Decisions under Uncertainty

EFI

The Economic Research Institute at the Stockholm School of Economics
Address: Sveavägen 65, Box 6501, S-113 83 Stockholm, tel 08-736 01 20

Basic Orientation

The Economic Research Institute (EFI), at the Stockholm School of Economics is devoted to the scientific study of problems in management science and economics. It provides research facilities for scholars belonging to these diciplines, training advanced students in scientific research. The studies to be carried out by the Institute are chosen on the basis of their expected scientific value and their relevance to the research programs outlined by the different sections of the Institute. The work of the Institute is thus in no way influenced by any political or economic interest group.

Research Divisions:

A Working-Life, Organizational and Personnel
B Accounting and Managerial Finance
C Managerial Economics
D Marketing, Structural Economics and Marketing Policy
F Public Administration
G Economic Geography
I Applied Information Systems
P Economic Psychology
S Economics

Independent Research Program:

Program for Participation and Organizational Development
Division for Applied Research in Business Administration

Additional information about research in progress and published reports is described in our project catalogue. The catalogue can be ordered directly from The Economic Research Institute, Box 6501, S-113 83 Stockholm, Sweden.

DECISIONS UNDER UNCERTAINTY

THE USEFULNESS OF AN INDIFFERENCE METHOD FOR ANALYSIS OF DOMINANCE

Anders
Hederstierna

EFI THE ECONOMIC RESEARCH INSTITUTE
STOCKHOLM SCHOOL OF ECONOMICS

A Dissertation for the Doctor's Degree in Business Administration
Stockholm School of Economics 1981

UDK 65.012.2
ISBN 91-7258-138-7
GOTAB, Stockholm 1981

Acknowledgments

Many persons have contributed in different ways to the work underlying this report. I am very grateful for their support and help.

Professor Bertil Näslund, my teacher and adviser, has with much insight and care guided me in all aspects of my studies.

Professors Erik Ruist and Karl-Erik Wärneryd have generously provided comments on earlier drafts of the report.

My collegues at the research division of the Economic Research Institute, as well as Civ. ek. Helén Anderson and Civ. ek. Ingalill Holmberg, have read and commented various parts of earlier manuscripts. Dr. Ingolf Ståhl has encouraged my work in the initial phase, and Acting Professor Nils-Göran Olve has made helpful arrangements in the final phase.

Several persons have generously spent their time and effort to participate in the empirical investigations. I cannot mention all, but wish to especially acknowledge the help from Civ. ing. Christer S. Nilsson and Civ. ing. Per Åkenäs, who made the empirical study reported in Chapter 4 possible.

Mrs. Harriet Lundh and Mrs. Birgitta Mössberg have efficiently typed earlier manuscripts of the report, and Mrs. Ingrid Rosenvinge has excellently typed the final version. Fil. kand. Eleonore Holm has expertly supervised the production of the report.

Svenska Handelsbanken's Foundation for Social Science Research has financially supported the work.

Stockholm, October 1981

Anders Hederstierna

Contents

1 Position, Purpose, Plan

Uncertainty plays a vital part in all decision making. In relatively unimportant choice situations, decisions are made without pondering over the uncertainty. For example, when deciding whether or not to bring the umbrella for the evening walk, the consequences of a bad decision seem not large enough to call for an investigation if it will rain. In many managerial decision situations, the consequences of a bad decision may be so large that it seems reasonable to think hard about the consequences of implementing the available choice alternatives, and the uncertainty involved. It not only seems reasonable to think hard, but also to think in such a way that the chosen alternative corresponds with the preferences and the judgment of the uncertainty.

Despite the fact that formal methods for analyzing decisions under uncertainty have existed for more than two decades, and that their potential usefulness has been described for several types of choice situations, they have not had the great impact on practice as many perhaps anticipated. One reason for this is, to be specific, that the methods have not sufficiently recognized the decision maker's inability to transform information about uncertain objects into precise as well as reliable probability estimates.

General support for the position that the decision maker is not a perfect processor of information can be found in

Simon's (1955) intention with the concept of bounded rationality:

> ... the task is to replace the global rationality of economic man with a kind of rationality that is compatible with the access of information and the computational capabilities that are actually possessed by organisms, including man ... (p. 99)

There is a large body of research demonstrating man's incapacity to correctly assess uncertain quantities, and especially to express the uncertainty in reliable probability estimates:

> The major advance in descriptive research over the last five years has been the discovery that people systematically violate the principles of rational decision-making when judging probabilities, making predictions, or otherwise attempting to cope with probabilistic tasks. Biases in judgments of uncertain events are often large and difficult to eliminate. The source of these biases can be traced to various heuristics or mental strategies that people use to process information.
> (Slovic, Fischoff & Lichtenstein, 1976, pp. ii-iii)

If the decision maker's lack of information and limitations on computational and introspectional capability not are accounted for when analyzing decisions under uncertainty, the confidence in the probability assessments, and hence, the usefulness of the analysis, will certainly be questionable. To quote Roberts (1963):

> People often feel that their judgments about probabilities are too vague, ambiguous, volatile, uneasy, or shaky to be taken seriously as a basis for a formal analysis.
> (p. 328)

A way to make the analysis more useful in this sense would be to allow for imprecision in the probability information, or to put it differently, allow the decision maker to be uncertain about the probability estimates.

Usefulness is, however, also related to the conclusiveness of the analysis, i.e. a useful analysis should enable the decision maker to identify the optimal choice among the available alternatives.

The dilemma resulting from these two aspects of usefulness is that, the more (less) imprecise input allowed for in the analysis, the less (more) likely that the analysis will be conclusive.

Even if the dilemma has not been discussed, methods have been suggested for approaching it. For example, Watson, Weiss & Donnell (1979) have suggested a fuzzy-sets approach to handle imprecise judgmental input. Tani (1978) has suggested approximations of probability estimates for critical states of the uncertain object. Isaacs (1963), Fishburn, Murphy & Isaacs (1968) have suggested an approach to investigate the sensitivity of the outcome of the analysis to misjudgment of the probability (see also Pierce & Folks, 1969). Ulvila & Brown (1978) have suggested an interactive step-through simulation method using Monte-Carlo sampling for possible reductions of the required probability estimates. Elvestedt (1979) has suggested an interactive approach for analysis of dominance using successively more precise information. Fishburn (1964, 1965) has suggested an approach for analysis of dominance by considering imprecise probability information at any given point in time.

To my knowledge, studies of the usefulness of the suggested methods have not been reported and is therefore difficult to assess. Fishburn's approach must, however, be considered as the most straight forward line of analysis and will therefore provide the basis for this study.

PURPOSE

The purpose of the study is to investigate the possibility of increasing the usefulness of formal analysis of decisions made under uncertainty, by suggesting a new indifference method for analysis of dominance, and examining its usefulness both theoretically and empirically.

PLAN

Chapter 2 provides a formal statement of the choice situation under consideration and a treatment of the concepts of utility and personal probability. The chapter concludes with a review of methods for analysis of dominance given different imprecise probability measures.

In Chapter 3 a new indifference method for analysis of decisions under uncertainty is suggested. The chapter is concerned with defining the method and discussing its characteristics.

Chapter 4 contains an investigation of the confidence in the type of personal probability estimates required by the indifference method relative to point estimates, bounded interval estimates, and ordinal estimates. The investigation includes theoretical propositions as well as data from an empirical study of twenty-four professional probability assessors.

Chapter 5 contains a real-life application of the indifference method to the problem of determining the size of weekly magazine editions. The model and methods suggested and tested for solving the problem are described. The experience gained from the application is discussed with respect to the participants' perceptions of the usefulness of the suggested approach.

Chapter 6 contains a summary of the report and a few preliminary ideas about future research.

2 Decisions under Uncertainty

The aim of this chapter is to set the stage for the following chapters by defining the choice situation under consideration. It shall also provide definitions of the key concepts of decisions made under uncertainty and methods for analyzing dominance when the probability information is imprecise.

FORMAL STATEMENT OF THE PROBLEM

Before stating the problem, a few terms used in the following pages have to be clarified. The definitions are adopted from Savage (1954).

Table 2:1. Definitions of a few used terms

Term	Definition
The world	The object about which the person is concerned
A state	A description of the world, leaving no relevant aspect undescribed
The true state	The state that does "in fact" obtain

The thruth of the true state may be seen as a mental construction by the decision maker which need not necessarily be validated by others.

For an illustration of the terms, suppose a company is planning an investment in a foreign country. The country has an unstable political situation, which may lead to a nationalization of all foreign assets. The world could be defined as the future political situation in the country, and the states be described as whether foreign assets will be nationalized or not. A priori, the decision maker would be uncertain about which of the states that will obtain, i.e. the true state.

The choice situation under consideration will be defined as follows:

1. The decision maker wants to choose an action from a finite set of n actions $\{a_1, a_2, \dots, a_n\}$.

2. The world is exhaustively described in a finite set of m mutually exclusive states $\{s_1, s_2, \dots, s_m\}$, where one, and only one, is the true state.

3. The decision problem has been summarized in a nxm matrix in which the element u_{ij} represents the utility if the decision maker chooses action a_i and state s_j obtains.

4. The decision maker wants to choose the action which maximizes expected utility, i.e.

$$\operatorname*{Max}_{i} E(a_i) \tag{2-1}$$

where $E(a_i) = \sum_j p_j u_{ij}$, but is uncertain about the personal probability of the m states $\{p_1, p_2, \dots, p_m\}$.

The world is exhaustively described, meaning that the states represent a complete description of the world. The states are also mutually exclusive. From this follows that the sum of the probability of the states amounts to unity: $\sum_j p_j = 1$.

It should be pointed out here that I shall assume that the assessor of the uncertain probability and the decision maker is the same person. If they are different persons, some difficulties may arise, especially what regards the decision maker's belief in the assessor's probability estimates, and, hence, in the analysis. However, the rational reason for choosing another person than the decision maker to make the assessments would be that this other person has better knowledge and information about the uncertain world. If the decision maker recognizes this argument, the confidence in the probability judgments should be as large as it possibly can (cf. Morris, 1979; Spetzler, 1968).

A further assumption in the analysis is that the assessor of the probability is one single person. Even if several persons can judge the probability of the states, the implications from the model and methods to be reviewed and suggested do not change. As long as the most knowledgeable and informed person makes the assessments, and the other participants agree to this, the belief in the judgments should be the highest. If such agreement can not be reached, methods for combining group judgments could be used (see e.g. Raiffa, 1968; Winkler, 1968; Morris, 1977), but perhaps needless to say, it is not possible to determine in advance if they will result in consensus between the group members (see also Staël von Holstein, 1970).

OPTIMAL ACTION AND DOMINANCE BETWEEN ACTIONS

The optimal action is defined as the one which maximizes expected utility, and if the choice of action is consistent with this model, the decision maker has a rational pattern of preferences and expectations as assumed by the axioms of utility theory and probability theory (see von Neumann & Morgenstern, 1947; Luce & Raiffa, 1957).

The fact that the axioms do not always describe actually observed behavior (in experimental settings) has been pointed out by e.g. Allais (1953), Ellsberg (1961), MacCrimmon (1968). To conclude from this that the theory is not "good", because it does not map individuals' actual behavior in choice situations, would only be reasonable if the theory claimed to be descriptive and predictive. Since the theory subscribed to here is normative, such conclusions must be considered as misdirected.

To compute the expected utility of all actions requires that we know the precise number of the probability p_j for all m states. Thus, it requires that the decision maker is able to express his/her uncertainty in precise estimates. As stated in Chapter 1, the position is that this may be difficult.

If we allow for imprecise information about the probabilities, the ambitions have to be lowered, and instead of trying to identify the optimal action we shall have to try and find non-optimal actions and exclude them from the list of considered actions. Thus, we want to identify actions with lower expected utility than any other action and dominate them. Stated differently, if $\{a_k, a_i\} \subset \{a_1, a_2, \ldots, a_n\}$ and $i \neq k$, then we shall say that action a_k dominates a_i if

$$E(a_k) \geq E(a_i) \tag{2-2}$$

In a strict sense, action a_k dominates a_i if

$$E(a_k) > E(a_i) \tag{2-3}$$

leaving out the indifference case.

The difference, or if you prefer, the similarity between optimality and dominance is quite straightforward. The optimal action dominates all other actions. If all actions are dominated by one action, this action is optimal. Formally, if $E(a_k) \geq E(a_e)$, where a_e denotes all actions in the set of actions except a_k then $\max_i E(a_i) = E(a_k)$, where $i = 1, 2, \ldots, n$.

It should be stressed that the dominance I here and in the following shall refer to is from an expected utility point of view. The use of the term "dominance" in this sense is in line with Fishburn's (1964, 1965) terminology, but does often, particularly in textbooks, refer to the case when, for example, $u_{kj} \geq u_{ij}$ for all $j = 1, 2, \ldots, m$, i.e. when action a_k is preferred to a_i regardless of the probability of the states. As we shall see later, this case will be referred to in this study as dominance when given "null" measures of the probability of the states.

Analysis of dominance does not ensure us to find the optimal action. In such cases where the result of the analysis is not one optimal action, and if the decision maker is not able or willing to give more precise probability information, a secondary criterion for choosing among the non-dominated actions could be used, such as mixed acts (see Fishburn, 1964, 1965; Savage, 1954) or decision rules such as maximin, minimax regret, Hurwicz rule etc. (See e.g. Baumol, 1977; Savage, 1954; Luce & Raiffa, 1957.)

In this study I will only consider the primary criterion of expected utility. Because of the non-repetitive character of most managerial decisions, it is likely that the notion of mixed acts, i.e. actions which are chosen with a probability not equal to one or zero, is cognitively questionable. If the choice of action is unique, the decision maker would probably find it meaningless to consider the notion of implementing an action in a non-deterministic way. This is also pointed out by, for example, Baumol (1977) and Lee (1971).

UTILITY

This study is primarily concerned about the probability aspect of decisions under uncertainty. However, since the concept of utility is a fundamental element in the underlying theory of decisions under uncertainty, a review of the most relevant parts will be provided here. For a more complete discourse on the subject, see von Neumann & Morgenstern (1947), Savage (1954), Fishburn (1964), Luce & Raiffa (1957).

Without any loss of generality, suppose three values A_i, A_j, A_r with the preference relation $A_j \gtrsim A_i \gtrsim A_r$ and, for transitivity, that $A_j \gtrsim A_r$, where "$\gtrsim$" denotes "preferred or indifferent to". The axiom of continuity then states that there exists a value u_i, between zero and one, such that the person is indifferent between A_i and the lottery $\{u_i A_j, (1-u_i) A_r\}$, where A_j and A_r are the prizes in the lottery.

Further, the axiom of monotonicity states that the lottery $\{u_i A_j, (1-u_i) A_r\}$ is preferred or indifferent to the lottery $\{u_i' A_j, (1-u_i') A_r\}$ if, and only if, $u_i \geqslant u_i'$.

If we assume a transitive preference relation over a set of lotteries of the kind described, and if to each lottery L there is assigned a number u(L) such that the magnitudes of the numbers reflect the preferences, i.e. $u(L) > u(L')$, if, and only if, L is preferred to L', then there exists a real-valued mapping u over the lotteries called the "utility function".

The expected utility hypothesis states that if the utility has the property that $u[pL, (1-p)L'] = pu(L) + (1-p)u(L')$, for all probabilities p and lotteries L and L', then the utility function is said to be linear (or that the person is risk neutral to money).

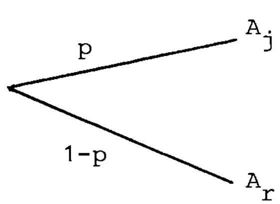
Measuring utility

Even if it may seem possible to measure utility by direct questions, it would demand much from the asked person. Above all, it would require that the person has a good understanding and an operational interpretation of the utility concept. Instead, indirect methods by using the device of binary lotteries seem more useful in real settings. Using the introduced notations, the general form of binary lotteries is

$$u(A_i) = pu(A_j) + (1-p)u(A_r) \tag{2-4}$$

where the prize A_i usually is referred to as the certainty monetary equivalent, CME. Thus $u(A_i)$ is the utility of receiving the prize A_i with certainty (p=1).

The right-hand side of the expression (2-4) may be illustrated in a binary lottery:



The feature of the lottery is that the person wins A_j with probability p and A_r with probability 1-p.

To measure the person's utility of A_i, he/she is asked to state the certain prize (CME = A_i) required for indifference between this prize and the lottery with specified prizes and probability. A variant is to specify A_i, A_j, and A_r and instead ask for the probability p required for indifference. Evidently, also this variant enables us to solve the equation in (2-4).

PERSONAL PROBABILITY

The terms "subjective" and "personal" probability have been used interchangeably in the literature. Here I shall use the latter, since the term "subjective" may have connotations of both inactivity and an arbitrary quantity (cf. Polanyi, 1958).

Definitions of personal probability have been given by several authors. de Finetti (1937), Koopman (1940), Good (1950), Kraft, Pratt & Seidenberg (1959) have presented an intuitive conception of personal probability, i.e. in terms of an a priori "degree of belief" that a proposition is true.

Ramsey (1931), Savage (1954), de Finetti (1937), Suppes (1955), Davidson & Suppes (1956), Pratt, Raiffa & Schlaifer (1964) have provided definitions of personal probability with reference to decision making. A definition with reference both to decision making and the frequency view of probability has been given by Anscombe & Aumann (1963), and because of its directness, I shall adopt their formulation.

Suppose we have a roulette lottery with the utility $f_1u_1(A_1) + \ldots, f_ru_r(A_r)$, where $u_1(A_1)$ denotes the utility of the prize A_1 and f_1 is the chance of winning A_1 and so on. f_1 is a known chance from a frequency point of view (for example a roulette wheel). Let Z be the set of all these roulettes lotteries, and $Z_1, Z_2, \ldots, Z_s$ denote each lottery in Z.

Suppose further that we have another lottery where the outcome (the states of the world) is not associated with any known chance (Anscombe and Aumann call it a "horse lottery"). Let $h_1, h_2, \ldots, h_s$ denote the mutually exclusive states of this horse lottery.

Each horse lottery is compound, meaning that if h_1 obtains ("horse no. 1 wins"), you will receive a ticket to the roulette lottery Z_1 in Z and so on. Such a compound lottery will be denoted as $[Z_1, Z_2, \ldots, Z_s]$. The set of all compound horse lotteries is denoted as H.

Let there also be a set of roulette lotteries whose prizes are compound horse lotteries and that the utility function on this set is symbolized by u^*.

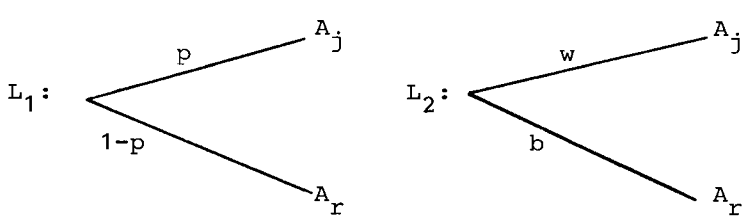
The theorem then states that there exists a set of s non-negative numbers $p_1, p_2, \ldots, p_s$, summing to 1, such that for all $[Z_1, Z_2, \ldots, Z_s]$ in H,

$$u^*[Z_1, Z_2, \ldots, Z_s] = p_1 u(Z_1) + \ldots + p_s u(Z_s) \qquad (2\text{-}5)$$

The number p_i is called the personal probability of the state (outcome) h_i of the horse lottery.

Interpreted, p_i is equal to the chance such that you would be indifferent between receiving a prize of $1 with probability p_i and $1 with equal physical chance. For a description of the underlying assumptions as well as the proof of the theorem, the reader is referred to Anscombe & Aumann (1963).

An illustrative way to describe the concept of personal probability is by using binary lotteries (standard gambles). Consider the situation where you are asked which of the two lotteries L_1 and L_2 you would prefer to play:



Lottery L_1 gives you a prize A_j with probability p and a prize A_r with probability 1-p. Lottery L_2 gives you the same prize A_j, if a white ball is drawn from an urn containing white (w) and black(b) balls, and A_r if a black ball is drawn from the same urn.

Thus, p is unknown from a frequency point of view. The chance of drawing a white or black ball from the urn is known from the proportion of white and black balls in the urn.

The personal probability p of the state yielding A_j in L_1, such that you are indifferent between L_1 and L_2, would then be equal to some proportion of white balls in the urn. Thus, there exists a number between zero and one, called the personal probability p, such that you are indifferent between playing the lotteries L_1 and L_2 (see e.g. Raiffa, 1968).

Measuring personal probability

Personal probability may be measured by using direct or indirect methods. For reviews of suggested techniques, see e.g. Huber (1974), Chesley (1975), Spetzler & Staël von Holstein (1975). Note, however, that most of these techniques do not refer to the decision oriented interpretation of personal probability.

Direct methods require the probability assessor to answer the questions in numerical form. By this, they also demand that the assessor is able to interpret the personal probability concept, which perhaps is a surmountable problem if the person is adequately trained. They also demand that the estimator can and is willing to tell the truth, i.e. give estimates in accordance to his/her true beliefs (see further Edwards, 1968). Experimental psychologists have approached the problem of obtaining truthful assessments by suggesting scoring rules, by which the subject receives (or rather expects to receive) a performance score dependent upon the difference between the assessment and the actually observed event or obtained state (see e.g. Staël von Holstein, 1970).

Indirect methods infer the personal probability from choices in real or hypothetical situations. The device used for hypothetical situations is the binary lottery, already described. Instead of using urns for calibrating the probability, Spetzler & Staël von Holstein (1975) have suggested "probability wheels" with the purpose of giving a more visual feeling of the proportions, i.e. the physical chance.

So far, we have implicitly been concerned with the problem of measuring precise personal probability estimates. To close in on the subject matter of this study, let us see how slight revisions of the binary lottery questions may enable the assessor of the personal probability to give imprecise estimates.

Suppose there are three states for which we want to elicit the probabilities p_1, p_2, p_3 that they will obtain. Three standard gambles L_1, L_2, L_3 could then be used for elicitation: $L_1 = p_1A_j + (1-p_1)A_r$, $L_2 = p_2A_j + (1-p_2)A_r$, $L_3 =$ $= p_3A_j + (1-p_3)A_r$.

To elicit ordinal estimates, i.e. a ranking of the magnitudes of the three probabilities, the assessor may be asked to give a preference ranking of the lotteries. For example, if it is assessed that $u(L_3) \geqslant u(L_2) \geqslant u(L_1)$, we may infer the ordinal ranking $p_3 \geqslant p_2 \geqslant p_1$.

Another imprecise estimate is inequality sets, meaning a ranking of the magnitudes of the probability for subsets and elements of the set of all probabilities. For example, if it is assessed that $u(L_1) + u(L_2) \geq u(L_3)$, it is possible to infer the inequality set $p_1 + p_2 \geq p_3$.

To elicit bounded interval estimates, i.e. judgments that the personal probability lies somewhere between two values in [0,1], two binary lotteries could be used: $L_1 = p_1 A_j + (1-p_1)A_r$ and $L_2 = wA_j + bA_r$, where, for example, w is the proportion of white balls in an urn containing white and black (b) balls. The uncertain assessor may then say that indifference between L_1 and L_2 occurs when $r^k \leq w \leq r^{k+\alpha}$. Thus, from this it is possible to infer that the personal probability p_1 lies in the bounded interval $[r^k, r^{k+\alpha}]$. I will explain the meaning of r^k and $r^{k+\alpha}$ more precisely in Chapter 4. At present it will suffice to say that they are values in [0,1].

ANALYSIS OF DOMINANCE GIVEN IMPRECISE PROBABILITY MEASURES

How can dominance between actions be concluded, when given different imprecise probability measures? In this section I will review conditions of dominance given earlier for "null" measures, ordinal rankings, bounded intervals and sets of inequalities. The analyses are based on Fishburn (1965) and later suggestions on his work.

Without any loss of generality, dominance will be analyzed between actions a_k and a_i, where $\{a_k, a_i\}$ is a subset of the set of n available actions $\{a_1, a_2, \ldots, a_n\}$ and $k \neq i$.

The difference in utility between actions a_k and a_i is denoted d_j, i.e.

$$d_j = u_{kj} - u_{ij} \qquad (2\text{-}6)$$

for all states $j = 1, 2, \ldots, m$.

"Null" measures

The so called "null" measure is the extreme imprecise probability measure in that the only information it contains is that any of the states may obtain but none with certainty. Thus, the assessment is that the probability lies somewhere in the open interval]0,1[.

Action a_k dominates a_i if the difference d_j is non-negative for all states, i.e. if

$$d_j \geqslant 0 \qquad (2\text{-}7)$$

for every $j = 1, 2, \ldots, m$ (Fishburn, 1965).

Obviously, the (lack of) information in "null" measures implies that a conclusive analysis of the type here considered, is only likely to be achieved in trivial decision settings. To take an example, suppose the following matrix summarizes a decision problem under uncertainty:

	s_1	s_2	s_3
a_1	8	5	0
a_2	7	5	-2

The decision maker has only been able to give "null" estimates of p_1, p_2, and p_3, such that they all lie somewhere between zero and one. As can be seen, $d_1 = 1$, $d_2 = 0$, $d_3 = 2$ and thus, since $d_1, d_2, d_3 \geqslant 0$, we conclude that action a_1 dominates a_2.

Ordinal measures

The ordinal, or rank order, measure is another imprecise piece of information about the decision maker's personal probability.

Given an ordinal estimate of the m states such that $p_1 \geqslant p_2 \geqslant \ldots \geqslant p_m$, it is possible to conclude that action a_k dominates a_i, if

$$\sum_{j^*=1}^{j} d_{j^*} \geqslant 0 \tag{2-8}$$

for all j = 1, 2,..., m (Fishburn, 1965).

For an illustration, suppose the decision maker has given the probability ranking $p_1 \geqslant p_2 \geqslant p_3$, and that the problem is as below.

	s_1	s_2	s_3
a_1	2	0	2
a_2	1	0	4

According to (2-8), the differences are +1, +1 + 0, +1 + 0 - 2. Since all summations are not greater than or equal to zero, we cannot conclude that action a_1 dominates a_2. The question then is if a_2 dominates a_1. The sums of the differences are -1, -1 + 0, -1 + 0 + 2. Since all are not greater than or equal to zero, we cannot say that action a_2 dominates a_1.

Kmietowicz & Pearman (1979) point out that these demanding conditions are unlikely to be fulfilled in most realistic problems. They have instead provided a definition of "weak" dominance when given ordinal estimates of the probability. The definition goes as follows: maximum and minimum expected differences are evaluated by taking the partical averages of the differences, i.e.

$$\bar{d}_j = \frac{1}{j} \sum_{j^*=1}^{j} d_{j^*} \tag{2-9}$$

for all j = 1, 2,..., m.

The largest of $\bar{d}_j$ is the maximum and the smallest is the minimum expected difference. Thus, it is plain that

$$\text{Max}\left[E(a_i) - E(a_k)\right] = -\text{Min}\left[E(a_k) - E(a_i)\right] \qquad (2\text{-}10)$$

and

$$\text{Min}\left[E(a_i) - E(a_k)\right] = -\text{Max}\left[E(a_k) - E(a_i)\right] \qquad (2\text{-}11)$$

where $E(a_k)$ represents the expected utility of action a_k and so on.

Now, if the ordinal ranking of the probability is $p_1 \geqslant p_2 \geqslant \ldots \geqslant p_m$, and if

$$\text{Max}\left[E(a_k) - E(a_i)\right] > \text{Max}\left[E(a_i) - E(a_k)\right] \qquad (2\text{-}12)$$

then

$$\text{Min}\left[E(a_k) - E(a_i)\right] > \text{Min}\left[E(a_i) - E(a_k)\right] \qquad (2\text{-}13)$$

and they conclude that action a_k "weakly" dominates a_i.

Kmietowicz & Pearman (1979) claim that Fishburn's (1965) definition of dominance as stated above, is a special "strict" case of "weak" dominance, i.e. when (2-12), (2-13) hold and, in addition, when

$$\text{Min}\left[E(a_k) - E(a_i)\right] \geqslant 0 \qquad (2\text{-}14)$$

To examplify "weak" dominance in this respect, suppose we have the same problems as for the illustration of Fishburn's definition:

	s_1	s_2	s_3
a_1	2	0	2
a_2	1	0	4

and as before that $p_1 \geq p_2 \geq p_3$. The expected differences defined in (2-9) become $1/1 \cdot 1$, $1/2(1+0)$, $1/3(1+0-2)$. Thus, since $+1 > 1/3$ and consequently $-1/3 > -1$, the conclusion would be that action a_1 dominates action a_2.

As we have already seen, this conclusion could not be made with Fishburn's definition above. Evidently, Kmietowicz and Pearman's dominance is justly called "weak", since the conclusion does not hold for the case when $p_1 = p_2 = p_3 = 1/3$ in the example above.

Bounded interval measures

Suppose the decision maker has been able to give the probability information as bounded interval estimates:

$$r_j^k \leq p_j \leq r_j^{k+\alpha} \tag{2-15}$$

for all $j = 1, 2, \ldots, m$, and $\alpha > 0$.

Two methods for analyzing dominance, given bounded interval measures, have been presented. Fishburn (1965) uses a linear programming approach as follows.

$$\text{Min } Z = \sum_j p_j d_j \tag{2-16}$$

$$\text{s.t.} \quad r_j^k \leq p_j \leq r_j^{k+\alpha}$$

$$\sum_j p_j = 1 \qquad (j = 1, 2, \ldots, m)$$

If the solution on $Z \geq 0$, action a_k dominates a_i. A perhaps more illustrative approach has been suggested by Sarin (1978), who has stated the following knap-sack algorithm:

1. Set $p_j = r_j^k$ for all $j = 1, 2, \ldots, m$.

2. Remaining "weight", $1 - \sum_j p_j$, is assigned successively to the smallest d_j within the given interval. The now remaining "weight" is assigned to the next smallest d_j within the interval, and so on, until all "weight" has been assigned.

With the assigned p_j to all states, it is possible to conclude that action a_k dominates a_i if $E(a_k) \geq E(a_i)$, which is evidently the same as $\sum_j p_j d_j \geq 0$.

To illustrate, consider the example where the probability of three states has been estimated as:

$$0.1 \leq p_1 \leq 0.2$$
$$0.5 \leq p_2 \leq 0.7$$
$$0.2 \leq p_3 \leq 0.5$$

and that the matrix below summarizes the problem.

	s_1	s_2	s_3
a_1	8	3	1
a_2	1	3	11

If we want to analyze if action a_1 dominates a_2, Fishburn's (1965) linear program would be:

$$\text{Min } Z = 7p_1 + 0p_2 - 10p_3$$

$$\text{s.t. } 0.1 \leq p_1 \leq 0.2$$
$$0.5 \leq p_2 \leq 0.7$$
$$0.2 \leq p_3 \leq 0.5$$

$$p_1 + p_2 + p_3 = 1$$

In this case, $Z < 0$, and we cannot conclude that action a_1 dominates a_2. To analyze if action a_2 dominates a_1, the goal function in the LP above would be: Min $Z = -7p_1 + 0p_2 + 10p_3$. The solution on $Z > 0$, and hence, action a_2 dominates a_1.

For an illustration of the knap-sack algorithm, let us analyze if action a_1 dominates a_2:

1. Set $p_1 = 0.1$, $p_2 = 0.5$, $p_3 = 0.2$

2. Remaining "weight", $1 - (0.1 + 0.5 + 0.2)$ is assigned to p_3, and the final assignment becomes: $p_1 = 0.1$, $p_2 = 0.5$, $p_3 = 0.4$. Since the difference in expected utility $(0.1 \cdot 7 + 0.5 \cdot 0 + 0.4 \cdot -10) < 0$, it cannot be concluded that action a_1 dominates a_2.

The analysis if action a_2 dominates a_1 is as follows:

1. Set $p_1 = 0.1$, $p_2 = 0.5$, $p_3 = 0.2$

2. Remaining "weight" (0.2) is first assigned to p_1 within the given bounds. The now remaining "weight" (0.1) is assigned to p_2, and the final assignment becomes: $p_1 = 0.2$, $p_2 = 0.6$, $p_3 = 0.2$. Since the difference $(0.2 \cdot -7 + 0.6 \cdot 0 + 0.2 \cdot 10) > 0$, action a_2 dominates a_1.

Inequality sets

Fishburn (1965) has analyzed dominance given inequality sets of the probability of the states, by using a method of equating coefficients. The following is, however, adopted from Abramson (1967), who has provided a more direct analysis by using a technique for substituting elements from the given set of inequalities.

By inequality sets it is meant that the decision maker has assessed that

$$p_{j+1} + p_{j+2} + \cdots + p_{j+\alpha} + p_{j+\alpha+1} \geqslant \tag{2-17}$$

$$\geqslant p_j \geqslant p_{j+1} + p_{j+2} + p_{j+3} + \cdots + p_{j+\alpha}$$

for some $\alpha \geqslant 1$ and $j = 1, 2, \ldots, m-1$, and/or that

$$p_j \geqslant p_{j+1} + p_{j+2} + \cdots + p_m \tag{2-18}$$

The analysis of dominance consists of three cases (Abramson, 1967):

Case 1: If $d_1 \geqslant 0$, replace p_1 by its lower bound. If then $d_j \geqslant 0$ for $j = 2, 3, \ldots, m-1$, it is possible to conclude that a_k dominates a_i.

Case 2: If $d_1 < 0$, and the inequality for p_1 is expressed as in (2-17), then replace p_1 by its upper bound. If $d_j \geqslant 0$ for $j = 2, 3, \ldots, m-1$, then a_k dominates a_i.

Case 3: If $d_1 < 0$, and the inequality for p_1 is given by (2-18), it is not possible to conclude that action a_k dominates a_i.

For an illustration, suppose the following matrix represents the choice problem:

	s_1	s_2	s_3
a_1	0	9	3
a_2	1	6	1

and that the following inequalities have been elicited:

$$p_2 + p_3 \geq p_1 \geq p_2 \tag{2-19}$$

$$p_2 \geq p_3 \tag{2-20}$$

$$p_3 \geq 0 \tag{2-21}$$

To analyze if action a_1 dominates a_2, it is noted that $d_1 < 0$. By using (2-19), it is possible (case 2) to rewrite the expected difference as

$$E(a_1) - E(a_2) \geq -1p_2 - 1p_3 + 3p_2 + 2p_3$$

where $p_2 + p_3$ is the upper bound of p_1.

Since d_2 and d_3 are non-negative, it is possible to conclude that action a_1 dominates a_2.

CONCLUSION

The reviewed analyses of dominance have concretely established one side of the dilemma regarding the usefulness of formal analysis of decisions under uncertainty namely, that the more imprecise probability information about the states of the world, the less likely that dominance, and hence, optimality, can be concluded.

In fact, we see from the conditions that if the decision maker is elicited "null" estimates, ordinal estimates, or estimates resulting in inequality sets as expressed in (2-17), (2-18), the analysis is only likely to be conclusive in trivial choice situations. Since bounded interval estimates can be more or less imprecise, it is not possible to say anything general about the likeliness of a conclusive result, except that, the wider the intervals are, the less likely that the analysis will provide a conclusive result for non-trivial choice problems.

Thus, even if it is reasonable to expect the decision maker to have considerably more confidence in the described imprecise estimates than in precise estimates, the analysis will not be useful in the sense that, in many non-trivial settings, it will not provide the decision maker with a definite guide for choosing among the available actions.

It is difficult to conceive any feasible way to completely solve the dilemma between conclusiveness and confidence, and hence, a trade-off has to be made between the two aspects. In the next chapter such a trade-off will be proposed.

3 An Indifference Method for Analysis of Dominance

In this chapter I shall suggest a method for analyzing decisions under uncertainty. The approach may be viewed as reversed from Fishburn's analysis described in Chapter 2, in that it proposes a method for determining the probability information required for concluding dominance, instead of treating the information as given prior to the analysis. The aim of the method is to increase, relative to earlier suggestions, the possibility of reaching a conclusive result in non-trivial settings, without demanding the decision maker to give more precise probability estimates than necessary for conclusiveness.

DEFINITION OF THE INDIFFERENCE METHOD

The reasoning behind the method is quite straightforward. Suppose we want to analyze if action a_1 dominates a_2, and that we for the moment do not have any information about the personal probability of the states. The question we may then ask is: how much of the total "weight" (personal probability) 1 has to be assigned to the states for which a_1 is preferred to a_2 and to the states for which a_2 is preferred to a_1, if the decision maker is to be indifferent between the two actions? Assuming a "worst case" distribution of the personal probability among the states of the world, it is possible to compute such "weights" or values of indifference, between the least preferred state if action a_1 is implemented, and every state for which a_1 is preferred to a_2. To conclude dominance, the decision maker is asked to assess if the personal probability is larger than the computed indifference value.

Without any loss of generality, dominance will be defined between actions a_k and a_i, where $\{a_k, a_i\} \subset \{a_1, a_2, \ldots, a_n\}$ and $k \neq i$.

Furthermore, dominance will be given a "strict" definition: action a_k dominates a_i if

$$E(a_k) > E(a_i).$$

The differences $d_j (= u_{kj} - u_{ij})$ are possible to form into two sets, F_t and G, such that

$$F_t = \{d_j : d_j > 0\} \text{ for } t = 1 \qquad (3\text{-}1)$$

$$G = \{d_j : d_j < 0\} \qquad (3\text{-}2)$$

where t is an iteration index which shall be explained presently. Hence, the elements contained in F_t represent the states for which action a_k is preferred to a_i and the elements in G represent the states for which action a_i is preferred to a_k. Note that the states for which $d_j = 0$ shall not be considered in the analysis since they do not have any effect on dominance.

The minimum differences in F_t and G are identified and denoted as:

$$f_t = \underset{j \in F_t}{\mathrm{Min}} \; d_j \qquad (3\text{-}3)$$

$$g = \operatorname*{Min}_{j \in G} d_j \tag{3-4}$$

For every set F_t in iterations $t > 1$, the minimum difference f_t in set F_{t-1} is excluded, or stated differently:

$$F_t = \{d_j : d_j > f_{t-1}\} \quad \text{for } t > 1 \tag{3-5}$$

The minimum difference g in set G will not be influenced by the iterations and is therefore not indexed by t.

Considering only the states represented by f_t and g, an indifference value I_t between zero and one is computed, such that the decision maker should be indifferent with respect to expected utility between actions a_k and a_i, if the probability of the state represented by f_t equals I_t and the probability of the state represented by g equals $1-I_t$, i.e.

$$I_t = g/(g-f_t) \tag{3-6}$$

The decision maker is now asked if it can be asserted that the personal probability of all states represented in F_t is larger than the computed indifference value. Thus, if it can be asserted that

$$\sum_{j \in F_t} p_j > I_t \tag{3-7}$$

for any $t = 1, 2, \ldots, c$, where c equals the number of states with different d_j represented in F_1, then it can be concluded that action a_k dominates a_i (a proof is provided in the appendix to this chapter).

For completeness, it shall be noted that if the decision maker asserts that

$$\sum_{j \in F_t} p_j = I_t \tag{3-8}$$

for any $t = 1, 2, \ldots, c$, then action a_k dominates a_i if

$$p_g < 1 - I_t \tag{3-9}$$

where p_g is the personal probability of the state represented by g.

If it cannot be concluded that action a_k dominates a_i, i.e. if (3-7) or (3-8), (3-9) are not asserted, the analysis continues to investigate if action a_i dominates a_k. Since this is the mirror image of the above described procedure, the analysis should be evident by redefining d_j as $u_{ij} - u_{kj}$. The total number of such possible pair-wise comparisons is $n(n-1)/2$.

NUMERICAL ILLUSTRATION

For an illustration of the indifference method, suppose a decision problem under uncertainty has been summarized in a 2x5 matrix:

	s_1	s_2	s_3	s_4	s_5
a_1	-4	0.5	-1	1	0
a_2	2	-0.5	1	-1	0.5

Let us analyze if action a_2 dominates a_1.

The difference d_j over all states is computed as:

$$\begin{aligned} d_1 &= 6 \\ d_2 &= -1 \\ d_3 &= 2 \\ d_4 &= -2 \\ d_5 &= 0.5 \end{aligned}$$

and formed into the sets F_1 and G:

$$F_1 = \{6,\ 2,\ 0.5\}$$
$$G = \{-1,\ -2\}$$

The minimum difference in each set is identified as:

$$f_1 = 0.5 \qquad (\in F_1)$$
$$g = -2 \qquad (\in G)$$

In the following iterations, the minimum differences are:

$$f_2 = 2 \qquad (\in F_2)$$
$$f_3 = 6 \qquad (\in F_3)$$

The indifference values are then computed in each iterations as defined in (3-6):

$$I_1 = -2/(-2\ -0.5) = 0.80$$
$$I_2 = -2/(-2\ -2) = 0.50$$
$$I_3 = -2/(-2\ -6) = 0.25$$

The decision maker is now asked to respond to these indifference values. The subsets of states represented by the differences contained in F_1, F_2, and F_3 are $\{s_1, s_3, s_5\}$, $\{s_1, s_3\}$, and $\{s_1\}$, respectively. Hence, it can be concluded that action a_2 dominates a_1 if

$$p_1 + p_3 + p_5 > 0.80$$

or if

$$p_1 + p_3 > 0.5$$

or if

$$p_1 > 0.25$$

As for the extreme case, when, for example, the decision maker asserts that

$$p_3 + p_1 = 0.5$$

it is possible to conclude that action a_2 dominates a_1 if

$$p_4 < 1-0.5$$

where p_4 is the probability of the state represented by the minimum difference ($g = -2$) in G.

If it cannot be concluded that action a_2 dominates a_1, we may want to analyze if a_1 dominates a_2 (which the analysis of course could have started with).

The signs of the differences d_j over all states are then changed and, by following the same procedure as already illustrated, the decision maker is asked to assess if

$$p_2 + p_4 > 0.85$$

or if

$$p_2 > 0.75$$

If any of these inequalities holds, it is concluded that action a_1 dominates a_2.

Suppose the extreme case when, for example

$$p_2 = 0.75$$

Action a_1 then dominates a_2 if

$$p_1 < 1 - 0.75$$

CONCLUSION AND COMMENTS

Conclusiveness of the analysis

Generally speaking, it is clear that the indifference method does not ensure a conclusive result, i.e. it may not enable the decision maker to identify the optimal action. This is, however, not a unique characteristic of the indifference method but a general feature of analysis of dominance. For inconclusive cases, and if the decision maker is not willing or able to make more precise probability judgments, a secondary criterion may be used for choosing among the non-dominated actions.

Note that when the world is described in a dichotomy (m=2), the indifference method is conclusive. Analysis of dominance for the dichotomous case has already been defined by e.g. Sarin (1978).

What can be said about the relation between the indifference method and earlier suggested methods?

There are two obviously extreme cases when the indifference method does not require any information about the probability: (1) if one of the sets F_1 and G is empty, dominance can be concluded, (2) if both sets are empty at $t = 1$ ($d_j = 0$ for all states), the decision maker should be indifferent between the analyzed actions. As mentioned in connection with "null" measures, only trivial problems can be solved without any information about the probability of the states.

The decision maker's assessments of indifference values result in interval measures. For concluding dominance, the required probability estimate is that the personal probability of the states represented in F_t lies in the half-open interval $]I_t, 1]$, or stated differently, that

$$I_t < \sum_j p_j \leq 1 \quad (j \in F_t) \tag{3-10}$$

The relation between the analysis of dominance given bounded interval measures, and the indifference method, may be illustrated using either Fishburn's (1965) linear program in (2-16) or Sarin's (1978) knap-sack algorithm. Let us look at the LP.

If the constraints in (2-16) are written as:

$$p_f = I_t \tag{3-11}$$

$$\sum_j p_j = 1 \qquad (j = 1, 2, \ldots, m)$$

where p_f denotes the personal probability of the state represented by f_t as defined in (3-3), the solution of Z is zero and the decision maker should be indifferent between the two partially analyzed actions.

If the constraint

$$p_g < 1 - I_t \tag{3-12}$$

is added to the constraints in (3-11), the solution on Z is larger than zero and, hence, dominance is concluded.

If the constraints in the LP in (2-16) instead are written as:

$$I_t < \sum_j p_j \leqslant 1 \qquad (j \in F_t) \qquad (3\text{-}13)$$

$$\sum_j p_j = 1 \qquad (j = 1, 2, \ldots, m)$$

the solution on Z is larger than zero and dominance is concluded.

In summary, the indifference method presents a way to compute the widest possible interval $]I_t, 1]$ for concluding dominance. This imprecise estimate is allowed to be assigned to the largest possible number of states (all states represented in F_t), in order to maximize the possibility of a conclusive result.

What can be said about the relationship between I-estimates and ordinal estimates? If we look at the numerical illustration of the indifference method, it is clear that, regardless of the ranking order between the probabilities of the five states, action a_1 cannot be concluded to dominate a_2.

Action a_2 can only be concluded to dominate a_1 if p_1 is assessed to be larger than all the other probabilities.

Generally speaking, the ordinal estimate $p_1 \geqslant p_2 \geqslant \ldots \geqslant p_m$ implies that

$$p_1 \geqslant 1/m \qquad (3\text{-}14)$$

and that

$$p_j \leqslant 1/j \qquad \text{for all } j = 2, 3, \ldots, m \qquad (3\text{-}15)$$

From (3-14) and (3-15) we see that ordinal estimates may provide more or less imprecise probability information, depending on the number of states. However, since this imprecision is unrelated to the decision problem, it seems, at least relative to I-estimates, unlikely that ordinal estimates will supply information sufficient for a conclusive analysis of dominance in non-trivial decision settings.

Utility imprecisely measured

The definition of the indifference method was based on the assumption that the utility information is precisely measured. With one slight revision, it is valid for situations when utility is measured in bounded intervals as well. Suppose the utility information is given as

$$\Psi_{ij} \leqslant u_{ij} \leqslant \phi_{ij} \tag{3-16}$$

for all actions a_i, $i = 1, 2, \ldots, n$, and for all states s_j, $j = 1, 2, \ldots, m$.

If we want to analyze if action a_k dominates a_i as before, the difference d_j has to be redefined from $u_{kj} - u_{ij}$ to

$$d_j = \Psi_{kj} - \phi_{ij} \tag{3-17}$$

for all m states. The analysis could then be carried out as already described in (3-1) through (3-9).

Evidently, to analyze if action a_i dominates a_k, d_j has to be redefined as $\Psi_{ij} - \phi_{kj}$ (cf. Fishburn, 1964).

Eliciting indifference estimates indirectly

Implicit in the definition of the indifference method was that the responses to the indifference values I_t were elicited by direct questions. Another possibility would be to use indirect methods, such as the binary lottery technique.

Suppose that we have two lotteries: $L_1 = I_t A_j + (1-I_t)A_r$ and $L_2 = wA_j + bA_r$, where $A_j \gtrsim A_r$. L_1 yields the prize A_j with probability I_t if the states in F_t obtain. w denotes the proportion of white balls in an urn containing only white and black (b) balls (or, for example, the equivalent for a "probability wheel").

To elicit I-estimates, the proportion of white balls in the urn is set at I_t, i.e. $I_t = w/(w+b)$. If the decision maker prefers to play L_1 to L_2, it can be concluded that the personal probability of the states represented in F_t is larger than I_t and, hence, dominance can be concluded.

Personal probability estimates for unions of states

The suggested indifference method requires the decision maker to give personal probability estimates for unions of states (except in noted cases), and it is critical that they represent the sum of the independent states.

In an experimental study, Beach & Peterson (1966) found that the subjects' personal probability estimates for unions of states were approximately equal to the sum of their estimates of the states in the unions. However, they refer to other studies showing conflicting results (e.g. Lindman, 1965; Phillip, Hays & Edwards, 1966; Organist, 1964). In view of this, it may seem reasonable to start the elicitation by presenting the indifference value for the single state, i.e. the state in F_c, and then the states in F_{c-1}, and so on, in order to explicate the required summations. However, there is also a reported tendency for subjects to overestimate the probability of single states, so the problem of overestimation does not seem to be specific for unions of states (see Peterson & Beach, 1967; Teigen, 1974).

There are other, more general problems related to the decision maker's ability to give unbiased probability estimates (see e.g. Kahneman & Tversky, 1974; Bar-Hillel, 1973; Lyon & Slovic, 1976; Wyers, 1976; for reviews, see Peterson & Beach, 1967; Slovic, Fischoff & Lichtenstein, 1977; Hogart & Makridakis, 1981). I shall not pursue these matters any further here but note that the problems have to be recognized and accounted for when using the indifference method, as well as other methods for analyzing decisions under uncertainty.

Robustness of the indifference method

Robustness may be defined as how well a method meets logical requirements (Little, 1970) or how sensitive (rather: insensitive) the outcome of the analysis is to misjudgments of the probability (Isaacs, 1963; Fishburn, Murphy & Isaacs, 1968).

In Little's sense of the word, the suggested indifference method is robust in two respects. First, the indifference values are defined to lie in the open interval]0, 1[and second, they are defined to become smaller over the iterations (see appendix). Both these characteristics represent logical demands which could be placed upon the indifference method.

In the other sense, the robustness of the indifference method is not quite as easy to investigate. It could be claimed, however, that indifference values have a "built-in" robustness in that misjudgments are only likely to occur when the personal probability lies close to the point of indifference. The closer they are, the more likely are misjudgments, but at the same time, the more reasonable it is that the decision maker should be indifferent between the actions. This point of view is perhaps best conceived for the case when the world is described in a dichotomy, but the robustness becomes more evident for the case of non-dichotomous worlds. If action a_k is concluded to dominate a_i based on a misjudgment of the relation between the personal probability and the indifference value, it does not necessarily have to be the "wrong" conclusion since the indifference values are based on a "worst case" distribution. Further, even if it in fact is the "wrong" conclusion, we have already noted that the actions should be close to indifference.

APPENDIX

In this appendix two proofs shall be given in order to verify the characteristics of the suggested indifference method.

First, it will be shown that the indifference value I_t does not increase with t, i.e. that

$$I_t > I_{t+1} \qquad \text{for all } t = 1, 2, \ldots, c-1 \qquad \text{(A3-1)}$$

If (A3-1) would not hold, the decision maker could be asked if a subset of states has a probability greater than or equal to a larger set of states containing this subset. This would obviously be an irrelevant question which may encourage inconsistency in the judgments.

From (3-1), (3-3) it is seen that $f_t > 0$, and from (3-5) that $f_t < f_{t+1}$. (3-2), (3-4) imply that $g < 0$ and that g is constant for t = 1, 2,..., c. From this it is possible to conclude that

$$g/(g-f_t) > g/(g-f_{t+1}) \qquad \text{(A3-2)}$$

which is equivalent to saying that (A3-1) holds, as can be seen from (3-6).

Second, we shall show that the conclusion following (3-7) holds, i.e. if

$$\sum_{j \in F_t} p_j > I_t \qquad \text{(A3-3)}$$

for any t = 1, 2,..., c, then action a_k dominates a_i. Since the world is fully described, including the states for which $d_j = 0$, it follows from (A3-3) that

$$\sum_{j \notin F_t} p_j < 1-I_t \qquad \text{(A3-4)}$$

(3-3) implies that

$$f_t < d_j \tag{A3-5}$$

for $j \in F_t$ and $f_t \neq d_j$. Similarly, (3-4) implies that

$$g < d_j \tag{A3-6}$$

for $j \notin F_t$ and $g \neq d_j$.

Hence, the proposition following (A3-3) holds since (A3-4) through (A3-6) imply that

$$\sum_{j \notin F_t} - d_j p_j < \sum_{j \in F_t} d_j p_j \tag{A3-7}$$

which is equivalent to $E(a_i) < E(a_k)$.

As for the special case in (3-8) and (3-9) it shall be noted that the elicited information only says something about the sum of the probability of the states represented in F_t. Therefore, it must be assumed that this sum is assigned to the state represented by f_t. Hence, if (3-9) can be asserted, it follows from (A3-6) that action a_k dominates a_i.

4 A Study of the Relative Confidence in Assessments of Indifference Values

The decision maker's confidence in the probability assessments is critical to the usefulness of formal analysis of decisions in real settings. The aim of this chapter is to investigate well trained assessors' relative confidence in the probability information required by the suggested indifference method. The study consists of theoretically derived propositions as well as findings from an empirical test.

A FEW KEY TERMS

Table 4:1 summarizes a few key terms which will be used in the following pages.

Table 4:1. Key terms related to the indifference method

Term	Definition
I-value	A point within the interval [0,1] denoted as I_t
p_a	The assessor's personal probability when responding to an I-value
I-estimate	The assessor's response to an I-value, i.e. $p_a \gtreqless I_t$
I-interval	The two half-open intervals $[0,I_t[$ and $]I_t,1]$ resulting from an I-estimate

USEFUL METHODS AND THE CONFIDENCE IN PROBABILITY ASSESSMENTS

Let me try to explain what in this chapter will be meant by the usefulness of methods for analyzing decisions made under uncertainty, and how this is related to the decision maker's assessments of personal probability.

The elements of formal analysis of decisions under uncertainty and their usual procedural relationship may be illustrated in a figure:

Model

$\boxed{\text{Input} \longrightarrow \text{Method} \longrightarrow \text{Output}}$

Figure 4:1. The elements of formal analysis of decisions made under uncertainty

The input element consists of the available actions and possible states, the consequences expressed in utility, and the personal probability of the states. The method should identify the optimal action among the available actions and produce this as the output of the analysis. According to the model, the optimal action is the one which maximizes expected utility. All these elements are more or less critical to the usefulness of the analysis in real settings.

To discuss usefulness, we must start to look at the output side, i.e. the outcome of the analysis. A necessary condition for usefulness is that the decision maker believes or has confidence in this outcome. It seems reasonable that, if the decision maker does not trust the result of the analysis, it is of no value as a guide for action at that point in time and within the specific decision context.

The analysis itself could of course be useful to the decision maker as an instrument for structuring, communicating, and stressing the critical parts of the problem, and describing the input element in an explicit way. Relative to earlier suggestions for analyzing dominance, the indifference method should not be different from them in this respect except, of course, for the input of personal probability. In this chapter I will only be concerned with the usefulness of the outcome of the analysis as a solution to the choice problem.

Related to this necessary condition is the aim of the analysis: it should help the decision maker to identify the optimal action. This is not a necessary condition for usefulness, but rather a criterion secondary to the confidence criterion. Nevertheless, we may say that the usefulness increases with the number of actions which can be dominated in the analysis. Again, if none of the possible actions can be excluded by dominance, the analysis is restricted to being a vehicle for structuring the problem and explicitly stating the input element of the problem.

These two requirements describe what shall be meant by usefulness in this chapter. In summary: the outcome of the analysis is not useful if the decision maker does not have confidence in it. If the decision maker has confidence in the outcome, then the degree of usefulness depends on the number of actions which can be excluded from the list of possible actions. The most useful method is the one which produces the optimal action and, at the same time, the decision maker believes this to be the best way to proceed.

Now, what determines whether the decision maker will have confidence in the outcome of the analysis or not? It was claimed that all the elements in Figure 4:1 are critical, and to be more specific, three requirements must be met. First, the decision maker must believe or have confidence in the input element. Second, the decision maker must accept the model as being the best way for choosing among actions. Third, the decision maker must accept the method which transforms input to output. It should be stressed that I have given the terms "acceptance" and "confidence" broad meanings in this context. If all other conditions are met, a decision maker who accepts a method (or model) is willing to use it for analyzing the decision and act upon the output from the analysis. Similarly, if all other conditions are met, a decision maker who has confidence in the input is willing to act according to the output from the analysis.

In the scope of this chapter I shall assume that these three requirements are met, except for the input of the personal probability of the states. With this assumption, the factor pivotal to the decision maker's confidence in the outcome of the analysis, and accordingly, to the usefulness of the method of analysis, is the confidence which the decision maker has in the personal probability assessments.

CONFIDENCE AND PRECISION IN PROBABILITY ESTIMATES

The methods described in Chapter 2 for analyzing dominance take the probability information as given, elicited prior to the analysis as illustrated by the input-method sequence in Figure 4:1.

This information can of course be more or less precise, depending on the decision maker's knowledge about the states under consideration and ability to process and transform this knowledge into probability estimates.

The advantage of eliciting the probabilities prior to the analysis is that the decision maker should be able to give judgments conforming to the existing knowledge about the states, and thereby obtain confidence in the estimates. Thus, by allowing a "free" choice of the precision in the probability estimates, one would expect that the decision maker should have confidence in the outcome of the analysis.

However, due to the unfortunate fact that the more imprecision being accepted in the probability estimates, the less likely that the optimal action can be identified, the analyst would be tempted to "force" the assessor to be more precise than knowledge admits.

The extremes on the precision-imprecision continuum are point estimates and null estimates. A point estimate is a single number in the interval [0,1], representing the assessor's personal probability that a certain state obtains. For example: "the probability of rain is 0.32". If point estimates are given for all states of the world, I shall say that the obtained probability measure is precise. The other extreme is the "null estimate"; an estimate that any of the considered states may obtain but none with certainty.

The dilemma may be stated in extreme terms as: if point estimates are given for all states, the optimal action can easily be identified, but the decision maker is not likely to have confidence in the outcome. On the other hand, if "null" estimates are given for all states, the decision maker will have confidence in the outcome, but the optimal action can only be identified for trivial decision problems.

The "null" estimate may be viewed as the interval estimate]0,1[. Depending on knowledge, this interval can be narrowed by the assessor, up to the extremely tight bounded interval [p,p], where p is a precise (point) estimate. Thus, bounded (and open) interval estimates may be viewed as more or less imprecise. Ordinal rankings and assessments of indifference values may also be viewed as more or less imprecise probability estimates, as seen in Chapter 3.

The indifference method does not allow the decision maker a "free" estimate of the personal probability, but "forces" him/her to respond to computed points within the interval [0,1]. With this reversed order of the input-method sequence in Figure 4:1, it becomes especially interesting to study the decision maker's confidence in the required estimates.

THE MEANING OF CONFIDENCE: ASSUMPTIONS

So far I have not explained the term "confidence" but only given synonyms such as "trust" and "belief". Other used terms in the literature are "feelings of doubt" and "attitude intensity" (see e.g. Brim, 1955; Koriat, Lichtenstein & Fischoff, 1980).

Research within experimental psychology has investigated the relation between personal probability judgments and confidence ratings, and it has been suggested that confidence reflects the assessors' knowledge about which they make the estimates (see Brim, 1955; Pitz, 1967; Beach & Wise, 1969). It has also been found that people tend to be overconfident in evaluating the correctness of their knowledge and that confidence depends on the amount and strength of evidence supporting chosen assessments (Koriat, Lichtenstein & Fischoff, 1980).

Another way of discussing confidence is to relate it to the "tightness" of the personal probability distribution, i.e. the more confidence, the tighter the distribution (see Hirschleifer & Riley, 1979).

In the next section I shall propose a few elementary relations between I-estimates and other types of estimates, based upon a set of hopefully reasonable (at least with respect to well trained probability assessors) assumptions about the confidence in the estimates. My assumptions are stated below as A1, A2, A3, and A4. In the assumptions, all probability assessments concern the same state or the same subset of the states of the world $\{s_1, s_2, \ldots, s_m\}$, and to simplify the notation, I will therefore leave out the subscript on the probability estimates denoting the state.

Let $\{r^1, r^2, \ldots, r^n\}$ be the set of all discrete and precise personal probability estimates in $[0,1]$, which the probability assessor may consider.

Let p^x and p^y be two different personal probability estimates (precise or imprecise), resulting in the measures:

$$r^k \leqslant p^x \leqslant r^{k+\alpha} \tag{4-1}$$

$$r^\ell \leqslant p^y \leqslant r^{\ell+\alpha} \tag{4-2}$$

where $\{r^k, r^{k+\alpha}, r^\ell, r^{\ell+\alpha}\} \subset \{r^1, r^2, \ldots\ldots, r^n\}$, and $\alpha > 0$.

<u>Assumption A1</u>

The probability assessor perceives a judgmental uncertainty function $f(r^i)$ of the precise estimate r^i, where $\{r^i\} \in \{r^1, r^2, \ldots, r^n\}$, such that

$$\sum_{i=1}^{n} f(r^i) = 1 \tag{4-3}$$

and $f(r^i) \geqslant 0$ for all $i = 1, 2, \ldots, n$.

Assumption A2

There exists an order relation (transitive binary relation) C, where C means "is estimated with more confidence than", such that

$$p^x C p^y \qquad \text{if} \qquad (4\text{-}4)$$

$$\sum_{i=k}^{k+\alpha} f(r^i) > \sum_{i=\ell}^{\ell+\alpha} f(r^i) \qquad (4\text{-}5)$$

Assumption A3

There exists an indifference relation EC, where EC means "is estimated with equal confidence as", such that

$$p^x EC p^y \qquad \text{if} \qquad (4\text{-}6)$$

$$\sum_{i=k}^{k+\alpha} f(r^i) = \sum_{i=\ell}^{\ell+\alpha} f(r^i) \qquad (4\text{-}7)$$

EC does not necessarily have to be a transitive relation. That is, if $r^j \leq p^z \leq r^{j+\alpha}$, where $\{r^j, r^{j+\alpha}\} \subset \{r^1, r^2, \ldots, r^n\}$, and if

$$p^x EC p^y EC p^z \qquad (4\text{-}8)$$

it may be that

$$p^x C p^z \qquad (4\text{-}9)$$

because the difference in consideration of the judgmental uncertainty between p^x and p^y and between p^y and p^z may not be perceived, but a difference may be perceived when p^x is compared with p^z.

Assumption A4

If

$$p^x C p^y \tag{4-10}$$

then the assessor will choose p^x as the personal probability estimate.

Also, if

$$p^x E C p^y \tag{4-11}$$

then the assessor will be indifferent between giving p^x and p^y as the personal probability estimate.

Somewhat dependent upon A4, it will also be assumed that, if the assessor considers all elements in $\{r^1, r^2, \ldots, r^n\}$ as possible personal probability estimates, then a "freely" chosen bounded interval estimate p^x, i.e. $[r^k, r^{k+\alpha}]$, should exactly and fully contain the judgmental uncertainty distribution. By "fully" it is meant that

$$\sum_{i=k}^{k+\alpha} f(r^i) = 1 \tag{4-12}$$

By "exactly" it is meant that if

$$f(r^i) = 0 \tag{4-13}$$

then r^i should not be included in p^x, for all $i = 1, 2, \ldots, n$.

THE RELATIVE CONFIDENCE IN INDIFFERENCE ESTIMATES: PROPOSITIONS

From assumptions A1, A2, A3, and A4, it is possible to derive a few theoretical propositions regarding consistent assessments of I-values (proposition P1 below) and the confidence ranking between I-estimates (P2), I-estimates and precise estimates (P3), I-estimates and bounded interval estimates (P4), and I-estimates and ordinal estimates (P5). Formal proofs of the propositions are not presented but I will try to make each proposition seem plausible by referring to the relevant assumptions. In all propositions except P5, the probability estimates concern the same state or the same subset of the states of the world. The reader is referred to Table 4:1 for definitions of the terms related to the indifference estimates.

<u>Proposition P1</u>

a: (i) If the assessor is presented with q number of I-values such that $I_1 > I_2 > \ldots\ldots > I_q$ and

(ii) if the assessor states that $p_a > I_t$ for any $t = 1, 2, \ldots, q-1$

then it should follow that $p_a > I_{t+1}$.

b: (i) If the assessor is presented with q number of I-values such that $I_1 < I_2 < \ldots\ldots < I_q$ and

(ii) if the assessor states that $p_a < I_t$ for any $t = 1, 2, \ldots, q-1$

then it should follow that $p_a < I_{t+1}$.

Proposition P1 is implied by (4-10) and (4-4), (4-5).

Proposition P2

Let p_a^t denote the personal probability estimate when responding to the indifference value I_t.

a: (i) If $I_t > I_{t+1}$ and

(ii) if the assessor states that $p_a^t > I_t$ and $p_a^{t+1} > I_{t+1}$

then it should follow that $p_a^{t+1} C p_a^t$ or, if the difference in considered judgmental uncertainty is not perceived, that $p_a^{t+1} EC p_a^t$.

b: (i) If $I_t < I_{t+1}$ and

(ii) if the assessor states that $p_a^t < I_t$ and $p_a^{t+1} < I_{t+1}$

then it should follow that $p_a^{t+1} C p_a^t$ or, if the difference in considered judgmental uncertainty is not perceived, that $p_a^{t+1} EC p_a^t$.

Proposition P2 is implied by (4-10) and (4-4), (4-5).

Proposition P3

Assume that the assessor states a precise probability estimate $p^x = r^i$.

a: (i) If $f(r^i) < 1$

then it should follow that $p_a^t C p^x$ or, if the difference in considered judgmental uncertainty is not perceived, that $p_a^t EC p^x$, for any I_t.

b: (i) If $f(r^i) = 1$

then it should follow that $p_a^t EC p^x$ for any I_t.

Proposition P3, part a, is implied by (4-10) and (4-4), (4-5). Part b is implied by (4-10) and (4-6), (4-7).

Proposition P4

Assume that the assessor states ("freely") a bounded interval estimate p^x as defined in (4-1).

a: (i) If $I_t \leqslant r^k$ or if $I_t \geqslant r^{k+\alpha}$

then it should follow that $p_a^t ECp^x$.

b: (i) If $r^k < I_t < r^{k+\alpha}$

then it should follow that $p^x Cp_a^t$ or, if the difference in considered judgmental uncertainty is not perceived, that $p^x ECp_a^t$.

Proposition P4, part a, is implied by (4-10), (4-12) and (4-6), (4-7). Part b is implied by (4-10), (4-12) and (4-4), (4-5) and (4-6), (4-7).

Note that part a of P3 and part b of P4 imply that the judgmental uncertainty is not fully contained in the precise and bounded interval estimate, respectively.

Proposition P5

Assume that the assessor states the ordinal estimate $p_1 \geqslant p_2 \geqslant \ldots\ldots \geqslant p_m$ for all m states of the world.

Changing the notation slightly, let I^1 denote the indifference value related to state s_1 at any t, and let p_a^1 denote the assessor's response to the indifference value I^1.

a: (i) If $I^1 < 1/m$

then it should follow that $p_a^1 Cp_1$ or, if the difference in considered judgmental uncertainty is not perceived, that $p_a^1 ECp_1$.

b: (i) If $I^j > 1/j$

then it should follow that $p_a^j C p_j$ or, if the difference in considered judgmental uncertainty is not perceived, that $p_a^j ECp_j$, for $j = 2, 3, \ldots, m$.

c: (i) If the judgmental uncertainty distribution is exactly and fully contained in [1/m, 1] for p_1 and

(ii) if $I^1 > 1/m$

then it should follow that $p_1 C p_a^1$ or, if the difference in considered judgmental uncertainty is not perceived, that $p_1 ECp_a^1$.

d: (i) If the judgmental uncertainty distribution is exactly and fully contained in [0, 1/j] and

(ii) if $I^j < 1/j$

then it should follow that $p_j C p_a^j$ or, if the difference in considered judgmental uncertainty is not perceived, that $p_j ECp_a^j$, for $j = 2, 3, \ldots, m$.

Proposition P5 is implied by Proposition P4, since the ordinal estimate $p_1 \geq p_2 \geq \ldots\ldots \geq p_m$ results in the bounded interval measure [1/m, 1] for p_1 and [0, 1/j] for p_j, where $j = 2, 3, \ldots, m$. It should be noted that parts c and d of P5 depend on the most unlikely condition that the judgmental uncertainty is exactly contained in the interval measures resulting from the ordinal estimate.

GENERAL INFORMATION ABOUT THE EMPIRICAL STUDY

The purpose of the empirical study was to compare the confidence in I-estimates with three other probability assessments: point estimates, bounded interval and ordinal estimates. There were three interrelated reasons for choosing these types of estimates. First, they represent varying degrees and types of imprecise probability estimates, and the confidence in them when compared with I-estimates should be different. Second, I wanted to empirically investigate the soundness of the theoretically proposed relative confidence in I-estimates. It should be mentioned that the assumptions and propositions, stated in the two previous sections, were not fully explicated at the time of the empirical study. Because of this, the proposed relative confidence between I-estimates and ordinal estimates, as expressed in P5, was not empirically tested. Third, they are types of estimates which have been described, as seen in Chapter 2, for analyzing dominance.

For symmetry reasons, the participants in the study were divided into three equally large groups, and the analysis of the results will consequently be divided to some extent into three parts.

First, let me give some general information about the participants, design, instructions, and elicited probability estimates.

Participants

Twenty-four male persons, employed at two Swedish companies in a certain industry, participated in the study on a voluntary basis. The profession which all the participants represented is very special and, at the time of the study, less than 100 persons were working in the profession in Sweden. The available resources for the study constrained me geographically to Stockholm. In Stockholm, at the two companies' headquarters, sixteen plus eight persons, respectively, were formally working in the profession at the time of the study. All these persons participated in the study except one, who did not find time for the questions.

He was replaced by a person who had extensive training in the profession but had recently moved to another department within the company.

The participants' work aimed at identifying technical sources of possible damages (here called "the world") to their clients and to prevent, control, and reduce the effects of such damages. A part of their work consisted of identifying a specific state (event) and estimating the probability that this state would obtain. The states in the world differ in one respect: the magnitude of the consequences to the company's clients if the states obtain. To give an example: suppose the world is defined as the amount of precipitation in Stockholm tomorrow. The states of the world may then be ranging from "dry" to "flood". In this analogy, the professionals participating in the study were mainly concerned about the state called "flood". Their professional experience was, however, related to all states of the world.

The participants were academically trained (with perhaps one or two exceptions) in the probability concept and professionally trained in assessing probabilities. Using Winkler & Murphy's (1968) terminology, the participants were anticipated to be both substantively and normatively "good" in assessing personal probabilities. Hence, they were assumed to have undisputable expertise in the domain in which the assessments were made as well as sufficient knowledge about the probability concept. In this respect they may be said to represent "ideal" probability assessors from a practical decision making viewpoint. My intention was not to make statistical inference to the population of "ideal" probability assessors, since the possibility of constructing a randomized and sufficiently large sample of well trained probability assessors did not seem feasible or even meaningful. Instead, the explorative results should be viewed as indications of these well trained assessors' confidence in different personal probability assessments.

General design and instructions

The interviews took place at the two companies. Each interview was carried out individually at the participant's desk during regular working hours, and lasted between a quarter to half an hour. Due to some planning trouble, all participants in the 1st group were chosen from one of the two companies. The participants from the other company were randomly assigned to the 2nd and 3rd groups.

Generally, such a non-randomized assignment would be improper from a statistical inductive point of view. However, my aim was not to make statistical inference to the three groups as a whole or between the groups, since they were formed for different and not comparative purposes. Because of the necessarily small number of participants in each group, I will not make statistical inference to each group itself, and the possibility of the obtained results being accidental will not be statistically tested. Apart from this, it seems worth noting that there were no indications of any critical systematic differences between the participants from the two companies, such as educational background, type of work, way of working or professional experience.

The subjects were informed that there would be no attempts to evaluate the elicited estimates by comparing them with existing empirically based probability information, or in any other way test the substantive content in their assessments.

They were also informed that the research was of principal interest and was not going to have any foreseeable effects on their current way of working.

With one exception, all subjects seemed interested in the problem, and my impression was that they were reasonably motivated and took pains to give thoughtful judgments. This is perhaps not surprising since the assessment tasks were directly related to their professional interest.

Each subject was asked to choose an object in which he had the longest professional experience concerning the world. By this choice, the substantive knowledge (measured in time) would be as large as possible. As can be seen from Table 4:2, the participants had, generally speaking, extensive experience in the self-chosen object.

Table 4:2. The participants' length of experience in the self-chosen objects

Length of experience in years	NUMBER OF SUBJECTS IN THE		
	1st Group	2nd Group	3rd Group
- 1	2	-	-
1 - 5	2	2	6
5 - 10	1	3	-
10 - 20	3	2	1
20 -	-	1	1

All probability assessments concerned the self-chosen object and states within the world. The assessments did not refer to the extreme state which the subject was most familiar to work with. Take again the example with precipitation: if the extreme states are "dry" and "flood", the assessment concerned "10 mm precipitation". Even if they were not most familiar with this state, it was, to them, a both well defined and well known state.

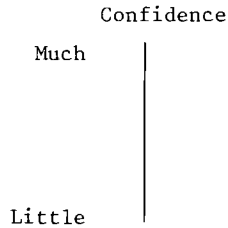
The reason for not choosing the extreme state was that I anticipated the probability to be extremely small, and the aim of the study would in that case have been difficult to achieve, which will be apparent when describing the elicited estimates later.

After the choice of the object and the definition of the state (in the 3rd Group, three different states were defined), the subject was asked to estimate his personal probability that the state(s) would obtain in the object within one year. In the 1st Group, point estimates were elicited. In the 2nd Group, bounded interval estimates were elicited. In the 3rd Group, point estimates and ordinal estimates were elicited. Using the respective estimates for reference, the subjects were asked to respond to different I-values. These I-values were chosen by me and how it was done will be described presently. Since the subjects were considered to be sufficiently trained in probability assessments, and also to have enough self interest and motivation to give truthful assessments, all estimates were elicited in a direct way by asking the subject direct questions.

To prevent as much as possible the subjects from having biased a priori feelings toward the types of probability judgments, they were not told which type I was working with in my method for analysis of decisions.

After giving the probability estimates, the subject was asked to place points on a "confidence scale" (illustrated in Figure 4:2). The points should represent the subject's relative confidence in the probability estimates.

The reason for using this relatively simple way of measuring the subjects' confidence was that I wanted to minimize the influence on the confidence of translating the internal



<u>Figure 4:2.</u> "Scale" used for confidence ranking of probability assessments

"feelings of doubt" to the explicit assessment of this doubt (cf. Koriat, Lichtenstein & Fischoff, 1980). I also viewed ordinal assessments of confidence as sufficient for the purpose of the study.

The term "confidence" was explained to the subjects as: "how much do you trust or how certain are you about the probability estimate you have just given?".

<u>Elicited probability estimates</u> (see also the appendix to this chapter)

<u>The 1st Group: Point estimates and chosen I-values</u>

Point estimates were elicited by asking each subject to give a number p within [0,1], representing his personal probability of the defined state. For example; "I would like you to give an estimate, as a point between zero and one, that the described state will obtain".

After this, four I-values were chosen: I_1, I_2, I_3, I_4, where the subscripts indicate the order of the questions. In all cases but one the I-values were computed as:

I_1: p+0.05
I_2: p+0.01
I_3: p-0.01
I_4: p-0.05

In one case, the subject gave 0.05 as p and, to avoid triviality, 0.01 was chosen as I_4. Suppose the subject gave 0.75 as the point estimate. The question related to I_1 then was, for example: "would you say that the probability of the described state is larger than, smaller than or equal to 0.80?".

The 2nd Group: Bounded interval estimates and chosen I-values

Each subject was asked to give the bounds of an interval within [0,1], representing his personal probability of the defined state. To simplify the notations, let us denote the lower and upper bound of the elicited interval as a and b, respectively (a and b correspond to r^k and $r^{k+\alpha}$, respectively, earlier defined in (4-1)).

After the elicitation of this interval, the subject was asked to respond to five I-values, all chosen within the given bounds. The I-values were chosen as follows, where the subscripts indicate the order of the questions:

I_1: b-0.01
I_2: ((a+b)/2)+0.01
I_3: (a+b)/2
I_4: ((a+b)/2)-0.01
I_5: a+0.01

When applicable, the I-values were rounded off to two decimals. To give an example of the questions, suppose that the subject estimated that the probability was somewhere between 0.2 and 0.6. The question related to I_2 then was: "would you say that the probability of the described state is larger than, smaller than or equal to 0.41?".

The 3rd Group: Ordinal estimates and chosen I-values

In order to have a numerical reference point for the choice of I-values, a point estimate within [0,1], representing the subject's personal probability of one defined state, was elicited. The subsequent I-estimates all referred to this same state.

I had been informed that the subjects were familiar with ordinal estimates of probability and therefore anticipated that they could have biased feelings toward such estimates. The computation of the I-values was not determined in advance as I wanted to choose them significantly below the point estimate but still not ask for trivial assessments of I-values. In three cases, two I-values were presented in order to avoid triviality. In all other cases, three I-values were chosen and presented.

For the ordinal estimates, the state defined for the point- and I-estimates was divided into three different states: s_1, s_2, and s_3. The subject was asked to respond to four ordinal statements regarding these states (for coherence with earlier notations, questions 1, 2, 3 are reserved for the I-values).

4. The probability of state s_1 is larger than, equal to, smaller than the probability of state s_2.

5. The probability of s_1 is larger than, equal to, smaller than the probability of s_3.

6. The probability of s_1 and s_2 is larger than, equal to, smaller than the probability of s_3.

7. The probability of s_1 and s_3 is larger than, equal to, smaller than the probability of s_2.

ANALYSIS OF THE EMPIRICAL RESULTS

In the following analysis of the empirical study, it will be assumed that the personal probability assessments were consistent over the time of elicitation, i.e. that learning did not take place between the probability estimates. As long as the information about the states was the same between the assessments, this is not an unreasonable assumption. Inconsistency over time would then only have occurred because the subjects were given more time for introspection. In the first group of the study, the participants were asked after the estimations if they would, in retrospect, change their estimates. In all cases but one they held on to their estimates.

Before turning to the empirical results on the theoretical propositions, let us look at the elicited point estimates, bounded interval estimates, I-estimates, and their relations.

The relation between point estimates and I-estimates

One purpose with the choice of I-values in the 1st Group was to test if the I-estimates would be such that they always contained the point estimate, i.e. if

$$I_3,\ I_4 < p_a < I_1,\ I_2 \qquad (4\text{-}14)$$

It may seem that the responses to the four values would be self-evident and always contain the point estimate as in (4-14). However, this order would only be obvious if the subject used the same criterion for both the assessment of p and p_a. That is, the order in (4-14) would be expected if p represented the mean, and p_a reflected the mass of the subject's judgmental uncertainty distribution, or if p represented the mode or median, and p_a reflected proportions of the uncertainty distribution. The order in (4-14) would be expected even if the uncertainty distribution was not symmetrically distributed around p.

The results summarized in Table 4:3 show that, out of 7 consistent subjects, 4 estimated that $p_a > I_1, I_2, I_3, I_4$ and 2 estimated that $p_a < I_1, I_2, I_3, I_4$. Only 1 subject gave estimates according to (4-14).

Table 4:3. I-estimates containing the point estimates in the 1st Group

Number of subjects	Estimates
1	$p_a < I_1, I_2$ and $p_a > I_3, I_4$
2	$p_a < I_1, I_2$
4	$p_a > I_3, I_4$
1	other

In 1 case, the subject gave I-intervals such that only the I_2-interval contained the point estimate. As we shall see presently, this subject gave inconsistent I-estimates.

After the assessments the subjects were asked if they found in retrospect their point estimates to be over- or under-estimations. One of the subjects who estimated that $p_a > I_1, I_2, I_3, I_4$ said that he considered his point estimate to be an underestimation. In all other cases they held on to their estimates.

Clearly, different criteria for estimates of p and p_a were used by most of the subjects. The study was not aimed and designed to investigate which criteria the subject used for the assessments of p and p_a, and it is therefore not possible to make a conclusive explanation of the results. However, from a cognitive point of view, it does not seem likely that the subject performed the difficult task of assessing the mean or expected value of his uncertainty distribution, and then not used the same criterion for the relatively easier assessment of p_a.

Instead, a more plausible explanation of the results would be that the point estimate represented the mode or median of the subject's judgmental uncertainty distribution and the assessment of p_a reflected the mass or proportion of this distribution on both sides of p. There is also a possibility that the subject provided a compromise between two or three of the central tendency measures for the point estimate, but this does not change the implications of the results on the use of precise measures for analysis of decisions under uncertainty.

The results imply that skewed uncertainty distributions were not rare instances. Only 1 of the subjects gave estimates of p_a according to (4-14) and thus indicated a symmetrical distribution at least within $\pm$ 0.05 from p. The use of point estimates when analyzing decisions under uncertainty may seem convenient but could turn out to be paradoxical: by aiming at identifying the optimal action, the solution may turn out to be non-optimal. A positive skewness would lead to a lower expected utility and a negative skewness would lead to a higher expected utility than if the judgmental uncertainty was symmetrically distributed around the mean or if the mean was given as point estimate. Skewness does not have to result in a different order of the expected utility between the analyzed actions. This clearly depends on the actual figures of the problem and how extreme the skewness is.

A more robust approach for analyzing decisions under uncertainty would be to allow for imprecise probability measures and try to identify dominance between actions. By this, it is not certain that the optimal action can be determined but at least it avoids to some extent the described problem with precise measures by instead considering proportions of the judgmental uncertainty distribution.

In the 3rd Group, 7 out of 8 subjects contained the point estimate with the I-intervals. Thus, this result seems quite contradictory to the result in the 1st Group. However, there are two reasonable explanations of this. First, the I-values chosen in the 3rd Group were all below the elicited point estimates, and we see from the result in the 1st Group that there was a tendency to contain the point estimate from below (5 out of 7 consistent subjects). Second, all I-values chosen in the 3rd Group were considerably below the point estimates relative to the 1st Group.

The relation between bounded interval estimates and I-estimates

Table 4:4 shows how the subjects in the 2nd Group responded to the five presented I-values.

Table 4:4. Assessments of the I-values in the 2nd Group

Number of subjects	Estimates
1	$p_a > I_t$ for all $t = 1 \to 5$
3	$p_a < I_t$ for all $t = 1 \to 5$
2	$p_a < I_t$ for all $t = 1 \to 4$ $p_a > I_5$
1	$p_a < I_1$ $p_a > I_t$ for all $t = 2 \to 5$
1*	$p_a < I_t$ for all $t = 1 \to 3$

* In this case it was only meaningful to present the first three I-values because of inconsistent answers which will be explained presently.

As we see from the table, 4 out of 7 consistent subjects indicated an extreme skewness of the uncertainty distribution within the bounded interval, by assessing that p_a was either below all five I-values or above all five presented I-values. The other 3 subjects indicated a slight skewness of the uncertainty distribution by estimating that p_a was either below all the first four presented I-values or above them.

This observed skewness is notable but, provided that the whole judgmental uncertainty distribution is contained in the bounded interval estimate, the use of analysis of dominance, as defined in Chapter 2, would still be valid from an expected utility standpoint. More will be said about this presently.

Consistency with propositions

Table 4:5 summarizes the number of subjects consistent with the propositions P1 through P4, relative to the number of applicable cases. As already mentioned, proposition P5 was not empirically tested.

Table 4:5. Relative number of subjects in each group consistent with the propositions

	RELATIVE NUMBER OF SUBJECTS IN THE		
Proposition	1st Group	2nd Group	3rd Group
P1	5/6	2/2	7/7
P2	8/8	7/8	8/8
P3	6/8	na	na
P4	na	2/7	na
P5	na	na	nt

na = not applicable
nt = not tested

It should be pointed out that internal consistency not necessarily implies that the estimates reflect the subject's "true beliefs" (Tversky & Kahneman, 1974; cf. Staël von Holstein, 1970). With this in mind, let us take a closer look at the results behind Table 4:5.

The 1st Group: Empirical results on P1

As already described, four I-values were presented where $I_1 > I_2 > I_3 > I_4$. In 5 out of 6 applicable cases, the subject's estimates were consistent with proposition P1. One subject estimated that $p_a > I_1$ and $p_a < I_2, I_3, I_4$ which was not consistent with P1. The other 2 subjects estimated that $p_a < I_1, I_2, I_3, I_4$ and P1 is not applicable in these cases.

The 2nd Group: Empirical results on P1

Five I-values were presented where $I_1 > I_2 > I_3 > I_4 > I_5$. In 2 out of 2 applicable cases the I-estimates were consistent with P1. As can be seen from Table 4:4, the other 6 subjects estimated in such a way that P1 is not applicable.

The 3rd Group: Empirical results on P1

Three I-values were presented where $I_1 > I_2 > I_3$, and 7 subjects estimated that p_a was larger than all the I-values. These 7 subjects were consistent with P1. In 1 case, the subject gave 0.03 as the point estimate and only two I-values were presented: 0.02 and 0.01. This subject estimated that p_a was less than both I-values and, hence, P1 is not applicable in this case.

The 1st Group: Empirical results on P2

Table 4:6 shows the confidence ranking given by the subjects who estimated that p_a was larger than two or more I-values. The numbers in the table represent the subscripts of the assessed I-values respectively. As seen, the subjects were consistent with P2.

In Table 4:6 and in the following tables, numbers written in parenthesis indicate that the subject found it impossible or not meaningful to discriminate in confidence between the corresponding estimates, i.e. the difference in consideration of the judgmental uncertainty was not perceived.

Table 4:6. Confidence ranking by subjects who estimated that p_a was larger than two or more I-values

Number of subjects	Much			CONFIDENCE			Little
1			4		3		
1	4		(3		2		1)
1		4		3		2	
1	(4		3		2		1)
1	4		3		2		1

Table 4:7 shows the confidence ranking given by the subjects who estimated that p_a was less than two or more I-values.

Table 4:7. Confidence ranking by subjects who estimated that p_a was less than two or more I-values

Number of subjects	Much			CONFIDENCE			Little
1			1		2		
1		1		2		3	
1	1		2		3		4
1		2		3		4	

We see that the confidence ranking was consistent with P2.

In summary, all subjects in the 1st Group were consistent with P2.

The 2nd Group: Empirical results on P2

Table 4:8 shows the confidence ranking given by the subjects who estimated that p_a was larger than two or more I-values.

Table 4:8. Confidence ranking by subjects who estimated that p_a was larger than two or more I-values

Number of subjects	CONFIDENCE Much				Little
1	(5	4	3	2)	
1	5	4	3	2	1

Both subjects were consistent with P2

Table 4:9 shows the confidence ranking given by the subjects who estimated that p_a was smaller than two or more I-values.

Table 4:9. Confidence ranking by subjects who estimated that p_a was smaller than two or more I-values

Number of subjects	CONFIDENCE Much				Little
2	1	2	3	4	5
2	1	2	3	4	
1	1	2	(3	4	5)
1*	3	(1		2)	

* This subject was clearly inconsistent. He explained his ranking by saying that he found it impossible to discriminate between the estimates: "They are all nothing but wild guesses". With this attitude he placed his estimates haphazardously on the confidence scale, and the I_4- and I_5-values were therefore not presented to him.

Apart from the noted exception, all subjects in the 2nd Group were consistent with P2.

The 3rd Group: Empirical results on P2

Table 4:10 shows the confidence ranking given by the subjects who estimated that p_a was larger than two or more I-values.

Table 4:10. Confidence ranking by subjects who estimated that p_a was larger than two or more I-values

Number of subjects	CONFIDENCE Much			Little
4	3		2	1
2*		2		1
1	(3		2)	1

* These subjects found the I_1- and I_2-estimates rather obvious and to prevent them from regarding the assessments as meaningless, the I_3-values were not presented.

Evidently, the estimates were consistent with P2.

As already mentioned, 1 subject gave an extremely small point estimate and was only presented with two I-values. He estimated that p_a was smaller than these two I-values and was consistent in his confidence ranking with P2.

In summary, all 8 subjects gave estimates consistent with P2.

Empirical results on P3

In the 1st Group 6 out of 8 subjects gave probability estimates and confidence rankings consistent with P3. To give an example of inconsistent assessments, 1 subject gave 0.10 as point estimate. When responding to the I_2-value, in this case 0.11, he chose the I-interval not containing the point estimate, i.e.]0.11, 1] and assessed less confidence in this I-estimate than in the point estimate. By instead choosing the I-interval [0,0.11[, he should have obtained at least as much confidence in the I-estimate as in the point estimate.

A plausible explanation of the observed inconsistency would be the effect of learning. One of two inconsistent subjects said that he found in retrospect his point estimate to be an underestimation. The other subject held on to his estimate.

Empirical results on P4

Tabel 4:11 shows the full confidence ranking given by the subjects in the 2nd Group. ab denotes the bounded interval estimate.

Table 4:11. Full confidence ranking by the subjects in the 2nd Group

Number of subjects	CONFIDENCE Much					Little
1	1	(ab	5	4	3	2)
1	ab	5	4	3	2	1
1	ab	1	2	3	4	5
2	1	2	3	4	5	ab
1	5	1	2	3	4	ab
1	1	2	(3	4	5)	ab
1	inconsistent					

As we see, only 2 out of 7 consistent subjects gave a confidence ranking consistent with P4, i.e. had more or at least as much confidence in the bounded interval estimates as in the I-estimates (all I-values were chosen within the bounds).

In all other cases, 5 out of 7, the confidence ranking was not consistent with P4.

The plausible explanation of these results would be that the inconsistent subjects' judgmental uncertainty distributions were not fully contained in the given bounded interval estimates. There is in fact a strong indication of this, in that 4 out of 7 subjects implied an extremely skewed uncertainty distribution within the bounded intervals, and that 3 out of these 4 subjects also belonged to the 5 cases which showed inconsistency with P4. Thus, it seems reasonable to say that the subjects gave too narrow bounded interval estimates, not completely reflecting their uncertainty. Similar effects of "overcertainty" or "overconfidence", especially among experts, have been reported elsewhere (see e.g. Staël von Holstein, 1970; Pickhardt & Wallace, 1974; Lyon, Fischoff & Slovic, 1978, and for a review; Slovic, Fischoff & Lichtenstein, 1977).

Even if it does not seem likely, there is a possibility that the whole judgmental uncertainty distribution was contained within the bounded interval, but that it was extremely skewed, i.e. extremely close to one of the bounds. However, even if this could be a possible explanation of the I-estimates, it is not a likely explanation of the differences between proposed and observed confidence ranking. Another explanation of the results would be that learning occurred between the different estimates.

The implications on the validity of using bounded interval estimates for analysis of dominance, without accounting for the possibility that the estimates do not reflect the assessor's whole uncertainty, would be that the approach, described in Chapter 2, could result in conclusions seeming more robust than they actually are. If not even the mean of the judgmental uncertainty distribution is contained within the bounded interval estimate, the analysis could result in conclusions not consistent with the expected utility model.

Ordinal estimates and I-estimates

Let us turn again to the assessments elicited from the subjects in the 3rd Group. First, we shall briefly look at the subjects' consistency with the applicable rules of transitivity. Second, the relative confidence between I-estimates and ordinal estimates shall be presented.

Transitive ordinal estimates

Transitivity is a requirement for an ordinal measure. Below is listed the possible combinations of ordinal estimates regarding the four statements (see p. 61) and the implications for transitivity that should hold because of additivity of indpendent states. p_1 denotes the personal probability of state s_1 and so on. It is assumed that p_1, p_2, $p_3 > 0$.

1. If $p_1 > p_2$ and $p_1 > p_3 \Rightarrow p_1 + p_2 > p_3$ and $p_1 + p_3 > p_2$
2. If $p_1 = p_2$ and $p_1 > p_3 \Rightarrow p_1 + p_2 > p_3$ and $p_1 + p_3 > p_2$
3. If $p_1 > p_2$ and $p_1 = p_3 \Rightarrow p_1 + p_2 > p_3$ and $p_1 + p_3 > p_2$
4. If $p_1 = p_2$ and $p_1 = p_3 \Rightarrow p_1 + p_2 > p_3$ and $p_1 + p_3 > p_2$
5. If $p_1 < p_2$ and $p_1 = p_3 \Rightarrow p_1 + p_2 > p_3$
6. If $p_1 = p_2$ and $p_1 < p_3 \Rightarrow$ $p_1 + p_3 > p_2$
7. If $p_1 > p_2$ and $p_1 < p_3 \Rightarrow$ $p_1 + p_3 > p_2$
8. If $p_1 < p_2$ and $p_1 > p_3 \Rightarrow p_1 + p_2 > p_3$

Table 4:12 below shows the number of subjects who were consistent and not consistent with the transitivity rules above.

We note that 6 out of 8 subjects gave ordinal estimates which were consistent with the applicable transitivity requirement.

Table 4:12. Number of subjects in the 3rd Group who were consistent/ not consistent with the transitivity rules

Transitivity rule number	NUMBER OF SUBJECTS WHO WHERE: Consistent	Not consistent
1	1	1
2	1	-
3	-	-
4	-	1
5	1	-
6	2	-
7	-	-
8	1	-

Relative confidence between ordinal estimates and I-estimates

Table 4:13 shows the full confidence ranking given by the subjects in the 3rd Group. As before, 1, 2, 3 correspond to the subscripts of the assessed I-values. 4, 5, 6, 7 correspond to the ordinal statements.

Table 4:13. Full confidence ranking by the subjects in the 3rd Group

Number of subjects	CONFIDENCE Much						Little
1	3	(6	7)	5	2	1	4
1	3	(2	4	5	6	7)	1
1	3	(2	7)	(5	6)	4	1
1	(3	2	5	7)	(1	6)	4
1	(4	5	6	7)	3	2	1
1	7	2	1	(5	6)	4	
1	7	(4	5	6)	2	1	
1	(1	4	5	6	7)	2	

We see that in 5 cases out of 8, the subjects had at least as much confidence in an I-estimate as in all ordinal estimates. In all these 5 cases, however, it was only the extreme I-estimate (I_3 and in 1 case, I_1) which attained at least as much confidence as in all ordinal estimates. In only 3 out of 8 cases, the extreme I-estimate received strictly more confidence than in all ordinal estimates. We also note that in 5 cases out of 8 cases, an I-estimate (I_1 and/or I_2) received the least confidence of all probability assessments in the 3rd Group.

CONCLUSION

The empirical results indicate that it was feasible to obtain I-estimates consistent with the elementary proposition P1. (Even if it was not investigated, it seems reasonable to expect difficulties, with respect to consistency between I-estimates, when the number of required I-estimates for many different subsets of states becomes large.)

The empirical data also indicate that the subjects' relative confidence in different I-estimates was consistent with the elementary proposition P2.

In P3 it was proposed that it is always possible to give an I-estimate with at least as much confidence as in a precise (point) estimate. Most of the subjects gave estimates which supported this proposition. Learning or a misunderstanding of the task was implicitly suggested to be the plausible and rational explanations of the observed inconsistencies with P3.

The empirical results indicate further that the subjects did not use the same criterion for giving precise estimates and I-estimates. It was suggested that, from a cognitive point of view, it did not seem likely that the elicited precise estimates represented the mean of the subjects' judgmental uncertainty function. Hence, the validity of the expected utility model when using precise estimates was questioned. Since the reason for eliciting precise estimates is to ensure that the analysis will be conclusive, the paradox was stated: by aiming at identifying the optimal action, the solution may turn out to be non-optimal. (Naert and Leeflang (1978) have suggested different types of questions for eliciting different measures of central tendency of a distribution. It would perhaps be interesting to empirically investigate the expediency of their suggestions with respect to judgmental uncertainty distributions.)

The proposed relative confidence between interval estimates and I-estimates, as stated in P4, was not supported by the empirical data. Most likely, the results indicate that, despite the subjects' "free" choices of the bounded intervals reflecting their uncertainty, proportions of their judgmental uncertainty were in most cases outside the given bounds. Based on this observation, it was suggested that, in most cases, the subjects gave too narrow interval estimates, not completely reflecting their uncertainty, and consequently, that the use of bounded interval estimates for analysis of dominance could turn out to be less robust than perhaps expected.

The relative confidence between ordinal estimates and I-estimates, given estimates for all states of the world, was theoretically proposed in P5. The empirical study, however, was designed to investigate the relative confidence between these types of estimates for only a partial description of the world. The empirical results indicate that, in many cases, the I-estimates had to be perceived as "easy" to make to obtain more confidence in them than in the ordinal estimates. In many cases, the more "difficult" I-estimates received less confidence than in all the elicited ordinal estimates. (These results should be viewed in the light of the subjects' prior acquaintance with ordinal estimates and the risk of leniency towards the type of estimates used by them in practice.)

APPENDIX

To illustrate how the probability estimates and the confidence assessments were elicited in the empirical study reported in this chapter, the following is a transcript of the written instruction guide used in the 1st Group. The instructions were verbally given to each subject and the guide does therefore describe more of the flow of the instructions rather than all of the actually communicated words.

1. The purpose of this experiment is to compare two methods for assessing probabilities. The methods will be compared with respect to the confidence which you have in the assessments. I will return to this in a while.

2. First of all, I would like you to choose ...(an object). I want you to choose the one in which you have the longest experience.

   Object:

   How long experience do you have in this object with respect to ...(the world)?

   Experience (in years):

3. I am now going to ask you to give your personal assessment of the probability or likeliness that a specific ...(the world) will obtain in this object within one year.

   I want to point out that I am not looking for the "right", "true", "objective" probability. Furthermore, I will not try to judge if your assessment is "right", "good", or in any other way corresponds well with empirical, historical data.

4. First, I would like you to estimate the probability that ...(a state of the world) will obtain in the object within one year. I want you to express your assessment as a single number, i.e. a point between zero and one (or, if you prefer, between zero and one hundred).

   Point estimate:

5. Would you say that the probability of exactly the same ...(object) and ...(state of the world) obtaining within one year is

   larger   smaller   equal   ....(I-value)

6. I am now going to ask you to compare your assessments with respect to your confidence in them, i.e. how much you trust the respective assessment or how certain you are about them.

   (Present the confidence scale)

7. (Go back to 5)

8. How much do you trust this assessment, i.e. how certain are you about it?

   (Present the confidence scale)

9. (Go back to 5 and 8)

10. In retrospect do you consider your first assessment, i.e. the point estimate, to be

    an overestimate   an underestimate   none of these?

5 Determining the Size of Magazine Editions: A Real-Life Application of Formal Analysis of Decisions and the Indifference Method

The aim of this chapter is to take a broader view on the usefulness of the suggested indifference method and on formal analysis of decisions under uncertainty in general, by reporting on a real-life application. The case provides, I hope, an especially interesting angle, not much earlier considered but certainly important, as several persons at different organizational levels participated in the application.

BACKGROUND

The Weekly Magazine Division of the Publishing Company issued during 1980 eight different magazines in Sweden and neighbouring countries on a weekly basis. The total weekly circulation of these eight magazines during 1980 was on the average 1286 thousand copies, of which 69.5 percent represented sales of non-subscribed copies.

During the late 1970's the circulation had stagnated, and in the late fall of 1980 the Company was facing financial difficulties. In their efforts to overcome these difficulties, the top management of the Company and the Division had, among other actions, decided to investigate the possibility of determining the number of printed and distributed copies for non-subscribed sales in a more profitable way. It goes perhaps without saying that the determination of the size of magazine editions for non-subscribed sales was very important since it in turn determined much of the Company's profit. The top management considered one of the big drains on the Company to be the cost of printed and distributed but returned unsold copies. They estimated that these costs amounted to about $5 million per year. The average number of returned copies represented 25 percent of the total number of distributed non-subscribed copies for 1980.

The problem put forth was: "Given that we want to have the best possible financial outcome, how many non-subscribed copies should be issued?".

The persons participating from the Company in the attempts to solve this problem were, at the time of the study:

- The Financial Manager of the Publishing Company who had been working in the Company for about two years. He was 35 years old and had a university degree in business administration and economics.

- The Manager of the Weekly Magazine Division who had been with the Company for about 15 years. He was 43 years old and was educated in engineering, business administration and law.

- The Distributions Manager of the Weekly Magazine Division. He had been working within the Company for 25 years and was 63 years old.

- The Model User (intended) and the person determining the size of magazine editions at the Weekly Magazine Division. He had been working 15 years with the Company. He was 38 years old and educated in business administration.

The Financial Manager, together with the Division Manager (I shall refer to them collectively as the "top management"), initiated the contact with the Economic Research Institute at the Stockholm School of Economics.

Since my aim was primarily to study the usefulness of formal analysis of decisions under uncertainty and the indifference method, and to collect information about this in specific, I did not wish to be bounded by a consulting arrangement between the Company and me. Therefore, we came to an agreement such that I would become "free" to carry out the application in the way which I regarded as proper for the research purpose, without any formal constraints on the design of the study or any restrictions on the publication of the results (except some financial data). The choice not to reveal the name of the Company was mine.

INTUITIVE APPROACH

The approach used for determining the size of magazine editions was based on available reports of past sales and the Model User's fingerspitzgefühl. Using one dimension to characterize his way of reasoning for reaching the decision, we may say that it was a heuristic procedure, based upon common sense and intuition (see Huysmans, 1970), or as Doktor & Hamilton (1973) put it:

> A propensity to reason by means of broad rules of thumb, attempting to synthesize and transfer from one experience to the next. (p. 886)

There are obvious difficulties to elicit decision makers' thought processes and to validate verbal descriptions of them (cf. Mintzberg, Raisinghani & Théoret, 1976; McArthur, 1980). I tried to elicit the Model User's considerations and thoughts by first participating in an actual procedure for reaching the decision and taking notes from his "loud thinking", and second, after a pre-test of the suggested model, participating in yet another actual decision and comparing my notes with his once again verbal description of the procedure.

Still, it was not possible to receive an exact description and explanation of how the final decision was reached, but let us at least look at the different steps of the decision and the ingredients in this procedure as elicited from the Model User. Approximately four weeks before the actual week of sales, the Model User received a draft of the magazine's front page. The outlined cover story and other main features (if outlined) of the number were registered by the Model User. All magazines had a more or less seasonal variation of sales, and the Model User considered actual sales of the magazine for the corresponding week of the previous year. In fact, for non-exceptional numbers, the reported sales in the corresponding week the previous year became the anchor of the decision. From this anchored level, smaller or larger adjustments were made depending on any significant difference between the previous year's number and the present.

The Model User adjusted the anchor if the total level of sales in the present year was different from the previous year for the specific magazine.

Returning to the rough copy of the front page, the Model User checked if the same or very similar main feature had appeared in any previous number (which frequently happened). If so, the reported sales for that number gave him an indication of the market's response to, and interest in, the feature. If sales were significantly different from a "normal" number, the anchor was adjusted.

Another step in the procedure for non-exceptional numbers was to review the present number of subscribers relative to the situation for the corresponding week of previous year. If any significant difference, the anchor was adjusted by the Model User.

At last, when the Model User had assessed the potential number of buyers by the described procedure into one, deterministically decided level of demanded copies, he added an extra amount of copies to cover the potential buyers who would not otherwise be able to buy a copy because of a skewed distribution to the local vendors. I will explain this presently.

For what might be called exceptional numbers, the described procedure was not completely carried out. If special sales promotion activities were planned for the number, the Model User was informed about the type and extent of these activities. The marketing manager of the magazine gave in these cases his/her feelings about the expected level of demand, and the Model User performed a compromise between his own beliefs about the likely level of demand and the marketing manager's beliefs.

In extreme cases, the marketing manager conferred with the Distributions Manager and they together made the decision of the size of the edition, with little involvement from the Model User. This procedure was mostly referred to as being necessary for promotional reasons.

SUGGESTED MODEL

The type of problem which the Company described has been extensively analyzed in the literature under the titles of "The Single Period Inventory Problem With Stochastic Demand" and "The Newsboy Problem" (see e.g. Hadley & Whitin, 1963; Wagner, 1969; Peterson & Silver, 1979). Several models and methods have been suggested (see Kottas & Lau, 1979; Atkinson, 1979; Lau, 1980 for recent suggestions), and some have been applied to real-life situations (see e.g. Brandes, 1967, 1971; Palsternack, 1980). Most of the models deal with the problem of determining optimal sizes of local inventories ("newsboys"), where local demand is assumed to be known in frequency distributions. As we shall see, I decided to approach the problem by analyzing how to determine the total size of non-subscribed editions to be printed and distributed to the Company's local "newsboys".

From a theoretical and methodological standpoint, it would have seemed proper to suggest a few different approaches and comparatively evaluate them with respect to their perceived usefulness for solving the problem. For practical reasons this could not be done, and the case should therefore be looked upon as an investigation of the feasibility of applying formal analysis of decisions made under uncertainty, the indifference method, and other methods, in reality, and the perceived usefulness of the suggested approach.

As a way of beginning, the reasoning behind the suggested model was outlined in a matrix:

	SD	MD	LD
SE	1	2	2
ME	3	1	2
LE	3	3	1

SD, MD, and LD denote possible levels of demand for the analyzed magazine: "Small Demand", "Middle Demand", and "Large Demand", respectively. SE, ME, and LE denote different sizes of magazine edition: "Small Edition", "Middle Edition", and "Large Edition", respectively. If, for example, there will be a Small Demand and the decision is to issue a Small Edition, demand and supply would ideally meet, i.e. $E = D$. The consequences of the decision would then be $P \cdot E$, where P denotes the net-profit from each sold copy and E denotes the total number of issued copies. These ideal states of affairs are identified by the number 1 in the matrix above. If $E>D$, i.e. the issued edition is larger than demand, the consequences would be $P \cdot D - CR(E-D)$, where CR denotes the cost of printed and distributed but unsold copies. The 3's in the matrix above represent these situations. If $D > E$, the consequences would be $P \cdot E - CS(D-E)$, where CS denotes the cost of shortage (opportunity cost). These consequences are identified by the number 2 in the matrix.

This is a simplification of the real problem. The simplification lies in the assumption that the distribution of magazines to the local vendors ("newsboys") was optimal. Suppose we have three local vendors, V_1, V_2, and V_3, and that the local demand at these vendors is as illustrated in the figure below.

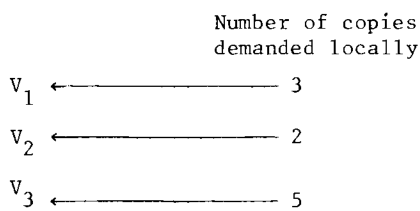


Figure 5:1. Example of demand at local vendors

For an ex post optimal distribution, the company should distribute 3, 2, and 5 copies to the three vendors V_1, V_2, and V_3, respectively. Ex ante, it is of course extremely difficult to meet local demand in this way. Instead, some vendors experience a shortage and others an excess of copies.

The next step was therefore to reduce the assumption of an ex post optimal distribution. The difference between an optimal and a non-optimal distribution may perhaps best be conceived in a figure:

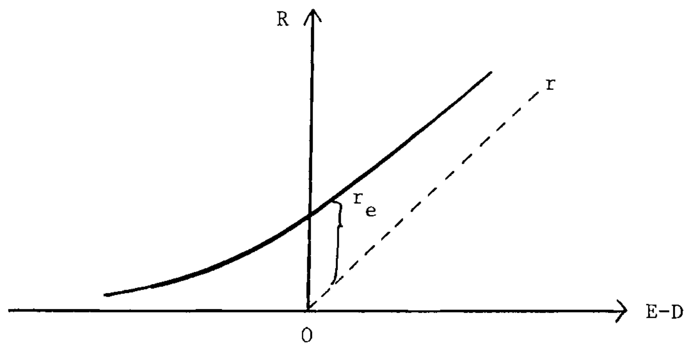


Figure 5:2. Relationship between number of demanded (D), issued (E), and returned (R) copies

The number of returned copies, given an ex post optimal distribution to the local vendors, is E-D for the case when $E > D$, and zero when $E = D$ and $D > E$. The number of returned copies given this nice condition is denoted by r in Figure 5:2.

Due to a skewed distribution, an additional return, identified as r_e in the figur, will occur. The sum of r and r_e is the total number of returned copies; R in the figure.

Given R and the decision variable E, we may state the logical relationship between shortage, level of demand, size of edition and number of returned copies as: $S=D-(E-R)$, where S is the total number of copies which is short at the local vendors. Obviously, S must be non-negative.

Also by definition, the total number of sold copies, F, is equal to E-R.

The consequences of a decided size of edition may then be expressed as:

$$P(E-R) - CR \cdot R - CS(D-(E-R)) \tag{5-1}$$

which may be rewritten as:

$$P \cdot F - CR \cdot R - CS \cdot S \tag{5-2}$$

Let us denote the sizes of editions for non-subscribed sales as the finite set $\{a_1, a_2, \ldots, a_n\}$, the levels of demand as the finite set $\{s_1, s_2, \ldots, s_m\}$, the personal probability of the levels of demand as $\{p_1, p_2, \ldots, p_m\}$, and the utility of the consequences in (5-2) of issuing edition a_i if demand s_j obtains as u_{ij}. The model which I suggested to the Company was to find the size of edition maximizing the expected utility, i.e.

$$\max_i \sum_j p_j u_{ij} \tag{5-3}$$

The model was presented to the participants who all said that, at face value, it seemed to be a proper but perhaps unconventional description of the problem. Especially the introduced uncertainty element was perceived as unusual which I shall comment on later.

However, they wanted "hard facts" that the decisions made with the help of the model would be more profitable than the decisions made in the usual, intuitive way. I will return to this request later when discussing the experience from the application. In order to provide these "facts", a test of the model for four different magazines was conducted during November and December in 1980.

USED METHODS FOR THE TEST

For conducting the test of the model in (5-3), I needed to obtain estimates of the total number of returned copies, the Company's utility function, the profit and costs, the possible levels of demand and editions, and the Model User's personal probability of the different levels of demand.

Estimates of profit and costs

The profit from each sold copy was determined as the net-selling price minus the variable cost of production and distribution. The cost of returned copies was estimated as the variable cost of production and distribution. There was no direct cost of returning copies, but it was included in the independent distributor's price for distribution. The cost of shortage was considered as an opportunity cost, i.e. the profit which the Company would have made from the demanded but non-supplied copy, and was therefore estimated as the profit component above.

The profit and costs were estimated for an interval of possible sizes of editions for each tested magazine and on a unit basis.

Estimates of total amount of returned copies

From historical data it would seem possible to estimate the number of returned copies for different levels of demand and editions. Unfortunately, it is of no help for determining how many copies that are actually demanded. Thus, by knowing the relationship between E and R, it would not have been any help for estimating R as a function of E-D. Instead, I used the Model User's knowledge and experience by eliciting his personal estimates of R. He was asked direct questions as in the following example:

Q: Suppose the total number of demanded copies is 150 thousand. How many copies would you expect to be returned if we issued an edition of 150 thousand copies?

A. Well, I would say about 35 thousand.

Q: Let us still suppose that the total number of demanded copies is 150 thousand. How many copies would you then expect to be returned if we issued an edition of 155 thousand copies?

A. I say that of these additional 5 thousand copies, we would only sell 1 thousand and the rest would be returned.

Q: You are saying that the total number of returned copies would increase to 39 thousand copies?

A: Yes.

The questions were asked for D equal to normal (as specified by the Model User) demand for each magazine. E was chosen above and below D at levels corresponding to the levels of E which the Model User considered. The ranges of (E-D), for which estimates of R were elicited, were determined by either when the increase in R was estimated to be equal to the increase of E (that is, all added copies were estimated to be returned) or when the range of (E-D) corresponded to the possibly considered difference between E and D.

First, the estimates were elicited before the test. The elicitation took about one hour for all tested magazines. After this, the estimates were plotted as a graph and presented to the Model User who, after some minor revisions, confirmed the estimates as being his true beliefs. After a pre-test consisting of four tested numbers, the estimates of R were once again elicited from the Model User in the same way as already described. It turned out that these estimates did not differ from the estimates used for the pre-test.

As I mentioned above, four different magazines (A, B, C, D) were tested, and the Model User's estimates of R were elicited for each magazine in the way described.

Estimates of the Company's utility function

At the time of the study, three persons were viewed as the policy makers of the Company in all respects except the editorial policy. Two of them were the Financial Manager and the Division Manager participating in the application. I decided to elicit their utility functions independently and, if different from each other, to try and find consensus between them or otherwise use the average of their judgments.

For all tested magazines, the consequences in (5-1) or (5-2) were estimated to lie somewhere between \$5 thousand and \$100 thousand and, thus, I wanted to estimate their utility function between these extreme monetary values. The device used for the elicitation was binary investments. The reason for using investments instead of lotteries as described in Chapter 2, was that I anticipated investments to be more realistic and familiar to the participants.

The investments were presented as:

> Let us say that you have two investment alternatives: i_1 and i_2. If you choose i_1, the Company will receive a net profit of ..... (+A) with the probability of 0.5 and zero profit with the same probability. If you choose i_2, the Company will receive, at the same point in time as in i_1, a profit of X with certainty. What does X have to be if you should feel indifferent between the two alternatives?

I started to elicit the Financial Manager's utility function with a number of investment choices with +A ranging from \$10 thousand to \$250 thousand in intervals of \$10 thousand. The results were that X = 0.9EV, where EV denotes the expected monetary value of i_1.

Quite independently and without knowing the Financial Manager's answers, the Division Manager was asked the same questions (as a matter of fact, it was the Financial Manager who asked the questions while I was taking notes). Surprisingly, it turned out that also the Division Manager wanted X to be equal to 0.9EV. Thus, it was possible to construct a utility function, identical for them both.

Since the function was linear in the relevant interval, I used the monetary values to express the utility of the consequences in (5-2).

Estimates of possible levels of demand and edition

During about six weeks I tested the model for nineteen different numbers of the four magazines. The Model User determined the actual number of copies to be issued in his usual way on Fridays and Tuesdays for magazines A, B, and C, D respectively. The days before I asked the Model User for his estimates of the intervals within which the demand was certain to fall, and the different sizes of editions which he considered as possible. For example, he said that the demand would certainly turn out to be somewhere between 175 and 220 thousand copies and that he considered it as reasonable that the edition should be somewhere in this interval as well (the additional copies compensating the skewed distribution excluded). He was then asked to divide these intervals into discrete levels in any way he wished. For magazines A, B, and C he wanted to consider both demand and edition as points of 5 thousand, e.g. 175, 180, 185,... etc. For magazine D he was used to consider points of 2 thousand. The reason for using this "dirty" approach was that I wanted to "copy" the Model User's way of reasoning and make it easier to understand and make the required judgments in a less time-consuming way. Also, it was convenient from a computational point of view.

After receiving this information I ran a small computer program to compute the consequences (in utility = monetary values) of each edition given the possible levels of demand. The interval of alternative editions was extended to include the extra number of copies, usually added to compensate for the skewed distribution. The result from this information collection was summarized in a matrix, exemplified in Figure 5:3.

		DEMAND									
		175	180	185	190	195	200	205	210	215	220
E	175	274	288	302	315	329	343	350	356	363	362
D I	180	268	289	303	316	330	344	358	365	371	378
T	185	262	283	304	318	331	345	359	373		
I O	190	256	277	298	319	333	346				
N	195	250	271	292							

Figure 5:3. Part of a matrix describing the consequences (in utility) of different levels of demand and sizes of edition (fictitious data)

After this, I met with the Model User and presented the matrix. He had then already determined the size of edition and the reason for making the "model decisions" after the "non-model decisions" was that I wanted him to collect the required information and make the necessary introspection for reaching the "non-model decision", and to use this in his judgments of the demand.

Estimates of the personal probability of the levels of demand

Indifference values were computed between pair-wise compared edition alternatives. For example, to analyze if an edition of 175 thousand copies dominated an edition of 180 thousand copies in the matrix above, the indifference value 0.73, i.e. (-16/(-16-6)), was computed. The Model User was then asked if his personal probability of the level of demand being 175 thousand copies was larger than 0.73. If so, we concluded dominance and deleted the edition alternative of 180 thousand copies from further consideration. If the answer was no, he was asked if his personal probability of the level of demand being different (larger) than 175 thousand copies was larger than 0.86, i.e. (-6/(-6-1)). If so, we concluded dominance and deleted the 175 thousand edition alternative from further consideration. If the answer was no again, he was asked if the probability of the level of demand being larger than 205 thousand was larger than 0.43, i.e. (-6/(-6-8)). If yes, we deleted the 175 thousand alternative and so on (see Chapter 3 for a definition of the method).

For the first four tested numbers, the Model User's responses to the indifference values were elicited indirectly by the binary lottery technique described in Chapter 3. For the rest of the test, his judgments were elicited by direct questions for reasons which will be explained later.

The first four tested numbers were viewed as a pre-test of the model and for further refinement and confirmation of the estimated variables as well as for training in the assessment of indifference values.

ANALYSIS AND DISCUSSION OF THE PERCEIVED USEFULNESS OF THE SUGGESTED APPROACH

The study in Chapter 4 aimed at investigating the usefulness of the indifference method in a restricted way, by testing the assessors' relative confidence in the method's characteristic requirement on input. In the following pages, based upon interviews with the participants and observations during the application, I will discuss and analyze the usefulness of formal analysis of decisions and the indifference method in a more extensive way. In the last sections of this chapter, a few general observations will be discussed.

Designing research on real-life applications presents a dilemma, since real-life settings restrict the possibility of determining the factors to be studied and controlled a priori. For practical reasons, the application must in most cases be aimed at being successful, requiring the researcher (often acting as a consultant) to be sensitive and adaptive to anticipated and observed effects of different factors. The difficulty in interpreting the results from such studies is evident.

On the other hand, real-life studies are important in order to shed light on different aspects of the usefulness of models and methods in more realistic environments than in perhaps controlled but often artificial settings (cf. Winkler & Murphy, 1973).

Another quite different aspect of research on real-life applications is the ethical implications of the management scientist's role as interventionist, and the responsibility for suggestions on changes in the decision making and decision making processes. A satisfactory treatment of this important issue would require a far-reaching discussion and, because of the limited scope of this study, I shall refrain from this. The reader is referred to Duncan & Zaltman (1975) and Howard (1980).

There is a considerable amount of factors influencing understanding, acceptance, confidence, and actual use of Management Science models in general. Some of these factors are related to, for example, organizational conditions (Brown, 1970; Rubenstein, et al., 1976), characteristics of the management scientist and managers involved in the application and implementation (Radner, Rubenstein & Tansik, 1970; Hammond, 1974), the working relationship and understanding between the management scientist and the manager (Churchman & Schainblatt, 1965; Narasimhan & Schroeder, 1979), the manager's cognitive (decision) style (Doktor & Hamilton, 1973; Huysman, 1970; Henderson & Nutt, 1980), characteristics of the model itself (Little, 1970; Souder, et al., 1975), the type of analyzed problem (Harvey, 1970; Huysman, 1975). A survey and discussion of recent research in this field is provided by Hildebrandt (1980).

Many of these factors, as well as situational variables, influenced to a more and less extent the participants' views on the usefulness of the suggested model and methods. Some of the factors were directly related to the characteristics of the model and the methods, others indirectly or not at all related. Some of them were anticipated by me to be important, others turned out to be important during the application.

It seemed quite clear though that the influence of the indifference method on the perceived usefulness was overshadowed by the participants' more basic concerns about the usefulness of formalizing and systematizing the decision problem, and explicitly introducing uncertainty and financial aspects into the decision. An exception was the Model User who, at least during the test, was faced with the practical problems of explicating his judgments of the uncertain demand. During the test the feasibility of eliciting indifference estimates and the usefulness of the indifference method was indicated both by the Model User's ability to provide relatively trusty indifference estimates and by the number of conclusive analyses.

In accordance with the discussion of usefulness in Chapter 4, the immediately following sections will contain an analysis of the real-life application with respect to my observations on the participants' confidence in the output, acceptance of the model, confidence in the input, and acceptance of the methods. As in Chapter 4, the terms "acceptance" and "confidence" will be broadly interpreted as the willingness to act upon a model/method and input/output, respectively.

It should be pointed out that the evaluation of the estimated financial outcome of the model was only presented to the participants twice: a partial evaluation during the test and a complete evaluation after the test. Ideally, I would have preferred to supply the participants with successive feed-back about the results in order to enable them to assess the performance of the model during the test. However, all necessary reports of sales upon which the financial evaluation was based were not available to me before the test was completed.

Confidence in the output

An interesting aspect of the conducted test was to see if the Model User had such a degree of confidence in the output that he adopted the model decisions. If so, we would expect convergence of the non-model editions towards the model editions during the test. The Model User had, after the pre-test, a fairly good understanding of the model and methods, which must be regarded as a pre-requisite for confidence in the output. If he would not have been able to assess the expediency of the model and methods, then acceptance and, hence, confidence in the output, could hardly have been expected.

Table 5:1 shows the relative difference (in edition sizes) between the non-model decisions and model decisions for all nineteen tested numbers and for each tested magazine.

Table 5:1. Relative differences between model edition and non-model edition for the tested numbers (computed as: 100 (model edition - non-model edition)/non-model edition)

	TESTED NUMBERS				
Magazine	1	2	3	4	5
A	-23	na	-25	-22	-21
B	-19	-29	-17	-25	-21
C	-12	-14	-22	-13	-18
D	nc	nc	-15	-13	-23

na = not available
nc = not conclusive

If the Model User had sufficient confidence in the output, we would expect a decreasing difference over time, between the non-model decisions and model decisions. It is noted from Table 5:1 that the difference did not significantly decrease as the test proceeded. If we, for example, measure the difference for the first and fifth tested numbers, we see that for all magazines except A, the difference is larger for the fifth tested number than for the first.

One plausible explanation of this result may be that the model decisions were such that the difference did not decrease. That is, the size of the editions determined with the help of the model decreased, but the non-model decisions did not decrease to the same extent.

Another explanation may be that the Model User was not able to anticipate what the model decisions would be, because they changed in a seemingly unpredictable way.

Every number is more or less unique. Some numbers in the test were what the Model User called "normal", meaning that they did not contain anything exceptional from the magazine's usual content and that no special marketing activities were planned for the numbers. Therefore, we may instead look at the non-model decisions reached for normal numbers. Since all model decisions were consistently below the non-model decisions, we would expect, if the Model User had sufficient confidence in the output, the number of issued copies to decrease over time.

Table 5:2 shows the non-model editions for normal numbers in absolute figures and the difference between them. For example, tested numbers 1 and 5 of magazine A were perceived as similar. The decisions reached without the model were 215 thousand for both numbers and, hence, there was no difference between them during the test.

Table 5:2. Non-model editions and differences over time for normal numbers (figures in thousand)

	TESTED NUMBERS					
Magazine	1	2	3	4	5	Difference
A	215				215	0
A			235	235		0
B	105	105				0
B			125	125	120	-5
C	210	210				0
D	85	85				0
D			105		95	-10

From Table 5:2 we note that for similar numbers there was a decrease in two out of seven "normal" observations (magazines B, D), but that: (1) these decreases occurred after many tested numbers, in fact, in the last tested numbers, (2) the decreases were relatively small, and (3) they ocurred only once for the two magazines.

When asked, the Model User said that he did not consciously adopt the lower model decisions, and from the observations it is not possible to conclude otherwise. Plausible explanations of this non-adoptive behavior are that he (1) was inert to change his usual way of determining the size of the editions, (2) was willing to adopt the model editions but that he was influenced by his superior, the Distributions Manager, who expressed his doubts about the usefulness of the model and the difficulties to issue small editions from a competitive standpoint and with respect to the motivation of the editorial staff of the magazines, (3) did not sufficiently understand the model and/or methods, and (4) did not accept the model and/or methods, and/or did not have confidence in the input.

The other interesting side of this aspect is to analyze if the Model User did not trust the output from the test to such a degree that he also did not want others to trust it. For example, if he controlled the model decisions such that the difference between model and non-model would be as large as possible. Loosely speaking, to control the output such that "bad" decisions were reached with the model. If so, we would expect divergence between the non-model decisions and model decisions. From Table 5:1 we see that there is in fact a slight divergence between both normal and not normal numbers. From Table 5:2 we note that the non-model decisions did not increase between "normal" numbers during the test, which otherwise could have explained some of the observed divergence. Therefore, two other explanations are plausible: (1) the Model User controlled the model output so as to make the difference between model and non-model as large as possible but within some limits, since many probability assessments were obvious or evident because of a felt need from the Model User to be consistent. However, the Model User claimed that he did not quite know how the output could be controlled and, even if he did, he said that he did not understand why it should be controlled, (2) the Model User's assessments of the probability of demand changed between the numbers, even between "normal" numbers, due to further refinement and understanding of the probability judgments required, and also further introspection of the potential demand for each number. This seems to be the most plausible explanation of the implied divergence.

Acceptance of the model

It may perhaps not have been surprising if the Model User had been reluctant to my attempts to model his decisions. If he had seen the model as a competitor to his position, it would probably have been difficult to have him describe his way of solving the problem and to participate actively in the test (cf. McArthur, 1980).

He showed signs of reluctancy in the beginning of my attempts. For example, he did not give an exhaustive description of how he made the decisions, and said that he did not have much time to spare for the test (in the beginning, each tested number demanded about half an hour of his attention and, at the end of the test, a quarter of an hour).

As the test proceeded, his initial reluctancy softened and he gave more time for the test, answered questions about his usual approach for reaching the decisions and showed signs of trying to fully understand the model. This change of attitude was most likely due to both the fact that the Distributions Manager showed interest in the model (who in turn was influenced by the top management's interest), and that there was a certain chance that the model could be put in use, and the Model User would in that case be operating it. Furthermore, the Model User's understanding of the model increased during the test, and he saw that it could be useful for deciding editions when the potential demand was "extremely uncertain", as he put it.

Acceptance of the introduced uncertainty

At the outset of the problem formulation and through the model presentation, the participants (except the Model User) indicated that they did not view the introduced uncertainty element, i.e. uncertain demand, as critical to the problem. Instead, my impression was that they conceived, or had expected a treatment of the problem in a deterministic fashion. There are of course numerous plausible reasons for preferring certainty to uncertainty, and I cannot pinpoint the most likely explanations in this case. It has been implied by, for example, Mathes (1969), Little (1970), and Hull (1980) that managers in general tend to accept deterministic treatments more likely than stochastic treatments of matters.

Even if the uncertainty element was not appreciated, it was not rejected by the top management and the Distributions Manager. Partly, we may say that this was due to the Model User's confirmation of uncertain demand situations, partly because they wanted to see if the model could be useful and, if so, to put it strongly, it did not matter how the model was constructed.

The "historical" call for certainty had probably influenced the Model User in such a way that, when asked to assess the probability of different levels of demand, he gave the already intuitively determined demand an extremely large probability. As the test proceeded, the probability estimates were more spread on the different levels of demand. This does not necessarily imply that his uncertainty increased during the test, but more likely that it was an effect of training and that he recognized and accepted the possibility of expressing his "true beliefs".

Another way of looking at the Model User's acceptance or non-acceptance of the uncertainty component in the model would be to study if the intervals of considered levels of demand (within which the Model User was certain that the "true" demand would fall) increased or decreased during the test. Table 5:3 shows the difference in given interval lengths between the tested numbers.

Table 5:3. Relative increase(+) and decrease(-) in considered intervals of demand between the tested numbers (computed as: 100 (interval length at t+1 - interval length at t)/ interval length at t)

	TESTED NUMBERS				
Magazine	1 and 2	2 and 3	3 and 4	4 and 5	(1 and 5)
A	0	+33	-13	+29	+50
B	0	+100	0	-17	+67
C	0	-17	+80	0	+30
D	0	+100	0	-25	+50

The indicated acceptance could reflect that the Model User changed the interval lengths because the numbers were not alike. Comparing Table 5:3 with Table 5:2, we see that in only one case (magazine A; numbers 1 and 5) did the interval length increase between normal numbers. For all other tested normal numbers the interval lengths did not increase. In fact, out of the seven combinations of normal numbers, the interval lengths decreased three times and remained unchanged three times.

Thus, the observations indicate that the Model User accepted the introduced uncertainty element by spreading the personal probability estimates among the levels of demand, but that he did not accept it to such an extent that he increased the interval lengths of considered levels of demand. There are of course other possible factors not here considered, which could have influenced the Model User's treatment of uncertainty during the test.

Acceptance of the model results

My attempts to convince the participants about the usefulness and merits of the model without evaluating it financially was not successful (similar problems have been described by e.g. Grayson, 1960). Especially the Financial Manager was not prepared to assess the usefulness of the model without such an evaluation. He meant that the intuitive approach could allow the Model User to use some undefinable irrational element, making the decisions profitable, and that this irrationality was not allowed for in the model.

The importance of the quality of the model, as viewed by the participants, faced me with two problems. As emphasized by e.g. Howard (1980), Watson & Brown (1978), there is no meaning of analyzing the result of a decision a posteriori, since the judgments of the uncertain world (demand) which determine the outcome are made a priori.

When formal analysis of decisions is used for solving single choice problems, with no apparent alternative way for making the decision, this argument becomes convincing. In this application, however, a procedure for determining the sizes of the editions already existed, and it is therefore conceivable that the model decisions should show a better performance than the non-model decisions.

The other problem was a methodological one: how to estimate the outcome of the not implemented model decisions and compare this with the implemented non-model decisions. (For general suggestions, see Taylor & Iwanek, 1980.) It was pointed out to the participants that the evaluation had to be made with the assumption that the Model User's estimate of returned copies (R) was a correct description of "reality", and that the perceived validity of these estimate should determine how much confidence they should have in the results from the evaluation (cf. Ackoff, 1979).

Two weeks after the actual week of sales, the Company received a report of sales based upon a sample, which was considered to provide estimates close to the actual number of sold copies. This report contained information about the number of sold copies (F) and the number of returned copies (R). Thus, by knowing both F, R, and E for the non-model edition, it was possible to estimate the total level of demand according to the given estimates of R. From this estimate of D (demand) I was able to estimate the number of sold, returned, and short copies for the model decision. Table 5:4 shows the estimated effects of the model decisions relative to the non-model decisions for the tested numbers (all figures rounded off to the nearest integer).

As we see, the total average effect of using the model for determining the size of the editions was estimated to have

Table 5:4. Estimated effects of model decisions relative to non-model decisions (computed as: 100 (model - non-model/ non-model)). See text about the validity of the estimates.

	Non-subscribed sales	Returned copies	Net profit
Magazine A: No. 1	-7	-76	+16
2	na	na	na
3	-12	-86	+1
4	-7	-75	+11
5	-7	-72	+10
Average Magazine A	-8	-77	+10
Magazine B: No. 1	-7	-46	+4
2	-12	-60	+1
3	-6	-55	+1
4	-12	-66	+15
5	-7	-57	+3
Average Magazine B	-9	-57	+5
Magazine C: No. 1	-3	-44	+2
2	-4	-55	+2
3	-6	-57	+14
4	0	-38	+11
5	-2	-54	+9
Average Magazine C	-3	-50	+8
Magazine D: No. 1	nc	nc	nc
2	nc	nc	nc
3	-5	-46	+3
4	-1	-36	+9
5	-10	-56	+1
Average Magazine D	-7	-46	+4
Total average	-6	-58	+7

na = not available
nc = not conclusive

been a 6 percent decrease in non-subscribed sales, a 58 percent decrease in the number of returned copies, and a 7 percent increase in the net-profit (defined as the net-selling price of the sold copies minus the total cost of production and distribution for the edition).

The participants' immediate reaction to the presented evaluation was that the estimated results seemed consistently and convincingly in favor of the model. The Model User and the Distributions Manager expressed, however, their doubts about the advisability of accepting the apparently smaller model editions, considering the possible long-term effects on circulation, advertising income, and motivation of the editorial staff. The top management indicated less concern about this; before discussing implementation of the model, and long-term implications, they wanted to know how much confidence they could have in the estimated effects from the test.

Confidence in the input

Generally speaking, the participants' confidence in the input to the test was related to the perceived "degree" of subjectivity in the estimates. None of the participants showed any lack of confidence in the estimates of profit and costs (which were obtained from the Company's accounting records). They did not express any doubts about the input of utility, which could have most likely been a source of disbelief if the utility function had not been linear, or if the Financial Manager's and the Division Manager's utility functions had been different with respect to linearity/non-linearity.

The top management was mainly concerned about the Model User's estimates of returned copies (R), and how much faith they could put in them. This was perhaps not surprising since the validity of these estimates was critical to the financial evaluation of the model.

The Distributions Manager indicated less confidence in the Model User's personal probability estimates and the estimated levels of possible demand than in the estimates of R. His relative disbelief in the Model User's personal probability estimates seemed not related to the precision or imprecision of the estimates, but rather to the notion of treating demand as uncertain in the first place.

The Model User was only explicit about disbelief in the input of personal probability estimates. In four cases out of nineteen tested numbers, the indifference method did not lead to one conclusive decision. In two of these four cases I asked the Model User to give point estimates of his personal probability in order to identify one decision. He said that he was extremely uncertain about these estimates, and that he did not trust them very much. (The sum of the point estimates for all possible levels of demand exceeded in both cases 1 and was close to 2. Even with revisions and further introspection it was not feasible to reduce the sum of the estimates to exactly obtain 1.) Relative to the elicited point estimates, he showed considerably more confidence in the indifference estimates.

Acceptance of the methods

It seemed quite clear that the indirect method (binary lotteries) for assessing indifference values was useful in the beginning of the test before the Model User had a sufficient understanding of how to express his judgments. When this level of understanding was reached, he regarded the indirect questions as too time-consuming and preferred instead to answer direct questions. In effect, the indirect method was a contributing factor to the Model User's understanding and acceptance of the indifference method. Most likely, the usefulness of the indirect method in this sense was not related to the indifference method per se, but to his acceptance of formalizing his judgments of the uncertain demand into personal probability estimates in general.

An intriguing aspect of the indifference method was that the Model User showed signs of not accepting the method (or at least confusion about it) when the indifference estimates were perceived as very easy, i.e. when the indifference values were far away from his "best guesses". In some way he expected the required estimates to be "difficult".

Related to the participants' acceptance of the methods was the aspect of quickness in obtaining estimates (data) for the test. My experience corresponds well with Lee's (1971): "Collecting data (information) is usually thought to be a chore and an expense rather than a joy" (p. 255). All participants in the application seemed to appreciate that, once the model was constructed and accepted for a test, the required data should be quickly collected and, at that point in time, not much attention was given the fact that the data were "dirty". They considered it to be more fruitful to test the model quickly than to collect data in a large, clean, and time-consuming way. The Distributions Manager pointed out that he had experience with consultants who had collected data for a model for such a long time that all parties eventually lost their interest in the model.

The participants viewed the suggested approach as unconventional and especially the treatment of R. Even if they agreed that the suggestion seemed logically sound as a whole, the method used for obtaining the estimates of R was only accepted for the test and not as a basis for assessing the overall usefulness of the approach after the test.

An alternative method for measuring R would have been to conduct a field experiment, which would probably have resulted in more confidence in the estimates and acceptance of the method also after the test. On the other hand, I anticipated that the time requirements for such an investigation would reduce the participants' interest in the study, and hence, reduce the usefulness of the suggested approach in this sense.

A FEW GENERAL OBSERVATIONS ON APPLYING FORMAL MODELS TO REAL-LIFE

Little's (1970) article on characteristics of "good" models and Churchman & Schainblatt's (1965) article on the relationship between managers and management scientists have been most influential pieces of research on implementation of Management Science models. Based upon observations from the real-life study, a few comments about these aspects of usefulness will be provided in the following two sections.

Characteristics of a useful model

After the pre-test I asked the Model User, Distributions Manager, and Division Manager independently to give a rank order regarding the importance of eleven characteristics of any model handling the Company's problem of determining weekly magazine editions. The used characteristics have been explicitly and implicitly proposed by Little (1970) and others, as being influential on the perceived usefulness of formal models in practice. The purpose of the ranking was to investigate if the participants had different views on the importance of the characteristics.

Table 5:5 shows the obtained ranking: 1 denotes the least important and 11 the most important characteristics, as viewed by the respective participant. The rankings clearly

Table 5:5. Ranking of model characteristics. 1 denotes the least important and 11 the most important characteristics as viewed by three of the participants.

Characteristics	Model User	Distri-butions Manager	Division Manager
A: The model includes all information usually considered	5	6	8
B: The model's construction is simple	10	5	2
C: The model is easy to understand	2	4	5
D: It is possible to control the model such that any solution may be obtained	4	1	1
E: Minor misjudgments do not influence the result	9	8	9
F: The model is adaptive to new situations	6	3	10
G: It is not time consuming to operate the model	11	10	6
H: The outcome of the model is the optimal decision	8	11	11
I: The judgments are easy to make	7	9	7
J: The outcome feels intuitively right	3	7	3
K: It is possible for others to see and understand how the result has been obtained	1	2	4

indicate that the participants had different views on the importance of many model characteristics, which should have interesting implications for successful implementation of models in practical settings. It is of course not possible to draw any general conclusions from the results because of the specificness of the analyzed decision problem, the participants, the organization, etc. Bearing these limitations in mind, let us briefly look at a few of the ranked characteristics.

Direct and indirect value of a model

Little (1970) has suggested that useful models should provide optimal results, at least in a first stage ("embryonic") sense. Others have stressed that the value of a model is primarily indirect, i.e. it should serve as a mean for improved communication between members of the organization (see Watson & Brown, 1978).

All three participants' rankings of the characteristics reflecting the direct (characteristic H) and indirect (K) value of a model, indicate that the optimization aspect was regarded as considerably more important than the communication aspect. It seems reasonable to expect, however, that these rankings would have been different if the decision problem had been more unstructured.

Time requirements for operating a model

Little's (1970) suggestions imply that a useful model should enable the user to perform quick analyses. According to Hammond (1975), this aspect is often overlooked by model builders and is therefore a potential source for unsuccessful implementation.

All three participants had at the time of the ranking almost the same information about the time requirements for operating the suggested model, but only the Model User had practical experience from the pre-test. He and the Distributions Manager viewed other tasks, assigned to the Model User, as strong competitors to the task of determining the size of editions. Hence, they considered the time-for-analysis characteristic (G) among the most important. The

Division Manager, however, looked upon this as a relatively less important aspect and pointed out that, if the use of a model could increase the profitability of the editions, it would be a matter of redirecting time and effort toward this more rewarding activity.

Simplicity and ease of understanding the model

According to Little (1970), a useful model should be simple in such a way that unimportant phenomena should be left out. The primary reason for simplicity is, again according to Little, that it promotes understanding.

There is a clear difference between the Model User's and Division Manager's ranking of the characteristic describing simplicity of a model (B). It is also obvious that the Model User did not regard this characteristic as positively correlated with characteristic C: ease of understanding the model. A plausible explanation of these rankings would be that the Model User's ranking of simplicity reflected his concern about the time requirements for operating the model, and that a simple model would imply a quicker handling. The Division Manager regarded it as relatively unimportant that the model should be simple, which seems consistent with his view of having a model for obtaining optimal decisions, i.e. he did not want to trade-off optimality for simplicity. The Distributions Manager would lie somewhere between these two views.

Relationship between user/manager and researcher

When studying the perceived usefulness of a model and/or method in real-life settings, it seems clear that the results are influenced by factors not related to the model and method per se, but rather to successful implementation in general. One such important factor is the relationship between user/manager and researcher.

Churchman & Schainblatt (1965) have suggested a classification of different positions to the problem of effective relationship between managers and researchers. Depending on if it is presupposed that the manager understands the researcher, i.e. reacts efficiently to what the researcher is trying to do (A), the researcher understands the manager (B), and the corresponding negations (A', B'), the positions may be summarized in a matrix:

	B	B'
A	Mutual understanding	Communication
A'	Persuasion	Separate functions

On principal grounds, Churchman & Schainblatt argue that, for a successful union of manager and researcher, the position of mutual understanding must be taken. This has also been suggested on empirical grounds (see Dyckman, 1967; Duncan, 1974).

Related to the positions are three different concepts of model building (see e.g. Schultz & Slevin, 1975):

1. Traditional model building, meaning that the model is constructed independently from the user and organization. Referring to Churchman & Schainblatt's classification, this concept would seem to correspond to a position of separate functions in the model building phase.

2. Evolutionary model building, characterized by informal interaction between the user and researcher in such a way that successively more complexity is introduced into the model. Adoption of this approach seems to imply a position of communication.

3. Behavioral model building, emphasizing learning and formal interaction, with the purpose to construct a model which is adjusted to behavioral characteristics of the user/organization. To suggest this approach would seem to also suggest that mutual understanding is necessary for successful implementation.

Schultz & Slevin (1975) imply in their discussion that behavioral model building is the appropriate way to increase knowledge about behavioral aspects of the user/organization, and that it presumably would produce models with high organizational validity, i.e. high compatibility with the user/organization.

Since the primary aim of the study reported in this chapter was to investigate the usefulness of a specific model and method, the suggested approach for determining weekly magazine editions was constructed independently from the Model User and participating managers. Even if informal interaction existed between the Model User and me (especially during the test), it did not have any significant influence on the methods used for the test.

In other words, the position of separate functions was taken in all phases of the study. That is, the objective was not to reach understanding in such a way that the Model User should necessarily accept the suggestions and/or that the suggestions should necessarily match the Model User. Understanding, in the sense of grasping, was, however, considered as important for an assessment of the usefulness of the model and the indifference method. The test, which was intended to provide a basis for this assessment, was possible to carry out mainly due to the top management's strong awareness of a gap between the intuitive way of determining the size of editions and the more systematic way in which they regarded the decisions should be made, and their strong motivation for trying to close this gap. From what has been said earlier about the participants' confidence in the input and output, and acceptance of the model and methods, it seems likely that the position of separate functions influenced the perceived usefulness of the approach and especially the method for obtaining estimates of R.

CONCLUSION

The reported real-life application shows that it was feasible to approach the problem of determining the size of weekly magazine editions, by using a formal analysis including the indifference method.

With respect to the perceived usefulness of the suggested approach, the observations indicate that the participants' concerns about the model and methods were overshadowed by their concerns about the expediency of systematizing and formalizing the decision.

The Model User's intuitively reached decisions indicate that he did not, during a test of the suggested approach, accept the model and/or methods, and/or did not have confidence in the input, to such an extent that he had enough confidence in the output for adoption of the model editions.

The observations indicate that the indifference method was useful, in the sense that the Model User showed relatively much confidence in the elicited indifference estimates, and that the analyses were conclusive in most cases.

After the test, the perceived usefulness of the approach seemed strongly related to the estimated effects of using the model and the participants' confidence in these estimates. This, in turn, seemed dependent upon the participants' confidence in the Model User's personal estimates of the relation between the size of editions, demand, and the number of returned copies.

A general observation shows that the participants had in some important respects different views on the characteristics of a "good" model for solving the problem. A general remark about different ways of understanding between the manager (participant) and the researcher implies that the observations should be viewed in the light of the seemingly necessary position taken by the researcher conducting real-life studies of specific models and/or methods.

EPILOGUE

When this is written in September 1981, an analysis is under way to investigate the validity of the estimated effects of using the model (as presented in Table 5:4). A small field experiment has been conducted, and the obtained data indicate a reasonable correspondence with the Model User's personal estimates of R, which were used in the test. It now seems that the participants from the Company are seriously considering to use the suggested approach for determining the size of weekly magazine editions. In October this year, the suggested approach will be tested for implementation, in such a way that the described model and methods will be used for actually determining the size of editions for two of the Company's magazines.

6 Summary and Future Research

The aim of this chapter is to provide a review (or preview) of the contents of the study and a few preliminary suggestions for further investigation.

SUMMARY

Four positions serve as the basis for the study. The first states that uncertainty plays a vital role in all important decision making, and should therefore, together with the decision maker's preferences, be included and analyzed in the process of choice, in such a way that the implemented action corresponds to the preferences and the judgments of the uncertainty, consistent with the expected utility model. The second position expresses that a useful analysis of decisions under uncertainty must recognize the decision maker's inability to give precise as well as reliable estimates of the world. The third position posits a dilemma between allowing for imprecision and the likeliness of identifying one optimal action, and that a trade-off has to be made between these two aspects of a useful analysis. The fourth position states that the most straight forward line of development seems to be analysis of dominance, assuming imprecise knowledge about the world at any given point in time.

To set the next stage of the study, the decision situation under consideration is defined as consisting of a finite set of available actions, a finite set of states of the world, a mapping of preference into utility, and an expression of uncertainty into personal probability. Definitions of utility and personal probability, as well as how to elicit these values, are provided. A review of earlier suggestions for analysis of dominance with imprecise probability information concretely establishes that, in most cases, conclusive analyses can only be achieved in trivial decision settings.

My intention is to increase the usefulness of formal analysis of decisions made under uncertainty, by suggesting and defining an indifference method. The proposed method presents a way to compute the most imprecise probability measure required for a conclusive analysis, i.e. an analysis which results in one optimal action. To obtain conclusiveness, the decision maker is asked to express the uncertainty as an assessment if the personal probability of a specific state or subset of states is larger than a computed (indifference) value between zero and one.

The primary criterion of a useful analysis is proposed to be the decision maker's confidence in the output from the analysis. This, in turn, is suggested to be dependent upon the decision maker's confidence in the input, acceptance of the model, and acceptance of the method. The type of input of probability estimates (indifference estimates) is the characteristic of the suggested indifference method, and the next step in the study is an investigation of the confidence in indifference estimates relative to point estimates, bounded interval estimates, and ordinal estimates. This investigation, consisting of propositions as well as data from an experiment in which twenty-four professional probability assessors participated, provides three main results: first, a few elementary and theoretically derived conditions, defining the relative confidence in indifference estimates. Second, empirical indications of the paradoxical use of point estimates for analysis of decisions, implying that the obtained optimal action could turn out to be non-optimal. Third, empirical indications of extremely skewed judgmental uncertainty distributions within the elicited bounded interval estimates, implying that the use of such estimates for analysis of decisions could turn out to be less robust and less reliable than perhaps earlier assumed.

To provide empirical observations on the usefulness of the indifference method in a realistic setting and in an extensive way, the next, and last, step in the study contains a real-life application of the expected utility model and the indifference method to the problem of determining the size of weekly magazine editions. It is observed that it was feasible to define and suggest a model and to use the suggested methods in a test of the approach. Another observation is that the participants' concerns about the uncertainty aspect in general, and the indifference method specifically, were overshadowed by their concerns about the expediency of analyzing the decisions in a formal manner, and how much confidence they could have in the estimated and presented effects (on sales and profits) of using the proposed approach. Furthermore, it is noted that the indifference method resulted in conclusive analyses in most cases, and that the intended user of the model indicated relatively much confidence in the indifference estimates. The observations also indicate that the intended model user did not have confidence in the output from the model, to such a degree that he adopted the "model editions". The analysis and discussion of the application concludes with a general observation regarding the participants' views on important characteristics of a "good model, and a general remark about different ways of understanding between managers and researchers.

FUTURE RESEARCH

In Chapter 3 I commented in general terms on the conclusiveness of the indifference method relative to Fishburn's (1964; 1965) and others' methods for analysis of dominance. Since conclusiveness is one of the aspects of usefulness of formal analysis of decisions made under uncertainty, it would seem interesting to study this further.

As a way of beginning, I have written a program for computer use named INSIM, which simulates S number of randomized decision problems and with M number of states of the world (see Hederstierna, 1981a). (With minor alterations, it could also serve as a program for interactive computer use of the indifference method in real and/or experimental situations.)

In INSIM, p_j, u_{kj}, u_{ij} are randomly (with uniform probability distributions) generated in]0,1[for j=1,...,M, such that $\Sigma_j p_j=1$. The decision problem is defined to be conclusive if the sum of the randomized p_j (represented in F_t) is larger than the randomized indifference values, in any iteration t. This corresponds to the definition of conclusiveness given in Chapter 3. Considering the results from the empirical study reported in Chapter 4, p_j may be seen as the simulated mean of a judgmental uncertainty distribution.

Table 6:1 shows the results from INSIM with a run of 20 and 200 randomized decision problems, respectively, and with the number of states ranging from 2 to 10. From the table we see that when M becomes large, the number of conclusive results seems to converge toward 50 percent of the total number of simulated decision problems. This may not be surprising, since it would be equally likely that the indifference value is larger respectively smaller than the sum of p_j represented in F_t, when M and S are sufficiently large.

Table 6:1. Simulation of conclusiveness of the indifference method. Print-out from an INSIM run with 20 and 200 simulated decision problems, respectively, and with the number of states of the world ranging from 2 to 10

NUMBER OF SIMULATED DECISION PROBLEMS = 20	
Number of states	Number of conclusive results
2	20
3	17
4	14
5	9
6	13
7	11
8	9
9	11
10	13
NUMBER OF SIMULATED DECISION PROBLEMS = 200	
2	200
3	178
4	145
5	128
6	108
7	111
8	97
9	100
10	97

I feel that it would be worthwhile to study the aspect of conclusiveness of the indifference method and other suggestions, perhaps in a similar way as in INSIM, but probably with non-uniform probability distributions of the randomized numerical values.

In connection to this, there is an intriguing difference between the number of conclusive results in the test of the indifference method in the real-life application reported in Chapter 5, and the results from INSIM. Even if the results are not directly comparable, it is worth noting that the number of conclusive decisions in the real-life study was larger than perhaps to be expected (15 conclusive results out of 19 possible). Could one explanation of this be that the analyst and the decision maker (Model User) want the solution to be conclusive, and that the probability assessor therefore adjusts the states of the world (which in many cases are flexible in interpretation) to include more and less of the described world? If so, what would the implications on the validity and usefulness of the analysis be?

For reasons of bias, irrationality, emotion, etc, one may question the scientific advisability of investigating the validity of one's own theoretical suggestions (see e.g. Ståhl, 1972). On the other hand, one may agree with Mitroff (1972), who implies that a scientist has to provide evidence, supporting or not supporting suggested positions, hypotheses, or theories. Both views imply a need for further theoretical and empirical investigations of the characteristics of useful analyses of decisions made under uncertainty, how to obtain such analyses, and, more specifically, the relative usefulness of the indifference method (as, for example, suggested by the theoretical propositions in Chapter 4).

In the experimental study and the real-life application, it was indicated that the professionals seemed to prefer certainty to uncertainty, by giving too narrow bounded interval estimates of the probability (Chapter 4) and by not appreciating the explicit introduction of uncertain demand (Chapter 5). With respect to different imprecise probability estimates, ordinal and indifference assessments seem to have a relative advantage over bounded interval assessments (not measures), since the assessor does not "reveal" the judgmental uncertainty when only assessing that the probability is larger or smaller than an indifference value, or that, for example, p_1 is larger or smaller than p_2.

Wide bounded interval estimates, however, seem to "reveal" greater uncertainty than narrow interval estimates. I think that an investigation of this matter could provide important knowledge about the complex and vital problem of decisions under uncertainty.

References

ABRAMSON, L.R., 1967, Analysis of Dominance with Inequality Sets. Operations Research, Vol. 15, pp. 363-365.

ACKOFF, R.L., 1979, The Future of Operational Research is Past. Journal of the Operational Research Society, Vol. 30, pp. 93-104.

ALLAIS, M., 1953, Fondements d'une théorie positive des choix comportant un risque et critique des postulats et axioms de l'école americaine. Econometrie, Vol. 40, pp. 257-332 (English translation in Centre National de la Recherche Scientifique, No. 3182, 1975).

ANSCOMBE, F.J. & AUMANN, R.J., 1963, A Definition of Subjective Probability. Annals of Mathematical Statistics, Vol. 34, pp. 199-205.

ATKINSON, A.A., 1979, Incentives, Uncertainty, and Risk in the Newsboy Problem. Decision Sciences, Vol. 10, pp. 341-353.

BAR-HILLEL, M., 1973, On the Subjective Probability of Compound Events. Organizational Behavior and Human Performance, Vol. 9, pp. 396-406.

BAUMOL, W.J., 1977, Economic Theory and Operations Analysis. Prentice-Hall, Englewood Cliffs.

BEACH, L.R. & PETERSON, C.R., 1966, Subjective Probabilities for Unions of Events. Psychonomic Science, Vol. 5, pp. 307-308.

BEACH, L.R. & WISE, J.A., 1969, Subjective Probability Estimates and Confidence Ratings. Journal of Experimental Psychology, Vol. 79, pp. 438-444.

BRANDES, O., 1967, Optimeringsproblem inom tidningsdistributionen. Handelshögskolan i Göteborgs Skrifter, Nr. 4. Gothenburg School of Economics, Gothenburg.

BRANDES, O., 1971, Supply Models. An Empirical Study of Adaptation and Innovation in the Firm (dissertation). Gothenburg Studies in Business Administration, No. 12. Gothenburg School of Economics, Gothenburg.

BRIM, O.G.Jr., 1955, Attitude Content-Intensity and Probability Expectations. American Sociological Review, Vol. 20, pp. 68-76.

BROWN, R.V., 1970, Do Managers Find Decision Theory Useful? Harvard Business Review, May-June, pp. 78-89.

CHESLEY, G.R., 1975, Elicitation of Subjective Probabilities: A Review. The Accounting Review, Vol. 50, pp. 325-337.

CHURCHMAN, C.W. & SCHAINBLATT, H., 1965, The Researcher and the Manager: A Dialectic of Implementation. Management Science, Vol. 11, pp. B69-B87.

DAVIDSON, D. & SUPPES, P., 1956, A Finitistic Axiomatization of Subjective Probability and Utility, Econometrica, Vol. 24, pp. 264-275.

DOKTOR, R.H. & HAMILTON, W.F., 1973, Cognitive Style and the Acceptance of Management Science Recommendations. Management Science, Vol. 19, pp. 884-894.

DUNCAN, R.B. & ZALTMAN, G., 1975, Ethical and Value Dilemmas in Implementation. In Schultz, R.L. & Slevin, D.P. (eds.) Implementing Operations Research/Management Science. American Elsevier, New York.

DUNCAN, W.J., 1974, The Researcher and the Manager: A Comparative View of the Need for Mutual Understanding. Management Science, Vol. 20, pp. 1157-1163.

DYCKMAN, T.R., 1967, Management Implementation of Scientific Research: An Attitudinal Study. Management Science, Vol. 13, pp. B612-B620.

EDWARDS, W., 1968, Conservatism in Human Information Processing. In Kleinmuntz, B. (ed.), Formal Representation of Human Judgment. Wiley, New York.

ELLSBERG, D., 1961, Risk, Ambiguity, and the Savage Axioms. Quarterly Journal of Economics, Vol. 75, pp. 643-669.

ELVESTEDT, U., 1979, Beslutsanalys - en interaktiv ansats (dissertation). The Economic Research Institute at the Stockholm School of Economics, Stockholm (English summary).

de FINETTI, B., 1937, La prévision: ses lois logiques, ses sources subjectives. Annales de l'Institut H. Poincaré Vol. 7, pp. 1-68 (English translation in Kyburg, H.E. & Smokler, H.E. (eds.), Studies in Subjective Probability. Wiley, New York 1964).

FISCHOFF, B. & SLOVIC, P., 1978, A Little Learning....: Confidence in Multicue Judgment Tasks. Research Paper from Decision Research; A Branch of Perceptronics, Oregon.

FISHBURN, P.C., 1964, Decision and Value Theory. Wiley, New York.

FISHBURN, P.C., 1965, Analysis of Decisions with Incomplete Knowledge of Probabilities. Operations Research, Vol. 13, pp. 217-237.

FISHBURN, P.C., MURPHY, A.H. & ISAACS, H.H., 1968, Sensitivity of Decisions to Probability Estimation Errors: A Reexamination. Operations Research, Vol. 16, pp. 254-267.

GOOD, I.J., 1950, Probability and the Weighing of Evidence. Griffin, London.

GRAYSON, C.J.Jr., 1960, Decisions under Uncertainty: Drilling Decisions by Oil and Gas Operators. Harvard University, Boston.

HADLEY, G. & WHITIN, T.M., 1963, Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs.

HAMMOND, J.S., 1974, The Roles of the Manager and Management Scientist in Successful Implementation. Sloan Management Review, Vol. 16, pp. 1-24.

HARVEY, A., 1970, Factors Making for Implementation Success and Failure. Management Science, Vol. 16, pp. B312-B321.

HEDERSTIERNA, A., 1980, Beslut under osäkerhet. Experimentformulär. Unpublished Research Paper. The Economic Research Institute at the Stockholm School of Economics, Stockholm.

HEDERSTIERNA, A., 1981a, INSIM: A Computer Program for Simulating Conclusiveness of the Indifference Method. Unpublished Research Paper. The Stockholm School of Economics, Stockholm.

HEDERSTIERNA, A., 1981b, Decisions under Uncertainty: A Note on Skewed Uncertainty Distributions. Omega, Vol. 9, pp. 442-443.

HEDERSTIERNA, A., (forthcoming), Model Characteristics: Understanding Differing Views of Relative Importance. Interfaces.

HENDERSON, J.C. & NUTT, P.C., 1980, The Influence of Decision Style on Decision Making Behavior. Management Science, Vol. 26, pp. 371-386.

HILDEBRANDT, S., 1980, Implementation - the Bottleneck of Operations Research: The State of the Art. European Journal of Operational Research, Vol. 6, pp. 4-12.

HIRSCHLEIFER, J. & RILEY, J.G., 1979, The Analytics of Uncertainty and Information - An Expository Survey. Journal of Economic Literature, Vol. 17, pp. 1375-1421.

HOGART, R.M. & MAKRIDAKIS, S., 1981, Forecasting and Planning: An Evaluation. Management Science, Vol. 27, pp. 115-138.

HOWARD, R.A., 1980, An Assessment of Decision Analysis. Operations Research, Vol. 28, pp. 4-27.

HUBER, G.P., 1974, Methods for Quantifying Subjective Probabilities and Multi-Attribute Utilities. Decision Sciences, Vol. 5, pp. 430-458.

HULL, J.C., 1980, The Evaluation of Risk in Business Investment. Pergamon, London.

HUYSMANS, J.H.B.M., 1970, The Implementation of Operations Research. Wiley, New York.

HUYSMANS, J.H.B.M., 1975, Operations Research Implementation and the Practice of Management. In Schultz, R.L. & Slevin, D.P. (eds.), Implementing Operations Research/ Management Science. American Elsevier, New York.

ISAACS, H.H., 1963, Sensitivity of Decisions to Probability Estimation Errors. Operations Research, Vol. 11, pp. 536-552.

KAHNEMAN, D. & TVERSKY, A., 1972, Subjective Probability: A Judgement of Represenativeness. Cognitive Psychology, Vol. 3, pp. 430-454.

KMIETOWICS, Z.W. & PEARMAN, A.D., 1979, Decision Theory and Weak Statistical Dominance. Journal of the Operational Research Society, Vol. 30, pp. 1019-1022.

KOOPMAN, B.O., 1940, The Axioms and Algebra of Intuitive Probability. Annals of Mathematics, Vol. 42, pp. 269-292.

KORIAT, A., LICHTENSTEIN, S. & FISCHOFF, B., 1980, Reasons for Confidence. Journal of Experimental Psychology: Human Learning and Memory, Vol. 6, pp. 107-118.

KOTTAS, J.F. & LAU, H-S., 1979, A Decision Model with Stochastic Cost and Demand Curves. Omega, Vol. 7, pp. 80-81.

KRAFT, C.H., PRATT, J.W. & SEIDENBERG, A., 1959, Intuitive Probability on Finite Sets. Annals of Mathematical Statistics, Vol. 30, pp. 408-419.

LAU, H-S., 1980, The Newsboy Problem under Alternative Optimization Objectives. Journal of the Operational Research Society, Vol. 31, pp. 525-535.

LEE, W., 1971, Decision Theory and Human Behavior. Wiley, New York.

LINDMAN, H.R., 1965, The Simultaneous Measurement of Utilities and Subjective Probabilities. Unpublished dissertation. University of Michigan, Ann Arbor.

LITTLE, J.D.C., 1970, Models and Managers: The Concept of a Decision Calculus. Management Science, Vol. 16, pp. B466-B485.

LUCE, R.C. & RAIFFA, H., 1957, Games and Decisions. Wiley, New York.

LYON, D. & SLOVIC, P., 1976, Dominance of Accuracy Information and Neglect of Base Rates in Probability Estimation. Acta Psychologica, Vol. 40, pp. 287-298.

MATHES, R.S., 1969, "D" People and "S" People. Science, Vol. 164, p. 630.

MACCRIMMON, K.R., 1968, Descriptive and Normative Implications of the Decision-Theory Postulates. In Borch, K. & Mossin, J. (eds.), Risk and Uncertainty. St. Martins Press, New York.

MCARTHUR, D.S., 1980, Decision Scientists, Decision Makers, and the Gap. Interfaces, Vol. 10, pp. 110-113.

MINTZBERG, H., RAISINGHANI, D. & THÉORET, A., 1976, The Structure of "Unstructured" Decision Processes. Administrative Science Quarterly, Vol. 21, pp. 246-275.

MITROFF, I.I., 1972, The Myth of Objectivity or Why Science Needs a New Psychology of Science. Management Science, Vol. 18, pp. B613-B618.

MORRIS, P.A., 1977, Combining Expert Judgments: A Bayesian Approach. Management Science, Vol. 23, pp. 679-693.

NAERT, P. & LEEFLANG, P., 1978, Building Implementable Marketing Models. Martinus Nijhoff, Leiden.

NARASIMHAN, R. & SCHROEDER, R.G., 1979, An Empirical Investigation of Implementation as a Change Process. In Doktor, R., Schultz, R.L. & Slevin, D.P. (eds.), The Implementation of Management Science. North-Holland, Amsterdam.

von NEUMANN, J. & MORGENSTERN, O., 1947, Theory of Games and Economic Behavior. Princeton University Press, Princeton.

ORGANIST, W.E., 1964, Use of Subjective Probabilities as a Response Technique in Multiple-Choice Behavior. Paper read at meetings of the Psychometric Society.

PALSTERNACK, B.A., 1980, Filling out the Doughnuts; The Single Period Inventory Model in Corporate Pricing Policy. Interfaces, Vol. 10, pp. 96-100.

PETERSON, C.R. & BEACH, L.R., 1967, Man as an Intuitive Statistician. Psychological Bulletin, Vol. 68, pp. 29-46.

PETERSON, R. & SILVER, E.A., 1979, Decision Systems for Inventory Management and Production Planning. Wiley, New York.

PHILLIPS, L.D. HAYS, W.L. & EDWARDS, W., 1966, Conservatism in Complex Probability Inference. IEEE Transactions in Human Factors in Electronics, Vol. HFE-7, pp. 7-18.

PICKHARDT, R.C. & WALLACE, J.B., 1974, A Study of the Performance of Subjective Probability Assessors. Decision Sciences, Vol. 5, pp. 347-363.

PIERCE, D.A. & FOLKS, J.L., 1969, Sensitivity of Bayes Procedure to the Prior Distribution. Operations Research, Vol. 17, pp. 344-349.

PITZ, G.F., 1967, Sample Size, Likelihood, and Confidence in a Decision. Psychonomic Science, Vol. 8, pp. 257-258.

POLANYI, M., 1958, Personal Knowledge: Towards a Post-Critical Philosophy. Routledge & Kegan Paul, London.

PRATT, J.W., RAIFFA, H. & SCHLAIFER, R., 1964, The Foundations of Decision under Uncertainty: An Elementary Exposition. Journal of the American Statistical Association, Vol. 59, pp. 353-375.

RADNOR, M., RUBENSTEIN, A.H. & TANSIK, D.A., 1970, Implementation in Operations Research and R-D in Government and Business Organizations. Operations Research, Vol. 18, pp. 967-991.

RAIFFA, H., 1968, Decision Analysis - Introductory Lectures on Choices under Uncertainty. Addison-Wesley, Reading.

RAMSEY, F.P., 1931, Truth and Probability. In by the same author, The Foundations of Mathematics and Other Logical Essays. Routledge & Kegan Paul, London.

ROBERTS, H.V., 1963, Risk, Ambiguity, and the Savage Axioms: Comment. Quarterly Journal of Economics, Vol. 77, pp. 327-336.

RUBENSTEIN, A.H., RADNOR, M., BAKER, N.R., HEIMAN, D.R. & McCOLLY, J.B., 1967, Some Organizational Factors Related to the Effectiveness of Management Science Groups in Industry. Management Science, Vol. 13, pp. B508-B518.

SARIN, R.K., 1978, Elicitation of Subjective Probabilities in the Context of Decision-Making. Decision Sciences, Vol. 9, pp. 37-48.

SAVAGE, L.J., 1954, The Foundations of Statistics. Dover, New York.

SCHULTZ, R.L. & SLEVIN, D.P., 1975, A Program of Research on Implementation. In by the same authors (eds.), Implementing Operations Research/ Management Science. American Elsevier, New York.

SIMON, H.A., 1955, A Behavioral Model of Rational Choice. Quarterly Journal of Economics, Vol. 69, pp. 99-118.

SLOVIC, P., FISCHOFF, B. & LICHTENSTEIN, S., 1976, Behavioral Decision Theory. Technical Report DDI-7. Oregon Research Institute, Oregon (quoted in Howard, 1980).

SLOVIC, P., FISCHOFF, B. & LICHTENSTEIN, S., 1977, Behavioural Decision Theory. Annual Review of Psychology, Vol. 28, pp. 1-39.

SOUDER, W,E., MAHER, P.M., BAKER, N.R., SHUMWAY, C.R. & RUBENSTEIN, A.H., 1975, An Organizational Intervention Approach to the Design and Implementation of R&D Project Selection Models. In Schultz, R.L. & Slevin, D.P. (eds.), Implementing Operations Research/Management Science. American Elsevier, New York.

SPETZLER, C.S., 1969, The Development of a Corporate Risk Policy for Capital Investment Decisions. IEEE Transactions on Systems Science and Cybernetics, Vol. SSC-4, pp, 279-300.

SPETZLER, C.S. & STAËL VON HOLSTEIN, C-A.S., 1975, Probability Encoding in Decision Analysis. Management Science, Vol. 22, pp. 340-358.

STAËL VON HOLSTEIN, C-A.S., 1970, Assessment and Evaluation of Subjective Probability Distributions (dissertation). The Economic Research Institute at the Stockholm School of Economics, Stockholm.

STÅHL, I., 1972, Bargaining Theory (dissertation). The Economic Research Institute at the Stockholm School of Economics, Stockholm.

SUPPES, P., 1955, The Role of Subjective Probability and Utility in Decision Making. Proceedings of the 3rd Berkeley Symposium on Mathematical Statistics and Probability, Vol. 5, pp. 61-73. University of California Press, Berkeley.

TANI, S.N., 1978, A Perspective on Modeling in Decision Analysis. Management Science, Vol. 24, pp. 1500-1506.

TAYLOR, P.B. & IWANEK, R., 1980, Intuitive versus Optimal Solutions to Some Problems in Distribution. Omega, Vol. 8, pp. 183-192.

TEIGEN, K.H., 1974, Overestimation of Subjective Probabilities. Scandinavian Journal of Psychology, Vol. 15, pp. 56-62.

TVERSKY, A. & KAHNEMAN, D., 1974, Judgement under Uncertainty: Heuristics and Biases. Science, Vol. 185, pp., 1124-1131.

ULVILA, J.W. & WATSON, R.V., 1978, Step-Through Simulation. Omega, Vol. 6, pp. 25-31.

WAGNER, H.M., 1969, Principles of Operations Research. Prentice-Hall, Englewood Cliffs.

WATSON, S.R. & BROWN, R.V., 1978, The Valuation of Decision Analysis. Journal of the Royal Statistical Society, A, Vol. 141, pp. 69-78.

WATSON, S.R., WEISS, J.J. & DONNELL, M.L., 1979, Fuzzy Decision Analysis. IEEE Transactions on Systems, Man and Cybernetics, Vol. SMC-9, pp. 1-9.

WINKLER, R.L., 1968, The Consensus of Subjective Probability Distributions. Management Science, Vol. 15, pp. 61-75.

WINKLER, R.L. & MURPHY, A.H., 1968, "Good" Probability Assessors. Journal of Applied Meteorology, Vol. 7, pp. 75-85.

WINKLER, R.L. & MURPHY, A.H., 1973, Experiments in the Laboratory and the Real World. Organizational Behavior and Human Performance, Vol. 10, pp. 252-270.

WYERS, R.S.Jr., 1976, An Investigation of the Relations Among Probability Estimates. Organizational Behavior and Human Performance, Vol. 15, pp. 1-18.

List of Reports Published since 1978 by the Economic Research Institute at the Stockholm School of Economics

Unless otherwise indicated, these reports are published in English.

1978

ASPLING, A. & LINDESTAD, L., 1978, In-company training. Reflection of three cases. Stockholm.[1]

EKMAN, E.V., 1978, Some dynamic economic models of the firm. A micro-economic analysis with emphasis on firms that maximize other goals than profit alone. Stockholm.

FJAESTAD, B. & JULANDER, C-R., 1978, Retailers' view of the future. A study of retailers in Norrköping. Report No. 6 from the research program "The retailing in change". Stockholm. (Mimeographed)[1]

FJAESTAD, B., JULANDER, C-R. & ANELL, B., 1978, Consumers in shopping centers in Norrköping. Report No. 4 from the research program "The retailing in change". Stockholm. (Mimeographed)[1]

FORSBLAD, P., SJÖSTRAND, S-E. & STYMNE, B.(eds.) 1978, Man in organizations. A view of co-determination and leadership. Stockholm: EFI/Liber Förlag.[1]

HEMMING, T., 1978, Multiobjective decision making under certainty. Stockholm.

HOLMLÖV, P.G., 1978, News in local press and municipal policy. Stockholm.[2]

HULTCRANTZ G., LINDHOFF, H. & VALDELIN, J., 1978, Chinese economic planning: Characteristics, objectives and methods - an introduction. China's developmental strategy: 7. Stockholm (Mimeographed)

JULANDER, C-R. & EDSTRÖM, E., 1978, Information for the design of product information about flats. An interview study with tenants in Husby/Akalla. Stockholm. (Mimeographed)[1]

1) Only published in Swedish.

2) Published in Swedish with an English summary.

JUNDIN, S., LINDQVIST, A., 1978, Telephone as a mail-box. A study of 100 tele facsimile users. Stockholm.[1)]

LINDQVIST, A., JULANDER, C-R. & FJAESTAD, B., 1978, Development of indicators of saving behavior - Report No. 2 from the project "Savings behavior". Stockholm. (Mimeographed)[1)]

SCHWARZ, B. & SVENSSON, J-E., 1978, Transports and transports research. Stockholm.[2)] (Off-print from Transportdelegationen's publication 1978:7.)

STJERNBERG, T., 1978, Retail employees in Linden. A study of employees influence and experiences in connection with the establishment of a shopping center. Report No. 3 from the research program "The retailing in change". Stockholm. (Mimeographed)[1)]

TELL, B., 1978, Capital budgeting in practice. Stockholm.[1)]

1979

AHLMARK, D. & BRODIN, B., 1979, The book trade in the future. Marketing and distribution. Stockholm.[1)]

AHLMARK, D. & BRODIN, B., 1979, State subsidies for the production and distribution of books - an economic analysis. Stockholm.[1)]

AHLMARK, D. & LJUNGKVIST, M-O., 1979, The financial analysis and management of publishing houses. Studies of development and behavior during the 1970s. Stockholm.[1)]

ANELL, B., 1979, When the store closes down. Report No. 5 from the research program "The retailing in change". Stockholm. (Mimeographed)[1)]

ANELL, B., 1979, Consumers and their grocery store. A consumer economic analysis of food buying patterns. Stockholm.[1)]

BERTMAR, L., 1979, Wages profitability and equity ratio, Stockholm.[1)] (Off-print from SOU 1979:10.)

BORGENHAMMAR, E., 1979, Health care budgeting, goals, structure, attitudes. Stockholm.

ELVESTEDT, U., 1979, Decision analysis - an interactive approach. Stockholm.[2)]

ENGLUND, P., 1979, Profits and market adjustment. A study in the dynamics of production, productivity and rates of return. Stockholm.

ETTLIN, F.A., LYBECK, J.A., ERIKSON, I., JOHANSSON, S. & JÄRNHÄLL, B., 1979, The STEP 1 quarterly econometric model of Sweden - the equation system. Stockholm.

FALK, T., 1979, The retail trade in Norrköping. Structure and location 1977. Report No. 7 from the research program "The retailing in change". Stockholm. (Mimeographed)[1)]

1) Only published in Swedish.

2) Published in Swedish with an English summary.

FALK, T., 1979, Retailing in Norrköping. Structural and locational changes 1951 - 1977. Report No. 9 from the research program "The retailing in change". Stockholm. (Mimeographed)[1]

HEDEBRO, G., 1979, Communication and social change in developing nations - a critical view. Stockholm: EFI/JHS. (Mimeographed)

HOLMLÖV, P.G., FJAESTAD, G. & JULANDER, C-R., 1979, Form and function in marketing. Household appliencies and kitchen carpentry: sales product development and advertising rhetoric 1961 - 1976. Stockholm. (Mimeographed)[1]

JANSSON. J.O. & RYDÉN, I., 1979, Cost benefit analysis for seaports. Stockholm.[1]

JANSSON, J.O. & RYDÉN, I., 1979, Swedish seaports - economics and policy. Stockholm. (Mimeographed)

JULANDER, C-R. & FJAESTAD, B., 1979, Consumer purchasing patterns in Norrköping and Söderköping. Report No. 8 from the research program "The retailing in change". Stockholm. (Mimeographed)

JUNDIN, S., 1979, Children and consumption. Stockholm.[1]

MAGNUSSON, Å., PETERSSOHN, E. & SVENSSON, C., 1979, Non-life insurance and inflation. Stockholm.[1]

Marketing and structural economics, 1979, (ed. Otterbeck, L.). Stockholm: EFI/IIB/Studentlitteratur.[1]

PERSSON, M., 1979, Inflationary expectations and the natural rate hypothesis. Stockholm.

von SCHIRACH-SZMIGIEL, C., 1979, Liner shipping and general cargo transport. Stockholm.

ÖSTERBERG, H., 1979, Hierarchical analysis of concepts - a technique for solving complex research problems. Stockholm: EFI/Norstedts.

1980

ELVESTEDT, U., 1980, Currency issues in Swedish firms. Stockholm.[1]

FORSBLAD, P., 1980, Chief executive influence in decision-making - some attempts at identification and description. Stockholm.[2]

JANSSON, J.O., 1980, Transport system optimization and pricing. Stockholm.

JONSSON, E., 1980, Studies in health economics. Stockholm.

LINDQVIST, A., 1980, Household saving and saving behavior. Report no. 3 of the project "Development of behavioral scientific indicators of saving". Stockholm.[1]

SJÖGREN, L., 1980, Cost control of building design. Stockholm.[2]

Shipping and ships for the 1990's, 1980, (eds. Rydén I. and von Schirach-Szmigiel, C.). Stockholm.

ÖSTMAN, L., 1980, Behavioral accounting. Stockholm.[1]

---

1) Only published in Swedish.

2) Published in Swedish with an English summary.

1981

KIRSTEIN, K. & JULANDER, C-R., 1981, Scanners in the supermarket: consequences for the consumers. Stockholm.1)

LEKSELL, L., 1981, Headquarter-subsidiary relationships in multinational corporations. Stockholm.

LINDQVIST, A., 1981, Household saving - behavioural studies of households' saving behaviour. Stockholm.2)

RUDENGREN, J., 1981, Peasants by preference? Stockholm.

SCHWARZ, B. & LEKTEUS, I., 1981, Energy futures - multiple objectives - new technology. Stockholm.1)

ÖSTMAN, L., 1981, Solidity and financial costs - some measuring problems. Stockholm.1)

1) Only published in Swedish.

2) Published in Swedish with and English summary.