PROFITS AND MARKET ADJUSTMENT

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Peter Englund

PROFITS AND MARKET ADJUSTMENT

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1. SUMMARY AND CONCLUSIONS

In a market economy the rate of return on investment plays a decisive role for resource allocation. In one of the most thoroughly elaborated parts of economic theory it is shown how resources will be allocated in a way that makes the marginal rate of return equal on all types of investments. It is further shown how such a situation fulfills the Pareto-criterion for efficiency. This theory is based on the assumption that there is equilibrium in the sense that all firms maximize profits subject to prices which equal demand and supply for all products.

The assumption of equilibrium is probably far from being fulfilled in reality. Equilibrium theory is studied, however, because it is believed that there is a tendency in the real economy to approach equilibrium. This belief is strengthened by the analysis of adjustment processes which are shown to lead towards equilibrium. The treatment of the dynamic adjustment is normally fairly rudimentary, however, and the main analytical interest is attached to the equilibrium properties of the model.

This study makes an attempt to contribute to the theory of market adjustment in a world which is dynamic in the sense that demand and technological conditions are changing continuously. The interest will not solely be focused on long-run properties of adjustment processes, but equally on short-run properties.

POINTS OF DEPARTURE

Chapter 2 presents some points of departure of the study. One is the empirical observation that rates of profit in reality differ quite widely across firms and across industries. That this is so at any particular time is of course nothing remarkable. The economic system is continuously subject to shocks, which will affect relative rates of return. But central economic theory teaches - under certain assumptions about technology, basically constant returns to scale - that there is a long run tendency towards equalization of rates of return. The empirical evidence indicates, however, that even in the long run do there remain considerable profitability differences. One among many possible interpretations of this is that the dynamic adjustment mechanisms do not work as standard theory presumes. This is one of the main reasons why I am interested in the formal modelling of an adjustment process.

A standard explanation of long-run profits above the normal level is monopoly. We will make a brief review of the empirical literature concerned with testing the hypothesis that the rate of return is positively correlated with the concentration ratio or other measures of the degree of monopoly. A basic problem with the theoretical framework underlying these studies is that productivity is assumed to be constant across all firms. In any real market economy, however, one of the most important disequilibrating factors is the introduction of new techniques of production. These are typically unevenly distributed across firms and industries; innovations create profitability differences. But causality runs in the other direction as well. High profits enable firms to undertake more R & D and, hence, to cut costs faster. On the other hand, high profits may lead to the growth of slack. This notion, that the rate of productivity change is an integral part of dynamic adjustment processes, has an origin in the work of Joseph Schumpeter. His theoretical perspective is one of the points of departure of this study.

The Schumpeterian view has, however, been little used for formal analysis. A reason for this may be that it has proved very difficult to formalize in terms of maximizing behaviour. This supposition will lead to a brief methodological discussion of the role of maximizing assumptions in economic models, profit-maximization in particular. There are three types of arguments that can be advanced in the favour of using profit maximizing assumptions: (i) the assumption gives a reasonable approximate description of actual behaviour, (ii) only firms behaving *as if* they were profit-maximizers will survive in the long run, (iii) the assumption serves to simplify the modelling of a complicated problem. I will claim that the first argument has been refuted by facts, and that the second argument only is valid under restrictive assumptions. The third argument is quite strong in many cases. I think, however, that it can often be reversed in cases where dynamic problems are involved; non-maximizing assumptions may lead to a model that is easier to handle for analytical purposes.

The use of non-maximizing assumptions of firm behaviour is associated with the behavioural theory of the firm due to Simon, Cyert, March and others. This type of theory has recently been revived by the work of Richard Nelson and Sidney Winter on what they label "evolutionary theory". The model we are going to study can be seen as a special case within this general class of theories.

Ours is a model of a dynamic adjustment process, where the rate of profit plays a central role. It has two functions. One function is to stimulate expansion. High profits lead to an increased rate of entry of new firms into the industry, and stimulate expansion of existing firms. Profits also have a function in affecting the rate of productivity development. There are two mechanisms at work here. One is that, in a world with imperfect capital markets, high profits mean that firms can afford to undertake more research and development. Another mechanism is that low profits tend to force firms to reduce slack.

By this we have presented the two basic behavioural assumptions underlying the adjustment process analyzed in this study. They can be formalized

$$\frac{\dot{q}}{q} = \alpha_q \pi + \delta_q \tag{1.1}$$

$$\frac{\dot{c}}{c} = \alpha_c \pi - \delta_c \tag{1.2}$$

where q is quantity produced, c is unit costs and π is the rate of profit. We assume that the expansion reaction coefficient α_q is positive, whereas the cost reaction coefficient α_c will be positive if the slack reduction effect dominates, but negative if the R & D effect is strongest. By adding a demand function to the behavioural assumptions we get a model of a market adjustment process. We will assume that price is continuously adjusted so as to equal demand with supply. By assuming the demand curve to have constant elasticity we have

$$\frac{\dot{p}}{p} = - \gamma \frac{\dot{q}}{q} + \delta_p \tag{1.3}$$

where p is unit price and γ is the inverse demand elasticity. For analytical convenience the rate of profit is defined as a sales margin

$$\pi = \frac{p - c}{c} \tag{1.4}$$

THEORETICAL BACKGROUNDS TO THE BEHAVIOURAL ASSUMPTIONS

Chapter 3 is devoted to analyzing some dynamic maximization problems that are not inconsistent with the assumptions of our model. That the behavioural assumptions are formulated in non-maximizing terms does of course not necessarily mean that they are in conflict with maximizing behaviour. Indeed, most behaviour can be seen as the outcome of the maximization of some utility function only the relevant restrictions are chosen.

The main purpose of the chapter is to show the effect on the individual firm's rates of expansion and cost reduction of variations in the exogenously given product price. This will provide us with interpretations of the reaction coefficients α_q and α_c, i.e. we will see what background factors that may affect the size of these coefficients.

We will study three different model structures. One is based on the assumption that there are costs associated with the adjustment of some factor of production. The second model is based on the assumption that the interest rate paid on loans is a function of the debt/equity ratio of the firm. The third is a model of a management-run company, where the attitudes of the management display disutility with respect to activities conducive to cost reduction.

Costs of adjustment may be of different types. One type is due to the fact that factor markets are monopsonistic. This means that the more new machines are purchased (or workers are hired) per unit of time, the higher price will have to be paid. Another type of adjustment cost arises when resources have to be taken from current production in order to train new workers or to install new machinery. The effects of a price change will be different depending on which type of cost is studied. In the former case we can say unambiguously that a higher price will lead to a higher rate of expansion. In the latter case, however, the short-run effect may be a decrease in production. In the long run, however, the effect will also in this case be to speed up expansion.

The dynamic behaviour of firms is largely determined by conditions in the capital market. The interest rate on borrowed capital is normally an increasing function of the debt/

/equity ratio of the firm. This relation will be taken into account by the owners when making investment decisions. We will analyze this within two models. First, we will take the cost-of-adjustment model of the previous section and introduce borrowing into it. It will then be seen that the properties of the interest rate function affect the optimal rate of investment, but they have no effect on the impact of a changed product price on investment, i.e. they have no effect on the expansion reaction coefficient α_q. Second, we will study a model of a management-run company where the management's preferences display diminishing marginal utility with respect to profits. In this case there will be an effect on α_q. To the extent that capital market conditions have an effect on fixed investment they also have an effect on investment in research and development. This means that the models of financial behaviour also can be interpreted in terms of the cost reaction coefficient α_c.

In the third, and final, section of the chapter we will analyze a model designed to elucidate how high profits may lead to growth of slack. This is done in a model of a management-run company, where the maximand is a utility function in profits and an activity labelled "effort". The input of effort leads to cost reduction and has no costs in monetary terms, but gives disutility to the management. This implies that a change in the product price will have an effect on the product price. The direction of this effect will depend, among other things, on the curvature of the utility function in profits. If the marginal utility of profits is constant, a price increase will increase the rate of cost reduction, whereas the opposite effect would follow if the utility function is strongly concave in profits.

The models analyzed in Chapter 3 treat the optimal behaviour of a firm, where the market price is assumed exogenous. Despite the fact that the market interaction between firm behaviour and demand is not treated, the analysis becomes, at times, quite involved. In order to come to grips with the market adjustment process it seems that one has to resort to less sophisticated behavioural assumptions like those we have made above in (1.1) and (1.2).

THE BASIC MODEL

Chapter 4 is devoted to a detailed analysis of the market adjustment model (1.1)-(1.4) above. The linearity of the system of differential equations means that it is possible to find an explicit solution. This expresses the values of the endogenous variables - quantity produced, unit cost, price and rate of profit - as functions of the initial values of these variables, the parameters of the model, and time.

The working of the model can be illustrated by a series of simple examples. First, consider the case when the demand curve and the level of unit cost are fixed ($\alpha_c=\delta_c=\delta_p=0$) and the rate of expansion is an increasing function of the profit margin ($\alpha_q>0$, $\delta_q=0$). This is the standard Marshallian adjustment mechanism. It is easily seen that it converges towards a zero-profits equilibrium in the long run.

Second, regard a case where demand is continuously increasing, i.e. the demand curve is gradually pushed outwards ($\delta_p>0$). In such a case there will also be convergence towards a long-run equilibrium, but this will imply positive profits unless there is expansion of production when the rate of profit is zero ($\delta_q>0$).

Third, regard the case when profits also affect the rate of cost reduction ($\alpha_q\neq 0$). The effects will then depend on the sign of the reaction parameter α_c. If the slack-reduction aspect dominates ($\alpha_c>0$), a rate of profit above the equilibrium level will be eliminated by a combination of an increase in unit costs and a fall in price due to expansion of production. In the opposite case when the R & D effect dominates ($\alpha_c<0$), one can no longer be sure that rates of profit above equilibrium will be eliminated at all. For if the effect on cost reduction is strong, the result may be that unit costs fall faster than price, in which case an explosive movement away from equilibrium will start.

In general, we will see that, assuming the process to be stable, the rate of convergence towards equilibrium will depend on the two adjustment coefficients α_c and α_q. A main lesson is, however, that these coefficients also affect the properties of the equilibrium of the model. This will be so in all cases, except when the equilibrium rate of profit is zero. This will come about under two quite different types of circumstances. One is when adjustment is very fast (α_q or α_c have high positive values). Another is when the trend parameters (δ_c, δ_q and δ_p) combine in a way that makes the rate of price change equal the rate of unit cost change when the rate of profit is zero. Neither of these cases seem on a priori grounds particularly realistic.

The main part of Chapter 4 is devoted to a detailed comparative dynamic analysis of the effects of variations of the parameters of the model. Some of these parameter shifts will be interpreted in terms of tax and subsidy changes. When the R & D effect dominates ($\alpha_c<0$) there will emerge some seemingly paradoxical results; an investment subsidy leads to slower expansion in the long run, a price increase leads to a lower price level in the long run, etc.

THE SENSITIVITY OF THE RESULTS TO ALTERNATIVE MODEL SPECIFICATIONS

In *Chapter 5* it will be studied whether the main conclusions from the analysis of Chapter 4 still hold if the model is respecified in alternative more realistic ways.

The first modification is the formulation of a model of many firms, which differ with respect to parameter values. A main question is whether there is any tendency for firms of a certain type, i.e. firms characterized by certain reaction coefficients, to expand at the expense of others. If this were so, it would strengthen the case for analyzing a model where all firms are equal, like that of Chapter 4. The results are not favourable to this idea, however. What types of firms survive in the long run depends on whether the trend parameters imply positive or negative equilibrium profits.

The second modification is the analysis of a non-linear model. When the rate of cost reduction is a linear function of the rate of profit, as in Chapter 4, there is no bound to the rate of cost reduction possible only profits take on a value that is extreme enough. Here it is instead assumed that there are upper and lower limits to the rates of cost reduction possible. This means that there will be a possibility of multiple equilibria. Which of the equilibria will be approached depends on initial conditions, and a marginal change in an exogenous variable may bring about a large change of the corresponding equilibrium.

A third modification is to let the rate of cost reduction depend on the level of production, apart from on the rate of profit. This implies that there will in general be two locally stable equilibria. One implies expansion of production and will be approached if initial size and/or initial rate of profit is high enough. The other implies contraction of production and will be approached in the opposite case.

The fourth modification is to let behaviour depend on expectations of the discounted value of future profits. If expectations are formed adaptively, this implies that there will be oscillations along the adjustment path. The model will still be locally stable under the same conditions as those holding for the basic model of Chapter 4.

EMPIRICAL EVIDENCE

In *Chapter 6* some empirical evidence is presented to study the relevance of the basic model formulation.

One of the points of departure stated in Chapter 2 was the observation that differences in rates of return seem to be persistent. This assertion is analyzed in more detail in this chapter, both by a couple of published studies and by my own analysis of Swedish data. Studying cross-sections of firms and industries we will see that there is a significant correlation between the rates of return to capital for two different years even if these are distant in time from each other. Actually, it seems that the correlation coefficient, rather than tending towards zero, approaches a positive limit value in the long run.

The rest of Chapter 6 deals with the two behavioural assumptions. As regards the relation between expansion and profits there is a multitude of studies to draw information from. Both studies of investment behaviour and studies of the expansion of turnover indicate that a likely value of α_q may lie in the range 0.2-1.0. This is based on studies across firms. To this figure should be added the effect on the rate of entry of new firms. Here much less is known, largely due to the lack of data. It appears, however, that entrance is of minor importance compared with expansion of existing firms.

The relation between cost reduction and profits is much less studied than that between expansion and profits. We will discuss two types of studies from the literature. One is studies of the impact of profits on research and development. The evidence is somewhat conflicting. But there appears to be a positive correlation between the rate of profit and R & D spending. This would indicate that $\alpha_c < 0$. In another type of study one tries to estimate separate rates of return on investment financed from different sources. From these studies it seems that the rate of return on retained earnings is considerably lower than that on externally financed investment. This may indicate that $\alpha_c > 0$. Thus it is easy to find evidence with different implications for the sign of α_c. In principle it should be possible to resolve this conflict by making a direct estimation of the rate of unit cost change on profits. I have done this on Swedish firm data and got a significantly negative regression coefficient, i.e. the R & D effect seems to dominate.

My study has its roots in a theoretical tradition in economics which goes back to the works of Alfred Marshall and Joseph Schumpeter. This is a tradition which emphasizes the analysis of dynamic processes rather than equilibrium situations. Another root is the behavioural theory of the firm due to Herbert Simon and others. This emphasizes that firms should be regarded as operating according to fixed decision rules that are only changed gradually. These should be taken as given and observable rather than being deduced from profit maximization.

Though these theoretical traditions have had general appeal to many economists, they have remained outside the core of economic theory and they have only been formalized very incompletely. The recent work by Nelson and Winter on "evolutionary theory" suggests that much headway can be made by simulation methods. This study represents an attempt to formulate a model of a similar type that is so simple that it can be treated analytically. I hope that I will convince some readers that this leads up a path that is well worth treading.

2. POINTS OF DEPARTURE

> There is no more important proposition in economic theory than that, under competition, the rate of return on investment tends toward equality in all industries. Entrepreneurs will seek to leave relatively profitable industries, and with competition there will be neither public nor private barriers to these movements. This mobility of capital is crucial to the efficiency and growth of the economy: in a world of unending change in types of products that consumers and businesses and governments desire, in methods of producing given products, and in the relative availabilities of various resources - in such a world the immobility of resources would lead to catastrophic inefficiency.
>
> (George J. Stigler, Capital and Rates of Return in Manufacturing Industries, Princeton 1963, p. 54.)

Under competition the marginal rates of profit tend to become equalized across industries. George Stigler is certainly not unique in ranking this among the "facts" that are rooted firmly in most economists' minds. In micro-economic theory the equilibrium proposition that marginal rates of profit *are* equal is central. And this theory frequently lies, explicitly or implicitly, behind policy recommendations made by economists. It is then something of a paradox that the *process of competition* which presumably leads to this equalization of rates of profit plays such a peripheral role in today's economics.

The paradigm of general competitive equilibrium analysis leaves the competitive process out of the picture completely. Behind the scene of the models firms are entering and leaving industries, consumers are searching for good buys and firms are adapting their price offers to attract customers. But this is taking place in some sub-model only aimed at showing that the variables, as time goes to infinity, will approach their equilibrium values. The questions asked and answered by the theory solely relate to what is happening on stage, i.e. in equilibrium.

This is of course a natural theoretical simplification which in many cases is analytically very fruitful. It is striking however how completely it has come to dominate formal modelling in economics. In the verbal version of microeconomic theory that is expounded to undergraduates and laymen, on the other hand, the dynamic processes of competition often play a more central role. In this verbal tradition, which goes back to Marshall, it is stressed how the rate of profit plays a dual role. On the one hand it is a *driving force* for change; differences in rates of return tend to lead to reallocation of resources. On the other hand it is the *outcome* of this dynamic process. In the long run it is presumed that there is a tendency for differences in rates of profit to be eliminated.

In the Marshallian tradition differences in the rate of profit are seen as a driving force mainly for quantity adjustment. In reality they perform another equally important function in that they affect the rate of productivity change. There are two main mechanisms here. First, low profits force firms to reduce slack and search for new technical solutions. Second, high profits provide firms with the financial means to increase their expenditures on research and development. These mechanisms imply that productivity change can largely be understood in disequilibrium terms. This view can be said to originate in the work of Schumpeter.

The central purpose of this study is to develop a formal model of a market adjustment process where profits is the central driving force for change both of production and productivity. There are several reasons to be interested in such a model. A main question will be whether this dynamic process leads to an equilibrium where the rates of profit are equal in all industries. We will see that this is not so in general. Equally important questions concern the general dynamic properties of the adjustment process. Is it stable? Does it converge fast or slowly? Are there oscillations on the way towards equilibrium? Answers to these questions are important in assessing the dynamic efficiency of a market economy. They are also important for public policy purposes. Is e.g. a profits tax neutral with respect to resource allocation and technical progress? What are the effects of a sudden demand increase? What will be the impact of an investment subsidy?

In the next section of this chapter we will take a brief look at the empirical evidence on profitability differences between industries and concentrate on to what extent these seem to be persistent. There are several possible explanations of the persistence of differences in rates of profit. We will look somewhat closer at one of these, that high profits are due to the exercise of monopoly power. There are two reasons for concentrating on this. First, it is the explanation that has been subject to by far the largest body of empirical studies. Second, the discussion and critique of the theoretical paradigm behind these studies will give us a background to our own model.

As an alternative to this paradigm we will then discuss the model structure associated with Schumpeter. This has, however, been little used for formal analysis. The reason for this, I will claim, is to a large extent that it is very difficult to formalize it in maximizing terms. For this reason we will discuss a theoretical framework, due to Nelson and Winter, where behaviour is modelled in non-maximizing terms. Our own model, which will be briefly sketched at the end of this chapter, can be seen as a highly specialized case within this framework. This model will be the main object of this study.

DIFFERENCES IN RATES OF PROFIT, THE EMPIRICAL PICTURE

By the rate of profit we will throughout this study mean a net rate of return on total capital employed. This is not an obvious choice. In many cases the rate of return on equity, perhaps together with the debt/equity ratio, is probably a more important determinant of firm behaviour. To keep the basic model simple, we have however chosen not to discuss the capital structure of the firm.[1]

Available statistics show that the rates of profit, defined in this way, do in fact differ between industries. In the tables based on the Swedish Enterprise Statistics in Appendix B we see that over the period 1953-1976 the yearly average across all industries, in nominal terms, has ranged from highs around 10 % for some years in the 1950's and the boom-year of 1974 to lows around 5 % in the 1970's (there are as yet no official figures for the post-war bottom year of 1977). There has been a considerable spread around these averages; the standard deviation across the industries has fluctuated between highs of 4 and 5 % for the years with highest averages and lows around 2 %.[2]

Data from other countries give similar evidence. George Stigler shows[3] that the average rate of return across U.S. manufacturing industries varied between 3 and 11 % over the twenty years period 1938-1957, whereas the standard deviation fluctuated between 1.9 and 4.3 %.

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[1] There is an exception to this in the model of financial behaviour of the firm in Chapter 3.

[2] These standard deviations are calculated around the unweighted averages of the industries presented in Tables 1 and 2 of Appendix B.

[3] Stigler (1963) p. 37.

There is of course nothing very remarkable in the observation that profits differ widely between industries during any single year. Unexpected events will occur in the economic environment all the time and one year is presumably much too short a time for complete adjustment. Indeed, it should probably be regarded as important that profits are allowed to vary, because this provides the necessary signals for reallocation of resources.

However, the empirical evidence again shows that even over the long term there is a considerable, though somewhat lower, dispersion. The tables for Sweden of Appendix B show that the standard deviation around the industry mean over the whole period 1953-1968 is 2.7 %. This is not much lower than the average of the yearly standard deviations, which is 3.5 %. For the years 1969-1976 the corresponding figures are 1.8 % and 2.7 %.[1]

For the United States Stigler calculates standard deviations of 1.6 % for 1938-1947 and 2.1 % for 1947-1956 across his population of manufacturing industries.[2] For England a study by Whittington gives a standard deviation of 2.4 % across 21 manufacturing industries for the period 1948-1960.[3]

Whereas it may be claimed that the periods considered are not long enough to allow profits to equalize fully, it seems clear that these observations call for an explanation. The hypothesis that has attracted most attention is that high profits are due to the exercise of monopoly power, and we will now consider some of the evidence brought to bear on this hypothesis.

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[1] In Chapter 6 we will discuss this evidence somewhat closer in terms of correlation coefficients between rates of return in year t and year t+x. It will be seen that there is a significant correlation even when x is more than ten years.

[2] Stigler (1963) p. 58.

[3] My calculations based on Table 2.3, p. 24 in Whittington (1971).

PROFITS AND THE DEGREE OF CONCENTRATION

That a monopoly position enables a firm to raise its price above its costs and thereby reap extra profits is an observation that goes back at least to Adam Smith.

A main problem associated with empirical tests of the monopoly explanation of profits is that "monopoly power" is not an easily observable variable. The classical proxy variable used has been concentration, normally measured either by the four-firm concentration ratio or the Herfindahl index.[1] The first such study was Bain (1951)[2] and a fairly recent survey, Weiss (1971), lists 31 subsequent studies testing basically the same hypothesis. With one prominent exception, the aforementioned study by Stigler (1963), these studies all conclude that profits are positively related to concentration. And Weiss states in concluding his survey that he thinks "that practically all observers are now convinced that there is something to the traditional hypothesis".[3] But the relationship found is typically quite weak, with an average R^2 ranging around 0.1-0.2. So there certainly remains uncertainty as to how much and exactly what there is to "the traditional hypothesis".

The type of study referred to above is explicitly concerned only with explaining the inter-industry variation in profitability. Still it is well established that there is a

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[1] This is defined as the sum of the squares of each firm's share of the total industry. The use of this index has a theoretical underpinning; in Cournot equilibrium the mark-up of price over variable costs is directly proportional to the Herfindahl index. See Cowling & Waterson (1976) and Hause (1977).

[2] The father of the research programme identifying causality as running from market structure to performance, such as profits, is Edward Mason. See Mason (1939). For the intellectual history of industrial organization studies see Phillips and Stevenson (1974).

[3] Weiss (1971) p. 371.

considerable intra-industry variation.[1] And from the point of view of oligopoly theory there is no reason to believe that all firms in the same industry should have the same rate of return; a large firm should be in a stronger bargaining position and should consequently be able to obtain a larger share of the monopoly profits pertaining to the group as a whole.

There are comparatively few studies using firm data instead of industry data. Hall and Weiss (1967) in what seems to be the first such study find a significantly positive effect of size as measured by turnover on profits along with the conventional positive effect of concentration. Subsequently a couple of studies have included the market share alongside with size and market concentration. Shepherd (1972) finds that profits rise quite significantly with market share, whereas the four-firm concentration ratio is insignificant and the asset size seems to affect profits negatively. Gale (1972) argues that there are more profits to be reaped from a high market share when an industry is concentrated; the more concentrated the industry is the easier is collusion, and the larger market share a firm has the more can it benefit from collusion relative to other firms in the industry. This hypothesis is tested using dummy variables, and it turns out that only in highly concentrated industries[2] does market share increase profits to a statistically significant degree.

All the results referred to so far are consistent with a situation where large firms in concentrated industries earn profits significantly above all other firms among which no significant differences are discernible. This seems indeed to be roughly consonant with the empirical picture. Demsetz (1973) has grouped firm rates of return data after the size

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[1] In order to be able to make this distinction we must know what constitutes an industry. In theory all firms in the same industry should produce the same homogeneous good. But the industry classification used in practice is made on a hotch-potch of criteria. This is a difficulty which should be kept in mind.

[2] Defined by a four-firm concentration ratio greater than the mean over all industries plus 0.15 standard deviation.

of the firm and the four-firm concentration ratio of the industry to which it belongs. It then turns out that the rate of return for large firms (assets above $50,000,000) belonging to highly concentrated industries (four-firm concentration ratio above 60 %) is almost twice that for any other class. The only notable difference between the other classes is that small firms (assets below $500,000) have rates of return a couple of percentage points below all other firms.[1] These data are certainly consonant with all the regression results referred to above; the rate of return is higher in concentrated industries as in the Bain-type studies, it is higher among firms with a large market share as found by Shepherd and finally it has exactly the interaction effect of Gale.

The interpretation of this empirical picture need, however, have little to do with the exercise of market power. An alternative view is outlined by Demsetz (1973) and Mancke (1974). Regard an initial hypothetical state of the world where all firms have the same market share. Assume then either, as Mancke does, that the outcome of the investments undertaken is uncertain or, as Demsetz does, that firms differ with respect to ability in a way that is not properly discounted into factor prices. This will then lead the firms to differ with respect to costs and profits. Assume further that there is a positive link between profitability and expansion. Then we will after a while observe a positive correlation between size (or market share) and profitability.

In the case outlined correlation has nothing whatever to do with market power. This does of course not preclude that market power once attained is also being used. Basically, however, causation has run in precisely the opposite direction to what is presumed in the Mason-Bain paradigm,

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[1] This may perhaps be explained by the exclusion of managerial compensation to the owner of the firm from the statistics.

from performance to structure. From the data commonly employed it is quite difficult to distinguish between these two interpretations. This at least is my conclusion from the interchange between Caves et al (1977) and Mancke (1977). To the extent that such a conclusion is warranted it appears unclear how much the type of econometric studies presented above has taught us about why profits differ between different firms and between different industries.

THE SCHUMPETERIAN VIEW

There is, however, a deeper problem involved. Assume that we know that in a certain sector of the economy high profits are associated with the exercise of monopoly power. Certainly we do know that this is so in some industries. Is it then very interesting to say that monopoly is the cause of high profits? And, if we answer that question in the affirmative, can we then conclude that the breaking down of the monopoly will lead to increased efficiency? The answer given to this question will depend on the economic model we have in the back of our minds. In a static general equilibrium world economists have long taught that the answer, with certain qualifications, is yes. However, as soon as it is admitted that the problem is essentially dynamic, the answer will be less clear-cut.

That monopolies are not everlasting has been explicitly recognized by economists at least since Schumpeter.[1] In his world entrepreneurs continuously try to create monopoly positions for themselves by means of technical, organizational, and market innovations. However, the monopoly position gained in that way will normally only be temporary, since other entrepreneurs will attempt to duplicate the successful innovations. In such a world it makes more sense to ascribe profits to the innovations made, which in turn reflect chance

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[1] His theory is first expounded in Schumpeter (1911).

or superior ability in the Mancke and Demsetz senses, than to ascribe it to monopoly power. To do this is of course not to deny that the temporary monopoly position will be exploited by withholding output. Although this Schumpeterian view of the world is on a general level accepted by most economists today, it has proved difficult to integrate into the central economic theory. More strikingly, its impact on the empirical industrial organization literature concerned with profitability differences has been virtually nil.

An exception to this sweeping statement is the work of Dahmén (1970). The main task of that study is to analyze the development of Swedish industrial firms in the interwar period. This is done mainly by classifying industries according to dynamic development processes into progressive, stagnating and regressive. Progressive industries are further divided between demand-pull and supply-push processes. One of the principal questions asked relates to the explanation of differences in profitability. This is the answer in succinct summary

> Rates of return normally differed significantly between the progressive industries, on the one hand, and stagnating and especially regressive industries on the other hand - particularly when regression in the latter case was a symptom of the negative development component .. The profits of pioneer firms were generally distinctly above average ... demand-pull processes normally yielded higher and more evenly distributed profits."[1]

Note how this emphasizes the explanation of profitability differences in terms of dynamic processes, not equilibrium positions as in the Mason-Bain paradigm.

Other examples of Schumpeterian influences may be found in different industry studies. Take the Phillips (1971) study

[1] Dahmén (1970) pp. 124-125.

of the aircraft industry as an example. A main characteristic of this industry has been the very large scale of research and development necessary. Success in the development of one generation of aircraft has given financial resources both for expansion and for development of a superior technology for the next generation. In this way concentration has been continuously growing. The increased concentration has enabled the surviving firms to exercise some monopoly power, which in turn, at least temporarily, has made it possible for some inferior firms to survive on the "competitive fringe" of the market. Here causation has run from basic technological conditions via the process of innovation to market structure, profitability and the rate of technical progress.

It is a basic premise of the present study that what we have here referred to as the Schumpeterian vision is essential for the understanding of inter-firm and inter-industry profitability differences. But all the empirical studies relating profits to concentration ratios, firm size, and market share are crippled in this respect in that they merely establish a correlation between two variables that are both endogenous for Schumpeter.

Related to the observation that the Schumpeterian perspective has been little used as a basis for empirical studies[1] is the observation that it is very poorly articulated formally. The main reason for this is no doubt that such an articulation has been judged to be very difficult. Let us for a moment consider the components that should be included in a "complete" model:

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[1] The reader may disagree with this judgement, having the studies on the determinants of research and development in mind. For a review see e.g. Mansfield (1968) and Kamien & Schwartz (1975). These studies tend, however, notoriously to overstress one aspect of Schumpeter's model such as testing the hypothesis of positive correlation between size and R & D. Fisher & Temin (1973) show that these tests seldom test what they claim to test. Moreover Nelson (1975) points out that what they set out to test does not have much to do with Schumpeter.

(a) each firm decides on production techniques to use and quantities to produce, on prices to charge, and on the search for new production techniques.

(b) these choices have to be made taking into account knowledge of present prices and production techniques available and expectations about future prices, the possibilities of finding other production techniques and the behaviour of other firms.

(c) the development of the system can then be described as the interplay between firm behaviour according to (a) and (b) and consumer behaviour.

That the formal modelling of such a system is a huge task goes without saying. Using the theoretical style which dominates today's economic theory, it would involve the following steps. First behaviour would be set up as a dynamic optimization problem under uncertainty. One would be interested not only in the decisions actually taken, such as the search rules followed, but also in the outcome of these decisions in the form of the distribution of new techniques found, etc. Further one would have to assume some expectations formation mechanism. Finally, account would have to be taken of the market interplay between these decisions. This seems to be what a formalization of Schumpeter should have to amount to, a complicated stochastic process.

The art of model-building consists of making the fruitful simplifications. Economic theorists have not been very successful with the Schumpeterian system in this respect. Instead, attention has been focused on parts of the system. Gaskins (1971) has studied the pricing strategy employed by a temporary monopolist facing potential competition from new entrants. Further there is a growing number of papers on the role of rivalry between firms in determining the rate of inventive activity and the introduction of new production techniques; see the survey by Kamien & Schwartz (1975).[1] Both these areas of research are certainly interesting, but since market structure is again treated as exogenous they fall short of catching the essence of the Schumpeterian vision. They treat parts of it in depth rather than simplifying far enough to be able to study the whole system.

THE ROLE OF MAXIMIZING ASSUMPTIONS

I will now put forward a hypothesis that the lack of progress with the attempts to formalize the Schumpeterian system in part stems from the inclination towards formulating firm behaviour in maximizing terms.

Often in economics one is confronted with the attitude that all behaviour can be expressed as the result of rational choice over a well defined choice set. It is then known that this, under fairly unrestrictive assumptions, can be represented by the maximization of a utility function. Maximization is according to this view seen not as a deliberate simplification, but as the only way to describe behaviour. The search for "microfoundations" of different theories typically takes the view that behaviour "should" be described in maximizing terms. For instance one is not content with a behavioural assumption stating that a firm encountering excess demand will tend to raise the price of its product, but one seeks to express this price change as the outcome of profit maximization under uncertainty.[2]

Introspection can make anybody hesitant about the transitivity of one's revealed preferences and hence of the possibility of expressing one's behaviour in maximizing terms. This holds especially when it comes to behaviour under uncertainty, as was shown by the famous tests made by Allais (1953, a, b), where a number of prominent statisticians and economists exposed intransitive preferences. And a recent

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[1] For more recent work see Kamien & Schwartz (1978 a, b).

[2] See e.g. Barro (1972) and Kirman (1975) for two quite different approaches to this problem with the same basic attitude.

Nobel laureate, Simon (1978) p. 32, summarizes the empirical knowledge as of today by stating that "the conclusion seems unavoidable that the SEU (maximization of subjective expected utility) does not provide a good prediction - not even a good approximation - of actual behavior".[1]

Against such evidence it may be claimed that it is all an information problem. Basically people are consistent; it is only that they have to take into account the cost of being consistent. While this argument has some force, it also shows drastically that for any practical application the "real" maximization problem must be simplified quite far. In that situation, when one has to simplify, it is far from evident that there are always advantages to modelling behaviour in maximizing terms.

On the other hand, it is of course obvious that assumptions of maximization often mean a tremendous simplification. The equivalence theorems of general equilibrium theory would be completely impossible without maximization assumptions. Paul Samuelson is perhaps the most eloquent spokesman for the view that a maximization assumption can simplify the most complicated problem, in economics and elsewhere.[2]

Let us now go over from discussing in general terms to discussing the theory of the firm. First it should be noted that while individual behaviour may be reasonably consistent there is much less reason to believe that group behaviour is consistent. And firm decisions are largely made by groups of people.

During the history of economics there has been a sporadic debate between skeptics and adherents of maximization in general and profit-maximization in particular. The skeptics

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[1] A main reference for this conclusion of Simon's is Kahneman & Tversky (1973).

[2] See e.g. his Nobel Lecture, Samuelson (1970).

have presented evidence of seemingly gross deviations from maximization in the form of administrative pricing, extremely rough investment calculations, X-inefficiency, etc. The adherents have typically answered by showing that it is possible to find the right combination of maximand and constraints to explain the perceived empirical puzzle. By such testing of the conclusions of a model it is, however, quite impossible to discriminate between alternative behavioural assumptions capable of explaining the same empirical phenomena. If one observes that companies stick to the same price for a long time and then suddenly make a large price change, this can as well be described as "irrational" as "satisfying" or as "profit-maximizing subject to concave costs of adjustment".[1]

Another argument that has been put forward in favour of profit-maximization as a reasonable approximation to actual behaviour is that of natural selection. The basic reference is to Alchian (1950). The argument was subsequently championed by Friedman (1953). In very broad lines they say that maximizers will tend to expand faster than non-maximizers, and they will expand to such an extent that profits will be forced down below zero for the non-maximizers. Since the owners of these firms will not infuse new capital indefinitely into a firm that steadily incurs losses there will be a tendency for profit-maximizers to survive at the expense of non-maximizers.

The logic of the Alchian-Friedman proposition is penetrated by Winter (1964).[2] His general result can be interpreted to imply that it holds provided (a) there are some profit-maximizers in the market, (b) the influx of new non-maximizers is not important in comparison with the total,

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[1] The latter interpretation follows from the aforementioned article by Barro.

[2] See also Samuelson (1978) for a recent profession of skepticism about the possibilities of imposing the language of maximization onto problems of Darwinian survival.

(c) external conditions vary sufficiently to discriminate between the profit-maximizers and those who just happen to be efficient under the ruling conditions and (d) the profit-maximizers continue to be maximizers even after the others are out of the market. Whether these conditions are met in reality is open to question, but it does not seem to me that it gives a very compelling reason to model companies as profit-maximizers. This is the view of Koopmans (1957):

> Here a postulate about individual behaviour is made more plausible by reference to the adverse effect of, and hence penalty for, departures from the postulated behaviour... But if this is the basis for our belief in profit maximization, then we should postulate that basis itself and not the profit maximization which it implies in certain circumstances ... Such a change in the basis of economic analysis would seem to represent a gain in realism attributable to a concern with the directly perceived descriptive accuracy of the postulates. It would lead us to expect profit maximization to be most clearly exhibited in industries where entry is easiest and where the struggle for survival is keenest, and would present us with the further challenge to analyze what circumstances give to an industry that character. It would also prevent us, for purposes of explanatory theory, from getting bogged down in those refinements of profit maximization theory which endow the decision makers with analytical and computational abilities and assume them to have information-gathering opportunities such as are unlikely to exist or be applied in current practice. It seems that nothing is lost, and much may be gained, in thus broadening the postulational basis of economic theory.[1]

AN EVOLUTIONARY PERSPECTIVE

Such a "broadening of the postulational basis" has recently been attempted in a series of papers by Richard Nelson and Sidney Winter[2] on what they label evolutionary theory:

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[1] Koopmans (1957) pp. 140-141.

[2] See Winter (1975) p. 100 for an acknowledgement of the affinity with Koopman's views.

> The first major commitment of the evolutionary theory is to a "behavioural" approach to individual firms. The basic behavioural premise is that a firm at any time operates largely according to a set of decision rules that link a domain of environmental stimuli to a range of responses on the part of firms. While neoclassical theory would attempt to deduce these decision rules from maximization on the part of the firm, the behavioral theory simply takes them as given and observable.[1]

The consequence of this commitment is that it should always be possible to test the basic assumptions of any specific evolutionary model. This should be contrasted with the methodological view expounded by, among others, Friedman (1953) and Machlup (1967) according to whom it suffices to test the conclusions of a theory.[2]

A basic principle of the evolutionary perspective is that firms will at any time operate according to *decision rules* that are inherited from the past. By decision rules are meant production techniques used (or set of production techniques from which a choice is made plus a rule for making the choice), pricing rules, etc. At any time a firm will by using these rules make routine decisions about production, pricing, etc. But the rules will also be changed under certain conditions, which are specified by the model. This change of behaviour will be related to the fulfilment of the *goals* of the company. A main goal is normally assumed to be that of making profits, though this is not formulated in maximizing terms. The profit goal may be seen as one of several goals, alongside with e.g. liquidity and solidity, derived from a fundamental goal of surviving.

There is of course nothing about this that is inconsistent with maximization. But the decision rules are taken as basic postulates of the theory and are not justified on the ground of maximizing behaviour.

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[1] Nelson & Winter (1974) p. 891. The intellectual debt to the Carnegie School of Simon, Cyert, March and others is evident.

[2] For a discussion of these methodological issues see further Winter (1975).

One may distinguish three general components of behaviour: production technique, quantity produced and price charged. For the present purpose, as in most specific models by Nelson and Winter, the "competitive" assumption is made that pricing is out of control of the single firm.

The non-fulfilment of a goal will start a process of *search* over a vaguely defined set of different ways of doing things. The relation between goal fulfilment and search may be discontinuous as proposed in models of satisficing behaviour; if profits, or whatever the goal may be, are above a certain level there is no search for alternatives. However, there may just as well be a continuous relation between profitability and the intensity of search.

Decisions about *expansion* or contraction *of production* are also made against profitability considerations. The more profitable a firm is, given the technique it is currently employing, the faster will it expand. This also holds for potential entrants into the industry, which are assumed either to make profitability calculations based on some technique they know of or simply to regard, or form expectations of, the profitability of firms currently operating in the industry. These assumptions of course bear a close affinity to the Marshallian view of market dynamics, though this is couched in terms of the "representative firm".

> Let us call to mind the "representative firm", ... and ... assume that the normal supply price of any amount of that commodity may be taken to be its normal expenses of production ... That is, let us assume that this is the price the expectation of which will just suffice to maintain the existing aggregate amount of production; some firms meanwhile rising and increasing their output, and others falling and diminishing theirs; but the aggregate production remaining unchanged. A price higher than this would increase the growth of the rising firms, and slacken, though it might not arrest, the decay of the falling firms; with the net result of an increase in the aggregate production. On the other

> hand, a price lower than this would hasten the decay of the falling firms, and slacken the growth of the rising firms; and on the whole diminish production: and a rise or fall of price would affect in like manner though perhaps not in an equal degree those great joint-stock companies which often stagnate, but seldom die.[1]

The rules governing the change of *production techniques* can be more varied. On the one hand, the intensity of managerial search for better techniques will presumably fall when profits increase.[2] This assumption underlies Winter (1971). On the other hand the resources for R & D, i.e. organized search for better techniques, will increase with profits. This assumption underlies some of the other specific evolutionary models, see e.g. Nelson and Winter (1977 a, 1978). Which of these two mechanisms dominates is an open question, the answer to which will probably differ from case to case. Note also that other factors than profits such as size and financial position of the company may be important determinants of the search for new methods of production.

So far we have only been concerned with the intensity of search for new production methods. In a more sophisticated model there will also be mechanisms governing the direction of search, e.g. according to factor-saving bias. There is also an important intertemporal dimension. Perhaps low profits are likely to lead to search for cost reduction mainly in the short run at the expense of R & D investments with a long run impact, whereas the opposite holds for a high profit situation. Another aspect which might be incorporated within an evolutionary perspective is diffusion of innovations. Finally account should be taken of the role of new firms entering into the industry. In Schumpeter's theory they were seen as a major vehicle of introducing new technology.

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[1] Marshall (1949) pp. 285-286.

[2] This view was already that of Adam Smith. See Rosenberg (1974).

Summarizing the evolutionary perspective we can say that it attempts to model the evolution of an economy or part of it, as a dynamic process. In doing this it is in keeping with older traditions in economics.[1] In particular Austrian economists have always insisted that competition must be understood as a process.[2] The evolutionary theory brings together two strands of thinking on the dynamics of a competitive economy, the Marshallian view stressing how profit signals stimulate expansion and contraction of output and the Schumpeterian view stressing how profit signals alert the search for new production methods. It is important to note that in general these two perspectives are not only added to each other but are really interacting. For the Marshallian mechanism means that there will be a *selection* effect in that some firms expand at the expense of others. And, at least if all firms follow the same expansion behaviour, these will be the firms with the most efficient techniques of production.

In a couple of papers Nelson and Winter deal with the question whether standard propositions from neoclassical theory still hold within an evolutionary framework. Regard the effects of a parameter change. These can now be decomposed into three parts: (a) effects through given decision rules, (b) effects of changed decision rules, and (c) effects of a changed composition of decision rules due to the selection effect. Even if the existing decision rules have qualitatively the same properties as profit-maximization, one cannot in general be certain that standard neoclassical propositions hold. Regard as an example the effect of increasing the wage in a two-factor world. This effect may by ordinary substitution among known production techniques (a) reduce the labour intensity, but it will also (b) induce inten-

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[1] See Stigler (1957) and McNulty (1968).

[2] Apart from Schumpeter see Kirzner (1973) and Littlechild (1978).

sified search for new techniques which may equally well be capital-saving as labour-saving, and it will (c) in general lead to a changed level of production which will affect firms differently according to the labour-intensity of the techniques they use. Hence, it is clear that in order to guarantee that the standard proposition would hold in an evolutionary model, more specific assumptions about "factor-intensity bias of search" and about expansion behaviour of different firms are needed.[1]

In other papers[2] Nelson and Winter analyze questions about technical progress, concentration, profits and "Schumpeterian" competition. There they resort to using simulation models. This facilitates the inclusion of many facets of reality into the model. But it also means that the results may be difficult to interpret. And one may sometimes be at odds whether a particular result is due to a particular set-up of parameter values or is valid more generally. But these problems should in principle not be so much different from the problems of seeing in any particular analytical model which results are due to which specific assumptions.

OUTLINE OF THE MODEL STUDIED

The point of departure of the present study is the evolutionary framework. But the models we are going to study are simplified very far as compared with the Nelson and Winter simulation models referred to above. Our focus of interest is on the interrelatedness among profits, the rate of expansion and technical progress. We keep the basic behavioural commitment from the evolutionary framework; firm behaviour is modelled in non-maximizing terms and in a way that makes it possible to test. The behavioural assumptions made

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[1] For such conditions see Nelson and Winter (1975). For a more general presentation of the "comparative statics" of an evolutionary model see Nelson and Winter (1977 b).

[2] Nelson and Winter (1977 a, 1978).

are very simple indeed; the rate of expansion and the rate of cost reduction are expressed as functions of profitability (current or expected).

The first of the behavioural assumptions is quite standard. It can be regarded as one possible way of formulating the Marshallian adjustment mechanism. The idea that the rate of cost reduction is a function of profits deserves some more comments. As pointed out above there may actually be two kinds of mechanisms present, both possible to trace back to Schumpeter. On the one hand, high profits will make it possible for the firm to undertake more research if the capital market is imperfect or if information is asymmetric. On the other hand, high profits may make the firm less eager to cut costs. We will close the model by coupling the behavioural assumptions about expansion and cost reduction with an inverse demand equation, expressing price as a function of quantity produced.

Before outlining the basic working of the model it is appropriate to point out some of the many simplifications made. First, in contrast to all of Nelson and Winter's models, we will consider a deterministic model. This means that we will not analyze the distribution of profit rates and other variables across firms in the same industry. Second, technical progress will be expressed as the rate of change of unit costs. There is no explicit treatment of factor price changes and factor substitution. Third, there is no direct interdependence between individual firms. The rates of cost reduction and expansion of any single firm are only affected by its own profitability. Even if firms use different techniques low-productive firms do not imitate high-productive ones. When deciding to develop new techniques firms do not take explicit account of other firms' behaviour. Decisions to expand are taken without direct regard to demand conditions. The lack of interdependence between firms in the same industry is to my mind one of the most serious restrictions of the model. Fourth, we disregard financial variables such as solidity and liquidity, which may be very important in affecting firms' behaviour. Fifth, there is no discussion of lags between the time when a measure is decided and the time when its effects are seen. This is probably a quite serious assumption as regards the rate of cost reduction. A decision to reduce slack, which will typically be a reaction to low profits, is likely to have effects fairly quickly, whereas it will take many years for a decision to spend more on R & D to affect costs in production. Sixth, pricing behaviour is treated very superficially. We simply assume that a price to clear the market will somehow always be established.

These simplifications are made in order to get a model highlighting this picture of reality. Profits provide signals to firms. High profits give incentives and financial means for expansion and R & D, and low profits force firms to try to reduce slack. Firms are myopic and simpleminded in not caring about any information except their own rate of profits. These reactions of the firms affect the rate of change of price (which is the same for all firms) and the rate of change of unit costs. This means that the rate of profits will be affected and the changed rate of profit feeds back into firms' behaviour.

The working of the model is easiest to illustrate by starting with a very simple example, where we assume that there is only one firm.[1] Fig. 2.1 illustrates the case where unit costs (c) are assumed constant irrespective of the rate of profit (π). Production (q) increases when profits are positive and falls when they are negative, and demand is constant over time. Formally we have

[1] Alternatively, it can be regarded as a model of an industry behaving as though there is only one firm, the representative firm of Marshall.

$$c = \bar{c}$$

$$\hat{q} = \frac{\dot{q}}{q} = \alpha_q \pi \qquad \alpha_q > 0 \qquad (2.1)$$

$$p = f(q)$$

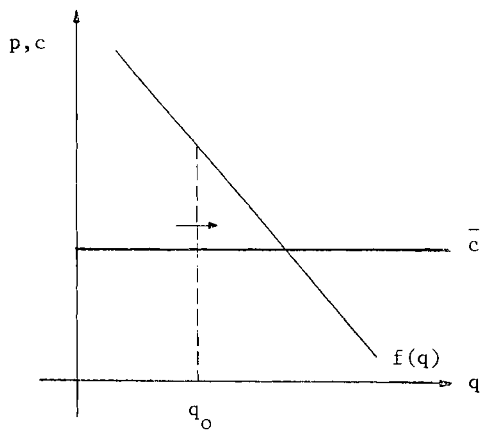

Figure 2.1

With initial quantity produced equal to q_o, profits will be positive and production will tend to increase. Under standard assumptions about the shape of the demand function profits will approach zero in the long run. This is nothing but the familiar Marshallian adjustment mechanism.[1]

Assume now that demand increases over time. With an expansion function like that of model (2.1) production will increase only when profits are positive, but in order to keep profits at zero in the long run it is necessary that production increases even when profits are zero. Starting in a situation like that of Fig. 2.1 production will increase continuously but it will never catch up with the quantity

[1] A disaggregate model similar to this one has been analyzed by Quandt and Howrey (1968). They assume entry to be determined by an equation like in (2.1) and existing firms to be profit-maximizers subject to increasing marginal costs. It is shown that there is a globally stable equilibrium, where the number of firms is fixed and the rate of profit zero, provided that the demand curve is fixed. See also Myers and Weintraub (1971) and Okuguchi (1972).

demanded at $p = \bar{c}$. There will exist a long run solution to this model characterized by a constant rate of profit and production growing at the same rate as demand. These equations formalize this model.

$$
\begin{aligned}
c &= \bar{c} \\
\hat{q} &= \alpha_q \pi \\
p &= q^{-\gamma} e^{\delta_p t} \; ; \quad \hat{p} = -\gamma \hat{q} + \delta_p \\
\pi &= \frac{p - c}{c}
\end{aligned}
\tag{2.2}
$$

Here we have parametrized the demand function with an inverse price elasticity γ and a constant growth rate δ_p. Further we have defined profits as a sales margin. Assume that there exists a steady-state where $\pi = \pi^*$ is constant. This means in this model that the rate of change of price is zero, i.e.

$$\hat{p}^* = -\gamma \hat{q}^* + \delta_p = -\gamma \alpha_q \pi^* + \delta_p = 0$$

We get out this simple expression for steady-state profits in terms of the parameters of the model

$$\pi^* = \frac{\delta_p}{\alpha_q \gamma}$$

Steady-state profits are thus an increasing function of the rate of growth of demand and a decreasing function of the inverse demand elasticity and of the reaction parameter α_q. Only if the reaction coefficient α_q is very high or demand is constant over time ($\delta_p=0$) will the profit margin in the long run approach zero. This is the very simplest version of the model possible. It shows that the Marshallian adjustment mechanism does not automatically lead to zero profits in the long run in a dynamic world. Clearly it would be possible to introduce an intercept term into the expansion function which would yield $\pi^* = 0$.

$$\hat{q} = \alpha_q \pi + \delta_q$$

implies

$$\pi^* = \frac{\delta_p - \delta_q \gamma}{\alpha_q \gamma}$$

If $\delta_q = \delta_p/\gamma$ steady-state profits would indeed be zero. It is very unclear however what mechanisms there are in real world markets that would make it natural to assume this.

This is the first leg of the theory, the idea that profits may be explained by the inconsonance between the growth behaviour of firms and the rate of expansion of the market.

The second leg is the idea that profits also affect costs. Let us now first go back to a world of constant demand. This model is depicted in Fig. 2.2.

$$\begin{aligned} \hat{c} &= \alpha_c \pi & \alpha_c &\gtrless 0 \\ \hat{q} &= \alpha_q \pi & \alpha_q &> 0 \qquad (2.3) \\ p &= f(q) \end{aligned}$$

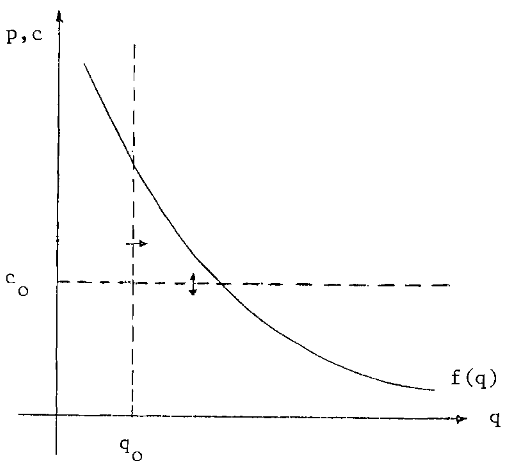

Figure 2.2

If $\alpha_c > 0$ we see that this leads to $\pi^* = 0$ in the long run. Not only will production increase to press price down, but costs will also rise. Hence, profits will be squeezed in two ways. But if $\alpha_c < 0$ it is not equally obvious what will happen. Then it is quite possible that costs will fall faster than prices and there will not exist any stable steady-state.

Introducing a cost reduction equation and intercepts into the model with changing demand.

$$\begin{aligned} \hat{c} &= \alpha_c \pi - \delta_c \\ \hat{q} &= \alpha_q \pi + \delta_q \\ \hat{p} &= - \gamma \hat{q} + \delta_p \\ \pi &= \frac{p - c}{c} \end{aligned} \tag{2.4}$$

This model will be analyzed in detail in Chapter 4. There we will look not only at its steady-state properties but also at the path from any given initial values of price and unit costs. The many simplifying assumptions made in the model (2.4) make it possible to find an explicit solution to that equations system, thereby enabling a more detailed analysis of the properties of the model outside steady-state. In Chapter 5 we will relax some of the more restrictive assumptions and see to what extent the results of the simple model are sensitive to this. In Chapter 6, finally, we will try to test the assumptions of the model empirically.

Before going into the analysis of the basic model we will, in Chapter 3, discuss some of the factors that lie behind the behavioural assumptions. This will be done by specifying some dynamic maximization models of individual firms adapting to changed external conditions. The analyses of these models will have a two-fold purpose. They will help us to interpret the behavioural parameters α_c and α_q, but they will also help to give an illustration of why it is so difficult to base a model where dynamic firm behaviour interacts with demand, like (2.4), on explicit maximizing assumptions. The reader who accepts the behavioural assumptions as they have been stated in (2.4) can without loss of continuity skip Chapter 3 and proceed directly to Chapter 4.

3. THEORETICAL BACKGROUNDS TO THE BEHAVIOURAL ASSUMPTIONS

In Chapter 2 we put forth a model of a market adjustment process. This is the basic model of this study. It will be analyzed in detail in Chapter 4, and in Chapter 5 we will investigate the sensitivity of the main results of this analysis to some alternative model specifications. Our model is based on two behavioural assumptions: (a) the rate of expansion of production is an increasing function of the rate of profit, and (b) the rate of change of unit costs is a function of the rate of profit. For simplicity these functions are assumed to be linear

$$\hat{q} = \dot{q}/q = \alpha_q \pi + \delta_q \qquad \alpha_q > 0 \tag{3.1}$$

$$\hat{c} = \dot{c}/c = \alpha_c \pi - \delta_c \qquad \alpha_c \gtrless 0 \tag{3.2}$$

q = total quantity produced in the industry

c = unit cost

$\pi = \frac{p-c}{c}$ = the profit margin

p = unit price.

The behavioural assumptions apply to an industry producing a homogeneous good. This means that (3.1) and (3.2) reflect a combination of the behaviour of single firms, the changed composition of firms and the entry of new firms.

In this chapter we will attempt to go behind the behavioural assumptions at the level of the individual firm. This will be done by studying some dynamic maximization models, where the firm is assumed to be maximizing the discounted value of future costs and revenues, or in some cases, the utility of future costs and revenues. The solutions to these models specify the entire future development of production as a function of the prices of the product and factors of production, the discount rate and other variables that are treated as exogenous. This means that we can study the effects of a price change on the rate of expansion or the rate of cost change, i.e. we can derive partial derivatives with respect to the product price (or, which in general is equivalent, with respect to the price of some factor of production).

We will interpret these partial derivatives in terms of the reaction coefficients α_c and α_q of our model. There are some problems inherent in doing this. First, the rate of profit is an endogenous variable in the optimization models and it is a priori quite possible that $\partial\pi/\partial p$ is negative. In such a case the sign of the derivative with respect to p would be opposite to that of the derivative with respect to π. In general this will not be so, but this should be ascertained for each particular model. Second, there is in general no presumption at all that the relation in question is a linear one as we have assumed in (3.1) and (3.2). This is a somewhat less fundamental problem, however, since we can at any instance interpret our behavioural assumptions as linear approximations around the steady-state of the model. We will consequently not make any attempts to discuss whether the maximization models of this chapter lead to globally linear functions.[1]

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[1] A study with this ambitious purpose is Treadway (1974), which shows how an accelerator model of investment under certain conditions can be derived from a model of convex costs of adjustment.

We will study three types of dynamic optimization problems. In the first section it will be assumed that one factor of production is fixed in the sense that it is only adjustable at a cost. The costs of adjustment may be of different types, arising out of monopsonistic situations in the factor markets or costs due to the training of new employees etc. Costs-of-adjustment models are hence fairly general and build on a well established literature.

A main influence on firms' dynamic behaviour stems from the capital market. This will be analyzed in the second section. The interest rate on borrowed capital will normally be a falling function of the firm's equity/debt ratio. This can be so for several reasons. We will study a simple model, where this effect arises in a competitive equilibrium situation in the loan market because lenders run a default risk and there is a gap in information between the lenders and the borrowing firms. In such a world there may be a link from the firm's current rate of profit via capital costs to the optimal rate of investment, both in buildings and machinery and in research and development. We will develop two dynamic models of the firm to study to what extent these capital market conditions affect the link between profits and the rate of investment. One model is a variant of the basic cost-of-adjustment model. The other is a model of a management run company.

The effect on R & D via capital market imperfections is one of the main determinants of the cost reduction coefficient α_c. Another important mechanism, that may work in the opposite direction, is that high profits may be conducive to the growth of slack. In the third section of this chapter we try to formalize this view by a model where the firm is assumed to maximize a utility function which displays disutility to activities leading to a high rate of cost reduction.

It is not intended to provide an exhaustive list of models of aspects of firm behaviour that are related to α_q and α_c. Rather, we have selected three areas that on a priori grounds seem to be the most relevant ones and try to analyze these in some depth. We noted in Chapter 2 that there are two ways to modify the simple profit maximization model. One can either introduce different restrictions or another type of maximand. In this chapter we will see examples of both ways. The cost of adjustment models build on the restriction that it is costly to adjust fast. The financial restrictions are analyzed both in a model with discounted dividends as maximand and one with a utility function that is concave in dividends. The managerial model of cost reduction in the final chapter also builds on a special utility function.

COSTS OF ADJUSTMENT

The standard way to justify the Marshallian adjustment model focuses on the differences between short-, medium- and long-run supply curves. This reflects that there are costs associated with changes in the inputs of certain factors of production.

A fairly general representation of a maximization problem taking account of costs of adjustment may be stated

$$\max \int_{0}^{\infty} \pi(t)e^{-\rho t}dt \tag{3.3}$$

$$\text{s.t. } \pi(t) = p(t)F(K,L,I,t) - w(t)L(t) - C(I,t) \tag{3.4}$$

$$\dot{K}(t) = I(t) - \delta(t)K(t) \tag{3.5}$$

The firm is assumed to maximize the integral of discounted future net income π. There are two factors of production, one of which, labour (L), is freely variable and one of which, capital (K), is only variable at a cost. In practice, of course, it may be equally costly to vary the employment of certain types of workers; labour is used here simply as a synonym for a freely variable factor. (3.4) expresses net income as the receipts from selling the product minus labour costs minus investment costs. (3.5) defines the rate of change of the capital stock as investment minus depreciation.

Investment costs occur in two places in (3.4). First it is assumed that the price paid for investment goods may vary with the amount of investment, i.e. C need not be a linear function of I. This reflects that many firms are more or less monopsonists in the market for investment goods. This is an external cost of adjustment.

Another type of adjustment cost stems from the fact that some factors of production, e.g. certain groups of skilled workers, may be firm-specific, i.e. they cannot be purchased in the open market but need some training inside the company before being of any use. When there is no expansion of production all employed skilled workers will devote their time to current production, but when production expands and new workers are taken on these will have to be trained by the old workers. Resources for expansion are then, in effect, taken from current production, as is reflected by the inclusion of investment as an argument in the production function. Such adjustment costs are labelled "internal".

The capital market is assumed to be perfect. The discount rate ρ can hence be interpreted as a fixed interest rate at which the firm can lend and borrow freely.

Convex Adjustment Costs

In order to reach any more specific conclusions we must specify the optimization problem more than is done in (3.3)-(3.5). This must be done in two dimensions. We must decide whether to regard external or internal adjustment costs, and we must make an assumption about the form of the adjustment cost function.

Consider the case of external adjustment costs, C(I). This function may be linear, which only says that there are no special adjustment costs. It may be concave, in which case there are decreasing marginal costs of fast adjustment, $C'' < 0$. This will imply a discontinuous adjustment pattern, either no change at all or a large adjustment once and for all. Finally it may be convex, $C'' > 0$. This makes the maximization problem well determined, and the optimal policy will typically consist of gradually adjusting the capital stock towards the long-run equilibrium size.

In this section we will concentrate on convex costs of adjustment, and leave the case of concave costs to the next section. There we will analyze a model where there is a fixed cost associated with each adjustment irrespective of the size of the change.

Let us then start by regarding a model where there are solely *external* adjustment costs. This was done in the seminal study in the area, Eisner & Strotz (1963).[1] They analyzed the model

$$\max \int_0^\infty \pi e^{-\rho t} dt \tag{3.6}$$

where

$$\pi = pF(K,L) - wL - C(I) \qquad C'' > 0 \tag{3.7}$$

$$\dot{K} = I - \delta K \tag{3.8}$$

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[1] There is by now a well established theory. For a survey of subsequent developments, see Söderström (1976).

Prices are assumed to remain constant forever and the production function is linearly homogeneous. This problem can be solved by calculus of variations yielding as necessary conditions

$$F_L = \frac{w}{p} \tag{3.9}$$

$$F_K = \frac{(\rho+\delta)C'(I)}{p} - \frac{\dot{I}C''(I)}{p} \tag{3.10}$$

It can be shown that with constant returns to scale there is one unique solution satisfying the transversality condition. This solution implies $\dot{I} = 0$,[1] and so

$$F_K = \frac{(\rho+\delta)C'(I)}{p} \tag{3.11}$$

Since the production function is linearly homogeneous (3.9) determines the factor intensity. And, since K is fixed in the short run, it thereby determines the level of production. With the factor intensity determined so is F_K, and (3.11) then gives the investment level and, consequently, the rate of change of production.

To see the effects of variations in the product price, p, on the optimal values, L* and I*, we differentiate (3.9) and (3.11) simultaneously.

$$\frac{\partial L^*}{\partial p} = \frac{F_L}{-pF_{LL}} > 0 \tag{3.12}$$

$$\frac{\partial I^*}{\partial p} = \frac{F(1,\ell)}{(\rho+\delta)C''(I)} > 0 \tag{3.13}$$

where $\ell = L/K$, and we have made use of Euler's theorem in deriving (3.13). Hence, an increase in the exogenously given price, i.e. an increase in the rate of profit, leads both to an instantaneous *shift upwards* of production due to the

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[1] See the phase-diagram in Fig. 3.1 on p. 75 and Treadway (1969) p. 232.

employment of more labour and to a *faster increase* of production due to larger investments.

From this it seems that the presence of convex external adjustment costs, i.e. monopsonistic factor markets, implies that the rate of expansion depends positively on the rate of profitability, i.e. that α_q of (3.1) is positive. It should be noted, however, that there will always be an instantaneous shift effect if any factor of production is freely variable. Hence, when we assume in (3.1) that it is the *rate of change* of production which depends on the level of profitability, strictly speaking this means that no factors of production are freely variable, or that there are no substitution possibilities.

With this reservation we can interpret $\partial I^*/\partial p$ as corresponding to α_q. The value of this partial derivative is seen from (3.13) to depend on the second derivative of the cost of adjustment function and on the factor intensity. It is a decreasing function of C''. This is quite obvious since a linear function ($C''=0$) would mean that there are no special adjustment costs in which case the optimization problem would be undetermined. $\partial I^*/\partial p$ will also depend on the properties of the production function. It is increasing with the labour intensity, i.e. the more important the freely variable factor of production is, the swifter will be the optimal response to price changes.

These straightforward results depend on the specification of adjustment costs as purely external. In many cases it is more realistic to assume *internal* adjustment costs, where resources for adjustment have to be taken from inside the firm, i.e. from resources otherwise devoted to production. This leads to a somewhat different picture. In particular the direction of the short-run effect on the rate of expansion of production will now be ambiguous.

This case may be formulated

$$\max \int_0^\infty \pi e^{-\rho t} dt \tag{3.14}$$

$$\text{s.t. } \pi = p\big(F(K,L) - C(I)\big) - wL \tag{3.15}$$

$$I = \dot{K} + \delta K \tag{3.16}$$

where, to facilitate the exposition, adjustment costs are assumed *separable* instead of the more general formulation in (3.4).[1] The only formal difference from (3.7) is then that investment costs here are multiplied by the product price.[2] Note, however, that the interpretation of C(I) differs between the models. In (3.7) it is the amount paid out to suppliers of new equipment. Here it is the revenue foregone from diverting productive resources to installment of new equipment and training new employees.

The first-order conditions are

$$pF_L = w \tag{3.17}$$

$$pF_K = p\big((\rho+\delta)C'(I) - \dot{I}C''(I)\big) \tag{3.18}$$

where assuming constant returns to scale again implies, by the transversality condition, $\dot{I} = 0$. Hence,

$$F_K = (\rho+\delta)C'(I) \tag{3.19}$$

Comparing this with (3.11) we see that p has dropped out. The reason for this is obvious. All resources for expansion now have to be taken from current production. When

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[1] On non-separable internal adjustment costs see Uzawa (1969) and Treadway (1970).

[2] It would be more realistic to have both external and internal adjustment costs in the same model. Certainly there are in most cases some adjustment costs that are external, e.g. expenditures for new machinery. This would call for adding a linear term, vI, to (3.15) representing the costs for the purchase of investment goods on a perfect market. This would give us the model analyzed in Treadway (1969).

the price of output increases so does the opportunity cost of expansion. As before, however, a price change will affect the optimal factor intensity and, hence, the production level. Differentiating (3.17) and (3.19) simultaneously we get

$$\frac{\partial L^*}{\partial p} = -\frac{F_L}{pF_{LL}} > 0 \tag{3.20}$$

$$\frac{\partial I^*}{\partial p} = \frac{\ell F_L}{p(\rho+\delta)C''} > 0 \tag{3.21}$$

Hence, as soon as there is some variable factor there will even in this case be an increase in investment. This is due to the fact that a higher price will increase the opportunity cost of capital and thereby lead to substitution in current production. This will leave room for devoting resources to expansion.

In contrast to the case of external adjustment costs it is no longer possible to determine unambiguously the sign of the instantaneous effect on quantity produced. The increase in labour input implies by itself increased production, but this effect is countered by the fact that resources for increased investment are taken away from current production. The slope of the short-run supply curve will hence be ambiguous.[1]

Whereas the instantaneous effect on the level of q is thus ambiguous, the effect on the rate of change of q is unambiguously positive. We have just seen that a higher product price leads to a higher rate of investment, thereby increasing the rate of change of the capital stock. This also means that L will increase faster along the optimal path. Consequently, a higher product price leads to a faster increase of production.

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[1] See Söderström (1976) p. 383.

On balance it seems that cost-of-adjustment theories give a theoretical basis, in terms of profit-maximizing behaviour, for the Marshallian view that high profits spur expansion, and consequently for an assumption that $\alpha_q > 0$.[1] There are some important qualifications, however:

(a) As soon as there are some factors which are freely variable, in the sense of having a linear cost-of-adjustment function, and the production function allows for substitution between factors of production, changes in profitability will lead to instantaneous shift effects.

(b) These instantaneous effects on production may well be in the opposite direction to the long-run effect, when resources for adjustment have to be taken from current production.

(c) If adjustment costs are purely internal and there are no freely variable factors, the product price will not affect the production of the individual firm at all.

Note that we have consistently been talking about expanding firms. This is not a sheer coincidence. In making use of the first order conditions we have implicitly assumed an interior maximum. But, since gross investment must be non-negative, there will be a corner solution with $\dot{K} = \delta K$ for sufficiently low values of p. Then the only effect of a further decrease in p will be the shift effect of a decrease in L, which follows from the optimality condition (3.9).

The cost-of-adjustment models briefly surveyed here take only a first step towards treating the whole process of industry responses to changed market conditions. In order to take any further steps the perspective must be widened from

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[1] Indeed, it is a main result of cost of adjustment theories that, under certain conditions, the rate of change of the capital stock may be expressed as a function of the gap between the steady-state capital stock and present capital stock, i.e. this type of model lends support to the accelerator model of investment. On this see Eisner & Strotz (1963), Lucas (1967), Gould (1968) and Treadway (1974).

studying a single firm adjusting to a fixed price to studying interacting firms making investment decisions based on expectations about future prices which are determined endogenously within the model.

The full dynamic behaviour of any such model will be quite complicated.[1] It will be instructive to give some attention to the specification of long-run equilibrium in a cost-of-adjustment model with many firms. We will see that assuming price-taking behaviour is not sufficient to guarantee that profits in the long run will be zero.

We study an industry composed of n identical firms producing the same homogeneous good. Assume that their behaviour can be described by the optimization problem (3.6)-(3.8), i.e. they all maximize discounted profits under convex external adjustment costs. Price is assumed to be a decreasing differentiable function of total production, $p(nF)$. We can then rewrite the first-order optimality conditions

$$p(nF)\cdot F_K = (\rho+\delta)C'(I) \quad (3.22)$$

[2]

$$p(nF)\cdot F_L = w \quad (3.23)$$

assuming static expectations throughout.

Together with the equation of motion

$$\dot{K} = I - \delta K \quad (3.24)$$

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[1] The closest to anything like this that I have seen is a paper by Brock (1972). This is not, however, based on an explicit cost-of-adjustment model of firm behaviour.

[2] This formulation assumes that C is solely a function of the individual firm's purchase of investment goods, and independent of the level of investment in the industry. It must hence be interpreted as an assumption that each firm is a monopsonist in a local market for its own investment goods.

(3.22) and (3.23) give the dynamics of this system. Steady-state is defined by $\dot{K} = 0$. We are interested in the size of profits in steady-state, where we define

$$\pi = pF(K,L) - wL - C(\delta K)$$

Inserting the optimality conditions we have, making use of Euler's theorem,

$$\pi^* = pF_L L^* + (\rho+\delta)K^*C'(\delta K^*) - pF_L L^* - C(\delta K^*) =$$

$$= (\rho+\delta)K^*C'(\delta K^*) - C(\delta K^*)$$

By the assumed convexity of C(I)

$$\delta K^*C'(\delta K^*) > C(\delta K^*)$$

Hence

$$\pi^* > 0.$$

This shows that in an industry with a given number of identical firms each of which behaves as a price-taker in the product market, profits will not be eliminated even in the long run. It has been claimed that "the theoretical problem of combining constant returns to scale with pure competition at any particular points is resolved ... by reference to convex adjustment costs ...".[1] But it seems that this is accomplished at the expense of allowing positive profits.

The above argument presupposes a fixed number of firms. It can be shown, however, that $\partial K^*/\partial n < 0$, and in the limit with free entry there will be an infinite number of infinitesimal firms. In the limit π^* also approaches zero. This can

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[1] See Söderström (1976) p. 385.

be compared with the standard theorem of general equilibrium theory, which says that profits will be zero when all firms act as price-takers and product in cost-mimimum. Here cost-minimum means an infinitesimal scale of production.

In the analysis of the basic model of the present study in Chapter 4 we will see that the rate of profit will in general not tend towards zero in the long run. The upshot of the analysis in this section is that a similar property holds as well for the standard cost-of-adjustment model.

Lump-Sum Adjustment Costs

It is not a priori obvious whether the case of convex adjustment costs as treated above is the rule or an exception. Especially when costs of decision-making are important the adjustment cost function may instead be concave. In this section we will confine ourselves to analyzing the extreme case where adjustment costs are solely of a lump-sum character, i.e. there are no variable costs, only a fixed cost no matter the size of the adjustment made. This fixed cost is the same for expansion as for contraction.

Then, as is generally the case with concave adjustment costs, there will be a discontinuous relation between profitability and the rate of change of production; either there is no change at all or a large change. In this section we will switch perspective and analyze a model of a group of firms facing identical cost and demand conditions with the exception of a stochastic component. We will then be able to express the average rate of change as a function of the average rate of profit.[1]

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[1] Alternatively one might stick to a deterministic model and make an assumption of the initial size distribution of firms. Then one would also get an approximately continuous average reaction.

Barro (1972) has developed a model to study how the optimal rate of price change is a function of excess demand, i.e. to derive the Walrasian market adjustment function. This same model will here be inverted to illuminate the Marshallian problem of determining the optimal rate of expansion.

The model is based on the assumption that each firm acts as a monopolist in the product market, and that the total cost curve is quadratic. This stands in contrast to the price-taking behaviour and constant returns to scale that were assumed in the previous section. Either of these assumptions is necessary, however, since there would otherwise be no bound to the size of the optimal adjustment once a change was made.

Let us regard a group of identical firms, each acting as a monopolist and each acting as though it were facing the downward sloping linear demand curve (3.25). β, the price sensitivity of demand, can be thought of as reflecting the market share in a Cournot-equilibrium.

$$q_t = \alpha - \beta p_t + u_t \qquad \alpha,\ \beta > 0 \tag{3.25}$$

or, inversely

$$p_t = \frac{\alpha}{\beta} - \frac{q_t}{\beta} + \frac{u_t}{\beta} \tag{3.26}$$

where u takes account of all price-independent demand variations. For the time being we regard it as deterministic and known by the firms.

The method of analysis followed is to first study the optimal behaviour of a firm in the absence of adjustment costs. We then compare the profits resulting from continuous adjustment according to this optimality condition with the profits resulting from no adjustment at all. This difference, which is a function of the demand shift term u, is a measure of the cost of not adjusting production. This is then compared with the lump-sum cost of adjustment. As soon as the former exceeds the latter an adjustment is made.

The decision problem in the case of continuous adjustment is then to maximize

$$\pi = pq - C(q) \tag{3.27}$$

subject to (3.26), where we assume C(q) to be quadratic

$$C = a + bq + cq^2 \qquad a,\ b,\ c > 0 \tag{3.28}$$

The first-order condition for a maximum gives

$$q^* = \frac{\alpha - b\beta + u}{2(1+c\beta)} \tag{3.29}$$

We will assume that this gives positive values for p^* and q^*, the necessary and sufficient condition for which is that

$$\left.\frac{\partial\pi}{\partial q}\right|_{q=0} = \frac{\alpha + u}{\beta} - b > 0 \;^{1)} \tag{3.30}$$

Variations in the demand-shift term u will lead to variations in profits. If adjustments are continuously made so as to fulfil (3.29) maximum profits, π^*, are a function of u and we have

$$\frac{d\pi^*}{du} = \frac{\alpha - b\beta + u}{2\beta(1+c\beta)} \tag{3.31}$$

The total change in profits due to variations in u from 0 to u_1 is then

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[1] That this really implies $p^* = p(q^*) > 0$ can be seen by substituting (3.29) into (3.26) and making use of (3.30).

$$\Delta\pi_{(o,u_1)} = \int_o^{u_1} \left(\frac{d\pi^*}{du}\right) du \qquad (3.32)$$

Evaluating this integral, we get, taking (3.31) into account[1]

$$\Delta\pi_{(o,u)} = \frac{\alpha - b\beta}{2\beta(1+c\beta)}\, u + \frac{1}{4\beta(1+c\beta)}\, u^2 \qquad (3.33)$$

We are interested in the profits foregone by not adjusting production according to demand variations. To see this we calculate the change in profits as a function of variations in u given that u = 0 is substituted into the optimality condition (3.31)

$$\Delta\pi'_{(o,u)} = \frac{\alpha - b\beta}{2\beta(1+c\beta)}\, u \qquad (3.34)$$

and

$$\Delta\pi_{(o,u)} - \Delta\pi'_{(o,u)} = \frac{1}{4\beta(1+c\beta)}\, u^2 > 0 \qquad (3.35)$$

is then the profits foregone by not changing the quantity produced expressed as a function of the shift in the demand curve, u.

If the demand curve were fully known by all firms and there were no adjustment costs, production would be varied continuously with u according to the profit-maximizing condition (3.29). We now introduce a lump sum cost of making adjustment, which is γ irrespective of the size and direction of the change in production. Let us further assume that the firms all regard the demand-shift variable u as the outcome of a stochastic process. The decision problem facing

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[1] This way of deriving (3.33) is actually simpler than the more direct way of substituting (3.29) into (3.26)-(3.28). Note also that we drop the subscript from u_1 of (3.32).

firms will then have certain similarities with the problem of optimal inventory policy when sales is regarded as the result of a stochastic process. When the stochastic movement goes in one direction only, as is the case for the inventory problem, one can prove that the optimal policy is to select a fixed floor and a fixed quantity of adjustment, so that each time the remaining stock reaches the floor one purchases a fixed amount of goods. There does not appear to be any general proof when the stochastic movement goes in both directions, but Barro supports his use of this type of policy by reference to numerical proofs.[1]

Be that as it may, we postulate that the problem for the company is to select a ceiling (h_c) and a floor (h_f) such that once u reaches the ceiling/floor the company changes its quantity produced to a value that is optimal given that value of u.

It is now assumed that the process generating u is a symmetric random walk. This means formally that

$$u_{t+1} = u_t + \varepsilon_t$$

where ε_t is a stochastic variable

$$\mathrm{pr}(\varepsilon_t = +1) = \mathrm{pr}(\varepsilon_t = -1) = 1/2$$

The important thing about the assumption is that $E(u_{t+1} | u_t = \bar{u}) = \bar{u}$. While this facilitates the analysis it is of course quite restrictive.

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[1] See Barro (1972) p. 21 note 2.

Ceiling and floor values are now chosen in order to minimize the expected cost of being out of equilibrium per unit of time over some time interval t. m is the number of adjustments over the interval. The expected cost per time period is given by

$$E\left(\frac{\text{cost}}{\text{time}}\right) = \gamma E(m/t) + E(\Delta\pi - \Delta\pi') = \gamma E(m/t) + \theta E(u^2) \quad (3.36)$$

where

$$\theta = \frac{1}{4\beta(1+c\beta)}$$

from (3.35).

It can now be shown[1] that the values of h_c and h_f that minimize the expected cost per time, (3.36), are

$$\hat{h}_{c,f} = \pm\sqrt{\sigma\sqrt{\frac{6\gamma}{\theta}}} \quad (3.37)$$

where σ^2 is the variance of ε_t per unit of time which for this process is a constant over time.

If we know where we start the process, i.e. the value of u at the outset, we can compute the probabilities that the next adjustment will be at the ceiling or at the floor.[2]

$$pr(\hat{h}_c) = \frac{1}{2}\left(1 + \frac{u}{\hat{h}_c}\right) \qquad pr(\hat{h}_f) = \frac{1}{2}\left(1 + \frac{u}{\hat{h}_f}\right)$$

From (3.29) we can calculate the quantity change at each adjustment. It will be

$$\frac{\partial q^*}{\partial u} \cdot \hat{h}_{c,f} = \frac{\hat{h}_{c,f}}{2(1+c\beta)}$$

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[1] For details, see Barro (1972) pp. 22-23. On the mathematical theory behind it, see Feller (1968) ch. XIV.

[2] See Feller (1968) p. 345.

and the expected adjustment at the next adjustment time, given that the process is now at u, is

$$E(\Delta q) = \frac{\hat{h}_c}{2(1+c\beta)}\left[pr(\hat{h}_c) - pr(\hat{h}_f)\right] = \frac{1}{2(1+c\beta)}\, u \qquad (3.38)$$

The expected duration of the random walk originating at u can be shown to be

$$D_u = \frac{\hat{h}^2 - u^2}{\sigma^2} \qquad (3.39)$$

Then, since we are considering a large number of firms identical in all respects except that they face different values of the stochastic variable u, one may speak of the average quantity change per unit of time by dividing (3.38) over (3.39)

$$\frac{\Delta q}{\Delta t} \approx \frac{E(\Delta q)}{D_u} = \frac{\sigma^2 \cdot u}{\hat{h}^2 - u^2} \cdot \frac{1}{2(1+c\beta)}$$

where u now must be understood as an average value over all firms.

This may in turn be approximated by assuming u^2 to be small relative to $\hat{h}^2$, i.e. assuming that present production is close to optimal on average, and

$$\frac{\Delta q}{\Delta t} \approx \frac{\sigma^2}{2(1+c\beta)} \cdot \frac{u}{\hat{h}^2} \qquad (3.40)$$

where $\hat{h}^2$ is given by (3.37). This gives us the rate of change of production as a function of the stochastic variable u. We are interested in transforming this into a function of π. To do this we use (3.34), which gives

$$u = \frac{2\beta(1+c\beta)}{\alpha - b\beta}\,(\pi - \pi^*)$$

where π^* is the profit when u = 0 and q is adjusted according to the optimality condition (3.29).

$$\frac{\Delta q}{\Delta t} \approx \frac{\sigma^2\beta}{\hat{h}^2(\alpha-b\beta)} (\pi-\pi^*) = k(\pi-\pi^*) \tag{3.41}$$

$$k = \frac{\sigma\beta}{(\alpha-b\beta)\ \sqrt{24\gamma\beta(1+c\beta)}}$$

When interpreting this expression one must bear in mind that π is not an exogenous variable but determined by the same parameters of the cost and demand function which are the ingredients of the adjustment parameter k. From the assumption that there exists a solution to the maximization problem of the firm which gives $q^* > 0$ for $u = 0$ it follows that $\alpha - b\beta > 0$; see (3.30). From this we see that $k > 0$. The rate of change of production is thus an increasing function of profits.

The coefficient k is the equivalent of this model to α_q. Let us now look at the partial derivatives of this item with respect to the parameters of the model. The impact of variations in π on the rate of expansion will, ceteris paribus, be stronger

- the lower the adjustment cost, $\partial k/\partial\gamma < 0$. This merely affects the average time between consecutive adjustments.
- the lower the rate at which marginal cost increases with the level of production, $\partial k/\partial c < 0$. If returns to scale are close to being constant, production will be varied much more in response to demand variations.
- the higher the price sensitivity of demand, $\partial k/\partial\beta > 0$. This is the outcome of two offsetting forces. On the one hand a given change in the stochastic component of demand, u, will induce smaller and less frequent quantity adjustments when demand is sensitive to price changes, by (3.40). On the other hand variations in u will have comparatively little impact on profits by (3.34). And the latter effect dominates. Remember that β refers to the demand curve perceived by firms. In a Cournot-equilibrium it will be related to the number of firms in the industry; many firms make for rapid adjustment.

- the higher the variance of demand, $\partial k/\partial\sigma^2 > 0$. A high variance implies frequent adjustments, an effect which is only partially offset by the fact that it will also affect the ceiling and floor values.

The model structure of this section is different from that of the previous one in a number of respects. The purpose has been to analyze a model with concave adjustment costs where we chose the special case of lump sum costs. This implies that the single firm will make discrete quantity adjustments. But the industry reaction pattern may be approximately continuous if the firms differ from each other. We have assumed differences to arise out of stochastic variations in the demand curve perceived by each firm. We have further assumed that each firm acts as a monopolist and that there are decreasing returns to scale. As we have just seen all these factors have an impact on the reaction coefficient k.

This completes the discussion of the effects of adjustment costs. This is quite a broad concept which covers costs arising for different reasons. Depending on the types of costs these may be external or internal, convex or concave. We have in this section stressed these differences in the functional form in which the adjustment costs enter into the firm's decision problem, rather than underlying economic reality in the form of monopsonistic factor markets, costs of decision making, costs of training new employees etc., which give rise to these characteristics. In the next section we will take a closer look at the special costs which arise out of imperfections in the capital market. These can only be formalized in adjustment cost terms in special cases, which do not include the case we will consider in the next section.

FINANCIAL RESTRICTIONS

It is often observed that one of the most important factors restricting the rate of expansion of the firm is its financing possibilities. The firm may perhaps not be subject to outright credit rationing, but at all instances its capital costs will rise with the amount of investment being undertaken. This observation stands in some contrast with the neoclassical paradigm where it is typically imagined, though in applications normally surrounded with reservations, that each firm can borrow whatever it needs at a fixed interest rate. This is a natural first assumption in a general equilibrium model in order to avoid the complexities of modelling an imperfect capital market.

In other model structures the natural way of avoiding these complexities may be to assume the total absence of capital markets.[1] This means that the firm has the choice between ploughing back its profits and giving dividends to its share-holders. This assumption rules out the possibility of any capital being invested in the company from outside, thereby begging the question of how companies get started. But it is not immediately clear that it is more unrealistic as a simplest possible assumption than assuming perfect capital markets. If we further make the admittedly arbitrary assumption that dividends are fixed at a certain percentage of profits it immediately follows that the rate of expansion of production is directly proportional to the level of profitability. The same general conclusion will of course still hold if we allow for loans and new equity from outside but require that these stand in some fixed proportion to the firm's equity.[2]

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[1] In evolutionary models this is the typical first assumption. See Winter (1975) p. 107.

[2] The assumption of a link between profits and investment in macro models goes back to Kalecki (1939) and is used in much of Cambridge, U.K. models. See also Wood (1975).

In this section we will, however, try to develop a model of the loan market that is somewhat less mechanical. The first step in the analysis will be to treat the supply of loans. We will show how, under certain conditions, the market interest rate can be expressed as a function of the size of the loan and the equity of the borrowing firm. The reason why we go into a formal derivation of this relation is that most existing dynamic models of firm behaviour under financial restrictions have simply made ad hoc assumptions about the nature of the financial constraints. These assumptions have in general been different from the relation derived here and must hence be based on other assumptions about the functioning of the loan market.

Our model of the supply of loans assumes a market imperfection arising out of a gap in information between the borrowing firm and the bank. The firm acts under subjective certainty about the future, whereas the bank faces a probability distribution of the firm's default risk. This is the only imperfection assumed. The loan market is competitive in the sense of the banks' expected profits on all types of loans being zero. There is no risk aversion involved.

A loan will in general incur default risk on the bank. This is a risk to the bank because the equity of a firm that goes bankrupt may be negative, and the bank will then have lost part of the loaned capital. The default risk will depend on the economic situation of the company, especially the size of the company's equity and the profitability of its investment opportunities. But it will of course also depend on the size and the interest rate of the loan offered.

The bank's supply of loans will then be an increasing function of the size of the equity of the borrowing firm and an increasing function of the market interest rate. In the absence of legal restrictions on the rate of interest charged there will in market equilibrium exist an array of interest rates depending on the combination of equity of the firm and the size of the loan. From the assumption of zero profits it follows that this market rate of interest in general increases with the size of the loan and decreases with the size of the firm's equity. In the special case of constant returns to scale of the firm's investments the market interest rate can be expressed as a function of the debt/equity ratio.

The assumption of constant returns to scale is made when we go over to study the firm's demand for loans. The firm is confronted with a dynamic maximization problem not too dissimilar from the one arising from costs of adjustment. Given its expectations of future price and cost developments it has to decide on how much of its earnings to give as dividends, how much to plough back into the firm and how much to borrow from outside. In doing this it must take account of the fact that the interest rate will rise with the size of the loan and with the amount of dividends paid, since the latter will reduce equity.

The Supply of Loans

Banks' supply of loans has been treated in the literature on credit rationing.[1] Let us consider the decision of a single bank treating the loan application from a firm. For simplicity we assume that all loans are given for one period of time, a year, only. At the beginning of the next year all loan conditions will be renegotiated. We will assume that the bank regards the borrowing firm's initial equity, E, as fixed and that the size of the firm's capital stock at the beginning of the period consequently is

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[1] See Hodgman (1960), Freimer and Gordon (1965), Jaffee and Modigliani (1969), Jaffee (1971), Koskela (1976) and for a recent survey Baltensperger (1978). The model used here is essentially that of Jaffee pp. 57-62.

$$K = E + M$$

where M is the size of the loan.

This is what in the literature is called open-end investment opportunities. In the credit-rationing discussion the case of a fixed-size investment opportunity, where K is fixed and the equity thus varies inversely with the loan, has been more frequently treated, see e.g. Jaffee and Modigliani (1969). That case is clearly not relevant here.

The value of the firm's assets at the end of the year, K_1, is uncertain. We assume that this can be written

$$K_1 = K\sigma(K)y$$

The rate of return, σy, is assumed to be affected by a deterministic component $\sigma(K)$ which adjusts the rate of return for the size of the investment. For $\sigma \equiv 1$ we have constant returns to scale, whereas $\sigma' < 0$ implies decreasing returns to scale. y is a stochastic variable with a density function $\phi(y)$. This can be regarded as representing the subjective probability distribution held by the bank. We assume that this is such that there exist subjectively certain minimum (v) and maximum (V) outcomes, i.e. that

$$\phi(y) = 0 \quad \text{for} \quad y \leq v \quad \text{and} \quad y \geq V, \quad \text{where} \quad v \geq 0$$

$$\int_v^V \phi(y)dy = 1$$

We also assume that there is nothing to prevent the firm from doing away with more than its equity. In reality there are bankruptcy laws intended to protect the interest of the lenders. It is clear, however, that these laws are not fully effectual in doing this. And it does not seem very unrealistic not to put any restrictions on the size of the end of period equity other than those made about the density function $\phi(y)$.

The bank is assumed to be neutral with respect to risk. This can be argued to be realistic on the ground that the bank is lending money to a large number of clients, the outcomes of whose investments are largely statistically independent. Further, assuming risk aversion would only serve to complicate the analysis but reinforce the conclusion.[1] Indeed, it may be a fairly striking result at first glance that the market interest rate depends on the size of the loan on a competitive market without risk aversion. The explanation of this is of course that the default risk will rise with the size of the loan and with the height of the interest rate, given the size of the equity. And hence, at a certain point it will simply not pay in terms of expected profits to increase the size of the loan offered.

The interest rate charged, which is treated by the bank as market determined, is r. The full contractual repayment $(1+r)M = RM$ will be made at the end of the period if $K_1 = K\sigma(K)y > RM$. If this is not so, the bank will take over all the firm's assets, K_1. We can then write the bank's expected profits as

$$\pi = MR \int_{\beta}^{V} \phi(y)dy + K\sigma \int_{v}^{\beta} y\phi(y)dy - IM \tag{3.42}$$

where we have defined $\sigma\beta = MR/K$ as the rate of return necessary for the firm to be able to pay back principal plus interest. The first term of (3.42) is the expected revenue provided that full repayment will be made. The second term is expected repayment when end-of-period assets fall short of RM. The third term is the opportunity cost to the bank of granting the loan, where we assume that the opportunity cost of capital is less than the market interest rate

$$I < R \tag{3.43}$$

[1] Risk aversion would mean that the default risk would be given extra weight. And hence it would be optimal for the bank to give a smaller loan.

By adding and subtracting $MR \int_v^\beta \phi(y)dy$ in (3.42) we get

$$\pi = M(R-I) - RM\Phi(\beta) + (E+M)\sigma \int_v^\beta y\phi(y)dy \qquad (3.44)$$

where we have substituted (E+M) for K. $\Phi(y)$ is the primitive function of $\phi(y)$, i.e. the cumulative function of the probability distribution. From (3.44) we see that profits will be zero if the interest rate is equal to the opportunity cost of capital (R=I) and the loan is sufficiently small as to make the probability of default zero ($\beta<v$). Then all terms of (3.44) are zero. With $R \leq I$, π can never be positive irrespective of the size of the loan, whereas with $R > I$ profits will be positive if the default probability is sufficiently small. The assumption (3.43) thus serves to make the problem formulation interesting by allowing positive profits for some M.

The market for loans is assumed to be competitive. This means that the profits on all types of loans are zero. This condition determines the market interest rate, marked by an asterisk below, as a function of the size of the loan and the equity of the borrowing firm.

By (3.44) zero profits implies

$$M(R^*-I) - R^*M\Phi(\beta) + (E+M)\sigma \int_v^\beta y\phi(y)dy = 0 \qquad (3.45)$$

It is obvious that this equation will have a unique solution R^* for any finite values of M and E, since by (3.44) $\partial\pi/\partial R > 0$, $\pi(R=0) = -IM < 0$ and $\pi \to \infty$ when $R \to \infty$. Implicit differentiation yields,

$$\frac{\partial R^*}{\partial M} = - \frac{\left[(E+M)\sigma' - \frac{E}{M}\sigma\right] \int_v^\beta y\phi(y)dy}{M\left(1 - \Phi(\beta)\right)} \qquad (3.46)$$

where I has been substituted away by (3.45).

This shows that the equilibrium interest rate will normally increase with the size of the loan. An exception to this would arise if there were strongly increasing returns to scale; with $\sigma' > 0$ the parenthesis in the numerator may be positive. The explanation of this effect is obvious. If there are increasing returns to scale, an increase in the size of the loan may make bankruptcy less likely instead of more likely.

The effect of an increase in equity is unambiguous

$$\frac{\partial R^*}{\partial E} = - \frac{\left(\sigma + (E+M)\sigma'\right) \int_v^\beta y\phi(y)dy}{M\left(1 - \Phi(\beta)\right)} < 0 \qquad (3.47)$$

The reader may wonder why there do not appear any marginal conditions. Does not the bank maximize expected profits by adjusting the size of the loan until the marginal profit from doing so goes down to zero? The answer to this question is that there are no decision parameters left for the bank in equilibrium. There is nothing that parallels the quantity variable of standard economic theory; varying the quantity loaned means that another good is sold which has other costs of production. In an ordinary equilibrium model with constant returns to scale the firm size is indeterminate and industry production is determined by quantity produced given the costs of production. Here the quantity of a good of given quality (R and E) sold by a single bank is indeterminate and the total quantity of such loans is determined by demand.[1]

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[1] It would be possible as an alternative to make the assumption that the interest rate is fixed, by convention or by usury laws. Then the bank would act as a monopolist and determine the size of the loan, i.e. the "quality" of the good sold, so as to maximize profits.

$$\partial\pi/\partial M = R\left(1-\Phi(\beta)\right) - I + (\sigma+K\sigma') \int_v^\beta y\phi(y)dy = 0$$

defines the optimal loan size, M^*, as a function of R and E, provided $\pi(M^*) \geq 0$. By implicit differentiation one can show that $\partial M^*/\partial E >$ > 0 unless there are strongly increasing returns to scale in which case one may again get an opposite effect.

An assumption of more or less rigid interest rates may seem to conform better with the stylized facts of real life than perfectly flexible interest rates. Certainly banks do not, and are often not allowed to, vary their interest rates widely between different customers. And certainly banks seem to practise credit rationing.[1] On the other hand it must be kept in mind that there are numerous other conditions of a loan contract that can be varied, e.g. amortization schemes, collateral, establishing of a customer relationship between the bank and the firm, etc. All these factors certainly call for an analysis of their own.[2] This would lead us too far here and we will instead stick to the assumption made that interest rates are adjusted freely.[3] Combining this with the assumption of perfect competition we have seen that the market interest rate a firm will have to pay is a function of the size of the loan and the size of the equity of the firm. Note that the interest is here only a function of the stocks of equity and loan, not of the rates of change. This means that it does not represent an adjustment cost similar to the case analyzed in the previous section.

To simplify the analysis of the next section we will now assume that the bank treats every loan applicant as though there are constant returns to scale of the firm's investments, i.e. we have $\sigma \equiv 1$ and $\sigma' = 0$. This means that the market interest rate can be expressed as a function only of the ratio between equity and borrowed capital, $r(E/M)$.[4] The properties of this function are

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1 This may even be consistent with profit-maximization if there are costs associated with the screening of customers to assess the probability distribution of their rates of return. On this see Jaffee and Russell (1976).

2 See e.g. Barro (1976) on the economics of collateral and Kane and Malkiel (1965) on why banks treat prime customers differently from non-prime customers.

3 The implications of a fixed interest rate and credit rationing have however been analyzed in some optimal control models. See e.g. Lesourne (1973) and Ylä-Liedenpohja (1976).

4 That this is so is easily ascertained by observing that $R'(E/M) = M\partial R^*/\partial E = -M^2/E \cdot \partial R^*/\partial M$. Remember also that $r = R-1$.

$$r'(E/M) = M\cdot\partial R^*/\partial E < 0 \tag{3.48}$$

$$r''(E/M) = M^2\cdot\partial^2 R^*/\partial E^2 =$$

$$= \frac{\partial\beta/\partial E}{M(1-\Phi)^2}\left[-\ (1-\Phi)\beta\phi\ -\ \phi\int_v^\beta y\phi(y)dy\right] > 0 \tag{3.49}$$

i.e. the market interest rate will be a falling convex function of the ratio of equity to debt.

The Demand for Loans

The analysis that follows depends crucially on the assumption that there is a gap in information between the bank and the borrowing firm. We have just seen how the bank's behaviour is governed by its subjectively held probabilities of the rate of return of the firm. We will now assume that these probabilities are completely unaffected by the parameters that enter into the firm's decision problem. In particular, following the main theme of this chapter, we will study the impact on the firm's borrowing and investment decisions of a price change. When doing this we will assume that the price change leaves the bank's assessment of the firm's future rate of return unaffected. Certainly this is to assume the banks being a bit more ignorant than they really are.[1] But it serves to focus the analysis on the

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[1] At least the bank will be able to read the firm's income statements. The profits figures given there should lead the bank to revise its probability distribution over the future, thereby functioning as signals in the terminology of Spence (1974). In such a case the link between current profits and the rate of investment would be strengthened. Not only will a price rise lead to increased investment because the marginal revenue product of capital rises, but also because the marginal cost of borrowing falls.

effects of the difference in information between the bank and the firm which certainly is important in reality. In order to simplify the analysis we will even assume that the firm has perfect knowledge, or at least subjectively certain expectations.

The impact of these particular loan market conditions on firm behaviour will be analyzed within two types of optimization models. We will first take the model with convex external adjustment costs[1] and introduce borrowing explicitly. It will then be seen that the properties of the interest rate function will affect the optimal level of investment. But it will not affect the partial derivative $\partial I/\partial p$. The second model studied is one where the maximand is a concave utility function. In this case the properties of the interest rate function will have an effect on $\partial I/\partial p$.

A Cost-of-Adjustment Model

As we pointed out it is generally assumed in the cost-of-adjustment model, like (3.6)-(3.8), that the capital market is perfect. The discount rate ρ can then be interpreted as the market rate of interest at which the firm can borrow and lend freely. This interest rate must also be thought of as the rate of return on equity demanded by the owners of the firm. There is in this model no need to distinguish between the firm's profit maximization and the owners' maximization of the dividends from the firm.

As soon as we give up the perfect capital market assumption there is no longer any reason why the interest rate at which the firm can borrow should be equal to the rate of return demanded by the owners. In such a case it makes a difference if we specify a problem where the owners maximize

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[1] It would be immaterial for the conclusions we will reach if we instead took the model with internal adjustment costs.

the discounted value of future dividends or a problem where the firm (the management) maximizes the discounted value of future profits. The latter case may be formalized by substituting the market interest rate for ρ in (3.6).

$$\text{Max} \int_{0}^{\infty} \left(K(pF-w\ell) - C(I)\right) e^{-r(E/M)t} dt \qquad (3.50)$$

Assuming that E is fixed this is a control problem with one state variable K (or M) and one control variable I. The necessary optimality conditions form a system of non-autonomous differential equations, i.e. a system where t enters explicitly. This implies that only under certain conditions will the system have any stationary point.[1]

Here we will instead assume that the owners maximize the discounted value of future dividends

$$\text{Max} \int_{0}^{\infty} D(t) e^{-\rho t} \, dt \qquad (3.51)$$

The discount rate is determined by the owners' alternative cost of capital. If the dividends are positive and the owners do not invest any new capital into the firm, this cost is the rate of return on alternative investment opportunities. It is not unreasonable to regard this rate as fixed. If on the other hand the owners are asked to invest capital in the firm ($D < 0$), they may have to go into the loan market themselves, and the more they invest the higher interest rate do they have to pay. This indicates that it may be natural to assume that ρ is a falling function of D or D/E. The latter assumption has been made in some steady-state models of firm growth; see Gordon (1962) and Eriksson (1975). As we show in Appendix A, p. 207, this assumption leads to a non-autonomous system of differential equations, which does not have any steady-state, at least not of the type analyzed in these studies.

[1] This would be so if there was a range where $r' = 0$, in which case the necessary conditions would degenerate to a system of (locally) autonomous differential equations.

Here we will instead make the simplifying assumption that ρ is fixed, but restrict our attention to cases when $D \geq 0$. Only then does it seem reasonable to treat ρ as a constant. We now define

$$D = K\{pF(1,\ell) - w\ell\} - C(I) - S \geq 0 \tag{3.52}$$

where the new symbol S stands for cash flow, amortization plus interest, from the firm to the bank. The firms' owners now maximize (3.51) under the restrictions of (3.52) and the two equations of motion

$$\dot{K} = I - \delta K \tag{3.53}$$

$$\dot{M} = rM - S \tag{3.54}$$

where $r = r(E/M)$; the properties of which are given by (3.48) and (3.49). This is a dynamic optimization problem with two state variables, K and M, and two control variables, I and S. By definition $E = K - M$. This means that when we are considering changes in one of the state variables, holding the other one constant, E will always be changed by the same amount. The problem is most conveniently handled by the maximum principle.[1] Form the present-value Hamiltonian

$$H = K\{pF(1,\ell) - w\ell\} - C(I) - S + \tilde{p}(I-\delta K) +$$

$$+ \tilde{\lambda}(rM-S) + \mu\{K(pF-w\ell) - C(I) - S\}$$

where $\tilde{p}$ and $\tilde{\lambda}$ are costate variables and μ a Kuhn-Tucker multiplier. The necessary conditions for a maximum are

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[1] For an introductory exposition see Arrow-Kurz (1970). My mathematical treatment here is not completely strict in order not to burden the exposition. I have, e.g., not stated the transversality conditions. Sufficient conditions are shown in appendix.

$$\frac{\partial H}{\partial I} = 0 \ ; \ \tilde{p} = (\mu+1)C'(I) \tag{3.55}$$

$$\frac{\partial H}{\partial S} = 0 \ ; \ \tilde{\lambda} = -1 - \mu \tag{3.56}$$

$$\dot{\tilde{p}} = \rho\tilde{p} - \frac{\partial H}{\partial K} = \tilde{p}(\rho+\delta) - (pF-w\ell) - \tilde{\lambda}r' \tag{3.57}$$

$$\dot{\tilde{\lambda}} = \rho\tilde{\lambda} - \frac{\partial H}{\partial M} = \tilde{\lambda}\left(\rho - \frac{\partial(r\cdot M)}{\partial M}\right) \tag{3.58}$$

$$\mu \geq 0 \quad \mu\left(K(pF-w\ell) - C(I) - S\right) = 0 \tag{3.59}$$

Apart from these dynamic conditions, we also have the static profit maximization condition

$$pF_{\ell} = w$$

This means that, for given p and w, the labour intensity ℓ will be constant along the solution path.

Let us now first proceed on the assumption that $D > 0$. By (3.59) this implies $\mu = 0$. We then see that, since $\tilde{\lambda}$ is constant by (3.56), (3.58) instantaneously determines the equity/debt ratio; $\dot{\tilde{\lambda}} = 0$ implies

$$\rho = \frac{\partial(rM)}{\partial M}$$

i.e. the ratio of equity to debt is adjusted so as to make the marginal interest rate equal to the discount rate ρ. This means that the equity/debt ratio is a constant along the solution path. Since $\tilde{\lambda}$ also is a constant, we can study the properties of the solution in a phase diagram in $\tilde{p}$-K space. From (3.57) we have

$$\tilde{p}\Big|_{\dot{\tilde{p}}=0} = \frac{pF - w\ell - r'}{(\rho + \delta)} \tag{3.60}$$

and by (3.55) and (3.53)

$$\left.\frac{\partial K}{\partial \tilde{p}}\right|_{\dot{K}=0} = \frac{1}{C''\delta} > 0 \tag{3.61}$$

Further

$$\frac{\partial \dot{\tilde{p}}}{\partial \tilde{p}} = \rho + \delta > 0 \tag{3.62}$$

and

$$\frac{\partial \dot{K}}{\partial \tilde{p}} = \frac{1}{C''} > 0 \tag{3.63}$$

We will hence get the phase diagram of Fig. 3.1.

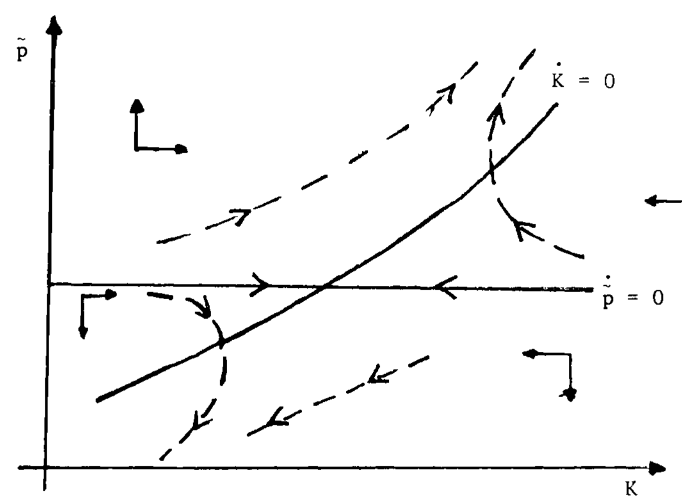

Figure 3.1

This phase diagram is the same as that which arises from the ordinary cost of adjustment model. The steady-state is a saddle point.[1] The diverging paths can be discarded since they do not fulfil the transversality conditions. There is hence a unique optimal path characterized by a constant $\tilde{p}$ and consequently a constant investment level.

That the model with a perfect capital market is only a special case of this model is seen by substituting $\rho = r$ into the optimality condition. We then see that the parenthesis of (3.58) is zero, i.e. the debt-equity ratio is now undetermined. Further we have from (3.55) and (3.60)

$$C'(I) = \frac{pF - w\ell}{\rho + \delta}$$

which is exactly the Euler equation (3.11). In our model this equation instead is

$$C'(I) = \frac{pF - w\ell - r'}{\rho + \delta} \tag{3.64}$$

This says that investment should be adjusted so as to make the discounted marginal benefits of a larger capital stock, net of depreciation, equal to the marginal cost of investment. These marginal benefits are here of two kinds: gross profits in production will rise and the interest rate will go down.

From this we see that the conditions in the capital market will affect the rate of expansion chosen by the firm. Comparing with the case of a perfect capital market, this implies that the growth rate is higher, which may seem counterintuitive. But this result is simply a consequence of the assumption that the discount rate is fixed whereas the interest rate paid on loans depends on the equity/debt ratio.

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[1] This is shown formally in Appendix A.

We have analyzed the necessary conditions on the assumption that $D > 0$. It is now time to see whether this is reasonable. Substituting from the optimality conditions into the definition of D, assuming that the Kuhn-Tucker multiplier is zero, we have

$$D = K\left(pF - w\ell\right) - C\left(\delta K^*\right) - r\left(\frac{M}{K}\right)^* K + \delta\left(\frac{M}{K}\right)^*\left(K^* - K\right) \quad (3.65)$$

Let us now assume $D^* > 0$, which certainly is a reasonable assumption if the problem is to be meaningful. Indeed if there is nothing to prevent the owners from withdrawing their equity, we must have $D^*/E^* \geq \rho$. $D^* > 0$ means

$$D^* = K^*\left(pF - w\ell\right) - C\left(\delta K^*\right) - r\left(\frac{M}{K}\right)^* K^* > 0 \quad (3.66)$$

Subtracting we have

$$D - D^* = (K - K^*)\left[(pF-w\ell) - \left(\frac{M}{K}\right)^*(\delta+r)\right]$$

and substituting from (3.66),

$$\frac{D - D^*}{K - K^*} > \frac{C(\delta K^*)}{K^*} - \delta\,\frac{M^*}{K^*} \geq 0 \quad (3.67)$$

The last inequality depends on an assumption not explicitly made before, namely that $C' \geq 1$, i.e. that the cost of acquiring an extra dollar's worth of capital goods is always at least one dollar. (3.67) then says that along the optimal path D will be above its steady-state value, and hence positive, if $K > K^*$. We can thus conclude that, under the assumptions made, D will be positive if the firm is contracting. But if K is below its steady-state value by a wide enough margin, D would be negative in the absence of the non-negativity restriction.

This concludes the discussion of the model (3.51)-(3.54). We have found that if the interest rate on loans to the firm is a function of the equity/debt ratio and the discount rate is fixed, the investment rate will, ceteris paribus, be higher than if the interest rate were fixed and equal to the discount rate. On the other hand, the link between the product price and investment will be unaffected, as is easily seen by differentiating (3.64) with respect to p and I and comparing this with (3.13). A reason for this implication is that the capital market imperfection introduced here does not represent an adjustment cost; r depends on the stocks of E and M, not on the flows $\dot{E}$ and $\dot{M}$.

A Managerial Model

Even if we keep the assumption that r is a function only of the stocks of E and M, the properties of this function may affect the link between the product price and investment if the maximand is assumed to be a *concave utility function of profits*. This is the case to which we now turn. We will interpret this as a model of a management-run company. The management derives utility from the dividends given to the share-holders. This is so, because if dividends are high the managers gain in prestige, their salaries may rise and so on, whereas, if dividends are low for a long time the managers may run the risk of losing their jobs. For high values of D it is likely that the marginal utility from increased dividends is quite low in comparison with the case when D is low. Hence, it is reasonable to assume that the utility function is concave in dividends. ρ is now the subjective rate of discount of the management. As in the previous model we will assume that $D \geq 0$. This is so because it is after all the owners' money that would be invested if $D < 0$. And while it may seem likely that the owners let the managers determine the size of the dividends as long as these are positive, it seems much less likely that they will uncritically invest new money in the company.

In order to keep the analysis reasonably simple we assume that there are no adjustment costs, i.e. $C(I) = I$. In order that the problem have a well determined steady-state, it is then necessary to assume that there are diminishing returns to scale. We do this by expressing gross profits, $pF(K,L) - wL$, as a function of price and the capital stock $\pi(K,p)$. The properties of this function follow from profit maximization.

Differentiating the first order condition $pF_L = w$ gives

$$\frac{\partial L}{\partial K} = -\frac{F_{LK}}{F_{LL}} \qquad \frac{\partial L}{\partial p} = -\frac{F_L}{pF_{LL}}$$

We then have the following derivatives of the gross profits function, where the sign of π_{KK} follows from the assumption of decreasing returns to scale.

$$\pi_K = pF_K > 0 \qquad \pi_p = F > 0$$

$$\pi_{KK} = p\,\frac{F_{KK}F_{LL} - F_{KL}^{2}}{F_{LL}} < 0$$

$$\pi_{pp} = -\frac{F_L^{2}}{pF_{LL}} > 0$$

$$\pi_{pK} = \frac{F_K F_{LL} - F_L F_{LK}}{F_{LL}} > 0$$

This gives us the maximization problem (3.68)-(3.71), where the properties of $\pi(K,p)$ are given above and those of $r(E/M)$ by (3.48) and (3.49).[1] We demand that the state va-

[1] The model is akin to that of Hochman et al (1973). The main difference is that they assume the interest rate to depend solely on the size of debt. This means that the interest rate is an increasing function of the size of equity, given the debt/equity ratio, which seems quite unrealistic.

riables be adjusted gradually. There is an exception to this, however, in that we allow the capital stock to be varied initially by varying the loan, M. This means that we require the equity to be fixed in the short run. This can be seen as a consequence of the assumption that the owners never invest any money in the company, $D \geq 0$. Note, however, that it constitutes an essential difference from the cost of adjustment model to allow the capital stock to make a sudden jump. The assumption that equity is fixed is attractive because it means that there will be no costs of making the initial adjustment; the maximand is a utility function in cash-flow to the share-holders and the initial shift will imply no cash-flow. We can then solve this dynamic maximization problem

$$\text{Max} \int_0^\infty U(D)e^{-\rho t}\, dt \tag{3.68}$$

$$U' > 0 \qquad U'' < 0$$

$$\dot{K} = I - \delta K \tag{3.69}$$

$$\dot{M} = rM - S \tag{3.70}$$

$$E = K - M$$

$$D = \pi(K,p) - I - S \geq 0 \tag{3.71}$$

Necessary conditions are obtained forming the present-value Hamiltonian

$$H = U(\pi-I-S) + \tilde{p}(I-\delta K) + \tilde{\lambda}(rM-S) + \mu\left(\pi(K,p)-I-S\right) \tag{3.72}$$

$$\frac{\partial H}{\partial I} = 0 \ ; \quad \tilde{p} = U' \tag{3.73}$$

$$\frac{\partial H}{\partial S} = 0 \ ; \quad \tilde{\lambda} = -U' \tag{3.74}$$

$$\dot{\tilde{p}} = \tilde{p}(\rho+\delta) - U'\pi_K - \tilde{\lambda}r' = \tilde{p}\big((\rho+\delta) - \pi_K + r'\big) \qquad (3.75)$$

$$\dot{\tilde{\lambda}} = \tilde{\lambda}\big(\rho - r + r'(1+E/M)\big) \qquad (3.76)$$

$$\mu\big(\pi(K,p) - I - S\big) = 0 \qquad \mu \geq 0 \qquad (3.77)$$

For the moment we assume that we are on the optimal path where the state variables only change gradually. For a discussion of the initial jump see p. 84 below. We further assume that D > 0. Since the two adjoint variables are equal, the properties of the optimal path can be illustrated in a phase diagram in $\tilde{p}$ - K space taking account of the interdependency between K and M. Since $\dot{\tilde{p}} = -\dot{\tilde{\lambda}}$ we have from (3.75) and (3.76)

$$\pi_K - \delta - r' = r - r'(1+E/M) \qquad (3.78)$$

On the left hand of the equality sign we have the marginal benefits of a larger capital stock: increased gross profits minus increased replacement investments plus lower interest costs. On the right hand we have the marginal costs of a larger loan in the form of a higher interest rate. These two should be equal. Simplifying we get

$$\pi_K - \delta = r - r'E/M \qquad (3.79)$$

By this we know the equity/debt ratio as a function of the capital stock along the optimal path. Differentiating this we get

$$\frac{\partial E/M}{\partial K} = - \frac{\pi_{KK}}{r''E/M} > 0$$

i.e. if the optimal path involves expansion (contraction) the equity/debt ratio will be rising (falling) all along the way towards the steady-state. In calculating slopes of phase-lines etc., equation (3.79) must always be taken into account.

We start with the phase-line for $\dot{K} = 0$. This fulfils (3.80) where $\dot{K} = \dot{M} = 0$ have been substituted into (3.73)

$$U'(\pi - \delta K - rM) = \tilde{p} \qquad (3.80)$$

Differentiating (3.78) and (3.80) gives

$$\begin{bmatrix} \pi_{KK} + r''E/M^2 & -r''(1+E/M)E/M^2 \\ U''\left(r - r'(1+E/M)\right) & U''\left(r'(1+E/M) - r\right) \end{bmatrix} \begin{bmatrix} dK \\ dM \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tilde{p}$$

where we have substituted from (3.79) into the differential of (3.80). By Cramer's rule we then have

$$\left.\frac{\partial K}{\partial \tilde{p}}\right|_{\dot{K}=0} = \frac{r''(1+E/M)E/M^2}{U''\left(r'(1+E/M) - r\right)\left(\pi_{KK} - r''E^2/M^3\right)} < 0 \qquad (3.81)$$

$\dot{\tilde{p}} = 0$ gives by (3.75)

$$\pi_K = \rho + \delta + r' \qquad (3.82)$$

and the value of K for $\dot{\tilde{p}} = 0$ is given by (3.79) and (3.82).

Next we differentiate (3.75) and (3.79) which gives

$$\begin{bmatrix} 1 & \tilde{p}r''1/M(1+E/M) \\ 0 & r''E/M^2(1+E/M) \end{bmatrix} \begin{bmatrix} d\dot{\tilde{p}} \\ dM \end{bmatrix} = \begin{bmatrix} \tilde{p}(r''1/M - \pi_{KK}) \\ r''E/M^2 + \pi_{KK} \end{bmatrix} dK$$

and

$$\frac{\partial \dot{\tilde{p}}}{\partial K} = -\tilde{p}\pi_{KK}(1+M/E) > 0 \qquad (3.83)$$

Finally we must derive $\partial\dot{K}/\partial\tilde{p}$. To do this we differentiate (3.79) with respect to time and substitute from (3.69) and (3.70)

$$(\pi_{KK} + r''E/M^2)(I - \delta K) - r''(1 + E/M)E/M^2(rM - S) = 0$$

Differentiating this and (3.73) yields

$$\begin{bmatrix} -U'' & -U'' \\ \pi_{KK} + r''E/M^2 & r''(1+E/M)E/M^2 \end{bmatrix} \begin{bmatrix} dI \\ dS \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} d\tilde{p}$$

and

$$\frac{\partial I}{\partial \tilde{p}} = \frac{\partial \dot{K}}{\partial \tilde{p}} = \frac{r''E/M^2(1 + E/M)}{U''(\pi_{KK} - r''E^2/M^3)} > 0 \qquad (3.84)$$

By (3.81)-(3.84) we can now draw a phase diagram, Fig. 3.2.

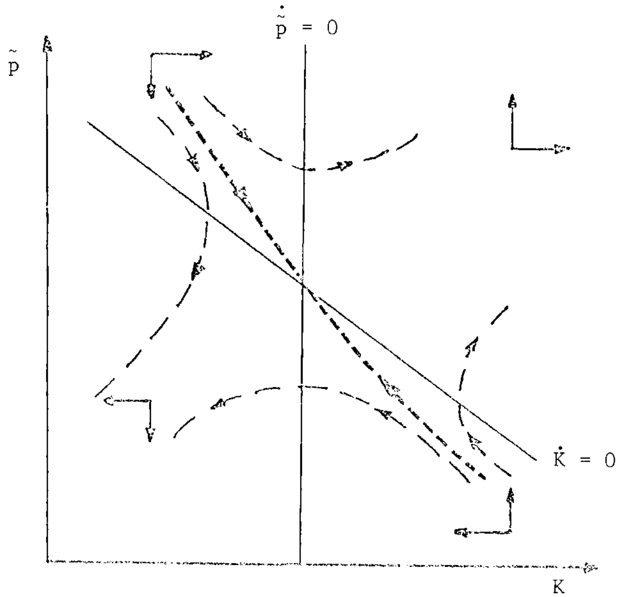

Figure 3.2

This shows that the steady-state is a saddle point. All paths but one that satisfy the necessary conditions diverge and can be discarded on the ground that they do not satisfy the transversality conditions.

Let us now see what happens if there is a sudden change in the product price, p. From (3.79) we see that this implies that either or both of the capital stock and the equity/debt ratio will be changed instantaneously. By the assumption that equity is fixed, we have

$$\left.\frac{\partial K_o}{\partial p}\right|_{dK_o = dM_o} = \frac{\pi_{Kp}}{r''E^2/M^3 - \pi_{KK}} > 0 \qquad (3.85)$$

The instantaneous effect of a price increase will be an increase of the capital stock accomplished by a debt increase. The size of this effect depends on the properties of the capital market as they affect the size of r". The effect will be larger if the capital market is close to perfect ($r'' \approx 0$). With a constant marginal interest rate ($r'' = 0$) the instantaneous effect (3.85) will be equal to the steady-state effect (3.86). This is a result that we should expect. With an imperfect capital market the loan increase will be kept back by the fact that the interest rate will go up. Let us now regard the effects on the new steady-state.

Differentiating (3.79) and (3.82) gives, where asterisks denote steady-state values,

$$\begin{bmatrix} \pi_{KK} & -r'' \\ \pi_{KK} & r''E/M \end{bmatrix} \begin{bmatrix} dK^* \\ dE^*/M^* \end{bmatrix} = \begin{bmatrix} -\pi_{Kp} \\ -\pi_{Kp} \end{bmatrix} dp$$

$$\frac{\partial K^*}{\partial p} = - \frac{\pi_{Kp}}{\pi_{KK}} > 0 \qquad (3.86)$$

$$\frac{\partial E^*/M^*}{\partial p} = 0 \qquad (3.87)$$

which shows that the steady-state capital stock will increase with the product price. The size of this effect solely depends on the properties of the production function and has nothing to do with the conditions of the capital market. The equity/debt ratio is in the long run always adjusted so as to make the marginal interest rate equal to the discount rate, as is easily seen from (3.76) if $\dot{\lambda} = 0$. It is hence independent of the product price.

Since equity is fixed in the short run the capital stock will increase instantaneously. Comparing (3.85) and (3.86), we see that

$$\frac{\partial K^*}{\partial p} > \left.\frac{\partial K_o}{\partial p}\right|_{dK_o = dM_o}$$

This means that if the price change occurs in steady-state or in a phase of expansion of the capital stock, the adjustment path after the initial shift will be one of continuous expansion. The opposite case may occur, however, if the firm is initially in a phase of disinvestment. The price increase may then lead it to increase the capital stock, by a sudden increase in debt, but after that reenter in a process of continuous contraction.

Both the models we have studied have the property that the steady-state value of the equity/debt ratio is determined by the discount rate and the properties of the interest rate function. Given this value of E^*/M^*, conditions in the markets for inputs and output determine K^*. In the cost-of-adjustment model, this separation can also be made outside steady-state. This is not so in the managerial model. Here we see that the size of the instantaneous adjustment of K, which is effected by changing M, depends on the properties of $r(E/M)$. The closer this is to being linear, the larger share of the total adjustment of the capital stock will be made instantaneously.

The different adjustment patterns are illustrated in Figs. 3.3 (the cost-of-adjustment model) and 3.4 (the managerial model). In the former case we are interested in the change in slope of the curve $(\partial I^*/\partial p)$. In the latter case we look at the size of the jump $(\partial K_o/\partial p)$. Though the latter is a shift effect we may, somewhat loosely, translate it into our α_q coefficient.

Both the models studied have been models with two production factors, labour and fixed capital. Consequently, the results have been interpreted in terms of the impact on the rate of expansion. In reality, however, capital market conditions probably have a greater impact on the rate of investment in R & D, since these are judged riskier by the banks. Hence, to the extent that the results of this section indicate that $\alpha_q > 0$, they also indicate that $\alpha_c < 0$.

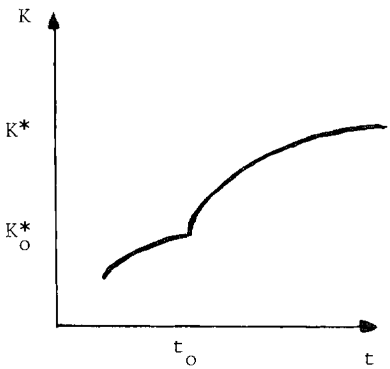

Figure 3.3

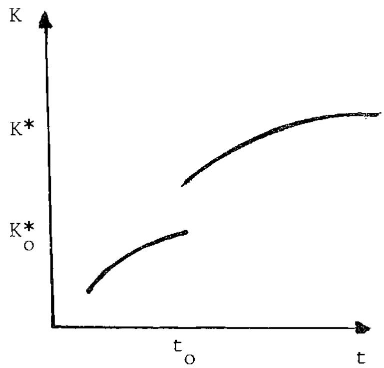

Figure 3.4

This concludes the discussion of the impact of capital market imperfections on the firm's investment behaviour. We have focused on a special type of imperfection, basically arising from a difference in information between the bank and the firm. We have excluded sticky interest rates and monopolistic conditions in the loan market from the analysis. Though both these phenomena certainly exist, they are perhaps of less fundamental importance. We have also excluded the impact of risk aversion by assuming the bank to be risk-neutral and the firm to act under subjective certainty. A world with uncertainty but equal information to all investors may be analyzed by the capital asset pricing model due to Lintner, Mossin and Sharpe.[1] In order to get anything out of such a model we would, however, need some hypothesis as to how a price change affects the uncertainty of future profits.

A MANAGERIAL MODEL OF COST REDUCTION

Newspaper accounts of what happens when a company runs into a crisis often focus on the cutting down on conspicuous expenses. One may recall the case of the leading Swedish producer of mechanical calculators, which over a range of years gave economic support to the local football team. When the electronic calculators entered the market the company incurred heavy losses and the support to the football team was taken away; within four years the former Swedish champions were relegated to a lower division.

Such examples are legion, and I do not think one can question that the example just given is fairly representative. But its quantitative importance may of course be questioned. For the overall rate of cost reduction it was much more important that the crisis forced the company to learn and introduce the new technique of electronic calculators faster than would otherwise have been the case.

A natural frame of reference if one wants to model this type of behaviour in maximizing terms is Williamson's work on managerial objectives in the theory of the firm.[2] As in the managerial model of financial decisions above, it is assumed that the management is strong enough in comparison with the owners to be able to pursue its own goals. This may only be compatible with survival in an industry where competition is not perfect in the standard sense. The management

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1 See Mossin (1973) for a textbook exposition.

2 See in particular his dissertation, Williamson (1964).

then maximizes a utlity function where profits are only one of many arguments. A prototype of this is

$$U(S,\pi),\ \frac{\partial U}{\partial S} > 0 \quad \frac{\partial U}{\partial \pi} > 0 \quad \frac{\partial^2 U}{\partial \pi^2} < 0 \quad \frac{\partial^2 U}{\partial S^2} < 0 \quad \frac{\partial^2 U}{\partial S \partial \pi} \geq 0 \qquad (3.88)$$

where S is short for staff. Williamson regards the expansion of staff beyond what is motivated by profitability considerations as typical of the modern management-run company. But of course we can let S represent any kind of "unproductive" expenses such as football teams.

Profits are defined by

$$\pi = R - S \qquad (3.89)$$

where R are profits derived from ordinary production and it is assumed that staff is a pure expense with no effect whatever on production.

The first order condition for a maximum of (3.88) subject to (3.89) is

$$\frac{\partial U}{\partial S} = \frac{\partial U}{\partial \pi}$$

This implicitly defines the optimal staff as a function of profits net of staff expenses, $S^*(R)$. Differentiating this gives

$$\frac{dS^*}{dR} = \frac{\frac{\partial^2 U}{\partial \pi^2} - \frac{\partial^2 U}{\partial S \partial \pi}}{\frac{\partial^2 U}{\partial \pi^2} + \frac{\partial^2 U}{\partial S^2} - 2\frac{\partial^2 U}{\partial S \partial \pi}} > 0$$

With the assumptions made - diminishing marginal utility both to profits and to staff, non-negative cross-effects and an interior maximum - it is clear that a rise in profits net of staff leads to the employment of a larger staff and to a decrease in productivity as customarily understood.

This is however a purely static effect, the level of profits determines the level of costs, and a change in the level of profits will have no effect whatever on the rate of change of costs. But just as the typical management derives utility from certain types of expenses such as a large staff, it is indeed likely that the rate of change of costs affects utility. The process of cutting costs will typically involve organizational changes, retraining of certain groups of employees, search for the implementation of new techniques of production, etc. All of this will probably in general be regarded as cumbersome and unpleasant by the management. This setting can be formalized by introducing activities leading to productivity change as an argument in the utility function. Since the rate of cost cutting of today will affect the levels of costs in the future, the maximand will now be the integral of discounted utility over all future.

We assume a production function with two inputs: labour (L), which is freely mobile, and "knowledge" (a), which can only be changed at a cost. The rate of change of the stock of "knowledge" is what we assume to be associated with disutility. We will label it "effort". To simplify the analysis we assume that there are no costs in monetary terms associated with "effort". It only enters as an argument in the utility function. We assume that the production function is separable in a and L and exhibits decreasing returns with respect to labour and knowledge, but we do not make any assumption about returns to scale. Profits can hence be written

$$\pi = pG(a)F(L) - wL \qquad F' > 0 \quad F'' < 0 \qquad (3.90)$$
$$G' > 0 \quad G'' < 0$$

and the maximand is

$$\int_{0}^{\infty} U\left(\pi(a,L),\dot{a}\right)e^{-\rho t}dt \tag{3.91}$$

where we assume the marginal utility of profits to be positive but decreasing ($U_1 > 0$, $U_{11} < 0$) and the marginal utility of "effort" to be negative and decreasing, i.e. the marginal disutility is increasing ($U_2 < 0$, $U_{22} < 0$). It is difficult to make any a priori assumption about the cross effect (U_{12}), so we will simplify the analysis by assuming $U_{12} = 0$, i.e. that the utility function is separable.

This is an optimal control problem with one state variable a and one control variable $\dot{a}$. It can be solved by the maximum principle. We form the Hamiltonian

$$H = U(\pi,\dot{a}) + \tilde{p}\dot{a} \tag{3.92}$$

where $\tilde{p}$ is the adjoint variable and π is given from (3.90). By ordinary static profit maximization we have the optimal input of labour, L*

$$F'(L^*) = \frac{w}{pG(a)} \tag{3.93}$$

and

$$\frac{\partial L^*}{\partial a} = -\frac{F'G'}{F''G} > 0$$

Before proceeding, we should check if an increase of a will lead to a fall in unit costs, since the aim of the exercise of this section is to throw some light on the cost reduction coefficient α_c. Unit costs are defined by

$$c = \frac{wL}{F(L)G(a)}$$

The partial derivative of this item with respect to a, taking (3.93) into account, is

$$\frac{\partial c}{\partial a} = \frac{w}{(GF)^2}\left[GF\,\frac{\partial L^*}{\partial a} - L\left(G'F + GF'\,\frac{\partial L^*}{\partial a}\right)\right] =$$

$$= \frac{wG'}{(GF)^2}\left[\frac{-FF'}{F''} + \frac{LF'^2}{F''} - LF\right]$$

It is certainly not possible to guarantee that this is negative in general. For $F = L^{\alpha}$, e.g., we get $\partial c/\partial a = 0$. This will also be so for any production function that is linearly homogeneous in a and L.

An obvious case when $\partial c/\partial a$ in general will be negative is when there is some fixed factor of production. We might e.g. assume that there are costs associated with changing the input of L. Another possibility, which we will now analyze briefly, is that F(L) really is the function of two inputs where the other one, which may be thought of as management, is fixed. Assuming F to be a CES production function we have

$$F(L) = \left(A + L^{-\rho}\right)^{-1/\rho}$$

where the elasticity of substitution is $1/(1+\rho)$. After some algebra we then get

$$\frac{\partial c}{\partial a} = \frac{wG'\rho L^{-(1+\rho)}}{(GF)^2F''}\left(A + L^{-\rho}\right)^{-(1+2/\rho)}\left[1 - \left(A + L^{-\rho}\right)^{-1} L^{-\rho}\right] < 0$$

This expression is obviously negative for any $A > 0$ irrespective of the value of ρ, except when $\rho=0$.[1] We have by this shown that when there is a fixed factor of production and a constant elasticity of substitution between this and labour, we are entitled to translate a partial derivative of

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1 In this case the production function is of Cobb-Douglas type. Then we get $\partial c/\partial a = 0$.

"effort" with respect to price, $\partial\dot{a}/\partial p$, into a partial derivative of the rate of change of unit costs, $\partial\dot{c}/\partial p$.

We can then go back to the maximization problem (3.91). The two necessary conditions for a maximum with respect to $\dot{a}$ are

$$\frac{\partial H}{\partial \dot{a}} = U_2 + \tilde{p} = 0; \quad \tilde{p} = - U_2 \tag{3.94}$$

$$\dot{\tilde{p}} = \rho\tilde{p} - \frac{\partial H}{\partial a} = \rho\tilde{p} - U_1 pFG'(a) \tag{3.95}$$

Sufficient conditions are derived in Appendix A, p. 210.

With the assumptions made we can now draw a phase diagram in a - $\tilde{p}$ space. The optimality conditions (3.94) and (3.95) imply

$$\tilde{p}\Big|_{\dot{a}=0} = - U_2(\pi,0) = \Phi(a,p)$$

$$\tilde{p}\Big|_{\dot{\tilde{p}}=0} = \frac{U_1 pFG'}{\rho} = \Psi(a,p)$$

$$\frac{\partial\Phi}{\partial a} = 0$$

$$\frac{\partial\Psi}{\partial a} = \frac{U_1 p}{\rho}\left(G''F - \frac{(G'F')^2}{GF''}\right) + \frac{U_{11}}{\rho}(pG'F)^2 < 0$$

$\partial\Phi/\partial a$ is zero because we have assumed the cross-derivative to be zero. The sign of $\partial\Psi/\partial a$ follows from the sufficient condition for a maximum
Further

$$\frac{\partial\dot{\tilde{p}}}{\partial\tilde{p}} = \rho > 0$$

$$\frac{\partial \dot{a}}{\partial \tilde{p}} = - \frac{1}{U_{22}} > 0$$

The phase diagram will then look like Figure 3.5.

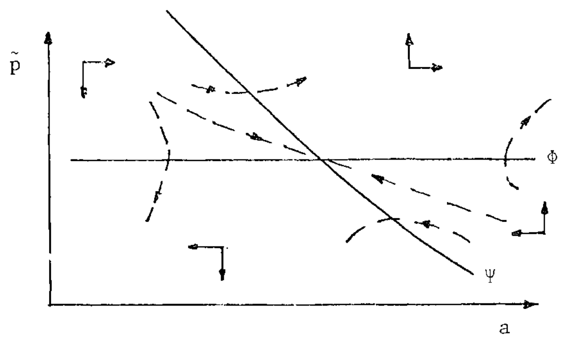

Figure 3.5

The figure shows that there is a unique optimal path satisfying the transversality conditions. There is an equilibrium level of a which will be approached monotonically in the long run. From (3.95) we see that in steady-state

$$U_2(\pi^*,0) = \frac{U_1(\pi^*,0)}{\rho} pFG'(a^*)$$

This says that the marginal disutility of increasing "effort" should in the steady-state be equal to the discounted marginal utility of having a larger capital stock.

Let us now make our usual experiment and regard the effect of a change of the exogenously given product price, p. We then have

$$\frac{\partial \Phi}{\partial p} = 0$$

$$\frac{\partial \Psi}{\partial p} = \frac{U_1 G' \left(F - \frac{F'^2}{F''}\right) + U_{11} p F^2 G G'}{\rho} \gtrless 0$$

The price change will thus leave Φ, the curve for $\dot{a} = 0$, unchanged, whereas Ψ, the curve for $\dot{\tilde{p}} = 0$, in general will change. The direction of this change is ambiguous, however. As is seen, the sign depends on several factors. Apart from the properties of the production function, it depends on the curvature of the utility function. If the positive first partial derivative of U with respect to π is large in absolute value relative to the negative second derivative we have $\partial\Psi/\partial p > 0$. Figure 3.6 illustrates the opposite case, where the second derivative is strongly negative and $\partial\Psi/\partial p < 0$.

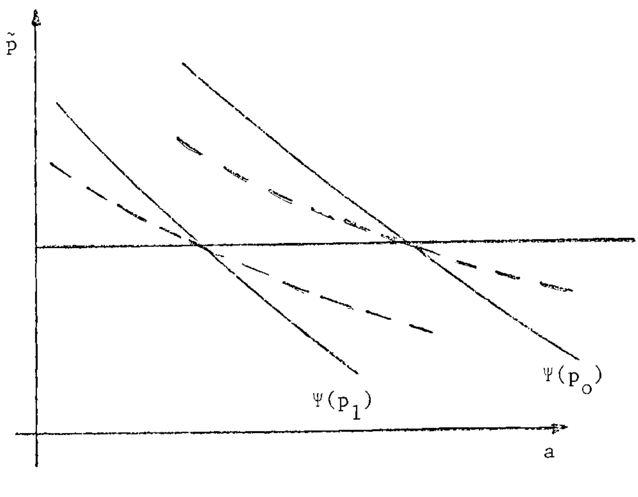

Figure 3.6 $p_1 > p_o$

In this case, since a is fixed in the short run, an increased price leads to a fall in $\tilde{p}$ and hence, by the optimality condition (3.94), to a fall in $\dot{a}$. The equilibrium level reached in the long run will also fall as a result of the price increase. In the opposite case, $\partial\Psi/\partial p > 0$, a price increase will instead reduce the instantaneous rate of cost reduction and raise the long-run level of unit costs.

The interpretation of this result is that there are two forces at work. On the one hand, a price increase means that the gains from a high level of "knowledge" will increase. This can be seen as one interpretation of the suggestion due to Schmookler (1962), among others, that the rate of technical progress will increase with demand. On the other hand the price increase leads by itself to increased profits, which means that the marginal utility of profits will fall. This means that, ceteris paribus, there will be a tendency for the firm to put in less "effort". Slack and X-inefficiency will, hence, be growing.

CONCLUDING COMMENTS

In this chapter we have studied the dynamic behaviour of individual firms in response to changed market conditions. Throughout the chapter firm behaviour has been modelled in maximizing terms. In the introductory chapter we discussed the pros and cons of such assumptions in general and profit maximization assumptions in particular. We claimed that a main reason why maximization is such a dominant assumption is that it helps to facilitate the analysis of otherwise very complicated problems. When it comes to problems of dynamic adjustment these advantages are not equally obvious.

To analyze the dynamics of a model where firms simultaneously decide on expansion and cost reduction and this interacts with demand is a difficult task. From one angle this chapter can be seen as an attempt to elucidate how difficult this would be. The models we have considered try to take up some of the informal arguments lying behind the rough behavioural assumptions (3.1) and (3.2). In most models in this chapter we have treated the product price as exogenously given, and constant over all future. Any interaction between firm decisions and demand is absent; we have only been concerned with firm behaviour, not with the theory of the market.[1] Despite this and despite the simplification deriving from our not considering cost and quantity adjustments in the same model, the treatment of the simple questions considered in this chapter is at times quite involved.

I hope however that this chapter also shows some of the advantages of maximization models. If profit maximization is accepted as the "normal" behavioural assumption, all content of a particular model lies either in the particular restrictions imposed on the maximization problem or in the deviation from profit maximization resulting from the introduction of a specific utility function. In this chapter we have investigated the consequences of restrictions arising from adjustment costs and imperfect information in the capital market. And we have studied the effects of utility functions that are concave in dividends or profits and which attach disutility to activities conducive to cost reduction. The conclusions of these models have provided us with some interpretations of the reaction coefficients α_c and α_q of the model to be analyzed in the next chapter.

The primary focus of interest has been on the effect of a sudden price change. Depending on the particular model under study this will instantaneously affect either or both the *rate of change* and the *level* of pro-

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[1] The model of a group of monopolists with lump-sum adjustment costs is, at least partially, an exception to this statement. See pp. 53-61.

duction (or costs). The latter case when there is a sudden jump in the variable means that the corresponding reaction coefficient will assume an infinite value. But it should be possible to interpret these coefficients as representing some kind of average reaction in a world with stochastic differences between companies as is done in the model with lump-sum adjustment costs. This model shows, however, that it is not in general admissible to identify the size of the reaction, e.g. the jump size of (3.38), with the expected adjustment per unit of time. Consequently, one should be a bit wary to translate e.g. the shift effects arising from an imperfect capital market (3.85) to α_q.

There is a more fundamental reason to be cautious in translating the different partial derivatives of this chapter into the reaction coefficients. Dynamic maximization models picture a much more sophisticated firm behaviour than the crude assumptions (3.1) and (3.2). In this chapter firms decide about the optimal path over the entire future. This may involve rapid initial expansion which gradually slows down as the steady-state is approached. Or it may involve heavy investment but little expansion of production at first and a more rapid expansion later on. Such intertemporal considerations are certainly not pictured by our simple behavioural assumptions. Still I believe that there is some justification in using the models of this chapter for throwing light on factors behind the reaction parameters α_c and α_q. This will be done more systematically in the introduction to the next chapter.

Finally, it should be noted that the models gathered in this chapter do not cover more than a few of the mechanisms that will affect α_c and α_q. Among the conspicuous omissions is the investigation of other utility functions, e.g. maximization of growth under some non-negative profits restriction. Another omission is the impact of uncertainty and the presence of risk aversion. A third omission is the analysis of the research and development process.

4. THE BASIC MODEL

In this chapter we will analyze the model that was put forth at the end of chapter 2. It is restated here for convenience.

$$\hat{q}_t = \frac{\dot{q}_t}{q_t} = \alpha_q \pi_t + \delta_q \tag{4.1}$$

$$\hat{c}_t = \frac{\dot{c}_t}{c_t} = \alpha_c \pi_t - \delta_c \tag{4.2}$$

$$p_t = a q_t^{-\gamma} e^{\delta_p t} \tag{4.3}$$

which gives

$$\hat{p}_t = \frac{\dot{p}_t}{p_t} = -\gamma \hat{q}_t + \delta_p \tag{4.4}$$

π_t is defined by

$$\pi_t = \frac{p_t - c_t}{c_t} \tag{4.5}$$

$$\alpha_c \gtrless 0 \quad \alpha_q > 0 \quad \gamma > 0 \quad \delta_c,\ \delta_q,\ \delta_p \gtrless 0$$

c_t = unit cost at time t

q_t = quantity produced

p_t = unit price

π_t = profit margin

The model applies at the level of an industry producing a single homogeneous good. (4.1) and (4.2) represent the behavioural assumptions, and (4.3) is a market equilibrium condition. The model is formulated in continuous rather than discrete time, since it will be treated analytically and not by numerical simulation.

This model builds on a number of restrictive assumptions: (i) it is formulated directly on the industry level, i.e. we are disregarding all aggregation problems; (ii) behaviour is guided by current profits, i.e. problems of expectations formation are disregarded; (iii) there is no interrelatedness between unit costs and quantity produced other than that they both depend on profits; (iv) the model is linear.

These assumptions help to render the model mathematically very tractable. It is possible to find the explicit solution to the system of differential equations. This will give us the values of the endogenous variables - q, c, p and π - as functions of the initial values of these variables and time. When, in the next chapter, we remove some of the simplifying assumptions this is in general no longer possible. We can then only study general dynamic properties of the system and the steady-state values of the variables.

The analysis in this chapter will proceed as follows. First the interpretation of the parameters of the model will be discussed. Then the general features of the model will be outlined. Attention will be concentrated on the stability properties of the system and on the steady-state values of the variables. In particular we will look at the determinants of the steady-state profit margin. After that we will present the explicit solution to the system of differential equations. Using this, we will study the effects all along the solution path of varying initial conditions and different parameters.

THE EQUATIONS OF THE MODEL

In assuming that the behavioural equations are applicable to a whole industry we have assumed away any discussion of systematic differences between firms within the same industry; if there are more firms than one, they must all be equal. We can however allow the entry of new firms by making the following simple assumption, which fits well with the mathematical structure of (4.1): the production increase stemming from new firms, expressed as a fraction of the level of production from existing firms, is a linear function of the profitability of the existing firms.[1] The expansion of existing firms is then

$$\frac{\dot{q}_i}{q_i} = \alpha_{qi}\pi_i + \delta_{qi} \tag{4.6}$$

and that of new firms[2]

$$\frac{\dot{q}_j}{q_i} = \alpha_{qj}\pi_i + \delta_{qj} \tag{4.7}$$

It follows immediately that we can express the coefficients of (4.1) as sums of the coefficients of (4.6) and (4.7)

$$\alpha_q = \alpha_{qi} + \alpha_{qj}, \quad \delta_q = \delta_{qi} + \delta_{qj}$$

As soon as the new firms have entered we assume that they behave exactly as all other firms. This means that the parameters of the cost reduction equation (4.2) simply reflect the behaviour of existing firms.

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1 The empirical support for this assumption is discussed briefly in Chapter 6.

2 Note that there is no exit from the industry. Since the behaviour of existing firms is described by (4.6) these will only contract gradually and never exit abruptly.

Let us now discuss what lies behind the parameters of the behavioural equations. The *expansion reaction coefficient* α_q (= $\alpha_{qi} + \alpha_{qj}$) will normally be positive. This will be assumed throughout this chapter, mainly in order to keep the number of possible alternative cases down. To start with the behaviour of new entrants, α_{qj} should most certainly be positive. The higher this parameter is, the more competitive, in the standard sense of the word, may we say that the industry is, for a high value of α_{qj} means that many firms will enter in response to a slight increase in profitability.

As regards the behaviour of existing firms, the analysis of the previous chapter indicates a number of factors that will have an impact on α_{qi}. Adjustment costs, be they convex or concave, are important. Their importance will of course depend on the intensity of the production factor that is not freely adjustable; it is an observation that goes back to Marshallian dynamics that adjustment will be slower in capital intensive industries. Furthermore, there are the effects stemming from imperfections in the capital market. As was shown in Chapter 3, we cannot generally infer from the fact that the interest rate depends on the equity/debt ratio that changed profits will lead to a changed rate of expansion. In the main this will be so if the utility function is concave in profits, otherwise not.[1] In this case the size of α_q will depend positively on the second-derivative of the interest rate as a function of the equity/debt ratio. The size of α_q of course also depends on whether management conceives of any effects on price of its output decisions as was the case in the monopolistic adjustment model.

The parameter representing the *rate of spontaneous expansion*[2] δ_q can have any sign. It is perhaps natural to

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[1] See Chapter 3 pp. 70-88.

[2] The term is perhaps not ideal. But a term like trendwise expansion would be misleading since profits even in the long run may deviate from zero.

assume that no new firms enter at zero profits, at least if all potential entrants expect the present level to hold forever. Looking at existing firms, we found in Chapter 3 that if there are increasing marginal costs of gross investment, there will be contraction when $\pi = 0$.[1] On the other hand there are also special costs associated with reducing the scale of production, such as costs for dismissing employees etc. On balance there may perhaps be a tendency to keep the number of employees constant when profits are zero. If there is technical progress, such behaviour implies $\delta_q > 0$. Finally, it must be recognized that it is difficult to maintain the hypothesis that δ_q is independent of the other parameters of the model, in particular δ_c and δ_p. We will come back to that issue later in this chapter when we discuss the conditions which make the steady-state profit margin equal to zero.

The *cost reaction coefficient*, α_c, can be either positive or negative. According to one view, associated with "the behavioural theory of the firm", high profits tend to make firms less eager to devote effort to search for more productive techniques of production. We tried to picture this hypothesis in the model of a firm maximizing a concave utility function of profits and "effort".[2] We found that if this function is strongly concave in profits, higher prices will lead to slower cost reduction. If this is so, α_c will be positive. If the utility function is close to being linear, the opposite effect follows. The intensity of search for new techniques will then increase with profits, and α_c will tend to be negative.

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[1] This can be inferred from the market equilibrium model of pp. 51-52. There we showed that production will be constant for $\pi = \pi^* > 0$. Since we know that the rate of expansion is increasing with π there must be contraction for $\pi = 0$.

[2] See Chapter 3 pp. 88-96.

Furthermore, productivity increases normally stem largely from R & D, and the conditions in the capital market will be equally important as is the case for fixed investments. Probably the imperfections are stronger for R & D investments, which are normally judged to be riskier than investments in building and machinery. If this effect via the imperfect capital market is strong, α_c will be negative. The same caveat applies with respect to this conclusion, however, as with respect to the impact of capital market conditions on α_q.

δ_c, the rate of *spontaneous cost reduction*, can be interpreted by assuming that the rate of productivity increase in the economy in general is reflected in the development of the factor prices of the industry analyzed. If this is so, δ_c will be positive (negative) if, at zero profits, productivity increases faster (slower) in this industry than in the economy as a whole.

The inverse *demand equation* (4.3) can be thought of as a reduced form from a larger system of demand equations. The choice of functional form follows from the wish to make the model linear in rates of change. To illustrate the interpretation of the *inverse demand elasticity* γ, let us consider this two-equation system,[1]

$$q_1 = Q^{\beta_1} p_1^{-\gamma_{11}} p_2^{\gamma_{12}}$$

$$q_2 = Q^{\beta_2} p_1^{\gamma_{21}} p_2^{-\gamma_{22}}$$

where Q is the income, β_i are the income elasticities, and γ_{ij} the own-price and cross-price elasticities. Transforming

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[1] This should be regarded as a partial system of demand equations. We will hence allow the income elasticities β_i to deviate from unity.

the system to rates of change and treating the quantities produced as exogenous variables, we obtain the reduced form equation[1]

$$\hat{p}_1 = \frac{\gamma_{22}}{\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22}} \hat{q}_1 - \frac{\beta_2\gamma_{12} + \beta_1\gamma_{22}}{\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22}} \hat{Q} +$$

$$+ \frac{\gamma_{12}}{\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22}} \hat{q}_2 \qquad (4.8)$$

where

$$\frac{\gamma_{22}}{\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22}} = -\gamma$$

of equation (4.3). We see that if the cross-derivatives are zero $\gamma = 1/\gamma_{11}$.

It is seen that the value of γ will be higher if there is considerable substitutability. The *price trend* δ_p corresponds to the sum of the two other terms. In a growing economy δ_p will be positive on average. It will be negative when the good in question is subject to sharp competition, say from imported goods. This is reflected in high values of both γ_{12} and $\hat{q}_2$.

If the industry produces a homogeneous good for the world market, γ reflects the industry's market share. Regarding a good produced by n countries, we have

$$p = a \left(\sum_{i=1}^{n} q_i \right)^{-\gamma'}$$

which after transformation to rates of change reads

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[1] Note that this is only admissible if the denominator of (4.8) is not zero. Assuming the own-price elasticities to be larger in absolute value than the cross elasticities, the denominator will always be negative, however.

$$\hat{p} = -\gamma' \sum_{i=1}^{n} \tilde{q}_i \, \hat{q}_i \tag{4.9}$$

where $\tilde{q}_i = q_i/\Sigma q_i$ is the country's share of the world market.

Hence, γ of equation (4.3) is the inverse demand elasticity for the product on the world market times the country's market share. The trend factor, δ_p, represents, in addition to the factors represented in (4.8), the effect of the expansion of other countries' production.

Note that we disregard all problems associated with establishing the equilibrium price. We simply assume that price will somehow adjust to the inelastic short-run supply.

The rate of profit, finally, is by (4.5) defined as a sales margin, π. This stands in a constant proportion to the rate of return on capital if the cost reduction process is Hicks-neutral.

The properties of the model will of course depend on all parameters of the equations. The reaction parameters α_c and α_q are particularly crucial. As we have seen they largely reflect what is usually meant by competitiveness of an industry: the importance of new entrants and the extent to which firms behave as profit-maximizers. It will hence be useful to label three main cases. These will imply different results of the comparative dynamic experiments conducted in this chapter, as can be seen from table 4.1 on p. 117. A *competitive* industry is characterized by swift adjustment of production to profitability variations either by established or new firms ($\alpha_q >> 0$). The sign of α_c is then largely immaterial, if α_c is not very large. In a *quasi-competitive* industry α_q is small but firms behave roughly as profit-maximizers which, with the interpretation given above, means $\alpha_c < 0$. In a *non-competitive* industry α_q is also small but there is a tendency of growth of slack when profits are high ($\alpha_c > 0$).

THE WORKING OF THE MODEL

We start the analysis with knowledge of the current values of all variables of the model. History has led to a certain production level, q_o, and a unit cost level, c_o. q and c are the state variables of the system. They can only be changed gradually as determined by the behavioural assumptions about cost reduction and expansion. The unit price, p_o, is determined by quantity produced via the demand equation. It can, however, vary freely in the short run if any of the parameters of the demand equation change. The price and unit cost also give us the initial profit margin. This will now determine the rate of change of the state variables by the behavioural assumptions. This development of q and c will in turn mean that the profit margin will tend to change. As this happens, and it can only happen gradually, it will affect the rates of change of c and q, which in turn will feed back to profits, and so on. As time goes on, the system will, if stable, approach a long run equilibrium path. As the system is constructed, this steady-state is characterized by a constant rate of change of c, q and p and a constant level of π.

A primary purpose of the analysis is to study why profits can differ between industries even over prolonged periods. The solution to the model expresses the profit margin at any particular time as a function of the initial values, c_o and q_o, and the parameters of the behavioural equations and the demand equation. Differences in profitability can hence, in terms of the model, be given three types of explanations. In the *short run*, profits will depend mainly on initial conditions. Demand may have shifted, which has led to super-normal profits, and firms have not had time to expand production so as to bring profits back towards their steady-state level. In the *medium run*, initial conditions will become gradually less important. How fast their impact vanishes depends on the parameters of the model. A possible explanation of seemingly long run profitability differences is that the adjustment process is very slow. In the very *long run*, however, initial conditions will be of no importance, provided the model is stable. Then only the six parameters of (4.1), (4.2) and (4.4) will affect profitability.

Let us illustrate the working of the model with a simple example. Assume that we start the analysis just after having observed a sudden increase in the rate of spontaneous cost reduction, δ_c, due to, say, some recent factor-saving innovation. The ensuing profits rise will stimulate expansion of production, which will tend to push the price down. At the same time costs will be affected. If α_c is positive, the pace of cost reduction will slow down, so the reactions both on the cost side and on the supply side will tend to press profits down. And as time goes on, the system will approach a steady-state situation where the profit margin remains constant over time. The steady-state profit margin π^* gives the steady-state rate of cost reduction $\hat{c}^*$ by (4.2) and the steady-state rate of expansion $\hat{q}^*$ by (4.1), which in turn by (4.4) gives the steady-state rate of price change $\hat{p}^*$. Since π^* is constant, by definition of the steady-state, $\hat{c}^*$ and $\hat{p}^*$ must be equal. From this condition we can determine π^*. It will in general depend on all parameters of the model in a way that we will see shortly.

It is intuitively clear that the process sketched above is stable. With a positive α_c, both reaction parameters work to eliminate super-(sub-)normal profits. If instead $\alpha_c < 0$, i.e. the increase in profits leads to more R & D and faster cost reduction, and if this effect is strong enough, we may instead get an explosive movement away from steady-state; costs will decrease faster and faster, and the profit margin will increase without bound.

PROPERTIES OF THE STEADY-STATE

Formally the development of the profit margin over time is given by differentiating the definition (4.5) with respect to time

$$\dot{\pi}_t = \frac{p_t}{c_t}\left(\hat{p}_t - \hat{c}_t\right) = \left(\pi_t + 1\right)\left(\hat{p}_t - \hat{c}_t\right) \tag{4.10}$$

and substituting from (4.1), (4.2) and (4.4) we have

$$\dot{\pi}_t = \left(\pi_t + 1\right)\left[-(\alpha_c + \alpha_q\gamma)\pi_t + \delta_c + \delta_p - \delta_q\gamma\right] \tag{4.11}$$

We define the steady-state as a situation where $\dot{\pi} = 0$. This gives us the steady-state profit margins

$$\pi^* = \frac{\delta_c + \delta_p - \delta_q\gamma}{\alpha_c + \alpha_q\gamma} \tag{4.12}$$

$$\pi^{**} = -1 \tag{4.13}$$

where it should be noted that π^{**} implies $p^{**} = 0$. We assume that the parameter values are such that $\pi^* > -1$ $(p^* > 0)$.

The steady-state profit margins in turn give the steady-state rates of change of unit costs, price and quantity produced, the first two being equal by the steady-state condition. We disregard the values associated with π^{**}.

$$\hat{c}^* = \hat{p}^* = \alpha_c\pi^* - \delta_c = \frac{\alpha_c\delta_p - \alpha_c\delta_q\gamma - \alpha_q\delta_c\gamma}{\alpha_c + \alpha_q\gamma} \tag{4.14}$$

$$\hat{q}^* = \alpha_q\pi^* + \delta_q = \frac{\alpha_q(\delta_c + \delta_p) + \alpha_c\delta_q}{\alpha_c + \alpha_q\gamma} \tag{4.15}$$

The stability properties of π^* and π^{**} are most easily seen in a phase-diagram. (4.11) gives $\dot{\pi}_t(\pi_t)$ as a quadratic function which equals zero for $\pi_t = \pi^*$ and $\pi_t = \pi^{**}$. The curvature is given by

$$\frac{d^2\dot{\pi}_t}{d\pi_t^2} = -2(\alpha_c + \alpha_q\gamma)$$

Depending on the sign of this item we have the two cases shown in the phase-diagram, Figure 4.1.

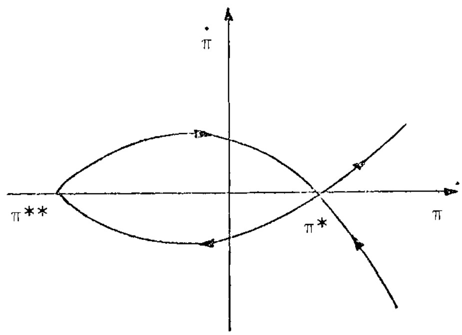

Figure 4.1

Hence π^* is globally ($p > 0$) stable if

$$\alpha_c + \alpha_q\gamma > 0 \qquad (4.16)$$

This formally restates the intuitive conclusion from above that the steady-state (i.e. the interesting one with $p > 0$) is stable if both reaction parameters are positive, whereas it is unstable if α_c is negative enough. π^{**} will be locally stable (for $\pi_o < \pi^*$) if π^* is unstable.

THE PROFIT MARGIN IN THE STEADY-STATE

Let us now see what determines π^* and start by seeing under which conditions it will be close to zero. This will happen either when the denominator takes on high positive values or when the numerator of (4.12) is close to zero.

The first case implies that either α_q or α_c is very high. A high α_q stands for what we have defined as a competitive industry. A high positive α_c on the other side means also that super-normal profits will be eliminated, but now by the growth of slack. We thus see that intense competition will indeed yield profits close to zero. But so will the opposite case, where there is little reaction to profitability variations on the expansion side (low α_q), but a strong reaction on the cost side (high positive α_c). In the former, competitive, case profits will be wiped away by increased production and reduced price. In the extreme non-competitive case they are instead eliminated by increased costs, which certainly conforms with the stereotype view of the vices of monopoly.

More likely in reality is perhaps the quasi-competitive case where α_c is negative, which follows if the company is profit-oriented and cost reduction is mainly a result of R & D. If α_q is not very high, i.e. the industry is not competitive, the value of the denominator of (4.12) will not be very high either. Then the burden of ensuring zero profitability will fall on the coefficients appearing in the numerator of (4.12). Of these, δ_p, which represents demand conditions, is outside the control of the industry in question, whereas δ_c and δ_q reflect both the behavioural characteristics of the firms in question and things such as factor price developments which are outside their control. Perhaps it is natural to regard δ_c as containing relatively more of factors that are outside the control of the industry. We may then express the value of δ_q which gives $\pi^* = 0$ as a function of δ_c, δ_p and γ:

$$\delta_q^* = \frac{\delta_c + \delta_p}{\gamma} \rightarrow \pi^* = 0 \qquad (4.17)$$

The meaning of this statement can be seen by firstly separating δ_c into a productivity trend δ_b and a price of production factors trend δ_w. If we secondly assume away all cross-effects in the demand equation, δ_p can by (4.8) be expressed as $\gamma\beta\hat{Q}$, where β is the income elasticity and $\hat{Q}$ stands for the development of real income in the economy as a whole.[1] $\hat{Q}$ can, for a one factor economy with constant population, be regarded as equal to δ_w so that (4.17) can be rewritten

$$\delta_q^* = \frac{\delta_b - \delta_w + \gamma\beta\delta_w}{\gamma} \tag{4.18}$$

Let us consider an industry that is average in the sense that $\delta_b = \delta_w$ and $\beta = 1$. This gives $\delta_q^* = \delta_w$, i.e. the steady-state profit margin will be zero if production is expanded at the same rate as productivity grows in the industry in question and in the economy as a whole. This implies that the industry employs a constant amount of production factors, which, as we conjectured in the discussion about the sign of δ_q on p. 103, may be a fairly natural first assumption. Hence, in an industry that is average in the above sense such a very crude behavioural rule will make (4.18) fulfilled.

In a non-average industry π^* will be zero if firms determine their rate of spontaneous expansion from projections of the expression in (4.18) for the market as a whole and try to keep their own market share unchanged. Casual empiricism may suggest that such behaviour is common. But in a market with many small firms it is not immediately apparent that there are any incentives for the individual firm associated with it. Further, it demands that all firms know the parameters of the demand equation.

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[1] This follows from (4.8), since δ_p is equal to the sum of the last terms in (4.8). The assumption of zero cross-effects ($\gamma_{12} = 0$) means that the last term is zero, and the second term is $\hat{Q}\cdot\beta_1/\gamma_{11} = \gamma\beta\hat{Q}$.

Summing up, we have distinguished two cases which will make $\pi^* = 0$. Either the numerator of (4.12) is zero or the denominator takes on very high values. The upshot of the discussion above is that there is little reason to believe that the numerator will be zero in general. Therefore the main conclusion seems to be that only in a world with a large number of potential new entrants operating under the same cost conditions as existing firms, and eager to take even the very slightest opportunity to make profits, are there any strong indications that profits even in the long run will approach zero.

Let us now instead ask the related question: how do the parameters of the model affect the steady-state profit margin? It is easily seen that the trend parameters unambiguously work in the expected direction; fast growth of demand and fast reduction of costs will increase profits, fast expansion will decrease profits. High positive values of the reaction coefficients tend to bring profits closer to zero but can never affect the sign of steady-state profits.

The elasticity of price with respect to quantity produced plays a more ambiguous role.

$$\frac{\partial \pi^*}{\partial \gamma} = \frac{-\alpha_c \delta_q - \alpha_q (\delta_c + \delta_p)}{(\alpha_c + \alpha_q \gamma)^2} = - \frac{\hat{q}^*}{\alpha_c + \alpha_q \gamma} \tag{4.19}$$

This says that the effect of the elasticity will depend on whether production is increasing or decreasing in the steady-state. If $\hat{q}^* > 0$ profits will be higher the more insensitive price is to this expansion.

The cases of extremely high and low price elasticities are worth pointing out. If $\gamma = 0$, i.e. the industry's share of the world market is very small, the profit margin is determined by the trends of price and costs and the reaction parameter with respect to cost reduction:

$$\pi^* \ (\gamma = 0) = \frac{\delta_c + \delta_p}{\alpha_c}$$

Note that for this steady-state profit level to be determinate and the process to be stable, α_c must be positive.

At the other extreme is the case of an inelastic demand for the product. Then the profitability performance is determined by the parameters of the expansion equation

$$\lim_{\gamma \to \infty} \pi^* = - \frac{\delta_q}{\alpha_q}$$

In summary, this shows that in a static world ($\delta_c = \delta_q = \delta_p = 0$) profits will always tend towards zero in the long run provided firms' reaction pattern is such that the model is stable ($\alpha_c + \alpha_q\gamma > 0$). But as soon as we leave the static world we see that this condition is no longer sufficient. Then, zero profits can come about if firms adjust their spontaneous rate of expansion to the underlying demand and productivity trends. If this is not so, profits may still approach a value close to zero if the industry is competitive ($\alpha_q >> 0$).

COMPARATIVE DYNAMICS

We have just investigated what determines profits in steady-state. If the adjustment process is slow, the steady-state values will be of less interest in comparison with shocks coming from outside. We will in this section consider the effects of shocks both in terms of shifts in parameter values and shifts in initial conditions. In order to study the effects of this all along the solution path, we need the explicit solution to the system of equations. For derivation, see Appendix A.

$$c_t = c_o e^{\hat{c}^* t} \Omega_t^{\Lambda c} \tag{4.20}$$

$$p_t = p_o e^{\hat{c}^* t} \Omega_t^{-\Lambda p} \tag{4.21}$$

$$q_t = q_o \left[e^{(\delta_p - \hat{c}^*)t} \Omega_t^{\Lambda_p} \right]^{1/\gamma} \tag{4.22}$$

$$\pi_t = (\pi_o + 1)\Omega_t^{-1} - 1 \tag{4.23}$$

where

$$\Omega_t = \frac{\pi_o + 1}{\pi^* + 1} + \left(1 - \frac{\pi_o + 1}{\pi^* + 1}\right) e^{-(\alpha_c + \alpha_q \gamma)(\pi^* + 1)t} \tag{4.24}$$

$$\Lambda_c = \frac{\alpha_c}{\alpha_c + \alpha_q \gamma} \qquad \Lambda_p = \frac{\alpha_q \gamma}{\alpha_c + \alpha_q \gamma}$$

The solution given above is quite complicated and can be presented in different ways. The auxiliary variables employed here are constructed to enable the division of the expressions of the solution path into one term (c_o) representing initial conditions, one term representing the steady-state path ($e^{\hat{c}^* t}$), and one term representing the impact of initial conditions ($\Omega^{\Lambda c}$).

The variable Ω can be interpreted as a measure of the impact of the initial deviation of the profit margin from its steady-state value. $\Omega_o = 1$, which says that there cannot be any impact at $t = 0$; $c(0) = c_o$ etc. As time goes to infinity, the second term in (4.24) goes to zero, and

$$\lim_{t \to \infty} \Omega_t = \Omega^* = \frac{\pi_o + 1}{\pi^* + 1}$$

which says that the ultimate impact on the *levels* of c, p and q depends on the initial deviation of the profit margin from its steady-state value.

We also see that the rate at which Ω will approach Ω^* depends on $\alpha_c + \alpha_q\gamma$, i.e. the same sum of the reaction coefficients which appears in the denominator of π^*. From this it is also clear that the model will only be stable if $\alpha_c + \alpha_q\gamma > 0$, for otherwise the second term in the expression for π^* will go to infinity with time.

Λ_c and Λ_p can be said to represent the share of adjustment that comes from the cost and price (supply) side respectively.

c(t) and p(t) are illustrated in Fig. 4.2. Since this is drawn in logarithms, the vertical distance between the curves shows the profit margin; $\ln p - \ln c = \ln(\pi+1)$.

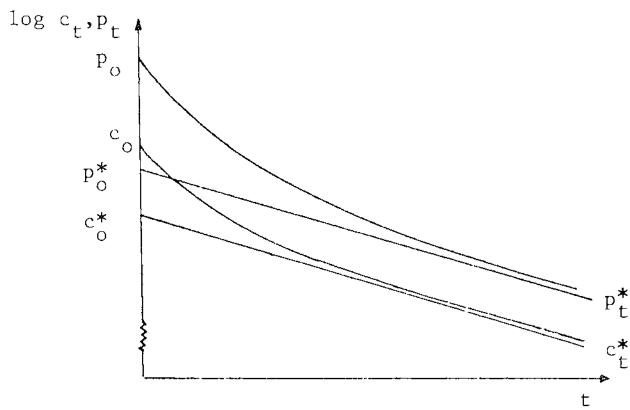

Figure 4.2

The solution paths will approach the steady-state paths $p^*(t)$, $c^*(t)$ monotonically. The equations for the latter are given by substituting Ω^* for Ω_t in (4.20) and (4.21). Whether the steady-state paths will be approached from above or below depends on initial conditions;

$$\frac{\pi_o + 1}{\pi^* + 1} > 1 \Rightarrow p_t > p_t^*,\ \alpha_c c_t < \alpha_c c_t^*$$

and vice versa. As the figure is drawn $\pi_o > \pi^*$, i.e. p_t will approach the steady-state from above; supernormal profits stimulate expansion and consequent price reduction. c_t will also approach the steady-state path from above if the industry is quasi-competitive ($\alpha_c < 0$).

Effects of Parameter Shifts

We will now in turn consider the effects of shifts of all parameters of the model. We will look at effects on the steady-state values as well as effects all along the solution path. The signs of the partial derivatives are given in Table 4.1.

Table 4.1. Signs of the Effects on the Endogenous Variables of Parameter Variations

	π^*	$\pi_{t\mid t<\bar{t}}$ 1)	c^*	$c_{t\mid t<\bar{t}}$ 1)	p^*	$p_{t\mid t<\bar{t}}$ 1)	q^*	$q_{t\mid t<\bar{t}}$ 1)
α_c	$-\pi^*$	$-\pi_o$	π^*	π_o	π^*	π_o	$-\pi^*$	$-\pi_o$
α_q	$-\pi^*$	$-\pi_o$	$-\alpha_c\pi^*$	$-\alpha_c\pi_o$	$-\alpha_c\pi^*$	$-\pi_o$	$\alpha_c\pi^*$	π_o
δ_c	+	+	–	–	–	–	+	+
δ_q	–	–	$-\alpha_c$	$-\alpha_c$	$-\alpha_c$	–	α_c	+
δ_p	+	+	α_c	α_c	α_c	+	+	+
γ 2)	$-\hat{q}^*$	$-\hat{q}_o$	$-\alpha_c\hat{q}^*$	$-\alpha_c\hat{q}_o$	$-\alpha_c\hat{q}^*$	$-\hat{q}_o$	$-\hat{q}^*$	$-\hat{q}_o$

Comments: The entrances in the table give the signs of the partial derivative in question, i.e. α_c in the column for c and row for δ_p means $\text{sgn}\,\frac{\partial c}{\partial \delta_p} = \text{sgn}\,\alpha_c$.

1) In some cases the sign of the short run effect can be different from the sign of the long run effect. For these cases we define a $t=\bar{t}$ such that $\text{sgn}\left(\frac{\partial}{\partial}\right)_{t<\bar{t}} \neq \text{sgn}\left(\frac{\partial}{\partial}\right)_{t>\bar{t}}$.

2) The derivatives with respect to γ have been calculated holding p_o constant. This presumes that the coefficient of the level of demand a is varied so as to compensate for the effect of a change in γ on p_o.

For derivations, see Appendix A.

All the derivatives of the table can be intuitively understood by inspecting the original equations of the model. Looking at (4.20-4.24) and Fig. 4.2, we see that the effects can be decomposed into three parts:

a) $d\hat{c}^*(d\hat{p}^*)$, the effect on the rate of change along the steady-state path. This effect will clearly dominate in the long run.

b) $dc_o^*(dp_o^*)$, the effect on the level of the steady-state path.

c) $d\{(\alpha_c + \alpha_q\gamma)(\pi^* + 1)\}$, the effect on the rate at which the gap between the actual path and the steady-state path will be closed.

This decomposition should be used cautiously since the three parts are closely interrelated; a change in any of the parameters will affect all three components.

We will now talk our way through the model to see how a variation of any of the six parameters affects costs, prices, quantities produced and profits in the short and in the long run. After that we will also analyze, and discuss the interpretation of, variations in initial conditions. To try to blow some life into the partial derivatives we will specify examples of changes in the real world to which the parameters of the model are constructed to correspond. The parameters can partly be varied by policy measures such as profits taxes and investment subsidies. We will conclude the comparative dynamic analysis by a discussion of such policy changes.

δ_c - The Rate of Spontaneous Cost Reduction

Consider a situation in which an important input factor is starting to become gradually cheaper, i.e. $d\delta_c > 0$. Assume further that the situation is one of subnormal profits, i.e. $\pi_o < \pi^*$.

Since we are only considering a shift of rates of change of costs, and not levels, nothing will happen instantaneously. The first effect will be on c, which after some time will be a bit lower than otherwise. This in turn will increase profits, and hence increase $\hat{q}$. The increase in profitability will also affect c according to the sign of α_c. If $\alpha_c > 0$ this secondary effect will take away some of the initial effect on unit costs. The net effect will always remain in the initial direction, however. And the effect on π will increase gradually until the new steady-state is reached. The effect on $\hat{c}$ will remain negative in steady-state. $\partial\hat{c}^*/\partial\delta_c$ will be larger than unity (in absolute value) if the industry is quasi-competitive ($\alpha_c < 0$). The size of the effect on π^* depends on the rate at which steady-state is approached ($\alpha_c + \alpha_q\gamma$). If this term is very high, a change in δ_c, or any of the other trend parameters, will have very little effect on profits. It will be wiped away either by never letting the increased rate of spontaneous cost reduction considerably affect the realized rate of cost reduction (if $\alpha_c >> 0$), or in a competitive industry ($\alpha_q\gamma >> 0$), by immediately increasing the rate of expansion to let price fall so as to match the increased rate of cost reduction.

The effect on c is shown in Fig. 4.3. As we have just seen ($d\hat{c}^* < 0$), i.e. the new steady-state path will slope steeper downwards. If $\alpha_c > 0$ and $\pi_o < \pi^*$, then $c_o > c_o^*$ and $dc_o^* < 0$, i.e. we will get the picture of Fig. 4.3. This shows that the rate of cost reduction goes faster at first and then, under the influence of increased profits, gradually approaches its steady-state rate of change.[1]

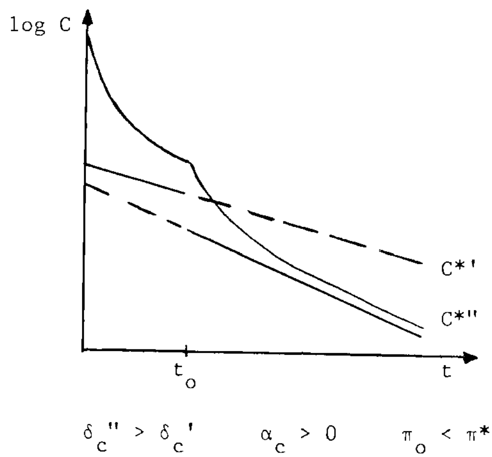

Figure 4.3

Summing up, all effects of the lower rate of wage increase are unambiguous under the assumptions made about the signs of the parameters. Profits will increase from their sub-normal level faster than they would otherwise do and the rates of cost reduction and expansion of production will be sped up in the short as well as in the long run. The higher is α_q, the more will expansion rise and the faster will costs fall. In the competitive case of a low α_c and a high α_q, the whole change in δ_c will carry over to steady-state, the rate of price change will be affected equally and π^* will remain unaffected and close to zero.

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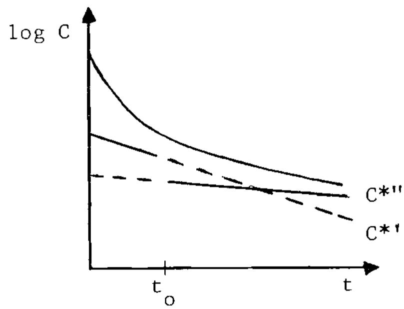
[1] To keep the diagram simple it only shows the path of c. That of p has, in this case, qualitatively the same properties.

δ_q - The Rate of Spontaneous Expansion

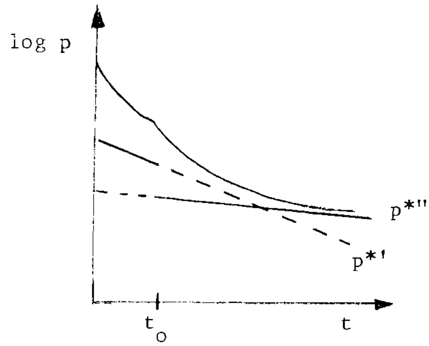
When discussing the conditions for $\pi^* = 0$ on p. 111 we saw that δ_q plays a crucial role. We then noted that it is pretty artificial to regard it as a parameter independent of other parameters. Still, we are now going to consider the effects of varying δ_q without varying any of the other parameters. We will consider $d\delta_q > 0$ and think of it as a sudden increase in the rate of entrance of new firms into the industry at zero profits. This can be thought of as a sudden change of new firms' expectations of the future level of the profit margin in the industry. This change of expectations is assumed to happen in a situation where profits are above the steady-state level, $\pi_o > \pi^*$. We will for this case also assume the industry to be quasi-competitive ($\alpha_c < 0$), i.e. a reduction of profits will also reduce the rate of cost reduction. We choose this case since it gives some results that may sound counter-intuitive.

The initial effect will of course be that production will increase faster than it would otherwise have done. This will in turn press the price down and hence profits will be lower. The fact that profits are lower on the new path will, since $\alpha_c < 0$, tend to make costs higher and profits still lower, which in turn will tend to reduce the rate of expansion of production. Hence, here we have two conflicting forces. The increase in δ_q speeds up the expansion of production, but since it thereby leads to lower profits, costs will be higher and profits still lower, and all this makes for a lower rate of expansion. As time passes, the effect on profits will have an increasingly negative effect on expansion. And as we approach steady-state, the effect of increasing the spontaneous rate of expansion is seen to be to slow down the rate of expansion, to increase the rates of change of costs and price and to press down the profit margin. The effects on the development of p and c are illustrated in Figures 4.4 and 4.5.



$\delta_q'' > \delta_q'$ $\alpha_c < 0$ $\pi_o > \pi^*$

Figure 4.4



$\delta_q'' > \delta_q'$ $\alpha_c < 0$ $\pi_o > \pi^*$

Figure 4.5

δ_p - The Rate of Demand Increase

Let us now consider a sudden fall in the trend of world market prices. Let this happen in a situation of initial equilibrium. Assume further that the industry is quasi-competitive. This means $d\delta_p < 0$, $\pi_o = \pi^*$ and $\alpha_c < 0$.

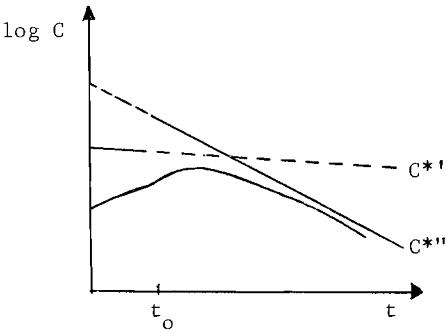
This case closely parallels that of a change in δ_q. The reader is referred back to Figures 4.4 and 4.5 for an illustration. The only difference is that here the initial effect is on the rate of price change, whereas in the case of a change in δ_q the initial effect was on expansion of production which via the demand equation affected price. The initial price reduction will now lead to a lower rate of expansion and a lower rate of cost reduction. The net effect will be to reduce profits and hence to further reduce expansion. After a while the effect of profits on expansion will be so strong as to imply a higher price level at the new path though the world market determined price trend has been shifted downwards.

This may sound paradoxical. But the logic of the model is simply that the reduced rate of cost reduction which follows upon the fall in profits must ultimately be matched by a reduced rate of price reduction which will come about by a reduction of the rate of expansion. It must be kept in mind that the example given here is only reasonable if the demand elasticity remains constant throughout the adjustment process. This will not be the case if the foreign competitors produce exactly the same good as the domestic producers, for then, by (4.9), γ will reflect the market share. The process described must hence be thought of as one where the two products are substitutes, due to e.g. quality differences, and the demand functions are such that γ remains constant.

α_c - The Cost Reaction Coefficient

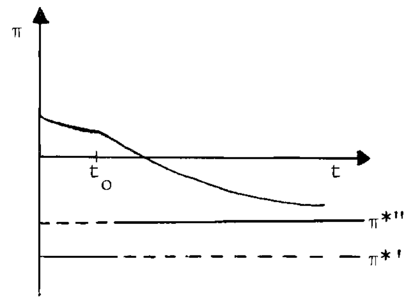
It has been seen in the examples just studied how α_c plays a crucial role in the model. We are now going to study the effects of changes in α_c. Consider an industry where management's preferences suddenly become more important relative to those of the owners. This means in terms of the managerial model of cost reduction of Chapter 3 that $d\alpha_c > 0$. Suppose that this happens in a situation of positive profits, $\pi_o > 0$, but that the trend factors work against the industry, $\pi^* < 0$.

Since profits are positive, the initial effect will be to slow down cost reduction. This means lower profits, which means a lower rate of expansion, and the negative effect on profits will be counteracted. As long as profits remain positive, part of the initial negative effect on costs will remain. But profits will become negative sooner or later. Since α_c has increased, this point of time will now come sooner. Once $\pi_t < 0$, the system will work the other way around. Cost reduction and expansion will be stimulated and π^* will not be so far below zero as it would otherwise have been. See Figures 4.6 and 4.7.



$\alpha_c'' > \alpha_c' > 0, \quad \pi_o > 0 > \pi^*$

Figure 4.6



$\alpha_c'' > \alpha_c' > 0, \quad \pi_o > 0 > \pi^*$

Figure 4.7

α_q - The Expansion Reaction Coefficient

α_q has been interpreted as an indicator of the degree of competitiveness in the industry, because it says how fast super- (or sub-) normal profits will be eliminated by expansion or contraction of production. Let us now consider the effect of having a sudden increase in the number of potential entrants. This will mean $d\alpha_q > 0$. Let this happen where $\pi_o > 0$, $\pi^* < 0$ and $\alpha_c < 0$.

The initial effect is to speed up expansion. Hence, price and profits will tend to fall faster than otherwise. This will now imply slower cost reduction since $\alpha_c < 0$. But since the model is stable, and $\pi^* < 0$, the profit margin will fall and reach zero after a while. When $\pi < 0$, the higher α_q instead leads to faster contraction and ultimately a steady-state will be approached with π^* closer to zero. The ultimate effect on $\hat{q}$ and $\hat{c}$ depends on the sign of α_c, which here is assumed to be negative. Initially, there will be slower cost reduction, but when profits turn negative the rate of cost reduction will instead be faster along the new path. And this must in steady-state be matched by a higher rate of price reduction and hence by faster increase of production. Hence, despite the fact that $\pi^* < 0$ and α_q is increased, the effect of this change is to increase $\hat{q}^*$; π^* changes so much as to more than offset the direct effect of the increase in α_q.

γ - The Inverse Demand Elasticity

Consider now the effect of a suddenly increased substitutability between goods produced by the industry in question and other goods. This will mean that the inverse demand elasticity, i.e. the elasticity of price with respect to quantity produced, increases, $d\gamma > 0$.[1] Let this happen when $\hat{q}_o > 0$, $\hat{q}^* > 0$ and $\alpha_c < 0$.

Since production is expanding initially the increase in γ means that price will tend to be lower immediately after the parameter shift. This in turn means lower profits, which, since $\alpha_c < 0$, lead to higher costs and less production. As time passes, the difference between the profit margin along the two paths will gradually increase. This means that the effects on cost reduction and expansion will be further strengthened. But as steady-state is approached, the effects on cost and price reduction will tend to equalize. Hence, the effect on price, which at first was negative, will tend to be positive in the long run. So the increased competition from close substitutes tends to press price down in the short run, but in the long run costs will go up and then prices must follow costs in order to prevent profits from falling further.

This completes the exposition of the effects of parameter variations on the solution paths of the model. We have

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[1] We are making the experiment of changing γ, holding p_o unchanged. This requires that the parameter representing the level of demand, a, be changed so as to offset the impact of $d\gamma$ on p_o. If we would not allow this, the distinction between a change in the demand level and the price elasticity would be blurred.

noted how the effects of any particular experiment depend on the other parameters of the model, in particular as they affect the signs of the profit margin and the rate of expansion, initially and in steady-state, and on the sign of the cost reaction parameter α_c.

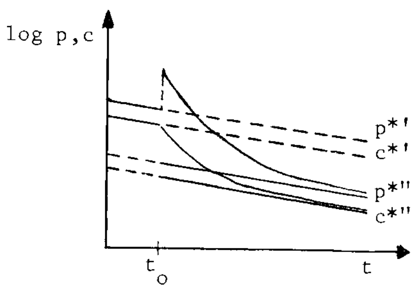
Effects of Changed Initial Conditions

Whereas we asked above, among other things, what initial conditions mean for the effect on the solution paths of varying the parameters of the model, we will now instead ask what the parameters of the model mean for the effect of varying initial conditions. Mathematically of course this comes down to the same thing, since the equations are continuous as required for both cross-derivatives to be equal. Conceptually, however, the experiments are different.

The question we now will be considering is: what is the effect of a shock in the form of a sudden change of price or unit cost? We will compare two hypothetical paths for the same industry. One path is continuous, as given by the solution to the equation system, whereas along the other path, there occurs a shift in a state variable at $t = t_o$. It may be a sudden price change of an input factor which causes c_o to shift, a sudden start of a new factory which causes q_o to shift, or a sudden demand change which causes p_o to shift.

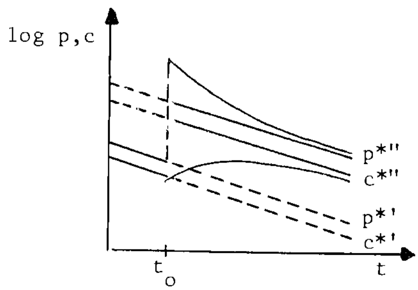
p_o - Initial Price

A shift in p_o can be due either to a shift in q_o or to a change of the value of a, the level of demand. The effect of an increase in p_o from a steady-state situation is illustrated in Figures 4.8 and 4.9. The increase in π_o leads initially to an increased rate of expansion and thereby a higher rate of price reduction than in steady-state. If the industry is quasi-competitive ($\alpha_c < 0$) as in Fig. 4.8, the rate of cost reduction will be sped up. As time goes on, the new steady-state paths will be approached and π^* restored by a combination of lower price and lower unit costs than if the economy had moved undisturbed along the steady-state path. Since $\alpha_c < 0$, not only will all the initial price increase be taken away, but finally the industry will settle on a lower price and cost level than otherwise. The less reaction there is on the expansion side, the more will costs and price have time to fall before the model reaches steady-state again; as is shown in Appendix A, p. 223, the limit value of the elasticity of p and c with respect to a change in p_o is Λ_c. This looks like a strange and remarkable result; one way to keep prices down in the long run is to force them up in the short run. But of course it is a result that follows directly from the assumption that high profits foster R & D and hence lead to reduced costs. Fig. 4.9 illustrates the opposite case of a non-competitive industry.



$dp_o > 0 \qquad \alpha_c < 0$

Figure 4.8



$dp_o > 0 \qquad \alpha_c > 0$

Figure 4.9

c_o - Initial Unit Costs

The working of the model has been demonstrated so many times that there is little need for repetition. The elasticity of steady-state costs and price with respect to a change in initial costs equals Λ_p. Since we assume $\alpha_q\gamma$ to be positive throughout, Λ_p is always positive. With $\alpha_c > 0$, it is less than unity. Then part of the initial cost rise will be taken away in the process of restoring steady-state. However, part of it will remain, and this will be matched by a price rise following from a lower level of production.

TAX CHANGES - AN INTERPRETATION OF PARAMETER SHIFTS

We will now use the model to analyze the effects of various types of tax changes. This will lead to interpretations that differ somewhat from those ordinarily expounded in the partial equilibrium literature.

The imposition of an *excise tax* is equivalent to $da < 0$, if p is interpreted as the unit price after deduction of the tax. In the competitive case ($\alpha_q >> 0$) we will get the standard result that neither costs nor price net of tax are affected and the only effect is that the production level is decreased. The whole tax burden is carried by the consumers. In the non-competitive case ($\alpha_c > 0$), a fall in the long-run level of unit costs and price net of tax will result. The equilibrium quantity will here again decrease as a result of the tax. In the quasi-competitive case ($\alpha_c < 0$), we will have just the opposite effect. Since the size of the market decreases, less resources will be devoted to R & D, and costs will not be reduced as fast as otherwise. Hence, the reduction of q* will be larger than in the competitive case.

A *profits tax* means that the gains (losses) derived from (not) adjusting to market conditions will be smaller. A profits tax applied in practice is normally assymetrical, however, since it does not affect the size of the losses, only the size of the gains. However, let us assume that π^* is above what for tax purposes is defined as zero,[1] and that

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[1] The accounting conventions of the tax authorities need not coincide with those relevant for this model. The former may e.g. not take inflation and implicit interest on equity into account.

we do not consider the effects on the development for $\pi < 0$. The coefficients α_c and α_q give the reaction to variations in the rate of profit before tax. This means that the introduction of a profits tax can be interpreted to mean that $d\alpha_c < 0$[1] and $d\alpha_q < 0$. The general effect of a profits tax will then be to reduce the rate of adjustment to shocks and to make the industry less competitive according to the definition we have employed in this chapter. If $\pi^* = 0$, there will be no effect on π^*, $\hat{q}^*$ and $\hat{c}^*$.

An *investment subsidy* means that, ceteris paribus, it will be more profitable to expand for any given level of π, i.e. δ_q will increase. As we have seen above, we cannot be sure that this will increase the rate of expansion in the long run. Actually, this will only be so if the industry is non-competitive ($\alpha_c > 0$). If the industry is quasi-competitive ($\alpha_c < 0$), an investment subsidy will only stimulate expansion in the short run, but since it implies a decreased profit margin, it leads to a slower rate of expansion in the long run.

INTERDEPENDENCE BETWEEN EXPANSION AND COST REDUCTION

The behavioural assumptions (4.1) and (4.2) refer to the rate of change of unit costs and output. But much of the argumentation behind these assumptions referred to unit costs and output only indirectly. In discussing adjustment costs in Chapter 3 we showed that a higher product price leads to more investment. With a *given* production function we could infer from this, with some reservations for the case of internal adjustment costs, that the rate of expansion would increase. But the argument behind (4.1), the equation governing the rate of change of unit costs, implies on the contrary that the production function is continuously changed. In this section we will take account of this and respecify the model to let the rate of change of

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[1] This presumes $\alpha_c > 0$. If the R & D effect dominates ($\alpha_c < 0$) there will be the opposite effect, $d\alpha_c > 0$.

input depend on π. This will lead to a reinterpretation of α_q and δ_q and a consequent reinterpretation of the properties of the model.

The cost reduction equation has been discussed in terms of slack and R & D. However, the rate of change of unit costs may also depend directly on expansion by returns to scale and learning by doing. This is a second factor, which may imply that q and c are directly interrelated. By making the basic behavioural assumption about a scale-independent efficiency factor instead of unit costs we will be able to reinterpret α_c and δ_c.

Finally in this section, we will turn the direction of causality around. Still assuming increasing returns to scale, we will assume that the desired rate of cost change can only be effected by adapting the scale of production. This may be seen as a highly stylized picture of an investment strategy of expanding out of a cost crisis. Casual empiricism suggests that such an attitude towards investment decisions is not uncommon in reality.

The modifications studied in this section have three related properties in common. First, one can no longer be certain about the sign of the expansion reaction coefficient α_q. Second, it is no longer easy to judge on a priori grounds whether the model is stable or unstable. Third, the steady-state values of the endogenous variables will tend to be sensitive to variations in the trend parameters.

The implications of formulating the reaction equation (4.1) in terms of input instead of output are easiest to illustrate by assuming a one-factor production function with constant returns to scale. Denoting the production factor L and its price w we have

$$q = bL$$

and unit cost

$$c = \frac{w}{b}$$

or rewriting in rates of change

$$\hat{q} = \hat{b} + \hat{L} \qquad (4.25)$$

$$\hat{c} = \hat{w} - \hat{b} \qquad (4.26)$$

Making behavioural assumptions about the rate of change of the efficiency factor, b, and input means

$$-\hat{b} = \alpha_b \pi - \delta_b \qquad \alpha_b \gtrless 0 \qquad (4.27)$$

$$\hat{L} = \alpha_L \pi + \delta_L \qquad \alpha_L > 0 \qquad (4.28)$$

Substituting from (4.27) and (4.28) into (4.25) and (4.26) we then have

$$\hat{q} = \alpha_q \pi + \delta_q$$

$$\hat{c} = \alpha_c \pi - \delta_c$$

where

$$\alpha_q = \alpha_L - \alpha_b \qquad \delta_q = \delta_L + \delta_b$$

$$\alpha_c = \alpha_b \qquad \delta_c = \delta_b - \hat{w}$$

From this we see that the sign of α_q is ambiguous. If there is a strong slack-reduction effect on the cost side ($\alpha_b >> 0$) this may well dominate the input reaction coefficient, and $\alpha_q < 0$.

The steady-state profit margin will now be

$$\pi^* = \frac{\delta_p - \hat{w} + \delta_b(1-\gamma) - \delta_L \gamma}{\alpha_b(1-\gamma) + \alpha_L \gamma} \qquad (4.29)$$

and the model is stable if

$$\alpha_b(1-\gamma) + \alpha_L\gamma > 0$$

If $\alpha_b > 0$ a sufficient stability condition is that $\gamma < 1$, i.e. that demand is elastic. If, however, demand is inelastic we see that the stability condition may be violated. At any instance the denominator of (4.29) may be close to zero, which means that π^* will be quite sensitive to variations in the trend parameters.

The case of *returns to scale* can be illustrated by this unit cost function

$$c = sq^{-\sigma}$$

where $-\sigma$ is the elasticity of unit costs with respect to quantity produced. This can be thought of either as ordinary economies of scale at the firm level or as external effects between firms at the industry level. $\sigma > 0$ means increasing returns to scale and $\sigma < 0$ decreasing returns to scale.

Transforming to rates of change gives

$$\hat{c} = \hat{s} - \sigma\hat{q}$$ [1]

We will now make the behavioural assumptions about q and the efficiency factor s.

$$\hat{q} = \alpha_q\pi + \delta_q \tag{4.30}$$

$$\hat{s} = \alpha_s\pi - \delta_s \tag{4.31}$$

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[1] This equation can also be thought of as arising from learning by doing.

which gives

$$\hat{c} = \alpha_c \pi - \delta_c$$

where

$$\alpha_c = \alpha_s - \sigma\alpha_q$$

$$\delta_c = \delta_s + \sigma\delta_q$$

This shows that with strongly increasing returns to scale, the rate of cost reduction may be positively affected by increased profits even if $\alpha_s > 0$.

In steady-state

$$\pi^* = \frac{\delta_s + \delta_p - (\gamma-\sigma)\delta_q}{\alpha_s + (\gamma-\sigma)\alpha_q} \qquad (4.32)$$

which shows that the elasticity of unit costs with respect to quantity, σ, plays exactly the same role as the elasticity of price with respect to quantity. If these two elasticities are equal, an increase in expansion will have no stabilizing effect since it will affect the rates of price change and cost change equally. In such a case the model will only be stable (the denominator positive) if $\alpha_s > 0$.

In this model cost reduction was seen as an unanticipated by-product of expansion. This is not perfectly convincing as a picture of causality in the real world. Certainly firms are to some extent aware of the presence of economies of scale and allow this to affect their decisions about expansion. In such a case it is possible that causation runs in the opposite direction; low profits necessitate cost reduction which can only be effected by increasing production. As an extreme case of this let us regard the case where costs are unalterable with the exception of scale effects ($\hat{s} = 0$), and quantity changes are solely determined by the desire to cut costs. This means

$$\hat{c} = -\sigma\hat{q} = \alpha_c\pi - \delta_c \tag{4.33}$$

and consequently

$$\hat{q} = -\frac{\alpha_c}{\sigma}\pi + \frac{\delta_c}{\sigma} \tag{4.34}$$

giving

$$\pi^* = \frac{\delta_c(1-\gamma/\sigma) + \delta_p}{\alpha_c(1-\gamma/\sigma)} \tag{4.35}$$

If $\alpha_c > 0$, this model will be unstable when $\gamma > \sigma$, i.e. when price is affected more than costs by changes in production. In such a case the firm is chasing a target that becomes ever more remote; it is impossible to improve profits by increasing production.

The modifications of the model assumptions considered in this section serve to highlight certain inconsistencies in firm behaviour. The first model depicts a firm whose decisions with respect to input are taken without direct regard to decisions with respect to cost change. The second model shows the case where expansion by itself implies cost reduction, but the firm does not explicitly take this into account. The firm in the third model is aware of the relation between expansion and cost reduction. But it is only concerned about cost reduction, and the rate of expansion is adapted so as to make it possible to achieve the desired rate of cost reduction.

All these models depict behaviour that can be characterized either by lack of knowledge or irrationality. Still it seems that such behaviour is not uncommon in the real world. In particular, I think that one can find many examples of the latter type of behaviour, where a situation of low profits is met not by contraction, but by expansion, so as to reap the cost benefits of a larger scale. Not seldom it seems that these benefits are bought at the expense of lower prices.

CONCLUDING COMMENTS

Adjustment processes in standard equilibrium theory are generally analyzed solely in order to ascertain that the equilibrium under study is stable. If one believes that the adjustment process is fairly rapid one may then be justified in concentrating attention on the properties of the equilibrium position of the model. The model studied in this chapter is based on the presumption that adjustment processes often are sluggish. In such a case it is natural to analyze the process in more detail and push the equilibrium position to the background.

A main lesson of this chapter is that the parameters which affect the speed of adjustment also affect the properties of the equilibrium. With very fast adjustment the model resembles an ordinary equilibrium model with constant returns to scale; profits are close to zero, exogenous shocks will be wiped away fast and leave little effect on costs, the rate of cost reduction is determined exogenously, an excise tax will be shifted to the consumers, a profits tax will have little effect on anything, etc.

The reaction parameters which affect the rate of adjustment will also affect the steady-state values of the variables, if the trend factors do not make the numerator of the expression for π^* (4.12) zero. In general, for given values of the numerator, the slower adjustment goes, the more will steady-state profits deviate from zero. The sign of π^*, however, depends solely on the trend parameters of the model which can largely be seen as exogenous. This leads to the general conclusion that a policy which induces firms to react slowly to profit signals from the market will lead to large variations between different sectors of the economy. This may be suggestive as one explanation of the increasing spread in profitability in the Swedish economy in the seventies (see Tables B.1 and B.2 in Appendix).

The model emphasizes the fact that firms will react to variations in profitability by adapting their rate of cost reduction. By this the model contains a rudimentary theory of productivity change. It shows the critical importance for many questions of the sign of α_c. Does the X-inefficiency or the R & D effect dominate? The case of a strong R & D effect ($\alpha_c < 0$) accounts for some paradoxical results; an investment subsidy leads to slower expansion in the long run, a slower demand increase leads to higher price in the long run, and a shock in the form of a sudden price increase will lead to a lower price in the long run.

The model is deliberately made very simple. Still it seems that it focuses on some mechanisms that are quite relevant for policy purposes in a dynamic setting. To some extent these are mechanisms that tend to be obscured by an ordinary equilibrium kind of analysis. The simple variations in the interpretation of the parameters of the model considered in the final section of this chapter show that it is possible to adapt the model to contain further aspects of firm behaviour.

5. THE SENSITIVITY OF THE RESULTS TO ALTERNATIVE MODEL SPECIFICATIONS

The model analyzed in Chapter 4 is very simple. Indeed, it was deliberately formulated so as to make it possible to find an explicit solution to the system of differential equations. This made it straightforward to analyze both long-, medium- and short-run properties of the model and in particular questions of the rate of convergence towards steady-state. The purpose of this chapter is to investigate whether the major conclusions from this analysis will remain unchanged if the model is reformulated in alternative more realistic ways. The modifications considered are the following: (a) firms differ with respect to parameter values and initial unit cost levels (b) the rate of cost reduction is a non-linear function of the profit margin (c) the rate of cost reduction depends both on the profit margin and the level of production (d) the rates of expansion and cost reduction depend on the discounted sum of future expected profit margins, not only on the current profit margin.

A DISAGGREGATED MODEL

In the introduction to Chapter 4 we assumed that all existing firms were equal both with respect to parameter values and initial unit costs. We further assumed that entrance of new firms was governed by the profits of existing firms, and that once having entered new firms behaved exactly like preexisting ones. The purpose of this section is to investigate the properties of this model with n different firms.

$$\hat{q}_i = \alpha_{qi}\pi_i + \delta_{qi} \qquad \dot{q}_i = 0 \text{ for } q_i = 0 \qquad i=1,\dots,n \qquad (5.1)$$

$$\hat{c}_i = \alpha_{ci}\pi_i - \delta_{ci} \qquad i=1,\dots,n \qquad (5.2)$$

$$\hat{p} = -\gamma\Sigma\tilde{q}_i\hat{q}_i + \delta_p \qquad (5.3)$$

$$\tilde{q}_i = \frac{q_i}{\Sigma q_i} \qquad i=1,\dots,n \qquad (5.4)$$

The expansion equation (5.1) has been rewritten so as to be well defined also for companies with no production. In effect it now says that there is no entry into the industry. But firms that do not produce are still characterized by a unit cost level, and hence, by a profit margin. This is slightly artificial and leads to some peculiarities. In particular, if $\alpha_{ci} < 0$, the profit margin may be growing forever but the firm will nevertheless not enter. The interpretation of this property will be discussed below.[1]

Ideally we would like to be able to see the model that was analyzed in Chapter 4 as an aggregated version of model (5.1) - (5.4). This creates problems however. In (4.1) q was meant to be total quantity produced in the industry, $q = \Sigma q_i$. Differentiating this with respect to time and inserting from (5.1) gives

$$\hat{q} = \frac{\Sigma\dot{q}_i}{\Sigma q_i} = \Sigma\tilde{q}_i(\alpha_{qi}\pi_i + \delta_{qi})$$

---

[1] A more reasonable model would be one where firms enter, at a certain entrance size, as soon as the profit margin exceeds a certain level. The steady-state properties of such a model would probably be basically the same as those of the model analyzed here. The dynamic properties would be more complicated however.

This shows that the aggregate expansion equation of Chapter 4 can be seen as an average of the individual expansion equations weighted by the market shares. However, if firms are different, the market shares will change over time.

SELECTION OF FIRMS

In the evolutionary models language of Nelson and Winter equations (5.1) and (5.2) represent the impact of the *search* behaviour of single firms in reaction to signals provided by variations in profits. At the same time there will in the aggregate also be a *selection* effect because certain firms grow at the expense of others. An interesting question is how this selection works. Is there a stable steady-state where only firms of a certain type survive? This question is related to the discussion initiated by Alchian (1950) about the survival of profit-maximizers.

The selection problem will be analyzed first by looking at the steady-state solution to the model (5.1) - (5.4), where all n firms differ from one another with respect to at least one of the behavioural parameters. After having done that we will look into the stability properties of these steady-state solutions.

By *steady-state* we mean a situation where the market shares of all firms are constant. This implies

$$\dot{\tilde{q}}_j = \tilde{q}_j \left(\hat{q}_j - \sum_{i=1}^{n} \tilde{q}_i \hat{q}_i \right) = 0 \qquad j=1,\ldots,n \tag{5.5}$$

Further we require that the profit margin is constant for all firms

$$\dot{\pi}_j = (\pi_j+1) \left(-\gamma \sum_{i=1}^{n} \tilde{q}_i \hat{q}_i + \delta_p - \hat{c}_j \right) = 0 \quad j=1,\ldots,n \tag{5.6}$$

This means that we demand that π is constant even for a firm with no production. We will discuss the interpretation of this requirement below.

The first question we will pose is under what conditions there will be more than one firm with a positive market share in steady-state. For any pair of firms j and k with $\tilde{q}^*_{j,k} > 0$, (5.5) and (5.6) then imply that $\hat{q}^*_j = \hat{q}^*_k$ and $\hat{c}^*_j = \hat{c}^*_k = \hat{p}^*$. This means

$$\alpha_{qj}\pi^*_j + \delta_{qj} = \alpha_{qk}\pi^*_k + \delta_{qk} \tag{5.7}$$

$$\alpha_{cj}\pi^*_j - \delta_{cj} = \alpha_{ck}\pi^*_k - \delta_{ck} \tag{5.8}$$

$$\alpha_{cj}\pi^*_j - \delta_{cj} = -\gamma\left(\alpha_{qj}\pi^*_j + \delta_{qj}\right) + \delta_p \tag{5.9}$$

For given values of all parameters, this gives three equations in two unknowns, π^*_j and π^*_k. This means that except by chance there will not exist any steady-state where more than one firm has a positive market share. Note that the market shares do not appear in (5.7) - (5.9). So even if these equations happen to be consistent with each other the solution leaves the market shares indeterminate.[1]

Let us disregard the special cases when there is a solution to (5.7) - (5.9) and concentrate on the case when one firm captures the whole market. This implies by (5.6)

$$\tilde{q}_j = 1 \qquad j=1,\dots,n \tag{5.10}$$

$$\pi^*_j = \frac{\delta_{cj} + \delta_p - \delta_{qj}\gamma}{\alpha_{cj} + \alpha_{qj}\gamma} \qquad j=1,\dots,n \tag{5.11}$$

$$\tilde{q}_i = 0 \qquad i=1,\dots,n \quad i\neq j \tag{5.12}$$

$$\pi^*_i = \frac{-\gamma\left(\alpha_{qj}\pi^*_j + \delta_{qj}\right) + \delta_{ci} + \delta_p}{\alpha_{ci}} \qquad i=1,\dots,n \quad i\neq j \tag{5.13}$$

---

[1] The property that one firm will capture the whole market would no longer hold in general if the behavioural equations were non-linear. Neither would it hold if the costs of production of one firm would affect the costs of production of other firms due to, e.g. learning.

The stability properties of these n steady-states are formally investigated in the Appendix. We find there that any equilibrium characterized by (5.10) - (5.13) is stable if the following three conditions hold.

(i) $\hat{q}_j^* > \hat{q}_i^*$, $i \neq j$. The rate of increase of production for the dominant firm must be larger than that of the other firms if their production were infinitesimal.[1]

(ii) $\alpha_{cj} + \alpha_{qj}\gamma > 0$. The same stability condition holds for the dominant firm as in the model of Chapter 4.

(iii) $\alpha_{ci} > 0$, $i \neq j$. The firms with zero market share must have non-negative cost reaction coefficients.

Starting with the third condition we see that this applies only to the firms with no production. For such firms the only reaction is on the cost side. Let us consider an initial situation where π_i is above π_i^*. If $\alpha_{ci} < 0$, this will induce the firm to cut its costs faster. Since price is determined by the behaviour of firms with a positive market share, it is unaffected and π_i will increase, leading to further accelerated cost cuts etc.

This illuminates how strange it is to assume $\alpha_{ci} < 0$ for a firm of infinitesimal size. The main argument behind a negative α_{ci} is that high profits enable firms to devote more resources to R & D. But certainly this mechanism cannot be of great importance for an infinitesimal firm. Later in this chapter we will investigate a model where the rate of cost reduction is determined, apart from the profit margin, also by the scale of production. In this section we can, however, only note that stability condition (iii) to some extent reflects a deficiency in the model formulation.

Let us then go over to stability condition (i). Making use of (5.13) and (5.1) we see that this implies

---

[1] $\hat{q}_i^*$ is the rate of expansion that follows from (5.1) if $\pi_i = \pi_i^*$ and $q_i > 0$.

$$\hat{q}_j^* > -\frac{\alpha_{qi}\gamma}{\alpha_{ci}}\hat{q}_j^* + \alpha_{qi}\frac{\delta_{ci} + \delta_p}{\alpha_{ci}} + \delta_{qi} \tag{5.14}$$

Provided that stability conditions (ii) and (iii) are fulfilled, this gives

$$\frac{\alpha_{qj}\left(\delta_{cj} + \delta_p\right) + \alpha_{cj}\delta_{qj}}{\alpha_{cj} + \alpha_{qj}\gamma} > \frac{\alpha_{qi}\left(\delta_{ci} + \delta_p\right) + \alpha_{ci}\delta_{qi}}{\alpha_{ci} + \alpha_{qi}\gamma} \tag{5.15}$$

i.e.

$$\hat{q}_j^* \; (\tilde{q}_j = 1) > \hat{q}_i^* \; (\tilde{q}_i = 1) \tag{5.16}$$

(5.16) states the intuitively reasonable result that the only stable steady-state is that which implies the fastest rate of growth of production. Note, however, that this conclusion hinges on the stability condition $\alpha_{ci} < 0 \; \forall \; i$ except possibly for the dominant firm.

(5.16) gives an answer to the question of survival of the fittest. The fittest are those who grow fastest. (5.15) says the same thing in terms of the parameters of the model. This expression can be made more transparent by assuming that firms differ only with respect to one parameter. It then implies, where j as before denotes the surviving firm and i any other firm,

$$\delta_{cj} > \delta_{ci}$$

$$\delta_{qj} > \delta_{qi}$$

$$\alpha_{qj} \gtrless \alpha_{qi} \quad \text{as} \quad \pi^* \gtrless 0$$

$$\alpha_{cj} \lessgtr \alpha_{ci} \quad \text{as} \quad \pi^* \gtrless 0$$

The first two inequalities are quite obvious. The fittest are those with the highest rates of spontaneous cost reduction and expansion. The other two sets of inequalities show that the type of firm which survives in steady-state depends on the sign of π^*. Remember that this sign only depends on the trend parameters, which in these comparisons are equal for all firms. If these make steady-state profits positive firms with high α_q and low α_c will survive, and vice versa if $\pi^* < 0$. A tentative interpretation of this is that if $\pi^* > 0$ there will be a tendency for the selection mechanism to make the industry more competitive in the sense of the word suggested in Chapter 4 ($\alpha_q >> 0$, $\alpha_c \approx 0$).

The general stability conditions above only guarantee local stability. Since we propose that the aggregate model is also applicable for situations far from steady-state it is of considerable interest to get some feeling for the global dynamic properties of the disaggregated model. As a simple example of this we analyze in Appendix A, p. 229, a model of two firms which differ only with regard to α_q. If both firms fulfil the stability condition (ii), there will be one globally stable steady-state fulfilling condition (i).[1] The main features of the adjustment process are shown in the phase-diagram, Fig. 5.1, where the axes show the market share of one firm and the profit margin, which is equal for both firms. The figure shows that π^* will now not be approached monotonically, but there will first be a phase of increasing profits followed by a decrease towards π^*.

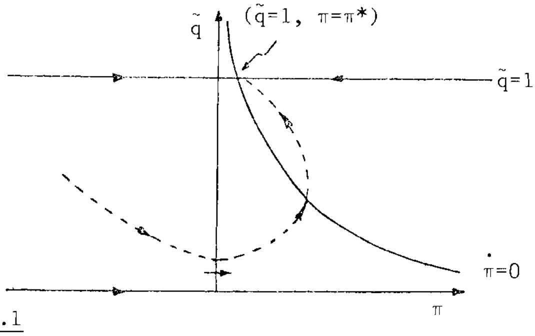

Figure 5.1

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[1] Condition (iii) does not apply since we assume that α_c and c are equal in both firms.

If one of the firms does not fulfil stability condition (ii), the steady-state will in general only be locally stable. Fig. 5.2 shows that if the initial profit margin and the market share of the dominant firm are low enough, the stable firm may be competed out of the market.

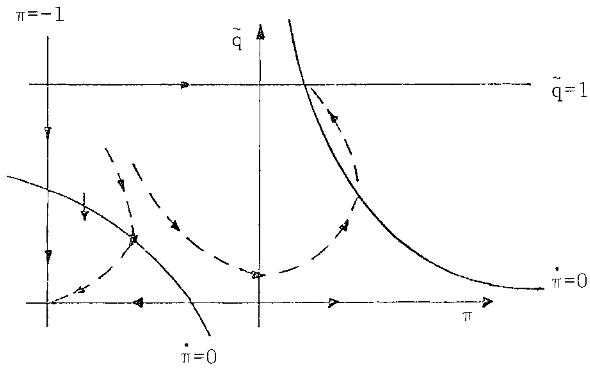

Figure 5.2

The main conclusion of this section is that it is not easy to infer that only firms with certain parameter values will survive in the long run. First, we have shown that the steady-state will be stable only if $\alpha_{ci} < 0$ for all except possibly one firm. Second, even if this holds we have seen that the type of firm which survives depends on what type of shocks are imposed on the system and the values of the trend parameters. This is a conclusion, though reached within a quite different model framework, which resembles that of Winter (1964) in his critique of Friedman's position with respect to survival of profit maximizers.

NON-LINEARITIES AND MULTIPLE EQUILIBRIA

The simple cost reduction equation (4.2) is obviously unrealistic for at least two reasons. As we just noted in discussing the disaggregated model it is quite unreasonable that $\hat{c}$ only depends on the profit margin, i.e. that it is independent of the scale of production. This problem will be taken into account in the next section.

Another reason why (4.2) is unrealistic is that it is linear. This means that there is no bound to the rate of cost reduction possible if only profits take on a value that is extreme enough. In this section we will consider cost reduction functions that approach limits as π approaches infinity and also functions the slope of which may change. A general conclusion from the analysis is that there may be multiple equilibria, some of which are unstable. This implies that a marginal change of a parameter may lead to a large change of the steady-state values, and the steady-state actually reached will depend not only on the parameter values but also on the particular initial conditions.

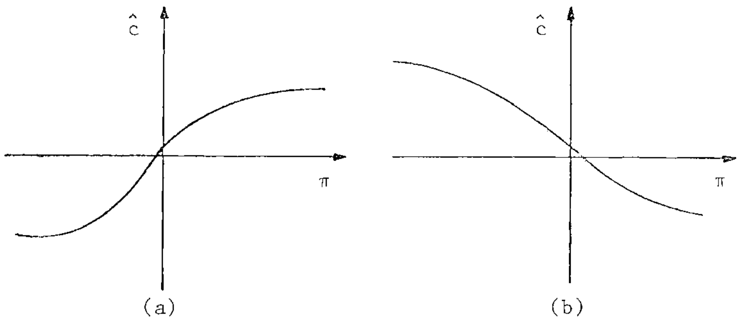

Figure 5.3

Fig. 5.3 shows two curves with asymptotes as π approaches $\pm\infty$.[1] The curves have been drawn with an inflection point around $\pi = 0$. For the case of $\alpha_c > 0$ this reflects that the

[1] Strictly speaking, the equations are not defined for $\pi < -1$.

pressure to reduce slack is strongest when the figures in the accounts are close to changing between black and red. Casual empiricism suggests that this may be a reasonable assumption. It is less easy to make any a priori judgement as regards the case $\alpha_c < 0$. This is so in particular because it is difficult to separate the effects arising from a high profit margin from those stemming from large scale.

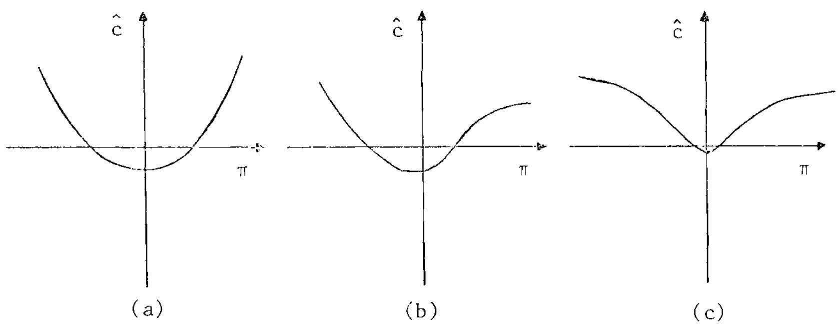

Figure 5.4

In Fig. 5.4 we have allowed the sign of the slope of the cost reduction function to vary. In particular, it seems likely that as profits become sharply negative, the effect of a further worsening will mainly be to bring about reactions of a short-term character which will rather make things worse in the longer run, whereas it is unlikely that there can be any strong impact on the motivation to reduce slack; indeed, too much adversity may even kill the motivation to search for improvement.

The curves are all drawn on the presumption that the maximum rate of cost reduction obtains at zero profits. This is of course not necessarily so. If we believe the motivational effects to be strong perhaps the maximum positive slope should be around zero and the curves of Fig. 5.4 be shifted leftwards. If, on the other hand, we think that it takes a considerable amount of R & D to get any results we may want to shift the curves rightwards and have the maximum rate of cost reduction at a positive profit margin.

The consequences of different types of non-linearities are conveniently shown graphically. Assuming the expansion function to be linear and calling the non-linear part of the cost reduction function $f_c(\pi)$, where $f_c(0) = 0$, we have this equation for the phase-curve.[1]

$$\hat{\pi} = \frac{\dot{\pi}}{\pi+1} = \hat{p} - \hat{c} = -\alpha_q\gamma\pi - \delta_q\gamma + \delta_p - f_c(\pi) + \delta_c \qquad (5.17)$$

We first show the case where f_c is linear as an illustration

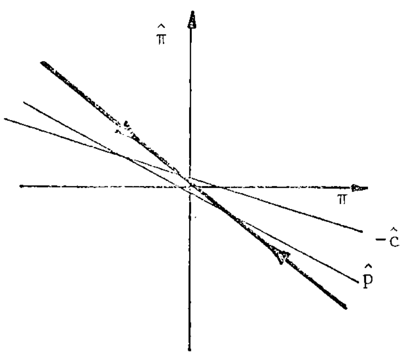

Figure 5.5

Both curves being linear $\hat{\pi}(\pi)$ will also be linear and there will only be one steady-state. It will be stable if $\hat{\pi}(\pi)$ is downward-sloping, i.e. if

$$\frac{df_c}{d\pi} + \alpha_q\gamma > 0$$

a condition which we recognize as the old stability condition $\alpha_c + \alpha_q\gamma > 0$.

[1] Note that the use of the symbol (^) is inconsequent, since it is not a relative rate of change here. It is nevertheless convenient and should not cause confusion.

Figures 5.6 and 5.7 show the phase diagrams for the cost reduction functions illustrated by Figures 5.3 and 5.4.

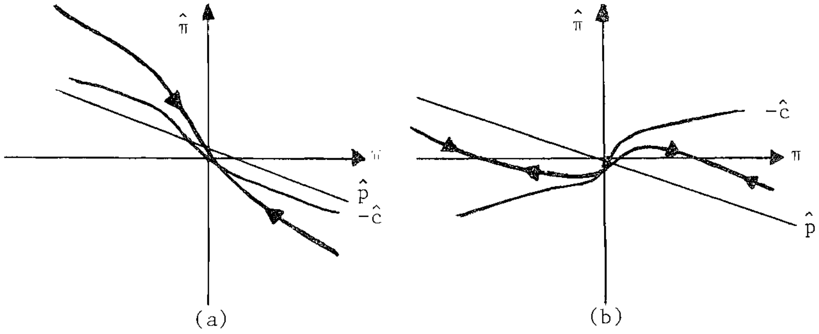

Figure 5.6

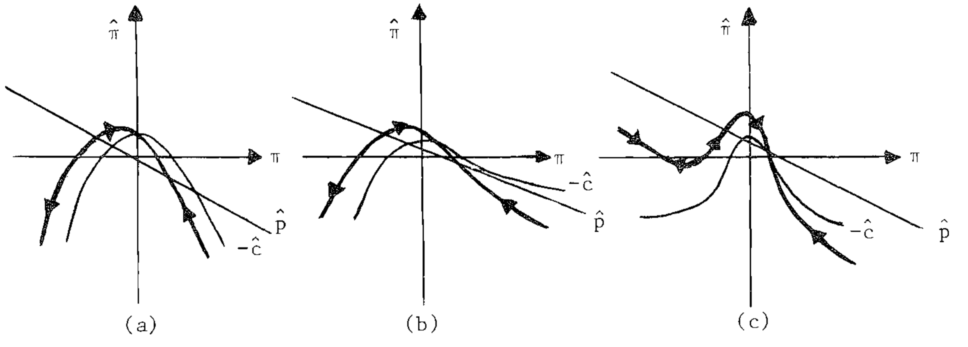

Figure 5.7

We will now be interested in answering the following questions. How many steady-states, if any, are there? Are they stable? Are they associated with positive or negative profits?

The *number* of steady-states is basically determined by the number of sign changes of the slope of the phase-curve.

No sign change, Fig. 5.6 (a), gives one steady-state. One sign change, Fig. 5.7 (a) and (b), can give two. And two sign changes, Figures 5.6 (b) and 5.7 (c), give a maximum of three steady-states. Whether we will really get the maximum possible number of steady-states for the different cases also depends on the sign of $\hat{\pi}$ at the turning points, which basically depends on the values of the trend parameters; it is easily seen that the number of intersections with the π-axis will vary as the curves are shifted upwards and downwards.

The *stability* properties of the steady-states follow from the sign of $\hat{\pi}$ as $\pi \to \pm\infty$. If $\hat{\pi} < 0$ when $\pi \to +\infty$, as is the case in all our examples, the steady state associated with the highest profit margin will be stable; profits will fall when they are above it and rise when they are below it. The other steady-states will be alternatingly unstable and stable.

The *sign* of the steady-state profit margins finally can be seen to depend on the sign of $\hat{\pi}(0)$. If, e.g. $\hat{\pi}(0) > 0$ and $\hat{\pi}(+\infty) < 0$, there is at least one positive and stable steady-state.

Looking at the different cases in somewhat greater detail, we see that the basic properties of the linear case remain as long as the slope of the phase-curve is everywhere the same. This is so if the marginal cost reducing effect of increased profitability is nowhere stronger than the marginal price reducing effect ($\frac{\partial \hat{c}}{\partial \pi} - \frac{\partial \hat{p}}{\partial \pi} > 0$ $\forall \pi$, or vice versa for the unstable case). Fig. 5.6 (a) is an example of this.

In 5.6 (b), the slope of the phase-curve shifts. For a range around zero, $\frac{\partial \hat{p}}{\partial \pi} - \frac{\partial \hat{c}}{\partial \pi} > 0$, whereas for high positive or negative profits, the rate of cost reduction will not change much and the effect on the rate of price change will dominate. If we have these sign changes, and $\hat{\pi}(0)$ is close to zero, there will be three steady-states. The slope being "right" for high and low profits, it is also seen that the middle steady-state will be unstable and the other two stable.

The cases where the rate of cost reduction reaches a maximum are depicted in 5.7 (a)-(c). In 5.7 (a) it is assumed that the curve is everywhere convex. Then the profit margin will be falling both for high negative and high positive values. The lower steady-state will be unstable and the higher one stable. The sign of this is as usual determined by the sign of the trend parameters. As the curve is shifted downwards, the value of π^* in the stable steady-state falls and the range of stability diminishes. For some parameter values there will not exist any steady-state at all; profits will be falling at all levels.

5.7 (b) differs from 5.7 (a) only in that it is based on the more realistic assumption that the rate of change of costs will approach some limit as profits increase. This will not make any important difference since the only modification is made on the side of positive profits where the downward-sloping effect from the price change still dominates.

In 5.7 (c) is made the same type of modification for highly negative profit margins. This leads to the same picture as in 5.6 (b). We again get three steady-states provided the curvature around zero is strong enough and the trend parameters are "right".

All the cases considered have the property in common that the highest steady-state is stable. This reflects that it indeed seems likely that sooner or later the negative effect on profits from expansion will outweigh the possibly positive effect from larger R & D funds. On the other hand, it seems quite conceivable that the lowest steady-state is unstable, since this means that too much adversity can kill an industry. It seems equally realistic to describe the death process of an industry in disequilibrium terms - a collapse with ever faster increasing costs and falling profit margins - as to describe it in equilibrium terms - exponentially decreasing production and a constant profit margin.

Going through all possible quasi-dynamic experiments in the non-linear multiple equilibria case would be tiresome both for the author and the reader. The interesting thing is hardly the effects on these complicated adjustment paths of marginal parameter variations, but rather the basic features delineated above.

A particularly interesting implication of these models is that a small shock may initiate a disequilibrium process towards a steady-state far removed from the original one. Regard any of Figures 5.7 and assume that the industry is initially close to the stable steady-state with $\pi^* > 0$. The effect of a sudden fall in demand will be a fall in π. If this is large enough we see from (a) and (b) that this will start a process which will wipe away the whole industry. In 5.7 (c) it will instead tend to bring the industry towards the other stable steady-state. If we instead regard a shift in the rate of growth of demand (δ_p), this means that there may no longer be any steady-state. It seems that this kind of analysis may offer one explanation of the fact that seemingly minor changes in external conditions sometimes lead to a drastically changed situation for an industry.

SCALE DEPENDENT COST REDUCTION

It is a well established fact that the inputs into R & D activities increase more than proportionately with the size of the firm. At least this holds up to some limit. The evidence is more ambiguous as regards the relation between size and innovative output. But the bulk of it seems to indicate that output, normally measured by patents, increases more than proportionately with size up to a limit.[1]

We will now introduce a scale-dependence of cost reduction in a way that takes account of the fact that $\hat{c}$ does not change with firm size after a certain limit. To do this we choose the logistic functional form (5.18)

$$\hat{c} = \alpha_c \pi - \frac{\zeta}{1 + e^{-\eta q}} - \delta_c \tag{5.18}$$

$$\alpha_c,\ \zeta,\ \eta > 0 \qquad \delta_c \gtrless 0$$

α_c can now be interpreted as representing the slack reduction effect and will hence be positive. (5.18) introduces two new parameters, ζ and η. From (5.19) and (5.20) we see that ζ is a measure of (twice) the difference between the maximum and minimum rate of cost reduction obtainable with a given level of the profit margin.

$$\hat{c}(q=0) = \alpha_c \pi - \frac{\zeta}{2} - \delta_c \tag{5.19}$$

$$\lim_{q \to \infty} \hat{c} = \alpha_c \pi - \zeta - \delta_c \tag{5.20}$$

(5.18) has an inflection point at $q = 0$,[2] i.e. $\partial^2 \hat{c}/\partial q^2 > 0$ for all positive q. η can then, from (5.21), be seen as a measure of the maximum value of $\partial \hat{c}/\partial q$, given the value of ζ.

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[1] See Markham (1965), Kamien & Schwartz (1975) and, for a Swedish study, Johannisson and Lindström (1971).

[2] We could of course choose a formulation of the logistic function which did not restrict the inflection point to q=0. It does not seem, however, that this would mean an important gain in realism.

$$\left.\frac{\partial \hat{c}}{\partial q}\right|_{q=0} = -\frac{\eta\zeta}{4} \tag{5.21}$$

In the basic model of Chapter 4 the general dynamic properties could be analyzed in π-space, since $\hat{q}$ and $\hat{c}$ were functions only of π. Now that $\hat{c}$ is also a function of q, we need a phase diagram in π-q space.

$$\dot{q} = q(\alpha_q \pi + \delta_q) \tag{5.22}$$

$$\dot{\pi} = (\pi+1)\left[-(\alpha_c+\alpha_q\gamma)\pi + \delta_p + \delta_c - \delta_q\gamma + \frac{\zeta}{1+e^{-\eta q}}\right] \tag{5.23}$$

We now define steady-state as a situation where $\dot{\pi}=0$. in accordance with the definition employed in Chapter 4. This gives[1]

$$\left.\pi\right|_{\dot{\pi}=0} = \frac{\delta_p + \delta_c - \delta_q\gamma + \frac{\zeta}{1+e^{-\eta q}}}{\alpha_c + \alpha_q\gamma} \tag{5.24}$$

This is a function of q and will only be constant if $\dot{q}=0$ or if $q\to\infty$. $\dot{q}=0$ gives

$$\pi = -\frac{\delta_q}{\alpha_q} \qquad \text{or} \qquad q = 0 \tag{5.25}$$

Further,

$$\frac{\partial \dot{q}}{\partial \pi} = \alpha_q q > 0 \tag{5.26}$$

$$\left.\frac{\partial \dot{\pi}}{\partial \pi}\right|_{\dot{\pi}=0} = -(\alpha_c+\alpha_q\gamma)(\pi+1) < 0 \tag{5.27}$$

where we have assumed that the stability condition of the basic model, $\alpha_c + \alpha_q\gamma > 0$, is fulfilled. The phase diagram will then look like Figure 5.8.

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[1] The steady-state $\pi^{**} = -1$ which implies p=0 will be kept out of the discussion.

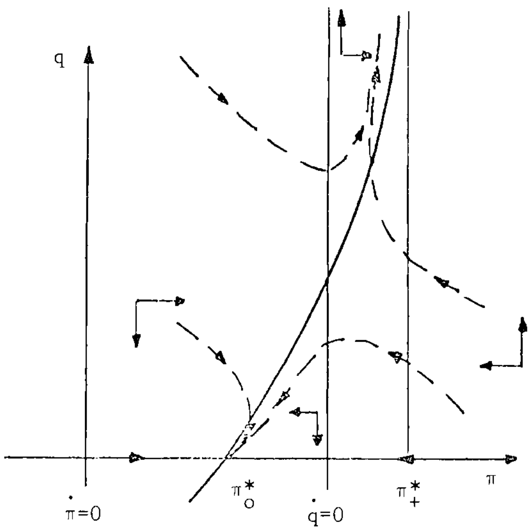

Figure 5.8

This shows that there are two locally stable steady-states. Depending on initial conditions the industry will approach either

$$\pi^*_o = \frac{\delta_p + \delta_c - \delta_q\gamma + \frac{1}{2}\zeta}{\alpha_c + \alpha_q\gamma}$$

$$q^*_o = 0$$

or

$$\pi^*_+ = \frac{\delta_p + \delta_c - \delta_q\gamma + \zeta}{\alpha_c + \alpha_q\gamma}$$

$$\hat{q}^*_+ = \frac{\alpha_q(\delta_p+\delta_c+\zeta) + \alpha_c\delta_q}{\alpha_c + \alpha_q\gamma} > 0$$

There is also an unstable steady-state at the intersection of $\dot{q}=0$ and $\dot{\pi}=0$. In general we cannot be sure that this intersection exists for real values of both variables and positive q. There will be a real solution to (5.24) and (5.25) with q>0 if

$$\pi_o^* < -\frac{\delta_q}{\delta_q} < \pi_+^*$$

If these inequalities do not hold, either of the steady-states will be globally stable. This is intuitively very reasonable. Let us for a moment regard the case of $\delta_q=0$, i.e. when there is no expansion at zero profits. This makes the phase-line $\dot{q}=0$ coincide with the q-axis as in figures 5.9 and 5.10. If the trend factors work in favour of the industry, in the sense that demand is increasing ($\delta_p>0$) and costs are reduced at zero profits irrespective of how much is produced $\left(\delta_c + \frac{1}{2}\zeta > 0\right)$, we will get the picture of Fig. 5.9. No matter what are the initial values, the steady-state with positive profits and expansion of production will be approached. Fig. 5.10 depicts the opposite situation of an industry always dying in the long run no matter which are the initial conditions.

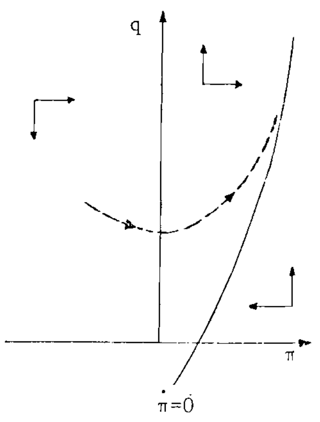

Figure 5.9

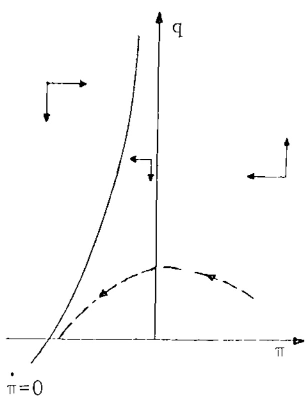

Figure 5.10

In the intermediate cases, Fig. 5.8, the initial conditions determine which steady-state will be approached. A small initial production level, combined with a low profit margin, makes for long run extinction. But if q_o and π_o are large enough, the steady-state with expansion will be approached. For long run survival it is, hence, important that the industry gets an initial stimulus of high profits in order to start growing and be able to reap the scale advantages in R & D.

From this result we can draw the conclusion that the model just analyzed will in the long run behave approximately like the model of Chapter 4, if it approaches the steady-state where production is expanding. Further, when we have separated out the scale effect on R & D, like in (5.18), it is more natural to assume $\alpha_c > 0$. This means that we have somewhat less reason to be bothered about the instability associated with $\alpha_c < 0$ noted in the discussion of the disaggregated version of the model.

One of the main arguments in favour of $\alpha_c < 0$ is that R & D has to be financed largely by profits that are plowed back into the company. By profits is here meant the total amount of profits in crowns or dollars, not a profit margin. This is another argument for the importance of scale in the cost reduction equation. However, if we accept this argument, it is no longer possible to separate q from π.

The amount of profits, $(p-c)q$, can by definition be written πqc. Inserting this into the same type of logistic function as (5.18) gives

$$\hat{c} = \alpha_c \pi - \frac{\zeta}{1 + e^{-\eta \pi q c}} - \delta_c \tag{5.28}$$

To illustrate this function we assume that there is no slack reduction effect ($\alpha_c = 0$). We then get a family of curves, one for each level of total cost. (See Fig. 5.11.) The higher total cost (qc) is, the more dramatic will be the effect of variations in the profit margin when this is close to zero.[1]

[1] Again we have assumed the inflection point to be at zero.

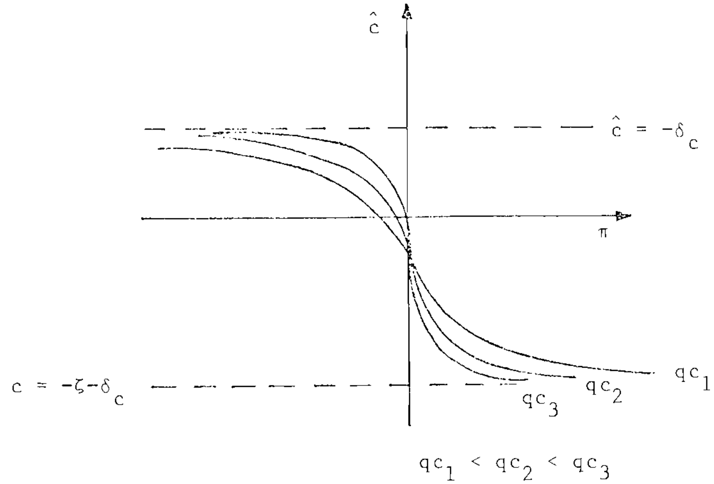

Figure 5.11

Even a general dynamic analysis of a model with a cost reduction function like (5.28), where both q and c enter multiplied by π, will be quite complicated. Depending on whether qc is increasing, decreasing or constant in the steady-state we now have three possible steady-states:

(i) $\hat{q}^*_+ + \hat{c}^*_+ > 0$

$$\pi^*_+ = A_+ = \frac{\delta_p + \delta_c - \delta_q\gamma + \zeta}{\alpha_c + \alpha_q\gamma} > 0$$

(ii) $\hat{q}^*_o + \hat{c}^*_o < 0$

$$\pi^*_o = A_o = \frac{\delta_p + \delta_c - \delta_q\gamma + \frac{1}{2}\zeta}{\alpha_c + \alpha_q\gamma} \gtrless 0$$

(iii) $\hat{q}^*_- + \hat{c}^*_- > 0$

$$\pi^*_- = A_- = \frac{\delta_p + \delta_c - \delta_q\gamma}{\alpha_c + \alpha_q\gamma} < 0$$

Note that $A_+ > A_o > A_-$. The existence of the steady-states depends on the specific parameter values. In general, we can neither be sure of the existence nor of the stability of any of the steady-states.

In the appendix we analyze the case where πq is substituted for πqc. This is a considerably simpler case because it can be analyzed diagrammatically. The steady-states of that model will be the same as those of the model above, but depend on the sign of $\hat{q}^*$ instead of $\hat{q}^* + \hat{c}^*$.

Depending on the parameter values there are 16 possible cases. This is an attempt to summarize the conclusions.

(i) If production is contracting even at $\pi = A_+$, π_o^* is a globally stable steady-state.

(ii) If production is increasing even at $\pi = A_-$, either of π_-^* and π_+^* will be approached. If $A_+ < 0$, π_-^* is globally stable, and if $A_- > 0$, π_+^* is globally stable. Otherwise it depends on initial conditions.

(iii) In the intermediate cases, where production is increasing at $\pi = A_+$ and decreasing at $\pi = A_-$, π_-^* will never be a steady-state.

(iv) If there is contraction at $\pi = A_o$ but expansion at $\pi = A_+$, either of π_o^* or π_+^* are approached in steady-state.

(v) If there is expansion at $\pi = A_o$, but contraction at $\pi = A_-$ either π_-^* or that profit margin which gives $\hat{q} = 0$ will be a steady-state. The latter will however be locally unstable if the change of curvature of the cost reduction function, η, is strong enough.

The main conclusion from this is that if the rate of cost reduction is assumed to depend on the amount of profits, it becomes of crucial importance whether the steady-state profit margin is positive or negative. It is this property that accounts for most of the complexities with this model.

The simple picture of the model based on the cost reduction function (5.18), figures 5.8-10, will now only be obtained when $A_- > 0$. For then π^*_- is not a feasible steady-state for positive q. But this conclusion does not hold in general when $A_- < 0$. Regard, as an example, Fig. 3 of the Appendix. As before, initial conditions determine which steady-state will be approached. But no matter how far the industry has come along the expanding steady-state, a shock that leads to losses will start a contraction towards $q^* = 0$. The explanation for this seemingly counterintuitive result is that there is an area - for $0 < \pi < \pi^*_+$ and q high enough - where production is increasing and prices falling but where this affects cost reduction positively to such an extent that profits are rising. But a shock that brings about a negative profit margin makes the rate of cost reduction much lower by (5.28), and thereby starts a process of contraction of production.

From the model based on the cost reduction function (5.18) we concluded that the linear model of Chapter 4 could be understood as a long-run approximation. This is no longer possible in general with equation (5.28).

THE ROLE OF EXPECTATIONS

In all the variants of the model considered until now we have let current profits be the variable influencing cost reduction and expansion. However, most of the arguments advanced in favour of the basic behavioural assumptions apply both to current profits and expected future profits.

In all the dynamic maximization models of Chapter 3, it is assumed that firms expect the current price to be ruling forever. From this follows that no distinction is made between a change in current price and a change in expected future price. Qualitatively, however, we would have got the same effects, i.e. the same sign of the partial derivatives, on expansion and cost reduction if we had instead studied the effects of an expected future price change.

In this section, we will introduce expectations into the basic model by substituting the integral over all discounted expected future profit margins, $\int_t^\infty \pi_\tau^e e^{\rho(t-\tau)} d\tau$, for π_t in (4.1) and (4.2). ρ is the firm's subjective rate of discount and π_τ^e is the profit margin expected at time t to be ruling at time τ. It is open to discussion whether one should instead weigh the profit margins by expected future production. A firm that is contracting rapidly will e.g. probably have less incentive to cut costs than a firm which expects to expand production. Taking this into account would however complicate the analysis unnecessarily, and we will hence analyze this system.

$$\hat{c} = \alpha_c \int_t^\infty \pi_\tau^e \; e^{\rho(t-\tau)} \; d\tau - \delta_c \tag{5.29}$$

$$\hat{q} = \alpha_q \int_t^\infty \pi_\tau^e \; e^{\rho(t-\tau)} \; d\tau + \delta_q \tag{5.30}$$

$$\hat{p} = -\gamma\hat{q} + \delta_p \tag{5.31}$$

The question is now how expectations about the future are formed. What determines π_τ^e, the profit margin that is expected at time t to be ruling at time τ? Basically, there are two approaches to expectations formation currently used in economic theory, adaptive expectations and rational, or consistent, expectations.

According to the hypothesis of *adaptive expectations* there is a constant guessing going on. At each time the agent in question looks at the guesses he has made earlier and compares them with the value which turned out to be true. In the light of this information he then revises the previous guesses about the future. In discrete time this process can be formulated[1]

$$x^e(t+\tau,t) - x^e(t-h+\tau,t-h) = a(h,\tau)[x(t) - x^e(t,t-\tau)] \tag{5.32}$$

[1] The following general discussion owes much to the treatment in Burmeister and Turnovsky (1976).

where $x^e(t+\tau,t)$ is the value of x expected at time t to be ruling at time $t+\tau$ and $x(t)$ is the actual value at t. $a(h,\tau)$ is the rate of adaptation which varies with the time interval between predictions (h) and the time horizon over which the forecast is made (τ).

In order to formulate an expression in continuous time which corresponds to (5.32) we divide both sides by h and let h go to zero. We then get in the limit

$$\frac{\partial x^e(t+\tau,t)}{\partial t} = \bar{a}(\tau)[x(t) - x^e(t,t-\tau)] \tag{5.33}$$

where $\bar{a}(\tau) = \lim\limits_{h\to 0} \frac{a(h,\tau)}{h}$.

In principle one would need an infinite number of such equations, one for each forecast horizon.

Letting also the forecast horizon approach zero we get

$$\frac{dx^e(t,t)}{dt} = \bar{a}(0)[x(t) - x^e(t,t)] \tag{5.34}$$

This implies that, in order that $x^e(t,t)$ be changing over time, $x(t) \neq x^e(t,t)$. This says that the forecaster is not supposed to know the value presently ruling of the variable in question. So the common formulation of adaptive expectations applied to levels of the variable being forecasted leads to an inconsistency in this respect.

One may instead let adaptive expectations be tied to the rate of change of x. Then we get as an analogue of eq. (5.33)

$$\frac{\partial \dot{x}^e(t+\tau,t)}{\partial t} = a'_\tau[\dot{x}(t) - \dot{x}^e(t,t-\tau)] \tag{5.35}$$

It may be argued that there is less of an inconsistency involved in allowing $\dot{x}(t) \neq \dot{x}^e(t,t)$, and that hence (5.35) makes somewhat more sense than (5.33). Now we can assume that the agent forms expectations about the rate of change for any future time and uses these expectations to compute the expected level.

$$x^e(t+T,t) = x_t + \int_0^T \dot{x}^e(t+\tau,t)d\tau \qquad (5.36)$$

Unfortunately this looks quite complicated with (5.35) being a mixed differential-difference equation.

For the sake of simplicity we will instead assume $\dot{x}^e(t+\tau,t)$ to be a simple function of $\dot{x}^e(t,t)$. Here this function will be assumed to be exponential

$$\dot{x}^e(t+\tau,t) = \dot{x}^e(t,t)e^{-s\tau} \qquad s > 0 \qquad (5.37)$$

Inserting this into (5.36) and evaluating we get

$$x^e(t+T,t) = x_t + \dot{x}^e(t,t)\,\frac{1}{s}\,(1-e^{-sT}) \qquad (5.38)$$

This implies that if $\dot{x}^e > 0$, x is assumed to be growing forever. The formulation does consequently not cover cases where growth today leads to expectations of decline tomorrow. It seems natural to assume $s > 0$. The opposite, $s < 0$, would mean that not only is the value of the variable expected to continue to increase, but it is even expected to increase at an increasing rate.

Making use of this formulation in (5.29) and (5.30) we get the basic equations

$$\hat{c}_t = \frac{\alpha_c}{\rho}\,\pi_t^E - \delta_c \qquad (5.39)$$

$$\hat{q}_t = \frac{\alpha_q}{\rho}\,\pi_t^E + \delta_q \qquad (5.40)$$

where

$$\pi_t^E = \rho \int_t^\infty \left[\pi_t + \dot{\pi}_t^e \frac{1}{s}\left(1-e^{s(t-\tau)}\right)\right] e^{\rho(t-\tau)} d\tau = \pi_t + \frac{\dot{\pi}_t^e}{\rho+s} \tag{5.41}$$

and the hypothesis of adaptive expectations says

$$\frac{d}{dt}\dot{\pi}_t^e = a(\dot{\pi}_t - \dot{\pi}_t^e) \tag{5.42}$$

From (5.41) we see that we get back to the model formulation of Chapter 4 if $s = \infty$. For then firms do not take any notice of the expectation that profits are changing, but assume the current level to be ruling forever.

The dynamics of the profit margin and the discounted sum of all future profits are given by

$$\dot{\pi} = (\pi+1)\left[-\frac{\alpha_c + \alpha_q\gamma}{\rho}\pi^E + \delta_c + \delta_p - \delta_q\gamma\right] \tag{5.43}$$

$$\begin{aligned} \dot{\pi}^E &= \dot{\pi} + \frac{a}{\rho+s}(\dot{\pi}-\dot{\pi}^e) = \\ &= \left[1 + \frac{a}{\rho+s}\right](\pi+1)\left[-\frac{\alpha_c+\alpha_q\gamma}{\rho}\pi^E + \delta_c + \delta_p - \delta_q\gamma\right] - a(\pi^E-\pi) \end{aligned} \tag{5.44}$$

The steady-state values of the system will of course not have been affected by the introduction of expectations[1]

$$\pi^* = \pi^{*E} = \frac{\delta_c + \delta_p - \delta_q\gamma}{\alpha_c + \alpha_q\gamma}\rho$$

$$\pi^{**} = \pi^{**E} = -1$$

---

[1] Remember that we have changed the dimension of the profit variable from an instantaneous rate in Ch. 4 to an integral. That is why the discount rate ρ appears in the expression for π^*. It could of course have been suppressed in the coefficients α_c and α_q.

But the dynamic properties are different now. To see this, we draw a phase diagram in the π-π^E plane. From (5.43) and (5.44) we have

$$\pi^E\Big|_{\dot{\pi}=0} = \pi^*$$

$$\pi^E\Big|_{\dot{\pi}^E=0} = \frac{\left(1 + \frac{a}{\rho+s}\right)(\delta_c+\delta_p-\delta_q\gamma)(\pi+1) + a\pi}{\left(1 + \frac{a}{\rho+s}\right)\left(\frac{\alpha_c+\alpha_q\gamma}{\rho}\right)(\pi+1) + a}$$

Inspection and some calculus show that $\pi^E\Big|_{\dot{\pi}^E=0}$ is continuous, differentiable and positively sloping in the interval $-1 < \pi^E < \infty$.

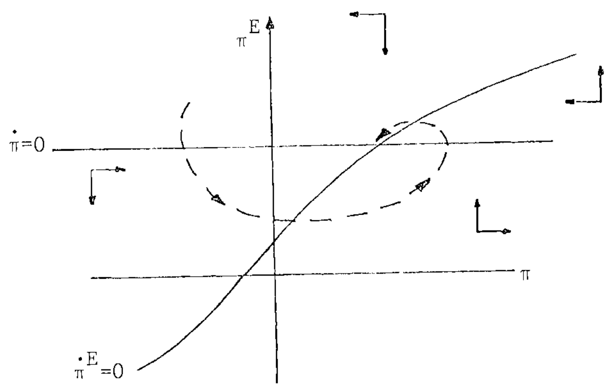

Figure 5.12

As is shown by Fig. 5.12, the steady-state has the form of a spiral sink.[1] In contrast with the model of Chapter 4, where expectations are static, actual, as well as expected,

[1] On the classification of stationary points, see Hirsch & Smale (1974) p. 96.

profits will be oscillating as the steady-state is approached. The diagram is drawn under the implicit assumption that the steady-state is stable. There is no reason to believe that this is so regardless of starting point. But by linearizing equations (5.43) and (5.44) around $\pi = \pi^E = \pi^*$ we can show that this steady-state is locally stable under the same stability condition as for the model with static expectations, $\alpha_c + \alpha_q\gamma > 0$.

Taking the partial derivatives and evaluating at steady-state we get

$$\frac{\partial \dot{\pi}}{\partial \pi} = 0$$

$$\frac{\partial \dot{\pi}}{\partial \pi^E} = -(\pi^*+1)\frac{\alpha_c + \alpha_q\gamma}{\rho}$$

$$\frac{\partial \dot{\pi}^E}{\partial \pi} = a$$

$$\frac{\partial \dot{\pi}^E}{\partial \pi^E} = -\left(1 + \frac{a}{\rho+s}\right)(\pi^*+1)\frac{\alpha_c + \alpha_q\gamma}{\rho} - a$$

The stability conditions are

$$-\left(1 + \frac{a}{\rho+s}\right)(\pi^*+1)\frac{\alpha_c + \alpha_q\gamma}{\rho} - a < 0$$

$$a(\pi^*+1)\frac{\alpha_c + \alpha_q\gamma}{\rho} > 0$$

which are seen to hold, under the assumptions made about ρ, s and a, if $\alpha_c + \alpha_q\gamma > 0$. The steady-state is locally stable.

This shows that with adaptive expectations we can no longer be sure that the model is stable regardless of starting point. And even if it is stable, there will in general be oscillations on the way towards steady-state. But all the steady-state properties remain the same as in the basic model, and if we only consider starting points close to steady-state, we can be sure about stability.

The hypothesis of adaptive expectations implies that firms do not have any knowledge of the working of the economic system that determines the variable being forecasted. They solely look at the past values of the variable. Real life firms are probably somewhat less ignorant. They have at least a superficial knowledge of demand conditions and the behaviour of other firms. As an extreme case we may think of firms with a complete knowledge of the world they live in. Then they will base their behaviour on expectations of the future which will turn out to be realized exactly. We may talk of this as the hypothesis of *consistent expectations*.[1]

The idea of introducing such a sophisticated expectations formation mechanism into a model where firm behaviour in other respects is so crude is admittedly quite strange. But it is not primarily done because it is supposed to give an interesting picture of real life behaviour, but rather because it may give some ideas about how the path towards, or away from, steady-state would look if expectations were always to be fulfilled.

In the appendix it is shown that the steady-state in this case will have the form of a saddle-point.[2] There is an infinite number of paths along which expectations are everywhere fulfilled, but only one of them leads to steady-state; the steady-state is unstable. One may argue, however, that having assumed consistent expectations, it is hardly extra restrictive to assume these expectations not to imply $\pi \to +\infty$ or $\pi \to -1$ in the long run. Then there is only one solution path that satisfies these assumptions,

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1 This leaves the term rational expectations to a stochastic model.

2 It seems to be a quite general property of models with consistent expectations in different model settings that the steady-state is a saddle point. Cf. e.g. Tobin (1965) and Mussa (1978).

the path which approaches steady-state. Along this trajectory profits are monotonically increasing or decreasing. In this respect it resembles the trajectory arising from static expectations closer than the one resulting from adaptive expectations.

The main conclusion from this section is that the role of expectations in general is to complicate the adjustment pattern. Adaptive expectations will lead to oscillations and it cannot be guaranteed that the process is stable except locally. With consistent expectations there is need for an extra assumption, which is not very different from assuming stability, to show that the model is stable.

CONCLUDING COMMENTS

In this chapter we have studied the consequences of modifying the behavioural assumptions of the basic model in different ways. All the cases have a general property in common. They lead to considerable complications of the general dynamic properties of the process including in some cases new and different stability conditions. However, it is of course possible in all cases to interpret the steady-states as arising out of a locally linear model of the type studied in Chapter 4.

The first modification made was the formulation of a disaggregated model where firms differ with respect to the parameter values. This is an important modification, since the whole study is based on the view that firm behaviour is potentially observable and can only to a certain extent be deduced from deeper principles such as profit maximization. If this view is accepted there is no strong a priori reason that all firms should have even roughly the same values of the behavioural parameters. An interesting question is then whether there is some tendency for firms of a certain type to expand at the expense of others. If this were so, it would strengthen the case for analyzing a model where all firms are equal. The analysis does not treat this idea very kindly. One conclusion is obvious. Firms with high rates of spontaneous cost reduction and expansion will tend to survive. But what types of values of the reaction parameters will be selected by the market environment depends on whether the trend parameters imply positive or negative steady-state profits.

In studying stability properties of the disaggregated model we find that the cost reduction coefficient α_C plays a crucial role; the model is only stable if α_C is positive for all firms. This restrictive stability condition reflects to some extent that a cost reduction function that is linear in the profit margin is overly simplified. In two sections we analyze, reverting to the aggregate model, the consequences of making the cost reduction function non-linear and of introducing the scale of production into it. Both of these cases lead in general to multiple steady-states. Initial conditions determine which steady-state will be reached, and marginal changes in the exogenous variables may bring about large changes of the steady-state. Both these cases lead to quite complicated dynamic patterns in an aggregate model and it does not seem easy to say much about the dynamics of a disaggregate model built on the same types of cost reduction functions. Still it seems likely that the introduction of scale into the cost reduction function, as in (5.18), would lead to stability in a disaggregate model; if all parameters are equal a firm with an initial scale advantage would ultimately capture the whole market.

The final modification considered is to let firm behavior depend on expected future profits instead of current profits. With adaptive expectations this implies that profits will oscillate along the solution path. Steady-state is still locally stable in an aggregate model. It seems likely however that one needs stronger conditions to guarantee stability in a disaggregate model with adaptive expectations, for then firms may have different expectations from each other, which may be destabilizing.

6. EMPIRICAL EVIDENCE

The main purpose of this study has been to develop and analyze a model of a market adjustment process where profits play a central role as a driving force for change. In this chapter some empirical evidence to study the relevance of the model is presented. The evidence will be gathered from rather diverse fields of empirical studies, that are more or less directly related to our model. The survey of these fields will, for obvious reasons, be highly selective. The discussion of earlier empirical work will be complemented by some original studies on Swedish firm and industry data.

In the first section we will take up again the question touched upon in Chapter 2 of the *persistence of interindustry profitability differences*. We will study cross-sections of firms and industries and find that there is a significant correlation between the rates of return to capital for two different years even if these are distant in time from each other. Actually one does not find any clear tendency for correlation to approach zero even in the long run. Rather it seems that correlation reaches a positive limit value after only a few years. By this we have indicated that the empirical problem that was singled out as a starting point of this study is not a mere short-run phenomenon.

In the two following sections we will take up the behavioural assumptions of our model. First, we will discuss the relation between the rate of return to capital and the rate of *growth of production*. Growth can stem either from entry of new firms or expansion of already established firms. We will first look at the determinants of entry. As regards growth behaviour of established firms, we will present both studies of investment behaviour and studies of expansion of production. We will see that the results are sensitive to whether the profit variable is lagged or not and whether the variables are averaged over a period of years or not. This is probably due to the fact that business cycle variations tend to be the most important factor in explaining changes from one year to the other. Finally in addition to these studies of firm data we will make a regression on data across industries of growth on profits.

In the third section we will look at the rate of *cost reduction*. We will use two types of information sources. One is studies of the determinants of R & D. The other is a group of studies where one tries to estimate separate rates of return depending on the source of financing used. We will also present a regression of the rate of cost change on the rate of return to capital using firm data.

THE PERSISTENCE OF PROFITABILITY

In Chapter 4 we defined a competitive industry as one where the expansion reaction coefficient α_q is very large. In such industries the rate of profit will converge rapidly towards zero, i.e. towards a state where firms earn a normal rate of return on invested capital. If one were to observe two cross-sections of the same set of competitive industries at two different points of time, there would probably be some correlation if the two points were close. But if enough time for adjustment is given there would be no correlation left at all.

On the other hand, in non-competitive industries where α_q and the cost reaction coefficient α_c are low in relation to the trend parameters, it will take longer time for disturbances to be wiped away, and there may even remain systematic differences between the rates of return in different industries in the long run.

Differences Across Industries

Stigler (1963) has studied the rates of return for a sample of 57 industries for the period 1947-1957. Rate of return is defined as a net rate of return after taxes on total assets.[1] The industries are divided into concentrated, unconcentrated and ambiguous, according to the four-firm concentration ratio.[2]

Table 6:1 Correlation Coefficient Between Rates of Return in year t and t+x, U.S. Manufacturing Industries 1947-1957

Industry structure	$r_{t,t+1}$	$r_{t,t+2}$	$r_{t,t+3}$	$r_{t,t+4}$	$r_{t,t+5}$	$r_{t,t+6}$	$r_{t,t+7}$	$r_{t,t+8}$	$r_{t,t+9}$	$r_{t,t+10}$
concentrated	.74	.72	.66	.69	.71	.57	.57	.53	.35	.40
unconcentrated	.72	.61	.57	.53	.47	.36	.26	.27	.20	.11
ambiguous	.77	.70	.64	.64	.70	.57	.38	.38	.37	.05

Comment: $r_{t,t+1}$ is an average over the ten pairs of consecutive years possible, $r_{t,t+2}$ over nine years etc.

Source: Stigler (1963) p. 71.

Table 6:1 is illustrated in Figure 6.1. We see that in the unconcentrated group the correlation coefficient goes down quite rapidly and is close to zero after ten years. But in the group of concentrated industries r remains around 0.4

[1] For details see Stigler (1963), appendix A.

[2] Concentrated industries are defined as those in which the four leading firms produce 60 per cent or more of the value added, and for which the market is national. Unconcentrated industries meet one of two conditions: (1) the market is national, and the concentration ratio is less than 50 per cent; (2) the market is regional and the concentration ratio is less than 20 per cent. The rest are classified as ambiguous. Stigler's study also covers the period 1938-1947. The results for this period point to even more striking differences between concentrated and unconcentrated industries.

even after ten years. It should be kept in mind, however, that the estimates of $r_{t,t+x}$ presented in Table 6:1 are subject to larger errors as x grows, since they are averages over gradually fewer pairs of years.

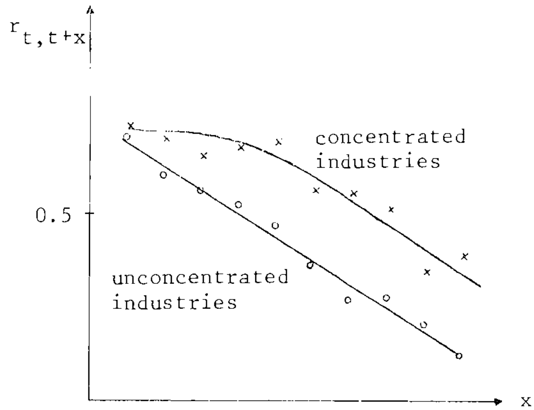

Figure 6.1

The concentration ratio can to some extent be seen as an indicator of α_q. High concentration normally means that new firms play a minor role and that existing firms may act as monopolists. If we can make this interpretation, we see that Stigler's findings fit quite nicely with the results of our model; there is less correlation for industries with high α_q, i.e. the unconcentrated industries.[1]

Stigler is, to my knowledge, the only one who has studied the persistence of differences in rates of return at the industry level. At the firm level there are a few studies, a couple of which will be discussed shortly. Before doing that let us see what evidence Swedish data show.

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[1] There are problems associated with the interpretation of correlation coefficients as indicators of the degree of competition. One is that there may be systematic measurement errors. This is probably so for the dairy and slaughtering industries in the Swedish data used below, as is pointed out in the appendix. Another problem is that business cycle movements are uncoordinated across industries. This should lead to higher correlation coefficients at four years intervals, which perhaps is visible in the tables. If long-run profits were equalized, business cycle movements should also lead to negative correlation coefficients for some years. However, with one exception, we do not find this.

In a small open economy like the Swedish one a standard proxy variable for competitiveness is some measure of the industry's dependence on international trade. The use of this proxy variable rests on the presumption that markets for traded goods, which are world-wide, in general are less concentrated than markets that are mainly supplied by local producers. This implies that we would expect a higher value of α_q for industries selling a large share of production abroad and having only a small share of the home market. In Appendix B are presented tables of the net rates of return on total assets for Swedish industries 1953-1976. We now group these industries according to the share of exports in total output (x/q) in 1970.[1] Industries with x/q < 20 % are classified as sheltered, the rest as non-sheltered. Tables 6:2 and 6:3 present the correlation coefficients of the rates of return for different years.

Table 6:2 Correlation Coefficients Between Rates of Return in year t and t+x, Swedish Manufacturing Industries 1953-1968

Industry structure	$r_{t,t+1}$	$r_{t,t+2}$	$r_{t,t+3}$	$r_{t,t+4}$	$r_{t,t+5}$	$r_{t,t+6}$	$r_{t,t+7}$	$r_{t,t+8}$	$r_{t,t+9}$
Sheltered (11 industries)	.86	.72	.65	.64	.63	.62	.59	.55	.49
Non-sheltered (18 industries)	.78	.70	.66	.62	.55	.46	.42	.43	.42

Industry structure	$r_{t,t+10}$	$r_{t,t+11}$	$r_{t,t+12}$	$r_{t,t+13}$	$r_{t,t+14}$	$r_{t,t+15}$
Sheltered (11 industries)	.39	.28	.22	.29	.41	.35
Non-sheltered (18 industries)	.44	.45	.35	.37	.35	.32

Source: Appendix B, Tables 3 and 4, and, for data on export-shares, Ohlsson (1977).

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[1] Other measures of the degree of dependence on international trade - imports as a share of consumption, or imports plus exports as a share of production - will not change the classification markedly.

Table 6:3 Correlation Coefficients Between Rates of Return in year t and t+x, Swedish Manufacturing Industries 1969-1976

	$r_{t,t+1}$	$r_{t,t+2}$	$r_{t,t+3}$	$r_{t,t+4}$	$r_{t,t+5}$	$r_{t,t+6}$	$r_{t,t+7}$
Sheltered (12 industries)	.67	.56	.45	.31	.27	.36	.17
Non-sheltered (21 industries)	.46	.19	.32	.34	.13	.17	.32

Source: Appendix B, Tables 5 and 6, and Ohlsson (1977).

The Swedish data show roughly the same tendency as Stigler's for the U.S.; the correlation coefficients fall gradually with time, but even after 15 years r is around 0.3. There is little difference, however, between sheltered and non-sheltered industries in Sweden, contrary to the difference between concentrated and unconcentrated industries found for the U.S. Probably it is more difficult to find a good proxy for competitiveness in an open than in a closed economy. The degree of dependence on international trade may be misleading in many cases.

There appears to be a difference between the two periods for Sweden. The r values are considerably lower for 1969-1976 than for 1953-1968. This may be interpreted as a sign of increased competitiveness. But it may also be the result of an increased randomness in the general economic environment; certainly the oil crisis of 1974[1] affected the relative profitability of some industries drastically. When comparing the figures one should also keep in mind that the composition of industries differs between the two periods.[2]

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[1] We see from Table 6 of Appendix B that the correlation coefficients between 1974 and 1975 and any other year are very low.

[2] This is the reason why the data are divided into two periods; industry classification was changed between 1968 and 1969.

The general conclusion seems clear, however. There are long-run inter-industry profitability differences, at least if by long run we mean a time perspective of no more than 10-15 years. This conclusion should be qualified somewhat taking into account the data for 1969-1976. It seems likely, however, that the low correlation coefficients found for part of this period to some extent is due to external shocks.

Differences Across Firms

The same question as was asked above with respect to industry data has been put with respect to firm data by *Mueller* (1977). One rationale for using firm data instead of industry data is that the classification of firms into industries is necessarily somewhat arbitrary. Firms make many different products, which are all to some extent differentiated from each other. The world is rather one of a large number of producers of close substitutes than one of a limited number of isolated markets for homogeneous goods.

Mueller uses a sample of 472 U.S. manufacturing firms with 24 years of data from 1949 to 1972. The profits variable is net income after taxes divided by total assets. For each year the sample is divided into 8 groups of 59 firms according to the rate of return; the firms with highest profits in group 1, etc. From these data is then computed the share of the firms initially in any group i that at time t are in group j. This share can be interpreted as the estimate of a transition probability, pr_{ijt}. The competitive environment hypothesis can then be said to be that all pr_{ijt} are equal if t is large enough. Mueller tests this by regressing each of the pr_{ij} on the reciprocal of time for the 24 years of observations available

$$pr_{ijt} = \alpha_{ij} + \beta_{ij}/t + u_{ijt}$$

α_{ij} is thus the long-run probability that a firm initially in group i will be in group j. If this is independent of the initial group, all α_{ij} will be 1/8. Mueller can however in general reject this hypothesis. 43 out of 64 estimated coefficients are significantly different from 1/8. See Table 6:4.

Table 6:4 Long-Run Probability Lines on Intergroup Mobility

(Each entry gives the projected probability that a firm which started in the ith profit class in 1949 would eventually be in the jth class, i.e. the α_{ij}.)

	j							
i	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	0.34**	0.17**	0.12	0.10*	0.06**	0.06**	0.07**	0.08**
(2)	0.21**	0.13	0.20**	0.14	0.10**	0.08**	0.06**	0.08**
(3)	0.14*	0.19**	0.08*	0.13	0.15*	0.12	0.11	0.09**
(4)	0.08**	0.14	0.14	0.09	0.19**	0.18**	0.12	0.07**
(5)	0.08**	0.13	0.15**	0.17**	0.08	0.14	0.14	0.11
(6)	0.07**	0.09**	0.13	0.14	0.11	0.07*	0.21**	0.19**
(7)	0.04**	0.07**	0.08**	0.13	0.15*	0.20**	0.13	0.20**
(8)	0.04**	0.08**	0.10**	0.11	0.15*	0.15*	0.17*	0.19**

* Significantly different from 0.125, two-tailed test. 0.05 confidence level.

** Significantly different from 0.125, two-tailed test. 0.01 confidence level.

Source: Mueller (1977) p. 373.

Most striking is the high probability of remaining in the extreme groups. α_{11} has by far the highest value of all. It is perhaps more surprising that the probability of remaining in the lowest class, α_{88}, is also significantly above average. The high probabilities of remaining in the extreme groups is partly a statistical artifact since the intragroup dispersion is probably much larger in these groups.[1] This can to some extent be corrected for by adding the contiguous elements[2] to the diagonal elements. We then see that these in all cases except one ($\alpha_{87}+\alpha_{88}$) sum to more than 3/8. This confirms the general conclusion that profitability differences tend to persist.

A study on British data is *Whittington* (1971). The data used are the published consolidated accounts of all companies engaged primarily in manufacturing or distribution in the United Kingdom and having debt or equity stocks quoted on a United Kingdom stock exchange. The study covers a total of 1,955 companies which continued an independent existence throughout the period from 1948 to 1960. The period is divided into two parts and this regression equation is estimated across firms in each of 21 different industries.

$$\pi_{1954-1960} = \beta_o + \beta_1 \pi_{1948-1954}$$

π is the pre-tax net rate of return on net assets.[3] The estimate of β_1 in this equation may be interpreted as a measure of the inverse of the rate of convergence of a single firm towards the industry mean; if $\beta_1 = 1$ there is no convergence at all, if $\beta_1 = 0$ there is complete convergence. Interpreted within the basic model of Chapter 4 this is related to the cost reaction coefficient α_c. For in a cross-section across firms in the same industry the rate of price change can be assumed to be the same for all firms. Linearizing (4.11) around steady-state we then have

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[1] I have used Mueller's technique on the Swedish industry data presented in Appendix B and got similar results. One should also note that, since the observations of the independent variable (1/t) take on the values 1, 1/2, 1/3 etc., the computed one year transition probability will have a disproportionate impact on the slope of the regression line.

[2] I.e. $\alpha_{11} + \alpha_{12}$, $\alpha_{21} + \alpha_{22} + \alpha_{23}$ etc.

[3] Defined as the balance-sheet total minus current liabilities and provisions.

$$\dot{\pi} = -\alpha_c(\pi^*+1)(\pi-\pi^*)$$

All estimates of β_1 but one are significantly larger than zero at the five per cent level. The average value is 0.54. This means that it takes six years to eliminate half of any initial inter-firm profitability differences. If we accept the assumption that the rate of price change is the same across all firms in the same industry, we can infer that α_c is positive, since $\beta_1 < 1$.[1]

A related study is *Day* (1973). It uses basically the same data and definitions as Whittington, but concentrates on two industries, building materials and vehicles, and only studies the change of profits between two consecutive years. As one would expect this leads to a larger value of β_1 than for the six year interval of Whittington. Regressing

$$\pi_{t+1} = \beta_o + \beta_1\pi_t$$

and using data for all twelve pairs of years between 1948 and 1960, Day gets an estimate of β_1 around 0.8 for both industries.

Finally a study of Swedish data by *Bertmar and Molin* (1977) should be mentioned. This is based on the same data of the Swedish Financial Statistics that underlie the tables of industry figures in Appendix B. A subset of firms comprising roughly the 431 largest firms in the manufacturing industries is analyzed for the period 1966-1972. The study treats a multitude of aspects of the development of firms with an emphasis on financial variables. For the present purpose let us only note the relation between rates of return of different years.

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[1] This hypothesis is not tested by Whittington but it appears likely that almost all β_1 are significantly smaller than unity. The inference that $\alpha_c > 0$ can of course only be made if we accept the assumptions of the basic model of Chapter 4. Otherwise a possible interpretation is that efficient techniques are diffused from firms with high profits to those with low profits.

The population covers firms from all types of manufacturing industries. This means, according to our theory, that we will expect fast expansion and consequent price fall for firms in industries with high rates of profit. It is consequently not admissible to treat the rate of price change as constant across the firms. This should lead one to expect a more rapid rate of convergence of profits towards some mean than for populations where all firms produce the same homogeneous good. And indeed, comparing with the studies of Whittington and Day the values are considerably lower.

Profits are defined as the net rate of return on total assets.[1] A regression for consecutive years like that of Day then gives an average value of the regression coefficient over the six pairs of years of 0.48, i.e. slightly above half the value found by Day. And this regression

$$\pi_{1972} = \beta_o + \beta_1 \pi_{1966}$$

gives a β_1 value of 0.13,[2] which is exactly half the lowest value found by Whittington for a six-year interval.

Let us summarize this section. The main theme has been whether differences in rates of return persist over time. This question has been approached across industries, across firms in general, and across firms in the same industry. We have tried to interpret the results so as to shed some light on the likely values of the reaction coefficients of the basic model of Chapter 4. As we have pointed out, data across firms producing the same homogeneous good reflect adjustment only on the cost side, whereas data across different industries or across firms from different industries

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[1] For exact variable definitions, see Bertmar & Molin (1977) ch. 5.

[2] These regression coefficients are not given by Bertmar & Molin, but have been calculated for this study making use of the data of standard deviation and correlation coefficients in Bertmar & Molin (1977) pp. 172 and 173.

give information of adjustment both on the cost and production (price) side.

We have seen that it is empirically verified that deviations of rates of return from the average tend to be reduced over time, but the equalization of rates of return across firms or industries seems to be quite a slow process. Indeed, it is open to considerable doubt whether this process leads to ultimate equalization at all. We tried to interpret this evidence within our basic model. Of course there are numerous other models that would give rise to the same empirical picture. Perhaps one would even be able to make a convincing case that much of what we see is due to measurement errors.[1]

EXPANSION AND PROFITS

Expansion of output can originate from two sources: entrance of new firms and expansion of already established firms. In the first part of this section we will look at what determines entry. As regards the behaviour of existing firms we will present results from studies of investment behaviour and studies of the rate of expansion of turnover. Finally, we will also discuss some evidence on the relation between expansion and profitability at the industry level.

Establishment of New Firms

There have been comparatively few empirical studies of the determinants of entry; *Du Rietz* (1979) in a recent work lists only eight studies in all. Among these only a couple include some measure of the rate of return among the independent variables.

[1] This is not the place to discuss such problems. It should be pointed out however that the study by Bertmar & Molin (1977), which provides the data for some regression estimates in the present section, has made a thorough attempt to remove measurement errors from the data.

Du Rietz studies the rate of entry into Swedish manufacturing industries 1954-1968. He finds that companies which started during this period had a share of total employment in 1968 varying between 18.6 % in the plastics industry and 0.6 % in the basic metal industries. Expressed as a share of the change of employment during the period, new firms in some industries accounted for more than 50 %.[1] This shows that, at least in some industries, the behaviour of potential entrants is equally important as that of existing firms. The regression analyses made by Du Rietz stress, beside market growth, average plant size and other proxy variables of minimum efficient scale as the most important determinants of the rate of entry.

Mansfield (1962), in what appears to be one of the first studies, uses data on entry and exit in four U.S. industries - steel, petroleum, tires, and autos - over different ten year periods.[2] Entry (E) is defined as the number of new firms that entered during each ten year period and survived until the end of the same period as a proportion of the original number of firms. E is regressed on $\pi/\bar{\pi}$, the ratio of the average rate of return in the industry to that in all manufacturing, and C, the investment required to establish a firm of minimum efficient size. Assuming that the relation is linear in logarithms gives

$$\ln E = 0.43 + 1.15 \ln (\pi/\bar{\pi}) - 0.27 \ln C$$

where both coefficients are significant at the five per cent level. It is instructive to compute the effect on entry of an increase in the relative rate of return from this equation.

[1] See Du Rietz (1979), table A:1. In the boat-building industry (ISIC No. 38412), where there is a considerable exit, this share was even above 100 %. This illustrates that this measure is difficult to interpret. In industries with decrease of employment it becomes undefined.

[2] The ten-year periods are of different age. The first one starts in 1916, and the last one ends in 1959.

For the automobile industry, which has the highest minimum investment requirement, $\pi = 1$ gives $E = 0.29$ and $\pi = 2$ (i.e. twice the overall average) gives $E = 0.64$. For the tire industry the corresponding figures are 0.70 and 1.55. This shows that over a ten years period the number of firms may more than double, and a doubled rate of return leads to a somewhat more than doubled rate of entry. Remember, however, that entry is measured by the share of the number of firms, not by the share of output.

A more recent study is *Orr* (1974). This is based on a sample of 71 Canadian industries for the period 1963-1967. The dependent variable is the logarithm of the number of new firms having entered during the period. Independent variables are the average rate of return over the previous four year period and variables that can be considered as proxies for barriers to entry - capital requirements, advertising intensity, and concentration ratio. The estimated coefficient for the rate of return is positive but insignificant, whereas all the barriers-to-entry variables exert a significant influence on entry.

Summing up, it appears that little is known about the strength of the reaction of potential entrants to variations in the rate of return. More is known about the importance of barriers to entry.[1] Differences in minimum efficient scale give significant effects in all studies. In interpreting this result one should bear in mind that according to our theory rates of return should be higher for industries with low barriers to entry. Hence, there is a problem of multicollinearity between the two variables, and one should not make too much of the finding that barriers to entry are significant and the rate of return is insignificant.

Translating the results to our model, one may conjecture that new firms normally have a rather limited impact on

[1] All results should be interpreted with care due to data problems; Mansfield defines all firms with a new name as entrants and Orr defines entry as one fourth of the increase in the number of firms above a certain minimum size. Du Rietz, who really measures new entrants, does not include the rate of profit among his explanatory variables.

the parameters of the expansion function (4.1). But this is probably quite different from industry to industry, largely depending on minimum efficient scale.

Investment Studies

The hypothesis underlying the present study, that profits breed expansion of production, is closely related to an investment theory associated with the works of Tinbergen (1939), Kalecki (1949-1950) and Klein (1951). The influence of profits on investment has undergone numerous tests. A seminal study is *Kuh* (1963). One of the main aims of that study is to compare the investment theory, where profits are the main explanatory variable, with different kinds of accelerator models where current and lagged sales stimulate investment.

Kuh analyzes yearly data for a sample of 60 U.S. firms from 1935 to 1955. The tests are carried out by ordinary regression methods using different functional forms, different variable definitions, and different lag structures. All equations contain a capital intensity index and a capital stock measure. Further sales and/or profits are included lagged or unlagged.

By profits is meant gross retained earnings, defined as net income after taxes plus depreciation minus dividends.

When either sales or profits are included they turn out to be almost equally significant, but with the sales hypothesis slightly favoured.[1] For cross-sections three coefficients out of four are significantly different from zero at the one per cent level, while for time series only about half are significant at the ten per cent level. When both sales and profits are included, sales are clearly more significant but

[1] By "favoured" I mean that the sales variable is significant more often than the profits variable.

neither sales nor profits are significant at the ten per cent level in more than two regressions out of three.

Neither functional form nor lag structure are of very great importance. Below are given the median values of 15 estimated cross-section regression equations

$$\frac{I_t}{K_t} = 0.03 + 0.29\,\frac{(\pi_t+\pi_{t-1})/2}{K_t} + 0.02\,\frac{S_{t-1}}{C\cdot K_{t-1}} \qquad R^2 = 0.45$$

where C is a capital intensity measure, K is the capital stock, π is gross retained earnings and S is total turnover.[1]

On one tail tests at the five per cent level the regression coefficient for π/K is significant 11 times out of 15. This is a fairly representative regression. Most of the estimates of the regression coefficient of I on π, irrespective of functional form or lag structure, give values in the range 0.2-0.3.[2] This value can be taken as an indicator of the expansion reaction coefficient α_{qi} of (4.6).

Some more recent studies of profits and investments have taken account of the fact that investment decisions are taken simultaneously with other decisions. The pioneer study with this approach is *Dhrymes and Kurz* (1967). They estimate by the three-stage least-squares method equations determining simultaneously dividends, external borrowing and investments. The coefficient of the profit variable is significant at the five per cent level in five regressions out of a total of ten. Its average value is around 0.1.

The simultaneous equations approach has also been used in two studies by *Mueller* (1967) and *Grabowski and Mueller* (1972). These are of particular interest because one of the

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[1] Kuh (1963) p. 265.

[2] It should be noted, however, that estimates using ratios which tend to remove heteroscedasticity generally make the profits variable significant more often.

dependent variables, apart from investment and dividends, is R & D spending. The profit variable used is profits before taxes divided by sales. The dependent variable, gross fixed investment, is also deflated by sales. The coefficients for profits are in the range 0.2-0.4 and barely significant with t-values usually below 2.[1]

In the studies presented above there appears to be a reasonably well established positive relationship between the ratio of investment to the capital stock and the lagged rate of return to capital, when estimating on samples across firms. The estimated coefficients are uniformly below 0.5.

Expansion of Turnover Across Firms

For the purpose of throwing light on the likely magnitude of α_q of our model one might wish to substitute the rate of change of production (or turnover or value-added) for the rate of investment as the dependent variable.

The use of investment has one distinct advantage. It is related to the long-run change of production capacity, at least if we disregard problems of differences in the rate of scrapping across industries, and production capacity is what q stands for in our model. Production measures, on the other hand, are invariably affected by changes in capacity utilization due to the ups and downs of the business cycle. There are problems, however, associated with interpreting investment as a proxy for long run production change. First, the rate of investment varies widely from year to year, especially when studying firm data. This is largely due to technical indivisibilities. Second, the capital intensity varies across

[1] The main purpose of the Grabowski & Mueller (1972) study is to compare two investment theories, one "managerial" where lagged profits is one explanatory variable and one "stockholder welfare" model, where a measure of risk enters as a proxy for the cost of capital to the firm. The analysis favours the managerial model. Indeed, the cost of capital term even has the wrong sign.

industries and firms. A higher rate of increase of the capital stock may hence not be associated with a faster increase of production capacity.

For these reasons it is also of interest to study the relation between the rate of expansion of production and the rate of return to capital. Studies estimating this relation have mostly been concerned with long-run problems, and the regressions have been made on averages over periods of at least five years. This has in some studies been motivated by explicit statements that the model under study is a steady-state model. Eriksson (1975) is an example of this.

If we do not make the steady-state interpretation, a problem is that theoretical considerations say that the profit variable should be lagged, at least if lagged profits represent expectations about future profitability. But in most studies growth rates and rates of profit are typically averaged over the same period. Provided that the time period is long enough this should not cause any serious bias however. Another effect is that the variance across the observations will be smaller. Since this is largely due to the elimination of business cycle effects, it may be more of an advantage than a disadvantage.

One of the studies regressing growth on profits is *Singh and Whittington* (1968). They use basically the same data as the study by Whittington (1971) mentioned above. Compound growth rates of turnover and average pre-tax rates of return on net assets are calculated for every company for the periods 1948-1954 and 1954-1960, respectively.

An ordinary linear regression of growth on profits is performed industrywise for both periods. The regression coefficients, which in all cases are significant at the 5 % level, vary from 0.35 to 0.61 and the intercepts, which can be interpreted as δ_q of our model, from -3.22 to 1.28. These values of the regression coefficients also vary considerably between the two periods for the same industry.

It is interesting to note that it turns out that big firms (with a book value of net assets above £ 2 million) generally have a higher regression coefficient, i.e. are quicker to adapt to variations in rate of return.

A problem with the interpretation of a regression of growth on profits is that causality runs in both directions. Profits spur growth, but on the other hand growth may lead to special costs and thereby a lower rate of profit. Judging from the evidence presented in this section the former effect dominates the latter in ordinary cross-section studies.[1]

The problem of direction of causality has been taken account of in a study by *Eriksson* (1975). He is mainly interested in the impact of growth on the rate of return. By employing the two-stage least-squares method he is able to show that there is a significant negative effect in this direction, whereas with the ordinary least-squares method data show a significant positive relationship.

The data used are from 56 enterprises in industries producing fabricated metal products, machinery, and equipment, quoted on the Stockholm stock exchange. The rate of growth of turnover ($\Delta pq/pq$) and profitability, measured as a pre-tax rate of return on total capital (π_T) or on production capital (π_p),[2] are averaged over the period 1963-1972 for each company. Different regressions are then performed on this cross-section material. The results of a linear regression of profits on growth are shown below.[3]

$$\pi_p = 0.040 + 0.32 \frac{\Delta qp}{qp} \qquad R^2 = 0.20$$

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[1] For a summary of some other studies, mainly on British data, which show basically the same picture, see Eatwell (1971).

[2] Production capital is defined as total capital excluding financial assets.

[3] Eriksson (1975) p. 182.

$$\pi_T = 0.038 + 0.24 \frac{\Delta qp}{qp} \qquad R^2 = 0.24$$

The coefficients in both equations are significantly different from zero at the one per cent level. Letting the variables change place yields the equivalent of our expansion equation[1]

$$\frac{\Delta qp}{qp} = \beta + \frac{0.20}{0.32} \pi_p = \beta + 0.63 \pi_p$$

$$\frac{\Delta qp}{qp} = \beta + \frac{0.24}{0.24} \pi_T = \beta + 1.00 \pi_T$$

I have redone the same type of regression on data originally collected for the aforementioned study be Bertmar and Molin (1977).[2] The data cover the period 1969-1976 and comprise firms from all manufacturing industries. This equation has been estimated.

$$\frac{\Delta qp}{qp} = \beta_o D + \beta_1 \pi$$

qp is turnover, D is a vector of dummy variables, one for each two digit manufacturing industry, and π is the rate of return on total capital. The variables are averaged over all years and the parameters of the regression equation are estimated across all 480 firms.

The estimate of β_1 is 0.27 (standard error = 0.05, $R^2 = 0{,}08$) which is notably lower than the figure obtained by Eriksson (1.00) on similar data. Three of the dummy variables are

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1 The regressions coefficient when regressing x_1 on x_2 is equal to the coefficient of determination divided by the regression coefficient when regressing x_2 on x_1.

2 Details about the data used are presented in Appendix B. The time period used for my estimates (1969-1976) differs from the one that Bertmar and Molin (1977) analyzed (1966-1972). This is so because data for later years have been added to their original data.

significant,[1] but the inclusion of dummies does not raise the explanatory power of the equation to a significant extent.

It is of interest to investigate what difference it makes if the regression instead is based on yearly figures with the rate of profit lagged one year. I have hence also estimated this equation

$$\frac{qp_{t+1} - qp_t}{qp_t} = \beta_o D + \beta_1 \pi_t$$

These are the estimates of β_1.

Year	1969	1970	1971	1972	1973	1974	1975	1976
β_1	0.04	-0.11	-0.15	-0.06	-0.10	-0.13	-0.18	-0.26
(standard error)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.08)

This shows that, except for the first year, there is a *negative* effect of profits on growth, when making the regression on yearly data with lagged profits. Four coefficients are significant at the five per cent level. This can be contrasted with the results obtained when estimating this equation

$$\frac{qp_t - qp_{t-1}}{qp_{t-1}} = \beta_o D + \beta_1 \pi_t$$

The difference is that this is a regression of expansion on profits during the same year, not lagged on year. This gives

[1] By this is meant that the intercept is significantly different from the intercept for the manufacture of fabricated metal products (ISIC 38). Significance is obtained for manufacture of wood products (ISIC 33) + 3.1, manufacture of paper products (ISIC 34) + 1.2 and manufacture of non-metallic mineral products (ISIC 36) - 2.6.

Year	1969	1970	1971	1972	1973	1974	1975	1976
β_1	0,28	0,29	0,23	0,13	0,35	0,53	0,29	0,27
(standard error)	(0,07)	(0,07)	(0,07)	(0,07)	(0,07)	(0,07)	(0,08)	(0,07)

This discrepancy between a regression with lagged and one with unlagged profits is probably largely due to business cycle movements. In years of boom, capacity utilization and prices increase as a result of exogenous demand increases. This will increase profits as well as the rate of growth of turnover and these two variables will hence be correlated. If the growth rates of industry demand are negatively correlated between consecutive years, this is compatible with the finding of negative regression coefficients for the equation with lagged π.

We have in this section looked at these four types of regression equations

	(i)	(ii)	(iii)	(iv)
Dependent Variable	$\frac{K_{t+1} - K_t}{K_t}$	$\sum_t \frac{\Delta qp}{qp}$	$\frac{qp_{t+1} - qp_t}{qp_t}$	$\frac{qp_t - qp_{t-1}}{qp_{t-1}}$
Independent Variable	π_t	$\sum_t \pi_t$	π_t	π_t

The investment equation (i) and the expansion equation based on averages over several years (ii) both give significantly positive regression coefficients, whereas the sign of the effect of profits on expansion using yearly data, (iii) and (iv), depends on whether π is lagged or not. As we have pointed out, (iii) and (iv) are largely influenced by business cycle factors and are consequently of less interest for our purposes. The coefficient estimates of (i) and (ii) both lie in the range 0.2-1.0. On the basis of this we may say as a rough guess that a likely value of the firm expansion reaction coefficient α_{qi} of (4.6) is around one half if time is measured in years. The evidence on the impact of the industry's rate of profit on entry is less easy to translate into α_{qj} of (4.7), but there does not seem to be much reason to believe that figures above 0.2-0.3 are likely. Hence a reasonable guess of the value of α_q ($= \alpha_{qi} + \alpha_{qj}$) would rather be on the low side of unity.

Expansion of Turnover Across Industries

We have interpreted a regression coefficient of the rate of change of turnover on the rate of profit as an indicator of α_q. This is not in general admissible however, since α_q refers to the rate of expansion of production. It can only be done if the rate of price change can be regarded as unaffected by the rate of expansion. This may be reasonable when the unit of observation is the firm, at least if all firms are from the same industry.

In this section we will estimate the same regression equation on observations across industries. Then we can of course no longer assume that price is independent of quantity produced. From (4.1) and (4.4) of the basic model we have

$$\hat{p} + \hat{q} = \alpha_q(1-\gamma)\pi + \delta_q(1-\gamma) + \delta_p \tag{6.1}$$

In order to get reliable estimates of α_q we would need to know the price elasticity of the demand facing the industries in question. In the absence of this we have to look for some proxy variable for demand elasticity. One such variable is the share of output that is exported. An industry that sells a large share of its output on the world market may even be regarded as facing an infinitely elastic demand schedule ($\gamma = 0$). This can motivate the same division of the industries into sheltered and non-sheltered that has been used before in this chapter.[1] The hypothesis would then be

[1] Note, however, that here the export share is used as a proxy for the elasticity of demand facing a particular industry, whereas above it was used as a proxy for the industry expansion reaction coefficient α_q.

that the regression coefficient for the non-sheltered group would be positive and larger than that for the sheltered group. In the latter case one cannot hold any particular hypothesis a priori. Further, since the demand elasticity also enters into the other terms of (6.1), one may add the export share (x/q) as a separate explanatory variable.

The regressions are estimated on the same data from the Swedish Enterprise Statistics that were analyzed in the section on the persistence of profitability. The figures are yearly averages for the period 1969-1976. For details on the data, see Appendix B. These are the results:[1]

Sheltered industries ($x/q < 20$ %)

$$\frac{\Delta qp}{qp} = \underset{}{12.08} - \underset{(0.64)}{0.20\,\pi} + \underset{(0.20)}{0.025\,x/q} \qquad R^2 = 0.01, \quad n = 12$$

Non-sheltered industries ($x/q > 20$ %)

$$\frac{\Delta qp}{qp} = \underset{}{1.70} + \underset{(0.48)}{0.85\,\pi} + \underset{(0.05)}{0.10\,x/q} \qquad R^2 = 0.32, \quad n = 21$$

The difference between the two groups supports our hypothesis. The regression for the group of sheltered industries is void of explanatory value, whereas both the rate of profit and the export share are significant for the non-sheltered group (one-tailed test, 5 per cent level).

The magnitude of the coefficient we have estimated on industry data is higher than most of the coefficients that we found on firm data. This is in accordance with our expectations[2] since the estimated coefficient in this case in-

[1] qp is defined as value-added. Data are from the Swedish Official Statistics on manufacturing (SOS: Industri 1969, 1976).

[2] At least if we believe that $\gamma = 0$ for the non-sheltered industries.

cludes the effect of entrance of new firms. Hence we may repeat the conclusion that a reasonable guess of the value of α_q would rather be on the low side of unity.

COST REDUCTION AND PROFITS

Empirical studies directly trying to estimate a relation between profitability and cost reduction are scarce. We will here take a look at two types of studies that are related to this question. One type is studies making regressions of R & D on profits. The other type is studies trying to estimate the rate of return on retained earnings. Finally, by using the same set of firm data as those employed above, we will make a direct regression of costs on the rate of return.

Research and Development and Profits

In a recent survey of the literature on market structure and innovation *Kamien & Schwartz* (1975) try to evaluate the evidence on the relation between R & D and profitability.[1] Their summary statement is that "the empirical evidence that either liquidity or profitability are conducive to innovative effort or output appears slim". The majority of the studies quoted - five out of eight - find a significant positive effect, however.

The aforementioned studies by *Mueller* (1967) and *Grabowski and Mueller* (1972) estimate a model where fixed investment, R & D, and dividends are simultaneously determined. With the dependent variable defined as R & D outlays as a share of sales, and with profits before taxes divided by sales as one of the independent variables, both studies get significant regression coefficients in the range 0.1-0.25. It is interesting to note that profits have a more signifi-

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[1] See Kamien & Schwartz (1975) pp. 24-26.

cant effect on R & D (t = 2.5-4) than on fixed investment (t = 1-2).

In a study of firms in the chemical, drug and petroleum industries, *Grabowski* (1968), has also found a significant effect of internal funds - after-tax profits plus depreciation and depletion allowances - on R & D expenditures.

On the other hand, studies by *Hamberg* (1966), *Scherer* (1965) and *Johannisson and Lindström* (1971) fail to find any significant relationship, though the estimated coefficients generally are positive.

On balance I think, however, that the data give some support to the hypothesis that there is a positive correlation at the firm level between profitability and R & D expenditures. It is not obvious, however, that one can infer from this that the rate of cost reduction increases with profits. Indeed the two studies using a measure of R & D output, patents, as the dependent variable - Scherer (1965) and Johannisson and Lindström (1971) - do not get significant coefficients.[1]

Return to Retained Earnings

Other indirect evidence is related to the conjecture, originally due to *Little* (1962), that ploughback of profits into a company appears to have little effect on future profits. If higher profits imply that a larger share of the capital stock growth is accounted for by ploughback and if the product price can be treated as exogenous to the individual firm, this conjecture can be interpreted within our model to indicate that $\alpha_c > 0$.

[1] Scherer does not get this for R & D spending either, and the Johannisson and Lindström study has problems with multicollinearity and heteroscedasticity. One should consequently not make too much of the observation that the results seem to differ depending on whether R & D is defined in input or output terms.

A study by *Baumol et al* (1970) attempts to estimate separate rates of return for different financial sources of new capital - ploughback (P), new equity (N), and new debt (D). They use yearly data for 900 industrial corporations for the 25 years period 1946-1970. The increase in profits for a company between two years can be accounted for as the sum of the returns from these three sources

$$\pi_1 - \pi_o = \Delta\pi_1 = r_P P_o + r_N N_o + R_D D_o$$

After t years the corresponding relation is

$$\pi_t - \pi_o = \Delta\pi_t = r_P(P_o+..+P_t) + r_N(N_o+..+N_t) + r_D(D_o+..+D_t)$$

In total this gives 25 equations. These are summed

$$\sum_{t=1}^{25} \Delta\pi_t = r_P \sum_{t=0}^{24} (25-t)P_t + r_N \sum_{t=0}^{24} (25-t)N_t + r_D \sum_{t=0}^{24} (25-t)D_t$$

This equation is then used to estimate r_P, r_N, and r_D across the sample of firms. Various definitions of the variables are tried as well as various time lags implying that ploughback in year t is not allowed to affect earnings until year $t+\tau$. The results of the regressions are quite striking. The estimates for the different equations give rates of return ranging from 14.5 per cent to 20.8 per cent on equity capital, from 4.2 to 14 on debt and from 3.0 to 4.6 per cent on ploughback.

The Baumol et al study has been critized on statistical grounds by *Friend and Husic* (1973). Two main conclusions emerge: (i) when one allows for heteroscedasticity the differences will diminish but remain considerable, and (ii) when one makes estimates on the sub-sample of firms that have issued new equity at some time, there is no significant difference between returns on ploughback and new equity. The same general conclusion is reached by *Whittington* (1972) on English data. He argues that it is the mere act of going to the stock market that imposes discipline and leads to productivity increases.

The idea that there really are different types of companies is also pursued by *Grabowski & Mueller* (1975). They argue that one would expect over-investment to occur in old firms making use of a mature product technology and having an old product structure, i.e. firms on the Schumpeterian negative side of development. As mature they define those companies that were in existence prior to the end of World War II and which had more than half of their sales in industries or products existing prior to that time.

With companies accordingly being classified into new and mature, rates of return are estimated from the same type of equation as that of Baumol et al, only that no account is taken of different modes of financing. Not unexpectedly the rates of return among new firms are much higher, 13.7-26.3 %, than among mature firms, 9.2-12.5 %.[1]

The Grabowski & Mueller analysis is appealingly simple. However, one can suspect that the sample of firms is biased. The new firms in the sample are probably the most successful ones from the population of new firms and their rates of return cannot be regarded as representative; in any case those who have gone bankrupt are not included in the sample. This observation unfortunately casts some doubt on the whole question. Is it so that the differences noted by Little and Baumol et al are simply explained by the non-randomness of the sample of firms?

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[1] This finding may seem to conflict with results from the product-cycle theory, which say that profits increase in the early phase and start to fall only in a late phase of the life-cycle. Probably the "new" firms of the Grabowski & Mueller sample have products in a fairly late stage of the product life-cycle.

The Rate of Cost Reduction Across Firms

The two types of studies presented above have different implications for the likely sign of α_c. The fact that a higher rate of profit seems to be accompanied by more R & D indicates that α_c may be negative. The result that the rate of return on retained earnings seems to be lower than the rate of return on capital from external sources indicates on the other hand a positive α_c.

We will now present an estimate of a regression coefficient of unit costs on the rate of return to capital. The set of data used is the same as that used above to estimate the expansion equation. This contains data on total cost (qc) and turnover (qp). This means that we should use $\hat{qc} - \hat{qp} = \hat{c} - \hat{p}$ as the dependent variable, for interpreted within our model we have

$$\hat{qc} - \hat{qp} = \alpha_c \pi - \delta_c - \hat{p}$$

Let us as before assume that $\hat{p}$ is not affected by the behaviour of single firms. We can then interpret the regression coefficient of $(\Delta qc/qc - \Delta qp/qp)$ on π as an estimate of α_c. Regressing on the same population of 480 firms for the period 1968-1976 as was used in the regression of expansion on profits gives a regression coefficient of -0.10 (standard error = 0.03, R^2 = 0.07).

This result thus indicates that a higher rate of profit on balance may be conducive to faster cost reduction. It should be pointed out, however, that there is not much empirical evidence in this field. Hence we can only conclude that on the Swedish firm data we have used here, we have found evidence that $\alpha_c < 0$.

CONCLUDING COMMENTS

The dynamics of firm behaviour is a complicated matter. The model analyzed in this study is based on two strategic behavioural relations. By this it deliberately abstracts from many other aspects. In this chapter we have presented some empirical evidence about the behavioural assumptions made.

In presenting the evidence we have tried to avoid getting bogged down in details and refinements. The theoretical model is a simple one, and what is really most interesting is to see whether the behavioural assumptions hold when account is *not* taken of any of the aspects of reality that the model abstracts from. As has been pointed out before, the behavioural relations can be seen as approximations or reduced forms of more complicated models.

This chapter gives us some indications about the likely magnitude of the reaction coefficients. As to expansion behaviour, there are a host of studies from which we have concluded that a reasonable guess may be a value of α_q around or somewhat below unity. As to α_c, it is much more difficult to make a guess, but the evidence we do have favours a negative value, say -0.1.

These figures should be interpreted as likely average values. The "true" values will differ widely from industry to industry. Let us still use $\alpha_q = 1$ and $\alpha_c = -0.1$ for a simple numerical example to look at the rate of convergence of differences in the rate of profit.

To study the rate of convergence, let us take equation (3.11)

$$\dot{\pi} = (\pi+1)\left(-(\alpha_c+\alpha_q\gamma)\pi + \delta_c + \delta_p - \delta_q\gamma\right)$$

Rewriting this in discrete time assuming $\pi^* = 0$, and approximating around steady-state, we have

$$\pi_{t+1} - \pi_t = -(\alpha_c + \alpha_q \gamma)\pi_t$$

The rate of convergence obviously depends on the inverse demand elasticity. Let us take two cases: unitary elasticity (γ=1) and infinitely elastic demand (γ=0). $\gamma = 1$ gives $\alpha_c + \alpha_q\gamma = 0.9$ implying that almost all of a deviation of π from its steady-state value will disappear in one year. $\gamma = 0$ on the other hand gives $\alpha_c + \alpha_q\gamma = -0.1$, i.e. the model is unstable since the sum is negative. This case may seem a bit exaggerated; certainly even export industries face downward-sloping demand curves. Taking $\gamma = 0.2$ instead we see that this gives $\alpha_c + \alpha_q\gamma = 0.1$, which means that only 10 per cent of any deviation will be taken away in one year.

Let me stress again that this example was only intended to give an idea of the magnitudes involved. The model is a theoretical model. As such it is deliberately made simple, much too simple to be of direct use for empirical analysis.

APPENDICES

A. MATHEMATICAL DERIVATIONS

THE DYNAMIC OPTIMIZATION PROBLEM (3.51) - (3.54)

Proof that the Stationary Point is a Saddle Point

By the necessary conditions we have this system of differential equations in the state variables, K and M, and the adjoint variables, $\tilde{p}$ and $\tilde{\lambda}$.

$$\dot{K} = C'^{-1}(\tilde{p}) - \delta K$$

$$\dot{M} = C'^{-1}(\tilde{p})\frac{M}{K} - \delta M$$

$$\dot{\tilde{p}} = -(pF - w\ell) + (\delta + \rho)\tilde{p} + r'$$

$$\dot{\tilde{\lambda}} = 0$$

Linearizing around the steady-state we get

$$\dot{K} = -\delta(K - K^*) + \frac{1}{C''}(\tilde{p} - \tilde{p}^*) \tag{A.1}$$

$$\dot{M} = -\delta\frac{M}{K}(K - K^*) + \frac{M}{K}\frac{1}{C''}(\tilde{p} - \tilde{p}^*) \tag{A.2}$$

$$\dot{\tilde{p}} = r''\frac{1}{M}(K - K^*) - r''\frac{1}{M}\frac{K}{M}(M - M^*) + (\delta + \rho)(\tilde{p} - \tilde{p}^*) \tag{A.3}$$

$$\dot{\tilde{\lambda}} = 0 \tag{A.4}$$

The characteristic equation is

$$\begin{vmatrix} -\delta-\lambda & 0 & 1/C'' & 0 \\ -\delta\cdot M/K & -\lambda & M/K\cdot 1/C'' & 0 \\ r''\cdot 1/M & -r''\cdot 1/M\cdot K/M & \delta+\rho-\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{vmatrix} =$$

$$= -\lambda\left\{(-\delta-\lambda)\left[(\delta+\rho-\lambda)(-\lambda) + r''/C''\cdot 1/M\right] + 1/C''\left[\delta r''\cdot 1/M + \lambda r''\cdot 1/M\right]\right\} =$$

$$= \lambda^2(\delta+\lambda)(\delta+\rho-\lambda) = 0$$

The characteristic roots are

$$\lambda_{1,2} = 0 \qquad \lambda_3 = -\delta \qquad \lambda_4 = \delta+\rho$$

which means that the solution to the system of linear differential equations will be

$$K - K^* = a_{11} + a_{12}t + a_{13}e^{-\delta t} + a_{14}e^{(\delta+\rho)t} \tag{A.5}$$

$$M - M^* = a_{21} + a_{22}t + a_{23}e^{-\delta t} + a_{24}e^{(\delta+\rho)t} \tag{A.6}$$

$$\tilde{p} - \tilde{p}^* = a_{31} + a_{32}t + a_{33}e^{-\delta t} + a_{34}e^{(\delta+\rho)t} \tag{A.7}$$

$$\tilde{\lambda} - \tilde{\lambda}^* = a_{41} + a_{42}t + a_{43}e^{-\delta t} + a_{44}e^{(\delta+\rho)t} \tag{A.8}$$

This system will have a stationary point, which is a saddle, if $a_{i1} = a_{i2} = 0$, $i=1,\dots,4$ and $\delta, \delta+\rho > 0$. The latter follows from assumptions made in the text. Let us now show that $a_{i1} = a_{i2} = 0$ by substituting the solution into the differential equations

$$\dot{K} = a_{12} - a_{13}\delta e^{-\delta t} + a_{14}(\delta+\rho)e^{(\delta+\rho)t} =$$

$$= -\delta\left(a_{11} + a_{12}t + a_{13}e^{-\delta t} + a_{14}e^{(\delta+\rho)t}\right) +$$

$$+ \frac{1}{C''}\left(a_{31} + a_{32}t + a_{33}e^{-\delta t} + a_{34}e^{(\delta+\rho)t}\right)$$

$$\dot{M} = a_{22} - a_{23}\delta e^{-\delta t} + a_{24}(\delta+\rho)e^{(\delta+\rho)t} =$$

$$= -\delta\frac{M}{K}\left(a_{11} + a_{12}t + a_{13}e^{-\delta t} + a_{14}{}^{(\delta+\rho)t}\right) +$$

$$+ \frac{M}{K}\cdot\frac{1}{C''}\left(a_{31} + a_{32}t + a_{33}e^{-\delta t} + a_{34}e^{(\delta+\rho)t}\right)$$

$$\dot{p} = a_{32} - a_{33}\delta e^{-\delta t} + a_{34}(\delta+\rho)e^{(\delta+\rho)t} =$$

$$= \frac{r''}{M}\left(a_{11} + a_{12}t + a_{13}e^{-\delta t} + a_{14}e^{(\delta+\rho)t}\right) -$$

$$- \frac{r''}{M}\cdot\frac{K}{M}\left(a_{21} + a_{22}t + a_{23}e^{-\delta t} + a_{24}e^{(\delta+\rho)t}\right) +$$

$$+ (\delta+\rho)\left(a_{31} + a_{32}t + a_{33}e^{-\delta t} + a_{34}e^{(\delta+\rho)t}\right)$$

From this we get

$$a_{12} = -\delta a_{11} + \frac{1}{C''}a_{31} \qquad (A.9)$$

$$a_{12} = \frac{1}{\delta C''}a_{32} \qquad (A.10)$$

$$a_{33} = 0 \qquad (A.11)$$

$$(\delta+\rho)a_{14} = -\delta a_{14} + \frac{1}{C''}a_{34} \qquad (A.12)$$

$$a_{22} = -\delta\frac{M}{K}a_{11} + \frac{M}{K}\cdot\frac{1}{C''}a_{31} \qquad (A.13)$$

$$\delta\frac{M}{K}a_{12} = \frac{M}{K}\cdot\frac{1}{C''}a_{32} \qquad (A.14)$$

$$-\delta a_{23} = -\delta\frac{M}{K}a_{13} + \frac{M}{K}\cdot\frac{1}{C''}a_{33} \qquad (A.15)$$

$$(\delta+\rho)a_{24} = -\delta\frac{M}{K}a_{14} + \frac{M}{K}\cdot\frac{1}{C''}a_{34} \tag{A.16}$$

$$a_{32} = \frac{r''}{M}a_{11} - \frac{r''}{M}\cdot\frac{K}{M}a_{21} + (\delta+\rho)a_{31} \tag{A.17}$$

$$\frac{r''}{M}a_{12} = \frac{r''}{M}\cdot\frac{K}{M}a_{22} - (\delta+\rho)a_{32} \tag{A.18}$$

$$a_{13} = \frac{K}{M}a_{23} \tag{A.19}$$

$$a_{14} = \frac{K}{M}a_{24} \tag{A.20}$$

where we have made use of (A.11) in writing (A.19). We see that (A.12), (A.16) and (A.20) form a system in a_{14}, a_{24} and a_{34}, and (A.11), (A.15) and (A.19) form a system in a_{13}, a_{23} and a_{33}. The determinants to both systems can be shown to be zero. Hence, they have non-trivial solutions.

Multiplying (A.9) by M/K and subtracting from (A.13) we get

$$a_{22} = a_{12}\frac{M}{K} \tag{A.21}$$

This implies by (A.18) that $a_{32} = 0$, and hence, by (A.14) and (A.21), $a_{12} = a_{22} = 0$. Further, from (A.9), (A.13) and (A.17), $a_{11} = a_{21} = a_{31} =$ $= 0$.

By this we have shown that the stationary-point is a saddle-point.

Sufficient Conditions for a Maximum

A Sufficient Condition for the necessary condition to define a maximum is that $H^*(K,M,\tilde{p},\tilde{\lambda},t) = \underset{I,S}{\text{Max}}\, H$ is a concave function of K and M for given values of $\tilde{p}$ and $\tilde{\lambda}$.

$$\frac{\partial H^*}{\partial K} = pF - w\ell - \tilde{p}\delta + \tilde{\lambda}r'$$

$$\frac{\partial H^*}{\partial M} = -\tilde{\lambda}r'\frac{K}{M} + \tilde{\lambda}r$$

$$\frac{\partial^2 H^*}{\partial K^2} = \tilde{\lambda} r'' \frac{1}{M}$$

$$\frac{\partial^2 H^*}{\partial M^2} = \tilde{\lambda} r'' \frac{K^2}{M^3}$$

$$\frac{\partial^2 H^*}{\partial K \partial M} = -\tilde{\lambda} r'' \frac{K}{M^2}$$

and

$$\frac{\partial^2 H^*}{\partial K^2} \cdot \frac{\partial^2 H^*}{\partial M^2} - \left(\frac{\partial^2 H^*}{\partial K \partial M}\right)^2 = 0$$

H* is thus a concave function and the necessary conditions do define a maximum.[1)]

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1 Note that the theorem does not demand strict concavity, see Arrow and Kurz (1970) proportion 6, p. 45.

SOLUTION TO THE OPTIMIZATION PROBLEM WHERE $\rho = \rho(D/E)$ (p. 72)

$$\text{Max} \int_0^\infty De^{-\rho(D/E)t}$$

$$\text{s.t. } D = K\big(pF(1,\ell) - w\ell\big) - C(I) - S$$

$$\dot{K} = I - \delta K$$

$$\dot{M} = rM - S \qquad r = r(E/M)$$

$$K = E + M$$

$$H = \big(K(pF-w\ell) - C(I) - S\big)e^{-\rho t} + \tilde{p}(I - \delta K) + \tilde{\lambda}(rM - S)$$

Necessary optimality conditions are

$$\frac{\partial H}{\partial I} = 0 \ ; \ \tilde{p} = (1 - \rho' tD/E)C' e^{-\rho t}$$

$$\frac{\partial H}{\partial S} = 0 \ ; \ \tilde{\lambda} = (\rho' tD/E - 1)e^{-\rho t}$$

$$\dot{\tilde{p}} = -\frac{\partial H}{\partial K} = \left(-(pF-w\ell) + \frac{D\rho' t}{E}(pF-w\ell-D/E) - (r'+\delta C')(\rho' tD/E-1)\right)e^{-\rho t}$$

$$\dot{\tilde{\lambda}} = \left(\rho' t \,\frac{D^2}{E^2} + (1 - \rho' tD/E)\right)e^{-\rho t}$$

Let us assume that there exists a steady-state to this model characterized by $\dot{K} = \dot{M} = 0$, as is done in the works by Gordon (1962) and Eriksson (1975). It is directly seen that, unless $\rho' = 0$, this implies that $\dot{\tilde{p}} \neq 0$, $\dot{\tilde{\lambda}} \neq 0$. With $\rho' = 0$ it would be possible by the standard variable transformation $\tilde{p}^* = \tilde{p}e^{-\rho t}$, $\tilde{\lambda}^* = \tilde{\lambda}e^{-\rho t}$ to get an autonomous system in $\tilde{p}^*$ and $\tilde{\lambda}^*$. Otherwise this is not so. Hence the presumed steady-state is not a stationary point to this maximization problem.

SUFFICIENT CONDITIONS FOR A MAXIMUM OF THE OPTIMIZATION PROBLEM (3.68) - (3.70)

We have this dynamic optimization problem

$$\text{Max} \int_0^{\infty} U\left(\pi(K) - I - S\right)e^{-\rho t}dt$$

$$\dot{K} = I - \delta K$$

$$\dot{M} = r(E/M)M - S$$

$$E = K - M$$

$$U' > 0, \quad U'' < 0, \quad \pi_K > 0, \quad \pi_{KK} < 0, \quad r' < 0, \quad r'' > 0$$

Necessary conditions follow from

$$H = U(\pi - I - S) + \tilde{p}(I - \delta K) + \tilde{\lambda}(rM - S)$$

$$\frac{\partial H}{\partial I} = 0 \ ; \quad \tilde{p} = U'$$

$$\frac{\partial H}{\partial S} = 0 \ ; \quad \tilde{\lambda} = -U'$$

$$\dot{\tilde{p}} = \tilde{p}(\rho + \delta + r' - \pi_K)$$

$$\dot{\tilde{\lambda}} = \tilde{\lambda}\left(\rho - r + r'(1 + E/M)\right)$$

A sufficient condition for these necessary conditions to define a maximum is that H^*, defined by

$$H^*(K,M,p,\tilde{\lambda},t) = \underset{I,S}{\text{Max}}\ H$$

is a concave function of K and M for given values of $\tilde{p}$ and $\tilde{\lambda}$. Note that we can treat I + S as one variable. In differentiating H^* account must be taken of the necessary condition $\tilde{p} = U'$, which gives

$$\frac{\partial(I+S)}{\partial K} = \pi_K$$

We then have

$$\frac{\partial H^*}{\partial K} = \tilde{p}\left(\pi_K - \delta - r'\right)$$

$$\frac{\partial H^*}{\partial M} = \tilde{p}\left(r'(1+E/M) - r\right)$$

$$\frac{\partial^2 H^*}{\partial K^2} = \tilde{p}\left(\pi_{KK} - r'' \frac{1}{M}\right) < 0$$

$$\frac{\partial^2 H^*}{\partial M^2} = - \tilde{p}r'' \; 1/M(1+E/M)^2 < 0$$

$$\frac{\partial^2 H^*}{\partial K \partial M} = \tilde{p}r'' \; 1/M(1+E/M) > 0$$

and

$$\frac{\partial^2 H^*}{\partial K^2} \cdot \frac{\partial^2 H^*}{\partial M^2} - \left(\frac{\partial^2 H^*}{\partial K \partial M}\right)^2 = -\tilde{p}^2 r'' \; 1/M(1+E/M)^2 \pi_{KK} > 0$$

H* is thus a concave function and the necessary conditions do define a maximum.

SUFFICIENT CONDITIONS FOR A MAXIMUM OF THE OPTIMIZATION PROBLEM (3.90) - (3.91)

The optimization problem gives this Hamiltonian

$$H = U(\pi,\dot{a}) + \tilde{p}\dot{a}$$

where

$$\pi = pG(a)F(L) - wL$$

and the first-order conditions demand

$$pG(a)F'(L^*) = w$$

$$\tilde{p} = - U_2$$

A sufficient condition for the necessary conditions to define a maximum is that $H^* = H(L^*,\dot{a}^*)$ is a concave function of a when $\tilde{p}$ and t are held constant

$$\frac{\partial H^*}{\partial a} = U_1 pG'F$$

$$\frac{\partial^2 H^*}{\partial a^2} = U_{11}(pG'F)^2 + U_1 p\left(G''F - \frac{(G'F')^2}{GF''}\right)$$

The sufficient condition for a maximum demands that this second-derivative be negative.

THE SOLUTION TO THE BASIC MODEL (p. 115)

We seek the solution to this system of two differential equations

$$\frac{\dot{c}_t}{c_t} = \alpha_c \pi_t - \delta_c \tag{A.22}$$

$$\frac{\dot{q}_t}{q_t} = \alpha_q \pi_t + \delta_q \tag{A.23}$$

where $\dot{c}_t$ denotes $\frac{dc_t}{dt}$ etc. The equations are linked together by

$$p_t = aq_t^{-\gamma} e^{\delta_p t} \tag{A.24}$$

i.e.

$$\frac{\dot{p}_t}{p_t} = -\gamma \frac{\dot{q}_t}{q_t} + \delta_p \tag{A.25}$$

and

$$\pi_t = \frac{p_t - c_t}{c_t} \tag{A.26}$$

Using (A.22)-(A.26) we get

$$\frac{d}{dt}\left(\frac{\dot{c}}{c}\right) = \alpha_c \frac{p}{c}\left(\frac{\dot{p}}{p} - \frac{\dot{c}}{c}\right) = \alpha_c\left(\frac{\frac{\dot{c}}{c} + \alpha_c + \delta_c}{\alpha_c}\right)\left(-\alpha_q\gamma\frac{\frac{\dot{c}}{c} + \delta_c}{\alpha_c} - \delta_q\gamma + \delta_p - \frac{\dot{c}}{c}\right) =$$

$$= -\left(1 + \frac{\alpha_q\gamma}{\alpha_c}\right)\left(\frac{\dot{c}}{c}\right)^2 + \left(\delta_p - \delta_c - \delta_q\gamma - \alpha_c - \alpha_q\gamma - 2\frac{\alpha_q\delta_c\gamma}{\alpha_c}\right)\frac{\dot{c}}{c} +$$

$$+ \alpha_c\delta_p + \delta_c\delta_p - \delta_c\delta_q\gamma - \alpha_q\delta_c\gamma - \alpha_c\delta_q\gamma - \frac{\alpha_q\delta_c^2\gamma}{\alpha_c}$$

Define

$$\kappa = -\left(1 + \frac{\alpha_q\gamma}{\alpha_c}\right) \tag{A.27}$$

$$\nu = \delta_p - \delta_c - \delta_q\gamma - \alpha_c - \alpha_q\gamma - 2\frac{\alpha_q\delta_c\gamma}{\alpha_c} \tag{A.28}$$

$$\eta = \alpha_c\delta_p + \delta_c\delta_p - \delta_c\delta_q\gamma - \alpha_q\delta_c\gamma - \alpha_c\delta_q\gamma - \frac{\alpha_q\delta_c^2\gamma}{\alpha_c} \tag{A.29}$$

Hence,

$$\frac{d}{dt}\left(\frac{\dot{c}}{c}\right) = \kappa\left(\frac{\dot{c}}{c}\right)^2 + \nu\,\frac{\dot{c}}{c} + \eta$$

This is the Ricatti-equation.

Define

$$-\frac{1}{\kappa}\frac{\dot{v}}{v} = \frac{\dot{c}}{c} \tag{A.30}$$

$$\frac{d}{dt}\left(-\frac{1}{\kappa}\frac{\dot{v}}{v}\right) = \frac{d}{dt}\left(\frac{\dot{c}}{c}\right) \quad \text{gives}$$

$$-\frac{1}{\kappa}\,\frac{v\ddot{v} - \dot{v}^2}{v^2} = \kappa\left(-\frac{1}{\kappa}\frac{\dot{v}}{v}\right)^2 + \nu\left(-\frac{1}{\kappa}\frac{\dot{v}}{v}\right) + \eta$$

$$-\frac{1}{\kappa}\frac{\ddot{v}}{v} = -\frac{\nu}{\kappa}\frac{\dot{v}}{v} + \eta$$

$$\ddot{v} - \nu\dot{v} + \kappa\eta v = 0$$

The solution to this second order differential equation is

$$v = \tilde{A}e^{\lambda_1 t} + \tilde{B}e^{\lambda_2 t}$$

where

$$\lambda^2 - \nu\lambda + \kappa\eta = 0$$

Inserting the original parameters from the definitions (A.27)-(A.29) gives after some tedious algebra

$$\lambda_1 = \delta_p - \delta_q\gamma - \frac{\alpha_q\delta_c\gamma}{\alpha_c} \tag{A.31}$$

$$\lambda_2 = -\delta_c - \alpha_c - \alpha_q\gamma - \frac{\alpha_q\delta_c\gamma}{\alpha_c} \tag{A.32}$$

By (A.30)

$$\frac{\dot{c}}{c} = -\frac{1}{\kappa}\frac{\dot{v}}{v}$$

$$c = (Cv)^{-\frac{1}{\kappa}} = (C\tilde{A}e^{\lambda_1 t} + C\tilde{B}e^{\lambda_2 t})^{-\frac{1}{\kappa}} = (Ae^{\lambda_1 t} + Be^{\lambda_2 t})^{-\frac{1}{\kappa}} \tag{A.33}$$

The values of the constants A and B are given by inserting the initial conditions p_o and c_o. (A.33) gives

$$c_o = (A+B)^{-\frac{1}{\kappa}} \tag{A.34}$$

And from (A.22) we have

$$p_o = \frac{\dot{c}_o + (\alpha_c+\delta_c)c_o}{\alpha_c} \tag{A.35}$$

where, from (A.33)

$$\dot{c}_o = -\frac{1}{\kappa}(A+B)^{-\frac{1}{\kappa}-1}(\lambda_1 A + \lambda_2 B)$$

Inserting this into (A.35) and using (A.34)

$$A = \frac{(\alpha_c+\alpha_q\gamma)p_o c_o^{\frac{\alpha_q\gamma}{\alpha_c}}}{(\alpha_c+\alpha_q\gamma+\delta_c+\delta_p-\delta_q\gamma)} \tag{A.36}$$

$$B = \frac{(\alpha_c+\alpha_q\gamma+\delta_c+\delta_p-\delta_q\gamma)c_o^{\frac{\alpha_c+\alpha_q\gamma}{\alpha_c}} - (\alpha_c+\alpha_q\gamma)p_o c_o^{\frac{\alpha_q\gamma}{\alpha_c}}}{(\alpha_c+\alpha_q\gamma+\delta_c+\delta_p-\delta_q\gamma)} \tag{A.37}$$

So (A.33), (A.36), (A.37), (A.31), (A.32) and (A.27) now give $c(t, p_o, c_o)$. This can be expressed in a form that makes it easier to interpret.

First, define

$$\Lambda_c = \frac{\alpha_c}{\alpha_c + \alpha_q \gamma} \tag{A.38}$$

$$\Lambda_p = \frac{\alpha_q \gamma}{\alpha_c + \alpha_q \gamma} \tag{A.39}$$

These are introduced mainly for notational convenience. They can be interpreted as the share of the adjustment of the profit margin towards equilibrium that comes from the cost side (Λ_c) and the price side (Λ_p) respectively.

A can be rewritten, by making use of the expression for steady-state profits (4.12)

$$A = \frac{p_o c_o^{\frac{1-\Lambda_c}{\Lambda_c}}}{\pi^* + 1} = c_o^{\frac{1}{\Lambda_c}} \frac{\pi_o + 1}{\pi^* + 1} \tag{A.40}$$

and, by (A.37) and (A.40)

$$B = c_o^{\frac{1}{\Lambda_c}} - A = c_o^{\frac{1}{\Lambda_c}} \left(1 - \frac{\pi_o + 1}{\pi^* + 1}\right) \tag{A.41}$$

c_t can now be written as

$$c_t = c_o \left[\frac{\pi_o + 1}{\pi^* + 1} e^{\lambda_1 t} + \left(1 - \frac{\pi_o + 1}{\pi^* + 1}\right) e^{\lambda_2 t}\right]^{\Lambda_c} \tag{A.42}$$

From the expressions for $\hat{c}^*$ and $\hat{c}^{**}$ (p.109) it is seen that

$$\lambda_1 = \frac{\hat{c}^*}{\Lambda_c} \tag{A.43}$$

$$\lambda_2 = \frac{\hat{c}^{**}}{\Lambda_c} \tag{A.44}$$

$\lambda_1 > \lambda_2$ if $\pi^* > -1$ and stable. Making use of (A.43) and (A.44), (A.42) can now be rewritten as

$$c_t = c_o e^{\hat{c}^* t} \Omega_t^{\Lambda_c} \tag{A.45}$$

where $\Omega_t = \frac{\pi_o+1}{\pi^*+1} + \left(1 - \frac{\pi_o+1}{\pi^*+1}\right) e^{(\lambda_2-\lambda_1)t}$

Ω_t can be seen as a measure of the impact of the deviation from steady state.

$$\Omega_o = 1, \; \lim_{t\to\infty} \Omega_t = \frac{\pi_o+1}{\pi^*+1} \; .$$

The expression for p(t) is given by inserting (A.45) into (A.22). After some manipulations

$$p_t = p_o e^{\hat{c}^* t} \Omega_t^{-\Lambda_p} \tag{A.46}$$

By (A.24) and (A.46) we have

$$q_t = q_o \left(e^{(\delta_p - \hat{c}^*)t} \; \Omega_t^{\Lambda_p} \right)^{1/\gamma} \tag{A.47}$$

(A.26), (A.45) and (A.46) give

$$\pi_t = \frac{\pi_o + 1}{\Omega_t} - 1 \tag{A.48}$$

(A.45)-(A.48) give the solution to the system (A.22)-(A.26).

COMPARATIVE DYNAMICS OF THE BASIC MODEL

Steady-State Effects

$$\pi^* = \frac{\delta_c + \delta_p - \delta_q\gamma}{\alpha_c + \alpha_q\gamma}$$

$$\frac{\partial\pi^*}{\partial\alpha_c} = -\frac{\delta_c + \delta_p - \delta_q\gamma}{(\alpha_c + \alpha_q\gamma)^2} = -\frac{\pi^*}{\alpha_c + \alpha_q\gamma}$$

$$\frac{\partial\pi^*}{\partial\alpha_q} = -\frac{\gamma(\delta_c + \delta_p - \delta_q\gamma)}{(\alpha_c + \alpha_q\gamma)^2} = -\frac{\gamma\pi^*}{\alpha_c + \alpha_q\gamma}$$

$$\frac{\partial\pi^*}{\partial\delta_c} = \frac{1}{\alpha_c + \alpha_q\gamma}$$

$$\frac{\partial\pi^*}{\partial\delta_q} = -\frac{\gamma}{\alpha_c + \alpha_q\gamma}$$

$$\frac{\partial\pi^*}{\partial\delta_p} = \frac{1}{\alpha_c + \alpha_q\gamma}$$

$$\frac{\partial\pi^*}{\partial\gamma} = -\frac{\alpha_c\delta_q + \alpha_q(\delta_c+\delta_p)}{(\alpha_c + \alpha_q\gamma)^2} = -\frac{\alpha_q\pi^* + \delta_q}{\alpha_c + \alpha_q\gamma} = -\frac{\hat{q}^*}{\alpha_c + \alpha_q\gamma}$$

$$\hat{q}^* = \alpha_q\pi^* + \delta_q = \frac{\alpha_q(\delta_c+\delta_p) + \alpha_c\delta_q}{\alpha_c + \alpha_q\gamma}$$

$$\frac{\partial \hat{q}^*}{\partial \alpha_c} = \alpha_q \frac{\partial \pi^*}{\partial \alpha_c} = -\frac{\Lambda_p}{\gamma}\pi^*$$

$$\frac{\partial \hat{q}^*}{\partial \alpha_q} = \alpha_q \frac{\partial \pi^*}{\partial \alpha_q} + \pi^* = \Lambda_c \pi^*$$

$$\frac{\partial \hat{q}^*}{\partial \delta_c} = \alpha_q \frac{\partial \pi^*}{\partial \delta_c} = \frac{\Lambda_p}{\gamma}$$

$$\frac{\partial \hat{q}^*}{\partial \delta_q} = \alpha_q \frac{\partial \pi^*}{\partial \delta_q} + 1 = \Lambda_c$$

$$\frac{\partial \hat{q}^*}{\partial \delta_p} = \alpha_q \frac{\partial \pi^*}{\partial \delta_q} = \frac{\Lambda_p}{\gamma}$$

$$\frac{\partial \hat{q}^*}{\partial \gamma} = \alpha_q \frac{\partial \pi^*}{\partial \gamma} = -\frac{\Lambda_p}{\gamma}\hat{q}^*$$

$$\hat{c}^* = \hat{p}^* = \alpha_c \pi^* - \delta_c = \frac{\alpha_c \delta_p - \alpha_c \delta_q \gamma - \alpha_q \delta_c \gamma}{\alpha_c + \alpha_q \gamma}$$

$$\frac{\partial \hat{c}^*}{\partial \alpha_c} = \pi^* + \alpha_c \frac{\partial \pi^*}{\partial \alpha_c} = \Lambda_p \pi^*$$

$$\frac{\partial \hat{c}^*}{\partial \alpha_q} = \alpha_c \frac{\partial \pi^*}{\partial \alpha_q} = -\Lambda_c \gamma \pi^*$$

$$\frac{\partial \hat{c}^*}{\partial \delta_c} = \alpha_c \frac{\partial \pi^*}{\partial \delta_c} - 1 = -\Lambda_p$$

$$\frac{\partial \hat{c}^*}{\partial \delta_q} = \alpha_c \frac{\partial \pi^*}{\partial \delta_q} = - \Lambda_c \gamma$$

$$\frac{\partial c^*}{\partial \delta_p} = \alpha_c \frac{\partial \pi^*}{\partial \delta_p} = \Lambda_c$$

$$\frac{\partial \hat{c}^*}{\partial \gamma} = \alpha_c \frac{\partial \pi^*}{\partial \gamma} = - \Lambda_c \hat{q}^*$$

Effects Outside Steady-State

The effects on π_t outside steady-state are easy to show graphically. We first take $\frac{\partial \dot{\pi}_t}{\partial \alpha_c}$ etc.

$$\dot{\pi}_t = (\pi_t+1)[-(\alpha_c+\alpha_q\gamma)\pi_t + \delta_c + \delta_p - \delta_q\gamma]$$

$$\frac{\partial \dot{\pi}_t}{\partial \alpha_c} = - \pi_t(\pi_t+1)$$

$$\frac{\partial \dot{\pi}_t}{\partial \alpha_q} = - \pi_t(\pi_t+1)\gamma$$

$$\frac{\partial \dot{\pi}_t}{\partial \delta_c} = \pi_t + 1$$

$$\frac{\partial \dot{\pi}_t}{\partial \delta_p} = \pi_t + 1$$

$$\frac{\partial \dot{\pi}_t}{\partial \delta_q} = - \gamma(\pi_t+1)$$

$$\frac{\partial \dot{\pi}_t}{\partial \gamma} = - (\pi_t+1)(\alpha_q\pi_t+\delta_q) = - (\pi_t+1)\hat{q}_t$$

From this it is clear that, $\forall t > 0$,

$$\frac{\partial \pi_t}{\partial \delta_c} > 0, \quad \frac{\partial \pi_t}{\partial \delta_p} > 0, \quad \frac{\partial \pi_t}{\partial \delta_q} < 0$$

For α_c, α_q and γ the sign of the partial derivative depends on initial conditions. Below it is shown how the phase-curve $\dot{\pi}(\pi)$ is affected by a change in α_c (or α_q) for $\pi^* > 0$ (Fig. A.1) and $\pi^* < 0$ (Fig. A.2). $\alpha''_c > {}'_c$

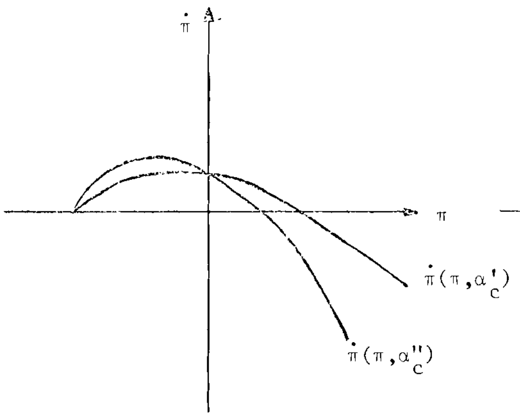


Figure A.1

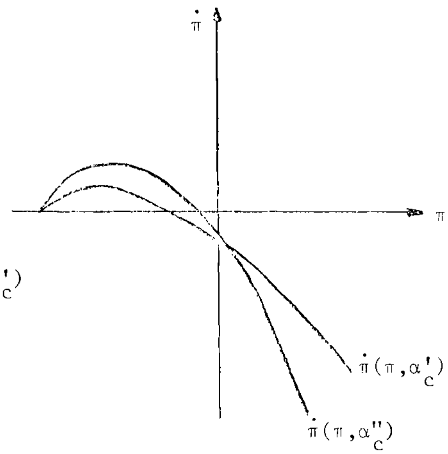


Figure A.2

From this we see that in all cases where $\text{sgn}\ \pi_o \neq \text{sgn}\ \pi^*$ there is a $t = \bar{t}$ such that $\text{sgn}\ \frac{\partial \pi_t}{\partial \alpha_c} \neq \text{sgn}\ \frac{\partial \pi_\tau}{\partial \alpha_c}\ \forall\ t > \bar{t},\ \tau < \bar{t}$. Further $\text{sgn}\ \pi_{\bar{t}} \neq \text{sgn}\ \pi_o$.

Likewise in all cases where $\text{sgn}\ \hat{q}_o \neq \text{sgn}\ \hat{q}^*$ there is a $t = \bar{t}$ such that $\text{sgn}\ \frac{\partial \pi_t}{\partial \gamma} \neq \text{sgn}\ \frac{\partial \pi_\tau}{\partial \gamma}\ \forall\ t > \bar{t},\ \tau < \bar{t}$.

From (A.45) we have

$$c_t = c_o e^{\hat{c}^* t} \Omega_t^{\Lambda_c}$$

$$\frac{\partial c_t}{\partial \delta_c} = c_o e^{\hat{c}^* t} \Omega_t^{\Lambda_c - 1} \Lambda_c \left[\frac{\pi_o + 1}{\pi^* + 1} \frac{\partial \pi^* / \partial \delta_c}{\pi^* + 1} \left(e^{(\lambda_2 - \lambda_1)t} - 1 \right) - \right.$$

$$\left. - t \left(1 - \frac{\pi_o + 1}{\pi^* + 1} \right) e^{(\lambda_2 - \lambda_1)t} \right] - c_o e^{\hat{c}^* t} \Omega_t^{\Lambda_c} \Lambda_p t =$$

$$= \frac{c_t}{\Omega_t} \left\{ \Lambda_c \frac{\pi_o + 1}{\pi^* + 1} \frac{1}{\lambda_1 - \lambda_2} \left(e^{(\lambda_2 - \lambda_1)t} - 1 \right) - \Lambda_c t \left(1 - \frac{\pi_o + 1}{\pi^* + 1} \right) e^{(\lambda_2 - \lambda_1)t} - \Lambda_p \Omega_t t \right\} =$$

$$= \frac{c_t}{\Omega_t} \left\{ (1 - \Lambda_p) \frac{\pi_o + 1}{\pi^* + 1} \frac{1}{\lambda_1 - \lambda_2} \left(e^{(\lambda_2 - \lambda_1)t} - 1 \right) - t \left(1 - \frac{\pi_o + 1}{\pi^* + 1} \right) e^{(\lambda_2 - \lambda_1)t} - \right.$$

$$\left. - \Lambda_p \frac{\pi_o + 1}{\pi^* + 1} t \right\}$$

taking into account that $\Lambda_c + \Lambda_p = 1$.

$$\lim_{t \to \infty} \frac{\partial c_t}{\partial \delta_c} = - c_t \Lambda_p t$$

i.e. in the long run the effect on the slope of the steady-state path will of course dominate. In the short run, however, it seems conceivable that $\frac{\partial c_t}{\partial \delta_c} > 0$, since the sign of two other terms within the parenthesis may be positive. To prove that this is <u>not</u> so take

$$\frac{\partial \{\}}{\partial \Lambda_p} = \frac{\pi_o + 1}{\pi^* + 1} \frac{1}{\lambda_1 - \lambda_2} \cdot \left(1 - e^{(\lambda_2 - \lambda_1)t} \right) - t \frac{\pi_o + 1}{\pi^* + 1} < 0 \ \forall \ t > 0$$

This means that if $\frac{\partial c_t}{\partial \delta_c}$ is positive for any Λ_p it must be for Λ_p infinitely close to zero (by assumption $\Lambda_p > 0$). But this means that we can regard c_t as the solution to eq. (A.22) with p_t constant and it is obvious that

$$\frac{\partial c_t}{\partial \delta_c} < 0 \quad \forall\ t > 0$$

$$\frac{\partial c_t}{\partial \delta_p} = \frac{c_t}{\Omega_t}\left\{\Lambda_c\left[\frac{\pi_o+1}{\pi^*+1}\,\frac{1}{\lambda_1-\lambda_2}\,(e^{(\lambda_2-\lambda_1)t}-1) - \right.\right.$$

$$\left.\left. - \left(1-\frac{\pi_o+1}{\pi^*+1}\right)te^{(\lambda_2-\lambda_1)t}\right] + \Lambda_c\Omega_t t\right\} =$$

$$= \frac{c_t}{\Omega_t}\,\Lambda_c\,\frac{\pi_o+1}{\pi^*+1}\left\{\frac{1}{\lambda_1-\lambda_2}\,(e^{(\lambda_2-\lambda_1)t}-1) + t\right\}$$

The parenthesis is positive for all positive t, i.e.

$$\Lambda_c\,\frac{\partial c_t}{\partial \delta_p} > 0 \quad \forall\ t > 0$$

$$\lim_{t\to\infty} \frac{\partial c_t}{\partial \delta_p} = c_t\Lambda_c t$$

$$\frac{\partial c_t}{\partial \delta_q} = -\gamma\,\frac{\partial c_t}{\partial \delta_p}\ , \text{ i.e.}$$

$$\Lambda_c\,\frac{\partial c_t}{\partial \delta_q} < 0 \quad \forall\ t > 0$$

$$\lim_{t\to\infty} \frac{\partial c_t}{\partial \delta_q} = -\ c_t\gamma\Lambda_c t$$

$$\frac{\partial c_t}{\partial \alpha_c} = \frac{c_t}{\Omega_t}\left\{\Lambda_c\left[\frac{\pi_o+1}{\pi^*+1}\,\frac{\pi^*}{\lambda_1-\lambda_2}\,(1-e^{(\lambda_2-\lambda_1)t}) - \left(1-\frac{\pi_o+1}{\pi^*+1}\right)te^{(\lambda_2-\lambda_1)t}\right] + \right.$$

$$\left. + \frac{\Lambda_p}{\alpha_c+\alpha_q\gamma}\,\Omega_t \log \Omega_t + \Lambda_p\pi^*\Omega_t t\right\}$$

$$\lim_{t\to\infty} \frac{\partial c_t}{\partial \alpha_c} = c_t\Lambda_p\pi^* t$$

Whereas the long-run effect thus depends on the sign of steady-state profits, the short run effect depends on initial profits. To see this clearer, take

$$\frac{\partial}{\partial t}\left(\frac{\partial c_t}{\partial \alpha_c}\right)\Bigg|_{t=0} = c_o\left\{\Lambda_c\,\frac{\pi_o+1}{\pi^*+1}\,\pi^* - \Lambda_c\left(1-\frac{\pi_o+1}{\pi\ +1}\right) + \right.$$

$$\left. + (\lambda_2-\lambda_1)\,\frac{\Lambda_p}{\alpha_c+\alpha_q\gamma}\left(1-\frac{\pi_o+1}{\pi^*+1}\right) + \Lambda_p\pi^*\right\} =$$

$$= c_o\left\{\Lambda_c\,\frac{\pi_o+1}{\pi^*+1}\pi^* - \Lambda_c\left(1-\frac{\pi_o+1}{\pi^*+1}\right) + \Lambda_p\pi_o\right\} = c_o\pi_o$$

$$\frac{\partial c_t}{\partial \alpha_q} = c_t\Lambda_c\left\{\frac{1}{\Omega}\left[\frac{\pi_o+1}{\pi^*+1}\,\frac{\gamma\pi^*}{\lambda_1-\lambda_2}\,(1-e^{(\lambda_2-\lambda_1)t}) - \gamma\left(1-\frac{\pi_o+1}{\pi^*+1}\right)te^{(\lambda_2-\lambda_1)t}\right] - \right.$$

$$\left. - \frac{\gamma}{\alpha_c+\alpha_q\gamma}\log\Omega - \gamma\pi^* t\right\}$$

$$\lim_{t\to\infty} \frac{\partial c_t}{\partial \alpha_q} = -\ c_t\Lambda_c\gamma\pi^* t$$

$$\frac{\partial c_t}{\partial \gamma} = c_t \left\{ \frac{\Lambda_c}{\Omega} \left[\frac{\pi_o + 1}{\pi^* + 1} \cdot \frac{\alpha_c \delta_q + \alpha_q(\delta_c + \delta_p)}{\lambda_1 - \lambda_2} \left(1 - e^{(\lambda_2 - \lambda_1)t}\right) + \right.\right.$$

$$\left.\left. + (\delta_q - \alpha_q)\left(1 - \frac{\pi_o + 1}{\pi^* + 1}\right) t e^{(\lambda_2 - \lambda_1)t} \right] - \alpha_q \Lambda_c \log \Omega - \Lambda_c \hat{q}^* t \right\} \quad 1)$$

$$\lim_{t \to \infty} \frac{\partial c_t}{\partial \gamma} = - c_t \Lambda_c \hat{q}^* t$$

$$\frac{\partial c_t}{\partial c_o} = e^{\hat{c}^* t} \Omega^{\Lambda_c} \left[1 - c_o \frac{\Lambda_c}{\Omega_t} \frac{p_o}{c_o^2(\pi^* + 1)} \left(1 - e^{(\lambda_2 - \lambda_1)t}\right) \right] =$$

$$= \frac{c_t}{c_o} \left[1 - \Lambda_c \frac{\frac{\pi_o + 1}{\pi^* + 1}\left(1 - e^{(\lambda_2 - \lambda_1)t}\right)}{\frac{\pi_o + 1}{\pi^* + 1}\left(1 - e^{(\lambda_2 - \lambda_1)t}\right) + e^{(\lambda_2 - \lambda_1)t}} \right]$$

$$\lim_{t \to \infty} \frac{\frac{\partial c_t}{\partial c_o}}{\frac{c_t}{c_o}} = 1 - \Lambda_c = \Lambda_p$$

The limit value of the elasticity will be approached from above (below) as Λ_c is positive (negative).

$$\frac{\partial c_t}{\partial p_o} = \frac{c_t}{c_o} \cdot \frac{\Lambda_c}{\pi^* + 1} \cdot \frac{1 - e^{(\lambda_2 - \lambda_1)t}}{\Omega_t}$$

$$\lim_{t \to \infty} \frac{\frac{\partial c_t}{\partial p_o}}{\frac{c_t}{p_o}} = \Lambda_c$$

---

1) Note that is has been implicitly assumed that $\frac{\partial \pi_o}{\partial \gamma} = 0$. This presumes that a be varied such as to compensate for the change in p_o due to the change in γ.

From (A.46) we see that the expression for p_t formally looks very much like the expression for c_t

$$p_t = p_o e^{\hat{c}^* t} \Omega_t^{-\Lambda_p}$$

Nothing of interest will be added by showing the expressions for all the partial derivatives. So we will confine ourselves to show the effects of varying initial conditions

$$\frac{\partial p_t}{\partial p_o} = e^{\hat{c}^* t} \left[\Omega_t^{-\Lambda_p} - \Lambda_p \Omega_t^{-\Lambda_p - 1} \frac{\pi_o + 1}{\pi^* + 1} (1 - e^{(\lambda_2 - \lambda_1)t}) \right] =$$

$$= \frac{p_t}{p_o} \left[1 - \Lambda_p \frac{\frac{\pi_o + 1}{\pi^* + 1} (1 - e^{(\lambda_2 - \lambda_1)t})}{\frac{\pi_o + 1}{\pi^* + 1} (1 - e^{(\lambda_2 - \lambda_1)t}) + e^{(\lambda_2 - \lambda_1)t}} \right]$$

$$\lim_{t \to \infty} \frac{\frac{\partial p_t}{\partial p_o}}{\frac{p_t}{p_o}} = \Lambda_c$$

$$\frac{\partial p_t}{\partial c_o} = p_o e^{\hat{c}^* t} \Lambda_p \Omega_t^{-\Lambda_p - 1} \frac{p_o}{c_o^2} \cdot \frac{(1 - e^{(\lambda_2 - \lambda_1)t})}{\pi^* + 1} =$$

$$= \frac{p_o}{c_o} \Lambda_p \frac{\frac{\pi_o + 1}{\pi^* + 1} (1 - e^{(\lambda_2 - \lambda_1)t})}{\frac{\pi_o + 1}{\pi^* + 1} (1 - e^{(\lambda_2 - \lambda_1)t}) + e^{(\lambda_2 - \lambda_1)t}}$$

$$\lim_{t \to \infty} \frac{\frac{\partial p_t}{\partial c_o}}{\frac{p_t}{c_o}} = \Lambda_p$$

STABILITY ANALYSIS OF THE DISAGGREGATED MODEL (pp. 137-144)

n firms, local stability

When all parameters are allowed to differ between the firms there can only by chance exist a steady-state where $\tilde{q}_i > 0$ for more than one firm, as is shown in the main text, p. 140. This possibility will be disregarded and we will only look at a steady-state where $\tilde{q}_1 = 1$, $\tilde{q}_i = 0$, $i = 2, \ldots, n$. Linearizing around steady-state we get 2n equations in equally many unknowns. But, since $\sum_{i=1}^{n} \tilde{q}_i = 1$, there is a linear dependence and we have to take away one of the equations. For convenience we skip $\dot{\tilde{q}}_1$.

$$\dot{\tilde{q}}_i = \tilde{q}_i(\hat{q}_i - \sum_{k=1}^{n} \hat{q}_k\tilde{q}_k) \qquad i = 2, \ldots, n$$

$$\dot{\pi}_i = (\pi_i+1)\left[-\gamma \sum_{k=1}^{n} \tilde{q}_k(\alpha_{qk}\pi_k+\delta_{qk}) + \delta_p - \alpha_{ci}\pi_i + \delta_{ci}\right] \qquad i = 1, \ldots, n$$

$$\frac{\partial\dot{\tilde{q}}_i}{\partial\tilde{q}_j} = \tilde{q}_i(\hat{q}_1 - \hat{q}_j) \qquad \begin{array}{l} i = 2, \ldots, n \\ j = 2, \ldots, n \\ j \neq i \end{array}$$

$$\frac{\partial\dot{\tilde{q}}_i}{\partial\tilde{q}_i} = \hat{q}_i - \sum_{k=1}^{n} \hat{q}_k\tilde{q}_k + \tilde{q}_i(\hat{q}_1 - \hat{q}_i) \qquad i = 2, \ldots, n$$

$$\frac{\partial\dot{\tilde{q}}_i}{\partial\pi_j} = \tilde{q}_i\left(\frac{\partial\hat{q}_i}{\partial\pi_j} - \sum_{k=1}^{n} \tilde{q}_k \frac{\partial\hat{q}_k}{\partial\pi_j}\right) \qquad \begin{array}{l} i = 2, \ldots, n \\ j = 1, \ldots, n \end{array}$$

$$\frac{\partial\dot{\pi}_i}{\partial\tilde{q}_j} = -(\pi_i+1)\gamma(\alpha_{qj}\pi_j+\delta_{qj}-\alpha_{q1}\pi_1-\delta_{q1}) \qquad \begin{array}{l} i = 1, \ldots, n \\ j = 2, \ldots, n \end{array}$$

$$\frac{\partial\dot{\pi}_i}{\partial\pi_j} = -(\pi_i+1)\gamma\tilde{q}_j\alpha_{qj} \qquad \begin{array}{l} i = 1, \ldots, n \\ j = 1, \ldots, n \\ j \neq i \end{array}$$

$$\frac{\partial \dot{\pi}_i}{\partial \pi_i} = -\ (\pi_i+1)(\gamma \tilde{q}_i \alpha_{qi}+\alpha_{ci})\ -$$

$$-\ \gamma \sum_{k=1}^{n} \tilde{q}_k(\alpha_{qk}\pi_k+\delta_{qk}) + \delta_p - \alpha_{ci}\pi_i + \delta_{ci} \qquad i = 1,\ \ldots,\ n$$

Evaluating these partial derivatives at $\tilde{q}_1 = 1$, $\tilde{q}_i = 0$ $(i=2,\ldots,n)$, $\pi_i = \pi_i^*$ $(i=1,\ldots,n)$, we see that all terms at the right hand side of the diagonal of this matrix vanish

$$\begin{array}{l|cccccc}
 & /\partial\tilde{q}_2 & \cdots & /\partial\tilde{q}_n & /\partial\pi_1 & \cdots & /\partial\pi_n \\
\hline
\partial\dot{\tilde{q}}_2/ & & & & & & \\
\vdots & & & & & & \\
\partial\dot{\tilde{q}}_n/ & & & & & & \\
\partial\dot{\pi}_1/ & & & & & & \\
\vdots & & & & & & \\
\partial\dot{\pi}_n/ & & & & & &
\end{array}$$

But the value of the determinant of such a matrix is simply the product of the diagonal terms. And hence the 2n - 1 roots of the corresponding characteristic equation are the 2n - 1 diagonal elements. The stability condition is that all these roots are negative. This means, taking into account that $\dot{\pi}_i^* = 0$,

$$\hat{q}_i^* - \hat{q}_1^* < 0 \qquad i = 2,\ \ldots,\ n$$

$$(\pi_1^*+1)(\alpha_{c1}+\alpha_{q1}\gamma) > 0$$

$$(\pi_i^*+1)\alpha_{ci} > 0 \qquad i = 2,\ \ldots,\ n$$

Two firms, all parameters equal, local stability

$$\hat{c}_{it} = \alpha_c \pi_{it} - \delta_c \qquad i = 1, 2 \tag{A.49}$$

$$\hat{q}_{it} = \alpha_q \pi_{it} + \delta_q \qquad i = 1, 2 \tag{A.50}$$

$$\hat{p}_t = -\gamma[\tilde{q}_t \hat{q}_{1t} + (1-\tilde{q}_t)\hat{q}_{2t}] + \delta_p \tag{A.51}$$

where

$$\tilde{q}_t = \frac{q_{1t}}{q_{1t} + q_{2t}} \tag{A.52}$$

Steady-state is defined by $\dot{\tilde{q}} = \dot{\pi}_1 = \dot{\pi}_2 = 0$

$$\dot{\tilde{q}} = \tilde{q}(1-\tilde{q})\alpha_q(\pi_1 - \pi_2) \tag{A.53}$$

$$\dot{\pi}_1 = (\pi_1+1)\{-\gamma[\tilde{q}(\alpha_q\pi_1+\delta_q) + (1-\tilde{q})(\alpha_q\pi_2+\delta_q)] + \delta_p - \alpha_c\pi_1 + \delta_c\} \tag{A.54}$$

$$\dot{\pi}_2 = (\pi_2+1)\{-\gamma[\tilde{q}(\alpha_q\pi_1+\delta_q) + (1-\tilde{q})(\alpha_q\pi_2+\delta_q)] + \delta_p - \alpha_c\pi_2 + \delta_c\} \tag{A.55}$$

$\dot{\pi}_1 = \dot{\pi}_2 = 0$ gives

$$\pi_1^* = \pi_2^* = \frac{\delta_c + \delta_p - \delta_q\gamma}{\alpha_c + \alpha_q\gamma}$$

$$\pi_1^{**} = \pi_2^{**} = -1$$

By (A.53) $\pi_1 = \pi_1^*$ and $\pi_2^* = \pi_2$ implies $\dot{\tilde{q}} = 0$. $\tilde{q}$ is hence indeterminate in steady-state.

We now investigate local stability properties by linearizing the equations (A.53)-(A.55) around steady-state. Evaluating the partial derivatives at $\pi_1 = \pi_2 = \pi^*$ gives

	$/\partial\tilde{q}$	$/\partial\pi_1$	$/\partial\pi_2$
$\partial\dot{\tilde{q}}/$	0	$\tilde{q}(1-\tilde{q})\alpha_q$	$-\tilde{q}(1-\tilde{q})\alpha_q$
$\partial\dot{\pi}_1/$	0	$-(\alpha_q\gamma\tilde{q}+\alpha_c)(\pi^*+1)$	$-\alpha_q\gamma(1-\tilde{q})(\pi^*+1)$
$\partial\dot{\pi}_2/$	0	$-\alpha_q\gamma\tilde{q}(\pi^*+1)$	$-[\alpha_q\gamma(1-\tilde{q}) + \alpha_c](\pi^*+1)$

The characteristic equation is

$$(-\lambda)\left(-(\alpha_q\gamma\tilde{q}+\alpha_c)(\pi^*+1) - \lambda\right)\left(-[\alpha_q\gamma(1-\tilde{q}) + \alpha_c](\pi^*+1) - \lambda\right) -$$

$$- (-\lambda)[-\alpha_q\gamma(1-\tilde{q})(\pi^*+1)][-\alpha_q\gamma\tilde{q}(\pi^*+1)] = 0$$

$$(-\lambda)\{\lambda^2 + (2\alpha_c+\alpha_q\gamma)(\pi^*+1)\lambda + \alpha_c(\alpha_c+\alpha_q\gamma)(\pi^*+1)^2\} = 0$$

$$\lambda_1 = 0$$

$$\lambda_{2,3} < 0 \quad \text{if} \quad \alpha_c(\alpha_c+\alpha_q\gamma) > 0$$

$$2\alpha_c + \alpha_q\gamma > 0$$

Two firms, $\alpha_{q1} \neq \alpha_{q2}$, global stability

Assuming $\pi_{1t_o} = \pi_{2t_o}$ and $\alpha_{c1} = \alpha_{c2}$, $\delta_{c1} = \delta_{c2}$, it follows that $\pi_{1t} = \pi_2$ for any t, since the price will always be the same for both firms. It is then possible to study the stability properties in a phase-diagram in $\pi - \tilde{q}$ space, where $\tilde{q}$ is the market share of firm 1.

$$\dot{\tilde{q}} = \tilde{q}(1-\tilde{q})(\alpha_{q1}-\alpha_{q2})\pi$$

$$\dot{\pi} = (\pi+1)\{-\gamma[\tilde{q}(\alpha_{q1}\pi+\delta_q) + (1-\tilde{q})(\alpha_{q2}\pi+\delta_q)] + \delta_p - \alpha_c\pi + \delta_c\}$$

$$\dot{\tilde{q}} = 0 \Rightarrow \tilde{q} = 0,\ \tilde{q} = 1,\ \pi = 0$$

$$\dot{\pi} = 0 \Rightarrow \tilde{q} = \frac{-(\alpha_c+\alpha_{q2}\gamma)\pi + \delta_p + \delta_c - \delta_q\gamma}{(\alpha_{q1}-\alpha_{q2})\gamma\pi},\ \pi = -1$$

$$\left.\frac{\partial\tilde{q}}{\partial\pi}\right|_{\dot{\tilde{q}}=0} = -\frac{\delta_p + \delta_c - \delta_q\gamma}{\gamma(\alpha_{q1}-\alpha_{q2})\pi^2}$$

$\dot{\tilde{q}} = \dot{\pi} = 0$ gives two steady-states

$$\tilde{q}_1^* = 1 \qquad \pi_1^* = \frac{\delta_c + \delta_p - \delta_q\gamma}{\alpha_c + \alpha_{q1}\gamma}$$

$$\tilde{q}_2^* = 0 \qquad \pi_2^* = \frac{\delta_c + \delta_p - \delta_q\gamma}{\alpha_c + \alpha_{q2}\gamma}$$

We first illustrate the case of $\delta_c + \delta_p - \delta_q\gamma > 0$, $\alpha_{q1} > \alpha_{q2}$, $\alpha_c + \alpha_{q1}\gamma > 0$, $\alpha_c + \alpha_{q2}\gamma > 0$. This means $\left.\frac{\partial\tilde{q}}{\partial\pi}\right|_{\dot{\pi}=0} < 0$ and $\pi^* > 0$, and we get the picture of Figure A.3.[1]

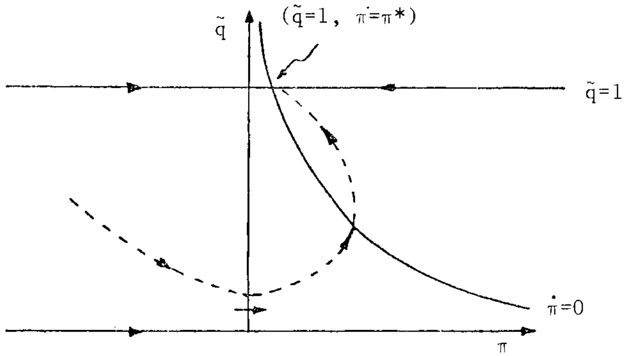


Figure A.3

So provided the parameters fulfil the stability conditions for each firm taken in isolation there will also be a stable steady-state with two firms acting simultaneously. If $\pi^* > 0$ the firm with the highest α_q will capture the market.

It can be of interest to see what happens if one of the firms shows an unstable behaviour. Let us regard the case $\delta_c + \delta_p - \delta_q\gamma > 0$, $\alpha_{q1} > \alpha_{q2}$, $\alpha_c + \alpha_{q1}\gamma > 0$, $\alpha_c + \alpha_{q2}\gamma < 0$. This means as before $\left.\frac{\partial q}{\partial\pi}\right|_{\dot{\pi}=0} < 0$. But there is now the possibility that $\left.\tilde{q}\right|_{\dot{\pi}=0}$ will be positive somewhere in the interval $-1 < \pi < 0$. This will be so if

$$\left.\tilde{q}\right|_{\dot{\pi}=0,\pi=-1} = \frac{\alpha_c + \alpha_{q2}\gamma + \delta_p + \delta_c - \delta_q\gamma}{(\alpha_{q2}-\alpha_{q1})\gamma} > 0$$

---

[1] Note that $\left.\tilde{q}\right|_{\dot{\pi}=0} < 0$ for $-1 \leq \pi < 0$.

which holds if the negative "instability term" $(\alpha_c+\alpha_{q2}\gamma)$ is strong enough relative to the positive "trend term" $(\delta_p+\delta_c-\delta_q\gamma)$. In such a world there is always the risk, as shown in Figure A.4 below, that the unstable firm will dominate and the steady-state $\pi^{**} = -1$, $\tilde{q} = 0$, will be approached.

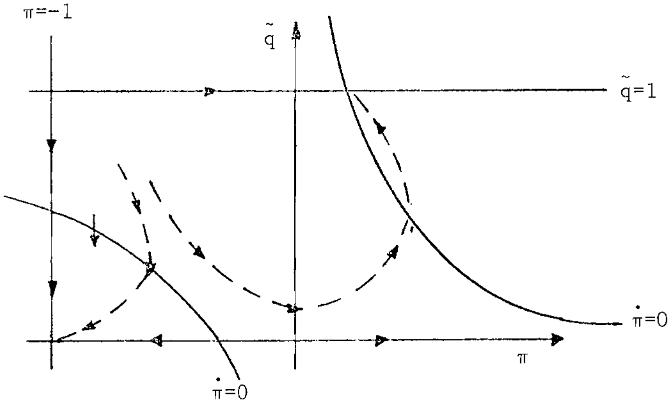


Figure A.4

SCALE DEPENDENT COST REDUCTION (pp. 151-159)

$$\hat{c} = \alpha_c \pi - \frac{\zeta}{1 + e^{-\eta q \pi}} - \delta_c \tag{A.56}$$

$$\hat{q} = \alpha_q \pi + \delta_q \tag{A.57}$$

$$\hat{p} = - \gamma \hat{q} + \delta_p \tag{A.58}$$

We wish to analyze the properties of this system by means of a phase diagram in the π - q plane

$$\dot{\pi} = (\pi+1)\left[- (\alpha_c+\alpha_q\gamma)\pi + \delta_c + \delta_p - \delta_q\gamma + \frac{\zeta}{1 + e^{-\eta q \pi}}\right] \tag{A.59}$$

$$\dot{q} = q(\alpha_q \pi + \delta_q) \tag{A.60}$$

$\dot{\pi} = 0$ gives

$$e^{-\eta q \pi} = \frac{\zeta}{(\alpha_c+\alpha_q\gamma)\pi - (\delta_p+\delta_c-\delta_q\gamma)} - 1$$

$$q\Big|_{\dot{\pi}=0} = \frac{\ln\left[\frac{\zeta}{(\alpha_c+\alpha_q\gamma)\pi - (\delta_p+\delta_c-\delta_q\gamma)} - 1\right]}{- \eta \pi} \tag{A.61}$$

$q\Big|_{\dot{\pi}=0}$ will take on real values for

$A_- < \pi < A_+$

where

$$\pi_-^* = A_- = \frac{\delta_p + \delta_c - \delta_q\gamma}{\alpha_c + \alpha_q\gamma}$$

$$\pi_+^* = A_+ = \frac{\delta_p + \delta_c - \delta_q\gamma + \zeta}{\alpha_c + \alpha_q\gamma}$$

The steady-state π_-^* exists if $A_- < 0$ and $\hat{q}(A_-) > 0$. π_+^* exists if $A_+ > 0$ and $\hat{q}(A_+) > 0$. In the sequel we will use $\pi_{+,-}^*$ instead of $A_{+,-}$. The reader should bear in mind that this does not imply that they exist as steady-states.

From (A.61) we further see

$$\lim_{\pi\to\pi_-^*} q\Big|_{\dot{\pi}=0} = \pm\infty \quad \text{as} \quad \pi_-^* \lessgtr 0$$

$$\lim_{\pi\to\pi_+^*} q\Big|_{\dot{\pi}=0} = \pm\infty \quad \text{as} \quad \pi_+^* \gtrless 0$$

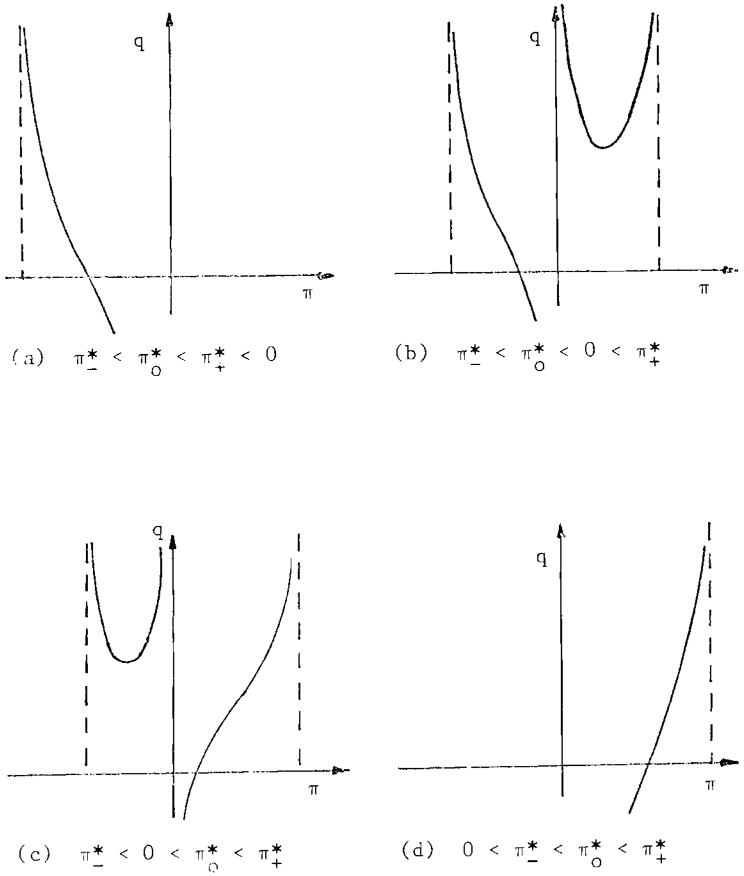
$$q\Big|_{\dot{\pi}=0} = 0 \;\Rightarrow\; \pi = \pi_o^* = \frac{\delta_p + \delta_c - \delta_q\gamma + \frac{1}{2}\zeta}{\alpha_c + \alpha_q\gamma}$$

$$\lim_{\pi\to+0} q\Big|_{\dot{\pi}=0} = \pm\infty \quad \text{as} \quad -\frac{\pi_+^*}{\pi_-^*} \lessgtr 1$$

$$\lim_{\pi \to 0} q \Big|_{\dot{\pi}=0} = \pm \infty \quad \text{as} \quad -\frac{\pi_+^*}{\pi_-^*} \gtrless 1$$

$$\left.\frac{dq}{d\pi}\right|_{\dot{\pi}=0} = \frac{1}{\eta\pi^2}\left\{ \frac{\zeta\pi}{(\alpha_c+\alpha_q\gamma)(\pi_+^*-\pi)(\pi-\pi_-^*)} + \ln\left[\frac{\zeta}{(\alpha_c+\alpha_q\gamma)\pi - (\delta_p+\delta_c-\delta_q\gamma)} - 1\right]\right\} \quad (A.62)$$

From (A.61) and (A.62) we see that $q(\pi)\big|_{\dot{\pi}=0}$ is a well-defined differentiable function everywhere in the interval $\pi_-^* < \pi < \pi_+^*$ except at $\pi = 0$. Depending on the parameter values it will look like either of Figs A.5 (a)-(d).



(a) $\pi_-^* < \pi_o^* < \pi_+^* < 0$

(b) $\pi_-^* < \pi_o^* < 0 < \pi_+^*$

(c) $\pi_-^* < 0 < \pi_o^* < \pi_+^*$

(d) $0 < \pi_-^* < \pi_o^* < \pi_+^*$

Figure A.5

Further we have from (A.59)

$$\frac{\partial\dot{\pi}}{\partial q} = (\pi+1)(\omega_c+\alpha_q\gamma)^2\frac{\eta}{\zeta}(\pi-\pi^*_-)(\pi^*_+-\pi) \gtrless 0 \quad \text{as} \quad \pi \gtrless 0 \tag{A.63}$$

where the derivative is evaluated at $\dot{\pi} = 0$.

This gives $\dot{\pi}$ for $\pi^*_- < \pi < \pi^*_+$. And $\dot{\pi}$ for other values of π are given directly from (A.59).

From (A.60) we have

$$\pi\Big|_{\dot{q}=0} = -\frac{\delta_q}{\alpha_q} \tag{A.64}$$

The dynamic properties will depend on the signs of π^*_-, π^*_o and π^*_+ as is evident from Figs A.5 and on the value of $\pi\Big|_{\dot{q}=0}$. Table A.1 gives the values and stability properties for the steady-states of all possible cases. Figs A.6 and A.7 show the complete phase-diagrams for a couple of these.

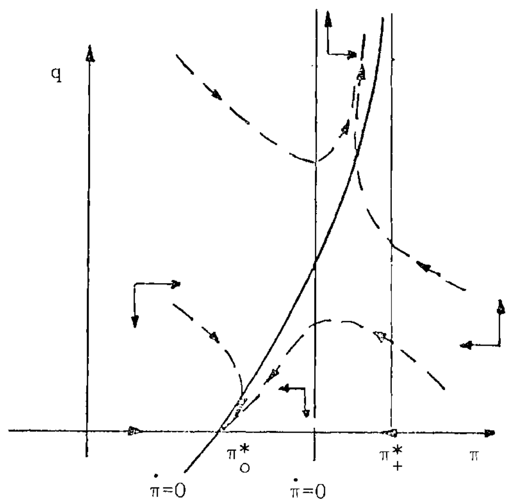


Figure A.6 $\quad 0 < \pi^*_- < \pi^*_o < -\frac{\alpha_q}{\delta_q} < \pi^*_+$

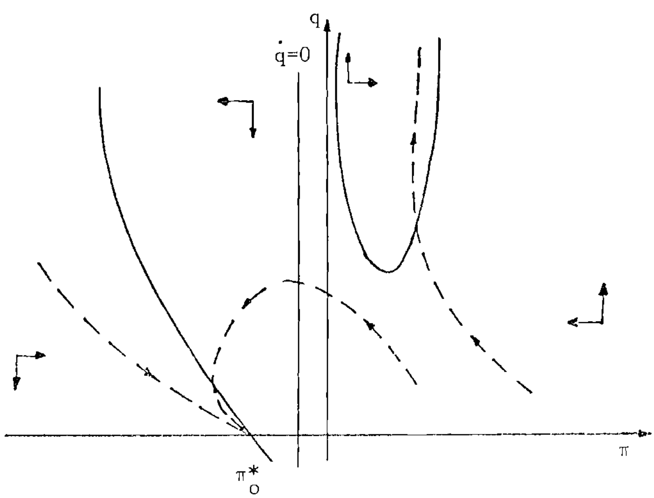


Figure A.7 $\quad \pi^*_- < \pi^*_o < -\frac{\delta_q}{\alpha_q} < \pi^*_+$

Table A.1. Values and Stability Properties of Steady-States

	$\pi^*_+ < -\frac{\delta_q}{\alpha_q}$	$\pi^*_o < -\frac{\delta_q}{\alpha_q} < \pi^*_+$	$\pi^*_- < -\frac{\delta_q}{\alpha_q} < \pi^*_o$	$-\frac{\delta_q}{\alpha_q} < \pi^*_-$
$\pi^*_- < \pi^*_o < \pi^*_+ < 0$	π^*_o, $q^*=0$ globally stable	π^*_o, $q^*=0$ globally stable	$\pi^*=-\frac{\delta_q}{\alpha_q}$, $q^*=q(\dot{\pi}=0, \pi=-\frac{\delta_q}{\alpha_q})$ stability depends on parameter values	π^*_-, $\hat{q}^*_-$ globally stable
$\pi^*_- < \pi^*_o < 0 < \pi^*_+$	π^*_o, $q^*=0$ globally stable	π^*_+, $\hat{q}^*_+$ π^*_o, $q^*=0$ both locally stable	π^*_+, $\hat{q}^*_+$ locally stable $\pi^*=-\frac{\delta_q}{\alpha_q}$, $q^*=q(\dot{\pi}=0, \pi=-\frac{\delta_q}{\alpha_q})$ stability depends on parameter values	π^*_+, $\hat{q}^*_+$ π^*_-, $\hat{q}^*_-$ both locally stable
$\pi^*_- < 0 < \pi^*_o < \pi^*_+$	π^*_o, $q^*=0$ globally stable	π^*_+, q^*_+ π^*_o, $q^*=0$ both locally stable	π^*_+, $\hat{q}^*_+$ globally stable	π^*_+, $\hat{q}^*_+$ π^*_-, $\hat{q}^*_-$ both locally stable
$0 < \pi^*_- < \pi^*_o < \pi^*_+$	π^*_o, $q^*=0$ globally stable	π^*_+, $\hat{q}^*_+$ π^*_o, $q^*=0$ both locally stable	π^*_+, $\hat{q}^*_+$ globally stable	π^*_+, $\hat{q}^*_+$ globally stable

From the phase diagrams we can make statements about stability properties in most cases. In many cases we will have two steady-states, which both are stable and either of which will be reached depending on the starting point. In two cases, however, the phase diagram does not help us, namely when $\pi^*_- < -\frac{\delta_q}{\alpha_q} < \pi^*_o < 0$. Then we will get the picture of Fig. A.8 (where $\pi^*_+ < 0$).

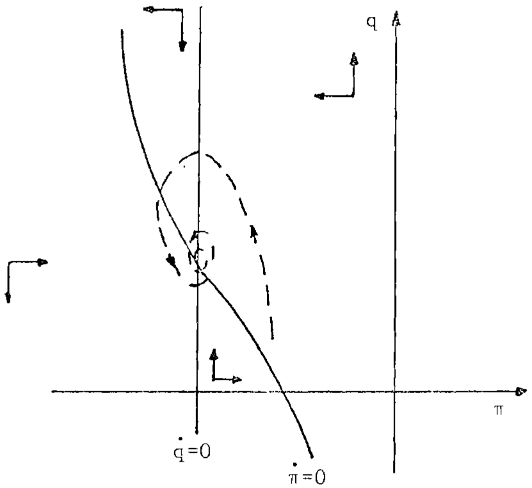


Figure A.8 $\quad \pi^*_- < -\frac{\delta_q}{\alpha_q} < \pi^*_o < \pi^*_+ < 0$

As the figure is drawn the steady-state $\pi = -\delta_q/\alpha_q$ is stable. But judging from the phase-diagram it could as well be unstable. It is also possible that it is locally stable around steady-state but unstable if we start further away from steady-state. The local stability properties can be seen by linearizing around steady-state:

$$\frac{\partial \dot{q}}{\partial q} = \alpha_q \pi + \delta_q = 0$$

$$\frac{\partial \dot{q}}{\partial \pi} = \alpha_q q > 0$$

$$\frac{\partial \dot{\pi}}{\partial q} < 0 \quad \text{(from (A.63)}$$

$$\frac{\partial \dot{\pi}}{\partial \pi} = (\pi+1)(\alpha_c+\alpha_q\gamma)[-1 + \frac{\eta q}{\zeta}(\alpha_c+\alpha_q\gamma)(\pi_+^*-\pi)(\pi-\pi_-^*)]$$

And the characteristic equation reads

$$\lambda^2 + (\pi+1)(\alpha_c+\alpha_q\gamma)[1 - \frac{\eta q}{\zeta}(\alpha_c+\alpha_q\gamma)(\pi_+^*-\pi)(\pi-\pi_-^*)]\lambda + (+) = 0$$

The corresponding differential equations are locally stable around steady-state if the real parts of both roots of this equation are negative, i.e. if

$$1 - \frac{\eta q}{\zeta}(\alpha_c+\alpha_q\gamma)(\pi_+^*-\pi)(\pi-\pi_-^*) > 0$$

where $\pi = -\frac{\delta_q}{\alpha_q}$ and q is given from (A.61). From this we see that the steady-state will be locally stable if

(i) it is sufficiently close to π_o^*, $q = 0$

(ii) η is close to zero, in which case the change of curvature of the logistic cost reduction equation around zero will be less pronounced.

CONSISTENT EXPECTATIONS (p. 166)

$$\hat{c}_t = \alpha_c \int_t^\infty \pi_\tau \, e^{\rho(t-\tau)} \, d\tau - \delta_c \tag{A.65}$$

$$\hat{q}_t = \alpha_q \int_t^\infty \pi_\tau \, e^{\rho(t-\tau)} \, d\tau + \delta_q \tag{A.66}$$

$$\hat{p}_t = -\gamma \hat{q}_t + \delta_p \tag{A.67}$$

This leads to

$$\dot{\pi}_t = (\pi_t + 1)\left[-(\alpha_c + \alpha_q\gamma) \int_t^\infty \pi_\tau \, e^{\rho(t-\tau)} \, d\tau + \delta_c + \delta_p - \delta_q\gamma\right] \tag{A.68}$$

$$\ddot{\pi}_t = -(\pi_t + 1)(\alpha_c + \alpha_q\gamma)\left(\rho \int_t^\infty \pi_\tau \, e^{\rho(t-\tau)} \, d\tau - \pi_t\right) + \frac{\dot{\pi}_t}{\pi_t + 1} \tag{A.69}$$

And substituting from (A.68) into (A.69) we get

$$\ddot{\pi}_t = \pi_t(\pi_t + 1)(\alpha_c + \alpha_q\gamma) + \rho\left[\dot{\pi}_t - (\pi_t + 1)(\delta_c + \delta_p - \delta_q\gamma)\right] + \frac{\dot{\pi}_t}{\pi_t + 1} \tag{A.70}$$

$\ddot{\pi} = 0$ gives

$$\dot{\pi} = (\pi + 1)^2 \frac{\rho(\delta_c + \delta_p - \delta_q\gamma) - (\alpha_c + \alpha_q\gamma)\pi}{\rho(\pi + 1) + 1} \tag{A.71}$$

And from $\dot{\pi} = \ddot{\pi} = 0$ we have the ordinary steady-state values

$$\pi^* = \frac{\rho(\delta_c + \delta_p - \delta_q \gamma)}{(\alpha_c + \alpha_q \gamma)}$$

$$\pi^{**} = -1$$

From (A.70) and (A.71) we have

$$\dot{\pi}\Big|_{\ddot{\pi} = 0} (-1 < \pi < \pi^*) > 0$$

$$\dot{\pi}\Big|_{\ddot{\pi} = 0} (\pi > \pi^*) < 0$$

$$\frac{\partial \ddot{\pi}}{\partial \dot{\pi}} = \rho + \frac{1}{\pi + 1} > 0$$

This gives us the phase-diagram of Fig. A.9 which shows that $\pi = \pi^*$ is a saddle-point.

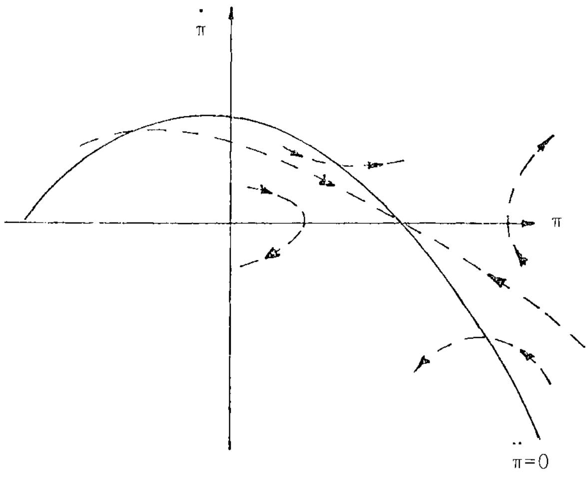


Figure A.9

B. STATISTICAL DATA, SOURCES AND DEFINITIONS

INDUSTRY RATES OF RETURN

The calculation of rates of return for Swedish industries used in Chapters 2 and 6 is based on the Swedish Official Statistics on Enterprises (SOS: Företagens intäkter, kostnader och vinster 1951--1964, SOS: Företagen 1965-1976). This series was started in 1951 and is based on earnings statements from companies. It covers all companies with more than 50 employees and, after 1969, a sample of smaller ones.

The rate of return measure we present in Tables B1 and B2 is a net rate of return on total assets employed. It is defined as the result after depreciation allowances and financial income (lines 8 + 9 + 10 + 11 + + 12 of the income statement in SOS: Företagen) divided by the balance-sheet total (line 1 of the balance-sheet).

A special problem is created by the fact that only income statements and no balance-sheets are presented in the statistics up until 1964. The problem of obtaining capital stock measures for the years before 1964 was solved in the following rough way. For each of the four years 1965-1968 the ratio between the depreciation allowances made and the balance-sheet total was calculated. This ratio was averaged over this period, thereby obtaining an index of the rate of depreciation of the capital stock for each industry. This index was then applied to the depreciation allowances actually made for the years 1951-1964. Depreciation allowances tend to vary quite widely from one year to the other due to tax considerations. It was therefore found necessary to average the true allowances made over a five-year period. To obtain the capital stock of year t, the average of the depreciation allowances made from t-2 up until t+2 was then multiplied by the index for the depreciation rate for the period 1965-1968. The inaccuracy of this procedure is obvious. It adds considerably to all other problems deriving from basing rate of return measures on companies' financial statements.

A few minor industries have been left out because their size has been drastically changed from one year to the other due to reclassification between industries. Reclassifications create problems in some other industries as well, especially due to the five-year averaging of depreciation allowances. With one exception (see foot-note 2 of Table B1) we have disregarded this problem.

In 1969 the system for classification of industries was changed. It does not seem meaningful to try to link the series; this can be done for some industries but is impossible for a large number. Table B2 presents rate of return figures for 1969-1976 according to the new classification system (ISIC). Unfortunately it is difficult to interpret a comparison of averages and standard deviations between the two periods.

Since the figures are based on earnings statements from companies they are almost certainly subject to measurement errors. It is very difficult to correct for this. One fairly obvious example of a measurement error is the low rate of return found for the slaughtering and dairy industries. These are probably to some extent due to internal pricing within the farmers' cooperatives.

Table B1: Net rate of return on total assets employed (%), 1953 - 1968

	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	Yearly average 1953 - 1968
Iron, steel and other metal mills	4.2	2.9	13.5	11.1	9.5	8.2	10.4	10.0	7.3	1/	1.8	2.8	2.0	2.5	4.2	3.8	6.3
Manufacture of hardware	11.7	12.6	13.5	12.8	12.9	9.1	10.4	11.0	10.1	9.7	9.4	10.4	9.4	7.8	7.2	7.3	10.3
Manufacture of non-ferrous metal products	12.5	11.3	14.5	13.3	14.0	13.3	11.0	12.0	10.2	9.3	7.9	7.7	9.9	9.2	7.6	8.3	10.8
Shipyards and boat building	3.1	3.8	4.5	4.1	5.5	6.2	5.5	6.0	4.6	3.7	2.4	1.2	3.7	2.5	2.9	4.1	4.0
Manufacture of electrical machinery	12.1	13.1	12.7	9.5	11.3	10.6	10.3	10.3	8.6	8.9	9.6	10.4	9.7	9.5	8.9	9.2	10.3
Not specified metal industries	16.4	17.5	19.2	15.7	11.3	15.6	22.9	14.2	12.7	12.5	11.0	11.8	11.1	10.3	10.4	9.7	13.9
Manufacture of concrete prod.	10.4	10.1	7.4	7.7	9.0	13.7	8.3	8.1	8.3	8.2	6.4	6.6	7.0	4.4	3.7	4.4	7.7
Manufacture of bricks and tiles	10.9	10.5	13.4	9.6	10.8	10.2	9.0	6.3	6.8	4.3	8.5	8.5	3.2	-1.2	-0.2	1.4	7.0
Manufacture of china, pottery and earthenware	4.6	7.0	6.6	5.3	6.5	5.3	8.0	10.5	3.1 2/	5.4	4.4	3.4	6.1	2.3	5.6	8.3	5.8
Manufacture of glass and glass-products	5.8	7.0	9.6	2.5	11.7	3.1	3.7	4.2	7.0 2/	8.8	4.6	5.6	5.3	5.4	4.0	6.7	5.9
Non specified non-metallic industry	5.3	6.3	5.2	7.2	10.7	11.0	16.8	15.3	15.4 2/	13.8	11.1	10.7	8.1	5.5	6.2	5.2	9.6
Saw mills and planning mills	3.2	6.0	9.7	4.4	6.1	9.7	5.4	3.3	4.1	3.5	3.2	3.0	4.9	3.7	3.6	4.5	4.9
Manufacture of furniture	11.0	9.0	6.6	7.3	8.7	9.4	6.1	7.7	7.9	8.1	6.1	5.2	5.5	7.4	8.3	6.3	7.5
Wood pulp mills	9.6	10.8	11.5	8.5	5.1	4.1	5.4	6.2	2.7	1.3	2.6	5.4	5.3	4.5	4.7	4.5	5.8
Paper mills	9.3	9.5	7.9	6.0	9.5	5.7	6.1	5.5	4.3	4.2	3.6	5.4	4.6	5.0	4.5	4.1	6.0
Other pulp and paper industries	6.0	12.2	13.5	11.4	8.8	6.4	5.8	9.6	2.9	2.5	5.4	4.6	7.7	4.7	3.5	4.4	6.8

Printing of newspapers	10.0	10.1	5.8	0.7	4.1	9.1	10.1	8.4	3.4	2.8	1.5	1.5	3.2	4.3	9.9	10.3	6.0
Bakeries	6.8	9.0	6.1	4.8	5.3	6.7	4.3	8.2	11.2	9.5	9.6	9.4	8.8	8.6	9.9	11.3	8.1
Manufacture of chocolate and sugar confection.	6.2	5.5	3.9	3.9	4.7	2.9	2.5	3.1	2.9	1.6	6.0	5.4	4.9	5.1	5.7	4.7	4.3
Manufacture of dairy products	5.8	3.2	3.4	2.7	2.8	2.5	3.7	2.3	2.2	2.3	2.2	2.3	2.8	2.3	2.3	1.7	2.8
Slaughtering and manufacture of meat products	5.2	3.3	4.3	4.5	5.1	2.9	2.5	2.4	3.1	2.5	2.9	3.6	3.5	4.0	4.6	3.2	3.6
Breweries and soft drink manufact.	11.0	10.9	10.7	8.1	9.5	9.8	9.6	7.2	8.7	11.4	9.7	10.4	9.2	11.7	9.6	11.2	9.9
Other food ind.	12.2	15.8	14.7	10.9	16.4	13.4	13.8	17.4	11.6	8.9	9.4	10.6	11.3	8.2	9.6	9.5	12.1
Wool industry	6.3	7.3	8.6	8.4	9.7	5.0	8.7	11.7	9.3	6.9	8.3	7.9	8.2	10.5	9.6	8.8	8.5
Cotton industry	9.9	7.6	6.6	9.8	13.8	6.4	8.2	9.6	8.7	3.1	3.4	4.6	7.0	3.5	7.7	3.3	7.1
Knitting mills	5.4	4.8	7.6	2.7	7.5	7.4	9.7	11.3	14.2	11.3	9.7	7.7	10.5	7.6	9.6	7.5	8.4
Manufacture of wearing apparel	7.7	7.5	6.7	6.4	7.4	9.0	7.7	9.8	9.3	7.9	7.9	6.3	5.6	6.2	7.2	5.6	7.4
Manufacture of footwear	8.0	7.6	7.5	6.7	6.9	7.2	7.2	3.3	4.8	5.8	5.3	3.2	3.3	3.0	2.0	2.9	5.3
Manufacture of rubber products	22.2	20.9	14.4	14.0	12.7	13.0	13.5	9.2	9.2	7.9	7.0	9.3	6.5	5.2	4.8	5.9	11.0
Unweighted average	8.7	9.1	9.3	7.6	8.9	8.2	8.5	8.4	7.4	6.7	6.2	6.4	6.5	5.6	6.1	6.1	7.5
Standard deviation	4.2	4.3	4.1	3.8	3.4	3.6	4.4	3.8	3.7	3.6	3.0	3.1	2.7	3.0	2.9	2.8	2.7
The whole industry	11.1	10.8	10.7	9.7	10.1	9.0	7.9	8.4	7.4	6.2	5.9	6.7	7.6	6.2	6.6	7.0	

1/ This year the metal mills were presented jointly with ore mines.

2/ Between 1960 and 1961 some firms were reclassified from manufacture of china and glass respectively to non specified non-metallic industry, thereby reducing the size of the china industry by 2/3 and that of the glass industry by 20% and increasing the size of the non specified manufacturing by 25%. This made it necessary to calculate the depreciation for 1959 as the three years average 1958-60, for 1960 as the one year figure for 1960, for 1961 as the one-year figure for 1961 and for 1962 as the three years average 1961-63.

Table B2: Net rate of return on total assets employed (%), 1969 - 1976

Industry (ISIC nr) Year	1969	1970	1971	1972	1973	1974	1975	1976	Average 1969-76
Slaughtering and manufacture of meat prod. (3111)	3.3	3.4	3.5	3.7	3.1	3.5	5.0	4.8	3.8
Manufacture of dairy products (3112)	1.5	2.6	2.8	3.8	4.9	3.2	4.0	4.5	3.4
Bakeries (3117)	9.4	9.5	7.7	8.9	9.4	8.2	7.7	8.3	8.6
Manufacture of chocolate and sugar confect. (3119)	3.4	3.1	3.7	5.7	5.2	4.5	2.2	9.5	4.7
Manufacture of beverages (313)	10.2	5.7	5.0	6.8	8.9	6.7	9.2	4.8	7.2
Knitting mills (3213)	5.2	2.6	3.9	8.5	6.0	8.5	7.1	3.7	5.7
Manufacture of textiles except knitting mills (321 excl. 3213)	3.6	3.5	4.5	7.2	7.6	7.8	3.3	5.5	5.4
Manufacture of wearing apparel (322)	4.0	2.9	5.0	5.4	5.9	5.8	4.1	3.7	4.6
Tanneries and manufacture of leather products (323)	3.8	4.4	1.9	5.4	8.4	2.5	7.1	3.8	4.7
Saw mills (33111)	3.7	3.9	2.5	6.5	12.2	15.4	9.3	6.6	7.5
Manufacture of wooden houses (33112)	5.3	4.0	3.4	6.2	9.1	9.1	6.1	8.0	6.4
Manufacture of wooden furniture (332)	5.7	4.8	5.5	7.3	7.9	7.3	7.9	9.6	7.0
Wood pulp mills (34111)	6.1	8.3	5.4	3.6	8.4	19.5	11.7	6.3	8.7
Paper mills (34112)	6.5	6.9	4.2	5.0	6.4	12.5	10.6	1.6	6.7
Combined pulp and paper mills (3411 excl. 34111 and 34112)	6.2	6.7	5.2	4.2	8.1	19.1	12.6	5.9	8.5
Manufacture of packages of paper and card board (3412)	4.0	4.2	4.1	5.9	14.3	17.5	14.8	6.7	8.9
Printing industries (342)	6.7	4.3	1/	4.0	6.0	4.5	4.1	5.9	5.1 1/
Manufacture of chemicals, textiles and plastics (351)	5.1	3.4	5.6	7.2	12.2	21.8	9.0	10.3	9.3

Paint industry (3521)	6.5	5.1	5.8	6.0	8.8	11.6	9.1	8.7	7.7
Drug industry (3522)	10.3	8.7	9.2	9.6	9.0	10.0	8.1	2/	9.3 2/
Other manufacture of chemical products (352 excl. 3521 and 3522)	7.8	3.9	4.9	6.2	7.7	10.8	10.9	2/	7.5 2/
Manufacture of rubber products (355)	4.7	6.5	4.9	4.1	5.1	8.5	4.8	1.4	5.0
Manufacture of plastics products (356)	6.0	4.3	8.9	8.5	9.6	15.5	6.0	6.1	8.1
Manufacture of china and earthenware (361)	1.6	2.9	2.2	5.1	6.6	5.5	3.4	8.7	4.5
Manufacture of glass and glass products (362)	7.6	10.2	4.0	7.8	10.1	4.1	1.6	6.3	6.5
Manufacture of bricks and concrete products (369)	5.4	5.0	5.5	6.4	9.4	7.2	6.3	8.7	6.7
Iron and steel mills (37101)	6.2	5.9	4.1	2.1	8.5	11.9	6.3	-1.0	5.5
Iron and steel foundries (37102)	2.4	3.7	5.4	2.7	4.8	7.9	4.4	-3.2	3.5
Manufacture of metal products (381)	4.6	5.0	5.4	6.2	7.2	9.1	8.0	7.1	6.6
Manufacture of machinery (382)	5.0	4.1	4.6	6.2	7.8	9.3	8.8	5.8	6.5
Manufacture of electrical machinery (383)	8.1	6.4	6.5	8.4	9.9	9.4	8.7	8.2	8.2
Shipyards and boat building (3841)	3.5	4.0	4.3	3.5	4.4	4.5	3.8	5.7	4.2
Means of transport industry other than shipyards (384 excl. 3841)	10.0	5.7	5.3	9.4	9.2	7.7	6.8	7.8	7.7
Unweighted average	5.6	5.0	4.8	6.0	8.0	9.4	7.1	5.8	6.5
Standard deviation	2.3	2.0	1.7	2.0	2.4	5.0	3.1	3.0	1.8
The whole industry	5.9	5.2	4.9	5.6	7.8	10.0	6.9	5.2	

1/ The printing industry was not presented separately for 1971. The average is calculated over seven years, excluding 1971.

2/ The drug industry was not presented separately for 1976. The averages are calculated over the seven years period 1969–1975.

Table B3: Correlation Coefficients Between Rates of Return in Year t and τ, Sheltered Industries 1953 - 1968

t\τ	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968
1953	1	.89	.80	.66	.79	.90	.88	.70	.54	.55	.44	.49	.46	.22	.24	.35
1954		1	.85	.66	.84	.89	.91	.89	.74	.64	.62	.67	.70	.36	.43	.57
1955			1	.85	.92	.75	.83	.70	.63	.51	.68	.76	.54	.11	.05	.22
1956				1	.93	.68	.56	.58	.71	.62	.78	.81	.60	.18	-.08	.00
1957					1	.78	.77	.79	.70	.58	.67	.74	.66	.24	.13	.20
1958						1	.87	.80	.71	.66	.51	.54	.55	.22	.23	.35
1959							1	.84	.55	.49	.39	.46	.50	.21	.31	.43
1960								1	.79	.62	.54	.57	.73	.42	.53	.56
1961									1	.91	.86	.83	.84	.56	.45	.53
1962										1	.78	.77	.83	.74	.32	.61
1963											1	.97	.76	.48	.24	.37
1964												1	.82	.48	.23	.41
1965													1	.78	.61	.68
1966														1	.85	.81
1967															1	.93
1968																1

Comment: The following industries are classified as sheltered: manuf. of concrete prod., manuf. of bricks and tiles, manuf. of furniture, printing of newspapers, bakeries, manuf. of chocolate and sugar confectionary, manuf. of dairy prod., slaughtering and manuf. of meat prod., breweries and soft drink manuf., other food ind. and manuf. of wearing apparel.

Source: Table B1.

Table B4: Correlation Coefficients Between Rates of Return in Year t and τ, Non-Sheltered Industries 1953 - 1968

t\τ	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968
1953	1	.92	.65	.73	.59	.62	.54	.26	.29	.30	.40	.63	.41	.44	.33	.32
1954		1	.72	.73	.47	.58	.48	.25	.14	.24	.42	.63	.47	.45	.26	.37
1955			1	.80	.42	.60	.48	.29	.15	.21	.30	.48	.39	.50	.30	.42
1956				1	.59	.67	.62	.52	.25	.18	.34	.52	.39	.41	.33	.25
1957					1	.46	.44	.47	.55	.47	.45	.60	.48	.49	.46	.32
1958						1	.82	.52	.57	.58	.59	.61	.50	.45	.39	.37
1959							1	.77	.74	.73	.72	.74	.56	.50	.58	.47
1960								1	.74	.64	.72	.69	.72	.56	.74	.57
1961									1	.88	.81	.76	.67	.61	.71	.46
1962										1	.89	.80	.67	.63	.59	.63
1963											1	.90	.86	.80	.72	.70
1964												1	.81	.81	.72	.66
1965													1	.83	.85	.76
1966														1	.82	.80
1967															1	.77
1968																1

Source: Table B1.

Table B5: Correlation Coefficients Between Rates of Return in Year t and τ, Sheltered Industries 1969 - 1976.

t\τ	1969	1970	1971	1972	1973	1974	1975	1976
1969	1	.82	.60	.63	.38	.18	.33	.17
1970		1	.62	.71	.44	.21	.37	.38
1971			1	.87	.44	.53	.17	.26
1972				1	.59	.54	.35	.46
1973					1	.87	.86	.29
1974						1	.75	.19
1975							1	.14
1976								1

Comment: The following industries are classified as sheltered: 3111, 3112, 3117, 3119, 313, 322, 332, 3412, 342, 3521, 356, 369.

Source: Table B2.

Table B6: Correlation Coefficients Between Rates of Return in Year t and τ, Non-Sheltered Industries 1969 - 1976.

t\τ	1969	1970	1971	1972	1973	1974	1975	1976
1969	1	.63	.63	.55	.36	.15	.33	.32
1970		1	.45	.09	.19	.10	.10	-.00
1971			1	.32	.06	.28	.27	.15
1972				1	.48	-.12	.00	.63
1973					1	.44	.29	.56
1974						1	.73	.19
1975							1	.20
1976								1

Source: Table B2.

DATA AND DEFINITIONS FOR REGRESSIONS ACROSS FIRMS

The regression equations across a population of Swedish firms that are presented in Chapter 6 are based on data gathered for the Swedish Official Statistics on Enterprises (SOS: Företagen). Data for the largest of these firms in the mining and manufacturing industries - those with a balance-sheet total of over 20 million Swedish kronor in 1970 - were put on a data file for the study by Bertmar and Molin (1977). This data file is continuously being up-dated.

For the regression estimates presented in Chapter 6 I have employed data from this file for the period 1969-1976. The following definitions have been used. For details see Bertmar and Molin (1977) Ch. 5.

Turnover (qp): gross operating income

Total cost (qc): gross operating costs minus historic cost depreciation

Total capital (K): bank and cash in hand balances + stock and inventories + machinery and equipment + real estate + new plants under construction + financial assets.

The rate of return on capital π is defined by

$$\pi = \frac{qp - qc}{K}$$

From the population of firms were deleted observations of three types: (i) firms that had grown by take-overs, (ii) firms with a growth rate of turnover between any pair of consecutive years above 75 % or below -25 %, (iii) firms with a value of π above 40 % or below -25 %. By this procedure the number of observations was reduced by 8-16 per cent as compared with the original population. The remaining number of firms then varied between 503 (1971) and 379 (1976).

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