Model Integration and the Economics of Nuclear Power

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Model Integration and the Economics of Nuclear Power

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STOCKHOLM SCHOOL OF ECONOMICS
THE ECONOMIC RESARCH INSTITUTE

A Dissertation for the Doctor's Degree
in Economics
Stockholm School of Economics 1985

ISBN 91-7258-192-1
UDK 621.039
330.35
minab/gotab, Stockholm 1985

Preface

A research programme devoted to the economics of energy, natural resources and the environment has been pursued at the Stockholm School of Economics since 1973. The main emphasis has been on the construction and application of quantitative models for studies of energy policy and energy markets.

My dissertation is a product of this research programme. The idea to link models to one coherent structure in order to better utilize the information spread among them, had been around for some time within the programme, when Karl-Göran Mäler in the fall of 1980 suggested that I should look closer at this issue. I got started in 1981. Especially a stay at the International Institute for Applied Systems Analysis (IIASA) in the summer of 1981 turned out to be very rewarding.

During my thesis work I have received help and constructive comments from many friends and colleagues. I wish to thank them all. In particular I owe a great deal to Lars Bergman and Karl-Göran Mäler, who have been my thesis advisors. I have also benefited from help and comments by Isak Assa, Clas Bergström, Anders Carlsson, Anthony Fisher, Richard Gilbert, Per-Olov Johansson, Ragnar Lindgren, Bertil Näslund, Andras Por, Claes Thimrén and Ernö Zalai. Also, I wish to thank Onerva Lahdenperä, Kerstin Niklasson and Monica Peijne, who skilfully have typed and re-typed the successive versions of this book.

Financial support from the Energy Research Commission and from Jacob Wallenbergs Fond is gratefully acknowledged. I am also grateful to the Swedish Committee for Systems Analysis for the financial support of my stay at IIASA.

Stockholm, March 25, 1985

Stefan Lundgren

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1 Applied General Equilibrium Analysis and Model Integration

1.1 INTRODUCTION

The aim of the study

This is a study in applied, or computable, general equilibrium analysis, with applications to the economics of energy, especially electricity. Applied general equilibrium analysis refers to the use of numerical and, generally, computer implemented Arrow-Debreu-Walras models for economic analysis and policy evaluations. Although it is not an entirely new approach - - its roots go back to Leontief (1941, 1953) and Johansen (1960) - it has been a very active field of research in the last few years.[1)] It is also during these years that the approach has become known as applied, or sometimes computable, general equilibrium modeling.

The basic theme of the present study is an issue which I have called model integration. By this I mean the merger of two or several originally independent models to one coherent model. In particular I have in mind the integration of a detailed, numerical model of a single sector in the economy with a computable general equilibrium of the whole economy.

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1 Dervis, de Melo and Robinson (1982), Scarf and Shoven (1984) and Shoven and Whalley (1984) provide introductions to this field as well as surveys of major works.

The model integration issue arises because of the existence of different types of models used for different purposes in applied economic analysis. One type is the applied general equilibrium models which focus primarily at the allocation of resources between various parts of the economy. Another type is the models of individual sectors which focus at the allocation of resources within a sector. The former type of models have in most cases a fairly aggregated and compact representation of individual sectors. They are usually represented by single production or cost functions, or functional relations derived from such functions. The sector models on the other hand often have a more disaggregated and explicit representation of, inter alia, the production technology.

It is often fruitful to use different types of models for different purposes. But model integration is of interest for at least two reasons.

One reason is to extend the range of applications for an applied general equilibrium model by exploiting the richer information of a sector model. An integrated model allows for instance explicit analyses of the general equilibrium effects of restrictions on the use of various concrete sectoral production technologies.

A second reason is to evaluate the validity of the partial equilibrium exercises in which the sector models usually are employed. Through model integration the sector model is embedded in a general equilibrium framework. It is then possible to check whether the exogenous variables in the sector model are approximately unaffected by its endogenous variables when potential general equilibrium effects are accounted for. Moreover an integrated model also allows the estimation of the effects on the rest of the economy, and of the induced feedback effects, which result from changes within the sector model.

This study has three distinct purposes. The first is to propose and motivate a particular approach to model integration. The approach is based on the use of optimization methods for the solution of the integrated model. It is particularly useful when the sectoral model is an activity analysis model, which is a very common case.

The second purpose is to illustrate the proposed model integration approach with an application to a model of the Swedish electricity and heat sector and a general equilibrium model of the Swedish economy.

This particular illustration is motivated by the third purpose of this study which is to analyse some aspects of the role of nuclear power in the Swedish electricity market and in the Swedish economy. The electricity power system has changed rather dramatically in the last decade, from a power system predominantly based on hydro power to a mixed thermal-hydro system with nuclear power as the major thermal technology. And with the present nuclear power discontinuation policy further large changes seem to lie ahead since the nuclear power plants shall be replaced by other thermal power technologies.

The analysis of the role of nuclear power is devoted to two issues: the economic consequences of a nuclear power discontinuation and the extent to which the investments in nuclear power have contributed to an excess power capacity during the 1980's. These issues are best analyzed in a framework where the choice of different electricity production technologies and its effects on the electricity market and the rest of the economy can be explicitly assessed. It is for this reason I have applied the model integration approach to a sector model of electricity and heat production and a computable general equilibrium model of the Swedish economy.

An outline of the study

Chapter 2 is concerned with the first of these three purposes. First model integration is defined more precisely and different possible approaches are briefly discussed. It is stressed that model integration primarily is a computational issue. Then the suitability for model integration of four different solution methods for general equilibrium models is discussed. The use of optimization methods in this context is then proposed. The optimization methods are however based on the equivalence between competitive equilibrium allocations and efficient allocations. This approach therefore appears less useful when the models to be used contain elements which destroy this equivalence property, for instance if they contain commodity taxes and similar distortions. In the final part of Chapter 2 I show that even though commodity taxes and similar distortions destroy the equivalence property, the optimization approach may still be used, provided that a slight modification is introduced. This result extends considerably the usefulness of the optimization approach.

In Chapter 3 the approach proposed in Chapter 2 is applied to a sectoral model of electricity and heat production and a computable general equilibrium model of a small open economy. Both models are applied to Swedish data.

Chapters 4 and 5 contain two exercises with the integrated model. The first, presented in Chapter 4, is an evaluation of the present Swedish nuclear power policy. The objective is to shed some light on how the nuclear power discontinuation policy will affect electricity production and electricity prices and on whether it will have any stronger repercussions on the rest of the economy.

The second exercise, contained in Chapter 5, is an evaluation of the large nuclear power investment programme in Sweden. Between 1972 and 1985 twelve nuclear power reactors have been built and taken into operation. Their total production capacity amounts to roughly half the current electricity demand. It has been argued that the massive nuclear power investments have resulted in an excess power capacity. This exercise is an attempt to investigate the extent and likely duration of the excess capacity as well as its effects on the electricity market and on the rest of the economy.

Chapter 6 contains a summary of the main results of the study and some suggestions for future research.

Since this is a study in applied general equilibrium analysis the remaining part of Chapter 1 is devoted to a discussion of this approach and of its applications, including applications to energy economics. I conclude this chapter with some comments about some other work on model integration.

1.2 APPLIED GENERAL EQUILIBRIUM MODELING

The Walrasian general equilibrium model of the competitive economy, generalized and definitely formulated by Arrow and Debreu, is one of the central tools of modern economic theory. Although it is mainly employed in theoretical inquiries, it has also begun to be more frequently used in applied, quantitative resource allocation studies. There are in particular three areas - development studies, tax analysis and energy economics - where applied general equilibrium models are much in use. They are in fact becoming so established as working tools that a new speciality in economics - applied, or computable, general equilibrium modeling - seems to gradually emerge.[2]

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[2] Several works have recently been published about applied general equilibrium modeling which are at least as much concerned with problems and approaches in modeling as with the results of model applications. See for instance Dervis, de Melo and Robinson (1982), Ginsburgh and Waelbroeck (1980) and Scarf and Shoven (1984). Johansen's book on his multisectoral growth model (Johansen (1960, 1974)) is one of the earliest examples of this kind of studies.

The Arrow-Debreu-Walras general equilibrium model has five essential ingredients.[3] (1) It distinguishes between two categories of economic agents: consumers (or households) and producers. (2) The households are characterized by their initial endowments of commodities and factors of production and by their preferences. (3) The producers are characterized by their production possibility sets. (4) The fundamental behavioural assumptions are profit maximization by producers and utility maximization by households - in both cases the prices are outside the agents' own control. (5) The equilibrium prices are such that, for each commodity and factor of production, the total supply is at least as large as the total demand.

Although there hardly exists a precise, generally agreed upon, definition of an applied, or computable, general equilibrium model, it can broadly be characterized as a model which is based on these five essential ingredients and where in addition the model representations of preferences and production possibilities have been parameterized and where model parameters and variables have been assigned numerical values. Consequently the equilibrium allocation and the equilibrium prices can be computed numerically. The models are usually relatively large in terms of sector and commodity disaggregation so the numerical calculations must be performed on a computer.

The Arrow-Debreu-Walras model is concerned with the real side of the economy; the allocation of production factors between different sectors, the determination of the production of various commodities, the distribution of this production between households and other final users, for instance the government sector, and the establishment of a set of relative prices which makes the equilibrium allocation compatible with decentralized behaviour based on these prices. The applied general equilibrium models have consequently been applied to resource

[3] The standard references for the Arrow-Debreu-Walras model and the associated theory are Debreu (1959) and Arrow and Hahn (1971).

allocation issues, often in a medium term or long run perspective. Typical examples are studies of economic growth and the structure of the economy (often in the context of developing countries), studies of the incidence of different tax systems, studies of tariffs and international trade, and studies of the economic impacts of disruptions on the supply side, like for instance higher energy prices.

The general equilibrium framework is useful in these contexts because it emphasizes two essential aspects which are important and common to all of them. The first is the interdependence between the different parts of the economy, which implies that a change somewhere, such as a change in the tax system, in general will transmit itself through the whole economic system and also induce feed-back effects. The second is that it provides some content to the concept of "market forces". Producers and consumers make their economic decisions on the basis of market prices. These are endogenously determined so that decentralized economic decisions become compatible with each other. Moreover, the equilibrium prices will reflect the marginal returns to commodities in different uses and the economic agents' optimization behaviour tends to equalize these returns.

With this emphasis the general equilibrium framework can provide insights in many resource allocation problems. It can do this partly because the institutional setup of the model economy is kept quite simple which makes the model analytically manageable. Markets work smoothly and economic agents are fully informed and behave competitively, i.e. as price takers. The general equilibrium model thus stresses the economy as an interdependent system and the key role of relative prices, while other aspects are given a simplified treatment or perhaps are neglected.

Of course this also means that while the general equilibrium framework may be a useful analytical tool in certain contexts, it is not universally applicable. Similarly, in those cases where it can be usefully applied it cannot be expected to account for all relevant aspects of the problem. It is important to bear this in mind when these models are applied and when the results of the model applications are interpreted.

Most constructors of applied general equilibrium models allow themselves to deviate from the strict principles of the Arrow-Debreu-Walras model. The empirical data which provide the data base for an applied general equilibrium model often have some features which are hard to reconcile with the theoretical Arrow-Debreu-Walras prototype model. This is often true for savings and investment data and for data on foreign trade. To get what one considers to be reasonable results for such variables, more or less ad hoc model elements are often introduced. It is important to remember however that if this is carried too far one of the attractive features of the general equilibrium approach may soon vanish. It is based on a well-developed theory which makes it comparatively easy to understand and to interpret the results of model applications. A lot of ad hoc elements will obscure the theoretical understanding of the model and make it more difficult to interpret and assess the results.

The origins of applied general equilibrium models

The applied, or computable, general equilibrium models are the latest development in a tradition that originated with the static and dynamic input-output analysis introduced by Leontief (Leontief 1941, 1953). The input-output analysis can in principle be regarded as a general equilibrium model albeit with a quite peculiar demand side (essentially completely inelastic final demand functions). The main motivation for the input-output analysis was however the interdependencies in production between different sectors and not the equilibrating role of relative prices. In this capability the input--output framework still remains an important building-block in most applied general equilibrium models.

Johansen (1960, 1974) extended the basic input-output framework by relaxing the fixed proportions assumption for capital and labor. Instead he allowed substitution between capital and labor and employed sectoral neoclassical production functions. He retained however the fixed coefficients assumption for intermediate inputs. Johansen also introduced a more flexible demand side by incorporating price dependent demand functions for private consumption. However, to be able to estimate the parameters of his model, Johansen was confined to a very limited set of functional parameterizations. He essentially used a single data point - a set of national accounts for a single year - for the estimation and as a result only a limited set of parameters could be estimated. This estimation procedure was of course also a part of the input-output tradition, since input-output coefficients in general are obtained from the flow of intermediate deliveries during a single period of time.

Many of the properties of Johansen's model have been preserved in the majority of the modern applied general equilibrium models. The production technology is generally modelled as an input-output system for intermediate inputs, whereas capital and labor (possibly with some disaggregation into different types of capital and labor) are flexible inputs and the substitution between them is modelled by neoclassical production functions. In models designed for energy studies energy is usually also treated as a flexible input.

Similarly, on the demand side, the demand for private consumption is represented by a set of price dependent demand functions consistent with utility maximization. In contrast to Johansen, who worked with a single aggregated household sector, several of the modern applied general equilibrium models distinguish between different types of consumers or households. This is particularly typical of tax incidence models and models of international trade (where each country is represented by at least one household).

A third similarity with the Johansen approach is the model parameter estimation procedure employed in most applied general equilibrium models. It is generally referred to as "model calibration". It means that except for a few key parameters, typically substitution elasticities, rates of technical change in the production functions and demand elasticities, the model parameters are chosen so that the model is able to replicate a given base year allocation.[4] The latter is generally assembled from the national accounts of the base year, although they often must be amended and modified to account for all exogenous and endogenous variables, as well as to comply with the equilibrium conditions of the general equilibrium model. The "model calibration" approach makes it necessary to impose very restrictive assumptions on technology and preferences so that the parameter estimation is feasible. In practice it means that production functions and preference functions almost exclusively are parameterized as Cobb-Douglas or CES-functions. With a Cobb-Douglas production function, for instance, all parameters can be estimated from information about the cost shares of the production factors and a choice for the unit measurement of output. For the somewhat more general CES-functions the distribution and scale parameters can also be estimated from the same data, provided however, that some value has been assigned to the elasticity of substitution. The latter can namely not be inferred from a base year allocation. Instead, one has to rely on other sources of information, such as econometric estimates reported in the literature. In general one can say that the base year allocation data determine a point through which the production function (or preference function etc.) shall pass, but to the extent the chosen functional form allows several possibilities for the slope or curvature in that point, one must make additional assumptions about the numerical values of, typically, elasticities.

[4] The model calibration approach is thoroughly discussed by Mansur and Whalley (1984). They also discuss some problems encountered in econometric estimations of general equilibrium models.

The merits of applied general equilibrium models

The use of applied general equilibrium models has two aspects. The first is the competitive general equilibrium approach. Its usefulness in applied analyses stems from its emphasis of the economy as an interdependent system and of the key role of relative prices. The former in a sense means that the analysis is complete. In principle no variables are kept constant outside the analysis, as in a partial equilibrium analysis, unless they truly are independent of all the endogenous variables. Of course, in any applied analysis, the partition into exogenous and endogenous variables must be based on practical feasibility, which makes the distinction between partial and general equilibrium analysis less transparent. But the gist of the latter is still that most major variables shall be determined within the model.

The competitive economy framework also gives the applied general equilibrium approach a sound theoretical underpinning. It makes it possible to understand and interpret the mechanisms at work in the model and thus to explain the results of model applications. At the same time the theoretical body imposes a degree of intellectual restraint on model construction and applications so that too much of ad hoc analysis can be avoided.

The second aspect of applied general equilibrium models is that they are quantitative, i.e. they are numerically specified. The obvious reason is that they are supposed to provide quantitative estimates of the effects of, for instance, a policy experiment. These estimates are then typically used to support arguments concerning the consequences for the actual economy of the same policy. But there are also other good reasons to work with a numerically specified model. It makes it possible to extend the realm of experiments in comparative statics. With a non-numerical, non-parameterized model comparative static exercises are in general confined to small perturbations of an initial equilibrium. The applied general equilibrium model in contrast facilitates the evaluation of non-marginal changes in exogenous variables or policy variables. Furthermore, qualitative comparative static results are sometimes indeterminate unless more specific assumptions about key parameters, for example elasticities, are introduced. The applied general equilibrium model is in this respect a comfortable tool for assessing the implications of different assumptions.

Another advantage with a numerically specified and computer-based model is that it facilitates a disaggregated analysis and thus can provide better information concerning effects on the pattern of resource allocation, the structure of relative prices and perhaps also income distribution.

Thus it is not correct to regard the applied general equilibrium models as something to use only for forecasts or projections. This can in some cases be an important task. But it should be kept in mind that it is difficult to assess the descriptive reliability of a model if its performance has not been compared to the performance of the actual economy which it is a model of. Such a comparison can be accomplished either if the model is econometrically estimated, and thus also tested against actual data, or if it is used to simulate historical events, so that the simulations can be evaluated against what actually happened. But few of the present generation of applied general equilibrium models have been econometrically estimated.[5] There is also a striking lack of attempts to assess the models' descriptive reliability by counterfactual simulations.[6] As

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[5] The model developed by Hudson and Jorgenson (1975) is one of the few exceptions. Another is the later versions of Johansen's multisectoral growth model of the Norwegian economy (see Bjerkholt et al. (1983)).

[6] Such counterfactual simulations have been carried out by Kelley, Williamson and Chetham (1972), Kelley and Williamson (1980) and Karlström (1980). These authors were however concerned with analyses of historical economic developments and not primarily with model reliability.

a result it can be difficult to relate projections of the development of the model economy to the actual economy which it is a model of.

Against this background it would be overly naive to consider this type of models to be tools which primarily provide conditional projections, or forecasts, i.e. answers to the following sort of questions: If the exogenous variables would have the following realizations, what would be the outcome of the policy experiment (or some other disturbance which we are interested in)? Instead, one should be clear about the fact that these models emphasize certain properties of an economy by abstracting from several others. The perfectly legitimate reason for this is to provide insights through abstraction and simplification. Comparative static analyses with the model, complemented with a generous number of sensitivity analyses and tests of alternative model parameter assumptions, should provide insights into the effects and quantitative importance of these properties for a certain policy experiment or some other exogenous disturbance. Since also the properties which the model abstracts from can affect the final outcome, the model projections cannot unqualified be interpreted as projections for the actual economy. On the other hand, when the model is applied in a certain context, and we believe that the properties it does emphasize are particularly important, the results produced by the model should also convey information about likely effects for the actual economy.

Applications

The modern computable general equilibrium models have mainly been applied to three categories of problems:

- issues in international trade, growth, economic structure and income distribution
- issues in public finance
- issues in energy economics.

The first category contains single-country models of both developed and developing countries as well as multi-country models which explore issues on trade flows and their impact on single countries or regions. The single-country models are mostly devoted to resource allocation issues; growth and structural change, the role of foreign trade in the growth process and in several cases with the distribution of income. Representative examples of such models for developed countries are the now classical model by Johansen (1960, 1974), the ORANI model for Australia (Dixon, Parmenter, Sutton and Vincent (1982) and Dixon, Parmenter and Rimmer (1984)) and Zalai (1982) for Hungary. There is a large collection of models in this category which are concerned with development strategies in developing countries. Examples are several studies by Dervis, de Melo and Robinson (Dervis (1975) (Turkey), de Melo (1977) (Colombia), Adelman and Robinson (1978) (Korea), Dervis and Robinson (1978) (Turkey), de Melo (1979) (Sri Lanka), de Melo and Robinson (1980) (Colombia) and Dervis, de Melo and Robinson (1982)). Other examples are Taylor and Black (1974) (Chile), Serra-Puche (1984) (Mexico), Feltenstein (1980) (Argentina), Lysy and Taylor (1980) (Brazil) and also the works by Kelley, Williamson and Chetham (1972) and Kelley and Williamson (1980).

Another type of models in this category is the multi-country models which focus on trade issues, particularly the effects of international tariff agreements or proposals. Examples are Deardorff and Stern (1981), Brown and Whalley (1980), Miller and Spencer (1977) and Ginsburgh and Waelbroeck (1980).

The models in the second category, i.e. the public finance models, are all descendants of the general equilibrium tax incidence analysis introduced by Harberger (1962). He supplemented his theoretical analysis with a numerical application to the United States. To be able to handle the model analytically, he was confined to a two commodity, two factor model and to study marginal changes in the corporate income tax. Shoven and Whalley (1972, 1973) and Shoven (1976) were the first to use the Scarf simplicial search algorithm to numerically solve disaggregated versions of the Harberger model. With numerical solution methods it is also possible to consider non-marginal tax changes. The early contributions by Shoven and Whalley have now been further extended and elaborated by several modellers, for example Piggott and Whalley (1976, 1982), Fullerton, Shoven and Whalley (1983), Ballentine and Thirsk (1979), Keller (1980), Slemrod (1983), Auerbach and Kotlikoff (1983) and Ballard, Fullerton, Shoven and Whalley (1983). Shoven (1983), Fullerton, Henderson and Shoven (1984) and Shoven and Whalley (1984) are useful surveys and overviews of these works. Several of these studies have given a lot of attention to the potential efficiency gains from an integration of the U.S. corporate and personal income tax, probably because Harberger's original contribution examined the incidence and efficiency loss of the U.S. corporate income tax. But several other topics such as the replacement of income taxes with a consumption tax, the size of the total efficiency cost of present tax systems, the efficiency of various transfer systems, such as housing subsidies and so on, have also been studied.

The models designed for energy economic analyses, and which form the third category of general equilibrium models distinguished above, are mainly focused at what has been called "energy-economy interactions". After the first oil price shock in 1973-74 much of the attention was directed to the impact of higher energy prices in general, and higher oil prices in particular, on the performance of the economy. Since energy in one form or the other is an essential and important input in all production activities, large increases in the cost of energy, or substantial reductions in the availability of various energy commodities, could have potentially serious effects on economic activity, growth and the standard of living. It became important to explore these issues, qualitatively and quantitatively. Another, but related, reason for the interest in energy-economy interactions arose from the observation that partial equilibrium analyses of energy markets now had to be done more carefully than before. The standard approach in these studies had been to take GNP, the prices of non-energy commodities and other indicators of the performance of the non-energy part of the economy as independent of energy prices and then to make projections of energy demand, energy production and energy prices. This may have been a quite satisfactory approach in the fairly stable environment of the fifties and the sixties, but with the more volatile energy markets we have experienced since the early seventies, it is necessary to appreciate that there may also be strong feed-back effects from the energy sectors to the rest of the economy.

Broadly speaking, one may distinguish two branches of research on energy-economy interactions. In one the role of the energy markets is studied in a framework of full utilization of factors of production, flexible, market clearing relative prices and an emphasis on the real side of the economy.

The other branch is more oriented towards macroeconomic adjustments following "supply shocks" like higher energy prices. Gordon (1975), Findlay-Rodriguez (1977) and Phelps (1978) represent early contributions to the theory of macroadjustments and macroeconomic policy responses to supply shocks. Other contributions include Buiter (1978), Obstfeld (1980), Nordhaus (1980), Rasche and Tatom (1981), Sachs (1982) and Darby (1982). Macroeconomic simulation models have also been extended to include energy aspects. A sample of such modeling efforts is collected in Mork (1981).

The first branch of research has extensively used the computable general equilibrium approach. One of the earlier contributions was the model by Hudson and Jorgenson (1975, 1978) of the U.S. economy. In contrast to most computable general equilibrium models the Hudson and Jorgenson model is econometrically estimated. The production sectors, including several distinct energy sectors, are represented by econometrically estimated translog functions.[7] This model of producer behaviour means that the input coefficients for intermediate as well as primary inputs are price-dependent. In particular, there are substitution possibilities between different types of energy as well as between energy and other production factors.

Also the final demand side of the model is econometrically estimated and price dependent. The Hudson and Jorgenson model thus allows the analysis of energy-economy interactions in a context where different forms of energy are substitutable against each other and against non-energy commodities in the intermediate demand as well as directly and indirectly in the final demand.

Other well-known models of energy-economy interactions in the U.S. economy are the ETA-MACRO (Manne (1977)) and the PILOT (Dantzig et al. (1978)) models. They are both intertemporal general equilibrium models. The ETA-MACRO model represents the economic system in a cruder way than PILOT. It is a neoclassical, aggregated one-sector growth model linked to a fairly detailed process model of the energy sector. In the PILOT model, on the other hand, the energy sector submodel is linked to a dynamic input-output model of the economy, which gives a richer description of the economic system, but in this case at the cost of maintaining fixed coefficients in the production technology.

In both models the energy sector is given a quite detailed representation which, in contrast to the Hudson and Jorgenson model, makes them suitable for energy technology assessments. They also explicitly deal with the U.S. domestic energy resource base and the exhaustibility of certain forms of primary energy.

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7 The translog function was introduced by Christensen, Jorgenson and Lau (1971, 1973).

Other models of energy-economy interactions in the U.S. economy can be found in Hitch (1977), EMF (1977) and EPRI (1979).

Models of energy-economy interactions have also been constructed for other countries. Far from all of these can be considered to be compatible with Walrasian general equilibrium analysis. In many cases they are hybrid models which combine elements of neoclassical production theory, input-output analysis and income-expenditure models of the final demand side. Examples of such models can for instance be found in Strub (1979). But there are also examples of the computable general equilibrium approach.

For Norway, for instance, a recent version of Johansen's multi-sectoral growth model has been extended to incorporate energy aspects (see Bjerkholt et al. (1983)). In particular the supply and demand of electricity have been given a detailed treatment in the extended model.

Another example is the system of computable general equilibrium models of a small open economy, developed by Bergman and Por (Bergman and Por (forthcoming) and Bergman (1982)). It has been used to analyse energy-economy interactions and energy policy issues in the Swedish economy.

The issue of model integration

It is especially in connection with energy economic studies that model integration has been of interest. Different aspects of the supply and demand of energy have often been thoroughly dealt with in separate models. The supplies of different forms of energy and the use of various energy conversion technologies are for instance dealt with in special energy supply models. The level of technological detail and the commodity disaggregation can be quite substantial in these models. Similarly, the demands of different forms of energy for different purposes have been dealt with in special energy demand models.

For the study of the interconnections between the demands and supplies of different forms of energy and the relations between the energy markets and the rest of the economy it can often be useful to bring together the different aspects collected in the various energy supply and demand models. One way of doing this is to integrate different models to a coherent structure. This has also been done by some modellers.

Thus, to improve the treatment of the energy supply in the Hudson and Jorgenson model it was linked to the Brookhaven Energy System Optimization Model, a very detailed model of the U.S. energy system (Hoffman and Jorgenson (1977)).[8] The Hudson and Jorgenson model has also been linked to a model of natural gas supply (see Bernanke and Jorgenson, 1975).

The ETA-MACRO and PILOT models can be viewed as models where detailed energy supply models have been integrated with models of the rest of the economy. Another case is the various supply and demand models developed within the Project Independence Evaluation System (PIES) in the U.S. (FEA (1974)). They were integrated through an energy market equilibrium model to an integrated model structure.

These are a few examples of model integrations. In the case of the Hudson and Jorgenson model the integrations were achieved by formalized iterations between the general equilibrium model and the sector models. For the PIES the market equilibrium was also computed through iterations between the various energy demand and supply models and a special integrating model which monitored the iterative process. For the ETA-MACRO and PILOT models, however, the energy sector models were directly integrated with a general equilibrium model to one common model structure. As a result it was not necessary to iterate between the original models.

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8 The Brookhaven model is described by Hoffman (1973) and Cherniavsky (1974).

The latter approach is also the one I adopt in the following. The main reason is that it, in contrast to the iterative approach, must not be uniquely designed for the particular models under consideration. The basic approach can be employed for many different sectoral models and thus provides more flexibility.

2 Model Integration and the Use of Optimization Methods for Computing General Equilibria

2.1 INTRODUCTION

The primary aim of this chapter is to propose a specific computational approach to model integration, i.e. the merger of two or several originally separate models. In particular, the approach is intended to be used to integrate a linear programming type of sector model with a general equilibrium model.

The applied general equilibrium models discussed in Chapter 1 often have a relatively aggregated representation of, for instance, individual production sectors. They are typically described by single, neoclassical production or cost functions, or by supply and demand functions derived from the former. This is quite natural since these models have been designed to emphasize the allocation of resources between different sectors and not primarily the allocation within individual sectors.

At the same time more detailed sectoral models are constructed for partial equilibrium sector studies. Such models often have a more disaggregated and explicit representation of, inter alia, the production technology.

It is useful and often necessary to use different models for different purposes. Through model integration it is however also possible to jointly apply several models to the same issue. Especially it makes it possible to determine the allocation of resources within a sector model simultaneously with the allocation of resources in a general equilibrium model. This is valuable for at least two reasons.

Sometimes there can arise a need to check a partial equilibrium analysis in a general equilibrium context. In certain cases it is important to examine whether the exogenous variables in a partial equilibrium model really are approximately unaffected by its endogenous variables when the potential general equilibrium effects are accounted for. And if they are not unaffected, it would obviously be useful to estimate the feedback effects on the endogenous variables as well as the impact on the rest of the economy. One way of doing this is to integrate the general equilibrium and the sector models.

Another reason for model integration is to exploit the richer information of a sector model. An integrated model can extend the range of applications for a general equilibrium model, for instance by allowing explicit analyses of restrictions on the use of various concrete sectoral production technologies.

The chapter is organized as follows. In Section 2.2 model integration is first defined more precisely and then different possible approaches are briefly discussed. It is stressed that model integration does not involve any principal or conceptual difficulties. It is primarily a computational issue. For this reason it is useful to briefly review various computational methods for computing general equilibria and their suitability in the context of model integration. This is done in Section 2.3. One possible computational approach, which also seems to be useful in the context of model integration, is the optimization approach. It means that numerical optimization algorithms are applied in order to identify efficient allocations in the general equilibrium model. According to the fundamental equivalence theorems of competitive analysis, the equilibrium allocation can then be found among these. However, the reliance on the equivalence theorems is also a major limitation of the optimization approach. It is not unusual that applied general equilibrium models contain features which imply that the equivalence theorems are not valid, i.e. the competitive equilibrium is not necessarily an efficient allocation. One important example is the existence of commodity taxes and similar distortions in the general equilibrium model. In Section 2.4 I show however that the optimization approach still may be applied in this case, provided that a slight modification is introduced. The result extends considerably the usefulness of the optimization approach.

In the remainder of this section a fairly general, but quite simple, formal representation of the general equilibrium model is introduced. This formal structure is the central workhorse in Sections 2.2 and 2.3.

The general equilibrium model

Let $z^T(p) = (z_1(p), z_2(p), \ldots z_n(p))$ be a vector of excess demands at the prices $p^T = (p_1, p_2, \ldots, p_n)$, let $v^T = (v_1, v_2, \ldots, v_n)$ be a vector of the total initial endowments of the economy and let A be an n×k activity matrix and $y^T = (y_1, y_2, \ldots, y_k)$ a vector of activity levels. The prices p^* are equilibrium prices and the activity levels y^* are equilibrium activities if

$$
\begin{aligned}
& z(p^*) - Ay^* \leq v \\
& p^{*T} \cdot [z(p^*) - A \cdot y^* - v] = 0 \\
& A^T \cdot p^* \leq 0 \quad \text{and} \quad y^{*T} A^T \cdot p^* = 0 \\
& y^* \geq 0 \quad \text{and} \quad p^* \geq 0
\end{aligned} \tag{1}
$$

The first two conditions are that the total supply of each commodity shall be at least as large as the total demand and if the excess supply for some commodity is strictly larger than zero, the corresponding price shall be zero.[1] The third condition is that no activity shall make a positive profit and if an equilibrium activity is strictly positive, its profit shall be zero. Finally, only nonnegative values for y and p are permissible equilibrium values.

An activity analysis part is incorporated in (1) because it is useful to be able to explicitly consider linear production technologies in the discussions of model integration and computational methods later on.

The representation in (1) is sufficiently general to allow several different interpretations with respect to the more precise structure of the underlying general equilibrium model. To illustrate some possibilities, let d(p) denote the consumers' excess demand vector at the prices p and let x(p) denote the firms' excess supply vector at the prices p. One possibility is that the excess demand functions z(p) summarize the consumers' demand decisions, i.e. z(p) = d(p), while the activity matrix represents the production technology of the whole economy so that x = - Ay.

---

[1] As usual, negative entries in A denote net inputs and positive entries net outputs.

Another possible interpretation is that the functions $z(p)$ summarize both the individual consumers' demand decisions and the producers' output and input decisions, i.e. $z(p) = d(p) - x(p)$. In this case the activity matrix representation is not required. This is of course still compatible with the structure in (1) if we let A contain elements which are all zero. In this case, the presumption is that the production decisions are single-valued functions of the commodity prices. At first glance this may appear rather restrictive, since the important class of models with constant returns to scale in the production technology cannot yield single-valued output and input decisions. But this complication can be circumvented. In these models the commodities can often be divided into commodities which are produced, on the one hand, and primary factors of production, such as capital and labor on the other. Let (p_1, p_2) be the corresponding partition of the n commodity prices and let $c(p_1, p_2)$ be the vector valued cost function for the produced commodities. Suppose that the consumption demand is such that there must be a positive production of all produced commodities in equilibrium. As a result, the relations $p_1 = c(p_1, p_2)$ must hold. They determine the prices p_1 as functions of the prices p_2, say $p_1 = f(p_2)$. Now, because of the constant returns to scale, the outputs of the produced commodities are completely demand determined, say as $x_1(p_1, p_2)$ where $x_1(p_1, p_2)$ now denotes the <u>gross</u> outputs of the produced commodities. Suppose finally, that the input demands for the non-produced commodities per unit of output of the produced commodities are given by the input matrix $a(p_1, p_2)$, where each element is a single-valued function of the commodity prices. If $d_2(p_1, p_2)$ denote the consumers' demands for the non-produced commodities, the excess demand functions can now be defined as $z(p_2) = a(f(p_2), p_2) \cdot x_1(f(p_2), p_2) + d_2(f(p_2), p_2)$.

Thus also this class of models can in general be fitted in the formal structure of the conditions in (1). In fact, for these models the search for equilibrium prices is often greatly facilitated since it can be confined to a less-dimensional subspace of R^n. In the not uncommon cases where n is of the order of 20 to 40 whereas the number of non-produced commodities is only two (labor and capital for instance) this is a substantial advantage.

A third possible interpretation is that the activity analysis matrix only pertains to some firms. Their excess supply vector is thus given by $- Ay$. If we now let $x(p)$ denote the excess supply vector of the remaining firms $z(p)$ is in this case given by $z(p) = d(p) - x(p)$. The discussion of model integration in the next section will be concerned with this third interpretation.

Before we proceed there are two things to note about the conditions in (1). Since each consumer is constrained by his budget constraint, i.e. the value of his share of the initial endowments and his share of the total profits in the economy, the value of the excess demands must be equal to the value of initial endowments plus profits (at given activity levels), i.e. $[z(p) - A \cdot y - v]^T \cdot p = 0$. This is of course Walras' law, which holds for all permissible price vectors. Consequently the second condition in (1) is in fact implied by Walras' law and it does not impose any additional constraint on the equilibrium prices. As usual Walras' law implies that the market equilibrium conditions are not independent: if the values of the total excess demands are zero in each of n-1 markets it must also be zero in the n:th market. Thus, it is always possible to exclude one equilibrium condition in, for instance, numerical computations of equilibrium prices.

The excess demand functions are homogeneous of degree zero in prices. Consequently, if p^* is an equilibrium price vector so is λp^* for all $\lambda > 0$, which is easily verified from the conditions in (1). As usual, a general equilibrium model with n commodities only determines n-1 relative prices. Thus one commodity may be chosen as a numeraire for the price system or the search for equilibrium prices may be confined to a subset of the positive orthant in R^n, for instance the unit simplex defined by $S_n = \{p: \sum_j p_j = 1\}$.

2.2 MODEL INTEGRATION

The general equilibrium model and sector models

Suppose that the general equilibrium model is such that it is possible to single out a subset of producers which produce - and supply to the rest of the economy - a limited number, say m, of the n commodities in the economy. These producers are the only ones who can produce these m commodities and they cannot produce any of the remaining n-m commodities. Then this group of producers can unambiguously be treated as a distinct sector of the economy, where the sector is defined by its m outputs. The activity analysis production technology is now assumed to pertain to this sector only, while the excess demand functions z(p) in (1) summarize the demand and supply behaviour of the rest of the producers and of the consumers.

To make this structure more apparent it can be explicitly introduced in the equilibrium conditions in (1). For this purpose let p^m denote the prices of the m sector outputs and p^n the prices of the remaining commodities. In a similar way the excess demand functions can be partitioned into the two sets $z^m(p)$ and $z^n(p)$ for the two groups of commodities and the initial endowments in the two sets v^m and v^n. The activity technology matrix can be partitioned into A^m and A^n, where A^m is the m×k matrix containing the rows of the sector commodities and A^n the (n-m)×k matrix containing the rest of the rows. The equilibrium conditions in (1) can now be rewritten as

$$
\begin{aligned}
& z^m(p^m, p^n) - A^m \cdot y \le v^m \\
& z^n(p^m, p^n) - A^n \cdot y \le v^n \\
& p^{m^T}[z^m(p^m, p^n) - A^m \cdot y - v^m] = 0 \\
& p^{n^T}[z^n(p^m, p^n) - A^n \cdot y - v^n] = 0 \\
& \begin{bmatrix} A^{m^T}, & A^{n^T} \end{bmatrix} \begin{bmatrix} p^m \\ p^n \end{bmatrix} \le 0 \qquad y^T \begin{bmatrix} A^{m^T}, & A^{n^T} \end{bmatrix} \begin{bmatrix} p^m \\ p^n \end{bmatrix} = 0 \\
& y \geqq 0 \qquad p^m \geqq 0 \qquad p^n \geqq 0
\end{aligned}
\tag{1'}
$$

In this framework z^m is the total demand in the economy for the m sector outputs, $A^m \cdot y$ is the supply and $A^n \cdot y$ is the sector's input demand for commodities from the rest of the economy.

Consider now the prices p^m and p^n as given and concentrate on the choice of activities y. Now $\pi \equiv A^{m^T} \cdot p^m$ is the revenue vector and $c \equiv - A^{n^T} \cdot p^n$ is the cost vector when all activities are operated at unit levels. Thus we have the following simple sectoral model of activity choices: $\max(\pi - c) \cdot y$. Of course this is a meaningful problem only if $\pi_i \leqq c_i$, $i = 1,2,\ldots,k$, i.e. no activity yields a strictly positive profit. If this condition is satisfied, however, the equilibrium values for the activities in (1') are also one of the solutions to this sectoral model.

The sectoral model can be used to analyse the choice of activities y for different assumptions about the revenue and cost parameters π and c. Of course, the sectoral model becomes really interesting first when there are explicit interrelations between activities and restrictions on their utilization, both which may be introduced as linear constraints on the choices of activity levels y. Such constraints are quite compatible with the conditions in (1'). One can, for instance, have the case where a subset of the n-m commodities not produced within the sectoral model are not proper commodities in the usual sense, but rather represent specific resources, or requirements, for the production within the sectoral activity analysis technology, for instance capacity requirements. In this case the corresponding excess demands from the rest of the economy are by definition always zero and the corresponding elements in v^n represent initial capacities. If we let $\bar{A}^n$ and $\bar{v}^n$ denote the rows in A^n and the elements in v^n which represent such special commodities, the sectoral model can now be written as

$$\begin{aligned} &\max\ (\pi - c)\cdot y \\ &\text{subject to } -\bar{A}^n \cdot y \leqq \bar{v}^n \end{aligned} \tag{2}$$

Note that in order to write the sector model in this way, the prices related to the special commodities shall be dropped in the calculation of the cost vector c. These prices are now instead given by the shadow values for the constraints of the sectoral model.

The model given by (2) is a fairly general form of the sort of linear programming models which frequently are used in sectoral studies.

So far I have, through a particular interpretation of the equilibrium model (1) shown how a sectoral model of a linear programming type can be deduced from it. It is now time to turn to the issue of model integration, i.e. the merger of a model like the one in (2) and a general equilibrium model as in (1).

There is no principal difficulty in doing this. Precisely as the model (2) can be deduced from the general equilibrium model (1') it is of course always possible to set up a structure like (1') around the partial equilibrium model so that prices and activity levels are consistently determined.

In practice, however, the option is rarely that of creating a general equilibrium structure around a given sectoral model. Instead, the model user has available a set of models designed for different purposes, such as sectoral models for more detailed studies of individual sectors and economy-wide models of a general equilibrium type which primarily focus on the interdependencies between individual sectors. The latter ones often have a more aggregated and compact representation of individual sectors, for example as single supply and demand functions. In such a more compact form the general equilibrium model (1') could for instance be expressed as

$$
\begin{aligned}
& z^m(p^m, p^n) - s^m(p^m, p^n) \leq v^m \\
& z^n(p^m, p^n) - s^n(p^m, p^n) \leq v^n \\
& p^{m^T} [z^m(p^m, p^n) - s^m(p^m, p^n) - v^m] = 0 \\
& p^{n^T} [z^n(p^m, p^n) - s^n(p^m, p^n) - v^n] = 0 \\
& p^m \geqq 0, \; p^n \geqq 0
\end{aligned}
\tag{1"}
$$

where now the net supply functions $s^m(p^m, p^n)$ and $s^n(p^m, p^n)$ have replaced the activity analysis sectoral representation.

The model integration issue can then be stated as follows: Given models (1") and (2), how can one ensure that the equilibrium prices and quantities in (1") are consistent with the prices and chosen activities in (2)?

This is obviously tantamount to a requirement that the sectoral model (2) and $[s^m(p^m, p^n), s^n(p^m, p^n)]$ shall be the same in the sense of giving the same sectoral supply and demand responses to the same set of prices.

Direct integration

The obvious way of ensuring mutually consistent solutions in the two models is to directly integrate the sector model (2) in (1"), letting it replace the net supply functions $s^m(p^m, p^n)$ and $s^n(p^m, p^n)$. There is no principal difficulty in doing this. The potential problems are practical. The general equilibrium model must be such that it is practically feasible to apply numerical solution methods which can handle weak inequalities and complementary slackness. Such methods are discussed in Section 2.3.

Another practical problem which can arise has to do with different commodity classifications in the sector model (2) and in the general equilibrium model. Since the two models usually have been more or less independently developed one rarely has exactly the same commodities in the two models. Typically, the sector model has a more disaggregated commodity list since it is especially designed for a more detailed analysis of a single sector in the economy. In order to incorporate the sector model in the general equilibrium model it is necessary to introduce a commodity aggregation interface between the two models.

In the general equilibrium model the commodities can be divided into two groups depending on whether they are sector outputs or not. The first group consists of the m sector outputs while the second group consists of the n-m remaining commodities.

To integrate the two models the demands for the m aggregated sector outputs derived from the general equilibrium model must be transformed to demands for the more disaggregated outputs of the sector model.

In a similar way the sector model's demands for inputs from the rest of the economy must be transformed to the n-m more aggregated commodities of the general equilibrium model.

The simplest way of accomplishing these transformations is to use a fixed coefficient aggregation/disaggregation. In this case there are two possible aggregation principles. The first is that of strictly complementary commodities, i.e. aggregated good is composed of single goods in fixed proportions. The second is the Hicksian aggregation i.e. the relative prices of the single goods are constant so that the aggregated good can be measured as a weighted sum of the single goods using the relative prices as weights.[2] Thus, in this case the single goods are perfect substitutes in the construction of the aggregated good.

Consider now first the case of transforming the demands for the m sector outputs of the general equilibrium model to the demands for the sector model's outputs. In this case the fixed proportions aggregation/disaggregation seems most reasonable. Then the aggregated sector demands in the general equilibrium model can easily be transformed to demands for the sector model's outputs and the latter will be produced in fixed proportions to each other. Of course to the extent that there are substitution possibilities in the demand for the sector model's outputs, the integrated model will exaggerate the inflexibility of the demand when the fixed proportions aggregation is used. On the other hand with the opposite extreme, the Hicksian aggregation, the sector model's outputs would be treated as perfect substitutes, i.e. any combination of them which sums to the aggregated demand of the general equilibrium model would do. With a linear model this would in general

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2 See Hicks (1946) pp. 312-313.

lead to extreme output specializations. This would obviously exaggerate the flexiblity of the final demand for the sector model's outputs. Moreover, in this case it would be necessary to use a set of constant relative demand prices in order to construct the aggregated goods and thus the relative prices of the sector model's outputs would be exogenous in the integrated model.

Consider next the transformation of the sector model's input demands to the n-m commodities of the general equilibrium model which are not sector outputs. In this case the Hicksian aggregation seems more appropriate. The aggregated commodities in the general equilibrium model can hardly be considered to consist of the sector model's inputs in fixed proportions since the input proportions of the sector model in general are endogenous. It is instead better to regard the n-m aggregated commodities supplied by the general equilibrium model as Hicksian aggregates of goods which are produced at constant relative costs. The input demands of the sector model can then be aggregated to these Hicksian commodities using the relative costs as weights.

To illustrate the aggregations/disaggregations discussed above let m as before be the number of the sector outputs in the general equilibrium model but let now m' denote the corresponding number for the sector model's outputs. Similarly, let n' be the number of the inputs to the sector model.

Let B^m be a m'xm matrix which disaggregates the m aggregated sector commodities of the general equilibrium model to the m' outputs of the sector model. A column in B^m thus contains the fixed proportions of the single goods which belong to the corresponding aggregated commodity. Let B^n be a (n-m)xn' matrix which aggregates the sector model's input demands to the n-m aggregated commodities in the general equilibrium model which are not sector outputs. A row in B^n thus contains the weights (i.e. the relative costs) of the single goods which belong to the corresponding aggregated commodity.

The sector model can now be incorporated in the equilibrium model through the two matrices B^m and B^n, yielding the following commodity balance relations

$$\begin{aligned} B^m (z^m - v^m) - A^{m'} y &\leq 0 \\ z^n - v^n - B^n A^{n'} y &\leq 0 \end{aligned} \qquad (3)$$

This approach of direct integration, using a fixed coefficient aggregation interface is the one I use in Chapter 3. The particular solution method I adopt is to state the general equilibrium model as an efficiency problem and to apply a general nonlinear optimization algorithm. This approach, both in terms of model integration and solution method, is quite similar to the one used by Manne (1977). He integrates a detailed energy sector model (see Manne (1976)) with an intertemporal, one-sector general equilibrium model of the U.S. economy.

Other approaches

If it is not practically feasible to directly integrate the sector model in the general equilibrium model there are other approaches to ensure consistency between the two models.

One possibility is to use the sector model (2) to estimate the supply and demand relations $s^m(p^m, p^n)$ and $s^n(p^m, p^n)$. By solving (2) many times for different sets of prices, a data set of output and input decisions is generated. It can then be used to econometrically estimate supply and demand relations which can be fitted into the general equilibrium model. Such estimations have been carried out by Griffin (1972, 1977, 1979) and Kopp and Smith (1981) (but for different purposes than model integration). A somewhat related approach was used by Bernanke and Jorgenson (1975) to incorporate natural gas supply functions derived from the natural gas model by MacAvoy and Pindyck (1973) in the Hudson and Jorgenson general equilibrium model of the U.S. economy (Hudson and Jorgenson (1975)).

Yet another approach is to indirectly integrate the two models through iterative solutions and modifications rather than to actually integrate them into a common formal structure. This was, for instance, the approach adopted by Hoffman and Jorgenson, in integrating the Hudson & Jorgenson model and the Brookhaven Energy System Optimization Model, a very detailed model of the U.S. energy system.[3] Such an iterative procedure could be anything from a quite sophisticated computerized scheme, as in the Hoffman and Jorgenson case, to simple model comparisons "by hand" by the model user himself.

2.3 SOLUTION METHODS FOR COMPUTABLE GENERAL EQUILIBRIUM MODELS

In the previous section I have discussed model integration. I have emphasized that it is a computational issue. For this reason I shall here briefly discuss four major computational methods which are commonly used to solve general equilibrium models. They are the standard iterative methods, for instance Newton-methods, for solving systems of nonlinear equations, the fixed-point method introduced by Scarf, optimization methods which essentially compute efficient allocations and then among these identify the equilibrium allocation(s) and finally a method based on a sequential linear complementarity algorithm.

The purpose of this discussion is to convey the basic ideas behind the methods and to identify those of them which are suitable in the context of model integration. In the case of a direct integration of an activity analysis sector model

[3] The Brookhaven model is described by Hoffman (1973) and Cherniavsky (1974).

with a general equilibrium model it is necessary to use a computational approach which can handle weak inequalities and complementary slackness. Since an integrated model also tends to be rather large, it is also important that the method of computation is reasonably efficient.

Iterative solution methods for nonlinear equations

For some equilibrium models, weak inequalities and complementary slackness in the equilibrium conditions (1) are not very relevant. The latter are, by model construction, known to hold as equalities at strictly positive prices, for instance. In these cases, the equilibrium conditions in (1) may be expressed as a set of nonlinear equations

$$z(p) = 0 \qquad (4)$$

To compute the equilibrium prices, i.e. to numerically solve such a set of nonlinear equations, there exists a multitude of iterative solution algorithms which can be used.[4] For an economist, a natural candidate would perhaps be the Walrasian tâtonnement process given by

$$p^{k+1} = p^k + \Phi[z(p^k)] \qquad (5)$$

where Φ: $R^n \rightarrow R^n$ is a sign-preserving function and p^k denote the prices at the k:th iteration.[5] But the economist would also be well aware of the rather discouraging results regarding the stability of the tâtonnement process.[6] One of the few

4 An exhaustive presentation of such methods can be found in Ortega and Rheinboldt (1970).

5 Since the model only determines n-1 relative prices, the iterations should really be defined only for n-1 prices while the remaining price is used as a numeraire. But I neglect this minor aspect here to avoid additional notation.

6 See Arrow and Hahn (1971) chapters 11 and 12.

positive results is that the price adjustment rule given by (5) is globally stable if the excess demand functions display gross substitutability between all commodities at all positive prices.[7] This is a fairly strong condition and for more complex models where the excess demand functions cannot be explicitly represented it is also hard to verify. Although the rule (5) is appealing to the economist's intuition, there are no strong arguments for it as a computational approach.

From a computational view the tâtonnement rule (5) is a particular version of a one-step Jacobi-Newton iterative method.[8] This is, for the system of excess demand equations (4), given by

$$p_i^{k+1} = p_i^k - \frac{\partial z_i(p^k)}{\partial p_i} z_i(p_k) \qquad i = 1, 2, \ldots, n \quad (6)$$

If all the own-price partial derivatives are negative then (6) is precisely a tâtonnement process with $\Phi_i(z) = (-\partial z_i/\partial p_i)\cdot z_i$, $i = 1, 2, \ldots, n$. But for computational purposes there is no reason to limit the process to sign-preserving Φ_i-functions, i.e. to increase (decrease) the price if the corresponding excess demand is positive (negative). Nor is it necessary to associate the price adjustment of the i:th price to the i:th excess demand equation. Whether the i:th price adjustment should be associated with the i:th or some other excess demand equation should rather be based on how it affects the performance of the iterative process.

A major advantage with the Jacobi iterative methods is their relative simplicity. The computational requirements are limited to evaluations of the function values z(p) and perhaps the diagonal elements of the Jacobian. It is not necessary to

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7 Two goods, i and j, i≠j, are gross substitutes at p when $\partial z_i(p)/\partial p_j > 0$ (see Arrow and Hahn (1971) p. 221).

8 See Ortega and Rheinboldt (1970 pp. 220-221).

compute all the elements in the Jacobian, nor is any matrix inversion involved.

This is however in general required for the Newton methods which often are used to compute solutions to equilibrium models. In the present context the Newton method is given by

$$p^{k+1} = p^k - Dz(p^k)^{-1} \cdot z(p^k) \tag{7}$$

where Dz(p) is the Jacobian for z(p).

The idea of the Newton method is thus to make a linear approximation of the excess demand functions at a point p^k and then to solve for the new price iteration p^{k+1} by setting the linear approximations equal to zero. Compared to Jacobi iterations the Newton methods use more information about the (local) properties of the equation system. Consequently, it is also computationally more demanding since the whole Jacobian and its inverse are required.

Jacobi iterations and Newton methods are only two among a multitude of iterative solution methods which may be used to numerically solve a system of nonlinear equations, but they seem to be the most commonly used in the context of equilibrium models.[9] They are fairly simple to use and the modeller can tune them to the needs of his specific model. On the other hand, there is no guarantee that they always will converge to a solution.[10] They are also not well suited to handle models where inequalities and complementary slackness conditions can be important. Thus these methods are not suitable for a model integration where the sector model is an activity analysis model.

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9 Ginsburgh and Waelbroeck (1980 ch. 7 and 8) discuss at length the use of Jacobi iterations and related methods in equilibrium models, and Dervis, de Melo and Robinson (1982, Appendix B) briefly outline the use of Newton methods.

10 The work on global analysis and regular differentiable economies by Smale and others (see for instance Smale (1976, 1981) and Debreu (1976)) indicates however that it is possible to devise a "global Newton" method for general equilibrium models which always will converge to an equilibrium provided that it exists.

Scarf methods

The equilibrium model given by (1) is homogeneous of degree zero in prices. This means that the search for equilibrium prices can be confined to a subset of R^n, for instance the unit simplex. The Scarf methods are a class of algorithms designed to systematically search over a simplex for equilibrium prices.[11] Unlike most of the other solution methods discussed in this section, their outstanding property is guaranteed convergence to a point which can be chosen to be as close as desired to an equilibrium. And this property holds for all conceivable equilibrium models provided only that certain fairly weak conditions are satisfied.[12]

The main drawback of the Scarf methods is that they are in general not as computationally efficient as several of the other solution methods discussed in this section, at least not at their present stage of development. But this is an area of quite active research which could lead to improved efficiency.[13]

It is not easy to give a brief presentation of the basic ideas of the Scarf methods, mainly because it is necessary to introduce a whole terminology concerning simplicial subdivisions and their properties. The essential ideas may however be conveyed through a simple and concrete example of an exchange economy.

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[11] The basic reference is Scarf (1973). His presentation is however conducted in terms of primitive sets and not in terms of simplicial subdivisions, a concept which is easier to grasp by geometric intuition and which now is the standard approach in presentations of the Scarf methods. It is also adopted by Scarf himself in later presentations of his methods, Scarf (1981), (1984). A thorough review of these methods including the developments since Scarf's original contribution is given by Todd (1976). See also Todd (1984).

[12] The Scarf methods are applicable whenever the equilibrium existence theorems relying on fixed point arguments, like the Brouwer or Kakutani theorems, are valid. These methods are namely algorithms for computing fixed points of a continuous mapping (or an upper hemicontinuous correspondence) and they will converge to an approximate fixed point provided such a point exists.

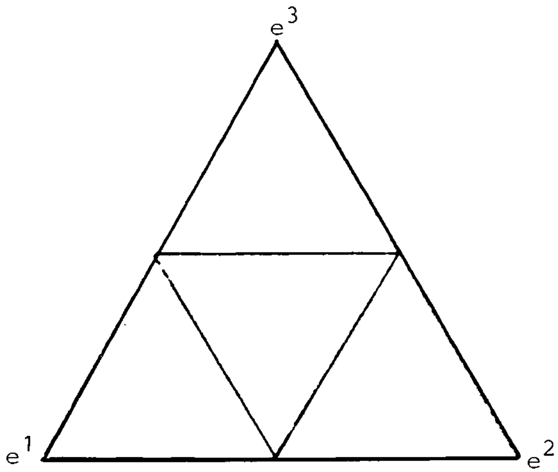
[13] Some of the results of this research are summarized and discussed in Todd (1984).

Consider thus an exchange economy with three commodities and a fixed, but arbitrary, number of consumers whose demand behaviour is summarized by the excess demand functions z(p). The equilibrium conditions are, in explicit form

$$
\begin{aligned}
&z_1(p_1, p_2, p_3) - v_1 \leq 0 \qquad p_1\cdot(z_1-v_1) = 0 \\
&z_2(p_1, p_2, p_3) - v_2 \leq 0 \qquad p_2\cdot(z_2-v_2) = 0 \qquad\qquad (8) \\
&z_3(p_1, p_2, p_3) - v_3 \leq 0 \qquad p_3\cdot(z_3-v_3) = 0
\end{aligned}
$$

The search for equilibrium prices can now be confined to the two-dimensional unit simplex defined by $S^2 = \{p\colon p = \sum_{j=1}^{3} \alpha_j \cdot e^j,\ \alpha_j \geq 0, \sum_{j=1}^{3} \alpha_j = 1\}$ and where e^1, e^2 and e^3 are the unit vectors for the three-dimensional Euclidean space. The simplex S^2 can be further divided into subsimplices as is illustrated in Figure 2.1.[14]

Figure 2.1 A simplicial subdivision of S^2



[14] To be a proper simplicial subdivision the collection of subsimplices should satisfy two conditions. First the original simplex should be contained in the union of all the subsimplices. Secondly, the intersection of any two subsimplices should be either empty or a full common side of both of them. (Scarf (1981) p. 1017.)

Next the idea is to assign labels to each of the vertices of this simplicial subdivision so that the labels carry some information about the mapping under consideration. For our three-commodity example we can use the scalar labels 1, 2 and 3. For a vertex p we assign an integer label i for which $z_i(p) - v_i \leq 0$. By Walras' law the excess demands cannot be strictly positive for all three commodities so this rule enables us to label all vertices. Furthermore, for any price vector p there must exist at least one price $p_i > 0$ for which $z_i(p) - w_i \leq 0$ because otherwise every price $p_i > 0$ would be associated with $z_i(p) - w_i > 0$ which clearly would violate Walras' law. So for a vertex p we can choose among the possible labels (i.e. those i for which $z_i(p) - v_i \leq 0$) one for which the corresponding price is strictly positive.

The relation between the vertex labels and a fixed point (or a point of equilibrium prices) is given by a combinatorial lemma known as Sperner's lemma.[15] It says that if the labeling rule is such that for each vertex the assigned label i is one of the indices for which the corresponding coordinate (or price) p_i is strictly positive, then there exists a simplex in the subdivision whose vertices have distinct labels. For this <u>complete simplex</u> it holds that at the vertex indexed by 1, $z_1(p^1) - v_1 \leq 0$, at the vertex indexed by 2 $z_2(p^2) - v_2 \leq 0$ and at the remaining vertex $z_3(p^3) - v_3 \leq 0$.

Now, by choosing a sufficiently fine grid for the simplicial subdivision, the three vertices p^1, p^2 and p^3 will be close to each other and by continuity the excess demands for all three commodities should remain strictly negative or be non-negative, but approximately zero, for all price vectors contained in the complete simplex. Thus the complete simplex constitutes an approximation of the equilibrium price vector.

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[15] See Scarf (1981) p. 1021.

The Scarf algorithm is a systematic search over the simplicial subdivision for the complete simplex. It is easy to demonstrate that, provided that certain initial conditions are satisfied, the Scarf search must terminate at a complete simplex in a finite number of iterations (see e.g. Scarf (1981) pp. 1019-1024).

The Scarf algorithm is easily extended to the n commodity case. It is applicable to continuous point-to-point mappings of the simplex into itself. Many general equilibrium models, however, yield point-to-set mappings, for instance because of an activity analysis production technology. In this case the simple scalar labelling illustrated above must be replaced by vector labels for each vertex and the mechanics of the Scarf algorithm become more complicated. But its basic ideas illustrated above are still valid.

It is thus possible to handle weak inequalities and complementary slackness with the Scarf algorithms and in principle they could be employed in the context of direct model integration. However, their computational inefficiency make them less attractive.

Optimization methods[16]

One of the fundamental results of competitive economic analysis is that an efficient allocation can, under proper conditions, be sustained as a competitive equilibrium. Conversely, the realized competitive equilibrium is also an efficient allocation. These results have a direct computational significance. It is possible to compute a competitive equilibrium by first computing the set of efficient allocations and then among these identify the equilibrium allocation and the associated

[16] I am here not concerned with solution methods for nonlinear equation systems based on optimization algorithms (see Ortega and Rheinboldt (1970) ch 8).

price system.[17] This is often relatively simple when there is, explicitly or implicitly, only one consumer in the equilibrium model. In partial equilibrium models this is often the case (i.e. there is a single market demand curve) and there is a voluminous literature on such models which are solved by maximizing the sum of consumers' and producers' surpluses.

With several consumers the computational problem is more complicated. Consider an economy with M consumers and N producers. Each consumer is represented by a utility function $u^i(x^i)$ (where x^i is consumer i's consumption vector), a vector v^i of initial endowments and a vector s^i of shares in the profits of the N producers. The producers are represented by their production possibility sets Y_j and y^j is the net output vector of the j:th producer. Efficient allocations can now be computed for this economy by solving the following mathematical programming problem for different values of the welfare weights α_i

$$\text{Max} \sum_{i=1}^{M} \alpha_i \, u^i(x^i)$$

subject to

$$\sum_{i=1}^{M} x^i - \sum_{j=1}^{N} y^j \leq \sum_{i=1}^{M} v^i \tag{9}$$

$$y^j \in Y_j \qquad \forall j$$

As usual the Kuhn-Tucker multipliers associated with the commodity balance constraints can be interpreted as prices. Their values are functions of the chosen welfare weights α. Denote them by $p(\alpha)$. For an arbitrary choice of α there is of course no reason to expect each consumer to satisfy his budget constraint, i.e. to expect that

[17] This is tantamount to solving the conditions in (1), where they now are interpreted as the first order conditions of the efficiency problem. The optimization approach is characterized by the application of general purpose optimization algorithms to the optimization problem which yields these first order conditions.

$$p(\alpha)^T \cdot x^i(\alpha) \leq p(\alpha)^T \cdot v^i + \sum_{j=1}^{N} s_j^i \, p(\alpha)^T \cdot y^j(\alpha) \quad \forall i \qquad (10)$$

where $x^i(\alpha)$ and $y^j(\alpha)$ are the solution to (9) given α. Only if $M = 1$ will the budget constraint (10) be automatically satisfied since it then is equivalent to the commodity balance constraint in (9). In this case the set of efficient allocations is equal to the set of competitive equilibrium allocations.

When $M > 1$ it can however be shown (Negishi (1960)) that there exists a set of welfare weights α such that the efficient solution of (9) also is a competitive equilibrium where the budget constraints in (10) are all satisfied. Ginsburgh and Waelbroeck (1980) discuss in detail various approaches to compute these weights, or equivalent parameters, within the computational framework given by (9) and (10). All these approaches involve computing a sequence of mathematical programming problems of a similar type as (9). Also Manne, Chao and Wilson (1980) and Chao, Kim and Manne (1982) propose related computational approaches involving a sequence of mathematical programming problems.

But the optimization approach is still most attractive in the case of a single consumer when it is possible to avoid computing a whole sequence of optimization problems. The main drawback is of course that the identification of efficient allocations with equilibrium allocations makes this approach less useful whenever the equilibrium model contains distortions, such as taxes, which destroy the efficiency properties of the competitive equilibrium. In Section 2.4 I will however discuss how a slight modification of the problem (9) allows the incorporation of certain types of commodity taxes and similar distortions.

To solve (9) one can apply nonlinear mathematical programming algorithms. They can of course also cope with linear technology constraints as in the sector model (2). Thus the optimization approach can, particularly for one consumer models, be used for model integrations where an activity analysis sector model is involved.

Sequences of linear complementarity problems

The equilibrium conditions given by (1) constitute a nonlinear complementarity problem. In general form the complementarity problem (CP) is[18]

$$\text{find } x \in R^n \text{ that solves } F(x) \geq 0,\ x \geq 0$$
$$\text{and } x^T F(x) = 0 \qquad \text{(CP)}$$

This mathematical problem is obviously relevant for instance in solving the first order conditions (i.e. the Kuhn-Tucker conditions) for a mathematical programming problem. It is a generalization of the linear complementarity problem given by

$$\text{find } x \in R^n \text{ that solves } q + Mx \geq 0,\ x \geq 0$$
$$\text{and } x^T[q+Mx] = 0 \qquad \text{(LCP)}$$

For the LCP numerical algorithms are available.[19] By linearizing F(x) in the CP they are also applicable to that problem. This is the basis for sequential LCP:s which have been suggested by Eaves (1978) and Robinson (1979) for computing stationary points of mathematical programming problems. This method has been applied by Mathiesen (1983) to computable general equilibrium models with very encouraging results.

[18] See Mathiesen (1983).

[19] See Cottle and Dantzig (1968).

Let $x^T \equiv (y^T, p^T)$. By making a first order Taylor expansion around $p = p'$, the equilibrium conditions in (1) can be expressed as the following linear complementarity problem

find $(y^T, p^T) \geq 0$ that solves

$$u \equiv \begin{pmatrix} 0 \\ v - (z(p')-Dz(p')p') \end{pmatrix} + \begin{bmatrix} 0 & -A^T \\ A & -Dz(p') \end{bmatrix} \begin{bmatrix} y \\ p \end{bmatrix} \geq 0 \qquad (11)$$

and

$$(y^T, p^T)\cdot u = 0$$

where $Dz(p')$ is the Jacobian of the excess demand functions evaluated at $p = p'$.

The idea of Mathiesen's method is to solve the approximation (11) of the original equilibrium model, using a LCP algorithm due to Lemke. The approximative solution for p is then substituted in the original excess demand functions and the approximation errors are evaluated. When they are sufficiently small according to some predefined norm, the solution to (11) is also close to the solution to (1).

There are two potential problems with Mathiesen's method. First, Lemke's algorithm is known to process the LCP (i.e. to compute a solution or show that none exists) only for certain types of matrices M. The matrix in (11) does not belong to this class of matrices other than perhaps in exceptional cases. Thus, there is no guarantee that the algorithm will find a solution to (11). Secondly, provided that the LCP algorithm finds a solution at each iteration, there is no guarantee that the sequence of LCP:s will converge to a solution of the nonlinear complementarity problem.

In spite of these potential problems, Mathiesen has applied his method to a variety of equilibrium models without running into these difficulties. Also in terms of computational efficiency his results seem very encouraging. For most of the equilibrium models Mathiesen tested, his sequential LCP method performed considerably better than these models' original solution methods which included Newton-methods, Scarf-algorithms and optimization codes.

It should be noted that near a solution to (1) the solution basis in (11) remains unchanged and only the values of the basic variables change. In this phase the sequential LCP method actually is equivalent to a Newton process.

Mathiesen's sequential LCP method is useful for equilibrium problems where it is important to account for weak inequalities and complementary slackness. In a sense it can be viewed as an extension of Newton methods to include these aspects.

Solution algorithms and model integration

Whether one computational approach is better than another depends on the particular model under consideration. In the case of model integration, where an activity analysis sectoral model shall be directly integrated with a general equilibrium model, it is necessary to use a computational approach which can handle weak inequalities and complementary slacks. This is possible with the Scarf methods, with the optimization approach and with the complementarity methods. Solution methods based on iterative methods for nonlinear equations are not useful in this respect. The Scarf methods are not very efficient and for this reason not very attractive in the case of model integration. In the choice between the optimization approach and the complementarity method the latter seems more promising since Mathiesen's results so far indicate that it is more efficient than at least some optimization codes. In the model application in Chapter 3 I have however adopted the optimization approach. The reason is simply that Mathiesen's work is quite recent (1983). At that time my work on model integration using the optimization approach was well on its way. The optimization approach also makes it possible to employ standard algorithms, while Mathiesen's algorithm is not yet widely available.

2.4 GENERAL EQUILIBRIUM MODELS, COMMODITY TAXES AND OPTIMIZATION METHODS

The discussion in the previous sections has, inter alia, suggested that optimization methods can be a useful computational approach to model integration. It is also the approach which I use in the model integration application in Chapter 3, where a sector model of electricity and heat production is integrated with a general equilibrium model. This integrated model is then employed in Chapters 4 and 5 to analyze the role of nuclear power on the Swedish electricity market.

The optimization approach to the computation of competitive equilibria rests on the equivalence between efficient and competitive equilibrium allocations. This equivalence can break down for several reasons, for instance if the model contains nonconvex production technologies or preferences. Another, and more important, case is when the model contains commodity taxes (and also distortions which could be expressed equivalently as commodity taxes). The fundamental equivalence theorems of competitive analysis are then in general not valid. This is a serious limitation of the optimization approach. It is important to be able to account for taxes and similar parameters in applied general equilibrium models even when they are not designed especially for tax studies. The model data bases are often calibrated so that the models can replicate a base year allocation based on actual national accounts, which contain taxes, trade margins, sectoral wage differences etc. To do this it is necessary to use tax parameters in the model. This is for instance the case for the general equilibrium model which is used in the model integration application in Chapter 3.

However, with unit taxes (and/or distortions which could formally be expressed as unit taxes) it is possible to compute the equilibrium at given taxes by a slight modification of the optimization approach. In its original form the optimization approach means that a standard Pareto problem is solved, i.e. a maximum of a welfare function is computed subject to a number of market equilibrium and production technology constraints. The modification which is required to handle commodity taxes is that an expression for total tax receipts should be subtracted from the objective function of the Pareto problem. Otherwise it should remain exactly the same as in the standard case without taxes.

The general result is proven and discussed below. It extends considerably the usefulness of the optimization approach. As an introduction, and to better understand the interpretation of the result, a simple two-good illustration is first considered.

A two-good illustration

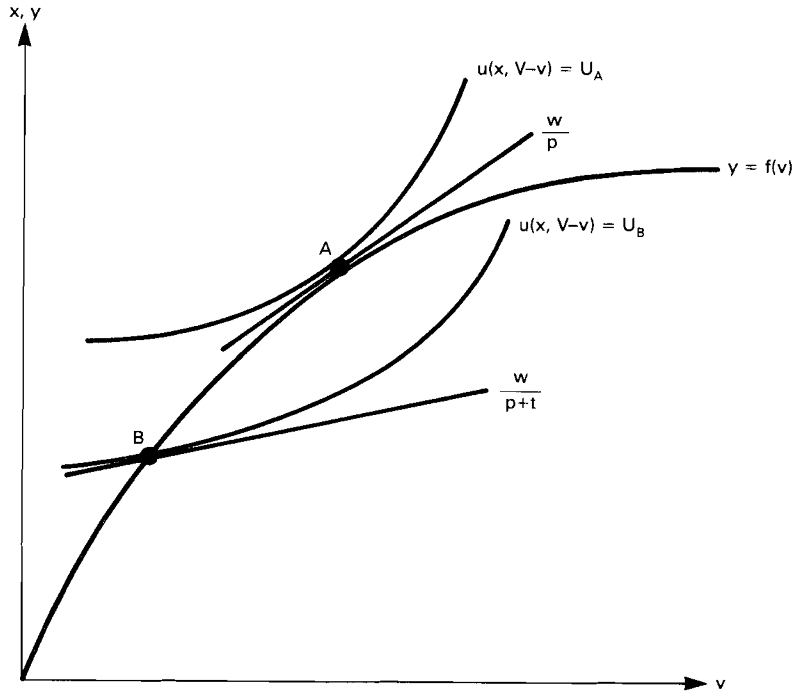
Consider a simple two-good economy where one good, denoted by y, is produced using the other good, denoted by v, as input. The production possibilities are given by the concave production function

$$y = f(v) \tag{12}$$

There is a single household, or consumer, represented by the utility function $u(x, V-v)$, where V is the household's endowment of the input good v and consequently $V-v$ is its net consumption (i.e. leisure if we think of v as labour). The consumption of the output good is denoted by x.

Without taxes the equilibrium allocation is at a point like A in Figure 2.2 (assuming an interior solution in the supply of the input good). The producers and the consumer face a common price system, denoted by p for the output good and w for the input good. As usual, the scale of the prices is indeterminate and the equilibrium relative price, w/p, is given by the slope of the tangent common to the production function and the indifference curve at A.

Figure 2.2 Equilibrium with and without a commodity tax



Let us now introduce commodity taxes. It can be done by unit taxes or by ad valorem taxes. Of course, for given prices the choice is arbitrary: a unit tax can always be reexpressed as an ad valorem tax and vice versa. However, when the model is used for experiments in comparative statics, at a given set of taxes, the choice does matter. With unit taxes, the tax income, in terms of some numeraire, per unit of the taxed good is kept constant during the experiment while the tax rate (i.e. the unit tax divided by the price of the taxed good) is endogenous. The converse is true for ad valorem taxes; the tax rate is kept constant while the unit tax is endogenous. In general, the result of the experiment is not independent of how taxes are represented.

The choice of how taxes should be represented should thus be guided by the structure of actual taxes in the economy which the model is intended to say something about. Actual taxes, however, are often a mixture of unit as well as ad valorem taxes. Moreover, applied general equilibrium models are by necessity based on aggregated national data and a limited number of model tax variables shall account for net tax flows caused by a large variety of actual taxes. Thus there is no evident reason to place one representation of taxes before the other. The modification of the optimization approach which is discussed here is however only possible for the case of unit taxes. For this reason the discussion in the following is based on this type of taxes.

Commodity taxes create wedges between consumer and producer prices. It is well known from the theory of taxation that the producer price system, on the one hand, and the consumer price system, including any lump-sum income such as distributed profits etc., on the other, may be independently normalized. Consequently, there is a degree of arbitrariness in the tax variables as long as they are not related to some numeraire. One common practice is to have an untaxed good, which serves as a common numeraire for both price systems.

Actually an untaxed good does not imply any loss of generality. The impact of commodity taxes on the equilibrium allocation is determined by the relative distortions they imply, i.e. the extent to which the ratio of the consumer relative price of any good to the producer relative price of the same good, deviates from unity. Since in an economy with n goods there are n-1 relative prices, n-1 tax variables are also sufficient to generate any set of relative tax distortions. The only case which cannot be treated when one good is untaxed is a uniform relative tax rate on all goods, which leaves the relative prices unaffected. However, as is also well-known from the theory of taxation, this is a quite uninteresting case.[20]

Returning to our two-good example, let the input good be the untaxed numeraire, i.e. $w = 1$, and let there be a specific tax $t > 0$ levied on the output good. The equilibrium, given the tax, is now at a point such as B in Figure 2.2.

The zero tax equilibrium A can be computed by the standard Pareto problem, i.e. by maximizing the utility function subject to the production constraint (12). But is it also possible to compute the tax equilibrium corresponding to point B in an analogous manner?[21]

Let us begin by examining the equilibrium given the tax t (expressed in terms of the input good). The expenditure function $e(p + t,w,u)$ gives the least lump-sum income (again expressed in the input good) required to achieve the utility u.[22] Let $\pi(p,w)$ be the maximum profits at the producer price p.[23] In equilibrium the utility level is determined by the equation

$$e(p^* + t,w,u^*) = \pi(p^*,w) + t \cdot x^* \tag{13}$$

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[20] Atkinson and Stiglitz (1980) provide a textbook treatment of the theory of taxation. On proportional taxes, see e.g. Sandmo (1976, pp. 44-45).

[21] Ginsburgh and Waelbroeck (1981) consider briefly, in the context of international general equilibrium models with tariffs, a similar approach to the one I discuss in the following.

[22] That is, the expenditure function is defined by

$$e(p+t,w,u) \equiv \min_{x,v} \{(p+t)x-wv: u(x, V-v) \geqq u\}.$$

For a textbook discussion of the expenditure function and its properties see Varian (1978) ch. 3. Dixit and Norman (1980) provide a thorough discussion of its usefulness in general equilibrium analysis.

[23] When there are constant returns to scale, the profit function is not well defined. In this case the equilibrium profits must be zero and only the expression for total tax receipts will enter the right-hand side of equation (13).

where the '*' indicates equilibrium values. Thus, it is assumed that tax receipts are redistributed to the consumer as a lump-sum transfer.[24] At given consumer prices the expenditure function is a utility function. It is sometimes referred to as the direct (income) compensation function $m(p^* + t, w; x, v) \equiv$ $\equiv e\left(p^* + t, w, u(x, V-v)\right)$.[25] It is easy to show that, among all feasible (x, v), i.e. those for which $x \leqq f(v)$, the tax equilibrium allocation (x^*, v^*) must maximize the direct compensation function <u>less the total tax receipts</u>. This is immediately evident from inspection of equation (13). The value of the direct compensation function less the total tax receipts equals the value of total profits. Since the latter have been maximized there cannot exist some other feasible allocation which yields higher profits and thus a larger value of the direct compensation function net of total tax receipts.

To show the result formally, suppose (x^*, v^*) does not maximize the direct compensation function net of tax receipts. Then there would exist some feasible allocation $(x', v') \neq (x^*, v^*)$ such that[26]

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[24] This assumption is also implicit in the illustration of the tax equilibrium at point B in Figure 2.2.

[25] See for instance Varian (1979).

[26] Since $w \equiv 1$, it is suppressed in the following in order to save notation.

$$m(p^* + t;\, x',\, v') - t\cdot x' = e\left(p^* + t,\, u(x',\, v')\right) - t\cdot x' =$$

(since the expenditure function is linearly homogeneous in prices)

$$= (p^* + t)\cdot e_q + e_w - t\cdot x' > (p^* + t)\cdot x^* - v^* - t\cdot x^* =$$

$$= m(p^* + t;\, x^*,\, v^*) - t\cdot x^{*}\text{[27]} \tag{14}$$

which implies

$$t\cdot(e_q - x') > p^*(x^* - e_q) - (v^* + e_w) \tag{15}$$

From the definition of the expenditure function we have

$$(p^* + t)\, x' - v' \geqq (p^* + t)e_q + e_w = e\left(p^* + t,\, u(x',\, v')\right) \quad (16$$

which implies

$$p^*\cdot(x' - e_q) - (v' + e_w) \geqq t(e_q - x') \tag{17}$$

Combining (15) and (17) we have

$$p^*\cdot(x' - e_q) - (v' + e_w) > p^*(x^* - e_q) - (v^* + e_w)$$

or

$$p^*\cdot x' - v' > p^*\cdot x^* - v^* \tag{18}$$

But (x^*, v^*) is the tax equilibrium allocation and thus maximizes profits, which contradicts the strong inequality in (18). Thus it can be concluded that (x^*, v^*) indeed must maximize the direct compensation function less the total tax receipts.

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[27] e_q and e_w denote the partial derivatives $\frac{\partial e(q,w,u)}{\partial q}$ and $\frac{\partial e(q,w,u)}{\partial w}$ respectively, where $q = p+t$ is the consumer price. The derivatives are evaluated at $q = p^*+t$, $w = 1$ and $u = u(x',v')$ in (14) - (17). The endowment term V is suppressed henceforth.

Thus, if we knew the equilibrium price we could compute the equilibrium allocation by maximizing the direct compensation function net of tax receipts.[28] This is hardly very helpful, however, since it requires information about parts of what we are trying to compute. But it suggests that the tax equilibrium may, in principle, be computed by maximizing some positive transformation of the utility function less an expression for the total tax receipts.

From the argument above we know that

$$e\left(p^* + t,\ u(x^*,\ v^*)\right) - tx^* \geqq e\left(p^* + t,\ u(x,\ v)\right) - t\cdot x \tag{19}$$

for all x, v such that $x \leqq f(v)$.

By expanding $e(p^* + t,\ u)$ around $u = u(x^*,\ v^*) \equiv u^*$, keeping the price system fixed, (19) can be reexpressed as

$$e\left(p^* + t,\ u(x^*,\ v^*)\right) - t\,x^* \geqq e(p^* + t,\ u^*) +$$

$$+\ e_u\cdot\left(u(x,\ v) - u^*\right) + R\left(u(x,\ v),\ u^*\right) - t\cdot x$$

or

$$e_u\cdot u(x^*,\ v^*) - tx^* \geqq e_u\cdot u(x,\ v) - tx + R\left(u(x,\ v),\ u^*\right) \tag{20}$$

where $e_u = \dfrac{\partial e(p^* + t,\ u^*)}{\partial u}$ and $R\left(u(x,\ v),\ u^*\right)$ is the second-order remainder term in the Taylor expansion.

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28 Obviously, if the equilibrium price were known, the most straightforward approach would be to compute the profit maximizing allocation directly. But for the sake of the argument we focus on the direct compensation function.

I will now assume that the utility function is concave. It means that the expenditure function is convex in u. Consequently the second-order remainder term is nonnegative and (20) then implies

$$e_u \cdot u(x^*, v^*) - tx^* \geqq e_u \cdot u(x, v) - tx$$

Thus (x^*, v^*) must be a global maximum for the following maximization problem:

$$\max_{x,v} e_u \cdot u(x,v) - tx \qquad \text{s.t.} \qquad x \leqq f(v) \tag{21}$$

A strictly concave utility function is sufficient to make the solution to (21) unique and in this case it must be (x^*, v^*), the tax equilibrium allocation. Thus the equilibrium allocation can be computed by solving (21), provided that we know the value of e_u, the inverse of the marginal utility of income, evaluated at the tax equilibrium. This variable is still an unknown part of what we want to compute, but the informational requirement has now been reduced from the whole vector of equilibrium prices required to compute the direct compensation function, to a single variable, e_u.

Suppose we are able to guess the correct value of the marginal utility of income. Solving (21) yields the equilibrium allocation (x^*, v^*). Moreover, from the objective function in (21) it is clear that the Kuhn-Tucker multiplier associated with the constraint is equal to the equilibrium producer price p^*.

But where would we end up if the guess on e_u is wrong? Let α denote the arbitrary guess on e_u. We thus solve the problem

$$\max \alpha u(x, v) - tx \quad \text{s.t.} \quad x - f(v) \leqq 0 \tag{22}$$

Denote the solution to (22) by $(\bar{x}, \bar{v})$, and the associated Kuhn-Tucker multiplier by $\bar{p}$. The first-order conditions are[29]

$$\alpha u_x(\bar{x}, \bar{v}) - t = \bar{p} \qquad (23\ a)$$

$$-\alpha u_v(\bar{x}, \bar{v}) = \bar{p}\ f_v(\bar{v}) \qquad (23\ b)$$

Define $\bar{w} = -\alpha u_v(\bar{x}, \bar{v})$. Clearly (23 b) is the profit maximization condition given the producer price system $(\bar{p}, \bar{w})$. Similarly, (23 a) and the definition of $\bar{w}$ correspond to the first-order conditions for utility maximization, given the consumer price system $(\bar{p} + t, \bar{w})$. The implied budget is $(\bar{p} + t)\bar{x} - \bar{w}\bar{v} = \pi(\bar{p}, \bar{w}) + t\bar{x}$ which is the actual budget at these prices. Consequently the allocation $(\bar{x}, \bar{v})$ is a tax equilibrium given the tax $t/\bar{w}$, expressed in terms of the numeraire good. The normalization of $\bar{p}$ and $\bar{w}$ in (23) is such as to make the marginal utility of income equal to $1/\alpha$, our arbitrary guess. To renormalize the price system in terms of the original numeraire, i.e. $w = 1$, all prices and the tax variable have to be divided by $\bar{w}$. The value of the marginal utility of income then becomes $\bar{w}/\alpha$. Only if $\bar{w}$ happens to equal unity is the solution $(\bar{x}, \bar{v})$ a tax equilibrium relative to the tax t (expressed in the numeraire good). In this case $1/\alpha$ also is the corresponding value of the marginal utility of income, so the initial guess must then have been correct.

To summarize: The solution to (22) is a tax equilibrium but the value of the tax is endogenous and depends on the chosen value of α. On the other hand, when we want to compute the tax equilibrium relative to a given tax value t, we know from (21) that there exists a value of α, namely $\alpha = e_u(p^* + t, u^*)$ such that the solution to (22) is the desired equilibrium. Thus by variation of the value of α in (22) it should be possible to compute the equilibrium allocation and price system given the unit tax t. The required value of α, the inverse of the marginal utility

[29] $u_x(x,v)$ and $u_v(x,v)$ are the partial derivatives $\frac{\partial u(x,v)}{x}$ and $\frac{\partial u(x,v)}{v}$ respectively.

of income, is a function of the prices and of the consumer's income. For the types of utility functions which are commonly used in applied models it is relatively easy to derive this function. The model user has in general also a rather good idea of the relevant range for the equilibrium prices and incomes. Thus it is not too difficult to make a good initial guess on α. The experience with the Cobb-Douglas utility function of the integrated model in Chapter 3 is that only a few variations of α (usually two or three) are required to find the correct value.

A more general result

In this section, the results illustrated above are extended to a more general environment. The assumption about a one-consumer economy is retained, however. When there are several consumers in the model economy, i.e. when the model shall account for distributional aspects, the optimization methods involve a sequence of optimization problems. The results for the one consumer case considered here can easily be extended to such a sequence.

Consider thus an economy with one consumer, N producers and a public sector. There are n private commodities. The following assumptions are made about the consumer:

C1: The preferences are represented by a continuous, strictly increasing and strictly concave utility function $u(x)$, where x is the consumption vector.

C2: $u(x)$ is defined on the closed and convex consumption set X.

C3: The consumer maximizes $u(x)$ subject to the consumption set and the budget constraint

$$q_c'x \leq q_c'v + \pi + T$$ [30]

[30] The prime indicates that u' is the transpose of the column vector u and thus u'v equals the inner product of the vectors u and v.

where q_c is the vector of consumer prices, v the vector of initial endowments, π the total distributed profits and T a net lump sum transfer from the public sector.

The producers are indexed by j. The production plan is the vector y_j where, as usual, positive elements are net outputs and negative elements are net inputs. The following assumptions are made:

P1: The production technology is given by the closed and convex production set Y_j.

P2: The producers maximize their profits $q_j'y_j$ subject to the production set. q_j is the vector of producer prices for producer j. Every producer is owned by the consumer, i.e. any profit income is distributed to the household.

The problem is to identify competitive equilibria for this economy for a given set of unit taxes. The exogenous taxes are defined by the vector t_c for the consumer and the vectors t_j, j=1,2,...,N, for the producers. The consumer and producer price vectors are related by the following requirement

$$q_c - t_c = p = q_j - t_j, \qquad j = 1,2,\dots,N \tag{24}$$

where p is the net price system in the economy. The values of the tax parameters are, to be meaningful, expressed in a common numeraire, which is taken to be

$$\sum_{i=1}^{n} p_i = 1.$$

In order to define a competitive equilibrium for this economy, given the tax structure, the use of the raised tax revenue must first be considered. The way the economy has been defined, neither corrective taxation nor redistribution of income are warranted. Thus, the only reason for taxes is to finance some form of public activity. Since only equilibria relative to an exogenous tax system are considered, the level of public activity should really adjust to whatever tax revenue is raised. Alternatively, the level of public activity could also be fixed exogenously, but then one must implicitly assume that any difference between the financial requirements of the public sector and the equilibrium tax revenue is covered by a lump-sum transfer from the household. To have both distortive commodity taxes and lump-sum transfers in the model may perhaps be disturbing to the theoretical purist. Nevertheless, this is a not unusual practice in computable general equilibrium models.[31] The tax variables are often quite crude representations of the actual tax system, since one is only interested in capturing certain aspects of it, such as energy taxes or tariffs. It seems reasonable in these cases to allow lump-sum transfers to cover any difference between the cost of public activities and the tax revenues and to interpret these lump-sum transfers as the net result of tax mechanisms which are not explicitly considered in the model.

The definition of an equilibrium introduced below will allow for both practices. In generalizing the two good example, I will however only consider the case with a fixed level of public activity.

Let g be a vector of public activities, such as production of public goods and services. For simplicity, it is assumed that the public sector does not produce any of the n private goods in the economy. But to supply its activities, the public sector requires private goods as inputs. Specifically, the feasible input vectors, for any given g, are given by the public input requirement set $Y_p(g)$. The following assumptions are made about the public sector:

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[31] This is for instance the case for the general equilibrium model used in the model integration application in Chapter 3. In applied general equilibrium models especially designed for tax studies, however, the public budget constraint plays an important role as an equilibrium condition excluding lump-sum financing.

G1: $Y_p(g)$ is closed and convex. To conform with the sign convention for the private net output vectors, the elements in the public input vectors are all nonpositive, i.e. $y_p \leq 0$.

G2: The public sector minimizes its cost $-p'y_p$ subject to the input requirement set $Y_p(g)$.

Since the taxes are exogenous, it is reasonable to treat the composition of the public activities in the same way. Therefore, let g^o be a vector which defines the exogenous mix of public activities. Their level is then determined by the scalar γ, i.e. $g = \gamma \cdot g^o$.

The consumer's budget constraint (see assumption C3) can be written

$$p'(x - \sum_j y_j - v) = t_c'(v - x) + \sum_j t_j'y_j + T \qquad (25)$$

where (24) and the definition $\pi \equiv \sum_j (p + t_j)'y_j$ have been used. The public budget constraint is

$$t_c'(v - x) + \sum_j t_j'y_j + T - p'y_p = 0 \qquad (26)$$

From (26) it is seen that $t_c'(x - v) - \sum_j t_j'y_j$ are the total tax revenues and they shall be equal to the cost of the public production plus the lump sum net transfer to the consumer.

Thus, positive elements in t_c imply that the corresponding consumption is taxed. Positive elements in t_j imply that the corresponding commodity is taxed if it is an input, while it is subsidized if it is an output.

Combining (25) and (26) yields

$$p'(x - y_p - \sum_j y_j - v) = 0 \qquad (27)$$

This is Walras' law in the present context. As a result, if, in equilibrium, the excess demand for some commodity, say i, is negative, the corresponding net price, p_i, must be zero.

A competitive equilibrium relative to a set of given taxes can now be defined as follows.

Definition 2.1. The allocation x^*, y_p^*, $\{y_j^*\}$,[32] the public activity level γ^*, the lump-sum transfer T^* and the net prices p^* constitute a competitive equilibrium relative to the taxes t_c, t_j, $j = 1,2,\ldots,N$ if the following conditions hold:

i) $$x^* - \sum_j y_j^* - y_p^* - v \leq 0 \tag{D2:1a}$$

$$p^{*\prime}[x^* - \sum_j y_j^* - y_p^* - v] = 0 \tag{D2:1b}$$

$$\sum_{i=1}^{n} p_i^* = 1 \quad \text{and} \quad p^* \geq 0 \tag{D2:1c}$$

ii) $$(p^* + t_j)'\cdot y_j^* \geq (p^* + t_j)'\cdot y_j \quad \text{for all } y_j \in Y_j,\; j = 1,2,\ldots,N \tag{D2:1d}$$

iii) $u(x^*) \geq u(x)$ for all $x \in X$ which satisfy the budget constraint

$$(p^* + t_c)'x \leq \sum_j (p^* + t_j)'y_j^* + (p^* + t_c)'v + T^* \tag{D2:1e}$$

iv) $$p^{*\prime}y_p^* \geq p^{*\prime}y_p \quad \text{for all } y_p \in Y_p(\gamma^*\cdot g^o) \tag{D2:1f}$$

$$p^{*\prime}y_p^* - \sum_j t_j' y_j^* - t_c^*(v - x^*) - T^* = 0 \tag{D2:1g}$$

where either γ^* is exogenous and T^* is defined by (D2:1g) or γ^* is endogenous and determined so that $T^* = 0$.

v) $x^* \in X$, $y_j^* \in Y_j$, $j = 1,2,\ldots,N$ and $y_p^* \in Y_p(\gamma^*\cdot g^o)$.

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[32] The notation $\{y_j\}$ is shorthand for the collection of vectors $y_1, y_2, \ldots, y_N$.

Definition 2.1 is a straightforward extension of the standard concept of a general competitive equilibrium. Condition (D2:1a) is the usual feasibility requirement and (D2:1b) implies that any good in excess supply has a zero net price. Condition (D2:1c) is imposed to have prices and tax parameters expressed in comparable units. Profit maximization by producers and utility maximization by the consumer are required by conditions (D2:1d-e).

Finally, (D2:1f) implies cost minimization in the public sector while (D2:1g) defines the public budget. The latter can be given two alternative interpretations: Either the equilibrium value of T^* is required to be zero (lump-sum transfers are not feasible) in which case γ^* must be endogenously determined so that (D2:1g) is satisfied for $T^* = 0$. Or γ^* is set exogenously in which case (D2:1g) is simply a definition of the required net lump sum transfer between the consumer and the public sector.

In the previous section I showed, with a simple example, that a competitive equilibrium relative a given tax maximized a certain monotone transformation of the utility function minus an expression for total tax receipts. This observation led to the formulation of the maximization problem (22) by which, given a proper choice of the coefficient α, the tax equilibrium could be computed.

The extension of (22) to the more general model presented above is given in the following definition.

<u>Definition 2.2.</u> The modified Pareto problem is[33]

$$\max_{x,\{y_j\},\,y_p} \quad \alpha u(x) + \sum_j t_j' y_j - t_c' x \qquad \text{(D2:2a)}$$

subject to

$$x - \sum_j y_j - y_p \leq v \qquad \text{(D2:2b)}$$

$$x \in X,\ y_j \in Y_j,\ j = 1,2,\ldots,N$$

$$\text{and } y_p \in Y_p(\gamma\cdot g^o) \text{ where } \gamma \text{ is fixed}$$

It is intuitively obvious that there can exist a solution to this problem only for some values of γ: the permissible values of γ are restrained by the economy's capacity to produce, which ultimately is determined by the endowment vector v. To ensure that the set of feasible allocations is not empty, the following restriction on the choice of γ is imposed:

R1: γ is such that there exist some $y_p^o \in Y(\gamma\cdot g^o)$ and some $y_j^o \in Y_j$, for $j = 1,2,\ldots,N$, for which $v + y_p^o + \sum_j y_j^o$ is in the interior of X.

When γ satisfies this restriction it is possible to find an $x^o \in X$ such that $x^o << v + y_p^o + \sum_j y_j^o$. It is clear that x^o, y_p^o and y_j^o for $j = 1,2,\ldots,N$ are a feasible allocation, so the feasible set cannot be empty.

The mathematical program in definition 2.2 is concave. When γ satisfies R1, the Slater constraint qualification is also satisfied, since $x^o - \sum_j y_j^o - y_p^o << v$. Thus, the Lagrange function for this problem is well defined and a solution to the modified Pareto problem also maximizes the associated Lagrangean.

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[33] The constant term $t_c'v$ is for simplicity dropped from the objective function.

The results illustrated by the simple example in the previous section are generalized in theorems 2.1 and 2.2.

<u>Theorem 2.1</u>. Let x^*, y_p^*, $\{y_j^*\}$, γ^*, T^* and $p^* \geq 0$ be a competitive equilibrium relative to the taxes t_c, t_j, $j = 1,2,\ldots,N$. Let α^* be the inverse of the value of the marginal utility of income at this equilibrium. Then x^*, y_p^* and $\{y_j^*\}$ are a solution to the modified Pareto problem with $\alpha = \alpha^*$ and p^* as the Kuhn-Tucker multipliers associated with the commodity balance constraints, and $\gamma = \gamma^*$.

<u>Proof:</u> x^*, y_p^*, $\{y_j^*\}$ and p^* are a solution if the following conditions hold:

i) $p^* \geq 0$

$$x^* - \sum_j y_j^* - y_p^* \leq v$$

$$p^{*\prime}[x^* - \sum_j y_j^* - y_p^* - v] = 0$$

ii) $\alpha^* u(x^*) + \sum_j t_j' y_j^* - t_c' x^* - p^{*\prime}[x^* - \sum_j y_j^* - y_p^* - v] \geq$

$$\geq \alpha^* u(x) + \sum_j t_j' y_j - t_c' x - p^{*\prime}[x - \sum_j y_j - y_p - v]$$

for all $x \in X$, $y_p \in Y_p(\gamma^* \cdot g^o)$ and $y_j \in Y_j$, $j = 1,2,\ldots,N$.

That condition i) holds follows directly from (D2:1a-b). Condition ii) holds if

a) $\alpha^* u(x^*) - (p^*+t_c)'x^* \geq \alpha^* u(x) - (p^*+t_c)'x$
for all $x \in X$

b) $p^{*\prime} \sum_j y_j^* + \sum_j t_j' y_j^* \geq p^{*\prime} \sum_j y_j + \sum_j t_j' y_j$

for all $y_j \in Y_j \quad j = 1,2,\ldots,N$

c) $p^{*\prime} y_p^* \geq p^{*\prime} y_p$

for all $y_p \in Y_p(\gamma^* \cdot g^o)$

The inequality a) follows from (D2:1e) (utility maximization) and the fact that α^* equals the inverse of the marginal utility of income. Similarly, (D2:1d) (profit maximization) implies the inequality b) and (D2:1f) (cost minimization in the public sector) implies inequality c).

Q.E.D.

Theorem 2.2. Let $\bar{x}$, $\bar{y}_p$, $\{\bar{y}_j\}$ be a solution to the modified Pareto problem and let $\bar{p}$ be the associated Kuhn-Tucker multipliers. Then $\bar{x}$, $\bar{y}_p$ and $\{\bar{y}_j\}$ with the net price system $\lambda\bar{p}$ constitute a competitive equilibrium relative to the taxes λt_c, λt_j; $j = 1,2,\ldots,N$, where $\lambda =$

$$= 1/\sum_{i=1}^{n} \bar{p}_i .$$

Proof: According to the Kuhn-Tucker theorem, the following relations hold

$$\bar{x} - \sum_j \bar{y}_j - \bar{y}_p \leq v$$

$$\bar{p}'(\bar{x} - \sum_j \bar{y}_j - \bar{y}_p - v) = 0$$

By the definition of λ, $\sum_{i=1}^{n} \lambda p_i = 1$. The two relations above are not affected by this scalar multiplication of $\bar{p}$. Thus conditions (D2:1a-c) are fulfilled.

The second of these relations can be rewritten as

$$(\bar{p}+t_c)'\bar{x} = (\bar{p}+t_c)'v + \sum_j(\bar{p}+t_j)'\bar{y}_j + \bar{p}'\bar{y}_p - t_c'(v-\bar{x}) -$$

$$- \sum_j t_j'\bar{y}_j = (\bar{p}+t_c)'v + \sum_j(\bar{p}+t_j)'\bar{y}_j + \bar{T} \qquad (28)$$

which can be interpreted as the consumers' budget constraint given that a lump-sum transfer $\bar{T}$ finances the net public budget deficit.

Furthermore, since $\bar{x}$, $\bar{y}_p$ and $\{\bar{y}_j\}$ maximize the Lagrangean, the following inequality holds:

$$\alpha u(\bar{x}) + \sum_j t_j'\bar{y}_j - t_c'\bar{x} - \bar{p}'(\bar{x} - \sum_j \bar{y}_j - \bar{y}_p - v) \geq$$

$$\alpha u(x) + \sum_j t_j'y_j - t_c'x - \bar{p}'(x - \sum_j y_j - y_p - v) \qquad (29)$$

for all $x \in X$, $Y_p \in Y_p(\gamma \cdot g^o)$ and $y_j \in Y_j$, $j = 1,2,\ldots,N$.

Let $x = \bar{x}$, $y_p = \bar{y}_p$ and $y_j = \bar{y}_j$ for all j except j = s. Then the inequality (29) implies

$$(\bar{p}+t_s)'\bar{y}_s \geq (\bar{p}+t_s)'y_s \quad \text{for all } y_s \in Y_s.$$

Since this is true for all s, (D2:1d) (profit maximization) is fulfilled.

Next, let $y_p = \bar{y}_p$ and $y_j = \bar{y}_j$, $j = 1,2,\ldots,N$. Then the inequality implies

$$\alpha\left(u(\bar{x}) - u(x)\right) \geq (\bar{p}+t_c)'(\bar{x}-x) \quad \text{for all } x \in X.$$

For all x which satisfy the budget constraint, the right-hand side is nonnegative so $u(\bar{x}) \geq u(x)$ since $\alpha > 0$. This proves that (D2:1e) (utility maximization) is fulfilled.

Finally, let $x = \bar{x}$ and $y_j = \bar{y}_j$, $j = 1,2,\ldots,N$. Then the inequality implies

$$\bar{p}'\bar{y}_p \geq \bar{p}'y_p \quad \text{for all } y_p \in Y_p(\gamma\cdot g^o)$$

so (D2:1f) (cost minimization in the public sector) is also fulfilled.

Thus it can be concluded that $\bar{x}$, $\bar{y}_p$, $\{\bar{y}_j\}$, the net price system $\bar{p}$ and the corresponding consumer prices $\bar{p} + t_c$ and producer prices $\bar{p} + t_j$, $j = 1,2,\ldots,N$ satisfy the conditions (D2:1d)-(D2:1f). It remains only to consider the normalization of prices and taxes.

Note first that if we maximize u(x) subject to the budget constraint $(\bar{p}+t_c)'x \leq \bar{R}$ where $\bar{R}$ is the value of the right-hand side in (28), then the marginal utility of income, i.e. the Kuhn-Tucker multiplier associated with the budget constraint, will be equal to $1/\alpha$. That is, the net price system $\bar{p}$, defined by the Kuhn-Tucker multipliers in the modified Pareto problem, is normalized so that the implied marginal utility of income is $1/\alpha$. To obtain the normalization

$$\sum_{i=1}^{n} p_i = 1,$$

the net price system $\bar{p}$ and all tax variables have to be multiplied by λ.[34]

To summarize: the allocation $\bar{x}$, $\bar{y}_p$, $\{\bar{y}_j\}$ and the net price system $\lambda\bar{p}$ constitute a competitive equilibrium relative to the set of taxes λt_c and λt_j, $j = 1,2,\ldots,N$, where the taxes are expressed in the numeraire

$$\lambda \sum_{i=1}^{n} \bar{p}_i = 1.$$

Q.E.D.

The level of public activity γ, is kept constant in Theorem 2.2. Consequently, the public budget constraint is closed by implicit lump-sum taxes. It may be possible to find a value of γ such that the implicit lump-sum taxes are zero, by successively solving the modified Pareto problem for different values of γ. As always with the optimization approach, a sequence of Pareto problems is required to be solved when the general equilibrium model contains more than one budget constraint.

Theorem 2.2 says that a solution to the modified Pareto problem is a competitive tax equilibrium. But the scale of the tax parameters cannot be exogenously controlled, i.e. the taxes are not t_c and t_j, $j = 1,2,\ldots,N$ but λt_c and λt_j, $j = 1,2,\ldots,N$ where the value of λ depends on the value assigned to α. On the other hand, theorem 2.1 says that if a competitive equilibrium relative to the tax system t_c, $\{t_j\}$ exists, then there is a value of α such that the equilibrium allocation and the net price system solve the modified Pareto problem. By varying the value of α it should then be possible to find the competitive equilibrium which corresponds to a given tax system, provided that such an equilibrium exists. These results extend the range of applicability of the optimization approach in computations of general equilibria.

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[34] This is equivalent to a multiplication of the objective function (D2:2a) by λ. The solution of the modified Pareto problem, apart from the scale of the Kuhn-Tucker multipliers, is of course invariant to such a multiplication.

3 Model Integration: An Application of the Optimization Approach

The main purpose of this chapter is to apply the optimization approach to model integration, which I discussed and developed in Chapter 2. In particular, the purpose is to integrate a detailed sector model of electricity and heat production with a general equilibrium model. This is done in order to create a framework which is suitable and useful for analyses of issues related to the Swedish electricity and heat markets, and in particular to the role of nuclear power.

Since the first nuclear power reactor began to operate in 1972 there has been a substantial increase of the nuclear power capacity. Today it accounts for almost half of the current electricity production. The large and rapid build-up of the nuclear power capacity stirred, however, a lot of controversy about the safety and environmental hazards of nuclear power. As a result the earlier promotion of nuclear power has now been replaced by a nuclear power discontinuation policy: the existing nuclear power plants shall be used for their remaining economic life time - but not any longer than 2010 - and new investments in nuclear power are not allowed.

Two issues which are related to this development are investigated later on in this study. One is whether the nuclear power investments have led to an overcapacity in the electricity production system. If this should be the case, what can then be said about its size and likely duration and about its consequences for the electricity market and the rest of the economy? Such an investigation is of interest both in order to learn from the past, i.e. in order to understand the causes of the overcapacity, as well as to get a better understanding of the likely development of the markets for electricity and heat over the coming years.

The second issue is the consequences of the nuclear power discontinuation for the electricity market and the rest of the economy. This is of course of interest as part of a general policy evaluation, especially since the nuclear power discontinuation probably shall not commence until the middle of the 1990's. Thus, it has not yet led to any real and binding commitments.

Now, to study these issues it is necessary to be able to analyse how different restrictions on the use of nuclear power affect the cost and supply of electricity. It shall be possible to identify the most competitive alternative to nuclear power and the main factors which determine this alternative. For these reasons we need a model where the choice of different electricity production technologies is treated in some detail. The sector model of electricity and heat production satisfies this requirement. It also includes the production of heat which is an advantage since it is closely related to the production of electricity. Electricity and heat can for example be jointly produced in combined power and heat plants and heat may be produced from electricity, e.g. in heat pumps or electric boilers.

Through the model integration the electricity and heat sector model is embedded in a general equilibrium structure. The basic reason for this is the standard one for general equilibrium analysis. The prices and quantities established on one market in the economy are in general not independent of the prices and quantities established on the remaining markets. For this reason the prices and quantities on all markets should be simultaneously determined in a general equilibrium. In the present context it means that the prices and quantities of electricity and heat should be determined simultaneously with the prices and quantities on all other markets in the economy. And this is precisely what is achieved through the model integration. With the integrated model the use of various production processes within the electricity and heat sector can be determined simultaneously and consistently with the allocation of resources in the rest of the economy.

Although it is in principle necessary to determine the equilibria for all markets simultaneously it is only important to do so in an applied analysis if the mutual interdependencies between different markets are strong enough to significantly affect the outcome of the analysis.

In the case of electricity it is often claimed that at least the links from the electricity sector to the rest of the economy are strong and thus important to capture. Electricity is for instance an intermediate input in most other sectors. It is thus claimed that changes in the price of electricity will significantly affect the cost of production in these sectors and consequently their international competitiveness. It is also claimed that the cost of electricity is important enough to affect the rewards which the firms can afford to pay for other factors of production, for instance labor. The price of electricity is also claimed to be important enough to influence the overall performance of the economy as measured by the GNP or the disposable income of the households.

If it is true that there are strong links from the electricity market to the rest of the economy it is also reasonable to expect non-negligible feedback effects from the rest of the economy to the electricity sector. The electricity sector uses various intermediate inputs and primary factors of production. If their prices change they affect the cost of the electricity supply, i.e. the supply schedule shifts. Similarly when the prices of other commodities than electricity change, and perhaps also the total income in the economy, the electricity demand schedule will in general also shift. These shifts will of course affect the final outcome on the electricity market.

It is the presumed existence of these mutual interconnections which motivates the general equilibrium approach, and thus the model integration in the present context. A detailed sector model is required in order to capture the effects of various restrictions on the use of certain production technologies such as nuclear power. And the sector model should be integrated with a general equilibrium model because it seems important to determine the production of electricity simultaneously with the allocation of resources in the rest of the economy.

As the two main building blocks for the integrated model, two existing models have been chosen. The first is one of the model versions in the system of computable general equilibrium models for a small, open economy developed by Bergman and Por (see Bergman and Por (forthcoming) and Bergman (1982)). The second is a model of the electricity and heat sector, developed by Bergman and Carlsson (1984). The models are applied to data for the Swedish economy and for the Swedish electricity and heat sector respectively.

There are several reasons why these two models have been chosen. One is that the present work about model integration arose in the context of these and related models. This study is a part of a broader research programme at the Stockholm School of Economics concerned with the development of quantitative models for analyses of energy markets, energy-economy interactions and energy policy. The idea of model integration arose as a way to join the information spread among different models by merging some of them to a common model structure which could be used in certain applications. Thus, it was quite natural to turn to these familiar models in the first place.

These two models have also been used for similar purposes as the integrated model. By using them as the main building blocks for the integrated model the results of applications of the latter can be compared with the results of applications of the original models. In this way it becomes possible to evaluate the gains from the integration of the models. For this reason the two original models are incorporated in the integrated model without any essential modifications.

By using already existing models, the model development effort for the integrated model is also greatly reduced. These two existing models are also well suited for the present purpose. The electricity and heat model contains sufficiently detailed information about different electricity and heat production technologies. The general equilibrium model is especially designed for studies of the relations between the energy sector and the rest of the economy. Both models are also reasonably consistent in terms of time perspective and the treatment of sector capital stocks.

There is, however, also a cost associated with the use of already existing and not modified models. They have been developed independently of each other without consideration of the particular requirements of model integration. For the models used here it is primarily the commodity classification and the treatment of investments which are hard to reconcile within an integrated structure. The way these aspects actually have been reconciled shall be regarded as a first step only. There is a scope for improvements. In particular it seems as if the optimization approach in itself can make it possible to improve the treatment of investments in the general equilibrium model more easily than its original solution algorithm would allow.

The integrated model is used in Chapter 4 to assess the consequences of the present Swedish nuclear power policy. In Chapter 5 it is used to evaluate the large nuclear power investment programme carried out during the 1970's and the first half of the 1980's.

A second purpose of this chapter is to provide the necessary model background for these applications. Thus, the various parts of the integrated model are also presented and discussed here.

The model of electricity and heat production is an activity analysis model and thus formulated as a mathematical programming problem. The general equilibrium model, however, essentially consists of a set of price-dependent excess demand equations for goods and factors of production. An important part of this chapter is to show how this general equilibrium model can be equivalently formulated as the Pareto-type of mathematical programming problem which is required for the optimization approach.

In Section 3.1 the general equilibrium model is presented and it is in particular shown how it can be stated as such a mathematical programming problem. The sectoral model of electricity and heat production is briefly presented and discussed in Section 3.2. The integration of the two models is discussed in Section 3.3 and Section 3.4, finally, contains a discussion of the computer implementation and the data base for the integrated model.

3.1 THE GENERAL EQUILIBRIUM MODEL

In this section the structure and the properties of the general equilibrium model used for the model integration are discussed in order to provide the necessary background both for the model integration in Section 3.3 and for the applications in subsequent chapters. The presentation emphasizes how this model can be formulated as a Pareto-type mathematical programming problem.

The system of computable general equilibrium models constructed by Bergman and Por is a set of resource allocation models especially designed for analysis of problems related to national energy policies in a small open economy. The latter concept refers to an economy with a large trade-exposed sector and with a limited influence on its terms of trade. The models are designed for energy policy analysis in the sense that two energy categories, electricity and fuels, are explicitly distinguished and energy input coefficients are endogenously determined (in contrast to other intermediate goods whose input coefficients are exogenous).

The models have been constructed in the spirit of Walrasian general equilibrium theory. Thus, they are exclusively concerned with the real side of the economy. Quantity variables like the production in various sectors, the allocation of labor and capital, the exports and imports of different goods and the demand for private consumption are functions of the relative prices of goods and factors of production. These prices are determined so as to equate supply and demand in all markets and the supply and demand relations are explicitly, derived from the representative producers' and consumers' optimization behaviour.[1] Several important variables are, however, exogenous. This is the case for public expenditures and for the supply of labor and capital. In one version net investments are exogenous, while the gross savings ratio is exogenous in other versions.

The model version which is merged with the electricity and heat sector model is a medium term model characterized by sector specific capital stocks and by a vintage structure for existing capital.[2] In some contexts this particular model is referred to as the ELIAS model (an abbreviation of the keywords Energy, Labor, Investment; Allocation and Substitution). I will adhere to this name convention here, since it is extremely convenient to use a brief label when refering to the model.

The ELIAS model is based on a distinction between the ex ante production function and the ex post production function. The ex ante production function represents the technological constraints which apply when new production units are designed. Once a production unit has been designed and built, certain technological decisions are irreversible. Thus, ex post the production opportunities, represented by the ex post production function, are more limited than those ex ante.

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[1] In some versions it is possible to keep the wage rate exogenous and instead let the excess supply of labor be endogenous.

[2] In Bergman's (1982) presentation of the models it is referred to as the "dynamic model", presumably because the sectoral distribution of capital can only be gradually changed over time through net investments in different sectors. Thus, the model exhibits gradual adjustments to external changes. It is referred to as a medium term model because the sectoral allocation of capital does in general not correspond to a long run equilibrium. On the other hand it is not a short run model because it assumes sufficient price flexibility to clear all markets.

As a result there will at a given point in time exist a set of capital stocks of different sizes, each characterized by the decisions concerning how much to invest, and the technological design of that investment, at earlier points in time. In this way the capital stock will have a vintage structure.

The capital is furthermore assumed to be sector specific. Once capital goods have been allocated to one of the model sectors they cannot be reallocated to another. The sectoral composition of the total capital stock in the economy can thus only change gradually over time through the flow of new investments directed to different sectors and through the rates of depreciation which may differ between the sectors.

Although this model usually is used to simulate a development over time for the model economy, it is not an intertemporal general equilibrium model. It is a one period model where the producers decide on current production and consumers decide on current consumption only. The only variables with intertemporal implications are total savings and investments, but they are essentially exogenous. In the original version of ELIAS, there is a constant savings ratio and the investments are equal to domestic savings minus the value of the net current account surplus. In the version used for the model integration in Section 3.3, the total investments are instead exogenous while the total savings will adjust to a level consistent with the exogenous investments.

The development over time is simulated by a sequence of one period equilibria. They are linked by allocating the total investments between the different sectors, which yields a new set of vintage sectoral capital stocks. Given these, and with all exogenous variables updated, a new one period equilibrium is established. Again the investments are allocated between sectors, exogenous variables are updated and yet another one period equilibrium is established. In this way the model economy follows a development determined by a sequence of one period equilibria.

The remaining part of this section is concerned with a rather detailed presentation and discussion of the production technology and producer behaviour, the treatment of foreign trade, the representation of the aggregated household sector and the allocation of sectoral investments in ELIAS.

The production technology and producer behaviour

There is no joint production and thus there is a one-to-one correspondence between the classification of goods and production sectors. The production sectors are indexed by j = 1, 2, ... n. Of these m produce tradeable goods. Because public sector production is a nontradeable output, there is at least one nontradeable sector and m is strictly less than n.[3] The public sector is given the index $j = n$ and the two energy sectors, representing production of electricity and of petroleum products, are given the indices $j = 1$ and $j = 2$, respectively. There is also a book-keeping sector, $j = n+1$, where different goods are aggregated into one single capital good.

The ex ante technology exhibits constant returns to scale, and in each sector, capital, labor, fuels and electricity are substitutable factors of production. The use of produced nonenergy inputs is determined by fixed input-output coefficients.

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[3] The indexation used here does not correspond exactly to that in the original presentations of the ELIAS model. In the latter there are at most two nontraded sectors. Also the index ordering of the two energy-sectors is reversed here, for convenience in the presentation of the integrated version in section 3.3.

The ex post production function for each sector is derived from the corresponding ex ante production function by the following two assumptions: First, once capital is invested in a sector it cannot be reallocated to some other sector. Secondly, once a new production unit has been built the energy input coefficients are no longer variable. Thus, ex post the use of energy is determined in the same way as for produced nonenergy inputs. Since capital is a fixed factor ex post, the ex post production function exhibits decreasing returns to scale.

Since energy input coefficients associated with capital invested in different time periods may not be the same, a vintage structure for capital should be distinguished. Also the productivity of labor employed with capital of different vintages will in general vary, making it necessary to use a vintage structure. The differences in labor productivity are partly due to technical progress, which shift the ex ante production function over time. They are also due to the differences in energy input coefficients. Since there is some degree of substitutability between labor and energy, the productivity of labor is not independent of the values of the energy input coefficients.

At any point in time then, there is a history of investment and associated technology decisions in the form of production units of different vintages in each sector. In each vintage the production opportunities are given by the ex post production function. Let $I_j(v,t,\delta_j)$ denote the capital invested in sector j in period v which remains in period t+1. This is obviously a function of the rate of depreciation in sector j, δ_j. Production in vintage v in sector j can now be written

$$X_{vj} = f_{vj}(L_{vj}, I_j(v,t,\delta_j)) \quad v=0,\ 1,\ \ldots t;\ j=1,\ 2,\ \ldots\ n \tag{1}$$

where X_{vj} is the output from vintage v in sector j, L_{vj} is the use of labor in vintage v in sector j and $f_{vj}(\cdot)$ is the ex post production function for vintage v in sector j. The production functions are continuous and concave.

Total output in sector j, X_j, then is

$$X_j = \sum_{v=0}^{t} X_{vj} \qquad j=1, 2, \ldots, n \tag{2}$$

The output of sector j is used for domestic purposes (i.e. private consumption, investments and as an intermediate input) and it is exported provided it is a traded good.[4] If the sector output was interpreted literally as a single good and if for tradeables, domestically produced goods were considered identical to those produced in the rest of the world, the law of one price should apply. Moreover, as the model describes a small open economy, the prices of tradeables should be world market determined. Then a traded good would be either exported or imported depending on whether its excess supply were positive or negative at the given world market prices.[5]

The model is, however, applied to aggregated data, where goods with the same commodity classification are both exported as well as imported. To allow for such intra-industry trade the model explicitly makes a distinction between exports, imports, domestic supply and domestic use of a good with a given sectoral classification. The domestic production of traded commodities is considered to consist of aggregates of goods for the domestic market and goods for the export market, all with the same aggregate sector classification. The distribution of the total production between the

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[4] Traded goods are given the first m indices, i.e. j=1, 2, ..., m, whereas nontraded goods are indexed as j=m+1, ..., n.

[5] Since, ex post, there are diminishing returns to scale in each production sector, there would in general be a positive domestic production of all traded goods.

two markets depends on the home market price relative to the world market price, and there is assumed to be a diminishing marginal rate of transformation in reallocating the aggregate output between the two markets. Formally this is accomplished by introducing a set of sectoral transformation functions

$$h_j(N_j^S, Z_j) - X_j \leqq 0 \qquad j=1, 2, \ldots, m \qquad (3)$$

where N_j^S is the supply to the domestic market of commodity j, Z_j is the export of commodity j and $h_j(\cdot)$ is a convex, linearly homogenous function. With a proper parameterization of (3), exports of each traded good can be allowed for in the model.

There is also a distinction between the use of domestically produced goods and of imports. Domestic output and imports with the same sectoral classification are assumed to be imperfect substitutes for domestic users. For simplicity the preferences for domestic goods and imported goods are assumed to be the same for all users independent of whether they are consumers or producers. The preferences are represented by the functions $g_i(N_i^D, M_i)$ $i=1, 2, \ldots, m$, where N_i^D is the domestic demand for domestic production of commodity i, M_i is the imports of commodity i and $g_i(\cdot)$ is a concave, linearly homogenous function. Again with a proper parameterization of the functions g_i, $i=1, 2, \ldots, m$, imports of each traded good can be allowed for. In this way, intra-industry trade can occur.

Intra-industry trade is thus incorporated in ELIAS by introducing a certain element of product differentiation. Imported goods and domestically produced goods supplied to the domestic market, and with a common commodity classification, are treated as similar, but different commodities. This has been a fairly common approach to foreign trade in computable general equilibrium models of open economies.[6] It is often referred to as the Armington assumption since the hypothesis was originally put forward in Armington (1969) as an explanation for intra-industry trade. In models with constant returns to scale, and more commodities than factors of production, the Armington assumption has the further advantage of ruling out overspecialisation, i.e. non-zero domestic production only foras many commodities as there are factors, something which in general is hard to reconcile with actual production patterns.

If the Armington assumption is applied not only to the home country, but also to the rest of the world, it implies the existence of export demand functions for the home country's domestic production. The export demand would be a function of the domestic commodity price relative to the corresponding world market price for each traded commodity. This appraoch is, however, not consistent with the small country assumption for the home country. The export prices received by domestic producers would depend on the quantities exported. Consequently the home country's terms of trade are endogenous and not exogenous. In order to avoid this the transformation functions (3) are introduced instead. They imply a set of export <u>supply</u> functions since the domestic producers decide on how much to produce for the domestic market and how much to export. In these supply functions the export prices received by domestic producers are exogenous and consequently so are also the terms of trade.

Product differentiation may be an important phenomenon in many instances and it is probably a crucial determinant for intra-industry trade. But then it seems unduly restrictive to only consider differentiation between domestic and foreign

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6 See, for instance, Dervis, de Melo and Robinson (1982 ch. 7) and Dixon et al. (1982).

producers as in the Armington hypothesis. If domestic producers are able to differentiate their products from foreign ones, they should also be able to differentiate them between each other. Thus, it would be motivated to allow product differentiation between all producers, domestic as well as foreign. But then one would also have to leave the competitive framework and enter a world of imperfect competition.

If one wants to stick with the competitive model it does not seem very meaningful to interpret the Armington hypothesis literally. Intra-industry trade is not necessarily incompatible with a competitive framework, since it can also be explained as something which results from studying aggregates of commodities rather than single commodities. The Armington hypothesis could then be viewed as a convenient way to account for the behaviour of such commodity aggregates, with which an applied general equilibrium model by necessity must be concerned.

In the model all users of commodity i always pay the same relative price for domestic goods and imports. Consequently, since the $g(\cdot)$-functions are linearly homogenous, they will all choose the same cost-minimizing "import-intensity" M_i/N_i^D. This fact allows the following convenient "as if"-interpretation: The $g(\cdot)$-functions define m composite goods available for domestic use. The composite goods are "produced" by m dummy sectors so that the cost of each of them is minimized, i.e. the dummy sectors solve the following problem

$$\min_{N_i^D,\, M_i} \quad P_i N_i^D + P_i^{WI} M_i$$

subject to $\qquad i=1, 2, \ldots, m \qquad (4)$

$$g_i(N_i^D, M_i) \geqq D_i$$

$$N_i^D \geqq 0, \; M_i \geqq 0$$

where D_i is the total domestic demand for the composite good i, P_i is the price of the domestic supply of commodity i and P_i^{WI} is the import price for imports of commodity i.

The relations (1), (2) and (3) together with a set of fixed coefficients for produced intermediate goods, complete the representation of the model economy's production technology. The intermediate inputs consist of the composite goods defined by the $g_i(\cdot)$-functions and whose prices are given by the Kuhn-Tucker multiplier associated with the constraint in (4). Given product and factor prices, producers are assumed to maximize profits subject to the technology constraints.

Thus, for sector j producing a traded good the supply of output and the demand for inputs are determined by the solution to the profit maximization problem (5).[7]

$$\max \Pi_j \equiv (P_j-\Theta_j)\cdot N_j^S + P_j^{WE}\cdot Z_j - \sum_{i=1}^{2}\sum_{v=0}^{t} P_{ij}a_{vij}X_{vj} -$$

$$-\left[\sum_{i=3}^{m} P_i^D a_{ij} + \sum_{i=m+1}^{n-1} P_i a_{ij}\right]X_j - W_j \sum_{v=0}^{t} L_{vj} \qquad (5a)$$

7 For the energy sectors, j=1, 2, one more term, representing complementary imports, should be added to the profit expression (5a). This term can be written $-P_j^C b_j X_j$ for j=1, 2, where b_j is the use of complementary imports per unit produced in sector j and P_j^C is the price of complementary imports to sector j. Note also that the output of the public sector is not used as an intermediate input anywhere, i.e. $a_{ij}\equiv 0$ for i=n and all j.

subject to $\qquad j=1, 2, \ldots, m \qquad (5)$

$$X_{vj} - f_{vj}(L_{vj}, I_j(v,t,\delta_j)) \leqq 0 \qquad (5b)$$

$$X_j - \sum_{v=0}^{t} X_{vj} \leqq 0 \qquad (5c)$$

$$h_j(N_j^S, Z_j) - X_j \leqq 0 \qquad (5d)$$

$$N_j^S \geqq 0,\ Z_j \geqq 0,\ X_j \geqq 0,\ L_{vj} \geqq 0,\ X_{vj} \geqq 0, \quad \forall\ j, v$$

where P_j is the domestic market price of good $j=1,2, \ldots, n$, P_j^{WE} the export price of good $j=1, 2, \ldots, m$, P_i^D the market price of the composite goods $i=1, 2, \ldots, m$, P_{ij} the price paid by sector j for the composite good $i=1, 2$ and W_j is the wage paid by sector j.[8] The input-output coefficients a_{ij}, are the use of commodity i for the production of one unit of good j. For the energy goods (i.e. $i=1, 2$) they are vintage dependent.

For the nontraded sectors (i.e. $m<j\leqq n$) the profit maximization problem (5) has to be modified. The export output term in the profit expression (5a) should be deleted and the transformation constraint (5d) should be replaced by

$$N_j^S - X_j \leqq 0 \qquad (5e)$$

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[8] Θ_j is a tax parameter which creates a wedge between the market price P_j and the price received by the producers, $P_j - \Theta_j$. The purchasing prices, P_{ij}, for the energy commodities $i=1, 2$ may also differ from the corresponding market prices P_i^D, because of taxes on energy use. Taxes in the model are further discussed below.

Preferences and household behaviour

All households are represented by an aggregated household sector. The labor supply is exogenous so the household sector is completely represented by a system of demand equations for private consumption. They are derived from utility maximization subject to a budget constraint. The utility function

$$U(Q_1, Q_2, \ldots\ldots, Q_S) \tag{6}$$

is defined for a set of specific consumption goods and Q_s is the private consumption of consumption good $s=1, 2, \ldots, S$. The utility function is assumed to be continuous and strictly concave.

The consumption goods are defined as linear combinations of the sector outputs. Consequently, the private consumption demand for sector output j, denoted by C_j, is given by

$$C_j = \sum_{s=1}^{S} t_{js} Q_s \qquad j=1, 2, \ldots, n-1 \tag{7}$$

where t_{js} is the amount of commodity j used in one unit of the private consumption good s.[9]

In the same way as domestically produced goods and imported ones, with the same sectoral classification, are imperfect substitutes as intermediate inputs, they are also imperfect substitutes in private consumption. Again the $g_j(\cdot)$-functions, which are assumed to be the same for all domestic users, represent how domestic outputs and imports can be "transformed" into domestic use, in this case private consumption. Consequently the private consumption demands for tradeables relate

9 The output of the public sector is not an ingredient in any of the consumption goods. Consequently, $t_{ns}=0$ for all s and $C_n\equiv 0$.

to the composite goods available at prices P_j^D, $j=1, 2, \ldots, m$.[10] The demand functions are determined by the solution to the utility maximization problem (8).

$$\max_{Q_s} U(Q_1, Q_2, \ldots, Q_S)$$

subject to (8)

$$\sum_{j=1}^{2} P_{jc} \sum_{s=1}^{S} t_{js} Q_s + \sum_{j=3}^{m} P_j^D \sum_{s=1}^{S} t_{js} Q_s + \sum_{j=m+1}^{n-1} P_j \sum_{s=1}^{S} t_{js} Q_s \leqq Y$$

$$Q_s \geqq 0 \quad \forall\ s$$

where Y is the disposable income of the household sector. The disposable income is the amount the household sector can spend on private consumption. In equilibrium it is equal to the total factor income, i.e. the sum of labor income and total profits, minus the value of gross domestic investments, the net surplus of the current account (i.e. foreign investments) and the value of public production. This means that the public sector is financed, at least partially, with lump sum taxes. Although the ELIAS model contains a set of tax variables (further discussed below), they are all exogenous and so is also the public sector production. There is no public sector budget constraint, so the net public deficit, determined once the equilibrium allocation and prices are known, must implicitly be financed by lump sum taxes.

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10 For the energy commodities, $j=1, 2$, the price paid by the households is P_{jc}, which may deviate from P_j^D due to taxes levied on the household's energy use.

Taxes and similar distortions

In Chapter 2 it is shown that an equilibrium in an economy distorted by unit commodity taxes can be computed by a modification of the standard Pareto problem. From the original objective function an expression for the total tax receipts shall be subtracted. This result provides the basis for the integration of the ELIAS model and the electricity and heat supply model. For this reason the commodity taxes and similar distortions in the ELIAS model shall be briefly discussed.

There are two types of taxes, a set of indirect commodity taxes and a set of taxes related to energy use. The indirect commodity taxes denoted by Θ_j, j=1, 2, ..., n, create a wedge between the producer price and the price paid by domestic users. The energy taxes relate to the use of electricity and fuels and are of two types. First, there are two general taxes on electricity and fuel use, denoted by τ_1 and τ_2, respectively. Secondly, there are also sector specific taxes on energy use, so that different sectors, including the aggregate household sector, may pay different prices for electricity and/or fuels. These sector specific taxes are denoted by ξ_{1j} and ξ_{2j}, j=1, 2, ..., n, c, for electricity and fuels, respectively.[11]

There is also a set of import duties and indirect taxes levied on imports. They are denoted by the parameters ϕ_i, i=1, 2, ..., m.

Beside these taxes, there is a similar "distortion" in the form of an exogenous wage structure, which is used to roughly account for the actual sectoral differences in labor costs which are registered in the national accounts. These diffe-

[11] The index j=c represents energy taxes levied on the household sector's direct use of electricity and fuels.

rences may be due to labor heterogeneity, temporary disequilibrium phenomena on the labor market etc. The wage paid by sector j is $W_j = \omega_j W$, where W is a general wage index and the ω_j's the parameters of the wage structure. Obviously, these parameters work exactly the same way as a set of sector specific labor taxes would. It will in the following be convenient to treat the wage structure as just another set of taxes.

In the ELIAS model all taxes, and the wage structure, are expressed ad valorem. In the optimization version presented here they are unit taxes, that is expressed in terms of some numeraire. The model taxes do not replicate any actual tax rates, but are simply devices to account for tax flows in the national accounts, which constitute the major data base for the model. These tax flows are the result of many different types of taxes (and subsidies) and thus there is really no a priori reason to use ad valorem instead of unit taxes. For the optimization version presented here, and which is required for the integrated model, unit taxes are the only possible ones.

With unit taxes, and treating also the parameters of the wage structure as taxes, the following expression for total "tax receipts" is obtained.

$$T(N^S, L_v, X_v, Q, M) \equiv \sum_{j=1}^{n} \Theta_j N_j^S + \sum_{i=1}^{2} (\tau_i + \xi_{ic}) \sum_{s=1}^{S} t_{is} Q_s + \\ + \sum_{i=1}^{2} \sum_{j=1}^{n} \sum_{v=0}^{t} (\tau_i + \xi_{ij}) \cdot a_{vij} X_{vj} + \sum_{j=1}^{n} \omega_j \sum_{v=0}^{t} L_{vj} + \sum_{i=1}^{m} \phi_i \cdot M_i \tag{9}$$

where

$$N^S \equiv (N_1^S, N_2^S, \ldots N_n^S)$$

$X_v \equiv (X_{01}, X_{11}, \ldots X_{tn})$

$Q \equiv (Q_1, Q_2, \ldots Q_s)$

$M \equiv (M_1, M_2, \ldots M_m)$

$L_v \equiv (L_{01}, L_{11}, \ldots L_{tn})$

The optimization version of ELIAS

The supply and demand functions implied by the profit maximization problems (5), the composite good cost minimization problems (4) and the utility maximization problem (8) constitute the behavioural equations of the ELIAS model.

The small open economy assumption means that the world market prices for exported and imported goods are exogenous. At given domestic prices, i.e. given the domestic output prices P_j, j=1, 2, ..., n, the prices of the domestically used composite goods, P_i^D, i=1, 2, ..., m and the wage rate W, the solutions to the profit maximization problems (5) determine, for each production sector, the demand for labor and its allocation between different capital vintages, the production from each vintage and the total production. In sectors producing tradeables, the allocation of this production between the domestic market and the export market is also given by the profit maximization solutions. Since intermediate inputs, including energy, are used in fixed proportions to production, the flows of intermediate deliveries are also determined.

The demands for the private consumption goods are determined by the solution to the utility maximization problem (8) given domestic prices and the disposable income of the household sector. Since the consumption goods are defined as linear combinations of the sector outputs, the demands for consumption goods can be directly translated to demands for the domestically used composite goods.

The gross investment, and thus the demand for the capital good, is exogenous. The capital good is also defined as a linear combination of the sector outputs and consequently the investment demand can also be directly translated to demands for the domestically used composite goods.

The total demands for the domestically used composite goods are thus made up of the demands for intermediate deliveries, the demands for private consumption and the investment demand. The cost minimization problems (4) then determine the amount of imports and the amount of domestically produced goods required to statisfy these demands at the least cost, given domestic and world market prices.

The optimization problems (4), (5) and (8) consequently give, for each of the n commodities, the supplies to the domestic market and to the export market, as well as the domestic demands for domestically produced goods and the import demands. To determine the equilibrium prices and the disposable income of the household sector, a set of market equilibrium conditions must be imposed in order to close the model. The supplies from domestic producers to the domestic market shall be equal to the total domestic demands, and the total demand for labor shall be equal to the exogenous supply. This yields n+1 equilibrium conditions which determine the output prices P_j, j=1, 2, ..., n, and the wage rate W.[12]

The world demand for the model economy's exports is infinitely elastic at the given world market export prices and similarly the supply of imports is infinitely elastic at the

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[12] The exogenous world market prices serve as the numeraire for the domestic price system.

given world market import prices. Consequently equilibrium conditions for export and import goods are not required. The net surplus of the current account shall, however, in equilibrium be equal to the exogenously determined net surplus requirement. This equilibrium condition determines the disposable income of the household sector.

Finally, the prices P_i^D, i=1, 2, ..., m shall in equilibrium be equal to the unit costs of the domestically used composite goods.

Suppose for the moment that the taxes and the exogenous wage structure are not used, that is these model parameters are set to zero. It should by now be clear that with a proper interpretation of commodities the ELIAS model can easily be fitted into the Arrow-Debreu model of a competitive economy.[13] By the fundamental theorems of competitive analysis the equilibrium allocation of the ELIAS model must, if it exists, also be an efficient allocation, i.e. it yields a maximum of the utility function (6) subject to the constraints set by the production technology (including the dummy "production functions" for the domestically used composite goods), the market equilibrium constraints and a condition implied by the required current account net surplus, that there shall be a net lump sum transfer to the rest of the world.

Since the production technology and the preferences in ELIAS are convex, the converse result also holds: An efficient allocation, i.e. a solution to the maximization problem just described, can be sustained as a competitive allocation.

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[13] One may think of ELIAS as a model which describes an economy with 4*m+n+1-m commodities. For each of the m tradeable sector outputs there is one commodity supplied to the domestic market and another supplied to the world market. There is furthermore an internationally produced good which may be imported and then, fourthly, there

Since there is only one household in the model it can furthermore be sustained without any reallocation of initial endowments.

The second property is the useful one as far as computations of competitive equilibria are concerned. By setting up the standard Pareto problem associated with the undistorted ELIAS model the competitive equilibrium can, at least in principle, be computed by applying algorithms for nonlinear optimization problems.

The results in Section 2.4 of Chapter 2 allow the same computational approach to be used also when unit taxes and the exogenous wage structure are introduced, provided that the objective function of the original Pareto problem is slightly modified.

The optimization version of ELIAS is presented in Table 3.1. The objective function consists of the utility function for the household sector, multiplied with the parameter α, minus the sum of total tax receipts. It is maximized subject to first the production technology constraints, which describe the production possibilities in the different vintages and

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13 (cont.)

is the domestically used composite good. Then there are finally n-m nontraded commodities and the capital good. Each of these commodities are supplied by price-taking and profit-maximizing producers. The composite goods are"produced" by the m dummy sectors and the cost minimization implied by (4), together with the equilibrium condition that their market prices are equal to their unit costs, is equivalent to profit maximization. The capital good production can be handled completely analogous to the composite goods.

The final demands for these commodities are derived from price-taking utility maximization behaviour or they are, for investments and the public sector output, exogenous. It is not necessary to treat the determinants of the export demand explicitly since the prices of export goods are exogenous and the demands are perfectly elastic at these prices.

how the total production in each of the sectors producing tradeables may be allocated between the domestic market and the export market. Secondly, there is a number of market equilibrium constraints which require the supply of each good, and of labor, to be at least as large as the demand. For tradeable goods, there are two sets of such market equilibrium constraints. The first of these applies to the domestically used composite goods whereas the second set requires the supplies of domestically produced goods to the domestic market to be at least as large as the domestic demands for these goods. For nontradeable commodities only one set of market equilibrium constraints is required since it is not necessary to distinguish between domestic use and domestic supply of goods with the same sectoral classification.

The final restriction is the current account constraint which requires the total value of exports minus the total value of imports to be at least as large as an exogenous given net transfer to the rest of the world.

From the concavity assumptions for the production functions and the composite goods functions $g_j(\cdot)$, $j=1, 2, \ldots, m$ and the assumed convexity of the transformation functions $h_j(\cdot)$, $j=1, 2, \ldots, m$, it follows that the set of feasible allocations is convex. The utility function is assumed to be continuous and strictly concave and consequently a solution to the maximization problem in Table 3.1 is always the unique global solution.[14] This is obviously an extremely valuable property since all methods for actually solving nonlinear maximization problems identify local rather than global maxima.

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[14] See Intriligator (1981). This means that the equilibrium allocation is unique, but this is not necessarily the case for the equilibrium price system, since the Kuhn-Tucker multipliers may not be unique.

The set of feasible allocations defined by the constraints is closed and bounded. Provided that the exogenous variables I, G, D and L^S, which represent the total investments, the public sector production, the current account requirement and the labor supply, respectively, and the input-output parameters are carefully and correctly specified it is also nonempty. Consequently, there also exists a global maximum for the problem in Table 3.1. Provided finally that a suitably strong constraint qualification is satisfied, there also exists a set of nonnegative Kuhn-Tucker multipliers associated with the constraints. In fact, with a correctly specified input-output system, i.e. one where a positive net output is always feasible in all model sectors, the set of permissable values for the exogenous variables I, G, D and L^S may be defined as those which permit the Slater constraint qualification to be satisfied.[15] By confining the choice of exogenous variables to this permissable set, the existence of the Kuhn-Tucker multipliers is guaranteed. Economically this means that the exogenous variables should be selected so that the demands for commodities implied by the investment, public sector and current account requirements do not completely drain the model economy's productive capacity as determined by the labor supply. This is of course a reasonable condition to impose on the choice of exogenous variables.

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15 For the maximization problem max $F(x_1, x_2, \ldots, x_n)$ subject to $g_i(x_1, x_2, \ldots, x_n) \leqq c_i$, i=1, 2, ..., m and $x_j \geqq 0$ j=1, 2, ..., n, the Slater constraint qualification requires that there exists a set of x_j-values $x_j^o \geqq 0$ j=1, 2, ..., n such that all constraints hold with strict inequalities, i.e. $g_i(x_1^o, x_2^o, \ldots, x_n^o) < c_i$, i=1, 2, ..., m.

As usual the Kuhn-Tucker multipliers indicate the sensitivity of the objective function to a change in the right-hand side of the constraints. In the standard Pareto problem this means that they can be interpreted as prices. From the results in Section 2.4 of Chapter 2 it follows that this is also true for the modified Pareto problem in Table 3.1.

Table 3.1 The optimization version of the ELIAS model. (See Appendix B for definitions of symbols.)

$$\text{Max} \quad \alpha U(Q_1, Q_2, \ldots, Q_S) - T(N^S, L_v, X_v, Q, M)$$

Production technology constraints:[16]

$$X_{vj} - f_{vj}(L_{vj}) \leqq 0 \qquad v=0, 1, \ldots, t; \quad j=1, 2, \ldots, n$$

$$X_j - \sum_{v=0}^{t} X_{vj} \leqq 0 \qquad j=1, 2, \ldots, n$$

$$h_j(N_j^S, Z_j) - X_j \leqq 0 \qquad j=1, 2, \ldots, m$$

$$N_j^S - X_j \leqq 0 \qquad j=m+1, \ldots, n$$

Market equilibrium constraints:[17]

$$\sum_{s=1}^{S} t_{js}Q_s + \sum_{i=1}^{n} \sum_{v=0}^{t} a_{vji}X_{vi} - g_j(N_j^D, M_j) \leqq 0 \qquad j=1, 2$$

$$\sum_{s=1}^{S} t_{js}Q_s + \sum_{i=1}^{n} a_{ji}X_i + a_{jI}I - g_j(N_j^D, M_j) \leqq 0 \qquad j=3, \ldots, m$$

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16 The expression for the remaining capital stock in sector j is from now on not explicitly written out in the production functions in order to save on the use of symbols.

17 Total gross investments, I, and public production, G, are exogenously determined.

$$\sum_{s=1}^{S} t_{js}Q_s + \sum_{i=1}^{n} a_{ji}X_i + a_{jI}I - N_j^S \leqq 0$$

$$j=m+1, \ldots, n-1$$

$$G - N_n^S \leqq 0$$

$$N_j^D - N_j^S \leqq 0 \qquad j=1, 2, \ldots, m$$

$$\sum_{j=1}^{n} \sum_{v=0}^{t} L_{vj} \leqq L^S$$

The current account constraint

$$\sum_{j=1}^{m} \left[P_j^{WE} Z_j - P_j^{WI} M_j - P_j^C b_j X_j \right] \geqq D$$

Non-negativity constraints

$$Q_s \geqq 0 \; \forall s, \; N_j^D \geqq 0, \; N_j^S \geqq 0, \; Z_j \geqq 0, \; M_j \geqq 0, \; X_j \geqq 0, \forall j$$

$$X_{vj} \geqq 0, \; L_{vj} \geqq 0, \; \forall \, v,j$$

The solution to the maximization problem in Table 3.1 and the associated Kuhn-Tucker multipliers, determine the equilibrium allocation and the price system for a single period. Since the new vintage capital stocks which are created by the current sectoral investments by assumption cannot be productively employed until the next period, the equilibrium is independent of them and also of the chosen energy input coefficients for the new vintage. Consequently, these matters can be determined recursively, once the equilibrium solution is known, within a separate investment allocation and technological design submodel. In this respect the optimization version of ELIAS does not differ from the original formulation of the model. But in order to make the model presentation selfcontained this section is concluded with a discussion of this part of the ELIAS model.

The sectoral allocation of investments and the technological design of new vintages

Dual to the ex ante production functions there exist unit cost functions $K_j(W_j, P_{1j}, P_{2j}, Q_j)$ where W_j is the price of labor, P_{1j} the price of electricity, P_{2j} the price of fuels and Q_j the user price of capital services.[18] In equilibrium the unit cost shall be equal to the price of the output, that is

$$K_j(W_j, P_{1j}, P_{2j}, Q_j) = P_j^* \qquad j=1, 2, \ldots, n \qquad (10)$$

where P_j^* is the net output price in sector j.[19] Consequently, the equation (10) implicitly determines Q_j, the cost of capital services which sector j can afford, given the equilibrium price system. The Q_j:s constitute the basis for the sectoral allocation of the total investments.[20]

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18 More precisely, these functions give the minimum cost of labor, capital services and energy for a unit of output. Thus they represent the unit cost net of the cost of non-energy intermediate inputs.

19 The net output price is defined as

$$P_j^* = P_j - \Theta_j - \sum_{i=3}^{m} P_i^D a_{ij} - \sum_{i=m+1}^{n-1} P_i a_{ij} - P_j^C b_j ,$$

i.e. the producer price net of the unit cost of non-energy, intermediate inputs.

20 An alternative is to use expectations about future values of W_j, P_{1j}, P_{2j} and P_j^* in (10) instead of the equilibrium price system. This is possible in ELIAS by using exogenous point expectations about the price variables.

Based on the user cost of capital services a sectoral rate of return, R_j, can be computed from the relation

$$Q_j = P_I(\delta_j + R_j) \tag{11}$$

where δ_j is the rate of depreciation in sector j and P_I the price of the capital good.

The sectoral investments, I_j, are now determined by the following relations

$$I_j = \begin{cases} \delta_j X_j \dfrac{\partial K_j}{\partial Q_j} \left[\dfrac{R_j}{R}\right]^{\varphi_j} & ; \quad R_j \geqq R \quad j=1, 2, \ldots, n \\ 0 & \quad R_j < R \end{cases} \tag{12}$$

where R is the market rate of interest, φ_j is an elasticity parameter and the expression $\delta_j X_j {}^{\partial K}j/\partial Q_j$ is an approximation of the depreciation of capital in sector j (note that ${}^{\partial K}j/\partial Q_j$ is the capital input coefficient per unit of output in the new vintage). If the marginal rate of return in a sector is equal to the market rate of interest, the sector investment is exactly sufficient to replace the depreciated capital while if it is larger the capital is increased. The basic idea is that there shall be a positive investment in a sector only if its marginal rate of return is at least as high as the market rate of interest. The size of the sectoral investment shall be an increasing function of the ratio of the marginal rate of return and the interest rate. The market rate of interest is determined by the condition $\sum_{j=1}^{n} I_j = I$.

From Shephard's lemma the energy input coefficients are given by

$$a_{t+1;1j} = \frac{\partial K_j}{\partial P_{1j}} \qquad j=1, 2, \ldots, n$$

$$a_{t+1;2j} = \frac{\partial K_j}{\partial P_{2j}} \qquad j=1, 2, \ldots, n \qquad (13)$$

in those sectors where $I_j > 0$.

In this manner a set of new vintage capital stocks is created. When also the exogenous variables have been updated the model can be solved again for the following period and in this way a sequence of temporary equilibria can be computed.

The investment allocation is ad hoc in the sense that it is not derived from, and thus may not be viewed as a reduced form of, a more complete model. It is a weak part in ELIAS and there are strong reasons to be sceptical about it. It does not, for instance, make the marginal rates of return to new capital equal in the different sectors. It is also doubtful whether it tends to do so in the long run. Even if there are relatively more investments in sectors with high marginal rates of return, which in itself perhaps may tend to even out differences, the investment allocation will also affect the prices and it is not clear what the net result will be. In general, the dynamic properties introduced in ELIAS by this investment allocation model have not been fully investigated by the model constructors. It was originally adopted because the solution algorithm, which was especially designed for ELIAS, could not handle production associated with new capital invested in the current period. Thus, the sectoral investments could not be determined simultaneously with the other variables. Instead they had to be computed recursively.

In spite of its deficiencies I have retained the original investment allocation also in the integrated model in order to make it comparable to the original ELIAS model and to make it possible to compare the results of the integrated model with the results of the corresponding applications with ELIAS. However, this also leads to a problem in the integrated model. In contrast to the other sector investments, the investments in electricity and heat production technologies are determined simultaneously with the rest of the equilibrium allocation in the integrated model. The simultaneous determination of prices, production and investments in the electricity and heat sector is a very attractive feature of the original electricity and heat supply model and it is maintained in the integrated model. But these investments are based on capital costs calculated at a given interest rate. In the integrated model this interest rate must remain exogenous since the marginal rates of return to new capital in the other sectors are determined recursively first when the one-period equilibrium has been computed.

This means for instance that the rate of interest used to calculate the capital costs of the different electricity and heat production technologies is independent of the market rate of interest R obtained in the one-period equilibrium. Although I have adopted the same variable definition as in the original ELIAS model and refer to the variable R as "the market rate of interest", it should, however, be noted that this variable is somewhat peculiar. As can be seen from (12) its value is always less than or equal to the lowest sectoral marginal rate of return among those sectors where there are positive investments. As a result it can happen that the value of R becomes quite low and in particular considerably lower than an average of the marginal rates of return in the sectors with positive investments.

The serious problem is thus not that the rate of interest used to calculate the capital costs of the electricity and heat technologies is independent of R but that it is independent of the marginal rates of return in the other sectors. The experience from the use of the model is, however, that the latter have been fairly stable. Thus, it has been possible to use an exogenous interest rate for the capital costs which is close to an average of the sectoral marginal rates of return.

It should also be noted that the optimization solution approach, which I use for the integrated model, in principle can be modified to allow current investments to be productively employed in the same period. It can be done by incorporating the ex ante production functions among the production technology constraints. The ex ante production functions then give the production possibilities for newly invested capital. In this way the current investments could be productively employed in the same period. The sectoral allocation of investments would then be determined simultaneously with the current equilibrium prices and thus also with the sectoral rates of return on capital.[21]

This completes the presentation of ELIAS, the first of the two building-blocks of the integrated model which is constructed in Section 3.3. The aim has been to provide the necessary background for the model integration and for the applications of the integrated model in subsequent chapters. It should be clear from the presentation that I have adopted

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21 Some preliminary experiments on the incorporation of the ex ante production functions have, however, shown that there are certain numerical difficulties, at least with the present parameterization of the ex ante production functions. They are highly nonlinear which has made the optimization code I use quite unstable.

the ELIAS model unchanged, except for a few minor modifications. In contrast to the original presentations of it, the discussion here has emphasized that ELIAS may be formulated as a Pareto-type optimization problem. As a result an optimization solution approach can be used to solve it. This is the basis for the construction of the integrated model.

3.2 THE ELECTRICITY AND HEAT SUPPLY MODEL

The second building-block of the integrated model is the electricity and heat supply model. It is taken from a model of the Swedish markets for electricity and space heat energy constructed by Bergman and Carlsson (1984).[22] It is basically an activity analysis model which describes the energy markets at a given time period. For this period the initial capacities for electricity and heat production are given from the past, but may be augmented by new investments. The model determines a competitive equilibrium for this period and since the capacities may be adjusted through new investments this period should be interpreted to be long enough to allow such capacity changes. Thus, it is not a short run model, but nor is it a long run model since the initial capacities very much affect the equilibrium solution. Precisely as ELIAS, it is best viewed as a medium term model.

In the following the general structure of the model is presented but without penetrating all its details. A complete statement of the model is given in Appendix A. Both the presentation below and the Appendix closely follow Bergman and Carlsson's (1984) description of the model. It is included here in order to make the discussion of the integrated model selfcontained and in order to provide the necessary model background for the applied analysis in the next two chapters.

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22 A presentation (in Swedish) of the model can also be found in Bergman and Mäler (1983).

General characteristics

The model explicitly treats four domestic energy markets, namely the nation-wide electricity market and three regional markets for heating energy. The latter set of markets will be referred to as "heat markets". The demand for heating energy is essentially the residential and commercial demand for space heating and hot water.

The heat markets are defined as aggregates of areas, urban as well as rural, with similar characteristics in terms of heat demand per km^2 and coverage of interconnected district heating networks. Thus, the first heat market covers the Stockholm, Göteborg and Malmö regions, i.e. Sweden's biggest cities, as well as the medium-sized towns Uppsala and Västerås, where the coverage and degree of integration of the district heating network make the potential demand for district heat relatively high. The second heat market is an aggregate of all other areas where integrated district heating networks exist or are planned. On this heat market, however, each individual, local heat market is considerably smaller than on the first heat market. The third heat market, finally, consists of the remaining areas of the country.

The first heat market represents the potential market for heating energy delivered from large heat generating plants as well as combined power and heat plants. The second heat market is the potential market for small units, i.e. less than 100 MW, for heat and combined power and heat generation. On the two first heat markets there is competition between district heating and various kinds of individual furnaces based on oil, electricity or various kinds of solid fuels. On the third heat market only the latter, the individual furnaces, are feasible heat production technologies.

On each heat market, as well as on the national electricity market, an extensive list of available and potentially available supply technologies is defined (see Appendix A). Each one of these technologies is represented by an activity vector, i.e. the input of fuels and other resources per unit of the main output. There is also joint output of electricity and heat in activities representing back pressure technologies, i.e. combined power and heat production. As electricity and heat thus are jointly produced in some processes, and electricity can be used to produce, or be a substitute for, heat in heating systems, there is a high degree of interdependence between the electricity and heat markets.

The model is a partial equilibrium model in the sense that it determines equilibrium price-quantity configurations on the nation-wide electricity market and the regional heat markets at given prices of all inputs. Mathematically it is designed as an optimization model. Thus, the sum of consumers' and producers' surpluses, added across the four markets explicitly treated in the model, is maximized. By the equivalence theorem the resulting solution can be interpreted as a competitive equilibrium on the markets in question.

For the integrated model the demand side of the electricity and heat markets model is not needed. The demands for electricity and heat are instead derived from the general equilibrium model. The presentation here is therefore concentrated to the commodity structure and the production technology, i.e. the supply side, in the model. The demand side is treated as exogenous and only a cost minimization version of the electricity and heat model is considered.

Final output and final demand

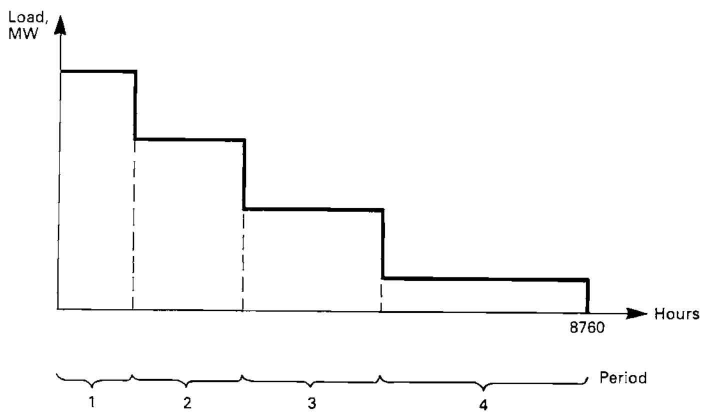
In the model two types of final output are distinguished. The first is electricity, which is also an intermediate input. That is, electricity is also used as an input within the energy sector. The other final output is space heating energy delivered from water-borne heating systems, i.e. district heating systems or individual oil furnaces, electric boilers or heat pumps.

As the individual space heating systems in general are operated by the households, the definition of final output implies that at least some of the households are final users as well as producers of water-borne space heating energy. It also implies that the household and commercial sectors to some extent demand electricity as an input in the production of water-borne space heating energy.

Production decisions

Suppose now that the annual demands for electricity and for water-borne heat are given (for instance derived from the general equilibrium model).

In order to represent the time-pattern of energy demand, the annual energy demand levels are allocated over four time-segments by means of fixed coefficients. Thus, on each of the four energy markets explicitly treated in the model, a "stylized" load-duration curve of the following type is generated.



The subdivision of the year into time-periods adds some complexity to the treatment of production and investment decisions in the model. But it also adds reality. It is a typical feature of electricity and heat markets that a significant share of the annual demand occurs during a relatively small number of hours. Accordingly the need for capacity varies substantially during the year, as well as during the week and the day. Thus, part of the available energy production capacity is fully utilized only during peak periods, while other parts may be continuously fully utilized.

In order to make a reasonable trade-off between realism and complexity, the feasible production decisions are defined in the following way in the model:

For large scale production units such as power plants and district heating plants four modes of operation are defined. Thus, the variable X_{kj} represents the rate of production under mode k in power plants of type j. When k=1 the plant is utilized at the rate X_{kj} during period 1, and then kept idle during the rest of the year. When k=2, the plant is utilized at the rate X_{kj} during period 1 <u>and</u> period 2, and then kept idle, and so on. Thus, k=4 means that the plant in question is continuously used the entire year.

For large electric boilers and heat pumps supplying heat to district heating systems the model allows a higher degree of flexibility. Thus, the variable Q_{tsh} is the rate of production during period t in plant type s on heat market h, and the variable is free to take any value which is compatible with the capacity constraint. This means that plants of this type can be used during, say, period 1 and period 4 and kept idle during periods 2 and 3. Such a production pattern is obviously not possible for the large-scale plants where the set of feasible production decisions is confined to the four modes of operation.

For small production units such as individual oil furnaces, electric boilers and heat pumps the set of feasible production decisions is confined to one single output level. Thus, the variable H_{rh} represents the production rate in individual heat production units of type r on heat market h. Once the production level has been chosen, it is assumed that this production level is kept during τ_r hours per year and that the distribution of "production hours" exactly corresponds to the distribution of heating energy demand over the four time periods.

The demands for electricity and heat can be satisfied in several ways by different combinations of the various production activities in the model. Each activity represents a specific power or heat technology operated in a certain mode. There is also a set of investment activities through which the initial capacities of the different production technologies can be augmented. The utilization of each of these activities is associated with a cost, which for the production activities mainly consists of the cost for fuel inputs and for the investment activities of the capital cost for new capacity.

The electricity and heat model determines a combination of production and investment activities which minimizes the cost of satisfying the given demand. The feasible set of activities is defined by a set of linear constraints. They are of two types. First,there is a set of market equilibrium constraints. They require the supply of each of the energy outputs to be at least as large as the corresponding demand. Secondly, there is a set of capacity constraints which require that the total production from each of the available production technologies is less than or equal to the initial capacity plus new capacity additions.

For the nation-wide electricity market there are four market equilibrium constraints; one for each of the four subperiods of annual electricity demand. For each such subperiod the electricity supply shall be at least as large as the demand during that period, including the intermediate demand for electricity in water-borne heat producticcn. The electricity is supplied from the model's various power technologies and from imported electricity.

In a similar manner there are,for each of the local heat markets, market equilibrium constraints for each of the four time segments. But there is also an additional dimension in the heat market equilibrium conditions. The users of water-borne heat are initially connected to a district heat system or they have an individual furnace based on oil or electricity. It is possible to convert from one type of water-borne heating to another in the model, but at a cost through a specific type of investment activities. To account for the initial adherence to one of the three types of water-borne heating systems, the demand for water-borne heat is divided into three parts by exogenous market-shares for district heat, for individual oil heating and for individual water-borne electric heating. In this way there are, for each local heat market and for each time segment, three market equilibrium conditions for water-borne heat (except for the third local heat market where there are only two, since district heating is not feasible).

The demands for the three forms of water-borne heat then consist of the initial demands determined by the market shares plus conversions to, minus conversions from, each type.

The second set of constraints in the model is the capacity constraints. They are quite straightforward. For each available production technology its use shall not exceed the initial capacity plus new capacity additions (including for water-borne heating technologies, any conversion to or from such a technology). There are also distribution capacity constraints on the exports and imports of electricity. Finally, there is also a distribution capacity constraint on district heat. For the two local heat markets where district heat is feasible, the share of total water-borne heat demand satisfied by district heat shall not exceed a certain proportion. In this way the market share for district heat has an upper bound and these constraints bound the possible amount of conversions to district heat.

The model is completely described in Appendix A which also contains a list of the technologies included in the model. As with ELIAS it will in the following be convenient to use a brief label to refer to it. Since it is a model of energy markets, it has been labelled ENMARK.

3.3 THE INTEGRATED MODEL

The electricity and heat model, which was described in the previous section, treats the production of electricity and heat in a quite considerable detail. But it is a partial equilibrium model, i.e. it takes the prices and the allocation of resources outside the electricity and heat sector as given and unaffected by events on the electricity and heat markets. In general, however, one can expect that such events have an impact on the rest of the economy and that this impact induces feedback effects from the rest of the economy to the electricity and heat sector. The partial equilibrium approach is then valid only if these general equilibrium effects are small. If they are not small, then the allocation of resources within the electricity and heat sector should be studied in a general equilibrium framework.

On the other hand, the general equilibrium model, which was described in Section 3.1, treats the production of electricity and heat simultaneously with the allocation of resources in the rest of the economy. But the representation of the electricity and heat sector is very aggregated and it is for instance not possible to study the effects of different electricity production technology choices. By integrating the two models it becomes possible to do this, as well as to study other aspects of different electricity and heat production technologies in a general equilibrium perspective. In this way the range of possible applications of the general equilibrium model is extended. It also becomes possible to evaluate the partial equilibrium approach, since the electricity and heat sector model now is embedded in a general equilibrium structure.

In this section the integration of the two models is described. First I discuss the linkage of the final outputs of ENMARK to the corresponding commodities in ELIAS and then the linkage of the inputs of ENMARK to the commodities of ELIAS. Then the complete integrated model is presented and finally I discuss the treatment of investments in the integrated model.

The production sector indexed as j=1 in the ELIAS model represents the production of electricity and district heat.[23] Thus, it largely corresponds to the ENMARK model. The integ-

[23] It corresponds to SNI 4 (electricity, gas and district heat utilities).

rated model is created by replacing this sector's production technology constraints in ELIAS with the ENMARK model.

If the latter had been completely comparable to the electricity and heat sector in ELIAS, i.e. if exactly the same commodities had been involved in both models, this would have been a simple and straightforward operation. The demand for the electricity and heat output in ELIAS would now instead have been directed to the corresponding output in the ENMARK model. The use of various intermediate inputs in ENMARK would have been treated as an additional demand source for the corresponding outputs of the other production sectors in ELIAS. In this way the two models would have been completely linked.

But the ENMARK supply part is not quite comparable to the corresponding sector in ELIAS, i.e. the commodities involved are not exactly the same. In order to merge the two models, the commodities in ENMARK must first be related to those in ELIAS.

As described in the previous section, the final outputs in ENMARK are electricity and space heating energy delivered from water-borne heating systems, i.e. district heat, individual oil furnaces or individual electric boilers. The final electricity demand consists of the demand for direct electric heating (i.e. in radiators) and the electricity demand for other purposes than heating (in addition, there is also the intermediate electricity demand from the heat sector). The two final electricity demand categories are treated separately because their annual distributions are different.

In ELIAS the output of the first energy sector is electricity and district heat, treated as a single aggregated commodity. The second energy sector, the "fuels" sector, represents the production of refined oil products. Thus, there is no explicit treatment of water-borne heating energy nor is there any distinction between different uses of electricity, other than that it is used in different sectors of the model economy. The commodities in ELIAS are furthermore measured in constant prices and not, as the final outputs in ENMARK, in physical units.

Similarly, on the input side there are also differences in commodity aggregation and in types of inputs used in the two models. Thus, the inputs in ENMARK are various fuels, such as nuclear fuel, oil, coal and domestically available fuels (peat, wood chips etc.), and more unspecified costs such as distribution costs and capital costs. In ELIAS on the other hand, the inputs are non-energy intermediate inputs corresponding to the various non-energy sectors, energy inputs from the "fuels" sector, complementary imports and the labor input.

In order to link the two models it is necessary to make the different sets of commodities compatible in some manner as was discussed in Chapter 2. Consequently, some rules on how to aggregate and disaggregate between the commodities in the two models must be introduced.

In Chapter 2, Section 2.2 I discussed how such rules based on fixed coefficients could be constructed. There are two possible approaches, Hicksian aggregation or a fixed proportions aggregation. In the former case the separate goods which belong to an aggregated commodity are assumed to be perfect substitutes, i.e. they have constant exchange ratios. In the second case there is a strict complementarity between the separate goods, i.e. they are used in fixed proportions. In Chapter 2 I also argued that the most reasonable approach for the linkage of the sector model's outputs to the commodities of the general equilibrium model is the fixed proportions aggregation/disaggregation. An Hicksian aggregation would in the present case imply that electricity and water-borne heat of different types are perfect substitutes, that the relative prices of electricity and water-borne heat are constant and that the electricity and heat sector could produce any combination of electricity and heat as long as their value measured at the constant relative prices sum to the aggregated demand for electricity and heat derived from the rest of the economy.

These implied assumptions of the Hicksian aggregation seem quite unreasonable. Instead I have used the fixed proportions approach to link the aggregated demand for electricity and heat, which is derived from ELIAS, to the outputs of ENMARK. The strict complementarity which is implied by this approach exaggerates the inflexibility in the demand for electricity and water-borne heat. There are at least some substitution possibilities between these two commodities. But in contrast to the Hicksian aggregation, the fixed proportions approach means that electricity and water-borne heat are treated as distinct commodities.

For the linkage of the input demands in ENMARK to the commodities of ELIAS, the Hicksian aggregation approach seems, as I argued in Chapter 2, more appropriate. The composition of the input demand in ENMARK is endogenous. It would be unnecessarily restrictive to match this against a supply of different inputs in fixed proportions from the rest of the economy which would be implied by the fixed proportions approach. Instead it is more reasonable to implicitly assume that the supply from the rest of the economy adjusts, at constant exchange ratios, to the input demand from the electricity and heat sector.

Unavoidably, these rules have a considerable degree of arbitrariness in them. Commodity aggregation can only be strictly justified under extremely stringent conditions. Whether the rules actually employed in the integrated model are plausible or not can only be judged by examining their implications for the supply and demand behaviour in the integrated model and their importance for the results in the tasks in which the model is used.

Linking the final outputs of ENMARK to ELIAS

The demand for the aggregated electricity and district heat commodity in ELIAS is assumed to consist of demandsfor electricity and for water-borne heating energy in fixed proportions. Let X_{1j} denote the j:th production sector's demand for this commodity, i.e., X_{1j} is given by the relations

$$X_{1j} = \sum_{v=0}^{t} a_{v1j} \; X_{vj} \qquad\qquad j=2, \ldots, n$$

Let E_j denote the electricity demand and W_j^d the demand for water-borne heating energy. Then the two latter are determined as

$$\begin{aligned} E_j &= e_j \cdot X_{1j} \\ & \qquad\qquad\qquad\qquad j=2, 3, \ldots, n^{24} \qquad (14) \\ W_j^d &= f_j \cdot X_{1j} \end{aligned}$$

where e_j and f_j are exogenous parameters. They are sector specific for two reasons. First,they are calculated from each sector's actual use of electricity and heat in the base year and their proportions may obviously differ between sectors. Secondly, the electricity and heating energy outputs in ENMARK are measured in physical units, whereas the commodities in ELIAS are measured in base year constant prices. Thus, these parameters also convert the demand into physical units. Since the data on energy use in physical units and in constant prices imply different implicit prices in different sectors, the conversion factors must also be sector specific.

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24 The variable X_{11}, i.e. the intermediate own use in sector 1, is not used in the integrated model. Instead the intermediate own use is now determined within the ENMARK part of the integrated model.

The electricity and heat output in ELIAS is not used in the capital good book-keeping sector nor is it used directly for private consumption. Instead the electricity and heat which in fact are used by the households are accounted for as intermediate inputs in a model sector producing residential services.[25] Also the households' use of fuel for heating purposes is treated as an intermediate use in the residential services sector and not as direct private consumption. Thus, there is no direct private consumption demand for heating energy.

Finally, the export- and import variables for electricity in ELIAS are replaced by the export- and import activities for electricity in ENMARK. This means that the product differentiation introduced in ELIAS by the transformation function and the domestic composite good definition is abandoned for the electricity sector in the integrated model. In principle there may still be both export and import of electricity because of the time segmentation of the electricity production and demand. Electricity could be exported in one time segment while imported in another for instance.

The equations (14) are not the only demand links from ELIAS to ENMARK. The fuel use in ELIAS mainly consists of fuels for transportation and fuels for heating purposes, where the latter essentially is oil for individual oil furnaces. Thus, it corresponds to a demand for water-borne heating energy in

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25 Depending on the degree of sectoral aggregation which the model user wishes to have, production of residential services may be a sector of its own or only a part of a more aggregated sector containing also other types of services production. The latter case has been used in the model applications in Chapters 4 and 5. In this section, for brevity in the discussion of how the two models are linked, the sector containing residential services production is referred to as the residential services sector.

the ENMARK terminology. In ENMARK, however, the demand for water-borne heating energy essentially is the residential demand for heating energy and for this reason it is not too implausible to make the following simplification: For the residential services sector, a certain proportion of the fuel demand is assumed to represent the demand for water-borne heating energy, while the remaining proportion represents the fuel demand for other purposes than heating. The intermediate fuel demands in all other ELIAS sectors are assumed to correspond only to a demand for other purposes than water-borne heating and shall thus not be linked to the ENMARK final output.

For sector j the intermediate fuel demand, X_{2j}, is determined as $X_{2j} = \sum_{v=0}^{t} a_{v2j}X_{vj}$. For the residential services sector, j=r, this is now modified as $X_{2r} = a_r \sum_{v=0}^{t} a_{v2r}X_{vr}$ where a_r is the non-heating proportion of the fuel demand. Thus,

$$W_r^o = c_r(1 - a_r) \sum_{v=0}^{t} a_{v2r} \cdot X_{vr} \tag{15}$$

represents the demand for water-borne heating energy. The parameter c_r is a conversion factor to physical units.

To complete the linkage on ENMARK's final output side, the distinction between electricity for heating and for non-heating purposes should be accounted for. For each production sector a certain proportion b_j of its electricity demand is taken to be electricity demanded for heating purposes and consequently the remaining proportion $1-b_j$ is electricity demanded for non-heating purposes. These assumptions may be stated more succinctly as follows. Let U_j be the electricity demand for non-heating purposes in sector j and V_j the electricity demand for direct electric heating. Then

$$U_j = (1 - b_j)E_j \qquad j=2, 3, \ldots, n \tag{16}$$

$$V_j = b_jE_j \qquad j=2, 3, \ldots, n \tag{17}$$

When these linkage relations were implemented some further simplifications were introduced. Since the heating energy in ENMARK predominantly corresponds to residential heating, electricity for heating purposes is assumed to be demanded only by the residential services sector. Thus, $b_j = 0$ for all j except j = r.

The total annual demands are obtained by summing over sectors, i.e.

$$U = \sum_{j=2}^{n} U_j = \sum_{j=2}^{n} (1-b_j)E_j = \sum_{\substack{j=2 \\ j \neq r}}^{n} E_j + (1-b_r)E_r \tag{18}$$

$$V = \sum_{j=2}^{n} V_j = \sum_{j=2}^{n} b_j E_j = b_r E_r \tag{19}$$

where U is total annual demand for non-heating electricity and V total annual electricity demand for direct electric heating. The total annual demand for water-borne heating energy, W, is given by

$$W = \sum_{j=2}^{n} W_j^d + W_r^o \tag{20}$$

Finally, the total demand for water-borne heating energy must be divided between the three local heat markets. Let λ_h, h= 1, 2, 3, be the fixed share of total heat demand which originates from local heat market h. Then the annual demand for heating energy in this market is given by

$$W_h = \lambda_h \cdot W \qquad h=1, 2, 3 \tag{21}$$

Linking the inputs in ENMARK to ELIAS

The inputs required in ENMARK's production activities are - besides the intermediate use of electricity - uranium, coal, oil and three types of domestically available solid fuels; wood chips,peat and waste. It is fairly straightforward to link these commodities to the final outputs of ELIAS.

The use of oil in the ENMARK part is linked to the fuel sector in ELIAS (i.e. sector j=2 which represents the production of petroleum products). Uranium and coal are not domestically produced but are treated as complementary imports to the electricity and heat sector.

The domestically available solid fuels are linked to the considerably more aggregated final outputs in ELIAS by assuming the latter to be Hicksian aggregates of various single commodities including those inputs which may be used in ENMARK. Thus, the use of wood chips, for instance, is linked to the sector in ELIAS which represents forestry production using the base year price of wood chips as the constant exchange ratio which transforms the demand for wood chips in ENMARK to a demand for forestry products in ELIAS.

Since the ENMARK model only contains energy and capital costs whereas in the corresponding ELIAS part non-energy intermediate inputs and labor use also are incorporated, the latter are in the linked model added to the production activities for electricity and district heat. Non-energy intermediate inputs and labor are used in fixed proportions to the production of electricity and district heat.

Distribution costs are incurred in some production activities in ENMARK. They are not linked into the general equilibrium relations of the integrated model, but they are still maintained in the model. It is important that they are included in order for the production and investment decisions to properly reflect the relevant costs. Formally they are treated as taxes and thus they affect production and investment decisions but do not correspond to actual resource requirements.

The integrated model

The complete integrated model is summarized in Table 3.2. In order to avoid a morass of model detail, the ENMARK part is kept very condensed.

The objective function is essentially the same as in Table 3.1. The only difference is the linear cost term $c^T x$ associated with the ENMARK activity vector x. It contains taxes, or parameters formally treated as taxes, which are imposed directly on the activity levels.

As for the non-integrated version in Table 3.1, the constraints in the modified Pareto problem are production technology constraints and market equilibrium constraints. For the former the first four set of constraints depicted in Table 3.2 are exactly the same as for the non-integrated model except that they no longer apply to sector $j=1$. This sector is now represented by the ENMARK model. The remaining production technology constraints give the production possibilities of the latter.

The production technology of the ENMARK model is compactly summarized by the technology matrix B. A subset of the rows of B pertain to the capacity constraints and are denoted by the submatrix B_k. Another subset of rows in B, denoted by the submatrix B_b, simply represent any other types of connections between different activities which the model may contain. The initial capacity vector g and the constraint vector b for these other types of constraints then jointly bound the set of feasible activity levels in the electricity and heat sector.

The remaining part of the matrix B is the rows corresponding to commodities which are exchanged between the two models. For each activity vector x there will be a certain input demand, given by $\sum_{k \in K} b_{jk} x_k$ for commodity j and these terms do now enter the market equilibrium constraints. The first five sets of these constraints in Table 3.2, and the labor market constraint, are essentially the same as in the non-integrated model in Table 3.1. The main difference is that the original intermediate demand from sector j=1 is now replaced by the activity input requirements just mentioned.

The market equilibrium constraint for the sector j=1 is replaced by several market equilibrium constraints pertaining to electricity and water-borne heating energy for the four time segments and for the different local heat markets. The terms $\sum_{k \in K} b_{e_\tau k} x_k$ and $\sum_{k \in K} b_{w_{\tau h} k} x_k$ are the supplies of electricity and of heat in time segment τ and local heat market h, respectively. The definitions of the electricity and water-borne heating energy demand variables, given by equations (14)-(21), are also included in Table 3.2 in their complete forms.

Finally, the current account constraint is now augmented by a set of trade variables related to the ENMARK model. First, there are the export and import of electricity in different time segments which may result from the chosen set of production activities. These terms now replace the original export and import variables pertaining to sector j=1. Secondly, the last two terms in the current account constraint represent the complementary imports of coal and uranium to the electricity and heat sector.

Table 3.2 The integrated model. (See Appendix B for definitions of symbols.)

$$\text{Max} \quad \alpha U(Q_1, Q_2, \ldots, Q_S) - T(N^S, L_v, X_v, Q, M) - c^T x$$

subject to

Production technology constraints

$$X_{vj} - f_{vj}(L_{vj}) \leqq 0 \qquad v=0, 1, \ldots, t \quad j=2, 3, \ldots, n$$

$$X_j - \sum_{v=0}^{t} X_{vj} \leqq 0 \qquad j=2, 3, \ldots, n$$

$$h_j(N_j^S, Z_j) - X_j \leqq 0 \qquad j=2, 3, \ldots, m$$

$$N_j^S - X_j \leqq 0 \qquad j=m+1, \ldots, n$$

$$B_k x \leqq \bar{g}$$

$$B_b x \leqq b$$

Market equilibrium constraints

$$\sum_{s=1}^{S} t_{2s} Q_s + \sum_{i=2}^{n} \sum_{v=0}^{t} a_i a_{v2i} X_{vi} + \sum_{k \in K} b_{2k} x_k - g_2(N_2^D, M_2) \leqq 0$$

where $a_i = 1$ for all i except i = r

$$\sum_{s=1}^{S} t_{js} Q_s + \sum_{i=2}^{n} a_{ji} X_i + a_{jI} I + \sum_{k \in K} b_j x_k - g_j(N_j^D, M_j) \leqq 0$$

$$j=3, 4, \ldots, m$$

(cont.)

$$\sum_{s=1}^{S} t_{js}Q_s + \sum_{i=2}^{n} a_{ji}X_i + a_{ji}I + \sum_{k\in K} b_{jk}x_j - N_j^S \leqq 0$$

$$j=m+1,\ \ldots,\ n-1$$

$$N_j^D - N_j^S \leqq 0 \qquad j=2,\ 3,\ \ldots,\ m$$

$$G - N_n^S \leqq 0$$

$$\sum_{i=2}^{n} \sum_{v=0}^{t} L_{vi} + \sum_{k\in K} b_{Lk} \cdot x_k \leqq L^S$$

$$u_\tau U + v_\tau V - \sum_{k\in K} b_{e_\tau k} \cdot x_k \leqq 0 \qquad \tau=1,\ 2,\ 3,\ 4$$

$$w_\tau W_h - \sum_{k\in K} b_{w_{\tau h}k} \cdot x_k \leqq 0 \qquad \tau=1,\ 2,\ 3,\ 4$$

$$h=1,\ 2,\ 3$$

where the electricity and heat demand variables are defined by the relations

$$U = \sum_{\substack{i=2\\ i\neq r}}^{n} e_i \sum_{v=0}^{t} a_{v1i} X_{vi} + (1-b_r)e_r \sum_{v=0}^{t} a_{v1r} X_{vr}$$

$$V = b_r e_r \sum_{v=0}^{t} a_{v1r} X_{vr}$$

$$W_h = \lambda_h \left[\sum_{i=2}^{n} f_i \sum_{v=0}^{t} a_{v1i} X_{vi} + c_r(1-a_r) \sum_{v=0}^{t} a_{v2r} X_{vr} \right]$$

$$h=1,\ 2,\ 3$$

(cont.)

Table 3.2 (cont.)

The current account constraint

$$\sum_{i=2}^{m} \left[P_i^{WE} Z_i - P_i^{WI} M_i - P_i^C b_i X_i \right] + \sum_{\tau=1}^{4} P_{e\tau}^{WE} \cdot b_{z_\tau k} \cdot x_k -$$

$$- P_{e\tau}^{WI} \cdot b_{m_\tau k} \cdot x_k -$$

$$- P_c^{WI} \sum_{k \in K} b_{ck} x_k - P_u^{WI} \sum_{k \in K} b_{uk} x_k$$

Non-negativity constraints

$$Q_s \geqq 0 \ \forall s, \ N_j^S \geqq 0, \ N_j^D \geqq 0, \ Z_j \geqq 0, \ M_j \geqq 0, \ X_j \geqq 0 \ \forall_j$$

$$X_{vj} \geqq 0, \ L_{vj} \geqq 0, \ \forall v, \ j, \ x_k \geqq 0, \ \forall k$$

The treatment of investments in the electricity and heat sector

Precisely as for the non-integrated model in Table 3.1, the solution to the maximization problem in Table 3.2, and the associated Kuhn-Tucker multipliers, determine the equilibrium allocation and the equilibrium price system, now for the integrated model. The convexity properties of the maximization problem are not affected by the modifications introduced in the integrated model, since all new terms are linear.

The equilibrium settled in this way is as before a one period equilibrium. In order to solve the model for a number of consecutive time periods, the gross investments must be allocated between the different sectors and the energy input coefficients of the new vintage must be determined. For all sectors except the electricity and heat sector this is done in exactly the same way as for the non-integrated model. The electricity and heat sector, represented by the ENMARK model, must, however, be treated differently.

In the original ENMARK model there is a set of initial capacities for the different production technologies. The production decisions are, however, not necessarily bounded by these initial capacities. They can be augmented through investment activities, which "produce" new capacity at a certain capital cost. The capital costs are expressed as annuities and are thus calculated from the investment outlay for a unit of capacity, at a given interest rate and a certain lifetime of the investment.

In ELIAS the investments in the current period are not productively available until the next period. A similar treatment of the ENMARK investments in the integrated model would mean that the investment activities could not be used and thus that the production activities would be definitely bouned by the initial capacities.

This would be very awkward, however, for two reasons. First, the model would have strict limits on its productive capacity for electricity and heat which could lead to difficulties in the numerical solutions. It would be harder to avoid situations where the set of feasible allocations turns out to be empty and, when this would not be the case, one could get rather absurd price effects when the solution is squeezed into the existing capacities.

Secondly, the investments in the electricity and heat sector would instead have to be determined simultaneously with the sectoral allocation of the total gross investments. But the investment activities cannot be fitted into the original investment allocation framework in any plausible way and one would also miss the joint determination of production and investment decisions which is a very attractive property of the original ENMARK model.

For these reasons the investment activities are instead included in the integrated model. Thus, for the electricity and heat sector, current investments are also productively employed in the same period.

Whether a positive utilization of a certain investment activity is economically justifiable or not obviously depends on the capital cost. Thus, they must be incorporated in the integrated model in order to base the investments on correct decision criteria. On the other hand the real resource requirement of the capital good which is implied by these investment decisions is of course not given by the annuitized capital costs but by the total investment outlays. To get a proper treatment of these two aspects of the investment activities they have been incorporated in the integrated model in the following way.

The capital costs are, like the distribution costs, formally treated as tax parameters. Thus, they affect the cost of utilizing investment activities but they do not correspond to any real resource requirement. Given the solution to the integrated model the chosen investment activities (if any) imply a certain total investment in the electricity and heat sector. This is deducted from the exogenous total gross investments in the model and the remaining part is then allocated between the other sectors by the original investment allocation submodel in ELIAS. The energy input coefficients for the new vintage in these sectors are still determined by equations (13).

Thus, for the integrated model, the sectoral allocation of investments is determined in two steps. First, the investments in the electricity and heat sector are determined from the one period equilibrium solution, i.e. the solution to the maximization problem 3.2. After they have been deducted from the exogenous total gross investments, the remaining part is allocated between the rest of the model sectors.

The capital costs in ENMARK's investment activities are calculated at a given, exogenous interest rate. They have to be calculated in this way as long as the marginal rates of return on new capital in the other sectors are determined recursively when the one period equilibrium has been computed.

Investment decisions in the electricity and heat sector are based on this exogenous rate of interest. Whenever new capital invested in electricity and heat production can match this rate of return requirement, the investments will be made. The supply of new capital goods to the electricity and heat sector is perfectly elastic at this rate of return. When the investment demands in the electricity and heat sector have been satisfied, the remaining part of the exogenous gross investments is inelastically supplied to the other sectors and allocated between them according to the ELIAS investment allocation submodel discussed in Section 3.1. Thus, regardless of what the rates of return to new capital are in these sectors, the investment demands in the electricity and heat sector are always satisfied first, provided that the electricity and heat prices are such that the exogenous rate of return requirement in this sector is fulfilled.

This asymmetric treatment of the electricity and heat sector on the one hand, and the remaining sectors on the other, is a consequence of the assumption that the current investments in electricity and heat production technologies are also available in the same period, whereas the current investments in the other sectors are not productively available until the next period.

This is not a very satisfactory treatment of the sectoral allocation of investments. It has only been adopted in order to retain the investment allocation model of ELIAS, while still allowing for the simultaneous determination of production and investment decisions in the electricity and heat sector. A more satisfactory treatment could be achieved by assuming the current investments in all sectors to be productively available in the same period. Then the production and investment decisions would be simultaneously determined in all sectors. In particular, the marginal rates of return to new capital could be endogenously determined within the one period equilibrium for all sectors, including the electricity and heat sector. Then it would no longer be necessary to use an exogenous interest rate to compute the capital costs in the latter. However, this cannot be done without further model development work.

It should be noted, finally, that as long as the exogenous interest rate in the electricity and heat sector is not too different from the marginal rates of return to new capital in the other sectors, the chosen approach is fairly reasonable. It is of course always possible for the model user to choose an interest rate which does not deviate too much from the sectoral rates of return.

3.4 IMPLEMENTATION AND THE DATA BASES

In order to implement the model, it is necessary to specify the production functions, the functions defining the composite goods for domestic use, the transformation functions between domestic and export markets and the utility function for the households.

The utility function is linear in the logarithms of the consumption variables. Thus, it yields a linear expenditure system for private consumption.

The ex ante production functions are nested Cobb-Douglas-CES functions. There is a constant elasticity of substitution between a composite capital-labor input, defined by a Cobb-Douglas function, and a composite fuels-electricity input, defined by a CES function. The ex post production functions are derived from the ex ante functions by fixing the capital stock and the energy input coefficients. Explicitly, they are expressed as

$$X_{vj} = A_{vj} \cdot I_j(v,t,\delta_j)^{\alpha_j} \cdot L_{vj}^{(1-\alpha_j)} \qquad j=1,\ 2,\ \ldots,\ n \qquad (22)$$

where

A_{vj} = a constant, whose value inter alia depends on the ex post fixed energy input coefficients and the technical change parameters.

α_j = the capital share of value added in sector j.

Both the transformation functions between domestic and export markets, and the functions defining the composite goods for domestic use, are specified as CES functions.

$$h_j(N_j,\ Z_j) = (n_j^S N_j^{\varepsilon_j} + z_j Z_j^{\varepsilon_j})^{\frac{1}{\varepsilon_j}}, \quad \varepsilon_j > 1 \qquad j=1,\ 2,\ \ldots,\ m \qquad (23)$$

$$g_j(N_j, M_j) = (n_j^D N_j^{\mu_j} + m_j M_j^{\mu_j})^{\frac{1}{\mu_j}} \qquad \mu_j < 1$$

$$\mu_j \neq 0$$

$$j=1, 2, \ldots, m \qquad (24)$$

where

n_j^S, z_j, n_j^D, m_j are distribution parameters in the CES functions

ε_j, μ_j are elasticity parameters in the CES functions.

For simplicity I have dropped the supply and demand superscripts for the N_j-variables. When the elasticity parameter ε_j is greater than one the corresponding transformation function is strictly convex. This parameter is closely related to the export supply elasticity with respect to the domestic price. The supply elasticity is given by the expression $(1-\varepsilon)^{-1}$. Similarly, when the elasticity parameter μ_j is less than one the corresponding composite good function is strictly concave. The import demand elasticity with respect to the domestic price is given by $(1-\mu_j)^{-1}$.

The main data source for the ELIAS part of the integrated model is input-output data. Given assumptions on some key parameters - the ex ante production function substitution elasticities, the sectoral rates of technological change and the foreign trade elasticities - the remaining parameters are estimated from a base year input-output data set. Here the base year is 1979. The 1979 input-output table is not a complete one, however. Except for the energy input coefficients it was constructed on the assumption that the 1975 input-output coefficients (which is the latest complete input-output table) were valid also for 1979. But some important revisions in the national accounts have been taken into account in the 1979 table, which makes it a better representation of the actual state of the economy than the 1975 table.

The sectoral classification used is shown in Table 3.3. It is the same which frequently has been adopted in simulations with the ELIAS model. It reflects an attempt to keep the number of sectors at a minimum, given a desire to catch the main effects on the industrial structure of changes in energy prices in the trade-dependent Swedish economy. There are several reasons to have as few sectors as possible, given the requirements of the issue being studied. One is that input-output relations tend to be more stable over time for aggregates of sectors than for individual industries. Another is that the possibilities of getting good estimates of exogenous variables (such as world market prices, technical change etc) for a large number of sectors are quite limited. A third reason is that the cost of solving the model is quite sensitive to its size. This third reason is particularly relevant for the integrated ELIAS-ENMARK model.

To set up and run the model for a number of periods it is necessary to make assumptions about exogenous variables and about the key parameters mentioned above. The assumed values of these parameters are shown in Table 3.4. The values of the foreign trade elasticities and of the production function substitution elasticities are the same as the ones usually used in simulations with the ELIAS model. They are "guesstimates" based partly on information from econometric studies, partly on judgements on economic reasonableness. Thus, the production function substitution elasticities are compatible with the econometric evidence in Pindyck (1980) and the foreign trade elasticities are partly based on the econometric information in Hamilton (1980) and in Restad (1981).

The most essential data for the ENMARK supply part are the operating and investment costs of the various activities. They are summarized in Table 3.5 and 3.6. Further information about parameter values for the ENMARK part is given in Appendix A, pages A8-A10.

Table 3.3 Model sectors

Sector		SNI classification
1	Electricity and heat production (represented by the ENMARK model)	4
2	Fossil fuels production	353, 354
3	Mainly import competing industries	11, 13, 31, 32, 33, 3412, 3419, 342, 355, 361, 362
4	Mainly exporting energy intensive industries	12, 2, 3411, 351, 37101, 37102
5	Other mainly exporting industries	352, 356, 3699, 37103, 372, 38, 39
6	Sheltered industries and service production	3691, 3692, 5, 6, 7, 8, 9 (priv.)
7	The public sector	
8	The capital good sector	

Table 3.4 Key parameter values

Sector	Production function substitution elasticities[26] $(1-\rho_j)^{-1}$	$(1-\gamma_j)^{-1}$	Foreign trade elasticities Export $(1-\varepsilon_j)^{-1}$	Import $(1-\mu_j)^{-1}$
2	.25	.25	-	0.5
3	.75	.75	-2.0	4.0
4	.75	.75	-5.0	0.5
5	.75	.75	-4.0	2.0
6	.75	.75	-2.0	0.5
7	.75	.75	-	-

Table 3.5 Investment and operating costs. (1981 prices)

	Investment cost SEK/kW_e	Operating cost SEK/kWh_e
Nuclear power	6000	.052
Fossil fuelled condense power		
coal	4230	.143
oil	2170	.257
Gas turbines	1400	.641
Combined power and heat production		
coal $\geq$ 200 MW_e	3600	.179[27]
coal < 200 MW_e	4500	.179[27]
oil $\geq$ 200 MW_e	2650	.322[27]
oil < 200 MW_e	3200	.322[27]

(cont.)

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26 $(1-\rho_j)^{-1}$ is the substitution elasticity between the composite capital-labor input and the composite electricity-fuels input and $(1-\gamma_j)^{-1}$ is the elasticity of substitution between electricity and fuels.

27 SEK per kWh_e + 1.75 kWh heat

Table 3.5 (cont.)

Fossil fuelled heat production		
coal $\geq$ 100 MW_e	670	.063
coal < 100 MW_e	935	.063
oil $\geq$ 100 MW_e	275	.109
oil < 100 MW_e	335	.109
Secure	2590	.030
Electricity based district heat production		
electric boilers	255	1.09 kWh_e/kWh heat
heat pumps	1367	.35 kWh_e/kWh heat

Table 3.6 Conversion and operating costs of individual heating systems. (1981 prices)

Conversion

From	To	Cost
Oil heating	water-borne electric heating	1040 SEK/kW_e
Oil heating	district heating	920 SEK/kW heat
Water-borne electric heating	district heating	920 SEK/kW heat

Operating cost

Oil heating	.231 SEK/kWh heat
Water-borne electric heating	1.05 kWh_e/kWh heat

APPENDIX A The Energy Market Model (ENMARK)[28]

1 Definitions

1.1 Indices

$h = 1, 2, 3$	Different heat markets ($h = 1$: Larger cities such as Stockholm, Gothenburg and Malmö; $h = 2$: Smaller towns with a potential for district heat plants; $h = 3$: All other areas).
$i = 1, 2, \ldots, 6$	District heat plants, except electricity based ones.
$j = 1, 2, \ldots, 12$	Electricity power plants except hydro power plants.
$k = 1, 2, 3, 4$	Utilization of a plant. When $k = 1$, the plant is utilized during time period $t = 1$, when $k = 2$ the plant is utilized during time period $t = 1$ and $t = 2$, etc.
$n = 1, 2, 3$ $r = 1, 2, 3$	Indices used for individual heating systems.
$s = 1, 2$	Electricity based district heat plants.
$t = 1, 2, 3, 4$	Part of the year (defined on the basis of demanded heating power. The largest demand for heating power is in period $t = 1$).

[28] This description is taken from Bergman and Carlsson(1984). A number of variables and expressions related to environmental emissions have been deleted, since they are not used in the present version of the integrated model. The version presented here is a cost minimization model with an exogenous demand requirement for electricity and heat.

v Hydro power plants.

z Export.

m Import.

1.2 Variables and parameters

1.2.1 Endogenous variables (System symbols in brackets)[29]

H_{rh} (Hrh) Use of power in individual heating system of type r in heat market h. GW_e when r = 2 and GW_v when r = 1, 3

Q_{tsh} (Qtsh) Use of power during period t in electricity based district heat plants of type s in heat market h. GW_e

X_{kj} (Xkj) Use of power under utilization k in electricity power plants of type j. GW_e

X_{tv} (Xtv) Use of power in hydro power plants during time period t. GW_e

Y_{kih} (Ykih) Use of power under utilization k in district heat plants of type i (except electricity based ones) in heat market h. GW_v

Z_t (Zt) Export of electricity during period t. GWh

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29 The term GW_e denotes GW electricity while GW_v denotes GW heat.

M_t (Mt) — Import of electricity during period t. GWh

$\Delta\bar{H}_{nrh}$ (DHnrh) — Conversion from individual heating system n to individual heating system r, in heat market h.

$\Delta\bar{X}_j$ (DXj) — Addition of capacity in electricity power plants of type j. GW_e

$\Delta\bar{Y}_{ih}$ (DYih) — Addition of capacity in district heat plants of type i in heat market h (except electricity based ones). GW_v

$\Delta\bar{Q}_{sh}$ (DQsh) — Addition of capacity in electricity based district heat plants of type s in heat market h. GW_e

1.2.2 Exogenous variables

U — Demand for electricity, except for radiators, heat pumps and electric boilers. GWh_e

V_h (Vh) — Demand for electricity for electric radiators in heat market h. GWh_e

W_h (Wh) — Demand for heat from water-borne heating systems in heat market h. GWh_v

E_v — Maximum annual energy production in hydro power plants during a normal year.

$\bar{H}_{rh}$ — Installed power capacity in individual heating systems of type r in heat market h. GW_e when r=2 and GW_v when r=1,3

$\overline{Q}_{sh}$	Installed power capacity in electricity based district heat plants of type s in heat market h. GW_e
$\overline{X}_v$	Installed power capacity in hydro power plants. GW_e
$\overline{X}_j$	Installed power capacity in electricity power plants of type j. GW_e
$\overline{Y}_{ih}$	Installed power capacity in district heat plants of type i in heat market h (except electricity based ones). GW_v
$\overline{Z}$	Maximum power transmission capacity to Denmark. GW_e
$\overline{M}$	Maximum power transmission capacity to Denmark. GW_e

1.2.3 Parameters

a_t	Number of hours in period t.
b_k	Number of hours in utilization k.
c_j	Cost of operation in electric power plants of type j. Millions of SEK/GWh_e.
d_{ih}	Cost of operation in district heat plants of type i on heat market h (except electricity based ones). Millions of SEK/GWh_v.

e_r Cost of operation of individual heating systems of type r. Millions of SEK/ GWh_v when r = 1, 3; millions of SEK/ GWh_e when r = 2.

f_s Cost of operation in electricity based district heat plants of type s.

g_{hj} Delivery of hot water to heat market h from electricity power plant of type j. GWh_v/GWh_e

ℓ_s The ratio between the output and the input power in electricity based district heat plants of type s. GW_v/GW_e

F_s Annuitized capital cost for additional power capacity in electricity based district heat plants of type s.

G_j Annuitized capital cost for additional power capacity in power plants of type j.

K_{ih} Annuitized capital cost for additional power capacity in district heat plants of type i in heat market h.

L_{nr} Annuitized capital cost for conversion from individual heating system of type n to type r.

P_{zt} The export price of electricity in period t.

P_{mt} The import price of electricity in period t.

α_j Accessibility of power plants of type j.

β_i Accessibility of district heat plants of type i.

σ Distribution losses.

τ_r Normal utilization expressed in hours of individual heating systems of type r.

θ_h Share of the demand for heat in heat market h allocated to district heat.

λ_h Share of the demand for heat in heat market h allocated to individual oil heating.

μ_t Share of the annual demand for electricity, except for electric radiators, heat pumps and electric boilers, which occur during period t.

ν_t Share of the annual demand for electricity for electric radiators which occur during period t.

ω_t Share of the demand for water-borne heat which occur during period t.

φ_h The maximum market share of district heat in heat market h.

2 Production plants

2.1 Plants for electricity production

Index	Type
v	: Hydro power
j = 1	: Nuclear power
j = 2	: Back pressure, oil, heat market 1
j = 3	: Back pressure, coal, heat market 1
j = 4	: Back pressure, wood chips, heat market 1
j = 5	: Back pressure, peat, heat market 1
j = 6	: Back pressure, oil, heat market 2
j = 7	: Back pressure, coal, heat market 2
j = 8	: Back pressure, wood chips, heat market 2
j = 9	: Back pressure, peat, heat market 2
j = 10	: Condense, oil
j = 11	: Condense, coal
j = 12	: Gas turbines

2.2 Plants for production of hot water in district heating systems, except electricity based ones.

Index	Type
ih = 11	: Heat plant, oil, heat market 1
ih = 21	: Heat plant, coal, heat market 1
ih = 31	: Heat plant, wood chips, heat market 1
ih = 41	: Heat plant, peat, heat market 1
ih = 51	: Heat plant, waste, heat market 1
ih = 61	: Secure, nuclear, heat market 1
ih = 12	: Heat plant, oil, heat market 2
ih = 22	: Heat plant, coal, heat market 2
ih = 32	: Heat plant, wood chips, heat market 2
ih = 42	: Heat plant, peat, heat market 2
ih = 52	: Heat plant, waste, heat market 2

2.3 Electricity based district heat plants

Index	Type
sh =	11 : Electric boilers, heat market 1
sh =	21 : Heat pumps, heat market 1
sh =	12 : Electric boilers, heat market 2
sh =	22 : Heat pumps, heat market 2

2.4 Individual heating systems

Index	Type
rh =	11 : Oil boiler, heat market 1
rh =	21 : Electric boiler, heat market 1
rh =	31 : Heat exchanger, heat market 1
rh =	12 : Oil boiler, heat market 2
rh =	22 : Electric boiler, heat market 2
rh =	32 : Heat exchanger, heat market 2
rh =	13 : Oil boiler, heat market 3
rh =	23 : Electric boiler, heat market 3

3 Parameter values

a) Time periods (hours)

t	a_t	k	b_k
1	1512	1	1512
2	2520	2	4032
3	2688	3	6720
4	2016	4	8736

b) Demand shares

t	ω_t	ν_t	μ_t
1	0.28	0.28	0.19
2	0.39	0.39	0.30
3	0.25	0.25	9.31
4	0.08	0.08	0.20

c) Maximum market share of district heat

h	φ_h
1	0.8
2	0.6

d) Accessibility

j	α_j	i	β_i
1	0.70	1	0.90
2	0.85	2	0.90
3	0.85	3	0.90
4	0.85	4	0.90
5	0.85	5	0.90
6	0.85	6	0.90
7	0.85		
8	0.85		
9	0.85		
10	0.80		
11	0.80		
12	0.80		

e) Distribution losses

σ_1 = Losses in distribution of electricity, $\sigma_1 = 0.10$

σ_2 = Losses in distribution of hot water from district heat plants, $\sigma_2 = 0.06$

4 The model

4.1 The objective function

$$\text{Max} \quad \sum_{t=1}^{4} P_{zt} Z_t - \sum_{t=1}^{4} P_{mt} M_t - \sum_{j=1}^{12} c_j \sum_{k=1}^{4} b_k \alpha_j X_{kj} -$$

$$- \sum_{h=1}^{2} \sum_{i=1}^{6} d_{ih} \sum_{k=1}^{4} b_k \beta_i Y_{kih} - \sum_{s=1}^{2} f_s \sum_{h=1}^{2} \sum_{t=1}^{4} a_t Q_{tsh} -$$

$$- \sum_{r=1}^{3} e_r \sum_{h=1}^{3} \tau_r H_{rh} - \sum_{j=1}^{12} G_j \Delta\overline{X}_j - \sum_{h=1}^{2} \sum_{i=1}^{6} K_{ih} \Delta\overline{Y}_{ih} -$$

$$- \sum_{s=1}^{2} F_s \sum_{h=1}^{2} \Delta\overline{Q}_{sh} - \sum_{\substack{n=1 \\ n \neq r}}^{2} L_{nr} \sum_{\substack{r=2 \\ r \neq n}}^{3} \sum_{h=1}^{3} \Delta\overline{H}_{nrh}$$

The objective is to minimize the cost of satisfying the exogenous demand for electricity and heat. The cost consists of the operating costs in the power plants, heat plants and the individual heating systems plus the capital costs for new capacity, or conversions from one capacity to another. There is also a possibility to import electricity at an exogenous import price. The export of electricity at an exogenous export price is a revenue source for the domestic electricity system.

4.2 Restrictions

a) Market equilibrium for electricity, period t (EL_t)

$$(1-\sigma_1)a_t \sum_{k=t}^{4} \sum_{j=1}^{12} \alpha_j X_{kj} + (1-\sigma_1)a_t X_{tv} +$$

$$+ (1-\sigma_1)M_t - a_t \sum_{s=1}^{2} \sum_{h=1}^{2} Q_{tsh} - \omega_t \sum_{h=1}^{3} \tau_2 H_{2h} - Z_t \geq$$

$$\geq \nu_t \sum_{h=1}^{3} V_h + \mu_t U \qquad t = 1, 2, 3, 4$$

The demand for electricity is divided into five parts:

i) V represents the demand for direct electric heating in electric radiators.

ii) U is the demand for electricity for non-heating purposes.

These two demand components are exogenous in the cost minimization version of ENMARK.

iii) Q is the electricity demand from electricity based district heat plants.

iv) H is the demand from electric boilers installed in individual heating systems.

v) Z, finally, is the export of electricity.

These three demand components are endogenously determined.

b) Market equilibrium in heat market h, period t

i) District heat ($HEAT_{th}$)

$$(1-\sigma_2)a_t \sum_{s=1}^{2} \ell_s Q_{tsh} + (1-\sigma_2)a_t \sum_{k=t}^{4} \sum_{i=1}^{6} \beta_i Y_{kih} +$$

$$+(1-\sigma_2)a_t \sum_{k=t}^{4} \sum_{j\in CHP} g_{hj}\alpha_j X_{kj} - \omega_t \sum_{r=1}^{2} \tau_r \Delta\bar{H}_{r3h} \geq \omega_t \Theta_h W_h$$

CHP = {j:j = 2, 3, ... 9}

t = 1, 2, 3, 4;

h = 1, 2

The exogenous heat demand is represented by the term $\omega_t \Theta_h W_h$, which is the share of the heat demand in period t which in the beginning of the period is allocated to district heat. The term $\omega_t \sum_{r=1}^{2} \tau_r \Delta\bar{H}_{r3h}$ is the conversion to district heat during the period.

ii) Individual oil furnaces ($HIND_{th}$)

$$\omega_t \sum_{r=2}^{3} \tau_1 \Delta\bar{H}_{1rh} + \omega_t \tau_1 H_{1h} \geq \omega_t (1-\Theta_h) \lambda_h W_h$$

t = 1, 2, 3, 4;

h = 1, 2, 3

The term $\omega_t(1-\Theta_h) \lambda_h W_h$ is the share of the heat demand allocated to individual oil heating in the beginning of the period. The term $\omega_t \sum_{r=2}^{3} \tau_1 \Delta\bar{H}_{1rh}$ is the conversions from oil heating to either individual electric heating or district heating during the period.

iii) Individual electric furnaces ($HINDEL_{th}$)

$$\omega_t \tau_2 \Delta\overline{H}_{23h} - \omega_t \tau_1 \Delta\overline{H}_{12h} + \omega_t \tau_2 H_{2h} \geq$$

$$\geq \omega_t (1-\Theta_h)(1-\lambda_h) W_h \qquad t = 1, 2, 3, 4; \quad h = 1, 2, 3$$

$\omega_t(1-\Theta_h)(1-\lambda_h)W_h$ is the share of the heat demand allocated to individual electric boilers in the beginning of the period. $\omega_t \tau_1 \Delta\overline{H}_{12h}$ is the conversion to, and $\omega_t \tau_2 \Delta\overline{H}_{23h}$ is the conversion from, individual electric heating during the period.

c) The distribution capacity for district heat in heat market h, period t ($LOKDIS_{th}$)

$$(1-\sigma_2) a_t \sum_{k=t}^{4} \sum_{i=1}^{6} \beta_i Y_{kih} + (1-\sigma_2) a_t \sum_{k=t}^{4} \sum_{j \in CHP} g_{hj} \alpha_j X_{kj} +$$

$$+ (1-\sigma_2) a_t \sum_{s=1}^{2} \ell_s Q_{tsh} \leq \varphi_h \omega_t W_h \qquad t = 1, 2, 3, 4; \quad h = 1, 2$$

This restriction imposes an upper bound on the market share of district heat in heat markets 1 and 2.

d) The distribution capacity for export and import of electricity ($EXDIS_t$ and $IMDIS_t$)

$$Z_t \leq a_t \overline{Z}; \qquad t = 1, 2, 3, 4$$

$$M_t \leq a_t \overline{M}; \qquad t = 1, 2, 3, 4$$

e) Capacity constraints

i) Hydro power (VATEF$_t$ and VATEN)

$$X_{tv} \leq \overline{X}_v \qquad \sum_{t=1}^{4} a_t X_{tv} \leq E_v \; ; \qquad t = 1, 2, 3, 4$$

ii) Nuclear power and back pressure (JEF$_j$)

$$\sum_{k=1}^{4} X_{kj} - \Delta\overline{X}_j \leq \overline{X}_j \; ; \qquad j = 1, 2, \ldots, 12$$

iii) District heat plants, except electricity based ones, in heat market h (IEF$_{ih}$)

$$\sum_{k=1}^{4} Y_{kih} - \Delta\overline{Y}_{ih} \leq \overline{Y}_{ih} \; ; \qquad i = 1, 2, \ldots, 6$$

$$h = 1, 2$$

iv) Electricity based district heat plants in heat market h (QEF$_{tsh}$)

$$Q_{tsh} - \Delta\overline{Q}_{sh} \leq \overline{Q}_{sh} \; ; \qquad s = 1, 2$$

$$h = 1, 2$$

$$t = 1, 2, 3, 4$$

v) Individual heating systems in heat market h (HEF$_{rh}$)

$$H_{rh} + \sum_{\substack{n=2 \\ n \neq r}}^{3} \Delta\overline{H}_{rnh} - \sum_{\substack{n=2 \\ n \neq r}}^{3} \Delta\overline{H}_{nrh} \leq \overline{H}_{rh}$$

$$r, n = 1, 2, 3$$

$$h = 1, 2, 3$$

ΔH_{r1h} not defined.

APPENDIX B. List of symbols[30]

A. Endogenous variables

X_{vj}	gross output in sector j=1, ..., n, vintage v=0, 1, ..., t.
X_j	total output in sector j=1, ..., n.
X_{ij}	use of commodity i=1, 2, ... n in sector j=1, 2, ... n.
I_j	gross investments in sector j=1, ..., n.
C_i	household consumption of commodity i=1, ..., n.
Y	total household consumption expenditures.
Z_i	export of production sector output i=1, 2, ..., m.
M_i	import of goods competing with production sector output i=1, ..., m.
N_j^S	Domestic supply of domestic production of commodity j=1, 2, ..., n.
N_j^D	domestic demand for domestic production of commodity j=1, 2, ..., n.
Q_s	private consumption of the consumption good s=1, 2, ... S.
L_{vj}	use of labor in sector j=1, 2, ..., n, vintage v=0, 1, ..., t.
P_i	price of production sector output i=1, ..., n.

[30] This list contains definitions of variables and parameters used in the ELIAS model, presented in section 3.1, and in the integrated model, presented in sections 3.3 and 3.4.

P_{ij}	user price of energy of type i=1, 2, in sector j=1, ..., n.
P_i^D	user price of commodity i=1, ..., n.
P_{ic}	user price of energy of type i=1, 2, in the household sector.
W_j	wage rate in sector j=1, ..., n.
W	index of the level of wages in the economy as a whole.
R_j	expected rate of return on investments in sector j=1, ..., n.
Π_j	total profits in sector j=1, 2, ..., n.
P_I	price of the investment good.
x_k	level of activity k.
x	the vector of activity levels.
D_j	demand for the domestically used composite good j=1, 2, ..., m.
Q_j	gross return on new capital in sector j=1, 2, ..., n.
P_j^*	unit cost net of non-energy intermediate inputs in sector j=1, 2, ..., n.
R	equilibrium rate of return
E_j	electricity demand in TWh_e from sector j=2, 3 ..., n.[31]

[31] TWh_e denotes TWh electricity while TWh_v denotes TWh heat.

W_j^d water-borne heat demand in TWh_v from sector $j=2,3, \ldots, n$.

W_r^o oil heating demand in TWh_v from the residential services sector (i.e. sector $j=r$).

U_j electricity demand for non-heating purposes in sector $j=2, 3, \ldots, n$. TWh_e

V_j electricity demand for direct electric heating in sector $j=2, 3, \ldots, n$. TWh_e

U total annual electricity demand in TWh_e for other purposes than heating.

V total annual electricity demand in TWh_e for direct electric heating.

W_h demand for water-borne heating energy in local heat market $h=1, 2, 3$. TWh_v

W total annual demand for water-borne heating energy. TWh_v

$N^S \equiv (N_1^S, N_2^S, \ldots, N_n^S)$

$X_v \equiv (X_{01}, X_{11}, \ldots, X_{tn})$

$Q \equiv (Q_1, Q_2, \ldots, Q_S)$

$M \equiv (M_1, M_2, \ldots, M_m)$

$L_v \equiv (L_{01}, L_{11}, \ldots, L_{tn})$

B. Exogenous variables in period t

L^S	supply of labor.
G	public consumption.
D	surplus on the current account expressed in foreign exchange
P_i^{WI}	price level of imported goods competing with domestically produced commodity i=1, ..., m. Since imports are measured in the base year domestic market prices, this price corresponds to the c.i.f. import price plus the base year import duties.
P_i^{WE}	f.o.b. price level on foreign markets where domestic producers of commodity i=1, 2, ..., compete.
P_j^C	c.i.f. price level of complementary imports used as inputs in sector j=1, 2.
τ_i	tax on energy of type i=1, 2.
ξ_{ij}	tax on energy of type i=1, 2 used in sector j=1, ..., n.
ξ_{ic}	tax on energy of type i=1, 2 used in the household sector.
I	total gross investments.
$P_{e\tau}^{WE}$	export price for electricity in time segment τ=1, 2, 3, 4.
$P_{e\tau}^{WI}$	import price for electricity in time segment τ=1, 2, 3, 4.
P_c^{WI}	import price for coal.
P_u^{WI}	import price for nuclear fuel.

C. Parameters[32]

a_{vij} — input of energy i=1, 2, per unit of output in vintage v=0, 1, ..., t in sector j=1, ..., n.

a_{ij} — input of commodity i=3, ..., n per unit of output in sector j=1, ..., n.

a_{jI} — input of commodity i=3, ..., n per unit of the capital good

b_j — input of complementary imports per unit of output in sector j=1, 2.

t_{is} — relative weight of commodity i=1, ..., n in consumer commodity group s=1, 2, ..., S.

ω_j — index of the relative wage rate in sector j=1, ..., n.

Θ_i — indirect tax on commodity i=1, ..., n.

ϕ_i — custom duty and indirect tax on the imports of commodities competing with commodity group i=1, ..., m.

δ_j — annual rate of depreciation of capital in sector j=1, ..., n.

μ_i — elasticity parameter between domestically produced and imported units of commodity i=1, ..., m.

n_i^D, m_1 — distribution parameters in the CES-function representing the trade-off between imported and domestically produced goods of type i=1, ..., m.

φ_j — the elasticity of investments in sector j=1, ..., n with respect to the expected excess profit ratio.

n_i^S, z_i — distribution parameters in the CES-function representing the transformation possibilities between the domestic and the export markets for commodities i=1, 2, ..., m.

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[32] The difference between "exogenous variables" and "parameters" is that the former may have different values in different periods, while the latter are constant over time.

ε_i elasticity parameter between the domestic and the export markets for commodities i=1, 2, ... m.

u_τ the share of total annual electricity demand for non-heating purposes which occur during time segment τ.

v_τ the share of total annual electricity demand for direct electric heating which occur during time segment τ.

w_τ the share of the annual demand for water-borne heating energy which occur during time segment τ.

$B = \begin{bmatrix} B_s \\ B_k \\ B_b \end{bmatrix}$ the technology matrix for the electricity and heat supply model where

B_s is the submatrix containing the commodity rows,

B_k is the submatrix of capacity utilization coefficients,

B_b is an auxiliary constraint submatrix.

$b_{e_\tau k}$ output or input of electricity in time segment τ in activity k.

$b_{w_{\tau h} k}$ output of heat in time segment τ on local heat market h from activity k.

b_{2k} input of oil in activity k.

b_{ck} input of coal in activity k.

b_{uk} input of nuclear fuel in activity k.

b_{jk} input of other intermediate commodites in activity k.

b_{Lk} input of labor in activity k.

$b_{z_\tau k}$ export of electricity from activity k in time segment τ.

$b_{m_\tau k}$ import of electricity from activity k in time segment τ.

g a vector of initial capacities in the electricity and heat sector.

b a right-hand side vector for the auxiliary constraints.

c^T a vector containing taxes, distribution costs and capital costs associated with the activities x.

e_j conversion to electricity demand in TWh_e per unit of the electricity/district heat composite good demanded in sector j=2, 3, ...,n.

f_j conversion to water-borne heating energy demand in TWh_v per unit of the electricity/district heat composite good demanded in sector j=2, 3, ...,n.

a_r the share of non-heating fuel demand in total fuel demand in the residential services sector (i.e. sector j=r).

c_r conversion to water-borne heating energy demand in TWh_v per unit of heating fuel demand in the residential services sector (i.e. sector j=r).

b_j the share of electricity for heating purposes in total electricity demand in sector j=2, 3, ...,n.

α_j the share of capital in value added in sector j=1, 2, ... n.

A_{vj} a time- and vintage dependent coefficient in the ex post production functions of sectors j=1, 2, ... n.

λ_h the share of local heat market h=1, 2, 3 in total annual heat demand.

D. Indices and index sets

K the set of activities.

n total number of sectors (and thus commodities)

m the number of traded commodities.

S the number of private consumption goods.

t the number of capital stock vintages.

4 The Consequences of a Nuclear Power Discontinuation

In February 1972 the first commercial nuclear power reactor in Sweden began to operate. It had an installed effect of 440 MW. Ten years later, nine nuclear power reactors were in operation with a total effect of 6425 MW. This means that the average annual growth rate of nuclear power capacity was 27 per cent during this period. Three more reactors are scheduled to be on line in 1985, bringing an additional nuclear power capacity of 3025 MW. With an average availability of 70 per cent, the energy capacity of the nuclear power system will be around 60 TWh, which is as much as is available from hydro power. Today the hydro power system has an energy capacity of around 60 TWh and only minor additions are planned for the coming decade.

The development of thermal power capacities in Sweden in the last twenty years is shown in Table 4.1. It can be seen that there also was a considerable buildup of peak- and reserve-capacity in gas turbines during the sixties and the first half of the seventies. Also, combined power and heat production capacity increased significantly.

The explanation for the buildup of the thermal power capacity is partly revealed by Figure 4.1. For a long time, the domestic hydro resources predominated the Swedish power system. Between 1950 and 1965 the production from the hydro power system grew at approximately the same rate as the total electricity demand, i.e. a little more than 6 per cent annually. As can be seen in Figure 4.1, the expansion in the hydro power system has been markedly slower since 1965. The average annual rate was 1.6 per cent between 1965 and 1981, whereas the rate of growth in electricity demand averaged 4.4 per cent.

The emerging difference in the two growth rates reflects the fact that the Swedish hydro resources were gradually becoming more or less fully exploited in the mid 1960's.[1] Thermal power had to supplement the hydro power to an increasing extent. And nuclear power was considered to be the most competitive alternative.

Table 4.1 Thermal capacity (MW) 1961/62-1981/82

	Nuclear	Conventional condense	Industrial back pressure	District heat back pressure	Gas turbines	Total
1961/62	-	1232	426	223	87	1968
1971/72	440	2741	575	820	710	5286
1976/77	3172	3418	706	1834	1719	10849
1981/82	6425	2973	882	2346	1775	14401

Source: CDL

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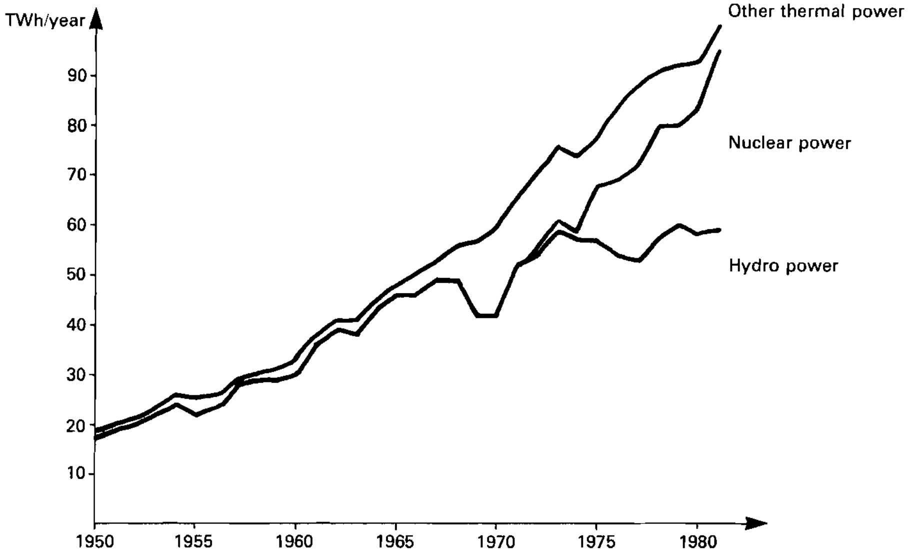
1 It is of course not meaningful to think of a well-defined, fixed upper limit, determined by technological and hydrological conditions, of hydro power. Marginal additions can be made available, but at increasingly higher costs, including environmental amenity values of unexploited waters. Decreasing returns in domestic hydro exploitation had reached a point where thermal alternatives became quite competitive.

Figure 4.1 The total electricity production 1950-1981 and its distribution among hydro power production, nuclear power production and other thermal power production



Source: CDL

During the seventies the power industry's nuclear power expansion plans came under increasingly fiercer attacks from groups concerned about the safety and environmental hazards of nuclear power reactors. Their arguments had a notable influence on the public opinion and the debate culminated in the 1980 referendum on the future use of nuclear power. Since a parliament majority supported alternatives where the use of nuclear power was to be abandoned - sooner or later - the referendum was not primarily on whether nuclear power should be allowed or not, but rather on the rate at which it should be discontinued.

The outcome of the referendum, manifested in the present nuclear power policy, was that the twelve nuclear power reactors expected to be in operation in 1985 shall be used during their remaining lifetime, but that no new investments in nuclear power capacity shall be allowed. Since the conventional estimate of the lifetime of a nuclear power reactor is twenty-five years, this means that after 2010 nuclear power shall not be used in Sweden. Even if the economic lifetime of a reactor should turn out to be longer than 25 years, the present policy still is that no nuclear power shall be allowed after 2010.

This chapter is devoted to a cost evaluation of the nuclear power discontinuation policy. The purpose is to discuss the impact of this policy on the cost and the structure of the electricity production and in particular to what extent it is likely to have any impact on the overall performance of the Swedish economy over the next thirty years. The integrated ELIAS-ENMARK model, presented in the previous chapter, is used to quantitatively illustrate the effects of the nuclear power policy.

The costs of a nuclear power discontinuation in Sweden have previously been analysed in Bergman (1981) and in Bergman and Mäler (1983). The first of these studies was done in connection with the nuclear power referendum and mainly focused on a quite fast and early discontinuation of existing nuclear power capacity. This is not a relevant alternative any longer. Instead the cost analysis in this chapter concerns the nuclear power policy adopted after the referendum. In the second study, Bergman and Mäler look at the same discontinuation alternative as in this chapter, but within the partial equilibrium framework of the ENMARK model. With the development of the integrated ELIAS-ENMARK model, there exists a more suitable tool for analyses of quantitative restrictions in the economy. Thus this chapter is a complement to their analysis, since it also considers the potential general equilibrium effects of the nuclear power policy.

A cost analysis is of interest as a part of a general policy evaluation. It is of course important to investigate whether there is a cost, and if so, how large, of pursuing the present nuclear power policy. The remaining lifetime of the present nuclear power plants is still quite long and the present policy has not yet imposed any real commitments regarding future power technologies. It is probably not necessary to make any larger investments in power capacity until around 1990. Thus, there is still an option to refrain from the nuclear power policy if its costs should turn out to exceed the value of the gains in security and environmental hazards, which motivate the present nuclear power policy.

It is also important to assess the credibility of the discontinuation policy. Present decisions on investments in durable equipment, and the technological design of that equipment, are based on expectations about future electricity prices. Presumably they could be expected to be higher if the current nuclear policy is carried out than if it were not. But if people do not regard the nuclear power policy as credible, then it will hardly influence their expectations about future electricity prices. In this case it might happen that the discontinuation cost becomes unnecessarily high because investments are too electricity-intensive.

Presumably the nuclear power policy would have a low credibility if the costs associated with it are high, or are believed to be high. In this case it is possible that some people find it hard to believe that the nuclear power discontinuation will be carried out after all. A quantitative cost analysis gives some insights into what the costs may be. If the analysis indicates that the costs are high, then the credibility could be in doubt. If the discontinuation policy in fact still shall be carried out then it may be necessary to consider signals which could convey this message to the various agents in the economy.

The chapter is organized as follows. The next section contains a brief introductory discussion of why and how a nuclear power discontinuation has a cost associated with it. Section 4.2 describes the main assumptions employed in the analysis and the results are presented in Section 4.3. The chapter ends with some concluding remarks in Section 4.4.

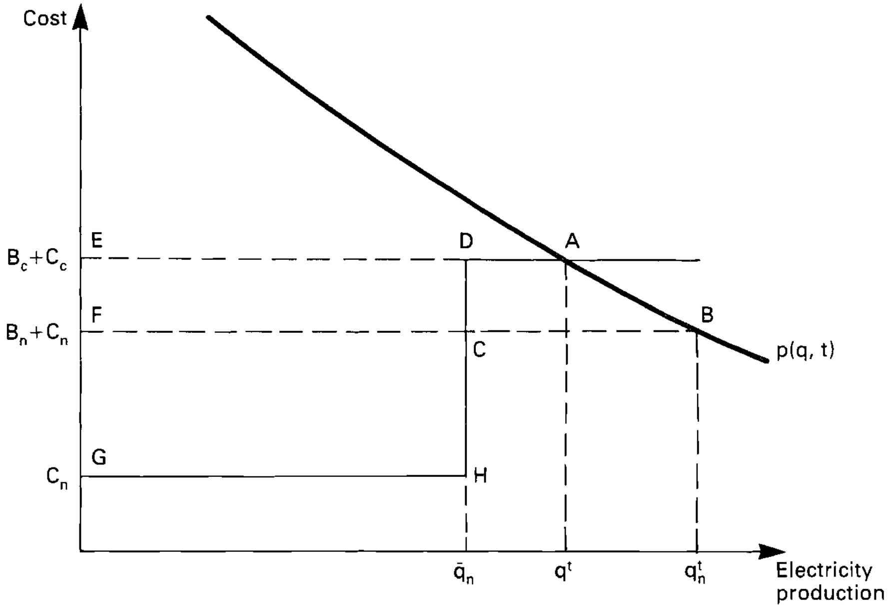
4.1 THE COST OF A NUCLEAR POWER DISCONTINUATION

The nuclear power policy, which the Swedish parliament decided on after the nuclear power referendum in 1980, consists of two parts. Apart from the twelve reactors expected to be on line in 1985, no further investments in nuclear power capacity are allowed. Secondly, since no production from nuclear power plants is allowed after 2010, it could mean that existing nuclear power plants are closed down even though it would be possible to continue to operate them for an additional number of years.

In stylized terms the situation around 2010 can be characterized as follows. There is an existing nuclear power production potential, denoted by $\bar{q}_n$ GWh. It is available at an operating cost of C_n per GWh. New nuclear power capacity can be built to produce electricity at the unit cost B_n+C_n, where B_n is the annuitized capital cost. Since the hydro resources are fully exploited, the alternative to nuclear power is conventional thermal power. It is assumed to be available at the unit cost B_c+C_c where C_c is the operating cost and B_c the annuitized capital cost of new conventional thermal power.

The cost situation is depicted in Figure 4.2. A demand function for the future time period t is also shown. It is the demand for electricity net of the supply from hydro power plants. Figure 4.2 thus refers only to thermal power production.

Figure 4.2 The cost of nuclear power discontinuation



The total electricity production is q^t as determined by the intersection of the inverse demand function p(q, t) and the unit cost line for conventional thermal power. Thus we have a situation where the electricity demand has grown sufficiently to motivate investments in new capacity. Had investments in new nuclear power capacity been allowed, the equilibrium production would have been q_n^t. The cost of not allowing new nuclear power capacity is thus given by the area ABCD if the initial capacity is still used. If the initially existing nuclear power capacity is closed down, and it actually is technologically obsolete, the additional discontinuation cost is the area EFCD, the incremental cost of conventional thermal replacements. If, however, the initial nuclear power capacity is closed down before it is technologically obsolete, there is a further cost, represented by the area CFGH, for the remaining lifetime.

Obviously, if conventional thermal power is competitive with nuclear power, i.e. if $B_c+C_c \leqq B_n+C_n$, then there cannot be a cost associated with the ban on new nuclear power capacity. In this case the discontinuation policy implies a cost only if the nuclear power plants are taken out of operation prematurely. This cost depends on the difference between the replacement cost and the operating cost of existing nuclear power capacity as well as on the remaining lifetime of the nuclear power reactors.

The cost difference between nuclear and conventional thermal power 15-20 years from now is of course highly uncertain. The most likely alternative to new nuclear power is coal fuelled condense power and/or combined power and heat production. The cost difference is primarily determined by relative capital costs and the relative fuel costs. It can be argued that since nuclear fuel and coal are predominantly used for the same purpose, i.e. power and heat production, there should not be any cost differences in the long run. If this is correct the discontinuation policy involves a cost only if nuclear power reactors are taken out of operation prematurely.

With a pronounced cost advantage for nuclear power, for instance, one would expect it to be the dominating technology in new power investments around the world. This should eventually tend to increase the price of nuclear fuel relative to the price of coal and thus to diminish the cost advantage of nuclear power and perhaps even eliminate it. But there are also good arguments to expect a persistently lower cost of nuclear power relative to coal power, at least in some parts of the world. Transportation costs are a substantial part in the delivered price of coal.[2] The construction costs may also be different in different parts of the world. Similarly, environmental regulations of both nuclear and coal power may differ around the world. Thus in certain areas coal power may be quite competitive with nuclear power while it in other areas remains more costly. IEA, for instance, has

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[2] See Sargent (1983).

claimed that coal power has a slight cost advantage in the U.S.A., whereas nuclear power is considerably more competitive in Japan. It is thus not unreasonable that coal power may remain more expensive than nuclear power in a coal-importing country like Sweden.

Then there is the widespread opposition in many countries against nuclear power. It has probably contributed to make coal power a more interesting alternative even if it should be somewhat more expensive.

It is thus quite plausible that nuclear power, at least in Sweden, may continue to have a lower cost than coal power.

The quantitative analysis in this chapter has been carried out under the assumption that nuclear power has a cost advantage compared to coal condense power. In the model calculations the marginal cost of electricity produced in new nuclear power plants is 70 per cent of the corresponding marginal cost of coal condense power. This relation is consistent with several recent reports on the costs of new nuclear power.[3]

The purpose of the analysis in this chapter is to estimate the cost of the present nuclear power policy given the specific assumption that nuclear power remains to have a cost advantage over coal power also in the future. In a sense this represents one "extreme" alternative. The other "extreme" is that the cost advantage of nuclear power would vanish fairly soon. In that case there would obviously not be any cost at all associated with the present nuclear power policy (except if the nuclear power plants are closed down even if they could be used beyond 2010). The general equilibrium model presented in the previous chapter is used to provide quantitative cost estimates. With this analytical tool, the electricity demand

[3] FTEE, January 1983.

can be derived from, and thus related to, the development of the rest of the economy. Moreover, it makes the impact of the discontinuation policy on the rest of the economy explicit. The potential discontinuation cost can thus be expressed in terms of the effect on the pattern of resource allocation in the economy.

4.2 DATA AND THE MAIN ASSUMPTIONS OF THE ANALYSIS

The cost analysis is carried out with the integrated ELIAS--ENMARK model described in the previous chapter. Its main data were described in Section 3.4 in Chapter 3. In this section the assumptions regarding exogenous variables and some additional parameters are described.

The sectoral rates of technological change are assumed to be around 2.3 % annually with somewhat higher values for the export-competing sectors. By convention, the productivity growth in the public sector is set to zero. The main motivation for these values is that they yield (together with the exogenous investments) an average annual GNP growth of around 2 % over the next thirty years. This growth rate is quite arbitrary but in view of historical growth rates it does not seem too unreasonable. For the fifty years 1930-1980 the average annual growth in Swedish GNP was 3.0 % and for the post-war period 1950-1980 it was 3.3 %.[4] The last figure does however hide the fact that the GNP growth rate in the seventies slowed down considerably compared to the previous two decades and in particular compared to the sixties. The presumption behind the two per cent growth rate is thus that the poor performance recently will not be a lasting phenomenon but that the Swedish economy hardly will return to the unique growth figures of the sixties.

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4 These figures are taken from Bentzel (1983).

The exogenous variables are the world market prices, the labor supply, the gross investments, the public sector production and the current account requirement. The following assumptions are made:

- the labor supply decreases with 0.2 % annually during the period 1980-2010
- the gross investments increase with 3.0 % annually for the same period
- the public production, and the public capital stock, increase by 0.9 % annually for the same period
- the current account improves considerably during the eighties and external balance is reached in 1995
- for the non-energy real world market prices the prices for the relatively labor-intensive import competing goods (sector 2) and the natural resource based export industries (sector 3) are assumed to decrease somewhat (0.2 % annually) and the prices for traded service production to increase somewhat (0.2 % annually).

The world market energy prices, i.e. nuclear fuel, coal and oil prices are also important exogenous variables. For these the following assumptions are made:

- for all three the actual development until 1983 is accounted for
- for the rest of the 1980's all three real prices are assumed to remain constant
- from 1990 and onwards the oil price is assumed to increase by 2 % annually while the coal price is assumed to increase by 1 % annually.

There is some degree of arbitrariness in these price assumptions. But in view of the large oil price increases 1979-1980, and that households and firms still are adjusting to them, it does not seem too implausible that the current period with stagnating oil prices will continue for a time.[5] Both oil and coal, however, are exhaustible natural resources so sooner or later one would expect their real prices to begin to rise.

The capital costs of the power and heat technologies in the electricity and heat sector have been calculated with a six per cent rate of discount. It is approximately equal to the average of the marginal rates of return which are realized in the remaining sectors in the simulations below.

4.3 THE MODEL SIMULATIONS AND THEIR RESULTS

The integrated ELIAS-ENMARK model is employed to simulate the development of the Swedish economy over the thirty-year period 1980-2010 under different assumptions about the use of nuclear power.[6] Two main alternatives for the nuclear power policy and for the economic lifetime of the nuclear power reactors are considered. In the first alternative there is no restriction at all on the use of nuclear power. Investments in new nuclear power capacity are allowed and the nuclear power reactors built up to 1985 are gradually taken out of operation after serving their full economic lifetime. In the second alternative, nuclear power investments are not allowed after 1985. But still the twelve existing reactors can be operated during their full economic lifetime.

As for the lifetime of the reactors the two alternatives twenty-five and forty years are used. In all, this makes four investigated alternatives, as shown by the following matrix:

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5 Such a view of the current international oil market is confirmed by a number of model simulations in a comprehensive study by Energy Modeling Forum at Stanford. See EMF (1982). The model simulations generally result in rather stable oil prices until around 1990.

6 In some cases the simulations are extended to 2020.

Operating time for nuclear power reactors / Nuclear power policy	A. 25 years of operation	B. 40 years of operation
1. Nuclear power investments allowed	1 A	1 B
2. Nuclear power investments not allowed	2 A	2 B

The consequences of closing down nuclear power reactors after twenty-five years of operation even if it would be possible to continue to run them longer, say for forty years, can be indirectly assessed by comparing the alternatives 2 A and 2 B.

These are not the only possible ways to pursue the discontinuation policy. Another way, for example, could be to allow the existing nuclear power reactors to operate for more than twenty-five years, but not any longer than until 2010. The economic consequences of the nuclear power discontinuation policy obviously depend on how it more exactly is carried out. Since this is unclear today, the main simulations focus on a gradual phasing out of nuclear power reactors, starting around 1995 when the first built reactor has been operating for twenty-five years, and continuing until 2010, when all nuclear power shall be abolished. The effects of a more concentrated discontinuation around 2010 will also be considered, however.

In order to keep the number of solutions of the model at a manageable level, it is solved for five eight-year periods. It would have been preferable to have shorter period-lengths (three or four years) because the sectoral investments and their technological design can then respond more frequently to the changes in the economic environment. With longer period-lengths the model tends to get somewhat inflexible in this respect. In Chapter 5 the period 1979-1995 is analyzed in more detail using a four-year period-length. A comparison of the results for this period for the two different period--lengths shows that the differences are rather small but not negligible. In particular, the longer period-length used in this chapter results in a higher electricity demand, because the response to rising electricity prices occurs with a lag. However, a shorter period-length would have increased the number of solutions and thus the solution costs. As always, one has to strike a balance between the preferable and the practical and in this case it meant using the eight-year period-length.

The presentation of the results starts with the development of the macroeconomic variables in the four main simulations. The impact of the nuclear power discontinuation in terms of losses in GNP and total private consumption is summarized in total cost measures. After some comments on the effects on the resource allocation pattern in the model economy, the impact on the electricity and heat sector is considered in more detail.

The macroeconomic impact

The impact on the model economy's macroeconomic performance of not allowing nuclear power investments is quite marginal. The development over the studied period for some macroeconomic key figures is given in Table 4.2.

Table 4.2 Macroeconomic key figures in the four main simulations. 1979=100.

	1995				2010			
	1 A	1 B	2 A	2 B	1 A	1 B	2 A	2 B
GNP	142.1	142.1	142.3	142.2	209.1	210.3	207.7	209.4
average annual growth % since 1979	2.2	2.2	2.2	2.2	2.4	2.4	2.4	2.4
Private consumption	130.4	130.4	130.6	130.6	202.3	204.3	198.4	201.2
average annual growth % since 1979	1.7	1.7	1.7	1.7	2.3	2.3	2.2	2.3
Exports	173.8	173.8	174.0	174.4	266.2	268.2	267.5	272.0
average annual growth % since 1979	3.5	3.5	3.5	3.5	3.2	3.2	3.2	3.2
Imports	139.1	139.1	139.0	139.6	211.4	212.9	210.6	214.2
average annual growth % since 1979	2.1	2.1	2.1	2.1	2.4	2.4	2.4	2.5

1 A = Nuclear power investments allowed, 25 years lifetime
1 B = " " " " , 40 " "
2 A = Nuclear power investments not allowed, 25 years lifetime
2 B = " " " " " , 40 " "

It is not until the turn of the century that a ban on nuclear power investments at all makes an impact. For 1995 it seems, somewhat surprisingly, that alternative 2 (i.e. no nuclear investments) is slightly more beneficial in terms of GNP and private consumption. This is due to the fact that the gross investments in the electricity and heat sector are twenty-six per cent higher in alternative 1 than in alternative 2 in the period prior to 1995.[7] Consequently, since the total investments in the economy are exogenous, the capital formation in the other industrial sectors is correspondingly lower. Since the investment decisions in the model are very short-sighted, being based on current prices only, there seems to

7 Due to investments in nuclear heat reactors (Secure), which are not allowed in the second nuclear power policy alternative. In the latter case, the model instead invests in coal fired heat plants which means lower capital outlays (see Table 3.5 in Chapter 3).

be too much investments in heat capacity prior to 1995, since the electricity prices later rise and the heat prices decline. In nuclear power policy alternative 1, capital outlays on heat capacity are somewhat larger. Thus, in spite of the restrictions on the use of the nuclear power technologies in alternative 2, it turns out to be temporarily slightly more beneficial. But the difference is very small indeed.

After the year 2000 the differences in terms of GNP and private consumption under the two nuclear power policies are more marked, but still small. In 2010 the GNP-loss incurred by the ban on nuclear power investments is 0.7 per cent if the operating time for nuclear power reactors is twenty-five years and 0.4 per cent if it is forty years. In the latter case the GNP loss also increases to 0.7 per cent when all nuclear power capacity which existed in 1985 has been taken out of operation (which in the simulations happened in 2020). The corresponding figures for the loss in private consumption are 2.0 per cent and 1.5 per cent respectively in 2010 and 2.0 per cent in 2020. Thus, the relative impact on private consumption is larger than for GNP. Because total investments, public expenditures and the current account net surplus are exogenous, most of the model economy's adaptation to changes must fall on private consumption.

The main reason for the lower GNP in the discontinuation alternatives is the productivity loss, which is caused by the higher cost of electricity production. The productivity loss occurs in two ways. First, more of the domestic resources must be used, directly or indirectly, for the production of electricity. The higher cost of electricity mainly manifests itself in a higher level of complementary imports. Thus more of the domestic resources are required in order to pay for the higher imports. Secondly, the higher electricity price induces a substitution away from electricity and instead the domestic production becomes relatively more labor and capital intensive. Thus, for given domestic factor supplies, these substitution effects also make GNP lower.

The sectoral allocation of the total exogenous investments is not so important for the GNP effects. In the first half of the simulation period the investments in the electricity and heat sector are somewhat higher in the discontinuation alternatives, whereas they in the second half are somewhat lower. This is mainly caused by a different timing of the investments in combined power and heat plants. They occur somewhat later in the alternatives where nuclear power is allowed and where thus the electricity price is lower. But the differences in the investment profiles are not very large. Moreover, the rate of return to new capital in the electricity and heat sector, which is six per cent by assumption, is approximately equal to the average of the marginal rates of return in the remaining sectors. Thus there are no pronounced productivity effects of differences in the sectoral allocation of the total investments.

Even if the effects on GNP and private consumption are small in relative terms, and for a single year, they may still amount to a considerable value in absolute terms, especially when the effects for several years are added to a total cost measure of the nuclear power discontinuation policy.

The theoretically most satisfactory way to calculate such total cost measures would be to compare the present value of the time paths for GNP, or private consumption, over an infinite time horizon. The computer simulations must however be carried out for a finite number of time periods. To at least partially account for the cost in terms of GNP losses and reduced private consumption which occur after the final simulation period, the total cost measures presented below have been calculated on the assumption that the relative losses incurred in the final simulation period are permanent. Assuming a steady state growth of 2.2 per cent the present values of these perpetual losses can be calculated.

They are, however, probably overstatements of the discontinuation costs. The model economy has not completely adjusted to the higher long run electricity equilibrium prices in the final simulation period: with further adjustments the losses should be reduced. Furthermore, it is perhaps not plausible to expect nuclear power to have a cost advantage over coal power eternally. For these reasons, the losses incurred during the simulation period only are also given.

The effect on the total private consumption expenditures is a measure of the real income effect of the households. If the relative prices of the consumption goods differ between the simulations it is however a problematic measure since it compares consumption baskets using different weights (i.e. relative prices). Therefore it is better to use standard measures of welfare changes such as the compensating variation or the equivalent variation. The latter is preferable when more than two alternatives shall be compared, since it preserves transitivity. However, if the relative prices of the consumption goods are the same in the different alternatives then the differences in total private consumption coincide with the equivalent variations (as well as the compensating variations). In the simulations presented here, this actually turns out to be the case, making it possible to focus directly on the changes in the private consumption expenditures, measured in current prices. To achieve some comparability with the estimated GNP-losses, which are measured in constant base year prices, the effects on private consumption reported below are however instead measured in constant base year prices. The private consumption losses measured in current prices are somewhat lower.

The total present values of GNP and private consumption for the four simulation alternatives are given in Tables 4.3 and 4.4. They are expressed as deviations from alternative 1 B, the most favourable one. To illustrate how sensitive these cost measures are to the chosen rate of discount, they have been calculated for three different discount rates, four, six and ten per cent. The six per cent discount rate corresponds approximately to the average of the marginal rates of return in the different sectors.

Table 4.3 Present value in 1983 of GNP. Deviations from alternative 1 B. Billion SEK. The figures in brackets are the losses which occur between 1980 and 2020.

		Operating time	
	Discount rate	25 years A	40 years B
Nuclear power investments allowed 1	4 %	- 161.7 (- 39.3)	-
	6 %	- 45.4 (- 20.5)	-
	10 %	- 8.2 (- 6.1)	-
Nuclear power investment not allowed 2	4 %	- 284.6 (- 83.4)	- 119.4 (- 33.8)
	6 %	- 93.8 (- 46.5)	- 43.4 (- 17.6)
	10 %	- 18.3 (- 14.6)	- 9.7 (- 5.8)

Table 4.4 Present value in 1983 of private consumption. Deviations from alternative 1 B. Billion SEK. The figures in brackets are the losses which occur between 1980 and 2020.

	Discount rate	Operating time 25 years A	Operating time 40 years B
Nuclear power investments allowed 1	4 %	- 169.0 (- 36.7)	-
	6 %	- 47.5 (- 19.3)	-
	10 %	- 8.4 (- 5.6)	-
Nuclear power investments not allowed 2	4 %	- 341.5 (- 86.1)	- 164.1 (- 38.5)
	6 %	- 100.6 (- 46.2)	- 46.8 (- 20.1)
	10 %	- 19.5 (- 14.1)	- 8.4 (- 5.7)

In terms of the present value of accumulated GNP-losses, the cost of the nuclear power discontinuation policy is, using a six per cent discount rate, 43.4 billion SEK if nuclear power stations can be operated for forty years, or 48.4 billion SEK if they have a lifetime of twenty-five years (48.4 is the difference between the alternative where nuclear power investments are not allowed and the one where they are allowed).

If, as a result of the nuclear power discontinuation policy, the existing nuclear power stations are taken out of operation after twenty-five years even if they could be operated for forty years, the GNP-loss increases to 93.8 billion SEK. In this case the major source of the cost is the premature scrapping of nuclear power capacity. It accounts for 50.4 billion SEK, i.e. 54 per cent of the total cost, since the cost of only banning new nuclear power investments is 43.4 billion SEK.

In terms of private consumption losses the corresponding figures are 46.8 billion SEK for forty years lifetime, 53.1 billion SEK for twenty-five years lifetime and 100.6 billion SEK if nuclear power capacity is prematurely scrapped.[8]

If the losses in GNP and private consumption after the final simulation period are disregarded the discontinuation cost is considerably reduced. In terms of GNP-losses it is 17.6 billion SEK with forty years lifetime of the nuclear power stations, it is 26.0 billion SEK with twenty-five years lifetime and 46.5 billion SEK if the nuclear power stations are prematurely scrapped. In terms of losses in private consumption it is 20.1 billion SEK (instead of 46.8 billion SEK) with forty years lifetime, it is 26.9 billion SEK (instead of 53.1 billion SEK) with twenty-five years lifetime and 46.2 billion SEK (instead of 100.6 billion SEK) with a premature scrapping.

Tables 4.3 and 4.4 clearly illustrate how sensitive the numerical values of the losses in GNP and private consumption are to the value of the discount rate. Thus the calculated cost figures should be used and interpreted carefully. Although the results in Table 4.2 indicate that the yearly relative losses due to the nuclear power policy are quite marginal, the accumulated cost in absolute terms could be substantial, particularly if also distant future losses are valued highly (i.e. if the discount rate is low).

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8 The losses in terms of private consumption exceed the losses in terms of GNP because of what may be called a terms-of-trade effect. The losses are measured in constant base year prices. In the simulation alternatives where nuclear power is discontinued there are in particular larger coal imports. Since the import price of coal increases over time relative to the other export and import prices (except the price of petroleum products) the required current account surplus is also higher in these simulations when measured in the constant base year prices. Consequently the decrease in the private consumption, measured in constant base year prices, is larger than for GNP.

The sectoral impacts

The impact of the nuclear power discontinuation policy on the resource allocation pattern in the model economy is not very dramatic, at least not with the level of aggregation used here. The relative prices, except the electricity price, are very similar across the alternatives.

For sector 4, i.e. the mainly exporting energy intensive industries, there is a slightly higher producer price in the latter part of the simulation period in the discontinuation alternatives 2 A and 2 B compared to the alternatives where nuclear power is allowed. Otherwise the output prices are unaffected by the nuclear power policy. The wage rate is however lower from 1995 and onwards in the discontinuation alternatives. In the case with a reactor lifetime of twenty-five years it is in 1995 1.4 per cent lower, in 2003 1.5 per cent lower and in 2011 2.5 per cent lower. In the case with a reactor lifetime of forty years the effect on the wage rate is similar but occurs later because the nuclear power plants are now used for a longer time.

The impact on the sectoral pattern of production is shown in Table 4.5. Prior to 1995 there are no effects at all. The greatest impact is again on the energy intensive export industries (sector 4). After the turn of the century the production is around 3 per cent lower in this sector with the ban on nuclear power than without it. The lower production in sector 4 is mainly the result of lower exports from this sector, which is illustrated in Table 4.6. The exports are 10-13 per cent lower after year 2000. The impacts on the remaining sectors are fairly small. There is, in the discontinuation alternative, a higher production and higher exports from sector 5 (other mainly exporting industries) whereas the production in sector 3 (mainly import competing industries) decreases as do also its exports. The imports are lower for all commodities in the discontinuation alternative.

Table 4.5 The effects of the nuclear power discontinuation policy on the sectoral distribution of production. The deviation of alternative 2 A (nuclear power not allowed) from alternative 1 A (nuclear power allowed). Per cent.

Year Sector	1995	2003	2011
2	+ 0.4	- 1.0	± 0
3	+ 0.4	- 0.2	- 1.1
4	- 1.0	- 3.0	- 3.1
5	+ 0.3	+ 0.5	+ 0.8
6	+ 0.2	+ 0.2	- 0.8

Table 4.6 The effects of the nuclear power discontinuation policy on exports and imports. The deviation of alternative 2 A (nuclear power not allowed) from alternative 1 A (nuclear power allowed). Per cent.

Year	1995		2003		2011	
Sector	Exports	Imports	Exports	Imports	Exports	Imports
2	± 0	+ 0.4	± 0	- 1.1	± 0	± 0
3	+ 0.4	- 0.9	± 0	- 0.4	- 0.7	- 3.0
4	- 2.7	± 0	-10.2	± 0	-13.0	± 0
5	+ 0.5	- 0.1	+ 0.8	- 0.1	+ 1.3	- 0.8
6	+ 1.1	± 0	+ 0.4	- 0.5	+ 1.1	- 1.1

Table 4.7 The effects of the nuclear power discontinuation policy on the sectoral use of energy. The deviation of alternative 2 A (nuclear power not allowed) from alternative 1 A (nuclear power allowed). Per cent.

Year	1995		2003		2011	
Sector	Fuels	Elec.	Fuels	Elec.	Fuels	Elec.
2	+ 0.5	+12.5	- 0.6	± 0	± 0	± 0
3	+ 0.3	+ 0.5	- 0.3	- 4.8	- 1.1	-14.5
4	- 0.9	± 0	- 2.9	- 6.8	- 2.0	-17.4
5	+ 1.1	+ 0.6	+ 0.5	- 4.4	+ 0.7	-18.5
6	+ 0.4	+ 0.4	- 0.4	- 3.3	- 1.2	-14.7
7	± 0	± 0	± 0	- 1.6	- 0.6	- 7.4

Table 4.7, finally, shows the effects on the energy use of the nuclear power discontinuation policy. In the discontinuation alternative the use of electricity decreases dramatically after the turn of the century when the electricity price has reached the long run marginal cost of coal condense power. Also the use of fuels is then lower (except in sector 5).

The impact on the cost and production of electricity

The electricity production, the producer price of electricity and the structure of the electricity and heat sector differ significantly between the four main simulations. Table 4.8 shows the producer prices of electricity and of district heat as well as the total consumption of electricity and district heat in the four alternatives.

The large nuclear power investment programme increases the nuclear power capacity from 3710 MW to 9440 MW during the first half of the 1980's, i.e. a 150 per cent increase, concentrated to a few years. The result is an excess electricity production capacity which lasts until the mid 1990's when the electricity demand has grown enough (a growth partly induced by low electricity prices caused by the excess capacity) to meet the capacity. The producer price of electricity actually decreases down to the short run marginal cost of nuclear power in the latter part of the 1980's. In 1995 it has risen again and is in all the four simulations equal to, or close to, the long run marginal cost of nuclear power.[9] In the A-simulation 1010 MW (i.e. 11 per cent) of the nuclear power capacity is scrapped in 1995. In alternative 1, where new nuclear power investments are allowed, the scrapped nuclear power capacity is replaced by new nuclear power and in fact the installed

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9 The capital costs of the power and heat technologies in the ENMARK part of the integrated model have been calculated with a rate of discount of six per cent.

Table 4.8 Consumption of electricity and heat and the production costs of electricity and district heat in the four simulations.

Year	Simulation[1]	Electricity production cost SEK/kWh	Electricity consumption TWh	District heat production cost SEK/kWh[2]		District heat consumption TWh
1987	1 A	.05	110.7	.10	.10	52.9
	1 B	.05	110.7	.10	.10	52.9
	2 A	.05	112.9	.10	.11	52.9
	2 B	.05	112.9	.10	.11	52.9
1995	1 A	.14	143.7	.06	.07	65.6
	1 B	.14	143.7	.06	.06	65.6
	2 A	.16	143.8	.05	.06	66.0
	2 B	.13	144.2	.07	.08	65.9
2003	1 A	.14	141.4	.08	.08	74.4
	1 B	.08	145.5	.09	.10	74.5
	2 A	.21	136.0	.04	.04	74.3
	2 B	.10	146.8	.09	.10	75.0
2011	1 A	.14	161.8	.08	.08	85.1
	1 B	.14	186.6	.08	.08	88.7
	2 A	.23	135.8	.03	.03	86.1
	2 B	.23	171.3	.03	.03	88.5

[1] 1 A = Nuclear power investments allowed, 25 years lifetime
1 B = " " " " , 40 " "
2 A = Nuclear power investments not allowed, 25 years lifetime
2 B = " " " " " , 40 " "

[2] The first figure refers to district heat market 1 (i.e. the largest urban areas) whereas the second figure refers to district heat market 2.

nuclear power capacity increases somewhat. In the second alternative the constraint on new nuclear power investments instead forces the electricity price to increase above the long run marginal cost of nuclear power.

The electricity price remains equal to the long run marginal cost of nuclear power in the simulations where new nuclear power investments are allowed. With the nuclear power investment constraint it continues to rise, however, as the electricity demand grows over time and as more existing nuclear power capacity is scrapped. Around 2000 it reaches the long-run marginal cost of coal condense power which now replaces nuclear power as the fundamental base load capacity of the model economy's electricity sector.[10]

In case 2 B, with the longer lifetime of nuclear power reactors, the electricity price rises slower and reaches the long run marginal cost of coal power first around 2010.

The consumption of electricity is very similar in the four simulations until the turn of the century. Following the low electricity prices in the mid 1980's the rate of growth in electricity demand increases, mainly because of a more intensive use of electricity in the various production processes of the model economy. But after the electricity price increases in the mid 1990's, the electricity demand stagnates and in two of the simulations it actually decreases.

Around 2010 the electricity consumption differs quite a lot between the four simulations. In the case of a nuclear power discontinuation and a lifetime of twenty-five years for nuclear

[10] In the alternatives 1 B and 2 B the electricity price dips in 2003. This is due to the shortsightedness in the model's investment decisions. In 1995 there are substantial investments in coal based combined power and heat plants. As a result of the higher electricity price, the electricity demand stagnates in the following period. The district heat demand continues to grow, however, and as a result there is in 2003 too much base load power capacity relative to the capacity for combined power and heat production. The result is a lower electricity price and higher district heat prices, except in the A-simulations where these effects are counteracted by the scrapping of existing nuclear power capacity.

power reactors, the electricity consumption remains constant at 136 TWh for the first decade of the next century. In the cases where new nuclear power investments are allowed, the electricity consumption ranges between 162 and 187 TWh around 2010. With a nuclear power discontinuation but allowing nuclear power plants to be operated for forty instead of twenty-five years, the electricity consumption increases to 171 TWh in spite of the high electricity price. But due to the vintage structure of the model, higher prices also have a lagged effect and continuing this simulation alternative one additional period, the result is a decline to 159 TWh.

The district heat markets are also, to some extent, affected by the nuclear power discontinuation policy. The total production levels are largely the same in all the four simulations over the whole simulation period, but the district heat prices differ, especially in the latter half of the studied period. As a result of the high electricity price (equal to the long run marginal cost of coal condense power) the marginal cost of heat is low and consequently the district heat prices are low. Around 2010 they are less than half of the district heat prices in the simulations where the electricity price is equal to the long run marginal cost of nuclear power.

Tables 4.9-4.12 show how the production of electricity and heat is distributed among different categories of power and heat technologies. In all the four simulations there is a large increase of combined power and heat production. Except for some marginal quantities it comes from coal fuelled combined power and heat plants. This large scale introduction of coal based electricity and heat production is however not a consequence of the nuclear power discontinuation policy, but rather of the oil price increases in 1979-1980. They result in substantial conversions from individual oil heating to district heat in all the four simulations.

In the nuclear power discontinuation policy alternatives the reductions in nuclear power capacity are replaced by electricity demand reductions, by production in coal condense plants and, to a minor extent, by higher production in coal based combined power and heat plants. Thus, in the case with twenty-five years lifetime (i.e. the simulations 1 A and 2 A) the 57 TWh of nuclear power production in the case where nuclear power is allowed are with the discontinuation policy replaced by 28 TWh of coal condense power, a little less than 3 TWh of coal based combined power and heat production and by 26 TWh lower total electricity demand. Almost half of the nuclear power capacity can thus be compensated by electricity demand reductions.

Similarly, in the case with forty years lifetime, the 84 TWh of nuclear power production around 2020 are in the discontinuation alternative replaced by 47 TWh of coal condense power production, by one TWh of coal based combined power and heat production and by 36 TWh lower total electricity demand.

The lower electricity demand in the discontinuation alternatives is the result of lower electricity input coefficients associated with capital invested in 1995 and later and of less electricity used as an intermediate input in the electricity and heat sector. Despite the fact that there are no ex post substitution possibilities in electricity use in the model economy (except within the electricity and heat sector) the electricity demand is quite sensitive to the electricity price.

Finally it can be noted that the investments in combined power and heat plants do not take place until the mid 1990's. During the 1980's there are instead investments in plants which produce heat only, which is due to the excess electricity production capacity and the resulting low electricity price. In the alternatives where nuclear power investments are allowed it is, in the large urban areas (i.e. in local heat market 1) profitable to invest in nuclear heat plants (the Secure reactors). However, because of the short-sightedness in the model's investment decisions, this profitability is questionable. Given the electricity and heat prices in the 1980's, the Secure reactor seems economically justifiable. But these prices are considerably influenced by the overcapacity in the electricity system. When the overcapacity is eliminated and the electricity price increases to a long run equilibrium value, heat can be produced at a lower marginal cost in combined power and heat plants. Thus the equilibrium price of district heat decreases and is no longer sufficient to cover the long run marginal cost of the Secure reactor.

Table 4.9 Electricity and heat production by different technologies. TWh. Simulation 1 A.

	1987	1995	2003	2011
Electricity				
Hydro power	61.5	61.5	61.5	61.5
Nuclear	46.5	55.7	46.9	56.8
Combined power & heat	2.7	26.5	33.0	43.5
Coal condense	-	-	-	-
Heat				
Combined power & heat	4.7	46.4	57.8	76.1
Other heat plants	48.2	19.2	16.6	9.0

Table 4.10 Electricity and heat production by different technologies. TWh. Simulation 2 A.

	1987	1995	2003	2011
Electricity				
Hydro power	61.5	61.5	61.5	61.5
Nuclear	48.2	46.4	31.6	-
Combined power & heat	3.2	35.9	40.5	46.1
Coal condense	-	-	2.4	28.2
Heat				
Combined power & heat	5.6	62.8	70.9	80.7
Other heat plants	48.3	3.2	3.4	5.4

Table 4.11 Electricity and heat production by different technologies. TWh. Simulation 1 B.

	1987	1995	2003	2011	2019
Electricity					
Hydro power	61.5	61.5	61.5	61.5	61.5
Nuclear	46.5	55.7	55.7	81.6	84.1
Combined power & heat	2.7	26.5	28.3	43.5	49.4
Coal condense	-	-	-	-	-
Heat					
Combined power & heat	4.7	46.4	49.5	76.1	86.5
Other heat plants	48.2	19.2	25.0	12.6	6.9

Table 4.12 Electricity and heat production by different technologies. TWh. Simulation 2 B.

	1987	1995	2003	2011	2019
Electricity					
Hydro power	61.5	61.5	61.5	61.5	61.5
Nuclear	48.2	52.0	52.0	31.6	-
Combined power & heat	3.2	30.7	33.3	48.3	50.5
Coal condense	-	-	-	29.9	47.2
Heat					
Combined power & heat	5.6	53.7	58.3	84.5	82.6
Other heat plants	48.3	12.2	16.7	4.0	1.4

Summary of the simulation results

The effects of the nuclear power discontinuation policy in the model economy can be summarized as follows:

- The long run equilibrium producer price of electricity increases from .14 SEK per kWh (the long run marginal cost of nuclear power) to .23 SEK per kWh (the long run marginal cost of coal condense power), i.e. a sixty-four per cent price increase.

- As a result of the massive, and in time concentrated, nuclear power investment programme in the seventies and the first half of the eighties, there is an excess capacity in electricity production which lasts until the mid 1990's. As long as the electricity excess capacity prevails, the electricity price is below the long run marginal cost of nuclear power and in fact, in the latter half of the 1980's, decreases down to the short run marginal cost of nuclear power.

- The electricity demand in the model economy is quite sensitive to changes in the electricity price, and the higher electricity prices which result from the nuclear power discontinuation lead to a significantly lower electricity demand.

- The discontinued nuclear power capacity is to almost fifty per cent compensated by a lower electricity demand. The rest is replaced by coal condense power and, to a minor extent, by increased production in coal fuelled combined power and heat plants.

- With the level of aggregation employed in the model economy, there are no dramatic effects on the structural composition of production and foreign trade. The most noticeble effects are a lower production and lower exports from the energy-intensive exporting industries (sector 4) and a considerably lower intermediate use of energy, particularly electricity. The relative prices, except the electricity and district heat prices, are largely unaffected. The wage rate is 1.5-2.5 per cent lower.

- There is a total productivity loss to the model economy caused by the higher cost of electricity following the nuclear power discontinuation. GNP is thus lower and most of the loss shows up in a lower private consumption. In relative terms the losses seem modest; 0.7 per cent lower GNP per annum and 2.0 per cent lower private consumption expenditures per annum. In absolute terms, measured as the present value of the future streams of GNP and private consumption, these losses are 26-48 billion SEK and 27-53 billion SEK, respectively.[11]

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[11] Calculated with a six per cent discount rate.

- If the nuclear power discontinuation policy implies that nuclear power plants are taken out of operation before it is economically and technologically motivated, the loss figures increase significantly; to 47-94 billion SEK for GNP and to 46-101 billion SEK for private consumption.[11]

Sensitivity analysis

The simulation results just summarized are of course contingent on many assumptions regarding model parameters, exogenous variables and the specific design of the simulations. It is thus important to try to assess how sensitive these results are to alternative assumptions and simulation designs. For that purpose three categories of sensitivity analyses have been carried out.

The first is concerned with a more concentrated nuclear power discontinuation policy. Instead of closing the nuclear power plants after twenty-five years of operation, they are kept in production until around 2010 (assuming, of course, that this is technologically feasible). But as before, nuclear power is not allowed after 2010.

The second category investigates the consequences of a higher electricity demand. In a first case the higher electricity demand is caused by a higher sectoral productivity growth than in the base case simulations, and thus a higher growth rate in the model economy. In a second case, the higher electricity demand is the result of larger conversions to electric heating than in the base case simulations.

In the third sensitivity analysis, the effects of other expectations regarding the future electricity prices are considered. The agents in the model economy have, as in the base case simulations, static expectations, but now an electricity tax is levied in the discontinuation case in order to prevent low electricity prices and to encourage lower electricity input coefficients in new capital equipment.

The effects of postponing the nuclear power discontinuation

If the nuclear power plants are allowed to operate until around 2010, the discontinuation is postponed as much as possible given that no nuclear power shall be allowed after 2010. Since the existing nuclear power capacity is utilized longer, the discontinuation cost should in this case be lower. In the base case simulations the additional costs of closing the nuclear power plants after twenty-five years of operation, although it would be possible to continue to use them for forty years, are 50.4 billion SEK in terms of the present value of GNP and 53.8 billion SEK in terms of the present value of private consumption.[12] By postponing the discontinuation, the cost in terms of GNP losses can be somewhat reduced. The additional cost of a too early closedown is now 30.6 billion SEK. That is, by allowing those nuclear power plants which in 2010 have not been operated for forty years to continue to produce electricity also for their remaining lifetime yields an additional benefit of 30.6 billion SEK.

The cost in terms of private consumption is reduced from 53.8 to 38.8 billion SEK. Although there is a productivity gain for the economy by allowing the nuclear power plants to operate for more than twenty-five years, it also means that the electricity price between 1995 and 2010 now is equal to the long

[12] These figures are the differences between the simulations 2 A and 2 B in Tables 4.3 and 4.4, respectively. They are calculated with a six per cent discount rate.

run marginal cost of nuclear power instead of gradually rising towards the long run marginal cost of coal condense power. As a result the electricity demand is higher. When the nuclear power plants are closed they are replaced by mainly coal condense power and consequently coal imports are higher. This explains the rather modest gain of the later discontinuation of nuclear power. It should be stressed, however, that if the expectations about the future electricity price are not completely static, but people to some extent in their investment decisions account for the future price effects of the nuclear power discontinuation policy, then the gain would probably be higher.

Higher electricity demand

In the base case simulations the exogenous sectoral rates of productivity growth are around 2.3 per cent. To check how the discontinuation costs are affected by a higher economic growth in the model economy they were raised to around 3.5 per cent. The average annual growth in GNP over the simulation period then increased from 2.3 per cent to 3.3 per cent. Consequently, the electricity demand is also higher, as can be seen in Table 4.13.

The development of the electricity price over the simulation period is largely the same as in the base case simulations. There is still a considerable excess base load (i.e. nuclear) capacity until the first half of the 1990's. The producer price of electricity still drops down to the short run marginal cost of nuclear power during the latter part of the 1980's. The only difference from the base case simulations is in the nuclear power discontinuation alternative, where the electricity price increases to the long run marginal cost of coal condense power already in 1995 instead of in 2003.

Table 4.13 Electricity consumption under different productivity growth assumptions. TWh.

		1987	1995	2003	2011
Base case simulations GNP-growth	1 A	110.7	143.7	141.4	161.8
2.3 %	2 A	112.9	143.8	136.0	135.8
Higher growth simulations GNP-growth	1 A	115.9	167.0	170.9	204.8
3.3 %	2 A	117.8	161.4	154.2	187.4

The relative impact of the discontinuation policy on GNP and private consumption is now less than in the base case simulations. In the final simulation period GNP is 0.3 per cent lower if nuclear power is not allowed (in the base case simulations it was 0.7 per cent lower in the discontinuation case). The private consumption is 0.6 per cent lower while it was 2.0 per cent lower in the base case simulations. In absolute terms, the present value of the accumulated losses is 33.7 billion SEK for GNP and 43.1 billion SEK for private consumption (again using a six per cent discount rate). These figures shall be compared to 48.4 billion SEK and 53.1 billion SEK, respectively, in the base case simulations.

In the second simulation the higher electricity demand is instead the result of larger conversions from oil heating to electric heating than in the base case simulations. In the latter the oil price increases in 1979-80 lead to substantial conversions from oil heating to district heat. There are only minor conversions to the use of electricity for heating purposes and then mainly through investments in heat pumps. In order to check the effects of larger conversions to electric heat, this simulation is based on a lower cost of converting to electric heating.

The result is still the same amount of conversions to district heat but now there are also substantial conversions to electric heating. In fact the demand for electricity for heating purposes more than doubles.

The effects on the total electricity consumption and the production cost of electricity are shown in Table 4.14. The total electricity consumption is now significantly higher than in the base case simulations. The electricity price is still low in the 1980's as a result of the excess nuclear power capacity, but because of the higher electricity demand it does stay somewhat above the short run marginal cost of nuclear power. From 1995 and onwards it is equal to the long run marginal cost of the base load capacity (i.e. nuclear power and coal condense power respectively).

Table 4.14 The consumption of electricity and the electricity production cost. Low conversion cost for electric heating.

		1987	1995	2003	2011
Electricity consumption	1 A	120.7	146.3	153.6	216.6
TWh	2 A	129.6	155.9	152.6	164.6
Production cost of	1 A	.08	.14	.14	.14
electricity SEK/kWh	2 A	.08	.20	.21	.24

The higher electricity demand increases, in this case, the discontinuation cost. In the final simulation period GNP is 1.1 per cent lower in the discontinuation case (compared to 0.7 per cent in the base case simulations) and the private consumption is 3.2 per cent lower (2.0 per cent in the base case simulations). In absolute terms, the present value of the GNP-losses is 67.0 billion SEK (48.4 billion SEK in the base case simulations) and the present value of the losses in private consumption is 118.2 billion SEK (53.1 billion SEK in the base case simulations).

A tax on electricity use

In the case with a low conversion cost to electric heating, the resulting higher electricity demand meant a larger discontinuation cost. But this relation does not allow the conclusion that measures taken to reduce the electricity demand would lead to a lower discontinuation cost. This is illustrated by a simulation where a tax is imposed on the use of electricity. It is imposed in the nuclear power discontinuation case in order to prevent the electricity price to fall below the long run marginal cost of the base load capacity (i.e. nuclear power) in the 1980's and to make it equal to the long run marginal cost of coal condense power already from 1995. The effects on the electricity consumption, GNP and private consumption are shown in Table 4.15.

Table 4.15 The effects of an electricity tax on the consumption of electricity, GNP and private consumption in the nuclear power discontinuation case. Deviation from the base case simulation 2 A.

	1987	1995	2003	2011
Differences in electricity consumption. TWh	-8.9	-23.7	-10.6	-10.2
GNP[1)]	-0.1	- 0.5	0.3	- 0.2
Private consumption[1)]	-0.3	- 0.7	0.8	-

1) Percentage deviation from simulation 2 A.

As can be seen the tax results in a lower electricity consumption. But it also has a negative impact on both GNP and private consumption. The only exception is that GNP and private consumption in 2003 are somewhat higher with the tax than without. In the simulation with the tax there are lower coal imports as a result of the lower electricity production. This tends to increase private consumption and also GNP. At the same time, however, the tax on electricity use creates a distortion which has a negative effect on GNP. For 2003 the distortionary effect is rather small, so the benefit of less coal imports dominates in the net effect.

But the overall effect of the tax on electricity use is to increase the cost of the nuclear power discontinuation, both in terms of GNP-losses and losses in private consumption. This is also what should be expected. By imposing the tax, the most important effect is that the model economy cannot reap the full benefits of the low marginal cost of electricity prior to at least 1995. There is one way, however, that the tax could have a positive effect in the model economy and that is through the expectations formation. With static expectations, one effect of the tax is that the future increase in the production cost of electricity is accounted for already in earlier periods. But judging from the results in Table 4.15 this does not seem to be a quantitatively important aspect of the tax.

4.4 CONCLUDING REMARKS

The preceding analysis illustrates the consequences of a nuclear power discontinuation in the model economy under certain, explicitly stated circumstances. The interesting question is of course what conclusions one may draw from it regarding the consequences for the Swedish economy of the present nuclear power policy.

As always the point with a model exercise is to investigate the effects of economic mechanisms which we from theoretical investigations, and perhaps also empirical experiences, find important for the issue under consideration. The results of the model exercise can then be used as arguments about the likely consequences of a similar experiment also in the real world.

In this respect I think the preceding model analysis at least supports the following conclusions:

The long run marginal cost of electricity will increase substantially - around fifty to sixty per cent - if the current cost ratio of coal condense power relative to nuclear power persists or deteriorates somewhat. The electricity price will increase with a similar amount. This means that the marginal cost of producing heat in combined power and heat plants decreases and as a result the competitiveness of plants which produce heat only, as for instance the Secure nuclear reactor, deteriorates.

Even without particularly high price elasticities for electricity within each single period, due to the ex post fixed energy input coefficients, the demand response to a permanently higher electricity price is quite significant. The lower electricity demand accounts for around half of the discontinued nuclear power capacity. The rest is mainly replaced by coal condense power.

The higher cost of electricity production implies a productivity loss for the economy. In relative terms the losses seem modest; 0.7 per cent lower GNP per annum and 2.0 per cent lower private consumption expenditures in the base case simulations.

With a higher level of electricity demand, the productivity loss is larger in one case, but lower in the other. When the higher electricity demand is the result of a higher growth rate for the economy, the discontinuation cost is lower. In contrast, when the higher electricity demand is the result of a more electricity-intensive heat production rather than a higher economic growth, the discontinuation cost is higher. But these simulation results still indicate relative annual productivity losses of the order of magnitude of 0.3-1.0 per cent of GNP and a couple of percentage points (1.0-3.0) of private consumption. The present values of the accumulated losses are (with a six per cent discount rate) of the order of magnitude of around 50 billion SEK.

The results summarized so far relate to the case where the lifetime of the nuclear power reactors is 25 years and they are gradually taken out of operation from 1995 and onwards. If the lifetime should be longer but they still are closed down after 25 years of operation, the discontinuation cost increases significantly in the base cases, from 50 billion SEK to 90 billion SEK in terms of the present value of the accumulated GNP losses and from 50 billion SEK to 100 billion SEK in terms of the present value of the private consumption losses. Thus the nuclear power discontinuation policy will be particularly costly if it implies a premature scrapping of nuclear power capacity, which is also confirmed by the simulation where the discontinuation is postponed until around 2010. In this case the GNP-losses were reduced and also the losses in private consumption were lower.

A comparison with other studies

The consequences of a nuclear power discontinuation have previously been analysed in Bergman (1981) and Bergman and Mäler (1983). The first of these studies was carried out in connection with the nuclear power referendum. It was concerned with a fast discontinuation of the existing nuclear power capacity during the 1980's although it had a long remaining economic lifetime. Thus the studied discontinuation policy was quite different from the ones analysed in this chapter. Since the nuclear power reactors are allowed to be operated during their full economic lifetime in the discontinuation alternatives studied in this chapter one should expect the discontinuation costs reported here to be less than the ones reported in Bergman (1981).

Several nuclear power discontinuation alternatives were studied in Bergman (1981) but the one which is most similar to the ones in this chapter meant that sufficient amounts of coal condense power were introduced to replace nuclear power. As a result the price of electricity increased with roughly 50 per cent. The resulting reductions in GNP and private consumption were in 1990 and 2000 of the order 0.3 per cent and 2-3 per cent, respectively. These effects are of the same order as the corresponding effects of the discontinuation alternatives studied in this chapter. Bergman (1981) also estimated the present value in 1980 of the loss in private consumption between 1980 and 2000 to around 70 billion SEK (calculated at a 4 per cent rate of discount). The present values in 1983 (also calculated at a 4 per cent rate of discount) of the corresponding losses for the discontinuation alternatives of this chapter are around 40-50 billion SEK.[13] It is quite natural that the costs reported in this chapter are lower since the nuclear power discontinuation studied by Bergman implied that all nuclear power plants were closed down during the 1980's. In the simulations in this chapter the nuclear power capacity can be used for a longer time, which of course reduces the discontinuation cost. The impacts on the sectoral allocation of resources reported in Bergman (1981) were quite small, which is also the case in this chapter.

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[13] See Table 4.4.

Bergman and Mäler (1983) studied nuclear power discontinuation alternatives very similar to the ones of this chapter but they confined themselves to a partial equilibrium study of the electricity and heat sector. Their results also indicate an electricity price close to the short run marginal cost of nuclear power during the 1980's. In their simulations it increases in the 1990's, but even in the nuclear power discontinuation alternatives it still remains below the long run marginal cost of nuclear power. Not until after the turn of the century does the electricity price become equal to the long run marginal cost of nuclear power, or in the discontinuation alternative, to the long run marginal cost of coal condense power.

The electricity production levels reported in Bergman and Mäler (1983) are from the mid 1990's and later generally lower than those of this chapter. The difference is rather large, around 20 TWh. One reason for this result is that the low electricity price in the 1980's leads to a relatively more electricity-intensive production in the ELIAS-ENMARK model. Since the energy input coefficients are fixed ex post, they remain relatively high also later on in the simulation period when the electricity price increases. In the model used by Bergman and Mäler, on the other hand, the electricity demand is represented by conventional demand functions, which means that the price effects on the electricity demand are reversible. Another reason for the differences in the electricity production levels is that the period-lengths in the simulations of this chapter are longer than those in Bergman's and Mäler's study. This means that, compared to their simulations, the electricity price responds with a lag in the simulations of this chapter, which reinforces the demand effect of the low electricity price in the 1980's. The difference in electricity production explains to some extent why the total discontinuation cost (calculated as the present value of the losses in consumers' and producers' surpluses) reported in

Bergman and Mäler (1983) is only 6 billion SEK whereas it is estimated to be 25-50 billion SEK in terms of the present value of the GNP-losses in this chapter.

Conclusion

The results of this chapter, as well as the ones obtained by Bergman and Mäler, hardly warrant the conclusion that there will be serious or dramatic economic consequences of the present nuclear power discontinuation policy. Although it is reasonable to expect a substantially higher electricity price, and consequently a significant reduction in the demand for electricity, the repercussions on the rest of the economy seem quite modest. Within the level of aggregation used in the simulations of this chapter, there is no dramatic impact on the pattern of resource allocation. But there is a loss in total productivity in the economy, due to the higher cost of electricity production. The productivity loss, manifested in a lower GNP compared to the simulation alternatives where nuclear power is allowed, mainly results in a lower private consumption, but this is somewhat arbitrary. Gross investments, public production and the current account requirement are assumed to be the same in all simulations, so the private consumption must bear most of the adjustments.

The integrated ELIAS-ENMARK model has made it possible to explicitly impose restrictions on the use of nuclear power and to study how the electricity and heat markets, as well as the rest of the economy, adjust to these restrictions. In contrast to the partial equilibrium version of the ENMARK model, the electricity and heat demand responses of the integrated model reflect the complete general equilibrium adjustments to the restrictions on the use of nuclear power. Moreover, with the integrated model the effects on the pattern of resource allocation in the rest of the economy can be explicitly assessed. Although these effects, in this particular case, are not very dramatic, they are still significant enough to affect the final equilibrium on the electricity and heat markets.

5 The Nuclear Power Investments and the Electricity Market in the 1980's

The analysis in Chapter 4 clearly indicates that for a quite substantial period from the early 1980's and until around 1995, the Swedish energy market is characterized by an excess capacity in electricity production. It is natural to ask how much the massive investments in nuclear power capacity, briefly described in the beginning of Chapter 4, have contributed to the excess capacity. In particular, one would like to know how that investment programme differs, if at all, from one which would have been, in some sense, optimal. To the extent that it is different, the question is how this has affected the cost of electricity production and the structure of the electricity (and heat) production system. Another issue is whether the difference is large enough to also make a noticeable impact upon the performance of the rest of the economy.

The present chapter is devoted to an analysis of these issues. In particular the integrated ELIAS-ENMARK model is employed to provide quantitative insights. The actual nuclear power investment programme is compared to one in which nuclear power investments only occur if the electricity price is sufficiently high to cover the long run marginal cost of nuclear power. The implications of the difference between these two investment programmes for the electricity and heat sector and for the allocation of resources in the rest of the economy, are then assessed.

The analysis below is an ex post evaluation of the nuclear power investments and, as such, has the benefit of at least partial hindsight. The result that there has been an overinvestment in nuclear power capacity in relation to the realized electricity demand does not necessarily mean that the ex ante decisions were bad ones. They were based on certain expectations about, among other things, future energy prices and electricity demand. Since resources have to be committed before the ex post realizations of the uncertain variables are known, the ex post optimal decisions will in general differ from the ex ante optimal ones. The crucial question is of course whether, given the existing information at the time of the investment decision, the actual decisions were the best ones or not.

An ex post evaluation, as in this chapter, can give some insights indirectly, by identifying important factors which contribute to the difference between the actual and the ex post optimal decisions. These factors can then serve as a starting point for a discussion of whether they were sufficiently accounted for at the investment date or not.

The chapter is organized as follows. The next section contains a theoretical discussion of the notion of optimal capacities in an electricity power system. The purpose is to define and clarify certain concepts which are used in the subsequent applied analysis. Section 5.2 presents the major assumptions in the quantitative analysis and the results are presented in Section 5.3. Finally, Section 5.4 contains some concluding remarks.

5.1 OPTIMAL POWER CAPACITIES

By its nature, electricity is consumed at the same instant as it is produced. Thus, in one sense, there is an excess capacity in the power system whenever the production and distribution capacities, in terms of installed effect, are more than sufficient to meet the peak load demand. But this is not the only relevant capacity concept. The power system should also be able to maintain a sufficient electricity production over time. In a hydro power system, for instance, the installed effect in the power stations should be sufficient to handle the peak load demand. But the water storage capacities (and the water availability) should also be sufficient to meet the cumulated electricity demand over time. Also for a thermal power system it is necessary to distinguish between the load capacity of the system and its energy capacity, where the latter depends on the average availability of the power stations over a period of time.

In order to define optimal power capacities more precisely, and to obtain operational criteria for whether existing power capacities are optimal or not, a simple model of optimal investments in a mixed hydro and thermal power system is analyzed. The investments are optimal in the standard sense of maximizing the sum of consumers' and producers' surpluses. The theoretical framework of the model is the extension of the standard theory of marginal cost pricing to the case of pricing a non-storable commodity with periodic demand fluctuations. Classical articles in this area are Boiteux (1960) and Williamson (1966). Drèze (1964) provides a valuable review of the theory. For various applications to electricity power production, see Turvey (1968), Turvey and Anderson (1977), Strøm (1983) and Rødseth (1983).

The model

Let there be n available thermal production technologies (nuclear, coal and oil condense, gas turbines etc.). The operating cost of the j:th technology is C_j, j = 1, 2, ..., n, per effect hour.[1]

[1] A more general cost function, $C(q_t)$, would not change anything essential in the following analysis. In fact all derived conditions for efficient prices and capacities hold for the more general case if the C_j-coefficients are interpreted as the marginal costs at the efficient solution. The results below about the merit ordering of different technologies in the optimal solution are however only correct in the case of constant operating costs.

The annuitized capital cost per unit of capacity (in terms of installed effect) for the j:th thermal technology is B_j, $j = 1, 2, \ldots, n$.[2] Hydro power is assumed to be available at a zero operating cost.[3] It is also assumed that there exists an initial hydro capacity and that no further additions are possible.

The demand for electricity, at a given price, varies over the year. Suppose the year can be divided into T time segments with the lengths Θ_t, $t = 1, 2, \ldots, T$, such that within each time segment the electricity demand at a given price is constant. Let $p_t(q_t)$ denote the inverse demand function in time segment t, where q_t is the demand, in energy terms, during this interval.[4]

The problem is to choose a set of thermal capacities, $\bar{q}_j$, $j = 1, 2, \ldots, n$, and their rate of utilization in different time segments so that the sum of consumers' and producers' surpluses is maximized. That is, the task is to solve the following problem:

$$\max_{q_t, q_{vt}, q_{jt}, \bar{q}_j} \sum_{t=1}^{T} \left[\int_0^{q_t} p_t(q)dq - \sum_{j=1}^{n} C_j \cdot \Theta_t \cdot q_{jt} \right] - \sum_{j=1}^{n} B_j \cdot \bar{q}_j \tag{1a}$$

subject to the constraints (the variables in brackets are the Kuhn-Tucker multipliers associated with the constraints).

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2 This is a conventional approach in the literature on efficient pricing and investments. By treating the capital cost as an annuity the complete intertemporal investment problem is transformed to a simpler atemporal problem. But implicitly this approach assumes that prices and operating costs, as well as the investment costs, remain the same over time. If they can be expected to change, the annuitized capital cost may not be the relevant concept for the efficient pricing and investment criteria.

3 The important assumption is that the hydro power operating cost is less than $\min_j\{C_j\}$ which hardly is controversial.

To let it be zero just simplifies the notation.

4 To keep the analysis as simple as possible, cross-price effects between time segments are disregarded. They are not an important feature in the numerical model which is used for the subsequent applied analysis, so this simplification is quite harmless in the present context.

$$q_t - \Theta_t(q_{vt} + \sum_{j=1}^{n} q_{jt}) \leqq 0 \qquad t = 1,2,\ldots,T \ (p_t) \qquad (1b)$$

$$q_{jt} - \bar{q}_j \leqq 0 \qquad \begin{array}{l} t = 1,2,\ldots,T \\ j = 1,2,\ldots,n \ (\lambda_{jt}) \end{array} \qquad (1c)$$

$$q_{vt} \leqq \bar{q}_v \qquad t = 1,2,\ldots,T \ (\lambda_{vt}) \qquad (1d)$$

$$\sum_{t=1}^{T} \Theta_t q_{vt} \leqq h \qquad (\mu) \qquad (1e)$$

$$q_{vt} \geqq 0 \ \forall t, \quad q_{jt} \geqq 0 \ \forall j,t \quad \bar{q}_j \geqq 0 \ \forall j$$

where q_{vt} is the hydro load supply in time segment t and q_{jt} is the load supply in time segment t from thermal technology j. The capacity in terms of installed effect in the hydro system is $\bar{q}_v$ while its energy capacity is h effect hours per year. The latter is a very crude way to account for the restrictions on hydro production set by the average annual water flows. A more elaborate model would explicitly account for the variability in the water flows over the year and the possibility to shift water supply between different time segments by water storage.[5]

From the Kuhn-Tucker first-order conditions we first note that

$$p_t = p_t(q_t) \qquad t = 1,2,\ldots,T \qquad (2)$$

[5] There are three aspects of the hydro power capacity. First there is the water availability during a given period of time. Secondly there is the capacity to store water in dams and thirdly there is the capacity of turbines. The constraint (1d) refers to the third aspect, the effect capacity of the existing turbines. The constraint (1e) accounts for the energy capacity determined by the water availability and the storage capacity during a normal year. The variable h refers to the maximal available hydro energy during such a normal year. Whether all this hydro energy can be transformed into electricity or not, depends on the turbine capacity which is given by the constraint (1d).

assuming a strictly positive electricity consumption in each time segment.[6] For the utilization q_{jt} of the j:th thermal technology in time segment t we have

$$p_t \leqq c_j + \lambda_{jt}\,\theta_t^{-1} \quad \text{and} \quad (p_t - c_j - \lambda_{jt}\,\theta_t^{-1})q_{jt} = 0 \tag{3}$$

and similarly for hydro power

$$p_t \leqq \mu + \lambda_{vt}\,\theta_t^{-1} \quad \text{and} \quad (p_t - \mu - \lambda_{vt}\,\theta_t^{-1})q_{vt} = 0 \tag{4}$$

<u>Only hydro production</u>

For the thermal capacities, $\bar{q}_j$, the Kuhn-Tucker first-order conditions are

$$\sum_{t=1}^{T} \lambda_{jt} \leqq B_j \quad \text{and} \quad \left[\sum_{t=1}^{T} \lambda_{jt} - B_j\right]\bar{q}_j = 0 \tag{5}$$

Suppose there are strict inequalities in (5) for all j. Thus $\bar{q}_j = 0$ for all j and electricity is produced in hydro power plants only. In this case

$$\sum_{t=1}^{T} \theta_t(p_t - c_j) < B_j \tag{6}$$

i.e. the differences between the electricity revenues and the total variable operating costs are not sufficient to cover the capital costs for the thermal capacities. If the given hydro load capacity $\bar{q}_v$ is sufficient, the available hydro energy capacity is allocated between the T time segments so that the electricity price (i.e. the marginal valuation of a kWh) is the same in each

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[6] A sufficient condition for a positive electricity demand in each time segment is that $\lim_{q_t \to 0} p_t(q_t) = \infty$ for all t.

segment. The common price, μ, is finally determined by the requirement that[7]

$$\sum_{t=1}^{T} q_t(\mu) = h \tag{7}$$

Positive thermal capacities

Let $J = \{j:\bar{q}_j>0\}$. Because of (5), $\lambda_{jt} > 0$ for at least some t for all $j \in J$. Let T_j be the set of time periods for which the shadow value of thermal capacity j is strictly positive. For all $t \in T_j$ it must be the case that $q_{jt} = \bar{q}_j > 0$ so $\lambda_{jt} = (p_t - C_j)\Theta_t$.

Consequently

$$\sum_{t=1}^{T} \lambda_{jt} = \sum_{t \in T_j} \lambda_{jt} = \sum_{t=T_j} (p_t - C_j)\ \Theta_t = B_j \quad \forall j \in J \tag{8}$$

During the time periods when thermal technology j is operated at full capacity it generates a revenue surplus which is sufficient to cover its capital cost. In time periods when technology j is operated, but not at full capacity, $p_t = C_j$, that is the electricity price is equal to the operating cost.

Obviously, if all thermal technologies had the same ranking in terms of both operating and capital costs there would be a unique least cost technology. Then it could never be optimal to invest in any technology but this one. But in fact there are thermal technologies, such as nuclear power, with high capital costs

7 If, for some t, $q_t(\mu) > \Theta_t\bar{q}_v$, a peak charge $\lambda_{vt}\cdot\Theta_t^{-1}$ would be added to the common price in that period, where λ_{vt} is determined by the condition that $q_t(\mu + \lambda_{vt}\cdot\Theta_t^{-1}) = \Theta_t\bar{q}_v$.

and low operating costs, and there are technologies, such as gas turbines, with low capital costs but high operating costs. Given the variability in demand it may then very well be optimal to have a mix of different thermal technologies.[8]

To illustrate, suppose there are positive investments in two thermal technologies, call them r and s. Then

$$\sum_{t \in T_r} \theta_t (p_t - c_r) = B_r$$

$$\sum_{t \in T_s} \theta_t (p_t - c_s) = B_s$$

From the definition of the sets T_r and T_s it follows that $p_t > c_r$ for all $t \in T_r$ and $p_t > c_s$ for all $t \in T_s$. Suppose $c_s > c_r$. Then, for all $t \in T_s$, it must be the case that $p_t > c_s > c_r$, i.e. $T_s \subseteq T_r$. Let $\Gamma_{sr} \equiv T_s^c \cap T_r$, i.e. the set of time periods in T_r which do not belong to T_s (T_s^c denotes the complement of the set T_s). Then the two conditions above may be reexpressed as

$$\sum_{t \in T_s} \theta_t p_t = B_r - \sum_{t \in \Gamma_{sr}} \theta_t (p_t - c_r) + c_r \sum_{t \in T_s} \theta_t$$

$$\sum_{t \in T_s} \theta_t p_t = B_s + c_s \sum_{t \in T_s} \theta_t$$

or

$$B_r - B_s - \sum_{t \in \Gamma_{sr}} \theta_t (p_t - c_r) = (c_s - c_r) \sum_{t \in T_s} \theta_t > 0 \tag{9}$$

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8 This argument is strengthened if one also accounts for uncertainty in the demand for power and the availability of plants. Such uncertainty generally makes it profitable to complement base load capacity by reserve capacity in, for instance, gas turbines.

Despite its higher operating cost the s-technology is competitive if its capital cost is sufficiently lower than that of the r-technology to compensate both the higher operating cost as well as the additional net value of the r-capacity in periods where $C_r < p_t \leq C_s$.

It should be clear from these considerations that the lengths of operation of the different thermal technologies are uniquely determined by their variable operating costs. During a period when technology s is operated so are also all technologies for which $C_j < C_s$ (if it is at all profitable to have a positive capacity of them). Among the technologies with positive capacities, the one with the lowest operating cost is used in all time segments, i.e. it is a base load technology, while the one with the highest operating cost is used only during peak demand. This is sometimes referred to as a merit-order dispatching of the power technologies.

Equation (8) is the required condition for optimal thermal capacities. It can be reexpressed as

$$\sum_{t \in T_j} \frac{\Theta_t}{\sum_{t \in T_j} \Theta_t} p_t = C_j + \frac{B_j}{\sum_{t \in T_j} \Theta_t} \qquad \forall j \in J \tag{10}$$

The right-hand side is the long run marginal cost of thermal technology j. It is the sum of the operating cost (which corresponds to the short run marginal cost) and the capital cost. Note that the capital cost per effect hour is defined by dividing by the number of hours for which the capacity is <u>fully operated</u> and not the total number of hours of operation.[9] This is of course a reflection of the optimality condition that during

[9] Since $p_t = C_j$ when the technology is operated at a positive rate but at less than full capacity, the capital cost can also be defined on total hours of operation, but then the left hand side in (10) should be the weighted electricity price for all periods in which the technology is used. Thus, it is important to be clear about which capital cost concept one uses.

the periods of full capacity utilization the technology should recover its capital cost, while in the remaining periods of positive but not full capacity utilization the electricity price should be equal to the short run marginal cost, and thus does not contribute anything to the capital cost.

The power capacity is optimal if the weighted electricity price during the full capacity periods is equal to the long run marginal cost. Consequently, there is an excess capacity of power technology j if its long run marginal cost is greater than the weighted electricity price. If it is less, on the other hand, it is profitable to expand the j:th capacity.

The role of hydro power

Suppose that the hydro power effect capacity $\bar{q}_v$ is sufficiently large to be a non-binding constraint in all time segments. From (4) it can be seen that the price of electricity must be the same (i.e. $p_t = \mu$) in all time segments with a positive hydro production. Furthermore, in those time segments in which there is no hydro production, $p_t \leqq \mu$. Consequently, hydro power will only be dispatched during those time segments where the marginal valuation of electricity is highest, i.e. in the peak demand segments.

Let $T_v = \{t: q_{vt} > 0\}$, i.e. the set of time segments with a positive hydro power production. Suppose j* is the thermal technology with the lowest merit order, i.e. the shortest operating time. Then $T_v \subseteq T_{j*}$. To see this, remember that when technology j* is operated at full capacity $p_t > C_{j*}$. Since hydro power is dispatched only when the electricity price is highest it must hold that $\mu > C_{j*}$. Consequently there cannot exist a time segment $t' \in T_v$ (and thus $p_{t'} = \mu$) such that $t' \notin T_{j*}$, because it would imply that $p_{t'} = \mu > C_{j*}$ and $\lambda_{j*t'} = 0$, which contradicts the Kuhn-Tucker first-order condition (3). Since j* has the shortest operating time it follows that T_{j*} must be a subset of all other

T_j for all $j \in J$ and thus so must T_v. Taking account of these facts the optimal capacity condition (10) can be expressed as

$$\sum_{t \in \Gamma_{vj}} \frac{\Theta_t}{\sum_{t \in T_j} \Theta_t} p_t + \mu \frac{\sum_{t \in T_v} \Theta_t}{\sum_{t \in T_j} \Theta_t} = C_j + \frac{B_j}{\sum_{t \in T_j} \Theta_t} \quad \forall j \in J \tag{11}$$

If the hydro energy capacity h is sufficiently large relative to the demand, so that $T_v = \{1,2,\ldots,T\}$, i.e. some hydro power is produced in all time segments, then (11) simplifies to

$$\mu = C_j + \frac{B_j}{\Theta} \quad \text{for } j \in J \tag{12}$$

where $\Theta \equiv \sum_{t=1}^{T} \Theta_t$. Then J will only contain (at most) one thermal technology, the least costly base load technology and the electricity price shall be equal to the long run marginal cost of this technology.

Note that the larger the share of hydro power is in the system, the more it tends to decrease the price variability between different time segments.

Although the hydro power capacities $\bar{q}_v$ and h were kept constant, the previous analysis also yields criteria for their optimal levels. The value of a marginal addition of hydro power effect capacity is given by

$$\sum_{t \in T_v} \lambda_{vt} = \sum_{t \in T_v} \Theta_t (p_t - \mu) \tag{13}$$

This is positive if the hydro power capacity $\bar{q}_v$ is a binding constraint for some time periods. If the marginal valuation is equal to the long run marginal cost of expanding the hydro power effect capacity, then $\bar{q}_v$ is at its optimal level. Finally, the electricity price $p_t = \mu$ for $t \in T_v$ is also the marginal valuation of an additional kWh of hydro storage capacity. Consequently, the storage capacity h is optimal if μ is equal to the long run marginal cost of storage additions.

The interdependence between electricity production and heat production

The preceding analysis led to definitions of, as well as operationally useful criteria for, optimal power capacities in a mixed hydro and thermal power system. Essentially the same analytical framework is employed in the subsequent quantitative analysis. However, in the latter one has to account for the fact that there are strong interconnections between production of electricity and production of heat. First, electricity and heat may be jointly produced in combined power and heat plants. Second, since electricity can also be used for heating purposes, there is some degree of substitutability between the two in the demand for electricity and heat.

These interdependencies do not, however, affect the principles derived in the preceding analysis. For pure power technologies the optimal capacity conditions are still given by equation (10). For pure heat technologies an analogous condition - the weighted heat price shall be equal to the long run marginal cost of the heat technology - can be derived. The equilibrium electricity and heat prices will of course be dependent on each other.

For combined power and heat technologies the optimum capacity condition (10) has to be slightly modified. Let, for these technologies, C_j refer to the operating cost per kWh of electricity produced and B_j the capital cost per kW of electricity effect capacity. For each kWh of electricity produced there is a joint output of b_j kWh of heat. Denote the heat price in time segment t by z_t. Then the optimum capacity condition for the combined power and heat technology may be written as

$$\sum_{t\in T_j} \frac{\Theta_t}{\sum_{t\in T_j} \Theta_t} (p_j + b_j z_t) = C_j + \frac{B_j}{\sum_{t\in T_j} \Theta_t} \qquad (14)$$

Note that the short run marginal electricity cost for the joint output technology is $C_j - b_j z_t$ and, similarly, the short run marginal heat cost is $(C_j - p_t)/b_j$. Consequently, an electricity price increase will, ceteris paribus, lower both the short run marginal heat cost and the corresponding long run marginal cost and thus reduce the competitiveness of pure heat technologies.

Another implication of joint electricity and heat production should also be noted. It was shown above that if the hydro energy and effect capacities were sufficiently large to allow hydro power production in all time segments, the electricity price would be the same in all periods and hydro power production would be complemented with at most one (base-load) thermal technology. The last proposition is not necessarily true when there is joint production of electricity and heat. This is easily seen in the case that heat has to be produced in combined power and heat plants because pure heat technologies are too expensive. Then, if electricity also were to be produced only in the combined power and heat plants, the electricity and heat prices have to be such that electricity and heat are demanded in the same proportion as they are jointly produced. Since the latter is relatively fixed, it may very well be the case that such prices fail to exist or that the required electricity price is sufficiently high to make it profitable to invest in a pure power technology. By investing in both a pure power and a combined power and heat technology, the range of feasible electricity and heat output ratios increases.

Uncertainty and indivisibilities

The model used above is of course a particular simplification of an actual power and heat system chosen in order to highlight the implications for efficient pricing and optimal investments of a periodically fluctuating demand. There are at least two other important aspects, namely uncertainty and indivisibilities, which could require certain modifications of the preceding analysis.

The optimum capacity conditions are based on marginal capacity additions. In general, such additions are not economically feasible; there is some least cost scale for new plants. Consequently, the conditions (10) will not hold exactly, except by sheer coincidence. Instead, the investment criterion must be based on a comparison of the net impact on the sum of consumers' and producers' surpluses of adding an optimal scale plant or not.[10] However, if the optimal plant scales are reasonably small compared to the total demand, the conditions (10) are probably good approximations. The subsequent quantitative analysis is conducted as if marginal additions were possible, but in interpreting the results the indivisibility qualification should be borne in mind.

In actual investment decisions one also has to account for the uncertainty which surrounds many of the variables and parameters of the planning problem (1). The demand in different time segments at a given price is not known exactly. The water availability for hydro power plants is a random variable. There is some degree of uncertainty regarding the availability of plants because of unscheduled maintenance. Even the cost parameters can be uncertain.

In principle it is possible to convert problem (1) to a planning problem under uncertainty by, for instance, choosing a set of capacities which maximizes the expected sum of consumers' and producers' surpluses. However, the quantitative models used in this chapter are simply not designed for decision problems under uncertainty. Consequently, uncertainty aspects are neglected in the following applied analysis. As has already been pointed out, it is necessary to remember that the quantitative analysis below thus proceeds from a comparison of actual power and heat capacities with ex post optimal ones.

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[10] See Williamson (1966).

Optimal capacities in a general equilibrium framework

The ENMARK part of the integrated ELIAS-ENMARK model essentially corresponds to the theoretical framework of the planning model (1). By well-known arguments (see Chapter 2, pp. 42-44, and also Section 2.4) the planning solution can also be interpreted as an equilibrium in a competitive power and heat market. Since the applied analysis is performed with the integrated ELIAS-ENMARK model, it extends the partial equilibrium framework of the preceding theoretical discussion to a general equilibrium setting. In particular, it means that the demand for electricity and heat is explicitly related to the allocation of resources in the rest of the economy and that the impact of changes within the electricity and heat sector on that allocation can be studied. This is useful because some of the applied analysis is concerned with how the optimal power capacities depend on the development in the rest of the economy and the extent to which differences between actual and optimal capacities are likely to have any consequences for the resource allocation in other sectors.

5.2 AN OUTLINE OF THE SIMULATIONS

The first nuclear power plant in Sweden was started in 1972. In 1979 there were six nuclear power reactors in operation with a total installed effect of 3,710 MW. At that time, four additional ones were under construction, and almost completed. The construction of the last two of the planned twelve had just begun. Today all twelve plants have been completed and are producing, or are about to start to produce, electricity.

This investment programme shall be compared with one where the nuclear power investments are such that the electricity price (properly weighted over demand loads) equals the long run marginal cost of nuclear power. The comparison is made within the integrated ELIAS-ENMARK framework. Power and heat capacities which are such that they make the electricity and heat prices equal to the marginal costs in the utilized power and heat technologies are referred to as efficient capacities below.

Ideally the model simulations should start from a base year like 1972 or possibly earlier. However, the base year of the integrated ELIAS-ENMARK model is 1979. An earlier base year would have required major revisions in the data bases for the model, and in particular the ENMARK part, something which definitely is beyond the scope of the present study. Instead 1979 is the starting point for the simulations and the period 1979-1995 is studied in more detail than was the case in Chapter 4.

The model simulations start from the following hypothetical situation: Suppose that the capital invested in the nuclear power reactors not yet in operation in 1979 is not sunk, but could, if desired, be shifted to its best alternative use. In the base year 1979 there is then an inherited nuclear power capacity of 3,710 MW. To increase this capacity the nuclear power capital cost has to be paid. In the first simulation alternative, referred to as alternative A below, nuclear power investments are equal to the actual ones. In the second one (alternative B) the nuclear power capacity is increased only if it is profitable, i.e. if the electricity price is sufficient to cover the sum of operating and capital costs.

The chosen approach is reasonable if the actual nuclear power capacities between 1972 and 1979 largely corresponded to efficient ones. Then the two investment alternatives would be the same and, since they are the only difference between the two simulation alternatives, so would the simulated developments of the model economy for this period. It would thus be correct to start in 1979 with a common base year allocation and a common nuclear power capacity. If the efficient nuclear power investments deviated from the actual ones up till 1979, but the differences only had a minor and negligible impact on the allocation of resources in the rest of the economy, it is still reasonable to start the simulations in 1979 from a common base year allocation. In this case the initial nuclear power capacity in simulation alternative B should be modified, though.

The efficient nuclear power capacity for the base year allocation has been calculated. It is around 3,200-3,300 MW, i.e. below the actual 3,710 MW, but not with very much. Since a single nuclear power reactor means a capacity addition of around 1,000 MW (the lumpiness in power investments discussed above) it seems reasonable to claim that the actual nuclear power capacity in 1979 was fairly close to an efficient one.

The assumptions regarding the values of parameters and exogenous variables are largely the same as in Chapter 4. In two respects they are different, however.

First, since the analysis is confined to the years 1979-1995, that period has been studied in more detail than in the previous chapter. Thus, the whole period is divided into four four-year sub-periods and the ELIAS-ENMARK model is solved for each of them. Since the vintage capital stocks and their energy input coefficients now are determined more frequently, there is somewhat more flexibility in the model simulations in this chapter compared to those in Chapter 4.

Secondly, the actual development of the exogenous variables between 1979 and 1983 has been accounted for. This is more important here than in Chapter 4, since the actual investment programme is evaluated with the benefit of hindsight and because these years now are relatively more important when the whole time horizon of the analysis is shorter.

As a background to the presentation of the simulation results in the next section, Table 5.1 summarizes the development of certain macroeconomic variables over the simulation period.

Table 5.1 Annual average growth rates 1979-1995 for selected macroeconomic variables. Simulation alternative A.

Gross national product	2.3 %
Private consumption	2.1 %
Exports	3.5 %
Imports	1.9 %

Against the background of the discussion in Section 5.1 the following excess capacity criterion is used: Whenever the electricity price (properly weighted over demand loads) is less than the long run marginal cost of a power technology, there is an excess capacity. The long run marginal cost consists of an operating cost and a capital cost. Since the latter is an annuity it depends, apart from the investment outlay for an additional unit of capacity, on the rate of discount and the lifetime of the investment.

The capital costs used in the simulations were calculated at a six per cent rate of discount. The lifetime assumptions differ between the various power and heat technologies but for nuclear power a lifetime of twenty-five years was used. The six per cent rate of discount corresponds to a weighted average of the sectoral marginal rates of return in the first half of the simulations A and B (the sectoral rates of return were weighted with the sectors' production shares). In the latter part of the simulations, around 1990, the average of the sectoral marginal rates of return increases, however, to around eleven per cent. Thus, there arises a discrepancy between the rate of return in the electricity and heat sector and the rates of return in the rest of the economy. This means that there is a bias towards too much investments in the electricity and heat sector.

5.3 THE SIMULATION RESULTS

One of the conclusions of the theoretical analysis in Section 5.1 is that if the hydro energy and effect capacities are not too small relative to the demand, hydro power will be produced in all time segments. Thus it will also equalize the price of electricity between the different parts of the year.

This is actually the case for both the main simulation alternatives, not unexpectedly, since the hydro power resources are predominant in the Swedish power system. The equalization of the electricity producer price over the year is however also due to the fact that only four time segments are distinguished in the model. As a consequence, extreme demand peaks, like for instance the winter morning hours, are averaged out and thus the model underestimates the electricity demand variability. But for the purpose of this chapter - an evaluation of the nuclear power (i.e. base load) investment programme - this is not a serious limitation. It means however that characteristic peak and reserve capacities like gas turbines are really not required by the model, a result which obviously cannot be interpreted literally.

Another conclusion of Section 5.1 is that with a positive hydro power production in all time segments, there should in the optimal solution be at most one positive base load power capacity and one positive combined power and heat capacity. This is also confirmed by the simulation results. In both simulations hydro power is supplemented by base load nuclear power and by coal fuelled combined power and heat plants.[11] Although the model inherits positive oil condense and oil fuelled combined power and heat capacities from the past they are almost never used, i.e. the electricity and heat prices are not sufficiently high

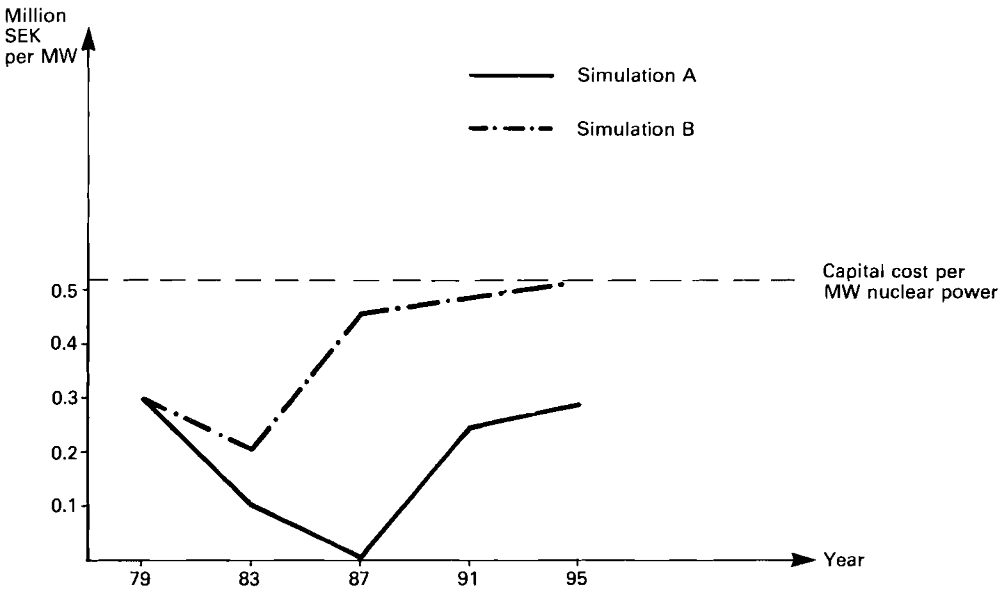
[11] Except for the first simulation period where existing oil fuelled combined power and heat plants are also used in addition to the coal fuelled ones. This is due to the fact that the 1979-80 oil price increases do not fully hit the model until the second simulation period.

to cover their operating costs.[12] The production of electricity and district heat by different technologies is shown in Tables 5.2 and 5.3.

The marginal value of nuclear power capacity

Figure 5.1 shows the marginal valuation of nuclear power capacity. In simulation A, i.e. with actual nuclear power investments, the marginal value of nuclear power capacity drops to zero as the nuclear power capacity increases with 150 % around 1985. Then as the economy, and thus the demand for electricity, grows the marginal value increases again. But even in 1995 it is only slightly more than half the capital cost of nuclear power. The simulations presented in Chapter 4 indicate, however, that the marginal value of nuclear power capacity reaches the nuclear power capital cost during the latter half of the 1990's.

Figure 5.1 The marginal value of nuclear power effect capacity



[12] There is one exception for oil fuelled combined power and heat plants; see footnote 11.

Table 5.2 Electricity and heat production by different technologies. Net of distribution losses. TWh. Simulation A.

	1979	1983	1987	1991	1995
<u>Electricity</u>					
Hydro power[1)]	61.50	61.50	61.50	61.50	61.50
Nuclear	20.44	40.44	49.58	52.01	52.01
Combined power & heat[2)]	12.08	4.58	4.58	17.55	20.50
<u>Heat</u>					
Combined power & heat[2)]	21.14	8.02	8.02	30.71	35.88
Other heat plants	9.63	40.67	43.33	26.84	26.25

Table 5.3 Electricity and heat production by different technologies. Net of distribution losses. TWh. Simulation B.

	1979	1983	1987	1991	1995
<u>Electricity</u>					
Hydro power[1)]	61.50	61.50	61.50	61.50	61.50
Nuclear	20.44	20.44	20.44	20.44	22.61
Combined power & heat[2)]	12.08	18.42	21.99	24.96	28.96
<u>Heat</u>					
Combined power & heat[2)]	21.14	32.24	38.48	43.66	50.68
Other heat plants	9.63	16.87	12.75	11.06	8.55

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1) The hydro power capacities are exogenously kept constant for the whole simulation period.

2) Coal fuelled ones except for 1979 when 45 per cent comes from oil fuelled ones.

In simulation B, where nuclear power capacity additions only take place if the electricity price is at least equal to the long run marginal cost of nuclear power, the marginal value of nuclear power capacity is rather close to the capital cost except in the first half of the 1980's. In 1995 it is equal to the capital cost and there is a minor investment in new nuclear power. The fall in the early eighties is due to an increase in the district heat demand caused by conversions from individual oil heating. The result is a somewhat higher market price for district heat which lowers the marginal cost of electricity production in the combined power and heat plants and hence the market electricity price.

As the electricity demand later increases the electricity price increases again (see Table 5.5 below). At the same time the model has invested in a quite substantial amount of coal fuelled plants producing heat only, which are competitive given the relatively high heat price and relatively low electricity price in 1983.[13] As the increase in the electricity demand motivates new investments in combined power and heat plants the heat production capacity increases further, which contributes to lower district heat prices again.

Table 5.4 shows what the deviations between the marginal nuclear power value and the nuclear power capital cost mean in terms of MW capacities. The efficient nuclear power capacities reported in the right hand column are the ones which make the electricity price equal to the long run marginal cost of nuclear power. As can be seen the 1979 capacity was only slightly above the efficient level. Apart from the rather dramatic fall in 1983, the efficient nuclear power capacity remains around 3200-3400 MW until 1995 when it increases to 4100 MW.[14] One may conclude

13 As was discussed in Chapter 3 the investment decisions are very short-sighted since they are based on current market prices only.

14 The small value of the efficient nuclear power capacity in 1983 shall not be taken too seriously. It reflects the temporary relatively low electricity price combined with the temporary increase in district heat prices. This price configuration makes it preferable, if it would be lasting, to expand substantially the electricity production capacity in combined power and heat plants at the expense of nuclear power capacity.

that simulation B almost corresponds to an efficient development for nuclear power capacity.

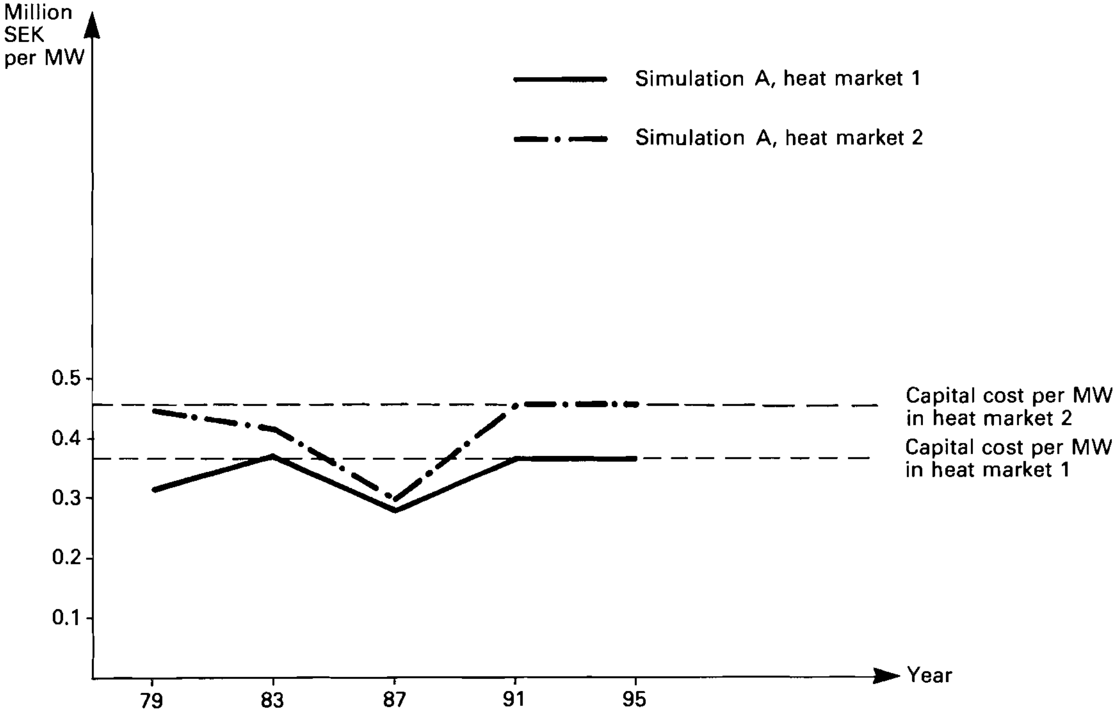
Table 5.4 Nuclear power capacities in simulations A and B and the efficient ones. MW.

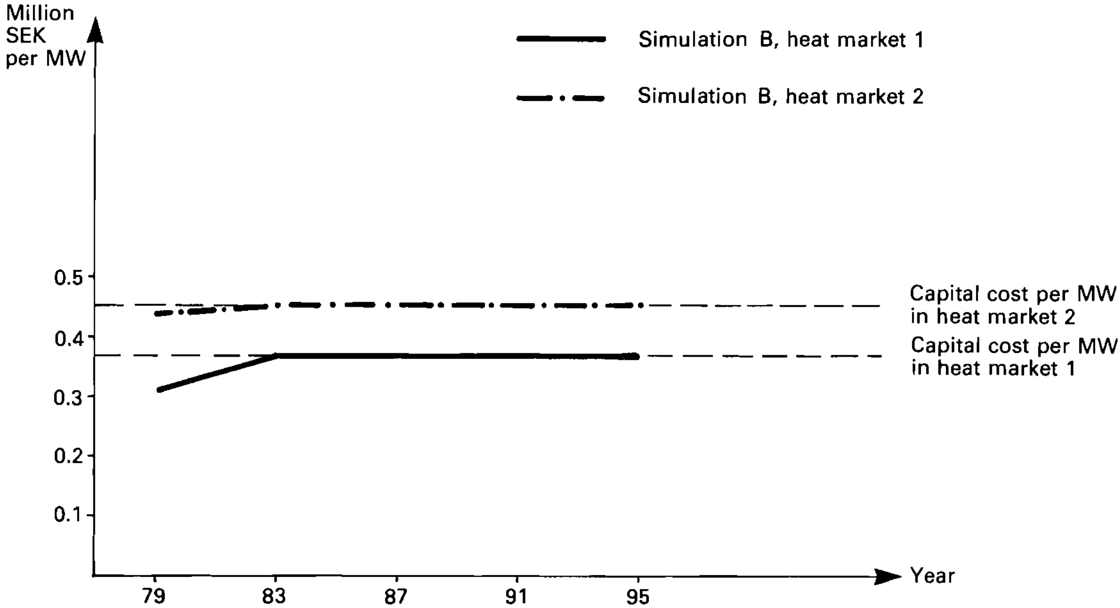
	A	B	Efficient capacities
1979	3710	3710	3439
1983	7340	3710	760
1987	9440	3710	3239
1991	9440	3710	3444
1995	9440	4105	4105

The marginal valuations of power from coal fuelled combined power and heat plants in the two district heat markets are illustrated in Figure 5.2. In simulation B the marginal value reaches the capital cost as early as 1983. In alternative A there is however a fall in the marginal value in the mid 1980's before it reaches the capital cost. The fall is caused by the nuclear power capacity increase of 2100 MW in 1985 which drives down the electricity market price.

These results imply a substantial overcapacity of nuclear power capacity in simulation A, i.e. as a result of the actual nuclear power investment programme. In the model economy it would have been preferable to keep the nuclear power capacity at the 1979 level of 3710 MW at least until the mid 1990's. Instead the power investments should have been directed to combined power and heat plants. As can be seen from Tables 5.2 and 5.3 the nuclear power production in simulation B is less than half of that in simulation A while the electricity production in combined power and heat plants is considerably larger.

Figure 5.2 The marginal value of power from coal fuelled combined power and heat plants





The cost and the production of electricity and district heat

The development on the electricity and district heat markets is summarized in Table 5.5. There are significant differences between the two simulations in terms of the production and the producer price of electricity, whereas for district heat both the production levels and the producer prices are rather similar. In simulation B the electricity demand grows very slowly (on average 1.2 per cent annually). The electricity producer price increases from .11 SEK per kWh to .14 SEK per kWh (i.e. an increase of 27 per cent) over the simulation period. In contrast, in simulation A, i.e. with the actual nuclear power investments, the electricity producer price is the same at the end of the simulation period as in the beginning, but drops down to the short run marginal cost of nuclear power plants in the middle of the 1980's. As a result the electricity production is larger than in alternative B.

The impact of the excess capacity on the rest of the economy

Finally, there is the question of whether the differences between the two simulations are significant enough to make a noticeable impact on the allocation of resources in the rest of the economy. As can be seen from Table 5.6 the aggregated macroeconomic development is similar in both simulations. But there is a slightly better performance in simulation B in terms of GNP and private consumption during the eighties. It is due to lower investments in the electricity and district heat sector which result in a higher capital stock in the other sectors of the model economy. Thus there is a more efficient sectoral allocation of the capital stock in simulation B.

There are also some notable differences in the sectoral production pattern between the two simulations which can be seen in Table 5.7. The higher electricity price in simulation B particularly affects sector 4, the energy intensive export industries. Between

Table 5.5 The cost and production of electricity and district heat 1979-1995 in the simulations A and B. The production figures are net of distribution losses.

Year	Electricity production cost SEK/kWh		Electricity production TWh		District heat production cost SEK/kWh 1)		District heat production TWh	
	A	B	A	B	A	B	A	B
1979	.11	.11	94.0	94.0	.08 .09	.08 .09	30.8	30.8
1983	.07	.09	106.5	100.4	.10 .11	.09 .10	45.3	49.1
1987	.05	.13	115.7	103.9	.10 .11	.07 .08	48.0	51.2
1991	.09	.14	131.1	106.9	.09 .09	.07 .08	57.6	54.7
1995	.10	.14	134.0	113.1	.09 .09	.07 .08	62.1	59.2

1) The first figure refers to district heat market 1 (i.e. the largest urban areas) whereas the second figure refers to district heat market 2.

Table 5.6 Macroeconomic performance in the simulations A and B. 1979 = 100[1)]

	1983		1987		1991		1995	
	A	B	A	B	A	B	A	B
GNP	109.9	110.6	116.5	117.1	128.9	128.7	143.0	143.1
Private consumption	110.2	111.5	113.4	114.6	125.6	125.4	140.0	140.4
Exports	117.1	118.1	128.8	129.5	150.6	148.9	174.8	173.4
Imports	101.5	101.1	107.6	107.3	119.8	118.6	134.6	134.1
Public consumption[2)]	105.8		109.7		113.7		117.9	
Gross investments[2)]	94.3		106.5		119.9		135.2	

1) The results for the base year 1979 are identical in both simulations so the table values for simulations A and B are directly comparable to each other.

2) Exogenous variable in the simulations.

1987 and 1995 the production in this sector is 4-5 per cent lower in simulation B than in simulation A, primarily as a result of a lower export supply.

Table 5.7 The difference in the sectoral production levels between simulation A and simulation B. Per cent

	1983	1987	1991	1995
Sector				
2	+0.9	+0.5	-1.3	-2.0
3	+1.7	+1.7	-0.1	+0.1
4	-0.8	-5.2	-4.1	-3.7
5	+1.2	+2.0	-0.2	-0.1
6	+0.9	+0.8	-0.2	±0

Even though the results for the electricity and district heat sector differ considerably between the simulations, particularly in terms of electricity production and prices, the differences do not have any dramatic impact on the allocation of resources in the rest of the economy. This is not so surprising since the share of electricity and district heat production in the total production of the model economy is of the order of two or three per cent and the cost share of electricity ranges between one and five per cent in the different model sectors.

Even though the overcapacity in nuclear power is substantial in simulation A, it thus mainly manifests itself in the electricity price, which decreases to the short run marginal cost of nuclear power, and the total electricity production, which is significantly higher than in simulation B. The district heat prices are somewhat higher but the district heat production is largely the same in both simulations. But because of the large nuclear power capacity, heat is to a larger extent produced in pure heat plants instead of in combined power and heat plants.

Sensitivity analysis

With the nuclear power investments in simulation A the model economy has a notable excess capacity in nuclear power at least until the mid 1990's. In other words, the realized electricity demand is too low in relation to the power investments. A higher electricity demand could have come about in two ways. A higher level of activity in general in the economy would have raised the electricity demand and so would higher, price or cost induced, electricity input coefficients in the various sectors.[15]

In order to check the effects of other electricity demand developments on the extent of the nuclear power overcapacity some sensitivity analysis has been carried out. As far as the electricity input coefficients are concerned it does not seem motivated to check alternative assumptions about them. Compared to the time when the nuclear power investment decisions were made there has hardly been any energy price or cost change which has led to unexpectedly lower electricity input coefficients. On the contrary, the most dramatic energy price change which has ocurred since then - the oil price increase - has, through substitution away from oil probably increased the electricity input coefficients. But for one sector, namely the electricity and heat sector, it may be worthwhile to look closer at the intermediate use of electricity.

In both the main simulations the higher oil prices trigger a conversion from individual oil heating to district heating. Primarily in simulation A, which results in lower electricity prices, some of the district heat is produced from electricity, for instance, in heat pumps. To some extent the model however exaggerates the profitability of the district heat investments.

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15 A third way would be to export electricity. This alternative is however not considered here. Such an analysis would be of interest only if the integrated ELIAS-ENMARK model contained a better representation of the foreign electricity demand than the present infinitely elastic one. Export considerations have also never been an important argument for domestic power investments.

The district heat costs in the model are constant average costs. But in reality the marginal cost seems to increase markedly as the heat market area expands. With a large increase in the model's district heat demand the constant average cost assumption could lead to an underestimation of the district heat cost. It is thus of interest to investigate how a higher district heat cost would affect the demand for electricity.

A 40-50 per cent increase in the cost of converting from individual oil heating to district heating makes such conversions uneconomical in the model. But it does not mean that it is beneficial to convert to individual electric heating instead. In fact, no conversions at all take place in these simulations. In the model the cost advantage of electric over oil heating is not large enough to motivate the conversion cost.

To check the consequences of larger conversions to individual electric heating one sensitivity analysis was carried out with a 50 per cent lower cost of converting to individual electric heating while at the same time the cost of converting to district heating from individual oil heating was doubled. The results in terms of electricity production, the producer cost of electricity and the marginal value of nuclear power capacity are shown in Table 5.8.

The altered relative conversion costs lead to a large expansion of individual electric heating in 1983 and 1987. In these years there are no conversions to district heating, but they now instead occur in 1991 and 1995. As a result of the increased electric heat demand, the total electricity production is larger compared to simulation A. But the difference is less than the electricity required for electric heating because the higher electricity price resulting from the higher demand also leads to offsetting demand reductions in other parts of the model economy.

Table 5.8 The effects of larger conversions to individual electric heating. Corresponding results for simulation A are shown in brackets.

	1983	1987	1991	1995
Electricity production TWh Net of distribution losses	112.3 (106.5)	124.4 (115.7)	135.3 (131.1)	143.8 (134.0)
Producer cost of electricity, SEK/kWh	.09 (.07)	.08 (.05)	.11 (.09)	.13 (.10)
Individual electric heating TWh Net of distribution losses	17.7 (7.6)	23.4 (8.0)	25.7 (9.0)	27.9 (9.7)
Marginal value of nuclear power Million SEK/MW	.18 (.11)	.17 (.00)	.31 (.25)	.44 (.28)

Since the electricity price increases, the marginal value of nuclear power is now higher than in simulation A. But it is still well below the nuclear power capital cost at least until the mid 1990's. So the excess capacity remains in spite of the considerable increase in the electric heat demand.

A second sensitivity analysis is concerned with the general level of activity in the model economy. By doubling the rate of productivity growth the GNP growth rate is increased from an annual average rate of 2.3 per cent to 4.0 % in simulations A and B. The effects of the higher growth rate on electricity production, the producer cost of electricity and the marginal value of nuclear power are shown in Table 5.9.

Table 5.9 The effects of higher productivity growth. Corresponding results for simulation A are shown in brackets

	1983	1987	1991	1995
Electricity production TWh Net of distribution losses	109.9 (106.5)	125.2 (115.7)	147.9 (131.1)	155.3 (134.0)
Producer cost of electricity SEK/kWh	.07 (.07)	.07 (.05)	.14 (.09)	.14 (.10)
Marginal value of nuclear power Million SEK/MW	.12 (.11)	.11 (.00)	.50 (.25)	.52 (.28)

The electricity production is now markedly higher compared to simulation A since the higher growth rate in the economy increases the electricity demand. As a consequence the electricity price also increases and in fact reaches the long run marginal cost of nuclear power around 1990. Even though the marginal value of nuclear power is well below the capital cost for most of the 1980's the rapid economic growth means that this overcapacity will be more or less eliminated by 1990. Because the nuclear power capacity additions are concentrated to the first half of the 1980's a temporary overcapacity is unavoidable, but the high growth rate in the economy makes the overcapacity period quite short in this case.

The nuclear power overcapacity is in the preceding discussion defined relative to a nuclear capital cost of .52 million SEK/MW, calculated on the basis of 25 years of expected operating time and a discount rate of six per cent. An expected operating time of 40 years reduces the capital cost to .44 million SEK/MW. A four per cent discount rate reduces the capital cost to .43

million SEK/MW and with a four per cent discount rate and 40 years of expected operating time it is further reduced to .34 million SEK/MW.

In the last case, the nuclear power investment programme still leads to a rather substantial overcapacity during the 1980's but it is now eliminated in the early 1990's rather than around the year 2000. In the other two cases, with a nuclear power capital cost around .43 - .44 million SEK/MW, the nuclear power overcapacity still remains in the mid 1990's although it is less than in simulation A.[16] With larger conversions to electric heating, as in the first sensitivity analysis, nuclear power would now cover its capital cost as early as in the first half of the 1990's and with a more rapid economic growth the over-capacity would be a very temporary phenomenon due to the fact that the capacity additions are concentrated to a few years around 1985.

5.4 SOME CONCLUDING REMARKS

The most important conclusions from the simulations can be summarized as follows:

- the nuclear power investments, which increase the installed capacity from 3710 MW in 1979 to 9440 MW in 1985, are in the model economy too large and too early in the sense that they lead to equilibrium electricity prices which are below the long run marginal cost of nuclear power until around the year 2000

- the efficient nuclear power capacity is in the model economy around 3500 MW until the mid 1990's when it increases to around 4100 MW

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[16] These are however only partial observations. A lower rate of interest is consistent with the general equilibrium model only if the total gross investments are higher. This would lead to a higher electricity demand and thus a further reduction of the excess power capacity.

- the nuclear power overcapacity is mainly manifested in lower electricity prices and a higher production of electricity compared to the alternative with an efficient nuclear capacity. The district heat markets are affected to some extent since lower electricity prices lead to higher district heat prices.

- the nuclear power overcapacity has an impact on the rest of the economy, although it is not dramatic. Because of the too high nuclear power investments, too much capital is allocated to the electricity and heat sector. The result is an efficiency loss in GNP. The low electricity price particularly leads to a higher production, and a higher export supply, in the energy intensive export industries compared to the case where the electricity price is equal to the long run marginal cost of nuclear power.

One factor which contributes to the nuclear power excess capacity is the conversions from individual oil heating to district heating caused by the higher oil prices after 1979-80. The larger district heat market makes it more attractive to produce a larger share of the total electricity production in combined power and heat plants rather than in nuclear power plants. As was argued above it is possible that the model exaggerates the profitability of conversions to district heat. But also with modified assumptions which make conversions to electric heating relatively more profitable, most of the excess nuclear power capacity remains. It is, however, eliminated earlier in the 1990's.

A more rapid economic growth in the model economy reduces the length of the overcapacity period. But even with a quite rapid growth, some period of excess capacity is almost inevitable since the capacity increments are so concentrated in time. Apparently, the general level of activity in the economy is the most crucial factor for the size of the nuclear power excess capacity. The large nuclear power capacity increments seem defendable only if they were based on an expected economic development with a higher level of activity than the one realized in the main model simulations.

The integrated model has in this case made it possible to study how the electricity and heat sector, as well as the rest of the economy, adjust to different investment programmes for the power system. In particular it has been possible to compute efficient power capacities given an electricity demand which captures the full general equilibrium response to changes in the supply of electricity and heat.

The raison d'être of the model simulation experiments is of course that they are carried out in a model which quantitatively resembles the Swedish economy and that they are inspired by problems and features in the Swedish economy. The insights in how the model economy responds and adapts to different disturbances, such as the nuclear power investment programme, should give some guidance on how the Swedish economy is likely to react to the same disturbances. Although the results from the model economy simulations shall be interpreted with care when they are applied to the Swedish economy, they should at least be helpful in identifying crucial adjustments and the order of magnitudes of various responses.

The simulations indicate that a large part and perhaps all of the 5730 MW of nuclear power which are added to the Swedish electricity system between the years 1980 and 1985 ex post are economically dubious. The most important reason is a much lower realized electricity demand than the investment decisions seem to have presumed. This in turn is mainly due to a slowdown in the growth of the Swedish economy. It is of course important to remember that the term "realized development" refers to the actual development for the first years of the 1980's and a general consensus in downward revisions of the growth perspectives for the Swedish economy for the next ten to fifteen years. To what extent these downward revisions will turn out to be correct is of course impossible to know but, as the sensitivity analysis indicates, also with a growth rate that exceeds what is now generally expected, the nuclear power overcapacity remains until around 1990. Consequently, one can claim that these nuclear power investments were made at least ten years too early.

It should be noted that one effect of the nuclear power discontinuation policy analyzed in Chapter 4 is to increase the profitability of the existing nuclear power plants. They will capture some of the rent created when the electricity price increases, as a result of not allowing new nuclear power investments, above the long run marginal cost of nuclear power. The definition of an efficient power capacity on which this chapter is based, is that the electricity price equals the long run marginal cost. This criterion is, strictly speaking, correct only if not only the current but also the future electricity prices are equal to the (constant) long run marginal cost.[17] It is a reasonable hypothesis if one expects the long run marginal cost to be stable over time and the power capacities to follow an efficient path. But the effect of not allowing new nuclear power is to increase the long run marginal cost of the electricity system and the electricity price will gradually adjust to this higher level. Consequently, the now existing nuclear power plants will in their later years earn higher profits than they would have if new nuclear power investments had been allowed. This will to some extent counterbalance the initial losses due to the overcapacity in the 1980's and perhaps also the 1990's. In fact, if the electricity price during the 1990's starts to increase towards the long run marginal cost of coal condense power and reaches this level around the year 2000, then also the last built nuclear power reactor has a positive present value (at a six per cent discount rate).

[17] See also note 2 on p. 208.

6 Summary and Suggestions for Future Research

The basic purpose of this dissertation has been to extend the range of possible applications of applied general equilibrium models in the field of energy economics. By integrating detailed models of individual energy sectors with a general equilibrium model of the whole economy, the richer information of the former can be exploited in applications of the latter. In this way it becomes for example possible to explicitly assess the general equilibrium consequences of different energy production technology choices, as well as the effects of other events on the energy markets.

It is often claimed that changes - be they policy induced or market induced - on the energy markets, have strong repercussions on the rest of the economy, through their effects on the cost and the supply of energy. Such claims can be properly evaluated only if the general equilibrium effects can be assessed, for instance within an integrated model. By model integration it is also possible to capture the feedback effects from the rest of the economy to the energy sectors, and thus to evaluate the partial equilibrium exercises in which single sector models often are employed.

Although the model integration idea was initiated by the needs of a particular application, namely to study the Swedish electricity market in a general equilibrium context, the approach developed and applied in the previous chapters is also intended to have a wider use. Sector models are developed in many contexts and not only in energy economics. In many cases it would be valuable to use the information contained in these models also in applied general equilibrium exercises. One way to do this is through a model integration like the one which has been illustrated in the previous chapters.

In this final chapter, the contents of the previous ones are briefly summarized and their main results and conclusions are pointed out. The chapter is concluded with some suggestions for future research.

Summary, main results and conclusions

Applied general equilibrium modeling has been a very active field of research in recent years. It has proven to be a useful approach in applied economic analysis. Chapter 1 contains a brief introduction to this topic. The main merit of the applied general equilibrium approach is its emphasis of the mutual interdependencies among different parts of the economy and of the key role of relative prices. Another merit is its thorough theoretical underpinning, which makes it relatively easy to understand and interpret the results of model exercises. A major weakness - at least at its present state of development - is its empirical reliability. The applied general equilibrium approach emphasizes economic mechanisms which we from theoretical and empirical considerations find important. But the empirical basis for the parameterizations and the numerical specifications of these mechanisms is still, in many cases, quite underdeveloped. But the models can still, however, in the form of quantitative examples, provide valuable illustrations of the importance and the orders of magnitudes of these mechanisms. Thus they constitute a powerful analytical tool for numerical comparative static exercises.

In Chapter 2 the idea of model integration is presented in some detail. It is stressed that model integration does not involve any principal or conceptual difficulties, but that it primarily is a computational issue. For this reason some common computational methods for computing general equilibria, and their suitability in the context of model integration, are reviewed. One possible computational approach, which also is suitable in the context of model integration, is the optimization approach. It means that numerical optimization algorithms are applied in order to identify efficient allocations in the general equilibrium model. According to the fundamental equivalence theorems of competitive analysis, the equilibrium allocation can then be found among these. However, the reliance on the equivalence theorems is also a major limitation of the optimization approach. It is not unusual that applied general equilibrium models contain features which imply that the equivalence theorems are not valid, i.e. the competitive equilibrium is not necessarily an efficient allocation. One important example is the existence of commodity taxes and similar distortions in the general equilibrium model. In the final part of Chapter 2 it is shown, however, that the optimization approach still may be applied in this case, provided only that a slight modification is introduced. In its original form the optimization approach means that a welfare function is maximized subject to a number of market equilibrium and production technology constraints. The required modification is a change of the objective function. The new maximand shall consist of the original welfare function, multiplied by a scalar parameter, minus an expression for the total tax receipts. In Chapter 2 it is proven that a solution to this modified optimization problem can be interpreted as a competitive equilibrium in an economy with given unit commodity taxes, provided that the value of the scalar parameter by which the welfare function is multiplied, is equal to the inverse of the marginal utility of income.

This result extends considerably the usefulness of the optimization approach. It is important to be able to account for taxes and similar parameters in applied general equilibrium models even when they are not designed especially for tax studies. The models are usually calibrated against a base year allocation based on actual national accounts, which contain taxes, trade margins, sectoral wage differences, etc. To do this it is necessary to use tax parameters in the model.

The optimization solution approach is useful in model integrations because it makes it possible to account for weak inequalities and complementary slackness. This is important because the sector models which are relevant to use in this context are often activity analysis models.

In Chapter 3 the optimization approach to model integration is illustrated by an application to an activity analysis model of the Swedish electricity and heat sector and a general equilibrium model of the Swedish economy. This particular illustration was chosen in order to create a framework which is suitable for general equilibrium studies of the Swedish electricity market, and especially the role of nuclear power. By construction, the integrated model contains a detailed representation of the electricity and heat sector, which makes it possible to capture the effects on the rest of the economy of restrictions on the use of, for instance, nuclear power. And since the sector model is embedded in a general equilibrium structure, the production of electricity and heat is determined simultaneously with the allocation of resources in the rest of the economy. Thus, the integrated model makes it possible to assess the mututal interconnections between the energy sector and the rest of the economy.

The integrated model is a medium term model of resource allocation, characterized by sector specific capital stocks and a vintage structure for the existing capital. The vintage structure is due to the fact that the electricity and fuel input coefficients associated with past investments are fixed, while they are variable prior to that an investment has been made. Given the distribution of the capital stock over the different sectors and vintages, the integrated model determines the sectoral allocation of production and factor use, the flow of intermediate deliveries and the composition of the final demand (private and public consumption, gross investments and foreign trade) as well as a set of market clearing commodity and factor prices. For the electricity and heat sector it determines the utilization of the different electricity and heat production technologies during four subperiods.

The integrated model can be used to simulate a development over time for the model economy, by solving it for a sequence of consecutive time periods. The only intertemporal linkage between the periods is the amount of total gross investments. They are exogenous and their sectoral allocation is determined recursively once the one period equilibrium has been computed. The same recursive model also determines the energy input coefficients of the new vintage which is created when the total gross investments are allocated between the different model sectors.

The electricity and heat sector is, with respect to investments and energy input coefficients, treated differently. The investments in different electricity and heat production technologies are not determined recursively, but simultaneously with the one period equilibrium. In this way the joint determination of production, prices and investments in the electricity and heat sector, which is a very attractive feature of the original sector model, is preserved also in the integrated model. The vintage capital structure applies to all sectors except the electricity and heat sector. The own use of energy within the latter is determined by the chosen mix of production activities. Thus the energy input coefficients are always variable in this sector.

The ex post fixed energy input coefficients mean that the demand for energy within a single period is quite inflexible. This is further reinforced by the way the original electricity and heat sector model has been integrated with the general equilibrium model. The energy demand derived from the latter is disaggregated by a set of fixed coefficients to demands for the various outputs of the electricity and heat model. Both these properties probably mean that the integrated model underestimates the substitution possibilities, both between different energy commodities, as well as between energy and non-energy commodities. This means that the model tends to exaggerate the consequences of disturbances on the energy markets. On the other hand, since it is a Walrasian general equilibrium model, all markets adjust smoothly and costlessly to such disturbances. There are thus no frictions or temporary disequilibrium phenomena, which perhaps can occur on the markets of the actual economy.

Chapters 4 and 5 contain two exercises with the integrated model. In Chapter 4 the consequences of the present Swedish nuclear power policy are investigated. According to this policy no additions to the present nuclear power capacity are allowed and the twelve nuclear power plants which exist today shall have been taken out of operation at the latest in 2010. The objective of the analysis in Chapter 4 is to shed some light on how the nuclear power discontinuation policy will affect electricity production and electricity prices and on whether it will have any stronger repercussions on the rest of the economy.

The consequences of the nuclear power policy are assessed by comparing the development of the model economy in two different main simulations; one where nuclear power is allowed and one where it is discontinued according to the present nuclear power policy. The main simulations are then carried out in some different versions with alternative assumptions about certain crucial variables.

The results indicate that the long run producer price will increase from the long run marginal cost of nuclear power to the long run marginal cost of coal condense power. The latter thus replaces nuclear power as the base load capacity of the electricity production system. In fact, the actual electricity price increase will be even more pronounced, since the electricity price during the 1980's falls below the long run marginal cost of nuclear power. This is a result of the massive, and in time concentrated, nuclear power investments in the seventies and the first half of the eighties, which lead to an excess capacity in the electricity power system.

In spite of the rather inflexible energy demand within each single period, the demand response to a permanently higher electricity price is quite significant. The discontinued nuclear power capacity is to almost fifty per cent compensated by a lower electricity demand. The rest is replaced by coal condense power and, to a minor extent, by an increased production in coal fuelled combined power and heat plants.

There are no dramatic effects on the structural composition of production and foreign trade. The most noticeable effects are a lower production and lower exports from the energy intensive export industries and a lower intermediate use of energy, particularly electricity. The relative prices, except the electricity and district heat prices are largely unaffected. The wage rate is 1.5-2.5 per cent lower.

The higher cost of electricity production implies a productivity loss for the economy. In relative terms the losses seem modest; 0.7 per cent lower GNP per annum and 2.0 per cent lower private consumption expenditures in the base case simulations.

The integrated model has made it possible to explicitly impose restrictions on the use of nuclear power and to study how the electricity and heat markets, as well as the rest of the economy, adjust to these restrictions. Although the effects on the rest of the economy, in this particular case, are not very dramatic, they are still significant enough to affect the final equilibrium on the electricity and heat markets. The electricity and heat demand responses thus reflect the complete general equilibrium adjustments to the restrictions on the use of nuclear power.

Chapter 5 contains an evaluation of the large nuclear power investment programme in Sweden. It has been argued that these investments have resulted in an excess power capacity. In Chapter 5 an attempt is made to investigate the extent and likely duration of the excess capacity as well as its effects on the electricity market and on the rest of the economy. The actual nuclear power investment programme is compared to one in which nuclear power investments only occur if the electricity price is sufficiently high to cover the long run marginal cost of nuclear power. The implications of the difference between these two investment programmes for the electricity and heat sector and for the rest of the economy are then assessed.

The simulations indicate that a large part, and perhaps all, of the nuclear power capacity which was added to the Swedish electricity system between the years 1980 and 1985 ex post are economically dubious. The most important reason is the slowdown in the growth of the Swedish economy, which resulted in a much lower electricity demand than the nuclear power investment decisions seem to have presumed. Another reason is that the oil price increases have made district heating more competitive. As a result the efficient allocation of the electricity production between the nuclear power plants, on the one hand, and the combined power and heat plants, on the other, has changed in the direction of a larger share of the latter.

The nuclear power excess capacity is mainly manifested in a lower electricity price and a higher production of electricity compared to the alternative with an efficient nuclear power capacity. The simulations indicate that the excess capacity lasts until the latter half of the 1990's.

The excess capacity has an impact on the rest of the economy, although it is not dramatic. Because of the too high nuclear power investments, too much capital is allocated to the electricity and heat sector. The result is an efficiency loss in GNP. The low electricity price particularly leads to a higher production, and a higher export-supply, in the energy intensive export industries compared to the case where the electricity price is equal to the long run marginal cost of nuclear power.

The results of Chapter 5 show that there is a nonnegligible social cost associated with the excess investments in nuclear power. The results of Chapter 4, on the other hand, show that there is a social cost of abstaining from the nuclear power capacity. At first glance this may perhaps seem paradoxical, but these results are of course quite consistent with each other. In the case of the excess investments there is a social cost because the capital invested in the nuclear power plants could have earned a higher return elsewhere in the economy. In the case of the nuclear power discontinuation there is a social cost for two reasons. First, given that capital has been invested in nuclear power plants, and cannot be reallocated to any other use, there is of course a cost if it is prematurely scrapped. Secondly, when the excess capacity in the electricity production system has been eliminated and new power investments are required, then there is of course a cost of abstaining from nuclear power if the alternative is a more expensive power technology.

Suggestions for future research

The suggestions for future work relate to further model development work.

It is in particular the treatment of the investments in the integrated ELIAS-ENMARK model which is high on the agenda. As was emphasized in Chapter 3, the sectoral allocation of the gross investments and the asymmetric treatment of the electricity and heat sector compared to the remaining sectors, are unsatisfactory. It is in principle possible to improve this part of the model by incorporating also the ex ante production functions among the production technology constraints in Table 3.2 and thus to allow the current investments to be productively available also in the current period. The sectoral allocation of the investments would then be determined simultaneously with the current equilibrium prices. The marginal rates of return to new capital would then be endogenously determined in all sectors and the sector investments would be allocated so that these rates of return become equal. With this approach it would no longer be necessary to use an exogenous rate of interest to compute the capital costs of the electricity and heat sector.

It should be mentioned that this modification of the investment allocation in principle can be carried out more easily within the optimization solution approach than with ELIAS' original solution algorithm. But it is still a nontrivial modification if the ex ante production functions are highly nonlinear.

Another potential model development is to make the model explicitly intertemporal. This would allow a better treatment of the allocation of investments, not only within each single period, but also between the different periods.

Another unsatisfactory part of the present version of the integrated ELIAS-ENMARK model is the derivation of the heat demand in the general equilibrium model ELIAS. The various heat outputs of the ENMARK model do not have a proper counterpart in ELIAS. For this reason it would be useful to explicitly introduce a heating energy demand in the ELIAS part of the integrated model, at least for the residential services sector.

Finally, it seems worthwhile to test Mathiesen's sequential linear complementarity algorithm. Mathiesen's results indicate that this algorithm may be more efficient than the optimization code I have used to solve the integrated ELIAS-ENMARK model.

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