargo regime with an optimal stock.

Note that S^* is implicitly given by

$$\lambda_0(S^*, \theta_0, \eta, p, r) = \frac{r + \theta_1}{\theta_1} p, \qquad (3.15)$$

which is just a rewriting of the optimal storage condition (3.10). Using (3.15) we can illustrate S^* in a diagram. This is done in Figure 3.5 where the curve $\lambda_0 = \lambda_0(S, \theta_0, \eta, r, p)$ is the lower of the λ_0 curves in Figure 3.2, that is, the one that corresponds to the reference case.

Figure 3.5 The optimal size of emergency reserves (reference case)



Some comparative-static results are immediate from Figure 3.5. Since $(r + \theta_1)p/\theta_1$ is decreasing in θ_1, a decrease in θ_1 means that the point of intersection between the λ_0 curve (which is left untouched when θ_1 is changed) and the line $(r + \theta_1)p/\theta_1$ will move to the left:

$$\frac{\partial S^*(\theta_0, \theta_1, \eta, r, p)}{\partial \theta_1} > 0.$$

This means that if the periods of free trade become longer on the average (that is, the probability that an embargo will be imposed at any given moment decreases) it will be wise to reduce the reserves.

The effect of a change in θ_0 is also rather evident. Here the line $(r + \theta_1)p/\theta_1$ is unchanged and from Section 3.3 we know that a decrease in θ_0 (that is, an increase in the average duration of the embargo) shifts the λ_0 curve upward. The optimal response to an increase in the average duration (that is, the probability that the embargo will end, given that it has been imposed, decreases) is thus to increase the emergency reserves. Hence:

$$\frac{\partial S^*(\theta_0, \theta_1, \eta, r, p)}{\partial \theta_0} < 0.$$

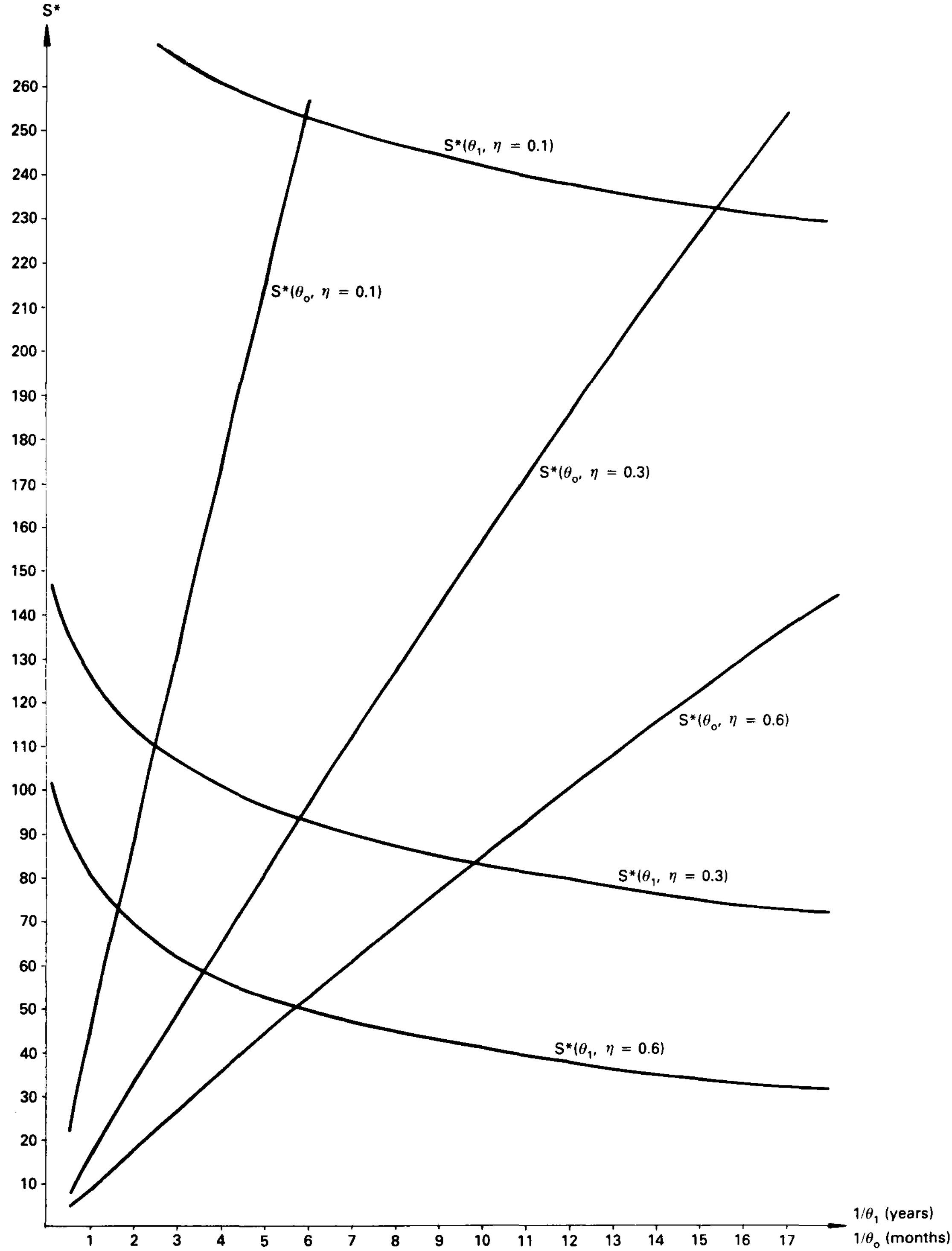
The effect of a change in the elasticity of demand η is also rather immediate from Figure 3.5 and the discussion of Section 3.3. Recall that an increase in the absolute value of the elasticity of demand η will shift the λ_0 curve downward. This in turn implies that the point of intersection between the λ_0 curve and the line $(r + \theta_1)p/\theta_1$ will move to the left:

$$\frac{\partial S^*(\theta_0, \theta_1, \eta, r, p)}{\partial \eta} < 0.$$

The economic reason behind this result is clear. An increase in η means that the country becomes less sensitive to embargoes, and the optimal response to this is of course to reduce the emergency stock.

Figure 3.6 illustrates the orders of magnitude involved in the comparative-static derivatives $\partial S^*/\partial\theta_1$, $\partial S^*/\partial\theta_0$ and $\partial S^*/\partial\eta$. The curves denoted by $S^*(\theta_1, \eta = 0.1)$, $S^*(\theta_1, \eta = 0.3)$ and $S^*(\theta_1, \eta = 0.6)$ display the impact on optimal reserves S^* of changes in the transition probability θ_1 for the indicated values on the elasticity of demand. The consequence of an increase in the absolute elasticity of demand is quite a substantial

Figure 3.6 Optimal levels S^* for different values of θ_o, θ_1 and η



displacement of the $S^*(\theta_1, \cdot)$ curve downward. If the elasticity is as low as 0.1, the optimal reserves are between 3.5 and 6.7 times larger (in the interval of an expected duration of the free-trade regime ranging from 1 year to 15 years) than what optimal inventory behavior would require if the elasticity were as high as 0.6. Likewise, the curves denoted by $S^*(\theta_0, \eta = 0.1)$, $S^*(\theta_0, \eta = 0.3)$ and $S^*(\theta_0, \eta = 0.6)$ illustrate the effect on optimal storage of changes in the hazard rate θ_0. Figure 3.6 shows that the effect of a decrease in the elasticity of demand is to raise the slope of the $S^*(\theta_0, \cdot)$ curve, which in turn implies a higher reserve requirement. If the transition probability θ_0 is halved (that is, if the expected duration of the embargo has doubled) the increase in the emergency reserves is quite dramatic.

The effect of a change in the rate of interest is immediately evident from Figure 3.5. From Section 3.3 we know that an increase in r will cause the λ_0 curve of Figure 3.5 to shift downward, while the line $(r + \theta_1)p/\theta_1$ will shift upward. The intersection S^* will then move to the left:

$$\frac{\partial S^*(\theta_0, \theta_1, \eta, p, r)}{\partial r} < 0.$$

So the qualitative result is that a higher interest rate implies that the optimal emergency stock falls: the attractiveness of storage is reduced since the cost of storage is increased. The orders of magnitude involved: if the rate of interest is increased from 6 per cent to 10 per cent the optimal reserve is reduced from 97 to 85.

The effect of a change in the price p in free trade is not immediately evident from Figure 3.5. A higher p will shift the λ_0 curve upward, but at the same time the line $(r + \theta_1)p/\theta_1$ will shift upward too. The intersection S^* may therefore move in either direction. We therefore have to rely on the differential calculus to determine which effect dominates. In the Appendix (A.3.13) it is shown that

$$\frac{\partial S^*(\theta_0, \theta_1, \eta, p, r)}{\partial p} < 0$$

must hold. So an increase in p causes the initial purchase to be more expensive, reducing the desired stockpile, but also makes the excess inventories at the end of the embargo more valuable, increasing the optimal inventory. The calculus shows that the former effect dominates the latter.

3.5 THE EFFECTS OF UNCERTAINTY ON STOCK WITHDRAWALS AND STOCKPILING

There has been an extensive discussion in the finance literature about whether increasing uncertainty about for example, income, returns and lifetime, will cause the consumer and investor to save more or less for the future. (See, for example, Levhari and Srinivasan (1968), Hahn (1970), Yaari (1970) and Levhari and Mirman (1977)). Mirrlees (1974) and Bismut (1975) illuminate a similar problem in the framework of a stochastic growth model. These models have shown that the effects of an increase in risk depends on the particular parametrization employed. This ambiguous result is in fact what intuition suggests. For example, if the uncertainty is specified as imperfect knowledge of returns to investment in financial assets, "increased uncertainty, will either lower saving because 'a bird in the hand is worth two in the bush' or raise it because a risk averse individual, in order to insure his minimum standard of living, saves more in face of increased uncertainty" (Rothschild and Stiglitz (1971:69).

In a study of the effect of uncertainty in the arrival date of a new technology on the rate of depletion of natural resources, Dasgupta and Stiglitz (1981) make a counter-intuitive conjecture: uncertainty leads to a faster initial depletion rate if the initial stock is small and to a slower depletion rate if it is large.

In this section we will pose a similar question: What is the effect of uncertainty about the date at which the embargo will end on the rate of stock withdrawals? We will also answer the question: What is the effect of the same uncertainty on the desired level of emergency reserves? In order to study these problems in a meaningful way we have to compare two embargo situations having the same expected duration but differing as to the distribution of the probability of the length of the embargo. For a justification of this approach in studies of the effect of uncertainty with regard to lifetime on optimal consumption decisions and for a discussion of the seemingly contradictory results that follow a comparison admitting arbitrary probability distributions of lifetimes, see Levhari and Mirman (1977). A more general discussion of the effects of changes in the degree of uncertainty is to be found in Rothschild and Stiglitz (1970). Applications may be found in Rothschild and Stiglitz (1971).

In order to be able to relate our results to some previous work on trade disruption and stockpiling characterized by a 2-period setting (e.g., Nordhaus (1974), Kuenne, Blankenship and McCoy (1979), Hogan (1982)) we will only make a comparison between a perfectly certain duration of the embargo and an uncertain duration.[16)] To make the comparison well defined, the considerations in the preceding paragraph suggest that the duration in the certain case must be equal to the expected duration in the risky case. Then the comparison clearly belongs to the wider class of problems in which one compares two random variables with the same expected value. The only difference is that we assume the probability distribution of the duration in one of the cases to be degenerate.

According to the discussion in Section 3.2, to be efficient when the embargo length is exponentially distributed an optimal stock withdrawal policy must satisfy the price path

$$\frac{\dot{\lambda}_t}{\lambda_t} = r + \theta_0 \, (\lambda_t - p)/\lambda_t \tag{3.16}$$

and the feasibility requirement

$$\int_0^\infty q_t \, dt = \int_0^\infty \lambda_t^{-\eta} \, dt = S. \tag{3.17}$$

Equations (3.16) and (3.17) give the efficient price of the oil stock, $\lambda_0 = \lambda_0(S, \theta_0, \eta, p, r)$. The optimal size of the emergency reserves $S^* = S^*(\theta_0, \theta_1, \eta, p, r)$ is given explicitly by:

$$S^* = \int_0^\infty \left[\left[\frac{r + \theta_1}{\theta_1} - \frac{\theta_0}{r + \theta_0} \right] p e^{(r+\theta_0)t} + \frac{\theta_0}{r + \theta_0} p \right]^{-\eta} dt. \tag{3.18}$$

Let us now derive the corresponding expression for the efficient price λ_0^c and the optimal stock S^{*c} when the random duration of the embargo regime equals its expected value. Since the duration of an embargo τ_0 is a random variable with probability density

$$f_0(\tau_0) = \theta_0\, e^{-\theta_0 \tau_0},$$

its expected value is (by partial integration) equal to

$$E[\tau_0] = \int_0^{\infty} \tau_0\, \theta_0\, e^{-\theta_0 \tau_0}\, d\tau_0 = \frac{1}{\theta_0}.$$

Since the date τ_0 is assumed to be perfectly known and equal to $1/\theta_0$ the problem (3.4) can be simplified to:

$$\bar{\bar{V}}(0, S) = \max_{\{q_t\}} \int_0^{1/\theta_0} e^{-rt} \frac{\eta}{\eta - 1} q_t^{\frac{\eta-1}{\eta}}\, dt - e^{-r/\theta_0}\, p\left[S^* - S_{1/\theta_0}\right] \qquad (3.19)$$

$$\text{subject to} \quad \int_0^{1/\theta_0} q_t\, dt = S_0;$$

$$S_t,\ q_t \geq 0.$$

To locate the condition that the optimal stock withdrawal policy must satisfy when the duration is known to be $1/\theta_0$, we let λ_t^c denote the price of oil removed from the stock and π_t denote the price of oil remaining in the reserves. Since $e^{-r/\theta_0}\, p$ is the cost of removing one unit of oil from stock we must have that $\lambda_t^c = e^{-r/\theta_0}\, p + \pi_t$. The marginal cost of removing oil is constant so we can write

$$\lambda_t^c = e^{-r/\theta_0}\, p + \pi_0\, e^{rt},$$

where π_0 is the initial royalty price. This in turn implies that

$$\frac{\dot{\lambda}_t^c}{\lambda_t^c} = \frac{r\pi_0 e^{rt}}{e^{-r/\theta_0}\, p + \pi_0\, e^{rt}}, \qquad (3.20)$$

that is, λ_t^c rises at a variable rate less than r.

Equation (3.16) says that $r \leq \dot{\lambda}_t/\lambda_t \leq r + \theta_0$ while equation (3.20) gives $\dot{\lambda}_t^c / \lambda_t^c < r$. So, if we confine the analysis of the two embargo situations to involve only a comparison between the rates of increase in the spot price, it might be thought that less is unambiguously saved in the risky situation. Increased uncertainty causes the spot price to grow at a higher rate. This means that the initial price must be lower if the same resource constraint is imposed in the two problems. The conjecture that follows from this preliminary observation is that the rate of stock withdrawals is higher in the risky situation and this in turn implies lower savings. However, what makes the problem a tricky one is the fact that the two problems have different resource constraints. And more important, the constraint in the risky problem calls, ceteris paribus, for more conservation (saving) than the constraint in the certain problem. Remember that inventories are allocated in the risky situation so as to ensure consumption at each date in the interval $[0, \infty]$, while there is no point in having redundant stocks at date $1/\theta_0$ in the certain situation. So there are two forces that work in opposite directions. "A bird in the hand is worth two in the bush" and "insurance against a cold winter" are both appealing arguments.

Let us examine numerically which of the two effects is larger. To find an expression for the initial value λ_0^c of the price path we solve the differential equation

$$\dot{\lambda}_t^c = (\lambda_t^c - e^{-r/\theta_0}\, p)r:$$

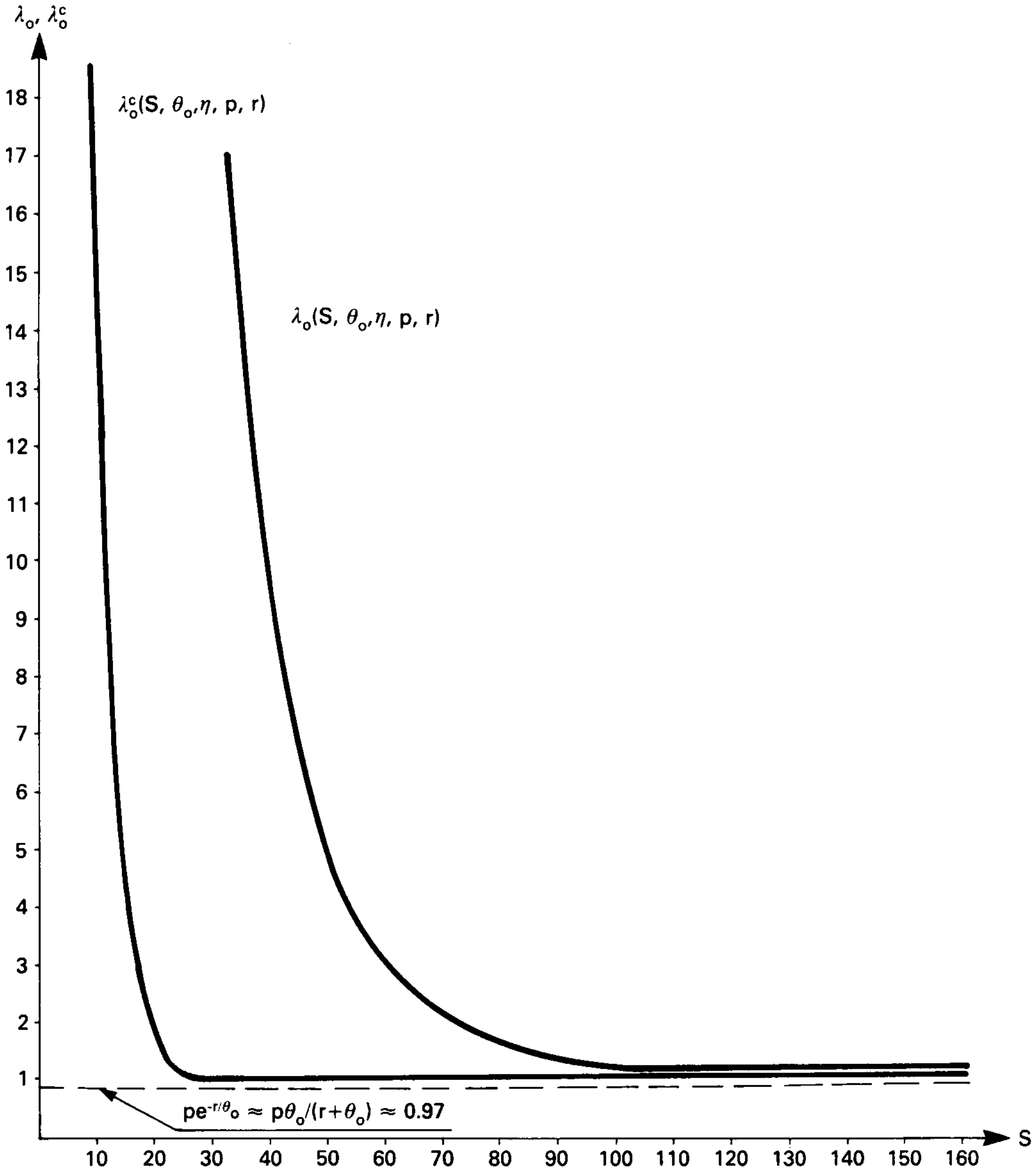
$$\lambda_t^c = (\lambda_0^c - e^{-r/\theta_0}\, p)\, e^{rt} + e^{-r/\theta_0}\, p, \tag{3.21}$$

where λ_0^c is given by the resource constraint

$$\int_0^{1/\theta_0} \left[(\lambda_0^c - e^{-r/\theta_0}\, p)\, e^{rt} + e^{-r/\theta_0}\, p\right]^{-\eta} dt = S. \tag{3.22}$$

Equation (3.22) is analogous to (3.9) which gives the initial price λ_0 in the risky situation. Figure 3.7 compares (for the reference case parameters) the solution of (3.22) and (3.9) for different values of emergency reserves on hand when the embargo starts. The λ_0 curve is the lowest of the three curves in Figure 3.2. Notice that the λ_0^c curve approaches the value $e^{-r/\theta_0}\, p$ asymtotically from above (cf. our identification of the stock withdrawal problem with the natural resource problem in Section 3.3). The conclusion that follows is that uncertainty leads to a higher initial price and thereby to a slower initial stock withdrawal rate.

Figure 3.7 The effects of uncertainty about the date at which an embargo will end on the initial royalty price



In order to analyze optimal stockpiling during a free-trade regime we have to take account of the fact that the date an embargo will be imposed is - in addition to the date, once imposed, it will end - a random variable. The value of a unit of oil at the beginning of this future embargo is $\lambda_0^c(S)$, where λ_0^c is the initial price when the random duration of the embargo is assumed to equal its expected duration and where we have indicated the dependence of λ_0^c on S explicitly. Since the embargo occurs at the random date τ_1, the discounted value is $e^{-r\tau_1}\lambda_0^c(S)$. The date τ_1 is exponentially distributed with parameter θ_1. Hence the expected discounted marginal benefit is equal to $\lambda_0^c(S)\theta_1/(r + \theta_1)$. The cost of storing one additional unit of oil under free trade is equal to p. Hence the optimal stock S^{*c} when the random duration of the embargo is equal to its expected value is given implicitly by

$$\lambda_0^c\,(S^{*c}) = \frac{r + \theta_1}{\theta_1}\,p. \tag{3.23}$$

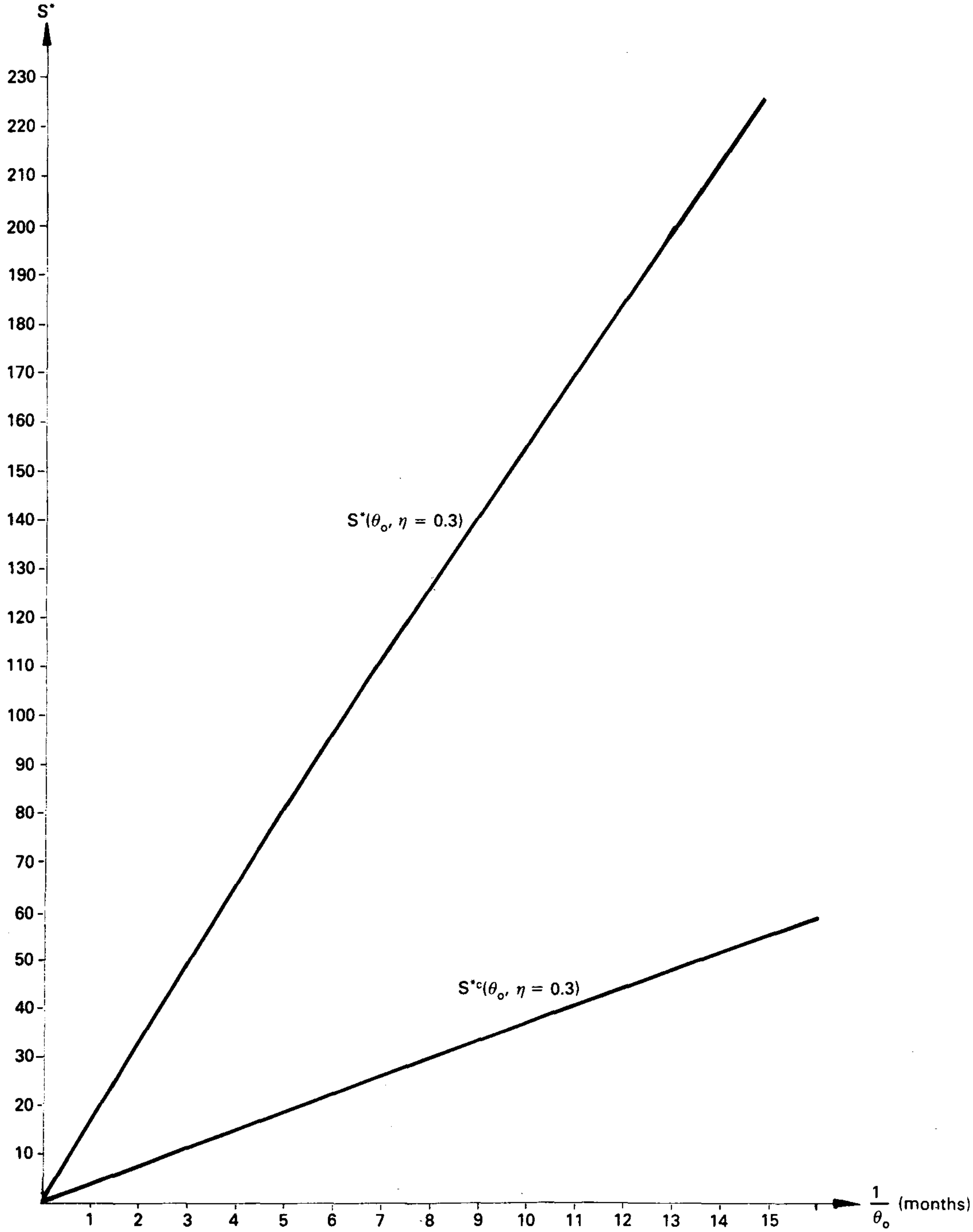
That is, equation (3.23) says that the marginal cost of storage should be equal to the expected discounted marginal benefit of storage.

Inserting (3.23) into (3.21) and substituting the solution for λ_t^c into the resource constraint at $S = S^{*c}$ gives us an explicit expression for the optimal stock:

$$S^{*c} = \int_0^{1/\theta_0} \left[\left(\frac{r + \theta_1}{\theta_1} - e^{-r/\theta_0}\right) pe^{rt} + e^{-r/\theta_0}\, p\right]^{-\eta} dt. \tag{3.24}$$

Equation (3.24) is the parallel to expression (3.18) which gives the optimal stockpile S^* when the duration of the embargo is uncertain. In order to examine whether or not the fact that the duration of the embargo is uncertain is an argument for holding larger emergency reserves, we compare the solution of (3.24) with that of (3.18). This is done in Figure 3.8. The figure reveals that the bias in stockpiling that arises when the random date at which an embargo will end is replaced by its expected value is quite substantial. The numerical calculations also show that the lower the (absolute) elasticity of demand, the larger is the difference between S^* and S^{*c}. Models characterized by a 2-period setting in which the duration of the embargo is assumed known with certainty and equal to its expected value can indeed be inadequate.

Figure 3.8 The effects of uncertainty about the date at which an embargo will end on the optimal level of emergency reserves



APPENDIX 3

3:1 Properties of the $\lambda_0(S, \theta_0, \eta, p, r)$ function

The initial price λ_0 is given implicitly by the resource constraint

$$\int_0^\infty \lambda_t^{-\eta}\, dt = S \tag{A.3.1}$$

where

$$\lambda_t = (\lambda_0 - \frac{\theta_0}{r + \theta_0} p)e^{(r+\theta_0)t} + \frac{\theta_0}{r + \theta_0} p.$$

By the implicit function theorem we can derive the following partial derivatives:

(i) Differentiating the resource constraint (A.3.1) with respect to λ_0 and S and rearranging the terms yields

$$\frac{\partial \lambda_0}{\partial S} = \frac{1}{\int_0^\infty q'(\lambda_t)e^{(r+\theta_0)t}dt} < 0, \tag{A.3.2}$$

where

$$q'(\lambda_t) \equiv - \eta\, \lambda_t^{-(\eta+1)}.$$

We also have

$$\frac{\partial^2 \lambda_0}{\partial S^2} = - \frac{\int_0^\infty q''(\lambda_t)\left[e^{(r+\theta_0)t}\right]^2 dt}{\left[\int_0^\infty q'(\lambda_t)e^{(r+\theta_0)t}dt\right]^3} > 0, \tag{A.3.3}$$

where

$$q''(\lambda_t) \equiv \eta(\eta + 1)\lambda_t^{-(\eta +2)}.$$

(ii) Differentiating (A.3.1) with respect to λ_o and θ_o and rearranging terms results in

$$\frac{\partial \lambda_o}{\partial \theta_o} = - \frac{\int_0^\infty q'(\lambda_t) \frac{\partial \lambda_t}{\partial \theta_o} dt}{\int_0^\infty q'(\lambda_t) e^{(r+\theta_o)t} dt} < 0. \tag{A.3.4}$$

The derivative in (A.3.4) is strictly less than zero since

$$\frac{\partial \lambda_t}{\partial \theta_o} = te^{(r+\theta_o)t}\left[\lambda_o - \frac{\theta_o}{r + \theta_o} p\right] + \frac{rp}{(r + \theta_o)^2}\left[1 - e^{(r+\theta_o)t}\right] > 0.$$

The sign of $\frac{\partial \lambda_t}{\partial \theta_o}$ follows from

$$\frac{\partial \lambda_t}{\partial \theta_o} = 0 \quad \text{for} \quad t = 0,$$

and

$$\frac{\partial^2 \lambda_t}{\partial t \partial \theta_o} = e^{(r+\theta_o)t}\left[t(r + \theta_o)[\lambda_o - \frac{\theta_o}{r + \theta_o} p] + [\lambda_o - p]\right] > 0,$$

since $(\lambda_o > p)$.

(iii) Differentiating (A.3.1) with respect to λ_o and η yields after rearranging terms,

$$\frac{\partial \lambda_o}{\partial \eta} = \frac{\int_0^\infty q(\lambda_t) \ln \lambda_t \, dt}{\int_0^\infty q'(\lambda_t) e^{(r+\theta_o)t} dt} < 0, \tag{A.3.5}$$

since $\lambda_t > p = 1$.

(iv) The expression for $\partial\lambda_0/\partial p$ can be obtained by differentiating the resource constraint (A.3.1) with respect to λ_0 and p:

$$\frac{\partial\lambda_0}{\partial p} = - \frac{\int_0^\infty q'(\lambda_t)\frac{\partial\lambda_t}{\partial p}\,dt}{\int_0^\infty q'(\lambda_t)e^{(r+\theta_0)t}dt} > 0. \tag{A.3.6}$$

The sign of $\partial\lambda_0/\partial p$ follows from

$$\frac{\partial\lambda_t}{\partial p} = \frac{\theta_0}{r + \theta_0}\left[1 - e^{(r+\theta_0)t}\right] < 0.$$

(v) Differentiating the resource constraint with respect to λ_0 and r yields:

$$\frac{\partial\lambda_0}{\partial r} = - \frac{\int_0^\infty q'(\lambda_t)\,\frac{\partial\lambda_t}{\partial r}\,dt}{\int_0^\infty q'(\lambda_t)e^{(r+\theta_0)t}\,dt} < 0, \tag{A.3.7}$$

where

$$\frac{\partial\lambda_t}{\partial r} = \frac{\theta_0}{(r + \theta_0)^2}\,p\,e^{(r+\theta_0)t} + \left[\lambda_0 - \frac{\theta_0}{r + \theta_0}\,p\right]t\,e^{(r+\theta_0)t}$$

$$- \frac{\theta_0}{(r + \theta_0)^2}\,p > 0.$$

The sign of $\partial\lambda_t/\partial r$ follows from the fact that for all $t > 0$, the first term is larger than the third term, and that $(\lambda_0 - \theta_0 p/r + \theta_0)$ is always positive.

3:2 Properties of the $S^*(\theta_0, \theta_1, \eta, p, r)$ function

The optimal stock S^* is given by the following equation

$$\lambda_0(S^*, \theta_0, \eta, p, r) = \frac{r + \theta_1}{\theta_1} p. \tag{A.3.8}$$

By the implicit function theorem we can derive the following partial derivatives:

(i) $$\frac{\partial S^*}{\partial \theta_0} = - \frac{\partial \lambda_0}{\partial \theta_0} \left[\frac{\partial \lambda_0}{\partial S^*}\right]^{-1} < 0 \tag{A.3.9}$$

The sign of $\partial S^*/\partial \theta_0$ follows from (A.3.2) and (A.3.4).

(ii) $$\frac{\partial S^*}{\partial \theta_1} = - \frac{rp}{(\theta_1)^2} \left[\frac{\partial \lambda_0}{\partial S^*}\right]^{-1} > 0, \tag{A.3.10}$$

where the sign follows from (A.3.2).

(iii) $$\frac{\partial S^*}{\partial \eta} = - \frac{\partial \lambda_0}{\partial \eta} \left[\frac{\partial \lambda_0}{\partial S^*}\right]^{-1} < 0. \tag{A.3.11}$$

The sign of $\partial S^*/\partial \eta$ follows from (A.3.2) and (A.3.5)

(iv) $$\frac{\partial S^*}{\partial p} = \left[\frac{r + \theta_1}{\theta_1} - \frac{\partial \lambda_0}{\partial p}\right]\left[\frac{\partial \lambda_0}{\partial S^*}\right]^{-1} \qquad (A.3.12)$$

Since $\frac{\partial \lambda_0}{\partial p} > 0$ the sign of $\frac{\partial S^*}{\partial p}$ is yet indeterminate.

Making use of (A.3.2) and (A.3.6) we can rewrite (A.3.12) as

$$\frac{\partial S^*}{\partial p} = \int_0^\infty \left[\frac{r + \theta_1}{\theta_1} e^{(r+\theta_0)t} - \frac{\theta_0}{r + \theta_0}\left[e^{(r+\theta_0)t} - 1\right]\right] q'(\lambda_t)dt < 0 \qquad (A.3.13)$$

where the sign follows from the fact that

$$\frac{r + \theta_1}{\theta_1} > \frac{\theta_0}{r + \theta_0} .$$

(v) $$\frac{\partial S^*}{\partial r} = \left[\frac{p}{\theta_1} - \frac{\partial \lambda_0}{\partial r}\right]\left[\frac{\partial \lambda_0}{\partial S^*}\right]^{-1} < 0, \qquad (A.3.14)$$

the sign of which follows from (A.3.2) and (A.3.7).

FOOTNOTES

1) The superscript "o" in the variable λ_t^0 that were used in Chapter 2 to indicate that it was associated with an embargo regime, is dropped here. Since we assume instantaneous stock adjustment we have $\lambda_t^1 \equiv p$ where the superscript "1" indicated a free trade regime. So there is no risk of confusing the variables between the two regimes.

2) For the special case $\eta = 1$ we define instead $u(q_t) = \ln q_t$.

3) Recall that if no embargo is now in effect, the maximum value function $V(1, S)$ denotes the maximum expected net utility to be derived from optimal behavior with an initial stock S in storage. The $V(1, S)$ function is completely defined by (2.5) of Section 2.2.

4) The Pontryagin necessary conditions for maximization of (3.5) are

$$\lambda_t = q_t^{-1/\eta}$$

$$\dot{\lambda}_t = (r + \theta_o)\lambda_t - \theta_o \, p$$

$$\dot{S}_t = -q_t$$

Since $u'(q_t) = q_t^{-1/\eta}$ goes to infinity as $q_t \to 0$ there will always be strictly positive values for S_t and q_t. Hence we can disregard the shadow prices associated with the non-negativity constraints $S_t, q_t \geq 0$.

5) The following integration formulae have been applied:

Case $\eta = 1$

$$z = \int_0^\infty \frac{dt}{1 + ye^{xt}} \Rightarrow z = \frac{1}{x} \ln \frac{1 + y}{y} \Rightarrow y = \left[e^{xz} - 1\right]^{-1}$$

Case $\eta = \frac{1}{2}$

$$z = \int_0^\infty \frac{dt}{\left[1 + ye^{xt}\right]^{\frac{1}{2}}} \Rightarrow z = \frac{2}{x} \ln \frac{1 + \left[1 + y\right]^{\frac{1}{2}}}{y^{\frac{1}{2}}} \Rightarrow y = 4e^{xz}\left[e^{xz} - 1\right]^{-2} .$$

6) See footnote 5.

7) The probability that an embargo regime will go into a free-trade regime during the interval of time, $0 \leq t \leq 1$ is

$$\int_0^1 \theta_0 e^{-\theta_0 t} dt = 1 - e^{-\theta_0} \approx \theta_0.$$

8) Since the parameters r and θ_0 are specified on a weekly basis, this means that what we have in storage when the embargo starts amounts to 75 weeks of normal consumption.

9) The numerical solutions in this chapter were obtained with the aid of some algorithms from the NAG (Numerical Algorithms Group) Library. Svante Johansson wrote the calling programs for the NAG routines.

10) The underlying transition probabilities (i.e., the probabilities that the country will be in a free-trade regime next week) for the curves $\lambda_0(S, \theta_0(6))$, $\lambda_0(S, \theta_0(9))$ and $\lambda_0(S, \theta_0(12))$ are 0.042, 0.027 and 0.021, respectively.

11) Note that

$$\frac{\theta_0}{r + \theta_0} p = E_0\left[e^{-r\tau_0} p\right],$$

where the expectation operator $E_0[\cdot]$ is defined by the density function $f_0(\tau_0)$ (cf. Section 3.2). In words this means that the "extraction cost" in the embargo model is equal to the expected discounted marginal replenishment cost.

12) For a linear demand curve there is a price so high that demand will be choked off. In the constant elasticity case such as ours, demand will never be choked off no matter how high the price.

13) Cf. footnote 11.

14) Cf. footnote 9.

15) To be exact, θ_1 is equal to 0.0038. Neglecting higher power terms in $e^{-\theta_1}$ (cf. footnote 7) we can interpret the parameter θ_1 as the probability that the economy will enter an embargo regime in the time interval $0 \leq t \leq 1$. Of course, $1 - e^{-\theta_1} \approx \theta_1$ is only a good approximation for small values of θ_1.

16) In fact, with the exponential distribution we are confined to this type of comparison. This is so since with the exponential distribution we cannot vary the variance without changing the mean (cf. Dasgupta and Stiglitz (1981)).

REFERENCES

Bismut, Jean-Michel. "Growth and Optimal Intertemporal Allocation." Journal of Economic Theory 10 (April 1975):239-257.

Dasgupta, Partha S., and Stiglitz, Joseph E. "Resource Depletion Under Technological Uncertainty." Econometrica 49 (January 1981):85-104.

Hahn, Frank H. "Savings under Uncertainty." Review of Economic Studies 37 (January 1970):21-24.

Hogan, William W. "Import Management and Oil Emergencies", in: D.A. Deese and J.S. Nye, eds., Energy and Security. Cambridge, Massachusetts: Ballinger Publishing Company, 1981.

Kuenne, Robert E., Blankenship, Jerry W., and McCoy, Paul F. "Optimal Drawdown Patterns for Strategic Petroleum Reserves." Energy Economics 2 (January 1979): 3-13.

Levhari, David, and Mirman, Leonard J. "Savings and Consumption with an Uncertain Horizon." Journal of Political Economy 85 (April 1977): 265-281.

Levhari, David, and Srinavasan, T.N. "Optimal Savings under Uncertainty." Review of Economic Studies 36 (April 1969):153-163.

Mirrlees, James A. "Optimal Accumulation under Uncertainty", in: J.H. Drèze, ed., Allocation under Uncertainty: Equilibrium and Optimality. London: MacMillan, 1974.

Nordhaus, William D. "The 1974 Report of the President's Council of Economic Advisors: Energy in the Economic Report." Americal Economic Review 64 (September 1974):558-565.

Numerical Algorithms Group Library Manual (FORTRAN) Mark 10 (NAG). Oxford, U.K. Numerical Algorithms Group Ltd.

Rothschild, Michael, and Stiglitz, Joseph E. "Increasing Risk. I: A Definition." Journal of Economic Theory 2 (September 1970):225-243.

Rothschild, Michael, and Stiglitz, Joseph E. "Increasing Risk. II: Its Economic Consequences." Journal of Economic Theory 3 (March 1971):66-84.

Yaari, M.E. "Uncertain Life Time, Life Insurance and the Theory of the Consumer." Review of Economic Studies 32 (April 1965):137-152.

4. Stockpiling and Stock Withdrawals in a Market Economy

4.1 INTRODUCTION

The results on optimal stockpiling and depletion of emergency reserves presented in Chapters 2 and 3 were derived in the context of a planning model. Now, is there any reason to believe that a market economy will supply the socially optimal level of storage under free trade and deplete the emergency reserves at a socially desirable rate during embargoes?

Over the past decade, rationales for government intervention in economies facing embargo threats have been discussed extensively. In Chapter 1 a brief account of this debate was given. In the present chapter we shall concentrate on one particular assumption that is critical to the validity of the welfare theorem, namely, the assumption that there must exist a complete set of risk and future markets.

An argument frequently seen in the economic literature is that actual market economies do not allocate resources efficiently because private agents will not undertake sufficient risk. For instance, Arrow and Lind (1970) conclude that the misallocation will take the form of under-investment.[1)] Relying more on general arguments than formal analysis, the conclusions drawn in the recent discussion of the implication for economic policy of embargo threats point in the same direction. A common argument for competitive inefficiency in this debate is that speculators (inventory holders) will not undertake sufficient risk in order to supply an optimal amount of storage.[2)] Also, in the natural resource literature, which deals with a problem that is similar to our problem of stock withdrawals during embargoes, it has been concluded that risk aversion together with uninsurability (due to an incomplete market structure) will imply less than an optimal amount of conservation.[3)]

In the capital theory literature there has been a lot of concern with potential misbehavior in the form of capital overaccumulation in actual economies. The phenomenon of capital overaccumulation was first observed by Malinvaud (1953). It is associated with the inability of market economies to foresee infinitely far into the future. The focus is here solely on the infinite duration aspect of time. Other aspects such as uncertainty about future events are not considered. In the natural resource literature the implication of myopic foresight for the intertemporal allocation of exhaustible resources has been analyzed by, e.g., Stiglitz (1974) and Dasgupta and Heal (1979). They show, also without any consideration of risk, that there is a tendency in market economies with natural resources either to use the resources (permanently) too slowly or (temporarily) too quickly.

In the present chapter we will analyze the efficiency implications of the absence of a complete set of markets in economies facing embargo threats. In Section 4.2 we will examine stockpiling and stock withdrawals in a competitive sequence economy assuming risk-neutral inventory holders (speculators) that have myopic (conditional) perfect foresight. It is found that a market economy with such a formation of expectations that faces embargo threats generated by a two-state stationary Markov process exhibits a kind of long-run instability similar to that described in the natual resource literature. However, in our framework with intermittent interruptions of oil imports, it is not only the indeterminacy of the initial price in embargo regimes that leads to intertemporal inefficiency. Also, there is no mechanism endogenous to the model to ensure that the competitive economy will keep an optimal stockpile under free trade.

In Section 4.3 we discuss the effects on stock withdrawals and stockpiling of introducing inventory holders that are risk averse rather than risk neutral. In a 2-period framework that resembles, in some aspect or the other, some of the models examined in the natural resource literature, the efficiency implications of risk-averse agents are analyzed. The results derived from this exercise combine the results suggested in the stockpiling and natural resource literature referred to above: Competitive depletion of emergency reserves during embargoes is excessive, and the economy's defense against embargoes will be too weak if we rely on private stockpiling. However, the model on which these results are based suffers from three weaknesses.

First, the measure of welfare employed is the expected consumers' surplus. This measure implies that the consumers are risk neutral in prices. We infer that stockpiling and stock withdrawal decisions based on risk aversion are incompatible with the maximization of the expected consumers' surplus. Hence, the argument for competitive inefficiency derived from these models is inconclusive. Second, there is only one risky asset in the model. In such a framework it comes as no surprise that risk-averse investors demand less (stockpile smaller amounts and deplete them at a faster rate) of this asset than risk-neutral investors. When there is more than one risky asset we know from, e.g., the Capital Asset Pricing Model (CAPM) that there are circumstances in which even risky assets are very attractive to risk-averse investors. We end Section 4.3 with a discussion of the public cost of risk-bearing vs. the private cost of risk-bearing. If the equilibrium expected returns on securities in the capital markets behave in accordance with the CAPM, it has been suggested that risk should not be treated differently for public as opposed to private investments (cf. Bailey and Jensen (1972) and Lind (1982)). However, the partial-equilibrium nature of CAPM (taking the investment level and production plans as fixed) together with the strong assumptions on preferences and beliefs "assumes away" parts of the problem dealt with in Arrow and Lind (1970). Remember the considerable stress they put on the incompleteness of existing capital markets. In fact, it is quite likely that the actual market economy will not attain a Pareto efficient inventory holding. Does this mean that governmental intervention can bring about a Pareto improvement? Not necessarily. The third weakness with the models we have employed in Section 4.3 is that the comparison between the planner and the market is not fair. By this we mean that the cost of and obstacles to market institutions that prevent the set of markets from being complete might also create difficulties for the planner.

It is mainly this last consideration that is taken up in Section 4.4. The idea is to place the planner and the market in a symmetric position with respect to the "transaction technology". In this approach one restricts the planner to operate only on the markets that do exist, i.e., one introduces the notion of constrained Pareto efficiency (see, e.g., Diamond (1967), Grossman (1977), Newbery and Stiglitz (1982) and Stiglitz (1982)).[4] The question we raise is the following: given the set of markets that do exist, will the market economy be efficient? The answer is that there is no presumption that even a rational expectations market equilibrium will lead to a constrained Pareto optimal amount of storage.

4.2 STOCKPILING AND STOCK WITHDRAWALS IN A COMPETITIVE SEQUENCE ECONOMY: THE CASE OF MYOPIC FORESIGHT AND RISK NEUTRALITY

Once we recognize that there is no such thing as a complete set of future and risk markets we have to consider a sequence of markets at successive dates. Also, while there is no need to consider price expectations in the atemporal Arrow-Debreu market, the prices at which the market is cleared at any one date in the sequence economy will depend on how expectations about future prices are formed and how they are revised in the light of new information. In this section we will examine stockpiling and stock withdrawals in such a sequence economy. We shall consider a partial-equilibrium framework. The focus of attention is on the stockpiling industry which by assumption owns the society's total endowment of oil at any time t. We will assume that the number of inventory holders is sufficiently large to warrant pricetaking behavior. We will take it that the inventory holders (speculators) have myopic (short-run) conditional perfect foresight so that expectations about the price that will prevail next period, given that a particular event occurs, are realized. To simplify we shall assume that the inventory holders are risk neutral. Hence risk markets are redundant and we can focus on the implications of an incomplete set of future markets.

Following the planning framework of Chapters 2 and 3 we will assume that periods of free trade come and go, interrupted by embargoes in an infinite sequence. We identify the state $x_t = 0$ with an embargo and the state $x_t = 1$ with a free-trade regime, both at time t. The stochastic process generating the sequence of regimes is assumed to be a stationary Markov process. We shall consider a discrete time model with time periods of unit length. Since we assume that the inventory holders have short-run (myopic) conditional perfect foresight their expectations about the price p_{t+1} that will prevail in the next period, given the market regime at $t + 1$, are realized. Let us denote these conditional spot prices as $(p_{t+1}|x_{t+1} = 1) = p$ and $(p_{t+1}|x_{t+1} = 0) = \lambda_{t+1}$, respectively. We will also assume that the inventory holders agree on the transition probabilities θ_0 and θ_1. The parameter θ_0 is the probability that the country will be in a free-trade regime in the next period, given that there is an embargo going on now. Likewise, if no embargo is in effect, θ_1 denotes the probability that the economy will be in an embargo regime in the next period.

The Sequence of Momentary Equilibria during Embargoes

We start the analysis by considering competitive behavior during an embargo regime. When an embargo regime is in effect at time t the spot price is λ_t. Consider now the time period $(t, t + 1)$. At time t the inventory holders assign the probability θ_0 that the embargo will end at time $t + 1$. If this happens the economy enters a free-trade regime in which it is possible to import oil at the known world market price p. If the embargo does not terminate - the probability of that is $1 - \theta_0$ - the price at time $t + 1$ will be λ_{t+1}. Let the numeraire be a risk-free asset, the rate of return on which is r. Hence λ_t units of the safe asset is worth $\lambda_t(1 + r)$ at time $t + 1$. If speculators are risk neutral then asset market equilibrium is given by

$$\theta_0 p + (1 - \theta_0)\lambda_{t+1} = (1 + r)\lambda_t. \tag{4.1}$$

Subtracting λ_t from each side of equation (4.1) and dividing by λ_t yields the movement of the spot price

$$\frac{\lambda_{t+1} - \lambda_t}{\lambda_t} = r + \theta_0 \frac{(\lambda_{t+1} - p)}{\lambda_t}. \tag{4.2}$$

Under competitive conditions the rate of return of holding the embargoed good in stock consists of two components, the rate of interest plus the expected rate of capital loss from holding oil in stock should the embargo expire, i.e., the expected rate of appreciation of the oil inventories equals the interest rate.[5)]

Along the price path determined by (4.2) the inventory holders will be indifferent at the margin between stock withdrawing and holding at each instant. The individual inventory holder's supply from storage is not uniquely determined. To determine the overall rate of stock withdrawals q_t we have to introduce the demand side. The consumers are assumed to use what they purchase at the instant of purchase, that is, their demand functions are functions of the current price only. Let $q(p_t)$ denote the market demand function. For simplicity we assume that it does not shift over time and that the demand is positive no matter how high the price. The condition for flow equilibrium in the market for oil is that the rate of stock withdrawals equal demand at the current price λ_t, i.e.:

$$q_t = q(\lambda_t). \tag{4.3}$$

So, given that an embargo is in effect, conditions (4.2) and (4.3) describe a sequence of momentary equilibria where in each time period both the asset market and the market for the oil flow are in competitive equilibrium.

Asset and Flow Market Conditions under Free-Trade Regimes

Consider now the situation when the embargo has expired and a free-trade regime has begun in which the country is able to buy a steady flow of oil at the constant price p. However, this flow is subject to possible curtailments. Consider again the time interval of unit length (t, t + 1). At time t speculators (inventory holders) assign the probability θ_1 that the free-trade regime will terminate in period t + 1. If the economy enters an embargo regime the speculators firmly believe that the spot price of oil in the next period will be λ_{t+1}. If the free-trade regime does not end - the probability of that event being $1 - \theta_1$ - the price in the next period will be the constant world market price p. Assuming again that the speculators are risk neutral, asset market equilibrium implies

$$\theta_1 \lambda_{t+1} + (1 - \theta_1)p = (1 + r)p, \tag{4.4}$$

that is, the expected value of a unit of oil in storage (LHS of (4.4)) must equal the value which is assured if one holds the numeraire asset (RHS of (4.4)). Or, alternatively, the current price p has to be equal to the discounted expected price in the next period. Rearranging terms in (4.4) yields

$$p = \frac{\theta_1}{r + \theta_1} \lambda_{t+1}, \tag{4.5}$$

which implies that the arbitrage equation also means that the cost of storing an additional unit (LHS of (4.5)) is just equal to the expected discounted marginal value of having an additional unit in stock when the embargo comes (RHS of (4.5)).

The condition for flow equilibrium in the market for oil at time t states that the imports of oil M_t plus the stock on hand as the economy enters period t S_t must equal flow demand $q(p_t)$ plus the amount carried over to the next period S_{t+1}; i.e.,

$$M_t + S_t = q(p_t) + S_{t+1}. \tag{4.6}$$

To summarize: The dynamics of stockpiling and stock withdrawals in a market economy lacking a complete set of markets can be described as a sequence of momentary equilibria. Competitive behavior ensures that both the market for the oil flow and the asset market equilibrate at each instant of time. During the embargo, competitive arbitrage would enforce the asset market equilibrium condition (4.2). Should free trade prevail instead, the corresponding equilibrium condition for the asset market would be (4.5). Flow equilibrium in the market for oil, should imports of oil be disrupted, implies (4.3) and should free trade be in effect we obtain the flow equilibrium condition (4.6).

The Inefficiency of Stock Withdrawals and Stockpiling in the Sequence Economy

We now wish to contrast the competitive resource allocation with the Pareto-optimal allocation derived in Chapters 2 and 3. We first make a comparison of the allocations during the embargo regime. We have seen that as long as the embargo lasts, the dynamic behavior of the market economy is described by the momentary stock equilibrium in the market for assets (4.2) and the momentary flow equilibrium (4.3).

The arbitrage equation (4.2) corresponds to the Euler-Lagrange equation (2.8) associated with the planning problem (2.4) of Chapter 2.[6] The flow equilibrium equation (4.3) is equivalent to the first-order condition (2.7) associated with the planning problem (2.4). So, apparently our market economy depletes the oil reserves efficiently. However, as we noted in Chapter 2 the system of differential equations for λ_t and $\dot{S}_t$ (see, (2.10) and (2.11)) has infinitely many solutions $\{\lambda_t\}$ and $\{S_t\}$. In the planning problem it was therefore necessary to impose initial conditions on the two paths. The initial condition $S_0 = S$ (here we assume that we commence at $t = 0$) is self-evident for both types of allocative mechanisms (i.e., trading in competitive markets and central planning). However, in the planning problem it was also necessary to impose the resource constraint (2.13) to obtain the initial price λ_0. Here an equivalent procedure would be to impose the resource constraint

$$\sum_{t=0}^{\infty} q(\lambda_t) = S, \tag{4.7}$$

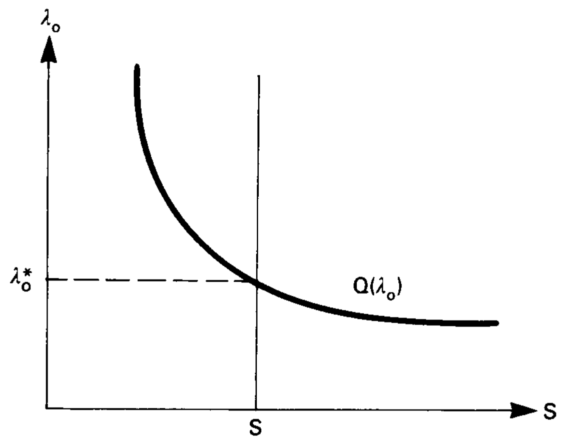
where λ_t is the solution to the difference equation (4.2), i.e.,

$$\lambda_t = \left[\lambda_0 - \frac{\theta_0}{r + \theta_0} p\right]\left[\frac{1 + r}{1 - \theta_0}\right]^t + \frac{\theta_0}{r + \theta_0} p.$$

Starting with the initial price λ_0^* determined from (4.7), the equilibrium price system (4.2) and (4.3) ensures that the oil inventories are allocated so as to ensure consumption in each period without leaving redundant stocks asymptotically.

If we were to assume that future markets extending infinitely far into the future clear (i.e., if we were to postulate an intertemporal competitive equilibrium), the correct initial price would be determined in the market economy too. Alternatively, if the inventory holders have long-run perfect foresight of demand during an embargo, the market equilibrium yields the optimal intertemporal allocation of the oil inventories. The left hand side of (4.7) can be interpreted as the long-run demand function for the embargoed good based on the initial price λ_0. Figure 4.1 illustrates the long-run optimality condition. The figure displays the long-run demand curve $Q(\lambda_0)$. The long-run supply in the embargo regime is completely inelastic at S, the stock on hand when the embargo starts. At the initial price λ_0^* the long-run demand equals long-run supply.[7)]

Figure 4.1 The long-run optimality condition for embargo regimes



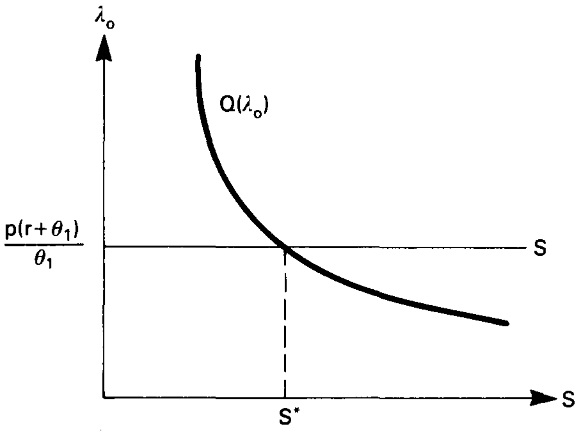
However, in the sequence economy we have described here there are no institutions or behavioral assumptions that ensure that the initial price is equal to the correct price λ_0^*. What happens in the market economy is that either redundant stocks are left asymptotically or that the assumption of perfect short-run conditional foresight is violated. In both cases the outcome is inefficiency.

Consider the case where the initial price in the embargo regime is too high, i.e., $\lambda_0 > \lambda_0^*$. The arbitrage condition (4.2) ensures that the spot prices along this modified Hotelling path will be too high at each instant of time. Along the entire path risk-neutral inventory holders are indifferent between holding and withdrawing from stocks and the rate of stock withdrawals is determined by flow demand. This demand will at each instant be too low (the price is always too high) and redundant stocks will be left over asymptotically. The bidding up of prices at a rate equal to $r + \theta_0(\lambda_{t+1} - p)/\lambda_t$ can last forever since nobody's expectations will be disappointed. So although the market economy behaves in a locally efficient manner during embargo regimes, the allocation of inventories is intertemporally inefficient.

If the initial price is too low, i.e., $\lambda_0 < \lambda_0^*$, the movement of the spot price according to the modified Hotelling rule (4.2) will again not be sufficient for optimality. For a while the economy might follow the modified Hotelling path even though the rate of stock withdrawals along that path is inconsistent with overall availability. However, as the bottom of the tanks begin to appear and a severe shortage is anticipated, speculators will sense an opportunity for profits. They will wish to buy in order to build up an oil stockpile and inventory holders will wish to hold the stock off the market. This will drive the current price up and reduce the excessive rate of stock withdrawals. Hence the story is quite different from the one in which $\lambda_0 > \lambda_0^*$. In that case the economy followed an errant path forever. When the initial price in an embargo regime is lower than the correct price the expectations will sooner or later be disappointed. How soon the economy will leave the errant path, that is, at what time myopic conditional perfect foresight is violated, depends on the precise manner in which expectations are revised in the light of new information about such things as the current inventory levels.[8)]

Turning now to the competitive allocation under free trade we find, of course, that the initial price is equal to the world market price p. However, there is nothing that guarantees that the market economy will keep an optimal stockpile S^* in free trade. Consider the asset (storage) condition (4.5) under free trade. It says that in equilibrium the price of oil in the beginning of an embargo exceeds the free trade price by a factor $(r + \theta_1)/\theta_1$. This condition looks similar to the optimal storage condition (2.31) derived in Chapter 2. However, from the analysis of Chapter 2 and 3 we know that $p(r + \theta_1)/\theta_1$ is identical to the marginal valuation of oil in storage only in case the country carries an optimal stock S^* when the embargo starts.[9)] In the competitive framework the optimal stock S^* would be determined if the inventory holders have long-run perfect foresight of demand during embargoes. This case is depicted in Figure 4.2 below, where $Q(\lambda_0)$ is the long-run demand curve. The supply of storage S is completely elastic at the price $p(r + \theta_1)/\theta_1$.

Figure 4.2 The long-run optimality condition for free-trade regimes



But none of the arguments establishing the conditions for asset and flow market equilibrium in the sequence economy described above allow us to determine the optimal stock S^*. Unless there are inventory holders having long-run perfect foresight or there exists a complete set of markets, there is nothing endogenous to the model that ensures that the stock will be optimally determined in the market economy.

The assumption of myopic expectations on which this literature is based may with reason be considered somewhat unrealistic. Stiglitz's argument for myopic expectations is not very convincing: "Since the individual can always sell his stock next period (in a truly competitive economy) at whatever price that prevails on the market next period, he needs only look one period ahead" (Stiglitz, 1974:140). It is true that he is myopic in the sense that his decisions depend only on his expectations about the price in the next period. However, this does not mean that he only needs to look one period ahead. A rational speculator translates into price forecasts all information available about the structure within which the price is determined. So his expectations about the price next period is based on his expectations about the entire sequence of future prices. A rational expectations equilibrium, in which the actual prices and the anticipated prices have the same probability distribution, is then conceivable.[10)]

The optimality of a rational expectations equilibrium in our model requires infinitely lived agents. Moreover, the rational expectations assumption places heavy demands on the rationality of the agents. To quote Radner: "This approach seems to require of the traders a capacity for imagination and computation far beyond what is realistic. An equilibrium of plans and price expectations might be appropriate as a conceptualization of the ideal goal of indicative planning, or of a long-run steady state toward which the economy might tend in a stationary stochastic environment." (1982:942). These considerations point to an approach which Radner calls the bounded rationality approach. The assumption of agents that look only one period ahead is an extreme example of this approach. Another example would be inventory holders that plan as if there existed a certainty equivalent date of the termination of the embargo. Instead of assuming that inventory holders have complete knowledge of the $Q(\lambda_o)$ function, i.e., the LHS of (4.7), we can assume that they approximate the long-run demand curve with

$$\bar{Q}(\lambda_o) = \sum_{t=o}^{1/\theta_o} q(\lambda_t),$$

where $1/\theta_o$ is the expected duration of an embargo. From the analysis in Chapter 3 we know that the $\bar{Q}(\lambda_o)$ function lies entirely to the left of the $Q(\lambda_o)$ function. Then, using the Figures 4.1 and 4.2 we can conclude that bounded rational inventory holders bring about less than an optimal amount of stockpiling in free-trade periods and too fast a depletion of oil inventories during embargoes.

In principle, the problem of intertemporal efficiency can be circumvented by a planning board that enforces a contingent intertemporal price structure of the kind derived in Chapters 2 and 3. In the natural resource literature some authors raise serious doubts about the ability of existing institutions in market economies to bring about an optimal exhaustion of resources. The thought of a planning board to guide resource depletion sometimes crosses their minds.[11)] Of course, one also recognizes that the assumption of an omniscient planner might be just as unappealing as postulating the existence of a complete set of markets.

To sum up: in the real world there is neither such a thing as a planning board having unlimited knowledge nor a complete set of markets. Behavioral assumptions involving rational expectations may be made. In some cases the optimality of rational expectations equilibria can be established. However, one may feel dubious about whether the rational expectations equilibrium concept is a useful notion for descriptive economic models. In order to be able to draw policy conclusions in this context, probably the only sensible approach is to introduce a weaker definition of optimality. In a somewhat different context (i.e., where risk markets are missing) in Section 4.4 we will make use of such a weaker definition of optimality. In the next section we shall discuss the possibility of competitive inefficient stockpiling and stock withdrawals when the inventory holders are risk averse.

4.3 ON THE POSSIBILITY OF COMPETITIVE INEFFICIENT STOCKPILING AND STOCK WITHDRAWALS IN A 2-PERIOD MODEL: THE CASE OF RISK-AVERSE INVENTORY HOLDERS

The question we pose in this section is the following: what are the efficiency implications of inventory holders (speculators) that do not behave as maximizers of expected profits but have aversion towards risk? An argument frequently seen (e.g., Arrow and Lind (1970)) is that risk aversion together with uninsurability (due to the incomplete market structure) will imply under-investment. Also, in the natural resource literature it has occasionally been put forward that risk aversion will imply less than an optimal amount of conservation. We will scrutinize these arguments and see whether the conclusions carry over to the stock-piling and the stock withdrawal problem.

The Effects of Risk Aversion on Stock Withdrawals and Stockpiling

We shall first look at the effects on stock withdrawals and stockpiling of introducing inventory holders (speculators) that are risk averse rather than risk neutral. For expository reasons we discuss the problem in a 2-period setting. In this way we can disregard any possible inefficient outcome due to the lack of terminal date.

Let us begin by examining the behavior of a representative risk-averse inventory holder during an embargo. As before we distinguish between two states of the world: either the embargo will end at the beginning of the next period (period 1) or it will remain in effect during that period too. Using our previous notation, we identify the state $x_1 = 0$ with an embargo in the next period and $x_1 = 1$ with a free-trade regime. The probability of $x_1 = 1$ is assumed to be θ_0 and the probability that the embargo will not expire (i.e., the probability that $x_1 = 0$) is, of course, $1 - \theta_0$. The conditional prices of oil in the next period are denoted by $p = (p_1|x_1 = 1)$ and $\lambda_1 = (p_1|x_1 = 0)$, respectively. The current price is denoted by λ_0 (i.e., $\lambda_0 = (p_0|x_0 = 0)$). Let S_0 denote the stock held by the inventory holder at the beginning of the first period, and let q_0 denote the amount of the good withdrawn in the first period. For stock withdrawal q_0 and market regimes $x_1 = 1$ and $x_1 = 0$, total revenue is given by

$$\pi(q_0|x_1 = 1) = \lambda_0 q_0 + (1 + r)^{-1} p(S_0 - q_0)$$

and

$$\pi(q_0|x_1 = 0) = \lambda_0 q_0 + (1 + r)^{-1} \lambda_1(S_0 - q_0),$$

respectively.

We assume that the representative inventory holder is a Von Neuman-Morgenstern utility maximizer with the utility function $u(\pi(q_0))$, such that $u'(\pi(q_0)) > 0$ and $u''(\pi(q_0)) < 0$. So the inventory holder's problem is to choose the q_0 that maximizes the expected utility of profits:

$$E[u(\pi(q_0))] = \theta_0 \, u(\pi(q_0|x_1 = 1)) + (1 - \theta_0) \, u(\pi(q_0|x_1 = 0)). \tag{4.8}$$

Differentiating (4.8) with respect to q_0 we obtain the following first-order condition for the maximum:

$$\frac{\theta_0 \left[d\pi(q_0|x_1 = 1)/dq_0\right]}{(1 - \theta_0)\left[d\pi(q_0|x_1 = 0)/dq_0\right]} = - \frac{u'(\pi(q_0|x_1 = 0))}{u'(\pi(q_0|x_1 = 1))} \tag{4.9}$$

To compare the solution of (4.9) (i.e., the stock withdrawal q_0 in the initial period) when the inventory holders are risk averse with that when they are risk neutral, we notice first that the marginal utility of profits is constant in the latter case. So, if the inventory holders are risk neutral the RHS of (4.9) is equal to -1 and the whole equation is identical to (4.1). If the inventory holders are risk averse, their utility functions are increasing in profits at a diminishing rate. This implies that, since $\pi(q_0|x_1 = 1) < \pi(q_0|x_1 = 0)$, the RHS of (4.9) is greater than -1. The LHS of (4.9) is equal to

$$\frac{\theta_0 \quad [\lambda_0(nq_0) - (1 + r)^{-1} \, D]}{(1 - \theta_0)[\lambda_0(nq_0) - (1 + r)^{-1} \, \lambda_1(nq_0)]}, \tag{4.10}$$

where the functional dependence of λ_0 and λ_1 on the total amount of stock withdrawals in the first period nq_0 (where n is the number of inventory holders) is indicated explicitly. Equation (4.9) obviously cannot be satisfied for the stock withdrawal nq_0^N where q_0^N is the optimal amount of stock withdrawal in the first period by the representative risk-neutral inventory holder. Let q_0^A denote the optimal stock withdrawal made by the representative risk-averse inventory holder. Since (4.10) is increasing in q_0 we must have that $q_0^A > q_0^N$ in order to guarantee equilibrium in the oil market at the same time as the profit maximization condition (4.9) is satisfied. Since (4.10) is greater than -1, the expected rate of appreciation of the oil price exceeds the risk-free rate of interest when the inventory holders are risk averse.

The above results are, of course, in accordance with what would be expected. Risk averse inventory holders are more cautious than those that are risk neutral. In order to reduce the likelihood of large capital losses from holding inventories should the embargo expire, they give up some expected profit by increasing the stock withdrawal now.

Consider now the situation in which there is free trade in the first period. Our representative speculator now buys an inventory of S_0 at price p with the intention of reselling it in the next period at a profit. For storage choice S_0 and market regime $x_1 = 0$ (i.e., an embargo is imposed in the next period), profit is given by

$$\pi(S_0|x_1 = 0) = - pS_0 + (1 + r)^{-1} \lambda_1 S_0$$

Should instead free trade prevail in the next period (i.e., $x_1 = 1$) profit is given by

$$\pi(S_0|x_1 = 1) = - pS_0 + (1 + r)^{-1} pS_0.$$

If θ_1 is the probability that an embargo will be imposed in the next period, the expected utility of profits can be written as

$$E[u(\pi(S_0))] = \theta_1 u(\pi(S_0|x_1 = 0)) + (1 - \theta_1) u(\pi(S_0|x_1 = 1)). \tag{4.11}$$

Differentiating (4.11) with respect to S_0 we obtain as necessary condition for a maximum:

$$\frac{\theta_1 d\pi(S_0|x_1 = 1)/dS_0}{(1 - \theta_1) d\pi(S_0|x_1 = 0)/dS_0} = - \frac{u'(\pi(S_0|x_1 = 0))}{u'(\pi(S_0|x_1 = 1))} . \tag{4.12}$$

To compare the solution of (4.12) when the inventory holders are risk averse with that when they are risk neutral we can follow our treatment in the comparison between these two cases for the stock withdrawal problem. The comparison shows immediately that the representative risk-averse speculator stockpiles smaller quantities of oil than the risk-neutral speculator.

We can summarize as follows: In a competitive economy, the inventories of oil during embargoes will be depleted faster if the stockholders display aversion towards risk than if they are risk neutral. During free-trade regimes, risk-averse investors will be more prudent in stockpiling oil than risk-neutral agents and the emergency reserves will accordingly be smaller in this case.

The Efficiency Implications of Risk Aversion

Now we come to the critical question: What are the efficiency implications of inventory holders (speculators) that do not behave as expected value decision makers but have aversion towards risk? The conclusion drawn in the natural resource literature is that risk aversion together with uninsurability (due to the incomplete market structure) will imply less than an optimal amount of conservation (see, e.g., Weinstein and Zeckhauser (1975), Heal (1975) and Hoel (1977, 1980)). It is tempting to reinterpret this result and make the following statements: Competitive depletion of emergency reserves during embargoes is inefficient if the inventory holders are averse to risk. Also, the economy's defense against embargoes will be too weak if we rely on private risk-averse inventory accumulation during free-trade periods.

Using a welfare criterion that is similar to the one used in the natural resource literature referred to above, we will produce these results and examine whether the underlying arguments for competitive inefficiency are correct or not. To this end let us assume that the emergency reserves are socially managed. We start with the planner's problem of allocating the emergency reserves S_0 when there is an embargo in effect in the first period. Imports of oil in period 1, M_1, is a random variable that takes the value zero should there be an embargo in the next period, that is, $(M_1 | x_1 = 0) = 0$. Should free trade prevail in the next period instead, $(M_1 | x_1 = 1)$ is given by $p(S_0 - q_0 + M_1) = p$, where $p(\ )$ denotes the inverse demand. We take as the measure of social welfare the expected net social surplus defined as the Marshallian consumers' surplus less the total costs of imports. Assuming that the planner assigns the same probability to the event that the embargo will end as the private stockholders, the welfare enjoyed by the country can be written as

$$W = \int_0^{q_0} p(q)dq + (1 + r)^{-1}\Big[\theta_0\Big[\int_0^{S_0-q_0+M_1} p(q)dq - pM_1\Big] + \qquad (4.13)$$

$$+ (1 - \theta_0)\Big[\int_0^{S_0-q_0} p(q)dq\Big]\Big],$$

where $p(q)$ denotes the inverse demand curve. Differentiating (4.13) with respect to q_0, we obtain as a necessary condition for maximum:

$$(1 + r)p(q_0) = \theta_0 p(S_0 - q_0 + M_1) + (1 - \theta_0)p(S_0 - q_0). \qquad (4.14)$$

Now, since we assume that imports of oil in period 1 should free trade prevail are given by $p = p(S_0 - q_0 + M_1)$, (4.14) can be written as

$$(1 + r)\lambda_0 = \theta_0 p + (1 - \theta_0)\lambda_1$$

where $p(q_0) = \lambda_0$ and $p(S_0 - q_0) = \lambda_1$. This expression is identical to the representative risk-neutral inventory holder's condition for a maximum.

Finally we shall look at the case in which free trade prevails in the first period. The problem is that of stockpiling and we take as a measure of social welfare the expected consumers' surplus in the next period minus the acquisition cost pS_0. Hence the planner's problem is to maximize

$$W = - pS_0 + (1 + r)^{-1}\Big[\theta_1 \int_0^{S_0} p(q)dq \qquad (4.15)$$

$$+ (1 - \theta_1)\Big[\int_0^{S_0+M_1} p(q)dq - pM_1\Big]\Big].$$

Differentiating (4.15) with respect to S_0 we have the condition for a maximum:

$$(1 + r)p = \theta_1 p(S_0) + (1 - \theta_1)p(S_0 + M_1), \qquad (4.16)$$

which, using the fact that M_1 in case the embargo expires is given by $p = p(S_0 + M_1)$ and that $p(S_0)$ is equal to λ_1, can be written as

$$(1 + r)p = \theta_1\lambda_1 + (1 - \theta_1)p.$$

This, in turn, is identical to the competitive storage condition of the representative risk-neutral speculator.

So it seems that we have demonstrated that if stockholding and stock depletion are undertaken by agents that are risk averse, the market equilibrium will not be socially optimal. However, this conclusion is misleading.

Just as in Arrow and Lind (1970), which we shall return to later, the approach taken here is that the planner's objective function should measure benefits and costs in terms of individuals' preferences. Now, the problem with the analysis above is that stockpiling and stock depletion decisions taken with the objective of maximizing the expected utility of profits (with $u''(0) < 0$) are inconsistent with the maximization of our particular welfare function. Optimality in our present embargo model has been judged by the same welfare function that was employed in the natural resource literature referred to above. This welfare function is special in at least two respects. First, in order to justify it we have, of course, to assume that it is valid for all individuals in the economy. That is, we have to assume a homogeneous consumer economy and argue in terms of the representative consumer. In a heterogeneous agent economy (in which consumers and inventory holders are different agents) the social welfare function would be some sort of function that aggregates the individual utility functions. In the homogeneous consumer economy the stockpiling and stock depletion must be undertaken by firms owned by the identical individuals or, alternatively, all inventories are held by the identical individuals directly. This implies that for consistency we have to specify objectives for stockpiling and stock withdrawals that are compatible with our social utility function.

A second special property of our welfare function is that the measure of expected consumers' surplus does imply price risk neutrality for consumers.[12)]In case the stockholding is undertaken by firms owned by the identical individuals, the owners would impose on these firms their own (price-risk-neutral) utility functions. In case the inventories are held directly by the identical individuals it is even more clear that introducing inventory holders that are risk averse is inconsistent with our particular welfare function.

So our results that risk-averse inventory holders will deplete the reserves faster than what is socially optimal suffers from a defect common to the conclusion drawn in the natural resource literature we referred to above. Also, the inconsistency between the objectives for stockholding and the welfare function implies that the argument that private stockpiling diverges from what is socially optimal is inconclusive.[13)]

Does the Government Have a Superior Position with Respect to Risk?

Now, although the argument for competitive inefficiency put forward above rests on incompatible assumptions, there might be other arguments that do not. There has been an extensive controversy about the question whether the government has a superior position with respect to risk. The position taken by, for example, Arrow and Lind (1970) is that the public sector should ignore uncertainty in evaluating investments. Their argument runs as follows. If the returns to a particular public investment are not correlated with other components of the national income and if the returns are shared by the entire population, the total cost of risk-bearing becomes negligible. So, Arrow and Lind suggest using the risk-free interest rate in evaluating government investment.

This is in contrast to the individual investment decisions under uncertainty we observed above. We noted that, as long as the investors are risk averse, the price of oil in storage must be adjusted so that its expected rate of return is greater than the risk-free rate of interest. Or, to put it another way, the risk premium must be positive in this case. In our framework, with a one-period investment that has a positive expected return, the appropriate adjustment for risk can be made by raising the risk-free discount rate.[14)]

So the conclusion that can be drawn if we apply the Arrow-Lind theorem to our problem is the following. The private investors under-invest, that is, stockpile too little and deplete the reserves too fast. The government, since it has a superior position with respect to risk, should either subsidize private stockholding, or, it should undertake the stockpiling by itself and thereby transfer risk to the public at large.

Two assumptions are essential for this result to hold. First, that the stockmarket and other markets that perform some of the functions of risk markets are incomplete to a substantial degree. Second, that the returns to a project (private or public) are uncorrelated with other sources of income. The Capital Asset Pricing Model (CAPM) provides a framework for a discussion of both these aspects. An extra bonus for the CAPM is that it allows us to stress the insurance aspects (rather than the riskiness) of either public or private stockpiling.

As is well known, the CAPM assumes either that investors have quadratic utility functions or that the asset returns have a joint normal distribution. A further characteristic of the model is that investors have homogeneous beliefs about asset returns. The basic result of the model tells us that the expected rate of return on, say, oil in storage is equal to the risk-free return plus a risk premium. Formally, the CAPM can be written

$$E[R_{oil}] = r + \mu Cov(R_{oil}, R_m) \tag{4.17}$$

where $E[R_{oil}]$ is the expected rate of appreciation of the oil inventories and r is the risk-free rate of interest. The risk premium is the market price per unit of risk μ multiplied by the covariance between returns on oil in storage R_{oil} and the market portfolio R_m.[15)] So, the appropriate measure of risk for oil in storage is its contribution to the market risk. Or, in terms of the consumption CAPM (see, e.g., Rubinstein (1976), Breeden (1979) and Leroy (1982)), the covariance between the rate of return on oil in storage and the marginal utility of income. The basic result of the consumption CAPM tells us that the risk premium on oil in storage, that is, the difference between the expected rate of appreciation of oil inventories and the risk-free rate, has a sign that is the opposite of the sign of the covariance between the rate of capital gain on oil and the marginal utility of consumption.

Now the rate of return on oil in storage is highly negatively correlated with the return to investments in general and the national income. For instance, should an embargo be imposed, the oil in stock would yield a high return while at the same time the income and returns on most other investments would be low. Likewise, if we are already being exposed to an embargo, there is a risk of a low (negative) return should the embargo expire. But at the same time, the income and return on other investments would be relatively higher in the following free-trade period. When the income is low (high) the marginal utility of income is high (low). Consequently, the risk premium on oil in storage is negative. This makes sense since oil in storage is very attractive to risk-averse investors because it provides not only diversification opportunities but also insurance against random fluctuations of income. So, the conclusion that risk-averse investors are more prudent in stockpiling and that they deplete the stocks at a faster rate than risk-neutral investors must be recast.

For the hypothetical world of CAPM, Bailey and Jensen (1972) and Lind (1982) conclude that risk should not be treated differently for public as opposed to private investments. So, given that the returns on oil in storage (publicly or privately owned) is negatively correlated with other components of the national income, the risk premium on oil should be negative. Hence, any stockpiling project should be evaluated using the risk-free discount rate adjusted for the (negative) risk allowance.

The result that the choice of discount rate for public investments should be adjusted for risk is not in conflict with the Arrow-Lind theorem. This theorem is based on the assumption of uncorrelated returns. However, the fact that the typical private inventory holder faces risks equal to those faced by the public stockpile manager is not in the spirit of Arrow and Lind (1970). In their paper they put considerable stress on the fact that many risk markets are lacking in an actual market economy. But according to CAPM, the portion of risk of an individual asset that is uncorrelated with the rest of the economy can be diversified away. So according to the CAPM, existing capital markets allocate risk as efficiently as perfect markets for claims contingent on states of the world. Using the state-preference approach, Arrow-Lind show that there is no discrepancy in this case between public and private cost of risk-bearing.

Hence, the original problem of incomplete markets dealt with in Arrow-Lind is "assumed away" in the framework of the CAPM. How reasonable is this? Several objections can be made to the specific assumptions incorporated in the CAPM. Certainly, the assumption on preferences might be relaxed. The CAPM belong to a set of alternative structures of the capital market that are less extensive than the Arrow-Debreu market, but that result in a Pareto optimal allocation of risk among investors. Their common assumptions are homogeneous expectations and the investors' utility functions belong to the HARA (Hyperbolic Absolute Risk Aversion) family (cf. Mossin (1977) Hakansson and Ohlson (1985)). The quadratic utility assumed in CAPM is, of course, a member of the HARA family. The above assumptions seem quite restrictive, especially since all utility functions must belong to the same HARA utility. Moreover, one essential feature of the CAPM is that it is only a partial equilibrium model of the capital market. The prices are determined under the assumption that the quantities of all assets are given. Neither the consumption-saving decisions of consumers nor the investment decisions of firms are considered in the model. So even if we accept the assumptions on expectations and preferences in the CAPM, the question of investment efficiency in an actual economy remains to be answered. To put it another way, the potential misallocation in the form of underinvestment, i.e., the point of departure for the Arrow-Lind paper, cannot be revealed in the CAPM. In fact, an investment allocation determined by the maximization of the market value of a firm according to CAPM is not necessarily Pareto optimal (cf. Mossin (1977), Baron (1979) and Merton (1982)).

One possible way of summing up is to say that the conclusion that private and public cost of risk-bearing are of equal size is less than obvious. In fact, some strong assumptions have to be satisfied to ensure that a market economy lacking a complete set of risk markets will allocate the resources Pareto optimally. However, one may ask oneself whether a government can do any better. We turn to this question in the next section.

4.4 STOCKPILING AND THE OPTIMALITY OF MARKET EQUILIBRIUM WITH RATIONAL EXPECTATIONS: THE CASE OF HETEROGENEOUS AGENTS

It has been shown that a competitive economy with homogeneous consumers that form rational price expectations will allocate the resources Pareto optimally.[16)] Now, though the homogeneous consumer economy is sometimes a useful construct from a pedagogical point of view, it is obviously of no use in analysing the effect of missing risk markets. In such an economy risk markets are, of course, redundant.

In a heterogeneous agent economy lacking risk markets there is nothing that guarantees equality across the individuals in the marginal rate of substitutions between income in different states of nature. So a market economy will not in general allocate the resources Pareto optimally and consequently the government may find it expedient to intervene. However, one may wonder whether this inefficiency can be removed by a central decision maker. To quote Grossman: "If for some reason competitive markets are incomplete, then the same reason might make it difficult for a central planner to coordinate allocations across time and states of nature" (1977:14). We will keep this doubt about the planner's ability in mind as we discuss the welfare properties of private stockpiling in two different types of market economies, one without any risk markets whatsoever and one with a stock market.

An Economy Without Risk Markets

In the first market economy, two types of participants are considered in the domestic oil market: consumers and suppliers (i.e., importers and inventory holders). The consumers' demand for oil in each time period is based only on that period's price and income. We assume that there are m identical consumers. Let us describe the representative consumer's choice with the indirect utility function $V(p, z)$ where p denotes the price of oil and z is income. We assume that the consumer's income is independent of the price of oil and that the prices of other goods are constant. Roy's identity gives the demand for oil, $q = -V_p/V_z$. Total demand is given by $mq = D(p, z)$.

In a free-trade regime, k identical suppliers are able to buy a steady flow of oil at a constant world market price $\bar{p}$.[17)] For notational reasons we denote the free-trade price with $\bar{p}$ instead of p that was used to denote the free-trade price in the previous chapters. Imports are $kM(x = 1) = D(\bar{p}, z) + kS$ where kS is the amount of oil stored in anticipation of a capital gain (see below) and where we identify the state of oil imports $x = 1$ with a free-trade regime. In the next period, the state of oil imports $\tilde{x}$ is random and it may take any number in the unit interval [0, 1]. For example, we identify $x = 0$ with a situation in which the imports of oil have been completely cut off and $x = 0.5$ when imports of oil are restricted to 50 per cent of $kM(x = 1)$. We will assume that $\bar{p}$ will be paid for imported oil.

In the next period, the domestic supply of oil consists of the available imports, plus the amount carried over by the suppliers from the previous period. The total cost of storing an amount S is $\bar{p}S + c(S)$ where $\bar{p}S$ is the purchasing cost and $c(S)$ is the cost of physical storage services. For simplicity it is assumed that the interest rate is zero. While the physical transfer of oil is not subject to uncertainty, the profitability of storage is, since the price in next period $\tilde{p}$ is random. The representative supplier is competitive, so he takes the distribution of prices in the next period as given. Below we shall see how the price distribution is generated and how it depends on the amount of oil carried over, the state of oil imports, and the income of the consumers, i.e., $\tilde{p} = p(S, \tilde{x}, z)$.

The profit in the next period of a supplier when he has stored S is thus

$$\pi = p(S, \tilde{x}, z)(S + M(\tilde{x})) - \bar{p}(S + M(\tilde{x})) - c(S).$$

Profits from imports is in our setting exogenously given. We assume that the supplier maximizes the expected utility of profits from storage, i.e.,

$$\max_{S} \; E[U(\pi)],$$

where the expectations are taken with respect to $p(S, \tilde{x}, z)$.

The representative supplier chooses S so that

$$E\left[U_{\pi}(p(S, \tilde{x}, z) - \bar{p} - c'(S))\right] = 0, \tag{4.18}$$

The condition (4.18) can be solved for the optimal level of storage S^{*}.

Market equilibrium with rational expectations is a price function $p(S, \tilde{x}, z)$ such that if S^{*} solves (4.18) then

$$k(M(x) + S^{*}) = D(p(S^{*}, x, z), z), \tag{4.19}$$

for all values of x.

In our model there are no risk markets that guarantee equality across the individuals in the marginal rate of substitutions between income in any pair of states of the oil imports. Unless

$$\frac{V_z(x)}{V_z(\hat{x})} = \frac{U_\pi(x)}{U_\pi(\hat{x})}, \tag{4.20}$$

for any pair of x and $\hat{x}$, the market equilibrium with rational expectations will not lead to a Pareto optimal amount of storage.

The natural question to ask is if governmental intervention might improve welfare. Two main types of approaches can be suggested. One possibility is to establish risk markets. Perfectly competitive markets in contingent claims on commodities would result in (4.20). One problem with this approach is that setting up risk markets may be very costly. Taking these costs into account, it might very well be so that the market economy provides the correct set of markets. So it will perhaps not be appropriate to regard the actual (incomplete) set of markets as *prima facie* evidence of inefficiency. Furthermore, Hart (1975) has shown that, unless the opening of new markets makes the market structure complete, the result of introducing a new security might in fact make everybody worse off. Quoting Hart: "In this respect, an economy with incomplete markets is like a typical second best situation. Only if all imperfections are removed, that is, in this case all markets are opened up, can we be sure that there will be any overall improvements" (1975:442).

The other approach is to ask: given the set of markets that do exist, will the market economy bring about a Pareto optimal level of storage? And - if the market economy fails this minimal test of efficiency - what intervention is required? In order to explore the efficiency with which the market stockpiles, we rest on Newbery and Stiglitz (1982) and their definition of constrained Pareto optimality. In their paper, they contrast the market allocation with the optimal choice of techniques in the face of technological uncertainty, assuming that the planner is restricted to operate only on the markets that do exist (i.e., he can not open up risk markets) but that he can directly control the choice of techniques and he can make lump-sum transfers which are not state dependent.[18] In our model we will assume that the planner can directly control the level of storage S and that he can make state independent lump-sum transfers.

To characterize the constrained Pareto optimal allocations we form the Lagrangian

$$L = E\big[V(p(S, \tilde{x}, z), z) + \lambda \frac{k}{m} (U(p(S, \tilde{x}, z)(S + M(\tilde{x}))$$
$$- \bar{p}(S + M(\tilde{x})) - c(S))\big].$$

Choosing S yields the first-order condition:

$$\frac{\partial L}{\partial S} = E\big[(V_p + \lambda \frac{k}{m} U_\pi(S + M(\tilde{x}))) \frac{\partial p}{\partial S} \tag{4.21}$$
$$+ \lambda \frac{k}{m} U_\pi(p(S, \tilde{x}, z) - \bar{p} - c'(S))\big] = 0.$$

Competitive equilibrium ensures that the second term in (4.21) is zero (cf. the storage condition (4.18)). So, using Roy's identity and the fact that $k(S + M(\tilde{x})) = mq$, the first-order condition for a constraint Pareto optimum is

$$E\left[(- V_z + \lambda U_\pi) q \frac{\partial p}{\partial S}\right] = 0. \tag{4.22}$$

Conditions (4.22) is similar to condition (18) for constrained Pareto optimum in Newbery and Stiglitz (1982). Condition (4.22) can be thought of as a generalization of the condition for full Pareto optimality. The condition for full Pareto optimality reads $V_z(x) = \lambda U_\pi(x)$ for some value of λ and for all possible realizations x of the oil imports $\tilde{x}$. This simply says that for any two individuals the marginal utilities of income stand in the same proportion to each other in all states of the world. Or, alternatively, the condition for unconstrained Pareto optimality requires that consumers and suppliers have the same marginal rate of substitution between incomes in any two pairs of states x and $\hat{x}$, i.e., $V_z(\hat{x})/V_z(x)/U_\pi(\hat{x})/U_\pi(x) = 1$. Now, rewrite condition (4.22) as

$$E\left[(\rho - 1)V_z q \frac{\partial p}{\partial S}\right] = 0 \tag{4.23}$$

where

$$\rho = \frac{\dfrac{V_z(\hat{x})}{V_z(x)}}{\dfrac{U_\pi(\hat{x})}{U_\pi(x)}}$$

and $\hat{x}$ is the state where $V_z(\hat{x})/V_z(x) = \lambda$. So from (4.23) we see that the condition for constrained Pareto optimality is that the weighted average of marginal rates of substitutions be the same for all individuals.

Newbery and Stiglitz (1982) establish that the necessary conditions for the rational expectations equilibrium to be a constrained Pareto optimum are exactly the same as the conditions for redundancy of risk markets. They find, in addition to the trivial case of no risk, two alternative conditions for risk market redundancy. They both require that V_z and U_π be constant, irrespective of the realization of the state of the world. One case in which this is fulfilled is when the suppliers are income risk neutral and the consumers are risk neutral in prices. The restriction on the suppliers' utility function is of course that $U_{\pi\pi} = 0$. For the consumers we have to assume that $V_{zp} = 0$. If this holds, their marginal utility of income is the same in all states of oil imports. For example, this condition holds for the direct utility function $U(q, z) = u(q) + z$ we employed in Chapters 2 and 3 and in Section 4.3. The price of oil does not here affect the marginal utility of the background commodity. For the logarithmic indirect utility function it also holds that $V_{zp} = 0$. The second condition Newbery and Stiglitz find for the redundance of risk markets is that consumers have this kind of indirect utility function. The logarithmic indirect utility function generates demand curves which have unitary price elasticity. In their model this implies that producers face no income risk. Hence, their condition (18) is satisfied irrespective of the producers' attitudes towards risk. For reasons of, e.g., differences in the cost structure, this second condition for risk market redundancy is not applicable in our storage model. Therefore, unless $V_{zp} = U_{\pi\pi} = 0$, (4.23) will not be satisfied and the market will not bring about a constrained Pareto optimal amount of storage.

We did not consider markets for risksharing at all in the model we just looked at. In that model, trading in competitive markets will lead to constrained Pareto optimality only under some fairly restrictive assumptions on the preferences. These conditions are in fact the same as those that are required for risk market redundancy. Now it may be argued that - just as the assumption of a complete market structure is unrealistic since the actual market structure is less extensive than a complete Arrow-Debreu market - it is empirically irrelevant to assume away any set of risk markets. Although the risk markets that do exist (e.g., stockmarkets and insurance markets) are incomplete, they play an important role in allocating riskbearing.

A Stockmarket Economy

In the following we shall analyse the optimality of a competitive market equilibrium supply of storage in a stockmarket economy. Our description of such an economy is similar to the one in Helpman and Razin (1978). The attempt to explore the efficiency with which such an economy stockpiles oil in the face of an embargo threats bears a close resemblance to Stiglitz's (1982) more general exploration of optimality of a stockmarket economy in the presence of technological uncertainty.

Consider a two-sector economy which stockpiles oil and produces a background commodity. Each sector is composed of a large number of identical firms. The background good industry is assumed to be a risk-free industry. The output of background commodities of the representative firm in the second period Z is a deterministic function of the investments in that firm. To keep things simple we write this as

$$Z = (1 + r)I_z,$$

where r is a given number and I_z is the level of investments made in the first period. The corresponding relation for the representative oil stockpiling firm is the following

$$S = I_s/\bar{p},$$

where S denotes the stock of oil in the next period, I_s is the level of investments in the stockpiling firm and $\bar{p}$ is the free-trade price of oil. Since imports of oil are random, the price of oil in the next period is random too, and so is the value of the stock $\tilde{p}$ S, where $\tilde{p}$ denotes the random price of oil in the next period.

We will make the traditional assumption (e.g., Diamond, 1967) that firms maximize their net stockmarket values. We assume that the firm's investment costs are financed directly by their initial owners so that their equity is the market value of the firm. It follows that the net value of a firm is equal to its stockmarket value minus investment costs, i.e.,

$$v_s^N = v_s - I_s, \tag{4.24}$$

and

$$v_z^N = v_z - I_z, \tag{4.25}$$

where v_z^N and v_s^N are the net value of the representative background good firm and stockpiling firm, respectively. v_s and v_z denote the stockmarket values of the two firms.

The firms choose a level of investment that maximizes the stockmarket value of their initial shareholders, i.e., they maximize (4.24) and (4.25), respectively. For example, suppose that a stockpiling firm has chosen to stockpile S^o and that this has resulted in an (observed) stockmarket value of v_s^o, i.e., v_s^o is the price of the random income $\tilde{p}S^o$ inherent in the stockmarket. Since the firm is small it believes that it faces a perfectly elastic demand schedule for its shares. Under this condition the perceived stockmarket value of a firm in the stockpiling industry is $v_s^o \cdot S/S^o$. The firm, in evaluating alternative storage levels, thus solves the following problem:

$$\underset{S}{\text{Max}} \quad v_s^o\, S/S^o - \bar{p}S. \tag{4.26}$$

Note that because of the constant returns to scale in storage, while the aggregate amount of storage is determinate, individual firms' stockpiling (and the number of firms' engaged in stockpiling) are not. Since there are constant returns to scale in the firm's operations and since all firms in each sector are identical we can aggregate and interpret $I_s(= \bar{p}S)$ and I_z as the total investment levels in the two industries and S and Z as the output of each sector. Then the decision problem (4.26) is identical to that of the stockpiling industry. This means that in equilibrium

$$v_s = \bar{p}S \ ,$$

that is, in equilibrium the stockmarket value of the stockpiling industry is equal to its investment costs. Likewise, for the background good industry we obtain

$$v_z = I_z.$$

As to the consumer side of the economy, it is assumed that there are m consumers (indexed i = 1,...,m). For simplicity, the consumption decisions in the first period are left out of the model. Assume instead that after individual i has decided on the current period's consumption of the two commodities, he has W^i available to invest in the two industries. We assume that shareholdings in the two industries are the only investment opportunities available. In the second period, when the transformation of investments into output and a particular state of the imports of oil have been realized, he buys oil and the background commodity using his returns from ownership holdings in the two industries.

The returns accruing to individual i depend on the price of oil next period $\tilde{p}$ and his allocation of W^i in the current period. If individual i invests a fraction α^i of his total financial holdings in the background good industry and the remainder in the stockpiling industry, his position next period in state x is given by

$$y^i(x) = \left[\alpha^i(1 + r) + (1 - \alpha^i) \frac{p(x)}{\bar{p}}\right]W^i, \tag{4.27}$$

where p(x) is the price the next period in state of oil imports x.

Each individual takes the distribution of prices $p(\tilde{x})$ as given. Individual i will make his portfolio choice so as to maximize expected utility. That is, he will choose α^i to maximize

$$E\left[V^i(p(\tilde{x}), y^i(\tilde{x}))\right],$$

where $V^i(\cdot, \cdot)$ is individual i's indirect utility function and where the expectations are taken with respect to $p(\tilde{x})$. The result of this maximization is that α^i must satisfy the necessary condition

$$E\left[V_y^i(1+r)\right] = E\left[V_y^i\ p(\tilde{x})/\bar{p}\right], \tag{4.28}$$

where V_y^i is the marginal utility of income. Equation (4.28) is a condition on individual i's allocation of his financial holdings between the risky stockpiling industry and the risk-free background good industry. It simply states that at the optimum a marginal increase of shareholdings in the background good industry will yield the same increase in the expected utility of income as a unit increase in shareholdings in the stockpiling industry.

The rational expectations market equilibrium is given by:

i) $$1 + r = \frac{E\left[V_y^i\ p(\tilde{x})/\bar{p}\right]}{E\left[V_y^i\right]} = \frac{E\left[V_y^j\ p(\tilde{x})/\bar{p}\right]}{E\left[V_y^j\right]} \quad \text{all } i,\ j \tag{4.29}$$

which states that the expected returns from investments in the stockpiling industry weighted by the expected marginal utility of income must be the same for all individuals and equal to the risk-free return in the background good industry.

(ii) $$\sum_{i=1}^{m} \beta_z^i = 1 \qquad \text{and} \qquad \sum_{i=1}^{m} \beta_s^i = 1 \tag{4.30}$$

where $\beta_z^i \equiv \alpha^i W^i/I_z$ and $\beta_s^i \equiv (1 - \alpha^i)W^i/I_s$ are the fraction of the background good industry and the stockpiling industry, respectively, owned by individual i; that is, the conditions (4.30) are market clearing conditions for the allocation of shares.

(iii) $v_z = I_z$ and $v_s = I_s$,

i.e., in equilibrium the net value of each industry (i.e., v_s^N and v_z^N) is equal to zero.

(iv) Finally, a rational expectations equilibrium is a price distribution $p(\tilde{x})$ that simultaneously satisfies

$$S + M(x) = \sum_{i=1}^{m} q^i(x)$$

for all x, where S + M(x) is the aggregate supply and $\sum_{i=1}^{m} q^i$ (where $q^i = V_p^i/V_y^i$) is aggregate demand, both in state x.

So far we have looked at the rational expectations equilibrium in a competitive stockmarket economy in which the firms choose levels of investment in order to maximize the stockmarket value of the shares of their initial owners and where the subsequent distribution of income among the individuals is effected by means of securities in a competitive stockmarket.

Let us now consider a planned economy with the same set of markets and the same technology as in the market economy. We assume that the planner has full control of the allocation of ownership rights $\{\beta_z^i\}$ and $\{\beta_s^i\}$ and the investment level in each industry, I_z and I_s. As in the market economy the total level of investments is fixed, i.e., $I_z + I_s = I = \sum W^i$. The aim is to contrast the market allocation with that allocation arising from a constrained Pareto optimum. The constraint here is that the planner is restricted to operate within the same set of markets that exists in the market economy.

A feasible allocation $\{\beta_z^{i*}\}$ $\{\beta_s^{i*}\}$ and I_s^* is a constrained Pareto optimum if and only if $\{\beta_z^{i*}\}$ $\{\beta_s^{i*}\}$ and I_s^* solves the following problem for j = 1, 2,...,m:

$$\max_{\beta_z^j,\, \beta_s^j,\, I_s} \quad E\left[V^j(p(\tilde{x}),\ y^j(\tilde{x}))\right]$$

$$\text{subject to} \quad \sum_{i=1}^{m} \beta_z^i = 1;$$

$$\sum_{i=1}^{m} \beta_s^i = 1;$$

$$E\left[V^i(p(\tilde{x}),\ y^i(\tilde{x}))\right] = E\left[\bar{V}^i(p(\tilde{x}),\ y^i(\tilde{x}))\right] \quad (i = 1,2,\ldots,m;\ i \neq j),$$

where

$$y^i(\tilde{x}) = \beta_z^i\ (1 + r)(I - I_z) + \beta_s^i\ p(\tilde{x})\ I_s/\bar{p} \quad (i = 1,2,\ldots,m),$$

and where $E[\bar{V}^i\ (p(\tilde{x}),y(\tilde{x}))]$ are given expected utility levels for all other individuals, i.e., for $i = 1,2,\ldots,m;\ i \neq j$.

To characterize the constrained Pareto optimal allocations we form the Lagrangian:

$$L = \lambda^j\ E\left[V^j(p(\tilde{x}),\ y^j(\tilde{x}))\right] + \sum_{i \neq j} \lambda^i\ E\left[V^i(p(\tilde{x}),\ y^i(\tilde{x})) - \bar{V}^i(p(\tilde{x}),\ y^i(\tilde{x}))\right]$$

where

$$y^i(\tilde{x}) = (1 - \sum_{j \neq i} \beta_z^j)(1 + r)(I - I_s) + (1 - \sum_{j \neq i} \beta_s^j)p(\tilde{x})\ I_s/\bar{p}$$

$$y^j(\tilde{x}) = \beta_z^j(1 + r)(I - I_s) + \beta_s^j\ p(\tilde{x})\ I_s/\bar{p}$$

λ^i are the Kuhn-Tucker multipliers for the utility constraints, $i=1,\ldots,m;\ i \neq j$;

$\lambda^j = 1$ (for symmetry).

Differentiating L with respect to β_z^j, β_s^j and I_s gives

$$\lambda^j\ E[V_y^j(1 + r)(I - I_s)] \qquad (4.31)$$

$$- \lambda^i\ E[V_y^i(1 + r)(I - I_s)] = 0 \quad (i=1,\ldots,m,i \neq j)$$

$$\lambda^j\ E[V_y^j\, p(\tilde{x})\ I_s/\bar{p}] - \lambda^i\ E[V_y^i\ p(\tilde{x})\ I_s/\bar{p}] = 0 \quad (i=1,\ldots,m,i \neq j) \qquad (4.32)$$

$$\sum \lambda^j\ E\left[V_y^j(\beta_s^j\ p(\tilde{x})\ I_s/\bar{p} - \beta_z^j(1 + r))\right] \qquad (4.33)$$

$$+ \sum \lambda^j E\left[(V_p^j + V_y^j \beta_s^j I_s/\bar{p}) \frac{dp(\tilde{x})}{dI_s} = 0.\right.$$

By assumption we have set $dp(\tilde{x})/d\beta_z^i = dp(\tilde{x})/d\beta_s^i = 0$ for all i in (4.31) and (4.32), respectively. Thus we have assumed that any change in the distribution of ownership rights does not change the distribution of prices. With this assumption the first-order conditions for the constrained Pareto problem with respect to β_z^j and β_s^j look like (4.31) and (4.32), respectively, and these in turn correspond exactly to the condition (4.29) for the competitive economy. So, for a given amount of stockpiling, the efficiency of the distribution of income by means of ownership rights is not at stake if we are willing to accept the circumstance that the allocation of ownership holdings does not affect the price distribution.[19)]

Exchange efficiency implies that the condition for constrained Pareto optimality with respect to investments in the stockpiling industry reduces to

$$\sum \lambda^j E\left[(V_p^j + V_y^j \beta_s^j I_s/\bar{p}) \frac{dp(\tilde{x})}{dI_s} = 0,\right. \tag{4.34}$$

or, using Roy's identity, $(q^i = V_p^j/V_y^j)$ condition (4.34) can be written as

$$\sum \lambda^j E\left[(V_y^j(\beta_s^j I_s/\bar{p} - q^j)) \frac{dp(\tilde{x})}{dI_s} = 0.\right. \tag{4.35}$$

Hence, a rational expectations market equilibrium is a constrained Pareto optimum only if (4.35) holds. In addition to the uninteresting case in which stockpiling has no bearing on the price distribution, that is, when $dp(\tilde{x})/dI_s = 0$, there are two conditions under which the market's allocation of investments between the background good and the stockpiling industry implies (4.35).

One possibility is that of a homogeneous-consumers economy. As mentioned earlier, for this case the Pareto optimality of a competitive rational expectations equilibrium has been shown. That the market allocation is efficient in this case can also be seen from (4.35). Individual j's implicit ownership of the economy's oil reserves is equal to $\beta_s^j I_s/\bar{p}$ and q^j is his consumption of oil. So the difference $\beta_s^j I_s/\bar{p} - q^j$ denotes his net trade. Now if all individuals are identical there is no trade in oil, and consequently (4.35) is satisfied. The other possibility in which the market's allocation of investment among the two industries will be efficient is that of a constant marginal utility of income. In this case the LHS of (4.35) can be written

$$V_y^j \, E\left[\sum_{j=1}^{m} (\beta_s^j \, I_s/p - q^j) \frac{dp(\tilde{x})}{dI_s}\right],$$

which is equal to zero since the sum of net trades must be zero.[20]

The results above are quite negative. There are some very special circumstances in which the market economy can be expected to bring about a constrained Pareto optimal amount of storage. However, the market economy will, in general, fail to do so. In principle, Pareto improvements can be achieved by a planner. By deciding on the size of the emergency reserves he can change the price distribution, i.e., the size and distribution of risk. In this way the difference across individuals in the marginal rate of substitution between incomes in different states can be reduced. Consequently, some direct controls or some taxes or subsidies can improve welfare. But in order to be able to proceed with that discussion one would like to know the direction of the bias and the orders of magnitude involved.[21] Also, the information requirement, which seems to be particularly demanding in this type of framework, has to be considered. Further research is necessary before we will be able to recommend the proper, if any, governmental intervention.

FOOTNOTES

1) They state: "Perhaps one of the strongest criticisms of a system of freely competitive markets is that the inherent difficulty in establishing certain markets for insurance brings about a sub-optimal allocation of resources. If we consider an investment as an exchange of certain present income for uncertain future income, then the misallocation will take the form of under-investment" (1970:374).

2) Adelman (1982), who has forcefully claimed that the only defence against an embargo lies in stockpiling, expresses a strong disbelief in the sufficiency of private stockpiling. He says: "Although private inventory accumulation should not be discouraged, it should not be relied on" (1982:10). He arrives at this conclusion by postulating that investment in emergency reserves is too risky to attract private speculators. This view is also supported by Nordhaus (1974). He points out: "The problem is serious, however, because there are no broad futures markets for petroleum in United States," and he continues: "It might be argued that development of such markets would be a better way of ensuring an efficient hedge against supply disruptions. Until such markets exist, however, it seems that the government has no alternative to ensuring sufficient storage in a direct manner" (1974:565). Nichols and Zeckhauser (1977) follow this line of reasoning. As they put it: "The great risk inherent in such a speculation, particularly if economics of scale make small stockpiles infeasible, might deter private parties from stockpiling sufficiently" (1976:68). There is a passage in Keynes (1938) which shows that this disbelief in private stockpiling is not new: "The competitive system abhors the existence of stocks, with a strong a reflex as nature abhors a vacuum, because stocks yield a negative return in terms of themselves" (1938:449).

3) See, e.g., Weinstein and Zeckhauser (1975) and Hoel (1977, 1980).

4) An alternative way to make a "fair comparison" between the planner and the market is to put a restriction on the information available at the planning board. Such a restriction is imposed on the planner in Chapter 5.

5) For obvious reasons our arbitrage conditions during embargo regimes bear a close resemblance to those dealing with the depletion of a natural resource under uncertainty. For instance, competitive arbitrage conditions when there is uncertainty about the arrival of a new technology have been examined by Dasgupta and Stiglitz (1981) and Stiglitz and Dasgupta (1981). The arbitrage equation for a competitively owned resource under threat of expropriation has been considered by Dasgupta, Eastwood and Heal (1978) and Dasgupta (1981). The equilibrium condition for the asset market (4.2) is a discrete and simplified version of the one obtained in Dasgupta and Stiglitz (1981). In their terminology, p is the fall back price of oil (in our interpretation independent of the oil reserves) after the new technology has been introduced.

6) The arbitrage condition (4.2) is the discrete-time version of the continuous Euler-Lagrange equation (2.8). The continuous formulation of the arbitrage equation (4.2) can be derived in the following way. If the time period is a small time interval of length v equation (4.1) can be written

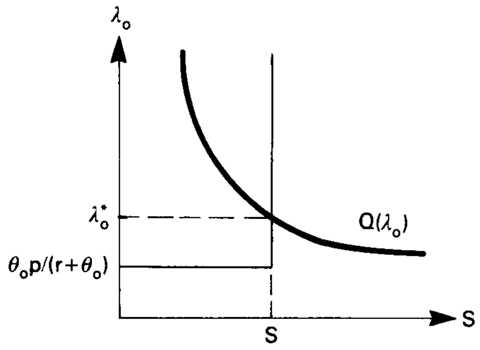
$$\theta_o vp + (1 - \theta_o v)(\lambda_t + d\lambda_t) = (1 + rv)\lambda_t \tag{4.1'}$$

Dividing (4.1') by v and taking the limit as $v \to o$ yields the movement of the spot price

$$\frac{\dot{\lambda}_t}{\lambda_t} = r + \theta_o \frac{(\lambda_t - p)}{\lambda_t}, \tag{4.2'}$$

which is identical to (2.8) in case $V_s(1, S_t) = p$, i.e., in case instantaneous stock adjustment is possible.

7) In order not to confuse the issue we are dealing with here, we have simplified the Figure 4.1. It is not entirely correct. With our identification of the stock withdrawal problem with the problem of depleting an exhaustible resource with a constant marginal extraction cost $\theta_o p/(r + \theta)$ (cf. Section 3.3), it is easy to see that the proper appearance of the figure is the following (cf. Weinstein and Zeckhauser (1975) for a similar figure in the context of a deterministic natural resource model):



8) Our analysis of the cases in which $\lambda_0 \neq \lambda_0^*$ is similar to the one in Stiglitz (1974) and Dasgupta and Heal (1979). For a discussion of the implication of myopic foresight for the intertemporal allocation of exhaustible resources, see these works. A related capital theoretic discussion is to be found in Shell and Stiglitz (1967).

9) The explicit expression for the optimal stock is the discrete time version of equation (2.32):

$$S^* = \sum_{t=0}^{\infty} q\left[\left[\frac{r + \theta_1}{\theta_1} - \frac{\theta_0}{r + \theta_0}\right] p\left[\frac{1 + r}{1 - \theta_0}\right]^t + \frac{\theta_0}{r + \theta_0}\, p\right] .$$

10) Kohn (1978) has shown that the existence and uniqueness of a rational expectations equilibrium in a competitive commodity market is ensured if speculative demand is small relative to consumer demand. Samuelson (1971) has shown that competitive arbitrage under rational expectations would enforce intertemporal equilibrium conditions that are identical to the conditions obtained from maximizing the expected discounted utility of consumption of the good.

11) In the language of Dasgupta and Heal (1979): "Short of this there is no guarantee whatsoever that a competitive environment will not result in the economy pursuing an errant programme" (1979:242).

12) As pointed out by, for example, Turnovsky (1976) and Newbery and Stiglitz (1981) consumers' surplus is an accurate measure of welfare if and only if

$$V_{p_i y} \equiv \frac{\partial^2 V(p_1 \ldots . p_n, y)}{\partial y \partial p_i} = 0$$

where V( ) is the indirect utility function, p_i is a price that is random and y is income. Using Roy's identity, i.e., $V_{p_i} = - q_i V_y$, and differentiating with respect to y we have

$$V_{p_i y} = V_y \, q_i (R - \eta_i)/y$$

where R is the coefficient of relative risk aversion and η_i is the income elasticity of good i. Hence, consumers' surplus is a correct measure when the coefficient of relative risk aversion is equal to the income elasticities of those commodities that are subject to random fluctuations in price.

13) Similar implausible assumptions in international trade models and natural resource models are dealt with in Kemp and Ohyama (1978) and Kemp and Long (1982). In the former paper there is a discussion of the Batra-Russel (1974) international trade model. This model is a two-good, two-country model of international trade in which the terms of trade are random. The producers maximize expected profits and social welfare is measured as the expected value of a concave utility function of both commodities. Kemp and Ohyama conclude that Batra and Russel have imposed two inconsistent assumptions on their model. In the latter paper there is a discussion of incompatible assumptions that is similar to ours.

14) As observed by previous authors (e.g., Lind (1981)) the raising of the discount rate to account for risk is only well-defined under the special circumstances given above.

15) The market price per unit of risk μ is defined as

$$[E(R_m) - r]/\text{var}\,(R_m).$$

16) See, e.g., Prescott and Mehra's (1980) introduction of capital accumulation into the asset pricing model of Lucas (1978). In their model economy there are homogeneous and immortal consumers with time separable utility functions. Each industry consists of many small firms, each producing capital and consumption goods. The source of risk is that output depends not only upon the capital invested but also upon random shocks generated by a stationary Markov process. Under these assumptions, together with the assumption of rational expectations, Prescott and Mehra show that a competitive equilibrium exists and that it is Pareto optimal. Notice that the formal structure of this model bears a close resemblance to the planning model in Chapter 2. Therefore, a competitive economy in the spirit of Prescott and Mehra's model of a market economy would probably "solve" the planner's problem in Chapter 2.

17) The assumption that the number of inventory holders k is fixed can, of course, only be justified by the simplification it affords.

18) If state-dependent transfers were possible, the planner would be able to achieve full Pareto optimality, i.e., (4.20) would be satisfied.

19) A satisfactory justification for prices to be independent of the distribution of ownership rights is that all consumers have identical, homothetic indifference maps. Notice that even if we assume that the consumers have identical, homothetic indifference maps they can still have different attitudes towards risk. That is, the stockmarket can play a role in allocating riskbearing even under this restriction on preferences.

20) Stiglitz (1982) reports another condition for the constrained Pareto optimality of the market. This condition is at hand when all individuals have Cobb-Douglas utility functions. This condition is not applicable for our model.

21) Diamond (1980) discusses public policies in the presence of incomplete risk markets.

REFERENCES

Adelman, Morris A. "Coping with Supply Insecurity." Energy Journal 3 (April 1982):1-17.

Arrow, Kenneth J., and Lind, Robert C. "Uncertainty and the Evaluation of Public Investment Decisions." American Economic Review 60 (June 1970):364-378.

Bailey, Martin J., and Jensen, Michael C. Risk and the Discount Rate for Public Investment. In: Studies in the Theory of Capital Markets. Ed.: M.C. Jensen. New York: Praeger Publishers, 1972.

Baron, David P. "Investment Policy, Optimality and the Mean-Variance Model." The Journal of Finance 34 (March 1979):207-232.

Batra, Raveendra N., and Russel, William R. "Gains from Trade under Uncertainty." American Economic Review 64 (December 1974):1040-1048.

Breeden, Douglas T. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." Journal of Financial Economics 7 (September 1979):265-296.

Dasgupta, Partha S., and Heal, Geoffrey M. Economic Theory and Exhaustible Resources. Cambridge: Cambridge University Press, 1979.

Dasgupta, Partha S., and Stiglitz, Joseph E. "Resource Depletion under Technological Uncertainty." Econometrica 49 (January 1981):85-104.

Dasgupta, Partha S., Eastwood, P.K., and Heal, Geoffrey M. "Resource Management in a Trading Economy." Quarterly Journal of Economics 92 (May 1978):297-306.

Dasgupta, Partha S. "Resource Pricing and Technological Innovations under Oligopoly: A Theoretical Exploration." Scandinavian Journal of Economics 83 (April 1981):149-177.

Diamond, Peter A. "The Role of a Stockmarket in a General Equilibrium Model with Technological Uncertainty." American Economic Review 57 (September 1967): 759-776.

Diamond, Peter A. "Efficiency with Uncertain Supply." Review of Economic Studies 47 (July 1980):645-651.

Grossman, Sanford I. "A Characterization of Equilibrium in Incomplete Markets." Journal of Economic Theory 15 (June 1977):1-15.

Hakansson, Nils and Ohlson, James. The Economics of Financial Markets and Public Information. Pitman Publishing Company. (Forthcoming, 1985.)

Hart, Oliver D. "On the Optimality of Equilibrium when the Market Structure is Incomplete." Journal of Economic Theory 11 (December 1975): 418-443.

Heal, Geoffrey M. Economic Aspects of Natural Resource Depletion. In: The Economics of Natural Resource Depletion. Ed.: D.W. Pearce. London: Macmillan, 1975.

Helpman, Elhanan, and Razin, Assaf. A Theory of International Trade Under Uncertainty. New York: Academic Press, 1978.

Hoel, Michael. Five Essays on the Extraction of an Exhaustible Resource which has a Non-Exhaustible Substitute. Memorandum from Institute of Economics, University of Oslo (October 1977).

Hoel, Michael. Extraction of an Exhaustible Resource under Uncertainty. In: Mathematical Systems in Economics. Eds.: W. Eichborn and R. Henn. Vol. 51. Meisenheim am Glan, 1980.

Kemp, Murray C., and Ohyama, Michihiro. "The Gain from Free Trade under Conditions of Uncertainty." Journal of International Economics 8 (February 1978): 139-141.

Kemp, Murray C., and Long, Ngo V. The Efficiency of Competitive Markets in a Context of Exhaustible Resources. In: Economic Theory of Natural Resources. Eds.: W. Eichborn, R. Henn, K. Neumann, and R.W. Shephard. Würzburg-Wien: Physica-Verlag, 1982.

Keynes, John M. "The Policy of Government Storage of Foodstuff and Raw Materials." Economic Journal 48 (September 1938):449-460.

Kohn, Meir. "Competitive Speculation." Econometrica 46 (September 1978): 1061-1076.

LeRoy, Stephen F. "Expectations Models of Asset Prices: A Survey of Theory." Journal of Finance 37 (March 1982):185-217.

Lind, Robert C. A Primer on the Major Issues Relating to the Discount Rate for Evaluating National Energy Options. In: Discounting for Time and Risk in Energy Policy. Ed.: E.C. Lind. Baltimore: Johns Hopkins University Press for the Resources for the Future, 1982.

Lucas, Robert E. Jr. "Asset Prices in an Exchange Economy." Econometrica 46 (November 1978):1429-1445.

Merton, Robert C. On the Microeconomic Theory of Investment under Uncertainty. In: Handbook of Mathematical Economics, Volume II. Eds.: K.J. Arrow and M.D. Intriligator. Amsterdam: North-Holland, 1982.

Mossin, Jan. The Economic Efficiency of Financial Markets. Lexington, Massachusetts: Lexington Books, 1977.

Malinvaud, Edmund. "Capital Accumulation and Efficient Allocation of Resources." Econometrica 21 (April 1953):233-268.

Newbery, David M.G., and Stiglitz, Joseph E. The Theory of Commodity Price Stabilization - A Study of the Economics of Risk. Oxford: Clarendon Press, 1981.

Newbery, David M.G., and Stiglitz, Joseph E. "The Choice of Techniques and the Optimality of Market Equilibrium with Rational Expectations." Journal of Political Economy 90 (April 1982):223-246.

Nichols, Albert L., and Zeckhauser, Richard J. "Stockpiling Strategies and Cartel Prices." Bell Journal of Economics 8 (Spring 1977):66-96.

Nordhaus, William D. "The 1974 Report of the President's Council of Economic Advisers: Energy in the Economic Report." American Economic Review 64 (September 1974):558-565.

Prescott, Edward C., and Mehra, Rajnish. "Recursive Competitive Equilibrium: The Case of Homogeneous Households." Econometrica 48 (September 1980): 1365-1379.

Radner, Roy. Equilibrium under Uncertainty. In: Handbook of Mathematical Economics, Volume II. Eds.: K.J. Arrow and M.D. Intriligator. Amsterdam: North-Holland, 1982.

Rubinstein, Mark. "The Valuation of Uncertain Income Streams and the Pricing of Options." Bell Journal of Economics 7 (Autumn 1976):407-428.

Samuelson, Paul A. "Stochastic Speculative Price." Proceeding of the National Academy of Sciences 68 (February 1971):335-337.

Shell, K., and Stiglitz, Joseph E. "The Allocation of Investment in a Dynamic Economy." Quarterly Journal of Economics 81 (November 1967):592-609.

Stiglitz, Joseph E. "Growth with Exhaustible Resources: The Competitive Economy." Review of Economic Studies (Symposium on the Economics of Exhaustible Resources 1974):139-152.

Stiglitz, Joseph E. "The Inefficiency of the Stock Market Equilibrium." Review of Economic Studies 49 (April 1982):241-261.

Stiglitz, Joseph E., and Dasgupta, Partha S. "Market Structure and Resource Extraction under Uncertainty." Scandinavian Journal of Economics 83 (1981):178-193.

Turnovsky, Stephen I. "The Distribution of Welfare Gains from Price Stabilization: The Case of Multiplicative Disturbances." International Economic Review 17 (February 1976):133-148.

Weinstein, Milton C., and Zeckhauser, Richard J. "The Optimal Consumption of Depletable Natural Resources." Quarterly Journal of Economics 89 (August 1975):371-392.

5. Demand Uncertainty and the Allocation of Emergency Reserves
– A Cake Eating Problem with Unknown Appetite and Horizon

5.1 THE PROBLEM

As found in the preceding chapter, there is no presumption that a market economy will attain an efficient coordination of the flows and stocks of an embargoed good or of a good that is subject to possible future curtailments in supply. In principle, the problem of an incomplete market structure can be circumvented by a planning board that calculates and implements a contingent intertemporal price structure of the kind derived in Chapters 2 and 3. However, in the real world there is no such thing as a planning board having unlimited knowledge. So the assumption of an omniscient planner is just as unappealing as postulating the existence of a complete set of markets.

The purpose of this chapter is to relax the unrealistic assumption of an omniscient planner. To this end we will put a restriction on the information available to the planning board. Note that such a constraint on the planner is different from the one imposed in Chapter 4. There we constrained the planner to operate only on the markets that do exist. To be more precise, in this chapter we will relax the unrealistic assumption introduced in Chapter 2 that the planner knows with certainty the demand conditions for the good in question. In all other essential respects the models of Chapter 2 and 3 remain intact. However, throughout the analysis we will focus solely on the situation in which the imports of oil have been completely cut off. Given the emergency reserves then available, the task of the planner is to deplete this stock in an optimal fashion.

The additional source of uncertainty or informational difficulty makes the task of the stockpile manager more involved. In particular, we shall see that when the demand conditions are not perfectly known by the planner, there will be a difference between using quantity controls and price incentives to regulate the rate of depletion of the emergency reserves. This is in contrast to a basic theme of microeconomics when there is perfect information, viz. the equivalence of planning by quantities and planning through prices. It is also quite separate from what we can infer from the planning models used hitherto. We cannot discover any difference between the two planning instruments in these models. On the contrary, in implementing the optimal stock release, the stockpile manager in Chapter 2 can equally well adhere to a quantity scheme for direct control as to an allocation attained by announcing an appropriate price path.[1) 2)]

However, when the stockpile manager has insufficient information concerning the demand conditions the following questions become crucial. First, should he announce a price, leave the consumers to make their own decisions and accomodate the resulting demand from the reserves or is a quantitative regulation of the reserves the best way to control the stock depletion. Or, to put it another way, should the stockpile manager set a price and allow the stock depletion to be random or should he decide in advance how much to be released and allow the price (marginal valuation of oil) be random? Second, as the embargo unfolds, might it be appropriate to switch between the two control modes? In our analysis we will make two different assumptions with respect to the quantity mode: either the stock withdrawal is (inefficiently) allocated by means of an allotment scheme without transferable rations or efficiently allocated through a competitive market for rationing coupons.[3)]

The chapter is organized as follows. In Section 5.2 we extend the models in Chapter 2 and 3 to include the imperfect knowledge of the demand conditions. The insufficient information concerning the demand together with the unknown duration of the embargo amounts to saying that the problem facing the stockpile manager is that of eating a cake when appetite and horizon are unknown.

Using dynamic programming arguments we convert the stochastic cake-eating problem into the problem of determining the solution of a particular stochastic functional equation. Manipulating that equation yields a maximization problem that in form bears a close resemblance to the one in Weitzman (1974). The (timeless) setting for the problem in Weitzman's paper is that the output of a firm must be regulated by a planning authority which faces uncertainty about the costs and benefits involved. His result is that the uncertain planning environment renders the conclusion of equivalence between the two extreme control modes - controlling the output by a quantity order and setting a price for the producer - unjustified. In particular he concludes that "quantities are the preferred planning instruments if and only if the benefits have more curvature than costs" (1974:485).[4)]

In Section 5.3 we characterize the solution of the stochastic functional equation. This provides some insights into the characteristics of an optimal decision within a given time period of the embargo. Especially, the relative merits of the two alternative modes for controlling the depletion of the reserves are examined. The basic analysis here is inspired by the model devised by Weitzman (1974).

While Section 5.3 focuses on a welfare comparison of a price allocation and a rationing scheme with transferable ration coupons, we investigate in Section 5.4 the relative desirability of the two control modes in case the stock withdrawal decided in advance under the quantity mode is inefficiently allocated.

The concluding section suggests some extensions.

5.2 THE MODEL

Let $q_t = q(\lambda_t, \varepsilon_t)$ denote the market demand function where q_t is the quantity of oil demanded at price λ_t and ε_t is a summary measure of other conditions affecting demand, each at time t.[5) 6)] Assuming $\partial q(\lambda_t, \varepsilon_t)/\partial\lambda_t < 0$ for all λ_t we can write the inverse demand function $\lambda(q_t, \varepsilon_t)$. For simplicity we assume that demand does not shift over time. We also assume that $\lim_{q_t \to 0} \lambda(q_t, \varepsilon_t) = \infty$ for all t. Moreover, we take it that consumers do not have storage capabilities or, equivalently, that they face infinite storage costs. So, consumers use what they purchase (or the amount they are allotted) in the same period as the purchase (or the allotment).

However, it is fairly unrealistic to assume that the stockpile board knows precisely the market demand conditions. A more realistic representation of its perception of that function would be $q(\lambda_t, \tilde{\varepsilon}_t)$, where the variable $\tilde{\varepsilon}_t$ is meant to reflect the incomplete knowledge of the demand conditions ε_t at time t. We assume that the error $\tilde{\varepsilon}_t$ is distributed independently and identically over time. With this specification there is no scope for learning about the demand conditions (See further Section 5.5).

We take it that the discounted gross benefit enjoyed by the country (as perceived by the stockpile board) at time t when q_t is the rate of stock depletion is given by[7)]

$$\int_0^{q_t} (1 + r)^{-t}\lambda(q, \tilde{\varepsilon}_t)dq \equiv (1 + r)^{-t}u(q_t, \tilde{\varepsilon}_t), \tag{5.1}$$

where r is the discount rate.[8)]

Since we assume a complete discontinuation of oil imports, we have to rely solely on the emergency reserves S on hand when the embargo starts. The relation of stock depletion takes the form

$$S_{t+1} = S_t - q_t, \tag{5.2}$$

where S_t is the stock of emergency reserves on hand in time period t and q_t is the stock withdrawal (consumption) in the same period.

The way q_t is determined depends on the control mode adopted. Either the planner selects the quantity mode and decides in advance to release, say $q_t = \hat{q}_t$, or he decides to announce a price, say $\hat{\lambda}_t$. In the latter case the quantity demanded (rate of stock withdrawal) is given by the ex ante unknown quantity, $q_t = q(\hat{\lambda}_t, \tilde{\varepsilon}_t)$. It is exactly the choice between these two alternatives that we shall discuss at length in Section 5.3.

As in the analysis in Chapters 2 and 3 we will assume that the duration τ of an embargo is unknown. Here we will take it that the duration is distributed according to the geometric distribution. Let θ denote the probability that the country will be in a free-trade regime in the next period, given that there is an embargo going on now.[9] Then the probability that τ is the last embargo period is equal to

$$f(\tau) = (1 - \theta)^{\tau}\theta.$$

After the embargo has expired in (the unknown) period τ the stockpile has to be refilled in order to have reserves available should a new embargo be initiated. We will assume that an instantaneous adjustment of the emergency reserves is possible in the following free-trade period $\tau + 1$. Denote the constant acquisition price by p. Then the total discounted replenishment cost is equal to $(1 + r)^{-(\tau+1)}\, p(S^* - S_{\tau+1})$, where S^* denotes the optimal level of storage.[10] Note that with our assumptions of a geometrically distributed duration of the embargo, of an identically and independently distributed error $\tilde{\varepsilon}_t$ and of instantaneous stock adjustment, the problem of the optimal management of emergency reserves is autonomous. That is, the problem does not depend on time explicitly.

For any time period t in which there is an embargo in effect, let J(S) denote the maximum expected welfare to be derived from optimal behavior with a stock $S_t = S$ in storage. Assume that the country is in an embargo regime in time period t = o. Then the maximum value function is defined by:

$$J(S) \equiv \underset{\{q_t\}}{\text{Max}}\ E\Big[\sum_{\tau=0}^{\infty}(1-\theta)^{\tau}\theta\Big[\sum_{t=0}^{\tau}(1+r)^{-t}u(q_t, \tilde{\varepsilon}_t) \qquad (5.3)$$

$$-(1+r)^{-(\tau+1)}\,p(S^* - S_{\tau+1})\Big]\Big]$$

$$\text{subject to } S_{t+1} = S_t - q_t;$$

$$S_0 = S;$$

$$S_t,\ q_t \geq 0,$$

where the function E takes the expected values of the variables $\tilde{\varepsilon}_t$ and where the operator "Max over $\{q_t\}$" means that either q_t, or λ_t with $q_t = q(\lambda_t, \tilde{\varepsilon}_t)$, should be chosen at each t so as to maximize the objective function. Changing the order of summation we find that the problem (5.3) facing the stockpile board can be equivalently written as [11)]

$$J(S) \equiv \underset{\{q_t\}}{\text{Max}}\ E\Big[\sum_{t=0}^{\infty}\Big[u(q_t, \tilde{\varepsilon}_t) \qquad (5.4)$$

$$-\theta(1+r)^{-1}p(S^* - S_{t+1})\Big](1-\theta)^{t}(1+r)^{-t}\Big]$$

$$\text{subject to } S_{t+1} = S_t - q_t;$$

$$S_0 = S;$$

$$S_t,\ q_t \geq 0.$$

The solution to 5.4 gives the optimal depletion of a given stock S when the duration of the embargo is uncertain and when the stockpile board only has insufficient information concerning the utility function.[12)] If we exclude the information gap $\tilde{\varepsilon}_t$ in the utility function in (5.4), the problem resembles the value functions dealt with in Chapters 2 and 3.[13)] In the case in which only the duration of the embargo was uncertain it was found, however, that it is sufficient to rely on an open-loop depletion control. Solving the value functions (2.4) or (3.5) gave us the time path of conditional spot prices $\{\lambda_t^*\}$ or, equivalently, the stock release path $\{q_t^*\}$ to be followed until the embargo expires. There, the controls were chosen once and for all immediately after the embargo had begun.

In (5.4) where the utility function is not perfectly known it is suboptimal to decide in advance an entire sequence $\{\lambda_t\}$ or $\{q_t\}$. The optimal control can not be specified as a function of time only, but must also be formulated in terms of the emergency reserves on hand at any date t. To see this, assume that the stockpile manager decides to announce a price, say λ_t. Then it is clear that the movement of the emergency reserve is not deterministic but subject to stochastic disturbances transmitted from the demand function, i.e., $S_{t+1} = S_t - q(\lambda_t, \tilde{\varepsilon}_t)$. This fact renders the open-loop technique employed in the previous models inapplicable. Instead we must rely on a model for sequential decision making where the depletion policy is selected sequentially after observing the result of the previous policy.

The only formulation that offers the possibility of revising decisions in light of the new information embodied in the current level of emergency reserves is the method of dynamic programming. Using the principle of optimality in dynamic programming we can write the recurrence relation associated with (5.4) as:

$$J(S_t) = \underset{0 \le q_t \le S_t}{\text{Max}} \; E\Big[u(q_t, \tilde{\varepsilon}_t) - \theta(1 + r)^{-1} p(S^* - S_{t+1}) + (1 - \theta)(1 + r)^{-1} J(S_t - q_t)\Big] \tag{5.5}$$

In words, (5.5) means that the expected welfare enjoyed by the country, if there is an embargo at time t, can be thought of as the expected welfare over time period t, plus the expected welfare that accrues from continuing optimally - whatever the size of reserves and release decisions in period t - from t+1 onward with the resulting stock $S_t - q_t$.

So the recurrence relation (5.5) converts the problem (5.4) into a series of one-period problems. However, it provides no insights into the characteristics of an optimal decision for any given time period. We see that both the utility in the current period t and future welfare are affected by the control q_t which is to be chosen optimally. From now on we will restrict our attention to the characteristics of an optimal decision within a given time period of the embargo. For this purpose we shall assume that we know the $J(S_t - q_t)$ function in the sense that its values are known for all $S_t - q_t$.

Let us rewrite (5.5) in a form that will be more convenient in the sequel. Subtracting $(1 - \theta)(1 + r)^{-1}J(S_t)$ from each side of equation (5.5) yields the following expression:

$$J(S_t) - (1 - \theta)(1 + r)^{-1}J(S_t) \tag{5.6}$$

$$\equiv \max_{o \leq q_t \leq S_t} E\left[u(q_t, \tilde{\epsilon}_t) - c(q_t)\right]$$

where

$$c(q_t) = (1 + r)^{-1} \left[\theta p(S^* - S_t + q_t) + (1 - \theta)(J(S_t) - J(S_t - q_t))\right]$$

The function $c(q_t)$ represents the expected discounted opportunity cost, i.e., benefits forgone in later periods owing to a stock release in the current period. This can be seen in the following way. Should the embargo expire at time t + 1 - the probability of that is θ - the cost of refilling the reserves is $p(S^* - S_{t+1}) = p(S^* - S_t + q_t)$. If the embargo does not terminate - the probability of that event is $1 - \theta$ - the cost of a stock release q_t in period t is equal to $J(S_t) - J(S_t - q_t)$, that is, the additional welfare that can be realized starting period t + 1 with a stock S_t instead of a stock $S_t - q_t$.

The J(S) function, being derived from an optimization problem with a strictly concave maximand subject to a linear constraint, is strictly concave in S. Then it follows immediately that c(q) is a strictly convex function, so we can conclude that the maximization problem in (5.6) is a concave program. Moreover, since the infinite slope of $u(q, \varepsilon)$ at $q = 0$ will always ensure strictly positive values for q, we know that the optimal stock withdrawal in the initial period lies in the open interval (0, S).

5.3 ALLOCATION OF EMERGENCY RESERVES IN THE INITIAL PERIOD

The planning problem in the initial period (period t = o) can now be stated as the problem of maximizing the expected net benefit enjoyed by the country from stock withdrawals during that period. From (5.1) and (5.6) we have that the problem facing the stockpile manager is to solve

$$\underset{q_0}{\text{Max}}\ E\left[\int_0^{q_0} \lambda(q, \tilde{\varepsilon}_0)dq - c(q_0)\right]. \tag{5.7}$$

The stockpile manager considers either of two modes of control for regulating the reserve depletion. The question we ask is, should he rely on market allocation by announcing an appropriate price and accomodating the resulting demand from the reserves or should he decide in advance how much of the reserves are to be released? Let us examine these two options one at a time before we bring them together in a comparison.

Let us start with the quantity control. In this section we will assume that the quantity withdrawn from stocks is efficiently allocated using an allotment scheme with transferable rations. So the optimal stock withdrawal is that quantity q_0^* which maximizes (5.7), that is;

$$E\left[\int_0^{q_0^*} \lambda(q, \tilde{\varepsilon}_0)dq\right] - c(q_0^*) = \underset{q_0}{\text{Max}}\ E\left[\int_0^{q_0} \lambda(q, \tilde{\varepsilon}_0)dq - c(q_0)\right]. \tag{5.8}$$

The solution is characterized by the first order condition

$$E\left[\lambda(q_0^*, \tilde{\varepsilon}_0)\right] = c'(q_0^*). \tag{5.9}$$

Needless to say, this means that the optimal stock withdrawal equates the expected marginal willingness to pay and the marginal opportunity cost.

Now, consider the case where the stockpile board announces a price to control the reserve depletion. When the price λ_0 is charged, the quantity demanded is given by $q_0 = q(\lambda_0, \tilde{\varepsilon}_0)$. Taking the demand function into account, the stockpile manager seeks that price λ_0^* which maximizes (5.7), so that

$$E\left[\int_0^{q(\lambda_0^*, \tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq - c(q(\lambda_0^*, \tilde{\varepsilon}_0))\right]$$

$$= \max_{\lambda_0} E\left[\int_0^{q(\lambda_0, \tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq - c(q(\lambda_0, \tilde{\varepsilon}_0))\right].$$

The following condition must characterize the optimal price λ_0^*:

$$E\left[\lambda(q(\lambda_0^*, \tilde{\varepsilon}_0), \tilde{\varepsilon}_0)\ \partial q(\lambda_0^*, \tilde{\varepsilon}_0)/\partial\lambda_0\right] \tag{5.10}$$

$$= E\left[c'(q(\lambda_0^*, \tilde{\varepsilon}_0))\ \partial q(\lambda_0^*, \tilde{\varepsilon}_0)/\partial\lambda_0\right].$$

The quantity demanded when the optimal λ_0^* is charged is an unknown quantity $q(\lambda_0^*, \tilde{\varepsilon}_0)$ that we denote with $q_0(\tilde{\varepsilon}_0)$. So, the expected value of welfare when the optimal price λ_0^* is charged will be

$$E\left[\int_0^{q_0(\tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq - c(q_0(\tilde{\varepsilon}_0))\right]. \tag{5.11}$$

We have noted that the stockpile manager can either charge a price

λ_0^* or regulate the stock withdrawal at q_0^*. However, the actual demand price and imputed marginal value of the emergency reserve will most likely diverge under either type of policy. Hence, the comparison of the two policies boils down to the question under which control mode for reserve depletion the difference between the total benefits of consumption and the (intertemporal) cost of stock withdrawals is largest on average.

Bringing together (5.8) and (5.11) we notice that the following formula measures the net advantage of charging the price λ_0^* over the option of regulating the stock release at q_0^*:

$$\Delta = E\left[\int_0^{q_0(\tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq \ - c(q_0(\tilde{\varepsilon}_0))\right] \tag{5.12}$$

$$- \left[E\left[\int_0^{q_0^*} \lambda(q, \tilde{\varepsilon}_0)dq_0\right] - c(q_0^*)\right].$$

Naturally, when Δ is positive, there is an advantage of using price incentives. Of course, the conclusion is the opposite when the sign of Δ is negative. The Δ-expression in its present form is by no means easy to analyze. To help in seeing what Δ depends on, we devote an appendix to a transformation of expression (5.12) into a mathematically more tractable form. This derivation follows the technical restrictions imposed by Weitzman (1974), viz. that the variance of $\tilde{\varepsilon}_0$ is sufficiently small to justify a second-order Taylor approximation and the random variable only affects the intercept of the demand schedule and not its slope.

The result of this transformation is the following expression for the net advantage of charging a price over regulating the reserves quantitatively:

$$\Delta = \frac{-\sigma^2}{2(\lambda^1)^2}(\lambda^1 + c^{11}), \tag{5.13}$$

where

λ' is the slope of the inverse demand curve;[14)]

c'' is the slope of the marginal opportunity cost curve;

σ^2 is the variance of the inverse demand curve, i.e.,

$$\sigma^2 = E\Big[\{\lambda(q_0, \tilde{\varepsilon}_0) - E[\lambda(q_0, \tilde{\varepsilon}_0)]\}^2\Big].$$

An alternative formulation that expresses (5.13) in terms of the variance of the quantity demanded is[15)]

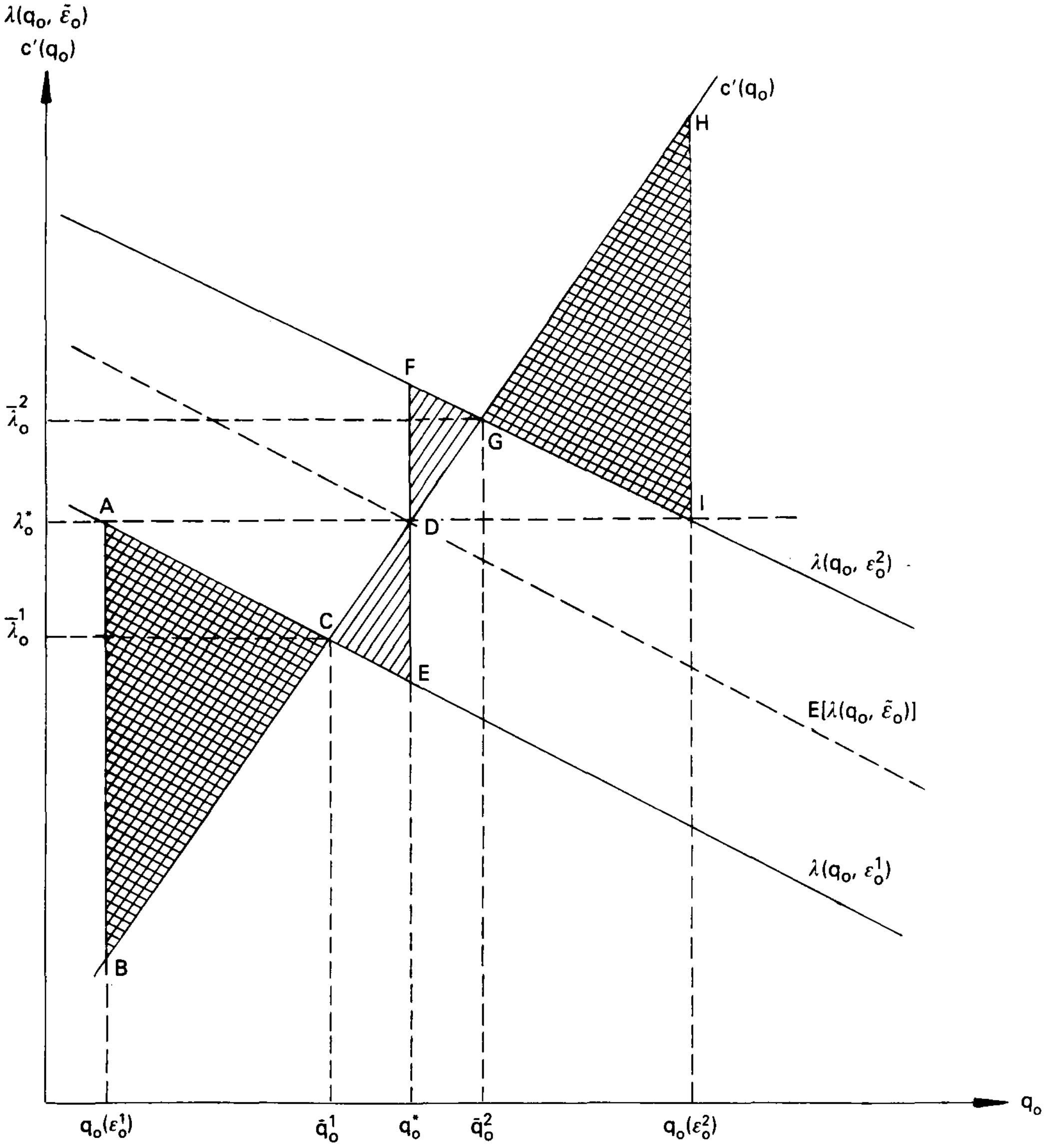
$$\Delta = -\frac{1}{2}(\lambda' + c'')\mathrm{var}(q_0(\tilde{\varepsilon}_0)). \tag{5.14}$$

The net advantage index lends itself to graphical analysis. In Figure 5.1 we have drawn two demand curves corresponding to the states of the world ε_0^1 and ε_0^2, respectively. The dotted curve $E[\lambda(q_0, \tilde{\varepsilon}_0)]$ shows the expected demand curve. The marginal opportunity cost curve is denoted by $c'(q_0)$. Notice that in Figure 5.1 we have made use of the simplifying assumptions mentioned above, viz. that the randomness of the demand schedule affects only its intercept but not its slope, together with the linear approximation of the schedule around the optimal quantity mode q_0^*.

In Figure 5.1 both the quantity q_0^* and the price λ_0^* have the property of equating the expected marginal willingness to pay to the marginal opportunity cost. For q_0^* this follows directly from (5.9) while (5.10) together with the assumption that $\partial q(\lambda_0, \tilde{\varepsilon}_0)/\partial\lambda_0$ is nonstochastic yields the property for λ_0^*. Despite this common property of the two modes for controlling stock withdrawals we can, in the case illustrated in Figure 5.1, establish a preference for the quantity mode.

Let us check this. If the price λ_0^* is announced, a quantity $q_0(\varepsilon_0^1)$ is demanded when the state ε_0^1 occurs and $q_0(\varepsilon_0^2)$ when ε_0^2 occurs. If the demand curve had been perfectly known, the stockpile manager would have charged the price $\bar{\lambda}_0^1$ or, equivalently, decided on the stock withdrawal $\bar{q}_0^1$ if the actual demand curve had been the one denoted with $\lambda(q_0, \varepsilon_0^1)$. If instead ε_0^2 had been the true description of the world, the optimal instruments

Figure 5.1 A case where the quantity mode is superior to the price mode



would have been $\bar{\lambda}_0^2$ or $\bar{q}_0^2$. The welfare forgone by setting the optimal ex ante price λ_0^* instead of the optimal ex post price $\bar{\lambda}_0^1$ in state ε_0^1 is the area ABC. The welfare loss caused by the decision to withdraw the optimal ex ante quantity q_0^* from stocks instead of the optimal ex post withdrawal $\bar{q}_0^1$, is the area CDE. Since ABC is larger than CDE, the deviation from the maximum expected welfare is larger when the price λ_0^* is charged compared with the opposing mode of control. A similar computation for the state ε_0^2 results in the areas GHI and DFG. Their relative sizes indicate the same results as above. Hence, the quantity q_0^* is the preferred planning instrument. It can also be seen from Figure 5.1 that a ceteris paribus decrease in the slope of the marginal opportunity cost curve will create a corresponding increase in the net advantage of using the price λ_0 as the device for implementing the stock release.

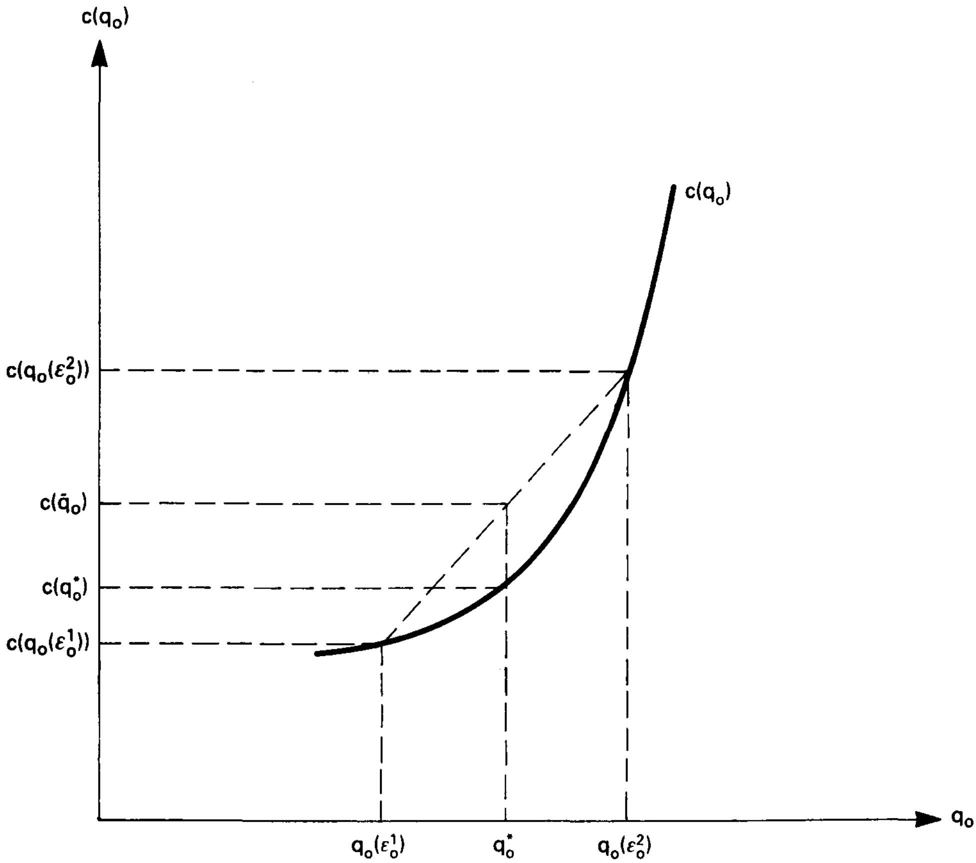
So, the graphical outcomes confirm the analytical results reflected in the net advantage formula. The sign of Δ depends on the absolute slopes of the demand and marginal opportunity cost curves. When the absolute slopes of the two curves are equal, they cancel each other, implying that both methods for controlling the stock withdrawal are equally good. If the demand curve is relatively flat and the slope of the marginal opportunity cost curve is large, then the net advantage of regulating the stock withdrawal at q_0^* will be large. Naturally, the conclusion is the opposite when the marginal opportunity cost increases more slowly then the marginal benefit around the optimal quantity mode q_0^* decreases. In this case it will be more efficient to charge the price λ_0^*.

Whatever the sum of λ^1 and c^{11}, the larger the variance around the expected demand curve (or, the larger the variance of the quantity demanded) the larger will be the expected loss from employing the wrong method of managing the stockpile. If the demand curve is perfectly known, then $\sigma^2 = 0$. In such a case we can not establish a preference for either of the two modes of control.

Figure 5.2 displays the cost function $c(q_0)$. If the price mode is in operation, the quantity demanded will be $q_0(\varepsilon_0^1)$ and $q_0(\varepsilon_0^2)$ in states

ε_o^1 and ε_o^2 , respectively. Let us for illustrative purposes assume that other states are out of the question and that $q(\varepsilon_o^1)$ and $q_o(\varepsilon_o^2)$ will occur with the same probability. As usual q_o^* in Figure 5.2 denotes the optimal quantity control.

Figure 5.2 The intertemporal cost side of the net advantage index



Due to the strict convexity of the cost function, the opportunity cost of releasing q_0^* with certainty $c(q_0^*)$ is less than the expected cost achieved when the price λ_0^* is announced. That latter cost is denoted with $c(\tilde{q}_0)$ and is equal to 0.5 $c(q_0(\varepsilon_0^1))$ + 0.5 $c(q_0(\varepsilon_0^2))$ so this part of the Δ-formula always records a positive bias for regulating the stock withdrawal with quantity decisions. The more convex the opportunity cost function, that is, the steeper the marginal cost curve, the larger is the ceteris paribus welfare forgone by using the price option. A little experimentation with the Figure 5.2 also reveals that the larger the variance in the quantity demanded $\mathrm{var}(q_0(\tilde{\varepsilon}_0))$, the larger is the deviation from the cost of withdrawing q_0^* with certainty. Geometrically, the larger the variation in the quantity demanded, the higher is the chord drawn between the two points on the cost curve, raising $c(\tilde{q}_0)$ but leaving $c(q_0^*)$ untouched.

Since the gross benefit function is strictly concave, one might suspect that a similar bias against the price option is felt on the benefit side of the net advantage formula: a certain stock withdrawal of size q_0^* under the quantity mode is preferred to the uncertain quantity $q_0(\tilde{\varepsilon}_0)$ achieved when the price λ_0^* is charged. This is verified in the appendix (see A.5.18). The bias for deciding in advance how much to withdraw (q_0^*) is there reflected in the negative term

$$E\left[\frac{1}{2}\lambda^1(-\lambda(\tilde{\varepsilon}_0)/\lambda^1)^2\right] = \sigma^2/2\lambda^1 = \frac{1}{2}\lambda^1\mathrm{var}(q_0(\tilde{\varepsilon}_0)).$$

However, in (A.5.18) we also have the positive term

$$E\left[-(\lambda(\tilde{\varepsilon}_0))^2/\lambda^1\right] = -\sigma^2/\lambda^1 = -\lambda^1\mathrm{var}(q_0(\tilde{\varepsilon}_0)).$$

The net result $-\sigma^2/2\lambda^1$ (or, alternatively, $-1/2\,\lambda^1\mathrm{var}(q_0(\tilde{\varepsilon}_0))$, which is always positive, is recorded in the first term in the Δ-formula (cf. (5.13) or (5.14)). So the negative effect of price induced variations in demand (due to the concavity of the gross benefit function) is counterbalanced by the efficiency gain of letting demand move in the appropriate direction relative to the actual valuation of oil.

5.4 THE RELATIVE DESIRABILITY OF THE TWO CONTROL MODES WHEN THE STOCK WITHDRAWAL UNDER THE QUANTITY MODE IS INEFFICIENTLY ALLOCATED

In this section we will drop the assumption that the stock withdrawal decided in advance under the quantity mode is efficiently allocated. Inefficient allocations would result if the consumers were prevented from selling and buying ration coupons, or if the coupon market were to function imperfectly. To gain insights into the relative desirability of the two control modes when the allocation of oil is inefficient under the quantity mode we assume that the stock withdrawal is allocated by means of an allotment scheme without transferable ration coupons. The allotment to any consumer is based on the stockpile board's imperfect knowledge of that particular consumer's demand conditions.

Let us formalize such an allotment scheme. Suppose the economy is composed of n consumers. Let $\lambda_j(q_o, \tilde{\varepsilon}_{oj})$ denote the stockpile manager's perception of the jth consumer's demand function. Now we define the optimal allotment scheme as the set of rations $\{q^*_{oj}\}^n_{j=1}$ such that

$$E\left[\sum_{j=1}^{n} \int_{0}^{q^*_{oj}} \lambda_j(q_j, \tilde{\varepsilon}_{oj})dq_j\right] - c\left(\sum_{j=1}^{n} q^*_{oj}\right) \tag{5.15}$$

$$= \underset{\{q_{oj}\}}{\text{Max}}\; E\left[\sum_{j=1}^{n} \int_{0}^{q_{oj}} \lambda_j(q_j, \tilde{\varepsilon}_{oj})dq_j - c\left(\sum_{j=1}^{n} q_{oj}\right)\right].$$

The optimal set of rations $\{q^*_{oj}\}^n_{j=1}$ is then characterized by

$$E\left[\lambda_j(q^*_{oj}, \tilde{\varepsilon}_{oj})\right] = c'\left(\sum_{j=1}^{n} q^*_{oj}\right) \quad (j = 1, 2, \ldots n). \tag{5.16}$$

Equations (5.16) imply, of course, that the optimal set of rations equates the expected marginal willingness to pay across consumers and the marginal opportunity cost.

With such an allotment scheme the generalization of (5.13) is given by[16]:

$$\Delta_n = - \sum_{j=1}^{n} \frac{\sigma_j^2}{2(\lambda_j^1)^2} (\lambda_j^1 + c^{11}) - \sum_{j=1}^{n} \sum_{i \neq j}^{n} \frac{\sigma_{ij}}{2\lambda_j^1 \lambda_i^1} c^{11} \tag{5.17}$$

where

λ_j^1 is the slope of the inverse demand schedule for consumer j (cf. footnote 14);

c^{11} is the slope of the marginal opportunity cost schedule (cf. footnote 14);

σ_j^2 is the variance of the inverse demand for consumer j, i.e.;

$$\sigma_j^2 = E\left[\{\lambda_j(q_{oj}, \tilde{\varepsilon}_{oj}) - E[\lambda_j(q_{oj}, \tilde{\varepsilon}_{oj})]\}^2\right];$$

σ_{ij} is the covariance between the inverse demands of consumer i and consumer j, i.e.,

$$\sigma_{ij} = E\left[\{\lambda_i(q_{oi}, \tilde{\varepsilon}_{oi}) - E[\lambda_i(q_{oi}, \tilde{\varepsilon}_{oi})]\} \cdot \right.$$
$$\left. \cdot \{\lambda_j(q_{oj}, \tilde{\varepsilon}_{oj}) - E[\lambda_j(q_{oj}, \tilde{\varepsilon}_{oj})]\}\right].$$

An alternative formulation that expresses (5.17) in terms of the variances and covariances of the quantity demanded is given by (5.18), which corresponds to formula (5.14) in Section 5.3.

$$\Delta_n = - \frac{1}{2} \sum_{j=1}^{n} (\lambda_j^1 + c^{11}) \mathrm{var}(q_{oj}(\tilde{\varepsilon}_{oj})) \tag{5.18}$$
$$- \frac{1}{2} \sum_{j=1}^{n} \sum_{i \neq j}^{n} c^{11} \mathrm{cov}(q_{oj}(\tilde{\varepsilon}_{oj}), q_{oi}(\tilde{\varepsilon}_{oi})).$$

The first two terms in (5.18) are much like the corresponding terms in formula (5.14). The first term,

$$-\frac{1}{2}\sum_{j=1}^{n} \lambda_j^1 \mathrm{var}(q_{oj}(\tilde{\varepsilon}_{oj})),$$

sums over all the consumers the net efficiency gain of the demand variation allowed by price incentives. The second term

$$-\frac{1}{2}\sum_{j=1}^{n} c^{11} \mathrm{var}(q_{oj}(\tilde{\varepsilon}_{oj}))$$

reflects the bias against price induced variation in total demand that causes the expected opportunity cost of stock withdrawals to be above that achieved where $\sum_{j=1}^{n} q_{oj}^*$ is depleted with certainty.

An important difference between (5.14) and (5.18) is found in the expression with the double summation in (5.18). Whenever the covariance between the demand curves of (any pair of) consumer i and j is positive, the variations in individual demands induced by prices are in the same direction, increasing the variance of total demand as compared with the situation when $\mathrm{cov}(q_{oj}(\tilde{\varepsilon}_{oj}), q_{oi}(\tilde{\varepsilon}_{oi}))$ is zero. Hereby a reduction of the competitiveness of the price option is recorded on the cost side of the net advantage formula (cf. the discussion accompanying Figure 5.2).

In proceeding to trace the effects of the nonexistence of a market for ration coupons on the net advantage of charging a price over the allotment scheme we make some simplifying assumptions. First, let us assume that the n consumers have linear demand curves with identical slopes and means. Let us denote the aggregate demand curve with $\lambda(\cdot, \cdot)$; the slope of this curve is related to the slope of an individual demand curve as $\lambda^1 = \lambda_k^1/n$ where k is any one of the individual consumers. Let us further assume that $\sigma_i^2 = \sigma_j^2 = \sigma^2$ for all i and j and that $\sigma_{ij} = \rho\sigma^2$ for all $i \neq j$, where ρ is the coefficient of correlation between the demand curves of different consumers.

Incorporating these additional assumptions into the Δ-formula of (5.17) yields:

$$\Delta_n = -\frac{\sigma^2}{2(\lambda^1)^2}\left[\lambda^1 + \frac{c^{11}}{n}(1 + \rho(n - 1))\right]. \tag{5.19}$$

If there is a perfect correlation between the demand curves, that is, if $\rho = 1$, then expression (5.19) simply represents the net advantage formula applicable when the quantity mode is combined with transferable ration coupons. This is what one would expect: with our assumptions, perfectly correlated consumers place the same marginal value on the allotment q_0^*/n, no matter what state of the world occurs. Nobody wants to buy or sell coupons. So there can not be any difference between convertible and inconvertible rationing in this case.

In order to proceed let us first look at the special case with two consumers that are perfectly inversely correlated. In this case, that is, when $\rho = -1$ and $n = 2$, the sum of the second and third terms in (5.19) is equal to zero. This in turn implies that the net advantage of the price option over the scheme of nontransferable ration coupons is positive. This result is fairly natural. The trouble with the price option in case the aggregated demand curve is unknown is that, though efficient allocation among the consumers is guaranteed, the total demand (stock withdrawal) might settle down at an undesirable level. However, in the case of two consumers with the same expected demand that are perfectly inversely correlated, the total demand is perfectly known whereas the individual demand curves are not. In other words the variation in total demand is zero, and the desirable stock withdrawal is known. The best way to allocate this quantity is, of course, through price incentives.

Now if there are more than two consumers, they cannot all be perfectly inversely correlated. We notice that

$$\mathrm{var}\left[\sum_{j=1}^{n} \lambda_j(q_{0j}, \tilde{\varepsilon}_{0j})\right] = \sum_{j=1}^{n} \sigma_j^2 + \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i \neq j}}^{n} \sigma_{ij}.$$

Inserting $\sigma_j^2 = \sigma^2$ for all j and $\sigma_{ij} = \rho\sigma^2$ for all $i \neq j$ we have

$$\text{var}\Big[\sum_{j=1}^{n} \lambda_j(q_{oj}, \tilde{\varepsilon}_{oj})\Big] = n\sigma^2 + n(n - 1)\rho\sigma^2, \tag{5.20}$$

which in turn implies that

$$\rho \geq \frac{1}{1 - n}.$$

For the special case $\rho = 1/(1 - n)$, we obtain a generalization of the case with two perfectly inversely correlated consumers. In this case the variance of aggregate demand is zero implying that an allotment scheme without transferable rations is unmistakable inferior to price incentives.

In the case where consumer's demand are uncorrelated, that is, when $\rho = 0$, the net advantage formula reads:

$$\Delta_n = - \frac{\sigma^2}{2(\lambda^1)^2}\Big[\lambda^1 + \frac{c^{11}}{n}\Big]. \tag{5.21}$$

Expression (5.21) implies that an increase in the number of consumers creates an increase in the advantage of the price option. The reason behind this result is, of course, that the larger the number of consumers, the larger are the expected losses from inconvertible rationing.

5.5 CONCLUDING REMARKS

For the sake of ease of exposition we have throughout concentrated on the choice between the two extreme modes for controlling the depletion of reserves. Various forms of complementary usage of both types of control have been considered in the literature on optimal planning instruments under uncertainty, e.g., Roberts and Spence (1976), Weitzman (1978) and Dasgupta, Hammond and Maskin (1980). An obvious extension of the present model is to consider planning instruments for controlling the stock withdrawal based on both price incentives and quantity targets.

Another extension is worth a brief reflection. In the present model the subjective assessment of the distribution of any given $\tilde{\varepsilon}_t$ made at t = 0, remains fixed throughout the embargo. So there is no scope for learning. However, the stockpile board is likely to learn from experience. The description of the information structure may take any of several forms. The simplest possible is when the information emerges autonomously with the passage of time. The anticipation of the information to be derived and the possibility of reacting to the emerging information flow must be taken into account. This version of the model could be used to explore the notion of option values (see, e.g., Arrow and Fisher (1974)) in the context of emergency reserves.

A more complex information structure is at hand if the amount of learning about the demand conditions is deliberately directed by decisions, i.e., if active learning is possible. For instance, information can be gained as a result of experimenting with the planning instruments. Since the information passed back to the stockpile board might not be the same under the two control modes the information content has to be distinguished. To cope with these considerations a Bayesian expectation revision mechanism has to be incorporated in the model. Hereby, the welfare comparison of the two control modes would include the net information advantage of the price option over the quantity mode.

Using dynamic programming arguments we have converted the original stochastic stock withdrawal problem (5.4) into the problem of determining the solution of a stochastic functional equation (5.6). The characteristics of an optimal decision in any given time period t, given that the maximum value function J(S) is known, have been obtained. However, we have not obtained an analytical solution to the problem, that is, we have not determined a closed-form expression for the J(S) function. Unfortunately, this means that we can only make a rough comment on the changes over time of the net advantage of the price option over quantity regulation. It is clear that as we get closer to the "bottoms of the tanks" in our reserves the marginal opportunity cost curve becomes steeper, implying an increasing ceteris paribus advantage of using quantitative regulations. However, this is counterbalanced by the increasing slope of the marginal benefit curve as the quantity released diminishes. Of course, a numerical solution to the functional equations would expose the actual time pattern of control modes for sustaining the optimal depletion.

APPENDIX 5

THE NET ADVANTAGE INDEX

5:1 The net advantage index applicable when the quantity mode is used in combination with a competitive market for ration coupons.

The problem facing the stockpile manager is to maximize

$$E\left[\int_0^{q_0} \lambda(q, \tilde{\varepsilon}_0)dq - c(q_0)\right], \tag{A.5.1}$$

where E[ ] is the expected value operator.

Under the quantity mode the optimal stock withdrawal is that quantity q_0^* which maximizes (A.5.1) so that

$$E\left[\int_0^{q_0^*} \lambda(q, \tilde{\varepsilon}_0)dq\right] - c(q_0^*) = \underset{q_0}{\text{Max}}\ E\left[\int_0^{q_0} \lambda(q, \tilde{\varepsilon}_0)dq - c(q_0)\right]. \tag{A.5.2}$$

The solution q_0^* is characterized by the first order condition

$$E\left[\lambda(q_0^* \ \tilde{\varepsilon}_0)\right] = c'(q_0^*). \tag{A.5.3}$$

If instead a price λ_0 is charged the demand reads $q_0 = q(\lambda_0, \tilde{\varepsilon}_0)$. The stockpile manager selects that price λ_0^* which maximizes (A.5.1) given the demand function $q(\cdot, \cdot)$ so that

$$E\left[\int_0^{q(\lambda_0^*, \tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq - c(q(\lambda_0^*, \tilde{\varepsilon}_0))\right] =$$

$$= \underset{\lambda_0}{\text{Max}}\ E\left[\int_0^{q(\lambda_0, \tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq - c(q(\lambda_0, \tilde{\varepsilon}_0))\right].$$

The following condition characterizes the optimal price λ_0^*:

$$E[\lambda(q(\lambda_0^*, \tilde{\varepsilon}_0), \tilde{\varepsilon}_0) \cdot \partial q(\lambda_0^*, \tilde{\varepsilon}_0)/\partial\lambda_0] = \tag{A.5.4}$$

$$= E[c'(q(\lambda_0^*, \tilde{\varepsilon}_0)) \cdot \partial q(\lambda_0^*, \tilde{\varepsilon}_0)/\partial\lambda_0].$$

To obtain an expression for the optimal price λ_0^* we first notice that when the price λ_0 is announced the utility maximizing demand is $q(\lambda_0, \tilde{\varepsilon}_0)$. This in turn implies that $\lambda_0 = \lambda(q(\lambda_0, \tilde{\varepsilon}_0), \tilde{\varepsilon}_0) = E[\lambda(q(\lambda_0, \tilde{\varepsilon}_0), \tilde{\varepsilon}_0)]$, i.e., when the price λ_0 is announced the stockpile manager knows that the marginal valuation of oil is λ_0. Then we can we rewrite (A.5.4) as

$$\lambda_0^* = \frac{E[c'(q(\lambda_0^*, \tilde{\varepsilon}_0)) \cdot \partial q(\lambda_0^*, \tilde{\varepsilon}_0)/\partial\lambda_0]}{E[\partial q(\lambda_0^*, \tilde{\varepsilon}_0)/\partial\lambda_0]}. \tag{A.5.5}$$

At the price λ_0^* the quantity demanded is $q(\lambda_0^*, \tilde{\varepsilon}_0)$ which can be expressed as a function of $\tilde{\varepsilon}_0$, say, $q_0(\tilde{\varepsilon}_0)$. So the expected welfare when the price λ_0^* is charged is given by

$$E\left[\int_0^{q_0(\tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq - c(q_0(\tilde{\varepsilon}_0))\right]. \tag{A.5.6}$$

Using (A.5.2) and (A.5.6) we can express the net advantage of the price mode over the quantity mode as:

$$\Delta = E\left[\int_0^{q_0(\tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq - c(q(\tilde{\varepsilon}_0))\right]$$

$$- \left[E\left[\int_0^{q_0^*} \lambda(q, \tilde{\varepsilon}_0)dq\right] - c(q_0^*)\right]. \tag{A.5.7}$$

Now we make a quadratic approximation of the area under the demand function around the optimal quantity q_0^* to obtain

$$\int_0^{q_0} \lambda(q, \tilde{\varepsilon}_0)dq \approx \int_0^{q_0^*} \lambda(q, \tilde{\varepsilon}_0)dq + \left[E\left[\lambda(q_0^*, \tilde{\varepsilon}_0)\right] + \lambda(\tilde{\varepsilon}_0)\right](q_0 - q_0^*) \quad (A.5.8)$$

$$+ \frac{1}{2} \lambda^1 (q_0 - q_0^*)^2,$$

where

$$\lambda(\tilde{\varepsilon}_0) \equiv \lambda(q_0^*, \tilde{\varepsilon}_0) - E\left[\lambda(q_0^*, \tilde{\varepsilon}_0)\right]$$

and

$$\lambda^1 \equiv \frac{\partial\lambda(q_0^*, \tilde{\varepsilon}_0)}{\partial q_0} \quad ^{1)}$$

Likewise, the quadratic representation of the cost function is written

$$c(q_0) \approx c(q_0^*) + c'(q_0^*)(q_0 - q_0^*) + \frac{1}{2} c^{11}(q_0 - q_0^*)^2, \qquad (A.5.9)$$

where

$$c^{11} \equiv c''(q_0^*). \quad ^{2)}$$

Expressions (A.5.8) and (A.5.9) look exactly like ordinary second-order Taylor approximations of the consumers' surplus and cost function. Now we make the assumption that the coefficient λ^1 is a fixed (nonrandom) coefficient. Moreover, without loss of generality, we take it that $E\left[\lambda(\tilde{\varepsilon}_0)\right] = 0$. On differentiating (A.5.8) with respect to q_0, the implication of these assumptions becomes clear. Then we have the following expression for the demand curve:

$$\lambda(q_0, \tilde{\varepsilon}_0) \approx E\left[\lambda(q_0^*, \tilde{\varepsilon}_0)\right] + \lambda(\tilde{\varepsilon}_0) + \lambda^1(q_0 - q_0^*). \qquad (A.5.10)$$

---

1) We use λ^1 instead of $\lambda_1(q_0, \tilde{\varepsilon}_0) \equiv \partial\lambda(q_0, \tilde{\varepsilon}_0)/\partial q_0$ to denote that the slope of the demand curve is deterministic by assumption.

2) Since $c(q_0)$ is deterministic there is no need to distinguish between $c''(q_0) = d^2c(q_0)/dq_0^2$ and c^{11}. For reasons of symmetry, however, we use c^{11}.

Thus, by assumption, the slope of the demand curve is perfectly known. The random variable $\tilde{\varepsilon}_0$ only affects, via the stochastic function $\lambda(\tilde{\varepsilon}_0)$, the intercept of the demand schedule, the expected value of which is given by $E[\lambda(q_0, \tilde{\varepsilon}_0)]$. Differentiating (A.5.9) with respect to q_0 we receive:

$$c'(q_0) = c'(q_0^*) + c''(q_0 - q_0^*). \tag{A.5.11}$$

Rearranging terms in (A.5.10) yields

$$q(\lambda_0, \tilde{\varepsilon}_0) \approx q_0^* + \frac{\lambda(q_0, \tilde{\varepsilon}_0) - E[\lambda(q_0^*, \tilde{\varepsilon}_0)] - \lambda(\tilde{\varepsilon}_0)}{\lambda'}. \tag{A.5.12}$$

Replacing q_0 in (A.5.11) by the expression for $q(\lambda_0^*, \tilde{\varepsilon}_0)$ from (A.5.12) and taking the expected value we obtain

$$E\left[c_1(q(\lambda_0^*, \tilde{\varepsilon}_0))\right] \approx c_1(q_0^*) + \frac{c''}{\lambda'}\left[\lambda_0^* - E[\lambda(q_0^*, \tilde{\varepsilon}_0)]\right]. \tag{A.5.13}$$

By assumption $\partial q(\lambda_0^*, \tilde{\varepsilon}_0)/\partial\lambda_0$ is nonstochastic. Then by (A.5.5), the left hand side of expression (A.5.13) is equal to λ_0^*. And by (A.5.3) this in turn implies that (A.5.13) can be written as:

$$\lambda_0^* = E[\lambda(q_0^*, \tilde{\varepsilon}_0)] \tag{A.5.14}$$

Then from (A.5.12), (A.5.14) and the definition $q_0(\tilde{\varepsilon}_0) \equiv q(\lambda_0^*, \varepsilon_0)$ we have

$$q_0(\tilde{\varepsilon}_0) = q_0^* - \frac{\lambda(\tilde{\varepsilon}_0)}{\lambda'}. \tag{A.5.15}$$

Setting q_0 equal to $q_0(\tilde{\varepsilon}_0)$ in the expression (A.5.8) and (A.5.9) we obtain:

$$\int_0^{q_0(\tilde{\varepsilon}_0)} \lambda(q, \tilde{\varepsilon}_0)dq \approx \int_0^{q_0^*} \lambda(q, \tilde{\varepsilon}_0)dq + \left[E[\lambda(q_0^*, \tilde{\varepsilon}_0)] + \lambda(\tilde{\varepsilon}_0)\right]\left[\frac{-\lambda(\tilde{\varepsilon}_0)}{\lambda'}\right] + \frac{1}{2}\lambda'\left[\frac{-\lambda(\tilde{\varepsilon}_0)}{\lambda'}\right]^2 \tag{A.5.16}$$

and

$$c(q_0(\tilde{\varepsilon}_0)) = c(q_0^*) + c_1(q_0^*)\left[\frac{-\lambda(\tilde{\varepsilon}_0)}{\lambda^1}\right] \tag{A.5.17}$$

$$+ \frac{1}{2} c^{11}\left[\frac{-\lambda(\tilde{\varepsilon}_0)}{\lambda^1}\right]^2,$$

respectively.

Inserting (A.5.16) and (A.5.17) into the Δ-formula (A.5.7) yields

$$\Delta = E\Big[-(\lambda(\tilde{\varepsilon}_0))^2/\lambda^1 + \frac{1}{2}\lambda^1(-\lambda(\tilde{\varepsilon}_0)/\lambda^1)^2 \tag{A.5.18}$$

$$- \frac{1}{2} c^{11}(-\lambda(\tilde{\varepsilon}_0)/\lambda^1)^2\Big] =$$

$$= -\frac{\sigma^2}{2(\lambda^1)^2}(\lambda^1 + c^{11})$$

where

$$\sigma^2 = \mathrm{var}(\lambda(\tilde{\varepsilon}_0)) = E\Big[\{\lambda(\tilde{\varepsilon}_0) - E[\lambda(\tilde{\varepsilon}_0)]\}^2\Big].$$

Noting that $\mathrm{var}(q_0(\tilde{\varepsilon}_0)) = E[(-\lambda(\tilde{\varepsilon}_0)/\lambda^1)^2] = \sigma^2/(\lambda^1)^2$ it follows that we can just as well express the Δ-formula in (A.5.18) in terms of the variation in demand:

$$\Delta = -\frac{1}{2}(\lambda^1 + c^{11})\mathrm{var}(q_0(\tilde{\varepsilon}_0)). \tag{A.5.19}$$

5:2 <u>The net advantage index applicable when the quantity mode is based on an allotment scheme without transferable ration coupons.</u>

Suppose the economy is composed of n consumers. Let $\lambda_j(q_{0j}, \tilde{\varepsilon}_{0j})$ denote the stockpile manager's perception of the jth consumer's demand curve. Then the generalization of (A.5.7) is

$$\Delta_n = E\Big[\sum_{j=1}^{n} \int_0^{q_{oj}(\tilde{\varepsilon}_{oj})} \lambda_j(q, \tilde{\varepsilon}_{oj})dq - c\big(\sum_{j=1}^{n} q_{oj}(\tilde{\varepsilon}_{oj})\big)\Big] \tag{A.5.20}$$

$$- \Big[E\Big[\sum_{j=1}^{n} \int_0^{q^*_{oj}} \lambda_j(q, \tilde{\varepsilon}_{oj})dq\Big] - c\big(\sum_{j=1}^{n} q^*_{oj}\big)\Big].$$

Using the same simplifying assumption that we made in case I, we can transform the Δ_n-formula into a form that is more easily interpreted. Construct quadratic representations of the functions

$$\int_0^{q_{oj}} \lambda_j(q, \tilde{\varepsilon}_{oj})dq \quad (j = 1 ,2,\ldots,n) \quad \text{and} \quad c\big(\sum_{j=1}^{n} q_{oj}\big)$$

around the optimal allotment scheme defined by

$$\{q^*_{oj}\}^n_{j = 1},$$

such that

$$E\big[\lambda_j(q^*_{oj}, \tilde{\varepsilon}_{oj})\big] = c'\big(\sum_{j=1}^{n} q^*_{oj}\big) \qquad (j = 1, 2,\ldots, n).$$

On inserting

$$q_{oj} = q_{oj}(\tilde{\varepsilon}_{oj}) = q^*_{oj} - \frac{\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}$$

into these approximations we get

$$\int_0^{q_{oj}(\tilde{\varepsilon}_{oj})} \lambda_j(q, \tilde{\varepsilon}_{oj})dq \approx \int_0^{q^*_{oj}} \lambda_j(q, \tilde{\varepsilon}_{oj})dq \tag{A.5.21}$$

$$+ \left[E\left[\lambda_j(q^*_{oj}, \tilde{\varepsilon}_{oj})\right] + \lambda_j(\tilde{\varepsilon}_{oj})\right] \left[\frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right]$$

$$+ \frac{1}{2} \lambda^1_j \left[\frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right]^2$$

and

$$c\left(\sum_{j=1}^{n} q_{oj}(\tilde{\varepsilon}_{oj})\right) \approx c\left(\sum_{j=1}^{n} q^*_{oj}\right) + c'\left(\sum_{j=1}^{n} q^*_{oj}\right) \left[\sum_{j=1}^{n} \frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right] \tag{A.5.22}$$

$$+ \frac{1}{2} c'' \left[\sum_{j=1}^{n} \frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right]^2 .$$

Inserting (A.5.21) and (A.5.22) into (A.5.20) yields

$$\Delta_n = E\left[\sum_{j=1}^{n} \left[E\left[\lambda_j(q^*_{oj}, \tilde{\varepsilon}_{oj})\right] + \lambda_j(\tilde{\varepsilon}_{oj})\right] \left[\frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right]\right.$$

$$+ \sum_{j=1}^{n} \frac{1}{2} \lambda^1_j \left[\frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right]^2 - c'\left(\sum_{j=1}^{n} q^*_{oj}\right)\left[\sum_{j=1}^{n} \frac{\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right]$$

$$\left. - \frac{1}{2} c'' \left[\sum_{j=1}^{n} \frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda^1_j}\right]^2\right].$$

Noting that

$$- \frac{1}{2} c^{11} E \left[\sum_{j=1}^{n} \frac{- \lambda_j(\tilde{\varepsilon}_{oj})}{\lambda_j^1} \right]^2 =$$

$$- \frac{1}{2} c^{11} E\left[\sum_{j=1}^{n} \left[\frac{- \lambda_j(\tilde{\varepsilon}_{oj})}{\lambda_j^1} \right]^2 \right] - \frac{1}{2} c^{11} E\left[\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\lambda_j(\tilde{\varepsilon}_{oj})\lambda_i(\tilde{\varepsilon}_{oj})}{\lambda_j^1 \lambda_i^1} \right],$$

Δ_n can be further simplified to yield

$$\Delta_n = - \sum_{j=1}^{n} \frac{\sigma_j^2}{2(\lambda_j^1)^2} (\lambda_j^1 + c^{11}) - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\sigma_{ij}\, c^{11}}{2\lambda_j^1 \lambda_i^1}, \tag{A.5.23}$$

where

$$\sigma_j^2 = \mathrm{var}(\lambda_j(\tilde{\varepsilon}_{oj})) = E\left[\{\lambda_j(\tilde{\varepsilon}_{oj}) - E[\lambda_j(\tilde{\varepsilon}_{oj})]\}^2\right],$$

$$\sigma_{ij} = \mathrm{cov}\left[\lambda_j(\tilde{\varepsilon}_{oj}), \lambda_i(\tilde{\varepsilon}_{oi})\right] =$$

$$= E\left[\{ \lambda_j(\tilde{\varepsilon}_{oj}) - E[\lambda_j(\tilde{\varepsilon}_{oj})]\} \cdot \{\lambda_i(\tilde{\varepsilon}_{oi}) - E[\lambda_i(\tilde{\varepsilon}_{oi})]\}\right].$$

Since

$$\mathrm{Var}(q_o(\tilde{\varepsilon}_{oj})) = E\left[\left[\frac{-\lambda_j(\tilde{\varepsilon}_{oj})}{\lambda_j^1}\right]^2\right] = \frac{\sigma_j^2}{(\lambda_j^1)^2},$$

and

$$\mathrm{cov}\left[q_{oj}(\tilde{\varepsilon}_{oj}), q_{oi}(\tilde{\varepsilon}_{oi})\right] = \frac{\sigma_{ij}}{\lambda_j^1 \lambda_i^1}$$

expression (A.5.23) can equivalently be written as

$$\Delta_n = -\frac{1}{2}\sum_{j=1}^{n}(\lambda_j^1 + c^{11})\ \mathrm{var}(q_{oj}(\tilde{\varepsilon}_{oj})) \qquad \text{(A.5.24)}$$

$$-\frac{1}{2}\sum_{j=1}^{n}\sum_{i\neq j}^{n} c^{11}\ \mathrm{cov}\left[q_{oj}(\tilde{\varepsilon}_{oj}),\ q_{oi}(\tilde{\varepsilon}_{oi})\right].$$

FOOTNOTES

1) In Chapter 2 we derived a price path $\{\lambda_t^o\}$ that should be announced by the planning board in order to sustain the optimal rate of depletion of emergency reserves. Since the utility function $u(q_t)$, where q_t denotes the rate of stock withdrawal, was assumed to be perfectly known in that model, the planning board could equally well use the quantity mode $\{q_t\}$ where $q_t = u'^{-1}(\lambda_t^o)$

2) It is not only so that the fact that the demand function is incompletely known renders the conclusion of the equivalence of the two control modes unjustified. Also, the foregoing discussion suggests that it is necessary to derive stock withdrawal policies in a feedback form which is different from the one in Chapter 2. In that chapter it was sufficient to rely on feedback control specified only as a function of the regime (embargo or free trade) that was going on at any date t. However, when the demand conditions are not perfectly known, the optimal control must also be stated in terms of the emergency reserves on hand at any date t. (See further Section 5.2.)

3) We shall only be concerned with the planning instruments indicated above. Various forms of complementary usage of price incentives and quantity controls in regulating the stock withdrawal will not be analyzed. (See further Section 5.5.)

4) Since Weitzman's first paper, "Prices vs. Quantities", increasing attention has been devoted to the topic of regulating firm's output when costs and benefits are uncertain. There are naturally many applications of "P vs. Q" in the field of pollution control (e.g., Yohe (1976), Fishelson (1976) and Dasgupta, Hammond and Maskin (1980). In the context of trade theory, Young and Anderson (1980, 1982), for example, have compared the use of P and Q (i.e., tariffs and quotas). For extensions and critical notes on Weitzman's original article, see e.g., Laffont (1977) Ireland (1977), Malcomson (1978), Yohe (1978) and Dasgupta (1982).

5) The superscript "o" to the variables (e.g., λ_t^o) that were used in Chapter 2 to indicate that they were associated with an embargo regime, is dropped here. Since we are only dealing with embargo regimes in the present chapter, this should not be a source of confusion.

6) We will assume that $q_t = q(\lambda_t, \varepsilon_t) > 0$ for all λ_t.

7) We assume, of course, that the marginal utility of income is constant. However, we have also implicitly assumed that individual demands can be aggregated to an aggregate demand function uniquely. Samuelson (1956) pointed out two circumstances in which this can be justified: (i) the government is continually distributing income optimally, or (ii) the preferences are restricted to be identical and homothetic for all consumers. Criterion(ii) can be relaxed somewhat (see Chipman (1974)). For a discussion of assumptions that relax criterion(i), see Varian (1984).

8) In (5.1) we need to assume that $0 < \eta = \lim_{q_t \to 0} \{-\lambda_1(q_t, \tilde{\varepsilon}_t)q_t/\lambda(q_t, \tilde{\varepsilon}_t)\} < \infty$.

9) The subscript "o" to the probability θ and the duration τ that has been used in the preceding chapters can be dropped here. Cf. note 5.

10) In Chapter 2, the analysis takes into account both the uncertainty with the respect to the date the embargo will expire and the date a new embargo will be imposed. The latter uncertainty is, of course, also relevant here since it might affect the depletion policy during the present embargo (See Section 2.3). In this chapter we follow the simplifying assumption made in Chapter 3 that instantaneous adjustment of the emergency reserves to the optimal level S^* is possible immediately after the embargo has terminated. Hence, we can disregard the uncertainty with respect to the date a new embargo will be imposed. Moreover, it implies that the marginal value of oil in storage immediately after the embargo has expired is equal to the world market price p. Cf. Section 5 in Chapter 2. The optimal stock S^* is, in the present framework, exogenously given. Under certain stationary assumptions for the following free-trade regime the stock S^* is a constant. Cf. Chapter 2.

11) Problem (5.4) can be derived from (5.3) in the following way: rewrite

$$\sum_{\tau=o}^{\infty} (1 - \theta)^{\tau}\theta\Big[\sum_{t=o}^{\tau} (1 + r)^{-t}\, u(q_t, \tilde{\varepsilon}_t)\Big] \qquad (*)$$

in (5.3) as:

$$\theta\, u(q_o, \tilde{\varepsilon}_o) +$$

$$+ (1 - \theta)\, \theta(u(q_o, \tilde{\varepsilon}_o) + (1 + r)^{-1}\, u(q_1, \tilde{\varepsilon}_1)) +$$

$$+ (1 - \theta)^2\, \theta(u(q_o, \tilde{\varepsilon}_o) + (1 + r)^{-1}\, u(q_1, \tilde{\varepsilon}_1) + (1 + r)^{-2}\, u(q_2, \tilde{\varepsilon}_2)) +$$

$$\vdots$$

$$+ (1 - \theta)^t\, \theta(u(q_o, \tilde{\varepsilon}_o) + (1 + r)^{-1}\, u(q_1, \tilde{\varepsilon}_1) \ldots (1 + r)^{-t}\, u(q_t, \tilde{\varepsilon}_t)) +$$

$$\vdots$$

By changing the order of summation this can be rewritten as

$$u(q_o, \tilde{\varepsilon}_o)(\theta + (1 - \theta)\,\theta + \ldots (1 - \theta)^t\,\theta \ldots) +$$

$$+ u(q_1, \tilde{\varepsilon}_1)(1 + r)^{-1}((1 - \theta)\,\theta + \ldots(1 - \theta)^t\,\theta \ldots) +$$

$$\vdots$$

$$+ u(q_t, \tilde{\varepsilon}_t)(1 + r)^{-t}((1 - \theta)^t\,\theta + (1 - \theta)^{t+1}\,\theta + \ldots) +$$

$$\vdots$$

Now, using the fact that

$$(1 - \theta)^{\tau}\,\theta + (1 - \theta)^{\tau+1}\,\theta + \ldots = (1 - \theta)^{\tau}$$

the double summation (*) in (5.3) can be rewritten as:

$$\sum_{t=o}^{\infty} u(q_t, \tilde{\varepsilon}_t)\left[\frac{1 - \theta}{1 + r}\right]^t. \qquad (**)$$

We also have that

$$\sum_{\tau=0}^{\infty} (1 - \theta)^{\tau} \theta(1 + r)^{-(\tau+1)} p(S^* - S_{\tau+1}) \quad \text{in (5.3) equals}$$

$$\sum_{t=0}^{\infty} \left[\frac{1 - \theta}{1 + r}\right]^{t} (1 + r)^{-1} \theta p(S^* - S_{t+1}) \qquad (***)$$

Combining (**) and (***) yields (5.4). Cf. Heal (1973), Levhari and Mirman (1977) and Johansen (1978) for treatments of an uncertain time horizon in the discrete time case.

12) We assume an optimal policy exists. In the following we will not actually solve the problem. We will only make use of dynamic programming arguments to decompose problem (5.3) into a sequence of simpler maximization problems. For this reason we will sidestep the sophisticated mathematics needed for the analysis of limiting behavior and for the treatment of the stochastic aspects of our infinite horizon model. For a mathematically rigorous treatment of stochastic infinite horizon models, see Bertsekas (1976).

13) Value function (5.4) is identical to Value function (3.5) except for the facts that the (parametrized) utility function in (3.5) is perfectly known, that time enters as a continuous rather than discrete variable and that the duration τ is exponentially rather than geometrically distributed.

14) We use the notation λ^1 instead of $\lambda_1(q_0, \tilde{\varepsilon}_0) \equiv \partial\lambda(q_0, \tilde{\varepsilon})/\partial q_0$ to denote that the slope of the demand curve in the Δ-formula is nonstochastic by assumption. Since the $c(q_0)$ function is nonstochastic there is no need to distinguish between $c''(q_0) \equiv d^2c(q_0)/dq_0$ and c^{11}. For reasons of symmetry, however, we use the notation c^{11} instead of $c''(q_0)$.

15) See the Appendix.

16) See Appendix 5:2 for a complete derivation.

REFERENCES

Arrow, Kenneth I., and Fisher, Anthony C. "Environmental Preservation, Uncertainty and Irreversibility." Quarterly Journal of Economics 88 (May 1974): 312-319.

Bertsekas, Dimitri P. Dynamic Programming and Stochastic Control. New York: Academic Press, 1976.

Chipman, John S. "Homothetic Preferences and Aggregation." Journal of Economic Theory 8 (May 1974):26-38.

Dasgupta, Partha S., and Heal, Geoffry M. "The Optimal Depletion of Exhaustible Resources." Review of Economic Studies (Symposium on the Economics of Exhaustible Resources 1974):3-28.

Dasgupta, Partha S., and Heal, Geoffrey M. Economic Theory and Exhaustible Resources. Cambridge: Cambridge University Press, 1979.

Dasgupta, Partha S., and Stiglitz, Joseph E. "Resource Depletion under Technological Uncertainty." Econometrica 49 (January 1981): 85-104.

Dasgupta, Partha S.; Hammond, Peter L., and Maskin, Erik A. "On Imperfect Information and Optimal Pollution Control." Review of Economic Studies 47 (October 1980):857-860.

Dasgupta, Partha S. The Control of Resources. Oxford: Basil Blackwell, 1982.

Fishelson, Gideon. "Emission Control Policies under Uncertainty." Journal of Environmental Economics and Management 3 (October 1976):189-197.

Heal, Geoffrey M. The Theory of Economic Planning. Amsterdam: North-Holland, 1973.

Ireland, N.I. "Ideal Prices vs. Prices vs. Quantities." Review of Economic Studies 44 (February 1977):183-186.

Johansen, Leif. Lectures on Macroeconomic Planning. Part 2: Centralization, Decentralization under Uncertainty Planning. Amsterdam: North-Holland, 1978.

Laffont, Jean J. "More on Prices vs. Quantities." Review of Economic Studies 44 (February 1977):177-182.

Levhari, David, and Mirman, Leonard J. "Savings and Consumption with an Uncertain Horizon." Journal of Political Economy 85 (April 1977):265-281.

Malcomson, James M. "Prices vs. Quantities: A Critical Note on the Use of Approximations." Review of Economic Studies 45 (February 1978):203-207.

Roberts, Marc J., and Spence, Michael. "Effluent Charges and Licenses under Uncertainty." Journal of Public Economics 6 (April-May 1976):193-208.

Samuelson, Paul A. "Social Indifference Curves and Aggregate Demand." Quarterly Journal of Economics 70 (February 1956):1-22.

Varian, Hal R. "Social Indifference Curves and Aggregate Demand. "Quarterly Journal of Economics XCIV (August 1984):403-414.

Weitzman, Martin L. "Prices vs. Quantities." Review of Economic Studies 41 (October 1974):477-491.

Weitzman, Martin L. "Optimal Rewards for Economic Regulation." American Economic Review 68 (September 1978):683-691.

Yohe, Gary W. "Substitution and the Control of Pollution." Journal of Environmental Economics and Management 3 (December 1976):312-324.

Yohe, Gary W. "Towards a General Comparison of Price Controls and Quantity Controls under Uncertainty." Review of Economic Studies 45 (June 1978):229-238.

Young, Leslie, and Anderson, James E. "The Optimal Policy for Restricting Trade under Uncertainty." Review of Economic Studies 47 (October 1980):927-932.

Young, Leslie, and Anderson, James E. "Risk Aversion and Optimal Trade Restrictions." Review of Economic Studies 49 (April 1982):291-305.

6. Centralized Quantitative Allocation

6.1 THE PLANNING PROBLEM

Oil emergency policies in Sweden consist largely of plans for government intervention with rationing during supply disruptions. The form of organization presently planned for the allocation of oil to the production system can be characterized as a temporary planned economy. The method, with a few exceptions, is based on rationing quotas or rations that are not transferable. The main purpose of this chapter is to discuss the losses of efficiency that arise in rationing programs with nontransferable rations.

Two main principles for such a centralized quantitative allocation can be distinguished. The first - undifferentiated allocation - allocates a given percentage, the same for all users, of the consumption in the base period. This method places little demand for information on the rationing authority. All it needs to know is the quantity available and the consumption during the base period for all the firms. Of course, the ease of implementation does not outweigh the considerable losses in efficiency caused by such a program. In other words, there are cogent reasons for choosing differentiated quotas. This second type of rationing requires the rationing authority to have access to considerably greater amounts of information.

The task of the rationing authority is to determine an allocation of oil that minimizes the damage done to the economy. If the problem is to be well defined and manageable, it will be necessary to specify the economic consequences of the shortfall, not only in terms of the decrease in the supplies of oil and the expected duration of the embargo etc., but also in terms of the short-term capabilities of the economy to adapt to the situation and of the mutual dependencies among the various sectors of the economy. In addition, the definition of the problem must include the set of criteria by which the effects on the economy are to be measured and evaluated. Strictly interpreted, this means that the rationing authority must have information not only on the available amount of oil, but also on the production and demand conditions for all the goods and, in addition, a welfare function which indicates how the income distribution effects are to be evaluated.

These requirements are by no means easy to satisfy. A great deal of the information is simply not available to the rationing authority. Therefore it has to resort to simplified methods of determining a program of rationing quotas. In Section 6.2 we shall outline a planning model which satisfies requirements that are somewhat less ambitiously formulated. This model is intended to be used by the National Industrial Board (SIND) - the responsible authority for industry during oil supply crises - to compute rationing quotas for the allocation of oil within the production system. The model is basically an input-output model of the conventional kind, and the objective is to maximize private consumption with given restrictions on, e.g., other demand components, the composition of private consumption and resource availabilities.

In Sections 6.3 and 6.4 we apply the model. Using the available information about production technology, patterns of interindustry deliveries, final demand conditions and resource availabilities, we approximate in Section 6.3 an efficient allocation of oil. This program of differentiated quotas is, of course, a coarse approximation to an efficient allocation. Among other things, it is assumed that the allocation of the good in short supply <u>within</u> the branches of industry is done proportionally, not taking consideration of the differences in production technology, product assortment and possibilities of adaptation. In addition, the specification of the various types of substitution possibilities, e.g., in the final demand, is very much simplified. The unavoidable fact is simply that the extremely detailed planning model that would be needed to determine an efficient allocation of oil quotas is not statistically or computably feasible.

It is, of course, very hard to quantify the losses of efficiency that arise in such a simplified, aggregated rationing program. For instance, we can not, owing to the lack of disaggregated data, measure the "transfer gains" that would be achieved in an efficient allocation through reallocating oil within the branches. In Section 6.4 we try to estimate the orders of magnitude of these losses by looking at programs less differentiated than the planning model permits. The idea is that these losses are of the same orders of magnitude as the gains in a rationing system that allows the use of more (decentralized) information (e.g., a system of transferable ration coupons). Our investigation of such rationing systems (e.g., the division of the branches of industry into 10 main groups that is used by the National Board of Economic Defense) shows that the losses of efficiency increase considerably if one does not take advantage of the possibilities for substitution within the branches of industry. This indicates that the losses of efficiency are very large in systems with nontransferable rations even though the quotas are based on the highest degree of disaggregated data that is statistically feasible.

6.2 A SHORT-TERM PLANNING MODEL

As the authority responsible for industry, one of the National Industrial Board's (SIND) duties is to allocate oil to industry in case the supply is disrupted. In this section we shall provide a brief exposition of the planning model which is to be used by SIND to compute rationing quotas. Complete descriptions of the model and the data sources have been presented elsewhere.[1)]

Ideally, the planning model would be an extended version of the models developed in the previous chapters, with a detailed consumption and production side with substitution possibilities between oil and other consumption goods and factors of production. Strictly interpreted, requirements of such completeness would lead to extremely complex model systems. Therefore, one has to refrain from formally introducing uncertainty and temporal aspects into the planning model. If empirically based results are to be attained it is necessary to resort to additional simplifications and use rather rough descriptions of the conditions of production, consumption, foreign trade and the resource availabilities.

As for the production system, i.e., the technical conditions of production and interdependences, the most important empirical materials used are input-output statistics, the national accounts and energy statistics. For the present, the most current input-output table available at any significant degree of disaggregation is a matrix with 88 sectors for 1975.[2)]

The possibilities for factor substitution within the individual production processes are ignored in the input-output description of the production conditions. For short-term analyses with given capital equipment, this is less troublesome than for long-term applications. More important for short-term analysis are the possibilities for substitution that cannot be described within the model owing to the aggregation of goods and processes. The problem arises when there are differences among the individual processes that make up the aggregate process with regard to production technology and goods produced. In such a case there are possibilities for substitution in the production function for the aggregate process even though each individual process can be characterized by fixed proportions of inputs and outputs. This means that the results of applying the model only include the consequences of a uniform cutback in the various plants that make up a branch of industry. In some cases a more efficient adjustment would be to close down those plants with the greatest oil requirements per unit of production.[3)]

Final demand is split into three categories: private consumption, public consumption and investment. Public consumption and the demand for investment goods are considered exogenous to the model. Where private consumption is concerned, it is assumed that the composition of the "private consumption bundle" is given. Hence, changes in the composition of consumption towards products that require less of the imported oil to produce are not taken into account.

The changes in the condition for foreign trade are extremely difficult to judge. To do so would require knowledge of the supply situation in other countries and of the adjustments in those countries to the supply crisis. In light of these difficulties, the simplest possible assumptions about foreign trade are made. We assume that total imports and exports as well as the composition of imports and exports are exogenously given.

There are three types of factors of production. The first type is specific to each sector and consists of the production capacity connected with the various production processes (branches). The second is labor which is a restriction common to the entire production system. The third group consists of the inventories of goods. We also include these as factors of production since they are not produced during the current period but have entered the system in some other way (from a previous period); more on this later.

To calculate the effects of a supply disruption we first develop a reference case which provides a picture of the economy at a given point of time when there are no disturbances in oil supplies. This reference case is merely an attempt to describe the Swedish economy in the context of the 88-sector planning model. Especially, the purpose is to establish the oil requirements and the capacity of production in the various sectors of the economy at the point of time under consideration. The reference case is based on the most recent fairly detailed structural data for Sweden (those for 1975), the national accounts and energy statistics. In the reference case the value of the composite private consumption good is maximized under restrictions that the supply of goods be at least as great as the demand for goods and that the exogenous supply of labor be at least as great as the demand for labor. Exports, imports, public consumption, deliveries of investment goods as well as the composition of the private consumption bundle are estimated from the most recent of the statistical sources we referred to above. It is assumed that inventories will neither be added nor used in the reference case.

In the second stage, the effects of a supply disruption are studied against the background of the image of the undisturbed economy provided by the reference case. The analysis in the sequel always assumes a given reduction of supply of oil to the production system, an amount that corresponds to a given percentage of the requirements in the reference case. In most other respects the disruption cases are assumed to satisfy the same requirements or restrictions as the reference case. The deviations are the result of economization with oil through modification of the oil coefficients for the production plants and through changes in inventories of final goods and semimanufactures. To take care of these factors, some of the restrictions that were assumed to apply in the reference case are modified.

The treatment of the modification of the oil coefficients of the production plants is based on estimates of fixed and variable coefficients and of the slack in the firms' use of oil. An account of these estimates and how they have been used in the model system is given in Appendix 6:2.

The possibilities for using the inventories of finished goods and semimanufactures are determined by the difference between the existing inventories and the lowest measured level during the 1970's and of an assumed greatest possible rate of use of inventories. The lowest level during the 1970's thus represents the level of inventories necessary for "transactions" purposes, a level which, by assumption, cannot be reduced without causing serious disturbances in the supply system. Appendix 6:3 provides an account of the treatment of inventories of final goods and semimanufactures.

6.3 A CRUDE APPROXIMATION OF THE EFFICIENT ALLOCATION

Using the planning model it is possible to approximate, roughly, an efficient allocation with a differentiated rationing program. The calculations that will be referred to here are based on conditions during the autumn of 1979 and the beginning of 1980. In the reference case we assume that certain adjustments of the oil coefficients in industry can be made (cf. Appendix 6:2). In addition, we assume that it is possible to decrease investments in building by 30% and that a certain amount of finished goods and semimanufactures in inventory can be used. The possibilities for using goods in inventory have been estimated according to the principles referred to in Appendix 6:3. As the main alternative, we have assumed that the inventory buffer is used at a rate that will lead to depletion in six months. Since we also assume that the disruption is expected to last only two or three months at the most, this is not an especially restrictive assumption.

The macroeconomic results of cutbacks in supplies of oil of 10%, 15% and 25% are summarized in Table 6.1. The percentages that are shown for employment effects, however, hardly provide a realistic picture of the effects in the short term, since the firms cannot (or do not desire to) reduce the number of their employees at the rate with which "the need for labor" decreases.

Table 6.1 Effects of shortfalls of supplies of oil on employment, production and final demand

Decrease in per cent from the reference case	Shortfall of oil in per cent of the oil requirements in the reference case		
	10%	15%	25%
Employment in the production system	0.5	6.7	20.5
Private consumption	0	0	22.3
Gross investments	0	12.6	16.1
Gross investments, incl. changes in inventories	2.5	25.7	32.8
Contribution of the production system to GNP	0.2	5.9	20.5

The adjustments of the oil coefficients and the drawdown of inventories mitigate the effects of the supply disruption. For moderate cutbacks in the supply of oil there are no major effects on production and employment. Cutbacks exceeding 10% cause considerable effects on the production possibilities. The supply of goods to the households is facilitated by the utilization of the inventories of semimanufactures and finished goods that are on hand when the embargo starts. These also permit some of the most oil intensive production processes to be reduced relatively much. Production in the consumer goods industry and private consumption can be maintained at a normal level even when the cutback in oil supplies is 15%. However, production in commerce and industry decreases by about 6%. Our estimates indicate that a 25% shortfall would have very serious consequences for production, employment and the possibilities for consumption. Production possibilities in commerce and industry decrease by roughly a fifth (about SEK 50 billion on an annual basis in 1975 kronor).

The total reduction of production in the economy is very unevenly distributed among the various sectors. In Appendix 6:4 this is illustrated for two cases in which the cutback in oil supplies is 15%. In one of the cases only adaptations of the oil coefficients of the firms are possible. The other case corresponds to the reference case above. The restructuring of production is especially noticeable for the reference case. The use of inventories and the cutback in construction activities lead to heavy cutbacks in production in the industries producing building materials and in sectors with large inventories. Several of these sectors are among the oil-intensive processing industries, where the inventory situation permits cutbacks in production without affecting exports and deliveries of intermediate goods to other industries. In terms of loss of production, the effect of the disruption will be about 9 percentage points higher in the case where only adjustments in the oil coefficients are possible than in the reference case.

The picture of costs shown in Table 6.1 and the changes in the production structure reported in Appendix 6:4 reflect a considerable change in the price system. Major increases in prices are mainly to be noted in oil-intensive sectors that do not have large inventories. Table 6.2 shows the sectors that experienced the largest increases in the prices of their products relative to those of the reference case. Those sectors whose prices at least doubled for a 15% decrease in supplies are included.

Table 6.2 Some extreme price increases for a 15 percent shortfall of oil

No	Industry/Product	Percent increase
52	Structural clay products	350
53	Cement, lime and plaster	344
48	Rubber products	202
54	Other non-metal mineral products	195
60	Non-ferrous metal casting	126
4	Iron ore mining	120
5	Non-ferrous ore mining	114

All of the extreme values shown in the table are for products involved in the early stages of production. All are for products from oil intensive sectors with small or nonexistent buffer stocks around the beginning of 1980. The increase in prices for iron and steel and for pulp and paper are considerably less, about 30% to 40%. The main reason for this is the large inventories that were available at that time.

Naturally, the prices of the goods and services reflect the price of oil. The price of oil in the various supply situations is quite high, several times greater than the market price in a normal supply situation. The values imply a very steep demand curve for oil. The implicit price elasticities vary between -0.02 (for a shortfall of 10 per cent) and -0.28 (for a cutback of 25 per cent).[4)]

The allocation programs for oil that produce these macroeconomic results show a great deal of differences in the quotas for the different branches. In Table 6.3 we present the way in which the 70 branches of industry are distributed with regard to rationing quotas. The table refers to the case in which the deliveries of oil to the production system have been cut back by 15%.

What are the characteristics of the sectors that receive large rations when the differentiation of the quotas is at the 88-sector level? In Table 6.3 we see that most of these sectors are involved in the production of consumer goods. This comes as no surprise. We have already noted that private consumption is unaffected by a 15% shortfall of oil. The sectors whose products are in stock in large quantities suffer relatively large cutbacks when oil is rationed. In addition, those sectors that manufacture building material (bricks, cement, wood products, etc.) receive quotas that are lower than the average.

Table 6.3 Rationing quotas at a fully differentiated system

Industry/Product	SNI	Rationing quota %
Plastics and synthetic fibers	35131	98
Slaughtering, and preparation of meat	3111	95
Oils and fats	3115	
Bakery products	3117	
Paper and board	34112	94
Grain mill production	3116	93
Leather and shoes	232/4	
Plastic products	3560	
Other paper and board products	3419	92
Pottery	3610	
Spinning and weaving	3211	91
Rubber products	3550	
Dairy products	3112	90
Canning of fruit and vegetables	3113	
Canning of fish	3114	
Sugar	3118	
Prepared animal foods	3122	
Beverage industries	3130	
Tobacco manufacturing	3140	
Hosiery and knitted goods	3213	
Clothing	3220	
Packaging products	3412	
Pharmaceutical chemicals	3522	
Other food	3121	89
Textiles, other than clothing	3212	
Printing	34201/3	
Publishing	34202	
Detergents	3523	
Instruments	3850	
Other manufacturing	3900	
Chocolates	3119	
Carpets, rugs	3214/9	
Fiberboard manufacturing	34113	86
Glass	3620	85
Motor vehicles	3843	
Bicycles and motorcycles	3844	
Furniture	3320	84
Other chemical products	3529	
Iron and steel casting	37103	
Electrical machinery	3831	
Electronics and telecommunications	3832	
Shipyards	3841	
Aircraft	3845	
Iron ore mining	2301	
Wooden packaging products	3312/9	
Semifinished plastic products	35132	
Lubricating oils, greases	3540	
Mechanical engineering	382	
Pulp manufacturing	34111	82
Cement, lime and plaster	3692	
Other nonmetal mineral products	3499	
Rail road equipments	3842	
Paints	3521	81
Domestic electrical appliances	3833	
Other electrical goods	3839	
Nonferrous metal casting	37204	80
Fertilizers and pesticides	3512	79
Iron and steel manufacturing	37101	
Other wooden materials	33119	78
General chemicals	3511	77
Other mining and quarrying	2900	75
Wooden building materials	33112	
Ferro-alloys manufacturing	37102	
Semifinished nonferrous metal products	37203	
Sawing and planing of wood	33111	72
Structural clay	3691	71
Other vehicles	3849	
Nonferrous ore mining	2302	70
Other metal goods	381	

By way of summary, we can state that the oil-intensive process industries are hit relatively hard by the rationing, but the simple rule to direct the cutbacks to where they - per cubic meter of oil - have little effect on the production in a particular plant has many exceptions. The high quota for the plastic and synthetic fibers industry illustrates this.

The results of the sensitivity tests indicate that the consequences of the supply disruption can be considerably different when the composition of domestic demand and foreign trade changes. Also, the possibilities for firms to adjust their oil coefficients in the short term and the possibilities of reducing inventories of finished goods and semimanufacturees are decisive for the effects of an oil supply disruption (cf. Appendix 6:4). Given the margin of error that exists in the estimates of these data, the risk of efficiency losses are very high in systems with nontransferable rations.[5)]

6.4 OTHER RATIONING SYSTEMS

Up to now we have assumed that the government authorities choose to allocate oil using a rationing program that is differentiated at a 88-sector level. Discussions with the National Board of Economic Defense and the Swedish Fuel Commission have indicated that difficulties of implementation and information would make simpler allocation formulae desirable.

A rationing system with purely proportional quotas turns out to cause unreasonably large negative effects on the production possibilities of the economy. An exception must be made for the plastics and synthetic fiber industry, which has very special conditions both regarding oil requirement and pattern of deliveries.

We begin by comparing the two extremes, i.e., a completely undifferentiated system (with the exception of plastics and synthetic fibers) and a fully differentiated system down to the sector level of a subdivision into 88 sectors. The differences in effects on production and consumption of these two systems can be seen in Table 6.4.

Table 6.4 Effects of a 15 percent shortfall of oil

Decrease in per cent from the reference case	A completely undifferentiated system (except for plastics and synthetic fibers)	A fully differentiated system
Contribution of the production system to GNP	17	6
Private consumption	24	0

The extreme effects on production of the undifferentiated system are mainly the result of bottlenecks that arise in different parts of the production system. The allocation of oil will not be efficient in this case. Some firms will not even be able to utilize their full rations owing to a shortage of intermediate goods. The amount of oil that is not used corresponds to about 5% of the total amount that was intended for use in the production system.

These estimates show that the effect of the disruption on the production possibilities will be about three times as great if undifferentiated quotas are used than if fully differentiated quotas are used. As can be seen in Table 6.4, it is not possible to "protect" private consumption by efficient use of the available inventories of goods. Only a third of the amount of stocks that is used in connection with differentiated quotas can be used in connection with undifferentiated quotas.

The advantages of an undifferentiated system of quotas - less complex administration and small requirements for information - hardly outweigh the losses that are indicated here. It is therefore urgent to try to employ a differentiated system. In this way consideration can be taken to the variation in sensitivity of the different sectors to cutbacks and to the risks for bottlenecks in the production system. For cases when the fully differentiated system - down to the 88-sector level - is considered too difficult to manage, we have studied different forms of quotas that lie between the two extremes.

They involve either groupwise differentiation or different alternatives with limited numbers of exceptions from the completely undifferentiated system. The total cutback in oil supplies is, of course, the same for all cases.

One case that is of special interest is the division of the branches of industry into 10 main groups used by the National Board of Economic Defense. A rationing system based on this classification differentiates among the following groups:

No	SNI	Industry/Products
1	2	Mining and quarrying
2	31	Food, beverage and tobacco
3	32	Textiles, clothing and leather
4	33	Wooden materials
5	34	Paper, board and pulp
6	35	Chemicals, rubber and plastic
7	36	Nonmetal mineral products
8	37	Iron, steel and metal
9	38	Manufacturing
10	39	Other manufacturing

but with the same quotas for all industries within each group.

Applying such a system strictly leads to great imbalances in the production system. It seems that making an exception for the chemical industry cannot be avoided and that the quotas for the branches included in that group be fully differentiated.

The possibility of making a few exceptions from a system of otherwise undifferentiated rations means making exceptions for those sectors for which the shadow price of oil is greatest. In this way priority is given to branches in which an additional quantity of oil leads to the greatest increases in production for the system as a whole.

The different in-between forms we will investigate here are

1. Differentiation at the SNI two-digit level with the exception of the chemical industry where the division into 11 branches is retained. This means a total of 20 sectors.

2. Undifferentiated rationing with exceptions - i.e., cutbacks of less than 15% - for 7 sectors.

3. Undifferentiated rationing with exceptions for 14 sectors.

Appendix 6:5 shows the sectoral classifications of these three "in-between" alternatives.

In Table 6.5 these alternatives are compared with the undifferentiated and "completely" differentiated cases insofar as effects on production are concerned.

Table 6.5 Effects on the contribution of the production system to GNP of a 15 percent shortfall in the supply of oil

Allocation method	Decrease in the contribution of the production system, per cent
Completely undifferentiated system (except plastics and synthetic fibers) (Alt 0)	17
Differentiation at the SNI two-digit level (except the chemical industry) (Alt 1)	10-14*
Undifferentiated rationing with 7 dispensations (Alt 2)	11
Undifferentiated rationing with 14 dispensations (Alt 3)	10
"Complete" differentiation, 88 sectors	6

* The interval represents slacks in the balance equations for commodities. These can be interpreted as unintended increases in inventories. The lower limit of the interval refers to the case where such inventory increases are valued at full price, while the upper limit represents the case where the increases are of no value.

It can be noted that the exceptions lead to a considerably more efficient allocation of resources than the undifferentiated case. However, quite a lot of the inefficiency owing to bottlenecks and differences in the marginal value of oil in the different sectors remains. For example, in alternative 2 about 2% of the oil supply cannot be used despite the fact that 7 branches have received quotas that are greater than 85%. In terms of production the effect of the disruption is almost twice as great in cases 1 through 3 as for "complete" differentiation.

From the estimates it can be seen that differentiation at the SNI two-digit level has hardly any advantage over undifferentiated quotas with exceptions. With regard to effect on productions, alternative 1 is about equivalent to alternative 2, despite the fact that 1 represents a greater degree of differentiation. The explanation for this is that in alternative 2 greater consideration is taken of the extreme differences in the marginal value of oil than in alternative 1, which does not consider the differences within the main groups. This holds especially for groups 5 and 7. It is apparent from Appendix 6:5 that the allotment to the various branches of the pulp, paper and graphics industry varies between 85% and 96% in alternative 2, but they receive a common quota of 89% in alternative 1. The corresponding figures for the nonmetallic mineral industries (SNI 36) are from 85% to 94% and 86%.

Of the three "in-between" alternatives, alternative 3 appears to be the most efficient. The effect on production is about 10% in that case. There is also equilibrium between the supply and demand for oil. The 14 sectors that receive special quotas are shown in Appendix 6:5. Most of them are in the earlier stages of production. Four are in the pulp and paper industries, three in the chemical industries and three in the nonmetallic mineral industries.

6.5 CONCLUDING REMARKS

What conclusions can be drawn from our study of centralized quantitative allocation of resources at the branch level? The results clearly indicate that an erroneous allocation policy can lead to a tripling of the damages of an oil supply disruption. Large efficiency losses may arise owing to the interdependences among the various sectors of the economy. First of all it is necessary to prevent bottlenecks involving intermediate goods. Not until such problems have been cleared up is it possible to limit the effects of a disruption on real income, employment and consumption.

Furthermore, we can assert that a system of dispensations - in which exceptions are made for branches in which additional oil will give the greatest increase to production - leads to a considerably more efficient allocation of resources during a crisis than a system with completely uniform quotas to all branches. However, even so, there still remain considerable losses in efficiency owing to the fact that full consideration has not been taken of the differences among branches with regard to short-term possibilities for adaptation. In terms of loss of production, the effect of the disruption will be almost twice as great in cases 1 through 3 (cf. Table 6.5) than in a completely differentiated system. This may involve a difference on the order of SEK 3 billion per month. This indicates that additional reallocation should be done.

However, even in the rationing program that is "fully" differentiated at the 88-sector level, there remain efficiency losses since it is assumed that the rationed good is allocated proportionally within the sectors. Once it has been decided that rationing will be used during an embargo, it is important to increase the efficiency in the allocation of oil. The only way to do this is to decentralize the decisions about the use of oil. This can be accomplished through some form of market system (e.g., transferable rations).

APPENDIX 6

6:1 List of goods and services

No.	SNI	Nomenclature
1	11	Agriculture and hunting
2	12	Forestry and logging
3	13	Fishing
4	2301	Iron ore mining
5	2302	Nonferrous ore mining
6	2900	Other mining and quarrying
7	3111	Slaughtering and preparation of meat
8	3112	Dairy products
9	3113	Canning of fruit and vegetables
10	3114	Canning of fish
11	3115	Oils and fats
12	3116	Grain mill production
13	3117	Bakery products
14	3118	Sugar
15	3119	Cocoa, chocolates and sugar confections
16	3121	Other food
17	3122	Prepared animal feeds
18	3130	Beverage industries
19	3140	Tobacco manufacturing
20	3211	Spinning and weaving
21	3212	Textiles, other than clothing
22	3213	Hosiery and knitted goods
23	3214/9	Carpets, rugs etc.
24	3220	Clothing
25	323/4	Leather and shoes
26	33111	Sawing and planing of wood
27	33112	Wooden building materials
28	33119	Other wooden materials
29	3312/9	Wooden packaging products
30	3320	Furniture and bedding
31	34111	Pulp manufacturing
32	43112	Paper and board manufacturing
33	34113	Fiberboard manufacturing
34	3412	Packaging products of paper, board etc.
35	3419	Other paper and board products
36	34201/3	Printing
37	34204	Publishing
38	3511	General chemicals
39	3512	Fertilizers and pesticides
40	35131	Plastics and synthetic fibers
41	35132	Semifinished plastic products
42	3521	Paints
43	3522	Pharmaceutical chemicals and preparations
44	3523	Soap, detergents and toilet preparations
45	3529	Chemical products
46	3530	Petroleum refining
47	3540	Lubricating oils, greases etc.
48	3550	Rubber products
49	3560	Plastic products
50	3610	Pottery
51	3620	Glass and glass products
52	3691	Structural clay products
53	3692	Cement, lime and plaster
54	3699	Other nonmetal mineral products
55	37101	Iron and steel products
56	37102	Ferroalloys manufacturing
57	37103	Iron and steel casting
58	37201/2	Nonferrous metal casting
59	37201	Semifinished nonferrous metal products
60	37204	Nonferrous metal casting
61	381	Other metal goods
62	382	Mechanical engineering
63	3831	Electrical machinery
64	3832	Electronics and telecommunication
65	3833	Domestic electrical appliances
66	3839	Other electrical goods
67	3841	Ship and boat building
68	3842	Manufacturing of railroad equipment
69	3843	Motor vehicles and parts
70	3844	Bicycles and motorcycles
71	3845	Manufacturing and repair of aircraft
72	3849	Other vehicles
73	3850	Instruments and photo equipment

6:1, continued

No.	SNI	Nomenclature
74	3900	Other manufacturing
75	4101, 4103	Electricity and district heating
76	4102	Gas manufacture and distribution
77	4200	Water supply
78	5000	Construction
79	61/62	Distributive trades
80	63	Restaurants and hotels
81	71	Transport and storage
82	72	Communication
83	81/82	Financial institutions and insurance
84	831Bo	Housing
85	831An	Management of other real estate
86	832/3	Business services
87	9511/3	Repair of cars and household goods etc.
88	other	Personal services

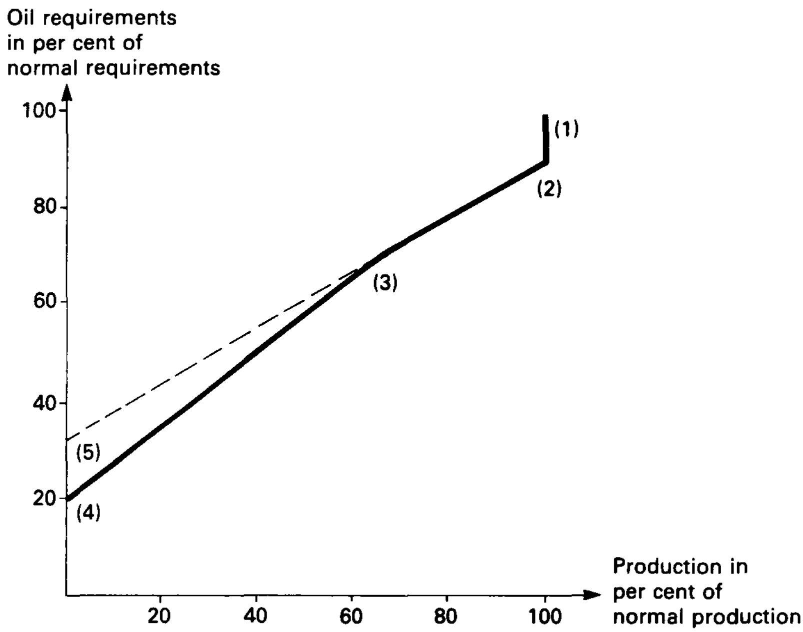
6:2 Adaptation of oil coefficients in various branches of industry

On the basis of questionnaires it has been shown that there are considerable possibilities of using better control and other methods of improving "efficiency" to reduce the requirements of oil without affecting the level of production (SIND PM, 1974:1 and SIND, 1979:2 (appendix)). The existence of such slack means that the firms do not function as efficiently as they might under normal conditions. This type of inefficiency is usually called X-inefficiency (Leibenstein, 1966).

Leibenstein presents a number of reasons why firms choose a higher cost level during successful periods to avoid the unpleasantness of greater effort and checking. Increased competition leads to pressure to use the factors of production more efficiently. During a supply crisis the reaction to price increases and threats of rationing may be to tune the production processes and reduce carelessness in the use of energy.

The concept of X-inefficiency is hardly free from objections. For example, it is possible to maintain that this is a case of the "ordinary" substitution process, i.e., a movement from a point on the production function that is less profitable considering the supplies and relative prices of the factors of production such as oil and labor (monitoring) to a point that gives maximum profits. Since a surplus of labor generally arises in the crises cases, it is possible to interpret the possibilities for "savings" shown in the questionnaire studies for the majority of industrial branches as the substitution of labor for oil. If oil supplies are reduced even more, then production must be reduced. The studies referred to also include the relationship between the volume of production and the supply of oil for such short-term responses. The companies involved were requested to estimate the situation for oil reductions corresponding to points 2, 3 and 4 of Figure A. On the basis of this figure we will relate how we have treated the results of the most recent questionnaire (SIND, 1979:2 (appendix)).

Figure A An example of the relationship between production and the requirements for oil in an individual company



* The coordinates for the points (1)-(5) are: (1) 100/100, (2) 100/90, (3) 65/70, (4) 0/20, (5) 0/33.

The numbers in the figure are interpreted as follows:

1. Oil consumption at full capacity production.

1-2. Reduction of oil consumption possible at continued full capacity production (neither the level nor composition of production is affected).

2. Oil consumption at full capacity production after elimination of the slack in oil consumption.

2-3. The decrease in production with optimal adaptation as oil supplies decrease.

3. The point at which production is no longer profitable (applies to some companies, but not to the one represented in the figure).

3-4. The decrease in production for reduced oil supplies in the cases where it is still possible to produce.

4. The consumption of oil when production is zero according to the questionnaire.

5. The consumption of oil when production is zero according to our approximation.

The figures given as coordinates in Figure A are a weighted average of responses from the various companies in the pulp industry. We make a correction of the normal oil coefficient on the basis of the slope of the line between points 2 and 3. The fixed consumption at point 5, which is 33% of the normal requirements for this branch of industry, is represented as an exogenous resource requirement. Thus we assume that input of oil and production follow the relationship that applies between points 2 and 3. This means that the nonbinding equation in the reference case

$$\sum_j e_j x_j^{ref} - L_{ref} = 0,$$

where

e_j is the normal coefficient of oil in sector j

L_{ref} is the supply of oil to the production system in the reference case

x_j^{ref} is the production in sector j in the reference case

is replaced by the following restriction in the crises cases

$$\sum_j d_j x_j \leq - \sum_j f_j x_j^{ref} + (1-\alpha)L_{ref},$$

where

x_j is the production in sector j

d_j is the corrected oil coefficient

f_j is the fixed coefficient for oil in sector j

α is the relative shortfall of oil.

The fixed and marginal coefficients for oil for the various branches of industry are shown below.

No.	Nomenclature	Oil		Minimum * inventory ratios
		Marginal coefficients	Fixed coefficients	
1	Agricultural and hunting	0.70	0.20	0.12
2	Forestry and logging	0.70	0.20	0.12
3	Fishing	0.70	0.20	0
4	Iron ore mining	0.81	0.13	0.11
5	Nonferrous ore mining	0.81	0.13	0.13
6	Other mining and quarrying	0.81	0.13	0.13
7	Slaughtering and preparation of meat	0.50	0.45	0.13
8	Dairy products	0.50	0.40	0.13
9	Canning of fruit	0.87	0.03	0.13
10	Canning of fish	0.87	0.03	0.13
11	Oils and fats	0.80	0.15	0.13
12	Grain mill production	0.74	0.19	0.13
13	Bakery products	0.77	0.18	0.13
14	Sugar	0.78	0.12	0.13
15	Cocoa, chocolates and sugar confections	0.84	0.04	0.13
16	Other food	0.79	0.11	0.13
17	Prepared animal feeds	0.79	0.11	0.13
18	Beverage industries	0.71	0.19	0.13
19	Tobacco manufacturing	0.71	0.19	0.13
20	Spinning and weaving	0.65	0.27	0.13
21	Textiles, other than clothing	0.70	0.20	0.13
22	Hosiery and knitted goods	0.70	0.20	0.13
23	Carpets, rugs etc.	0.70	0.20	0.13
24	Clothing	0.70	0.20	0.13
25	Leather and shoes	0.73	0.20	0.13
26	Sawing, planing and preserving of wood	0.70	0.15	0.13
27	Wooden building materials	0.70	0.15	0.13
28	Other wooden materials	0.70	0.15	0.13
29	Wooden packing products	0.70	0.15	0.13
30	Furniture and bedding	0.70	0.15	0.13
31	Pulp manufacturing	0.71	0.19	0.04
32	Paper and board manufacturing	0.83	0.12	0.11
34	Packaging products of paper, board etc.	0.85	0.08	0.13
35	Other paper and board products	0.85	0.08	0.13
36	Printing	0.58	0.32	0.13
37	Publishing	0.70	0.20	0.14
38	General chemicals	0.85	0.10	0.14
39	Fertilizers and pesticides	0.80	0.10	0.14
40	Plastics and synthetic fibers	0.24	0.76	0.14
41	Semifinished plastic products	0.80	0.10	0.14
42	Paints	0.80	0.10	0.13

* For a definition, see Appendix 6:3.

43	Pharmaceutical chemicals	0.80	0.10	0.13
44	Soap, detergents and toilet preparations	0.80	0.10	0.13
45	Chemical products	0.80	0.10	0.13
47	Lubricating oils, greases	0.82	0.18	0.13
48	Rubber products	0.64	0.29	0.13
49	Plastic products	0.55	0.40	0.13
50	Pottery	0.66	0.32	0.21
51	Glass and glass products	0.74	0.23	0.21
52	Structural clay products	0.96	0.01	0.09
53	Cement, lime and plaster	0.96	0.01	0.09
54	Other nonmetal mineral products	0.96	0.01	0.09
55	Iron and steel manufacturing	0.72	0.24	0.37
56	Ferro-alloys manufacturing	0.72	0.24	0.37
57	Iron and steel casting	0.72	0.24	0.37
58	Nonferrous metals	0.75	0.15	0.17
59	Semifinished nonferrous metal products	0.75	0.15	0.17
60	Nonferrous metal casting	0.75	0.15	0.17
61	Other metal goods	0.70	0.15	0.12
62	Mechanical engineering	0.70	0.15	0.12
63	Electrical machinery	0.65	0.20	0.12
64	Electronics and telecommunications	0.65	0.20	0.12
65	Domestic electrical appliances	0.65	0.20	0.12
66	Other electrical goods	0.55	0.30	0.12
67	Ship and boat building and repair	0.55	0.30	0.12
68	Manufacturing and railroad equipment	0.70	0.15	0.12
69	Motor vehicles and parts	0.70	0.15	0.12
70	Bicycles and motorcycles	0.70	0.15	0.12
71	Manufacturing and repair of aircraft	0.70	0.15	0.12
72	Other vehicles	0.70	0.15	0.12
73	Instruments and photo equipment	0.70	0.20	0.13
74	Other manufacturing	0.70	0.20	0.13
77	Water supply	0.70	0.20	0
78	Construction	0.70	0.20	0
79	Distributive trades	0.70	0.20	0
80	Restaurants and hotels	0.70	0.20	0
81	Transport and storage	0.70	0.20	0
82	Communication	0.70	0.20	0
83	Financial institutions and insurance	0.70	0.20	0
84	Housing	0.70	0.20	0
85	Management of other real estate	0.70	0.20	0
86	Business services	0.70	0.20	0
87	Repairs of cars and household goods etc.	0.70	0.20	0
88	Personal services	0.70	0.20	0

6:3 The treatment of inventories of finished goods and semimanufactures

The upper limit for the drawdown of inventories is written as

$$\Delta L_i^{max} = (S_i - B_i x_i^{ref}) n_i$$

where

ΔL_i^{max} is the maximum drawdown for good i

S_i is the existing inventories of good i

B_i is the minimum inventory ratio for good i during the 1970s, i.e.,

$$B_i = \text{Min}\left(\frac{S_i^t}{x_i^t}, \ t \in \text{1970s}\right)$$

x_i^{ref} is the production in sector i in the reference case

n_i is a coefficient giving the "drawdown rate" for good i.

For example, $n_i = 1$ implies that the inventories of good i can be reduced at a rate that corresponds to complete depletion during one year. Then, if the duration of the supply disruption is four months, a third of the inventories will be used during the disruption period. This is a rather restrictive assumption. In most of the cases, which refer to a disruption with a duration of three months, we assume $n_i = 2$ for all i. That is, we assume that the inventories will be drawn down at such a rate that half will be used during the period of disruption. To illustrate the sensitivity of this assumption, we have also considered cases in which all of the inventories would be used in three months. The value of n_i is 4 in those cases. The minimum inventory ratios for the various groups of goods are shown in Appendix 6:2.

6:4 Changes in the production structure at a 15 per cent shortfall of oil

No.	Industry/Products	Case* A	Case* B
1	Agriculture and hunting	71	100
2	Forestry and logging	96	89
3	Fishing	72	100
4	Iron ore mining	99	86
5	Nonferrous ore mining	88	70
6	Other mining and quarring	84	77
7	Slaughtering and preparation of meat	75	100
8	Dairy products	75	100
9	Canning of fruit	71	100
10	Canning of fish	66	100
11	Oils and fats	65	100
12	Grain mill production	72	100
13	Bakery products	78	100
14	Sugar	61	100
15	Cocoa, chocolates	73	100
16	Other food	72	99
17	Prepared animal feeds	68	100
18	Beverage industries	73	100
19	Tobacco manufacturing	74	100
20	Spinning and weaving	67	98
21	Textiles, other than clothing	80	99
22	Hosiery and knitted goods	68	100
23	Carpets, rugs etc.	75	97
24	Clothing	73	99
25	Leather and shoes	59	100
26	Sawing, planing and preserving of wood	97	82
27	Wooden building materials	96	86
28	Other wooden materials	94	90
29	Wooden packaging products	89	95
30	Furniture and bedding	86	99
31	Pulp manufacturing	99	88
32	Paper and board manufacturing	96	99
33	Fiberboard manufacturing	98	85
34	Packaging products of paper, board etc.	85	97
35	Other paper and board products	91	98
36	Printing	86	98
37	Publishing	82	98
38	General chemicals	81	79
39	Fertilizers and pesticides	61	86
40	Plastics and synthetic fibers	82	92
41	Semifinished plastic products	86	91
42	Paints	88	89
43	Pharmaceutical chemicals and preparations	76	99
44	Soap, detergents and toilet preparations	73	99
45	Chemical products	87	92
47	Lubricating oils, greases	82	79
48	Rubber products	82	97
49	Plastic products	82	96
50	Pottery	87	91
51	Glass and glass products	82	83
52	Structural clay products	93	73
53	Cement, lime and plaster	93	84
54	Other nonmetal mineral products	94	84
55	Iron and steel manufacturing	95	77
56	Ferro-alloys manufacturing	92	70
57	Iron and steel casting	94	83
58	Nonferrous metals	89	67
59	Semifinished nonferrous metal products	92	80
60	Nonferrous metal casting	92	86
61	Other metal goods	92	75
62	Mechanical engineering	96	97
63	Electrical machinery etc.	96	98
64	Electronics and telecommunications	90	99
65	Domestic electrical appliances	82	94
66	Other electrical goods	91	95
67	Ship and boat building and repair	90	99
68	Manufacturing and repair of railroad equipment	92	96
69	Motor vehicles and parts	87	100
70	Bicycles and motorcycles	66	100
71	Manufacturing and repair of aircraft	100	99
72	Other vehicles	94	99
73	Instruments and photo equipment	86	99
74	Other manufacturing	78	98

* In case A only short term adjustments in the oil coefficients are possible. In case B (the reference case) it is also assumed that construction activities can be cut down by 30% and that inventories of finished goods and semimanufactures can be used at a rate that will deplete them in 6 months.

6:4, continued

77	Water supply	86	98
78	Construction	96	83
79	Distributive trades	82	95
80	Restaurants and hotels	77	99
81	Transport and storage	88	96
82	Communication	84	98
83	Financial institutions and insurance	84	96
84	Housing	74	100
85	Management of other real estate	87	97
86	Business services	90	96
87	Repair of cars etc.	81	97
88	Personal services	80	100

6:5 Alternative rationing programs

a) Alt. 0 Undifferentiated rationing (excl. plastics and synthetic fibers)

The rationing quota for plastics and synthetic fibers manufacturing is 95 %, for all other industries around 85 %.

b) Alt. 2 and alt. 3 Undifferentiated rationing (85 %) with 7 and 14 dispensations, respectively.

Industry/product	SNI	Rationing quota Alt. 2	Rationing quota Alt. 3
Iron ore mining	2301	( )	89
Slaughtering/preparation of meat	3111	89	90
Bakery products	3117	( )	87
Pulp manufacturing	34111	( )	89
Paper and board manufacturing	34112	( )	91
Fiber board manufacturing	34113	96	90
Other paper and board products	3419	86	89
Plastics and synthetic fibers	35131	96	97
Rubber products	3550	( )	86
Plastic products	3560	88	88
Structural clay products	3691	( )	86
Cement, lime and plaster	3692	90	91
Other nonmetal mineral products	3699	94	95
Iron and steel casting	37103	( )	87

c) Alt. 1 Differentiation of the SNI two-digit level (SNI 2) excl. manufacturing of chemicals

Industry/product	SNI 2	Rationing quota
Mining	2	85
Food, beverage and tobacco	31	86
Textiles and clothing	32	80
Wooden materials	33	78

Paper, board and pulp	34	89
Chemicals: General chemicals	3511	72
Fertilizers and pesticides	3512	63
Plastics and synthetic fibers	35131	97
Semifinished plastic products	35132	80
Paints	3521	81
Pharmaceutical chemicals	3522	75
Soap, detergents	3523	50
Chemical products	3529	80
Lubricating oils	3540	83
Rubber products	3550	86
Plastic products	3560	89
Nonmetal mineral products	36	86
Iron, steel and metal goods	37	85
Manufacturing	38	84
Other manufacturing	39	79

FOOTNOTES

1) Methodologically, the model derives to a high degree from the operational general equilibrium model developed by Werin (1965) and the input-output study done by Werin for the National Industrial Board (SIND) in connection with the oil supply disruption of 1973-74. The input-output model was used to develop rules for the allocation of oil within the production system. The model and a number of numerical examples for the oil supply situation during the winter of 1973-74 were published in Werin (1974).

   The rationing model was also the point of departure for some work in connection with the Energy Preparedness Commission. In "Prismekanismen som förbrukningsregulator" ("The Price Mechanism as a Consumption Regulator", SOU 1975:61), Mats Persson compares the effect on employment within different branches of industry when prices are increased with the effect of a rationing program that minimizes the effects of the crisis on employment. The analysis was accomplished using different formulations of the objective, in which the value of final demand, profits in the production system and employment, respectively, were maximized.

   An extended version of the rationing model was developed by the Energy Systems Research Group at the Stockholm University under the auspices of the Swedish Energy Research Commission (EFN) and the National Industrial Board (SIND). An account of the model and a number of simulations based on the situation at the end of 1978 were presented in SIND (1979) and in Bergström (1980). Since that time the model has been extended and improved in a number of respects. It is now based on a division of the production system into 88 sectors as compared to the 35 used earlier. The treatment of the adaptation of oil coefficients and the possibilities of using inventories of semimanufactures and final goods has been improved.

2) The classification of the goods in the 88-sector model is shown in Appendix 6:1.

3) To improve the description of the possibilities for adaptation in the production system the work that has been done in Sweden on activity models and other studies of process industries could be used. (See, e.g., Lundgren (1983), Hultkrantz (1982) and Hjalmarsson (1977)). In addition, the model we use in this chapter does not permit varying the composition of energy supplies in the short run. This precludes increasing the supply of electricity at the cost of fuel or vice versa. The basis for such a decision should be the prices for electricity and fuels during the embargo and the substitution ratios in the various plants that transform energy. The basis for short-run mechanisms for the transformation of electricity and hot water can be found in Bergman and Carlsson (1985), Bergman (1977) and Bergman, Bergström and Björklund (1976). In the last reference there is also a description of the substitution between petroleum products that can be achieved by modifying production in the refineries and cracking plants.

4) Short-term price elasticities for the Irsaeli economy, estimated using a method similar to the one used here, are of the same magnitude as ours. The absolute values of these elasticities increase with the size of the shortage, from -0.09 for a small shortage to -0.28 for a decrease in imports corresponding to approximately 20% of the normal imports (Fishelson (1981)).

5) For instance, the heterogeneity of the output of the branches of industry, i.e., the fact that the planner has to work with a rather coarse division of industry into branches, is a factor that leads to uncertainty regarding the use of inventories. It is likely that the flexibility provided by the inventories is overestimated at such a high level of aggregation. In other words, it is not certain that the goods that are in stock for a certain aggregate group of goods actually correspond to the goods that are demanded in later stages of production and for final use.

REFERENCES

Bergman, Lars, and Carlsson, Anders. The Impact of Sulphur Emission Control Policies: Projections of Swedish Energy Markets 2001. EFI, Research Report. Stockholm: EFI, 1985.

Bergman, Lars. Energy and Economic Growth in Sweden - An Analysis of Historical Trends and Present Choices. Stockholm: EFI, 1977.

Bergman, Lars, Bergström, Clas and Björklund, Anders. An Energy Forecasting Model for Sweden. Stockholm: EFI and SIND (1976:6), 1976.

Bergström, Clas. "A Planning Model for Acute Energy Shortages." International Journal of Energy Research 3 (February 1980):267-270.

Fishelson, Gideon. "Possible Consequences of Restricting the Import of Oil." Resources and Energy 3 (April 1981):93-103.

Hjalmarsson, Lennart. Substitutionsmöjligheter mellan energi and andra produktionsfaktorer. Stockholm: Ds I 1977:17, 1977.

Hultkrantz, Lars. Skog för nutid och framtid - en samhällsekonomisk analys av det lämpliga virkesuttaget. Stockholm: EFI, 1982.

Leibenstein, Harvey. "Allocative Efficiency Vs. X-Efficiency." American Economic Review LVI (June 1966):392-415.

Lundgren, Stefan. A Model of Energy Demand in the Iron and Steel Industry. Research Paper No. 6254, Stockholm: EFI, 1983.

Persson, Mats. Prismekanismen som förbrukningsregulator. In: Energiberedskap för kristid. SOU 1975:6, Bilaga 7.

SIND PM 1974:1. Energiransonering i industrin. Konsekvenser för sysselsättning och varuförsörjning. Stockholm: SIND, 1974.

SIND 1979:2. Akut energibrist - en planeringsmodell. Stockholm: SIND, 1979.

Werin, Lars. A Study of Production, Trade and Allocation of Resources. Stockholm Economic Studies. Stockholm: Almqvist & Wiksell, 1965.

Werin, Lars. "Energiransonering i industrin - konsekvenser för sysselsättning och varuförsörjning." SIND PM 1974:1. Stockholm: SIND, 1974.

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---

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1985

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