Aspects of Trade Credit

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Aspects of Trade Credit

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Preface

When I started working on this thesis during the fall of 1979 my intention was to study trade credit in a macroeconomic context. To get ready to tackle the macroeconomic problems in this field I decided to start from the micro level in order to learn about how and why single firms use trade credit. Shortly after the work had begun I became more and more engrossed in microeconomic trade credit issues. The chapters in this book show that the microeconomics of trade credit kept my mind occupied to such an extent that I never reached the macro level. My from micro to macro strategy became a pure microeconomic approach and Aspects of Trade Credit shows my attempts to plough new furrows in this field.

During my years at the Economics Department at the Stockholm School of Economics I have accumulated debts to many friends and colleagues. I wish to thank all those who have helped me in my work with this book. My thesis adviser, Johan Myhrman, sowed the seed to this thesis and at various formal and informal seminars I have benefitted from comments by him but also by Lars Bergman, Peter Englund, Göran Eriksson, Anders Hagelberg, Lars Hörngren, Jonas Lind, Ragnar Lindgren, Karl-Göran Mäler, Mats Persson, Torsten Persson, Staffan Viotti, Anders Vredin and Pehr Wissén. Whenever confronted with probabilistic problems Håkan Lyckeborg patiently helped me to sort out my thoughts. Also, I wish to thank Onerva Lahdenperä, Kerstin Niklasson, Monica Peijne and Marianne Widing who expertly raised my typing to readability. The main burden fell on Onerva Lahdenperä and Monica Peijne.

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To Deborah, my wife, and to our daughter Elin, who silently closed the door, I want to express my gratitude for understanding and enduring.

Stockholm in April, 1984

Stefan Ingves

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1. Trade Credit – The Concept and its Dimensions

1.1 INTRODUCTION

The objective of this study is to present various aspects of how and why firms use trade credit. This is done with the help of several different models that make it possible to concentrate on one, or a few, trade credit aspects at a time. Within the different model worlds it is possible to give plausible explanations of some of the trade credit "stylized facts" presented in this chapter, and it is possible to identify different variables that are important determinants of how and why firms use trade credit. In turn, these variables are of interest if one wants to carry out econometric studies of trade credit on the micro level. However, such studies lie beyond the scope of the work presented here.

In Chapters 2-7 the analysis is carried out with the help of the tools and methods provided by ordinary microeconomic theory. This means that I present a bridge between two of the main strands of the trade credit literature. I go more into detail than in macroeconomic studies, but at the same time the analysis is less detailed, and more abstract than in the descriptive cash management literature. Thus, readers familiar with either, or both, of the two strands will hopefully find some new and interesting aspects of the use of trade credit, while readers unfamiliar with the subject get a more varied picture of trade credit than in many other studies.

Mostly I discuss various microeconomic aspects of the use of trade credit, but in order to give some background to the contents in the chapters that follow it is initially of interest to introduce the trade credit concept and some "stylized facts" in a more general setting. In this introductory chapter I define what a trade credit is, I describe what the closest substitutes are, and I present a number of different trade credit dimensions, all of which are closely connected with either the financial or the goods market. In highly aggregated models trade credit can be seen as a factor which affects the velocity of money. Analogously with the theory of the demand for money this leads to the conclusion that trade credit can be seen as a lubricant that makes the transaction system run smoother or a financial variable in a portfolio context. When I move from the macro to the micro level I discuss the empirical trade credit picture in the Swedish corporate sector, and the role of trade credit in the liquidity portfolio of Swedish firms. I also comment on what empirical studies tell about the use of trade credit. Conclusions drawn from the Swedish data and the empirical studies are summarized in a number of "stylized facts". I also ask within what model framework it is possible to study financial and goods market aspects of trade credit, and I reach the conclusion that it is transaction costs and the transaction technology that make it possible to distinguish trade credit from other assets and liabilities. Finally at the end of the chapter I give a brief outline of the contents in Chapters 2-7.

1.2 THE MACRO PERSPECTIVE

1.2.1 *A macro approach*

Trade credit is usually an integrated part of ordinary business transactions among non-financial firms, and it is often an important part of both current assets and liabilities of most firms. When I in this section talk about trade credit it is simplest to let the term represent those items that are included under accounts payable and accounts receivable on the balance sheet of non-financial firms. This is also the common way of defining trade credit in empirical studies.

In a macro economic context trade credit has traditionally been seen as a variable which affects the velocity of circulation of money. The importance of trade credit in this respect has long been well understood. Both Wicksell (1898)[1] and Fisher (1931) give many examples of how the use of trade credit (book credit, merchandise credit) can increase the velocity of circulation. A short example from Fisher's The Purchasing Power of Money explains what the issue is all about. In the excerpt below Fisher defines the velocity of money as, "the quotient of the amount spent to the amount on hand".

> "For instance, a laborer receiving and spending $7 a week, if he cannot "charge", must make his week's wages last through the week. If he spends $1 a day, his weekly cycle must show on successive days at least as much as $7, $6, $5, $4, $3, $2, and $1, at which time another $7 comes in. This makes an average of at least $4. But if he can charge everything and then wait until pay day to meet the resulting obligations, he need keep nothing through the week, paying out his $7 when it comes in. His weekly cycle need show no higher balances than $7, $0, $0, $0, $0, $0, $0, the average of which is only $1.
>
> Through book credit, therefore, the average amount of money or bank deposits which each person must keep at hand to meet a given expenditure is made less. This

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1 Wicksell's Geldzins und Güterpreise was published in 1898. When I throughout this study refer to Wicksell I have used the English translation, Interest and Prices, from 1936.

means that the rate of turnover is increased; for if people spend the same amount as before, but keep smaller amounts on hand, the quotient of the amount spent divided by the amount on hand must increase."

When Wicksell and Fisher discuss trade credits and the velocity of circulation it is in my opinion remarkable with what clarity of vision they comment various aspects of the use of trade credit. In some cases their contributions have passed unnoticed when authors of today have "rediscovered" old trade credit issues.

During the most recent twenty five years trade credit has usually been debated in a monetary policy context. Trade credit has been seen as a variable which has a potential neutralizing effect on monetary policy. The debate about trade credit and the efficiency of monetary policy was partially initiated by the discussion in the Radcliffe Report (1959) about how trade credit can act as a counteracting buffer against a restrictive monetary policy. From a Swedish perspective there has, since the mid 50's, been a continuous debate about to what extent "the grey credit market", to which trade credits belong, neutralizes regulation of financial intermediaries.[2]

In a seminal paper about the macroeconomic aspects of trade credit Brechling and Lipsey (1963) summarize different trade credit theories and explain under what conditions an expansion of the use of trade credit can offset a contractionary monetary policy. They make a distinction between gross and net credit theories. *Gross theories* hold that it is possible to counteract a restrictive monetary policy through equal increases in accounts receivable and accounts payable. *Net theories* hold that trade credit can neutralize a contractionary monetary policy only if there is an increase in net credit. (I discuss the net credit concept in greater detail in Section 1.3.1, which deals with various trade credit dimensions.)

[2] "The grey credit market" can be defined as those parts of the Swedish credit market which are not regulated by the central bank. For a more detailed description see Lundberg and Senneby (1959) and Ingves (1981).

Without getting deeper into the issue of gross vs net theories next I briefly discuss the role of trade credit in a macro economic monetary policy context. This is done by the help of a very simple model which makes it possible to point at some important aspects of trade credit with respect to monetary policy. Assume that the supply and demand for money is determined in the following way:

(1.1) $$M^s = mB + B_{TCtr}$$

(1.2) $$M^d = (1/V)Y.$$

The supply of money is a function of the monetary base, B, which includes base money controllable by the authorities and base money which cannot be controlled, for example base money determined by transactions with persons outside the currency area in an open economy, a multiplier, m, which is determined by required and desired reserve ratios, and B_{TCtr}, which represents transferable trade credit. The demand for money is determined by the velocity of money, V, and nominal national income, Y. The demand for money represents the need for transactions balances in order to maintain a given level of activity in the economy. Setting $M^s = M^d$ and solving for Y gives

(1.3) $$Y = mVB + VB_{TCtr}.$$

In this simple world the authorities can produce a contractionary monetary policy in two different ways. The controllable part of the base can be decreased or the multiplier can be decreased. Trade credit, in turn, can offset such policies in three different ways: *Firstly*, if trade credit is transferable, trade credit I.O.U.:s are universally accepted as a mean of payment by all firms and households. In this case trade credit is a substitute for money and a reduction of the base can be completely offset by an increase in trade credit. When monetary policy is represented by its effect on nominal income there is complete offsetting if

$$(1.4) \qquad \frac{dB_{TCtr}}{dB}_{/dY=0} = -m \ .$$

This result is the effect of an implicit assumption in (1.1) that trade credit is transferable but it cannot be used as a deposit within the financial system, and consequently B_{TCtr} does not affect the multiplier process. However, trade credit is generally not transferable and therefore it seems reasonable to set $B_{TCtr} = 0$. *Secondly*, if a decrease in the controllable part of the monetary base is counteracted by an inflow of foreign exchange caused by changed trade credit terms there is complete offsetting when the inflow of currency equals the reduction in the controllable part of the base. In this way changes in international trade credit flows can counteract monetary policy. Other international flows of capital can naturally have the same effect. *Thirdly*, the monetary base is controllable in a closed economy with neither transferable trade credit nor a foreign component of the base. In this case changes in B can be counteracted by changes in velocity. The variation in velocity needed to neutralize a decrease in B is

$$(1.5) \qquad \frac{dV}{dB}_{/dY=0} = -\frac{V}{B} \ .$$

An increase in velocity means that firms and households hold less money in relation to their transactions than previously. Such an increase in V can be generated by an increase in gross credit if all firms conduct their business as usual but extend the length of the credit period, but sooner or later cash payments have to be made and the need to hold money increases and velocity falls. Hence, an increase in gross credit can only temporarily offset monetary policy. If there is an increase in net credit it increases velocity permanently if the net credit givers finance additional net credit by reductions in money holdings and net credit takers use it to maintain the level of production. In order to determine whether an increase in net trade credit is expansionay a more complex model is needed than the one used here. The balance sheet adjustments following an increase in net credit given must be known in order to determine whether money circulation is increased or if other financial assets and liabilites adjust instead.

An example of a more detailed macroeconomic approach than the one sketched above is Koskela's (1979) "Trade credits, credit rationing and the short run effectiveness of monetary policy". The article represents a connection between the trade credit literature and monetary macroeconomics. Trade credit and monetary policy is treated solely within the financial sector. Monetary policy has a direct effect on the banking sector and outside this sector there is a trade credit market which is only indirectly affected by monetary policy. Koskela studies the interaction between the two markets under various types of monetary policy and concludes that trade credit can counteract monetary policy but the trade credit market does not always act as a buffer. The market outside the banking sector is called the market for trade credit but, the way the model is constructed, it can be any kind of financial intermediation which is not regulated by those who conduct monetary policy. This means that on this level of aggregation there is a point of tangency between the literature about trade credit and the literature about financial intermediation in general. It brings trade credit close to the discussion about financial intermediation in for example Gurley and Shaw (1961), Tobin and Brainard (1963) and Guttentag and Lipsey (1969). A central theme in this literature is whether unregulated financial intermediaries can offset monetary policy or not.

Despite the simplicity of the model above it shows some important macroeconomic aspects of trade credit. In the transferable credit and open economy cases it is a direct substitute for base money. In the closed economy (or fully controlled international capital movements) no transferable credit case the trade credit effects are summarized by their impact on velocity. This conclusion gets empirical support in a study by Zahn and Hosek (1973). In a model with the same structure as the one used above they found that velocity in the U.S. economy was better predicted when trade credit was included in the demand for money function than when it was not. In conventional monetary theory the central determinant of velocity is the demand for money. Having reached the conclusion that trade credit also affects velocity it lies close at hand to find clues about the use of trade credit by looking at the demand for money function. There are two basic ways to look at the use of money. *Firstly*, money can be seen as a lubricant which makes it easier to carry out transactions. *Secondly*, it is also a store of wealth. It is an asset in the portfolio of firms and individuals. In a similar way trade credit plays a double role. *Firstly* it increases the efficiency of the transaction system since it is one way to separate goods transactions from simultaneous money transactions. *Secondly*, it is also a financial asset (or liability) and as such it is a part of the wealth portfolio of firms and households. One function does not exclude the other. A trade credit may well simultaneously fill its asset and transaction function. The conclusion to be drawn from this discussion about trade credit in terms of velocity and the demand for money is that in order to understand the use of trade credit it is necessary to study both *financial trade credit* and *transactions trade credit*. In order to do this I leave the macroeconomic perspective and in the chapters that follow I study trade credit on a more disaggregated level. The velocity approach represents a kind of "black box" model. My objective is to study some aspects of the use of trade credit in order to get a somewhat more detailed picture of trade credit as a part of "the machinery" in the box.

1.2.2 *The empirical picture*

A conclusion which follows from the discussion above is that the use of trade credit is a function of both financial and transaction variables. This is a fact that has been taken into account by most authors who have studied trade credit in an emppirical context. Generally formulated trade credit demand and supply functions can thus, for example, be written

$$(1.6) \qquad TC^d = AP = f(r_{TC}, r_s, P, m) \ ,$$

$$(1.7) \qquad TC^s = AR = g(r_{TC}, r_s, S, m) \ ,$$

where AP represents accounts payable, r_{TC} is the trade credit rate of interest, r_s is the cost of close substitutes, P represents purchases, and m other market specific variables. In (1.7) AR represents accounts receivable, and S is sales. Here both P, S and m can signify the use of trade credit in a transactions context. In some cases AP and AR are treated as functions of sales and purchases only. Examples of empirical studies with the latter type of equations are Brechling and Lipsey (1963), Eliasson (1967), Nadiri (1969) and Ferris (1981). Various formulations of such equations give both in time series and cross section studies the same unambiguous result. Trade credit described in terms of accounts payable or accounts receivable is closely correlated with the level of purchases or sales.

Kanniainen (1976)[3] has set forth a special case of the generally formulated demand function in (1.6). The demand for trade credit, which is represented by accounts payable, is

$$(1.8) \qquad AP = f(r_{TC}, r_s, m)P,$$

where P represents purchases over the planning horizon, τ. Next assume that purchases per unit of time, p, are roughly constant over the planning period. Then

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3 Kanniainen (1976), page 44.

(1.9) $Tp = f(r_{TC}, r_S, m)\tau p,$

when T is the length of the credit period. Division of both sides by τp gives,

(1.10) $\frac{AP}{P} = \frac{T}{\tau} = f(r_{TC}, r_S, m).$

Consequently in this special case the f(..) function represents the ratio of accounts payable to purchases and the length of the credit period measured as a proportion of the planning horizon. Often empirical data only contain information about AP and P. Given that the assumptions above are reasonable this then is an indirect method to estimate variations in the length of the credit period. Another property of this model formulation is that the elasticity of accounts payable with respect to purchases is equal to one. If P represents both a connection to the transaction system and a proxy for firm size this is thus a way to express the view that the transactions technology is the same among large and small firms. It is the financial variables that determine the relative size of accounts payable when one compares firms of different size. A similar reasoning applies of course also to a trade credit supply function of the same type as the function in (1.8).

In the empirical literature the financial aspects of trade credit have received much more attention than the transactions aspects. One way to study reactions to financial market conditions is to study the error terms in time series equations where accounts receivable or payable are only functions of sales and purchases respectively. Variations between actual and predicted AR and AP values then represent the influence of financial market variables on the use of trade credit. Using this technique Brechling and Lipsey (1963) came to the conclusion that among British firms in the 50's both accounts receivable and accounts payable increase during periods with a restrictive monetary policy.[4] Eliasson (1967),on the other hand,did not reach this conclusion when he used roughly the same method analyzing the Swedish manufacturing sector. The other alternative is to estimate regression equations where accounts receivable and accounts payable are functions of various financial variables. Working with U.S. data Laffer (1970) reached the conclusion that the ratio AP/(AP+M), where M is the money supply, increases when the short term interest rate rises. Nadiri (1969) found that both AR and AP depend positively on the short term interest rate and Ferris (1981) found the same kind of relationship between AR, AP and the yield on corporate bonds. In a study of British firms Davis-Yeomans (1974) reached the conclusion that both AR and AP increase during periods with a restrictive monetary policy. That accounts receivable and accounts payable move in the same direction is not surprising since accounts receivable of one firm represents accounts payable of other firms. When aggregated data are used the close connection between AR and AP is more pronounced the higher the intrasectoral and the lower the intersectoral trade. The empirical picture of trade credit in a financial market context is more ambiguous than the close connection between AR and sales, and AP and purchases. However, this is not the right place for a detailed discussion of the conclusions drawn in various empirical studies of financial trade credit. Here it is sufficient to note that in the studies mentioned above there are significant regression coefficients which relate trade credit to variables that are considered important in a financial market context.

This conclusion is also supported by a simple time series analysis of accounts payable and loans from financial institutions of the sector non-financial enterprises in the aggregated Swedish financial accounts during the 70's. Since the 50's

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4 The conclusions of Brechling and Lipsey have been questioned by White (1964) and Eliasson (1967).

Swedish monetary policy has been very availability oriented. The quantity of loans from the banking sector has been a central goal variable. During the 70's this policy was conducted by the use of recurrent loan ceilings. With excess demand for credit in the bank loan market firms try to borrow elsewhere. One close substitute to short term borrowing from commercial banks is trade credit. Thus, when bank loans lie below normal, in terms of a long run time trend, trade credit ought to lie higher than normal and vice versa. Trade credit can act as a buffer during periods with a restrictive monetary policy. In Figure 1.1 I have plotted the residuals of the deflated time trend of accounts payable and loans from financial institutions during the years 1970-79.[5] (The aggregated financial accounts do not contain information about purchases of non-financial enterprises, therefore a time trend approach has been used instead. An alternative is to use GNP as a proxy for purchases. Then the magnitudes of the residuals differ compared to the figure but the signs of the residuals are exactly the same.) The two curves in the figure show that a positive accounts payable residual corresponds to a negative loans from the financial sector residual and vice versa. The only exception is the residuals for 1970. Thus, by simple eye inspection it seems as if, from a monetary policy point of view, trade credit has been used as an offsettings substitute to loans from the financial sector. Here the conclusions are based on stock data, but similar conclusions can also be drawn from flow data which include not only trade credit but also other types of borrowing

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5 The accounts payable observations (AP) are in current prices, billions of SEK, 39.430, 39.953, 43.050, 54.216, 62.558, 68.081, 76.874, 81.244, 87.479, 99.194, the loans from the financial sector observations (L) are 71.709, 78.459, 83.812, 90.739, 99.603, 109.341, 122.874, 154.018, 168.994, 190.230, and the gross domestic product in purchasers' values price index series with 1970 as base year is, 100, 107.6, 115.2, 123.6, 134.4, 154.1, 172.1, 191.1, 209.9, 224.8. The regression equations are with deflated values

$$AP = 38.5886 + 0.659t, \text{ and } L = 68.6519 + 1.2084t.$$

Sources: The National Bureau of Statistics, Statistical Reports, Financial Accounts 1977-79 N 1980:17, Appendix to N 1980:17, and National Accounts N 1981:2.1.

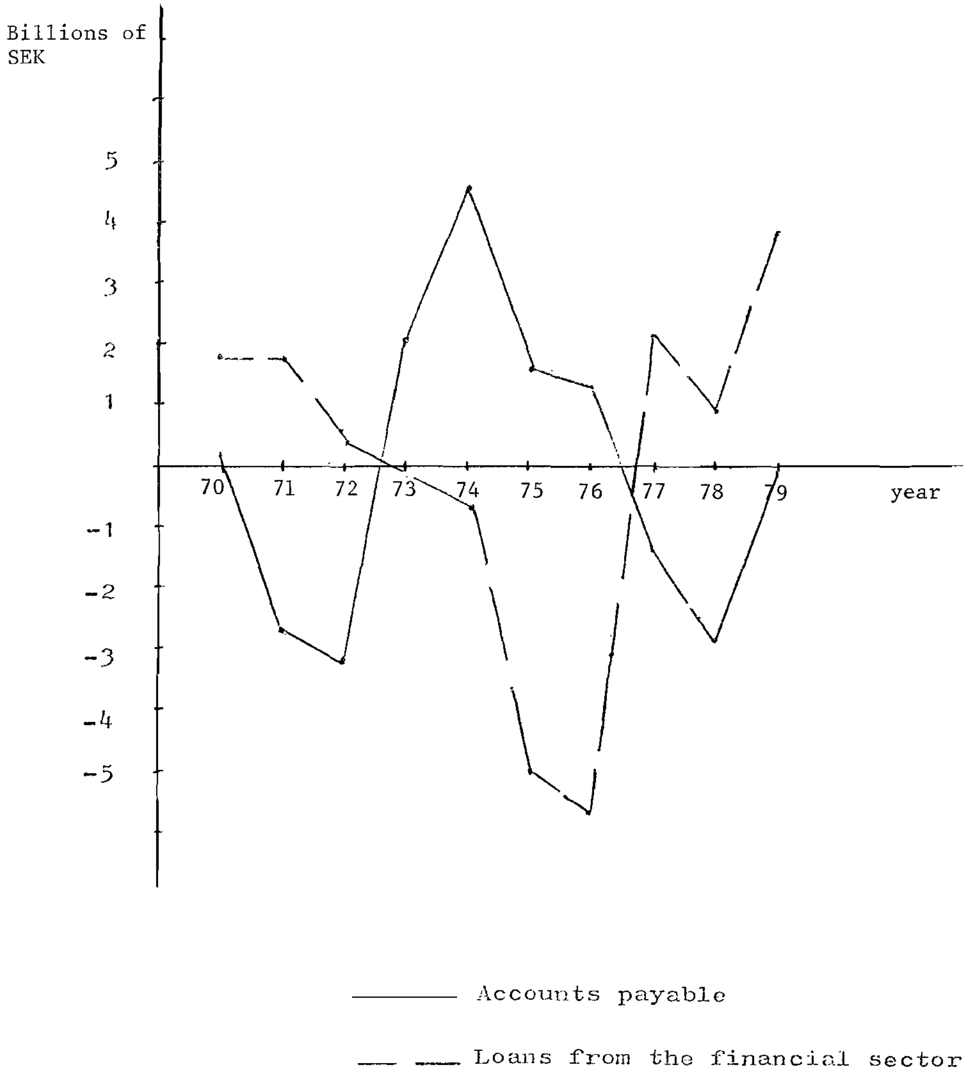

Figure 1.1 Residuals of accounts payable and loans from the financial sector, non-financial enterprises, 1970-79.

that cannot be directly controlled by the Central Bank.[6]

The curves in Figure 1.1 represent a gross trade crecit approach to study how the use of trade credit reacts to different types of monetary policy. However, it is much more common to use variations in net credit (accounts payable minus accounts receivable) instead. An increase in net credit indicates that purchasing power is redistributed from net credit givers to net credit takers. Meltzer (1960) studied firms with various size of total assets within the U.S. manufacturing sector and found that firms with large assets were more likely, than small firms, to increase net credit faster than sales during periods with "tight money". Also working with U.S. manufacturing sector data with firms classified according to asset size Jaffee (1970) found that firms with large assets extended and small firms received net credit during periods with high credit rationing in the bank loan market. (Jaffee used the ratio of risk free loans to total loans as a proxy for credit rationing.) In a study based on a similar rationing proxy technique Kanniainen (1976) found that in the Finnish manufacturing sector net credit increased among large firms and decreased among small firms (large and small in terms of turnover) during periods with abnormal credit rationing. The previously discussed paper by Brechling and Lipsey also indicated that net trade credit tends to offset monetary policy. Their statistical data included a high proportion of very large firms. These studies represent what can be called a *financial or asset market view* of trade credit.[7] The goods market and transactions aspects are left in the background. Except in the Brechling and Lipsey paper the statistical data have been classified according to various measures of firm size. An important conclusion is that during periods of "tight money" large firms supply more net credit than small firms. Behind the classification of firms lies an assumption that small firms are

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6 See Ingves (1981), section 5.2.

7 This is what Davis Yeomans (1974) call the orthodox or Radcliffe view of trade credit, see Davis-Yeomans page 71. It is called the Radcliffe view because this is the way the Radcliffe Committee (1959) envisaged trade credit neutralization of monetary policy.

more often liquidity constrained than large. Small firms have less access to different parts of the financial market either because they do not have the information that is necessary or because there is credit rationing which discriminates against small firms. With this view of the world it is natural to see net trade credit as a variable which is used to redistribute and alleviate the effects of a restrictive monetary policy. Nadiri (1969) and Ferris (1981) found that changes in net credit did not work the way it is envisaged by the financial market view. One possible explanation to their empirical results is that they use data which are not classified according to firm size. It seems, for example, unlikely that discrimination in the financial market is based on product group classification of subsectors of the U.S. corporate sector, which is the disaggregation used by Ferris.

That different classifications of firms lead to different results stresses the importance of choice of disaggregation if one wants to track the effects of various types of monetary policy. An alternative to firm size is to classify firms according to some measure of goods market strength, when strength represents the ability of a firm to dictate the trading conditions on the markets where the firm operates, or the degree of monopsony and monopoly power on the input and output markets respectively. Assume there is a period with "tight money". Then on the input side monopsony power can be used to force suppliers to grant all the credit the firm needs and on the output side monopoly power can be used to control how much credit is granted. This implies that it is the combination of monopoly and monopsony power that affects the distribution of net credit. Strong firms can use all the credit they want and they can refuse to supply credit, while the opposite holds for weak firms. Thus, firms that are strong both on the input and output side are more likely to have low net credit than weak firms. With different strong/weak combinations the size of the net credit position is unclear. In a study of British manufacturing corporations Davis and Yeomans (1974) reached the conclusion that during a period with a very restrictive monetary policy the impact of "tight money" partly was shifted from large to small firms. In their sample large firms were also firms with considerable monopoly or oligopoly power and small firms were firms with little market power. Hence, the financial market view of net credit seems to be incompatible with the goods market view, when financial market strength is highly correlated with goods market strength. In the financial market case the credit flow goes from strong to weak firms and in the goods market case the flow is reversed. One way to make the two views compatible is to look at the degree of monetary tightness. Strong firms need their goods market power to excert pressure in the financial market first when monetary policy is such that their liquidity position is affected. When there is financial market discrimination in favour of strong firms this is likely to occur first under periods with a very harsh monetary policy. Consequently a financial market view of net credit does not necessarily exclude goods market strength aspects as a determinant of how net credit is used to redistribute purchasing power among firms.[8] The introduction of a goods market view makes empirical studies of monetary policy effects on trade credit more difficult since then goods market conditions also have to be taken into account. One complicating factor, indicated by the discussion above, is that goods and financial market variables can both work in the same and in opposite direction with respect to the effects of monetary policy on trade credit.

In this section I have discussed some empirical studies that have focused on various trade credit issues. The main con-

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8 Davis and Yeomans call a trade credit distribution based on goods market strength a Galbraithian pattern. On the one hand Galbraith (1957) argues that firms in competitive markets are more vulnerable to variations in economic policy than firms in oligopolistic markets. With this as a starting point Davis and Yeomans reach their conclusion about net credit based on market strength. On the other hand, however, Galbraith also argues that there is financial market discrimination against competitive firms. This again leads to a financial market view of the distribution of net credit, except when monetary policy is restrictive enough to also affect firms in oligopolistic markets. Since, in my opinion, Galbraith's arguments are not unambiguously in favour of a distribution of net trade credit based on goods market strength it is better to call this view of the world something else than a Galbraithian pattern.

clusions to be drawn from the survey can be summarized in the following way:

- There is a close relationship between accounts receivable and sales, and between accounts payable and the level of purchases.
- Both accounts receivable and payable are affected by monetary policy and financial market conditions.

These "stylized facts" are not in conflict with the earlier conclusions that there are both financial and transactions aspects of trade credit.

1.3 THE MICRO PERSPECTIVE

1.3.1 *The trade credit concept*

At the beginning of this chapter I noted that a trade credit is usually an integrated part of ordinary business transactions among non-financial firms, and in the survey of the empirical literature trade credit was represented by accounts receivable and accounts payable on the asset and liability side respectively. When I in this section move from the macro to the micro level it is time to give a more precise definition of what I mean when I use the term trade credit. Henceforth a trade credit is a financial arrangement which fits the following definition.

Definition: A trade credit is a loan agreement between non-financial firms or between non-financial firms and households. Further, in order to be a trade credit the financial agreement must be intimately connected with the sale or purchase of goods and services. The credit must originate from one of the traders, and at the time of repayment the debt must be settled directly between the traders.

The use of trade credit means that the payment date is separated from the date of delivery or purchase. The credit flow goes from seller to buyer if payment is made after delivery (post payment), which is the case represented by accounts receivable and payable. When payment is made prior to delivery (pre payment) the credit flow is reversed and goes from buyer to seller. These are the two major payment alternatives but various mixed payment patterns are not unusual. The definition is also broad enough to include credits which usually are not considered trade credits. An example is monthly wage payments, which with the definition above is a form of trade credit since it is a post payment of services delivered continually during the month.

Figure 1.2 describes the time profile of a no default post payment agreement. The total length of the credit period has been divided into three separate parts. The first part is called routine credit. This is an extra credit period due to the fact that it takes time for the seller to record and send out the bill. If the bill is dated from the day of delivery regardless of when the bill is received there is of course no routine credit. The second part of the credit period is the payment deferral agreed by seller and buyer. Finally, the third part is the extra credit gained if payment is delayed after the final payment date. If there is no routine credit and if payment is made at the last payment date the total length of the credit period is the same as the payment deferral offered by the seller. It is of course also possible to draw a picture of the time profile of a trade credit when there is pre payment instead of post payment. The main difference between such a picture and Figure 1.2 is that the direction of the credit flow is reversed

A conventional seller to buyer trade credit is often a two part offer: a discount for payment within a relatively short discount period and a longer net period, at the end of which full payment is required. The time/price offer is often expressed as

(1.11) P_1, d/D, net T,

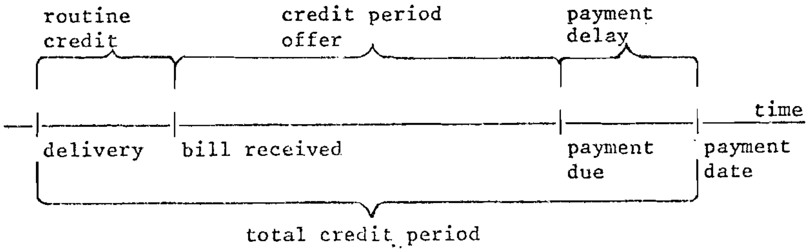

Figure 1.2 The time pattern of a post payment trade credit agreement, c.f. Peterssohn (1976).

where P_l = list price, which is the price that has to be paid at the end of the credit period, d = the percent discount for payment within the discount period, D = the length of the discount period, and T = the length of the net credit period in days. If no discount period is offered the terms are simply net T and $(1-d)P_l$ represent the cash payment price. The buyer's cost of borrowing by using trade credit is calculated as an internal rate of return, where the cash price has to equal the present value of the list price. With the notation in (1.11) r is such that

$$(1.12) \qquad (1-d)P_l = e^{-r(T-D)}P_l \ ,$$

and solving for r gives

$$(1.13) \qquad r = -\ln(1-d)/(T-D) \ ,$$

when interest is assumed to be continuously compounded. For terms of 2/10, net 30, r is 0.00101/day or 0.369/annum continuously compounded; net 60 gives 0.00045 and 0.164 respectively. Hence, it is clear that for a constant list price and a constant discount period the borrowing cost falls when the credit period is lengthened.[9]

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[9] This example comes from Schwartz and Whitcomb (1979).

An alternative way of looking at trade credit is to compare different price/time payment arrangements. Trade credit alters the effective price paid by the buyer and the effective price received by the seller. The effective price to the buyer (P_{be}) is the lesser of the present value of the discount price to be paid in D days or the present value of the list price to be paid in T days:

$$(1.14) \qquad P_{be} = \min((1-d)P_1\exp(-r_bD),\ P_1\exp(-r_bT)),$$

where r_b = the maximum rate of interest the buyer is willing to pay for the credit. If there is no discount period the effective price is the lesser of the cash price or the present value of the list price. Both the internal rate of return and the present value approach make it possible to compare trade credit with other types of borrowing or lending.

In conventional cash management literature it is common to compare different trade credit strategies when the investment in accounts receivable is measured in terms of production costs. Kim and Atkins (1978) have shown that such a formulation of how to compare different trade credit strategies does not capture the total opportunity expense of deferred payment.[10] With the help of an example they show that this approach does not take the time value of trade credit properly into account. An alternative is to compare the present value of different trade credit alternatives. Then the time value is included in a proper way. The present value approach makes it possible to compare changes in both revenue and costs at different points in time. This is important since usually the length of the credit period both for credit given and credit taken varies. One further advantage of the present value approach is that the method is one of the building blocks in orthodox capital budgeting

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[10] For a longer and more detailed discussion about trade credit in the conventional cash management literature, see Kim and Atkins (1978).

theory. Hence, it becomes possible to compare investments in accounts receivable with other investment alternatives. Trade credit can be included in a wealth maximization framework.

1.3.2 *Some different dimensions*

The amount of trade credit given by a firm is summarized in accounts receivable and trade credit taken in accounts payable. These two accounts represent what can be called the stock dimension of trade credit. In the pre payment case the stock dimension is represented by advances to suppliers on the asset side and advances from customers on the liability side. Accounts receivable at time t is the sum of all past credit sales which have not yet been paid. The composition of accounts receivable (AR) is determined by the integral in (1.15), R(u) represents revenue from sales at time u and T is the length of the credit period,

$$(1.15) \qquad AR_t = \int_{t-T}^{t} R(u)du.$$

With constant sales the integral can be written

$$(1.16) \qquad AR_t = TR,$$

and with this simple formulation of accounts receivable it is easy to see that trade credit has a *goods market dimension* and a *time dimension*. There is a goods market dimension because R is determined within the goods market and is affected by the level of output and the sales price,[11] and there is of course a time dimension because payment is deferred T periods. Both (1.15) and (1.16) represent an accounting valuation of trade credit. The fact that there is an inflow of revenue at different moments in time has not been taken into account.

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[11] This is what Petersohn (1976), page 111, calls the monetary dimension of trade credit.

Usually statistical data do not contain explicit information about the time dimension. A common way of calculating the length of the credit period from data about accounts receivable (or accounts payable) and annual sales ($S = \tau R$, τ = the length of the sales period) (or purchases) is to divide accounts receivable with average sales. (This is the method I used when I discussed the trade credit demand function set forth by Kanniainen (1976).) With constant sales and credit period the method is exact and the credit period is

(1.17) $$\frac{AR}{S/\tau} = T ,$$

and with variable sales and credit period it is an approximate measure of T. The relevant numbers for the aggregated corporate sector in Sweden were 1980:[12] AR = 101.2 billion SEK, S = 902.9 billion SEK and τ = 250, which is the approximate number of days a firm operates during one year. Insertion of these numbers in (1.17) gives an approximate credit period of 28 days. However, if the credit period offered by a firm with constant sales is T days AR = TR only if the firm operates every day during the credit period. If this is not the case the number of no-sales days (h) must be subtracted and AR = (T-h)R, in this case (1.17) underestimates the credit period, h has to be added to get the true credit period. Consequently the approximate credit period for the aggregated corporate sector is probably somewhat longer than 28 days.

The length of the credit period combined with the list price gives a trade credit offer a financial or *credit market dimension*. Another aspect of the credit market dimension is that a trade credit represents a loan with an upper limit R,

(1.18) $$L \leqq R .$$

This limits the extent to which trade credit can be used as a substitute for other types of loans. A perhaps subtle difference between trade credit and other types of loans, for example

12 See Table 1.1, page 32.

a bank overdraft facility, is that trade credit with post payment does not represent a transfer of general purchasing power from seller to buyer. A buyer without money of his own cannot use a trade credit from firm X to buy goods from firm Y. In the pre payment case this does not hold. Then trade credit also represents general purchasing power.

In the section where I discussed macroeconomic aspects of trade credit it became clear that net credit has been considered an important variable when one tries to explain how non-financial firms use their financial resources and how these are redistributed among firms without having to pass via the financial sector. Next I take a closer look at net credit than in the section about empirical studies of trade credit and monetary policy. Net credit further widens the multidimensionality of the trade credit concept. So far I have mostly talked about trade credit given. Since a trade credit given by one firm is a trade credit taken by some other firm or private person the different dimensions I have discussed apply both to accounts receivable and payable. When one discusses net trade credit it is necessary to simultaneously take both trade credit given and trade credit taken into account.

Net trade credit is defined as the difference between accounts receivable (AR) and accounts payable (AP). A firm has net trade credit given if accounts receivable exceed accounts payable, and net credit taken if the reverse is true. If

(1.19) $AR - AP > 0$,

AP is the credit which is passed on from the suppliers of the firm to its customers. The difference AR - AP is additional credit contributed by the firm itself. If the inequality is reversed

(1.20) $AR - AP < 0$,

AR is passed on to the customers of the firm and AR - AP is withdrawn from the credit system. In this case trade credit is used as a net source of finance. If

(1.21) $AR > 0$ and $AP = 0$,

the firm only makes additions to the trade credit system, there is no passing on, and if

(1.22) $AR = 0$ and $AP > 0$,

there is no passing on of trade credit either but in this case the firm uses all the trade credit by itself. In the simplest of cases when the use of inputs, sales, prices and the length of the credit period are constant (1.16) can be combined with a similar expression for accounts payable to get net trade credit,

(1.23) $T_1P_QQ - T_2P_XX$,

T_1 = credit period credit given, T_2 = credit period credit taken, Q = output, X = input and P_Q, P_X the price of output and input respectively. With this formulation net trade credit depends on input and output prices and quantities and the length of the credit periods. If the net credit framework is further simplified and it is assuemd that the credit periods are of the same length a positive or a negative net credit only depends on if there is a positive or a negative profit. With a positive profit there is both passing on and an addition to the credit system, with zero profit there is only passing on, and with negative profit both passing on and withdrawal from the system. With a positive profit it is the profit per unit of time times the length of the credit period which is the additional trade credit generated by the firm.

Naturally it is also possible to discuss net trade credit in a similar way when there is pre payment of goods. Then the credit flow is reversed and goes from buyer to seller. A firm that wants to maximize its use of trade credit can do so if it sells its output with prepayment and inputs are bought with post payment. This firm receives trade credit both on the input and on the output side.

When *financial intermediation* is discussed one usually thinks about the intermediation that takes place when different financial firms are involved but trade credit can also be said to be a part of the intermediation process. The net trade credit concept can be used to show this intermediation function. When a firm passes on trade credit it acts more like a bank than a broker since both accounts receivable and payable are included on the balance sheet of the firm. Trade credit is a part of the firm's own assets and liabilities in the same way as deposits and loans for a bank. In order to show when there is intermediation assume that there are three different firms which use trade credit, the flow of goods goes from firm 1 via firm 2 to firm 3, and the firms use post payment. The chain of trade credit is depicted in (1.24).

(1.24) $$AP_1 = AR_1 = AP_2 < AR_2 = AP_3 = AR_3$$

$\longrightarrow$ goods

$\longleftarrow$ payments

First take the case when $AP_2 = 0$ and $AR_2 > 0$. Then there is no flow of credit from firm 1 to firm 3. This is not financial intermediation. There is a difference between being an originator and an intermediary in unspent income.[13] Here firm 2 is an originator of unspent income since it does not use any trade credit itself. Second when both AP_2 and $AR_2 > 0$ firm 2 acts as an intermediator. Credit is passed on from firm 1 via firm 2 to firm 3. In addition to the "passing on" firm 2 can also act as an originator or final user of unspent income in the way it has

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13 This distinction between being an originator or an intermediary in unspent income comes from Gurley and Shaw (1960) pages 199-202.

been described in (1.19) and (1.20). Here firm 2 is the link that connects the credit extended by firm 1 with the financial transactions of firm 3. It is also clear why firm 2 does not act just like a broker. If firm 2 were a broker firm 3 would stand directly in debt to firm 1 instead of to firm 2. Schwartz and Whitcomb (1979) argue that trade credit in general is a form of financial intermediation. This is not necessarily true. From (1.24) it is clear that there is trade credit intermediation if both accounts payable and accounts receivable are greater than zero; another type of intermediation takes place if for example bank borrowing is passed on in the form of trade credit. If on the other hand a firm has not borrowed any of its money capital with accounts receivable greater than zero it does not imply financial intermediation. In this case the firm is instead an originator in unspent income.

One of the important functions of financial intermediaries is their ability to *transform financial assets*. This transformation can be made in two different ways.[14] *Firstly* they can change the *time perspective* and *secondly* the *risk structure* of various assets. These transformation possibilities are also open to firms that use trade credit. A time transformation has for example taken place if the credit period for accounts receivable is longer than the credit period for accounts payable. A risk transformation takes place if those who have supplied the firm's debt consider this debt a less risky investment than the financial resources invested by the firm in its accounts receivable. It is naturally also possible to change both the time and risk structure at the same time. To further investigate why these asset transformations can be beneficial both to the single firm and to the financial system as a whole is beyond the scope of this chapter. Here it suffices to note that asset transformation is not solely reserved for financial firms, sometimes it applies to trade credit as well. It is one more addition to the various trade credit dimensions I have discussed.

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[14] See Niehans (1978) chapter 9.

Now let me summarize my discussion about the trade credit concept. I have defined what a trade credit is, and I have discussed several different credit dimensions. In order to qualify as a trade credit it must be intimately connected with the flow of goods and services. This gives trade credit a *goods market dimension*. I noted that there is a *stock dimension* which is summarized in accounts receivable and advances to suppliers on the asset side and accounts payable and advances from customers on the liability side. The stock dimension depends partly on the length of the credit period, the *time dimension* of credit. Since trade credit is a financial arrangement it naturally also has a *credit market dimension*. I also discussed *net credit* and noted that it is a useful concept in a macroeconomic context. Finally, I argued that trade credit also can be a part of *financial intermediation* and that this intermediation process can include *transformation of financial assets*.

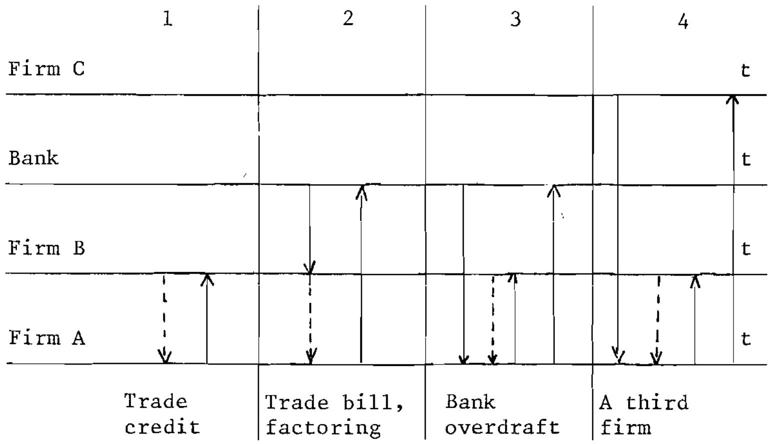
1.3.3 *Substitutes*

In the preceding section I have commented on some different trade credit dimensions. Now in order to complete the picture of what a trade credit is I briefly discuss the difference between trade credit and some close substitutes both on the demand and the supply side. This is done by the help of Figures 1.3 and 1.4 on page 30.

In Figure 1.3, which depicts the demand side, firm A buys goods from firm B. Different credit arrangements that can be combined with the goods transaction are shown in the vertical direction, and the time profile of the different payment alternatives in the horizontal direction. A has four different possibilities to separate the date of delivery from the date when all debts in connection with the goods transaction have been settled. Column one shows an ordinary trade credit agreement. Payment is deferred, and A pays B at the end of the credit period. A very close alternative to this payment pattern is shown in column two. Instead of having to wait for payment B can require that A uses a commercial bill, which is handed over to B and then immediately discounted in a bank. At the payment date A pays the bank instead of firm B. From A's point of view the payment pattern in column two is equivalent to the one in column one, but from B's point of view it is different. When the commercial bill is discounted B receives immediate payment from the bank. The credit arrangement is moved from B to the bank. A payment flow equivalent to the one described in column two is the payments that take place if B's trade credit is handled by a factoring company. Then both A and B stand in debt to the factoring company until the end of the credit period. A because the trade credit bill is made payable to the factor, and B because the payment to B is made in the form of a loan from the factoring company with the trade credit bill as collateral. In both cases the credit arrangement between the two traders is broken in the sense that a settlement of the debt does not mean that A pays B at the end of the credit period. A financial intermediary, a bank or a factoring company takes over the credit part of the goods transaction. The use of commercial bills or factoring, the way it was described above, is for two reasons more closely related to an ordinary trade credit than other types of borrowing. *Firstly*, it is buyer and seller that make the original credit agreement and *secondly*, commercial bills and factoring contracts include right of recourse clauses. Thus, if the buyer cannot pay at the end of the credit period the seller must repay the bank or factor instead. The seller is not freed from credit risk. A further step away from the trade credit case is shown in column three. Assume that A has an overdraft facility with a bank. When the goods are delivered from B to A the overdraft facility is used to pay B, and at a later date A pays back its loan from the bank. Here the goods market transaction is totally separated from the credit market. A payment pattern similar to the one in column three is depicted in column four. The only difference is that now A borrows from a non-financial firm instead of from a bank or a factoring company. Thus, borrowing from a non-financial firm is not necessarily a trade credit. However, if C buys output from A and there is pre payment from C to A this is also a trade credit. (Then the last money flow arrow in Figure 1.3 is replaced by a goods flow arrow.) Consequently the demand for trade credit is represented by post payment on the input side and pre payment on the output side. Figure 1.3 makes it clear that A has several different possibilities to separate the goods transaction from the date when all debts in connection with the purchase have been settled. The number of substitutes is further increased if various types of long term debt are taken into account too.

I have showed that trade credit has several close substitutes on the demand side. The same is also true when trade credit is seen from the supply side. Some alternatives are presented in Figure 1.4. Instead of seeing goods and cash flows from A's point of view I now look at firm B. Instead of investing in accounts receivable, column one, it can choose alternatives within the financial market such as ordinary bank deposits, certificates of deposit, short term bonds or other investment opportunities available within the financial sector, column two. Another possibility is to make various financial agreements with non-financial firms, column three. A special case is given in the fourth column, which shows pre payment of inputs. Here the payment pattern is reversed compared to the demand side. Thus, trade credit supply is represented by post payment on the output side and pre payment on the input side. A fifth alternative, not included in the figure, is to use the funds within the firm itself, for example to increase advertising or inventories. Hence the supply of trade credit has to be compared with other types of investments both within the financial sector, the non-financial sector, and within the firm itself.

The alternatives given in Figures 1.3 and 1.4 represent the closest trade credit substitutes on the demand and supply side respectively. The figures make clear what the alternatives are but they say nothing about the importance of the different alternatives. In order to shed some light on the use of trade credit compared with other assets and liabilities I next comment on some balance sheet data from non-financial firms.



---------- goods flow, ———— money flow, t = time

Figure 1.3 Trade credit and its closest substitutes, the demand side

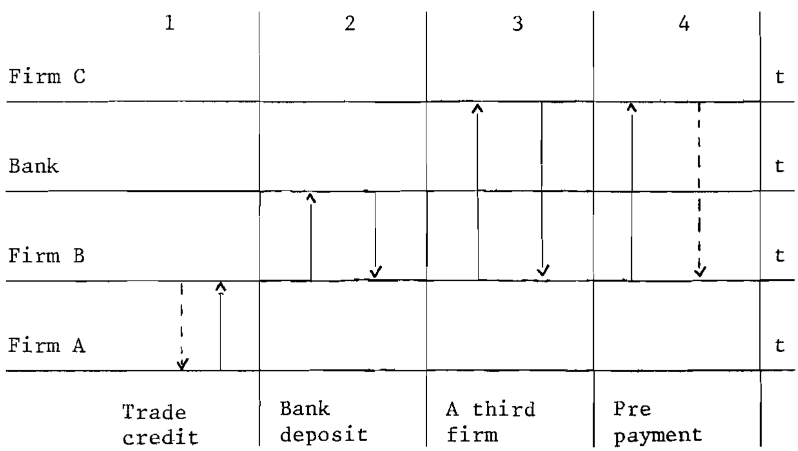


Figure 1.4 Trade credit and its closest substitutes, the supply side

1.3.4 *The empirical picture, some Swedish data*

Table 1.1 is an aggregated balance sheet of the Swedish corporate sector. The balance sheet entries above the broken line are roughly equivalent to the lending and borrowing alternatives given in Figures 1.3 and 1.4. Together with remaining current assets and liabilities they represent the liquidity portfolio of Swedish firms. I use the term liquidity portfolio to represent assets and liabilities which have a time profile such that they more easily can be transferred into cash, if need be, compared to other assets and liabilities.[15] In a trade credit context it is of interest to concentrate on the liquidity portfolio instead of on all balance sheet items. It seems reasonable to assume that trade credit is a closer substitute to various current assets and liabilities than for example to physical capital or long term loans. The liquidity portfolio in Table 1.1 contains some interesting information about trade credit compared with the other assets and liabilities. Both accounts receivable and accounts payable are much larger than advanced to suppliers and advances from customers respectively. This means that trade credit with post payment agreements is much more common than pre payment. This is probably also why trade credit in most empirical studies is represented by accounts receivable and accounts payable. Accounts receivable plus advances to suppliers were 103,025.4 million SEK or 25 % of current assets and accounts payable plus advances from customers were 98,864.7 million SEK or 37.9 % of current liabilities. Consequently trade credits were the second largest current asset and the largest current liability. Note also that trade credits were much larger than loans among non-financial firms. This means that it was the most common way to transfer purchasing power among non--financial firms without first having to pass via the financial sector. The same year the total stock of loans from commercial

15 This is not the only way to compose an aggregated liquidity portfolio. For an alternative combination of assets and liabilities see for example Kanniainen (1976).

Table 1.1 The liquidity portfolio of the Swedish corporate sector, 1980.

With the official statistical publication of the Swedish corporate sector as a starting point it is not possible to divide the balance sheet entry Loans into Loans to/from Financial Firms and Loans to/from Non-financial Firms. In the table below Loans to Financial firms is represented by 80 % of the entry Loan and accomodation bills on the asset side and 80 % of the entry Short term loans on the liability side, the remaining 20 % have been added to Receivables from group companies and Payables to group companies on the asset and liability side respectively. This has been done in order to get an approximation of Loans to/from Non-financial Firms. In reality the unspecified entries Other Short Terms Assets and Liabilities also contain assets and liabilities originating between non-financial firms. At this level of aggregation it has however not been possible to make a more accurate division of the items in the liquidity portfolio.
Source: National Bureau of Statistics, Enterprises 1980.

ASSETS	Millions SEK	%	LIABILITIES	Millions SEK	%
Cash and Bank	44,850.2	11.1			
Accounts Receivable	101,217.3	25.1	Accounts Payable	78,150.8	30.1
Trade Bills Receivable	4,124.9	1.0	Trade Bills Payable	4,444.0	0.2
Advances to Suppliers	1,808.1	0.4	Advances from Customers	20,313.9	7.8
Loans to Financial Firms	12,765.6	3.2	Loans from Financial Firms	30,465.5	11.7
Loans to Non-financial Firms	43,875.3	10.9	Loans from Non-financial Firms	43,164.4	17.8
Inventories	157,950.7	39.2			
Other Short Terms Assets	36,439.2	9.0	Other Short Term Liabilities	83,322.5	32.1
Current Assets	403,031.3	99.9 (61.4)	Current Liabilities	259,861.1	99.7 (39.6)
Long Term Loans	107,423.1	(16.4)	Long Term Loans	287,893.7	(43.9)
Physical Capital	145,845.8	(22.2)	Net Worth	108,545.4	(16.5)
Total Assets	656,300.2	(100.0)	Total Liabilities	656,300.2	(100.0)

banks to the business sector was 94,983[16] million SEK, or roughly the same size as the stock of trade credits in the corporate sector. Although mere size alone is not enough one conclusion is then that trade credit is a potential buffer against variations in the supply of loans from commercial banks. This conclusion was also supported by the analysis of the trade credit and bank loan residuals in Figure 1.1 on page 13.

In order to broaden the empirical picture I have disaggregated the liquidity portfolio and show parts of it when the corporate sector has been divided into 9 subsectors. See Table 1.2 (page 34). Receivables, commercial bills and pre payment from suppliers are measured in percent of current assets and payables, commercial bills and pre payment from customers are measured in percent of current liabilities. The disaggregated picture shows that there are substantial differences in both accounts receivable and accounts payable between different sectors. The spread between the highest and the lowest percentage number is roughly the same for receivables and payables and a large relative share of accounts receivable is not necessarily combined with a large relative share of accounts payable. Consequently it seems as if accounts receivable and payable have been used in different ways in different sectors. Pre payment is quite low for most sectors and this is true both on the asset and the liability side. One notable exception is the business services sector which has a very large part pre payment from customers compared to the other sectors. Commercial bills are low both on the asset and the liability side.[17] One exception is the agriculture, forestry and fishing sector which has much more commercial bills on the liability side than the other sectors. Finally, there are large differences in net trade credit. The agriculture, forestry and fishing sector is the largest receiver of net credit

16 Source: The National Bureau of Statistics, Financial Accounts, N 1982:11.

17 This conclusion does not change dramatically if factoring is included too. In 1980 the total factoring assets of finance companies were 3,131 million SEK. (Source: Financial Accounts, N 1982:11).

Table 1.2 Trade credit in different subsectors of the Swedish corporate sector, 1980.

	ASSETS			LIABILITIES			
Sector*	Receivables	Prepayment Suppliers	Commercial Bills	Payables	Prepayment Customers	Commercial Bills	Net Credit**
	(% of current assets)				(% of current liabilities)		
1	22	5.6	0,3	42	7,3	9,4	-14.2
2	14,1	0.1	2,7	25	0,1	1,7	- 4.8
3	20,6	0.5	1	25,2	9,3	1,1	10,8
4	23,1	1,1	0	33.8	0,3	0,1	10,4
5	41	0,4	0,1	40.7	0,7	2,8	21,8
6	29.9	0,2	1,5	40,6	1,5	2,1	4,7
7	34,9	0,3	0,1	27,8	0,8	4,3	8,1
8	24.5	0	0,2	16,6	25,7	0,2	14
9	26.6	0,1	3.5	24,1	1,3	2.4	14,9

* *Sectors:* 1 = Agriculture, forestry and fishing, 2 = Mining and quarrying, 3 = Manufacturing, 4 = Electricity, gas and water, 5 = Construction, 6 = Wholesale and retail trade, restaurants and hotels, 7 = Transport and storage (postal services and telecommunications are excluded), 8 = Business services (financial firms and property management are excluded),9 = Social and personal services (government agencies, police, defence etc. are excluded).
Source: Enterprises 1980.

** % of current liabilities.

(given the way it is measured in Table 2.2), and the construction industry is the largest supplier of net credit.

Net trade credit is used as a measure of how purchasing power is transferred between different firms or sectors under various types of monetary policy. In this context there may well be sector classifications other than the ones given here that give a more accurate picture of how net credit redistributes purchasing power. This is so since net credit within each sector cancels, (in the extreme case with a closed system there can be no aggregated net credit, accounts receivable must equal accounts payable) and a sector division based on types of production does not distinguish net credit givers from net credit takers.

Finally, instead of comparing different product sectors in Table 1.3 I present firms of different size in the manufacturing sector. Firm size is represented by number of employees. The table shows that large firms both have a lower proportion of accounts receivable to current assets and accounts payable to current liabilities than small firms. In the inter firm credit case the reverse holds. Large firms have more inter firm credit than small firms, and this is the case both on the asset and on the liability side. One conclusion from these observations is then that if trade credit is seen solely in a credit market context trade credit, and consequently also other parts of the financial market, is not used in the same way by small and large firms. The inter firm credit example in the table above confirms this. If on the other hand trade credit is seen solely as a goods market phenomenon it must be related to sales and purchases. The last two columns in Table 1.3 show the proportion of accounts receivable to sales and accounts payable to purchases. The ratios show less variation than the corresponding ratios with respect to assets and liabilities. Another interesting observation is that the ratios are falling. Large firms use less trade credit in proportion to sales and purchases

Table 1.3 Trade credit and inter firm credit for different size firms within the manufacturing sector. Swedish firms 1980. Percent.

Number of employees	AR/CA	IF/CA	AP/CL	IF/CL	AR/S	AP/P *
20 - 49	30.3	6.1	46.9	5.1	13.1	13.2
50 - 99	29.7	6.6	40.2	8.9	13.4	11.6
100 - 199	25.8	11.1	34.6	12.1	13	13.7
200 - 499	28.9	9.2	31.5	13.6	12.6	8.4
500 - 999	23.6	15.6	27.2	16.4	12	8.6
1000 -	15.6	17.7	18.2	20.1	11.5	8

* AR = accounts receivable, S = sales, AP = accounts payable, P = purchases, CA = current assets, IF = inter firms credit, CL = current liabilities. All ratios are given in percent. Inter firm credit is represented by credit among firms within conglomerates (group companies), since there are also other types of inter firm credit inter firm credit has a downward bias in the table above.
Source: Enterprises 1980.

than small firms. If trade credit is seen as a lubricant that makes it easier to carry out transactions this may be an indication that there are some kind of transactions returns to scale. Another explanation is that large firms have better access to the availability oriented and quantity regulated credit market and therefore use less trade credit.

In a time series study of the Swedish manufacturing sector, 1950-63, Eliasson (1967) found that the AP_t/P_t ratio was rising, (S_t is used as a proxy for P_t) and he concludes that for a single firm there are no trade credit returns to scale. However, a secular increase in the AP_t/P_t ratio does not necessarily exclude returns to scale. A cross section study at each t may well reveal a falling AP/P ratio, large firms use less trade credit than small firms. This is equivalent to saying that there is a downward sloping AP/P schedule which shifts upward over time.

Table 1.3 concludes my description of trade credit in the Swedish corporate sector. The findings in the three tables can be summarized in the following way:

- Accounts receivable and payable make up a substantial part of the liquidity portfolio, and, compared with loans from commercial banks, they are large enough to be of interest from a monetary policy point of view.

- Large firms use less trade credit than small firms, both measured in proportion to current assets or liabilities, and sales or purchases.

- Commercial bills and factoring are used to a much lesser extent than ordinary trade credit.

- Post payment is by far more common than pre payment. One exeption is business services, which uses much more pre payment than the other parts of the corporate sector.

These micro level "stylized facts" represent a broadening of the picture of trade credit compared to my conclusions based on macroeconomic studies. They show that when one moves down to the micro level different firms in different parts of the Swedish corporate sector use trade credit in different ways. Thus, various credit and goods market conditions have to be taken into account when one studies how and why individual firms use trade credit.

1.4 THE INDIFFERENCE PROPOSITION

1.4.1 *The indifference proposition*

My description of what a trade credit is, and my discussion about the stylized facts lead to one fundamental question. Why is there a particular type of credit called trade credit, with the qualities summarized by the stylized facts, or alternatively, what factors determine the existence of trade credit? There are two different ways to find an answer to this question. One alternative is to start with a thorough descriptive investigation and ask why firms use trade credit. Another alternative, which I have chosen here, is to start with some well known economic models and determine if it is meaningful to introduce trade credit within a particular model framework. This approach gives of course no exact answer to the existence issue but it shows what financial and goods market conditions are of importance if trade credit is to be distinguished from other types of credit. It is assumptions about how different markets work that determine if trade credit does, or does not, have perfect substitutes. With perfect substitutes trade credit does not have unique product characteristics, and then it is not meaningful to discuss a special kind of credit called trade credit.

A common way to introduce inter temporal transfer of purchasing power is to present Walras' well-known general equilibrium model in an inter temporal context (see for example Hirshleifer (1970)). In such a model the preferences of the consumers, endowments and the production opportunities of firms are given. There is no uncertainty, everybody has perfect information, markets are perfect and complete. Finally, and from a trade credit point of view most important, there is a non-resource using frictionless transaction system. Everybody can lend or borrow at the same rate of interest.[18] There is indifference between the

18 For a more detailed discussion about an asset market without frictions and uncertainty, see Ivarsson (1980) chapter 2.

demand and supply of trade credit and other types of financial transactions. Since trading in the goods market is costless there can also be no special goods market motive to use trade credit. Note, however, that these conclusions do not mean that trade credit cannot exist in a world of this kind. Trade credit may well exist, but there are perfect substitutes and nothing that distinguishes it from other financial arrangements.

Hirschleifer (1970), among others, has extended the intertemporal model with what he calls *productive uncertainty* or "ignorance as to the outcóme of exogenous natural events that would be of concern even to a Robinson Crusoe isolated from all market opportunities" (will the harvest be good or bad?). Preferences, endowments and production opportunities for all states of the world and all dates are known. Transactions can still be carried out without resource using frictions and there is no *exchange uncertainty*. Everybody knows all exchange opportunities. The introduction of productive uncertainty does not change the conclusion that trade credit cannot produce services that do not have perfect substitutes.

A model which deals exclusively with the financial market is the capital asset pricing model (CAPM). It is designed to show how equilibrium is determined in the financial market when there is uncertainty about the outcome of investments. In its basic version it is a one period model where investors maximize the utility of wealth. Uncertainty is expressed in terms of means and variances, all investors have the same subjective estimates of investments opportunities, trading is costless, and all assets are marketable in a perfect competitive market. It can be shown that in equilibrium the expected return on asset j is

$$(1.25) \qquad E(\overline{R}_j) = R_F + \lambda \frac{\text{cov}(\overline{R}_j, R_M)}{\sigma(\overline{R}_M)} ,$$

where R_F is a riskless rate of interest, $\lambda = (E(\overline{R}_M) - R_F/\sigma(R_M))$ is a market risk premium per unit of risk, $E(\bar{R}_M)$ is the expected return on the market portfolio, $\sigma(\bar{R}_M)$ is the standard deviation of return on the market portfolio and cov $(\bar{R}_j,\bar{R}_M)$ is the covaraince between the return on asset j and the return on the market portfolio. The "-" denotes random variables.[19] The expected return of asset j is determined by the riskless rate of interest and a constant times the covariance between the return on j and the return on the market portfolio. Now, assume that asset j is sold by firm j in order to finance a purchase of goods for its production, (1.25) shows that the return on asset j, and consequently also its price, will be the same regardless of to whom it is sold. Hence, the firm is indifferent between various financing alternatives. In this kind of world there is nothing that distinguishes trade credit from, for example, the other financing alternatives I have discussed earlier in this chapter. When Copeland and Khoury (1980) discuss trade credit in a CAPM context they reach the conclusion that a firm will supply trade credit if the return on the investment is higher or equal to the market determined return on investments with the same risk. However, in a frictionsless CAPM model it is impossible to supply trade credit if the return is higher than the market determined return, nobody will demand trade credit with such conditions. Trade credit seems to be an interesting investment because nothing is mentioned about other assets. The indifference problem is lost.

The three models, which I have described very briefly, have not been designed with trade credit in mind. Despite this they are interesting from a trade credit perspective. In each model trade credit has perfect substitutes. The models include too stringent assumptions about how the markets operate in order to include a special demand for trade credit. This conclusion can be summarized in an indifference proposition.[20]

19 For a proof of this result see for example Jensen (1972).

20 A quite similar indifference proposition has been used by Schwartz and Whitcomb (1979). However, they do not take the goods market into account. Trade credit is treated only in a financial market context.

Proposition: In frictionless perfect markets (goods and financial markets) there are no unique trade credit qualities that distinguishes trade credit from other payment arrangements.

This proposition shows the central problem when one discusses the determinants of trade credit. When markets are perfect trade credit does not produce a unique service, or services, which cannot be produced at the same cost in alternative ways. Consequently, a study of trade credit must develop within a model framework which includes various market frictions. It is both financial and goods market frictions which determine how trade credit is used. Exact statements can be made first when the transactions technology and its cost/revenue relationships are known. The indifference proposition is of course valid not only for trade credit but also for other types of financial "products". If markets are perfect it is for example impossible to motivate the existence of financial intermediaries. It has long been understood that financial intermediaries develop as a response to resource using market frictions.[21] They are a part of the transaction technology which is absent in the models above. The proposition above and the preceding discussion have shown that this is also true for trade credit.

1.4.2 *Trade credit and transaction costs*

The exchange system does generally not operate without costs in a world with geographical distances, communication costs, and various types of uncertainty. A catch all, and very loose, term for costs generated by market frictions is transaction costs.

21 Recently several authors have discussed different motives for the existence of financial intermediaries. In some cases the conclusions are also applicable in a trade credit context. The statement about financial intermediaries and resource using market frictions comes from Campbell and Kracaw (1980). They discuss financial intermediaries as producers of information.

These costs can be defined as costs associated with the transfer of ownership titles from one individual (or firm) to another.[22] Transaction costs can be divided into different kinds of exchange costs both within the goods market and within the financial market. A few examples of the costs involved are the time and trouble it takes to carry out exchanges (trips to the bank, negotiations about contracts), costs to keep records, costs to enforce contracts, and taxes levied on transactions. Another type of transaction cost is various types of information costs.[23] When information is not ubiquitous and free resources are needed to gather and process information about which exchanges are advantageous. An introduction of exchange costs limits the set of profitable trading possibilities. Mayshar (1978), and others, have shown that in the CAPM model the introduction of exchange costs limits the number of assets held by each investor. Investors have "preferred habitats" within which trading takes place.[24] The financial market is divided into several sub-markets with assets especially designed to meet the needs of a limited group of transactors.

When resources are needed not only to produce goods and services but also to trade it is natural to find that some firms specialize in trading. In the financial market it is various kinds of financial intermediaries which represent this specialization. Consequently, the existence of transaction costs is one way to motivate the existence of financial intermediaries.[25]

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22 This definition comes from Niehans (1969); quite similar definitions have also been used by Demsetz (1968) and Faith and Tollison (1981).

23 I use transaction costs to represent all kinds of exchange costs. Another alternative is to treat transaction and information costs as two different types of costs.

24 See Coates (1976) page 88.

25 See Benston and Smith (1976). They use a transaction cost approach when they discuss why there are financial intermediaries.

In the same way it is possible to think of trade credit as a payment arrangement that under certain conditions has lower transaction costs than other alternatives. This implies that in a world with transaction costs the indifference proposition need not hold. Different firms are not necessarily indifferent between trade credit and other payment arrangements. Trade credit does not have perfect substitutes.

When trade credit is treated in a financial market context I have shown that regulation of commercial banks seems to affect the use of trade credit. Recurrent loan ceilings force firms out of the bank loan market. The efficiency of such a policy requires that high transaction costs keep liquidity constrained firms from moving to other parts of the financial market. Without transaction costs a loan ceiling within a limited part of the market has no effect since there is immediate adjustment within the unregulated parts of the market. If, in such a situation, trade credit expands more than other types of credit it is important to remember that it is not the regulation per se which causes this expansion. It is the existence of different transaction costs among various financial assets that can make the use of trade credit favourable. It seems, for example, reasonable to assume that the transaction costs among firms which have traded with each other for a long time are low compared to if a firm has to turn to new parts of the financial market to aquire credit. Similarly, if there is an asymmetrical spread of information financial intermediaries can ration credit regardless of monetary policy.[26] Then rationed firms can use trade credit instead, if those who supply trade credit have more information about those who demand credit than the financial intermediaries, or if they are willing to accept more risk. These are but general comments about how one can explain the use of trade credit in a financial market context. In order to be more explicit it is necessary to know in what kind of an environment the single firm works. Then it is possible to show

26 See Jaffee and Russel (1976).

why trade credit does, or does not, belong to the financial market "preferred habitat".

There are different ways to explain the use of trade credit in a goods market context. Trade credit is a part of the transactions technology and it can lower trading costs in different ways. In a seminal paper about the theory of the firm Coase (1937) argues that firms are established because of transaction costs. Resources within the firm can be used without the market mechanism. This is more efficient than having "firms" which consist of many small markets. Then one way to extend the region within which markets are feasible is to use trade credit. If payment always has to be made on delivery it is costly to monitor all transactions. Efficiency can be gained from specialization if arrivals and shipments of goods are separated from payment routines. When payment patterns are flexible the transaction cost of using a market is lowered. Trading becomes more efficient and the need to internalize transactions within firms is lessened. A fairly similar argument is used by Clower and Howitt (1978) when they discuss the transaction demand for money. They use the term "bunching costs" to represent costs associated with rapid clearance of payments when there is a simultaneous in- and outflow of money. The difference between my argument and "bunching costs" is that the separation of simultaneous goods and money flows lower transaction costs regardless of if there is a simultaneous in- and outflow of money.

Another, quite different, goods market explanation of the use of trade credit has been set forth by Faith and Tollison (1981). They note that in a world with uncertainty, information costs and asymmetrical information between buyer and seller it is costly to negotiate and enforce complete contingent contracts. One way to avoid high contract costs is to use simple payment rules insted. These rules are designed in such a way that they restrain shirking. For example, if a firm produces some good especially designed and ordered by a customer the firm can require x percent of the price as pre payment to ensure that the buyer fulfils his commitments. Here market structure is also of importance since if the market is "thin" a good ordered by a customer can perhaps not be sold to somebody else, if, on the other hand, there is a large market, failure by some customer to buy what he has ordered need not affect the firm negatively. Post payment schemes are conceivable when there is uncertainty about the quality of a good. With post payment the buyer can check the quality of the goods before he pays. The feasibility to use trade credit in order to lower transaction costs is determined by what kind of good or service is traded,and how information is distributed between buyer and seller. In order to be more explicit about what payment patterns emerge one has to have detailed information about the market which is studied. The important difference between the Hirshleifer uncertainty model, which led to the indifference proposition, and the world envisaged here is that now everybody does not have the same kind of information. The buyer, the seller, or both, can plan to deceive the other party. It is this transactions uncertainty together with contract costs that can make the use of trade credit worthwhile.

In summary the discussion has made it clear that if trade credit is to be distinguished from other types of lending or borrowing, it is necessary to assume that there exist transaction costs. This conclusion about trade credit and transaction costs is perhaps trivial, but the existence of trade credit has in many cases been taken for granted without comments. Therefore it has, in my opinion, been valuable to point out the importance of transaction costs when trade credit does not have perfect substitutes. The upshot of the discussion is that transaction costs both within the financial and the goods market affect the use of trade credit. It is the absence of transaction costs that gives rise to the indifference proposition. This implies that when trade credit is introduced in theoretical models it is necessary, either explicitly or implicitly, to introduce assumptions about various transaction costs. Since there are such costs both in the financial and in the goods market there are numerous possibilities to study different trade credit cases with various combinations of (no) uncertainty, (no) goods market transactions costs, and (no) financial market transactions costs. These can in turn be combined with various assumptions about market structure.

1.5 AN OUTLINE OF THE STUDY

The stylized facts have shown that the use of trade credit is affected by financial market conditions, sales and purchases, and by the type of good or service that is being traded. In addition to this the indifference proposition showed that if trade credit is to be distinguishable from other types of credit it is essential that there are various types of market frictions. Different frictions give rise to different motives to use trade credit. The upshot of the discussion is that when one looks at trade credit on the micro level there is a number of different aspects of how and why firms use trade credit under various financial and goods market conditions. This observation summarizes the idea behind this study. My objective is to move behind macroeconomic "black box" approaches in order to make a more detailed study of various aspects of the use of trade credit. This is done with the help of several different theoretical models, which make it possible to concentrate on one, or a few, trade credit aspects at a time. Within the different model worlds it is possible to give plausible explanations of some of the stylized facts, and to comment on different institutional conditions that affect the use of trade credit. A micro economic approach also makes it possible to identify different variables that are important determinants of how and why firms use trade credit. Identification of these variables is of interest if one wants to make empirical studies of trade credit on the micro level.

In the chapters that follow a trade credit is categorized either as a *financial trade credit* or a *transactions trade credit*. The former represents credits that include profitable interest arbitrage when goods market conditions and payment costs have not been taken into account. The latter represents the use of trade credit as a mean to reduce transaction costs in order to facilitate trade. Transactions trade credit can in turn be divided into two types. *First*, credits that are used in order to reduce the cost of making payments, or to reduce the cost of making transactions from assets to money. Here trade credits are used to reduce what can be called conventional transaction costs. *Second*, another type of transactions trade credit is various pre and/or post payment arrangements designed to reduce uncertainty when the traders do not know the ability, or intention, of the opposite party to fulfil his commitments. With this dichotomy trade credits can be divided into three different categories. In this way various aspects can be discussed one at a time. However, in reality both financial and transactions aspects simultaneously determine how and why firms use trade credit.

Chapters 2-5 contain a discussion about various aspects of financial trade credit. Transaction costs are not taken into account, and the analysis rests on the assumption that the interest rate structure is such that profitable interest arbitrage is possible. *In Chapter 2* I introduce financial trade credit in a neoclassical theory of the firm framework, and I show how profitable interest arbitrage affects the quantity of output, prices, and the distribution of output between different markets, I assume that the seller knows the interest rate of the buyer. I also discuss some methodological aspects of multiperiod models, and I present such a model, which can be used to study profit maximization when the firm faces a cash flow constraint.

In Chapter 3 I assume that only the distribution of interest rates among buyers is known. I show how the seller chooses that trade credit rate of interest which maximizes profit and I discuss how changes in the interest rate distribution affect trade credit, output, and price decisions. I also analyze a special case when the quantity of demand among different customers depends on the difference between the trade credit rate of interest and the interest rate of the buyer.

In Chapter 4 I reintroduce known interest rates, and instead I add uncertainty in terms of possible payment failure by the trade credit customers. I discuss how attitudes toward risk affect trade credit decisions, output, and the distribution of output between different markets, both when payment failure is treated in isolation, and when it is combined with demand uncertainty. One important conclusion in the chapter is that different attitudes toward risk is one motive for a varied supply of trade credit contracts.

In Chapter 5 I use the model structure set forth in Chapter 4 to discuss under what conditions trade credit is preferred to ordinary bank loans, and what the driving forces are behind the markets for factoring services, commercial bills, sale of trade credit bills and credit risk insurance. I also discuss some institutional conditions that affect the use of trade credit and give rise to an interest rate spread between various borrowing alternatives. In Chapters 2-4 I only discuss the supply side of the market. In Chapter 5 the demand side is also to some extent taken into account.

In Chapters 6 and 7 I concentrate on transactions aspects of trade credit. In these chapters trade credit is used because it reduces the cost of making both goods and financial market transactions. *In Chapter 6* I use the model structure set forth in Chapter 4, and I add uncertainty about the quality of the good that is being traded. I show under what conditions there will exist Pareto sanctioned pre or post payment contracts compared to cash payment. Here trade credit is used as a kind of insurance contract or a guarantee, and the analysis rests on the assumption that it is cheaper to use trade credit than other substitutes. I also discuss a special case which can give rise to contracts including partial cash payment. With the help of the model structure set forth in this chapter it is finally possible to present a plausible explanation of the fact that pre payment is common in the business services sector. The contents of Chapter 6 lies somewhat besides what is discussed in ordinary transaction cost literature.

In Chapter 7 I use conventional transaction cost approaches to explain why firms use trade credit. I deal with both goods and financial market transaction costs. When there is a fixed goods market payment cost I show how this generates an accumulation of debt and payment at fixed intervals when a firm purchases a continuous flow of goods or services, and when bills arrive at random I use queue theory to show how a firm determines the optimal size of its payment system. One important conclusion is that when there are fixed goods market payment costs trade credit produces a unique service compared to the substitutes. I analyze goods market transaction costs both when there is certainty and uncertainty. The same approach is used when I deal with financial transaction costs. (The cost of making conversions of assets to money.) First I introduce trade credit in the well known saw tooth model of the transactions demand for money. I discuss how this affects optimal money holdings, the length of the money holding and the trade credit period, and I discuss the determinants of the optimal size of accounts payable. As long as the cost of using trade credit is not infinitely high it pays to defer payments. This reduces money holdings compared to the no trade credit case. In the last part of the chapter I discuss how trade credit can be used to give firms some leeway when unforeseen transactions occur. By using trade credit a firm can change the timing but not the magnitude of payments. This reduces the need to make rapid unplanned asset transfers. In this context trade credit can be used as a buffer which reduces the precautionary demand for reserves. One nice property of the models in Chapter 7 is that they lead to explicit expressions of the optimal size of accounts payable as a function of interest rates, transactions costs, and purchases. Thus I present models that are easy to test empirically.

This is, briefly, the coverage of topics in the study. Aspects of trade credit is a broad subject and my choice of topics naturally includes some sins of omission, some planned, and undoubtedly also others that I am not aware of. I have, of necessity, selected a limited number of aspects. In most of the chapters I use a microeconomic approach which is very partial. I discuss one market, and one side of the market. Thus I have chosen not to treat trade credit in an equilibrium context neither on the micro nor macro level. Further I discuss one aspect at a time, and this is done by the help of several different types of models. I have made no attempt to construct a model that encompasses as many trade credit aspects as possible. Finally, my comments about the stylized facts are based on theoretical conclusions. I point out many variables that are of interest in an empirical context, but empirical tests of the relevance of my conclusions are left as a topic for future research.

2. Trade Credit in a Theory of the Firm Context

2.1 INTRODUCTION

In the preceding chapter I reached the conclusion that both goods and financial market conditions affect the use of trade credit, and the indifference proposition summarized the view that if it is to be possible to distinguish trade credit from its closest substitutes it is necessary to introduce frictions within either the financial, the goods, or within both markets. In this chapter I do not deal further with market frictions. Here I simply assume that they exist and the use of trade credit is taken for granted. I discuss the profit maximizing behavior of a firm which supplies trade credit and I mostly limit myself to a discussion about the interaction between the use of trade credit and the supply of goods when trade credit is seen solely as a financial investment. It is a partial approach, the functioning of the financial system as a whole, and the behavior of other firms is left in the background. *First*, I introduce trade credit in a profit maximization framework when the demand for goods is presented in terms of product characteristics, and I show how the use of trade credit is a part of a "combination policy" problem. *Then*, I discuss the relationship between the list price (the price paid at the end of the credit period) and the length of the credit period, and the interrelationship between the level of output and the supply of trade credit by a profit maximizing firm with monopoly power. *Finally*, I extend the model in a multiperiod context and I comment on some methodological problems in connection with trade credit in multiperiod model. I also present a model which can be used to study the interaction between trade credit, other financial variables, and the goods market. *In summary*, in this chapter trade credit is presented in a neoclassical theory of the firm context.

2.2 TRADE CREDIT AND THE THEORY OF THE FIRM - THE GENERAL CASE

In the introductory chapter i argued that trade credit is usually an integrated part of ordinary business transactions among non-financial firms. In the post payment case the supply of trade credit is equivalent to a supply of loans by the seller of goods or services to the buyer, and in the pre payment case the supply/demand relationship is reversed. The trade credit loan goes from buyer to seller. In all chapters, except Chapter 6, I mainly discuss trade credit with post payment, which is the most common trade credit arrangement. A conventional seller-to-buyer trade credit offer consists of two parts: a discount for payment within a relatively short period after delivery, and a longer net period, at the end of which full payment is required. A trade credit offer is often expressed as

$$(2.1) \qquad P_l,\ d/D,\ \text{net } T\ ,$$

where P_l = list price, which is the price that has to be paid at the end of the credit period, d = the per cent discount for payment within the discount period, D = the length of the discount period in days, and T = the length of the net period in days. (Net period = the length of the credit period if trade credit is accepted.) If no discount period is offered, payment is made either on delivery, or at the end of the credit period T. Then the terms are

(2.2) P_1, net T.

In order to simplify the exposition this is the case I study in this and the chapters that follow. The list price has to be compared with the cash on delivery price, P_o. If $P_1 = P_o$ with $T > 0$, it is always favorable for the buyer to accept a credit offer. In this case trade credit costs nothing. If $P_1 > P_o$, the difference between the two prices represents the implicit interest included in the trade credit offer.

In the conventional neoclassical theory of the firm literature there is usuallay a demand function of the type

(2.3) $Q = f(P_o, P)$.

Demand depends only on the cash on delivery price of the product, P_o, when the cash prices, P, of all other products are kept constant. In order to study the behavior of firms when there is trade credit a more general demand function is needed. Both financial and goods market aspects of trade credit must be taken into account. One way to go about it is to use a demand function which includes more parameters than just the cash price of the product itself. Trade credit can be seen as a part of a package of different product characteristics which affect demand. Such a demand function can include both financial and goods market characteristics of trade credit. This is a good example of how other variables than the cash price are included in the multidimensional demand functions discussed by for example Rasmussen (1972) and Rosen (1974). Rasmussen notes explicitly that trade credit is one parameter which is likely to affect demand besides the cash price.[1] A

[1] See Rasmussen (1972) page 170. The demand function (2.4), which implies some degree of monopoly power, has more in common with the demand function set forth by Rasmussen than the one set forth by Rosen (1974). The latter author discusses a case with perfect competition where the demand curve is horizontal for a given combination of price and product characteristics.

general formulation of a multidimensional demand function which includes trade credit is

$$(2.4) \qquad Q=Q(P_1,T,A),\ Q'_{P_1} < 0,\ Q'_T,\ Q'_A > 0,\ Q''_{P_1},\ Q''_T,\ Q''_A < 0,$$

where T is the length of the credit period and A represents other product characteristics that also affect demand, advertising for example. Here T can represent both financial and goods market aspects of trade credit.

Now assume that:

- There is a firm which produces one output and maximizes the present value of profit.
- The firm faces a demand function (2.4) which is negatively inclined with respect to the list price and positively inclined with respect to the length of the credit period and other demand affecting characteristics.
- Payment is made at the end of the credit period.
- There is a convex cost function.
- There is no uncertainty.

Given these assumptions there is a profit function

$$(2.5) \qquad \pi = e^{-r_s T} P_1 Q(P_1,\ T,\ A) - C(Q(P_1,\ T,\ A)),$$

which is to be maximized, where r_s is the interest rate of the seller.[2] The first order conditions are

$$(2.6) \qquad \pi'_{P_1} = e^{-r_s T}(Q(P_1,\ T,\ A) + P_1 Q'_{P_1}(P_1,T,A)) - C'_Q Q'_{P_1}(P_1,T,A) = 0,$$

[2] For a similar type of model formulation, see Kim and Atkins (1978).

$$(2.7) \qquad \pi_T' = e^{-r_sT} P_1 Q_T'(P_1,T,A) - r_s e^{-r_sT} P_1 Q(P_1,T,A) - $$

$$- C_Q' Q_T'(P_1,T,A) = 0,$$

$$(2.8) \qquad \pi_A' = e^{-r_sT} P_1 Q_A'(P_1,T,A) - C_Q' Q_A'(P_1,T,A) = 0,$$

(2.6), (2.7) and (2.8) state that the marginal revenue following a change in the list price, the length of the credit period and advertising respectively has to equal the marginal cost. An increase in the length of the credit period gives rise to two types of marginal costs. *First*, production costs increase because of higher demand and *second*, there is a present value reduction because the credit period is lengthened. The maximization problem is no longer only an output or price decision. The first order conditions determine a "combination policy" where the whole package of demand affecting variables are combined in an optimal way.

The generally formulated demand function (2.4) is not the only way to introduce trade credit in a profit maximization context. Nadiri (1969) uses a demand function where trade credit is treated as if it were an advertising expense. In the extreme case when the demand effect from trade credit is equivalent to the demand effects from advertising (2.7) is equivalent to (2.8). To treat trade credit as an advertising expense has been criticized by Bitros (1979). He points out that an advertising expense does not give rise to a claim by the firm on its customers, but there is such a claim when a firm supplies trade credit. Schwartz (1974) discusses initially a demand function where

$$(2.9) \qquad Q = f(P_1 = P_o | T > 0),$$

which is the case when trade credit cannot be seen as a financial investment since the implicit interest rate is zero. One way to bypass the direct goods market demand effects is to separate trade credit from the goods market. Lindsay and Sametz (1967) treat trade credit as a two good offer, the seller's product and his credit. In the literature about foreign trade credit the goods market is usually absent. The quantities produced and sold are determined outside the models and the optimal payment patterns are determined solely as portfolio problems. Two authors who use this approach are Lietaer (1971) and Soenen (1979).

The objective function in (2.5) and the marginal conditions represent a general formulation of trade credit in a theory of the firm context. The objective function is too general to say much about why firms use trade credit. Much of the work in this study represents attempts to use less general objective functions in order to shed light on various aspects of the use of trade credit. To do this different aspects are mostly treated one at a time. Next I present an alternative model formulation which concentrates on financial aspects of trade credit. It is also, more or less, a stepping-stone to the models and analysis in Chapters 3-6.

2.3 THE SCHWARTZ - WHITCOMB MODEL[3]

2.3.1 *The prices*

In order to set forth a less general model it is first necessary to discuss the connection between the cash price, P_0, and the list price, P_1, (the price paid at the end of the credit period) in some greater detail than I have done so far. Assume that trade credit is seen solely in a financial context. There are no transactions or other goods market motives to use trade credit. The seller prefers trade credit to cash payment if the

[3] See Schwartz and Whitcomb (1979).

present value of the list price exceeds the cash price, and the buyer prefers trade credit to cash payment if the present value price is lower than the cash price. Now, since I have assumed that there is no discount period the list price can be written as a function of the cash price, a trade credit rate of interest r_{tc}, and the length of the credit period T,

$$(2.10) \qquad P_1 = e^{r_{tc}T} P_o .$$

With this notation both buyer and seller are better off with trade credit than cash payment if the inequalities in (2.11) hold.

$$(2.11) \qquad P_{pvb} = e^{(r_{tc}-r_b)T} P_o < P_o < e^{(r_{tc}-r_s)T} P_o = P_{pvs} .$$

Looking at the terms on both sides of the inequality sign it is possible to distinguish two different cases

$$(2.12\ a) \quad r_b > r_t > r_s \Rightarrow P_1 > P_o, \quad T > 0,$$

$$(2.12\ b) \quad r_b < r_t < r_s \Rightarrow P_1 < P_o, \quad T < 0.$$

First, when the interest rate of the buyer is higher than the interest rate of the seller it pays to set T > 0, post payment, and with the list price set according to (2.10) it will be higher than the cash price. *Second*, when the interest rate inequality is reversed there is pre payment, which can be interpreted as T less than zero. In this case the seller is willing to set a list price which lies below the cash on delivery price. The buyers get a discount for payment in advance. The empirical picture of the Swedish corporate sector showed that trade credit with post payment is much more common than trade credit with pre payment. This implies that when the direction of the credit is seen solely in a financial market context trade credit generally goes from firms with low interest rates to firms which do not have easy access to the credit market and therefore have to pay higher interest rates. However, it is important to remember that here I only discuss trade credit from a financial market point of view. If goods market reasons for trade credit use are also taken into account it is possible to show situations when $T < 0$, even if $r_b > r_s$. (See Chapter 6.) The inequalities in (2.11) and (2.12) show that the gain from trade credit stems from profitable interest arbitrage. The division of the arbitrage profit can vary between two extreme price policies. With

$$(2.13) \qquad P_1 = e^{r_b T} P_o, \text{ (buyer price compensation)}$$

the buyer is indifferent between P_o and P_1 and the whole arbitrage profit accrues to the seller, and with

$$(2.14) \qquad P_1 = e^{r_s T} P_o, \text{ (seller price compensation)}$$

the seller is indifferent between the two prices and the profit accrues to the buyer.

A *third* case occurs when there is interest rate equality between buyer and seller

$$(2.15) \qquad r_b = r_{tc} = r_s \Rightarrow P_1 \gtreqless P_o,\ T \gtreqless 0.$$

Then both buyer and seller are indifferent between different trade credit arrangement compared to cash payment. The list price can both be higher or lower than the cash price, the direction of the credit is indeterminate, and buyer price compensation equals seller price compensation. This is also a summary of proposition one in Schwartz-Whitcomb (1979). It is an indifference proposition with regard to the choice of credit policy. In a credit market without interest rate differentials there is no financial motive for trade credit. Thus, if it is to be meaningful to discuss financial trade credit it is necessary to add the assumption that $r_b \neq r_s$. Such interest rate differentials can arise either because various information and transaction costs within the financial market give rise to situations where different firms have different information and different accessto various parts of the market, or if there are administrative regulations, e.g. ceilings on loans from commercial banks and interest rate ceilings. When such ceilings are binding some firms will be liquidity constrained in the bank loan market and they have to turn to other sources of finance. Although an interesting topic in itself an in depth discussion about the functioning of a credit market with various inefficiencies lies beyong the scope of this study. In this chapter I limit myself to note that interest rate differentials is a prerequisite for financial trade credit, and given that such differentials exist, I study how this affects the behavior of profit maximizing firms.

2.3.2 *The model*

Assume that all buyers have the same cost of capital, $r_b > r_s$, and r_b is known by the buyer. The seller produces Q, and demand is determined by its present value or effective price function

$$(2.16) \qquad Q = Q(e^{(r_{tc}-r_b)T} P_o).$$

Given this demand function the question is then what list price will the seller choose?[4] In order to sell any given output with trade credit the list price can be increased until the effective price equals the cash price. This happens when the seller uses trade credit with buyer price compensation, or to make sure

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[4] The determination of the optimal relationship between the list price and the cash on delivery price is also discussed in a paper by Hill and Riener (1979). They use a present value model and start with the non-optimal case when $P_1 = P_o$, then they show different cases with $P_1 > P_o$. A limitation of their discussion is that they do not take the present value demand function and the interest rate of the buyers explicitly into account. Consequently, with their formulation it is impossible to determine why customers accept trade credit in the first place and if there is post payment or pre payment.

that all buyers accept trade credit when r_{tc} lies slightly below r_b. With *buyer price compensation* the demand function in (2.16) equals a normal cash price demand function and the objective function of the firm can be written

$$(2.17) \qquad \pi = e^{(r_b - r_s)T} P_o Q(P_o) - C(Q(P_o)) .$$

From this formulation of the objective function it is obvious that since $r_b > r_s$, the optimal trade credit policy is to set $T = +\infty$. This is so since the price compensation scheme has the effect that when the credit period is lengthened the list price changes too, and with a constant difference between r_b and r_s the present value of revenue increases indefinitely. In order to bring the model more in line with reality, with finite credit periods, it is necessary to make some assumption about how the use of trade credit affects the interest rates, - both r_b and r_s are differentiable functions of T, $r_b(T)$ and $r_s(T)$. The interest rate functions represent the time adjusted cost of funds or the return on investment of the buyer and seller respectively. How are these credit period dependent interest rates then to be interpreted? Although I here have not taken more than one investment cycle into account assume that history repeats itself. Then, ignoring the implicit interest payment, accounts payable equals TR(Q). Thus, the longer the credit period the higher is the stock of loans. If in this case the seller has to increase his borrowing and if the buyer uses trade credits to replace other sources of funds it seems reasonable to assume that both interest rates increase with the length of the credit period, $r_b'(T)$, $r_s'(T) > 0$. If, on the other hand, the buyer faces a decreasing marginal efficiency of investment function his interest function will be falling, $r_b'(T) < 0$. Disregarding the stock effects following an increase in T the interest rate functions will be rising if there is a liquidity premium on long term credit. One case when this occurs is when the interest rates are risk adjusted and the probability of default increases with the length of the credit period. (The implications of this assumption are studied in Chapter 4.) With the interest rate functions included in the objective function in (2.17) I have presented a model which is almost equivalent to the model set forth by Schwartz and Whitcomb (1979). However, they do not derive the objective function from a more general model with a present value demand function, and they do not discuss the choice of price compensation policy.

With the new interest rate function added to (2.17) the decision problem of the firm is to maximize the present value of profit with respect to T and P_o. Differentiation with respect to T gives the first order maximum condition

$$(2.18) \qquad \pi'_T = [(r'_b(T)T+r_b(T))-(r'_s(T)T+r_s(T))]e^{(r_b(T)-r_s(T)T)} P_oQ(P_o)=0,$$

which can be rewritten

$$(2.19) \qquad MI_b(T) - MI_s(T) = 0 \ ,$$

where $MI_b(T)$ and $MI_s(T)$, the expressions within the two parentheses in the brackets in (2.18), represent the marginal interest of the buyer and seller respectively. Consequently, at the optimum the length of the credit period has to be chosen such that the marginal interest rates are equal. From (2.19) it is clear that the choice of the optimal credit period is independent of the level of output. The choice of T is separated from output decisions because of the buyer price compensation scheme. The second order conditions are π''_{TT}, $\pi''_{P_oP_o} < 0$ and $\pi''_{TT}\pi''_{P_oP_o} - (\pi''_{TP_o})^2 > 0$. Schwartz and Whitcomb (1975)[5] show that these conditions can be summarized with the following inequality

$$(2.20) \qquad M'_b(T) - MI'_s(T) < 0,$$

given the normal assumption that $\pi''_{P_oP_o} < 0$.

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[5] They use $P_o(Q)Q$ as the no trade credit revenue function, but the result holds also with the inverse function $Q(P_o)$.

In the most common case with a positive T this implies that the marginal interest rate of the seller increases faster than the marginal interest rate of the buyer. At the optimum the seller's marginal interest schedule must intersect the buyer's schedule from below.

The length of the credit period is independent of the level of output but the reverse does not hold. Differentiation with respect to the cash price gives,

$$(2.21) \qquad \pi'_{P_o} = e^{(r_b(T)-r_s(T))T}(P_oQ'(P_o)+Q(P_o))-C'(Q(P_o))Q'(P_o) = 0.$$

The level of output is affected by the choice of T because T affects the present value of revenue. For example, with $r_b(T) - r_s(T) > 0$ and $T > 0$ revenue increases compared to the case with no trade credit. The cash price falls and output increases until the marginal condition holds. Consequently, there is interdependence between the financial and the goods market. Financial market conditions affect the level of output. Now assume that the firm starts with profit maximization and no trade credit and the optimal cash price is $\bar{P}_o$. In the model above the optimal cash price with trade credit is such that

$$(2.22) \qquad P^*_o < \bar{P}_o,$$

when (2.21) holds. This is a ficticious cash price which is used to calculate the optimal list price, P^*_1. This list price must lie below the original full buyer price compensation list price, which is used when the output effect is not taken into account, because

$$(2.23) \qquad P^*_1 = e^{r_b(T)T}P^*_o < e^{r_b(T)T}\bar{P}_o = \bar{P}_1.$$

This also implies a present value price below $\bar{P}_o$. Consequently, the buyers are better off and everybody accepts trade credit. A comparison of the no trade credit cash price with the final trade credit list price shows that when there is a downward sloping demand schedule it is not optimal to use complete buyer price compensation to eliminate all demand effects.

The additional credit due to the output effect cannot be used to replace other alternative sources of funds. If a trade credit offer is accepted and if Q_o is the quantity bought without trade credit funds the size of P_oQ_o are available for alternative use during T periods. The increase in supply, dQ, does not make more funds available to the buyer. Consequently, the quantity effect cannot be used to finance retirement of other debts. It fills a financial function only if dQ is resold immediately and the revenue from the sale is used during the folloging T periods. High transaction costs in connection with rapid transfer of Q makes such a behavior unlikely.

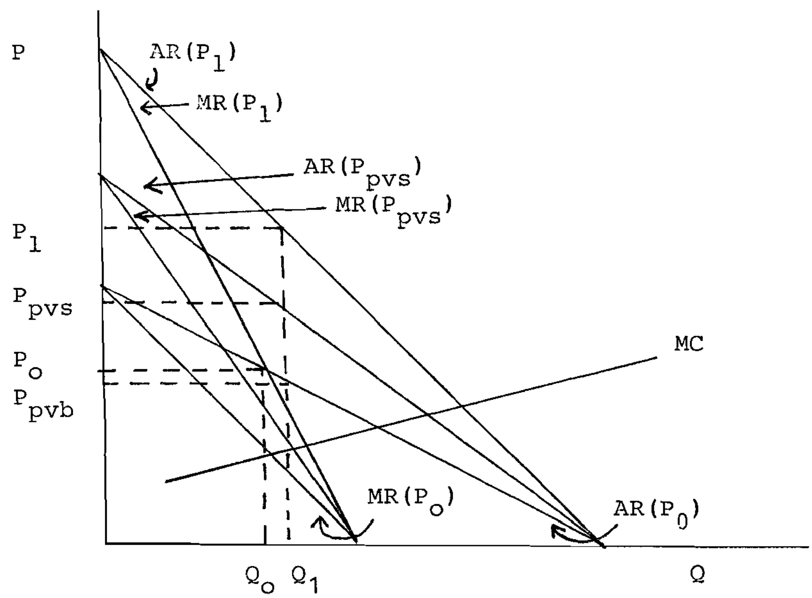

Figure 2.1 Profit maximization with trade credit, T > 0.

It is also possible to give the maximization results a graphical interpretation. This is done in Figure 2.1, which shows the case with post payment. In order to understand the figure it is first necessary to say a few words about the three different average and marginal revenue functions that are included. The AR(P_o) curve shows the demand function when there is no trade credit. The AR(P_1) shows the demand function, in list price terms, when there is trade credit with buyer price compensation. The vertical distance between the two curves is the size of the price compensation. The AR(P_{pvs}) shows the average present value revenue of the saller. From the buyer's point of view, AR(P_{pvb}) = AR(P_o). The three marginal revenue curves are interpreted in the same way as the AR-curves. The list price demand schedule lies to the right of the present value demand schedule and since $r_b > r_s$ the present value average revenue schedule of the seller lies between the two curves. Profit is maximized when $MR_{pv} = MC$, which gives a list price of P_1 and the present value price of the buyers is lower than the quantity price combination with no trade credit. MR_{pvs} to the right of the original MR ensures that the profit of the firm increases. Equality of interest rates implies that the present value AR--schedule of the firm concides with the present value AR-curve of the customers; consequently, in this case there is no present value price difference and no gains from trade credit are possible.

2.3.3 *Some comments and applications*

Monetary policy interpreted in terms of the optimum condition in (2.19) deserves a short comment. If a period with "tight money" increases the general interest rate level and the interest rate functions of buyer and seller shift upward with the same proportion the optimal length of the credit period is not affected. However, it is important to keep in mind that here trade credit is seen only as a financial investment, if other reasons for trade credit are also taken into account the conclusion need not necessarily hold. It is first when the interest rate functions do not shift in the same way that the optimal credit period changes. Non-proportional changes mean that the interest rate effects are not evenly spread in the economy. This is particularly the case when there is a binding loan ceiling in the market for bank loans. Such recurrent loan ceilings have been common in Sweden during the 70's. Rationed firms have to look for funds elsewhere and an increased use of trade credit is a substitute which lies close at hand. Several empirical studies[6] have shown that both accounts receivable and accounts payable increase during periods with a restrictive monetary policy. This observation fits with the model above if the interest rate function of the buyer shifts upward so that the length of the credit period increases. Shifts in the functions can also lead to trade credit reversals. A phenomenon which has received no attention in the empirical literature.

The ordinary textbook example of a price discriminating producer with negatively inclined demand functions shows how the same product is sold in several well separated markets at different prices. A separation of the credit market into several different markets has a similar effect. The producer sells his product with various credit terms depending on what interest rate function determines the time value of money in each market. The result is that the list price in different markets is a function of both the cash price demand conditions and the interest rate functions. The profit function with n separate markets and buyer price compensation is

$$(2.24) \qquad \pi = \sum_{j=1}^{n} e^{\left(r_{b_j}(T_j) - r_{s_j}(T_j)\right)T_j} P_{oj}(Q_j)Q_j - C\left(\sum_{j=1}^{n} Q_j\right).$$ [7]

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[6] See for example Davis and Yeomans (1974).

[7] This is the case when the trade credit contracts are refinanced one by one. Thus, the discount factor r_s can vary between different markets. Another alternative is to assume that the refinancing is treated as a lump sum. In this case r_s can e.g. be based on the average length of the credit period, when the proportions of sales to different markets have been used as weights.

In (2.24) separate cash price demand functions are not necessarily combined with different interest rate functions. The $r_{b_j}(T_j)$ function can very well be the same in several markets. With the profit function (2.24) the trade credit decision, just as in the case with one market, is independent of the decision about how to distribute output between different markets. Differentiation of (2.24) with respect to T_j gives an optimum condition equivalent to (2.19) for each market. However, the reverse is not true, the distribution of output between the different markets is not independent of the choice of optimal T:s. In order to show how output is divided between different markets assume that there are two markets with identical cash price demand functions but different interest rate functions. The interest rate function of one buyer is equivalent to that of the seller and the cost function of the producer is such that marginal cost is constant. In the no trade credit case output is distributed between the two markets in such a way that $MR_1 = MR_2 = \overline{MC}$. In the trade credit case, when T has been set in accordance with (2.19), and the interest rate function in market one is such that $r_1(T_1) = r_s(T_1) < r_2(T_2)$ for all $T > 0$, the first order condition for an optimum is

$$(2.25) \qquad e^{(r_2(T_2)-r_s(T_2))T_2} MR_2 = MR_1 = \overline{MC}.$$

The exponent in front of MR_2 is greater than one and the equality condition implies that $MR_2 < MR_1$. The difference between the interest rates has the effect that the quantity supplied is going to increase in the market with the higher rate of interest.

A credit market with many different interest rates can be the result of many types of information and transaction costs. Another factor which drives a wedge between different parts of the credit market (given the existence of information and transaction costs) is different types of regulation. If there is credit rationing in the bank loan market due to a ceiling on bank loans unsatisfied demand will be directed towards other parts of the credit market. This will push up interest rates outside the bank loan market. If in this case the seller of Q is a non-rationed firm it can profitably supply rationed firms with trade credit. The example above shows that supply is going to increase to customers with the higher rate of interest. If also equal interest rate functions represent cases with T = 0, the introduction of regulations makes it profitable to set T > 0. The size of T depends on the shape of the interest rate functions. If these are linear in T it is easy to show that the optimal T depends on the initial interest rate differential between buyer and seller. The larger the differential the longer the credit period. It can also be shown that the larger is the exponent in front of MR_2 above. This in turn implies that the interest rate differential determines the size of the quantity effect. Consequently, credit rationing can give the result that the supply of goods and trade credit increases and it is directed towards those who are rationed in the regulated part of the market. It is naturally also possible to think of examples when the firm which produces Q is rationed. Non--rationed customers are then offered pre payment arrangements. One way to avoid these side effects of regulation is to regulate more, so that those firms which supply trade credit cannot choose their own profit maximizing credit policy. These conclusions about trade credit and the distribution of output between different markets are interesting because they show how a substitution process which is unwelcome by the regulating authorities can be profitable for a single firm. Credit within the non-financial sector decreases the efficiency of the regulation.

An alternative to the profit function (2.24), with separated markets, is to assume that there is perfect competition in the goods market. The cash on delivery price is fixed, seen from the point of view of the single producer. In this case profit is maximized if the firm picks out that segment of the market which gives

$$(2.26) \qquad \max e^{(r_{b_j}(T_j)-r_s(T_j))T_j}.$$

If customers with the highest interest rate do not buy all output the best strategy is to divide output in decreasing order, starting with that part of the market which has the highest value in (2.26).

Another way to model the goods market is to assume that there is a cash price which depends on the level of output, but which has to be the same for all customers. In this case the profit function is[8]

$$(2.27) \qquad \pi = \sum_{j=1}^{n} e^{(r_{b_j}(T_j)-r_s(T_j))T_j} P_o\left(\sum_{j=1}^{n} Q_j\right)Q_j - C\left(\sum_{j=1}^{n}\right)Q_j,$$

and profit is maximized when each T_j is set according to (2.19).

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[8] Schwartz and Whitcomb argue that the profit function represents the case with the same list price for all buyers. This cannot be so. In the case when the firm is free to set each T_j according to (2.27)

$$P_{1_j} = e^{r_{b_j}(T_j)T_j} P_o\left(\sum_{j=1}^{n} Q_j\right).$$

The exponent in front of P_o will vary for different interest functions. The same is true if the firm is restricted to set $T_j = T$ for all customers. If the list price is to be the same for all customers, one way to formulate the revenue function is to write

$$R = \sum_{j=1}^{n} e^{-r_s(T_j)T_j} P_1 Q_j \left(e^{-r_{b_j}(T_j)T_j} P_1\right).$$

Here profit is maximized with respect to T_j and P_1.

(This is a case studied by Schwartz and Whitcomb (1979).) However, they do not discuss the distribution of output between different customers. In (2.27) neither the cash price nor the cost function depends on the distribution of output between different customers. This implies that the optimal policy must be to determine each T_j in (2.27), choose that T_j which gives (2.26), and sell all output to this (or these) customer(s). If those who are not offered any Q are willing to pay more, and if this is recognized by the seller, the demand function is separable and profit can be maximized by using the formulation in (2.24). One way out of this dilemma with the demand function is to assume that the seller sets the same P_o for all, e.g. because the information costs are too high to make it worth-while to find the separate demand functions. On the other hand, in the credit market the information costs must be low enough to make it profitable to determine the $r_{b_j}(T_j)$ functions. Another interpretation of (2.27) is to assume that the quantities have been chosen in such a way that

$$(2.28) \qquad P_o\left(\sum_{j=1}^{n} Q_j\right) = P_o(Q_1) = P_o(Q_2) = \ldots$$

In this case the marginal conditions need not hold. Profit can be increased if output is redistributed until the present value of marginal revenue is equal in each market. If the quantities are not chosen such that P_o is the same for all but the firm uses the same P_o anyway some customers may be rationed. Jaffee and Russell (1973) have shown that such a price policy cannot be optimal if it is possible to discriminate among different markets. The best policy is to maximize (2.24).

When different customers accept different trade credit offers it means that the customers do not find it profitable to arrange borrowing among themselves. They use different sources of finance. If the buyers supply trade credit or other types of credit among themselves this will tend to decrease the interest rate differentials. The trade credit offers become more similar. The same happens if there are many suppliers of trade credit. These two credit supply processes can eliminate the interest rate differentials among buyers. To what extent this actually happens depends on how easy it is to substitute trade credit for other types of credit. The higher the degree of substitutability the closer is the situation when it is not meaningful to talk about a particular type of credit called trade credit.

2.4 TIME INTERDEPENDENCE - SOME METHODOLOGICAL COMMENTS

2.4.1 *Time interdependence*

The model I have discussed above represents a one period decision problem which can include pre or post payment. In this section I extend the model and include a planning period over which profit is maximized. I take two additional complications into account. *Firstly,* I discuss time interdependence and *secondly,* I introduce a budget or cash flow constraint. I show how trade credit can be treated when there is a more complex production and financial system.

In Section 2.2 I discussed the determination of the best "combination policy" when the firm has an objective function formulated in terms of various product characteristics. Here I present the same type of model in a planning period context, when there is interdependence between the decision variables both within and between time periods.[9] Assume that

[9] The idea to this model structure comes from Shiff and Lieber (1974).

a profit maximizing firm with some monopoly power has a demand function of the same type as in (2.4). Demand depends on the list price $P_1(t)$, the length of the credit period $T(t)$, the level of inventory $I(t)$, good will from advertising $A(t)$, and a shift variable t. Then with instantaneous production the objective function over the planning interval $0 \leq t \leq \tau$ can be written

$$(2.29) \qquad \pi = \int_0^{\tau} e^{-rt} \left(e^{-r_s(T(t))T(t)} P_1(t)Q(P_1(t),T(t),I(t),A(t),t) - \right.$$

$$\left. - C(O(t),I(t),a(t)\right)dt,$$

with $Q'_{P_1} < 0$, $Q'_{T,I,A} > 0$, $Q''_{P_1,T,I,A} < 0$ and $C'_{O,I,a} > 0$, $C''_{O,I,a} > 0$. I assume that the interest rate, r, which is used to discount profit is constant. This simplifying assumption is introduced to avoid a discount problem.[10] There is a convex cost function which includes the level of output $O(t)$, inventory (inventory holding costs), and the level of advertising $a(t)$. Here trade credit can represent both financial and goods market or transactions trade credit. With this model

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10 If r varies over time the profit at every t has to be discounted by the sum of all different interest rates during all the short time periods in the interval $0 \leq t \leq t_1$. (For a discussion about different discount problems, see Hischleifer (1971), Chapter 4, page 110.) Then the discount factor is

$$e^{-\int_0^{t_1} r(t)dt}.$$

It is further not possible to use the $r_s(T)$ function since this is not the discount rate which is used for two t:s lying next to each other. Some time adjusted interest rate has to be used instead. It is of course possible to assume that this time adjusted interest rate is a function of the length of the credit period. However, I have judged it not to be worthwhile to take into account the additional complexity following an introduction of r which is not constant.

formulation the role of inventory is twofold. *First,* there is a "service effect". By holding inventory the firm can better service its customers. This gives a direct demand effect. *Second,* there is a "transaction effect". If inventory and backlogging are not accumulated production must equal demand. High fluctuations are introduced in production, resulting in relatively high production costs. The change in inventory (2.30) is determined by the difference between the rate of production and the rate of demand, and the change in advertising good will (2.31), which affects demand, is determined by the rate of advertising minus a depreciation factor d(t).

$$(2.30) \qquad \frac{\partial I}{\partial t} = O(t) - Q(P_1(t), \ldots, t),$$

$$(2.31) \qquad \frac{\partial A}{\partial t} = a(t) - d(t).$$

Consequently, the demand effect of advertising which is not repeated is decreasing. Now the objective function has been formulated as a dynamic optimization problem with control variables T(t), O(t), $P_1(t)$ and a(t), and the state variables I(t) and A(t). Then a necessary condition for maximization of the objective function is that the Hamiltonian function, for all $0 \leq t \leq \tau$, is maximized with respect to the control variables. Given the objective function above the Hamiltonian is [11]

$$(2.32) \qquad H = e^{-rt}\Big(e^{-r_s(T(t))T(t)} P_1(t)Q(P_1(t),T(t),I(t),A(t),t) - C(O(t),I(t),a(t)) + \lambda_1(t)(O(t) - Q(P_1(t),\ldots,t)) + \lambda_2(t)(a(t) - d(t))\Big).$$

[11] For an introduction to the mathematics of dynamic optimization, see for example Intriligator (1971) part IV.

Differentiation of H with respect to T gives the first order condition

$$(2.33) \qquad e^{-(rt+r_s(T(t))T(t))} P_1(t) Q_T' =$$

$$= (r_s' T(t) + r_s(T(t)) e^{-(rt+r_s(T(t))T(t))} P_1 Q + \lambda_1 Q_T'.$$

This expression is easy to interpret. T(t) is to be chosen in such a way that the present value marginal revenue due to an increase in T(t), the lhs of (2.33), has to equal marginal cost. The expression for marginal cost, the rhs of (2.33), includes two types of costs. The present value cost and cost of a change in the level of inventory. With unchanged output an increase in demand in period t is going to decrease inventory with Q_T', and since inventory is included in demand function demand will decrease in t+dt. The value of this demand effect is determined by $\lambda_1(t)$, which can be interpreted as the shadow price of inventory.[12] It is this indirect effect on demand which introduces trade credit time interdependence in the model. A trade credit decision in one period affects demand in the next period and consequently it must also affect future decision about trade credit, output, the level of inventory, and advertising. Similarly it is easy to show that changes in the level of output have an indirect effect on demand, and the marginal value of an increase in inventory, the shadow price of inventory, has to equal the marginal present value cost of an increase in output. Finally, advertising is used in such a way that its marginal cost is equal to the shadow price of an increase in the demand affecting stock of advertising good will.

In a numerical simulation of a model with a structure similar to the one described here Shiff and Lieber (1974)

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[12] See Intriligator (1971) page 352.

show that the length of the credit period varies inversely with demand. The use of trade credit as an instrument to smooth demand fluctuations hinges critically on their assumption that the list price is exogenous to the model and unaffected by variations in the length of the credit period. With a fixed list price it seems natural that demand depends positively on the length of the credit period. A price policy such that $P_1 = \bar{P}_1$ means that the firm has chosen not to, or is restricted not to, take advantage of a possible financial motive to supply trade credit.[13] One can also think of a fixed list price in terms of an oligopolistic market. If the competitors do not react to variations in the length of the credit period, as long as the list price does not vary, it can represent a kind of hidden price concession. The longer the credit period, the larger the price concession, and the larger the effect on demand.[14] Another alterntive price policy is to supply trade credit with the financial market as a point of reference. In this case it is optimal to use buyer price compensation to exploit interest rate differentials. However, if the firm is free to set prices any way it wants it is a priori difficult to say whether the two extreme price policies mentioned here are preferred to some "middle way" solution.

2.4.2 *A budget constraint*

In the different versions of the trade credit model I have discussed so far I have not said anything about cash flows. One way to interpret this is that funds are always instantaneously available to the firm as long as it is willing to pay r_s,

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[13] Shiff and Lieber (1974) do neither discuss different motives for trade credit nor different price policies. They assume that demand depends only on the length of the credit period and the level of inventory.

[14] For a more detailed discussion of the use of trade credit in this way, see Schwartz and Whitcomb (1978).

whatever r_s happens to be. The introduction of a budget constraint makes it possible to include cases when it is not just the cost but also the availability of funds that affects the behavior of the firm. In this context trade credit is of interest since different trade credit policies affect both the size and the time profile of the flow of funds.

A reasonable assumption about a financial restriction which has to be taken into account is that cash balances cannot be negative. If the cash balances become negative, it means that the firm cannot meet its payment obligations, bankruptcy follows and its activities have to cease. In a model with the same type of objective function as the one in (2.29) the introduction of a budget constraint means that a new state variable has to be added to the model. This state variable is the level of money holdings, M(t), and taking account of the bankruptcy assumption it is constrained to non-negative values over the planning interval. A difficulty with the introduction of M(t) as a new state variable is that the equation of motion of the money stock includes various lags when there is trade credit. (Either just trade credit given, trade credit taken, or both.) The change $\partial M/\partial t$ does not only depend on what happens in t but also on what happened when trade credit decisions were made further back in the past, e.g. with a constant credit period the change in M is

$$(2.34) \qquad \frac{\partial M}{\partial t} = R(\ldots, t-T) - C_t(I(t), a(t)) - c_t(0(t-T_1), T_1(t-T_1))$$

where T_1 represents the credit period of trade credit taken, the cost of trade credit has been included in the c_f function. Inventory and advertising expenses are still assumed to be paid immediately when they occur. Such lags make the model mathematically more complex than the model with inventory and advertising good will as the only state variables. It is no longer possible to use ordinary dynamic programming to find a solution to the optimization problem. In order to circumvent the problems caused by the introduction of lags in the budget constraint I have reformulated the model and present a discrete time version of it below.

Assume that the objective of the firms is to maximize the sum of net cash receitps over the planning period and the cash equivalent value of net assets at the end of the planning period. This type of objective function eliminates discount problems, but instead it must be possible to determine the cash equivalent values of the stock of assets and liabilities at the end of the planning period. If such prices exists the beginning (present value) and end of period maximization approaches are equivalent.[15] Combined with a cash constraint, a discrete time, end of planning period, version of the objective function in (2.29) can be written

$$(2.35) \qquad \pi = \sum_{t=1}^{\tau-T} R(..,t) - \sum_{t=1}^{\tau} C\left[I(t),\ a(t)\right] - \sum_{t=1}^{\tau-T_1} c\left[O(t),\ T_1(t)\right] + $$

$$+ m(I)I(\tau) + m(T) \sum_{t=\tau-T}^{\tau} R(..,t) - m_1(T_1) \sum_{t=\tau-T_1}^{\tau} c\left[O(t),\ T_1(t)\right],$$

subject to,

$$(2.36) \qquad M_o + \sum_{t=1}^{t-T} R(..,t) - \sum_{t=1}^{t} C\left[I(t),\ a(t)\right] - \sum_{t=1}^{t-T_1} c\left[O(t),\ T_1(t)\right] =$$

$$= M(t) \geqq 0, \text{ for } 0 \leq t \leq \tau.$$

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15 A model with such an end of period maximization approach has been used by Damon and Schramm (1972). Their model is designed to show the interaction between production, marketing and finance. It is a partial model in the sense that all long run financial and investment decisions are constant. Alternative trade credit policies and possible rationing is not discussed. The approach is interesting since it is easy to adapt this type of model to various demand, production and financial conditions.

The first term in (2.35) represents the inflow of revenue during the planning period, the second is the outflow of money due to inventory and advertising costs, and the third term represents the sum of paid production expenses, where $c'_{T_1} > 0$. These terms show the accumulation of cash balances over the planning interval. The remaining three terms represent the cash equivalent value of end of period inventory, accounts receiveble and accounts payable respectively, with asset conversion prices m(I), m(T) and $m_1(T_1)$. The τ budget constraints (2.36) are written in terms of accumulated cash flows. In (2.35) and (2.36) the credit periods are constant but this is not really necessary. The important point is that (2.35) represents accumulated in- and outflows of money regardless of when the sale and/or purchase of goods has taken place.[16] With the objective function (2.35), the constraints

16 When both sales price and the length of the credit period can vary, accounts receivable contain information about future changes in the inflow of funds and it lies in the interest of the managers of the firm to use this information as efficiently as possible. There is also a quite large literature about the information aspects of accounts receivable. Analysis of accounts receivable can be used to gather information about changes in the payment behavior of customers and to estimate changes in the inflow of money in the future due to variations in the choice of credit policy. When a firm sells its output to a large number of customers with different payment habits and different credit periods it becomes necessary to aggregate information in order to make it easily accessible to the decision makers. Such information is nowadays included in standard programs of both large and small computers. Peterssohn (1969) and (1976) gives a thorough description of various both theoretical and empirical issues in this field. Another interesting approach is the one used by Hansen (1961). He discusses variations in the in- and outflow of foreign exchage due to variations in the payments for exports and imports. His results refer to a country but they are also applicable when one discusses a budget constraint and various trade credit policies for a single firm. Hansen's mathematical treatment of lagged payment flows has passed unnoticed by most authors who do not explicitly discuss foreign trade credit.

Models that are used to process information about changes in the flow of funds represent sub-models which can be a part of larger models where both trade credit, production and finance are integrated. They represent an additional cost function, the management of accounts receivable and payable, which can be added to the model above. However, it is beyond the scope of this chapter to go further into detail, here it suffices to note that lagged payments lead to a number of information problems. This is particularly the case if there is uncertainty about future payments.

(2.36) and additional non-negativity constraints $T(t)$, $T_1(t)$, $O(t)$, $a(t) \geq 0$, the reformulation of the model is completed. The trade credit non-negativity constraints are of course a drawback since they exclude trade credit with pre payment. Now the model has been reformulated as a non-linear programming problem of the type[17]

(2.37) Maximize $\pi = f(x)$
subject to $g_t(x) \leq r_t, \quad 1 \leq t \leq \tau, \; x \geq 0.$

Such a maximization problem has a global maximum if the Kuhn--Tucker conditions hold, and the objective and constraint functions are concave. Unfortunately I have not been able to show that the concavity conditions hold. Consequently I do not use the Kuhn-Tucker conditions to characterize a global maximum. Damon and Schramm (1972) argue that a similar type of model does not fulfil the concavity conditions. They use a numerical solution algorithm developed by Fiacco and McCormick (1968) to find local maxima. This algorithm is also applicable to the model above. By varying the initial conditions it is possible to check whether there are several different local maxima or if the model converges to a global maximum which determines the optimal choice of trade credit, output and advertising policy.

Now assume that the model has been solved with the help of some solution algorithm and there is reason to believe that a global maximum has been found. Then (2.35) represents the best possible cash equivalent position of the firm at τ. The objective function can be rewritten in terms of a balance sheet which shows the optimal stocks of assets and liabilities, given that these stocks have been accumulated in such a way that the cash expenditure constraint has been fulfilled in

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17 See Damon and Schramm (1972), page 168 and Chiang (1974), page 722.

every period during the planning interval. Taking account of the budget constraint at τ the accumulated net cash flow in (2.35) can be written,

$$\text{(2.38)} \qquad \sum_{t=1}^{\tau-T} R_t - \sum_{t=1}^{\tau-T_1} C_t - \sum_{t=1}^{\tau} c_t = M(\tau) - M_o.$$

Insertion of this expression in the objective function gives

$$\text{(2.39)} \qquad \pi = M(\tau) - M_o + AR + I - AP$$

where AR, I and AP represent accounts receivable, inventory and accounts payable respectively, when all three are evaluated with their cash equivalent values. This expression for profit can be interpreted as a balance sheet,

Assets	Liabilities
$M(\tau)$	AP
AR	M_o
I	π

with cash balances $M(\tau)$, accounts receivable and inventory on the asset side, and accounts payable, initial money holdings, which can represent money invested by the owners of the firm, and the balancing item π, which represents accumulated net worth over the planning period, on the liability side.

Since this end of period balance sheet includes only a few assets and liabilities it is a kind of "bare bones" description of the production and financial activities of most firms. It has been designed especially to give an example of how trade credit together with other demand affecting variables can be included in a multiperiod profit maximization framework when there is a budget constraint that can restrict the activities of the firm. The demand function, which is general enough to include both financial and goods market trade credit, and the budget constraint together reflect the repercussions of different trade credit policies. With the same kind of model structure it is of course also possible to include a wider spectrum of financial assets and liabilities. One alternative is to let the balance sheet of the firm be represented by the liquidity portfolio I discussed in Chapter 1. If also cost functions representing various transaction costs in connection with goods and financial transactions are included it is possible to study the interaction of trade credit with other financial variables, and how variations in the financial environment affect the composition of the liquidity portfolio and production decisions of the firm. However, it is beyond the scope of this chapter to build and solve a full scale numeric model. In this section with methodological comments I have had the more limited objective to discuss what problems and possibilities one encounters when profit maximization with trade credit is not just a one period decision problem but a problem which involves maximization over some prespecified planning interval.

In the chapters that follow I do not use a programming approach to study trade credits. Instead I set forth several different models designed to deal with different aspects of trade credit, and I make no attempt to present a general model encompassing my multitude of aspects. One possible step in the latter direction is to use the programming approach presented here, but with a more complex structure. Then, by the help of numerical methods, it is possible to study how various goods and financial market conditions affect the use of trade credit and the composition of the optimal liquidity portfolio of the firm. The virtue of such an approach is the ease with which many aspects can be handled simultaneously, and the vice, which I have deemed more important, its lack of analytical clarity.

2.5 SUMMARY

In this chapter I have discussed trade credit in a theory of the firm context when the existence of trade credit has been taken for granted. I started with a demand function based on goods characteristics, and I showed how the supply of trade credit is a part of a "combination policy" problem. Then I discussed the interdependence between the list price and the length of the credit period when trade credit was treated as a financial investment, $r_b > r_s$ implied post payment and a list price higher than the cash on delivery price, and $r_b <$ r_s implied pre payment with a discount. The "stylized facts" have earlier shown that post payment is much more common than pre payment. This means that if the interest rates are seen as the only determinants of the direction and the duration of the credit, trade credit flows from low to high interest firms or consumers dominate. Another explanation is that it is not only the $r_b \gtrless$ relationship that determines the distribution of credit with pre or post payment. I used the Schwartz--Whitcomb model to show that trade credit with buyer price compensation has an output effect and that the introduction of trade credit affects the distribution of output between different markets, and I reached the conclusion that changes in monetary policy affect the credit period only if the interest rate functions do not shift in the same way. This occurs when the effects of monetary policy are unevenly spread in the economy. In the section with methodological comments I discussed some problems in connection with multiperiod maximization. The introduction of a budget constraint gave rise to lags that make it difficult to use ordinary dynamic optimization. One way out of this dilemma is to maximize the end of period cash equivalent position of the firm. I described a "bare bones" model which led to an optimal end of period liquidity portfolio when the budget constraint had been taken into account. A model framework similar to the one set forth here can also be made more complex with more assets and liabilities, it is also possible to introduce a transaction system which gives rise to transaction costs such that the composition of the liquidity portfolio is affected. Thus, the model can be extended in various ways if one wants to study numerically how the liquidity portfolio is affected by varying financial and goods market conditions.

3. Trade Credit with Unknown Interest Rates

3.1 INTRODUCTION

In the preceding chapter I discussed profit maximization in a model where the seller knew the interest rates of the customers, and he took advantage of this knowledge by supplying trade credit with buyer price compensation. The interest rate implicit in the credit offer was set equal to the interest rates of the buyers. However, in many cases it is unlikely that the seller has such perfect knowledge about the interest rates of the customers. High information costs make it more likely that the seller knows, for example, that a large proportion of the customers have interest rates that are higher than his own cost of capital, but he does not know the interest rate of each individual customer. In this chapter I take uncertainty about the interest rates of the buyers into account. Here the analysis rests on the assumption that high information costs make it uneconomical to have perfect knowledge. The seller does not know the interst rate of each individual customer, but he knows the distribution of interest rates, and his own proposition in the distribution. One prerequisite for the discussion in this chapter is that there exists a distribution of interest rates. The existence of such a distribution is a question of both theoretical and empirical interest. However, to make a thorough empirical investigation is beyond the scope of this chapter. I limit myself to the observations in Table 3.1, which shows minimum and maximum rates of interest of some common types of short term loans. Since the spread is from ten

Table 3.1 The distribution of interest rates on three types of short term loans, Sweden 1982. Source: Sveriges Riksbank, Statistical Yearbook, 1982, and ICA--Kuriren, No. 13, 1983.

Commercial Banks, credit on account	10-25 %
Finance Companies, credit cards	24-30 %
Trade Credit (installment credit)	27-50 %

to fifty percent the table supports the idea that there exists a distribution of interest rates. Credit on account at ten percent is usually reserved for a few prime business customers, and trade credits between firms also include interest rates below twentyseven percent. These figures indicate that a seller which supplies trade credit will meet customers with very varying interest rates. The dispersion depends of course also on what type of good or service that is being traded. Consumers generally pay higher interest rates than firms. Thus, a firm which sells both to the business and household sector is likely to meet much more interest variation than for example a firm which is a subcontractor to large corporations. There are many ways to explain the existence of the interest rate distribution presented above. Loans to different customers are more or less risky, different market participants have different information and search costs, commercial bank loan ceilings and interest guide lines give rise to credit rationing, and rationed customers are forced to search for unregulated loan channels without interest guide lines. Further in the market for loans to the household sector the subjective rate of time preference and the income tax structure matters. Interest payments are deductible from gross income when the personal income tax is determined. The effect of this is that persons with different marginal income tax can accept different pre tax interest rates with the same post tax rate of time preference.

When the seller only knows the distribution of interest rates he can still make a profit from interest arbitrage, but now it is impossible to use buyer price compensation to set the list price. Assume that the seller uses the formula

$$(3.1) \qquad P_1 = e^{rT}P_o \ ,$$

to calculate the list price P_1, when r is an implicit interest rate which relates the list price to the cash price P_o, and T is the length of the credit period. Then both seller and buyer are better off with than without trade credit if the inequality in (3.2) holds

$$(3.2) \qquad e^{(r-r_s)T}P_o > P_o > e^{(r-r_b)T}P_o \ ,$$

where r_s and r_b represent the interest of seller and buyer respectively. The present value of the seller has to be higher than the cash price and the present value price of the buyer has to be lower than the cash price. Now, the seller chooses r, which in turn determines the list price. Assume that when the interest rate of an individual customer is not known the seller makes the same credit offer to all his customers. A low r, and consequently a low list price, means that a large number of customers will accept the credit offer, but the interest arbitrage profit will be low. A high r means a high profit from interest arbitrage, but a low number of customers will accept the credit offer. There is consequently a trade off between the size of r and the number of trade credit customers. It is the consequences of this trade off that I study in this chapter. It is the distribution of interest rates and the position of the seller's own interest rate in this distribution that govern the choice of trade credit policy. External factors such as fiscal and monetary policy affect the use of trade credit by their effect on the interest rate of the seller and the distribution of interest rates.

The analysis that follows is of the same partial type as in the preceding chapter. In the *first* section I set forth a model which presents the supply of trade credit by a single firm, with a constant cost of capital, and a known distribution of interest rates among its customers. The model fits, for example, a retail store with a brand name monopoly. I show that there exists an optimal combination of trade credit rate of interest and cash price. In the *second* section I analyse the comparative statics of the model, and I comment on how changes in the interest rate distribution affects the supply of trade credit, output, and prices. One of the conclusions is that an increase in the rate of interest of the seller is not wholly passed on to the customers (when the interest rates are normally distributed). I also show that the results are distribution specific. In the *third* section I extend the model and include a quantity effect. Among trade credit customers, the quantity demanded depends both on the cash price and the difference between their own rate of interest and the trade credit rate of interest. I show that this demand effect lowers the optimal trade credit rate of interest. In order to solve the model I assume that the interest rates are exponentially distributed.

3.2 THE MODEL[1]

3.2.1 *Profit maximization without a direct quantity effect*

The model presented below is based on the following assumptions:

- There are n different customers with identical demand functions, $Q(P_o)$, $Q'(P_o) < 0$, $Q''(P_o) \leq 0$.
- There is no direct trade credit effect on demand. Those who find the use of trade credit favorable pay the list price instead of the cash price, but the quantity demanded is not altered. (This assumption will be relaxed later.)
- The cost of producing, or buying, one unit of Q is constant, C.
- The length of the credit period T = 1.
- The seller does not know the interest rate, r_b, of individual customers, but he knows the cumulative distribution function, F(r), of interest rates.
- The interest rate of the seller is constant, r_s.

With these assumptions the expression for profit is

$$(3.3) \qquad \pi = nQ(P_o)\left[P_o\left(e^{r-r_s}\big(1-F(r)\big) + F(r)\right) - C\right],$$

where r is the implicit interest rate used to calculate the list price, F(r) represents the proportion of customers such that $r_b < r$, and $\big(1-F(r)\big)$ is the proportion of customers with $r_b > r$, those who use trade credit. The decision problem

[1] I wish to thank Torsten Persson and Pehr Wissén who sowed the seed to this approach. They bear no responsibility for the model formulations I have chosen here. Errors and obscurities are entirely my own.

of the seller is to find r and P_o such that (3.3) is maximized. This model formulation fits, for example, a retail store with a local brand name monopoly when the aggregated demand for credit is known at the same time as it costs too much to determine the interest rate of each individual customer. Since the firm cannot discriminate between each customer's willingness to pay for a trade credit loan everybody is offered the same list price. The revenue part of the profit function above can also represent the decision problem of a bank that lends X SEK to each customer when its own cost of borrowing is constant. In the no trade credit case the expression within brackets reduces to P_o - C. This occurs first when r is set high enough to exclude all trade credit customers, then (1-F(r)) is zero, and second when $r = r_s$, the case when the seller makes no profit from interest arbitrage. From (3.4) it is clear that the choice of r is independent of the choice of cash price, while the reverse does not hold. Thus, the optimal r and the proportion of trade credit and no trade credit customers are found by maximizing the $K(r,r_s)$ function in (3.4):

$$(3.4) \qquad K(r_s,r) = e^{r-r_s}\left[1-F(r)\right] + F(r)$$

The assumption that T = 1 has been included to simplify the model. If T were included in the maximization it is still necessary to include an assumption that the interest rates are functions of the length of the credit period, otherwise both T and (3.4) go to infinity if there is some interest rate $r_b > r$.

Before I discuss specific distributions I give a heuristic proof that (3.4) has a maximum, $r^x > r_s$, if the distribution is finite in the sense that F(r) = 1 before r approaches infinity.

Proof. *First*, at $r = r_s$, $K(r_s,r)$ equals one. *Second*, when $r > r_s$, the sum of F(r) and $[1-F(r)]$ is equal to one, at the same time as $[1-F(r)]$ is multiplied by a positive number greater than one. Thus, in this case $K(r_s,r)$ must be greater than when $r = r_s$.

Third, when the distribution is finite $K(r_s,r)$ is again equal to one when r reaches its upper boundary, or if the distribution does not have an upper boundary when F(r) approaches one before r goes towards infinity. Consequently $K(r_s,r)$ must have a maximum in the interval $r_s < r < \bar{r}$, where $\bar{r}$ represents $F(r) = 1$.

Given that there exists a maximum the $K(r_s,r)$ function has a first order condition which is easy to interpret in terms of marginal revenue and marginal cost,

$$(3.5) \qquad e^{r-r_s}\left[1-F(r)\right] = (e^{r-r_s} - 1)F'(r).$$

The lhs of (3.5) represents the marginal revenue following an increase in r. The interest arbitrage profit per trade credit customer increases. The rhs of (3.5) represents the marginal cost of increasing r, when this cost is expressed in terms of the loss of trade credit customers. Thus, this is a summary of the interest arbitrage profit, number of trade credit customers, trade off I mentioned in the introduction. MR is positive and MC is equal to zero when r equals r_s, and MC exceeds MR if

$$(3.6) \qquad \frac{e^{r-r_s}}{e^{r-r_s}-1} - \frac{F'(r)}{1-F(r)} < 0,$$

where (3.6) is a rewriting of (3.5). When r gets large the limit value of the inequality becomes[2]

$$(3.7) \qquad -\frac{f'(r)}{f(r)} > 1 \quad \text{or,} \quad r < \varepsilon_r,$$

where ε_r represents the interest rate elasticity of the density function, f(r). The upshot of this is that there exists a maximum with MR = MC if the distribution is such that in the right tail there exists interest rates such that the elasticity of the density function exceeds the interest rate. Two distributions with this right tail property are the normal and the exponential. With the former the inequality becomes

[2] To derive this expression I have used L'Hôpital's rule and $F'(r) = f(r)$, $F''(r) = f'(r)$, when f(r) represents the density function of r.

$$r > m + \sigma^2 ,$$

and with the latter

(3.9) $$1/m > 1 ,$$

where m is the average rate of interest and σ^2 is the variance. (The distributions are defined on pages 92 and 104.[3]) Thus, in both cases there exist solutions to (3.4).

The discussion above has shown that the problem how to choose the optimal interest rate can be solved. Next, I show how this affects the choice of cash price and the level of sales. With $K(r_s,r)$ from (3.4) inserted in (3.3) the expression for profit is

(3.10) $$\pi = nQ(P_o)[P_oK(r_s,r) - C].$$

After some reshuffling of terms maximization with respect to P_o gives the first order condition

(3.11) $$\pi'_{P_o} = K(r_s,r)[P_o + Q(P_o)/Q'(P_o)] - C = 0.$$

Now assume that r is at its optimum $r^x > r_s$, which implies that $K(r_s,r^x) > 1$, and let P_o represent the no trade credit cash price which maximizes profit. Then the lhs of (3.11) must be positive and P_o must fall until (3.11) holds. The introduction of trade credit consequently reduces the cash price and increases the quantity demanded compared to the no trade credit case. This conclusion is analogous to the conclusion about the quantity of sales in the no uncertainty case, when the seller knew the interest rates of the buyers. The second order condition with respect to P_o can after some reshuffling of terms be written

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3 For a detailed description of the properties of these distributions, see Blom (1970).

(3.12) $$\pi''_{P_oP_o} = 2K(r_s,r^x)Q'(P_o) + \left(K(r_s,r^x)P_o - C\right)Q''(P_o) < 0,$$

which always holds, given the normal assumptions about the shape of the demand function. Finally

(3.13) $$\pi''_{P_or} = K'_{r^x}(r_s,r^x)\left(P_o + Q(P_o)/Q'(P_o)\right) = 0,$$

because $K'_{r^x}(r_s,r^x) = 0$, and the second order condition

(3.14) $$\pi''_{P_oP_o}\pi''_{rr} - (\pi''_{P_or})^2 > 0$$

holds because the fact that $K(r_s,r)$ has a maximum implies $\pi''_{rr} < 0$, (3.12) is negative and (3.13) is equal to zero. Consequently, there exists an optimal combination of r and P_o, which together determine the list price $P_1 = (\exp r^x)P_o$, such that expected profit is maximized when some of the customers use trade credit. Furthermore the quantity of sales is not independent of this solution.

3.2.2 *Normally distributed interest rates*

3.2.2.1 A numerical example

In order to make a closer analysis of how the firm reacts to various changes in the exogenous variables it is necessary to have information about the shape of the F(r) function and the underlying probability density function. In this section I assume that the interest rates are normally distributed. *First*, in a numerical example I determine the optimal trade credit rate of interest, and with the help of Figure 1 I explain the profit maximizing behavior of the firm in terms of how to choose the most favorable position on a loan demand curve. In the *second* section I make a detailed analysis of the comparative statics of the model.

With this assumption the cumulative distribution and its derivatives are the following functions:

$$(3.15) \qquad F(r,m,\sigma) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{(r-m)/\sigma} e^{-\xi^2/2} d\xi \ ,$$

$$(3.15') \qquad F'_r(r,m,\sigma) = \frac{1}{\sqrt{2\Pi}\ \sigma} e^{-\frac{(r-m)^2}{2\sigma^2}} \ ,$$

$$(3.15'') \qquad F'_r(r,m,\sigma) = - F'_m(r,m,\sigma) = - (\frac{\sigma}{r-m}) F'_\sigma(r,m,\sigma) \ ,$$

$$(3.15''') \qquad F''_{rr}(r,m,\sigma) = - \frac{1}{\sqrt{2\Pi}\ \sigma}((r-m)/\sigma^2) e^{-\frac{(r-m)^2}{2\sigma^2}} \ .$$

With normally distributed interest rates (3.5) cannot be solved analytically to find r^x because $F(r,m,\sigma)$ is represented by a non-elementary integral, but it is relatively easy to solve the equation numerically. In Figure 3.1 I have depicted a numerical solution to (3.5), when r_s=0.1, m=0.25 and σ=0.071. The marginal revenue function represents the increase in interest arbitrage profit, given that the number of trade credit customers is unchanged, and the marginal cost function shows the loss of interest arbitrage profit due to a loss of trade credit customers when r rises. The loan demand function, D_L, is based on the cumulative distribution function. When $r=r^x$ all customers with an r exceeding r^x accept a trade credit offer. There are $n(1-F(r,m,\sigma,))$ such customers and each customer borrows P(Q)Q. When r is high enough nobody demands trade credit, and when r goes to zero all customers accept credit. The decision problem of the firm is to find the optimal position on this demand curve. Given the assumptions about the size of the interest rate of the seller, the average interest rate, and the standard deviation, the optimal trade credit rate of interest becomes $r^x \approx 0.2265$, and $(1-F(r,m,\sigma)) \approx 0.633$. Consequently, in this example, roughly

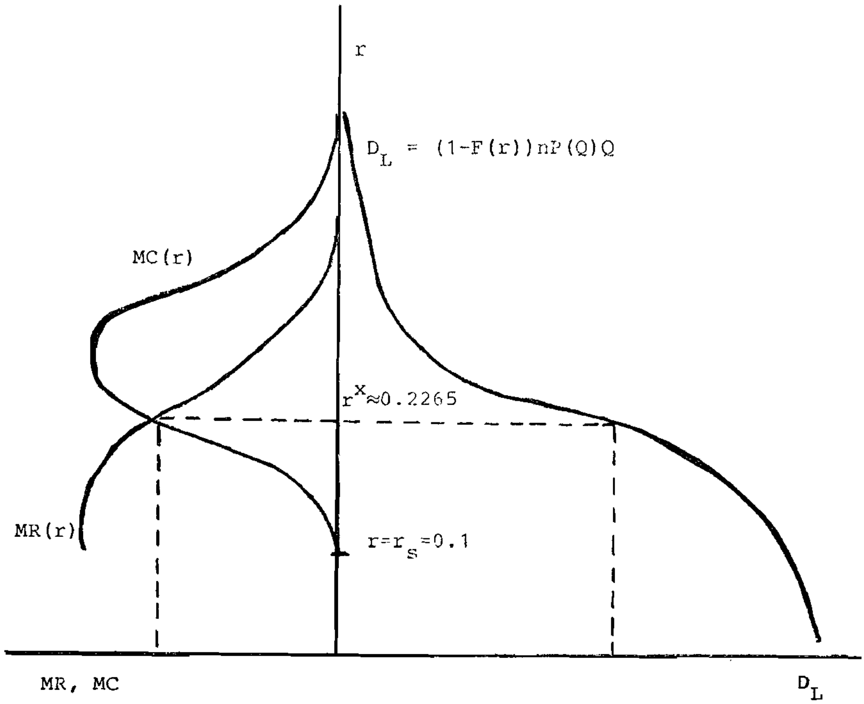

Figure 3.1 *The choice of the optimal trade credit rate of interest $r^x \approx 0.2265$, with $r_s = 0.1$, $m = 0.25$ and $\sigma = 0.071$.*

63 percent of the customers accept the optimal credit offer, since they have a rate of interest exceeding r^x. To some extent my choice of the shape of the normal distribution reflects the dispersion of the interest rates in Table 3.1. With m = 0.25 and $\sigma = 0.071$ the probability of an r outside the interval $0 < r < 0.5$ is for all practical purposes equal to zero. The probability that r exceeds 0.1 is roughly 0.98. Thus, most of the firm's customers are potential trade credit customers, and $r_s = 0.1$ represents roughly the commercial bank return on deposits during the period in question. Eightyfour percent of the customers have interest rates in the interval $0.15 < r < 0.35$, which means that my example largely reflects the interest structure among household. Thus, my numerical example can represent a firm which sells consumer durables.

3.2.2.2 The comparative statics of the model

In this section I assume that a maximum has been found, and I discuss how changes in the exogenous variables r_s, m, and σ affect r^x, the number of trade credit customers, the cash price, the quantity demanded, and the list price. With normally distributed interest rates a short form of the objective function in (3.4) is $K(r_s,r,m,\sigma) = 0$.

Implicit differentiation of the first order condition with respect to r and t_s gives[4]

$$(3.16) \qquad \frac{dr}{dr_s} = \frac{-F_r'(r,m,\sigma)}{K_{rr}''(r_s,r,m,\sigma)} > 0,$$

where $K_{rr}''(r_s,r,m,\sigma)$ is negative, and $F_r'(r,m,\sigma)$ is positive, see (3.15'). This means that an increase in the cost of capital of the seller, (cet.par.), raises the interest rate which is used to calculate the list price. A higher r means that $[1-F(r,m,\sigma)]$ falls, the proportion of trade credit customers falls. A result which confirms the intuitive conclusion that a firm with a high cost of capital is unlikely to have a large number of trade credit customers.

Next, assume that the whole distribution shifts to the right, dm > 0. For example, there is a restrictive credit policy aimed at the household sector. The households are rationed

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[4] The numerator in (3.16) is

$$e^{r-r_s}[1-F(r)] \quad - \quad e^{r-r_s} F_r'(r)$$

Add and subtract $F_r'(r)$,

$$\left[e^{r-r_s}[1-F(r)] + F'(r)\left[1 - e^{r-r_s}\right]\right] - F_r'(r).$$

The expression in brackets is K', which is equal to zero. The numerator can consequently be written $-F_r'(r)$.

in the bank loan market and have to turn to other sources of finance. This is a recurring phenomenon in the Swedish bank loan market, where banks often are requested to be very restrictive with loans to the household sector, while all available funds are channelled either to the public sector or to industry. Implicit differentiation of (3.5) with respect to r and m gives

$$(3.17) \qquad \frac{dr}{dm} = \frac{-K''_{rm}(r_s,r,m,\sigma)}{K''_{rr}(r_s,r,m,\sigma)} > 0.$$

(3.17) is positive because both the denominator and the numerator are negative.[5] Hence an increase in the average interest rate of the customers increases the implicit list price interest rate. The effect on the proportion of trade credit customers is

$$(3.18) \qquad -\frac{dF(r,m,\sigma)}{dm} = \frac{K''_{rm}(r_s,r,m,\sigma)}{K''_{rr}(r_s,r,m,\sigma)} F'_r(r,m,\sigma) - F'_m(r,m,\sigma) > 0.$$

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5 The denominator is negative and consequently the sign of (3.14) is determined by the sign of $-K''_{rm}(r_s,r,m,\sigma)$ which can be written

$$- F'_r(r,m,\sigma)\left[e^{r-r_s} + \frac{r-m}{\sigma^2}\left(1 - e^{r-r_s}\right)\right].$$

This expression is negative if the bracketed expression is positive, which it is if the following inequality holds:

$$r < m + \sigma^2(1/1-e^{r_s-r}).$$

At $r = r^x$ it holds because then (3.8) cannot hold, and $r < m + \sigma^2$. Since the expression within parenthesis is greater than one the conclusion about the inequality follows. Hence, the bracketed expression is positive, and the numerator in (3.17) is negative.

(3.18) is proved below.[6] This means that an increase in the average rate of interest also increases the proportion of trade credit customers. Trade credit is thus one way to channel demand for credit from a regulated bank loan market. This of course given that the customers are willing to pay a higher rate of interest.

Finally, I study the effects of variations in the dispersion around the mean, $d\sigma > 0$. Differentiation of (3.5) with respect to r and σ gives

$$(3.19) \qquad \frac{dr}{d\sigma} = - \frac{K''_{r\sigma}(r_s,r,m,\sigma)}{K''_{rr}(r_s,r,m,\sigma)} \gtrless 0.$$

I have found it difficult to determine the sign of (3.16) analytically. Therefore I have used the numerical example discussed earlier to calculate $K''_{r\sigma}$ when $r = r^x = 0.226$, $m = 0.25$ and $\sigma = 0.071$. In this special case $K''_{r\sigma}$ is positive, and since K''_{rr} is negative, this implies that

$$(3.19') \qquad \frac{dr}{d\sigma} > 0.$$

Consequently, an increase in the standard deviation increases the list price interest rate. The effect on the proportion of trade credit customers is[7]

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6 Taking account of the equalities in (3.15") it is possible to write

$$F'_r(r,m,\sigma)\left[\frac{K''_{rm}(r_s,r,m,\sigma)}{K''_{rr}(r_s,r,m,\sigma)} + 1\right] > 0.$$

F'_r is positive, hence the inequality holds if the bracketed expression is positive. This requires that the ratio of second order partial derivatives, which is negative (see (3.17)), is greater than -1. This ratio can after some reshuffling of terms be written

$$\frac{F'_r\left[e^{r-r_s} + \left((r-m)/\sigma^2\right)\left[1-e^{r-r_s}\right]\right]}{-F'_r\left[e^{r-r_s} + \left((r-m)/\sigma^2\right)\left(1-e^{r-r_s}\right) + 1\right]} > -1.$$

It is evident that the absolute value of the numerator is lower than that of the denominator. Consequently, the inequality holds and the lhs of the first expression must be positive. This proves that (3.18) is positive.

7 To derive (3.20) I have used the equalities in (3.15").

$$(3.20) \qquad \frac{-dF(r,m,\sigma)}{d\sigma} = F_r'(r,m,\sigma)\left[\frac{K''_{r\sigma}(r_s,r,m,\sigma)}{K''_{rr}(r_s,r,m,\sigma)} + \frac{r-m}{\sigma}\right].$$

Since in the numerical example $r^x < m$, both expressions within the brackets are negative. Hence, in this case (3.18) must be negative. An increase in the standard deviation decreases the proportion of trade credit customers.

One way of looking at the results in (3.19') and (3.20) is to let them represent what happens when more and more customers discover some new credit alternative, for example credit card credit. If this is the main alternative to trade credit the interest rate distribution becomes more centered around these interest rates, and in turn, if this is interpreted as a decreasing σ the trade credit rate of interest falls and the proportion of trade credit customers increases. An alternative way of treating the results above is to interpret variation in the dispersion around the mean to be the result of changes in the marginal income tax structure in the household sector. In Sweden interest payments are deductible from gross income when the personal income tax is determined. The effect of this is that persons with a high income and a high marginal income tax can tolerate high interest rates and the reverse is true for low income groups. This implies that households with a high marginal income tax and/or a high rate of time preference lie in the right tail of the interest rate distribution and those with a low marginal income tax lie in the left tail. In recent years the income tax schedules have been revised. The objective has been to give the majority of Swedish wage earners a marginal income tax of roughly fifty percent. With unchanged income tax revenue such a reform means that the marginal income tax of high income households is lowered, while the reverse holds for low income households. This means that the effect of the tax reform on the interest rate distribution is likely to be a reduced standard deviation. In the numerical example above the consequence of this is that the list price interest rate falls and the proportion of trade credit customers increases. Thus, the interest rate effect is in accordance with what seems intuitively reasonable. This is a highly simplified example of how the tax structure can affect the use of trade credit in the household sector. To be more realistic it is possible that higher moments have to be taken into account if variations in the tax structure also affect the skewness of the interest rate distribution. However it is beyond the scope of this chapter to explore the intricacies of the Swedish tax system in greater detail.

So far, my analysis of the model has dealt with the effects of changes in the exogenous variables, r_s, m, and σ on the list price interest rate and the proportion of trade credit customers. It remains to analyse the effects on the cash price and the quantity of output. The price and quantity effect is indirect and goes via the $K(r_s,r,m,\sigma)$ function.

The cash price effect following an increase in r_s is[8]

$$\text{(3.21)} \qquad \frac{dP_o}{dr_s} = \frac{-\left[P_o Q'_{P_o}(P_o) + Q(P_o)\right]\left[-e^{r-r_s}\left(1-F(r,m,\sigma)\right)\right]}{\pi''_{P_oP_o}} = \frac{-(-)(-)}{-} > 0$$

[8] To derive (3.21), first differentiate (3.10) with respect to P_o, then differentiate the resulting expression with respect to P_o and K to get

$$dP_o = \frac{-\left[P_o Q'_{P_o}(P_o) + Q(P_o)\right]}{\pi''_{P_oP_o}} \, dK(r_s,r,m,\sigma)$$

where dK is

$$dK = K'_{r_s}\, dr_s + K'_r \frac{-F'_r}{K''_{rr}}\, dr_s .$$

Now, $K'_r = 0$ at $r = r^x$, and it is possible to write

$$dK = -e^{r-r_s}(1-F(r))dr_s .$$

The first parenthesis in the numerator is negative because otherwise the first order condition $\pi'_{P_o} = 0$ is violated. That the second parenthesis is negative is clear from footnote 8, and the denominator, (3.12), is negative. The result is a positive dP_o/dr_s. An increase in the interest of the seller raises the cash price, and consequently also reduces the quantity of output. Hence, not only those who use trade credit but all customers are affected by a change in r_s. This price effect taken together with the fact that $dr/dr_s >$ > 0 means that an increase in the interest rate of the seller has a double effect on the list price. First the list price increases because r increases and second it increases because $dP_o/dr_s > 0$ and P_o is used to calculate the list price, P_1.

The effect of an increase in the average rate of interest on the cash price is[9]

$$(3.22) \qquad \frac{dP_o}{dm} = \frac{-\left[P_o Q'_{P_o}(P_o) + Q(P_o)\right]\left[F'_r(r,m,\sigma)\left[e^{r-r_s} - 1\right]\right]}{\pi''_{P_o P_o}} = \frac{-(-)\,(+)}{-} < 0,$$

where the second parenthesis in the numerator is positive because F'_r is positive and $r > r_s$. Hence, an increase in the average rate of interest decreases the cash price. A rightward shift in the interest rate distribution increases the profit from trade credit. Then, in order to increase sales, the cash price must fall. The effect on the list price is ambiguous.

$$(3.23) \qquad \frac{dP_1}{dm} = e^r\left[P_o \frac{dr}{dm} + \frac{dP_o}{dm}\right] \lesseqgtr 0,$$

[9] The second parenthesis in the numerator is

$$K'_m = -e^{r-r_s}F'_m + F'_m = F'_r\left(e^{r-r_s} - 1\right) > 0.$$

see (3.17) and (3.21). On the one hand, there is a positive interest rate effect, $dr/dm > 0$, it pays to charge a higher list price when the interest rates of the customers rise. On the other hand, there is a negative cash price effect, $dP_o/dm < 0$, when profit increases it pays to increase sales. The final effect on the list price depends on the strength of these two forces which work in opposite directions.

Finally, the effect of an increase in the dispersion around the mean is[10]

$$(3.24) \qquad \frac{dP_o}{d\sigma} = \frac{-\left\{P_o Q'_{P_o}(P_o) + Q(P_o)\right\}\left\{\frac{r-m}{\sigma} F'_r\left(e^{r-r_s} - 1\right)\right\}}{\pi''_{P_o P_o}} = \frac{-(-)(+,-)}{-} \gtrless 0.$$

The sign is ambiguous, but it is clear that in those cases when $r < m$, $dP_1/d\sigma$ is positive, as in the numerical example I have commented on earlier, and negative when $r > m$. In the numerical example there is consequently no ambiguity about the effect on the list price. Since both $dr/d\sigma$ and $dP_o/d\sigma$ are positive, an increase in σ increases the list price. This means that the marginal income tax effect, which was represented by a falling σ, has a double reducing effect on the list price. First, there is the interest effect and second, there is the effect on the cash price. Further $dP_o/d\sigma >$ > 0 implies that a falling σ has a positive effect on the quantity of sales.

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10 The second parenthesis in the numerator is

$$K'_\sigma = - e^{r-r_s} F'_\sigma + F'_\sigma.$$

Then, taking account of (3.15") on page 92, it is possible to write

$$K'_\sigma = \frac{r-m}{\sigma} F'_r\left(e^{r-r_s}-1\right) \gtrless 0,$$

the sign of which depends on if r lies to the left or right of the averge interest rate.

The comparative statics of the model are summarized in Table 3.2.

Table 3.2 The comparative statics of the trade credit model with normally distributed interest rates. The circled inequality signs in the third column represent the numerical example $r^x = 0.226$, $m = 0.25$, $r_s = 0.10$ and $\sigma = 0.071$.

$dr/dr_s > 0$	$dr/dm > 0$	$dr/d\sigma \underset{<}{\textcircled{>}} 0$
$-dF/dr_s < 0$	$-dF/dm > 0$	$-dF/d\sigma \overset{>}{\textcircled{<}} 0$
$dP_o/dr_s > 0$	$dP_o/dm < 0$	$dP_o/d\sigma \underset{<}{\textcircled{>}} 0$
$dQ/dr_s < 0$	$dQ/dm > 0$	$dQ/d\sigma \overset{>}{\textcircled{<}} 0$
$dP_1/dr_s > 0$	$dP_1/dm \gtrless 0$	$dP_1/d\sigma \underset{<}{\textcircled{>}} 0$

Note that the effects on $-F$, P_o and Q following a shift in r_s are opposite those of shifts in m. This is the equivalent of shifts in either the $r_s(T)$ or the $r_b(T)$ function in the case with known interest rates in the preceding chapter. Here, either the position of r_s in the interest distribution is altered, or the position of the whole distribution is altered. Note also that the circled inequality signs in the $d\sigma$ column are equivalent to those in the dr_s column. Consequently, in the numerical example with $r^x < m$ the effect of variations in σ always go in the same direction as variations in the interest rate of the seller. A closer analysis of the $d\sigma$ column shows that $r^x \gtrless m$ is an important determinant of the signs of the differentials. A consequence of this is that the position of r_s in the interest rate distribution matters when the effects of variations in σ are analysed.

The effects of dr_s and dm in the table above show what happens if for example variations in monetary policy discriminate against either the seller or the buyers. It remains to analyse the case when, for example, a tight monetary policy is such that the interest rate distribution shifts to the right at the same time as the position of the interest rate of the seller is maintained in the distribution. Then

$$(3.25)\qquad \frac{dr}{dm}\Big/_{dm=dr_s} = \frac{dr}{dm} + \frac{dr}{dr_s}\ ,$$

and by using (3.16), (3.17), footnotes 3, and 5, it is possible to write,

$$(26)\qquad \frac{dr}{dm}\Big/_{dm=dr_s} = \frac{\left(-K''_{rr_s}(r_s,r,m,\sigma) - K''_{rm}(r_s,r,m,\sigma)\right)}{K''_{rr}(r_s,r,m,\sigma)}\ ,$$

which is

$$(3.27)\qquad \frac{dr}{dm}\Big/_{dm=dr_s} = \left[\frac{-F'_r(r,m,\sigma)\left(e^{r-r_s} + \frac{r-m}{\sigma^2}\left(1-e^{r-r_s}\right)+1\right)}{-F'_r(r,m,\sigma)\left(e^{r-r_s} + \frac{r-m}{\sigma^2}\left(1-e^{r-r_s}\right)+1\right)}\right] = 1.$$

Hence in this case the increase in the trade credit interest rate is equivalent to the increase in m and r_s. Further since both dr/dm and dr/dr_s are positive it follows that

$$(3.28)\qquad 0 < \frac{dr}{dm} < 1, \quad 0 < \frac{dr}{dr_s} < 1.$$

Consequently, when either the average interest rate of the customers or the interest rate of the seller, but not both, increases, the whole interest rate effect is not passed on to the implicit trade credit interest rate.

The cash price, and quantity effect, following a $dm = dr_s$ shift of the distribution goes via its effect on the $K(r_s,r,m,\sigma)$ function.

$$(3.29)\quad dK(r_s,r,m,\sigma)_{/dm=dr_s} = (K'_{r_s} + K'_m)dm + 2K'_r(\frac{dr}{dm})dm.$$

Since $K'_r = 0$, by using footnote 7, and (3.15"), it is possible to write

$$(3.30)\quad \frac{dK(r_s,r,m,\sigma)}{dm}_{/dm=dr_s} = -\left[e^{r-r_s}\left(1-F(r,m,\sigma)\right) + F'_r(r,m,\sigma)\left(1-e^{r-r_s}\right)\right] =$$

$$= - K'_r(r_s,r,m,\sigma) = 0.$$

Hence, the $K(r_s,r,m,\sigma)$ function is left unchanged, which means that when $dm = dr_s$, the cash price and the quantity of sales is left unaffected. The results in (3.27) and (3.30) are intuitively easy to grasp. The position of the distribution is changed but not its shape. Similarly, the in a profit maximization context relevant part of the interest arbitrage, number of trade credit customers, trade off function, $K(r_s,r,m,\sigma)$, is shifted, but its shape is left unchanged. Its new maximum must therefore also change with the same scale factor as the interest rate shift, and since the interest margin is unchanged $K(r_s,r,m,\sigma) =$ $= K(r_s+dm,\ r+dm,\ m+dm,\ \sigma)$. *In summary*, a symmetric shift of the interest rate distribution representing a monetary policy, which is such that it only affects the general level of interest rates, affects the choice of trade credit policy only by an increase in r such that $dr=dm=dr_s$.

3.3 EXPONENTIALLY DISTRIBUTED INTEREST RATES

In the model above the assumption about normally distributed interest rates made it impossible to derive the optimal trade credit rate of interest analytically. However, with the help of a numerical example I presented a solution which was not altogether unrealistic, given the interest rate dispersion in

Table 3.1. The shape of the distribution of interest rates is ultimately an empirical issue. In many cases the upper and lower bounds probably lie much closer than in Table 3.1. However, it is theoretically interesting to show how a change of distribution affects some of the results presented above. One distribution which suits the model particularly well is the exponential distribution, which with the same notation as in the preceding section is

$$(3.31) \qquad F(r,m) = 1 - e^{-\frac{r}{m}}, \quad r \geq 0,$$

$$(3.31') \qquad F'_r(r,m) = f(r,m) = \frac{1}{m}e^{-\frac{r}{m}},$$

$$(3.31'') \qquad E(r) = m = \sigma.$$

I do not claim that exponentially distributed interest rates are a realistic description of the Swedish interest rate structure, but with exponentially distributed interest rates it is easy to derive an expression for r^x analytically. Since this distribution is easy to work with I also use it in the next section where I extend the model. Now, the $K(r_s,r,m)$ function, see (3.4), which is to be maximized, is

$$(3.32) \qquad K(r_s,r,m) = e^{r(1-(1/m))-r_s} + 1 - e^{-r/m} .$$

Differentiation with respect to r gives the first order maximum condition

$$(3.33) \qquad K'_r(r_s,r,m) = \left(1-\frac{1}{m}\right)e^{r(1-(1/m))-r_s} + \frac{1}{m}e^{-r/m} = 0 ,$$

and by solving for r the optimal trade credit interest rate is

$$(3.34) \qquad r^x = r_s - \ln(1-m) ,$$

and it is reasonable to assume that m is less than one. Consequently $-\ln(1-m)$ is positive and $r^x > r_s$. Insertion of the numbers from my numerical example ($r_s = 0.1$ and $m = 0.25$) in (3.34) gives an r^x which is roughly 0.388. In this case 33 percent of the customers have interest rates below 0.1 and 13 percent have interest rates exceeding 50 percent. It can further be shown that the second order condition

$$(3.35) \qquad K''_{rr}(r_s, r, m) < 0$$

holds at $r = r^x$.[11]

Differentiation of (3.34) with respect to r_s and m gives

$$(3.36) \qquad \frac{dr}{dr_s} = 1, \; \frac{dr}{dm} = \frac{1}{1-m} > 1.$$

Consequently a rise in the interest rate of the seller is wholly passed on to the trade credit customers, and an increase in the average interest rate of the customers raises the trade credit interest rate by more than the change in the average rate. From (3.34) and (3.36) it also follows that when $dm = dr_s$, and both change simultaneously,

$$(3.37) \qquad \frac{dr}{dm}_{/dm=dr_s} = 1 + \frac{1}{1-m} > 1 ,$$

---

[11] Proof of (3.35)

$$K''_{rr} = e^{-r/m}\left[\left(1-\frac{1}{m}\right)^2 e^{r-r_s} - \frac{1}{m^2}\right] < 0,$$

divide by $e^{-r/m}$, use (3.33), and the fact that $e^{r_s - \ln(1-m) - r_s} = \frac{1}{1-m}$ to get

$$\left(1-\frac{1}{m}\right)^2 \frac{1}{1-m} - \frac{1}{m^2} < 0,$$

which reduces to

$$-m < 0$$

and the inequality holds because m is positive.

which when interest rates were normally distributed was equal to one, see (3.27). That this result is not the same as previously depends on the fact that with exponentially distributed interest rates variations in m change both the position and the shape of the distribution. (dm also equals $d\sigma$ since $E(r) = m = \sigma$.) The observations in (3.36) and (3.37) make it clear that the effect on the trade credit interest rate following variations in r_s and/or m are greater compared to the case with normally distributed interest rates. In the preceding section a simultaneous change in m and r_s did not affect the cash price and the quantity of sales. The equivalent to (3.30) is in the case studied here[12]

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[12] Proof of (3.38). Insertion of $r^x = r_s - \ln(1-m)$ in the bracketed expression gives

$$\frac{1}{1-m} + \frac{r_s - \ln(1-m)}{m^2}\left(1 - \frac{1}{1-m}\right),$$

which can be simplified to,

$$\frac{1}{m(1-m)}\left(m - r_s + \ln(1-m)\right).$$

First, if $r_s \geq m$ this expression is negative because $\ln(1-m) < 0$ in the interval $0 < m < 1$. Second, if $r_s < m$ it is still negative if $g(m)=m+\ln(1-m) < 0$ in the interval under consideration. Now

$$g'(m) = 1 - \frac{1}{1-m} < 0$$

and

$$g''(m) = \frac{1}{(1-m)^2} > 0.$$

Thus, if $g(m) < 0$ close to $m = 0$ the inequality holds in the whole interval. At $m = 0.01$, $g(0.01) = -0.00005$. Consequently the bracketed expression in (3.38) is negative independently of $r_s \gtrless m$, and the whole expression above must be positive.

$$(3.38)\quad \frac{dK(r_s,r,m)}{dm}\bigg/_{dm=dr_s} = -e^{-r/m}\left[e^{r-r_s} + \frac{r}{m^2}(1-e^{r-r_s})\right] > 0,$$

at $r = r^x$. The $K(r_s,r,m)$ function varies, which means that the cash price and the level of sales is affected. Use footnote 7 and (3.38) to get

$$(3.39)\quad \frac{dP_o}{dm}\bigg/_{dm=dr_s} = \frac{-\left[P_oQ'_{P_o}(P_o) + Q(P_o)\right]\left[-e^{-r/m}(e^{r-r_s} + \frac{r}{m^2}\ (1-e^{r-r_s}))\right]}{\pi''_{P_oP_o}}$$

$$= \frac{-(-)(+)}{-} > 0,$$

which in turn implies a positive quantity effect.

In summary, a simultaneous increase in the interest rate of the seller and the average interest rate of the customers raises the trade credit interest rate by more than the change in r_s, and lowers the cash price, which means that the effect on the list price is ambiguous. None of these effects are equivalent to the corresponding results in the case with normally distributed interest rates. A weakness of the model is thus that the choice of interest distribution is of great importance.

3.4 THE QUANTITY EFFECT - AN EXTENSION OF THE MODEL

3.4.1 *Revenue and cost functions*

The model used so far has not included a direct effect from trade credit on demand. In this section I extend the model and include trade credit in the demand function. To do this I need one additional assumption:

- The choice of trade credit policy affects demand if $r_b \geq r$, then $Q = Q(P_o,r,r_b)$ with $Q'_r < 0$ and $Q'_{r_b} > 0$, if $r_b < r$, then $Q = Q(P_o)$.

This extension introduces a new direct quantity effect between the choice of trade credit policy and the level of sales. Initially I present a general formulation of the extended model and then I use exponentially distributed interest rates when I show that there exists an optimal combination of r and P_o, which in turn determines the list price, aggregated demand and the division between trade credit and no trade credit customers. I use exponentially distributed interest rates because it simplifies the analysis considerably, while it still is possible to compare the model with its simplified version in the preceding section.

When there are n customers with identical demand functions total revenue is

$$(3.40) \quad TR = nP_o\left[e^{r-r_s}\int_r^{\infty} Q(P_o,r,r_b)f(r_b,m)dr_b + Q(P_o)F(r,m)\right] ,$$

where, in turn, the bracketed expression reduces to (3.4), the no quantity effect case, if $Q(P_o,r,r_b)$ is replaced by $Q(P_o)$. With this formulation of the model there are three different revenue effects following an increase in the trade credit rate of interest. *First*, the interest arbitrage profit per unit trade credit sale increases, *second*, the number of trade credit customers falls, and *third*, the quantity demanded by remaining trade credit customers falls. From a revenue point of view it pays to increase r as long as the first positive effect outweighs the two negative effects.

When I retain the assumption that the seller buys each unit of Q at a constant price C the cost function is

$$(3.41) \quad TC = nC\left[\int_r^{\infty} Q(P_o,r,r_b)f(r_b,m)dr_b + F(r,m)Q(P_o)\right] ,$$

which reduces to $nCQ(P_o)$ when the trade credit demand effect is eliminated from the demand function. The expressions (3.40) and (3.41) taken together constitute the general fomulation of the extended model. To analyse the effects of the introduction of trade credit in the demand function it is further necessary to make an assumption about the size of the quantity effect.

When I discussed trade credit and profit maximization in the known interest rates case I used a demand function

$$(3.42) \qquad Q(e^{r-r_b}P_o) \ ,$$

which reduced to

$$(3.43) \qquad Q(e^{r_b-r_b}P_o) = Q(P_o) \ ,$$

because I assumed that the seller used buyer price compensation to calculate the list price. In the case with unknown interest rates it is impossible to use buyer price compensation, for at least some customers it is likely that $r < r_b$ as long as r_s is not too high. This means that there is a quantity effect,

$$(3.44) \qquad Q(e^{r-r_b}P_o) > Q(P_o).$$

The analysis is simplified if the lhs of (3.44) is easily integrable. Therefore, I make the additional assumption that the trade credit demand function is homogeneous with respect to the exponent of the positive difference between the interest rates. The trade credit demand function is

$$(3.45) \qquad e^{r_b-r}Q(P_o) \ ,$$

which means that the quantity effect depends on the size of the difference between the interest rate of a given customer and the trade credit interest rate. The difference between the present value approach, (3.42), used in the no uncertainty case, and the demand function in (3.45) has been depicted in Figure 3.2.

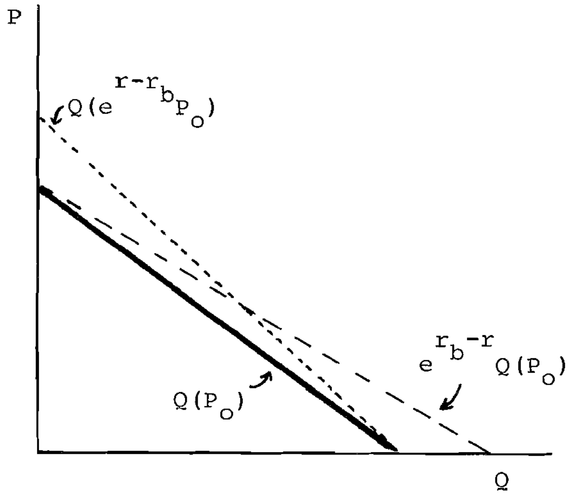

Figure 3.2 The difference between the two trade credit demand functions.

The figure has been constructed in the following way. The bold line represents the no trade credit demand schedule. Assume that the firm introduces trade credit without altering the cash price, whatever it happens to be, and $r < r_b$. The two demand functions shift out in the way depicted in the figure. The shifts on the vertical and horizontal axis need not be identical. As long as P_o is either not very high or very low the two demand schedules lie close to each other. The homogeneity assumption simplifies the exposition because to integrate it is now not necessary to know the shape of the cash price part of the demand schedule, $Q(P_o)$

in (3.45) can be moved in front of the integral sign in (3.40) and (3.41). Taking account of the assumption about the trade credit demand function, the profit function is

$$(3.46) \qquad \pi = nQ(P_o)\Big[P_o\big(e^{-r_s}\int_r^\infty e^{r_b} f(r_b,m)\,dr_b + F(r,m)\big) - \\ - C\big(e^{-r}\int_r^\infty e^{r_b} f(r_b,m)\,dr_b + F(r,m)\big)\Big].$$

3.4.2 *The exponential distribution*

Despite the simplifications it is still cumbersome to analyse the model the way it is formulated above. With a general, or a normal, density function it is also not possible to determine r^x analytically. To finally reach a manageable model formulation I assume that the interest rates of the buyers are exponentially distributed, see (3.31)-(3.31"), with this assumption the integral in (3.46) is

$$(3.47) \qquad \int_r^\infty \frac{1}{m} e^{r_b(1-\frac{1}{m})} dr_b = \frac{1}{1-m} e^{r(1-\frac{1}{m})} \quad ,$$

and insertion of (3.47) and (3.31) in (3.46) gives the final expression

$$(3.48) \qquad \pi = nQ(P_o)\Big[P_o\Big(\frac{1}{1-m} e^{r(1-\frac{1}{m})-r_s} + (1-e^{-\frac{r}{m}})\Big) - \\ - C\Big(\frac{1}{1-m} e^{-\frac{r}{m}} + (1-e^{-\frac{r}{m}})\Big)\Big] \; ,$$

where the quantity effect is represented by the term 1/1-m, compared to the corresponding model in the preceding section, see (3.32). By comparing (3.30) and (3.48) it is also clear that the choice of trade credit policy no longer is an independent sub-problem, which can be solved irrespective of the cost function, and the choice of cash price. The first order maximum condition with respect to r can after some manipulation be written

$$(3.49) \qquad \pi'_r = nQ(P_o)\,\frac{1}{m}\,e^{-\frac{r}{m}}\Big[P_o(1 - e^{r-r_s}) - C(1 - \frac{1}{1-m})\Big] = 0,$$

with the second-order condition,

$$(3.50) \qquad \pi''_{rr} = -\,e^{r(1-\frac{1}{m}) - r_s} < 0.$$

Hence, given P_o, there exists an r that maximizes π. Solving (3.49) gives

$$(3.51) \qquad r^x_Q = r_s + \ln\Big(1 - \frac{C}{P_o}(1 - \frac{1}{1-m})\Big).$$

The subscript Q has been added to denote that it is the optimal trade credit interest rate when there is a quantity effect. The logarithm is positive because the expression following the cost price ratio is negative. Consequently, also in this case the trade credit rate of interest exceeds the interest rate of the seller, but now the choice of r is not independent of the cost function and the choice of cash price. Further[13]

[13] Proof of (3.52). The two interest rate expressions are

$$r^x_Q = r_s + \ln\Big(1 - \frac{C}{P_o}\,(1 - \frac{1}{1-m})\Big) < r_s - \ln(1-m) = r^x.$$

This expression can be rewritten

$$\ln\left[\frac{(1-m)}{(1-m)}\;\Big(1 - \frac{C}{P_o}(1 - \frac{1}{1-m})\Big)\right] < -\ln(1-m),$$

which is equal to

$$\ln\Big(1 - m - \frac{C}{P_o}\,(1-m-1)\Big) < 0,$$

or

$$1 - m + \frac{C}{P_o}\,m < 1,$$

which reduces to

$$P_o > C.$$

(3.52) $r_Q^x < r^x$, if $P_o > C$.

Hence, if the latter inequality holds, which intuitively seems reasonable, the introduction of a quantity effect reduces the trade credit interest rate, because now a high profit from interest arbitrage is counteracted by its negative demand effect on all trade credit customers. With an assumption of a one hundred percent mark up (this assumption is of course a gross simplification since P_o is one of the endogenous variables of the maximization problem), and the same assumptions about r_s and m as before ($r_s = 0.1$, and $m = 0.25$), $r_Q^x = 0.254$, which is to be compared with $r^x = 0.338$ in the no quantity effect case.

The first-order maximum condition with respect to P_o can be written

(3.53) $$\pi'_{P_o} = \frac{k(r_s,r,m)}{l(r_s,r,m)} \left(P + \frac{Q(P_o)}{Q'(P_o)}\right) - C = 0,$$

where[15]

(3.54) $$\frac{k(r_s,r,m)}{l(r_s,r,m)} > 1.$$

The size of this ratio implies that also in this case the optimal cash price will be lower than the cash price in the case when there is no trade credit at all.

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$$\frac{k(r_s,r,m)}{l(r_s,r,m)} = \frac{\frac{1}{1-m}\, e^{-\frac{r}{m} + r - r_s} + 1 - e^{-\frac{r}{m}}}{\frac{1}{1-m}\, e^{-\frac{r}{m}} + 1 - e^{-\frac{r}{m}}} > 1$$

because

$$e^{r-r_s} > 1.$$

The second-order condition is

$$(3.55) \qquad \pi''_{P_oP_o} = Q''_{P_oP_o}(P_o)\left[\frac{k(r_s,r,m)}{l(r_s,r,m)}P_o - C\right] + \\ + \frac{2k(r_s,r,m)}{l(r_s,r,m)}Q'_{P_o}(P_o) < 0$$

and the inequality holds, given the assumption about the shape of the demand function and the assumption about the latter inequality in (3.52). The third second-order condition, $\pi''_{rr}\pi''_{P_OP_O} - (\pi''_{rP_O})^2 > 0$, is a long unwieldy expression the sign of which I have been unable to determine. Hence, here I simply assume that it holds.

The first-order condition $\pi'_{P_O} = 0$ in (3.53) can also be written in terms of the cash price elasticity of demand, ε_{P_o}

$$(3.56) \qquad P_o\frac{k(r_s,r,m)}{l(r_s,r,m)}(1-\varepsilon_{P_o}) + \varepsilon_{P_o}C = 0.$$ [15]

Hence, also in this case the conventional price elasticity condition $\varepsilon_{P_o} > 1$ is fulfilled. Further if the firm makes a positive no trade credit profit at $\varepsilon_{P_o} = 1$ it also holds that

$$(3.57) \qquad C < P_o \quad \text{at} \quad \varepsilon_{P_o} > 1 ,$$

and the inequality requirements $C < P_o$ and $C < \left(k(r_s,r,m)/\right.$ $\left./(r_s,r,m)\right)P_o$ in (3.52) and (3.55) hold.

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$$\varepsilon_{P_o} = -Q'_{P_o}\frac{P_o}{Q},$$

and (3.53) is equivalent to

$$\frac{k(r_s,r,m)}{l(r_s,r,m)}\left[\left(\frac{P_oQ'_{P_o}}{Q}\right)Q + Q\right] + \left[-\frac{P_oQ'_{P_o}}{Q}\right]\frac{Q}{P_o}C = 0,$$

which reduces to

$$P_o\frac{k(r_s,r,m)}{l(r_s,r,m)}(1-\varepsilon_{P_o}) + \varepsilon_{P_o}C = 0.$$

This concludes my presentation of the extended model with a quantity effect. I do not repeat the comparative statics of the preceding section. Calculations I have made show that, despite my simplified demand function and choice of interest rate distribution, it is difficult to compare the results from this model with those from the model without a quantity effect, without using numerical methods. The important difference between the two models is that the quantity effect reduces the trade credit rate of interest, which, in turn, means that the number of trade credit customers increases. Quantity effects other than the one presented here are naturally also conceivable. The size, and design, of the effect determines whether the optimal r lies close to r^x in (34) or r_s. The lower the quantity effect, the closer the solution is to r^x.

3.5 SUMMARY

In this chapter I have studied the behavior of a profit maximizing firm which supplies trade credit when only the distribution of interest rates is known. I have shown how the trade off between a high interest rate arbitrage profit per customer or a large number of trade credit customers is solved, both with and without a specific trade credit demand effect. Given that the results are distribution specific the analysis has shown that there exists combinations of a trade credit rate if interest, a cash, and a list price such that over all profit is maximized. The comparative statics in the first model, and the quantity effect in the extended model, have shown that also in this world there is an interdependency between the choice of trade credit policy, cash price, and output. Changes in the shape of the interest rate distribution were interpreted as changes in the marginal income tax rates or the effects of monetary policy, which in turn had implications for the choice of trade credit policy. Some other interesting results were that, when the interest rates are exponentially distributed an increase in r_s is wholly passed on to the trade credit customers, while this is not the case with normally distributed interest rates. Finally I showed that the introduction of a quantity effect reduced the trade credit rate of interest if the firm makes a profit on cash sales. In addition to uncertainty about the interest rates of the customers there is usually also uncertainty about payment at the end of the credit period. This type of uncertainty and its implications for the choice of trade credit policy is my topic in the next chapter.

4. Trade Credit and Payment Uncertainty

4.1 INTRODUCTION

In this chapter I continue my discussion about trade credit in a financial market context. Extended versions of the trade credit model in Chapter 2 are used to shed light on some additional aspects of the use of trade credit. The important difference between this chapter and the preceding one is that now I drop the assumption about unknown interest rates and reintroduce the assumption that the seller knows the interest rates of the buyers. Instead, I assume that there is uncertainty about payment at the end of the credit period.

In Section 4.2 I introduce uncertainty about payment at the end of the credit period, and I discuss the trade credit and output decisions of a credit granting firm in terms of a simple mean--variance utility function. One way to look at the utility function is to let maximization of expected profit represent firms with a large number of owners that can diversify their investments while, for example, mean-variance maximizers can represent small firms where the owner(s) has (have) few possibilities to diversify. I show that the introduction of uncertainty reduces both the length of the credit period and the quantity of output compared to the no uncertainty case, and this effect depends on the degree of risk aversion. In Section 4.3 I extend the model and include more than one market. I show how the correlation between the payment behavior in different markets affects the decisions of the firm. Here different markets can represent both different types of customers within a country or sales to many different countries. I also introduce uncertainty about the position of the demand function. The introduction of double uncertainty increases the complexity of the model, now both payment uncertainty and uncertainty about the position of the demand function has to be taken into account. This is, for example, a situation describing the decision problems of an exporting firm when trade credit contracts include a foreign invoice currency. In Section 4.4 I discuss why firms use down payment agreements and I show that a firm which maximizes expected profit will require down payments only if this produces additional information about the payment behavior of the customers. If this is not the case the supply of trade credit will be an all or nothing decision. However, I also show that this need not hold if the firm has risk aversion. Risk aversion is one motive to supply trade credit with a down payment regardless of whether the down payment produces additional information or not. Finally, in Appendix I I show that a safety first decision rule leads to results similar to the ones presented earlier in the chapter, and I also show that the same conclusion holds with respect to a changed payment behavior, when complete default is replaced by uncertainty about the payment date. In Appendix II I compare my choice of credit granting rule with an other common alternative which rests on the assumption that a refusal to grant credit eliminates demand completely.

4.2 TRADE CREDIT AND BAD DEBTS

4.2.1 *The utility function*

The objective of this chapter is to study the behavior of a firm of the same type as in Chapter 2, but now I also include uncertainty about payment at the end of the credit period. Before I introduce this type of uncertainty in this section I first present a mean-variance utility function, with the help of which it is possible to study how uncertainty and different attitudes towards risk affect the bahavior of a credit granting firm. The use of a utility function means that either the firm is owned by a single person, and that all decisions are determined by his utility function, or that the firm is operated by individuals sufficiently similar to justify the use of a "representative" utility function.[1] In the case with a local brand name retail store, as in the preceding chapter, the assumption about a utility function, which represents the attitudes towards risk, seems reasonable. As far as I know, nobody has yet discussed trade credit decisions within a country or currency area in these terms. The introduction of uncertainty and a utility function adds new aspects to the maximization problem from Chapter 2 and the connection between trade credit and output decisions. The analysis that follows is based on two main building blocks. It is a mixture of the theory of the firm under uncertainty and an analysis of attitudes towards risk familiar from portfolio theory. In two seminal articles about uncertainty and the theory of the firm Sandmo (1971) and Leland (1972) use a general von Neumann-Morgenstern utility function. Instead I have chosen to carry out my analysis with the help of a mean-variance utility function, which also is linear in variance. Of course, this places rather stringent restrictions on under what conditions my conclusions are strictly applicable. However, my choice of utility function makes it easy to adapt the model in Chapter 2 to payment uncertainty, and it is easy to carry out the analysis in terms of risk adjusted interest rates. A property that I use both in Chapters 5 and 6. Thus, between the choice of analytical generality and elegance or a simplified exposition I have chosen the latter. Consequently a topic for future research is to study to what extent the results presented in this chapter hold with a more general utility function.

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1 See Sandmo (1971).

Foreign trade credit, or rather various lending and/or borrowing arrangements when different currencies are involved, has been studied with the help of a utility function in a number of articles,[2] which mostly include a predetermined level of output. All financial decisions are separated from the real side of the firm, which acts more like a bank than a producer or seller of goods. One exception is Ethier (1973) who studies the payment behavior of an importing firm. An alternative to my use of a personified utility function is to study wealth maximization in a capital asset pricing context.[3] However, such an approach is not applicable here since one of the cornerstones of the CAPM model is lack of market frictions, an assumption that does not hold in the world I have in mind in this study.

To derive a mean-variance utility function linear in variance first expand the general function $U(\pi)$ about its mean

$$(4.1) \qquad U(\pi) = U(\bar{\pi}) + U'(\bar{\pi})\ (\pi-\bar{\pi}) + \frac{1}{2}U''(\bar{\pi})\ (\pi-\bar{\pi})^2 + R_o\ ,$$

$$R_o \approx 0\ ,$$

where π stands for profit and $\bar{\pi}$ expected profit. Taking expectation of both sides (4.1) can be written

$$(4.2) \qquad E(U(\pi)) = U(\bar{\pi}) + \frac{1}{2}U''(\bar{\pi})V(\pi).$$

One alternative is to stop here and use (4.2) as the relevant utility function, where $U''(\pi)/2$ represents the curvature of the utility function and a measure of the attitude towards risk. This is the approach taken by among others Dhrymes (1964), when he discusses the theory of a monopolistic multiproduct firm under uncertainty, and Farrar (1962). However, I have chosen

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2 See for example Kenen (1965) and Branson (1968).

3 See Hite (1979).

to work along the same lines as Pulley (1981), when he discusses wealth and the return on financial assets, and take one more step before I arrive at a final formulation of a utility function. To do this I also expand $U(\pi)$ about π_o, where π_o represents the level of profit when there is no trade credit, and assume that the remainder R_1 is close to zero.

$$(4.3) \qquad U(\bar{\pi}) = U(\pi_o) + U'(\pi_o)(\bar{\pi}-\pi_o) + R_1 , \qquad R_1 \approx 0 .$$

Further assuming that

$$(4.4) \qquad U''(\bar{\pi}) \approx U''(\pi_o) ,$$

and inserting (4.3) and (4.4) in (4.2) it is after some reshuffling of terms possible to write

$$(4.5) \qquad \frac{E(U(\pi)-U(\pi_o))}{U'(\pi_o)} + \pi_o \equiv G[E(U(\pi))] = \bar{\pi} + \frac{1}{2}\frac{U''(\pi_o)}{U'(\pi_o)} V(\pi) ,$$

or, with $k = \frac{1}{2}\frac{U''(\pi_o)}{U'(\pi_o)}$,

$$(4.6) \qquad G[E(U(\pi))] = \bar{\pi} + kV(\pi) .$$

Now, the utility function in (4.5) is equivalent to (4.2) as a measure of utility because, except $E(U(\pi))$, the terms on the lhs of (4.5) are constants with respect to maximization decisions with trade credit. The constant k is equal to minus half the coefficient of absolute risk aversion in the Pratt-Arrow sense.[4] Consequently, utility is here measured in terms of expected profit plus a constant, representing the attitude towards risk times the variance of profit. A decision maker with a negative k has risk aversion and a risk lover has a positive k.

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[4] The Pratt-Arrow measure of absolute risk aversion is defined as $-U''(\pi)/U'(\pi)$. See Arrow (1964) and Pratt (1964).

The derivations above make clear that one way to look at the utility function in (4.6) is to see it as an approximation which is sufficiently accurate to replace a general function. This is the view taken here. The approximation is good if the profit is normally distributed and the credit risk is small. With numerical experiments based on asset market data, Pulley (1981) has recently shown, that such a linear mean-variance function is a good approximation of more general utility functions if the trading interval is short. In his study a short trading interval is thirty days, which happens to be the ordinary length of the credit period in many trade credit agreements.

4.2.2 *Payment uncertainty*

In this chapter I also study the behavior of a firm that supplies trade credit with profitable interest arbitrage. Now, in order to present a model with the help of which it is possible to study the effects of an introduction of uncertainty and bad debts, some assumptions about within what kind of environment the firm operates are needed in addition to the utility function set forth above. The model assumptions give rise to an extended version of the buyer price compensation model set forth in Chapter 2.

- There are no goods market motives to use trade credit
- The seller uses buyer price compensation, and the interest rate functions of buyer and seller are $r_b(T)$ and $r_s(T)$ respectively.
- There are n customers with identical demand functions, $Q(P_o)$, $Q'(P_o) < 0$, $Q''(P_o) < 0$.
- The cost function is a linear function of the quantity of sales, $C(Q) = CQ$.
- When the firm supplies trade credit, there is uncertainty about payment at the end of the credit period.

- Each credit agreement can be treated as a Bernoulli trial. Either payment is made fully at the end of the credit period, with probability Pr(T), or there is complete default with probability (1-Pr(T)), Pr'(T) < 0.

- The customers are identical with respect to the payment probability and the "payment lotteries" are independent.

The third assumption is needed because if $Q(P_o)$ represents the whole market, the choice of a 0/1 alternative with respect to payment at the end of T implies that either all trade credit customers pay or default at the same time, and such a behavior seems unlikely. The assumption about complete payment or default simplifies the exposition. In Appendix I I show that a credit period of stochastic length leads to results similar to the ones presented here. The assumption about complete default means that the seller cannot use the good he has sold as collateral or that the good cannot be taken back e.g. because it has been changed in some production process, or that the resale value of used goods is low compared to the initial sales price. (For example, in parts of the market for consumer durables, the market value of goods falls rapidly as soon as the good has been taken out of the store.) There are naturally also many cases when the seller can salvage some part of the debt, but I have judged it not to be worthwhile to introduce this additional complexity in the model. The probability of default function represents the information the seller has about his customers. The Pr(T) function can represent either some subjective probability measure or it can be based on elaborate statistical calculations. There is a whole literature which deals with how to construct and use credit scoring applications.[5] What this literature essentially does is to describe different statistical methods with the help of which it is possible to make judgements about whether credit should be granted or not, which is equivalent to judgements about

[5] See Altman, E.I., Avery, R.A., Eisenbeis, J.F. and Sinley, J.R. (1981).

payment probabilities such as the Pr(T) function. To go into greater detail lies beyond the scope of this chapter. Here it suffices to note that there are many ways to determine the shape of the Pr(T) function and there are information costs included in the process. These costs I do not take into account in the model below. This way of introducing uncertainty rests on the assumption that the longer the credit period the more things can happen to make the trade credit customers unable to pay when payment is due. This is also the view taken by the Installment Payment Committee when they discuss the length of the credit period offered to households.[6] They argue that the shorter the credit period, the easier it is for a consumer to correctly judge his ability to fulfil an installment plan and the lower is the probability that preferences will change. In some cases the introduction of uncertainty is unlikely to change the results compared to Chapter 2. This happens when there are lasting customer relationships with frequent trade. One way to look at the models presented below is to see them as examples of markets where the buyers make few purchases with long intervals between each trip to the market. For example, the market for consumer durables or some other market for capital goods.

With continuously compounded interest rates the exposition is simplified if the probability of payment function also is exponential. Thus, from now on I use the function $Pr(T) = \exp(-dT)$, where d is the critical information variable determined by the seller. With this final simplification and the other assumptions above total revenue is binomially distributed and expected profit can be written

$$(4.7) \qquad E(\pi) = ne^{(r_b(T)-r_s(T)-d)T} P_oQ(P_o) - nCQ(P_o) \; .$$

When $P_oQ(P_o) = P_o(Q)Q = R(Q)$ and $(r_b(T) - r_s(T) - d)T = M(T)$ a shorter version of (4.7) is

[6] See The Installment Payment Committee (1975) Appendix 2, p. 312.

(4.8) $E(\pi) = n(e^{M(T)}R(Q) - CQ)$.

It is further necessary to add that I study cases with $T > 0$. In the pre payment case $d = 0$, because there is no goods market uncertainty and pre payment eliminates bad debts. The variance of profit or equally the variance of revenue from one customer is

(4.9) $$V(\pi) = V(R) = Pr(T)(1-Pr(T))e^{2(r_b(T)-r_s(T))T}R(Q)^2$$

and since, with respect to payment at the end of T, there are n independent customers, the utility function can be written

(4.10) $$U(\pi) = n(e^{M(T)}R(Q) - CQ + kPr(T)(1-Pr(T))e^{2(r_b(T)-r_s(T))T}R(Q)^2$$ [7]

where from now on $U(\pi)$ represents $G[E(U(\pi))]$ in (4.6). Or shorter, when the n variable has been dropped

(4.11) $$U(\pi) = e^{M(T)}R(Q) + kV(\pi(T,Q)) - CQ\ .$$

The expression in (4.11) represents my final utility of profit formulation. Provided that n is sufficiently large the binomially distributed profit will be approximately normally distributed, which makes it possible to use a mean-variance utility function. In addition to this I have made the simplifying assumption that the utility of profit is linear in variance.

4.2.3 *The comparative statics of the model*

Differentiation of (4.11) with respect to T and Q gives the first order optimum conditions

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[7] This follows directly from insertion of the expression for π in the variance formula

$$V(\pi) = \Sigma f(\pi)(\bar{\pi}-\pi)^2\ ,$$

when the frequence distribution only takes on two values.

$$(4.12) \qquad U'_T(\pi) = (MI_b(T) - MI_s(T) - d)e^{M(T)}R(Q) + kV'_T(\pi(T,Q)) = 0,$$

$$(4.13) \qquad U'_Q(\pi) = e^{M(T)}R'(Q) + kV'_Q(\pi(T,Q)) - C = 0 \ ,$$

where $MI_b(T)$ and $MI_s(T)$ represent the same marginal interest rate expressions as in Chapter 2. In (4.12) it is no longer enough to choose T in such a way that the marginal interest rates are equal. The marginal increase in the probability of default and the disutility of increased uncertainty also has to be taken into account. Similarly the marginal revenue following an increase in Q is reduced because a positive MR_Q also increases the variance. The introduction of the variance expression leads to second order conditions the signs of which are difficult to interpret. Fortunately, the model still has something to tell because at k = 0 there exists second order conditions that are identical to those in Chapter 2 except for the d variable which does not affect the conclusions about the signs of the derivatives. Consequently, at k = 0 there exists a unique maximum and it is possible to study the effects of small variations in k in the vicinity of k = 0. Thus, it is possible to compare the behavior of risk neutral and risk averse firms.

From (4.12) it is clear that at k = 0 the length of the credit period is determined by

$$(4.14) \qquad MI_b(T) - d = MI_s(T) \ ,$$

which states that the marginal risk adjusted interest rate of the buyer has to equal the marginal interest rate of the seller, where d is the risk adjustment factor of the seller. The $MI_b(T)$ and $MI_s(T)$ functions may well also include various risk adjustment factors that have been determined on the credit market outside the model. Taking account of the first order conditions implicit differentiation of (4.12) with respect to T and d, and (4.13) with respect to Q and d gives

$$(4.15) \qquad \frac{dT}{dd}_{/k=0} = \frac{1}{MI_b'(T) - MI_s'(T)} < 0 \ ,$$

$$(4.16) \qquad \frac{dQ}{dd}_{/k=0} = \frac{TR'(Q)}{R''(Q)} < 0 \ .$$

Since these conditions also hold at d = 0, the conclusion is that the introduction of uncertainty, in terms of bad debts, reduces the length of the credit period and the quantity of output. The optimal credit period falls because the introduction of uncertainty reduces the marginal interest rate of the buyer, $MI_s(T)$ must fall and this occurs when T falls. An increase in d at d = 0 reduces marginal revenue, therefore Q is reduced until the condition marginal revenue equals marginal cost holds. Uncertainty also introduces an asymmetry in the model. When there is post payment, the length of the credit period is determined by (4.14), T > 0, and $r_b > r_s$, but when there is pre payment, there is no uncertainty that reduces the length of the credit period, T is determined by

$$(4.17) \qquad MI_b(T) = MI_s(T) \ ,$$

T < 0 and $r_b < r_s$. Interest rate equality is no longer the dividing line between post payment, pre payment or no trade credit at all. In Chapter 6 I study the implications of a removal of this asymmetry with respect to uncertainty. In that chapter I introduce product or quality uncertainty which in the pre payment case gives rise to risk adjustment calculations by the buyer.

Implicit differentiation of (4.12) with respect to T and k gives

$$(4.18) \qquad \frac{dT}{dk}_{/k=0} = - \frac{V_T'(\pi(T,Q))}{M''(T)e^{M(T)}R(Q)}$$

where M"(T) is negative and equivalent to the second order condition in Chapter 2. Hence the sign of (4.18) is determined by the sign of $V'_T(\pi(T,Q))$ which is

$$(4.19)\qquad V'_T(\pi(T,Q)) = 2((MI_b - MI_s - \tfrac{1}{2}d)e^{2(r_b(T)-r_s(T)-\frac{1}{2}d)T} - (MI_b - MI_s - d)e^{2(r_b(T)-r_s(T)-d)T})R(Q)^2 .$$

At k = 0 (4.19) reduces to

$$(4.20)\qquad V'_T(\pi(T,Q)) = 2(MI_b - MI_s - \tfrac{1}{2}d)e^{2(r_b(T)-r_s(T)-\frac{1}{2}d)T}R(Q)^2 ,$$

which is positive since then (4.14) holds. Consequently an increase in the length of the credit period increases the variance and it is possible to draw the conclusion:

A producer with risk aversion has an optimal T which is lower than the optimal T of a risk neutral producer.

Differentiating (4.13) implicitly with respect to Q and k it is possible to write

$$(4.21)\qquad \frac{dQ}{dk}_{/k=0} = -\frac{V'_Q(\pi(T,Q))}{e^{M(T)}R''(Q)} .$$

R"(Q) is negative by assumption and $V'_Q(\pi(T,Q))$ is

$$(4.22)\qquad V'_Q(\pi(T,Q)) = 2R(Q)R'(Q)Pr(T)(1-Pr(T))e^{2(r_b(T)-r_s(T))T} > 0,$$

which is positive because R'(Q) is positive. Hence, the numerator (4.21) is positive and it is possible to conclude:

When a risk averse producer supplies trade credit he produces a lower output than a risk neutral producer.

The upshot of this is that the length of the credit period and the level of output (sales) will vary among producers (retailers) with different attitudes towards risk. The introduction of risk aversion also has the effect that the spread between the interest rate of buyer and seller must be wider than previously if there is to be a supply of trade credit. The latter conclusion can be shown in the following way:

Assume that the seller has chosen some $T > 0$. Then he is better off with than without trade credit if the interest rate functions in the exponential expression are such that

$$r_b(T) - d' > r_s(T) , \tag{4.23}$$

where d' represents a risk and risk aversion interest rate adjustment factor which increases as the level of risk aversion increases.[8] Hence, the higher the risk aversion the higher the observed interest differential $r_b(T)-r_s(T)$ before it pays to supply trade credit. This means that the choice between trade credit or no trade credit will vary among firms with different risk aversion.

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[8] The risk aversion adjustment factor d' has been derived in the following way: The utility from revenue with trade credit is

$$e^{(r_b(T)-r_s(T)-d)T} R(Q) + kV(\pi(T,Q)) = e^{(r_b(T)-r_s(T)-d')T} R(Q),$$

where the rhs is equivalent to the lhs when the equation has been solved for d', which represents a risk aversion adjustment factor. Solving for d' gives

$$d' = d - \ln\left[1 + k \frac{V(\pi(T,Q))}{e^{(r_b(T)-r_s(T)-d)T}R(Q)}\right]/T$$

with risk aversion k is negative. Hence, the expression within brackets is less than one, the logarithmic expression must be negative, and d' rises as k falls.

4.2.4 *A graphical interpretation*

The maximization problem also has a graphical interpretation, which has some similarities with the wellknown utility maximization problem in portfolio theory.[9] Both $E(\pi)$ and $V(\pi)$ are functions of T and Q and holding Q constant $(Q=\bar{Q})$ it is possible to draw a curve in terms of $E(\pi)$ and $V(\pi)$, which shows combinations of $E(\pi)$ and $V(\pi)$ when T varies. Such a curve exists for every level of output and its slope is

$$(4.24) \qquad \frac{E'_T(\pi(T,\bar{Q}))}{V'_T(\pi(T,\bar{Q}))} =$$

$$= \frac{M'(T)e^{M(T)}R(\bar{Q})}{2((MI_b-MI_s-\tfrac{1}{2}d)e^{2(r_b(T)-r_s(T)-\frac{1}{2}d)T}-(MI_b-MI_s-d)e^{2(r_b(T)-r_s(T)-d)T})R(\bar{Q})^2}$$

The curve, the slope of which is represented by (4.24), starts on the $E(\pi(T,\bar{Q}))$ axis at π_o, the level of profit when T = 0 (see Figure 4.1). Initially it has a positive slope because both $M'(T)$ and $V_T'(\pi(T,\bar{Q}))$ are positive in the interval $0 > T > T^x$, when T^x represents the optimal T with k = 0. The $(E(\pi(T,\bar{Q})), V(\pi(T,\bar{Q})))$ function reaches a maximum at $M'(T^x) = 0$, and when T gets large the curve must have a positive slope because then both the expected profit and the variance are reduced. As T goes to infinity the expected profit approaches $-C\bar{Q}$ and the variance approaches zero. It is further an advantage if the curve is concave $(\partial(E_T'/ /V_T')/\partial T < 0)$. At k = 0 the sign of the second order derivative of the $(E(\pi(T,\bar{Q})), V(\pi(T,\bar{Q})))$ function is determined by the expression

$$(4.25) \qquad M''(T^x)e^{M(T^x)}R(\bar{Q})V_T'(\pi(T^x,\bar{Q})) < 0 \quad ,$$

which is negative because $M''(T^x)$ is negative and $V_T'(\pi(T^x,\bar{Q}))$ positive. Hence, the function is concave close to its maximum.

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[9] See for example Sharpe (1970), Chapter 4.

Unfortunately it is difficult to make some general statement about the shape of the curve in the whole interval $0 \leq T \leq T^x$. Concavity follows if the interest arbitrage effect is weaker than the uncertainty effect represented by an increased variance. Now, the slope of an indifference curve to the utility function in (4.11) is -k. Since a negative k represents a utility function with risk aversion, -k indicates that in this case the indifference curves have a positive slope. A higher variance has to be compensated by a higher expected profit. Utility is maximized when T is chosen in such a way that an indifference curve is a tangent to the mean-variance curve

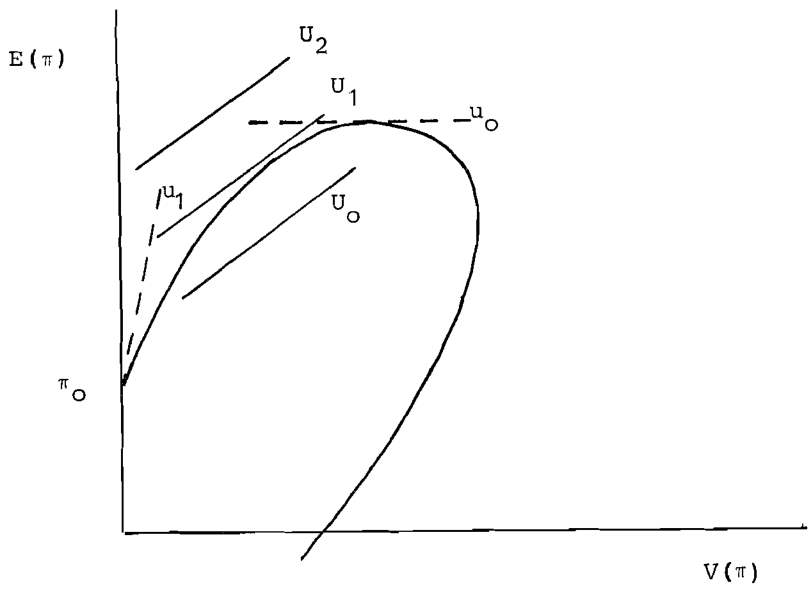

Figure 4.1 Maximization of utility in terms of mean and variance.

$$(4.26)\qquad -k = \frac{E'_T(\pi(T,Q))}{V'_T(\pi(T,Q))} .$$

If further Q is chosen optimally the highest indifference curve when there is risk aversion lies as far to the "north-west" as possible in the figure. The indifference curve U_1 in the figure represents the optimal choice of T. When the mean-variance curve is rising T is also rising. Given this relationship, it is in the figure also clear how risk aversion affects the choice of the length of the credit period. A risk neutral producer has a horizontal indifference curve (u_o) and consequently he chooses a T which is larger than the T represented by the point of tangency between U_1 and the mean-variance curve. If the risk aversion is high enough the indifference curve u_1 is steeper than the mean-variance curve and it is optimal to set T = 0. The mean-variance curve looks the same when the whole market has been taken into consideration. The only difference is that the scale on each axis has to be multiplied by n. The mean-variance curve in Figure 4.1 looks almost like the wellknown efficiency locus from portfolio theory. However, the curve in the figure is not equivalent to the efficiency locus. It demonstrates the optimal design of one asset, a trade credit contract, while the efficiency locus shows different combinations of many different assets, the characteristics of which are exogenous to the model.

4.3 SOME EXTENSIONS

4.3.1 *More than one market*

Until now may analysis of trade credit with bad debts has concerned a firm which sells its output in one market, where the customers have identical demand functions. In this section I extend the model and include many different markets. Differences between two markets can depend both on differentno trade credit demand functions,different interest rate functions,and on different assessments of the probability of default. I retain the assumption that the payment behavior of one customer in one market is independent of the payment behavior of other customers in the same market, but I add an assumption that there can be interdepencence between different markets. Thus, the model extensions set forth here can represent the trade credit/ output decisions of a firm that sells its output to different categories of customers within a country or to customers in different countries, when the over all credit risk is dependent on the distribution of trade credits between different markets. *First*, I make a general formulation of an objective function with many markets. *Then*, I discuss the connection between the type of model presented here and ordinary portfolio theory, and *finally*, I discuss how the introduction of many markets affects the choice of T and Q in one market. In this way the model can be compared with the results in the preceding sections of this chapter.

In (4.27) the same utility function as I have used before has been extended to include i = 1,..., m different markets with n_i customers in each market. Utility is to be maximized with respect to the length of the credit period, T_i, and the quantity and division of output, Q_i. With the same notation as before, this means that the objective function is[10]

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10 The expression for variance, the second and third row of (4.27), is a direct application of the formula for the variance of a sum of stochastic variables, X_i,

$$V\left\{\sum_{i=1}^{m} X_i\right\} = \sum_{i=1}^{m} V(X_i) + 2\sum_{i<j}^{m} C(X_i,X_j),$$

where the covariance, $C(X_i,X_j)$ also can be written

$$C(X_i,X_j) = \rho_{ij}V(X_j)^{1/2}V(X_j)^{1/2}$$

when ρ_{ij} is the coefficient of correlation between i and j. See also for for example Blom (1970).

$$(4.27) \qquad U(\pi) = \sum_{i=1}^{m} n_i R_i(T_i,Q_i) - C\left[\sum_{i=1}^{m} n_i Q_i\right] +$$
$$+ k \left[\sum_{i=1}^{m} n_i V_i(T_i,Q_i) + 2\sum_{i<j}^{m} \rho_{i,j}(n_i V_i(T_i Q_i)\; n_j V_j(T_j,Q_j))^{\frac{1}{2}}\right]$$

The first row of (4.27) shows expected profit summed over all m markets. The second and third rows show the risk aversion coefficient times overall variance, which is the sum of the variance in each market plus twice the sum of the covariance between different markets, $\rho_{i,j}$ in the variance expression is the coefficient of correlation between market i and j, $-1 < \rho_{i,j} < 1$. In the special case when $k = 0$, risk neutrality, the model represents maximization of expected profit and it can be treated as a sum of the different revenue functions. With $k \neq 0$ the model has much in common with portfolio choice models. The important difference is that here the portfolio is not solely determined in terms of financial assets. The important part of the portfolio choice is the distribution of output between different markets and the combination of output and trade credit contracts. It is also the output effect that makes the model different from portfolio models of firms engaged in international trade.[11] In those models the choice is not between the distribution of output and trade credit contracts but between different payment arrangements, including trade credit, when there is uncertainty about the future spot exhange rate.

The general model formulation in (4.27) and the two market case discussed below are related to the mean-variance portfolio choice model of portfolio theory.[12] However, my model represents a more complicated structure since here both output and trade

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[11] See Lietaer (1971), Makin (1978) and Soenen (1979).

[12] The basic portfolio model occurs in a large number of textbooks. One good introduction is for example Sharpe (1970), Chapters 3 and 4.

credit decisions directly affect expected revenue and variance. In order to compare the two models I have rewritten the original portfolio model, which describes how the optimal composition of a portfolio of securities is determined in terms of revenue and cost functions comparable to my trade credit model above, see (4.28). In each market the expected price (expected return in a portfolio model) is constant and independent of the distribution of output between different markets. Total output is also constant. ($C\bar{Q}$ is the amount which is invested in a portfolio model. A bar ($\bar{\ }$) in (4.28) represents a variable which is exogenous to the model.) With these restrictions all that is left to determine is the distribution of output between different markets (in the case with securities the distribution of funds between different securities). The maximization problem is reduced to the determination of the Q_i:s in (4.28) given that they sum to $\bar{Q}$.

$$(4.28) \qquad U(\pi) = \sum_{i=1}^{m} n_i E(\bar{P}_i) Q_i - C\bar{Q} + k \left\{ \sum_{i=1}^{m} n_i V_i (\bar{P}_i) Q_i^2 + 2 \sum_{i<l}^{m} \rho_{i,j} (n_i V_i (\bar{P}_i) Q_i^2 n_j V_j (\bar{P}_j) Q_j^2)^{1/2} \right\}$$

$$\text{s.t.} \sum_{i=1}^{m} n_i Q_i = \bar{Q}, \qquad i = 1, \ldots, m$$

Now the maximization problem in (4.27) is equivalent to the one in (4.28) if total output is given, the length of the credit period is fixed, and demand is independent of the quantity offered for sale. Then (4.27) reduces to a problem of how to distribute output between the different markets. If these assumptions do not hold, (4.27) represents a more complicated problem where the values of the separate expected revenue and variance functions are not predetermined. Every redistribution of output affects both the separate expected revenue and the variance functions. Output and trade credit decisions become more complicated than in (4.28). If the number of markets is large, it is, in order to carry out the calculations, necessary to know a large number of variances and correlation coefficients. Here I have assumed that these coefficients are known, but this is actually a problem about investment in information. However, it is beyond the scope of this chapter to include an investment cost function and investment in information. With many markets it is perhaps reasonable to assume that many correlation coefficients are zero or small enough to be of no importance. Then the optimization problem can be reduced to manageable proportions, but the problem how to determine these coefficients still remains, of course.

To make the general formulation of my model comparable with the results in the one market case assume that the firm sells its product in two markets with one customer in each market and that it uses buyer price compensation. A utility function based on these assumptions is given in (4.29)

$$(4.29) \qquad U(\pi) = e^{M_1(T_1)} R_1(Q_1) + e^{M_2(T_2)} R_2(Q_2) - C(Q_1+Q_2)$$

$$+ k\Big[V_1(R_1(T_1,Q_1)) + V_2(R_2(T_2,Q_2))$$

$$+ 2\rho_{1,2}[V_1(R_1(T_1,Q_1))V_2(R_2(T_2,Q_2))]^{1/2}\Big].$$

The first order conditions with respect to T_1 and Q_1 are

$$(4.30) \qquad U'_{T_1}(\pi) = M'_1(T_1)e^{M_1(T_1)} R_1(Q_1) + kV'_{T_1}R_1(T_1,Q_1) \ .$$

$$\cdot\left[1 + \rho_{1,2}\left(\frac{V_2(R_2(T_2,Q_2))}{V_1(R_1(T_1,Q_1))}\right)^{1/2}\right] = 0 \ ,$$

$$(4.31)\qquad U'_{Q_1}(\pi) = e^{M_1(T_1)} R'_1(Q_1) + kV'_{Q_1}(R_1(T_1,Q_1)) \cdot$$

$$\cdot \left[1 + \rho_{1,2}\left\{\frac{V_2(R_2(T_2,Q_2))}{V_1(R_1(T_1,Q_1))}\right\}^{1/2}\right] - C = 0$$

If $\rho_{1,2}$ is different from zero the credit period and the quantity of sales is no longer determined solely within the market itself. The payment correlation between different markets has to be taken into account too. At $k = 0$ all cross derivatives are zero, which means that the markets are independent and the second order conditions can be summarized by $M''(T_{1,2}) < 0$, which is the same condition as in the one market case. Hence, at least in this case the utility function in (4.29) has a unique maximum. At $k = 0$ the effect of an increase in risk aversion is determined by

$$(4.32)\qquad \frac{dT_1}{dk}_{/k=0} = \frac{V'_{T_1}(R_1(T_1,Q_1))\left\{1 + \rho_{1,2}\left\{\frac{V_2(R_2(T_2,Q_2))}{V_1(R_1(T_1,Q_1))}\right\}^{1/2}\right\}}{-U''_{T_1T_1}(\pi)},$$

$$(4.33)\qquad \frac{dQ_1}{dk}_{/k=0} = \frac{V'_{Q_1}(R_1(T_1,Q_1))\left\{1 + \rho_{1,2}\left\{\frac{V_2(R_2(T_2,Q_2))}{V_1(R_1(T_1,Q_1))}\right\}^{1/2}\right\}}{-U''_{Q_1Q_1}(\pi)},$$

where both V'_{T_1} and V'_{Q_1} are positive. Note that both expressions are equivalent to their corresponding one market expressions if $\rho_{1,2}$ is equal to zero. Consequently, if, on the one hand, $\rho_{1,2}$ is positive both the length of the credit period and the quantity of sales to market one is reduced, and this effect is stronger than if market one is the only market where the firm operates and the effects of an increase or decrease of risk aversion are enhanced. If, on the other hand, $\rho_{1,2}$ is negative the effects are smaller than before and in some cases, depending on the size of $\rho_{1,2}$ and the $V_1(\ldots)$, $V_2(\ldots)$ ratio, the results can be reversed. Finally, assume that k is different from zero, and that the second order conditions hold. Then an increase in the correlation coefficient affects the credit period in such a way that

$$(4.34) \qquad \frac{dT_1}{d\rho_{1,2}} = \frac{kV'_{T_1}(R_1(T_1,Q_1))\left[\frac{V_2(R_2(T_2,Q_2))}{V_1(R_1(T_1,Q_1))}\right]^{1/2}}{U''_{T_1T_1}(\pi)} .$$

With the assumptions about V'_{T_1} and $U''_{T_1T_1}$ above this implies that the sign of (4.34) is determined by the sign of k. Hence, an increase in the correlation coefficient reduces T_1, if the managers of the firm have risk aversion. It can also be shown that a similar conclusion holds with respect to the effect on Q.

4.3.2 *Double uncertainty*

I have gradually introduced more complex versions of the basic model. So far there has only been one type of uncertainty. That is uncertainty about payment at the end of the credit period depending on the length of the credit period.In this section I introduce an additional uncertainty factor, uncertainty about the position of the demand function, and I show how it affects decisions about output and credit.

An exporter is one example of a firm facing the kind of double uncertainty described below. If the firm supplies trade credit denominated in a foreign currency, without hedging future receipts, then there is uncertainty both about whether the customers will pay or not at the end of the credit period, and uncertainty about the future spot exchange rate. Different hedging strategies can of course also be included. Such strategies, but without quantity effects, have been discussed by several authors. (See for example Kenen (1965) and Branson (1968).) Hence, I do not inted to go into detail here. The assumption about defaults adds, however, a new complication to hedging problems, because if a customer does not pay at the end of the credit period the firm has to make additional transactions to meet its obligations in the forward foreign exchange market. As far as I know, no one has yet studied the effects of these extra transactions on hedging decisions.

Assume that regardless of whether trade credit is granted or not there is also uncertainty about the position of the cash price demand function.[13] Then a firm which supplies trade credit has to take into account both uncertainty about the cash price and about payment at the end of the credit period. This means that for a firm which supplies trade credit and uses buyer price compensation the demand function is

$$P(Q,\varepsilon) = P(Q) + \varepsilon, \quad E(\varepsilon) = 0, \quad V(\varepsilon) > 0, \tag{4.35}$$

where ε is a stochastic variable that shifts the position of the demand function. To derive the utility function when this additional assumption has been added I use Gauss' approximation formula to get an approximate expression of the mean and variance when there is double uncertainty. The present value revenue function is a product of two stochastic variables

$$g(X_1,X_2) = X_1,X_2 , \tag{4.36}$$

where

$$X_1 = \begin{cases} 1 , & P(1) = e^{-dT} \\ 0 , & P(0) = 1-e^{-dT} \end{cases} \tag{4.37}$$

$$E(X_1) = e^{-dT}, \; V(X_1) = e^{-dT}(1-e^{-dT}) = V(Pr(T)) , \tag{4.38}$$

$$X_2 = e^{(r_b(T)-r_s(T))T} R(Q,\varepsilon) , \tag{4.39}$$

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[13] This is the most common way of combining uncertainty and the theory of the firm. See for example the previously cited articles by Sandmo (1971) and Leland (1972).

(4.40) $$E(X_2) = e^{(r_b(T)-r_s(T))T} R(Q), V(X_2) = e^{2(r_b(T)-r_s(T))T} Q^2 V(\varepsilon).$$

By using Gauss' approximation formula[14] the mean and variance of the $g(X_1X_2)$ function can be written

(4.41) $$E(X_1X_2) \approx E(X_1)E(X_2),$$

(4.42) $$V(X_1X_2) \approx V(X_1)(E(X_2))^2 + V(X_2)(E(X_1))^2 + \\ + 2\rho_{1,2}V(X_1)^{1/2}V(X_2)^{1/2}E(X_1)E(X_2).$$

Insertion of (4.38) - (4.40) in (4.41) and (4.43) gives

(4.43) $$E(X_1)E(X_2) = e^{M(T)}R(Q) \quad ,$$

(4.44) $$V(X_1)(E(X_2))^2 = V(Pr(T))e^{2(r_b(T)-r_s(T))T}R(Q)^2 = \\ = V(\pi(T,Q|\varepsilon = 0) \quad ,$$

(4.45) $$V(X_2)(E(X_1))^2 = V(\varepsilon)e^{2M(T)}Q^2 = V(\pi(T,Q,\varepsilon)) \quad ,$$

(4.46) $$(V(X_1)V(X_2))^{1/2}E(X_1)E(X_2) = [V(\pi(T,Q|\varepsilon = 0))V(\pi(T,Q,\varepsilon))]^{1/2},$$

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14 According to this formula the mean and variance of a function with several stochastic variables, $g(X \;, \ldots, X_n)$ is approximately

$$E(g(X_1, \ldots, X_n)) \approx g(\overline{X}_1, \ldots, \overline{X}_n),$$

$$V(g(X_1, \ldots, X_n)) \approx \sum_{i=1}^{n} V(X_i)(g'_{\overline{X}_i})^2 + 2\sum_{i<j} C(X_i,X_j)g'_{\overline{X}_i}g'_{\overline{X}_j}$$

where $\overline{X}_i$ represents the expected value of variable i and $C(X_iX_j)$ is the covariance between X_i and X_j. For a longer discussion about these approximation formula see Blom (1970), Chapter 5.

where (4.44) is the same trade credit variance discussed before and (4.45) is an additional variance following from the uncertainty about the shift factor in the demand function. Finally, by using (4.43) - (4.46) the mean-variance utility function is

$$(4.47)\qquad U(\pi) = e^{M(T)}R(Q) - CQ + k[V(\pi(T,Q|\varepsilon = 0)) + V(\pi(T,Q,\varepsilon)) + 2\rho[V(\pi(T,Q|\varepsilon = 0))V(\pi(T,Q,\varepsilon))]^{1/2}] ,$$

where I have omitted the scale variable n, the number of customers, and ρ is the coefficient of correlation between demand shifts and payment behavior. The use of the approximation formula simplifies the calculation of means and variances. Particularly if $\rho = 0$ the overall variance of two stochasti variables is reduced to the sum of two variances. An extension to more than two types of uncertainty or more than one market is straightforward. Then more new variances and covariances must be taken into account. Thus, the approximation formula can be used to generate mean-variance expressions for many different kinds of uncertainty. In the utility function above it is obvious that the introduction of demand uncertainty gives rise to additional disutility terms. The demand variance and the covariance between demand and payment behavior. This will naturally also affect the choice of T and Q. The first order maximum conditons are

$$(4.48)\qquad U'_T(\pi) = M'(T)e^{M(T)}R(Q) + k\Bigg[V'_T(\pi(T,Q|\varepsilon = 0)) + V'_T(\pi(T,Q,\varepsilon)) + \rho\Bigg[V'_T(\pi(T,Q|\varepsilon = 0))\frac{V(\pi(T,Q,\varepsilon))^{1/2}}{V(\pi(T,Q|\varepsilon = 0))^{1/2}} + V'_T(\pi(T,Q,\varepsilon))\frac{V(\pi(T,Q|\varepsilon = 0))^{1/2}}{V(\pi(T,Q,\varepsilon))^{1/2}}\Bigg]\Bigg] = 0 ,$$

$$(4.49) \qquad U_Q'(\pi) = e^{M(T)}R'(Q) - C + k\Big[V_Q'(\pi(T,Q|\varepsilon = 0)) + V_Q'(\pi(\overline{T},Q,\varepsilon))$$

$$+ \rho\Big[V_Q'(\pi(T,Q|\varepsilon = 0))\frac{V(\pi(T,Q,\varepsilon))^{1/2}}{V(\pi(T,Q|\varepsilon = 0))^{1/2}}$$

$$+ V_Q'(\pi(T,Q,\varepsilon))\ \frac{V(\pi(T,Q|\varepsilon = 0))^{1/2}}{V(\pi(T,Q,\varepsilon))^{1/2}}\Big]\Big] = 0 .$$

With k = 0 these expressions reduce to their counterparts in the basic model used earlier. Hence, at k = 0 there exists an optimal combination of T and Q. Implicit differentiation of (4.48) and (4.49) with respect to T, Q and k at k = 0 gives

$$(4.50) \qquad \frac{dT}{dk}_{/k=0} = \frac{V_T'(\pi(T,Q|\varepsilon = 0))\ \left[1 + \rho\,\frac{V(\pi(T,Q,\varepsilon))^{1/2}}{V(\pi(T,Q|\varepsilon = 0))^{1/2}}\right]}{-\ U_{TT}''(\pi)} ,$$

$$(4.51) \qquad \frac{dQ}{dk}_{/k=0} = \frac{\left[\begin{array}{l} V_Q'(\pi(T,Q|\varepsilon = 0)) + V_Q'\ (\pi(T,Q,\varepsilon)) \\[2ex] + \rho\left[V_Q'(\pi(T,Q|\varepsilon = 0))\frac{V(\pi(T,Q,\varepsilon))^{1/2}}{V(\pi(T,Q|\varepsilon=0))^{1/2}}\right. \\[2ex] \left. +\ V_Q'(\pi(T,Q,\varepsilon))\frac{V(\pi(T,Q|\varepsilon = 0))^{1/2}}{V(\pi(T,Q,\varepsilon))^{1/2}}\right] \end{array}\right]}{-\ U_{QQ}''(\pi)} ,$$

where the denominator in both expressions is positive. In (4.50) I have also taken account of the fact that $V_T'(\pi(T,Q,\varepsilon)) = 0$ at k = 0 because then $M'(T) = 0$. In (4.50) the effect of an increase in k on the length of the credit period is identical to the effect in the one-market, one-type-of-uncertainty case when $\rho = 0$. An increase in risk aversion reduces the credit period, and this decision is independent of $V(\varepsilon)$. If $\rho \neq 0$ the conclucions are similar to those in the example with two markets. This is so since the approximation formula makes the variances additive, as in the two-market case. The first order variance derivatives in (4.51) are positive, because marginal revenue is positive. Hence, an increase in risk aversion reduces the quantity of output as long as the effect of a negative correlation coefficient is not too strong, and with a positive correlation the quantity effect is stronger than in the one-market, one-type-of-uncertainty case. Note also that when there is no payment uncertainty (4.51) reduces to

$$(4.52) \qquad \frac{dQ}{dk} = \frac{V_Q'\ (\pi(Q,\varepsilon))}{-\ U_{QQ}''(\pi)} > 0 \ ,$$

which is the wellknown output effect from the general theory of the firm under uncertainty. Schwartz and Whitcomb (1979) argue that in their model (here the model set forth in Chapter 2) an introduction of uncertainty in terms of a shifting demand schedule does not change the conclusions about the choice of trade credit policy. This is also true here if there is no payment uncertainty. However, with payment uncertainty the exposition above makes it clear that it is both types of uncertainty that simultaneously determine the choice of T and Q.

4.4 PARTIAL TRADE CREDIT - A WAY TO REDUCE UNCERTAINTY

4.4.1 *Expected profit and partial trade credit*

In the different trade credit models I have discussed so far the granting of credit has been an all or nothing decision. A credit request is either granted in full or not at all. Most authors who discuss trade credit have modelled the supply of trade credit in this way. One exception is Bierman and Hausman (1970).

Especially in the market for consumer durables a common practice is to supply trade credits that are lower than the amount requested by the customer. For example 20 percent down payment and the rest to be paid at the end of the credit period. In this section I discuss why it can be optimal for a firm to supply partial trade credit. I only study the simple case with partial cash payment and partial payment at the end of the credit period. More elaborate payment patterns are not included, they do not change the basic idea of the model. The credit granting decision, with either a supply of trade credit such that $L_{TCs} = PQ$, as in the all or nothing case, or $L_{TCs} < PQ$, belongs to a wider set of problems which deals with reasons for the existence of supply and demand for credit between non-financial firms. Here $L_{TCs} = PQ$ is an upper boundary on the supply of loans from one firm to another. Naturally, this need not alwaysbe the upper bound, but here, as in earlier chapters, I take the existence of trade credit for granted and limit myself to the more narrow task of discussing the profit maximizing behavior of a firm which supplies trade credit. Just as in the preceding sections I limit the discussion to financial market reasons for $L_{TCs} < PQ$. In some cases goods market conditions can also generate partial credit. (See Chapter 6.)

One way to introduce partial credit is to assume that there is a demand function such that demand is affected by both the cash price/list price combination and the division of payment between cash payment and payment at the end of the credit period.

$$(4.53) \qquad Q = Q(ce^{-r_b(T)T}P_1 + (1-c)P_o) \ ,$$

where c is a constant between zero and one indicating the proportion of the total payment to be paid at the end of the credit period. Consequently, the quantity demanded depends on the average price, where the division between cash sales and credit sales has been used to weigh the two prices. This is the situation facing a customer who buys Q units when (1-c)Q units are paid immediately and cQ units are paid at the end of the credit period.[15] From here it is possible to proceed by including (4.53) in the objective function and maximize with respect to T, P_1, c and P_o. However, I stick to the assumption about a list price based on buyer price compensation. Then (4.53) reduces to

$$(4.54)\qquad Q = Q(P_o) ,$$

because $P_1 = e^{r_b(T)T} P_o$. This means that the objective function can be written

$$(4.55)\qquad E(\pi) = e^{M(T)} cR(Q) + (1-c)R(Q) - CQ ,$$

$$\text{s.t. } 0 \leq c \leq 1 ,$$

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15 In (4.53) the cash price and the list price can differ. Another alternative is to assume that the supplier of trade credit uses a price policy with only one price, the list price P_1, which is to be paid partly on delivery and partly at the end of the credit period. In this case the demand function is

$$Q = Q(ce^{-r_b(T)T} P_1 + (1-c)P_1) .$$

From the buyers' point of view, it is now, of course, best to get an offer with c = 1.

Here I only discuss optimization with mixed payment patterns when there are two payment dates. The approach used here can also be extended to include cases with many payment dates. Then the demand function has to be adjusted accordingly. One possibility is to write

$$Q = Q\left[\sum_{n=0}^{N} c_n e^{-r_b(T_n)T_n} P_1\right] \text{ with } \sum_{n=0}^{N} c_n = 1.$$

Demand is a function of the weighted present value price, where c_n presents the proportion of the price to be paid at T_n.

when risk aversion has not been taken into account. Again I have dropped the n variable because it does not affect the analysis. From (4.53) it is clear that it is still not possible to have partial trade credit. The model generates all or nothing solutions. When M(T) is positive it always pays to set c equal to one and if M(T) is negative c is equal to zero. Consequently, firms that maximize expected profit and use buyer price compensation have no motive to supply partial trade credit if $c < 1$ does not yield any additional information.

The assumption that there are customers who default means that when the seller grants credit he cannot know whether a customer is going to pay or not at the end of the credit period. To determine whether a customer is a good or a bad risk he uses available information to calculate the P(T) function. In addition to this the firm can make a customer alter his probability of default if the customer in some direct way can signal his intention or ability to honor a credit agreement. One such signal is the size of the down payment. The higher the down payment, the lower the probability of default, because the lower is the amount to be paid at the end of the credit period. Another different aspect of the use of down payment is the following. Assume there are "good" and "bad" customers among those who use trade credit. A "good" customer always honors his payment obligations while a "bad" customer has no intention of doing so. Then the size of the down payment can be used as a way to screen "good" and "bad" customers. The higher the down payment, which is a signalling cost to the customer, the lower the gain for "bad" customers. At some point the signalling cost becomes too high and the "bad" customers withdraw from the market. Then "good" customers are the only ones who signal and accept the down payment.[16] This means that the size of the down payment also in this case is likely to affect the probability of payment. There is a fundamental difference between the two aspects of down payment above.

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16 This is an application of the market signalling concept introduced in the seminal work about market equilibrium with uncertainty by Spence (1974).

In the first case, the fact that some customers default can be seen as a game against nature. For example, some customers loose their jobs and their ability to pay is reduced. In the second case the down payment can be seen as a part of a game between buyer and seller, when the seller does not know the buyer's true motive for a credit agreement. In this case more complicated models are needed than the ones set forth here. However, both ways of looking at the role of the down payment lead to the same conclusion. The size of the down payment affects the probability of payment at the end of the credit period. This conclusion is supported by empirical data from the Installment Credit Committee. They found a negative correlation between the size of the down payment and the number of purchases where the customers had difficulties to meet their payment obligations. They also show that in many cases there is a considerable signalling cost. For example, in the market for yachts the down payment is said to be roughly fifty percent of the sales price.[17]

The upshot of this discussion about the role of the down payment is that the missing link in (4.55) above is that the down payment variable, c, is not included as an argument in the probability of payment function. To include the down payment in the probability function I assume that

$$(4.56) \qquad P(T,c) = e^{-dT-gc} \ .$$

The introduction of additional uncertainty in terms of c in (4.56) clearly rests on an ad hoc assumption. Naturally, uncertainty about payment at the end of T can also be introduced in alternative ways. This is ultimately an empirical question determined by what information the firm has about its customers. One alternative to (4.56) is to use $\exp(-dT-gcP_1R(Q))$, in this case it is the total payment at the end of T which is important. Insertion of (4.56) in the objective function gives

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[17] See The Installment Credit Committee (1975), p. 171.

$$(4.57) \qquad E(\pi) = e^{N(T,c)}cR(Q) + (1-c)R(Q) - CQ \ ,$$

$$\text{s.t. } 0 \leq c \leq 1 \ ,$$

where $N(T,c) = M(T) - gc$. Maximization of (4.57) with respect to T, Q and $\bar{c}$ gives the first order conditions

$$(4.58) \qquad E'_T(\pi) = M'(T)e^{N(T,c)}cR(Q) = 0$$

$$(4.59) \qquad E'_Q(\pi) = R'(Q)(ce^{N(T,c)} + 1 - c) - C = 0 \ ,$$

$$(4.60) \qquad E'_c(\pi) = R(Q)((1-gc)e^{N(T,c)} - 1) = 0 \ ,$$

where $M'(T) = (MI_b - MI_s - d)$, hence the length of the credit period must be the same as before. With a positive trade credit effect, $N(T,c) > 0$, in (4.59) the expression within the parenthesis must be positive. This implies that, compared to the no trade credit case, there is a positive output effect, which in turn, means that the cash price must be lower than when profit is maximized with $T = 0$. Finally, if there exists an interior solution to (4.57), (4.60) (the marginal gain from a redistribution of cash and trade credit payments) must be equal to zero somewhere in the interval $0 \leq c \leq 1$. At $c = 0$ (4.60) is positive if

$$(4.61) \qquad e^{M(T)} > 1 \ ,$$

which is the condition that there has to be a gain from trade credit. The whole analysis in this chapter rests on the assumption that this condition holds. At $c = 1$ (4.60) is negative if

$$(4.62) \qquad e^{M(T)} < \frac{e^g}{1-g} \ ,$$

and I assume that this condition holds. Consequently, then there must exist some c between zero and one such that (4.60) is equal to zero. The inequality in (4.63) places an upper bound on the gain from trade credit if there is to be an interior solution to the maximization problem. The inequality in (4.63) implies that g must lie between zero and one, g cannot be negative because this means that a decrease of the down payment reduces uncertainty, which seems unreasonable, and g must be less than one because exp(M(T)) cannot be negative. It can also be shown that, given the assumptions in (4.61) and (4.62), the second order maximum conditions

$$(4.63) \qquad E''_{QQ}(\pi) < 0,\ E''_{QQ}(\pi)E''_{TT}(\pi) > 0,\ E''_{QQ}(\pi)E''_{TT}(\pi)E''_{cc}(\pi) < 0\ ,$$

hold.[18] Hence, there exists an optimal combination of credit period,

---

[18] Proof of (4.63). The second order derivatives are

$$\text{(i)} \quad E''_{TT}(\pi) = e^{N(T,c)}cR(Q)(M''(T) + M'(T)^2) < 0\ ,$$

$$\text{(ii)} \quad E''_{QQ}(\pi) = R''(Q)(ce^{N(T,c)} + 1 - c) < 0\ ,$$

$$\text{(iii)} \quad E''_{cc}(\pi) = e^{N(T,c)}gR(Q)(gc-2) < 0\ ,$$

$$\text{(iv)} \quad E''_{Tc}(\pi) = M'(T)e^{N(T,c)}(1-gc)R(Q) = 0\ ,$$

$$\text{(v)} \quad E''_{TQ}(\pi) = M'(T)R'(Q)ce^{N(T,c)} = 0\ ,$$

$$\text{(vi)} \quad E''_{Qc}(\pi) = R'(Q)((1-gc)e^{N(T,c)} - 1) = 0\ .$$

(i) is negative because $M'(T) = 0$ and $M''(T)$ is negative; (ii) is negative because $R''(Q)$ is negative and $N(T,c) > 0$ by assumption; (iii) is negative because the assumption that $0 \leq g \leq 1$ ensures that the expression within parenthesis is negative. Finally, the cross derivatives are zero *first*, because $M'(T) = 0$, and *second*, because the expression within parenthesis in (vi) is equal to zero according to (4.60). The second order condition for a maximum when there are several variables is that the principal minors of the matrix with second order derivatives alternate in sign, the odd numbered minor being negative. Now, since all the cross derivatives are zero, this condition reduces to an evaluation of the diagonal elements in the matrix. Then the principal minors can be written as in (4.63) and they alternate in sign with odd numbered being negative. Consequently, the second order maximum conditions are fulfilled.

output, and down payment policy such that $0 \leq c^x \leq 1$.

Changes in g and d show the effects of increased risk. Assume that the firm receives new information about the customers, both g and d increase. The effect on credit period, output and down payment is given by the comparative statics of the model.[19] Implicit differentiation of the first order conditions with respect to T, Q, and g gives

$$(4.64) \qquad \frac{dT}{dg} = \frac{c^2 M'(T) e^{N(T,c)} R(Q)}{E''_{TT}(\pi)} = \frac{0}{-} = 0 \ ,$$

$$(4.65) \qquad \frac{dQ}{dg} = \frac{c^2 e^{N(T,c)} R'(Q)}{E''_{QQ}(\pi)} = \frac{+}{-} < 0 \ ,$$

$$(4.66) \qquad \frac{dc}{dg} = \frac{-e^{N(T,c)} R(Q)(gc-2)c}{E''_{cc}(\pi)} = \frac{-(-)}{-} = < 0 \ .$$

Hence, an increase in risk with respect to the down payment does not affect the length of the credit period but reduces both the quantity of output and the proportion of sales paid at the end of the credit period. The effects of an increase in d on T and Q have the same sign as in (4.15) and (4.16). Increased payment uncertainty reduces both T and Q. In addition to this an increase in risk with respect to the length of the credit period also reduces the proportion of sales paid at the end of the credit period

$$(4.67) \qquad \frac{dc}{dd} = \frac{T(1-gc) e^{N(T,c)} R(Q)}{E''_{cc}(\pi)} = \frac{+(+)}{-} < 0 \ .$$

---

[19] The sign of the partial derivatives are given by the discussion in footnote 20.

These results taken together show that increased risk drives the model towards the no trade credit solution. This is so since both T and c are reduced.

The possibility that it is optimal for the firm to supply partial trade credit has much in common with that part of the credit rationing literature where rationing decisions are based not on the inability to charge different interest rates, but on the connection between the probability of default and loan size.[20] The supply of trade credit from seller to buyer is less that P_0Q because an increase in loan size also represents an additional uncertainty cost which must be taken into account. In terms of a risk adjusted interest rate the exponential expression can be written

$$(4.68) \qquad \left[r_b(T) - (d + \frac{gc}{T}) - r_s(T)\right]T ,$$

where d + gc/T represents the risk adjustment factor which has to be subtracted from the observed interest rate of the buyer. To generate the partial trade credit result above it is essential that c is one of the risk adjustment components if the objective function of the firm represents maximization of expected profit.

4.4.2 *Partial trade credit with risk aversion*

When a firm maximizes expected profit the analysis above has shown that the use of partial trade credit is justified if this yields additional information about the payment probability at the end of the credit period. In this section I show that partial trade credit can be used without such an information requirement if the managers of the firm have risk aversion. When the seller has risk aversion the utility function that replaces (4.55) can be written

$$(4.69) \qquad U(\pi) = e^{M(T)}cR(Q) + (1-c)R(Q) + kc^2V(\pi(T,Q)) - CQ$$

$$\text{s.t. } 0 \leq c \leq 1 ,$$

[20] See the survey by Baltensperger (1978) and Hodgman (1960).

where the variance expression has been added compared to (4.55). The marginal utility with respect to c is positive as long as

$$(4.70) \qquad -k < \frac{R(Q)(e^{M(T)}-1)}{2cV(\pi(T,Q))} .$$

Since the lhs of (4.70) represents absolute risk aversion the inequality implies that the risk aversion must be small anough if an increase in c is to be worthwhile. If the producer is not infinitely risk averse it is possible to conclude that since the rhs of (4.70) goes to infinity as c approaches zero there must exist some c small enough to justify the inequality in (4.70). Thus, one can draw the conclusion:

A producer with finite risk aversion supplies at least partial trade credit when the expected revenue with trade credit is higher than without trade credit.

Note also that here it is possible to have $0 < c < 1$ without having to include the cash payment in the probability of default function. This was not possible earlier when I discussed partial trade credit in terms of maximization of expected profit. Hence, risk aversion is an additional motive to use partial trade credit.

The conclusion about risk aversion and partial trade credit also has a graphical interpretation, see Figure 4.2 below. With $c < 1$, the slope of a curve with partial trade credit is

$$(4.71) \qquad \frac{E_T'(\pi(T,Q))}{cV_T'(\pi(T,Q))} > \frac{E_T'(\pi(T,Q))}{V_T'(\pi(T,Q))} ,$$

and it is steeper than a curve with $c = 1$ at the same T. It is also evident from the expression for expected profit and variance that

$$(4.72) \qquad cE(\pi(T,Q) < E(\pi(T,Q)) ,$$

$$(4.73) \qquad c^2V(\pi(T,Q)) < V(\pi(T,Q)) .$$

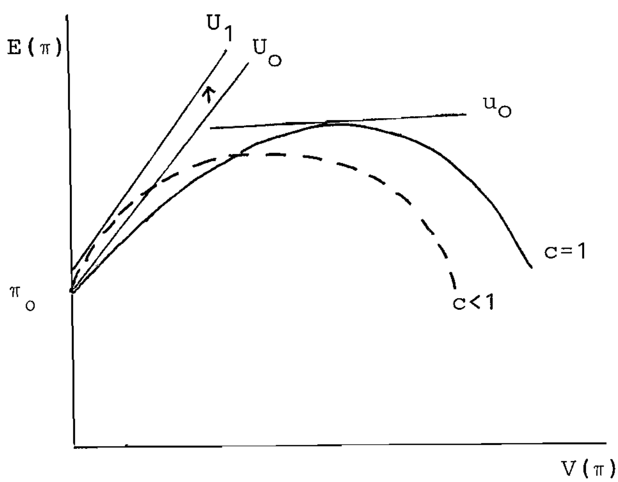

Figure 4.2 Utility maximization with partial trade credit.

This means that the highest $cE(\pi(T,Q))$ point must lie below $E(\pi(T,Q))$ and the highest $c^2V(\pi(T,Q))$ point must lie to the left of $V(\pi(T,Q))$. This, (4.71), and the fact that all curves start at the same point implies that initially the $c < 1$ curve must lie above and then below the $c = 1$ curve. A consequence of this is that if the indifference curves are not vertical (infinite risk aversion) there must exist some curve with $c < 1$ such that $U_o < U_1$. Another consequence is that the shape of the $c < 1$ curve excludes solutions with $c < 1$ if the slope of the indifference curves (or equivalently risk aversion) is small enough, because then the point of tangency between the highest indifference curve and a mean-variance curve must lie "north-east" of the $c < 1$ curve. It is also evident that with no risk aversion the $c = 1$ solution is preferable. Then it is necessary to introduce some additional assumption about the role of a down payment to justify the use of partial trade credit. The upshot of the analysis above is that if risk aversion differs between firms there will exist many different trade credit contracts. For example, a small firm with a single owner (high risk aversion) supplies, cet. par., partial trade credit while a large corporate firm (low or no risk aversion) uses "all or nothing" solutions. To what extent this is an observation that is in line with reality remains a topic for future empirical research.

4.5 SUMMARY

In this chapter I have descussed trade credit issues that are of concern to firms facing the possibility of payment failure at the end of the credit period. The only motive to supply trade credit has been risk adjusted interest arbitrage, and also here this gave rise to an interdependence between trade credit decisions and the quantity and distribution of output between different markets. First I showed that the introduction of uncertainty reduces the quantity of output and the length of the credit period, and these two effects were strengthened with increasing risk aversion. Another consequence of the introduction of payment failure and risk aversion was that the higher the risk aversion the wider is the interest margin before the seller is better off with than without trade credit. When I introduced more than one market it became clear that the interdependence, in uncertainty terms, between different markets has to be taken into account. It was a formal way of giving answers to questions such as: "What is the optimal way of spreading credit risks among markets in different countries?" The introduction of double uncertainty made it possible to combine trade credit uncertainty with other uncertainty aspects. With this approach it is possible, for example, to take both payment and exchange rate uncertainty into account. I have also showed that expected profit maximizers have no reason to supply trade credit with down payment, unless the down payment produces some additional information about the payment behavior of the customers. This differs from the result in the expected utility maximization case, when I showed that risk aversion can give rise to down payment credit contracts.

APPENDIX I (Two alternative model formulations.)

I.1 The safety first decision rule

In the preceding parts of this chapter I have discussed different aspects of trade credit when the objective function was a mean-variance utility function. Here, I discuss the behavior of a firm which uses a safety first decision rule instead. The safety first decision rule is formulated such that the firm minimizes the probability of some prespecified disaster event. In a model based on a profit function it seems reasonable to let the disaster event be represented by a level of profit less than or equal to zero, $D \leq 0$. In order to formulate a safety first objective function I start with the Tchebycheff inequality, which states that the probability that a stochastic variable X deviates from its mean $\bar{X}$ by more than $bV(X)^{1/2}$ is[1]

$$\text{(A.I.1)} \qquad P\left(|X-\bar{X}| \geq bV(X)^{1/2}\right) \leq \frac{1}{b^2}, \qquad b > 0,$$

which with D = 0 can be written[2]

$$\text{(A.I.2)} \qquad P(X \leq 0) \leq \frac{V(X)}{\bar{X}^2} .$$

Now, let X represent the level of profit. Then (A.I.2) is

$$\text{(A.I.3)} \qquad P\left[e^{(r_b(T)-r_s(T))T} R(Q,\varepsilon) - CQ \leq 0\right] \leq \frac{V\left(\pi(T,Q|\varepsilon=0)\right)+V\left(\pi(T,Q,\varepsilon)\right)}{\left(e^{M(T)}R(Q) - CQ\right)^2},$$

---

[1] See Blom (1970) chapter 5, p. 21, and Philippatos (1979).

[2] The inequality in (A.I.2) has been derived in the following way: Set $bV(X)^{1/2} = \bar{X} - D$, then with D = 0, the rhs of (A.I.2) is equivalent to $1/b^2$. Further

$$P(|X-\bar{X}| \geq \bar{X} - D) = P(|X-\bar{X}| \geq \bar{X}) = P(-X + \bar{X} \geq \bar{X}) = P(X \leq 0),$$

when $X \leq \bar{X}$.

where the notation is the same as in (4.37) and (4.43)-(4.45). To simplify the exposition I have also assumed that ρ in (4.42) is equal to zero. If trade credit is the only source of uncertainty, $V\big(\pi(T,Q,\varepsilon)\big) = 0$, minimization of the probability of default implies that the rhs of (A.I.3) has to equal zero, which occurs when T = 0. With no risk tolerance at all there is no supply of trade credit, and this holds even if there is a large positive interest rate differential. In the case when there is double uncertainty the conclusion about the length of the credit period need not hold. The overall variance is not zero at T = 0, and it pays to increase T as long as the rhs of (A.I.3) is falling. Differentiation of the rhs of (A.I.3) with respect to T gives the condition that T is increased if

$$\text{(A.I.4)} \qquad M'(T)e^{M(T)}R(Q) \geq \left[\frac{e^{M(T)}R(Q) - CQ}{2\left[V\big(\pi(T,Q|\varepsilon=0)\big) + V\big(\pi(T,Q,\varepsilon)\big)\right]}\right](\ldots)$$

$$(\ldots) = V_T'\big(\pi(T,Q|\varepsilon=0)\big) + V_T'\big(\pi(T,Q,\varepsilon)\big).$$

It pays to increase the length of the credit period as long as the marginal increase in profit is larger than the marginal increase in variance, when the latter has been weighted with half the ratio of expected profit to overall variance. Hence (A.I.4) holds as long as the increase in trade credit variance is small, because then the positive revenue effect dominates the negative variance effect. I have shown earlier that when a firm maximizes expected profit M'(T) = 0. This implies that, in this case, the inequality in (A.I.4) cannot hold. M'(T) becomes positive if T falls. Consequently, the credit period is shorter in the safety first case than if a firm maximizes expected profit. When the owners of the firm have a mean-variance utility function the first order condition with respect to T can be written

$$\text{(A.I.5)} \qquad M'(T)e^{M(T)}R(Q) = -k\Big[V_T'\big(\pi(T,Q|\varepsilon=0)\big) + V_T'\big(\pi(T,Q,\varepsilon)\big)\Big],$$

which means that if -k is equivalent to the bracketed expected profit variance ratio in (A.I.4) the two objective functions give the same solution. If, on the other hand, -k is smaller (less risk aversion) the mean-variance approach gives a longer credit period than the safety first criterion. Here, I have only discussed the length of the credit period, but similarly, it is easy to show the output effects with expressions analogous to (A.I.4) and (A.I.5).

I.2 Alternative payment behavior

My discussion about various aspects of trade credit under uncertainty has rested on the assumption that at the end of the credit period there is either complete default or full payment. This is of course only one of several possible assumptions about the payment behavior of the customers. Another alternative is to assume that nobody defaults but the actual length of the credit period can differ from the agreed length. With this assumption the length of the credit period can be treated as a stochastic variable

$$\bar{T} = T + \theta, \tag{A.I.6}$$

where T is the agreed length of the credit period and $\bar{T}$ is the real length. This means that the discount function now is

$$e^{\left(r_b(T)-r_s(T)\right)T - r_s(T)\theta} = e^{N(T) - r_s(T)\theta}, \tag{A.I.7}$$

where, in order to simplify the exposition, I have assumed that the interest rates are based on the agreed credit period and there is no penalty for late payment. Default is in this case equivalent to a large θ, because then the present value approaches zero. To analyze this type of payment behavior, it is necessary to make some assumption about the distribution of θ. This is an empirical issue. Here I assume that θ has an exponential distribution, which means that $\theta \geq 0$. Payment is made some time after the agreed credit period.

This is a simplification. It is probably more likely that θ has a skewed distribution, with most of its mass to the right of $\theta = 0$. Payment prior to T is less likely than payment after T. When payments are exponentially distributed among n independent customers with identical demand functions, the utility function can be written[3]

$$(A.I.8) \qquad U(\pi) = n\left[e^{N(T)}R(Q)\,\frac{1}{1+r_s(T)m} - CQ + \right.$$

$$\left. + \, ke^{2N(T)}R(Q)^2\left[\frac{1}{(1+2r_s(T)m)} - \frac{1}{(1+r_s(T)m)^2}\right]\right].$$

With first order conditions, at k = 0

$$(A.I.9) \qquad N'(T) - \frac{mr_s'(T)}{1+r_s(T)m} = 0,$$

---

[3] The exponential distribution has the frequency function

$$f(\theta) = \begin{cases} \frac{1}{m}\, e^{-\frac{\theta}{m}} & \theta \geq 0 \\ 0 & \theta < 0, \end{cases}$$

with mean m. Then the expected present value of revenue is

$$E(R_{pv}) = e^{N(T)}R(Q)\frac{1}{m}\int_0^\infty e^{-\theta(\frac{1}{m} + r_s(T))}\,d\theta = e^{N(T)}R(Q)\,\frac{1}{r_s(T)m+1}$$

The variance is $V(R_{pv}) = E(R_{pv}^2) - (E(R_{pv}))^2$, where the former expression is

$$e^{2N(T)}R(Q)^2\int_0^\infty \frac{1}{m}\, e^{-\theta(\frac{1}{m} + 2r_s(T))}\,d\theta = e^{2N(T)}R(Q)^2\,\frac{1}{1+2r_s(T)m}$$

The expressions taken together give the variance expression

$$V(R_{pv}) = V(\pi(T,\ Q,\ m)) = e^{2N(T)}R(Q)^2\left[\frac{1}{1+2r_s(T)m} - \frac{1}{(1+r_s(T)m)^2}\right]$$

$$\text{(A.I.10)} \quad R'(Q)\,\frac{e^{N(T)}}{1+r_s(T)m} - C = 0$$

Looking at (A.I.9) T must be shorter than in the no uncertainty case, since then $m = N'(T) = 0$. A similar conclusion holds with respect to the quantity of output. The present value marginal revenue is higher with $m = 0$. Hence, output must be higher in the no uncertainty case. Consequently, a worsening of the payment habits of the customers reduces both the credit period and the quantity of output. It can further be shown that the second order conditions hold, provided that the average payment delay is small. A large m eliminates the interest arbitrage profit, and the seller is better off without than with trade credit. Again at $k = 0$ the effect of an increase in risk aversion[4] is

$$\text{(A.I.11)} \quad \frac{dT}{dk}\Big/_{k=0} = \frac{V_T'\left(\pi(T,Q,m)\right)}{-\,U_{TT}''(\pi)} > 0,$$

---

[4] The sign of the first order derivative of the variance in (A.I.8), with respect to T is determined by the derivative of the two ratios

$$\frac{e^{2N(T)}}{\left[1+2r_s(T)m\right]} - \frac{e^{2N(T)}}{\left[1+r_s(T)m\right]^2}.$$

The sign of the derivative of the first ratio is determined by

$$N'(T) - \frac{r_s'(T)m}{1+2r_s(T)m} > 0.$$

That this expression is positive follows from (A.I.9). The derivative of the second ratio is

$$2e^{2N(T)}\left[\frac{N'(T) - \dfrac{r_s'(T)m}{1+r_s(T)m}}{\left[1+r_s(T)m\right]^2}\right] = 0$$

and it equals zero because the numerator is equivalent to (A.I.9). Hence, $V_T'\left(\pi(T,Q,m)\right)$ is positive at $k = 0$.

$$\text{(A.I.12)} \quad \frac{dQ}{dk}_{/k=0} = \frac{V'_Q\left[\pi(T,Q,m)\right]}{-\;U''_{QQ}(\pi)} > 0,$$

because the increase in variance is in both cases positive. Hence, also in this case a risk averse producer has a lower output and a shorter credit period than a risk neutral producer.

APPENDIX II (A note on the credit granting decision.)

My discussion about different aspects of trade credit when transaction costs and goods market uncertainty has not been taken into account has rested on the implicit assumption that the firm supplies trade credit if it is better off with than without trade credit. With simplified notation expressed in terms of expected profit, the decision criterion is grant credit if

(A.II.1) $P(T)R(T) - C(T) > R(Q) - C(Q)$,

where P(T) is the probability of payment, R(T) and C(T) the present value of revenue and cost of production respectively, and the rhs of (A.II.1) represents no trade credit net profit. This decision criterion has not been used by all authors who discuss the use of trade credit. Without commenting on underlying market conditions Mehta (1968), (1970), Bierman and Hausman (1970), Alton and Gruber (1975) and Copeland and Kohoury (1979) use some version of the condition; grant credit if

(A.II.2) $P(T)R(T) - C(T) > 0$,

that is, credit should be granted if the present value of revenue is higher than the cost of production. The authors are primarily not interested in the type of trade credit theory I have discussed. Except the article by Copeland and Kohoury the main topic in their articles is trade credit in a multiperiod context, when credit granting decisions are combined with investment in and processing of information about the customers of the firm. The decision rule given in (A.II.3) is the starting point of their articles, and it must therefore affect the results. It is thus of interest to compare this decision rule with the one I have chosen. The two decision criteria are equal if the no--trade-credit alternative is such that the firm makes a zero profit either because $R(Q) = C(Q) = 0$, or because no trade credit means no sales and no production. From the discussion in the articles it is clear that the authors have the latter case in mind. This is a very stringent assumption, which is stated without comments by the authors, since it means that trade credit is a dominating component in the demand function. If no trade credit means no demand trade credit policies other than those that rest on risk adjusted interest arbitrage are of course also possible. However, I want to argue that my decision criterion is preferable to (A.II.2) because (A.II.2) can include cases when the expected net profit with trade credit is lower than in the cash sales case.

5. Risk, Information and Some Other Aspects of Trade Credit

5.1 INTRODUCTION

In this chapter I broaden the perspective. Trade credit is not treated in isolation. Here I comment on how attitudes toward risk, information among financial market participants, and institutional factors affect the use of trade credit and some close substitutes. When I discuss attitudes toward risk and information I use parts of the model structure set forth in the preceding chapter. The risk adjusted interest rate of different market participants is a central variable in the analysis, and quantity effects, the way they were described in the preceding chapter, are not taken into account. I still concentrate on interest arbitrage aspects of different credit alternatives. Various transaction cost aspects are discussed in Chapters 6 and 7. In the sections about institutional aspects I use some simple new model formulations without uncertainty. Here some demand effects are also taken into account.

In Section 5.2 I argue that the risk adjustment factor, which is a part of the risk adjusted interest rate, differs among the market participants because of different information about the probability of default, different types of security, different attitudes toward risk, and different opportunities to diversify. I then show under what conditions a financial intermediary supplies a loan in a roundabout way, via a loan to the seller and a trade credit between seller and buyer, instead of giving a loan directly to the buyer. I discuss under what conditions both seller, buyer and financial intermediary accept crecit card credit, and finally, in a short note I argue that the combination of loan and interest rate ceilings turn commercial banks into risk minimizers. A behavior which leaves the field open for non-risk minimizers to supply various types of short term credit.

In Section 5.3 I discuss three alternatives to simple trade credit. *First*, I show how commercial bills or factoring are used to lower the probability of default compared with unsecured loans. This is thus one way to reduce risk when a firm or household cannot offer traditional types of mortgages as security. Since commercial bills have no transaction cost advantages compared to simple trade credit, the risk aspect is possibly one way to explain the stylized fact that commercial bills play such a minor role on the balance sheet of the aggregated Swedish corporate sector. *Second*, I discuss two alternatives which affect the credit risk taken by the seller. I show under what conditions there is likely to exist markets for sale of trade credit bills, documentary credit, and credit risk insurance. Three alternatives that from the sellers' point of view can eliminate trade credit risk completely. The upshot of the discussion is that the three markets have the same driving forces. Interest rate differentials, different information about credit risk, different attitudes toward risk, and different opportunities to diversify can, one at a time or simultaneously, give rise to situations when a sale of trade credit bills can make both seller and buyer of bills better off. The same holds with respect to credit risk insurance.

In Section 5.4 I look at some important institutional aspects of different borrowing alternatives. In Sweden consumer trade credits include, by law, as a minimum a twenty percent down payment requirement, while credit card credit includes no such rule. I use a simple two period utility function to show that this difference between trade credit and credit cards makes credit cards competitive also in cases when the credit card rate of interest exceeds the trade credit rate. This is one possible explanation of the rapid spread of credit cards in the seventies. I also discuss bank loans with compulsory savings requirements, and show why credit card borrowing is preferred to this type of loan arrangement when there is a marked interest rate differential in favor of bank loans.

Finally, in Section 5.5 I comment on the use of credit risk insurance and interest subsidies as a way to boost exports to high risk developing countries. By the help of the well known kinked demand curve from oligopoly theory I show how interest rate subsidies combined with seller price compensation give rise to positive demand effects if competing countries do not use the same kind of export subsidy. In this context trade credit is used as a way to grant buyers more or less hidden price concessions.

5.2 TRADE CREDIT, THE BANK'S VIEWPOINT

5.2.1 *Loans from a third party*

So far I have only discussed trade credit from the seller's point of view. Alternative financial arrangements have not been taken into account. Here I compare trade credit with loans from some financial intermediary. There are three transactors involved: the buyer and seller of goods and a financial intermediary, e.g. a commercial bank. The bank can lend either directly to the buyer or finance the seller's supply of trade credit. In this section I discuss the two lending alternatives from the bank's point of view and I still concentrate on financial and risk aspects. First I present the risk adjusted interest rate concept in some greater detail than in the preceding chapter.

In Chapter 4 I showed that if the seller has a mean--variance utility function, the utility of revenue can be written

$$(5.1)\quad e^{\left(r_b(T)-r_s(T)-d\right)T}R(Q) + kV\left(\pi(T,Q)\right) = e^{\left(r_b(T)-r_s(T)-d'\right)T}R(Q),$$

where d' is a risk/risk aversion interest rate adjustment factor,[1]

$$(5.2)\quad d' = d - \ln\left[1 + k(1-e^{-dT})e^{\left(r_b(T)-r_s(T)\right)T}R(Q)\right]/T.$$

In (5.2) the risk aversion coefficient, k, is multiplied by the expected loss of revenue in case of payment failure. This product represents the disutility of uncertainty. With risk aversion the logarithmic expression is less than one and the higher the risk aversion the higher is d'. To facilitate a comparison between the use of trade credit and some of its substitutes I assume that the credit period is predetermined, T = 1, and, if not differently stated, quantity effects are not taken into account. Thus, there is no connection between profitable interest arbitrage and the choice of cash price. This means that a firm which has a choice to supply or not to supply trade credit is better off with trade credit if

$$(5.3)\quad e^{r_b - r_s - d'}R(Q) - CQ > R(Q) - CQ,$$

where the utility of trade credit alternative is represented by the risk/risk aversion interest rate adjustment formulation. (From now on I use the short form, risk adjustment, to represent the d'-variable.) The inequality in (5.3) reduces to

$$(5.4)\quad r_b - d' > r_s,$$

the risk adjusted lending rate must exceed the borrowing rate. The reformulation of my treatment of risk aversion is summarized by the risk adjustment coefficient d', which in the inequality above can be interpreted as a rate of interest formulation of

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[1] When (5.1) is solved for d', k is multiplied by the ratio of the variance and the expected revenue. When common terms have been cancelled this ratio reduces to the expression following k above.

the price of risk. Consequently the more risk averse the seller the higher is the price of risk and the higher is the lending rate before it pays to supply trade credit.

Now the analysis in this chapter rests on the assumption that the credit market, with its various sub-markets, is not characterized as a frictionless market with ubiquitous and free information and zero search and contract costs. Instead I think it is appropriate to describe the credit market as a market where there is a non-uniform distribution of information about the characteristics of different assets and of attitudes towards risk. Different market participants have, at a given moment and position in time and space, different opinions about various lending or borrowing opportunities, and it is costly to obtain more information about different trading alternatives.[2] Now assume further that lenders other than the supplier of trade credit can, e.g. by methods similar to the one sketched above, calculate risk adjusted interest rate expressions similar to the one in (5.4). Then an important factor which determines whether a credit offer is made or not is the d'-variable. Different lenders can have different d':s when they determine whether to supply or not to supply credit to a given customer. (This holds of course also with respect to the beliefs about what r_b is, but to keep the analysis simple I do not take unknown interest rates into account.) *First*, different lenders can have different information about the probability of default. In many cases, a long-lasting customer relationship can practically eliminate the probability of default, while when a firm enters a new market there can be considerable uncertainty about the customers' ability to pay. It is in the latter case that financial intermediaries play an important role as producers and transmitters of information. *Second*, different types of

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2 For a more elaborate exposition of these ideas about how markets operate, see Alchian (1970) and Brunner and Meltzer (1964).

security affect the probability of default. There is a whole array of different types of secured loans that can be used as substitutes for trade credit, but a detailed description lies beyond the scope of this chapter. Here, it is easiest to think of trade credit as an unsecured loan which is compared with other short term loans of the same type. (Trade credit agreements can of course also include various reservations of title clauses.) *Third*, different transactors have different attitudes towards risk. *Fourth*, if the covariance between different borrowers is different from zero, as in the two market case in the preceding chapter, this also affects the size of d' because different lenders have different opportunities to diversify. A supplier of trade credit with customers that have a high covariance with respect to the probability of default is likely to have a higher d' than for example a bank with a greater opportunity not to put all eggs in the same basket. This is particularly the case if the transaction costs incurred in finding alternative financial investment opportunities are high.

To motivate the existence of different types of credit it is necessary to study what conditions are likely to generate a particular credit flow. The choice of market conditions discussed here is naturally somewhat arbitrary, but despite this, I think they can shed some light on factors that give rise to the large flora of different credit arrangements observed in reality, when the assumptions about a frictionless credit market are dropped. The characteristic feature of a trade credit agreement is that there is a simultaneous goods and financial transaction between the traders. In the introductory chapter I showed that close substitutes to trade credit involve a third party which handles the financial part of a given goods transaction. Now, assume that a financial intermediary can supply credit either directly to some buyer or to the seller, which in turn can supply trade credit to the buyer. The latter alternative is chosen if

$$(5.5) \qquad r_s - \delta_s > r_b - \delta_b > r,$$

where δ_s and δ_b represent the risk adjustment coefficients calculated by the bank of the seller and buyer respectively, and r is the bank's cost of borrowing. Further, if $r_s < r_b$ it is necessary that $\delta_s < \delta_b$. The bank must have more "positive" information about the seller than about the buyer. Combining (5.4) and (5.5) a roundabout credit flow takes place if the following row of inequalities holds

$$(5.6) \qquad r_b - d' > r_s > r_s - \delta_s > r_b - \delta_b > r,$$

which implies that $d' < \delta_b$. The seller must have a lower risk adjustment coefficient than the bank. If a roundabout supply of credit takes place δ_s is not completely independent of δ_b. The greater the seller's expected ability to pay, regardless of the buyer's behavior, the less important is this interdependence. One situation when the inequality in (5.6) is likely to hold is when the financial intermediary has less information about the customers than the seller, because then the risk adjustment coefficient of the bank is likely to be large. This is, e.g. the situation facing a small local bank. Such a bank supplies credit to local firms but not to their customers if they lie outside the region within which the bank has reliable information. In this case, the seller acts as a financial intermediator instead of the bank.

In (5.6) both the bank and the seller are better off with roundabout credit. Another case when both are better off with direct credit to the buyer is when

$$(5.7) \qquad r_s + d' > r_b > r + \delta_b.$$

Now, if $r_s < r_b$ the seller sees his customer as a high risk while the bank does not do so. This is a situation that can occur if the bank and the seller have different objective functions. A large corporate bank or finance company probably maximizes expected profit because both the firms and their shareholders have opportunities to diversify, while a single owner of a small firm takes other risk aspects than the expected outcome into account and has fewer opportunities to diversify. The ability to process information can also be important. A finance company operating a credit card system can generally be expected to be able to use available information more efficiently than some small producer primarily concerned with production of goods instead of granting of credit. The producer can have very little information about a particular customer while the finance company has good information about the payment behavior of the population to which the customer belongs. Then it is likely that there is a situation such that (5.7) holds.

There exist many different payment arrangements between buyer, seller and some external financier. The distribution of credit risk between the three parties is one way to distinguish one payment alternative from another. Here, I comment on some further aspects of the simple direct credit agreement between buyer and financier. In a subsequent section I also discuss some alternative, more complicated three party goods transaction credit agreements. A typical example of direct credit from a third party to the buyer is different types of purchases when the buyers use credit cards. Then the credit card company supplies the credit and takes the credit risk. Between 1970 and 1980 the credit card debt to Swedish finance companies increased ten times in real terms, while during the same period refinancing of trade credit contracts by the same finance companies not even doubled, in real terms.[3] Since the sellers of goods accept credit cards as a mean of payment, they must be better off with than without credit cards. Otherwise, they would probably have developed competitive payment alternatives and the rapid spread of credit cards would have been slower.

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[3] For a detailed description of different credit card payment alternatives, see Finance Companies, SOU 1977:97.

Further, an indication that the sellers gain from the use of credit cards is that they pay a fee to the finance company each time they accept a credit card purchase. Now assume that the seller finds the supply of trade credit too risky, given the interest rate that the buyer accepts, but a finance company is of a different opinion. Then, if the finance company charges the buyer a credit card interest rate equivalent to the buyer price compensation interest rate described in preceding chapters, the buyer present value price of a particular good will be equivalent to the cash price charged by the seller. With this interest rate policy the seller cannot be made better off and there is no reason for the seller to pay the finance company a credit card fee. If, on the other hand, the finance company charges an interest rate such that $r < r_b$, there will be a positive demand effect because the present value price of the buyer falls

$$(5.8) \qquad Q(P_o) < Q\left(e^{r-r_b} P_o\right).$$

This means that the seller can increase the cash price without reducing demand as long as

$$(5.9) \qquad P \leq e^{r_b - r} P_o,$$

when P is the new cash price. The demand function shifts out and the seller must be better off with than without credit card credit. When the inequality in (5.9) holds all three parties gain. A seller which supplies trade credit switches to credit card sales if a positive quantity effect plus the gain in utility from the elimination of uncertainty outvies the loss of interest arbitrage profit from trade credit. The important point here is that if the finance company has less risk aversion, different information or access to other parts of the credit market than the seller, it can give the buyer a credit offer such that the present value price will be lower than the cash price. This generates a positive demand effect which makes it profitable for the seller to participate in the three party goods transaction credit arrangement.

The information/risk aspect of trade credit, in comparison with other credit arrangements, differs much between different markets. In the rural district in Finland where I grew up, it is common that the local population pay their accumulated grocery bills in the local grocery store at the end of each month. In this case, in a small quite stationary society the sellers have practically perfect information about all local customers and there is no need to use some risk reducing outside financier. A seller of consumer durables in a big city faces a totally different situation. He is likely to meet his customers only once and he has little information about who they are and what their payment habits are. In this case it can be worthwhile to include a third party that specializes in information processing and risk taking.

5.2.2 *Bank regulation - a digression*

A recurrent objective of Swedish monetary policy has been to discourage lending to the household sector by commercial banks. The objective has been to avoid interest rate crowding out in the loan market. High marginal tax rates combined with income taxes calculated net of interest payments have made most households relatively insensitive to high nominal interest rates. When households are rationed in the commercial bank loan market the inequalities in (5.6), disregarding the $r_b - \delta_b$ variables, is a condition which states under what circumstances there will be a roundabout credit flow, via goods producing firms to the household sector. The longer such a policy is in force, the more likely it is that goods producers gather reliable information about their customers, and the more likely it is that indirect credit flows take place. In the long run this can eliminate the rationing effects completely. Another monetary policy tool which discriminates against commercial banks is interest rate ceilings, which also can be combined with lending restrictions. Without regulation it is reasonable to assume that banks seek to maximize the margin between the risk adjusted lending rate and the borrowing rate. In the example discussed above, the objective is to find

$$(5.10) \qquad \max(r_b - \delta_b - r,\ r_s - \delta_s - r),$$

when in a stable market both alternatives in the long run ought to give roughly the same result. If, however, there is a binding interest rate ceiling on bank loans the objective becomes

$$(5.11) \qquad \max(\bar{r} - \delta_b - r,\ \bar{r} - \delta_s - r),$$

which means that in this case the best policy is to minimize the risk, choose a δ which is as small as possible. An interest rate ceiling will redistribute credit from high risk to low risk customers and it leaves the field open for alternative channels to supply high risk customers with credit. In Sweden, interest rate ceilings have been determined in terms of average lending rates, but this does not change the conclusion about risk minimization. Assume that a bank has lent 1 SEK to the seller, which also is assumed to be the low risk customer, the average interest rate has to equal $\bar{r}$ and the bank has to decide whether to lend one additional SEK either to the seller or to the buyer. The objective is to find

$$(5.12) \qquad \max\left(r_s - \delta_s + r_b - \delta_b - 2r,\ 2(\bar{r} - \delta_s - r)\right),$$

but since $r_s + r_b = 2\bar{r}$ this reduces to

$$(5.13) \qquad \max\left(2(\bar{r} - r) - \delta_b - \delta_s,\ 2(\bar{r} - r) - 2\delta_s\right),$$

which implies risk minimization. Choose customers such that the sum of the risk adjustment coefficients is as small as possible. In this case, lend an additional SEK to the seller. This type of risk minimization is one way to explain the rapid increase in loans from unregulated finance companies during the seventies.

5.3 THREE ALTERNATIVES TO SIMPLE TRADE CREDIT

5.3.1 *Commercial bills or factoring*

With the same model as in Section 5.2 it is possible to discuss different risk, risk aversion and information aspects of some alternatives to simple trade credit. A simple trade credit arrangement is the case when the seller grants the buyer permission to defer payment, when the payment at the end of the credit period is made directly to the seller. The other alternatives commented on here are the use of commercial bills (or its substitute factoring), the use of documentary credit (or alternatively, a sale of trade credit bills to a finance company), and finally trade credit combined with credit risk insurance.

Simple trade credit involves only seller and buyer, while if a commercial bill is used, and if it is discounted, a third party, a commercial bank, is included too. When seller and buyer agree to use a commercial bill, the seller is paid immediately by the bank when the bill is discounted, and the buyer pays the bank at the end of the credit period. If the bank has discounted a commercial bill, and the buyer fails to pay at the end of the credit period, the bank can take action of recourse against the seller, which then has to pay the bank. Thus, from the seller's point of view, the risk is the same as in the simple trade credit case. However, the bank is from a risk perspective not indifferent between commercial bills and other types of loans. This is particularly the case when a commercial bill is compared with an unsecured loan either to the seller or to the buyer. (This is shown below.)

An alternative to commercial bills is factoring. In this case seller and buyer make an ordinary trade credit agreement and the trade credit bills are forwarded to a finance company. The seller gets a loan from the finance company, which keeps the trade credit bills as collateral until the buyer pays the finance company, and the debt between finance company and seller is cancelled. If the buyer fails to pay the finance company has a right to take action of recourse against the seller. This is what makes commercial bills and factoring almost equivalent. A difference between the two is that commercial bills are discounted one by one, while if the seller uses factoring the finance company establishes an upper borrowing limit based on the loan collateral value of the stock of trade credit bills. This means that from a transaction cost point of view factoring is more efficient than commercial bills.[4] One of the stylized facts commented on in the introductory chapter was that simple trade credit is much more common than commercial bills. Commercial bills made up only 1 percent of short term assets and 0.2 percent of short term liabilities of the aggregate Swedish corporate sector, while the corresponding percentages for accounts receivable and accounts payable were 25.1 percent and 30.1 percent respectively. (The percentages do not change dramatically if factoring is included too.) One possible explanation of this fact lies in the risk structure of commercial bills compared with other types of credit. Since the seller, from a risk point of view, is indifferent between simple trade credit and commercial bills, it is the risk seen from those who supply either seller or buyer with credit that matters. One way to explain these risk aspects is to extend the simple probability of payment approach I have used so far. (There are also transaction cost aspects of the choice of different two or three party arrangements, but they lie outside the scope of this chapter.)

Assume that there is a bank which can give an unsecured loan either to the seller, who uses the loan to finance his trade credit to the buyer, or directly to the buyer. The two loan alternatives are still of the zero-one type, either full repayment or complete default. This means that the seller's

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[4] For a more detailed description of juridical aspects of commercial bills and factoring contracts, see Karnell (1972) and Finance Companies (1977).

ability to pay depends on his own financial position and on whether the buyer defaults or not. To express these two loan alternatives in probability terms I need the following notation:

Sp = payment by the seller,

Bp = payment by the buyer,

Bnp = no payment by the buyer (default),

Scb = payment when the bank uses a commercial bill,

P(..) = probability of payment including one or several of the arguments above.

Then the probability of payment by the seller is[5]

(5.14) $P(Sp) = P(Sp \cap Bp) + P(Sp \cap Bnp)$,

where,

(5.15) $P(Sp \cap Bp) = P(Bp)P(Sp|Bp)$,

(5.16) $P(Sp \cap Bnp) = \big(1 - P(Bp)\big)P(Sp|Bnp)$.

Insertion of these expressions in (5.14) gives

(5.17) $P(Sp) = P(Bp)[P(Sp|Bp) - P(Sp|Bnp)] + P(Sp|Bnp)$.

This is the probability of payment when the bank grants the seller an ordinary unsecured loan. A priori it is impossible to say whether $P(Sp) \gtrless P(Bp)$. This depends on what information the bank has about the seller and the buyer. Now, assume that the bank uses a commercial bill instead (or if the bank is replaced by a finance company there is a factoring agreement between finance company and seller). Then

(5.18) $P(Sp|Bp) = 1$,

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5 This is also a formalization of the thoughts in Wicksell (1898), Chapter 6. The probability formula in (5.14) is a direct application of the theorem of total probability in Chou (1975), p. 101.

because if the buyer pays, he pays directly to the bank instead of to the seller. In this case (5.17) becomes

$$(5.19) \qquad P(Scb) = P(Bp) + \big(1 - P(Bp)\big)P(Sp|Bnp),$$

and the difference between (5.19) and (5.17) is

$$(5.20) \qquad P(Scb) - P(Sp) = P(Bp)\big(1 - P(Sp|Bp)\big).$$

Similarly, the difference between the probability of payment with a commercial bill and payment by the buyer can be written

$$(5.21) \qquad P(Scb) - P(Bp) = P(Sp|Bnp)\big(1 - P(Bp)\big).$$

This means that if none of the payment probabilities above take on the extreme values zero or one, the probability of payment is higher when the bank uses a commercial bill compared to either an unsecured loan to the seller or to the buyer. Hence, from the bank's point of view, the main function of a commercial bill is to reduce risk. In terms of the notation used earlier with

$$(5.22) \qquad P(Sp) = e^{-\delta_s}, \; P(Bp) = e^{-\delta_b},$$

the use of a commercial bill gives

$$(5.23) \qquad P(Sp), P(Bp) < P(Scb) \Rightarrow \delta_s, \delta_b > \delta_{cb}.$$

From (5.20) and (5.21) it is clear that if the probabilities of payment are high, the use of a commercial bill gives only a small increase in the probability of payment. In this case the risk aspects of the two alternatives are unimportant. With, for example, $P(Bp) = P(Sp|Bp) = 0.975$, the increase in the probability of payment in (5.20) is only 0.00244. If both seller and buyer are considered high risk borrowers, the use of a commercial bill gives a considerable increase in the probability of payment. With $P(Bp) = P(Sp|Bp) = 0.5$, the increase in the probability of payment in (5.20) is 0.25.

These risk aspects are possibly one way to explain interest rate differences between discounted commercial bills and the supply of unsecured credit on account by commercial banks. (The interest rates are given in Table 5.1.) With competition among commercial banks, if not perfect, at least some competition, the lending rates ought to approach the borrowing rates plus some transaction cost and risk compensation. With this view of the world, and taking account of the discussion about the probability of payment above, the interest rates charged on unsecured credit ought to be higher than the rates on discounted commercial bills. Table 5.1 shows that this is also the case with respect to the average rates. The minimum rates are quite close to each other, which in terms of the discussion here means that there is little uncertainty, the payment probabilities lie close to one. However, there is a considerable difference between the maximum rates. This is possibly a reflection of the fact that a high risk loan arranged as a commercial bill always has a higher probability of payment than an unsecured credit.

Table 5.1 Interest rates, commercial banks 1980, r = average rate, $\underline{r}$ = minimum rate, $\bar{r}$ = maximum rate. Source: Bank of Sweden, Yearbook 1980.

	r	$\underline{r}$	$\bar{r}$	%
Credit on account unsecured	13.89	10.10	22.00	
Discounted bills	13.81	10.25	17.50	

The comparison above has concerned unsecured credit and commercial bills. Another alternative is to supply some type of secured credit which can eliminate uncertainty almost completely. An advantage of secured credit is that it reduces the need to be informed about how the borrower uses the money that is being lent to him. This is important if information costs are high. In a credit market with uncertainty, information and transaction costs, the type of security that the borrower can offer affects his cost of capital. The better the security, the lower is the risk compensation required by the lender. It is in this risk/security perspective that the use of commercial bills traditionally has played an important role to facilitate trade when other types of security are not available, or if it is too expensive to arrange other types of securities. When a firm cannot offer traditional mortgages as security, either because it is too heavily in debt, or because it has an asset structure badly adapted to traditional securities such as mortgages on plant, stock or equipment, it has to resort to high risk borrowing from firms that take higher risks than commercial banks. In those cases when the risk aspect is an important determinant of the borrowing rate, high risk firms or households will pay high interest rates. This is possibly one contributing explanation of the fact that commercial bills play such a minor role on the balance sheet of the aggregated Swedish corporate sector. Further, from a transaction cost point of view, there are no advantages to use commercial bills compared to simple trade credit, and the transactions motive is probably the most important explanation of why firms use trade credit (for a more detailed discussion see Chapter 7).

Table 5.2 Assets and liabilities in the aggregated Swedish corporate sector and in the subsector Garages with more than twenty employees, year 1980. Source: Enterprises, 1980.

	Aggregate corporate sector	Garages 20 -		Aggregate corporate sector	Garages 20 -
Receivables / Current assets	0.25	0.12	Payables / Current liab.	0.30	0.19
Commercial bills / Current assets	0.01	0.12	Commercial bills / Current liab.	0.002	0.05
Cash and bank dep. / Current assets	0.11	0.01	Cash and bank dep. / Current liab.	0.17	0.02
Debt / Equity	7.77	12.50			

Having plowed through disaggregated data of the nine sub-sectors of the Swedish corporate sector I have found that the sub-sector garages use far more commercial bills than other Swedish corporations. Table 5.2 shows that 1980 commercial bills were only one percent of current assets in the aggregated corporate sector, while garages had twelve percent commercial bills. Garages have a higher debt equity ratio and they have a very low liquidity, measured as the ratio of cash and bank deposits to current assets and liabilities. Hence, in terms of these ratios, garages belong to a high risk group compared to the aggregated corporate sector. (There are naturally differences within the sector. A more interesting classification is to look at data based on different balance sheet characteristics, but such data are not available without reclassification of the information supplied by the firms in the corporate sector.) Now, as shown above, a commercial bank prefers a commercial bill to an unsecured loan to a high risk customer. This is possibly one explanation of why garages have a larger proportion of commercial bills than other sectors. Further, looking at the customer side, it is probably difficult for the average car owner to get a bank loan to have his car fixed. Then, a commercial bill is one way to supply credit at the same time as the risk is reduced, since the probability of default falls compared to an unsecured credit either to the buyer of garage services or to the garage itself.

5.3.2 *Documentary credit or sale of trade credit bills, and credit risk insurance*

In the preceding section the use of a commercial bill did not affect the credit risk taken by the seller. Here, I discuss cases when the seller grants credit at the same time as a third party, a risk taker, is included in the transaction to eliminate the credit risk taken by the seller. The easiest way to think of these cases is to see the elimination of credit risk as a two step procedure. First, the seller calculates the optimal length of the credit period, and then he compares the result with the no risk alternatives. In Figure 5.1, U_o represents the utility which can be reached with credit risk. Risk reducing payment alternatives must bring the seller to a higher indifference curve than U_o. Note that with risk aversion the seller is willing to accept a lower profit than the simple trade credit expected profit, if the credit risk is eliminated. Hence, there is a trade-off between the value of a reduction of uncertainty, here represented in terms of the variance of profit, and a reduction of the level of profit below the optimal trade credit expected profit. It is this margin that gives rise to trade in trade credit contracts. In this section I show under what conditions there will exist markets for risk reducing alternatives such that the seller can move from U_o to U_1. The alternatives I comment on here are a sale of trade credit bills to a factoring company, or the commercial bank substitute, the use of documentary credit, and insurance against credit risk. A factoring company that buys trade credit bills cannot take action of recourse against the seller, if the buyer defaults. Thus, the credit risk is moved from seller to the finance company. A difference

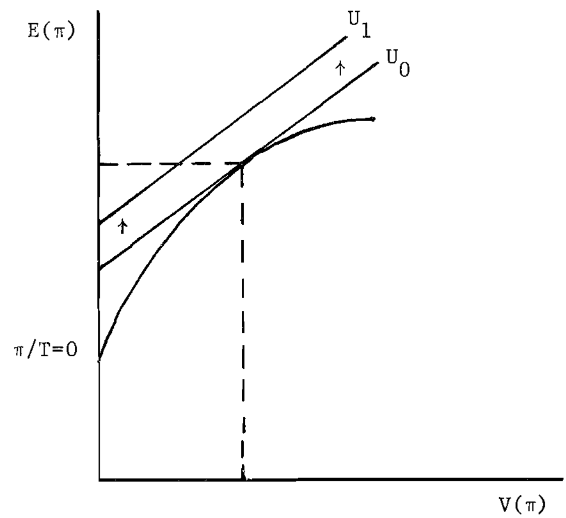

Figure 5.1 Trade credit and a sale of trade credit bills, or credit risk insurance.

between documentary credit and a a purchase of trade credit bills is that documentary credit usually is arranged by the buyer, who informs the seller to which bank he can send the documents of a purchase agreement to get paid, while a sale of trade credit bills is arranged by the seller.[6] Here, I do not go into detail about the two alternatives. The important point is that they eliminate the credit risk taken by the seller. Documentary credit is the traditional way of using commercial banks as risk takers and information specialists in international trade. It is one way to eliminate the double uncertainty I discussed in the preceding chapter, because such a payment arrangement eliminates both credit and exchange rate risk.

Assume that the seller is better off with than without trade credit, and next he is considering a sale of his trade credit bills to a factoring company, assume further that T = 1. The seller is better off when a finance company buys the trade credit bills if

$$(5.24) \qquad e^{r_b - r_s - d'} R(Q) - CQ < e^{r_b - r_f} R(Q) - CQ \ ,$$

where d' is the familiar risk adjustment factor and r_f is the discount rate used by the finance company. The finance company is better off if

$$(5.25) \qquad e^{r_b - r - \delta_b'} R(Q) > e^{r_b - r_f} R(Q) \ ,$$

where δ_b' represents the beliefs about the probability of payment by the buyer and the risk aversion of the finance company, and r is the finance company's sort of capital. The inequalities in (5.24) and (5.25) give the trading condition

$$(5.26) \qquad r_s + d' > r_f > r + \delta_b' \ .$$

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[6] For a detailed discussion about the juridical aspects of documentary credit, see Hellner (1974), Section 7.7.

Both parties are better off if (5.26) holds. If neither finance company nor seller have risk aversion, the market condition can also be written

$$(5.27) \qquad e^{r-r_s} < \frac{P_f(Bp)}{P_s(Bp)} ,$$

where $P_f(Bp)$ and $P_s(Bp)$ represent the probability of payment by the buyer perceived by the factoring company and seller respectively. Note that it is not necessary that either one of the two probabilities in some sense is a true probability. A transaction at a given moment in time is determined by what the seller and the finance company believe, regardless of whether they have correct or false information. Repeated trading generates new information, which affects the calculation of the probabilites in the future. From (5.27) it is possible to draw the following two conclusions: *First*, if seller and finance company have the same cost of capital and the same information about the probability of payment, there will be no trading. *Second*, trading is possible with $r > r_s$ if $P_f(Bp) > P_s(Bp)$ and there is a large enough difference between the two probabilities, and with $r < r_s$ as long as $P_f(Bp)$ is not too small. Thus, the ability to process and use available information is an important driving force in this market. Now, looking at (5.26) it is clear that risk aversion will also affect trading possibilities. A seller with a high risk aversion will have a large d', and this widens the trading opportunities. *Finally*, if the finance company has greater opportunities to diversification and risk pooling than the seller this will also widen the "trading interval" in (5.26). *In summary*, the market for documentary credit and sale of trade credit bills is driven by interest rate differentials, different information about credit risk, different attitudes towards risk, and different opportunities to diversify. All four factors, taken one at a time or simultaneously, determine the trading opportunities in the market.

According to the Finance Company Committee (1977) sale of trade credit bills to finance companies is unusual in domestic trade. Several of the larger Swedish finance companies have commercial banks as parent company and they have excellent access to and information about the Swedish credit market. Therefore it is reasonable to assume that in most cases $r \leq r_s$. Then no sale of trade credit bills, interpreted in terms of (5.27), means that $P_s(Bp) > P_f(Bp)$. The finance companies have no information advantage in domestic trade, and then it does not pay for the sellers to eliminate risk by selling trade credit bills to finance companies. This method of eliminating risk is much more common in international trade. Then, finance companies and commercial banks can take advantage of their information channels to gather reliable information about markets abroad and they can offer small and mid-size firms competitive risk eliminating discounts. A domestic market where these conclusions about information and risk elimination do not hold is the market for consumer durables. When I discussed cases when the seller is better off with direct credit to consumers instead of trade credit, I derived a market condition equivalent to (5.26), see (5.7). Hence, in this domestic market finance companies seem to be able to use their information processing skills better than in intra firm trade. (This is of course not the only explanation of the rapid spread of credit card credit. Credit rationing is equally important.)

So far, I have discussed the use of credit card credit and the use of documentary credit or sale of credit bills as three alternatives to eliminate seller uncertainty about payment at the end of the credit period. A fourth alternative is to buy credit risk insurance. Assume that a seller who grants trade credit can buy credit risk insurance which covers losses to one hundred percent, and there is no excess. Then, with the same zero-one payment alternative as before, the seller is better off with insurance if

$$(5.28) \qquad e^{r_b - r_s} R(Q)(1-e^{-d'}) > Pre.$$

The disutility adjusted expected loss must exceed the insurance premium, Pre. An insurance company is willing to sell an insurance policy if

$$(5.29) \qquad Pre > e^{r_b - r_i} R(Q)(1 - e^{-\delta_{bi}}),$$

where r_i is the cost of capital to the insurance company and $e^{-\delta_{bi}}$ is the by the insurer perceived buyer probability of payment. The premium must be higher than the expected default payment. With zero profit from insurance as a lower bound for the supply of insurance policies (5.28) and (5.29) give an insurance market condition which can be written

$$(5.30) \qquad e^{r_i - r_s} > \frac{1-e^{-\delta_{bi}}}{1-e^{-d'}} = \frac{P_i(Bnp)}{P_s(Bnp)},$$

where $P_i(Bnp)$ is the probability of default perceived by the insurer, and $P_s(Bnp)$ is the corresponding utility, or risk aversion, adjusted probability of the seller. This expression is the insurance market counterpart to (5.27), which showed the trading condition for a sale of trade credit bills. The conclusions following from (5.30) are equivalent to those following from (5.27). This is so since, if for example, $r_s \geq r_i$, (5.30) gives,

$$(5.31) \qquad P_i(Bnp) < P_s(Bnp) \Rightarrow P_i(Bp) > P_s(Bp),$$

which is the same conclusion that can be drawn from (5.27). Thus, the market for credit risk insurance is driven by the same factors that drive the market for documentary credit or sale of trade credit bills. From the seller's point of view both fill the same risk reducing function and can, in this respect, be seen as perfect substitutes. Then, the conclusion that firms engaged in domestic interfirm trade do not have an information disadvantage compared to finance companies implies that the credit risk premiums ought to be low. This is so since there is little reason to believe that insurance companies are better at gathering and processing information than finance companies. In 1980, the total sum of credit risk premia was 1,0336 million SEK, some of which probably did not concern trade credit transactions.[7] The aggregated accounts receivable in the corporate sector was at the end of 1980 101,217.3 million SEK. This means that, when there is roughly a 30 days average credit period, the proportion of insurance premia to trade credit assets was (1,0336/12)/101,217.3 = 0.008.

5.4 SOME BUYER ASPECTS OF DIFFERENT CREDIT ALTERNATIVES

5.4.1 *Trade or credit card credit*

So far, the credit arrangements I have commented on have been seen from the creditors'or sellers' point of view. Next, I look at the same problem from the buyers' side and I discuss some features of the credit market that I think have contributed to the rapid expansion of credit from finance companies as an alternative to trade credit or bank borrowing in the household sector. I still concentrate on financial and time preference aspects, and I do not take into account that different payment/credit alternatives also differ from a transaction cost point of view.

I start with a discussion about a consumer's choice between trade or credit card credit, but the approach is also applicable to many small firms which finance their purchases by using trade credit or some close substitute. Think of a consumer facing the in Sweden not uncommon situation that he because of a tight monetary policy is rationed in the commercial loan market. If he wants to redistribute consumption from the future to today he can either buy goods with trade or credit card credit. If he uses trade credit he is, according to Swedish law, required to make a minimum down payment of twenty percent of the value of the credit purchase.[8] However, if he uses credit card credit there is no

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[7] Source: The National Bureau of Statistics, K:1981, 3.3.

[8] A 20 percent down payment requirement was not unusual before the law was passed. There are examples of many markets where the down payment requirement is considerably higher than 20 percent. See The Installment Credit Committee (1975).

down payment. This is another explanation than the risk/information aspects in the preceding section to the rapid expansion of credit card credit. To show the difference between the two types of credit I study a two period utility maximization problem. In reality most credit agreements are more complicated and stretch over more than two periods, but the simple cases studied here are sufficient to bring out the major points of interest. To analyze the choice between the two borrowing alternatives I need the following assumptions:

- An individual receives an income $I_t = I$ in periods 0 and 1.
- In his effort to redistribute consumption in favor of consumption in period 0 he is offered to buy goods, L, in addition to his income I_0.
- The goods can be paid at t = 1 either with a credit card credit, with interest rate r_{cc}, or with a trade credit, the interest rate of which is r_{tc} and which includes a down payment cL.
- He has a utility function which is linear in consumption and which has a constant rate of time preference $(1+r_b)$.
- $r_b > r_{tc}, r_{cc}$.

$$(5.32) \qquad U_{cc} = I + L + \frac{1}{1+r_b}(I - (1+r_{cc})L)$$

$$(5.33) \qquad U_{tc} = I + L - cL + \frac{1}{1+r_b}(I - (1-c)(1+r_{tc})L).$$

A utility function which is linear in consumption and which has a constant rate of time preference gives corner solutions with $r_b \neq r_{tc}, r_{cc}$, if the consumer is free to choose the size of L. (The consumer either borrows or lends as much as possible.)[9] Despite this I have chosen such a utility function because it simplifies the algebraic exposition, and corner solutions is not a problem since L is given by the seller. Finally, $r_b > r_{tc}, r_{cc}$

9 For a thorough discussion about utility functions with a constant rate of time preference, see Deaton and Muellbauer (1980) Chapter 12.

in order to exclude no credit alternatives. Then the credit card alternative is preferred if

(5.34) $U_{cc} - U_{tc} > 0$,

which implies,

(5.35) $cr_b + (1-c)r_{tc} > r_{cc}$.

The credit card rate of interest must not exceed the weighted average of the consumer and the trade credit rate of interest, when the down payment and end of period proportions have been used as weights. Looking at this inequality it is clear that the credit card alternative is preferred if $r_{cc} < r_{tc}$. Another, and more interesting observation, is that if $r_{cc} > r_{tc}$ there are still situations when the inequality holds. Hence, the credit card alternative is competitive also in cases when the credit card rate of interest is higher than the trade credit rate of interest. This can be one contributing explanation of the rapid expansion of credit card borrowing. From (5.35) it is also clear that the size of the tolerable interest rate margin depends on the size of the down payment requirement. The higher c, the higher is the accepted interest rate margin $r_{cc} - r_{tc}$. A numerical example with $r_b = 0.3$, $c = 0.2$ (the minimum down payment ratio) and $r_{tc} = 0.25$ gives that (5.35) holds if $r_{cc} < 0.26$ which seems reasonable because the interest rate on credit card debts ranges, in most cases, between twenty to thirty percent and trade credit interest rates lie in the same interval or higher.[10]

The conclusions drawn from the algebraic example also have a graphical interpretation, and then there is no need to use a utility function with linear indifference curves (see Figure 5.2) With incomes I_0 and I_1 the highest utility without borrowing is U_0. With borrowing the consumer moves down along the bold line if he uses trade credit. The slopes of these lines indicate

[10] See Finance Companies (1977) Chapter 7.

the rates of interest. The steeper the slope, the higher the rate of interest. Here, I have set $r_{tc} = 0$ and $r_{cc} > 0$. When the consumer is offered to buy L with credit card credit he reaches U_{cc}, and when he is offered trade credit with down payment cL he only reaches U_{tc}, which in this case is lower than U_{cc}, despite the fact that r_{cc} is higher than r_{tc}. It is also evident that if c falls there must be some point when the individual is indifferent and then better off with trade credit than with credit card credit as long as $r_{tc} < r_{cc}$.

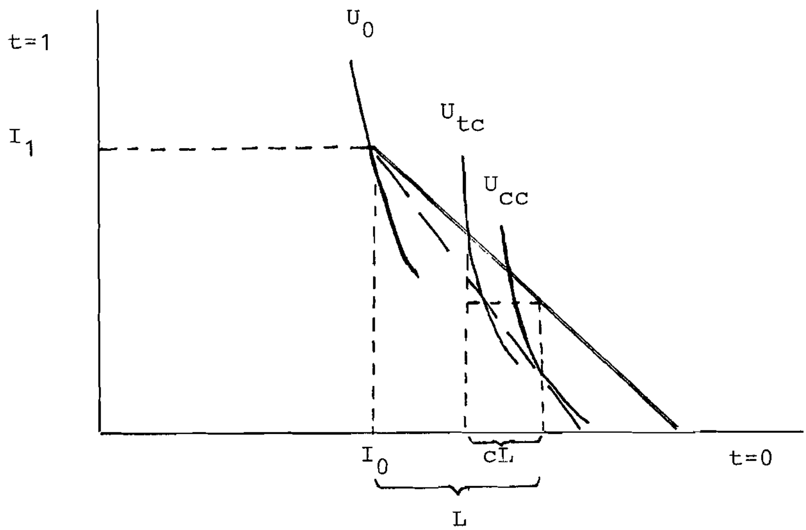

Figure 5.2 A comparison of trade credit with down payment and credit card credit, $r_{cc} > r_{tc} = 0$.

If the interest rates are equal, the interest rate lines coincide, and the trade credit alternative will never be preferred if $c > 0$. *In summary*, the upshot of the analysis is that down payments work heavily in favor of credit card instead of trade credit.

5.4.2 *Compulsory saving schemes*

An alternative to trade or credit card credit is an ordinary loan from a commercial or a savings bank. An important difference between trade and credit card credit and bank loans is that the former are granted immediately when the buyer applies for a loan, while it is unusual that an average household is granted a bank loan immediately. The ordinary procedure is that a bank customer first in some way has to signal his ability to pay, for example by saving X SEK during a given period of time before he is granted a loan. A common bank loan agreement is to save X SEK during a minimum period of nine months in order to be allowed to borrow 2X SEK at the end of the saving period. This type of "save first, borrow later" arrangement has always been exempt from regulation and it is often the only type of bank loan available to the average consumer. Another important difference between this type of bank loan and trade or credit card credit is the rate of interest.

Credit card and trade credit interest rates are generally roughly ten percent higher than a corresponding bank credit interest rate. Since the three types of credit exist side by side there must be some explanation to this large interest rate differential. One possible explanation is that, because of more limited signaling, trade and credit card credit is more risky. Finance companies and firms supplying trade credit cannot use deposits as risk reducing measures while commercial banks are allowed to do so. In this case the interest rate margin is a form of risk compensation. Another explanation, similar to the one in the comparison between trade and credit card credit, is that the implicit interest rate of the only type of bank loan available to otherwise rationed consumers is higher than the nominal borrowing rate because first the borrower has to go through a period of "forced saving", which means that consumption possibilities must be deferred into the future. To show the effect of the "save first, borrow later" phenomenon I use the same model approach as in the preceding section. Here it is a three period model which is simple enough to give some insights by using algebraic manipulation, and I limit myself to a comparison of credit card and bank credit. A similar analysis is of course also possible between trade and bank credit. Then the down payment ratio must be taken into account too. The model assumptions are:

- There is an individual with the same utility function as in the preceding model, and he receives the income $I = I_t$ in three consecutive periods.
- The first borrowing alternative is: borrow L with credit card credit in period 0, repay the loan fully in period 1 with interest rate r_{cc}.
- The second borrowing alternative is: save S during period 0 in a commercial bank, with interest rate r_{cb}, borrow $L - (1+r_{cb})S$ in period 1, and repay the loan, with interest rate r_{cc}, in period 2.
- $r_b > r_{cc}, r_{cb}$.

The size of the bank loan is here based on the assumption that the redistribution of consumption is the same in both alternatives, but it is moved one period in the bank borrowing case. An example, borrow and buy a TV set this year, or save first buy the TV next year, and borrow a part of what it costs and repay the loan the year after. With these assumptions the utility in the credit card and bank credit alternatives is, respectively

$$(5.36) \qquad U_{cc} = I + L + \frac{1}{1+r_b}(I - (1+r_{cc})L) + \frac{1}{(1+r_b)^2} I ,$$

$$(5.37) \qquad U_{cb} = I - S + \frac{1}{1+r_b}(I + (1+r_{cb})S + L - (1+r_{cb})S) + $$

$$+ \frac{1}{(1+r_b)^2}(I - (1+r_{cb})(L - (1+r_{cb})S)).$$

The credit card alternative is preferred if

$$(5.38) \qquad U_{cc} - U_{cb} = L\left[1 - \frac{1+r_{cc}}{1+r_b} - \frac{1}{1+r_b} + \frac{1+r_{cb}}{(1+r_b)^2}\right] + $$

$$+ S\left[1 - \frac{(1+r_{cb})^2}{(1+r_b)^2}\right] > 0 ,$$

which after some manipulation can be written[11]

$$(5.39) \qquad r_{cb} + \left[r_b(r_b-r_{cc}) + \frac{S}{L}((1+r_b)^2 - (1+r_{cb})^2)\right] > r_{cc}.$$

Now since $r_b > r_{cc}$, r_{cb} the bracketed expression is positive, and it is possible to draw the following conclusions. *Firstly*, in the trivial case when $r_{cc} < r_{cb}$ a credit card credit is always preferred to a bank loan. *Secondly*, there must also exist interest rates $r_{cc} > r_{cb}$ such that the inequality in (5.39) holds. This is consequently one explanation of why there is a spread between the bank loan and credit card rates of interest. *Thirdly*, when there is no saving requirement (S/L=0), but some other time consuming signaling process, there still exist $r_{cc} > r_{cb}$ such that credit card credit is preferred, but in this case the maximum interest rate margin before the individual is indifferent between the two alternatives is smaller than in the case with saving. A numerical example with $r_b = 0.3$, $r_{cb} = 0.143$, the average saving loan bank rate during the first half of 1980,[12] and S/L=1/3, which is a standard S/L ratio, gives the result that (5.39) holds if $r_{cc} < 0.278$. Since the margin between credit card and bank loan interest rates is roughly ten percent this numerical example fits the facts remarkably well. If there is no saving requirement the interest rate margin is

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[11] To derive (5.39) multiply through by $(1+r_b)^2$ in (5.38), divide by L and simplify.

[12] Source: Bank of Sweden, Yearbook (1980).

considerably reduced. In this case, (5.39) holds if $r_{cc} < 0.179$. There are of course also other factors that affect the choice between trade, credit card and bank credit, but I think that what I have shown here is a plausible explanation of the rapid expansion of the use of credit card credit and of observed interest rate differentials. Consumers with a high rate of time preference are willing to pay a high rate of interest, and still be better off, in order to avoid down payment requirements and compulsory saving schemes.

5.5 TRADE CREDIT AND EXPORT SUBSIDIES

Finally, before I in the two chapters that follow move focus from financial to transactions aspects of trade credit, I briefly discuss the use of trade credit as an export subsidy. I comment on interest and quantity effects. However, to deal with the interesting issue whether this type of export subsidy is good or bad for a nation as a whole is beyond the scope of this chapter.

Just as in the case with documentary credit, there are more opportunities to take advantage of favorable insurance terms among firms engaged in international trade. One company which supplies such insurance is the government-owned Swedish Export Guarantee Board (EGB). Roughly five percent of the value of completed export agreements include EGB guarantees.[13] One reason that EGB insurance is more common than domestic insurance is because the premiums are lower than the zero profit premium given by (5.29). It is in many cases a more or less well hidden export subsidy. The premiums have an upper limit of roughly five percent of the value of an export contract.[14] With $Pre = 0.05e^{r_b}R(Q)$ in (5.28) this means that a risk neutral seller with a one year interest rate of twenty percent is better off with insurance if the probability of payment is less than ninety four percent. At the same time EGB accepts insurance contracts where the probability of payment is considerably lower. Thus, the premium policy is incompatible with the goal that EGB is

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[13] See Bohm (1979), page 4.

[14] See Bohm (1979), page 38.

to be run on a commercial basis, without subsidies.[15]

During the latter part of the 70's most OECD countries have had trouble with unemployment and the external balance. A common goal has been to increase exports. One way to prop up export sales, without getting too much attention by competing exporting countries, has been to supply subsidized export credit risk insurance of the type discussed above. The idea behind this policy is that a reduction of uncertainty increases sales, and this in turn increases employment. This conclusion is also supported by the models I have set forth. In the preceding chapter I showed that a reduction of uncertainty increases output. Another in a trade credit context important export promoting measure is interest rate subsidies given to exporters who also supply trade credit. In many cases the credit terms have become an important part of international competition. This is particularly true with respect to exports to many developing countries with more or less serious solvency problems. In Sweden the two alternatives to subsidize exports are closely connected since a prerequisite to receive an interest rate subsidy is that the export agreement is covered by credit risk insurance. Thus, a positive quantity effect is generated by a combination policy. Low cost insurance eliminates uncertainty and interest rate subsidies lower the cost of capital. Here I do not comment on the effect of an elimination of uncertainty, this I have done earlier, but I make a few comments about the quantity effect following a reduction of the cost of capital. In 1982 the interest rate subsidies were roughly one billion SEK. About eight percent of the total of Swedish exports and fifty percent of the export of capital goods included interest rate subsidies. The main receivers of these subsidies were shipyards and eight to ten large corporations which produce capital goods.[16] These firms are highly specialized and they produce goods such that, at least in the short run, they face downward sloping demand schedules from foreign and domestic buyers. Several of

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[15] There are actually a number of reasons why the EGB is making losses. This is, however, not the right place to go into detail.

[16] Source: Affärsvärlden (1982) No. 47.

these firms operate world wide on markets which can be characterized as oligopolistic.[17] With this interpretation of the market structure it is not surprising that these firms claim that the interest rate subsidies are important. They can be used as a way to grant hidden price concessions, which, as long as the other competitors do not follow, have a positive quantity effect.

The objective of a government, which supplies subsidized credit to an exporting firm, is to generate a positive output effect and thus an increase in employment. The size of this output effect depends on what list price policy the receiver of interest rate subsidies uses. With the same model structure as I have used so far the revenue function is

$$(5.40) \qquad e^{r_b - r_{ss}} P_o Q(e^{r_b - r_b} P_o,\ P_c),$$

with buyer price compensation and,

$$(5.41) \qquad e^{r_{ss} - r_{ss}} P_o Q(e^{r_{ss} - r_b} P_o,\ P_c)$$

with seller price compensation, when r_{ss} is the subsidized interest rate of the seller, P_o is the cash price set by the seller, P_c is the cash price set by the competitors, and $T = 1$. From (5.40) it is clear that if there is cash price rigidity and P_o is fixed, for example because there is a collusion price $P_o = P_c$, and the seller uses buyer price compensation there is no direct quantity effect. The only effect, following a reduction of r_{ss}, is an increase in the present value revenue of the seller. Consequently, if the seller uses some kind of collusion sales price an interest rate subsidy has no quantity effect. This also holds with respect to other subsidies. The only thing that happens is that the profit of the seller increases. If P_o can vary there will be an indirect quantity effect in the case with buyer price compensation. A falling r_{ss} increases the present value of revenue, it becomes profitable to increase sales and

[17] This is what McKinnon (1979) calls tradables I in his interesting description of trade credit in different international markets.

this happens when the list price is reduced. (This is the same result as I have derived in the chapter about trade credit when there was no uncertainty.) If, on the other hand, the seller uses seller price compensation[18] as in (5.41), a reduction of r_{ss} gives a direct demand effect because the present value price of the buyer falls. In Figure 5.3 I show the quantity effect of an interest rate subsidy when the seller uses seller price compensation in a market with oligopolistic competition. In the diagrammatic exposition I use the from early oligopoly theory wellknown kinked demand curve in a trade credit context. Assume that at (P_o, Q_o) there is interest rate equality between buyer and seller. There is no financial motive to supply trade credit. Assume also that P_o happens to be the collusion price in the market for Q and its closest substitutes. Now, the government wants the seller to increase output and to generate an output effect, the government supplies subsidized credit such that $r_{ss} < r_b$, if the seller uses seller price compensation. Assume further that in the short run the competitors do not change their prices as long as P_o is fixed. A list price $P_1 = e^{r_{ss}} P_o$ can thus be interpreted as a hidden price concession. These assumptions mean that there is a quantity effect as long as $r_{ss} < r_b$. As r_{ss} falls there is a movement down along the D_o demand curve in the figure. From the seller's point of view a fixed cash price combined with seller price compensation means that the present value revenue is constant (see also (5.41)). $P_o = AR = = MR$, the bold horizontal line in the figure. Now a profit maximizing producer increases output until MR = MC, which in this case means that the interest rate subsidy must be such that demand is Q_1. Then, there is clearly a positive demand effect. This is a situation which prevails if the competitors do not change their prices. However, it is a wellknown fact that interest rate subsidies are used by all OECD countries and by other countries too. This means that, in terms of Figure 5.3., the demand Q_1 is not likely to prevail with a given subsidy. As other countries follow and introduce interest rate subsidies, or other ex-

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[18] The important point is that the list price must fall when r_{ss} falls and this regardless of whether the seller uses seller price compensation or some other list price policy.

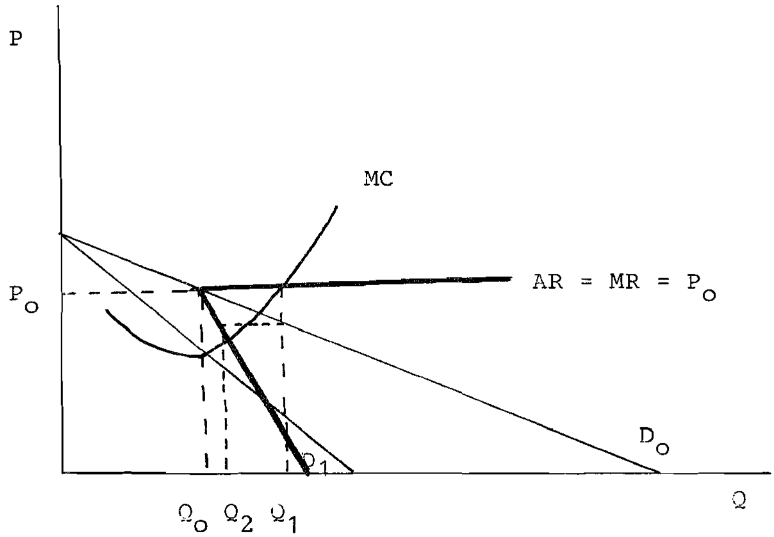

Figure 5.3 Subsidized seller interest rates on a market with oligopolistic competition.

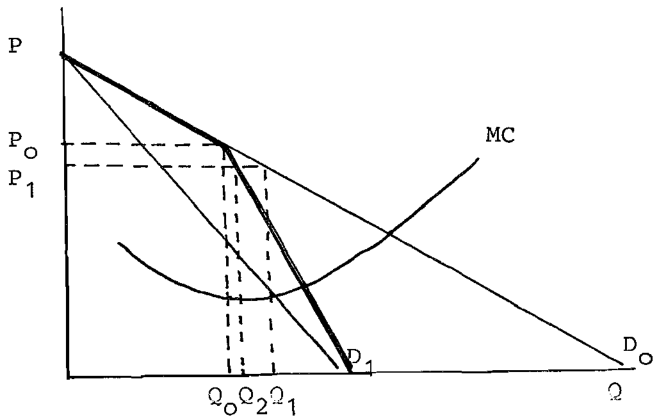

Figure 5.4 Trade credit as a hidden price concession on a market with oligopolistic competition.

port promoting measures, the long run demand curve will shift inwards to D_1. Then, with an unchanged subsidy, demand falls to Q_2. To maintain demand at Q_1 the subsidy must increase. In order to increase or to maintain output at a level higher than normal the government must participate in a game among nations, a game which generates increasing interest rate subsidies. The Swedish experience shows that to participate in such a game can be quite expensive. This is particularly true if the long run quantity effects are small. Consequently, this is probably one explanation of why governments within the OECD collude and stipulate a so-called consensus interest rate which is the lower bound, below which the subsidized seller interest rate is not allowed to fall. Non-OECD members are still free to choose whatever interest rate subsidies they want. This leads e.g. to a demand for increased government subsidies by Swedish shipyards.

Finally, trade credit can also be used to generate another type of hidden price concession. The discussion above rested on the assumption that it was the sellers' cost of capital that determined the size of the price concession. Another alternative is to combine a varying length of the credit period with a fixed list price. This type of price policy can also be used by a firm operating on an oligopolistic market within a country, and it has many similarities with the interest rate subsidy example.

Figure 5.4 shows the demand situtation facing a producer on an oligopolistic market.[19] Assume that (P_oQ_o) is the collusion price/output combination when no one supplies trade credit. Assume further that the seller supplies trade credit, with $P_o = P_1$, when initially none of the competitors follow with a similar trade credit offer. The D_o demand curve represents the present value price of the seller, and an increase in the length of the credit period represents a movement down along D_o, because with $P_o = P_1$ the present value price of the buyers falls.

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[19] This way of treating trade credit was first used by Schwartz and Whitcomb (1978).

If the seller can use the length of the credit period as a hidden price concession the length of the credit period is chosen in such a way that demand is equal to Q_1 and the seller is better off than at Q_o. Just as in the interest rate subsidy case the demand effect is reduced if the competitors answer with similar price policies. Then with a fixed credit period demand drops to Q_2. Here I have presented two ways to grant the customers a hidden price concession. A third alternative is, for example, to combine the fixed list price case with an interest rate subsidy.

5.6 SUMMARY

In this chapter I have, in a rather informal way, discussed the use of trade credit and some close substitutes in a financial market context. With the help of the utility function approach set forth in Chapter 4 I have argued that the risk adjusted interest rate is a central variable which determines the choice of different lending or borrowing alternatives. Another important factor is various institutional rules and regulations, which give rise to profitable interest arbitrage opportunities. In the section about some buyer aspects of different credit alternatives I showed how trade credit with minimum down payment requirements, and bank loans with compulsory savings schemes, could not compete with credit card loan offers in cases when the credit card rate of interest exceeded the trade credit or bank loan cost of borrowing. Thus, this was one way of explaining the rapid spread of credit cards during the seventies. Another conclusion based on the institutional set up was that interest and loan ceiling turn commercial banks into risk minimizers, which, of course, is detrimental to their competitiveness. Besides my comments about institutional factors I have argued that information and risks are important. When I discussed commercial bills emphasis was put on the probability of payment, I showed that a commercial bill is always less risky than an unsecured loan either to seller or buyer. This is of minor importance when the probability of payment is high, but in high risk cases a commercial bill can increase the probability of payment considerably. Seen in this context commercial bills become an instrument to supply high risk loans by commercial banks. This can be one reason why commercial bills play such a minor role on the aggregated balance sheet of the Swedish corporate sector. All firms are not high risk firms, and it is not always that both seller and buyer are high risk borrowers. In the section about sale of trade credit bills, documentary credit, and credit risk insurance I showed that the factors that are the driving forces of these markets, and that give rise to transactions where both parties gain, are interest rate differentials, different information about credit risks, different attitudes toward risk, and different opportunities to diversify. Take e.g. a large corporate finance company which buys trade credit bills from a small firm with a risk averse owner. In this case it is likely that the critical variables mentioned above are such that both buyer and seller of the bills will be better off. The seller eliminates the credit risk at the same time as the finance company can take advantage of its skill as a specialized risk taker and processor of information. The conclusions about the choice of various credit flows have been derived by the help of a very partial approach, which rests on the assumptions that market participants have different values and information, and that it is costly to gather, process, and use information. In a less partial, more general equilibrium oriented, approach the issue is how markets behave when there is uncertainty and information costs. This is a rich subject in itself, and it lies beyond the scope of this chapter. Finally, I showed how trade credit can be used as a hidden price concession in order to increase exports. The conclusion was that quantity effects are likely to be small in the long run. Here I have only commented on to what extent interest rate subsidies give rise to positive quantity effects on exports. Another interesting issue, which lies besides the central theme in this study, is whether a country should use this type of subsidy or not. Will interest rate subsidies with resulting generous trade credit offers make a country better off or not?

6. Post Payment, Pre Payment and Reduction of Buyer or Seller Uncertainty

6.1 INTRODUCTION

In the preceding chapter I have discussed various aspects of trade credit and uncertainty when the central motive to use trade credit has been risk compensated interest arbitrage. In addition to payment uncertainty I now also introduce uncertainty about the quality of the product or service that is traded.[1] I show under what conditions the use of trade credit makes both buyer and seller better off compared to cash payment, and I show how the distribution of uncertainty and risk aversion between seller and buyer generates either pre or post payment. In order to isolate the effects of product and payment uncertainty I mostly assume that the interest rates of buyer and seller are equal. The use of trade credit when interest arbitrage or financial trade credit motives have been eliminated stems from the fact that in a world with uncertainty and an uneven distribution of information it is costly to negotiate and enforce contracts. The choice of the payment date in relation to the date when a good or service has

1 There is a large literature about both price and quality uncertainty (see for example the reference list in Soeria-Atmadja (1983) but as far as I know it is only Faith and Tollison (1981) that discuss separation of the payment date from the date when a good or service is transferred from seller to buyer as one way to reduce uncertainty. However, Faith and Tollison do not use the term trade credit and their article has no connection with the rest of the trade credit literature.

been delivered can then be one simple informal agreement with a contract or transaction cost low enough to facilitate trade compared to cash payment. Thus, under some circumstances trade credit can be one way to avoid market failure resulting in no trade at all or in intra instead of inter firm trade. In this context other ordinary financial arrangements cannot fill the same function as a trade credit. A loan from a third party does not eliminate product uncertainty, while trade credit with post payment can do so. This does not mean, however, that there are no substitutes to trade credit, but one has to look outside the financial market to find them. Other risk reducing measures are e.g. to rent instead of to buy, various guarantees, truthful advertising, free samples, licensing, and government quality controls.

In Section 6.2 I discuss under what conditions both seller and buyer are better off with post compared to cash payment. I discuss contracts with partial cash payment and finally I comment on how trade credit can generate price differentials between high and low quality goods. In Section 6.3 I show under what conditions pre payment will be likely and with the simple model structure as a background I try to explain my stylized fact observation that pre payment is very unusual in all subsectors of the Swedish corporate sector, except among firms supplying various types of business services. One of the limitations of my analysis is that I only study under what conditions there are Pareto sanctioned trade credit agreements compared to cash payment. The next step is to study the existence or non-existence of market equilibria when there is both product and payment uncertainty, but such an extension lies beyond the scope of this chapter.

6.2 PRODUCT UNCERTAINTY AND POST PAYMENT

6.2.1 *Post payment*

Throughout the preceding chapters I have assumed that there is no uncertainty with respect to quality or other product characteristics. This is a situation which pertains when highly standardized products are traded (minerals, crude oil, wheat, paper products etc.) or when repeat purchases from a limited number of sellers have reduced uncertainty to a minimum. However, there are also a host of situations when the buyer does not have full information about what he is buying at the moment when the good or service is transferred from seller to buyer. In many cases it takes both time and trouble to decide whether a product has the desired technical characteristics and if the buyer has enough information prior to a purchase there is still uncertainty because the seller might not deliver what he has promised. For example, it is difficult to know the quality of a used car or the quality of the service that a lawyer, a doctor, or a travel agency will provide. In such cases the buyer has, of course, a much stronger position if he uses post payment instead of cash or pre payment. Below I study a highly simplified case when trade credit can be used to reduce uncertainty. This does not mean that there are no other alternatives available. An interesting issue is to study why different uncertainty reducing measures are used in different markets. What distinguishes markets with guarantees from markets with free samples or post payment? Although an interesting subject this is not the right place to go into detail. Here I limit myself to discuss under what conditions post payment is preferred to cash payment.

Assume, as in the two preceding chapters, that there is payment uncertainty, the cash price is given, there are no quantity effects and the length of the credit period is equal to one. Assume further that in those cases when a discussion about risk aversion is relevant both buyer and seller have mean-variance utility functions of the same type as the seller utility function in the preceding chapters. Now imagine a world where the good that is being traded has two qualities, high or low. The buyer uses the good as an input in his production. When the quality is high it can immediately be used in the production process, while if the quality is low it can be used first after some adjustments, which give rise to an extra cost of ε per unit of input. There are many sellers which supply either high or low quality goods. When a batch of goods is transferred from seller to buyer the buyer cannot distinguish a batch of high quality from low quality goods. The prevailing cash payment liability rule is complete buyer liability (caveat emptor). Thus, cash payment represents a situation with buyer uncertainty because the final production cost is not known. Assume further that if the buyer uses trade credit he can determine the quality of the good before he pays. The buyer is honest so that only if the quality happens to be bad the extra adjustment cost is deducted from the payment, a measure which is accepted by the seller. Hence, in this world trade credit is one way to eliminate product uncertainty compared to cash payment. Caveat emptor is transformed to caveat venditor.

With these assumptions the cash payment utility of the buyer is[2]

(6.1) $$U(\pi_b) = R'(X) - PQ - \bar{\varepsilon}Q + k_b Q^2 V(\varepsilon) = R(X) - e^{\beta} PQ,$$

(6.2) $$\beta = \ln\left(1 + \frac{\bar{\varepsilon} - k_b Q V(\varepsilon)}{P}\right),$$

where R(X) is the revenue of the buyer, PQ is the known cost of the input Q, $\bar{\varepsilon}Q$ is the expected additional bad quality cost, and $k_b Q^2 V(\varepsilon)$ represents the disutility of bad quality. The utility function has been rewritten in terms of a risk/risk aversion adjustment factor, β, which increases with the size of the expected adjustment cost and the size of the risk aversion coefficient k_b. Now, cet.par., the buyer is better off with trade credit if the cost of input is lower with trade credit than the risk adjusted cost of input with cash payment. Thus, the buyer prefers trade credit to cash payment if the inequalities in (6.3) hold, where r_t and r_b represents the trade credit and the buyer rate of interest respectively.

(6.3) $$e^{r_t - r_b} PQ < e^{\beta} PQ \Rightarrow r_b + \beta > r_t ,$$

when I assume that the adjustment cost does not exceed the cost of a trade credit purchase.

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2 The utility function above is a special case of a more general model used by Blair (1974) to study quantity effects with random input prices.

Trade credit is used if the risk adjusted interest rate of the buyer is higher than the cost of trade credit, or alternatively, if r_b is moved to the rhs of the inequality sign, the willingness to eliminate risk, expressed in interest rate terms, must exceed the net cost of credit. Next let a high quality producer represent the seller side of the market. Earlier I have shown that the seller is better off with than without trade credit if

(6.4) $r_t > r_s + d'$.

The risk adjusted interest rate of the seller must be lower than the trade credit rate of interest. The inequalities in (6.3) and (6.4) taken together then show that both seller and buyer are better off with trade credit if the buyer's risk adjusted interest rate is higher than the seller's,

(6.5) $r_b + \beta > r_s + d'$.

When the inequality in (6.5) holds the buyer is willing to pay the seller an insurance premium which is high enough to compensate the payment uncertainty borne by the seller. The seller is the insurer and the buyer the insured. Note that an insurance premium based on (6.5) differs from ordinary insurance premia in one important respect. The size of an ordinary insurance premium is based on, among other things, opinions about the occurence of some given event. However, the inequality in (6.5) is based on opinions about the occurence of two different events. The buyer is interested in the additional cost of buying low quality goods and the decision of the seller is based on the perceived probability of payment by the buyer. There are also other interesting comments that can be made about the inequality above. So far a prerequisite for trade credit with post payment has been that the interest rate of the buyer is higher than the interest rate of the seller. In (6.5) this need not necessarily hold. If beta is large enough r_b can be lower than r_s and both buyer and seller can still be better off with than without trade credit. When $r_b = r_s$ there is no financial motive for trade credit, but the risk reduction motive can still lead to post payment with trade credit interest rates higher than r_b. This is consequently one alternative way of explaining the fact that trade credit often costs more than for example bank borrowing. The size of β relative to d' is determined by several factors. The expected adjustment cost and the probability of payment affect β and d' regardless of whether the parties are risk averse or not. In addition to this the attitudes toward risk are important. Thus, when the financial motive is eliminated, post payment can be expected to be used in markets where buyers have high risk aversion and high expected bad quality adjustment costs, and sellers have low risk aversion and believe there is little payment uncertainty.

To give an idea of when such situations occur I give some examples. This approach is one way to explain why post payment is prevalent in the labor market. In most cases the individual who sells his labor services is likely to have a very high payment probability, b' is small,[3] and when this is combined with a high expected cost of low performance or shirking post payment results. It is well known that trade credit interest rates are very high in the market for used cars (sometimes thirty to fifty percent). In the preceding chapter my interpretation of this fact was that the customers are rationed in the bank loan market. In light of the discussion above an alternative explanation is high risk aversion and/or a high expected bad quality cost possibly combined with low payment probabilities perceived by the sellers. This combination also gives rise to a high trade credit rate of interest compared to bank borrowing. The buyers are willing to pay a high insurance premium in order to avoid the extra cost of buying a lemon. A third example of markets where this type of credit offer is used is the sale of various consumer products by mail. In this case a sale offer can be "pay within ten days or return to the seller". Such an offer eliminates buyer uncertainty, which can be very important particularly if the buyer has ordered some "no name

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3 This is especially true if wage payments are insured against default.

brand".[4] Finally, I want to stress the fact that when trade credit is treated in a reduction of risk context there are also close substitutes. Thus, the inequality in (6.5) shows only under what conditions this type of trade credit can exist, and this does not necessarily mean that it also always will exist. Some substitute, for example various guarantees, might be a more efficient way of redistributing uncertainty compared to caveat emptor cash payment.[5]

6.2.2 *Partial trade credit revisited*

In the preceding section both buyer and seller could be better off with full post payment compared to cash payment. In this section I show under what conditions Pareto sanctioned solutions including partial trade credit are possible. In Chapter 4 I discussed such solutions when only the seller side of the market was taken into account. (The buyer was indifferent between partial trade credit or cash payment.) Here I also include the buyer side of the market and I show that both buyer and seller can prefer partial credit to cash payment. I also show that sometimes partial credit is the only possible solution. In order to concentrate on the insurance premium product uncertainty aspects of the use of trade credit I assume that the interest rates of buyer and seller are equal. The financial trade credit motive is eliminated.

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4 My example belongs to the most consumer oriented. Another alternative is to combine cash payment with a "if not satisfied money back" guarantee. In this case the seller is compensated for the risk reduction by a zero interest on his short term loans from dissatisfied customers, or by positive quantity effects compared to a caveat emptor cash payment offer.

5 When discussing product guarantees Soeria-Atmadja (1983), page 232, notes that in most cases guarantees are included regardless of whether an individual consumer is willing to pay for the guarantee or not. Consequently it is difficult to estimate the price of guarantees by consumers. In this context the trade credit rate of interest can perhaps be used as a proxy. It is a direct measure of the cost to reduce uncertainty and it fills roughly the same function as a guarantee.

Assume that Q units are traded, (1-c)Q units are paid in cash, cQ after one period ($0 \leq c \leq 1$), and cash payment is still a caveat emptor purchase. Thus, if c is less than one the buyer still faces the risk of having to pay $(1-c)\varepsilon Q$ in quality adjustment expenses.[6] With these assumptions the buyer is better off with at least partial credit if the inequality in (6.6) holds.

$$(6.6) \qquad R(X) - (1-c)(P+\bar{\varepsilon})Q - e^{r_t - r_b} cPQ + k_b(1-c)^2 Q^2 V(\varepsilon)$$
$$> R(X) - (P+\varepsilon)Q + k_b Q^2 V(\varepsilon).$$

When common terms have been cancelled this inequality can after some reshuffling of terms be written

$$(6.7) \qquad \left[\frac{\bar{\varepsilon} - P(e^{r_t - r_b} - 1)}{-k_b QV(\varepsilon)}\right] + 2 \geq c \quad .$$

In Figure 6.1, page 210 I show under what conditions this inequality holds. *First*, when $r_t \leq r_b$ the bracketed expression is always positive, given that the buyer has risk aversion. Thus, the buyer is better off as long as c is greater than zero. *Second*, $\bar{\varepsilon} = P(\exp(r_t - r_b) - 1)$ represents the "all or nothing" interest limit, when the buyer has no risk aversion. *Third*, when r_t rises further the bracketed expression sooner or later becomes negative, a point will be reached where there is a trade off between r_t and c. When r_t is high enough the interest cost outvies the value of reduced uncertainty and c must fall. Thus, for some large enough r_t, c will fall to zero. *Fourth*, regardless of the size of c the buyer will be better off if r_t falls, because the cost of reducing uncertainty is lowered. Consequently, the area below the buyer indifference line represents such combinations of c

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6 This assumption restricts the options of the buyer. Another alternative is to study the case when the whole adjustment cost can be deducted from the trade credit payment. An analysis of this case is more complicated than the analysis above, but in the end it also leads to a figure very similar to Figure 6.1.Consequently, added complexity adds very little extra insight compared to the model presented above.

and r_t where the buyer is better off with than without trade credit.[7] Next consider the decision problem facing a good quality seller. He prefers a contract with a post payment component if his utility is higher than with cash payment. This occurs when the inequality in (6.8) holds

$$(6.8) \quad PQ - C < (1-c)PQ + ce^{r_t - r_s - d}PQ + k_s c^2 (PQ)^2 e^{2(r_t - r_s) - d}(1-e^{-d}) - C,$$

where the expression following k_s is the same variance as in Chapter 4. After some reshuffling of terms this inequality can be reformulated in terms of an inequality similar to (6.7). Thus, trade credit is preferred if c is chosen in such a way that the inequality in (6.9) holds, given that c is equal to or less than one.

$$(6.9) \quad \frac{(e^{r_t - r_s - d} - 1)e^{-2r_t + 2r_s + d}}{-k_s PQ(1-e^{-d})} \geq c .$$

From (6.9) it follows that a buyer indifference function starts at c = 0, with $r_t = r_s + d$, and it is further easily shown that it is an increasing function of r_t with a maximum at $r_t = r_s + d + 0.69$.[8] All points above the seller indifference line in the figure represent combinations of c and r_t such that the seller is better off with than without trade credit. With $-k_s$ in the numerator of the lhs of (6.9) it is clear that risk aversion affects the shape of the seller indifference function. Thus, for high values of $-k_s$ it will reach a maximum before c equals one. A conclusion which is in accordance with my discussion about risk aversion and the size of c in Chapter 4. From (6.7), (6.9) and Figure 6.1. it then follows that in the special case when $r_b = r_s$ there exists various combinations of c and r_t such that both buyer and seller prefer a contract which includes some type of trade credit compared to cash payment. (There are of course also

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7 No product uncertainty is a special case, $\bar{\varepsilon} = V(\varepsilon) = 0$. Then the inequality reduces to $r_b \leq r_t$, where I assumed equality in the preceding chapter.

8 The equality represents a maximum because it can be shown that the second order condition holds as long as $r_t < r_s + d + 1{,}386$.

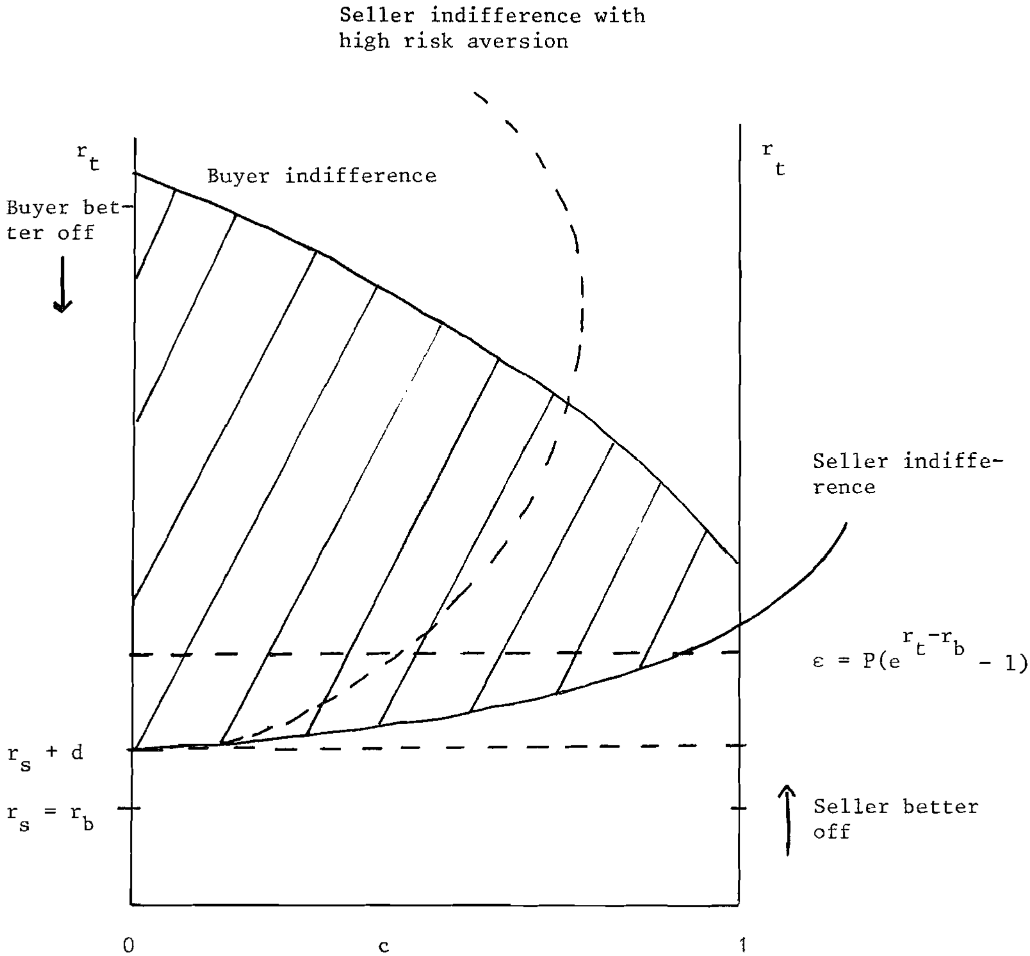

Figure 6.1 Various Pareto sanctioned combinations of cahs payment and trade credit interest rates when r_b *equals* r_s.

other combinations of r_b and r_s that can give rise to Pareto sanctioned contracts.) Figure 6.1 also shows that partial trade credit solutions can only occur when the risk aversion of buyer and seller are such that either the seller indifference line crosses the right r_t axis above the buyer indifference curve or it has a maximum before c equals one. A special case is when both k_b and k_s are equal to zero. Then the "bargaining area" lies between the two dotted lines in the figure.

Eventually the figure only shows possible Pareto optimal solutions without indicating which one will be chosen by the traders. To be able to say something about different solutions within the marked area it is either necessary to know something about the bargaining power of buyer and seller, or some social utility function has to be used to give social utility values to different combinations of r_t and c. As an example of the latter approach I here use a Benthamite social utility function and I discuss where in the figure buyer and seller will end up when the objective is to choose a Pareto optimal trade credit contract such that the combination of c and r_t is a point where the sum of the utility of buyer and seller is maximized.[9] Thus, the objective function is

$$(6.10)\quad \underset{r_t,\, c}{\text{Max}} \quad R(X) - (1-c)(P+\bar{\varepsilon})Q - e^{r_t - r_b} cPQ + k_b(1-c)^2Q^2V(\varepsilon)$$

$$+ (1-c)PQ + e^{r_t - r_s - d} cPQ + k_s c^2 (PQ)^2 e^{2(r_t - r_s)-d}(1-e^{-d}) - C,$$

subject to the inequalities in (6.6) and (6.8).

Differentiation of (6.10) with respect to r_t gives, when common terms have been cancelled, the marginal social utility condition

$$(6.11)\quad e^{-d} - 1 + 2k_s cPQe^{r_t - r_s - d}(1-e^{-d}) < 0,$$

which is always negative when the seller has risk aversion. Con-

9 For a discussion about properties of social utility functions see Atkinson and Stiglitz (1980).

sequently when r_t falls the cost reduction of the buyer is always greater than the loss of utility of the seller, which means that whatever the size of r_t it always pays to let r_t fall until it reaches the seller indifference line. Thus, in this special case (6.10) represents a very buyer oriented social welfare function.

In order to characterize one relatively simple solution along a seller indifference line assume that the seller maximizes expected profit, $k_s = 0$. A social optimum then lies somewhere along the broken line at $r_s + d$ in the figure. When $r_s + d = r_t$ is inserted in (6.10) and when (6.10) is differentiated with respect to c, it follows after some manipulation that the marginal social utility is positive or equal to zero as long as the inequality in (6.12) holds.

$$(6.12) \qquad \frac{\bar{\varepsilon} - P(e^{d}-1)}{-k_b QV(\varepsilon)} + 2 \geq 2c.$$

Given the assumptions about k_s and the interest rates the only difference between (6.12) and (6.7) is that in (6.12) c is multiplied by two. Then it follows that if the buyer is very risk averse the optimal solution equals or is close to one, and if the buyer indifference line cuts the corresponding line of the seller at some point $\hat{c} < 1$ the optimum is given by the interest rate/trade credit combination $(r_s + d, \hat{c}/2)$. The trade credit rate of interest equals the risk adjusted rate of interest of the buyer and c equals half of the largest c that is accepted by the buyer.

The upshot of this discussion about partial trade credit is that given my assumptions about utility functions, interest rates, and type of buyer and seller uncertainty it has been possible to show that there can exist a wide range of Pareto sanctioned r_t/c combinations when the pure interest arbitrage motive has been eliminated, and in some cases partial trade credit is the only possible solution. A future challenge is to study trade credit models which include alternative utility functions and which allow more complex contracts than the ones sketched here. Then it is perhaps possible to show, for example, how contracts which include both pre, cash, and post payment can emerge.

6.2.3 *Trade credit as a quality signal*

In his seminal article about markets with product uncertainty Akerlof (1970) argues that when the buyers cannot distinguish good quality from bad quality the market price will reflect the average quality of the good that is being traded. If then good quality also means higher production costs there is no incentive to produce high quality goods. Bad quality drives out good, a situation which ultimately can lead to complete market failure. Thus, if buyers cannot distinguish good from bad good quality producers have incentives to find various signaling or information methods in order to make potential customers aware of quality differences before a purchase is completed. This can be done in many different ways and on the preceding pages I have argued that the use of trade credit is one way to convey information from seller to buyer. One important simplification in my analysis is the fact that quantity effects are not included. With quantity effects it is possible to construct cases when the seller is better off with than without trade credit when r_t does not include complete risk compensation. This is for example the case when no trade credit means market failure.

When I have discussed the design of various trade credit contracts the seller side of the market has so far always been represented by a good quality seller. However, it is also possible that a bad quality seller is willing to supply trade credit. This occurs if the interest arbitrage profit exeeds the bad quality adjustment cost. A bad quality seller without risk aversion supplies trade credit if

$$(6.13) \qquad e^{r_t - r_s - d} PQ - C - e^{-d} \varepsilon Q > PQ - C,$$

where once again quantity effects have not been taken into account. The lhs of (6.13) represents the expected profit with trade credit, when the last term is the expected bad quality adjustment cost which is deducted from the agreed payment. When this inequality is rewritten in terms of interest rates trade credit is preferred when

$$(6.14) \qquad r_t > r_s + d + \ln(1 + \frac{e^{-d}\varepsilon}{P}).$$

The trade credit rate of interest must exceed the risk adjusted interest rate plus a positive term which represents the quality adjustment cost. Thus, compared to a high quality seller the quality adjustment cost term has been added. (See (6.4) where d' is replaced by d when the seller has no risk aversion.) This means that when both types of sellers have the same interest rate and the same believes about payment failure by the buyers a good quality seller can make trade credit offers that are more favourable than those of bad quality sellers. If in this case competition among sellers pushes down the trade credit rate of interest only good quality sellers will make competitive credit offers, and a low interest credit offer becomes a perfect signal which distinguishes good quality from bad quality sellers. Further, if this fact can be communicated to presumptive buyers they do not have to accept credit offers in order to reduce uncertainty, it is enough to know what offers different sellers make. Finally, if quantity effects are also taken into account an introduction of trade credit will lead to a fall in the demand for goods from low quality sellers. There will be price adjustments until the cash price of a bad quality seller plus the expected adjustment cost equals the present value price of a trade credit purchase. Low quality goods will sell at a lower cash price than high quality goods. Thus, in this highly stylized example, the introduction of trade credit leads to price differentiation, which is a necessary condition to generate a supply of high quality goods, if it costs more to produce high than low quality. Post payment can consequently be one way to avoid market failure.

6.3 PRE PAYMENT

The stylized facts in the introductory chapter showed that pre payment is very unusual when the Swedish corporate sector is studied as an aggregate. On a disaggregated level business services is the only subsector where pre payment from customers equals to or is larger than accounts payable. An interesting issue is then why pre payment is common in the business services sector and so unusual elsewhere. In this section I discuss under what conditions the use of pre payment is likely to arise, and why pre payment is common in the business services sector. Within the same model framework as in the preceding sections it is possible to show under what conditions both seller and buyer prefer pre to cash payment. I do not take quantity effects into account and I only study cases with full pre payment. Partial pre payment can be studied in a way similar to my discussion about Pareto optimal partial post payment contracts.

Pre payment implies a reversal of the distribution of uncertainty between seller and buyer compared to post payment. Thus, when there is both product and payment uncertainty pre payment means that it is the buyer who takes the risk, since he does not know what he gets when a good or service is delivered. This risk is accepted only if the buyer is properly compensated. Hence, in this case it is the buyer who is the insurer and the seller the insured. Now, assume that cash payment implies no buyer uncertainty and pre payment represents a caveat emptor contract. (Another alternative is to assume that only post payment implies no product uncertainty, such an assumption does not alter the conclusions presented below in a fundamental way. Consequently, I have chosen the simpler version when cash payment represents no product uncertainty.) When a seller delivers some good to a buyer three things can happen. *First*, the buyer accepts the good and pays cash. *Second*, the buyer refuses to complete the purchase because he is not satisfied, or *third*, he is satisfied but cannot pay. When either one of the two types of payment failure occurs the seller has goods on hand which he must sell to somebody else. If the seller produces some standardized staple product this type of payment failure has little or no consequences. Then the product can be sold to somebody else at the same cash price. If, on the other hand, the seller has produced some custom made good with such characteristics that alternative buyers are willing to purchase the good first after a considerable price reduction the consequences can be very severe. In this case a broken contract represents consequences very similar to payment failure in the post payment case. The seller has produced a good or service and receives little or no revenue. This is particularly the case with services. A service which has been completed cannot be resold. (Think, for example, of the nature of the services supplied by lawyers, doctors and plumbers.)

Next I formulate these observations about the consequences of payment failure in terms of a risk adjustment coefficient which reflects the seller's willingness to pay in order to reduce risk. The more severe the consequences of payment failure the higher is the risk adjustment coefficient. Thus, payment is most likely to occur in markets where the seller produces services or custom made goods. A seller prefers pre to cash payment if the pre payment price at the cash payment date exceeds the risk adjusted cash price. When this is formulated in terms of the utility function used earlier a seller prefers pre payment if the inequality in (6.15) holds

$$(6.15) \qquad e^{r_s - r_t}PQ - C > e^{-d'}PQ - C,$$

where d' represents the risk aspects of cash payment and the attitude towards risk. Written in terms of interest rates the inequality becomes

$$(6.16) \qquad r_s + d' > r_t.$$

Pre payment is preferred if the risk adjusted interest rate of the seller exceeds the trade credit rate of interest. The buyer, in turn, is better off with pre payment if the risk adjusted present value cost is lower than the cash price. That is when

$$(6.17) \qquad e^{r_b - r_t + \beta} PQ < PQ,$$

where β represents the risk adjustment factor derived in (6.2). The inequalities in (6.16) and (6.17) imply that pre payment is preferred by both seller and buyer when the seller's risk adjusted interest rate exceeds the buyer's,

$$(6.18) \qquad r_s + d' > r_b + \beta.$$

Here (6.18) is an inequality similar to the post payment inequality in (6.5), but compared to the post payment case the inequality sign has been reversed. When there is not uncertainty (6.18) reduces to the pre payment interest arbitrage condition discussed in Chapter 2, and when there is interest rate equality the inequality summarizes combinations of both buyer and seller notions of the consequences of risk and attitudes toward risk that can generate pre payment. Thus, $r_s > r_b$ need not be a necessary condition to generate pre payment. Here I have shown how interest rate differentials and imperfect information combined with risk reduction motives can lead to pre payment. A third reason is market power, which has been stressed by Faith and Tollison (1981). A well known lawyer or doctor with a longstanding good reputation can for example use some of his monopoly power to demand pre payment with a small or no price reduction. A price policy which is difficult to use by less well known lawyers and doctors. Market power does of course not exclude the other reasons for pre payment. Below I give an example of when all three reasons can work in the same direction.

When Klein, Crawford and Alchian (1978) (K.C.A.) discuss various leasing contracts they define the *quasi rent value* of a good as:

"the excess of its value over its salvage value, that is, its value in its next best use by another renter". With this terminology custom made goods or services represent products with high quasi rents. K.C.A. argue that in markets with high quasi rents there is a high risk of opportunistic behavior by the buyers (lessees). If a buyer resumes price negotiations when a good has been produced the seller has no trump on hand if the quasi rent is high compared to the sales price. They further go on and argue that in such cases vertical integration can be more efficient than costly contract negotiations. From the discussion presented above it follows that an alternative to vertical integration in quasi rent markets is pre payment agreements. This, of course, provided that it without too costly contract negotiations is possible to make offers such that the inequality in (6.18) holds. With pre payment the risk bearing is moved from seller to buyer. This, in turn, can give rise to opportunistic behavior by the seller. One way to avoid such temptations is to use partial pre payment, which divides the risk between buyer and seller. Various pre payment agreements are in some cases much more likely than vertical integration as a mean to reduce uncertainty. For example, in the market for consumer durables it is generally impossible for the average consumer to engage in vertical integration. Vertical integration eliminates uncertainty by eliminating the market whereas more or less complex payment schemes can contribute to avoid market failure, when information is imperfect with "lemons" on both sides of the market.

The trade credit literature contains no information about the price differences between various cash and pre payment agreements. Consequently very little is known about the implicit interest rates implied by (6.18). An empirical investigation is beyond the scope of this chapter and is left as a topic for future study. Here I limit myself to some short comments about a few observations of pre payment agreements that are readily available. One of the most common pre payment agreements with available price information is newspaper subscriptions. The largest morning newspaper in the Stockholm area is Dagens Nyheter. The cash price is 3 SEK and a one month pre payment subscription costs 60 SEK. The undiscounted difference between a thirty day subscription and thirty days of cash payment is 3 x 30 - 60 = 30 SEK. Thus, the subscription alternative includes a substantial price reduction. In this case pre payment represents an offer to pay 60 SEK in advance in order to receive a flow of 3 SEK worth of newspaper during thirty consecutive days. The implicit interest rate of this offer is given by (6.19)

$$(6.19) \qquad 60 = \sum_{t=0}^{29} ((1+r)^{-t})3 \ ,$$

with the solution $r \approx 0{,}03$ or on an annual basis $r \approx 1080\%$. Since DN's cost of capital probably lies between ten to twenty percent it is clear that risk reduction and possibly goods market transaction costs (see the next chapter) dominate completely. With the terminology used earlier one can say that d' is much larger than r_s in (6.18).[10] A newspaper producer clearly has risk reduction incentives to make favourable payment offers. Pre payment reduces uncertainty about future demand, which is valuable since new papers have extreme quasi rents if they are not sold say within twelve hours after printing.

In Table 6.1 I present some balance sheet ratios of the same type as the "stylized facts" in the introductory chapter, where I noted that the business services sector uses much more pre payment than other subsectors of the Swedish corporate sector. Among the categories within business services Architects and Building Consultants, and Technical Agencies and Institutions use by far much more pre payment than the other sub groups. Business services can in turn be compared with the other extreme, the Petrochemical Industry where pre payment is very rare. Thus, the table

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10 An alternative to the calculation of an implicit interest rate of the pre payment subscription in terms of a "package offer" is to calculate the implicit interest rate for 29 different credit offers, $2 = (1+r_1)^{-1}3$, $2 = (1+r_2)^{-2}3$, ..., $2 = (1+r_{29})^{-29}3$. In this case the interest rate will be falling, starting with $r_1 = 18.000\%$ and ending with $r_{29} = 507\%$.

Table 6.1 Pre and post payment in some sub sectors of the Swedish corporate sector. AR = accounts receivable, CA = current assets, Prep = pre payment from customers, CL = current liabilities, S = turn over. Source: The National Bureau of Statistics, Firms 1980, Appendices 1 and 6.

Corporate sector, number of employees	AR/CA	Prep/CL	AR/S	Prep/S
Aggregated Business Services, 20-	0.23	0.37	0.14	0.19
Architects and Building Consultants, 20-	0.30	0.27	0.18	0.13
Technical Agencies and Institutions, 50-	0.09	0.60	0.11	0.65
Petrochemical Industry, 20-	0.31	0.01	0.10	0.03

shows that pre payment ranges from sixty to one percent of current liabilities, (the percentage for the aggregated corporate sector is seven point eight, See Chapter 1 Table 1.1). One way to explain this marked difference between business services and other firms is to look at the type of products produced in the business services sector. For example, architects produce custom made, site specific, drawings, which in most cases, cannot be used by somebody else than the original buyer. Drawings represent a good with a high quasi rent, and the consequences of a broken contract are comparable with complete payment failure. Thus, architects have strong incentives to make pre payment offers. Some architects also have enough market power to demand pre payment, and finally, architects often work in small firms with less access to the commercial loan market than large building contractors. Thus, both the consequences of attitudes toward risk, market power, and interest rate differentials can motivate the use of trade credit with pre payment. The petrochemical industry, on the other hand, represents a market with almost diametrically opposite market characteristics, and consequently few pre payment motives. *First*, the products are highly standardized. The same product can be sold to a large number of different customers. There is no or only a low quasi rent. *Second*, with many suppliers selling identical products there is no scope for pre payment from a market power point of view. Post payment as a way to compete and to grant hidden price concessions is much more likely. *Finally*, many of the petrochemical companies are owned by large international corporations, with much better access to large parts of the financial market than many of their customers, r_b is likely to be larger than r_s, which also points towards post instead of pre payment.

6.4 SUMMARY

In this chapter I have discussed trade credit in a transaction or contract cost context. This is an approach that has received very little attention in the trade credit literature. It has been possible to single out some plausible explanations of why firms use pre, post, or mixed payment trade credit contracts. I have argued that trade credit can be used to reduce uncertainty in markets where frequent trade or customer relationships have not reduced uncertainty to a minimum. Pre payment eliminates payment uncertainty, post payment quality uncertainty, and mixed payment patterns reduce both types of uncertainty. In a seminal article about the theory of the firm Coase (1937) argues that firms emerge because some activities have lower transaction costs within than between firms. This argument also has a market transaction cost counterpart. When trade credit is used to reduce uncertainty it improves the functioning of the market. The transaction cost of using the market is lowered, and the need to internalize transactions is lessened. Thus, informal trade credit agreements can be one way to reduce contract costs. When trade credit is seen in this context its role is not the same as in the preceding chapters, and close substitutes are not found among the other financial alternatives I have discussed earlier. Instead trade credit is an alternative to other risk reducing measures such as guarantees, free samples, and government controls. I have showed that post payment is preferred by both seller and buyer when the risk adjusted interest of the buyer exceeds that of the seller. In this case the seller is the insurer and the buyer the insured. In markets where buyers have high risk aversion and where the cost of buying a "lemon" is high post payment is likely to emerge. I have also showed under what conditions partial pre payment is likely. In some cases this is the only possible solution. Further I drew the conclusion that there can exist a wide range of Pareto sanctioned interest/post payment combinations when the interest rates are equal. In the section about pre payment I argued that both seller and buyer benefit from such an agreement when the risk adjusted interest rate of the seller exceeds that of the buyer. Here the buyer is the insurer and the seller the insured. I also argued that such contracts are likely to be in use in markets where the quasi rent value of the product that is being traded is high. This is so because custom made products are hard to sell if the original buyer does not fulfil his commitments. One of my stylized facts in Chapter 1 was that pre payment is rather unusual compared to post payment. The only exception is the business services sector. Pre payment is common, for example, among architects, and given the discussion in this and preceding chapters there are three different ways to explain this. Interest differentials, market power, and the fact that they produce high quasi rent goods.

Given the model structure set forth in this and the preceding chapters it has been possible to single out some plausible explanations of why pre or post payment is used in different markets. One possible next step is then to make an empirical micro study of firms in different markets in order to find out whether interest rate differentials, attitudes toward risk and degree of quasi rent seems to explain the use of various payment schemes, and to what extent complex payment agreements are a substitute for vertical integration or complete market failure.

7. Some Transaction Cost Aspects

7.1 INTRODUCTION

The contents of Chapter 6 lie somewhat besides the mainstream transaction cost literature. In this chapter I return to conventional transaction cost theory, with its roots in inventory models, and with the help of conventional transaction cost and queue theory I show how transaction and interest costs can make it wortwhile to use trade credit instead of cash payment. Here both goods, and financial market transaction costs can motivate use of trade credits. The contents of this chapter is divided into two main sections. First I discuss goods market, and in the second section financial market transaction costs. Both the goods and financial market parts, in turn, have a similar structure insofar as they first include deterministic and then stochastic models, where in the latter the inflow of bills payable is uncertain.

In the section about goods market transaction costs I start with a deterministic model with a constant flow of goods or services, fixed goods market payment costs, and no financial market transaction costs. These assumptions give rise to a cost minimizing "saw tooth" payment pattern. Bills accumulate for t periods, and the whole debt is cleared periodically. With goods market payment costs other types of financial agreements cannot always fill the same function as trade credits. The important point is that the use of trade credit makes it possible to reduce the number of payments in the goods market. In an extension of the model I show that it is a combination of inventory storage costs and goods market payment costs that give rise to the use of trade credit. High storage costs is an incentive to make goods market transactions with short intervals, and trade credits reduce the number of payment occasions, which reduces the transaction cost. One of the drawbacks of "saw tooth" models is that accounts payable will be equal to zero at given payment intervals. This is a regularity uncommon to most firms. In the section about a queue theoretic approach to explain the use of trade credit this problem is remedied. I show how a firm's choice of bill processing capacity can give rise to a line of bills, when there is a trade off between the choice of capacity cost of operating the processing system, and the interest cost of bills awaiting and being processed. Here both the length of the credit period and the size of account payable will vary around stable long run averages. This approach leads to new square root formulas that are surprisingly similar to those set forth in the inventory theoretic section. In addition a new quantity effect is introduced since now not only the value but also the number of arriving bills matters.

In the section about financial transaction costs I first extend Baumol's (1952) classical "saw tooth" demand for money model. There is a constant rate of purchases, a fixed assets to money transaction cost, zero interest on money, and positive interest rates on other assets and trade credit. Instead of studying average money holdings I concentrate on the back log period (when the firm holds no transaction balances, and consequently accumulates debt) to determine the optimal size of accounts payable. The length of the credit period, as a proportion of a transaction cycle, is shown to be equal to the ratio of the asset rate of interest to the trade credit rate of interest. If the trade credit rate of interest goes toward infinity no backlogging is allowed, and the model reduces to the Baumol model. With reasonable assumptions about the size of the interest rates the interest elasticity of accounts payable is greater than one, and an increase in the general level of interest rates increases the use of trade credit. A conclusion which differs from the conclusion in Chapter 2, where only an uneven spread of interest changes could change the length of the credit period. Finally, in the second section about financial transaction costs I introduce stochastic net disbursements. Here I use a conventional demand for reserves model to study situations when the reserves are insufficient. That is when the firms use trade credit instead. The argument behind this approach is that by using short term credit the firm is given time to gather information while it is temporarily sheltered from the need to make rapid asset transfers of unknown size. The firm is given lee-way to plan its financial transactions. In order to derive analytical expressions for the optimal size of accounts payable I assume that net disbursements are exponentially distributed. Then it is possible to show that the use of trade credit as a buffer not necessarily eliminates demand for precautionary reserves. Also in this context the interest rate elasticity of accounts payable is greater than one, and an increase in the general level of interest rates increases average accounts payable. One property that all the models have in common is that they give rise to explicit formulas of average accounts payable. Thus, it is easy to test if the transaction cost approaches to trade credit presented here also get empirical support.

7.2 FIXED GOODS MARKET PAYMENT COSTS

In this section I discuss how the use of trade credit can lower goods market trading costs. The exposition that follows rests on two important assumptions. *First*, trading costs are represented by fixed payment costs. Each time a payment is made a fixed amount of resources has to be used to complete the transaction. *Second*, the firm or individual who uses trade credit purchases goods or services in a continuous flow. Some examples of such flows, which usually include trade credit, are wage payments, newspaper subscriptions, rent payments and payments of public utilities such as electricity, water and telephone services. One way to explain the use of trade credit in this context is the existence of payment costs. (This does, of course, not exclude the possibility that in these cases there can also be financial reasons to accept trade credit.) For example, it takes an employer more time and trouble to pay his employees every hour or daily compared to monthly wage payments, and no one pays his electricity bills every day. Similar arguments hold with respect to the other examples. The assumption that it costs something to make payments has a counterpart on the seller's side. Receiving payments also gives rise to transaction costs. (Payments have to be recorded and money transfers have to be attended to.) The seller who delivers a continuous flow of goods or services can also have incentives to receive payments at discrete intervals, and the problem is to find payment terms that simultaneously satisfy both seller and buyer. Here, however, I do not take the seller side of the market into account. I only study how goods market transaction costs affect the buyer's choice of payment intervals.

Fixed transaction costs and continuous flows of goods or services are two corner-stones of inventory theoretic models which determine the optimal size of inventories either in terms of goods or various financial assets. Here I use the same type of model approach to explain the use of trade credit in a goods market transaction cost context. Thus, the reader familiar with various square root formulas will find a number of new ones showing the optimal size of accounts payable. I start with a basic, continuous delivery of goods or services, model where financial assets and liabilities other than money and trade credit bills are not taken into account. Then I introduce goods inventory and intra payment period asset holdings, and finally I discuss how uncertainty about the return on asset holdings affects the choice of average accounts payable and money holdings.

7.2.1 *Trading cost minimization - the basic model*

Assume that in a world without uncertainty there is an individual or a firm who uses a constant daily quantity of some good or service. There is no inventory, the rate of delivery is the same as the rate of consumption. Each time the buyer makes a payment there is a fixed trading cost which measures the time and trouble it takes to make a payment. The buyer is free to choose to pay the purchases daily or with given intervals within a predetermined planning period. If the buyer uses trade credit with post payment, instead of cash payment, the seller is assumed to be properly compensated by the interest payments on accounts receivable. There is a constant inflow of money required to settle accounts payable, and interest is paid on money holdings. There are no other financial assets or, alternatively, with fixed financial market transaction costs, the planning period is not long enough to justify transactions between money and other interest bearing assets. With the terminology used by Haley and Higgins (1973) this view of the world represents a firm with surplus funds during the trade credit cycle, and it could represent the flow of goods or services examples mentioned above. Before I present a formal model based on these assumptions let me first introduce some notation which recurs throughout this section:

S = the daily rate of consumption of the goods or services purchased by the buyer,

c = a fixed payment cost,

i = the daily trade credit rate of interest,

i_M = the daily interest on money holdings,

t = the length of one trade credit cycle,

τ = the length of the planning period.

With this notation the total trading cost over the planning period is

$$(7.1) \qquad TC = \frac{\tau}{t} c + \frac{(i-i_M) tS\tau}{2} ,$$

where the first term shows the transaction cost, the number of payment cycles (integer constraints on τ/t are not taken into account) times the payment cost, and the second term shows the net interest on trade credit.[1] This is a model with the same structure as most basic inventory models. The difference is that instead of an inventory of goods (7.1) represents the cost of managing an inventory of trade credit bills. Other goods market payment cost models that have been developed within an inventory theoretic framework are those of Barro and Santomero (1974), Policano (1977) and Clower and Howitt (1978). These models cover many more aspects of transaction costs than the trade credit payment cost problem discussed here. One way to look at the model in (7.1) is to see it as a trade credit distillate of the larger models set forth by the authors mentioned here.

The objective is to strike a balance between transaction costs, which fall when t rises, and interest costs, which rise when t rises, such that the total trading cost is minimized. Differentiation of (1) with respect to t gives the first order condition for a cost minimum

$$(7.2) \qquad t = \left(\frac{2c}{(i-i_M)S}\right)^{\frac{1}{2}} ,$$

and, with constant daily purchases, (7.2) in turn determines an optimal saw tooth pattern of accounts payable. Knowing the op-

1 The second term has been derived in the following way: the accumulated interest cost during one cycle is

$$\int_0^t iS\Theta d\Theta = \frac{it^2S}{2} ,$$

when there is no discount period, and there are τ/t such interest payments. Similarly the interest on money holdings is $\frac{i_M t^2 S}{2}$.

timal cycle length it follows that average accounts payable over one cycle and over the planning period is

$$(7.3) \quad \overline{AP} = \frac{St}{2} = \left(\frac{cS}{2(i-i_M)}\right)^{\frac{1}{2}} .$$

With a constant inflow of money per unit of time (7.3) also represents the average money holding. From (7.3) it is clear that $\overline{AP}$ increases with increasing daily purchases, transaction costs and return on money holdings, and decreases with an increasing cost of trade credit.

The elasticities of $\overline{AP}$ with respect to i, i_M and S are,

$$(7.4) \quad \varepsilon_i = -\frac{1}{2}\frac{i}{i-i_M} , \quad \varepsilon_{i_M} = \frac{1}{2}\frac{i_M}{i-i_M} , \quad \varepsilon_S = \frac{1}{2} .$$

Both interest rate elasticities are functions of the net cost of trade credit. With $i = 2i_M$, which is not totally unlikely, ε_i equals one and ε_{i_M} one half, and when the interest rates are equal the elasticities are not defined. This reflects the fact that if the net cost of trade credit is zero the model degenerates. All transactions take place at the end of the planning period. If $i_M > i$, trade credit becomes a source of revenue and there is no incentive to pay before $t = \tau$. With $i_M > i$ the financial aspects dominate the transactions motive to use trade credit. This result is also in accordance with similar conclusions drawn from other models with fixed payment costs by Barro and Santomero (1974) and Policano (1977). The essence of my discussion about financial aspects of trade credit with post payment, which is the by far most common credit arrangement, was that there exists a financial motive to use trade credit if $i_M > i$. Here I have shown that in the case with $i_M < i$ there is still a motive to use post payment, because, although from a financial point of view there is no interest arbitrage profit to be made,

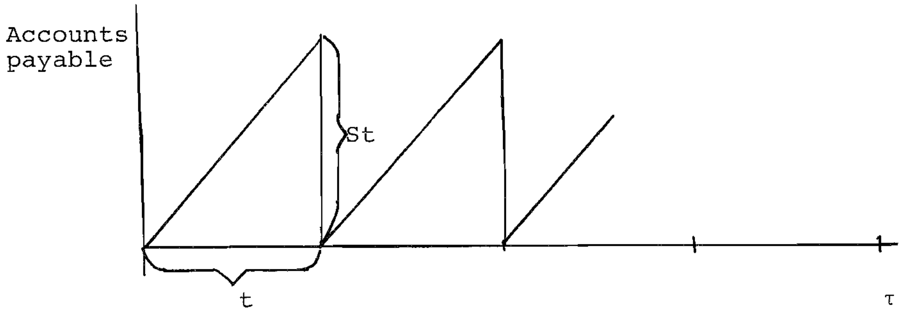

Figure 7.1 Accounts payable with fixed goods market transaction costs.

it is still sound to use trade credit to economize on goods market transaction costs. It pays to postpone payments as long as the sum of the net credit cost and the transaction cost over one payment cycle is lower than the cash payment transaction cost during the same length of time. It is also important to keep in mind that a credit from a third party does not generate the same reduction of payment costs. For example, if the firm uses a bank over draft facility instead of trade credit it still has to make daily payments to the seller, and these give rise to transaction costs. In this goods market transaction cost context trade credit yields a unique service which cannot be produced by other types of credit arranged by the buyer. Thus, this is one possible explanation of why firms use trade credit which includes implicit interest rates that are higher than the return on short term funds.

Finally, the ε_S elasticity shows that there is not a one to one relationship between the size of average accounts payable and the size of purchases. If transaction costs and interest costs are the same among large and small firms, this means that large firms ought to have a lower ratio of accounts payable to purchases than small firms. This is a hypothesis which is easily testable and it is supported by the stylized facts in Chapter 1. A word of warning is justified, however. Just as in the case with the square root formula of the transactions demand for money,[2] the model presented above probably rests on too stringent assumptions to be applicable in an analysis based on aggregated statements of earnings and balance sheet data. Transaction costs and interest rates need not be the same, and most firms also make a large number of irregular purchases which do not follow the constant flow pattern essential to the model above. These remarks do not mean, however, that the model lacks explanatory value. It shows one important stylized aspect of why it is meaningful to talk about a transactions demand for trade credit.

As an alternative to the case with a continuous inflow of money it is also possible to assume, perhaps more realistically, that the firm or consumer at the beginning of the planning period has recourse to a sum of money M, which is used to purchase a continuous inflow of goods or services in the interval 0 - τ. In Appendix I I present such a model where I also have included an assumption that all trade credit interest payments are incurred within the model, which means that trade credit interest payment reduces money holdings and thus also interest receipts from money holdings. This is an addition which most asset or money transactions cost models do not include. The most common approach is to let all costs be incurred outside the models. Interest payments within the model make it more complex than the one presented above. In the Appendix it is shown that the introduction of trade credit interest payments reduces the length of the credit period, but this reduction is small. Thus, the two models yield practically the same results.

One characteristic of both models is that when i_M exceeds i it is optimal to pay at the end of the planning period. Then the use of trade credit has ceased to be a cost; instead it is a source of revenue. In this case the fixed goods payment transaction cost approach to explain the use of trade credit

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2 See Brunner and Meltzer (1967).

breaks down. One way to remedy this drawback is to include interest rates which are not constants but functions of the length of the trade credit cycle. If both the cost of trade credit and the return on short term funds increase with the length of the trade credit cycle, it is possible to include cases when the interest on money holdings is higher than the trade credit rate of interest. Then it can be shown that, in the continuous cash inflow model one case when there exists a non-trivial optimal solution to the choice of the length of trade credit cycles is when the marginal interest cost of trade credit rises faster than the marginal interest revenue from money holdings. In this case it is possible to derive marginal interest rate conditions, with respect to the length of a trade credit cycle, of a type similar to the marginal interest conditions I discussed in the chapters which dealt with different financial aspects of trade credit. For example, there is a rising i'(t) function when a credit offer includes a discount period with zero interest, a normal credit period with an agreed interest rate, and finally, some kind of penalty payment for payment after the end of the agreed credit period. One case which gives rise to an increasing trade credit rate of interest is if the seller adds some type of increasing risk compensation if t rises above what he considers to be a "normal" transactions cycle.

7.2.2 *Trade credit with goods inventories and financial transaction costs*

In the basic model presented above continuous consumption was combined with a continuous inflow of goods or services. Such simultaneous flow patterns are observable particularly when goods or services are non-storable (for example labor services) or when storage costs are high (for example electricity or perishable food stuffs). If the continuous rate of consumption assumption is retained, there are also many situations where the buyer uses an inventory, with periodic replenishment, instead of continuous deliveries. In this section I extend the model and I discuss the case when the buyer uses trade credit at the same time as he holds a goods inventory. There is a continuous inflow of money, the customers pay cash, interest is earned on intra credit period bank deposits, and bank transactions include a fixed transaction cost. This type of payment/inventory/money holding scheme can, for example, describe a grocery store or a restaurant which receives fresh meat supplies once or twice a week while the bills are paid once or twice a month.[3] To derive a trading cost function based on these assumptions I first need some additional notation:

t_I = the length of an inventory cycle,

c_I = a fixed inventory set-up or order cost,

s_I = a daily inventory storage cost,

t_M = the length of a money holding cycle,

c_M = a fixed deposits to money transaction cost,

i_M = the daily interest on deposits.

The costs of holding a goods inventory are the traditional ones. There is a fixed inventory set-up or order cost, and there is a daily storage cost, no lead time and no backlogging. In the grocery store example, the latter cost can represent the cost of storing perishable food stuffs. These costs are the building blocks in standard inventory models and the total inventory cost over the planning period is

$$(7.5) \qquad TC_I = \frac{\tau}{t_I} c_I + \frac{s_I t_I S \tau}{2} ,$$

where S is the daily rate of sales and where the storage cost has been calculated in the same way as the interest cost in foot note 1. The next step is to calculate the trade credit cost. The pay-

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[3] As far as I know, the only other authors that have discussed the same type of inventory trade credit problem are Haley and Higgins (1973) and Policano (1977).

ment costs are the same as previously, but the introduction of inventory holdings changes the interest on accounts payable. Assume that inventory cycles and trade credit cycles are chosen in such a way that the ratio t/t_I is an integer. Payments are made after t/t_I inventory cycles and then the process starts over again until the end of the planning period is reached. (See Figure 2 where the interrelationship between accounts payable, the goods inventory, and deposits is depicted.) During one trade credit cycle the interest on accounts receivable is the sum of the interest paid on the batches of goods that have arrived. For example, assume that a trade credit cycle is made up of three inventory cycles. Then the interest of the first, second and third batch respectively is

$$(7.6) \qquad i(t_I S)t_I,\ i(t_I S)2t_I,\ i(t_I S)3t_I,$$

where $t_I S$ is the value of one batch, and the overall interest cost is the sum of the three terms. A general formulation of this sum can be written[4]

$$(7.7) \qquad iSt_I^2 \sum_{j=1}^{t/t_I} j = \frac{iSt_I^2}{2} \frac{(t+t_I)}{t_I} \left(\frac{t}{t_I}\right) = \frac{iS}{2}(t+t_I)t \ ,$$

and since there are τ/t trade credit cycles over the planning interval, the total interest cost is

$$(7.8) \qquad \frac{iS}{2}(t+t_I)\tau \ ,$$

which means that the total trade credit trading cost becomes

$$(7.9) \qquad TC_{TR} = c\frac{\tau}{t} + \frac{iS}{2}(t+t_I)\tau \ .$$

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[4] Where (7.7) is based on the fact that $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$.

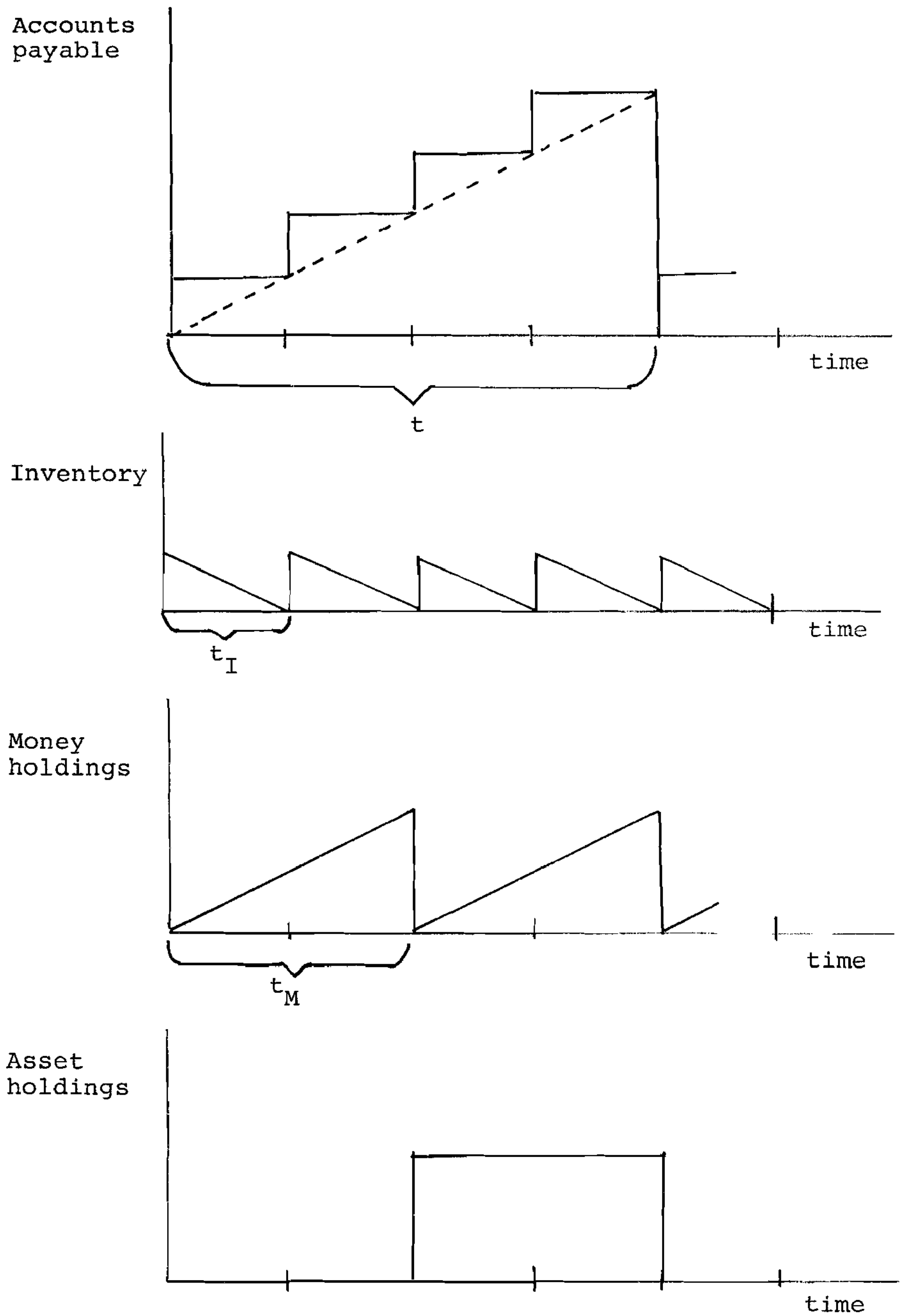

Figure 7.2 Accounts payable with fixed payment costs, inventory set-up and holdings costs, and financial transaction costs.

The firm accumulates deposits until a payment date occurs. The interest revenue during one trade credit cycle is

$$(7.10)\quad i_M t_M^2 (1+2+,\dots,+\frac{t}{t_M} - 1)S = i_M t_M^2 S \sum_{j=1}^{\frac{t}{t_M}-1} j = \frac{i_M(t-t_M)tS}{2},$$

where the upper boundary of the sum is $t/t_M - 1$ since St_M is accumulated before the first deposit is made. Thus, the gain from holding deposits is,

$$(7.11)\quad TR_{dep} = \frac{i_M(t-t_M)S\tau}{2} - c_M \frac{\tau}{t_M}.$$

Combining (7.5), (7.9) and (7.11) gives the final trading cost function

$$(7.12)\quad TC = c\frac{\tau}{t} + c_M \frac{\tau}{t_M} + c_I \frac{\tau}{t_I} + \frac{(i-i_M)tS\tau}{2} + \frac{i_M t_M S\tau}{2} + \frac{(s_I+i)t_I S\tau}{2},$$

where (7.11) is included with a negative sign. Minimization of (7.12) with respect to t, t_M and t_I gives the first order conditions

$$(7.13)\quad t = \left(\frac{2c}{(i-i_M)S}\right)^{\frac{1}{2}},\quad t_M = \left(\frac{2c_M}{i_M S}\right)^{\frac{1}{2}},\quad t_I = \left(\frac{2c_I}{(i+s_I)S}\right)^{\frac{1}{2}},$$

when I have tacitly ignored the fact that t/t_I, t/t_M, and τ/t have to be integers.[5] The length of the credit period is the same as in the basic model, and the money holding period is one version of Baumol's (1952) famous square root formula. Both are independent of the choice of inventory cycle. The length of an inventory cycle depends both on the "physical" inventory storage cost and on the trade credit rate of interest. In those cases where this model is applicable the inventory storage cost is likely to be much higher than the daily trade credit rate of interest. An annual interest rate of twenty percent gives i = = 0.000555, which seems to be a very low number if it is interpreted in terms of s_I.

[5] I have omitted the second order conditions; they hold and are easy to derive.

Further it follows that

$$(7.14)\quad \bar{M} = \frac{St_M}{2} = \left(\frac{c_M S}{2i_M}\right)^{\frac{1}{2}},$$

$$(7.15)\quad \bar{I} = \frac{St_I}{2} = \left(\frac{c_I S}{2(i+s_I)}\right)^{\frac{1}{2}},$$

$$(7.16)\quad \overline{AP}_I = (t_I S + 2t_I S + \ldots + \frac{t}{t_I} t_I S)\frac{t_I}{t} = \frac{S}{2}(t+t_I) = \overline{AP} + \bar{I} =$$

$$= \left(\frac{cS}{2(i-i_M)}\right)^{\frac{1}{2}} + \left(\frac{c_I S}{2(i+s_I)}\right)^{\frac{1}{2}},$$

$$(7.17)\quad \bar{A} = (t_M S + 2t_M S + \ldots + (\frac{t}{t_M} - 1)t_M S)\frac{t_M}{t} = \frac{S(t-t_M)}{2} =$$

$$= \overline{AP} - \bar{M} = \left(\frac{cS}{2(i-i_M)}\right)^{\frac{1}{2}} - \left(\frac{c_M S}{2i_M}\right)^{\frac{1}{2}},$$

where the "—" represents averages, $\bar{M}$ is money holdings, $\bar{I}$ inventory, $\overline{AP}_I$ accounts payable in the inventory model, and $\bar{A}$ deposits or asset holdings. Although the length of a trade credit cycle is the same as before average accounts payable has changed. This is also evident from Figure 2, now average accounts payable, $\overline{AP}_I$, is based on the step function in the figure, while earlier it was based on a continuous function, the dotted line in the figure. $\overline{AP}_I$ is the weighted sum of the different steps in the figure, when the ratio t_I/t has been used as weight. $\overline{AP}_I$ in (7.16) shows that now average accounts payable can be treated as the sum of accounts payable in the no inventory case and the average inventory holding. It is also evident that $\overline{AP}_I$ is a function of all the inventory and trade credit cost variables.

Thus, in this respect there is an interdependence between the choice of inventory and trade credit policy. It is also clear that if the inventory storage cost is high, $\bar{I}$ approaches zero. Hence, one way to look at the existence of continuous flows of goods or services is to see such flows as cases when the storage cost is infinitely high.[6] High storage costs give rise to short inventory cycles, there will be many payment occasions, and with high enough payment transaction costs this gives rise to the use of trade credit, which in turn makes it possible to make intra payment cycle deposits. Assume that the firm has chosen an inventory cycle of length t_I and that trade credit is used only over the same cycle. Then the firm is better off with an extended credit cycle if the inequality in (7.18) holds

$$(7.18) \qquad c\left(\frac{\tau}{t_I} - \frac{\tau}{t}\right) > \frac{(i-i_M)(t-t_I)S\tau}{2},$$

when $t > t_I$ does not change t_M. The inequality shows that it pays to increase t if the payment cost reduction exceeds the increased interest cost. The higher the payment cost and the shorter the inventory cycle the more likely it is that this inequality holds.

There are many ways to introduce uncertainty in a trade credit transaction cost context. Finally, before I leave this model approach to explain the use of trade credit, assume that there is an uncertain return on deposits, which alternatively can be interpreted as holdings of bonds or certificates of deposit. Assume further that the mean variance utility function set forth in Chapter 4 is a sufficiently good approximation of a general utility function. The objective is to maximize the expected utility of the return on deposits, and the objective function can be written[7]

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[6] See also Policano (1977), page 167.

[7] The idea to this model formulation comes from Buiter and Amstrong (1978).

$$(7.19)\qquad U = \frac{(i_M - i)tS\tau}{2} - \frac{i_M t_M S\tau}{2} - c\frac{\tau}{t} - c_M \frac{\tau}{t_M} + kV(i_M)\left(\frac{S\tau}{2}\right)^2 (t-t_M)^2,$$

where i_M represents expected return, $V(i_M)$ the variance of i_M, and the risk adjustment factor k is multiplied by the variance of the interest revenue over the planning horizon. The inventory cost terms from (7.5) have been omitted since they include no terms that are affected by the fact that i_M is assumed to be a random variable. Differentiation of U with respect to t and t_M gives the first order conditions

$$(7.20)\qquad U'_t = \frac{(i_M - i)S}{2} + \frac{c}{t^2} + \frac{1}{2}kV(i_M)S^2\tau(t-t_M) = 0$$

$$(7.21)\qquad U'_{t_M} = -\frac{i_M S}{2} + \frac{c_M}{t_M^2} - \frac{1}{2}kV(i_M)S^2\tau(t-t_M) = 0.$$

These two equations have no nice analytical solution except at k = 0, when the model reduces to the one presented above. The preceding analysis has shown that there exists a maximum if k = 0.

Now, assume that there is a small increase in risk aversion. Implicit differentiation of (7.20) and (7.21) gives

$$(7.22)\qquad \frac{dt}{dk}\Big/_{k=0} = \frac{t^3\, V(i_M)S^2\tau(t-t_M)}{4c} \geq 0, \text{ if } t \geq t_M,$$

$$(7.23)\qquad \frac{dt_M}{dk}\Big/_{k=0} = \frac{-t_M^3\, V(i_M)S^2\,\tau(t-t_M)}{4c_M} \leq 0, \text{ if } t \geq t_M.$$

Thus, an increase in risk aversion (a falling k, since k is negative number) reduces the trade credit cycle and increases the money holding period, provided that $t_M < t$. The reduction in t stems from the fact that this reduces interest revenue from asset holdings, the variability of which gives rise to disutility. From (7.22) and (7.23) it also follows that an increase in risk aversion reduces average accounts payable and asset holdings, and increases average money holdings.

7.3 A QUEUE THEORETIC APPROACH

In the preceding section I have used an inventory theoretic approach to explain the use of post payment trade credit in a goods market context. Two important assumptions were that there are fixed goods market payment costs and that the buyer uses the purchased goods or services in a deterministic steady flow. These assumptions made the models suitable to explain the payment patterns of supplies of goods and services that are impossible to store or have high storage costs. With a "saw tooth" pattern of accounts receivable firms hold no trade credit bills at the beginning of every cycle. This is a situation which is unfamiliar to most financial managers. Most firms use a large number of inputs of goods or services; of these some are non-storable of the type discussed above, some are purchased with recurring short intervals, and finally, some are purchased very infrequently. In addition to this, purchase patterns also include stochastic elements, because in an uncertain world it is not possible to know exactly the demand for various inputs and with information and search costs it can pay to change purchase dates, if one thinks that the price is right. Such a mixture of purchases gives rise to a variable inflow of trade credit bills, and these are not necessarily paid according to the all or nothing payment scheme given by the inventory theoretic models above. In this section I show that on transaction cost grounds one can also in this more general case justify the use of trade credit. Before I go further into details, let me give an example of why I think it is possible to use a queue theoretic approach to explain the use of trade credit in a goods market transaction cost context.

A firm which buys a large number of various inputs receives a stream of different bills. When a bill arrives it takes time and trouble to record purchases and to administer payments, and if deliveries of goods and bills go together it takes time to forward the bills to those who make payments. Rapid processing of arriving bills and making payments requires both personnel and other resources. With an irregular inflow of bills the capacity needed to process bills and handle payments will vary. If then operating costs (for example the opportunity cost of making payments instead of doing something else) rise with the size of the transactions capacity it can be economical to choose payment dates such that there will form a line of bills awaiting processing or being processed within the transaction system of the firm. Thus, if there is a trade-off between the cost of making immediate payments and trade credit costs when the capacity to process bills within the firm is reduced, this trade-off can give rise to use of trade credit with post payment. One can also give examples of similar trade-offs on the seller side. These give rise to a separation of dates of delivery and the dates when bills are sent to the buyers. Hence, both seller and buyer can have incentives to separate irregular goods flows from money flows in order to operate their internal transaction systems in an efficient way. Here, however, I only take the buyer aspects into account. A counterpart to the payment delays mentioned above is the time it takes to clear checks or other payments within the banking system. There the cost of accepting floating checks and payment delays is lower than the cost of managing a system which can handle all peak load transactions immediately.

7.3.1 *An example without uncertainty*

Before I introduce a queue model in terms of stochastically arriving bills let me first present a simple deterministic example. The inventory theoretic models in the preceding section rest on the assumption that the capacity to make payments at a given moment in time is independent of the cost of making payments. Thus, it was possible to pay all bills simultaneously at a constant transaction cost. Now assume instead that the choice of transaction speed affects the choice of payment system. A high transaction speed gives rise to a higher payment cost than a low transaction speed. When there is no uncertainty the rate of arriving bills must equal the rate of bills being serviced. Otherwise the firm has either excess capacity or there will be an endless line of bills awaiting payment. With a steady in- and outflow of bills the size of accounts payable is determined by the length of time a bill stays within the transaction system of the firm. When a bill arrives, it has to pass through book--keeping procedures. For example, it has to be recorded and a check has to be made whether the bill is in accordance with what has been supplied. One way of looking at these internal transaction routines is to treat them as a transportation system, where the transportation time affects the operating cost. If μ bills are rushed through the firm in one day it costs more than if they spend one week within the transaction system.

In (7.24) I have specified a cost function based on this notion of how such an intra firm transaction system works.

$$(7.24) \qquad TC = \left(\frac{C}{t}\right)\mu + iSt\mu.$$

The first part of the expression represents the cost of producing an output of μ paid bills during the unit time period, (C/t) is the cost of passing one bill through the system and since there is a stable flow of bills through the firm the output is μ bills per period. This part of the cost function rests on the assumption that (C/t) is an appropriate description of how operating costs per bill fall when the time in the system increases and that there are no returns to scale. An alternative is to start with a general cost function $C(t,\mu)$ and discuss what properties are needed if (7.24) is to have a unique minimum with respect to t. The functional form I have chosen has the property that although the basic model assumptions differ it gives rise to the same type of square root optimum as in the preceding models. The second term represents the unit period interest cost, the trade credit rate of interest is i, the value of one bill is S, and there are $t\mu$ bills in the system. The analogy with transportation costs is also clear in (7.24). Assume that perishable goods are transported to a distant market. Then the first part of the expression can represent the cost of different types of transportation, for example by air or rail, and the second part represents the cost of spoilage during transport. Minimization of (7.24) with respect to t gives the rewritten first order condition

$$(7.25) \qquad t = \left(\frac{C}{iS}\right)^{\frac{1}{2}},$$

which is the optimal processing time of one trade credit bill. Thus, there is a positive relationship between C, the cost of passing a bill through the internal transaction system in one period, and the length of the credit period, and a negative relationship with respect to the interest cost. Since there are $t\mu$ bills in the system, accounts payable is

$$(7.26) \qquad AP = t\mu S = \mu\left(\frac{SC}{i}\right)^{\frac{1}{2}},$$

a formula which has some similarities with the other accounts payable formulas presented so far in this chapter. One difference between this and the preceding models is that here accounts payable is constant. There is no "saw tooth" variation. Another, and more important, difference is that now purchases include both a price and a quantity effect, with the elasticities

$$(7.27) \qquad \varepsilon_S = \tfrac{1}{2}, \quad \varepsilon_\mu = 1.$$

The first one is a price elasticity, since S is given in monetary units, and the second one represents a quantity elasticity, because μ is the number of bills arriving in on period. Thus, if the value of purchases increases accounts payable increases proportionately with respect to the number of trade credit bills and less than proportionately with respect to the value of each bill. Just as in the "saw tooth" models it is further essential that the trade credit rate of interest really represents a net cost. If at the same time the firm can invest funds reserved to make payments, and these yield an interest i_M, it is the difference $i - i_M$, which represents the interest cost. From (7.26) it is then clear that the time in the system grows infinitely large when i_M approaches i from below. With i_M close to i it is no longer meaningful to discuss trade credit in terms of this queue model. Then the transaction system does not include a waiting cost. This is also a property of the models in the remaining parts of this section. Thus, I assume that i represents a positive net cost.

7.3.2 *Some rudimentary queue theory*[8]

The model above presents the choice of transaction system when there is no uncertainty. Next I take a closer look at the other extreme, when both the number of bills arriving in a given period of time and the time it takes to process a bill is random. The objective of the firms is to find a cost minimizing combination of queue length, bills in line, and transaction capacity within the firm. This problem can be solved by using well known results from queue theory. I have not encountered any applications of queue theory in monetary micro models. Therefore, I first give a brief introduction to the theory that lies behind the formulas I use to calculate average accounts payable. However, there are many operations analysis applications of queue theory, because both production of goods and services include a host of situations which give rise to queue theoretic problems. For example,how to determine the optimal number of free way toll booths, the capacity of harbor facilities, the optimal number of cash registers in supermarkets and so on.

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[8] The thoughts and formulas presented in this section are primarily based on Panico (1969), Chapters 1 and 3, and to a lesser extent on Cox and Walters (1961), Chapters 1 and 2, and Blom (1970, Chapter 9.

A queue forms when demand for service is greater than servicing capacity. For example, if it takes one minute for a bank teller, which works alone, to service one customer, and if the number of customers arriving every minute during the day is random, a queue will form, if in a given minute there happens to be more than one arrival. Queue theory deals with the queue characteristics of this type of situation. It gives answers to questions such as, what is the average number of customers in line, and what is the average waiting time in line? There is a large number of processes that can be described in terms of random arrivals combined with a limited capacity to service. There are also many different types of queues. The simplest case is the one line - one server setting as in the example above. Other types of queues are one line - - multiple servers and multiple lines - multiple servers, for example. In the next section I discuss trade credit in terms of a simple one line - one server model. One line represents the line of trade credit bills being serviced and awaiting service within the firm, and the internal transaction system taken as a unit is given the role of the single server. Trade credit can also be fitted into more complex queue systems, for example the multiple lines - multiple servers case, but this is not the right place to delve too deeply into queue theory. The simple one line - one server model suffices to describe the basic payment delay processes I have in mind. Next I present the one line - one server model and some queue formulas it gives rise to. The formulas are stated without proofs. Detailed proofs are given in Panico (1969) Section 3.7.2. First some new notation is needed:

μ = mean service rate - average number serviced in one unit of time,

λ = mean arrival rate - average number arriving in one unit of time,

$\overline{t}$ = expected time in system - expected time in line plus expected time required for service,

$\overline{n}$ = expected number being serviced plus waiting.

The size of a queue is governed by two processes, both of which are assumed to be stochastic when those who arrive stay in an ordered line, regardless of the length of the line or the foreseeable time in line. The first one is the arrival pattern, and the second one is the service pattern. Assume that the arrival pattern can be described by a discrete stochastic process in continuous time. That is, any time in the interval t+h - t there can be a discrete "jump". Such a "jump" is here interpreted as an arrival which is either serviced immediately or added to the line. Next assume further that the stochastic process has the following properties: 1) the probability of an arrival in different time intervals of equal length is constant, 2) the probability of an arrival during a time interval h is approximately proportional to h, provided that h is small, and 3) the probability of more than one arrival during any small interval h is approximately zero, when compared to the probability of a single change during that interval. Then it can be shown that the probability of x arrivals during the unit time interval is Poisson distributed with the probability density function

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad 0 \leqq x \leqq \infty, \tag{7.28}$$

and mean arrival rate λ. Thus, the stochastic process summarized by (7.28) describes the arrival pattern. Next the service time, or more generally, the time in the system, can either be fixed or stochastic. Fixed service time represents in the trade credit case, for example, a situation when all incoming bills are such that they require exactly the same time of processing. Variable service times represent the case when bills are of different types, for example, small or large payments, bills denominated in different currencies, and bills which are not in conformity with deliveries. Another way of looking at variable service times is to see it as a result of varying efficiency. Now in the special case when both the arrival rate and the service rate are Poisson distributed it can be shown that the expected total time in the system, time in line plus service time, is

(7.29) $\bar{t} = \frac{1}{\mu-\lambda}$,

and the expected number in the system, those being serviced plus those waiting, is

(7.30) $\bar{n} = \frac{\lambda}{\mu-\lambda}$.

These formulas hold, given the condition that

(7.31) $\frac{\lambda}{\mu} < 1$,

the average number arriving in one unit of time must be smaller than the average number being serviced.[9] An intuitive interpretation of this condition is that, if the inequality in (7.31) is reversed, during a unit time interval on average there will be more units entering the system than leaving it. Then two things can happen. Either the line grows without bound, which in most cases is a situation that does not make sense, or new units do not enter the line because it is considered too long. In the latter case the assumption about independent arrivals (assumption 2 above) does not hold, and some other queue model must be used instead. With this very short introduction to queue theory it is time to move back to the heart of the matter and discuss trade credit again.

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9 This ratio is sometimes called the Traffic Intensity or Clearing Ratio. Another term is the Erlang out of deference to the Danis engineer and mathematician A K Erlang, who was a pioneer investigator in congestion theory.

7.3.3 *Trade credit as a queue phenomenon*

Assume there is a firm that purchases goods and services in such a mixture that the Poisson assumptions about the inflow of bills and service hold. Think e.g. of a department store or a mail-order company, both buy a very large number of different items, which can give rise to a time-independent inflow of bills. The firm wishes to minimize the cost of operating its payment system when this procedure has no influence on goods purchases and the corresponding pattern of arriving bills. The relevant costs are the cost of having a given service/payment capacity and the cost of having a line of bills to be paid. To describe the cost of operating the payment system some more notation is needed:

c_c = the cost of having the capacity to pay one bill during the unit time period,

$\overline{S}$ = average purchase per bill,

i = the interest cost of trade credit during the unit time period.

Now, with this notation, and the notation given in the preceding section, the total average cost of operating the payment system during a time period can be written

$$(7.32) \qquad \overline{TC} = c_c\mu + i\,\overline{S}\,\overline{n}\ ,$$

where the first part of the rhs of (7.32) represents the average service cost, the service rate times the service cost, when I assume that the cost relationship is linear, and the second part is the interest cost. The one period average interest cost of one trade credit bill is $i\overline{S}$ and there are $\overline{n}$ bills in line and being serviced. To get this product I have assumed that the average purchase per bill is independent of the average number in line, $E(Sn) = E(S)E(n)$. Insertion of the formula for $\overline{n}$ from (7.30) gives the cost function

(7.32') $\overline{TC} = c_c\mu + i\overline{S}\,\frac{\lambda}{\mu-\lambda}$.

The decision variable of the firm is μ, the average number serviced in one unit of time, and the cost of operating the payment system is minimized when the first and second order conditions

(7.33) $c_c(\mu-\lambda)^2 - i\overline{S}\lambda = 0$,

(7.34) $\mu - \lambda > 0$,

hold. Thus, the clearing ratio condition in (7.31) is equivalent to the second order condition. Taking this inequality into account the solution to (7.33) is

(7.35) $\mu = \lambda + (\frac{i\overline{S}\lambda}{c_c})^{\frac{1}{2}}$,

and insertion of (7.35) in (7.29) gives the expected total time in the system, or with trade credit terminology, the average length of the credit period

(7.36) $\overline{t} = (\frac{c_c}{i\overline{S}\lambda})^{\frac{1}{2}}$.

Just as in the "saw tooth" pattern model, the length of the credit period increases with rising transaction costs and falls with increasing interest costs. When μ is known the average number of bills in the system is given by insertion of (7.35) in (7.30), and since average sales per bill is known the product of the two determines optimal average accounts payable

(7.37) $\overline{AP} = (\frac{c_c\overline{S}\lambda}{i})^{\frac{1}{2}}$.

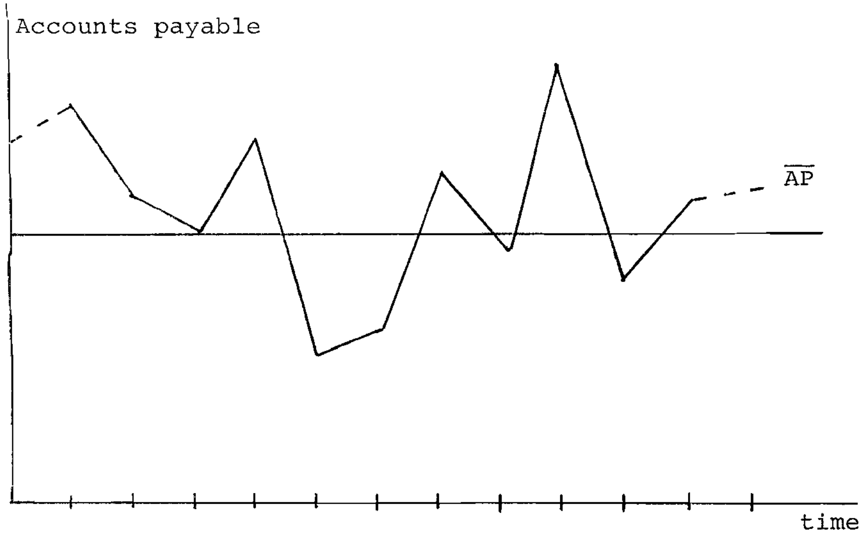

Figure 7.3 *A queue theoretic interpretation of accounts payable. The figure represents the development of accounts payable in an arbitrary time interval, when the payment system has been operating a long time.*

Hence, this approach also gives rise to a square root formula to describe average accounts payable.[10] However, here accounts payable will not show a systematic variation. In this respect the model is perhaps closer to reality than the "saw tooth" or systematic step functions in the fixed transaction cost model. In the deterministic transaction cost service time model AP was constant, for example $\overline{AP}$ in the figure above. With random arrivals and service times AP will vary around its mean and can also occasionally be equal to zero. The probability that there are no trade credit bills in the transaction system at

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[10] The formula in (7.37) above can also be written

$$\overline{AP} = \left(\frac{(2c_c)(\lambda\overline{S})}{2i}\right)^{\frac{1}{2}},$$

which is equivalent to the "saw tooth" formula, (7.3), if $2c_c = c$ (the fixed payment cost), when $\lambda\overline{S}$ represents daily purchases.

time t is given by the formula

$$(7.38) \qquad P(AP_t = 0) = 1 - \frac{\lambda}{\mu} ,$$

which, by using the formula in (7.30), can be written

$$(7.39) \qquad P(AP_t = 0) = \frac{1}{1+\bar{n}} .$$

The probability that accounts payable equals zero is given by the reciprocal of one plus the average number of bills in the transaction system.

$\overline{AP}$ in (7.37) has been derived under the assumption that the service times were random with mean μ. If the service time is fixed, it can be shown that

$$(7.40) \qquad \bar{n} = \frac{1}{2} \frac{\lambda}{\mu-\lambda} ,$$

and minimization with this formula inserted in the cost function gives average accounts payable equal to

$$(7.41) \qquad \overline{AP} = 2^{\frac{1}{2}} \left(\frac{c_c \bar{S} \lambda}{i}\right)^{\frac{1}{2}} ,$$

hence average accounts payable increases compared to the case with variable service times. The upshot of the results in (7.36) and (7.41) is that when payment costs rise with payment capacity there is a trade-off between payment costs and interest costs, and this trade-off can make it economical to choose a payment capacity such that there will form a line of trade credit bills.

Disregarding i_M and irrespective of whether the service time is constant or variable, the elasticities of average accounts payable with respect to i, $\bar{S}$, and λ are respectively

$$(7.42) \qquad \varepsilon_i = -\frac{1}{2}, \ \varepsilon_{\bar{S}} = \frac{1}{2}, \ \varepsilon_\lambda = \frac{1}{2} .$$

Thus, the three elasticities are constant, the interest rate elasticity is equal to the corresponding elasticity in the "saw tooth" model, and the quantity elasticity is less than one. Consequently in this case the size of average accounts payable does not rise linearly with an increase in the average number of bills arriving during the unit time period.

To represent the flow of bills through a firm I have used a single server - single line queue model, with a rate of interest independent of the average length of the credit period. In reality there are often more complex flows of bills. For example, from the date of arrival until the date of payment a bill can move through several different queues within the firm. Under certain conditions such a row of interdependent queues can be treated as a sum of several independent queues of the same type as the one described above.[11] I have further only discussed the buyer side of the market, but for reasons similar to the ones in the model above, the seller can also have incentives to separate the date when a product is sold and the date when the bill is sent to the buyer. In this case the payment cost is of the same type as above and the interest cost represents the opportunity cost of not receiving payment immediately. One example of such delays on the seller side is the following, which at the same time is a description of a more complicated queue structure than the one used above. For example, think of some small firm which sells plumbing services to a large number of different customers with random demand for service. Assume that in this firm there are only two types of queues. *First*, a queue of customers waiting for service, and *second*, a queue in terms of the paper work which goes together with each service occasion. The two queues are interdependent and if swift service is an important part of staying competitive, the queue with customers waiting to be served has priority over the queue with paper work.

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[11] For further details see Cox-Walter (1961), Section V.5.

In such a situation the paper work can be attended to during periods when there are no customers in line. On average, such a priority policy gives rise to a separation of the date when a service occasion is completed and the date when the bill is sent to the customer. *In summary*, one can thus conclude that when demand is stochastic, and when the costs of making payments vary with the capacity of the payment system, these costs together with interest costs generate a float of bills between completed service by the seller and payment by the buyer. Consequently, with this view of the world, the payment system can be seen as a large web of interdependent queues, with irregular flows of bills under way towards payment.

So far I have presented three different types of models that determine the optimal size of accounts payable in a goods market transaction cost context. The models can be called the "saw tooth", the non-stochastic queue, and the stochastic queue model. One common property of these is that they yield accounts payable formulas with a specific functional form. To test whether these formulas also get empirical support is a topic for future study. Before leaving the goods market transaction cost aspects of trade credit let me finally comment on what the models predict about the connection between firm size and the size of accounts payable. In Chapter 1 my stylized facts showed that large firms (large in terms of the number of employees) use less trade credit than small firms, when the use of trade credit is measured as the ratio of accounts payable to purchases. (See Table 1.3, page 36.) With the same notation as above the ratio of accounts payable to purchases over the planning horizon can be written

(7.43) "saw tooth" model, $\frac{\overline{AP}}{S\tau} = \frac{1}{\tau}\left(\frac{c}{2(i-i_M)S}\right)^{\frac{1}{2}}$,

(7.44) non-stochstic queue, $\frac{AP}{S\tau} = \frac{\mu}{\tau}\left(\frac{C}{(i-i_M)S}\right)^{\frac{1}{2}}$,

(7.45) stochastic queue, $\frac{\overline{AP}}{S\tau} = \frac{1}{\tau}\left(\frac{c_c{}^{\lambda}}{(i-i_M)S}\right)^{\frac{1}{2}}$, where I have added the i_M variable in the queue models. The question is then how firm size is to be treated in this context. In the "saw tooth" model firm size can be represented by S, the value of daily purchases. Thus, it is clear from (7.43) that the trade credit ratio will be falling. In the queue models firm size can be represented both by the number of bills arriving during the unit time period, μ and λ, and by the value of arriving bills, S and $\bar{S}$. When the variables increase the ratio is in this case falling if $\mu < S^{\frac{1}{2}}$, and $\lambda < \bar{S}$, in (7.44) and (7.45) respectively. Consequently there will be a falling trade credit ratio only if the quantity effect is smaller than the price effect. In reality there are also other factors that make a comparison of firms of different size more difficult than in the formulas given above. *First*, if increasing firm size is combined with an increasing number of identical transaction units (the payment units are not centralized) the trade credit ratio will be constant. *Second*, my comments above rest on the assumption that the interest rate structure is the same among large and small firms. If this is not the case there will be some firms where the goods market transaction aspects dominate and other firms where the financial motive to use trade credit is most important, and this will make it difficult to compare "pure" size effects. An interesting empirical issue is then to try to determine if it is the goods market transaction cost effects, the way they have been described in this chapter, or the fact that small firms often are discriminated in the financial market that give rise to a falling ratio of accounts payable to purchases.

7.4 TRADE CREDIT AND THE COST OF MAKING FINANCIAL TRANSACTIONS

In order to explain the use of trade credit in a transaction cost context I have so far dealt with various types of goods market transaction costs. The financial market has been in the background. In this section I bring the financial market into focus and I show how fixed financial market transaction costs affect the use of trade credit.[12] Here I do not take goods market transaction costs of the type discussed earlier into account. This separation of transaction costs is not necessarily in line with reality, but it simplifies the exposition and makes it possible to concentrate on the different trade credit aspects one at a time. A future challenge is to construct models which simultaneously encompass several different financial and goods market aspects of trade credit. The two main topics of this section have some similarities with the discussion in parts 7.2 and 7.3. *First*, I use an extension of the classical transaction demand for money approach to explain the use of trade credit in a steady rate of consumption, fixed financial transaction cost context. *Then*, when there is a stochastic cash flow, I show how trade credit can be used as a buffer to strike a balance between transaction costs and money shortage costs.

An important difference between this and the goods market section is that in the goods market transaction cost case trade credit produced a unique service by separating goods transfers from money transfers, but when there are only financial market transaction costs there are also other types of credit which can fill the same function as trade credit. Thus, when trade credit is seen solely in a financial market context there are close substitutes, while this is not so in the goods market transaction cost case.

7.4.1 *Fixed transaction costs - the certainty case*

In this section I use an inventory theoretic approach to show how fixed financial transaction costs can give rise to a demand for trade credit. I only take the demand side of the market into account; the trade credit rate of interest is assumed to be such

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12 I call the cost of switching from one asset to another, for example from bonds to money, a financial market transaction cost. The cost of making goods payments, from money to goods, is a goods market transaction cost.

that the supplier of trade credit is properly compensated, and I only discuss trade credit with post payment.

Assume that there is a world with fixed transaction costs when financial assets are transferred into money, only money buys goods, and it costs nothing to administer payments when there is a money to goods transfer. Assume further that there is a firm which over some planning period buys a constant stream of goods or services and that the firm at the beginning of the planning period has enough assets to finance these purchases. The assets are interest bearing and there is no interest on money holdings. Thus, the firm operates in a world represented by the assumptions needed to derive the classical "saw tooth" pattern of the transactions demand for money. The payment system sketched so far includes two types of costs. The asset transaction cost, and the opportunity cost of holding money. Cost minimization gives rise to stepwise falling asset holdings combined with a "saw tooth" pattern of money holdings. Now add the possibility that when money balances are zero the firm can continue its steady rate of purchases by accumulating trade debt (instead of making an asset transfer). When trade credit includes interest payments there is an additional cost trade-off to be taken into account. Regardless of whether the firm holds money or not post payment reduces asset transaction costs by reducing the number of asset conversions. This cost reduction then has to be weighted against the interest cost of trade credit. The objective of the firm is to minimize the cost of making payments over the planning period. To present this problem in terms of a cost function I first define the variables in the model below, and I give a graphic interpretation of the money holding/trade credit pattern I have in mind. Most of the notation is familiar from the preceding sections:

S = the daily rate of consumption of the goods or services purchased by the buyer,

C_a = a fixed asset market transaction cost,

i = the daily trade credit rate of interest,

i_a = the daily rate of interest on some financial asset,

t_1 = the length of one money holding cycle,

t_2 = the length of one trade credit cycle,

τ = the length of the planning period.

With these assumptions the size of money holdings and accounts payable during two consecutive cycles is depicted in Figure 7.4. below. The model approach is borrowed from inventory theory. The inventory theoretic counterpart to the payments in the figure is a lot size model with backlogging.[13] To state a cost function which can be used to determine a cost minimizing combination of t_1 and t_2 I first derive the different costs during one cycle. Before I do this let me add a final important model assumption. All costs are treated as financial charges that are incurred outside the model. Trade credit interest payments and transaction costs do not reduce interest earning assets during the planning period.

The opportunity cost of holding money during t_1 is the interest foregone on asset holdings. With a continuous liquidation of asset the interest revenue on a decreasing stock of assets of initial size St_1 is

$$(7.46) \qquad \int_0^{t_1} i_a S\Theta d\Theta = \frac{i_a S t_1^2}{2} .$$

This is the interest foregone on holding St_1 of money during one cycle. The cost of using trade credit during t_2 is

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[13] See, for example, Churchman, Ackoff and Arnoff (1957), Chapter 8.

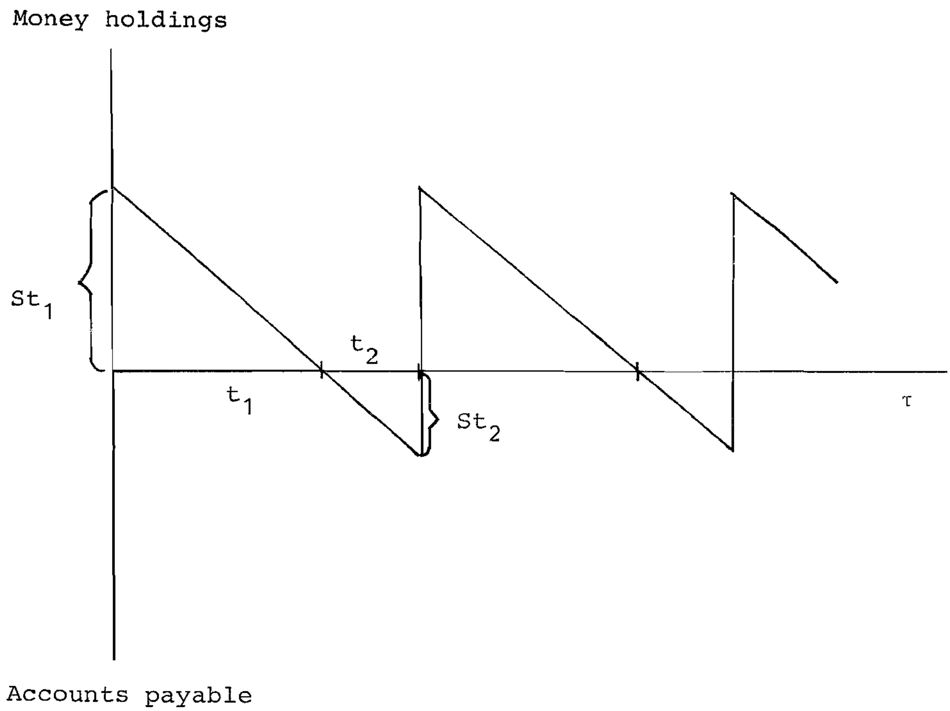

Figure 7.4 Money holdings and accounts payable with fixed financial market transaction costs.

$$\int_0^{t_2} iS\theta d\theta = \frac{iSt_2^2}{2} . \qquad (7.47)$$

However, the use of trade credit also includes a revenue part. During a trade credit cycle there are no asset transfers. Thus, on the one hand, there is an interest revenue from St_2 worth of assets during t_2 periods. If, on the other hand, the firm does not use trade credit, and it does not hold money, there is a continuous liquidation of assets and the interest revenue is given by an equation similar to (7.47). Therefore, the extra interest revenue due to the use of trade credit is the difference between the interest on assets when there is no liquidation and when there is a continuous liquidation during t_2. This difference is

$$(7.48) \qquad i_a S t_2^2 - \int_0^{t_2} i_a S \Theta d\Theta = \frac{i_a S t_2^2}{2} .$$

Finally, the cost of making an asset transfer at the beginning of a cycle is c. This means that when there are $\tau/(t_1+t_2)$ cycles the cost of operating the payment system becomes

$$(7.49) \qquad TC = \frac{i_a S t_1^2}{2} \frac{\tau}{t_1+t_2} + \frac{(i-i_a) S t_2^2}{2} \frac{\tau}{t_1+t_2} + c_a \frac{\tau}{t_1+t_2} .$$

Note that when t_2 is equal to zero the cost function is the same as Baumol's (1952) classical transactions demand for money model. An alternative way of deriving the cost function in (7.49) is to discuss in terms of averages. With this approach the opportunity cost of holding money can be explained in the following way: the average money holding during t_1 is $St_1/2$, thus the average money holding during one cycle is

$$(7.50) \qquad \frac{St_1}{2} \frac{t_1}{t_1+t_2} ,$$

and since there are τ days during the planning period, the interest loss on average money holdings is given by the first expression in (7.49). The remaining parts of (7.49) can be interpreted similarly.

The basic structure of the model set forth above is the same as that of a model used by Sastry (1970). However, the model presented here is both an extension and a reformulation of the Sastry model. *First*, Sastry only uses his model to discuss the size of the interest elasticity of the transactions demand for money. There are no comments about the size of the outstanding debt and its various elasticities. *Second*, the cost of accumulating debt during t_2 is presented as a general shortage cost and the fact that the use of credit generates an extra interest revenue is not taken into account. To derive a cost function of a type similar to (7.49) Sastry discusses in terms of average costs. In a critique of the Sastry model, Wrightsman and Terninko (1970) argue that an analysis based on a cost function such as (7.49) is in error, because the interest loss on average cash balances overstates the true opportunity cost of holding money. If their critique is correct (7.49) is also a badly stated cost function. However, I want to argue that their assertion can be refuted. Before I discuss the trade credit aspects of the model I make a short digression to argue why (7.49) correctly measures the opportunity cost of holding money.

To simplify I only discuss a case when the planning period includes one cycle. It can also be shown, however, that the arguments carry over to the n cycle case. To use the terminology of Wrightsman and Terninko (W-T) define the true opportunity cost of holding money as the interest earnings differential between holding no money and holding X SEK. Assume that the firm starts with $S(t_1+t_2)$ worth of assets, the money holding period is t_1, the credit period is t_2 and $\tau = t_1 + t_2$. Then W-T argue that the true opportunity cost is

$$(7.51) \qquad \frac{i_a S(t_1+t_2)^2}{2} - i_a S t_2(t_1+t_2) = \frac{i_a S t_1^2}{2} - \frac{i_a S t_2^2}{2} .$$

The first part of the expression represents the interest revenue when there is a steady outflow of assets and the second part is the interest revenue when t_1 worth of assets is transferred into money at the beginning of the planning period. The opportunity cost, the way it is stated in (7.51), is lower than the interest loss on average cash holdings which is

$$(7.52) \qquad \frac{i_a S t_1^2}{2}$$

with $\tau = t_1 + t_2$. I want to argue, however, that the expression in (7.51) is not a correct measuring rod of opportunity cost, bacause when the firm accumulates debt there is no outflow of assets. When this is taken into account the true opportunity cost can be written

$$(7.53) \qquad \left[\frac{i_a S t_1^2}{2} + i_a S t_2 (t_1+t_2)\right] - i_a S t_2 (t_1+t_2) = \frac{i_a S t_1^2}{2} ,$$

where the bracketed expression represents the case when no money is held. The opportunity cost is thus equal to the interest loss on the average cash balance over the planning period. By using averages Sastry does not overstate the opportunity cost, W-T understate it.

7.4.1.1 Cost minimization, the size of accounts payable and elasticities.

In order to derive the optimal size of accounts payable the cost function in (7.49) has to be minimized. This can be done in two different ways. One alternative is to restate the model in terms of conversion sizes St_1 and $S(t_1+t_2)$, and by the use of the properties of proportional triangles derive a new cost function, which then is minimized. This is the method used by Churchman et al. (1957).[14] The other alternative is to minimize the cost function as it stands, and since t_1 and t_2 are my decision variables, this is the method used here. Differentiation of (7.49) with respect to t_1 and t_2 gives the two first order conditions

$$(7.54) \qquad TC'_{t_1} = \frac{i_a S\tau}{2} \left(\frac{2t_1(t_1+t_2)-t_1^2}{(t_1+t_2)^2}\right) - \left(\frac{(i-i_a)S\tau}{2}\right) \frac{t_2^2}{(t_1+t_2)^2}$$

$$- c_a \frac{\tau}{(t_1+t_2)^2} = 0 ,$$

$$(7.55) \qquad TC'_{t_2} = -\left(\frac{i_a S\tau}{2}\right) \frac{t_1^2}{(t_1+t_2)^2} + \left(\frac{(i-i_a)S\tau}{2}\right) \left(\frac{2t_2(t_1+t_2)-t_2^2}{(t_1+t_2)^2}\right)$$

$$- c_a \frac{\tau}{(t_1+t_2)^2} = 0.$$

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[14] See Chapter 8, note 2 for details.

This is a non-linear equation system with two unknowns. After considerable manipulation it can be shown that there exists a solution[15]

$$(7.56) \qquad t_1 = \left(\frac{2c_a}{S}\right)^{\frac{1}{2}} \left(\frac{i-i_a}{ii_a}\right)^{\frac{1}{2}} ,$$

$$(7.57) \qquad t_2 = \left(\frac{2c_a}{S}\right)^{\frac{1}{2}} \left(\frac{i_a}{i(i-i_a)}\right)^{\frac{1}{2}} ,$$

and from this it follows that the average size of accounts receivable during t_2 and the average money holding during t_1 is respectively

$$(7.58) \qquad \overline{AP}_{t_2} = \frac{St_2}{2} = \left(\frac{Sc_a}{2}\right)^{\frac{1}{2}} \left(\frac{i_a}{i(i-i_a)}\right)^{\frac{1}{2}} ,$$

$$(7.59) \qquad \overline{M}_{t_1} = \frac{St_1}{2} = \left(\frac{Sc_a}{2}\right)^{\frac{1}{2}} \left(\frac{i-i_a}{ii_a}\right)^{\frac{1}{2}} .$$

However, these are not the long run averages which show average accounts payable and money holdings over the planning period. The weights $t_2/(t_1+t_2)$ and $t_1/(t_1+t_2)$ are missing. To derive the long run averages, and the elasticities that go with them, I first rewrite the trade credit and money holding proportions of an asset transfer cycle in terms of ratios of interest rates. In Appendix II I show that the trade credit and money holding proportions of one cycle can be written

$$(7.60) \qquad \frac{t_2}{t_1+t_2} = \frac{i_a}{i} , \quad \frac{t_1}{t_1+t_2} = 1 - \frac{i_a}{i} .$$

The ratio $t_2/(t_1+t_2)$ is equal to the ratio of the asset rate of interest to the trade credit rate of interest. From this it follows that if i becomes infinitely large t_2 will be equal to zero. I have earlier pointed out that when t_2 is equal to zero the model is equivalent to Baumol's (1952) model.

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[15] See Appendix II, where I also prove that the second order conditions hold if $i > i_a$.

One way to interpret his "saw tooth" pattern of the transactions demand for money is to see it as a special case, when the cost of running out of cash is infinitely high. If, on the other hand, $i = i_a$, it is t_1 which is equal to zero. In this case it costs nothing to use trade credit, while there still is an opportunity cost of holding money, and since it is asset transactions which give rise to transaction costs the optimal policy is to carry out all asset and payment transactions at the end of the planning period. Furthermore, if $i < i_a$, when the interest rates are independent of t_2 and t_1, the firm makes a profit from interest arbitrage and the optimal policy is to pay at $t = \tau$. In this case the transaction cost interest cost trade-off no longer exists and the transaction cost model breaks down. The interest arbitrage aspects dominate. This is also the reason why the trade credit rate of interest must be higher than the asset rate of interest if the cost function in (7.49) is to have a unique minimum. (The formal way of proving this is given in Appendix II, where it is shown that the second order conditions hold if $i > i_a$.) Hence, just as in the goods market transaction cost case, the financial tansactions motive is one way to justify the use of trade credit among firms which have a borrowing rate exceeding the lending rate. Note, however, that there is an important difference between goods market transaction cost trade credit and the type of credit discussed in this section. In the goods market case trade credit produced a unique service, while here other types of short term loans can fill the same function.

Finally, when the expressions for accounts payable and money holdings in (7.58) and (7.59) are multiplied by their respective cycle proportions in (7.60), long run average accounts payable and money holdings are given by

$$(7.61) \qquad \overline{AP} = \overline{AP}_{t_2} \frac{t_2}{t_1+t_2} = \left(\frac{Sc_a}{2}\right)^{\frac{1}{2}} \left(\frac{i_a^{\frac{3}{2}}}{i^{\frac{3}{2}}(i-i_a)^{\frac{1}{2}}}\right),$$

$$(7.62) \qquad \overline{M} = \overline{M}_{t_1} \frac{t_1}{t_1+t_2} = \left(\frac{Sc_a}{2i_a}\right)^{\frac{1}{2}} \left(1 - \frac{i_a}{i}\right)^{\frac{3}{2}} .$$

Just as in the models in the preceding sections there is a square root relationship between daily purchases, the transaction cost and accounts payable. In addition to this, accounts payable depends on the interest rates in a rather complicated way. The money holding expression is the familiar square root formula multiplied by an interest term. Note also that, in accordance with the comments about the Baumol (1952) model, (7.62) is identical to the square root formula when the trade credit rate of interest goes towards infinity, and no money is held when i equals i_a. With a trade credit interest rate in the interval $i_a < i < \infty$ it is evident from (7.62) that an introduction of trade credit reduces average money holdings compared with the original square root formula. Consequently, this is a minimization of transaction costs verification of Fisher's (1931) and Wicksell's (1898) assertions that trade credit increases the velocity of circulation, which was the starting point of my discussion about various trade credit dimensions in Chapter 1.

Both $\overline{AP}$ and $\overline{M}$ have an elasticity with respect to daily purchases equal to one half. The elasticities of accounts payable with respect to the asset and the trade credit rate of interest are

$$(7.63) \qquad \varepsilon_{i_a\overline{AP}} = \frac{3}{2} + \frac{1}{2}\frac{i_a}{i-i_a} ,$$

$$(7.64) \qquad \varepsilon_{i\overline{AP}} = -\frac{3}{2} - \frac{1}{2}\frac{i}{i-i_a} ,$$

where all calculations that these and the following formulas in this section are based on are presented in Appendix II. Since $i > i_a$, both elasticities are in absolute value greater than 3/2. Thus, the transactions demand for trade credit is in this context quite interest rate elastic. In addition to this the elasticities are not constants but depend on the interest rates. The closer i is to i_a the higher are the elasticities. It further follows that the elasticity with respect to the trade credit rate of interest is always greater than the elasticity with respect to the asset return. In the section about goods market transaction costs I used a numerical example based on the assumption that $i = 2i_a$. With this assumption the elasticities are

$$(7.65) \qquad \varepsilon_{i_a \overline{AP}} = 2 \ , \quad \varepsilon_{i\overline{AP}} = 2.5 \ ,$$

which is considerably higher than in the goods market transaction cost case. The transactions demand for money elasticities corresponding to (7.63) and (7.64) are

$$(7.66) \qquad \varepsilon_{i_a \overline{M}} = -\frac{1}{2} - \frac{3}{2}\,\frac{i_a}{i-i_a} \ ,$$

$$(7.67) \qquad \varepsilon_{i\overline{M}} = \frac{3}{2}\,\frac{i_a}{i-i_a} \ .$$

From (7.66) it is clear that the asset rate of interest elasticity always will have an absolute value greater than one half, which is the corresponding value in the Baumol model. The elasticity in (7.66) is also always greater in absolute value than the elasticity with respect to the trade credit rate of interest. With the assumption that $i = 2i_a$, as in the other examples, the elasticities have the numerical values

$$(7.68) \qquad \varepsilon_{i_a \overline{M}} = -2 \ , \quad \varepsilon_{i\overline{M}} = 1.5 \ .$$

The elasticity with respect to the asset rate of interest is thus in this case considerably higher than one half. Whether the elasticities given above are in accordance with the true behavior of firms or individuals is an open question left for future research.

Finally, let me comment on an interesting asymmetry property of the model. In Chapter 2, where I discussed financial or interest arbitrage aspects of trade credit, I showed that an increase in the general level of interest rates left the length of the credit period unchanged if both the interest functions of seller and buyer shifted in the same way. Similarly, in the chapter about unknown interest rates, I showed that a general increase in interest rates, in terms of a shift of a normal distribution of interest rates, left the behavior of the seller unchanged when the position of the interest rate of the seller was maintained in the distribution. Thus, if an increase in the general level of interest rates was to affect the use of trade credit, it was essential that such an increase spread unevenly throughout the economy. In the transactions demand for money and trade credit model above, shifts in either the asset or the trade credit rate of interest will, of course, affect the size of accounts payable, but the model does not have the same symmetry property as the models in the chapters about financial trade credit. Assume that there is a simultaneous and equal increase of the asset and the trade credit rates of interest, $di = di_a$. Then it can be shown that the change of accounts payable in (7.61) and money holdings in (7.62) can be written

$$(7.69) \qquad \frac{d\overline{AP}}{di}\Big/_{di=di_a} = \frac{3}{2}\left(\frac{Sc_a}{2}\right)^{\frac{1}{2}} i_a^{\frac{1}{2}}\, i^{-\frac{5}{2}} (i-i_a)^{\frac{1}{2}} > 0 ,$$

$$(7.70) \qquad \frac{d\overline{M}}{di}\Big/_{di=di_a} = -\frac{3}{2}\left(\frac{Sc_a}{2}\right)^{\frac{1}{2}} (i-i_a)^{\frac{3}{2}}\left(\frac{1}{3} + \left(\frac{i}{i_a}\right)^{\frac{1}{2}}\right) i^{-3} < 0 .$$

An increase in the general interest level increases accounts payable and reduces money holdings. This also implies that t_1 falls and t_2 rises. Consequently, a traditional through interest rates working monetary policy affects both the transactions demand for money and trade credit. The re-distribution from money to trade credit is a result of the fact that when $di = di_a$, the cost of using trade credit is left unchanged, while the opportunity cost of holding money increases. This uneven cost increase works in favor of an increased use of trade credit. This result stems from the fact that no interest is payed on money holdings. If money earns interest a general increase in the level of interest rates keeps the margin between different interest rates constant and the effects in (7.69) and (7.70) are eliminated.

7.4.2 *Stochastic cash flows*

In the preceding section I used a model originally designed to explain the transactions demand for money. Here I also use a demand for money model to show how trade credit can function as a buffer in order to reduce financial transaction costs when payments are stochastic. To do this I use a precautionary demand for money or bank reserve model, and instead of concentrating on precautionary money holdings I discuss the case when payments exceed cash immediately on hand. Just as before I only take the demand side of the market into account.

When net disbursements are not known with certainty the financial managers of a firm must decide how much money to hold in order to meet payment obligations within some given time period. If there is an opportunity cost of holding money and a penalty cost if disbursements exceed cash on hand there is a trade-off between interest foregone on money holdings and the penalty cost, where the latter can be interpreted as the cost of making unforeseen transfers of assets into money. In this context that trade credit can be one way to reduce the cost of making payments. This assertion rests on the assumption that by postponing payment temporarily the cost of making an asset transfer is lowered, when it cost more to make sudden unplanned than planned asset transfers. By using short term credit there is more time to gather information and to plan what financial transactions have to be made. Trade credit, and other types of short term borrowing, can in a world of uncertainty, of course not, eliminate stochastic disbursements but it eliminates the uncertainty about when payments have to be made. By separating tha date when bills are received from the date when bills are paid the firm is temporarily sheltered from the need to make unknown asset transfers and it is given lee-way to plan its financial transactions. Trade credit is not the only asset that fills this function, but the use of trade credit lies close at hand since in many cases the granting of trade credit is more or less automatic and a payment date which changes within a given time interval requires no extra negotiations. Trade credit can thus reduce the cost of making asset transfers but this does not necessarily mean that the demand for precautionary transactions balances is completely eliminated. There is still a trade-off between the opportunity cost of holding money and the cost of trade credit combined with the cost of making asset transfers at the end of the credit period. One situation when trade credit eliminates all uncertainty about payment is when constant daily orders of goods are combined with random transportation times between buyer and seller. This case has recently been studied in an interesting article by Ferris (1981). Below I give a simplified graphical interpretation of the mathematical model set forth by Ferris. Besides the mathematical model the article by Ferris has on a more general level provided food for thought about trade credit and financial transaction costs in an uncertainty context.

7.4.2.1 The Ferris approach

Assume that firm A delivers one unit of output a day to firm B, which uses one unit a day of the good in its production process. Assume further that the transprotation time between A and B is random. It takes 1-7 days to send one unit from A to B. Then during an isolated two week period one possible arrival pattern is given in Figure 7.5, where it is assumed that shipments are made only during the first week. With random arrivals firm B never knows with certainty how much of the input is available and when payments have to be made. If both running out of input and not having enough cash to make payments give rise to shortage costs (the cost of not being able to maintain a steady rate of production, and the cost of having to make unforeseen asset transfers) the firm has cost incentives to hold buffer stocks of both money and goods. Firm A will also have a random inflow of funds which can give rise to a need to hold precautionary cash balances. Now assume that instead of making cash payments firm B uses trade credit and informs A that payment will be made at the end of the two week period. This does, of course not, eliminate the need to hold a buffer stock of goods, but neither A or B need to hold precautionary cash balances if random transportation times between A nd B is the only type of uncertainty. Both the size and the time profile of the money flow between the two firms is known with certainty. (See the dotted pile in the figure. Fixed goods market payment costs, which Ferris do not take into account, will also contribute to such a payment pattern.) In this special case trade credit provides reduction of uncertainty benefits both to seller and buyer. This is due to the combination of a deterministic sale/purchase pattern with random transportation times. However, the reduction of uncertainty effect is often likely to lie solely on the buyer side of the market. In the example above the seller has information about the payment behavior of the buyer but in most cases the seller probably has no such exact information about when the buyer intends to pay. When this information is lacking it is only the buyer that can use trade credit to create new information in order to reduce financial transaction costs. In this case other types of short term borrowing can also fill the same buffer function as trade credit. In my brief presentation of payment uncertainty above the introduction of trade credit eliminated the demand for precautionary money holdings completely. Next I discuss the use of trade credit as a buffer in terms of a model where it can be optimal both to hold precautionary reserves and to use trade credit.

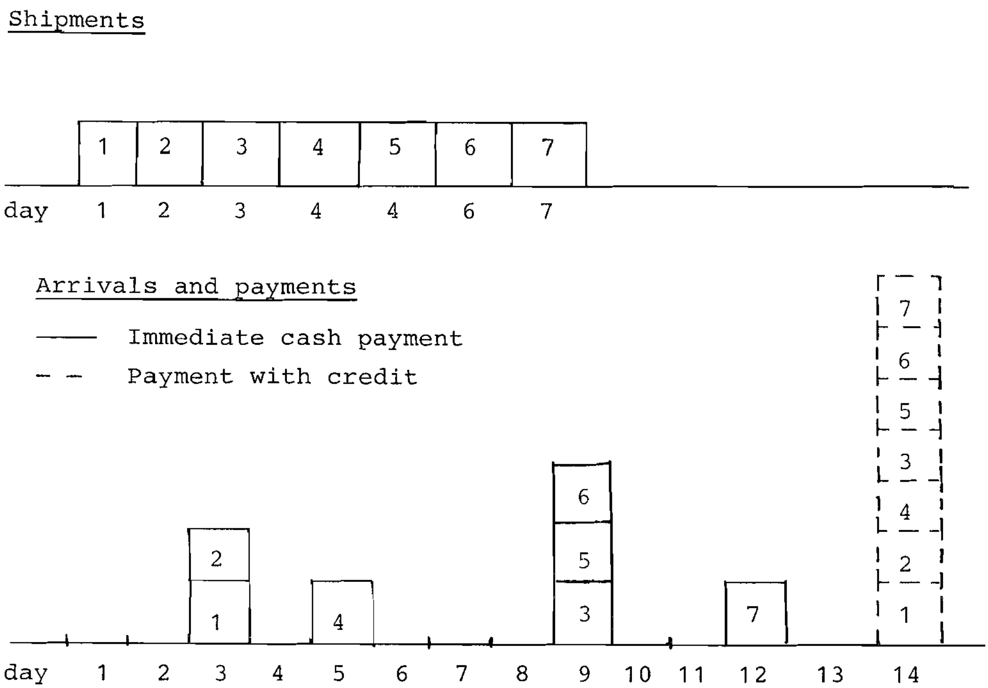

Figure 7.5 The Ferris approach. A hypothetical stochastic payment pattern during an isolated two week period with constant sales and random transportation times.

7.4.2.2 Trade credit, financial transaction costs and stochastic payments

Most firms have a mixture of largely deterministic and stochastic payments. One way to look at the discussion presented below is to see it as a sub-model, describing that part of accounts payable which is due to irregular payments, while the size of accounts payable due to regular payments has been determined elsewhere. Assume that the financial manager of a firm knows that in the time interval $0 - \tau$ there will be one net disbursement of unknown size. (I define a net disbursement as payments minus receipts, thus, a negative net disbursement is interpreted as a net inflow of funds.) When this date occurs, and if there is a positive net disbursement, the firm can either use cash on hand plus possible asset transfers to pay immediately, or if there is a shortage of cash, the firm can use trade credit to postpone necessary asset transfers until τ. One motive to clear payments a τ instead of some time in the interval $0 - \tau$ is that the transfer date is known with certainty and the firm is given time to plan its transactions. Furthermore then the firm knows the exact size of the transaction in advance. In the light of the discussion about the information producing aspects of using trade credit, or some other type of short terms borrowing, it is then reasonable to assume that the transaction cost of transferring assets into money is higher some time within the interval $0-\tau$ than at τ. In order to express these ideas in terms of transaction cost functions the following notation is needed:

M = money holdings,

$\tilde{D}$ = stochastic net disbursements,

$f(\tilde{D})$ = the probability density function of net disbursements,

$\bar{t}$ = the average time until a no trade credit disbursement occurs, the distribution function is assumed to be such that $g(t=\tau) = 0$,

τ = the date when a trade credit debt is repaid,

$c_{\bar{t}}$ = the cost of making an asset transfer within the interval,

c_{τ} = the cost of making an asset transfer at τ, $c_{\bar{t}} > c_{\tau}$,

i = the trade credit rate of interest,

i_a = the opportunity cost of holding money.

Assume further that the distribution of payment dates is independent of the distribution of net disbursements. Then the expected cost of making payments without trade credit can be written

$$(7.71) \qquad E(TC_M) = i_a \bar{t}M + c_{\bar{t}} \int_M^{\infty} f(\tilde{D})\,d\tilde{D} \ ,$$

where the firt term is the opportunity cost of holding money until the payment date and the second term is the expected asset transaction cost. When the firm uses trade credit the expected cost of making payments is

$$(7.72) \qquad E(TC_{tc}) = i_a\bar{t}M + c_{\tau} \int_M^{\infty} f(\tilde{D})d\tilde{D} + (i-i_a)(\tau-\bar{t}) \int_M^{\infty} (\tilde{D}-M)f(\tilde{D})d\tilde{D} \ ,$$

where again the first term is the opportunity cost of holding money, the second term is the expected transaction cost at τ, and the third term represent the expected interest cost of using trade credit. This cost function is of the same type as a cost function set forth in an Appendix to Whalen (1966). Whalen discusses the precautionary demand for reserves and with his terminology (7.72) represents: "A model with constant and proportional costs of illiquidity". The asset transaction cost term is the constant cost and the trade credit interest payment represents the variable cost term because it depends on the difference $\tilde{D}$-M. This type of model formulation is also common within the theory of the banking firm.[16] The cost functions in (7.71) and (7.72) rest on an implicit assumption, which is mostly not commented on by reserve theorists. A net inflow or an outflow of cash lower than M($\tilde{D}$<M) does not give rise to additional transaction and interest opportunity costs. Such costs can without greater difficulty be added to the model, but the objective function in (7.72) is sufficient to discuss the use of trade credit as a payment buffer. Note that in the special case when $\tilde{D}$ is known trade credit eliminates all payment uncertainty in the same way as in Figure 7.5. When the firm can choose between the two payment alternatives trade credit is preferred if

$$(7.73) \qquad (c_{\bar{t}} - c_{\tau}) \int_M^{\infty} f(\tilde{D})d\tilde{D} - (i-i_a)(\tau-\bar{t}) \int_M^{\infty} (\tilde{D}-M)f(\tilde{D})d\tilde{D} > 0 \ ,$$

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16 See, for example, Baltensperger (1980), Section 2.

when (7.73) represents (7.71) minus (7.72). When $i \leq i_a$ trade credit is always preferred bacause the use of trade credit does not represent an additional cost and the transaction cost is reduced compared to the cash payment alternative. It is first when $i > i_a$, and i is sufficiently large to neutralize the effect of a reduced transaction cost that cash payment is favorable. In order to reduce transaction costs the firm will consequently accept trade credit also when $i > i_a$ which is in accordance with the conclusions drawn from the other transaction cost models in this chapter.

Now assume that $i > i_a$ and that it is more favorable to use trade credit than full cash payment. Then it remains to determine what the cost minimizing size of average accounts payalbe will be. This is done in an indirect way by first determining the optimal size of money holdings up to the average payment date. The use of trade credit does not necessarily mean that payment costs are minimized when the firm holds no cash balances at all. In the cost function given in (7.72) the size of money holdings is the only decision variable. Two extensions is to include an information function which affects the variability of net disbursements and more than one asset that can be transferred into money. These extensions do not change the basic structure of the model, and in order to derive a relatively simple and explicit cost minimizing expression of average accounts payable I have chosen to omit such extensions.[17] The expected payment cost in (7.72) is minimized when M is chosen in such a way that

$$(7.74) \qquad i_a\bar{t} = c_\tau \; f(M) + (i-i_a)(\tau-\bar{t})\int_M^\infty f(\tilde{D})d\tilde{D}$$

holds.[18] The marginal interest gain of decreasing money holdings has to equal the marginal transaction cost. Equation (7.74) determines the optimal money holding and when this is known the

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17 For a detailed discussion about how such models can be constructed see Baltensperger (1974) and Baltensperger and Milde (1976).

18 See Appendix III.

size of average accounts payable is also indirectly determined. To get further, to derive an analytical expression for average accounts payable, it is necessary to have some information about the probability density function $f(\tilde{D})$. Money holdings are either zero or positive. Therefore it is only the positive part of the distribution which has to be taken into account. Furthermore in order to derive average accounts payable the distribution must be such that the integral in (7.74) is solvable. Such distributions are not necessarily in line with net disbursements in reality and the analysis that follows can best be seen as a relatively simple example of how the model works.

One distribution with the desired proporty is the exponential. The probability of a positive net disbursement of size $\tilde{D}$ is given by the function

$$(7.75) \qquad f(\tilde{D}) = \frac{1}{\alpha}\frac{1}{D}e^{-\frac{\tilde{D}}{D}}, \quad \tilde{D} \geq 0, \quad \alpha \geq 1,$$

which is based on the assumption that the probability of a positive net disbursement is equal to $1/\alpha$. Note that $E(\tilde{D})$ is equal to D with density function $(1/D)\exp(-\tilde{D}/D)$. The constant D in (7.72) is consequently the expected value of positive net disbursements. When (7.75) is inserted in (7.74), and the integral is solved, the first order condition is [19]

$$(7.76) \qquad i_a\bar{t} - \frac{1}{\alpha}e^{-\frac{M}{D}}((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}) = 0,$$

and solving for M the demand for money can be written

$$(7.77) \qquad M_{tc} = -D\ln\left(\frac{\alpha i_a\bar{t}}{(i-i_a)(\tau-\bar{t}) + c_\tau/D}\right).$$

From (7.77) it is clear that money holdings will be positive only if

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19 When (7.76) is differentiated with respect to M it is immediately clear that the second order condition always holds when $i \geq i_a$.

$$\text{(7.78)} \qquad \alpha i_a \bar{t} D < (i - i_a)(\tau - \bar{t}) D + c_\tau .$$

The net interest cost of using trade credit, the size of average positive net disbursements, plus the transaction cost have to be larger than α times the opportunity cost of holding D worth of money. The constant α on the lhs of (7.78) is the reciprocal of the constant that indicates the probability of a positive net disbursement. Hence, it follows that, cet. par., the lower the probability of positive net disbursements the lower is the precautionary demand for money. Average accounts payable during the period $(\tau - \bar{t})$ is given by the formula[20]

$$\text{(7.79)} \qquad \overline{AP} = \frac{1}{\alpha} \int_M^\infty (\tilde{D} - M) \frac{1}{D} e^{-\frac{\tilde{D}}{D}} d\tilde{D} = \frac{D}{\alpha} e^{-\frac{M}{D}} .$$

Consequently, when distribution function, transaction costs, and interest rates are such that the firm holds no money, average accounts payable will be equal to $1/\alpha$ multiplied by average positive net disbursements.

Next assume that the variables just mentioned are of such magnitudes that a cost minimizing firm uses some combination of money holdings and trade credit. When the expression for M in (7.77) is inserted in (7.79), which in turn is implified, average accounts payable can be written

$$\text{(7.80)} \qquad \overline{AP}_{/M > 0} = \left[\frac{i_a \bar{t} D}{(i - i_a)(\tau - \bar{t}) D + c_\tau} \right] D ,$$

which is the ratio between the opportunity cost of holding D worth of money and the transaction cost of using D worth of trade credit, multiplied by D. From the inequality in (7.78) it

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[20] See Appendix III.

follows that the ratio within brackets must be smaller than $1/\alpha$. Thus, average accounts payable will be smaller than $1/\alpha$ times average positive net disbursements. The probability of using trade credit is given by the formula

$$(7.81) \qquad P(\tilde{D}-M>0) = \frac{1}{\alpha}\int_M^\infty \frac{1}{D} e^{-\frac{\tilde{D}}{D}} d\tilde{D} = \frac{1}{\alpha} e^{-\frac{M}{D}}$$

and when M is greater than zero this probability is given by the bracketed ratio in (7.80)

$$(7.82) \qquad P(D-M > 0 \mid M > 0) = \frac{i_a\bar{t}D}{(i-i_a)(\tau-\bar{t})D + c_\tau} < \frac{1}{\alpha},$$

where again $1/\alpha$ on the rhs of the inequality sign stems from the assumption about the probability of a positive net disbursement being equal to $1/\alpha$. One minus the probability expression in (7.82) gives the so called "coefficient of security", which is used within the theory of the banking firm to describe the probability of having enough money on hand to avoid transaction and penalty costs.[21]

Minimization of the no trade credit cost function in (7.71) gives a demand for money equation

$$(7.83) \qquad M = -D\ln\left(\frac{\alpha i_a\bar{t}}{c_t/D}\right),$$

and provided that M is treater than zero M_{tc} is less than M if

$$(7.84) \qquad c_t - c_\tau - (i-i_a)(\tau-\bar{t})D > 0.$$

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[21] See Olivera (1971), page 1096.

Now (7.84) equals (7.73), which holds by assumption.[22] Thus, the introduction of trade credit as a buffer reduces the precautionary demand for money. Consequently, this is another cost minimization verification of the assertion that trade credit, or other types of short term credit, increases the velocity of circulation.

The interest elasticities of average accounts payable are

$$(7.85) \qquad \varepsilon_i = -\frac{i}{i-i_a + c_\tau/D(\tau-\bar{t})},$$

$$(7.86) \qquad \varepsilon_{i_a} = 1 + \frac{i_a}{i-i_a + c_\tau/D(\tau-\bar{t})}.$$

Both elasticities are functions of the interest rates and the ratio of the transaction cost to average positive net disbursements. (The derivation of these and the following expressions are given in Appendix III.) The elasticity with respect to the size of net disbursements is

$$(7.87) \qquad \varepsilon_D = 1 + \frac{1}{1 + (i-i_a)(\tau-\bar{t})D/c_\tau},$$

when net disbursements have been expressed in terms of D. Both ε_{i_a} and ε_D are always greater than one and ε_i is smaller than minus one if the return on D worth of assets during the credit period is higher than the transaction cost, $i_a(\tau-\bar{t})D > c_\tau$. When positive net disbursements are exponentially distributed an increase in D will also increase the dispresion of the distribution $(E(\tilde{D}) = V(\tilde{D}))$, and if this is interpreted as an increase in uncertainty one can say that, the elasticity of average accounts payable with respect to an increase in uncertainty lies between one and two. In the special case when $c_\tau = 0$, no transaction cost at the end of the credit period, $\overline{AP}_{/M>0}$ is

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[22] When the integrals given by (7.79) and (7.81) are inserted in (7.83) (7.84) follows immediately.

(7.88) $$\overline{AP}_{/M>0} = (\frac{i_a}{i-i_a})(\frac{\bar{t}}{\tau-\bar{t}})D.$$

Average accounts payable is proportional to average positive net disbursements, ε_D is equal to one, and with $i = 2i_a$, the same assumption as in all my numerical examples, ε_i is equal to minus two, which is the same as the corresponding elasticity in the deterministic transaction cost model, and ε_{i_a} equals two and a half, a half higher than in the other model. Hence, in this case both models predict that the demand for trade credit is quite interest elastic.

Finally, assume that the interest rates increase and $di = di_a$. Then it can be shown that average accounts payable also will increase

(7.89) $$\frac{d\overline{AP}_{/M>0}}{di_{/di=di_a}} = \frac{\overline{AP}_{/M>0}}{i_a} > 0 .$$

An increase in the general level of interest rates increases accounts payable. Consequently, this model has the same asymmetry properties as the deterministic model. The reason for this is also the same. When $di = di_a$ the cost of using trade credit is unchanged while the opportunity cost of holding money increases, M falls and accounts payable increases. The direction of the effects of changes in the exogenous variables are thus the same in both the financial transaction cost models. With regard to the model with stochastic net payments it is important to keep in mind, however, that the magnitudes of these effects are distribution specific. Alternative probability distributions lead to alternative formulations of average accounts payable.

7.5 SUMMARY

In this chapter I have shown how transaction costs can give rise to a demand for trade credit. In the section about goods market payment costs a constant rate of purchases combined with fixed payment costs gave rise to a "saw tooth" pattern of accounts payable. When I introduced inventory holdings it became clear that it is a combination of payment and storage costs that can make the use of trade credit favorable. In this context trade credit produces a unique service. It reduces the number of goods market payment occasions. A service function which, for example, an ordinary bank loan cannot fill. The queue model was more realistic since irregular purchases were allowed. It presented the view that payment delays are caused by an irregular inflow of bills combined with a fixed bill servicing capacity. The objective of the firm was to determine the size of its transaction system. I also argued that with this view of the world the payment system can be seen as a web of interdependent queues where irregular flows of bills are under way towards payment. In the section about financial market transaction cost I showed that trade credit, or other types of short term borrowing, can be treated as backlogging in an inventory model. In this context trade credit lengthenes the time between asset conversions and reduces average money holdings. In the section about unforeseen net disbursements I argued that trade credit can be used as a buffer which gives the firms time to plan its financial transactions. It eliminates uncertainty about the timing but not uncertainty about the size of payments. In the demand for reserves model I derived an explicit expression for accounts payable by assuming that net disbursements were exponentially distributed, and it became clear that introduction of trade credit as a buffer does not necessarily eliminate demand for precautionary reserves.

I have used several different model approaches to explain why cost minimizing firms find it worthwhile to use trade credit both in a goods and financial market transaction cost context. The models presented in this chapter have several features in common. *First*, all led to explicit formulas representing the determinants of average accounts payable. A natural next step is then to study to what extent the model formulations I have presented get empirical support. *Second*, with reasonable assumptions about interest rates the interest rate elasticities of accounts payable are quite high, and the elasticity with respect to purchases is less than one. The latter implies that if the size of purchases is a proxy for firm size the formulas predict that the ratio of accounts payable to purchases will be falling with increasing firm size, which is in accordance with the stylized facts in Chapter 1. *Finally*, the models degenerate if the cost of using trade credit is lower than the short run return on money or other assets. Then the interest arbitrage aspects dominate, and all payment are made at the end of the planning period. In addition to the financial motives to use trade credit, transaction costs can be one reason why firms accept trade credit rates of interest that are quite high. The seller is properly compensated at the same time as the buyer reduces his transaction costs.

APPENDIX I (Transactions trade credit when trade credit interest payments are incurred within the model.)

Assume, that the firm or consumer at the beginning of the planning period has recource to a sum of money M, which is used to purchase a continuous inflow of goods or services in the interval 0 - τ. Interest is paid on money holdings, there is a fixed payment cost to make goods payments, and there are no financial market transaction costs. To set up the cost function I use discrete time. When a payment date occurs, the cash payment to be made is

$$\text{(A.I.1)} \qquad St + \sum_{j=1}^{t-1} jiS = St + \frac{iSt(t-1)}{2} = St(1 + i\frac{t-1}{2}) = StX ,$$

where (A.I.1) is based on the fact that

$$\text{(A.I.2)} \qquad \sum_{j=1}^{n} j = \frac{n(n+1)}{2} .$$

St is the cost of goods over one trade credit cycle, and the sum represents the trade credit interest cost. The upper boundary of the sum is t-1, which reflects the assumption that there are not interest payments if t equals one. The interest revenue from falling money balances is

$$\text{(A.I.3)} \qquad i_M t_M + i_M t(M-StX) + i_M t(M-2StX) + \ldots + i_M t(M-(n-1)StX),$$

with $n = \tau/t$, or

$$\text{(A.I.4)} \qquad \sum_{1}^{n} i_M t_M - \sum_{j=1}^{n-1} ji_M t^2 SX ,$$

and by using the sum formula (A.I.4) can after some simplification be written

$$\text{(A.I.5)} \qquad i_M \tau M - \frac{i_M \tau S}{S}(\tau - t)(1 + i\frac{t-1}{2}),$$

where the latter part of the expression represents the reduction of the interest revenue due to an outflow of money during the planning period. With $t = \tau$ there is no outflow of money during the planning period, and interest revenue is $i_M \tau M$, and $t = 1$ represents the case with a continuous outflow of cash. When (A.I.5) is included with a negative sign, and after some reshuffling of terms, the trading cost function becomes

$$\text{(A.I.6)} \qquad TC = c\frac{\tau}{t} + \frac{(i-i_M)St\tau}{2} + \frac{(ii_M S(t-1)\tau}{4}(\tau - t) - \frac{iS\tau}{2} - $$

$$- i_M \tau (M - \frac{S\tau}{2}) \ ,$$

where the first two terms are equal to the cost function in (7.1). The in a cost minimization context important addition is the third term which shows the loss of interest on money holdings due to trade credit interest payments. This is an addition which most asset or money transaction cost models do not include. The most common approach is to let all costs be incurred outside the model. Minimization with respect to t gives the first order condition

$$\text{(A.I.7)} \qquad \frac{(i-i_M)S\tau}{2} - \frac{c\tau}{2t^2} + \frac{ii_M S\tau}{2}(\tau - 2t + 1) = 0 \ .$$

This equation has no "nice" analytical solution, but I have solved (A.I.7) by the help of a numerical example. Assume that $i_M = 0.000278$, $i = 0.000555$, annual interest rates of ten and twenty percent respectively, $\tau = 360$ and $c/S = 0.125$. With these assumptions $t = 30$ in equation (7.2) and the solution (A.I.7) is $t \approx 29$. Thus, the two interest on money models yield almost identical results. The trade credit interest payment effect is of no practical importance. Finally, if no interest is paid on money holdings the credit period is shortened and $t \approx 21$. In this case the trade credit interest rate elasticity is constant and equal to one half.

APPENDIX II (Mathematical derivations to the no uncertainty financial transaction cost model.)

In order to solve the equation system represented by (7.54) and (7.55) for t_1 and t_2, rewrite the equations so that (7.54) and (7.55) are respectively

$$(A.II.1) \qquad \left(\frac{i_a S}{2}\right)(2t_1t_2+t_1^2) - \left(\frac{(i-i_a)S}{2}\right)t_2^2 - c_a = 0 ,$$

$$(A.II.2) \qquad -\left(\frac{i_a S}{2}\right)t_1^2 + \left(\frac{(i-i_a)S}{2}\right)(2t_1t_2+t_2^2) - c_a = 0 ,$$

Next set $A = \frac{i_a S}{2}$ and $B = \frac{(i-i_a)S}{2}$, and a shorter version of the equation system is

$$(A.II.3) \qquad At_1^2 + 2At_1t_2 - Bt_2^2 - c_a = 0 ,$$

$$(A.II.4) \qquad -At_1^2 + 2Bt_1t_2 + Bt_2^2 - c_a = 0 ,$$

add the two equations and solve for t_2

$$(A.II.5) \qquad t_2 = \frac{c_a}{(A+B)t_1} .$$

Insert (A.II.5) in (A.II.3) and multiply through with t_1^2. This gives a forth degree equation,

$$(A.II.6) \qquad At_1^4 + \left(\frac{2Ac_a}{A+B} - c_a\right)t_1^2 - \frac{Bc_a^2}{(A+B)^2} = 0 ,$$

and by redefining t_1 in terms of x, $x = t_1^2$, (A.II.6) can be written as a second degree equation

$$(A.II.7) \qquad x^2 + \frac{c_a}{A}\left(\frac{2A}{A+B} - 1\right)x - \frac{Bc_a^2}{A(A+B)^2} = 0 ,$$

which has the solution

$$\text{(A.II.8)} \qquad c = -\frac{c_a}{A}\left(\frac{2A}{A+B} - 1\right)\frac{1}{2} \pm \left(\frac{c_a^2}{4A^2}\left(\frac{2A}{A+B} - 1\right)^2 + \frac{Bc_a^2}{A(A+B)^2}\right)^{\frac{1}{2}} .$$

This expression can, in turn, be rewritten

$$\text{(A.II.9)} \qquad x = \frac{1}{2}\frac{c_a}{A}\frac{1}{A+B}\left(- A + B \pm (A^2 - 2AB + B^2 + 4AB)^{\frac{1}{2}}\right) ,$$

which reduces to

$$\text{(A.II.10)} \qquad x = \frac{1}{2}\frac{c_a}{A}\frac{1}{A+B}\left(- A + B \pm (A+B)\right) ,$$

and because of the definition of x, the positive root is the final solution. Thus, t_1^2 is

$$\text{(A.II.11)} \qquad t_1^2 = \frac{1}{2}\frac{c_a}{A}\frac{2B}{A+B} ,$$

and after insertion of the definitions of A and B the optimal length of the money holding period can be written

$$\text{(A.II.12)} \qquad t_1 = \left(\frac{2c_a(i-i_a)}{Sii_a}\right)^{\frac{1}{2}} .$$

Finally, by using (A.II.5), the length of a trade credit cycle is

$$\text{(A.II.13)} \qquad t_2 = \left(\frac{2c_a}{S}\right)^{\frac{1}{2}}\left(\frac{i_a}{i(i-i_a)}\right)^{\frac{1}{2}} .$$

Using the expression for t_2 and t_1 in (7.56) and (7.57) the ratio of the trade credit period to the length of one asset conversion cycle can be written

$$(A.II.14)\quad \frac{t_2}{t_1+t_2} = \frac{(\frac{2c_a}{S})^{\frac{1}{2}}(\frac{i_a}{i(i-i_a)})^{\frac{1}{2}}}{(\frac{2c_a}{S})^{\frac{1}{2}}(\frac{i_a}{i(i-i_a)})^{\frac{1}{2}}+(\frac{2c_a}{S})^{\frac{1}{2}}(\frac{i-i_a}{ii_a})^{\frac{1}{2}}} .$$

When both numerator and denominator are multiplied by the square root of the interest expression in the numerator (A.II.14) can, after some simplification, be written

$$A.II.15)\quad \frac{\frac{i_a}{i-i_a}}{\frac{i_a}{i-i_a}+1} = \frac{i_a}{i} .$$

Similarly it can be shown that the ratio $\frac{t_1}{t_{1+}t_2}$ is equal to

$$(A.II.16)\quad \frac{t_1}{t_1+t_2} = 1 - \frac{i_a}{i} .$$

The first two second order conditions are

$$(A.II.17)\quad TC''_{t_1} = (\frac{i_aS}{S})\ 2(t_1+t_2) > 0 ,$$

$$(A.II.18)\quad TC''_{t_2} = (\frac{i_aS}{2})\ 2(t_1+t_2) > 0 .$$

Both hold when interest rates are positive and $i > i_a$. The cross derivative is

$$(A.II.19)\quad TC''_{t_1t_2} = i_aSt_1 - (i-i_a)St_2.$$

Then the condition $(TC''_{t_1})^2(TC''_{t_2})^2 > (TC''_{t_1t_2})^2$ can be written

$$(A.II.20)\quad (t_1+t_2)^2 i_a(i-i_a) > (i_a t_1-(i-i_a)t_2)^2 .$$

In (A.II.14) - (A.II.16) I have shown that $t_2 = (t_1+t_2)(i_a/i)$ and $t_1 = (1-(i_a/i)(t_1+t_2)$. Insertion of the expression for t_2 on the rhs of (A.II.20) gives

$$(A.II.21)\quad [i_a t_1 - (i-i_a)\frac{i_a}{i}(t_1+t_2)]^2 = i_a^2[t_1-(1-\frac{i_a}{i})(t_1+t_2)]^2$$

where the bracketed expression is equal to zero according to the t_1 equality above. The rhs of (A.II.20) is equal to zero and the inequality reduces to

$$(A.II.22)\quad i > i_a .$$

Thus, there exists a cost minimizing combination of t_1 and t_2, if there is no scope for profitable interest arbitrage.

The elasticities are

$$(A.II.23)\quad \varepsilon_{i_a\overline{AP}} \equiv \overline{AP}'_{i_a}\frac{i_a}{\overline{AP}} = \frac{(\frac{Sc_a}{2})^{\frac{1}{2}}(\ldots)i_a\, i^{\frac{3}{2}}(i-i_a)^{\frac{1}{2}}}{i_a^{\frac{3}{2}}(\frac{Sc_a}{2})^{\frac{1}{2}}} =$$

$$(\ldots)\quad = \frac{\frac{3}{2}i_a^{\frac{1}{2}}(i^{\frac{3}{2}}(i-i_a)^{\frac{1}{2}} - i_a^{\frac{3}{2}}i^{\frac{3}{2}}(-\frac{1}{2})(i-i_a)^{-\frac{1}{2}}}{i^3(i-i_a)} =$$

$$= \frac{3}{2}(\frac{i_a^{\frac{1}{2}}i^{\frac{3}{2}}((i-i_a)^{\frac{1}{2}} + \frac{1}{3}i_a(i-i_a)^{-\frac{1}{2}})}{i^3(i-i_a)})$$

$$= \frac{3}{2}(i_a^{\frac{1}{2}} i^{-\frac{3}{2}}(((i-i_a)^{-\frac{1}{2}} + \frac{1}{3}i_a(i-i_a)^{-\frac{3}{2}}))$$

(A.II.24) $$= \frac{3}{2} \frac{i_a i^{\frac{3}{2}}(i-i_a)^{\frac{1}{2}}}{i_a^{\frac{3}{2}}} \frac{i_a^{\frac{1}{2}} i^{-\frac{3}{2}}((i-i_a)^{-\frac{1}{2}} + \frac{1}{3}i_a(i-i_a)^{-\frac{3}{2}}}{} =$$

(A.II.25) $$= \frac{3}{2} + \frac{1}{2} \frac{i_a}{i-i_a}$$

(A.II.26) $$\varepsilon_{i\overline{AP}} = \overline{AP}'_i \frac{i}{\overline{AP}} = \frac{(\frac{Sc_a}{2})^{\frac{1}{2}}(\ldots) \; i i^{\frac{3}{2}}(i-i_a)^{\frac{1}{2}}}{(\frac{Sc_a}{2})^{\frac{1}{2}} \; i_a^{\frac{3}{2}}}$$

(...) $$= -i_a^{\frac{3}{2}}(i^{\frac{3}{2}}(i-i_a)^{\frac{1}{2}})^{-2}(\frac{3}{2}i^{\frac{1}{2}}(i-i_a)^{\frac{1}{2}} + i^{\frac{3}{2}} \frac{1}{2}(i-i_a)^{-\frac{1}{2}})$$

$$= -\frac{3}{2} \frac{i_a^{\frac{3}{2}} i^{\frac{1}{2}} \; ((i-i_a)^{\frac{1}{2}} + \frac{1}{3}i(i-i_a)^{-\frac{1}{2}})}{i^3(i-i_a)}$$

(A.II.27) $$-\frac{3}{2}(i_a^{\frac{3}{2}} \; i^{-\frac{5}{2}}((i-i_a)^{-\frac{1}{2}} + \frac{1}{3}i(i-i_a)^{-\frac{3}{2}} \; i_a^{-\frac{3}{2}} \; i^{\frac{5}{2}}(i-i_a)^{\frac{1}{2}})$$

(A.II.28) $$= -\frac{3}{2} - \frac{1}{2} \frac{i}{i-i_a} ,$$

(A.II.29) $$\varepsilon_{i_a\overline{M}} = \overline{M}'_{i_a} \frac{i_a}{\overline{M}} = (\frac{Sc_a}{2})^{\frac{1}{2}}(\ldots) \frac{i^{\frac{3}{2}} i_a^{\frac{1}{2}}}{(i-i_a)^{\frac{3}{2}}} (\frac{Sc_a}{2})^{-\frac{1}{2}}$$

$$(\ldots) \quad = \frac{-\frac{3}{2}(i-i_a)^{\frac{1}{2}}\, i_a^{\frac{1}{2}}\, i^{\frac{3}{2}} - \frac{1}{2} i_a^{-\frac{1}{2}}\, i^{\frac{3}{2}} (i-i_a)^{\frac{3}{2}}}{i_a i^3} =$$

$$= -\frac{3}{2}((i-i_a)^{\frac{1}{2}}\, i^{-\frac{3}{2}} (i_a^{-\frac{1}{2}} + \frac{1}{3}\, i^{-\frac{3}{2}} (i-i_a)))$$

$$\text{(A.II.30)} \quad -\frac{3}{2}((i-i_a)^{\frac{1}{2}}\, i^{-\frac{3}{2}} (i_a^{-\frac{1}{2}} + \frac{1}{3}\, i_a^{-\frac{3}{2}} (i-i_a)))\; (ii_a)^{\frac{3}{2}} (i-i_a)^{-\frac{3}{2}} =$$

$$\text{(A.II.31)} \quad = -\frac{3}{2}(\frac{i_a}{i-i_a} + \frac{1}{3}) = -\frac{1}{2} - \frac{3}{2}\frac{i_a}{i-i_a} ,$$

$$\text{(A.II.32)} \quad \varepsilon_{i\overline{M}} = \overline{M}_i' \frac{i}{\overline{M}} = (\frac{Sc_a}{2})^{\frac{1}{2}} (\ldots)\; i(i-i_a)^{-\frac{3}{2}}\, i^{\frac{3}{2}}\, i_a^{\frac{1}{2}}$$

$$(\ldots) \quad = \frac{\frac{3}{2}(i-i_a)^{\frac{1}{2}}\, i^{\frac{3}{2}}\, i_a^{\frac{1}{2}} - (i-i_a)^{\frac{3}{2}}\, i_a^{\frac{1}{2}}\, \frac{3}{2} i^{\frac{1}{2}}}{i_a i^3}$$

$$= \frac{3}{2}(i-i_a)^{\frac{1}{2}} (i^{-\frac{3}{2}} - (i-i_a)\, i^{-\frac{5}{2}})\, i_a^{-\frac{1}{2}}$$

$$\text{(A.II.33)} \quad \frac{3}{2}(i-i_a)^{\frac{1}{2}} (i^{-\frac{3}{2}} - (i-i_a)\, i^{-\frac{5}{2}})\; i_a^{-\frac{1}{2}}\, i(i-i_a)^{-\frac{3}{2}}\, i^{\frac{3}{2}}\, i_a^{\frac{1}{2}} =$$

$$= \frac{3}{2}(\frac{i}{i-i_a} - 1) = \frac{3}{2}\frac{i_a}{i-i_a} .$$

The effect of a simultaneous increase in the interest rates is

(A.II.34) $$\frac{d\overline{AP}}{di}{}_{/di=di_a} = \overline{AP}'_{i_a} + \overline{AP}'_i = (\frac{Sc_a}{2})^{\frac{1}{2}}(\ldots)$$

(...) $$= \frac{3}{2}(i_a^{\frac{1}{2}}\ i^{-\frac{3}{2}}((i-i_a)^{\frac{1}{2}} + \frac{1}{3}i_a(i-i_a)^{-\frac{3}{2}})$$

$$- \frac{3}{2}i_a^{\frac{3}{2}}\ i^{\frac{1}{2}}((i-i_a)^{\frac{1}{2}} + \frac{1}{3}i(i-i_a)^{-\frac{1}{2}})i^{-3}(i-i_a)^{-1} =$$

$$= \frac{3}{2}i_a^{\frac{1}{2}}\ i^{-\frac{5}{2}}(i-i_a)^{\frac{1}{2}}$$

(A.II.35) $$\frac{d\overline{AP}}{di}{}_{/di=di_a} = \frac{3}{2}(\frac{Sc_a}{2})^{\frac{1}{2}}\ i_a^{\frac{1}{2}}\ i^{-\frac{5}{2}}(i-i_a)^{\frac{1}{2}} > 0$$

(A.II.36) $$\frac{d\overline{M}}{di}{}_{/di=di_a} = \overline{M}'_{i_a} + \overline{M}'_i = (\frac{Sc_a}{2})^{\frac{1}{2}}(\ldots)$$

(...) $$= (-\frac{3}{2}(i-i_a)^{\frac{1}{2}}\ i^{-\frac{3}{2}}(i_a^{-\frac{1}{2}} + \frac{1}{3}i^{-\frac{3}{2}}(i-i_a))$$

$$+ \frac{3}{2}((i-i_a)^{\frac{1}{2}}(i^{-\frac{3}{2}} - (i-i_a)i^{-\frac{5}{2}})i_a^{-\frac{1}{2}}) =$$

$$= -\frac{3}{2}(i-i_a)^{\frac{3}{2}}\ i^{-\frac{3}{2}}(\frac{1}{3} + i^{\frac{1}{2}}\ i_a^{-\frac{1}{2}})$$

(A.II.37) $$\frac{d\overline{M}}{di}{}_{/di=di_a} = -\frac{3}{2}(\frac{Sc_a}{2})^{\frac{1}{2}}(i-i_a)^{\frac{3}{2}}(\frac{1}{3} + i^{\frac{1}{2}}\ i_a^{-\frac{1}{2}})i^{-\frac{3}{2}} < 0$$

APPENDIX III (The stochastic cash outflow model.)

To derive the first order condition in (7.74) differentiate (7.72) with respect to M by using the rule

$$\text{(A.III.1)} \quad \frac{d}{d\alpha}\int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)dx = \int_{a(\alpha)}^{b(\alpha)} f'_{\alpha}(x,\alpha)dx + f(b(\alpha),\alpha)\frac{db}{d\alpha} - f(a(\alpha),\alpha)\frac{da}{d\alpha} .$$

Then integral in (7.79) is solved by using integration by parts

$$\text{(A.III.2)} \quad \frac{1}{\alpha}\int_M^{\infty}(\tilde{D}-M)\frac{1}{D}e^{-\frac{\tilde{D}}{D}}d\tilde{D} = \frac{1}{\alpha}\left(-e^{-\frac{\tilde{D}}{D}}\tilde{D} - \int_M^{\infty} - e^{-\frac{\tilde{D}}{D}}d\tilde{D}\right) - \frac{1}{\alpha}\int_M^{\infty}\frac{1}{D}Me^{-\frac{\tilde{D}}{D}}d\tilde{D} =$$

$$= \frac{1}{\alpha}\left(Me^{-\frac{M}{D}} + De^{-\frac{M}{D}}\right) - \frac{1}{\alpha}Me^{-\frac{M}{D}} = \frac{De^{-\frac{M}{D}}}{\alpha} .$$

The elasticities are given in (A.III.3) - (A.III.8). All formulas represent the case with $\overline{AP}_{/M>0}$, but the subscript has been omitted below

$$\text{(A.III.3)} \quad \varepsilon_i = \overline{AP}'_i \frac{i}{\overline{AP}} = \frac{-i_a\bar{t}D(\tau-\bar{t})\ i\ \left((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}\right)}{\left((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}\right)^2 i_a\bar{t}D}$$

$$\text{(A.III.4)} \quad = -\frac{i}{i - i_a + \frac{c_\tau}{D(\tau-\bar{t})}} ,$$

$$\text{(A.III.5)} \quad \varepsilon_{i_a} = \overline{AP}'_{i_a}\frac{i_a}{\overline{AP}} =$$

$$= \frac{\left(\bar{t}D\left((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}\right) + i_a\bar{t}D(\tau-\bar{t})\right)\left((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}\right)i_a}{\left((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}\right)^2 i_a\bar{t}\,D} =$$

(A.III.6) $$= 1 + \frac{i_a}{i-i_a + \frac{c_\tau}{D(\tau-\bar{t})}}$$

(A.III.7) $$\varepsilon_D = \overline{AP}'_D \frac{D}{\overline{AP}} =$$

$$= \frac{(i_a\bar{t}((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}) + i_a\bar{t}D\frac{c_\tau}{D^2})((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D})D}{((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D})^2 i_a\bar{t}D} =$$

(A.III.8) $$= 1 + \frac{1}{1 + \frac{(i-i_a)(\tau-\bar{t})D}{c_\tau}} .$$

The effect of a simultaneous increase in the interest rates is

(A.III.9) $$d\overline{AP} = \frac{-i_a\bar{t}D(\tau-\bar{t})}{((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D})^2} di$$

$$+ \frac{\bar{t}D((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}) + i_a\bar{t}D(\tau-\bar{t})}{((i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D})^2} di_a ,$$

and with $di = di_a$ this expression reduces to

(A.III.10) $$\frac{d\overline{AP}}{di}\Big/_{di=di_a} = \frac{\bar{t}D}{(i-i_a)(\tau-\bar{t}) + \frac{c_\tau}{D}} = \frac{\overline{AP}}{i_a} > 0 .$$

REFERENCES

Affärsvärlden (1982), No. 47.

Akerlöf, G. (1970), "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism", Quarterly Journal of Economics, 84 (August).

Alchian, A. (1970), "Information Costs, Pricing and Resource Unemployment", in Phelps, E.S. et al., Microeconomic Foundations of Employment and Inflation Theory, W.W. Norton & Co, New York.

Altman, E.I., Avery, R.A., Eisenbeis, J.F. and Sinley, J.R. (1981), Applications of Classification Techniques in Business, Banking and Finance, JAI Press, Greenwich, Conn.

Arrow, K.J. (1964), "Aspects of the Theory of Risk-Bearing", The Yrjö Jansson Foundation, Helsinki.

Atkinson, A.B., Stiglitz, J.E. (1980), Lectures on Public Economics, McGraw-Hill Book Co., New York.

Baltensperger, E. (1974), "The Precautionary Demand for Reserves", The American Economic Review, 64 (March).

Baltensperger, E., Milde, H. (1976), "Predictability of Reserve Demand, Information Costs, and Portfolio Behaviour of Commercial Banks", The Journal of Finance, 31 (June).

Baltensperger, E. (1978), "Credit Rationing, Issues and Questions", Journal of Money, Credit and Banking, 10 (May).

Barro, J.R., Santomero, A.M. (1974), "Transaction Costs, Payment Periods, and Employment", in Johnson, H.G. and Nobay, A.R. (eds.), Issues in Monetary Economics: Proceedings of the 1972 Monetary Study Group Conference, Oxford University Press, London.

Baumol, W.J. (1952), "The Transactions Demand for Cash: An Inventory Theoretic Approach", Quarterly Journal of Economics, 66 (November).

Benston, G.J., Smith, C.W. Jr. (1976), "A Transaction Cost Approach to the Theory of Financial Intermediation", Journal of Finance, 31 (May).

Bierman, H., Hausman, W.R. (1970), "The Credit Granting Decision", Management Science, 16 (April).

Bitros, G.C. (1979), "Neoclassical Theory of Trade Credit; A Critique and a Reformulation", Econometrica, 47 (January).

Blair, R.D. (1974), "Random Input Prices and the Theory of the Firm", Economic Inquiry, 12 (June).

Blom, G. (1970), Sannolikhetsteori med tillämpningar, Studentlitteratur, Lund.

Bohm, P. (1979), "Exportkreditgarantierna i samhällsekonomisk belysning", Department of Economics, University of Stockholm, Skrift no. 1979:5. Stockholm.

Branson, W.H. (1968), Financial Capital Flows in the U.S. Balance of Payments, North Holland, Amsterdam.

Brechling, F.P.R., Lipsey, R.G. (1963), "Trade Credit and Monetary Policy", The Economic Journal, 73 (December).

Brunner, K., Meltzer, A.H. (1964), "Studies on Money and Monetary Policy", Journal of Finance, 19 (May).

Brunner, K., Meltzer, A.H. (1967), "Economies of Scale in Cash Balances Reconsidered", Quarterly Journal of Economics, 81 (August).

Buiter, W.H., Armstrong, C.A. (1978), "A Didactic Note on the Transactions Demand for Money and Behavior Towards Risk", Journal of Money, Credit and Banking, X (November).

Campbell, T.S., Kracaw, W.A. (1980), "Information, Production, Market Signalling, and the Theory of Financial Intermediation", Journal of Finance, XXXV (September).

Chiang, A.C. (1974), Fundamental Methods of Mathematical Economics, 2nd ed., McGraw-Hill, Kogakusha.

Chou, I.U. (1975), Statistical Analysis, Holt, Rhinehart and Winston, New York.

Churchman, C.W., Ackoff, R.L., Arnoff, E.L. (1957), Introduction to Operations Research, John Wiley & Sons, Inc., New York.

Clower, R.W., Howitt, P.W. (1978), "The Transactions Theory of the Demand for Money: A Reconsideration", Journal of Political Economy, 86 (June).

Coase, R.H. (1937), "The Nature of the Firm", Economica, New Series, Vol. IV.

Coates, R.C. (1976), The Demand for Money by Firms, Marcel Dekker, New York.

Copeland, T.E., Khoury, N.T. (1980), "Analysis of Credit Extensions in a World with Uncertainty", in Smith, K.V. (ed.), Management of Working Capital, 2nd ed., West Publishing Company, St. Paul, Minn.

Cox, D.R., Smith, W.L. (1961), Queues, Chapman and Hall, London.

Damon, W.W., Schramm, R. (1972), "A Simultaneous Decision Model for Production, Marketing and Finance", Management Science, 19 (October).

Davis, E.W., Yeomans, K.A. (1974), "Company Finance and the Capital Market - A Study of the Effect of Firm Size", University of Cambridge, Department of Applied Economics, Occasional Paper 39.

Deaton, A., Muellbauer, J. (1980), Economics and Consumer Behaviour, Cambridge University Press, Cambridge.

Demsetz, H. (1968), "The Cost of Transacting", Quarterly Journal of Economics, 82 (February).

Dhrymes, P.J. (1964), "On the Theory of the Monopolistic Multiproduct Firm Under Uncertainty", International Economic Review, 5(3).

Eliasson, G. (1967), Kreditmarknaden och industrins investeringar, Almqvist & Wiksell, Uppsala.

Elton, E., Gruber, M. (1975), Finance as a Dynamic Process, Prentice Hall, Englewood Cliffs, N.J.

Ethier, W. (1973), "International Trade and the Forward Exchange Market", American Economic Review, 63 (June).

Faith, R.L., Tollison, R.D. (1981), "Contractual Exchange and the Timing of Payment", Journal of Economic Behavior and Organization, 1 (December).

Farrar, D.E. (1962), The Investment Decision Under Uncertainty, Prentice Hall, Englewood Cliffs, N.J.

Ferris, S.J. (1981), "A Transactions Theory of Trade Credit Use", Journal of Political Economy, 89 (May).

Fiacco, A.W., McCormick, G.P. (1968), Nonlinear Programming; Sequential Unconstrained Minimization Techniques, John Wiley and Sons, New York.

Fisher, I. (1931), The Purchasing Power of Money, Macmillan, New York.

Galbraith, J.K. (1957), "Market Structure and Stabilization Policy", Review of Economics and Statistics, XXXIX (May).

Gurley, J.G., Shaw, E.S. (1960), Money in a Theory of Finance, The Brookings Institution, Washington.

Guttentag, J.M., Lipsey, R.G. (1967), The Uniqueness of Commercial Banks, The Brookings Institution, Washington.

Haley, C.W., Higgins, R.C. (1973), "Inventory Control Theory and Trade Credit Financing", Management Science, 20 (December).

Hansen, B. (1961), Foreign Trade Credits and Exchange Reserves - A Contribution to the Theory of International Capital Movements, North Holland, Amsterdam.

Hellner, J. (1973), Köprätt, P.A. Norstedt & Söner, Stockholm.

Hill, N.C., Riener, K.D. (1979), "Determining the Cash Discount in the Firm's Credit Policy", Financial Management, 8 (Spring).

Hirschleifer, J. (1970), Investment Interest and Capital, Prentice Hall, London.

Hite, G. (1979), "On the Theory of the Firm in a Capital Asset Pricing World", in Bicksler, J.L. (ed.), Handbook of Financial Economics, North Holland, Amsterdam.

Hodgman, R.D. (1960), "Credit Risk and Credit Rationing", Quarterly Journal of Economics, 74 (May).

ICA-Kuriren (1983), no. 13.

Ingves, S. (1981), Den oreglerade kreditmarknaden, Expert-rapport till Kreditpolitiska Utredningen, Ds:E 1981:13.

Ivarsson, L. (1980), "Bankernas portföljbeteende", Umeå Economic Studies.

Jaffee, D.M. (1970), Credit Rationing and the Commercial Loan Market, John Wiley & Sons, New York.

Jaffee, D.M., Russell, T. (1973), "A Simplification of Credit Rationing Theory in Terms of Insurance Market Failure", Princeton, unpublished manuscript.

Jaffee, D.M., Russel, T. (1976), "Imperfect Information, Uncertainty, and Credit Rationing", Quarterly Journal of Economics, 90 (November).

Jensen, M.C. (1972), "Capital Markets: Theory and Evidence", Bell Journal of Economics, 3 (Autumn).

Kanniainen, V. (1976), "The Demand for Money, Other Liquid Assets and Short Term Credit by Finnish Firms", The Finnish Economic Association, XXXV, Helsinki.

Karnell, G. (1972), Växel och checkrätt, Juridiska föreningens förlag, Stockholm.

Kenen, P.B. (1965), "Trade Speculation and the Forward Exchange Rate", in Baldwin, R.E., Trade Growth and the Balance of Payments: Essays in Honor of Gottfried Haberler, North Holland, Amsterdam.

Kim, Y.H., Atkins, J.C. (1978), "Evaluation of Investments in Accounts Receivable: A Wealth Maximizing Framework", Journal of Finance, XXXIII (May).

Klein, B., Crawford, R.G., Alchian, A. (1978), "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process", Journal of Law and Economics, 21 (October).

Koskela, E. (1979), "Trade Credits, Credit Rationing and the Short Run Effectiveness of Monetary Policy", Working Paper, Dept. of Economics, University of Helsinki.

Laffer, A. (1970), "Trade Credit and the Money Market", Journal of Political Economy, 78 (March/April).

Leland, H.E. (1972), "Theory of the Firm Facing Uncertain Demand", American Economic Review, 62 (June).

Lietaer, B.A. (1971), Financial Management of Foreign Exchange, MIT Press, Cambridge, Mass.

Lindsay, J.R., Sametz, A.W. (1967), Financial Management, An Analytical Approach, Homewood, Ill.

Lundberg, E., Senneby, B. (1956), "Den nya penningpolitikens dilemma", Skandinaviska Bankens Kvartalsskrift, No. 3.

Makin, J.H. (1978), "Portfolio Theory and the Problem of Foreign Exchange Risk", Journal of Finance, XXXIII (May).

Mayshar, J. (1979), "Transaction Costs in a Model of Capital Market Equilibrium", Journal of Political Economy, 87 (August).

McKinnon, R.I. (1979), Money in International Exchange, Oxford University Press, Oxford.

Mehta, D. (1968), "The Formulation of Credit Policy Models", Management Science, 15 (October).

Mehta, D. (1970), "Optimal Credit Policy Selection. A Dynamic Approach", Journal of Financial and Quantitative Analysis, 5 (December).

Meltzer, A.H. (1960), "Mercantile Credit, Monetary Policy, and Size of Firms", Review of Economics and Statistics, 42 (November).

Nadiri, M.I. (1969), "The Determinants of Trade Credit in the U.S. Total Manufacturing Sector", Econometrica, 37 (July).

National Bureau of Statistics, Enterprises (1980).

National Bureau of Statistics, Enterprises (1980), Appendices 1 and 6.

National Bureau of Statistics, Statistical Reports, K 1981:3.3.

National Bureau of Statistics, Statistical Reports, N 1980:17.

National Bureau of Statistics, Appendix to N 1980:17.

National Bureau of Statistics, Statistical Reports, N 1981:2.1.

National Bureau of Statistics, Statistical Reports, N. 1982:11.

Niehans, J. (1969), "Money in a Static Theory of Optimal Payment Arrangements", Journal of Money, Credit and Banking, 1 (November).

Niehans, J. (1978), The Theory of Money, Johns Hopkins University Press, Baltimore.

Olivera, J.H.G. (1971), "The Square-Root Law of Precautionary Reserves", Journal of Political Economy, LXXIX (September/October).

Panico, J.A. (1969), Queuing Theory, Englewood Cliffs, N.J.

Peterssohn, E. (1969), "Modeller för prognos av inbetalningar från kunder", Economic Research Institute, Stockholm School of Economics.

Peterssohn, E. (1976), "Kreditgivning mellan företag", Economic Research Institute, Stockholm School of Economics.

Philippatos, G.C. (1979), "Alternatives to Mean-Variance Portfolio Selection", in Bicksler, J. (ed.), Handbook of Financial Economics, North Holland, Amsterdam.

Policano, A.J. (1977), "An Inventory Theoretic Model of Trade Credit Transactions", Economic Inquiry, XV (April).

Pratt, J. (1964), "Risk Aversion in the Small and in the Large", Econometrica, 32 (January-April).

Pulley, L.B. (1981), "A General Mean-Variance Approximation to Expected Utility for Short Holding Periods", Journal of Financial and Quantitative Analysis, XVI (September).

Radcliffe Report (1979), The Committee on the Working of the Monetary System, Cmnd 827 HMSO.

Rasmussen, A. (1972), "Pristeori eller parameterteori", Skrifter fra Institutet for Afsaetningsøkonomi 16, Copenhagen.

Rosen, S. (1974), "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition", Journal of Political Economy, 82 (January).

Sandmo, A. (1971), "On the Theory of the Competitive Firm Under Price Uncertainty", American Economic Review, 61 (May).

Sastry, R.A S. (1970), "The Effect of Credit on Transactions Demand for Cash", Journal of Finance, 25 (September).

Schiff, M., Lieber, Z. (1974), "A Model of the Integration of Credit and Inventory Management", Journal of Finance, 29 (March).

Schwartz, R.A. (1974), "An Economic Model of Trade Credit", Journal of Financial and Quantitative Analysis, 8 (September).

Schwartz, R.A., Whitcomb, D.K. (1978), "Implicit Transfers in the Extension of Trade Credit", in Boulding, K., Wilson, T. (eds.), The Channels of Redistribution Through the Financial System: The Grants Economics of Money and Credit, Praeger, New York.

Schwartz, R.A., Whitcomb, D.K. (1979), "The Trade Credit Decision", in Bicksler, J.L. (ed.), Handbook of Financial Economics, North Holland, Amsterdam.

Sharpe, W.F. (1970), Portfolio Theory and Capital Markets, Mc Graw-Hill Book Co., New York.

Soenen, L.A. (1979), Foreign Exchange Exposure Management, Sijthoff & Noordhoff, Germantown, Maryland.

Soeria-Atmadja, H. (1983), "Product Guarantees and Liability Rules: An Economic Analysis", Dept. of Economics, University of Stockholm.

SOU (1975), "Om konsumentkreditlag m.m." (The Installment Payment Committee), SOU 1975:63, Liber, Stockholm.

SOU (1977), "Finansieringsbolag" (Finance Companies), SOU 1977:97, Liber, Stockholm

Spence, M.A. (1974), Market Signaling: Informational Transfer in Hiring and Related Screening Processes, Harvard University Press, Cambridge, Mass.

Sveriges Riksbank (1982), Statistical Yearbook, Stockholm.

Sveriges Riksbank (1980), Statistical Yearbook, Stockholm.

Tobin, J., Brainard, W.C. (1963), "Financial Intermediaries and the Effectiveness of Monetary Controls", The American Economic Review, 53 (May).

Whalen, E.L. (1966), "A Rationalization of the Precautionary Demand for Cash", Quarterly Journal of Economics, 80 (May).

White, W.H. (1964), "Trade Credit and Monetary Policy: A Reconciliation", The Economic Journal, LXXIV (December).

Wicksell, K. (1936), Interest and Prices, Macmillan, London.

Wrightsman, D., Terninko, J. (1971), "On the Measurement of Opportunity Cost in Transactions Demand Models", Journal of Finance, 26 (September).

Zahn, F., Hosek, W.R. (1973), "Impact of Trade Credit on the Velocity of Money and the Market Rate of Interest", Southern Economic Journal, 40 (October).

List of reports published

Unless otherwise indicated, these reports are published in English.

ANELL, B., Rational problem solving or organized anarchy? A case study of the establishment of a shopping center in central Norrköping. Report No. 10 from the research program "Retailing in transition". (Mimeographed).1)

BARK, A., MÅHLEN, A.K. & STJERNBERG, T., New techniques in the stores; Computers' effects and employees' influence. Stockholm. Arbetslivscentrum/EFI 1)

BJÖRKLUND, A., Studies in the dynamics of unemployment. Stockholm

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BRUNSSON, N., (Ed) Management research - truth or moral? On normativity in the science of management. Stockholm. Studentlitteratur. 1)

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---

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---

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---

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---

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