Wages and Growth in an Open Economy

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EFI THE ECONOMIC RESEARCH INSTITUTE
STOCKHOLM SCHOOL OF ECONOMICS

A Dissertation for the Doctor's Degree in Economics

Stockholm School of Economics 1982

UDK 330.35
330.341.4
338(485)
ISBN 91-7258-144-1
GOTAB, Stockholm 1982

Acknowledgements

No man is an island, entire of itself; every man is a piece of the continent, a part of the main. These often quoted words, written by John Donne in 1624, apply with particular force to the author of a doctoral thesis. I wish to thank all the friends and colleagues who have helped me in my work with this book.

My thesis adviser, Karl G. Jungenfelt, has played a much larger role than even the great frequency of references to his works suggests. He has supported my work from the start and has always been there to help and encourage me when I have had problems, great or small. My gratitude to him is very deep.

Many people have read drafts of the various chapters, giving comments which have influenced the final version. A particularly heavy load has fallen upon Lars Calmfors, Lars Hörngren and Stefan Lundgren, who have been constructive seminar discussants. The members of my thesis committee, which in addition to Karl G. Jungenfelt consists of Karl-Göran Mäler and Staffan Viotti, have also provided much detailed and constructive criticism.

Alberto Izquierdo drew the beautiful diagrams and Sarah McPhee considerably improved my English. She is not to blame for errors I have made when making last minute changes.

I wish to thank Margareta Blomberg, Kerstin Niklasson and Monica Peijne. Their contribution goes far beyond typing and retyping the various drafts. I wish to thank them for all their help and encouragement.

A six-month stay at the Institute of Economics at the University of Oslo provided inspiration at an early phase of this work.

Chapter 2 of the book has, in earlier versions, been presented at the Nordic Meeting of Economists 1981 in Kungälv and at the Workshop on Open Economy Macroeconomics in August, 1981 at the Institute of International Economic Studies at the University of Stockholm. I received many helpful comments on these two occasions.

Chapters 3 and 4 have, with minor alterations, earlier been published in Swedish. During parts of 1979 and 1980 I worked for the Ministry of Economic Affairs on the Medium Term Survey. That work resulted in a book, quoted among the references, which contains the present chapters 3 and 4. It was a stimulating experience to work at the Ministry, and I wish to thank all the people there who took an interest in my work.

The Economics Department at the Stockholm School of Economics has been a very stimulating place to work in. To all those I have not specifically mentioned, and they are many, I wish to express my sincere gratitude.

This work has received generous financial support from the *Humanistisk-Samhällsvetenskapliga Forskningsrådet* and the *Bankforskningsinstitutet*.

Stockholm in March, 1982

Pehr Wissén

Contents

1 An Introduction to the Study

1.1 INTRODUCTION

This study deals with economic growth in a small open economy. It contains a theoretical discussion of this theme and some empirical analysis. The aim of this work is to adapt neoclassical growth theory to the situation facing a small open economy like Sweden. Neoclassical growth theory has been extensively researched, but has only to a lesser extent been applied to the problems facing a small open economy.

By a small open economy I mean an economy producing a tradable good that can be bought and sold on the world market at a given price.

Neoclassical growth theory offers three equilibrium concepts.[1] *First*, instantaneous equilibrium, which I shall call a short-run equilibrium. In most of the following applications (although not all of them), I will assume that the labour market clears and when a nontraded good is included in the model, that its market also clears. A short-run equilibrium is attained when there is such a combination of the wage rate and the relative price that these two conditions are met. *Second*, there is the equilibrium growth path. This is a growth path where the short-run equilibrium

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1 Equilibrium concepts should be discussed with reference to a specified model. The reasoning that follows in this introduction is relevant for the models which are used in the present study, as will later become clear.

conditions are satisfied at all points in time. *Third*, there is the steady state growth path. This is an equilibrium growth path such that output, capital stock and labour input all grow at the same rate. The rate of growth on the steady state growth path is determined by some exogenous resource growth, such as exogenous technological progress and/or the growth of the labour force. In the present study, I will throughout assume technological progress.

This third concept, steady state growth, is central to the study. The focus of interest is the adjustment of a small open economy to steady state growth when, for some reason, it is not on the steady state growth path. Due to this focus, I will mainly be concerned with stability analysis in the theoretical discussion. I will not devote any interest to examining comparative statics on the steady state paths.

In the empirical analysis, I will assume that one steady state path exists which does not shift. My interest is to study how the economy adjusts to this path. When using the model, one could also assume that the economy is always in steady state, but that the path shifts. The choice between these two approaches is not immediately clear. One possible interpretation of the difference between these approaches is that the former rests on the implicit assumption that speeds of adjustment are slow, while the latter assumes that adjustment to steady state growth is immediate. In any case the two approaches can be viewed as complements rather than substitutes since they bring forth different aspects of economic growth.

The empirical problems which will be treated are inspired by the debate on economic policy in Sweden during the post-war period.

Problems of economic growth have long been at the centre of the debate on economic policy in Sweden. Especially the causal linkages between wages, profitability and real growth have been extensively deliberated. Indeed, in 1972, Erik

Lundberg wrote: "It seems to me that interrelations between short-term stabilisation issues of employment and prices on the one hand and developments of productivity and profits on the other have been examined more closely in the policy debate in Sweden than in many other countries." (Lundberg [1972]). In Sweden, wage negotiations are highly centralized. A core question at every round of wage negotiations is naturally the margin for wage increases.[1]

Certainly in any small open economy the problem of finding the equilibrium wage rate compared to the rest of the world is crucial. If the wage rate is increased excessively,[2] a partial effect is that supply of the traded good is decreased and the domestic demand for it is increased. This would normally result in a balance-of-trade deficit. If, however, the government can increase taxes to reduce domestic demand, and use the tax receipts to create employment for those laid off from the traded goods sector, then balanced trade and full employment can be restored. The shares of the sectors in the economy thus change. Furthermore, incentives to invest in the traded sector weaken since profitability decreases.

This more or less happened in Sweden in the post-war period. It seems, however, that another less expected thing happened in Sweden. It has been a common empirical observation that productivity, *of the already existing machines*, increased during the period. When discussing empirical observations concerning the growth of productivity in Sweden, Lundberg wrote: "It can thus be argued that the easy-going '50s implied an 'accumulation of inefficiencies' in the form of reserves of potential productivity gains for the '60s

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1 This may be one reason why the above mentioned problems are examined more closely "in Sweden than in many other countries" as Lundberg [1972] asserts.

2 "Excessively" must be given a precise meaning. In this section, before I have developed a model, I use expressions like "excessively", "too much", "too fast", etc. These expressions are, implicitly, comparisons with some equilibrium concept. Steady state growth, defined in the models that follow, will serve as the desired reference path.

so that when the squeeze occurred and the incentives to reduce costs and raise productivity grew strong, there was an unusual accumulated reserve of opportunities." (Lundberg [1972] p. 480).

It seems to have been a widespread notion in the late '60s and early '70s that a large portion of the increase in productivity during the '60s came from the elimination of a slack[1] that had gathered in the preceding decade. A model which has empirical relevance for Swedish post-war development should, I believe, take into account this phenomenon.

The observation about the slack was made in the period 1965-1975. In the middle of the 1970s, the traded goods sector started having problems. Output, investments and employment stagnated and even fell. It can be hypothesized in the light of the remarks above, that for a number of periods wages had increased too fast. This had not created balance of payments problems, since the demand policy was sufficiently restrictive. It did not cause unemployment for two reasons: *First*, the nontraded goods sector expanded, engaging those who were laid off in the traded goods sector. *Second*, and important for this analysis, the reduction of the slack took the form of increased output, and meant only a moderate decrease in employment in the traded goods sector. The rapidly rising wages depressed profitability and investments and also increased the scrapping of older machines. Thus the capital stock grew more slowly than would have been the case had wages not increased too fast. When ultimately there was no more slack to reduce, output as well as employment started to fall.

The presence of the slack may be significant, because it hides the fact that wages are too high. It provokes a lag, making it hard to determine the level of the steady state wage rate.

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[1] The word "slack" is used with different meanings in economics. One may e.g. distinguish between capacity slack (less than full capacity utilization) and efficiency slack (resources not efficiently utilized). In this work I will throughout assume a particular form of *efficiency slack*.

The slack mechanism has been adapted to a model and thoroughly analyzed at the industry branch level. In two studies Jungenfelt showed how these mechanisms have interacted in the printing industry (see SOU 1974:34) and in the textile, and the pulp and paper industries (Jungenfelt [1982]). The hypothesis has, however, not been examined completely at the macro level.

The purpose of the present study is twofold and could be seen against the background of what has been said above:

(i) The first aim is to design a theoretical framework within which the problems discussed above can be analyzed. The need to find an equilibrium wage rate in a long-run growth perspective was stressed above. Neoclassical growth theory is well-suited for defining this long-run (steady state) equilibrium wage rate. The theory has, however, not been developed very far in the case of a small open economy. In chapter 2, I will thus make some attempts to adapt generally accepted neoclassical growth models for closed economies to the case of the small open economy. Some work has been done with this ambition before,[1] but comparatively little.

(ii) The second aim of the present study is to try to adapt the hypothesis about the causes of the stagnation in Swedish industry to the framework developed in chapter 2. The mechanisms earlier formalized by Jungenfelt [1982] will thus be fitted into the macro growth model framework developed in chapter 2.

Simulations with this extended model will be conducted, illustrating its empirical relevance.

The nature of the present work is thus not to test a hypothesis, but rather to develop a theoretical framework within which the problems referred to above can be analyzed, and to illustrate the empirical relevance of this framework.

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1 See e.g. Kouri [1979], Bruno [1976], Korkman [1980].

In the next section, in accordance with the first aim of this work, I make a minor change in the assumptions behind Solow's famous one-sector growth model,[1] adapting it to the case of a small open economy. Using that framework I can then more clearly state the hypothesis about the mechanisms behind the Swedish growth experience. It will also be possible to describe the theoretical extensions that are made, by referring to the simple model.

1.2 A SIMPLE MODEL OF GROWTH IN AN OPEN ECONOMY

In this section, I will develop a one-sector growth model, as an introduction to the models that will be used in the rest of this study. This model can serve as a prototype for the models that follow. Its general equilibrium properties are discussed in the present section. In section 1.3, I discuss how the model, or rather some equations from it, can be used partially to comment on the traded goods sector. I will assume that wages are exogenous, and show how output, employment, investments and labour productivity respond to exogenously growing wages. In section 1.4, this is compared to the actual development of the above-mentioned variables. The comparison will serve as a basis for discussing the development of the model for empirical use.

The model can be characterized in the following way:

(i) It is a one-sector model. The good can be used either for consumption or as a capital good in production. The production function is homogeneous of degree one in capital and labour.

(ii) The economy is small and open. This means that the price of the good is exogenous. There is assumed to exist a world market rate of return (r*) on some asset (like an internationally traded bond). This rate of return is also

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[1] Solow [1956].

exogenous to the small country. Furthermore, the domestic market for the good does not necessarily clear. There can be imbalances in the trade with the rest of the world.

(iii) All labour income is instantaneously consumed and all profit income saved (though not necessarily invested).

(iv) Technological progress is Harrod-neutral.[1]

(v) Labour and capital are always fully employed.

The following notation will be used:

Q	output
K	the stock of capital
L	labour
λ	the rate of Harrod-neutral technological progress
w	the wage rate in terms of the good
r*	the internationally determined rate of return
r	the domestic rate of return, equal to the marginal physical product of capital
I	investments; the change in the capital stock
D	consumption demand

The production function can be written as:[2]

(1:1) $$Q = F\left(K,\ L \cdot \exp(\lambda t)\right)$$

It is assumed to have constant returns to scale. The assumption about Harrod-neutral technical progress makes it possible to write the function with labour measured in efficiency units.

The supply of labour is assumed to be constant:

(1:2) $$L^S = \bar{L}$$

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1 Harrod-neutral technological progress will be assumed throughout. I make the assumption to ensure the existence of a steady state growth path in the models.

2 For typographical reasons I will throughout denote $e^{f(t)}$ by $\exp\left(f(t)\right)$.

The domestic rate of return is assumed to be equal to the physical marginal product of capital:

$$(1:3) \qquad r = \frac{\partial Q}{\partial K}$$

Since firms are profit-maximizing, they will employ labour up to the point where the marginal product of labour is equal to the wage rate:

$$(1:4) \qquad w = \frac{\partial Q}{\partial L}$$

Under constant returns to scale, the marginal product is a function of the labour/capital ratio (with labour measured in efficiency units). Since $\partial^2 Q/\partial L^2 < 0$, (1:4) can be inverted to give the firm's demand for labour:

$$(1:5) \qquad L^D = n(\omega)\exp(-\lambda t)K$$

where $n \equiv \frac{L\cdot\exp(\lambda t)}{K}$ is the labour/capital ratio in efficiency units, and ω is the wage rate per efficiency unit of labour (i.e. $\omega \equiv w\cdot\exp(-\lambda t)$).

Since the marginal physical product of capital is a function of the labour/capital ratio, it can also be written as a function of ω; $r = r(\omega)$ with $r' < 0$.[1]

Consumption demand is, by assumption, equal to the total wage bill:

$$(1:6) \qquad D = w\cdot L$$

Investment demand is assumed to be determined by the following relation:

$$(1:7) \qquad I = \dot{K} = \left[\lambda + \varphi\left(r(\omega)-r^*\right)\right]K$$

where $\varphi(0) = 0$, $\varphi' > 0$. A thorough motivation for this formulation will be given in chapter 2. The intuitive idea behind it

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[1] This defines the factor-price frontier. See e.g. Samuelson [1962] or Burmeister, Dobell [1970] for a textbook treatment.

is simple, however. Costs of adjusting the capital stock are assumed to exist. This implies that if the capital stock is not optimal, it will only gradually adjust to its optimal value. A feature of a long-run, steady state equilibrium is that $r = r^*$, i.e. that the domestic rate of return is equal to the international rate of return. The expression (1:7) shows that if $r > r^*$ the capital stock will grow *faster* than λ, and vice versa. Since λ is the exogenous resource growth in the model, it is also the rate of steady state growth. Thus $r > r^*$ means that the capital stock grows faster than in steady state.

If the two demands for the good, consumption demand and investment demand, are added, it is seen that they do not necessarily add up to total income. The difference is explained by foreign lending or borrowing. This is the essential difference between the present model and Solow's [1956] one-sector growth model. Solow's model is reached, formally, by assuming that investment always adjusts to the level of savings, and by replacing the assumption about Harrod-neutral technological progress with the growth of the labour force (which is functionally equivalent). These changes make the model a closed-economy model.

In short-run equilibrium it will be assumed that there is full employment. Hence it will be assumed that for all points in time, the following holds:

$$(1:8) \qquad \bar{L} = n(\omega) \cdot \exp(-\lambda \cdot t) \cdot K$$

The wage rate adjusts to ensure full employment. An equilibrium growth path is a growth path such that there is always full employment.

What are the features of the steady state growth path? Since $r(\omega)$ must be equal to r^* on a long-run, steady state growth path, ω must obviously be constant. By definition ω is $w \cdot \exp(-\lambda t)$. Thus the wage rate in terms of the good must grow at the rate λ in steady state growth. As is seen from the investment demand equation, the capital stock also grows at the rate λ in steady state. Looking at (1:8) it is furthermore clear that there can be full employment in steady state.

Trade is balanced when total domestic supply is equal to total domestic demand. Trade is balanced in steady state if the following equality holds:

$$(1:9) \qquad q(\omega)\cdot K = \lambda\cdot K + w\cdot L$$

where $q \equiv Q/K$, i.e. the output/capital ratio. Under constant returns to scale it is a function of the labour/capital ratio, which in turn depends on ω. Hence it is constant in steady state growth. The left hand side of (1:9) is total output and the right hand side, total demand. Since K and w both grow at the rate λ, (1:9) can clearly be satisfied. From Euler's theorem follows that the condition can equivalently be written as:

$$(1:10) \qquad r^*\cdot K + w\cdot L = \lambda\cdot K + w\cdot L$$

Thus trade is balanced in steady state if and only if $r^* = \lambda$. Otherwise there is an accumulation or decumulation of bonds in steady state growth. This will be further analyzed in chapter 2.[1] Intuitively it is easy to see that if $r^* > \lambda$, i.e. if the rate of return abroad is larger than the rate of growth at home, it will be profitable to invest abroad, which means that more is saved than is invested domestically. Thus the country runs a balance of trade surplus, and aggregate supply is greater than aggregate demand. If $r^* < \lambda$, the converse is true. The country runs a balance of trade deficit and invests more than it saves.

Thus there exists a steady state growth path for this simple model. If $r^* = \lambda$, which is a relation between two exogenous parameters, trade is balanced in steady state.

The main emphasis in this study will be on the stability of steady state growth. What happens if the model is given

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[1] Appendix 2:2.

initial conditions not on the steady state path? Will the economy converge to steady state growth? To look at this, it is convenient first to define a new variable: $\bar{K} \equiv K \cdot \exp(-\lambda \cdot t)$. Using this variable, the full employment condition can be written:

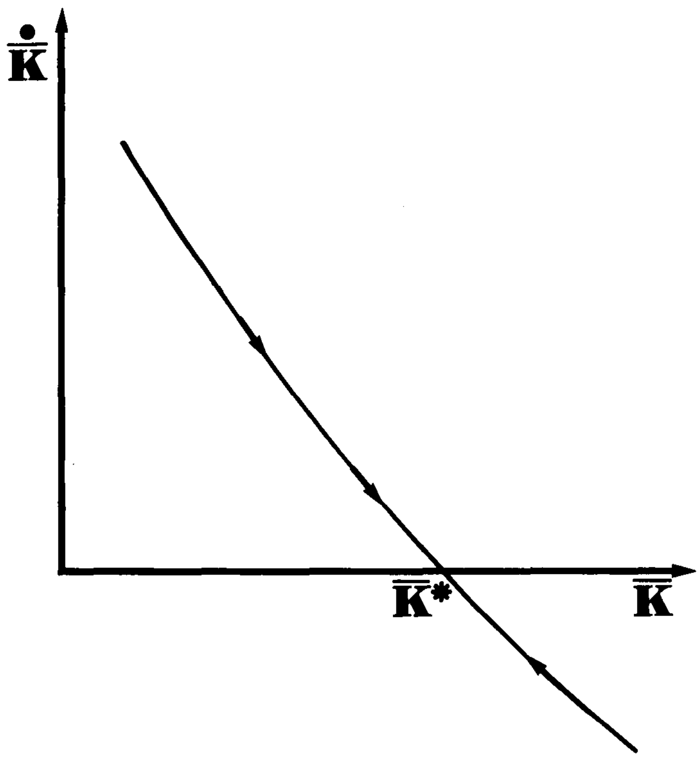
$$(1{:}11) \qquad \bar{L} = n(\omega) \cdot \bar{K}$$

This defines ω as a function of $\bar{K}$, since $n(\omega)$ can be inverted. Furthermore $\omega'(\bar{K}) > 0$. A larger capital stock must be associated with a higher real wage to clear the labour market. $\omega(\bar{K})$ can be substituted into the transformed differential equation (1:7):

$$(1{:}12) \qquad \dot{\bar{K}} = \varphi\left[r\left(\omega(\bar{K})\right) - r^*\right] \cdot \bar{K}$$

This is a nonlinear differential equation in $\bar{K}$. The phase diagram is shown in Figure 1.1.

Figure 1.1 The one-sector, full employment model



It is clearly seen from the diagram that the model is stable. The condition for local stability is that $d\dot{\bar{K}}/d\bar{K} < 0$ at the equilibrium point, or that:

$$(1{:}13) \qquad d\dot{\bar{K}}/d\bar{K} = \varphi' \cdot r' \cdot \omega' \cdot \bar{K}^* < 0$$

which is satisfied under the assumptions that have been made. $\bar{K}^*$ denotes the equilibrium value of the capital stock. Thus, if the value of $\bar{K}$ is close to the steady state value, it converges to the capital stock defined by the steady state path. To the left of $\bar{K}^*$ in Figure 1.1, the capital stock is small relative to steady state. Hence the wage rate is low and profits are high. This means that growth is fast. To the right of $\bar{K}^*$ the converse is true. Profits are low and growth is slow.

This model may serve as a prototype for the models that follow in subsequent chapters. When describing the outline of the study in section 1.5, I will refer to it to describe the models that follow. First I will, however, use some relationships from the model, although not the full model, to discuss some features of post-war growth in the traded goods sector in Sweden.

1.3 SOME PARTIAL RELATIONSHIPS

In this section I regard wages as exogenously given, and look at the effects of an exogenous development of wages on output, investment, employment and labour productivity. I do this in order to make comparisons with the actual development in the traded goods sector in Sweden.

Assume that there are two sectors in the economy, one producing a nontraded good and the other producing a traded good. In both sectors the production functions, the relationships derived from them, and the investment functions are of the form described in section 1.2. If I assume that wages are somehow exogenously determined, this means I can study the effects of the wage path on output, labour demand and investments. If wages were exogenously increased at the rate of λ, a steady state path like the one derived in section 1.2 could be reproduced.[1]

It is thus assumed that wages and the capital stock are functions of time, $\omega(t)$, $K(t)$. The levels of the wage rate and the capital stock are state variables.

Differentiating the investment function, (1:7), with respect to time yields:

$$\dot{I} = \left[\left[\lambda + \varphi\left(r[\omega(t)] - r^*\right)\right]^2 + \varphi' \cdot r' \cdot \dot{\omega}\right] \cdot K(t) \tag{1:14}$$

The change in investments at any point in time may be positive or negative depending on how rapidly wages rise. The faster wages increase, the slower investments increase. Note that if the *level* of wages is low relative to steady state, equation (1:14) is likely to be positive, regardless of whether $\dot{\omega}$ is high or not. In this case profits are high but decreasing. Both the level and the rate of change of wages are thus important to determine the change in investments.

If labour demand is differentiated with respect to time, (1:15) results:

$$\dot{L}^D = \left[n' \cdot \dot{\omega} + n \cdot \left[\lambda + \varphi\left(r[\omega(t)] - r^*\right)\right]\right] \cdot K(t) \tag{1:15}$$

The change in labour demand may be positive or negative. A higher $\dot{\omega}$ will, however, always mean a lesser incerase (or a decrease) in labour demand. A level of ω *lower* than in steady state means that $\dot{L}^D$ is likely to be positive, but with a smaller value for a higher $\dot{\omega}$.

The change in output is shown by (1:16):

$$\dot{Q} = \left[q' \cdot n' \cdot \dot{\omega} + q \cdot \left[\lambda + \varphi\left(r[\omega(t)] - r^*\right)\right]\right] \cdot K(t) \tag{1:16}$$

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[1] It takes a bit more to show this in a two-sector model. That will be postponed until chapter 2, however.

Output will increase more slowly the faster wages increase. If the *level* of wages is low relative to steady state, output will grow fast. This is explained by a high profitability which gives a rapid growth of the capital stock.

The change in labour productivity can, with a slightly different notation, be written as:

$$(1{:}17) \qquad (\dot{Q/L}) = Q \cdot \frac{K}{L} \cdot \frac{dL^D}{d\omega} \cdot \frac{\omega}{L^D} \cdot \left[\frac{dQ}{dL} \cdot \frac{L}{Q} - 1\right] \cdot \frac{\dot{\omega}}{\omega}$$

The expression is positive for $\dot{\omega} > 0$, since $dL^D/d\omega < 0$ and $0 < dQ/dL \cdot L/Q < 1$, from the assumption about linear homogeneity. Thus labour productivity can be expected to increase as wages increase. The simple mechanism behind this is the increase in capital intensity as the wage rate increases.

These simple relations thus show that if wages are regarded as exogenous, different combinations of the *level* of wages relative to steady state and the *rates of change* of wages will have different effects on the growth of the aggregates I, L^D, Q and Q/L. In the next section, I will see whether the observed development of the real wage in the traded goods sector in Sweden during the post-war period, has given rise to the paths of the variables that these relationships might lead one to expect.

1.4 THE SWEDISH POST-WAR DEVELOPMENT

When analyzing the data in terms of the relationships discussed in section 1.3, assumptions need to be made about the steady state path. In the foregoing section, it was noted that it is decisive for the time paths of the variables whether the wage rate is above or below the steady state path. I will be very brief in giving the reasons for my particular choice of a steady state path here. A more thorough justification will be made in chapter 4.

I will look at the Swedish post-war growth, and treat real wages in terms of the traded good as an exogenous variable whose time path determines the time paths of output (value added), employment (which I shall treat as equal to labour demanded by the firms), investment and labour productivity.

The rate of steady state growth is equal to λ, the rate of Harrod-neutral technological progress. I assume that it is equal to 5 %. The growth rates of the variables studied have been above as well as below this value during the post-war period. The choice of 5 % growth makes it, in my opinion, possible to make the most convincing analysis of post-war growth. The *exact* figure, 5 %, is, however arbitrary.

It will be assumed that the steady state level of wages was 20 % *above* the actual value in 1950 and that the capital stock was *not smaller than* in steady state the same year.

The reasoning behind the assumption about the wage level will be developed in chapter 4. Briefly, there are two major reasons for this assumption: *First*, it seems to be a widespread notion that the Swedish devaluation, with around 15 %, in 1949 was excessive.[1] It created an international margin for growth in the Swedish industry. *Second*, Sweden's competitive position was strong due to the fact that the country was outside World War II. While other countries were rebuilding, Sweden had the productive capacity to supply the investment goods that were demanded. In my view, both these arguments can be used to defend the assumption that in 1950 the steady state wage path was above the actual real wage.

Empirical evidence for the development of the capital stock is unreliable due to measurement problems. To assume that it is larger than or equal to the steady state value, implicitly means that I assume that the wage level before 1950 was below or at the steady state level for a sufficiently long period.

Given these assumptions about the steady state, the development of w is shown in Figure 1.2.

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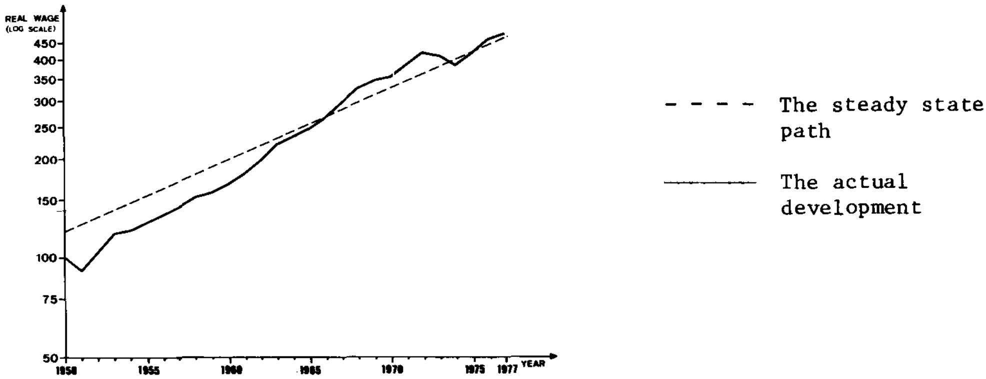
[1] See e.g. Lundberg [1971].

Figure 1.2 The real wage rate in the traded goods sector in Sweden 1950-1977



Data source: The National Accounts.

The real wage rate is defined as the wage per hour divided by a price index for the sector's value added.

The traded sector includes: forestry, fishing, mining plus the whole industry sector excluding import-sheltered food manufacturing and the beverage and tobacco industries. This is the aggregation used in the EFO [1973] study.

Note that the diagram is logarithmic-linear. In the diagram is drawn, besides the path for the real wage, a steady state path with a 5 % growth rate. The level of the steady state wage is 20 % above the actual real wage in 1950.

Given the assumptions behind the diagram, it illustrates two stylized facts:

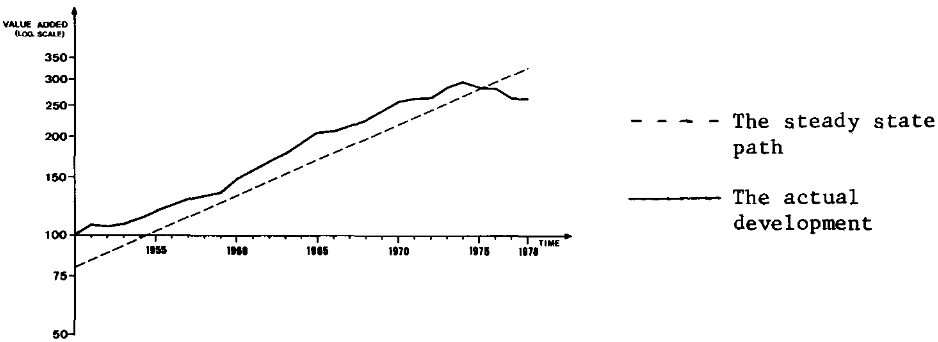
(i) The post-war period can be divided into two periods: the period before 1965, when the real wage was *below* the steady state level, and the period after 1965, when it was (with the exception of 1974) *above* the steady state level.

(ii) There was a marked acceleration in the rate of growth of the real wage between 1953 and 1968. During the latter part of that period the real wage grew considerably faster than in steady state.

These two stylized facts should, according to the growth model of the previous section, have certain effects on the other variables. Does the simple growth model help to explain what has happened?

Figure 1.3 shows value added and an assumed steady state path between 1950 and 1977.

Figure 1.3 Value added in the traded goods sector in Sweden 1950-1977



Data source: The National Accounts.

In Figure 1.3 it has been assumed that the steady state level of value added in 1950 was 20 % *below* the realized level. The assumptions already made about w and K necessarily mean that Q was larger than in steady state in 1950. Since the model is not specified numerically, I do not know whether 20 % is an acceptable number or not. I have chosen the number for symmetry purposes. It is, however, arbitrary.

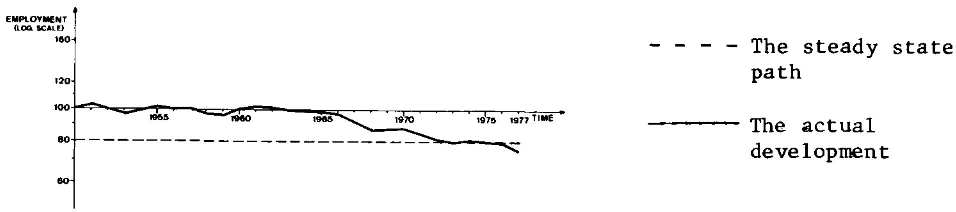
Expression (1:16) shows that output grows more slowly as wages increase faster, because rapidly rising wages cause the firms to employ progressively less labour at the given capital stock. Lower rates of employment decrease output. A high wage level depresses profits and hence decreases investments, so the increments to the capital stock decrease. A high wage level will thus also depress the growth of output.

In Figure 1.3, output does not respond to wages as would otherwise be expected. The stylized facts mentioned earlier imply that output should grow slower and slower from 1953 to 1968, and after 1968 at a roughly constant rate. The path for output must not inevitably cross the steady state path in 1965, since it takes time for the capital stock to adjust. It seems, however, unlikely in the light of the relationships in section 1.3, that it should take ten years for output to fall below the steady state level.

The conclusion from this interpretation of the growth of output is that the reaction to the exogenously rising wages arrived some ten years later than might be expected from the relationships in section 1.3.

As for the development of employment (in physical units), it is assumed to be constant in steady state. Furthermore, the steady state level is taken as 20 % *below* the actual employment level in 1950. This relationship is analogous to the placement of steady state *output* 20 % below actual output in 1950. Theoretically, employment should be above the steady state level. I use the number 20 % again for symmetry purposes. Figure 1.4 demonstrates this assumption.

Figure 1.4 Employment (hours worked) in the traded goods sector in Sweden 1950-1977



Data source: The National Accounts.

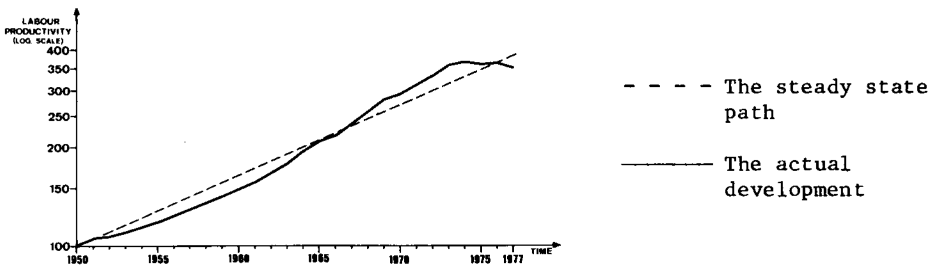
The model implies qualitatively the same reaction in terms of employment as in terms of production. Expression (1:15) shows how labour demand changes in the model when the real wage changes. Between 1950 and 1968, considering the stylized facts, I expect employment to increase initially although at a decreasing rate, and eventually maybe to decrease. After 1968 employment should be roughly constant.

Once again, this is not the case. Employment is more or less constant up to 1965. After that it decreases and reaches the steady state level around 1975.

This is similar to the pattern for production. The reaction to the wage increase appears later than expected.

Since I assume the steady state levels of both output and employment to be 20 % below the actual levels and since labour productivity is the ratio between them, labour productivity should be at its steady state level in 1950. Figure 1.5 shows its development.

Figure 1.5 Labour productivity in the traded goods sector in Sweden 1950-1977



Data source: The National Accounts

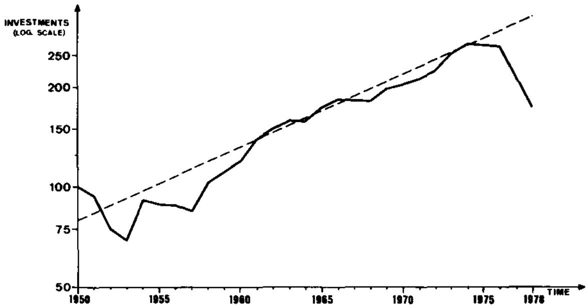
Equation (1:17) shows how labour productivity can theoretically be assumed to develop.

Given the stylized facts, it should accelerate up to 1968 and then grow at an approximately constant rate. This is roughly what happened. Since, in the expression defining labour productivity, neither the denominator nor the numerator behaved as theory predicted they would, the resulting conformity to theory may come as a surprise.

In the period 1953-1968 labour productivity theoretically should have accelerated relative to steady state as a result of a faster decrease in employment than in output. In reality, labour productivity accelerated relative to steady state as a result of increasing output and approximately constant employment. The theory, as it is presented, does not satisfactorily explain this.

Finally, the theory can be applied to the development of investments. Figure 1.6 shows investments in machinery.

Figure 1.6 Investments in machinery in the traded goods sector in Sweden 1950-1978



- - - - The steady state path

——— The actual development

Data source: The National Accounts

Equation (1:14) shows how investments are determined to behave by the model. The larger $\dot{\omega}$ is, the smaller $\dot{I}$ should be. The lower the wage *level* is, the higher $\dot{I}$ should be. Given the stylized facts, I would expect investments to grow at a decreasing rate up to 1968 and then grow at an approximately constant rate. In Figure 1.6, I have assumed that in 1950 the steady state level was 20 % below the actual level. The reasons for this are the same as the reasons for the corresponding assumptions in the earlier diagrams.

The actual development in Figure 1.6 does not correspond closely to the theoretical prediction. There is no sign of investments growing at a decreasing rate (at least not until 1975).

One possible explanation for the inaccuracy of the theoretical prediction for investments is the static expectations assumption used here. If investments reacted to wages with a lag, it might improve the fit between the curves. The introduction of lags in the investment function might also help to explain the other series.

One would, however, need lags of about ten years' length in the investment function in order to match the curves. *A priori* this seems to be too long a time for a plausible investment function. Still, an econometric study of the lags in the investment function and its effects on other variables may be worthwhile. Another means to improve the results is to change the model, and this study is an elaboration of that approach.

1.5 OUTLINE OF THE STUDY

In the preceding two sections, a simple growth model has been used to discuss post-war growth in the Swedish economy. Against this background it is now possible to describe the theoretical and empirical problems which are treated in this study.

In chapter 2 the simple growth model from the present chapter is extended in various ways. The focus of the analysis is on various aspects of adjustment to steady state growth in a small open economy.

The assumption about market-clearing wages is relaxed in section 2.3. Instead, a wage rigidity is assumed. The presence of a wage rigidity in the model causes cycles in profits and the rate of growth of the capital stock. These cycles may have a certain empirical relevance.

In section 2.4 the model is extended to a two-sector growth model with one traded and one nontraded good. The labour market and the market for the nontraded good are assumed to clear in short-run equilibrium. The rates of technological progress are assumed to be equal in both sectors, and capital is assumed to be non-shiftable. Non-shiftable capital implies that once an investment is made in one sector, the piece of capital stays there forever. The stability analysis shows various cases of non-steady state growth. Stability conditions are given in terms of elasticities on the production function and the demand function.

In section 2.5, the wage rigidity is introduced in the model from section 2.4. This gives a richer dynamic analysis. Again, the emphasis is on stability analysis. One interesting result is that a short-run disequilibrium, say a wage shock, will have real consequences that can be expected to last for a long time (though not infinitely), unless the disequilibrium is quickly corrected by measures of economic policy.

Finally, the model from section 2.4 is changed so that the rates of technological progress are assumed to be different in the two sectors. This assumption is introduced because it gives the model a certain resemblance to the so--called "Scandinavian model", or "EFO model". The EFO model is a model for studying inflation. It has played an important role in the Swedish debate and possibly also in the wage negotiations, and is therefore of considerable interest. I show that in order for a steady state to exist, both the community utility function and the production function must be Cobb-Douglas. Thus very restrictive assumptions must be placed on the model, if it is to accomodate this particular assumption.

In an appendix to chapter 2, the accumulation of bonds in the economy is explicitly treated. This primarily shows that unless one assumes that $\lambda = r^*$, the model gives unacceptable conclusions. Thus it should be assumed that the domestic rate of steady state growth, λ, is equal to the world market rate of interest, r^*. The latter could be interpreted as the world market rate of steady state growth.

The role of chapter 2 is to further develop the framework from chapter 1. It gives various insights into problems of growth in a small open economy. In particular, the chapter analyzes various growth paths in the adjustment to steady state growth.

There are, however, at least two problems which these models do not treat. *The first one* is the question of lags. It has been noted that Sweden experienced a considerable lag between a change in wages and its effects on real variables, like employment and production. A lag can be built into the model in at least two ways. The first way is to recognize that expectations about future prices and wages must enter into decisions about real variables. Expectations may adjust slowly to the development of wages and prices, and this adjustment may be sufficient to explain the lags. From the cursory examination of the data that I have carried out in this chapter, it seems as if the lags would have to be up to ten years long.

The second way to formalize the lag, consists in assuming that there is a slack in production at each point in time.[1] I will assume that the firms do not operate their machines with the technically maximal efficiency. They do, however, increase efficiency at each point in time. The reason for this may be what has in the literature been termed learning-by--doing.[2] The basic idea is simply that it takes time for the firms to learn to operate their machines in the most efficient way. I will use what I call an *efficiency function* to describe this process. The assumption that efficiency increases faster as labour costs increase more rapidly, can help to explain the lag. If efficiency can, initially, respond to increasing wages, there need be no immediate reaction to the increasing wages. It seems reasonable, however, that the potential for increases in efficiency may sooner or later be exhausted if wages continue to rise. Thus, eventually, production and employment should stagnate, if wages continue to be above their steady state value for a long time. In this way, one may design a fundamentally different mechanism that provokes a lag.

---

[1] The slack concept is discussed by Leibenstein [1966].

[2] The original reference is Arrow [1962].

In chapter 3 I will use both a simple type of lag in, for example, the investment function, and the slack mechanism to create a more realistic growth model.

The second important aspect which makes the model more realistic, is the scrapping of machines. The scrapping of capital in industry was large in Sweden after 1965, when real wages were above the steady state level. Since there were disincentives to invest, the size of the capital stock became a significant problem. These problems are highlighted if the model allows for the scrapping of machines.

A *vintage* growth model allows for a much richer analysis, including endogenous scrapping of machines. It does, however, also complicate the analysis. Rendering the model more realistic does carry the disadvantage of increased complexity.

In chapter 3 I will thus further develop the model of growth in an open economy. Two important features are added:

(i) Capital is assumed to be heterogeneous in the sense that there are different vintages of capital. The model is a vintage growth model of what has in the literature been termed a putty-clay type.

(ii) The two types of lags which were discussed above will be incorporated into the model. Particular attention will be paid to the efficiency function.

The basic structure of the model will, however, be the one used in chapter 2. I will distinguish between a traded and a nontraded good. The existence of a steady state growth path will be shown. My main interest with the vintage model will be on *growth outside steady state*. The steady state path is, however, important since some relationships in the model can be derived as deviations from the steady state path. In the investment function, I will start with steady state investments, which are easy to compute, and then specify how investments may deviate from this path. Also the development of the product real wage, which I will use as the exogenous variable, will be described in terms of its deviation from the steady state path. The aim of chapters 3 and 4 will thus be accomplished by deriving the steady state path, and studying deviations from it. Thus, the steady state path will play a central role, although I am not primarily interested in steady state growth *per se*.

Chapter 4 will subsequently contain numerical simulations with the model.[1] I will show how production, employment, investments, the life of capital and the distribution of efficiency over vintages vary when wages are exogenously varied around the steady state path as in Figure 1.1. The purpose of chapter 4 is to illustrate that the model of chapter 3 is capable of simulating, in a satisfactory way, the actual development in the traded goods sector.

In the Swedish policy debate in the 1970s, much attention was given to what is known as the "cost crisis". Sweden's wage costs deviated considerably from those of its trading partners in the mid-'70s. In the debate, the cost crisis was given an important role in the explanation of the stagnation.[2]

If the explanation of the stagnation in the '70s in this study is true, it has important policy implications. A short-run wage shock can be quickly corrected and growth restored. The stagnation pictured in the present study takes a longer time to reverse. This is, of course, essential when choosing measures of economic policy. The present study (i) shows that the stagnation would have come regardless of the "cost crisis" and (ii) explains why the effects of the wage shock were so severe.

A short discussion of the implications of this model for economic policy, as well as a summary and some remarks about its empirical relevance, will be given in chapter 5.

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1 Numerical simulations with a vintage growth model, applied to the Swedish experience, have been carried out by Bentzel [1978]. It differs from the present study in a number of respects, notably in that it analyzes a longer time period (1870 to 1975) and in that it is a one-sector model. While the present study treats problems in the medium run, Bentzel's study treats long-run problems.

2 An example of this is a government committee, the so-named "Bjurel-group" (See *Vägar till ökad välfärd* [1979]).

2 Growth Models for Open Economies with Non-Shiftable, Malleable Capital and Nontraded Goods

2.1 AN OVERVIEW OF THE CHAPTER

The general purpose of this study is to analyze the adjustment to steady state growth in a small open economy. An initial, simple model designed for this purpose was presented in chapter 1. *Adjustment* in that model consists of the capital stock adjusting to the steady state path. The basic process is, that if the capital stock is smaller than in steady state, wages are lower and hence profits are higher. Higher profits are assumed to provide incentives to increase the capital stock rapidly. Hence, the capital stock approaches its steady state value.

The assumption that only the capital stock adjusts slowly to its equilibrium value, while all other variables adjust instantly, describes poorly the dynamics of the adjustment process. In reality, most variables do not immediately find their equilibrium values. There are many examples in economic theory where the interaction of different variables, as they adjust to their equilibrium values, gives rise to cyclical developments or even to instability. The theme of this chapter is to study the interaction of two, or more, variables as they adjust to their long-run equilibrium values.

The time period needed for a variable to find its equilibrium value varies between different variables. In this study I am concerned with medium- or long-run adjustments. Consequently, I am not interested in those variables that adjust to their equilibrium values within short time periods (within a year for example).

Referring to the model in chapter 1, it seems to me that the assumption of a clearing labour market should be relaxed. Thus, there need not be any short-run equilibrium, using the terminology from chapter 1. It is an empirical fact that there may be excess supply or excess demand in the labour market for long periods of time. This may be because the wage rate is slow to react to a state of excess supply or excess demand. Wages may be *rigid* in the short run and only gradually adjust to their equilibrium value.[1] In this chapter, I will try to analyze some effects of rigid wages on the adjustment to steady state growth in a small open economy.

In the debate on growth problems for the Swedish economy, the question of adjusting the *structure* of the economy has been central. Particularly the shares of the traded goods sector and the nontraded goods sector in GNP have been debated. To be able to analyze structural adjustment in a model, at least two sectors are needed. I will analyze a two-sector model with a traded and a nontraded good, where capital is *non-shiftable*[2] between the sectors. I will thus assume that once an investment is made in a sector, the machine remains in that sector. It cannot be transferred to another use. This has the consequence that rates of return may differ in the two sectors in the short run. Since capital cannot be transferred between the sectors, rates of return can only be equalized

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[1] On rigid wages in a growth model for a small, open economy, see Kouri [1979] or Bruno [1981].

[2] The term "non-shiftable" is used by e.g. Inada [1966]. Sometimes the term "sector specific" is used as a synonym for "non-shiftable"; see e.g. Neary [1978].

if one sector expands faster than the other.[1] This may be regarded as a rigidity in the sectoral composition of the economy.

The consequences of the presence of these three rigidities for the adjustment to steady state growth will be the theme in this chapter. It is, however, complicated to study the interaction of both the capital stock adjustment, a wage rigidity and the adjustment of the relative sizes of the sectors at the same time. For a clearer exposition, this chapter will proceed step by step.

In *section 2.3*, I will introduce a wage rigidity in the model used in chapter 1. It is thus assumed that the wage rate is given in the short run. In the long run the rate increases, relative to steady state, if there is excess demand in the labour market, and the rate decreases if there is excess supply. The main concern in the section will be the interaction between the capital stock adjustment and the labour market adjustment. It will be shown that the rate of growth of wages and that of the capital stock *may* adjust cyclically to steady state.

To make an analysis of structural adjustment possible, a two-sector model is introduced in *section 2.4*. In order to add only one complication at a time, I will assume that the labour market clears. Thus, full employment is assumed. In this model, two types of disequilibria may occur at any point in time. Firstly, the capital stock is not necessarily in equilibrium in any one sector. Investment functions will be the same as in chapter 1. When investors compare the profitability of investing in a sector to investing in an international bond, a difference between the two rates of return leads to a decision to adjust the capital stock. That is one adjustment taking place in the model. Rates of return may however also differ between the two sectors of the economy.

---

[1] For models with *traded goods* where there is non-shiftable capital in the short run but shiftable capital in the long run, see Neary [1978], Mayer [1974], Mussa [1974] and Mussa [1978].

If the model is stable, different rates of return must give such incentives to investments in the two sectors so that in the long run, the rate of return is equalized between the sectors. That is the other type of adjustment taking place in the model. The interaction of these two types of adjustment will be analyzed and conditions for stability will be given. Here, too, different types of adjustment paths (cyclical or asymptotical) can be demonstrated.

The simultaneous treatment of all the three types of adjustment mentioned above is the topic of *section 2.5*. Thus, at the same time, the capital stocks may not be adjusted, so that the rate of return in physical capital is not equal to the return on the international bond, rates of return may differ between the sectors, and there may be unemployment. Obviously, this is a complicated case. Conditions for the economy to approach steady state growth in the long run will be given. As an example of how the adjustments may interact, I will also discuss the case of a wage shock. If the economy is in a state of steady state growth and for some reason the wage rate is increased, unemployment will result. If the wage level is immediately restored, the wage shock need not have any effects in the medium run. If it is not restored, however, investment can be expected to take place at different rates in the two sectors. All the three types of adjustment in the economy will then start affecting each other. Subsequently it may take a long time before the economy returns to steady state growth again.

Finally, in *section 2.6*, I make a minor digression from the topic of adjustment to steady state growth. In the Swedish policy debate the EFO model or Scandinavian model has played a great role.[1] It is a model for studying inflation. It defines an equilibrium growth path for wages (called a "main course" or "corridor", see Lindbeck [1979] p. 18), such that

[1] See e.g. Lindbeck [1979], Edgren, Faxén, Odhner [1973], Aukrust [1977].

the growth rate of wages is equal to the sum of the growth rate of world market prices and of productivity in the traded goods sector. The model assumes that rates of technological progress are *not* equal in the two sectors of the economy. The EFO model is not specified as a general equilibrium model. In section 2.6 I investigate whether the equilibrium growth path of the EFO model could be interpreted as the steady state growth path for a two-sector growth model of the kind used here.

I conclude that it can be interpreted as a steady state growth path, *if* the production functions in both sectors are Cobb-Douglas and *if* the community utility function is also Cobb-Douglas. These are very restrictive assumptions.

2.2 SOME BASIC RELATIONSHIPS

In this section some relationships will be given that are used throughout this chapter, regardless of the differences in specifications of the models that may follow.

The production functions are written as:

$$Q = Q\big(K, L\cdot\exp(\lambda\cdot t)\big) \tag{2:1}$$

where Q denotes output, K the amount of capital, L labour and λ is the rate of Harrod-neutral technological progress.[1]

The production functions are assumed to have constant returns to scale. Given the assumption of Harrod-neutral technological progress, they can be written as (2:1) with the labour input measured in efficiency units.[2]

---

[1] All notation can be found in Appendix 2:1.

[2] Harrod-neutral technological progress is assumed in order to ensure that steady state growth exists in the models that follow. Alternatively growth of the labour force could have been assumed. The two assumptions are functionally equivalent but technological progress seems like a better description of at least Swedish reality.

From an assumption of profit maximization and a given capital stock, it follows that the firms hire labour up to the point where the value of its marginal product is equal to the wage rate:

$$(2{:}2) \qquad w = \frac{\partial Q}{\partial L}$$

The assumption of linearly homogeneous production functions implies that production can be written as a function of the labour/capital ratio in efficiency units. Profit maximization further means that these labour/capital ratios are chosen as functions of the ratio between the wage paid to an efficiency unit of labour and the price of output,[1] thus:

$$(2{:}3) \qquad n = n(w \cdot \exp(-\lambda t)) \equiv n(\omega) \qquad n' < 0$$

where $n \equiv (L/K) \cdot \exp(\lambda t)$ and $\omega \equiv w \cdot \exp(-\lambda t)$. The demand for labour follows directly from (2:3):

$$(2{:}4) \qquad L^D = n(\omega) \cdot \exp(-\lambda t) \cdot K$$

Under constant returns to scale the output/capital ratio too is a function of the labour/capital ratio, thus:

$$(2{:}5) \qquad Q = q(n(\omega)) \cdot K \qquad q' > 0$$

(2:5) gives output as a function of the wage/price ratio and the stock of capital.

The domestic rate of return is assumed to equal the physical marginal product of capital:

$$(2{:}6) \qquad r = \frac{\partial Q}{\partial K}$$

---

[1] From (2:2) is seen that profit maximization implies $w = (\partial Q/\partial L)$. The assumptions on the production functions mean that $(\partial Q/\partial L)$ is a function of $(L/K) \cdot \exp(\lambda t) \equiv n$. Furthermore, $(\partial^2 Q/\partial L^2) < 0$ so the function can be inverted giving (2:3).

The supply of labour is assumed to be exogenously given and constant

(2:7) $L^S = \bar{L}$

Throughout this chapter I assume that all wage income is instantaneously consumed. If D denotes consumption, this assumption is written:

(2:8) $D = w \cdot \bar{L}$

Savings are assumed to equal total capital income.[1] If B denotes the total holdings of bonds, r* the rate of return to bond holdings and K the physical capital stock, the savings assumption can be written:

(2:9) $S = r \cdot K + r^* \cdot B$

All savings are not necessarily invested in domestic capital. This is another expression for a possible surplus or a deficit in the balance of trade.

Investment demand will be made up of one equilibrium and one disequilibrium part. In steady state, the capital stock will grow at the rate λ. Outside steady state, the rate of growth of the capital stock (which in the absence of depreciations is equal to gross investments) will depend on the rate of return of capital compared with the rate of return to the alternative asset; r*.

---

[1] The alternative assumption concerning savings/consumption, is to assume that a constant fraction of gross national (or gross domestic product) is consumed. If I had made that assumption, I would have had to include the stocks of bonds and capital in the consumption function. That would have considerably complicated the analysis that follows. I wished to avoid that complication.

(2:10) $I(t) = \dot{K}(t) = [\lambda + \varphi(r(t)-r^*)]\circ K(t)$

where $\varphi(0) = 0$ and $\varphi' > 0$.

In capital market equilibrium the rate of return of capital must be equal to r^*. This means that, according to (2:10), $\dot{K}/K = \lambda$.

The disequilibrium part (i.e. the φ-function) can be derived in the following manner:[1]

When a firm decides whether or not to make an investment, it considers the discounted value of the stream of incomes the investment will generate. Income from the investment will be equal to the physical marginal product of capital. The discounted value of the future income stream will be called the demand price of capital.

The supply price of capital is always 1. Thus, equality between the supply price and the demand price at time t' gives:

(2:11) $\int_{t'}^{\infty} r(t)\cdot\exp(-r^*(t-t'))dt = 1$

If (2:11) is assumed to hold at all points in time, it can be differentiated with respect to t'. That yields equality (2:12):

(2:12) $r(t') = r^*$

The equation (2:12) thus shows that in capital market equilibrium the rate of return of capital is equal to r^*. In the investment equation (2:10) investment depends on the difference between r and r^*. This follows if one assumes *stationary expectations*, i.e. that the investor assumes that today's rate of return will last forever. If $r(t) = r$ is substituted in (2:11) the integral can be solved yielding $r/r^* = 1$. This means that if $r > r^*$, the demand price of capital

[1] See Stein [1971] p.60, or Sargent [1979].

is greater than the supply price and investments should be expected to increase. If $r < r^*$ the reverse is true. Stationary expectations will be assumed throughout this chapter.

If no cost is attached to adjustment of the capital stock, adjustment will be made instantly. If some cost is associated with adjusting the capital stock, then an investment function like (2:10) will exist.

This gives the rationale for formulating the investment function the way it is formulated in (2:10).

In all the models that follow, disequilibrium in the balance of trade gives rise to accumulation or decumulation of bonds. From the assumption that all capital income, including bond income, is saved, and since saving does not affect any other variable, it follows that the accumulation of bonds does not have any feedback on any other variables. Thus, bondholdings can be solved recursively in the models that follow.

Bondholdings in steady state are analyzed in Appendix 2.2. There it is shown that, unless it is assumed that $\lambda = r^*$, the model gives unacceptable conclusions. When $\lambda = r^*$, i.e. when the domestic rate of steady state growth is equal to the world market rate of interest, there is balanced trade in steady state. This was shown already in chapter 1. Thus, there is no accumulation or decumulation of bonds. There may well be positive or negative holdings of bonds in steady state, however.

If it is assumed that $\lambda > r^*$, for example, more will be invested than what is saved in steady state. This means that the country has a balance of payments deficit in steady state. It is demonstrated in Appendix 2.2 that this means that the whole capital stock is owned by foreigners in steady state. Conversely, if $r^* > \lambda$ the small economy will gradually take over the whole world. It will continually accumulate bonds in steady state. In this case the small country assumption could be questioned.

These conclusions are formally derived in Appendix 2.2. Since bond holdings can be solved recursively in all the models that follow, and since what has been said above is valid for all the models that follow, I will not have to comment much more on the accumulation of bonds in steady state. Note, finally, that these conclusions rest upon the particular specification of the model that is chosen here. If, for example, consumption had depended on wealth, the model would have functioned differently.

2.3 THE ONE-SECTOR MODEL WITH RIGID WAGES

Long periods of either excess demand or excess supply in the labour market is a feature of most economic systems. There are, at least, two possible, not mutually exclusive, explanations for this. Either wages adjust slowly to a full employment equilibrium, or the labour market is subject to many shocks which keep it under a state of disequilibrium for long periods. I will here investigate the effects of a slow adjustment of the wage rate to excess supply or excess demand in the labour market. I will call this *a wage rigidity*, meaning that wages are fixed in the short run but adjust over time. This process resembles the well-known Phillips curve in its simplest form.

It will be shown that the addition of a wage rigidity to the one-sector model may give rise to cycles, where high profitability leads to rapid investments. These investments drive up the wage rate, reduce profitability and growth until wages fall, and cause profits to increase again and the process thus repeats itself. The interesting feature of the model that follows is that it can generate these cycles.

The dynamic behaviour of the model is determined by the wage adjustment equation and the investment function:

(i) Wages are rigid in the short run and increase as a function of the difference between the supply of labour and the demand for labour. The equation describing wage adjustment can thus be written:

$$\dot{w} = [\lambda + \psi(\bar{L} - L^D)]w \tag{2:13}$$

$$\psi' < 0, \ \psi(0) = 0$$

where L^D,[1] as was shown in section 2.2, is determined by:

$$L^D = n\big(w \cdot \exp(-\lambda t)\big) \cdot K \cdot \exp(-\lambda t)$$

(ii) The investment function is as described in section 2.2. The assumption that expectations are stationary, i.e. that today's value of r is expected to remain forever, is important for the stability properties of the model.[2]

As in chapter 1, I define:

$$\bar{K} \equiv K \cdot \exp(-\lambda t)$$

so that the capital stock is measured in efficiency units. Using this notation the model can be written

$$\begin{cases} \dot{\bar{K}} = \varphi\big(r(\omega) - r^*\big)\bar{K} \\ \dot{\omega} = \psi(\bar{L} - L^D)\omega \end{cases} \tag{2:14}$$

---

[1] $\psi = 0$ *could* be interpreted as giving the natural rate of unemployment. $\bar{L}$ need thus not mean a binding constraint on the labour market.

[2] Constancy of the physical marginal product is a feature of steady state. Thus it may be said that expectations are such that the agents believe they are in steady state. This further means that they will have rational expectations in steady state - but only in steady state.

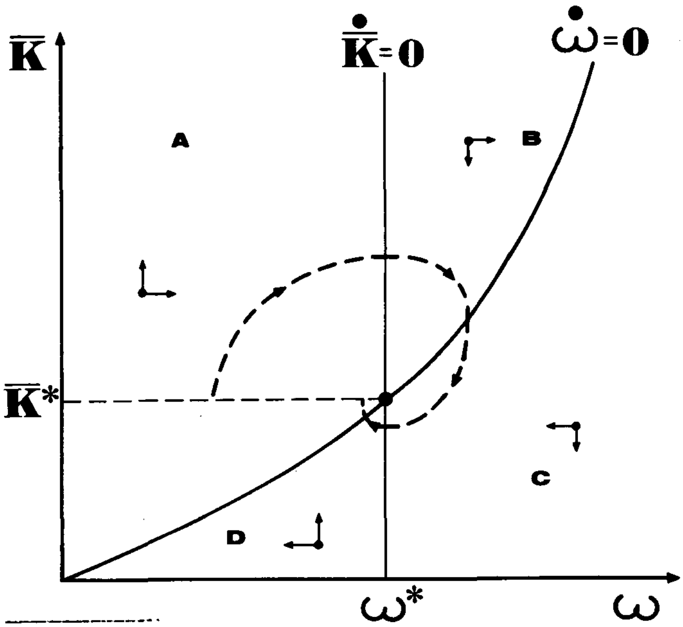
where ω as before is the wage rate per efficiency unit of labour in terms of the good $\left(\omega \equiv w \cdot \exp(-\lambda t)\right)$. The short-run equation solves:

$$(2{:}15) \qquad \bar{L} = n(\omega)\bar{K} + u$$

and give as solution $u(\bar{K},\omega;\bar{L})$, where u is unemployment. An increase in the wage rate will increase unemployment, while an increase in the capital stock will decrease it. This is a system of two differential equations in ω and $\bar{K}$.[1]

The phase diagram for the model is given by figure 2.1:

Figure 2.1 Phase diagram for the one-sector model with a wage rigidity



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[1] In principle this is the model that Kouri [1979] analyzes. He has two goods, one traded and one nontraded. The nontraded good does not use capital, however. The relative price between the traded and the nontraded good determines what part of wages is spent on which good. Since Kouri chooses units so that labour productivity in the nontraded goods sector is equal to unity, demand for the good is also equal to demand for labour in the sector. Thus the relative price also determines the share of the labour force allocated to the production of the nontraded good.

The slope of the $\dot{\omega} = 0$ line is found by implicit differentiation of the Phillips curve equation (2:13).[1] From the phase diagram is seen that the model may give rise to oscillations. It is not possible to tell from the figure alone whether or not the model is stable. To determine local stability one may linearize the model and look at the characteristic roots of the coefficient matrix for the linear system at the equilibrium point.[2]

Linearization of the system at the equilibrium point yields:

$$(2:16)\quad \begin{cases} \dot{\omega} = -\psi' \cdot n' \cdot \bar{K}^* \cdot (\omega-\omega^*) - \psi' \cdot n \cdot \omega^*(\bar{K}-\bar{K}^*) = 0 \\ \dot{\bar{K}} = \varphi' \cdot r' \cdot \bar{K}^* \cdot (\omega-\omega^*) + \varphi \cdot (\bar{K}-\bar{K}^*) = 0 \end{cases}$$

where an asterisk denotes the value of a variable in equilibrium. The characteristic equation is thus:

$$(2:17)\quad \mu^2 + \psi' \cdot n' \cdot \bar{K}^* \cdot \omega^* \cdot \mu + \psi' \cdot n \cdot \omega^* \cdot \varphi' \cdot r' \cdot \bar{K}^* = 0$$

where μ is the root. The condition for local asymptotical stability is that the characteristic roots have negative real parts. Hence the model is stable if

$$(2:18)\quad \begin{cases} \psi' \cdot n' \cdot \bar{K}^* \cdot \omega^* > 0. \\ \psi' \cdot n \cdot \omega^* \cdot \varphi' \cdot r' \cdot \bar{K}^* > 0 \end{cases}$$

---

1 The second derivative of that line is as depicted in Figure 2.1 in the Cobb-Douglas case. For more general cases I have not been able to determine the sign of the second derivative.

2 See Sydsaeter [1978] p. 261 for this stability theorem that is due to Liapunov.

which is satisfied under the assumptions made. The model is thus locally stable.[1]

What is the interpretation of the disequilibrium growth path in Figure 2.1?[2] Assume that the economy is initially in equilibrium with the wage and the capital stock growing at the rate λ in $(\omega^*, \bar{K}^*)$. The wage is suddenly decreased (e.g. following a devaluation). In region A there are high profits and low wages relative to steady state. This means that the capital stock is increasing faster than in steady state but also that there is excess demand in the labour market. Wages are thus increasing. The increasing wages squeeze profits and the growth rate of the capital stock decreases. As the economy enters region B the capital stock starts increasing more slowly than the rate of technical progress. This means that the labour/capital ratio measured in efficiency units starts to increase. The economy enters region C. Here excess supply has developed in the labour market, wages start decreasing relative to steady state and profits begin to increase again. When the economy enters region D the capital stock once more starts increasing faster than in steady state and the cycle repeats itself.[3]

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1 One needs a restriction on the higher order terms in the Taylor series expansion (2:16) for the stability theorem to be applicable. I will simply assume that the functions used are such that the stability theorem is applicable. The stability theorem, with a proof, can be found in Coddington, Levinson [1955], theorem 1.1, page 314.

Throughout this chapter I will assume that the functions used are such that the stability theorem is valid without commenting further on it.

2 The existence of cycles (or spiral points in the terminology of Coddington, Levinson [1955], page 374) may in principle be checked by looking at the discriminant of the linear system. Doing this one can conclude that there are cycles if:

$$\psi' \cdot (n')^2 \cdot \bar{K}^* \cdot \omega^* > n \cdot \varphi' \cdot r' \cdot 4$$

Thus the larger (numerically) are ψ' and n', and the smaller are φ' and r', the more likely is the solution of the model to have cycles.

This result is also subject to restrictions on the linearization. See Coddington, Levinson [1955], chapter 15.

3 The interpretation of this cycle is done in greater detail by Kouri [1979]. He also discusses the empirical relevance of the cycle for Finland.

The assumption about the formation of expectations is crucial in order for the cycles to occur. A model with rational expectations would probably not yield cycles.

Finally, one can look at the behaviour of the balance of trade in the model. As noted in section 2.2, it can be solved recursively and does not have any feedback effects on any other variable in the model.

A change in the wage level affects both savings and investments in the same direction. In order to say something about the development of the balance of trade, one has to make an assumption about which effect is greater.

The difference between domestic output and domestic consumption plus investment of the good is equal to the balance of trade, which equals the accumulation or decumulation of bonds:

$$(2{:}19) \qquad \dot{B} = \left[q\left(n(\omega)\right)\cdot\bar{K} - \omega\cdot n(\omega)\cdot\bar{K} - [\lambda + \varphi\left(r(\omega) - r^*\right)]\cdot\bar{K}\right]\cdot\exp(\lambda\cdot t)$$

(2:19) is written in terms of the variables ω and $\bar{K}$. Setting $\dot{B} = 0$, it is seen that one value of ω exists that satisfies the equation. The solution is however independent of $\bar{K}$.

It is shown in Appendix 2.2 that steady state with balanced trade can only exist if $r^* = \lambda$. Thus, looking back at Figure 2.1, the line separating balance of trade deficits from surpluses must be the same as $\dot{\bar{K}} = 0$. Will the balance of trade improve or deteriorate when wages increase? That depends on whether investments decrease more or less than savings when the wage rate increases. Setting (2:19) equal to zero and differentiating with respect to ω yields:

$$(2{:}20) \qquad [q'\cdot n'\cdot\bar{K}^* - n\cdot\bar{K}^* - \omega^*\cdot n'\cdot\bar{K}^* - \varphi'\cdot r'\cdot\bar{K}^*] \gtreqless 0$$

This is greater or less than zero depending on whether

(2:21) $$q'\cdot n'\cdot \bar{K}^* - n\cdot \bar{K}^* - \omega^*\cdot n'\cdot \bar{K}^* \gtreqless \varphi'\cdot r'\cdot \bar{K}^*$$

The left hand side of (2:21) can in turn be rewritten[1]

(2:22) $$-n\cdot \bar{K}^* \gtreqless \varphi'\cdot r'\cdot \bar{K}^*$$

The left hand side of (2:22) is the change in savings when the wage rate increases and the right hand side the change in investments. The assumption that the effects on investments are always greater than the left hand side, yields definite results in Figure 2.1. This assumption means that regions B and C are always associated with a balance of trade deficit. The opposite is true of regions A and D. Thus one could in some growth phase have unemployment and a balance of trade *deficit* (region C). From a policy point of view, this is a difficult case. The other policy dilemma is in region A, where there is excess demand in the labour market and a *surplus* in the balance of trade.

2.4 A TWO-SECTOR MODEL WITH CLEARING LABOUR MARKET

The next step in the present analysis of growth in open economies is the introduction of a second sector, which produces a nontraded good. The model is similar to Kouri's [1979] model, which also is a two-sector, traded/nontraded-goods model. There are two important differences between this model and Kouri's model, however. Kouri has capital only in one sector (the tradable goods sector), and he assumes rigid wages. In

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[1] The left hand side can be written $n'\cdot\bar{K}^*(q'-\omega^*) - n\bar{K}^*$. q' is the marginal product of an efficiency unit of labour which under profit maximization is equal to ω^*. The two terms vanish because a higher wage creates unemployment. This decreases production and consumption by exactly the same amount since all labour income is consumed and workers are paid the value of their marginal physical product.

the model in this section there is a labour market which clears.

This model is a complement to the analysis that will follow in chapters 3 and 4, which explains its development here. In chapters 3 and 4 the growth of the traded goods sector will be studied within a two-sector framework. The production functions become, though, much more complicated, by allowing for different vintages of capital. The present model has the advantage of a considerably simpler description of the production structure. It therefore permits a general equilibrium view of the problems. The stability analysis will give insight into growth outside steady state when the relative price and the product real wage are endogenous. It is also, in a fashion, a generalization of Kouri's [1979] model.

The basic assumptions underlying the present model are the following (which were also discussed in section 2.2):

(i) Technology and the investment function in both sectors are as described in section 2.2. The investment function that is used implies, as before, stationary expectations, which is important for the stability analysis.

(ii) As before, all capital income is saved, though not necessarily invested in domestic capital, and all wage income is consumed.

(iii) Capital is assumed to be non-shiftable both in the short and the long run. Once a unit of capital is invested in one sector it stays there. The investment decisions in the model would have been more complicated if existing capital could be moved from one sector to another. I have wanted to avoid that complication.

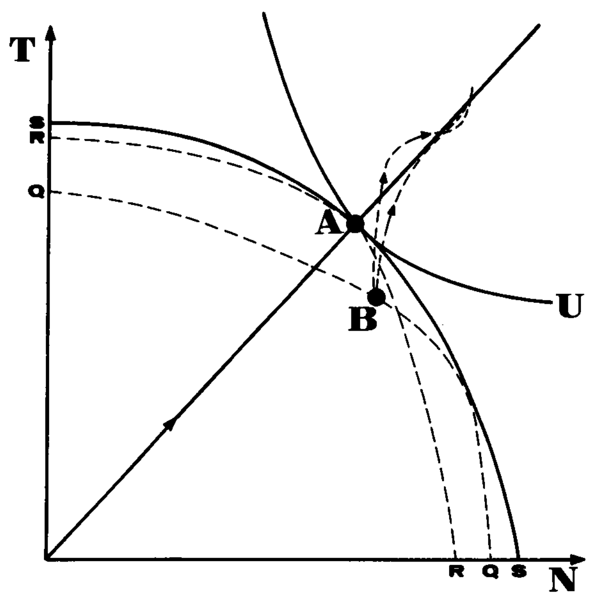
(iv) The traded good is both a consumption and an investment good. The nontraded good can only be consumed. The traded goods sector will be indexed with T and the non-traded with N.

(v) The labour market is assumed to clear.

(vi) The rate of Harrod-neutral technological progress is assumed to be the same in both sectors (it will be denoted by λ).

The problem analyzed in this section can be illustrated by means of Figure 2.2.

Figure 2.2 The two-sector model with a clearing labour market



SS is the long run (or steady state) transformation curve between the traded and the nontraded good. RR is one short run transformation curve, defined by a particular allocation of capital between the two sectors. RR is the short run curve consistent with steady state growth along the ray through the origin and the point A in Figure 2.2. Point A is a situation of short run and long run equilibrium with balanced trade. The first task in this section is to show the existence of such a short and long run equilibrium which is called a steady state. The other task is stability analysis. Assume that the economy, for some reason, is not in steady state growth. Instead it produces in point B in Figure 2.2.[1] This is a situation of short run equilibrium, which means that there is full employment and that the market for the nontraded good clears. It is however not a long run equilibrium. The question is whether or not the economy will move towards steady state growth in the long run. It will be shown that restrictions must be put on the utility function and the production functions to ensure stability. If the utility function and the production functions satisfy the stability conditions, then the economy will approach steady state growth in the long run, as shown in Figure 2.2. The adjustment to steady state growth may be in the form of damped cycles around the ray from the origin (shown by the dotted line).

For the formal analysis of the model, two new variables are defined:

$$\bar{K}_T \equiv K_T \cdot e^{-\lambda \cdot t}$$

$$\bar{K}_N \equiv K_N \cdot e^{-\lambda \cdot t}$$

---

[1] The consumption point may be different from the production point, in which case there is disequilibrium in the balance of trade. The diagram only deals with the production points.

Using these definitions the investment functions can be written as:

$$(2:23)\qquad \begin{cases} \dot{\bar{K}}_T = \varphi_T\left(r_T(\omega) - r^*\right)\bar{K}_T \\ \dot{\bar{K}}_N = \varphi_N\left(p \cdot r_N(\omega/p) - r^*\right)\bar{K}_N \end{cases}$$

where p is the relative price (the price of the nontraded in terms of the traded good). The product $p \cdot r_N$ is thus the value (in terms of the traded good) of the marginal physical product in the nontradable goods sector.

The equilibrium conditions in the labour market and in the market for the nontraded good are:

$$(2:24)\qquad \begin{cases} \bar{L} = n_N(\omega/p)\cdot\bar{K}_N + n_T(\omega)\cdot\bar{K}_T \\ q_N\left(n_N(\omega/p)\right)\cdot\bar{K}_N = \Omega(p)\cdot\omega\cdot\bar{L} \end{cases}$$

In (2:24) the consumption demand for the nontraded good has been given a form which implies unitary income elasticity. As will later be shown, this is necessary for the existence of a steady state. $\Omega(p)$ is some function of the relative price. (2:24) solves $\omega(\bar{K}_N,\bar{K}_T;\bar{L})$ and $p(\bar{K}_N,\bar{K}_T;\bar{L})$.

The two equations in (2:24) determine ω, the wage rate per efficiency unit of labour in terms of the traded good, and p, the price of the nontraded good in terms of the traded good. The first equation says that the labour market clears at all points in time. The second equation says that the market for the nontraded good clears at all points in time. From the definition of ω it is clear that when $\dot{\omega}/\omega = 0 \Rightarrow \dot{w}/w = \lambda$; i.e. the wage rate in terms of the traded good increases at the rate λ. If the solution to (2:24) is substituted into (2:23), one gets a system of differential equations: $\dot{\bar{K}}_T(\bar{K}_T,\bar{K}_N;\ \bar{L},r^*)$, $\dot{\bar{K}}_N(\bar{K}_T,\bar{K}_N;\ \bar{L},r^*)$ where $\bar{L}$ and r^* are exogenous parameters in the model.

The stationary point for this system is obtained when $\dot{\bar{K}}_T = 0$ and $\dot{\bar{K}}_N = 0$. It has the property that both equations in (2:23) are equal to zero and both equations in (2:24) are satisfied at all points in time, or that:

$$\begin{cases} r_T(\omega^*) = r^* & (2{:}25) \\ p^* \cdot r_N(\omega^*/p^*) = r^* & (2{:}26) \\ n_N(\omega^*/p^*)\bar{K}_N^* + n_T(\omega^*)\bar{K}_T^* = \bar{L} & (2{:}27) \\ \Omega(p^*) \cdot \omega^* \cdot \bar{L} = q_N(\omega^*/p^*) \cdot \bar{K}_N^* & (2{:}28) \end{cases}$$

The stars indicate that the variables have their equilibrium values.

The first two equations say that the rate of return (in terms of the tradable good) should be equal in both sectors, and equal to r^*. This defines capital market equilibrium. In order for (2:25) to hold, obviously ω^* must be constant, or, in other words, the real wage (in terms of the traded good) must increase at the rate λ.

Turning to (2:27) it is clear that there can be full employment[1] only when p, the relative price, is constant. As ω and $\bar{K}_T$ are both constant, the second term on the left hand side in (2:27) (the demand for labour in the traded goods sector) must be constant. Thus the first term (labour demand in the nontraded goods sector) must also be constant, which occurs only if p is constant.

[1] Full employment is strictly not necessary. One may allow some degree of unemployment (e.g. some "natural rate of unemployment") in steady state.

Given the constancy of ω and p, (2:26) can be satisfied, i.e. the rate of return on the nontraded good will be constant in steady state.

Turning to the last equation, (2:28), which gives the condition for the market for the nontraded good to clear, output will increase at the rate λ. If the condition is to be satisfied at all times t, demand must also increase at the rate λ. As p is constant and labour income increases by λ, it follows that the demand functions must have income elasticities equal to one (the utility function must be homothetic). This has already been assumed when choosing the form of the demand function. No restriction need, however, be placed on the price elasticity.

Concerning the steady state, it can finally be noted that trade is balanced in steady state only when $\lambda = r^*$. The analysis of Appendix 2:2 is in this respect relevant for the present model, too.

Equations (2:24) are the short-run part of the model. They are assumed to be satisfied at each point in time and to solve p and ω. It is apparent that (2:24) is homogeneous in wages and prices. This means that a change in the world market price (p_W), which is the unit of measurement, would change no real variables in the model. It would simply make the wage rate and the price of the nontradable good, measured in some monetary unit of account, change proportionally, leaving ω and p unaltered. As p_W cannot affect any real variables, a change in the exchange rate cannot affect any real variables either. Changes in the state variables, $\bar{K}_N$ and $\bar{K}_T$, may however change ω and p, but the state variables are given in the short run. To make an interesting short-run macro model of equations (2:24) some assumption about wage- or price-rigidities would have to be made.

One way to break the homogeneity of the short-run model is to make the wage rate a state variable and thus given in the short run. This could be done e.g. by introducing a Phillips curve in the model. With the Phillips curve, full employment cannot be expected. The effects of adding a Phillips curve to the present model will be shown in the next section.

To study the stability of the present model, one needs to know the partial derivatives on the functions that give the solution to (2:24), i.e. one needs to know how changes in $\bar{K}_N$ and $\bar{K}_T$ affect the real wage and the relative price in short--run equilibrium.

To determine this, (2:24) has been differentiated with respect to $\bar{K}_N$, $\bar{K}_T$, ω and p. The result is:

$$(2:29) \qquad \frac{d\omega}{d\bar{K}_T} = -\frac{1}{\Delta} \cdot \frac{Q_N \cdot L_T}{p \cdot \bar{K}_T} \cdot \left[\theta_{L,N} \cdot \eta_{L,\omega/p,N} + \varepsilon_p\right] > 0$$

$$(2:30) \qquad \frac{dp}{d\bar{K}_T} = -\frac{1}{\Delta} \cdot \frac{Q_N \cdot L_T}{\omega \cdot \bar{K}_T} \cdot \left[\theta_{L,N} \cdot \eta_{L,\omega/p,N} - 1\right] > 0$$

$$(2:31) \qquad \frac{d\omega}{d\bar{K}_N} = -\frac{1}{\Delta} \cdot \frac{Q_N \cdot L_N}{p \cdot \bar{K}_N} \cdot \left[\varepsilon_p + \sigma^N_{K,L}\right] \gtreqless 0$$

$$(2:32) \qquad \frac{dp}{d\bar{K}_N} = \frac{1}{\Delta} \cdot \frac{Q_N}{\omega \cdot \bar{K}_N} \cdot \left[L_T \cdot \eta_{L,\omega,T} + L_N \cdot (\eta_{L,\omega/p,N} \cdot \theta_{K,N} + 1)\right] \gtreqless 0$$

and

$$(2:33) \qquad \Delta \equiv \frac{Q_N}{p \cdot \omega} \cdot \left[L_T \cdot \eta_{L,\omega,T} \cdot (\theta_{L,N} \cdot \eta_{L,\omega/p,N} + \varepsilon_p) + L_N \cdot \right.$$
$$\left. \cdot \eta_{L,\omega/p,N} \cdot (\varepsilon_p + 1)\right] > 0$$

where the derivatives have been expressed in terms of elasticities and factor shares:

$$\eta_{L,\omega/p,N} \equiv \frac{\partial L^D_N}{\partial(\omega/p)} \cdot \frac{\omega/p}{L^D_N} = \frac{\partial Q_N/\partial L_N}{L_N \cdot \partial^2 Q_N/\partial L^2_N} < 0$$

which is the elasticity of labour demand with respect to the wage/price ratio in the nontraded goods sector,

$$\eta_{L,\omega,T} \equiv \frac{\partial L_T^D}{\partial \omega} \cdot \frac{\omega}{L_T^D} = \frac{\partial Q_T/\partial L_T}{L_T \cdot \partial^2 Q_T/\partial L_T^2} < 0$$

which is the elasticity of labour demand with respect to the wage rate, in the traded goods sector,

$$\varepsilon_p \equiv \frac{\partial D_N}{\partial p} \cdot \frac{p}{D_N} = \frac{\Omega' \cdot \omega \cdot \bar{L} \cdot p}{D_N} < 0$$

which is the price elasticity of demand,

$$\theta_{L,N} \equiv \frac{w \cdot L_N}{p \cdot Q_N} = \frac{\partial Q_N}{\partial L_N} \cdot \frac{L_N}{Q_N} \qquad 0 < \theta_{L,N} < 1$$

which is labour's share of output in the nontraded goods sector,

$$\theta_{K,N} \equiv \frac{r_N \cdot \bar{K}_N}{Q_N} = \frac{\partial Q_N}{\partial \bar{K}_N} \cdot \frac{\bar{K}_N}{Q_N} \qquad 0 < \theta_{K,N} < 1$$

which is the share of capital in output from the nontraded sector. Note that due to linear homogeneity (Euler's theorem): $\theta_{L,N} + \theta_{K,N} = 1.$

$$\sigma_{K,L}^N \equiv \frac{\partial \log (\bar{K}_N/L_N)}{\partial \log (\partial Q_N/\partial L_N \big/ \partial Q_N/\partial \bar{K}_N)} = - \theta_{K,N} \cdot \eta_{L,\omega/p,N} > 0$$

This is the elasticity of substitution between capital and labour in the nontraded goods sector. It is defined so as to be positive.

In (2:33) the determinant has been given a positive sign. It is not immediately obvious from the expression why it should be positive. In Appendix 2:3 the determinant (2:33) is analyzed in some detail. There it is shown that the system of differential equations $\dot{\omega}(\omega,p)$, $\dot{p}(\omega,p)$ which gives the *tâtonnement* equations for the labour market and the market for the nontraded good is stable if and only if the determinant (2:33) is positive. This is an application of the so-called correspondence principle.

Since there are four different elasticities involved in expression (2:33), there are a number of different cases that could give stability. It is noteworthy that a sufficient, although not necessary, condition for $\Delta > 0$ is that the price elasticity, ε_p, is smaller than -1 (larger than unity in absolute value). The appendix shows the case where $\varepsilon_p = 0$ and both production functions are Cobb-Douglas. These assumptions greatly simplify the expression.

Given that (2:33) is positive, it is evident that $d\omega/d\bar{K}_T$ and $dp/d\bar{K}_T$ ((2:29) and (2:30)) must be positive. This means that an increase in the capital stock in the traded goods sector must unambiguously increase the real wage rate (since it increases labour demand) and the relative price (since it increases supply in the traded goods sector). The corresponding effects of an increase in $\bar{K}_N$ are not immediately visible, however.

From (2:31) is seen that a necessary and sufficient condition for $d\omega/d\bar{K}_N \geq 0$, is that $-\varepsilon_p \geq \sigma^N_{K,L}$, i.e. that the numerical value of the price elasticity is greater than the value of the elasticity of substitution in the nontradable goods sector. If the relation between the elasticities is reversed, the sign of the derivative changes.

The sign of $dp/d\bar{K}_N$, finally, is determined by the following relation:

$$(2:34) \qquad \frac{dp}{d\bar{K}_N} \lesseqgtr 0 \Leftrightarrow -L_T \cdot \frac{\sigma^T_{K,L}}{\theta_{K,T}} \lesseqgtr L_N \cdot (\sigma^N_{K,L} - 1)$$

Since this is a relation between three elasticities, a number of sign combinations are possible. A *sufficient condition* for $dp/d\bar{K}_N < 0$ is that $\sigma^N_{K,L} \geq 1$. Thus if the production functions are Cobb-Douglas, it is guaranteed that $dp/d\bar{K}_N < 0$, which may be what one would expect.

Assuming that $d\omega/d\bar{K}_N > 0$ and $dp/d\bar{K}_N < 0$, the stability of the long-run growth model will now be analyzed.

Substituting the solution to the short-run system into the dynamic equations yields:

$$(2:35)\quad \begin{cases} \dot{\bar{K}}_T = \varphi_T\left[r_T(\omega(\bar{K}_N,\ \bar{K}_T)) - r^*\right]\bar{K}_T \\ \dot{\bar{K}}_N = \varphi_N\left[p(\bar{K}_N,K_T)\cdot r_N(\omega(\bar{K}_N,\bar{K}_T)/p(\bar{K}_N,\bar{K}_T)) - r^*\right]\bar{K}_N \end{cases}$$

This is a system of two differential equations: $\dot{\bar{K}}_T(\bar{K}_T,\bar{K}_N)$, $\dot{\bar{K}}_N(\bar{K}_T,\bar{K}_N)$. A stationary point for the system is a point $(\bar{K}_T^*,\bar{K}_N^*)$ such that $\dot{\bar{K}}_T = \dot{\bar{K}}_N = 0$. A stationary point is *locally stable* if the characteristic roots of the coefficient matrix for the linear system have negative real parts.[1] After linearizing the two equations in (2:35) and computing the characteristic equation, the conditions that its roots have negative real parts can be written:

$$(2:36)\quad \begin{cases} \bar{K}_T^*\cdot\varphi_T'\cdot r_T'\cdot\dfrac{d\omega}{d\bar{K}_T} + \bar{K}_N^*\cdot\varphi_N'\cdot\left[\dfrac{dp}{d\bar{K}_N}\cdot r_N + r_N'\cdot\left(\dfrac{d\omega}{d\bar{K}_N} - \dfrac{dp}{d\bar{K}_N}\cdot\dfrac{\omega}{p}\right)\right] < 0 \\ r_T'\cdot\left(r_N - r_N'\cdot\dfrac{\omega}{p}\right)\cdot\left[\dfrac{d\omega}{d\bar{K}_T}\cdot\dfrac{dp}{d\bar{K}_N} - \dfrac{d\omega}{d\bar{K}_N}\cdot\dfrac{dp}{d\bar{K}_T}\right] > 0 \end{cases}$$

The conditions in (2:36) are satisfied if:

(i) the assumptions concerning the production functions and the φ_i-functions which were made in section 2.2 are also made here.

(ii) the own price elasticity of demand is negative.

(iii) (2:33) is satisfied, i.e. the determinant is positive. As mentioned above, a *sufficient condition* for this is $-\varepsilon_p \geq 1$.

(iv) $d\omega/d\bar{K}_N > 0$. A *necessary and sufficient condition* for this is $-\varepsilon_p \geq \sigma_{K,L}^N$.

---

[1] Using the same theorem by Liapunov that was used earlier. See footnotes on pages 39 and 40 for references.

(v) $dp/d\bar{K}_N < 0$. A *sufficient condition* for this is $\sigma^N_{K,L} \geq 1$, as shown in (2:34).

Thus assuming (i)-(v) above, this two-sector growth model is stable.

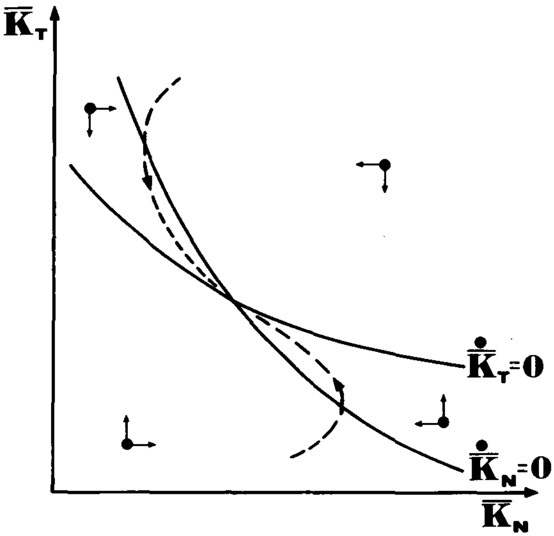
Setting the two equations in (2:35) equal to zero gives two implicit functions in $\bar{K}_N$ and $\bar{K}_T$. Differentiating these yields:

$$(2{:}37) \qquad \left.\frac{d\bar{K}_T}{d\bar{K}_N}\right|_{\dot{\bar{K}}_T=0} = -\frac{d\omega/d\bar{K}_N}{d\omega/d\bar{K}_T} < 0$$

$$(2{:}38) \qquad \left.\frac{d\bar{K}_T}{d\bar{K}_N}\right|_{\dot{\bar{K}}_N=0} = -\frac{\dfrac{dp}{d\bar{K}_N}\cdot r_N + \dfrac{\omega\cdot r'_N}{\bar{K}_N}\left[\dfrac{d\omega}{d\bar{K}_N}\cdot\dfrac{\bar{K}_N}{\omega} - \dfrac{dp}{d\bar{K}_N}\cdot\dfrac{\bar{K}_N}{p}\right]}{\dfrac{dp}{d\bar{K}_T}\cdot r_N + \dfrac{\omega\cdot r'_N}{\bar{K}_T}\left[\dfrac{d\omega}{d\bar{K}_T}\cdot\dfrac{\bar{K}_T}{\omega} - \dfrac{dp}{d\bar{K}_T}\cdot\dfrac{\bar{K}_T}{p}\right]} \lesseqgtr 0$$

(2:37) is the slope of the locus, in $\bar{K}_T,\bar{K}_N$-space, where $\dot{\bar{K}}_T = 0$. It is unambiguously negative. The slope of the line where $\dot{\bar{K}}_N = 0$ could be either positive or negative. The numerator is negative (i.e. $d\dot{\bar{K}}_N/dK_N < 0$) but the denominator could be positive or negative. Figure 2.3 shows the case when the denominator is negative:

Figure 2.3 Phase diagram for the two-sector model with full employment



Since the model is stable, the absolute value of the slope of the $\dot{\bar{K}}_N = 0$ locus is greater than the slope of the $\dot{\bar{K}}_T = 0$ locus.[1] It is seen from Figure 2.3 that in this case adjustment to long-run equilibrium takes place without any cyclical behaviour in the growth rates of the two sectors.

The other case, when the slope of the $\dot{\bar{K}}_N = 0$ locus is positive, is depicted in Figure 2.4.

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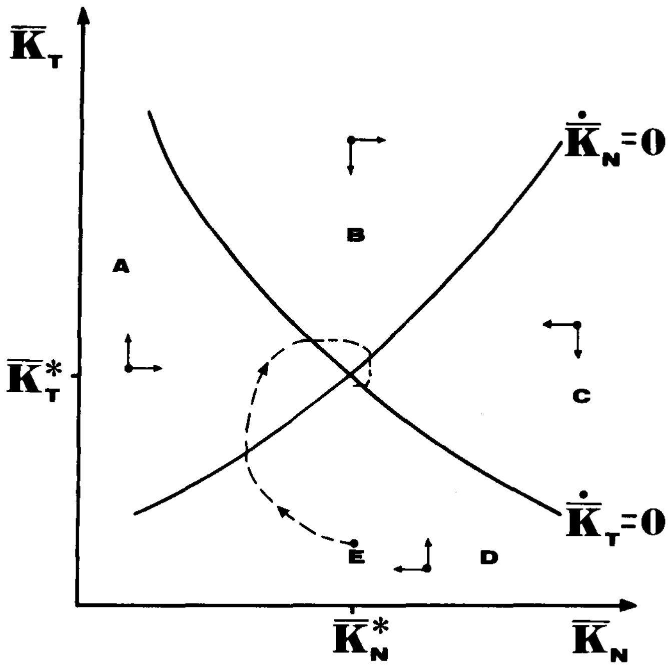
[1] It is easy to show that this is the case.

Figure 2.4 Phase diagram for the two-sector model with full employment



In this case cycles appear.

Starting at point E in Figure 2.4, the capital stock in the nontradable goods sector is at its steady state value, while that in the tradable goods sector is below its steady state value. This, one can imagine, may be the case after some exogenous change in international competitiveness has made part of the capital stock in the traded goods sector obsolete. With the assumptions made, there are disincentives to invest in the N-sector and incentives to invest in the T-sector. This makes $\bar{K}_T$ grow and $\bar{K}_N$ decrease relative to the steady state rate of growth. This is the situation in region D in Figure 2.6. The growing capital stock in the T-sector pushes up the relative price p, which increases incentives to invest in the N-sector. Thus the economy enters region A, where capital stocks increase (relative to steady state growth) in both sectors. Region A is a region of rapid growth. The fast increase in the capital stocks pushes wages up, which discourages further growth by reducing profitability. The capital stock in the T-sector starts decreasing as the economy enters region B. Still, the capital stock grows in the N-sector, as the relative price increases. The more output expands in the N-sector, the less the relative price increases. The economy then enters region C, the slow growth region, where the capital stocks in both sectors decrease relative to steady state. This makes wages increase slower and increases profitability, the economy enters region D, and the cycle repeats itself.

The sign of the denominator in (2:38) determines whether one obtains a growth path without cycles, as in Figure 2.3, or growth with cycles as in Figure 2.4. The expression (2:38) can be rewritten as follows:

$$(2:39) \qquad \frac{1}{\Delta^2} \cdot \frac{Q_N \cdot L_T \cdot Q_N \cdot L_N}{\bar{K}_T \cdot p \cdot \bar{K}_N} \cdot \left[\frac{r_N}{\omega} \cdot \left(\theta_{L,N} \cdot \eta_{L,\omega/p,N} - 1 \right) + \frac{r'_N}{p} \cdot (\varepsilon_p + 1) \right] \cdot$$

$$\cdot (\varepsilon_p + \sigma^N) \gtreqless 0$$

where Δ is the determinant defined by (2:33). Three elasticities are involved in determining the sign of this expression, so again a number of different cases are possible. It is seen however that if $\varepsilon_p \geq -1$, (2:39) is unambiguously positive. This is the case where there may occur cycles. Thus at least it is true that the lower (numerically) the price elasticity of demand, *ceteris paribus*, the greater the likelihood that the model will show cycles in the adjustment to long-run equilibrium.

The locally stable case analyzed so far has built on three assumptions[1], namely:

(i) that the determinant Δ in (2:33) is positive. A *sufficient* condition for this was shown to be that $\varepsilon_p \leq -1$. Necessary conditions are more complicated.

[1] Given the assumptions about technology and the demand functions which are employed throughout the chapter.

(ii) that $-\varepsilon_p \geq \sigma^N_{K,L}$, which implies that $d\omega/d\bar{K}_N \geq 0$.

(iii) that $dp/d\bar{K}_N < 0$. A *sufficient* condition to obtain this result is that $\sigma^N_{K,L} \geq 1$. Again, necessary conditions are more complicated.

Given (i)-(iii) above, the model has been shown to be locally asymptotically stable. Should it be the case that $0 \leq -\varepsilon_p \leq 1$, then the model may show cycles in the adjustment to equilibrium. For sufficiently high values of $-\varepsilon_p$, cycles may be ruled out.

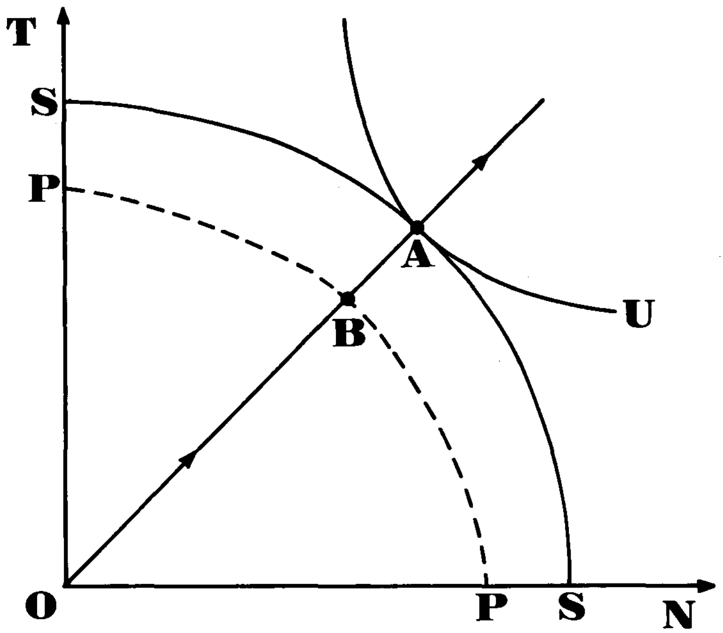
Referring back to Figure 2.2 at the beginning of this section, it has been shown that both paths from point B in the diagram are possible. Given (i)-(iii) above, the economy will however approach steady state growth in the long run, provided it does not start too far from it. The cycles would probably not appear in the solution to the model if rational expectations were assumed. The short-sightedness in the investors' expectations, coupled with wide swings in the relative price seem to be what gives rise to the cycles.

I have not been able to construct an unstable case. An unstable case would require a set of parameter values such that one of the inequalities in (2:36) is not satisfied, with the restriction that the parameters are such that Δ in (2:33) is positive. This is a problem involving two rather complex inequalities and four elasticities. The case when $\varepsilon_p = \sigma^N_{K,L} = 0$ is a possible candidate for an unstable case. It makes $d\omega/d\bar{K}_N = 0$. The condition that $dp/d\bar{K}_N > 0$, which leads to instability, is however only satisfied if $\Delta < 0$, which is ruled out by assumption. These parameter values can thus not give instability, provided that $\Delta > 0$ holds. This does not prove that the model cannot be unstable for some set of parameter values. I have however not succeeded in finding any set of parameters which makes it unstable.

2.5 A TWO-SECTOR MODEL WITH RIGID WAGES

The model in section 2.4 is always in short-run equilibrium, in the sense that there always is full employment. The focus in section 2.4 was on the adjustment to long-run equilibrium (steady state growth) from an initial situation of long-run disequilibrium. In this section the model from section 2.4 will be extended somewhat, by allowing for short-run disequilibrium, i.e. unemployment. That gives the possibility to study three types of situations: short-run disequilibrium with the same proportion of capital in the two sectors as is present in steady state growth, short-run equilibrium and a different capital allocation than in steady state (i.e. the problem analyzed in section 2.4), or situations with both types of disequilibria at the same time. The first of these situations will be given some attention in this section. It is illustrated in Figure 2.5.

Figure 2.5 The two-sector model with unemployment



Point A in Figure 2.5 represents a full, long- and short--run, equilibrium. Capital is allocated between the sectors so that the rates of return are equalized, there is balanced trade and there is full employment. SS is the steady state transformation curve between the T- and the N-good. PP is the transformation curve when there is unemployment, and where the amounts of capital in the two sectors are the same as in steady state growth. Point B[1] can be thought of as the situation immediately after a wage shock that disturbes an economy in steady state growth. In this section I can conclude that in point B, the economy will not necessarily tend to grow along the ray OA. Consequently, if the disequilibrium persists, the capital stocks may grow in proportions other than those on the steady state path. Once that has happened, steady state takes time to regain. A short-run disequilibrium that is not corrected relatively fast may thus have consequences in the medium run. If the model is stable, it will however return to steady state growth in the long run.

Formally the model in this section will, with one major exception, have the same properties as the model in section 2.4. The exception is that the model in this section will have a rigid wage rate in the short run. Its movement over time will be governed by a Phillips curve relation, much like in section 2.3. Thus if the wage rate is too high, there will be less labour demanded in the T- and N-sectors than is supplied in the economy. This is assumed to exert a downward pressure on the real wage rate.

This model generalizes the Kouri model [1979] discussed earlier. The present model has capital in both sectors and a Phillips curve, while Kouri had capital in only one sector. The model in this section allows for a more thorough analysis of long- and short-run disequilibria, but the thoroughness comes at the cost of a more complicated model.

---

1 Point B is the production point. If there is imbalanced trade, the consumption point may be somewhere on a vertical line above B.

To simplify the model I will assume that those who are not employed in the T- or the N-sector will be paid the same wage as those who are employed, i.e. an unemployment compensation. This means that $\bar{L}$ workers will always be paid the wage rate w. The number of workers who are being paid an unemployment compensation, will be denoted L_G. The reason for this simplifying assumption is the following: Assume that the market for the nontraded good always clears and that $L_G = \bar{L} - L_N^D - L_T^D$. If the wage rate increases from one period to another it reduces employment, hence L_G increases. If no unemployment compensation is paid out, this has three partial effects in the market for the nontraded good: firstly, supply decreases because the wage rate increased; secondly, demand *increases* since the wage rate increased; but thirdly, demand *decreases* since employment decreased. The net effect of these three partial effects may be that the price of the nontraded good decreases, is unchanged or increases. If the price decreases, labour demand is further reduced, so unemployment goes up even more, and so on, in an unstable fashion. This could be avoided by placing restrictions on some elasticities in the model or by assuming that everyone is paid the same wage. Since the latter is simpler, I will make that assumption.

There is one disadvantage to this latter assumption: L_G cannot reasonably become negative. It is hard to interpret the model if L_G is allowed to be negative. For this reason the model that follows is restricted to situations of a too high wage level, i.e. $L_G \geq 0$.

The unemployment compensation could either be financed by a tax or by a government budget deficit. I will assume throughout that the government runs a budget deficit. I will assume that the government borrows abroad to finance the unemployment compensation that it pays out, since this is the simplest assumption.

The simplest way to introduce a tax in the model would be to write consumption demand as $\Omega(p)(1-\tau)\cdot w\cdot\bar{L}$, where τ is the tax rate. None of the signs of the partial derivatives in the short-run system that follow would be changed by this. Hence no dynamics of the model would be changed either. If the government budget is assumed to be balanced at all times, it follows that $\tau\cdot w\cdot\bar{L} = w\cdot L_G$, or $\tau = L_G/\bar{L}$. The tax rate is equal to the proportion of workers who receive unemployment compensation. At a given wage rate such a system is, in terms of the present model, equivalent to paying no unemployment compensation at all. A fully-financed unemployment compensation scheme merely transfers purchasing power from the employed to the unemployed. The total wage bill is, at a given wage rate, unaffected by the operation. Thus this alternative is, for the workings of the present model, equivalent to giving no unemployment compensation at all.

Formally the model is the following:

$$(2{:}40)\quad \begin{cases} \dot{\bar{K}}_T = \varphi_T\left(r_T(\omega) - r^*\right)\bar{K}_T \\ \dot{\bar{K}}_N = \varphi_N\left(p\cdot r_N(\omega/p) - r^*\right)\bar{K}_N \\ \dot{\omega} = \psi\left(\bar{L} - L_N^D - L_T^D\right)\omega \end{cases}$$

where $\psi(0) = 0$, $\psi' < 0$,

and the short-run system solves

$$(2{:}41)\quad \begin{cases} q_N\left(n_N(\omega/p)\right)\cdot\bar{K}_N = \Omega(p)\cdot\omega\cdot\bar{L} \\ \bar{L} = n_N(\omega/p)\cdot\bar{K}_N + n_T(\omega)\cdot\bar{K}_T + L_G \end{cases}$$

where

$$\bar{K}_T \equiv K_T \cdot \exp(-\lambda t)$$

$$\bar{K}_N \equiv K_N \cdot \exp(-\lambda t)$$

$$\omega \equiv w \cdot \exp(-\lambda t)$$

as before.

Because of the Phillips curve this short-run system is not homogeneous in wages and prices, as was the case in section 2.4. An exogenous change in p_T (the world market price of the traded good), which may be brought about by e.g. a devaluation, will therefore have effects on real variables.

To see the workings of the short-run system, (2:41), it has been differentiated with respect to ω, p, $\bar{K}_N$, $\bar{K}_T$ and L_G. The result is the following:

(2:42) $$\frac{dp}{d\bar{K}_N} = -\frac{q_N}{\Delta} < 0$$

(2:43) $$\frac{dp}{d\bar{K}_T} = 0$$

(2:44) $$\frac{dp}{d\omega} = \frac{-q_N' \cdot n_N' \cdot p^{-1} \cdot \bar{K}_N + \Omega \cdot \bar{L}}{\Delta} > 0$$

(2:45) $$\frac{dL_G}{d\bar{K}_N} = -n_N - \frac{q_N \cdot n_N' \cdot \omega \cdot p^{-2} \cdot \bar{K}_N}{\Delta} \gtreqless 0$$

(2:46) $$\frac{dL_G}{d\bar{K}_T} = -n_T < 0$$

(2:47) $$\frac{dL_G}{d\omega} = -n_N' \cdot p^{-1} \cdot \bar{K}_N - n_T' \cdot \bar{K}_T + \frac{n_N' \cdot \omega \cdot p^{-2} \cdot \bar{K}_N \cdot (-q'_N \cdot n_N' \cdot p^{-1} \cdot \bar{K}_N + \Omega \cdot \bar{L})}{\Delta} \gtreqless 0$$

(2:48) $$\Delta \equiv -q_N' \cdot n_N' \cdot \omega \cdot p^{-2} \cdot \bar{K}_N - \Omega' \cdot \omega \cdot \bar{L} > 0$$

When the capital stock in the N-sector is increased, the relative price must fall to clear the market, as shown by (2:42). An increase in $\bar{K}_T$ does not affect p however (2:43), as opposed to the case in the last section when the labour market was assumed to clear. $d\bar{K}_T$ does not affect the wage rate since it is fixed in the short run. Neither does it affect supply or demand in the market for the nontraded good. The sign of $dp/d\omega$ is positive as seen by (2:44). Since that particular derivative will be used later, it is rewritten for easier interpretation:

$$(2:49) \qquad \frac{dp}{d\omega} = \frac{p}{\omega} \cdot \frac{1 - \theta_{L,N} \cdot \eta_{L,\omega/p,N}}{-\varepsilon_p - \theta_{L,N} \cdot \eta_{L,\omega/p,N}}$$

From (2:49) is seen that if $-\varepsilon_p = 1$, i.e. if the price elasticity of demand is unity[1], then the elasticity of the relative price with respect to the wage rate is also equal to unity. This property will be convenient later.

The sign of $dL_G/d\bar{K}_N$ is ambiguous, as seen from (2:45). This is because there are two counteracting tendencies. An increase in $\bar{K}_N$ directly increases labour demand in the N-sector and hence tends to reduce L_G. An increase in $\bar{K}_N$ does however also decrease p, the relative price, which reduces labour demand in the N-sector. That effect works in the opposite direction from the first one. To derive a condition that determines the sign, (2:45) may be rewritten as:

$$(2:50) \qquad \frac{dL_G}{d\bar{K}_N} = - \frac{L_N}{\bar{K}_N \cdot (\theta_{L,N} \cdot \eta_{L,\omega/p,N} + \varepsilon_p)} \cdot \left[\sigma^N_{K,L} + \varepsilon_p\right]$$

It can thus be concluded that $dL_G/d\bar{K}_N \lesseqgtr 0 \Leftrightarrow \sigma^N_{K,L} \lesseqgtr -\varepsilon_p$.[2]

The sign of $dL_G/d\bar{K}_T$ is unambiguously negative as seen from

[1] This means that the utility function is Cobb-Douglas, since the income elasticity is equal to one by assumption.

[2] This can be recognized as the condition determining the sign of $d\omega/d\bar{K}_N$ in section 2.4.

(2:46). If the capital stock increases in the T-sector, it decreases unemployment.

The sign of $dL_G/d\omega$ is finally ambiguous. On the one hand an increasing wage rate lowers labour demand in both sectors, which tends to increase unemployment. On the other hand a higher wage means that the demand curve for the N-good shifts to the right and the supply curve to the left. This unambiguously raises p, which has the effect of *increasing* labour demand in the N-sector. The largest of these two effects on labour demand determines the sign of the derivative. (2:47) can be rewritten as:

$$\text{(2:51)} \qquad \frac{dL_G}{d\omega} = - \frac{1}{\omega \cdot (\theta_{L,N} \cdot \eta_{L,\omega/p,N} + \varepsilon_p)} \cdot \Big[L_T \cdot \eta_{L,\omega,T} \cdot (\theta_{L,N} \cdot \eta_{L,\omega/p,N} + \varepsilon_p) + L_N \cdot \eta_{L,\omega/p,N} \cdot (\varepsilon_p + 1) \Big]$$

The condition which renders $dL_G/d\omega$ positive or negative is thus exactly the same as that which renders the determinant in section 2.4 larger or smaller than zero. A sufficient, but not necessary, condition for positive $dL_G/d\omega$ is that $-\varepsilon_p \geq 1$.

Substitution of the solution to (2:41) into (2:40) gives a system of three differential equations in $\bar{K}_N$, $\bar{K}_T$ and ω. As it is a system of three variables it cannot easily be represented in a phase diagram. In order to determine the local stability of the system, the equations are linearized around the equilibrium point and the characteristic roots of the coefficient matrix are computed.

It turns out that necessary and sufficient conditions for stability are quite complex. Some simple sufficient conditions will however be derived. It will be shown below that if the conditions for stability that were used in the full employment model are satisfied, and if furthermore the price elasticity of demand is sufficiently large, then the present model is stable.

The Routh-Hurwitz theorem[1] gives necessary and sufficient conditions in order for the roots of a polynomial equation to lie strictly in the left half of the complex plane. If the theorem is applied to the characteristic equation, three conditions result which, if they are satisfied, guarantee stability.

The conditions will be expressed in terms of the derivatives on the short-run system. To simplify the expressions a new concept will be introduced, namely:

$$\rho_N(\bar{K}_N,\bar{K}_T,\omega) \equiv p(\bar{K}_N,\bar{K}_T,\omega)\cdot r_N\left(\omega/p(\bar{K}_N,\bar{K}_T,\omega)\right)$$

ρ_N is the value of the physical marginal product of capital in the N-sector, in terms of the traded good.

Using this concept, the stability conditions can be written:

$$\varphi_N' \cdot \frac{\partial\rho_N}{\partial\bar{K}_N} \cdot \bar{K}_N^* + \psi' \cdot \frac{dL_G}{d\omega} < 0 \tag{2:52}$$

$$\varphi_T' \cdot r_T' \cdot \bar{K}_T^* \cdot \psi' \cdot \frac{dL_G}{d\bar{K}_T} \cdot \omega^* \cdot \varphi_N' \cdot \frac{\partial\rho_N}{\partial\bar{K}_N} \cdot \bar{K}_N^* > 0 \tag{2:53}$$

$$\begin{aligned}(2{:}54)\qquad & \left[\varphi_N' \cdot \frac{\partial\rho_N}{\partial\bar{K}_N} \cdot \frac{\bar{K}_N^*}{\omega^*} + \psi' \cdot \frac{dL_G}{d\omega}\right] \cdot \left[\frac{dL_G}{d\bar{K}_N} \cdot \varphi_N' \cdot \frac{\partial\rho_N}{\partial\omega} \cdot \bar{K}_N^* - \right.\\ & \left. - \varphi_N' \cdot \frac{\partial\rho_N}{\partial\bar{K}_N} \cdot \bar{K}_N^* \cdot \frac{dL_G}{d\omega}\right] + \frac{dL_G}{d\omega} \cdot \varphi_T' \cdot r_T' \cdot \bar{K}_T^* \cdot \psi' \cdot \frac{dL_G}{d\bar{K}_T} < 0\end{aligned}$$

An asterisk on a variable indicates an equilibrium value.

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[1] See e.g. Gandolfo [1971] page 240.

When the short-run system was differentiated, the signs of $dL_G/d\bar{K}_N$ and $dL_G/d\omega$ could not be determined. If two assumptions are made, they can be given definite signs however:

(i) Assume that $\sigma^N_{K,L} < -\varepsilon_p$. This implies that $dL_G/d\bar{K}_N < 0$, i.e. that unemployment decreases as more capital is employed in the N-sector. This is the intuitively expected result. The assumption about the relation between the elasticities is that which was made in section 2.4 to satisfy the stability conditions.

(ii) Assume that expression (2:51), $dL_G/d\omega$, is positive, i.e. that unemployment increases after an increase in the real wage rate. Once again this is the result one would expect. The condition on the elasticities which gives this result is that which guarantees that the determinant in section 2.5 is positive. The necessary condition is complex. A sufficient condition is that $-\varepsilon_p \geq 1$.

To evaluate the stability conditions the partial derivatives on $\rho_N(\bar{K}_N, \bar{K}_T, \omega)$ are needed:

$$(2:55)\quad \begin{cases} \dfrac{\partial \rho_N}{\partial \bar{K}_N} = \dfrac{dp}{d\bar{K}_N} \cdot r_N - p^{-1} \cdot r_N' \cdot \omega \cdot \dfrac{dp}{d\bar{K}_N} < 0 \\ \dfrac{\partial \rho_N}{\partial \bar{K}_T} = 0 \\ \dfrac{\partial \rho_N}{\partial \omega} = \dfrac{dp}{d\omega} \cdot r_N + r_N' \cdot \left[1 - \dfrac{dp}{d\omega} \cdot \dfrac{\omega}{p}\right] \gtreqless 0 \end{cases}$$

To derive a stable case, I shall assume that $\partial \rho_N/\partial \omega < 0$, i.e. that the value of the marginal physical product of capital in the N-sector decreases as the real wage rate increases. It is seen from (2:55) that the sign of $\partial \rho_N/\partial \omega$ is not determined. The first term in the expression is positive. The second term is negative if and only if $dp/d\omega \cdot \omega/p < 1$ (since $r_N' < 0$). It is shown in (2:49) that $dp/d\omega \cdot \omega/p < 1$ if and only if $-\varepsilon_p > 1$. The second term in the expression determining $d\rho_N/d\omega$ must be sufficiently large numerically to outweigh the first positive term. Thus, a third assumption is needed:

(iii) Assume that $-\varepsilon_p > 1$. The value of the price elasticity of demand must be *sufficiently greater than one*, so that $\partial\rho_N/\partial\omega < 0$.

Under assumptions (i)-(iii) the model is locally stable. The assumptions give *sufficient conditions* for stability.

In summary, if the full employment model of section 2.4 is locally stable, the present model is locally stable too, provided that the price elasticity of demand is sufficiently large.

This result illustrates how the present model is a development of the previous model.

Finally in this section I will show that a wage shock, disturbing an economy in steady state growth, is likely to lead to unbalanced growth in the readjustment to steady state. This is the case illustrated in Figure 2.5. The impact of a wage disturbance on the rates of growth of the capital stocks in the two sectors, can be computed as:

$$(2{:}56) \qquad \frac{d\left(\dot{\bar{K}}_T/\bar{K}_T - \dot{\bar{K}}_N/\bar{K}_N\right)}{d\omega} = \varphi_T' \cdot r_T' - \varphi_N' \cdot \frac{d\rho_N}{d\omega} \gtreqless 0$$

$d\rho_N/d\omega$, the effect of a wage increase on the value of the marginal physical product of capital in the N-sector, is defined in (2:55). It is clearly seen from (2:56) that there is no reason to expect balanced growth (i.e. that $\dot{\bar{K}}_T/\bar{K}_T = \dot{\bar{K}}_N/\bar{K}_N$) following a wage disturbance. $d\rho_N/d\omega$ was assumed to be negative in the stability analysis above.

Not even if the investment functions (the φ_i-functions) are identical in the two sectors is there any reason to expect that (2:56) is equal to zero.

It can be concluded here then, that if a wage disturbance in steady state is not corrected through some economic policy measure (such as a devaluation), its effects are likely to be longlasting. If the disturbance is immediately corrected (or at least corrected so fast that investments are not affected by the disequilibrium wage rate), then the economy can continue to grow in a balanced fashion. If it is not corrected however, then (2:56) shows that the capital stocks will grow at different rates in the two sectors. In terms of Figure 2.5, the economy will not grow along the ray from the origin. It has been shown above that the model may be stable, but will adjust to long run equilibrium in some other fashion than along the ray through the origin in Figure 2.5. Such adjustment is likely to take time, since it involves investments in physical capital in different rates in the two sectors. This seems to be one important motive not to let short run disequilibria persist too long. Of course there are other motives. A too high wage rate means unemployment, or at least necessitates government policy to combat unemployment (represented by L_G in the present model). Even if the unemployed are somehow compensated, the present analysis shows that there are good reasons to quickly correct a too high wage rate.

The present section has introduced a wage rigidity into the model from section 2.4. Thus full employment is no longer guaranteed in the model. Essentially two things have been demonstrated:

(i) The model with the wage rigidity is stable under the same conditions as the model with a flexible wage rate provided that the price elasticity of demand is sufficiently high.

(ii) In the model with rigid wages, a wage or price disturbance may have real effects. The model in section 2.4 was homogeneous in wages and prices. It was argued in this section that a wage disturbance may have quite longlasting effects on the economy.

2.6 A TWO-SECTOR MODEL WITH CLEARING LABOUR MARKET AND DIFFERENT RATES OF TECHNOLOGICAL PROGRESS IN THE TWO SECTORS: THE STEADY STATE

In the Swedish post-war debate on wage formation and inflation, the "EFO" or "Scandinavian" model[1] has played rather a large role. Since a part of the present study deals with the Swedish post-war experience, it may be interesting to see whether the EFO assumptions may be made compatible with the models used here. Could the equilibrium growth path in the EFO-model be interpreted as the steady state solution to the kind of model that is being used here? That is the problem that is analyzed in this section.

If different rates of Harrod-neutral technological progress in the sectors (λ_T and λ_N) is assumed, the model in section 2.4 resembles somewhat the "Scandinavian" or "EFO" model. The Scandinavian model is a model for studying inflation. Thus it was constructed for a different purpose than the present model. It has, however, also been used to define an equilibrium wage path. One feature of the Scandinavian model is that the difference between the rate of price increase in the traded goods sector[2] and the nontraded goods sector is equal to the difference between the rate of productivity increase in the two sectors. Assuming that the same wages are paid in both sectors, that wage increases are equal to price increases plus productivity increases, and that the wage is determined by the tradable goods sector, one obtains a theory for the overall rate of inflation. In this section I will show that these assumptions are compatible with steady state growth in the two-sector model. I will, however, make a point of showing that rather heavy restrictions must be placed on the model for this steady state to exist.[3]

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1 See e.g. Edgren, Faxén, Odhner [1973] or Lindbeck [1979].

2 In the EFO model the sectors are usually termed sheltered (S-sector) and competitive (C-sector). I will continue to use the terms nontraded and traded, meaning the same thing.

3 A similar point has been made by Kierzkowski [1976], in a somewhat different framework.

The model analyzed in the present section is identical with the one in section 2.4 with the single difference that $\lambda_T \neq \lambda_N$.

The purpose of the present section is to show the effects of this particular assumption for the existence of a steady state in the model.

I will define a steady state as a state where there is full employment, the market for the N-good clears, there is balanced trade and capital market equilibrium. By capital market equilibrium is meant that the value of the marginal product of capital (in terms of the traded good), is the same in both sectors and equal to r*. The following five equations must thus be satisfied at all points in time in a steady state.

$$\bar{L} = n_N\left(w^* \cdot \exp(-\lambda_N \cdot t)/p^*\right) \cdot K_N^* \cdot \exp(-\lambda_N \cdot t) + n_T\left(w^* \cdot \exp(-\lambda_T \cdot t)\right) \cdot K_T^* \cdot \exp(-\lambda_T \cdot t) \tag{2:57}$$

$$q_N\left[n_N\left(w^* \cdot \exp(-\lambda_N \cdot t)/p^*\right)\right] \cdot K_N^* = \Omega(p^*) \cdot w^* \cdot \bar{L} \tag{2:58}$$

$$q_T\left[n_T\left(w^* \cdot \exp(-\lambda_T \cdot t)\right)\right] \cdot K_T^* = \xi_N \cdot K_N^* + \xi_T \cdot K_T^* + \gamma_T \cdot w^* \cdot \bar{L} \tag{2:59}$$

$$r^* = r_T\left[n_T\left(w^* \cdot \exp(-\lambda_T \cdot t)\right)\right] \tag{2:60}$$

$$r^* = p^* \cdot r_N\left[n_N\left(w^* \cdot \exp(-\lambda_N \cdot t)/p^*\right)\right] \tag{2:61}$$

The stars indicate equilibrium values. $\xi_N \cdot K_N^*$ is investment demand in steady state in the N-sector, ξ_N is a constant (the rate of growth of the capital stock in the N-sector in steady state). $\xi_T \cdot K_T^*$ is investment demand in the T-sector in steady state and $\gamma_T \cdot w^* \cdot \bar{L}$ is consumption demand for the traded good. γ_T is a constant (the budget share of the T-good).

The problem now is whether exponential paths exist for the variables with asterisks in (2:57)-(2:61), such that the five equations are satisfied.

The exponential paths are denoted by:

$$w^*(t) \equiv w_o \cdot \exp(c \cdot t)$$

$$p^*(t) \equiv p_o \cdot \exp(a \cdot t)$$

$$K_i^*(t) \equiv K_{o,i} \cdot \exp(d_i \cdot t) \qquad i = T,N$$

I use the following elasticities:

$$n_N' \cdot \frac{w \exp(-\lambda_N \cdot t)}{n_N \cdot p} \equiv \eta_{L,\omega/p,N}'$$

$$n_T' \cdot \frac{w \exp(-\lambda_T \cdot t)}{n_T} \equiv \eta_{L,\omega,T}'$$

$$q_N' \cdot \frac{n_N}{q_N} \equiv \theta_{L,N}$$

$$q_T' \cdot \frac{n_T}{q_T} \equiv \theta_{L,T}$$

$$\frac{\Omega'(p) \cdot p}{\Omega(p) \cdot w \cdot \bar{L}} \equiv \varepsilon_p$$

$$r_N' \cdot \frac{n_N}{r_N} \equiv \frac{\theta_{L,N}}{\sigma_{K,L}^N}$$

The prime in $\eta_{L,\omega/p,N}'$ and $\eta_{L,\omega,T}'$ denote that the elasticities differ somewhat from those used e.g. in section 2.4. Here the elasticity is defined with the wage rate per efficiency unit measured relative to the rate of technological progress occurring in the respective sector.

There are four growth rates in the exponential functions above. These will be expressed in terms of the parameters of the model.

It is immediately obvious from (2:59) that the following equality must hold:

(2:62) $$d_T = d_N = c$$

i.e. the growth rate of the capital stock in the two sectors must be equal, and equal to the rate of growth of the wage rate.

(2:60) implies the following equality:

(2:63) $$c = \lambda_T$$

i.e. the rate of growth of the wage rate must equal the rate of Harrod-neutral technological progress in the traded goods sector.

Thus, three of the four growth rates are determined in terms of technological parameters. The rate of change of the relative price in steady state remains to be determined.

Given (2:62) and (2:63), (2:57) is satisfied if and only if:

(2:64) $$\eta'_{L,\omega/p,N} \cdot (\lambda_T - \lambda_N - a) + \lambda_T - \lambda_N = 0$$

(2:58) is satisfied if and only if:

(2:65) $$\theta_{L,N} \cdot \eta'_{L,\omega/p,N} \cdot (\lambda_T - \lambda_N - a) + \lambda_T = \varepsilon_p \cdot a + \lambda_T$$

and (2:61) is satisfisfied if and only if:

(2:66) $$a + \frac{\theta_{L,N}}{\sigma^N_{K,L}} \cdot \eta'_{L,\omega/p,N} \cdot (\lambda_T - \lambda_N - a) = 0$$

These are three equations from which the parameter a can be solved. Solution of a from (2:64) yields:

$$(2{:}67) \qquad a = \frac{(1+\eta'_{L,\omega/p,N})\cdot(\lambda_T-\lambda_N)}{\eta'_{L,\omega/p,N}}$$

(2:65) gives:

$$(2{:}68) \qquad a = \frac{(\eta'_{L,\omega/p,N}+\sigma^N_{K,L})\cdot(\lambda_T-\lambda_N)}{\eta'_{L,\omega/p,N} + \sigma^N_{K,L} + \varepsilon_p}$$

and if (2:66) is solved for a it yields:

$$(2{:}69) \qquad a = \frac{(\sigma^N_{K,L}+\eta'_{L,\omega/p,N})\cdot(\lambda_T-\lambda_N)}{\eta'_{L,\omega/p,N}}$$

If (2:67) and (2:69) are to be satisfied at the same time, it is clear that the following must hold:

$$(2{:}70) \qquad \sigma^N_{K,L} = 1$$

i.e. the elasticity of substitution must be equal to unity. Thus the production function is Cobb-Douglas.

Substituting this into (2:68), it is seen that the following also must hold:

$$(2{:}71) \qquad \varepsilon_p = -1$$

i.e. the price elasticity of demand must be equal to -1. Thus, the community utility function is Cobb-Douglas.

This proves that, among the CES family of production functions, only the Cobb-Douglas production function is compatible with steady state growth when there are different rates of technological change in the two sectors. Furthermore, the community utility function must be Cobb-Douglas. The rate of change of the relative price is given by (2:67).

It can also be shown that among the class of linearly homogeneous production functions, and assuming Harrod-neutral technological progress, only those with a constant elasticity of substitution are compatible with steady state growth in this particular case. The expression for the elasticity of substitution in the nontraded goods sector can be written as a function of the labour/capital ratio, which in turn is a function of time in steady state. Differentiation of the expression for the elasticity of substitution with respect to time, can be shown to yield:[1]

(2:72) $$\dot{\sigma}^N_{K,L} = \dot{n} \cdot \frac{d\sigma}{dn}$$

This must be equal to zero in steady state. Since $\dot{n} \neq 0$, this can only be possible if $d\sigma/dn = 0$, which per definition means that the production function must be CES.

It is easy to show[2] that when the production function is Cobb-Douglas, the rate of change of the relative price is given by

(2:73) $$a = (\lambda_T - \lambda_N) \cdot (1 - \alpha_N)$$

where α_N is the output elasticity of capital in sector N. This can be compared with the result from the EFO model which says that the relative price should change by $\lambda_T - \lambda_N$.

---

[1] The elasticity of substitution can be expressed in terms of the production function and its first- and second-order derivatives (see e.g. Layard, Walters [1978]). Assuming linear homogeneity and Harrod-neutrality, these are functions of the labour/capital ratio in efficiency units. The elasticity of substitution can be written:

$$\sigma^N_{K,L} = -\frac{q'\big(n(t)\big) \cdot \big[q\big(n(t)\big) - q'\big(n(t)\big) \cdot n(t)\big]}{q\big(n(t)\big) \cdot q''\big(n(t)\big) \cdot n(t)}$$

i.e. $\sigma^N_{K,L}\big(n(t)\big)$. Differentiating this expression with respect to time yields (2:72).

[2] This is shown at the end of chapter 3 where the steady state for the vintage model is derived.

It has been shown that if the rates of Harrod-neutral technological progress are different in the two sectors, $\lambda_N \neq \lambda_T$, certain restrictions must be placed on the utility functions and production functions for a steady state to exist. Both the utility functions and the production functions must be Cobb-Douglas for the steady state to exist. This is rather a heavy restriction to put on the model.

A similar point has, as already mentioned, been made by Kierzkowski [1976], who writes: "The Scandinavian model is consistent with a general equilibrium model in which ... the community preferences are homothetic, and price elasticity of demand is equal to 1."[1] He does not reach the conclusion that the production functions must be Cobb-Douglas, since he does not take into account what I above called capital market equilibrium.

Now I can return to the question posed at the beginning of this section regarding the Scandinavian model. As long as the assumption $\lambda_N \neq \lambda_T$ is made, the equilibrium growth path of the Scandinavian model is a steady state solution to this two-sector growth model only if production functions and utility functions are Cobb-Douglas. Furthermore, when the production functions are Cobb-Douglas, the criterion emerging from the Scandinavian model defining an alleged equilibrium wage path is not capable of discriminating between different wage paths. Profit maximization and a Cobb-Douglas production function implies that the following relation holds:

$$(2:73) \qquad w = (1-\alpha)\cdot p \cdot \frac{Q}{L}$$

where α is the output elasticity of capital. Thus it is *always* true that the rate of change of wages is equal to the sum of the rate of change of price plus the rate of change of labour productivity. This is a feature of *any* wage path.

---

[1] Kierzkowski [1976] pages 239-240.

The criterion from the Scandinavian model can thus *not* discriminate between various wage paths to find one equilibrium wage path, in the case when the production function is Cobb-Douglas.

In a two-sector growth model, it seems better to assume that $\lambda_N = \lambda_T$. As an empirical fact the increase in rates of productivity may of course differ between the sectors in spite of this. That is however endogenously determined and not a feature of steady state growth.

APPENDIX 2:1 NOTATION USED IN CHAPTER TWO

T	index for the traded good
N	index for the nontraded good
$Q_i(t)$	output of the good from sector i; i = T, N
$K_i(t)$	amount of capital in sector i; i = T, N
$L_i(t)$	amount of labour allocated to sector i; i = T, N
L_s	total supply of labour
λ_i	the rate of Harrod-neutral technological progress in the production function in sector i; i = T, N
$D_i(t)$	demand for good i for consumption purposes; i = T, N; in some sections the demand function will be given the form: $\Omega(p)\cdot w\cdot L$, where $\Omega(p)$ is some unspecified function of the relative price.
$w(t)$	the wage rate in terms of the tradable good
$p(t)$	the price of the nontradable good in terms of the tradable good
$r_i(t)$	the marginal physical product of capital in sector i; i = T, N
r^*	the rate of return on the international asset
$I_i(t)$	investments in sector i; i = T, N
S	total savings
B	holdings of bonds
b	holdings of bonds per efficiency unit of labour
k	the capital/labour ratio in efficiency units $\left(\frac{K}{L\cdot\exp(\lambda t)}\right)$, i.e. the inverse of n.
$u(t)$	the number of people who are unemployed at time t
p_W	the world market price
L_G	the number of workers who are being paid an unemployment compensation
τ	the income tax rate

The following definitions are used:

$q_i(t)$	$\equiv \dfrac{Q_i(t)}{K_i(t)}$	the output/capital ratio in sector i; i = T, N
$n_i(t)$	$\equiv \dfrac{L_i(t)}{K_i(t)} \cdot \exp(\lambda_i t)$	the labour/capital ratio in efficiency units; i = T, N
$\omega(t)$	$\equiv w(t) \cdot \exp(-\lambda t)$	the wage rate per efficiency unit
$N(t)$	$\equiv L(t) \cdot \exp(\lambda t)$	the amount of labour in efficiency units
$\bar{K}(t)$	$\equiv K(t) \cdot \exp(-\lambda t)$	
$\bar{K}_T$	$\equiv K_T \cdot \exp(-\lambda t)$	
$\bar{K}_N$	$\equiv K_N \cdot \exp(-\lambda t)$	
$\eta_{L,\omega/p,N}$	$\equiv \dfrac{\partial L_N^D}{\partial(\omega/p)} \cdot \dfrac{\omega/p}{L_N^D}$	the elasticity of labour demand with respect to the wage/price ratio, in the nontraded goods sector
$\eta_{L,\omega,T}$	$\equiv \dfrac{\partial L_T^D}{\partial \omega} \cdot \dfrac{\omega}{L_T^D}$	the elasticity of labour demand with respect to the wage rate, in the traded goods sector
ε_p	$\equiv \dfrac{\partial D_N}{\partial p} \cdot \dfrac{p}{D_N}$	the price elasticity of demand for the nontraded good
$\theta_{L,N}$	$\equiv \dfrac{w}{p} \cdot \dfrac{L_N}{Q_N}$	labour share of output in the nontraded goods sector
$\theta_{K,N}$	$\equiv r_N \cdot \dfrac{K_N}{Q_N}$	the share of capital in output from the nontraded goods sector
$\sigma_{K,L}^N$	$\equiv \dfrac{\partial \log (K_N/L_N)}{\partial \log (\partial Q_N/\partial L_N / \partial Q_N/\partial K_N)}$	the elasticity of substitution between capital and labour in the nontraded goods sector
ρ_N	$\equiv p \cdot r_N$	the value in terms of the traded good of the marginal physical product of capital in the N-sector

APPENDIX 2:2 A MODEL WHERE BONDS ARE EXPLICITLY TREATED

In all the growth models for a small open economy treated in this study, I assume that there exists an exogenously given rate of return, r^*, to some internationally-traded asset (e.g. a bond). In other sections I will not treat the accumulation of this bond explicitly. For this reason I shall in this appendix look at a *one-sector model* where the accumulation of the bond can be treated.[1]

The model that is used in this appendix is the same as in section 2.2. The basic features of the model are recapitulated below:

(i) There exists one good which is tradable and can be used either for consumption or for investment purposes. It is in perfectly elastic supply in the world market.

(ii) When the small country runs a deficit in the balance of trade, it sells its holding of the internationally traded bond (B) whose rate of return is r^*. Bond holdings will be negative if the country is a net debtor, in which case the country pays $r^* \cdot B$ on the debt.

(iii) All wage income is assumed to be instantaneously consumed. All capital income (including positive or negative income from bond holdings) is saved.

(iv) The only difference from the basic relationships in section 2.2 is that capital will be assumed to adjust instantaneously. This means that at all points in time, $r(\omega) = r^*$ and $\dot{K}/K = \lambda$, where λ is the rate of Harrod-neutral technological progress. As will become clear, this assumption does not affect the analysis of bondholdings in steady state or of the stability properties.

The equations of the model are the following:

$$\text{(A:2:1)} \qquad \dot{K} = \lambda \cdot K$$

[1] I am indebted to Professor Michael Hoel for useful suggestions on this subject.

(A:2:1) says that the capital stock grows at the rate of technological progress, which is the rate of steady state growth in the economy.

(A:2:2) $D = w \cdot L^D$

(A:2:2) says that all wage income is spent on consumption (D) of the good.

As before, labour demand can be written

(A:2:3) $L^D = n(\omega) \cdot K \cdot \exp(-\lambda t)$

where $\omega = w \cdot \exp(-\lambda t)$ and w is the wage rate in terms of the good. Labour supply is exogenously given. Full employment in steady state thus requires that the capital stock and the wage rate both grow at the rate λ.

Total wealth is equal to

(A:2:4) $B + K \equiv N(b+k)$

where B is the amount of bonds, K is the physical stock, N is labour measured in efficiency units, b is bonds per efficiency unit of labour and k is the capital/labour ratio in efficiency units (i.e. the inverse of n).

Savings are equal to changes in total wealth:

(A:2:5) $\dot{B} + \dot{K} = N(r^* \cdot k^* + r^* \cdot b)$

where k* is the steady state value of the capital/labour ratio in efficiency units.

The left hand side of (A:2:5) can be written in terms of efficiency units of labour:

(A:2:6) $\left(\frac{\dot{b}}{b} + \lambda\right)b + \left(\frac{\dot{k}^*}{k^*} + \lambda\right)k^* = r^* \cdot k^* + r^* \cdot b$

Instantaneous adjustment of the capital stock means that $\dot{k}^*/k^* = 0$, so (A:2:6) can be written:

$$\dot{b} + \lambda b + \lambda k^* = r^* \cdot k^* + r^* \cdot b \tag{A:2:7}$$

This is a differential equation in b, whose general solution is

$$b^* = (b_o + k^*) \cdot \exp(r^* - \lambda)t - k^* \tag{A:2:8}$$

There are obviously three cases:

(i) $\lambda > r^*$. In this case the model is stable and the holdings of bonds in steady state will be $-k^*$. The entire capital stock in the country will be owned by foreigners. This is so because capital grows faster when invested in the capital stock of the small country than in bonds. Thus foreigners will buy the whole capital stock or, put another way, the country will have balance of trade *deficits* because it invests more than it saves.[1]

(ii) $\lambda = r^*$. In this case bondholdings are b_o, i.e. they are determined by some initial conditions. Thus it is seen that bondholdings may well be non-zero in steady state, although they do not change value.

(iii) $\lambda < r^*$. In this case the model is not stable. The country will go on accumulating bonds as the return to bonds is greater than the rate of growth at home. Less is therefore invested than is saved or in other words, the country runs balance of trade surpluses.[2]

---

[1] Maybe Sweden, in the period when the railroads were built and growth was fast, is an example of this case.

[2] This could maybe be called the OPEC-case. At least some OPEC-countries have surpluses in the trade balance, higher return to bonds than growth possibilities at home, and they accumulate financial claims on the rest of the world.

It has been assumed that capital adjusts instantaneously. Would it change the conclusions if the investment function were included? Clearly not, as there is no feedback from the bond-holdings to the investment function. Thus, if a model with two differential equations in b and k were formulated, it would be recursive; $\dot{k}(k)$, $\dot{b}(b,k)$. This recursiveness is present in all the models in this chapter. If the equation for the accumulation of bonds were added, it could be solved recursively, after the others were solved. This explains the absence of the equation which shows the accumulation of bonds in the applications that follow.

It has thus been shown that bondholdings can be solved recursively. This is due to the savings assumption. Because of this analytical conveniency, the savings assumption is used throughout this chapter.

It has also been shown that unless one assumes $\lambda = r^*$, either the entire capital stock in the small country will be owned by foreigners (the case when $\lambda > r$), or the small country will finally own the whole world. In this case (when $\lambda < r$) one can question the small country assumption. From this I conclude that $\lambda = r$ is the only reasonable assumption.

The third conclusion is that when $\lambda = r^*$, the holdings of bonds may be non-zero. There is thus nothing inconsistent about the presence of bondholdings in steady state. Bondholdings are however necessarily constant in steady state.

APPENDIX 2:3 STABILITY OF THE TÂTONNEMENT PROCESS

In order to determine the sign of the determinant that appears in expressions (2:29)-(2:32), I will here show that stability of the short-run system (defined below) demands that the determinant be positive.

Adjustment to short-run equilibrium is instantaneous. Nevertheless one may imagine that the short-run equilibrium is brought about by a successive adjustment of ω and p. In economic theory it is usually assumed that excess demand in a market causes the price to increase and vice versa. Thus the wage and price adjustment equations (the *tâtonnement* equations) for the present model can be written:

$$\text{(A:2:9)} \quad \begin{cases} \dot{p} = f\left\{q_N\left(n_N(\omega/p)\right)\cdot\bar{K}_N - \Omega(p)\cdot\omega\cdot\bar{L}\right\} \\ \dot{\omega} = g\left(\bar{L} - n_T(\omega)\cdot\bar{K}_T - n_N(\omega/p)\cdot\bar{K}_N\right) \end{cases}$$

It is assumed that $f' < 0$, $g' < 0$ and $f(0) = 0$, $g(0) = 0$. By stability of the short-run system I mean that the system of differential equations $\dot{p}(p,\omega)$, $\dot{\omega}(p,\omega)$ in (A:2:9) is stable for given values of $\bar{K}_N$, $\bar{K}_T$ and $\bar{L}$. I will show that the condition in order for (A:2:9) to be stable is that the determinant Δ defined by (2:33) is positive.

Local stability of (A:2:9) can be determined by linearizing the expressions around the equilibrium point and examining the characteristic roots. The linear system is:

$$\text{(A:2:10)} \quad \begin{cases} \dot{p} = f'\cdot\left[-q_N'\cdot n_N'\cdot\omega^*\cdot\bar{K}_N\cdot p^{*-2} - \Omega'\cdot\omega^*\cdot\bar{L}\right]\cdot(p-p^*) \; + \\ \qquad + \; f'\cdot\left[q_N'\cdot n_N'\cdot p^{*-1}\cdot\bar{K}_N - \Omega\cdot\bar{L}\right]\cdot(\omega-\omega^*) \\ \dot{\omega} = g'\cdot n_N'\cdot\omega^*\cdot p^{*-2}\cdot\bar{K}_N\cdot(p-p^*) \; + \\ \qquad + \; g'\cdot(-n_T'\cdot\bar{K}_T - n_N'\cdot p^{*-1}\cdot\bar{K}_N)(\omega-\omega^*) \end{cases}$$

where ω^* and p^* are the equilibrium values. The characteristic equation can be written:

$$\text{(A:2:11)}\quad \mu^2 - \Big[f'\cdot\left(-q_N'\cdot n_N'\cdot\omega^*\cdot\bar{K}_N\cdot p^{*-2} - \Omega'\cdot\omega^*\cdot\bar{L}\right) -$$

$$g'\cdot\left(-n_T'\cdot\bar{K}_T - n_N'\cdot\ p^{*-1}\cdot\bar{K}_N\right)\Big] \cdot \mu + f'\cdot g'\cdot\left(-q_N'\cdot n_N'\cdot\omega^*\cdot\bar{K}_N\cdot p^{*-}\right.$$

$$\left. - \Omega'\cdot\omega^*\cdot\bar{L}\right) \cdot \left(-n_T'\cdot\bar{K}_T - n_N'\cdot p^{*-2}\cdot\bar{K}_N\right) - f'\cdot\left(q_N'\cdot n_N'\cdot p^{*-1}\cdot\bar{K}_N -\right.$$

$$\left. - \Omega\cdot\bar{L}\right) \cdot g'\cdot n_N'\cdot\omega^*\cdot p^{*-2}\cdot\bar{K}_N = 0$$

where μ is the characteristic root.

Necessary and sufficient conditions for (A:2:11) to have roots with negative real parts are:

$$\text{(A:2:12)}\quad -f'\cdot\left(-q_N'\cdot n_N'\cdot\omega^*\cdot\bar{K}_N\cdot p^{*-2} - \Omega'\cdot\omega^*\cdot\bar{L}\right) -$$

$$- g'\cdot\left(-n_T'\cdot\bar{K}_T - n_N'\cdot p^{*-1}\cdot\bar{K}_N\right) > 0$$

$$\text{(A:2:13)}\quad \left(-q_N'\cdot n_N'\cdot\omega^*\cdot\bar{K}_N\cdot p^{*-2} - \Omega'\cdot\omega^*\cdot\bar{L}\right) \cdot \left(-n_T'\cdot\bar{K}_T - n_N'\cdot p^{*-1}\cdot\bar{K}_N\right) -$$

$$- \left(q_N'\cdot n_N'\cdot p^{*-1}\cdot\bar{K}_N - \Omega\cdot\bar{L}\right) \cdot n_N'\cdot\omega^*\cdot p^{*-2}\cdot\bar{K}_N > 0$$

The first of these two conditions, (A:2:12), is satisfied because of the assumptions on the production functions, the demand function and the tâtonnement equations already made. The second condition, (A:2:13), is however not necessarily satisfied. It can be rewritten in the following way:

$$\text{(A:2:14)}\quad \left(\frac{\partial Q_N}{\partial L_N}\cdot\frac{\partial L_N^D}{\partial p} - \frac{\partial D_N}{\partial p}\right) \cdot \left(-\frac{\partial L_T^D}{\partial \omega} - \frac{\partial L_N^D}{\partial \omega}\right) +$$

$$+ \left(\frac{\partial Q_N}{\partial L_N}\cdot\frac{\partial L_N^D}{\partial \omega} - \frac{\partial D_N}{\partial \omega}\right) \cdot \frac{\partial L_N^D}{\partial p} > 0$$

Taking account of the fact that

$$\frac{-\partial L_N^D}{\partial p} \cdot \frac{p}{L_N^D} = \frac{\partial L_N^D}{\partial \omega} \cdot \frac{\omega}{L_N^D} = \eta_{L,\omega/p,N}$$

(A:2:14) can again be rewritten as:

$$\text{(A:2:15)} \quad \frac{Q_N}{\omega^* \cdot p^*} \cdot \Big[\left(-\theta_{L,N} \cdot \eta_{L,\omega/p,N} - \varepsilon_p\right) \left(-\eta_{L,\omega,T} \cdot L_T^D - \right.$$

$$\left. - \eta_{L,\omega/p,N} \cdot L_N^D\right) - \left(\theta_{L,N} \cdot \eta_{L,\omega/p,N} - 1\right) \cdot \eta_{L,\omega/p,N} \cdot L_N^D \Big] > 0$$

which can further be simplified to

$$\text{(A:2:16)} \quad \frac{Q_N}{\omega^* \cdot p^*} \cdot \Big[L_T \cdot \eta_{L,\omega,T} \cdot \left(\theta_{L,N} \cdot \eta_{L,\omega/p,N} + \varepsilon_p\right) +$$

$$+ L_N \cdot \eta_{L,\omega/p,N} \cdot \left(\varepsilon_p + 1\right) \Big] > 0$$

which is exactly the same expression as (2:33). Thus it has been shown that the condition for the short-run system to be stable, in the sense defined above, is the same as the condition that the determinant (2:33) be positive. From this I conclude that the determinant (2:33) must be assumed to be positive.

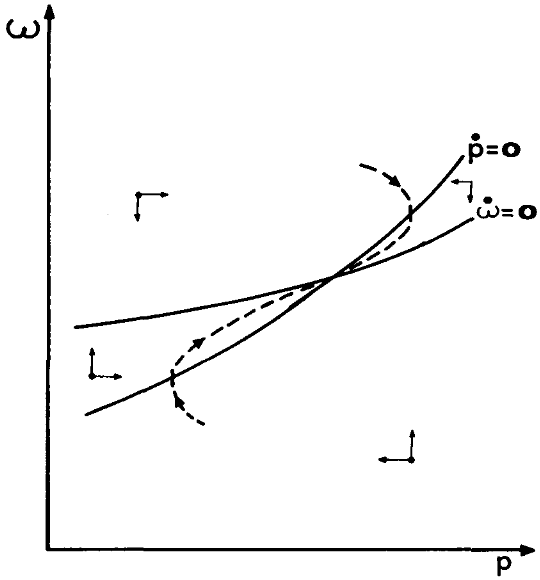
To interpret this condition one may draw the phase diagram for the system (A:2:9). To do that, the slopes of the two equilibrium locuses are needed:

$$\left.\frac{d\omega}{dp}\right|_{\dot{p}=0} = \frac{q_N' \cdot n_N' \cdot \omega \cdot p^{-2} \cdot \bar{K}_N - \Omega' \cdot \omega \cdot \bar{L}}{q_N' \cdot n_N' \cdot p^{-1} \cdot \bar{K}_N - \Omega \cdot \bar{L}} > 0$$

$$\left.\frac{d\omega}{dp}\right|_{\dot{\omega}=0} = \frac{n_N' \cdot \omega \cdot p^{-2} \cdot \bar{K}_N}{n_T' \cdot \bar{K}_T + n_N' \cdot p^{-1} \cdot \bar{K}_N} > 0$$

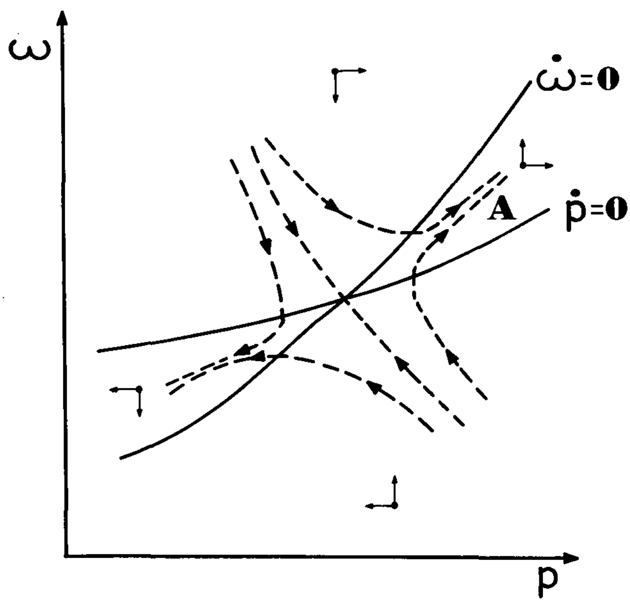
The stable case, when the $\dot{p}=0$ line has a greater slope than than the $\dot{\omega}=0$ line, is shown in figure A.2.1.

Figure A.2.1 Phase diagram for the tâtonnement process



No cycles can occur in this case. The unstable case is the case when the slope of the $\dot{\omega}=0$ line is greater than the slope of the $\dot{p}=0$ line. It is shown in figure A.2.2.

Figure A.2.2 Phase diagram for the tâtonnement process



This is a saddle-point case. The possibility that the model is unstable comes from a complex interplay between the four elasticities in expression (A:2:16). To fully describe this interplay in words is complicated. One may however get some intuitive feeling for what occurs by considering e.g. point A in figure A.2.2. At that point there is, for the given wage and relative price, excess demand in both the labour market and the market for the nontraded good. Excess demand in the labour market causes the real wage rate to increase. This has two effects in the market for the nontraded good: first, it *increases* demand (shifts the demand curve to the right), and second, it *decreases* supply (it shifts the supply curve to the left). This will unambiguously raise the relative price p. If the effect on the relative price is large, the net effect may be to *increase* labour demand in the nontraded goods sector. If furthermore this sector is large in the economy, total labour demand may increase (in spite of the fact that it must decrease in the traded goods sector). Thus, for a certain combination of values of the elasticities, an increasing real wage (excess demand in the labour market) may be associated with an *increasing* excess demand in the labour market. This obviously implies instability.

One way to simplify expression (A:2:16) is to assume that $\varepsilon_p = 0$, i.e. demand is perfectly inelastic with respect to the relative price, and that technology in both sectors is Cobb--Douglas. If α_i is the output elasticity of capital in sector i (i = T, N), it is easy to show that

$$\text{(A:2:17)} \quad \Delta \lessgtr 0 \Leftrightarrow \frac{L_N}{1 - \alpha_N} \gtrless \frac{L_T}{\alpha_T}$$

Thus the larger the share of capital in output, *in both sectors*, the more likely the model is to be unstable. Note that an extreme assumption about the price elasticity is made, however. A sufficient, although not necessary, condition for stability is that the price elasticity is smaller than -1 (larger than unity in absolute number).

3 A Vintage Growth Model with an Efficiency Function

3.1 INTRODUCTION

In this and the following chapter, the empirical questions introduced in chapter 1 will be taken up. The analysis will remain within the general theoretical structure that was used in chapter 2, but some changes in the specification of the production functions will be made, as motivated by the empirical problems.

Starting with the model from section 2.5 (which had two sectors and a Phillips curve), two important changes will be made: *first*, the assumption about malleable capital will be dropped. From now on it will be assumed that once an investment is made, the technique of production cannot be changed. The growth model will thus be what is called a vintage model, with a putty-clay technology.

Second, this vintage model will differ from conventional[1] vintage models as I assume that an efficiency function exists. It will be described thoroughly below. Briefly, it means that the productivity of a machine increases over time. It will be assumed that the speed with which efficiency increases depends on the rate of growth of wages. This mechanism is crucial for explaining the lag-phenomenon discussed in chapter 1.

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1 An example of a "conventional" vintage model is Phelps [1963].

Essentially these two modifications of the model from section 2.5 mean a certain refinement of the description of technology at the micro level (the level of the individual machine). Above all these modifications make the description of the short-run macro production function much more complex.[1] As will be demonstrated below, output at the sector level depends in a complex way on the existing vintage structure.

This allows for a much richer analysis, but complicates the mathematical formulation of the model considerably. With the use of this complex macro production function, the full two-sector model in section 2.5 no longer yields intelligible solutions. For this reason a partial analysis will be made. Only the traded-goods sector will be analyzed.[2]

When the traded-goods sector is analyzed partially, wages will be treated as an exogenous variable.

Consequently, the disadvantage of availing of the more interesting macro production function is that the model has to be used in a partial way, and hence wages have to be treated as exogenous.

Apart from these changes in the basic model, the investment function will be derived differently than in chapter 2. That will however not change the workings of the model in any fundamental way.

Since the algebraic solutions to the model are complicated, its workings will be illustrated with numerical simulations. This will be done in chapter 4.

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1 The concept of a "short-term macro production function" is used by Johansen [1972], with the same meaning as here but in a slightly different context.

2 With the exception of the derivation of a steady state for the model. In steady state things are so simple that both sectors can be handled.

In the models used so far I have assumed that labour and capital can be smoothly substituted for each other both *ex post* and *ex ante* the investment. For long-run analysis this may be a reasonable assumption to make. If the focus is on the behaviour of the economy as it adjusts to long-run equilibrium, the vintage assumption may however give more empirically interesting conclusions. For comparisons across steady states, the traditional neoclassical assumption is better suited. The gain in terms of realism, from bringing in vintages of capital seem fairly obvious. The losses in terms of analytical convenience is however not negligible. Indeed analytical convenience necessitates smooth substitutability if general equilibrium results are desired, as in chapter 2.

The vintage growth model has only produced simple analytical results for steady states.[1] The main impression from the analysis of the vintage model *in steady state* is that it behaves much like a neoclassical growth model.[2] For this reason it has been of a limited interest for purely analytical purposes.[3]

The conclusion to be drawn from the existing literature on vintage growth models is that one should only expect simple analytical results about steady state growth. In chapter 1 I argued that the growth of the traded goods sector in Sweden during the post-war period can be described as an adjustment to a steady state growth path. A vintage growth model cannot

1 The putty-clay assumption, i.e. the assumption that technology allows capital and labour to be substituted for each other *ex ante* the investment but not *ex post*, was brought into the literature on economic growth in a classical article by Johansen [1959]. The properties of growth models with this kind of technology was subsequently analyzed in a number of articles, e.g.: Phelps [1963], Kemp and Thanh [1966], Sheshinski [1967], and Solow [1962].

2 Solow [1962] explicitly makes the point that the putty-clay growth model in certain respects behaves like a neoclassical model. A vintage growth model with a *clay-clay* assumption is analyzed in detail in Solow, Tobin, von Weizsäcker, Yaari [1966].

3 One exception is Bardhan [1970] who uses two countries, both with vintage capital in the production functions. He studies patterns of trade under different assumptions. All his analysis concerns steady states. See also Petith [1972].

thus be expected to yield simple analytical results about the Swedish growth experience. It can however give numerical results, and that is how it will be used here.

In the following sections I will give the equations of a vintage growth model. The parameters of the model will, in chapter 4, be assigned values that I believe have some empirical relevance and the model will be fed with input data which are exogenous to it. The aim here is to analyze the vintage growth model as it adjusts to a steady state growth path. The model will simulate output, employment and investments as well as the distribution over vintages of output and employment.[1] It will also give the age of the oldest existing machine.

These results are interesting by themselves as they show the dynamic behaviour in the adjustment to steady state growth. If the results seem to correspond to historical data, they suggest the empirical relevance of the model. Of course it does not *prove* that the model is empirically relevant. Good simulations do, however, not allow a rejection of the hypothesis that the model's mechanisms are empirically important.

The vintage assumption renders the model more realistic. For simulation purposes it seems, however, that the vintage growth model, as it is derived by e.g. Kemp and Thanh [1966], needs to be modified.[2]

With rising real wages, a vintage growth model would predict that the latest vintage of machines has the highest labour productivity, since it uses the best known technology. Empiri-

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[1] A simulation study with a vintage growth model has been done by Solow [1963]. It had, however, a quite different purpose than the present study. Solow used a vintage model, outside of steady state growth, to generate data that were used for making regressions using models which assume smooth substitutability. The errors that follow from using the wrong assumption about technology in the regression were then studied.

[2] The case for modifying the vintage growth models to make them more realistic is argued by Jungenfelt [1982].

studies have indicated,[1] however, that the latest vintage of machines is *not*, at a given point in time, generally the most efficient. A vintage growth model built to be empirically relevant should take this into account.

The superficial look at the Swedish experience in chapter 1 also indicated that there can be considerable lags between the change in wages and their effects on aggregate employment and production. A realistic model should allow for these lags.

These considerations have led me to specify an efficiency function in the model.[2] The basic idea, which is well established in economic theory, is quite simply that, the longer a good is produced, the more efficient the production tends to be.[3]

The formulation most frequently used (formulated by Arrow [1962]) is that "learning", i.e. efficiency, is a function of cumulative gross investment. In the present model I will instead make "learning" a function of time and also make the speed with which efficiency increases a function of the rate of wage increase. Thus I will depart from Arrow's formulation.

What is gained in the model by introducing this efficiency function? Its introduction has two advantages:

(i) The newest vintage will no longer, at any given point in time, be the most efficient one. This is in accordance with the empirical studies already mentioned and should make the model more realistic.

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1 See e.g. Gregory, James [1973].

2 This idea comes from Jungenfelt [1982].

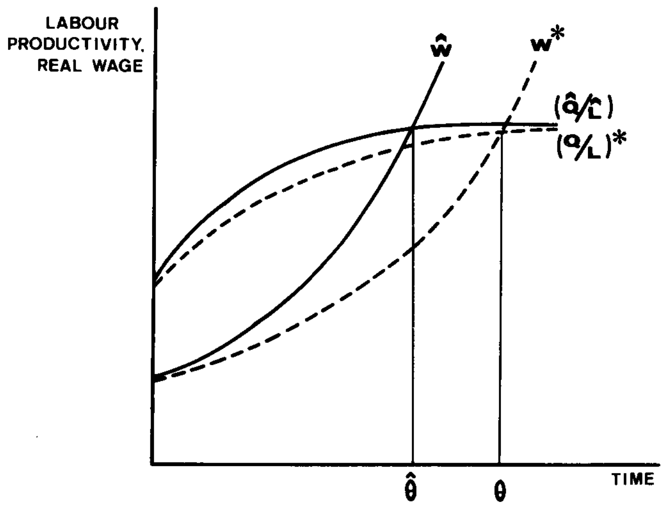
3 Lundberg [1961] observed that in the ironworks of Horndal efficiency increased although no investments were made. This was named the "Horndal--effect". Several authors (e.g. Haavelmo [1954]) have treated the phenomenon from different angles. The most often quoted analysis is Arrow [1962] who introduced the assumption that cumulative gross investment affects output in a formalized clay-clay growth model. He called it "learning-by-doing", which is the term several authors following Arrow (e.g. Levhari [1966]) have used.

(ii) It will give rise to a lag-phenomenon. When wages increase rapidly (like the period 1950-68 in Figure 1.2), firms respond by increasing efficiency faster. Thus initially, nothing need happen with the growth of aggregate output or employment after an acceleration in wages. Sooner or later the room for increases in efficiency should however be exhausted. At that time, the effects on employment and production will show. The efficiency function may in this way prove to be the mechanism that gives rise to the lags already discussed.

Just as in chapter 2, I will assume that the investors have *stationary expectations*.[1] I will assume that they base their expectations about the future wage rate on the level and rate of change they observe at the time the investment is made. This is crucial for the dynamics of the model.

A simple figure illustrates the basic point about bringing vintages and the efficiency function into the model.

Figure 3.1 Wages and the efficiency function



[1] As in chapter 2, expectations could be termed *steady state expectations* in this chapter, too. Investors will be assumed to form their expectations as if they always believed the economy was in steady state growth.

The x-axis in Figure 3.1 measures time and the y-axis labour productivity (Q/L) and the real wage rate (w). The broken line, w*, shows the expected development of the real wage rate at the time the investment is made. If investors have full information about the efficiency function, which will be assumed, thev also know how efficiency will develop, given the expected wage path. The expected path for labour productivity is denoted $(Q/L)^*$ in figure 3.1. When the machine no longer covers its variable costs, it is scrapped. The time when it is expected to be scrapped is denoted by θ in figure 3.1.

Assume now that the realized wage rate will increase faster than expected. Then labour productivity will also increase faster than expected, under the assumptions in the present model. $(\hat{Q}/\hat{L})$ shows the realized development of labour productivity. In spite of the fact that labour productivity increases faster than expected, the life of the machine will be shorter than expected. $\hat{\theta}$ denotes the realized life of the machine.

From this follows that if the wage rate increases faster than was expected, two things occur: (i) In the time period up to $\hat{\theta}$, output is greater than was initially expected.[1] (ii) In the time period between $\hat{\theta}$ and θ, capacity is lower than was expected, since the machine is scrapped.

In the simulations with the model these two effects will be important. Note however that these are micro effects. Output and employment on the macro level depend in a more complicated fashion on the existing vintage structure. The two simple effects above will be clearly visible even on the macro level, however.

Figure 3.1 also demonstrates why the assumption about stationary expectations is important. If an assumption about rational expectations (perfect foresight) had been made, no unanticipated events can occur. Thus none of the effects described above would be present.

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1 It could also be that employment is lower. That will, however, be ruled out by assumption in the applications that follow.

The general structure of the model in section 2.5, and a technology with vintages of capital and the efficiency function, constitute the model that will be used in this and the following chapter.

In the present chapter I will give the equations of this vintage growth model with an efficiency function. First it will be shown how the efficiency function is formalized and how the firms choose technology. Then aggregate investments will be derived in a slightly different fashion than in earlier chapters, and finally I will show how the steady state in a two-sector model with this kind of technology works. It will be shown to work similarly to the steady states already analyzed in chapter 2.

3.2 THE EFFICIENCY FUNCTION

Formally the *ex ante* micro production function[1] consists of a conventional Cobb-Douglas production function, multiplied by the efficiency function.

It is assumed that a new machine always starts at efficiency level x_o.[2] As time goes by, the machine becomes more efficient. The improvement of machine efficiency over time can be a result of the technological characteristics of the machine or of the learning period required for the crew to operate the machine efficiently. Regardless of the underlying causes, in the model a machine becomes more efficient over time.

A feature of the efficiency function is that a maximum attainable level of efficiency is assumed to exist, called x_1.[3] Effi-

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1 The concept of an ex ante micro production function is taken from Johansen [1972]. The function gives the production possibilities facing a micro unit before the investment is made.

2 x_o will later be assumed to vary between different machines.

3 x_1 too will be assumed to vary between different machines.

ciency level x_1 is the highest that the given state of technology permits the machine to reach. Furthermore, I assume that a constant fraction of the gap between x_1 and the efficiency level at time t is closing at each point in time. This means that the efficiency of the machine asymptotically approaches x_1. Otherwise stated, it means that the more learning that has already taken place, the harder it is to learn more.

A third assumption is that the efficiency of the machine increases faster as the product real wage increases faster. This will prove to be quite an important assumption for the numerical results. The assumption has been used earlier by Jungenfelt [1982]. I state it here simply as an assumption and not as the outcome of some optimizing behaviour on the micro level. As I am primarily interested in the consequences on the macro level of such an assumption, I have not probed deeper into how this feature may be explained.[1]

The level of efficiency at time t is assumed to be a function of the compound rate of growth of the product real wage from the date the investment in the machine is made up to time t.

With these considerations in mind, the efficiency function is formulated as (3:1):

$$x(t,v) = x_1 - (x_1 - x_o)\exp\left(-\hat{c}(t,v)\cdot z\cdot (t-v)\right) \quad (3:1)$$

1 One possible explanation of a positive relation between the wage rate and efficiency can be found in Englund [1979], pp. 90-96. He has a utility maximizing firm that can pursue its interests (partly) independently of its owners. Its utility depends on profits (positively) and on "effort" (negatively). Englund shows that it may be consistent with utility maximization for this firm to increase effort (which increases labour productivity) when wages increase. The model provides possible microfoundations for the present model.

where x is a measure of efficiency on a particular machine

t is time

v is an index of vintage (i.e. the date the machine was installed)

$\hat{c}(t,v)$ is the compound rate of growth of the product real wage between time v and time t.[1] $(t \geq v)$.

z is a parameter.

Substituting the expression for the compound rate of growth into (3:1), it can be written equivalently as:

$$x(t,v) = x_1 - (x_1 - x_o) \cdot [w(v)/w(t)]^z$$

The function (3:1) is in accordance with the assumptions already made, thus:

$$x(v,v) = x_o$$

i.e. at time v, when the investment in machines of vintage v is made, the efficiency level is x_o.

$$\lim_{t\to\infty} x(t,v) = x_1$$

provided that $z > 0$, $\hat{c} > 0$, $\forall t,v$. This means that as time goes to infinity, efficiency goes to the level x_1.

$$\frac{d}{dt}\left(x(t,v) \dot{-} x_1\right) \cdot \left[x(t,v) - x_1\right]^{-1} = \hat{c}(t,v) \cdot z$$

This states that, as long as the rate of change of the product real wage is constant, the percentage of the gap between existing efficiency and full efficiency being closed at any time is constant. It also shows that relatively more of the gap closes if the rate of change of the product real wage is higher.

[1] This means that if $w(t)$ is the wage level at time t, then $\hat{c}(t,v) = [\ln w(t) - \ln w(v)]/t-v$.

It will later be convenient to write (3:1) as

$$x(t,v) = x_1\left[1 - (1-H)\exp\left(-\hat{c}(t,v)\cdot z\cdot(t-v)\right)\right] \tag{3:2}$$

where $H \equiv \frac{x_o}{x_1}$ is a parameter that will be varied in the numerical simulations.

This exact formulation of the efficiency function has to my knowledge not been treated elsewhere in the literature,[1] although Jungenfelt [1982] employs a very similar assumption.

Compared to Arrow's formulation of learning-by-doing, this formulation differs in that here "learning" is a function of time (not accumulated investments) and here is tied to new machines and not the production of a new good. The rate of learning also depends here on the product real wage.

The micro foundations could be explored more than is done here. However, this study is mainly concerned with the macro consequences of the efficiency function.

3.3 THE CHOICE OF TECHNOLOGY AND THE EXPECTED LIFE OF MACHINES

In order to derive the macro- (or sector-) production function at any point in time, t', the labour intensities and investment volumes in all existing vintages v ($v \leq t'$) must be known. I assume that technology is of the putty-clay type, i.e. that capital and labour are freely substitutable before the investment is made, and that factor proportions are fixed *ex post* the investment. Thus the expectations held by the investors at all times $t \leq t'$ are important for actual production and employment at time t'. This occurs because the expectations at, for example, time t^o (where $t^o < t'$) determined the labour-intensity and volume of investment made at time t^o. If this unit of capital is

1 In an article by Joskow and Rozanski [1979] the existence of learning-by-doing in nuclear plant operations is tested. Their model differs from the present one in that learning is *not* a function of the rate of change of the product real wage, and that it is a function of accumulated output (and not time). They do however get significant results for an exponential shape of the learning curve, i.e. the shape of the efficiency curve that I assume here.

still in existence at t', it plays a part in determining the aggregate production and employment at t'.

One machine will be described by *two* indices. It will firstly be described by v, i.e. its vintage. This is natural as a machine installed at a certain point of time embodies the technology available at that time and also uses the factor proportions chosen at the time the investment was made. I also index the machine with x_1. For reasons later to be explained, a distribution of values of x_1 on the machines in the sector is assumed to exist at any point in time. I come back to the reasons for making this assumption in the next chapter on the investment function. For the present, the only consequence of the distribution over x_1 is that the appropriate functions below will be indexed by x_1. An index x_1 thus denotes one particular machine in a vintage v.

Taking account of this in the indexation, the production possibilities facing a potential investor are described by a slightly modified Cobb-Douglas function:

$$Q^*(t,v,x_1) = I^{\alpha}(v,x_1) \cdot L^{1-\alpha}(v,x_1) \cdot \exp(\lambda v) \cdot x^*(t,v) \tag{3:3}$$

where $Q^*(t,v,x_1)$ denotes the expected output at time t from machines of vintage v with final efficiency x_1 (throughout this chapter a star denotes expected values)

$I(v,x_1)$ is the investment volume in vintage v in machines with final efficiency x_1[1]

$L(v,x_1)$ is the amount of labour employed on machines of vintage v with final efficiency x_1

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[1] Note that, unlike e.g. Johansen [1959] or Kemp and Thanh [1966], I(v,x) is *not* a function of time, i.e. there is no physical depreciations in the model. It would have been easy to add, but I concentrate the interest to economic obsolescence and implicitly assume that economic obsolescence always dominates physical.

λ is the rate of embodied technological progress[1]

$x^*(t,v)$ is the expected level of efficiency at time t with a machine of vintage v.

The production function (3:3) can be viewed as the planning function the firm utilizes. When making an investment the firm must commit itself for a certain number of years ahead. It must then make calculations for this future time period, utilizing (3:3) and the expected time path of the product real wage.

To calculate the expected development of gross profits, the firm must form expectations about the future wage path. In this and the following chapter it will be assumed that firms always use the correct *level* of wages and prices when forming their expectations and that they expect the *rate of change* of wages and prices always to remain at that of the planning moment.[2]

So, if at time t' the product real wage rises with $c^*(t')$ per cent, the typical firm expects that the future wage path will be

$$w^*(t) = w(t')\cdot\exp\left(c^*(t')\cdot(t-t')\right)$$

Consequently, in times of accelerating wage increases, the firm will underestimate the future wage increase.

It is assumed that the firm's objective is to maximize discounted gross profits over the expected life of the machine. I denote the expected life by $\theta(v)$. If the rate of discount, which, for simplicity, is assumed to be constant, is denoted by r; if

[1] Note that there is no disembodied technological progress. This too would have been easy to add with a term $\exp\left(\gamma(t-v)\right)$, but would not contribute enough results to justify its introduction.

[2] In the numerical calculations in chapter 4, the levels and rates of change *two periods ago* have been used when expectations have been calculated. I have done this because I believe that lags of various sorts, information lags and planning lags, are important in practice. In this chapter, when developing the model, I simplify the expressions by writing them as if the wage at time t was used when forming expectations at time t. Thus I drop the two-period lag in this chapter.

the expected, discounted gross profits on vintage v, machine x_1 is denoted by $\pi(v,x_1)$; and if the capital intensity[1] is denoted by $y(v,x_1)$, then the maximization problem can be written:

$$(3:4) \qquad \underset{y(v,x_1),\theta(v)}{\text{Max}}\ \pi(v,x_1) = \int_0^{\theta(v)} \left[Q^*(t,v,x_1) - w^*(\tau)L(v,x_1)\right]\exp(-r\tau)\,d\tau$$

where $\tau \equiv t - v$.

If the production function (3:3) and the assumption about the development of the product real wage rate are substituted into (3:4), it can be maximized with respect to $\theta(v)$ and $y(v,x_1)$.

When calculating the optimal values of $\theta(v)$ and $y(v,x_1)$ it has proved convenient to assume that there is an H such that $H = \frac{x_o}{x_1}$. As mentioned I will later assume that a distribution over x_1 exists. The assumption that $x_o = H\cdot x_1$ which simplifies the computations considerably, obviously means that x_o has the same distribution as x_1.

Employing this last assumption, the optimization gives two equations in two unknowns, $\theta(v)$ and $y(v,x_1)$:

$$(3:5) \qquad \exp\left(c^*(v)\cdot\theta(v)\right) = \frac{w^e(v)\cdot[1 - (1-H)\exp\left(-z\cdot c^*(v)\cdot\theta(v)\right)]}{x^e(v)\cdot(\alpha-1)}$$

$$(3:6) \qquad y(v,x_1) = \left[\frac{w(v)\cdot w^e(v)}{(1-\alpha)\cdot\exp(\lambda\cdot v)\cdot x_1\cdot x^e(v)}\right]^{\frac{1}{\alpha}}$$

where the variables $w^e(v)$ and $x^e(v)$ are defined by:

$$w^e(v) \equiv \int_0^{\theta(v)} \exp\left(c^*(v)-r\right)\tau\ d\tau$$

$$x^e(v) \equiv \int_0^{\theta(v)} \left[1 - (1-H)\exp\left(-z\cdot c^*(v)\cdot\tau\right)\right]\exp\left(-r\cdot\tau\right)d\tau$$

---

[1] I.e. the ratio between the amount of capital and labour; $\frac{I(v,x_1)}{L(v,x_1)}$.

The variables so defined are expressions for the discounted product real wage and the discounted efficiency over the life of the machine.

What is the economic interpretations of this solution?

(3:5) is the equation that determines $\theta(v)$. It cannot be solved explicitly for $\theta(v)$, which is a major obstacle to finding explicit solutions to the model.[1] Equation (3:5) could have been derived, instead of maximizing (3:4) with respect to $\theta(v)$, by solving the following equation:

$$L(t',x_1)\cdot w^*(t'+\theta) = Q^*(t'+\theta,t',x_1) \tag{3:7}$$

(3:7) is written for the vintage installed at $t = t'$. It says that at time $t' + \theta$ the total wage bill paid should be equal to total revenue, i.e. gross profits (or quasi rents) should be zero. The solution to (3:7), with respect to θ, is the same as (3:5). It could also be noted that (3:5) can be written as:

$$\frac{dx^e}{d\theta} \cdot \frac{\theta}{x^e} = \frac{dw^e}{d\theta} \cdot \frac{\theta}{w^e} (1 - \alpha) \tag{3:8}$$

which says that the value of θ that maximizes gross profits is such that an increase in θ increases efficiency by the same percentage as it raises wages (weighed by labour's exponent in the Cobb-Douglas function; $(1 - \alpha)$). This means that the expected gain from prolonging the life of the machine in terms of increased efficiency is exactly offset by the expected loss in terms of increased wage costs.

The expression showing the optimal capital intensity looks just like the conventional static expression with the ex-

[1] The same problem with solving for θ is present in the articles by Kemp and Thanh [1966] and Phelps [1963], since their models are in many respects similar to the present model. In both these articles graphical analysis is used to ascertain that only one root $\theta(v)$ exists in the equation corresponding to (3:5). As their models are not exactly the same as the one used here, their results cannot be used to infer that only one root exists to (3:5). In no numerical simulations with (3:5) has, however, more than one root been found in the expected life interval 0-100 years.

ception that expected wages and expected efficiency paths are included. Note especially that the higher the present (and/or expected) wage rate, the higher will be the optimal capital intensity.

The values of $\theta(v)$ and $y(v,x_1)$ are the smallest building blocks in the model. They describe the technology a firm will choose at any time, given the parameters of the model and given the present and expected wage rate and efficiency. These parameters and variables are exogenous inputs in the model.

I have interpreted the choice of technology as if the firm faces a continuum of possible technologies and chooses the one that maximizes expected discounted profits. Another possible interpretation is that the capital goods producers make the calculations above. They calculate the optimal technologies and supply only these to the market. With such an interpretation, there is no choice open to the capital-using firm. This is purely a matter of interpretation, however.

To give a complete description of a vintage of machines, i.e. of the machines installed at a certain date, the only thing missing is the volume of investment at that date. Its determination is the task of the next section.

3.4 THE INVESTMENT FUNCTION

The easiest solution when making historical simulations with a vintage capital growth model would be to keep the total amount invested in each vintage exogenous.[1] That will not be done here. Instead an investment function will be derived, making investments a function of the level and expected rate of growth of wages and prices.

The basic idea behind the investment function in this chapter is similar to the concept presented in chapter 2. In steady state, investments are assumed to grow at the exogenously

[1] This is the approach chosen by Jungenfelt [1982] and by Solow [1963].

determined rate of steady state growth. In this section I derive a function describing how investments may deviate from the steady state path when wages, which are assumed to be exogenous, deviate from steady state.

The similarity with the investment function employed in chapter 2 is thus that in the present chapter, too, the investment function consists of one steady state part and one part describing the deviations from steady state. In this chapter, too, higher wages mean lower investments. The mechanisms behind the functions are, however, different. Here I assume that machines differ with respect to efficiency when used by different potential investors, and that only some investments are profitable at a certain wage level. The mechanism resembles the Keynesian textbook MEC-function (marginal efficiency of capital function).[1] Investments can be ranked with respect to profitability, and only some are carried through.

To derive this I will assume that at any point in time a distribution of values of x_1 exists, called $f(x_1)$.

The reason why this distribution exists may be that, although the capital goods are in all aspects identical when they leave the capital-goods-producing industry, they are more or less productive depending on the firm in which they are installed. This may be due to differing managerial skills, affecting the factual productivity of a unit of capital. It may also follow from the fact that different firms (not specified in the model) may consist of capital goods (machines) of different vintages. A new machine, bought by some firm, is supposed to work together with the machines already in use in that firm. Clearly its efficiency could depend on which these other machines are, with regard to their vintages.

It is not hard to imagine that there may be differences in efficiency for a new investment, depending on the firm in which it is made. The assumption employed here specifically

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[1] For a discussion of the MEC-function see e.g. Leijonhufvud [1968], pp. 157-185.

means that in some firms a new unit of capital will start at a higher level of efficiency (x_o) and also end at a higher level of efficiency (x_1) than in others. Efficiency will however increase from x_o to x_1 at the same speed in all firms.[1] Given that one wants to represent in a model differences between the firms with regard to efficiency, there are clearly a number of assumptions one could make. As I see no strong *a priori* arguments in favour of any single assumption, I have chosen a distribution on x_1 plus the assumption that $x_o = H \cdot x_1$, since this is computationally simple and produces differences between the firms with respect to efficiency.

It is assumed that the firms know their own efficiency exactly, i.e. they know the values x_o and x_1 take for their particular unit of capital.

If it is assumed that the distribution exists, then it seems reasonable that some *lowest* value of x_1 should exist, at which an investment is made (this value will in the following be denoted by $\underline{x}_1$). The higher the wage, the higher it seems that $\underline{x}_1$ would have to be to make the investment meet some target for the rate of return, r. The principle behind the investment function derived in this section will be that deviations from steady state cause a larger or smaller part of the distribution $f(x_1)$ to be used, thereby varying the volume of investment relative to steady state.

The first problem when deriving such an investment function, is the question of the shape of the $f(x_1)$-distribution. A complete theory for the determinants of the distribution could of course give the answer, but no such theory will be attempted here. Instead, I simply make an assumption about the shape of the distribution. A restriction is that x_1 should be bounded away from infinity. No machine should be allowed to be infinitely efficient. The distribution should also take only positive values. Negative efficiency does not make sense.

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[1] Jungenfelt [1982] assumes that there is a distribution on z, the speed at which efficiency increases, in the model. Clearly this is an equally realistic assumption to make as the one employed here.

With these considerations in mind, and for simple computation, I choose a rectangular distribution, i.e. a distribution described by two parameters:

$$(3:9) \qquad f(x_1) = \frac{1}{\beta-\gamma} \qquad 0 < \gamma < x_1 < \beta$$

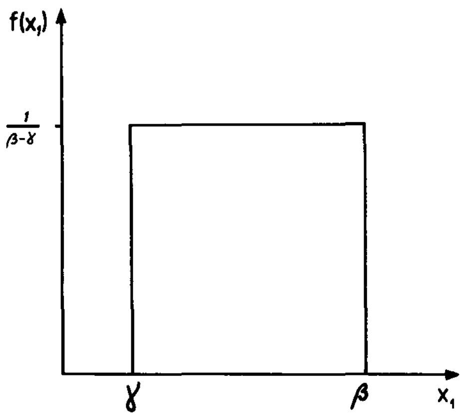
which means that:

$$(3:10) \qquad F(x_1) = \int_{\gamma}^{\beta} \frac{1}{\beta-\gamma}\, dx_1 = 1$$

where $f(x_1)$ is the frequency of machines with efficiency x_1 and $F(x_1)$ is the cumulative frequency distribution.

The distribution is pictured in figure 3.2:

Figure 3.2 A rectangular distribution



If the price of the capital good is equal to one, it is clear that the following relation must hold:

$$(3:11) \qquad I(v,x_1) \leq \pi^*(v,x_1)$$

i.e. the costs for an investment should be less than or equal to the expected, discounted profits $(\pi^*(v,x_1))$, over its expected lifetime. At the margin, (3:11) holds with equality.

From (3:11) it is clear that pure profits exist in the model. The efficiency distribution has the consequence that, unless some firms pay more for their machines than others, they earn more money than others. Pure profits are present throughout this chapter. If the differences in efficiency are interpreted as the result of differences in managerial skills between firms, the profits may be regarded as the payments to different qualities of management.

(3:11) can be solved for the marginal value of x_1, here denoted $\underline{x}_1(v)$:

$$(3:12) \qquad \underline{x}_1(v) = \frac{[w(v) \cdot w^e(v)]^{1-\alpha}}{\exp(\lambda v) \cdot x^e(v)} \cdot B$$

where $B \equiv \frac{1}{(1-\alpha)} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}$ is a constant.

It is seen that $\underline{x}_1(v)$ is higher as the wage rate at time v is higher. This value, $\underline{x}_1(v)$, cuts the distribution in figure 3.1 somewhere between γ and β, and all investments with values of $\underline{x}_1$ larger than $\underline{x}_1(v)$ are carried through.

In steady state $\underline{x}_1(v)$ will take some value, called $\tilde{\underline{x}}_1$, which, as is shown later, is independent of v. An investment function can then be written as:

$$(3:13) \qquad I(t) = \int_{\underline{x}_1(v)}^{\beta} \frac{1}{\beta - \tilde{\underline{x}}_1} dx_1 \cdot \tilde{I}(t) = \frac{\beta - \underline{x}_1(v)}{\beta - \tilde{\underline{x}}_1} \cdot \tilde{I}(t)$$

where I(t) is actual investments at time t and $\tilde{I}(t)$ is investments in steady state at time t. It is assumed that the investment volume calculated from (3:13) is spread evenly over the x_1-distribution between $\underline{x}_1(v)$ and β.

The interpretation of (3:13) is simple. In steady state $\underline{x}_1(v) = \tilde{\underline{x}}_1$ and so $I(t) = \tilde{I}(t)$. Outside steady state a certain share of $\tilde{I}(t)$ is realized, larger or smaller than $\tilde{I}(t)$ depending on whether $\underline{x}_1(v) \gtrless \tilde{\underline{x}}_1$.

The part of (3:13) that describes investment behaviour outside steady state would obviously appear differently if another shape of the x_1-distribution was used.

If the investment function is solely viewed as some function giving a relation between the wage rate and investments, then (3:13) behaves much like the functions used in chapter 2. The derivation of it is, however, quite different.

The optimal capital intensity, the expected life of a machine, and the volume of investment are all variables whose values are calculated from the *expected* paths of wages and efficiency. They are *ex ante* variables. The planned investment volume is assumed always to be carried through and of course the optimal capital intensity is chosen. If the wage path turns out to be different than expected, the life of the machine will then be other than what was expected at the time the investment was made. Expectations will not be fulfilled, affecting aggregate employment and output. The next section will give the expressions for the *realized* aggregate output, employment and life of the machines. Only in steady state are all expected and realized magnitudes equal.

3.5 THE REALIZED LIFE OF MACHINES, EMPLOYMENT AND OUTPUT

If the wage path develops differently from what was expected when an investment was made, the life of the machine and the realized efficiency will also be different than expected. This, in turn, implies that the profitability from the investment will, *ex post*, be different from what was expected *ex ante* the investment.

The realized life of a machine is a strategic variable in the model. It is denoted by $\hat{\theta}(t)$ at time t and can be calculated from:

$$(3{:}14) \qquad \frac{Q\left(t,\hat{\theta}(t),x_1\right)}{L\left(t,\hat{\theta}(t),x_1\right)} = w(t)$$

(3:14) says that the marginal machine[1] is the machine that just covers its variable costs, i.e. whose quasi-rent is zero.

The optimality conditions, (3:5) and (3:6), can be substituted into (3:14) giving (3:15):

$$\text{(3:15)} \qquad \frac{w\left(t-\hat{\theta}(t)\right)\cdot w^{e}\left(t-\hat{\theta}(t)\right)}{(1-\alpha)\cdot x^{e}\left(t-\hat{\theta}(t)\right)} \cdot \left[1 - (1-H) \cdot \left(w(v)/w(t)\right)^{z}\right] = w(t)$$

The ratio on the left hand side in (3:15) reflects the expectations the investor had when the machine was installed. It governed the choice of capital intensity at time $\left(t-\hat{\theta}(t)\right)$. The second factor affecting $\hat{\theta}$ is the term in brackets in (3:15), i.e. the realized development of efficiency for the machine. It is seen that the higher is the product real wage, the greater efficiency is. This increased efficiency leads to a longer life for the machine.

From (3:15), it is also seen that the productivity of labour is independent of x_1. Thus, all machines in a certain vintage have the same labour productivity. From the expression for optimal capital intensity, (3:6), it is seen that an investor with a higher x_1 chooses a lower capital intensity and vice versa. Labour productivity is therefore the same for all investments in that particular vintage, regardless of x_1. This property comes from the assumption that an investment with a higher x_1 also has a higher x_o, so that $x_o = H\cdot x_1$. If this had not been the case, labour productivity would have been a function of x_1 making all expressions and computations considerably more complicated. The fact that a constant H implies constant labour productivity in a given vintage is the main reason for this simplifying assumption.

It is apparent from (3:15) that the higher w(t) is, the greater the labour productivity on the marginal machine. Thus, a positive relationship exists between the level of wages and labour productivity in the model. In the empirical applications this is of great significance.

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1 Generally the marginal machine should be identified by its efficiency x_1, its vintage v and the time t at which it is the marginal machine.

When the realized life of a machine is determined, it is an easy matter to determine realized output and employment at any point in time:

$$(3:16) \qquad Q(t) = \int_{t-\hat{\theta}(t)}^{t} \int_{\underline{x}_1(v)}^{\beta} Q(t,v,x_1) \cdot 1/[\beta-\underline{x}_1(v)]dx_1 dv$$

$$(3:17) \qquad L(t) = \int_{t-\hat{\theta}(t)}^{t} \int_{\underline{x}_1(v)}^{\beta} L(t,v,x_1) \cdot 1/[\beta-\underline{x}_1(v)]dx_1 dv$$

(3:16) is the aggregate production function[1] and (3:17) the aggregate demand for labour at time t.

If the optimality conditions, (3:5) and (3:6), are substituted into (3:16) and (3:17) and the integrals evaluated to the extent possible, the aggregate production function and the aggregate demand for labour can be written in terms of the exogenous variable and the parameters of the model:

$$(3:18) \qquad Q(t) = C \cdot \int_{t-\hat{\theta}(t)}^{t} \Bigg[\left\{1 - (1-H)\cdot\left(w(v)/w(t)\right)^{z}\right\} \cdot \tilde{I}(v) \cdot \left(w(v)\cdot w^{e}(v)\right)^{\frac{\alpha-1}{\alpha}} \cdot \left(x^{e}(v)\right)^{\frac{1-\alpha}{\alpha}} \cdot \exp\left(\frac{\lambda}{\alpha}\cdot v\right) \cdot \left(\beta^{\frac{1+\alpha}{\alpha}} - \underline{x}_1^{\frac{1+\alpha}{\alpha}}(v)\right)\Bigg] dv$$

$$(3:19) \qquad L(t) = D \cdot \int_{t-\hat{\theta}(t)}^{t} \tilde{I}(v) \cdot \left(\frac{w(v) \; w^{e}(v)}{\exp(\lambda\cdot v) \; x^{e}(v)}\right)^{-\frac{1}{\alpha}} \cdot \left(\beta^{\frac{1+\alpha}{\alpha}} - \underline{x}_1^{\frac{1+\alpha}{\alpha}}(v)\right) dv$$

---

[1] The short-run macro-production function in the terminology of Johansen [1972].

where $C \equiv \dfrac{\alpha \cdot (1-\alpha)^{\frac{1-\alpha}{\alpha}}}{(1+\alpha)(\beta-\tilde{\underline{x}}_1)}$

and $D \equiv C \cdot (1-\alpha)$

are two constants.

These expressions give output and employment for all machines of all vintages existing at time t.

Expressions (3:18) and (3:19) give a more detailed description of technology than the models in chapter 2 were able to give. It is, however, apparent from (3:18) and (3:19) that this detail requires complicated functions. Generally one cannot expect to get analytical results out of expressions like the ones above. The model does, however, work in a simple fashion in steady state.

The next section will show that a two-sector model, even with this complex representation of technology, behaves in a simple manner in steady state growth.

3.6 THE STEADY STATE

In this section a two-sector growth model, similar in structure to the model in chapter 2 (section 2.5), will be constructed. The main differences between the two-sector model analyzed here and the one in chapter 2 is that technology will here be described by the vintage structure, and that investment functions will be like those derived in section 3.4. I will show that a steady state path exists for such a model. The questions of stability and uniqueness of this path will however not be analyzed.[1]

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[1] Sheshinski [1967] shows stability for a one-sector vintage growth model with a similar but not identical description of technology.

The basic outline of the model is the following:

(i) There are two sectors of production in the economy: one producing a traded good (T) and the other a nontraded good (N). The traded good can be bought at the world market at a given price. It can be used either for consumption or as a capital good, whereas the nontraded good can be used only for consumption purposes. The aggregate production functions for the two goods are given by expression (3:18) with the parameters indexed by T and N.

(ii) The investment demand functions are given by (3:13).

(iii) It is assumed that all wage income is consumed and that all capital income is saved.

(iv) Consumption demand is given by:

$$D_N(t) = \Omega_N \cdot \frac{w(t) \cdot L(t)}{p(t)} \tag{3:20}$$

$$D_T(t) = \Omega_T \cdot w(t) \cdot L(t)$$

where $w(t)$ is the wage rate in terms of the traded good and $p(t)$ the relative price, i.e. the price of the non-traded in terms of the traded good. This demand function has an income elasticity equal to one and an own price elasticity equal to minus one.

(v) The total supply of labour in the economy is inelastic, and given by:

$$L^S = \bar{L} \tag{3:21}$$

(vi) There is an exogenously given rate of interest r, used as the rate of discount by the firms when calculating the profitability from an investment. The rate, r, can be imagined as the return to some international asset, like the parameter r* in chapter 2. Since it is used by all firms as the rate of discount, it must be considered as an alternative rate of return open to all investors in the economy. It does not have the same influence as r* did in chapter 2, however. In the present model, no investment will be carried through that is not expected at least to earn r as a rate of return. The difference between r and the actual rate of return does not govern investments as in chapter 2, however

(vii) A fixed exchange rate is assumed.

(viii) The rates of technological progress are assumed to be different and denoted by λ_T and λ_N. The case when $\lambda_T = \lambda_N = \lambda$ is shown at the end of the section.

Given this basic structure of a two-sector model, I am going to show that exponential growth paths exist for the wage rate and the rate of investment in the two sectors, such that three equilibrium conditions are satisfied at all points in time. This I shall call the steady state growth path. I will also show the paths for output in the two sectors in steady state.

To begin with I shall *assume* that the realized as well as the expected life of machines is constant in steady state. Later I will show that this must indeed be the case.

The technique is to assume that steady state paths occur for the wage rate (in terms of the traded good), for the rate of investment in the two sectors and for the relative price. I then derive the value which these rates of growth must take if the three equilibrium conditions are to be satisfied. I thus assume the following steady state growth paths for the wage rate, for investments in the two sectors and for the relative price:

$$(3{:}22) \qquad \tilde{w}(t) = \tilde{w}(0)\cdot\exp(\tilde{c}\cdot t)$$

$$(3{:}23) \qquad \tilde{I}_T(t) = \tilde{I}_T(0)\cdot\exp(\tilde{d}_T\cdot t)$$

$$(3{:}24) \qquad \tilde{I}_N(t) = \tilde{I}_N(0)\cdot\exp(\tilde{d}_N\cdot t)$$

$$(3{:}25) \qquad \tilde{p}(t) = \tilde{p}(0)\cdot\exp(\tilde{a}\cdot t)$$

where the symbol " ˜ " denotes a variable in steady state.

The equilibrium conditions that must be satisfied at all points in time are:

$$(3{:}26) \qquad Q_N(t) = D_N(t)$$

$$(3{:}27) \qquad Q_T(t) = D_T(t) + I_T(t) + I_N(t)$$

$$(3{:}28) \qquad L^S = L_N^D(t) + L_T^D(t)$$

(3:26) says that the market for the nontraded good clears. (3:27) says that trade is balanced or, equivalently, that savings are equal to investments. (3:28), finally, is the condition for full employment.

Full employment can obviously be achieved only if the employment in the respective sectors is constant, i.e. if $L_N^D(t) = \tilde{L}_N$ and $L_T^D(t) = \tilde{L}_T$.

If the assumed steady state paths for wages, investment, and the relative price (3:22)-(3:25) are substituted into the expression for aggregate labour demand, and if the integral is solved, the result is:

$$(3{:}29) \qquad \tilde{L}_N^D = L_N^O \cdot \exp\left[\left(\tilde{d}_N + \frac{\lambda_N+\tilde{a}-\tilde{c}}{\alpha_N}\right)\cdot t\right]$$

$$(3{:}30) \qquad \tilde{L}_T^D = L_T^O \cdot \exp\left[\left(\tilde{d}_T + \frac{\lambda_T-\tilde{c}}{\alpha_T}\right)\cdot t\right]$$

Here L_N^O and L_T^O are constants that are too complex to be of any interest.

The conditions for the right hand sides of (3:29) and (3:30) to be constant are:

$$\text{(3:31)} \qquad \tilde{d}_N = \frac{\tilde{c} - \lambda_N - \tilde{a}}{\alpha_N}$$

$$\text{(3:32)} \qquad \tilde{d}_T = \frac{\tilde{c} - \lambda_T}{\alpha_T}$$

which gives two relations in the four unknowns $\tilde{d}_N$, $\tilde{d}_T$, $\tilde{c}$ and $\tilde{a}$. For a steady state to have full employment, (3:31) and (3:32) must be satisfied.

Turning to the aggregate production functions, substitution of (3:22)-(3:25) into (3:18) gives:

$$\text{(3:33)} \qquad \tilde{Q}_N(t) = Q_N^O \cdot \exp\left[\left(\tilde{d}_N + \frac{(\tilde{c}-\tilde{a})(\alpha_N-1)+\lambda_N}{\alpha_N}\right)\cdot t\right]$$

$$\text{(3:34)} \qquad \tilde{Q}_T(t) = Q_T^O \cdot \exp\left[\left(\tilde{d}_T + \frac{\tilde{c}(\alpha_T-1)+\lambda_T}{\alpha_T}\right)\cdot t\right]$$

where Q_N^O and Q_T^O are constants.

The conditions for equilibrium in the goods market can now be written as:

$$\text{(3:35)} \qquad Q_N^O \cdot \exp\left|\left(\tilde{d}_N + \frac{(\tilde{c}-\tilde{a})(\alpha_N-1)+\lambda_N}{\alpha_N}\right)\cdot t\right| = \Omega_N \cdot \frac{\tilde{w}(0)\cdot\exp(\tilde{c}\cdot t)\cdot\bar{L}}{\tilde{p}(0)\cdot\exp(\tilde{a}\cdot t)}$$

$$\text{(3:36)} \qquad Q_T^O \cdot \exp\left[\left(\tilde{d}_T + \frac{\tilde{c}(\alpha_T-1)+\lambda_T}{\alpha_T}\right)\cdot t\right] = \Omega_T\cdot\tilde{w}(0)\cdot\exp(\tilde{c}\cdot t)\cdot\bar{L} +$$

$$+ \tilde{I}_T(0)\cdot\exp(\tilde{d}_T\cdot t) + \tilde{I}_N(0)\cdot\exp(\tilde{d}_N\cdot t)$$

which are to be satisfied for all t. If the values for $\tilde{d}_N$ and $\tilde{d}_T$ obtained in (3:31) and (3:32) are substituted into (3:35) and

(3:36), the result is:

$$Q_N^O \cdot \exp\left((\tilde{c}-\tilde{a})\cdot t\right) = \Omega_N \cdot \frac{\tilde{w}(0)\cdot\bar{L}}{\tilde{p}(0)} \cdot \exp\left((\tilde{c}-\tilde{a})\cdot t\right) \tag{3:37}$$

$$Q_T^O \cdot \exp(\tilde{c}\cdot t) = \Omega_T\cdot\tilde{w}(0)\cdot\bar{L}\cdot\exp(\tilde{c}\cdot t) + \tilde{I}_T(0)\cdot\exp(\tilde{d}_T\cdot t) + \tilde{I}_N(0)\cdot\exp(\tilde{d}_N\cdot t) \tag{3:38}$$

Obviously the market for the nontraded good can clear as long as output in that sector grows at the rate $\tilde{c}-\tilde{a}$.

The market for the traded good will clear if

$$\tilde{d}_T = \tilde{d}_N = \tilde{c} \tag{3:39}$$

that is, if the rates of growth of the capital stocks in the two sectors are equal and furthermore equal to the rate of growth of the wage rate in terms of the traded good.

If (3:39) is substituted into (3:31) and (3:32), the rates of growth of output in steady state in the two sectors and the rate of growth of the relative price can be expressed in terms of technological parameters:

$$\tilde{c} = \frac{\lambda_T}{1-\alpha_T} \tag{3:40}$$

$$\tilde{c}-\tilde{a} = \frac{\lambda_T}{1-\alpha_T} \cdot \alpha_N + \lambda_N \tag{3:41}$$

$$\tilde{a} = \lambda_T\cdot\left(\frac{1-\alpha_N}{1-\alpha_T}\right) - \lambda_N \tag{3:42}$$

This two-sector economy is thus capable of steady state growth if the allocation of labour between the two sectors is constant, if output from the traded goods sector grows at the rate determined by (3:40) and if the capital stocks in both sectors and the wage rate in terms of traded goods grow at the same rate. Output from the nontraded goods sector should grow at the rate determined by (3:41), and the relative price as shown by (3:42).

Additionally, if (3:41) is added to (3:42), expression (3:40) results. This means that the *value* of the output from the nontraded goods sector will grow at the same rate in steady state as the value of output from the traded goods sector. In steady state the ratio between the two sectors, in value terms, is therefore constant.

These are similar results to those obtained in chapter 2 and they show that the vintage model is not too different from an ordinary neoclassical model in steady state.[1]

If the rates of Harrod-neutral technological progress are equal in the two sectors, as was the case in section 2.5, then $\lambda_T/(1-\alpha_T) = \lambda_N/(1-\alpha_N)$. (See footnote below.) If this assumption is substituted into (3:42) above, the relative price is constant in steady state. This is the result already derived in chapter 2.

Only two things are left to be demonstrated concerning the steady state path, the constancy of the life of machines and the constancy of $\underline{x}_1(v)$.

[1] The fact that the steady state rate of growth in this chapter is $\lambda_T/1-\alpha_T$, while in section 2.6 it was λ_T, does not have any relation to the vintage assumption. It is merely a result of a somewhat confusing notation. In section 2.6 λ_T was assumed to be *the rate of Harrod-neutral technological progress*. In this chapter the vintage production function is written:

$$Q_T = K_T^{\alpha_T} \cdot L_T^{1-\alpha_T} \cdot \exp(\lambda \cdot t) \qquad \text{or equivalently}$$

$$Q_T = K_T^{\alpha_T}\left[L_T \cdot \exp\left\{\left(\lambda_T/1-\alpha_T\right)\cdot t\right\}\right]^{1-\alpha_T} \Leftrightarrow Q_T = \left(\frac{K_T}{L_T \cdot \exp\left((\lambda_T/1-\alpha_T)\cdot t\right)}\right)^{\alpha_T} \cdot L_T \cdot \exp\left((\lambda_T/1-\alpha_T)\cdot t\right)$$

From this is seen that the growth rate of labour *in efficiency units* is $\lambda_T/1-\alpha_T$ which is the parameter that I called λ_T in chapter 2. There is thus no difference in the rates of steady state growth in the two models.

In steady state, the expected rate of wage increase will always be equal to the realized rate. From expression (3:5), which determines the expected life, it is clear that θ must be a constant if the expression is to hold with a constant c. If any of the parameters should change then θ will change too. The same conclusion follows of course from (3:15); the expression that determines the realized life. It can only be satisfied in steady state if θ is a constant.[1] From this I conclude that θ is constant in steady state.

Finally, it remains to be shown that $\underline{x}_1$ is a constant in both of the sectors in steady state. For the nontraded goods sector, the marginal investment should be such that $I_N(v,x_1) = \pi_N^*(v,x_1)$ where gross profits are measured in terms of the traded good. Taking this into account, it can be shown that the marginal value of x_1 in each of the two sectors in steady state can be written as:

$$(3{:}43) \qquad \underline{x}_{1,T}(v) = \frac{[\tilde{w}(0)\cdot\exp(\tilde{c}\cdot t)\cdot w^e(v)]^{1-\alpha_T}}{\exp(\lambda_T\cdot v)\cdot x^e(v)} \cdot B_T$$

$$(3{:}44) \qquad \underline{x}_{1,N}(v) = \frac{[\tilde{w}(0)\cdot\exp(\tilde{c}\cdot t)\cdot w^e(v)]^{1-\alpha_N}}{\exp(\lambda_N\cdot v)\cdot x^e(v)\cdot\tilde{p}(0)^{(1-\alpha_N)}\cdot\exp(\tilde{a}\cdot v)} \cdot B_N$$

where B_T and B_N are constants, as defined after the expression (3:12).

Since $\tilde{c} = \lambda_T/1-\alpha_T$ in steady state it is clear that $\underline{x}_{1,T}$ is indeed a constant in steady state. If the steady state conditions (3:40)-(3:42) are substituted into (3:44), $\underline{x}_{1,N}$ will also be a constant. This shows that the investment functions as they have been derived here are compatible with steady state growth.

---

[1] Kemp and Thanh [1966] succeed to derive an explicit expression for θ in steady state. Their vintage model is in a number of aspects simpler than the one used here. Of course the model used here could be simplified so that their results are obtained. Without those simplifications I have not been able to solve explicitly for θ, even in steady state.

Thus I have shown that a steady state growth path, as it has been defined here, does exist in the vintage model. The growth rates for the different variables in terms of the parameters of the model have been presented.

This vintage growth model can give a relatively detailed idea of what happens at the micro level when the development at the macro level is described as an adjustment to a steady state growth path. This gives other aspects of the growth process than the models from chapter 2. As already mentioned, this realism comes at a cost. The model is so complicated that I cannot get any explicit analytical solutions from it. Thus I have to make numerical simulations.

In the next chapter the simulations for the traded goods sector will be shown. I will compare them with historical data in an attempt to assess the empirical relevance of the simulations.

4 The Empirical Application of the Vintage Model

4.1 INTRODUCTION

In chapter 1, I presented a hypothesis attempting to explain the cease in growth in the traded goods sector in Sweden in the middle of the 1970s. Wages were treated as exogenous, and as *below* their steady state level before the mid-1960s. (1966 to be exact) and as *above* their steady state level after that time. This may have produced higher growth rates than the steady state growth rate until around 1966 and eventuallly lower growth rates and stagnation thereafter. I maintained that considerable lags and inertia in the system may have delayed the effects of the high wage levels on the real variables.

The purpose of this chapter is to demonstrate the mechanisms which generate this interpretation of the historical data. The vintage growth model in the previous chapter was designed to empirically analyze growth in the traded goods sector during the period mentioned. In this chapter, I will use the model to make a partial analysis of the traded goods sector. The parameters of the model and the initial conditions are assigned values that seem reasonable from an empirical point of view. No attempt is made, however, to use econometric methods to determine parameter values. The simulations using the model can be regarded as numerical examples which hopefully have empirical relevance.

The exogenous variable in the simulations is the product real wage. The endogenous variables are aggregate output and employment as well as the distribution of output and employment over vintages, the realized and expected life of machines and, finally, the amount of investment. The values of all the variables in steady state are calculated.

There are a number of reasons why I have considered it worthwhile to conduct these simulations:

(i) First, the simulations provide one test of the empirical relevance of the model. The simulations are compared with the actual development of output, employment and investments. This may allow the rejection of the hypothesis that the model is empirically relevant. As indicated above, the distribution of the endogenous variables over the vintages as well as the amount of slack at any time are also simulated and analyzed for their empirical accuracy, and subsequent relevance.

(ii) Second, the simulations give a number of results of sheer theoretical interest, since they show how a vintage capital model develops outside steady state growth. The results give in a certain sense both the macro development (represented by the aggregated variables) and the micro development (represented by the variable distributions over the vintages) during a given growth phase.

(iii) Third, particular interest is devoted to the model's aggregate slack during the presentation of the results. Since it is assumed that firms do not always operate their machines at the technically maximal efficiency, a certain amount of slack exists in the model at any given time.[1] The amount of slack at any point in time depends on how fast the product real wage has risen historically, since the slack is reduced when the product real wage increases fast.

---

[1] This aggregate slack resembles what Leibenstein [1966] termed x-inefficiency.

The hypothesis advanced here about the stagnation in the seventies is critically dependent upon the existence of this aggregate slack. This slack cushioned the effect of the rapidly increasing wages around the middle of the 1960s. However, the product real wage rate persisted above its steady state level, and eventually towards the middle of the 1970s stagnation appeared. This explanation is in contrast to other attempts to explain the stagnation after 1975, which have generally placed a greater emphasis on what is known in the Swedish policy debate as the cost--crisis between 1975 and 1977.[1] Other explanations focus in a certain sense more on the short run, and hence lead to other policy conclusions. These I will come back to in chapter 5.

In the simulations I try to find out to the extent possible whether it is reasonable to regard the stagnation as a delayed response to the product real wage remaining above its steady state level after 1966. I therefore devote special interest to the amount of slack in the sector at different points in time.

In the next section, 4.2, I describe the *a priori* considerations when giving values to the parameters of the model. The parameters I discuss include α_T and λ_T, which together imply the rate of steady state growth as was shown in chapter 3.

In section 4.3, the initial conditions used in the simulations are shown. I have chosen 1953 as the starting year in the simulations. The initial conditions include the distribution over the existing vintages of the capital stock in 1953. The position of the economy relative to the steady state path, or, more specifically, the value of investments and the product real wage relative to their steady state values, are also specified.

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[1] The debate on the stagnation in Swedish industry is briefly summarized in my report to the Medium Term Survey, Wissén [1980].

Once values for the parameters are chosen and once the initial conditions are stated, the model can simulate the development of the sector given an exogenous path for the product real wage. The realized path for the product real wage is discussed in section 4.4. In that section, expectations formation and lags in the model are also discussed. The results from one simulation with the model for the period 1953-1978 are shown in section 4.5. In section 4.6, the sensitivity of the solution analyzed in section 4.5 is discussed. Its sensitivity with regard to variations in the wage level during different years and to variations in the parameters H and z is shown. Variations in H and z illustrate the importance of the slack with regard to the solution.

4.2 THE CHOICE OF PARAMETER VALUES

The rate of technological progress in the traded goods sector, λ_T, and the output elasticity of capital in the *ex ante* micro production function, α_T, are strategic parameters in the model. The expression $\lambda_T/1-\alpha_T$ is, as has already been shown, the rate of Harrod-neutral technological progress. This is the growth rate of output, investment and the product real wage in steady state. One way to approach the question of which values these parameters should reasonably be assigned, may be to look at the value of the actual rates of growth of these variables during the period. Table 4.1 shows the year-to-year growth rates for value added, investments and the product real wage in the traded goods sector in Sweden for the period 1953-1978:

Table 4.1 *Growth rates of value added, investments and the product real wage in the traded goods sector in Sweden 1953-1978*

Year	Rate of growth of value added Percent	Rate of growth of investments Percent	Rate of growth of product real wage Percent
1953	0.9	-8.3	12.6
54	4.6	28.8	2.5
55	5.2	-3.3	5.6
56	4.1	-1.1	5.3
57	4.8	-3.4	5.1
58	1.5	19.2	6.8
59	3.0	8.4	3.9
60	9.9	6.9	6.1
61	6.5	16.1	7.5
62	6.1	6.2	9.0
63	5.2	5.2	11.9
64	8.6	-0.6	4.8
65	6.5	10.3	5.8
66	1.4	5.6	7.8
67	3.3	-1.0	10.6
68	4.5	0	9.0
69	7.2	8.4	5.4
70	6.4	3.5	2.3
71	1.9	4.3	9.0
72	1.1	5.5	8.0
73	7.2	11.7	-2.7
74	3.7	7.2	-6.1
75	-3.4	0	8.3
76	-1.4	-1.8	9.9
77	-7.3	-20.2	4.1
78	0	-22.4	
Average 53-78	3.5	3.6	5.7 (average 53-77)

Data source: The National Accounts.

The actual growth rates ought not to be, on the average, too far from the steady state rates. In Table 4.1, a considerable dispersion appears in the growth rates from year to year. Of course, this has partly to do with business cycles, from which I abstract. Over the wole period, value added increased by 3.5 % per year, the volume of investments by 3.6 % and the product real wage by 5.7 %. Since none of these variables is assumed to have been on its steady state path for the whole period, the figures above need not represent the steady state rate of growth. If they approach the steady state path, with value added and investments starting *above* the path and the product real wage rate *below* it, then 3.5 % and 5.7 % represent the reasonable limits of the interval where the steady state growth rate lies. Any growth rate in this interval would seem equally feasible. A combination of assumptions about the intercept and the growth rate of the steady state path is required, however. I have settled for a 5 % rate of growth in steady state. This was shown in chapter 1 in Figures 1.2 - 1.6. If anything, the figure chosen, 5 %, is on the high side, but the exact number is not of crucial importance for this study. The choice of 5 % growth in steady state places a restriction on the combination of possible values of α_T and λ_T.

In a conventional neoclassical equilibrium model, α_T is equal to the share of capital income in total income. Since the present model is a vintage model, α_T is not necessarily equal to the share of capital. In a neoclassical model, the share of labour income is one minus the share of capital income if linear homogeneity is assumed. For an existing vintage, v (I omit the index x_1), the share of labour income in output at time t can be computed as:

$$(4:1) \qquad \frac{w(t)\cdot L(v)}{Q(t,v)} = (1-\alpha)\cdot\frac{w(t)}{w(v)}\cdot\frac{x^e(v)}{w^e(v)}\cdot\left[1-(1-H)\cdot\left(\frac{w(v)}{w(t)}\right)^z\right]$$

From this it is seen that at the time the investment is made, i.e. when t = v, labour's share of output is equal to:

$$(4:2) \qquad \frac{w(v)\cdot L(v)}{Q(v,v)} = (1-\alpha)\cdot\frac{x^e(v)}{w^e(v)}\cdot H$$

which is greater or less than (1-α) depending on whether $x^e(v)/w^e(v)\cdot H$ is greater or less than one. Since H is necessarily less than one, it is sufficient that $w^e(v) > x^e(v)$ for labour's share to be less than (1-α) at the time the investment is made. It has already been shown, in (3:7), that at the marginal vintage, labour's share of output is equal to unity. So labour's share of output goes from the value shown in (4:2) to unity. It is seen from (4:1) that it must increase monotonically in steady state. The share of labour in the aggregate (sector) output at time t is given by:

$$(4:3) \qquad \frac{w(t) \cdot \int_{t-\hat{\theta}(t)}^{t} \int_{\underline{x}_1(v)}^{\beta} L(v,x_1) \cdot 1/[\beta-\underline{x}_1(v)]dx_1dv}{\int_{t-\hat{\theta}(t)}^{t} \int_{\underline{x}_1(v)}^{\beta} Q(v,x_1) \cdot 1/[\beta-\underline{x}_1(v)]dx_1dv}$$

which is constant in steady state. It can be shown that labour's share in the sector output is equal to $(1-\alpha_T)$ if $r = \lambda_T/1-\alpha_T$, i.e. if the rate of interest is equal to the growth rate in steady state.[1] If $r > \lambda_T/1-\alpha_T$, Kemp and Thanh [1966] show that, in their model, labour's share is greater than $(1-\alpha_T)$ and vice versa. If $r = \lambda_T/1-\alpha_T$, the labour share is less than $(1-\alpha_T)$ for younger machines, which is exactly counterbalanced by a labour share larger than $(1-\alpha_T)$ for older machines. On the average the share is $(1-\alpha_T)$.

In the empirical applications of the model which follow, it will always be true that the rate of interest, r, will be greater than the rate of growth in steady state. Consequently, $(1-\alpha_T)$ is expected to have a *lower* value than labour's share of the income generated in the sector. In other words, the value of α_T should be higher than the share of capital income in total income.

The development of labour's share of value added, as measured by the data used for this study, is given in Table 4.2:

---

[1] This is shown in Appendix 4:1 for the present model. Kemp and Thanh [1966] show a corresponding theorem for their model.

Table 4.2 *Labour's share of value added in the traded goods sector in Sweden 1950-1975*

1950	57.3 %
1955	62.8 %
1960	64.6 %
1965	67.9 %
1970	68.5 %
1975	65.8 %

Data source: The National Accounts.

There is a rising trend in labour's share of value added in the traded goods sector. Approximately 0.65 seems reasonable as an average. This means that 0.35 can be taken as a lower limit of the interval where α_T could be found.[1] In the simlations that follow, α_T will be set equal to 0.44. It is clearly possible to change that value in either direction.

Given that I have assumed the rate of steady state growth to be equal to 5 % and α_T to 0.44, it follows that λ_T must be equal to 0.028. The rate of embodied technological progress in the *ex ante* production function is assumed to be 2.8 %. Values of at least ±1 % would seem equally acceptable.[2]

The rate of discount, r, used by the firms could, as mentioned, be interpreted as the rate of return to some international asset. In the simulations this parameter has

---

[1] In the present model I only consider machine capital. The 35 % is payments to all capital involved in production. Jungenfelt [1982] assumes that machine capital has 60 % of the output elasticity of total capital. This means that 0.21 should be taken as the lower limit of α_T.

[2] Without going into details two estimations for Swedish industry during the post-war period can be given. Yngve Åberg [1969] estimated the rate of embodied technological progress for Swedish industry 1947-1964 to 1.19 %. Lars Heikensten [1977], using Solow's [1959] method, got 3.5 % for the period 1950-1971. This does give an interval between approximately 1 % and 4 % within which there are reasons to expect to find λ_T.

not appeared to influence the results very much. I have set r equal to 0.11.[1]

The parameter β is the upper limit in the rectangular distribution of $x_1; f(x_1)$. It appears in the expressions for aggregate output and aggregate labour demand, due to its presence in the investment function. Obviously β must be set so that it is greater than $\underline{x}_1(v)$. Otherwise investments would be zero or negative. β does not have any measurable economic interpretation, so its value is based on how closely the investment curve has fitted actual data for different values of β. A very high value of β gives a flat investment path, given the development of the product real wage. This is seen from the investment function (3:13). A given variation in w makes the change in $\underline{x}_1$ *small* as a percentage of β, when β is given a high value. If β is low, a change in the product real wage causes $\underline{x}_1$ to change a lot relative to β, and brings about a great change in investments. So, the value of β affects the sensitivity of investments to changes in the wage rate. I have chosen $\beta = 50$, on the grounds that the simulated path for investment looks sound with that value. I have found no other way to determine β.

Four parameters remain to be determined: $I(0)$, $w(0)$, H and z. Two of these, $I(0)$ and $w(0)$, will be discussed in the next section as they are initial conditions given to the model for the year 1953.

The values of H and z both contribute to determine the aggregate slack existing in the model at any time. As already suggested, they are crucial for the hypothesis advanced concerning Swedish post-war growth. For this reason I will vary these two parameters in a later section, illustrating the hypothesis.

---

[1] There is a computational reason for choosing this value: If $\hat{c}$, the realized rate of growth of the product real wage, is equal to r, a number of expressions get denominators equal to zero. 0.11 is higher than the highest value of $\hat{c}$ ever realized in the simulations. It was chosen on this ground.

The other parameters discussed in this section will have throughout the values which have been determined here.

4.3 THE INITIAL CONDITIONS

To make simulations for the period 1953 to 1978, the model must be given initial conditions. They include the position relative to the steady state path, of the product real wage and investments. They also include the distribution over vintages of the existing capital stock in 1953.

In chapter 1, the assumption has already been made that the steady-state level of the wage rate in the beginning of the 1950s was 20 % *above* the realized level. In 1949, Sweden devalued the *krona* with, on the average, 15 %. The effects of this on the product real wage depend, as was discussed in chapter 2, on the model used. It has been argued, however, that the 1949 devaluation strengthened the competitiveness of Swedish industry considerably.[1] Whether the devaluation placed the product real wage above or below the steady state path is not evident. Since there seem to have been lasting positive effects of the devaluation on competitiveness, I think it may be fair to assume that it placed the product real wage *below* the steady state level.

There are other arguments which favour this assumption. The assumption that the product real wage is below its steady-state level means that it can be expected to increase faster than the rate of steady state growth for a period. This phenomenon indicates a certain growth potential in the sector. What reasons are there in the beginning of the 1950s to believe that the traded goods sector in Sweden had a growth potential in this sense?

Firstly, Sweden was one of the few countries which were not involved in the Second World War. The labour force and the capital stock were intact and could supply goods

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[1] Erik Lundberg, for instance, has argued that the 1949 devaluation created an "international margin for expansion" [1971] p. 13.

demanded in Europe. Sweden's relatively high production of investment goods put the country in a favourable position to meet post-war demand.

Secondly, Swedish industry was over-diversified. Having been in autarky during the War, Sweden had productivity gains to reap from specializing in the goods where the country had a comparative advantage. This also meant that growth could be faster than in steady state during a transitional phase.

Thirdly, it has been argued[1] that all West-European countries, including Sweden, were technologically behind the U.S.A. This happened already between the two World Wars, and the gap increased during the Second World War. As the European countries lagged behind technologically, and as technology was relatively easy to imitate, these countries could experience faster than steady state growth in the period when they closed the gap.

I have considered these arguments when deciding the intercept for the product real wage. The constant w(0) is set equal to 120 and the realized product real wage, which is exogenous to the model, is set equal to 100 in 1953. Thus I assume that the steady state level of the product real wage is 20 % above the realized level in 1953. The exact number, 20 %, is of course arbitrary.

If the steady state level of the product real wage was 20 % above its realized level, it follows that realized investments most likely were *above* the steady state path for investments. As is described later, investments are held exogenous until 1953. Thus, the relation between the realized investment in 1953 and steady state investments must be determined. I assume that I(0), the value of investments in steady state in 1953, is 90, and the value of the realized

[1] See Abramovitz, M. [1977].

investments is 100,[1] i.e. that steady state investments are 10 % below the realized level. The exact number, 10 %, is arbitrary.

The intercepts of the other variables, i.e. the relation between realized and steady state values for output and employment, are endogenous in the model.

The distribution of the capital stock over the existing vintages in 1953 is among the initial conditions. To derive this distribution one needs to know the amount invested on all existing vintages, from the oldest existing vintage in 1953, to vintage v = 1953, as well as the capital-labour ratios that were chosen on these vintages. Both the amount invested in a vintage and the capital-labour ratio could be simulated by the model. The volume of investment could, however, also be kept exogenous. Some of the years involved, especially the War years, were unusual and could not be expected to be simulated well by the model. Thus I have kept the volume of investments for the vintages before 1953 exogenous.

The development of investment in machinery in Swedish industry 1900-1978, as well as the functions used in the simulations to approximate the curve for the years 1900--1953, is shown in Figure 4.1.

---

[1] Strictly speaking investments are exogenous up to 1953, and are endogenously determined by the model from 1953 and on. Thus realized investments are 100 just before 1953, and determined by the simulation thereafter.

Figure 4.1 Investment in machinery in Swedish industry 1900-1978

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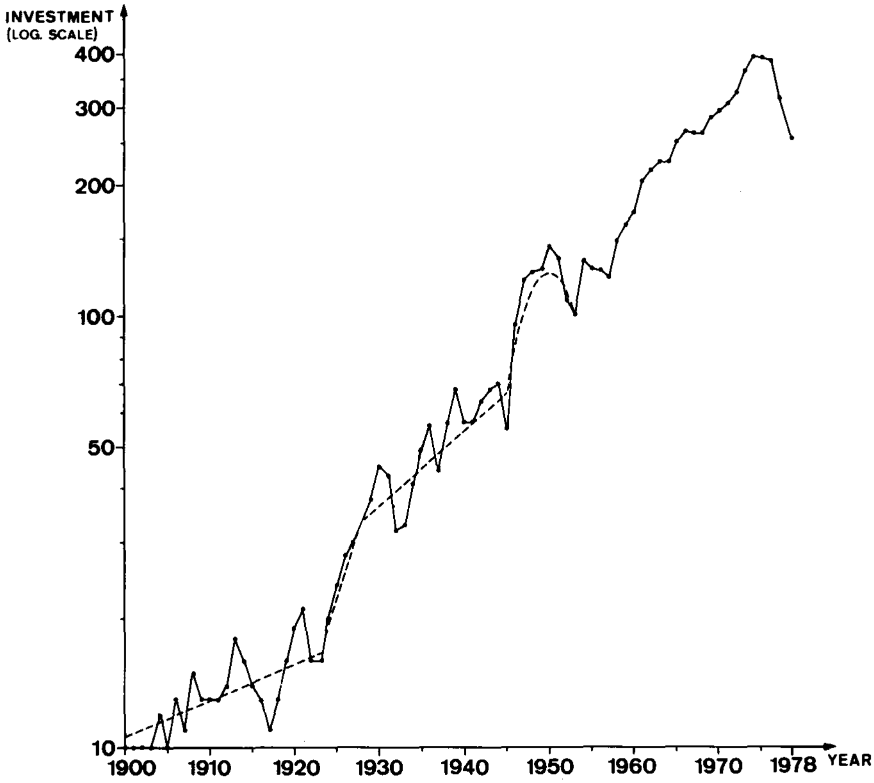
1953=100



Data source: For the period 1900-1953, Johanson, Ö. [1967]
for the period 1953-1978, the National Accounts

———— Actual development

- - - - Function used as approximation in the simulations

The functions used to approximate the actual development of investments are:

$I(v) = 100 \cdot \exp(-1.18 + 0.02 \cdot v)$ $1900 \leq v \leq 1923$

$I(v) = 100 \cdot \exp(2.42 + 0.14 \cdot v)$ $1923 \leq v \leq 1928$

$I(v) = 100 \cdot \exp(-0.08 + 0.04 \cdot v)$ $1928 \leq v \leq 1945$

$I(v) = 100 \cdot \exp\left((-0.15 - 0.025 \cdot v) \cdot v\right)$ $1945 \leq v \leq 1953$

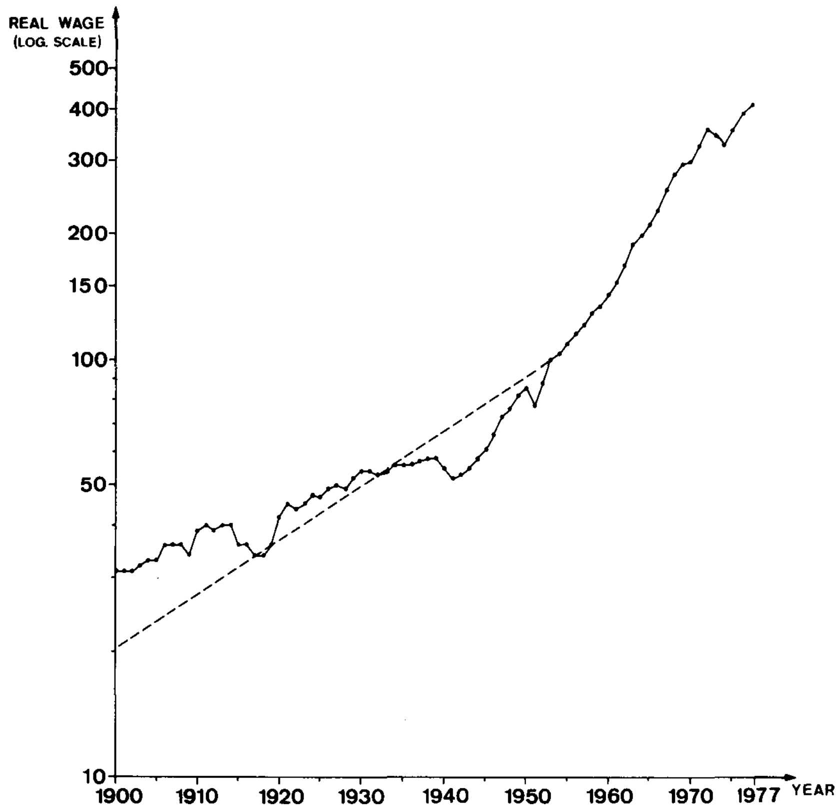
The age of the oldest existing machine is endogenous in the model, and in many of the simulations it has been around fifty years in 1953. This means that the oldest existing machine had been installed around the turn of the century.

The development of the product real wage between 1900 and 1977, and the function used to approximate it, is shown in Figure 4.2.

Figure 4.2 The product real wage in Swedish industry 1900-1977

Index

1953=100



Data source: For the period 1900-1953, Johansson, Ö. [1967] and Jungenfelt, K.G. [1959].
For the period 1953-1977, the National Accounts.

——————— Actual development

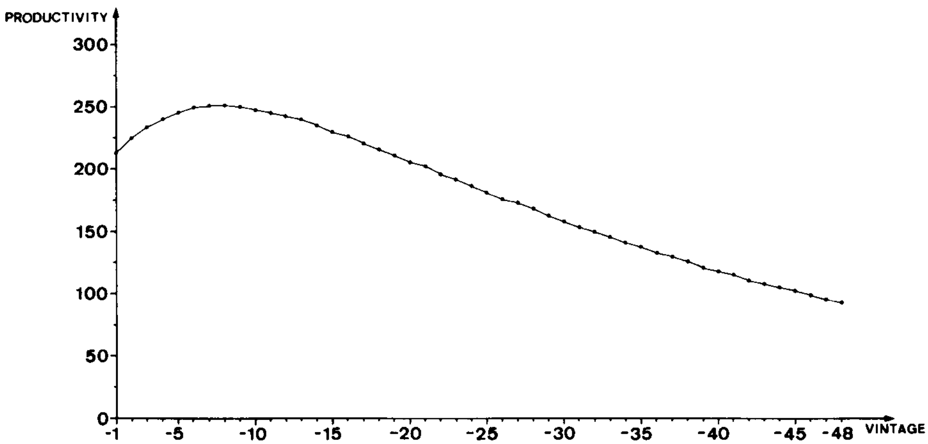
\- - - - - Function used as approximation in the simulations

The function used to approximate the actual development of the product real wage is:

$$w(t) = 100 \cdot \exp\left(0.03 \cdot (t - 1953)\right) \qquad t \leq 1953$$

If values are assigned to all the parameters of the model and the development of the product real wage is given as in Figure 4.2, the realized labour productivity on different vintages in 1953 can be calculated. The result is shown in Figure 4.3.

Figure 4.3 Simulated labour productivity for existing vintages in 1953



Values of the parameters in the simulation:

$\alpha = 0.44$	$H = 0.5$	$\beta = 50$
$r = 0.11$	$z = 4$	$I(0) = 90$
	$\lambda = 0.028$	$w(0) = 120$

From Figure 4.3 it can be seen that the oldest existing machine in 1953, in this simulation, was 48 years old; it was installed in 1905. It had a labour productivity equal to w(1951). Without lags in the model, the marginal machine should have had a labour productivity equal to w(1953), which is 100. In all the simulations a two-period lag has been used, however. It is assumed that the wage level from two years earlier is used when the decision to scrap a machine is taken. In other words, the machine is run with some loss a period before it is scrapped.

The curve is initially increasing, illustrating that machines become more efficient the longer they are used. This effect dominates, for the younger vintages, over the effect of increasing capital-labour ratios. After vintage $v = -8$ the curve starts decreasing. This reflects that, historically, the product real wage has increased leading to lower capital-labour ratios the older a machine is. The shape of this curve is in rough accordance with the empirical findings cited earlier,[1] which stated that the youngest vintage does not necessarily have the highest labour productivity.

Figure 4.3 is a variant of the famous Salter diagram.[2] It is seen that a higher product real wage would, *ceteris paribus*, have made a number of vintages obsolete. The flatter the curve, the larger number of vintages will, *ceteris paribus*, become obsolete for a given increase in the product real wage.[3]

4.4 THE REALIZED AND THE EXPECTED PRODUCT REAL WAGE PATHS

To carry out simulations with the model, the realized and the expected product real wage still remain to be determined at all points in time from 1953 onward. There are actually *three* different concepts for the product real wage in the model: the *realized* product real wage, the *expected* product real wage, and the product real wage in *steady state*. The latter of these has already been determined. It is described by an exponential function having the value 120 in 1953 and increasing by 5 %.

The realized product real wage is the exogenous variable in the model. Its development is shown in Figure 4.4.

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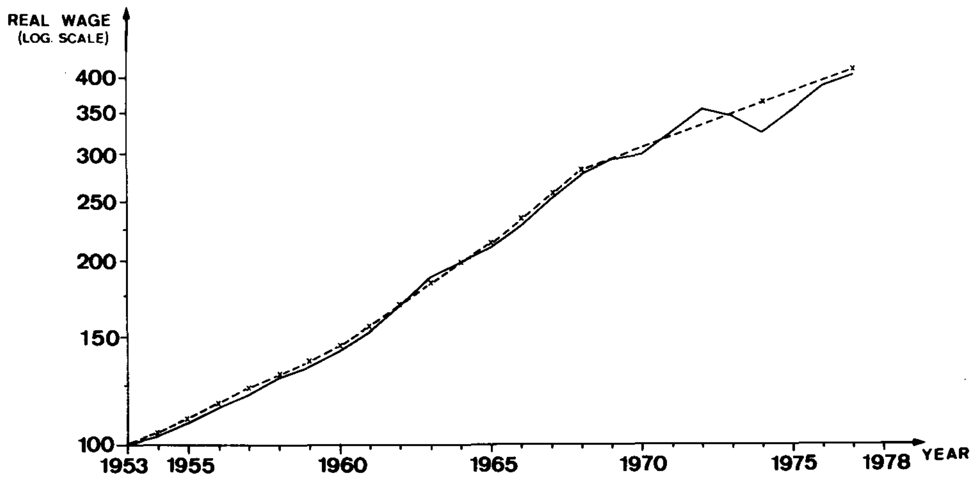
[1] E.g. Gregory, James [1973].

[2] See Salter [1966].

[3] Since there is an endogenous response in productivity following an increase in the product real wage in the model, things would not, even in the model, be quite this simple. This will be illustrated later.

Figure 4.4 The product real wage in the traded goods sector in Sweden 1953-1978



Data source: The National Accounts

——————— Actual development

- - - - - Function used as approximation in the simulations

The functions used to approximate the development of the product real wage are:

$$\hat{w}(t) = 100 \cdot \exp\left[\left(0.04 + 0.002 \cdot (t-1953)\right) \cdot (t-1953)\right] \quad 1953 \leq t \leq 1968$$

$$\hat{w}(t) = 100 \cdot \exp\left(1.05 + 0.04 \cdot (t-1968)\right) \quad 1968 \leq t \leq 1978$$

Two phenomena are worth noticing in Figure 4.4. The first is the acceleration in the product real wage in the period 1953-1968. Especially the last years in this period were characterized by very high growth rates for the product real wage. This created problems within the sector because it squeezed profit margins. Nevertheless, the sector continued to grow despite these years of rapid increases in the product real wage. The results of the simulations are trying to explain this apparent paradox.

The second element to note in Figure 4.4 is that the development after 1968 has been smoothed, in the sense that the *decrease* in the product real wage 1972-1974 is ignored. Instead, there is an adjustment towards steady state from 1968 onward, since the product real wage grows by 4 % in the simulations. This is lower than the 5 % rate on the steady state path, which means that the product real wage approaches the steady state path from above. The model is not very well suited to deal with a decreasing product real wage and this justifies the neglect of the decrease in the product real wage. Among the consequences of a decrease in w are that firms will start investing in machines with progressively lower capital-labour ratios, and machines which were earlier obsolete will become profitable again. The efficiency function, as it has been modelled here, shows that the firms forget what they had earlier learnt, when the product real wage decreases. Should w start rising again after a period of decrease, it might happen that some newly invested machines become obsolete. Thus "holes" might be created in the vintage structure. All of this could of course be handled theoretically, but it would require additional work on the model. I have not considered that work to be justified in the present case, and therefore the dip in the product real wage has been smoothed out.

The *expected* product real wage during the life of a machine is assumed to be determined by the level and rate of change of the product real wage two periods before the investment was made. The firms use the level and rate of change of w two periods ago and calculate the expected future product real wage as: $w^*(\tau) = \hat{w}(v-2) \cdot \exp\left(\hat{c}(v-2) \cdot \tau\right)$, where $\tau \equiv t - v$.

This assumption means that expectations are stationary, in the sense that the expected rate of change of the product real wage is assumed to be constant, even though it may accelerate in the simulations. Consequently, the future product real wage is underestimated in periods when it is accelerating. The assumption that investors expect the rate of change of the product real wage to remain constant means, in a sense, that they always believe they are in steady state, where the rate of change of w is constant. This is in conformity with the assumptions made in chapter 2, where I also assumed that expectations were formed such that investors always thought they were in steady state.[1] It may be noted that in steady state the investors will have rational expectations. The significance of assuming adaptive expectations instead of rational expectations in the present analysis was discussed in chapter 3, section 3.1.

The two-period lag is employed throughout. The model works as if the present wage level could never be observed, but only the level two periods ago. The lag is used in the calculations of realized investments, the optimal capital-labour ratio, the realized life of machines and the realized efficiency of a machine.

Lags are accounted for because it ordinarily takes time from, for example, the time of the decision to invest in a new machine until the real economic consequences of the decision appear. The decisions are taken using the product real wage at the time of the decision and are assumed to have their effects (e.g. the machine is delivered and installed) two periods later. Whether this is a reasonable assumption, whether the lag should be two years long and whether it should be applied to all the decisions mentioned above, is of course not self-evident.

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[1] One may use the term "steady state expectations" for expectations formed in this way. As that term has not to my knowledge been used elsewhere I will not use it here.

4.5 A SIMULATION OF STYLIZED POST-WAR GROWTH

In this section, I will give the results from one simulation of the model. This simulation gives, in my opinion, a good representation of the stylized post-war growth in the Swedish traded goods sector. The simulation is made to illustrate various mechanisms in the model and not to reproduce certain exact numbers. For this reason, I have not used any sophisticated methods to determine whether this really is the set of parameters that minimize the deviations between the simulated and the actual data.

The presentation of this particular simulation, which may be viewed as a reference solution for later comparisons, is quite detailed. I first show how aggregate output, employment and investments are simulated. It turns out that the investment function behaves badly and the reasons for this are discussed. Then I show the development of some variables for which no actual data exist. Here the results from the simulations can be of particular interest. The development of the age of the oldest existing machine, the aggregate amount of slack, the distribution of labour productivity over different vintages and the size of different vintages relative to steady state are shown. These are all variables for which no actual data exist (or at least only proxies) and where the model tells more about the development than the aggregate data do.

In sections 4.2 and 4.3, I discussed the choice of parameter values for all the parameters except H and z.[1] In this section, z and H are given values[2] such that the simulated paths for value added, employment and investments resemble the realized paths. In the next section, I vary z and H to study their effects on the simulated paths.

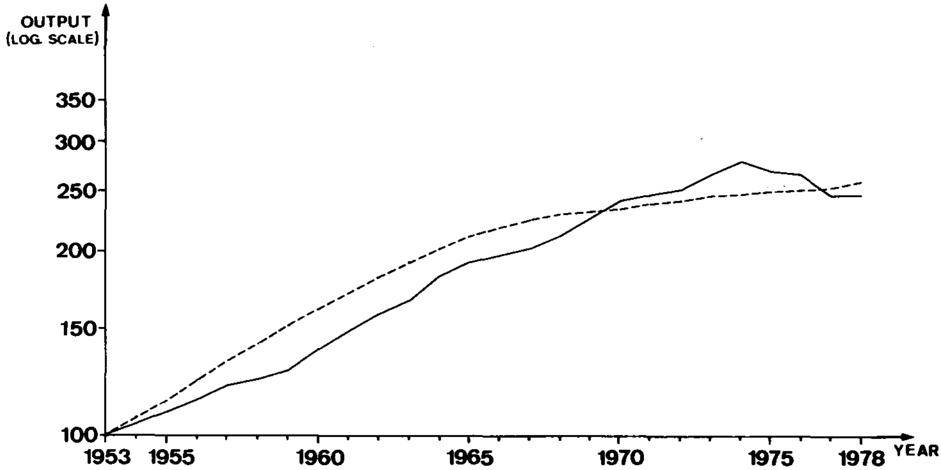
---

[1] H is the relation between the initial efficiency and the final efficiency (x_0/x_1) for a machine and z is the speed with which the gap between x_0 and x_1 is closed, given the rate of wage increases.

[2] z will be set equal to 4 and H to 0.5.

How does the model simulate value added in the traded goods sector?[1] Figure 4.5 shows the result:

Figure 4.5 Simulation of value added in the traded goods sector 1953–1978



——— Realized path

- - - - Simulated path

Parameter values:

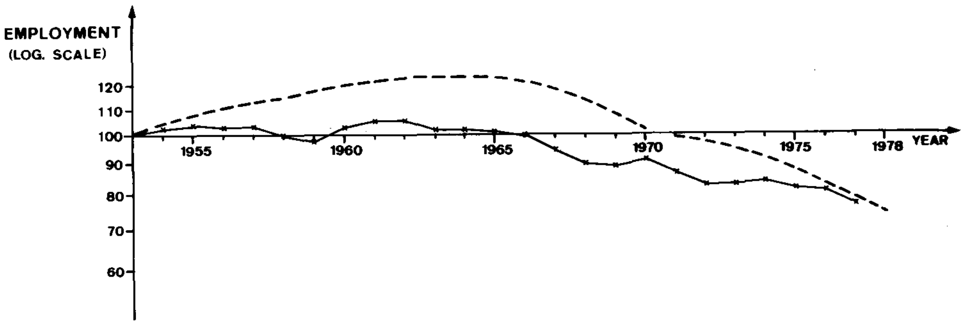
$\alpha = 0.44$	λ	=	0.028
$r = 0.11$	$I(0)$	=	90
$H = 0.5$	$w(0)$	=	120
$z = 4$	β	=	50

Output grows faster in the simulation than on the realized path during the 1950s and '60s and the stagnation in the '70s is more marked in the simulation than on the realized path. This last fact can, at least partially, be explained by short-run business cycle phenomena such as the raw-materials boom in the first half of the '70s. The important thing is that the simulations show growth in the first two decades and stagnation in the third decade.

[1] The program I used is written in BASIC, see Wissén [1982].

The same qualitative difference between the realized path and the simulated one is apparent when looking at employment in Figure 4.6.

Figure 4.6 Simulation of employment in the traded goods sector 1953-1978



————	Realized path
- - - -	Simulated path

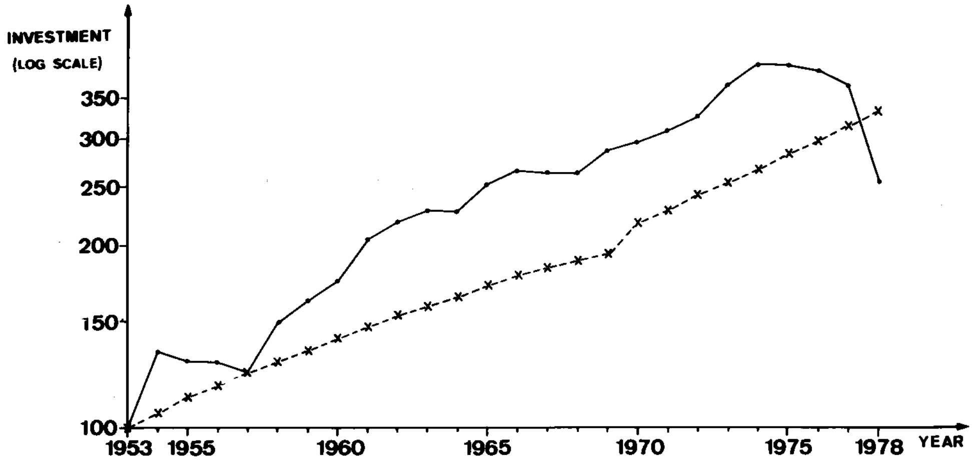
Parameter values:

α = 0.44	λ	=	0.028
r = 0.11	I(0)	=	90
H = 0.5	w(0)	=	120
z = 4	β	=	50

Here employment grows too fast during the first two decades (in fact it should not grow at all) and falls more dramatically in the '70s than the realized path. Once again, however, the curve is in rough accordance with the stylized facts. It may be noted that from 1966 and on, i.e. from the year when the product real wage is assumed to cut the steady state path, employment does fall markedly both on the realized path and on the simulated one. Although employment decreases rapidly, output does not decrease because there is a sharp, endogenous increase in labour productivity, preventing output from decreasing.

The simulation of investments in machinery is shown in Figure 4.7:

Figure 4.7 Simulation of investments in machinery in the traded goods sector 1953-1978



————	Realized path	Parameter values:	
-x-x-x-	Simulated path	$\alpha = 0.44$	$\lambda = 0.028$
		$r = 0.11$	$I(0) = 90$
		$H = 0.5$	$w(0) = 120$
		$z = 4$	$\beta = 50$

The simulated path here seems farther from the realized path than in the cases of employment and value added.

The result from the model, as shown in Figure 4.7, depends on the exogenous path for the product real wage. The deviation of investments from the steady state path depends on the deviation of the product real wage from its steady state path as shown by (3:13). The product real wage influences investments through its effects on $\underline{x}_1(v)$. It can be seen from (3:12) that if the product real wage grows at a constant rate (so that θ is constant), different from the steady state rate, then $\underline{x}_1(v)$ will grow over the vintages as in (4:4).

$$\underline{x}_1(v) = F \cdot \exp\left((\hat{c}-\tilde{c})\cdot(1-\alpha)\cdot v\right) \tag{4:4}$$

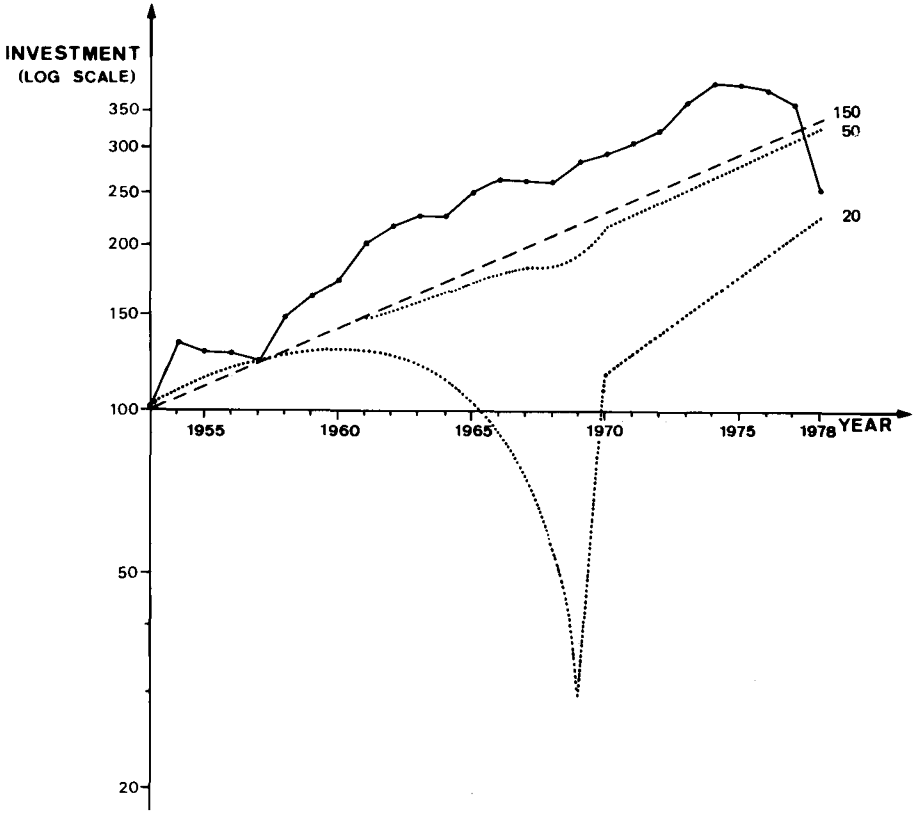
where

$$F \equiv \frac{\left(w(0)\ w^e\right)^{1-\alpha}}{(1-\alpha)x^e\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}$$

F is a constant when $\hat{c}$, the rate of change of the product real wage, is constant. It is noteworthy that investments deviate from the steady state path for investments as a simple function of the difference between $\hat{c}$ and $\tilde{c}$.

The effect of a given wage increase on investments depends on the parameter β (i.e. the upper limit in the distribution of x_1). The lower β is relative to the values of x_1 that are realized, the greater will be the proportional effects of e.g. a wage increase on investments. This is illustrated in Figure 4.8 where β is varied in the reference solution of the model.

Figure 4.8 *Simulations of investments in machinery in the traded goods sector 1953-1978. The effects of variations in β*



————	Realized path	Parameter values:	
- - - -	Simulated paths	α = 0.44	λ = 0.028
		r = 0.11	I(0) = 90
		H = 0.5	w(0) = 120
		z = 4	β is 150, 50 and 20, respectively

It is apparent in Figure 4.8 that the lower β is, the larger is the depressing effect on investments of the rapidly increasing product real wage, the last years in the 1960s. In the case where β = 20, it is very marked. All the simulations have been given index = 100 in 1953 to facilitate comparisons.

I prefer the value 50 for β since it gives the simulated curve some similarity with the realized path. Since β does not really have an economic interpretation, it is not of very great interest, however.

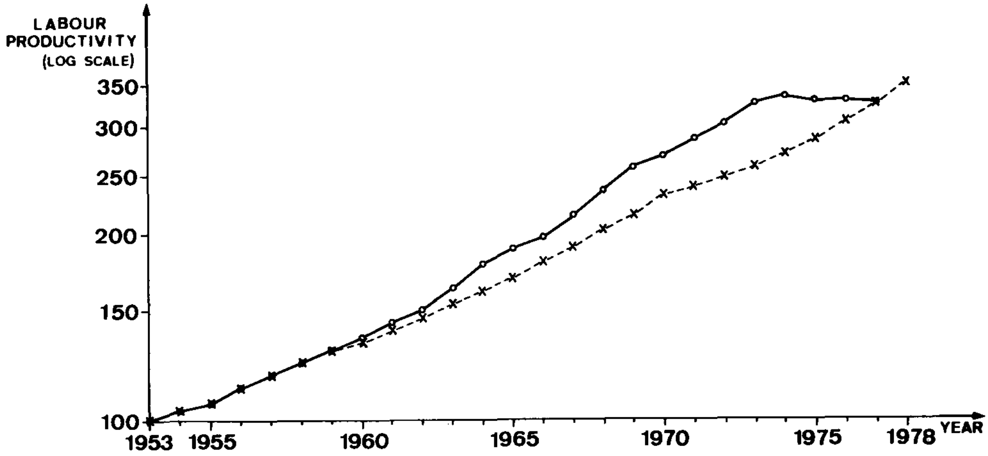
There seems to be no way to make the model simulate investments better than in Figure 4.7. With wages increasing faster than in steady state from the late '50s to the late '60s, the investment function employed here should not be expected to reproduce the rapid increase in investments from the late '50s to the early '60s, or the increase in the early '70s that actually took place. The reasons for this are difficult to uncover. Perhaps the investor had very optimistic expectations during these periods, and expected extremely good profitability in the future. These expectations for profitability may have surpassed those assumed in the model and the profitability finally realized.

The model does not catch the sizeable decrease in investments after 1975 because the increase in the product real wage in that period has been smoothed away when using the model.

The result from simulating investments suggests that the mechanism employed here is too simple. Maybe more elaborated expectations and/or lags in the investment function could improve the model. This study does not explore that question further.

Aggregate labour productivity is, by definition, the ratio between aggregate output and aggregate employment. The fact that output did not decrease during the last part of the period, in spite of the fact that employment decreased, is explained by the increase in labour productivity. Figure 4.9 shows the realized as well as the simulated path for aggregate labour productivity:

Figure 4.9 Simulation of aggregate labour productivity in the Swedish traded goods sector 1953-1978



————	Realized path	Parameter values:	
-x-x-x-x	Simulated path	α = 0.44	λ = 0.028
		r = 0.11	I(0) = 90
		H = 0.5	w(0) = 120
		z = 4	β = 50

If the solution of the model is compared with reality, it is seen that the '50s and the '60s are well simulated. In reality, productivity increased even faster than in the simulation in the late '60s. The stagnation in aggregate labour productivity from 1972 could not, however, be captured by the model. With increasing wages, aggregate labour productivity must increase in the model. The explanation for the stagnation after 1972 has to be found in factors outside the model employed here. It might be that the relatively large programs for subsidizing industry have played a role. The aim of the support to industry has been to prevent the closing-down of non-profitable firms, as this would have created unemployment. An effect of this policy has obviously been to dampen the increase in aggregate labour productivity that would otherwise have occurred. Another explanation of the stagnation in aggregate labour productivity after 1972 may be that capacity utilization fell, partly due to cyclical phenomena. Labour hoarding became more important in this recession than had earlier been the case, at least partly because of changed laws. Neither of these two possible explanations is captured in the model. They *could* explain the difference between the model and reality.

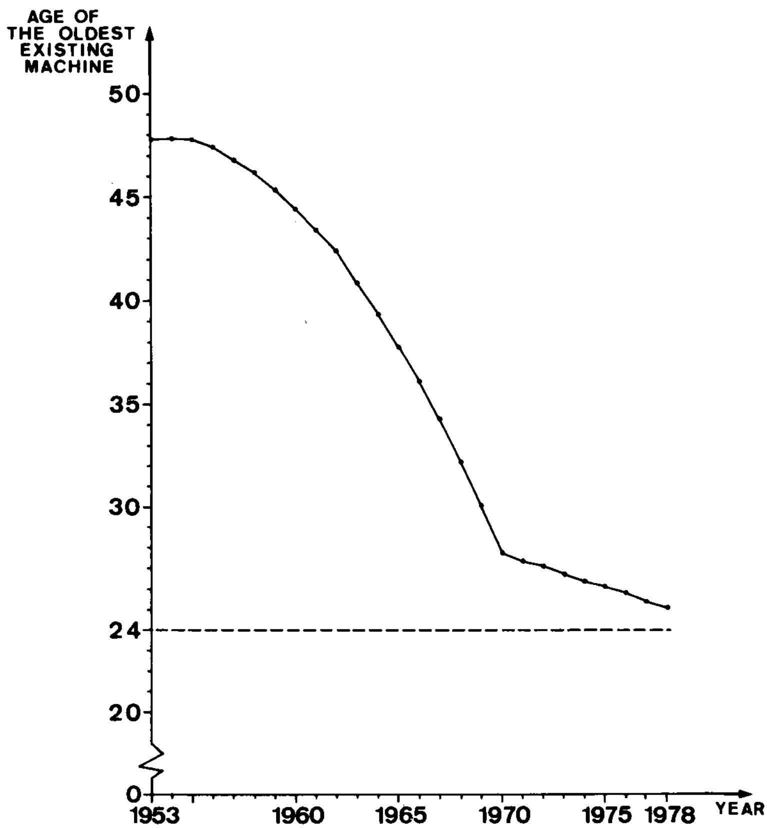
It should also be noted that, in the simulation, output was more or less constant during the '70s while employment decreased rapidly, producing the increase in productivity. In reality, employment did not decrease that much, possibly due to the factors mentioned above, and output actually fell during the years when labour productivity was constant. This produced constant productivity.

Why then does aggregate labour productivity increase in the model? Four sources for productivity increases can be identified:

(i) *The exogenously-determined technological progress in the ex ante micro production function.* Since technological progress is embodied, its impact on aggregate productivity depends on the volume of investment. Thus the impact of exogenously-determined technological progress on productivity is to some extent endogenous in the model.

(ii) *Scrapping of machines.* When the product real wage increases, a number of machines with low labour productivity become obsolete. This increases aggregate labour productivity. This is one endogenous channel through which productivity increases. The development of the age of the oldest existing machine in the simulation is shown in Figure 4.10.

Figure 4.10 Simulation of the age of the oldest existing machine in the traded goods sector 1953-1978



——— Simulated path

- - - - Steady state value

Parameter values:

$\alpha = 0.44$	$\lambda = 0.028$
$r = 0.11$	$I(0) = 90$
$H = 0.5$	$w(0) = 120$
$z = 4$	$\beta = 50$

The figure clearly shows how the age of the oldest existing machine decreases, from 48 years in 1953, to 25 years in 1978.[1] The decrease in the age of the oldest existing machine contributes to the rapidly increasing aggregate labour productivity during the period.

[1] These figures may be high compared to reality but that is of no great importance for the present use of the model.

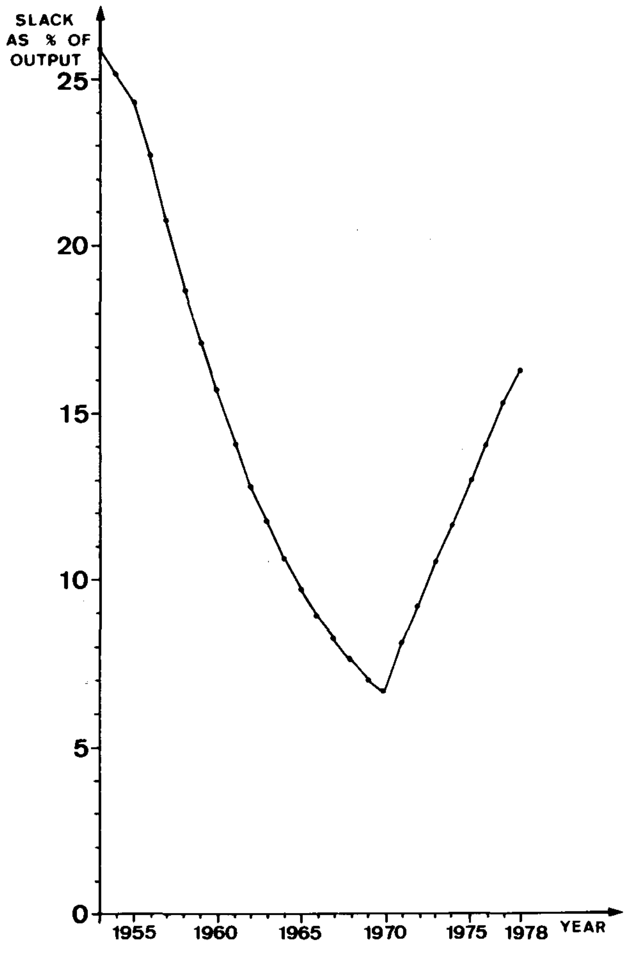
The age of the oldest existing machine was computed using expression (3:15).

(iii) *Changes in the capital-labour ratio on the latest vintage*. As was shown in expression (3:6), a higher product real wage means a higher capital-labour ratio on the latest vintage. This leads to a higher aggregate labour productivity than if wages had been lower. This is a second channel through which productivity increases *endogenously* as wages increase.

(iv) *The reduction of aggregate slack*. At each point in time some slack exists in the model. It can be measured by the amount of production attained if the efficiency variable x took the value x_1 on all existing machines, instead of the amount actually produced. Aggregate slack is a function of the level of the product real wage, since the realized efficiency at time t is assumed to depend on the level of wages at time t, as shown in (3:1). Reduction of slack is a third channel through which productivity increases *endogenously* as wages increase.

The ratio between the simulated slack and simulated output has been computed and is shown in Figure 4.11.

Figure 4.11 Aggregate slack as a percentage of actual output in the simulation



Parameter values:

α = 0,44	λ =	0,028
r = 0,11	I(0) =	90
H = 0,5	w(0) =	120
z = 4	β =	50

It is seen in Figure 4.11 that in 1953 the aggregate slack was 26 % of output. This implies that if firms could somehow have instantly reached maximal efficiency on all existing machines, then total output in the sector would have been 26 % higher.

A decrease in aggregate slack increases aggregate labour productivity. In the simulation, the slack decreased from 26 % of output to around 7 % of output in 1970. This is one explanation why output did not fall around 1970, when the product real wage increased fast.[1] The simulations in the next section, where H and z are varied, illustrate the importance of the aggregate slack for the results in the simulations. In steady state, aggregate slack is a constant fraction of output (both increasing at the rate $\lambda_T/(1-\alpha_T)$). The increase in aggregate slack after 1970 in Figure 4.11 comes from wages increasing less than in steady state (with 4 % compared to 5 % in steady state).

A similar occurrence is represented by Figure 4.12, which shows aggregate slack, measured in the same way, compared to what it would have been in steady state growth.

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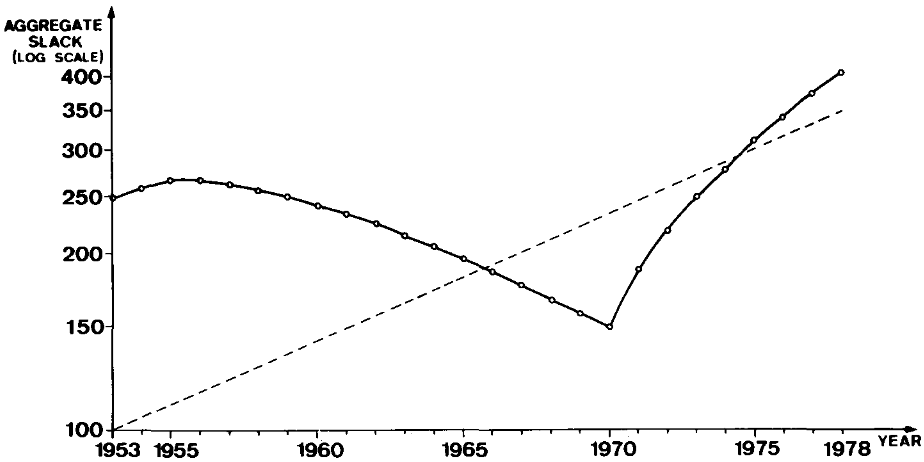
[1] If the slack had not been there, aggregate productivity would have risen because the age of the oldest existing machine would have decreased even further. This could even have led to a decrease in output, and of course an even greater decrease in employment.

Figure 4.12 Aggregate slack in the simulation and in steady state growth



————	Aggregate slack in the simulation
- - - -	Aggregate slack in steady state

Parameter values:

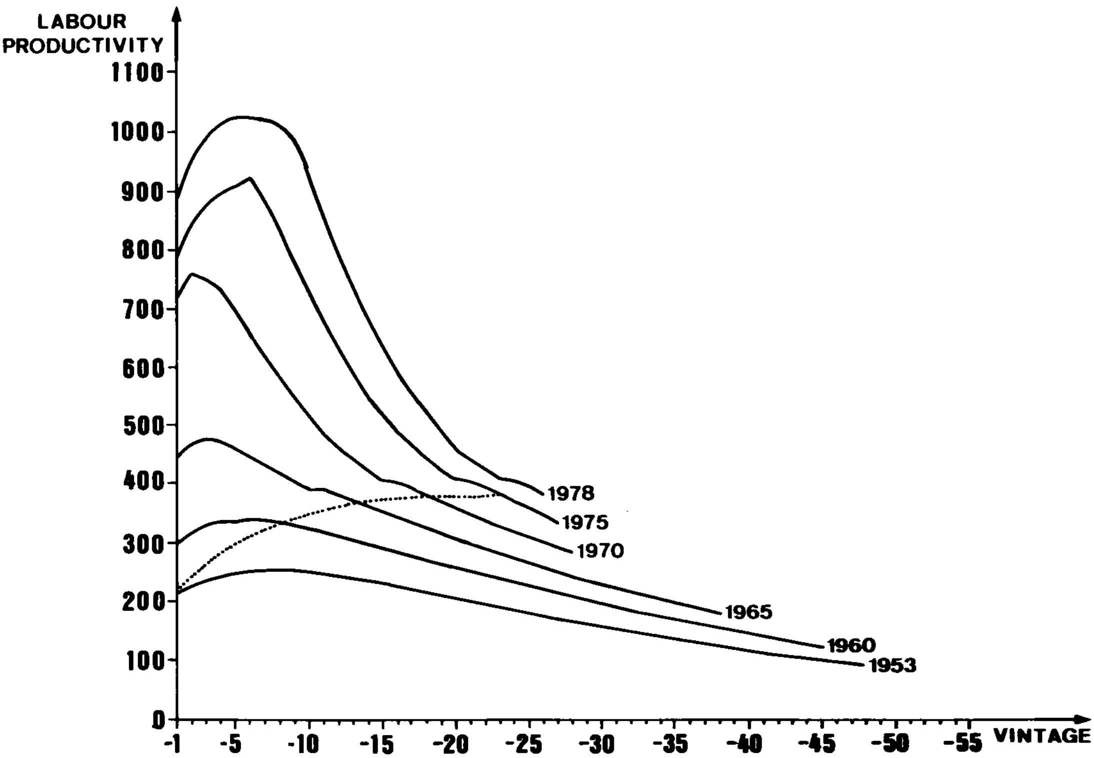
α = 0.44	λ = 0.028
r = 0.11	I(0) = 90
H = 0.5	w(0) = 120
z = 4	β = 50

The figure shows that the aggregate slack was 2.5 times as high as in steady state in 1953, and was below its steady state value between 1966 and 1974. As early as 1975, slack had been accumulated in excess of the steady state value. The empirical relevance of this particular result may be disputed, since this result is linked to the smoothed curve for the product real wage that is used.

These four factors, the technological progress, the decreasing life of machines, the increasing capital-labour ratio on new vintages, and the decreasing slack, explain the rapidly rising productivity during the period.

More insight into what happens in the model may be gained by looking at Figure 4.13. It shows how productivity was distributed over the existing vintages in the years 1953, 1960, 1965, 1970, 1975 and 1978, respectively.

Figure 4.13 Labour productivity, as simulated by the model, on different vintages in 1953, 1960, 1965, 1970, 1975 and 1978



Parameter values:

α = 0.44 λ = 0.028
$\ddot{r}$ = 0.11   I(0) = 90
H = 0.5   w(0) = 120
z = 4 β = 50

On the horizontal axis in Figure 4.13 are vintages, numbered -1, -2, ... For the curve showing the distribution in 1970, vintage -5 is then the machine that was installed in 1965. Labour productivity is measured on the vertical axis. The labour productivity on the oldest existing vintage at any time is the same as the product real wage two years earlier (since I have assumed that the firm uses the product real wage from two periods earlier when it decides to scrap a machine). In the figure one can also follow the history of any particular vintage. The development over time of vintage 1952 is shown with a dotted line in the figure as an example.

There are four observations to be made from Figure 4.13:

The first observation concerns the shape of the distributions. It is clear that the latest vintage does not have the highest labour productivity, as it would have in a conventional vintage growth model with increasing wages.[1] This is due to the efficiency function built into the model. Labour productivity increases as a function of time and the rate of change of wages. This produces the shape of the curves in Figure 4.13.

Second, the number of existing vintages in Figure 4.13 gets smaller over time. In 1953 the oldest machine was 47.8 years old, while in 1978 it was only 25.1 years old. As mentioned, this is a consequence of the rapidly increasing product real wage during the period.

A third observation is that the distributions become steeper over time. This reflects two facts: the first one is that the firms choose higher capital/labour ratios when the product real wage increases. As the product real wage increased faster in the sixties than earlier, the differences between

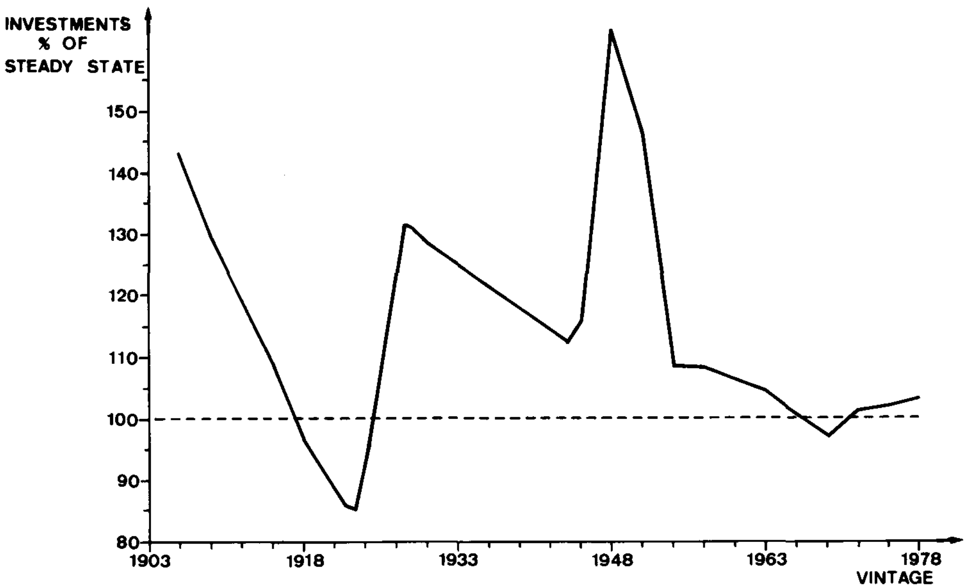
[1] For instance in Phelps [1963] or Kemp, Thanh [1966].

successive vintages installed in the 1960s are large. This produces steep curves. The second thing that produces the steep curves is the *ex post* productivity response to increasing wages. The younger the vintage, the more capacity it has to increase its efficiency *ex post* when the wage increases.

The fourth observation from Figure 4.13 concerns the dotted line, i.e. the line showing the development over time of the vintage installed in 1952. During its first ten years in existence, vintage 1952 increased its labour productivity rather drastically (it actually increased by about 50 % during the first ten years). After that it increased only slowly and it ceased to cover its variable costs some time between 1975 and 1978, which of course means it was scrapped. When the investment was made in 1952, the expected life of the machine was 38 years according to the simulation. This was based on an expected increase in the product real wage of 3 %. Since wages increased faster than was expected, the machine was not profitable as long as was expected.

Figure 4.13 says nothing about the size of the different vintages. Through the investment function, size too, is affected by the development of the product real wage. In steady state the size of the vintages (the amount invested in each vintage) increases exponentially at the rate $\lambda_T/(1-\alpha_T)$. In Figure 4.14 the size of the vintages at different points in time is compared to what they would have been in steady state.

Figure 4.14 *Simulation of the ratio between the amount invested in a vintage and the amount invested in steady state growth*



Parameter values:

$\alpha = 0.44$	$\lambda = 0.028$	————	Simulated investments as percentage of steady state investments
$r = 0.11$	$I(0) = 90$		
$H = 0.5$	$w(0) = 120$		
$z = 4$	$\beta = 50$	- - - - - -	Steady state investments

Up to 1953, investments are kept exogenous in the model, so up to that date, Figure 4.14 shows how the exogenous investments compare with the assumed steady state path. The capital stock that existed in 1953 was apparently larger than it would have been in steady state. This is in accordance with the assumptions in chapter 1, that employment and output were *above* the steady state level in 1953. Especially the capital stock consisted of more new capital than the capital stock in steady state.

Looking at the last year that has been simulated, 1978, the oldest machine was of vintage 1953. Thus from Figure 4.14 the capital stock appears greater in 1978 than in steady state. It did not, however, have the same composition with respect to vintages that the steady state path had. The simulation shows relatively too many new vintages and also relatively too few vintages around ten years old.

Whether or not this result is reasonable, that the capital stock in 1978 may have been larger than in steady state, will be discussed in chapter 5. It seems quite clear that in 1978, there were too few new vintages in the traded goods sector in Sweden compared to what there would have been with a more balanced growth. It also seems to be an important policy problem to compensate for these vintages that should have been installed in the late 1970s but were not. I will come back to the policy problems and the way in which they can be analyzed with the help of this model in the last chapter.

The point in showing Figure 4.14 is that it illustrates the fact that generally the existing vintages are not as evenly distributed as in steady state growth. When too little has been invested for a number of years, like the last years in Swedish industry, it may be desirable to compensate for the lost investments.

4.6 THE EFFECTS OF VARIATIONS IN THE LEVEL OF THE PRODUCT REAL WAGE AND IN THE PARAMETERS H AND z

In this section I will show how the solution of the model is affected when the level of the product real wage and the parameters H and z are changed. The sensitivity of the solution to the model to these parameters and to the wage rate is shown in order to describe certain developments in the sector. A full understanding of how the model behaves numerically would demand variations in all the parameters and possibly simultaneous variations in two or more of the parameters. That has not been attempted. No parameters other than H and z will be varied systematically.

Variations in the level of the product real wage has, among other things, an effect on the level of employment (in the model this effect is strictly on labour demand, which is assumed to equal employment). When showing the effects of variations in the level of W, I deal only with the effects on employment. The effects of W on output and investment have the same sign.

The elasticity of employment with respect to the real wage in an open economy has been analyzed, as afore mentioned, by Drèze and Modigliani [1981]. Although their approach is different from the one here, some comparisons can be made. They write: "To study the trade-off between real wages and employment, we treat external balance as a binding constraint on demand management. An exogenous increase in real wages, affecting adversely the competitiveness of domestic producers on the export and import markets, impairs external balance. The impact of the wage increase on output and employment is evaluated through the reduction in domestic demand required to restore external balance. At the empirical level, we endeavour to evaluate separately the influence on exports and imports of domestic costs at unchanged capacity levels, and of capacity levels themselves. And we endeavour to evaluate the influence of real wages on capacity levels through scrapping and investment. All evaluations rely on the foreign trade equations of econometric models of the Belgian economy." (Drèze, Modigliani [1981] p. 1.) They make a distinction between short-run and medium-run elasticities of employment with respect to the real wage. The concept used in the present study best resembles what they call a medium-run elasticity (i.e. taking account of capacity adjustments).

In the following, I proceed to take the complete solution of the model for e.g. 1953 and to vary the level of the product real wage in that particular year by - 60 %, - 40 %, - 20 %, 20 %, 40 %, 60 %, 80 % and 100 %. The variation in the product real wage, which is assumed to be unexpected, gives rise to three reactions:

(i) New investments are affected with regard both to volume and capital intensity.

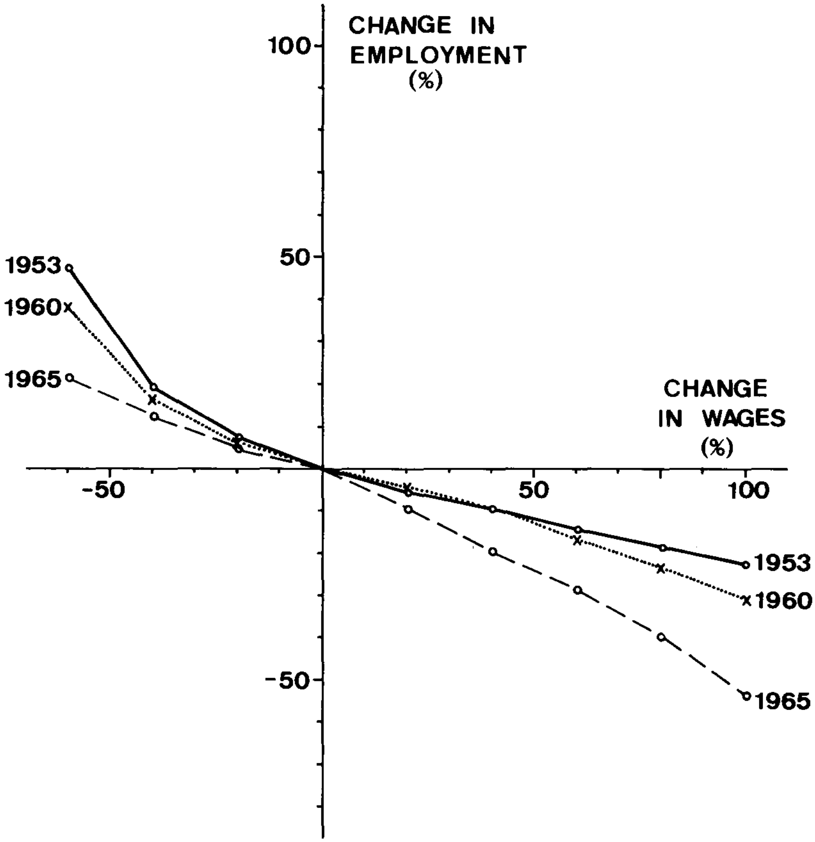
(ii) A number of machines are either scrapped (if the wage rate is raised) or put back into operation after having been unprofitable earlier (if the wage rate is lowered).

(iii) Productivity on each machine increases (if wages are increased) through the efficiency function. With regard to decreases in the wage rate, I have put the restriction on the model that firms cannot forget what they have already learned. Thus when a mechanical use of the efficiency function would evoke *falling* efficiency (a falling value of x), I have set the change in x equal to zero.

With the introduction of a wage shock, for example, that the product real wage suddenly increases by 60 %, the experiment then answers the question of how many jobs are lost during the following x months, if a devaluation is not made restoring the original wage level.

The result of the experiment is shown in Figures 4.15a and 4.15b.

Figure 4.15a Simulations of the effect on employment of changes in the product real wage 1953, 1960 and 1965[1]



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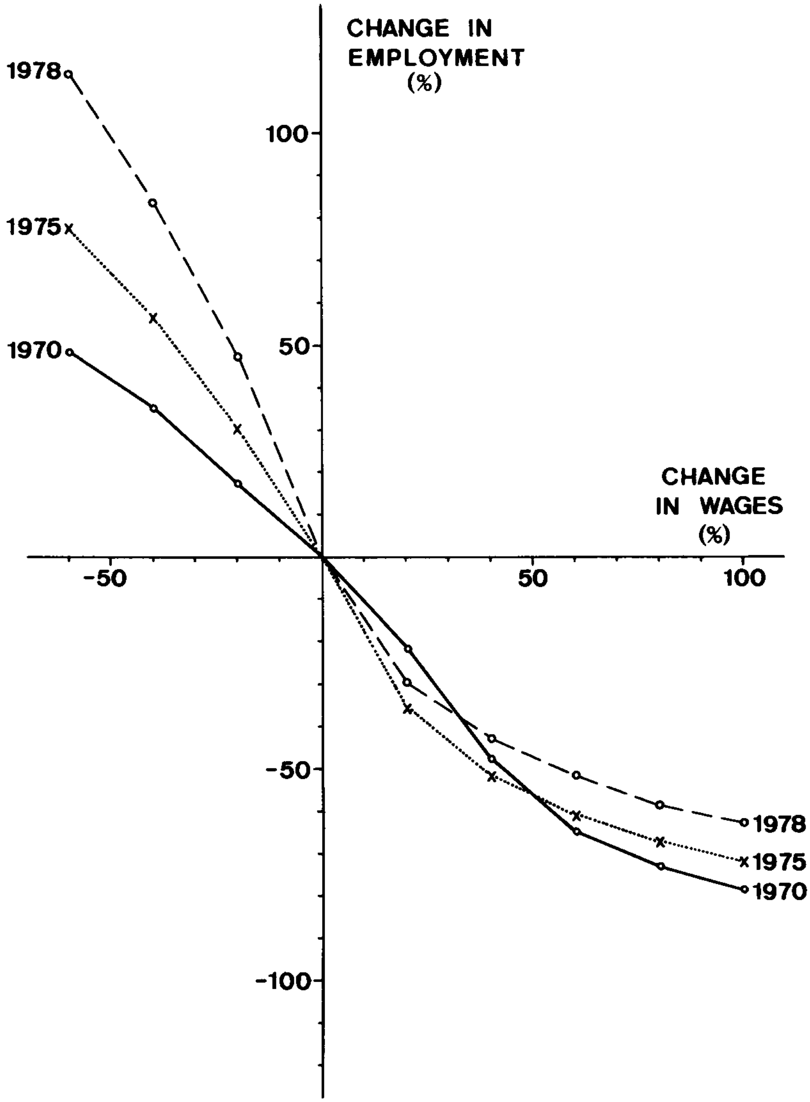
1 The percentages are measured with the values of W and L before the change as base.

Figure 4.15b Simulations of the effect on employment of changes in the product real wage 1970, 1975 and 1978[1]



[1] The percentages are measured with the values of W and L before the change in the base.

Looking at Figures 4.15a and b it is apparent that the curves have gradually become steeper.[1] Their increasing slope indicates that the sector has become more vulnerable to shocks of various sorts.

Referring back to Figure 4.13, this may seem a bit paradoxical. Figure 4.13 shows the distribution of labour productivity over different vintages. The diagram shows how this distribution gets steeper over time. *Ceteris paribus*, less vintages become obsolete toward the end of the period, after a given increase in the product real wage. In the model there is, however, the possibility of increasing productivity in response to an increasing product real wage. As is shown in Figure 4.11, the slack is relatively greater in 1953 than in 1975. Larger slack made employment less sensitive to changes in the product real wage in 1953 than in 1978.

In Figure 4.15a, the elasticity is nowhere greater than one in absolute number, while in 4.15b it is greater than one for large intervals of changes in w, and for all the three years. In both figures the ranking of the curves suffers a reverse, going from the northwest to the southeast quadrant. Thus a 60 % decrease in the product real wage increases employment more in 1978 than in 1975 or 1970. A 100 % increase in the product real wage does at the same time decrease employment less in 1978 than in 1975 or 1970. The same reversal of order is apparent in Figure 4.15a. Aggregate slack is clearly an important contributing factor. Slack was larger in 1978 than either in 1975 or in 1970 (see Figure 4.12). In Figure 4.15a slack was larger in 1953 than in 1960 or in 1965. The years when slack was large, employment was less sensitive to wage increases.

When considering the validity of the elasticities it should be pointed out that when w is decreased, a number of

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1 The ratio between the two values on the curve is the elasticity of employment with regard to the product real wage. Note, though, that I use the values *before the change* when calculating the percentages. This means that with a 100 % increase in the product real wage the elasticity cannot, as a matter of definition, become numerically greater than 1.

machines that were earlier obsolete are suddenly operational again. The empirical relevance of this can be questioned.

It is of additional interest that if a labour union wished to maximize the total wage bill it would have demanded wage increases in the '50s and '60s, and then wage cuts in the '70s. This has some correspondence to the behaviour of the labour unions during the last years of the '70s as compared with the earlier periods.

How do the results in Figures 4.15a and b compare to Drèze, Modigliani's results? They summarize their work in the following way: "The conclusions from our empirical investigation are first that esitmates of the trade-off between real wages and employment in Belgium are subject to considerable imprecision; second that the short-run elasticity of employment with respect to real wages keeping capacity constant is probably quite small (like -0.2), and definitely less than unity in absolute value; third that the corresponding medium-run elasticity taking into account capacity adjustments is probably sizeable (like -2), and definitely larger than unity in absolute value; ..." (Drèze, Modigliani [1981], pp. 8-9).

A comparison between Drèze, Modigliani and this work inspires the following observations: The elasticity computed in this study, which in Drèze, Modigliani's terminology would be called a medium-run elasticity, is also greater than unity in the '70s. It is nowhere as large as two but that is less significant. They conclude also that the estimates are "subject to considerable imprecision". If empirical estimates of this elasticity are computed from different time periods, one would expect different results. Even given different magnitudes of change in w, one would expect different values for the elasticity. This seems to be an important conclusion from the present study: the elasticity of employment with respect to the product real wage in an open economy varies over time in a systematic fashion as the structure of industry, and the slack in the industry, varies. In this model, even the medium-run elasticity was clearly less than 1 in absolute value in the '50s and '60s. The model lends an explanation for the change in the value of the elasticity over time.

It is unlikely that all the effects considered above of a wage increase would occur instantaneously. The effects on investment, scrapping and the efficiency function discussed above are slow processes. The effects on employment could, alternatively, have been computed after the product real wage had increased by a certain percentage per year for a specified number of years. In this way, I could have taken into account the fact that these effects take time. I could also have compared the impact of a change in w when it is foreseen and when it is not foreseen. These comparisons would complicate the computation, however, whereas the point is to show that employment is less sensitive to changes in the real wage in the beginning of the period than it is at the end. This point is not altered by calculating the elasticity in another way.

Since the slack plays a great role in all the results, the parameters which determine slack, z and H, have been varied for result comparison.

The effects on output and employment of varying H and z, respectively, are shown in Figures 4.16, 4.17, 4.18 and 4.19.

Figure 4.16 Simulations of the effects on output of variations in H.

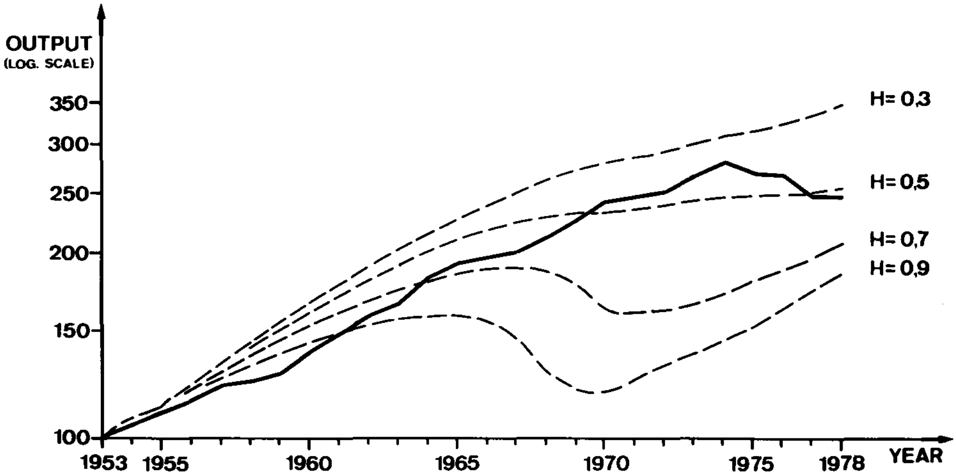


Figure 4.17 Simulations of the effects on employment of variations in H.

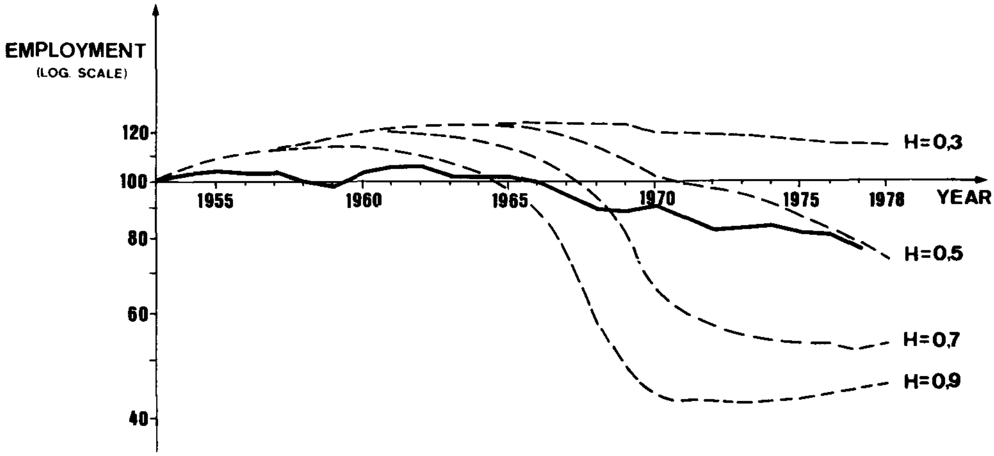


Figure 4.18 Simulations of the effects on output of variations in z

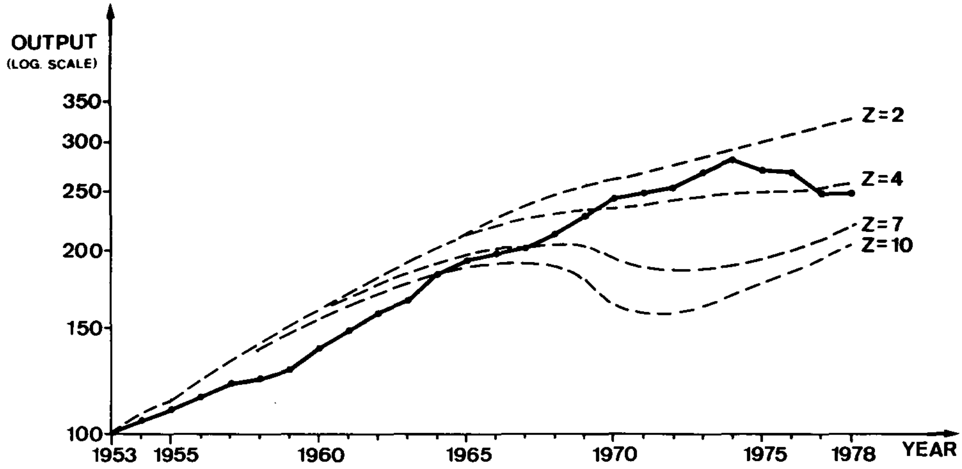
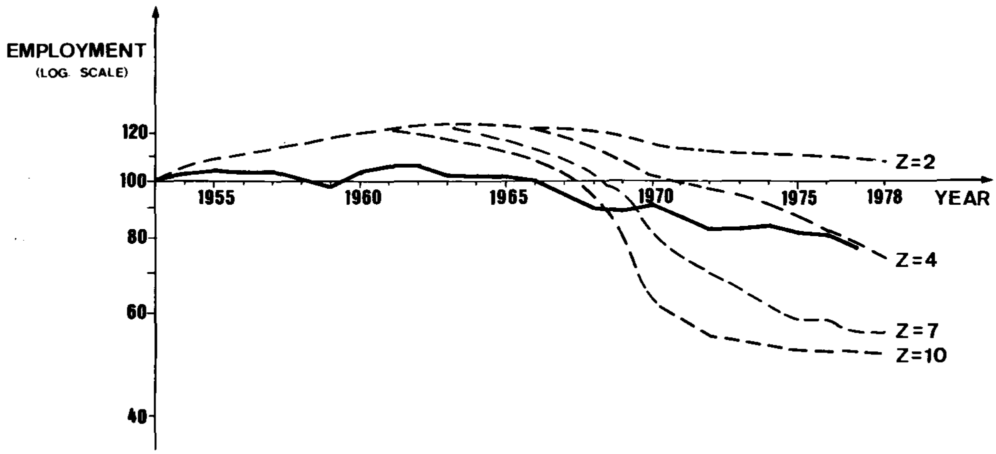


Figure 4.19 Simulations of the effects on employment of variations in z



It is clear that the greater z and H become, the sooner and more dramatically stagnation arrives. When either of the parameters has a sufficiently high value, output will actually decrease already in the late 1960s. When the values of the parameters are low, stagnation is pushed further into the future, or never arrives. Apparently, the sector would have gone through a severe crisis already in the '60s, if there had been room for an increase in productivity by reducing slack as a response to the extremely high wage increases.

Aggregate slack in the sector can be written as:

$$(4:5) \qquad G \cdot \int_{t-\hat{\theta}(t)}^{t} (1-H) \cdot \left[\frac{w(v)}{w(t)}\right]^{z} \cdot \psi(v)\,dv$$

where

$$G \equiv \frac{\alpha \cdot (1-\alpha)^{\frac{1-\alpha}{\alpha}}}{(1+\alpha)(\beta-\underline{x}_1^*)}$$

$$\psi(v) \equiv \tilde{I}(v) \cdot \left[w(v) \cdot w^e(v)\right]^{\frac{\alpha-1}{\alpha}} \cdot \left[x^e(v)\right]^{\frac{1-\alpha}{\alpha}} \cdot$$

$$\cdot \exp\left(\frac{\lambda}{\alpha} \cdot v\right) \cdot \left|\beta^{\frac{\alpha+1}{\alpha}} - \underline{x}_1(v)^{\frac{\alpha+1}{\alpha}}\right|$$

In expression (4:5) H and z are seen to influence aggregate slack in two ways: directly, so that a higher value of z and H makes slack smaller; also indirectly as H and z determine $\hat{\theta}(t)$ and $\theta(t)$ (this is shown in (3:5) and (3:15)). $\theta(t)$ in turn determines $w^e(v)$, $x^e(v)$ and $\underline{x}_1(v)$. The net effect is unclear. Figures 4.20 and 4.21 show the effects on the aggregate slack of the variations in z and H:

Figure 4.20 *Simulations of the ratio between aggregate slack and aggregate output. Effects of variations in H.*

Figure 4.21 *Simulations of the ratio between aggregate slack and aggregate output. Effects of variations in z.*

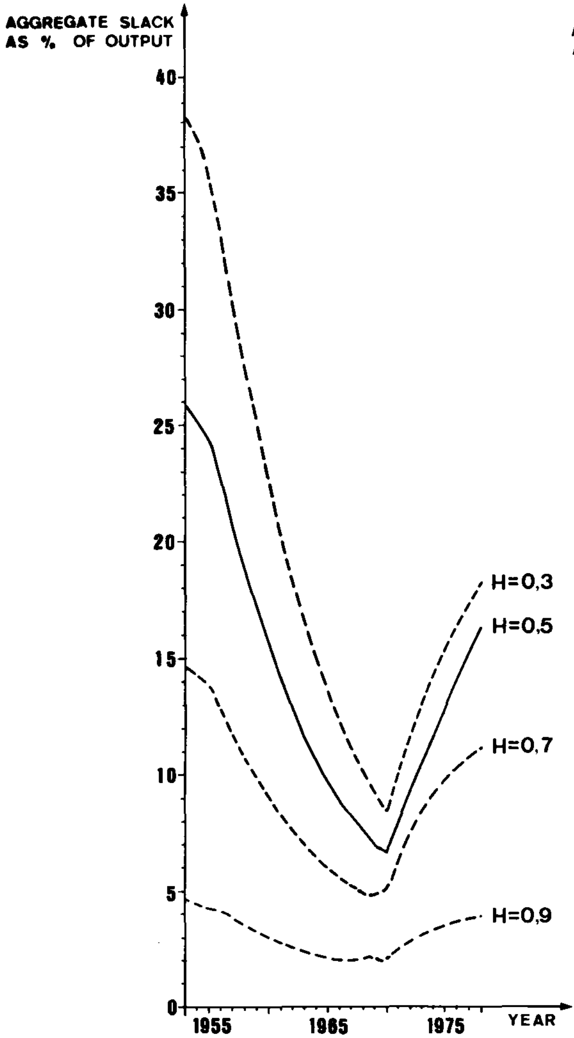


Figure 4.20

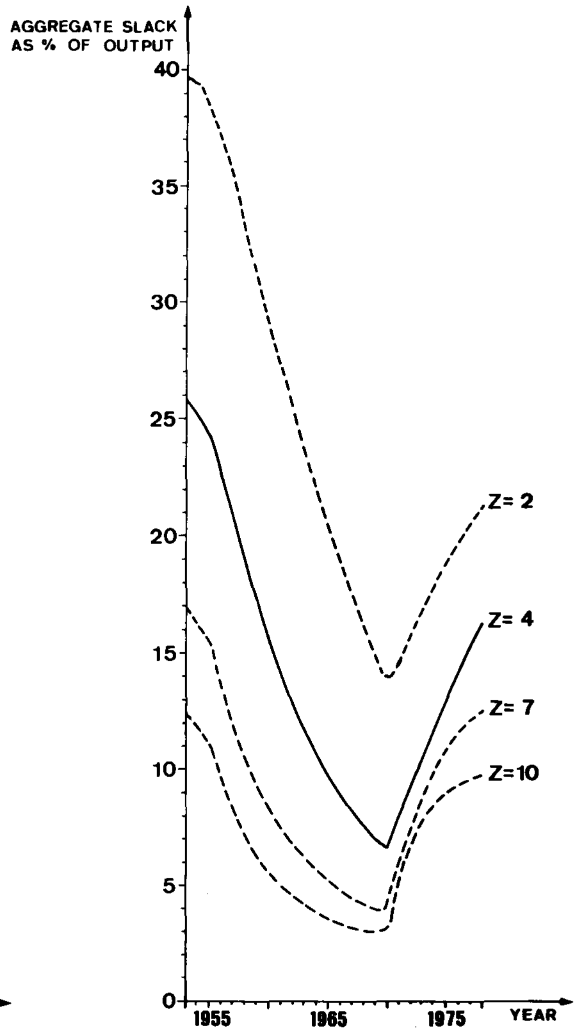


Figure 4.21

It is evident from the figures that higher values of z and H really do reduce aggregate slack, at least with the set of parameter values used here.

How can this result be interpreted? Conceivably there was a threatening stagnation in Swedish industry in the late '60s. Profits were squeezed by the rapidly increasing product real wage rate. This *could* have rendered a large number of machines obsolete, which would have led to a decrease in output. This is what happens in the simulations when z and H are sufficiently large. However, there was room to compensate for the *ex ante* profit squeeze with a decrease in the slack. This may have been brought about by the firing of redundant employees in firms, by mergers or similar measures. In this way, output could increase through increasing productivity. Aggregate productivity would anyway have risen without such measures but in such a case through the intensified scrapping of machines.

APPENDIX 4:1 A PROPERTY OF STEADY STATE GROWTH FOR THE VINTAGE MODEL

In this appendix I will show that if $r = \tilde{c} = \lambda/(1-\alpha)$ in steady state, i.e. if the rate of interest is equal to the rate of steady state growth, labour's share of output in the sector is equal to $1-\alpha$. This is a result already derived by Kemp and Thanh [1966] for their vintage growth model. Since the present model differs somewhat from theirs (by the presence of the efficiency function), I show here that this proposition is also valid for the present model.

An expression for $w(t)\cdot L(t)/Q(t)$ in steady state growth is needed. First, I derive the expressions for $L(t)$ and $Q(t)$.

Aggregate output can generally (see expression (3:18)) be written as:

$$\text{(A:4:1)} \qquad Q(t) = C \cdot \int_{t-\hat{\theta}(t)}^{t} \Big[\left(1 - (1-H)\cdot[w(v)/w(t)]^{z}\right) \cdot \tilde{I}(v) \cdot \left[w(v)\cdot w^{e}(v)\right]^{(\alpha-1)/\alpha} \cdot \left[x^{e}(v)\right]^{(1-\alpha)/\alpha} \cdot \exp(\lambda/\alpha\cdot v) \cdot \left[\beta^{(1+\alpha)/\alpha} - \underline{x}_1(v)^{(1+\alpha)/\alpha}\right] \Big] dv$$

where

$$C \equiv \frac{\alpha\cdot(1-\alpha)^{(1-\alpha)/\alpha}}{(1+\alpha)\cdot(\beta-\underline{x}_1^*)}$$

In steady state growth, assuming that $\tilde{c} = r$, it is true that

$$w^{e}(v) = \theta$$

This is seen from the definition of $w^{e}(v)$. Furthermore x^{e} is a constant, which I denote by $\tilde{x}^{e}$.

Thus Q(t) may be written:

$$\text{(A:4:2)} \qquad \tilde{Q}(t) = C \cdot \int_{t-\theta}^{t} \Bigg[\Big(1 - (1-H) \cdot \exp\big(-\tilde{c} \cdot z(t-v)\big)\Big) \cdot I_o \cdot$$

$$\cdot \exp(\tilde{d} \cdot t) \cdot \Big[w_o \cdot \exp(\tilde{c} \cdot v) \cdot \theta\Big]^{(\alpha-1)/\alpha} \cdot$$

$$\cdot (\tilde{x}^e)^{(1-\alpha)/\alpha} \cdot \exp(\lambda/\alpha \cdot v) \cdot$$

$$\cdot \Big[\beta^{(1+\alpha)/\alpha} - \underline{x}_1^*(v)^{(1+\alpha)/\alpha}\Big]\Bigg] dv$$

Taking account of the fact that $\tilde{d} = (\tilde{c}-\lambda)/\alpha$ in steady state, the integral in (A:4:2) may be solved, yielding:

$$\text{(A:4:3)} \qquad \tilde{Q}(t) = C \cdot I_o \cdot (w_o \cdot \theta)^{(\alpha-1)/\alpha} \cdot (\tilde{x}^e)^{(1-\alpha)/\alpha} \cdot$$

$$\cdot \Big(\beta^{(1+\alpha)/\alpha} - \underline{x}_1^{*(1+\alpha)/\alpha}\Big) \cdot$$

$$\cdot \exp(\tilde{c} \cdot t) \cdot \Bigg[\frac{1 - \exp(-\tilde{c}\theta)}{\tilde{c}} - \frac{1 - H}{\tilde{c}(z+1)} \cdot$$

$$\cdot \Big\{1 - \exp\big(-\tilde{c}(z+1) \cdot \theta\big)\Big\}\Bigg]$$

Turning to aggregate employment, (3:19) shows that it can be written as:

$$\text{(A:4:4)} \qquad L(t) = C \cdot (1-\alpha) \int_{t-\hat{\theta}(t)}^{t} \tilde{I}(v) \cdot \left(\frac{w(v) \cdot w^e(v)}{\exp(\lambda \cdot v) \cdot x^e(v)}\right)^{-\frac{1}{\alpha}} \cdot$$

$$\cdot \Big(\beta^{(1+\alpha)/\alpha} - \underline{x}_1(v)^{(1+\alpha)/\alpha}\Big) dv$$

Once more accounting for $\tilde{d} = (\tilde{c}-\lambda)/\alpha$ and $w^e(v) = \theta$ in a steady state where $\tilde{c} = r$, (A:4:4) can be written as:

$$(A:4:5) \qquad \tilde{L}(t) = C \cdot (1-\alpha) \cdot I_o \cdot \left(\frac{w_o \cdot \theta}{\tilde{x}^e}\right)^{-1/\alpha} \cdot$$

$$\cdot \left(\beta^{(1+\alpha)/\alpha} - \underline{x}_1^{*(1+\alpha)/\alpha}\right) \cdot \theta$$

The expression for labour's share in output can be written, after cancelling terms:

$$(A:4:6) \qquad \frac{\tilde{w}(t) \cdot \tilde{L}(t)}{\tilde{Q}(t)} = \frac{(1-\alpha) \cdot \left(\frac{1-\exp(-r\cdot\theta)}{r} - (1-H)\frac{1-\exp\left(-(\tilde{c}z+r)\,\theta\right)}{\tilde{c}z + r}\right)}{\frac{1-\exp(-\tilde{c}\theta)}{\tilde{c}} - (1-H)\,\frac{1-\exp\left(-\tilde{c}(z+1)\cdot\theta\right)}{\tilde{c}\cdot(z+1)}}$$

Substitution of $\tilde{c} = r$ into (A:4:6) yields:

$$(A:4:7) \qquad \frac{\tilde{w}(t) \cdot \tilde{L}(t)}{\tilde{Q}(t)} = 1 - \alpha$$

which is the desired result.

Thus it has been shown that if $r = \tilde{c}$, i.e. if the rate of interest is equal to the rate of steady state growth, labour's share of output is then equal to $1 - \alpha$.

5 Summary and Conclusions

5.1 A SUMMARY OF THE STUDY

This chapter firstly summarizes the study, and secondly briefly discusses the empirical results. Following the latter discussion, I will also draw conclusions for economic policy.

The theme of the study is adjustment to steady state growth in a small open economy. Steady state growth is growth such that all inputs, and consequently output, grow at the same rate. This growth rate is determined by the exogenous growth of some resource, in this case in the form of Harrod--neutral technological change.[1] In steady state growth, a number of equilibrium conditions are satisfied, making steady state growth compatible with full employment, clearing goods- and capital-markets and balanced trade. The dynamic behaviour of a small open economy as it adjusts to steady state growth is the central topic of this study.

In *chapter 1*, after a verbal introduction to the problem, a simple one-sector neoclassical growth model

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[1] Population growth can equally well be assumed.

for a small open economy is presented. It is a minor reformulation of Solow's [1956] famous one-sector growth model for a closed economy. This model adds an investment demand function to Solow's model, and relaxes the assumption that savings are necessarily equal to investments. All wage income is assumed to be consumed, which means that if more is saved than is invested, more is also produced than is demanded and hence, there is a surplus in the balance of trade. A disequilibrium in the balance of trade is assumed not to have any repercussions on the economy, since a surplus leads the citizens to accumulate bonds, which pay a given rate of return. This income is saved together with other income from capital. It does not affect consumption. This simplifying assumption is employed throughout the study.

The model is stable. If the capital stock is initially not growing at the steady state rate of growth, it is demonstrated that it will approach that growth rate. If it is supposed that the capital stock is growing slower than in steady state, then, under full employment conditions, wages also grow slower than in steady state. A lower growth rate of wages means progressively higher profits and larger investments than in steady state growth. Hence, the growth rate of the capital stock approaches the steady state rate. A mechanism similar to this one appears in the other models in the study which assume full employment. The simple one-sector model sketched here serves as a prototype for the models in chapter 2.

The concept of a steady state growth path and some of the relationships from the one-sector growth model are then used to discuss the post-war growth experience in the traded goods sector in Sweden. As I discuss only the traded goods sector, I keep wages exogenous.

The comparison of historical data with an assumed steady state path gives rise to what I call two stylized facts. The *first* stylized fact is that the post-war period can be divided into two periods. The first period, up to the mid '60s, was characterized by a level of the product real wage *below* the steady state level. During the second period, after the mid '60s, the level of the product real wage was *above* the steady state level. The *second* stylized fact is that the product real wage (i.e. the ratio between the wage rate and the price of output) grew very rapidly, relative to the assumed steady state path, between 1953 and 1968. In the simple one-sector model, these stylized facts are associated with certain paths for output, employment, investments and labour productivity.

When comparing actual data with the development indicated by the model, it seems that the reaction to the rapidly increasing wages in the '60s somehow came ten years late. It looks as though there would be a lag of approximately ten years from the time of the change in the wage rate until its effects on the real variables. One important task in the empirical part of the study is to explain this phenomenon.

It has been a common notion among Swedish economists that the wage increases in the late '60s were met with the reduction of slack existing in the firms. It is argued that during the "easy-going '50s" (Lundberg [1972]) slack accumulated in the firms. This was consumed to increase productivity when wages accelerated. Thus, the growth of output did not cease although the product real wage rate increased by around 10 % per year. This did, however, conceal the fact that the scrapping of machines increased. It may also have been the case that investments were insufficient to keep capacity growing fast enough to compensate for scrapping. Thus one may roughly say that the traded goods sector went into a crisis without it being noticed. This hypothesis has been analyzed at the industry branch level by Jungenfelt (SOU 1974:34 and [1982]). It has not, however, been explored before at the macro level.

With this background in mind, the purpose of the present study is:

(i) *to develop a framework within which problems of growth in a small open economy are studied.* Particularly, the aim is to develop a neoclassical growth model for a small open economy where the adjustment to steady state growth can be analyzed. To some extent this has been done in the existing literature, but it can be further developed.

(ii) *to analyze the hypothesis about the Swedish post-war development discussed above in terms of the framework presented.* It can then be shown that the process mentioned above, empirically observed by Lundberg and other Swedish economists, can be represented in a consistent manner in a neoclassical growth model. The adaption of the historical data to the model in my opinion strengthens the hypothesis regarding post-war growth. Since a number of results can be extracted from the model, this work also gives further insights into the adjustment of a small open economy to steady state growth.

In *chapter 2*, the simple model from chapter 1 is developed further. The model in chapter 1 is used to analyze the adjustment of the capital stock to steady state growth. It represents an extremely simple description of the adjustment process. In reality, a number of variables are adjusting towards their long-run equilibrium values, influencing each other. This may give rise to various complicated growth paths. In chapter 2, the number of types of adjustments occuring in the model is increased.

First of all, I assume that the wage rate does not immediately adjust such that it clears the labour market. In most industrialized countries the labour market is in a state of excess supply or excess demand for longer periods of time. One way to represent this in a model is to assume that it takes time for the wage rate to adjust to its equilibrium value. Wages are assumed to be *rigid*, meaning they are given in the short run, but adjust in response to excess supply or excess demand. In *section 2.3* it is shown that wage rigidity coupled with a slowly adjusting capital stock may cause a cyclical adjustment of both the wage rate and of the capital stock to steady state growth. This result has earlier been shown by Kouri [1979]. This outcome rests upon an assumption of stationary expectations in the investment function.

In *section 2.4*, a two-sector model with full employment is introduced, distinguishing one traded and one non-traded good. Capital is *non-shiftable* between sectors, implying that once an investment is made, that particular machine stays where it is installed, forever. This gives rise to a rigidity in the sectoral composition of the economy. Rates of return may differ between the sectors in the short run. In steady state, rates of return are equalized between the sectors. Thus, if the model is stable, growth must equalize rates of return. I demonstrate under which conditions the model is stable. Cyclical adjustments are again possible. This model has to my knowledge not been analyzed in the literature before.

In *section 2.5*, all the three rigidities treated in chapter 2 are joined in one model, in order to analyze their mutual interaction. Thus, a rigidity appears in the adjustment of the capital stock meaning that the rate of return in a sector may differ from the internationally given rate of return. A wage rigidity is present, implying that the labour market does not necessarily clear, but that the wage rate reacts to states of excess demand or excess supply. Finally, there is a rigidity in the sectoral composition of the economy, stemming from the assumption about non-shiftable capital. This implies that the rates of return may differ between the sectors. The conditions under which this model is stable are shown. I demonstrate how the different adjustments may interact and create various growth paths. This model has not previously been analyzed in the literature either.

Finally, in chapter 2, *section 2.6*, I leave the topic of adjustment to steady state growth for a minor digression. The Scandinavian model, or EFO model, of inflation has been much used in the policy debate in Sweden and also in the Swedish wage negotiations. It defines an equilibrium path for the development of the wage rate. One feature of the model is that it assumes that the exogenous rates of technological progress are different in the two sectors of the economy. The purpose of section 2.6 is to investigate whether the equilibrium wage path of the EFO model could be interpreted as the steady state growth path of a two-sector neoclassical growth model of the kind used in section 2.4.[1] It is concluded that the two models are comparable, but only if the community utility function is assumed to be Cobb--Douglas and the production functions in both sectors are assumed to be Cobb-Douglas as well. These are very restrictive assumptions, but they must be placed on the model here if it is to be consistent with these features of the EFO model.

In *chapter 3*, the two-sector model from chapter 2 is developed further. Two essential features are introduced into the model: vintages of capital and what I call an efficiency function. These are introduced in order to increase the empirical relevance of the model. The choice of these specifications is governed by two demands: *Firstly*, I want to include endogenous scrapping of machines, as I believe this is an important factor in the Swedish experience.

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[1] A similar problem has been treated by Kierzkowski [1976].

Its inclusion required the use of a vintage growth model. *Secondly*, I want a mechanism that can give rise to an endogenous productivity increase on already existing machines. This calls for the efficiency function. In chapter 3 I use the framework developed in chapter 2, but employ this more elaborate description of the technology of production. It is noteworthy that these mechanisms only give interesting conclusions outside steady state growth. In steady state growth they do not add very much to the explanatory power of the models.

Vintage growth models are well developed in growth theory (references are given in chapter 3). Most analyses with vintage growth models have, however, been confined to steady state growth. This is so because it is generally not possible to get intelligible solutions from a vintage growth model outside steady state growth. Because the use of a vintage growth model is more complicated, it necessitates two restrictions in the empirical analysis. *First*, the analysis is partial; only the traded goods sector is treated. I have considered a full general equilibrium analysis outside steady state growth with a vintage model too complicated to handle. I hold wages exogenous, in the absence of general equilibrium solutions for the wage rate. *Second*, I cannot rely on analytical solutions of the model. I have to use numerical examples to demonstrate its qualitative and quantitative properties.

To incorporate the endogenous increase of productivity on the already existing machines, the vintage growth model is modified somewhat by the introduction of the efficiency function mentioned above. The concept is adapted from Jungenfelt [1982]. Productivity is assumed to increase over time on a machine. The speed at which productivity increases is assumed to depend on how fast wages increase. The concept of the efficiency function is related to, though different from, "learning-by-doing" as used by e.g. Arrow [1962]. Formally the efficiency function is incorporated in the model by multiplying the micro production function by a function describing the development of efficiency over time.

The presence of the efficiency function means that there exists a slack in the economy. The size of the slack depends on how fast wages have increased historically. If wages have increased fast, there is little slack left, making the sector vulnerable to further wage increases. This mechanism is important in the empirical application of the model.

In chapter 3, after an introduction in *section 3.1*, this vintage growth model (with putty-clay technology) with an efficiency function is presented. It is derived by starting with the characteristics of the individual machine, aggregating up to the sector level to show output and employment at the sector level, and finally deriving the steady state growth path for a two-sector model.

In *section 3.2*, the efficiency function is discussed. I give the reasons why the function has its particular form. Briefly, each machine starts at some exogenously determined level of efficiency and approaches asymptotically some likewise exogenously determined higher efficiency level. The speed at which the machine approaches the terminal efficiency depends on how much wages increase during the life of the machine.

In *section 3.3*, the choice of technology is derived. It is assumed that the individual investor has stationary expectations concerning rates of change, meaning that he or she expects the current rate of change of wages and prices to continue over the expected life of the machine. Assuming the investor maximizes profits, the capital-intensity and the expected life of the machine is determined.

To derive output and employment in a sector, it is necessary to know how much is invested in each vintage. The derivation of an investment function is the task in *section 3.4*. It is derived differently than in chapter 2. The general idea is, however, similar to that of chapter 2. Thus I assume that a steady state path exists where the capital stock grows at the exogenously determined rate or resource growth. The investment function shows how the growth of the capital stock deviates from steady state when wages deviate from their steady state path. The investment function rests on two concepts; the steady state path and an assumed exogenously determined distribution for the efficiency of the machines in a vintage. If wages are higher than in steady state, a lower investment volume is realized, because at a higher wage level a higher efficiency is required for an investment to break even. This gives rise to a negative relationship between the wage level and the volume of investment. The existence of efficiency differences between the firms has the consequence that there are pure profits.

All the variables determined so far in the chapter are based on *ex ante* information. The agents' expectations need to be known to compute the variables. In *section 3.5, ex post* variables are computed. The expressions determining realized output, employment and the realized life of a machine are derived. They depend on the given vintage structure as well as on the realized wage rate.

Thus, the functions showing labour demand, output and investments on the sector level are derived. In *section 3.6*, these functions are used in a two-sector macro model to derive a steady state growth path. The derived path is shown to be very similar to the steady states in chapter 2. The life of the machines is constant and the variables grow at the same rates as in the steady state in the model in e.g. section 2.4. Chapter 3 thus demonstrates that a slack--phenomenon can be represented in a consistent way in a two--sector vintage-growth model.

To be able to analyze the vintage growth model outside steady state growth, and to demonstrate its empirical relevance, numerical simulations have been carried through. These are reported in *chapter 4*. Numerical simulations are a substitute for theoretical analysis of the kind made in chapter 2, since the functions here are too complicated to derive explicit solutions. The values of the parameters and the initial conditions are furthermore chosen so as to simulate Swedish post-war development. The simulation gives insight into how the industrial structure, as represented by the vintage structure, adjusts during a growth phase like the one Sweden has experienced in the post-war years.

In *sections 4.2, 4.3 and 4.4*, it is discussed how the values of the parameters of the model have been chosen, how the initial conditions have been determined and how the exogenous variable, the product real wage rate, is represented in the model. The crucial assumptions, already introduced in chapter 1, are that the product real wage increases by 5 % on the steady state path,[1] and that in 1953 the product real wage in steady state was 20 % above the actual product real wage. These two assumptions lead to what I called the stylized facts in chapter 1.

As reported in *section 4.5*, the model simulates output, employment and aggregate labour productivity in a way which corresponds closely to actual data. Investments are, however, not very convincingly simulated. One possible explanation of this may be that expectations are not correctly represented in the investment function.

The simulation of aggregate labour productivity is fairly accurate. The data shows a marked increase in aggregate labour productivity in the late '60s. This was caused by the rapid increase in the product real wage. In the model, increasing labour productivity can be explained by four factors:

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[1] In the steady state solution, this is determined by two parameters: α and λ. They are given values consistent with 5 % growth in steady state.

(i) *The exogenous technological progress in the ex ante micro production function.* Technological progress is assumed to be embodied, which implies that the impact of technological progress in the aggregate is partially endogenous since it depends on the volume of investment.

(ii) *The endogenous scrapping of machines.* When the oldest machines are scrapped, aggregate labour productivity increases since they have the lowest labour productivity. The more the wage rate increases, the more extensive the scrapping will be.

(iii) *The increasing capital-intensities on the new vintages.* When wages increase, a higher capital intensity is chosen on the latest vintage. This serves to increase aggregate labour productivity.

(iv) *The reduction of slack on the existing machines.* When the wage rate increases fast, as it did in the late '60s, aggregate slack is reduced. The scope for reducing aggregate slack naturally depends on how much slack there existed when wages started increasing fast. This, in turn, depends on the past history of the sector.

The understanding of the interplay of these four sources of productivity growth is vital for the understanding of the workings of the model. Particularly (ii) and (iv) play important roles in the simulation of Swedish post-war growth. This is brought out clearly when the parameters that determine the size of the aggregate slack are varied in *section 4.6*. It seems as if the reduction of slack was an important source of productivity growth in the late 1960s and early 1970s. Gradually, however, scrapping of machines became relatively more important. The roles of the different sources of productivity growth thus changed during the period.

If the size of the slack is, hypothetically, assumed to be small at the beginning of the period, the buffer is not large enough to meet the rapidly increasing product real wage in the late '60s. Then scrapping increases drastically already in the late '60s. This causes capacity to decrease and output and employment to fall. Due to the presence of slack, stagnation was, in the simulation which resembles actual data, cushioned and delayed. It is noteworthy that the aggregate labour productivity increases in either case, regardless of the size of the slack. In the first case it is through increased scrapping of machines and in the second through decreased slack, however. To decrease the slack can be considered a desirable thing. It means increased real wages without using any resources. The problem with growth through the reduction of slack, is that it is hard to know exactly when wages are increased too much, leading to stagnation.

The difference caused by various amounts of slack is demonstrated by some further simulations in section 4.6. For different years I vary the level of the real wage rate, increasing or decreasing it by different amounts. Then I study the effects on employment in the sector due to a variation in the wage rate. This gives the elasticity of employment with respect to wages. In the simulation, this elasticity increased during the period. Thus, in this sense one may say that the sector grew progressively more sensitive to wage increases. The sensitivity comes to a large extent from the decreased slack during the period. The elasticity of employment with respect to the real wage rate can thus be expected to vary over time, as explained by the model.

5.2 CONCLUDING COMMENTS ON THE EMPIRICAL RESULTS

In this last section I discuss two problems concerning the empirical relevance of the simulation in chapter 4. I also discuss the interpretation of Swedish post-war growth supported by the simulation with the model.

The first problem concerning the empirical results concerns the reduction in aggregate slack. It plays a central part in explaining the Swedish growth experience. Slack is, however, a variable which is hard to operationalize and quantify. One may ask whether there are any indications of large productivity increases on the already existing machines in the late 1960s. I have not made any systematic attempts to investigate this, although it may well be worthwhile. It seems to me, however, that *mergers* are one possible source of productivity gains on existing machines. Mergers may be motivated by financial considerations, but also by the possibilities to reap productivity gains by combining existing machines in a new way. The number of mergers did increase rapidly in the late '60s.[1] A study of mergers in Swedish industry in the period 1946-69 may be quoted: "The upswing was especially steep during the latter part of the 1960's ... Prior to 1960 the proportion of employees per year in acquired firms to all industrial employees exceeded 1 percent in one year only. Since 1965 that rate has normally exceeded 2 percent, and in 1969 it came to 4.4 percent or nearly 38,000 employees" (Rydén [1971] page 209). To the extent that mergers actually do increase productivity on existing machines, this observation at least does not contradict the results derived in the model. Further support for my hypothesis about the reduction in slack is given by the observations made by Swedish economists, as exemplified by the quotation from Erik Lundberg in chapter 1.

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[1] See Rydén [1971] and, for data for the '70s, SIND [1978].

The second problem with the simulation results concerns the size of the capital stock relative to steady state. In Figure 4.14 in chapter 4, it looks as if the simulated capital stock in 1978 was larger than its corresponding steady state value.[1] This probably contradicts the intuition of most Swedish economists.

When considering the apparent contradiction between this result and reality, there are three aspects of the model worthy of comment. The first characteristic is that there is a certain degree of arbitrariness in the determination of the intercept of the steady state wage path. It can be lowered to e.g. 110 instead of 120. As seen from Figure 1.1, this would mean that the product real wage is above its steady state level for a longer period. Simulations with that assumption (not reported here) show that is does not drastically affect the results. It does, however, decrease the size of the capital stock relative to steady state. A change in the value of this parameter may thus alter this result. Secondly, the wage path in the simulation has been smoothed after 1968. Thus, the "cost crisis" in the mid-'70s and the subsequent dip in investments are not accounted for in the simulations. In this way the simulation overstates the capital stock in 1978. The third point is that the capital stock, measured as past investments, really may be larger than in steady state. It could, however, be that it is tied to uses that are considerably more capital-intensive than those on the steady state path. In the '60s expected wage increases were, according to the model, very high. Subsequently, capital intensities were chosen which were higher than those on the steady state path. In this way the total capital stock, measured by past investments, may be larger than in steady state, but still insufficient for e.g. balance of payments equilibrium, since it is tied to high capital intensities.

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[1] I have not tried to compute a measure for the capital stock. Simple inspection of Figure 4.14 suggests, however, that since investments were larger than in steady state for most parts of the period, the capital stock was larger than in steady state in 1978.

The work reported here has inspired the following, brief, analysis of post-war growth in the Swedish traded goods sector. The analysis is a bit speculative, in the sense that not everything follows directly from the model. In my opinion it is however suggested by the model in chapter 3 and the simulations with it in chapter 4.

In the 1950s and the early 1960s a growth potential existed for the traded goods sector for two, related, reasons. *Firstly*, the level of the product real wage was below the steady state level, which meant that it could, adjusting to steady state growth, increase faster than on the steady state path. *Secondly*, there was a margin for productivity increases on the existing machines above what was the case in steady state growth. This also became a source of rapid growth.

It seems that this growth potential was recognized in the Swedish debate.[1] There were discussions about e.g. the productivity gains that could be reaped by scrapping the least efficient machines and moving labour to new, efficient ones. As already mentioned, productivity gains on the existing machines were also observed. Thus I think one may say that the growth potential was well known and also gradually used up. The product real wage increased very rapidly in the mid '60s and late '60s. This created no immediate employment or balance of payments problems, for at least two reasons: (i) the nontraded goods sector expanded rapidly, employing those who were displaced in the traded goods sector and (ii) the productivity increase on the existing machines resulted in increased output and only a moderate decrease in employment in the traded goods sector.

This smooth but deceptive process did, however, hide the fact that the capital stock in the traded goods sector

1 Some important contributions to the debate, by for instance Gösta Rehn, can be found in Turvey [1952]. The debate is also accounted for by Lundberg [1972] and Lindbeck [1975].

was growing at a decreasing rate. The scrapping of machines was considerable and investments were not large enough to compensate for scrapping. Maybe the new investments also were characterized by a too high capital intensity.

At a time when the slack was small, in the middle of the 1970s, the cost crisis hit the traded goods sector. The cost crisis consisted in the wage cost per hour in Sweden increasing by around 15 % relative to our major trading partners. This could not, on account of the fact that slack already was small, be met with productivity increases. Furthermore, the debt/equity ratio was high in most firms. It had been increasing a long time, making firms financially vulnerable. The effect of the wage shock was the scrapping of machines, or subsidies from the government, which aimed at preventing scrapping. The government program of subsidies to ailing industries in this period (roughly 1975-1978) is probably one explanation why labour productivity did not increase as it would have done if the scrapping had taken place. When there were cost crises earlier, in the '50s or '60s, productivity on the existing machines could be improved. This alleviated the effects that later in the 1970s were disastrous.

It may seem curious that this was allowed to happen. Why did wages increase so drastically that they created stagnation? Since I have not tried to model wage formation in the empirical chapter, I have no well-founded answer. The possible role of a simplistic use of the EFO model is worth mentioning, however. If a higher expected productivity increase than the underlying rate of technological progress is used, when deciding upon wage increases using the EFO model, this forecast will be self-fulfilling. It was clearly shown in chapter 4 how wage increases create productivity increases. Thus, if in the central wage negotiations the rapid increase in labour productivity in the late '60s was expected to be a lasting phenomenon, high wages would obviously be decided upon. *Ex post* it also looked as if these were matched by a productivity increase. The empirical analysis in chapter 4 clearly shows why this was *not* consistent with long-run equilibrium growth. Thus, one can suspect that an uncritical use of the EFO model may have been a factor contributing to wage increases that were not consistent with equilibrium growth.

The growth problems in the late 1970s and early '80s can, according to the model used here, not be expected to be solved quickly. The picture that emerges here is that the cost crisis appeared in an economy which was not in steady state growth. It might have been approaching this state, but it was not yet there. The capital stock in the traded goods sector was too small to accomplish balanced trade at the given wage level.The size of the capital stock in the traded goods sector demanded cost adjustment for a long time period, already before the wage shock had occurred. The effect of the cost crisis in this particular situation was to make the need for cost adjustment even more pronounced.[1] The model leads one to believe that still into the early 1980s there is need for cost adjustments which may have to continue over a longer time period.

The question which then arises is how the economy should approach steady state growth. Two possible ways are shown in Figure 5.1:

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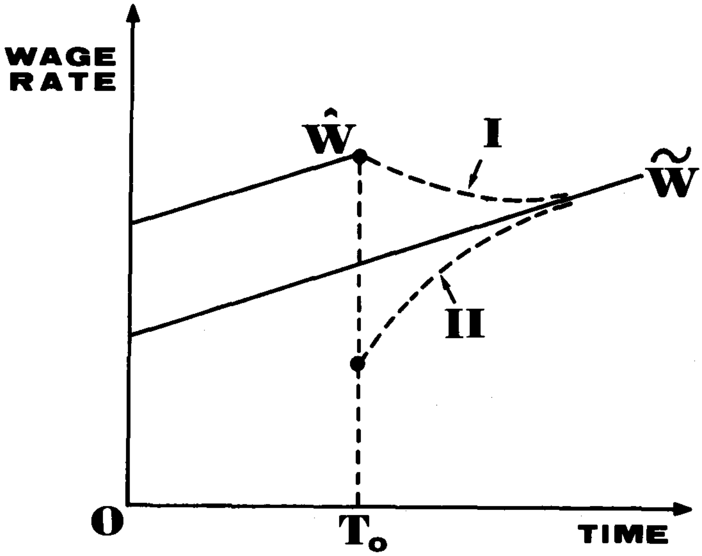
[1] The analysis in section 2.5 is relevant for this hypothetical case.

Figure 5.1 Adjustment paths for the product real wage



In Figure 5.1, $\tilde{w}$ denotes the steady state wage level and $\hat{w}$ is a realized wage path. At time t_o the product real wage is above the steady state level and has been so for a certain time period. The paths I and II show two possible adjustment paths, one approaching from above and one from below. Path I can be called the gradual-adjustment case and path II can be called the immediate-adjustment case. In the absence of simulations I can only speculate about the effects of choosing one or the other. It seems very likely that the gradual-adjustment case would take a much longer time to return to steady state growth than the immediate--adjustment case.[1] In the immediate-adjustment case the "lost"

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1 In a report to the Ministry of Economic Affairs (Wissén [1980]), I reported a simulation with the model from chapter 3, illustrating adjustment path I. If wages grow with 4 % per year from 1978 and on, it took, in the simulation, approximately 10 years to reach steady state growth. Maybe this can be seen as an indication of the length of the time period involved.

investments are, in a sense, compensated for, since the wage rate is lower than in steady state, following the period when it was too high. This leads to a period of over-investments which compensate for the underinvestments which occurred in the period when the wage rate was above the steady state level. A period of lower-than-steady-state wages *may* also give the firms a possibility to restore their debt/equity ratios. Lower wages imply higher profits and hence provide means for the firms to increase the equity. It also seems as though the adjustment could be made with balanced trade in the immediate-adjustment case, while it is more uncertain whether trade could be balanced on the gradual-adjustment path, due to the high wages.

High wages mean both high consumption demand for the traded good, a low rate of capital formation and continued large scrapping. All these three effects tend to deteriorate the balance of trade. The low rate of capital formation in the traded goods sector means decreased output but also low demand for the traded good for investment purposes. This last effect counterbalances the others with respect to the effects on the balance of trade. The net effect is thus not evident, although empirically it seems probable that higher wages should be associated with a deteriorating balance of trade.

I would, finally, like to mention some possible developments of the models used here, which have occurred to me while working with the study.

First, the representation of *expectations* in the models should be mentioned. I believe that there is more to be gained by altering the specification of expectations. In the models in chapter 2, the assumption about stationary expectations is crucial for the occurrence of cycles. Other types of adaptive mechanisms than the one in chapter 2, or rational expectations, could be tried. In the empirical parts, in chapter 4, there are reasons to suspect that the crude representation of expectations is one reason why the investment function does not perform very well. I think that a reasonable hypothesis is that investors had more optimistic expectations in the late '60s and early '70s than what is assumed in the model. This hypothesis is worthy of a deeper study to be fully clarified.

Related to the question of representing expectations in the models is the problem of deriving an investment function. In chapter 2, for example, in the two-sector models, there are investments in both sectors at the same time, although one of the sectors may have a higher rate of return than the other. This should be explained by the model, not merely assumed. The dynamics of the model would probably be changed if it is assumed that the investor compares the rates of return in the two sectors. Chapter 2 provides a framework for analyzing the effects of various hypotheses about investment behaviour in a small open economy.[1]

Concerning the empirical work, there are some extensions which may be worthwhile. The first, and maybe the most obvious extension consists in trying to make simulations with two sectors.[2] The simulation of the development in the two sectors creates complications. The vintage model, as it stands now, can be solved recursively. In order for a two-sector model to be manageable, it would probably have to be recursive too. There may be simplifying assumptions that retain the recursiveness.

Another empirical problem of interest concerns the presence of slack, which has played a large role in this work. It is, as mentioned before, hard to observe slack. Nevertheless there are ways to approach this problem. In my opinion

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1 There are results for closed economies which could be used, e.g. Shell, Stiglitz [1967].

2 For a theoretical extension of the vintage production model to many sectors, see Kemp and Thanh [1966].

and in view of the implications from the present work, it is of interest to try to measure the importance of productivity increases at the existing machines in the post-war period.

The present work has not been much concerned with policy problems. I have touched upon such questions only in this last chapter. This follows from the construction of the models. It could be worthwhile to introduce policy parameters (like various types of taxes) into the models. It could also be of interest, although I think it is difficult, to analyze the optimal adjustment to steady state growth. In the present Swedish situation such an analysis could lead to empirically valuable conclusions.

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