Lars Matthiessen

A STUDY IN FISCAL THEORY AND POLICY

Part Two. Corporate Taxation and Economic Growth

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CONTENTS OF PART TWO

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CHAPTER VI

DEPRECIATION ALLOWANCES, CAPITAL GROWTH AND THE EFFECTIVE TAX BURDEN

Summary[1]

In this article the relationship between accelerated depreciation, the rate of growth, the effective tax burden and the accumulation of concealed reserves in a firm is studied. If the decrease in the value of fixed assets caused by wear and tear and aging is smaller than the depreciation allowances the latter are called accelerated. The analysis is based on a number of simplifying assumptions. Thus for example it is assumed that the firm's net investment grows at a constant rate, that normal depreciation is made according to the declining balance method and that the price level is constant. Initial and normal depreciation of fixed capital and investment allowances as well as inventories write-downs are taken into account.

The first step in the analysis is to form an expression showing the dependence between the effective tax rate and the parameters considered in the case when the profit rate is exogenously given. The effective tax rate is of course the actual tax liability divided by the "true" profit. This expression shows that the effective tax rate will be smaller the higher are the rate of growth and the rates of depreciation. Further it appears that the effective tax rate will be higher the higher are the profit rate and the coefficient of wear and tear. If the rates of depreciation are changed the effective tax rate will, during a period of transition, converge toward a limiting value. Under certain conditions the speed of this convergence is determined by the normal rate of depreciation and the rate of growth. In the following section it is discussed how the effective tax rate is effected when the different types of depreciation allowances are combined in different ways.

After the theoretical analysis it may be of interest to study the problem in more quantitative terms. Sections 6–8 contains some numerical examples which show conceivable orders of magnitude for the effective tax rate under different "realistic" assumptions about the values of the parameters. Since the number of parameters considered is fairly large this numerical analysis is not very exhaustive. The numerical examples show among other things that the effective tax rate is rather sensitive to variations in the firm's rate of growth and profit rate and that the speed of the convergence is relatively high.

* Lecturer, Stockholm School of Economics.

[1] I am indebted to Professor Leif Johansen and Mr. Karl-Göran Mäler for many valuable suggestions and comments on earlier versions of this paper and to the Bankforskningsinstitut, which helped to finance the research for this article.

That the growing firm which uses the right to accelerated depreciation can keep its effective tax rate below the nominal tax rate is of course related to the fact that the firm is accumulating concealed reserves. Section 9 contains a discussion of what will happen to the firm's concealed reserves if the rates of depreciation, the growth rate or the profit rate change. It is shown that minor decreases in the rates of depreciation cannot bring about any (net) liquidation of the firm's concealed reserves. The same is true if the rate of growth decreases but remains non-negative.

In the first part of this paper the effective tax rate is studied under different conditions concerning the profit rate, the rate of growth, the rates of depreciation and other exogenously given parameters. The aim, thus, was not to discuss how the behavior of the firm may be influenced by the fact that accelerated depreciation is permitted. Section 10, on the other hand, contains some discussion of this very important but difficult problem. It is assumed that the profit rate (before taxes), r, other things being equal, is a function of the rate of growth, g, and that from a certain point $dr/dg < 0$. If the firm's required rate of yield and $r = r(g)$ are known, the effect on the rate of growth of an introduction (or change in an existent system) of accelerated depreciation can be demonstrated. If the firm raises its required rate of yield because of the accumulation of concealed reserves, the right to accelerated depreciation will increase the desired rate of growth less than otherwise would be the case. It is stressed that this second part of the analysis is very schematic and that a more complete analysis presupposes the introduction of a large number of relationships not considered.

Introduction

1. In two well-known articles,[1] Domar and Eisner have shown that the right to accelerated depreciation results not only in a partial tax deferral for all firms but also a continuing tax reduction for expanding firms. Domar and Eisner, however, refrained from introducing an explicit measure of the effective tax burden in their analyses. As Domar explains,

> "Ideally, we should want to know the effects of accelerated depreciation on investment decisions. Existing investment theory, however, is so inadequate that it has to be built anew for practically every purpose. Having no desire to attempt this here, we shall set ourselves a more modest assignment—to investigate the behaviour of the ratio between accelerated and normal depreciation allowances under dif-

[1] Evsey D. Domar "The Case for Accelerated Depreciation", *Quarterly Journal of Economics* 1953, p. 493–519 and Robert Eisner "Accelerated Amortization, Growth and Net Profits", *Quarterly Journal of Economics* 1952, p. 533–44. Domar's article has been republished in "Essays in the Theory of Economic Growth", New York 1957 and all references to Domar will relate to this edition.

ferent sets of conditions—in the hope that a high ratio will be conducive to investment and to development of new firms. This modesty obviates the need of specifying the exact nature of the income tax, ...".[1]

The problem has recently been treated by three Scandinavian economists: P. Nörregaard Rasmussen, Leif Johansen and K.-O. Faxén.[2] These authors all arrive at the same major conclusions as Domar and Eisner. However, while Nörregaard Rasmussen and Johansen employ largely the same analytical methods as Domar, Faxén introduces the effective tax rate directly into the analysis. In order to accomplish this certain assumptions must be made about the net profit tax and the profit rate of the firm. On the other hand, this method allows a study not only of the relation between growth and the effective tax rate but also between the latter and the profit rate. A common feature of Domar's, Eisner's, Nörregaard Rasmussen's and Johansen's analyses is that their models only (or mainly) cover fixed capital. While Faxén's model is not formally presented, as is demonstrated in the last section of the Appendix, his analysis can be applied either to inventories or to fixed capital. In the latter case, a special type of depreciation allowance must be assumed.[3]

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[1] Domar op. cit., p. 196–97.

[2] P. Nörregaard Rasmussen "En note om afskrivningar, skattepliktig indkomst og växst", *Nationalökonomisk Tidsskrift* 1962, p. 150–56, Leif Johansen, *Offentlig Ökonomikk*, Oslo 1964, p. 233–39 och K.-O. Faxén, *Skatter och ekonomisk utveckling*, printed 1963 in an S.N.S. publication under the same title. Faxén's analysis can also be found in the paper (written in cooperation with Leif Mutén) "Tax Policy and Economic Growth in Sweden" in the volume "Foreign Tax Policies and Economic Growth", 1966 published by the *National Bureau of Economic Research.* (U.S.)

[3] After this article was almost completed, my attention was drawn to an analysis of accelerated depreciation by Edgar O. Edwards. (See E. O. Edwards "Depreciation and the Maintenance of Real Capital" in J. L. Meij (ed.) *Depreciation and Replacement Policy*, Amsterdam 1961, p. 46–140, particularly p. 92–102 and p. 126–33.) Edwards investigates how after-tax profit rates are influenced by the use of different types of accelerated depreciation allowances. In other words he calculates (cf. the equation p. 94)

$$\frac{t^N(A_t - bK_t)}{S_t}$$

where the numerator is the tax saving made possible by accelerated depreciation. As is evident, this expression can be calculated without any assumptions about profitability. Edwards is somewhat laconic about the investment incentive. "Any depreciation policy which raises the after-tax profit rates of growing firms relative to less rapidly growing (or declining) firms is considered to provide a positive incentive in favour of growth. The size of the differential created is treated as an index of the size of the resulting incentive." (p. 96).

The aim of this article is to present a formal analysis of the relation between depreciation allowances, capital growth and the effective tax burden. Different types of depreciation allowances for fixed capital and inventories will be taken into account. In order to illustrate the problem quantitatively, a series of numerical examples will be presented. Finally, the question of whether accelerated depreciation allowances are growth-promoting will be touched upon.

The article is organised in the following manner. The second section sets out the simplifying assumptions upon which the rest of the analysis is based. In the third section, the effective tax rate is specified and the different types of depreciation allowances treated in the analysis are presented. In the fourth section, it is shown how the effective tax burden of a growing firm during the transition period and during subsequent periods is dependent on among other things, the firm's rate of growth, rate of profit and rate of depreciation. In the fifth section different types of depreciation allowances are compared. Sections 6 and 7 contain numerical illustrations. From these it appears that the effective tax rate formally can be negative under certain assumptions, and Section 8 discusses how these assumptions can be modified to avoid that result. Depreciation at an accelerated rate involves the accumulation of concealed reserves. Section 9 discusses the effect of changes in the rates of depreciation allowance, rate of growth and rate of profit on the firm's concealed reserves. In Section 10, the assumption of an exogeneously determined profit rate is abandoned in order to facilitate discussion of the significance of the depreciation allowance for the firm's investment decision. Finally, Section 11 contains some concluding remarks about the analysis.

In order to facilitate reading, certain derivations necessary to the analysis have been placed in an Appendix. These include the determination of the book value of fixed capital and the effective tax rate in the more general case when the rates of growth of net investment and the stock of capital are not necessarily equal. The Appendix is followed by a list of the most important symbols used in the analysis. In many cases, however, the symbols are explained when first introduced in the text.

Assumptions and Definitions

2. The analysis is based on the following assumptions:

1° The firm's net investment in fixed capital, I_t^F, and in inventories, I_t^L, increases at a constant rate, g.

2° Fixed capital, K_t, constitutes a constant proportion, k, of the total stock of capital, S_t.

3° The actual depreciation in the value of fixed assets, as a result of wear and tear and aging constitutes in every period a constant proportion, b, of the stock of capital, K_t.

4° Normal (or current) depreciation allowances constitute in every period a constant proportion, d, of the fixed capital's value at the beginning of the period (the declining balance method).

5° Wear and tear and aging of inventories is assumed to be insignificant and these are therefore not subject to depreciation.

6° For the sake of simplicity the right to write down inventories is assumed to apply to inventory investment (net) but not to already acquired inventories.

7° It is assumed that the firm actually depreciates to the extent that the law allows with respect to the firm's stock of capital, previous depreciation and new investment.

8° The corporate tax is proportional. Tax loss offset and carry over are not allowed.

9° The firm's profit rate, r, is a constant and independent of g.

10° The price level is constant.

Several comments should be added to these assumptions. Firstly, assumptions 7° and 9° are modified in Section 8 and Section 10 respectively. Assumption 2° means, of course, that the rates of growth of fixed assets and inventories are equal but not necessarily constant. Let us denote the rate of growth of fixed assets by g_t and assume that $g \neq 0$. By dividing (54) by (55) we obtain the following relation between g_t, g_0 and g.

$$g_t = \frac{g g_0 (1+g)^t}{g_0(1+g)^t + g - g_0} \tag{1}$$

From (1) it immediately appears that $g_t = g$ if $g = g_0$. Further, we can see that g_t converges towards g (zero) when $g > 0$ $(g < 0)$ as $t \to \infty$.[1] In what follows, when we assume that g is positive we shall also assume that $g = g_0$.

The assumption of a fixed proportion between the quantity of fixed assets and inventories (2°) has been chosen because of the rather long term character of part of the analysis. Had one alternatively assumed that net investment in fixed assets increased at a faster rate than investment in inventories, inventories as a proportion of the total stock of capital would obviously approach zero $(k \to 1)$. In the long run, one should thus have disregarded the existence of inventories. However, there are no difficulties in principle in working with different rates of increase for fixed asset investment and inventory investment.

Assumption 3° involves, inter alia, that we disregard the relation between the assets economic longevity and the type of tax allowance.[2]

To assume that normal depreciation can be carried out only according to the declining balance method (so that depreciation constitutes a geometrically decreasing proportion of the asset's acquisition cost), is somewhat arbitrary. If one had instead assumed the use of the straight line method or other appropriate methods—at the same time as one assume another development over time of capital wear and tear than 3°—the analysis would hardly have been affected in essentials, assuming that depreciation had actually been accelerated. But this would make the analysis more complicated.[3] For the sake of simplicity, we assume that an extra depreciation allowance is not given in conjunction with the retirement of the asset.[4] This means in principle that the firm never discontinues normal depreciation of its acquired assets.

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[1] If the volume of investment is constant and hence $g = 0$, instead of (1) we obtain:

$$g_t = \frac{g_0}{1 + g_0 t}$$

[2] For a penetrating discussion of this relation see Sven-Erik Johansson, *Skatt–investering–värdering*, Meddelanden from FFI, Stockholm 1961, Ch. 5.

[3] In his article, Domar analyses the straight line method as well as declining balance depreciation. Other methods are the "Sum-of-the-year's-digits" and the annuity method used in the U.S.A. For a discussion of all four methods, see E. Cary, Brown, "The New Depreciation Policy under the Income Tax: An Economic Analysis", *National Tax Journal* 1955, pp. 81–98.

[4] Domar also discusses the case of final allowances op. cit., p. 221.

The assumption that the corporation tax is proportional is practical to work with and moreover corresponds to the actual circumstances in a number of countries.[1] According to the prevailing Swedish rules, the municipal tax paid in a given year is deductible for national tax purposes in the following year. Under such a system, the total corporation tax, T_t, will be proportional only if (municipal) taxable profits, W_t, change at a constant rate or remains constant.[2]

The rate of profit before tax is equal to V_t/S_t. In the economic sense, profits are those available for taxation, dividends, new investment etc. after taking into account depreciation so that the net worth of the firm remains unchanged during the period. In other words, V_t is the profit retained when depreciation changes correspond to the decrease in the value of fixed assets as a result of wear and tear and aging. The assumption of a constant price level avoids any need to differentiate between nominal and real net worth in the determination of profits in the economic sense.[3] In this way one also evades "... the now popular subject of the alleged deficiency of normal depreciation charges in inflationary periods to finance replacements. ..."[4]

3. The taxable profits of a firm are equal to profits before depreciation less allowable depreciation charges i.e.

$$W_t = V_t^G - A_t \tag{2}$$

Since

$$V_t^G = V_t + bK_t \tag{3}$$

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[1] A comparison of the tax laws of different countries falls outside the framework of this article. Reference can be made to "The Role of Direct and Indirect Taxes in the Federal Revenue System", (Published by the *National Bureau of Economic Research* and the *Brooking Institute* 1964) which contains a description of corporation tax in the U.S.A., U.K., France, Italy and West Germany (particularly, pp. 229-30, 259-65).

[2] If we denote the national and municipal corporation tax rates respectively by t_S and t_L and the rate of increase of taxable profits by g_t^W, the total corporate tax rate t_t^N will be

$$t_t^N = t_L + t_S\left(1 - \frac{t_L}{1+g_t^W}\right)$$

t_t^N and g_t^W thus vary together but the covariation is relatively weak.

[3] Net worth is the value of the stock of capital valued at current ("objective") market prices. It is thus not the same as the subjective capital value which also includes goodwill. See e.g. Sven-Erik Johansson op. cit., pp. 50–52.

[4] Domar op. cit., p. 198.

(2) can be written

$$W_t = V_t + bK_t - A_t \tag{4}$$

The corporation tax is calculated as $T_t = t^N W_t$ (where t^N is the nominal tax rate) and the effective tax rate t_t^E will therefore be

$$t_t^E = t^N \left[1 - \frac{A_t - bK_t}{V_t}\right] \tag{5}$$

or

$$t_t^E = t^N(1 - B_t) \tag{6}$$

where

$$B_t = \frac{A_t - bK_t}{V_t} \tag{7}$$

If the depreciation charges correspond to asset wear and tear (i.e. $A_t = bK_t$) obviously $B_t = 0$, $W_t = V_t$ and $t_t^E = t^N$. If the firm, however, uses its right to depreciate at a rate exceeding asset wear and tear, the concealed reserves will increase in period t by $V_t - W_t = A_t - bK_t$ and this part of (the actual) profits will not be taxed in period t. And the reduction of the tax burden which the firm can achieve in period t through accelerated depreciation will be $t^N B_t$.

In the following discussion, the task will be to investigate how different types of depreciation allowances influence B_t and thereby t_t^E. In order to refine the analysis, we shall assume that the depreciation allowance at the start (and before) applies only to fixed assets and corresponds to asset wear and tear. From period zero it is assumed that a new and more advantageous type of depreciation allowance is enacted. The analysis, therefore, concerns the effects on the tax burden of the *introduction* of an accelerated depreciation allowance. Section 9 discusses the effect of a reduction in the extent of the accelerated depreciation allowance.

The following alternative types of depreciation allowances will be treated:

I	$A_t = dC_t + aI_t^G + hI_t^L$	(8)
II	$A_t = dC_t + aI_t^G$	(9)
III	$A_t = dC_t + hI_t^L$	(10)
IV	$A_t = dC_t$	(11)
V	$A_t = dC_t + I_t^G$	(12)
VI	$A_t = bK_t + aI_t^N$	(13)

As regards the meaning of the symbols, reference can be made to the list of symbols after the Appendix. The book value of the fixed assets is determined in the following way:

$$C_t = C_{t-1} + I^G_{t-1} - A^K_{t-1} \tag{14}$$

where A^K_t denotes the write down of fixed assets. C_t is equal to K_t if A^K_t corresponds to the continuous decrease in value (because of wear and tear) since the acquisition of the fixed asset. With accelerated depreciation allowances, however, $C_t < K_t$. In Cases I–V an accelerated depreciation allowance will entail diminished depreciation possibilities at a later date. In Case VI, aI^N_t are investment allowances which do not reduce normal depreciation allowances. One could therefore say that the total amount that can be depreciated on a given capital asset can amount to $100(1+a)\%$ of its investment cost in Case VI and 100 % in the first five cases.

The Effective Tax Rate in the Case When the Profit Rate is Exogenously Given

4. We can now more exactly determine and discuss the effective tax rate t^E_t. For the moment we assume that $g = g_0 > 0$. As has been mentioned, this means that both the stock of capital and net investment increase at a constant rate g. Furthermore, the rate of profit (before taxes) is considered as exogenously given. Finally, we start formally with Case I, i.e. where initial allowances and inventory as well as normal depreciation allowances are permitted. The analysis, however, also covers Cases II–V, since these may be considered special cases of Case I.

The assumption that the write down of fixed assets at the initial point (and before that) corresponds to the actual wear and tear means that $C_0/K_0 = x = 1$. If, in equation (75) in the Appendix, we set $g = g_0$ and $x = 1$ we get

$$B_t = \frac{\dfrac{d-b+a(g+b)}{g+d}\left[g+d\left(\dfrac{1-d}{1+g}\right)^t\right]k + gh(1-k)}{r} \tag{15}$$

A sufficient (but not a necessary) condition for $B_t > 0$ and thus for $t^E_t < t^N$ is $d > b$. From (6) and (15) it follows that we can express the ratio t^E_t/t^N thus

$$\frac{t_t^E}{t^N} = \alpha - \beta\left(\frac{1-d}{1+g}\right)^t \tag{16}$$

α is the limiting value t^E/t^N for t_t^E/t^N and β is the interval in which the ratio varies, i.e. $(t^E - t_0^E)/t^N$. Concerning t_0^E/t^N (or α–β) we have

$$\frac{t_0^E}{t^N} = 1 - \frac{(d-b+ag+ab)\,k + gh(1-k)}{r} \tag{17}$$

When t increases, t_t^E/t^N increases monotonically toward the limit

$$\frac{t^E}{t^N} = 1 - \frac{g}{r}\left[\frac{d-b+a(g+b)}{g+d}k + h(1-k)\right] \tag{18}$$

t^E is independent of g_0 but becomes lower the swifter is the rate of growth g. Naturally, it is this factor which is the real point in Domar's and Eisner's analyses of depreciation allowances even if these authors do not explicitly indicate a relation between t^E and g. t^E will be lower the greater are the rates of depreciation d, a and h, but higher the greater are the wear and tear coefficient b and the profit rate r.

The sensitivity of t^E to changes in g and r is indicated by the respective derivatives.

$$\frac{dt^E}{dg} = -\frac{t^N}{r}\left[\frac{d(d-b+ag+ab)+ag(g+d)}{(g+d)^2}k + h(1-k)\right] < 0 \tag{19}$$

$$\frac{dt^E}{dr} = \frac{t^N g}{r^2}\left[\frac{d-b+a(g+b)}{g+d}k + h(1-k)\right] > 0 \tag{20}$$

Concerning the covariation between t^E and k we have

$$\frac{dt^E}{dk} \gtreqless 0 \text{ as } h \gtreqless \frac{d-b+a(g+b)}{g+d} \tag{21}$$

We have seen that t_t^E is an increasing function of t and that it converges. That t_t^E doesn't equal its limiting value as soon as the new depreciation rules are introduced is due to the fact that during a transitional period (which is infinitely long, in principle, in our model) the firm still has assets which have been depreciated under the old rules. When the assets held at the start of period 0 have been totally depreciated, the transition period is over and $t_t^E = t^E$. It can be of interest to see the rate at which convergence occurs. A measure for this is given in equation (22).

$$\gamma_t = \sum_{i=1}^{t} \frac{t_i^E - t_{i-1}^E}{t^E - t_0^E} = 1 - \left(\frac{1-d}{1+g}\right)^t \tag{22}$$

γ_t denotes the increase in the effective tax rate from period 0 to period t, measured as a fraction of the largest possible increase $t^E - t_0^E$. According to the assumptions used here, this fraction will depend only on d and g, and the convergence will be quicker the greater, are g and d.

5. In this section we will compare the different types of depreciation allowances mentioned in Section 3. The limiting value of the effective tax rate in Case I (t_I^E) is immediately clear from (18). t_{II}^E, t_{III}^E, t_{IV}^E and t_V^E can be derived by subsituting the values for depreciation rates d, a and h, which apply in these four cases in (18). Finally, t_{VI}^E is clear from (84). One could ask, for example, whether the right to initial allowances and normal depreciation allowances, other things being equal, leads to a lower effective tax burden than that which accrues from normal and inventories depreciation allowances. Naturally enough, this depends on the composition of the stock of capital and the magnitude of the depreciation rates. But also the rate of asset wear and the rate of growth are of significance. To be more exact, the following conditions apply:

$$t_{III}^E \gtreqless t_{II}^E \quad \text{to the extent that} \quad \frac{1-k}{k} \lesseqgtr a\frac{c_1}{h} \tag{23}$$

where

$$c_1 = \frac{g+b}{g+d} \tag{24}$$

The greater g is the more probable it is, ceteris paribus, that t_{II}^E will be less than t_{III}^E. Continuing with the comparisons we obtain the conditions.

$$t_I^E \gtreqless t_V^E \quad \text{as} \quad \frac{1-k}{k} \lesseqgtr (1-a)\frac{c_1}{h} \tag{25}$$

$$t_{III}^E \gtreqless t_V^E \quad \text{as} \quad \frac{1-k}{k} \lesseqgtr \frac{c_1}{h} \tag{26}$$

$$t_{III}^E \gtreqless t_{VI}^E \quad \text{as} \quad \frac{1-k}{k} \lesseqgtr \frac{c_1-(1-a)}{h} \tag{27}$$

$$t_{IV}^E \gtreqless t_{VI}^E \quad \text{as} \quad c_1 \gtreqless 1-a \tag{28}$$

Table 1. *Ten combinations of parameter values (in %).*

	k	b	d	a	h	g	r
A	80	20	30	30	60	10	12.5
B	—	—	—	—	—	5	—
C	—	—	—	—	—	2	—
D	—	—	—	—	—	—	16 2/3
E	—	—	—	—	—	—	20.0
F	40	—	—	—	—	—	—
G	—	25	—	—	—	—	—
H	—	—	50	—	—	—	—
I	—	—	—	50	—	—	—
J	—	—	—	—	20	—	—

The (ten) comparisons of t^E in different cases which can be made in addition to those mentioned above give obvious results which are independent of the values of the parameters chosen. Completely free depreciation (Case V) gives the lowest tax burden of the six cases if $(1-k)/k < c_1(1-a)/h$. As has earlier been mentioned, total possible depreciation exceeds the investment cost of a capital asset in Case VI (but not in the other cases). In spite of this, Case VI gives a greater tax burden for firms than Cases I, II and V. If $c_1 < 1-a$, Case VI will be quite simply the case which gives the greatest effective tax rate.

Some Numerical Examples

6. It would be suitable to illustrate the analysis performed in the preceding section by some numerical examples. Since a fairly large number of parameters are included in the analysis, a few numerical examples can only vaguely suggest conceivable orders of magnitude of the effective tax burden. Table 1 presents a number of combinations of "realistic" values for the relevant parameters.

Wherever in Table 1 no parameter value is given, the same value as in combination *A* applies. This combination, which can be used as a reference combination, approximates current Swedish conditions as regards rates of depreciation *d*, *a* and *h*.[1]

[1] According to the so called "completion rule" machinery and equipment need not be evaluated at the end of the accounting year more highly than the amount which would be obtained if the annual depreciation allowance had consistently amounted to 20 % of the original acquisition cost. As is known, Swedish firms can also choose a form of linear depreciation where the depreciation percentage depends on the calculated durability of the assets.

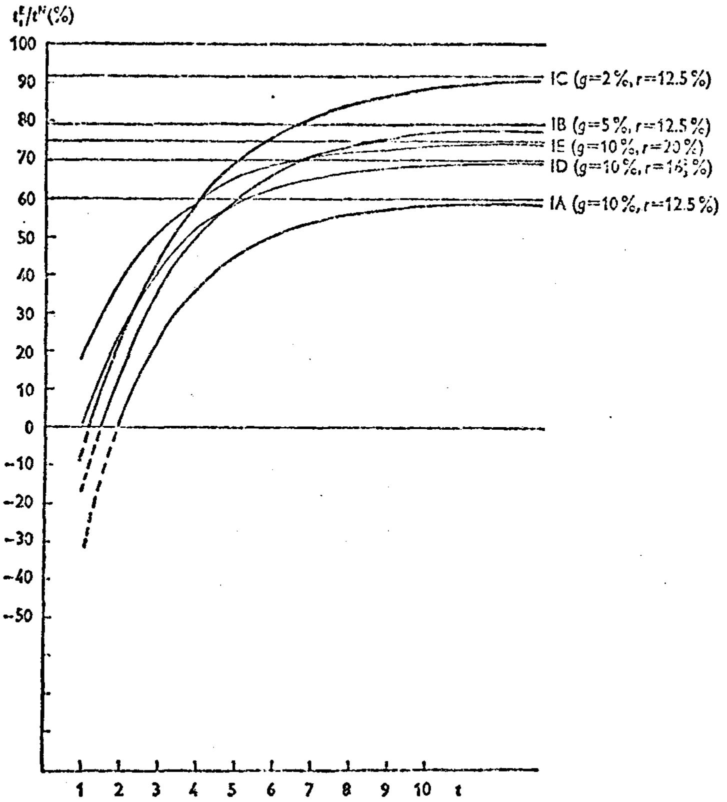

Fig. 1.

In order to demonstrate how variations in g and r respectively can affect the tax burden, we substitute in (15), each of the parameter combinations A, B, C, D, and E and then calculate t_t^E/t^N. We then obtain the functions (29)–(33) below.

Case	t_t^E/t^N (%)	Equation
I A	$60-91.2\,(\frac{7}{11})^t$	(29)
I B	$79.2-96\,(\frac{2}{3})^t$	(30)
I C	$91.44-99.6\,(\frac{35}{51})^t$	(31)
I D	$70-68.4\,(\frac{7}{11})^t$	(32)
I E	$75-57\,(\frac{7}{11})^t$	(33)
II A	$69.6-91.2\,(\frac{7}{11})^t$	(34)
III A	$74.4-48\,(\frac{7}{11})^t$	(35)
IV A	$84-48\,(\frac{7}{11})^t$	(36)
V A	$36-192\,(\frac{7}{11})^t$	(37)

Table 2. *Numerical Examples showing the Effect of the Introduction of Accelerated Depreciation Allowances on t_t^E/t^N.*

	Case											
t	IA (1)	IB (2)	IC (3)	ID (4)	IE (5)	IIA (6)	IIIA (7)	IVA (8)	VA (9)	IAa (10)	IAb (11)	IAc (12)
0	−31.2	−16.8	−8.2	1.6	18.0	−21.6	26.4	36.0	−156	−31.2	−31.2	0
1	2.0	13.2	23.0	26.5	38.7	11.6	43.9	53.5	−86.2	7.2	4.6	0
2	23.1	36.5	44.5	42.3	51.9	32.7	55.0	61.6	−41.8	32.2	27.8	21.8
3	36.5	50.8	59.2	52.4	60.3	46.1	62.0	71.6	−13.5	49.0	43.3	35.7
4	45.0	60.2	69.3	58.8	65.7	54.6	66.5	76.1	4.5	60.7	53.8	44.5
5	50.5	66.6	76.3	62.9	69.0	60.1	69.4	79.0	16.0	69.0	61.0	50.2
6	53.9	70.8	81.0	65.5	71.2	63.5	71.2	80.8	23.2	75.2	66.2	53.7
7	56.1	73.6	84.3	67.1	72.6	65.7	72.4	82.0	27.9	79.7	70.0	56.0
8	57.5	75.5	86.5	68.2	73.5	67.1	73.1	82.7	30.8	83.1	72.9	57.5
9	58.4	76.7	88.0	68.8	74.0	68.0	73.6	83.2	32.7	85.8	75.1	58.4
∞	60.0	79.2	91.4	70.0	75.0	69.6	74.4	84.0	36.0	100.0	100.0	60.0
Equation	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(43)

Note: The three last columns are variations of the parameter combination A, the variations being: in Case IAa, $g = -10\,\%$ and $g_0 = 10\,\%$; in Case IAb, $g = 0$ and $g_0 = 10\,\%$; in Case IAc a variable rate of depreciation, a_t, has been used (cf. Section 8).

Columns (1)–(5) of Table 2 indicate the values we obtain for t_t^E/t^N in the five cases when t varies, while the corresponding curves are plotted in Figure 1.[1] Figure 1 demonstrates rather clearly that the effective tax rate is noticeably sensitive to variations in the rate of increase of net investment and variations in the firm's profit rate. Further, the convergence can be seen to be relatively quick. In Case IB, for example, $(1-d)/(1+g)$ equals $\frac{2}{3}$ and already in period 3, γ_t (compare equation (22)) amounts to approximately 70 % while $\gamma_4 =$ 80 %, $\gamma_6 = 91$ % and $\gamma_9 = 97$ %. In Case IC, the convergence is somewhat slower and in the other cases faster than in Case IB.

We shall now provide examples of how the tax burden can vary with the types of depreciation allowances permitted. We insert parameter combination A in each of the four variations of (15) which are applicable to Cases II–V and then calculate t_t^E/t^N. In this way we obtain equations (34)–(37). These generate the values given in Columns (6)–

[1] If one wishes to obtain t_t^E explicitly in the numerical example instead of t_t^E/t^N a numerical assumption for t^N is required. Under current Swedish rules, t^N is about 50 % ($t_S = 40$ % and t_L averages approximately 16 %, compare note 2, p. 214).

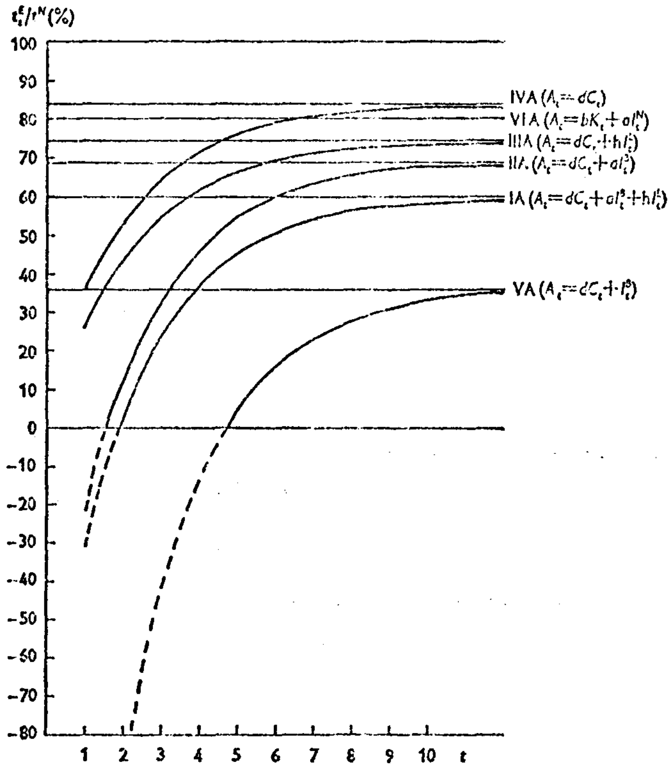

Fig. 2.

(9) in Table 2 and in Figure 2, in which Cases IA and VIA have also been admitted. With this combination of parameter values the case of entirely free depreciation allowances (V) will be the most favourable from the viewpoint of corporate taxation ($t_V^E/t^N = 36$ %) while the tax burden will be highest in the case of only normal depreciation allowances ($t_{IV}^E/t^N = 84$ %).

Finally, Table 3 presents the limiting values t^E/t^N when we use each of the ten parameter combinations in the six cases. The table shows that t^E/t^N will be lowest in Case V for all parameter combinations except F for which Case I gives the lowest tax burden. t^E/t^N will be largest in Case VI with combinations C and H and largest in Case IV with the other combinations.

7. Section 13 in the Appendix shows that the effective tax rate converges towards t^N when $g \leqslant 0 < g_0$. In order to illustrate these possibilities we can assume that $g_0 = 10$ %, $g = -10$ % and $g = 0$. We otherwise

Table 3. t^E/t^N *in* %.

	I	II	III	IV	V	VI
A	60	69.6	74.4	84	36	80.8
B	79.2	84	86.1	90.9	68	90.4
C	91.44	93.36	94.08	96	87.2	96.16
D	70	77.2	80.8	88	52	85.6
E	75	81	84	90	60	88
F	56	84.8	63.2	92	68	90.4
G	65.6	75.2	82.4	92	36	80.8
H	48.8	58.4	58.4	68	36	80.8
I	50.4	60	74.4	84	36	68
J	66.4	69.6	80.8	84	36	80.8

start from the parameter combination A and the norm for comparison used earlier ($x = 1$). In the case of decreasing net investment (Case IAa) we then obtain

$$\frac{t_t^E}{t^N} = 100 - \frac{80 \cdot 0.7^t + 51.2 \cdot 0.9^t}{2 - 0.9^t} \tag{38}$$

while the case of a constant volume of net investment gives

$$\frac{t_t^E}{t^N} = 100 - \frac{2624 \cdot 0.7^t + 1312}{30 + 3t} \tag{39}$$

The values generated by (38) and (39) have been shown in Table 2 (Columns 10 and 11). The effective tax rate in period 0 is naturally independent of the rate of increase of net investment and therefore t_0^E is equally large in the two cases. Even firms which grow at a decreasing rate derive a tax advantage from the introduction of accelerated depreciation allowances but the advantage is diminishing since t_t^E converges towards t^N. No net disaccumulation of concealed reserves occurs in this case.[1]

8. In the numerical illustrations presented in Diagrams 1 and 2, the parameters were such that t_t^E/t^N in certain cases was negative for low values of t. Since we have assumed that loss offset is not allowed, t_t^E

[1] When Leif Johansen analyses the case of decreasing investment (op. cit., p. 238) he obtains the result that the tax burden at the start will be less than, but after a while, greater than it would have been without accelerated depreciation allowances. To this must be added the fact that Johansen, as well as Domar, uses the rate of increase of *gross* investment while g in this article is the rate of increase of *net* investment. Consequently, different problems have been studied.

clearly cannot be negative. By utilizing depreciation allowances in every period to the full extent allowed under the law—compare assumption 7°—clearly the firm will write down assets to an extent "unnecessary" from the viewpoint of taxation. It is hardly realistic to assume that a firm actually behaves in this manner. In this section we shall discuss this aspect of the problem.

Whenever assumption 7° applies, a conversion (in period 0) from depreciation allowances in accordance with $A_t = bK_t$ to depreciation allowance in accordance with, for example, (8) will be accompanied by "unnecessary" write downs from the tax viewpoint up to and including period t_z[1] where

$$t_z = \frac{\log \frac{\alpha}{\beta}}{\log \frac{1-d}{1+g}} \tag{40}$$

Let us modify assumption 7° by assuming that the value of the rate of depreciation a for example, is a maximum and that the firm uses a variable rate of depreciation $a_t \leqslant a$ in order to avoid taxation in as many periods as possible after the change in the rates of depreciation allowance. In the event, the same variable rate of depreciation must be used in the determination of C_t (cf. (53)). In Case IA, which we can select as an example, t_0^E is negative. If we set $a_0 = 13.75\ \%$ and $a_1 = 26.60\ \%$, $t_0^E = t_1^{E*} = 0$. In order then to obtain $t_2^E = 0$, it is necessary that a_2 must be about 38 % and when at most 30 % is allowed, $t_2^E > 0$. Since during period 2 and afterwards, there is no reason for a firm to depreciate at a rate lower than 30 %, assumption 7° will apply during and after this period. The limiting value for t_t^E is independent of the value for C_t/K_t in the initial period. The limiting value, t^E/t^N, will therefore be the same either when a is constant during the entire process or variable in the beginning and then constant. Let t_y denote the final period in which the firm can entirely avoid taxation by depreciating at a variable rate. During and after period t_y,

$$\frac{t_t^E}{t^N} = \alpha\left[1 - \left(\frac{1-d}{1+g}\right)^{t-t_y}\right] \tag{41}$$

[1] If t_z is not a whole number, it designates the largest whole number which is less than t_z.

The reduction, D_t, in t_t^E/t^N that the firm can achieve by depreciating at a variable rate instead of a constant rate will be

$$D_t = \left[\alpha - \beta\left(\frac{1-d}{1+g}\right)^{t_y}\right]\left(\frac{1-d}{1+g}\right)^{t-t_y} \tag{42}$$

The expression within the parenthesis on the left is t_t^E/t^N when $t = t_y$ (cf. (16)). With a variable depreciation rate, therefore, we obtain a curve for t_t^E/t^N which lies under the curve which would be generated were the depreciation rate constant but in both cases t_t^E/t^N converges towards the same limiting value. In Case IA equation (41) will appear in the following way

$$\frac{t_t^E}{t^N} = 60\left[1 - \left(\frac{7}{11}\right)^{t-1}\right] \tag{43}$$

In Table 2 (columns 1 and 12) these variations of case IA can be compared in greater detail.

Naturally, the assumptions can be modified in another way, for example, by assuming that losses may be carried forward to later period and deducted. Whether we modify the assumptions in one or the other way would probably be of little significance in the present context.

The Effects of Changes in Depreciation Rates, the Rate of Growth and the Rate of Profit

9. That the growing firm, utilizing its right to accelerated depreciation allowances can hold the effective tax rate below the nominal tax rate, is of course related to the fact that the firm accumulates net concealed reserves.[1]

[1] If we consider a particular asset, for example a machine, the difference—disregarding initial allowances—between accelerated depreciation allowances and the reduction in the asset's value due to wear (i.e. the increase in the concealed reserve in the machine in period t) will be

$$\Delta R_t^t = [d(1-d)^t - b(1-b)^t]\,K^t$$

where K^t is the original cost. For low (high) values of t ΔR_t^t will be positive (negative) assuming that $d > b$. Whether the firm accumulates net concealed reserves in fixed capital depends therefore on whether the accumulation in the newer part of fixed capital exceeds the disaccumulation in the older part.

It can be interesting to see the implications for a growing firm continuously accumulating concealed reserves of a change in its rate of growth, profit rate or the rates of depreciation allowances. Assume that the firm has utilized accelerated depreciation allowances for such a long time that the effective tax rate has reached (or approached) its limiting value. Let us begin with a decrease in the *rates of depreciation allowance*. If we assume that $g = g_0 > 0$, B_t will be (cf. (75)):

$$B_t = \frac{(x - k_1)\, dk\left(\frac{1-d}{1+g}\right)^t + g[(1-k_1)\,k + h(1-k)]}{r} \tag{44}$$

x and k_1 respectively are the limiting values for C_t/K_t at the old and the new rates of depreciation allowance. Since these have decreased, $x - k_1 < 0$ (cf. (58)). Whether or not B_t is negative—which means a net liquidation of concealed reserves and therefore $t_t^E > t^N$—depends on whether the numerator's second positive term is numerically greater than the first negative term. This in turn depends on, inter alia, how large is the decrease in the depreciation allowance rates and how large is g. If the change in the rates of depreciation allowance entails that accelerated depreciation is no longer allowed at all, clearly new concealed reserves cannot be formed and a subsequent liquidation of the firm's total concealed reserves will be unavoidable. In that event, for all t, $B_t < 0$ and $t_t^E/t^N > 1$ decreases approaching 1. If the new depreciation rules permit accelerated depreciation allowances to a certain extent, B_t will be positive sooner or later. Thus, in this case only during a limited number of periods can net disaccumulation of concealed reserves *possibly* occur. In order to illustrate this, we assume that the change in the rules of depreciation allowance entails that initial allowances are discontinued but d and h remain unchanged. In Case IA, t_t^E/t^N (calculated in percent) will then be:

$$\frac{t_t^E}{t^N} = 74.4 + 43.2\,(\tfrac{7}{11})^t \tag{45}$$

This change in a means that t_t^E/t^N immediately increases from 60 % (cf. Table 3) to 117.6 % and then decreases to 74.4 %. If we instead assume a decrease in a from 30 % to 20 %, t_t^E/t^N will be:

$$\frac{t_t^E}{t^N} = 64.8 + 14.4\,(\tfrac{7}{11})^t \tag{46}$$

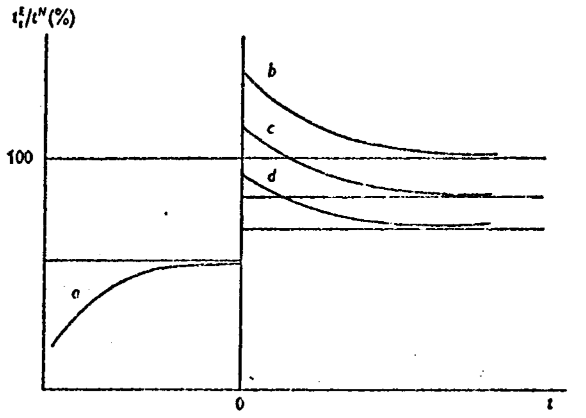

Fig. 3.

With this relatively modest decrease in a, $t_i^E/t^N < 1$ and net disaccumulation of concealed reserves will not take place. The different possibilities have been shown in Figure 3.

The change in the rate of depreciation allowance occurs at point 0 and curve a represents the previous development. Curve b assumes that accelerated depreciation allowances are altogether discontinued. Curves c and d represent respectively a strong and less strong reduction in the rate of depreciation allowance, which reduces the acceleration but does not eliminate it.

We shall now investigate the effects of a decrease in the firm's *rate of growth* on its tax burden. We can again use (44) as our point of departure. x and k_1 are now respectively the limiting values for C_t/K_t before and after the change in g. Since g decreases, $x - k_1 > 0$ (cf. (58)). Thus, we see that a decrease in the rate of growth cannot result in a net disaccumulation of concealed reserves (but only in a decrease in net accumulation). This applies even if g becomes zero and the firm is stationary. If in equation (80) in the appendix we insert $x = k_1$ according to (58) we obtain:

$$B_t = \frac{gk(1-a)(d-b)(1-d)^t}{r(g+d)} \tag{47}$$

In (47) g denotes the rate of growth before the firm became stationary. If we start with Case IA and assume that g decreases from 10 % to 5 %, t_i^E/t^N will be:

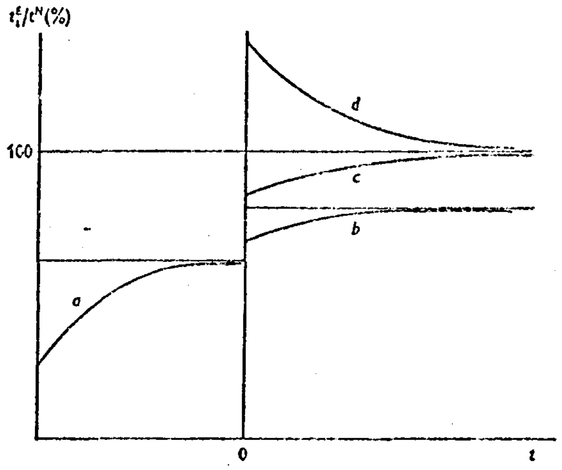

Fig. 4.

$$\frac{t_t^E}{t^N} = 79.2 - 4.8\left(\tfrac{2}{3}\right)^t \tag{48}$$

t_t^E/t^N rises immediately from 60 % to 74.4 % and then converges towards 79.2 %. Had g instead decreased to zero we would have obtained

$$\frac{t_t^E}{t^N} = 100 - 11.2 \cdot 0.7^t \tag{49}$$

If, however, the capital stock began to decrease after the period of growth and accelerated depreciation—as appears in the Appendix—disaccumulation of concealed reserves would have been unavoidable. Assume that $g_0 = -10$ % and $g = -20$ %. This means that the capital stock decreases at a decreasing rate. Assume further that the values in combination A apply for the other parameters. After inserting these values in (75), we obtain t_t^E/t^N.

$$\frac{t_t^E}{t^N} = 100 + \frac{147.2 \cdot 0.8^t - 112 \cdot 0.7^t}{1 + 0.8^t} \tag{50}$$

As is apparent in this case, $t_t^E > t^N$ and t_t^E converges towards t^N. The cases have been shown in Figure 4.

Curve b (d) assumes, obviously, that the firm grows (declines) after the decrease in the rate of growth, while c represents the stationary case.

Finally, we shall consider the case where the *profit rate* changes after the firm during a period of growth has utilized accelerated depreciation allowances. If r increases, as has earlier been pointed out, t^E/t^N increases but t^E/t^N evidently can never exceed 1. Thus, net dis-accumulation of concealed reserves cannot be brought about in this way.

The Profit Rate a Function of the Rate of Growth

10. The problem on which the preceding analysis has attempted to shed some light has been the following. If accelerated depreciation is allowed in some form or other, how large will be the effective tax burden (t_t^E/t^N) for a firm which grows at a given, constant rate and has a profit rate that is given and constant. In other words, we have not attempted to study the way in which the firm's *behaviour* possibly can be influenced by the introduction of accelerated depreciation allowance but rather to study t_t^E/t^N under alternative assumptions about, inter alia, r and g. In the present section, the interesting but complicated question of whether accelerated depreciation allowances provide an investment incentive for a firm will be discussed.

To assume an exogeneously given profit rate r is of course equivalent to assuming that profits V_t change at the same rate (g_t) as the stock of capital, i.e. that the marginal efficiency of investment is independent of the size and the speed of the changes in the capital stock. If the actual rate of profit after taxes $\bar{r} = (1 - t^E)r$ exceeds the firm's required rate of profit after taxes r^*, the distance between $\bar{r}$ and r^* will further increase if the rate of growth becomes higher. With the help of (18) we can quite simply calculate that rate of growth—at given values for d, a, h, b, k and r—which gives $t^E = 0$ (or another value) and therefore $\bar{r} = r$. The growth rate which the firm will choose can hardly be determined without introducing many other factors into the analysis. By way of example, limited possibilities of financing may set a limit upon the rate of growth. Or aversion to increased debt or the greater risks that may possibly accompany an increased growth rate can limit g. These and other often decisive factors are not taken into account in the present analysis. However, we shall now abandon the rigid assumption that the profit rate is independent of the rate of growth.

Assume that, ceteris paribus

$$r = r(g) \tag{51}$$

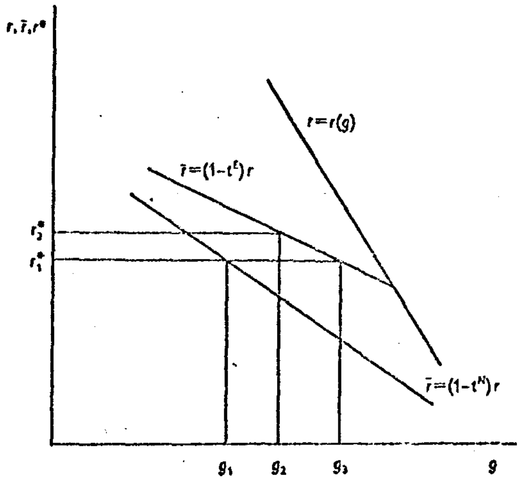

Fig. 5.

and that $r' < 0$ in the relevant interval. The profit rate after taxes will now be $\bar{r} = (1 - t^N)r$ if accelerated depreciation is not allowed and $\bar{r} = (1 - t^E)r$ if accelerated depreciation is allowed.[1] These three functions are shown in Figure 5.[2]

If g and r are positive, $0 \leq t^E < t^N < 1$ and therefore

$$(1 - t^N)\, r < (1 - t^E)\, r \leq r \tag{52}$$

Assume now that the firm's required yield (after taxes) is r_1^*. If accelerated depreciation is not allowed, a firm, under our assumptions, will select the growth rate g_1. Given the possibility of accelerated depreciation allowances, however, the firm will choose the higher growth rate g_3. One can conclude in this case that accelerated depreciation allowances, by increasing the profit rate after taxes, provide an investment incentive for the firm. In this case the result was that the firm's rate of growth increased from g_1 to g_3.[3] This result assumes, inter alia,

[1] We do not consider the transition period in the following discussion.

[2] With our assumptions the function $\bar{r} = (1 - t^E)r$ is in fact non-linear.

[3] Obviously this reasoning is also applicable to the capitalized current value of future profits. Under our assumptions, the capitalized value of profits before taxes in (the beginning of) period 0 will be (if $r^* > g$)

$$N_0 = \frac{r(g)}{r^* - g} K_0$$

Assume that N_0 is first an increasing and then a decreasing function of g with maximum when $g = g_1$. The current value of profits after taxes is thus $\bar{N}_0 = (1 - t^N)\, N_0$ if accelerated that the firm's required yield is not influenced by the fact that concealed reserves are formed when depreciation allowances are utilized at an accelerated rate. If the accumulation of concealed reserves has the effect that the firm's management raises their yield requirement to r_2^*, the effect of acceleration on the rate of growth will not be g_3-g_1 but rather g_2-g_1. Conceivably, the required rate of profit can be raised so high that no effect on g results.

One can now ask whether management has reason to raise its required rate of profit because concealed reserves are being formed by the firm. Since concealed reserves correspond to untaxed profits, a firm with concealed reserves has, in a sense, a latent tax debt. If the firm is compelled to liquidate its reserves, the tax falls due and this can occur at a time which is disadvantageous from the viewpoint of the firm—for example, when liquidity is low or because the firm's dividend policy in a given case cannot be pursued. The conclusion should therefore be that an aversion to concealed reserves can be reasonable. The relevance of this reasoning is of course entirely dependent on the probability that a liquidation of concealed reserves will be necessary. There has perhaps been a tendency in a part of the literature to overestimate the risks in this context.[1] We can refer to the analysis in Section 9. There it was shown that a net liquidation of concealed reserves was not necessitated by the fact that a growing firm began to increase at a decreasing rate or became stationary. If, however, the firm's stock of capital begins to *decrease*, a liquidation of concealed reserves will be unavoidable. The same will be true if the rate of depreciation allowance decreases pronouncedly and liberal transitional regulations do not exist. Consequently, it appears reasonable to assume that a liquidation of

depreciation is not allowed. The firm then will choose the rate of growth that entails that $d\overline{N}_0/dg = 0$, i.e. g_1. If accelerated depreciation allowances are permitted the current value of profits after taxes will be $\overline{N}_0 = (1-t^E)N_0$. We now seek the value for g which in this case makes $d\overline{N}_0/dg = 0$. This value for g is determined by

$$\frac{dN_0}{dg} = \frac{N_0}{1-t^E}\frac{dt^E}{dg}$$

A calculation of dt^E/dg (cf. equation (18)) shows that if in addition to our previous assumptions we assume that $r' < 0$, $dt^E/dg < 0$. In this case $dN_0/dg < 0$. This is true when $g > g_1$. The rate of growth g_3 which the firm chooses in this case is therefore greater than g_1.

[1] See, e.g., Robert Eisners review of a study by Osmo V. Jaskari in *Econometrica* 1963, pp. 619–20.

concealed reserves in the manner discussed will be necessary only in relatively extreme cases.

At the sale of the firm the sale price will probably be lower the greater are the concealed reserves. In passing it can be mentioned that this circumstance is obviously not necessarily a reason for the firm's management to attempt to avoid accumulation of concealed reserves. If the tax saving, say, is continuously invested in productive capital, the result of the formation of concealed reserves can conceivably be an increase in the market value of the firm.

Undoubtedly, the analysis based on Diagram 5 is unsatisfactory in a number of respects. Firstly, the firm's choice obviously doesn't lie among different *constant* rates of growth but rather among different variants of the function $g = g(t)$. In the second place, nothing has been said about the determinants of the shape of the function, $r = r(g)$. A fruitful discussion of this subject presupposes the introduction of a rather large number of hitherto unconsidered factors into the analysis. In the present discussion, this will not be attempted. Rather, we shall indicate the extent of the problem with a few somewhat randomly chosen examples of such factors. Firstly, it will be necessary to make certain assumptions about the future market demand for the firm's products. Secondly, the nature of the market and the reactions of competing producers must be specified. Thirdly, the costs of financing investment occasioned by the alternative growth rates should be discussed. Fourthly, it will be reasonable to consider explicitly technological development and its effects (along with that of the assumed wage development) on the costs of the firm.

Some Concluding Remarks

11. In this paper, the term accelerated depreciation has been taken to mean depreciation at a faster rate than the reduction in value of the asset as a result of wear and tear and aging. It is not necessary to emphasize that a specification and utilization of this concept are fraught with exceptional difficulties. Among these are the difficulties of defining the capital concept meaningfully and operationally and of measuring the physical wear and tear of the asset as well as the reduction in value as a result of technological and organizational change. That most

authors have been disinclined directly to define accelerated depreciation and have avoided introducing an explicit measure of the effective tax burden into their analyses of accelerated depreciation undoubtedly relates to these difficulties. Moreover, it is quite possible to draw conclusions about the direction of the change in the effective tax burden caused by changes in the type and rates of depreciation allowance merely by comparing the amount of depreciation allowances under different sets of conditions.

In this article, firstly, an attempt has been made—under ideal assumptions—to indicate the actual tax burden that *can* result from different types of depreciation allowance. In order to keep its scope within reasonable proportions we have not discussed how the firm finances investment and the related dividend policy. Nor have we considered the possibilities of transfers to investment reserves funds or to internal pension funds which can be supposed to influence the profit rate after taxes in a manner similar to accelerated depreciation allowances.[1]

Secondly, the investment incentives provided firms by different types of depreciation allowances have been discussed. That this second part of the analysis is of a more experimental character than the first is readily apparent. The idea was not so much to arrive at precise and definitive conclusions but rather to point to some factors which—everything else being equal—could be relevant to such a discussion. It appears not unfair to assert that the theoretical literature until now has contributed relatively little to elucidate the question of accelerated depreciation and investment incentives. Also, empirical studies have been rare.[2]

In the Swedish debate over gross and net profits taxation, one argument put forward *against* net profits taxation has been that it discri-

[1] The extent to which Swedish firms have utilized depreciation allowances or transfers to various funds can perhaps be indicated by the fact that during the years 1953–62 paid taxes (including investment tax) fluctuated in the interval between 0.8–1.0 thousand million Swedish Kronor, while gross profits (including changes in inventory reserves) in the same period increased rather evenly from 2.7 to 5.2 thousand million Swedish Kronor according to calculations carried out by Jaak Järv. These data are presented in Faxén's article in the above mentioned publication from the *National Bureau of Economic Research*. How this development should be interpreted must remain an open question in the present article.

[2] See D. V. Corner and Alan Williams "The Sensivity of Businesses to Initial and Investment Allowances", *Economica* 1965, p. 35.

minates against firms which earn larger profits and can therefore be assumed to be the most efficient and the most worthy of expansion.[1] This, of course, would be quite correct as long as actual profits were those taken into account for tax purposes. Such an assumption, however, is somewhat unrealistic. For, given accelerated depreciation allowances, net profits taxation will favour the highly profitable and expanding firms and therefore probably provides an incentive to growth. The exact effects of gross profits taxation in this respect appear to be somewhat uncertain but would be well worth thorough analysis.

APPENDIX

Calculation of the Book value of the Fixed Capital and the Concealed Reserves

12. In order to calculate the effective tax rate the book value of the fixed capital, C_t, must be expressed in terms of predetermined magnitudes. This step shall be carried out in this section. Once this has been accomplished, we can immediately obtain an expression for the concealed reserves in fixed capital.

a) Assume firstly that $g \neq 0$. When initial allowances are permitted (14) can be written

$$C_t = (1-d)\, C_{t-1} + (1-a)\,(I_{t-1}^N + bK_{t-1}). \tag{53}$$

Since $g \neq 0$

$$I_t^N = I_0^N (1+g)^t \tag{54}$$

$$K_t = K_0 \left[1 - \frac{g_0}{g} + \frac{g_0}{g}(1+g)^t\right] \tag{55}$$

which inserted in (53) gives

$$C_t = (1-d)\, C_{t-1} + (1-a)\, K_0 \left[\frac{g_0}{g}(g+b)(1+g)^{t-1} + b\left(1 - \frac{g_0}{g}\right)\right]. \tag{56}$$

The assumption that, at the start, the depreciation allowance corresponds to the actual decrease in the value of capital due to wear and aging means that $C_0 = K_0$. However, we shall not assume any special norm for comparison here and therefore we set $C_0 = xK_0$. In the event, the solution of the difference equation (56) will be

$$C_t = \left[(x - k_3)(1-d)^t + k_1 \frac{g_0}{g}(1+g)^t + k_2\left(1 - \frac{g_0}{g}\right)\right] K_0 \tag{57}$$

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[1] See, e.g. Rehn, Gösta, "Bruttobeskattning", *Framtidens företagsbeskattning*, S.N.S. publication Stockholm 1960, pp. 31–37.

where
$$k_1 = (1-a)\frac{g+b}{g+d} \tag{58}$$

$$k_2 = (1-a)\frac{b}{d} \tag{59}$$

$$k_3 = k_2 + (k_1 - k_2)\frac{g_0}{g}. \tag{60}$$

The ratio between C_t and K_t will be

$$\frac{C_t}{K_t} = \frac{(x-k_3)(1-d)^t + k_1\frac{g_0}{g}(1+g)^t + k_2\left(1-\frac{g_0}{g}\right)}{1-\frac{g_0}{g}+\frac{g_0}{g}(1+g)^t}. \tag{61}$$

If the stock of capital is increasing (i.e. $g_0 > 0$) C_t/K_t will converge towards the following limiting values

$$g>0: \frac{C}{K} = k_1 < 1 \tag{62}$$

$$g<0: \frac{C}{K} = k_2 < 1. \tag{63}$$

That g_0 and g are positive, implies that C_t, K_t, L_t and total concealed reserves, R_t, gradually (after the transition period) will increase at the rate, g. If, on the other hand, $g_0 > 0 > g$, C_t, K_t, L_t and R_t will tend to become stationary.

If the stock of capital decreases ($g_0 < 0$) C_t/K_t converges towards k_1 if $g = g_0$ and towards k_2 if $g < g_0$.

Concealed reserves in fixed capital, $R_t^K (= K_t - C_t)$, can be expressed thus

$$R_t^K = \left[\left(1-\frac{g_0}{g}\right)(1-k_2) + \frac{g_0}{g}(1-k_1)(1+g)^t - (x-k_3)(1-d)^t\right]K_0. \tag{64}$$

Concealed reserves in inventories, R_t^L, are

$$R_t^L = h\frac{g_0}{g}[(1+g)^t - 1]L_0. \tag{65}$$

The change in the total concealed reserve during period t is therefore

$$\Delta R_t = (x-k_3)\,dK_0(1-d)^t + g_0[(1-k_1)K_0 + hL_0](1+g)^t. \tag{66}$$

b) We assume now that $I_t^N = I_0^N > 0$, i.e. $g = 0$. Instead of (55), we will then have

$$K_t = K_0(1+g_0 t) \tag{67}$$

which, substituted in (53), gives

$$C_t = (1-d)C_{t-1} + (1-a)[g_0 + b + b(t-1)g_0]K_0. \tag{68}$$

The solution of (68) is

$$C_t = [(x-k_4)(1-d)^t + k_2 g_0 t + k_4] K_0 \tag{69}$$

where
$$k_4 = k_2 + \frac{g_0}{d}(1-k_2-a). \tag{70}$$

From (67) and (69), it appears that with constant positive net investment we obtain the same limiting value for C_t/K_t (namely k_2) as when we assumed that $g_0 > 0 > g$ (or $g < g_0 < 0$). In that case, R_t^K, R_t^L and ΔR_t will be

$$R_t^K = [1 - k_4 + (1-k_2) g_0 t - (x-k_4)(1-d)^t] K_0 \tag{71}$$

$$R_t^L = t h g_0 L_0 \tag{72}$$

$$\Delta R_t = [g_0(1-k_2) + d(x-k_4)(1-d)^t] K_0 + h g_0 L_0. \tag{73}$$

Calculation of the Effective Tax Rate When the Profit Rate is Exogenously Given

13. We start from the premise that initial allowances and inventory write-downs as well as normal depreciation allowances (Case I) are permitted. Substitution of (8) in (7) gives

$$B_t = \frac{dC_t + aI_t^N + hI_t^L - b(1-a) K_t}{rS_t}. \tag{74}$$

We shall view in turn the case of a firm with a growing, a stationary and a declining stock of capital.

a) *The growing firm* ($g_0 > 0$). Assume first that $g \neq 0$. If in (74) we take into account (54), (55) and (57) and the expressions for I_t^L and S_t corresponding to (54) and (55) we obtain

$$B_t = \frac{(x-k_3)\, dK_0(1-d)^t + g_0[(1-k_1) K_0 + hL_0](1+g)^t}{rS_0\left[1 - \frac{g_0}{g} + \frac{g_0}{g}(1+g)^t\right]}. \tag{75}$$

The numerator in (75) is of course equal to ΔR_t according to (66). ΔR_t can also be written

$$\Delta R_t = (x-k_2) K_0 d(1-d)^t + hg_0 L_0(1+g)^t$$
$$+ g_0 K_0(1-k_1)[(1+g)^t - (1-d)^t] + g_0 K_0 a(1-d)^t. \tag{76}$$

Parameters d, a, h and b are of course positive and less than 1. Moreover, we have assumed that d is not less than b. If now r as well as g and g_0 are positive (cf. (76)) clearly $x > k_2$ is a sufficient (but not a necessary) condition for $B_t > 0$ or, in other words, for $t_t^E < t_0^N$. When $g > 0$, as appears from (75) the limiting value for B_t will be

$$B = g\frac{(1-k_1) K_0 + hL_0}{rS_0} > 0. \tag{77}$$

The limiting value, t^E, for the effective tax rate will therefore be

$$t^E = t^N\left[1 - \frac{g}{r}(1-k_1)k - \frac{g}{r}h(1-k)\right]. \tag{78}$$

If net investment is positive but declining—i.e. $g < 0 < g_0$—it can be shown that $x > k_2$ is a sufficient but not a necessary condition for $t_t^E < t^N$.[1] When $g < 0$, the limiting value for B_t will be zero (cf. (75)) and therefore $t^E = t^N$.

We assume now a constant volume of investment i.e. $g = 0$. (67), (69) and (70) are inserted in (74):

$$B_t = \frac{(x-k_2)\,dK_0(1-d)^t + (1-k_2)\,g_0K_0[1-(1-d)^t] + ag_0K_0(1-d)^t + hg_0L_0}{rS_0(1+g_0t)} \tag{79}$$

In this case the numerator is equal to ΔR_t according to (73). From (79) it appears that B_t is positive if $x > k_2$. Further, it can be seen that B_t converges towards zero when $t \to \infty$ and therefore that we also obtain $t^E = t^N$.

b) *The Stationary Firm.* ($g_0 = 0$). We obtain B_t by setting $g_0 = g = 0$ in (79) or in (75)

$$B_t = \frac{(x-k_2)\,dK_0(1-d)^t}{rS_0}. \tag{80}$$

It can be seen immediately that $B_t > 0$ if $x > k_2$ and B_t converges monotonously towards zero when $t \to \infty$.

c) *The Declining Firm* ($g_0 < 0$). It appears reasonable to start from the premise that fixed capital cannot decline in value at a faster rate than it wears out. If $g < g_0 < 0$, the stock of capital declines at a decreasing rate and converges towards the value $S_0(1 - g_0/g)$ (cf. (55)). We assume $x > k_2$. If, for example, we have $g + d > 0$, then $k_2 > k_3$ (cf. (58)–(60)). In that case, the first term in the numerator in (75)—as well as obviously the denominator-is positive while the second term is negative. It is therefore conceivable that B_t is positive for low values for t. However, B_t will gradually become negative since the positive term is reduced at a faster rate than the negative term. B_t converges towards zero. If $g = g_0 < 0$, the stock of capital converges towards zero at a constant rate. Also, in that case, B_t can be positive for low values of t if $x > k_2 > k_3$. B_t decreases monotonically towards the limiting value (77) which in this case is negative.

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[1] We can show this with (76) as the starting point. Of the four terms in this expression, under the present assumptions, the two first terms and the last term are positive just as is the factor g_0K_0. If $|g| < d$ the third term is positive because $1-k_1$ and $[(1+g)^t-(1-d)^t]$ are both positive. If $d < |g| < (d-b+ab)/a$, $1-k_1$ and $[(1+g)^t-(1-d)^t]$ are negative and the third term is therefore positive. If $|g| = (d-b+ab)/a$, $1-k_1 = 0$. If, finally, $|g| > (d-b+ab)/a$, $1-k_1 > 0$. Since

$$a-(1-k_1) = -\frac{(1-a)(d-b)}{g+d} > 0$$

the sum of the third and fourth terms will be positive. The case where $|g| = d$ shall not be analysed here.

Finally, if $g > g_0$, K_t will be zero after a finite number of periods t_z where

$$t_z = \log\left(1 - \frac{g}{g_0}\right) \Big/ \log(1 + g). \qquad (81)$$

The limiting value for B_t has no economic meaning in this case, and therefore shall not be subjected to further analysis.

Case VI

14. In this section, we will calculate the effective tax rate in Case VI. As in Section 13, the profit rate is assumed to be exogenously given.

Substitution of (13) in (7) gives

$$B_t = \frac{aI_t^N}{rS_t} \qquad (82)$$

or

$$B_t = \frac{akg_t}{r}. \qquad (83)$$

If $g > 0$, g_t converges towards g (cf. (1)) and t_t^E approaches

$$t^E = t^N\left(1 - \frac{ag}{r}k\right). \qquad (84)$$

If we assume that $g = g_0$, then t_t^E will be constant and equal to the limiting value just indicated. If $g = 0$, then B_t converges towards zero and we have $t^E = t^N$.

Faxén's analysis is not rigourously formalized and therefore the assumptions he has used are not evident in certain respects. The diagram in which Faxén summarizes his analysis[1] is based on an expression taking the form

$$t_t^E = t^N\left(1 - \frac{g}{r}s\right) \qquad (85)$$

where s is a rate of depreciation allowance. t_t^E is thus equally large in all periods. As can easily be seen, we can arrive at this result in two ways. Firstly, we can start from Case VI and assume that the analysis only includes fixed capital ($k = 1$) and that $g = g_0$. We then obtain (85) where $s = a$. Alternatively, we can simply assume that the analysis applies only to inventory investment. If we set $k = 0$ in (15) and then calculate t_t^E, we obtain (85) where, now, $s = h$.

[1] See Faxén, op. cit., p. 61.

List of Symbols

The following list indicates the most important symbols used in the analysis.

K_t the *actual* value of fixed capital
C_t the *book* value of fixed capital
R_t^K concealed reserves in fixed capital ($R_t^K = K_t - C_t$)
L_t the *actual* value of inventory
R_t^L concealed reserves in inventories
S_t the firm's total stock of capital ($S_t = K_t + L_t$)
R_t the total concealed reserves in capital ($R_t = R_t^K + R_t^L$)
k fixed capital as a proportion of the total capital stock ($k = K_t/S_t$)
x the relation between the book value and the actual value of fixed capital in the initial position ($x = C_0/K_0$)
b the proportion of fixed capital which wears or ages per time period.
I_t^G gross investment in fixed capital
I_t^N net investment in fixed capital
I_t^L net investment in inventories
A_t total depreciation allowances (including possible inventory write-downs)
V_t profits—in the economic sense—*before* taxes
V_t^G profits before depreciation allowances ($V_t^G = V_t + bK_t$)
W_t taxable profits ($W_t = V_t^G - A_t$)
T_t the corporation tax ($T_t = t^N W_t$)
t^N the nominal corporate tax rate
t_t^E the effective tax rate ($t_t^E = T_t/V_t$)
r the firm's profit rate *before* taxes ($r = V_t/S_t$)
$\bar{r}$ the profit rate *after* taxes, i.e. $\bar{r} = (V_t - T_t)/S_t$
r^* the firm's required rate of yield *after* taxes
d the normal (or current) rate of depreciation allowance for fixed capital ($d \geqslant b$)
a the proportion of gross (or net) investment which can be depreciated immediately.
h the ratio indicating the allowed write-down of inventory investment (calculated net)
g the rate of increase of I_t^N and I_t^L
g_t the rate of growth of the stock of capital ($g_t = I_t^N/K_t = I_t^L/L_t$)

Index t alludes to period t wherever flow quantities are concerned and to the beginning of period t wherever stock quantities are involved.

CHAPTER VII

THE NORDIC INVESTMENT FUNDS SYSTEMS. A BRIEF OUTLINE

Development of the Swedish IF-system[1]

1. The Swedish investment funds system was introduced as early as 1938 with the explicit aim of inducing firms to cut investment activity in boom periods and stimulate it during recessions. The 1938 legislation as well as the revised act from 1947 were more or less experimental but in 1955 the Swedish parliament approved a new law containing the investment fund provisions in their final form.

Three basic features of the Swedish investment funds system have been unchanged through all three decades. The first is that the right to make allocations to an investment fund (IF for short) is the prerogative of corporations and unincorporated associations whereas other unincorporated firms are not entitled to do so. Since 1963 even savings banks have been allowed to make allocations. Secondly, an allocation to an investment fund reduces the firm's taxable profits and thus tax liabilities for the year the allocation is made. Thirdly, investment funds can only be used to finance assets specified in the law. This specification has, on the other hand, changed a great deal. Generally the tendency has been to permit more and more types of investment to be financed with released investment funds. Thus in addition to expenditure on plants and durable equipment, outlays for repair and maintenance of industrial buildings and most investments in forestry were included from 1955. And in 1963 special provisions covering investments in inventories were added to the law (see section 4). If, finally, it is considered important, investment funds may in special cases be used for e.g. road construction, water and waste pipes and sales promotion efforts abroad (provided that the goods are produced in Sweden).

2. The 1947 rules were very much criticized on the ground that an IF-allocation increased the firm's liquidity by the amount of taxes that otherwise had to be paid. Thus in a boom year allocations made it in fact possible for firms to finance considerably more investment expenditure than if the allocations had been subject to taxation. A Business Tax Committee therefore proposed that a certain part of IF-allocations should be deposited in the Central Bank. In order to stimulate allocations the deposit rate should be

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1) More detailed presentations of the Swedish IF-legislation can be found in e.g. [10] pp. 9-15, [13] pp. 23-31, [14] pp. 221-35, [15] chap. 3 and [18] pp. 9-21.

somewhat lower than the corporate tax rate. At that time (1954) the average corporate tax rate was about 47 per cent (cf. Table VII:1) and thus a deposit rate of 40 per cent was suggested by the committee.[2)]

This view was evidently accepted in the government's IF-proposition submitted to parliament early in February 1955 and approved late in May. Before this approval, however, the government had decided to take further steps against the rapidly developing inflationary pressures. Among the measures proposed late in April and enacted in early June was an increase of the state corporate tax rate from 40 to 50 per cent. Thus, the gap between the corporate tax rate and the deposit rate became much wider than was originally suggested. This probably increased the attraction of IF-allocations considerably and reduced the impact of the corporate tax increase.[3)] Another effect of the widening of the gap between the corporate tax rate and the deposit rate was to make it easier for the firms to adapt themselves to the more restrictive depreciation rules proposed by the Business Tax Committee and enacted in June 1955. It is not clear whether this widening was deliberately intended or simply a result of incomplete coordination of the use of different instruments in a stormy period for economic policy. After a lowering of the state corporate tax rate from 1960 and an increase from 40 to 46 per cent of the corporate deposit rate at the same time the gap between the two was drastically reduced as appears from Table VII:1. Since 1960 the gap has been increasing due to the steady rise in municipal tax rates and it now (1973) exceeds 8 percentage points.[4)]

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2) See [14] p. 244.

3) When discussing the proposed increase in corporate taxation in the Supplementary Bill in April 1955 (Proposition No. 190, p. 36) the minister of finance Sköld stated: "To the extent that the firms wish partly to avoid the effects of increased profit taxation through tax-free allocations to investment funds this would be a development which is altogether desirable from a conjunctural point of view." (My translation) This argument seems to rest on the rather dubious assumption that a deposition in the Central Bank of 40 per cent of a given profit will reduce the firm's investment outlays as much as a profit tax of 56 per cent (cf. Table VII:1).

4) For (most) unincorporated associations the state tax rate is only 32 per cent which gives a combined tax rate 1972 of 43.2 per cent. The deposit rate has consequently been set at 40 per cent and the gap between the two is thus of the same size as that of corporations and saving banks. Before 1970 saving banks had the same standing as unincorporated associations in this respect.

3. The 1955 legislation involved changes of the original rules in four other important respects. The first concerns time limits for tax-free allocations. According to the older provisions allocations which were not used within six (later ten) years would be subject to taxation. This time limit was abolished in 1955. In the second place the share of profits which could be allocated to investment funds was raised from 20 per cent (the main rule) to 40 per cent. Concerning forestry the maximal allocation was set at 10 per cent of gross income from 1955. Thirdly, a so-called "free sector" was introduced. If allocated funds have not been released by the authorities within five years after allocation the firms are free to withdraw a certain part of them (30 per cent) for investment purposes. But the normal procedure would still be (at least until recently) that the government authorizes the Labour Market Board, a central administrative agency, to grant firms permission after application to use their investment funds during a limited period of time in which total demand otherwise would be insufficient to secure full employment. Such "general releases" may apply to all or certain firms and types of investment and further conditions may be prescribed. The government may also in special cases permit releases of past and future allocations to investment funds to finance desirable projects, the completion of which might take more than two years. Releases on a selective basis might also be granted by the government to individual firms in order to stimulate investments in certain regions or in general. When funds are set free by the government releases can neither exceed 75 per cent of existing (or future) investment funds nor 75 per cent of the cost of the project.

Finally an extra investment deduction amounting to 10 per cent of releases was introduced in 1955. This extra deduction is not given if releases are granted directly by the government (whatever the purpose is) or if funds are withdrawn from the free sector.

4. In the boom year 1960 special IF-arrangements were made in order to check private investments.[5] Firms were invited to deposit in the Central Bank not 46 per cent but 100 per cent of profits. The voluntary part of the deposits would be repaid at the end of 1961.[6] As a compensation for the

5) See [13] p. 31.

6) To avoid repayments at the end of 1961 the firms were offered a deduction amounting to 10.5 per cent of extra deposits which were not withdrawn before the end of 1962.

Table VII:1. Nominal corporate tax rate and investment fund deposit rate in Sweden 1954-1972 (percentage points)

	Nominal corporate tax rate 1)			IF-deposit rate	$t^N-\beta$	$\frac{\beta}{t^N}$
	State t_S	Municipal t_L	Total t^N	β		
1954	40	12.40	47.4	-	-	-
1955	45	12.24	51.7	40	11.7	77.4
1956	50	12.37	56.2	40	16.2	71.2
1957	50	12.61	56.3	40	16.3	71.0
1958	50	13.69	56.8	40	16.8	70.4
1959	50	14.20	57.1	40	17.1	70.1
1960	40	14.62	48.8	46 2)	2.8	94.3
1961	40	14.99	49.0	46	3.0	93.9
1962	40	15.24	49.1	46	3.1	93.7
1963	40	15.46	49.3	46	3.3	93.3
1964	40	16.49	49.9	46	3.9	92.2
1965	40	17.25	50.4	46	4.4	91.3
1966	40	18.29	51.0	46	5.0	90.2
1967	40	18.71	51.2	46	5.2	89.8
1968	40	19.34	51.6	46	5.6	89.1
1969	40	20.24	52.2	46	6.2	88.1
1970	40	21.00	52.6	46	6.6	87.5
1971	40	22.54	53.5	46	7.5	86.0
1972	40	23.78	54.3	46	8.3	84.7
1973	40	23.94	54.4	46	8.4	84.6

1) The municipal tax rate is the national average tax rate for all municipalities. The total tax rate is calculated as $t^N = t_S + t_L(1-t_S)$ since the municipal tax paid in a given year is deductible for state tax purposes the following year. Under such a system, the total tax rate is, strictly speaking, constant only if municipal taxable profits grow at a constant rate.

2) The increase became effective from July 1st, see [13] p. 27.

Source: Årsbok för Sveriges Kommuner 1970 (SOS) table 18 and Central Bureau of Statistics.

extra deposits the firms were given extra deductions from taxable income corresponding to 12 per cent of the voluntary deposits made before August 1st 1960 and 8 per cent of voluntary deposits made after that date but before the first of November. A similar arrangement was made in 1961.

In its report presented in 1962, the IF-committee considered the possibility of making such special rules a permanent part of the IF-legislation. The committee concluded, however, that if a temporary profit sterilization is desirable in a boom, then it is preferable that the provisions are designed in accordance with the special circumstances prevailing during that particular boom.[7] The special rules were consequently not made permanent.

In 1963 the IF-legislation was extended in different respects. One main purpose was to stimulate firms to make investments in inventories during recessions. According to the new provisions the Labour Market Board may release investment funds for inventory investments. When this happens, deposited funds are paid out by the Central Bank in the usual proportion and an extra investment deduction of 10 per cent of the release-financed stock increase is also given. In addition to that, IF-financed inventories may be written down to the same extent as inventories financed in other ways. After four years, released funds are subject to taxation but if the firm prefers it, they may instead be reallocated to the investment fund. Such a reallocation does not diminish the scope for normal allocations. In this case allocations may in other words exceed 40 per cent of profits. If released funds eventually are reallocated, the system simply implies that the firms obtain four year interest-free loans.[8] In what follows we shall disregard inventories altogether.

Another change concerned the right to transfer investment funds. Since the IF-system naturally cannot give tax benefits to firms without investment funds (e.g. all new firms) concerns have been allowed from 1963 to transfer investment funds from one corporation to another within the concern.

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7) See [17] pp. 52-53.

8) For further details, see [18] chap. 7.

Two interpretations of the IF-system

5. The IF-system may be conceived of in different ways and this has probably caused some confusion in the literature on the subject. According to one interpretation allocations may be regarded as depreciation allowances in advance of the purchase of plant and equipment. When the investment funds some years later are released and the investments actually are made, the book value of the capital stock (and thus the base for future current depreciation) is immediately reduced by an amount equal to the IF-financed cost of the new capital goods. This depreciation is of course not deductible since a deduction was allowed when the allocation was made. The use of the IF-system implies in other words that depreciation allowances reduce taxable profits and the book value in different periods and not simultaneously as is normally the case. With this conception the IF-system is viewed as a variant of accelerated ("pre-initial") depreciation combined with a deposit arrangement.

When seen as a depreciation scheme the IF-system thus offers two tax benefits compared with other ways of financing investments. The first is the tax credit involved in admitting depreciation allowances at the time of allocation instead of during the normal life-time of the capital good. With regard to a single investment project the "repayment" of this tax credit is made during the active years of the asset when otherwise normal depreciation allowances cannot be claimed. This benefit is of course larger for a building with a long writing-down period than for machinery with a short one. The maximum benefit is obtained in cases where depreciation for tax purposes is not admitted at all (e.g. road construction before the change of the rules in 1969). Similarly, the tax credit benefit is smallest when immediate depreciation normally is possible (which is true of cost for e.g. repair and maintenance). The second IF-benefit is the extra 10 per cent deduction mentioned earlier. From these two benefits should be deducted (in a proper present value calculation) the interest forgone on deposited funds from allocation to release.

According to another interpretation, allocations to investment funds are regarded as a temporary sterilization of profits for tax purposes. When the funds later are released they are consequently subject to taxation. On the other hand the firm would now be entitled to immediate and deductible depreciation allowances corresponding to the investments financed by released funds. Thus the firm's taxable income is not affected by the "desterilization" of profits and the immediate depreciation since these two transactions cancel out.[9] The temporary profit sterilization is also combined with a deposit arrangement.

It is clearly immaterial whether the IF-system is seen as a device aiming at a reallocation in time of depreciation allowances or of taxable profits. In all periods the book value of fixed capital as well as taxable profits are independent of whether one or the other interpretation is chosen. The discussion in this paper will be based on the first and, as it seems, more straightforward interpretation which is also the one accepted in the law containing the IF-previous.[1] It is also accepted in the study of Johansson-Edenhammar.[2]

6. That the IF-system in some cases has been presented in a partly confusing way may be due to a failure to keep the two interpretations strictly apart. In his survey Wickman stresses that "... allocations to such reserves are not taxable at the time when they are set aside ... The tax benefits come when the reserves are utilized: the building, machinery and other permissible costs may be written off at once and an extra 10 per cent of the cost of investment may be deducted from taxable income."[3] Similar descriptions of the tax benefits of the IF-system are given by e.g. Canarp, Lundberg and OECD.[4] The immediate writing-down of IF-financed investments is - strangely enough - also seen as a tax benefit by the IF-committee reporting in 1962, by the minister of finance (Sträng) in the 1963 proposition and by the Labour Market Committee reporting in 1965.[5] Also Hansen and Thunholm in their presentations of the Swedish IF-rules fail to say anything about the consequences for the firm's future tax burden which releases (and withdrawals) lead to.[6] As pointed out above, the immediate writing off

9) The taxable profit in years of release is of course reduced by the extra investment deduction.

1) See Law No. 1963:215 paragraph 15 where it is stated that the book value must be reduced by an amount corresponding to the IF-release and that this depreciation is not deductible. See also the comments to the law in Adolf Lundewall Skattehandbok, Stockholm 1961, pp. 965-66.

2) See [18] pp. 16-17.

3) See [35] p. 4-5. It should be pointed out, however, that the numerical analysis in Appendix A is clear and cannot be misunderstood in this respect.

4) See [9] p. 34, [22] pp. 227-29 and [29] pp. 26-29.

5) See [17] pp. 82-83, [13] p. 10 and [1] p. 159.

6) See [16] pp. 354-57 and [34] pp. 302-05.

in connection with a release is not a tax benefit because either it is not deductible (interpretation one) or it is matched by an increase in taxable income due to "desterilization" of profits (interpretation two).

In the literature it is often stressed that it is more advantageous for a firm to use investment funds to finance investments in buildings than in machinery.[7] The reason is not that the tax benefit obtained through immediate depreciation is larger when long-term rather than short-term assets are written down. Since the depreciation cannot be deducted from taxable income, no tax benefit is involved at all. The reason is instead that the value of the increased future tax costs (resulting from the loss of future tax base after the immediate depreciation) is smaller the longer the durability of the IF-financed asset is.

The IF-systems in the Nordic countries

7. After this brief presentation of the development and present form of the Swedish IF-system it is probably of some interest to compare the Swedish variant with the corresponding systems in the other Nordic countries.[8] In order to make a long story short, the main features of all four systems are presented in Table VII:2. In that table the countries are placed according to the degree of similarity with the Swedish IF-system. Sweden and Denmark are thus the extreme cases.

The Swedish system is par excellence an instrument for countercyclical (and regional) policies. This is so even more than Table VII:2 shows since in Sweden the so-called free sector (the 30 per cent of allocations which may be withdrawn freely after 5 years) has played an insignificant role in the sense that only rather limited amounts of funds have been set free through this channel. The Finnish investment funds and the Norwegian A-system (cf. Table VII:2).have also been designed to serve countercyclical purposes but 4-5 years after allocations are made all funds automatically become part of the free sector and may be withdrawn according to the firm's wishes.

The basic aim of the Norwegian B-system and the Danish EF-legislation is to stimulate industrial growth and not to influence conjunctural developments. Thus, with regard to these funds, the authorities cannot make releases at

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7) In chapter IX it is shown that this rule is not valid under all circumstances.

8) The Finnish IF-law can be found in [3] 1969 pp. 50-55. The Danish rules are presented in [27] while the Norwegian system is outlined in [8] and [26]. The Nordic rules are also compared in [15] chap. 3.

Table VII:2. Main features of the present IF-systems in the Nordic countries

	Sweden	Finland[1]	Norway[2] A	Norway[2] B[3]	Denmark
1. Year of introduction	1938	1955	1962	1952	1957
2. Passing of present legislation	1963	1969	1967	1969	1965
3. IF used mainly as an instrument for stabilization policies (SP), regional policies (RP) or growth policies in general (GP)	SP+RP	SP	SP	RP	GP
4. Can unincorporated firms participate?	no	yes	yes	yes	yes
5. Maximal share of profits (before taxes) which may be allocated to IF, % (α)	40[4]	30	18 3/4 (25)	13 3/4-27 1/2 (25-50)[5]	20
6. Share of allocations which must be deposited, % (β)	46	48[6]	133 1/3 (100)	90.9[7] (50)	50
7. Present nominal corporate tax rate, % (t^N)	54.4[8]	48[9]	ca. 50[10]	ca. 50	36[11]
8. Deposits as a percentage of the alternative tax payment $[\beta/t^N(1+\gamma_A)]$	84.6	100	200	100	1389
9. Deposit received by the Central Bank (CB) or private banks (PB)?	CB	CB	CB	PB	PB
10. Interest rate on deposits, % (r^*)	0	3	3	current	current
11. Interest subject to taxation?	-	no	no	yes	yes
12. Number of years after which firms may withdraw funds freely (m)	5	5	4	1-5	1-12
13. Share of allocations which firms may withdraw freely, % (δ)	30	100	100	100	100
14. Releases and/or withdrawals limited by current investment outlays?	yes	yes	no	no	yes

continued ....

	Sweden	Finland[1)]	Norway[2)] A	Norway[2)] B[3)]	Denmark
15. Extra deductions given as a percentage of allocations, % (γ_A)	0	0	33 1/3 (25)	81.8 (45)	0
16. Extra deductions given as a percentage of releases, % (γ)	10[12)]	6	0	0	0
17. Addition to taxable income when funds are not used according to rules, % (of misused funds)[13)]	10	5	-	-	5-60

1) The Finnish rules apply only to state taxation.

2) In the official Norwegian terminology the extra deduction (see line 15) is considered part of the allocation. In order to make the Norwegian and the other Nordic IF-systems directly comparable, we consider as IF-allocations only the funds which can be transferred back to the firm if they are used to finance investments (as specified in the law). For this reason the extra Norwegian deduction is not included in allocations. This explains the somewhat odd figures for Norway in some cases. The "nice" figures within parentheses are the official ratios based on the assumption that the extra deduction is part of allocations.

3) With regard to the extra deduction the IF-system for development areas distinguishes between Northern Norway and the remaining development areas. In the former area this deduction is 81.8% (officially 45%) and in the latter area it is 53.8% (officially 35%). The figures in column B apply to Northern Norway. The figures for the remaining development area would be 16 1/4 - 32 1/2 in line 5, 76.9% in line 6 and 53.8% in line 15.

4) In forestry the limit is 10% of sales.

5) Allocations (including the extra deduction) may exceed 25% of profits to the extent that taxable profits thereby are not reduced below the average for the two preceding years.

6) Entrepreneurs subject to progressive taxation are required to deposit 38%.

7) Instead of depositing, the firm may present a private bank guarantee of the stipulated deposit.

8) For calculation, see note to Table 1.

9) The normal (state) rates are 47 per cent on distributed profits (in excess of dividends received) and 49 per cent on other profits. 1968-70 the rates were increased on a temporary basis (1970 the temporary surtax was 4 per cent). Municipal tax rates vary between 9-18 per cent. Municipal taxes are not deductible for state tax purposes.

10) The state tax rate is 30 per cent whereas municipal tax rates vary between 16-19 per cent. Municipal taxes are not deductible for state tax purposes.

11) Danish corporations are allowed to deduct from profits 2.5 per cent of the (nominal) value of stocks. This deduction cannot exceed 50 per cent of profits. The corporate tax is collected by the state which transfers 15 per cent of it to the municipalities.

12) This deduction is not admitted when funds are withdrawn by the firm or released selectively by the government.

13) Furthermore interest on taxes which would have been due in the absence of allocations are payable according to the Norwegian B-system (4%) and the Finnish IF-law (5%).

all, and these two variants are thus pure free sector systems.[9] The following comments to Table VII:2 will, as far as Norway is concerned, be based on the general counter-cyclical IF-system outlined in column A.

The maximal share of profits which may be allocated to investment funds is twice as high in Sweden as it is in Denmark and Norway with Finland in between. The deposit provisions in Sweden and Finland are more liberal than in the other two countries. In the former countries the rule is that deposits should not exceed the amount of taxes that otherwise should have been paid. Danish firms with compulsory bookkeeping have to deposit 50% of allocations or about 140% of taxes otherwise due. Other Danish firms must deposit an amount equal to the allocation.[1] Under the Norwegian system deposits of the same size as allocations (proper) and the extra deduction (one third of allocations) are required. With a nominal tax rate of about 50% this implies deposits which are twice as great as the alternative tax payments.

With regard to the purposes for which allocated funds may be used the Norwegian system is the most liberal, since it permits that withdrawn (or released) funds are used to write down old plant and equipment as well as new investments. The amount of investments currently undertaken does not, in other words, limit the firms' ability to use fund as is the case in the other three countries. Another implication of this Norwegian rule is that the freedom of the withdrawing enterprise to choose assets with a favourable durability for down-writing purposes - i.e. long term assets - is likely to be greatly enhanced. Under the Norwegian system a firm finally has the option to make withdrawn funds subject to taxation without penalty and this too is a unique feature of the Norwegian IF-system (cf. line 17 in Table VII:2).

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9) The Danish tax committee, which outlined the original IF-system, ruled out, strangely enough, the possibility that Danish investment funds should be used as a tool for short-term demand management. The main argument appears to be that this would involve undue regulation of private enterprises. Consequently the possible forms of such a system were not investigated at all. See [7] pp. 140-41.

1) As indicated in Table 2, deposits in Denmark are made in private credit institutes. It is of course possible that a bank may agree to finance a deposit through a new loan - especially since the bank can do that without paying out cash. As pointed out by Mossin [27] the firm's ability to finance investments is hardly influenced by the IF-allocation in such a case.

Let us finally explain and compare the extra deduction given. The 6 per cent deduction in Finland is similar to the 10 per cent extra investment deduction allowed automatically in Sweden when funds are set free through a general release. The Finnish deduction is however given only after a discretionary government decision.

In Norway allocations to investment funds are seen as temporary profit sterilizations (interpretation 2). When funds are used only 75 per cent of the released or withdrawn funds are added to taxable income but at the same time equally large extra (deductible) depreciations on plant and equipment may be made. In this way the firm escapes taxation altogether of 25 per cent of all allocated funds. This is the way the Norwegian system normally is described. In terms of the Swedish system the extra tax benefit given in Norway can be seen as a deduction from taxable income amounting to one third of allocations proper. The word proper is added to indicate that we have not considered the deduction a part of allocations (in which case the meaning of allocations becomes the same as in Sweden). This unofficial point of view, which has only formal significance, has been adopted in Table VII:2 as explained in note 2 after the table. An important difference with regard to the extra tax benefit is evidently that Norwegian firms, contrary to Swedish and Finnish, get the extra deduction regardless of whether funds are released from the tied sector or withdrawn from the free sector. Under the Swedish system withdrawals imply, in other words, an extra (opportunity) cost which is equal to the value of the investment deduction not obtained.

No extra tax benefits are given according to the Danish IF-rules.

Investment funds in operation

8. Before the formal analysis it may be useful to present data which will give the reader an idea of the extent to which the IF-systems have actually been used in the past. Such descriptions have appeared elsewhere and it is therefore hardly necessary to be very detailed here.[2)] We shall deal mainly with Sweden, but data for the other Nordic countries will be discussed briefly in section 15.

2) See for instance [18] pp. 21-24, [13] pp. 32-36, [9] pp. 34-38 and [15] chap. 4.

The Swedish investment funds played only a minor role before 1955 for a number of reasons. In the first place the pre-1955 provisions regulating depreciation allowances and inventory evaluation were rather generous and little incentive was left to make allocations to investment funds. Secondly the older rule, that investment funds not released within 10 years would be subject to taxation, reduced interest in the IF-system. Also the fact that no funds were released before 1958 contributed probably to the corporations' lack of interest. Finally, in order to reduce profits after taxes and hold back private investments, provisional legislation actually precluded firms from allocating funds during 1952-54.[3)]

Since 1955, however, investment funds have become much more important. This is mainly due to the less favourable depreciation rules enacted in that year and to some of the new features in the 1955 IF-act (e.g. the free sector and the 10 per cent extra investment deduction). Table VII:3 gives yearly data for allocations, releases, deposits and the number of corporations involved in Sweden since 1955.

9. The number of <u>allocating</u> firms has increased significantly since the final legislation was passed but the number is still relatively small or less than 4 per cent of all Swedish corporations. This is of course an advantage from an administrative point of view since the system is based on fairly close cooperation between the firms and the authorities. The largest corporations (with more than 1000 employees) have shown more interest in the IF-system than smaller firms in the sense that the former in most years have allocated a significantly larger share of value added to investment funds than the latter.[4)] Of the total accumulated funds at the end of 1960, 70 per cent had come from corporations with more than 1000 employees, 15 per cent from firms with 300-1000 employed and the remaining 15 per cent from corporations with fewer than 300 employed. There has, however, been a clear tendency toward greater participation by small enterprises and the corresponding figures for 1970 were 51, 14 and 35 per cent respectively.

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3) See propositions 1951:33 and 1952:200. See also [11] pp. 195-96.

4) This has been shown by Södersten, see [33] pp. 328-29.

	(1) Number of firms with investment funds	(2) Allocations A_t^G	(3) Releases and withdrawals $R_t^G + F_t^G$	(4) Accumulated investment funds S_t^G	(5) Accumulated funds as a percentage of private gross investment (excl. permanent residential housing) S_t^G/I_t^P	(6) Deposits in the Central Bank A_t^N	(7) Repayments by the Central Bank $R_t^N + F_t^N$	(8) Accumulated deposits in the CB S_t^N	(9) Manufacturing industry allocations to IF as a percentage of gross profits
1955	771	167	1	414	9.8	7	-	7	4.3
1956	846	125	1	539	11.7	71	-	78	3.2
1957	973	215	-	754	15.4	80	1	156	4.4
1958	1192	419	30	1143	20.2	114	29	242	10.0
1959	1289	530	309	1364	22.1	176	136	282	13.4
1960	1639	1063	381	2046	28.5	958	97	1143	21.9
1961	1819	520	167	2394	29.3	425	338	1230	8.5
1962	2099	439	171	2663	30.3	122	455	897	6.8
1963	2304	522	645	2539	27.5	209	241	866	8.1
1964	2584	684	314	2910	29.2	238	136	968	9.4
1965	2789	662	228	3345	30.1	318	147	1140	8.0
1966	2949	530	303	3572	27.9	304	167	1276	7.0
1967	3126	779	536	3814	29.2	289	576	989	11.2
1968	3112	908	1421	3301	26.5	424	530	882	11.3
1969	3235	1034	730	3605	27.7	444	252	1073	10.4
1970	3517	755	369	3991	28.0	507	330	1250	7.4
1971						406	605	1051	
1972						463	467	1047	

Sources: [19] (1965 p. 72 and 1970-71 p. 105), the Labour Market Board, [5], Bank of Sweden, Enterprises 1970 Table E, SOS 1972 and National Accounts 1950-71 Statistical Reports N 1972:93 published by the Central Bureau of Statistics (CBS).

Note to the table. The Labour Market Board data (columns 1-5) are based on taxation statistics. Since deposits are made after the accounting year (which may differ from the calendar year), the Labour Market Board data are not directly comparable with the Central Bank Statistics which are on a calendar year basis. The figures include the allocations 1960 and 1961 which - according to the special ad hoc rules - assumed 100 per cent deposits. The special allocations 1960 amounted to 770 million crs. and of the 54 per cent excess deposits 300 million crs. were repaid in December 1961 and 115 million crs. in December 1962. In 1961 290 million crs. were allocated according to the special rules and the 157 million excess deposits were repaid in December 1962. (See [13] pp. 31-32.) The CBS-data in column 9 cover only larger firms, that is from 1955-64 firms with more than 25 employees and thereafter firms with more than 50 employees. Gross profits are adjusted for changes in inventory reserves but do not include the balance of financial incomes and expenses.

As appears from Table VII:3, total allocations rose at a rapid rate after 1955. The very large allocations in 1960 (and to some extent even 1961) were due to the special ad hoc incentives mentioned in section 4. Since the beginning of the sixties, yearly allocations in Sweden have varied between 400 and 1000 million crs. It would of course be interesting to relate allocations to gross profits but the latter data are not readily available. The Central Bureau of Statistics has, however, presented estimated figures for manufacturing industry which give at least a rough idea of the size of gross profits and allocations. Accordingly, to the CBS data (presented in column 9 in Table VII:3) less than 5 per cent of gross profits (adjusted for changes in inventory reserves) were allocated to investment funds 1955-57. The sharp rise during the recession 1958-59 is explained by declining profits as well as by increased allocations. After 1960 (when as much as 22 per cent of gross profits were allocated) The ratio has varied around 7-11 per cent. Taxable profits have also been kept down through accumulation of hidden reserves in buildings, machinery and inventories and (in the beginning of the period considered) through allocations to pension trusts.[5] On a trend basis allocations to IF in manufacturing industry have risen at a faster rate than current depreciation allowances and after 1960 the former expressed as a percentage of the latter have varied between 14 and 24 per cent.

At this point it may also be illustrative to indicate some results of Södersten's recent analysis of Swedish corporations 1953-68.[6] During this period the profit margins[7] of Swedish corporations have been squeezed significantly (from about 34 to 26 per cent). The corporations have, however, been able to maintain an almost unchanged gross saving ratio in the period due in particular to the declining tax ratio. The latter development is of course partly explained by the profit squeeze but to a considerable extent it is also the result of larger depreciation allowances and allocations to IF (in relation to value added).

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5) Of all these transactions aimed at reducing taxable income, allocations to IF accounted for 11-17 per cent 1955-57 and for 27-29 per cent 1958-62 with the exception of 1960 when the share was 62 per cent. These ratios are implied in the data for manufacturing industry 1953-62 which have been presented by Mutén-Faxén, see [28] p. 360.

6) For more details, see [33] pp. 323-32.

7) Gross profits as a share of value added. Even the saving and the tax ratios as used by Södersten are measured as percentages of value added.

It is also of interest, finally, to compare the accumulated investment funds with private fixed investments (net of residential construction). This is done in Table VII:3 column 5. The percentage ratio rose significantly after 1955 and in all years since 1960 it has exceeded 25 per cent. If, in other words, all corporations were asked to use their investment funds and actually did that, an anticipated 25 per cent decline in private investments in plants and equipment could be avoided. This is naturally an extreme case and actual releases have been far smaller than that and have taken place only after application by the firm.

10. The first general release of investment funds took place in the recession 1958-59.[8)] From May 1958 to September 1959 the Labour Market Board approved of almost all applications for releases. The dominating part of the applications was for funds to finance buildings and plants. In order to secure employment during the winter, compliance with a construction time table approved by the local labour market authorities was a condition for release. The funds could only be used within a specified period of time which usually was 12 or 18 months, in a few cases 2 years. The Labour Market Board permitted the release of 695 million crs. - about one half of existing funds - to a total of 418 corporations, approximately one third of those with investment funds. At the same time another 315 millions were released for long-term projects by the government. It has been estimated that about 90 per cent of the above mentioned funds was actually used by the firms. To judge the net impact of these releases it would of course be necessary to know to what extent investments now financed by investment funds would have been made if no releases had been allowed. No thorough empirical investigation of the net effect of the 1958-59 releases has been made. It has, however, been argued that a large part of the expansionary effects came too late and stimulated private investment activity well into 1960 when the new boom had begun.[9)]

The next large scale release came in 1962. In May the Labour Market Board was authorized to release funds for investments in buildings. A condition

8) For an account of conjunctural development and economic policy in Sweden in the post-war period, see [24].

9) See The Swedish Economy, November 1960, p. 40 and [13] p. 33. This experience influenced the designing of the subsequent releases.

was that construction had been started before November 1962 and only costs incurred before May 1 1963 would be covered by the release. The latter provision was changed in April 1963 so that outlays made between November 1 1963 and March 31 1964 would also be covered. Approximately 700 million crs. were released to 554 corporations in this way but the estimated costs for the projects in question were considerably higher (1150 million).

Late in 1962 it was decided to release funds for investments in machinery (and ships) as well. In this case it was stipulated that orders had to be made before May 1 1963 and delivery should take place so early that the main employment effect would occur during 1963. About 300 million were released for this purpose.

In the empirical study made by Eliasson [10], the aim was to measure the net impact of the 1962-63 release. According to his study the result of the release was a substantial and well-timed net effect of about 300 million crs. on gross industrial construction during the 10 month release period from July 1 1962 to April 30 1963. This amounts to about 15 per cent of total <u>industrial</u> construction during the fiscal year 1962/63. The share of <u>total</u> construction would of course be much smaller. Eliasson points out also, however, that a non-desired backlog effect remained during the summer of 1963 and that the geographical distribution of the net effects was less favourable since the strongest impact was found in regions with no particular unemployment problems. As far as the release for investments in machinery and equipment is concerned a net increase in orders placed at about 160 million crs. was registered. An estimated 30 per cent of these orders was placed abroad.

11. A third round of general releases began in 1967. In May it was announced that investment funds could be used to finance investments in manufacturing industry and mining. Concerning investments in buildings, costs incurred during the last quarter of 1967 and the first quarter of 1968 could be written off against IF if construction had started after May 19, 1967. With regard to investments in machinery a condition was that purchases were made during the period May 19 - September 30, 1967 and that delivery took place before March 31, 1968. Late in 1967 this offer was extended to cover purchases in the period December 8, 1967 - March 31, 1968 if the equipment was delivered before July 1, 1968. The estimated total cost of the investment projects covered by these two releases was 1850 million crs.

In October 1967 it was decided to release IF for inventory investments made in the period November 1, 1967 - March 31, 1968. About 140 million crs. were released for this purpose.

The last release to check the 1966-68 recession was announced in April 1968. Again investments in buildings outside manufacturing industry and mining were not eligible whereas no such restriction was prescribed this time for investments in machinery.[1] Orders for machinery had to be placed between May 1 and September 30, 1968 and delivery should be made before April 1969. Investments in buildings could be written off against investment funds if construction started after May 1, 1968 and the costs were incurred between October 1, 1968 and March 31, 1969. The total cost of the projects which benefitted from the April 1968 release was approximately 880 million crs. This brings the total for the general releases during the 1967-68 recession to 2870 million crs. Of this investment cost only a part was IF-financed.

The two general releases announced in May and December 1967 have been the subject of an empirical investigation [31] similar to the one made by Eliasson [10]. The new study, undertaken by Rudberg-Öhman, deals mainly with investments in machinery whereas Eliasson's main field of interest was construction activity. According to the new study the net effect of the two releases was to increase outlays for machinery and equipment in manufacturing industry and mining by more than 7 per cent (measured at an annual rate) during the last quarter of 1967 and the first quarter of 1968.[2] The corresponding figure for the December 1962 release was 5 per cent. It has, however, been estimated that the import leakage was somewhat larger 1967-68 than 1962-63 (42 per cent and 30 per cent respectively).[3]

The fourth round of counter-cyclical releases was undertaken during the 1971 recession (the most severe in Sweden in the post-war period). A general

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1) It may be pointed out here that a temporary 25 per cent tax on building investments was in force from April 1967 to September 1968. The tax was not deductible and therefore prohibitive. Among the sectors exempted from the (selective) tax were e.g. manufacturing and mining, residential construction and the Development Area.

2) See [31] pp. 54-55.

3) See [31] pp. 66-67. Another difference between the releases was that the net effect as a percentage of the gross effect was greater in 1962 (53 per cent) than in 1967 (35-43 per cent).

release for inventory investments during the rest of the year was announced in July 1971. A total of 480 million crs. was used in this way. Apart from that no general releases were authorized 1971-72. Instead, the government decided to rely for the first time on selective releases for conjunctural purposes. According to Grundberg 918 million crs. were released by the government during 1971 for projects outside the development area.[4] Whether the new way of handling releases should be seen as a permanent feature or not is not clear at the present time. If all future releases are going to be selective, enterprise attitudes towards investment funds would be changed for at least two reasons. In the first place expectations concerning the likelihood of releases would probably be influenced. Secondly (as pointed out earlier) the extra investment deduction is not admitted when releases are selective.

12. It should, finally, be pointed out that the Swedish IF-system since 1963 has been used not only to fight recessions but also to encourage investments within the development area.[5] The first major regional release was a general one from mid-1963 to mid-1965. During this period about 370 million crs. were set free for investments in Northern Sweden. The subsequent regional releases have all been selective. Two types can be distinguished. In the first group we find releases to be used exclusively within the development area. During the years 1964-71 releases of this type amounted to about 760 million crs. The second group consists of so called combined releases which means that funds are made available for investments outside the development area as a reward for using investment funds within this area. The combined releases 1964-71 anounted to about 2.5 milliard crs. According to Grundberg's estimate only 14 per cent of this sum was used <u>within</u> the development area. The combined releases were rather large 1964-65 and 1969-71 but of modest size 1966-68. Although this evidence is not conclusive it seems probable that these releases had some undesirable inflationary effects during the boom years 1964-65 and 1969-70. On the other hand the combined releases 1971 reinforced the general anti-recession policy.

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4) See [15] Table 17 p. 141.

5) For a discussion of regional releases in Sweden, see [2] and [15].

With regard to the free sector it has earlier been pointed out that withdrawals from that sector have been relatively unimportant in Sweden until now.

13. Let us finally take a brief look at allocations in the other Nordic countries. These are shown in Table VII:4. As appears, allocations in Norway and Denmark have been quite substantial in recent years whereas investment funds have played a minor role so far in Finland. Allocations in Norway rose sharply after the counter-cyclical IF-law was changed in 1967 (column A) and the passing two years later of the new regional IF-provisions (column B). At the end of 1971 accumulated A-deposits in Bank of Norway amounted to 1486 million crs. which corresponds to 7 per cent of private fixed capital formation 1970.[6)] Allocations in Denmark were more than quintupled between 1958 (the first year of operation) and 1967. The decline 1967-68 was probably to a large extent due to the anticipated introduction of a pay-as-you-go tax collecting system from 1970.[7)]

Table VII:4. Allocations to IF in Norway, Denmark and Finland 1951-71 (Million Norwegian and Danish crowns and Finnish marks)

	Norway		Denmark	Finland
	A	B		
1951	-		-	-
1952	-		-	-
1953	-		-	-
1954	-		-	-
1955	-	717 (1951–1959)	-	6
1956	-		-	10
1957	-		-	6
1958	-		113	4
1959	-		113	8
1960	-	86	133	15
1961	-	100	165	12
1962	58	125	216	11
1963	83	93	230	-
1964	106	127	260	-
1965	114	162	412	11
1966	148	145	463	14
1967	268	130	605	17
1968	352	150	365	13
1969	373	255		-
1970	410	342		83
1971				30

Sources. Denmark: [27] p. 80 and Statistiske efterretninger, Norway: [15] p. 94 and p. 114, Finland: [15] p. 173.

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6) For a detailed survey of operations under the Norwegian regional IF-system, see [15] pp. 93-120.

7) The reform implied that taxes on incomes earned in 1968 and payable in 1970 would not be collected.

CHAPTER VIII

AN IF-MODEL FOR A GROWING FIRM

Assumptions Underlying the Formal Analysis

14. In this chapter we shall present a model which may serve as a starting point at an evaluation of the tax benefits which a growing firm may obtain through an IF-system. The approach is essentially the same that was used in chapter VI to analyse verious forms of accelerated depreciation. This means that the effective tax rate is the basic concept in the analysis. It also follows that profitability effects will be our main concern and that liquidity constraints on the firm's investment activity will be disregarded. We investigate how the effective tax rate is affected when a firm with an exogenous growth rate makes yearly allocations to investment funds while funds are released by the authorities or withdrawn by the firm according to a certain time pattern. The analysis is, in other words, not concerned with individual investment projects but with a growing flow of such projects. The assumption about an exogenous growth rate (and therefore investment decisions which are independent of the IF-system) is essential in the first step of the analysis. Ultimately the aim is of course to arrive at a more or less precise evaluation of how the IF-systems may influence the behaviour of firms and capital formation in the private sector of economy.

As we have seen in Chapter VII the Swedish IF-system contains most of the features of the IF-systems of the other Nordic countries. In order to be specific we shall therefore use the Swedish provisions as a starting point for the formal analysis. The Swedish rules which are most important for the analysis can be summarized as follows:

1° Each year corporations in Sweden are allowed to set aside in an investment fund a maximum of 40 % of profits before tax. The allocation is deductible for tax purposes. We shall suppose that our firm annually makes allocations A_t^G amounting to the share α of profits, i.e.

$$A_t^G = \alpha V_t \qquad (1)$$

2° Of the allocation 46 % must be deposited in a blocked account in the Central Bank which is not interest-bearing. Let β denote that share of the allocation which is deposited in the Central Bank while A_t^N is the deposited sum in a given year. Thus

$$A_t^N = \beta A_t^G \tag{2}$$

3° If total effective demand for goods and services is considered insufficient to secure full employment the authorities may grant firms permission to use their investment funds during a certain period of time. We shall assume that such releases of investment funds occur regularly with an interval of p years between the releases. We shall further suppose that the release constitutes a certain fraction f_B of the firm's gross investment during the year of release. As an alternative to this, the releases are assumed to amount to a certain fraction, q, of the allocations during the p years before the release. If the first allocation is made in year zero and the first release takes place in year p then $t = np$ $(n=1,2,\ldots)$ will be the years with releases. The two alternative assumptions about the authorities' release behavior may therefore be written

$$R_{np}^G = f_B I_{np}^G \tag{3}$$

$$R_{np}^G = q \sum_{i=0}^{p-1} A_{(n-1)p+i}^G \tag{4}$$

The amount paid out by the Central Bank in connection with the release is

$$R_{np}^N = \beta R_{np}^G \tag{5}$$

4° If the release is granted by the labor market authorities firms are entitled to a so called investment deduction. This is an extra deduction from taxable income of 10 % of the (actual) release at the tax-assessment of the same year. If the investment deduction is denoted H_{np} and the fraction of the release which may be deducted γ we have

$$H_{np} = \gamma R_{np}^G \tag{6}$$

5° When funds have been blocked in an IF-account during at least five years (including the year when the allocation took place) 30 % of such funds may be used freely for investment purposes. If, however, a part of the allocation in question previously has been released with the authorities permission the free use of that allocation is reduced correspondingly. That part which actually can be used freely constitute the so called "free sector" while the remaining part will be referred to as the "tied sector". Withdrawals from the free sector can be written in the following way

$$F_t^G = \delta A_{t-m}^G \qquad t \geq m \tag{7}$$

m is the number of years between the allocation and the withdrawal and δ is the share of the allocation in year t-m which can be withdrawn by the firm. The part of the withdrawal which is paid out in cash by the Central Bank is as before

$$F_t^N = \beta F_t^G \tag{8}$$

6° When funds are released from the tied sector or withdrawn from the free sector the new investments must be written down by an amount which is equal to the investment funds used. These immediate depreciation allowances are not deductible. The firm's volume of investment puts in other words a limit to the total use of investment funds in a given year. Let D_t^n denote depreciation allowances which are not deductible. Thus we have

$$D_t^n = F_t^G + R_{np}^G \tag{9}$$

and

$$D_t^n \leq I_t^G \tag{10}$$

15. To the rules of the IF-system should be added a number of simplifying assumptions.

7° We shall assume that the rate of growth of the firm's capital stock K_t is a predetermined function of time. This means of course that in the present model the firm's investment decisions are not affected by the existence of the IF-system. Two possibilities will be considered. In the first place the rate of growth is assumed to be constant and equal to θ . Secondly we shall suppose that the growth rate is G in years with releases from the tied sector and g in other years where $G>g$. For a full IF-cycle of p years the rate of growth is in other words constant and given by

$$K_{np} = \left(\frac{1}{1-G}\right)\left(\frac{1}{1-g}\right)^{p-1} K_{(n-1)p} \tag{11}$$

8° The actual depreciation in the value of the capital stock as a result of wear ant tear and aging constitutes in every period a constant proportion, b, of the stock of capital at the end of the period.

9° Normal (or current) depreciation allowances constitute in every period a constant proportion, d, of the book value of the capital stock at the end of the period (the declining balance method). The extent to which conventional depreciation is accelerated can be measured by the difference d-b which is assumed to be non-negative. Actual differences between depreciation due to wear and tear and aging on the one side and current (or normal) depreciation allowances on the other are consequently attributable to the working of the IF-system as well as conventional accelerated depreciation. At the end of this chapter it is shown that the size of the tax benefits which a firm may obtain by using the IF-system depend upon the degree of acceleration of current depreciation. It is for this reason that no attempt has been made in the formal analysis to separate the effects of the IF-system from effects of conventional accelerated depreciation.

10° Net profit taxation is proportional and constant.

11° The firm's rate of profit before taxation, r, is constant and independent of the size of the capital stock, its rate of growth and the firm's capital structure (the shares of internal financing and debt). Given the firm's dividend policy the rate of profit is furthermore assumed to be large enough to permit the formation of concealed reserves through IF-allocations and accelerated depreciation which occurs.

12° Available internal funds (including repayed deposited funds) and the firm's borrowing potential are sufficient to rule out that investments are restricted by liquidity considerations.

13° The use of accelerated depreciation and other schemes influencing taxable profits are not affected by the use of the IF-systems.

14° All relevant prices are constant.

15° The model is deterministic: expectations about future receipts and payments are single valued.

These assumptions are very similar to these used in the study by Johansson-Edenhammar.[1/]

Some implications

16. After the main assumptions underlying the formal analysis have been presented it may be useful to indicate some implications of these assumptions. This will be done in this and the following section.

Let us start with the tied sector and point out in what way assumptions (3) and (4) are related to each other. From assumption 7° it follows that

$$I^G_{np} = (G+b)K_{np} \tag{12}$$

According to assumption 11° the amount of profit before taxes is

$$V_t = rK_t \tag{13}$$

We insert equations (1) and (11)-(13) into (4). After summation (4) can be written

$$R^G_{np} = q\frac{\alpha r(1-G)\left[1-(1-g)^p\right]}{g(G+b)} I^G_{np} \tag{14}$$

Equation (14) shows that the assumption that the release from the tied sector every p'th year amounts to 100 f_B per cent of the firm's gross invesment is equivalent to the assumption that the release constitutes a constant fraction q of the allocations during the p years before the release. This is hardly surprising with the assumptions we have made. q and f_B are thus related in the following way

$$f_B = q\,\frac{\alpha r(1-G)\left[1-(1-g)^p\right]}{g(G+b)} \tag{15}$$

In the subsequent analysis the way in which different parameters enter depends upon whether assumption (3) or (4) is used. It is for that reason that they will be used alternatively. If $q(f_B)$ is regarded as the authorities parameter of action then $f_B(q)$ is a dependent variable determined by r, G, g, b, p, , and $q(f_B)$ as indicated by (15).

1/ See [18] pp. 56-62.

q is the part of the allocations which is actually released. It is of course possible that the authorities permit the release of a larger part of the allocations than the firm can or is prepared to invest in new machinery and buildings within the prescribed time limit. In that case the actual value of q is lower than the maximum value. As a rule we shall suppose that q in these two senses are equal. Although it is conceivable that a release from the tied sector exceed the allocations during the previous p years it is appropriate in the present context to assume $q \leqq 1$.

We now turn to the free sector. Since there is no a priori reason to believe that withdrawals from this sector will affect the time pattern of investments in any particular way, we shall consider only the possibility of a constant growth rate as far as the free sector is concerned. Let f_F denote the ratio between withdrawals and gross investment in a given year (i.e. F_t^G/I_t^G).

If equations (1), (7) and (11)-(13) are taken into account f_F can be expressed as

$$f_F = \frac{\alpha\delta r(1-\theta)^m}{(\theta+b)} \tag{16}$$

From equations (9) and (10) it follows that

$$f = f_B + f_F \leqq 1 \tag{17}$$

Equation (17) in combination with (15) and (16) define the combinations of parameter values which are consistent with the condition that in any given year the total use of funds cannot exceed the firm's gross investment. We shall therefore refer to equation (17) as the investment restriction. At the same time the total use of I-funds naturally cannot be in excess of previous allocations. For simplicity this condition is stated in this way

$$q + \delta \leqq 1 \tag{18}$$

δ is the part of the free sector which is actually released. This part may be less than the maximum fraction permitted in the law if releases from the tied sector are extensive or gross investments are small.

17. We shall finally indicate the firm's cumulated investment fund S_t^G. We have

$$S_t^G = \sum_{i=0}^{t} A_i^G - \sum_{i=1}^{n} R_{ip}^G - \sum_{i=m}^{t} F_i^G \tag{19}$$

If the rate of growth is constant and if t is a year with release from the tied sector ($t = np$) we get after summation

$$S_t^G = \frac{\alpha r}{\theta} K_t \left[1 - q(1-\theta) - \delta(1-\theta)^m - (1-q-\delta)(1-\theta)^{t+1}\right] \tag{20}$$

The cumulated deposited IF is thus

$$S_t^N = \beta S_t^G \tag{21}$$

With the assumption used here (that releases from the free sector is possible only after m years while releases with the authorities permission never occur earlier than the year after the allocation) a growing enterprise cannot clear its investment funds completely, even if $q + \delta = 1$. The accumulation of funds is evidently smallest when $q = 1$ and $\delta = 0$, other things being equal. In that case $S_t^G = A_t^G$.

Determination of the Current Tax Ratio

18. We are now ready to determine the firm's effective tax rate in a situation with an operating investment funds system. Introduction of what is here called the tax ratio is postponed to section 22. As pointed out earlier the growth rate is assumed to be constant as far as the free sector is concerned.

Taxable profit W_t is gross profit less deductible depreciation allowances, allocations to investment funds and - in years with releases from the tied sector - investment deductions. According to assumption 9° current depreciations allowances D_t^d are

$$D_t^d = dC_t \tag{22}$$

Index d indicates that these allowances are deductible for income tax purposes. We may thus write

$$W_t = V_t + bK_t - dC_t - A_t^G - H_{np} \tag{23}$$

In the following the firm's cash payments to the Central Bank in connection with allocations to investment funds are regarded as tax payments while repayments from IF-accounts in the Central Bank are treated as transfer payments (negative taxes). Accordingly the firm's total tax payment in a given year will be

$$T_t = t^N W_t + A_t^N - R_{np}^N - F_t^N \tag{24}$$

t^N is the statutory corporate tax rate. The <u>effective</u> tax rate is defined as

$$t_t^E = T_t / V_t \tag{25}$$

which - after insertion of (23) and (24) - becomes

$$t_t^E = \frac{t^N(V_t + bK_t - dC_t - A_t^G - H_{np}) + A_t^N - R_{np}^N - F_t^N}{V_t} \tag{26}$$

Substitution of equations (1)-(3), (5)-(8), (12), and (13) into (26) gives the following expression for the effective tax rate

$$t_t^E = t^N \left[1 - \alpha - \frac{d-b}{r} + \frac{d}{r}(1 - \frac{C_t}{K_t}) \right] + \alpha\beta -$$

$$- \left[\frac{f_B(t^N\gamma+\beta)(G+b)}{r} \right]_{t=np} - \left[\alpha\beta\delta(1-\theta)^m \right]_{t \geq m} \tag{27}$$

With all terms included (27) is valid for $t = np \geq m$. If withdrawals from the free sector have not yet been possible because $t < m$ then the term inside the paranthesis with the subscript $t \geq m$ is zero. If t is a year without release from the tied sector ($t = np+i$ where $i=1,2,\ldots,p-1$) then the term inside the paranthesis with the subscript $t = np$ is equal to zero. In what follows this notation will be used.

19. In order to develop expression (27) we now have to determine the book value of the capital stock C_t. Since the change of the book value is the firm's gross investment less depreciation allowances C_t can be expressed as

$$C_t = C_{t-1} + I_t^G - D_t^d - D_t^n \tag{28}$$

or, after inserting (9) and (22)

$$(1+d)C_t = C_{t-1} + I_t^G - R_{np}^G - F_t^G \tag{29}$$

From this point on it seems convenient to keep the tied-sector and the free-sector systems apart. The solution of the difference equation (29) for the former respectively the latter system is[1/]

$$\frac{C_{np+i}}{K_{np+i}} = 1 - f_B(1-x-n)\frac{1-z^{n}}{1-z}\, y^i - \frac{u}{1-y}\left[1-y^{i+1} + x(1-y^p)\frac{1-z^{n}}{1-z}\, y^i\right] \qquad (30)$$

$$\frac{C_t}{K_t} = 1 - \frac{u}{1-v}(1-v^{t+1}) - \left[\frac{\alpha\delta r(1-\theta)^m}{\theta+d}\,(1-v^{i+1})\right]_{t \geqq m} \qquad (31)$$

where

$$u = \frac{d-b}{1+d} \qquad (32)$$

$$v = \frac{1-\theta}{1+d} \qquad (33)$$

$$x = \frac{1-G}{1+d} \qquad (34)$$

$$y = \frac{1-g}{1+d} \qquad (35)$$

$$z = xy^{p-1} \qquad (36)$$

The time variable is composed of the following parts

$$m+j = t = np + i \qquad (37)$$

where $i = 0,1,2,\ldots,p-1$; $n = 0,1,2,\ldots$ and $j = 0,1,2,\ldots$. j is thus the number of years which have gone since the <u>first</u> release from the free sector. i, on the other hand, is the number of years which have elapsed since the <u>latest</u> release from the tied sector. As long as no funds whatsoever have been used (in which case t is less than m and p while $n = 0$) and in the absence of normal accelerated depreciation (which means that $d = b$), $C_t = K_t$ as equations (30) and (31) show. With $f_B = \delta = 0$ and a constant rate of growth θ (so that $v = x = y$ and $z = x^p$) any difference between C_t and K_t would be due to accelerated depreciation. In that case both (30) and (31) would be reduced to

$$\frac{C_t}{K_t} = 1 - \frac{d-b}{\theta+d}\left[1 - \left(\frac{1-\theta}{1+d}\right)^{t+i}\right] \qquad (38)$$

1/ For a derivation, see Section A in the appendix to this chapter.

20. Having got the solutions for C_t we may now calculate the effective tax rate. Instead of dealing with t_t^E, however, it appears convenient to consider the <u>tax ratio</u> t_t defined as the effective tax rate as a ratio of the statutory tax rate or

$$t_t = t_t^E / t^N \tag{39}$$

t_t may of course also be seen as the ratio between tax liabilities when depreciations are accelerated (one way or the other) and tax liabilities under a system without acceleration.[1/] We insert in turn (30) and (31) in (27), divide by t^N and get

$$t_{np+i} = 1 - \alpha\left(1 - \frac{\beta}{t^N}\right) - \left[f_B \frac{(G+b)(\gamma + \frac{\beta}{t^N})}{r} \right]_{t=np} +$$

$$+ f_B \frac{d}{r}(1-x-u)\frac{1-z^n}{1-z} y^i - \frac{d-b}{r} + \frac{ud}{r(1-y)} \left[1 - y^{i+1} + x(1-y^p)\frac{1-z^n}{1-z} y^i \right] \tag{40}$$

$$t_t = 1 - \alpha\left(1 - \frac{\beta}{t^N}\right) - \alpha\delta(1-\theta)^m \left[\frac{\beta}{t^N} - \frac{d}{\theta + d}(1 - v^{j+1}) \right]_{t \geq m} -$$

$$- \frac{d-b}{r} + \frac{ud}{r(1-v)}(1 - v^{t+1}) \tag{41}$$

or, for the sake of brevity

$$t_{np+i} = 1 - A - \left[B^I \right]_{t=np} + B_t^{II} - D^I + D_t^{II} \tag{40a}$$

$$t_t = 1 - A - \left[F^I - F_t^{II} \right]_{t \geq m} - D^I + D_t^{II} \tag{41a}$$

In equations (40) and (41) the firm's current tax ratio has been expressed as a function of the parameters of the IF-system α, β, γ, δ, m, p and f_B and the tax rate t^N, the wear-and-tear coefficient b, the depreciation allowance rate d, the profit rate r and the rate of growth. Instead of f_B we may think of q as the authorities' parameter of action. If q is substituted for f_B according to (15) the terms B_I and B_{II} in (40) are changed into

1/ It is also true that $t_t = W_t / V_t$.

$$B^I = \left[q \frac{\alpha(1-G)(\gamma + \frac{\beta}{t^N})\left[1-(1-g)^p\right]}{g} \right]_{t=np} \tag{42}$$

$$B_t^{II} = q \frac{\alpha dx\left[1-(1-g)^p\right](1-z^n)y^i}{g(1-z)} \tag{43}$$

Expressions (42) and (43) contain α but not r and b while the opposite is true for the same terms in equation (40). We thus see that the choice between f_B and q as action parameter determines, in our model, whether or not the firm's tax ratio is an explicit function of the rate of profit and the depreciation rate.

21. In this and the following section the solution for the tax ratio will be discussed in some detail. More precisely we shall interpret the solution and say something about the size of the tax ratio and its development over time after the introduction of the IF-system in year zero.

As is evident from equations (40a) and (41a) each of the two tax ratio expressions consists of six components. Let us start with the tied sector and (40). The second term A shows how allocations change the tax ratio. This effect is of course due to the fact that the deposit rate β is not equal to the rax rate t^N. The third term B^I shows the lowering of the tax ratio every p'th year which is the result of the transfer of funds from the blocked account in the Central Bank and the extra investment deductions in connection with releases from the tied sector. B^I is consequently zero in years without such release (i.e. when $t = np+i$ where $i = 1,2,\ldots,p-1$). The fourth term in (40) B_t^{II} indicates how it affects the current tax ratio that previous releases have reduced the depreciation base (the book value) and thus current depreciation allowances. This term is therefore equal to zero as long as releases have not yet occurred (i.e. when $t < p$ so that $n = o$).

The fifth and sixth terms in (40) reflect the influence on the tax ratio of normal accelerated depreciation and in the absence of such acceleration (i.e. when $d = b$) these terms will of course disappear. D^I represents the reduction of the tax ratio which would have been brought about by the current excess write-down if $C_t = K_t$. D^{II} shows the increasing effect on the tax ratio which is due to the fact that previous accelerated depreciation has reduced the current depreciation base.[1/]

We next come to the tax ratio expression under a free-sector IF-system. Evidently the interpretation of each term in (41) is quite similar to that of the corresponding term in (40). F^I indicates how much the tax ratio is lowered when deposited funds are paid out from the free sector account in the Central Bank. F_t^{II}, on the other hand, reflects the size of the tax ratio which is due to the decrease of the depreciation base following from previous withdrawals from the free sector. F^I and F_t^{II} are of course equal to zero before year m when funds may be withdrawn for the first time.

22. We shall now discuss the development over time of the tax ratio. With normal accelerated depreciation, a constant growth rate and no IF-provisions ($\alpha = f_B = \delta = 0$). The current tax ratio would be

$$t_t = 1 - \frac{d-b}{r(\theta+d)}\left[\theta+d\left(\frac{1-\theta}{1+d}\right)^{t+1}\right] \tag{44}$$

After a sharp decline after the introduction of accelerated depreciation in year zero the tax ratio would thus in this case increase monotonically towards the limiting value $1-\theta(d-b)/r(\theta+d)$ which is less than unity provided $r > \theta$.

The case with a free sector IF-system is similar to accelerated depreciation in the sense that the tax ratio converges. With no accelerated depreciation the tax ratio is equal to 1-A from year zero (when the first allocations are made) to the last year before the first withdrawal (i.e. year m-1), cf. equations (41) and (41a). For t=m we have

$$t_m = 1 - \alpha\left(1-\frac{\beta}{t^N}\right) - \alpha\delta(1-\theta)^m\left(\frac{\beta}{t^N} - \frac{b}{1+b}\right) \tag{45}$$

t_m is smaller than 1-A provided $\beta/t^N > b/(1+b)$. As t increases beyond m, t_t rises monotonically towards the limiting value (indicated by the absence of a t-subscript)

$$t^{FS} = 1 - \left(1-\frac{\beta}{t^N}\right) - \alpha\delta(1-\theta)^m\left(\frac{\beta}{t^N} - \frac{b}{\theta+b}\right) \tag{46}$$

1/ In a given year an excess of current depreciation allowances over wear-and-tear reduces the tax ratio by

$$\frac{dC_t - bK_t}{V_t} \quad \text{or} \quad \frac{(d-b)K_t - d(K_t - C_t)}{rK_t}$$

If the free-sector IF-system is combined with accelerated depreciation the tax ratio still converges towards a limiting value which is given by equation, (41). If this limiting value is denoted t^{FSAD} it can be shown that

$$t^{FSAD} \lesseqgtr t^{FS} \quad \text{as} \quad f_F \lesseqgtr 1 \tag{47}$$

where f_F is defined by (16). In terms of the limiting value of the tax ratio it is in other words advantageous for the firm to combine a free-sector IF-system with conventional accelerated depreciation provided the free sector is smaller than 100 per cent of allocated funds.

Even under a pure <u>tied sector</u> system the tax ratio is $1-A$ as long as no releases have been granted, i.e. from year zero to year $p-1$. If $\beta < t^N$ (as is the case in Sweden but not in the other Nordic countries, cf. Table VII:2 and if the firm for one reason or another chooses not to accept releases of funds, then the tax ratio is constant, less than unity and equal to $1-A$.

How does the tax ratio change after the releases have started? The successively increasing loss of depreciation base in connection with the releases implies a decrease in the book value in relation to the value of the capital stock. That this is so can be seen in equation (30) where the factor $(1-z^n)$ in the second term increases monotonically towards unity as $t \to \infty$ since z is positive and less than one. This means, as (27) and (40) show, that the tax ratio increases over time. The tax ratio does not, however, converge monotonically but fluctuates cyclically around the rising trend. The cyclical fluctuations are of course due to the assumption about periodical releases from the tied sector. In a formal sense the fluctuations can be ascribed to the factor y^i in the term B_t^{II} in equation (40) which gives fluctuations for the whole term. Besides it is only every p'th year that B^I in the same expression is not zero. Under a <u>pure</u> tied sector IF-system and with our assumptions we may therefore say that the tax ratio t_{np+i} $(i=0,1,\ldots,p-1)$ converges towards p different limiting values when t approaches infinity. If $i=0$ the limiting value is

$$t_{(i=0)} = 1 - \alpha\left(1 - \frac{\beta}{t^N}\right) - f_B \frac{G+b}{r} \left[\gamma + \frac{\beta}{t^N} - \frac{b}{(1+b)(1-z)}\right] \tag{48}$$

If $i=1,2,\ldots,p-1$ the limiting values will be

$$t_{(i\neq 0)} = 1 - \alpha\left(1 - \frac{\beta}{t^N}\right) + f_B \frac{b(1-x)y^i}{r(1-z)} \tag{49}$$

Other things being equal the numerical value of $t_{(i\neq 0)}$ is greatest when $i=1$ and smallest when $i=p-1$.

If the tied sector system is combined with accelerated depreciation even the last term in equation (40) would contribute to cyclical fluctuations around a using trend.

The Average Tax Ratio

23. With a fluctuating current tax ratio it is of course somewhat difficult to survey how the tax burden is influenced by the use of the IF-system. As far as monotonically converging systems are concerned one possibility would be to base an evaluation on the limiting value of the tax ratio. An obvious difficulty when alternative limiting values are compared is, however, that the pace at which convergence occur, may differ significantly.

In this section we shall therefore carry the analysis a further step forward through the introduction of a measure which sums up the tax effects of the IF-system. This measure is simply the mean value of the tax ratio. We first calculate the average effective tax rate:

$$\overline{t_T^E} = \frac{\sum_{t=0}^{T} \frac{T_t}{D^t}}{\sum_{t=0}^{T} \frac{V_t}{D^t}} = \frac{\sum_{t=0}^{T} t_t^E \frac{V_t}{D^t}}{\sum_{t=0}^{T} \frac{V_t}{D^t}} . \tag{50}$$

D equals $1+r^d$ where r^d is the discount rate of interest which the firm considers relevant. It should be noted that when this subjective discount rate is determined taxation is taken into account.[1/] We assume that the discount rate is greater than the average rate of growth.[2/] In the numerator of (50) we have the sum of the discounted tax payments - defined in accordance with equation (24) - and the denominator is the sum of the discounted profits. $\overline{t_T^E}$ is thus a weighted average of the firm's effective tax rates between year zero and year T and the weight is for every year the discounted value of profits. If in equation (13) V_t is expressed in terms of K_o the average tax ratio can be written

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1/ The discount rate of interest is also "after" taxes in Johansson's and Edenhammar's study, see [18] p. 60. For a discussion of the relationship between the discount rate of interest before and after taxes, see [18A] p. 72.

2/ Formally this means that $D^p > (\frac{1}{1-G})(\frac{1}{1-g})^{p-1} = (\frac{1}{1-\tilde{\theta}})^p$ where $\tilde{\theta}$ the average rate of growth. If $G=g=\theta$ the condition is $D(1-\theta) > 1$.

$$\bar{t} = \frac{\overline{t_t^E}}{t^N} = \frac{\sum_{t=0}^{T} t_t w_t}{\sum_{t=0}^{T} w_t} \tag{51}$$

where

$$w_t = \left[\frac{1}{D(1-G)}\right]^n \left[\frac{1}{D(1-g)}\right]^{n(p-1)+i} \tag{52}$$

In order to simplify let us assume that the firm's time horizon T is not limited. If we use (40) and (41) to calculate the average tax ratio letting t approach infinity we obtain for the tied-sector system1/

$$\bar{t} = 1 - \alpha(1 - \frac{\beta}{t^N}) - \frac{\alpha q\left[1-(1-g)^p\right]\left[D(1-g)-1\right]}{gD\ D^p(1-g)^p-1}\left[\gamma + \frac{\beta}{t^N} - \frac{Dd}{D(1+d)-1}\right] - \frac{d-b}{r}\left[1 - \frac{Db}{D(1+d)-1}\right] \tag{53}$$

and for the free-sector system1/

$$\bar{t} = 1 - \alpha(1 - \frac{\beta}{t^N}) - \frac{\alpha\delta}{D^m}\left[\frac{\beta}{t^N} - \frac{Dd}{D(1+d)-1}\right] - \frac{d-b}{r}\left[1 - \frac{Dd}{D(1+d)-1}\right] \tag{54}$$

or, for the sake of brevity

$$\bar{t} = 1 - A - \bar{B} - \bar{D} \tag{53a}$$

and

$$\bar{t} = 1 - A - \bar{F} - \bar{D} \tag{54a}$$

In equation (53) q and not f_B is considered as the authorities parameter of action. If f_B is substituted for q in accordance with (15) $\bar{B}$ is changed into

$$\bar{B} = f_B \frac{(G+b)\left[D(1-g)-1\right]}{rD(1-G)\left[D^p(1-g)^p-1\right]}\left[\gamma + \frac{\beta}{t^N} - \frac{Dd}{D(1+d)-1}\right] \tag{53}$$

1/ For a derivation, see Section B in the appendix to this chapter.

If f_B is the parameter of action $\bar{B}$ is a function of b and r but not α. If q is the action parameter the opposite is true. In this respect the B-terms in equation (40) and equation (53) behave not surprisingly in the same manner. It is interesting to point out, however, that with q as the exogenous and f_B as the endogenous variable the mean value calculation entails the elimination of G. This is not the case when f_B is the action parameter. Thus $\bar{B}$ in equation (55) is a function of g and G while $\bar{B}$ according to (53) contains only the growth rate g. It should also be observed that the growth variable disappears from the free sector expression when the tax ratio is transformed into the average tax ratio, cf. equations (41) and (54).

24. In this section some comments to the solutions for the average tax ratio will be made. When equations (53) and (54) are seen in relation to (40) and (41) it is obvious that $\bar{B}$ is the mean value of $(B_t^I - B_t^{II})$ and reflects the impact of tied sector releases on the average tax ratio. $\bar{F}$, on the other hand is the mean value of $(F_t^I - F_t^{II})$ and shows the effects of withdrawals from the free sector. $\bar{D}$, finally, indicates the effects of conventional accelerated depreciation in the absence of an IF-system and is thus the mean value of $(D^I - D_t^{II})$. It should be stressed, however, that the presence or absence of accelerated depreciation is of importance for the extent to which the IF-system influences the tax ratio. This is easily seen in equation (53) and (54). The depreciation allowance rate d affects $\bar{B}$ and $\bar{F}$ only through the term $E = Dd/(D+Dd-1)$ which varies positively with d. This means - all other things being equal - that the reduction of the average tax ratio brought about by the IF-system is smaller the greater the depreciation allowance rate is (or - given the value of the depreciation coefficient - the greater the absolute acceleration d-b is).

D is positive provided $d > b$. Concerning the size of A, $\bar{B}$ and $\bar{F}$ we see that

$$A \gtreqless 0 \text{ as } \frac{\beta}{{}_tN} \lesseqgtr 1 \qquad (56)$$

$$\bar{B} \gtreqless 0 \text{ as } \gamma + \frac{\beta}{{}_tN} \gtreqless \frac{Dd}{D(1+d)-1} \qquad (57)$$

$$\bar{F} \gtreqless 0 \text{ as } \frac{\beta}{{}_tN} \gtreqless \frac{Dd}{D(1+d)-1} \qquad (58)$$

Given our assumptions use of the IF-system will evidently lead to an average effective tax rate below the statutory tax rate if A, $\overline{B}$ and $\overline{F}$ are all positve. Needless to say, this condition is sufficient but not necessary. In Sweden at present $\beta = 0.46$, $\gamma = 0.10$ and $t^N = 0.544$ (national average). This means that $\beta/t^N = 0.846$ and (as mentioned earlier) that $A > 0$. The term E on the right side of condition (57) and (58) is positive and less than one. As we have seen $dE/dd > 0$. Furthermore $dE/dD < 0$. Thus the smaller d is and the greater D is, the more probable it is, other things being equal, that the effective tax rate is less than the statutory tax rate. If $d = b = 0.30$ and $D = 1.10$ then $E = 0.77$. With these parameter values the average effective tax rate for a growing firm is less than the statutory tax rate whatever numerical values are chosen for the other parameters appearing in equations (53) and (54). All parameters are of course important in determining the <u>size</u> of the average effective tax rate.

Another (and less special) statement that can be made is the following. Since we have assumed that $D > 1/(1-\bar{\theta})$ - where $\bar{\theta}$ is the average rate of growth, cf. note on p. VIII:14 - A, $\overline{B}$ and $\overline{F}$ are positive if the following condition is fulfilled

$$\frac{d}{\bar{\theta}+d} < \frac{\beta}{t^N} < 1 \qquad (59)$$

Since $d/(\bar{\theta}+d)$ is larger than the term E provided $D > 1/(1-\bar{\theta})$, conditions (57) and (58) are fulfilled if (59) is valid.

As is evident (59) is a sufficient but not a necessary condition for $\bar{t}_t^E < t^N$. A very interesting implication of condition (59) is that the higher the rate of growth is the more probable it is that the use of the IF-system leads to an average effective tax rate which is lower than the nominal tax rate. If for instance $d = 0.30$, $\bar{\theta} = 0.07$, and $\beta/t^N = 0.846$ condition (59) is fulfilled.

25. One may ask if it would be possible to increase the tax benefits from the IF-system by investing more than otherwise in years in which releases are granted. We shall look at two different situations. In the first place it is assumed that the rate of growth in release years is increased from θ to G without changing the rate of growth in other years. Secondly we consider the case where the increased growth rate in release years is matched by a reduction of the growth in other years which leaves the growth rate for the full IF-cycle of p years

unchanged. In the latter case the average growth rate $\bar{\theta}$ is determined by

$$(1-\bar{\theta}) = (1-G)(1-g)^{p-1} \qquad (60)$$

Let f_B and not q be the authorities parameter of action. As appears from equations (53) and (55) it is only the $\bar{B}$-term of the average tax ratio which depends on the rate of growth. We can therefore use (55) to discuss the two situations. The case with an increased average growth rate is very straightforward. If the initial growth rate is θ an increase of the growth rate in release years would result in a $\bar{B}$-value which would be $(G+b(1-\theta)/(\theta+b)(1-G)$ times larger than the initial $\bar{B}$-value. Since this factor is larger than unity the increased growth rate would lead to a lower average tax ratio.

The same would be true in the second case with a changed time pattern for investments but unchanged average growth rate. This has been demonstrated in section C in the mathematical appendix to this chapter. We have thus seen that - whether the average growth rate is increased or not - a concentration of the firm's investments to release years will lower the tax burden. The IF-system provides in other words incentives to change the time pattern of private investments in accordance with the authorities' wishes and to increase the growth rate. Needless to say a number of other circimstances will have to be considered in order to evaluate past developments and make predictions about the future.

26. In the formal analysis we ended up with expressions showing how the current and average tax ratios depend upon the parameters of the model. Tax ratios are, however, somewhat indirect indicators of profitability effects. It may therefore be helpful to relate the tax ratio to the firm's profitability after taxes. To answer the question how the firm's profitability after taxes is changed by the use of the IF-system knowledge about the firm's after-tax-profitability in the absence of the IF-system is of course needed. It is assumed that the only alternative to making IF-allocations is taxation. If $\bar{r}_a$ and $\bar{r}_a^*$ denote the average rate of profit after taxes when the IF-system is used, respectively when it is not used, we have

$$\bar{r}_a = r(1-\bar{t}^E) \qquad (61)$$

and

$$\bar{r}_a^* = r(1-t^N) \qquad (62)$$

The relative change of profitability after taxes which is due to the working of the IF-system is thus

$$\frac{\bar{r}_a - \bar{r}_a^*}{\bar{r}_a^*} = \frac{t^N}{1-t^N}(1-\bar{t}) \qquad (63)$$

Alternatively (63) may be said to indicate the present value of all tax saving due to the use of the IF-system measured as a percentage of the present value of profits after taxes, With a statutory tax rate of approximately 50 per cent the after-tax profitability effect would (in relative terms) simply be $1-\bar{t}$. Quite analogous expressions may of course be obtained in terms of the corresponding current variables.

APPENDIX III

MATHEMATICAL APPENDIX TO CHAPTER VIII

A. Determination of the Book Value

a. The Tied Sector

1. The determination of the book value for the tied sector IF-system is complicated a little bit by our assumption of a non-constant growth rate. As a result of this the difference equation (29) - when written in the ratio form C_t/K_t - is bound to contain "constant" terms which are functions of time. The rate of growth is constant, however, in any p-year period from one release to another, as equation (12) shows. We shall consider this fact when we solve (29). We thus express the book value C_t at the end of year np as a function of $C_{(n-1)p}$ p years before, gross investments and depreciations during the p years and the release in year np or

$$C_{np} = C_{(n-1)p} + \sum_{i=1}^{p-1} I^G_{(n-1)p+i} + I^G_{np} - d \sum_{i=1}^{p-1} C_{(n-1)p+i} - dC_{np} - R^C_{np} \qquad (1^o)$$

Divide by K_{np} and let Y_{np+i} represent the ratio C_{np+i}/K_{np+i}. After having considered (3), (11) and

$$I^G_{np+i} = (g+b)K_{np+i} \qquad (i \neq 0) \qquad (2^o)$$

(1^o) may be written

$$Y_{np} = Y_{(n-1)p} \frac{(1-G)(1-g)^{p-1}}{1+d} + \frac{g+b}{1+d} \frac{\sum_{i=1}^{p-1} K_{(n-1)p+i}}{K_{np}} + \frac{G+b}{1+d}(1-f_B) -$$
$$- \frac{d}{1+d} \frac{\sum_{i=1}^{p-1} C_{(n-1)p+i}}{K_{np}}$$

or, when (32), (34) and (35) are taken into account

$$Y_{np} = Y_{(n-1)p} xy^{p-1}(1+d)^{p-1} + (1-y-u) \frac{\sum_{i=1}^{p-1} K_{(n-1)p+i}}{K_{np}} +$$
$$+ (1-x-u)(1-f_B) - \frac{d}{1+d} \frac{\sum_{i=1}^{p-1} C_{(n-1)p+i}}{K_{np}} \qquad (3^o)$$

2. Calculate the first sum in (3^o) while taking into account equation (12) and

$$K_{(n-1)p+i} = \frac{K_{(n-1)p}}{(1-g)^i} \qquad (i \neq 0) \qquad (4^o)$$

We thus get

$$\sum_{i=1}^{p-1} K_{(n-1)p+i} = K_{np} \frac{(1-G)\left[1 - (1-g)^{p-1}\right]}{g} \qquad (5^o)$$

3. The next step is to calculate the second sum in (3^o). From (28) it follows that

$$C_{(n-1)p+i} = C_{(n-1)p+i-1} + I^G_{(n-1)p+i} - dC_{(n-1)p+i} \qquad (6^o)$$

Divide by $K_{(n-1)p+i}$ and consider (2^o) and $K_{(n-1)p+i}(1-g) = K_{(n-1)p+i-1}$ (since $i \neq 0$) and we get

$$Y_{(n-1)p+i} = Y_{(n-1)p+i-1} \frac{1-g}{1+d} + \frac{g+b}{1+d}$$

or after using (32), (34) and (35)

$$Y_{(n-1)p+i} = yY_{(n-1)p+i-1} + (1-y-u) \qquad (7^o)$$

(7^o) is a difference equation of the first order with the solution

$$Y_{(n-1)p+i} = Y_{(n-1)p}y^i + (1-y-u) \frac{1-y^i}{1-y} \qquad (8^o)$$

In order to get $C_{(n-1)p+i}$ we multiply by $K_{(n-1)p+i}$. We also consider (4^o) and (35). We get

$$C_{(n-1)p+i} = k^* \left(\frac{1}{1+d}\right)^i + \frac{1-y-u}{1-y} K_{(n-1)p} \left(\frac{1}{1-y}\right)^i \qquad (9^o)$$

where

$$k^{*} = C_{(n-1)p} - \frac{1-y-u}{1-y} K_{(n-1)p} \qquad (10^{o})$$

We now calculate the sum or

$$\sum_{i=1}^{p-1} C_{(n-1)p+i} = k^{*} \frac{1 - \left(\frac{1}{1+d}\right)^{p-1}}{d} + K_{(n-1)p} \frac{1-y-u}{1-y} \frac{\left(\frac{1}{1-g}\right)^{p-1} - 1}{g} \qquad (11^{o})$$

Multiply by $d/(1+d)$ and divide by $K_{np} = K_{(n-1)p}/(1-G)(1-g)^{p-1}$ and we obtain

$$\frac{d}{1+d} \frac{\sum_{i=1}^{p-1} C_{(n-1)p+i}}{K_{np}} = \frac{k^{*}}{K_{(n-1)p}} \frac{\left[1 - \left(\frac{1}{1+d}\right)^{p-1}\right](1-G)(1-g)^{p-1}}{1 + d} +$$

$$+ \frac{1-y-u}{1-y} \frac{1 - (1-g)^{p-1}}{g} \frac{d(1-G)}{1+d}$$

or after using (10^{o}), (34) and (35)

$$\frac{d}{1+d} \frac{\sum_{i=1}^{p-1} C_{(n-1)p+i}}{K_{np}} = \frac{C_{(n-1)p}}{K_{(n-1)p}} xy^{p-1} \left[(1+d)^{p-1} - 1\right] +$$

$$+ \frac{x(1-y-u)}{1-y} \left[d \frac{1 - (1-g)^{p-1}}{g} - (1-g)^{p-1} + y^{p-1}\right] \qquad (12^{o})$$

4. We now insert (5^{o}) and (12^{o}) into (3^{o}) and get

$$Y_{np} = Y_{(n-1)p} xy^{p-1} + (1-y-u) \frac{(1-G)\left[1 - (1-g)^{p-1}\right]}{g} + (1-x-u)(1-f_{B}) -$$

$$- \frac{x(1-y-u)}{1-y} \left[d \frac{1 - (1-g)^{p-1}}{g} - (1-g)^{p-1} + y^{p-1}\right]$$

or

$$Y_{np} = Y_{(n-1)p}xy^{p-1} + \frac{1-y-u}{1-y} x\left[1 - (1-g)^{p-1}\right] -$$

$$- \frac{1-y-u}{1-y} x\left[y^{p-1} - (1-g)^{p-1}\right] + (1-x-u)(1-f_B)$$

or since $z = xy^{p-1}$

$$Y_{np} = Y_{(n-1)p}z + \phi_1 \qquad (13^o)$$

where

$$\phi_1 = (1 - \frac{u}{1-y})(x-z) + (1-x-u)(1-f_B) \qquad (14^o)$$

The solution of (13^o) is

$$Y_{np} = Y_0 z^n + \phi_1 \frac{1-z^n}{1-z}$$

or since according to (29) and (32) $Y_0 = (1+b)/(1+d) = 1 - u$

$$Y_{np} = (1-u)z^n + \phi_1 \frac{1-z^n}{1-z} \qquad (15^o)$$

5. We are now able to determine Y_{np+i}. From (8^o) it follows that

$$Y_{np+i} = Y_{np}y^i + (1-y-u) \frac{1-y^i}{1-y} \qquad (16^o)$$

or after inserting (14^o) and (15^o)

$$Y_{np+i} = (1-u)y^i z^n + \frac{1-z^n}{1-z} y^i \left[(1 - \frac{u}{1-y})(x-z) + (1-x-u)(1-f_B)\right] +$$

$$+ (1 - \frac{u}{1-y})(1-y^i) \qquad (17^o)$$

or after dissolving the two last terms in (17^o)

$$Y_{np+i} = 1 - f_B(1-x-u) \frac{1-z^n}{1-z} y^i + (1-u)y^i z^n + \frac{1-z^n}{1-z} y^i (x-z+1-x) -$$

$$- \frac{1-z^n}{1-z} y^i\left[(x-z) \frac{u}{1-y} + u\right] - y^i - \frac{u}{1-y} (1-y^i)$$

or

$$Y_{np+i} = 1 - f_B(1-x-u)\,\frac{1-z^n}{1-z}\,y^i + (1-u)y^i z^n + (1-z^n)y^i -$$

$$- \frac{1-z^n}{1-z}\,y^i\,\frac{u}{1-y}\,(x-z+1-y) - y^i - \frac{u}{1-y}\,(1-y^i) \qquad (18^o)$$

Terms number three, four and six in (18^o) are reduced to $- uy^i z^n$ and after dissolving term five we get

$$Y_{np+i} = 1 - f_B(1-x-u)\,\frac{1-z^n}{1-z}\,y^i - uy^i z^n - \frac{1-z^n}{1-z}\,y^i\,\frac{u}{1-y}\,(x-y) -$$

$$- (1-z^n)\,y^i\,\frac{u}{1-y} - \frac{u}{1-y}\,(1-y^i)$$

or

$$Y_{np+i} = 1 - f_B(1-x-u)\,\frac{1-z^n}{1-z}\,y^i - \frac{u}{1-y}\left[y^i z^n(1-y) + \frac{1-z^n}{1-z}\,y^i(x-y)+(1-z^n)y^i+1-y^i\right]$$

or

$$Y_{np+i} = 1 - f_B(1-x-u)\,\frac{1-z^n}{1-z}\,y^i - \frac{u}{1-y}\left[1 - y^{i+1}z^n + \frac{1-z^n}{1-z}\,y^i(x-y)\right]$$

Add and subtract y^{i+1} within the bracket and we finally get

$$Y_{np+i} = 1 - f_B(1-x-u)\,\frac{1-z^n}{1-z}\,y^i - \frac{u}{1-y}\left[1 - y^{i+1} + x(1-y^p)\,\frac{1-z^n}{1-z}\,y^i\right] \quad (19^o)$$

(19^o) is identical with (30).

Ab. Determination of the Book Value. The Free Sector

6. This case is quite straightforward since the rate of growth is assumed to be constant. Until the first withdrawal is made in year m, C_t will differ from K_t only because of accelerated depreciation. We therefore calculate the book value for the last year before the first withdrawal or rather the ratio $Y_{m-1} = C_{m-1}/K_{m-1}$. From (29) it follows that for $t < m$

$$(1+d)C_t = C_{t-1} + I_t^G$$

Since $I_t^G = (\theta+b)K_t$ and $K_{t-1} = (1-\theta)K_t$ we have after considering (32) and (33)

$$Y_t = vY_{t-1} + (1-v-u) \qquad (20^o)$$

The solution of this difference equation for year m-1 is

$$Y_{m-1} = Y_0 v^{m-1} + (1-v-u)\,\frac{1-v^{m-1}}{1-v} \qquad (21^o)$$

Since according to (29) $Y_0 = (1+b)/(1+d) = 1-u$ (21°) may be reduced to

$$Y_{m-1} = 1 - u\,\frac{1-v^m}{1-v} \qquad (22^o)$$

7. For $t \geqq m$ - or what is the same $j \geqq 0$ cf. (37) - we have according to (29), (7), (1) and (13)

$$(1+d)C_j = C_{j-1} + I_j^G - \alpha\delta r K_j (1-\theta)^m$$

which is changed into

$$Y_j = vY_{j-1} + \phi_2 \qquad (23^o)$$

where

$$\phi_2 = 1-v-u - \frac{\alpha\delta r(1-\theta)^m}{1+d} \qquad (24^o)$$

The solution of the difference equation (23°) is (since the number of years between year m-1 and year j is j+1)

$$Y_j = Y_{m-1}v^{j+1} + \phi_2\,\frac{1-v^{j+1}}{1-v}$$

which after insertion of (22°) and (24°) is reduced to

$$Y_j = 1 - u\,\frac{1-v^{t+1}}{1-v} - \frac{\alpha\delta r(1-\theta)^m}{\theta+d}\,(1-v^{j+1}) \qquad (25^o)$$

(25°) is identical with (31).

B. Calculation of the Average Tax Ratio

a. The Tied Sector

8. According to (50) we have

$$\bar{t} = \frac{\overline{t_t^E}}{t^N} = \frac{\sum_{t=0}^{\infty} w_t t_t}{\sum_{t=0}^{\infty} w_t} \qquad (26^{o})=(50)$$

As weight we use the discounted value of profits or

$$w_t^* = \frac{V_t}{D^t} \qquad (27^{o})$$

where $D = 1+r^d$ is the discount factor and r^d the relevant discount rate of interest. We get

$$w_t^* = \frac{rK_t}{D^t} = \frac{r}{D^{np+i}} \frac{K_0}{(1-G)^n (1-g)^{n(p-1)+i}}$$

or

$$w_t^* = rK_0 \left[\frac{1}{D(1-G)}\right]^n \left[\frac{1}{D(1-g)}\right]^{n(p-1)+i}$$

or

$$w_t^* = rK_0 A^n B^{n(p-1)+i} \qquad (28^{o})$$

where

$$A = \frac{1}{D(1-G)} \qquad (29^{o})$$

$$0 < B = \frac{1}{D(1-g)} < 1 \qquad (30^{o})$$

and $0 < AB^{p-1} < 1$ so that

$$\bar{t} = \frac{\sum_{t=0}^{\infty} A^n B^{n(p-1)+i} t_t}{\sum_{t=0}^{\infty} A^n B^{n(p-1)+i}} \qquad (31^{o})$$

9. Calculate the sum of the weights $w_t = w_t^*/rK_0$:

$$\sum_{t=0}^{\infty} w_t = \sum_{t=0}^{\infty} A^n B^{n(p-1)+i} = \sum_{t=0}^{\infty} (AB^{p-1})^n B^i = \sum_{n=0}^{\infty} \sum_{i=0}^{p-1} (AB^{p-1})^n B^i =$$

$$= \sum_{n=0}^{\infty} (AB^{p-1})^n \sum_{i=0}^{p-1} B^i.$$

$$\sum_{t=0}^{\infty} w_t = \frac{1}{1 - AB^{p-1}} \frac{1 - B^p}{1 - B} \qquad (32^o)$$

or after inserting (29^o) and (30^o)

$$\sum_{t=0}^{\infty} w_t = \frac{D(1-G)\left[D^p(1-g)^p - 1\right]}{\left[D(1-g)-1\right]\left[D^p(1-G)(1-g)^{p-1}-1\right]} \qquad (33^o)$$

10. The current tax ratio is

$$t_{np+i} = 1 - A - \left[B^I\right]_{t=np} + B_t^{II} - D^I + D_t^{II} \qquad (34^o)=(40a)$$

which inserted into (26^o) gives

$$\bar{t} = \frac{\sum_{t=0}^{\infty} w_t(1 - A - \left[B^I\right]_{t=np} + B_t^{II} - D^I + D_t^{II})}{\sum_{t=0}^{\infty} w_t}$$

or

$$\bar{t} = 1 - A - D^I - \frac{\sum_{n=1}^{\infty} w_{np} B^I - \sum_{t=p}^{\infty} w_t B_t^{II} - \sum_{t=0}^{\infty} w_t D_t^{II}}{\sum_{t=0}^{\infty} w_t} \qquad (35^o)$$

We shall in turn calculate each of the three terms which depend on time.

11. The weighted average of B^I is

$$\sum_{n=1}^{\infty} w_{np} B^I = B^I \sum_{n=1}^{\infty} (AB^{p-1})^n = B^I (AB^{p-1}) \sum_{n=1}^{\infty} (AB^{p-1})^{n-1} =$$

$$= B^I (AB^{p-1}) \frac{1}{1 - AB^{p-1}} \qquad (36^o)$$

The weighted average of B^I is thus (36^o) divided by (32^o) or

$$\overline{B^I} = B^I \frac{(1-B)AB^{p-1}}{1 - B^p} \qquad (37^o)$$

We insert (29^o) and (30^o) into (37^o) and obtain

$$\overline{B^I} = B^I \frac{D(1-g) - 1}{D(1-G)\left[D^p(1-g)^p - 1\right]} \qquad (38^o)$$

Using (42) $\overline{B^I}$ becomes

$$\overline{B^I} = \frac{\alpha q(\gamma + \frac{\beta}{N_t})\left[1 - (1-g)^p\right]\left[D(1-g) - 1\right]}{gD\left[D^p(1-g)^p - 1\right]} \qquad (39^o)$$

12. The weighted average of B_t^{II}. According to (40) B_t^{II} may be written

$$B_t^{II} = k_1(1-z^n)y^i \qquad (40^o)$$

where

$$k_1 = f_B \frac{d(1-x-u)}{r(1-z)} \qquad (41^o)$$

$w_t B_t^{II}$ is thus

$$w_t B_t^{II} = k_1 (AB^{p-1})^n B^i (1-z^n) y^i$$

or

$$w_t B_t^{II} = k_1\left[(AB^{p-1})^n - (zAB^{p-1})^n\right](yB)^i \qquad (42^o)$$

so that

$$\sum_{t=p}^{\infty} w_t B_t^{II} = k_1 \sum_{n=1}^{\infty} \left[(AB^{p-1})^n - (zAB^{p-1})^n \right] \sum_{i=0}^{p-1} (yB)^i =$$

$$= k_1 \left[\frac{AB^{p-1}}{1 - AB^{p-1}} - \frac{zAB^{p-1}}{1 - zAB^{p-1}} \right] \frac{1 - (yB)^p}{1 - yB} =$$

$$= k_1 \frac{(1-z)AB^{p-1}}{(1-AB^{p-1})(1-zAB^{p-1})} \frac{1 - (yB)^p}{1 - yB}$$

Since

$$Ax = By \tag{43°}$$

it follows that $(1-zAB^{p-1}) = (1-y^p B^p)$ so that

$$\sum_{t=p}^{\infty} w_t B_t^{II} = k_1 \frac{(1-z)AB^{p-1}}{(1-AB^{p-1})(1-yB)} \tag{44°}$$

The weighted average of B_t^{II} is thus (44°) divided by (32°) or

$$\overline{B_t^{II}} = k_1 \frac{(1-z)AB^{p-1}}{1-yB} \frac{1 - B}{1 - B^p} \tag{45°}$$

or, after using (29°), (30°) and (35)

$$\overline{B_t^{II}} = k_1(1-z) \frac{(1+d)\left[D(1-g) - 1\right]}{(1-G)\left[D(1+d) - 1\right]\left[D^p(1-g)^p - 1\right]} \tag{46°}$$

From (15) and (41°) follows that

$$k_1 = \frac{\alpha q d(1-G)\left[1 - (1-g)^p\right]}{g(1-z)(1+d)} \tag{47°}$$

which inserted into (46°) gives

$$\overline{B_t^{II}} = \frac{\alpha q\left[1 - (1-g)^p\right]}{g} \frac{d\left[D(1-g) - 1\right]}{\left[D(1+d) - 1\right]\left[D^p(1-g)^p - 1\right]} \tag{48°}$$

13. The weighted average of D_t^{II}. According to (40) D_t^{II} may be written

$$D_t^{II} = k_2 + k_2 \ (k_3-y)y^i - k_2 k_3 z^n y^i \tag{49°}$$

where

$$k_2 = \frac{ud}{r(1-y)} \tag{50°}$$

$$k_3 = \frac{x(1-y^p)}{1-z} \tag{51°}$$

so that

$$\sum_{t=0}^{\infty} w_t D_t^{II} = k_2 \sum_{t=0}^{\infty} w_t + k_2(k_3-y) \sum_{t=0}^{\infty} y^i w_t - k_2 k_3 \sum_{t=0}^{\infty} y^i z^n w_t \tag{52°}$$

We calculate the last two terms in turn:

$$L_2 = k_2(k_3-y) \sum_{t=0}^{\infty} y^i w_t = k_2(k_3-y) \sum_{t=0}^{\infty} (AB^{p-1})^n (yB)^i =$$

$$= k_2(k_3-y) \sum_{n=0}^{\infty} (AB^{p-1})^n \sum_{i=0}^{p-1} (yB)^i$$

$$L_2 = k_2(k_3-y) \frac{1}{1-AB^{p-1}} \frac{1-y^p B^p}{1-yB} \tag{53°}$$

$$L_3 = k_2 k_3 \sum_{t=0}^{\infty} z^n y^i w_t = k_2 k_3 \sum_{t=0}^{\infty} (zAB^{p-1})^n (yB)^i =$$

$$= k_2 k_3 \sum_{n=0}^{\infty} (zAB^{p-1})^n \sum_{i=0}^{p-1} (yB)^i$$

$$L_3 = k_2 k_3 \frac{1}{1-zAB^{p-1}} \frac{1-y^p B^p}{1-yB} \tag{54°}$$

As mentioned earlier it follows from (43°) that $(1-zAB^{p-1}) = (1-y^pB^p)$ and (54°) may be reduced to

$$L_3 = k_2k_3 \frac{1}{1-yB} \tag{55°}$$

The weighted average of D_t^{II} is obtained when (53°) and (55°) are inserted into (52°) which is then divided by (32°). We get:

$$\overline{D_t^{II}} = k_2 + \frac{k_2(k_3-y)\dfrac{1-y^pB^p}{(1-AB^{p-1})(1-yB)} - k_2k_3\dfrac{1}{1-yB}}{\dfrac{1-B^p}{(1-AB^{p-1})(1-B)}}$$

or

$$\overline{D_t^{II}} = k_2 + k_2\frac{1-B}{(1-yB)(1-B^p)}\left[(k_3-y)(1-y^pB^p) - k_3(1-AB^{p-1})\right] \tag{56°}$$

The expression within brackets in (56°) may be written

$$[\quad] = k_3B^{p-1}(A-y^pB) - y(1-y^pB^p)$$

Since according to (43°) $A-y^pB = A(1-z)$ we get after insertion of (51°)

$$[\quad] = x(1-y^p)AB^{p-1} - y(1-y^pB^p) = xAB^{p-1} - y - y^pB^{p-1}(xA-yB)$$

or, considering (43°)

$$[\quad] = -\ y(1-B^p) \tag{57°}$$

which inserted into (56°) gives

$$\overline{D_t^{II}} = k_2\frac{1-y}{1-yB} \tag{58°}$$

After combining (58°) with (30°), (50°), (32) and (35) we obtain

$$\overline{D_t^{II}} = \frac{d-b}{r} \frac{Dd}{D(1+d) - 1} \tag{59°}$$

14. We now add terms together to get the average tax ratio (for the tied sector). The average tax ratio according to (35°) may be written

$$\bar{t} = 1 - A - (\overline{B^I - B_t^{II}}) - (D^I - D_t^{II}) \tag{60°}$$

We combine (60°) with (39°), (48°) and (59°) and get

$$\bar{t} = 1-A- \frac{\alpha q\left[1-(1-g)^p\right]}{gD} \frac{D(1-g)-1}{D^p(1-g)^p-1}\left[\gamma+\frac{\beta}{t^N} - \frac{Dd}{D(1+d)-1}\right] - \frac{d-b}{r}\left[1- \frac{Dd}{D(1+d)-1}\right] \tag{61°}$$

(61°) is identical with (52).

Bb. Calculation of the Average Tax Ratio. The Free Sector

15. The average tax ratio is

$$\bar{t} = 1-A-D^I- \frac{\sum_{t=m}^{\infty} F^I w_t - \sum_{t=m}^{\infty} F_t^{II} w_t - \sum_{t=0}^{\infty} D_t^{II} w_t}{\sum_{t=0}^{\infty} w_t} \tag{62°}$$

In this case the sum of the weights becomes, cf. (32°)

$$\sum_{t=0}^{\infty} w_t = \frac{1}{1-E} \tag{63°}$$

where

$$0 < E = \frac{1}{D(1-\theta)} < 1 \tag{64°}$$

We thus have

$$\sum_{t=0}^{\infty} w_t = \frac{D(1-\theta)}{D(1-\theta) - 1} \tag{65°}$$

16. The weighted average of F^I. According to (41) we have for $t \geqq m$

$$F^I = \alpha\delta(1-\theta)^m \frac{\beta}{t^N} \tag{66°}$$

$$\sum_{t=m}^{\infty} F^I w_t = F^I \sum_{t=m}^{\infty} E^t = F^I \frac{E^m}{1-E} \tag{67°}$$

$\overline{F^I}$ is thus (67°) divided by (63°). After considering (64°) we get

$$\overline{F^I} = \frac{\alpha\delta \frac{\beta}{t^N}}{D^m} \tag{68°}$$

17. The weighted average of F_t^{II}. According to (41) we have for $t \geqq m$

$$F_t^{II} = k_4(1-v^{j+1}) = k_4(1-\frac{1}{v^{m-1}} v^t) \tag{69°}$$

where

$$k_4 = \alpha\delta(1-\theta)^m \frac{d}{\theta+d} \tag{70°}$$

Thus

$$\sum_{t=m}^{\infty} F^{II} w_t = k_4\left[\sum_{t=m}^{\infty} E^t - \frac{1}{v^{m-1}} \sum_{t=m}^{\infty} (vE)^t\right] = k_4\left[\frac{E^m}{1-E} - \frac{1}{v^{m-1}} \frac{v^m E^m}{1-vE}\right]$$

or

$$\sum_{t=m}^{\infty} F^{II} w_t = k_4 \frac{(1-v)E^m}{(1-E)(1-vE)} \tag{71°}$$

We divide (71°) by (63°) and obtain after using (64°) and (70°)

$$\overline{F_t^{II}} = \frac{\alpha\delta}{D^m} \frac{Dd}{D(1+d) - 1} \tag{72°}$$

18. The weighted average of D_t^{II}. As (41) shows

$$D_t^{II} = \frac{ud}{r(1-v)} (1-v^{t+1}) \tag{73°}$$

so that

$$\sum_{t=0}^{\infty} D_t^{II} w_t = \frac{ud}{r(1-v)} \left[\sum_{t=0}^{\infty} E^t - v \sum_{t=0}^{\infty} (vE)^t\right] = \frac{ud}{r(1-v)}\left[\frac{1}{1-E} - \frac{v}{1-vE}\right]$$

or

$$\sum_{t=0}^{\infty} D_t^{II} w_t = \frac{ud}{r} \frac{1}{(1-E)(1-vE)} \tag{74°}$$

We insert (32), (33) and (64°) and divide by (63°) to get $\overline{D_t^{II}}$.

$$\overline{D_t^{II}} = \frac{d-b}{r} \frac{Dd}{D(1+d) - 1} \tag{75°}$$

19. We now add terms together to get the average tax ratio (for the free sector system). The average tax ratio according to (62°) may be written

$$\bar{t} = 1-A-(\overline{F^I}-\overline{F_t^{II}}) - (D^I-\overline{D_t^{II}}) \tag{76°}$$

We insert (68°), (72°) and (75°) and get

$$\bar{t} = 1-A- \frac{\alpha\delta}{D^m}\left[\frac{\beta}{t^N} - \frac{Dd}{D(1+d)-1}\right] - \frac{d-b}{r}\left[1 - \frac{Dd}{D(1+d)-1}\right] \tag{77°}$$

(77°) is identical with (53).

C. The Effect of a Changed Growth Pattern

20. In this section we shall compare the average tax ratios resulting from a constant and a non-constant growth rate when the growth rate for the full IF-cycle of p years is the same in the two cases. The latter condition implies the following relationship between the growth rates $g<\theta<G$:

$$(1-\theta)^p = (1-G)(1-g)^{p-1} \qquad (78^o)$$

We assume that f_B and not q is the authorities' parameter of action. As appears from equations (53) and (55) only the $\overline{B}$-term of the average tax ratio depends on the growth rate. We can therefore use equation (55) to make the comparison. Let $\overline{B}_{Gg}$ and $\overline{B}_\theta$ denote the value of $\overline{B}$, when the growth rate is non-constant, respectively constant. Using (55), the ratio $\overline{B}_{Gg}/\overline{B}$ may now be written

$$\frac{\overline{B}_{Gg}}{\overline{B}_\theta} = \frac{(G+b)(1-\theta)}{(\theta+b)(1-G)}\left(\frac{1+D(1-\theta)+...+D^i(1-\theta)^i+...+D^{p-1}(1-\theta)^{p-1}}{1+D(1-g)+...+D^i(1-g)^i+...+D^{p-1}(1-g)^{p-1}}\right) \qquad (79^o)$$

The numerator of the expression within brackets is now multiplied by $1-\theta$ while the denominator is multiplied by 1-G. This makes the last term in the numerator and the denominator equal, cf. (78^o). We now compare terms number i+1 in the numerator and the denominator by calculating their difference d_{i+1}.

$$d_{i+1} = D^i\left((1-\theta)^{i+1} - (1-G)(1-g)^i\right) \qquad (80^o)$$

or

$$d_{i+1} = \frac{D^i}{(1-g)^{p-1-i}}\left((1-\theta)^{i+1}(1-g)^{p-1-i} - (1-G)(1-g)^{p-1}\right) \qquad (81^o)$$

From condition (78^o) and (81^o) it is now evident that $d_1, d_2, \ldots, d_{i+1}, \ldots, d_{p-1}$ are all positive. Since it is also true that $(G+b)/(\theta+b)>1$ it follows that

$$\overline{B}_{Gg} > \overline{B}_\theta \qquad (82^o)$$

If the firm increases its growth rate in years with releases and lowers it in other years without changing the growth rate over the IF-cycle, this would result in a reduced average tax ratio.

CHAPTER IX

SIMULATIONS WITH THE MODEL. THE TIED SECTOR

27. The formal analysis carried out so far could not, of course, give the reader a precise idea of the extent to which the IF-system is capable of reducing the tax burden of participating firms. In this chapter as well as in the two following chapters a series of simulations will be presented to meet this obvious need. As we have seen, the numbers of parameters is fairly large despite our simplifying assumption. From this follows that the numerical analysis will have to be a partial one, that is, a few parameters will be varied at a time while the rest is kept constant. Evidently, tax ratios calculated in this way will be more or less sensitive to the choice of numerical values for the constant parameters. It is therefore essential to choose numerical values which are empirically important. Needless to say, it is far from evident which parameter values are empirically important. In three important respects we shall add further simplifying assumptions to those introduced in the previous chapter. It is in the first place supposed that the growth rate is constant and equal to θ even when the tied sector is considered. In section 25 it was shown that with a given share f_B of investments financed by releases a higher growth rate in release years would result in a lower average tax ratio. This is true whether the <u>average</u> growth rate is constant or increased. Regarding f_B as the authorities parameter of action, the assumption of a constant growth rate would thus prevent some of the potential tax benefits inherent in the IF-system from appearing in numerical simulations.

We shall secondly assume that conventional accelerated depreciation does not occur of that d=b. As pointed out in section 24 the tax benefits offered by the IF-system are smaller the larger d-b is, given the value of b. By assuming d=b we have consequently introduced a bias in our numerical calculations which tend to exaggerate the tax benefits which may be obtained from the use of investment funds. Finally, q and not f_B will be seen as the authorities release parameter.

In this chapter we shall deal with an IF-system consisting of a tied sector of the Swedish type. The simulations in the next chapter illustrate the working of a free sector system and all the four Nordic IF-systems will be considered. In Chapter XI finally, some numerical comparisons with other tax systems (accelerated depreciations, investment allowances and an investment tax credit) will be presented.

In a tied-sector system the average tax ratio would be $\bar{t} = 1 - A - \bar{B}$, cf. equation (53). Setting $G = g = \theta$ and $d = b$ we get

$$\bar{t} = 1 - \alpha\left(1 - \frac{\beta}{t^N}\right) - q\alpha\frac{\left[1 - (1-\theta)^p\right]\left[D(1-\theta) - 1\right]}{\theta D\left[D^p(1-\theta)^p - 1\right]}\left[\gamma + \frac{\beta}{t^N} - \frac{Db}{D(1+b) - 1}\right] \quad (64)$$

In the following sections we shall isolate and discuss the importance of variations of the release rate q, the allocation rate α, the deposit ratio β/t^N, the investment deduction rate γ, the interval p between each release, the depreciation rate b and the discount factor $D=1+r^d$. The significance of growth variations will be considered in the next section where q is the variable parameter.

28. With all other parameters constant the average tax ratio is a linearly decreasing function of q. Equation (64) may be rewritten as

$$\bar{t} = 1 - \alpha\left(1 - \frac{\beta}{t^N}\right) - q\alpha k_1 \quad (65)$$

where of course $k_1 = \bar{B}q\alpha$. Now as q increases so does f_B given by equation (15). Since releases cannot exceed gross investments currently undertaken, q-values which make $f_B > 1$ are not feasible. The relationship between f_B, q and α can be expressed as

$$f_b = q\alpha k_2 \quad (66)$$

where $k_2 = f_B/q\alpha$. Assume now that the allocated share of profits is varied such that $\alpha = 1/qk_2$. In that case we have $f_B = 1$ for all q. Insert $\alpha = 1/qk_2$ into (65):

$$\bar{t}^*_\alpha = \left(1 - \frac{k_1}{k_2}\right) - \frac{1}{q}\,\frac{1 - \frac{\beta}{t^N}}{k_2} \quad (67)$$

Equation (67) expresses the average tax ratio as a function of q when $\alpha = 1/qk_2$ such that $f_B = 1$. $\bar{t}^*_\alpha$ is thus the <u>lowest</u> attainable value of $\bar{t}$ when q is varied as an independent variable and α is varied dependently as indicated. Instead of α we could of course have chosen to vary another variable appearing in the f_B-expression. The $\bar{t}^*_\alpha$-function is a hyperbola with the asymptotes $q = 0$ and $\bar{t} = 1 - k_1/k_2$. With $k_2 > 0$ and $\beta/t^N < 1$ (as in Sweden) the hyperbola is positively sloping and upwardly convex.

Table 5 contains a number of equations generated by equations (65) and (67) through variation of α, θ and b while keeping β/t^N, γ, D, r and p constant. Some of these functions are shown in Figure 1.

As can be seen in the table $\bar{t}$ decreases when the rate of growth θ rises but this inverse coovariation is relatively insignificant. With the parameter values assumed in Table 5 the average tax ratio will change less than one percentage unit when the growth rate varies between 2.5% and 10%. The eight $\bar{t}$-functions in Figure 1 are all based on $\theta = 5\%$ and with the scale used it would make nearly no visible difference at all if the growth rate was decreased to 2.5% or increased to 10%.

Although the $\bar{t}$-function is insensitive to variations in the growth rate the same is not true of the $\bar{t}_\alpha^*$-function. The higher the rate of growth the lower the value of $\bar{t}_\alpha^*$ for any given value of q as Figure 1 shows. A higher growth rate will in other words make it possible to release a larger share of allocated funds without violating the investment constraint (implying that releases cannot exceed current investments). Such a development would at the same time reduce the firm's tax burden $\bar{t}$. Assume for instance that the firm's depreciation rate is 5%, that the permitted 40% of profits are allocated and that the fixed parameter values from Table 5 apply. With a growth rate of 2.5% only about 40% of allocations could be released without violating the investment restriction, cf. point A in Figure 1. With releases of this size the average tax ratio would be nearly 88%. Had the growth rate instead been 5%, about 57% of the firm's investment funds could be released giving an average tax ratio of about 84%, cf point B. Finally, with a growth rate equal to 10% practically all funds could be released and $\bar{t}$ would be as low as about 76%, as is indicated by point C in the figure.[1)]

It is also evident from Figure 1 that - although $\bar{t}$ is negatively related to q - a relatively small release rate combined with a large allocation rate may bring about a lower average tax ratio than a larger q combined with a smaller α. Compare for instance points B and E (in Figure 1) which are based on the growth and depreciation rates $\theta = b = 5\%$. With allocations corresponding to 40% of profits and releases of about 57% the average tax ratio is about 84% as shown by point B. With $\alpha = 100\%$ and (approximately) $q = 23\%$ the average tax ratio is about 78% cf point E.[2)]

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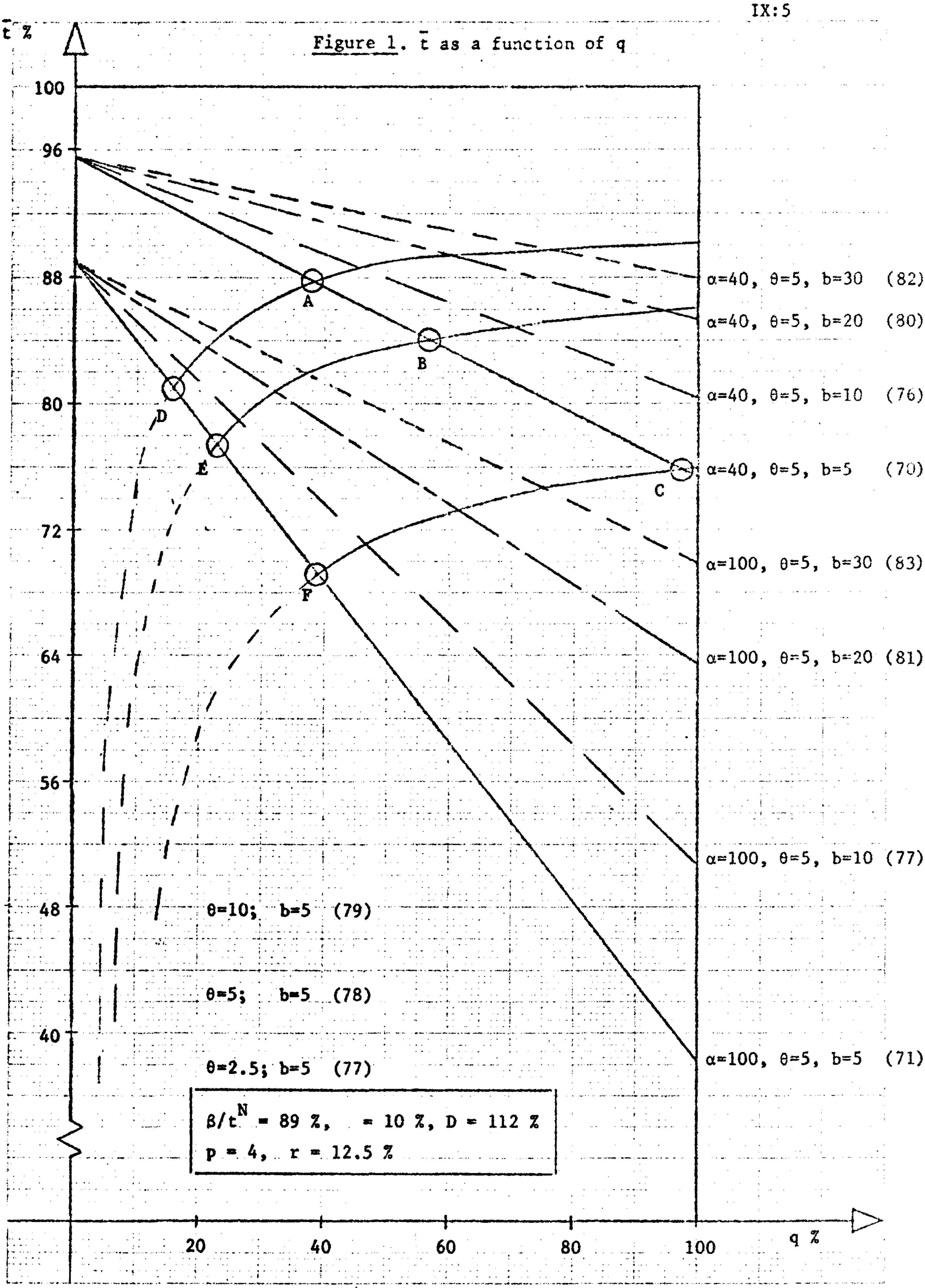
1) Point C is in fact the crossing point for the curves given by equations (72) and (86) while A is the crossing point for (68) and (84) but as pointed out the functions (68), (70) and (72) would nearly coincide in the diagram if they all had been shown.

2) Since movements along the $\bar{t}_\alpha^*$-curve imply inverse proportional changes of α and q the values of $\bar{B}$ and f_b are not affected. The changes of $\bar{t}$ is thus due entirely to changes of $\bar{A}$.

Table 5. The average tax ratio $\bar{t}$ as a function of q according to equations (65) and (67). Fixed parameters: β/t^N = 89%, γ = 10%, D = 112%, p = 4 and r = 12.5%. Percentage units.

b	θ	α	$\bar{t}$ or $\bar{t}^*_\alpha$	equation number
5	2.5	40	$\bar{t} = 95.60 - 0.2000q$	(68)
5	2.5	100	$\bar{t} = 89 - 0.5037q$	(69)
5	5	40	$\bar{t} = 95.60 - 0.2023q$	(70)
5	5	100	$\bar{t} = 89 - 0.5058q$	(71)
5	10	40	$\bar{t} = 95.60 - 0.2036q$	(72)
5	10	100	$\bar{t} = 89 - 0.5090q$	(73)
10	2.5	40	$\bar{t} = 95.60 - 0.1521q$	(74)
10	2.5	100	$\bar{t} = 89 - 0.3803q$	(75)
10	5	40	$\bar{t} = 95.60 - 0.1527q$	(76)
10	5	100	$\bar{t} = 89 - 0.3818q$	(77)
10	10	40	$\bar{t} = 95.60 - 0.1538q$	(78)
10	10	100	$\bar{t} = 89 - 0.3845q$	(79)
20	5	40	$\bar{t} = 95.60 - 0.0763q$	(80)
20	5	100	$\bar{t} = 89 - 0.1908q$	(81)
30	5	40	$\bar{t} = 95.60 - 0.0762q$	(82)
30	5	100	$\bar{t} = 89 - 0.1005q$	(83)
5	2.5	$\alpha = \frac{1}{q^{k_2}}$	$\bar{t}^*_\alpha = 92.01 - 176\ (1/q)$	(84)
5	5	$\alpha = \frac{1}{q^{k_2}}$	$\bar{t}^*_\alpha = 88.52 - 250\ (1/q)$	(85)
5	10	$\alpha = \frac{1}{q^{k_2}}$	$\bar{t}^*_\alpha = 80.27 - 427\ (1/q)$	(86)
10	2.5	$\alpha = \frac{1}{q^{k_2}}$	$\bar{t}^*_\alpha = 89.99 - 291\ (1/q)$	(87)
10	5	$\alpha = \frac{1}{q^{k_2}}$	$\bar{t}^*_\alpha = 86.99 - 375\ (1/q)$	(88)
10	10	$\alpha = \frac{1}{q^{k_2}}$	$\bar{t}^*_\alpha = 80.12 - 569\ (1/q)$	(89)

Figure 1. $\bar{t}$ as a function of q



From Table 5 and Figure 1 it appears not surprisingly that the average tax ratio is relatively sensitive to changes in the allocated share of profits α and the depreciation rate b, given the value of q. If $\alpha = 40\%$ we have $\bar{t} = 95.6\%$ if no releases occur (q=0). With $\alpha = 100\%$ the corresponding $\bar{t}$-value is 89%. If all funds are released (q=100%) and the depreciation rate is 30% $\bar{t}$ would be 87.98% with 40% of profits allocated and as low as 78.95% if all profits are allocated, cf. equations (82) and (83). Had the durability of the firm's plant and equipment been longer so that b = 5% instead of 30%, the corresponding $\bar{t}$-values would have been 75.37%, respectively 38.44%, see equations (70) and (71).

To what extent these hypothetical $\bar{t}$-values can be realized will of course depend upon the actual growth rate and the permitted allocation rate. In terms of Figure 1 only those $\bar{t}$-values which at the same time lie on or above the relevant $\bar{t}$-curve and the corresponding t^*_α-curve will be attainable. In Figure 1, b= 5% was assumed in the calculation of the three $\bar{t}^*_\alpha$-functions. With a larger depreciation rate the $\bar{t}^*_\alpha$-functions are likely to shift downward (as in Table 5).[1)]

29. Let us next regard the allocated share of profits α as the main independent variable. From equation (65) it is clear that $\bar{t}$ is a linear function of α, given the values of all other parameters. The negative slope of this straight line is $-(1 - \beta/t^N + qk_1)$ while the vertical intercept is unity. A number of such functions generated by equation (65) can be seen in Table 6 and in Figure 2. The largest share of allocations which can be released without making releases exceed current gross investments is $q = 1/\alpha k_2$, cf. equation (66). Substitute $1/\alpha k_2$ for q in (65) and we get the linear $\bar{t}^*_q$-function

$$\bar{t}^*_q = \left(1 - \frac{k_1}{k_2}\right) - \left(1 - \frac{\beta}{t^N}\right)\alpha \qquad (90)$$

Equation (90) indicates the (lowest attainable) values of the average tax ratio when α is varied and the release rate q is chosen such that releases precisely match gross investments ($f_B = 1$). Four functions generated by (90) through variation of the depreciation rate b are shown in Table 6 and Figure 2. In the figure A, B, C, D, E and F are the points where the t^*_q-curves cross the corresponding $\bar{t}$-lines. With complete release of funds (q = 100%), a depreciation rate of 5% and the other parameter values as assumed in Figure 2 an allocation rate of approximately 22% will give the lowest attainable average tax ratio which in this case is about 86%, cf. point A. If, however, q is lowered to 40% α can be increased to about 56 and in this way the average tax ratio goes down to about 82%, cf. point E.

---

1) In equation (67) k_2 varies negatively with b. It can further be shown that the ratio $k_1/k_2 = \bar{B}/f_B$ varies positively with b if $(\gamma + \beta/f^N)$ is "large". A value in the neighborhood of unity such as the present Swedish one would be sufficiently large to give a positive covariation between k_1/k_2 and b and thus a downward shift of (67) when b increases.

Consider now q as given and assume that the depreciation rate b is varied so as to secure $f_B = 1$. According to equation (15) b is thus determined as

$$b = \alpha qr(1-\theta)\,\frac{1-(1-\theta)^P}{\theta} - \theta = \alpha k_3 - \theta \qquad (109)$$

If this b-value is inserted into our $\bar{t}$-function (64) we get the following $\bar{t}_b^*$-function

$$\bar{t}_b^* = 1 - \alpha\left[1 - \frac{\beta}{t^N} + k_4(\gamma + \frac{\beta}{t^N})\right] + \alpha k_4 D\,\frac{\alpha k_3 - \theta}{D(1+\alpha k_3-\theta) - 1} \qquad (110)$$

where $\theta < k_4 < 1$ and

$$k_4 = \frac{q\left[1-(1-\theta)^P\right]\left[D(1-\theta) - 1\right]}{D\theta\left[D^P(1-\theta)^P - 1\right]} \qquad (111)$$

In Figure 2 the curves ABCD and EF are in fact generated by (110).[1] Using the assumed parameter values, ABCD and EF can be expressed as

$$\bar{t}_b^* = 1 - \alpha\,\frac{967 + 5.06\alpha}{640 + 49.34\alpha} \qquad (112)$$

and

$$\bar{t}_b^* = 1 - \alpha\,\frac{430 + 2.12\alpha}{640 + 19.74\alpha} \qquad (113)$$

respectively. As in Table 6 α is measured in percentage units in equations (112) and (113).

It is interesting to note that the derivative $d\bar{t}_b^*/d\alpha$ from equation (110) is

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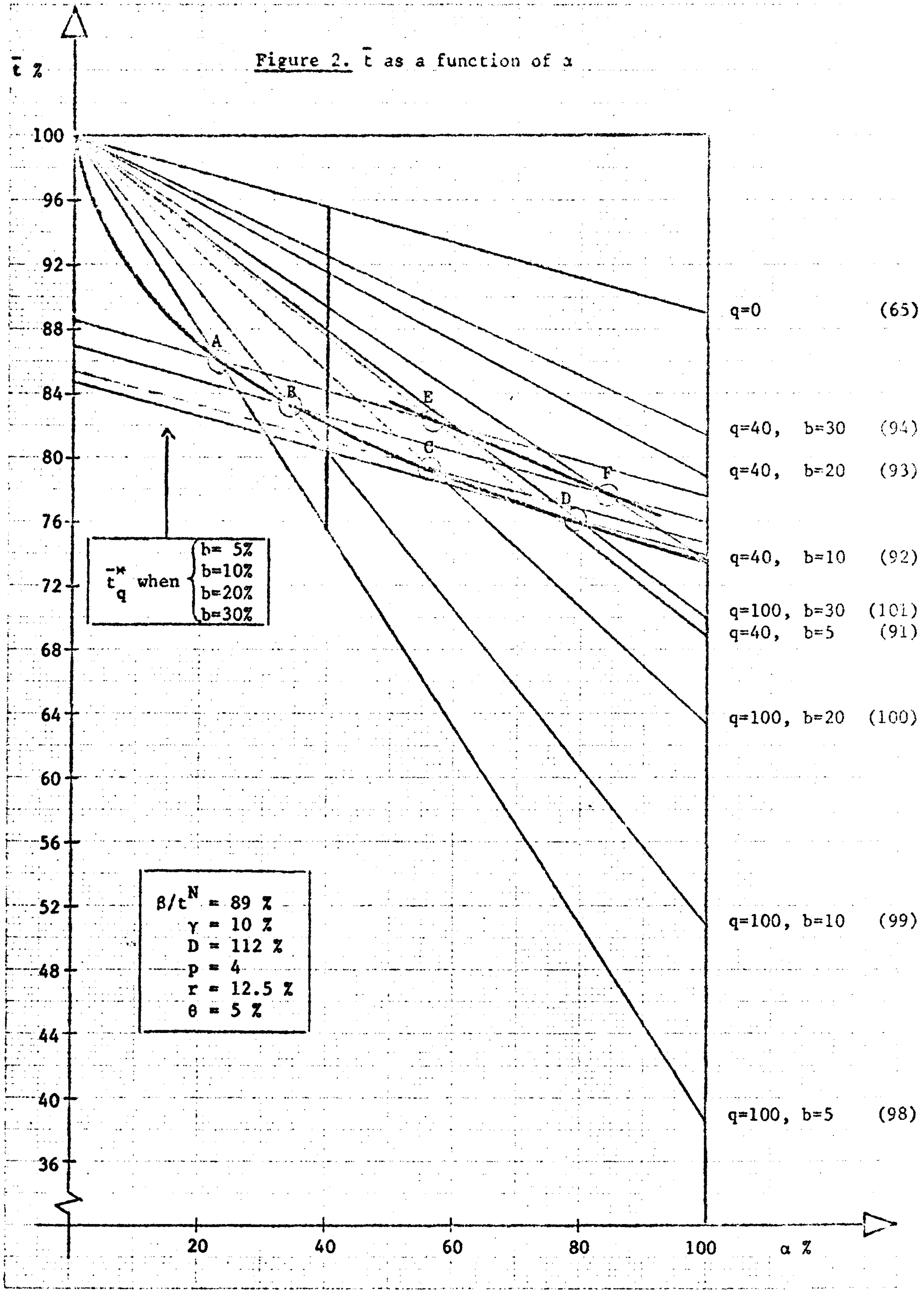
1) Since we are concerned only with positive values of the depreciation rate b it is clear that to values of α below a certain limit no corresponding positive b-values can be found which make $f_B = 1$. In such intervals (110) is not defined. For equation (112) - or ABCD - the lower limit to α is $\alpha = 11.5$ %, for equation (113) or EF it is $\alpha = 28.4$ %.

Table 6. The average tax ratio $\bar{t}$ as a function of α according to equations (65) and (90). Fixed parameters: β/t^N = 89%, γ = 10%, D = 112%, p = 4 and r = 12.5%. Percentage units.

q	θ	b	$\bar{t}$ or $\bar{t}_q^*$	equation number
40	5	5	$\bar{t} = 100 - 0.3123\alpha$	(91)
		10	$\bar{t} = 100 - 0.2628\alpha$	(92)
		20	$\bar{t} = 100 - 0.2120\alpha$	(93)
		30	$\bar{t} = 100 - 0.1863\alpha$	(94)
	10	5	$\bar{t} = 100 - 0.3135\alpha$	(95)
		10	$\bar{t} = 100 - 0.2638\alpha$	(96)
		20	$\bar{t} = 100 - 0.2128\alpha$	(97)
100	5	5	$\bar{t} = 100 - 0.6158\alpha$	(98)
		10	$\bar{t} = 100 - 0.4918\alpha$	(99)
		20	$\bar{t} = 100 - 0.3650\alpha$	(100)
		30	$\bar{t} = 100 - 0.3008$	(101)
	10	5	$\bar{t} = 100 - 0.6190\alpha$	(102)
		10	$\bar{t} = 100 - 0.4948\alpha$	(103)
		20	$\bar{t} = 100 - 0.3650\alpha$	(104)
$q = \frac{1}{\alpha k_2}$	5	5	$\bar{t}_q^* = 88.52 - 0.11\alpha$	(105)
		10	$\bar{t}_q^* = 86.99 - 0.11\alpha$	(106)
		20	$\bar{t}_q^* = 85.52 - 0.11\alpha$	(107)
		30	$\bar{t}_q^* = 84.85 - 0.11\alpha$	(108)

Figure 2. $\bar{t}$ as a function of α



negative.[1] Curves such as ABCD and EF are in other words downward sloping. Let us explain the meaning of that.

It is true - all other things being equal - that the tax burden $\bar{t}$ is lower the smaller the depreciation rate b is. This has been pointed out by nearly all writers who have discussed the investment funds. But it is also true, given the release rate q, that the greater the depreciation rate b is, the larger can the allocation rate α be without violating the investment restriction. And the larger α is the smaller will of course the average tax ratio be. We have in other words seen that with a <u>larger</u> depreciation rate (and a given release rate) it becomes possible for the firm to allocate more of its profits to investment funds (increase α) and that the lowest attainable tax burden $\bar{t}^*_\alpha$ is <u>reduced</u> in this way. Return to Figure 2 and assume that all allocated funds are released. (q = 100). The firm with capital of long durability (b = 5) can at most allocate 22% of profits and its tax burden will then be about 86%. An enterprise with equipment lasting fewer years (b = 30) would be able to allocate more than 80% of profits (disregarding the present 40 % limit in Sweden) and reduce its tax burden to less than 76 %.

Quite a similar argument can of course be made in terms of variable release and depreciation rates assuming α to be given. This will be postponed to section 33.

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1) We have

$$\frac{d\bar{t}^*_b}{d\alpha} = -\left[1 - \frac{\beta}{t^N} + k_4(\gamma + \frac{\beta}{t^N})\right] + k_4 k_5$$

where

$$k_5 = 1 - \frac{(D-1)\left|D(1-\theta) - 1\right|}{\left[D(1-\theta) - 1 + Dk_3\alpha\right]^2}$$

Having assumed that $D(1-\theta) - 1$ and $b = \alpha k_3 - \theta$ are both positive it follows that $0 < k_5 < 1$. Now k_4 is also positive and less than unity so that

$$\frac{1 - \beta/t^N}{k_4} + (\gamma + \frac{\beta}{t^N}) > (1 - \frac{\beta}{t^N}) + (\gamma + \frac{\beta}{t^N}) > k_5$$

and $d\bar{t}^*_b/d\alpha$ is negative.

In Figure 2 the intercept of the linear $\bar{t}^*_q$-function decreases as b increases. This is sufficient but not a necessary condition for the negative slope of (110).

The present maximal allocation rate of 40 % in Sweden means of course that in Figure 2 only tax rates to the left of or at α = 40 % and above or at the relevant $\bar{t}^{*}$-curve can actually be reached.

30. When the average tax ratio is seen as a function of β/t^N (the deposit ratio) equation (64) can be written

$$\bar{t} = \left[1 - \alpha - \alpha k_4(\gamma - \frac{Db}{D(1+b) - 1})\right] + \alpha(1 - k_4)\,\frac{\beta}{t^N} \qquad (114)$$

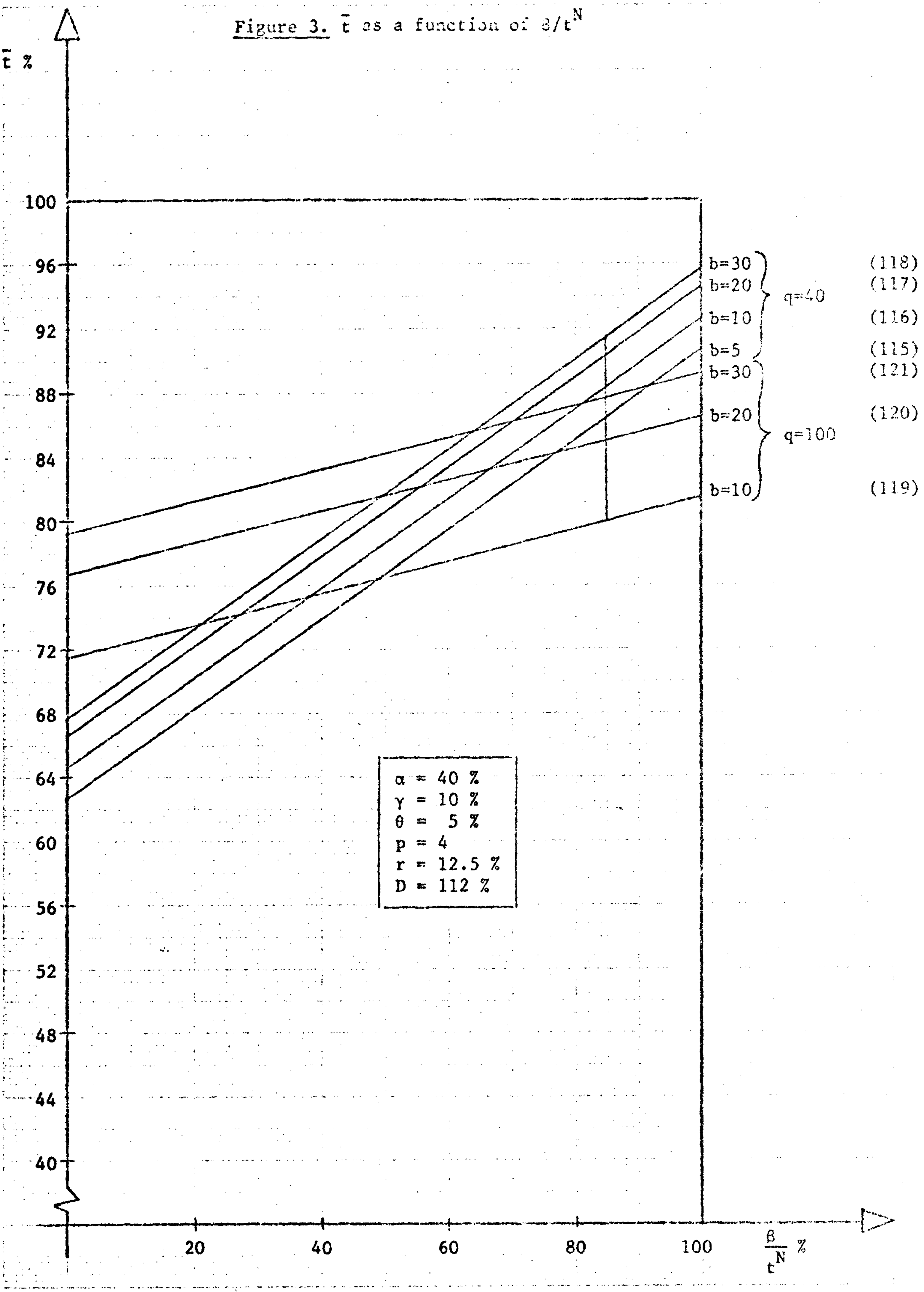
where k_4 is given by equation (111).

(114) is a straight line and the slope is positive since $0 < k_4 < 1$. In Table 7 and in Figure 3 a few equations based on (114) are presented. The slope is invariant for changes in b but varies negatively with q and positively with α. The intercept, on the other hand, varies positively with b as well as q but negatively with α. In Table 7 is also indicated the value of f_B corresponding to the combination of parameter values used in each case. f_B is of course independent of the value of β/t^N. With b = 10 %, q = 100 % and the values of the other parameters fixed as shown f_B = 117.48 %. Equation (119) is in other words not feasible because releases in that case would exceed current gross investments.

Table 7. The average tax ratio $\bar{t}$ as a function of β/t^N according to equation (114). Fixed parameters: α = 40%, γ = 10%, Θ = 5%, p = 4, r = 12.5% and D = 112%. Percentage units.

q	b	$\bar{t}$	f_B	equation number
40	5	$\bar{t} = 62.62 + 0.2796(\beta/t^N)$	70.49	(115)
40	10	$\bar{t} = 64.61 + 0.2796(\beta/t^N)$	46.99	(116)
40	20	$\bar{t} = 66.63 + 0.2796(\beta/t^N)$	28.20	(117)
40	30	$\bar{t} = 67.67 + 0.2796(\beta/t^N)$	20.14	(118)
100	10	$\bar{t} = 71.52 + 0.0990(\beta/t^N)$	117.48	(119)
100	20	$\bar{t} = 76.59 + 0.0990(\beta/t^N)$	70.49	(120)
100	30	$\bar{t} = 79.17 + 0.0990(\beta/t^N)$	50.35	(121)

Figure 3. $\bar{t}$ as a function of β/t^N



From Figure 3 can be seen how high the deposit rate would have to be compared to the tax rate in order to eliminate the tax benefits of the IF-system. With 40 % of allocations released and b = 30 % the average tax ratio would be 100 % if β/t^N = 116 % approximately. With b = 5 % the corresponding β/t^N-value would be about 134 %. If on the other hand all funds were released and b < 30 %, β/t^N would have to exceed 200 % before all tax benefits had disappeared.

Figure 3 may give some impression of the size of the tax benefit which is due to the fixation of the deposit rate below the nominal tax rate in Sweden. This tax benefit, which varies positively with the allocation rate and negatively with the release rate, is at most 4 percentage units with the functions shown in Figure 3 and the present β/t^N-value of 85 %. The diagram also indicates the very large tax benefit that would appear in a system without obligation to deposit a part of allocations such as the pre-1955 Swedish IF-system. If for instance b = 5 %, q = 40 % and β = 0 the average tax ratio would be about 63 % instead of 88 % with β/t^N = 89 %, cf. equation (115).

31. In this section the extra investment deduction rate γ is regarded as the variable parameter. For that purpose equation (64) can be written

$$\bar{t} = \left[1 - A - \alpha k_4\left(\frac{\beta}{t^N} - \frac{Db}{D(1+b) - 1}\right)\right] - \alpha k_4 \gamma \qquad (122)$$

Functions obtained from (122) are presented in Table 8 with the corresponding f_B-values, showing whether or not the particular combination of parameter values is feasible. Some of the feasible functions are shown graphically in Figure 4.

How large a reduction of the average tax ratio a given investment deduction rate can bring about depends of course on the size of the slope of (122), which in its turn depends on the allocation, release and growth rates in addition to D and p. The depreciation rate affects the intercept (positively) but not the slope. With the present Swedish value of γ = 10 % the extra investment deduction reduces the average tax ratio by a little more than three percentage units at most in the examples given in Table 8. The reduction of the average effective tax rate would thus be about 1.5 percentage units or less. The relative importance of this tax benefit will vary according to the size of the intercept. With 40 % allocations, all funds released and a depreciation rate of 20 % - as in equation (130) - about 20 % of the total tax reduction offered by the IF-system can be attributed to the investment deduction. In instead α = 40 %, q = 40 % and b = 5 % - as in equation (123) - the corresponding share is only about 10 %.

Table 3. The average tax ratio $\bar{t}$ as a function of γ according to equation (122). Fixed parameters: $\theta = 5\%$, $p = 4$, $r = 12.5\%$, $8/t^N = 89\%$ and $D = 112\%$. Percentage units.

α	b	q	$\bar{t}$	equation number	f_B
40	5	40	$\bar{t} = 88.71 - 0.1204\gamma$	(123)	70.49
		70	$\bar{t} = 83.56 - 0.2107\gamma$	(124)	123.35
	10	40	$\bar{t} = 90.69 - 0.1204\gamma$	(125)	46.99
		70	$\bar{t} = 87.02 - 0.2107\gamma$	(126)	82.24
		100	$\bar{t} = 83.33 - 0.3011\gamma$	(127)	117.48
	20	40	$\bar{t} = 92.72 - 0.1204\gamma$	(128)	28.20
		70	$\bar{t} = 90.57 - 0.2107\gamma$	(129)	49.34
		100	$\bar{t} = 88.41 - 0.3011\gamma$	(130)	70.49
	30	40	$\bar{t} = 93.75 - 0.1204\gamma$	(131)	20.14
		70	$\bar{t} = 92.38 - 0.2107\gamma$	(132)	35.24
		100	$\bar{t} = 90.98 - 0.3011\gamma$	(133)	50.24
100	10	40	$\bar{t} = 76.73 - 0.3011\gamma$	(134)	117.48
	20		$\bar{t} = 81.81 - 0.3011\gamma$	(135)	70.50
	30		$\bar{t} = 84.38 - 0.3011\gamma$	(136)	50.35
	30	70	$\bar{t} = 80.95 - 0.5268\gamma$	(137)	88.10

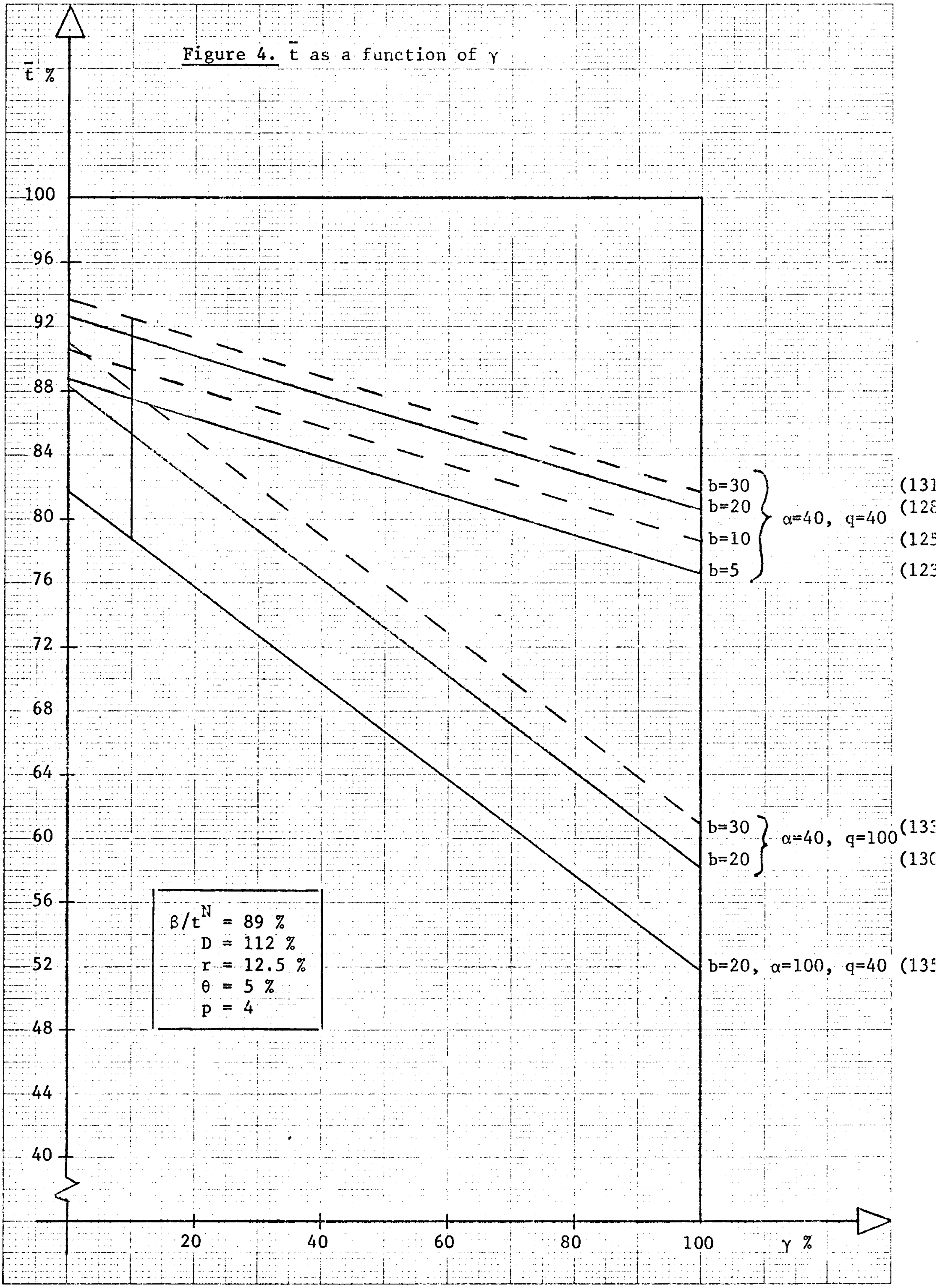


Figure 4. $\bar{t}$ as a function of γ
$\bar{t}$ %
100
96
92
88
84
80
76
72
68
64
60
56
52
48
44
40
20
40
60
80
100
γ %
b=30
b=20
b=10
b=5
α=40, q=40
(131
(128
(125
(123
b=30
b=20
α=40, q=100
(133
(130
b=20, α=100, q=40 (135
β/t^N = 89 %
D = 112 %
r = 12.5 %
θ = 5 %
p = 4

32. The significance for the average tax ratio of how often funds are released is illustrated in Table 9 and Figure 5. Whether $\bar{t}$ rises or is reduced when p increases depends upon the values of D, θ and p.[1)] With the parameter values assumed in Table 9 the covariation between $\bar{t}$ and p is positive. As appears $\bar{t}$ is more sensitive to variations of p the longer the durability of capital equipment and the larger the release rate is. With b = 5 % and a complete release of funds $\bar{t}$ increases by 4.90 percentage units when p goes up from 1 to 5. With b = 30 % and q = 40 % the corresponding increase is only 0.74 percentage units, cf. Table 9.

<u>Table 9.</u> Covariation between the average tax ratio $\bar{t}$ and the number of years between each release p. Fixed parameters: $\alpha = 40\%$, $\gamma = 10\%$, $\theta = 5\%$, $r = 12.5\%$, $\beta/t^N = 89\%$ and $D = 112\%$. Percentage units.

q	p	b = 5		b = 10		b = 20		b = 30	
		$\bar{t}$	f_B	$\bar{t}$	f_B	$\bar{t}$	f_B	$\bar{t}$	f_B
40	1	86.00	19.00	88.35	12.67	90.76	7.60	91.98	5.43
	2	86.53	37.05	88.75	24.70	91.03	14.82	92.18	10.59
	3	87.03	54.20	89.13	36.13	91.28	21.68	92.37	15.49
	4	87.51	70.49	89.47	46.99	91.52	28.20	92.55	20.14
	5	87.96	85.96	89.83	57.31	91.75	34.39	92.72	24.56
70	1	78.80	33.25	82.92	22.17	87.13	13.30	89.26	9.50
	2	79.73	64.84	83.63	43.26	87.60	25.94	89.61	18.53
	3	80.61	94.85	84.28	63.23	88.04	37.94	89.95	27.10
	4	81.44	123.35	84.91	82.24	88.46	49.34	90.26	35.24
	5	82.23	150.43	85.50	100.29	88.86	60.17	90.56	42.98
100	1	71.60	47.50	77.48	31.67	83.50	19.00	86.56	13.57
	2	72.92	92.63	78.51	61.78	84.17	37.05	87.06	26.46
	3	74.18	135.49	79.43	90.33	84.80	54.20	87.53	38.71
	4	75.37	176.22	80.33	117.53	85.40	70.49	87.98	50.34
	5	76.50	214.90	81.18	143.27	85.96	85.96	88.40	61.40

1) We have more precisely that

$$\Delta\bar{t} = \bar{t}_{p+1} - \bar{t}_p \gtreqless 0 \quad \text{as} \quad 1 - \theta \gtreqless \frac{D^p - 1}{D^{p+1} - 1}$$

Covariation between $\bar{t}$ and p

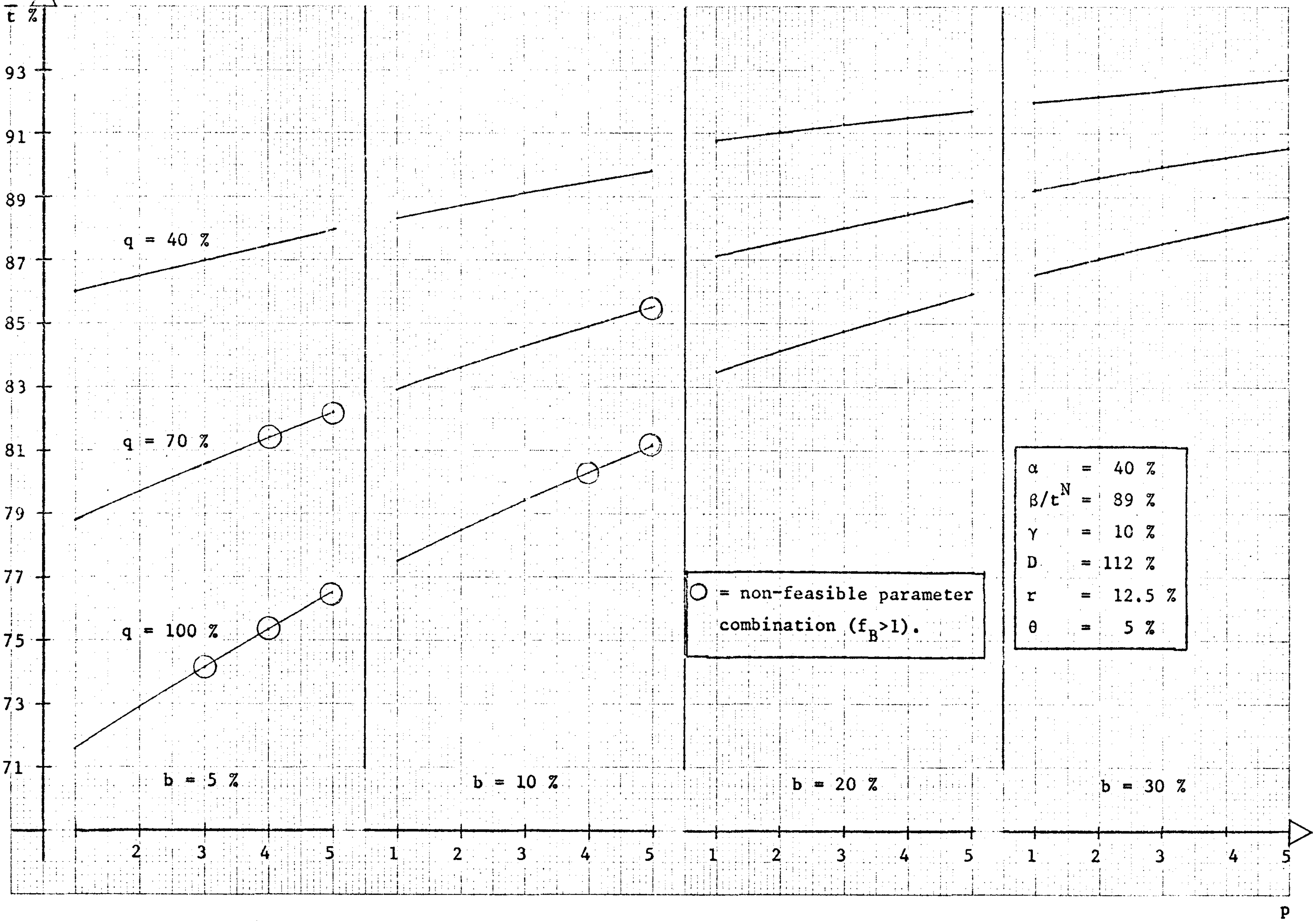


Figure 5.

33. Next in turn to be regarded as the main independent variable in the tied sector system is the depreciation rate b. Now equation (64) can be written as

$$\bar{t} = (1 - A - \bar{B}_1) + k_4 D \frac{b}{D(1+b) - 1} \tag{138}$$

According to (138) the average tax ratio is a rising function of b with the intercept $(1 - A - \overline{B^I})$. The slope is, however, decreasing so that the average tax ratio is more sensitive to changes in the depreciation rate when that rate is small than when it is large. Six functions based on expression (138) are shown in Table 10 and in Figure 6. The table also gives for each function the value of the depreciation rate which makes f_B equal to one. These b-values, which are all positive, have been marked as points A, B, C, D, E and F in Figure 6. The largest share of allocations which can be released without violating the investment restriction is determined by equation (15) as:

$$q = \frac{\theta(\theta+b)}{\alpha r(1-\theta)\left(1 - (1-\theta)^P\right)} \tag{145}$$

Inserting this expression into (64) gives us

$$\bar{t}^*_q = (1-A) - (\theta+b)k_6 \left(\gamma + \frac{\beta}{t^N} - \frac{Db}{D(1+b) - 1}\right) \tag{146}$$

where

$$k_6 = \frac{D(1-\theta) - 1}{rD(1-\theta)\left(D^P(1-\theta)^P - 1\right)} > 0 \tag{147}$$

Now curves such as ABC and DEF in Figure 6 are actually $\bar{t}^*_q$-curves generated by equation (146). Both ABC and DEF are downward-sloping. Under the present Swedish system this will be the case if the rate of growth is small compared to the firm's discount rate of interest and/or the depreciation rate is relatively high.[1)]

With a small depreciation rate the tax burden will, ceteris paribus, be relatively small, as we have seen. On the other hand only a relatively small part of allocated funds can be released in this case since releases are limited by the amounts invested. With a larger depreciation rate and thus a larger volume of gross investment more funds can be released and this tends of course to diminish the tax burden. Thus it may very well be so that the lowest attainable average tax ratio may be lower for a firm with a high depreciation rate and a large release rate than for a firm with a low depreciation rate and therefore a rather small (maximal) release rate.

In Figure 6 assume that maximal allocations are made ($\alpha = 40\%$). With releases amounting to 40% of allocations the lowest obtainable tax burden 85.61% will be reached with b = 2.05% (point A). If instead the depreciation rate is 12.65% all funds can be released without violating the investment restriction and the average tax ratio is lowered to 82.07% (point C).

---

1) The derivative of (146) with regard to b is

$$\frac{d\bar{t}_q^*}{db} = -k_6\left(\gamma + \frac{\beta}{t^N} - k_7\right)$$

where

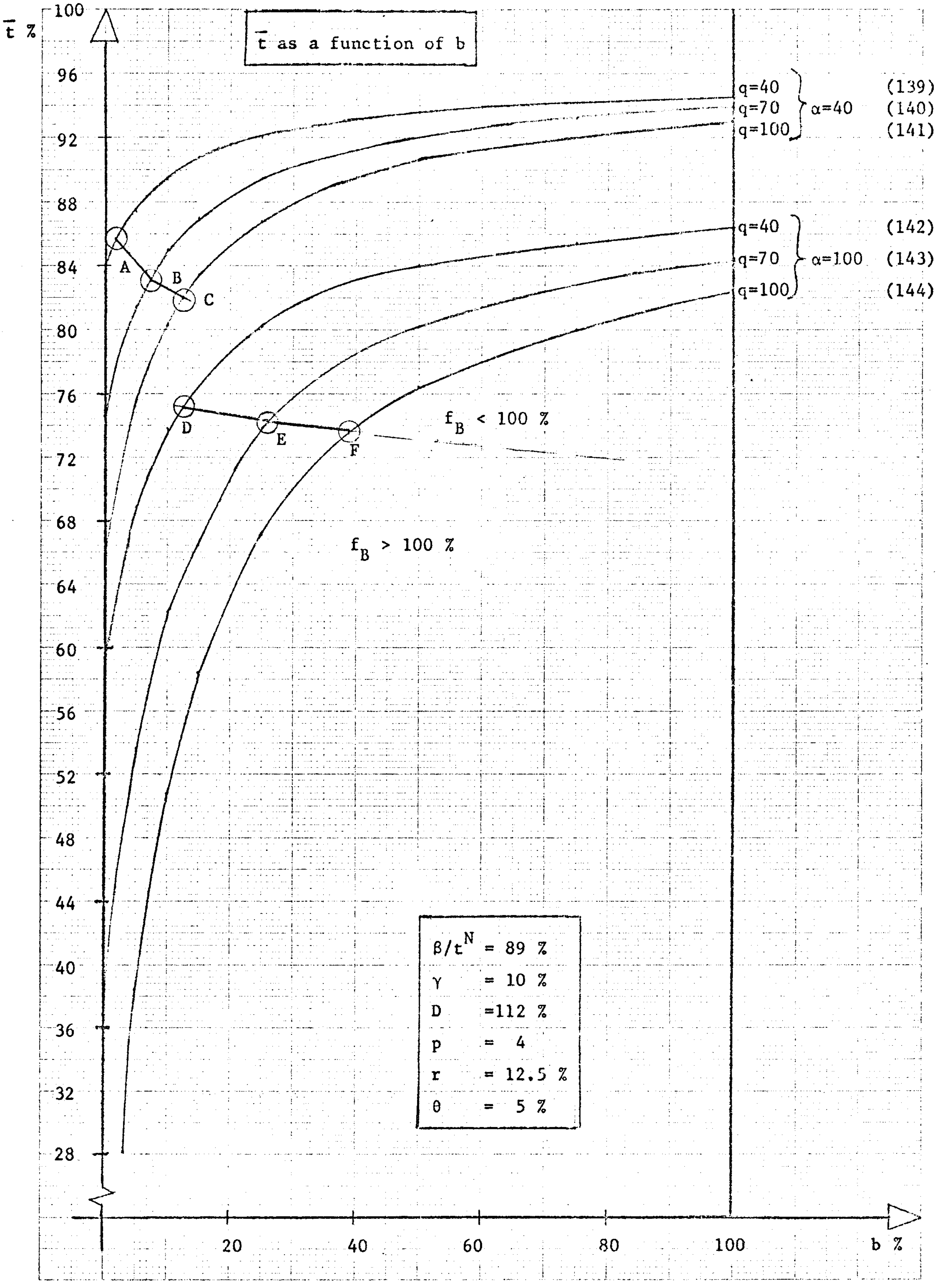
$$k_7 = 1 - \frac{(D-1)\left[D(1-\theta) - 1\right]}{\left[D(1+b) - 1\right]^2}$$

From assumption $D(1-\theta) - 1 > 0$ it follows that $0 < k_7 < 1$. Now $k_6 > 0$ and we have

$$d\bar{t}_q^*/db \gtreqless 0 \quad \text{as} \quad \gamma + \beta/t^N \lesseqgtr k_7$$

Assuming $\gamma + \beta/t^N = 0.99$ and $D = 1.12$, $b = \theta = 0.10$ gives $k_7 = 0.982$ while $b = 0.30$ and $\theta = 0.05$ implies $k_7 = 0.963$. With $b = 0.30$ and $\theta = 0.10$ we get $k_7 = 0.995$. $d\bar{t}_q^*/db$ would in other words be negative in the first two cases and positive in the last.

Figure 6.



t̄ as a function of b
t̄ %
100
96
92
88
84
80
76
72
68
64
60
56
52
48
44
40
36
32
28
20
40
60
80
100
b %
q=40
q=70
q=100
α=40
(139)
(140)
(141)
q=40
q=70
q=100
α=100
(142)
(143)
(144)
A
B
C
D
E
F
f_B < 100 %
f_B > 100 %
β/t^N = 89 %
γ = 10 %
D =112 %
p = 4
r = 12.5 %
θ = 5 %

Table 10. The average tax ratio $\bar{t}$ as a function of b according to equation (138). Fixed parameters: β/t^N = 89%, γ = 10%, Θ = 5%, r = 12.5%, D = 112% and p = 4. Percentage units.

α	q	$\bar{t}$	equation number	b-value which makes f_B equal to 1
40	40	$\bar{t} = 83.68 + \frac{13.48b}{1.12b + 12}$	(139)	2.05
	70	$\bar{t} = 74.74 + \frac{23.59b}{1.12b + 12}$	(140)	7.34
	100	$\bar{t} = 65.79 + \frac{33.71b}{1.12b + 12}$	(141)	12.62
100	40	$\bar{t} = 59.19 + \frac{33.71b}{1.12b + 12}$	(142)	12.62
	70	$\bar{t} = 36.85 + \frac{59.00b}{1.12b + 12}$	(143)	25.86
	100	$\bar{t} = 14.47 + \frac{84.30b}{1.12b + 12}$	(144)	39.05

34. Let us conclude the numerical analysis of the tied sector by considering very briefly the importance of variations in the discount factor D. Table 11 contains a few $\bar{t}$-values obtained by using the actual Swedish values of α, β/t^N and γ and assuming a growth rate of 5% and complete releases every fourth year while varying b and D as indicated. Whether the $\bar{t}$-values in the table are feasible or not is unimportant in this connection.[1)] According to the table the average tax ratio decreases 2-3 percentage units when D increases from 6 to 12 percent. The size of the interval in which $\bar{t}$ varies does not seem to depend on b in a simple way. Larger intervals are of course conceivable. If p = 1 and b = 10%, D = 1.06 and D = 1.12 would give $\bar{t}$ = 81.96%, respectively $\bar{t}$ = 77.49%. The interval would thus be about 4.5 percentage units.

1) This depends also on the profitability rate r. With r = 12.5% the parameter combinations with the two lowest b-values will in fact violate the investment restriction.

Table 11. The average tax ratio under different assumptions with regard to the discount factor D and the depreciation rate b. Fixed parameters: $\alpha = 40\%$, $\beta/t^N = 89\%$, $\gamma = 10\%$, $\theta = 5\%$, $q = 100\%$ and $p = 4$. Percentage units.

b	D			
	1.06	1.08	1.10	1.12
5	77.56	76.21	75.57	75.37
10	83.08	81.54	80.59	80.33
20	88.31	87.00	86.06	85.40
30	90.45	89.39	88.58	87.98

CHAPTER X

SIMULATIONS WITH THE MODEL. THE FREE SECTOR

Introduction

35. If we want to compare the tax-reducing capacity of tied-sector and free-sector IF-systems, the most convenient starting point seems to be the expressions for the average tax ratio given by equations (53) and (54). A few comparisons of that kind will be made in Chapter XI. A property of the average tax ratio in the absence of a tied sector is, as we have seen, that it is not a function of the growth rate with our assumptions whereas the current tax ratio depends on θ. This is of course related to the fact that the size of the weight attached to each year's current tax ratio when calculating the average tax ratio depends on the growth rate. In this chapter our main concern will be comparisons between the free-sector systems in the four Nordic countries and since the growth parameter is of some interest we shall take the limiting value of the current tax ratio as our point of departure.

The limiting value of the current tax ratio for a pure free-sector system is

$$t_\infty = 1 - A - F_\infty \qquad (148)$$

or

$$t_\infty = 1 - \alpha(1 + \gamma_A - \frac{\beta}{t^N}) - \alpha\delta(1-\theta)^m(K\frac{\beta}{t^N} - \frac{b}{\theta+b}) \qquad (149)$$

where

$$K = 1 + (1 - t^R)[(1 + r^*)^m - 1] \qquad (150)$$

Compared with (46) on p. VIII:12 the expression for t_∞ is here generalized so as to include the free sector systems in all Nordic countries as special cases. The role of the factor K is to take account of whether or not deposits are interest-bearing. If they are not, the rate of interest on deposits r^* is zero and K=1 as in Sweden. If payments of interest on deposits are tax-free, as they are in Finland and Norway (cf. Table VII:2), t^R=0 in equation (150) and $K = (1+r^*)^m$. Although the market rate on time deposits are paid in Denmark, K is probably smaller than in Norway and

Finland since m in Denmark may be as low as one while $t^R = t^N$.[1)] In (149) the special Norwegian investment deduction in connection with allocations has also been considered by adding the parameter γ_A. The limiting value t_∞, as given by (149), is finally based on the assumption d=b or that conventional accelerated depreciation does not occur.

As pointed out earlier a difficulty when limiting values are compared is that the pace at which convergence takes place may differ significantly. It may therefore be helpful to present the followint measure of the convergence rate

$$\frac{t_t - t_m}{t_\infty - t_m} = 1 - \left(\frac{1-\theta}{1+b}\right)^{t-m} \qquad (151)$$

This expression measures the increase in the current tax ratio from period m (when the convergence starts) to period t measured as a fraction of the largest possible increase $t_\infty - t_m$. With the assumption used here, this fraction will depend only on θ and b and the convergence will be quicker the greater are θ and b.[2)]

36. Under a free-sector system, the investment restriction is much less likely to be an active constraint than with investment funds of the tied-sector type. The reason is of course that intermittent releases in general are based on allocations during a number of years whereas withdrawals, made as soon as legally possible, consist of only one year's allocation. Given the use of investment funds over time it therefore tends to be easier to synchronize the flow of investment activity to withdrawals than to releases. Let us take a look at the probable size of the investment restriction $f_F = \alpha\delta r(1-\theta)^m/(\theta+b)$ - cf. equation (16) p. VIII:6 - with the present parameter values in the Nordic countries. f_F varies negatively with the growth rate θ and the depreciation rate b and we shall therefore choose the relatively small values θ=1 % and b=5 %. Calculating f_F for Sweden (where α=40 % and δ=30%) it then appears that only if

1) With a market rate of interest of 6 % K would be about 1.04 in Denmark. In Norway and Finland K is 1.13 and 1.16 respectively.

2) The same expression as (151) applies to the case of accelerated depreciation, see p. VI:11.

the profit rate exceeds 52.6 % will the investment restriction be violated. For Finland the corresponding border value of r is much lower (21 %) mainly because the size of the free sector δ is much larger (100 %). As mentioned earlier, the Norwegian system contains no investment restriction. Concerning Denmark, assume that funds are withdrawn after one year ($m=1$). In that case the border value of r which makes f_F equal to one is about 30 % (since $\alpha=20$ % and $\delta=100$ %). Higher m-values imply of course that the border value increases.

Since the calculated border values of r are much higher than the values of r considered in the numerical analysis, the investment restriction may in most cases be disregarded as far as free-sector systems are concerned.

The impact of parameter variations

37. It appears convenient to start the discussion of the impact of parameter changes on the limiting value of the tax ratio by indicating the relevant partial derivatives and their sign for each of the four countries. The derivatives are, cf. equation (149):

$$\frac{\partial t_\infty}{\partial \theta} = \alpha\delta(1-\theta)^m \left[\frac{m}{1-\theta}\left(K\frac{\beta}{t^N} - \frac{b}{\theta+b}\right) - \frac{b}{(\theta+b)^2}\right] \quad (152$$

$$\frac{\partial t_\infty}{\partial \alpha} = \frac{\beta}{t^N} - 1 - \gamma_A - \delta(1-\theta)^m \left(K\frac{\beta}{t^N} - \frac{b}{\theta+b}\right) \quad (153)$$

$$\frac{\partial t_\infty}{\partial (\beta/t^N)} = \alpha\left[1 - \delta K(1-\theta)^m\right] \quad (154$$

$$\frac{\partial t_\infty}{\partial \delta} = -\alpha(1-\theta)^m \left(K\frac{\beta}{t^N} - \frac{b}{\theta+b}\right) \quad (155)$$

$$\frac{\partial t_\infty}{\partial b} = \alpha\delta(1-\theta)^m \frac{\theta}{(\theta+b)^2} > 0 \quad (156$$

$$\frac{\partial t_\infty}{\partial m} = -\alpha\delta\left[\ln(1-\theta)\right]\left(K\frac{\beta}{t^N} - \frac{b}{\theta+b}\right) \quad (157)$$

Table 12. Sign of the t_∞- derivatives

Derivative of t_∞ with respect to	Sweden	Finland	Norway	Denmark
θ	+ or -	+ or -	+ or -	+ or -
α	+ or -	-	+ or -	+ or -
β/t^N	+	+ or -	+ or -	+ or -
δ	+ or -	-	-	-
b	+	+	+	+
m	+ or -	+	+	+

The growth rate θ appears twice in F_∞ giving rise to opposite tendencies. The factor $(1-\theta)^m$ reflects the importance of the "lag" after which withdrawals and loss of depreciation base occur. The faster the firm is growing the smaller is the net tax benefit from a withdrawal in relation to current profit, given the time period m after which funds may be withdrawn. The term $b/(\theta+b)$ represents the tax increasing effect of the loss of depreciation base in connection with withdrawals. For a given depreciation rate this tax increase is smaller the higher the growth rate is. Whether one tendency or the other will prevail, depends on the actual parameter values. The larger the value of the (initial) growth rate and the policy parameters m, K and β/t^N are, the less likely it is that the covariation between t_∞ and θ is negative.

With the parameter values used in Table 13 and 14 the Danish t_∞ and the growth rate vary not surprisingly in opposite directions whereas a strong positive correlation is found for Norway.[1] With regard to Sweden and Finland the covariation is not unambiguous but the sensitivity of t_∞ to growth variations seems to be relatively small. To be somewhat more precise it may be pointed out that a sufficient - but not necessary - condition for a positive covariation between t_∞ and θ under the Norwegian system is that the growth rate exceeds 2.5%. For Finland and Denmark the corresponding conditions are $\theta>8.4\%$, respectively $\theta>22.3\%$.[1]

---

1) Let us explain this briefly. Equation (152) may be written

$$\frac{\partial t_\infty}{\partial \theta} = -\frac{\alpha\delta(1-\theta)^{m-1}}{(\theta+b)^2} M$$

M is a quadratic function of b which is upwardly convex provided $K\beta/t^N>1$. If the discriminant D of this function is negative, M would be negative for all b. D is a quadratic function of θ with the roots

$$\theta = \frac{1 + mQ \pm m\sqrt{Q^2-1}}{1 + m^2 + 2mQ}$$

where

$$Q = 2K\frac{\beta}{t^N} - 1$$

D is downwardly convex if $K\beta/t^N>1$. Denoting the lower and upper roots θ_L and θ_U, it follows that $\theta_L<\theta<\theta_U$ would imply $D<0$ and thus $M<0$ and $\partial t_\infty/\partial\theta>0$. The figures given in the main text are θ_L-values. The corresponding θ_U-values for Norway, Finland and Denmark are 71.2% respectively 30.4% and 77.6%. It may be added that if D should be positive the sign of $\partial t_\infty/\partial\theta$ would depend upon the value of b. If either $b<b_L$ or $b>b_U$, $\partial t_\infty/\partial\theta$ would still be positive. This may be illustrated by the following pairs of (θ,b)-values where $\theta<\theta_L$.

	θ	b	$\partial t_\infty/\partial\theta$
Norway	0.5	10	-
	1	10	+
Finland	7	10	-
	7	5	+
Denmark	15	10	-
	15	5	+

Another sufficient but not necessary condition for $\partial t_\infty/\partial\theta>o$ (still assuming that $K\beta/t^N>1$) is that the depreciation rate b exceeds $1/m(K\beta/t^N-1)$. With the present parameter values it would be sufficient to assume b>12.5% for Norway. For DK and SF this condition would demand b-values in excess of 100%.

38. From Table 13 and 14 it also appears quite clearly that the tax reduction obtained from the free-sector system becomes smaller the shorter the durability of plant and equipment is. Contrary to what is the case under a tied system this pattern is not likely to be modified when the investment restriction is taken into account. As we have seen in Section 36 it is only in relatively extreme cases that this constraint becomes an active one with a free-sector system.

The way the allocation and withdrawal rates and m influence t_∞ is rather obvious. Provided allocations and withdrawals involve a net tax benefit (that is if $A + F_\infty > o$) the tax benefit is of course proportional to the share of profits allocated. Then it is also true that $\partial t_\infty/\partial\alpha<o$. Similarly if withdrawals are profitable ($F_\infty>o$), the benefit will, needless to say, be larger the shorter the "freezing" period m is and the larger a share of allocated funds that is withdrawn. As appears from Table 13 the sensitivity of t_∞ to changes in α varies considerably. An increase of α by 10 percentage units would in Norway reduce t_∞ by 5.8 percentage units if $\theta=5\%$ and b=10%, cf. equation (166). If, on the other hand, $(\theta,b)=(15,10)$ the reduction of t_∞ would be only 0.3 percentage units, cf. equation (168). The impact of changes in α also varies a great deal in Denmark depending on the values of θ and b whereas the impact of changes in α seems to be more even in Sweden and Finland. That t_∞ in Denmark and especially Norway are more sensitive to variations of the withdrawal rate than t_∞ would be in Sweden and Finland is of course due to the higher deposit ratio β/t^N in the former countries.

Whether the covariation between the tax ratio and the deposit ratio is a positive or a negative one depends on the size of the growth rate compared to the after-tax rate of interest paid on deposits. In Sweden with no interest on deposits $\partial t_\infty/\partial(\beta/t^N)$ is of course positive. With tax-free interest payments as in Finland and Norway $\partial t_\infty/\partial(\beta/t^N)$ would be positive if $\theta>r^*$. In Denmark the same would be the case if $\theta>(1-t^R)r^*$. [1]

1) The precise condition is $\frac{\partial t_\infty}{\partial(\beta/t^N)} \gtreqless o$ as $\theta \gtreqless \frac{(1-t^R)r^*}{1+(1-t^R)r^*}$

Table 13. The free-sector IF-system: t_∞-functions under the Nordic systems (Percentage units)

Variable parameters θ	b	SWEDEN	FINLAND	NORWAY	DENMARK
5	10	$t_\infty=100-0.1925\,\alpha$ (158)	$t_\infty=100-0.3817\,\alpha$ (162)	$t_\infty=100-0.5780\,\alpha$ (166)	$t_\infty=100-0.3500\,\alpha$ (170)
	30	$t_\infty=100-0.1484\,\alpha$ (159)	$t_\infty=100-0.2344\,\alpha$ (163)	$t_\infty=100-0.4229\,\alpha$ (167)	$t_\infty=100-0.1691\,\alpha$ (171)
15	10	$t_\infty=100-0.2099\,\alpha$ (160)	$t_\infty=100-0.3372\,\alpha$ (164)	$t_\infty=100-0.0308\,\alpha$ (168)	$t_\infty=100-0.4988\,\alpha$ (172)
	30	$t_\infty=100-0.1744\,\alpha$ (161)	$t_\infty=100-0.2189\,\alpha$ (165)	$t_\infty=100-0.1084\,\alpha$ (169)	$t_\infty=100-0.2721\,\alpha$ (173)
5	10	$t_\infty=66.19+0.3071\frac{\beta}{t^N}$ (174)	$t_\infty=85.47+0.0308\frac{\beta}{t^N}$ (178)	$t_\infty=85.18+0.0149\frac{\beta}{t^N}$ (182)	$t_\infty=92.67+0.0033\frac{\beta}{t^N}$ (186)
	30	$t_\infty=67.96+0.3071\frac{\beta}{t^N}$ (175)	$t_\infty=89.89+0.0308\frac{\beta}{t^N}$ (179)	$t_\infty=88.09+0.0149\frac{\beta}{t^N}$ (183)	$t_\infty=96.28+0.0033\frac{\beta}{t^N}$ (187)
15	10	$t_\infty=62.13+0.3468\frac{\beta}{t^N}$ (176)	$t_\infty=75.32+0.1456\frac{\beta}{t^N}$ (180)	$t_\infty=78.92+0.0769\frac{\beta}{t^N}$ (184)	$t_\infty=86.80+0.0232\frac{\beta}{t^N}$ (188)
	30	$t_\infty=63.55+0.3468\frac{\beta}{t^N}$ (177)	$t_\infty=78.87+0.1456\frac{\beta}{t^N}$ (181)	$t_\infty=81.53+0.0769\frac{\beta}{t^N}$ (185)	$t_\infty=91.33+0.0232\frac{\beta}{t^N}$ (189)
5	10	$t_\infty=94-0.0567\,\delta$ (190)	$t_\infty=100-0.1145\,\delta$ (194)	$t_\infty=125-0.3583\,\delta$ (198)	$t_\infty=107.8-0.1480\,\delta$ (202)
	30	$t_\infty=94+0.0022\,\delta$ (191)	$t_\infty=100-0.0703\,\delta$ (195)	$t_\infty=125-0.3293\,\delta$ (199)	$t_\infty=107.8-0.1118\,\delta$ (203)
15	10	$t_\infty=94-0.0799\,\delta$ (192)	$t_\infty=100-0.1012\,\delta$ (196)	$t_\infty=125-0.2559\,\delta$ (200)	$t_\infty=107.8-0.1777\,\delta$ (204)
	30	$t_\infty=94-0.0325\,\delta$ (193)	$t_\infty=100-0.0657\,\delta$ (197)	$t_\infty=125-0.2297\,\delta$ (201)	$t_\infty=107.8-0.1324\,\delta$ (205)
5	10			$t_\infty=95.42-0.1875\gamma_A$ (206)	
	30			$t_\infty=98.32-0.1875\gamma_A$ (207)	
15	10			$t_\infty=105.66-0.1875\gamma_A$ (208)	
	30			$t_\infty=108.29-0.1875\gamma_A$ (209)	
Fixed parameters					
α		40	30	18.75	20
β/t^N		85	100	266 2/3	139
δ		30	100	100	100
m		5	5	4	1
K		1	1.16	1.13	1.04
γ_A		–	–	33 1/3	–

Table 14. t_{∞}-values under the Nordic free-sector IF-systems. Percentage units.

θ	b							
	5	10	20	30	5	10	20	30
	SWEDEN				FINLAND			
3	91.68	93.17	94.20	94.61	86.24	89.95	92.54	93.55
5	90.75	92.30	93.54	94.07	84.70	88.57	91.66	92.98
10	90.34	91.52	92.70	93.29	85.37	88.32	91.27	92.75
15	90.80	91.60	92.52	93.02	87.90	89.89	92.17	93.44
	NORWAY				DENMARK			
3	85.55	87.95	89.61	90.27	91.92	94.72	96.67	97.43
5	86.80	89.34	91.38	92.25	89.88	93.04	95.58	96.66
10	92.18	94.23	96.28	97.30	87.82	90.82	93.82	95.32
15	98.07	99.54	101.22	102.15	87.51	90.06	92.98	94.60

Note. The values of the fixed parameters are as given in Table 13.

Tax Benefits under the Nordic Systems

39. In this and the next section rather sketchy illustrations of the tax reducing potential of the free-sector systems in the Nordic countries will be presented. A series of simulations are shown in Table 13 and 14 to which reference has been made already. Table 14 is based on the following t_{∞}-expressions

$$t_{\infty}^{S} = 0.94 - 0.12(1-\theta)^{5}(0.85 - \frac{b}{\theta+b}) \qquad (210)$$

$$t_{\infty}^{SF} = 1.00 - 0.30(1-\theta)^{5}(1.16 - \frac{b}{\theta+b}) \qquad (211)$$

$$t_{\infty}^{N} = 1.25 - 0.19(1-\theta)^{4}(3.00 - \frac{b}{\theta+b}) \qquad (212)$$

$$t_{\infty}^{DK} = 1.08 - 0.20(1-\theta)(1.44 - \frac{b}{\theta+b}) \qquad (213)$$

The values of the fixed parameters utilized in these four equations are indicated in Table 14. In the cases considered in Table 13 the investment restriction - equation (16) - is not violated unless the profit rate is very high. In the table withdrawals as a percentage of gross investments are highest when $(\theta,b)=(3,5)$. For Finland a profit rate in excess of 31% would be required to bring about a violation of the investment restriction. For Denmark and Sweden the border value of r would be even higher or 41%, respectively 75%. In Table 14 the investment restriction would in some cases be violated if the variable IF-parameter is given a high value and the profit rate is low. This would for instance be the case in Finland and Denmark with an allocation rate of 100% and a profit rate greater than 13% while at the same time $\theta=b=5\%$.

As pointed out earlier the term A in equation (148) reflects the tax effects of allocations. Since it is only in Sweden that the deposit ratio β/t^N is less than unity, it follows of course that, in contrast to Sweden, allocations <u>per se</u> are not profitable in the three other countries. For Norwegian and Danish firms maximal allocations would raise the tax ratio by 25%, respectively 7.8% (assuming that taxation is the only alternative to allocations). In Finland the deposit rate is equal to the statutory tax rate, as we have seen, and A is thus zero. A Swedish corporation, finally, making maximal allocations would at present (1973) obtain a 6% reduction of <u>total</u> tax liabilities.

The tax effects of withdrawals are summarized in the term F_∞. In Denmark, Finland and Norway F_∞ is always positive since $K\beta/t^N>1$. In Sweden F_∞ is positive unless the growth rate is very small relative to the depreciation rate.[1)]

Let us compare the size of F_∞ in the four countries when we in turn keep each of the IF-parameters equal in all countries. If the allocation rate α is the same the tax effects of withdrawals could be ranked as follows

$$F_\infty^S < F_\infty^{SF} < F_\infty^{DK} < F_\infty^N \qquad (213A)$$

1) $F_\infty^S < o$ if $\theta < b(t^N-\beta)/\beta$ or, with present parameter values, if $\theta<0.17\ b$.

A sufficient (but not necessary) condition for F_{∞}^{N} F_{∞}^{DK} is that the growth rate is smaller than 21.7%. Apart from that the ranking is unconditional. If we assume that the size of the free sector, δ, is the same in all four countries, F_{∞}^{S} would again be smallest and F_{∞}^{N} would be largest. The size of F_{∞}^{SF} compared to F_{∞}^{DK} depends upon the values of θ and b. Assuming that $b/(\theta+b) \geqq 0.20$ and $\theta > 3.7\%$ the ranking would again be the one given by (213A).

Next we take it for granted that m or the number of years after which withdrawals can be (or are) made is equal. Again the ranking (213A) holds provided $\theta < (2/3)b$ as far as Finland and Denmark are concerned. If $\theta > (2/3)b$ $F_{\infty}^{SF} > F_{\infty}^{DK}$ would be true. If the treatment of deposits is similar in all countries so that K would be equal the F_{∞}-ranking would once more be (213A). Sufficient (but not necessary) conditions in order to establish $F_{\infty}^{SF} < F_{\infty}^{DK}$, respectively $F_{\infty}^{DK} < F_{\infty}^{N}$ are $1.9\% < \theta < 9.7\%$ and $\theta < 17.8\%$. We finally suppose that $K\beta/t^{N}$ is the same in all countries. This changes the ordering and we have

$$F_{\infty}^{S} < F_{\infty}^{N} < F_{\infty}^{DK} < F_{\infty}^{SF} \tag{213B}$$

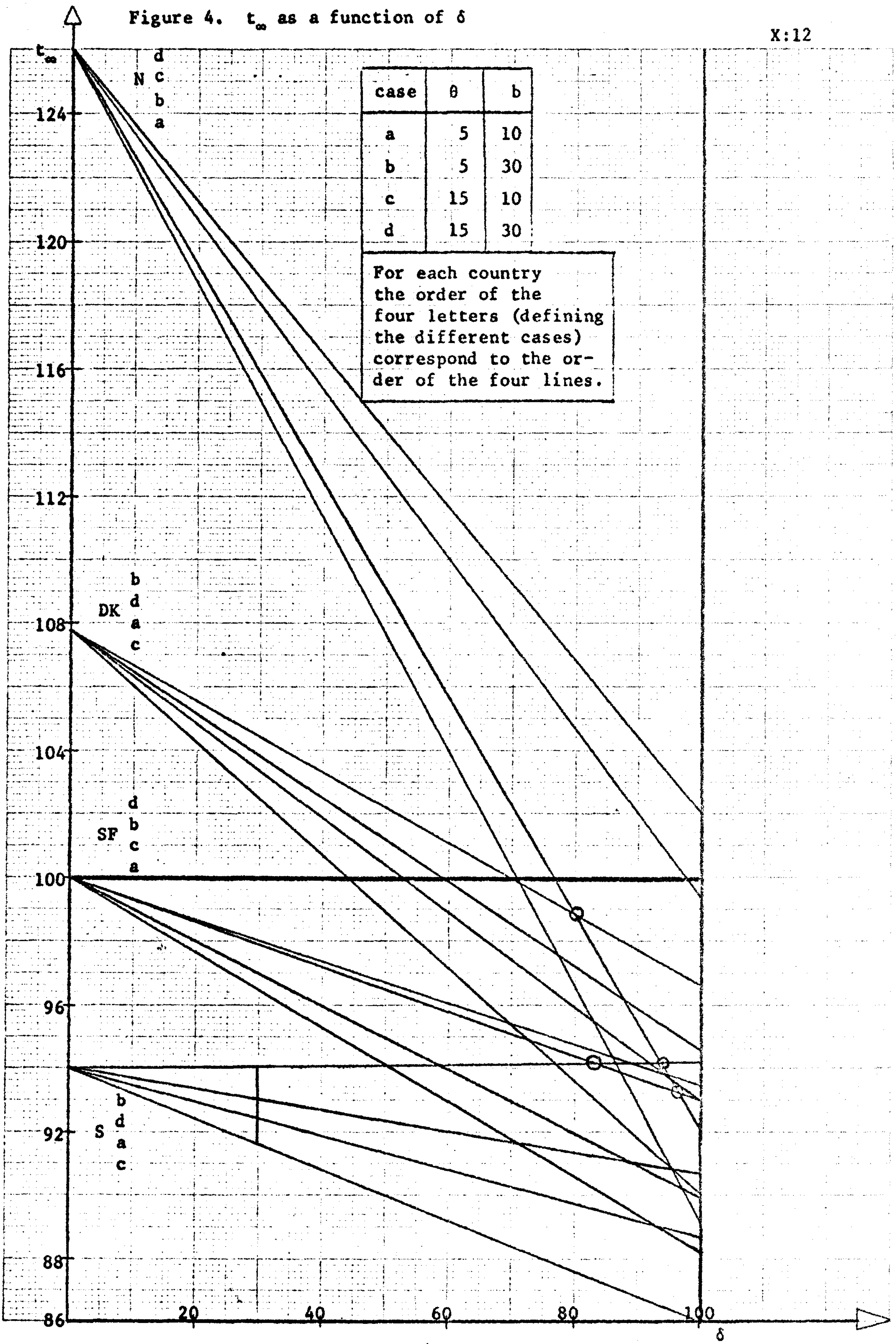
F_{∞}^{S} is thus still smallest but now $F_{\infty}^{SF} > F_{\infty}^{N}$ as long as $\theta < 37.5\%$. Concerning Denmark and Finland the ranking would be reversed if the growth rate exceeds 9.7%.

40. In the previous section we have discussed the size of A and F_{∞} separately but for a complete comparison between the four free-sector systems an evaluation of net effects would of course be needed. It does not seem to be possible to establish general and clear-cut conclusions about the tax-reducing potential of the different systems. Instead, as we have seen, the ranking (in terms of the limiting value of the tax ratio) will depend upon the durability of capital and the growth rate in addition to the values of the IF-parameters.

Some comments to the simulations in Table 13 and 14 should be made. With the same allocation rate α in all four countries and a growth rate of 5 per cent t_∞ would be lowest in Norway, lower in Denmark than in Finland and highest in Sweden. This is true whether b is 10 or 30 per cent, as Table 13 shows. If the growth rate had been as high as 15 per cent, t_∞ in Norway would instead have been the highest but apart from that the t_∞-ranking would have been the same as before. This deterioration of the Norwegian position is of course due to the fact that the covariation between t_∞^N and the growth rate is strongly positive. When we next assume that the deposit ratio β/t^N is variable the picture is somewhat less simple. Consider for instance the case where the growth rate is 15 per cent and the depreciation rate 30 per cent. With these values t_∞^S would be lowest provided the common deposit ratio is smaller than 66.7 per cent. If instead $66.7\% < \beta/t^N < 182.5\%$, t_∞^N would be lowest. Finally, if $\beta/t^N > 182.5\%$, t_∞^{DK} would be the smallest tax ratio. From the functions (174) - (189) it is also immediately seen approximately at what β/t^N-level the tax benefit disappears altogether. Assuming for instance $(\theta, b) = (15\%, 30\%)$ this would happen in the case of Sweden and Finland when β/t^N equals 105%, respectively 145%. For Norway and Denmark the corresponding figures are much higher or 273%, respectively 374%.

The tax ratios resulting from variations in the withdrawal rate δ are shown in Figure 4, which is based on Table 13. The vertical intercepts represent of course the term 1-A. Points marked by a circle indicate δ-values where the ranking of the t_∞-values for two countries is reversed. By drawing the line t_∞=100% it is easily seen in the diagram how large δ must be to secure a positive tax benefit for Danish and Norwegian firms under various conditions. With $(\theta, b) = (5\%, 30\%)$ a withdrawal rate in excess of 70 per cent would give a t_∞-value below 100 per cent in Denmark. The corresponding border value in Norway is δ = 76 per cent. For equal and low values of δ, the Swedish tax ratio would be much lower than t_∞ for the other countries in the cases considered in the diagram, but as δ rises the relative superiority of t_∞^S would tend to disappear.

Figure 4. t_∞ as a function of δ



case	θ	b
a	5	10
b	5	30
c	15	10
d	15	30

For each country the order of the four letters (defining the different cases) correspond to the order of the four lines.

Table 14 shows a handful of t_∞-values resulting from variation of the growth and depreciation rates when the IF-parameters are given and equal to the actual values. The sensitivity of the Norwegian tax ratio to growth rate changes is once more apparent. With θ equal to 3 per cent t_∞^N is lower than the tax ratios in the other countries whatever the value of b. t_∞^N is also lowest if θ=5 per cent and b is 20 per cent or more. On the other hand the Norwegian tax ratio is the highest for all b-values considered if θ is 10 or 15 per cent. It also appears that the Finnish tax ratio is smaller than the Swedish for all pairs of (θ,b) except (15%, 30%) and that $t_\infty^{SF} < t_\infty^{DK}$ in all cases except (θ,b) = (15%, 5%). Concerning Denmark and Sweden it is seen that $t_\infty^{DK} > t_\infty^{S}$ if θ is 3 per cent or if b is 20 per cent or more. It is also the case when (θ,b) = (5%, 10%). On the other hand a high growth rate combined with a low depreciation rate tends to make t_∞^{DK} lower than t_∞^{S}.

That withdrawals from the free sector in Sweden have been almost negligible is not surprising taking into account the small withdrawal rate (30 per cent) and the fact that the 10 per cent extra investment deduction is not admitted as far as the free sector is concerned. In Chapter VII we have also seen that allocations to the Finnish investment funds - in contrast to those in Denmark and Norway - have been rather unimportant (cf. table on p. VII:20).

CHAPTER XI

COMPARISONS BETWEEN INVESTMENT FUNDS AND OTHER TAX SYSTEMS

41. It is of course of considerable interest to compare an IF-system with other currently used tax systems which may give firms tax benefits similar to those offered by investment funds. This chapter is therefore devoted to an attempt to make such comparisons. In the present section the other tax systems are presented and in sections 42 and 43 the formal framework is established. In section 44 we discuss in which direction the tax ratios under different systems are changed when the parameters are changed. After that the absolute tax ratios, which the four considered depreciation systems may lead to, are compared. In section 46, finally, the tax burden under different IF-system is confronted with the tax burden obtained under the four depreciation systems.

We shall consider two kinds of accelerated depreciation. The first, accelerated normal depreciation, is assumed to amount to dC_t in each period. The difference between the depreciation rate for income tax purposes, d, and the actual depreciation rate, b (which reflects wear and tear and obsolescence), is a measure of the absolute acceleration.

The second kind is initial allowances. Under this system a certain fraction of gross investments may be written off immediately over and above normal depreciation but the base for future depreciation is reduced correspondingly.

Investment allowances differ from initial allowances since the depreciation base is not influenced as far as the former are concerned. An investment allowance is thus simply an extra deduction given when real investments are made and has nothing to do with accelerated depreciation. Initial and investment allowances have been used widely in Great Britain during the post-war period. Initial allowances were introduced in 1945 and the rates have varied between 5 and 40 per cent depending upon year and type of asset. Investment allowances with rates varying between 10 and 20 per cent for most investments[1] have been used more or less permanently between 1954

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1) For more details see (36) pp. 421-50 (especially pp. 424-25).

and 1966 when they were replaced with outright investment grants.[1] In Sweden a 10 per cent investment allowance is - as we have seen - a permanent feature of the IF-system. More generally, 10 per cent investment allowances (for state tax purposes only) have been granted for industrial investments in machinery in 1964 and for all investments in machinery and equipment 1968. During the recession 1971-73 a 10 per cent investment allowance was given to building investments and for machinery purchases the original investment allowance of 10 per cent was successively increased to 30 per cent (cf. Chapter IV).

The final tax device to be considered is the investment tax credit introduced by the Kennedy administration 1962 and ultimately repealed in April 1969.[2]

The immediate tax benefit amounted to 7 per cent of expenditures on machinery and equipment. It is thus a credit against tax liabilities and not a deduction from taxable income as initial and investment allowances. The investment tax credit, on the other hand, increases future tax liabilities since the depreciation base is reduced by the tax credit. In this respect the tax credit is similar to initial allowances. The right to claim investment tax credits was restricted in certain respects. Thus, only machinery and equipment with a durability of four years or more were considered "qualified investment" and if the durability was less than eight years only part of the investment outlay could be used as a base for the tax credit. The tax credit itself was limited to 25.000 dollars plus 25 per cent of taxes above that amount.[3]

42. Let a, h, and k denote the initial allowance rate, the investment allowance rate, and the investment tax credit rate respectively. Assume

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1) According to professor Robert Neild, a major reason for that change was the belief that a large number of British firms do not fully recognize the tax benefits offered by initial or investment allowances because profitability calculations either are made in terms of profits before taxes or otherwise are too crude. The investment grants have recently been replaced by rather liberal depreciation guidelines by the conservative government.

2) The Council of Economic Advisers argued that "The national priorities of the 1970's did not require or justify this special incentive", see The Annual Report of the Council of Economic Advisers, Washington D.C. February 1970, p. 31.

3) For further details see [13] p. 40. The importance of this limitation is discussed in section 43 below.

further that all investment outlays qualify for the tax benefits.[1] With all four depreciation systems in operation simultaneously the total deductible amount would be

$$D_t = dC_t + aI_t^G + hI_t^G \tag{214}$$

Taxable income W_t would therefore be

$$W_t = V_t + b\dot{K}_t - D_t \tag{215}$$

while tax liabilities (payments) T_t would be

$$T_t = t^N W_t - kI_t^G \tag{216}$$

cf. the corresponding expressions (22) - (24) for the IF-model. As before, the book value of fixed capital is increased in each period by investments and decreased by depreciation allowances so that

$$C_t = C_{t-1} + I_t^G - dC_t - (a+k)I_t^G \tag{217}$$

The solution of this difference equation is

$$C_t = K_t \frac{(1-a-k)(\theta+b) + \left(d-b+(a+k)(\theta+b)\right)\left(\frac{1-\theta}{1+d}\right)^{t+1}}{\theta + d} \tag{218}$$

After using this expression the current tax ratio $t_t = T_t/V_t t^N$ can be written

$$t_t = 1 - \frac{d - b + (a+k)(\theta+b)}{r(\theta+d)}\left[\theta + d\left(\frac{1-\theta}{1+d}\right)^{t+1}\right] - \frac{k(\theta+b)(1-t^N)}{rt^N} - \frac{h(\theta+b)}{r} \tag{219}$$

With d=b and a=h=k=o the tax ratio is of course 1. This is the case before period zero when the tax devices discussed here are supposed to be introduced. As a consequence of this the tax ratio is lowered considerably below unity but as time goes on t_t increases in a convergent manner. The limiting value is evidently:

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1) This is of course not a realistic assumption in some (if not most) cases. If a depreciation scheme (such as the investment tax credit) apply only to certain types of investment outlays and we assume that the qualified capital outlays' share of total investment expenditures is constant, this complication can be dealt with easily. We simply consider the depreciation rates a, h and k, not as the statutory rates but as the statutory rates multiplied by the appropriate ratios of qualified investments and total investments.

$$t_\infty = 1 - \frac{\theta\left(d - b + (a+k)(\theta+b)\right)}{r(\theta+d)} - \frac{k(\theta+b)(1-t^N)}{rt^N} - \frac{h(\theta+b)}{r} < 1 \qquad (220)$$

The final step is to calculate the average tax ratio in the same way as in section 23. This yields

$$\bar{t} = 1 - \frac{d - b + (a+k)\ (\theta+b)}{r} \quad \frac{D - 1}{D(1+d) - 1} - \frac{k(\theta+b)(1-t^N)}{rt^N} - \frac{h(\theta+b)}{r} \qquad (221)$$

In the following analysis we shall consider each of the four tax systems in turn and it is therefore assumed that only one system operates at a time. We assume further that profits before depreciation allowances are large enough to absorb all permitted allowances. The tax ratio is in other words always positive. The way the average tax ratio is influenced by parameter changes in the four cases is indicated in Table 15 which may be compared with Table 12. To give a rough idea of the potential profitability effects of the four systems, some simulations are presented in Table 16.

Table 15. Sign of Covariation Between the Average Tax Ratio and the Parameters Under different Tax systems

Tax System	Tax Function	Parameters					
		Depreciation parameter[1]	b	θ	r	D	t^N
Accelerated Normal Depreciation	$\bar{t}_d = 1 - \frac{d-b}{r}\frac{D-1}{D(1+d)-1}$ (222)	–	+	0	+	–	0
Initial Allowances	$\bar{t}_a = 1 - \frac{a(\theta+b)}{r}\frac{D-1}{D(1+b)-1}$ (223)	–	–	–	+	–	0
Investment Allowances	$\bar{t}_h = 1 - \frac{h(\theta+b)}{r}$ (224)	–	–	–	+	0	0
Investment Tax Credit	$\bar{t}_k = 1 - \frac{k(\theta+b)}{r}\left(\frac{D-1}{D(1+b)-1} + \frac{1-t^N}{t^N}\right)$ (225)	–	–	–	+	–	+

1) The depreciation parameters have been used as subscripts to the average tax ratio.

Table 16. The Average Tax Ratio Under Different Tax Systems.
Fixed parameters: $r=12.5\%$, $t^N=51.69\%$ and $D=1.12$. Percentage units

Common variable parameters		Accelerated Normal Depreciation			Initial Allowances		Investment Allowances		Investment Tax Credit	
θ	b	d	d−b	τ_d	a	τ_a	h	τ_h	k	τ_k
5		7.5	2.5	88.24	10	94.55	10	92	7	91.95
	5	10	5	79.31	25	86.36	15	88	10	87.07
		15	10	66.67	40	78.18	20	84	12	84.48
		12.5	2.5	90.77	10	93.79	10	88	7	87.80
	10	15	5	83.33	25	84.48	15	82	10	82.58
		20	10	72.09	40	75.17	20	76	12	79.09
		22.5	2.5	93.55	10	93.02	10	80	7	82.03
	20	25	5	88.00	25	82.56	15	70	10	74.33
		30	10	78.95	40	72.09	20	60	12	69.19
		32.5	2.5	95.04	10	92.63	10	72	7	76.52
	30	35	5	90.62	25	81.58	15	58	10	66.46
		40	10	83.10	40	70.53	20	44	12	59.75
10					10	91.82	10	88	7	86.42
	5				25	79.55	15	82	10	80.60
					40	67.27	20	76	12	76.72
					10	91.72	10	84	7	83.74
	10				25	79.31	15	76	10	76.77
					40	66.90	20	68	12	72.12
					10	91.63	10	76	7	78.44
	20				25	79.07	15	64	10	69.19
					40	66.51	20	52	12	63.03
					10	91.58	10	68	7	73.17
	30				25	78.95	15	52	10	61.67
					40	66.32	20	36	12	54.00

43. In the formal analysis we have not explicitly considered the investment tax credit limit mentioned earlier and a few words may be in order. According to this rule, the tax credit is maximized to (in thousands of dollars):

$$TC_{max} = 25 + 0.25(T_t - 25) \tag{226}$$

To simplify, let us here calculate tax payments as $T_t = t^N V_t = t^N r K_t$. This would in fact give a somewhat too <u>low</u> estimate of T_t (and thus of TC_{max}), since the loss of depreciation base caused by the investment tax credit is disregarded in this way. Suppose now that qualified investments amount to a certain share s of total gross investments $I_t^G = (\theta+b)K_t$. Without the limit the tax credit would then be $TS = ks(\theta+b)K_t$. The difference between TC_{max} and TC is thus

$$TC_{max} - TC = 18.75 + K_t\left(0.25t^N r - ks(\theta+b)\right) \tag{227}$$

A sufficient but not necessary condition for $TC_{max} > TC$ is (assuming t^N=50% and k=7%) that

$$s(\theta+b) < 1.785\ r \qquad (228)$$

This condition seems to indicate that the gross growth rate (θ+b) must be rather high relative to the profit rate (before taxes) to make the upper tax credit limit an active constraint. The highest gross growth rate considered in the numerical analysis is 40% while the profit rate is assumed to be 12.5%. With these parameter values the investment tax credit limit would be of no importance if less than 55% of all investment outlays were qualified expenditures. With a gross growth rate of less than 22%, the same would be true even if all investments were eligible. With a higher tax credit rate than 7% these borderline values would of course be lower.

44. In this and the next section we shall briefly comment on Tables 15 and 16 and make some comparisons with investment funds. We shall first discuss how the tax ratio is influenced by parameter changes under different systems and then - in section 45 - compare the absolute tax burden which the four depreciation systems may bring about.

With initial or investment allowances or an investment tax credit, the tax benefit becomes larger the greater the firm's <u>growth rate</u> is. Assuming that a=h=k the sensitivity of the average tax ratio to changes in the growth rate is smallest with initial allowances and greatest with the investment tax credit.[1] This is illustrated in Table 16 to some degree. As far as accelerated normal depreciation is concerned the average tax ratio is independent of the growth rate with our assumptions but if instead we look at the limiting value of the current tax ratio - cf. equation (220) - we shall find a negative correlation as in the three other cases.[2] With an IF-system of the Swedish tied sector type, we found a similar negativa covariation in the numerical analysis but a rather weak one as long as we disregard the investment restriction. Taking that into account, we saw that the possibilities to lower the tax burden increase as the growth rate rises.

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1) It is also necessary to assume that the factor within the brackets in equation (225) is greater than unity. This is the case in Table 16.

2) Letting t^d denote the limiting value of the tax ratio under accelerated normal depreciation we have $$\frac{dt^d_\infty}{d\theta} = -\frac{d(d-b)}{r(\theta+d)^2} < 0$$

A relatively short durability of plant and equipment (or a high rate of depreciation) means a relatively large tax ratio with accelerated normal depreciation simply because the absolute acceleration d-b is small in such a case. Under the three other systems the tax benefits become larger the shorter the durability of capital is. Again, as appears from Table 16, $\bar{t}_a$ is less sensitive to changes in b than $\bar{t}_h$ and $\bar{t}_k$, provided the initial depreciation rates are equal.[1] With an IF-system (whether of the tied or free sector type) the tax benefits are larger the longer the durability of capital is. It is, however, possible, as we have seen, that this conclusion will have to be reversed in cases when the investment restriction actually is a constraint. Needless to say, the differences between the tax systems with regard to the impact of capital durability on the tax ratio are highly important when it comes to an evaluation of the systems.

Other things being equal, changes in the depreciation rate imply of course changes in the volume of gross investment in each period in addition to changes in the (average) durability of new plant and equipment. It may therefore be of interest to look at the effects of variations in b while keeping the volume of gross investment constant. Let us assume opposite changes of the depreciation and the growth rate such that $\theta^G = \theta + b$ is constant. As appears from Table 15, this changes the sign of the covariation between $\bar{t}$ and b from minus to plus in the case of initial allowances and the investment tax credit whereas $\bar{t}_h$ becomes invariant to such changes in b. As far as $\bar{t}_d$ is concerned, it makes of course no difference whether θ or θ^G is kept constant since $\bar{t}_d$ is not a function of the growth rate.

Under all four systems the tax ratio is an increasing function of the profit rate r. We can in other words say that the use of one or more of these tax instruments changes a proportional corporate tax system into a progressive one. In Chapter VIII we saw that with an IF-system the average tax ratio is not a function of the profit rate if the release rates q and δ are considered exogenous.

Finally, it may be pointed out that only with an investment tax credit is the tax ratio a function of the statutory tax rate t^N and the covariation

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1) Whether $|d\bar{t}_h/db| \gtreqless |d\bar{t}_k/db|$ depends on whether $(D-1)(D-D\theta-1)/(D+Db-1)^2 + \frac{1-t^N}{t^N} \gtreqless 1$

is positive as the table shows. In the IF-system $\bar{t}$ varies positively with the deposit ratio β/t^N and thus negatively with t^N.

45. Let us now take a look at the absolute size of the average tax ratio under different tax systems. An investment allowance is obviously a more powerful tax reducing instrument than an initial allowance of the same size. With h=a=10%, a growth rate of 5% and the depreciation rate varying between 5 and 30% $\bar{t}_h$ lies in the interval 72-92% while $\bar{t}_a$ varies between 92 and 95%, cf. Table 16. Whether initial allowances bring about a lower or higher average tax ratio than accelerated normal depreciation depends upon the following condition:

$$\bar{t}_a \lesseqgtr \bar{t}_d \quad \text{as} \quad a \gtreqless \frac{d-b}{\theta+b}\,\frac{D(1+b)-1}{D(1+d)-1} \tag{229}$$

The corresponding condition obtained when accelerated normal depreciation and investment allowances are compared is

$$\bar{t}_h \lesseqgtr \bar{t}_d \quad \text{as} \quad h \gtreqless \frac{d-b}{\theta+b}\,\frac{D-1}{D(1+d)-1} \tag{230}$$

Let us finally compare the investment tax credit with the other three systems. The tax-reducing impact of the investment tax credit is clearly greater than that of initial allowances, given that the two rates are equal (k=a). The investment tax credit can in fact be seen as a <u>combination</u> of an initial allowance with the rate k and an investment allowance with the rate $k(1-t^N)/t^N$ as is evident from equations (223) - (225). With "realistic" parameter values it is also likely that the investment tax credit implies a lower average tax ratio than the investment allowance provided again that the rates are of the same size (k=h).[1] Concerning the power of the investment tax credit compared to that of accelerated normal depreciations it is true that

$$\bar{t}_k \lesseqgtr \bar{t}_d \quad \text{as} \quad k \gtreqless \frac{d-b}{\theta+b}\,\frac{D(1+b)-1}{D(1+d)-1}\,\frac{t^N(D-1)}{D-1+Db(1-t^N)} \tag{231}$$

The last two ratios are evidently positive and less than one. The higher the growth and depreciation rates are, the more probable it is that the tax

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1) The condition is of course that the factor within the brackets in equation (225) is greater than unity. This is the case if $t^N \leq 0.5$. It is also the case if for instance $b \leq 0.44$ and $D \geq 1.06$ while $t^N \leq 0.53$.

burden with accelerated normal depreciation will exceed the tax burden under any of the three other systems, cf. equations (229) - (231). With the combinations of parameter values used in Table 16 $\bar{t}_d$ is in fact smaller than $\bar{t}_a$, $\bar{t}_h$ and $\bar{t}_k$ when b is small but greater when b is large.

46. It is evidently not so simple to compare the tax ratios obtained under different depreciation schemes with the tax ratios which an IF-system may lead to because of the fairly large number of parameters involved. An attempt to make a comparison between a tied IF-system of the Swedish type, the four free-sector systems and the depreciation systems has been made in Table 17. In that table we have presented the values of the average tax ratio corresponding to 24 tied sector parameter combinations. For each of the four depreciation systems we have further calculated the values of the depreciation parameter which yields the same average tax ratio as each of the 24 IF-parameter combinations. We have in other words translated sets of IF-parameter values into the equivalent depreciation parameter values. For the four free-sector systems we have in a similar way calculated the allocation rates which - given the actual parameter values in the four countries - is needed to give the $\bar{t}$-values in the table. Concerning the tied IF-system, it has been assumed that the firm actually allocates 40% of profits to investment funds, that it faces a deposit ratio of 89% and that releases occur every fourth year. In addition it has been supposed that the investment deduction rate is 10% as presently and that the firm's rate of profit and discount rate of interest are 12.5% and 12% respectively. The growth, depreciation and release rates are varied as shown in the table. Releases as a percentage of gross investments are indicated in the f_B-column. If that figure exceeds 100%, the investment restriction is obviously violated.

With for instance a growth rate of 5%, a 20% depreciation rate and complete release (q=100%) the average tax ratio is 85.40%. In order to obtain that tax burden with accelerated normal depreciation, d has to be 26.43% and the absolute acceleration is hence 6.43 percentage units. If instead the same tax burden is to be secured with either initial allowances, investment allowances or an investment tax credit, the values of the depreciation parameters a, h and k will have to be 20.93%, 7.30%, and 5.69% respectively. It should of course be kept in mind at this point that the calculations are based on the assumption that all investment outlays are entitled to the tax benefits discussed here. If only a part of such outlays qualify the depreciation rates in Table 17 should be raised correspondingly. To illustrate this assume that 60% instead of 100% of investments are eligible for investment allowances. In that case the statutory allowance rate would have to be 12.17% (= 7.30/0.60) instead of 7.30% in order to give an average tax ratio of 85.40%.

General and unconditional statements about the profitability effects of investment funds compared to these of the other four tax systems can hardly be made and the content of Table 17 is therefore not easily summarized. Let us make some observations. In the first place it may be noted that $k<h<a$ for all values of θ and b considered in the table. This means of course that the tax reducing impact of the investment tax credit (disregarding the upper limit) is greater than that of an equally large investment allowance which in its turn exceeds the impact of an initial allowance of the same size.

Comparing tied-sector investment funds with an initial allowance of say 18% or an investment allowance of 9% or an investment tax credit of 6.5% (without upper limit) Table 17 shows the IF-system to be the most powerful in terms of tax impact if at least 70% of allocated investment funds are released and the wear and tear rate b does not exceed 10%. As the growth rate increases and the release rate becomes smaller the balance is changed against the IF-system (the above borderline value of b gets smaller). Alternatively it may be said that in all cases covered by Table 17 tied-sector investment funds are more powerful than an initial allowance of 11.5% or an investment allowance of 3% or a tax credit of 2.5% provided that 70% or more of investment funds are released.

To the extent that only certain types of investment outlays qualify for tax benefits under the three depreciation schemes discussed here, the balance is of course shifted in favour of the IF-system. The same is true if the tax credit limit is taken into account. When the gross growth rate is small the investment restriction (implying that releases cannot exceed gross investments) may on the other hand limit the release rate to values below these indicated in Table 17.

Table 17. Equally powerful combinations of parameter values under different tax systems. Percentage units.

Common parameters 1)		Swedish tied sector IF-system 2)			Accelerated normal depreciation		Initial allowances	Investment allowances	Investment tax credit	Free-sector IF-systems: α 3)			
θ	b	q	$\bar{t}$	f_B	d	d-b	a	h	k	Sweden	Finland	Norway	Denmark
5	5	40	87.50	70.49	7.68	2.68	22.92	15.63	9.67	60.33	26.21	33.51	20.33
		70	81.42	123.36	9.35	4.35	34.07	23.23	14.37	89.67	38.95	49.81	31.21
		100	75.34	176.22	11.35	6.35	45.22	30.83	19.07	119.02	51.70	66.11	40.10
	10	40	89.49	46.99	12.90	2.90	16.94	8.76	6.03	58.65	27.37	39.22	22.46
		70	84.91	82.24	14.43	4.43	24.31	12.57	8.66	84.21	39.30	56.31	32.24
		100	80.33	117.48	16.17	6.17	31.69	16.39	11.29	109.77	51.22	73.40	42.03
	20	40	91.52	28.20	23.37	3.37	12.15	4.24	3.30	56.31	29.34	52.67	26.67
		70	88.46	49.34	24.78	4.78	16.55	5.77	4.50	76.63	39.93	71.68	36.29
		100	85.40	70.49	26.43	6.43	20.93	7.30	5.69	96.95	50.52	90.68	45.91
	30	40	92.55	20.14	33.87	3.87	10.10	2.66	2.22	54.78	31.04	70.28	30.91
		70	90.26	32.25	35.22	5.22	13.22	3.48	2.90	71.56	40.58	91.89	40.41
		100	87.98	50.35	36.64	6.64	16.32	4.29	3.58	88.38	50.08	113.40	49.88
10	5	40	87.51	41.27			15.26	10.41	6.44				
		70	81.44	72.21			22.69	15.47	9.57				
		100	75.37	103.17			30.11	20.53	12.70				
	10	40	89.44	30.95			12.76	6.60	4.55				
		70	84.83	54.16			18.33	9.48	6.53				
		100	80.21	77.38			23.91	12.37	8.52				
	20	40	91.49	20.63			10.17	3.55	2.76				
		70	88.40	36.11			13.85	4.83	3.77				
		100	85.32	51.38			17.54	6.12	4.77				
	30	40	92.53	15.48			8.87	2.34	1.95				
		70	90.22	27.08			11.62	3.06	2.55				
		100	87.92	38.69			14.35	3.78	3.15				

1) Fixed parameters: r = 12.5% and D = 1.12.

2) Fixed parameters: α = 40%, β/t^N = 89%, γ = 10% and p = 4.

3) The assumed values of the fixed parameters are D = 1.12, β/t^N = 89% (for Sweden) and the parameter values shown in Table 13.

Let us finally confront the tied IF-system with accelerated normal depreciation. As Table 17 shows the profitability effects are largest under the former system if all funds are released and the absolute acceleration does not exceed 6 percentage units. With a release rate between 70 and 100 % the corresponding figure is 4 percentage units. Variations of the wear and tear rate in the interval 5-30 % do not upset these conclusions.

The last four columns of Table 17 show the values of the allocation rate α which are necessary under the four free-sector IF-systems to bring about the average tax ratio indicated in column 4. A few observations based on these figures may be made. In the first place we may note that for all four countries all α-values tend to be significantly higher than the actual α-values (shown in Table 16). This means of course that for the growth and depreciation rates considered here the Swedish tied-sector system tends to be a <u>more</u> powerful tax-reducing instrument than any of the free-sector variants. A few exceptions to this rule appear in Table 17. Thus the Finnish system (where at present α = 30 per cent) involves in fact a slightly lower average tax ratio than under the tied system provided firstly that only 40 per cent of allocations are released under the tied system and secondly that the depreciation rate is not as high as 30 per cent. In the very same cases the average tax ratio under the Danish system (where α = 20 per cent) is only slightly higher than under the tied system.

We may also note that the "necessary" value of α is lowest under the Danish and highest under the Swedish free-sector system. This means that in terms of the average tax ratio and given the values of all other IF-parameters the Danish system tends to be <u>potentially</u> the most powerful of the four free-sector variants. To this should be added that our selection of fixed IF-parameters is arbitrary and that the size of the actual tax benefits offered of course depends on the actual value of α.

CHAPTER XII

SOME CONCLUDING REMARKS TO PART TWO

47. The aim of the analysis in Part Two has been rather modest. An attempt has been made to build a model which can be used at an evaluation of the tax-reducing potential of investment funds and alternative tax systems available to a growing firm. To ensure that the model is manageable a series of simplifying assumptions has been accepted. First of all the firm's growth rate has been considered an exogenous variable. This means of course that the firm's investment activity is not influenced by the presence or absence of the discussed tax systems. Ideally, we would like to construct a model capable of explaining how the firm's investment and production behaviour is modified by the tax benefits offered through various corporate tax systems. What makes such an approach very difficult is of course the fact that our knowledge of the factors which determine private investments is still very far from being complete. Along the lines of Domar and Eisner we have instead accepted the far less ambitious task of analyzing the size of the effective tax rate under alternative assumptions with regard to the firm's growth rate, profitability, durability of fixed capital and the various tax parameters involved. Ultimately, we are of course interested in the relationship between the effective tax rate and the firm's volume of investment. Other things being equal, it seems reasonable to assume - as Domar did - that the larger the tax benefits involved in a certain taxation scheme the larger the volume of investments. A detailed numerical specification of such a relationship must of course be based on an econometric estimation.

It should be stressed again that all problems related to the financing of investments have been disregarded, not because they are considered less important but to simplify the analysis. This is in the first place tantamount to an assumption that investment activity is not restricted by mere unavailability of funds. As is well-known even the structure of financing may act as a constraint on investments to the extent that firms have strong preferences regarding their sources of finance. For a number of reasons it seems to be more or less generally accepted that firms are reluctant to accept external financing beyond a certain limit (for instance a certain debt ratio).[1] Empirical observations appear not to be inconsistent with this asumption.[2] This is naturally not the place to pursue this theme.

We have moreover not considered the role played by the firms' dividend policies. Here a distinction between larger and smaller enterprises is important since the earners of the latter in many cases will prefer to obtain their share of profits in the form of high wages and salaries. Concerning larger firms it is usually assumed that their dividend policies aim at a certain dividend level which is kept stable or rising at a cautious rate so that future dividend reductions can be avoided. For some time dividends may of course be paid out of previously retained earnings but it is moreover commonly supposed that firms hesitate to pay dividends without showing a corresponding amount as taxable profit. In that case the need to pay dividends may rule out a full utilization of the legal possibilities to write down assets or make allocations to investment funds. The firm's dividend policy prevents in other words a minimization of the current effective tax rate.[3]

It should also be emphasized that our analysis has been partial even in the sense that one tax scheme has been studied at a time. In a more realistic setting it would of course be necessary to consider all tax devices simultaneously. Our aim, however, has not been to discuss how firms weigh the properties of different tax systems against each other in a particular situation. Instead, an attempt has been made for each tax instrument to demonstrate its tax-reducing potential and other relevant properties.

48. Since the analysis in Part Two was summarized in Chapter I a second summary will not be given here. Instead we shall complete the study with a brief discussion of the usefulness of investment funds and some other

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1) In practice it is of course very difficult to determine to what extent an increased flow of internal funds is needed to accomplish an expansion of industrial investments and how far it is desired by the business community for distributional reasons. Needless to say this uncertainty leaves room for "tactical" arguments.

2) In the United States financing from internal sources has in recent years accounted for from about 80 to over 100 per cent of total capital expenditures, see [31A] p. 195. Although the share of internal financing has declined in Swedish manufacturing industry since the middle of the Fifties it was by the end of the Sixties still about 85 per cent or comparable to the American share. See [6] p. 309-10.

3) For a discussion of dividend policies, see e.g. Södersten [ ] p. 341-46.

fiscal parameters as instruments of stabilization and allocation policies. This far from exhaustive discussion can be seen as a complement to the instrument comparisons in Chapter XI. The other instruments to be considered are investment taxes (subsidies), initial and investment allowances and investment deductions. In the previous chapter it was pointed out that the investment tax credit may be seen as a combination of an initial allowance and an investment allowance. It is therefore not necessary to include the investment tax credit explicitly in the following comparisons.

Investment funds have been strongly criticized on the ground that they favor firms with large historical profits. Since it evidently is far from certain that the investment projects of these firms are the most profitable and with an imperfect capital market it is clear that an operating IF-system may result in an allocation of resources which is less efficient than it otherwise would have been. None of the other tax devices seems to involve a bias of this sort.

A similar bias is created if the levels of current profits vary considerably among firms. Introduction of initial and investment allowances or investment deductions (including those given in accordance with IF-provisions) will obviously not lead to an immediate reduction of the effektive tax rate unless profits - given a possible dividend restriction and the firm's normal depreciation policy - are sufficiently large to absorb these deductions. Increased investment incentives cannot, therefore, be created through these channels as far as firms without "excess" profits are concerned. This outcome is of course modified to some extent if loss offset is granted or losses may be carried forward. The tax benefits obtained through releases of investment funds (disregarding the extra investment deduction) or though outright subsidies are, on the other hand, independent of current profit and will thus, if used, create more general investment incentives than the various deductions. In a situation where restrictive measures are called for and investment taxes are introduced or raised varying levels of current profit may again involve a differential treatment of firms in favor of those with large profits. This is the case if the investment tax - as the one charged in Sweden 1952-53 and 1955-57 - is deductible (or part of the depreciation base). With a corporate tax rate of about 50 per cent the effective investment tax for a firm with current profit would in fact be only half as great as the investment tax facing a firm earning no current "excess" profit. If on the other hand the investment tax is not deductible - and this was true of the selective investment tax payable 1967-68 and 1970-71 in Sweden - no such bias is involved.

As we have seen earlier allocations to Swedish investment funds will increase the firm's liquidity and after-tax profitability if taxation is the only alternative. This does not mean necessarily that the Swedish IF-system is unable to bring about a reduction of investments in boom periods. Internally financed investments during a boom may be said to imply an opportunity cost equal to (the present value of) the tax benefits obtained if the project were financed by released funds in a later recession. It is therefore conceivable that a partial shifting of the firm's investment activities from boom to recession periods may appear advantageous and therefore will be carried out. The fact that investment funds are a permanent part of the corporate tax system makes it probably easier (less costly) to consider this possibility when investment plans are made. It may, on the other hand, be argued that uncertainty about future releases, the release conditions to be stimulated and future selling prices, wage costs, exchange rates etc. tend to eliminate the benefits from a changed time pattern for the firm's investment activity. It appears therefore reasonable to conclude that the working of the Swedish IF-systems is asymmetrical in the sense that it can stimulate investments in recessions but only to a very limited extent check investments in boom periods.[1) In this respect the Swedish IF-system is thus a much less efficient instrument for stabilization purposes than investment taxes/subsidies. The latter instrument appears also superior in the sense that an evaluation of the costs and benefits involved is fairly simple when investment taxes or subsidies are used but relatively complicated as far as investment funds are concerned. And unrecognized benefits can hardly act as incentives.

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1) We have here disregarded the special ad hoc benefits offered if allocations were made during the boom years 1960-61.

It has been pointed out earlier that the low share of firms with investments funds (about 4 per cent of all Swedish corporations) has made the administrative handling of the system fairly simple. Measured in terms of production or investments the share of participating firms is of course much higher or well in excess of 50 per cent. That the low participating rate may be seen as a disadvantage from equity and resource allocation points of view is quite clear; The low participating rate and the predominance of larger firms tend also, however, to make the IF-system less effective as far as stabilization is concerned. In the surveys made to assess the effects of the releases in 1962 and 1967-68 (see /10/ and /31/) the ratio between net and gross effects (i.e. the share of IF-financed investments which would not have been undertaken in the absence of the release) was considerably lower for large than for small firms.This difference is probably due to the fact that large enterprises, in contrast to smaller firms, generally have detailed and rather rigid long term investment plans.Although these plans are not easily changed it may be easy to switch to IF-financing when releases of funds are made.Since all firms would benefit from a negative investment tax or an extra investment deduction (provided profits admit it) these two tools would be - other things being equal - more powerful than an IF-release when investment stimulation is called for.

A final disadvantage with investment funds compared to the other tax instruments is that by and large it is not possible to vary the benefits when the release is general and the release period is given. This point has been stressed by Assar Lindbeck. To illustrate his point Lindbeck makes a comparison with credit policy and argues that it would be less useful if only zero and 10 per cent were allowed as values for the rate of interest.[1)]

Neddless to say, in a complete assessment of the merits and disadvantages of investment funds compared to other tax instruments it would be necessary to consider a number of further aspects such as the possibilities to obtain a precise timing of effects on investments and employment, to differentiate the effects between regions, to avoid evasions etc.

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1)See article in DAGENS NYHETER July, 24 1971("Fel på fonderna")

List of Symbols

The following list in alphabetical order indicates the most important symbols used in the analysis

a	the initial allowance rate
A	the reduction of the tax ratio due to allocations $[A = \alpha(1-\beta/t^N)]$
A_t^G	that part of the gross allocation which is deposited in the Central Bank $(A_t^N = \beta A_t^G)$
b	the proportion of the capital stock which wears or ages per time period
$\bar{B}$	the reduction of the average tax ratio due to releases from the tied sector
C_t	the book value of the firm's capital stock
d	the normal (or current) rate of depreciation allowance
D	the firm's discount factor $(D = 1 + r^d)$
$\bar{D}$	the reduction of the average tax ratio due to conventional accelerated depreciation
D_t^d	deductible depreciation allowances $(D_t^n = dC_t)$
D_t^n	non-deductible depreciation allowances $(D_t^n = R_{np}^G + F_t^G)$
f_B	release of funds from the tied (bound) sector as a fraction of gross investments $(f_B = R_{np}^G/I_{np}^G)$
f_F	withdrawal of funds from the free sector as a fraction of gross investments $(f_F = F_t^G/I_t^G)$
f	$f = f_B + f_F$
$\bar{F}$	the reduction of the average tax ratio due to withdrawals from the free sector
F_t^G	gross withdrawal of funds from the free sector $(F_t^G = \delta A_{t-m}^G)$
F_t^N	that part of the free sector withdrawal which is paid out in cash by the Central Bank $(F_t^N = \beta F_t^G)$
g, G	the rate of growth of the firm's capital stock (and investments)
h	the investment allowance rate
H_{np}	the extra investment deduction in connection with releases from the tied sector $(H_{np} = \gamma R_{np}^G)$

I_t^G	gross investments
I_t^N	net investments
i	the number of years which have elapsed since the <u>latest</u> release from the tied sector
j	the number of years which have gone since the <u>first</u> withdrawal (in year m) from the free sector
k	the investment tax credit rate
K_t	the <u>actual</u> value of the firm's capital stock
m	the number of years which must go before an allocation (partly or wholly) may be withdrawn from the free sector
n	the number of years in which releases from the tied sector have occured
p	the number of years between the periodical releases from the tied sector (the length of the IF-cycle)
q	the release from the tied sector as a fraction of the allocations during the previous p years
r	the firm's rate of profit <u>before</u> taxes ($r = V_t/K_t$)
r^d	the discount rate of interest which the firm considers relevant
r^*	the rate of interest paid on deposits
$\bar{r}_a$	the firm's average rate of profit <u>after</u> taxes ($\bar{r}_a = r(1-\bar{t}_t^E)$
R_{np}^G	gross release from the tied sector
R_{np}^N	the cash payments by the Central Bank in connection with tied sector releases ($R_{np}^N = \beta R_{np}^G$)
S_t^G	the firm's cumulated investment fund (gross)
S_t^N	the cumulated deposits in the Central Bank ($S_t^N = \beta S_t^G$)
t_t^E	the effective tax rate ($t_t^E = t_t/V_t$)
t^N	the statutory corporate tax rate (the state and municipal tax rates combined)
t^R	the tax rate to be applied on interest on deposits
t_t	the tax <u>ratio</u> ($t_t = t_t^E/t^N$)
$\bar{t}$	the average tax ratio
$\bar{t}_\alpha^*$	the lowest attainable value of the average tax ratio when (for instance) α is varied to secure that $f_B = 1$

t the time variable or index ($t = np + i = m + j$)

T_t the firm's tax payments (including cash payments to or from the Central Bank)

V_t the amount of profit before taxes ($V_t = rK_t$)

W_t taxable profit

α that part of profit before taxes that may be allocated to an investment fund

β the share of the gross allocation that should be deposited in the Central Bank

γ the part of a tied sector release that is approved as an extra investment deduction

γ_A the share of an allocation that is approved as an extra investment deduction

δ the fraction of an allocation that can be withdrawn from the free sector after m years

θ the rate of growth of the firm's capital stock (in the case of constant growth, i.e. $G = g = \theta$)

The time index t alludes to period t wherever flow variables are concerned and to the end of period t wherever stock variables are involved.

List of references

[1] Arbetsmarknadspolitik (Labour Market Policy), SOU 1965:9, Stockholm 1965.

[2] Balanserad regional utveckling (Balanced Regional Development), SOU 1970:3, Stockholm 1970.

[3] Bank of Finland: Yearbook 1969.

[4] Bank of Norway: Report and Accounts 1962-65, Annual Reports 1966-69.

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