Essays on Exchange Rates, Prices and Interest Rates

by

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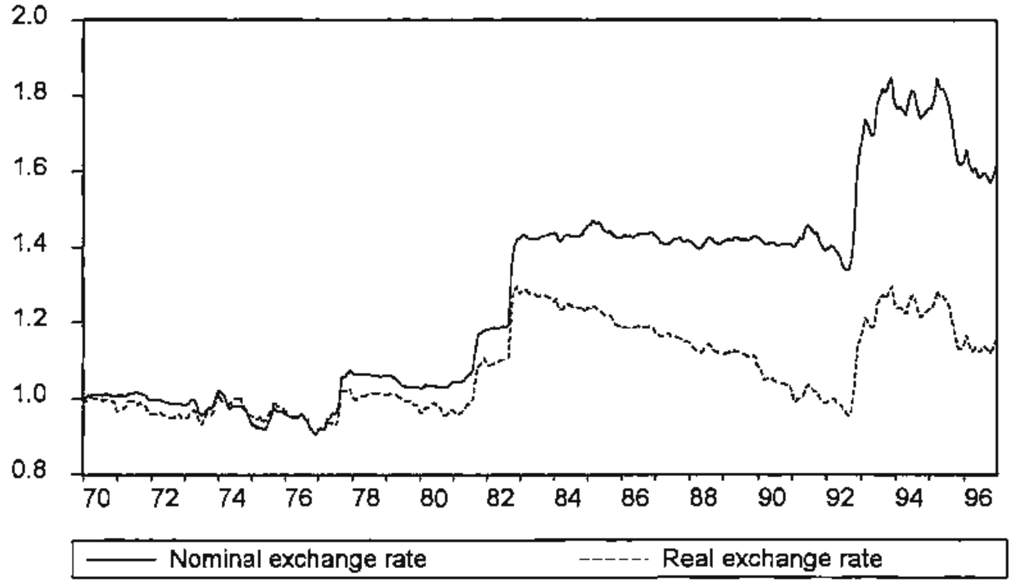
INTRODUCTION AND SUMMARY

While most economists seem to agree that monetary policy affects real variables in the short run but not in the long run, there is less consensus about how long the so called long run is, about the quantitative importance of short-run real effects or even about what mechanisms are at work. The four essays in this thesis are in one way or another concerned with the real effects of money. Chapter 2 is a theoretical model of how the welfare effects of simple inflation targets can be improved if they are supplemented by an escape clause to be evoked in case the economy is hit by a major shock. Chapters 3 and 4 report on empirical studies of deviations from monetary neutrality in the context of prices and exchange rates. This is a rewarding field for studying monetary neutrality since relationships between real variables on one hand, and nominal (as opposed to real) exchange rates on the other, clearly reveal non-neutralities. Chapter 5 addresses exchange rate risk premia on a number of European currencies against the German mark.

There are several channels through which monetary policy can affect real variables such as unemployment. If price adjustment is slow, changes in the nominal exchange rate will be accompanied by changes in the real exchange rate in the short run. By allowing a higher inflation rate than expected by private agents, the central bank may take actions which will lower the real wage and thereby increase employment. A third possibility could be that the actions of the central bank influence the variance of nominal variables like the exchange rate, which in turn has real effects. Other channels, like credit availability, are not discussed in this thesis.

Monetary authorities may influence the nominal exchange rate discretely by devaluing or revaluing the currency if the country has a fixed exchange rate or continuously by changing the interest rate if it has a floating exchange rate. With sticky prices, such measures will affect not just nominal but also real exchange rates. These real effects will persist until nominal prices have adjusted and restored the real exchange rate to its original equilibrium level. This is illustrated in Figure 1. It shows the effective real and nominal Swedish exchange rates from 1970 to 1996. The Swedish krona was devaluated in 1976, 1977, 1981, 1982 and subsequently allowed to float in 1992. During the years following each devaluation, the currency was obviously above its long-run average. Hence, Swedish products were inexpensive compared to the products of other countries and the competitiveness of Swedish exporting firms was temporarily improved. Each time, the competitive advantage was then gradually depleted as the Swedish inflation rate exceeded the foreign inflation rate.

Figure 1: Effective real and nominal Swedish exchange rates, 1970-1996



The effective exchange rates are weighted averages of the bilateral exchange rates against Sweden's 16 most important trading partners, using the MERM weights constructed by the IMF. The price levels used in the real exchange rates are consumer price indices. The higher the indices are, the weaker the exchange rates. Source: Sveriges riksbank

The simplest hypothesis about the behaviour of equilibrium real exchange rates is that they are constant. Chapter 3 reports on empirical tests of long-run purchasing power parity (PPP) on real exchange rate data from 16 OECD countries between 1960 and 1995. PPP was found to hold for less than half of the countries studied.[1] There is a tendency for it to hold better for small open countries whose economic development has been similar to that of their major trading partners. This makes sense since models of time varying equilibrium real exchange rates focus on differences among countries in the development of productivity, terms of trade and the current account. If a change in the nominal exchange rate (e.g. a devaluation) results in a change in the real exchange rate, money has had a real effect. The empirical results suggest that the adjustment to long-run equilibrium is rather slow. Half of the effect of a shock to real exchange rates has disappeared after 2-4 years.

Unless one is prepared to abandon the idea of a long-run equilibrium, equilibrium real exchange rates of the countries for which PPP did not hold must be changing over time. In this case, evidence that PPP does not hold is not evidence of monetary non-neutrality but of time varying equilibrium real exchange rates. An conceivable approach would be to model time varying equilibrium real exchange rates and test this model and monetary neutrality simultaneously. However, if disaggregated price data

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[1] PPP is usually rejected in empirical tests unless the sample period is extremely long, see e.g. Froot and Rogoff (1994).

are used, monetary neutrality may be tested in a more straightforward manner, without having to model the behaviour of real exchange rates.

If firms adjusted their prices to nominal exchange rate changes immediately and completely, monetary policy would not be able to influence real variables through the exchange rate. It is clear from Figure 1 that this is not the case. How exporting and importing firms choose to set their prices in response to exchange rate changes is obviously a key issue. For instance, exporters may prefer to reduce their profit margins and stabilise prices in the currency of the importing country in order to retain market shares when their domestic currency depreciates. While the aggregated consumer price indices studied in Chapter 3 are important to the economies, they reveal little information about the underlying behaviour of firms. In Chapter 4, disaggregated data are used to investigate how the price setting of Swedish export firms is affected by real and nominal exchange rate changes.

The concept of "pricing to market" implies that exporting firms set different prices in different countries of destination. There is ample documentation of such behaviour on the part of US, German and Japanese firms.[2] Since pricing to market is inconsistent with the small open economy assumption, Swedish exporting firms may be expected to behave differently from those in major industrial nations. Chapter 4 focuses on Swedish export prices of (ideally) identical products to Germany, France, United Kingdom, the United States and Japan from 1980 to 1995. If the marginal cost of production can be assumed to be the same irrespective of the destination country, information about the pricing behaviour of firms can be obtained from the prices charged in different national markets. A second advantage of this data set is that exchange rates, price levels and unemployment rates of the countries of destination may reasonably be assumed to be exogenous with respect to Swedish export prices. This is convenient since the simultaneous determination of prices, exchange rates and demand otherwise makes it difficult to disentangle deviations from monetary neutrality and pricing to market behaviour.

If firms respond to changes in nominal exchange rates that are not changes in real exchange rates, money is not neutral. If they respond to macroeconomic conditions in the countries of destination, they are engaged in pricing to market behaviour. If there is pricing to market behaviour but money is neutral, only real variables should influence relative export prices. Simple correlation coefficients show that relative export prices are related to both real and nominal exchange rates. The hypothesis that

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[2] See, for example, Knetter (1989), Marston (1990) and Kasa (1992).

nominal exchange rates should not have an independent influence on relative export prices over and above the effect through real exchange rates is testable as a short-run and as a long-run restriction on the data. Deviations from long-run monetary neutrality were found in about half of the 133 relative export prices studied. However, the sample period is only 15 years, which may be too short to test hypotheses about long-run restrictions. It may be more appropriate to interpret the results as evidence of persistent rather than permanent deviations from monetary neutrality. We also concluded that Swedish exporting firms appeared to be engaged in pricing to market since relative export prices were systematically related to real exchange rates and unemployment rates in the countries of destination.

Monetary policy may also affect real variables through real wages. If nominal wages are rigid, unexpected inflation created by the central bank lowers the real wage, which in turn induces firms to increase their labour demand. The central bank thereby has the possibility of influencing employment in the short run, until nominal wages have adjusted. The second chapter in the thesis belongs to the old but ever ongoing discussion about how this option should be used. It contains a theoretical model of how the welfare effects of simple inflation targets may be improved by adding an escape clause, to be evoked in case the economy is hit by a major shock.

A number of OECD countries including Sweden have recently introduced unconditional inflation targeting as the sole objective of monetary policy.[3] This implies that the development of real variables like unemployment has no weight in the central bank objective function. Such unconditional inflation targets are not easily motivated by the literature on optimal monetary policy since it has been shown that welfare can be improved by letting monetary policy respond to supply shocks, i.e. by allowing inflation to deviate from the target. On the other hand, the temptation to use monetary policy to reduce unemployment systematically creates an inflation bias. Various ways to solve this "credibility versus flexibility" problem by constructing objective functions for the central bank have been discussed in the literature. However, optimal central bank contracts appear to be difficult to implement in practice.[4]

There has been relatively little discussion concerning two related questions: What commitment technologies are available and, consequently, what types of central bank rules are feasible? For instance, the political system and the visibility of shocks

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[3] Among these countries are Canada, United Kingdom, Israel, Spain and New Zealand.

[4] Persson and Tabellini (1993) and Walsh (1995) have shown that the central bank can be induced to implement the optimal monetary policy if it minimises social loss plus a linear inflation punishment.

obviously constrain the complexity of contracts and the contingencies on which they may be conditioned. We observe only very simple monetary policy rules; perhaps simplicity is a key condition for feasibility. An escape clause to be evoked if and when a major shock hits the economy is one way to improve the welfare outcome of simple inflation targets.[5] It is shown in the paper that under certain circumstances, it is possible to calculate optimal trigger points below or above which the inflation targets should be abandoned in favour of output stabilisation.

In practice, central banks do not set the inflation rate as in the model. In most countries, they set (or target) a short interest rate. However, the options of central banks in small open economies are limited if capital is free to move internationally. As soon as the interest rate of the small open economy (say, Sweden) deviates from the interest rate of the neighbouring large economy (say, Germany), the exchange rate will move to bring expected returns to investments in the two currencies back into equilibrium. If the Swedish currency is perceived as risky and investors are risk averse, investments in Swedish krona may carry a risk premium over investments in German mark, meaning that investors demand a higher expected return in order to be willing to hold Swedish krona.

Although there may also be some political or default risk, measures of "risk" are often related to the volatility of asset returns. On second thought, it is obviously not simple volatility that matters but the covariance with consumption. An asset with low return in bad times, when consumption is low and the money is needed the most, is perceived as more risky than an asset with high return in bad times. Monetary policy may influence the variance of the exchange rate and/or the interest rate and hence the risk premium. For instance, it is often argued that participation in the European Monetary Union would reduce risk premia since nominal exchange rates become less volatile. Lower risk premia imply lower real as well as nominal interest rates. Such risk premia are studied in Chapter 5.

Using daily data on exchange rates and interest rates in Sweden, Norway, Finland, Italy, Spain, the United Kingdom and Germany from 1993 to 1996, we investigated whether expected Deutsche mark returns to investments in non-German currencies are higher in times of high risk.[6] We used two different measures of "risk": the conditional variance of returns and the conditional variances of unobservable risk factors in the

[5] Escape clauses have been discussed by Flood and Isard (1989) and Lohmann (1992).

[6] Two recent surveys of this literature are Engel (1996) and Lewis (1995).

economy.[7] Given specific assumptions about the statistical processes followed by the variables involved, the exchange rate risk premium can be obtained as a function of the conditional variances of unobservable risk factors. A Kalman filter was used to extract the risk factors from the observable *ex post* returns.[8] However, we found little evidence of risk premia in the sense that predictable returns are higher in times of higher risk was found.

To conclude, there appears to be considerable scope for monetary policy to affect real variables. The real effects of nominal exchange rate changes are so persistent that it is difficult to distinguish them from permanent effects using data sets covering several decades. A conclusion from the literature on optimal monetary policy has been that the costs of the option to pursue active monetary policy exceed the benefits. In this case, the hands of the policy maker should be tied to make it credible that she will not be tempted to use monetary policy to increase employment systematically.[9] However, it may be possible to exploit the scope of monetary policy to affect real variables in a manner that improves welfare. One way to do this would be to follow a rule according to which monetary policy may only be used in really bad times. Although this approach is not without practical problems, they appear to be manageable.[10] Since the real effects of monetary policy seem to be large, institutional design as well as policy making should take this into account.

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[7] The idea is similar to that of Baillie and Bollerslev (1990). They investigated whether foreign exchange rate risk premia are related to the variance of *ex post* returns and conclude that they are not.

[8] This method is adapted from King, Sentana and Wadhwani (1994).

[9] An example is Barro and Gordon (1983).

[10] There is actually one example of explicit escape clauses: in New Zealand, deviatious from the inflation target are allowed under a number of circumstances.

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INFLATION RULES
WITH CONSISTENT ESCAPE CLAUSES

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Abstract

Simple inflation targets may be supplemented with an escape clause to be evoked in case the economy is hit by a major supply shock. In this paper, consistent solutions to the Flood and Isard (1990) escape clause model are derived in the spirit of Lohmann (1990). She showed that Flood and Isard's assumption of symmetric boundary values of shocks, outside of which the zero inflation rule should be broken, is inconsistent if the output or employment target differs from the natural rate. This is quantitatively important since the optimal boundary values in the consistent model are highly asymmetric. The effects of unemployment persistence on the optimal escape clause are also investigated in a two period version of the model. In the second period, monetary policy should respond more often to supply shocks if unemployment is persistent. The first period effect may be of either sign.

1. INTRODUCTION

With the Kydland and Prescott (1977) concept of dynamic inconsistency, the debate on rules versus discretion in monetary policy took on a new dimension. The ideal monetary policy - to keep average or expected inflation at zero while responding to shocks - is not time consistent since the central bank has incentives to create surprise inflation. Since rules may be flexible, "price stability versus flexibility" may be a more appropriate description of the discussion than "rules versus discretion". The ideal policy may indeed be characterised as a (state contingent) rule that specifies a flexible response to shocks.

If fully state contingent rules could be implemented, there would be no need to discuss various second best solutions. The time inconsistency problem occurs because policy makers cannot commit credibly to the ideal monetary policy. No such commitment technology exists. Although this is the crucial issue, there is relatively little discussion in the literature about what kind of commitment technologies do exist and, consequently, what types of contracts with the central bank can be written. There seems to be widespread agreement that fully state contingent contracts are unfeasible, but it is not clear whether this is because the shocks are unobservable, unverifiable ex post or due to some other problem.

As discussed by McCallum (1994), "solutions" to the time inconsistency problem usually assume that a commitment technology exists outside the central bank, in the hands of society or the government. Thus, the time inconsistency problem caused by the lack of commitment technology is solved by assuming the existence of a commitment technology. On the other hand, one could argue that the original problem was that the central bank did not have incentives to pursue a zero inflation policy given its objective function. Most solutions to the problem imply other objective functions for the central bank that result in better time consistent outcomes. What is assumed to exist is a technology to design an objective function for the central bank.

If it is possible to create a credible commitment to a lower inflation rate, the inflationary bias can be reduced. Various proposals to this end have been discussed in the literature. Most of them imply a loss of flexibility to respond to shocks. This trade off between credibility and flexibility is one of the major dilemmas in monetary policy decision making. On one hand, monetary authorities wish to tie their hands in order to obtain credibility for anti-inflationary policies. For instance, a (credible) zero inflation rule completely eliminates the inflationary bias. On the other hand, if the hands of the monetary authorities are tied and the economy is hit by a major shock, the costs in terms of unemployment may be high. Flexibility is needed to mitigate the effects of shocks on the real economy.

With the central bank objective function designed by Rogoff (1985), society still faces a trade off between credibility and flexibility. He showed that social welfare can be improved by appointing a "conservative" central banker that dislikes inflation more than society does. Independent research by Persson and Tabellini (1993) and Walsh (1995) then showed that an incomplete contract that adds a linear inflation punishment to society's loss function mimics the outcome of the ideal complete or fully state contingent contract. The inflation bias is removed without changing the incentives to stabilise employment. If it were possible to implement an objective function where linear inflation punishment is added to the social loss function, the ideal outcome that combines zero average or expected inflation with full flexibility to respond to shocks could be achieved. The same outcome results if the unemployment target in the central bank objective function coincides with the natural rate of unemployment, or if the inflation target in the central bank objective function is lower than the inflation target in the social welfare function.[1]

Although the linear inflation punishment is optimal, it may not be feasible. What would such a punishment consist of? Central banks are normally financed by the public; making them pay a pecuniary fine would simply be a reshuffling of tax money. The cost of overriding the central bank or replacing its leadership is difficult to fine tune to a specific optimal value. No attempts to implement linear inflation punishments have been observed in practice.[2] Re specifying the inflation or unemployment targets in the central bank objective function also seems to be a rather abstract idea that would be difficult to implement. There appear to exist restrictions on the kind of central bank objective functions that can be implemented. What seems to have existed is discretionary policy, simple inflation or exchange rate targets (with implicit escape clauses) and one case of simple inflation rules combined with explicit escape clauses (New Zealand).

Possibly, escape clauses strike a feasible compromise between simple inflation rules (credibility) and discretionary policy (flexibility). The escape clause model was first discussed by Flood and Isard (1989b). It is an incomplete contract with only two states: normal times if a small shock is realized and bad times if a large shock is realized. The central bank follows a simple inflation rule in normal times and resorts to discretionary policy in bad times. Flood and Isard showed that a contract including an

[1] The former is shown by Persson and Tabellini (1990) and the latter by Svensson (1995).
[2] Walsh (1994) investigates whether the Reserve Bank act of New Zealand can be characterised as a linear inflation punishment and finds that is cannot.

escape clause to be evoked if the economy is hit by a major shock, is preferable to both simple inflation rules and fully discretionary policy. The problem is to specify for what shocks the escape clause should be evoked. Flood and Isard made the simplifying assumption that it should be evoked if the absolute value of the shock exceeds a certain value. This trigger value depends on the distribution of the shocks.

Lohmann (1990) subsequently showed that the Flood/Isard model is inconsistent. The simplifying assumption that the trigger values are symmetric around zero is indeed simplifying but also incorrect. As long as the output (or unemployment) target differs from the natural level, the optimal trigger points will be not be symmetric around zero. Responding to shocks that increase unemployment is more desirable than responding to shocks that decrease it. This seemingly subtle detail greatly complicates the calculations. While pointing out Flood and Isard's mistake and indicating how the problem could be handled consistently, Lohmann does not actually solve the model with asymmetric boundaries.

Obstfeld (1991) develops an exchange rate escape clause model in which the boundaries are not assumed to be symmetric. With a triangular distribution function for the disturbance terms, the model is not analytically solvable. Instead, Obsfeldt calibrates it and discusses numerical solutions given specific parameter values. For some parameter values, there is a unique equilibrium results in multiple equilibria.

In this paper, consistent solutions to an inflation escape clause model given a simple uniform distribution function are derived in the spirit of Lohmann (1990) and Flood and Isard (1989). The welfare effects of consistent escape clauses are analysed. In Section 3, unemployment persistence is added to a two period version of the model and the effects on the optimal boundary values are investigated. Section 4 contains conclusions and implications for policy making and institutional design.

2. A ONE PERIOD ESCAPE CLAUSE MODEL

With an escape clause contract, the central bank follows a simple inflation rule in normal times and resorts to discretionary stabilisation policy if the economy is hit by a major supply shock. The model consists of a social loss function and a Phillips curve that gives unemployment as a function of the inflation surprise and a supply shock. Private agents see through the decision problem of the central bank when forming their inflation expectations.

Two different formulations of escape clauses have figured in the literature. The contract may simply specify the interval of shocks $[\underline{z}, \overline{z}]$ within which the central bank keeps inflation at the target. Alternatively, society may leave the decision to the central bank and instead inflict a punishment on it for breaking the zero inflation rule. Obstfeld (1991) discusses both formulations. He calls the former "non-discretionary" and the latter "discretionary" escape clauses. Lohmann (1992) discusses a discretionary escape clause in which the central bank always adheres to the inflation target, but the government is able to override the central bank and pursue a discretionary policy at a certain cost. Flood and Isard (1989) primarily discuss non-discretionary escape clauses.

A non-discretionary escape clause contract may be thought of as follows: As long as the realized values of the supply shocks fall between $\underline{z}$ and $\overline{z}$, the central bank sticks to a zero inflation rule. If a supply disturbance that falls outside the band is realized, it breaks the rule and stabilise output optimally. The policy rule is:

$$(1) \quad \pi = \begin{cases} 0 & \text{if } z \in [\underline{z}, \overline{z}] \\ \arg\min L & \text{if } z \notin [\underline{z}, \overline{z}] \end{cases},$$

where L is the social loss function, z is the realized supply shock and π is the rate of inflation. As discussed by Obstfeld (1991), non-discretionary escape clauses presuppose a commitment technology that forces the central bank to follow the zero inflation rule in face of a minor disturbance. The policy rule in (1) is assumed to be perfectly credible even though the central bank always has incentives to pursue a discretionary policy. Non-discretionary escape clauses are not time consistent. The non-discretionary escape clause model is still useful for deriving the socially optimal boundaries of shocks within which the central bank should adhere to the zero inflation rule.

Given that non-discretionary escape clauses are time inconsistent, is it possible to design a time consistent escape clause contract that mimics the socially optimal outcome? A discretionary escape clause specifies that the central bank chooses at will when to break the zero inflation rule, but it has to pay a fixed cost if it chooses to do so. In equilibrium, the inflation expectations of private agents are consistent with the boundaries chosen by the central bank.

The discretionary escape clause model is time consistent in the sense that the central bank does not have incentives to renege on the contract ex post. However, it is vulnerable to the critique by McCallum (1994) that the time inconsistency problem is not solved but only transferred to a different level of the political system. Can it be made credible that "society" or the parliament will impose a cost on the central bank for decreasing unemployment in the short run? As in most solutions to the time inconsistency problem, the technology to determine an objective function for the central bank is assumed to exist.

The basic model is the same for both discretionary and non-discretionary escape clauses. The purpose of the exercise is to determine the optimal boundaries within which the central bank should stick to the zero inflation rule.

2.1. The non-discretionary escape clause

There are two states: a discretionary state D if a large disturbance occurs and a state R in normal times, where the central bank follows the zero inflation rule. Inflation in the discretionary state is derived as a function of the boundary values that determine expected inflation. This gives expected inflation and unemployment in both states. The inflation and unemployment rates are substituted into the expected social loss, which is minimised with respect to the boundary values.

The social loss function in each state is the same as usual in this literature:

(2) $$L_i = \left(U_i - \overline{\overline{U}}\right)^2 + \chi\pi_i^2$$

where U is the unemployment rate, $\overline{\overline{U}}$ is the target unemployment rate, π is the inflation rate and χ is the relative disutility of inflation. Defining q as the probability that the central bank will adhere to the rule, expected social loss is a weighted average of the expected losses in the two states.

(3) $$EL = qEL^R + (1-q)EL^D,$$

where L^R is social loss given that the central bank adheres to the zero inflation rule, L^D is social loss given that it breaks the rule and E is the expectation operator. As will be shown, the unemployment rates and the inflation rate in the discretionary state depend on the probability that the central bank adheres to the zero inflation rule.

The probability of adhering to the rule, q, is found by integrating the density function of the disturbance terms from the lower boundary $\underline{z}$ to the upper boundary $\overline{z}$:

(4) $q = prob(\underline{z} < z < \bar{z}) = \int_{\underline{z}}^{\bar{z}} f(z)dz$

A short run Phillips curve determines the unemployment rate as a function of the inflation surprise:

(5) $U = U^n - (\pi - E\pi) + z$.

where U^n is the natural unemployment rate, π is the inflation rate, $E\pi$ is expected inflation and z is a productivity shock.

The decision variables are the boundaries of the interval within which to adhere to the rule. In the original model by Flood and Isard (1989), the boundary values were assumed to be symmetric ($|\underline{z}| = |\bar{z}|$). It was subsequently shown by Lohmann (1990) that the objective function in (2) implies that the optimal bands are asymmetric ($|\underline{z}| > |\bar{z}|$) if the target level of unemployment is lower than the natural level. It is more desirable to respond to shocks that increase unemployment rather than to shocks that decrease unemployment. More precisely, she indicates that the bands would be symmetric around a constant rather than around the (zero) mean of the shocks.

To clarify the different assumptions about the boundaries, the probability that the central bank will adhere to the rule is defined as

$q = prob(|z| < \bar{\bar{z}}) = \int_{-\bar{\bar{z}}}^{\bar{\bar{z}}} f(z)dz$ with a symmetric band and

$q = prob(\underline{z} < z < \bar{z}) = \int_{\underline{z}}^{\bar{z}} f(z)dz$ with an asymmetric band.

All intervals have a midpoint and it will sometimes be convenient to express the asymmetric interval as symmetric around a non-zero constant:

$$q = prob(\bar{K} - \bar{\bar{z}} < z < \bar{K} + \bar{\bar{z}}) = \int_{\bar{K}-\bar{\bar{z}}}^{\bar{K}+\bar{\bar{z}}} f(z)dz .$$

The complications with an asymmetric interval arise from the non-zero conditional expected value of z given that the central bank breaks the zero inflation rule. With symmetric bands and symmetric distributions, the expectation of z given that it will fall outside the interval is zero. The probability that the central bank will break the zero inflation rule in response to a disturbance that decreases unemployment is as large as the probability that it will break the zero inflation rule in response to a disturbance that increases it. With asymmetric bands, it is more likely that the rule will be broken because of a supply shock that increases unemployment than because of a supply shock that decreases unemployment. The conditional expectation enters into expected inflation in both states and thereby into the Phillips curves and expected social loss.

While Lohmann shows that the Flood/Isard model with symmetric bands is inconsistent, she does not actually solve it with asymmetric bands. Since the use of asymmetric bands greatly complicates the calculations, several authors have sticked to the symmetry assumption[3]. The paper by Obstfeld (1991) is an exception in this respect.

Given the boundaries $\underline{z}$ and $\bar{z}$ and the resulting probability q that the central bank will adhere to the zero inflation rule, the model can be solved as usual, starting with the optimal discretionary inflation. It is found by substituting the Phillips curve (5) into the objective function (3) and minimising with respect to the discretionary inflation rate π_D. This yields:

$$(6)\ \pi_D = \frac{E\pi + U^n - \bar{\bar{U}} + z}{1+\chi}$$

The expected discretionary inflation rate is the expected value of (6):

$$(7)\ \ E\pi_D = E\left[\frac{E\pi + U^n - \bar{\bar{U}} + z}{1+\chi}\middle| z \notin \underline{z}, \bar{z}\right] = \frac{E\pi + U^n - \bar{\bar{U}} + E\left[z \middle| z \notin \underline{z}, \bar{z}\right]}{1+\chi}$$

Noting that $E\pi = (1-q)E\pi_D$, since $\pi_R = 0$, (7) may be solved for the expected inflation rate:

$$(8)\ \ E\pi = \frac{\left(U^n - \bar{\bar{U}} + E\left[z \middle| z \notin \underline{z}, \bar{z}\right]\right)(1-q)}{(q+\chi)}$$

Substituting (6) and (8) into the Phillips curve equation (5) gives the unemployment rates in the two states:

$$(9)\qquad U^R = U^n + E\pi + z$$

$$(10)\qquad U^D = U^n - (\pi_D - E\pi) + z$$

[3] One example is found in Persson and Tabellini (1990), p 30.

Finally, substituting equations (6) and (8) into (9), (10) and (3) gives social loss. As usual, all terms including Ez disappear in the expected social loss since the expected value of a shock is zero. The terms including $E\left[z^2\right]$ do not since the variance of the shocks is non-zero. Now there are two conditional variance terms that are functions of q: the variance of the shocks given that the central bank will adhere to the rule $\sigma_R^2(q)$ and the variance given that it will break the rule $\sigma_D^2(q)$.

(11)

$$EL = \frac{\left(U^n - \overline{\overline{U}}\right)^2 (1+\chi)}{\left(q(\underline{z},\bar{z}) + \chi\right)} + \frac{\chi}{1+\chi}\left(1 - q(\underline{z},\bar{z})\right)\sigma_D^2(\underline{z},\bar{z}) + q(\underline{z},\bar{z})\sigma_R^2(\underline{z},\bar{z}) + f\left(E\left[z \middle| z \notin \underline{z},\bar{z}\right]\right)$$

The first term of expected social loss is due to the labour market distortion that creates a wedge between the desirable and the natural rate of unemployment. There is the dead weight loss itself and an effect via expected inflation. The temptation to inflate depends (positively) on the labour market distortion. This term is decreasing in q since only in the discretionary state will there be a temptation to inflate. If q is equal to one, the central bank always sticks to the zero inflation rule and the first term of (11) consists only of the dead weight loss from the distortion. If q is less than one, expected inflation in the discretionary state is added as well.[4]

The second and third terms have to do with the variance of the shocks. Except for the term $\chi/(1+\chi)$, this is the probability weighted sum of the conditional variances and equal to the total variance of the shocks. What an escape clause does is that is reduces the total variance of unemployment by allowing stabilisation in response to large shocks. If the discretionary inflation rate (6) and expected inflation (8) are substituted into the unemployment rate in the discretionary state (10), it becomes clear that $\chi/(1+\chi)$ is the amount of stabilisation or response to a shock. Thus, $\chi/(1+\chi)\sigma_D^2(1-q)$ is the variance of unemployment under discretion, multiplied by the probability that the discretionary state will occur. Similarly, $q\sigma_R^2$ is the variance of unemployment under the zero inflation rule, multiplied by the probability that the non-discretionary state will occur. Since the zero inflation rule is broken when large shocks occur, the conditional variance in the discretionary state is much larger than the conditional variance under the zero inflation rule. Reducing the variance under

[4] As q approaches zero, the escape clause coincides with the discretionary regime, with the expected social loss $\frac{\left(U^n - \overline{\overline{U}}\right)^2 (1+\chi)}{\chi} + \frac{\chi}{(1+\chi)}\sigma_z^2$.. If q is equal to one, expected social loss will be $\left(U^n - \overline{\overline{U}}\right)^2 + \sigma_z^2$.

discretion therefore reduces the total variance considerably. Indeed, this is the whole point of an escape clause.

The last term, defined in equation (12), is a rather messy function of the expected value of the shock given that the central bank will stabilise optimally. This expected value enters into expected inflation, which enters into unemployment in both states and into the discretionary inflation (6). Under the simple inflation rule and under pure discretion, if the probability of adhering to the inflation rule is either zero or one, (12) is equal to zero since the conditional mean coincides with the unconditional mean, which is zero.

$$(12)\quad f\left(E\left[z \middle| z \notin \underline{z},\bar{z}\right]\right)=\frac{E\left[z \middle| z \notin \underline{z},\bar{z}\right](1-q)\left(E\left[z \middle| z \notin \underline{z},\bar{z}\right](1-q)+\left(U^{n}-\bar{\bar{U}}\right)(1+\chi)\right)}{(1+\chi)(q+\chi)}$$

Three terms in the expected social loss in (11) depend on the distribution of the shocks: the expected value of the shock given that the central bank will break the rule and the conditional variances in both states. Explicit solutions can only be obtained for specific distribution functions. Before returning to this issue in Section 2.3, two different ways to implement an escape clause will be discussed.

2.2. The time consistent discretionary escape clause

In the non-discretionary escape clause, the contract between society and the central bank specifies that the central bank should adhere to the zero inflation rule if the realized shock falls between the boundaries $\underline{z}$ and $\bar{z}$. The optimal interval within which the central bank should adhere to the rule is found by minimising expected social loss (11) with respect to $\underline{z}$ and $\bar{z}$. If the realized shock falls outside this interval, it stabilises employment optimally. As discussed in the introduction, the non-discretionary escape clause model is time inconsistent. Ex post, the central bank will always want to break the zero inflation rule.

Once the socially optimal boundaries of the interval are known, it may be possible to specify a fixed cost for breaking the rule that will induce the central bank to imitate the optimal behaviour. With a low cost of breaking the rule, it will of course choose to do so more often. If the fixed cost is chosen correctly, the central bank will apply the socially optimal boundaries. This is the discretionary escape clause.

Obstfeld (1991) applies such a model to the problem of adhering to a fixed exchange rate. He is not able to solve the model analytically. Instead, he calibrates it and discusses numerical solutions given specific parameter values. In some cases, the model results in multiple equilibria and instability. Expected depreciation is a function of how often the central bank will break the rule. The boundary values of the interval, in turn, are functions of the expected depreciation. If expected depreciation is higher, non-accommodation is more costly and the central bank will choose to break the rule more often. There are several combinations of boundary values and expected depreciation that are mutually consistent in the sense that expectations are rational given the boundary values chosen by the central bank and the boundary values are optimal given expected depreciation.

In this paper, analytical solutions to the discretionary escape clause model can be obtained given a simple uniform distribution function for the disturbance terms. With a uniform distribution, the equilibrium is unique as long as both the upper and the lower band fall within the area of possible outcomes of the shocks. A fixed cost for breaking the zero inflation rule that mimics the socially optimal outcome can be found under certain restrictions on the variance of the shocks. If the variance of the shocks is small enough, the optimal lower band falls outside the support of uniform distribution. In this case, there are multiple equilibria for some parameter values. As with the tent-shaped distribution used by Obstfeld, there are three equilibria. One equilibrium is always real and two may be real or imaginary depending on the parameter values.[5]

With a discretionary escape clause, the central bank makes a choice after the disturbance is realised. If it deviates from the rule and stabilises employment optimally, it has to pay the fixed cost C and gets the loss associated with L^D. If it sticks to the zero inflation rule, the loss is L^R. The objective function is identical to the social loss function except for the cost of breaking the rule:

$$(13)\quad EL = EqL^R + E(1-q)\left(L^D + C\right) = qE\left(U^R - \overline{\overline{U}}\right)^2 + (1-q)E\left[\left(U^D - \overline{\overline{U}}\right)^2 + \chi\pi_D^2 + C\right]$$

Given a realized shock, deviating is optimal if:

$$(14)\quad L^R(z) > L^D(z) + C$$

The derivation of optimal and expected inflation is exactly the same as above since the fixed cost of breaking the rule enters the objective function in a manner that does not

[5] Obstfeld considers only one sided escape clauses. In his model, the one sided exchange rate escape clause implies that revaluations of the currency are ruled out. If the fixed exchange rate rule is broken, the currency is always devalued. In terms of the model in this paper, this corresponds to an escape clause that allows the central bank to respond discretionarily only to large disturbances that increase unemployment.

influence the first order conditions which determine these variables. From the social losses (2) and unemployment rates in each state (9) and (10), we have:

$$(15) \quad L^R = \left(E\pi + U^n - \overline{\overline{U}} + z\right)^2$$

and

$$(16) \quad L^D = \left(E\pi - \pi^D + U^n - \overline{\overline{U}} + z\right)^2 + \chi\pi^{D2}$$

The latter expression can be simplified by substituting discretionary inflation from (6) into (16):

$$(17) \quad L^D = \left(\frac{\chi}{1+\chi}\left(E\pi + U^n - \overline{\overline{U}} + z\right)\right)^2 + \chi\left(\frac{1}{1+\chi}\left(E\pi + U^n - \overline{\overline{U}} + z\right)\right)^2 = $$
$$= \frac{\chi}{(1+\chi)}\left(E\pi + U^n - \overline{\overline{U}} + z\right)^2$$

The incentive constraint (14) can be simplified using (15) and (17). The central bank will prefer to deviate from the zero inflation rule and instead stabilise unemployment optimally if:

$$(18) \quad L^R - L^D = \frac{1}{(1+\chi)}\left(E\pi + U^n - \overline{\overline{U}} + z\right)^2 > C$$

If a sufficiently large or sufficiently small shock is realised, the benefits of stabilisation will exceed the cost that the central bank has to pay if it breaks the zero inflation rule.

The boundary values $\underline{z}$ and $\overline{z}$ as a function of the cost of breaking the rule are found by simultaneously solving equations (19) to (21) below. Equation (19) determines the upper boundary value of z for which the benefits of stabilisation equals the cost of breaking the inflation rule. Equation (20) determines the lower boundary, and equation (21) repeats expected inflation as function of the boundaries from equation (8). It is at this step that Obstfeld encounters multiple equilibria.

$$(19) \quad \frac{1}{(1+\chi)}\left(E\pi + U^n - \overline{\overline{U}} + \overline{z}\right)^2 - C = 0$$

$$(20) \quad \frac{1}{(1+\chi)}\left(E\pi + U^n - \overline{\overline{U}} + \underline{z}\right)^2 - C = 0$$

$$(21) \quad E\pi = \frac{\left(U^n - \overline{\overline{U}} + E\left[z \middle| z \notin \underline{z}, \overline{z}\right]\right)(1-q)}{(q+\chi)}, \text{ where } q = prob\left(\underline{z} < z < \overline{z}\right) = \int_{\underline{z}}^{\overline{z}} f(z)dz$$

As noted by Lohmann (1990), the boundaries are symmetric around a constant:

$$(22) \qquad \underline{z} = \overline{K} - \sqrt{K(C)} \text{ and}$$

$$\overline{z} = \overline{K} + \sqrt{K(C)}$$

where

$$(23) \quad K(C) = (1+\chi)C \text{ and}$$

$$(24) \quad \overline{K} = \frac{(1+\chi)\left(\overline{\overline{U}} - U^n\right) - (1-q)E\left[z \middle| z \notin \overline{z}, \underline{z}\right]}{\chi + q}.$$

A higher cost of breaking the rule results in a larger interval within which the central bank adheres to the rule. The midpoint of the interval is a function of the labour market distortion, the expected value of the shock given that the central bank breaks the zero inflation rule and, through the effects on expected inflation, the probability of adhering to the rule.

Given the boundary values that the central bank will choose for a given cost of breaking the rule, expected social loss can be expressed as a function of this cost. Equation (22) is substituted into the social loss function (11), which is minimised with respect to C. It is not a priori clear whether the socially optimal boundaries found by minimising expected social loss with respect to $\underline{z}$ and $\overline{z}$ coincide with the boundaries chosen by the central bank under a discretionary escape clause contract. The width of the interval in (23) is a continuos function of C, but the midpoint in (24) does not necessarily coincide with the midpoint of the socially optimal interval. This and several other interesting questions about the escape clause model can only be investigated for specific distribution functions of the supply shocks.

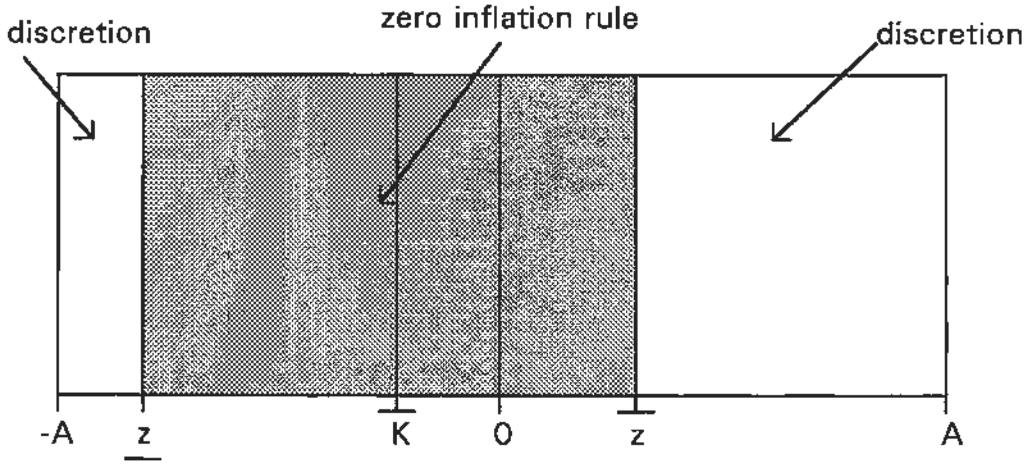
2.3. *Uniformly distributed shocks*

Explicit solutions to the escape clause model can only be obtained given an assumption about the distribution of the disturbance terms. Since the calculations get rather messy, it is preferable to start with the simplest possible distributional form: the uniform distribution.

A uniform distribution between A and $-A$ has the "height" $1/2A$. Its density function is:

$$(25) \qquad f(z)=\frac{1}{2A} \qquad z\in[-A,A]$$
$$f(z)=0 \qquad z\notin[-A,A]$$

Figure 1: A uniform distribution



With a uniform distribution, the conditional expectations of z take particularly simple forms. As is obvious from Figure 1, the expected value of z given that the central bank adheres to the rule is $\overline{K}$. It also follows from simple geometry that q can be expressed either as $\frac{\bar{z}-\overline{K}}{A}$ or as $\frac{(\bar{z}-\underline{z})/2}{A}$. The expected value of z given that the rule is broken can most easily be found by noting that the probability weighted sum of the conditional means equals the unconditional mean:

$$(26) \qquad qE\left[z\middle|z\in\underline{z},\bar{z}\right]+(1-q)E\left[z\middle|z\notin\underline{z},\bar{z}\right]=0 \quad \Rightarrow$$
$$E\left[z\middle|z\notin\underline{z},\bar{z}\right]=-\frac{q}{1-q}E\left[z\middle|z\in\underline{z},\bar{z}\right]$$

The conditional variances are connected in the same manner: the probability weighted sum of the conditional variances equals the unconditional variance.

$$(27a) \qquad \operatorname{var}\left[z\middle|z\in\left[\underline{z},\bar{z}\right]\right]=\sigma_R^2=\left(\frac{\bar{z}-\underline{z}}{2}\right)^2,$$

and

(27b) $$q\sigma_R^2 + (1-q)\sigma_D^2 = \left(\frac{A}{2}\right)^2 \Rightarrow$$
$$\operatorname{var}\left[z \middle| z \notin \left[\underline{z}, \overline{z}\right]\right] = \sigma_D^2 = \frac{A^2}{4(1-q)} - \frac{q}{(1-q)}\left(\frac{\overline{z}-\underline{z}}{2}\right)^2$$

The socially optimal interval or the non-discretionary escape clause is found by substituting equations (26) and (27) into expected social loss (11) and minimising it. It turns out to be convenient to minimise with respect to $\overline{K}$, the midpoint of the interval and $\overline{\overline{z}}$, the distance from the midpoint rather than with respect to $\underline{z}$ and $\overline{z}$. Both procedures result in explicit solutions, but all expressions except the one for $\overline{K}$ are algebraically extremely messy. It is the midpoint of the interval that is relevant for comparing the socially optimal boundaries to the boundaries from the discretionary escape clause model:

(28) $$\overline{K} = \frac{A(1+\chi)\left(\overline{\overline{U}} - U^n\right)}{2\overline{\overline{z}}}$$

Turning to the discretionary escape clause with uniformly distributed shocks, equations (19) and (20) together with expected inflation (9) are solved to get the boundary values as functions of the cost of breaking the rule:

(29) $$\underline{z}^* = \frac{(1+\chi)}{\chi}\left(\overline{\overline{U}} - U^n\right) - \sqrt{(1+\chi)C} \text{ and}$$
$$\overline{z}^* = \frac{(1+\chi)}{\chi}\left(\overline{\overline{U}} - U^n\right) + \sqrt{(1+\chi)C}$$

Now, expected social loss (11) given the conditional variances and conditional expected values from the uniform distribution can be minimised with respect to C. This gives the optimal cost of breaking the rule:

(30) $$C^* = \frac{A^2\chi}{3(1+\chi)} - \frac{4(1+\chi)\left(U^n - \overline{\overline{U}}\right)^2}{3\chi^2}$$

Some comparative static results can be obtained from (30). As expected, a higher variance of the shocks results in a higher cost of breaking the rule and a wider optimal interval. A larger labour market distortion shortens the interval and moves its midpoint further to the left. An increase in χ moves the midpoint of the interval to the right and increases the interval[6].

Expression (30) for C is positive if:

$$(31) \qquad A > \frac{2(1+\chi)}{\chi\sqrt{\chi}}\left(U^n - \overline{\overline{U}}\right)$$

Thus, if the variance of the shocks is large relative to the labour market distortion $\left(U^n - \overline{\overline{U}}\right)$ and society's relative disutility of inflation χ, there exists a double sided optimal escape clause. The central bank will give up the zero inflation rule if the economy is hit by large negative or large positive shocks. This happens because welfare is defined as a function of the squared deviation of unemployment from the desired rate. Very low unemployment is as undesirable as high unemployment.

The optimal non-discretionary escape clause can be compared to the optimal discretionary escape clause in (29) and (30). It is clear that the time consistent solution - to let the central bank decide when to break the zero inflation rule subject to a fixed cost of doing so - does not mimic the socially optimal boundaries. The optimal value of $\overline{\overline{z}}$ in the non-discretionary escape clause does not coincide with $\sqrt{(1+\chi)C^*}$ from the discretionary escape clause. The midpoints of the interval are also different since the optimal $\overline{\overline{z}}$ differs from $A\chi/2$.

In the calculations above, it was assumed that A is large compared to the labour market distortion so that both the upper and the lower boundary fall within the interval $[A,-A]$. In this case, it is optimal to let the central bank break the zero inflation rule and stabilise employment in response to shocks that decrease unemployment as well as to shocks that increase it. There are actually three possible types of solutions, depending on the relation between the optimal boundaries and the variance of the supply shocks. If the variance of the shocks is sufficiently small, both the optimal boundaries fall outside the interval $[A,-A]$ and the escape clause coincides with the simple zero inflation rule. For a middle interval of the variance of the shocks, the lower boundary hits $-A$ while the upper boundary is still below A and the escape clause will

[6] The partial derivative of the midpoint with respect to χ is $\left(U^n - \overline{\overline{U}}\right)/\chi^2$, which is positive. The partial derivative of the width of the interval, which can be either positive or imaginary, is:

$$\frac{A^2\chi + 8\left(U^n - \overline{\overline{U}}\right)^2(1+\chi)}{2\sqrt{A^2\chi - 4\left(U^n - \overline{\overline{U}}\right)^2(1+\chi)^2/\chi^2}}.$$

be one sided. These three cases apply to all distribution functions whose support is finite. With a normal distribution, for instance, the probability of an arbitrarily large or small shock is positive and it seems likely that a double sided escape clause will always exist[7].

The optimal one sided non-discretionary escape clause is found by minimising expected social loss with respect to $\bar{z}$ given $\underline{z} = -A$. There is a unique explicit solutions to this problem given a uniform distribution function, but it is algebraically extremely messy and not worth repeating here. The optimal discretionary one sided escape clause results in multiple equilibria with the uniform distribution. When (19) to (21) are solved for $\bar{z}$ given $\underline{z} = -A$, there are two equilibrium values of $\bar{z}$ for each cost of breaking the zero inflation rule. One equilibrium is always real valued and the other may be real or imaginary depending on the parameter values.

Obstfeld (1991) also found multiple equilibria when analysing one sided exchange rate escape clauses. If there are several equilibrium values of the upper band $\bar{z}$ for a given cost of breaking the zero inflation rule, private agents do not know what value to focus at when forming their inflation expectations. Since different inflation expectations lead to different boundary values for the central bank, the outcome is unstable unless there is a mechanism to coordinate the expectations to one of the equilibria.

To conclude this section, a unique solution to the discretionary escape clause model can be found if the variance of the shocks is large enough. However, the boundaries chosen by the central bank given the optimal cost of breaking the rule do not coincide with the socially optimal boundaries found by minimising expected social loss with respect to $\underline{z}$ and $\bar{z}$. The time consistent discretionary escape clause

2.4. Simple rules, discretion and escape clauses: A welfare comparison

Introducing an escape clause option to either the discretionary regime or the zero inflation regime cannot reduce welfare. However, it is possible to derive clear-cut conditions for an escape clause contract to be superior to simple rules and discretion.

Expected social loss under the simple zero inflation rule is found by substituting $E\pi = 0$ into (15) and taking the expected value of this expression. Alternatively, the probability of adhering to the rule may be set to one in the expected social loss (11):

[7] In principle, the model can be solved for any distribution function including the normal distribution. However, the computations turn out to be rather complicated with other than extremely simple distribution functions.

$$(32) \quad EL^R = \left(U^n - \overline{\overline{U}}\right)^2 + \sigma_z^2$$

Similarly, expected social loss under discretion is found by substituting (7) and (9) into (16) and taking the expected value. Alternatively, the probability of adhering to the rule and the expected value of the shock given that the rule will be broken in equation (11) may be set to zero.

$$(33) \quad EL^D = \left(U^n - \overline{\overline{U}}\right)^2 \frac{(1+\chi)}{\chi} + \frac{\chi}{1+\chi}\sigma_z^2$$

If social loss under the simple zero inflation rule exceeds social loss under discretion, the latter is preferable. This is the case if the variance of the disturbance terms is large relative to the labour market distortion and society's relative disutility of inflation:

$$(34) \qquad EL^R > EL^D \text{ iff } \sigma_z^2 > \frac{(1+\chi)}{\chi}\left(U^n - \overline{\overline{U}}\right)^2$$

The objective function in the escape clause models is:

$$(35) \quad EL = qEL^R + (1-q)EL^D = qE\left(U^R - \overline{\overline{U}}\right)^2 + (1-q)E\left[\left(U^D - \overline{\overline{U}}\right)^2 + \chi\pi_D^2\right]$$

Expected social loss as function of the boundaries was derived above and is repeated here for convenience:

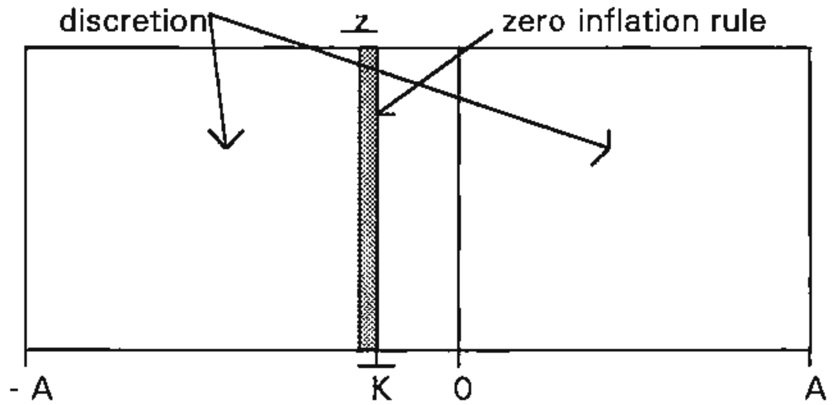
(36)

$$EL = \frac{\left(U^n - \overline{\overline{U}}\right)^2 (1+\chi)}{\left(q\left(\underline{z},\bar{z}\right) + \chi\right)} + \frac{\chi}{1+\chi}\left(1 - q\left(\underline{z},\bar{z}\right)\right)\sigma_D^2\left(\underline{z},\bar{z}\right) + q\left(\underline{z},\bar{z}\right)\sigma_R^2\left(\underline{z},\bar{z}\right) + f\left(E\left[z \middle| z \notin \underline{z},\bar{z}\right]\right)$$

The welfare effect of introducing a marginal region of zero inflation rule in the discretionary regime can be found by looking at the sign of the partial derivative of expected social loss with respect to the lower boundary $\underline{z}$ at the point where $\underline{z}$ and $\bar{z}$ are zero. Similarly, the welfare effects of introducing a marginal region of discretion in the zero inflation regime is determined by the sign of the partial derivative of expected social loss with respect to the upper boundary $\bar{z}$ at the point where $\underline{z}$ and $\bar{z}$ are minus and plus infinity.

Given a particular distribution of the disturbance terms, these partial derivatives can be analysed. The special case of a uniform distribution is investigated below. Introducing an escape clause in a purely discretionary regime would create a small interval of disturbances within which the central bank sticks to a zero inflation rule. Given the asymmetry that the labour market distortion creates, the section to be covered by the rule would be productivity shocks that decrease unemployment.

Figure 2: Introducing a small region of zero inflation rule in a discretionary regime



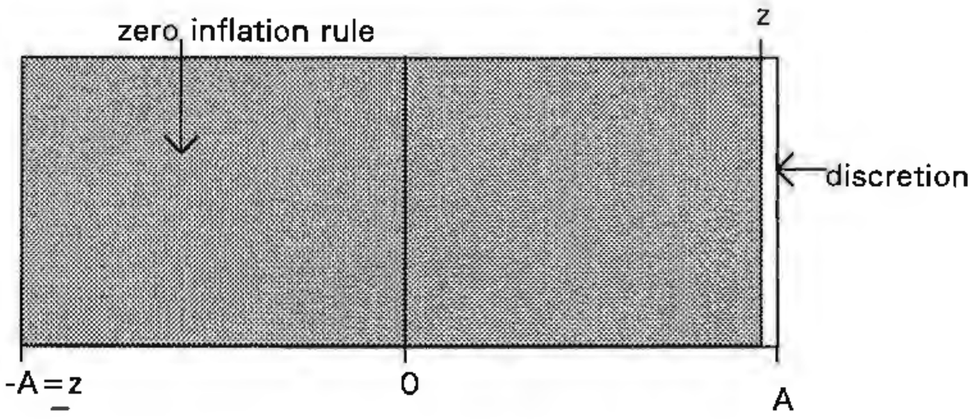
Pure discretion implies that both the upper and the lower boundary are zero so that there is no region within which the central bank adheres to the rule. The welfare effect of introducing an escape clause is captured by the derivative of expected social loss with respect to the lower boundary when both the boundaries are zero:

$$(37)\quad \left.\frac{\partial EL}{\partial \underline{z}}\right|_{\substack{\bar{z}=0\\ \underline{z}=0}} = \frac{(1+\chi)\left(U^n - \bar{\bar{U}}\right)^2}{2A\chi^2} > 0$$

The partial derivative of expected social loss with respect to the lower boundary at the point where both boundaries are zero is positive. This means that social loss can be decreased by decreasing the lower bound, thus introducing an escape clause or a section within which a zero inflation rule is implemented.

Similarly, going from a simple zero inflation rule to an escape clause means letting the central bank respond discretionary to the most severe realisations of the disturbance term. Unemployment is increased by positive shocks, so it is the largest shocks that are most destructive. Thus, the upper bound is marginally decreased when an escape clause is introduced.

Figure 3: Introducing a small discretionary region in a simple inflation rule regime



Now we are interested in the partial derivative of social loss with respect to the upper boundary at the point where the lower boundary is $-A$ and the upper is A:

$$(38)\quad \left.\frac{\partial EL}{\partial \bar{z}}\right|_{\substack{\bar{z}=A \\ \underline{z}=-A}} = \frac{A^2\left(15+22\chi+3\chi^2\right)+4A\left(U^n-\bar{\bar{U}}\right)\left(1+3\chi+2\chi^2\right)}{8A\left(1+\chi\right)^3}-\frac{4\left(U^n-\bar{\bar{U}}\right)^2}{8A\left(1+\chi\right)}$$

Equation (38) defines the parameter restrictions under which a one sided escape clause is superior to the simple zero inflation rule. This rather unintuitive expression is positive if the variance of the shocks is large enough compared to the labour market distortion. Thus, an escape clause model is always superior to pure discretion and superior to a simple inflation rule if the variance of the shocks is sufficiently large.

It is also interesting to investigate whether the welfare improvement from adding escape clauses to a simple inflation rule are quantitatively important. Given specific (and preferably realistic) parameter values, the expected social loss associated with different monetary policy rules can be calculated. Table 1 shows expected social loss for a few sets of parameter values given a simple zero inflation rule, discretionary policy, an escape clause and the optimal contract of Walsh (1995) and Persson and Tabellini (1992).

Table 1:
Expected social loss under different monetary policy rules and parameter values

A	$U^n - \overline{\overline{U}}$	χ	simple rule	discretion	escape clause	optimal contract
3	1	1	3.25	3.13	NA*	2.13
4	1	2	5.0	4.17	3.94	3.67
4	2	0.5	8.0	13.33	NA*	3.33
5	1	1	7.25	5.13	4.87	4.13
5	4	0.5	22.25	50.08	NA*	18.08
7	0.5	2	13.5	8.54	8.49	8.42
10	2	1	29	20.5	19.46	16.5

* For these parameter values, both the upper and the lower boundaries lie outside the possible outcomes of the shocks. Thus, the escape clause coincides with the simple rule.

A number of observations can be made from Table 1. First, for several fairly realistic sets of parameter values, the simple rule is not improved by an escape clause. The variance of the shocks has to be large in relation to the labour market distortion and the relative disutility of inflation for an optimal escape clause to exist. This is at least partly a consequence of the uniform distribution function. An escape clause is evoked only for major shocks. With a uniform distribution of the shocks between $-A$ and A, there are no extreme outcomes and an escape clause is not needed unless A is large.

Among the parameter sets chosen here, an optimal escape clause exists for approximately the same sets of parameter values for which discretion is preferable to a simple zero inflation rule. This is not a coincidence. The conditions for discretion to he superior to a simple zero inflation rule are similar to the conditions for an optimal (double sided) escape clause to exist. With a uniform distribution, the restriction becomes (39), which is indeed very similar but not identical to (37).

$$(39) \quad A > 2\sqrt{\frac{(1+\chi)}{\chi}\left(U^n - \overline{\overline{U}}\right)}$$

In the cases where an optimal escape clause exists, expected social loss is reduced by one third or more when the escape clause is added to the simple rule. Of course, other parameter values can also be found, for which the escape clause model results in exactly the same expected social loss as the simple rule. Discretion is preferable to a simple rule for many sets of parameter values. However, there are two factors that bias the results in favour of discretion compared to the simple rule. In order to ensure that major shocks occur with non-zero probability, the total variance has to be much larger with the present uniform distribution than with, for instance, a normal distribution. Also, the parameter values are partly chosen to illustrate the effects of an escape clause. With a uniform distribution function, it only exists for large values of A and small values of the labour market distortion.

3. ESCAPE CLAUSES IN TWO PERIODS WITH PERSISTENT UNEMPLOYMENT

The one period model in Section two may also be interpreted as a multiperiod model where there are no intertemporal links between the periods, so that all periods are identical. In particular, unemployment is only affected by a shock in one period. A conceivable intertemporal link between the time periods is to introduce unemployment persistence, so that parts of the effects of a shock in period t spill over into unemployment in period t+1.

In this section, unemployment persistence is introduced and added to the model. Since unemployment persistence is an intertemporal phenomenon, this necessitates at least a two period model. The optimal boundaries derived in Section 2 may change if unemployment is persistent. Intuitively, one would expect that it should be optimal to have a smaller interval of shocks within which the central bank adheres to the zero inflation rule if unemployment spills over to the next period. Expected unemployment and particularly its variance is higher the wider is the interval. Period t unemployment is more costly if it persists also in the next period. Society trades away some of this cost by letting expected inflation increase through a smaller interval within which the central bank follows the zero inflation rule.

The two period model with unemployment persistence is not analytically solvable even with the simple uniform distribution function. It can be calibrated and solved for particular parameter values, however. It turns out that the intuition above holds only for some parameter values.

3.1. Modelling unemployment persistence

There are several ways to model unemployment persistence. In this paper, a fraction λ of the deviation of first period unemployment from the natural rate carries over to the second period unemployment. It may be thought of as being added to the second period natural rate of unemployment. The first period Phillips curve is the same as in the previous section. In the second period, a weighted sum of the natural rate and first period unemployment takes the place of the natural rate.

(40) $U_1 = U^n - (\pi_1 - E\pi_1) + z_1$ and

(41) $$U_2 = (1-\lambda)U^n + \lambda U_1 - (\pi_2 - E\pi_2) + z_2 = \\ = U^n - \lambda(\pi_1 - E\pi_1 + z_1) - (\pi_2 - E\pi_2) + z_2$$

The social loss function is now the first period loss plus a discount factor β times the second period loss:

(42) $EL = EL_1 + \beta EL_2$

Expected social loss in each period is the same as in the last section:

(43) $$EL_i = q_i EL_i^R + (1-q_i)EL_i^D = q_i E\left(U_i^R - \overline{\overline{U}}\right)^2 + (1-q_i)E\left[\left(U_i^D - \overline{\overline{U}}\right)^2 + \chi E\pi_{iD}^2\right],$$

This problem is solved by backward induction. The second period optimal decision variables are expressed as functions of the first period variables and substituted into the objective function, which is minimized with respect to the first period decision variables. Since the second period is the last period, the decision problem is the same as in the one period model in the last section with one exception: The natural unemployment rate is replaced by the weighted sum of first period unemployment rate and the natural rate U^n. The second period unemployment rates in the two states are:

(44) $$U_2^R = (1-\lambda)U^n + \lambda U_1 + E\pi_2 + z_2$$

$$U_2^D = (1-\lambda)U^n + \lambda U_1 + E\pi_2 - \pi_2 + z_2$$

Thus, second period inflation and expected inflation are found by substituting $(1-\lambda)U^n + \lambda U_1$ for U^n in equations (7) and (9):

(45) $$\pi_{2D} = \frac{E\pi_2 + (1-\lambda)U^n - \overline{\overline{U}} + \lambda U_1 + z_2}{1+\chi}$$

The expected value of second period discretionary inflation at the point when the central bank decides on first period inflation (knowing the realisation of z_1) will be:

$$(46)\quad E_1\pi_{2D} = E_1\left[\frac{E_0\pi_2 + (1-\lambda)U^n - \overline{\overline{U}} + \lambda U_1 + z_2}{1+\chi}\middle| z_2 \notin \underline{z}_2, \overline{z}_2\right] =$$

Noting that $E_0\pi_2 = (1-q_2)E_0\pi_{2D}$, since $\pi_{2R} = 0$, (46) may be solved for the unconditional expected inflation in period two. q_2 is the probability that the central bank will adhere to the zero inflation rule in the second period or $\int_{\underline{z}_2}^{\overline{z}_2} f(z)dz$.

$$(47)\quad E_0\pi_2 = \frac{\left(U^n - \overline{\overline{U}} + E_0\lambda\left(q_1U_1^R + (1-q_1)U_1^D\right) + E_0\left[z_2 \middle| z_2 \notin \underline{z}_2, \overline{z}_2\right]\right)(1-q_2)}{(q_2+\chi)}$$

Optimal first period inflation given the effects in the second period is found by substituting (44), (45) and (47) into social loss (43) and minimising with respect to π_1^D:

$$(48)\quad \pi_1^D = \frac{(\chi+q_2)\left(U^n - \overline{\overline{U}} + z_1 + E_0\pi_1\right) + \lambda\beta(q_1-1)\left(U^n - \overline{\overline{U}} + \lambda(z_1 + E_0\pi_1)\right)}{\chi + \lambda^2\beta\left(1 - 2q_1 + q_1^{\,2}\right) + q_2}$$

Expected first period inflation will be $E_0\pi_1 = (1-q_1)E_0\pi_{1D}$ since inflation under the rule is zero. In principle, expected inflation and the discretionary inflation may be substituted into the Phillips curves to solve for the unemployment rates under discretion and under the rule. The unemployment rates may in turn be substituted into the objective function. Unfortunately, the expected social loss will be an extremely messy expression. It is difficult to solve this model analytically. However, it can be calibrated given a distribution function for the shocks and specific parameter values.

3.2. Uniformly distributed shocks

With uniformly distributed shocks, we know the second period boundaries and the optimal second period cost of breaking the rule from equations (29) and (30). All that is needed is that the natural unemployment is replaced by the weighted sum of first period unemployment and the natural rate. Society makes decisions about the second period boundaries (in the non-discretionary model) or the second period cost of breaking the rule (in the discretionary model) before it knows the realisations of the shocks. Thus, the boundaries $\overline{z}_2$ and $\underline{z}_2$ and the cost C_2 are based on expectations in period zero:

$$(49)\quad \underline{z}_2 = E_0\left[\frac{(1+\chi)}{\chi}\left(\overline{\overline{U}}-(1-\lambda)U^n-\lambda U_1\right)-\sqrt{(1+\chi)C_2}\right],$$

$$(50)\quad \overline{z}_2 = E_0\left[\frac{(1+\chi)}{\chi}\left(\overline{\overline{U}}-(1-\lambda)U^n-\lambda U_1\right)+\sqrt{(1+\chi)C_2}\right] \text{ and}$$

$$(51)\quad C_2^* = E_0\left[\frac{A^2\chi}{3(1+\chi)}-\frac{4(1+\chi)\left((1-\lambda)U^n+\lambda U_1-\overline{\overline{U}}\right)^2}{3\chi^2}\right]$$

It is clear from the above equations that the second period interval will be smaller than in the one period model - the central bank will accommodate shocks more often if unemployment is persistent. The denominator of the second term in (51) is larger with unemployment persistence unless the variance of the shocks is zero.[8] If the optimal cost of breaking the rule is larger, the resulting interval within which the central bank should adhere to the zero inflation rule is smaller. The intuition behind this is that persistent unemployment increases the variance of second period unemployment. Society dislikes variance in unemployment and trades some of it against increased expected inflation by reducing the interval within which the central bank adheres to its zero inflation rule. However, the midpoint of the interval is unchanged by unemployment persistence.

The optimal first period boundaries and/or the optimal cost of breaking the rule can in principle be found by minimising expected social loss after the second period variables have been substituted out. Unfortunately, the expressions get too complicated even with a simple uniform distribution. What can be done is to insert specific parameter values and minimise expected social loss numerically. This results in optimal first and second period intervals for each set of parameter values. An example is shown in Figures 4a-4c. The parameter values are not chosen to be realistic but to illustrate how the model works. The optimal first period interval in Figure 4a is rather asymmetric around zero. If the labour market distortion is increased further, the lower boundary will hit $-A$ and the optimal escape clause will be one-sided.

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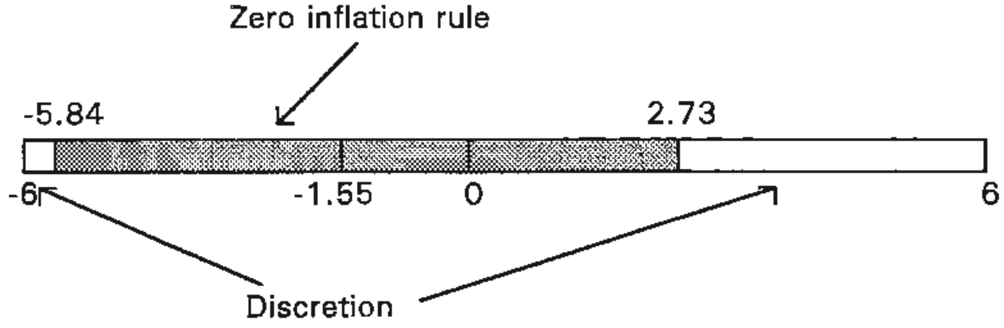
[8] If the second term is larger with persistent unemployment, C_2 is smaller than C. Comparing $E_0\left(U^n-\overline{\overline{U}}\right)^2$ to $E_0\left[(1-\lambda)U^n+\lambda U_1-\overline{\overline{U}}\right]^2$, the former is equal to $U^{n^2}+\overline{\overline{U}}^2-2U^n\overline{\overline{U}}$. Substituting (40) for U_1, expanding and taking expectations of the latter yields $U^{n^2}+\overline{\overline{U}}^2+\phi z^2-2U^n\overline{\overline{U}}$, where ϕ is a positive constant. Thus, C_2 is smaller than C unless z^2 is equal to zero.

Figure 4a: The optimal first period interval when $\chi = 1.7$, $\lambda = 0$, A=6 and $U^n - \overline{\overline{U}}$=1



From Figures 4b and 4c, it can be seen that the optimal interval is much smaller with unemployment persistence than without. For other sets of parameter values, the result will be the opposite.

Figure 4b: The optimal first period interval when $\chi = 1.7$, $\lambda = 1$, A=6 and $U^n - \overline{\overline{U}}$=1

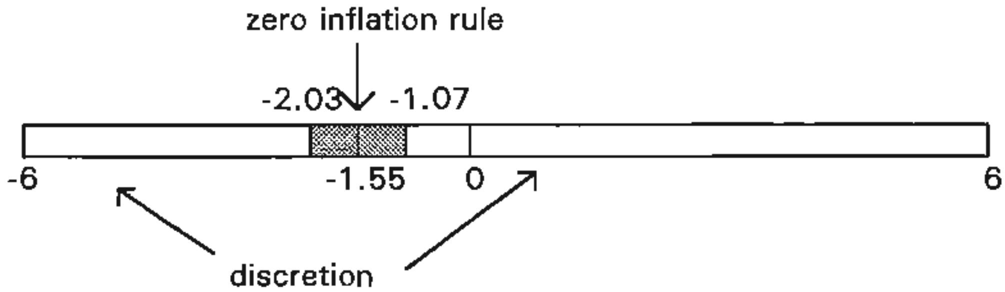
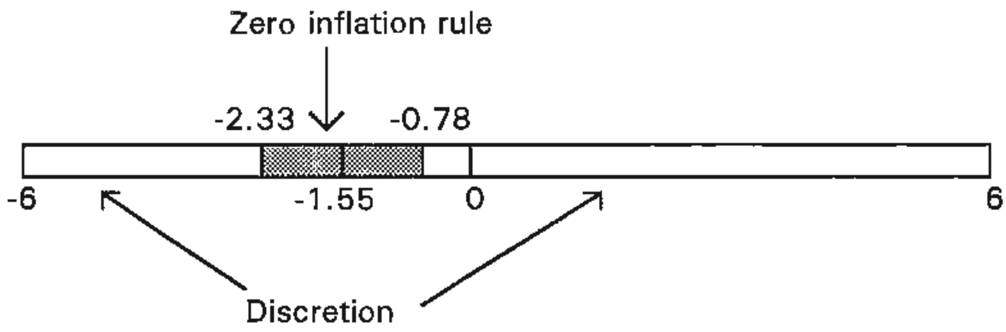


Figure 4c: The optimal second period interval when $\chi = 1.7$, $\lambda = 1$, A=6 and $U^n - \overline{\overline{U}}$=1



Intuitively, the optimal second period interval is smaller because as the variance of second period unemployment increases, a portion of it is traded for higher expected inflation. Concerning the first period, there are mechanisms at work in both directions. A smaller interval may be preferable since it is more costly not to stabilise when the unemployment created in the first period carries over to the second period. On the other hand, the temptation to inflate is very large with persistent unemployment. In order to avoid this temptation, a larger interval within which the central bank follows the zero inflation rule may be desirable. The relative importance of these two effects varies with the parameter values, and no general conclusion can be drawn.

The optimal first period intervals are found by numerical minimisation of the expected social loss for each set of parameter values. The expressions are extremely messy and it is not possible to show how the first period optimal boundaries behave in the general case.

A second observation from Figure 4 is that the optimal boundaries are indeed very asymmetric around zero. This property is increasing in the difference between the natural and the target rates of unemployment. Assuming that the boundaries are symmetric around zero is not a good approximation. The fact that it is optimal to cushion shocks that increase unemployment rather than shocks that decrease it is quantitatively important and can not be neglected.

3.3. Simple rules, discretion and escape clauses: Welfare with persistence

The counterparts of equations (37) and (38), which define under what conditions the optimal escape clause is superior to simple rules and discretion are analytically intractable in the two period model with persistent unemployment. Instead, the quantitative effects of adding unemployment persistence are illustrated in Table 2.

Table 2: Expected social loss when unemployment is persistent

A	$\left(U^n - \overline{\overline{U}}\right)$	χ	simple rule	discretion	escape clause
3	1	1	8.75	14.24	NA*
4	1	2	14.00	52.00	12.53
4	2	0.5	20.00	147.11	NA*
5	1	1	20.75	18.25	18.01
5	4	0.5	50.75	580.86	NA*
7	0.5	2	37.25	19.10	17.76
10	2	1	83.00	73.00	64.44

* For these parameter values, both the upper and the lower boundaries lie outside the possible outcomes of the shocks. Thus, the optimal escape clause coincides with the simple rule.

The most conspicuous feature of Table 2 is perhaps that expected social loss under discretion is very large if unemployment is persistent, even with a moderate labour market distortion. There are many parameter values for which the simple zero inflation rule cannot be improved by an escape clause. This kind of exercise would be more interesting in a model with more than two periods or with a distribution function with non-zero probability of extreme outcomes.

4. POLICY IMPLICATIONS

The normative literature on time-consistency has undoubtedly had a substantial real influence on monetary policy institutions in several countries. New Zealand is perhaps the most obvious case. The Governor of the central bank is subjected to a contract that entails his dismissal if inflation exceeds two percent. New Zealand is also the only country that uses explicit escape clauses that allow the central bank to change its priorities in special circumstances. Institutional reforms aiming at increased central bank independence are discussed in Sweden among other countries. Inflation targets have been adopted as the single official objective of monetary policy in a number of countries.

An inflation rule with an escape clause results in inflation outcomes that are either at the target, if the realized shock is minor, or far off the target, if a major shock is realized. This gives the following inflation rule:

$$(52) \quad \pi = \begin{cases} 0 & \text{if } z \in [\underline{z}, \overline{z}] \\ \arg\min L(\pi(z)) & \text{if } z \notin [\underline{z}, \overline{z}] \end{cases}$$

New Zealand alone specifies explicit escape clauses or circumstances under which the central bank may deviate from the inflation target. Some of these special circumstances refer to how the relevant price index is measured, while others are indeed supply shocks, as the model says that they should be. The inflation rate can be expected to move outside the permissible range in response either to significant changes in import or export prices or to a crisis such as a natural disaster or a major disease among livestock. Central bankers in other countries have occasionally hinted that they have implicit escape clauses (United Kingdom, Sweden)[9].

In contrast to the policy rule in (52), the monetary policy rule in a number of countries specifies a target rate of inflation and a permissible range of outcomes around the target:

[9] Mervyn King (1995, p 12) interprets the British inflation target to be conditional on special circumstances. The Governor of the Swedish Central Bank has expressed similar opinions in speeches.

(53) $\pi \in [\underline{\pi} \le \pi^* \le \overline{\pi}], \quad \forall z$

Table 3 shows some examples of inflation targets and ranges of permissible outcomes.

Table 3: Inflation targets and ranges of permissible outcomes

Country	target	range	for the years	comments
Canada	2%	1-3%	1995	A path of gradually lower targets has been used since 1992.
Spain	- -	below 3%	1996-99	The exchange rate may be more important under certain conditions
New Zealand	-	0-2%	1992 and on	Explicit escape clauses are in effect
Israel	-	8-11%	1995	Target ranges for the following year are announced each year in September
United Kingdom	2.5%	1-4%	1995	By 1997, the target rate is reduced to 1.75 % and the range will be 1-2.5 %
Finland	2%	-	1995 and on	
Sweden	2%	1-3%	1995 and on	

There are several ways to interpret such rules. $\underline{\pi}$ and $\overline{\pi}$ could correspond to $-\theta A$ and θA. In that case, the monetary policy rule implies a discretionary response to supply shocks, but the supply shocks are so small (or the range of permissible outcomes is so wide) that monetary policy is able to respond to the shocks while keeping inflation within the band. Given that the width of the band is usually only two percentage points while output and unemployment vary considerably more, this interpretation seems unlikely.

Alternatively, a range of permissible outcomes could be interpreted as an attempt to follow a simple inflation rule without using monetary policy to respond to shocks. The use of ranges instead of simply a target rate of inflation could be a consequence of the fact that central banks are unable to fully control the inflation rate. The width of the band then reflects the extent to which central banks perceive that they control the inflation rate. Since central banks attempt to steer the inflation rate subject to various disturbances, this seems to be a more reasonable interpretation.

A conceivable disadvantage of inflation bands is that the inflation target tends to be replaced by the upper boundary of the interval. For instance, if the inflation target is two percent per year and the band ranges from one to three percent, the actual goal of monetary policy may become to keep the inflation rate below three percent. It is in order to avoid this that the Bank of Finland has chosen to announce only the target inflation rate without specifying a range of permissible outcomes.

In the simple model discussed in this paper, the ideal inflation target is zero, since it is assumed that there are no benefits from expected inflation. If some small amount of inflation is assumed to have a positive effect on the economy, for instance by facilitating relative price changes, the non-zero inflation targets used by a number of central banks may be motivated. A non-zero inflation rule can easily be incorporated into the model, but deriving it endogenously necessitates a more complicated model with relative price stickiness or some other imperfection that creates benefits from inflation.

In the model, the central bank assesses the size of the realized shock and decides whether to stick to the zero inflation rule. In practice, shocks are not directly observable. However, the rate of unemployment is an observable variable. There is a one-to-one correspondence between the size of the disturbance and the change of the unemployment rate in the model. The optimal boundaries on the disturbance z are easily transformed to optimal boundaries on the unemployment rate U. Thus, even if the shocks are unobservable, the model can be rewritten in terms of an observable variable.

The reasoning above is valid if there are only supply shocks in the model. In practice, there are many different types of shocks. Demand shocks are seldom discussed within the context of optimal monetary policy, because they do not constitute a problem (as long as they are observable). A demand shock increases unemployment and decreases inflation. If the central bank creates surprise inflation in response to the shock, unemployment will decrease and inflation will increase. In this framework, it is always optimal (and possible) to offset demand shocks completely. In contrast, since a supply shock increases both unemployment and inflation, the policy makers are faced with a trade off between inflation and unemployment stabilisation. It is generally not desirable to offset supply shocks completely.

If there are several different types of shocks and the policy makers are unable to distinguish between them, boundaries on the unemployment rate are not equivalent to boundaries on the supply shocks. In practice, many shocks are directly observable and policy makers may have rather clear ideas about what shocks the economy has been hit by. For instance, oil price increases due to political events are clearly observable. In the case of New Zealand, the different types of shocks for which the escape clause is to be evoked are clearly specified but the trigger values are not. This suggests that quantitative uncertainty is more important than qualitative.

Another problem with the time consistent discretionary escape clauses is that they may be plagued by multiple equilibria. Obstfeld (1991) finds multiple equilibria in his one sided exchange rate escape clause model given a triangular distribution. In this paper, double sided escape clauses have a unique equilibrium given a uniform distribution of the shocks, while there are multiple equilibria even with uniformly distributed shocks if the variance of the supply shocks is so small that the escape clause should be one sided. With the tent shaped distribution used by Obstfeld, there may be multiple equilibria also in the double sided case. The alternative equilibria may involve high expected inflation that the central bank chooses to accommodate frequently, thus justifying the expectations. The models have only been solved for extremely simple and unrealistic distribution functions. If a discretionary escape clause was actually implemented, the economy could well end up in a bad equilibrium.

The discretionary escape clause model is vulnerable to the same criticism as the optimal linear inflation punishment discussed in the introduction: What exactly constitutes the punishment for breaking the zero inflation rule or creating excess inflation? Is it a pecuniary fine? Central banks are government institutions; imposing fines on them would simply imply a reshuffling of tax money. Is the fine taken from the salaries of the board or the governor? This seems to be a possible way to create incentives for price stability, but it would be very difficult to create an objective function where the preferences of society are supplemented by an additional marginal cost for creating inflation. If the salaries of the central bank management depend on inflation outcomes, they have no obvious incentives to take the society's preferences for low unemployment into account.

It seems more reasonable to interpret the cost of breaking the zero inflation rule as a loss of reputation. In a multiperiod model, the central bank's concern about future inflation expectations creates incentives not to inflate today. But the cost of loosing its reputation for low inflation is given. It is not possible to specify a correctly chosen cost that will induce the central bank to choose the optimal boundaries. The fact that exactly measured "punishments" may be difficult to implement is discussed in Walsh (1995). However, there is relatively little discussion about what kind of central bank objective functions are feasible. Only very simple monetary policy regimes have been observed in practice.

Except for New Zealand, the countries that pursue credible low inflation policies do not seem to posses any particular "commitment technology". For example, Germany and Switzerland have independent central banks and a track record of low inflation rather than special commitment technologies. What appears to be at work is reputational forces in a repeated game combined with simple monetary policy objectives. The way to implement an escape clause model would be to specify the socially optimal boundaries and give the central bank incentives to apply them through the threat of loosing its reputation. This is a non-discretionary escape clause. Given that clear formulations of the monetary policy objectives improve credibility, explicitly formulated escape clauses are clearly preferable to an implicit understanding that the inflation target will not be pursued at all costs.

McCallum (1995) criticises the literature on optimal monetary policy for overemphasising the problem of dynamic inconsistency. According to him, nothing stops the central bank from simply implementing the first best solution and set average inflation at zero. On average, it would achieve higher utility in this manner. However, the behaviour proposed by McCallum does not maximise the central bank's objective function as stated in his model. Abandoning the principle of maximising agents may not be the optimal response of the researcher. Perhaps multiperiod models with reputational equilibria provide the objective function that motivates far sighted central banks to avoid fruitless attempts to exploit the short run Phillips curve. The same mechanism could in principle be used to implement non-discretionary escape clauses.

The new findings of this paper can be summarised as follows: In a consistent escape clause model, the optimal boundaries are highly asymmetric around zero. The simplifying assumption that they are symmetric cannot be considered a good approximation. It is optimal to respond to shocks that increase unemployment rather than to shocks that decrease unemployment since society's target unemployment rate is lower than the natural rate.

Escape clauses have been shown to be preferable to simple inflation rules if the variance of the supply shocks is large enough compared to the labour market distortion and the relative disutility of inflation. It is unclear whether this condition is met in practice. The assumptions of this particular model bias it in favour of simple rules in two ways. First, there are no really bad outcomes with a uniform distribution function unless the total variance is very large. With a distribution function where the probability of extreme outcomes is non-zero, escape clauses may be preferable under more generous parameter restrictions. Second, the form of the social loss function implies that society is risk neutral. If risk aversion is assumed, escape clauses that reduce the severity of the really bad outcomes would be more valuable.

In a two period version of the model with persistent unemployment, the optimal boundaries are smaller than without persistence. Monetary policy should be more activist and respond to shocks more often if unemployment persists into future periods.

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LONG RUN REAL EXCHANGE RATES - A COINTEGRATION ANALYSIS*

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Abstract

Long run purchasing power is tested on 16 OECD countries using data from 1960 to 1994. PPP is rejected for some countries (Canada, Japan, Switzerland, Austria, Italy and Spain) and not rejected for other (Sweden, France, Holland and the United Kingdom). For the latter countries, impulse response functions show that half of a disturbance to the equilibrium real exchange rate disappears within three years. The method used is Johansen's maximum likelihood approach to cointegration. Simulations are used to obtain empirical critical values of the tests.

Keywords: Purchasing Power Parity, Real Exchange Rates

JEL Classification Number: F31

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1. Introduction

The idea of long run purchasing power parity (PPP) may be expressed as the hypothesis that the equilibrium real exchange rate is stationary. If PPP holds, the real exchange rate (defined as EP^*/P, i.e. the foreign price level times the nominal exchange rate divided by the domestic price level) fluctuates around an equilibrium value. Shocks to the real exchange rate have only temporary effects. If the real exchange rate is pushed below (above) its equilibrium level, it can be expected to rise (fall).

Empirical tests have tended to reject the hypothesis that real exchange rates are stationary. PPP is also often is said to lack solid theoretical underpinnings. Still, it is frequently used as a building block of models in international macroeconomics. It can be argued that the case for PPP is considerably better than its reputation, both when considering theoretical underpinnings and the results of empirical tests.

The simplest theoretical motivation for PPP is that if money is neutral in the long run, the real exchange rate should not be influenced by monetary disturbances. If shocks are monetary in nature, PPP holds in any model which is characterised by long run monetary neutrality. Furthermore, PPP always follows as a special case in models of equilibrium real exchange rates. For example, in models where the real exchange rate is determined by country differences in productivity or relative productivity in the tradable and non-tradable sectors, PPP holds if countries have the same productivity development.[1] If the real exchange rate is driven by the terms of trade, it will be stationary if the terms of trade are stationary.[2] These two approaches to the equilibrium real exchange rate have been combined in a single model by Gregorio and Wolf (1994). A third approach has been to derive an equilibrium real exchange rate from the condition that the current account has to balance in the long run.[3]

Models where the development of real variables determine the real exchange rate result in PPP in two cases: if the real variables in question are unchanged or if the real shocks affect the countries in a "symmetric" way that leaves the equilibrium real exchange rate between them unchanged.

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[1] The most popular models where the equilibrium real exchange rate is determined by relative productivity developments or relative productivity developments in the tradable and non-tradable sectors are Balassa (1964) and Samuelsson (1964).

[2] Models where the terms of trade or relative price movements within the tradable sector determine the real exchange rate include Frenkel and Razin (1992), Ostry (1988) and Greenwood (1984).

[3] Examples of models built on current account balance are Kimbrough (1982), Dornbush and Fisher (1980), Frenkel and Rodriguez (1975) and Kouri (1976).

PPP has been tested empirically numerous times and rejected more often than not. There is widespread agreement that it does not hold in the short run. The disagreement concerns whether PPP holds in the long run and how long the long run is. In Section 2 of this paper, it will be argued that whether PPP is rejected or not depends partly on the choice of countries, the length of the sample period and the econometric techniques used. If a sufficiently long sample period and an appropriate econometric technique is used, PPP is supported in many cases.

Earlier studies suggest that if there is mean reversion in real exchange rates, it is slow. Estimations indicate that half of the effect of a disturbance disappears within two to four years.[4] When PPP is tested on data covering only 10-15 years, mean reversion or stationarity may not be discovered. In their overview of empirical evidence of PPP, Froot and Rogoff (1994) construct an example with a true AR(1) coefficient of 0.981. In monthly data, this corresponds to a half life of disturbances to the equilibrium relationship of three years. They show that it would be necessary to have a data set covering 72 years to be able to reject a unit root at the 95% significance level, using the Dickey-Fuller test. Thus, either very long time series or more powerful tests would be needed to confirm that the real exchange rate does not contain a unit root. It is definitely relevant to be able to distinguish between a unit root and an AR-coefficient of 0.98 in monthly data. In the latter case, most of the disturbance to the equilibrium real exchange rate can be expected to disappear within a typical business cycle, while shocks have permanent effects if there is a unit root.

Most tests of PPP have focused on bilateral real exchange rates between major industrial nations like USA, Japan and Germany. A case can be made that since these countries have rather different economic structures, the real exchange rates between them are less likely to be stationary than the real exchange rates between more homogeneous European countries. For instance, asymmetric real shocks like different productivity developments and varying terms of trade may be more important when considering the real exchange rate between Japan and the United States than for the real exchange rate between Germany and Belgium. As will be discussed in the next section, there are rather few studies of whether the real exchange rates between European countries are stationary.

If the mechanism driving PPP has to do with international competitiveness, it may be more relevant to study multilateral PPP than bilateral. For a single country, a real appreciation against one partner country may offset a real depreciation against

[4] Froot and Rogoff (1994)

another. However, as shown by Nessén (1996), multilateral PPP cannot hold simultaneously for all countries involved unless bilateral PPP holds between all countries. Multilateral or effective real exchange rates are simply linear combinations of bilateral real exchange rates. It is not possible to form a full set of stationary linear combinations of bilateral real exchange rates unless all of them are stationary.[5] Thus, unless bilateral PPP holds between all the countries, at least one multilateral real exchange rate has to be non-stationary.

In this paper, multilateral PPP is tested on monthly data for 16 OECD countries from 1960 to 1994. The sample period, 35 years, is longer than in most studies of PPP except those using data since the turn of the century. The method used to test for cointegration is Johansen's (1988) maximum likelihood approach, which incorporates tests of linear restrictions on the cointegrating vectors. Since it has been shown that the finite sample distributions of the test statistics may differ considerably from the asymptotic distributions, even with several hundred observations, parametric bootstrap tests provide some additional information about the interpretation of the test results.[6] Empirical distributions and corresponding critical values given normally distributed residuals are generated. Since the residuals are not normally distributed but display excess kurtosis and skewness, simulations with resampled residuals are used to shed some light on how the tests work given the present distribution.

The paper is organised as follows: Section 2 is an overview of empirical tests of PPP. The data are described in Section 3, and Section 4 discusses the statistical method. The empirical results and the simulations of the tests are presented in Section 5, followed by the conclusions in Section 6.

2. An overview of the empirical literature on PPP

Before the virtual explosion of PPP tests that followed the introduction of econometric techniques designed to handle non-stationary data, it had become more or less a stylised fact that PPP was rejected in empirical tests. Standard textbooks in international economics like Krugman and Obstfeld (1986) concluded that PPP does not hold. Evidence presented by Adler and Lehman (1983) supported this view also for extremely long data sets (1900-1972). In contrast, it was shown by Frenkel (1978, 1981) that PPP could not be rejected for a number of hyperinflationary economies and

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[5] This is similar to the more familiar result that there cannot be k cointegrating vectors in a system of k I(1) variables unless all variables are stationary.

[6] Simulations of the Johansen procedure have been made by Jacobson, Vredin and Warne (1994), among others.

for the US versus Germany, Britain and France in the 1920's. These papers were exceptions to the rule.

Since the concepts of integration and cointegration became common knowledge, empirical tests of PPP have proliferated. The results are mixed and no final verdict has been reached. Studies covering less than 15 years of data almost always reject PPP, while those covering the entire century usually do not. Furthermore, rejections are much more frequent for the United States and Canada than for the European countries.

One approach has been to investigate whether the real exchange rates contain a unit root which is incompatible with PPP. As discussed in the introduction, the power of unit root tests to distinguish between AR-coefficients close to one and unit roots is questionable in small samples. There is a handful of studies that use very long sample periods. Kim (1990) and Abuaf and Jorion (1990) apply unit root tests to a data set that covers 1900-1972 and reject a unit root in the real exchange rate.[7] Froot and Rogoff (1994) discuss whether the fact that PPP seems to hold in the very long run is due to what they call a survivor bias: Data from the beginning of the century is only available for a handful of countries. Perhaps PPP holds better for these early industrialisers than for other countries for which data is unavailable.

Other studies of unit roots in real exchange rates include Bleaney (1992), who rejects a unit root only for the real exchange rate between Germany and France when investigating all bilateral rates between Germany, France, Italy, Belgium and Holland between 1979 and 1988. Edison and Fischer (1991) study the same countries but use data from 1973 to 1989. They reject a unit root (allowing for a structural break in the relationship in 1979) for six of 15 bilateral exchange rates. Mark (1990) studies five intra-European real exchange rates between 1973 and 1989 and concludes that they all contain a unit root. The Dickey-Fuller statistics exceed 2.0 in three cases, however. Given the low power of the test, it is unlikely that a true AR coefficient of, say, 0.95 would have been discovered by the ADF test.

A second approach has been to investigate whether nominal exchange rates and price levels are cointegrated. Studies using cointegration techniques have quite often found cointegration among nominal exchange rates and price levels. A problem with many of these studies is that the existence of a stationary linear combination of exchange rates and prices does not necessarily mean that PPP holds. According to PPP, it is the real exchange rate that should be stationary. This implies certain restrictions on the cointegrating vector(s). Unfortunately, it is difficult to test hypotheses on the

[7] This is the same data set as Adler and Lehmann (1983) used.

cointegrating vectors in several of the most commonly used cointegration procedures. Examples of studies that claim to report evidence supportive of PPP without testing whether the cointegrating vectors are compatible with PPP are Canarella, Pollard and Lai (1990) and Enders (1988), Heri and Theurillat (1990) and Kim (1990). The last two papers report point estimates of the cointegrating vectors close to those required by PPP.

A few papers do incorporate tests of restrictions on the cointegrating vectors. In a study that is often referred to as presenting evidence favourable to PPP, Fisher and Park (1991) reject the null hypothesis of no cointegration for all G-10 countries except the US and Canada on data from 1973 to 1988. The hypothesis that the real exchange rate is stationary is however rejected in 43 of 55 cases using CPI and 51 of 55 using wholesale prices. Nessén (1996) finds four cointegration vectors in the seven variable system of dollar exchange rates and price levels for the US, Britain, Germany and Japan. Tests of whether the real exchange rates are cointegrating vectors lead to rejections of the hypothesis, however. The data cover 1970-90. Cheung and Lai (1993) also find cointegration among exchange rates and price levels of the US versus Britain, France, Germany, Switzerland and Canada for the sample period 1974-1989 but reject the restrictions imposed by PPP.

The table in Appendix A compiles the results from a large number of empirical tests of PPP. Several observations can be made. Investigations of real exchange rates using US data have dominated the literature. The small European countries have largely been ignored, especially the Nordic countries. Second, PPP has fared worse in tests focusing on the United States. Rejections are fewer when studying real exchange rates between other countries.

In contrast to of the traditional time series approach to test PPP, Frankel and Rose (1995) use 45 years of panel data for 150 countries. Their results support purchasing power parity with an estimated half life of disturbances to the real exchange rate of four years. Another interesting recent contribution to the literature is the paper by Froot, Kim and Rogoff (1995). They test the law of one price between Holland and England for eight commodities on data covering 700 years. While they find "persistent" (century-long) deviations from the law of one price, the deviations are not characterised by long run trends.

Third, bilateral PPP has been tested much more often than multilateral PPP. Possible reasons for this are that the choice of weights is rather arbitrary and that the hypothesis of stationary effective real exchange rates is testable within multivariate systems of price levels and bilateral exchange rates. However, as the number of partner countries grows, the dimension of the multivariate system turns prohibitively large. Many of the European countries have diversified trade patterns and it would be unfortunate to restrict the number of partner countries so drastically that one multivariate system could be used. In this paper, as in Johansen and Juselius (1992), PPP is therefore tested within three variable systems of domestic prices, weighted averages of foreign prices and weighted averages of nominal exchange rates.

To sum up, most tests of PPP use short sample periods and focus on the bilateral real exchange rates between the USA and other major countries. In contrast, this paper investigates multilateral PPP, using a fairly long sample period and including a number of previously neglected small European countries.

3. The data

Monthly data from 1960 to 1994 on exchange rates and consumer price indices have been collected from OECD:s Main Economic Indicators. Other conceivable choices of price indices would be industrial prices and wholesale prices that contain a larger share of tradables. Logarithms of level data are used throughout.

Multilateral real exchange rates have been constructed using the IMF MERM weights. This weighting system considers not only the share of trade between any pair of countries but also the extent to which they compete on third markets. For instance, the direct trade between Japan and Sweden is rather small, but Swedish and Japanese products compete on third markets. This gives Japan a larger weight in the Swedish effective exchange rate than what is motivated by the trade volume between the countries.

Appendix B shows the real effective exchange rates for the 16 countries studied. Simply looking at the figures, the real exchange rates of Belgium, Finland, France, Italy, Sweden, United Kingdom and possibly the US and Germany appear to be stationary. The real exchange rates of Austria, Canada, Denmark, Holland, Japan, Norway, Spain and Switzerland appear to be non-stationary. As will he shown in section 5, this impression is only partly supported by the test results.

Before proceeding to cointegration analysis, it is useful to know the time series properties of the individual series. One usually considers three possibilities when performing unit root tests on nominal variables like price levels: data may be stationary around a deterministic time trend or have one or two unit roots. A second unit root means that the first difference of a variable - the inflation rate, for instance - is non-stationary, while the second difference is stationary.

A variable x_t is said to be integrated of order *d* if it has a stationary, invertible, non-deterministic ARMA representation after differencing *d* times. If a variable is integrated of order *d*, it has *d* unit roots. The number of unit roots in the time series on price levels and nominal exchange rates is investigated using the Dickey-Fuller test:

$$(1) \quad \Delta x_t = \rho x_{t-1} + \sum_{i=1}^{p} \Delta x_{t-i} + \mu t + \varepsilon_t$$

p is the number of lags in the AR-process. The presence of a unit root implies that ρ is equal to one. This hypothesis is tested against the alternative that ρ is less than one. Two unit roots in the level of x_t is equivalent to one unit root in the first difference of x_t, which is tested using (1) on differenced data. Table 1 reports the results for nominal exchange rates and price levels. A linear trend is included in the tests.

When the presence of two unit roots is suspected, Dickey and Pantula (1987) show that the hypothesis of a double unit root should be tested first. If it is rejected, the presence of a single unit root may be investigated as usual. The Dickey-Pantula test (not shown) indicates a single unit root in all variables, as does the Dickey-Fuller test without a deterministic trend.

Table 1: Unit roots in price levels and nominal exchange rates, t-statistics from the ADF-test

	CPI, levels	CPI, differences	Exchange rates, levels	Exchange rates, differences
Austria	-0.937	-7.935**	-0.124	-6.892**
Belgium	-0.785	-3.403**	-1.047	-6.254**
Canada	-0.556	-2.640*	-0.913	-5.984**
Denmark	-1.598	-4.655**	-1.860	-6.578**
Finland	-0.877	-3.940**	-0.732	-6.645**
France	-0.723	-2.643*	-1.349	-6.558**
Germany	-0.730	-6.180**	-0.463	-6.925**
Holland	-1.829	-5.382**	-0.166	-6.748**
Italy	-0.429	-2.711*	-0.714	-6.089**
Japan	-2.247	-3.862**	-0.594	-6.204**
Norway	-0.334	-4.491**	-0.062	-6.748**
Spain	-0.437	-3.513**	-1.860	-5.282**
Sweden	-0.718	-4.844**	-0.361	-6.312**
Switzerland	-0.792	-5.429**	-0.334	-6.881**
United Kingdom	-0.537	-4.110**	-1.349	-7.032**
USA	-0.152	-2.846*	-1.537	-6.254**

Critical values are taken from Fuller (1976)
* significant at 5%
** significant at 1%

All series have a first unit root but none of them contains an second unit root. Thus, they are all I(1), integrated of order one.

4. The statistical model

Cointegration tests look for linear combinations of I(1) time series that are stationary (or, more generally, linear combinations of I(d) time series that are integrated of an order lower than d). Since price levels and nominal exchange rates are integrated of order one, they are cointegrated if a linear combination of them is stationary.

The first generation of cointegration tests were based on stationarity analysis of the residuals from a cointegrating regression. When more than two variables are involved, Johansen's (1988) multivariate procedure is more suitable. It focuses on the rank of the Π-matrix in equation (3).

$$(3) \quad \Delta x_t = \mu + \sum_{i=1}^{p} \Gamma_i \Delta x_{t-i} + \Pi x_{t-p} + \varepsilon_t ,$$

When the variables are integrated of order one, their first difference is stationary. Thus, the right hand side of equation (3) must be stationary as well. If there is no cointegration between the variables, Π must be a zero matrix. If the Π-matrix has reduced rank, implying that $\Pi = \alpha\beta'$, the variables are cointegrated, with β as the cointegrating vector. If the variables are stationary in levels, Π would have full rank.

The maximum likelihood estimation of β boils down to solving an eigenvalue problem.[8] The rank of the system is determined by investigating how many of the eigenvalues are non-zero. A non-zero eigenvalue can loosely be interpreted as a positive correlation between the corresponding linear combination of x_t and (the stationary) Δx_t. Since the asymptotic correlation between a stationary and a non-stationary process is zero, a positive correlation means that this linear combination of x_t is stationary. With three variables, there will be three eigenvalues with corresponding eigenvectors. If the true rank is *r*, there will be *r* non-zero eigenvalues. Johansen (1988) has developed two tests of the number of non-zero eigenvalues.

[8] See Johansen (1988) for a full discussion of the procedure.

First, the eigenvalues are ordered from the largest to the smallest. The maximal eigenvalue statistic tests whether eigenvalue number s+1 is non-zero. It is computed as:

$$(4) \qquad \lambda_{max} = -T\ln(1-\lambda_{s+1})$$

The trace test investigates whether all eigenvalues from number s+1 to r are zero:

$$(5) \qquad trace = -T\sum_{i=s+1}^{r}\ln(1-\lambda_i)$$

Asymptotic distributions of the two cointegration rank tests are derived in Johansen (1988, 1991 a and b). The distributions are not invariant to various restrictions due to the treatment of constants. Critical values that take this into consideration have been simulated by Osterwald-Lenum (1992).

The Johansen procedure identifies the vector space spanned by the cointegrating vectors. Tests of linear restrictions on the cointegrating vectors investigate whether a certain vector belongs to this space. Linear restrictions on β are tested by comparing the likelihoods of the restricted and unrestricted β. Asymptotically, the likelihood ratio test statistic is χ^2 distributed. It is computed as:

$$(6) \qquad -2\ln Q = T\sum_{i=1}^{r}\left[\ln(1-\lambda_i^R) - \ln(1-\lambda_i)\right],$$

where λ_i are the estimated eigenvalues from the unrestricted model and λ_i^R are the eigenvalues when the restrictions are imposed. When only *r1* of the *r* cointegrating vectors are restricted, a slightly different test is used:

$$(7) \qquad -2\ln Q = T\left[\sum_{i=1}^{r1}\ln(1-\lambda_i^R) + \sum_{i=r1}^{r-r1}\ln(1-\lambda_i^E) - \sum_{i=1}^{r}\ln(1-\lambda_i)\right],$$

where λ_i^E are the estimated eigenvalues of the *(r-r1)* unrestricted cointegration vectors, given the restriction on the *r1* first vectors.

A cointegrated VAR-model as in equation (3) can be interpreted as an error correction representation with the α – parameters as the error correction parameters. It can also be rewritten as a moving average representation as in equation (8):

$$(8) \quad \Delta x_t = C(1)\mu + C(L)\varepsilon_t$$

Impulse response functions show how a variable reacts over time to a shock. They are obtained from the Wold moving average representation (8) of the cointegrated VAR-system in (3). The impulse response functions simply use the estimates of *C(L)*. The vector of shocks is set to one standard error in period t and zero in all other periods. There will be an immediate effect on Δx_t in period *t*, determined by the estimated coefficients of *C(0)*, a one period lagged effect, determined by the coefficients of *C(1)* and so on.

The moving average representation in (8) can be written as a common trends model. If there are K variables and r cointegrating vectors, there will be *K-r* stochastic trends that drive the system. Innovations to the stochastic trends have permanent effects on the variables. The error terms in equation (8) are linear combinations of the permanent and transitory shocks in the common trends representation. Given some identifying assumptions, the structural shocks can be distinguished. The identifying assumptions are usually obtained from economic theory. Given the lack of theoretical foundations of PPP, no attempt to identify the shocks will be made here.

5. Empirical results

After a few model specification tests, the cointegration rank, the cointegrating vectors and the dynamic adjustment mechanism will be investigated. Since the critical values from the asymptotic distribution are not necessarily appropriate for a small sample, simulations will be used to generate empirical critical values.

In a parametric bootstrap test, synthetic data sets are generated using the estimates of the parameters in the data generating process (3) and normally distributed disturbance terms. The tests for cointegrating rank and restrictions on the cointegrating vectors are applied to the data sets and the test statistics are collected. 10 000 samples are generated in each case. The resulting distributions of the test statistics under the true null hypothesis are used to calculate empirical critical values. The power of the tests can be investigated by testing a false hypothesis and see how often it is rejected. Non-parametric bootstrapping is used in a similar fashion except that resampled residuals are used instead of generated normally distributed error terms.

5.1. Model specification tests

The matrix x_t in the VAR-model consists of three timeseries for each country: The domestic price level, a weighted average of foreign price levels and the nominal effective exchange rate. First, the number of lags to be included has to be determined. Information criteria like AIC, BIC and LIL minimise a function in which the increase in explanatory power from additional lags is balanced by a punishment for the additional parameters that have to be estimated. In addition, autocorrelation or non-normality of the residuals may indicate that the specified number of lags is incorrect.

The information criteria are computed as:

$$(9) \qquad \begin{aligned} AIC &= \ln\left|\Sigma(p)\right| + 2pK^2 / T, \\ BIC &= \ln\left|\Sigma(p)\right| + 2 \ln TpK^2 / T, \text{ and} \\ LIL &= \ln\left|\Sigma(p)\right| + 2 \ln \ln TpK^2 / T, \end{aligned}$$

where $\sum(p) = E[\varepsilon_t(p)\varepsilon_t(p)'$ or the residual variance matrix when p lags are used, K is the number of variables and T is the number of observations.

Results from the model specification tests are presented in Table 2. In order to reduce the amount of statistical information, the multivariate LM test for first order autocorrelation and the Box-Ljung test for higher order autocorrelation are reported for the preferred lag length only. For fewer lags, at least one of the tests indicate serial correlation. In many cases, more lags are needed to remove higher order serial correlation than to remove first order serial correlation. Priority is then given to higher order autocorrelation.

The Bera-Jarque test for normality of the residuals (excess kurtosis and skewness) indicates non-normality at all lag lengths. This is not surprising given the data on fixed exchange rates with occasional realignments. The problem is less severe for the countries that have had flexible exchange rates since the break-down of the Bretton Woods system. A number of spikes in the nominal exchange rates coincide with devaluations. There are also spikes in the consumer prices that are due to changes of indirect taxes. Adding dummy variables removes these large residuals and reduces non-normality.[9] Although this is a notable improvement, significant non-normality remains in most cases. An indication of how the cointegration tests behave with non-

[9] The dummy variables are described in Appendix E

normal residuals may be found among the simulation exercises below. The Bera-Jarque statistics are reported for the preferred lag length only. The last column shows the preferred number of lags.

Table 2: Specification tests

	Indicated lag length:			LM(1),	Box-Ljung,	Bera-Jarque	Bera-Jarque	preferred
	AIC	LIL	BIC	p-val	p-val	preferred lag	w/ dummies	laglength
Canada	2	2	2	0.05	0.06	132.8	56.69 (0.00)	9
US	5	2	2	0.05	0.12	109.7	81.62 (0.00)	9
Japan	2	2	2	0.06	0.43	75.5	50.90 (0.00)	10
Austria	5	5	2	0.09	0.04	323.1	67.08 (0.00)	8
Belgium	7	3	2	0.10	0.07	352.3	69.20 (0.00)	5
Denmark	3	2	2	0.17	0.23	410.9	44.96 (0.00)	10
Finland	4	2	2	0.09	0.01	1136.7	89.43 (0.00)	2
France	5	2	2	0.06	0.01	351.9	36.13 (0.00)	4
Germany	5	2	2	0.22	0.01	285.0	30.20 (0.00)	7
Italy	7	2	2	0.13	0.05	391.6	106.35 (0.00)	5
Holland	6	2	2	0.05	0.99	514.8	30.39 (0.00)	7
Norway	6	2	1	0.05	0.91	437.2	39.77 (0.00)	9
Sweden	7	2	2	0.05	0.01	277.6	23.70 (0.00)	4
Switzerland	5	4	2	0.08	0.03	110.8	34.07 (0.00)	7
United Kingdom	7	2	2	0.05	0.11	192.0	43.56 (0.00)	9
Spain	5	2	2	0.17	0.00	291.2	43.34 (0.00)	5

In most cases, more lags than indicated by the information criteria have to be added to remove residual autocorrelation. In particular, 9 or 10 lags are needed for Norway and the flexible exchange rate countries United States, Canada, Japan and United Kingdom. It is well known that the Johansen procedure is sensitive to the chosen lag length, an issue that will be discussed further in the next section.

5.2. Cointegration rank

The testing procedure starts by investigating the null hypothesis that all the eigenvalues are zero, implying a zero Π-matrix and no cointegration. If $r=0$ is rejected, the largest eigenvalue must be non-zero. Next, the hypothesis $r=1$ or more than one non-zero eigenvalue is tested. Finally, if more than two eigenvalues are non-zero, the Π-matrix has full rank and the individual series must be stationary. Table 3 shows the results from the cointegration rank tests.

Table 3: Cointegrating rank

	λ_1	λ_2	λ_3	trace			λ-max			cointegrating
				r=0	r=1	r=2	r=0	r=1	r=2	rank (at 10 %)
critical values				21.63	10.47	2.86	15.59	9.52	2.86	
Canada	0.0323	0.0157	0.0091	23.23	10.04	3.68	13.20	6.36	3.68	1
USA	0.0351	0.0205	0.0005	22.94	8.58	0.20	14.35	8.39	0.20	1
Japan	0.1033	0.0700	0.0001	73.37	29.34	0.04	44.04	29.30	0.04	2
Austria	0.0277	0.0231	0.0066	23.38	12.07	2.66	11.31	9.41	2.66	2
Belgium	0.1079	0.0077	0.0040	51.11	4.77	1.63	46.35	3.14	1.63	1
Denmark	0.0603	0.0223	0.0001	34.01	9.08	0.03	24.94	9.04	0.03	1
Finland	0.0397	0.0131	0.0001	23.68	5.29	0.05	15.97	5.24	0.05	1
France	0.0375	0.0255	0.0001	26.15	10.58	0.05	15.57	10.53	0.05	2
Germany	0.0664	0.0099	0.0000	31.80	4.04	0.00	27.76	4.03	0.00	1
Italy	0.0858	0.0245	0.0000	46.48	10.06	0.00	36.41	10.06	0.00	1
Holland	0.0775	0.0340	0.0046	41.21	13.47	1.58	27.74	11.88	1.58	2
Norway	0.0269	0.0074	0.0011	14.40	3.45	0.45	10.96	2.99	0.45	0
Spain	0.0379	0.0159	0.0059	24.64	8.93	2.41	15.71	6.52	2.41	1
Sweden	0.0540	0.0025	0.0014	24.15	1.57	0.56	22.58	1.01	0.56	1
Switzerland	0.1080	0.0137	0.0053	51.14	7.81	2.16	43.33	10.96	2.16	1
United Kingdom	0.0369	0.0238	0.0054	27.01	11.89	2.19	15.12	9.70	2.19	1

The last column shows the number of cointegrating vectors using the 10 percent critical values from the asymptotic distributions. In a number of cases, the choice of lag length influences the indicated number of cointegrating vectors. In particular, for Canada and the United States, 9 and 10 lags are needed to remove autocorrelation. With fewer lags, the tests indicate that no cointegration is present. In most cases, however, the indicated rank does not change if a few lags are added or removed.

Three different cases emerge from Table 3. For Japan, Austria, France and Holland, there are two cointegration vectors. Since the number of common trends is the number of variables minus the number of cointegration vectors, this implies that there is only one stochastic trend in the system. For Norway, the tests indicate no cointegration in the three-variable-system of the effective nominal exchange rate and domestic and foreign price levels. For the remaining eleven countries, one cointegration vector is found, implying two stochastic trends. One interpretation of this would be that the system is driven by a domestic monetary policy trend and a foreign monetary policy trend.

The empirical distributions of the test statistics under various circumstances can be investigated by running the tests on a large number of synthetic data sets that contains a known number of cointegrating vectors. The data sets are created using the data generating process (3). The parameters are estimated conditional on the rank of Π. First, normally distributed errors with the estimated variance-covariance matrix are used. Since inclusion of dummy variables is known to change the critical values, data sets with dummy variables and normally distributed errors are generated as well. In order to investigate what happens under non-normality, bootstrapping with resampled residuals is used, both with the original highly non-normal residuals and with dummy variables and the slightly non-normal residuals. The empirical results presented in the tables refer to the latter case.

The results for Sweden are shown in Table 4. The results for the cointegration rank tests differ only slightly between countries and the remaining critical values appear in Appendix C.

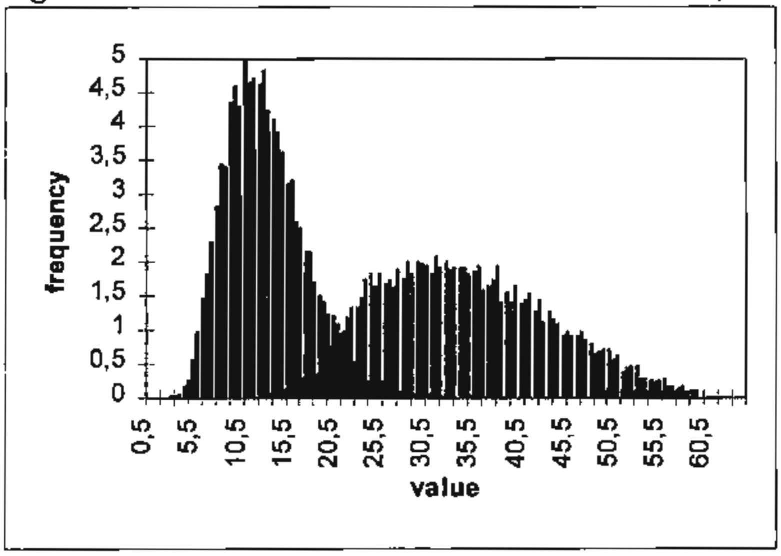
Table 4: Critical values from the empirical distributions: The case of Sweden

	95% percentiles for the empirical distributions							
	the normal distribution		actual distribution (bootstrapping)		normal distribution w/ dummy variables		actual distribution w dummy variables (bootstrapping)	
	trace	λ-max	trace	λ-max	trace	λ-max	trace	λ-max
r>0	30.43	21.49	52.68	41.47	29.36	20.12	60.27	51.39
r>1	13.66	11.71	20.03	17.07	11.72	10.47	12.31	10.63
r>2	5.92	5.92	4.93	4.93	3.17	3.17	4.53	4.53

Since the critical values from the empirical distribution are considerably larger than those from the asymptotic distribution, a true hypothesis is rejected too often using the latter. There are differences between countries, but they are rather small. The results are well in line with those of Jacobson, Vredin and Warne (1994) in a similar simulation exercise with about 100 observations in each sample. They also found empirical critical values that are about 20 percent higher than the asymptotic. Thus, the tests are oversized and too many cointegrating vectors tend to be found when asymptotic critical values are used.

For the first λ -max and trace-statistics, there is a large difference between the distributions of the test statistics with normal and with non-normal residuals for all countries. Figure 1 shows the distributions of the λ -max statistics for testing r=0 versus r=1 with normally distributed residuals and with resampled non-normal residuals in the case of Sweden. The distribution given non-normal residuals is located far to the right of the distribution given normal residuals. However, this is the case only for the first test statistics. When considering r=1 versus r=2 and r=2 versus r=3, both the trace test and the λ -max are distributed fairly similarly with normal and non-normal residuals.

Figure 1: The distributions of the λ -max test for r=0 vs r=1, normal and non-normal residuals



The cointegration rank test statistics from the simulations can also be used to investigate how often the tests find the correct number of cointegrating vectors. Table 5 shows the results for Sweden, using normally distributed residuals and no dummy variables. For instance, when the true rank of Π is one, the λ-max statistics indicate zero cointegration vectors in 48 percent of the cases, one in 34 percent and two in 14 percent. The results for different countries differ very little.

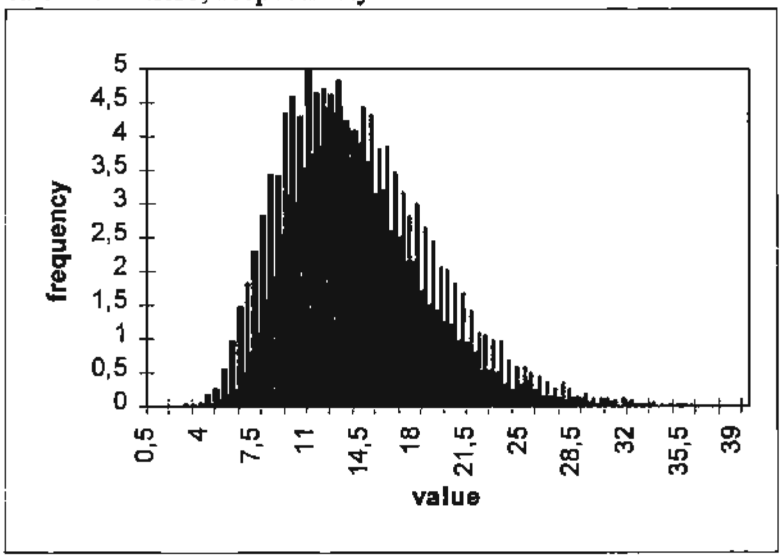
Table 5: Frequencies of preferred rank, using asymptotic critical values

	test result:r=0		test result:r=1		test result:r=2		test result: r=3	
true rank	trace	λ-max	trace	λ-max	trace	λ-max	trace	λ-max
r=0	**66.13**	**74.13**	24.85	22.21	5.65	3.10	3.37	0.01
r=1	48.41	60.88	**33.92**	**29.30**	11.73	8.17	5.94	1.65
r=2	43.31	57.17	34.92	30.55	**13.64**	**9.65**	8.13	2.63

Table 5 shows that the power of the tests is mediocre in the present setting. They tend to indicate far too often that there are no cointegrating vectors and pick the correct number of cointegrating vectors in a minority of the cases except when the true rank is zero. For instance, when the true rank is one, both the tests indicate that the rank is zero about half of the time. Even when the true rank is two, the tests very often result in zero cointegrating vectors.

Figure 2 illustrates the point. It shows the distribution of the largest λ-max test statistics given that the true rank is one and given that the true rank is zero. The power of the test depends on how difficult it is to distinguish between these two distributions. As is evident from Figure 2, the distributions differ little from each other and the power is consequently low. A conceivable reason for the observed low power is that although the true rank of Π is non zero, the cointegrating vectors may not be empirically significant. Furthermore, the parameter estimates may be imprecise.

Figure 2: Distrihutions of the λ-max test for r=0 vs r=1 with a true cointegrating rank of one and zero, respectively



As is evident from Table 6, the situation deteriorates further if the empirical critical values are used instead of the asymptotic. Both tests indicate zero cointegrating vectors most of the time even when the true number is two. The trace test performs marginally better than the λ-max test. Since the (absolute) power to distinguish between different hypotheses is so low when critical values from the empirical distributions are used, it does not seem appropriate to apply these critical values.

Table 6: Frequencies of preferred rank, using empirical critical values

	test result: r=0		test result: r=1		test result: r=2		test result: r=3	
true rank	trace	λ-max	trace	λ-max	trace	λ-max	trace	λ-max
r=0	**87.19**	**90.00**	6.73	8.72	2.46	1.18	0.01	0.00
r=1	78.93	82.71	**12.93**	**13.47**	6.17	3.44	1.97	0.00
r=2	74.20	79.31	14.98	15.47	**7.75**	**4.54**	3.06	0.01

When critical values from the asymptotic distribution are used, one cointegrating vector is found in most cases. The tests indicate two cointegrating vectors in five cases and zero in one (the United States). The simulations show that the asymptotic critical values are too small. Reconsidering the test results in Table 3 with this in mind, one might suspect that the number of cointegrating vectors is lower than what the test indicate. If the critical values from the empirical distributions given normally distributed errors are used, no cointegration (r=0) is found for Canada, the US, Austria, Finland, France, Spain and Sweden. Finally, using empirical critical values from the bootstrapping exercise with resampled residuals, cointegration is found only for Japan and Switzerland.

5.3. The cointegrating vectors

PPP implies not only that home prices, foreign prices and nominal exchange rates should be cointegrated but also that the cointegrating vector should be [-1, 1, 1]. The latter condition means that it is the real exchange rate that is stationary, not some other linear combination of exchange rates and price levels.

As with the tests for cointegrating rank reported above, the empirical distribution for the samples sizes and parameter values at hand have been simulated. The empirical distributions are located further to the right than the asymptotic distributions, which means that the critical values are higher than for the χ^2 distribution. Thus, a true hypothesis will be rejected too often when critical values from the asymptotic distribution are used. Critical values and p-values from the empirical distributions appear in Table 7 along with the estimated cointegrating vectors and the likelihood ratio test statistics.

Table 7: The cointegrating vectors

	Cointegration vector(s)	LR and p-value for $[-1, 1, 1] \in \beta$	5 % empirical crit. val. and p-value of LR	size
Canada	[-1.00, 1.017, 0.267]	30.95 (0.00)	20.746 (0.00)	51.01
USA	[-1.00, 0.871, -0.261]	11.14 (0.00)	13.158 (0.07)	28.34
Austria	[-1.00, 0.162, 0.065]	12.12 (0.00)	10.386 (0.01)	19.75
	[-1.00, 0.849, 0.213]			
Belgium	[-1.00, 1.142, 5.726]	9.23 (0.01)	14.745 (0.09)	30.96
Denmark	[-1.00, 1.025, 0.448]	6.32 (0.04)	12.690 (0.15)	25.83
Finland	[-1.00, 1.299, 1.192]	9.22 (0.01)	14.974 (0.09)	31.60
France	[-1.00, 1.216, -0.752]	2.40 (0.12)	8.863 (0.23)	11.14
	[-1.00, 1.046, 0.343]			
Germany	[-1.00, 0.979, 0.855]	6.36 (0.04)	9.957 (0.11)	16.70
Holland	[-1.00, 0.740, 0.986],	1.02 (0.31)	7.395 (0.42)	9.15
	[-1.00, 1.239, 1.463]			
Italy	[-1.00, -8.043, 14.641]	12.99 (0.00)	10.14 (0.03)	18.01
Japan	[-1.00, 1.125, 0.392]	13.28 (0.00)	9.251 (0.02)	15.33
	[-1.00, 1.621, -0.446]			
Spain	[-1.00, 4.890, -5.673]	11.57 (0.00)	13.436 (0.04)	27.41
Sweden	[-1.00, 0.755, 1.346]	6.12 (0.05)	13.182 (0.18)	26.25
Switzerland	[-1.00, 0.874, 0.331]	33.92 (0.00)	16.099 (0.00)	38.01
Britain	[-1.00, 1.210, 0.493],	1.65 (0.44)	12.767 (0.68)	26.16

The coefficients of the estimated cointegrating vectors have the expected signs except for the coefficients on the nominal exchange rate for the US and Spain and the coefficient on foreign prices for Italy. However, few of the point estimates are close to the [-1, 1, 1] required by PPP. The closest ones are [-1.00, 1.299, 1.192] for Finland and [-1.00, 0.979, 0.855] for Germany. Several of the exchange rate coefficients are smaller than one half: 0.267 for Canada, 0.331 for Switzerland, 0.448 for Denmark and 0.493 for the United Kingdom.

Using the asymptotic critical values at the five percent significance level, the hypothesis of a constant real exchange rate is not rejected for France, Holland, Sweden and the United Kingdom. The marginal significance of the test statistic for Germany and Denmark is 0.04. Relying instead on critical values from the empirical distributions, PPP may hold also for the US, Belgium, Denmark, Finland and Germany. The hypothesis of a stationary real exchange rate is rejected for Canada, Austria, Italy, Japan, Spain and Switzerland.

The test results can be compared to the visual impression of real exchange rates in Appendix A. For instance, the real exchange rates of Austria, Denmark and Holland look rather similar, but PPP is rejected for Austria while Holland is not even a borderline case. The real exchange rates of Italy, the US and possibly Belgium look more stationary than the real exchange rates of Holland and Denmark. Still, the tests reject PPP for the former but not for the latter. The result that PPP may hold seems more consistent with eye-ball econometrics in the cases of Finland, France, Germany, United Kingdom and Sweden. The real exchange rates of Austria, Canada, Norway, Japan, Spain and Switzerland do not look stationary and stationarity is indeed rejected.

Simulations may shed some light on the size and power of the likelihood ratio test. There are more pronounced differences between the countries when the likelihood ratio test is considered than was the case with the cointegration rank tests. The test is oversized in all cases. As Table 8 shows, many of the empirical critical values are two to three times higher than the corresponding values from the χ^2 distribution. A casual observation from Table 8 is that tests of whether the real exchange rate belongs to the space spanned by two cointegrating vectors seem less oversized than tests of whether a single vector is equal to [-1, 1, 1].

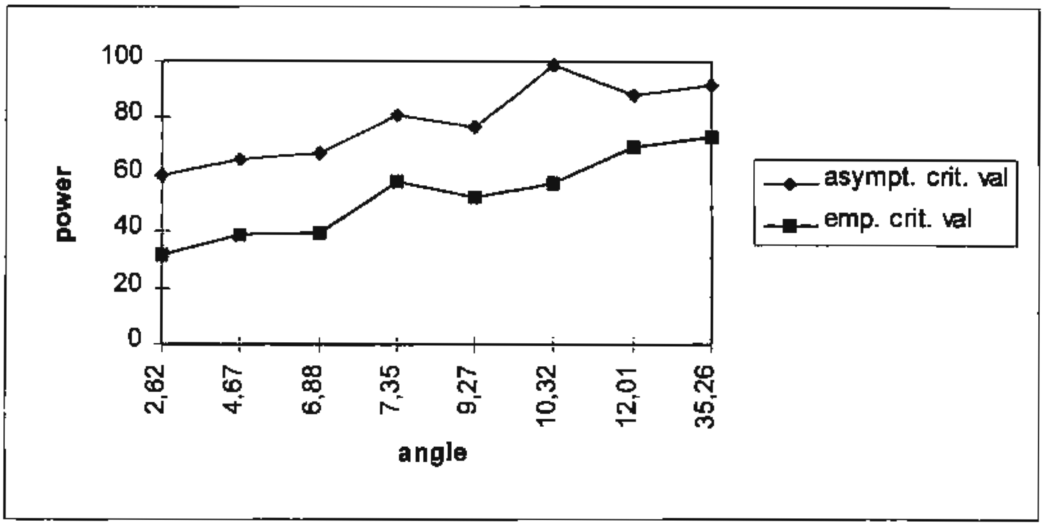
Table 8: Empirical distributions of the likelihood ratio test

	99%	95%	90%	50%	size
Asymptotic	9.21	5.99	4.61	1.39	5.00
Austria	15.56	10.39	7.88	2.37	19.75
Belgium	22.62	14.74	10.90	4.94	30.96
Canada	38.71	30.95	16.49	6.14	51.01
Denmark	18.61	12.69	9.90	3.22	25.83
Finland	22.65	14.97	11.47	3.68	31.60
France	14.51	8.86	6.08	2.37	11.14
Germany	15.25	9.96	7.67	2.32	16.70
Holland	10.11	7.39	5.41	2.24	9.15
Italy	14.73	10.14	7.57	2.35	18.01
Japan	14.03	9.25	7.25	2.49	15.33
Norway	20.04	13.12	10.23	3.31	27.12
Spain	20.71	13.44	10.43	3.22	27.41
Sweden	20.50	13.18	10.26	3.08	26.25
Switzerland	22.60	16.10	12.64	4.51	38.01
USA	21.16	14.52	11.85	3.11	28.34
Britain	19.93	12.76	7.38	3.62	26.16

The four first columns of Table 8 contain the empirical critical values at various significance levels. The final column shows the size of the likelihood ratio test when the asymptotic five percent critical value is used. The size varies between 9 and 51 percent. A true cointegrating vector of [-1, 1, 1] is rejected far too often when critical values from the asymptotic distribution are used.

The power of the likelihood ratio test can be investigated by feeding β-vectors different from [-1, 1, 1] into the data generating process, running the tests of whether they are equal to [-1, 1, 1] and calculating the rejection percentage. The power is an increasing function of the angle between the true and the false cointegrating vectors. However, it is also a function of the other parameters and their standard errors. No attempt is made to correct the numbers in Figure 3 for these other factors, which may be a reason why the power curves are not increasing monotonically in the angle between the true and the false cointegrating vectors.

Figure 4: Power of the test for linear restrictions on β using empirical critical values



For instance, the angel between a true cointegrating vector [-1.0, 0.8, 1.2] and [-1, 1, 1] is 9.27 degrees. The test rejects the restriction that this cointegrating vector is equal to [-1, 1, 1] in 51.89 percent if the five percent critical value from the empirical distribution is used. While it is difficult to evaluate the power of a test formally, the power of the likelihood ratio test for linear restrictions on the cointegrating vectors seems reasonable using empirical critical values.

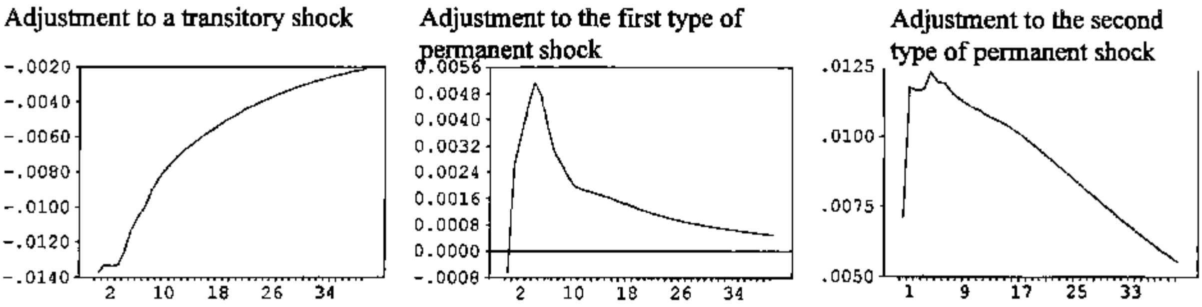
To conclude, PPP is rejected for some countries (Japan, Switzerland, Austria, Italy, Canada and Spain) and not rejected for several other countries (Sweden, France, Holland and the United Kingdom). The US, Belgium, Denmark, Finland and Germany are borderline cases where the hypothesis of a stationary real exchange rate is rejected using critical values from the asymptotic distribution but not rejected using critical values from the empirical distribution.

5.4. Impulse response functions

The dynamic adjustment towards PPP after a disturbance can be illuminated using impulse response functions. The polynomial C(L) in the moving average representation in equation (8) contains an infinite number of lags. Here, impulse responses are calculated up to 40 lags or almost three and a half years.

We are primarily interested in how the real exchange rate adjusts to disturbances. Equation (8) is easily transformed to levels of the variables. Levels of prices and exchange rates are in turn easily transformed into levels of real exchange rates. Figure 4 shows the impulse response functions for the Swedish real exchange rate. The corresponding figures for the other countries are presented in Appendix D.

Figure 4: Impulse response functions for the real exchange rate: Sweden



With one cointegration vector, there are two independent stochastic trends that drive the system. In this system of domestic and foreign price level and the nominal exchange rate, it seems natural to identify the trends with domestic and foreign monetary policy. Innovations to the stochastic trends have permanent effects on the price levels and the nominal exchange rate. In addition, there is a transitory shock that has transitory effects on the variables. One identifying assumption is needed to distinguish the two permanent shocks. The only obvious alternative is the small country assumption that the foreign price level is unaffected by the domestic price level. Only a few of the countries could be characterised as small. However, our interest is primarily in the speed of the adjustment process. It can be investigated without identifying the shocks.

The main conclusion from an examination of the impulse response functions is that the bulk of the adjustment is completed within 40 months or a little more than three years. There is an adjustment back to PPP in almost every case, even for the countries for which PPP was rejected. The cointegrating vector corresponding to PPP is imposed as a long run restriction when estimating the impulse response functions, but this does not force the short run parameters to imply a rapid adjustment to PPP. Indeed, there are a few cases where the impulse response functions do not show an adjustment back to PPP within 40 months: The real exchange rates of Belgium and United Kingdom do not seem to adjust to one of the permanent shocks.

6. Conclusions

Before the virtual explosion of PPP tests that followed the introduction of econometric techniques designed to handle non-stationary data, it had become more or less a stylised fact that PPP was rejected in empirical tests. One conclusion from the review of empirical evidence in Section 2 is that PPP has indeed fared badly when American data are used. However, focusing instead on the more homogeneous European countries, the hypothesis of a stationary real exchange rate is rejected less often. Excluding tests on short sample periods decreases the proportion of rejections further. There are few studies that include small European countries like Sweden, Norway, Denmark and Finland and use a long sample period. Also, almost all studies have focused on bilateral rather than multilateral real exchange rates.

In this paper, multilateral PPP has been tested on data from 1960 to 1994 on 16 OECD countries, using Johansen's maximum likelihood approach that incorporates tests of linear restrictions on the cointegrating vectors. The hypothesis of a stationary real exchange rate is rejected for Austria, Switzerland, Canada, Japan and Norway. However, it is not rejected for Sweden, Finland, Holland, France and the United Kingdom even using the asymptotic critical values that are known to reject a true hypothesis too often. If critical values from the simulated empirical distributions are used, PPP is not rejected for Germany, Denmark Belgium and possibly the US either.

A few of the test results are surprising in light of the figures on the real exchange rates. The real exchange rate of Italy looks rather stationary, but the tests reject stationarity. The real exchange rates of Denmark and Holland look non-stationary, but the tests do not reject stationarity. In most cases, however, eyeball econometrics and the formal test results reach similar conclusions.

Empirical distributions and critical values are simulated for the λ-max and trace rank tests as well as the likelihood ratio test for linear restrictions on the cointegrating vector. They show that there may be considerable discrepancies between asymptotic and empirical distributions of the test statistics. All the tests are oversized. The power of the cointegration rank tests is unacceptably low when critical values from the empirical distributions are used, while it seems acceptable for the test for linear restrictions on β.

Appendix A: Empirical tests of PPP

Table 10: The number of rejections of PPP and the number of times a pair of countries was tested

	Aut	Bel	Fra	Ger	Hol	Swe	Swi	Ita	Fin	Nor	Den	UK	US	Jap	Can
Aut	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Bel		-	0/1	0/1	0/1	1/2	0/1	0/1	-	-	-	2/2	2/2/**0/1**	1/2	-
Fra			-	2/3	1/2	1/2	0/1	0/1/**1/2**	-	-	-	1/1/**0/2**	5/5/**1/7**	1/2	1/2/**0/2**
Ger				-	0/1	0/1	1/2	3/3	-	-	-	1/1	4/4/**3/3**	1/2	3/3
Hol					-	0/1	1/2	1/1	-	-	-	1/1	1/1/**1/3**	1/2	1/1
Swe						-	1/2	-	-	-	-	-	-	1/2	1/1
Swi							-	2/2	-	-	-	-	3/3/**2/2**	2/3	-
Ita								-	-	-	-	**1/2**	**2/7**	**1/2**	-
Fin									-	-	-	-	-	-	-
Nor										-	-	-	**0/1**	-	-
Den											-	-	-	-	-
UK												1/1	3/4/**5/9**	1/1	1/2/**0/2**
US													**6/8**	2/2/**6/8**	4/5/**7/11**
Jap														-	-
Can															-
>20	0/0	0/1	3/13	3/3	1/3	0/0	2/2	5/13	0/0	0/1	0/0	6/15	35/60	6/8	8/17
excl*	0/0	0/0	1/6	0/0	0/0	0/0	0/0	3/6	0/0	0/0	0/0	1/6	0	0/0	0
>15	0/0	6/13	12/20	15/21	7/13	5/11	10/16	6/8	0/0	0/0	0/0	8/10	24/26	10/16	9/12
excl*	0/0	4/11	6/13	8/14	5/11	4/10	7/13	6/8	0/0	0/0	0/0	6/6	0	8/14	0

Only tests using sample periods longer than 15 years are included. Fat numbers denote studies on data covering more than 20 years. Tests of multilateral PPP are placed on the diagonal of the matrix. Searching through major data bases of journal articles and the references of the papers discussed above, all papers covering sufficiently long sample periods are included in Table 10. Among studies covering more than 20 years of data, this amounts to the following papers: Adler and Lehmann (1990), Kim (1990), Abuaf and Jorion (1990), McNown and Wallace (1990), Ardeni and Lubian (1991), Layton and Stark (1990) and Glen (1992). With sample periods of 15-20 years, the studies by Cheung and Lai (1993), Glen (1992), Nelson (1990), Fisher and Park (1991) and Johansen and Juselius (1992) are included.

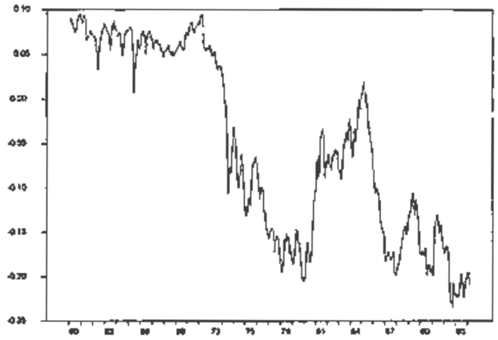
A few observations can be made:

- 60 of 72 tests focuse on the US real exchange rate.
- Of the tests using sample periods longer than 20 years, 35 of 60 tests on the US real exchange rate reject PPP. Among tests on other real exchange rates, only 3 of 13 reject it.
- Of tests using 15 to 20 years of data, 87 percent of the tests on the North American dollars reject PPP, compared to 51 percent of the tests on other currencies.

* Tests on real exchange rates against the United States and Canada have been excluded.

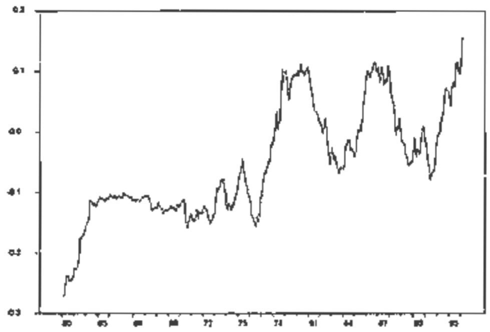
Appendix B: Figures on the real exchange rates

Austria

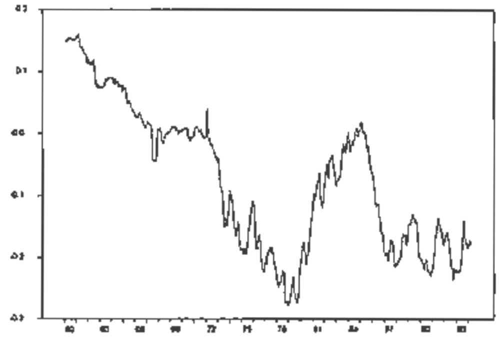


Belgium

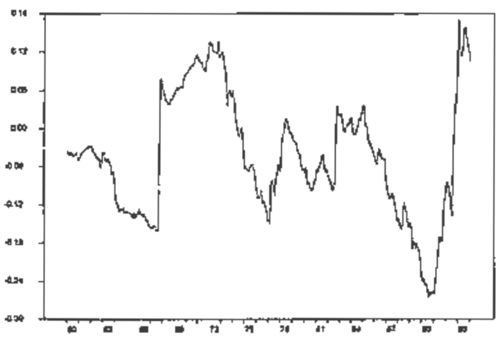
Canada



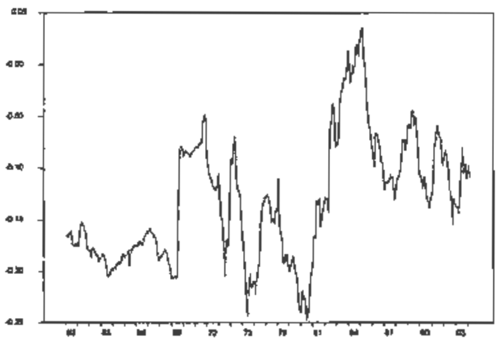
Denmark



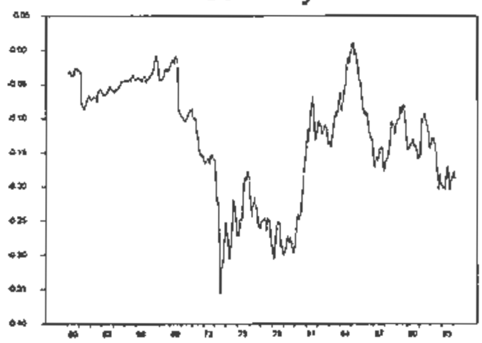
Finland



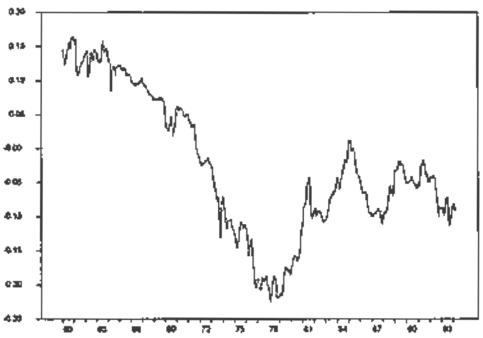
France



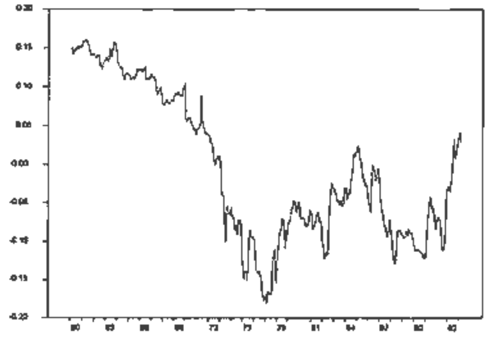
Germany



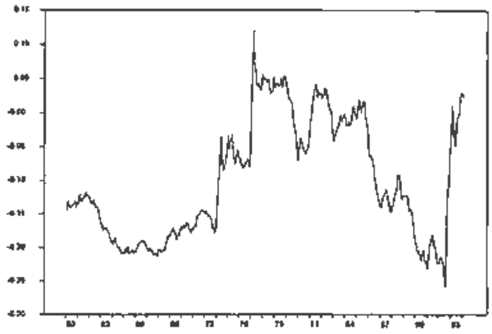
Holland



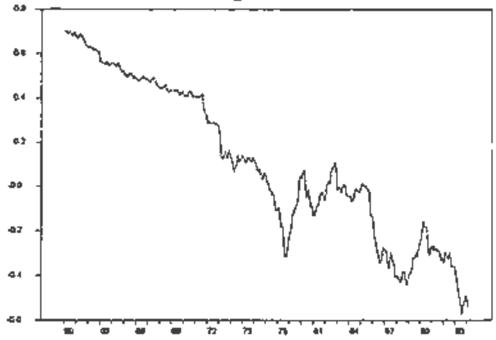
Norway



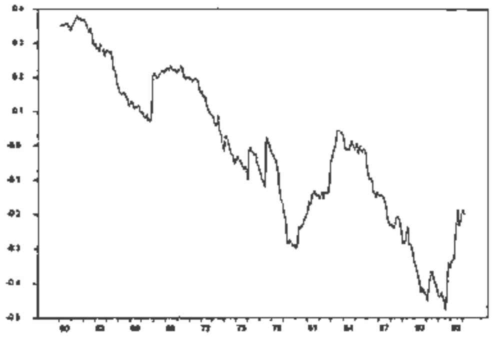
Italy



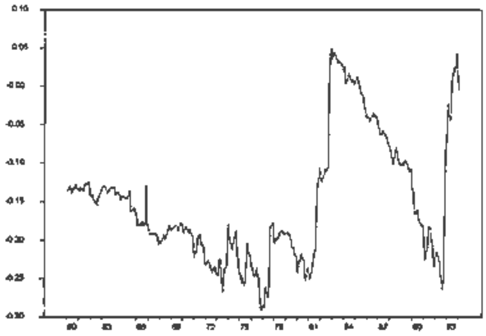
Japan



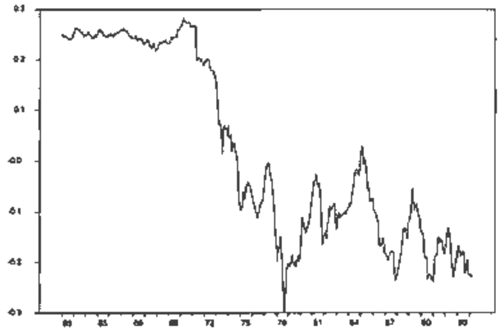
Spain



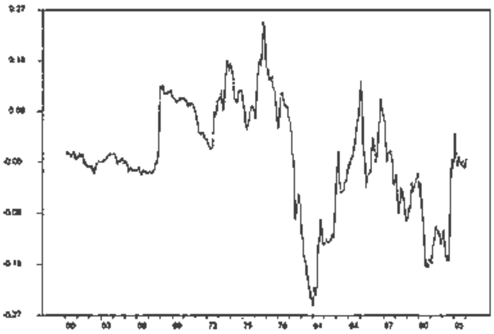
Sweden



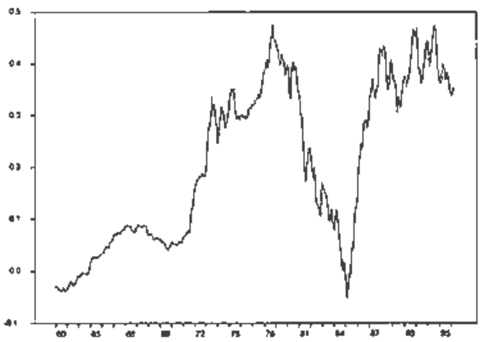
Switzerland



United Kingdom



USA



Appendix C: Critical values from the empirical distributions of the cointegration rank tests

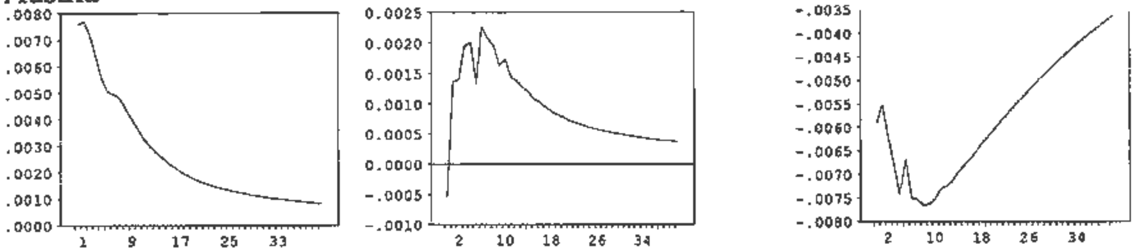
	95% percentiles for the empirical distributions							
	the normal distribution		actual distribution (bootstrapping)		normal distribution w/ dummy variables		actual distribution w dummy variables (bootstrapping)	
	trace	λ-max	trace	λ-max	trace	λ-max	trace	λ-max
Canada								
r>0	30.97	21.44	53.28	42.01	30.05	21.77	48.35	37.55
r>1	13.29	11.29	13.29	11.40	11.63	9.82	11.57	9.85
r>2	4.11	4.11	3.94	3.94	3.71	3.71	3.44	3.44
USA								
r>0	31.20	21.97	40.81	29.69	30.52	21.56	37.68	27.10
r>1	15.70	13.29	15.23	13.13	15.32	12.83	15.28	13.00
r>2	5.80	5.80	5.58	5.58	7.26	7.26	6.20	6.20
Japan								
r>0	31.11	21.84	43.48	29.43	31.75	22.44	40.65	29.66
r>1	15.96	13.80	15.19	13.59	16.60	14.02	16.98	14.60
r>2	7.56	7.56	7.05	7.05	9.14	9.14	8.49	8.49
Austria								
r>0	30.38	21.38	78.14	66.80	30.91	21.08	66.49	55.71
r>1	14.35	12.31	14.46	12.55	17.62	14.18	13.79	12.13
r>2	4.81	4.81	4.72	4.72	5.25	5.25	6.70	6.70
Belgium								
r>0	31.10	21.78	68.58	54.51	29.97	21.10	56.73	44.29
r>1	13.01	11.00	13.16	11.13	12.56	10.53	12.86	10.56
r>2	4.81	4.81	4.87	4.87	4.33	4.33	4.37	4.37
Denmark								
r>0	30.70	21.68	52.68	41.47	31.07	21.03	52.27	41.76
r>1	15.62	13.53	15.91	13.84	15.06	12.50	15.12	12.77
r>2	5.30	5.30	4.93	4.93	5.73	5.73	5.85	5.85
Finland								
r>0	31.09	21.51	48.19	36.88	29.85	20.51	46.36	36.54
r>1	13.37	11.19	13.19	11.37	11.70	10.53	12.08	10.75
r>2	5.56	5.56	5.27	5.27	4.54	4.54	4.99	4.99
France								
r>0	31.00	21.40	58.59	47.93	30.27	21.12	59.28	48.67
r>1	12.73	10.93	12.75	11.09	11.87	10.36	11.94	10.59
r>2	4.13	4.13			3.49	3.49	3.10	3.10

95% percentiles for the empirical distributions

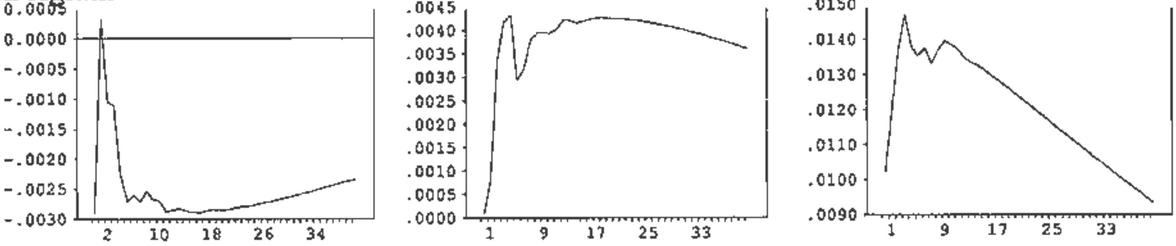
	the normal distribution		actual distribution (bootstrapping)		normal distribution w/ dummy variables		actual distribution w dummy variables (bootstrapping)	
	trace	λ-max	trace	λ-max	trace	λ-max	trace	λ-max
Germany								
r>0	31.04	21.61	55.18	42.39	32.61	22.77	51.39	37.48
r>1	14.45	12.29	14.66	12.48	13.93	12.28	13.78	12.20
r>2	4.84	4.84	4.78	4.78	4.68	4.68	4.21	4.21
Italy								
r>0	30.69	21.66	46.94	35.56	32.45	23.10	39.35	29.44
r>1	15.88	13.74	16.15	14.24	13.57	11.34	14.38	12.33
r>2	4.53	4.53	4.18	4.18	3.22	3.22	3.57	3.57
Holland								
r>0	30.68	21.52	67.58	54.65	34.87	26.36	75.36	62.34
r>1	16.37	14.67	15.74	13.88	14.67	12.94	15.60	13.53
r>2	4.71	4.71	4.96	4.96	4.87	4.87	4.97	4.97
Norway								
r>0	30.87	21.67	50.35	38.36	30.57	21.66	49.60	36.59
r>1	13.30	11.28	13.14	11.37	13.87	11.49	13.28	12.16
r>2	4.80	4.80	4.98	4.98	4.97	4.97	4.89	4.89
Spain								
r>0	31.22	21.91	48.27	36.25	31.02	21.68	45.59	34.83
r>1	12.73	10.94	12.61	11.07	13.18	11.13	11.39	10.35
r>2	3.90	3.90	3.67	3.67	4.02	4.02	2.36	2.36
Switzerland								
r>0	30.93	21.85	41.66	29.59	33.86	22.74	37.90	27.07
r>1	13.36	11.44	13.76	11.70	14.30	12.14	14.14	12.93
r>2	4.64	4.64	4.68	4.68	3.06	3.06	3.04	3.04
Britain								
r>0	30.65	21.65	48.10	36.83	29.08	19.91	49.08	38.34
r>1	12.99	10.93	13.38	11.29	12.42	10.52	13.02	10.66
r>2	4.97	4.97	3.66	3.66	4.94	4.94	5.42	5.42

Appendix D: Impulse response functions (response to a transitory shock and to the two permanent shocks)

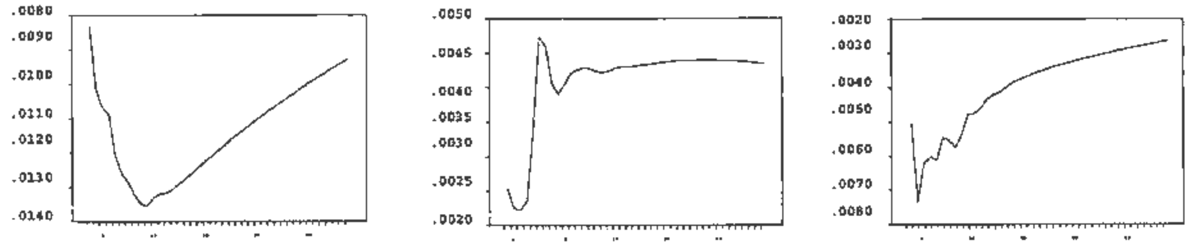
Austria



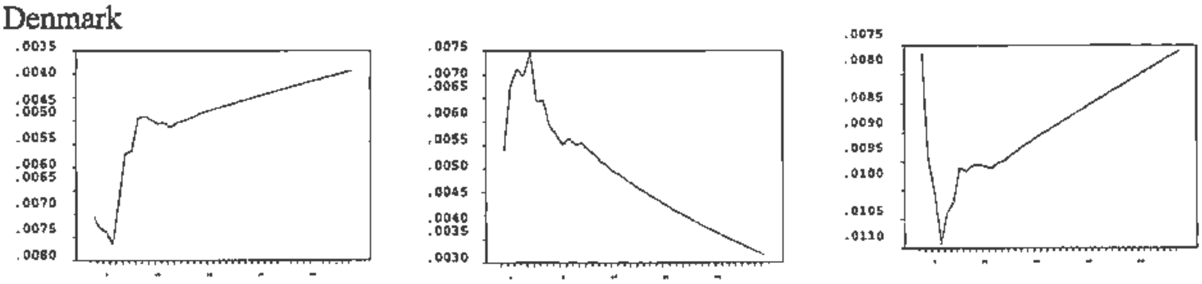
Belgium



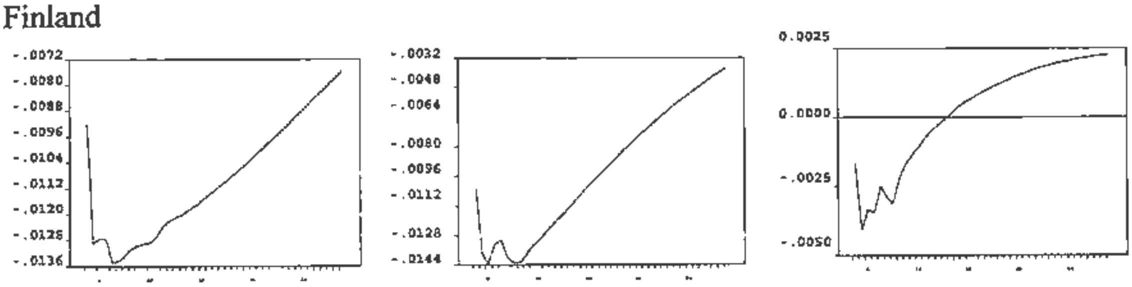
Canada



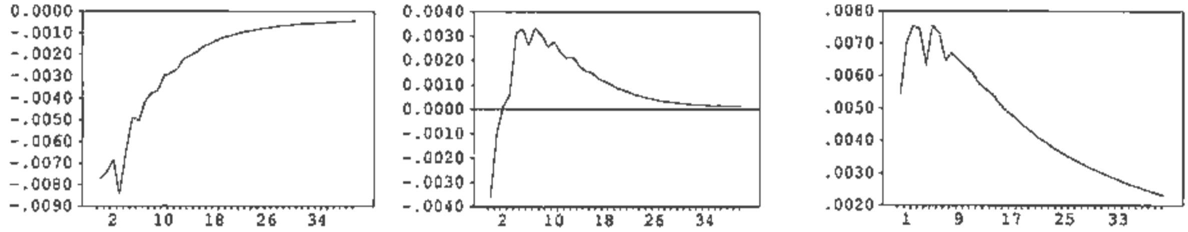
Denmark



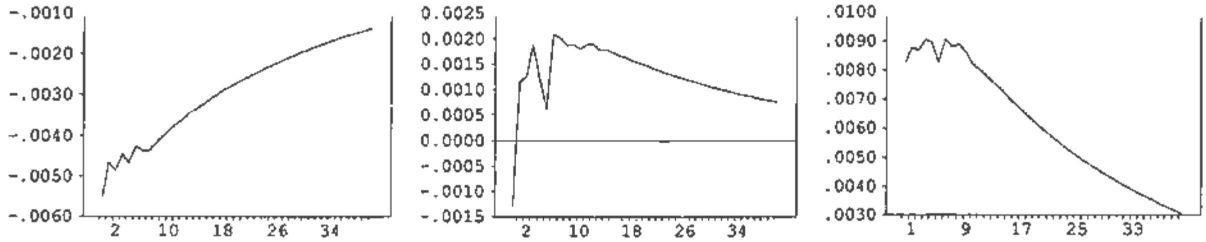
Finland



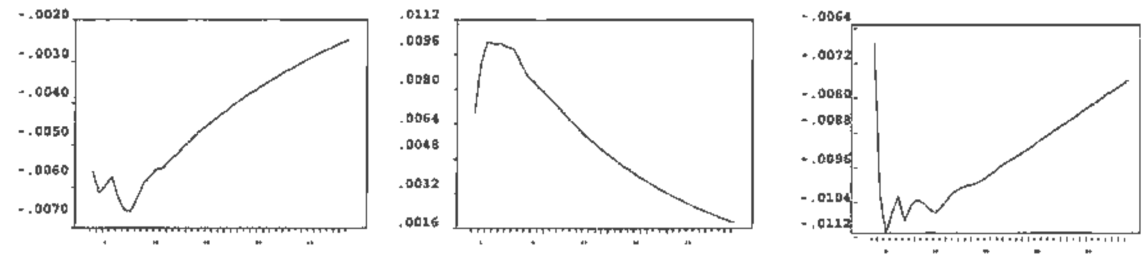
France



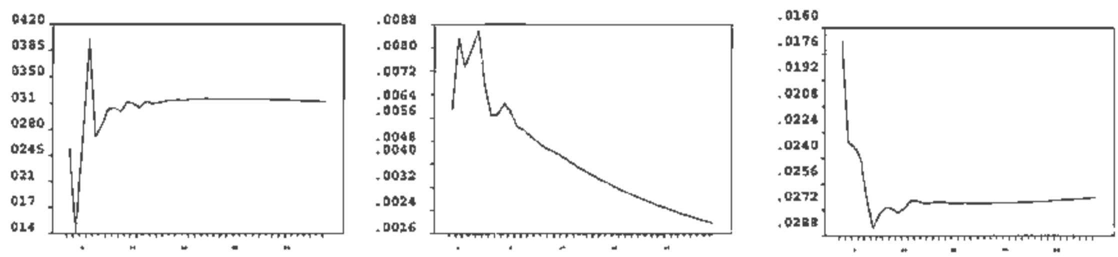
Germany



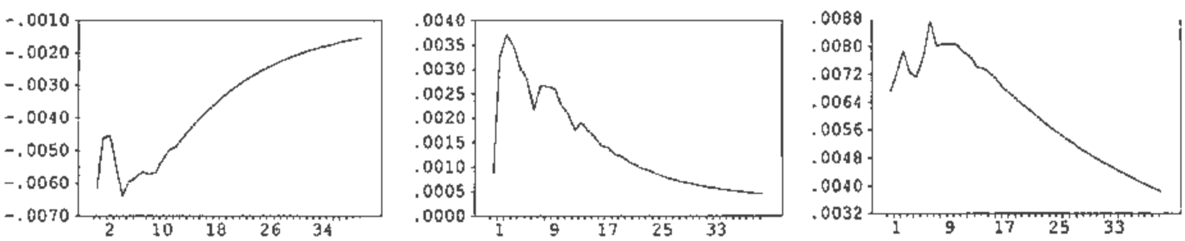
Italy



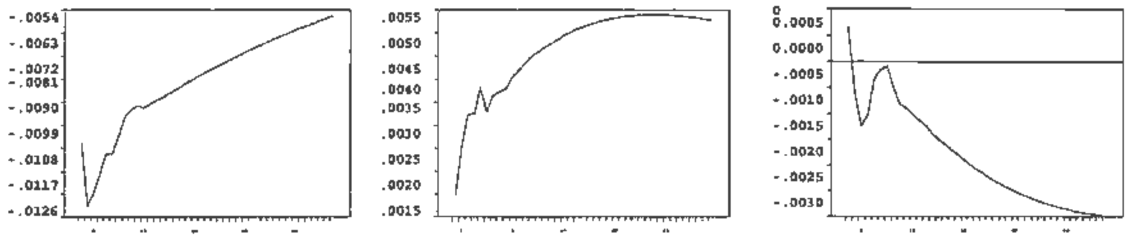
Japan



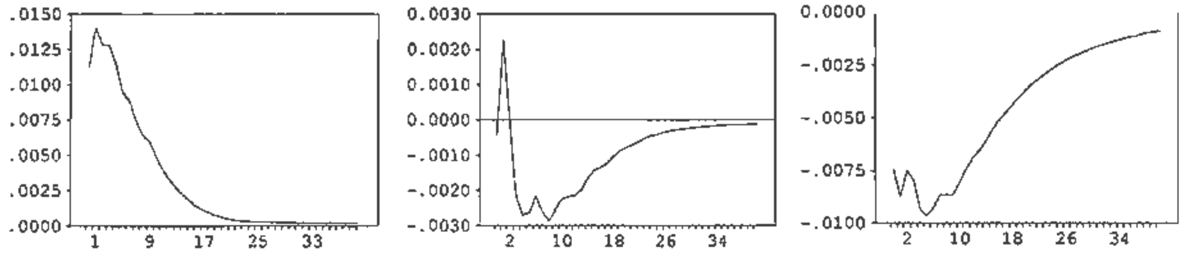
Netherlands



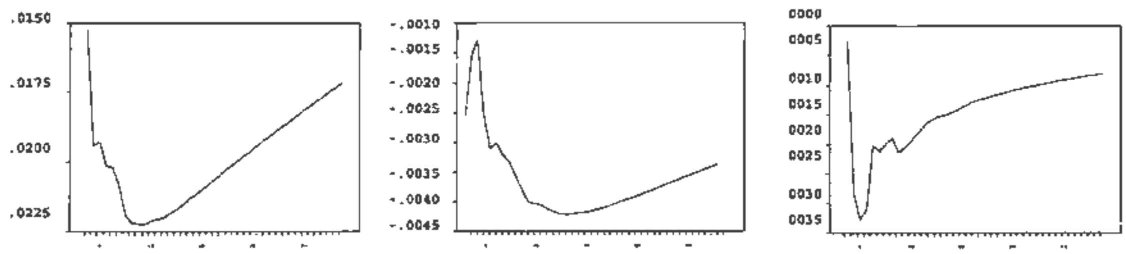
Norway



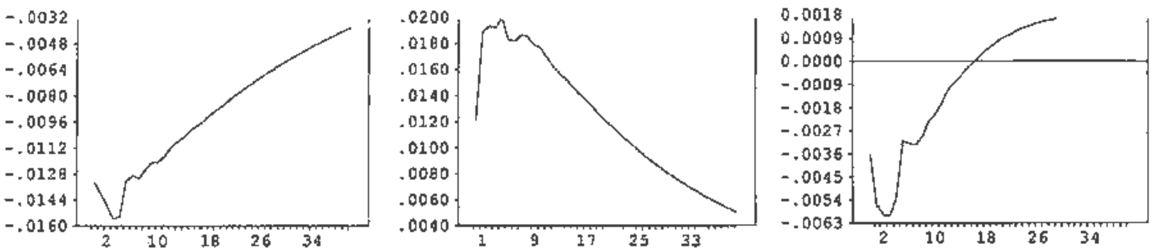
Switzerland



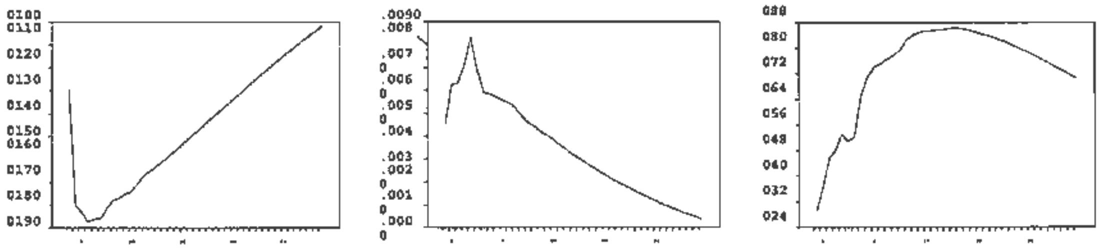
Spain



United Kingdom



United States



Appendix E: The dummy variables

Austria

1961:06, 1962:06, 1963:07, 1964:06, 1965:06, 1966:01, 1966:06, 1967:06, 1971:05, 1973:06, 1973:07, 1984:01, 1988:07, 1989:07, 1990:07, 1991:07, 1992:07, 1993:07

Belgium

1964:06, 1971:09, 1973:03, 1973:04, 1973:08, 1973:12, 1974:02, 1976:10, 1977:01, 1977:12, 1978:10, 1980:01, 1981:07, 1982:03

Canada

1961:07, 1970:06, 1973:02, 1973:08, 1974:01, 1975:01, 1980:01, 1990:01, 1991:01

Denmark

1964:05, 1965:05, 1966:05, 1966:05, 1967:05, 1967:07, 1967:12, 1970:07, 1972:02, 1972:03, 1973:02, 1973:03, 1973:06, 1973:08, 1974:01, 1974:09, 1975:10, 1976:04, 1979:07, 1979:12, 1980:03, 1981:02, 1981:12, 1992:11, 1993:09

Finland

1964:01, 1967:10, 1967:11, 1971:06, 1972:01, 1972:11, 1973:07, 1973:09, 1975:01, 1976:07, 1977:04, 1982:10, 1992:09, 1992:11

France

Germany

1961:03, 1962:08, 1965:06, 1066:01, 1968:01, 1969:10, 1971:01, 1973:03, 1973:06, 1973:08, 1975:07, 1980:01, 1982:06, 1991:07

Netherland

1961:03, 1968:01, 1969:01, 1969:08, 1970:01, 1973:04, 1973:05, 1973:09, 1973:11, 1974:02, 1975:07, 1976:05, 1977:12, 1978:10, 1980:01, 1980:10, 1981:07, 1981:09, 1985:04, 1987:01, 1987:11, 1989:01, 1991:07, 1993:07

Norway

1961:01, 1962:07, 1964:01, 1966:07, 1969:01, 1970:01, 1971:01, 1971:05, 1973:02, 1973:06, 1973:08, 1975:01, 1975:07, 1977:01, 1977:09, 1978:01, 1978:10, 1979:01, 1980:12, 1981:01, 1982:01, 1982:08, 1986:05, 1991:03, 1992:11

Sweden

1961:01, 1962:01, 1965:07, 1966:11, 1971:01, 1973:06, 1973:08, 1973:12, 1975:01, 1977:04, 1977:06, 1977:09, 1980:01, 1981:09, 1982:10, 1983:01, 1990:01, 1992:01, 1992:11, 1993:01, 1993:07, 1994:01,

Switzerland

1961:01, 1962:01, 1964:06, 1965:06, 1966:06, 1971:05, 1973:02, 1973:08, 1973:10, 1974:02, 1878:08, 1978:11, 1979:06, 1980:01, 1985:01

United Kingdom

1967:11, 1967:12, 1968:01, 1972:07, 1973:07, 1973:08, 1974:08, 1975:05, 1977:01, 1979:07, 1980:01, 1983:04, 1984:01, 1985:04, 1992:10, 1993:01

USA

1961:07, 1973:02, 1973:11, 1980:07, 1985:10, 1987:01, 1990:01

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Pricing–to–market in Swedish Exports[1]

by

Annika Alexius[2] and Anders Vredin[3]

December 13, 1996

Abstract:
In this paper fluctuations in prices of Swedish exports to five countries are investigated in order to test whether there are systematic differences between prices to different markets and whether relative export prices are systematically affected by macroeconomic conditions in destination countries. Simple correlations suggest that relative export prices are related to real exchange rates, but also to nominal exchange rates. Formal tests, based on an error–correction model, indicate that the deviations from no pricing–to–market and neutrality of money are quite common and persistent. Over a sample of 15 years, long run monetary neutrality is rejected in almost half of the cases. In most cases, the degree of pricing–to–market is also affected by aggregate demand (unemployment) in export markets.

Keywords: export prices, exchange rates, pricing–to–market, neutrality of money, exchange–rate pass–through.

JEL classification numbers: F14, F31, F41.

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1. Introduction

Following the nominal appreciation of the US dollar in the mid–1980's, and the concomitant real appreciation of the relative price of US products, much research has has been devoted to questions about relations between exchange rates and foreign trade. One strand of the literature tries to explain persistent deviations from purchasing power parity (PPP), i.e. is focused on the determinants and characteristics of fluctuations in real exchange rates (see Froot and Rogoff, 1995, for a survey).[4] Another research area is concerned with the issue whether temporary changes in exchange rates are followed by persistent changes in exports and imports (see e.g. Baldwin and Krugman, 1989). Yet another lively field deals with the "pass–through" of nominal exchange rates to export and import prices, terms of trade and real exchange rates (see e.g. Feenstra, 1989). It has also been examined whether changes in nominal and real exchange rates affect the relative price of a certain good in different markets, i.e. whether the degree of "pricing–to–market" is affected by exchange rate changes (see e.g. Knetter, 1989, 1993 and Kasa, 1992). All of this literature is clearly relevant to the situation of many European countries after the large jumps in nominal and real exchange rates associated with the 1990's "currency crises".

The questions about how a country's export and import prices are affected by changes in nominal exchange rates span over many, theoretically and empirically, complex issues. The answers will depend on such things as the degree of competition in export and import markets, whether the country is a "small" exporter and importer (that must take prices as given from the rest of the world), and the degree and persistence of nominal price and wage rigidity.

Figure 1 depicts the evolution of Sweden's nominal exchange rate, real exchange

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[4]The definition of the concept real exchange rate varies in the literature. In this paper we define the real exchange rate as the ratio (or the log of the ratio) between the consumer price indexes in two regions, expressed in common currency.

rate (the deviation from purchasing power parity, PPP) and (inverse of) terms of trade 1970 – 1995.[5] Nominal and real exchange rates behave in a manner which is typical for many countries (see Froot and Rogoff, 1995, Alexius, 1996, and Obstfeld, 1995). Changes in nominal and real exchange rates are correlated in the short run, but there is little evidence of a long run relationship. Changes in the nominal exchange rate seem to be permanent, while the real exchange rate appears to be mean–reverting. PPP thus seems to hold in the long run, and the deviations show about the same degree of persistence as in other countries. The terms of trade series is positively correlated with the real exchange rate (the correlation coefficient is close to 0.5), as one should expect it to be (see e.g. De Gregorio and Wolf, 1994). Terms of trade do not seem to be strongly affected by the jumps in the nominal exchange rate, which is consistent with the small–open–economy (SOE) hypothesis. This picture is however modified when one looks at disaggregated data.

Figure 2 shows aggregate export and import prices together with the nominal exchange rate 1980 – 1995. Both export and import prices did respond to the devaluations of the Krona in 1981 and 1982, and to the depreciation in 1992–93. But export and import prices moved less than the nominal exchange rate.[6] This suggests that nominal exchange rate changes affect the competitive position of Swedish firms, at least in the short run.

Figure 3 a – d show export prices on wood products (SNI 33) and transport equipment (SNI 384), together with the nominal exchange rate, after the large changes in the value of the Krona in 1981–82 and 1992–93.[7] Swedish export prices on wood

[5]The nominal exchange rate is a so called effective exchange rate calculated using the IMF's MERM weights. The real exchange rate is constructed in the same way using Swedish and foreign consumer price indices. (Source: Central Bank of Sweden.) The (inverse of the) terms of trade is the ratio between the Swedish aggregate import and export price indices. (Source: International Financial Statistics, IMF.) Data are monthly.

[6]Estimates reported by Adolfson (1996) suggest that about one third of a change in the nominal exchange rate passes through to the aggregate import price index in the short run (within two months).

[7]The price series are monthly and have been obtained from the National Institute for Economic Research (*Konjunkturinstitutet*).

products have not responded very strongly to exchange rate changes, which means that exchange rate changes have largely been "passed through" to foreign import prices. Export prices on transport equipment have however responded more strongly.[8]

Gottfries (1994) studies how the price of aggregate Swedish exports of manufactured goods is determined and finds results which do not appear to be consistent with the SOE hypothesis,[9] but with export prices in *Swedish* currency being incompletely adjusted to exchange rate changes. The international evidence on "pricing–to–market" behavior, on the other hand, is typically interpreted as consistent with "local currency price stability", i.e. with prices being set in the *buyers'* currencies and not immediately adjusted to e.g. exchange rate changes.

In this paper we study disaggregated data on prices of Swedish exports. Following the literature on how exporting firms "price to market", we analyse relative prices of (disaggregated and ideally identical) exports to different markets. It is assumed that relative prices of exports from a certain industry are not affected by changes in domestic costs of production (i.e. that domestic production costs are the same irrespective of export destination). Instead, fluctuations in relative prices are viewed as a result of exogenous changes in world market conditions. In particular, we want to test whether the degree of pricing–to–market (if any) is systematically affected by real variables only, such as real exchange rates and aggregate demand, or whether nominal exchange rates also influence firms' pricing behavior.

The rest of the paper is organized as follows. In section 2 we present the pricing–to–market concept and some descriptive statistics for our data on Swedish export prices. A model of a price discriminating exporter, borrowed from Kasa (1992), is presented and tested on Swedish data in Section 3. Kasa's analysis is a useful starting point since – unlike many other studies of pricing–to–market – it is based on a

[8]Interestingly, the SOE hypothesis fits prices on transport equipment relatively well also when one looks at Swedish import prices; see Adolfson (1996).

[9]Naug and Nymoen (1996) reach similar results in their study of prices of Norwegian imports.

theoretical model of a profit maximizing firm. The model however imposes neutrality of money, which implies that nominal exchange rates are not allowed (even in the short run) to affect relative export prices except via real exchange rates. Empirical results from an error correction model, which imposes less theoretical structure but allows for more general empirical relations, are presented in Section 4. Section 5 contains conclusions and suggestions for future research.

2. A description of pricing–to–market

To be able to discuss the issues clearly, we will use the following notation:

$P_m^{i,j,k}$	=	Price of imports from country j to country i, goods from industry k, in country i:s currency
$P_x^{j,i,k}$	=	Price of exports from country j to country i, goods from industry k, in country j:s currency
$E^{i,j}$	=	Price of currency j in terms of currency i

Abstracting from tariffs and transportation costs, we define[10]

$$(1) \qquad P_m^{i,j,k} = E^{i,j} P_x^{j,i,k}.$$

The hypothesis that exporters in country j do not price–to–market can be formally expressed as

$$(2) \qquad P_x^{j,i,k} = \bar{P}_x^{j,k} \qquad \forall\, i$$

[10]Time subscripts are suppressed in this section.

i.e. the price of exports from a certain industry is the same (in the exporting country's currency) on all export markets.[11]

Now, consider an extreme case of PTM: Pricing policies are such that exporters in country j keep import prices fixed in the importing country's currency, so called local currency price stability.[12] (Some may want to define this as a case of "no pass–through".) Then

$$(3) \qquad P_m^{i,j,k}/P_m^{l,j,k} \quad = \quad (P_x^{j,i,k}/E^{l,i}P_x^{j,l,k}) \qquad = \text{constant.}$$

This means that the relative price of exports to markets i and l $(P_x^{j,i,k}/P_x^{j,l,k})$ will be perfectly positively correlated with the *nominal* exchange rate $E^{l,i}$. The correlation between the relative price and the nominal exchange rate here reflects nominal price rigidity at the industry level, which implies that money is not neutral.

As an alternative, consider the following case of relative price rigidity: Market conditions are such that exporters in country j keep their price in market i proportional to the general price level in country i, $\overline{P}^i$, with the factor of proportionality being the same on all exports markets, i.e.

$$(4) \qquad P_m^{i,j,k} \quad = \quad \alpha^{j,k}\overline{P}^i.$$

The relative price of exports to markets i and l will then be

$$(5) \qquad P_x^{j,i,k}/P_x^{j,l,k} \quad = \quad E^{l,i}\overline{P}^i/\overline{P}^l.$$

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[11]Even if exporting firms price–to–market, (2) may hold if by chance market conditions are exactly the same in different export markets. We neglect this rather unlikely case, and interpret (2) as a definition of no pricing–to–market.

[12]See Donnenfeld and Zilcha (1991) and Friberg (1996) for analyses of export pricing. Macroeconomic implications of local currency price stability are considered by Betts and Devereux (1996).

This means that the relative price of exports to markets *i* and *l* will be perfectly positively correlated with the *real* exchange rate $Q^{l,i} = E^{l,i}P^i/P^l$. There are good theoretical and empirical reasons to expect $Q^{l,i}$ and $E^{l,i}$ to be correlated because of macroeconomic conditions.[13] These examples thus show that price discrimination in itself does not imply that relative export prices should be correlated with nominal exchange rates. It may give rise to correlations with real exchange rates, but some deviation from monetary neutrality – either at the macro or industry level – is also needed to produce correlations with nominal exchange rates.[14]

Evidence on PTM in the Swedish export industry will be presented in subsequent sections. The rest of this section contains information about our data set.

To test the PTM hypothesis one needs data over export prices from a certain firm or industry to different markets. Unfortunately, such data are not officially available. One way to proceed is then to collect price data directly from exporters or importers. The most common procedure is however to use official data on the value and volume of exports to calculate so called "unit values" (export value in current prices/ divided by export volume in physical units). Such data are available on disaggregated levels in foreign trade statistics. An obvious problem is that fluctuations in unit values occur for many other reasons than price changes. The severity of this measurement problem varies over industries, aggregation levels and also over time. Using data only on exports to quantitatively important destinations and aggregating data over goods and time have proven to be useful ways to avoid some of the inherent measurement problems with unit values (Börjesson, 1989).

We have chosen to look at Swedish exports to five countries – the US, Great Britain, France, Germany and Japan – and to work primarily with the SITC 4–digit

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[13]See e.g. Svensson (1985) for a theoretical argument and Obstfeld (1995) for empirical evidence.

[14]An analysis of PTM that tries to separate the influences from nominal rigidities and other market imperfections (price discrimination) has been performed by Giovannini (1988).

aggregation level.[15] Quarterly averages have been constructed from monthly data. Around 50 industries were selected on the grounds that they are quantitatively important for Swedish exports or similar to categories used in previous studies of PTM. The need to have consistent, reasonably long time series further limited our sample. In the end, Statistics Sweden provided us with monthly data on exports (values and volumes) from 1980 – 1994 for 29 4–digit SITC groups, five 5–digit SITC groups and six groups from the SNI classification (the six industries used by Marcusson, 1994).[16] After having dropped the industries with the most obviously serious measurement problems, we arrived at the sample of 18 industries reported in *Tables 1 – 3*.[17] Five destination countries give us ten relative export prices for each industry. The 180 series on relative unit values used to calculate the correlations in Tables 2 and 3 are not all devoid of measurement errors, but we will make further comments about such problems in connection with the regression analyses presented below.

Of the 180 correlation coefficients between the relative export price ($P_x^{j,i,k}/P_x^{j,l,k}$) and the *nominal* exchange rate ($E^{l,i}$) in Table 2, 70% are larger than the five per cent critical value (about 0.22).[18] Most of the correlation coefficients are positive, but some are negative. As noted above, there would be a perfect positive correlation if export prices were fixed in the importing countries' currencies. This does not seem to be a full explanation for the indications of PTM provided in Table 2, first because the correlations are not perfect, and second because the pattern is almost identical for the

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[15]Norway is a quantitatively more important export market for Sweden than the US, and some other countries are more important than France and Japan. Judging from IMF's MERM weights, however, French and Japanese companies are important competitors to Swedish firms.

[16]Mats Marcusson has kindly provided us with the series on regional export price series used in his study from 1994. Unfortunately, Statistics Sweden has no documentation over how these unofficial price series have been constructed, so we have chosen to work with the data on unit values for the same industries instead. Our data set also includes data on imports which are not used in the present study.

[17]It should be noted that one industry (SNI 38431, motor vehicles) contains one of the others (SITC 7812, passenger transport vehicles).

[18]The observations are not independent, so a standard significance test is not applicable. But since we will perform econometric analyses below, we will not dwell on that matter here.

relation between relative export prices and *real* exchange rates – see Table 3. Thus, the correlations in Table 2 do not necessarily imply that exporters' pricing decisions are influenced by nominal exchange rates *over and above* any influence from real exchange rates.

Figures 4 a – c show the relative unit values of exports to different countries for three industries. Normalised nominal exchange rates between the countries are also plotted in the figures. Figure 4 a gives an example of a strong positive correlation (0.903): the correlation between the relative price of cars (SITC 7812) exported to the US and France, and the nominal FRF/USD exchange rate. The deviations from no PTM are very large (between –30% and +70%), and they are systematically related to the nominal exchange rate. It seems very unlikely that this pattern could arise as an accidental result of various measurement errors.[19] In Figure 4 b we have a negative correlation (–0.552) between the relative price of containers of papers (SITC 6421) exported to the UK and France and the FRF/GBP exchange rate. Although the relative unit value series is very volatile and probably overestimates the fluctuations in the relative export price, there is a systematic negative relation with the nominal exchange rate. Finally, Figure 4 c provides an example where there is no correlation: the relative price of exports of kraft paper and board (SITC 6414) to the US and the UK and the GBP/USD exchange rate. The deviations from no–PTM appear to be quite large and non–random, but they are not contemporaneously correlated with the nominal exchange rate.

A natural question to ask about Table 2 and Figures 4 a – c is whether the correlations that appear to be significant are not in fact spurious, in particular since nominal exchange rates are often well described as "unit root processes" (or, equivalently, "integrated of order one" or governed by "stochastic trends"). The only

[19]Interestingly, the fluctuations in the relative price between Swedish cars sold in the US and France are of the same magnitude as those in the relative price between German cars sold in the US and Canada; see Kasa (1992). Flam and Nordström (1995) present more detailed results on relative prices of cars in Europe.

way to tell whether the correlations are spurious or genuine is of course to formulate a model of the relation between prices and exchange rates and to make this model subject to econometric analysis.[20]

3. A structural model of pricing–to–market

Many empirical studies of PTM are not based on theoretical models of rational profit maximizing firms. Some analyses are explicitly non–structural and rather atheoretical, while others are based on *ad hoc* assumptions about for instance nominal rigidities. The analysis by Kasa (1992) is an exception, and therefore serves as a useful starting point.

3.1 Kasa's theoretical model

Kasa presents a model of a monopolist in country *j*, who exports his products to two foreign markets. The exporter incurs costs of adjusting his sales in the two different markets. His problem is to choose sales volumes in the two markets, X_t^1 and X_t^2, in order to maximize expected real profits:[21]

$$\text{(6)} \qquad \max_{\{X_t^1,\, X_t^2\}} \quad EXP_0 \sum_{t=0}^{\infty} \beta^t \Big\{ Q_t^{j,1} P^1(X_t^1, Y_t^1) X_t^1 + Q_t^{j,2} P^2(X_t^2, Y_t^2) X_t^2$$
$$- (X_t^1 + X_t^2)c - (1/2)(X_t^1 / X_{t-1}^1 - 1)^2 h^1 - (1/2)(X_t^2 / X_{t-1}^2 - 1)^2 h^2 \Big\},$$

where c is the (constant and real) marginal cost of production, h^i measures the cost of

[20]Another natural question is whether the correlations arise from measurement errors, i.e. whether they would still be present in an analysis of genuine prices rather than unit values. Unfortunately, the reason for using unit values is that data on genuine prices are hard to get, so this question is difficult to answer. Friberg and Vredin (1996) look at relative export prices on *one* product and find correlations similar to those obtained from unit values.

[21]When they are not necessary in order to clarify the notation, superscripts denoting the exporting country (*j*) and industry (*k*) will be suppressed in this section.

adjusting sales in market i, $Q^{j,i}$ is the real exchange rate between the exporting country and country i ($E^{j,i}P^i/P^j$), and $P^i(.)$ denotes the (inverse) demand function in country i. The latter determines the ratio between the price of the imported good and the general price level in country i as a function of the imported quantity (X^i), a demand variable (Y^i), an intercept (A^i) and a demand elasticity (η^i):

$$(7) \qquad P_{m,t}^{i,j}/P_t^i \quad = \quad P^i(X_t^i, Y_t^i) \quad = \quad [A^i X_t^i Y_t^i]^{-(1/\eta^i)}.$$

Based on specific assumptions about the stochastic processes for the exogenous variables – the real exchange rates $Q^{j,i}$ and demand Y^i – and by imposing rational expectations, Kasa derives the following regression equation for the optimal relative export price (in the exporter's currency):

$$(8) \quad p_{x,t}^{j,1} - p_{x,t}^{j,2} \quad = \quad k + \lambda(p_{x,t-1}^{j,1} - p_{x,t-1}^{j,2}) + \gamma_1 \Delta q_t^{j,1} + \gamma_2 \Delta q_t^{j,2} + \gamma_3(p_{x,t-1}^{j,2} - \bar{p}_{t-1}^{j}) + \nu_t,$$

where small letters have been used to denote logs, k is a constant and ν depends on the innovations in the demand variables Y^1 and Y^2.

In Kasa's model, the relative export price $p_{x,t}^{j,1} - p_{x,t}^{j,2}$ is affected by its lagged value because of the costs of adjusting exports to each market. The higher the adjustment costs, the higher is λ. Real exchange rates affect relative prices because it may be optimal for the monopolist to charge a higher price in market i if the general price level in market i goes up. Since there are costs of adjustment, the monopolist has to form expectations about future real exchange rates. The extent to which changes in real exchange rates affect relative export prices is determined by the degree of autocorrelation in real exchange rates. The less persistent changes in real exchange rates are, the less willing to change the supply will the exporter be, and the larger becomes the effect on the relative price. Kasa notes that the presumption is that γ_1 is positive and that γ_2 is negative, unless there are strong cross–effects in the stochastic processes driving real exchange rates. He also notes that γ_3 – which is hard to interpret intuitively – is equal to zero if the costs of adjustment are the same in the two markets.

The extremely simple PTM rule given by (5) can be derived as a special case of (8) if adjustment costs are high, $\lambda = 1$, there are no demand shocks ($\nu = 0$), and adjustment costs and the stochastic processes driving real exchange rates are such that $\gamma_1 = -\gamma_2 = 1$ and $\gamma_3 = 0$. It is clear that there in general is little reason to expect the unconditional correlation between relative export prices and real exchange rates to be perfectly positive.

Kasa's model is consistent with neutrality of money (unless the demand shock ν derives from monetary policy). The exporter who solves the problem (6) is concerned about his real profits (in terms of the consumption basket of the exporting country). According to (7), demand is affected by the relative price of imports. There is no money illusion, although certain relative price rigidities are introduced through the assumptions that marginal production costs and adjustment costs are fixed in real terms. Nominal exchange rates exert no independent influence on the degree of PTM.[22]

3.2 Empirical results from applying Kasa's model to Swedish data

We have applied Kasa's regression model (8) to the 180 series on relative export prices, and the corresponding real exchange rates, that were discussed in section 2. We included dummies to take care of the seasonal pattern in unit values.

Most of the 180 models appear to be well behaved from a statistical point of view. Standard tests for residual autocorrelation and normality do not indicate serious misspecifications. To save space, diagnostic tests are not reported in detail, but are

[22]The restrictions on (8) implied by the hypothesis of monetary neutrality may however be tested by including the different components of the real exchange rate ($\Delta q^{j,i} = \Delta e^{j,i} + \Delta \bar{p}^i - \Delta \bar{p}^j$) as separate regressors.

available upon request.

Some empirical results are reported in *Tables 4 – 6*. In terms of R^2, the explanatory power is reasonably high (Table 4). There are systematic deviations from no–PTM in the data. The point estimates of λ are given in Table 5. About 2/3 of the estimated coefficients are significantly different from zero (according to a 5 % t–test).

Since a lagged dependent variable may be significant for many reasons (e.g. nominal rigidities, as suggested by Giovannini, 1988), the estimates of $\gamma_1 - \gamma_3$ probably tell more about the relevance of Kasa's real adjustment–cost model than the estimates of λ. The point estimates of γ_1 and γ_2 (not shown) typically have the expected signs. That is, an appreciation of Germany's real exchange rate vis–à–vis Sweden ($\Delta q^{SW,GE} > 0$) most often raises the export price on exports from Sweden to Germany in relation to export prices to other destinations ($\gamma_1 > 0$). The symmetry restriction that $\gamma_1 = -\gamma_2$ can only be rejected for 13 of the 180 regressions (according to a 5% t–test). However, as seen from Table 6, the hypothesis that $\gamma_1 = \gamma_2 = \gamma_3 = 0$ can only be rejected in 1/4 of the cases.

Our conclusion from applying Kasa's model to Swedish data is that there are persistent deviations from no–PTM, but that changes in real exchange rates do not provide a satisfactory explanation for the pricing–to–market behavior in most industries. A reasonable conjecture is that fluctuations in relative export prices must be explained in terms of a model which is based on less restrictive assumptions about nominal exchange rates and demand. As noted above, Kasa's model is characterized by monetary neutrality, and the demand variables Y^1 and Y^2 are only allowed to affect relative export prices through the white noise error term ν. Before turning to an analysis of these issues, two other potential problems with the results reported in this section should be mentioned.

First, the results from the different industries and markets may not all be equally important. A casual inspection of the time series on relative export prices suggests that the measurement problems associated with unit values are more severe in some cases than others. When the dependent variable is subject to measurement errors and included, lagged, as an explanatory variable, all coefficient estimates will be biased and inconsistent. We interpret the fact that the regression models usually passed conventional misspecification tests as reassuring, but we also believe that measurement problems may be considerable in around 1/4 of the 180 regressions. There seem to be two charateristic features of this group regarding the results reported above: Unit values for Swedish exports to Japan seem to be especially unreliable, in particular in the beginning of the sample; and measurement problems seem to make it harder to reject that $\gamma_1 = \gamma_2 = \gamma_3 = 0$.

Second, the results reported above show signs usually associated with spurious regressions: high autocorrelation in the dependent variable, high R^2's, and insignificant parameters. Without further analysis, we cannot tell whether the results in Tables 4 – 6 are any more genuine than the simple correlations in Table 2 – 3.

4. An error–correction model of pricing–to–market

In this section we will present and estimate an error correction model. We test for integration and cointegration using the maximum likelihood procedures suggested by Johansen (1991) and Johansen and Juselius (1990, 1992). The tests provide information about whether there is any pricing–to–market behavior, in the long as well as the short run, whether fluctuations in relative export prices are consistent with neutrality of money, and whether the degree of PTM is affected by aggregate demand (measured as the aggregate unemployment rate).

4.1 The case for an error–correction model

Giovannini (1988) derives and estimates an equation similar to (8) which however is interpreted quite differently. Giovannini assumes that the exporter has to set the price in nominal terms (in the exporting or importing country's currency) before nominal exchange rates, demand variables or prices of competing products can be observed. Giovannini uses his model to derive an estimate of how much of the degree of PTM that can be attributed to exchange rate surprises (in combination with preset prices) and *ex ante* price discrimination. The coefficient on the lagged relative price (λ in Kasa's model) is interpreted as an estimate of the length of price presetting. As in Kasa's analysis, lagged values of exogenous variables are included and motivated in terms of rational expectations. But Giovannini allows both for deviations from neutrality of money and for autocorrelation in the demand variables, so lagged values of more variables than real exchange rates are included.

It is generally accepted that nominal variables such as nominal exchange rates and price levels are typical examples of "$I(1)$ processes", i.e. series that are integrated of order one. (Some researchers have found them to be integrated of order two, $I(2)$.) Real exchange rates are possibly stationary, $I(0)$, but mean reversion is often so slow that the unit root hypothesis cannot be rejected (see e.g. Nessén, 1996, and Alexius, 1996).[23] If relative export prices are also integrated of order one, the correlations in Tables 2 and 3 are spurious, unless relative export prices are cointegrated with exchange rates (nominal or real, respectively). In either case, Kasa's model (8) is misspecified if relative export prices are $I(1)$.

Kasa's empirical analyses suggest that real exchange rates are governed by stochastic trends (are integrated of order one), while relative export prices are (implicitly) treated as stationary. Giovannini performs his analyses on data which are assumed to be stationary after deterministic trends have been removed. A model which is less structural than the models by Kasa and Giovannini, but encompasses both, and does not rest on the premise that relative export prices are stationary, is the following

[23]Tests for unit roots (augmented Dickey–Fuller and Johansen tests) suggest that real and nominal exchange rates, price levels and unemployment rates are $I(1)$ in most cases in our data set.

error correction model:

$$(9)\quad \Delta(p^{j,1}_{x,t} - p^{j,2}_{x,t}) = K + \Sigma^n_{s=1}\beta_{1s}\Delta p^{j,1}_{x,t-s} + \Sigma^n_{s=1}\beta_{2s}\Delta p^{j,2}_{x,t-s} + \lambda_1 p^{j,1}_{x,t-1} + \lambda_2 p^{j,2}_{x,t-1} + \Sigma^n_{s=0}\Gamma'_s\Delta Z_{t-s} + \Pi' Z_{t-1} + \epsilon_t$$

where Z is a vector including the exogenous variables, $Z' = [e^{j,1}, \bar{p}^j, \bar{p}^1, e^{j,2}, \bar{p}^2, y^1, y^2]$, and Γ_s and Π are parameter vectors with seven parameters each. Let γ_{is} and π_i denote the i:th elements of Γ_s and Π, respectively. Then, for example, Kasa's model (8) implies the following restrictions on the error correction model (9):

$$(10)\quad \begin{gathered} \gamma_{10} = \gamma_{30}\ (= \gamma_1 \text{ in Kasa's model});\\ \gamma_{40} = \gamma_{50}\ (= \gamma_2 \text{ in Kasa's model});\\ \gamma_{20} = -(\gamma_{10} + \gamma_{40})\\ \text{all other parameters} = 0, \text{ except};\\ \lambda_1\ (= \lambda - 1 \text{ in Kasa's model});\\ \lambda_2\ (= -\lambda + \gamma_3 + 1 \text{ in Kasa's model}); \text{ and}\\ \pi_2\ (= -\gamma_3 \text{ in Kasa's model});\\ \lambda_1 + \lambda_2 = -\pi_2. \end{gathered}$$

The assumption of neutrality of money, on the other hand, implies only the following restrictions:[24]

$$(11)\quad \begin{gathered} \beta_{1s} + \beta_{2s} + \gamma_{1s} + \gamma_{2s} + \gamma_{4s} = 0, \forall\ s\\ \lambda_1 + \lambda_2 + \pi_1 + \pi_2 + \pi_4 = 0,\\ \gamma_{1s} = \gamma_{3s} \text{ and } \gamma_{4s} = \gamma_{5s}, \forall\ s\\ \pi_1 = \pi_3 \text{ and } \pi_4 = \pi_5. \end{gathered}$$

[24]Monetary neutrality is here taken to mean that the relative export price is unaffected by a proportional change in all prices (of goods and currencies) denominated in a certain currency (i.e, the home currency (j) or any of the foreign currencies (1 or 2)).

It can be seen that (10) implies (11) but not vice versa.

The restrictions on fluctuations in relative export prices implied by neutrality of money, and the further restrictions suggested by Kasa, may thus be tested as restrictions on the more general error correction model (9). An even more general approach would be to specify a vector error correction model for the vector $[p^{j,1}_{x,t}, p^{j,2}_{x,t}, Z_t]'$. But since we are convinced that it is reasonable to treat the variables in Z as strictly exogenous when we look at disaggregated Swedish export prices, we will only present results from single–equation models.

In order to simplify the analysis and also save degrees of freedom, the tests reported in the next sub–section are based on the following modified version of (9):

$$(12)\quad \Delta(p^{j,1}_{x,t} - p^{j,2}_{x,t}) = \tilde{K} + \tilde{\beta}(\Delta p^{j,1}_{x,t-1} - \Delta p^{j,2}_{x,t-1}) + \tilde{\lambda}(p^{j,1}_{x,t-1} - p^{j,2}_{x,t-1}) + \Sigma^{1}_{s=0}\tilde{\Gamma}'_s \Delta V_{t-s} + \tilde{\Pi}' V_{t-1} + \tilde{\epsilon}_t$$

where $V' = [e^{2,1}, \bar{p}^1, \bar{p}^2, y^1, y^2]$. In this application y^1 and y^2 are unemployment rates in countries 1 and 2 (source: OECD Main Economic Indicators). Note also that $e^{2,1} = e^{j,1} - e^{j,2}$. Formally, (12) is a special case of (9) with the following restrictions:

$$(13)\quad \begin{gathered} \beta_{11} = -\beta_{21}\ (= \tilde{\beta}); \\ \text{all other } \beta_{is}\text{:s} = 0; \\ \lambda_1 = -\lambda_2\ (= \tilde{\lambda}),; \\ \gamma_{1s} = -\gamma_{4s}\ (= \tilde{\gamma}_{1s}),\ s = 0,\ 1; \\ \gamma_{is} = 0, \forall\ s > 1, \forall\ i; \\ \gamma_{2s} = 0, \forall\ s; \\ \pi_1 = -\pi_4\ (= \tilde{\pi}_1); \\ \text{and } \pi_2 = 0. \end{gathered}$$

It can be seen that (13) involves fewer restrictions than (10), i.e., than Kasa's model, but more restrictions than (11), i.e. than required by neutrality of money.

In comparison with the error correction model (9), the following restrictions have been imposed. First, we have set the lag order to 1.[25] Second, we have imposed the symmetry restrictions that $e^{j,1}$ and $e^{j,2}$ have the same coefficients, with opposite signs, and that the the relative export price $p_x^{j,1} - p_x^{j,2}$ is not affected by the domestic price level $\bar{p}^j$. To understand what these restrictions mean, note that imposing the symmetry restriction $\gamma_1 = -\gamma_2$ on Kasa's model (8), implies the restrictions $\gamma_{10} = \gamma_{30} = -\gamma_{40} = -\gamma_{50}$ and $\gamma_{20} = 0$ on the error correction model (9). This means that the relative export price $p_x^{j,1} - p_x^{j,2}$ is affected by the real exchange rate between the destination countries 2 and 1 ($e^{j,1} - e^{j,2} + \bar{p}^1 - \bar{p}^2$), but not by the real exchange rates between these countries and the exporting country. The symmetry restrictions implied by (12) are somewhat weaker than this, since they allow the coefficients on $e^{j,1} - e^{j,2}$, $\bar{p}^1$ and $\bar{p}^2$ to be different, and hence for deviations from neutrality of money. One may thus say that the model by Kasa, tested in the previous subsection, imposed monetary neutrality and allowed for a test of symmetry; while the approach taken in this subsection section is to impose symmetry and test for neutrality of money.

4.2 Monetary neutrality

If the cointegration vector for $[p_x^{j,1} - p_x^{j,2},\ V']'$ is proportional to $[1,\ a,\ a,\ -a,\ b,\ c]'$, then the following holds for the error correction model (12):

$$\tilde{\pi}_1 = \tilde{\pi}_2 = -\tilde{\pi}_3 \tag{14}$$

A corresponding restriction may be imposed on the short run dynamics:

[25] With two lags in the underlying VAR model (which implies 1 lag in the error correction model), the residuals pass the Box–Ljung test for serial correlation and the Doornik–Hansen test for non–normality. Furthermore, for a few arbitrarily selected regressions, information criteria indicate that two lags are appropriate.

$$\tilde{\gamma}_{1s} = \tilde{\gamma}_{2s} = -\tilde{\gamma}_{3s},\ s = 0,\ 1. \tag{15}$$

The restrictions in (13) – (15) together imply (11), i.e. neutrality of money (but not vice versa). The restrictions on the short run dynamics (15) may be interpreted as short run monetary neutrality, while the cointegration restriction (14) may be interpreted as long run monetary neutrality. If $p_x^{j,1} - p_x^{j,2}$ has a stochastic trend (is integrated of order one) which is in common (is cointegrated) with $e^{2,1}$, $\bar{p}^1$, and $\bar{p}^2$, and if the long run (cointegration) relation is proportional to [1, a, a, $-a$, b, c]', the relation holds between the relative export price, the real exchange rate and the other real variables (demand). Nominal exchange rates and prices exert no independent influence. A special case of long run monetary neutrality is if the relative export price $p_x^{j,1} - p_x^{j,2}$ is stationary, i.e. if [1, 0, 0, 0, 0, 0]' is a cointegration vector.

Two Johansen tests for cointegration applied to the error correction model (12) are reported in Tables 7 and 8. Among the 180 relative price series, there are 47 series for which the Johansen procedures cannot be applied because of missing values. The hypothesis that the relative export price is $I(0)$ (i.e. that [1, 0, 0, 0, 0, 0]' is a cointegration vector) is rejected in 103 of the remaining 133 cases; cf. *Table 7*. In 30 cases relative export prices thus appear to be stationary, i.e. there is no PTM in the *long run*. In most cases, however, relative export prices appear to have stochastic trends. This suggests that exporters often are able to price–to–market even in the long run. In 43% of the cases of long run price discrimination (44 out of 103), the hypothesis that the relative export price is cointegrated with the real exchange rate (and the other real variables) cannot be rejected; cf. *Table 8*.

The evidence on whether fluctuations in relative export prices are consistent with *long run* monetary neutrality is thus mixed. In about half of the cases, the relative export price is either stationary (30 cases) or cointegrated with real variables (44 cases). These results are consistent with long run monetary neutrality.[26] But in the remaining

[26]The test reported in Table 7 involves more restrictions than monetary neutrality,

(59) cases, the relative export price has a trend in common with nominal exchange rates and/or price levels. These cases are not consistent with long run monetary neutrality.

Even if neutrality of money holds in the long run, it may be violated in the *short run*. The restrictions (15) can be rejected in 40 of the 133 cases; cf. *Table 9*. It is surprising that short run monetary neutrality is rejected somewhat less often than long run monetary neutrality, since one expects money to be "more neutral" in the long run. However, it may be argued that our sample period (15 years) is too short to support long run neutrality. It is known that the slow mean reversion in real exchange rates makes it hard to reject the unit root hypothesis in short samples. The same argument may apply to our study. It is also known that the distribution of the Johansen test can be quite different in small samples compared with the asymptotic distributions used here (see e.g. Jacobson *et al.*, 1996).

4.3 Relative export prices and aggregate demand

The evidence of PTM in Swedish exports seems to be in line with results from earlier studies of other countries. Export prices differ, in domestic currency, between destination countries. The differences are sometimes related to movements in real exchange rates, sometimes to nominal exchange rates. Few studies have investigated whether relative export prices are also related to other determinants of aggregate demand. In Kasa's (1992) study demand shocks were assumed to be absorbed by the regression residual.

The error correction model (12) used in the above tests of monetary neutrality, was estimated using unemployment rates in destination countries as measures of the demand variables (y^1 and y^2). Although other measures could be used, e.g. industrial production or GDP, the estimated error correction model allows us to test whether aggregate demand influences relative export prices.

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because of the symmetry restrictions discussed above.

Tests of the hypothesis that the (normalized) cointegration vector has the form $[1, a, b, c, 0, 0]'$, i.e., that the unemployment rates do not enter the long run relation, are reported in *Table 10*. In 44 of 133 cases (33%) this hypothesis cannot be rejected. The degree of PTM seems to be affected by aggregate demand in most cases. Inspection of the point estimates of the cointegration vectors reveals that the effects from the unemployment rates vary. Sometimes an increase in unemployment in country 1 raises the relative price of exports to country 1, but in some cases higher unemployment is associated with a lower relative export price. The results point to a need for more theoretical (and empirical) analysis of the relation between PTM and aggregate demand.

Looking at the short run relations, changes in unemployment rates have no significant effect on changes in relative prices in 102 cases (according to a 5% F–test). The hypothesis that $\tilde{\gamma}_{4s} = \tilde{\gamma}_{5s} = 0$, $s = 0, 1$, is thus rejected in 31 cases (23%). These results may seem to suggest that unemployment has a stronger influence on relative export prices in the long run than in the short run. Whether unemployment has any long run trend or not is however a difficult issue, both theoretically and empirically (see e.g. Jacobson *et al.*, 1996). It cannot be ruled out that the "long run" relations between unemployment and relative export prices that we find reflect how relative export prices respond to "short run" fluctuations in aggregate demand.

5. Conclusions and suggestions for future work

If exporters have some market power and opportunities for arbitrage in goods market are limited, they may price–to–market, i.e. set different prices in different destination countries. Relative export prices may thus be correlated with e.g. real exchange rates and aggregate demand in importing countries because of price discrimination. If export prices are nominally rigid in the importers' currencies (so called local currency pricing), changes in nominal exchange rates are not fully "passed through", and relative export prices – in exporters' currencies – are correlated with nominal exchange rates.

Previous work by other researchers has suggested that exporters in the US, Germany and Japan price–to–market. One expects pricing behavior in a small economy like Sweden to differ from that of large industrialised countries. However, our results are consistent with price dicrimination in Swedish exports. Unit values of Swedish exports display persistent deviations from no pricing–to–market in most cases, and their short and long run fluctuations are related to real exchange rates and aggregate demand (unemployment) in destination countries. Relative export prices are also related to nominal exchange rates. Since real and nominal exchange rates are correlated at the macro level, the correlations between relative export prices and nominal exchange rates need not reflect nominal rigidities at the industry level. The hypothesis of monetary neutrality is tested in our empirical analysis. It turns out that in many industries relative export prices are affected by nominal exchange rates not only through real exchange rates. Nominal exchange rates and prices have independent influences on export prices, in the short as well as in the long run. We reject long run monetary neutrality in almost half of the cases.

Since we explore a data set which has not previously been used for this purpose, we consider this paper a pilot study of pricing–to–market in Swedish exports. It has been shown that data on unit values contain useful information. Like most of the earlier work in this field, our empirical analysis has been performed without strong restrictions from theoretical models of price discrimination. Much work therefore remains before we can provide a satisfactory explanation for the pricing–to–market behavior.

First, more general models of imperfect competition than the simple monopoly model should be applied to explain possible links between the degree of pricing–to–market and observable industry characteristics. Second, our results suggest that real phenomena like imperfect competition may not give a full explanation to fluctuations in relative export prices. Monetary issues, e.g. the phenomenon of local currency pricing, must also be better understood. The empirical analysis could likewise be improved in many ways. The degree of pricing–to–market could be related to market shares (e.g. along the lines of Gottfries, 1994, or Feenstra, Gagnon and Knetter, 1996), costs of imported inputs (see e.g. Gron and Swenson, 1996), and trade barriers (as in e.g. Flam and Nordström, 1995). The time series of export prices from different industries to different markets could be pooled in order to find relations which are common across industries and/or export destinations. This is also theoretically motivated since each industry exports to more than two markets. More careful tests – along the lines of e.g. Jacobson *et al.* (1996) and Naug and Nymoen (1996) – are also warranted. There is clearly a trade–off between the number of industries one decides to study and the depth of the empirical analysis. Further work could apply a more compact empirical framework and pay more attention to e.g. measurement errors, possible signs of misspecification and small sample properties.

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Figure 1:

Real/nominal exchange rates and inverted terms of trade, 1970–1995

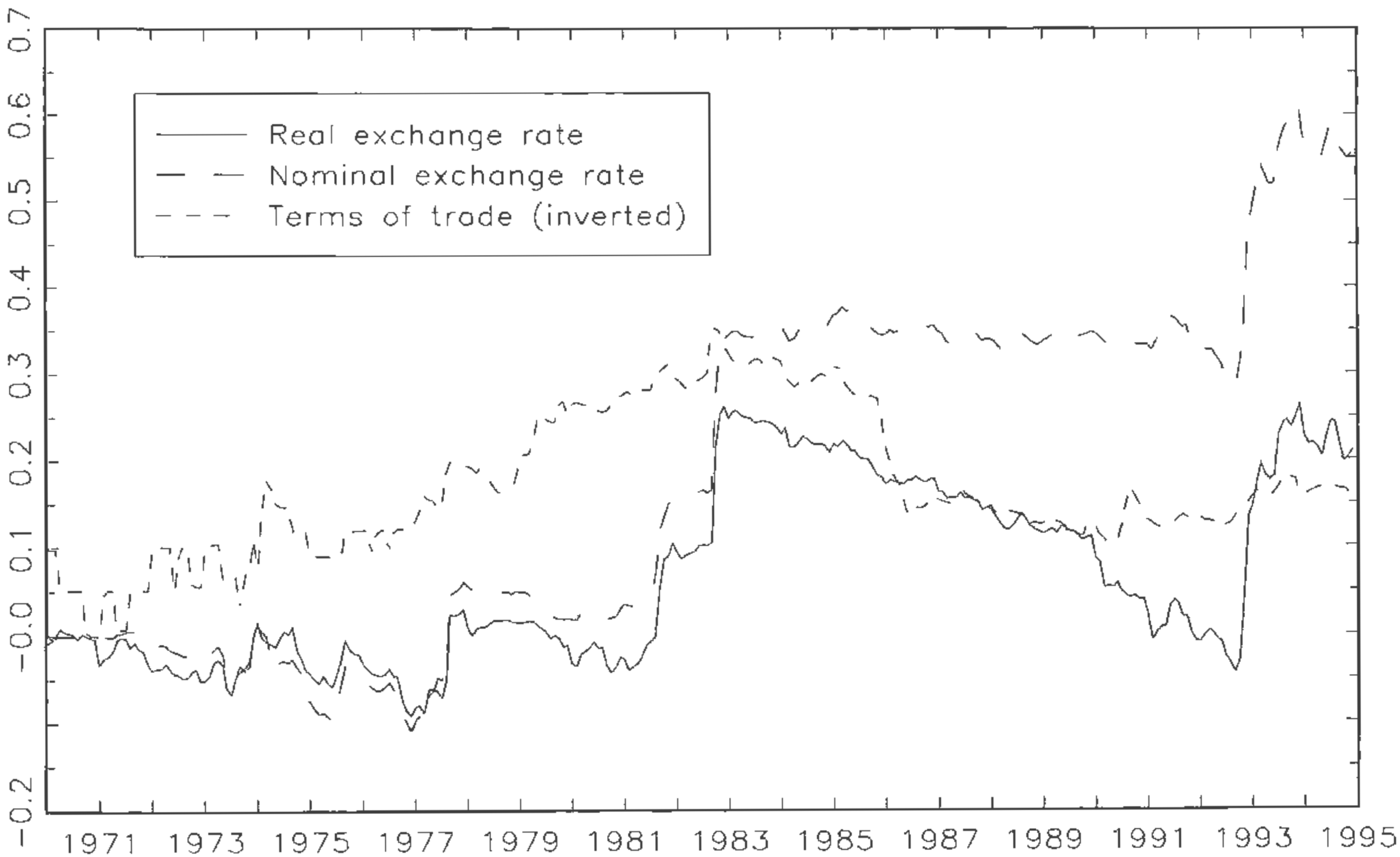

Figure 2: Nominal exchange rate and export/import prices, 1979–1995

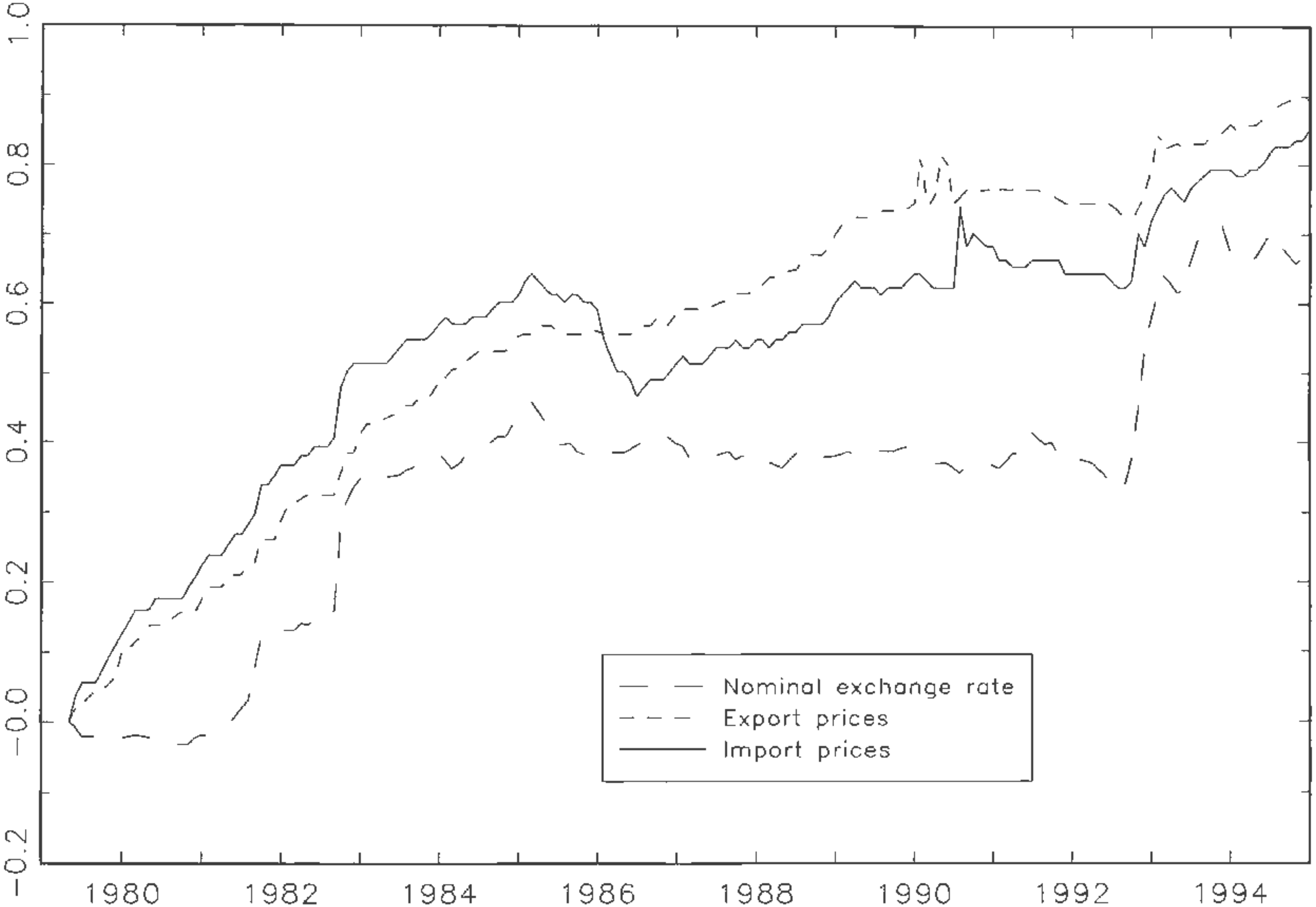

Figure 3a: The export price of wood products 1981–1985

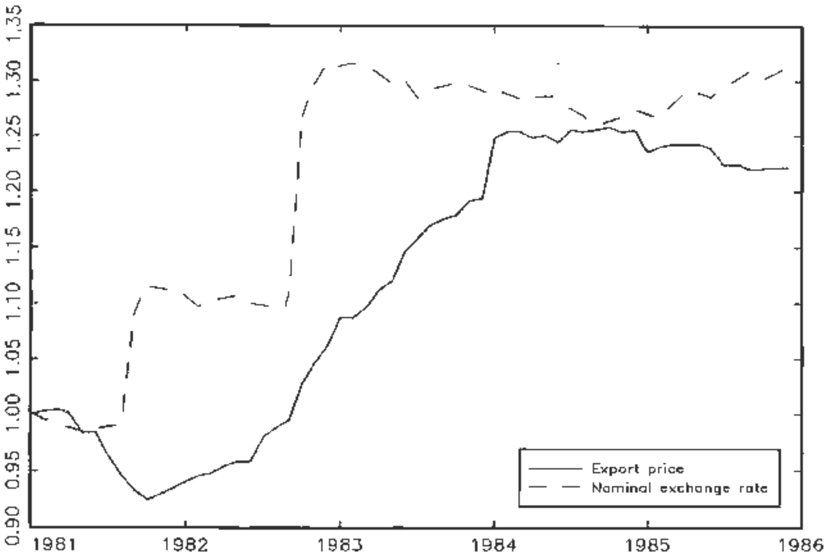

Figure 3b: The export price of wood products 1991–1995

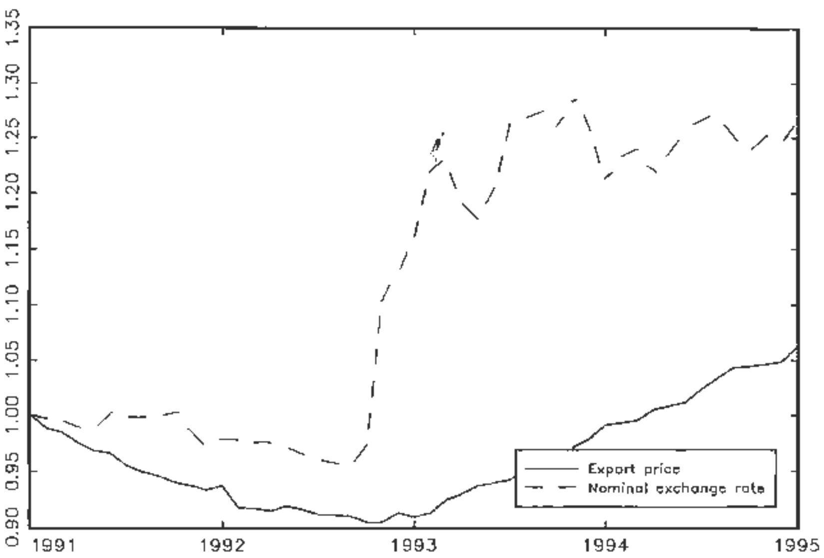

Figure 3c: The export price of transport equipment 1981–1985

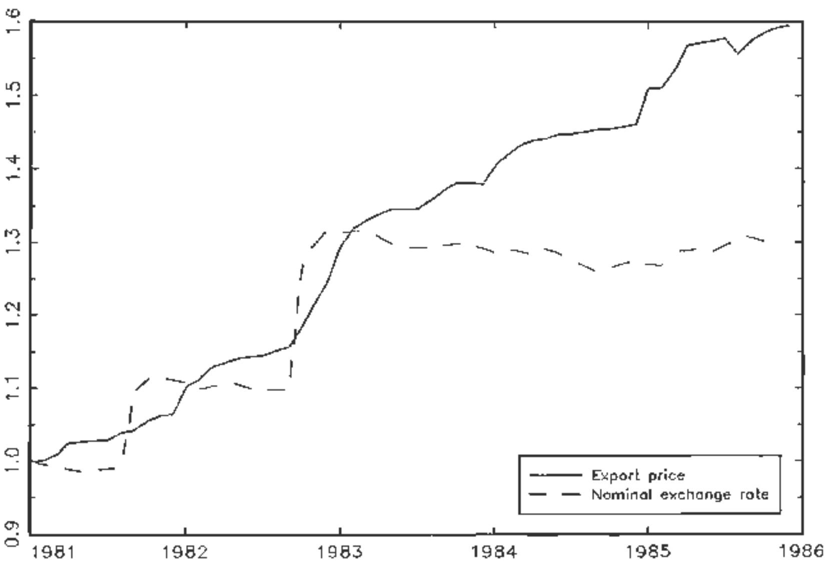

Figure 3d: The export price of transport equipment 1991–1995

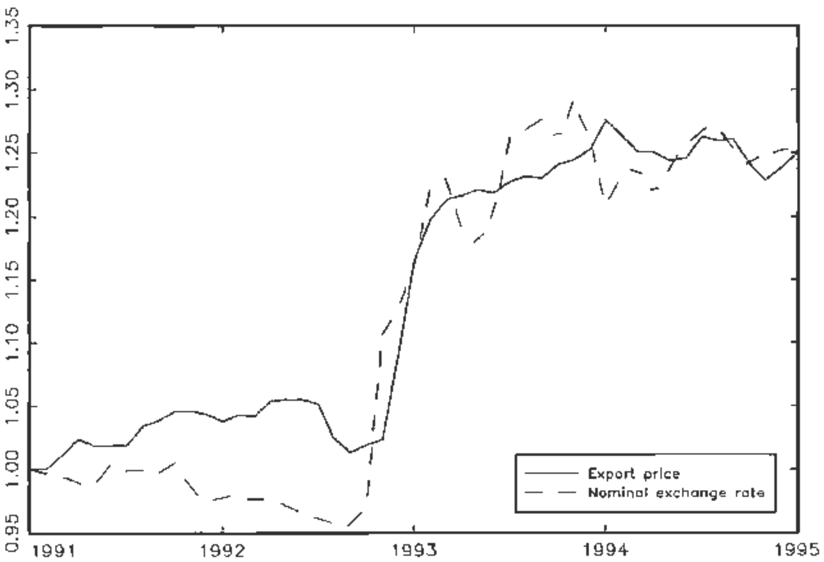

FIGURE 4a: PUS/PFR, PASSENGER TRANSPORT VEHICLES, AND FRF/USD

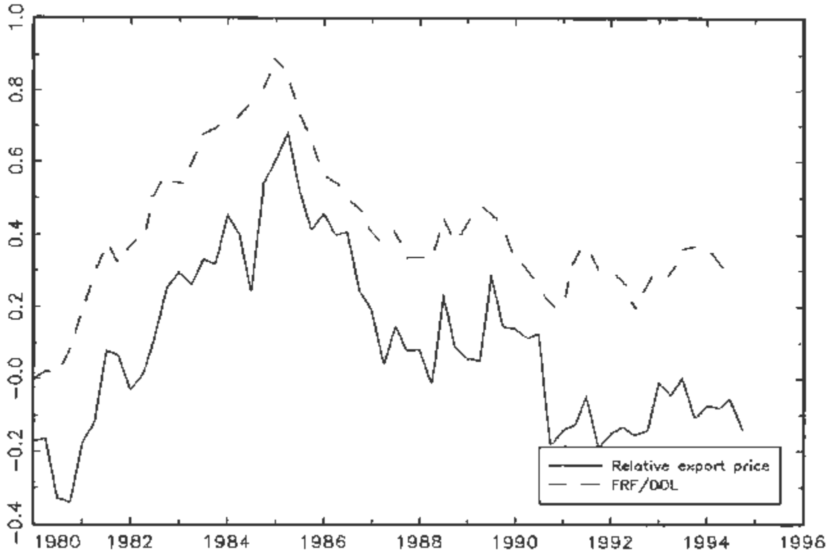

FIGURE 4b: PUK/PFR, CONTAINERS OF PAPER, AND FRF/GBP

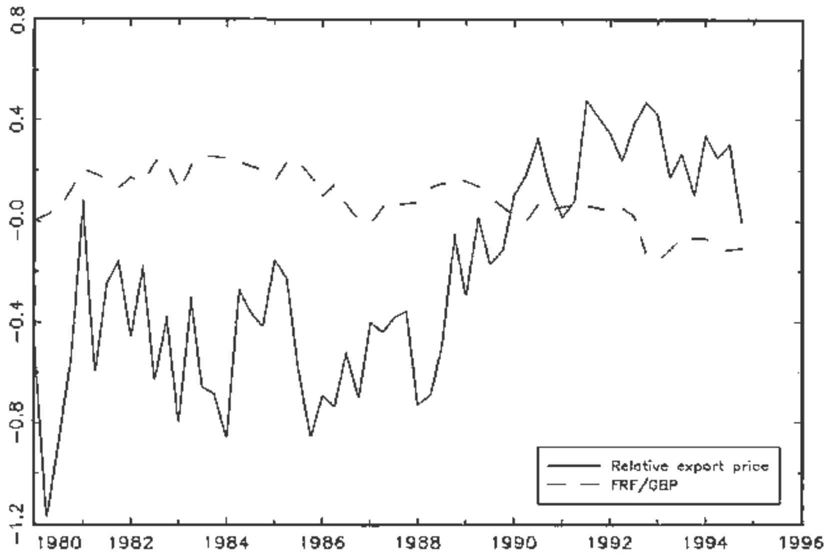

FIGURE 4c: PUS/PUK, KRAFT PAPER AND KRAFT BOARD, AND GBP/USD

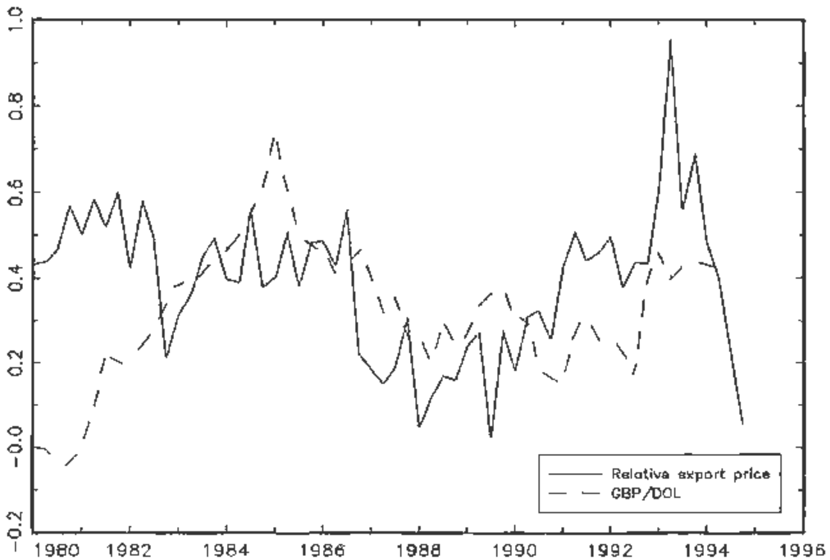

Table 1: Swedish industries included in study of pricing to market:

SITC 11248	Vodka	Vodka
SNI 33202	Ostoppade trämöbler	Non–upholstered wooden furniture
SNI 3811	Verktyg, redskap	Cutlery, hand tools
SNI 38291	Hushållsapparater	Household appliances
SNI 3832	Teleprodukter	Radio, television and communication equipment and apparatus
SNI 3833	Elektriska hushållsapparater	Electrical appliances and housewares
SNI 38431	Bilar och underreden	Motor vehicle and chassi manufacturing
SITC 6411	Tidningspapper i rullar eller ark	Newsprint, rolls, sheets
SITC 6414	Kraftpapper och kraftpapp, obelagda och obestrukna, i rullar eller ark	Kraft paper, board, uncoated
SITC 6421	Kartonger, säckar, påsar o. andra förpackningar av papper, papp m.m.	Containers, etc. of paper
SITC 68424	Aluminiumfolie	Aluminium foil
SITC 6954	Andra handverktyg (inkl. glasmästardiamanter), skruvstycken o.dyl.	Hand tools, etc.
SITC 6956	Knivar och skärstål för maskiner	Knives, cutting blades
SITC 7441	Truckar	Work trucks, tractors, etc.
SITC 7731	Isolerade elektr. ledare	Insulated wire, etc.
SITC 7812	Personbilar	Passenger transport vehicles
SITC 7843	Delar och tillbehör till motorfordon	Other parts, motor vehicles
SITC 8744	Instrument o. apparater för fysikalisk el. kemisk analys	Instruments, analysis, etc.

Table 2: Correlations between relative export prices and nominal exchange rates

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	-0.277[18]	0.612	-0.086[20]	-0.029[15]	0.426[20]	0.627[18]	0.582[15]	0.548[31]	-0.169[20]	0.110[23]
33202	0.165	0.483	0.452	-0.185	0.521	-0.240	0.376	0.522	0.463	0.399
3811	-0.173	-0.242	-0.261	0.472	-0.752	0.646	0.292	-0.427	-0.140	-0.432
38291	-0.290	0.453	-0.233[1]	-0.278	-0.127[1]	-0.021	0.182	-0.511[1]	-0.440[1]	-0.166
383200	-0.080	0.535	0.435	-0.134	0.658	-0.096	0.638	0.166	0.277	0.356
3833	-0.005	-0.146	-0.452	0.436	-0.236	0.117	-0.078	-0.387	-0.457	-0.295
38431	0.116	0.736	0.249	-0.284	0.511	0.808	0.901	0.369	0.058	0.260
6411	0.542	0.589[1]	0.395[12]	0.350	0.434[1]	0.500[1]	0.409[1]	0.586[12]	0.440[12]	0.286
6414	0.792	0.312	0.443[2]	-0.024	0.549[2]	0.000	0.034	0.726[2]	0.479[2]	0.671
6421	-0.303	0.399	0.359[4]	0.456	0.420[4]	0.227	0.203	0.267[4]	0.487[4]	-0.552
68424	0.407	0.117[14]	0.275[20]	-0.039	0.239[21]	0.134[14]	0.104[14]	-0.142[20]	0.366[20]	-0.001
6954	0.439	-0.255	0.276[6]	0.380	0.385[6]	0.036	-0.019	0.499[6]	0.457[6]	0.142
6956	0.603	0.768	-0.339	0.832	-0.129	0.770	0.417	0.174	0.520	-0.126
7731	0.363	-0.222[1]	0.527[13]	0.804	0.201[13]	0.510[1]	0.236[1]	0.704[13]	0.694[13]	-0.249
7441	0.251	0.287	0.298[6]	-0.006	0.113[6]	0.429	0.389	0.225[6]	0.254[6]	0.081
7812	0.124	0.852	0.063	0.345	0.534	0.727	0.903	0.084	0.285	0.180
7843	0.405	0.661	-0.513	0.512	-0.503	0.268	0.120	-0.643	-0.287	-0.469
8744	-0.199	0.367	0.295	-0.167	0.339	0.169	0.562	-0.086	0.322	-0.032

[#] denotes the number of missing values in the series

Table 3: Correlations between relative export prices and real exchange rates

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	-0.113[18]	0.491	-0.089[20]	0.030[15]	0.432[20]	0.683[18]	0.634[15]	-0.090[31]	-0.099[29]	0.076[23]
33202	-0.168	0.436	0.410	-0.362	0.438	-0.230	0.489	0.318	0.316	-0.046
3811	0.165	-0.028	-0.207	0.386	-0.688	0.517	0.141	-0.216	-0.055	-0.189
38291	-0.177	0.394	-0.161[1]	-0.208	-0.029[1]	0.221	0.346	-0.308[1]	-0.321[1]	0.010
383200	0.061	0.546	0.444	-0.319	0.687	0.087	0.610	0.202	0.352	-0.098
3833	0.186	-0.051	-0.437	0.333	-0.116	0.004	-0.199	-0.318	-0.368	-0.079
38431	0.401	0.820	0.240	-0.336	0.586	0.888	0.883	0.546	0.109	0.594
6411	0.744	0.587[1]	0.362[12]	0.534	0.427[1]	0.552[1]	0.439[1]	0.618[12]	0.514[12]	0.576
6414	0.688	0.206	0.396[2]	-0.237	0.463[2]	0.040	0.225	0.595[2]	0.387[2]	0.577
6421	0.119	0.412	0.316[4]	0.290	0.380[4]	0.390	0.160	0.233[4]	0.430[4]	-0.152
68424	0.413	0.041[14]	0.289[20]	0.059	0.097[21]	0.235[14]	0.036[14]	0.231[20]	0.211[20]	0.378
6954	0.220	-0.218	0.246[6]	0.187	0.380[6]	-0.185	-0.106	0.320[6]	0.372[6]	-0.120
6956	0.534	0.702	-0.297	0.492	-0.055	0.761	0.306	0.354	0.514	0.153
7441	0.025	0.333	0.332[6]	-0.056	0.123[6]	0.393	0.286	0.277[6]	0.315[6]	-0.068
7731	0.205	-0.031[1]	0.481[13]	0.412	0.128[13]	0.254[1]	0.025[1]	0.558[13]	0.510[13]	-0.401
7812	0.332	0.880	0.063	-0.111	0.599	0.887	0.843	0.239	0.309	0.203
7843	0.003	0.583	-0.481	0.229	-0.434	0.353	0.049	-0.482	-0.257	-0.225
8744	0.083	0.429	0.272	-0.198	0.322	0.336	0.598	-0.037	0.366	-0.003

[#] denotes tbe number of missing values in the series

Table 4:
R^2 from the Kasa regressions

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	0.162	0.802	0.332	0.409	0.637	0.586	0.710	0.304	0.386	0.441
33202	0.686	0.665	0.532	0.558	0.631	0.737	0.744	0.613	0.553	0.664
3811	0.585	0.573	0.735	0.294	0.838	0.494	0.722	0.841	0.746	0.597
38291	0.163	0.767	0.510	0.049	0.395	0.682	0.667	0.582	0.551	0.196
38320	0.473	0.648	0.595	0.510	0.566	0.483	0.678	0.311	0.623	0.614
3833	0.603	0.392	0.580	0.634	0.676	0.392	0.493	0.602	0.571	0.687
38431	0.633	0.833	0.111	0.294	0.518	0.902	0.818	0.472	0.042	0.645
6411	0.863	0.434	0.338	0.843	0.384	0.358	0.275	0.554	0.530	0.737
6414	0.834	0.521	0.210	0.608	0.552	0.449	0.514	0.543	0.170	0.809
6421	0.487	0.302	0.128	0.635	0.143	0.381	0.375	0.055	0.340	0.681
68424	0.529	0.288	0.191	0.211	0.253	0.190	0.219	0.180	0.245	0.547
6954	0.205	0.373	0.388	0.375	0.261	0.266	0.289	0.529	0.344	0.290
6956	0.429	0.563	0.607	0.621	0.266	0.652	0.263	0.427	0.171	0.308
7441	0.279	0.438	0.132	0.121	0.150	0.330	0.394	0.066	0.140	0.209
7731	0.384	0.443	0.384	0.635	0.101	0.504	0.721	0.424	0.578	0.588
7812	0.465	0.886	0.075	0.384	0.532	0.900	0.852	0.162	0.135	0.352
7843	0.602	0.555	0.600	0.764	0.268	0.344	0.602	0.363	0.315	0.687
8744	0.116	0.155	0.331	0.311	0.253	0.223	0.285	0.284	0.331	0.167

Table 5:
Estimates of λ and t-statistics

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	0.091	**0.675**	-0.097	**0.463**	**0.597**	**0.586**	**0.685**	0.260	-0.036	0.301
	0.421	4.494	-0.602	2.753	3.331	5.093	6.545	1.128	-0.157	1.373
33202	**0.733**	**0.434**	**1.720**	**0.892**	**1.239**	**0.812**	**0.898**	**0.468**	**0.408**	**0.819**
	3.830	2.470	3.955	6.564	5.371	7.789	11.289	3.790	3.153	9.649
3811	**0.629**	**0.431**	0.115	**0.426**	-0.037	**0.514**	**0.605**	**0.742**	**0.645**	-0.033
	6.094	4.050	0.619	3.333	-0.165	4.580	4.987	9.691	6.738	-0.224
38291	0.033	**0.428**	**-0.663**	0.023	**0.397**	**0.766**	**0.799**	**0.653**	**0.617**	-0.200
	0.562	2.701	-2.126	0.161	2.200	9.086	9.164	6.557	5.945	-1.152
38320	**0.682**	**0.926**	**0.369**	**0.607**	**0.486**	**0.622**	**0.618**	**0.486**	**0.720**	**0.653**
	5.073	7.563	2.678	3.556	2.629	3.456	3.302	3.782	5.541	3.800
3833	**0.706**	-0.142	0.529	0.275	**0.650**	**0.541**	**0.583**	**0.699**	**0.651**	**0.757**
	3.187	-0.218	0.890	1.664	4.927	4.663	5.434	6.867	7.051	7.154
38431	**0.773**	**0.880**	-0.182	**0.447**	**0.818**	**0.998**	**0.975**	**0.657**	0.163	**0.844**
	7.511	8.964	-0.864	2.978	7.122	18.33	13.623	6.179	1.117	7.737
6411	**0.823**	-0.034	**0.647**	**0.578**	**0.534**	**0.463**	**0.333**	**0.530**	**0.551**	**0.888**
	9.550	-0.169	3.039	4.294	3.843	4.762	3.356	3.364	3.453	11.595
6414	**0.849**	**0.703**	**0.521**	**0.757**	**0.762**	**0.683**	**0.732**	**0.548**	**0.308**	**0.849**
	12.075	3.982	3.148	7.663	5.720	5.742	6.609	4.286	2.335	13.119
6421	**0.457**	**0.957**	-1.040	1.190	0.523	**0.466**	**0.438**	0.193	0.095	**1.118**
	2.251	2.574	-1.415	5.412	1.656	3.157	3.314	1.267	0.512	7.406
68424	**0.488**	-1.836	-0.053	0.154	**0.417**	**0.385**	0.240	-0.173	-0.292	**0.795**
	3.423	-1.786	-0.132	1.857	2.594	2.166	1.516	-0.989	-1.655	6.347
6954	**0.501**	0.314	0.247	**0.673**	-0.231	**0.447**	**0.315**	**0.615**	**0.441**	**0.520**
	3.096	1.352	0.563	3.568	-0.932	2.787	2.091	4.904	2.991	2.161
6956	**0.819**	**0.493**	**0.746**	**0.716**	0.165	**0.899**	**0.369**	**0.683**	**0.372**	0.316
	3.583	2.764	4.026	3.783	1.057	7.826	2.355	5.456	2.696	1.983
7441	-0.033	0.090	0.060	0.170	0.579	**0.482**	**0.546**	0.064	0.051	0.278
	-0.191	0.213	0.058	0.980	1.864	3.987	4.676	0.395	0.345	1.916
7731	0.204	0.498	1.147	-0.164	0.133	**0.429**	**0.508**	**0.488**	**0.425**	-0.017
	1.393	1.034	1.531	-0.524	0.573	3.182	3.186	2.825	2.709	-0.062
7812	**0.684**	**0.981**	0.081	**0.614**	**0.845**	**0.985**	**1.001**	**0.363**	**0.348**	**0.580**
	5.756	13.682	0.438	5.132	7.195	18.523	14.591	2.680	2.625	4.783
7843	**0.830**	**0.738**	**1.139**	**0.966**	-0.012	**0.529**	**0.588**	**0.447**	-1.106	0.009
	6.111	3.739	3.728	4.861	-0.054	4.310	3.567	3.880	-0.995	0.037
8744	0.060	-0.034	-0.080	-0.069	**0.384**	**0.464**	**0.507**	**0.506**	**0.570**	0.100
	0.358	-0.206	-0.489	-0.459	2.496	3.061	3.322	3.879	4.441	0.695

Bold numbers indicate that λ is significantly different from zero using 5 percent critical values.

Table 6:
F-tests for the null hypothesis that $\gamma_1 = \gamma_2 = \gamma_3 = 0$, p-values

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	0.381	0.104	0.093	0.063	**0.004**	**0.016**	**0.001**	0.552	0.344	0.222
33202	0.938	**0.006**	0.058	0.495	0.147	0.347	0.154	0.058	**0.037**	0.250
3811	0.213	**0.003**	**0.002**	0.369	**0.000**	0.990	0.242	**0.039**	**0.045**	**0.000**
38291	0.592	**0.014**	**0.001**	0.815	0.342	0.071	0.761	0.073	0.084	0.257
38320	0.293	0.235	**0.005**	0.723	0.115	0.919	0.526	0.130	0.361	0.809
3833	0.138	0.195	**0.023**	**0.000**	**0.042**	0.127	0.192	0.056	0.058	0.592
38431	**0.044**	0.067	0.209	0.473	0.151	**0.005**	**0.006**	0.920	0.928	0.654
6411	**0.008**	**0.001**	0.497	**0.000**	0.391	0.171	0.042	0.243	0.210	**0.004**
6414	**0.001**	0.961	0.782	0.282	0.726	0.997	0.860	0.310	0.813	0.121
6421	0.492	0.587	0.404	**0.025**	0.767	0.559	0.353	0.971	0.078	**0.029**
68424	**0.030**	**0.054**	0.265	0.121	0.594	0.631	0.155	0.400	0.230	0.240
6954	0.415	0.684	0.524	0.715	0.094	0.959	0.178	**0.041**	0.577	0.540
6956	0.520	**0.031**	0.521	0.689	0.458	**0.042**	0.876	0.143	0.986	0.425
7441	**0.019**	0.150	0.294	0.541	0.781	0.222	0.138	0.857	0.213	0.256
7731	**0.003**	0.940	0.630	**0.030**	0.860	0.318	0.412	0.534	**0.051**	**0.016**
7812	0.110	**0.025**	0.529	0.437	0.139	**0.001**	**0.006**	0.846	0.884	0.685
7843	0.335	0.772	0.263	0.601	0.152	0.612	0.697	0.131	0.348	**0.012**
8744	0.704	0.110	**0.006**	**0.003**	0.809	0.268	0.234	**0.042**	**0.036**	0.134

Bold numbers indicate that the null hypothesis is rejected using 5 percent critical values.

Table 7:

Likelihood ratio statistics for the null hypothesis that the relative export price is stationary

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	MV	16.15	MV	MV	MV	MV	MV	MV	MV	MV
33202	20.38	21.45	22.87	25.37	12.60	14.61	16.26	13.52	19.29	15.82
3811	**3.06**	14.36	17.74	27.03	17.84	12.01	20.00	15.19	12.80	21.44
38291	20.13	**6.87**	MV	18.72	MV	7.77	12.18	MV	MV	16.07
38320	15.93	21.26	12.57	23.04	14.96	**9.90**	25.53	13.34	22.83	16.40
3833	**8.28**	**10.31**	15.07	19.41	12.37	18.03	11.59	11.69	24.64	**5.74**
38431	31.72	21.75	24.56	13.72	**4.00**	16.84	18.71	16.32	**11.01**	18.20
6411	12.05	21.59	MV	**9.68**	MV	15.49	14.05	MV	MV	**4.73**
6414	11.57	**6.51**	MV	21.57	MV	11.26	15.76	MV	MV	16.14
6421	12.37	24.60	MV	18.59	MV	28.29	33.28	MV	MV	26.71
68424	20.98	MV	MV	5.36	MV	MV	MV	MV	MV	22.23
6954	**8.94**	**10.66**	MV	13.63	MV	33.59	**8.09**	MV	MV	17.95
6956	25.98	19.79	30.92	39.65	23.07	29.39	16.15	17.30	34.49	**11.06**
7441	30.60	25.24	MV	**10.61**	MV	16.60	20.22	MV	MV	17.98
7731	15.82	MV	MV	26.60	MV	MV	MV	MV	MV	18.75
7812	12.36	18.11	19.90	15.52	**7.68**	**5.74**	11.42	13.99	**9.21**	**7.33**
7843	23.96	26.26	32.09	**7.32**	17.67	25.28	15.60	14.06	**8.95**	12.35
8744	**7.52**	**10.58**	15.74	**5.91**	**10.81**	26.78	**10.34**	**2.82**	14.56	**5.07**

The LR-statistic is $\chi^2(5)$, with a 5 percent critical value of 11.07.

MV means that there are missing values in the series.

Bold numbers indicate that the null hypothesis is not rejected using the 5 percent critical value.

Table 8:

Likelihood ratio statistics for the null hypothesis that the relative export price is cointegrated with the real exchange rate

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	MV	11.03	MV	MV	MV	MV	MV	MV	MV	MV
33202	**5.30**	**2.76**	18.38	21.27	**3.01**	**5.11**	**5.83**	**5.11**	15.04	8.70
3811	***	**5.93**	17.19	7.47	**5.65**	8.05	15.50	**2.14**	**3.88**	14.64
38291	16.80	***	MV	**0.85**	MV	***	8.71	MV	MV	**5.74**
38320	9.64	12.69	**0.38**	19.65	**4.85**	***	24.34	**0.65**	6.39	13.73
3833	***	***	**2.61**	13.35	**5.54**	**1.93**	6.10	**1.88**	8.22	***
38431	15.45	**4.02**	**0.45**	**3.59**	***	**4.67**	6.70	**2.00**	***	**0.55**
6411	**2.52**	8.24	MV	***	MV	**2.48**	9.92	MV	MV	***
6414	10.41	***	MV	11.48	MV	***	**1.12**	MV	MV	8.88
6421	12.37	8.99	MV	16.41	MV	20.29	23.98	MV	MV	15.75
68424	**1.78**	MV	MV	**5.36**	MV	MV	MV	MV	MV	11.71
6954	***	***	MV	7.00	MV	19.23	***	MV	MV	6.77
6956	**2.20**	13.01	26.86	29.12	**2.40**	9.11	8.88	**3.32**	9.03	***
7441	8.23	**3.59**	MV	***	MV	7.42	14.03	MV	MV	**3.26**
7731	**0.42**	MV	MV	23.73	MV	MV	MV	MV	MV	14.77
7812	10.83	**2.11**	**5.39**	7.97	***	***	7.72	**2.91**	***	***
7843	23.35	18.90	26.16	***	9.23	**1.40**	**4.96**	10.50	***	**3.76**
8744	***	***	15.37	***	***	**3.72**	***	***	**2.39**	***

The likelihood ratio statistic is $\chi^2(2)$, with a 5 percent critical value of 5.99.

*** means that the relative export price is stationary.

Bold numbers indicate that the null hypothesis is not rejected using the 5 percent critical value.

Table 9: F-tests of short run monetary neutrality, p-values

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	MV	0.127	MV	MV	MV	MV	MV	MV	MV	MV
33202	0.071	**0.003**	0.913	0.431	**0.028**	0.324	**0.006**	0.610	0.734	0.117
3811	0.106	0.929	0.514	**0.011**	**0.014**	0.208	**0.002**	0.279	0.509	0.326
38291	0.160	0.218	MV	**0.024**	MV	0.275	**0.002**	MV	MV	0.477
383200	0.173	0.075	0.959	0.192	0.150	0.412	0.566	0.057	0.309	0.626
3833	**0.017**	0.131	**0.001**	**0.001**	0.151	**0.011**	**0.029**	0.753	0.746	0.063
38431	0.105	**0.003**	0.804	0.257	0.087	**0.012**	0.108	0.180	**0.033**	0.787
6411	0.119	**0.008**	MV	**0.005**	MV	**0.049**	**0.005**	MV	MV	0.376
6414	**0.019**	0.148	MV	0.459	MV	0.118	0.178	MV	MV	0.405
6421	0.931	**0.030**	MV	**0.007**	MV	0.262	**0.001**	MV	MV	0.183
68424	**0.010**	MV	MV	**0.010**	MV	MV	MV	MV	MV	0.089
6954	0.115	0.250	MV	**0.013**	MV	0.109	MV	MV	MV	0.661
6956	0.222	0.991	0.053	0.054	**0.006**	**0.039**	**0.001**	0.119	0.647	0.624
7731	**0.006**	MV	MV	0.062	MV	MV	MV	MV	MV	0.316
7441	0.502	**0.001**	MV	0.072	MV	**0.001**	**0.045**	MV	MV	0.156
7812	**0.026**	**0.037**	0.289	0.119	0.187	0.461	**0.033**	0.192	0.273	0.747
7843	0.664	0.162	0.213	0.434	0.168	0.748	0.297	0.756	0.810	0.642
8744	0.360	0.782	**0.041**	0.140	0.474	0.178	**0.014**	0.672	**0.003**	0.385

Bold numbers indicate that the null hypothesis is rejected using the 5 percent critical values.

Table 10:

Likelihood ratio statistics for the null hypothesis that the unemployment rates do not enter into the cointegration relation

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	MV	**2.05**	MV	MV	MV	MV	MV	MV	MV	MV
33202	6.27	9.54	9.65	7.43	**4.22**	12.53	**0.03**	**4.91**	**3.24**	**0.03**
3811	***	**5.13**	7.25	6.33	**3.97**	**0.92**	9.12	**4.20**	8.77	**4.78**
38291	11.42	***	MV	6.32	MV	***	8.92	MV	MV	**2.44**
38320	7.62	16.19	**0.02**	19.42	7.85	***	**3.81**	**3.91**	11.36	**3.64**
3833	***	***	**4.45**	14.38	**2.97**	**2.26**	**6.11**	**0.38**	11.86	***
38431	30.69	11.95	11.72	**4.04**	***	15.76	7.70	7.84	***	18.09
6411	**2.03**	11.11	MV	***	MV	**2.47**	10.31	MV	MV	***
6414	**5.28**	***	MV	**2.60**	MV	***	11.99	MV	MV	10.63
6421	**2.43**	**5.01**	MV	**0.66**	MV	21.68	7.33	MV	MV	6.21
68424	**1.59**	MV	MV	***	MV	MV	MV	MV	MV	**4.59**
6954	***	***	MV	**2.97**	MV	33.20	***	MV	MV	**1.93**
6956	**4.10**	7.72	15.66	13.60	**0.71**	8.04	6.78	6.16	9.82	***
7441	7.27	**0.08**	MV	***	MV	15.81	**1.98**	MV	MV	**4.38**
7731	7.74	MV	MV	**1.83**	MV	MV	MV	MV	MV	8.85
7812	11.35		**5.37**	**3.64**	***	***	**4.75**	**2.60**	***	***
7843	11.25	17.00	9.75	***	**5.39**	16.05	9.71	**1.55**	***	8.23
8744	***	***	13.16	***	***	13.96	***	***	10.38	***

Bold numbers indicate that the null hypothesis is not rejected using the 5 percent critical value.

Table 11:

F-tests of whether changes in the unemployment rates are significant in the short run,

p-values

	GE-UK	GE-US	GE-JA	GE-FR	US-JA	US-UK	US-FR	JA-UK	JA-FR	UK-FR
11248	MV	0.255	MV	MV	MV	MV	MV	MV	MV	MV
33202	**0.016**	**0.000**	0.814	0.288	0.203	**0.028**	0.188	0.358	0.069	**0.020**
3811	**0.049**	0.356	0.066	0.200	0.070	0.957	0.404	**0.022**	0.299	0.271
38291	0.231	0.627	MV	0.215	MV	0.804	0.091	MV	MV	0.841
383200	**0.038**	0.216	0.441	0.193	0.819	0.889	**0.037**	0.054	**0.019**	0.553
3833	0.945	**0.012**	**0.020**	0.928	0.374	0.324	**0.024**	**0.005**	**0.005**	0.295
38431	0.073	0.283	0.133	0.288	0.055	**0.039**	0.537	0.055	0.494	0.904
6411	**0.046**	**0.001**	MV	0.171	MV	**0.049**	0.378	MV	MV	0.436
6414	0.404	**0.030**	MV	**0.001**	MV	**0.024**	**0.018**	MV	MV	0.342
6421	**0.025**	0.301	MV	0.397	MV	0.359	0.084	MV	MV	0.201
68424	0.445	MV	MV	0.385	MV	MV	MV	MV	MV	0.272
6954	0.560	0.186	MV	0.591	MV	**0.047**	0.100	MV	MV	0.270
6956	0.690	0.327	**0.000**	0.100	**0.030**	0.089	0.664	0.119	0.361	0.340
7731	0.730	MV	MV	0.266	MV	MV	MV	MV	MV	0.126
7441	0.430	0.303	MV	0.963	MV	0.814	0.482	MV	MV	0.487
7812	0.382	0.508	**0.032**	0.655	0.196	0.463	0.425	0.064	0.626	0.357
7843	**0.032**	0.073	**0.001**	0.951	**0.009**	0.111	0.083	0.084	0.370	0.054
8744	0.345	0.531	0.067	0.346	0.212	0.193	0.287	0.776	**0.045**	0.547

Bold numbers indicate that the null hypothesis is rejected using the 5 percent critical value.

A LATENT FACTOR MODEL OF EUROPEAN EXCHANGE RATE RISK PREMIA

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Abstract

The floating of a number of European currencies in 1992-93 created a new body of data on risk premia on floating exchange rates. In this paper, excess returns to investments in SEK, NOK, FIM, GBP, ITL and EPT against the DEM are investigated. We model the risk premia as functions of time varying second moments. First, univariate GARCH-M models are estimated for each currency. Excess returns are higher in times of higher conditional variance for five of the six currencies investigated. Then a latent factor GARCH model that takes common effects in the different currency markets into account is applied. We use a Kalman filter to identify the unobservable risk factors. Excess returns seem to be related to the factors. However, the factor risk does not appear to be priced.

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1. Introduction

Uncovered interest parity (UIP) remains a key assumption in international macroeconomic modelling in spite of a massive body of empirical research rejecting such a relation between nominal interest rates and exchange rates. If UIP does not hold, there must be either systematic forecast errors or risk premia (or both) in foreign currency markets. Lewis (1995), McCallum (1994) and Engel (1996) provide surveys of this literature. While UIP is almost always rejected in empirical tests, specific alternative models of how the risk premium is determined have not found much empirical support either.

Genuine risk premia reward the investor for taking on (non-diversifiable) risk. According to standard asset pricing models, the risk premium is determined by the variance of asset returns, the covariance with the market portfolio or the covariance with the marginal utility of consumption depending on what asset pricing model is used. In general equilibrium models like Lucas (1982), the source of risk is the covariance between monetary shocks and output shocks.

Empirical studies using consumption data to test Euler equations have had difficulties explaining the behaviour of exchange rate risk premia. A few examples are Hodrick (1989), Kaminsky and Peruga (1990) and Backus, Gregory and Telmer (1993). However, consumption does not vary enough to explain the variation in ex post returns unless consumers are implausibly risk averse. Macklem (1991) and Bekaert (1996) simulate versions of the Lucas (1982) general equilibrium model but are unable to mimic the behaviour of observed foreign exchange risk premia. In short, attempts to explain risk premia using data on the observable variables suggested by theoretical models have generally not been successful.

In the following, risk premia on investment in SEK, NOK, FIM, GBP, ITL and EPT against the DEM are modelled as functions of time varying second moments. First, we estimate univariate GARCH-M models for each currency in order to investigate whether the conditional variances are related to expected excess returns. The univariate models indicate that a multivariate specification would be appropriate since the innovations to the conditional variances of the different currencies are correlated. Multivariate GARCH-models are cumbersome since the number of parameters to be estimated is large. It is necessary to impose some structure on the covariances of the assets.

We use a latent factor GARCH-model. In this framework, risk premia on the different assets covary because they are driven by common risk factors. However, the factors are not directly observable. We use a Kalman filter to extract the factors from the observable excess returns. The idea is that the sources of risk in the economy are not directly observable but that risk premia are driven by movements in a limited number of risk factors. The modest questions that can be answered within this framework are whether common factors with time varying variances can be found in excess ex post returns and whether the excess ex post returns are higher when these conditional variances are high. Hence, the existence of a specific type of risk premium can be confirmed or rejected but the sources of the risk are not identified in terms of observable variables. We do not attempt to explain the negative correlation between interest rate differentials and exchange rate depreciation typically found in time series data.

In methodology, the paper is similar to Diebold and Nerlove (1989), who study the behaviour of nominal exchange rates using first a univariate GARCH model and then a latent factor model, applying a Kalman filter to identify the unobservable factor. However, they are not concerned with risk premia but rather in providing a good description of multivariate exchange rate movements. Among the numerous attempts to model foreign exchange risk premia as functions of time varying second moments, this paper is perhaps most related to the multivariate GARCH-M model of Baillie and Bollerslev (1990).

The paper is organised as follows. In the next section, the data are described and simple tests of UIP are performed. In Section 3, univariate GARCH-M models are estimated. Section 4 discusses a more general theoretical model and derives the risk premia as functions of the conditional variances of the risk factors. In Section 5, the multivariate model is estimated using a Kalman filter to identify the unobservable factors. Section 6 concludes.

2. Data

Since the countries in question have only had floating exchange rates for four years, we choose high frequency data to get a sufficient number of observations. Unless one finds a way to distinguish between term premia and exchange rate risk premia, it is necessary to use interest rates with non-stochastic returns, which rules out for instance holding period yields to government bonds. Data on overnight interest rates and daily exchange rates for Sweden, Norway, Finland, Spain, Italy and United Kingdom from January 4 1993 to April 9 1996 are obtained from the Swedish Riksbank. This gives us a sample of 800 observations. The ex post return to an investment in a specific currency over the return to an investment in DEM is defined as $r_t^* - r_t + s_{t+1} - s_t$ or the foreign interest rate minus the German interest rate from t to $t+1$ plus the log of the exchange rate (the price of foreign currency in terms of DEM) at $t+1$ minus the log of the exchange rate at t.

Figure 1 shows ex post excess returns on SEK and ITL compared to DEM for the period October 1 1994 to June 30 1995, annualised yields. It can be seen that they are positive on average (0.05 means five percentage points). There also seem to be volatility clusters in the sense that high volatility is followed by high volatility and low volatility is followed by low volatility. The variances of the two series appear to be positively correlated. This suggests that a multivariate GARCH model may be appropriate.

Figure 1: Ex post returns on SEK and ITL

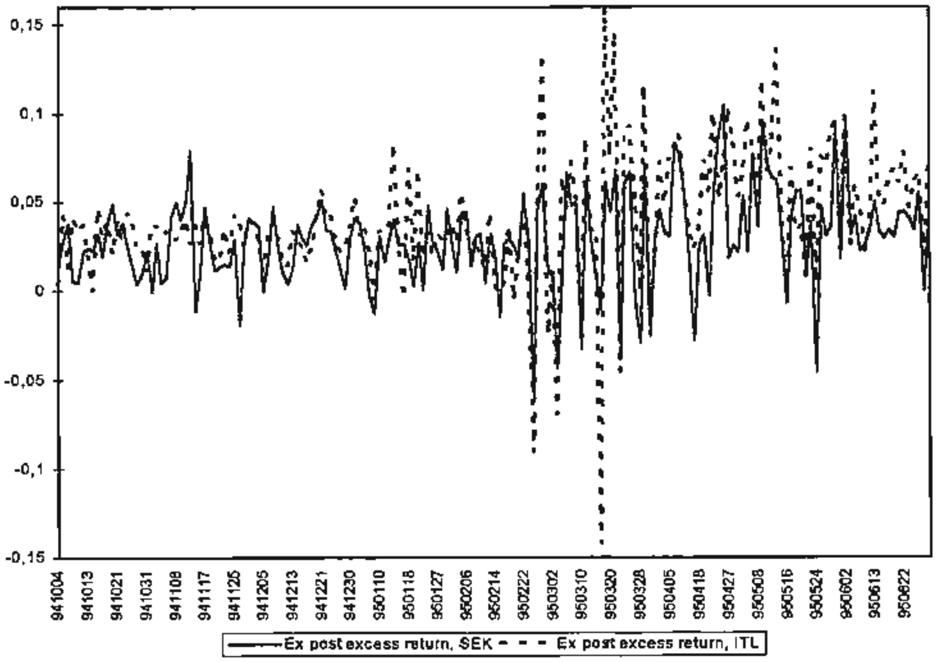


Table 1 shows some descriptive statistics on the ex post returns. The means are not significantly different from zero. There is no evidence of autocorrelation as indicated by the heteroskedasticity-consistent Box-Ljung statistics from Milhoej (1985) in the last column. As evident from the excess kurtosis and skewness measures, the excess returns are far from normally distributed.

Table 1: Statistics on ex post excess returns

	Mean	Variance	Excess Kurtosis	Skewness	Box-Ljung
Sweden	$4.347*10^{-5}$	$4.087*10^{-5}$	1.223	-0.286	10.267
	(0.75)		(0.00)	(0.00)	(0.42)
Finland	$6.553*10^{-5}$	$1.796*10^{-5}$	2.236	0.213	10.691
	(0.66)		(0.00)	(0.00)	(0.38)
Norway	$-1.042*10^{-5}$	$0.298*10^{-5}$	2.654	-0.308	16.134
	(0.86)		(0.00)	(0.00)	(0.09)
UK	$-13.35*10^{-5}$	$4.087*10^{-5}$	2.807	-0.298	13.155
	(0.85)		(0.00)	(0.00)	(0.22)
Italy	$5.221*10^{-5}$	$4.167*10^{-5}$	9.546	-1.096	8.542
	(0.819)		(0.00)	(0.00)	(0.58)
Spain	$-8.908*10^{-5}$	$1.837*10^{-5}$	10.631	-0.950	5.108
	(0.56)		(0.00)	(0.00)	(0.88)

p-values within parentheses

In addition, we run a standard test for uncovered interest parity in order to find out whether the empirical regularities noted by McCallum (1994) and others apply also to our data set. Uncovered interest parity is usually tested using a regression of the form

$$(1) \qquad s_{t+1} - s_t = \alpha + \beta(r_t - r_t^*) + u_t \ ,$$

where s_t is the spot rate in period t, r_t is the foreign interest rate and r_t^* is the German interest rate. A stylized fact is that the estimate of β is negative and large, usually below -1 and often below -3 while UIP predicts $[\alpha,\beta] = [0,1]$.[1] A negative estimate of β implies that the currencies of high interest rate countries tend to appreciate, not depreciate as predicted by UIP.

Running this regression on our data set results in point estimates of β that are small and negative in five out of six cases, three of which are significantly different from zero. The intercept is significantly positive for Italy, indicating a non-zero average excess return over the sample period. As evident from the third column, the joint hypothesis that the intercept α is zero and the slope β is one is rejected by the F-test in all cases. R^2 is very low, between 0.00 and 0.01.

[1] McCallum's (1994) average point estimate of β over three currencies against the dollar is -4, a result considered typical by Engel (1995).

Table 2: Results from tests of UIP

Country	α*	β*	$F(\alpha=0,\beta=1)$**	R^2	Box-Ljung**
Sweden	0.317 (1.771)	-0.110 (-2.014)	1955.93 (0.00)	0.004	13.22 (0.58)
Norway	0.019 (0.880)	-0.061 (-2.981)	3875.24 (0.00)	0.010	17.09 (0.31)
Finland	-0.021 (-0.290)	-0.021 (-0.403)	386.62 (0.00)	0.000	15.81 (0.39)
Spain	-0.138 (-0.782)	0.052 (1.073)	5241.36 (0.00)	0.003	8.05 (0.92)
Italy	0.407 (2.751)	-0.088 (-2.664)	2795.87 (0.00)	0.004	15.05 (0.45)
United Kingdom	0.054 (0.409)	-0.035 (-0.476)	216.37 (0.00)	0.000	21.03 (0.14)

* t-statistics using Newey-West autocorrelation and heteroscedasticity consistent standard errors within parentheses
** p-values within parentheses

Although the point estimates of β are negative, they are not below -1. Hence, UIP is rejected also for our data set but the tendency for currencies of high interest countries to appreciate is not as pronounced as in other studies.

3. Univariate Volatility and the Risk Premia

As indicated in Figure 1, Deutsche Mark returns to investments in the other currencies are characterised by periods of low and high volatility. Intuitively, the presence of exchange rate risk premia means that the expected return on investments in the currency is higher in times of high volatility of the return. This idea has been formalised e.g. in the risk-return model by Engle, Lilien and Robins (1987). The risk premium in their model is linearly related to the conditional standard deviation of the return.

Time varying conditional variances of financial time series can often be modelled as GARCH-processes, where the conditional variance is a function of the lagged squared residuals and the lagged conditional variances (See Bollerslev, Chou, and Kroner (1992), Bera and Higgins (1993), or Bollerslev, Engle, and Nelson (1994) for a survey of the literature on these types of time series models). The GARCH-M model discussed by Engle, Lilien and Robins (1987) is particularly relevant for models of risk premia, since it allows the conditional variance to affect the mean. In order to investigate whether the return is higher in times of high volatility we estimate univariate GARCH-M models for each country.

The following GARCH(p,q)-M model is estimated using Quasi Maximum Likelihood Estimation,

(2a) $$y_t = b_o + b_h h_t + u_t \quad , \qquad u_t \sim iid\left(0, h_t^2\right) \ ,$$

(2b) $$h_t^2 = a_0 + \sum_{i=1}^{p} a_i u_{t-i}^2 + \sum_{j=1}^{q} c_j h_{t-j}^2 \quad ,$$

where y_t is the observed excess return and h_t^2 is the conditional variance of the excess return. The coefficient b_h captures the effect of the conditional standard deviation on the asset return. We estimated the three specifications we thought most likely to match the daily returns data and the best specification is reported in Table 3 (the three candidates were GARCH(1,1)-M, GARCH(2,1)-M, and ARCH(5)-M).

As evident from Table 3, there are significant ARCH-effects in the variances of all currency returns. The Box-Ljung autocorrelation test statistics (with ten degrees of freedom) look reasonable for the standardised residuals, Q(10), and squared standardised residuals, Q2(10). They also pass the Engel (1982) LM-test for remaining ARCH effects on 1-10 lags (not reported). The Bera-Jarque tests (BJ) indicate non-normality of the standardised residuals in all the regressions. Hence, Bollerslev and Woolridge (1992) standard errors that are robust to non-normality have been used (shown within brackets). With the robust *t*-statistics few of the parameters in Table 3 remain significant.

The mean equation intercepts are not statistically significantly different from zero for any of the currencies and are not reported. The mean parameter b_h is not significantly different from zero for any currency return but it is positive as expected, except for Norway. This result is in line with the paper by Baillie and Bollerslev (1990), who do not find significant effects of the conditional variance on the mean return (using standard *t*-statistics). However, it is interesting to note that we get the highest t-values for those currencies (SEK, ITL, and ESP) that most observers would regard as the most risky investments during the period studied.

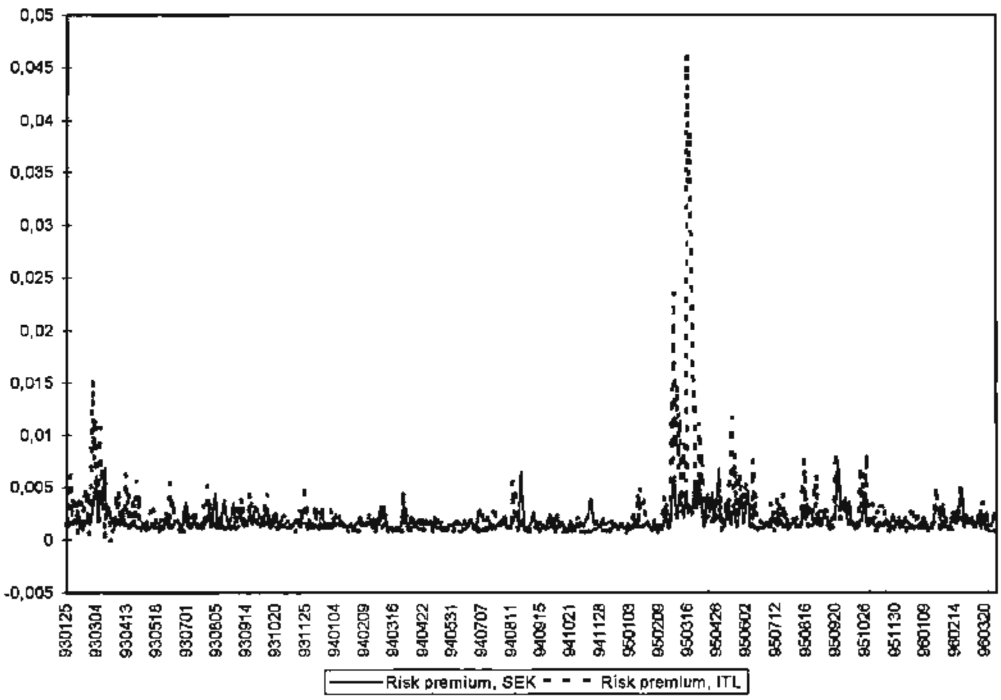
Table 3: Parameter estimates of the GARCH(p,q)-M model in (2)

	Sweden	Finland	Norway	UK	Italy	Spain
b_h	0.333	0.010	-0.094	0.272	0.157	0.150
	(1.147)	(0.53)	(0.70)	(0.87)	(1.72)	(1.38)
	[0.32]	[0.24]	[0.18]	[0.12]	[0.58]	[0.07]
a_0	$2.4*10^{-5}$	$0.6*10^{-5}$	$0.1*10^{-5}$	$9.1*10^{-5}$	$0.8*10^{-5}$	$0.7*10^{-5}$
	(8.98)	(7.48)	(9.24)	(8.93)	(5.96)	(6.95)
	[2.44]	[2.16]	[1.63]	[0.78]	[0.32]	[0.04]
a_1	0.060	0.198	0.270	0.058	0.129	0.432
	(1.85)	(4.14)	(4.09)	(1.25)	(2.43)	(6.17)
	[0.77]	[1.16]	[0.74]	[0.07]	[0.13]	[0.11]
a_2	0.131		0.073	0.060	0.227	
	(2.39)		(1.85)	(1.62)	(4.22)	
	[0.63]		[0.49]	[0.41]	[0.23]	
a_3	0.118		0.151	0.113		
	(2.46)		(3.77)	(2.54)		
	[0.90]		[1.18]	[0.34]		
a_4	0.036		0.079	0.058		
	(1.36)		(2.06)	(2.18)		
	[0.53]		[0.56]	[0.59]		
a_5	0.061		0.092	0.091		
	(1.96)		(2.88)	(2.34)		
	[0.72]		[0.97]	[0.51]		
c_1		0.426			0.485	0.280
		(7.44)			(10.99)	(4.70)
		[1.81]			[0.69]	[0.06]
BJ	45.54	28.77	109.8	392.1	711.0	11781.0
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Q(10)	10.36	17.51	13.02	13.11	12.57	9.72
	(0.41)	(0.06)	(0.22)	(0.22)	(0.25)	(0.47)
Q2(10)	5.18	10.48	13.13	18.40	13.61	15.07
	(0.88)	(0.40)	(0.22)	(0.05)	(0.06)	(0.13)

t-values using conventional standard errors within parentheses.
t-values using Bollerslev and Woolridge (1992) standard errors within brackets.
p-values for specification tests within parentheses.

The risk premia are obtained by multiplying the conditional variances with their effect on the mean b_h. For Sweden the risk premium is one third of one standard deviation of the return. Figure 2 shows the risk premia on SEK and ITL, converted to annual yields. It can be seen that they are quite large in times of high conditional volatility (the number 0.01 denotes one percentage point). They also seem to be correlated between the currencies. Figure 2 is of course based on point estimates that are not statistically significant. Although there is at best weak statistical evidence of risk premia in Table 3, this could be due to us not having measured risk in an approriate way. We have implicitly assumed that risk is non-diversifiable so that only the variance of the return matters.

Figure 2: Risk premia on SEK and ITL



In order to investigate whether there are cross effects between the currencies, we have calculated the correlations between squared standardised residuals (the innovations to the conditional variances) as proposed by Cheung and Ng (1996). Table 4 shows the contemporary correlations.

Table 4: Correlations between squared standardised residuals

	Swe	Fin	Nor	UK	Ita	Esp
Swe	1	0.417	0.400	0.306	0.345	0.066
Fin		1	0.157	0.138	0.189	0.052
Nor			1	0.139	0.299	0.130
UK				1	0.129	0.166
Ita					1	0.092

All correlations are positive and all but two are significant at the 5 percent level (the critical value is 0.087). There are clearly common effects in the variances of the different excess returns. Indeed, if the model above is generalised to take the existence of several risky foreign assets into account the covariances between their returns enter into the optimisation problem as well. We now turn to a more general asset pricing model that takes common effects into account.

4. A More General Model of the Risk Premia

The model above has several weaknesses. Each country is analysed in isolation, whereby cross effects between the currency markets are ignored. Second, more general models would not single out the variance of each asset return as the determinant of expected excess returns. They would rather depend on the covariance of the return with the intertemporal marginal rate of substitution, m_t. (However, the variance of excess returns $\mathrm{var}_t(y)$ may function as a proxy for the covariance $\rho_{ym}\sqrt{\mathrm{var}_t(y)}\sqrt{\mathrm{var}_t(m)}$ since they can be expected to be correlated). In this section, a more general theoretical model is derived. Given specific assumptions about the process followed by the pricing kernel, a convenient expression for expected excess returns can be derived.

Given general equilibrium and no arbitrage, a pricing kernel can be derived. Starting from the pricing kernels of domestic and foreign assets, we have

$$(3) \qquad E_{t-1}[m_t](1+r_{t-1}) = 1 \quad \text{and} \quad E_{t-1}[m_t^*](1+r_{t-1}^*) = 1,$$

where m_t is the domestic pricing kernel, r_t is the domestic nominal interest rate, m_t^* is the foreign pricing kernel and r_t^* is the foreign nominal interest rate. For instance, letting m_t denote the intertemporal marginal rate of substitution, (3) is the first order conditions of the consumption capital asset pricing model. Using the covered interest rate parity condition, we can price the foreign asset in terms of the domestic currency to obtain

$$(4) \qquad E_{t-1}\left[m_t\left(1+r_{t-1}^*\right)\frac{S_t}{S_{t-1}}\right] = E_{t-1}\left[\exp\left(\ln m_t + \ln\left(1+r_{t-1}^*\right) + \ln S_t - \ln S_{t-1}\right)\right] = 1.$$

Using (3), assuming log normality and taking logs, we get

$$(5) \qquad E_{t-1}\left[y_t\right] \equiv E_{t-1}\left[r_{t-1}^* - r_{t-1} + \ln S_t - \ln S_{t-1}\right] = -\operatorname{cov}_{t-1}\left(\ln m_t, y_t\right) - \frac{1}{2}\operatorname{var}_{t-1}\left(y_t\right).$$

The first term is the usual covariance between the marginal utility of consumption and the return on the asset. A risky asset with a positive risk premium has low return in times of high marginal utility (low consumption). The second term stems from Jensen's inequality. We will not put too much weight on this term for several reasons. It is difficult to give it an economically meaningful interpretation. If the home country and the foreign country are reversed, increased variance of the exchange rate will have the opposite effect on the risk premium. A number of papers have studied the size of the Jensen inequality term in this context and found it negligible.[2] In his survey of the literature, Engel (1996) concludes: "In the empirical literature on foreign exchange risk premia, the Jensen inequality term can be ignored because of their small size".

The general theoretical model in (5) has to be combined with specific assumptions about the processes followed by excess returns and the pricing kernel in order to derive a convenient solution for the risk premium. Excess returns are assumed to follow a K-factor linear model:

$$(6) \qquad y_{it} = \mu_{it} + \sum_{k=1}^{K}\beta_{ikt} f_{kt} + v_{it} \ ,$$

where y_{it} is the excess return to currency i, μ_{it} is the conditional mean, f_{kt} is the k:th factor and β_{ikt} is the corresponding factor loadings of currency i. v_{it} is the residuals or

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[2] Backus, Gregory and Telmer (1993), Hodrick (1989), Cumby (1988) and Engel (1984)

idiosyncratic risk, assumed to be white noise with variance σ_i^2.[3] Furthermore, the factors are assumed to be uncorrelated with each other and with the residuals and have zero expected value, i.e. $E_{t-1}[f_{kt}] = 0$.

The K factors represent risks that are common to all the assets, while the residuals represent diversifiable, asset specific risk. For instance, one factor could be productivity disturbances to the German economy that effects all the Deutsche Mark exchange rates, another could be changes in the prospects for a European Monetary Union. From a Lucas (1982) type of model, we would expect a productivity shock and a monetary policy shock in each country. However, the different monetary policies are unlikely to be linearly independent of each other or of the real shocks. Hence, the number of orthogonal risk factors in the data is an empirical question.

While the realisation of a particular type of common shock in period t is unpredictable given the information in period $t-1$, the variance of the shock is not as it follows a GARCH process. As we will see, this is what drives the risk premia or the conditional means μ_{it}.

We assume that the factor loadings β_{ikt} are constants β_{ik} and that the factors are characterised by time varying variances λ_{kt}. An alternative assumption of constant factor variances but time varying factor loadings would lead to the same conditional variance of excess returns, namely

$$(7) \qquad h_{it} = E_{t-1}\left[(y_{it} - \mu_{it})(y_{it} - \mu_{it})\right] = \sum_{k=1}^{K} \beta_{ik}\beta_{ik}\lambda_{kt} + \sigma_i^2 \ .$$

The pricing kernel is assumed to obey a K-factor linear process:

$$(8) \qquad \ln m_t = \nu + \sum_{k=1}^{K} \tau_k f_{kt} + w_t \ .$$

The following moment conditions are assumed to hold: $E_{t-1}[w_t] = 0$, $E_{t-1}[w_t v_t] = 0$, $E_{t-1}\left[w_t \middle| f_{1t,\dots,} f_{Kt}\right] = 0$ and $E_{t-1}\left[w_t^2\right] = \sigma_w^2$.

From (5), (6) and (8), ignoring the Jensen inequality term, the expected returns or conditional means can be derived as functions of the conditional variances of the

[3] It is not necessary to assume constant idiosyncratic risks. King, Sentana and Wadhwani (1994) apply a GARCH model also for asset specific risks and are thereby able to test whether the price of this risk is non-zero as expected from the theory.

factors λ_{kt}, the factor loadings β_{ik} and τ_k that will be interpreted as the price of k-factor risk.

(9) $$\mu_{it} = \sum_{k=1}^{K} \beta_{ik} \tau_k \lambda_{kt} .$$

5. Estimates of the Latent Factor GARCH Model

The idea behind the latent factor model is that excess returns are driven by movements in a limited number of risk factors of the economy. The main problem when estimating such models is that these factors are unobservable. There is a variety of methods for identifying the factors in (6). In Engel, Ng and Rotschild (1990), Engle and Ng (1993) and Sellin (1996), the factors are approximated by factor representing portfolios of the included assets. Korajczyk and Viallet (1992) use a principal components method that is most useful when the number of assets in the model is very large. Here, as in Diebold and Nerlove (1989), a Kalman filter is used to capture the movements of the unobservable factor(s) from the observable ex post returns of the assets. This technique is also used in King, Sentana and Wadhani (1994), where in addition some of the factors are connected to macro variables and are thus "partially observable". Such a connection will not be possible to make in the present paper since we use daily data.

Since the number of parameters to be estimated increases quickly with the number of factors included in the model, the number of factors has to be kept small. In several papers, only one factor is assumed to exist and the potential presence of additional factors is not investigated.[4] Korajczyk and Viallet choose to use five factors but find that their main results are robust to changes of the number. A likelihood ratio test can be used to determine whether an additional factor is needed or not. Hence, the model is first estimated with one factor. Additional factors are then included as long as they are significant.

Using the Kalman filter, the K unobservable factors and their conditional variances can be extracted from the observable excess returns. The processes followed by the observable and unobservable variables have to be specified, including the way in which the observable variables are related to the unobservable. The Kalman algorithm then constructs an optimal guess of the unobservable variables in each period, using the

[4] See for instance Diebold and Nerlove (1989).

observable variables and the parameters of the dynamic specifications. These parameters and the constructed time series of unobservable variables in turn imply values of the observable variables that can be compared to the actual values. Hence, each set of parameters is associated with a sequence of prediction errors. Maximum likelihood estimation is used to pick the set of parameters and the corresponding sequence of unobservable variables that minimise the prediction errors is estimated.

The state-space model of the Kalman filter is particularly simple in this context since the unobservable risk factors follow a process without serial correlation (by assumption). This reduces the state equation to white noise. Combining (6) and (9), the observation equation that links the factors to the observable returns can be written as

$$(10) \qquad y_{it} = \sum_{k=1}^{K} \beta_{ik}\lambda_{kt}\tau_k + \sum_{k=1}^{K} \beta_{ik} f_{kt} + v_{it}\ , \quad i = 1,\ldots,N .$$

where the y_{it} are the observed returns. The first term on the right hand side of (10) is the risk premium. It consists of the conditional variances of the factors λ_{kt}, multiplied by the constant prices of k-factor risk τ_k and the sensitivities of asset i to the factors β_{ik}. The second term is the same factor sensitivities β_{ik} multiplied by the innovations to the factors. The expected value of these innovations is zero. The factors are assumed to be uncorrelated with each other. Finally, v_{it} are the asset specific innovations, assumed to be uncorrelated with each other, over time and to have zero expected value.

The covariance matrix of excess returns H_t is $\beta\Lambda_t\beta^T + \Omega$, where Λ_t is the *KxK* diagonal matrix of conditional variances of the factors, β is the *NxK* matrix of factor loadings and Ω is the *NxN* time invariant variance-covariance matrix of the asset specific shocks. It is assumed to be diagonal, i.e. all correlation between asset returns stems from their joint dependence on the unobservable factors. An informal indication of whether the common effects have been removed from asset returns can be obtained from the correlations of the asset specific disturbances given the estimated factors. They will be zero if the factors as modelled actually capture the common effects.

The conditional variances of the factors are assumed to follow GARCH(p,q) processes,

$$(11) \qquad \lambda_{kt} = \Psi_{k0} + \sum_{i=1}^{p} \Psi_{ki} E\left[f_{k,t-i}^2 \middle| I_{t-i}\right] + \sum_{i=1}^{q} \Phi_{ki}\lambda_{k,t-i}\ , \quad k = 1,\ldots K$$

The conditional variances of the different factors are assumed to be independent of each other. The unconditional variance of factor k is given by

(12) $$\mathrm{V}[\mathrm{f}_{\mathrm{kt}}] = \frac{\Psi_{\mathrm{k0}}}{1-\sum_{\mathrm{i=1}}^{\mathrm{p}}\Psi_{\mathrm{ki}}-\sum_{\mathrm{j=1}}^{\mathrm{q}}\Phi_{\mathrm{kj}}} .$$

Since the scaling of the factors is irrelevant, we can normalise the unconditional variance to one by setting $\Psi_{k0} = 1-\sum_{i=1}^{p}\Psi_{ki}-\sum_{j=1}^{q}\Phi_{kj}$. Following King, Sentana and Wadhani (1994), the updating recursions for the Kalman filter in this case can be derived as

(13) $$f_{t|t} = E[f_t|I_t] = \Lambda_t\beta^T\left(\beta\Lambda_t\beta^T+\Omega\right)^{-1}\left(y_t-\beta\Lambda_t\tau\right)$$

for the factors and

(14) $$\Lambda_{t|t} = \mathrm{V}[\mathrm{f}_t|\mathrm{I}_t] = \Lambda_t - \Lambda_t\beta^{\mathrm{T}}\left(\beta\Lambda_t\beta^{\mathrm{T}}+\Omega\right)^{-1}\beta\Lambda_t$$

for the conditional variances of the factors. When combining the updating equations with (11), we use the fact that $\mathrm{E}[\mathrm{f}_{\mathrm{kt}}^2|\mathrm{I}_t] = \mathrm{E}^2[\mathrm{f}_{\mathrm{kt}}|\mathrm{I}_t]+\mathrm{V}[\mathrm{f}_{\mathrm{kt}}|\mathrm{I}_t]$.

The likelihood function to be maximised is

(15) $$\ln L = C-\frac{1}{2}\sum_{t=1}^{T}\left(\ln|H_t|+\left(y_t-\beta\Lambda_t\tau\right)^T H^{-1}\left(y_t-\beta\Lambda_t\tau\right)\right) .$$

The Kalman filter identifies the factors and their conditional variances. The GARCH-parameters Ψ_i and Φ_i, the variances of the asset specific shocks Ω_{ii}, the elements of the β-matrix of factor loadings or the sensitivities of the assets to movements in the factor and τ, the prices of factor risks, are estimated by maximum likelihood. The parameter estimates for the one-factor model are shown in Table 5. A GARCH(1,1) turned out to be sufficient as higher order terms were insignificant and did not result in significant increases of the likelihood function.

The factor loadings that relate the excess returns to the conditional variance of the factor are all positive as expected and highly significant. The Italian lira is most sensitive to movements in the factor and the Norwegian krone the least. The factor loadings are small in absolute value but this does not necessarily mean that the risk premia are small, since they depend on the product of the factor loadings, the price of factor risk and the conditional variance of the factor. The unconditional variance of the factor is normalised to 1.00, which makes it large compared to daily returns. Hence, we expect factor loadings to be small. The GARCH-effects in the factor variance are also highly significant. The only insignificant parameter is τ that can be interpreted as the price of factor risk. Hence, although there is evidence of a common factor it does not appear to be priced.

The assumption that Ω is diagonal or that the filtered asset returns are only correlated through their mutual dependence on the latent factors is used to identify the parameters and cannot be relaxed. However, the correlations between the asset specific residuals can be calculated as a check of whether the factor seems to capture common movements in the asset returns. It turns out that the residuals from the one factor model are correlated, which indicates that a second factor may be needed. As obvious from the last two rows in Table 5, there is autocorrelation in the residuals and the squared residuals. The residuals are also highly non-normal, so Bollerslev and Wooldridge (1992) robust standard errors reduce the t-statistics substantially.

Since the first factor did not remove all correlation between the asset specific shocks, we also estimate the two-factor model. The results are shown in Table 6. With two factors, half of the factor loadings of the first factor are insignificant and several of them are much smaller than the corresponding parameters in the one factor model. The factor loadings of the second factor are significant and of the same magnitude as in the one factor model. There are more pronounced differences between the factor loadings of the currencies here. For instance, returns to investments in SEK and ITL are not significantly affected by movements in the first factor. The results are similar to the one factor model in that the GARCH-parameters are significant but the prices of factor risk are not.

Table 5: Estimates from the one factor model

GARCH-parameters	Ψ_1		Φ_1			
	0.135		0.837			
	(4.63)		(22.97)			
	[0.22]		[0.91]			
	Sweden	Finland	Norway	UK	Italy	Spain
Factor loadings	β_1	β_2	β_3	β_4	β_5	β_6
	$4.51*10^{-3}$	$2.57*10^{-3}$	$1.43*10^{-3}$	$5.76*10^{-3}$	$5.89*10^{-3}$	$2.71*10^{-3}$
	(8.14)	(7.79)	(8.26)	(7.38)	(8.22)	(8.10)
	[0.45]	[0.44]	[0.45]	[0.39]	[0.40]	[0.45]
Standard deviations of country specific shocks	$\Omega_{11}^{1/2}$	$\Omega_{22}^{1/2}$	$\Omega_{33}^{1/2}$	$\Omega_{44}^{1/2}$	$\Omega_{55}^{1/2}$	$\Omega_{66}^{1/2}$
	$4.33*10^{-3}$	$3.37*10^{-3}$	$0.94*10^{-3}$	$10.48*10^{-3}$	$4.13*10^{-3}$	$3.34*10^{-3}$
	(41.36)	(39.07)	(27.42)	(50.99)	(62.91)	(42.62)
	[1.89]	[1.56]	[0.85]	[2.24]	[1.34]	[0.95]
Price of factor risk	τ					
	0.010					
	(0.28)					
	[0.01]					
Function value	19321					
Q(10)	17.01	25.66	29.58	21.73	30.53	12.49
	(0.07)	(0.004)	(0.001)	(0.02)	(0.001)	(0.25)
Q2(10)	177.7	212.1	103.3	92.8	31.9	44.7
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Q multivariate	625.5					
	(0.00)					
Bera-Jarque	891.3	462.7	468.1	242.1	5896.7	7637.4
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
B-J multivariate	15598.2					
	(0.00)					

t-values using conventional standard errors within parentheses.
t-values using Bollerslev and Woolridge (1992) standard errors within brackets.
p-values for specification tests within parentheses.

Table 6: Estimates from the two-factor model

GARCH-parameters	Ψ_{11}	Ψ_{21}	Φ_{11}	Φ_{21}
	0.069	0.929	0.085	0.866
	(2.93)	(39.63)	(3.56)	(22.77)
	[0.08]	[1.07]	[0.23]	[1.38]

	Sweden	Finland	Norway	UK	Italy	Spain
Factor loadings	β_{11}	β_{21}	β_{31}	β_{41}	β_{51}	β_{61}
	$-0.200*10^{-3}$	$0.967*10^{-3}$	$0.932*10^{-3}$	$2.694*10^{-3}$	$0.285*10^{-3}$	$6.481*10^{-3}$
	(-0.29)	(1.69)	(2.53)	(2.11)	(0.72)	(3.93)
	[-0.01]	[0.04]	[0.06]	[0.05]	[0.03]	[0.11]
	β_{12}	β_{22}	β_{32}	β_{42}	β_{52}	β_{62}
	$6.094*10^{-3}$	$2.863*10^{-3}$	$1.281*10^{-3}$	$4.743*10^{-3}$	$4.370*10^{-4}$	$1.858*10^{-3}$
	(12.01)	(11.59)	(9.61)	(8.25)	(1.74)	(6.39)
	[0.38]	[0.52]	[0.29]	[0.28]	[0.07]	[0.17]
Standard deviation of country specific shocks	$\Omega_{11}^{1/2}$	$\Omega_{22}^{1/2}$	$\Omega_{33}^{1/2}$	$\Omega_{44}^{1/2}$	$\Omega_{55}^{1/2}$	$\Omega_{66}^{1/2}$
	$2.503*10^{-3}$	$3.576*10^{-3}$	$1.750*10^{-3}$	$1.143*10^{-2}$	$6.826*10^{-3}$	$1.275*10^{-3}$
	(3.92)	(39.2)	(31.38)	(53.21)	(43.96)	(2.21)
	[0.09]	[1.38]	[1.06]	[2.18]	[1.86]	[0.06]
Price of factor risk	τ_1	τ_2				
	0.010	0.007				
	(0.19)	(0.20)				
	[0.00]	[0.00]				
Function value	20110					
Q(10)	19.96	4.57	8.92	12.93	33.92	33.93
	(0.068)	(0.971)	(0.071)	(0.374)	(0.001)	(0.001)
Q multivariate	471.1					
	(0.09)					
Bera-Jarque	13.071	61.73	105.02	192.57	214.2	314.3
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
B-J multivariate	901.4					
	(0.00)					

t-values using conventional standard errors within parentheses.
t-values using Bollerslev and Woolridge (1992) standard errors within brackets.
p-values for specification tests within parentheses.

The model with two factors did not improve the fit of the model as much as we might have hoped. Idiosyncratic risk seem to be very important and the Box-Ljung test of the squared residuals as well as Engle's test for ARCH (not reported) indicates that there are still ARCH effects. Hence, we next try a one factor model with time-varying idiosyncratic risk. More specifically we let the conditional variances of the country specific shocks vary over time as a GARCH(1,1) process,

$$\Omega_{it} = \phi_{i0} + \phi_{i1} E\left[v_{i,t-1}^2 \middle| I_{t-1}\right] + \phi_{i2}\Omega_{i,t-1} \quad (16)$$

The resulting parameter estimates are reported in Table 8. Our attempts to model the conditional variances for Norway and the U.K. were not successful, so we let these be constant over time. However, the GARCH modelling of the other four countries' idiosyncratic risk proved successful and results in a much better specified model. The Box-Ljung statistics for the levels look fine except for the U.K., while the Box-Ljung for the squares are O.K. for the series which we managed to model with time-varying idiosyncratic risk. However, the residuals are still far from normally distributed and robust t-values are reported within square brackets. The precision in the estimates of the factor loadings are better than in the previous models but the factor risk is still not priced.

Table 8: Estimates from the one factor model with time-varying idiosyncratic risk

GARCH-parameters	Ψ_1		Φ_1			
	0.149		0.840			
	(7.12)		(34.92)			
	Sweden	Finland	Norway	UK	Italy	Spain
Factor loadings	β_1	β_2	β_3	β_4	β_5	β_6
	$5.92*10^{-3}$	$3.33*10^{-3}$	$1.91*10^{-3}$	$7.94*10^{-3}$	$6.13*10^{-3}$	$3.23.71*10^{-3}$
	(21.50)	(16.73)	(26.63)	(18.97)	(21.10)	(20.89)
Conditional variance	ϕ_{10}	ϕ_{20}	ϕ_{30}	ϕ_{40}	ϕ_{50}	ϕ_{60}
of country specific	$2.1*10^{-6}$	$3.0*10^{-6}$	$0.9*10^{-6}$	$1.1*10^{-4}$	$0.7*10^{-6}$	$0.1*10^{-6}$
shocks	(3.54)	(5.73)	(14.08)	(20.31)	(3.36)	(2.91)
	ϕ_{11}	ϕ_{21}			ϕ_{51}	ϕ_{61}
	0.172	0.282			0.120	0.121
	(5.57)	(8.36)			(3.86)	(6.83)
	ϕ_{12}	ϕ_{22}			ϕ_{52}	ϕ_{62}
	0.750	0.478			0.861	0.900
	(16.30)	(11.55)			(28.45)	(78.95)
Price of factor risk	τ					
	0.029					
	(0.56)					
Function value	19585					
Q(10)	14.83	16.69	15.04	22.40	7.50	6.83
	(0.14)	(0.08)	(0.13)	(0.01)	(0.68)	(0.74)
Q2(10)	5.51	12.03	32.98	86.23	0.63	2.65
	(0.85)	(0.28)	(0.00)	(0.00)	(1.00)	(0.99)
Bera-Jarque	116.6	48.4	216.3	236.5	813.8	3590.2
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

t-values using conventional standard errors within parentheses.
t-values using Bollerslev and Woolridge (1992) standard errors within brackets.
p-values for specification tests within parentheses.

7. Conclusions

Ex post returns to investments in SEK, NOK, FIM, GBP, ITL and EPT against the DEM from 1993:01:01 to 1996:04:09 display a number of interesting features. Uncovered interest parity fares as badly on this data set as in most other studies. There are significant positive risk premia on average, while the tendency of currencies of high interest rate countries to appreciate instead of depreciate as predicted by the theory is less pronounced than generally found.

Our attempts to model the risk premia as functions of time varying second moments met with limited success. We first estimate GARCH-M models for each currency in isolation. The mean-parameters that relate the conditional variances to the asset returns are positive but not significant in five out of six cases. With robust standard errors it is evident that the parameters of these regressions are imprecisely estimated, because of severe non-normality in the residuals.

Not unexpectedly, the conditional variances of the six assets are correlated. Hence, a multivariate framework may be more efficient and provide additional information about the interactions between different currency markets. The latent factor model allows a multivariate specification while keeping the number of parameters to be estimated reasonably low. In this model, the risk premia are assumed to be driven by movements in one or more common sources of risk. However, the risk factor are not directly observable. We estimate the unobservable factors from the observable asset returns using a Kalman filter.

Both the one factor model and the two factor model fair reasonably well, especially after allowing for time-varying idiosyncratic risk. The factor loadings are significant using conventional standard errors, indicating that there are common sources of risk. However the risk does not seem to be priced in the market.

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