ESSAYS ON

STOCHASTIC FISCAL POLICY, PUBLIC DEBT AND PRIVATE CONSUMPTION

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PREFACE

And so the day of printing out the final version has come, and my thesis journey is coming to an end. It has sometimes been a journey with a very obscure ending that started out when my (head) thesis advisor, Karl Jungenfelt, suggested that I should apply to the doctoral program at the Stockholm School of Economics. I am very grateful to Karl for this suggestion, and for all the invaluable support and useful comments that he has provided during the journey. I know that I could not have found a better advisor anywhere else in academia. I am also greatly indebted to my two other advisors, Anders Paalzow and Anders Vredin. Anders P. has been extremely patient with all my ignorant questions, ranging from how to use the expectations operator to how to express certain ideas in words. He has also provided good company both at the office, at "Svea" and in other places. Anders V., on the other hand, has provided little company at "Svea", but has been invaluable in many other places, and for many other reasons. He has provided an abundant amount of comments and encouragement, and spent several buckets of red ink when reading through my drafts. Without my excellent advisor troika, I would definitely not be writing this preface today.

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Finally, to all of you, I will soon be working hard on improving my "external debt" position ♪

(I wonder how that will affect your "consumption paths"...are T Bonds net wealth? Sorry, I have probably worked on this thesis too long.)

Stockholm, May 1995

Torbjörn Becker

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Introduction and Summary of the Thesis

1. INTRODUCTION

One important aspect of fiscal policy is how the creation of public debt affects the private sector. This aspect is particularly vital when evaluating stabilization policies with an aim to affect aggregate demand. The Keynesian perspective on this issue is that, for a *given* path of government consumption, by decreasing taxes today, the private sector will increase its spending, and thus aggregate consumption increases. The reason for this conclusion, in its simplest version, is that private consumption today is determined by today's disposable income; with a tax cut, disposable income increases and so does private consumption. This conclusion implies that households regard their holding of government bonds as net wealth, i.e. the households do not discount any part of the future tax increases that are necessary to repay the public debt. A similar conclusion of the effects of debt policy is obtained in neoclassical models of the Yaari-Blanchard type (see Yaari [1965], Blanchard [1985], Weil [1989], and Buiter [1988]), where new households enter the economy in the future, and thus a part of the tax burden is evaded by the households that are presently part of the economy. In this type of model, households discount only a part of future tax payments, and thus a tax cut today increases private consumption, but by a smaller amount than in a Keynesian model.

The Ricardian story, on the other hand, is that, for a given level of government consumption, the level of public debt does not matter for the private sector. The reason for this result is that in Ricardian models households internalize the government's intertemporal budget constraint, i.e. households fully discount future tax payments, and thus government bonds do not represent any net wealth. Thus, increasing today's disposable income by postponing taxes does not lead to any changes in private consumption in the Ricardian world. One of the reasons for obtaining this conclusion is that there are no new agents entering the economy that will share the future tax burden, or alternatively, that the households alive today are altruistic in the sense that they care about the utility of the new entrants (see Barro [1974], Burbidge [1983], Abel [1987b], Kimball [1987], Weil [1987], and Jungenfelt [1991]).

The implications of these models with respect to the usefulness of debt policy as a way of affecting aggregate demand are of course radically different, with the Keynesians arguing that debt policy is a useful tool for affecting aggregate demand and the Ricardians saying that there is no point in using debt policy to affect aggregate demand.

This thesis focuses on the link between public debt and private consumption through the households' perception of their future incomes. However, there are other potential aspects of public debt policy that will not be discussed in the thesis, such as public debt management, intergenerational redistribution, and sovereign debt and exchange rates. The focus here is instead on how we can extend the analysis of debt policy to a stochastic world, since most of the previous literature discusses debt neutrality in models with perfect foresight. For empirical investigations of debt neutrality, it is both natural and essential to remove the assumption of perfect foresight to obtain reasonable interpretations of an estimated model. Furthermore, with uncertainty in a theoretical model, there is a possibility that the level of public debt affects the perceived risk, which in turn can have effects on private consumption if households care about risk.

The connection between public debt, or, more generally, the tax system, and the perceived income risk has been discussed to some extent in the tax literature. The view has then usually been that the tax system provides an insurance against risk in gross income, see for example Stiglitz [1969], Varian [1980], Smith [1982], Barsky et al. [1986]. However, discussions of risk created by the tax system are rare, although there are a few exceptions, see for example Chan [1983]. What is the reason for discussing tax uncertainty at all? First of all, the mere size of the public sector in many Western economies suggests that the public sector determines a large part of households' disposable income. Secondly, a large part of government spending in these economies is due to different transfer systems, the Swedish public sector being a very obvious example of both these points. If we add the fact that the political system very often changes the rules and rates involved in both the tax and transfer systems, we realize that a large part of disposable income will be uncertain for these reasons.

Furthermore, when uncertainty is discussed in connection with budget deficits, the conclusion in, for example, Barsky et al. [1986] is that the implied increase in future tax rates provides an additional insurance on the uncertain gross income, which basically implies that households will reduce their precautionary savings. Critical for this conclusion is, of course, that the tax and transfer system is not subject to uncertainty, and that it is actually possible to increase future tax revenues by increasing the tax rates. In economies that start out with very high levels of government spending and thus also tax rates, it is not obvious that the policy maker can actually rise tax rates in the future to balance the budget. It might equally well be the case that the transfers are reduced, and these transfers are in many cases what constitutes the main part of the insurance in the tax and transfer system. In other words, it is far from obvious that the households will regard the postponed taxes as an insurance. We can also note that a bad realization of, for example, the level of employment in many economies creates an additional incentive for the policy maker to reduce transfers, since the tax base is reduced and expenditures increased. This would then imply that a bad outcome in the labor market is accompanied by a bad outcome of transfers for the household, i.e. the public sector and market outcomes are positively correlated in this case.

The above discussion is the main motivation for the theoretical papers in the thesis, that focuses on risk created by the public sector rather than the market. At this stage the analysis is made under the assumption of no uncertainty from the market, to make the analysis of tax risk as transparent as possible. We then also have a vehicle for future studies, where the correlations of market risk and tax and transfer risk can be analyzed. This is, however, at this stage an area of future research.

2. SUMMARY OF THE THESIS

The thesis consists of five separate papers that analyze the effects of debt policy on private consumption. In the first paper, *"Government Debt and Private Consumption: Theory and Evidence"*, theoretical models giving rise to the Ricardian equivalence result, as well as models predicting deviations from debt neutrality, are reviewed. In general, the Ricardian models are based on assumptions which cannot be expected to hold strictly, such as infinite planning horizons, perfect capital markets and lump-sum taxes. The question is to what extent these assumptions, and hence the equivalence hypothesis, are reasonable approximations of the real world. This can only be established by empirical studies. To formulate a test of Ricardian equivalence, it is, however, vital to extend the standard analysis in deterministic models to stochastic models. In a stochastic environment, we need to incorporate the fact that agents have to make predictions about future levels of government consumption, and that public debt might be a useful predictor for this purpose. It is therefore necessary that an empirical study distinguishes between debt as a potential source of net wealth, which is the concern of the equivalence proposition, and debt's role as a signal of future levels of government consumption. Furthermore, the expected *duration* of a change in, for example, government consumption, is important, since it will affect the present value of the change, which is what matters in a Ricardian model. If we translate these observations to the empirical framework, we need to distinguish between expected and unexpected changes as well as permanent and transitory changes. It is argued that there are few empirical studies that make these distinctions, and in case the distinctions are made, the evidence is in favor of the Ricardian equivalence proposition. In other words, although the Ricardian equivalence hypothesis is burdened with unrealistic assumptions, it seems to provide predictions that are roughly consistent with data.

In the second paper, *"An Investigation of Ricardian Equivalence in a Common Trends Model"*, a VAR model with cointegrating constraints is estimated on US data, including gross national income, private consumption, government consumption and net taxes. This econometric framework has several advantages over the more standard single equation test of the Ricardian equivalence question. In a VAR model, we can distinguish between expected and unexpected changes, as well as between permanent and transitory shocks, which is vital for reasons discussed in the first paper.

We find that the estimated system has two co-integrating vectors, as suggested by no-Ponzi game conditions on the sectors' intertemporal budget constraints. The variables are driven by two common stochastic trends, interpreted as a technology trend and a public sector trend. There are thus two shocks with permanent effect, interpreted as a technology shock and a shock to the size of the public sector. The two temporary shocks are interpreted as a private demand and government financing shock, respectively.

A stronger version of the no-Ponzi game constraint is a solvency condition, which implies particular co-integrating vectors. These cointegration vectors are both rejected for the sample period, indicating that the public sector will not be able to repay its debt if the current policy is maintained. However, the private sector is at the same time accumulating wealth, which is consistent with predictions from a Ricardian model. Further, the equivalence theorem predicts that private consumption should be unaffected by financing shocks. Data, however, indicate that there is a significant short run effect on both income and private consumption from the financing shock, but the effect indicates that increasing taxes are accompanied by increasing private consumption, contrary to both standard Ricardian and Keynesian models.

This type of pattern cannot be explained by the standard theoretical models of perfect foresight, or in models where agents do not care about risk. How can we then explain this empirical puzzle? One potential explanation may be that budget deficits affect the households' perceived risk, which in turn can affect their consumption decision if they care about risk. We know from the theory of decision making under uncertainty that if households are prudent, i.e. their utility functions have a positive third derivative, they will consume less today if their future income becomes more risky. This type of behavior is labeled precautionary saving. The notion of precautionary savings has been used in recent macroeconomic models to explain empirical puzzles that arise when the standard version of the permanent income hypothesis is used to forecast actual consumption data, see for example Zeldes [1989] and Caballero [1990]. Most studies that discuss risk and precautionary saving focus on risk that is induced by the market, i.e. gross income is uncertain. In addition, the type of changes in uncertainty that are analyzed are often based on mean preserving spreads, i.e. the expected outcome is not changed, but the variance of the outcome is, which is one useful way of analyzing risk under these circumstances.

The last three papers in this thesis assume a different source of uncertainty, namely the government, or more specifically, the future taxes extracted by the government. Furthermore, the restriction on the future tax distribution is not a mean preserving spread constraint, but instead we let the government's intertemporal budget constraint impose restrictions on the future tax distribution. The purpose is to first investigate how uncertain taxes affect the households' consumption decisions, and then to analyze if tax cuts can make private consumption decrease, which is the empirical "puzzle" observed in the second paper.

The first theoretical paper, *"Risky Taxes, Budget Balance Preserving Spreads and Precautionary Savings"*, starts by extending the Sandmo [1970] paper on general capital income risk to the case of risky capital taxation. In his framework the concept of a mean preserving spread (MPS) is used for the risk analysis. In connection with risky taxes it is, however, possible to explicitly connect the tax risk with the government's budget constraint, and thus obtain a more natural restriction on the tax distribution than the MPS condition. In this paper the concept of a budget balance preserving spread (BBPS) is developed and used for the analysis of stochastic taxes. The paper is concluded with a comparison of the effects that a MPS and a BBPS in tax rates has on consumption decisions. It is shown that the comparative statics results for a BBPS can be different from the results obtained with a MPS. The reason is that the household's consumption choice affects the tax base, and thus the distribution for future tax rates must change to fulfill the government's budget constraint. In other words, the future (net) income distribution is not exogenous as it is in the mean preserving spread analysis presented in, for example, Sandmo [1970], but is partly determined by the households' consumption choice.

The fourth paper, *"Budget Deficits, Tax Risk and Consumption"*, investigates the effects of budget deficits on consumption when individual taxes are stochastic, i.e. the households make independent draws from a distribution of taxes or tax rates. The paper assumes that future taxes are riskier than taxes today, and provides a link between the amount of tax risk and the level of public debt. It is shown that the co-movements between budget deficits and private consumption will depend on how risk averse individuals are. In the case of lump-sum taxes, it is sufficient to assume that individuals have a precautionary savings motive to obtain the result that consumption today will decrease with decreased taxes today. Furthermore, if we use a time separable iso-elastic utility function, the standard analysis of capital income risk predicts (precautionary) savings to increase with increased risk if the coefficient of relative risk aversion is greater than one. This is no longer a sufficient condition when the risk is due to uncertain and distortionary capital income taxes. In general, the coefficient must be greater than one to obtain precautionary savings in response to greater risk implied by a budget deficit. This is due to the fact that capital income taxes are distortionary and have to change in response to changes in private consumption to fulfill the government's budget constraint. The results in the paper are consistent with Ricardian equivalence only for some specific utility function. However, in the same way, the results are consistent with models that display a positive relation between debt and private consumption only for certain utility functions. For individuals that are enough risk averse or prudent, we can equally well obtain the result that we have a *negative* correlation between first period consumption and public debt (or first period disposable income), without changing the expected value of government consumption. To summarize, if future taxes are uncertain, increased disposable income in the present period will decrease present consumption if households are prudent enough. This paper thus offers a theoretical explanation of the empirical puzzle observed for US data in the second paper.

The last paper, *"Budget Deficits, Stochastic Population Size and Consumption"*, analyzes the effects on present consumption of budget deficits under different assumptions about demographic factors. In the first part, birth and death rates are deterministic, and in the second part, birth rates are assumed to be stochastic. In the case of stochastic birth rates, future taxes will be stochastic due to uncertainty about the tax base, and thus public debt will make a larger proportion of taxes uncertain. Here, the realization of the *per capita* tax in the second period is the same for all households, so this is a case of aggregate tax risk, compared to the individual tax risk analyzed in the previous paper.

In this paper it is shown that present consumption will decrease when public debt is increased, both when the expected birth rate is zero, and when the expected population size is constant, if households are prudent and the uncertainty about the future population size is high enough. This is contrary to the standard models with deterministic population size, where an increase in public debt raises present consumption, if the birth rate is greater than zero. If the birth rate is zero in a deterministic model, we obtain the Ricardian result of debt neutrality. Since we here get the result that present consumption will decrease with a tax cut, we have provided a second explanation of the empirical puzzle presented above.

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Essay I

Government Debt and Private Consumption: Theory and Evidence*

Abstract

The Ricardian equivalence theorem has been widely debated since (at least) the seventies. The theorem states that households should not change their consumption path in response to changed timing of taxes, given the path of government consumption. In the paper, theoretical models giving rise to the equivalence result as well as models predicting deviations from debt neutrality are presented. In general, the Ricardian models are based on unrealistic assumptions, such as infinite horizons, perfect capital markets and lump-sum taxes. The issue of Ricardian equivalence is thus perhaps better viewed as a question concerning to what extent the equivalence hypothesis is a reasonable approximation of the real world. This could only be established by empirical studies. To formulate a test of Ricardian equivalence, it is however vital to extend the standard analysis in deterministic models to stochastic models. In a stochastic model we need to incorporate the fact that agents have to make predictions about future levels of government consumption, and that public debt might be a useful predictor for this purpose. It is therefore necessary that an empirical study distinguishes between debt as a potential source of net wealth, which is the concern of the equivalence proposition, and debt's role as a signal of future levels of government consumption, which is due to the stochastic nature of the world. It is argued that there are few empirical studies that make this distinction, and in case the distinction is made, the evidence is in favor of the Ricardian equivalence proposition, namely that public debt is not net wealth to households. Changing the timing of taxes will therefore not change private consumption. In other words, although the Ricardian equivalence hypothesis is burdened with unrealistic assumptions, it seems (historically) to provide a reasonable approximation of actual data.

1. Introduction

The Ricardian equivalence proposition states that for a given path of government consumption, the timing of taxes, or equivalently, the accumulation and decumulation of public debt, does not affect private consumption. In a closed economy, it therefore also leaves the interest rate, investments and output unchanged. If this is a valid prediction, the scope of fiscal policy as a stabilization tool of the macroeconomy is very limited. The proposition stands in sharp contrast to the basic Keynesian[1] perspective, where a tax reduction/public debt accumulation in one

* The author is grateful for comments and suggestions from the advisor trojka; Karl Jungenfelt, Anders Paalzow and Anders Vredin. All remaining erors are of course the author's sole responsibility.

[1] The label *Keynesian* is used for models that imply that decreased taxes today will increase consumption today, although the present value of government consumption is left unchanged. There are of course other models

period increases private consumption and thus also affects other macro variables like output and unemployment.

The debate about the equivalence proposition was revived with a famous paper by Barro [1974]. Barro argued that the private sector's holding of government bonds does not represent net wealth to the households, and has thus no effect on private consumption. Barro's analysis has been supported by other papers displaying the equivalence result, but there are also contributions to the literature which favor the Keynesian prediction.

In general, models that generate the equivalence result are burdened with some more or less implausible or at least questionable assumptions, where some of the most noted concern "infinite" planning horizons, perfect capital markets and lump-sum taxes. On the other hand, models that do not generate the equivalence result also depend on assumptions that can be questioned, for example regarding liquidity constraints or myopic individuals. As always, it is hard to judge from theory alone what the best/most reasonable description or approximation of the real world is.

Following Barro's paper, there have been several empirical studies aimed at describing what the effects of debt or changed timing of taxes are on private consumption and interest rates. Potentially, the empirical studies could provide a way of evaluating the different competing theories. However, testing Ricardian equivalence is in itself a complex task, where one first has to translate the theoretical predictions into testable hypotheses. Then the appropriate data has to be chosen, as well as a statistical method that generates valid tests or descriptions of data.

The aim of this paper is to establish a link between theoretical predictions and empirical tests of Ricardian equivalence. In the next section, some standard theoretical models, generating both Ricardian and non-Ricardian outcomes, are reviewed, and their crucial features are discussed. Then the concept of Ricardian equivalence in a stochastic world is analyzed, before the results and difficulties of some empirical studies are presented. Finally, in the concluding section, some suggestions for extensions and possible improvements of these empirical studies are discussed.

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than the ones presented by Keynes that display this feature, but in this context, the label Keynesian will be used for models where individuals regard their holdings of government bonds as net wealth.

2. THEORETICAL MODELS

There are two major types of models used to analyze the equivalence theorem. First there are models where agents have an "infinite" planning horizon, and secondly there are models where individuals for one reason or another are "myopic". Note however that the concepts "infinite" and "myopic" are only defined in connection with the planning horizon of the government. Infinite is then synonymous to agents having the same planning horizon as the government, and myopic refers to models where agents have a shorter planning horizon than the government. For example, in a model where both individuals and government plan for two periods, individuals are said to have "infinite" horizons, while in a model where individuals plan for ten periods and the government for eleven, individuals are said to be "myopic".

Two standard examples of these types of models are infinite horizon representative agent models (IRA:s) and overlapping generations models (OLG:s). In this section I will give an overview of some specific models, first a Keynesian model, then a basic IRA model, a basic OLG model, an OLG model with altruism and finally Blanchard's [1985] model. Blanchard's model could be regarded as a mix of an OLG and IRA model, since the time of death is stochastic, but due to the stochastic properties of the death rate, all agents have the same expected planning horizon. Crucial to all of these models is that they assume that the government's actions are know for all future periods, as is the income of households and the interest rate.

2.1 A KEYNESIAN CONSUMPTION FUNCTION

The most basic version of a Keynesian consumption function states that present consumption is a function of current disposable income, or more formally

$$C_t = \beta_0 + \beta_1 YD_t \quad , \tag{1}$$

where C is private consumption, β_0 is an intercept, β_1 is the propensity to consume out of current disposable income, and disposable income is defined as $YD_t \equiv Y_t - T_t$, i.e. total income minus (net) taxes. This type of consumption function has been criticized for lacking an explicit derivation from utility maximizing individuals, although there exist motivations for why this consumption function could result from utility maximizing individuals. The most straight forward motivation is that individuals are liquidity constrained, and the best they could do then is to simply consume all their current disposable income, implying that $\beta_0 = 0$ and $\beta_1 = 1$ in the above consumption function. It is obvious from this formulation alone that it is the amount of taxes individuals pay that matters for the private consumption decision, and not the amount of real resources consumed by the government. Furthermore, changing the timing of taxes would obviously change private consumption, since there are no forward looking elements present in the consumption function.

2.2 A BASIC IRA MODEL[2]

The agent's problem is to maximize his utility from period 0 to infinity according to the following equations

$$\begin{aligned} &\max_{c} \quad \int_0^{\infty} U(c_t)e^{-\delta t}dt \\ &s.t. \quad \dot{w}_t = rw_t + y_t - c_t - \tau_t \\ &\qquad \lim_{t\to\infty} w_t e^{-rt} = 0 \end{aligned} \qquad (2)$$

where $U(c_t)$ represents the momentary utility derived from consuming c_t, δ is the subjective discount factor, $w_t = b_t + d_t$, is total financial wealth consisting of corporate bonds (b) and government debt (d). Labor income (y) is determined exogenously as is the lump-sum tax (τ). The expression $\lim_{t\to\infty} w_t e^{-rt} = 0$ is the transversality or no-Ponzi game condition that restricts the agent from borrowing an infinite amount (i.e. letting w go to minus infinity) for consumption. More precisely it states that as time goes to infinity, the present value of the amount borrowed should go to zero. This has the implication that the agent cannot borrow resources in one period, and in future periods borrow to pay both the previous loan and the interest on this loan. Pursuing such a strategy would make the agent's debt grow at the same speed as the discount factor, and thus the product of the two would not go to zero as required by the imposed condition. If this condition (or a similar one) is not imposed, the maximization problem would not have a solution, since then the agent would choose to have an infinite consumption in each period, financed by an ever increasing personal debt (negative wealth).

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[2] See for example Blanchard and Fisher [1989], ch. 2.

The budget constraint, together with the transversality condition, can be integrated to yield the intertemporal budget constraint

$$\int_0^\infty c_t e^{-\delta t} dt = \int_0^\infty y_t e^{-\delta t} dt - \int_0^\infty \tau_t e^{-\delta t} dt + b_0 + d_0 \ . \tag{3}$$

The government obeys the following budget constraint

$$\begin{aligned} & g_t + rd_t - \dot{d}_t = \tau_t \\ & \lim_{t \to \infty} d_t e^{-rt} = 0 \ , \end{aligned} \tag{4}$$

where g_t is government consumption, and the second equation is again the No-Ponzi game assumption, that in this case restricts the government from running an ever growing debt. Integrating this yields the government's intertemporal budget constraint

$$\int_0^\infty \tau_t e^{-rt} dt = \int_0^\infty g_t e^{-rt} dt + d_0 \ . \tag{5}$$

In the model $\delta = r$, either as a consequence of maximization if we analyze a closed economy, or in the case of an open economy, by assumption, to prevent the economy from going to zero or infinity as time goes to infinity. Taking this into account and then substituting the government budget constraint into the agent's intertemporal budget constraint we get

$$\int_0^\infty c_t e^{-rt} dt = \int_0^\infty y_t e^{-rt} dt - \int_0^\infty g_t e^{-rt} dt + b_0 \ , \tag{6}$$

from which it is obvious that what matters for the agent is not the timing of taxes, but the total amount of resources used by the government.[3] In other words, by checking that the budget

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[3] Since d does not appear in the equation, it is obvious that the agent cannot perceive that government bonds are net wealth.

constraint remains unchanged when we conduct a policy experiment, we know that the consumption path of households will remain unchanged. In particular, this will be the case for the Ricardian experiment, where the timing of taxes is changed and the present value of government consumption is held constant.

The critical assumptions or features of the model giving rise to the equivalence result are quite a few. First of all, the individuals and the government face the same interest rate, implying perfect capital markets without, for example, liquidity constrained individuals. Also, since perfect foresight is assumed, the returns on b and d are the same, and government debt does not represent an additional investment opportunity. Further, there is no gain from tax-smoothing in the model, since only lump-sum taxes are considered.

In the model we also have a representative agent, and thus heterogeneity among individuals can obviously not be considered. Finally, the infinite horizon and the assumption of no population growth imply that there is no way for individuals to evade taxes, by dying and/or levy taxes on other generations. These are only some of the strands of criticism of this model, but maybe some of the more important ones.

In general, deviations from either of the above assumptions could make consumption respond to changes in the timing of taxes. The most discussed assumption regards the planning horizon, where the conclusion is that if individuals have shorter planning horizons than the government, they will in general regard their holdings of government bonds as net wealth, implying that a tax cut today (given government consumption) will increase consumption.

2.3 A BASIC OLG MODEL[4]

In this model individuals live for two periods, they only work in their first period as young, and there is no population growth. Furthermore, as young the agent maximizes utility according to

$$\begin{aligned} &\max_{c_t^i} \quad U\left(c_t^i, c_{t+1}^i\right) \\ &s.t. \quad c_t^i = y_t - w_t - \tau_t^i \\ &\qquad\quad c_{t+1}^i = (1+r)w_t - \tau_{t+1}^i \ , \end{aligned} \tag{7}$$

[4] See for example Diamond [1965] or Blanchard and Fisher [1989] , ch. 3.

where $U\left(c_t^i, c_{t+1}^i\right)$ is the utility function to be maximized by choosing consumption in the first (and thereby also second) period, c_t^i and c_{t+1}^i, where the subscripts t and t+1 stand for consumption as young and old, respectively, while the superscript i stands for the generation born in period i. Total wealth is again $w_t = b_t + d_t$, i.e. the sum of corporate bonds (b_t) and government debt (d_t), y_t is (fixed) labor income that is only received as young, and $\tau_{t,t+1}^i$ is the lump-sum tax paid in the two periods by generation i.

Without population growth and the possibility for the government to levy different taxes on different generations, the tax payments for a specific generation will be equal to half the tax receipts of the government. Total tax receipts for the government are then $T_t = \tau_{t+1}^{i-1} + \tau_t^i$ in period $t=i$ (when generation i is young). The Ricardian question is as usual what the effects are if the government changes the timing of taxes, but keeps the present value of taxes unchanged.

If we take the simplest case of postponing taxes for one period, the government budget constraint implies that $\Delta T_{t+1} = -\Delta T_t(1+r)$. Substituting this into the young person's budget restriction shows that the net effect is zero, thus not affecting his consumption path.[5] For the old generation the story is different, they will in fact increase consumption by the whole tax reduction they receive (in the case with undifferentiated taxes, half of ΔT_t). Thus the tax change will have an effect on aggregate consumption. The losers in this experiment are, of course, the not yet born generation $i+1$, that will have to pay for generation $i-1$'s increased consumption.

In other words, debt is not neutral in this model, since changing the timing of taxes will affect private consumption. Without specifying the production side, capital market and openness, we cannot say how this consumption change will affect other macro variables, but, in general, all variables endogenous to the economy will be affected.

We would of course obtain neutrality if the government could make sure that it is the generation which benefits from a tax cut that later pays the tax plus interest for deferring the

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[5] If we consolidate the budget constraint we get $c_t^i + c_{t+1}^i/(1+r) = y_t - \tau_t^i - \tau_{t+1}^i/(1+r)$, which makes the wealth effect of the considered policy obvious.

tax. However, this type of policy is in general not considered, since it is viewed as unrealistic. In his 1974 paper, Barro "saved" the neutrality result by introducing intergenerational links through altruism. Since then many articles have dealt with the issue. The following section presents a model with symmetric altruism in an OLG model that gives rise to debt neutrality.

2.4 AN OLG MODEL WITH SYMMETRIC ALTRUISM

In this type of model, the standard utility function is modified to allow children and parents to care about each other's utility. Several ways of modeling this has been proposed. Below I will consider the specification used by Burbidge [1983] where generations care about each other in a symmetric way. Other ways of treating altruism can be found in Barro [1974], Buiter [1979], Carmichael [1982], Weil [1987], Abel [1987], Kimball [1987] and Jungenfelt [1991], where most of the articles analyze when gifts or bequests are operative, in the sense that they actually will take place. In general, the altruistic motive can be too weak in some states of the economy, so that agents will then refrain from giving gifts or leaving bequests to neutralize intergenerational transfers implemented by the government. However, in the case of symmetric altruism, either gifts or bequests will be operative (unless the generations, by chance, have an efficient intergenerational allocation of wealth to start with). The way to set up the utility function is as follows, let utility be time separable and define total utility, v_t, for generation i that is born in period t, as

$$v_t = \left(\frac{1+n}{1+\delta}\right)^{-1} v_{t-1} + u\left(c_t^i, c_{t+1}^i\right) + \frac{1+n}{1+\delta} v_{t+1} \quad , \tag{8}$$

which states that total wealth for generation i is a weighted sum of their parents' utility v_{t-1}, the utility, $u\left(c_t^i, c_{t+1}^i\right)$, they derive from their own consumption as young and old, c_t^i and c_{t+1}^i, and their children's utility, v_{t+1}. The weights are the ratio of one plus the population growth, n, and one plus the subjective discount rate, δ. Note that parents' utility is "reversely" discounted compared to childrens', (and thus the label "symmetric" altruism, since parents live before children.) In general this is, of course, an arbitrary discounting rule that might or might not be an appropriate description of the real world, but has analytically nice properties, since gifts and bequests will always be operative, due to this assumption.

By substituting consecutively for the v_{t+i}'s and assuming[6] (arbitrarily) that $v_{t-1} = u_{t-1}$, this can be written as

$$v_t = \sum_{i=-1}^{\infty} \left(\frac{1+n}{1+\rho} \right)^{-i} u\left(c_t^i, c_{t+1}^i\right) \quad , \tag{9}$$

which now is close in spirit to the standard infinite horizon model, the only substantial difference being the underlying assumptions made to arrive at the formulation. In this case, we interpret the model as individuals maximizing their family's utility rather than their individual utility. This utility function is to be maximized subject to the (consolidated) budget constraint[7]

$$c_t^i + g^i + \frac{c_{t+1}^i}{1+r_{t+1}} + \frac{b^i}{1+r_{t+1}} = y_t + g^{i+1} \frac{1+n}{1+r_{t+1}} + \frac{b^{i-1}}{1+n} \quad , \tag{10}$$

where g^i now stands for gifts from generation i to generation $i-1$, and b^i stands for bequests from generation i to generation $i+1$. This maximization with altruism gives rise to additional first order conditions not present in models without altruism, of the form

$$\frac{\partial u\left(c_t^i, c_{t+1}^i\right)}{\partial c_t^i} = \left(\frac{1+r_{t+1}}{1+\rho} \right) \frac{\partial c\left(c_t^{i+1}, c_{t+1}^{i+1}\right)}{\partial c_t^{i+1}} \quad \text{for } i = t-1, t. \tag{11}$$

This represents the utility trade-off *between* generations. In general, this condition is not satisfied unless either gifts or bequests are operative.[8] This has also the implication that a policy that redistributes wealth across generations can and will be fully offset by individuals, by changing the amount of gifts or bequests. Therefore, a policy of the type considered in the preceding section will leave the consumption paths unchanged because no redistribution of wealth will take place, or rather, the redistribution implied by government debt policy will be fully offset by intergenerational transfers in the form of gifts or bequests.

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[6] The assumption is made to avoid the Hall of mirros problem, i.e. that we get an infinite recursion.
[7] The maximization requires $n < \delta$, otherwise the utility goes to infinity with i.
[8] Unless the generations already have a wealth distribution that implies that the first order condition is satisfied.

In the above analysis it was assumed that the utilities of parents and children were evaluated in a symmetric way, with children's utility discounted by $1/(1+\delta)$ and parent's "reversely" discounted by $1+\delta$. This assumption gives rise to the fact that gifts or bequests are always operative, which in turn is crucial for the neutrality result. This type of symmetry is abandoned in for example Abel [1987], where the case of different discount factors for children and parents is analyzed. Abel shows that some restrictions on these parameters are needed to get a meaningful maximization problem, and furthermore that there exists a range of allowable parameter values that imply that neither gifts nor bequests will be operative. In this range a redistributive fiscal policy will have an impact on individuals' wealth and thus on the consumption paths.

Jungenfelt [1991] shows that this range with inoperative gifts and bequests can be made smaller if a family introduces implicit contracts in the form of pension clubs. These clubs will survive in a sub game perfect equilibrium under certain restrictions on the parameters in the model, a crucial parameter being the set-up cost for a pension club, that cannot be too low.

To summarize the above discussion, the effect that changed timing of taxes could have on private consumption is due to wealth reallocations across generations. However, in a model with symmetric altruism, gifts or bequests will always be operative, so that any redistribution implemented by the government will be fully offset by individuals. For example, if a family to begin with has an efficient allocation of wealth between generations, and the government decreases taxes today and raises them in the future, implying that today's generation will be richer, today's generation will simply increase bequests to their children (or children will reduce their gifts) to reestablish the original optimal wealth allocation.

2.5 Blanchard's [1985] model

Blanchard's model is in a way a mix between an IRA-model and an OLG-model, where we have an infinite number of generations alive in every period. This would in general make aggregation impossible in the model, due to differences in the propensity to consume as well as in the wealth of the infinite number of generations in the model. The way this is handled in OLG models is to assume that there are only a few generations alive in any period, so it is simple enough to compute the consumption for each generation and then add them together. Blanchard, on the other hand, makes an assumption about the probability of death, namely that all individuals face the same probability (p) of dying at each point in time. This has the very useful implication that all individuals have the same expected remaining life-time and thus also the same propensity to consume out of wealth, so it does not matter who holds what parts of the wealth in the economy.

Due to the above, the economy behaves as if it had only one representative consumer. This feature makes aggregation possible despite the infinite number of generations. The setup of the model is as follows. At every point in time individuals face a constant probability of dying (p), but the population is held constant by setting the birth rate at p too.[9] A perfect annuity or life insurance market is functioning, so individuals do not face the risk of dying with wealth they could have consumed. This in turn makes the return on savings equal to $r+p$, rather than simply r. In other words, there will not be any involuntary bequests, but instead the savings of a dead individual goes to the insurance company that pays the extra return p on savings to those alive. Individuals born in period s are assumed to maximize expected utility in period t according to

$$\begin{aligned} &\max_{c} \quad E_t\left[\int_t^{\infty} \log c_{s,v} e^{\delta(t-v)} dv\right] \\ &s.t. \quad \dot{w}_t = (r_t + p)w_t + y_t - c_t - \tau_t \\ &\qquad \lim_{t\to\infty} w_{s,v} e^{\int_t^v (r_\mu + p) d\mu} = 0 \end{aligned} \qquad (12)$$

which states that individuals maximize the expected discounted sum of utilities over time, and the momentary utility function is the logarithm of the consumption flow. The maximization is subject to a budget constraint that states that the present value of consumption is equal to the present value of income net of lump-sum taxes (variables defined as before). Since the uncertainty comes completely from the unknown death date, this is equivalent to the following maximization problem

[9] An assumption that will actually be crucial for the results, since, as Buiter [1988] points out, zero birth rate is a necessary and sufficient condition for neutrality in this model, given of course perfect capital markets and lump-sum taxes.

$$\max_{c} \int_{t}^{\infty} \log c_{s,v} e^{(p+\delta)(t-v)} dv$$
$$s.t. \quad \int_{t}^{\infty} c_{s,v} e^{\int_t^v \left(r_\mu + p\right) d\mu} dv = w_{s,t} + h_{s,t} \quad , \tag{13}$$

where we can note that the effect of an uncertain life time is to change the effective discount factor from δ to $p+\delta$. Furthermore, $w_{s,t}$ is non-human wealth, again defined as $w_t \equiv b_t + d_t$, where b_t is now corporate bonds and d_t is public debt. Finally, $h_{s,t}$ is human wealth, defined as

$$h_{s,t} \equiv \int_{0}^{\infty} y_v e^{\int_t^v \left(r_\mu + p\right) d\mu} dv - \int_{0}^{\infty} \tau_v e^{\int_t^v \left(r_\mu + p\right) d\mu} dv \ . \tag{14}$$

The solution to this problem, due to the logarithmic utility, is of the simple form

$$c_{s,t} = (p+\delta)\left[w_{s,t} + h_{s,t}\right] , \tag{15}$$

so that consumption in each period is a (constant) fraction of total discounted wealth. The government is assumed to consume *G*, that does not affect individuals' marginal utility. The path of *G* is known and the government can in any period finance *G* with either lump-sum taxes (*T*) or debt (*D*). The government's budget constraint is

$$G_t + rD_t = \dot{D}_t + T_t \ , \tag{16}$$

which, as before, together with the transversality condition,

$$\lim_{t\to\infty} D_t e^{\int_t^v r_\mu d\mu} = 0 \ , \tag{17}$$

can be integrated to yield the intertemporal budget constraint

$$\int_t^{\infty} T_v e^{\int_t^v r_\mu d\mu} dv = \int_t^{\infty} G_v e^{\int_t^v r_\mu d\mu} dv + D_t \ . \tag{18}$$

Before characterizing the effects of tax reallocation, we have to specify the aggregates in the economy. The aggregate values over all individuals alive today are computed according to

$$X_t = \int_{-\infty}^{t} x_{s,t} p e^{p(s-t)} ds \ , \tag{19}$$

which implies that the aggregate (private) consumption function can be written as

$$C_{s,t} = (p+\delta)\left[W_{s,t} + H_{s,t}\right] , \tag{20}$$

where $W_t = D_t + B_t$, and

$$H_t = \int_0^{\infty} Y_v e^{\int_t^v (r_\mu + p) d\mu} dv - \int_0^{\infty} T_v e^{\int_t^v (r_\mu + p) d\mu} dv \ . \tag{21}$$

Aggregate wealth evolves according to

$$\dot{W}_t = rW_t + Y_t - C_t - T_t \ . \tag{22}$$

What happens in this model, if we for a given path of G reallocate taxes from period t to $t+\kappa$? The government budget constraint gives us the following condition

$$-e^{\int_t^{t+\kappa} r_\mu d\mu} dT_t = dT_{t+\kappa} \ . \tag{23}$$

The effect of this tax reallocation on aggregate consumption in period t is due to a change in human wealth described by

$$-dT_t - dT_{t+\kappa} e^{\int_t^{t+\kappa} \left(r_\mu + p \right) d\mu} , \tag{24}$$

which with (23) can be written as

$$-dT_t \left(1 - e^{-p\kappa} \right) . \tag{25}$$

In case p (or κ) is zero, this expression is equal to zero, but not otherwise. With p equal to zero, we are of course back in the IRA-model. For $p > 0$, the expression in parenthesis is between zero and one, so a tax decrease in t leads to a positive wealth effect and thus to raised consumption in t.

Why do we get this result, or what does $1 - e^{-p\kappa}$ represent? It is simply the probability that someone alive today "evades" the taxes in $t+\kappa$ by dying before that period. In general, this feature alone would not imply that debt is net wealth, since the agents who survive have a larger tax burden in the future if nothing more happens. An alternative way of analyzing the expression is to note that it can be decomposed into one part representing the differences in returns that the government and individuals have, and one part that represents the number of agents that share the debt burden. This is not transparent in the original models, since the birth rate is assumed to be equal to the death rate, p. If we for a moment instead separate these rates and call the birth rate q, and do not impose the restriction that $p = q$, what will the expression in (25) look like then? If we concentrate on the expression in parenthesis, and abstain from canceling out exponents, it is more transparent what the expression represents. Assuming for simplicity that the interest rate is constant, the expression becomes

$$1 - 1 e^{r\kappa} e^{-(r+p)\kappa} e^{p\kappa} e^{-q\kappa} . \tag{26}$$

We could note that by setting $p = q$, and canceling terms, we get back our original expression. The question is then what the parts in the current expression represent. The first term is simply the instant wealth effect from the tax cut, neglecting changes in future tax payments. The second term then picks up the change in future tax payments, and this is where all the action is. First of all, future tax payments will increase with the interest rate, since the government is borrowing today to make the tax cut ($e^{r\kappa}$). Secondly, if the agent saves this tax cut, his return is actually the interest rate plus the return received from the insurance policy, thus his liability decreases with this amount ($e^{-(r+p)\kappa}$). We then have left the effects from changes in the population size. Here we start with the effect from people dying, which implies that the *per capita* debt to pay will increase with the factor $e^{p\kappa}$. The goods news is, however, that there are new people entering the economy, and these entrants will be part of the tax base, which reduces the per capita debt with the factor $e^{-q\kappa}$.

What have we learnt from this decomposition? First of all, if government and individuals were using the same discount factor, the effect would be zero, but since individuals take into account their probability of dying, the agents' discount factor is larger than the government's. However, this effect would totally cancel if the population size was declining at the same rate as agents were dying, i.e. if birth rates were set to zero. This has been pointed out in Weil [1987] and Buiter [1988]. In other words, the individuals currently alive need new entrants into the economy for the wealth effect of postponing taxes to be present.

One final question is how large the change in human wealth would be in response to a reallocation of taxes in this model if we use actual death rates as an estimate for p, and assume like the model that the population size is constant. In Table 1 simple calculations of the value of $1-e^{-p\kappa}$ for two different values of κ and with actual death rates in different age groups, as well as for the total population, are presented. The calculations are based on death probabilities for Sweden in 1988, and since the death rates used are deaths per year, the values of κ should be regarded as the number of years that taxes are postponed.

	[1-e$^{-\kappa p}$] in percent	
Age group	$\kappa = 1$	$\kappa = 10$
20-24	0.07	0.69
40-44	0.19	1.83
50-54	0.44	4.27
60-64	1.16	11.0
75-79	5.11	41.2
Total	1.15	11.0

Table 1. *Percentage effect of tax reallocation on human wealth.*
Source: SCB [1990] and own calculations.

If we translate this into effects on consumption[10], assuming that δ, the subjective discount factor, is 0.10, these range from 0.007 percent to 4.1 percent, with an average for the total population of 1.1 percent for κ=10. In other words, creating a budget deficit of 1 billion SEK that is repaid in ten years would in this model generate, on average, an 11 million SEK rise in private consumption today.

In Figure 1, we illustrate the effects that the actual budget deficit would give rise to according to Blanchard's model in comparison to total actual consumption. The interpretation presented here assumes that the actual world is Ricardian, i.e. there are no wealth effects from postponing taxes, and this is then compared to forecasts made by the Blanchard model for actual government deficits. The Blanchard forecast takes the actual consumption path as the base line, and the wealth effect induced consumption from postponing taxes is then added to the base line.

Actual and predicted consumption

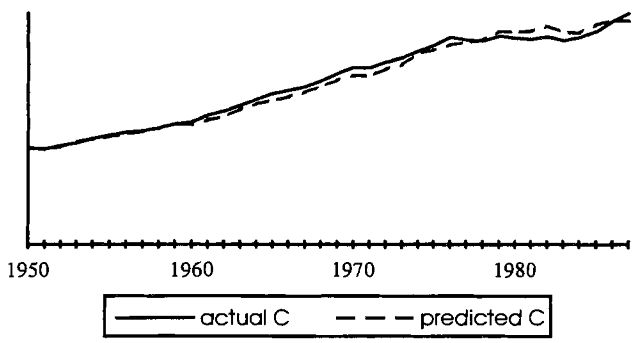

Figure 1. *The solid line represents the actual private consumption that is assumed to be generated by a Ricardian model, while the broken line represents predictions from the Blanchard model, with p for age group 74-79, κ=10 and δ=0.10.*

My conclusion after using eyeball econometrics of Figure 1 is that it seems rather unlikely that ordinary time series methods would be able to identify the three series depicted as being generated by different models. This indicates that ordinary tests of Ricardian equivalence might well accept the equivalence proposition if finite horizons is the only mechanism that creates deviations from it. Stated differently, the Blanchard model gives rise to numerically small

[10] The effect on aggregate consumption is achieved by multiplying with the factor $(p+\theta)$.

effects in this simple minded comparison, which would probably be hard to identify by investigating data. In a more sophisticated analysis of the US, Poterba and Summers [1987] also reach the conclusion that finite life times and deficits generate effects on consumption that are numerically small.

Figure 2 compares the changes in consumption predicted by the Blanchard model with actual consumption detrended in different ways (linear trend, exponential trend and first differences.) In other words, how well do predictions made by the model match actual consumption changes? Again, all background factors are assumed to be unchanged in the illustration, and the figures do not represent a test of the theory. However, the figures might tell us something about the relevance of the model's ability to explain consumption changes in response to changes in the government's budget. The model values are calculated with the actual Swedish government sector financial net savings data, using death probabilities for people at 74-79 years of age, tax postponement of 10 years and finally a discount factor of 10 percent, all parameters chosen to make the magnitude greater and the patterns more visible, without affecting the co-variance between the model prediction and actual consumption changes.

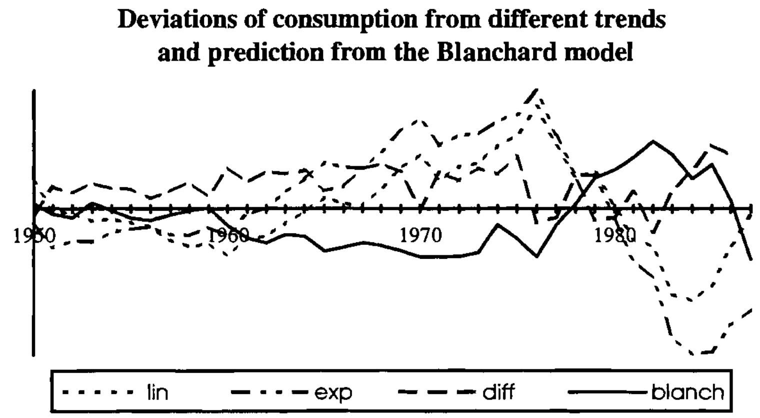

Figure 2. *Consumption changes in response to actual government financial net savings predicted by the model (blanch) compared to deviations of actual consumption from a linear trend (lin), an exponential trend (exp) and a stochastic trend, i.e. stationary first differences (diff).*

The conclusion after looking at these figures is that it seems most unlikely that we will be able to identify the *positive* relation between debt/financial net savings and consumption, predicted by most non-Ricardian models. Of course, the graphs do not take into account background factors that could influence one series negatively and the others positively, but it seems unlikely that we would be able to identify such strong background correlations that would totally change the correlation between the model's predictions and the actual data. These graphs are not intended to serve as a substitute for more precise econometrics, only as a guide to what we might expect to find.

To summarize the conclusions from the Blanchard model. First of all, to obtain a positive wealth effect from postponing taxes, which is the mechanism that creates deviations from Ricardian equivalence, finite horizons is not enough, but we need new entrants into the economy that will pay part of the postponed taxes. Secondly, the numerical magnitude of the effects generated by the model, if we use actual data as input in the model, is small. This suggests that even if the Blanchard model is the appropriate description of the real world, the Ricardian hypothesis will still be a very good approximation of the world. In other words, if we, for one reason or the other, are looking for a model that generates significant deviations from debt neutrality, the Blanchard model is probably not a good choice.

3. RICARDIAN EQUIVALENCE IN A STOCHASTIC WORLD

The above section described models where the path of future government consumption is known. What happens if we relax this rather unrealistic assumption? In the deterministic world, Ricardian equivalence is a well-defined concept; for a given path of government consumption, the timing of taxes or debt leaves private consumption unchanged. A general formulation of the equivalence proposition in the deterministic world could be made by starting with the following general consumption function

$$C_t = f\left(Y^p\right), \tag{27}$$

which simply states that consumption, C_t, is a function of permanent income, Y^p. At this stage we have not said anything about what the components of permanent income are. However, if we take the Ricardian view, we would not allow government debt or taxes to enter the permanent income, but instead the present value of government consumption will affect permanent income. On the other hand, the non-Ricardian view is that debt would represent net wealth to the agents, and is thus a part of permanent income. Here the distinction between Ricardian and non-Ricardian is clear-cut.

However, in the stochastic world, the concept of Ricardian equivalence is less clear cut, which will be discussed below. We particularly want to know how the equivalence proposition should be formulated when performing econometric tests of the proposition.

3.1 DEFINING AND TESTING RICARDIAN EQUIVALENCE IN A STOCHASTIC WORLD

In a stochastic world, we are, generally speaking, dealing with probability distributions or expected values of variables. In a study of Ricardian equivalence, we would study, for example, the probability distributions of private and public consumption as well as probability distributions of taxes, income and so on. The ultimate equivalence would then be that the probability distribution of private consumption does not change in response to changes in the probability distribution of taxes (or debt) in different periods, given that the probability distribution of government consumption remains unchanged.

If we now write down a general consumption function, it can be formulated as

$$C_t = g\left(Y_t^{pe} \middle| I_t\right), \tag{28}$$

which states that consumption today is a function of the *expected* permanent income, Y_t^{pe}, conditional on the information available today, I_t. There are now two possible interpretations of the equivalence proposition. The first would say that debt (or taxes) should not enter *neither* the expected permanent income, *nor* the information set used to make predictions of the permanent income. An important aspect of this formulation is that current debt is not allowed to be useful as a predictor of future levels of government consumption. This formulation is a very narrow definition of Ricardian equivalence, and it seems to be more restrictive than most researchers would like.

An alternative formulation is to define Ricardian equivalence as the case where debt does not have a direct effect on consumption by representing net wealth to the individuals, but is allowed to be a useful predictor of future levels of, for example, government consumption. In other words, we would define Ricardian equivalence as the case where debt is not allowed to enter the Y_t^{pe} measure directly, but is allowed to enter the information set, I_t.

Given that we actually think that the second definition of Ricardian equivalence is the most appropriate, i.e. debt is allowed to enter the information set, it is clear that an econometric study that is not able to separate between the direct and indirect effect from debt will be burdened with great interpretational difficulties. Alternatively, such a study would be occupied with the first definition of Ricardian equivalence in a stochastic world, which does not appear to be a natural extension of the Ricardian concept from the deterministic world. As the title of Barro's 1974 paper suggests, what we want to investigate is whether or not agents regard their holdings of government bonds as net wealth and thus part of their permanent income, not if it can be used as a predictor of future levels of government consumption (although this could, of course, be an interesting question for other discussions than the one of Ricardian equivalence.)

An empirically important aspect of having to make predictions about the permanent income is that in order to evaluate how a Ricardian consumer should respond to a changed taxation or debt creation in one period that potentially implies changed levels of government consumption, we need to distinguish to what extent these effects on government consumption are expected or unexpected (a "shock") and if the effects are permanent or transitory. In other words, it is important to realize that the entire future path of government consumption has to be forecasted, and it is essential for the interpretations with respect to the equivalence proposition to know if a particular shock has permanent or transitory effects on government consumption, since the Ricardian predictions would be substantially different in the two cases.

For example, if an increase in debt signals a permanent expected reduction in government consumption, this would in the Ricardian world imply that private consumption is increased by the same amount that government consumption is reduced. In case the effect on government consumption instead is viewed as temporary, we need to calculate the present value of this reduction, and spread its effects out on all future private consumption. In this case there will be a less than one-to-one substitution of private and government consumption. It is then obvious that if the statistical method that we use can make this distinction between permanent and transitory shocks, the interpretation of results with respect to the equivalence will be greatly facilitated.

Furthermore, if a change in debt or taxes is fully anticipated, its realization would not represent any new information and consumption would not change, whether the equivalence proposition is true or not. This stresses the importance of a econometric study being able to separate between expected changes and shocks.

The above discussion of the general formulation of Ricardian equivalence in a stochastic world points at three key issues. First, the concept of Ricardian equivalence has to be explicitly formulated, also for the case when the path of government consumption has to be forecasted. There are then at least two different interpretations of the Ricardian proposition. One where we state that deficits or debt creation do not have neither direct effects (through affecting individuals' perceived wealth), nor indirect effects (due to potential signaling effects) on private consumption. In the second, and more interesting, definition of Ricardian equivalence in a stochastic world, debt is allowed to have indirect effects but not a direct effect on private consumption.

The second issue is that if we use the latter formulation of the equivalence hypothesis, it is central to incorporate in an empirical study how debt signals future changes in government consumption. Neglecting to incorporate this effect will either imply that we use the first definition of the equivalence proposition (which is probably not what most researchers would consider to be the interesting formulation of the proposition), or that we have serious problems in interpreting the estimated coefficients. Finally, it is also important to clearly distinguish whether effects on, for example, government consumption are permanent or transitory, since again, this is crucial in determining the relevance of the Ricardian equivalence proposition.

4. PREVIOUS TESTS OF RICARDIAN EQUIVALENCE

Since the early seventies several studies have been performed in order to analyze the equivalence proposition. In the following section, these previous studies will be divided into four main categories. First, there are studies based on single equation consumption functions. Secondly, a two-equation model with rational expectations restrictions will be discussed. In the third section, a structural model aimed at estimating deep parameters central for the theoretical derivation of Ricardian equivalence is presented. Finally, studies aimed at investigating the effects of debt policy on the interest rate are discussed.

4.1 SINGLE EQUATION METHODS

Under this label we discuss both estimation of consumption functions of a Keynesian type and estimation based on Euler equations; studies in this spirit include Kochin [1974], Yawitz and Meyer [1976], Tanner [1979], Kormendi [1983], Feldstein [1982], and Bernheim [1987]. To start with the most basic Keynesian consumption function, the following relation is postulated[11]

$$C_t = \beta_0 + \beta_1 YD_t \ , \tag{29}$$

where C is private consumption and YD is disposable income. To estimate the coefficients in this model, we have to determine what the appropriate measures of C and YD are. As a first approximation, we could think of total consumption expenditure, and GNP minus taxes, respectively. An alternative definition of YD would be some measure of permanent income, which would fit more naturally into the equivalence world, but in most cases the discussion below would be the same. To test Ricardian equivalence, we then include public debt (D) or alternatively public deficit[12]. The following model will thus be estimated

$$C_t = \beta_0 + \beta_1 YD_t + \beta_2 D_t + \varepsilon_t \ . \tag{30}$$

The test of Ricardian equivalence is a test of $\beta_2 = 0$, which would imply Ricardian equivalence. If, on the other hand, $\beta_2 > 0$, the implication is that households regard their holdings of public debt as net wealth. The question is whether or not this is an appropriate test of the equivalence proposition.

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[11] Under some special circumstances, (e.g. liquidity constraints or "rules of thumb" near rational behvior), a similar consumption function could be derived also for utility maximizing individuals, but for the present analysis it is not really important how this formulation is obtained.

[12] It is a little odd to say that this is a test of Ricardian equivalence, since it uses a consumption function that would hardly be the result of a Ricardian model, but it could perhaps instead be viewed as a test of the magnitude of the wealth effect in a Keynesian model. However, since this type of estimation in many cases starts with an ad hoc formulation of a consumption function, there is perhaps little point in justifying it afterwards.

If we assume for the moment that we can actually estimate the debt coefficient in a statistically correct way[13], and that we know the probability distribution of interest to perform tests on estimated coefficients, can we then use this approach to draw conclusions about the validity of the equivalence proposition? In general, the answer is no!

This is due to the fact that this type of hypothesis testing is derived from the theoretical models above that assume perfect foresight with respect to (in particular) government consumption. The equivalence proposition states that for a *given path of government consumption*, changing the timing of taxes or debt does not affect private consumption. In reality it is, however, not plausible to assume that the households know the path of government consumption, but rather have to make forecasts of future levels of government consumption.

In the above testing, the role of debt as a predictor of future levels of government consumption is neglected. If, for example, households know that in general a deficit today will imply reductions of government consumption tomorrow, it is consistent with the Ricardian view that private consumption increases with higher debt, not because government bonds are regarded as net wealth, but rather because the expected present value of government consumption is reduced. This points out that it is crucial to take into account how expectations of government consumption are formed, which is more straightforward in a system of equations approach.

An alternative starting point for estimating a single equation is the Euler equation approach. The Euler equation is derived from utility maximizing agents as in, for example, the IRA model discussed in section 2.2, and is thus derived in a theoretically more satisfying (or at least explicit) way than the previously discussed consumption function. By using the consumption Euler equation, Hall [1978] derives the following relation between present and past consumption

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[13] There are potentially several statistical problems in estimating the postulated relation. To start with, there might be a problem of simultaneity bias, since it is likely that the explanatory variables are not exogenous with respect to private consumption. Another potential problem is that in practice, many more variables are introduced in the right hand side to capture different aspects of the income measure, which in turn is likely to introduce multicollinearity. Finally, issues of non-stationarity have often been neglected, which could make the inference invalid.

$$C_t = C_{t-1} + \varepsilon_t, \tag{31}$$

which is the, since then, well known random walk in consumption implied by the permanent income hypothesis. Although this formulation is valid under rather general conditions, there are some restrictions or approximations underlying this formulation. Either the individuals will have to be risk neutral or have quadratic utility functions, or the stochastic changes will have to be small enough to motivate a linearization of the underlying concave utility function. The good news from a statistical point of view are that we do not have to incorporate more variables in the right hand side, thus avoiding multicollinearity and simultaneity bias. However, we still have the question of how this formulation could be used when testing Ricardian equivalence.

At this stage it is vital to distinguish between the Ricardian and permanent income hypothesis, and although Ricardian equivalence implies that the permanent income hypothesis is true, the reverse is *not* true. The permanent income hypothesis states that households make predictions of all their future incomes and then try to smooth consumption in such a way that they expect to consume the same amount in every period. However, the permanent income hypothesis does not tell us what the components of permanent income are. This is on the other hand the central question in the Ricardian equivalence proposition: is government debt net wealth and thus a contributor to net income or not? In other words, if the permanent income hypothesis is correct, the Ricardian proposition could still be either true or false.

What does the distinction between the two hypotheses imply for tests of Ricardian equivalence? As a first thought, we might consider including *lagged* debt and then test whether or not its coefficient is equal to zero. However, the problem with such a test is that the coefficient will become zero if the permanent income hypothesis is true, irrespectively of the validity of Ricardian equivalence. The fact that *any lagged* variable will get a zero coefficient in a regression of the Euler equation if the permanent income hypothesis is true, is one of the central insights of Hall's paper. In turn this implies that *lagged* debt cannot be used to test the validity of Ricardian equivalence in an Euler equation, since its coefficient should be zero if the permanent income hypothesis is valid, irrespectively of the validity of the Ricardian hypothesis.

What about using contemporaneous debt in the Euler equation instead? This would make it possible to avoid accepting Ricardian equivalence due to an acceptance of the permanent income hypothesis. However, this creates new problems along the lines discussed for Keynesian consumption functions, namely that debt could then be useful for predicting future levels of government consumption, as well as future levels of income. In addition to this problem with interpreting the estimated coefficient, we have again introduced some of the statistical problems that the Euler equation could otherwise avoid compared to the Keynesian formulation.

To summarize, using a single equations approach when investigating the equivalence proposition seems burdened with serious limitations, both from a purely statistical point of view, and more importantly, from the point of designing a valid test of the Ricardian hypothesis, since we need to incorporate how expectations about future levels of government consumption are formed. Perhaps not very surprisingly, the evidence from this type of studies is mixed. Some authors claim to find support for the Ricardian hypothesis, while others reject the hypothesis. This may suggest not only that it is hard in general to test theory, but also that this particular testing strategy of the equivalence hypothesis is burdened with both statistical and interpretational difficulties.

4.2 RATIONAL EXPECTATIONS AND CROSS-EQUATIONS RESTRICTIONS

One study that combines utility maximizing individuals with a government sector in order to examine Ricardian equivalence is Aschauer [1985]. The model specified is based on rational expectations, where individuals derive utility from government consumption as well as private consumption. More formally, agents maximize with respect to effective consumption, C_t^*, defined as the weighted sum of government and private consumption, $C_t^* = C_t + \theta G_t$, where θ describes a constant marginal rate of substitution between private and government consumption. Assuming also a quadratic momentary utility function, Aschauer derives the following consumption function

$$C_t = \alpha + \beta C_{t-1} + \beta\theta G_{t-1} + \theta G_t^e + u_t \ , \tag{32}$$

which he combines with a forecasting equation for government consumption

$$G_t = \gamma + \varepsilon(L)G_{t-1} + \omega(L)D_{t-1} + v_t \ . \tag{33}$$

This forecasting equation uses past values of government consumption and deficits to make predictions of government consumption. Plugging (33) into (32) and rewriting yields

$$\begin{aligned} C_t &= \delta + \beta C_{t-1} + \nu(L)G_{t-1} + \mu(L)D_{t-1} + u_t \\ G_t &= \gamma + \varepsilon(L)G_{t-1} + \omega(L)D_{t-1} + v_t \end{aligned} \quad , \tag{34}$$

which implies the following set of cross-equation restrictions

$$\begin{aligned} &\delta = \alpha + \theta\gamma \\ &\nu_i = \begin{cases} \theta(\beta - \varepsilon_i) & for \quad i = 1 \\ -\theta\varepsilon_i & for \quad i = 2,\ldots,n \end{cases} \\ &\mu_j = -\theta\omega_j \qquad\qquad for \quad j = 1,\ldots,m \quad . \end{aligned} \tag{35}$$

Aschauer's interpretation of these cross-equation restrictions is then that if they do not hold, debt has an impact on private consumption which differs from the impact justified from the observed predictive power that debt has for future levels of government consumption. This interpretation of the Ricardian hypothesis is in line with the preferred definition of Ricardian equivalence in a stochastic world discussed in Section 3, where debt is allowed to enter the information set, but not the permanent income measure directly.

Another way of interpreting Aschauer is that he removes the part of debt that works as a signal of future levels of government consumption, and investigates if the remaining part of debt has an impact on private consumption. This would then be regarded as the wealth, or direct, effect debt has on private consumption. The null hypothesis of valid cross-equation restrictions, that is, no wealth effect, is interpreted as a test of Ricardian equivalence.

At this point it is vital to understand why this test will actually be able to separate between the permanent income and the Ricardian hypothesis. In the standard Euler equation presented in Section 4.1, we noted that we would *not* be able to separate between the two hypotheses.

However, in this study government consumption enters the utility function via the specification of effective consumption. In the definition of effective consumption, the parameter θ describes to what extent government consumption substitutes for private consumption. When we then solve the Euler equation (which is now defined for effective consumption) for private consumption, both lagged private *and* government consumption will be present in the right hand side, with lagged government consumption multiplied by the additional factor θ. This implies that as long as government consumption actually substitutes for private consumption, so that θ is non zero, this modified permanent income hypothesis does allow for an additional variable with non zero coefficient. The central role of θ to achieve identification of this model can also be seen in the cross-equation restrictions, that all will become unidentified if θ is set to zero.

In the estimation, it is therefore vital to test if θ is actually significantly different from zero, which Aschauer concludes it is, and the point estimate indicates that a dollar spent on government consumption is worth approximately twenty cents of private consumption in utility terms. Furthermore, Aschauer concludes that he cannot reject the joint hypothesis of rational expectations and Ricardian equivalence at conventional levels of significance. In other words, debt only plays a role in explaining private consumption to the extent that it is a useful signal for future levels of government consumption, but debt has no wealth effect on consumption.

Aschauer's formulation is one of the most rigorous ones for studies using Euler equations to test the equivalence proposition. The framework incorporates the forecast equation of the government consumption, and makes use of the reasonable rational expectations concept to derive testable hypotheses that do not suffer from interpretational vagueness.

4.3 ESTIMATING DEEP PARAMETERS

This section describes the model of Leiderman and Razin [1987], which is based on Blanchard's [1985] framework, where all individuals face a probability γ to survive ($\gamma = 1 - p$, where p is the death rate in Blanchard's model) to the next period. Further, they focus on consumption expenditure (X_t) as a flow into a stock of consumption goods (C_t), and it is from this stock that consumers derive their utility. Formally, individuals maximize expected utility according to

$$
\begin{aligned}
\max_{c} \quad & E_t \sum_{\tau=0}^{\infty} (\gamma\delta)^{-t} U(c_{t+\tau}) \\
s.t. \quad & c_t = (1-\phi)c_{t-1} + x_t \\
& x_t = b_t + y_t - (R/\gamma)b_{t-1} \\
& \lim_{t\to\infty} (\gamma/R)^t b_t = 0 \qquad ,
\end{aligned}
\tag{36}
$$

where $U(\cdot)$ is the momentary utility function, c_t and x_t are the per capita stock and flow of consumption goods (capital letters then represent the aggregates over households of the same variables). Moreover, the stock of consumption goods is depreciating with ϕ in each period. Again labor income is y_t and assumed to be exogenous, and $R = 1 + r$, where r is the constant interest rate. The subjective discount factor is δ, and finally, b_t are bonds issued by agents, i.e. the negative of wealth in previous models. The last line is the no-Ponzi game assumption that constrains the agents to have no remaining debt in present value terms as time goes to infinity.

In addition to the utility maximizing individuals with access to a perfect capital market, the authors allow for a part of the population $(1-\Pi)$ to be liquidity constrained according to

$$X_{c,t} = Y_{c,t-1} + v_t \ , \tag{37}$$

so that they use all of last period's income for consumption expenditure, except for a stochastic term v_t. Aggregate consumption expenditure is then

$$X_t = \Pi X_{u,t} + (1-\Pi)X_{c,t} \ , \tag{38}$$

where $X_{u,t}$ comes from unconstrained individuals who solve the maximization problem in (36). For the empirical implementation they also specify first order autoregressive processes for income (Y) and taxes (T) as

$$\Delta Y_t = \rho_Y \Delta Y_{t-1} + \eta_{Y,t} \tag{39}$$

$$\Delta T_t = \rho_T \Delta T_{t-1} + \eta_{T,t} \ . \tag{40}$$

In the last part of the empirical investigation they also include government consumption (G) in the same manner

$$\Delta G_t = \rho_G \Delta G_{t-1} + \eta_{G,t} , \tag{41}$$

and adjust the budget constraint. The maximization problem is also modified to allow for substitution between private and public consumption, where government consumption is assumed to substitute for private consumption with a factor θ, i.e. a dollar of government consumption is worth θ dollars of private consumption.

The interesting feature of this approach is that it estimates deep parameters that appear as critical assumptions in the derivation of the equivalence hypothesis; the death rate, $1-\gamma$, and the fraction of liquidity constrained individuals, $1-\Pi$, should both be equal to zero according to the standard assumptions used to derive debt neutrality. In other words, the authors have allowed for two potential sources for deviations from debt neutrality, and investigate whether or not data support these standard assumptions made in Ricardian models.

The advantage of this study is that the interpretation and test of Ricardian equivalence is very straightforward once the model is formulated and the parameters estimated (which is actually not totally trivial). However, the drawback of this very structural approach, is that we can only conclude that we will not have deviations from Ricardian equivalence due to violations of these particular assumptions, but there might at the same time be other sources that in the real world will invalidate the predictions of Ricardian models. If we then do not test *all* these potential sources that could create deviations from the equivalence result, we will not be able to tell if Ricardian equivalence is totally valid. In this study, for example, we can conclude that we do not have deviations from Ricardian equivalence due to finite lives or liquidity constrained individuals, but this will only be equivalent to accepting the equivalence hypothesis if there are no other factors that can create deviations from the equivalence proposition.

Another unfortunate feature of the model is that Leiderman and Razin do not have an explicit formulation of the government sector in the form of a budget restriction, which seems natural in a study of the equivalence proposition. For example, it seems plausible that there might be some restrictions on the processes for T, G and Y in terms of common trends that would provide additional restrictions that are now neglected.

Furthermore, theory (or common sense) places restrictions on γ, Π, θ, and ϕ, but none of these restrictions seems to have been included in the estimation. In their estimation some of these obvious restrictions are violated, but the authors seem to ignore this and proceeds with the analysis. Their conclusion is that the estimated coefficients support the equivalence hypothesis, or rather, that individuals do not act as if they have finite lives or are liquidity constrained. Another study that investigates the proportion of liquidity constrained individuals is Campbell and Mankiw [1991], who find that a substantially larger fraction of households are liquidity constrained for a cross-section of countries (not including Israel that is analyzed in the above study), which of course would imply a violation of a standard assumption in Ricardian models.

4.4 INTEREST RATES AND THE TERM STRUCTURE

Most studies of RE concentrate on the consumption function, but this is in general not the only variable that is assumed to be affected by budget deficits by opponents of RE. One other key variable is supposedly the interest rate, which is assumed to be positively correlated with deficits, usually via a crowding-out mechanism. This approach then hinges on the assumption that the interest rate is not given from a world capital market, but is determined endogenously within the country. In other words, the approach might be of more relevance in for example the US than in Sweden. In a closed economy, the mechanism would be working through changes in economy wide savings in response to a change in the government's budget stance. In a Ricardian world, dissaving in the public sector would be fully off-set by increased saving in the private sector, again of course postulating that the level of government spending is unaffected by the change in the government budget. In other words, economy wide savings will be kept constant, and the interest rate would be unaffected.

In Plosser [1982] and Plosser [1987] this potential deviation from RE is investigated by using a term structure model for interest rates, which is combined with an ad hoc macro model for explaining the spot interest rate. Below, the 1982 paper will be described. Other papers investigating the effect of deficits on the interest rate are Evans [1987], Boothe and Reid [1989], and Quigley and Porter-Hudak [1994].

The model for the equilibrium expected return to an n-period bond is

$$E_t\left[H_{n,t+1}\right] = R_{1,t} + \phi_{n,t} \ , \tag{42}$$

where $R_{1,t}$ is the spot interest rate and $\phi_{n,t}$ is a marginal liquidity premium. To this basic rational expectations model of the term structure macro variables are added in order to explain $R_{1,t}$ according to

$$R_{1,t} = a(L)' X_t + \eta_t \ , \tag{43}$$

where X_t could consist of any variables that could predict the spot rate. By defining $Z_t = \left[X_t \vdots \eta_t\right]$ and then specifying a process for Z_t according to $Z_t = D(L)u_t$ we get

$$H_{n,t+1} - E_t\left[H_{n,t+1}\right] = b_n'\left(Z_{t+1} - E_t\left[Z_{t+1}\right]\right) . \tag{44}$$

By further specifying a VAR model for X_t, we obtain the following model to be estimated

$$X_{t+1} = A(L)X_t + u_{t+1} \tag{45}$$

$$H_{t+1} - R_{1,t} = \phi + B\left[X_{t+1} - A(L)X_t\right] + \nu_t \ . \tag{46}$$

These equations state that the unexpected or excess return for bonds of different maturities is explained by a vector of liquidity premia ϕ and by unexpected changes in the policy variables in X, through the coefficient matrix B that is to be estimated. Furthermore, u_{t+1}, the innovations in X, are obtained after estimating the lag polynomial $A(L)$. The variables Plosser includes in X_t are government spending, government debt held by the public sector, and government debt held by the Federal Reserve (the monetized debt).

Plosser states that tests of the non-linear cross-equation restriction on $A(L)$ are joint tests of the market efficiency/rational expectations hypothesis and the expectations model of the term structure. To summarize Plosser's conclusions; he finds that the government spending variable is more important than the two debt variables in explaining movements in the interest rate. Further, only government consumption has a significant positive impact on the interest rate (although quantitatively the effect is small). In other words, Plosser's findings are consistent with a Ricardian model.

In his 1987 paper, Plosser uses the framework presented above, but uses new data. Further, a connection between debt shocks and *ex ante* real interest rates is analyzed, as well as the importance of expected future deficits. The vector of "policy" variables (X), includes industrial production per capita, the inflation rate, real per capita public debt, real per capita debt held by the Federal Reserve, and real per capita military outlays, which is used as a proxy for (temporary) Federal spending, all measured as growth rates. Finally, the one-period yield ($R_{1,t}$), is now also included in the "policy" vector.

The estimation again suggests a very small impact of the financing variables, on the interest rate, and the only noted effect is a *fall* in interest rates due to a positive debt shock. In other words, the result contradicts the conventional wisdom of a positive correlation between debt and interest rates.

The basic critique of Plosser's papers is the lack of clearly stated identifying assumptions and an ad hoc way of modeling the influence of the macro variables. What variables should be included, and what parameters can be given structural interpretations? These are issues central to the interpretation of the results. However, if we take the interpretations given in the paper, the study supports the equivalence hypothesis.

5. Conclusions and Extensions

The purpose of this paper has been to link the theoretical predictions of Ricardian equivalence to empirical tests. It is first noted that studies based on estimating a single equation consumption relation are burdened with both statistical and interpretational difficulties, and do not seem to provide a fruitful way for determining the validity of the equivalence theorem. The interpretational difficulties are to a large extent due to the fact that the implicit models underlying these tests all assume perfect foresight with respect to government consumption. In reality however, debt or deficits are likely to affect the *expected* value of future government consumption, which has to be taken into account when performing tests in a stochastic world.

Aschauer [1985] incorporates this potential signaling effect of debt when estimating a two equation model with cross-equations restrictions implied by rational expectations. The estimation methods could potentially be improved, to incorporate issues like non-stationarity and co-integrated time series. Furthermore, in the recent macro literature (see for example Zeldes [1989], Caballero [1990], or Weil [1993]), it has been popular to explain deviation from the predictions made by the standard permanent income model to risk averse individuals and precautionary savings. Since risk aversion creates an additional reason why lagged variables, for example lagged wealth, enter the Euler equation (see Sheshinski [1988]), this could be an interesting extension to Aschauer's model.

To summarize the theoretical and empirical relevance of Ricardian equivalence, there are few (if any) well formulated empirical studies that reject the equivalence proposition's predictions, although the theoretical models generating the equivalence are burdened with unrealistic assumptions. The interpretation of the evidence is that either these unrealistic assumptions cancel each other out, or the equivalence proposition is actually a decent approximation of the real world. The latter interpretation is to some extent also justified by the numerical example in Blanchard's non-Ricardian model, where the model generates modest deviations from the equivalence hypothesis.

However, there are other issues of debt policy which might not have been operating in the investigated economies in the past, since most studies deal with well developed countries. For example, when debt reaches extreme levels as proportion of GDP, and with a large amount of the debt placed outside the country, this could introduce other mechanisms than the ones we have focused on here, like exchange rate crises. It seems less likely that the Ricardian proposition would be a reasonable approximation in these cases, although it is a fair description of the real world at more moderate debt levels.

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Essay II

An Investigation of Ricardian Equivalence in a Common Trends Model*

Abstract

A common trends model for gross national income, private consumption, government consumption and net taxes is estimated on US data. The system has two cointegrating vectors and thus two common stochastic trends, interpreted as a technology trend and a public sector trend. The two temporary shocks are interpreted as a private demand and government financing shock, respectively. Theoretical models suggest that the two cointegrating vectors could be due to the private and public sectors' intertemporal budget constraints. We find two co-integrating vectors, as predicted by no-Ponzi game constraints on the sectors. However, a stronger version of the no-Ponzi game constraint is a solvency condition, which implies particular co-integrating vectors. These cointegration vectors are both rejected for the sample period, indicating that the public sector will not be able to repay its debt if the current policy is maintained. However, the private sector is at the same time accumulating wealth, which is consistent with predictions from a Ricardian model. Further, the equivalence theorem predicts that private consumption should be unaffected by financing shocks. Data, however, indicate that there is a significant short run effect on both income and private consumption from the financing shock, but the effect indicates that increasing taxes is accompanied by increasing private consumption, contrary to both standard Ricardian and Keynesian models. In the theoretical world, this type of pattern could be generated in models with risk averse individuals and uncertainty about future taxes.

1. Introduction

This paper investigates the Ricardian equivalence theorem. The theorem states that for a given path of government expenditures, the timing of taxes should not affect the consumption decision made by individuals paying the taxes. The simple idea behind the theorem is that substituting taxes today for taxes plus interest tomorrow, via debt financing, does not affect the wealth of individuals. This type of government action leaves the intertemporal budget constraint of individuals unchanged and will therefore not change consumption paths or investments. Thus, Ricardian predictions about the effects of fiscal policy that leave government consumption unchanged, are quite far from the (more intervention optimistic)

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Keynesian ones, partly because the Keynesian idea (in its most simple version) is that agents only care about present disposable income. There are of course other more sophisticated models that create some room for the type of stabilization policy implied by a simple Keynesian model, for example overlapping generations models without operative links between generations. However, to make the reference to these models simple in the following discussions, the label "Keynesian" will be used for models which imply that government debt is regarded as net wealth by today's generation (although the original model builders would in some cases object to this label). It is important to realize that there is a way to affect the households' consumption also in Ricardian models, but then the government has to change the present value of its consumption, and the private consumption change will be distributed over the entire planning horizon (giving little use of such a policy for stabilization purposes).

There are some common features of most models generating the equivalence result. The most noted are perhaps infinite lives, perfect capital markets and lump sum taxes.[1] In general we would not think that all of these assumptions would be true in the real world, which has perhaps been the reason for some economists to reject the equivalence proposition completely. The interesting question is, however, not if the equivalence hypothesis is true, but rather how good an approximation it is when we use aggregated data. Taking this a step further, are Ricardian or non-Ricardian models most useful as starting points for studies of fiscal policy on a macro level? In this respect the equivalence question is an interesting and valid question, although models that generate 100 percent tax discounting are burdened with some unrealistic assumptions.

Several empirical studies of the validity of the equivalence hypothesis have been conducted, with both different methodology and, not surprisingly, conclusions. For an overview, see for example Bernheim [1987], Seater [1993], and Becker [1995a]. In Aschauer [1985] and Becker [1995a] the importance of distinguishing between models with perfect foresight and stochastic models is stressed. The importance is due to the original formulation of the equivalence hypothesis, where changed timing of taxes should leave private consumption unchanged, *given* the path of government consumption. However, in a model that does not assume perfect

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[1] What we really mean by "infinite lives" is a bit less restrictive; we require the agent to have the same planning horizon as the government, which could be justified by saying that agents care enough about their children (altruism), or by assuming that also the government has a finite horizon. The crucial factor about the capital market is that the individuals and the government face the same interest rate, and can of course borrow or lend without binding constraints.

foresight, government consumption has to be forecasted by agents, and a reasonable assumption is that agents use all available information to make these forecasts. One part of the information set is then potentially government debt. Debt could thus affect private consumption through its forecasting ability of government consumption. This indirect, or signaling, effect is not a violation of a reasonably formulated equivalence proposition in a stochastic model. However, if debt has a direct, or wealth, effect on private consumption, this is regarded as a violation of Ricardian equivalence.

The stochastic formulation of the equivalence hypothesis also points at the importance of distinguishing between expected and unexpected changes in a variable. If for example government consumption increases, and this is a predicted change, we would not expect private consumption to change in this period, but in the period when the change became known. If, on the other hand, the change were unexpected (a ”shock”), we would expect private consumption to change if the world is Ricardian. The magnitude of the response will, however, be contingent on the expected duration of the shock. If the above increase in government consumption is expected to be permanent, the reduction in private consumption will be larger than if the increase is temporary. This makes the distinction between permanent and transitory shocks important when deciding on the validity of the equivalence proposition.

In this paper, a vector autoregressive system with cointegrated restrictions, or a *common trends* model, is used to investigate the validity of the equivalence theorem. Within this framework we cannot only explicitly handle the statistical issues of simultaneity and non-stationarity, but also the theoretically important aspects of expected changes versus shocks and distinguish between shocks that are transitory and permanent. This is desirable both from a methodological and a theoretical perspective.

In order to interpret a statistical model, we need to impose a certain amount of structure (or restrictions) on the empirical model. This structure is derived from the theory that underlies the empirical specification. There is always a trade-off between the amount of structure imposed, and how free data will be to determine the empirical relations. In many studies of the Ricardian equivalence hypothesis, there has either been very little structure imposed or a great deal of structure (see Becker [1995a] for an overview). The problem with the studies that lack structure is that they most often neglect the important aspects of a stochastic world, as discussed above. On the other hand, by specifying a complete structural model, there is little room for data to display features that lie outside the particular model used, and since the real world is by definition more complex than any model of the world, we might lose important insights by imposing too much structure.

The approach taken in this paper combines the important aspects of a stochastic world and its general implications about the equivalence proposition, without formulating and testing a particular theoretical model. The restrictions imposed in this study are therefore restrictions that are valid in many different stochastic models of Ricardian equivalence. In particular, co-integrating relations suggested by the intertemporal budget constraints associated with the private and public sectors are used, and the restrictions used to identify shocks are based on a general formulation of Ricardian equivalence. The purpose of this paper is thus to study the dynamic patterns of income, consumption and government activity, without performing a completely structural test of the Ricardian hypothesis. Instead we will compare the properties of the estimated model, with the properties of theoretical stochastic models, Ricardian as well as non-Ricardian.

The model is estimated with US quarterly data from CITIBASE ranging from 1960 to 1993. The variables included are GDP, private consumption measured as total consumption expenditures, government expenditures net of interest payments and transfers, and, finally, government receipts net of interest payments and transfers. All variables are in 1987 dollars and per capita. We can reject the stronger version of intertemporal budget balance for the period regarding both the private and the public sector, which implies that neither sector will end up with zero debt or wealth as time goes to infinity, if the present policy or behavior is maintained. The private sector accumulates wealth, while the government is running a deficit. However, although the theoretical co-integrating vectors can be rejected at very low risk levels, the point estimates of the freely estimated co-integrated vectors are close to the theoretical ones (the coefficients have the same sign and are of roughly the same magnitude), and the impulse responses generated with the two different sets of co-integrating constraints are very similar. Furthermore, the no Ponzi game condition, which is a weaker condition than the solvency condition, has not been rejected, since we find two co-integrating vectors.

From the Ricardian perspective, the most interesting relation to study is how consumption responds to a government financing shock. In the study, consumption is *positively* correlated with taxes. This suggests that neither standard Ricardian nor Keynesian models are supported by the results presented in this paper. A theoretical explanation of this correlation can be found in Chan [1983] and Becker [1995b]. These papers discuss how precautionary savings respond to lower taxes today, which are compensated by higher, but uncertain, taxes in the future.

The paper starts with a section discussing the methodology of the common trends framework, and connects this to co-integrating relations derived from the private and public sectors' intertemporal budget constraints. In the following section, the model is estimated and the co-integration vectors are tested. Further, impulse responses and variance decompositions are presented, to investigate the variables responses to different shocks. The paper ends with a discussion of some conclusions from, and limitations with, the present study.

2. Ricardian Equivalence and Cointegration

The Ricardian equivalence proposition states that private consumption today should be unaffected by both expected and unexpected changes in the financing of a given path of government consumption. If we, however, only study expected changes in the financing, this would be the case also in a stochastic overlapping generations model, since agents would then have changed their consumption at the time the change became known, and would not make additional adjustments today. It is therefore important to distinguish expected changes from unexpected ones in the empirical study. Furthermore, in the Ricardian world, an unexpected permanent increase in government consumption should crowd out private consumption one for one, if we assume that government consumption does not enter individuals' utility functions in a non-separable way.

The empirical analysis is based on a VAR model with co-integrating restrictions, as developed and used by for example Engel and Granger [1987], Lütkepohl [1989], Johansen and Juselius [1990], Warne [1990] and Englund, Vredin and Warne [1994]. This approach makes it possible to study how different variables respond to different transitory and permanent shocks, and thus we do not study effects from expected changes, since they do not discriminate between Ricardian and non-Ricardian models. A brief description of this method is presented below.

2.1 CO-INTEGRATION AND COMMON TRENDS

A general problem with time series studies of macro data is that the series are non-stationary or trending, implying that the mean and variance of a particular series change over time, which violates some basic assumptions made for standard statistical inference. In order to analyze series with this property, it was earlier common practice to detrend the series either by taking out a linear trend or by using first differences. Starting with Engel and Granger [1987], the notion of co-integration was introduced, which involves a slightly different approach to detrending. The concepts of co-integration, common trends, and related issues will now be discussed in connection with different representations of a vector autoregressive model. (A more formal analysis is presented in the appendix.)

Start by assuming that we have an n dimensional system of variables that can be represented by a structural model given by

$$X_t = B(0)X_t + B(L)X_{t-1} + \varphi_t \qquad (1)$$

where $X_t \equiv [x_{1t} \ \cdots \ x_{nt}]'$ is the $n \times 1$ vector of variables, $B(0)$ is an $n \times n$ matrix of coefficients of contemporaneous relations, $B(L)$ is a matrix of lagpolynomials, and $\varphi_t \equiv [\varphi_{1t} \ \cdots \ \varphi_{nt}]$ is an $n \times 1$ vector of normally distributed errors. We assume that $E[\varphi\varphi']$ is diagonal, so the shocks are independent, which motivates the label structural for this representation. In the following method discussion, this model will be presented in different forms (or representations), but it is important to note that these different representations can (under certain assumptions) be derived from the same underlying structural model.

The first representation we can use is the vector autoregressive model

$$\Pi(L)X_t = \rho + \varepsilon_t , \qquad (2)$$

which can be viewed as a reduced form of the structural model. $\Pi(L)$ is a matrix of lagpolynomials and ε_t is the (new) vector of disturbances. In this representation $E[\varepsilon\varepsilon']$ is a positive definite matrix and, in general, not diagonal, since these errors are linear combinations of the structural (independent) errors. In the appendix, the connection between the different representations is made explicit in terms of coefficient matrixes and errors.

To study co-integration we can use the error correction representation of this system, or

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \rho + \varepsilon_t \quad , \tag{3}$$

where the Γ's are $n \times n$ coefficient matrixes. Co-integration implies that the matrix Π can be written as $\alpha\beta'$, where α and β are $n \times r$ matrixes, and with $\beta' X$ stationary although X is not. β contains the co-integrating vectors, and α is referred to as the matrix of adjustment coefficients.

By investigating the rank of the coefficient matrix Π, we can determine the number of co-integrating vectors. If the rank(Π) = r is equal to the dim(X) = n, then the vector process X is stationary, if $r = 0$, X is an ordinary difference stationary process, while if $0 < r < n$ we have r co-integrating vectors.

To understand the applicability of co-integration for economic modeling, we can use the classical example of consumption and income, which are two series that can often be described as difference stationary if analyzed separately. If, however, we for some reason think that consumption in the long run will be a fixed proportion of income, a linear combination of the logs of income and consumption (or consumption over income) provides a stationary relationship between the levels. In this case consumption and income are co-integrated (and in the above notation, $n = 2$ and $r = 1$).

The Johansen and Juselius [1990] maximum likelihood procedure produces estimates of α and β, and also two tests of the number of co-integrating vectors. The first test is the so called trace test and the second the lambda max test[2]. These tests do not have ordinary χ^2

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[2] The trace test = $-T \sum_{i=r+1}^{p} \ln\left(1 - \hat{\lambda}_i\right)$

The lambda max test = $-T \ln\left(1 - \hat{\lambda}_{r+1}\right)$, distributions, but are multivariate versions of the Dickey-Fuller distributions and are for some cases tabulated in Johansen and Juselius [1990] and in Osterwald-Lenum [1992] .

The common trends representation of the system is:

$$X_t = X_0 + A\tau_t + D(L)\varepsilon_t \tag{4}$$

where $\tau_t = \mu + \tau_{t-1} + \psi_t$. τ_t represents the underlying common stochastic trends that drive the variables in the long run. Here we assume that $E[\psi\psi']$ is diagonal, so the errors driving the stochastic trends can be viewed as structural. Furthermore, these errors are permanent shocks if $A \neq \mathbf{0}$, since they affect the variables through the random walks, τ_t. As shown by Stock and Watson [1988], the number of common stochastic trends (and thus also the number of permanent shocks) is equal to $k = n - r$, i.e., the number of common trends is equal to the dimension of the system minus the number of co-integrating relations. In the consumption income example, k = 2-1 = 1, and thus we can conclude that income and consumption are driven by one common stochastic trend.

The vector moving average (VMA) representation of the system is written as:

$$\Delta X_t = \delta + R(L)\varphi_t \quad . \tag{5}$$

Here the error vector is defined as $\varphi_t \equiv [\psi_t' \nu_t']'$, where the first part contains the kx1 permanent shocks, and ν contains rx1 transitory shocks. The transitory shocks are only affecting the system for a limited time, and eventually the effects from this type of shock die out. Note, however, that in the short run, both permanent and transitory shocks will affect the system, but in the long run, only permanent shocks matter. The reduced form errors, ε, are linear combinations of the structural shocks, so $\varepsilon_t = F^{-1}\varphi_t$. To identify F , it is usually assumed that the structural shocks are independent.

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where $\hat{\lambda}$ are the estimated eigenvalues calculated when solving the eigenvalue problem providing the estimates of β.

The next step is to consider how we can use these representations. Starting with the error correction representation, we know that this can be regarded as a reduced form, which can be estimated without imposing more structure or identifying assumptions. First of all, this gives us an estimate of the rank of Π (which is equivalent to the number of co-integrating vectors) and an estimate of the space spanned by the corresponding β vectors[3]. It also provides an estimate of the co-variance matrix $\Sigma \equiv E[\varepsilon\varepsilon']$.

In principle, we could end the analysis here, if the only thing we are interested in is the co-integration properties of the model. If we, however, also want to estimate the effects of various structural shocks, we need to make some further identifying assumptions. For a formal treatment, see the appendix, or for a more comprehensive analysis, see for example, King et al [1987], Englund, Vredin and Warne [1994], or Keating [1990] in the case of identification with rational expectations.

From the error correction estimates, we can derive conditions for the identification of the CT and VMA models. Having estimated the co-integrating space, and assumed that the structural shocks are independent, we have made the distinction between permanent and transitory shocks. If we, however, have more than one permanent (or transitory) shock, we need to impose restrictions to distinguish between the different permanent (transitory) shocks. The identification of the permanent shocks is made by imposing restrictions on the A matrix, where we can first use the fact that $\beta' A = 0$ from the common trends representation (since $\beta' X_t$ should be stationary). This imposes rk restrictions on A. We also have $k(k+1)/2$ restrictions from the estimated covariances. To identify A exactly, $k(k-1)/2$ additional restrictions are required, since there are a total of nk parameters in A. From this stage of the identification process, we have defined the first part of the identification matrix F.

The second part of the identification process is to identify the transitory shocks. This can be done as in the standard VAR framework, where one puts (zero) restrictions on the contemporaneous effects on some of the variables from some of the temporary shocks (see Sims [1980] or Bernanke [1986]). Defining $R_{r,0}$ as the matrix containing the r final rows of R_0 in the VMA representation, i.e. the matrix containing the coefficients that belong to the

[3] If the β vectors are derived from economic theory, they can be viewed as structural also in this context.

contemporaneous effects of the transitory shocks, we have to impose $r(r-1)/2$ on $R_{r,0}$ to achieve exact identification of the transitory shocks. This provides the remainder of our identification matrix F.

2.2 THEORETICAL COINTEGRATION VECTORS

The literature on sustainable government debt suggests that some co-integrating restrictions are implied by the government's intertemporal budget constraint. The government has the following budget constraint and transversality condition

$$\begin{gathered} G_t + (1+r_t)D_t - D_{t+1} = T_t \\ \lim_{t\to\infty} E\left[(1+r_t)^{-t} D_t\right] = 0 \end{gathered} \quad , \qquad (6)$$

where G is government spending, D is public debt, T is taxes and r is the interest rate. The transversality, or no Ponzi game, condition says that the expected discounted value of government debt should go to zero as time goes to infinity. This implies that if the government borrows one dollar in period t and never repays the debt, it will grow with the interest rate, and when discounting it back with the interest rate, we will get the original one dollar debt, and thus violate the transversality condition. Another interpretation is that debt can grow over time, but this growth rate has to be lower than the interest rate to satisfy the transversality condition. A final interpretation is that the government cannot roll over an ever increasing debt and borrow to pay the accrued interest on the debt indefinitely.

Different assumptions about the variability of the interest rate together with the above formulation of the government lead to different restrictions on how the variables must behave for the government to obey its intertemporal budget constraint. Hansen and Sargent's [1991] assumption about the interest rate is that $E(r_t \mid I_{t-1}) = r$, i.e. the expected interest rate conditional on the available information is constant over time, which they use to show that the deficit must be a stationary series[4]. Together with the variable vector $[T_t \quad G_t \quad D_t]$ that is used

[4] The assumption that the expected interest rate conditional on the available informations is constant, is of course of vital importance, since the debt level is not allowed to influence the expected interest rate. In the present study, the assumption could be motivated in two ways. The first is from the theoretical analysis of Ricardian equivalence, where the assumption is a result of the analysis, and since this study is aimed at an empirical investigation of the equivalence proposition, the assumption is consistent with the theoretical models. Secondly, from a more practical point, we would think that a connection between the debt level and the interest rate reflects a risk premium associated with the default risk of the government. This is perhaps a non negligible

here, this implies that the cointegrating vector due to the government's intertemporal budget constraint should be $\begin{bmatrix}1 & -1 & -r\end{bmatrix}$.

Studies testing the validity of the US government's intertemporal budget constraint have been presented by Hakkio and Rush [1991] (who reject budget balance), and Hamilton and Flavin [1986], Trehan and Walsh [1988] and Bohn [1991](who do not reject budget balance). Smith and Zin [1991] reject budget balance for Canada. In the paper by Hamilton and Flavin, the authors discuss financing debt by printing money, since this creates seignorage and reduces the real value of debt. In this paper, the monetary aspect is ignored, since we are using a non-monetary model. The constraints on the government behavior are thus to some extent strengthened, since we do not allow for money creation to contribute to the revenue side of the government. However, using money creation (implying higher inflation) to finance a deficit is usually assumed to have other costs for the economy than the ones that appear in the government's expenditure side. It is hard to see how an empirical study could account for the costs associated with money creation, which is then only present on the government's income side. In this non-monetary model, we implicitely assume that the net value of money creation is zero, or alternatively, we can say that we here impose stronger restrictions on the government than the ones imposed in a monetary model.

Bergman [1995] make a distinction between violating the no-Ponzi game condition and "practical" solvency, where the first concept only imposes relatively weak constraints on the series, while the second requires that the inclusive of interest payments budget deficit/surplus is a stationary process with mean zero. The distinction between the concepts can be explained by the previous discussion of the transversality condition. We then noted that the no-Ponzi game assumption allows debt to grow, as long as it grows at a rate smaller than the interest rate. In other words, debt can "explode", but not as fast as it would if it were growing with the interest rate. However, this implies that at infinity the government still has debt, but the present value of this debt is zero, due to a positive discount factor.

At a first glance, this might give the impression that the no-Ponzi game constraint is of little importance, since the government can have a positive debt without violating this condition.

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risk in some less developed countries, but for more stable developed countries, the default risk would be negligible for determination of the interest rate.

However, it is important to realize that the interest rate that the government has to pay is exactly the same as the discount factor, so by running a deficit today and never running a surplus, the condition will be violated. If the government runs a large deficit today, implying that it has accumulated a debt that will grow with the interest rate, it "only" needs to run surpluses that make sure that the debt grows slower than the interest rate, i.e. some of the interest on the debt is paid with primary surpluses.

How large have the surpluses to be then? To answer this, we can imagine two polar strategies. The most straight forward is to say that in one period the primary surplus is big enough to repay all of the outstanding debt. This is however not the only valid strategy. Another option is to run very tiny primary surpluses, so that debt grows at a rate smaller than the interest rate. However, if these surpluses are not large enough to actually repay the debt, this policy will have to be maintained indefinitely, since by not running a tiny surplus in all future periods implies that debt would start growing with the interest rate and thus violating the no-Ponzi game condition.

In other words, the answer is not surprisingly a combination of the size of surpluses and the number of periods that we have to run these surpluses. Furthermore, we note that by using the second strategy, the government does not actually repay the principal, but only services (a part of) the accruing interest payments. Thus debt will grow indefinitely, but at a rate that is lower than the discount factor, making the present value of debt as time goes to infinity go to zero. In this case, the government obeys the no-Ponzi game constraint, but at the same time it is "insolvent" (to use Bergman's and Forslid's label), since it will not repay all its debt.

To formalize this discussion, we can use a continuous time model and a constant interest rate. The no-Ponzi game condition is then $\lim_{t\to\infty} D_t e^{-rt} = 0$. If we assume that debt is created at time zero, and the amount is D_0, debt will grow with the interest rate if there are no primary surpluses, i.e. $D_t = D_0 e^{rt}$. Taking the present value limit of debt as time goes to infinity, we get of course $\lim_{t\to\infty} D_0 e^{rt} e^{-rt} = D_0$, which violates the no-Ponzi game condition. If we instead assume that the government runs a surplus in each period such that $T_t - G_t = \varepsilon D_0$, and uses the surplus to service (part of) its interest payments on debt, debt will evolve according to $D_t = D_0 e^{(r-\varepsilon)t}$ and in the limit we have $\lim_{t\to\infty} D_0 e^{(r-\varepsilon)t} e^{-rt} = \lim_{t\to\infty} D_0 e^{-\varepsilon t} = 0$, as long as ε is a positive number. In this case, the no-Ponzi game constraint will be obeyed, but the principal of debt will not be repaid as long as $\varepsilon < r$. For the sake of the co-integration exercise, this implies that if the co-integrating vector is $[1 \quad -1 \quad -r]$, the no-Ponzi constraint is obeyed and debt will not grow over time. If $[1 \quad -1 \quad -\varepsilon]$ is instead the co-integrating vector, the no-Ponzi game assumption is still obeyed, but the principal of debt will not be repaid if $\varepsilon < r$. Note that ε can be arbitrarily small for the no Ponzi game constraint to be obeyed, and thus debt can grow at a rate that is arbitrarily close to the interest rate.

To my knowledge, tests of intertemporal budget constraints have been restricted to the government sector. However, in Ricardian models, we have the same type of long run restriction for the private sector. We can write the period by period budget constraint for the private sector and the associated no-Ponzi game condition (or transversality condition) as

$$W_{t+1} = (1 + r_t)W_t + YL_t - C_t - T_t \qquad (7)$$

$$\lim_{t \to \infty} E_0\left[(1 + r_{t-1})^{-1} W_t\right] = 0 \qquad (8)$$

where C is private consumption, W is total financial (i.e. non-human including the holdings of government bonds) wealth, and YL is labor income. We then get that total income, taxes, private consumption and private wealth should be cointegrated in the same way as receipts and expenditures were for the government sector above. For the system with total income, private consumption, government consumption, taxes, government debt and financial wealth, or $[YL \quad C \quad G \quad T \quad D \quad W]$, we would thus expect to find, at least, two cointegrating relations. The cointegrating vector associated with the private sector's budget constraint would be $\beta_1 = [1 \quad -1 \quad 0 \quad -1 \quad 0 \quad r]$, and the one associated with the government's budget constraint $\beta_2 = [0 \quad 0 \quad 1 \quad -1 \quad r \quad 0]$. The zero on D in the private sector's cointegrating vector is due to the definition of W, which includes public debt.

If we combine the private sector's income into one variable $Y \equiv YL + rW - rD$, and define $NT \equiv T - rD$, i.e. netting out interest payments on public debt, we get the system of variables $[Y \quad G \quad NT \quad C]$, and the cointegrating vectors $\beta_1' = [1 \quad 0 \quad -1 \quad -1]$ and $\beta_2' = [0 \quad 1 \quad -1 \quad 0]$. The co-integrating relations are thus the private sector's surplus (*PS*) and the government's deficit (*GD*) (the government's deficit and the private surplus are including interest payments on debt, since we have netted out the interest payments from taxes), i.e.

$$\begin{bmatrix} PS_t \\ GD_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Y_t \\ G_t \\ NT_t \\ C_t \end{bmatrix}, \tag{9}$$

should be stationary if the government and private sectors do not accumulate or decumulate wealth in a way that implies that they would end up with a non zero wealth as time goes to infinity. However, the no-Ponzi game only postulates that there exist co-integrating relations for the sectors, without the additional restriction that the co-integrating vectors should be of this specific form.

The existence of two cointegrating relations, as discussed above, could be viewed as necessary conditions for the Ricardian equivalence proposition. If we let the government violate its budget constraint from the start, there is not much left for testing with respect to the equivalence proposition. What happens if we reject that β_2 is a cointegrating vector for the data we are analyzing? Does this observation lead us to conclude that the government is violating its intertemporal budget constraint? The answer is no. We can only say that the government cannot pursue the same policy indefinitely, so in that sense the government policy used during the sample period is not "sustainable". This need not be interpreted as if the government is violating its intertemporal budget constraint, but rather as a signal that the policy will (have to) change in the future, if we think that the government will eventually repay its debt. Furthermore, the existence of two co-integrating vectors that are reasonably similar to the ones postulated here, would suggest that the no-Ponzi game assumption is obeyed, which again is a weaker condition than a condition stating that debt or wealth cannot grow at an exponential rate over time.

3. Empirical Results

In this section, data and model selection are discussed, before the results from the empirical analysis are presented.

3.1 Operationalization

In the theoretical model, Y represents total income for the private sector minus the interest payments on public debt. Further, depreciation of capital can be viewed as a negative investment, and is thus implicit in the change of total wealth in the model. Therefore, gross national income, which nets out interest payments between sectors and does not subtract the depreciation of the capital stock, is used. G is government expenditures net of transfer and interest payments. NT is total receipts minus transfers and interest payments. Note that at this stage transfers are regarded as negative taxes, which is in line with Ricardian modeling, where only lump-sum taxes are considered. Finally, total private consumption expenditures have been used as the measure of C.

In this study we clearly have to include durables in the consumption measure, or else net that out of the income measure in order to get the private sectors budget constraint right. There are some important objections against including durables in the analysis. However, excluding durables could also be unwise, unless we want to start out with a bias in favor of Ricardian equivalence, since durables are often considered more sensitive to income or wealth changes than are non-durables (if we for example imagine a decrease in income, it seems more likely that consumption of durables will be reduced more than consumption of non-durables, since durables are more often "luxury" goods). There have been different suggestions made about the appropriate measures for a study of Ricardian equivalence; for a more extensive discussion of this see for example Graham [1992] and Becker [1995a]. Graham discusses the definition of consumption measures and Becker analyzes the effects of aggregation in more dimensions than over groups of consumption goods with different durability.

The data set used in this paper consists of quarterly data for the US from 1960:1 to 1993:1, where the following variables from CITIBASE have been used: GDP (Y), government expenditures net of interest payments and transfers (G), taxes net of transfers and net interest payments (NT) and finally, total private consumption (C). All variables are in real (1987) dollars per capita, in levels (not logs). The series and transformations are described in the appendix.

3.2 MODEL SELECTION AND DIAGNOSTIC CHECKS

In the discussion above, the order of the VAR model was assumed to be known. In practice, however, we seldom know the order, but it has to be decided in connection with the estimation. There are several tests for deciding the VAR order and to investigate the properties of the estimated residuals.

Determining the VAR order

There are different information criteria that could be used to determine the lag length. The estimate used here are Schwarz' Bayesian Information Criterion (BIC) defined as

$$BIC = \ln\left|\hat{\Sigma}(p)\right| + \frac{2\ln T}{T} pn^2$$

and Hannan and Quinn's Law of the Iterated Logarithm (LIL) defined as

$$LIL = \ln\left|\hat{\Sigma}(p)\right| + \frac{2\ln\ln T}{T} pn^2,$$

where p is the lag length, T is the number of observations and n is the number of variables. The decision rule is to pick the VAR order (p) that minimizes either criterion. Both criteria point at a very low lag order (one or two lags), but the value is not much changed for an order of three or four.

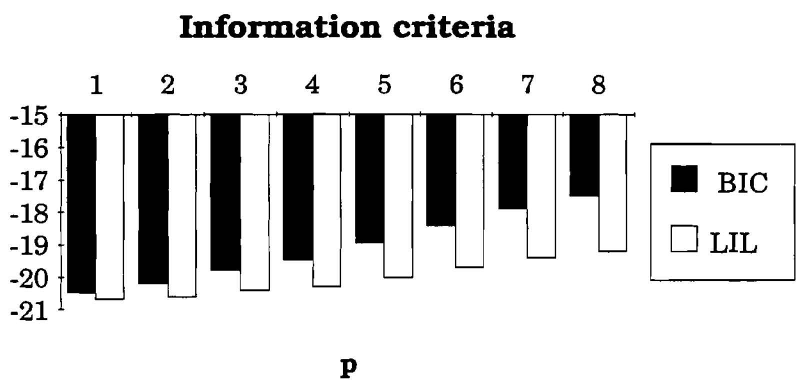


Figure 1. *Information criteria for determining lag order, choose lag order (p) that minimizes the criteria.*

Autocorrelation tests

To check for autocorrelation in the residuals the *LR-test* is used first. The test is defined as

$$\lambda_{LR}(i) = T(\ln|\hat{\Sigma}(p-i)| - \ln|\hat{\Sigma}(p-i+1)|) \sim \chi^2(n^2),$$

where $\hat{\Sigma}(p-i)$ is the estimated Σ for the hypothesis

$H_0^i\!:\Pi_{p-i+1} = 0$ against $H_1^i\!:\Pi_{p-i+1} \neq 0$ given $\Pi_p = \ldots = \Pi_{p-i+2} = 0$.

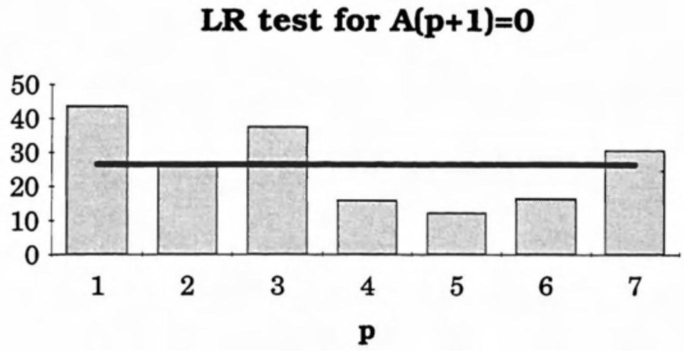


Figure 2. *Likelihood ratio test of autocorrelated residuals, the bars are observed values and the line is the critical value.*

The evidence from the likelihood ratio test points at either four or perhaps more than seven lags. In order to be as parsimonious as possible, a maximum of four lags is regarded to be the first choice, which is more in line with the above information criteria. Other diagnostic tools that check that the residuals are not autocorrelated are the Box-Pierce test with general lag length, which is a joint test of autocorrelations 0 to t being zero, and the likelihood ratio test for specifically first and fourth order autocorrelation. Both tests are distributed as χ^2 with degrees of freedom equal to the square of the number of constrained parameters. H_0 is that there is no autocorrelation. Using a VAR with four lags and a test size of five percent, there are no significant autocorrelations left in the residuals.

Normality of estimated residuals

The final test is to check if the residuals are normally distributed. This is done by investigating the skewness and kurtosis. The test statistic for skewness is defined as

$$\lambda_1 = Tb_1'b_1 / 6 \xrightarrow{d} \chi^2(n),$$

where $b_1 \equiv (b_{11},\dots,b_{n1})'$ with $b_{n1} = \frac{1}{T}\sum_t v_{nt}^3$, and the v's are the estimated standardized residuals (i.e. transformed to have mean zero and unit variance), and T is the sample length.

The test statistics for kurtosis is

$$\lambda_2 = T(b_2 - 3_n)'(b_2 - 3_n)/24 \xrightarrow{d} \chi^2(n),$$

where $3_n \equiv [3 \ \cdots \ 3]'$, is a $n \times 1$ vector, and $b_2 \equiv (b_{12},\dots,b_{n2})'$ with $b_{n2} = \frac{1}{T}\sum_t v_{nt}^4$. H_0 is that the residuals are normal.

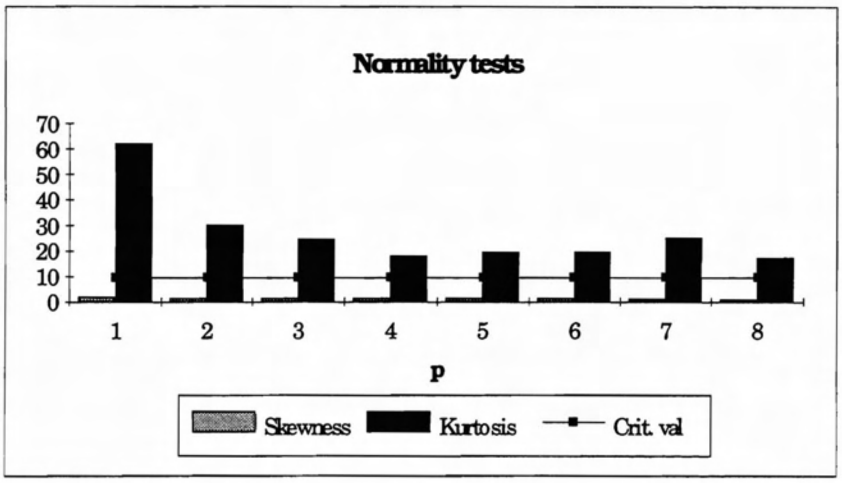


Figure 3. *Normality test of estimated residuals, the bars are the observed values and the line is the critical value.*

We note that skewness is not a problem at any lag length, but kurtosis is not entirely satisfactory for any lag length, which might indicate a problem with the current specification. The minimum observed value for the kurtosis test is obtained for $p = 4$.

The above tests make me conclude that four lags seems appropriate, since autocorrelation and normality of the residuals then seem OK and the information criteria do not indicate a need for more lags. The LR test might indicate that more lags should be included, and the kurtosis is not entirely satisfactory, but it is on the other hand not improved by including more lags. All other evidence is in favor of a maximum of four lags.

3.3 THE EMPIRICAL CO-INTEGRATING VECTORS

The number of empirical cointegrating vectors are examined with the method in Johansen [1991]. The null hypothesis is that we have r co-integrating vectors and the alternative is that we have exactly r+1 co-integrating vectors (the Lambdamax test) or that we have at least r+1 co-integrating vectors (the Trace test).

	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
LAMBDAMAX	25.494	16.494	8.233	0.021
"p-value" [5]	(0.09)	(0.18)	(0.4)	(>0.5)
TRACETEST	50.242	24.748	8.254	0.021
"p-value"	(0.03)	(0.18)	(0.4)	(>0.5)

Table 1. *Tests of the number of co-integrating vectors for the system* $X \equiv [Y, G, NT, C]$ *with four lags.*

The above LAMBDAMAX and TRACE tests indicate at least one and possibly two co-integrating vectors. I have chosen to work with two co-integrating vectors, since they can be motivated by the theoretical arguments presented above. However, in a purely data oriented analysis, using only one co-integrating vector might have seemed more appealing. Using the estimated β or co-integrating vectors, the cointegrating relations can be written as

$$\begin{bmatrix} Z_t^1 \\ Z_t^2 \end{bmatrix} = \begin{bmatrix} -2.276 & -6.559 & 7.572 & 3.793 \\ 7.121 & 0.399 & -3.471 & -8.785 \end{bmatrix} \begin{bmatrix} Y_t \\ G_t \\ NT_t \\ C_t \end{bmatrix}, \tag{10}$$

where the Z_t's should be stationary. These restrictions are then used when we estimate the common trends model. We also assume that the two remaining linear combinations are stationary in first differences, i.e., ΔZ_t^3 and ΔZ_t^4 are stationary, with Z_t^3 and Z_t^4 defined as linear combinations orthogonal to the cointegration vectors. In Figure 4 we have depicted the four relations that should be stationary. Inspection of the graphs does not indicate that non-stationarity should be a problem if we use these transformations. The tests of the number of co-

[5] The *p-values* are approximations based on Table 1 in Osterwald-Lenum [1992].

integrating vectors indicated that there might be only one co-integrating relation, but the figure does not indicate a problem with non-stationarity of the second co-integrating relation.

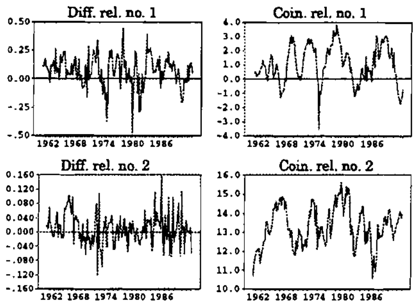


Figure 4. *The Empirical Stationary Transformations*

With empirical cointegration vectors we will have to use our imagination to make interpretations of what the relations represent, and since we can only estimate the space spanned by the co-integrating vectors, we have to do some (arbitrary) normalization. However, some normalizations might be easier to interpret than others. If, for example, we transform the co-integrating vectors presented above by a simple row reduction, i.e. use another normalization, we get $\beta_1''=[1 \quad 0 \quad -0.43 \quad -1.23]$ and $\beta_2''=[0 \quad 1 \quad -0.99 \quad -0.12]$, which is more easily compared to the theoretical co-integrating vectors derived above.

3.4 TESTING THE THEORETICAL COINTEGRATING VECTORS

The next step is to test the stronger version of the intertemporal constraint, i.e., will the government actually repay its debt or will it become insolvent if the current policy is conducted indefinitely? For the private sector the question turns out to be whether they actually will consume all its wealth in the end if the current consumption income pattern is maintained. This amounts to testing if $\beta_1'=[1 \quad 0 \quad -1 \quad -1]$ and $\beta_2'=[0 \quad 1 \quad -1 \quad 0]$ are in the co-integrating space. This is the test of a stronger transversality condition than the no-Ponzi game assumption discussed above, namely that the sectors do not have debt or assets that grow exponentially. In other words, if the government pursues its current policy, does this indicate that they will actually repay the pricipal of the debt and not only service part of the interest payments with primary surpluses?

Three tests are conducted, one on each of the two theoretic cointegrating restrictions separately, and one joint test on both vectors. The test statistic is distributed as $\chi^2(d)$ with the number of restrictions as the degrees of freedom, d. First, testing the restriction associated with the private sector's budget constraint, we get an observed statistic of 13.72, which for two degrees of freedom is significant at the one percent level, indicating that the behavior *over the sample period* is not in line with a behavior where all wealth is finally consumed. The same result is true for the public sector, with an observed statistic of 15.39, which for two degrees of freedom indicates that the stronger version of the transversality condition is not fulfilled. Finally, the joint test of the two cointegrating restrictions gives an observed statistic of 19.24, which for four degrees of freedom can also be rejected at any normal risk level.

What conclusions can now be drawn from these observations? The most straightforward conclusion would be to say that neither the public, nor the private sector are following their intertemporal budget constraints. However, a more careful interpretation of the results is that the tests provide an indication that both sectors will change their behavior in the future, if we think they have to repay all debt or consume all wealth. The behavior of the public sector is then not "sustainable" over an infinite horizon. However, if we think that money creation would significantly improve the government's long run budget stance, this may imply that the present policy is sustainable, although this non-monetary model does not predict that debt will eventually be repayed, if the present path is followed. On the other hand, the private sector is not violating the constraint in the sense that it is running a deficit, but it is instead accumulating more capital than is necessary for the constraint to be fulfilled. In the case of the household sector, accumulating wealth at the same rate as output grows could be consistent with optimizing agents along a balanced growth path.

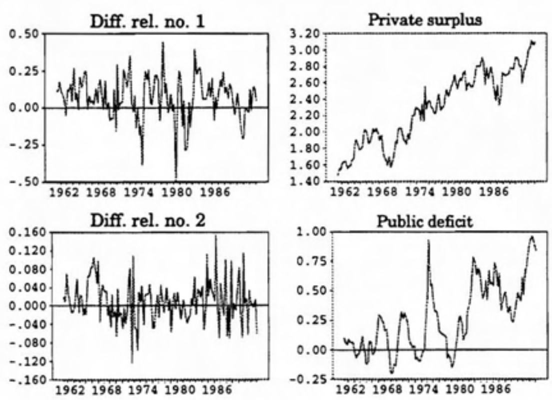


Figure 5. *The Theoretical "Stationary" Transformations*

The graphs in Figure 5 clearly show what we have learnt from the LR tests, that the theoretical cointegrating vectors do not appear to be stationary over the sample period. The first cointegrating relation is the private sector's budget *surplus,* while the second relation is the public sector's *deficit.* For both relations we can see an upward trend, indicating that the private sector actually has been saving more as the public sector has been dissaving. Of course we cannot make any causal conclusions from this observation, but the pattern is clearly one that could be associated with a world of Ricardian equivalence. This behavior could potentially be explained by a private sector that realizes that the government eventually has to increase taxes to repay the public debt, and thus the private sector has to accumulate savings to hedge future consumption from the higher expected tax payments.

We should thus be careful in totally rejecting these theoretical cointegration restrictions if we want to make long term forecasts. In the short run, however, the tests can be viewed as an indication of the way the behavior of the government sector will change, but then other short term considerations could be dominating, and thus invalidate the behavior predicted by the long run restrictions. In estimating the system, I will therefore use both the empirical and the theoretical cointegrating vectors, in order to see if they generate widely different forecasts for different horizons.

3.5 IDENTIFICATION OF THE COMMON TRENDS MODEL

In estimating the common trends and vector moving average models, we have to make some assumptions about how different variables respond to different shocks, to achieve identification of the coefficient matrixes. We will make identifying assumptions which are consistent with assumptions/conclusions in standard Ricardian models. This way of identifying the model could be regarded as having the Ricardian hypothesis as the null hypothesis.

With two co-integrating vectors ($r=2$) and four variables ($n=4$), we have two common stochastic trends, and thus two permanent and two transitory shocks. The first stochastic trend is viewed as a stochastic trend in GDP that gives rise to corresponding trends in consumption and taxes. The first permanent shock is thus regarded as a technology shock that could potentially affect all variables in the system indefinitely. We allow the public sector to be driven by another stochastic trend than the GDP trend, that could potentially change the GDP proportion of the public sector over time. The second permanent shock is thus interpreted as a public sector shock, that indicates the (preferred) level for the public sector.

To distinguish between the two permanent shocks, we need $k(k-1)/2=1$ identifying restriction. The way the permanent part is identified here is by not letting the public sector shock have any permanent effect on GDP, although there might be short run effects, (i.e., the element (1,2) in the matrix A in the common trends representation is set equal to zero). This is in line with the general Ricardian equivalence framework, since if G for example enters the production function this would complicate the connections between the private and public sector substantially. Here these possible long run connections between G and GDP are, by construction, instead captured in what is labeled a technological shock.

The first transitory shock is interpreted as a public finance shock, connected to short run decisions on how to finance the public sector, while the second transitory shock is interpreted as a private demand shock. In order to identify the transitory shocks, the public finance shock is not allowed to have any contemporaneous effect on GDP (i.e., the element (1,1) in the matrix $R_{r,0}$ from the vector moving average representation is set equal to zero.), which again is an assumption consistent with a Ricardian model. However, only a minimum amount of restrictions has been put on the system. For example, we still allow GDP to change in the short run in response to both temporary shocks (with the one exception that GDP does not respond contemporaneously to the public financing shock.)

We are now able to produce the impulse responses for both the two permanent and the two transitory shocks. By using the identifying assumption, we can also separate the two permanent shocks from each other as well as the two transitory shocks. Without these identifying assumptions, we could have separated between the permanent and transitory shocks. However, with the identifying assumptions made, we can also separate between the different permanent and transitory shocks. From a Ricardian perspective it is especially important to distinguish between a permanent shock to a trend in the public sector activity, and a transitory public sector financing shock. We also need to distinguish these shocks from other shocks that affect the economy. Here, the latter shocks are represented by a permanent technology shock and a transitory demand shock.

The estimated common trends model based on the empirical co-integration vectors is (with standard errors in parenthesis)

$$\begin{bmatrix} Y_t \\ G_T \\ NT_T \\ C_T \end{bmatrix} = X_0 + \begin{bmatrix} \underset{(0.0235)}{0.1069} & \underset{(0.0000)}{0.000} \\ \underset{(0.0204)}{0.0459} & \underset{(0.0109)}{0.0489} \\ \underset{(0.0199)}{0.0377} & \underset{(0.0115)}{0.0514} \\ \underset{(0.0157)}{0.0738} & \underset{(0.0041)}{-0.0181} \end{bmatrix} \begin{bmatrix} \tau_t^{TE} \\ \tau_t^{PE} \end{bmatrix} + u_t^E \ ,$$

where τ_t^{TE} and τ_t^{PE} represent the stochastic trends in GDP and public sector, respectively, and u_t^E is the residual vector. The estimated coefficient matrix multiplying the stochastic trends describes how the variables respond in the long run to the stochastic trends. We can note that the estimated coefficients are in line with the lables of the trends. For example, the trend in GDP associated with the permanent technology shock has the effect of increasing the levels of all variables, and GDP the most, while the public sector trend has the greatest effect on the public sector variables (by construction, the long run effect on GDP is zero, since this was an identifying assumption). The stochastic trends evolve according to

$$\begin{bmatrix} \tau_t^{TE} \\ \tau_t^{PE} \end{bmatrix} = \begin{bmatrix} 0.6027 \\ -0.3561 \end{bmatrix} + \begin{bmatrix} \tau_{t-1}^{TE} \\ \tau_{t-1}^{PE} \end{bmatrix} + \psi_t^E \ ,$$

where the first vector on the right hand side contains the drifts for the two stochastic trends and ψ_t^E is the innovation vector, containing the permanent technological and public sector shocks.

Using the theoretical co-integration vectors we instead get

$$\begin{bmatrix} Y_t \\ G_T \\ NT_T \\ C_T \end{bmatrix} = X_0 + \begin{bmatrix} \underset{(0.0553)}{0.1348} & \underset{(0.0000)}{0.000} \\ \underset{(0.0341)}{0.0569} & \underset{(0.0099)}{0.0363} \\ \underset{(0.0341)}{0.0569} & \underset{(0.0099)}{0.0363} \\ \underset{(0.0305)}{0.0779} & \underset{(0.0099)}{-0.0363} \end{bmatrix} \begin{bmatrix} \tau_t^{TT} \\ \tau_t^{PT} \end{bmatrix} + u_t^T \ ,$$

with the same notation as in the above system, except that the superscripts are now T (Theoretical co-integrating vectors), instead of E (Empirical dito).

$$\begin{bmatrix} \tau_t^{TT} \\ \tau_t^{PT} \end{bmatrix} = \begin{bmatrix} 0.4591 \\ -0.4662 \end{bmatrix} + \begin{bmatrix} \tau_{t-1}^{TT} \\ \tau_{t-1}^{PT} \end{bmatrix} + \psi_t^T .$$

We could note that there are a few differences between the predictions delivered by the two systems, which are due to the stronger constraints than the theoretical co-integrating vectors impose. First of all, we could sum the long run effects from the stochastic trends over the variables. In the "T" system, we have the result that if we sum the coefficients that private and public consumption have for the technology trend, the sum is equal to the GDP coefficient, i.e. the coefficients describe how an increasing GDP is divided between private and public consumption. Furthermore, the change in government consumption and taxes are equally large, implying that the growth does not generate a budget deficit. In the "E" system the corresponding coefficients sum to a number greater than the coefficient on GDP, i.e. the technology shock results in private and public consumption changes that are larger than the increase in GDP, and the coefficients on government consumption and taxes imply that this type of growth creates a budget deficit. These differences are only another way of viewing the effects of imposing the co-integrating constraints. In the "T" system we have imposed exactly these restriction on the coefficients, and we know from our tests of the co-integrating vectors that they are different from the empirically determined ones.

In the same way we see that the coefficients of public and private consumption belonging to the public sector trend sum to zero in the "T" system, since the GDP effect is zero, and that also here the government's budget balance is maintained. In the "E" system, there is not a crowding out of private consumption at the rate that government consumption is increasing, and at the same time, by summing the coefficients on taxes and government consumption, we note that the government will improve its budget stance due to this trend.

Finally, the relations describing the evolution of the stochastic trends show that the drift associated with the technology shock is positive, while somewhat surprisingly, the drift associated with the public sector trend is negative, implying that government consumption and net taxes have been declining as a proportion of GDP over the period. However, we have to remember that transfers are netted out in this study, since they are regarded as negative lump-sum taxes, so this observation is not equivalent to stating that total government activity has declined as a fraction of GDP.

However, at this stage we can conclude that the estimated systems do not differ very much in the sense that the signs of all coefficients are the same, and the values are of the same magnitude, although the underlying co-integrating vectors are significantly different. The observation that the systems seem to be very similar will be evident also in the impulse response charts generated by the VMA representation below.

3.6 IMPULSE RESPONSES

In Figures 6, 7, 8 and 9 the impulse-responses for the four shocks are presented for the system using both the empirical cointegration vectors and the theoretical ones. The figures show the responses with 95 percent asymptotic confidence bounds (see for example Lütkepohl [1989] and Warne [1990b], Theorem 2.3 for computation) to a one standard deviation shock. The comments will in general be made on the impulse responses for the empirical co-integration vectors, while the ones for the theoretical co-integrating vectors are presented for comparison only, since there is evidence that the model is not satisfactorly specified if we used the theoretical co-integrating vectors.

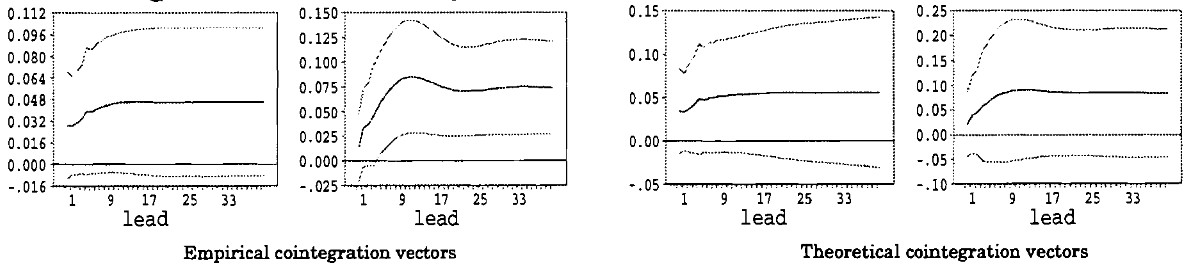
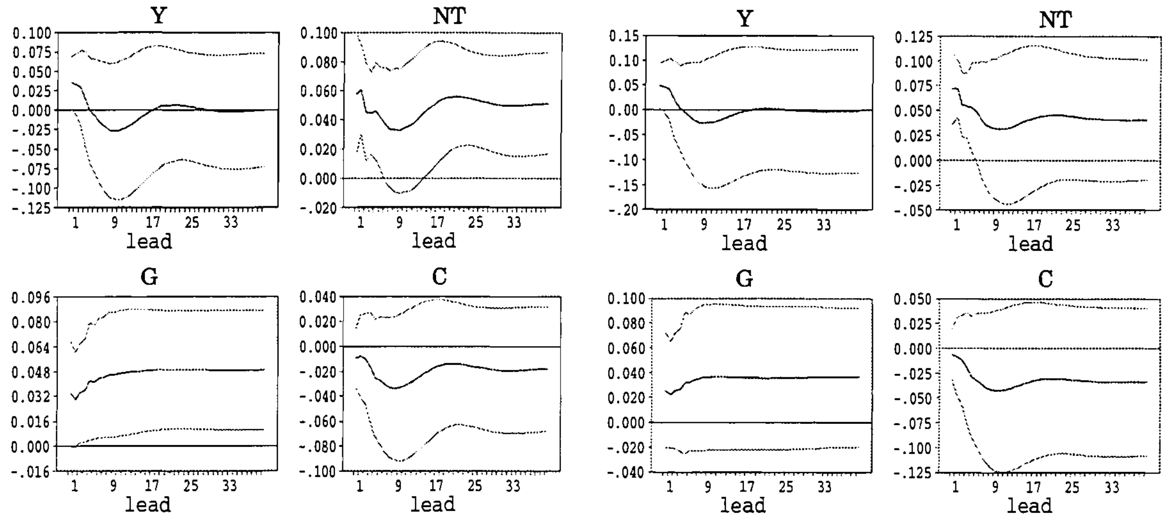
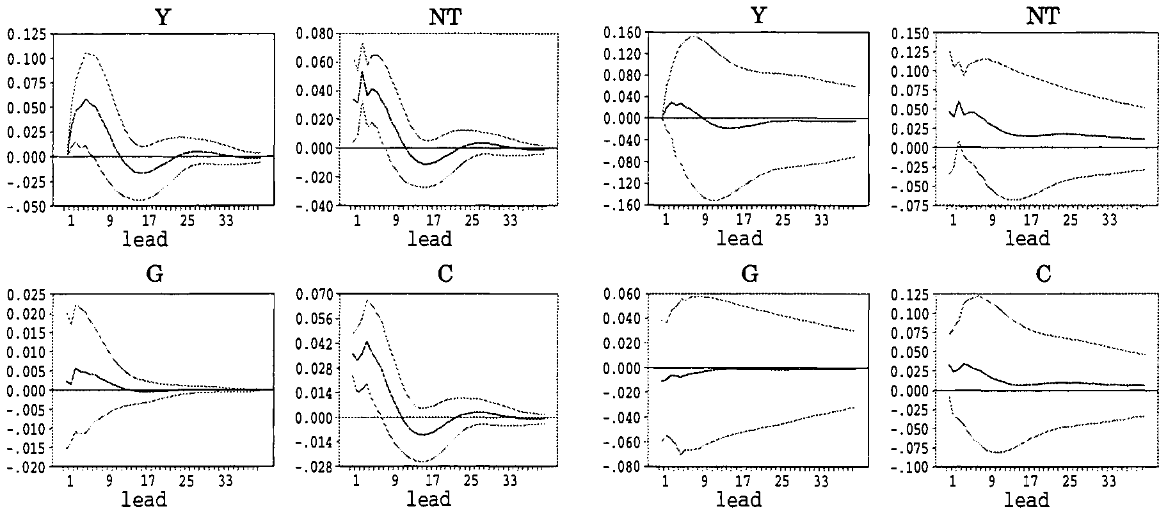


Figure 6. *Responses to the first permanent or technology shock*



Empirical cointegration vectors   Theoretical cointegration vectors

Figure 7. *Responses to the second permanent or public sector shock*



Empirical cointegration vectors   Theoretical cointegration vectors

Figure 8. *Responses to the first transitory or public financing shock*

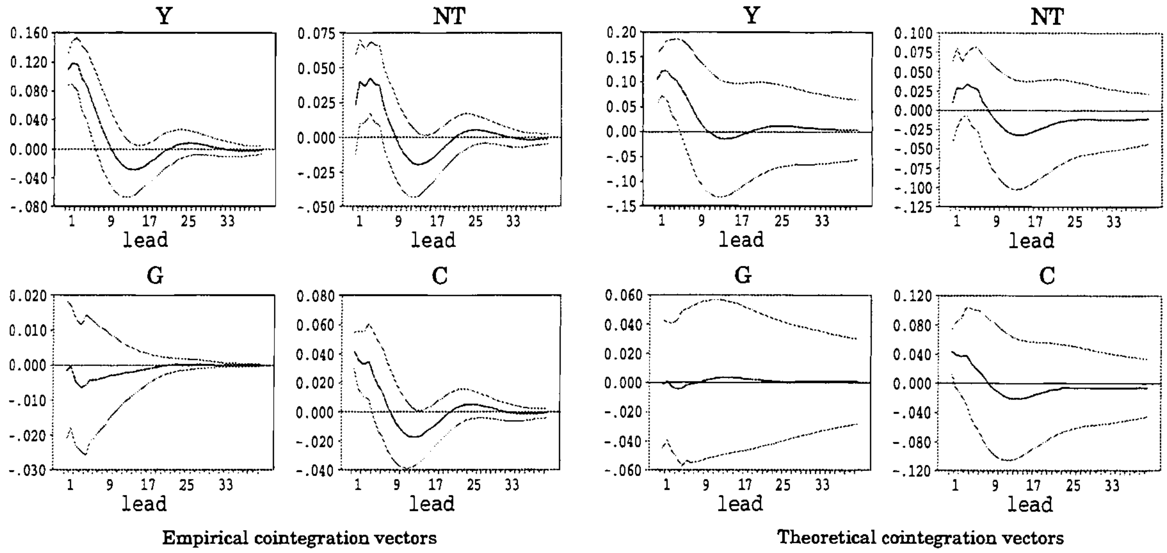


Figure 9. *Responses to the second transitory or demand shock*

In Figure 6 we have what we interpret as a technology shock, which has permanent positive effects on all variables. However, only the responses in Y and C are significant. This is clearly in line with a Ricardian model, which would predict that an increase in (net) income would be accompanied by increased consumption. However, this conclusion is also valid for a Keynesian model, and does not to allow a "rejection" or "acceptance" of either model.

In Figure 7 the responses to the public sector shock are displayed. As an identifying assumption, we did not allow this shock to have a permanent effect on Y. This does, however, not rule out short run effects on Y, but they are not significant at any forecast horizon. For G and NT, however, there are significant increases in response to the shock. The point estimates of the permanent increases in G and T are of roughly the same magnitude, which suggests that the budget deficit is not much affected by permanent changes in the size of the government sector. However, the confidence bounds are very wide. The effect on C would be expected to be negative if the shock represents a growing public sector, at least in a standard Ricardian model, where government consumption does not enter individuals' utility functions (in a non-separable way) or the production function. This observation is in line with the point estimates, but again, the confidence interval is rather wide and does not allow us to rule out positive effects. Again, we cannot discriminate between a Ricardian and a Keynesian model, since both government consumption and taxes are increased. The Ricardian story would be that consumption should be reduced in response to increased government consumption, but at the same time, a Keynesian model would make the same prediction for private consumption due to increasing taxes.

In Figure 8 we have the responses to the government financing shock. From a Ricardian perspective, such shocks would have no effects on consumption or production. Specifically, consumers would not regard the holdings of government bonds as net wealth, and therefore not change their consumption path in response to changes in timing of debt and taxes. The conflicting theories would (most often) state that *de*creases in the bond holdings, or, equivalently, *in*creases in taxes would reduce consumption, due to decreases in current disposable income. With the little amount of structure imposed on the system, it is hard to interpret the graphs in a very precise way, or regard them as actual tests of the equivalence theorem, since income in all but the first period is allowed to change and the net present value of all tax changes is not necessarily zero, which are two important assumptions we use when deciding if a theoretical model is Ricardian or not. However, the final assumption that is made in the theoretical experiment is to maintain government consumption at its original path, which is actually very close to what we observe in the impulse response diagram for G.

Nevertheless, it is rather interesting to note that the significant *rise* in taxes is accompanied by a significant short run *increase* in both Y and C when we use the empirical co-integrating vectors. One possible explanation for this is that individuals are risk averse and perceive that future incomes are more uncertain if taxes are postponed. Models that deal seriously with this type of uncertainty about both the production possibilities and government action are rare in the literature. Two exceptions, however, are Chan [1983] and Becker [1995b]. In these papers, it is shown that a tax increase may lead to an increase in private consumption, due to the fact that future taxes are riskier than taxes today.

In the last impulse response figure we have responses to the demand shock. The shock gives rise to significant short run positive effects on Y, NT and C, and insignificant effects on G. These responses seem to be in line with what are usually thought of as effects from a demand shock.

Finally, we can note that the responses to the two transitory shocks are almost identical, and without imposing more structure, it is difficult to interpret them separately. One interpretation of this observation is that we might have included too many co-integrating vectors in the estimation. In the Ricardian perspective, it is however interesting to note that for both shocks, taxes and consumption are positively correlated.

3.7 VARIANCE DECOMPOSITION

To what extent do fluctuations in the different variables come from one shock or another? This is the question answered by a variance decomposition. The decomposition can be made in two steps. First, a variance decomposition between permanent and transitory shocks, which is contingent on the identifying assumptions that only k shocks have permanent effects. With the additional identifying assumptions that impose restrictions on A and $R_{r,0}$, we can make a further decomposition into contributions from specific permanent and transitory shocks. The variance decompositions at various forecast horizons are shown in Figures, 10, 11 and 12.

In the short run, the transitory demand shock clearly dominates the variation in Y and also C for the very short forecast horizon. The public sector variables seem much less sensitive to transitory shocks, perhaps indicating that G has not been used as a policy instrument against transitory shocks. The variation in NT is to some extent explained by the financing shock, as expected, but the permanent public sector shock is clearly dominating.

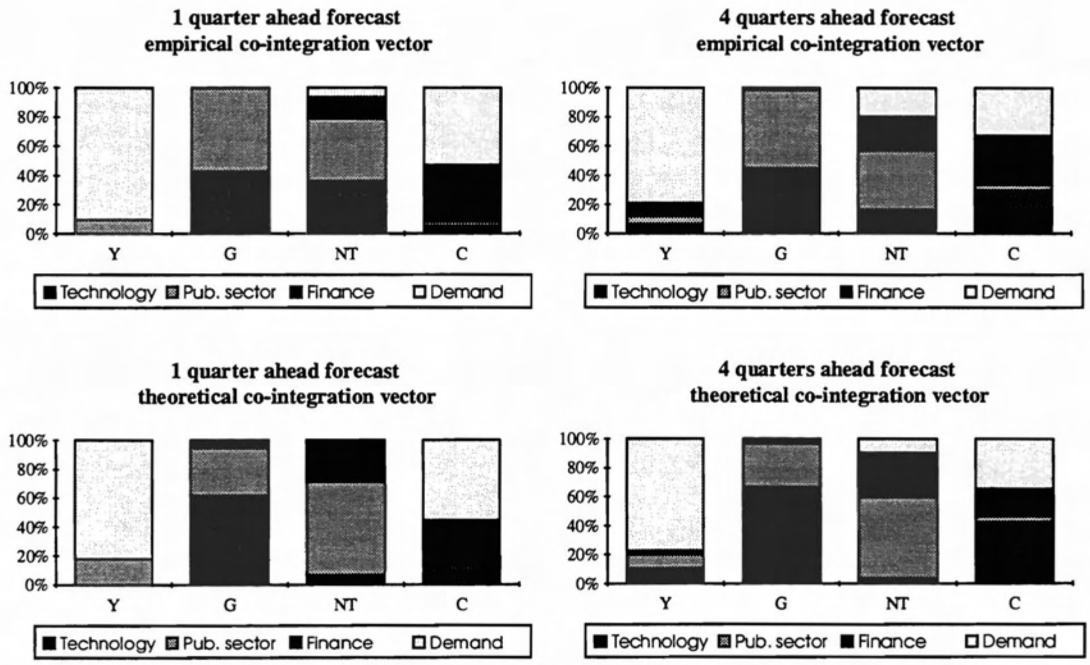


Figure 10. *Short run variance decomposition. The different sections of the bars for each variable represent the fraction of the forecast error that is due to the different shocks. (Note that the order from bottom to top of the segments on the bars is equivalent to the order from left to right of the keys.)*

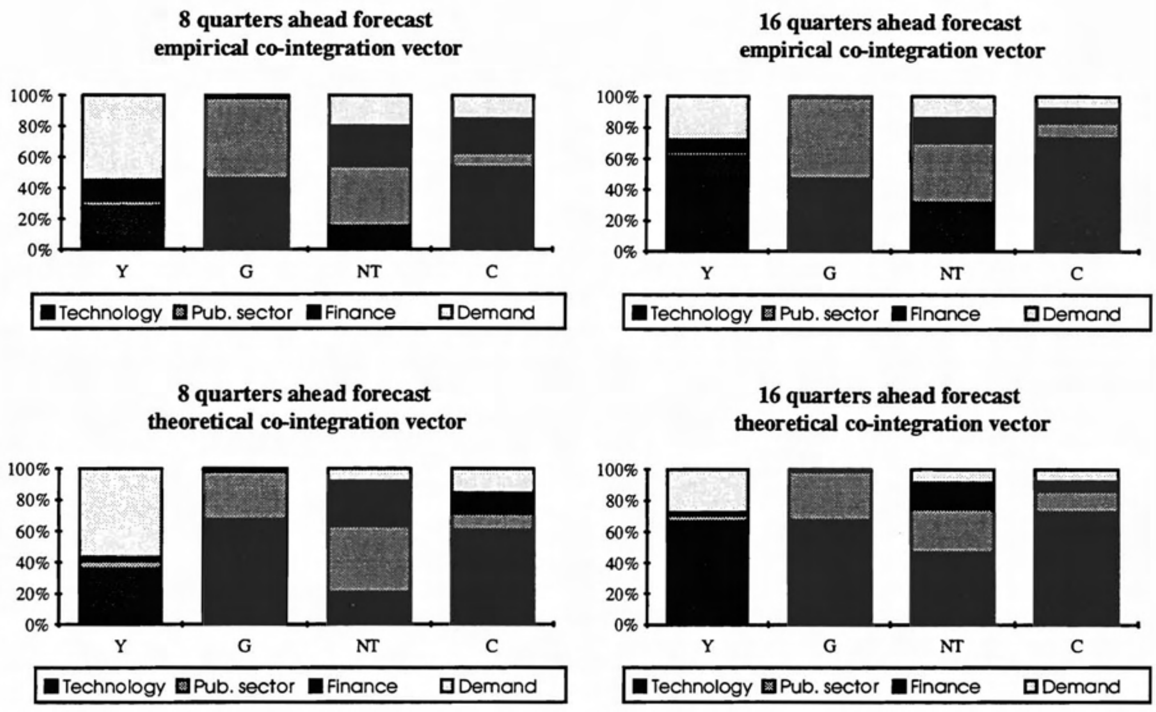


Figure 11. *Medium term variance decomposition.*

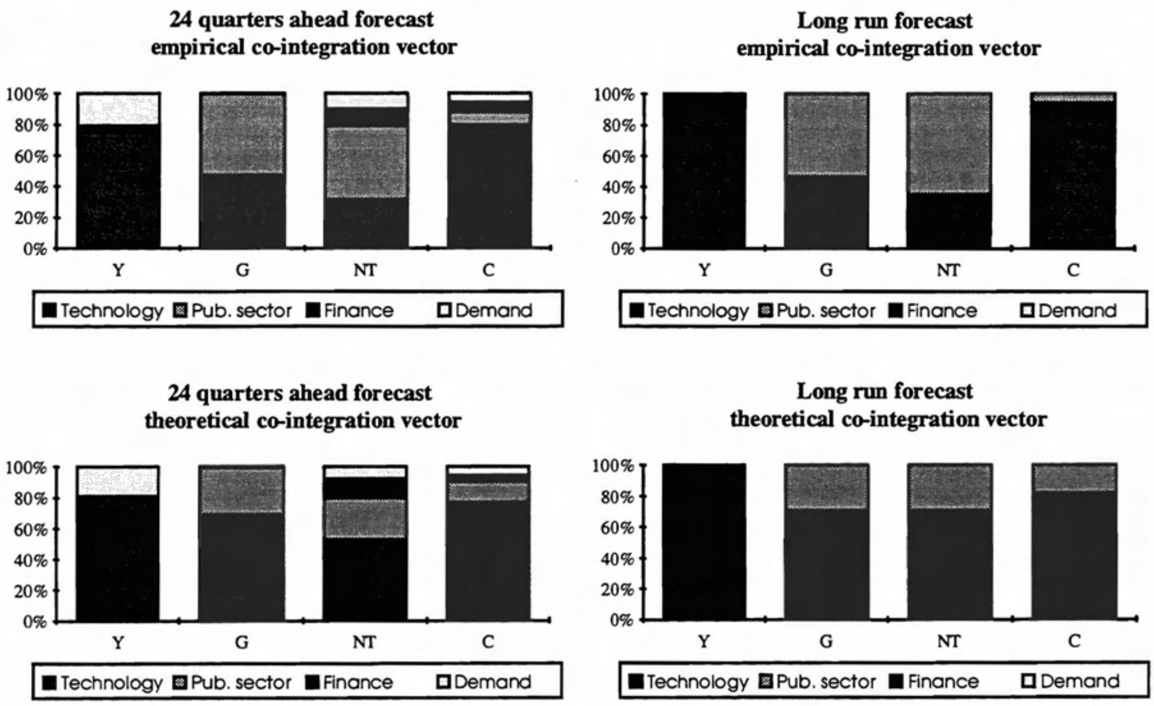


Figure 12. *Long run variance decomposition*

In the medium run, Y is more and more explained by the technological shock, and the point estimates of the influence of the public sector shocks on Y are almost zero. The same picture emerges for C, with even less influence from the transitory and the permanent public sector shocks. G is still explained in very much the same way as in the short run; the permanent shocks totally dominate. For NT the picture is a little more mixed, the two public sector shocks still being very important. This does to some extent support the interpretations or labels of the different shocks that have been used.

In the long run, the technology shock almost totally dominates the variation in C (and of course all the variation in Y by assumption), while the variation in the public sector variables are to a large extent determined by the public sector trend.

With theoretical cointegration vectors, most results remain unchanged. One result that is affected, however, is the long run importance of the different shocks for the public sector. Imposing theoretical cointegration constraints makes the point estimates of the proportions explained by the public sector shock fall from around 60-70 percent to only 25-30 percent. At the same time, the proportion of the variation in consumption explained by the public sector shock increases, and is of roughly the same magnitude as for the government variables. This reflects that there is less room for a separate public sector trend when the intertemporal budget constraint is obeyed, and also that the private and public sectors are more closely linked to changes in GDP in this case.

If what we identify is actually a financing shock, it is not supposed to have any effect on real variables like C or Y according to the equivalence theorem. For these data, there seems to be some evidence for an influence of the financing shock. We find that the responses in C and Y are actually positive and significant in the short run, and that the financing shock is important for the short run forecast errors of C. This would contradict both the equivalence theorem as well as simple Keynesian models, which would predict a negative correlation between taxes and consumption.

Further, the equivalence theorem predicts that permanent changes in G should crowd out C in a one to one fashion, since G does not (in general) enter the production function or the utility function in a non-separable fashion. This type of model observation has been part of tests for or against Ricardian equivalence. The conclusion drawn after observing a correlation between G and C that implies less than a one to one substitution, is to reject the equivalence theorem.

From the variance decomposition figures, however, we can conclude that the technology shock contributes substantially to the forecast errors in both G and C, and the impulse responses show positive effects on both C and G from this shock. An observation that G and C are positively correlated might thus be explained as a result of technology shocks, without any direct bearing on the equivalence theorem. Identifying this type of effect in single equation tests, which have been widely used, seems very hard.

Finally, the general impression is that the labeling of the shocks seems quite in line with the variance decomposition figures. Income and consumption is to a large extent explained by the permanent technology shock and in the short run also by the temporary demand shock, while the variation in public sector variables is to a large extent explained by what has been labeled public sector shocks.

4. Conclusions and Extensions

The common trends framework has some clear advantages over a single equation approach in that it distinguishes between expected changes and shocks to the variables, but also over the traditional VAR analysis in that it explicitly distinguishes between permanent and transitory shocks. Further, it handles the issues of non-stationarity, simultaneity and co-integration explicitly, which is often not the case when single equation models are used to investigate the equivalence theorem. For tests of Ricardian equivalence, it is crucial to make the distinctions between anticipated changes and shocks, as well as between temporary and permanent changes, which further motivates the use of the common trends framework for this particular study.

In the empirical investigation of US data, we have a model with four variables and thus four shocks. Having decided that the system has two co-integrating vectors (partly based on theory and partly on empirical tests), we can conclude that two of the shocks are temporary and two are permanent. The first permanent shock is interpreted as a shock to technology, and the second as a public sector shock. Another way of stating this is to say that the system has one stochastic technological trend and one stochastic public sector trend. Furthermore, the first transitory shock is interpreted as a public financing shock and the second as a demand shock.

The results from the estimation do not lend direct support to either the Ricardian equivalence theorem, or a simple Keynesian model. First, we reject the stronger versions of the intertemporal budget restrictions for the government and the private sector. Over the period, the public sector accumulates debt[6], and the public sector accumulates wealth. This could potentially be explained by a private sector that realizes that in the long run (longer than the sample period), the government will have to increase taxes to repay the outstanding debt.

In a way, we have to decide whether we regard tests of intertemporal budget constraints as a test of their relevance within a limited period of time, or as an indication of whether or not a certain policy is sustainable in the long run. If we take the latter view, we would conclude that the test of the government's intertemporal budget constraint only indicates that it will have to raise taxes in the future to finance a given level of government consumption, in which case it is very Ricardian of the private sector to accumulate wealth.

Secondly, in determining the validity of the equivalence theorem, we also investigate the effects on private consumption from a temporary government financing shock. The impulse responses indicate that consumption increases in response to increased taxes. Individuals thus discount more than 100 percent of the tax reduction. This is not an observation that is consistent with the theorem (which predicts exactly 100 percent tax discounting), but at the same time, it is also contrary to predictions from a Keynesian type of model (which in its crudest form does not have any tax discounting, or, in more sophisticated versions, a tax discounting between 0 and 100 percent). This finding can thus be regarded as evidence against the equivalence theorem, but for another reason than the standard disposable income argument. One possible explanation for this type of result could be that individuals perceive postponed taxes as a source of uncertainty, and thus engage in precautionary savings.

Furthermore, the two transitory shocks give rise to very much the same responses. On the one hand this makes the interpretation of one shock as a demand shock and the other as a financing shock somewhat arbitrary. On the other hand, they both provide us with a reasonable way of

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[6] As was discussed earlier, there might be monetary reasons that improve the government's budget, and these effects are not present in this non-monetary model.

comparing how consumption and taxes covary when government consumption remains constant. The drawback is that income reacts to the transitory shocks as well, making a clear-cut Ricardian conclusion hard. The interesting feature to note, however, is that consumption and taxes move in the same way for both shocks. With higher taxes (decreased debt), consumption increases. Again, this is an observation that is quite far from any Keynesian prediction, and also not entirely consistent with the Ricardian view. A reservation has to be made for the change in income, although we could still see that the change in consumption is larger than the increase in *net* income. In other words, we seem to have other effects than simply the change in net income (or wealth) that is analyzed in standard theoretical models.

Finally, the Ricardian proposition also predicts that private consumption should be crowded out one by one from permanent increases in government consumption. This does, however, not seem to be the case for the permanent public sector shock, which affects government consumption and taxes permanently, but does not affect private consumption significantly. Again, it is not in line with the Keynesian models either, since in such models an increase in taxes would affect private consumption negatively.

This paper has been aimed at exploring data, but to do so with a link to the theoretical world. First, the co-integrating vectors have been derived from theory, and secondly, the identifying assumptions used are in line with a Ricardian model. The estimated models are at this stage only exactly identified. In this way, we let data "speak for itself" to the largest possible extent. The price we pay for this is that interpreting the results in terms of validity of the Ricardian equivalence theorem becomes harder.

To conduct a formal test of the theorem, we will have to specify the exact model that we think is generating the equivalence result, and then test cross equation restrictions derived from a rational expectations intertemporal utility maximizing model. One study in this spirit is Aschauer [1985]. An area of future research would be to extend his model to include the co-integrating constraints derived here, and then to estimate and test a similar model in a common trends framework.

Furthermore (sligthly on the philosophical side), with enough data we will expect to reject basically any theoretical model, since they are all approximations of the real world, and if more data narrow confidence intervals, we will always reach a point where the estimates are significantly different from the model predictions. To reject a theory in this case is perhaps not desirable, since the model might still generate reasonable predictions and help us understand important economic mechanisms. This is certainly something to keep in mind when testing also the Ricardian proposition. As was shown, we could reject the stronger version of the intertemporal budget constraints here. Will we now stop considering governments eventually repaying their debt as important parts of economics because of this, or could this anyway be viewed as reasonable approximations and useful for long term forecasts?

Finally, theoretical work, perhaps in combination with simulation studies, could be used to shed some light on the question under what circumstances increases in present taxes could give rise to *in*creased consumption, and also to what extent uncertainty created by the government is different to uncertainty about the production process. Steps towards this can be found in for example Chan [1983] and Becker [1995b], where the latter paper analyzes the effects from uncertain future taxes and public debt in the framework of Sandmo [1970]. In these papers, there are cases where individuals with strong enough risk aversion reduce their consumption in response to decreased taxes.

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6. APPENDIX

6.1 REPRESENTING AND IDENTIFYING VECTOR TIME SERIES

In this section, we will start by showing how a vector time series can be written in different ways, or be given different representations. We will also discuss how the structural forms can be identified. This section is to a very large extent based on Englund, Vredin and Warne [1994], and for a more complete discussion, see their paper. For the reader with a more general interest in the method and related issues, see for example, Sims [1980], Cooley and LeRoy [1985], Engel and Granger [1987], King et al [1987], Johansen and Juselius[1990], Warne [1990], Keating [1990] and Johansen [1991].

6.1.1 REPRESENTATIONS OF A VECTOR TIME SERIES AND THEIR CONNECTIONS

Start with the structural vector autoregressive model

SVAR: $$X_t = B(0)X_t + B(L)X_{t-1} + \mu + \varphi_t, \tag{11}$$

with $E(\varphi\varphi') = I$, so that a shock to the system can be identified as coming from one (structural) equation.

From the above model we write the unrestricted VAR (UVAR) or reduced form as

UVAR: $$\Pi(L)X_t = \rho + \varepsilon_t, \tag{12}$$

where $E(\varepsilon_t \varepsilon_t') = \Sigma$ is positive definite.

The lagpolynomials and errors from the SVAR and UVAR are related according to

$$\begin{aligned} A(L) &= (I - B(0))^{-1} B(L) \\ \rho &= (I - B(0))^{-1} \mu \\ \varepsilon_t &= (I - B(0))^{-1} \varphi_t \\ E(\varphi\varphi') &= I = (I - B(0))\Sigma(I - B(0))' \end{aligned} \tag{13}$$

We can rewrite the above model in error correction form (EC) according to

EC: $$\Delta X = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-1} + \rho + \varepsilon_t, \quad (14)$$

which still can be regarded as a reduced form. In this representation, the matrix Π can be written in terms of the $n \times r$ matrices α and β as $\Pi = \alpha\beta'$, where β contains the r co-integrating vectors. Since β contains the co-integrating vectors, $\beta'X$ is stationary. With $r = n$, the system is stationary in levels, and with $r = 0$, we have a difference stationary process. To connect this to the UVAR representation we have that

$$\begin{aligned} \Gamma_i &= -\sum_{j=i+1}^{p} \Pi_j \\ \alpha\beta' = \Pi &= \left(I - \Pi_1 - \ldots - \Pi_p\right) = -\Pi(1) \end{aligned} \quad (15)$$

To get back to structural interpretations of the model we can use the common trends representation (CT) and the vector moving average (VMA) representation

CT: $$X_t = X_0 + A\tau_t + D(L)\varepsilon_t, \quad (16)$$

with $\tau_t = \mu + \tau_{t-1} + \psi_t$, being the common stochastic trends. In this representation we only have $n - r = k$ different stochastic trends, so that τ is k-dimensional and the loading matrix A is $n \times k$. From above we know that $\beta'X$ is stationary, and thus $\beta'A$ has to be equal to zero to remove the non-stationarity in the CT representation, which we will use later for identification of the model.

VMA: $$\Delta X_t = \delta + R(L)\varphi_t, \quad (17)$$

where $\varphi_t \equiv [\psi_t{}', \nu_t{}']'$ is a vector that contains both the k structural permanent shocks, ψ, and the r structural transitory shocks, ν. If we instead use the reduce form VMA representation we have

$$\Delta X_t = \delta + C(L)\varepsilon_t, \tag{18}$$

which could be written as

$$X_t = X_0 + \delta t + C(L)(1 + L + \ldots + L^t)\varepsilon_t \tag{19}$$

or

$$X_t = X_0 + C(1)\xi_t + D(L)\varepsilon_t, \tag{20}$$

where $\xi_t = \rho + \xi_{t-1} + \varepsilon_t$. Now ξ_t is a $n\times 1$ vector of stochastic trends (rather than a $k\times 1$ vector of common stochastic trends as was the case in the structural CT representation). Here $\beta'C(1)=0$ for the same reason as $\beta'A=0$ in the CT representation, since we then make the linear combinations of the series (described by the co-integrating vectors) stationary. Connecting the reduced form VMA and the CT representations, we have that

$$\begin{aligned} C(1)\rho &= A\mu \\ C(1)\varepsilon_t &= A\psi_t \\ C(1)\Sigma C(1)' &= AA' \end{aligned} \tag{21}$$

6.1.2 IDENTIFYING THE STRUCTURAL CT AND VMA

We will now discuss the assumptions needed to identify the matrixes in the structural representations. In particular, we would like to identify the matrix A in the CT representation and the lagpolynomial $R(L)$ in the VMA representation. By estimating the VAR or EC model, we get consistent estimates of $C(1)$ and Σ. Furthermore, from estimating the EC, we get an estimate of the co-integrating vector β. To start with the identification of the A matrix, we know that $\beta' X_t$ should be stationary. Premultiplying the CT representation with β' we see that we must have $\beta' A = 0$, in order to cancel out the influence of the stochastic trends. This condition provides rk restrictions on A. Furthermore, $C(1)\Sigma C(1)' = AA'$ adds $k(k+1)/2$ restrictions, and thus, since A has nk parameters, we need $k(k-1)/2$ additional restrictions to identify A exactly.

Identifying the VMA is a bit more complicated. First we define the identification matrix as $F \equiv [F_k' \ F_r']$, which connect the reduced form errors and the structural ones according to $F\varepsilon_t = \varphi_t$. Define the first part of the identification matrix as $F_k \equiv (A'A)^{-1}A'C(1)$, which identifies the permanent shocks. This is derived from the condition $C(1)\varepsilon_t = A\psi_t$. We know this part of the identification matrix after completing the first step described above. It now remains to determine the matrix F_r, that is, to identify the transitory shocks. From the VMA we have that $C(L)F^{-1} = R(L)$. We have also decided that the first k shocks should be the permanent and the r last the transitory. This implies that $R(1) = C(1)F^{-1} = [A \quad \mathbf{0}]$. We can now use $F_r = Q^{-1}\omega'\Sigma^{-1}$ to identify the transitory shocks (see Warne [1990b]), where $\omega = \alpha\gamma$ and $QQ' = \omega'\Sigma^{-1}\omega$. The Q matrix is chosen to make the co-variance matrix of the transitory shocks equal to the identity matrix, so we obtain structural errors. Given the Q matrix and the matrix of loadings α, we then choose the non-singular $r \times r$ matrix γ in a way that makes the contemporaneous responses of certain variables to the transitory shocks zero. This is analogous to the standard identification of VAR models, where we also impose a number of zero restrictions on contemporaneous responses.

To see more formally how restrictions on γ impose restrictions on the contemporaneous responses, we could use

$$\text{resp}(X_t) = R_{r,0} = \alpha\gamma(Q')^{-1}, \tag{22}$$

which in Warne [1990b] is shown to be the response to a one standard deviation shock in the transitory shocks, ν. The matrix $R_{r,0}$ contains the responses to the transitory shocks and is the last r columns of R_0 in the VMA representation. To achieve exact identification, it is sufficient to restrict $r(r-1)/2$ elements of $R_{r,0}$ to zero. However, to obtain this, the matrix γ has to be chosen carefully. Having done this, the identification of the model is completed.

6.2 DEFINITIONS OF VARIABLES

Variable names and data from CITIBASE, quarterly, SAAR, billion dollars, 1960:1 - 1993:1.

ggsr+ggfr = sum of federal, state and local government receipts, current prices.

ggst+ggft = -- " -- transfers, current prices.

ggsex+ggfex = -- " -- expenditures, c.p.

ggsint+ggfint = -- " -- interest payments, c.p.

gdge = deflator to 1987 prices.

gnpq = GNP in 1987 prices.

gcq = Total private consumption in 1987 prices.

gpop = Total population

My transformations

To 1987 prices:

nt = 100*(ggsr(t)+ggfr(t)-ggst(t)-ggft(t)-ggsint(t)-ggfint(t))/gdge(t)

g = 100*(ggsex(t)+ggfex(t)-ggst(t)-ggft(t)-ggsint(t)-ggfint(t))/gdge(t)

To per capita units:

y = gnpq(t)/gpop(t)

g = g(t)/gpop(t)

nt = nt(t)/gpop(t)

c = gcq(t)/gpop(t)

Essay III

Risky Taxes, Budget Balance Preserving Spreads and Precautionary Savings*

Abstract

The paper analyzes the effects on consumption from changes in the riskiness of taxes. It starts by reinterpreting the Sandmo [1970] paper on general capital income risk to the case of risky capital taxation. In his framework the concept of a mean preserving spread (MPS) is used for the risk analysis. In connection with risky taxes it is however possible to explicitly connect the tax risk with the government's budget constraint. In this paper the concept of a budget balance preserving spread (BBPS) is developed and used for the analysis of stochastic taxes. The paper is concluded with a comparison of the effects that a MPS and a BBPS has on consumption decisions. It is shown that the comparative statics results for a BBPS could be different from the results obtained with a MPS.

1. Introduction

Since, at least, von Neumann and Morgenstern's [1944] expected utility maximization, economics has not only dealt seriously with choice under certainty, but also with decision making in an uncertain world. Pratt [1964] and Arrow [1971] then introduced the concepts of absolute and relative risk aversion, (which makes it possible to compare the taste or aversion for risk between different utility functions with a unit free measure that is independent of affine transformations of the utility function). These concepts have proved to be very useful in the study of decision making under uncertainty.

When we use expected utility maximization for intertemporal choice, we need to distinguish between temporal and timeless risk, as discussed in for example Drèze and Modigliani [1972] and Machina [1984]. The concept of risk aversion is specified in the case of timeless risk, where no time passes before the uncertainty is resolved. One of the results from Drèze and Modigliani is that a temporal uncertain gamble will never be preferred to a timeless one if the outcomes have the same distribution. In, for example, Sandmo [1970] the assumption of

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decreasing temporal risk aversion is used (with time separable utility, this implies a positive third derivative of the utility function), as motivated by for example Arrow [1971], in addition to the standard assumptions of timeless risk aversion (which implies a negative second derivative of the utility function).

The effect of uncertain future income on consumption/savings decisions are analyzed in Leland [1968] and Sandmo [1970]. Leland shows, by using a Taylor expansion, that pure risk aversion will not in itself give rise to precautionary savings, but assumptions on the third derivative of the utility function can ensure a precautionary savings motive. Sibley [1975] generalizes Leland's two period analysis to a multiperiod model, and shows that the condition for precautionary savings is still a positive third derivative of the utility function. Sandmo analyzes a more straightforward case, where a specific parameter is introduced that describes the risk, or, equivalently, variance in his model. This simplification with respect to the type of change in risk considered leads to a more direct analysis of the conditions needed for the utility function. In the case of income risk, it is for example enough to state that individuals have decreasing temporal risk aversion in order to find precautionary savings as a result of increasing risk. Where increasing risk is defined as a mean preserving spread, i.e. the expected outcome is not affected, but the variance of the outcome is. An analysis of the conditions of the utility function in combination with more general assumptions about distributional changes for deriving clear-cut comparative statics results could be found in Ormiston and Schlee [1992]. (The reader interested in comparative statics for non-expected utility[1] is referred to Machina [1989].)

Since Leland's [1968] paper, the notion of precautionary savings has received attention both in theoretical and empirical work. In the theoretical world, Kimball [1990] formalizes the concepts of precautionary savings, and shows how the theory of risk aversion can be translated into a theory of "prudence", where prudence is a measure of the strength of the precautionary savings motive, rather than the strength of disliking risk. The latter concept can be summarized by the familiar measure of absolute risk aversion defined as $a \equiv -u''/u'$. In an analogous way, the measure of absolute prudence is defined as $\eta \equiv -u'''/u''$, where u is the utility of future consumption. Kimball shows that most theorems about risk aversion can be applied to precautionary savings by simply substituting $-u'$ for u. He also notes that the measure of

[1] Expected utility analysis has the implication of linearity in probabilities, while non-expected utility analysis instead assumes that preferences over probability distributions is smooth, and could locally be approximated by expected utility analysis.

absolute prudence will be larger(smaller) than the measure of absolute risk aversion when absolute risk aversion is declining(increasing). (This is for example the case for time separable iso-elastic utility, where we also always have a positive coefficient of prudence if we assume that individuals are risk averse.)

The important aspect of the theory of prudence is that we can analyze how the consumption/savings choice will change in response to changes in risk. We can thus derive conditions on distributional changes in combination with conditions on the utility function in terms of prudence that are needed for clear-cut results of comparative statics exercises that involve risk. For example, in the case of capital income risk, there will be two effects that affect the consumption choice. First, a substitution effect that says that if capital income becomes more risky, so does second period consumption, and thus the agent will shift consumption away from the second period. However, there is an additional effect, labeled income effect, that says that with increased uncertainty about second period income implied by increased uncertainty about the return on savings, the agent faces a risk of having a very low consumption in the second period, which implies a very high marginal utility in that period. Since the expected utility from consumption in the first and second period should be the same, the agent will save more to avoid having a very high marginal utility in the second period. With a mean preserving spread in capital income, there are thus two effects on first period consumption with opposite signs. However, Ormiston and Schlee [1992] tell us that with a coefficient of relative prudence that is greater(smaller) than two, the income(substitution) effect will dominate and first period consumption will decrease(increase). In the case of iso-elastic utility, this condition is equivalent to Sandmo's [1970] condition that the coefficient of relative risk aversion is greater than one.

There is empirical evidence that the inclusion of precautionary savings helps explain observed deviations from models without precautionary savings. For example, in studies of the permanent income hypothesis, there have been features of data that could not be well explained with models that do not allow for precautionary savings, most noted, underspending of the elderly (see Mirer [1979]), excess growth (see Deaton [1986]), excess smoothness (see Campbell and Deaton [1989]) and excess sensitivity (see Flavin [1981]) of consumption. To define the empirical puzzles in more detail; *underspending of the elderly* is based on cross-section data that shows that the elderly do not dissave during retirement, *excess sensitivity* says that consumption moves "too much" in response to anticipated changes in labor income, *excess growth* implies that aggregate consumption grows over time in a way that cannot be explained by the real interest rate being greater than the rate of time preference, and finally *excess smoothness* says that consumption responds "too little" to unanticipated changes in labor income.

Zeldes [1989] simulates a model with preferences that display constant relative risk aversion, implying that households are prudent, and assumes that labor income is an i.i.d. process. He can then conclude that the above empirical puzzles can be explained by using a model with preferences that display a precautionary savings motive instead of the standard models of the permanent income hypothesis that lack this feature. Furthermore, Caballero [1990] derives a theoretical model which he combines with different assumptions about the stochastic process that governs labor income, and concludes that he can explain a large part of the empirical puzzles in the US data discussed above. In Caballero [1991] he also shows that about 60 percent of US wealth accumulation can be due to precautionary savings.

However, in a micro based study estimating the coefficient of prudence by using data from the Consumer Expenditure Survey, Dynan [1993] concludes that the coefficient of prudence is very small, and actually so small that it is not consistent with normal assumptions about households' aversion of risk. Taken seriously, this would raise the question why micro and macro data yield so different results. In this paper I will not try to answer that, and since this model is aimed at macro phenomena, the importance of the precautionary motive will be stressed.

In this paper as well as in many of the above papers, the portfolio nature of savings analyzed in the finance literature is neglected. There are of course models that combine the consumption decision with portfolio decisions, the early references being Samuelson [1969], who analyzes consumption and portfolio decision in discrete time, and Merton [1969], who uses continuous time. This type of analysis then answers the question of what amount of risky and riskless assets to hold in the portfolio when risk changes. The results are determined by assumptions made on the shape of the utility function, and the conditions on the parameters in the utility function are closely related to the conditions derived in Sandmo's [1970] study of capital income risk without a portfolio decision. Drèze and Modigliani [1972] also discuss consumption and portfolio decision, and the separability between these decisions.

Risk in connection with taxation has been discussed since, at least, Domar and Musgrave [1944]. In Stiglitz [1969] the effects of different taxes on risk taking are discussed. More precisely, the amount of savings put into a risky asset rather than a safe one under different tax policies is analyzed. Another aspect from this part of the literature is that capital income taxation that is certain reduces the variance of net asset returns, since the uncertain gross returns are multiplied by a number smaller than one. If the tax system is then set up such that it also makes transfers in the case of negative returns, taxes act as an insurance or risk reducer. In the extreme, if the gross returns are symmetrically distributed around zero, the "tax" will of course only have the characteristics of an insurance and not a tax, since the variance is reduced but the tax revenue is zero. In more sensible cases, the expected gross return is above zero, and loss offsets are not total, and thus the tax system will work partly as an insurance and partly as a revenue raiser for the government. The insurance aspect could of course be one part of the tax system, but by assuming that the gross return is deterministic, we will abstract from this issue here.

In this paper, the focus will be on the stochastic features of the *tax system* rather than on the stochastic features of the market. This approach might require some motivation. The first part of the motivation is due to the increasing part of disposable income that is contingent on government policies. In Sweden, for example, the public sector turns over about 70 percent of GDP, a large part of the turnover being transfers to the private sector. Combining this observation with the observation that the tax and transfer system is consistently being "reformed", implies that a large part of the households' income is contingent on public policies that keep changing over time. In other words, a large part of the fluctuation in disposable income is due to the public sector rather than the market.

Furthermore, in real business cycle models, including stochastic features of government policy has helped to improve the performance of these models. If we read out the headings in McGrattan [1994], these say: "The standard [real business cycle] model's predictions improved slightly with indivisible labor and significantly with fiscal shocks", which further motivates studying a stochastic tax system. Finally, in the institutional economics literature, the value of stable rules and regulations has been stressed, and this paper can be a formalization of a certain aspect of the rule system, namely how agents are taxed. However, the aim of the paper is not to provide a normative analysis of the tax system, but rather a positive analysis of the effects that uncertain taxes can have on private consumption.

There have been some previous papers taking the view that taxes rather than the market are uncertain, for example Chan [1983], and Alm [1988]. In Chan, lump-sum taxes are analyzed with a government budget constraint, while Alm analyzes different aspects of taxation more in the spirit of optimal taxation, including uncertainty, but ignoring the government's budget constraint.

This paper analyzes uncertain *distortionary* taxes when the government's budget constraint is explicitly considered. It is, however, not in the spirit of the optimal taxation literature, but more in the spirit of general choice under uncertainty as discussed above. The framework is adopted from Sandmo's [1970] paper, where we have an explicit parameter representing changes in risk. Analyzing risk in this set-up facilitates the interpretation of the results, and simplifies the algebra, but the price to pay is that, in reality, riskiness could be reflected by other aspects than changes in the variance (see for example Rothschild and Stiglitz [1970, 1971] for definitions of increasing risk, Huang and Litzenberger [1988] for an overview of stochastic dominance, and Whitmore [1970] for the definition of third degree stochastic dominance).

The reason for including the government's budget constraint in the analysis is that with distortionary taxes, households can affect the tax base and thus also the expected value of the uncertain tax rate. In the cases where we are interested in models with uncertain distortionary taxation, it is shown that the comparative statics results are affected by the choice of mean preserving spread (MPS) or budget balance preserving spread (BBPS) analysis. If we instead analyze models with lump-sum taxes (which is the analog of Sandmo's [1970] analysis of income risk) as in Becker [1994], MPS and BBPS will obviously give the same results, since then, by definition, consumers cannot affect the tax base.

To summarize, the analysis of risky choice has a fairly long history in the economics literature, and in many cases the tax system is not (explicitly) considered. In the cases where the tax system is considered, however, the focus is with few exceptions on market induced risk, with the potential for taxes to act as an insurance against risk. This paper has a different starting point, namely that the tax system itself is the origin of uncertainty, and that its riskiness affects households' consumption decision. Further, one natural starting point when analyzing risk, namely mean preserving spreads, is now compared with budget balance preserving spreads, to take into account that the risk source is the government and the government has a budget constraint to obey. The budget balance preserving spread analysis is thus a more natural risk concept when we analyze risk created by the government rather than by the market.

It is shown in the paper that the BBPS risk concept modifies the assumptions needed on the utility function in order to derive clear comparative statics results with respect to risk and consumption. In particular, it is shown that, for *small* enough values of the relative *risk aversion* coefficient, as well as for values greater than one, the *precautionary savings* motive will dominate in the case of capital income uncertainty. In the usual mean preserving spread analysis this is the case only when the coefficient is larger than one.

The paper is organized as follows: in Section 2, two experiments with uncertain taxes are conducted. The first uses a mean preserving spread in the tax rate, while the second uses a budget balance preserving spread. In Subsection 2.4 the results of the two experiments are compared. The paper ends with a section containing a summary and some conclusions. Finally, in the appendix, changing a deterministic tax rate is analyzed, as well as Sandmo's original analysis of general capital income uncertainty within the notation and solution framework used in this paper.

2. TWO CONCEPTS OF TAX RISK

The general structure of the problem is that we study an economy that exists for two periods, with a government sector and an infinite number of households. In the first part, the government is simply an institution that keeps the mean of the expected tax rate constant, which is analogous to Sandmo's [1970] analysis of capital income risk, while in the second part the government obeys a budget constraint. In both cases, the households are expected utility maximizers that have labor income in the first period, and save in order to consume in the second period. Gross returns on savings are exogenous and known, while the tax rate on capital income is stochastic. In the problems below we will solve a two-equation system for two unknowns. The first equation is the representative household's first order condition, and the second equation is the mean preserving spread or the government's budget constraint, alternatively. In the case of a mean preserving spread, the system of equations approach is not really needed, since the system could be analyzed recursively, while in the second part the equations are interdependent. Using the system of equations approach also in the first case is motivated by the fact that we then derive the same type of conditions for both models, which facilitates comparisons.

2.1 HOUSEHOLDS' PROBLEM

Households are rational economic units that maximize their expected utility subject to a budget constraint. There are many (or an infinite number of) identical households that all solve the problem

$$\begin{aligned} \max_{c_1,c_2} \; & E\left[U(c_1,c_2)\right] = \int_{\Gamma} U(c_1,c_2) f(\gamma^i) d\gamma^i \\ \text{s.t.} \quad & c_1 = y_1 - \tau_1 - a_1 \\ & c_2 = (1 + r - r(p\gamma^i + \theta))(y_1 - c_1 - \tau_1) \,, \end{aligned} \tag{1}$$

where $U(.)$ is a von Neumann-Morgenstern utility function with $U' > 0$ and $U'' < 0$, i.e. agents are assumed to be risk averse, c_t is period t consumption, y_1 is an exogenous (labor) income that the individual has only in the first period, τ_1 is a lump-sum tax in the first period, r is the certain gross interest rate, a_1 is the savings from period one to two, p is the multiplicative policy parameter, γ^i is the individual's realized draw from the tax rate distribution Γ, and finally θ is the additive tax parameter. Since we are analyzing capital income tax rates, we only allow for realizations of the total tax rate, $p\gamma^i + \theta$, that are between zero and one, and therefore the average tax rate will also be between zero and one. We can interpret the relative magnitudes of p and θ as the relative weights of the stochastic and deterministic parts of the tax system, respectively. For example, a large p and a small θ indicate a relatively risky tax system in the sense that tax rates will have a large spread.

The assumption that we are analyzing an infinite number of households makes the probability distribution of tax rates independent of the individual's choice of consumption. In the mean preserving spread analysis, this is never a problem, but in the budget balance preserving spread analysis, we have a feed-back to the tax rates from household behavior. To avoid that the household internalizes this feed-back, we assume that there are many households, so the feed-back effect from the behavior of a particular household is negligible.

The first and second order conditions (FOC and SOC respectively) to this problem are

FOC: $$F^2 \equiv E[U_1 - R^i U_2] = 0 \tag{2}$$

SOC: $$S \equiv E\left[U_{11} - 2R^i U_{12} + R^{i^2} U_{22}\right] < 0 \ , \tag{3}$$

where $R^i \equiv \left(1 + r - \left(p\gamma^i + \theta\right)r\right)$ is the individual-specific realized net rate of return on capital that is stochastic due to the stochastic tax rate. In this model this is the only source of uncertainty. The first order condition states that the expected marginal utility of consumption in period one and two should be equal. The FOC is the second equation in the system of equations we will analyze.

2.2 MEAN PRESERVING SPREADS IN TAX RATES

This section uses the framework of Sandmo [1970] to analyze precautionary savings in response to stochastic taxes when using a MPS. In this experiment, the government is only choosing tax rates such that the mean tax rate on capital is preserved, but the spread is altered for different choices of p. In the following sections the government will be defined by a budget restriction rather than from solely being a risk experimenter that determines the spread of returns[2].

The tax system consists of one standard tax rate, γ, that is stochastic, one multiplicative policy variable, p, and one additive deterministic policy variable θ. Together these variables define the expected tax rate, $E\left[p\gamma^i + \theta\right] = p\bar{\gamma} + \theta$, that is assumed to be constant, since we analyze a mean preserving spread in this section. This specification is equivalent to the formulation used in Sandmo's [1970] framework presented in the appendix. Since the parameter p is

[2] I leave it to the reader to decide which of these assumptions about the government is most closely related to real world observations.

multiplicative, it affects both the spread and the mean of the total tax rate. θ, on the other hand, is additive, and only affects the mean. The restriction could be formalized as

$$F_M^1 \equiv p\bar{\gamma} + \theta - \text{constant} = 0 \ , \tag{4}$$

which will be the first equation of our two-equation system (subscript M for MPS). This condition could be totally differentiated as it is, to show how θ has to change for a given change in p, which is how Sandmo proceeds. Here we will use a more tedious system of equation approach in order to derive the analog of the equations obtained in the following experiment, where we have to take into account changes in the tax base. The tax parameter p is used for the comparative statics exercise, while θ and c_1 are the two endogenous variables that ensure budget balance for the government and that the representative household's first order condition is satisfied. Combining the FOC and the mean preserving spread condition yields the following system of equations

$$\begin{cases} F_M^1 = 0 \\ F^2 = 0 \ . \end{cases} \tag{5}$$

The system is totally differentiated and solved for the partial derivatives of interest (for a formal derivation, see the appendix). We start by investigating how the additive tax parameter has to change in response to an increase in the multiplicative tax parameter in order to satisfy the mean preserving spread condition

$$\frac{\partial \theta}{\partial p} = -\bar{\gamma} \ , \tag{6}$$

which says that if p is increased, the average tax will rise with $\bar{\gamma}$ and thus the additive tax parameter has to change equally much in the other direction. This condition could of course equally well be obtained by total differentiation of the first equation, which is the way it is done in Sandmo. Again, this unnecessarily tedious approach is used to conform to the steps needed in the following sections.

Now we want to investigate how consumption changes in response to a mean preserving spread. We have that

$$\frac{\partial c_1}{\partial p} = -\frac{1}{S} E\left[\Phi\left(\gamma^i - \bar{\gamma}\right)\right], \tag{7}$$

where $\Phi \equiv r\left(U_2 - a_1 U_{12} + R^i a_1 U_{22}\right)$. This is proportional to the expression describing how consumption changes in response to increased capital income risk in Sandmo's original paper, which is presented in the appendix. The difference between this expression and Sandmo's is due to the fact that we here analyze a stochastic capital income *tax rate* and not simply a stochastic capital income. The sign determination is equivalent to the capital income risk case, since a higher realized value of the tax rate means a lower capital income, while a higher realized value in the original case leads to a higher capital income. Thus the minus sign in front of the present expression.

The intuition for the sign determination could be thought of as follows. An increase in p makes second period consumption more risky, implying that households would like to substitute their second period consumption to first period consumption, thus the *positive substitution effect.* At the same time, with increasing p, households face the risk of having a very low second period income, which creates an incentive for precautionary savings, thus the *negative income effect.* In the following discussion we will continue to associate the label substitution effect with the component involving the first derivative, and the label income effect with the terms including the second derivatives of the utility function, although, as will be shown, the results associated with some cases of a dominating substitution effect will differ.

2.3 Budget Balance Preserving Spreads in Tax Rates

We are now ready to investigate the effects of explicitly incorporating the government's intertemporal budget constraint into the analysis of risky taxes. The tax system consists of lump-sum taxes in the first period, as well as government consumption in both periods. In the second period, there is a capital income tax rate that is stochastic from the individuals' perspective, but certain from the government's point of view, since there are many (or an infinite number of) identical households that pay taxes. Compared to the risk analysis above, the government now obeys a budget constraint, rather than a condition stating that the mean of the tax rate should be constant. In an analysis of risky taxes, the spread concept used in this section yields a more natural condition on the tax distribution. We write the budget constraint as

$$\bar{g} = g_1 + \frac{g_2}{1+r} = \tau_1 + \frac{E(\tau_2)}{1+r} = \tau_1 + (p\bar{\gamma} + \theta) r(y_1 - c_1 - \tau_1)/(1+r) , \qquad (8)$$

where $\bar{g}$ is the present value of the total resources extracted by the government[3], with g_t extracted in period t. Taxes in the second period, τ_2, are collected as a capital income tax, with the tax rate $p\bar{\gamma} + \theta$. Rewrite this, and now define the function F_B^1 (B for **BBPS**) as

$$F_B^1 \equiv (p\bar{\gamma} + \theta)(y_1 - c_1 - \tau_1) - (\bar{g} - \tau_1)(1+r)/r = 0 , \qquad (9)$$

A notational point of importance is again the distinction between the individual's realized value of the stochastic tax rate, denoted by γ^i, and drawn from a distribution of tax rates Γ, defined for positive numbers between zero and one, and the aggregate variable that the government faces, $\bar{\gamma}$, which is no longer a stochastic variable since we assume that there are sufficiently many individuals for the law of large numbers to work. The total tax rate on capital income for a particular individual is $(p\gamma^i + \theta)$, with $E[p\gamma^i + \theta] = p\bar{\gamma} + \theta$, which is the tax rate that appears in the government's budget constraint.

The experiment in this section is again to investigate how consumption changes in response to a change in p, but now the government's budget balance has to be preserved, rather than the mean of the tax rate. This is achieved by changing the additive tax parameter, θ. What we are analyzing here is thus not the "ordinary" mean preserving spread of the previous subsection, but rather a budget balance preserving spread of the tax rate.

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[3] In general this waste of resources is not a very good approximation of government activity, but for the purpose of keeping the analysis at a more transparent level, this characterization could perhaps be justified. A more generous interpretation of the present set-up is to consider the government sector activity as fixed, and with some additive separability assumption, this activity could be ignored in the present analysis.

The potential for this experiment to differ from the above analysis is that we have to take into account changes in the tax base in response to changes in risk. This implies that if the tax base actually changes, the mean of the tax rate will have to change in order to satisfy the government's budget constraint. However, this has the implication that for some levels of government spending in combination with some utility functions, there might not exist a tax rate that is between zero and one that satisfies the government's budget constraint. In the above section, it was sufficient to make the assumption that the tax rate has to be between zero and one. With a budget balance preserving spread, we can, however, start out in a situation where the government's budget is balanced, and end up in a situation where a tax rate equal to one is not sufficient for balancing the budget, due to the endogenous changes in the tax base as a response to changes in the tax risk. Only the cases where tax rates between zero and one is sufficient for achieving budget balance will be analyzed in this paper. However, it is still of importance to realize that the government could actually end up violating its budget constraint in this model, due to changes in the tax risk, without having changed its consumption.

If we use the household's first order condition and the government's budget constraint, we get the following system of equations

$$\begin{cases} F_B^1 = 0 \\ F^2 = 0 \, . \end{cases} \tag{10}$$

The system is totally differentiated, and we can then solve for the partial derivatives of interest. We start by investigating how the additive tax parameter has to change in order to satisfy the government budget constraint. The derivative can be written as

$$\frac{\partial \theta}{\partial p} = -\bar{\gamma}\,\frac{a_1 S + pE\left[\gamma^{\,i}\Phi\right]}{|J|} - \theta\,\frac{E\left[\gamma^{\,i}\Phi\right]}{|J|} \, , \tag{11}$$

where $\Phi \equiv r\left(U_2 - a_1 U_{12} + R^i a_1 U_{22}\right)$, and $|J| = a_1 S + \left(p\bar{\gamma} + \theta\right)E[\Phi] \neq 0$ (which is the condition that the Jacobian determinant has to be non-zero in order to use the implicit function theorem.) The sign of this derivative will in general be negative, since if we raise the expected value of the stochastic tax rate, the value of the deterministic tax rate can, in general, be decreased, to maintain budget balance for the government. There is, however, a possibility that the dervative is positive, which is basically to say that we are to the right of the maximum of the Laffer curve. In such a case, an increase in p, and thus the risky part of the tax, have very strong effects on the consumption choice of the household. This implies that the tax base then erode to the extent that the deterministic tax rate has to increase to compensate for the reduced tax base to maintain budget balance for the government. We will discuss the circumstances where this situation might appear after we have investigated how consumption will change.

To investigate how consumption changes in response to a budget preserving spread, we have the derivative

$$\frac{\partial c_1}{\partial p} = \frac{\bar{\gamma} a_1 E[\Phi] - a_1 E[\gamma^i \Phi]}{|J|} = -\frac{a_1}{|J|} E[\Phi(\gamma^i - \bar{\gamma})] \ . \tag{12}$$

The expression is (as expected) very similar to the result with a mean preserving spread, with the exception that we now have $a_1/|J|$ in the first factor rather than simply $1/S$. Again, the sign will be determined by the strength of the income and substitution effects, with the complication that we now do not always know that the sign of the denominator is negative, since the denominator can be positive in the case of "dominating" substitution effects and a small enough coefficient of relative risk aversion. The specific conditions determining the sign of the derivative between first period consumption and the multiplicative tax parameter will be discussed below.

2.4 Comparing BBPS with MPS

A question of interest at this point is to establish how the above results from the mean preserving spread analysis compare to the budget balance preserving spread in more detail. In the case of a mean preserving spread of the return to capital, we know that the consumption response to increased risk could be divided into three cases: first, the substitution effect could dominate, secondly, the income effect could dominate and, finally, the effects could be equal and thus leave consumption unchanged.

The BBPS analysis could differ from the MPS analysis due to changes in the tax base, and thus to endogenous changes in the mean tax rate. We therefore know that in the case where the income and substitution effects cancel, the analysis of an intertemporal budget balance preserving spread is equivalent to Sandmo's mean preserving analysis, since then the tax base remains unchanged, and thus the average tax rate has to be kept constant. This could be seen more formally in equations (11) and (12), which then reduce to the same expressions as in Sandmo's case, namely equations (6) and (7).

The two interesting cases to analyze are thus when either the income or substitution effect dominates, to use Sandmo's original labels. Start by writing out some terms in (12) and reshuffle a little to obtain

$$\frac{\partial c_1}{\partial p} = \frac{1}{S + \frac{p\bar{\gamma} + \theta}{a_1} E[\Phi]} E\left[\Phi\left(\bar{\gamma} - \gamma^i\right)\right] . \tag{13}$$

The sign is determined by two parts, firstly

$$E[\Phi] = E\left[r\left(\underbrace{U_2}_{+} \underbrace{- a_1\left(U_{12} - R^i U_{22}\right)}_{-} \right)\right] , \tag{14}$$

where the first term is positive and the second is negative,[4] and secondly

$$E\left[\Phi\left(\bar{\gamma} - \gamma^i\right)\right] = rE\left[U_2\left(\bar{\gamma} - \gamma^i\right) - a_1\left(U_{12} - R^i U_{22}\right)\left(\bar{\gamma} - \gamma^i\right)\right] . \tag{15}$$

In Sandmo's MPS case, the sign of the derivative is determined by the second factor, since the denominator is always negative in his case. The conclusion is then that if the substitution effect dominates the derivative is positive, and if the income effect dominates the derivative is

[4] The second term is negative if we assume that $\partial c_1 / \partial y_1 > 0$.

negative. For the BBPS, we have to sort out the relationship between $E\left[\Phi\left(\bar{\gamma}-\gamma^{i}\right)\right]$ and $|J|$. A proof of these signs for the case of time separable iso-elastic utility is presented in the appendix, to show how the size of the relative risk aversion coefficient enters the analysis. For a proof with a general utility function, see the appendix in Sandmo [1970].

The three cases are then:

- Income and substitution effects cancel out

  $\Rightarrow E\left[\Phi\left(\bar{\gamma}-\gamma^{i}\right)\right]=0 \;\Rightarrow\; |J|<0.$

- Income effect dominates substitution effect

  $\Rightarrow E\left[\Phi\left(\bar{\gamma}-\gamma^{i}\right)\right]>0 \;\Rightarrow\; |J|<0.$

- Substitution effect dominates income effect

  $\Rightarrow E\left[\Phi\left(\bar{\gamma}-\gamma^{i}\right)\right]<0 \;\Rightarrow\; |J|>0 \text{ if } -a_1 S<\left(p\bar{\gamma}+\theta\right)E[\Phi],$

  $|J|<0 \text{ if } -a_1 S>\left(p\bar{\gamma}+\theta\right)E[\Phi].$

Obviously the MPS and BBPS are equivalent in the first case, since then the BBPS analysis reduces to the MPS analysis. The intuition is, of course, that if we do not have a change in consumption patterns the tax base does not change, so the tax rate has to have the same mean also in the BBPS analysis.

The conclusion for a dominating income effect in the MPS analysis is that the derivative is negative. This is still valid, since then E[Φ] is negative and leaves the negative sign of the denominator $|J|$ unchanged, which combined with a positive numerator generates a negative derivative.

However, the magnitude of the derivative will be smaller in the BBPS analysis compared to the MPS analysis. The explanation is that as households save more, the tax base is larger and the expected tax rate on capital is thus reduced. This changes the relative price on first and second period consumption, but in a deterministic way through the additive tax parameter. From the appendix we know that for a deterministic increase in the net return on capital, due to a decrease in θ, and with what we here label a dominating income effect, first period consumption increases. This latter effect counteracts the initial effect of reduced first period consumption, but does not alter the sign of the derivative.

Finally, we have the case where the substitution effect dominates, which in the MPS analysis makes the derivative positive. In the BBPS this is no longer obvious. Instead it is possible to show that when the risk aversion coefficient is small enough, we will have a negative sign on the derivative also when the so-called substitution effect dominates. This will be the case when the Jacobian determinant is positive (which implies that $-a_1 S < \left(p\bar{\gamma} + \theta\right)E[\Phi]$). The conditions on the utility function and tax parameters needed to fulfill this condition can be found in the appendix. (It is then perhaps a misuse of language to call this a dominating substitution effect, but I will anyway keep Sandmo's original labels.)

The intuition for this is perhaps not so obvious, but one interpretation is that when the net return on capital becomes low, households do not want to save as much, which at the same time makes them more concerned about the spread in second period income and thus the spread in second period consumption. In other words, households face the risk of having a very low consumption in the second period if the spread increases, implying that they take precautionary actions against this by saving more, thus reducing first period consumption.

Finally, we also note that it is only when the substitution effect dominates that we can be to the right of the maximum of the Laffer curve, since we then know that first period consumption increases, implying that the tax base is smaller. However, this does not necessarily lead to the conclusion that the determininstic tax increases when the stochastic tax increases. It merely indicates that the total expected tax rate has to increase, i.e. the derivative between θ and p has to be greater than $-\bar{\gamma}$, and potentially, it can then also become positive. A formal condition of when the derivative is positive can be found in the appendix.

Two graphical summaries of the signs of the two derivatives for different values of the coefficient of relative risk aversion for MPS and BBPS are presented in Figure 1 and 2.

Mean preserving spread analysis

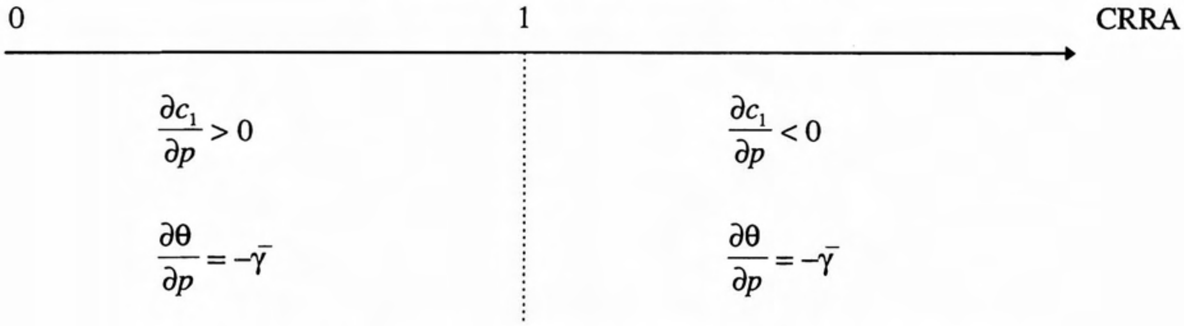

Figure 1. *The sign and magnitude of the derivatives for different values of the coefficient of relative risk aversion (CRRA) for a mean preserving spread in tax rates.*

Budget balance preserving spread analysis

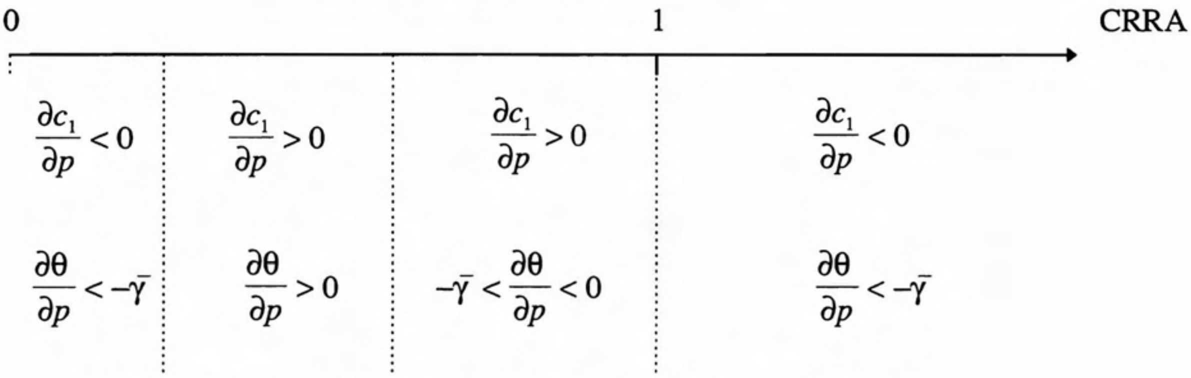

Figure 2. *The sign and magnitude of the derivatives for different values of the coefficient of relative risk aversion (CRRA) for a budget balance preserving spread in tax rates.*

3. Summary and Conclusions

This paper analyzes intertemporal consumption behavior in response to changes in risky capital income taxation. The framework used is adopted from Sandmo [1970], but instead of only using the risk concept of mean preserving spreads, the concept of budget balance preserving spreads is developed. The model is a simple two period model with a government sector and an infinite number of households. Risk is introduced in the model by assuming that individuals face a stochastic tax rate on capital income. In the case of a mean preserving spread of the tax rate, the effects on consumption are determined by the strength of the income and substitution effects; thus savings could either increase or decrease when additional risk is introduced. Using a time separable iso-elastic utility function, this is equivalent to assuming that the coefficient of relative risk aversion is greater (dominating income effect) or smaller (dominating substitution effect) than one.

When instead a BBPS is analyzed, the income effect will dominate also for small values of the risk aversion coefficient (or with high average tax rates), since the individual is then exposed to a high risk even before the tax risk is increased. This leads to the, at first, not so intuitive conclusion that the precautionary savings will take place both when the risk aversion coefficient is small enough and when it is greater than one. The crucial factor distinguishing the two cases of MPS and BBPS is that in the latter case the mean of the tax rate has to change in response to consumption changes, since the tax base then changes and the government has an intertemporal budget constraint to satisfy, which in turn affects the consumption choice.

The present analysis has focused on risk created by the tax system rather than the market/production side of the economy. This is of course not the full picture of risk in the real world, but it might serve as a starting point for understanding how risk created by the government could be different from (unspecified) market risk.

There are several ways in which the present analysis could be extended. One way could be to introduce uncertainty with respect to both gross returns and tax rates, in order to investigate how assumptions about the correlation between these would modify the results. Another extension would be to investigate portfolio decisions for the household, with different assets facing different tax and return risks.

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5. APPENDIX

5.1 LIST OF VARIABLES

θ	additive tax parameter
p	multiplicative tax parameter for comparative statics exercises
γ^i	the stochastic tax variable, the superscript denotes that it is individual specific
	$E[\gamma^i] = \bar{\gamma}$, (non-stochastic) is what the government uses,
	since there is a large (enough) number of households
g_t	government consumption in period t = 1,2
τ_1	first period lump-sum tax
U	von Neumann-Morgenstern utility function
U_{ij}	partial derivative with respect to elements *i, j*
c_t	private consumption in period t = 1,2
y_t	income in period t = 1,2
r	interest rate given from world market
a_1	household savings ($a_1 \equiv y_1 - c_1 - \tau_1$)
$f(x)$	density function of variable x defined over X

5.2 THE DERIVATION OF THE PARTIAL DERIVATIVES

The analysis in the following subsections starts from the points in the main text where the systems of equations are presented, and derives the partial derivatives.

5.2.1 THE MEAN PRESERVING SPREAD CASE

Totally differentiate the system in (5) to get

$$\begin{bmatrix} \frac{\partial F_M^1}{\partial \theta} & \frac{\partial F_M^1}{\partial c_1} & \frac{\partial F_M^1}{\partial p} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial c_1} & \frac{\partial F^2}{\partial p} \end{bmatrix} \begin{bmatrix} d\theta \\ dc_1 \\ dp \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{16}$$

The Jacobian matrix for this system is

$$\begin{bmatrix} \frac{\partial F_M^1}{\partial \theta} & \frac{\partial F_M^1}{\partial c_1} & \frac{\partial F_M^1}{\partial p} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial c_1} & \frac{\partial F^2}{\partial p} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \bar{\gamma} \\ E[\Phi] & S & E[\gamma^i \Phi] \end{bmatrix}, \tag{17}$$

where $\Phi \equiv r(U_2 - a_1(U_{12} - R^i U_{22}))$, and now with $R^i \equiv 1 + r - r(p\gamma^i + \theta)$.

In order to use the implicit function theorem we have to check that the Jacobian determinant of the endogenous variables is non-zero. Or

$$|J| = \begin{vmatrix} \frac{\partial F_M^1}{\partial \theta} & \frac{\partial F_M^1}{\partial c_1} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial c_1} \end{vmatrix} = S \neq 0 \ , \tag{18}$$

which is satisfied if we have a well defined optimization, because then $S < 0$, since this is the second order condition for a maximum.

Proceed by dividing both equations in the differentiated system by ∂p to obtain the system of equations[5]

$$\begin{bmatrix} \frac{\partial F_M^1}{\partial \theta} & \frac{\partial F_M^1}{\partial c_1} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial c_1} \end{bmatrix} \begin{bmatrix} \frac{\partial \theta}{\partial p} \\ \frac{\partial c_1}{\partial p} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F_M^1}{\partial p} \\ -\frac{\partial F^2}{\partial p} \end{bmatrix}. \tag{19}$$

[5] The interpretation of the derivatives will be as partials rather than totals, since the system contains more exogenous variables than p, thus the "∂ " rather than "d" in the remaining analysis.

From this we can solve for the derivatives of interest by using Cramer's rule. First we have the derivative of the additive tax parameter with respect to the multiplicative tax parameter

$$\frac{\partial \theta}{\partial p}=\frac{\begin{vmatrix} -\dfrac{\partial F_M^1}{\partial p} & \dfrac{\partial F_M^1}{\partial c_1} \\ -\dfrac{\partial F^2}{\partial p} & \dfrac{\partial F^2}{\partial c_1} \end{vmatrix}}{|J|}=-\bar{\gamma}\ . \tag{20}$$

Finally, we have the partial derivative between first period consumption and the multiplicative tax parameter

$$\frac{\partial c_1}{\partial p}=\frac{\begin{vmatrix} \dfrac{\partial F_M^1}{\partial \theta} & -\dfrac{\partial F_M^1}{\partial p} \\ \dfrac{\partial F^2}{\partial \theta} & -\dfrac{\partial F^2}{\partial p} \end{vmatrix}}{|J|}=-\frac{1}{S}E\left[\Phi\left(\gamma^i-\bar{\gamma}\right)\right]. \tag{21}$$

5.2.2 THE BUDGET PRESERVING SPREAD CASE

Totally differentiate the system in (10) to get

$$\begin{bmatrix} \dfrac{\partial F_B^1}{\partial \theta} & \dfrac{\partial F_B^1}{\partial c_1} & \dfrac{\partial F_B^1}{\partial p} \\ \dfrac{\partial F^2}{\partial \theta} & \dfrac{\partial F^2}{\partial c_1} & \dfrac{\partial F^2}{\partial p} \end{bmatrix}\begin{bmatrix} d\theta \\ dc_1 \\ dp \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}\ . \tag{22}$$

The Jacobian matrix for this system is

$$\begin{bmatrix} \dfrac{\partial F_B^1}{\partial \theta} & \dfrac{\partial F_B^1}{\partial c_1} & \dfrac{\partial F_B^1}{\partial p} \\ \dfrac{\partial F^2}{\partial \theta} & \dfrac{\partial F^2}{\partial c_1} & \dfrac{\partial F^2}{\partial p} \end{bmatrix}=\begin{bmatrix} a_1 & -(p\bar{\gamma}+\theta) & \bar{\gamma}a_1 \\ E[\Phi] & S & E\left[\gamma^i\Phi\right] \end{bmatrix}, \tag{23}$$

where $\Phi \equiv r\left(U_2-a_1\left(U_{12}-R^iU_{22}\right)\right)$.

In order to use the implicit function theorem we have to check that the Jacobian determinant of the endogenous variables is non-zero. Or

$$|J| = \begin{vmatrix} \frac{\partial F_B^1}{\partial \theta} & \frac{\partial F_B^1}{\partial c_1} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial c_1} \end{vmatrix} \neq 0 . \tag{24}$$

We have that

$$|J| = \begin{vmatrix} a_1 & -\left(p\bar{\gamma} + \theta\right) \\ E[\Phi] & S \end{vmatrix} , \tag{25}$$

which yields the condition

$$|J| = a_1 S + \left(p\bar{\gamma} + \theta\right) E[\Phi] \neq 0 . \tag{26}$$

Proceed by dividing both equations in the differentiated system by ∂p to obtain the system of equations

$$\begin{bmatrix} \frac{\partial F_B^1}{\partial \theta} & \frac{\partial F_B^1}{\partial c_1} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial c_1} \end{bmatrix} \begin{bmatrix} \frac{\partial \theta}{\partial p} \\ \frac{\partial c_1}{\partial p} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F_B^1}{\partial p} \\ -\frac{\partial F^2}{\partial p} \end{bmatrix} . \tag{27}$$

From this we can again solve for the derivatives of interest by using Cramer's rule. First we have the effect on the additive tax parameter

$$\frac{\partial \theta}{\partial p} = \frac{\begin{vmatrix} -\frac{\partial F_B^1}{\partial p} & \frac{\partial F_B^1}{\partial c_1} \\ -\frac{\partial F^2}{\partial p} & \frac{\partial F^2}{\partial c_1} \end{vmatrix}}{|J|} . \tag{28}$$

We get

$$\frac{\partial \theta}{\partial p} = -\bar{\gamma} \frac{a_1 S + pE[\gamma^i \Phi]}{|J|} - \theta \frac{E[\gamma^i \Phi]}{|J|} . \tag{29}$$

Secondly, we have the effect on first period consumption

$$\frac{\partial c_1}{\partial p} = \frac{\begin{vmatrix} \frac{\partial F_B^1}{\partial \theta} & -\frac{\partial F_B^1}{\partial p} \\ \frac{\partial F^2}{\partial \theta} & -\frac{\partial F^2}{\partial p} \end{vmatrix}}{|J|} . \tag{30}$$

This can be written as

$$\frac{\partial c_1}{\partial p} = \frac{\bar{\gamma} a_1 E[\Phi] - a_1 E[\gamma^i \Phi]}{|J|} = -\frac{a_1}{|J|} E[\Phi(\gamma^i - \bar{\gamma})] . \tag{31}$$

5.3 CONDITION FOR $|J| > 0$ WITH TIME-SEPARABLE ISO-ELASTIC UTILITY

We want to investigate if the Jacobian determinant could be larger than zero, or

$$|J| = a_1 E\left[U_{11} - 2R^i U_{12} + R^{i^2} U_{22}\right] + (p\bar{\gamma} + \theta) r E\left[U_2 - a_1 \left(U_{12} - R^i U_{22}\right)\right] > 0. \tag{32}$$

To obtain a more transparent condition than we could with a general utility function, we will use a time separable iso-elastic utility function, with the relative risk aversion equal to α, or $U(c_1,c_2) = u(c_1) + \beta u(c_2)$, with $u(c) = c^{1-\alpha}/(1-\alpha)$. The condition then becomes

$$a_1 E\left[-\alpha c_1^{-\alpha-1} - R^{i^2}\alpha\beta c_2^{-\alpha-1}\right] + (p\bar{\gamma} + \theta) r\beta E\left[c_2^{-\alpha} - a_1 R^i \alpha c_2^{-\alpha-1}\right] > 0 \ . \tag{33}$$

Use $c_2 = R^i a_1$, the FOC: $c_1^{-\alpha} = \beta E\left[R^i c_2^{-\alpha}\right]$, and then define $\bar{c}_2 \equiv a_1 \bar{R}$, where $\bar{R} \equiv E\left[R^i\right]$ and $\bar{c}_g \equiv \bar{c}_2 / c_1$ to get

$$E\left[-\alpha\beta\bar{c}_g R^i c_2^{-\alpha} - \alpha\beta\bar{R}R^i c_2^{-\alpha}\right] + \bar{R}(p\bar{\gamma} + \theta) r E\left[\beta c_2^{-\alpha} - \alpha\beta c_2^{-\alpha}\right] > 0 \ , \tag{34}$$

or equivalently

$$-\alpha\beta\left(\bar{c}_g + \bar{R}\right) E\left[R^i c_2^{-\alpha}\right] + (1-\alpha)\beta\left(p\bar{\gamma} + \theta\right) r\bar{R} E\left[c_2^{-\alpha}\right] > 0 \ . \tag{35}$$

The first order condition tells us that $E\left[R^i c_2^{-\alpha}\right] > 0$, so for $\alpha > 1$, the condition can never be fulfilled, and thus we know that for a dominating income effect, the Jacobian determinant will be negative. If, however, $\alpha < 1$, the condition can be fulfilled. The first term is still negative in this case, but the second term will be positive. To get a general idea of the magnitude of the two terms, we can use the covariance between the net return on capital and the marginal utility in the second period. We know that the covariance is negative for a risk averse agent, since with a higher return, second period income and consumption is higher, and thus the marginal utility is lower. The covariance is equal to $Cov\left(R^i, c_2^{-\alpha}\right) = E\left[R^i c_2^{-\alpha}\right] - \bar{R}E\left[c_2^{-\alpha}\right] < 0$, and thus we know that $\bar{R}E\left[c_2^{-\alpha}\right] > E\left[R^i c_2^{-\alpha}\right]$. If we assume that these two factors are instead equal, we obtain a sufficient, but not necessary condition on α that is

$$\alpha < \frac{r(p\bar{\gamma} + \theta)}{\bar{c}_g + \bar{R} + r(p\bar{\gamma} + \theta)} < 1 \ . \tag{36}$$

The condition contains three variables that are functions of α, and we cannot derive a condition in terms of exogenous parameters with less than deriving an explicit solution to the agent's problem, which we cannot do in general. However, we know that all the included variables are positive, and thus that the expression is positive and less than one. We have therefore showed that there exists an α smaller than one and greater than zero that make the Jacobian determinant positive.

5.4 PROOF OF SIGN OF $E\left[\Phi\left(\bar{\gamma}-\gamma^{i}\right)\right]$ AND $E[\Phi]$

I will restrict the proof to the case of dominating income effect, but the analysis is completely analogous for the case of dominating substitution effect. In section 2.4 it is stated that for the case of dominating income effect, $E\left[\Phi\left(\bar{\gamma}-\gamma^{i}\right)\right]>0$ while $E[\Phi]<0$. Sandmo [1970] presents a general proof of the first inequality for a general utility function, but here we also show the connection between the two. In this proof, we will restrict the analysis to a time separable iso-elastic utility function to display how the coefficient of relative risk aversion enters the analysis. Start with the second inequality

$$E[\Phi] \equiv rE\left[U_2 - a_1\left(U_{12} - R^i U_{22}\right)\right] < 0 \ . \tag{37}$$

Use a time separable iso-elastic utility function and $c_2 = a_1 R^i$ to get

$$\beta r E\left[(1-\alpha)c_2^{-\alpha}\right] < 0 \ , \tag{38}$$

which will be satisfied if $\alpha > 1$. We will now show that this is consistent with the first inequality being positive, or

$$E\left[\Phi\left(\bar{\gamma}-\gamma^{i}\right)\right] \equiv rE\left[\left(U_2 - a_1\left(U_{12} - R^i U_{22}\right)\right)\left(\bar{\gamma}-\gamma^{i}\right)\right] > 0 \ . \tag{39}$$

With the same utility function we have

$$\beta r E\left[(1-\alpha)c_2^{-\alpha}\left(\bar{\gamma}-\gamma^{i}\right)\right] , \tag{40}$$

which requires some additional analysis before we know the sign. For $\alpha > 1$ we have that

$$(1-\alpha)c_2^{-\alpha} > \left\{(1-\alpha)c_2^{-\alpha}\right\}_{\bar{\gamma}} \quad \text{if } \bar{\gamma} > \gamma^i \ . \tag{41}$$

Further $(\bar{\gamma} - \gamma^i) > 0$ if $\bar{\gamma} > \gamma^i$, so multiplying both sides of the equation will not change the inequality, and thus

$$(1-\alpha)c_2^{-\alpha}(\bar{\gamma}-\gamma^i) > \left\{(1-\alpha)c_2^{-\alpha}\right\}_{\bar{\gamma}}(\bar{\gamma}-\gamma^i) \quad \text{if } \bar{\gamma} > \gamma^i \ . \tag{42}$$

Taking expectations on both sides and noting that the factor in curly braces on the left hand side is non-stochastic we have

$$E\left[(1-\alpha)c_2^{-\alpha}(\bar{\gamma}-\gamma^i)\right] > \left\{(1-\alpha)c_2^{-\alpha}\right\}_{\bar{\gamma}} E\left[\bar{\gamma}-\gamma^i\right] = 0 \quad \text{if } \bar{\gamma} > \gamma^i \ . \tag{43}$$

To show that the left hand side is positive, it is enough to show that the right hand side is positive (or non-negative). If we write out the second factor we get $\bar{\gamma} - E\left[\gamma^i\right] = 0$. We have thus shown that the right hand side is zero, implying that the left hand side is positive, which is what we wanted to prove.

The proof was constructed for $\bar{\gamma} > \gamma^i$, but will hold also for a realized tax rate that is greater than the expected, since then the inequality in (41) is reversed, but will be reversed back after multiplying with $(\bar{\gamma} - \gamma^i) < 0$, so the inequality in (42) will still be valid.

5.5 Condition for $\partial\theta/\partial p > 0$

Start by writing the derivative as

$$\frac{\partial\theta}{\partial p} = -\frac{a_1 S\bar{\gamma} + (p\bar{\gamma} + \theta)E\left[\gamma^i\Phi\right]}{a_1 S + (p\bar{\gamma} + \theta)E[\Phi]} \ , \tag{44}$$

which will always be negative if the numerator and denominator have the same sign. This will for example be the case when the income effect dominates, since then $E[\Phi] < 0$, which implies that both the numerator and denominator is negative. However, when the substitution effect dominates, we have a possibility that the derivative is positive, if the tax base has eroded enough. This is the case when the numerator is positive, while the numerator is still negative. To make the numerator positive, the second term has to dominate the first, which is always negative. At the same time, the second term in the denominator is not allowed to dominate the first. Furthermore, we know that when the denominator, the Jacobian determinant, is positive, we have the case where the substitution effect is so strong that it makes first period consumption decrease with increased risk. In that case, we know that the derivative between the two tax parameters has to be negative, since the tax base has then become larger. In other words, to obtain a positive derivative between the two tax parameters, the substitution effect has to be strong, but it cannot be too strong.

5.6 DETERMINISTIC TAXES

We begin our analysis by investigating the effect that a purely deterministic tax rate has on consumption, without involving the government's budget constraint. This is basically the same as conducting a comparative statics analysis of the interest rate's effect on consumption paths. As a starting point for the main analysis, it will be useful to know what the effect on first period consumption is from an increase in a *known* tax rate. The experiment will be conducted as simply as possible, by only performing a comparative statics exercise on the individual's problem. Let the individual solve the problem

$$\begin{aligned} &\max_{c_1,c_2} U(c_1,c_2) \\ &\text{s.t.} \quad c_1 = y_1 - \tau_1 - a_1 \\ &\qquad\quad c_2 = y_2 + (1+r-\theta r)(y_1 - c_1 - \tau_1)\ , \end{aligned} \tag{45}$$

where θ is the deterministic tax rate on capital income. The first and second order conditions are then

FOC: $$F \equiv U_1 - (1+r-\theta r)U_2 = 0 \tag{46}$$

SOC: $$S \equiv U_{11} - 2(1+r-\theta r)U_{12} + (1+r-\theta r)^2 U_{22} < 0\ . \tag{47}$$

Define $R \equiv 1 + r - \theta r$, and differentiate the FOC with respect to the deterministic tax parameter to obtain

$$\frac{\partial F}{\partial \theta} = U_{11}\frac{\partial c_1}{\partial \theta} - U_{12}\left(R\frac{\partial c_1}{\partial \theta} + ra_1\right) + rU_2 - RU_{21}\frac{\partial c_1}{\partial \theta} + RU_{22}\left(R\frac{\partial c_1}{\partial \theta} ra_1\right) = 0 \ . \qquad (48)$$

Rewrite to get the partial derivative of interest

$$\frac{\partial c_1}{\partial \theta} = -\frac{r}{S}\left[\underbrace{U_2}_{+} \underbrace{-a_1(U_{12} - RU_{22})}_{-}\right] \ . \qquad (49)$$

If the first term within brackets dominates the second, the sign of the derivative is positive, while if the second term dominates, the expression is negative.

5.7 SANDMO'S GENERAL CAPITAL INCOME RISK

This section presents the analysis in Sandmo [1970] with the general approach and notation used in this paper. The two equations now represent a mean preserving spread in capital income and the first order condition of the households respectively.

Mean preserving spread

The idea is that the expected value of the return on capital should be kept constant (at κ), so that

$$F^1 \equiv E[px^i + \theta] = p\bar{x} + \theta - \kappa = 0 \ , \qquad (50)$$

where $\bar{x} \equiv E[x^i]$.

Households

The households are expected utility maximizers and solve

$$\begin{aligned} &\max_{c_1, c_2} \ E[U(c_1, c_2)] = \int_X U(c_1, c_2) f(x^i) dx^i \\ &\text{s.t.} \quad c_1 = y_1 - a_1 \\ &\qquad\quad c_2 = y_2 + (1 + (px^i + \theta))a_1 \ , \end{aligned} \qquad (51)$$

which gives rise to the first and second order conditions

FOC: $$F^2 \equiv E[U_1 - R^i U_2] = 0 \qquad (52)$$

SOC: $$S \equiv E\left[U_{11} - 2R^i U_{12} + R^{i2} U_{22}\right] < 0 \,, \tag{53}$$

where $R^i \equiv 1 + \left(px^i + \theta\right)$.

The first order condition is the second equation in our system, so now we have the system of equations

$$\begin{cases} F^1 = 0 \\ F^2 = 0 \,. \end{cases} \tag{54}$$

If we totally differentiate the system, the Jacobian matrix is

$$\begin{bmatrix} \dfrac{\partial F^1}{\partial \theta} & \dfrac{\partial F^1}{\partial c_1} & \dfrac{\partial F^1}{\partial p} \\ \dfrac{\partial F^2}{\partial \theta} & \dfrac{\partial F^2}{\partial c_1} & \dfrac{\partial F^2}{\partial p} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \bar{x} \\ E[\Lambda] & S & E\left[x^i \Lambda\right] \end{bmatrix}, \tag{55}$$

where $\Lambda \equiv -\left[U_2 - a_1\left(U_{12} - R^i U_{22}\right)\right]$.

In order to use the implicit function theorem we have to check that the Jacobian determinant of the endogenous variables is non-zero. Or

$$|J| = \begin{vmatrix} \dfrac{\partial F^1}{\partial \theta} & \dfrac{\partial F^1}{\partial c_1} \\ \dfrac{\partial F^2}{\partial \theta} & \dfrac{\partial F^2}{\partial c_1} \end{vmatrix} = S \neq 0 \,, \tag{56}$$

which is satisfied if we have a well defined optimization, since then $S < 0$.

Proceed by dividing both equations by ∂p to obtain the system of equations.

$$\begin{bmatrix} \dfrac{\partial F^1}{\partial \theta} & \dfrac{\partial F^1}{\partial c_1} \\ \dfrac{\partial F^2}{\partial \theta} & \dfrac{\partial F^2}{\partial c_1} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \theta}{\partial p} \\ \dfrac{\partial c_1}{\partial p} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial F^1}{\partial p} \\ -\dfrac{\partial F^2}{\partial p} \end{bmatrix}. \tag{57}$$

From this we can solve for the derivatives of interest by using Cramer's rule. We start by investigating how the additive return parameter, θ, has to change when p changes, in order to satisfy the mean preserving spread condition.

$$\frac{\partial \theta}{\partial p} = \frac{\begin{vmatrix} -\frac{\partial F^1}{\partial p} & \frac{\partial F^1}{\partial c_1} \\ -\frac{\partial F^2}{\partial p} & \frac{\partial F^2}{\partial c_1} \end{vmatrix}}{|J|} = -\bar{x} \ . \tag{58}$$

This condition could of course equally well be obtained by total differentiation of the first equation, which is the way it is done in Sandmo. Our unnecessarily tedious approach is used to conform to the steps needed in the following sections.

The next step is to investigate how consumption changes in response to a mean preserving spread. We have that

$$\frac{\partial c_1}{\partial p} = \frac{\begin{vmatrix} \frac{\partial F^1}{\partial \theta} & -\frac{\partial F^1}{\partial p} \\ \frac{\partial F^2}{\partial \theta} & -\frac{\partial F^2}{\partial p} \end{vmatrix}}{|J|} \ , \tag{59}$$

or,

$$\frac{\partial c_1}{\partial p} = -\frac{1}{S} E\left[\Lambda\left(x^i - \bar{x}\right)\right] = \underbrace{\frac{1}{S} E\left[U_2\left(x^i - \bar{x}\right)\right]}_{+} \underbrace{- \frac{1}{S} a_1 E\left[\left(U_{12} - R^i U_{22}\right)\left(x^i - \bar{x}\right)\right]}_{-} \ . \tag{60}$$

The first term, the substitution effect, can be shown to be negative if risk aversion is assumed. The second term, the income effect, will be negative if decreasing temporal risk aversion is assumed. Combining these assumptions renders the total effect an ambiguous sign. Proofs of these statements could be found in Sandmo's [1970] original paper.

Essay IV

Budget Deficits, Tax Risk and Consumption*

Abstract

This paper analyzes the effects of budget deficits on consumption when individual taxes are stochastic. It is shown that the co-movements between budget deficits and private consumption will depend on how risk averse individuals are. In the case of lump-sum taxes, it is sufficient to assume that individuals have a precautionary savings motive to obtain the result that consumption today will decrease with increased disposable income today. Furthermore, if we use a time separable iso-elastic utility function, the standard analysis of capital income risk predicts (precautionary) savings to increase with increased risk if the coefficient of relative risk aversion is greater than one. This is no longer sufficient when the risk is due to uncertain capital income taxes. In general, the coefficient must be greater than one to obtain precautionary savings in response to the greater risk implied by a budget deficit. The results in the paper are consistent with Ricardian equivalence only for some specific utility function, but not in general. However, in the same way, the results are consistent with standard "Keynesian" models that display a positive relation between debt and private consumption only for certain utility functions, and could equally well generate the opposite result for individuals that are enough risk averse or prudent, without changing the expected value of government consumption. In other words, if future taxes are uncertain, increased disposable income in the present period will decrease present consumption, if households are prudent enough.

1. INTRODUCTION

The effects of government budget deficits on private consumption is a central question for both macro economists and policy makers, the reason for this being that in recessions, stabilization policies have often aimed at increasing aggregate demand by increasing the households' current disposable income, without changing public spending, i.e. by creating a budget deficit. To obtain such a positive demand response to budget deficits, the public has, in general, to regard government bonds as net wealth. With Barro's famous [1974] paper, the debate on this issue (re)started, with the validity of the Ricardian equivalence proposition as the central theme. In

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short, the proposition states that private consumption will not change in response to changes in the timing of taxes, as long as government consumption remains unchanged.

There are of course theoretical models that support the equivalence proposition as well as models that create deviations from Ricardian equivalence. The focus in most models that create deviations from Ricardian equivalence has been on the potential wealth effect from postponing taxes. This is due to individuals regarding their holdings of government bonds as net wealth, i.e. they do not discount future tax payments one hundred per cent. However, the type of model, Ricardian or non-Ricardian, that best describes the real world can only be determined by empirical investigations.

There have been a number of papers trying to test the Ricardian proposition empirically, with mixed conclusions, see for example the studies in Yawitz and Meyer [1976], Kormendi [1983], Feldstein [1982] and Bernheim [1987], or for an overview Seater [1993]. When we use a stochastic model of Ricardian equivalence (which ought to be a natural starting point for an econometric study), one central issue is to sort out the effect current tax changes has on the expectations about future government consumption. This is vital, since the Ricardian prediction for postponing taxes is valid only for a given level of expected government consumption, so if a change in the timing of taxes leads to a revision of the expected value of government consumption, this will affect private consumption not because government bonds are perceived as net wealth but because the expected present value of government consumption has changed. An empirical study that explicitly handles this is Aschauer [1985], while many of the earlier empirical studies neglect this signalling role that debt can play. Other papers that discuss the importance of future government policies are Drazen and Helpman [1990], Bertola and Drazen [1993] and Giavazzi and Pagano [1990].

Giavazzi and Pagano illustrate the important link between fiscal contractions and expectations about future levels of government consumption with an example where increased taxes (and thus reduced disposable income) lead to increased private consumption. They explain this phenomenon by stating that when the level of future government consumption is expected to fall in periods of fiscal contractions, households revise their expectations about future net income in a positive direction. This change in expected future net income will then increase present consumption, although present disposable income is reduced. This argument is of course fully consistent with standard Ricardian models.

However, Becker [1995a] finds that even when the expected level of future government consumption is unchanged, present consumption rises with increased taxes. This is not a pattern that can be explained in standard models that deal with the question of Ricardian equivalence. In this paper it is argued that we can explain this observation with households that care about risk, in combination with uncertainty about future tax payments and thus uncertain disposable income. In other words, households do not only consider changes in expected disposable income, as stressed in most of this literature, but they are also concerned about the spread of their disposable income.

The idea that households care about both the spread and the expected values is familiar from the literature that analyzes expected utility maximization, starting with von Neumann and Morgenstern [1944]. The more formalized analysis of risk aversion originates from Arrow [1963], Pratt [1964] and Arrow [1971], where the widely used measure of absolute risk aversion is defined. This measure describes how much an agent dislikes risk, but it does not tell us how agents change their consumption over time to hedge themselves against risk.

With Leland [1968] and Sandmo [1970], a formal treatment of the intertemporal consumption choice in connection with uncertainty is developed, and the concept of precautionary savings is introduced. In brief, the conclusion is that with increased uncertainty about future income, agents will save more if certain conditions of the utility function are fulfilled, and this type of saving in response to risk is labelled precautionary savings. Individuals that display a precautionary savings motive are said to be prudent, and in the same way that risk averse individuals' preferences have a positive measure of absolute risk aversion, prudent individuals have a positive measure of absolute prudence, see Kimball [1990].

To formalize the above discussion, in an atemporal model, individuals that have preferences that satisfy $-U''/U'>0$ (i.e. the measure of absolute risk aversion is positive), where U' is the first derivative of the utility function, are said to be *risk averse*. In a two period model, individuals that have time separable utility and preferences over second period consumption that satisfy $-U'''/U''>0$ (i.e. the absolute measure of prudence is positive) are said to be *prudent*.

In the recent macro literature, the concept of precautionary savings has been used to explain observed deviations from Hall's [1978] formulation of the permanent income hypothesis. Hall's original paper implies that consumption can be described as a random walk, and that unexpected changes in income that are transitory should affect consumption relatively little, while changes in income that are permanent should give rise to larger changes in consumption. However, these predictions are based on a quadratic utility function, that displays the well known certainty equivalence outcome, and thus the precautionary savings motive is not present. If we instead allow preferences to display the precautionary savings motive, we are able to explain observed consumption patterns better than we can do with Hall's original formulation, see for example Zeldes [1989], Caballero [1990] and Weil [1993].

Most of the studies that analyze precautionary savings are concerned with risk in gross income or with general income risk, and do not incorporate uncertainty created by the government. However, the public sector determines a large part of households' disposable income, and the rules that determine taxes as well as transfers are subject to frequent revisions (or reforms to use the politically correct label). Thus it seems natural to investigate how risk created in connection with the tax system affects households, as a complement to the standard analysis of market risk.

One study that incorporates uncertainty about future taxes is Chan [1983]. He concludes that for some utility functions, we will get the result that increased taxes today can increase present consumption, without having changed the expected value of disposable income. Chan uses lump-sum taxes in his model, which we will also use in the first experiment below. The advantage of the present study is that we use a model similar to Sandmo [1970], which lets us explicitly formulate the conditions needed on the utility function to obtain this result. Not surprisingly, we need to assume that households' preferences are such that they display a precautionary savings motive to obtain the result that increased taxes today will increase present consumption.

The above result was achieved with lump-sum taxes, and thus the consumers cannot affect the tax base. In this paper, we also extend the analysis to the case of distortionary taxes. With distortionary taxes, the consumption choice affects the tax base, and thus we will need an endogenous response in tax rates to maintain the government's intertemporal budget balance. This formulation suggests from the start that we will get deviations from Ricardian equivalence, since we now use both distortionary and uncertain taxes. What the deviation will be is however an open question. In the paper it is shown that with strong enough risk aversion (or rather the implied precautionary savings motive), we will still get the result that increased taxes today will increase present consumption, without having changed the expected value of disposable income. We have thus presented a theoretical explanation of the empirical puzzle in Becker [1995a] for both lump-sum taxes and for a more realistic tax system with distortionary taxes.

2. A MODEL OF STOCHASTIC TAXES

Papers that describe how the consumption savings choice is affected by changes in general income and capital risk in two period models are, for example, Leland [1968] and Sandmo [1970], and Ormiston and Schlee [1992]. The two latter papers, among others, uses the concept of mean preserving spreads to define the changes in risk. The latter paper also provides conditions of the utility function for alternative distributional changes. In this specific model, we study an economy that exists for two periods, with one government sector, a large number of households, and the interest rate given by a world market. The government obeys an intertemporal budget constraint, and the households are expected utility maximizers. The methodology is to solve a two equation system for two unknowns, where the first equation is the government's budget constraint and the second equation is the (representative) household's first order condition.

The uncertainty we introduce here is, contrary to the optimal taxation literature, based on the assumption that the government could tax people differently although they have identical endowments and utility functions. This assumption requires some motivation, beside the purely technical aspect as a way to introduce uncertainty in the model. The most basic motivation is by simply observing past government actions and investigate if tax burdens carried by different individuals could be derived from their endowments and consumption patterns. In many cases this would probably require some imagination. Further, in the model used, we have aggregated both consumption and savings, but in reality people consume bundles of goods and have different assets in their portfolios, and, when making the consumption and investment decisions, the tax rules that will be used in the future are uncertain and potentially different for different assets and consumption goods. This would introduce uncertain net interest incomes.[1]

2.1 CHANGING THE TIMING OF LUMP-SUM TAXES

In this experiment, we assume that the government uses lump-sum taxes to finance its known consumption in both periods, that households have access to a capital market with a given interest rate, and that the planning horizon for the government and the households is the same. In a non-stochastic model, this would usually generate the Ricardian equivalence result, namely that changing the timing of taxes will not affect private consumption. In the present model, however, we introduce uncertainty about future taxes, but we keep the level of government consumption fixed. We will therefore not have deviations from Ricardian equivalence due to changed expectations about government consumption, which implies that the expected tax burden will be kept constant. This experiment is closely related to Sandmo's [1970] analysis of income risk, and to Chan's [1983] analysis of tax risk.

Government

The government obeys an intertemporal budget constraint according to

$$F^1 \equiv T_1 + \frac{T_2}{1+r} - G_1 - \frac{G_2}{1+r} = 0 , \qquad (1)$$

where r is the interest rate, and G_1 and G_2 are government consumption in the first and second period, that are assumed to be deterministic, and thus known by households in the first period. First and second period aggregate lump-sum taxes, T_1 and T_2, are also certain, although second period individual taxes are uncertain in a way described below.

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[1] One recent Swedish experience displaying the uncertain nature of the tax system is the "one-time" tax on private pension funds, not to mention all other constant revisions (or tax reforms) of the tax system. Furthermore, transition rules between different tax systems in Sweden have, for example, had the effect that some tax deductions have been worth between 20 and 250 per cent of the realized loss, depending on the composition of the remainder of the individual's capital income, as well as the timing of different portfolio decisions.

Households

We assume that we have n identical households that maximize expected utility according to

$$\max_{c_1, c_2} E[U(c_1,c_2)] = \int_{\Pi} U(c_1,c_2) f(\pi) d\pi$$
$$\text{s.t.} \quad c_1 = y_1 - \tau_1 - a_1 \qquad (2)$$
$$c_2 = y_2 - \tau_2 + (1+r)(y_1 - c_1 - \tau_1) \ ,$$

where $U(.)$ is a von Neumann-Morgenstern utility function, c_1 and c_2 are consumption in the first and second period, y_1 and y_2 are exogenous gross income in the first period and second period, τ_1 and τ_2 are lump-sum taxes in the first and second period, $a_1 \equiv y_1 - c_1 - \tau_1$ is savings and r is the given interest rate. The feature of the model that makes it stochastic is the specification of the tax system. Taxes in both periods are lump-sum, but first period taxes are certain and equal to $\tau_1 = T_1/n$, where T_1 is aggregate taxes, i.e. first period taxes are defined as a *given* proportion of total taxes determined by the known number of households. Second period taxes are instead assumed to be stochastic and equal to $\tau_2 = \pi T_2$, where π is a stochastic variable defined over the non-negative distribution Π with $E[\pi] = 1/n$. We then let each household make a draw of its share of taxes, π, from the probability distribution in the second period to get its second period tax payment. The household's expected share of total taxes over both periods is then simply the aggregate taxes divided by the number of households, while the realized tax can be smaller or greater than this. All the uncertainty in the model is due to the uncertain future taxes.

We also have to make sure that the realized taxes in the second period are such that the government's budget constraint is fulfilled. There are two alternative ways of handling this. First we can assume that all individuals except one make a draw from the distribution of taxes, and then the last individual pays the remaining tax necessary for the government to fulfill the budget constraint. An alternative interpretation is that there are sufficiently many households to make the law of large number apply, i.e. the realized mean of tax payments is arbitrarily close to the expected value of tax payments necessary to fulfill the budget constraint.

The first and second order conditions (FOC and SOC) for the maximization problem are

FOC: $$F^2 \equiv E\left[U_1 - (1+r)U_2\right] = 0 \tag{3}$$

SOC: $$S \equiv E\left[U_{11} - 2(1+r)U_{12} + (1+r)^2 U_{22}\right] < 0 \;, \tag{4}$$

where subscripts on the utility function denote partial derivatives with respect to consumption in period one and two. The first order condition states that households equalize the expected marginal utility of consumption in the two periods, while the second order condition ensures that we have a maximum.

By using the household's FOC and the government's intertemporal budget constraint we get the following system of equations

$$\begin{cases} F^1 = 0 \\ F^2 = 0 \,. \end{cases} \tag{5}$$

If we differentiate the system with respect to first and second period taxes and first period private consumption and assume that second period taxes are exogenous and the other two variables are endogenous, we can solve for the partial derivatives describing how first period consumption and taxes will change in response to a change in second period taxes. (The derivation of the partial derivatives can be found in the appendix.) We first have the derivative describing how first period taxes have to change in order to fulfill the government's intertemporal budget constraint when second period taxes are changed. It is simply

$$\frac{\partial T_1}{\partial T_2} = -\frac{1}{1+r} \;, \tag{6}$$

which is of course an expression of the fact that second period taxes are discounted by the interest rate, so by increasing future taxes by one dollar, we can lower present taxes by $1/(1+r)$.

To describe how first period consumption will change in response to a change in second period taxes to fulfill the first order condition, we have the somewhat more complicated expression

$$\frac{\partial c_1}{\partial T_2} = \frac{1}{S} E\left[\left(U_{12} - (1+r)U_{22}\right)\left(\pi - \frac{1}{n}\right)\right]. \tag{7}$$

This effect on first period consumption in response to a change in second period taxes is analogous to the effect obtained by Sandmo [1970] in the case of uncertain future income. From his analysis, we know that first period consumption will decrease with increased uncertainty about future income if households' preferences display decreasing temporal risk aversion. An equivalent assumption is that the household's utility function is such that the household has a precautionary savings motive, which is a widely used assumption that will be used also here.[2] The reason for the first period consumption to decrease with increased future taxes (and thus increased present disposable income) is here due to the fact that second period taxes are stochastic, while first period taxes are certain. Thus, by substituting taxes today for taxes in the future, individuals are exposed to a greater uncertainty and will engage in precautionary savings.

For a proof of the sign of the derivative between first period consumption and future taxes, see the appendix. Furthermore, we can note that it is not sufficient to assume that individuals are risk averse to determine how first period consumption changes in response to a changed timing of taxes, because risk aversion only determines how their utility will be affected and not how they will change their first period consumption when they are exposed to risk.

We have now derived a deviation from Ricardian equivalence in a model where the households and the government have the same planning horizon, they have access to a perfect capital market and the government uses lump-sum taxes to finance a given amount of government

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[2] Decreasing temporal risk aversion implies that $U_{112} - U_{222} < 0$, which with time separable utility says that the third derivative with respect to second period consumption is positive. Furthermore, the coefficient of absolute prudence is defined as $\eta = -U_{222}/U_{22}$, and when this coefficient is positive, individuals display a precautionary savings motive. Since the coefficient of absolute prudence will be positive for a risk averse individual with a positive third derivative of the utility function, the assumption of decreasing temporal risk aversion is equivalent to the assumption that individuals have a precautionary savings motive if we assume a time separable utility function. We can note that in the case of time separable iso-elastic utility, assuming that individuals are risk averse also implies that the third derivative of the utility function will be positive.

consumption. The deviation from Ricardian equivalence is due to the stochastic nature of the tax system in combination with individuals that are not risk neutral. If we further use the assumption that households' preferences are such that they have a precautionary savings motive, we get the result that today's consumption will decrease with increased disposable income in the first period. This is contrary to most models that display deviations from Ricardian equivalence, in particular to a Keynesian model, where present consumption is always increasing with increasing present disposable income.

Also non-Ricardian models with an intertemporal utility maximizing framework usually get the Keynesian type of deviation from Ricardian equivalence, since most of them have assumptions that generate a positive wealth effect for households from postponing taxes, and neglect the stochastic elements involved when shifting taxes over time. One exception is the paper by Chan [1983], where a model similar to the one presented here is used to obtain the result that postponing taxes can lead to decreased consumption today. However, in his paper it is less straightforward to derive explicit conditions on the utility function needed to get this effect.

2.2 Changing the Timing of Capital Income Taxes

In this experiment, we use distortionary capital income taxes in the second period, rather than the lump-sum taxes used in the previous analysis. In the first period, we still use lump-sum taxes, but the interpretation of these taxes can be in terms of a tax on an exogenous income, where the income could be due to already installed capital. In other words, we can think of the tax system as a system taxing capital income in both periods, the difference being that in the first period the tax is non-distortionary and non-stochastic, while in the future it is both distortionary and stochastic. With distortionary taxes, we do not expect the Ricardian hypothesis to be true, even if we remove the stochastic features of the model. It is, however, still interesting to investigate what the deviations from the equivalence propositions are. In other words, the question is now how present consumption changes if we substitute certain, non-distortionary, taxes today for uncertain, distortionary, taxes tomorrow.

The tax system could be interpreted as if the government uses a standard range of tax rates on capital income, defined in such a way that for a balanced first period budget, the second period is also expected to balance. Further, if we assume that when the government runs a deficit it multiplies the tax schedule for the second period with a constant greater than one, and when it runs a surplus it uses a coefficient less than one. Thus, if the government runs a deficit, this has both the effect of raising the expected value of the second period interest income tax and increasing its spread, and hence we have introduced a very simple connection between high debt and increased risk.

Government

The government obeys the following intertemporal budget restriction.

$$\bar{g} = g_1 + \frac{g_2}{1+r} = \tau_1 + \frac{E(\tau_2)}{1+r} = \tau_1 + \frac{p\bar{\gamma}ra_1}{1+r} \quad , \tag{8}$$

where $\bar{g}$ is the net present value of the total financing requirement, due to the resource use, $g_{1,2}$ in periods one and two, r is the exogenously given interest rate, τ_1 is first period lump-sum taxes, a_1 is households' savings, and p is a policy parameter that multiplies the average tax rate on capital income, $\bar{\gamma}$. Rewrite the government's budget constraint and define

$$H^1 \equiv \tau_1 + p\bar{\gamma}r(y_1 - c_1 - \tau_1)/(1+r) - \bar{g} \ , \tag{9}$$

which will be the first equation in the system analyzed.

Households

Households are assumed to be rational economic units that all solve the following maximization problem

$$\begin{aligned} &\max_{c_1,c_2} \ E[U(c_1,c_2)] = \int_{\Gamma} U(c_1,c_2) f(\gamma^i) d\gamma^i \\ &\text{s.t.} \quad c_1 = y_1 - \tau_1 - a_1 \\ &\qquad\ \ c_2 = (1 + r - p\gamma^i r)(y_1 - c_1 - \tau_1) \quad , \end{aligned} \tag{10}$$

where U is a von Neumann-Morgenstern utility function, c_t is period t consumption, y_1 is exogenous income in the first period[3], $a_1 = y_1 - c_1 - \tau_1$ is savings between the first and second period, and γ^i is an individual tax rate on capital that is drawn in period two from the tax distribution Γ, with $E[\gamma^i] = \bar{\gamma}$. Since the stochastic tax rate is multiplied by p, we have that the realized tax rate is $p\gamma^i$, which is present in the household's budget constraint, while the expected tax rate is $p\bar{\gamma}$, which is present in the government's budget constraint. Note that with the present formulation of the tax system, increasing p is equivalent with increasing risk. The expectations operator is due to the uncertain capital income tax rate alone, since the interest rate is assumed to be known. Since we also assume that there is an infinite number of households, no individual household will affect the expected tax rate by its own consumption choice. Furthermore, the households are identical, so we analyze only a representative household, and all variables are in *per capita* terms. The first and second order conditions to this problem are

FOC: $$H^2 \equiv E\left[U_1 - R^i U_2\right] = 0 \tag{11}$$

SOC: $$S \equiv E\left[U_{11} - 2R^i U_{12} + R^{i^2} U_{22}\right] < 0\ , \tag{12}$$

where $R^i = \left(1 + r - p\gamma^i r\right)$.

The second equation we use in our two equation system is the FOC, which gives us the following system of equations

$$\begin{cases} H^1 = 0 \\ H^2 = 0\ . \end{cases} \tag{13}$$

If we differentiate the system with respect to the multiplicative tax parameter, p, first period taxes, τ_1, and first period private consumption, c_1, and assume that p is exogenous and the other two variables are endogenous, we can solve for the partial derivatives describing how

[3] Second period labor income is assumed to be zero, since otherwise it is not obvious that households will actually save, and then the use of a capital income tax is of little interest.

first period consumption and taxes will change in response to a change in p. (The derivation of the partial derivatives can be found in the appendix.) We first have the derivative describing how first period taxes have to change in order to fulfill the government's intertemporal budget constraint when p is changed

$$\frac{\partial \tau_1}{\partial p} = -\frac{\frac{\bar{\gamma} r a_1}{1+r} S + \frac{p\bar{\gamma} r}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right]}{|J|}, \tag{14}$$

where $\Psi \equiv R^i\left(U_{12} - R^i U_{22}\right)$, and $|J| = \frac{\bar{R}}{1+r} S - \frac{p\bar{\gamma} r}{1+r} E[\Psi] < 0$, with $\bar{R} \equiv 1 + r - p\bar{\gamma} r$. We can start by noting that the expression is much more complex than the corresponding expression derived when we shifted lump-sum taxes, and the reason for this is that we now have to take into account the changes in private consumption, since these affect the tax base. For the moment we will only note that if $E\left[\gamma^i r a_1 \Psi / R^i\right] \geq E\left[\gamma^i r U_2\right]$ the expression will be negative. This condition implies certain restrictions on the utility function, and thus for some utility functions the derivative can be positive.

If the expression is positive, this implies that an increased expected capital tax rate in combination with increased spread of individual tax rates, i.e. an increase in p, could have the effect of decreasing tax revenues in the second period, so that we would be to the right of the maximum on the Laffer curve. In the appendix it is shown that we will be to the right of the maximum on the Laffer curve if we use a time separable iso-elastic utility function with a sufficiently small coefficient of relative risk aversion (RRA). The intuition for this condition is best understood if we consider the limiting case where RRA approaches zero so that the individual is risk neutral. With a linear utility function and the net interest rate equal to the discount factor, it does not matter in which period to consume, and in the first period there is no tax to pay on consumption, while in the second period, the individual will have to pay tax on savings and thus on second period consumption. In that case, the capital income tax will be fully avoided by consuming all wealth in the first period, and there is then nothing left to tax in the second period.

Finally, the effect on first period private consumption is

$$\frac{\partial c_1}{\partial p} = -\frac{1}{|J|}\left(\frac{\bar{R}}{1+r} E\left[\gamma^{\prime} r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right] + \frac{\bar{\gamma} r a_1}{1+r} E[\Psi]\right). \tag{15}$$

To determine the sign of the derivative we start by rewriting the expression as

$$\frac{\partial c_1}{\partial p} = -\frac{r}{|J|(1+r)}\left(E\left[a_1 \Psi\left(\bar{\gamma} - \frac{\bar{R}}{R^i}\gamma^{\prime}\right)\right] + \bar{R}E\left[\gamma^{\prime} U_2\right]\right). \tag{16}$$

The next step is to use $\Psi \equiv R^i\left(U_{12} - R^i U_{22}\right)$, and to add and subtract the term $R^i \bar{\gamma} U_2$ in the parenthesis. We then get

$$\frac{\partial c_1}{\partial p} = \frac{r}{|J|(1+r)} E\left[U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^{\prime}\right) - a_1\left(U_{12} - R^i U_{22}\right)\left(R^i\bar{\gamma} - \bar{R}\gamma^{\prime}\right) - \bar{\gamma} R^i U_2\right], \tag{17}$$

which could be analyzed in a way similar to Sandmo [1970] or Becker [1995b]. In this case we have three effects to consider, two that are standard when analyzing consumption choice with uncertain capital income (which are represented by the two first terms) and one due to the distortionary tax on capital income (represented by the last term). The two standard effects are analogous to the effects that Sandmo [1970] labels as the substitution and income effects, respectively.

The substitution effect (the first term) says that with increased uncertainty about the return on capital, second period consumption becomes riskier, and a risk averse individual will then prefer to consume in the risk free first period. However, the income effect (second term) goes in the other direction, and the reason for this is that the marginal utility of consumption in the second period will be very high if the individual has a bad realization that gives a low second period income, since we assume that the utility function is concave, i.e. individuals are risk averse. The income effect will hence make agents save more, i.e. consume less in the first period, to safeguard themselves against bad outcomes when risk increases.

A risk averse individual will thus have these two uncertainty effects to trade off. With a time separable utility function with iso-elastic utility in each period, we know from Sandmo [1970] that the strength of the income and substitution effects will depend on the coefficient of relative risk aversion (RRA), and with RRA equal to one these effects will cancel each other out. With RRA smaller than one the substitution effect will dominate, and with RRA greater than one the income effect will dominate. Ormiston and Schlee [1992] state a somewhat more natural condition, namely that the coefficient of relative prudence has to be greater(less) than two for the income(substitution) effect to dominate, and also develop conditions for other distributional changes. In the case of an iso-elastic utility function, it is straightforward to verify that if the coefficient of relative risk aversion is one, the coefficient of relative prudence is two.

In addition to the two "standard" effects discussed, we here have an additional term. This is the last term, that describes how the average tax rate on capital makes the agent save less than would be the case if there were no tax on capital. With an increase in the riskiness, we also have the effect that the average tax rate on capital increases, which makes second period consumption more expensive and provides an incentive for consumption in the first period. This effect reinforces the standard substitution effect, and reduces the range where the income effect dominates.

In this case with a distortionary tax, the above condition on RRA will only determine the sign of the total expression in the case of a dominating substitution effect, since in that case the effect of the last term will reinforce the first effect. Since we know that the denominator is always negative, and that the numerator is positive in the case of a dominating substitution effect, we then have the result that the derivative is positive. In other words, with increased risk about the net return on capital, i.e. increased disposable income in the first period, households will consume more in the first period.

For the case of a "dominating" income effect in the standard case, we can still get the result that first period consumption increases with increased risk here, if the last term is sufficiently large. In general, we can say that the income effect has to be stronger in this case than in the case without taxes to generate a negative derivative between first period consumption and the multiplicative tax parameter. This is due to the fact that the tax distribution is endogenous, and has to respond to changes in consumption to maintain the government's budget balance. In other words, to make present consumption decrease with increased disposable income in the first period, it is no longer sufficient to assume that the coefficient of relative prudence is greater than two. In the following subsection, we will provide a sufficient condition on the coefficient of relative risk aversion to obtain the result that first period consumption decreases with a tax cut in the first period.

2.2.1 THE CASE OF TIME SEPARABLE ISO-ELASTIC UTILITY

In this section, a time separable iso-elastic utility function is used to derive a sufficient condition on the coefficient of relative risk aversion to obtain a negative derivative between first period consumption and the multiplicative tax parameter, i.e. that first period consumption will decrease in response to increased disposable income in the first period. Multiplying the first and second factor in (17) by -1 and cancel terms, we get the condition

$$E\left[a_1\left(U_{12}-R^iU_{22}\right)\left(R^i\bar{\gamma}-\bar{R}\gamma^i\right)+\bar{R}\gamma^iU_2\right]<0 \ , \tag{18}$$

to obtain the result that first period consumption will decrease in response to increased risk.

Using an iso-elastic utility function $U(c_1,c_2)=u(c_1)+\beta u(c_2)$ with $u(c)=c^{1-\alpha}/(1-\alpha)$, where α is the coefficient of relative risk aversion[4] and β is the time preference, the condition becomes

$$E\left[a_1R^i\alpha\beta c_2^{-\alpha-1}\left(R^i\bar{\gamma}-\bar{R}\gamma^i\right)+\bar{R}\gamma^i\beta c_2^{-\alpha}\right]<0 \ , \tag{19}$$

or, by using $c_2=a_1R^i$ and canceling the rate of time preference, equivalently

$$E\left[\alpha\bar{\gamma}R^ic_2^{-\alpha}+(1-\alpha)\bar{R}\gamma^ic_2^{-\alpha}\right]<0 \ . \tag{20}$$

[4] The coefficient of relative prudence for this utility function is $\alpha+1$.

We know that the first term will always be positive, since the agents are assumed to be risk averse, i.e. $\alpha > 0$, we analyze taxes and not transfers, and finally the FOC implies that $R^i c_2^{-\alpha} > 0$.

The second term will be positive if $\alpha < 1$, since $Cov(\gamma^i, c_2^{-\alpha}) = E[\gamma^i c_2^{-\alpha}] - \bar{\gamma} E[c_2^{-\alpha}] > 0$ implies that $E[\gamma^i c_2^{-\alpha}] > 0$, and $E[\gamma^i c_2^{-\alpha}] > \bar{\gamma} E[c_2^{-\alpha}]$, which will be used later. Thus the expression is positive for $\alpha < 1$. In other words, for the case of a dominating substitution effect, we have demonstrated that first period consumption increases with increased tax risk, and thus increased disposable income in the first period.

The next question is to investigate whether there exists a coefficient of relative risk aversion that is large enough for the expression to be negative, implying that first period consumption decreases with a first period tax cut. Use $R^i \equiv 1 + r - p\gamma^i r$, and $\bar{R} \equiv 1 + r - p\bar{\gamma} r$ to rewrite the condition as

$$\alpha(1+r)\bar{\gamma}E[c_2^{-\alpha}] + (\bar{R} - \alpha(1+r))E[\gamma^i c_2^{-\alpha}] < 0 , \tag{21}$$

where we note that the first term will always be positive, but the second term will be negative for large enough values on α. In general, it is hard to answer the question of how large α has to be to fulfill the condition, since we need to know the relation between $E[\gamma^i c_2^{-\alpha}]$ and $\bar{\gamma}E[c_2^{-\alpha}]$ to establish this. However, to make the condition a little more transparent in terms of α, start by writing the condition as

$$\alpha(1+r) + (\bar{R} - \alpha(1+r))\Omega < 0 , \tag{22}$$

where $\Omega \equiv E[\gamma^i c_2^{-\alpha}] / (\bar{\gamma}E[c_2^{-\alpha}]) > 1$. If we then solve for α, we get

$$\alpha > \frac{\bar{R}\Omega}{(1+r)(\Omega - 1)} . \tag{23}$$

This condition on α only provides a hint to the exact condition on α, since Ω is a function of α. It thus still remains to see if this condition can actually be fulfilled, and we will now use a numerical example to show that it can actually be fulfilled. Start by assuming that $\alpha = 2$, $r = 1$, $p = 1$, and that the tax distribution is two-valued, with $\gamma^i = 0$ or 1 with equal probabilities. We further assume that savings, a_1, which is an endogenous variable, is equal to one. This is really an implicit assumption about the first period income and the level of government consumption. However, to make this an explicit assumption about the net income level, we would have to derive explicit solutions to the problem, which is complicated, and not necessary for demonstrating that the condition can be fulfilled. With this parametrization, we get that $\Omega = 1.6$ and that the left hand side in (23) is equal to 1.2, and since the calculation was done for $\alpha = 2$, we see that the condition is fulfilled. In other words, we have constructed an example where first period consumption decreases when the risk and average tax are increased, i.e. first period taxes are reduced.

3. Discussion and Extensions

This paper uses a simple two period framework with a government sector and a large number of households. A two-equation system is formulated to analyze the effects of different levels of risk, which are positively related to the level of public debt. The first equation is the government's budget constraint and the second is the household's first order condition for optimal consumption. We then analyze the effects on private consumption from changing the timing of taxes. Two different taxes are analyzed, first lump-sum taxes in the Ricardian tradition, and then the more realistic case of capital income taxes that are distortionary.

In the case of lump-sum taxes it is sufficient to assume that households' preferences display a precautionary savings motive to get the result that increased disposable income in the first period due to shifting taxes to the second period implies that households consume less in the first period. This is due to the assumption that taxes today are certain, while taxes in the future are uncertain. This result is analogous to Sandmo's [1970] result on income risk, and his mean preserving spread analysis uses the same assumptions about the utility function as the present analysis of an intertemporal budget preserving spread to show that present consumption will be reduced if risk is increased, i.e. first period taxes are reduced in the present model.

In the case of distortionary capital income taxes, we can still obtain the result that present consumption decreases with increased current disposable income, i.e. with increased uncertainty about future taxes. However, individuals must be more prudent in this analysis of an intertemporal budget preserving spread to obtain this result than they have to be in Sandmo's [1970] analogous mean preserving spread analysis of capital income risk. The paper can thus explain the empirical puzzle found in Becker [1995a], both for the case of non-distortionary and the case of distortionary taxes, if we make the appropriate assumptions on the utility function.

The analysis has provided a mechanism that links the level of risk to the government's budget deficit, and the condition on the utility function needed for this mechanism to provide the result that private consumption today is *reduced* in response to current tax cuts. However, depending on how prudent households are, the case of precautionary savings in response to tax cuts as well as a "Keynesian" demand injection could be generated by the model, although the reason for the demand injection would be far from the standard Keynesian explanations. It is worth noting that households now care about both the expected value and the spread of their tax payments, and that we do not alter the present value of government consumption. So the goods news for the advocate of active stabilization policy through changes of the government's financing policy is that it could actually change aggregate demand without changing government consumption. The bad news, however, is that one has to know the degree of risk aversion of households to know if aggregate demand increases or decreases in response to a tax cut, as well as how households perceive the risk induced by postponing taxes.

A justification for this type of tax schedule that lies outside this framework is to think of the government as the controller of inflation, and that the government uses a tax scheme based on nominal incomes rather than real. In this case, the way of raising extra tax revenues in case of a deficit is to create inflation, and thereby increase the real incomes as well as the spread of tax payments. The multiplicative tax system used here could potentially be regarded as a short-cut for the inflation story. It is interesting to think of this as a potential explanation for the excess burden of an inflation tax that we often think exists but has been little accounted for in the literature. An area of future research is to include a monetary side explicitly in the model, to combine the income side of creating inflation, with the cost in terms of increased uncertainty.

Finally, in this paper, gross returns on capital were assumed to be known. This implies that we neglect the potential of the tax system to act as an insurance of gross returns. The reason for making this simplifying assumption was to have a model of pure government uncertainty, to see how this alters the standard analysis of capital income uncertainty. However, if we want to come closer to the real world, it is reasonable to make also gross returns stochastic. We would then have to take into consideration the covariance between the gross returns and the tax rate. If there is a positive correlation between the return and the tax rate, this could potentially offset some of the precautionary savings created by the tax system in the present model.

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5. Appendix

5.1 List of variables

θ	additive tax parameter
p	multiplicative tax parameter for comparative statics exercises
γ^i	the stochastic tax variable, the superscript denotes that it is individual specific $E[\gamma^i]=\gamma$, (non-stochastic) is what the government uses, since there is a large (enough) number of households
g_t	government consumption in period t = 1,2
τ_1	first period lump-sum tax
U	von Neumann-Morgenstern utility function
U_{ij}	partial derivative with respect to elements *i, j*
c_t	private consumption in period t = 1,2
y_t	income in period t = 1,2
r	interest rate given from world market
a_1	household savings ($a_1 \equiv y_1 - c_1 - \tau_1$)
$f(x)$	density function of variable x defined over X

5.2 Derivation of derivatives

The analysis of both lump-sum taxes and capital income taxes below starts from the points in the main text where the equation systems have been formulated.

5.2.1 Lump-sum taxes

Totally differentiate the system of equations in (5) to get

$$\begin{bmatrix} \frac{\partial F^1}{\partial T_1} & \frac{\partial F^1}{\partial c_1} & \frac{\partial F^1}{\partial T_2} \\ \frac{\partial F^2}{\partial T_1} & \frac{\partial F^2}{\partial c_1} & \frac{\partial F^2}{\partial T_2} \end{bmatrix} \begin{bmatrix} dT_1 \\ dc_1 \\ dT_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \qquad (24)$$

The Jacobian matrix for the system is

$$\begin{bmatrix} \frac{\partial F^1}{\partial T_1} & \frac{\partial F^1}{\partial c_1} & \frac{\partial F^1}{\partial T_2} \\ \frac{\partial F^2}{\partial T_1} & \frac{\partial F^2}{\partial c_1} & \frac{\partial F^2}{\partial T_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{1}{1+r} \\ \frac{1+r}{n} E\left[(1+r)U_{22} - U_{12}\right] & S & E\left[\pi\left((1+r)U_{22} - U_{12}\right)\right] \end{bmatrix} . \tag{25}$$

To use the implicit function theorem, the endogenous variable Jacobian determinant has to be non zero, or

$$|J| = S \neq 0, \tag{26}$$

which is fulfilled since S is the left hand side of the second order condition and thus a negative number. Divide the system by ∂T_2 to get

$$\begin{bmatrix} \frac{\partial F^1}{\partial T_1} & \frac{\partial F^1}{\partial c_1} \\ \frac{\partial F^2}{\partial T_1} & \frac{\partial F^2}{\partial c_1} \end{bmatrix} \begin{bmatrix} \frac{\partial T_1}{\partial T_2} \\ \frac{\partial c_1}{\partial T_2} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F^1}{\partial T_2} \\ -\frac{\partial F^2}{\partial T_2} \end{bmatrix} . \tag{27}$$

Use Cramer's rule to get the partial derivatives of interest. The derivative between first and second period taxes is

$$\frac{\partial T_1}{\partial T_2} = \frac{\begin{vmatrix} -\frac{\partial F^1}{\partial T_2} & \frac{\partial F^1}{\partial c_1} \\ -\frac{\partial F^2}{\partial T_2} & \frac{\partial F^2}{\partial c_1} \end{vmatrix}}{|J|} = -\frac{\frac{\partial F^1}{\partial T_2}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^2}{\partial T_2}\frac{\partial F^1}{\partial c_1}}{|J|} , \tag{28}$$

which is equivalent to

$$\frac{\partial T_1}{\partial T_2} = -\frac{1}{1+r} . \tag{29}$$

The derivative between first period consumption and second period taxes is

$$\frac{\partial c_1}{\partial T_2} = \frac{\begin{vmatrix} \frac{\partial F^1}{\partial T_1} & -\frac{\partial F^1}{\partial T_2} \\ \frac{\partial F^2}{\partial T_1} & -\frac{\partial F^2}{\partial T_2} \end{vmatrix}}{|J|}, \tag{30}$$

or equivalently

$$\frac{\partial c_1}{\partial T_2} = \frac{1}{S} E\left[\left(U_{12} - (1+r)U_{22}\right)\left(\pi - \frac{1}{n}\right)\right]. \tag{31}$$

To determine the sign of this expression, we start by noting that the denominator is the SOC from a maximization problem, and is thus negative. Decreasing temporal risk aversion then implies that the numerator is positive, so that $\partial c_1/\partial T_2 < 0$.

Before presenting the proof of this statement, which is an adoption of the proof in Sandmo [1970] , we will evaluate another derivative, namely $\partial c_1/\partial y_1$. This will provide a condition that later will help us to determine the sign of $\partial c_1/\partial T_2$. To obtain $\partial c_1/\partial y_1$, we use implicit differentiation of the FOC to obtain

$$\frac{\partial c_1}{\partial y_1} = -\frac{\partial F^2}{\partial y_1} \Big/ \frac{\partial F^2}{\partial c_1} = -\frac{E\left[U_{12} - (1+r)U_{22}\right](1+r)}{S} \ . \tag{32}$$

It seems reasonable to assume that this derivative is positive, thus implying that $E\left[U_{12} - (1+r)U_{22}\right] > 0$, which is a condition that will be used in the following analysis.

A proof that decreasing temporal risk aversion implies $\partial c_1/\partial T_2 < 0$ will now be stated. Start by totally differentiating the absolute risk aversion function, $A = -U_{22}/U_2$, which yields

$$dA = \frac{\partial A}{\partial c_1} dc_1 + \frac{\partial A}{\partial c_2} dc_2 \ . \tag{33}$$

Along the budget line we have $dc_1 = -\frac{1}{(1+r)} dc_2$, which we substitute into the differential and divide by dc_2 to obtain

$$\frac{dA}{dc_2} = -\frac{1}{1+r}\frac{\partial A}{\partial c_1} + \frac{\partial A}{\partial c_2} < 0 \ , \tag{34}$$

where the sign is determined by the assumption of *decreasing temporal risk aversion.* Continuity assumptions let us write

$$\frac{\partial A}{\partial c_1} \equiv \frac{\partial}{\partial c_1}\left(-\frac{U_{22}}{U_2}\right) = \frac{\partial}{\partial c_2}\left(-\frac{U_{12}}{U_2}\right), \tag{35}$$

which is substituted into the inequality, so that we get the condition

$$\frac{\partial}{\partial c_2}\left(\frac{U_{12}-(1+r)U_{22}}{U_2}\right) < 0 \ . \tag{36}$$

In the next step this condition implied by decreasing temporal risk aversion is used to determine the sign of $\partial c_1 / \partial T_2$.

Start by defining the second period consumption for tax payments equal to the expected value $\bar{c}_2 = y_2 + (1+r)y_1 - (1+r)c_1 - T_2/n - T_1(1+r)/n$. By using the intertemporal budget restriction this could be written as $c_2 = \bar{c}_2 - \pi T_2 + T_2/n$. Decreasing temporal risk aversion could now be used to write

$$\frac{U_{12}-(1+r)U_{22}}{U_2} \geq \left\{\frac{U_{12}-(1+r)U_{22}}{U_2}\right\}_{\pi=1/n} \quad \text{if } \pi \geq 1/n \ , \tag{37}$$

where the right hand side is deterministic. Furthermore, $U_2(\pi - 1/n) \geq 0$ if $\pi \geq 1/n$. Multiplying both sides with this expression gives

$$\left(U_{12}-(1+r)U_{22}\right)(\pi - 1/n) \geq \left\{\frac{U_{12}-(1+r)U_{22}}{U_2}\right\}_{\pi=1/n} \mathrm{U}_2(\pi - 1/n) \quad \text{if } \pi \geq 1/n \ , \tag{38}$$

which we take expectations of to get

$$E\left(U_{12}-(1+r)U_{22}\right)(\pi - 1/n) \geq \left\{\frac{U_{12}-(1+r)U_{22}}{U_2}\right\}_{\pi=1/n} \mathrm{E}\left[\mathrm{U}_2(\pi - 1/n)\right] \quad \text{if } \pi \geq 1/n \ . \tag{39}$$

We want to show that the left hand side is positive, but a sufficient condition for this is that the RHS is positive, which is what we will show. The first part of the expression is positive due to the assumption that $\partial c_1/\partial y_1 > 0$. To show that $E[U_2(\pi - 1/n)] > 0$, we start by noting that $U_2 \geq \{U_2\}_{\pi=1/n}$ if $\pi \geq 1/n$, since $U_{22}<0$. Obviously, $\pi - 1/n \geq 0$ if $\pi \geq 1/n$, and thus $U_2(\pi - 1/n) \geq \{U_2\}_{\pi=1/n}(\pi - 1/n)$.

Taking expectations yields $E[U_2(\pi - 1/n)] \geq \{U_2\}_{\pi=1/n} E[\pi - 1/n] = 0$, which then implies that the LHS > 0 in (39). We have therefore proved that $\partial c_1/\partial T_2 < 0$, if we assume decreasing temporal risk aversion.

5.2.2 Capital Income Taxes

Totally differentiating the system (13) gives

$$\begin{bmatrix} \frac{\partial H^1}{\partial \tau_1} & \frac{\partial H^1}{\partial c_1} & \frac{\partial H^1}{\partial p} \\ \frac{\partial H^2}{\partial \tau_1} & \frac{\partial H^2}{\partial c_1} & \frac{\partial H^2}{\partial p} \end{bmatrix} \begin{bmatrix} d\tau_1 \\ dc_1 \\ dp \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{40}$$

The Jacobian matrix for the system is

$$\begin{bmatrix} \frac{\partial H^1}{\partial \tau_1} & \frac{\partial H^1}{\partial c_1} & \frac{\partial H^1}{\partial p} \\ \frac{\partial H^2}{\partial \tau_1} & \frac{\partial H^2}{\partial c_1} & \frac{\partial H^2}{\partial p} \end{bmatrix} = \begin{bmatrix} 1 - \frac{p\bar{\gamma}r}{1+r} & -\frac{p\bar{\gamma}r}{1+r} & \frac{\bar{\gamma}ra_1}{1+r} \\ -E[\Psi] & S & E\left[\gamma^i r U_2 - \frac{\gamma^i r a_1}{R^i}\Psi\right] \end{bmatrix} , \tag{41}$$

where $\Psi \equiv R^i(U_{12} - R^i U_{22})$, and $a_1 \equiv y_1 - c_1 - \tau_1$. In order to use the implicit function theorem we have to check that the Jacobian determinant of the endogenous variables in this problem is non-zero. Or

$$|J| = \begin{vmatrix} \frac{\partial H^1}{\partial \tau_1} & \frac{\partial H^1}{\partial c_1} \\ \frac{\partial H^2}{\partial \tau_1} & \frac{\partial H^2}{\partial c_1} \end{vmatrix} = \begin{vmatrix} \frac{\bar{R}}{1+r} & -\frac{p\bar{\gamma}_r r}{1+r} \\ -E[\Psi] & S \end{vmatrix} \neq 0 , \tag{42}$$

where $\bar{R} = 1 + r - p\bar{\gamma}_r r$. This is equivalent to

$$|J| = \frac{\bar{R}}{1+r} S - \frac{p\bar{\gamma}_r r}{1+r} E[\Psi] \neq 0 . \tag{43}$$

The expression will be negative if E[Ψ] ≥ 0, which we usually assume, since this implies that $\partial c_1/\partial y_1 > 0$. This connection between E[Ψ] ≥ 0 and $\partial c_1/\partial y_1 > 0$ was shown above.

We then proceed by dividing both equations in the differentiated system with ∂p to obtain the following system

$$\begin{bmatrix} \frac{\partial H^1}{\partial \tau_1} & \frac{\partial H^1}{\partial c_1} \\ \frac{\partial H^2}{\partial \tau_1} & \frac{\partial H^2}{\partial c_1} \end{bmatrix} \begin{bmatrix} \frac{\partial \tau_1}{\partial p} \\ \frac{\partial c_1}{\partial p} \end{bmatrix} = \begin{bmatrix} -\frac{\partial H^1}{\partial p} \\ -\frac{\partial H^2}{\partial p} \end{bmatrix} . \tag{44}$$

From this we can solve for the derivatives of interest by using Cramer's rule. For the government's budget to be satisfied, first period lump-sum taxes have to change according to

$$\frac{\partial \tau_1}{\partial p} = \frac{\begin{vmatrix} -\frac{\partial H^1}{\partial p} & \frac{\partial H^1}{\partial c_1} \\ -\frac{\partial H^2}{\partial p} & \frac{\partial H^2}{\partial c_1} \end{vmatrix}}{|J|} = -\frac{\frac{\partial H^1}{\partial p}\frac{\partial H^2}{\partial c_1} - \frac{\partial H^2}{\partial p}\frac{\partial H^1}{\partial c_1}}{|J|} . \tag{45}$$

This is equivalent to

$$\frac{\partial \tau_1}{\partial p} = -\frac{\frac{\bar{\gamma} r a_1}{1+r} S + \frac{p\bar{\gamma} r}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right]}{|J|} . \tag{46}$$

Finally, the effect on first period private consumption is

$$\frac{\partial c_1}{\partial p} = \frac{\begin{vmatrix} \frac{\partial H^1}{\partial \tau_1} & -\frac{\partial H^1}{\partial p} \\ \frac{\partial H^2}{\partial \tau_1} & -\frac{\partial H^2}{\partial p} \end{vmatrix}}{|J|} , \tag{47}$$

or equivalently

$$\frac{\partial c_1}{\partial p} = -\frac{1}{|J|}\left(\frac{\bar{R}}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right] + \frac{\bar{\gamma} r a_1}{1+r} E[\Psi]\right) . \tag{48}$$

5.3 PROOF OF SIGN DETERMINATION

Below a proof is constructed to verify the sign of the first term of the numerator in the derivative between p and c_1. What we want to determine the sign of is

$$E\left[\underbrace{U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)}_{-} \underbrace{- a_1\left(U_{12} - R^i U_{22}\right)\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)}_{+}\right] . \tag{49}$$

The first term we will show is negative, and the second term is positive if we assume decreasing temporal risk aversion. The proof is constructed along the lines of Sandmo's [1970] proof in the case of capital income risk (see the appendix of that paper). Starting with the first factor, we have that

$$U_2 < \{U_2\}_{\bar{\gamma}} \text{ if } \gamma^i < \bar{\gamma} , \tag{50}$$

since we assume that individuals are risk averse, so that $U_{22} < 0$, and c_2 increases with lower realized tax rates. We now multiply both sides of the inequality with $\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)$ to get

$$U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) < \{U_2\}_{\bar{\gamma}}\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) \text{ if } \gamma^i < \bar{\gamma} . \qquad (51)$$

Taking expectations on both sides, we obtain

$$E\left[U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)\right] < \{U_2\}_{\bar{\gamma}} E\left[R^i\bar{\gamma} - \bar{R}\gamma^i\right] \text{ if } \gamma^i < \bar{\gamma} . \qquad (52)$$

To see that the right hand side is zero, write out the expression as

$$E\left[R^i\bar{\gamma} - \bar{R}\gamma^i\right] = \bar{\gamma}E\left[1 + r - p\gamma^i r\right] - \left(1 + r - p\bar{\gamma}r\right)E\left[\gamma^i\right] = 0 . \qquad (53)$$

We have thus shown that the right hand side is zero, so the left hand side must be negative.

To show that the second term in (49) is positive, we assume that the utility function displays decreasing temporal risk aversion, or equivalently that $\left(U_{12} - R^i U_{22}\right)/U_2$ is decreasing in c_2. We can then write

$$\frac{U_{12} - R^i U_{22}}{U_2} < \left\{\frac{U_{12} - R^i U_{22}}{U_2}\right\}_{\bar{\gamma}} \text{ if } \gamma^i < \bar{\gamma} . \qquad (54)$$

Further, we have that

$$U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) > 0 \text{ if } \gamma^i < \bar{\gamma} . \qquad (55)$$

Multiplying both sides of (54) with this expression and taking expectations we get

$$E\left[\left(U_{12}-R^iU_{22}\right)\left(R^i\bar{\gamma}-\bar{R}\gamma^i\right)\right]<\underbrace{\left\{\frac{U_{12}-R^iU_{22}}{U_2}\right\}_{\bar{\gamma}}}_{+}\underbrace{E\left[U_2\left(R^i\bar{\gamma}-\bar{R}\gamma^i\right)\right]}_{-}\text{ if }\gamma^i<\bar{\gamma}\ . \qquad (56)$$

To show that the left hand side is negative, it is sufficient to show that the right hand side is negative. We know that the factor in braces is positive, and in the above proof we have showed that the last factor is negative; we can thus conclude that the right hand side is negative. To summarize, we have proved that under the assumptions of risk aversion and decreasing temporal risk aversion, the first term in (49) is negative and the second term is positive.

5.4 CONDITION FOR $\partial\tau_1/\partial p>0$

To be to the right of the maximum of the Laffer curve, we have the condition that

$$\frac{\partial\tau_1}{\partial p}=-\frac{\frac{\bar{\gamma}ra_1}{1+r}S+\frac{p\bar{\gamma}r}{1+r}E\left[\gamma^i r\left(U_2-\frac{a_1}{R^i}\Psi\right)\right]}{|J|}>0\ . \qquad (57)$$

Since the denominator is negative, the numerator has to be positive in order to make the expression positive. By using the definitions of S and Ψ we get

$$\frac{\bar{\gamma}r}{1+r}E\left[a_1\left(U_{11}-2R^iU_{12}+R^{i2}U_{22}\right)+p\gamma^i r\left(U_2-a_1\left(U_{12}-R^iU_{22}\right)\right)\right]>0\ . \qquad (58)$$

Using a time separable iso-elastic utility function (c.f. section 2.2.1) and $c_2=a_1R^i$ this becomes

$$-a_1\alpha c_1^{-\alpha-1}+E\left[\left(p\gamma^i r\beta-\alpha\beta\,(1+r)\right)c_2^{-\alpha}\right]>0\ . \qquad (59)$$

Letting α approach zero, the condition becomes $p\bar{\gamma}r\beta>0$, which will always be fulfilled if the return on capital is positive and we study taxes rather than transfers, since consumption in the first period increases with smaller α and thus $1/c_1$ goes to a positive number less than infinity, implying that the first term goes to zero.

Essay V

Budget Deficits, Stochastic Population Size and Consumption*

Abstract

This paper analyzes the effects on present consumption of budget deficits under different assumptions regarding demographics. In the first part, birth and death rates are deterministic, and in the second part, birth rates are assumed to be stochastic. In the case of a deterministic population size, an increase in public debt raises present consumption, if the (deterministic) birth rate is greater than zero, while with a zero birth rate we obtain debt neutrality. This is consistent with the results in Blanchard [1985] and Buiter [1988]. However, for the case of stochastic birth rates, it is shown that we can obtain the result that present consumption will *decrease* when public debt is increased, both when we have a zero expected birth rate, and when the expected population size is assumed to be constant, so that the expected birth rate is positive and equal to the death rate. The explanation is that with an uncertain birth rate, the future tax base is uncertain, which makes per capita taxes uncertain in the future. Shifting taxes to the future thus implies greater uncertainty about future net income, and induces precautionary savings.

1. Introduction

The Ricardian equivalence proposition states that households will not change their consumption path in response to changed timing of taxes, provided the level of government consumption is kept unchanged. This implies that households do not regard their holdings of government bonds as net wealth, and that in this context government deficits do not affect the real economy, see for example Barro [1974]. The model assumptions underlying the Ricardian debt neutrality are quite a few; access to a perfect capital market, the use of lump-sum taxes, and "infinite" planning horizons, to mention some of the most noted and questioned. In this paper, we will focus on the assumption of infinite planning horizons, where infinite planning horizons is really a collective label for the assumptions that the households and the government

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have the same planning horizon, and that there are no new entrants into the economy that will share future tax burdens. Stated differently, if we fix the government's planning horizon, Ricardian models usually assume that both the death and birth rates for households during the planning horizon are equal to zero, although it is sufficient to assume that the birth rate is equal to zero to obtain debt neutrality in a deterministic world with an annuity market.

In some influential papers on budget deficits and consumption, the focus of attention has also been on the planning horizon of individuals and population growth, see for example Barro [1974], Blanchard [1985], Weil [1989], Weil [1987a], and Buiter [1988]. The main point is that it is not, in fact, the length of the planning horizon of individuals but rather the new entrants to the economy that create deviations from Ricardian equivalence. The important aspect of demographics in these models is that the tax base in future periods is affected by the assumptions on death and birth rates, but the level of government consumption is fixed. This implies, for example, that the tax burden for a particular individual will be reduced if there are new tax payers born into the economy. Consequently, if the timing of taxes is changed, so that more of the tax burden is shifted to periods where the tax base is larger and/or different than today, this will have a positive wealth effect for the current generations. (As in the standard Ricardian analysis, perfect capital markets, lump-sum taxes, and an exogenously given level of government consumption are assumed). In Buiter [1988], productivity growth is added as a factor that creates additional room for the government to postpone taxes, and levy them on both later and more productive generations.

In the papers discussed above, there is only room for positive changes in the future. Either the aggregate economy can become more productive, or the tax base can be extended. Furthermore, the total population size in future periods is known, since both death and birth rates are deterministic, which implies that the tax base and thus the size of lump-sum taxes in the future are known. However, in the face of real world budget deficits today, it is not always obvious that households expect the future to be more rewarding than the past in terms of tax base and productivity. One interpretation would be to say that the size of the population is stochastic and that it could actually fall, so that there might actually be less people in the future to share the tax burden. This case has been analyzed to the extent that deaths are allowed, while the birth rate is set to zero. However, if we widen our interpretation of the birth rate to include migration, we have the possibility of people leaving the economy in the second period, and thus making the "birth" rate negative. In this paper we will allow a negative birth rate for this reason. This implies that the tax base can deteriorate both because agents can die and because they can leave the country.

A decreasing population is not something that we have experienced to a great extent, but if we instead of population think in terms of the tax base, which is basically people earning income, it seems more in line with the unemployment experiences over the last decades. The uncertainty could thus be derived from uncertainty about future employment levels. Furthermore, productivity could be stochastic and subject to negative shocks as well as positive, or in the case of a small open economy, the terms of trade could be stochastic and a source of uncertain future purchasing power. In the following analysis, this uncertainty will be labeled population size uncertainty to adhere to the Blanchard [1985] framework. However, this should only be regarded as a collective label for the types of uncertainty discussed here, and an alternative label could be *tax base uncertainty*. Finally, if we ignore these factors, the case of decreasing population due to emigration seems to be an important subject in the EU, where people are free to work (and be taxed) in any member country. Potentially, countries that have high taxes compared to the other countries without providing better public services, production possibilities or other benefits, will probably face a risk of a decreasing population due to emigration.

From the literature dealing with intertemporal choice under uncertainty, we know that agents change present consumption not only in response to changes in expected values of income, but also in response to changes in the risk (or "spread"), see for example Leland [1968], Sandmo [1970], Ormiston and Schlee [1992] for comparative statics results, Kimball [1990], for a formalization of the theory of precautionary savings and prudence, and Zeldes [1989] and Caballero [1990] for the importance of precautionary savings for explaining observed consumption patterns. In the present paper, we introduce risk in connection with future lump-sum taxes, since the tax base is stochastic, which in turn implies that the net income in the future is stochastic. By shifting more or less taxes to the future, the risk that the agent is facing changes. If we make the standard assumption that agents are risk averse, postponing taxes will not only have (potentially) the standard wealth effect on private consumption, but also an effect due to changed income risk in the future. The risk in this paper is an aggregate tax risk in the sense that all households have the same realization of the stochastic variable. This can be compared to Becker's [1995] analysis of individual tax risk, where instead each household experiences a specific realization of the uncertain tax.

There are of course other aspects of the tax system that will not be explicitly considered in this paper. For instance, in other parts of the tax literature, the tax system is regarded as an insurance of uncertain gross income, see Stiglitz [1969] and Varian [1980] for a general discussion, Barsky et al. [1986] for a discussion in connection to Ricardian equivalence, and Smith [1982] for a discussion of intergenerational redistribution and uncertain population size. The potential insurance aspect of a tax system is however not the only role taxes will play in an uncertain world, but in some cases the tax system in itself will create additional uncertainty. To focus on this latter aspect of the tax system and abstract from the insurance aspect, we will in this paper assume that gross returns are certain.

The purpose of this paper is to first summarize the effects of budget deficits on consumption when the death and birth rates are known, implying that individuals might not know if they live for one or two periods, but in aggregate the size of the second period population is known. This is the standard framework that is used in the papers described above. Put in another way, the life time of a particular individual is allowed to be stochastic, but second period population is deterministic. In the second part of the paper, a stochastic second period population size is modeled, by introducing a stochastic birth rate. The assumption of stochastic birth rates implies that the future lump-sum tax becomes stochastic, and there is thus a reason for prudent households to engage in precautionary savings when taxes are postponed.

2. DETERMINISTIC POPULATION SIZE

In this section, we will analyze the effect of budget deficits on present consumption in the cases of deterministically increasing, decreasing, and constant future population. It is important to make the distinction between the aggregate and individual levels here, since although individuals do not know for certain if they are alive in the next period, they still know what the future population size is. Put differently, we know the number of individuals that will be alive for sure, but we do not know who the survivors will be. In this way the model is deterministic, although it is stochastic for the individual household. The stochastic nature is, however, of a particularly simple kind from the problem solver's viewpoint, since it affects the discount factor in a deterministic way, and has no impact on future (lump-sum) income or taxes. The framework used is an economy that exists for two periods, with one government sector that obeys an intertemporal budget constraint, an infinite number of identical households that are expected utility maximizers, and a deterministic interest rate given from the world market. For the original analysis of a deterministic population size in continuous time, see Yaari [1965] and Blanchard [1985], and for extensions see, for example, Buiter [1988].

Population and insurance

In period one population is N_1 and in period two population is $N_2 = (1+q-\pi)N_1$, where π is the (deterministic) death rate, which is greater than zero. q is the (deterministic) "birth" rate, which is allowed to be negative as long as $1+q-\pi > 0$, since second period population cannot be negative.

Since individuals face a constant probability of dying, and we assume that the population is large, there is room for an insurance policy that will cost the individual his savings, a_1, if he/she dies, and pay a bonus of $a_1\pi*$, where $\pi* \equiv (1+r)\pi/1-\pi$, if the individual survives to the second period, see Yaari [1965] or Blanchard [1985] for this type of self-financed insurance arrangement. Because we are using a model in discrete time, we have to adjust the factor multiplying a_1 to account for the fact that there are less people alive in the second period to share the return from the insurance company, and that the savings earn interest. Thus we get $\pi*$ instead of simply π, which is the premium received at every moment in a continuous time model. In this way there will be no unintentional bequests in the economy, and we will later assume that individuals are not altruistic, so there will be no motive for intentional bequests. We could also note that this is a policy that would be provided by a perfect competition market of insurance where the insurance company has no transaction costs.[1] In other words, $1+r+\pi*$ will be the return on savings for the surviving individuals, due to the insurance arrangement, while the government only gets (or pays) $1+r$. However, as we will see in the following analysis, this difference in the return on savings will not in itself imply that public debt is net wealth to the households, since there is a counteracting effect in terms of the reduction in tax base implied by a positive death rate.

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[1] Adding transaction costs that are a fraction of the payments would of course reduce the bonus accordingly, but would not affect the present analysis in a significant way.

Government

The government obeys a budget constraint in the first and second period according to

$$\begin{aligned} T_1 &= G_1 - D_1 \\ T_2 &= G_2 + (1+r)D_1 \, , \end{aligned} \tag{1}$$

which states that first period aggregate taxes are equal to first period aggregate consumption minus aggregate debt, and in the second period, aggregate taxes must cover both consumption and debt repayment plus interest on the debt. Define per capita values of a variable X as $x_t \equiv X_t \,/\, N_t$. To make notation slightly easier, second period government consumption is set to zero. The per capita equivalent of the above budget restrictions then becomes

$$\begin{aligned} \tau_1 &= g_1 - d_1 \\ \tau_2 &= \frac{1+r}{1+q-\pi} d_1 \, , \end{aligned} \tag{2}$$

where of course second period per capita tax payments fall with increasing(decreasing) birth(death) rate.

Individuals

The identical individuals are expected utility maximizers that solve the problem

$$\begin{aligned} &\max_{c_1, c_2} E\left[U(c_1, c_2)\right] = U(c_1) + \frac{1-\pi}{1+\delta} U(c_2) \\ &\text{s.t.} \qquad c_1 = y_1 - \tau_1 - a_1 \\ &\qquad\qquad c_2 = y_2 + (1 + r + \pi^*) a_1 - \tau_2 \; , \end{aligned} \tag{3}$$

where U(.) is a von Neumann-Morgenstern utility function, y_t is exogenous income in period *t*, c_t is consumption in period *t*, τ_t is lump-sum taxes in period *t*, savings from period one to two is denoted by a_1, r is the deterministic and exogenous interest rate, δ is the subjective discount factor, and, finally, π is the probability of death between period one and two. Two things to note are the factor $1-\pi$ in front of second period utility, which is due to the probability of dying, and the appearance of $a_1\pi^*$ in the budget restriction, which is due to the insurance arrangement. Finally, to reduce notation slightly, the exogenous income in the second period is assumed to be zero in the following analysis, since this assumption does not alter any of the results.

By using the per capita version of the government's budget restrictions we can write second period consumption as

$$c_2 = (1+r+\pi\,*)(y_1 - c_1 - g_1) + \left((1+r+\pi\,*) - \frac{1+r}{1+q-\pi}\right)d_1 \,. \tag{4}$$

We can here make the observation that the level of government debt affects the individual, since it does not cancel out in the budget restriction of the individual when we substitute in the government's budget restriction for arbitrary choices of death and birth rates. The first term in the parenthesis displays the fact that individuals can save the period one tax cut when debt is increased by one unit, while the second term represents the (per capita) tax that has to be paid in period two to repay one unit of debt. The coefficient on public debt describes the wealth effect for the household from creation of public debt in the first period, which we will use in the following analysis.

The next step is to solve for optimal consumption. The first and second order conditions (FOC and SOC) for the individual's maximization in (3) are

FOC:

$$U'(c_1) - \frac{(1-\pi)(1+r+\pi\,*)}{1+\delta}U'(c_2) = 0 \tag{5}$$

SOC:

$$S \equiv U''(c_1) + \frac{(1-\pi)(1+r+\pi\,*)^2}{1+\delta}U''(c_2) < 0. \tag{6}$$

We now ask the question how an increased public debt affects first period consumption by implicit differentiation of the FOC. This yields

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{\underbrace{S}_{-}}\underbrace{U''(c_2)}_{-}\underbrace{\left(1+r+\pi\,* - \frac{1+r}{1+q-\pi}\right)}_{?}, \tag{7}$$

where $\kappa \equiv (1-\pi)(1+r+\pi^*)/(1+\delta) > 0$. We note that the first two factors are both negative, if we assume that the problem has a maximum and that individuals are risk averse, implying that $U''(c_2) < 0$. To determine the sign of the last factor, we must know the relationship between death and birth rates. There are three main cases of interest: constant population, decreasing population and increasing population. In the following analysis, we will note that the results will always be determined by the last factor, i.e. by the coefficient on public debt. (N.B.: the cases are all deterministic, so it is at this stage not an issue in what population regime an individual lives in.)

Case 1: Constant population

In this case, either no one enters or leaves the economy, or agents enter and leave the economy at the same rate. As will be demonstrated, it is not irrelevant for debt policy what the reason is for a constant population. From the literature on Ricardian equivalence, see for example Blanchard and Fisher [1989], we know that infinite horizon models without population growth differ from the model in Blanchard [1985], where agents die and are born at the same rate, with respect to the effects of debt policy.

a: $\pi = q = 0$ (constant population, no deaths or births)

This is the case of standard Ricardian models. By inspection of either second period consumption above or by inspection of the derivative between consumption and debt, it is clear that the timing of taxes/debt level has no effect on individuals. In this case, individuals cannot evade taxes in the second period by sharing them with new entrants to the economy.

b: $\pi = q > 0$ (constant population, with equal death and birth rates)

In the case of equal death and birth rates, second period consumption will be

$$c_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \pi^* d_1 , \tag{8}$$

where the coefficient on debt is simply the savings premium from the life insurance, which is always positive, so increasing public debt will make the current generation wealthier. The explanation is that the individuals of the current generation that survive to the second period have new taxpayers in the future to share the tax burden with.

To investigate how first period consumption changes in response to increased public debt, we have the derivative

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2)\pi^* > 0 . \tag{9}$$

The first two factors are negative and the last is positive, so the expression is positive. In other words, individuals raise their present consumption in response to higher public debt. This is in line with the result generated in Blanchard's [1985] paper of finite horizons. The explanation is that since individuals only care about their own consumption, they become wealthier when some of the tax burden is levied on "their" children. Furthermore, for the individuals to maximize their utility, a fraction of the additional wealth will be consumed in the first period, and the remainder in the second period.

Case 2: Decreasing population

This case is perhaps not the most noted when analyzing the effects of debt policy, although Buiter [1988] shows that in the case of a positive death rate and a zero birth rate, we will obtain debt neutrality. However, Buiter's case is only one case where we have a decreasing population, and the other cases that generate a decreasing population will be discussed below. The reasons for not emphasizing the case of decreasing population might have been obvious from historical observations of growing population, but in many parts of the industrialized world, and with the interpretation of employed population as the tax payers, this case seems rather relevant. In other words, today it is no longer obvious that the tax base cannot decline as well as increase, and this case is the case of declining tax base.

a: $\pi > 0$ and $q = 0$ (decreasing population, deaths but no births)

Start with consumption in period two, which is now

$$c_2 = (1 + r + \pi^*)(y_1 - c_1 - g_1) , \tag{10}$$

where we see that debt does not enter the expression determining consumption in the second period, i.e. the debt coefficient in (4) is zero. This is consistent with Buiter [1988], who concludes that a zero birth rate is sufficient to make the wealth effect equal to zero in a continuous time model of the Yaari-Blanchard type, (see Yaari [1965] and Blanchard [1985]).

The derivative between first period consumption and debt is now

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \cdot 0 = 0 \ , \tag{11}$$

which again is a statement of debt neutrality when the birth rate is zero. In other words, since debt does not affect the level of wealth, it will not affect the consumption decision in a deterministic world. This is the clearest example of that the assumption of finite horizons alone does not generate deviations from Ricardian equivalence. At first it might appear puzzling, since the return on savings for the households is larger than the interest the government has to pay on its outstanding debt. Therefore, we might think that the households will be better off if they save between periods rather than the government, i.e. introducing public debt would be regarded as net wealth by the households. The reason that debt is not net wealth is, of course, that the tax base deteriorates between the periods when we have a positive death rate. Another way of analyzing the effects of debt in this particular case where $q = 0$ is to rewrite the debt coefficient in (4) according to

$$1 + r + \pi * - \frac{1+r}{1-\pi} = \frac{1+r}{1-\pi} - \frac{1+r}{1-\pi} = 0 \ , \tag{12}$$

where the first term is again what the households get if they save a dollar's tax cut to the second period, and the second term is what they have to pay in extra tax in the second period. We can now see that the difference in returns on savings is completely wiped out when the government's savings rate is adjusted for the decreasing population. In other words, the government has a different return on savings at the aggregate level, but not at the per capita level, which is the key to understanding this result and Buiter's [1988] statement that a zero birth rate is a necessary condition for debt neutrality in a continuous time model.

b: $\pi > q > 0$ *(decreasing population, death rate higher than birth rate)*

Second period consumption is

$$c_2 = (1 + r + \pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1 + r + \pi^*)}{1 + q - \pi} \right) d_1 \ . \tag{13}$$

The coefficient on debt is positive, which implies that the current generation becomes wealthier if public debt is created. In this case the derivative between first period consumption and debt is

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \left(\frac{q(1 + r + \pi^*)}{1 + q - \pi} \right) > 0 \ . \tag{14}$$

In other words, with increased public debt, first period consumption will increase, since households regard their holdings of government bonds as net wealth. Again, this is a result of the new entrants in the second period, who will share the tax burden with the individuals from the current generation that survive to the next period.

c: $\pi = 0, q < 0$ *(decreasing population, no deaths but negative "birth" rate, i.e. emigration)*

Here we let the "birth" rate be negative, which implies that people leave the country in the second period. Second period consumption becomes

$$c_2 = (1 + r + \pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1 + r)}{1 + q} \right) d_1 \ . \tag{15}$$

The coefficient on debt is now negative, which implies that debt creation in the first period represents negative wealth to the surviving households that remain in the country. The derivative between first period consumption and debt is now

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \left(\frac{q(1 + r)}{1 + q} \right) < 0 \ . \tag{16}$$

Since the derivative is negative, first period consumption will be reduced in response to increased public debt, which is of course a consequence of public debt being net debt rather than net wealth for the households.

d: $\pi > 0, q < 0$ *(decreasing population, deaths and negative "birth" rate, i.e. emigration)*

Second period consumption is now

$$c_2 = (1 + r + \pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1 + r + \pi^*)}{1 + q - \pi} \right) d_1 . \tag{17}$$

The coefficient on debt is still negative, which implies that public debt is negative wealth to the households. The derivative between consumption and debt is

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \left(\frac{q(1 + r + \pi^*)}{1 + q - \pi} \right) < 0 . \tag{18}$$

The derivative tells us that first period consumption will decrease when public debt is created. We can also note that the coefficient on debt is a larger negative number than it is in case 2c, i.e. with negative birth rates, public debt represents more net debt when death rates are positive than when the death rate is equal to zero.

Case 3: Increasing population

This is the usual way of modifying the assumption of a constant population. Again, this can be motivated by historical observations of the population size, but with the interpretation of population as the equivalent of work-force, it is not totally obvious that this is the most relevant case in many economies today.

a: $\pi = 0$ *and* $q > 0$ *(increasing population, no deaths but births)*

With no deaths and positive birth rates, second period consumption can be written as

$$c_2 = (1 + r)(y_1 - c_1 - g_1) + \frac{(1 + r)q}{1 + q} d_1 . \tag{19}$$

In this case it is straightforward to see that the debt coefficient is positive, and thus that increased debt is regarded as net wealth.

To analyze how first period consumption changes in response to an increase in debt, we have the derivative

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \frac{(1+r)q}{1+q} > 0 , \qquad (20)$$

with $\kappa \equiv (1+r)/(1+\delta)$ since death rates are set to zero. The first and second factors are both negative, while the last factor is positive, implying that first period consumption increases with increases in the debt level. In other words, it here becomes obvious that we do not need the assumption of finite horizons to have the result that public debt is net wealth to the households, which in turn makes households consume more today when public debt is created.

b: $0 < \pi < q$ (increasing population, death rate lower than birth rate)

If we add positive death rates, the expression for second period consumption can be written as

$$c_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1+r+\pi^*)}{1+q-\pi} \right) d_1 , \qquad (21)$$

which gives a positive coefficient on debt, and thus implies that public debt is net wealth to the households alive in the first period.

The derivative between first period consumption and debt is now

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \left(\frac{q(1+r+\pi^*)}{1+q-\pi} \right) > 0 , \qquad (22)$$

where the first two factors are negative and the last is positive, implying a positive derivative. In other words, postponing taxes to period two increases consumption in period one. In this case we can note that the assumption of positive death rates makes the coefficient on public debt larger than in the case with increasing population without deaths, i.e. although a positive death rate in isolation does not make public debt net wealth, it reinforces the wealth effect of public debt when we have a birth rate that is non-zero.

To summarize the results of deterministic death and birth rates, we start by noting that a constant population will generate the Ricardian result if both death and birth rates are equal to zero, but else a positive relation between present consumption and budget deficits/the level of debt. When the population is increasing, we get the result that debt always represents net wealth to the households alive in the first period, and that first period consumption thus increases with higher debt levels. In the case of decreasing population, we get the result that individuals will consume less in the first period when public debt is increased, if "birth" rates are negative, i.e. some people emigrate.

In some discussions on the validity of Ricardian equivalence, one could get the impression that a positive death rate alone would make individuals regard government bonds as net wealth. However, as pointed out by Buiter [1988] in a continuous time model, and here in a model in discrete time, it is the assumption about birth rates that is central for the question of debt neutrality. We note that it is the sign of the birth rate that determines how first period consumption will change when debt changes, and with a positive/zero/negative birth rate we will get a positive/zero/negative wealth effect from the creation of public debt, which in turn determines how first period consumption will change. The existence of a positive death rate is, however, relevant when determining the magnitude of the wealth effect, although it is irrelevant for determining its sign. We can note that in the case where we have a positive birth rate, introducing a positive death rate makes the debt coefficient greater (see *3a* and *b*), while in the case where we have a negative birth rate and introduce a positive death rate, the debt coefficient becomes smaller (more negative). In other words, the existence of a positive death rate creates a leverage effect on the wealth effect (positive or negative) created by a non-zero birth rate.

3. STOCHASTIC POPULATION SIZE

With deterministic death and birth rates, second period population is deterministic, and thus also the tax base. This implies that the per capita tax in the future is known in the first period. The question in this section is what happens if the population size in the future is stochastic, which implies that the tax base is uncertain and thus also the *per capita* tax in the future. There are several ways of making second period population size stochastic, basically both death and birth rates could be stochastic, or we could make one rate stochastic and the other deterministic. In order to keep the analysis as simple as possible, we will introduce a stochastic birth rate together with a deterministic death rate, which makes the population size in the second period stochastic. In this way, the individual still knows the return on capital and by how much to discount future utility, but the second period *per capita* lump-sum tax, and thus disposable income, becomes stochastic. This uncertain tax will be equal for all households, so we are analyzing an aggregate tax risk. In Becker [1995], tax risk and budget deficits were analyzed under the assumption that individuals have different realizations of their second period tax payment, i.e. that paper analyzed individual rather than aggregate tax risk.

3.1 THE MODEL

There are still three sectors in the economy, one providing insurance, one government sector, that can finance its budget with either taxes or public debt creation, and finally, one household sector maximizing expected utility.

Population and insurance

Population and insurance are defined as in the previous section, with the vital distinction that in period two, population is now $\tilde{N}_2 = (1 - \pi + \tilde{q})N_1$, where π is the (deterministic) death rate, and $\tilde{q}$ is the stochastic birth rate. Since the birth rate is stochastic, so is second period population size, (thus the "tilde" over these variables, and other variables that are stochastic). The insurance system will be arranged as previously, and this is the first instance where the choice of having a deterministic death rate matters, since otherwise the return on savings would be stochastic under a self financed insurance system.

Government

The government still obeys a budget constraint in the first and second period according to

$$
\begin{aligned}
T_1 &= G_1 - D_1 \\
T_2 &= G_2 + (1+r)D_1 \ ,
\end{aligned}
\tag{23}
$$

with the per capita equivalent of the above budget restrictions now being

$$
\begin{aligned}
\tau_1 &= g_1 - d_1 \\
\tilde{\tau}_2 &= \frac{1+r}{1+\tilde{q}-\pi} d_1 \ .
\end{aligned}
\tag{24}
$$

Second period *per capita* tax payments fall with increasing(decreasing) birth(death) rate, and are now stochastic due to the uncertain birth rate.

Individuals

The identical individuals are expected utility maximizers with time separable utility that solve the problem

$$
\begin{aligned}
&\max_{c_1,c_2} E[U(c_1,c_2)] = U(c_1) + \frac{1-\pi}{1+\delta} E[U(\tilde{c}_2)] \\
&\text{s.t.} \qquad c_1 = y_1 - \tau_1 - a_1 \\
&\qquad\qquad \tilde{c}_2 = (1+r+\pi^*)a_1 - \tilde{\tau}_2 \ .
\end{aligned}
\tag{25}
$$

The variables are defined as before, the only difference being that variables with a "tilde" are stochastic. This is the other instance where letting the death rate still be deterministic matters, since we can now move the factor $1-\pi$ outside the expectations operator. By using the *per capita* version of the government's budget restrictions we can write second period consumption as

$$
\tilde{c}_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \left((1+r+\pi^*) - \frac{1+r}{1+\tilde{q}-\pi} \right) d_1 \ . \tag{26}
$$

Still the level of government debt affects the individual, since it does not cancel out in the budget restriction of the individual when we substitute in the government's budget restriction, for arbitrary values of death, birth and interest rates. We could also note that in this case of stochastic population, it is the presence of second period taxes through the debt level that creates the uncertainty about future consumption after conditioning on survival. Put differently, with deterministic death and birth rates, an individual knew how much taxes he had to pay if he survived. In the presence of stochastic birth rates, however, he also needs to know the realized birth rate to calculate the *per capita* tax and thus second period consumption. Finally, we note that the coefficient on debt is not linear in the realized birth rate, which has consequences that will be discussed below.

The first question to ask is how the coefficient on debt, or the wealth effect, is affected by having a stochastic birth rate rather than a deterministic. We know that in the deterministic model, the reason for the wealth effect is that individuals will regard their bond holdings as net wealth if they can levy future tax payments on new entrants to the economy, while they will experience a negative wealth effect if the birth rate is negative. In addition to this, with a positive death rate, agents have a higher return on their savings than the government, due to the life insurance system, which creates an additional leverage effect from the creation of public debt.

At first, it might seem natural to assume that the wealth effect will not be affected by having a stochastic birth rate rather than a deterministic. This is, however, not the case, since when we introduce uncertainty about the birth rate in the present model, we will not only make the debt coefficient stochastic, but we also get a smaller *expected* value for the debt coefficient compared to the value it would have if the expected birth rate were deterministic. The reason is that the coefficient on debt that determines the wealth effect is non-linear in $\tilde{q}$. In Figure 1, a plot of the debt coefficient, over a range of birth rates such that the expected value of the birth rate is equal to the death rate, is displayed.

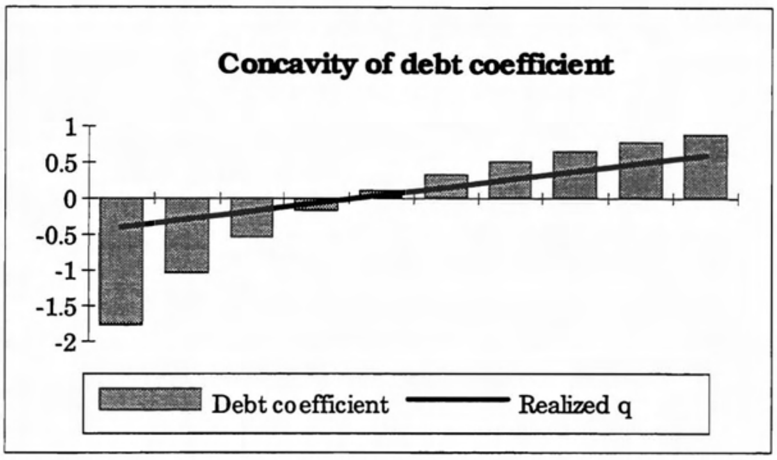

Figure 1. *The size of the debt coefficient for different values of the birth rate, with the expected birth rate equal to the death rate.*

The question is then what this picture implies for the expected debt coefficient, i.e. the debt coefficient implied by a postulated birth rate distribution. In general, we cannot determine the sign of the coefficient without making assumptions about the probability distribution of the birth rate, and the magnitude of the death and interest rates, i.e. a positive expected birth rate is not a sufficient condition for a positive wealth effect. If we in this example assume that the birth rate is uniformly distributed over the range, we could simply add the bars together and divide by the number of "observations" or birth rate classes to obtain the expected debt coefficient. The values of the expected debt coefficient for different values of the interest rate and the death rate are displayed in Table 1.

The numbers in the table are generated by assuming that birth rates are uniformly distributed, with a distribution starting at zero and with an expected value equal to the death rate. In the case of a deterministic birth rate equal to the death rate, the debt coefficients will be higher than the expected debt coefficients, (compare Table 1 and 2).

Expected debt coefficients

r	π 0.1	0.5	0.9
0.5	0.16	1.31	11.8
1	0.21	1.75	15.7
2	0.32	2.62	23.6

Table 1. *Expected debt coefficients for different values of the interest rate, r, and death rate, p, assuming that the birth rate distribution is uniform, starting at zero, with ten possible values, and with the expected birth rate equal to the death rate.*

Deterministic debt coefficient

r	π 0.1	0.5	0.9
0.5	0.17	1.5	13.5
1	0.22	2.0	18.0
2	0.33	3.0	27.0

Table 2. *Deterministic debt coefficient, death rate equal to birth rate. In this case (1b) we know that the debt coefficient is equal to π*.*

We can note that although the expected birth rate in the distribution of birth rates is equal to the death rate, the implied expected debt coefficient is smaller than it is when the birth rate is assumed to be a constant equal to its expected value, as in the case of a deterministic population size. As we mentioned above, the reason for this, perhaps non-intuitive, behavior of the debt coefficient is that it is a concave function of the birth rate (see Figure 1). This entails that the *expected* debt coefficient will be smaller than the deterministic coefficient. Furthermore, if we allow for negative birth rates, i.e. individuals leaving the country in period two, we can obtain a *negative* expected debt coefficient, although the expected birth rate is equal to the death rate. If, for example, the interest rate is equal to one and the death rate is equal to 0.5 and we use a distribution of birth rates that starts at -0.4, but has an expected value of 0.5, this will produce a debt coefficient of -0.25. The cases of negative expected debt coefficients are perhaps not the cases we are most interested in, but the point is that the standard deterministic debt coefficient will in some cases give a substantial overvaluation of the wealth effect if birth rates are stochastic. We can, however, note that if we have an expected birth rate that is equal to zero, the associated debt coefficient will always be negative, due to the concavity. This is a feature of the expected debt coefficient that will be used below.

The next step is to solve for optimal consumption. The first and second order conditions for the individual's maximization in (25) are

FOC: $$U'(c_1)-\frac{(1-\pi)(1+r+\pi^*)}{1+\delta}E[U'(\tilde{c}_2)]=0 \qquad (27)$$

SOC: $$S \equiv U''(c_1) + \frac{(1-\pi)(1+r+\pi^*)^2}{1+\delta} E[U''(\tilde{c}_2)] < 0. \tag{28}$$

We could now ask the question how an increased public debt affects first period consumption by implicitly differentiating the FOC, which now yields

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} E\left[U''(\tilde{c}_2)\left(\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi} \right) \right]. \tag{29}$$

where $\kappa \equiv (1-\pi)(1+r+\pi^*)/(1+\delta)$. Compared to the deterministic case, we now have an expectations operator in front of the last two factors, and thus the sign of this derivative could not be determined as easily as in the deterministic case. However, a way to handle this problem is to adopt a similar strategy to the one in Sandmo's [1970] analysis of income risk, which we will do below. We can start by noting that the first factor is always negative and that the derivative thus has the opposite sign of the factor inside the expectations operator.

To determine the sign of the factor inside the expectations operator and thus how first period consumption will change in response to an increase in public debt, start by writing second period consumption as

$$\tilde{c}_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \left(\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi} \right) d_1 . \tag{30}$$

We know that if the realized birth rate is equal to zero, the debt coefficient will be zero. If the realized birth rate is positive(negative) we know that the debt coefficient is positive(negative). From this define

$$c^* \equiv (1+r+\pi^*)(y_1 - c_1 - g_1) , \tag{31}$$

which will be the consumption in the second period if a zero birth rate is the realization. We also know that second period consumption will be larger than c^* if $\tilde{q} > 0$ when $d_1 > 0$.

Start by assuming that $U'''(\cdot) > 0$, which is the standard assumption if we want individuals to have a precautionary savings motive, see for example Leland [1968], Sandmo [1970] and Kimball [1990]. This gives us

$$U''(\tilde{c}_2) \geq U''(c^*) \quad \text{if } \tilde{q} \geq 0 \,. \tag{32}$$

Multiplying both sides with the debt coefficient, we get

$$U''(\tilde{c}_2)\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi} \geq U''(c^*)\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi} \quad \text{if } \tilde{q} \geq 0 \,. \tag{33}$$

Taking expectations on both sides yields

$$E\left\{U''(\tilde{c}_2)\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi}\right\} \geq U''(c^*)E\left\{\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi}\right\} \quad \text{if } \tilde{q} \geq 0 \,. \tag{34}$$

To show that the left hand side is positive (non-negative), which implies that first period consumption decreases when debt increases, it is sufficient to show that the right hand side is positive (non-negative). The first factor is obviously negative if we assume that individuals are risk averse, while the second factor in principle could have any sign. At this stage we need to know what the expected debt coefficient is to determine the sign. Clearly, if the expected debt coefficient is non-positive, the right hand side will be non-negative, and thus the left hand side must be non-negative. One example where we know that the expected debt coefficient is non-positive, from the above discussion on the wealth effect, is when the expected birth rate is zero.[2]

[2] An alternative way of deriving a condition on the expected debt coefficient that generates a negative derivative between first period consumption and public debt is as follows. Define $\theta \equiv \tilde{q}(1+r+\pi^*)/(1+\tilde{q}-\pi)$. We know that $Cov(U''(\tilde{c}_2)\theta) = E[U''(\tilde{c}_2)\theta] - E[U''(\tilde{c}_2)]E[\theta] > 0$ if $U''' > 0$. Furthermore, to show that $E[U''(\tilde{c}_2)\theta] > 0$, which is the condition for a negative derivative between consumption and debt, we note that this will always be true from the covariance expression if $E[\theta] \leq 0$, since then the last term is negative, and thus the first term has to be positive to make the covariance positive.

This implies that the sign of the derivative between first period consumption and debt is negative when the expected birth rate is zero, so that if debt increases, present consumption decreases. In other words, if we modify Buiter's [1988] zero birth rate condition to a condition that states that the *expected* birth rate is zero, we get precautionary savings in response to increased debt, instead of debt neutrality, which would be the result in the Yaari-Blanchard model with deterministic population size.

The above condition of a zero expected birth rate is sufficient but not necessary to obtain a negative derivative between present consumption and public debt, and below we will illustrate that we can obtain a negative derivative between public debt and present consumption also when we have a constant expected population. In Section 2, it was shown that with deterministic population size the derivative is positive, i.e. with increased public debt first period consumption rises. This is also the result obtained by Blanchard [1985] in a continuous time model with a constant population. There are two reasons why we can obtain the opposite result with a stochastic population size. First, the debt coefficient is concave here, and secondly, debt creation affects the distribution ("spread") of the future uncertain income, and thus induces precautionary savings if households are prudent.

To create an example where we have a constant expected population and still a negative derivative between debt and first period consumption, we assume the following two-valued probability distribution for the birth rate

$$\tilde{q} = \begin{cases} -x & \text{with probability } 1-\lambda \\ ((1-\lambda)x+\pi)/\lambda & \text{with probability } \lambda \end{cases}, \tag{35}$$

which implies that $E[\tilde{q}] = \pi$, and $Var[\tilde{q}] = (1-\lambda)(x+\pi)^2/\lambda$, so that the variance increases with smaller values of λ and with larger values of x and π. If we use this probability distribution to evaluate the derivative, we get

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S}\left\{-U''(c_{2l})\frac{(1-\lambda)x(1+r+\pi^*)}{1-x-\pi} - U''(c_{2h})\frac{-(x(1-\lambda)+\pi)(1+r+\pi^*)}{1+(x(1-\lambda)+\pi)/\lambda-\pi}\right\}, \tag{36}$$

where c_{2l} is the consumption in period two when we have a negative realization of the birth rate, and c_{2h} is when we have a positive realization of the birth rate, and thus $-U''(c_{2l}) > -U''(c_{2h})$ if $U'''(\cdot) > 0$, i.e. households are prudent. We also note that the first factor is negative, since $S < 0$. To obtain a negative derivative between first period consumption and debt, the factor in braces must be positive. A sufficient, but not necessary, condition for this can be obtained by assuming that $-U''(c_{2l}) = -U''(c_{2h})$, which yields the following condition for a negative derivative between debt and first period consumption

$$\frac{(1-\lambda)x(1+r+\pi^*)}{1-x-\pi} - \frac{(x(1-\lambda)+\pi)(1+r+\pi^*)}{1+(x(1-\lambda)+\pi)/\lambda-\pi} > 0, \tag{37}$$

which is simply to say that the expected debt coefficient is negative.

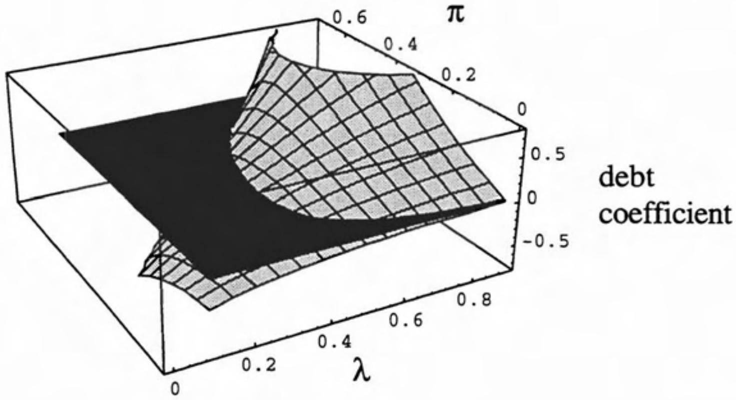

Figure 2. *Sufficient condition on the death rate and spread parameter of the birth rate distribution to determine the sign of the derivative between first period consumption and debt. The lighter surface below the darker implies a negative expected debt coefficient and thus a negative derivative, i.e. higher debt reduces first period consumption.*

In Figure 2, the expected debt coefficient (lighter surface) is plotted together with the surface representing a zero debt coefficient (darker surface). Where the lighter surface is below the darker, the expected debt coefficient is negative, and thus the above condition is fulfilled. The implication of this is again that we have a sufficient, but not necessary, condition for making first period consumption decrease with higher public debt. This will in general be the case for a small value of λ, which implies a high variance in the birth rate distribution. Furthermore, we note that the condition is more easily met when the death rate is either very high or very low.

As we have noted, the above condition is actually a condition that states that the expected debt coefficient is negative, so in a way we have at this stage analyzed the wealth aspect of shifting taxes over time, and not really the effect that is due to the perceived uncertainty. The precautionary savings effect, on the other hand, depends on the preferences that we assume, and we note that the above condition did not involve the magnitude of the precautionary savings motive, i.e. the size of the coefficient of relative prudence. However, since $-U''(c_{2l}) > -U''(c_{2h})$ if consumers are prudent, we know that we can set the parameters such that the debt coefficient is zero, and still make (36) negative, and with a stronger precautionary savings effect, the debt coefficient can be positive (or the spread of the birth rate distribution smaller), without making households consume more in the first period when the level of public debt is increased. This is obviously a result that depends on assumptions of the degree of prudence rather than solely the wealth effect from debt policy. We have thus demonstrated that first period consumption can decrease when debt is increased also in the case where the expected population is constant, i.e. the expected value version of the Blanchard [1985] case, without relying on a negative wealth effect.

4. SUMMARY AND CONCLUSIONS

The paper first discussed the effects of debt creation when the population size is deterministic. In general, increased debt will generate an increased consumption in the first period, due to a wealth effect for the agents alive in that period. However, for negative "birth" rates, i.e. emigration, the outcome is reversed, so that first period consumption is reduced in response to an increased debt.

In the second part of the paper, a model of stochastic population size was formulated to show that present consumption can *decrease* in response to public debt creation. To start with, we showed that if we use the assumption that the expected birth rate is zero, i.e. the expected value analogue of Buiter's [1990] condition of zero birth rate, we now get the result that increased debt reduces first period consumption, although Buiter's conclusion is that of debt neutrality. We also showed that the precautionary savings motive can make first period consumption decrease when debt is increased, even when the expected birth rate is equal to the death rate, if we assume that the variance in the birth rate is high. Furthermore, we can note that when the derivative is actually positive, we still have a precautionary savings effect, which reduces the consumption increase in the first period when debt is increased compared to the deterministic case. In the stochastic case, most of a (potential) positive wealth effect from debt will be consumed in the second period, which can be viewed as a modification of the deterministic case, originally analyzed in Blanchard [1985], where the wealth effect is evenly consumed over all periods. We also noted that if we allow for bad enough realizations of the birth rate, the expected wealth effect itself will become negative due to the concavity of the debt coefficient, although the population is expected to be constant. In other words, if a stabilization policy aims at increasing present private consumption by debt creation, this paper not only suggests that the effect on present consumption will be substantially smaller than the effect predicted by a deterministic model, but also that the effect can be negative.

The analysis of stochastic birth rates used the assumption that the "birth" rate could be negative. This is motivated in the paper by allowing a fraction of the population to emigrate in the second period. In the present analysis, the fraction that leaves the country is not dependent on the level of public debt. An area of future research is to connect the debt level to the emigration rate, by investigating a two-country model where people are free to move between countries. It seems quite plausible that a high level of public debt in one country will make people more inclined to emigrate to the other country in the second period (i.e. we get a higher probability of negative "birth" rates), since they would then avoid the higher tax in the future. However, if the public sector in the high debt country engages in activities that improve the income in the second period (or rather consumption possibilities), for instance by investing in pension funds or infra-structure, we will have an effect that counteracts the high tax. In such a model, we can discuss both the uncertainty effects of debt policy and "fiscal federalism" questions. An interesting question is to connect this analysis to the opening of borders within the EU.

Furthermore, in the present model, the interest rate was given from the world market. If we instead would like to determine the return on savings endogenously from the marginal product of capital, we note that this, in general, depends on the size of the labor force. In this model, it would be natural to interpret the young generation, i.e. new entrants, as the labor force in the second period. How would that affect the present analysis? If we use the standard assumption of decreasing marginal product on capital for a given number of workers, we realize that for a small realized birth rate, the labor force will be smaller, and thus the return on savings is reduced. At the same time, the per capita tax will be relatively high. In other words, in this case, the second period tax will not be an insurance against market risk, but instead increase the variation in second period net income.

Finally, the present analysis can be viewed as an aggregate tax risk analogue of Becker's [1995] analysis of individual tax risk. The essential feature of both these papers is that taxes in the future are stochastic, and if individuals display a precautionary savings motive, this can make present private consumption decrease with increased public debt, i.e. when the present taxes are shifted to the future, households engage in precautionary savings. This stochastic feature of debt policy is often ignored, but in case fiscal policy aims at affecting aggregate demand in the economy, it is vital to understand that households do not only respond to changes in present values but also to changes in the risk they perceive.

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