Nils-Erik Norén:

Long-range Decision Models in Mining

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Long-range Decision Models in Mining

THE ECONOMIC RESEARCH INSTITUTE

at the Stockholm School of Economics
Sveavägen 65, Stockholm, Sweden

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FOREWORD

This report, carried out at The Economic Research Institute will shortly be submitted as a doctor's thesis at the Stockholm School of Economics. The author has been entirely free to conduct his research in his own ways as an expression of his own ideas.

The institute is grateful for the financial support which has made this research possible.

Stockholm, March, 1969

THE ECONOMIC RESEARCH INSTITUTE
at The Stockholm School of Economics

Karl-Erik Wärneryd	Paulsson Frenckner
Director of	Program Director
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I was first confronted with the problem of determining optimum mining limits, optimum rates of production, and related problems in mining during my time with the mining company Luossavaara-Kiirunavaara Aktiebolag (LKAB). Especially, I am indebted to Mr. Hans Ahlmann, who initiated my first studies in this field by asking the right questions. In subsequent years I have frequently received impulses from LKAB in the forms of new challenging questions, and very generous information concerning new facts revealed and ideas developed within the company. I am very grateful to the management of LKAB for their generosity in this respect, and for their encouraging interest in my work.

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Important improvements in this study are also due to comments from and discussions with my colleagues and friends at LKAB and EFI. In particular, I am indebted to Messrs. Mårten Björnsson and Dan Nilsson, of LKAB, who have read the preliminary report and made many useful comments, and to Messrs. Jan Brytmark, Bo Hall, Ingemar Johansson, and Lennart Nilsson, of LKAB, and Göran Ahlberg, Björn Carlsson, Bengt Fornstad, Ingemund Hägg, Bertil Lindström, Ulf Lundman, and Ingolf Ståhl, of EFI, who have given of their time to read and discuss various parts of this study.

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Certain mathematical parts of the study have been read and commented by Mr. Gunnar Ribrant. Dr. Staffan Persson has been my adviser in questions concerning computer programming. Messrs. Sven Frenkel and Bengt Norén have aided in testing and correcting the computer programs and in correcting certain manuscripts. Messrs. Peter Docherty and Kay Risberg have read the English manuscript, and suggested many simplifications and other improvements. Mrs. Harriet Lundh has with great skill and endurance answered for the typing of the entire report including the preliminary versions. It is a pleasure to express my gratitude for all this assistance. If any error remains, the blame is mine.

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Nils-Erik Norén

The COMPUTER PROGRAMS described in this study are available from the Economic Research Institute at the Stockholm School of Economics.

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CHAPTER 1

1 Summary

11 The firm, the ore deposit, and the decision models

This is a report on a research project in which a number of decision models have been developed. They are intended to simplify or improve the decision making in a mining company in connection with the exploitation of ore deposits. The main results of the study are presented in this chapter. Naturally many important items regarding assumptions and methods as well as conclusions and results have been excluded in this summary. The subsequent chapters contain the full exposition.

A mining company is assumed to have one or more ore deposits at its disposal. The goal of the company is taken to be maximum profit in the long run. The capital value at the decision time[1] of future[2] payments to and from the firm as a consequence of the exploitation of the ore deposit, is used in the company as an operational criterion in choices between alternative actions, i.e. as a practical measure of the long-range profit of different ways of utilizing an ore deposit. The rate of interest is a given constant. All the data that are relevant for a decision are assumed to be known with certainty.

If the ore deposits are to be mined decisions must be made in a number of questions. The questions constitute decision problems and decision variables or decision parameters. The optimum values of the decision variables will be determined by means of decision models. The decision problems and decision variables, which are treated, are:

Problem 1) Which ore deposits should be mined?

Problem 2) How quickly should the ore be extracted? (The annual rate of production).

Problem 3) How extensively or carefully should the ore be recovered? (The mining limit, or the cut-off grade).

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1) The capital value at the decision time is the capital value discounted to the time of the decision, i.e. as a rule a present value. Compare Schneider (1944, p. 21).

2) Future as seen from the decision time.

<u>Problem 4)</u> Which machinery and technical methods should be used? (The technology).

<u>Problem 5)</u> How far should the crude product be refined? (The refinement level).

The market situation is assumed to be externally given, so that the price is uniquely determined when the above decisions have been made. Decisions concerning ore prospecting are presumed to be made independently, and are not treated here.

The decision models, which have been constructed as part of the project, are optimisation models, by means of which the optimum courses of action are determined. A general assumption in the optimization models is that all decision variables (or parameters) except one, namely the variable whose optimum value is to be determined, are given constants.

The second decision problem, the determination of the optimum rate of production, will be treated first, because it gives a good picture of the principal problems encountered in constructing the models, and how they are solved. The models will also prove useful in solving the first problem, i.e. if the ore deposit is worth mining.

Two models for optimizing the rate of production will be treated:

1) A simple model for determining the optimum rate of production in a mine, if the rate does not change during the production period of the mine. It is a static model, which has been treated by e.g. Massé (1959, pp. 348 ff.)[1].

2) A more developed model introducing a means of taking into account possible changes in the rate of production due to future decisions. It is assumed that optimum decisions are made also in the future. These decisions can be predicted and taken into account as the future is assumed to be known with certainty. The model thus optimizes successive interdependent decisions on principles generally used in dynamic programming, and is in this sense a dynamic model[2]. The model has been converted into a computer program (Appendix B).

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1) The complete references are given in the list of References.

2) The model was also described in a preliminary report on this project (Norén, 1967).

In order to facilitate this exposition, the assumption is introduced that the company owns only one ore deposit with a homogeneous ore and with clearly defined boundaries between the ore and the non-ore rock. The two models are founded on two different sets of assumptions concerning the properties of the deposit and the mining method. The mine of the first model may be visualized as an open pit with its ore-treating and transporting facilities, whereas the mine of the second model may be visualized as a mine with underground mining in a steep ore body utilizing some mining method with stepwise sinking to main haulage levels, from which the ore is subsequently hoisted to the surface for further treatment (Fig. 1:1). The ore extraction starts at the top of the ore body and proceeds downwards. The stepwise sinking constitutes zones, into which the ore deposit is partitioned. The second model can, however, be used for the first type of mine as well.

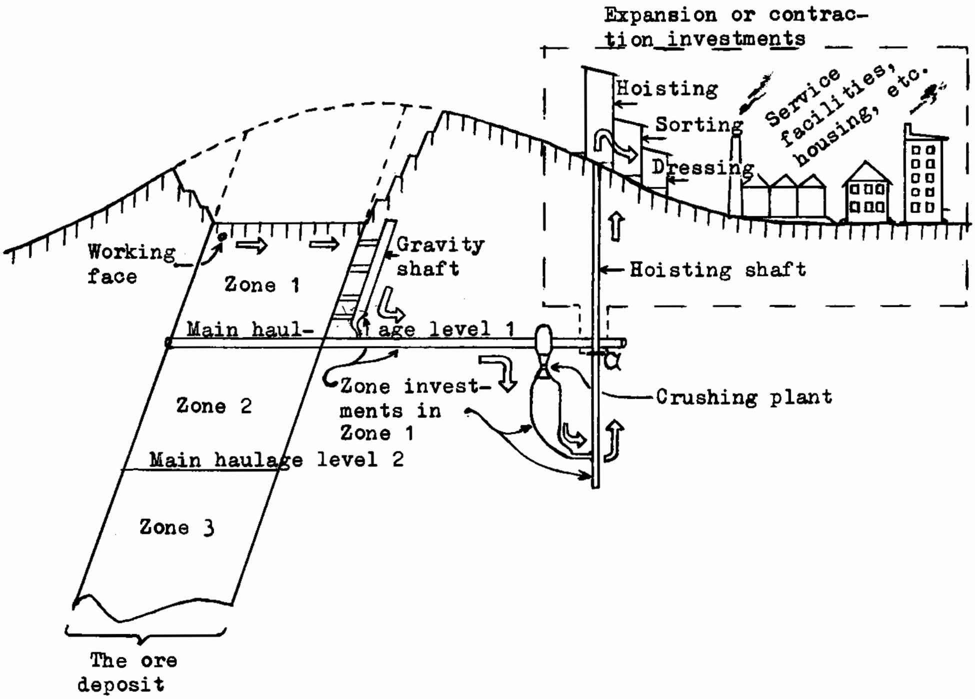

Fig. 1:1 Vertical section through a mine. The shafts, tunnels, machinery, etc. corresponding to three types of investments (expansion or contraction investments and zone investments; section 1313) are indicated. The working face is the point or points in the deposit, which are currently being mined.
⇨ The ore flow in mining

The third decision problem, i.e. to find optimum mining limits, is surveyed next. Two cases are treated:

1) An ore deposit consists of a main ore body (M in Fig. 1:2), surrounded by a number of smaller ore bodies, A, B, C, and D. Which should be mined and which should be left in place? An application of one of the models for optimizing the rate of production provides a solution of this problem.

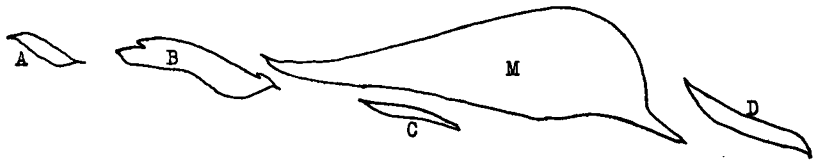

Fig. 1:2 Horizontal section through a main ore body and surrounding smaller ore bodies.

2) An ore deposit contains a rich principal vein surrounded by, or extending into, poorer ore as shown in Fig. 1:3. Now the boundaries of the ore body are no longer clearly defined. As successively poorer ore is recovered through extending the mining limits, the average grade decreases and the ore reserve increases. What ore should be mined, and what should be left in place? The ore deposit is partitioned into zones and subzones, and the mining limits are assumed to be determined successively as the mining proceeds downwards. The mining limits are represented by the average grades in the subzones. A dynamic model (see above) for optimizing the average grades in the subzones has been constructed and converted into a computer program (Appendix B).

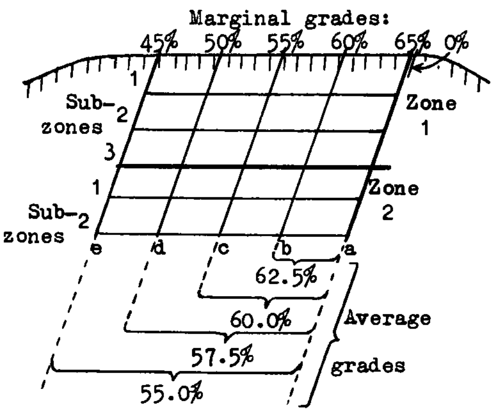

Fig. 1:3 Vertical section through an ore deposit demonstrating the concepts of a mining limit, a cut-off grade, a marginal grade, and an average grade.[1)2)]

1) In the computer programs the average grades are treated as fractions, i.e. %/100, e.g. 60 % is represented by 0.60.

2) Ore reserve: Each section a-b, b-c, c-d, and d-e is assumed to contain the (continued)

Finally, the fourth and the fifth decision problems, determining the optimum technology and the optimum refinement level, respectively, are solved by applying some of the other models.

12 Optimum rate of production: Massé's static model

121 Assumptions

An ore deposit contains 100 MT[1] of ore. As the ore is extracted payments will occur, which, after income taxes have been taken into account, amount to:

Ore price	20 KR/T[1]	
- Payments for current costs (only variable current costs are assumed to occur)	10 KR/T	
= Current net payments	10 KR/T	10 KR/T
Expansion investments (investments in machinery, buildings, etc. in connection with the opening of the mine, i.e. acquisition of production capacity, or expanding the rate of production (from 0); hence "expansion investments")		50 KR/T annual capacity

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(continued)

same tonnage of ore, e.g. 1 MT/section/subzone (MT: see below).
Marginal grades: The lines drawn upwards from a, b, etc. connect points in the ore deposit where the ore contains the indicated fractions of iron (or other useful substance). The marginal grade diminishes at an equal rate in the whole interval 65% - 45%.
Average grades: If the mining limits a and b are selected, the marginal grade is 60% and the average grade of the ore mined will be 62.5%. If a and c are selected, the marginal grade is 55% and the average grade 60%, etc. Note how the ore reserve and the marginal and the average grades are interrelated.
The cut-off grade is the mining limit expressed as a marginal grade or as an average grade.

1) Regarding measures, the following conventions are applied:

KR - Swedish "kronor" (the monetary unit)
KR/T - Swedish "kronor" per ton
MKR - Millions of Swedish "kronor"
MT - Millions of tons
T - Ton(s)

Capital letters are used as a concession to the printers of most computers in order to obtain a uniform standard in all parts of this report, including those concerned with the computer programs.

In calculations the company uses an annual rate of interest after tax of 5%. All prices and costs are assumed to be constant over time.

The expansion investment is supposed to be completed and paid for at the end of year 0. The resulting capacity is available from the beginning of the year 1. The amount invested is directly determined by the annual capacity (MT/year). The capacity is assumed to be fully utilized during the entire production period of the mine. The decision concerning the rate of production is thus made in year 0 as the capacity is determined. It is binding for the future, until all the ore has been recovered.

122 Capital-value model

In this case the capital-value model is extremely simple, and may be expressed in the following manner:

    (1) Capital value of annual current net payments[1)]
\- (2) Expansion investment
=   Capital value at time 0 of the ore deposit

The optimum rate of production and thus the optimum capacity is the rate which gives the highest capital value of the ore deposit.

123 Optimum annual rate of production

The table of Fig. 1:4 demonstrates how to calculate the capital value and its size at different rates of production. The table also shows the direct (and evident) connection between the rate of production and the production period of the ore deposit. The maximum capital value is obtained at a rate of production of about 5 MT/year. Thus the production period of the ore deposit will be 20 years. The example is graphically represented in Fig. 1:5.

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1) If the production period is T years and the rate of interest is 100·r %, the annual amount is multiplied by

$$\sum_{t=1}^{T}(1+r)^{-t}=\frac{1-(1+r)^{-T}}{r} \text{ for } t=1,2,\ldots,T.$$

Rate of production	Production period	Current net payments		Expansion investment	Difference = cap. value of deposit
		Annual	Cap.value at time 0		
MT/year	Years	MKR/year	MKR	MKR	MKR
0.1	1,000	1	20	5	15
0.5	200	5	100	25	75
1	100	10	198	50	148
2	50	20	365	100	265
3.33	30	33.3	512	167	345
5	20	50	623	250	373←
6.67	15	66.7	692	333	359
10	10	100	772	500	272
20	5	200	866	1,000	- 134
50	2	500	928	2,500	- 1,572
100	1	1,000	952	5,000	- 4,048

Fig. 1:4 Capital values at time 0 of the ore deposit at different annual rates of production (compare Fig. 1:5).

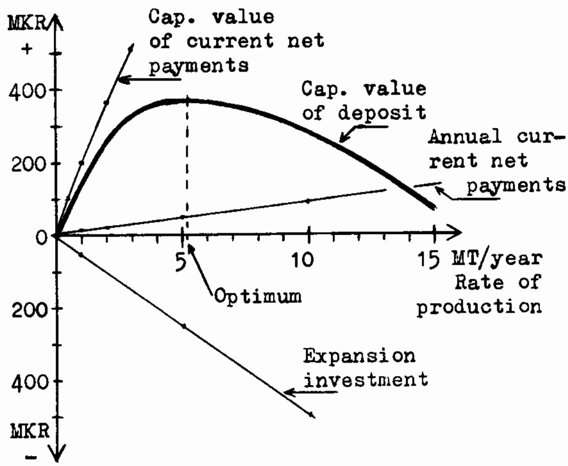

Fig. 1:5 Capital values at time 0 of the ore deposit at different annual rates of production (compare Fig. 1:4).

124 Massé's conclusions on production period and capacity utilization

Massé draws some interesting conclusions from his model. For example, making certain assumptions, primarily that the current net payments per ton of annual rate of production and the expansion investment per ton of increase in the annual rate of production (i.e. capacity) are constant over time, and equal for all rates of production, and that they are equal for all parts of the ore deposit, he deduces that the optimum action is to obtain production capacity at one single moment, i.e. at time 0, and to fully utilize the capacity throughout the production period.

In addition, using the same conditions, Massé states that the optimum rate of production varies proportionally with the size of the ore reserve of the deposit. This means that the optimum length of the production period is the same for a large ore deposit as for a small, ceteris paribus. He also demonstrates that the optimum rate of production increases as the current net payments per ton increase, and decreases as the expansion investment per ton expansion of the annual rate of production increases.

125 Further conclusions

Some further observations may be made in addition to Massé's conclusions. If the original assumptions <u>ex post</u> prove wrong, the decision concerning the rate of production may have to be revised. This will occur if the ore price falls unexpectedly. First, consider the case of a price fall of <u>limited duration</u>, i.e. the price is expected to resume its original level before the end of the produotion period. The price fall may be countered in several ways, e.g.
1) by producing as before, i.e. at capacity, and selling at the lower price, or
2) by stopping production for the duration of the recession and selling the ore thus saved in the future at the original price[1]. An analysis by means of the static model gives the following conclusions[2]:

1) The relative profitability of the various actions depends primarily on the extent of the fall in prioe and the remaining production period of the ore deposit.

2) If the remaining production period is long, i.e. if the price fall occurs some time during the beginning of the produotion period, a very considerable

1) The oompany is assumed to face a fixed price, which it cannot influence. The alternative of producing on stock is disregarded.

2) Norén (1968, pp. 6–8).

price reduction is required to make the alternative of cutting production preferable.

3) The closer the end of the production period, the smaller is the prioe reduotion necessary to motivate a temporary closing of the mine.

As opposed to the previous case, a <u>durable</u> prioe fall, i.e. a price fall that is expected to be permanent, does not imply any opportunity of selling the ore which is not immediately mined and sold at a higher price in the future. In this case the optimum action is to continue production at capacity, if the price fall is not so great as to force the oompany to close the mine. The latter alternativ will have to be considered in the example given in section 123, if the price decreases by more than 10 KR/T after tax.

The importance of the rate of interest may also be illustrated. A tempting intuitive way of reasoning is to say that the higher the rate of interest, the lower is the capital value at time 0 of future annual current net payments. This entails that the ore should be recovered more quickly, i.e. the annual rate of production should be greater. However, as may be seen from Fig. 1:6, this conclusion is valid only up to a certain limit, i.e. up to about 12% in the given case. Above the limit the optimum rate of production decreases as the rate of interest increases.

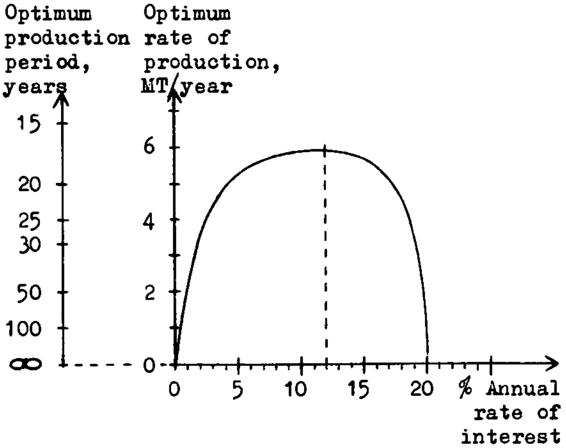

Fig. 1:6 Optimum rates of production and optimum production periods at different rates of interest. Other data according to the example of section 123.

13 Optimum rate of production: The dynamic model

131 Assumptions

1311 Successive interdependent decisions

The static decision model is in many instances too simple. It describes the reality in sufficient detail to give relevant information for decisions only in certain special cases. It is especially insufficient if the original assumptions are such that it is possible at the outset to predict that reasons for changing the rate of production determined originally will present themselves in the future. In this case it is not reasonable to hold on to the assumption of a constant rate of production during the whole production period of the ore deposit.

A principal reason why it could be favourable at a future point of time to change the rate of production determined originally, is that a considerable part of the plant, which determines the production capacity, must be renewed. For instance, a new main haulage level (see Fig. 1:1) may have to be built to replace an older one which has served a part of the ore deposit (a zone) that has been emptied of ore. For what capacity should the new main level be built? The optimum decision on that occasion depends on the rate of production in the past. Simultaneously the previous rate of production has been influenced by the assumptions in the past concerning the capacity of the main level in question. In this way a network of interrelations may exist, which is disregarded in the static model.

Another reason for changing the rate of production determined originally in a subsequent decision has already been discussed, i.e. changes in prices and costs over time. It was assumed that these were unpredictable at the time of the original decision. Therefore they were not, and could not be, taken into account in the original optimization of Fig. 1:4. If the changes can be predicted, the case is different. As the future has been assumed to be known with certainty, the variations in a complete analysis should influence the original decision, which may include taking variations in the rate of production into account.

A more realistic model should incorporate a system for considering the interdependencies of decisions at different points of time. It should also allow for variations in prices and costs with the time, including the effects of the technological evolution, and with the progress of the mining, e.g. that different parts of the mine have different properties, and that it is usually more expensive to extract ore from the lower levels of the mine than from the upper levels. Thus it seems necessary to develop a model for dynamic optimization[1], which also contains a more detailed description of the payments caused by the ore extraction.

The important difference between a static and a dynamic optimization may be stated thus: In a static optimization it is assumed that all decisions regarding the future are made at one single moment, i.e. the initial or actual decision time, whereas in a dynamic optimization it is assumed that the initial decisions are changed and completed at future decision times. In the dynamic optimization it is assumed that one decision is made first at the beginning of the production period, and that decisions are then made at certain time intervals, when the management of the company has to consider the possibilities of making changes. The decisions are optimum decisions. They are optimized under the conditions that have been determined through previous decisions, and the effects on future decisions are taken into account. Massé's static model is applicable in the special case where one single decision involving only a constant rate or production is made.

1312 An ore deposit and a mining technique - an example

An ore deposit contains 150 MT of ore. It is to be mined in three steps, i.e. zones, each containing 50 MT (compare Fig. 1:1). Each zone is served by a main haulage level, which must be completed before any ore can be extracted from the zone. A hoisting plant, a sorting plant, repair shops, transport facilities, an office building, housing, etc. are common to the three zones. The corresponding investment is the expansion investment. The deposit is not previously mined, which implies that an expansion investment has to be completed at the end of year 0. Then the resulting plant can be utilized from the beginning of year 1[2].

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1) The dynamic aspects have been observed by several authors, e.g. by Hotelling (1931).

Massé (1959) also approaches the dynamic aspects in his discussion of uncertainty in the estimate of the size of the ore reserve.

2) In the computer programs and, as a consequence, also in Appendix D where the various payments are defined, the investment is for technical reasons assumed to be completed at the end of year 1 and taken into production from the beginning of year 2. This complication is disregarded in the present discussion and in Appendix A, as digressing into such details would obscure the main issue.

Investments traceable to one zone are named zone investments, and are assumed to be completed at the starting time of the zone. The starting time of a zone is assumed to coincide with the moment of closing down the previously mined zone. The mining starts in zone 1 at the beginning of year 1. The rate of production in a certain zone is determined when the investments are being determined, i.e. immediately before the mining starts in the zone, and is assumed to be fixed for the entire production period of the zone.

The assumption of a constant rate of production during the production period of a zone is partly motivated by the conclusions of section 124 concerning the optimum capacity utilization, as each zone may be considered a mine having approximately the properties assumed there.

If the rate of production is changed at the starting time of a new zone, the works and utilities constituting the expansion investment at time 0 have to be changed. If the rate of production is increased, this causes further expansion investments. If it is decreased, this causes contraction investments. They are assumed to coincide in time with the zone investments.

Payments incurred as a consequence of the final closing of the mine are called close-down payments. They are assumed to be paid at the end of the production period of the deposit.

1313 Payment functions

In addition to the investments and close-down payments, which have already been discussed, current payments are introduced into the model. They are assembled into the annual current net payment which has three main components:

1) Current payments for products sold (+)
2) Current disbursements incurred in the current production (–)
3) Current reinvestments (not including the zone investments) (–)

These payments are expressed in MKR/year, and are assumed to constitute a constant flow during any single year if the interest is reckoned continuously. They are assumed to be paid at the end of the year if the interest is reckoned discontinuously. The discontinuous case has been chosen for this summary and for Appendix A[1].

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1) The method of continuous reckoning is applied in the computer programs.

All payments are assumed to be primarily functions of the decision variable, i.e. the rate of production, or, in certain cases, the change in the rate of production. However, there are other important factors. In order to make the payment functions more generally valid for different parts of the same deposit (zones and subzones), and for different deposits as well as for other decision problems, the payments are expressed as explicit functions of 1) the annual rate of production, which may vary from one zone to another, 2) the average grade of the ore mined in a subzone, which may vary from one subzone to another, 3) the size of the zone, 4) the depth of the current main haulage level, which is expressed in terms of the ore reserve of zone 1 for the first main level, of the ore reserve of zones 1 and 2 together for the second main level, etc., and 5) the point of time at which the payment is made, or at least the year during which it is made. All the five independent variables are explicitly considered in spite of the fact that only the rate of production is a decision variable in the optimization model, whereas the other variables are assumed to be constants, i.e. treated as parameters with given values.

The payment functions will not be treated in detail. They may assume practically any form whatever within the framework of the model. For instance, a number of complete payment models are given in the preliminary report[1], and a set of payment functions is described in Appendix D[2]. The former are simple as they are intended for manual solutions, whereas the latter is intended for computer solutions and rather flexible, as the set of functions contains about 70 coefficients, which can be used to achieve functions approximating reality over a comparatively wide range. The set of 70 coefficients may vary with the time, as different sets can be given for each of the years 1, 2, ..., A, where A stands for a data horizon which may be anything from 2 to 50 years. The payment functions can, in this way, be made to vary with the time.

Capital-value model

The fundamental idea of the capital-value model is simple. All payments that occur as a result of the exploitation of the ore deposit, are discounted at the given rate of interest to a common point of time, and added. This gives the

---

1) Norén (1967).

2) The payment functions have been constructed for hypothetical ore deposits. It is expected that the functions will in reality have to be "tailor-made" to fit the individual case. However, parts of the hypothetical functions, especially those of Appendix D, may be to a great extent directly applicable.

capital value at that point of time of the deposit. The capital-value model was formulated in this way in the static model. The capital-value model in the dynamic optimization model is constructed similarly, but the successive decisions concerning the rate of production causes other capital values than that at time 0 of the entire ore deposit to be of interest. It is now assumed that decisions are made so as to make the capital values at the decision times, i.e. at the starting times of zones 1, 2, or 3, respectively, of the ore remaining in the deposit at these points of time, as large as possible. Hence, the capital value at time 0 of the entire ore deposit is the decision criterion only when the rate of production is originally determined at the starting time of zone 1. As the rate of production is determined anew at the starting time of zone 2, the ore of zone 1 has already been mined. Consequently, only the ore reserves of zones 2 and 3 remain. Thus the capital value of zones 2 and 3 discounted to the starting time of zone 2 is the relevant criterion. Finally, the capital value at the starting time of zone 3 of the only remaining zone is the criterion for the last decision.

The relevant capital values can be calculated in several ways. One way is to do this in two steps. In the first step the capital values of the zones as separate units at their respective starting times are calculated. However, the technological interrelations between the zones are taken into account, e.g. as the investments required to alter the rate of production are part of the payments.

In the second step the capital values of the separate zones are discounted[1] and added to capital values which are relevant for the respective decisions, i.e. the capital values at the decision times of the ore remaining in the deposit at the respective times. The rates of production giving the highest capital values are the optimum rates, which are assumed to be selected in the decisions. The two steps are illustrated in Fig. 1:7 and Fig. 1:8. The following symbols are used:

B_n	Capital value at time T_n of zones n, n+1, n+2, ..., N.
B'_n	Capital value at time T_n of zone n.
Maximum B_n $Q_{n...N}$	The highest value of B_n as a function of Q_n, if Q_{n+1} is determined so as to make B_{n+1} as high as possible, if also Q_{n+2} is such that B_{n+2} is as high as possible, etc. up to

1) In the computer programs, and consequently also in Appendix D, all payments are, for technical reasons, discounted at once to time 0. No further discounting is needed in that case.

	and including Q_N and B_N. The notation thus implies optimization of Q_n, provided that Q_{n+1}, Q_{n+2}, ..., Q_N are similarly optimized.
N	Number of zones (in the example N=3).
$n=1,2,\ldots,N$	Subscript of zones. They are mined in the order 1, 2, ..., N.
Q_n	Rate of production in zone n.
$(1+r)^{-t}$	Present value of 1 KR payable in t years at a rate of interest of 100·r %.
T_n	Starting time of zone n.
Υ_n	Production period of zone n.

The capital values at the decision times of the future mining, i.e. B_1, B_2, and B_3 are measures of the profit of future mining as it will be experienced at the three respective decision times. It has been assumed that an optimum decision is made at each decision time, which implies that the decision is optimal under the conditions prevailing at the decision time[1]. These conditions are influenced by previous decisions. Available capacity is an example of such a condition. Assumptions concerning future decisions are also part of the assumptions for any current decision. What is decided in the future is in turn dependent on preceding decisions, thus also on the current one. The capital-value calculations must take all such interrelations into account.

The capital values at the respective decision times are easily calculated if the rates of production in all zones were known. This is taken as a starting point in determining the optimum rates of production: The rates of production in all the zones are given arbitrary values, which are then systematically varied. A number of sets of rates of production, each set consisting of one rate for each zone, is obtained, and the capital values (B_1, B_2, B_3) pertaining to each set may be calculated. For the nearest zone the rate of production which, when combined with the optimum rates in the subsequent zones, gives the highest capital value at the given decision time, is the optimum. In many cases the optimum set of rates of production is the one that makes the capital value of the entire ore deposit a maximum.

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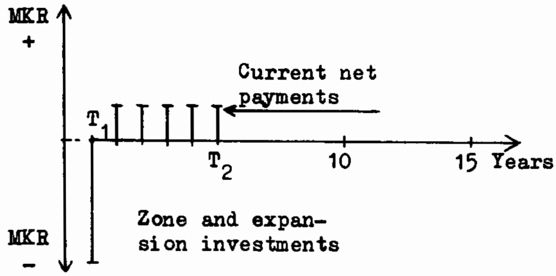
1) This is an application of the "principle of optimality". See e.g. Bellman (1957, p. 83).

<u>Zone 1</u>

+ (1) Capital value at time T_1 of current net payments
- (2) Zone investment in zone 1
- (3) Expansion investment at time T_1

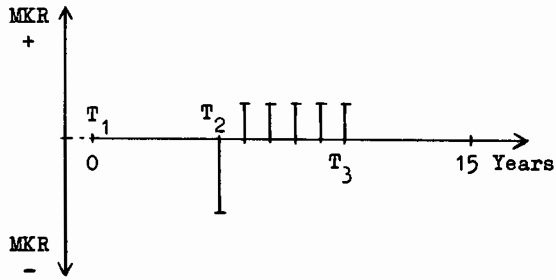
= Capital value at time T_1 of zone 1, i.e. B'_1



<u>Zone 2</u>

+ (1) Capital value at time T_2 of current net payments
- (2) Zone investment in zone 2
- (3) Expansion or contraction investment at time T_2

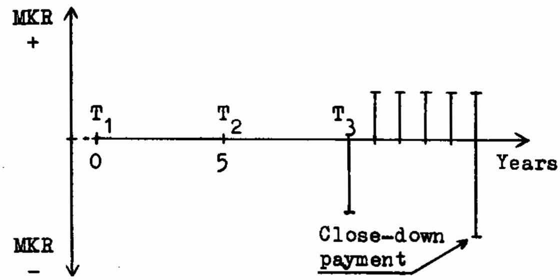
= Capital value at time T_2 of zone 2, i.e. B'_2



<u>Zone 3</u>

+ (1) Capital value at time T_3 of current net payments
- (2) Zone investment in zone 3
- (3) Expansion or contraction investment at time T_3
- (4) Close-down payment at time $T_3 + \mathcal{T}_3$

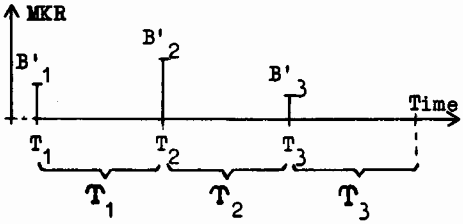
= Capital value at time T_3 of zone 3, i.e. B'_3



<u>Fig. 1:7</u> Step 1: The capital values of the zones at their respective starting times are calculated. The graphs illustrate the various payments which have been determined through the payment functions. The production period of each zone is assumed to be 5 years.

Decision at time T_1: Determine optimum Q_1

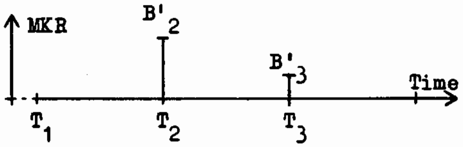
$$\underset{Q_{1\ldots3}}{\text{Maximum}}\ B_1 = B'_1 + B'_2\cdot(1+r)^{-\mathcal{T}_1} + B'_3\cdot(1+r)^{-(\mathcal{T}_1+\mathcal{T}_2)}$$



$B'_1 = f(Q_1)$. Q_1 is to be determined.
$B'_2 = f(Q_1, Q_2)$. Q_2 has not yet been determined.
$B'_3 = f(Q_2, Q_3)$. Q_2 and Q_3 have not yet been determined.

Decision at time T_2: Determine optimum Q_2

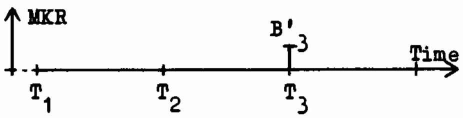
$$\underset{Q_{2\ldots3}}{\text{Maximum}}\ B_2 = B'_2 + B'_3\cdot(1+r)^{-\mathcal{T}_2}$$



$B'_2 = f(Q_1, Q_2)$. Q_1 is given and Q_2 is to be determined.
$B'_3 = f(Q_2, Q_3)$. Q_3 has not yet been determined.

Decision at time T_3: Determine optimum Q_3

$$\underset{Q_3}{\text{Maximum}}\ B_3 = B'_3$$



$B'_3 = f(Q_2, Q_3)$. Q_2 is given and Q_3 is to be determined.

Fig. 1:8 **Step 2: Capital values at the decision times of future mining are calculated and applied as decision criteria. Each decision directly concerns the rate of production in the nearest zone, but indirectly influences all the decisions.**

133 Optimization methods

1331 Dynamic optimization

The core of the optimization problem, as it has been formulated here, is to find a system for determining which sets of rates of production are to be examined. A traditional and generally valid method for solving dynamic programming problems, of which this problem is an example, may be stated concisely in three points in this case[1]:

1) Optimize Q_3 for all possible combinations of values of Q_1 and Q_2.
2) Apply the result in optimizing Q_2, thus also Q_3, for all possible values of Q_1.
3) Apply the result in optimizing Q_1, thus also Q_2 and Q_3.

As the method apparently requires a considerable amount of calculating, simplifications are desirable. It is also possible to find simpler optimization methods by utilizing some speoial characteristics of the present problem and introducing certain special assumptions. However, the fundamentals of the general method are retained.

1332 Assumptions for the simplified dynamic optimization

In order to simplify the calculations the number of interrelations between decisions at various points of time is reduced to a minimum. Therefore, the interdependencies between the zones are assumed to be exhaustively described in terms of:

1) The starting time of the current zone. This point of time is measured on a continuous time scale with the zero point at the starting time of zone 1[2]. The starting time of the current zone sums up in one single date the rates of production and the ore reserves actually extracted in the preceding zones, i.e. in the zones previously mined.

2) The rate of production in the preceding zone. It defines the production

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1) This is an application of Bellman's (1957, pp. 6-9) solution.

2) Compare the footnote in section 1312, which also applies here.

capacity available without further expansion or contraction investments when the mining of the current zone will start[1].

3) The mining limit, the technology, and the refinement level of the preceding zone. These variables or parameters define the properties of the existing works, equipment, service utilities, etc. in other dimensions than the rate of production.

4) The accumulated quantity of ore reserve used up from the starting time of zone 1[2]. Along with the number of the zone it defines how far the mining has proceeded, which part of the ore deposit is currently being mined, and the remaining ore reserve.

Further assumptions have already been mentioned or implied. Those which are most important for the optimization are:

1) The mining company is the decision-making unit. The capital value at the decision time of future[3] payments is the decision criterion. The goal of the mining company is to attain maximum capital value. The rate of interest is given and constant over time.

2) The company does not control any other ore deposits which are influenced by the decisions concerning the rate of produotion, and will not do so in the future either. Decisions concerning ore prospecting are presumed to be made independent of decisions concerning the deposit.

3) The size of the ore reserve and other properties of the ore deposit are known with certainty.

4) The mining limits, the partitioning of the deposit into zones and other dimensions of the technology, and the refinement level have already been determined. The zones are mined in the order 1, 2, and 3, or, more generally, 1, 2, ..., N.

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1) Disregarding that the production capacity may diminish as a consequence of the increasing depth (especially the hoisting capacity).

2) More exactly: the cumulative equivalent ore reserve. See section 2 of Appendix D.

3) Future as seen from the decision time.

5) The sizes of the payments are determined by a number of given payment functions. The payment functions may vary with the time. If that is the case the variations over time are known with certainty.

6) The rate of production may be changed only when the mining activities are transferred from one zone to another. It is thus constant during the production period of a zone. Changes cause expansion or contraction investments.

7) The rate of production always equals the production capacity.

1333 Two concrete methods

Two methods for solving the stated optimization problem have been developed. The first is a manual graphical method, which may be used if the number of zones is small, say 1 - 4. It is restricted also in other respects, as the payment functions, for practical reasons, must be very simple (compare section 1313), and constant over time (compare above, assumption 5)). This decreases the usefulness of the method, which is also influenced by the fact that a very large number of calculations are required to obtain a solution if the number of zones is increased above 1 or 2.

The technique of the graphical dynamic optimization has been taken as a starting point of the computer program for optimizing the rate of production. This program is the second method. The limitations of the graphical method have been relaxed, so that the assumptions listed in the previous section are entirely accurate. However, some subproblems, especially that of payment functions varying with the time, are solved through iterative optimizations. The tentative optimum of one optimization is thus utilized to define new assumptions for the next optimization. The iteration continues until a stated minimum increase in the capital value of the ore deposit is no longer obtained through further optimizations. The iteration is programmed to take place automatically. The iterative method suffers from some weaknesses, which affects the reliability of the optimization negatively. There are, though, possibilities of testing and controlling the results, which at least partially eliminates this disadvantage.

The computer program has been successfully tested on a few hypothetical and practical cases. However, more testing is required before it can be considered fully reliable.

The two optimization methods will not be described in detail in this summary[1]. The basic ideas of the optimization model will be discussed[2], though, in close relation to the graphical method shown in Appendix A, and as the model is applied in the case of three zones[3]. As previously mentioned (section 1331), the zones are optimized in the order 3, 2, and 1. Thus, consider first zone 3.

The capital value at time T_3 of zone 3 is shown in Fig. 1:9. For the curve B_3 it is assumed that $Q_2=Q_3$. Now, assume that Q_2 is a given constant, less than Q_3, and that the expansion investment is 50 KR for each ton the annual rate of production is expanded. It is evident that the expansion increases the capital value after the expansion investment has been deducted, i.e. it is profitable, if the capital value according to the curve B_3 increases by 50 KR, or more, as Q_3 increases by one ton. On the other hand, if the capital value increases by less than 50 KR an expansion is not advantageous. A tangent to the curve is constructed, which has the gradient 50 KR/T/year (Fig. 1:9, α_3, left). The rate of production at the point of tangency is the expansion limit. To the left of the limit the gradient of the curve is greater than 50 KR/T/year, and expansion up to the expansion limit is profitable. To the right of the limit it is not profitable to increase the rate of production.

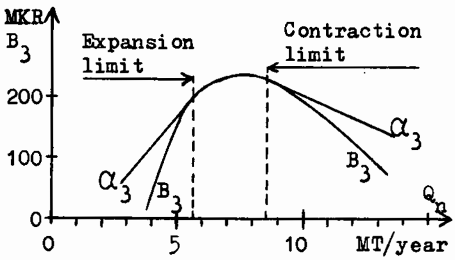

Fig. 1:9 Expansion limit and contraction limit in zone 3. B_3: see section 132.

Similarly, if Q_2 is assumed to be greater than Q_3, i.e. if the rate of production is decreased, and the contraction investment is 20 KR for each ton the annual rate of production is diminished, a tangent having the gradient −20 KR/T/year may be drawn (Fig. 1:9, α_3, right). The point of tangency indicates the rate of production which is the contraction limit. If Q_2 exceeds the

1) The simplified graphical dynamic optimization is described in Appendix A. The principles of the method used in the computer program are on the whole the same. The main difference is the iteration. The program is described in Appendices B, C, D, and E. The optimization methods are further discussed in following chapters.

2) As the assumptions of the two methods are not identical they represent two versions of the optimization model.

3) The variations over time of the payment functions are therefore disregarded for the moment.

contraction limit, the optimum decision is to decrease the rate to that limit. If Q_2 is in the interval between the expansion and the contraction limit, the optimum decision is to keep the rate of production constant, i.e. $Q_2=Q_3$ is optimum.

Next, assume that the expansion investment is a fixed amount, e.g. 20 MKR, plus 50 KR for each ton the annual rate of production is increased. The expansion limit still holds good in the sense that the rate is optimally increased up to that limit. However, Q_2 must now be so low that the increase in the capital value according to the curve B_3 also covers the fixed amount, 20 MKR. As may be seen from Fig. 1:10, this is the case if Q_2 is lower than the rate of production denoted as the lower constancy limit. Similarly, an upper constancy limit is constructed. It should be carefully noted that Fig. 1:9 represents a special case, where the lower constancy limit coincides with the expansion limit, and the upper constancy limit with the contraction limit.

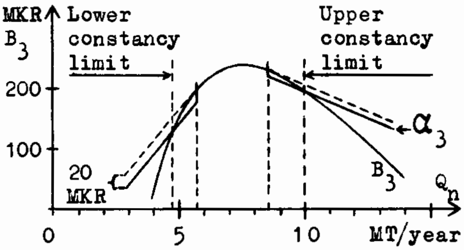

Fig. 1:10 Constancy limits in zone 3. B_3: see section 132.

In this way the four limits are determined for every zone except zone 1 for which the optimum rate is directly determined (Fig. 1:12). As B_3 is the criterion in zone 3, B_2 and B_1 are the criteria in zones 2 and 1, respectively (section 132).

The four limits and their implications are illustrated in Fig. 1:11. If the

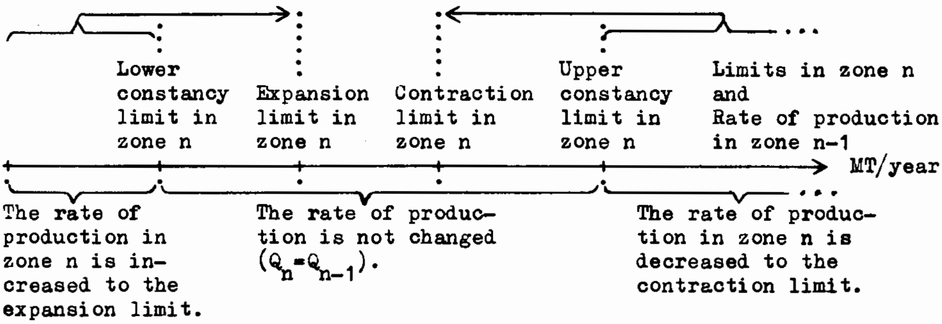

Fig. 1:11 The optimum rate of production in zone n for various rates of production in zone n-1.

rate of production in zone n-1 is lower than the lower constancy limit, the optimum decision for zone n is to increase the rate to the expansion limit. If the rate of production in zone n-1 exceeds the upper constancy limit, the optimum decision for zone n is to decrease the rate to the contraction limit. These decision rules will be applied to determine an optimum set of rates of production, or an optimal policy, for zones 1, 2, and 3.

1334 Optimum

The results of the partial optimizations in the zones, i.e. of determining and applying the four limits previously discussed, can be summarized as in Fig. 1:12. The curve α_3 is recognized from Fig. 1:9. The others are obtained in a similar way. In principle the figure illustrates the results of the graphical method as well as of the computer method. However, the output of the computer program is confined to key information necessary to interpret the results instead of the complete curves.

The decision model is primarily intended to supply information for the decision at time T_1. At this point of time it is determined whether the ore deposit should be mined. The rate of production in zone 1 is also determined. The curve α_1 summarizes this information. It shows the capital value at time T_1 of the entire ore deposit as a function of Q_1, which is measured on the "horizontal" axis. The highest capital value is obtained at Q_1=10 MT/year. This is the optimum rate of production in zone 1. The curve α_1 presupposes optimum rates in the future, i.e. optimum Q_2 and Q_3. In Fig. 1:12 the optima can be found by means of the curves α_2 and α_3 together with the expansion limits, the contraction limits, and the constancy limits.

In zone 2 the expansion limit and the lower constancy limit is 7.1 MT/year. Thus, if the rate of production in zone 1 is less than 7.1 MT/year, the optimum rate in zone 2 is 7.1 MT/year. The contraction limit and the upper constancy limit is 20 MT/year. If the rate exceeds 20 MT/year in zone 1, the optimum action at time T_2 is to cut production to that rate. Fianlly, if the rate in zone 1 is between 7.1 and 20 MT/year, the optimum action at time T_2 is to keep the rate constant.

Depending on the rate of production in zone 2, the optimum rate in zone 3 similarly varies between 5.7 and 8.5 MT/year.

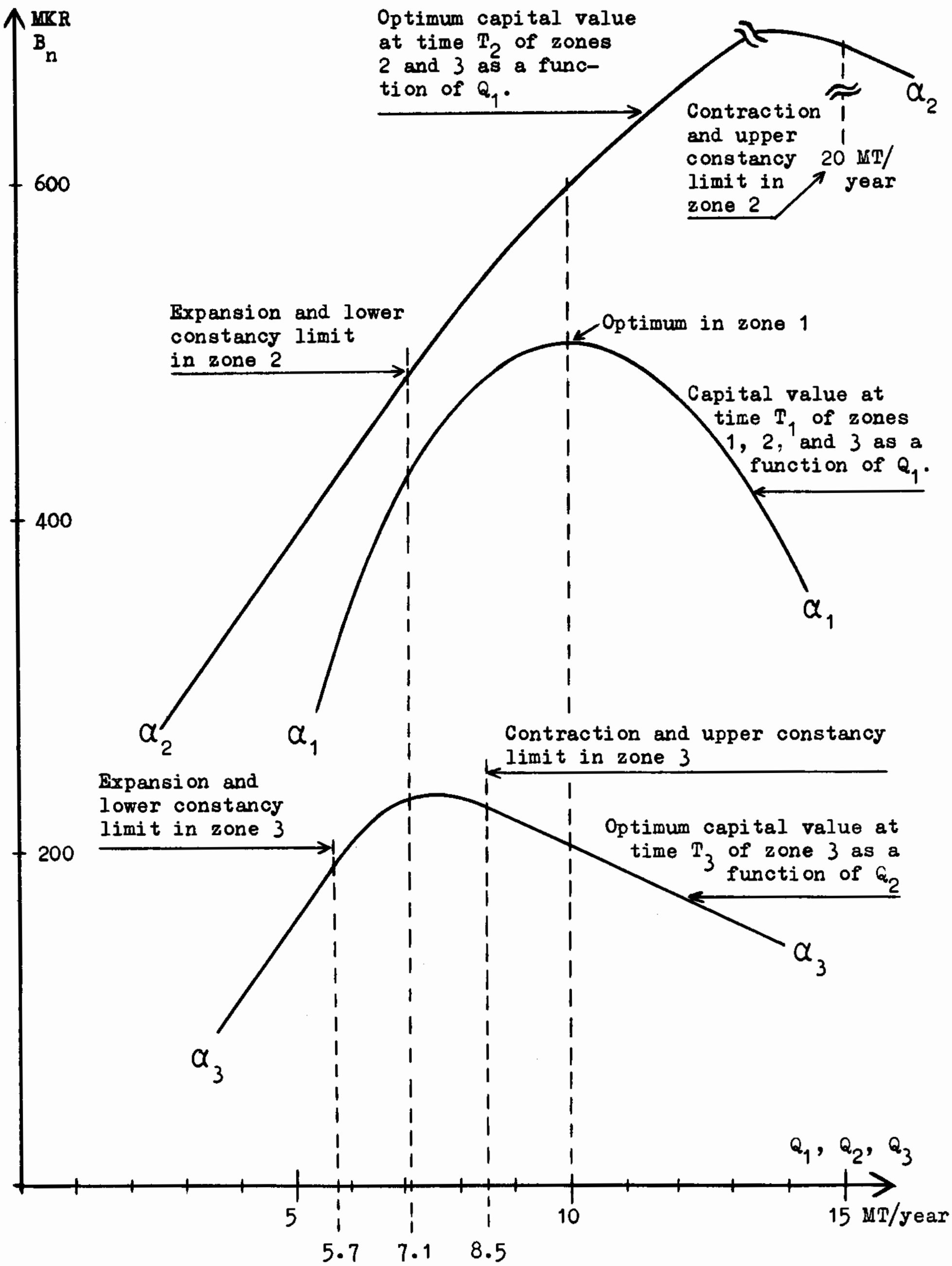

Fig. 1:12 Capital values at the times T_1, T_2, and T_3 of the zones which remain to be mined at each respective time. Compare Fig. 1:9, above, and Fig. A:6 in Appendix A.

Assuming optimum decisions in zones 2 and 3, the optimum rate of production in zone 1 is 10 MT/year. Then, according to Fig. 1:12, the optimum rate in zone 2 is also 10 MT/year, and in zone 3 it is 8.5 MT/year. The optimum rates in zones 2 and 3 are only predictions concerning future decisions, which are taken for granted in the actual decision, i.e. in determining the rate of production in zone 1. The optimization described does not provide the information on which the decisions at the starting times of zones 2 and 3 are based. At these points of time, i.e. at times T_2 and T_3, the model is employed to make new optimizations. New data reflecting the best available information at these respective decision times are used. The actual rate of production, or the capacity at the decision time is a part of the new data. The zones will also be renamed. At the starting time of the second zone this zone will be the first of the remaining ones, thus zone 1. The former zone 3 will become zone 2. The optimization method is then applied as before, only this time it is adapted to two zones instead of three.

The principles of the optimization methods are applicable irrespective of the number of zones[1].

Until now the payment functions have been assumed to be constant over time. They may vary. To take such variations into account, iterative optimizations are made. This is impracticable if the graphical method is used, but iterations are made automatically in the computer method.

134 Conclusions

According to the curve α_1 in Fig. 1:12 the capital value of the entire ore deposit is positive at optimum. Thus, the deposit should be mined. A negative capital value at all rates of production indicates that the deposit cannot be profitably mined.

The curve α_1 also reflects the situation before the start of the mining in another respect. The point of time at which all the ore has been extracted from the deposit is still very distant. The final closing of the mine constitutes an opportunity cost of ore. As this is still a cost in the distant future, its present value is low, which tends to increase the optimum rate of

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1) Compare the first paragraph of section 1333. The maximum number of zones is 14 in the computer programs.

production. The most important constraining factor is the investments which rapidly increase as the rate of production increases. Both the expansion investment and the zone investments exert this influence.

The curve α_2 illustrates some aspects of the situation when the construction of the mine including its works and machinery has been completed, and mining is well under way. If the rate of production in zone 1 is somewhere between the lower and the upper constancy limits in zone 2, the optimum action at time T_2 will be to keep the rate constant. It follows that it might prove extremely difficult to correct a previous erroneous decision.

As the production capacity is determined by previous investments, and cannot be profitably increased, it will be a constraint on the short-run production planning. The opportunity cost of ore is often entirely disregarded in usual criteria for short-run planning, such as different forms of annual surpluses. If such criteria are applied, the current capacity will be considered too low. Consequently, a strong drive towards improvements and rationalization aiming at increased production capacity is to be expected. In order to avoid errors in such decisions it may be useful to put the decisions into their proper context by means of the proposed decision models.

The situation changes towards the end of the production period of the mine, which is illustrated by the curve α_3. The closing of the mine is now near in time. The opportunity cost of ore is high enough to motivate a decrease in the rate of production. Excess capacity, which has to be disposed of, is typical of the situation. If the closing of the mine, even if it is made successively, incurs costs, e.g. for finding new employments for the personnel or for restoring nature in the vicinity of the mine, the cutting of production is hampered. It might be felt that the end of the production period approaches only too quickly.

High close-down payments, especially in combination with high production costs in the lowest parts of the mine, introduce another problem. The capital value of the remaining ore will sooner or later be negative. This situation will arise sooner, the greater the close-down payments are, ceteris paribus. From then on the mine operates at a loss in the long run, but the mining company cannot improve its situation by immediately closing the mine, as the close-down payments in that case would be incurred at once, which would increase their capital value at the current point of time. At the same time such a decision would deprive the company of the annual current net payments during the remaining production period. Thus, an earlier close-down would only make the capital value at the current point of time more negative. The production continues until the entire ore reserve has been extracted[1].

The financial implications of the situation is that future payments to the company do not cover future expenditures. The deficit will have to be paid at the end of the production period. Funds must have been accumulated in the company during previous years. A very restrictive and far-sighted financial policy is necessary.

14 Optimum mining limits

141 Multiple interrelated ore bodies

In section 11 the following problem was stated: Determine the optimum mining limits if the ore deposit is broken up into several separate parts, some of which may be left unexploited (Fig. 1:2). The problem is not explicitly formulated in the decision models discussed previously, but these are nevertheless useful in a simple solution of the present problem, which has been recommended by Carlisle (1954): Compare alternative mining limits, each at its optimum rate (or set of rates) of production.

The ore of each ore body of Fig. 1:2 is assumed to be homogeneous. The optimum rate of production is first determined on the assumption that only the ore body M is mined. It is then determined on the assumption that only M and A are mined, only M and B, etc. The parameters of the payment functions, the partitioning into zones, and the ore reserve have to be determined separately for each such combination of ore bodies. The optimum rate of production and the corresponding capital value are determined for every possible combination[2]. The combination giving the highest capital value is the optimum combination. The ore bodies, which are not members of the optimum combination, cannot be profitably exploited, and should be left in place.

1) This does not mean that the deepest part of an ore deposit should always be mined. This is another optimization problem, which is easily solved according to the principles put forward in section 141.

2) Some combinations can often be eliminated on the basis of more general considerations, e.g. the alternative M+A, if A and B contain the same sort of ore. In this case M+B is clearly superior to M+A, as it apparently involves lower preparation and transport costs.

Fundamental conditions for the practical use of the method is that an exhaustive list of mutually exclusive alternatives can be made, and that the number of alternatives is comparatively small.

142 Decreasing marginal grade in an ore deposit

1421 Assumptions

The second case in determining mining limits introduced in section 11 deals with an ore deposit where the ore grows successively poorer as the deposit is more extensively mined (Fig. 1:3).

The ore deposit is assumed to extend downwards and to consist of three zones, each partitioned into three subzones according to Fig. 1:3. The subzones are successively mined in the following order[1]: 11, 21, 31, 12, 22, ..., 23, 33.

Mining limits may be changed from one subzone to another, e.g. following the line indicating the marginal grade 60% in subzone 11, then be changed to the 55% line in subzone 21, etc. The marginal cut-off grades are then 60%, 55%, etc., respectively. However, for practical reasons the decision variable of the proposed model is the average grade in each subzone. Consequently, the measure of the mining limit is the average grade. As may be seen in Fig. 1:3 an average grade in a subzone implies a mining limit in the more direct sense.

The average grade is assumed to be determined for each subzone at the point of time at which the mining operations are transferred to it, i.e. at the starting time of the subzone, time $T_{n',n}$. The average grade is assumed to be kept constant during the production period of the subzone.

The payments connected with the ore extraction are essentially the same as those discussed in section 1313. In fact, the same payment functions are utilized in the computer program for optimizing the average grade as in that for optimizing the rate of production. However, one type of payment will appear, which has not yet been discussed: If the average grade is changed, a grade-change investment must be made, e.g. in order to adapt the ore-treatment

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1) The subscripts are written in the order: subzone, zone. Commas are inserted between the members of multiple subscripts only where they are necessary for the sake of clarity. Examples: Subzone 21 means subzone 2 of zone 1. Subzone 11,5 means subzone 11 of zone 5.

plant to the new ore, i.e. to the new average grade. The grade-change investment is assumed to be small in comparison with the expansion and the contraction investment.

As before, the values of other decision variables or parameters, such as the rate of production, are assumed to be given constants. However, it should be noted that the method mentioned in section 141 opens up a way of taking into account interdependencies between the various decision variables.

1422 Capital-value model

The capital-value model discussed in section 132 is utilized with the following additions:

1) The capital value of the zone, B'_n, is broken up into the capital values of the subzones, $B'_{n'n}$. Let $T_{n'n}$ be the production period of subzone n'n. Then, for example,

$$B'_1 = B'_{11} + B'_{21} \cdot (1+r)^{-T_{11}} + B'_{31} \cdot (1+r)^{-(T_{11}+T_{21})}.$$

2) The current payments occur in each subzone, as does the grade-change investment.

3) The zone investments as well as the expansion and the contraction investments occur only in the subzones 11, 12, and 13, i.e. in the first subzone of each zone. The close-down payments occur only in subzone 33.

4) $B_{n'n}$ is the decision criterion instead of B_n, the capital value at time T_n of future mining as seen from that point of time. $B_{n'n}$ is the capital value at time $T_{n'n}$ of subzone n'n and the subsequently mined subzones, e.g. B_{22} is the sum of the capital values B'_{22}, B'_{32}, B'_{13}, B'_{23}, and B'_{33}, all discounted to time T_{22}.

1423 Optimization method

The ore reserve increases as the mining limits are extended, i.e. as the average grade is made smaller. As the rates of production are given constants, the production period of the entire ore deposit as well as the production periods of the single subzones also increase. A decision concerning the average grade influences future decisions. The grade-change investment also implies that the current decision depends on the average grade in the preceding subzone which is just being finished at the decision time. Consequently, the decisions concerning the average grades in the various subzones are interdependent. This must be considered when the optimum average grades are determined. Therefore, a dynamic optimization method is applied.

The dynamic optimization methods described in sections 1333 and 1334 are not applied in this case, mainly for two reasons:

1) A simpler method is desired as the number of decisions is greater (optima are determined for the subzones instead of for the zones), primarily because the grade-change investment is very small in comparison with the expansion or the contraction investment.

2) The smallness of the grade-change investment makes the determination of limits similar to the expansion limits, the contraction limits, and the upper and the lower constancy limits less important. The limits would more or less coincide at the maximum of the capital value curve (compare Fig. 1:9).

An optimization method especially designed for determining optimum average grades has been developed in the form of a computer program[1]. It operates in the following manner:

The optimization starts from an initial guess comprising the average grades of all subzones of all zones. Then all the starting times, i.e. $T_{n'n}$ for n'=1,2,3 and n=1,2,3, are determined. The optimum average grade in the last subzone, 33, is then determined by systematically varying the grade until the average grade giving the highest capital value B_{33} is encountered. Next, the optimum in the penultimate subzone, 23, is determined similarly. The average grades of the previously mined subzones are still kept constant at the values given in the initial guess. In the subzone succeeding the current one, i.e. in the last subzone, the optimum average grade determined in the previous step is also kept constant. Thus, the capital value B_{23} can be calculated for every possible average grade in subzone 23, and its maximum can be established.

1) The program has been tested to the same extent as that for optimizing the rates of production.

In the third step the optimum in subzone 13 is found, given the initial guess for the preceding subzones, i.e. 11, 21, ..., 32, and the optima already determined for the subsequent subzones, i.e. 23 and 33. In this way optima are calculated for all subzones in the reverse order to that in which they are mined. However, if the optima do not coincide with the initial guess, the assumptions on which one subzone is optimized are rendered invalid by the subsequent optimizations of other subzones. Therefore, when the optimum average grade in subzone 11 has been determined, the process is repeated from the beginning, replacing the initial guess with the set of optimum average grades just determined. This is repeated until the capital value of the entire ore deposit, i.e. B_{11}, cannot be increased substantially.

The method is easily extended to any number of subzones and zones. The computer program handles a maximum of 20 subzones in each of a maximum of 14 zones.

1424 Conclusions

A few general observations on the behaviour of the optimum average grades can be made.

There is a tendency for the optimum average grade to decrease as the ore extraction proceeds. In the beginning of the production period only the best parts of the deposit are recovered, while towards the end the poorer parts are also worth mining[1)]. An explanation of this phenomenon is that the opportunity cost of ore is low in the beginning, when the closing of the mine still is in a distant future. As the end of the production period approaches, the opportunity cost increases, which makes the mining of successively poorer ore profitable.

The tendency towards decreasing average grades over time is traceable also within the zones, as in each zone the opportunity cost of ore increases as the next zone investment approaches in time.

The optimum decisions at the various decision times are not necessarily the same as the decisions which would give the maximum capital value of the entire ore deposit, i.e. make the value of B_{11} a maximum.

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1) This is a conclusion, which has also been reached by Henning (1963).

15 Optimum technology and optimum refinement level

151 Repeated application of the previous models

The optimum technology and the optimum refinement level can be determined with the method described in section 141. The model for optimizing the average grade can be used in this connection as an alternative to the model for determining the optimum rate of production. In any case, the optimization is made within the framework of the described capital-value models. However, as it is not always possible to make an exhaustive list of the possible alternatives the problems are only partially solved. The method only serves to select the best alternative among a given set of alternatives.

If the decision variable is continuous the method is useful in obtaining an approximate optimum. Various values of the decision variable are systematically selected and evaluated until an acceptable approximation of the optimum has been found.

152 Simplified methods

In sections 141 and 151 it has been understood that the rate of production or the average grade should be optimized for each given alternative. This apparently implies the assumption that the optimum values of these variables depend on the current alternative, i.e. on which ore bodies are mined, on the technology applied, etc. If experience from previous analysis shows that these optima are not influenced by, or are not sensitive to which alternative is decided on, the rate of production and the average grade can be taken for granted and be made equal in all alternatives. This will save a considerable amount of calculating work. The simplification may also be combined with alternating optimizations of the relevant decision variables, e.g. the rate of production and some technology variable, such as the sizes of the zones. The latest optimum of the most recently optimized variable is then inserted as a condition for a new optimization of the other variable.

The capital-value model which is integrated into the two previously mentioned computer programs, has been inserted into a third program, which is otherwise independent. The program can be used to calculate the capital values of a set of predefined alternatives.

16 Solutions of the five problems

Five problems were posed in section 11. Optimization models have been constructed for solving them. Two models are fundamental because either or both of them are applied in solving all the problems. Both models are based on the theory of dynamic programming. The two principal optimization models have been developed into computer programs:

EXRATE for determining optimum rates of production.

CUTOFF for determining optimum average grades (cut-off grades).

Problem 1) is to determine which ore deposits should be mined. Both principal models yield the capital value of the deposit, which can be used to determine whether a deposit is worth mining. More complicated decision problems with two or more deposits can also be solved, although the methods have not been discussed in this summary.

Problem 2) is to determine optimum rates of production, and is solved by means of EXRATE.

Problem 3) is to determine optimum mining limits. Mining limits which are able to be expressed as average grades, are optimized directly by means of CUTOFF. Other forms of mining limits are optimized by repeated application of EXRATE, or CUTOFF, or both these models.

Problem 4) is to determine the optimum technology, and is solved by repeated application of EXRATE, or CUTOFF, or both these models.

Problem 5) is to determine the optimum refinement level, and is solved in the same way as Problem 4).

The models are based on certain assumptions which limit their practical usefulness, especially concerning Problems 4) and 5).

It has been assumed in this summary that the mining company controls one single ore deposit. Problems involving multiple deposits are solved by repeated application of the two principal models (section 73).

17 Introduction to the following chapters

With the summary as a background the models will be discussed and explained in detail.

CHAPTER 2 The problems to be solved are stated more precisely and the purpose of the study is specified.

CHAPTER 3 General problems of model building are discussed with a view to the problems which are to be solved. Important assumptions are derived by a study of the boundaries of the models, and by specifying the types of decision models which are relevant for the decision problems treated in this study.

CHAPTER 4 The assumptions made in building the decision models are enumerated. Formal decision models are formulated.

CHAPTER 5 Methods are derived and discussed for solving the decision problems as they are formulated in the decision models.

CHAPTER 6 The formal decision models of CHAPTER 4 and the solutions derived in Chapter 5 are brought together to form complete decision models.

CHAPTER 7 The models treated in Chapters 3 to 6 are based on two major assumptions. First that the decision maker is able to carry out decided changes in the values of controlled variables, e.g. the rate of production, instantaneously whatever the size of the change. Secondly, that the decision maker controls only one ore deposit. The decision models are extended to cases where these assumptions are relaxed.

CHAPTER 8 Some conclusions are discussed.

CHAPTER 2

2 The study: Its problems and purpose

21 Exhaustible resources

Two principal groups of natural resources can be distinguished, renewable resources, e.g. growing forests, and exhaustible resources, e.g. mineral deposits. The boundary between the two groups is not quite distinct. Thus, in the short run a forest may be regarded as a limited stock of trees which can be used up[1]. According to Gray (1913, p. 469) the renewal problem is also an economic question: Does it pay to renew the resources? The magnitude of a mineral deposit as known at a certain time is a given constant, but the limits of the known deposit can be extended through prospecting. Prospecting in combination with the exploitation can thus result in renewal of the resources. This type of renewal is more apparent if a larger area is examined instead of a single deposit: New deposits are located as the previously known deposits are exploited[2].

Renewal in the form of locating unknown deposits does not result in a factual increase in the existing amount of the substance forming the resources. It only increases the possibilities of an acting agent to utilize the substance. In spite of this the renewal through exploration is important as the first condition for successful utilization is that the deposit is known. This subjectivity is implicitly assumed in terms such as "resources" and "deposit".

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1) Streyffert (1938, pp. 136 ff.), among others, discusses the forests as exhaustible resources and the optimization of the production period. His approach is similar to Massé's approach in the static model described in section 12. The assumptions concerning the payments differ, though, and Streyffert mainly discusses the effects of variations in unit exploitation costs as a result of variations in the length of the production period, and of price variations over time. Regarding the latter, he concludes that the optimum production period will increase as the price of the final product increases <u>over time</u> (ibid. p. 161).

2) A parallel may be found in forestry. For example, "Faustmann's formula" from 1849 gives the capital value of a forest on the assumption that the exploitation continues forever in identical replacement cycles (Streyffert 1938, p. 11). The formula is applied in determining the optimum production period, i.e. the length of the replacement cycle (ibid. p. 41). However, the parallel is not perfect as it presupposes that new deposits are found at the same rate as the older ones are exhausted. Instead, the rate of renewal of resources, minerals as well as forests, is rather an optimization problem (Gray 1913, p. 469).

This study is confined to the treatment of mineral deposits which have already been detected. In compliance with existing recommendations[1] a mineral deposit is defined as a geologic formation containing minerals which are or might become useful and extractable. The study is also restricted to exhaustible resources. Furthermore, in order to simplify the discussion it will be oonfined to ore deposits, which are defined in agreement with the above definition of mineral deposits. However, the analysis applies to other exhaustible resources as well.

The ore of an ore deposit can be exploitable or not exploitable in the economic sense, i.e. the exploitation can have a positive or a negative value according to some measure of value. Also, a deposit can be only partly exploitable. Exploitable ores constitute ore reserves[2]. The value of the exploitation of ore, and consequently the ore reserves depend on present technological and economic conditions and on present expectations concerning future conditions[3]. The ore that is not exploitable might become exploitable if the expectations are changed. Hence, it is called potential ore[4]. Together, the ore reserves and the potential ore of a deposit, a district, a country, etc. form the ore resources[4] of the deposit, the district, the country, etc., respectively.

The ore contents of an ore deposit can be known with a varying degree of certainty. Usually three degrees of certainty are distinguished, such as proved, probable, and possible ore[5]. In the present analysis ore reserves and ore resources will include all ore, irrespective of the degree of certainty of the estimate.

Some of the ore is lost in the processes of extracting and refining it. Ore losses incurred before the ore is loaded at the working face for transport to the gravity shafts, the crushing plant, the sorting plant, etc. (see Fig. 1:1),

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1) For example, in a survey of world iron-ore resources initiated by the United Nations (Survey 1955, p. 45 (Percival)) and by Svenska Gruvföreningen (1964, p. 18).

2) In discussing certain problems, e.g. the drawing of mining limits, the ore reserve is a variable. Then it should be interpreted as the exploitable ore if the suggested mining limit is the optimum limit.

3) In the industry the practice is to use similar definitions of ore reserves, except that the present conditions are often stressed to the exclusion of the future conditions, as e.g. in Survey 1955, p. 20 and p. 170 (Blondel and Lasky).

4) Ibid. pp. 20 and 173.

5) Ibid. pp. 20 and 171. Other classifications exist (ibid.).

may be distinguished from ore losses in subsequent stages, i.e. ore losses mainly incurred in the sorting and the dressing plant. It should be observed that the latter, i.e. the ore losses where waste is removed, need not necessarily mean losses of the valuable substance of the ore. Furthermore, in some mining methods, especially caving methods, waste rock originally situated outside the ore body will be mixed into the ore. The phenomenon will be called a negative ore loss (in the mine). Thus, e.g., of an initial amount of 1.0 MT of ore in its original, solid state only 0.9 MT will be loaded, transported, and treated. 0.1 MT is lost. In addition to this negative ore losses of, say, 0.05 MT occur, which increases the extracted quantity to 0.95 MT. Out of this perhaps only 0.8 MT will remain after sorting, dressing, and other treatment. The rest, i.e. 0.15 MT, has been removed as waste.

The tonnage which is transported and treated in the mine as well as the input tonnage of the sorting plant, will be 0.95 MT, i.e. the tonnage after the ore losses (positive and negative) in the mine have been deducted. The mining and the ore treatment facilities must be dimensioned for this tonnage. Thus, to simplify the following discussion all ore tonnages are defined as the tonnages after the tonnages, in absolute values, of positive ore losses in the mine have been subtracted, and of negative ore losses in the mine have been added, but before ore losses in subsequent stages of the process have been deducted. This applies to ore reserves as well as to production capacity and rate of production. Thus, the ore reserves will be measured in terms of extracted quantity of ore, extracted waste rock included[1]. The grade of the ore is defined correspondingly, i.e. as the grade of the mixture of ore and waste rock that is loaded at the working face.

22 The problems

221 Decision makers

At least two parties are traditionally considered to have an interest in the exploitation of ore deposits, i.e. the exploiting party or the mining company, and the community which is usually represented by those invested with the political power. Hotelling (1931), Ciriacy-Wantrup (1952), and Scott (1954), among others, discuss the opposing interests of the mining company and the community,

1) This is contrary to practice, e.g. in Sweden. See Svenska Gruvföreningen (1964, p. 20), where it is stated that the ore reserve should be measured as the ore in place, i.e. in its original, solid state.

their symptoms, their effects, their reasons, etc. The problem of exploiting ore deposits can be approached from an economic as well as from a political point of view, and on the macro as well as on the micro level. Only the economic aspects will be discussed, principally from the point of view of the mining company, i.e. its management. However, in a few exceptional cases the results of the managerial analysis will be superficially treated from the point of view of the community.

222 Goals of the decision maker

Decisions concerning the exploitation of ore deposits are made in the light of the goals of the mining company. The decisions are assumed to be rational in the sense that they result in a maximum degree of goal satisfaction within the restrictions on the freedom of action which may prevail. The decision maker thus optimizes his decisions[1]. Applying the reasoning of March and Simon (1958, pp. 137-138) optimization presupposes the following assumptions:

1) The decision maker has to make a choice from a given set of alternatives.

2) A set of consequences is attached to each alternative action. The consequences can be known with three different degrees of certainty:
   a) certainty, i.e. the outcome of each alternative is fully known
   b) risk, i.e. more than one outcome is possible (ex ante) and all these are fully known, together with an objective probability distribution of outcomes[2].
   c) uncertainty, i.e. more than one outcome is possible (ex ante), but the objective probability distribution is not known[2]. (Genuine uncertainty).

3) The decision maker is able to rank the outcomes in order of preference, from the most preferred to the least preferred.

4) The decision maker chooses the alternative to which the preferred outcome set is attached. In the case of certainty, the choice is unambiguous. In the case of risk, the optimum decision is usually considered to be to select the alternative for which the statistically expected utility (measured on some

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1) Satisficing behaviour is an alternative to optimizing behaviour. See e.g. March and Simon (1958, pp. 141 and 169).

2) March and Simon (ibid.) do not explicitly state the "ex ante" assumption, nor do they state that the probability distribution should be objective.

given scale) of the possible outcomes is as high as possible. In the case of uncertainty, the choice is more difficult, and will not be discussed here[1].

The present study will be confined to the case of certainty.

The goal of the mining company is assumed to be maximum profit for the company. Profit maximization is, however, not the only possible goal. This has been recognized by several authors, e.g. Frenckner (1953) and Johansson (1961, p. 3). The problem will not be dealt with here, except to take the opportunity to point out that even if the mining company has another goal, profit maximization offers a basis for determining the cost in terms of forgone opportunities, which the company is incurring in order to attain the other goal or goals. Thus, the goal of profit maximization is accepted for this study. However, the goal is not operational, as no unambiguous measure of profit exists (Frenckner 1953, pp. 17-19). For this reason it is, in accordance with the reasoning of March and Simon (1958, p. 156), replaced by an operational subgoal in the actual decision making.

An operational subgoal which is often, but not always, consistent[2] with the more general goal of profit maximization, is maximum capital value at the decision time of future payments occurring as consequences of the different alternatives. The term "payments" is defined so as to cover payments received by the company as well as expenditures, i.e. payments from the company or negative payments. In the case of risk, the corresponding operational goal is maximum statistically expected value of the possible capital values. The connection between a selected subgoal and a more general goal has been treated on a more general level, although in different specialized contexts, by March and Simon (1958), Danielsson (1963), Langholm (1964), and Hällsten (1966). The connection between the two specific goals, i.e. profit maximization and capital-value maximization, has been treated by e.g. Lutz (1951). The discussion will not be repeated here. It is sufficient to observe that in spite of certain weak points the capital value has been accepted as a measure of long-run profit where the profit is a result of a series of payments at various points of time, i.e. in investment problems, by several recent authors, e.g. Massé (1959) and Johansson (1961).

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1) Milnor (1954) and Luce and Raiffa (1957, pp. 278 ff.) make more comprehensive studies of various criteria in the case of uncertainty.

2) The "goodness" of criteria has been discussed by e.g. Hitch and McKean (1954). They state that the test for a "good" criterion is its consistency with a good criterion at a higher level (ibid. p. 179). Also, compare Hällsten's (1966, pp. 2-3) "global model" and "decision model".

One of the more difficult problems encountered in connection with capital-value maximization is to determine the rate of interest to apply in the calculations. In order to solve this problem, and also to eliminate other weak points of the pure capital-value maximization, certain programming methods have been developed. Recent examples of the latter approach are those presented by Charnes, Cooper, and Miller (1959), Albach (1962), Hållsten (1962), and Weingartner (1963). However, in decisions involving consequences over very long time intervals the latter approach does not differ substantially from the traditional capital-value approach[1]. As the decision problems which will be discussed here, are mainly of the long-range type the more easily handled traditional capital-value approach will be applied. The rate of interest is assumed to be a given constant.

To summarize, the operational goal of the mining company is assumed to be to obtain maximum capital value at the decision time of future payments. The capital value is the <u>decision criterion</u> and the <u>optimization criterion</u>, i.e. among the available alternative courses of action the company will select the one ensuring maximum capital value. The rate of interest is assumed to be given, and the future to be known with certainty.

The capital value is widely accepted as an optimization criterion in the literature concerning long-range mining decisions[2]. It is not always evident what types of payments are included in the capital-value computations, but at least Ciriacy-Wantrup and Massé (ibid.) discuss payments and points of time of the payments. Other criteria are also discussed in the literature[3]. If motives for the selected criteria are mentioned, the reasons commonly stated for one criterion or the other are practice in mining companies or difficulties in applying the capital-value criterion.

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1) The programming approach involves a horizon in time beyond which traditional capital-value methods are usually applied. The horizon is for practical reasons comparatively close to the decision time, e.g. 3 to 5 years after the latter. In decisions involving very long periods of time, the first few years are presumably not crucial to such an extent as to motivate a technically difficult discrimination in the method of evaluating the alternatives. However, the problem should be borne in mind in a sensitivity analysis.

2) The capital value is used as the final criterion by e.g. Gray (1913, pp. 474–475), Hotelling (1931, p. 140), Ciriacy-Wantrup (1952, p. 77), Carlisle (1954, p. 601), Herfindahl (1955, p. 131), Massé (1959, p. 350), Billiet (1959, p. 22), Uggla (1958, p. 202), and Albach (1967, p. B-554, however, on the assumption of a fixed time horizon). On the other hand, Hållsten (1966), and others, question the ability of the capital-calue criterion to reflect the interests of the owners of the company in investment decisions (of which the present decision problems are examples).

3) Henning (1963, p. 54) discusses capital-value criteria as well as other criteria.

For optimizations on communal levels a "communal capital-value criterion" is used by e.g. Hotelling (1931, p. 143) and Ciriacy-Wantrup (1952, p. 230). However, in order to make the criterion applicable in this context the payments as defined on the company level, have to be adjusted (ibid.)[1]. As the management of the mining company is often interested in knowing the social optimum as well as the optimum from their own point of view, and as they have a capital-value optimization method available, it is a natural step to try to use the same method in both instances. This is the principal reason for digressions to the communal levels in this study.

223 Decision variables

In the preceding section it has been referred to "decision problems" which have to be solved so as to maximize the capital value. The decision problems of this study will now be defined. Exhaustibility will be considered a principal property of ore deposits. Consequently, the problems connected with ore prospecting will be excluded, as prospecting from the point of view of the individual mining company is a means of renewing depleted ore deposits[2]. Thus, the ore reserve is considered the main restriction in the production model[3] of the mining company. The problem of the company is to exploit the available deposits in the best possible manner, i.e. so that the capital value of future payments due to the exploitation of the deposits is a maximum. Further, the attention will be concentrated on decisions which especially influence the ore reserves and the production periods of the deposits.

The market situation is assumed to be given so that the mining company faces a set of demand functions for its products. Thus, decisions concerning pricing and marketing will not be treated.

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1) From the point of view of the community a "communal capital-value" appears to be a reasonable measure of the economic consequences of alternative acts. See e.g. Werin (1968, p. 71). For developing countries it is, however, more disputable. See e.g. Kahn (1951) and Rollins (1955 and 1956).

2) The economics of prospecting has been treated by e.g. Grayson (1960) and Allais (1957). Herfindahl (1955) discusses the effects of prospecting on the optimum rate of production in a deposit when the company faces perfect competition on the ore market. He also investigates the market equilibrium with respect to exploration and exploitation on the assumption of a competitive industry, and concludes that for the industry as a whole the aspect of exhaustibility is relevant only at a late stage in the history of the resource, as exploration becomes too expensive in comparison with the value of new deposits (ibid. pp. 133-135).

3) Danø (1966, p. 10) describes a production model, which is a generalized form of the production function, as "a system of quantitative relationships expressing the restrictions which the technology of the process imposes on the simultaneous variations in the quantities of inputs and outputs".

The following decision problems remain to be solved as the deposits are exploited (compare the descriptions of various types of ore deposits in section 11):

Problem 1) Which ore deposits should be mined? or: Is the deposit exploitable?

Problem 2) At what rate should the ore be extracted? (The annual rate of production).

Problem 3) How extensively or carefully should the ore be recovered? (The mining limit, or the cut-off grade).

Problem 4) Which mining methods, machinery, technical methods, etc. should be used? (The technology)[1].

Problem 5) How far should the crude product be refined[2]? (The refinement level)[3].

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1) Technological efficiency is presumed. In production theory, e.g. according to Danø (1966, pp. 14-15), the relevant range of economic choice is restricted to technologically efficient alternatives, i.e. alternatives which are not "... technologically inefficient in the sense that it is possible to produce more of one output without having to produce less of any other output and without using more of any input, or that it is possible to produce the same amounts of all outputs with less of one input and not more of any other input. ...". Carlson (1939, pp. 14-15) holds a similar view.

However, the problem of determining optimum technology is not solved by this assumption, especially as the technological decisions have consequences during a considerable time period as well as an influence on the ore reserve (e.g. through ore losses) and the properties of the final products. Thus, e.g., different mining methods cause different ore losses which not only influence the ore reserve but also the properties of the final products, as the ore losses are often incurred in certain limited parts of the deposit (which influences the output if the deposit is not homogeneous), or are selective as regards the mechanical structure of the ore (the latter being an important aspect of the quality of the output, i.e. the final product). The problem of technology in production is penetrated more thoroughly by Frisch (1965, especially pp. 24-28 and 40 ff.).

2) The term "refinement" is used in a wide sense, including crushing, sorting, dressing, sintering, pelletizing, stockpiling, etc.

3) If the final product of a mine is one single product, the refinement level amounts to the same as the quality of the product. Danø (1966, p. 133) distinguishes between product quality from the point of view of production and product quality from the point of view of demand. Both aspects are covered by problems 4) and 5). Compare next page, footnote 1).

The output of the mine may, however, comprise a number of products which are produced in the same process. The relative quantities of the different products can be varied, at least within certain limits. Then, according to Danø (1966, p. 167) as well as Frisch (1965, p. 11), the case is one of joint production.

Finally, a part of the output can be processed further, e.g. transformed into pellets or sinter instead of being sold as ore. This may be considered a case of assorted production (Frisch 1965, p. 11).

All these aspects are covered by the concept of a refinement level.

The names of the <u>decision variables</u>, i.e. the variables concerning which decisions are to be made, pertinent to the problems, are written in brackets. Already at this stage of the discussion it should be noted that the names refer to single variables only in special cases. For instance, in the general case the rate of production refers to a set of rates, i.e. one rate in each zone of a deposit (compare sections 11 and 1312), or even several sets, i.e. one for each deposit, if the mining company controls more than one deposit in which the rates of production may be varied freely. Similarly, the other variable names may represent sets of variables.

The decision problems of questions 3, 4, and 5 are multi-dimensional also in other respects. According to section 11, Fig. 1:2 and Fig. 1:3, the mining limits may concern the selection of an optimum combination of interdependent ore bodies or the determination of mining limits within an ore body. Naturally, the two problems can appear jointly in one single mine. The technology is also multi-dimensional as it concerns decisions regarding mining methods, mining equipment, zone sizes, plant lay-out, etc. Finally, the refinement level is a collective notation for the number of qualitatively different final products, their metal percentages and fragmentation, the extent to which the ore is sintered, etc.[1].

The decision variables can be continuous as well as discontinuous. Examples of the former type are the rate of production and the special case of mining limits called the cut-off grade. Only these two decision variables will be formally treated as continuous variables in this study. Examples of the latter type are mining methods and the combinations of final products. Discontinuous decision variables are formed by discrete alternatives (Ackoff 1962, p. 112). The decision variables will be further specified as the specific decision problems are dealt with.

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1) The technology and the refinement level are interdependent as, e.g., different mining methods result in various properties of the output. In some respects it might even be stated that the one determines the other. Often a decision concerning one variable infringes the range of possible choices concerning the other. However, it appears impracticable always to fully distinguish between the two aspects, i.e. technology and refinement level. In such cases the decision problem can be treated as a technology problem if the input or production side of the problem appears most important to the decision maker, and as a problem of refinement level if the output or marketing side is more interesting. This does not exclude simultaneous optimization of two or more variables which are pertinent in both cases.

224 Decision models

Quite generally, a scientific model may be viewed as a simplified or idealized representation of states, events, and objects[1]. Problem situations have been described in the preceding sections, situations in which decisions have to be made. The corresponding class of models is decision models[2]. A decision model contains[3]:

1) A measure of the value to the decision maker of alternative decisions (the decision criterion of the company, i.e. the capital value).

2) Decision variables which are controlled by the decision maker (the decision variables of the mining company).

3) Parameters, i.e. variables and constants which are not controlled by the decision maker (e.g. the ore resources and the market situation).

4) A functional relationship between the decision criterion on one side, and the decision variables and the parameters on the other.

5) Constraints or restrictions on the decision variables (e.g. available techniques and that the annual production accumulated over the production period, equals the ore reserve).

Points 1) to 4) describe the goal function[4] or the objective function[5] of the company. Thus, the decision model consists of a goal function and a set of restrictions.

So far only the measure of the operational goal of the company, i.e. the decision criterion of the company, has been taken into account in the decision model. The model contains no statement of how the company is going to apply the criterion in its decisions. As the company is assumed to seek maximum value of the measure within the given constraints, i.e. to optimize, the process of obtaining the maximum should also be incorporated in order to complete the model. This form of decision model will be called an optimization model[6].

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1) The definition is essentially the same as those given by Churchman, Ackoff, and Arnoff (1957, p. 157) and Ackoff (1962, p. 108).

2) Ackoff (1962, p. 111).

3) The exposition is an application of Ackoff's (1962, p. 111) description of decision models.

4) A term used by Hållsten (1966, p. 1).

5) A term used by Danø (1966, p. 1).

6) An alternative goal is satisficing, i.e. attaining a satisfactory value of the measure of the operational goal (compare the first footnote of section 222, including the reference to March and Simon). The corresponding form of the decision model would be called a satisficing model.

The optimization model is a simplified representation of the decision problem. Approximations and omissions have been made. The actual decision situation is represented by a decision model where it is only partially described: The decision model represents a precisely defined and comparatively simple model situation which exists in an unstructured environment[1] (the actual decision situation), the influences of which are not taken into account[2]. The same holds true for the optimization model. Thus,

1) actual decision makers are represented by an idealized entity, i.e. the decision maker[3] or the mining company.

2) actual goals of the decision makers are represented by a single goal, i.e. the operational goal[4] or to attain maximum capital value.

3) the environment[5] of the decision model in other respects than those mentioned under 1) and 2) above, is represented by the parameters[6].

4) the actually available alternative actions and their consequences are represented by the decision variables and the value of the goal function[6].

5) the actual decision is represented by the optimum alternative[7].

These are important weak points in the optimization models, which should be considered when the models are constructed and applied.

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1) This view on a decision model is held by Danielsson (1963, p. 46) as well as Hållsten (1966, p. 3) and others.

2) Such influences are aspects pertaining to Hållsten's "global model" (ibid.).

3) The problem of determining who is decision maker in an organization is treated by e.g. March and Simon (1958, pp. 112 ff.).

4) Compare section 222. It is only an assumption, not a proved fact, that the operational goal is consistent with the more general goal of profit maximization. Further, the latter is not proved to be the actual goal of the decision maker.

5) The term is Hållsten's (1966, p. 1).

6) According to the preceding discussion, and Ackoff (1962, p. 11). Furthermore, various approximations generally encountered within decision models are discussed by Ackoff (1962, pp. 117 ff.).

7) The decision maker does not necessarily choose the optimum solution arrived at by means of the decision model, and the application of the optimum solution to the decision problem might deviate from the application presumed in the model (Ackoff 1962, p. 139).

The future is assumed to be known with certainty[1]. As this assumption is not a very realistic one, some reasons in favour of it will be given. Thus, it is a simplification which makes it easier to build a decision model and to find a solution to a decision problem as it is represented in the model. A model assuming certainty is useful in the study of other aspects of the problem than those connected with risk and uncertainty.

The certainty model may also be seen as a preliminary to a model where risk or uncertainty is taken into account. An example of this is that it can be used in a sensitivity analysis which establishes the importance of different parameters with respect to the goal assumed, i.e. their influence on the capital value[2]. Then it can be determined whether the certainty model combined with a sensitivity analysis is sufficient from the point of view of the decision maker or whether a more sophisticated model is desirable[3]. In the latter case

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1) The problems of exploiting ore deposits are undoubtedly problems of risk or uncertainty. The ore resources of large areas are usually not known with certainty, and neither are those of a single deposit, although the range of possible outcomes may not be as wide in the latter case as in the former. The former case has been studied by e.g. Allais (1957), who treats mining exploration in the Sahara as a problem of risk, applying the expected capital value of ore deposits as the main criterion (however, the dispersion is also taken into account). The principles should be applicable to prospecting for solid fuel or oil as well (ibid.). Drilling for oil has been treated as a decision problem under risk as well as uncertainty by Grayson (1960).

Decision problems under risk and uncertainty concerning a single ore deposit, have been treated by e.g. Billiet (1959) and Massé (1959). Ventura (1959) discusses and develops Billiet's model further. Their common problem is to determine optimum rates of production, i.e. rates giving maximum expected capital value, when the ore reserves are known as probability distributions. Albach (1967) also treats mining problems under risk. Billiet and Massé also examine the case of genuinely uncertain ore reserves, applying the criterion of "minimax regret" (see e.g. Milnor (1954) or Luce and Raiffa (1957, pp. 280 ff.) concerning this criterion).

The ore reserve of an ore deposit is not the only parameter which is known with some degree of uncertainty. The future market situation and, consequently, the prices or quantities sold of the final products, and the future investment and production costs, are also elements introducing risk or uncertainty.

2) The sensitivity analysis may be considered an experimentation on the decision model. Experimentation as a means of determining the relevance and the importance of variables upon one another has been discussed by e.g. Churchman, Ackoff, and Arnoff (1957, pp. 163-164 and 577 ff.) and Ackoff (1962, pp. 311 ff.).

3) This is a question of economic optimization, i.e. weighing the cost of an extended analysis against the gains of better decisions. Compare e.g. Churchman, Ackoff, and Arnoff (1957, p. 106) and Ackoff (1962).

important parameters can be detected by means of the sensitivity analysis. Thus the interest can be focused on relevant parameters in subsequent attempts to improve the decision model[1].

The building of a decision model assuming certainty can also be a first step in the construction of a decision model assuming risk. In this case probability distributions of parameter values are introduced into the certainty model at a subsequent stage[2]. Furthermore, those parts of the certainty model where alternative actions are evaluated in terms of the capital value, are necessary in determining conditional values, i.e. the capital values of alternative actions if the actual values of the probability-distributed parameters were given[3]. This applies also in the case of genuine uncertainty, where alternative actions have to be evaluated under various assumptions concerning the uncertain variable[4].

Because of these considerations the aspects of risk and uncertainty are not explicitly treated in this study. Instead it is observed that it should be possible to make an extensive sensitivity analysis and to evaluate predefined alternatives. It is assumed that this will enable each user of the models described in this study to take the most important uncertain factors of his special decision problem into consideration outside the models discussed. Thus, the decision situation which has been presented in preceding sections will be treated under the assumption of certainty. How can the corresponding optimization model or models be constructed, and the decision problems be solved?

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1) A relevant parameter is here a variable or constant which influences the capital value significantly from the point of view of the decision maker. The definition is in accordance with Ackoff's (1962, p. 311) more general definition.

2) This approach has been used by e.g. Billiet (1959) and Massé (1959). It should also be noted that dynamic programming (which will be used) permits a unified approach to the cases of certainty and risk (Bellman and Dreyfus 1962, p. 43).

3) The use of the conditional value in decisions under risk is discussed by e.g. Schlaifer (1959, pp. 24 ff.). He also gives a more general definition of the conditional value.

4) See e.g. Luce and Raiffa (1957, pp. 275 ff.).

23 The purpose

The purpose of this study can now be stated. In order to solve the decision problems stated in section 223 optimization models will be constructed and methods to determine optimum solutions will be found. The decision criterion is the capital value at the decision time of future payments. The rate of interest is a given constant. The study will be concentrated upon the types of ore deposits described in sections 11 and 21. The future is assumed to be known with certainty.

The study will mainly be confined to models for optimizing the rate of production and the cut-off grade in the single-deposit case. Hence, two principal optimization models will be constructed. Models for optimizing the other decision variables will be constructed only to such an extent that this can be done by applying the two principal optimization models or their main components.

A few characteristics of the decision situation will be varied, and the corresponding adaptations of the optimization models will be discussed:

1) Decisions at a single point of time versus a series of interdependent decisions at various points of time.
2) A single decision variable at each decision time versus simultaneous decisions concerning two or more decision variables.
3) Factor and product prices constant versus variable over time.
4) Unlimited versus restricted rate of expansion of the rate of production.
5) A single ore deposit versus two or more deposits.

Naturally, it is not possible to predict all decision situations which may appear in reality. Many more decision situations are met with in the mining company than those treated here. Therefore, the optimization models will not cover all possible cases. Instead, they are intended to be examples of how specific, realistic, and complicated decision situations can be simplified, and the decision problems solved accordingly.

A secondary purpose is to discuss the application of the decision models to decision problems as seen from the point of view of the community. This aspect will be only superficially treated, mainly by indicating discrepancies between the points of view of the mining company and the community.

CHAPTER 3

3 Model building

31 Some problems in model building

The relation between reality and the optimization model has been discussed on a general level in section 22. A model of the decision maker was combined with a general model of the decision situation. This has resulted in an operational goal and a measure of value consistent with the operational goal of the decision maker. Thus the operational goal is maximum capital value and the measure is the capital value associated with alternative actions. The general model of the decision situation was combined with the operational goal to form the concept of an optimization model.

The purpose of this study is to construct optimization models in order to solve the decision problems stated. To achieve this it is convenient to determine a few stages or steps which can be discussed successively. Thus the question whether the optimization model can be partitioned into submodels which can be treated separately will be examined.

After that, a few general problems regarding the optimization models must be taken into consideration before an attempt is made to solve the actual decision problems. A first problem is that a decision may have effects which are extended infinitely into the future. It may also have effects on the environment of the optimization model, which are not included in the model. Thus, which effects of a decision should be incorporated into the model must be determined. This is the problem of the boundaries of the model.

Decisions at various points of time exert influence upon one another. A second problem is thus to determine to what extent such interdependencies are relevant, and should be taken into account. These are the problems of interdependencies and static versus dynamic models.

A model may have to be tested. This constitutes a third problem.

32 Submodels in the optimization models

An optimization model may include submodels[1]. In the present case two sets of submodels can be distinguished. The first set consists of informal submodels in the general model of the decision situation describing the ore deposits, the mining operations, and the market situation. These models are called informal, because they are not developed into internally complete, systematic descriptions of what they represent. The models describing the ore deposits have been made comparatively complete, though, in section 11. The models of the mining operations have also been comparatively fully treated in sections 11 and 1312[2]. Otherwise the informal models are to a great extent implied in the assumptions of the formal models discussed below.

The informal models discussed are thus conceptual intermediaries between the formal models discussed below and reality. They can also be regarded as fragments of a global model[3]. The structure is illustrated in Fig. 3:1.

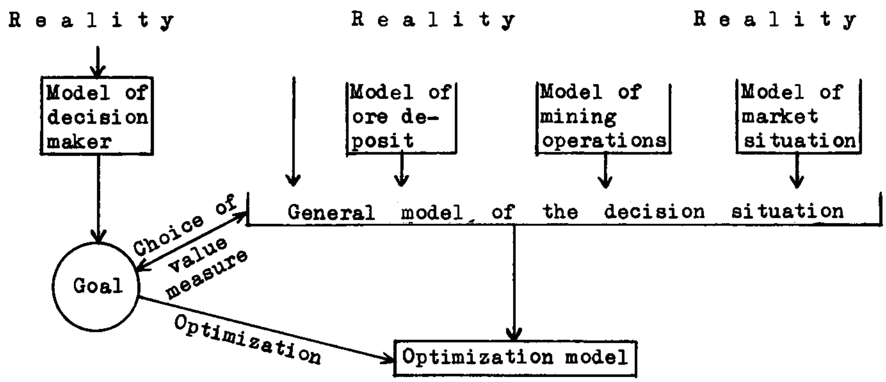

Fig. 3:1 The informal submodels of the optimization model.
= Informal model = Formal model
= Direction of influence

1) Ackoff (1962, p. 112).

2) The descriptions of the ore deposits and the mining operations are mainly based on the author's experiences of the iron-ore mines in northern Sweden and on an elementary mining textbook, Gruvkursen (1961).

3) Hållsten's (1966, p. 1) concept. Compare section 224.

The second set of submodels consists of formal submodels which will be construct- ed in this study. They are all concerned with one single ore deposit, and in order to treat more than one deposit a combination of one-deposit models will be used. The formal submodels for one deposit comprise an ore-reserve model, a set of payment models, and a capital-value model. The ore-reserve model expresses the ore reserve as a function of one of the decision variables, namely the cut-off grade or the average grade of the ore[1]. The set of payment models expresses payments at any given point of time as functions of certain decision variables[2] and parameters (the payment functions). The assumptions of the ore-reserve model and the payment models are derived from the general model of the decision situation and its submodels. The ore-reserve model and the payment models are submodels in the capital-value model which expresses the capital value of an ore deposit as a function of decision variables and the ore reserve. The values of the decision variables are assumed to be given. The model is subject to further assumptions derived from the general model of the decision situation and its submodels, especially the ore-reserve restriction, i.e. that the annual production accumulated over the production period, must equal the ore reserve.

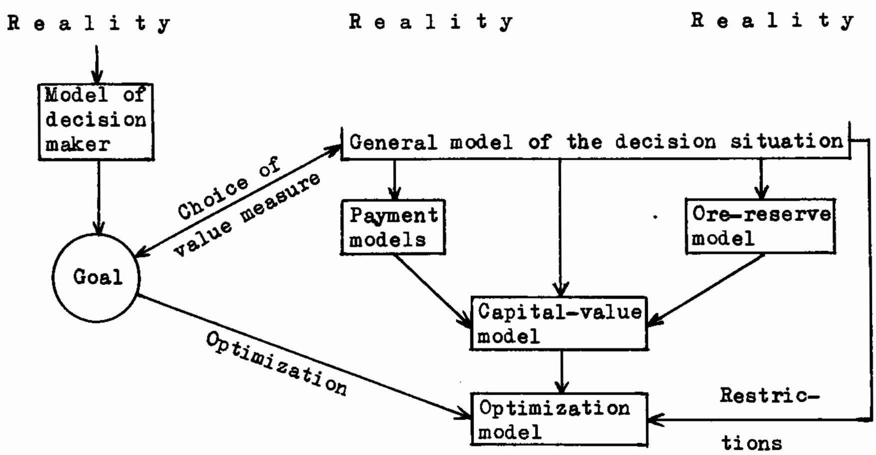

Fig. 3:2 The formal submodels of the optimization model.
Legend: See Fig. 3:1.

1) Compare Fig. 1:3.

2) The identity of the variables is irrelevant for the moment. They are discussed below and in section 4 of Appendix D.

In the special case where only one ore deposit is taken into consideration, the capital-value model also contains the goal function of the optimization models. In other cases the capital-value models of the relevant deposits have to be combined. This will be discussed in Chapter 7. Until then it will be assumed that the mining company is concerned with only one ore deposit. The optimization may be subject to further restrictions.

The structure of formal submodels is illustrated in Fig. 3:2.

33 Boundaries of an optimization model

331 Boundaries and assumptions

Each alternative action in a decision situation is characterized by its effect on the situation of the decision maker. Ideally, all effects should be taken into account in the optimization model. However, the model is a simplification, and only a selected set of effects is included. This has already been discussed in section 224. The simplifications determine the boundaries of the model[1]. Thus the boundaries of the optimization model have to be defined in various dimensions[2], such as e.g. which variables and parameters to include, the precision in the description of relationships, and the precision in the calculations. To the extent that these boundaries are recognized, they form the explicit assumptions of the model. Hence the boundary problem adds a new dimension to the discussion of formal and informal models in section 32, i.e. there exists a set of assumptions which completely defines the formal model. The assumptions may be implicit or explicit. This does not hold true for the informal model.

The specific assumptions of the optimization models will be treated in connection with the actual construction of the models. Regarding these assumptions the method used in determining the boundaries, i.e. in establishing the assumptions or determining what is taken into account in the model and not, will not be discussed further. Only the problem of defining a horizon in time will be discussed here, as time may be considered an additional dimension in the other

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1) Langholm (1960, p. 455).

2) Langholm (1960 and 1964) discusses the boundaries of decision models. He is primarily interested in time horizons, but as a starting point he treats the present more general delimitation problem (ibid. 1960, pp. 454-455 and 1964, pp. 12 and 18 ff.). The latter problem has also been discussed by e.g. Danielsson (1963, pp. 46 ff.) and Hållsten (1966, pp. 1 ff.).

assumptions. This is so because the effects of a decision have extension in the time dimension. Furthermore, many effects extend into the infinitely distant future. For instance, an ore deposit cannot be mined again, once it has been exhausted.

332 Time horizons

3321 Information horizon and model horizon

At the time of the decision the decision maker knows the consequences of his decision only up to a certain future time. The latter constitutes his information horizon[1]. A time horizon beyond which consequences are not taken into account can also be determined for an optimization model. This is the model horizon[2]. The model horizon is sufficiently distant if the optimum decision is independent of what happens beyond it. Langholm has derived criteria for determining if a model horizon is sufficiently distant. A sufficient criterion is that the optimum decision will be the same for all possible states at the horizon[3].

The choice of a model horizon will be discussed by applying a general method introduced by Savage (1954), and used by e.g. Danielsson (1963), Hållsten (1966), and Langholm (1960 and 1964) in their analyses of models. The method in question is to compare conclusions (i.e. optima) obtained by means of one model with the corresponding conclusions obtained by means of another more extensive model, i.e. a global model, which includes that which is represented by the former model as well as the context or environment of this model[4].

3322 The global model

Langholm (1960, p. 457) states that the model horizon is sufficiently distant in the general case if it is moved into the infinitely distant future. This

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1) Langholm (1964, p. 85). Langholm's definition is broader, but the definition given above suffices for the present discussion.

2) Langholm (1960, p. 455) and Langholm (1964, p. 19).

3) Langholm (1960, p. 458) and Langholm (1964, p. 53, 248, and 399). Other criteria are derived. These will not be applied here, as the criterion mentioned is simple, and yields conclusions that appear satisficing in practice.

4) Compare Danielsson (1963, p. 88), Hållsten (1966, p. 1), and Langholm (1960, p. 455).

is consequently assumed to be the case in the global model which will be discussed first. Smaller models with closer horizons will then be examined in order to establish whether the alternative horizons are sufficiently distant or not.

The ore deposit is mined during its production period. A certain part of the deposit is mined during the production period of the part in question. Only such parts of an ore deposit are discussed, which are mined one at a time. The entire deposit as well as each part is mined without interruption until the entire ore reserve of the deposit or the part of the deposit has been extracted. Thus, the production period is the period of time during which an ore deposit or a certain part of it is being exploited. The production period is delimited by the point of time when the mining of the relevant ore commences and the point of time when the mining of the ore in question ends. Unless otherwise stated, the production period is the production period of the entire ore deposit.

The ore reserve and the payments occurring as a consequence of the exploitation of the ore in a deposit have been assumed to be known with certainty (sections 23 and 32). The point of time when the ore reserve has been exhausted and the mine is being closed, i.e. the end of the production period, is in important respects an information horizon. Up to this moment the alternatives considered are fully defined in terms of the decision variables and the interdependencies between decisions at various points of time are determined by the ore-reserve model, the payment models, and the ore-reserve restriction. The payments are also determined as functions of the decision variables.

In addition, it is assumed that the decision variables, i.e. the rate of production, the average grade, etc., cease to have a direct influence upon subsequent decisions and events at the end of the production period. The assumption appears realistic as the ore reserve has been exhausted, the mine is closed and the haulage and sorting plants have been dismantled, sold, or transferred to other activities of the company, the personnel have been pensioned, dismissed, or transferred to other activities, etc. at this moment. The payments incurred in these close-down activities are explicitly taken into account in the payment functions[1].

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1) The close-down payments (section 1313 and section 41 of Appendix D). The capital value of activities beyond the end of the production period are included to a certain limited extent. This occurs, however, only if production factors are transferred to other activities within the company beyond the point of time mentioned. The capital values at the end of the production period of future payments due to the factors, are then included as "scrap values". As a consequence the information horizon is somewhat diffuse.

The period of time from the decision time to the end of the production period is the remaining production period. The capital value of payments during this period (close-down payments included) summarizes all the relevant consequences which occur during the period as a consequence of the decision. This capital value will be called the capital value of future mining. It is assumed to be discounted to the decision time at the given constant rate of interest.

The model horizon has been assumed to be infinitely distant. Then the activities beyond the information horizon (the end of the production period) must be explicitly taken into account in the optimization model. This can be done in the following manner. The rate of interest is assumed to equal the opportunity cost of money capital. Thus it is assumed that money capital can be obtained at the given rate of interest, and that excess money capital is placed at the given rate of interest within the model horizon[1]. The capital value at the decision time of future mining is discounted to the information horizon. This capital value is the amount of money capital which is to be used beyond the information horizon. It can be calculated for all alternatives under consideration. As the decision variables themselves have been assumed not to have any influence on events beyond the information horizon, all interdependencies between the period of time preceding the horizon and the time succeeding it, can be expressed in the point of time forming the information horizon and capital values discounted to this moment.

The capital value of activities before the information horizon (the end of the production period) is added to the capital value of those after the information horizon, all values being discounted to the information horizon. The sum of the capital values can then be discounted to some other common point of time, if desired. This is the optimization criterion of the global model used in evaluating smaller models with closer model horizons.

3323 Alternative model horizons

Decision models with closer horizons will be compared with the global model. It is first assumed that the model horizon of the smaller model is the information horizon in the alternative giving the longest production period, i.e.

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1) The assumptions are implicit in the use of a fixed rate of interest. See e.g. Johansson (1961, p. 14).

the most distant information horizon. The alternatives are those considered at the decision time. In all other alternatives the information horizon is closer. The activities in the time period between the information horizon and the model horizon, are in these alternatives assumed to give the capital-value increment 0. The capital values of the alternatives discounted to the model horizon are then obtained by multiplying their capital values at the decision time by a constant[1] which is equal for all alternatives. Thus, the amount of money capital which is to be used beyond the model horizon, is proportional to the capital values at the decision time of future mining in the alternatives considered. This relationship is the only route through which decisions before the model horizon can influence the activities beyond the horizon.

The activities beyond the fixed model horizon give a capital-value increment. The incremental capital value can equal 0 or either be positive or negative. It is not taken into account in the smaller model. In the global model, however, it is added to the capital calue of future mining, as seen from the decision time, discounted to the model horizon of the smaller model, i.e. it is added to the available money capital. Hence, in principle it affects the decision. It has been stated that the activities beyond the model horizon and consequently also the incremental capital value, are influenced by the decision only through the available money capital. If it is assumed that the incremental capital value increases, does not change, or decreases by less than one monetary unit for each additional monetary unit available at the model horizon, the sum of the available money capital and the incremental capital value increases. The sum decreases if the available money capital decreases.

The available money capital is the capital value at the decision time of future mining discounted to the model horizon. The sum of the capital values of the activities before and after the model horizon increases, on the assumptions made above, if the available money capital increases. Viewed from the decision time this signifies that the capital value of future mining and succeeding activities increases if the capital value of future mining increases, and decreases if the capital value of future mining decreases. The former capital value is the criterion of the global model. The latter capital value is the criterion of the smaller model, i.e. if the model horizon is the most distant information horizon among the alternatives considered.

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1) The compound amount factor for the period between the decision time and the model horizon.

The conclusion is thus that the optimal decision is found to be the same if the model horizon is placed at the most distant information horizon as if it is placed at a point infinitely distant in the future. The optimal decision is the same for all possible states at the closer model horizon on the assumptions specified above. Thus the closer model horizon discussed is sufficiently distant according to Langholm's criterion which was described in section 3321.

A second alternative model horizon can be discussed in the following way. The activities during the time between the information horizon and the model horizon have been assumed to yield an incremental capital value of 0. Hence, the capital value at the decision time, determined according to the previous model, does not change in any of the alternatives considered if the model horizon is put equal to the information horizon, i.e. if the model horizon is placed at the end of the production period of the ore deposit, instead of being fixed to the given point of time defining the previously discussed model horizon. The capital value discounted to the fixed point of time does not change either. On the assumptions given the end of the production period is consequently also a sufficiently distant model horizon.

A third model horizon to investigate is that determined as a fixed point of time preceding the most distant information horizon. In this case it is apparent that for the alternatives resulting in longer production periods some ore is left at the model horizon. The mining of this ore comes as a direct continuation of the activities before the model horizon[1]. Production factors obtained before the horizon, mining limits before the horizon, etc. may be of immediate importance to the activities beyond the model horizon.

If the model horizon is defined as for the present the ore-reserve restriction will not be effective in some of the alternatives considered. In other alternatives the restriction will be effective. The variations in the length of the production period will also be taken into account only partially. Thus, the optimization is made in such a way that the influence of the alternatives

1) In some special cases independence between activities before and after the model horizon (except for the available money capital and the incremental capital value obtained by means of the former) can be assumed, although the ore deposit is not exhausted at the model horizon. For example, the legal rights to the deposit may end at a given point of time, and they may be impossible to renew. From the point of view of the mining company this time determines the ore reserve. The point of time in question is then a sufficiently distant model horizon under the same conditions as those making the end of the production period a sufficiently distant model horizon.

on these factors are only partially taken into account. In the alternatives where the model horizon is closer than the information horizon this implies that the size of the ore reserve is unlimited as viewed within the time limit drawn by the model horizon.

If the ore reserve is considered infinitely large, as above, and thus inexhaustible, the optimum value of a decision variable is that which gives maximum capital value at the decision time of the activities before the infinitely distant model horizon. Assuming the same rate of interest, the annuity of the capital value calculated over the time between the decision and the horizon, has its maximum for the same value of the decision variable. Observing this, some consequences of the fixed model horizon in question can be deduced from some examples treated in the literature. They regard the optimum rate of production[1)] and the optimum mining limit[2)]. In the examples the change of the model horizon from the end of the production period to the closer fixed date resulted in a higher rate of production[3)] and a less extensive mining limit, i.e. a shorter production period. The conditions were not changed in other respects than those regarding the model horizon.

The conclusion is that the optimum is affected by the change to the third, closer, model horizon as the effects in the period of time between the second and the third horizon are not taken into account[4)], e.g. the effect that the length of

1) Gray (1913, pp. 473 and 475) and Carlisle (1954, p. 601). The rate of prodiction giving maximum annual profit is larger than that giving maximum capital value of the (remaining) ore. Thus, the annuity being a measure of annual profit, the fixed horizon in question implies a larger optimum rate of production.

2) Carlisle (1954, p. 606). The "level of recovery" giving maximum annual profit is less extensive than that giving maximum capital value of the (remaining) ore. Thus the closer horizon implies a less extensive mining limit (i.e. a higher average grade of the ore mined).

3) An illustration of this is also easily made by means of the example in sections 121-123 where the rate of production, according to Massé (1959, p. 350), is proportional to the ore reserve. If the ore reserve is not a restriction it is infinite from the point of view of the model, which would imply a rate of production approaching infinity.

4) The second model horizon was sufficiently distant on the assumptions given. It is used here as a basis of comparison instead of the global model. In other words, the model having the second model horizon is a global model in comparison with the model having the third model horizon.

the production period is influenoed. Consequently, a model horizon determined as a fixed point of time which precedes the information horizon, is not sufficiently distant for the main body of the decision problems studied[1)] (compare section 223).

A fourth horizon may be considered, where the horizon is the end of the production period of a part of the ore deposit, e.g. a zone. This means apparently that the size of the ore reserve is more or less arbitrarily determined. Considering Massé's conclusions (see section 124) this is equivalent to determining an arbitrary rate of production. In general this horizon would mean that the effects of decisions concerning the current zone on succeeding zones, are neglected. Thus e.g. the effects of the alternatives considered, which occur at the end of the production period of the deposit, are erroneously referred to the end of the production period of the part of the deposit. A change in the length of the production period is in itself such an effect. Optimizations of decisions which affect the production period are then dependent on the determination of this type of horizon. The conclusion is again that the horizon discussed here is not sufficiently distant for this study[2)].

3324 A sufficient model horizon

In summary it can be concluded that a model horizon coinciding with the end of the production period of the ore deposit is the closest horizon which is suf-

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1) To make decisions giving maximum annuity of the capital value at the decision time, when the annuity is calculated over the production period of an ore deposit containing a limited ore reserve, would yield similar nonoptimal decisions concerning identical decision problems. The reason is that the annuity criterion would imply a series of successively mined identical ore deposits, which would extend infinitely into the future. This would also imply an infinite ore reserve. The case can be compared with the chain of identical machine replacements treated in the traditional investment theory, e.g. by Preinreich (1940, p. 19), where the annuity of the capital value of the single project, calculated over the service life of the latter, is the criterion in determining the optimum chain of identical projects. The rate of interest being a given constant, this criterion is equivalent to the criterion of the capital value of the infinite replacement chain as the latter is obtained by multiplying the single-project capital value by the annuity factor divided by the constant rate of interest. The capital value of the infinite chain is used as the criterion by e.g. Schneider (1944, p. 84), Churchman, Ackoff, and Arnoff (1957, p. 485), and Massé (1959, p. 60).

2) An example of this case has previously been presented by the author (Norén 1967, Appendix 1). It clearly shows that the optimum strongly depends on the size of the arbitrarily selected part of the ore deposit (ibid. p. 1:24). However, it should be noted that the analysis is incomplete there, as only a special case has been treated, i.e. the value of the decision variable was assumed to be constant over the entire period considered.

ficiently distant according to the criterion stated in section 3321. The conclusion rests on some assumptions:

1) The decision variables cease to have a direct influence upon subsequent decisions and events at the end of the production period.

2) Activities between the end of the production period in the general case and the end of the production period in the alternative resulting in the longest production period among all considered alternatives, yield an incremental capital value of 0.

3) The capital value of activities beyond the most distant end of a production period (the same as in 2) above) increases, does not change, or decreases by less than one monetary unit if, at the point of time in question, one more monetary unit is placed in the activities after that time.

4) There is no time limit imposed on the mining company for the exploitation of the ore deposit.

The first assumption is a natural consequence of the fact that mines are usually abandoned when they have been exhausted. The second means that the capital invested[1)] is placed at the given rate of interest during the period of time adjacent to the period during which the consequences of the decisions have been assumed to be known. The third means that beyond the most distant model horizon marginal amounts invested may yield any return as long as it is better than if the marginal amount was given away or lost. The assumptions do not appear very restrictive from a practical point of view. The second assumption might be the most difficult one as it presupposes some precise knowledge of a time beyond the end of the production period of the ore deposit. If the fourth assumption is not fulfilled a fixed model horizon should be determined, which equals the time limit imposed.

The horizon discussed at present will be used in the models constructed in this study as it appears to be sufficiently distant to yield dependable optima at the same time as it is a natural information horizon. The optimization criterion can thus be defined more precisely as the <u>capital value at the decision time of the ore deposit</u> if the ore deposit is not yet being mined at the decision time, and the <u>capital value at the decision time of remaining ore</u> if the ore deposit is being mined. The two capital values are collectively named the <u>capital value</u>

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1) The capital value of remaining ore at the decision time discounted to the model horizon.

<u>of future mining</u>. The word "future" refers to the time after the decision time. The capital value is discounted to the decision time if no other date is indicated.

The horizon chosen is implied in several optimization models presented in the literature, e.g. Gray (1913), Hotelling (1931), Carlisle (1954), Billiet (1959), Massé (1959), and Henning (1963). Thus, the choice is very traditional.

It should be observed, however, that in special cases where the effects on the length of the production period and where other long-run effects after a given point of time, are not relevant for the decision, a closer horizon is appropriate. For example, the mining company may have obtained only a temporary legal right to exploit the deposit. Then a fixed model horizon is determined by this fact. The case of a fixed model horizon has been studied by Albach (1967).

The model horizon varies between alternative decisions considered at the same decision time. This is the case also in traditional investment theory if non-repetitive investment projects[1)] with different service lives constitute the available alternatives. The model horizon is then the end of the service life of the project. The decision criterion is the capital value of payments during this period[2)].

The model horizon coincides with the information horizon. It cannot be moved closer to the decision time without a considerable risk of errors. However, the future is known in less detail the more distant it is. Even if this is theoretically in conflict with the assumption that the future is known with certainty, it is a practically relevant observation. Thus, there is reason to consider how less detailed information concerning the more distant future can be introduced into the certainty model. The information horizon is thus resolved into subhorizons which are different for various types of information. These will be called <u>data horizons</u>.

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1) Including projects involving a finite number of repetitions of an investment, which have been treated by e.g. Preinreich (1940, pp. 15-16) and Schneider (1944, pp. 83-84).

2) The case has been treated by e.g. Preinreich (1940, pp. 12-15), Schneider (1944, pp. 74-83), Massé (1959, pp. 41-57) and Johansson (1961, pp. 16-22).

3325 Data horizons

The information horizon has been based on the technology of mining, i.e. it has been defined as the end of the production period of the ore deposit or the moment when the mine is finally closed due to the fact that the ore reserve is exhausted. However, as the ore reserve is an economic concept (section 21) the information horizon is not independent of economic facts and considerations.

The economic data used in the optimization model are the rate of interest which has been assumed to be a given constant and the coefficients of the payment functions. The coefficients[1)] determining the functions include factor and product prices as well as factor quantities. Thus, again there is no definite limit between economic and technological assumptions.

A practical problem is to find coefficients of the payment functions, which cover the period from time zero to the information and model horizon. These horizons are variables in the optimization models, which depend on the values of the decision variables. Thus, the coefficients of the payment functions must cover an indefinite period. To simplify the model it is assumed that the coefficients are known for all future occasions. Then infinity constitutes a most distant data horizon, i.e. the infinity data horizon, but only data concerning the time up to the model horizon are relevant for the model.

It is assumed that the information concerning the coefficients of the payment functions is successively less detailed for increasingly distant future times. Three periods with different degrees of detailed knowledge will be distinguished in the optimization models proposed in this study. The most detailed information is available for the first period which begins at time zero and ends at a point of time which will be called the data subhorizon. The data subhorizon is the end of a calendar year. Within the period all coefficients must be determined individually for each calendar year separately. The coeffioients of years 1 and 2 form the basis for the estimates of the coefficients. Those of the sucoeeding years up to the data subhorizon are also estimated individually, which assumes the same very high level of information as for years 1 and 2. The data subhorizon may occur at different times for different payment funotions, i.e. for different types of payments. Thus, there exists a set of data sub-

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1) These coefficients are also called parameters in the computer programs (compare PAR(JD,LT) in Appendix C).

horizons, which consists of one horizon for each payment function. The determination of the aotual dates of the data subhorizons is a part of the process of determining the coefficients themselves.

As the information becomes successively more difficult to obtain, the individual estimates are replaced by estimates of price-index numbers, one for each calendar year and payment function[1]. The other ooefficients are then kept unchanged from one year to another. The step from individual annual estimates of all coefficients determining a certain payment function to annual estimates of index numbers, is taken at the data subhorizon. Index numbers are specified annually up to a data horizon which is the end of the last calendar year for which such index numbers are obtained. The data horizon is equal for all payment functions. The period between a data subhorizon and the data horizon forms a second period for which the information concerning the coefficients is less detailed than for the first period, i.e. for the time preceding the data subhorizon.

A third period for which the information is still less detailed begins at the data horizon and extends to the infinity data horizon. For this period it is assumed that the coefficients, including the price-index numbers, of the data-horizon year form the best available estimate.

The data horizon and subhorizons have been established as a result of the decrease in knowledge as the distance in time between a decision and its consequences increases. The long-range forecast is assumed to be less detailed than the short-range forecast. The data horizon and subhorizons can also be determined from another point of view, namely the degree of approximation in the optima obtained, which the decision maker desires. Closer data horizons may be accepted for a rough estimate than for a more exact one. To what extent a given set of data horizons results in erroneous suboptimizations depends on circumstances in the particular cases, e.g. the rate of interest, the alternatives available, the values of parameters and coefficients in the optimization model, etc. For this reason the problem will not be treated here. The data horizons are instead regarded as factors to be examined in a sensitivity analysis.

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1) Compare sections 42 to 49 of Appendix D.

34 Interdependencies and static versus dynamic optimization models

341 Static versus dynamic optimization models

Statics is usually opposed to dynamics in economic theory, but the meaning of the terms is not unanimously agreed on. Baumol (1959, pp. 3 ff.) surveys a number of definitions used in various contexts. The distinction between economic dynamics and statics is in his own definitions that economic phenomena are studied in relation to preceding and succeeding events in the former case but not in the latter[1]. Comparative statics is an application of economic statics, which lies between statics and dynamics. Here, the situation immediately before a change in one or more of the conditions determining the situation, is compared with the situation immediately after the change[2]. A form of dynamic analysis is period analysis, where the events of one period of time are explained by events in preceding periods, external changes being allowed for[3]. The lengths of the periods are defined by the requirement that they must be so short that the plans existing at the beginning of a period are not changed during the period[4]. A period analysis comprising only one period is a special case, where the analysis is static in the sense defined above.

The principal ideas in the period analysis are useful in formalizing the discussion of the decision problems of this study. If committing[5] decisions concerning all relevant decision variables are made at one single point of time, the analysis of the decision problems is static as only one period of time is involved. The length of the period is the time elapsed between the decision time and the model horizon, i.e. the remaining production period[6]. The corresponding optimization model is <u>a static optimization model</u>. It is characterized by having only one decision time explicitly taken into account.

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1) Baumol (1959, pp. 4-5).

2) Ibid. pp. 4 and 118.

3) Ibid. p. 128.

4) Ibid. p. 134.

5) Committing is here used in the sense that a decision, by definition, binds the decision maker to the consequences of the decision, and that the decision cannot be changed once it has been made, except by making a new decision at a later point of time. Compare e.g. Drucker (1959, p. 239).

6) Note that this period may vary as the value of the decision variable varies (compare section 3323).

Decisions concerning the relevant decision variables can be made at different points of time. If the decisions are interdependent the analysis of the decision problems is dynamic, and is an instance of period analysis. The length of the period is the interval between the decisions. Depending on the decision variable examined, the length of the period may vary as the value of the decision variable varies, e.g. if the decisions are concerned with only a specific part of the ore deposit and influence its production period[1], or be a constant number of years. The corresponding optimization model is a dynamic optimization model. It is characterized by having more than one decision time explicitly taken into account. The decisions at the different decision times are interdependent[2].

A special type of decision problem involving more than one period is that which concerns independent decisions only. In that case the problems need not be studied in relation to one another. They are adequately treated in separate optimizations where a static optimization model is applied to the decision problem of each decision time.

Interdependencies between decisions have been found important in the choice of optimization models to be built. Before they are discussed, however, a few words will be said about dynamic programming, which is a mathematical theory especially designed for the treatment of interdependent decisions. The theory affects the model building.

342 Dynamic programming

Dynamic programming is the mathematical theory of multi-stage decision processes, i.e. sequence of decisions[3]. To a decision problem involving a sequence of decisions the solution obtained is an optimal policy, i.e. a sequence of optimum values of the decision variables[4]. The meaning of an optimal policy is specified through the principle of optimality which is interpreted in this study in the following way:

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1) Compare the production periods of zones and subzones discussed in Chapter 1.

2) A decision is assumed to be irrevocable in the static optimization model as it can be changed only by making a new decision, and only one decision time is involved. In the dynamic optimization model a decision is not irrevocable but assumed to be committing for the time being until the end of the period, i.e. until a new decision time has been reached.

3) Bellman (1957, p. vii).

4) Ibid. p. 17. Bellman also gives a more general definition (ibid. p. 82). Note also that the set of optimum values of the decision variables does not need to be unique (ibid. p. 85).

N decisions are to be made for N successive periods[1] in the order 1, 2, ..., N. The initial state, i.e. the state prior to decision 1, is given. It provides the background of decision 1. As a result of decision 1 the state at the end of period 1 has changed. The new state constitutes the background of decision 2. As a result of this decision a new state is obtained at the end of period 2, which forms the background of decision 3, and so on for the whole sequence of decisions. An optimal policy has the property that each decision is optimal against the background formed by the state resulting from preceding decisions, and on the assumption that the succeeding decisions are also optimal in this sense, i.e. the remaining decisions also form an optimal policy as seen from the end of the current period[2].

The principle of optimality is fundamental in dynamic programming. It leads up to a mathematical formulation of the decision problem, which is a second principal feature of dynamic programming, i.e. the recurrence relation (ibid. pp. 83 ff.). In order to demonstrate the latter some symbols are needed:

B_n	Capital value at the beginning of period[3] n of the activities during the periods n, n+1, n+2, ..., N.
B'_n	Capital value at the beginning of period n of the activities during the period as a function of the state at the beginning of the period and the relevant decision variable (Q_n), the value of which is determined through a decision at the beginning of the period. B'_n is independent of Q_i for $i=n+1, n+2, \dots, N$.
e	Base of natural logarithm.
$e^{-j \cdot T_n}$	Present-value factor for T_n years at the continuous rate of interest j.

1) Period as defined in section 341, i.e. the interval between decisions, thus a period of time during which the plans existing at the beginning of the period are not changed.

2) This should be compared with Bellman's (1957, p. 83) definition of the principle of optimality, where only the step from the initial state to the state at the end of the first period and the intermediate first decision, are explicitly treated (although it is prescribed that the remaining decisions must form an optimal policy). The objective of his analysis is to determine an optimal policy in the sense that it gives maximum N-stage (N-period) return (ibid. pp. 7 and 84). Then dynamic programming can be applied to other problems involving interdependent decisions than the problems involving period analysis discussed above (ibid. p. xi). The difference will be discussed later.

3) Note that the subscripts of the zones are designated by n and N elsewhere in this study. They can be interpreted as periods. Thus the common notation.

j	Continuous rate of interest (100·j %).
$\underset{Q_{n...N}}{\text{Maximum}}\ B_n$	Capital value B_n if the values of $Q_n, Q_{n+1}, \ldots, Q_N$ constitute an optimal policy.
N	Total number of periods. The periods appear in the order 1, 2, ..., N.
Q_n	Decision variable (e.g. rate of production) in period n.
T_n	Length of period n.

The decision problems of this study can then be formulated as in the following example where only one decision variable is determined at each decision time, i.e. at the beginning of each period. The optimization model is then the following recurrence relation:

$$\underset{Q_{n...N}}{\text{Maximum}}\ B_n = B'_n + \underset{Q_{n+1...N}}{\text{Maximum}}\ B_{n+1} \cdot e^{-j \cdot T_n} \tag{3.1}$$

where $n=1,2,\ldots,N$ and $B_{N+1}=0$.

Other restrictions determine T_n and influence B'_n but these need not be discussed at the present moment (see e.g. section 32).

To suit this optimization model the capital-value model is also written as a recurrence relation. Thus, for $n=1,2,\ldots,N$:

$$B_i = B'_i + B_{i+1} \cdot e^{-j \cdot T_i} \tag{3.2}$$

where $i=n,n+1,\ldots,N$ and $B_{N+1}=0$.

Other restrictions exist, but need not be discussed here.

The principle of optimality and the recurrence relations are used in determining optimum decisions at each decision time, i.e. at the beginning of each period. This is not always the same as to determine the combination of decisions which leads to maximum capital value at the beginning of the first period. The optimization problem can instead be formulated thus: Determine the combination of decisions which leads to maximum capital value at the beginning of the first period, subject to the restriction that each subsequent decision is optimal from the point of view of the current decision time. The restriction means that the second and following decisions shall be a combination leading to maximum capital value at the beginning of the second period (the first decision being given), subject to the restriction in question, that the third and following decisions shall be a combination leading to maximum capital value at the beginning of the third period (the first two decisions being given), subject to the restriction, etc. The restriction will be referred to as the <u>restriction in the principle of optimality</u>.

There is often no conflict between the optimization subject to the restriction described and an optimization without the restriction, i.e. to determine the combination of decisions giving maximum capital value at the beginning of the first period. The restriction exerts influence only where the optimum decision for a period n, as determined at the beginning of period n, differs from the optimum decision for period n in conjunction with decisions for preceding periods, the combined optimum being determined at the beginning of a period preceding n. Such a situation is conceivable, however, and an example will be given, although it might seem to be somewhat farfetched[1]. An optimization regarding two periods n=N and n=N-1, is considered. The length of each period, i.e. T_n, is a decision variable at each decision time T_n, i.e. the beginning of each period. Hence, the decision times T_i for $i=n+1,n+2,\ldots,N$ are variables too, which depend on the decision at time T_n. Payments for reinvestments and other costs for maintaining equipment in a useful condition (in short, reinvestments) are important for the decisions. The equipment is assumed not to need any reinvestments during the last years before time T_{N+1}, i.e. the end of period N. No adverse effects are expected from stopping the reinvestments during these years.

In the planning of the mining company it is assumed that a specific reinvestment policy is adopted: At any decision time the company cannot count on more than five years without reinvestments. The time for stopping reinvestments is determined at the beginning of the period during which the five-year limit is

1) The example is obtained as a result of a simplification made concerning the reinvestments in the mining company in the payment models of section 44 in Appendix D. There it is assumed that the reinvestments decrease linearly with the remaining production period after a point of time when the remaining production period has reached a certain given value ($c_{21,a}$). If less than $c_{21,a}$ years remain at a given decision time, the remaining production period can be varied freely, but the effect of such variations on reinvestments made before the current decision time, is ignored. Thus, the discussion of the example is also a discussion of assumptions and consequences of the simplification. The simplification might be inconsistent with rational decision making, and be an impracticable policy, but these aspects will be neglected here.

reached. If less than five years remain to time T_{N+1}, a decision increasing the number of the remaining reinvestment-free years up to five years is accepted although the decision might increase the total number above five. In addition it is assumed that $T_{N-1} < (T_{N+1}-5) < T_N$. The recurrence relation is applied, which gives the situation shown in Fig. 3:3 (rate of interest 0%).

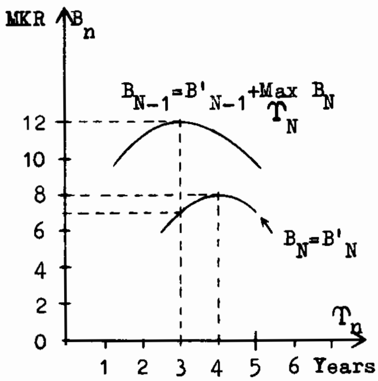

Fig. 3:3 Capital values at decision times T_{N-1} and T_N as functions of $\mathcal{T}_{N-1}$ and $\mathcal{T}_N$, respectively. $\max_{\mathcal{T}_N} B_N=8$.

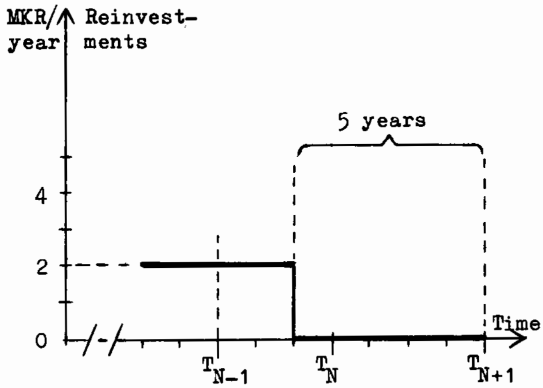

Fig. 3:4 Decision times and reinvestment policy according to the optima obtained from Fig. 3:3.

The optimum decision at time T_N is to make $\mathcal{T}_N=4$ years. Reinvestments are not made during this period. One reinvestment-free year remains for period N-1. The optimum decision at time T_{N-1} is to make $\mathcal{T}_{N-1}=3$ years. At the same time it is determined that reinvestments should be made during the first two years only. The decisions are illustrated in Fig. 3:4.

The situation will change if all decisions are assumed to be made simultaneously at time T_{N-1}. It can be decided that $\mathcal{T}_N=3$, which decreases B_N by 1 MKR (Fig. 3:3). However, this also makes it possible to decide that the reinvestments should be stopped one year earlier. According to Fig. 3:4 the reinvestments amount to 2 MKR/year. If it is assumed that no other effects will occur, the result is an increase in B'_{N-1}, and thus also in B_{N-1}, by 2 MKR. Consequently, B_{N-1} increases by 2-1=1 MKR to 13 MKR owing to the changes in comparison with the optimum of Fig. 3:4. Similar conclusions can be reached making other assumptions concerning the reinvestments, e.g. that they are being stepped down successively during the given number of years according to some given rule, instead of being stopped altogether.

A combination of values of the decision variables can thus be found, which yields a higher capital value at the beginning of period N-1 than do the optimum decisions at each separate decision time according to the recurrence relation. In spite of this the latter optimum is preferred in the optimization model because a deviation from it would imply that the decision maker acting at time T_{N-1} would make the decision for the decision maker aoting at time T_N, and have some means of enforcing his decision. This is assumed not to be the case.

A fundamental assumption for the principle of optimality in period analysis, as it is formulated here, can be stated against the background of the preceding discussion. A decision maker optimizes his decisions as the decision problems present themselves at the decision time on the assumption that future decision makers will do likewise. He does not consider what would have been better for a preceding decision maker if the latter could have trusted him to act in his favour[1].

The solution to the dynamic problem posed can be obtained by successive approximations as described by Bellman (1957, pp. 9 and 88-89), observing the implications of the interpretation of the principle of optimality. The first step, i.e. the initial approximation, is to determine the optimum decision in period N as a function of the variables defining the state at the beginning of period N (time T_N):

$$\underset{Q_N}{\text{Maximum}}\ B_N = B'_N \tag{3.3}$$

The second step is to determine the optimum decisions in the periods N-1 and N as a function of the variables defining the state at the beginning of period N-1 (time T_{N-1}). As the latter variables, together with Q_{N-1}, determine the

1) It follows that the dynamic programming formulation discussed here is not applicable in determining optimum values of decision variables which are determined simultaneously, i.e. at one single decision time. In this case the values of the decision variables giving the maximum capital value at time T_1 are desired.

state at time T_N, the optimum value of Q_N is determined through the first step, i.e. through (3.3). Thus, the result of the first step is used in the second[1]:

$$\underset{Q_{N-1\ldots N}}{\text{Maximum}}\ B_{N-1} = B'_{N-1} + \underset{Q_N}{\text{Maximum}}\ B_N \cdot e^{-j \cdot T_N} \tag{3.4}$$

The <u>third step</u> is to determine the optimum decisions in the periods N-2, N-1, and N as a function of the variables defining the state at the beginning of period N-2 (time T_{N-2}). As in step two the result of the preceding step is used to determine optima in the succeeding periods, i.e. optimum values of Q_{N-1} and Q_N. A <u>fourth</u>, a <u>fifth</u>, etc. step are taken similarly, the recurrence relation (3.1) being the general form of the functional relations (3.3) and (3.4), until the N:th step.

In the <u>N:th step</u> the state at the beginning of the period (time T_1) is given (see above). Hence, the optimal policy is determined when the optimum decision in period 1, i.e. the optimum value of Q_1, has been determined for this state. As before the result of the preceding step, i.e. step N-1, is used to determine optima in the succeeding periods, i.e. the optimum values of Q_2, Q_3, ..., Q_N.

343 <u>Interdependencies in the general model of the decision problem</u>

Two types of interdependencies between decision variables have been mentioned: those between simultaneously determined variables and those between decisions at different points of time. Both types appear together in the decision problems treated in this study. The interdependencies are numerous and complicated. They vary in form and importance from one deposit to another. For these reasons it would be impracticable to give a complete description of all conceivable interdependencies. Thus, only some examples will be discussed in this exposition.

A variable can influence another variable directly or indirectly. Direct influence occurs where the value of one variable affects the value of the other

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1) The reformulation of the principle of optimality implies that the term $\underset{Q_N}{\text{Maximum}}\ B_N \cdot e^{-j \cdot T_N}$ and the corresponding term in the more general formulation (3.1) are not necessarily absolute maxima. They are restricted maxima which merely indicate the capital values of activities after times T_N and T_{n+1}, respectively, if the decisions at these points of time are optimal.

without intermediate optimizations, e.g. as the rate of production influences payments received for products sold or the length of the production period. Indirect influence occurs where the value of one variable influences the value of another as a consequence of intermediate optimizations, e.g. where the prices of the final products influence the optimum rate of production or the remaining ore reserve influences the optimum mining limit, the optimum rate of production, etc. The direct influences form the structure of the capital-value model and the restrictions on the optimization.

Simultaneous decisions concerning different decision variables are to a great extent interdependent. For instance, the mining limit directly influences the size of the ore reserve. At the same time the optimum mining limit may be indirectly influenced by the remaining production period at the decision time, which depends on the remaining ore reserve[1]. Another example is that for a given ore reserve the production period is determined by the rate of production. The optimum rate of production is indirectly influenced by the size of the ore reserve[2] and occasionally by the production period[3]. The payments are directly influenced by all decision variables. At the same time the optima are indirectly influenced by the payment functions, i.e. the payments. The above examples and further relations are illustrated in Fig. 3:5. Compare also section 223, where interdependencies are discussed in footnotes to points 4) and 5).

Direct interdependencies between decisions at different times are to a great extent a result of the technology of the ore-extraction process. Production capacity[4] obtained after a decision in one period remains available for utilization in subsequent periods in so far as it is not confined to a single period[5]. In order to change the capacity, costs (investments) are incurred. The mining

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1) The influence of the remaining production period on the optimum mining limit is shown by e.g. Henning (1963, p. 57).

2) Massé (1959, p. 350). Compare section 124.

3) Hotelling (1931, p. 164), especially in cases with small fixed investments.

4) The measure of capacity used in this study is the annual rate of production. The maximum rate, or the capacity, is here defined as a long-range average rate where certain variations in the utilization of the fixed factors of production, are smoothed out. The variations smoothed out are those caused by circumstances which are not controlled by the decision maker, i.e. circumstances not constituting decision variables in the optimization models. For a discussion of fixed production factors and capacity see e.g. Frisch (1965, p. 15) and Danø (1966, pp. 8 ff.).

5) An example of production factors used only during one period are those obtained in connection with the zone investments (section 1312).

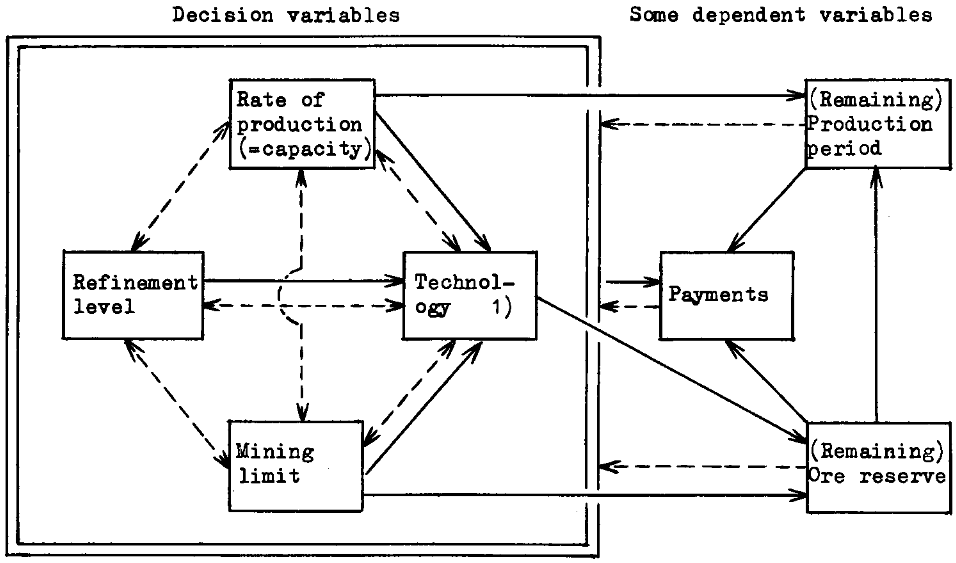

Fig. 3:5 Some of the most important interrelations between different variables determined simultaneously.
———> Direct influence - - - -> Indirect influence

limits may determine the utilization of certain plant or the shape of a main haulage level, etc. Thus, in order to change a mining limit, investments might be necessary to adapt the mine. The technology chosen and the refinement level can also often be changed if certain investments are made. Consequently, decisions at one time directly influence payments in later periods when changes are to be made. Furthermore, the remaining ore reserve is at any time directly influenced by the rates of production and the mining limits in all preceding periods, and by the mining limits in all subsequent periods.

Indirectly, by itself and by way of its direct influence on the remaining production period, the remaining ore reserve influences the optimum values of the decision variables. Also, the payments exert indirect influence upon the optima (Fig. 3.5). These indirect influences lead to indirect influences over time. For example, the choice of a rate of production during one period (the original

1) Only certain technological decisions influence the ore reserve directly. See e.g. the discussion of ore losses in section 21 and the footnote to point 4 in section 223.

decision) directly influences future payments (for changing the rate of production) which indirectly influence the optimum rates of future periods. The latter, again, indirectly influence the remaining production period at the original decision time, thus indirectly influencing the original decision.

Similar chains of interdependencies could be described for most decision variables and combinations of decision variables. Instead of this, however, simplified assumptions concerning the interdependencies will be discussed in order to make the decision problems manageable. It is apparent, however, that the decision problems discussed in this study involve interdependent decisions at different decision times. Hence a dynamic optimization model is relevant (section 341), and the theory of dynamic programming is applicable (section 342).

344 Decisions and interdependencies in the dynamic optimization models

3441 Elimination of simultaneous decisions

Two factors of importance in determining which interdependencies should be taken into account and how this should be done can be deduced from the discussion in the previous sections, namely the decision variables determined simultaneously, i.e. at the same decision time, and when decisions are made, i.e. the decision times themselves or, in the vocabulary of period analysis, the length of the periods. Regarding the first factor it is assumed that only one decision problem exists so that only one decision variable has to be determined for each period. The problem of simultaneous decisions will then be reopened at a later stage of the discussion. Thus, optimization models will be constructed on the assumption that all decision problems except one have already been solved. The solutions assumed are incorporated into the parameters and coefficients of the optimization models[1].

3442 Rate of production and average grade as decision variables in two principal models

The simplifying assumption leads to the problem of determining which decision variables should be selected for explicit treatment. It has been stated

1) It has already been assumed that only one ore deposit is involved (section 32). Influences from other deposits of the mining company are consequently not included. However, in principle such influences do not differ from other interdependencies. They can thus be treated in the same manner.

(section 223) that the interest was primarily to be concentrated on decision problems influencing the size of the ore reserve and the length of the production period. According to Fig. 3:5 the rate of production and the mining limits are such factors. Hence a model will be constructed for optimizing the rates of production during a set of periods. The mining limits are assumed to have been determined beforehand. Another model will be constructed for determining the optimum mining limits during a set of periods. There the rates of production are assumed to have been determined in advance. In order to simplify the latter model the average grade of the ore mined will be used as a decision variable for each period, as this is a continuous variable (section 223). The average grade was defined in section 11 (Fig. 1:3)[1].

As another simplification the two optimization models thus constructed or the capital-value models included in the optimization models will be used in determining the optimum values of other decision variables, including the refinement level, other forms of mining limits than that measured in average grade (sections 11 and 14), and the technology. According to Fig. 3:5 the latter two groups of decision problems may have a direct influence on the ore reserve. Consequently, it is especially important that the models can be used also for these problems without distorting the results.

3443 Decision times

The other factor of interest in discussing interdependencies is the lengths of the periods or the decision times (section 3441). They depend on the problem to be solved. The model horizon determines the maximum length of a period. The period or periods must also cover the whole of the time within the model horizon. The technology of the mining described in sections 11 and 1311 suggests that the production periods of the respective zones of the ore deposit are employed as periods in the period analysis that aims at determining the optimum rates of production. Together, the production periods of all zones form the production period of the ore deposit, the end of which constitutes the model horizon.

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1) Alternatively, the marginal grade could be used as a decision variable. A certain stability over time in the quality of the final products of the mine is assumed to be desirable. For this reason it is assumed that the short-run production planning aims at smoothing out fluctuations in the average grade of the ore produced, so that the average grade is constant during the production period of each subzone. The mine, the plant, etc. are assumed to be adapted to the average grade. Then the revenues and a considerable part of the costs of production depend more directly on the average grade than on the marginal grade. For this reason the consequences of the decision are assumed to be more easily described in terms of the average grade, and the latter is selected as a decision variable.

The period has been defined as a period of time during which the plans existing at the beginning of the period are not changed (section 341), i.e. the interval of time between decisions. If it is assumed that a change in the rate of production is made only after a decision and that the decision is implemented without loss of time, i.e. that the change occurs instantaneously at the decision time, the plan existing at the beginning of the period must be to operate at a constant rate during the period. Furthermore, it is assumed that the rate of production equals the production capacity (defined in section 343). The assumption of a constant rate of production during the production period, which equals the capacity, is reasonable against the background of Billiet's and Massé's analysis of the static (single-period) case[1]. It should be kept in mind that this is only an assumption made to simplify the model. Billiet (ibid.) discusses the conditions which must be fulfilled if the assumption is to represent an optimum[2].

The periods of the optimization model used in determining optimum average grades can be chosen in a similar way. There is, however, one important difference. In comparison with the rate of production the average grade is often very easily changed. Instead of investments in new machinery and equipment, minor adjustments may suffice. Then the reasons for keeping the average grade constant over long periods of time decrease in importance. Consequently, the lengths of the periods should be shorter. This is accomplished by subdividing the zones into subzones and using the production periods of the subzones as periods in the period analysis (section 11).

As a simplification it is assumed that the average grade is constant during the period and that any change in the average grade is preceded by a decision which is then implemented instantaneously at the decision time. Whether it is optimal to keep the average grade constant during the production period depends on the technological and economic conditions of each individual decision problem. In some cases the average grades obtained may have to be regarded as discontinuous

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1) Billiet (1959, pp. 22-24) and Massé (1959, pp. 348 ff.). See also section 12, where Massé's model and some of his conclusions are described.

2) The conditions are fulfilled in the special case discussed in section 12. The exact formulation of the conditions will not be repeated here. It may be enough to describe the conditions as restrictions on the variations over time of the payments involved in the optimization model (the ore reserve of the deposit is assumed to be constant).

approximations of a continuously changing average grade[1]. On the other hand, such conditions can also prevail that changes are not easily made. This does not involve any special problems as the optimum decisions will then be to choose the same average grades for two or more successive periods.

In summary, the periods of the dynamic optimization model for determining optimum rates of production are the production periods of the zones. The rate of production is constant during this period, and determined through a decision at the beginning of the period, i.e. the decision time is the starting time of the zone. The periods of the dynamic optimization model for determining optimum average grades are the production periods of the subzones. The average grade is constant during this period, and determined through a decision at the beginning of the period, i.e. the decision time is the starting time of the subzone. For simplicity it is also assumed that each zone is subdivided into the same number of subzones. It follows from the definitions that the starting time of a zone always coincides with the starting time of a subzone (the first subzone of the zone). It is also assumed that the partitioning of the ore deposit into zones and subzones is given and constant. A measure of the size of a zone or a subzone satisfying this condition will be discussed later on.

3444 The actual decision and future decisions

Only the first decision in a sequence of decisions involved in the dynamic optimization model, is actually implemented. This is an important facet of all long-range planning, which has been pointed out by e.g. Drucker (1959, p. 239), who emphasizes that decisions are made only in the present. Other decisions are nothing but intentions. Consequently, the actual decisions concern the rate of production in the first zone[2] considered in one model and the average grade in the first subzone considered in the other model. If the ore deposit is not previously mined at the actual decision time (the time of the first decision) the actual decision concerns the first zone or subzone, respectively, of the deposit. If the deposit is already being mined at the actual decision time the actual decision concerns the next zone or subzone, respectively, for which the decision variable is still a variable and not a constant determined in a previous decision.

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1) Another approximation has previously been suggested by Henning (1963): Annual decisions. A model based on this assumption has also been discussed by the author (Norén 1967, pp. 160–173). The present approximation has been chosen for this study because it permits a simple solution which applies in situations where decisions for the subzones are relevant as well (section 56).

2) The zones and subzones are mined successively in a predetermined order (sections 11 and 1421).

In a dynamic optimization model decisions at the beginning of each period are optimized. All decisions except the actual decision are future decisions as viewed from the actual decision time. The future decisions are only predictions of what will actually be determined in the future, made in order to provide predictions of the consequences of the actual decision when its influence on future decisions is taken into account. Of course, as the future has been assumed to be known with certainty, these predictions are also assumed to come true. However, if the certainty model is viewed as a simplified representation of a situation involving risk or uncertainty, no contradiction arises if it is also stated that when, after some time, the decisions that were originally future decisions become actual decisions, these decisions will probably turn out to differ from the predictions. It follows that the optimizations made in connection with the actual decision do not provide optima to be used in future decisions. New optimizations have to be made then, which utilize more up-to-date information.

However, in the discussion of the dynamic optimization models the future decisions will be treated as if they were actually implemented. The first decision, i.e. the actual decision, is assumed to be made at time zero[1]. The subsequent decisions (the <u>current decisions</u>) are then made at the <u>current decision times</u>, i.e. the starting times of new zones or subzones, respectively, depending on whether optimum rates of production or optimum average grades are being determined.

3445 Simplified intertemporal interdependencies

Interdependencies between simultaneous decisions have been assumed to be incorporated into the parameters and coefficients of the optimization models, all decisions except one at each decision time having been assumed to be made in advance. Also the interdependencies between decisions at different decision times have to be simplified if the optimization models are to be reasonably simple.

In the optimization model for determining the optimum rates of production the decision times are the starting times of the zones. The date of the starting

1) Time zero is replaced by time 1 (the end of the first year) in the models of the computer programs. All other decision times are consequently dated by the actual time plus one year. This complication will not be discussed here. Compare section 3 of Appendix D.

time summarizes the rates of production of all previously mined zones as all factors determining their ore reserves, e.g. the mining limits, are given and thus also the ore reserves of the zones. For this reason and in order to take purely temporal variations in consequences into account, the decision time itself is used as one of the variables defining the intertemporal relations.

The ore reserve of each zone is given. The remaining ore reserve at any decision time is often of importance for the optimum. It is known if the subscript number of the zone for which the optimization is currently being made (the current zone), is known. Hence the subscript number of the current zone is used as a variable defining intertemporal relations.

The production capacity in works, equipment, service facilities, etc. which are common to all zones (see sections 11 and 1312), is assumed to equal the rate of production. The common production factors are thus assumed to be adapted to the rate of production in the zone currently mined at any time. To achieve this, a change in the rate of production is assumed to cause an expansion or a contraction investment (section 1312). Hence, the rate of production in the previously mined zone defines the state of the existing common production factors at any decision time.

In the optimization models for determining the optimum average grades a corresponding trio of variables is selected, i.e. the decision time or the starting time of the current subzone (the subzone for which the optimization is currently being made), the subscript numbers of the current zone and subzone, and the average grade in the previously mined subzone.

The cost of hoisting as well as other costs of mining and transporting the ore from the working face to the surface (see Fig. 1:1) depend on the level of the zone currently mined. For simplicity it is assumed that this dependence is a function of the remaining ore reserve after the current zone has been exhausted[1] or the subscript number of the zone itself. Both are defined by means of the subscript number. The latter can also be used to define other properties of the ore deposit or the mining technology which are peculiar to individual zones. Some related detail problems are treated in section 633 (varying ore reserve per metre or foot vertical distance).

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1) More exactly, by the cumulative equivalent ore reserve of the first subzone of the zone mined immediately after the current zone. See RES(NS,N_n) in section 222 of Appendix B and assumptions 7) and 39) (the expression $\sum_{i=1} R_i$) in section 42 below.

In both models the following state variables[1] are thus assumed to fully specify the state at the decision time, i.e. at the starting time of the current zone or subzone:

1) The starting time of the current zone or subzone.
2) The subscript number of the current zone and subzone. If the rate of production is being determined, the current subzone is always subzone 1.
3) The rate of production in the zone preceding the current zone.
4) The average grade in the subzone preceding the current subzone.

The reinvestments form a further intertemporal interdependence as they depend on the remaining production period of the ore deposit (section 342). The rates of production and average grades of the zones and subzones, respectively, which are mined after the current zone or subzone, also constitute intertemporal interrelations as they determine the remaining production period and the capital value of future mining, both influencing the current optimization.

35 Model testing

The dynamic optimization models are normative[2] in their uses, i.e. they are used in order to deduce which alternative from among a set of alternative actions is optimal according to some given criterion. It is not relevant to test such a model against reality with respect to its normative functions, i.e. to compare the prescribed behaviour with the actual behaviour of the decision maker.

The optimization models, however, also contain important elements of description and prediction. They describe the ore deposit, the mining operations, and the economic consequences of alternative actions. The descriptions mostly refer to future states and actions, and are then predictions. From this point of view the models can in principle be tested. How accurately do the models describe the situation at the actual decision time, and how accurately do the models predict relevant future states and effects resulting from alternative actions in the present? These two questions should be answered, but there are certain difficulties involved, which prevent it from being done in this study. The models are being discussed on a more general level, especially with regard to their assumptions, instead of being rigorously tested.

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1) Compare e.g. Bellman (1957, p. 81).
2) Normative in the sense the word is used by e.g. Ackoff (1962, p. 31).

Some of the difficulties encountered if the models are to be tested systematically should be mentioned[1]. One is that the models refer to ore deposits in general and that they in actual applications are intended to be adapted to the specific situation, mainly by the selection of data, but also in some respects by changes in the structure of the models[2]. Accuracy in one application is not an indicator of accuracy in another application. A second difficulty arises as a consequence of the distance of the model horizon. The production period of an ore deposit is usually at least 10 to 20 years[3], and sometimes 50, or 100 years, or more. The final test of predictions cannot be made until the time for which the prediction is made, has been reached.

Both difficulties mentioned indicate that the testing of the optimization models preferably is postponed and made by actual users of the models[4]. For this reason the discussion on a more general level mentioned above may be considered enough for this study.

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1) Methods of model testing are described and discussed by e.g. Churchman, Ackoff, and Arnoff (1957, pp. 577 ff.) and Ackoff (1962, pp. 392 ff.).

2) Compare section 1313.

3) Allais (1957, pp. 292 and 321) uses 25 years as a normal minimum production period.

4) A sensitivity analysis can be used to locate crucial points in the model, thus reducing the drawbacks of not having a well-tested model. It can also be used to determine permissible sizes of possible errors, or to determine the effect of errors of given sizes. The use of sensitivity analysis where tests are impracticable is suggested by e.g. Churchman, Ackoff, and Arnoff (1957, pp. 590–591).

CHAPTER 4

4 Capital-value model and the two principal optimization models

41 Introduction

It has been stated in Chapter 3 that two main optimization models are to be constructed, one for determining optimum rates of production in the zones of a single ore deposit and one for determining optimum average grades in the subzones of the deposit. Dynamic optimization models making use of dynamic programming are suitable in solving the optimization problems discussed. A major formal submodel of the optimization model is the capital-value model, which will be constructed in this chapter.

The decision problems of the two main optimization models are interdependent. Although it has been declared that the two problems will be optimized separately at first, a joint optimization will be prepared from the beginning. For this reason a common capital-value model will be made where the capital value at any decision time of future mining is expressed as a function of both sets of decision variables, as well as of the variables specifying the state at the decision time with respect to both decisions. The two sets of decision variables are the sequence of rates of production in future zones and the sequence of average grades in future subzones.

As a starting point in the construction of the capital-value model the assumptions will be specified. Two submodels of the capital-value model will then be disoussed before the latter is finally obtained. In addition, the two dynamic optimization models will be formulated as problems of dynamic programming.

Optimization models of ore deposits have been constructed before this. They have frequently been referred to in this study, and some of the more interesting models found in the literature will finally be discussed and compared with the proposed models.

42 Assumptions

Assumptions concerning the decision problems studied have been made in Chapters 2 and 3. They will be enumerated[1] here together with a number of new assump-

1) For each assumption which has already been treated the section where the assumption is originally discussed is indicated.

tions on which the two optimization models rest. It should be noted that the assumptions refer to the optimization models, including the submodels first discussed, and not only to the latter.

1) The models are constructed and applied to decision problems as seen from the point of view of an idealized decision maker, i.e. the mining company. (Sections 221 and 224.)

2) The mining company has one single ore deposit at its disposal. (Section 32.)

3) Ore prospecting for new deposits does not take place, or is determined independently of the ore deposit of 2). If the prospecting results in new deposits they are assumed to be independent of the present deposit, and the latter is assumed to be independent of the new deposits. This is an interpretation of an assumption made in section 223, namely that problems connected with ore prospecting are excluded from the study.

4) The ore deposit is of one of the general types described in section 11, especially Fig. 1:1, Fig. 1:2, and Fig. 1:3. Thus, the ore reserve is known with certainty and it is determined as a single-valued function of all relevant mining limits. It includes proved, probable, and possible ore. The latter two classes of ore are treated as if they were known with certainty. (Sections 11 and 21.)

5) The ore deposit is partitioned into N zones, and each zone into N' subzones. The partitioning of the deposit into zones and subzones is made through decisions concerning the technology of mining, the decision variables being the sizes of the zones and subzones. If the partitioning is irrelevant for a certain deposit, N, or N', or both N and N' may equal 1[1]. The sizes are assumed to have been determined. See also assumption 7). (Sections 11 and 3443.)

6) If the partitioning into zones is irrelevant from the point of view of mining technology, an arbitrary partitioning into zones can be made in order to approximate the decision times of a series of decisions concerning other decision problems than the partitioning itself (compare section 3443). The same holds true for the partitioning into subzones if the latter is irrelevant from the point of view of mining technology.

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1) The optimization problem is a static problem if N=N'=1. The dynamic optimization model can, however, be applied also in this case, as it is simply a special case of period analysis, which involves only one period. Compare section 341.

7) The measure of the size of a subzone is the equivalent ore reserve in it. The <u>equivalent ore reserve</u> is the reserve if the average grade of the ore mined in the subzone assumes a certain given value which has been determined in advance[1], i.e. the <u>equivalent average grade</u>. The size of a zone is the sum of the equivalent ore reserves in the subzones of the zone. As a complement to assumption 5) it is assumed that the mining limits in other dimensions than the dimension average grade are given and constant in the two principal optimization models (compare e.g. Fig. 1:2 and the corresponding decision problem posed in section 11). On the assumptions made the sizes of the zones and subzones are given and constant.

8) As the size of a subzone is a given constant, the ore reserve of the subzone is assumed to be a single-valued function of the average grade of the ore mined in the subzone. This function is contained in the ore-reserve model, and expresses the ore reserve[2] in subzone n'n as a function which is fully determined by the following parameters and variables:

C_{in}	Technical coefficients relevant for zone n. i=1,2,3,4.
C_i	Technical coefficients relevant for all zones. i=5,6,7,8,9.
$\bar{\bar{h}}_n$	The equivalent average grade in zone n, which is assumed to be equal in all subzones of the zone.
$\bar{h}_{n'n}$	The average grade of the ore mined in subzone n' of zone n.
$\bar{R}_{n'n}$	The equivalent ore reserve in subzone n' of zone n.

9) The assumption 7) implies that the partitioning into zones and subzones must be reconsidered if the mining limits are altered, except if the latter are expressed as average grades. Coefficients and parameters which determine the relationships between the sizes of zones and subzones and the values of other variables, e.g. payments, must also be revised.

10) The subzones and zones are mined successively in the order[3] 11, 21, 31, ..., N'1, 12, 22, 32, ..., N'2, 13, 23, 33, ..., N'N. This order is implied where the ordinals of the zones, i.e. n=1,2,3,...,N, and the

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1) In principle the equivalent average grade can be determined arbitrarily, but some values are preferred for practical reasons. See e.g. sections 222 (HEQV(N) and PAR(JD,LT), especially PAR(4,LT)) and 513 (HNORM) of Appendix B. Note that the difference between HEQV(N) and PAR(4,LT) is significant in the payment functions (section 4 of Appendix D).

2) In this study it is also occasionally called the actual ore reserve in order to distinguish it from the equivalent ore reserve.

3) The notation has been explained in section 1421 (footnote).

ordinals of the subzones, i.e. n'=1,2,3,...,N', are referred to. (Section 1421.) It follows that the entire ore reserve is assumed to be extracted. (Section 21.) The starting time of a zone or a subzone coincides with the end of the production period of the preceding zone or subzone, respectively. (Sections 341 and 3443.)

11) The decision maker has to make a choice from a given set of alternatives. (Section 222.)

12) A set of consequences is attached to each alternative action. The consequences are known with certainty. (Section 222.)

13) The decision maker is able to rank the consequences in order of preference, and chooses the alternative to which the preferred consequences are attached, i.e. the optimum alternative. (Sections 222 and 224.)

14) The goal of the decision maker is to obtain maximum capital value of future mining. The consequences of the alternatives are thus measured in capital value. The model horizon is the end of the production period of the ore deposit, which implies the four assumptions enumerated in section 3324. Only the fourth will be repeated here: There is no time limit imposed on the mining company for the exploitation of the ore deposit. The importance of assumption 10) for the determination of the model horizon should be observed. (Sections 222, 224, and 3324.)

15) The decision problems are discussed on a general level in section 223. In the two optimization models they are represented by two sets of decision variables, i.e. the rate of production in each zone and the average grade in each subzone. The rate of production is not changed within a zone, and the average grade not within a subzone. All other decisions are assumed to have been made[1] so that the consequences of the alternatives considered are determined if the values of the decision variables in the two sets are determined. (Sections 223, 224, 3441, and 3442.)

16) The alternatives considered are defined by the values of the decision variables. The latter are controlled by the decision maker. (Section 224.)

17) In determining optimum rates of production the average grades are assumed to be given constants. In determining optimum average grades the rates of production are assumed to be given constants. Compare assumption 15), especially the footnote. (Sections 3441 and 3442.)

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1) Compare assumptions 5) and 7) where special cases of such decisions are treated. The assumptions are also more exacting there, as the decision variables involved (the sizes of the zones and the subzones and the mining limits except the average grades) are assumed to be given constants.

18) The decision times are the starting times of the zones and the subzones when the rates of production and the average grades, respectively, are determined. According to assumption 15) the values of the decision variables are not changed between decision times. (Section 3443.)

19) The capital value as an optimization criterion involves some specific assumptions[1]. One is that the consequences of each alternative can be fully described in terms of a flow[2] of future payments which expresses the expectations at the decision time[3]. The flow is single-valued.

20) A second specific assumption is that the rate of interest is a given constant. The rate of interest is assumed to express the minimum return on capital invested in order to induce the decision maker to invest capital in a project. The decision maker is assumed to be able to borrow or lend any amount desired at the given rate. (Section 3322.) Note: No "risk discount".

21) A third specific assumption is that the series of payments does not include interest payments, amortizations, and similar payments of a purely financial character.

22) A fourth specific assumption is that the production is assumed to be instantaneous in the sense that current inputs[4] are purchased, paid for, and used at the same instant of time as the resulting quantities of final products (various forms of ore) are sold and paid for by the buyer. The process is continuous and the current payments (section 1313) form a continuous flow over time. All investments are made simultaneously with the corresponding investment decisions, the equipment, machinery, etc. required being purchased, installed, paid for, and taken into production at the decision time[5]. The close-down payments are assumed to be made at the end of the production period of the ore deposit[6].

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1) The specific assumptions (19) - 22)) are made by direct application of the corresponding assumptions made by Johansson (1961, p. 19).

2) Johansson (1961, p. 19) assumes a series of payments when the interest is reckoned discontinuously. Here the interest is reckoned continuously. Then it is simpler to assume a flow of payments (ibid. pp. 196-197). The flow may contain discontinuities, e.g. the zone investments.

3) Compare sections 224, 32, and 3322 above where assumptions have been made to the same general effect.

4) Current inputs are e.g. materials, labour, energy, etc. (Danø 1966, p. 6).

5) Johansson (1961, p. 19) does not assume the interest to be reckoned continuously (assumption 23)). For this reason the assumptions here deviate from his. For example, he assumes that investments take place at the beginning of a year and that current payments are made at the end of the years. However, in an appendix he also studies the continuous case, where the fundamentals of the present assumptions are found (ibid. pp. 194 ff.).

6) Compare the salvage value in the investment theory (ibid. p. 196).

23) Interest is reckoned continuously.

24) The money value, i.e. the purchasing power of money, is constant over time. If the money value varies the models can be used provided that all payments are expressed in real units, i.e. in units of constant purchasing power, and that the rate of interest correspondingly is the real rate of interest[1].

25) Taxes are payments similar to all other payments in that they shall be taken into account in the evaluation of the alternatives to the extent that they are related to the latter[2]. In the present study other taxes than income taxes are assumed to be treated as other payments[3]. Income taxes are assumed to be taken into account directly as the sizes of other payments are determined. In order to facilitate this transformation of payments before tax into the corresponding payments after tax the following two assumptions are being made[4] (26) and 27)).

26) The flow of payments in each infinitesimal instant of time or, for simplicity, each single payment before tax is assumed to be transformed into the corresponding payment after tax independently of other payments. Payments of different types (assumption 37)) are also assumed to be transformed independently. The term "payments" is used to denote payments after tax.

27) The given and constant rate of interest is the rate after tax. The latter is thus assumed to be independent of other variables and parameters in the model.

28) In the general formulation of the optimization models the payments can be functions of time, the time being an independent variable, e.g. due to changes over time in the prices of production factors and final products, changes which are independent of the ore deposit and the mining company. The influence of time upon the sizes of the payments may be taken into account by inserting time as a variable in the functions determining the payments. This is not practiced in the models proposed[5]. Instead the

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1) See e.g. Johansson (1961, pp. 159–173 and 217–224) and Edwards and Bell (1961, especially pp. 31–69) for a discussion of methods of taking money-value variations into account and the measurement problems attached to such variations.

2) See e.g. Frenckner (1957) and Johansson (1961, pp. 5–6 and 62). The importance of various forms of taxes on mining decisions has also been discussed by Hotelling (1931, pp. 164 ff.) and Ciriacy-Wantrup (1952, Chapter 13).

3) Johansson (1961, p. 5).

4) Johansson (1961, p. 62–84) discusses how to treat income taxes in capital-value calculations, including the determination of the rates of interest before and after tax.

5) It would involve some computational problems to make such functional relationships flexible and fairly generally applicable. A oomputationally similar problem is that of discounting, which involves only one time relationship, equal for all payments and simple in form, i.e. a constant relative change.

influence of time is taken into account by varying the set of coefficients which determines the payment functions. A separate set is determined for each calendar year, beginning with year 1 and ending with the year which is defined as the data horizon. The payments are thus expressed as step functions of time. (Section 3325.)

29) Data subhorizons may exist within the data horizon. They delimit periods for which more detailed predictions exist from periods for which the predictions are more rough. Data subhorizons may coincide with the data horizon. Beyond the data horizon the payment functions of the data-horizon year are assumed to be relevant. (Section 3325.)

30) The technology changes over time, the markets of production factors and final products change, the know-how of management and other personnel of the mining company changes, etc. All conceivable and predictable changes over time are included in the variations determined by the step functions of time of assumption 28). This includes rationalization, i.e. investments necessary to improve the process of extracting and treating the ore, the subsequent reductions in expenditures and increases in revenues due to improvements in products and reduction of waste, etc. The present assumption might also be interpreted to mean that, at least to a certain limited extent, the decisions concerning technology and refinement level may be made at fixed time intervals, i.e. annually, as well as at intervals determined by the production periods of zones and subzones.

31) The rate of production is assumed to equal the production capacity. The former is the annual rate measured in tons according to section 21. The latter is measured in tons as the rate of production, and has been defined in section 343. According to assumption 15) the rate of production is constant during the production period of a zone. (Section 3443.)

32) The average grade of a subzone equals the average grades of the ore mined[1)] at each instant of time during the production period of the subzone. The average grade is measured according to Fig. 1:3 in relation to the tonnages of ore measuring the ore reserve according to section 21.

33) The mine, including equipment and machinery, sorting plant, service facilities, etc. is assumed to be adapted to the rate of production and the average grade prevailing at any point of time. (Sections 3443 and 3445.)

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1) "Mined" means produced and sold (assumption 22)).

34) The adaptation prescribed by assumption 33) takes place when a decision is made to make a change[1], i.e. at the starting times of the zones and subzones (assumptions 15) and 18)). The adaptation involves payments for changing the mine, including equipment and machinery, sorting plant, service facilities, etc. The payments are expansion investments if the rate of production is increased, contraction investments[2] if it is decreased, and grade-change investments[3] if the average grade is changed (increased or decreased). The current payments[4], the zone investments[5], and the close-down payments[5] are presumably influenced by the adaptation. This is assumed to have been taken into account in the coefficients determining the payment functions which define the payments in question. The adaptation is thus regarded as a decision problem pertaining to the technology or the refinement level[6], which is assumed to have been solved, so that the effects on the payments can be expressed as given functions of the other decision variables.

35) The adaptation is instantaneous at the decision time (assumption 22)). This also follows from assumptions 15) and 18) and is stated in section 3443. The size of a change is not limited in any way. It is e.g. assumed to be possible to acquire production capacity by infinitesimal increments and there is no upper limit to the size of an instantaneous increase in the production capacity and, consequently, in the rate of production.

36) The markets on which the company purchases production factors and sells the final products, are assumed to be given, so that the prices are single-valued functions of the amounts of factors purchased and products sold at any time.

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1) If a decision is not made to make a change, the production capacity of the mine is assumed to be constant during the entire production period. Usually the capacity declines as the mining proceeds to greater depths, e.g. because of increasing difficulties in the vertical transport of ore, machinery, material, men, etc. The capacity reduction is disregarded in the present models. The problem of decreasing capacity and a graphical solution of it have previously been discussed by the author (Norén 1967, pp. 96–101).

2) Compare the discussion of these payments in section 1312.

3) Compare the discussion of these payments in sections 1412 and 3443. The grade-change investments are in the final model assumed to be small in a sense defined in section 562.

4) Defined in section 1313.

5) Defined in section 1312.

6) For instance, by adjusting a given sorting plant or changing it, the final products from a given input of crude ore can be changed. Conversely, a change in the quantity or the average grade of the crude ore influences the final products. Input, sorting plant, and final products are all influenced by the adaptation. The rate of production and average grade refer to the crude ore.

The properties of the factors and products are determined through previous decisions (assumption 15)). Concerning the final products it is consequently assumed that the payments received for products sold can be expressed as a function of the rate of production, the average grade, and, according to assumption 39), some other variables of less importance from the point of view of the market.

37) Eight different types of payments are distinguished:

Current payments:	Current payments for products sold	(1)
	Current disbursements incurred in the current production	(2)
	Current reinvestments (not including zone investments)	(3)
Investments:	Zone investments	(4)
	Expansion investments	(5)
	Contraction investments	(6)
	Grade-change investments	(7)
Close-down payments		(8)

The current payments constitute a continuous flow which is measured per annum, i.e. in monetary units per annum or MKR/year. Together they form the (annual) current net payments which are defined as (1)-(2)-(3). Investments and close-down payments are paid at the points of time indicated in assumption 22) and are measured in monetary units (MKR). The payments of different types are additive if they occur simultaneously and otherwise if they are discounted to a common point of time at the given rate of interest.

38) The payments are measured against the alternative of not exploiting the ore deposit[1].

39) The payments are expressed in functions which are fully determined by the following parameters and variables:

c_{ia} Coefficients of the payment functions (assumption 28) and section 3325). The coefficients represent the environment of the decision model (section 224), including the consequences of the decisions which are assumed to have been made

1) Compare the investment theory, e.g. Johansson (1961, p. 19), where in connection with a machine investment the payments are measured against the alternative of not making the investment.

before the model is applied (assumption 15) with particular cases in assumptions 34) and 36)).
$i=1,2,\ldots,70$. $a=1,2,\ldots,A$.

a — The calendar year during which the payment or flow of payments occurs (assumption 28)). $a=1,2,\ldots,A$ where A denotes the data horizon. The value of a is determined

1a. if the starting time of the current zone is given when the rate of production is being optimized (section 3445) or

1b. if the starting time of the current subzone is given when the average grade is being optimized (section 3445) and

2. if the rates of production and average grades in the current zone and subzone and in all subsequently mined zones and subzones are known[1]. The principle of optimality (section 342) indicates one way of determining the latter.

$\sum_{i=1}^{n} R_i$ or $\sum_{i=1}^{n-1} R_i + \sum_{k=1}^{n'} R_{kn}$ where $k=1,2,\ldots,n'$ — The cumulative equivalent ore reserve of zone n+1, i.e. the sum of equivalent ore reserves R_i in the zones $i=1,2,\ldots,n$, or the cumulative equivalent ore reserve of subzone $n'+1,n$, respectively[2]. n' and n are the subzone and zone during the production period of which the payment is made. The values of the respective expressions are determined if the values of n or n' and n are given. The payments can be functions of n and n' (section 3445)[3]. The payments may also be functions of R_n or $R_{n'n}$.

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1) The equivalent ore reserve and the equivalent average grade of each subzone are given constants (assumption 7)). Then the corresponding actual ore reserve, or simply ore reserve, is determined as the average grades are known (above and assumption 8)). The chronology of the process is easily derived from this and assumption 10). This is done in the beginning of section 3 of Appendix D, and will not be repeated here.

2) Refers to the ore reserves which have been exhausted before the zone (or subzone) in question is being mined. Hence the zone n+1 and the subzone $n'+1,n$ are referred to. Thus the cumulative and the remaining equivalent ore reserves of a zone (or a subzone) added together equal the equivalent ore reserve of the deposit as viewed from the actual decision time.
Note that R_i is used in the payment functions. $R_i=\sum_{n'=1}^{N'} R_{n'i}$ where $n'=1,2,\ldots,N'$ and $R_{n'i}$ are given constants according to assumptions 7) and 8).

3) Some complications are treated in section 633.

Q_{n-1} — The rate of production in the zone preceding the zone during the production period of which the payment occurs. Q_{n-1} is known if the state at the current decision time is given (section 3445) and the rates in the subsequently mined zones are also known (point 2 under variable a above). Q_0 reflects the state at the actual decision time and may be equal to 0 or a given positive constant.

$\bar{h}_{n'-1,n}$ if $n' > 1$, $\bar{h}_{N',n-1}$ if $n'=1$ — The average grade in the subzone preceding the one during the production period of which the payment occurs. It is known under the conditions stated under Q_{n-1}, "average grades" replacing "rates". $\bar{h}_{N'0}$ is denoted $\bar{h}_{00}$ and is a given positive constant which reflects the state at the actual decision time. If the deposit is not being mined at this time $\bar{h}_{00}=0$. The average grade in the preceding subzone is in the final model assumed to have only a small influence on the payments during the production period of a subzone, especially if the average grades in the two subzones do not differ very much. The meaning of this is explained more precisely in section 562.

$T_{1,N+1}$ — The end of the production period of the ore deposit. It is known under the conditions stated under a, and determines the length of the remaining production period (section 3445).

Q_n — The rate of production in the zone during the production period of which the payment occurs.

$\bar{h}_{n'n}$ — The average grade in the subzone during the production period of which the payment occurs.

n' and n — The subzone and zone, respectively, during the production period of which the payment occurs. Observe that n' and n denote the current subzone and zone in other contexts.

40) The reinvestments are treated as current payments. This implies e.g. that the equipment and machinery acquired by an expansion investment are successively being replaced right from the instant they have been installed. Two important exceptions are possible whithin the framework of the model, i.e. to define an arbitrary development during the first years after the actual decision time and to define a rule according to which the reinvestments

decrease during the last years of the production period[1]. The former is defined by means of the relevant coefficients for the first years. The latter should be some simple function of the remaining production period. Examples are found in section 44 of Appendix D and in section 342. Thus, it is assumed that the reinvestment decisions are made according to a predefined rule causing a flow of payments as discussed above (assumption 15)).

41) In addition to those parameters and variables mentioned in assumptions 8) and 39) the capital value of future mining is also determined by:

j	The given and constant continuous rate of interest (assumptions 20) and 23)).
$\mathcal{T}_{n'n}$	The production period of subzone n' of zone n. See $T_{n'n}$.
$T_{n'n}$	The starting time of subzone n' of zone n. Denoting the current decision time $T_{\alpha\beta}$, the following starting times are needed[2]: $T_{\alpha\beta}$, $T_{\alpha+1,\beta}$, $T_{\alpha+2,\beta}$, ..., $T_{N'\beta}$, $T_{1,\beta+1}$, $T_{2,\beta+1}$, ..., $T_{N'N}$. The similarly subscripted production periods $\mathcal{T}_{n'n}$ are also used.

In addition, the subscript numbers of the current subzone and zone are used. They are denoted α and β, respectively, above[3].

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1) The development of reinvestments over time and other problems in determining reinvestments have been treated in the literature on the theory of production, e.g. by Frisch (1965, pp. 293 ff.), and the theory of investment, e.g. by Johansson (1961) and Näslund (1966). The latter authors discuss factors influencing the service life and the interdependence between service life and other payments, e.g. maintenance costs.

Frisch shows how reinvestments first increase, then fluctuate, and finally converge to a constant level if the service lives of the equipment forming the primary or initial investment are continuously distributed (a continuous or approximately continuous distribution is held to be realistic; ibid. pp. 298 and 302). New primary investments are assumed not to be made and the process is perpetual. The initial increase and fluctuations in the beginning of the production period of the deposit can be taken into account in the payment model. For the rest of the period the constant level is assumed to represent actual reinvestments. Reinvestments are accordingly treated as current payments.

If new primary investments are made later during the production period (e.g. expansion investments) a new period of fluctuations will occur before a new constant level is attained (ibid. pp. 304-308). This is disregarded in the payment model, where it is simply assumed that the new constant level is attained immediately after the change. This also holds true for other changes, e.g. reductions in the rate of production and changes in the average grade.

2) $T_{1,N+1}$ is also relevant, but has been mentioned in assumption 39).

3) The symbols are used for other purposes also.

42) The state at the current decision time is specified by $T_{\alpha\beta}$, α, β, $Q_{\beta-1}$, and $\bar{h}_{\alpha-1,\beta}$ (if $\alpha>1$) or $\bar{h}_{N',\beta-1}$ (if $\alpha=1$). (Section 3445). This assumption has been incorporated into the preceding assumptions. The values of the variables are given constants when the current decision is being made (section 342 in the definition of the principle of optimality). The values are provided by the optimization method.

43) In optimizing the principle of optimality is applied. This means here, loosely expressed, that future decisions are assumed to be optimal as viewed from the point of time when they are made. (Section 342.)

44) In determining alternatives to be evaluated in the optimization the ore-reserve restriction is observed, i.e. $T_{n'n} \cdot Q_n = R_{n'n}$ for $n=1,2,\ldots,N$ and $n'=1,2,\ldots,N'$. All of the ore reserve is mined (assumptions 10)). (Section 32.)

The assumptions listed proceed in general from reality, i.e. the decision problem, toward the optimization models, from the general to the specific, and from the ore deposit to the market. To some extent the assumptions follow from one another or a specific assumption may be contained in another more general one. In this way some consequences of an assumption have been stressed whereas others have not been mentioned at all. This has been done with a view to what may be important in applications of the models, and the resulting redundance should be more clarifying than confusing.

43 Ore-reserve model

The ore-reserve model describes how the ore reserve in a subzone varies with the average grade of the ore mined in the subzone. The prevailing assumptions are stated in section 42. Assumptions 4) to 10) are especially relevant for this model. The ore deposit illustrated by Fig. 1:3 is the informal model which forms the background. The ore-reserve model can be extended to cover deposits such as that of Fig. 1:2, but with successively decreasing marginal grades for more extensive mining limits in each or some of the ore bodies. It can also be extended to ore deposits where the marginal grade decreases in many or all directions from a central core. The subzones need not be horizontal layers as shown in Fig. 1:3. The principal condition is that there exists a decision rule combining an average grade with a physical mining limit in such a way that the ore reserve of a subzone of a given and constant size (a given equivalent ore reserve) is a single-valued function of the average grade. In addition the subzones must also be mined successively, one after another.

The ore-reserve model is not necessary for the deposit of Fig. 1:1, where the mining limits are given. However, the capital-value model and the optimization models containing the ore-reserve model are fully applicable in this case too.

Although the mining limits are given geometrically, mining limits exist in other dimensions, limits which are not given. An example may be given. In certain mining methods, e.g. sub-level caving, waste rock from outside the ore deposit is mixed into the ore in the process of blasting (in rounds) and loading at the working face. After each round the ore loaded first is usually pure, but as the loading continues an increasing quantity of waste rock appears, which must be loaded, transported, and treated with the ore. A mining limit exists here, which under certain assumptions[1] can be expressed as a marginal grade or an average grade. The mining limit is of course a rule for determining when to stop loading[2]. The presently discussed ore-reserve model is applicable if the rule gives a relation between the ore mined in a subzone[3] (waste rock included; see section 21), i.e. the ore reserve of the subzone, and the average grade of the same ore. The relation must express the ore reserve as a single-valued function of the average grade.

A general formulation of the ore-reserve model can be stated as follows ($R_{n'n}$ denotes the ore reserve in subzone n' of zone n; compare assumption 8)):

$$R_{n'n} = f(\bar{h}_{n'n}, \bar{a}_n, n, R_{n'n}) \qquad (4.1)$$

The general formulation must be specified if the model is to be applied to a specific decision problem. The specific formulation depends on the ore deposit and the technology of the particular problem. An example of a specific model is given in section 2 of Appendix D.

The specific model is part of the optimization models constructed in this study. The optimization model includes methods for determining the optima. As will be discussed in next chapter the optimization models are transformed into computer programs which yield the optima desired. The specific ore-reserve model formulated in Appendix D is consequently inserted in the programs. It is, however, not essential for the programs, as the latter are constructed so as to facilitate changes in the ore-reserve model within the limits defined by the assumptions, the general model, and the program (section 1 of Appendix D).

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1) The particular assumptions will not be discussed here.

2) The problem has been treated by e.g. Hillberg and Sjöstrand (1958), Hansagi (1959), Ahlmann (1963), and Norén (1963 and 1967).

3) For example, in the mining method mentioned above, a sublevel may constitute a suitable subzone.

44 Payment models

The payment models are concentrated in the payment functions, i.e. the payments expressed as functions by means of the parameters and variables listed in assumption 39) in section 42. The models are also in other respects constructed on the assumptions given in section 42. One payment function is defined for each type of payment (the types are defined in assumption 37)). It is, however, possible to reduce the number of functions within the framework of the optimization models. For example, one single payment function instead of three may define the current net payments.

No general formulation of the payment functions will be given, as the general functions would only contain the list of parameters (except c_{ia}) and variables, which has already been presented in assumption 39). A set of specific payment models is described in section 4 of Appendix D. Some of the models do not contain all the parameters and variables available, as it is assumed that the corresponding payments are insensitive to the parameters and variables excluded. As the specific ore-reserve model the specific payment models may be different for different specific situations. The payment models are also inserted into the computer programs, but in such a way that changes in the payment models are faoilitated within the limits defined by the assumptions and the program (section 1 of Appendix D).

The payment models as well as the ore-reserve model can be made specific in two different but interrelated steps. The first step is to <u>forecast</u> the consequences of the alternatives considered in terms of ore resources[1], ore reserves, and payments. The second step is to <u>translate</u> the forecasts into specific models, i.e. to express them as functions by means of the constants and variables enumerated in assumptions 8) and 39) in section 42.

The general outline of the forecasting is determined by the models, as the latter indicate the information needed to evaluate the alternatives considered in the optimization problem. The specific models also contain hypotheses concerning interrelations between variables. The hypothetical models are useful where all alternatives cannot be considered separately. Instead, a few alternatives can be selected, and the consequences of these alternatives estimated. The hypothetical models may indicate especially important alternatives or differences between alternatives. The hypothetical models can then be used to predict the

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1) Ore resources are defined in section 21. The equivalent ore reserve is a measure of ore resources.

consequences of further alternatives. Independent predictions for such alternatives are also made in the same way as for the original alternatives. A comparison between the two predictions yields a test of the hypothetical models[1].

The ore-reserve model and the payment models described in Appendix D may be useful as hypothetical models. Such models should be adaptable to a wide variety of problem situations, i.e. they should be flexible. The models are rather flexible. A comparatively large number of coefficients are used in addition to parameters such as $\bar{h}_n$ and $R_{n'n}$. The factors influencing the payments are mostly kept apart and the relationships are usually expressed as simple linear functions or exponential functions which are added to yield the total payments. Their relative importance is easily controlled by means of the values of the coefficients. In the computer programs they are also easily exchangeable as parts of the models can be removed or altered separately.

To translate the forecasts into specific models is to revise the hypothetical models so that they reflect the forecasts as well as possible. Approximations and simplifications are necessary, as predictions concerning the complicated reality and the relationships between the predictions have to be described as functions by means of the parameters and variables available[2]. Methods for revising the models and estimating the values of the coefficients will not be discussed here[3]. They are discussed to some extent in Appendix B (sections 222, 322, 422, 51, and 52) and Appendix C (section 1).

To facilitate the interpretation of the payment functions into payments under various assumptions a subsidiary computer program has been made, which produces graphs of the payment functions. Another program has been made to aid in revising the values of the coefficients of the payment functions (section 5 of Appendix B). The programs should be useful in the forecasting step in determining the values of the coefficients of the hypothetical models and in the

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1) The problems of forecasting (evaluation of variables) and testing of hypotheses will not be treated in this study. They have been treated by e.g. Ackoff (1962, pp. 140-309) and textbooks on statistics and managerial economics.

2) By changing the computer programs the number of coefficients can be increased. It may e.g. be useful to define a set of coefficients in the payment functions, which consists of a set for each zone or even for each subzone.

3) Curve fitting and estimation in general have been discussed by e.g. Ackoff (1962, pp. 251-281). The technique of regression analysis may be useful. See e.g. references in Ackoff (1962, pp. 329-332).

testing of the latter models, as well as in the translating step where the payment models to be used in the optimizations are established.

45 Capital-value model

The capital-value model determines the capital value of future mining if the ore-reserve model, the payment models, and the values of all decision variables are given. It is described in section 3 of Appendix D. The model is inserted into the computer programs and is formulated with a view to programming problems. Hence the capital value is discounted to time zero and the actual decision time is the end of the first year, i.e. $T_{11}=1$ (compare section 3444). This means that a capital value obtained by the capital-value model, i.e. $B_{\alpha\beta o}$, should be discounted to the current decision time in order to be equal to the capital value used as decision criterion, i.e. $B_{\alpha\beta}$, according to section 3324 and assumption 14) in section 42 (compare also section 342 where the recurrence relation is discussed). Thus

$$B_{\alpha\beta} = B_{\alpha\beta o} \cdot e^{j \cdot T_{\alpha\beta}} \qquad (4.2)$$

where $T_{\alpha\beta}$ is the current decision time, i.e. the starting time of subzone α of zone β, for which the optimum value of the decision variable is currently being calculated. However, for all alternatives considered at time $T_{\alpha\beta}$ the value of $e^{j \cdot T_{\alpha\beta}}$ is equal as $T_{\alpha\beta}$ is a constant (assumption 42)). Consequently, $B_{\alpha\beta}$ and $B_{\alpha\beta o}$ will rank the alternatives equally, and $B_{\alpha\beta o}$ can be used as criterion instead of $B_{\alpha\beta}$.

It is now apparent that the capital-value model of Appendix D is a recurrence relation similar to the relation (3.2) in section 342. The differences are explained by (4.2) and by the facts that the capital-value model is more specific regarding the payments during each period and that the periods are defined as the production periods of subzones[1]. The ore-reserve restriction (section 32) is taken into account in the definitions of the production period and the decision times.

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1) See footnote to $B'_{n'nO}$ in section 46 regarding close-down payments and $B_{1,N+1,O}$.

46 A general formulation of the optimization models

The capital-value model contains the goal function of the optimization models and the ore-reserve restriction. The optimization problems are dynamic and a suitable optimization method has been derived from the theory of dynamic programming (section 342 and the conclusion ending section 343). The optimization models are thus recurrence relations expressing the goal function and the principle of optimality, completed with the prevailing restrictions, i.e. in this case the ore-reserve restriction. The recurrence relations indioate a general method of solving the optimization problems (section 342), whioh is part of the optimization models.

In addition to previously defined[1] symbols the following ones are needed:

$B'_{n'n0}$	Capital value at time zero of payments occuring as a oonsequence of the mining of subzone n' of zone n, including the investments at time $T_{n'n}$, the current payments during the production period $\mathcal{T}_{n'n}$, and close-down payments[2] if n'n equals N'N.
$\underset{\bar{h}_{n'n...N'N}}{\text{Maximum}} \; B_{n'n0}$	Capital value $B_{n'n0}$ if the values of $\bar{h}_{n'n}$, $\bar{h}_{n'+1,n}$, $\bar{h}_{n'+2,n}$, ..., $\bar{h}_{N'n}$, $\bar{h}_{1,n+1}$, $\bar{h}_{2,n+1}$, ..., $\bar{h}_{N'N}$ constitute an optimal policy.
$\underset{Q_{n...N}}{\text{Maximum}} \; B_{1n0}$	Capital value B_{1n0} if the values of Q_n, Q_{n+1}, Q_{n+2}, ..., Q_N constitute an optimal policy (compare assumption 18) in section 42).

It has been stated that two optimization models were to be constructed in a first stage. A general formulation of the two models follows:

For determining optimum average grades:

$$\underset{\bar{h}_{n'n...N'N}}{\text{Maximum}} \; B_{n'n0} = B'_{n'n0} + \underset{\bar{h}_{n'+1,n...N'N}}{\text{Maximum}} \; B_{n'+1,n,0} \qquad (4.3)$$

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1) The symbols are also enumerated in Appendix F.

2) The close-down payments are forming the capital value $B_{1,N+1,0}$ in the capital-value model of Appendix D. They are a constant once the decision at time $T_{N'N}$ has been made and, consequently, not subject to any decision at time $T_{1,N+1}$. To simplify the formulation of the optimization model they are included in the capital value $B'_{N'NO}$, thus making $B_{1,N+1,0}=0$.

where[1] $n'=1,2,\ldots,N'$ and $n=1,2,\ldots,N$,

$B_{n',N+1,0}=0$, and

$1,n+1$ substitutes $n'+1,n$ if $n'=N'$.

<u>For determining optimum rates of production:</u>

$$\underset{Q_{n\ldots N}}{\text{Maximum}}\ B_{1n0} = \sum_{n'=1}^{N'} B'_{n'n0} + \underset{Q_{n+1\ldots N}}{\text{Maximum}}\ B_{1,n+1,0} \qquad (4.4)$$

where $n'=1,2,\ldots,N'$ and $n=1,2,\ldots,N$ and

$B_{1,N+1,0} = 0.$

(4.3) and (4.4) are <u>subject to common restrictions:</u>

$$R_{n'n} = f(\bar{h}_{n'n},\bar{a}_n,n,R_{n'n}) \text{ and} \qquad (4.1)$$

$$\mathcal{T}_{n'n}\cdot Q_n = R_{n'n} \text{ for } n'=1,2,\ldots,N' \text{ and } n=1,2,\ldots,N. \qquad (4.5)$$

$$T_{11} = 1 \qquad (4.6)$$

$$T_{n'+1,n} = T_{n'n}+\mathcal{T}_{n'n} \text{ for } n'=1,2,\ldots,N' \text{ and } n=1,2,\ldots,N \qquad (4.7)$$

where $1,n+1$ substitutes $n'+1,n$ if $n'=N'$ and

T_{11} and $T_{1,N+1}$ are the beginning and the end, respectively, of the production period of the remaining ore reserve at the actual decision time, i.e. T_{11}.

The general formulation indicates the general method of solving the two optimization problems. This method has been discussed previously. However, to complete the optimization models the method will be specified in detail. Optimization methods will be discussed in Chapter 5. Their application to the two problems treated now as well as to the other decision problems discussed in section 223 will be considered in Chapter 6.

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1) It should be observed that $n'n$ denotes the current subzone here, i.e. the same as the subscripts $\alpha\beta$ in sections 42 and 45 as well as in section 3 of Appendix D.

Before the optimization methods are expounded, however, some of the more interesting optimization models concerning ore deposits, which have been published previously, will be discussed. Such models have been referred to repeatedly in the preceding chapters. They do not treat exactly the same decision situations as those discussed here. A survey of the differences in this respect may be useful in determining whether to use some earlier model or one of those presented in this study for the solution of the practical decision problems facing an individual decision maker.

47 Some earlier optimization models in mining

This is a survey of the decision problems and decision situations treated by various authors, rather than a survey of the models themselves[1]. Furthermore, only differences between the problems and situations treated in these works and those dealt with in this study are discussed.

Except for Albach's (1967) model the models are concerned with one single ore deposit. Albach's problem can be stated as that of determining the optimum combination of rates of production in a given set of open-pit mines within a fixed time horizon ("planning horizon"). The rates are determined periodically, the length of the periods being a given constant, e.g. one year. The production capacity of each mine may not be exceeded, and the total production per unit of time in all mines must meet given delivery requirements. Risk plays an important role as the ore reserve of each deposit is known only as a stochastic variable.

One of the earlier models encountered in the literature was presented by Gray (1913). He determines optimum annual rates of production, provided that decisions are made annually, on the assumption that expansion and contraction investments equal zero regardless of the size of a change. In particular it is indicated that the variation over time of the optimum rate is extremely great under this assumption, as well as that the rate would be "more nearly uniform" if investments in fixed capital were necessary[2]. Gray also relates his decision model and discussion to the classical economic theory (especially Ricardo).

1) A survey of the models mentioned here (except those dated 1967) has previously been made by the author (Norén 1967, pp. 9-54).

2) Gray's model and discussion have been important to the author as they very distinctly indicate the dynamic nature of the optimization problem in question. Hotelling's (1931) and Henning's (1963) models have been of similar importance.

Hotelling (1931) determines the optimum rates of production as a function of time and assumes continuous decisions. As Gray he first assumes that expansion and contraction investments do not exist. The influence of various market forms (free competition, monopoly and duopoly) and forms of taxes are examined. In addition, the optimum course of exploitation from a social view is discussed. The influence of investments in fixed capital is also treated, although separately from the rest of the analysis. Hotelling determines the conditions for optima analytically (calculus of variations) whereas Gray has used simple numerical methods.

Ciriacy-Wantrup (1952) takes the discussion of conservation of natural resources as a starting point for a comprehensive treatment of problems regarding exploitation of ore deposits and related problems from the point of view of the individual mining company and that of the community. The decision situations of individual companies are mainly the same as those treated by Gray and Hotelling. Another problem of practical interest is discussed, which should be mentioned. This is the practical implementation of a theoretical optimization, and the relation of the optimization to the actual behaviour of the decision maker[1].

Herfindahl (1955) discusses primarily exploration and exhaustibility, using Gray's discussion and main assumptions.

Billiet (1959) and Massé (1959) have been discussed in sections 12, 224 (risk and uncertainty), and 3443. In the case of certainty they treat the problem of determining the optimum rate of production for an ore deposit which, in the terminology of this study, consists of one subzone in one zone (N=N'=1), i.e. a single-period or static problem. However, they also penetrate a dynamic aspect of the single-period problem by examining the conditions on which the optimal policy is to keep the rate of production constant during the period (the production period of the deposit).

Ventura (1964) should be mentioned together with Billiet as he reformulates Billiet's optimization problem under risk as a dynamic programming problem. He does this in a survey of mining problems in general, e.g. prospecting, evaluation of the ore reserves and average grades in ore deposits, exploitation decisions, and technical problems.

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1) Compare section 224 (the footnote to "weak point" 5).

Rice (1963) analyses and compares a couple of schematic formulae for evaluating ore deposits, "Hoskold's formula" and "Morkill's formula". "Hoskold's formula" proves to be inconsistent with the capital-value criterion under certain conditions, whereas "Morkill's formula" can be made to yield the same capital value as a straightforward capital-value calculation.

Carlisle (1954) treats the problems of determining an optimum rate of production, an optimum mining limit, and the optimum combination of rate and limit. The rate and the limit are assumed to be determined at the beginning of the production period of the deposit and to remain constant throughout that period. The basic analysis is made on the assumption that the rate of interest equals zero, or without discounting. Investments in fixed capital take place at the beginning of the production period. Tessaro (1960) demonstrates some of Carlisle's models with some numerical examples.

Henning (1963) treats the problems of determining an optimum rate of production, optimum mining limits, and the optimum combination of rate and limits. The optimization problems are treated on assumptions principally the same as those used by Carlisle. Regarding the mining limits the analysis is extended to a dynamic optimization model where the capital-value criterion (with a rate of interest exceeding zero) is used. The mining limit is expressed as a cut-off (marginal) grade. The ore deposit and the timing of the investments are such that the deposit can be described as consisting of one subzone in one zone. The grade-change investment equals zero regardless of the size of a change. The optimum cut-off grades are determined annually according to an approximate formula based on these assumptions.

Ruskin (1967) describes various computer programs for mining applications. One is a model of the ore deposit for determining e.g. ore reserves and average grades by zones. Up to nine zones can be distinguished by defining the shape and size of each zone. Another program uses the first program as a model of the ore deposit and, assuming open-pit mining, simulates the mining process in order to determine the optimum shape of the open pit. A further program calculates the capital value of the deposit for a given production plan. The program mentioned first provides necessary data concerning the ore deposit.

Although they entirely use other optimization criteria than the capital value, models presented by the following authors should be mentioned:

Bisdorf (1958 and 1962):	Mainly rate of production.
Callaway (1958):	Short-run rate of production and average grade by break-even analysis.
Dorstewitz, Wilke, and Bindels (1963):	Mainly mining methods.
Lane (1961):	Mining limits.
Manula and Kim (1967)	Short-run planning of production and stockpiling to meet a given demand. Computer program available.
Soukup (1963):	Mining limits.
Wilke (1962):	Mining methods.

CHAPTER 5

5 Optimization methods

51 Decision problems and solutions

The solution of a decision problem is the course of action to pursue. An optimization model is a guide in the search for the best course of action. The solution of the optimization problem posed in the model yields optimum values of the decision variables. The latter are solutions of the decision problem only in so far as the model adequately represents the decision problem[1]. The problem of determining the optimum values of the decision variables of the optimization model will be treated here. The term "solution" consequently refers to solutions of the optimization problems as they are posed in the optimization models.

This chapter will mainly deal with solutions of the problems of determining optimum rates of production and optimum average grades. However, a discussion of solutions of other optimization problems is also important, although it will be comparatively short. Optimization problems where only one decision variable is concerned at each decision time are simplest, and will be treated first. Solutions of the more complicated problems where two or more variables are determined at each decision time are discussed then.

There are two principal ways of solving the optimization problems, namely analytically and numeriacally[2]. Numerical solutions are preferred in the present case for several reasons. They appear to be simpler as there are a number of fairly complicated relationships between the variables involved, especially if parametric variables such as the equivalent ore reserve, the equivalent average grade, and the rate of production (if the average grade is being optimized) or the average grade (if the rate of production is being optimized), are retained in the functions. They are usually easier to explain, and yield solutions in a form which is often more suitable for presentation to the decision maker. If the numerical solution results in a direct evaluation of a set of alternatives, i.e. yields the capital values of the alternatives, the results

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1) Churchman, Ackoff, and Arnoff (1957, p. 169) and Ackoff (1962, p. 343).

2) Ackoff (1962, p. 343).

are not only easily presented, but the material for a simple form of sensitivity analysis is prepared with the solution. This should be useful for the decision maker in his evaluation of the solution. Thus, numerical optimization methods directly utilizing the capital values of a set of alternatives are preferred.

The importance of a sensitivity analysis has been stressed by Bellman (1957, p. 6). It has also been discussed in section 224, where the desire to facilitate an extensive sensitivity analysis was expressed, and in section 35. Two sensitivity problems may be considered. One is to determine the influence of a change in the assumed conditions on the optimum solution. The other is to determine the cost in terms of the difference in capital value between an optimum solution and a certain non-optimum solution, i.e. the cost of a certain deviation from the optimum. The direct evaluation of a set of alternatives mentioned above immediately solves the latter problem, at least partially. The former problem can be solved by repeated optimizations on varying conditions. For this reason the optimization model should be easy to apply repeatedly to the same optimization problem. In addition, there should be means available to determine capital values of single alternatives which have not been considered previously, partly as new alternatives may turn up during the process of finding and evaluating a solution, and partly as a means of roughly determining the importance of a given change in the conditions without carrying out the entire optimization procedure.

If the payment functions are very much simplified and certain other simplifications are introduced (discussed below), solutions can be obtained by manual calculations. In general, however, this is impracticable, especially if solutions are to be obtained repeatedly for varying conditions. It is obviously convenient to use an electronic computer. For this reason the optimization models are transformed into computer programs. The latter will be referred to as <u>programs</u>.

The purpose of this chapter and the next one is, accordingly (compare section 3442 and assumption 15)):

1) To introduce a program for determining optimum rates of production by zones according to the optimization model (4.4) in section 46. Name of program: EXRATE.

2) To introduce a program for determining optimum average grades by subzones according to the optimization model (4.3) in section 46. Name of program: CUTOFF.

3) To introduce a program for determining the capital value of the ore deposit according to the capital-value model described in section 45, if the values of all decision variables (and all parameters and coefficients) are given (compare above, especially the paragraph concerning sensitivity analysis). Name of program: CAPVAL.

4) To establish if and how the three programs can be utilized for optimizations if multiple-variable decisions are considered.

The programs are described in Appendix B and Appendix E. Appendix B is mainly intended to serve as a manual for a user of the programs. It also introduces the optimization methods although from a more technical angle. Appendix E contains the FORTRAN source programs which are the most exact and complete formulations that exist of the models and methods utilized.

The theory of dynamic programming provides the basis of the methods used in the two optimization programs. A quality of dynamic programming is that the methods used to arrive at a solution can be adapted to the specific properties of an individual optimization problem in such a way that the problem is more easily solved than by the general method described in section 342[1]. Thus, such properties will be looked for, and the specific optimization methods will be presented. To this end a simplified version of the problem of determining optimum rates of production will be treated first. The simplifications are that the importance of time is disregarded to a certain extent, and that payment and capital-value functions have certain properties. The importance of time is discussed first. The importance of other simplifications are then discussed in connection with the solution of the simplified version of the problem. Alternative methods are then discussed against the background of the available solution with a view to their use in digital computers.

First, the optimization of the rate of production will be discussed in sections 52-55. Next, the optimization of the average grade will be treated in section 56. Simultaneous optimization of rate of production and average grade will then be discussed in section 57. Finally, other decision problems will be treated in section 58.

A short summary of the optimization models will appear in Chapter 6.

The ambition is not to find the best method to arrive at a solution, but rather to find a passable method. The same limited level of efficiency is aimed at

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1) Bellman and Dreyfus (1962, p. viii).

as regards the computer programs. More efficient methods and programs may be developed than those presented in this study.

52 The importance of time

The state at each decision time is determined by the values of the five state variables $T_{\alpha\beta}$, α, β, $Q_{\beta-1}$, and $\bar{h}_{\alpha-1,\beta}$ or $\bar{h}_{N',\beta-1}$ (assumption 42) in section 42). Using the typographically simpler notations of section 46, α is replaced by n', and β by n (current subzone and zone, respectively). The dynamic optimization method determines in which order the zones and subzones are optimized, i.e. the reverse mining order (section 342 and assumption 10)). Thus, the values of n' and n are given at each decision time. It is assumed that only the rate of production is being optimized. Then the values of $\bar{h}_{n'-1,n}$ or $\bar{h}_{N',n-1}$ are given constants (assumption 17) and n'=1 (assumption 18)). The remaining variables are T_{1n} and Q_{n-1}. Their values depend on decisions before time T_{1n} concerning Q_1, Q_2, ..., Q_{n-1}. When the decision concerning Q_n is actually being made T_{1n} and Q_{n-1} are, of course, given constants, and the problem is reduced to determining the optimum value of Q_n on the assumption that Q_{n+1}, Q_{n+2}, ..., Q_N are also optimized at the corresponding decision times, i.e. $T_{1,n+1}$, $T_{1,n+2}$, ..., T_{1N}, respectively.

In the optimization model, however, the state at the decision time is fully specified only at T_{11}. At all other decision times T_{1n} and Q_{n-1} are not specified. They depend on optimizations which still remain to be done when Q_n is being optimized. Thus, the optimum value of Q_n must be determined for all possible combinations of values of T_{1n} and Q_{n-1} if the dynamic-programming solution described in section 342 is to be used. Within the limits determined by the ore-reserve restriction and the chronology of the mining process (the expressions (4.5), (4.6), and (4.7) in section 46) any combination of values is possible[1]. Owing to the expansion and the contraction investments the optimum value of Q_n may be strongly influenced by the value of Q_{n-1} (section 3443). In addition Q_{n-1} may also influence other payments (assumption 39) and section 44). Hence it is essential to determine the optimum value of Q_n as a function of Q_{n-1}.

The decision time, T_{1n}, remains to be discussed. The value of T_{1n} influences the capital value expressed as a function of the decision variable by way of

1) T_{1n} is a given constant if the value of Q_{n-1} is given in the special case where n=2.

dating the payments and the capital value at the decision time. The dating of the payments affects the discounting procedure as well as the size of the payments (assumption 28)). The dating of the capital value exerts influence on the discounting only.

The payments and, consequently, the capital value of future mining are discounted to time zero. The capital value discounted to the current decision time T_{1n} is an equivalent optimization criterion (section 45). Assumption 22) and the expression (4.7) in section 46 ensure that if the decision time is moved by any given amount of time, all future payments are moved in the same direction by the same amount of time, if the future decisions are assumed to be unaffected. As the rate of interest is constant over time (assumption 20)) the discounting procedure is independent of the value of T_{1n} if the capital value at the decision time T_{1n} is used as optimization criterion. Hence, T_{1n} can be disregarded in the discounting procedure if the latter is changed so that the optimization model is of the form expressed in (3.2) in section 342.

The dating of the payments affects their sizes. The value of T_{1n} determines the payment times. The effects on the optimization depends entirely on the specific assumptions of each particular case concerning the payment functions. The dating is irrelevant if the payment functions do not vary with the time, i.e. if the sets of coefficients c_{ia} (assumption 39); i=1,2,...,70) are equal for all values of a, and time is not inserted as a variable in the payment functions. Assuming this, T_{1n} can be disregarded also as the variable that determines the dating of the payments.

The decision time, i.e. the value of T_{1n}, can be disregarded in the optimization on the conditions specified, if the problem is to determine optimum rates of production. Q_{n-1} is then the only variable among the variables determining the state at the decision time, the value of which is not specified when the optimization is being made[1].

The simplified optimization problem will be solved next.

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1) Similar conclusions can be reached concerning the problem of optimizing average grades. $\bar{h}_{n'-1,n}$ or $\bar{h}_{N',n-1}$ is then the only state variable which must be varied in optimizing the decision at time $T_{n'n}$.

53 Dynamic optimization of rates of production: Graphical solutions

531 Simplifying assumptions

As an intermediate step in solving the optimization problems posed in section 46 (A general formulation of the optimization models) a simple example concerning the rate of production will be solved. The simplification rests on the following simplifying assumptions which have been discussed in section 52:

1) The capital value at the decision time of future payments is used as optimization criterion instead of the capital value in question discounted to time zero. This is in accordance with assumption 14), but a change in the models introduced in sections 45 and 46.

2) The payment functions are equal for all times and do not include time as a variable (a change in assumptions 28), 29), 30), 39), and 40)).

In order to confine the discussion of the direct influence of the value of Q_{n-1} on the optimum value of Q_n to expansion and contraction investments and in order to simplify the payment functions determining the investments, it is further assumed that:

3) Q_{n-1} is not included as a variable in any payment function but those determining the expansion and contraction investments (a change in assumption 39)).

4) The expansion and the contraction investments are continuous and linear functions of the expansion, i.e. Q_n-Q_{n-1} if $Q_n>Q_{n-1}$, and the contraction, i.e. $Q_{n-1}-Q_n$ if $Q_{n-1}>Q_n$, respectively. They equal zero if $Q_{n-1}=Q_n$. They are independent of the level of the rate of production, i.e. Q_{n-1} and Q_n taken separately. These are restrictions to be added to assumption 39).

5) $N'=1$, and the subzone subscript is deleted. Thus, $B_{1n}=B_n$, $T_{1n}=T_n$, etc.

6) The capital values at the decision times of future mining, i.e. B_n for $n=1,2,\ldots,N$ where n denotes the current zone, are expressed as continuous functions of the respective Q_n. For the sake of simplicity an unrealistic assumption is made here, which will be discussed later. Each function is first monotonously increasing at a decreasing rate, and then monotonously decreasing at an increasing rate, for increasing values of the Q_n in question if $Q_n=Q_{n-1}$.

532 Solution by a graphical method

A method of determining the optimum rates of production is described in Appendix A. The simplifying assumptions of section 531 are used. A summary has been given in section 1333 and the method will not be described here[1]. It is a graphical method and the computations are assumed to be made manually.

The method can be used for practical optimizations if the simplifying assumptions can be accepted as fairly representative of the actual situation. However, it involves a considerable amount of computation and curve drawing. The payment functions should be very simple, and the number of zones small (say 3 or 4) if the amount of computation is not going to be prohibitive for a manual solution.

The optimization would be considerably simpler if a static model could be used. The graphical method has been used in an analysis of static optimization models as approximations of the dynamic model discussed here[2]. In the static models the rate of production was assumed to be constant throughout the production period of the deposit. The optimization problem was that of the actual decision time, i.e. T_{11}, and the optimum according to the static model was compared with that of the first zone according to the dynamic optimization. The analysis was based on a number of hypothetical examples. In many cases the two models yielded identical optima, but differences of the order 20 % were also encountered[3]. The general conclusion was that a dynamic optimization was safest as no general rules could be given to determine whether the static model gave the correct solution[4] (in comparison with the dynamic model). The analysis was confined to the case where the payment functions are independent of time. It was, however, inferred that the static approximation was less reliable if the payment functions varied with the time[5]. It is concluded that a more efficient dynamic solution is desirable.

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1) The use of tangents to a curve to find optima and the interpretation of curves pertaining to one period as functions of variables pertaining to another period are essential ingredients of the solution. Methods of the former type have been used by e.g. Hirshleifer (1958). The latter type of methods is a direct application of the optimization method described in section 342.

2) Norén (1967, pp. 105–121).

3) Ibid. p. 111.

4) Ibid. p. 121.

5) Ibid. p. 120.

533 Extension of the graphical method

5331 Introduction

Apart from the computational difficulties, the shortcomings of the graphical method are due to the simplifying assumptions enumerated in section 531. According to section 52 optimum is not affected by the simplifying assumption 1). Thus, the simplification need not be discussed further. The simplifying assumption 2) is concerned with payment functions and time, and is often practically significant. Solutions where the payment functions may vary with the time will be discussed in section 542. 3) and 4) constitute restrictions on the payment functions. If they are removed certain complications arise, which will be illustrated in section 5332. 5) has been made for convenience only, and is deleted. 6) will be discussed in section 5333.

The symbol n denotes the current zone, and if not otherwise stated $n \neq 1$, as the first zone is treated in a special way (see Appendix A). This will be taken for granted in the subsequent discussion in this chapter.

5332 Nonlinear expansion and contraction investments

The simplifying assumption 3) can be removed if the capital value at the decision time of the payments mentioned in 3) are added to the expansion and contraction investments on the condition that the capital value in question meets the conditions specified in 4). Also, 3) can be removed unconditionally if 4) is removed. However, to simplify the discussion, only the expansion and contraction investments will be mentioned in the subsequent sections.

If the simplifying assumption 4) is removed the expansion and contraction limits are no longer easily determined points of tangency. These limits and the constancy limits may be multivalued, e.g. functions of Q_{n-1} or Q_n. Several cases can be distinguished. The expansion investment may be a degressively increasing function of the expansion $(Q_n - Q_{n-1})$, but independent of Q_{n-1} and Q_n taken separately. Fig. 5:1 is an application of the technique described in Appendix A, and demonstrates the resulting complications (compare Fig. A:2 in Appendix A). Expansion curves substitute the expansion lines. If $Q_{n-1} = \gamma_1$ expansion to $Q_n = \gamma_3$ is equally profitable as expansion to $Q_n = \gamma_4$. If $Q_{n-1} < \gamma_1$ expansion to γ_4 is preferred to expansion to γ_3. If $Q_{n-1} > \gamma_1$ expansion to γ_3 is pre-

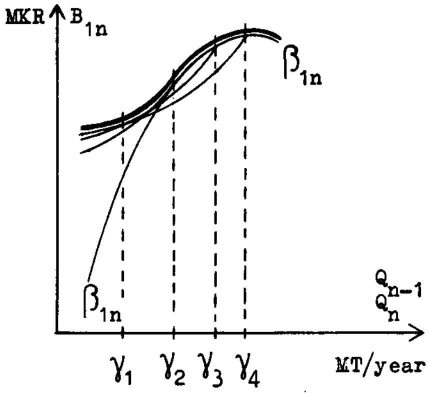


Fig. 5:1 Degressive expansion investment.
β_{1n}: see Appendix A (β_n), e.g. Fig. A:5. The other curves are expansion curves[1].

ferred. More exactly, from a given value of Q_{n-1} the optimum expansion is to the value of Q_n for which the slope of the curve β_{1n} is equal to the slope of the highest expansion curve at the given value of Q_{n-1}. The expansion limit is thus a function of Q_{n-1} and there may be values of Q_{n-1} for which the decision maker is indifferent to expansion to two or more alternative rates in zone n. For example, γ_1 is such a value of Q_{n-1} if no expansion curve exists at $Q_{n-1}=\gamma_1$, which yields a higher capital value than those to γ_3 and γ_4.

The lower constancy limit (γ_2) is the rate Q_n for which the slope of the curve β_{1n} equals the slope of the expansion curve at the origin of the latter, i.e. where $Q_n-Q_{n-1}=0$ (the point where the expansion curve joins the curve β_{1n}). It coincides with the lowest expansion limit. The optimum capital value B_{1n} expressed as a function of Q_{n-1} is the envelope of the expansion curves if Q_{n-1} is smaller than the lower constancy limit ($Q_{n-1} < \gamma_2$), and the curve β_{1n} if Q_{n-1} is larger (the heavy solid curve).

1) The expansion and contraction curves will appear in figures in the subsequent discussion without being explicitly identified as such. They should easily be recognized where they appear.

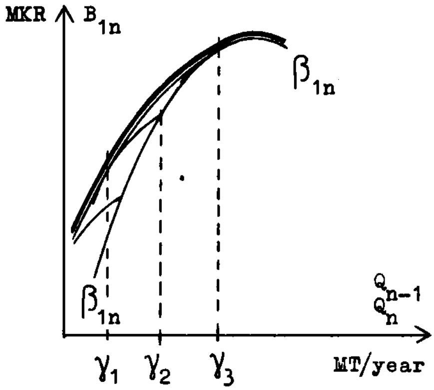


Fig. 5:2 Progressive expansion investment.

Similar conclusions may be drawn if the expansion investment is a progressively increasing function of the expansion (Q_n-Q_{n-1}). Fig. 5:2 contains an example. Here the lower constancy limit coincides with the highest expansion limit (γ_3).

The expansion investment has been assumed to be independent of the level of the rate of production, i.e. Q_n and Q_{n-1} taken separately, in the preceding examples. Fig. 5:3 demonstrates examples where the expansion investment is a linear (homogeneous) function of the expansion and a function of the rate of production after the expansion as well. In graph A a given expansion is more expensive the higher Q_n is after the expansion. In graph B a given expansion is less expensive the higher Q_n is after the expansion. The graphs do not need to be explained or commented further, as the previously described technique is utilized.

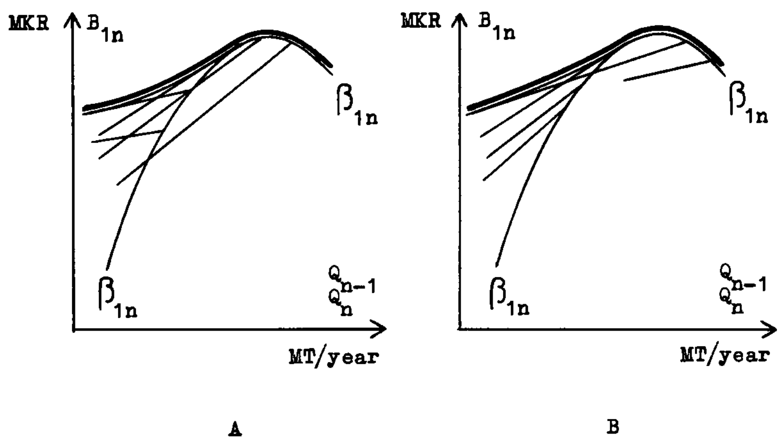
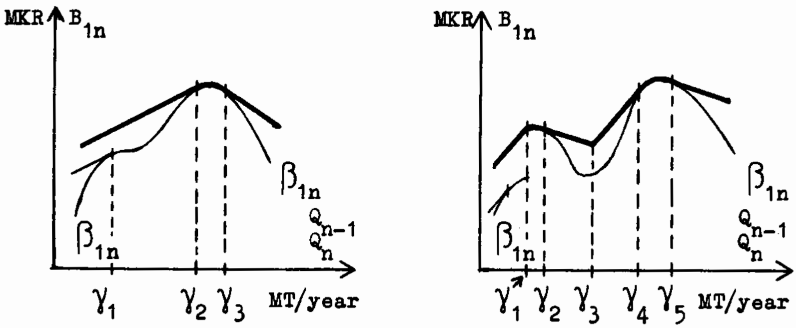


Fig. 5:3 Expansion investment dependent on the rate of production after the expansion.

Only expansion has been discussed but it is apparent that similar problems may occur regarding contraction and that similar graphical solutions can be found.

5333 Multimodality

If the simplifying assumption 6) is removed, situations as those illustrated in Fig. 5:4 may arise. Case A does not involve any serious difficulties. In case B, however, the expansion and contraction limits as well as the constancy limits depend on the value of Q_{n-1}.



A B

Fig. 5:4 Multiple tangential expansion and contraction lines.

It should be noted that 6) ensures that similar cases do not appear. Fig. A:6 (the curve β_2) and Fig. A:9 (the curve β_{N-1}) in Appendix A show, however, that the complications illustrated in Fig. 5:4 are inherent in the optimization method. The assumption cannot be accepted in a practical model. Furthermore, the capital value is composed of discounted payments which are added. The payment functions may be nonlinear functions of the decision variable. Hence, when they are added together (after discounting) they may result in a corresponding capital-value function which is multimodal with respect to the decision variable, i.e. a capital-value function which has more than one maximum (compare Wilde 1964, p. 6).

534 Conclusions

It is concluded that the graphical method can be extended and used in solving the more complicated problems which arise if the simplifying assumptions 3), 4), 5), and 6) are removed, although the amount of computations may become very large. In its original, simple form the graphical method is not general in application. Difficulties arise if the expansion and contraction limits and the constancy limits are not constant with respect to Q_{n-1}, i.e. if the decision rule yielding optimum decisions for zone n varies with the rate of production in the preceding zone (n-1). This problem has in principle been solved by extending the graphical method. Furthermore, the payment functions must be constant over time if the method is to be manageable (the simplifying assumption 2)). Finally, the graphical operations must be transformed into numerical operations in order to suit a program for a digital computer[1].

54 Iterative methods: Solution by repeated complete optimizations

541 Some definitions

The optimum rates of production are determined sequentially in the graphical method. The optimum values of Q_n were determined in the order n=N,N-1,N-2,...,1. The resulting values of Q_n for n=1,2,...,N form an optimal policy. The sequence of optimizations yielding an optimal policy will be called a complete optimization. The successive optimizations concerning the separate zones (or subzones if the average grade is being optimized) will be called partial optimizations. The optimum obtained for the current zone (or subzone) in a partial optimization will be called a partial optimum or the optimum of the current zone (or subzone). The results of the partial optimizations are the decision rules implied in the expansion and contraction limits and the two constancy limits.

542 Solution if payment functions vary with the time

5421 Choice of method

The problem to be discussed is how to introduce payment functions which vary with the time into the optimization. According to section 52 the problem is

1) The extension of the graphical method has previously been treated by the author (Norén 1967, pp. 91-96). Numerical solutions were not discussed there.

equivalent to re-introducing T_{1n} as a state variable[1]. A direct method of introducing T_{1n} into the optimization is to repeat the partial optimization for a number of alternative values of T_{1n} and use all the partial optima thus obtained as decision rules in subsequent optimizations. However, it is easily seen that the method grows complicated after a couple of partial optimizations, and leads to a large amount of computation.

Another method is to set arbitrary values to T_{1n} for n=2,3,...,N (T_{11} is a given constant) and determine the partial optima assuming that the decisions concerning the current zone are made at these arbitrary times. A decision rule is first obtained for n=N. The rule is used in the optimization of the decision rule for n=N-1. The decision rules of zones N and N-1 are used in the subsequent partial optimization, etc., until the first complete optimization has been carried out. The values of T_{1n} for n=2,3,...,N+1 are then determined from the optimal policy obtained, and a new complete optimization utilizing these values is performed. New complete optimizations are made similarly until B_{11}, i.e. the capital value of future mining at the actual decision time T_{11}, ceases to increase.

The method has certain properties common with the single-factor procedure for experimental[2] optimization suggested by Friedman and Savage in 1947 and discussed by e.g. Ackoff (1962, pp. 385-387) and Wilde (1964, pp. 124 ff.)[3]. In the single-factor procedure one controlled variable is optimized while all other controlled variables are held constant. Then the next controlled variable is optimized, all others again being held constant, etc. All the controlled variables are optimized in this way. The procedure is applied repeatedly until an optimum is found[4]. The method suggested differes from the single-factor procedure in an important respect. In reality one set of variables, the decision times, are fixed when the decisions are made concerning the other set of variables, the rates of production. Hence the procedure can only be used to a limited extent in that an optimum decision rule can be determined for a given

1) Note that the simplifying assumption 5) in section 531 has been removed. Thus, the subscript indicating the subzone has been re-introduced.

2) The optimization method is experimental as it involves experimentation on the model, i.e. simulation (Ackoff 1962, pp. 346 and 385).

3) Wilde calls the method the sectioning method. Reference: See Ackoff or Wilde (ibid.).

4) The method is ineffective for certain kinds of sharp ridges on the surface constituted by the capital-value function. Then the optimum cannot be reached by this method (Wilde 1964, p. 125). An example of such a ridge is found in Fig. 5:7 (section 5432), the diagonal between the two constancy limits.

decision time whereas it is irrelevant to try to determine the optimum decision time for a given decision rule (the restriction in the principle of optimality). In addition, it should be observed that the optimization criterion changes successively from one partial optimization to another as the remaining ore reserve changes. The goal function changes in content. Thus, the relevant iterations are those comprising the entire optimal policy, and not those made in order to determine an individual optimal policy.

5422 Description of the method

The second method outlined in the preceding section will be used, and needs to be described in more detail. The approach can be discussed by means of a hypothetical example. The contour map in Fig. 5:5[1] shows the contours of the surface formed by the capital-value function which determines B_{1n}. The optimization in zone n is first made on the assumption that $T_{1n}=\gamma_0$ (the arbitrary value). The decision rule which is to be optimized, i.e. the four limits, is represented by the variable $(\text{Rule})_{1n}$. The decision rule is assumed to be independent of Q_{n-1} and Q_n taken separately[2]. The contours indicate the capital value at time T_{1n} of future mining, i.e. B_{1n}, if the decision rule indicated by $(\text{Rule})_{1n}$ is followed in determining Q_n and the corresponding rules are followed in subsequent decisions at times $T_{1,n+1}$, $T_{1,n+2}$, ..., $T_{1,N}$.

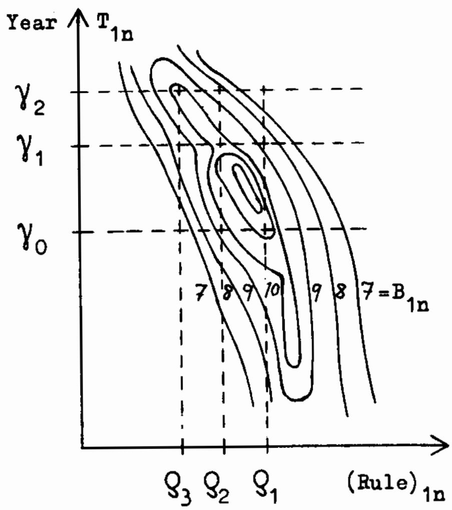


Fig. 5:5 Decision times taken into account through repeated complete optimizations.

In the partial optimization of zone n in the first complete optimization the decision rule Q_1 is found to be optimal. It is thus used to determine Q_n in the remaining optimizations where $(\text{Rule})_{1i}$ for i=n-1,n-2,...,2, and Q_1 are

1) This type of graphical representation has been used by e.g. Wilde (1964, p. 68) in his discussion of optimization methods.

2) The simplifying assumption 4) in section 531 and, according to the conclusion in section 5333, also the simplifying assumption 6).

determined. In these optimizations the value of T_{1n} varies as the values of the various Q_i are varied in the course of the experimentation. Q_1 is not optimal for all the cases which arise during the optimization procedure, but is used in spite of this. The errors thus introduced should be corrected in a second complete optimization.

The first complete optimization is assumed to have yielded an optimal policy where $T_{1n}=\gamma_1$. The tentatively optimal decision rule Q_1 is inoptimal as it yields the capital value B_{1n} indicated at the point (Q_1, γ_1), which could be increased to a maximum by using Q_2. The latter is the rule obtained and used in the second complete optimization. When the second complete optimization has been carried out three important questions must be answered: Did B_{11} increase? Is still $T_{1n}=\gamma_1$? Is the optimum approached?

5423 Behaviour of the capital value

B_{11} need not necessarily increase because the principle of optimality, as interpreted in this study, introduces a restriction on the optimizations (section 342). Except for such cases where the restriction is effective, however, B_{11} should increase if $\gamma_0 \neq \gamma_1$ (Fig. 5:5). The following discussion may clarify the matter. The partial optimizations in zones n and n-1 form a part of the second complete optimization where T_{1n} has been assumed to equal γ_1 when the second rule, i.e. $(\text{Rule})_{1n}=Q_2$, was determined. A change in $(\text{Rule})_{1,n-1}$ (an experiment) causes T_{1n} to change in value, e.g. to γ_2. This incurs a reduction[1] in the value of B_{1n}, as the decision rule leads to a choice giving the value of B_{1n} indicated at the point (Q_2, γ_2). The change in $(\text{Rule})_{1,n-1}$ is an improvement only if it results in an increase in $B_{1,n-1}$, i.e. if the decrease in B_{1n} discounted to time $T_{1,n-1}$ is more than balanced by an increase in the capital value of zone n-1[2]. Only then the alternative decision rule for zone n-1 can be optimal. The decision rule for zone n influences $B_{1,n-1}$ only in this way. Hence an improvement as seen from time T_{1n} in the decision rule $(\text{Rule})_{1n}$ can

1) An increase in B_{1n} may also occur. The increase is then smaller than it would have been if the optimal set of decision rules for the new value of T_{1n} had been used (compare a relatively small decrease in T_{1n} from γ_1 in Fig. 5:5).

It should be noted that B_{1n} must be recalculated for each new alternative decision rule in zone n-1. In Appendix A, where the payments were independent of the value of T_{1n}, a given curve expressing B_{1n} as a function of Q_{n-1} could be used (e.g. the curve α_3 in Fig. A:5). In the present case one such curve exists for every value of T_{1n}.

2) The interest on B_{1n} during the time between γ_1 and γ_2 is disregarded in this explanation.

only increase the capital value $B_{1,n-1}$. This holds true for successive values of n, i.e. for n=N,N-1,N-2,...,1. Thus, B_{11} increases for successive complete optimizations, or is constant, if the restriction in the principle of optimality is not effective (compare section 342).

The conclusion is not valid for B_{1n} if $n>1$. B_{1n} may be reduced if the reduction is compensated in zones mined previous to zone n, i.e. in zones 1, 2, ..., n-1. The step from Q_1 to Q_2 illustrates the point.

5424 Convergence

The question whether $T_{1n} = \gamma_1$ after the second complete optimization has been carried out, is a question of convergence[1]. It is pertinent also with respect to the decision rules. The first complete optimization yields the final solution in the simple case treated in sections 531 and 532 (see also Appendix A). Assuming that the payment functions vary discontinuously (annually) with the time and alternatingly towards increasing and decreasing payments (assumption 28)) no general answer appears possible[2]. Instead it will be accepted that the decision rules do not always converge and the computational methods, i.e. the computer programs, will accordingly be formed so that it is indicated whether the rules converge or not[3]. As B_{11} may decrease for successive complete optimizations (according to the restriction in the principle of optimality) no general rules for selecting an optimal policy can be given if the rules do not converge. The "optimization" is then merely explorative, and yields a basis for applying other methods, including the judgement of the decision maker.

An operational criterion of convergence is that the optimum rate of production and the four limits constituting the decision rule of each zone in one complete optimization are equal to the corresponding rates and limits in the preceding complete optimization. If the series of complete optimizations is interrupted

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1) The conditions for convergence in dynamic-programming solutions have been discussed by Bellman (1957).

2) A number of examples have been solved in testing the program EXRATE which contains the model. Converging decision rules have been found in all cases, usually after less than three complete optimizations. However, the testing is incomplete as the examples cover only a few possible situations.

3) Intermediate output data are printed by the programs. This enables the researcher to observe the performance of the programs at the different stages of the optimizations. See Appendix B.

prematurely[1], it may be possible to deduce whether the optimizations are convergent. A new effort may also be made by repeating the procedure using the last complete optimization as a starting point.

Certain new potentially optimal alternatives may have to be evaluated if the optimizing method fails. This can be done independently of the optimization programs by means of the program CAPVAL. In making conclusions from such alternatives it should be observed that the restriction imposed by the present application of the principle of optimality must be respected.

5425 Optimum and suboptima

If the decision rules do not converge the method generally does not yield an optimum. There remains to be established whether an optimum is actually obtained if the decision rules converge[2]. This is undoubtedly the case if the simplifying assumptions of section 531 are made (only the simplifying assumptions 2), 4), and 6) are practically important). The payments may also increase or decrease by a uniform fraction per unit of time, e.g. ± 3% per annum, as this is equivalent to a decrease or an increase, respectively, in the rate of interest[3]. The method is not affected by this.

If the payment functions vary discontinuously or fluctuate with the time, more than one capital-value maximum may exist, but only one is found. If the search technique is strictly rising, the outcome of a search for maxima on a multimodal capital-value surface (the response surface) depends on where the search starts (Wilde 1964, p. 88). The present technique is not strictly rising as B_{1n} may decrease (the movement from (ϱ_1, γ_0) to (ϱ_1, γ_1) in Fig. 5:5 owing to the first complete optimization) but this does not involve any systematic search for alternative maxima, and the outcome of the search depends on the starting point. For this reason the arbitrarily selected decision times used in the first complete optimization must be exchangeable. Then new series of

1) The interruption rules are described in section 222 (the variables DELTA3 and JTOTMX) of Appendix B.

2) The uniqueness of optima of dynamic-programming problems has been discussed by Bellman (1957).

3) If u is the relative increase per unit of time in payments (reckoned continuously) and j is the continuous rate of interest, a discounting rate p is derived from the present-value calculation as follows:

$$e^{-j\cdot t}\cdot e^{u\cdot t}=e^{-(j-u)\cdot t}=e^{-p\cdot t}$$

$$p=j-u$$

complete optimizations can be made for varying initial decision times, and compared. The importance of the problem of selecting the best optimum can be evaluated by means of the different optimizations, and the number of further experiments determined accordingly[1]. The solution giving the highest capital value B_{11} is selected as the best solution available.

The relevant value of B_{11} is that of the last complete optimization in each series of complete optimizations because this is the alternative which best meets the conditions imposed by the restriction in the principle of optimality (section 342). Convergence has been assumed. Then the partial optimizations in the preoeding complete optimizations are made on more erroneous assumptions concerning the state at each current decision time than the last complete optimization.

543 Solution if the decision rule for zone n depends on the decision in zone n-1

5431 Description of the method

The graphical methods described in section 533 made it possible to solve problems where the decision rule for zone n depends on the rate of production in the preceding zone. The simplifying assumptions 3) to 6) in section 531 could be removed. However, the graphical methods in question can only with difficulty be transformed into purely numerical methods. Some other solution is desirable.

The method of repeated complete optimizations offers an alternative. The arbitrarily selected decision times used in the first complete optimization imply a single-valued set of rates of production, or Q_n for n=1,2,...,N (expressions (4.5) - (4.7) in section 46). This set of rates will be called the initial guess. In the second complete optimization the initial guess is replaced by the first optimal policy, i.e. the optimal policy obtained in the first complete optimization. In a third complete optimization the second optimal policy serves in place of the initial guess, etc. To start with the more general case, a complete optimization which is not the first one will be treated in the beginning. The optimal policy determined in the latest complete optimization forms a natural starting point in the current complete optimization. The optimal policy contains an optimum value of Q_{n-1}. The decision

1) The problem of evaluating sequential experiments has been discussed by e.g. Ackoff (1962, pp. 361-363) from the point of view relevant for this question.

rule for the current zone n will be determined for the optimum value of Q_{n-1} to the extent this is possible. Another approximation will be used where this is not possible.

The method can be described by means of the type of graphs known from Appendix A. The curve β_{1n} in Fig. 5:6 shows the capital value B_{1n} if the values of $Q_{n+1}, Q_{n+2}, \ldots, Q_N$ are optimized, but on the assumption that $Q_n = Q_{n-1}$ (section 4 of Appendix A). The latest complete optimization has yielded $Q_{n-1} = \gamma_1$ as an optimum. This is the starting point. Only expansion is economically relevant here. Expansion to $Q_n = (EL)$ yields maximum capital value. EL is the expansion limit[1] if $Q_{n-1} = \gamma_1$.

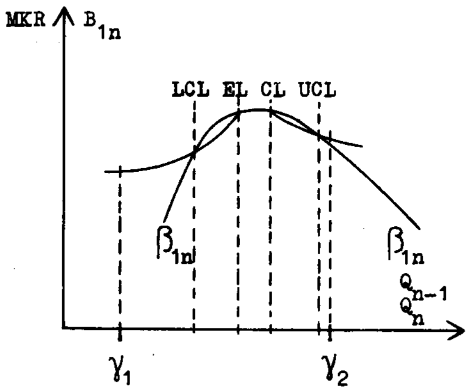

Fig. 5:6 Approximate decision rule for zone n.

The expansion curve to (EL) is relevant for expansion to (EL) from all values $0 \leq Q_{n-1} < (EL)$. Hence, the capital value increases only if $Q_{n-1} < (LCL)$, and LCL is the lower constancy limit if the expansion ends at $Q_n = (EL)$ independently of the actual value of Q_{n-1}.

The problems concerning contraction remain to be solved. As γ_1 is too low to permit efficient contraction of the rate of production, a new value of Q_{n-1} must be found. Assume for the present moment that a value $Q_{n-1} = \gamma_2$ has been found. The contraction limit is then CL, and the upper constancy limit UCL (Fig. 5:6).

The value selected for Q_{n-1} influences the optimization, i.e. the determination of CL and UCL. γ_1 is the best available estimate of the value Q_{n-1} will have when the value of Q_n is actually determined. It is therefore assumed that if contraction takes place, the contraction will be made from a fairly low value of Q_{n-1}. In order to be of interest, however, the latter should be greater than the contraction limit. The upper constancy limit determined in the latest complete optimization is greater than or equal to the contraction limit obtained in

1) (EL) denotes the value of Q_n which forms the expansion limit EL. Similarly: (LCL) and LCL, (CL) and CL, and (UCL) and UCL.

that complete optimization. Hence, in many cases it can be expected to have, or to be close to a value which has this property. The value of Q_{n-1} is thus set equal to the upper constancy limit of the complete optimization which precedes the current one, plus a given constant[1]. The constant added reduces the number of cases where the value selected lacks the specified property of being greater than the contraction limit which is searched for.

A corresponding rule is used if the optimum value of Q_{n-1} exceeds the upper constancy limit, the values of both variables being those of the latest complete optimization. Then the current determination of the contraction limit is made for the optimum value of Q_{n-1}, and the upper constancy limit is determined correspondingly. The contraction limit is determined for a value of Q_{n-1} which equals the lower constancy limit previously determined in the latest complete optimization, minus a given constant[2].

The starting-point values of Q_{n-1} corresponding to γ_1 and γ_2 in Fig. 5:6 are set equal to the previously determined lower constancy limit, minus the constant, and upper constancy limit, plus the constant, respectively, if the optimum value of Q_{n-1} should fall in between the two limits, or equal one of them. In all other cases either the expansion limit or the contraction limit is determined with Q_{n-1} equal to the most recent optimum value of Q_{n-1}. The constancy limits are then determined as described.

A previously determined optimum value of Q_{n-1} and previously determined constancy limits do not exist when the first complete optimization is made. The initial guess concerning Q_{n-1} is used as regards the determination of the expansion limit in zone n. For programming reasons a comparatively high arbitrary value of Q_{n-1} is selected for the determination of the contraction limit. The constancy limits are determined as before, i.e. along the expansion and contraction curves to the respective expansion and contraction limits which have just been determined. Furthermore, a test is performed to ensure that the two values of Q_{n-1} are reasonably distant from the expansion limit and the contraction limit obtained, and that the former values enclose the limits. A new, more distant value is tried if any of the values selected proves to be too close to the relevant limit. The procedure is repeated until satisfactory values have been found[3].

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1) The constant is 4·DELTA1. The parameter DELTA1 is specified in the input data of the program (sections 321 and 322 of Appendix B).

2) DELTA1.

3) The procedure is described in more detail in section 552 and in Appendix B (section 322, the variable M). Note that what is "reasonably distant" (above) is specified by the values of the parameters M and DELTA1.

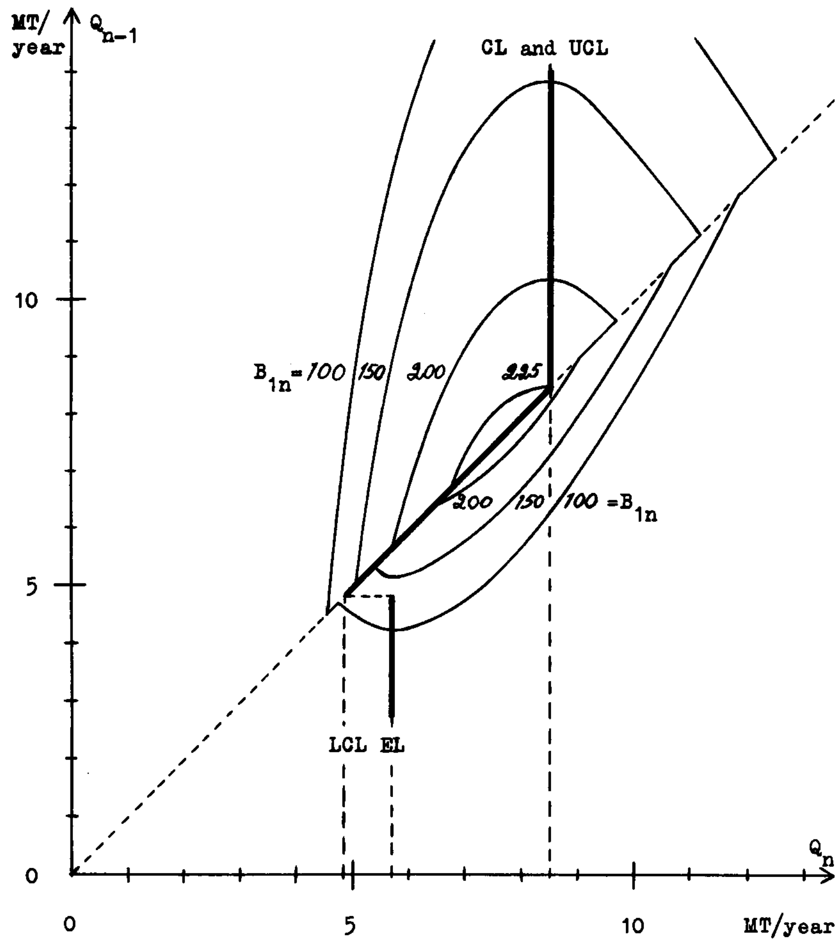
The method described yields an approximation of the graphical solutions given in Fig. 5:1, Fig. 5:2, Fig. 5:3, and Fig. 5:4. The expansion and contraction limits vary with Q_{n-1} there. They are here approximated by two fixed limits, i.e. EL and CL, respectively, in Fig. 5:6. The constancy limits are constants in both types of solutions, but they are only approximated here (LCL and UCL in Fig. 5:6).

Finally, it should be observed that the expansion and contraction investments have been treated as if they were the single types of payments depending on the value of Q_{n-1}. This has been made for convenience only. The method suggested easily copes with the extended interdependence (compare the beginning of section 5332).

5432 Nature and importance of the approximations

The nature and importance of the approximations introduced by the method described in section 5431 can be studied by means of contour maps. Fig. 5:7 shows the case treated in Appendix A. The rates of production and the capital-value amounts are obtained from Fig. A:3 as regards the contraction limit and the upper constancy limit, and from Fig. A:7 as regards the expansion limit and the lower constancy limit. The contours are the locus of the points where the capital values B_n are of the values indicated. The diagonal $Q_{n-1}=Q_n$ indicates the alternatives where the rate of production is not changed. Above and to the left of the diagonal is a contraction area. Below and to the right of the diagonal is an expansion area.

There is a sharp ridge along the diagonal. Where this ridge marks the highest capital values attainable for given values of Q_{n-1} the optimum decision is to make $Q_n=Q_{n-1}$. This occurs between the two constancy limits LCL and UCL. If $Q_{n-1}<(LCL)$ a higher capital value is attainable if the rate of production is increased. A maximum is reached at $Q_n=(EL)$. This is the expansion limit. However, it should be observed that the capital value may decrease first, as in the figure, for small increases in the rate of production, because it influences the search for the maximum. It is similarly found that contraction to the contraction limit CL is the optimum action if $Q_{n-1}>(UCL)$. The heavy solid lines indicate the optimum values of Q_n for different values of Q_{n-1}. The optima are reached if the decision rule stated in section 6 of Appendix A is used.



<u>Fig. 5:7</u> Capital values B_{1n} for expansion and contraction at time T_{1n}. Expansion and contraction limits and constancy limits if expansion and contraction investments are linear functions of the change in the rate of production and independent of Q_{n-1} and Q_n taken separately.

Owing to the simplifying assumptions made in section 531 the expansion limit is equal for all values of Q_{n-1} if $0<Q_{n-1}\leq(LCL)$ and the contraction limit is equal for all values of Q_{n-1} if $Q_{n-1}\geq(UCL)$. On these assumptions the contour map and the limits are also independent of the decision time, i.e. of the value of T_{1n}, and, consequently, independent of the subsequent partial optimizations in the current complete optimization (section 532 and Appendix A). The contours change with the value of T_{1n} if the simplifying assumption 2) is relaxed. The ensuing difficulties have been discussed in section 542.

The more complicated cases discussed in section 533 can be illustrated similarly. The capital values, the expansion limits and the lower constancy limits from Fig. 5:1 and Fig. 5:2 are reproduced in Fig. 5:8. The contraction limits and upper constancy limits are drawn arbitrarily, but the following discussion naturally applies also to more complicated cases of contraction.

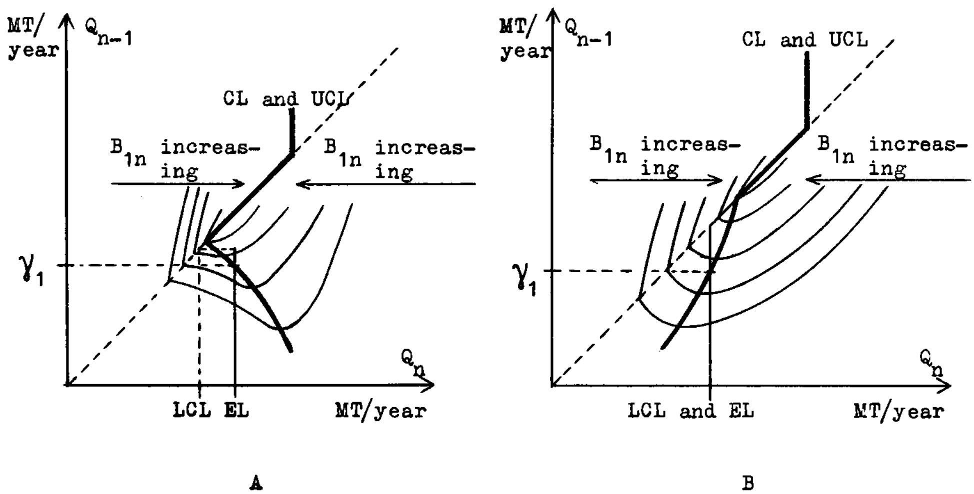


Fig. 5:8 Expansion limits in zone n dependent on Q_{n-1} according to Fig. 5:1 (graph A) and Fig. 5:2 (graph B).

The heavy solid lines indicate the optimum decisions concerning Q_n. The graphical solutions shown in section 533 yield a decision rule leading to these optima. The approximate solution described in section 5431 (Fig. 5:6) is based on the assumption that $Q_{n-1}=\gamma_1$. Approximate optima are determined with the decision rule defined by LCL and EL, the latter being the approximate lower constancy limit and expansion limit of Fig. 5:6. The line Q_n=(EL) for $0<Q_{n-1}\leq$(LCL) (graph A) and the diagonal line from the point ((LCL), (LCL)) to the point where the rates of production equal the correct lower constancy limit (light solid lines) indicate the decisions concerning Q_n which are made if the approximate decision rule is applied. Graph B is interpreted accordingly.

It is easily seen that the approximation is close to the actual optimum if Q_{n-1} is close to γ_1. As γ_1 is determined so as to be equal to or as close as possible to the optimum value of Q_{n-1} of the complete optimization preceding the current one[1], good approximations are obtained in the most important interval. Hence, the approximate decision rule does not appear to introduce any significant risk of erroneous optimizations or non-convergence if Q_{n-1} converges towards an optimum in repeated complete optimizations if the exact decision rule is used. Two areas may show tendencies of instability, i.e. where Q_{n-1} is close to LCL or UCL. Here small changes in Q_{n-1} may result in large changes in Q_n. However, the effect on subsequent partial optimizations is minor as by definition of LCL and UCL the capital value does not change in proportion[2]. For example, the capital value is equal for $Q_n=Q_{n-1}$ and Q_n=(EL) if Q_{n-1}=LCL (compare LCL and EL in Fig. 5:8, graph A).

The cases illustrated in Fig. 5:3 in section 5332 are of the same nature, and can be treated similarly. The same conclusions are reached.

The approximate solutions discussed refer to the cases of nonlinear expansion and contraction investments introduced in section 5332 where the decision rule for zone n depends on the decision in zone n-1. Such a dependence may also be caused by multimodality of the capital value B_{1n} when this is expressed as a function of Q_n with $Q_n=Q_{n-1}$. The case has been introduced in section 5333 and illustrated in Fig. 5:4 (graph B). The same type of approximate decision rule

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1) Except where the current complete optimization is the first one.

2) The subsequent partial optimizations are influenced only by the reduction in the capital value B_{1n} (in comparison with the actual optimum capital value).

is used to simplify the computational treatment of this problem as has been displayed above. Some new problems arise, which are illustrated in Fig. 5:9.

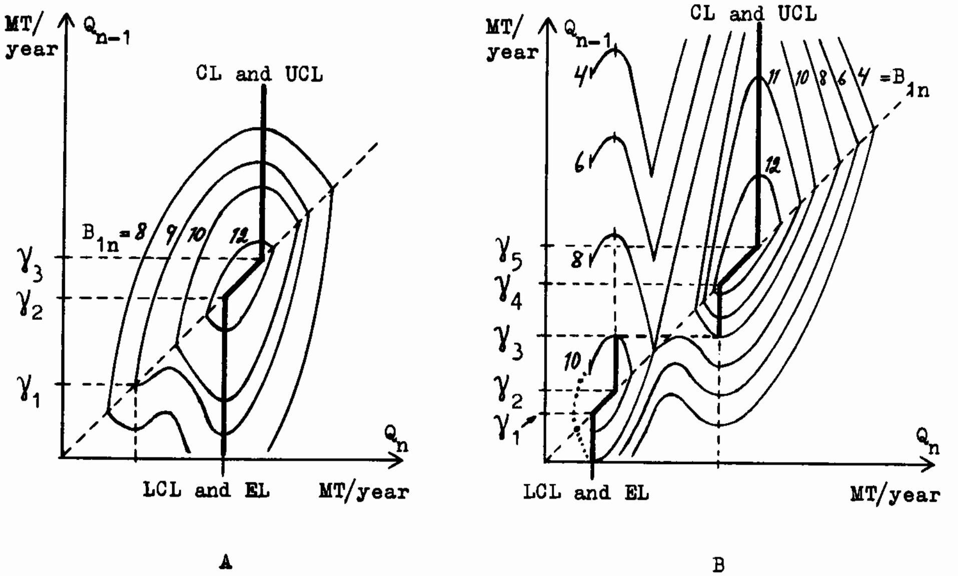

Fig. 5:9 Capital-value surfaces according to Fig. 5:4.

If $0 < Q_{n-1} < \gamma_1$ in graph A the capital value B_{1n} has two maxima for $Q_n > Q_{n-1}$, one at $Q_n = \gamma_1$ and the other at $Q_n = \gamma_2$. However, expansion to $Q_n = \gamma_2$ yields the higher capital value for all the relevant values of Q_{n-1}. γ_2 is the expansion limit. It is independent of the value of Q_{n-1}. New problems do not arise except that B_{1n} expressed as a function of Q_n may have two maxima. According to the method described in section 5431 (Fig. 5:6) the search for a maximum, i.e. for an expansion limit, proceeds from a preselected value of Q_{n-1}. The search must continue until all maxima have been found, and the highest maximum yields the desired limit. The heavy solid line indicates optimum.

The situation is more complicated in graph B of Fig. 5:9. The heavy solid lines indicate the true optima which are determined as before. It is found that the decision rule, i.e. the four limits, depend on the value of Q_{n-1}. One set of limits is found if the search is confined to the interval $Q_{n-1} > \gamma_3$, and another is obtained of $0 < Q_{n-1} < \gamma_3$. If both intervals are searched, both sets will be found. The approximate method will select only one set of limits, namely the limits LCL, EL, CL, and UCL in the figure[1]. Thus, the approximate decision rule will give $Q_n = Q_{n-1}$ for $\gamma_2 < Q_{n-1} < \gamma_4$ instead of the truly optimal values indicated by the heavy solid lines in this interval.

The approximation introduces an error into the optimization. Its influence on the capital value can be appreciated in the figure. The nature of the error will not change with repeated complete optimizations, unless the capital-value surface changes drastically. This may happen if the payment functions vary with the time. Then the approximate decision rule is selected more or less accidentally if a multimodel surface should be introduced after the first complete optimization in place of a formerly unimodal one, or if the true limits should be changed outwards (a considerably lower expansion limit or a considerably higher contraction limit). The choice depends on the previously determined decision rules.

However, the dependence on chance is mostly restricted to that complete optimization in which the drastic change is introduced. The limits are often automatically corrected in subsequent complete optimizations if no new drastic changes occur, except where gaps of the type represented by the difference between γ_1 and γ_4 or γ_2 and γ_5, are great. A tendency towards the extreme limits exists, although it is not to be relied on in all cases[2].

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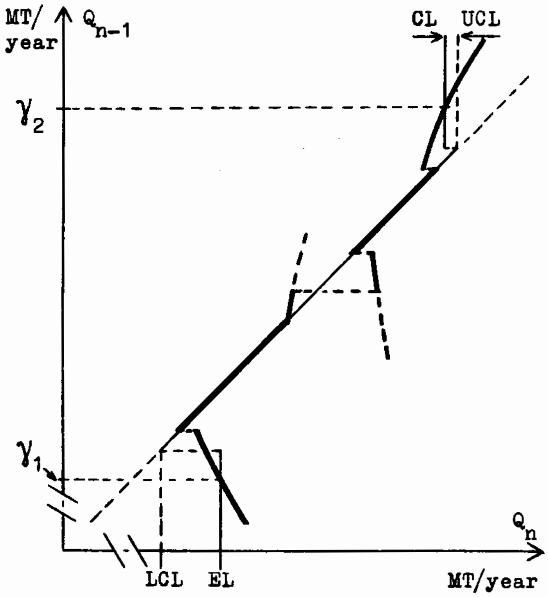
1) According to section 5431 a low value of Q_{n-1} is used in determining the approximate expansion limit, and a high value in determining the contraction limit. The constancy limits are determined by calculations based on the other limits. Hence the choice of the extreme limits. However, there is always the risk that the parameters M and DELTA1 are set too low. Then the program (EXRATE) will simply approximate the decision rule with one of the two sets of corresponding limits.

2) Compare Fig. 5:6 and observe the effect of $\gamma_1 >$ (LCL). Note that the new value of LCL after reduction by DELTA1 forms the γ_1 used in the partial optimization of zone n in the subsequent complete optimization.



The nature of the approximations made in the method suggested can be illustrated by a hypothetical example where various simplifications are brought together. Fig. 5:10 shows the true optima of such a case, and the approximate optima which are normally obtained if the simplified decision rules are utilized. The approximate limits are denoted LCL, EL, CL, and UCL, as before. The contour lines are excluded. By making the approximations it is possible to treat cases where the decision rules for zone n depend on the decision in the preceding zone, n-1.

Fig. 5:10 Approximations in the method suggested.
——— true optima
——— approximate optima
γ_1, γ_2: See section 5431 (Fig. 5:6)

55 Multimodality in the partial optimizations and computational methods

551 Maxima to search for

The technique of finding points of tangency and the other graphical techniques utilized in the solutions discussed in this chapter can be transformed into a search for maxima as regards the approximate expansion and contraction limits. This is easily seen from Fig. 5:7 and Fig. 5:8 if compared with Fig. 5:6. B_{1n} is the capital value at the decision time of future mining. For a given (low) value of Q_{n-1}, i.e. γ_1 in Fig. 5:6, the value of Q_n is determined for which B_{1n} is maximum. The value of Q_n is the approximate expansion limit. Then, for another given value of Q_{n-1}, i.e. the considerably higher value γ_2 in Fig. 5:6, another value of Q_{n-1} is determined for which B_{1n} is maximum. This is the approximate contraction limit.

The decision time T_{1n} has been assumed constant in the partial optimization with respect to the alternatives compared then, i.e. with respect to Q_n and the various values of Q_{n-1} forming the points of departure in the maximizations. Hence, the capital value B_{1n} can be substituted by B_{1n0} without affecting the optimization (section 45). Doing this the capital value maximized is recognized as that which is maximized in the optimization model (4.4) in section 46.

An approximate lower constancy limit is the rate of production $Q_n = Q_{n-1}$ which yields the same capital value (B_{1n} or B_{1n0}) as the expansion from that rate of production to the expansion limit[1]. Correspondingly, an upper constancy limit is the rate of production $Q_n = Q_{n-1}$ which yields the same capital value as the contraction from that rate to the contraction limit. A simple trial and error method is used in the search for constancy limits. It is described in section 321 (EX7) of Appendix B, and will not be treated here.

There is one optimization which has not been discussed, except for in Appendix A, i.e. to determine the optimum value of Q_1. Here the value of Q_{n-1}, i.e. Q_0, is a given constant (assumption 39)). Expansion limit, contraction limit, and constancy limits are not needed. It is sufficient to determine the value of Q_1 for which B_{110} is maximum. The value of Q_1 supplied by the initial guess or the latest complete optimization forms a natural starting point in the search for a maximum.

Three different problems involving a search for maxima have been stated, i.e. determining expansion limits, contraction limits and the optimum value of Q_1. In all cases (n=1 included) the maximum value of B_{1n0} expressed as a function of Q_n is desired. For this reason a common search method will be used. The capital-value functions in question may be multimodal with respect to Q_n. This complicates the search procedure as usual procedures for unimodal functions are not satisfactory[2]. The search is made in two stages, i.e. a first approximation and a second approximation.

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1) From here on the approximate limits are called the expansion limit, the contraction limit, and the constancy limits.

2) Wilde (1964, p. 13). Wilde discusses optimum methods for unimodal functions. No effort is made to optimize the search procedure in this study. The method suggested is the simultaneous method with uniform spacing (ibid. p. 22) repeated twice, i.e. in a two-stage sequential search. In addition, the number of experiments in the simultaneous method is determined by the outcome of the previous experiments, i.e. sequentially.

552 The first approximation

The principles of the first approximation are shown in Fig. 5:11. In optimizing Q_1 the value of B_{110} is first determined for Q_1 equal to the optimum value of Q_1 in the latest complete optimization or, if the current complete optimization is the first one, the value of Q_1 given in the initial guess. The value of B_{110} for lower values of Q_1 are then determined successively until a sequence of strictly decreasing values of B_{110} is obtained for successively decreasing values of Q_1. The process is then repeated from the original point of departure but for increasing values of Q_1. The search is here terminated when a sequence of strictly decreasing values of B_{110} has been obtained for successively increasing values of Q_1.

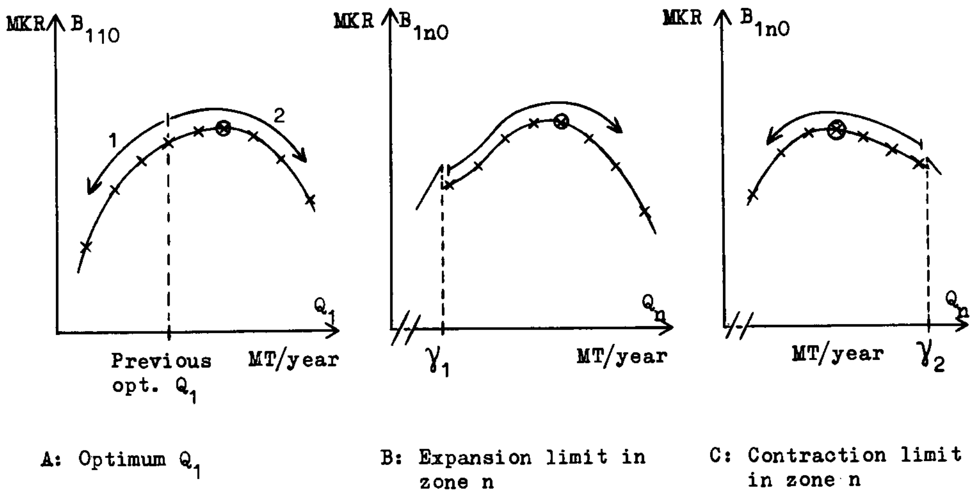

Fig. 5:11 Search for capital-value maxima.
γ_1 and γ_2 according to section 5431 (Fig. 5:6).
x observation ⊗ observed maximum.

The interval between the observations can be selected by the program user. It is the value of the parameter DELTA1. The number of observations yielding strictly decreasing values of B_{110} is another parameter to be determined by the researcher, i.e. M[1]. The parameters are discussed in sections 222 and 322 of Appendix B. Sections 221 and 321 in Appendix B (the subprograms CUT2 and EX2, respectively) contain further details of the method.

1) M=3 in Fig. 5:11.

The same method is used in determining the expansion and contraction limits. The search, however, proceeds only in one direction. The starting points in the search are $Q_n = \gamma_1$+DELTA2 and $Q_n = \gamma_1$-DELTA2, respectively. DELTA2 is a parameter whose value is small, which will be discussed in the next section. In addition, it is prescribed that at least the first M observations yield strictly increasing values of B_{110} in the first complete optimization (compare section 5431).

Complications arise owing to multimodality. The value of M must be great enough if the search is not to be interrupted prematurely. DELTA1 must also have a suitable value. Some types of errors are illustrated in Fig. 5:12. Two maxima will not be detected. The situation is not very probable but may exist. A remedy for these types of errors may be to make the optimization repeatedly, using various values of M and DELTA1. It should be noted that M and DELTA1 interact here. A smaller value of DELTA1 is desirable, but this may require a greater value of M. A great value of M causes extensive computation and makes the optimization procedure expensive[1].

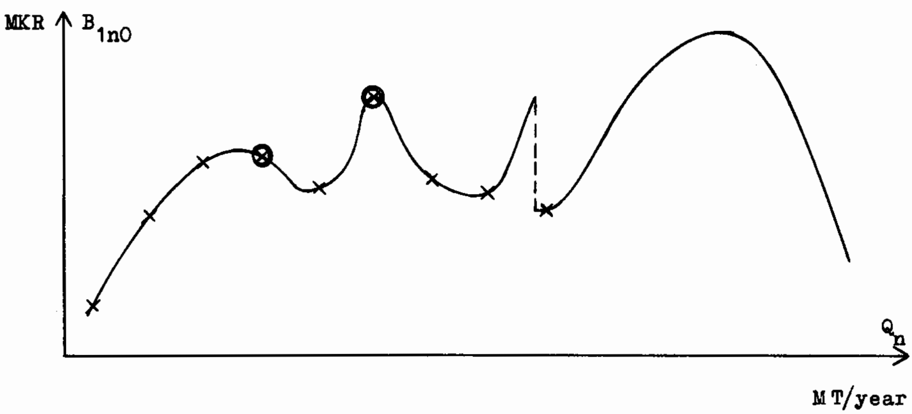

Fig. 5:12 Hypothetical example showing some errors which may be committed in the first approximation. M=3.

× observation ⊗ observed maximum

1) Recommendations as to the sizes of the parameters cannot be made here.

553 The second approximation

The second approximation is made on the principles illustrated in graph A of Fig. 5:11. The starting point is a maximum observed in the first approximation. The interval between observations is DELTA2 which is small in comparison with DELTA1 (say e.g. one-fifth or one-tenth), and can be determined by the program user (section 322 of Appendix B). The equivalent of M is a fixed constant in the program. (Section 225 of Appendix B, where the average grade is discussed, applies here.)

The second approximation is carried out around all maxima observed in the first approximation. The maximum giving the highest value of B_{1n0} is finally selected. The search for the maximum terminates here. Thus, DELTA2 determines the maximum precision in the optimization.

The discussion of the methods for determining optimum rates of production is now concluded. The methods described are used in the program EXRATE. A summary description will appear in Chapter 6. The program is described in section 3 of Appendix B.

56 Optimum average grade

561 Application of the optimization model for rate of production

The average grade can be optimized with methods similar to those proposed for the optimization of the average grade, provided that the interdependencies between decisions at various times are those specified in section 3445 and assumption 42), and that the other assumptions made in section 42 are valid as well. Expansion and contraction investments and other payments of similar character (compare the beginning of section 5332) are then replaced by the grade-change investments. The partial optimizations concern subzone n'n instead of zone n, $\bar{h}_{n'n}$ is the decision variable instead of Q_n, and $B_{n'n0}$ is the criterion instead of B_{1n0}.

The same optimization methods can accordingly be used. Only small changes in the program EXRATE would be necessary to adapt the program to the new decision problem. The preceding discussion of the optimization of the rate of production would apply. However, the present optimization problem can be solved by a simpler method. Hence, the necessary changes in the program EXRATE will not be discussed.

562 A simple solution

It was argued in section 3443 that the average grade is often more easily changed than the rate of production. Then the grade-change investments are small, at least for small changes in the average grade. Further, it was assumed in assumption 39) in section 42 that payments during the production period of one subzone are not very much influenced by the average grade in the preceding subzone, especially if the average grades in the two zones are fairly similar. The assumptions are based on the observation that the sorting plant, dressing plant, etc.[1] are normally flexible with respect to changes in the properties of the input ore within a certain interval. However, repeated small changes in one direction will accumulate, and the interval of flexibility will be left. Hence, the payments for one subzone depend on the average grade in several of the preceding subzones. In spite of this only the average grade in the preceding subzone is included in the payment model (assumption 39)). This yields a correct description of the reality behind the model if e.g. maintenance and replacements of parts of the plants can be utilized to achieve a successive adaptation of the plants to the changing average grade with small extra costs.

The payments influenced by the average grade are summarized in the equivalents of the expansion and contraction curves (compare section 5332). The small degree of dependence on the average grade in the preceding subzone[2], i.e. $\bar{h}_{n'-1,n}$, leads to the situation illustrated in Fig. 5:13. The optimum change is to $\bar{h}_{n'n} \approx \gamma_3$ if $\bar{h}_{n'-1,n}$ is close to γ_3, i.e. if the change is small. Then the equivalents of the expansion limit, the contraction limit, and the two constancy limits all coincide at the value γ_3. This will be utilized to simplify the optimization method.

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1) As the rate of production is measured in tons of crude ore, a change in the average grade of the ore does not influence the utilization of equipment for drilling, loading, hauling, crushing, hoisting, etc. to the extent that these activities concern the crude ore (compare Fig. 1:1). There are exceptions but they are assumed to be insignificant. For example, a lower average grade implies longer transports as the mining is more extensive (Fig. 1:3), the low-grade ore may have another volume/mass relation, another mechanical structure, etc. than a high-grade ore, which may cause extra payments if the grade is changed.

2) Henceforth the special case where $\bar{h}_{n'-1,n}$ is replaced by $\bar{h}_{N',n-1}$, i.e. where $n'=1$, will not be indicated explicitly.

If the four limits coincide at γ_3, the optimum decision is always to make $\bar{h}_{n'n}=\gamma_3$. This decision rule substitutes the more complicated decision rule used in the optimization of the rate of production. No other changes are made in the original method (section 561). An initial guess is made, which comprises $\bar{h}_{n'n}$ for n'=1,2,...,N' and n=1,2,...,N. The subzones are optimized successively, starting with subzone N'N and continuing with subzones N'-1,N, N'-2,N, etc. The optimum value of $\bar{h}_{n'n}$, i.e. the equivalent of γ_3 in Fig. 5:13, is determined for the value of $\bar{h}_{n'-1,n}$ which is given by the initial guess if the first complete optimization is being made, or by the preceding complete optimization in other cases. The method is the one described for optimizing Q_1 in section 552 (Fig. 5:11, graph A). Complete optimizations are made repetitively as described in sections 542 and 5431.

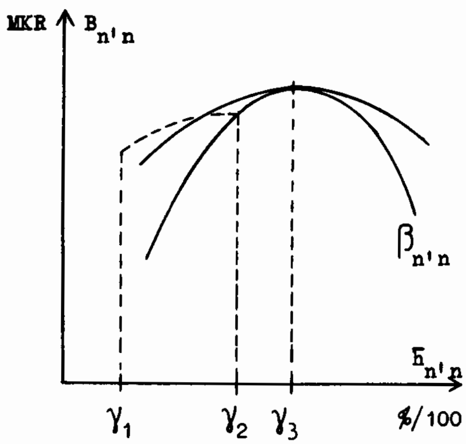

Fig. 5:13 Changes in average grade at time $T_{n'n}$. $\beta_{n'n}$: see β_n in section 4 of Appendix A (adjustments refer to grade here, not rate).

The program CUTOFF is constructed on the basis of the method indicated. It is described in section 2 of Appendix B.

563 Nature and importance of the approximations

The method yields approximate optima. If e.g. $\bar{h}_{n'-1,n}=\gamma_1$, the optimum value of $\bar{h}_{n'n}=\gamma_2$ (Fig. 5:13). $\bar{h}_{n'n}=\gamma_2$ will be kept throughout the current complete optimization, even if, e.g., the optimum value of $\bar{h}_{n'-1,n}=\gamma_3$ is obtained in the next partial optimization. Fig. 5:14 illustrates the matter. $B_{n'n}$ is calculated on the assumption that the optimum average grades have been determined for the subsequently mined subzones, and are implemented. The heavy solid line indicates the optimum decisions concerning $\bar{h}_{n'n}$ as a function of $\bar{h}_{n'-1,n}$. A

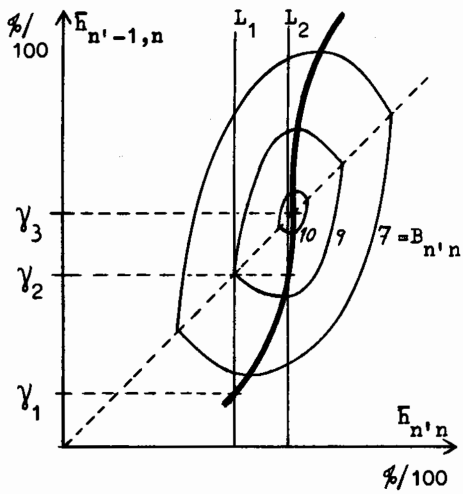

Fig. 5:14 Capital values $B_{n'n}$ for combinations of average grades in the subzones n'-1,n and n'n.

decision rule following this line would yield the true optimum. Instead, the approximate decision rule $\bar{h}_{n'n}=L_1$ is followed, which has been determined for $\bar{h}_{n'-1,n}=\gamma_1$ in the partial optimization of subzone n'n. This is correct if the next partial optimization yields $\bar{h}_{n'-1,n} \approx \gamma_1$. On the other hand, if the optimum value of $\bar{h}_{n'-1,n}$ proves to be e.g. γ_2, the rule causes an error. However, the error is corrected in the next complete optimization where the optimum is then found to be $\bar{h}_{n'n}=L_2$. Hence, on the given assumptions the optimization converges towards an optimum in the same manner as the optimization of the rate of production, and with the same exceptions.

If a change in the average grade is not as easily made as has been assumed, two separate constancy limits will exist, the lower and the upper. The optimization still converges towards an optimum, except for the case where there is a fixed payment to be made for each change or if a small change is very expensive, whereas greater changes incur comparatively small additional expenses (see e.g. Fig. 5:7). Such payments occurring in subzone n'n do not involve any difficulties in the optimization of $\bar{h}_{n'n}$ for a given value of $\bar{h}_{n'-1,n}$. The difficulty arises in the subsequent optimization of $\bar{h}_{n'-1,n}$ for a given value of $\bar{h}_{n'-2,n}$. The curve $\beta_{n'-1,n}$ will have a peak for $\bar{h}_{n'-1,n}=\bar{h}_{n'n}=\gamma_3$ as shown in Fig. 5:15. $\bar{h}_{n'n}=\gamma_3$ is the approximate optimum just obtained. The peak is artificial, as a correct decision rule for subzone n'n would make $\bar{h}_{n'n}$ equal to $\bar{h}_{n'-1,n}$ within the interval delimited by the constancy limits for subzone n'n. The optimum $\bar{h}_{n'-1,n}=\bar{h}_{n'n}=\gamma_3$ is incorrectly determined for subzone n'-1,n.

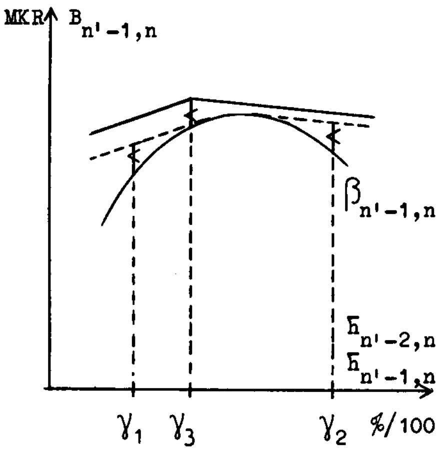

Fig. 5:15 Changes in average grade at time $T_{n'-1,n}$ if fixed payments occur for changes at time $T_{n',n}$. The fixed payments for changes at time $T_{n'-1,n}$ are ignored in drawing the equivalents of the expansion and contraction lines. ∤ The fixed payment.

The error may be corrected by repeated complete optimizations, but only if $\bar{h}_{n',n}$ and $\beta_{n'-1,n}$ change so much that the peak ceases to influence the optimization (illustrated in Fig. 5:15: if $\bar{h}_{n',n} < \gamma_1$ or $\bar{h}_{n',n} > \gamma_2$). If the fixed payment for a change at time $T_{n',n}$ or its equivalent in the case where a small change is comparatively expensive, is small, the interval $\gamma_1 - \gamma_2$ (Fig. 5:15) is small and the effects of the error correspondingly less serious. To avoid the error the original method (section 561) must be reverted to.

57 Simultaneous optimization of rates of production and average grades

The problem of simultaneous optimization of rates of production and average grades will here be solved by applying a method which is selected because it is obvious when the two programs EXRATE and CUTOFF are available. Many other methods can be used and some of them are probably superior to this one.

For EXRATE a given set of average grades is assumed to exist, and for CUTOFF a given set of rates of production (assumption 15) in section 42). In addition, the initial guess provides values of the respective decision variables of the two programs. To simplify the discussion, rates and grades are said to constitute the initial guess in both programs. The method suggested is to make the two optimizations alternately, each time inserting the latest optimal policy as a new initial guess[1]. This is continued until the capital value ceases to increase or increases at too small a rate to motivate further optimizations[2].

1) The method has been suggested by Carlisle (1954, p. 607) and by Henning (1963, p. 57) for the case where only one decision time and two decision variables are involved.

2) This is a problem of optimizing the search procedure, which will not be treated here. A similar problem has been discussed by Ackoff (1962, pp. 361-363).

The method can be viewed as an application of the single-factor procedure[1] with two decision variables, i.e. the optimal policy concerning the rate of production and the optimal policy concerning the average grade. This method is ineffective if sharp ridges of the type found in Fig. 5:7 (section 5432, the diagonal between the two constancy limits) exist on the capital-value surface formed by the capital value expressed as a function of the two policies. The optimization will terminate at either end of such a ridge or somewhere on the ridge depending on the initial guess. In addition to this weak point the method suffers the shortcomings of the two separate methods.

Whether ridges of the indicated type generally exist is not known. It may, however, be found out in each particular case by repeating the procedure with widely separated first initial guesses. Equal optima are arrived at if the source of error discussed is not at hand. If different optima are obtained initial guesses in between the original ones may be tried. This will yield a fairly good solution, but not the optimum one.

By using the method the dynamic aspects are not taken into account as regards the combination of rate and grade at each particular moment. In order to do this the method should be applied repeatedly in the following manner:

1) As if the <u>actual</u> decision time (section 3444) were T_{1N}, i.e. for zone N only.

2) As if the actual decision time were $T_{1,N-1}$, i.e. for zones N-1 and N only. The optimum decision rules obtained through execution of point 1 are used for zone N.

3) As 2) for $T_{1,N-2}$, and using the results of 1) and 2)

etc.

This method is not developed here as it would require so long computer times and so much administrative work that it would probably be impracticable in most situations. The simpler method yields only an approximate optimum. The nature and extent of the approximation will not be expounded in this study.

58 Other optimization problems

Remaining optimization problems are 1) those concerned with mining limits in other dimensions than the average grade, 2) technology, and 3) refinement

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1) Section 5421.

level. It is assumed that the number of alternatives considered is finite and small, or that the results obtained concerning a small number of alternatives yield a sufficient precision from the point of view of the decision maker.

An example of the latter case may be found in the problem of determining where to stop mining an ore deposit which extends to a great depth. The partitioning into zones being given, the following alternatives may be considered: Mine 14 zones, mine 13 zones, mine 12 zones, or mine 11 zones. If e.g. the mining of 13 or 12 zones proves to yield about the same capital values whereas the mining of 14 or 11 zones yields lower capital values, it is concluded that the optimum is somewhere in the neighbourhood of 12 and 13 zones. The decision maker is assumed to accept this precision. If he does not, which may happen where other decision variables are concerned, new alternatives are constructed and evaluated, alternatives which are close to the best ones according to the first optimization.

The method suggested is to optimize each alternative by means of EXRATE and CUTOFF and to select the alternative giving the highest capital value. If either the rate of production or the average grade is not influenced by the choice, CUTOFF or EXRATE, respectively, is used. If neither rate nor grade is influenced, the program CAPVAL can be used. CAPVAL calculates the capital value of one or more given alternatives (section 4 of Appendix B)[1].

The description of the alternatives must be made in terms of the coefficients defining the ore-reserve model and the payment models (sections 43 and 44) and the parameters for stating equivalent average grades (HEQV(N)), equivalent ore reserves (RES(NS,N)), the number of zones (NMAX), and the number of subzones (NSMAX). Hence, a new set of parameters and coefficients must be determined for each alternative.

The method has the disadvantages of the methods used in EXRATE or CUTOFF if either of these programs is used and, if they are combined, those mentioned in section 57. In addition, the often cumbersome determination of the coefficients of the payment functions must be made repeatedly. Only a few alternatives can be evaluated. It is practically impossible to make dynamic optimizations in this way. The advantages of the method are that the long-range effects on the ore

1) The method of combining the two main optimization models in evaluating the alternatives and the method of assuming rates and grades to be given and simply calculating the capital value of the alternatives, have previously been discussed by the author (Norén 1967, pp. 181-185) for solving similar optimization problems, although with considerably simpler models involved.

reserve, the production period of the deposit, and the timing of major investments (the zone investments) can be taken into account in the evaluation of the alternatives. Only three detailed models (the three programs mentioned) need to be constructed where more sophisticated solutions of the problems discussed at present would probably require as many detailed models as there are separate decision problems. It may be possible, however, to find a common procedure for solving three or more decision problems jointly. This problem is left for the future.

Simpler[1)] models can be used where the long-range effects are irrelevant for the decisions, e.g. in short-range production planning and minor investment decisions in a going concern.

Finally, some practical points may be made. CAPVAL should be used to a great extent in evaluating the alternatives, also where the rates of production and the average grades are decision variables. Several alternatives can be evaluated in one single run. CAPVAL uses far less time in the computer than EXRATE and CUTOFF. However, it yields only approximate evaluations as it does not optimize, but the nature and extent of the approximations are easily controlled by means of the two optimizing programs. It should also be observed that those optimization problems should be treated first, which are expected to be unsensitive to changes in the solutions of the other decision problems. The more sensitive optimizations can then be made for fairly accurate values of the more stable decision variables, which reduces the number of iterations[2)].

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1) Simpler with respect to the long-range effects. They may naturally be much more sophisticated in other respects.

2) It has been recommended by e.g. Ackoff (1962, p. 385) that the most important variables are optimized first in using the single factor procedure. This is an application of the same principle.

CHAPTER 6

6 Optimization models

61 Introduction

The decision problems to be solved have been stated in section 223. Different optimization models are needed for the various decision problems. They will be presented in this chapter. A full description of the optimization models contains the optimization methods as well as the general formulation. A general formulation of the two optimization models constructed for determining optimum rates of production and optimum average grades has been given in section 46. Solutions of these and other optimization problems have been derived and discussed in Chapter 5. Hence, this chapter is essentially a summary of Chapters 4 and 5.

The optimization models are based on the assumptions enumerated in section 42. Some important assumptions are that the mining company is the decision maker, that the company controls only one ore deposit, and that the rate of production, the average grade, and other decision variables may be changed instantaneously by any amount.

As a general rule it is recognized that the mining of an ore deposit is of advantage to the decision maker, and thus entered upon, if the capital value at the actual decision time is positive. If the deposit is already being mined at this point of time continued mining is advantageous if the capital value of future mining is higher than the capital value, discounted to the same time, of future payments occurring if the mine is closed immediately (compare assumption 38)).

62 Optimum rates of production

An optimum rate of production is determined for each zone according to the optimization model (4.4), subject to the restrictions (4.1) and (4.5) to (4.7), as described in section 46. A dynamic-programming solution is based on the graphical solution provided in Appendix A. The graphical solution is transformed into a numerical one according to section 55. By applying this solution (a complete optimization) iteratively[1], payment functions can be assumed, which

1) This is done automatically in the computer program.

vary with the time, and expansion limits, contraction limits, and constancy limits are permitted, which vary with the rate of production in the zone preceding the zone currently optimized (the current zone). The solution of these particular problems are discussed in section 54. The functions expressing the capital values of future mining as functions of the decision variables relevant at the various decision times[1] may be multimodal or contain discontinuities (sections 5432 and 55).

The optimization model is contained in the program EXRATE which is described in Appendix B.

The optimization model has some defects. It has not been established that the solution will always converge towards an optimum if the payment functions vary arbitrarily with time. In this case, if the solution converges towards an optimum, this optimum may be a suboptimum (section 542). Then, however, it may be possible to determine more than one optimum by varying the initial guess, and select the best optimum (section 5425). Multimodality in the capital-value functions (see above), drastic decreases in an expansion limit or a lower constancy limit, and drastic increases in a contraction limit or an upper constancy limit from one complete optimization to another may cause suboptimizations or errors (section 5432). The correct maximum is not always found where the capital-value function is multimodal. The risk of this depends to a great extent on the values of two parameters (M and DELTA1), which are determined by the program user for each particular optimization (section 552 and Appendix B).

The optimal policy determined by means of EXRATE is of the general shape shown in Fig. 6:1. The graph is based on a hypothetical example used in testing the program. The zones are all of the same size and the payment functions are constant over time.

The rate of production is constant during the production period of the first three zones. Massé's conclusion that the optimum action is to mine at a constant rate which equals the production capacity, is valid here (compare section 124). The rates decrease in the last zones. Gray's (1913, pp. 475–476) and Hotelling's (1931, p. 164) conclusion that the optimum rate decreases over time is valid here (compare section 47). Another of Massé's conclusions takes here

1) B_{1n} as a function of Q_n at time T_{1n} for n=1,2,...,N.

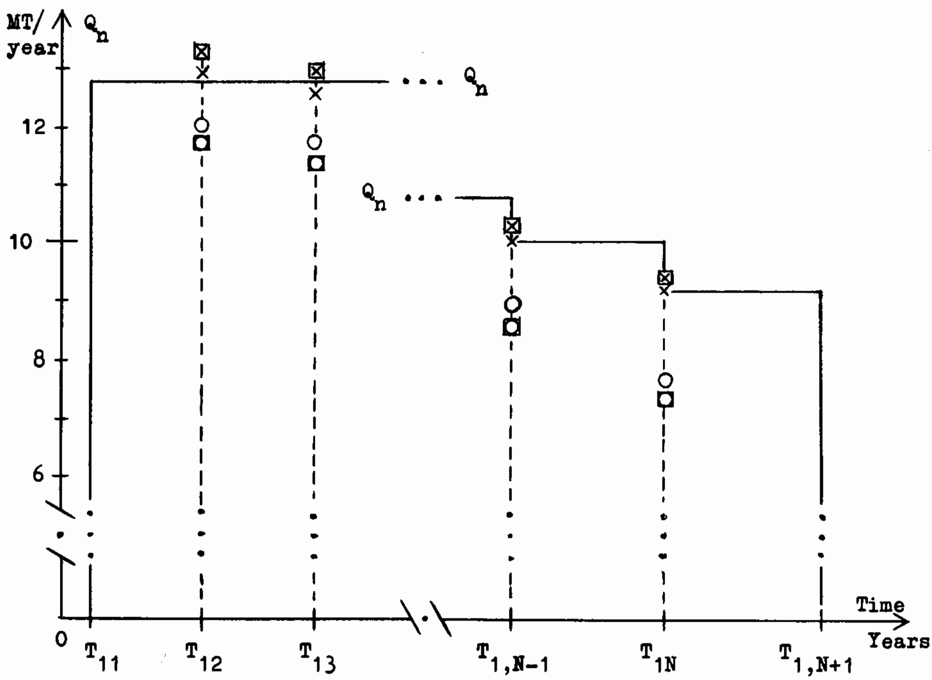

Fig. 6:1 General shape of the optimal policy: Rates of production

⊠ Upper constancy limit ◘ Lower constancy limit
× Contraction limit ○ Expansion limit

precedence over the one mentioned above, i.e. that a small ore reserve tends to yield a lower optimum rate of production than a larger reserve (section 124)[1].

1) Massé's conclusion is more precise and based on certain specific assumptions. The formulation given above is a generalization of his conclusion describing a mere tendency and the assumptions can be disregarded.

63 Optimum mining limits

631 Mining limits in various dimensions

Mining limits can be drawn in several dimensions. Measured in the average grade of the ore in a subzone the mining limit represents a geometrical delimitation as discussed in section 11 (Fig. 1:3), or a decision rule applied in loading at the working face (section 43), or some other form of continuous limit with the particular property of being able to be determined for each subzone.

An ore deposit may consist of more than one ore body or have narrow ore veins (section 11, Fig. 1:2). Which ore bodies and veins to be mined and which to be left in place is also a problem of determining mining limits. It may be termed the problem of subsidiary ore bodies.

An ore deposit may extend to a great depth. There may exist a level under which mining is not profitable. The mining limit to be determined is the optimum bottom level of the mine.

632 Optimum average grade

An optimum average grade is determined for each subzone of each zone according to the optimization model (4.3), subject to the restrictions (4.1) and (4.5) to (4.7), as described in section 46. A dynamic-programming solution is based on the principles indicated in section 62, i.e. those applied in optimizing the rates of production, except that a simplification is introduced. The simplification is based on the assumption that the average grade is comparatively easily changed from one subzone to another (section 562).

The optimization model is contained in the program CUTOFF which is described in Appendix B.

The optimization model has the shortcomings specified in section 62, except for that which concerns the four limits exclusively. In addition, if a fixed amount must be paid out each time the average grade is changed or if a small change is very expensive, the model may fail to yield a true optimum (section 563). The failure can be partly remedied by repeated applications with different initial guesses, or by using another model which is described in section 561, but which is not transformed into a computer program.

The optimal policy determined by means of CUTOFF is of the general shape shown in Fig. 6:2 which is based on the same example as Fig. 6:1[1]. There is a tendency for the average grade to decrease as time passes, i.e. a tendency towards more extensive mining limits. The tendency is in accordance with Henning's (1963, p. 57) results. The tendency is intensified in the later subzones of each zone,

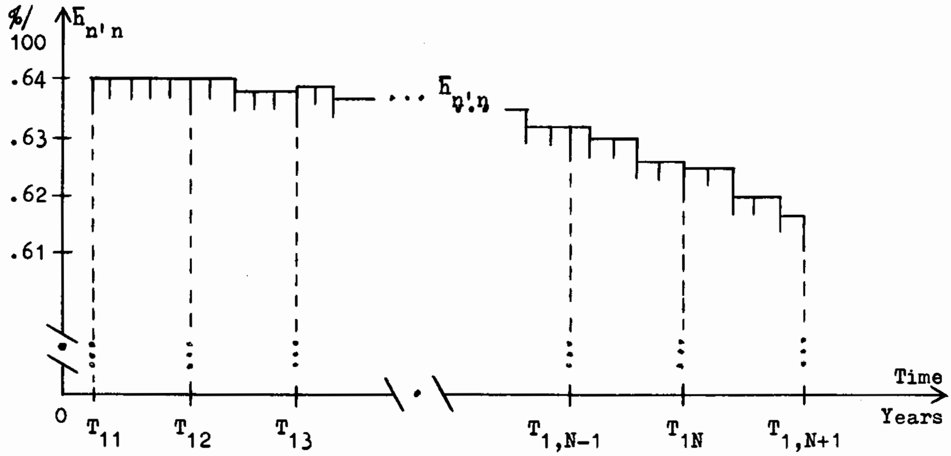

Fig. 6:2 General shape of the optimal policy: Average grades.

Starting time of a zone — Starting time of a subzone

which is in accordance with the general tendency. The latter is due to the fact that the ore reserve is used up so that the final closing of the mine, i.e. $T_{1,N+1}$, approaches as time passes. The opportunity loss of not being able to produce and sell an additional ton of ore at time $T_{1,N+1}$ is evaluated higher at the current decision time the closer $T_{1,N+1}$ is, because of the positive rate of interest. An additional ton of ore can be gained at time $T_{1,N+1}$ by mining an extra ton at the extensive limit at the current decision time. Thus, the increasing opportunity cost results in a decreasing average grade[2].

1) The two optimizations start from the same initial guesses as regards rates of production as well as average grades. The input data used in the actual computations contain certain systematical errors, the influence of which has been corrected in the graph.

2) In principle the argument follows Henning's argument (ibid. p. 56).

A similar argument can be put forward with respect to the ore reserve of each zone. The opportunity cost of ore at the end of the production period of the zone is influenced by the interest on the zone investment and consequently higher than the opportunity cost of ore immediately after the zone investment has been made. This explains why the tendency towards decreasing average grades is intensified in the later subzones of each zone, and reversed or very weak at the starting times of each zone.

633 Subsidiary ore bodies

An example of subsidiary ore bodies is given in Fig. 1:2. Another example is shown in Fig. 6:3. The problem is to determine which ore bodies to mine and which to leave in place. Some complementary assumptions must be made in addition to assumptions 4) and 5) in section 42.

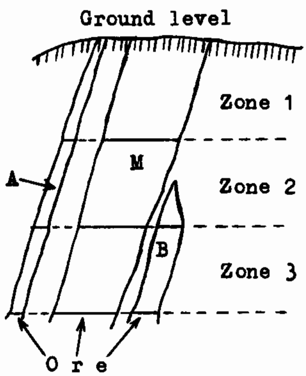

Fig. 6:3 Vertical section through main ore body M with the subsidiary ore bodies A and B.

The ore bodies are assumed to be partitioned into zones so that each zone contains a given and constant amount of equivalent ore reserve from each ore body. The amounts can be changed by changing the partitioning into zones or by excluding either a subsidiary ore body or a part of it, e.g. B in zone 2. It is thus assumed that the mining proceeds simultaneously in the various ore bodies currently being mined, and that the proportions of equivalent ore reserves used up in the various ore bodies are given constants for each zone. If this strong interdependence does not exist, the ore bodies should be treated as different ore deposits.

As an explanation of a point in section 3445 and in assumption 39) another problem may be treated in this context. The cumulative equivalent ore reserve is used as a measure of the distance between the ground level and the current main haulage level. This distance influences certain payments, especially those for hoisting the ore and for transporting men, machinery, and material to the working level (section 3445). The measure has been chosen as an approximation of more direct measures such as metres, feet, etc. In the cases illustrated in Fig. 1:1 and Fig. 1:3 the approximate measure is in proportion to the distances measured in metres or feet. This is not the case in the example shown in Fig. 6:3 if ore body B is included. This must be taken into account in estimating the payment functions in which the equivalent ore reserve is a variable, e.g. by eliminating the fluctuations of the equivalent ore reserve per metre or foot vertical distance by means of some corrective linear, exponential, sine, etc. function in the payment functions in question. The subscript number of the zone can be used as a measure if the distance in metres or feet between the top level and the bottom level of a zone is equal for all zones. It should be observed that this problem may arise in an ore deposit consisting of only one ore body as well.

The optimization model can be described as follows. With the zones given as shown in Fig. 6:3 a number of alternatives can be defined, for example:

Alternative	Zone 1	Zone 2	Zone 3
1	M	M	M
2	AM	AM	AM
3	AM	AM	M
4	AM	AM	ABM
5	AM	ABM	ABM

M, AM, etc. refer to including the parts of the ore bodies M, A and M, etc. which are relevant for the different zones. The list of alternatives is not exhaustive. It is assumed that other alternatives have been eliminated (compare section 141).

The optimization method to be used has been described in section 58. In order to find the optimum alternative the optimum in each alternative 1 to 5 is determined by means of EXRATE (the optimization model of section 62). The alternative yielding the highest capital value is the optimum desired. EXRATE and CUTOFF are used alternately if the average grades are decision variables in any of the ore bodies (section 57). CAPVAL can be used to evaluate the alternatives if the rates of production and the average grades are given in all alternatives. CUTOFF can be used if the rates are given in all alternatives, but not the average grades.

The solution suffers the imperfections of the other optimization models applied. They have been stated in sections 62 and 632 for cases where EXRATE or CUTOFF, respectively, are utilized. The weak points of both programs remain if they are oombined in a simultaneous optimization of rates and grades. Then, in addition, the danger of obtaining an inoptimal combination of rates and grades may exist, and, further, the dynamic aspects of the interdependencies between rate and grade at each moment are not observed. This means that the optima obtained for the actual decision time may be erroneous (section 57).

CAPVAL does not contain an optimization method and, consequently, the shortcomings of the optimization methods discussed so far are not relevant if this program is used. However, independently of the optimizations within each one of the enumerated alternatives (1 to 5) the optimization model used for selecting the optimum alternative has some disadvantages. The alternatives have to be defined in terms of a fairly large number of parameters and coefficients, which may be difficult to determine. Hence, only a small number of alternatives can be considered. This makes it practically impossible to make a dynamic optimization of the decision problem discussed.

634 Optimum bottom level

The problem of determining where to stop mining an ore deposit for which complete exhaustion of the ore resources does not set a definite end to the mining, will be treated here. It occurs where payments for current operating costs, investments, etc. increase as the mining proceeds to greater depths or in some other direction[1], to such a degree that the capital value of the deposit can be increased by a reduction in the equivalent ore reserve[2] (compare assumption 10)). The same effect may occur where the quality of the ore deteriorates as the mining proceeds. Similar problems may also arise if the prices of the final products are expected to decrease as time passes or the costs of extraction are expected to increase (ceteris paribus).

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1) An example where another direction is relevant is that a deposit (ore, gravel, clay, etc.) is a vein which is exploited cheaply at one end but increasingly costly towards the other end, e.g. because the ground is already exploited there for other purposes, such as housing or industrial buildings. (The zones are here vertical sections instead of horizontal sections of the deposit.)

2) Neither EXRATE nor CUTOFF permits a rate of production equalling zero. Nor can the average grade be so high that the ore reserve of a zone or a subzone equals zero. If the automatic optimizations lead to such results, the program execution is stopped before the optimizations are completed. Such a stop may very well be the first indication that a part of the ore deposit should be left in place.

A solution of the problem is obtained by the optimization method described in section 58. Thus, the relevant alternatives are defined (see example in section 58) and evaluated as indicated. The principal weak points of the model are the same as those of the model described in section 633.

64 Optimum technology

641 On the relevance of the long-range model

The optimization method to be used is the one described in section 58. There are many technological problems in mining. For many of them the method suggested is unsuitable as it lays much stress on the long-range consequences of considered alternatives, especially the influence of the alternatives on the ore reserve and on the production period of the ore deposit, which may be irrelevant for many problems. For other problems where the long-range consequences are relevant there may be difficulties in describing the alternatives considered in terms of the parameters and constants available.

In general, the optimization model in question is relevant where the alternatives directly (and differently) influence the equivalent ore reserve (compare sections 21 and 223), the average grade[1], and the rate of production in a working mine[2]. The production period of the deposit, the timing of zone investments, etc. are influenced in these cases, which motivates that the decision is made with a view to the long-range effects.

In other cases the alternatives considered merely influence certain payments, e.g. if an investment is made in order to reduce the payments for current production costs. In principle, such changes influence the long-range optimizations

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1) Compare e.g. sub-level mining described in section 43. A mining method where no waste rock is mixed into the ore would evidently yield other average grades.

2) The rate of production should be somewhere between the constancy limits, and should consequently not be changed, if the rate had previously been optimized. However, it is often possible to make changes in the technology used (mining methods, machinery, planning, etc.) which increase the overall capacity at much lower costs than those assumed in the optimization. This is a symptom of erroneous assumptions in the optimization (in assumptions 30), 31), 33), and 34)) but it will nevertheless most certainly happen in practice. The unique change in the production capacity may then be in effect throughout the production period of the zone, or longer, depending on whether the change is bound to a particular zone, and when a contraction in the rate of production will be optimal.

via the payment functions, but it may be safely assumed that this can be neglected where only small changes are concerned. Then the present model is irrelevant or at least unnecessarily complicated (compare section 58). The model may be relevant, however, if the projects considered are of a substantial size.

Two technological problems where the optimization method of section 58 is relevant will be treated separately. The first is to determine optimum zone sizes (see assumption 7) for definitions) and the second is to determine the optimum mining method in a special situation.

642 Optimum zone sizes

The size of a zone can be changed by moving the main haulage level upwards or downwards (Fig. 1:1) or the corresponding change where other factors constitute the relevant delimitations. Keeping to the case shown in Fig. 1:1 an increase in the size of one zone results in decreases in one or more other zones, or in a decrease in the number of zones[1]. A large zone is more expensive than a small one where zone investments are concerned. For example, the length of the gravity shafts and that part of the hoisting shaft which pertains to the particular zone[2] are longer and more expensive. The main haulage level, the crushing plant, etc. are built at a lower level which may make them more expensive. The quality of the final product may also be influenced as the natural crushing which takes place in gravity shafts increases with the length of the shaft. This influences payments for current operating costs or the prices of the final products in one direction or the other.

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1) A similar problem has been treated by Schneider (1944, pp. 83-84). His problem is there to determine the optimum service life of each machine in a finite chain of machine replacements. The number of machines in the chain is a given constant. This is a parallel to the case where the number of zones is given, and the optimum way of dividing the deposit into the given number of zones is sought. There is one essential difference between the two problems, namely that there is no restriction in Schneider's problem, which corresponds to the ore-reserve restriction.

2) The shaft from α to the bottom the shaft is part of the zone investment for zone 1 (Fig. 1:1). This part of the shaft is subsequently used in hoisting from lower zones, but it has to be completed before the mining starts in zone 1. For the new zone the shaft is prolongated by a length which corresponds to the distance between main haulage levels. These prolongations are parts of the respective zone investments. The delimitation α between expansion investments at time T_{11} and zone investments at time T_{11} is arbitrary, and has been made in order to make the zone investments of zone 1 comparable to those of subsequent zones.

The size of each zone is a variable in the payment functions (assumption 39) and the example in section 45 of Appendix D), as well as the level of the main haulage level. Hence, a single set of coefficients of the payment functions may suffice to describe all the alternatives considered. This facilitates the optimization substantially, as alternatives can be picked and evaluated with a minimum of work once the functional relationships have been established[1].

The optimization model is similar to that described in section 633 and the weak points of the latter are relevant, except for the possible improvements mentioned above[2].

643 Optimum mining method - An example

The ore deposit shown in Fig. 6:3 can be mined in several ways. One problem may be to decide to what extent the upper part of the deposit should be exploited by open-pit mining and from where the deposit should be exploited by underground mining. Assuming that the problems met in selecting the actual mining methods to be used in the open pit and in the underground mine, have been solved, the principles of a solution will be discussed here.

The two parts of the deposit are assumed to be mined successively from above. The step from open-pit mining to underground mining may be such a drastic change that the same payment functions cannot be used for both stages. An extension of the payment models presented in section 44 is needed, as there is very little room for defining different payment functions for different zones within the framework of these models. The necessary change has been mentioned in section 44, and is a very simple one: A set of coefficients is defined, c_{ian} or $c_{ian'n}$ where $i=71,72,\ldots$, $a=1,2,\ldots,A$, $n'=1,2,\ldots,N'$, and $n=1,2,\ldots,N$. The maximum value of i is the number of coefficients desired for each zone or subzone. It must be determined with a view to the memory capacity of the computer (especially if n' is also used as a subscript or if A is large).

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1) There are occasions when payments expressed as simple functions of size and level are not decisive factors, e.g. if the geological structure of the bedrock varies irregularly with the level. Then each alternative may have to be evaluated more elaborately by means of individual payment functions.

2) A graphical solution of a simplified version of the optimization problem has previously been shown by the author (Norén 1967, pp. 185-197). The computer programs being available, the solution is, however, too cumbersome to be practically useful. For this reason it is not shown in this study.

The new coefficients are used in the payment models as the coefficients c_{ia} (assumption 39) in section 42)[1]. With this extension different functions can be constructed for different zones or subzones within the framework of the models, especially within the programs EXRATE, CUTOFF and CAPVAL[2]. Naturally, the program user is free to use the whole or only a part of the new set of coefficients. It may e.g. be adequate to define one set of payment functions which is common to all zones exploited by open-pit mining, and another set of functions which is valid for all under-ground mining. Such short-cuts can be made within the subprograms containing the payment functions.

A way of optimizing the border line between the two parts of the deposit can be found by assuming that a number of zones, e.g. five, are mined as an open pit. The sizes of the zones are here determined by the mining method used. It may e.g. be useful to define zones so that periodically recurring removals of waste rock can be treated as zone investments[3]. The zones are defined as before in the section of under-ground mining. The models in their extended form are applied as described in section 58 in order to evaluate the alternative described above, and other alternatives obtained by varying the zone sizes and the number of zones detailed to each of the two main sections of the deposit. The optimization is simplified if the payments in each section can be expressed as simple functions of the zone size and the bottom level of the zone (see section 642).

The optimization model has the disadvantages described in section 633.

65 Optimum refinement level

So far the flow of ore from the working face in the mine to the port of shipping or the customer of the mining company has been studied from the mining side. The output has been assumed to be sold at a price determined by a given function (assumptions 36) and 39)). It has been pointed out that the quality (in a wide sense) of the output can be influenced by decisions concerning min-

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1) The set of coefficients c_{ia} cannot be removed without further changes in the programs.

2) The new coefficients must be defined in the programs (in COMMON) and data-input and data-output statements must be inserted. These changes are easily made.

3) The model discussed in section 642 can in that case be used to optimize waste-rock removal. It should be noted that the extension of the payment models is not necessary if the entire ore deposit is mined by open-pit mining.

ing limits and technology. It can also be influenced more deliberately by a decision maker. The problem has been called that of optimizing the refinement level.

A mine usually yields more than one final product. This is illustrated in Fig. 6:4. Each final product has its own market, which is, more or less,

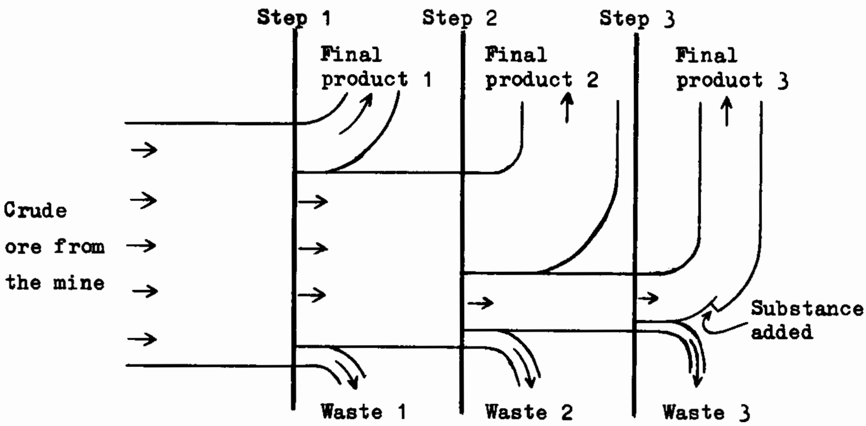

Fig. 6:4 The flow of ore through three steps of sorting and/or dressing. The rate of production is defined in tons of crude ore in this study.

independent of the markets of the other products. The waste may also in itself be or contain potential final products[1]. The payment models are exclusively concerned with the tonnages of crude ore from the mine. Thus, the payments received for the various final products are added and expressed as a function of the rate of production, i.e. the tonnage of crude ore which is the input to the sorting plant, the average grade of that ore (which equals the average grade of the currently mined subzone), and other variables and parameters (assumption 39)). The corresponding current operating payments include current operating payments in sorting plant, dressing plant, etc., among them payments for substances added in the process (step 3). The expansion investment, the contraction investment and the grade-change investment are assumed to include payments for obtaining and changing the necessary plants.

1) This has been observed by e.g. Frisch (1965, p. 11) and Danø (1966, p. 167).

In the terms of the models presented, a change in the refinement level is a change in some of the payment functions, e.g. an increase in the investments defined by the expansion-investment function combined with an increase in the current net payments. The rate of production, the average grade, etc. are influenced only indirectly, via the optimizations (section 343). Hence, the relevance of the long-range effects discussed in this study depend on the size of the change considered or the size (measured in payments) of the differences between the alternatives considered. Where the long-range effects are relevant the method described in section 58 can be used. Other cases will not be treated here[1].

The ore deposit has been assumed to be the main factor of production (assumptions 2), 3), 10), and 14), for example). This assumption can be invalidated by a change in the refinement level. If sinter plants, blast furnaces, etc. are built up around the deposit by the mining company, the latter may find itself in a situation where the existence of the ore deposit is of minor interest. The necessary ore can be obtained from other mines if the mine of the company is exhausted. There is reason to assume that the end of the production period of the deposit is not a sufficient model horizon (section 332) in such a case, and the optimization models may be inapplicable[2]. This should be kept in mind if the method of section 58 is used to evaluate more radical changes in the final products. In addition, the optimization model has the disadvantages stated in section 633.

66 Conclusion

According to section 23 the purpose of this study is to construct optimization models and find methods to determine optimum solutions of the decision problems stated in section 223. A limitation of the decision problems to be treated has been made in section 223. Only problems influencing the ore reserve or the production period of the deposit were to be treated. The task is now considered completed for the special case where only one ore deposit is concerned and where changes can be made unrestrictedly and instantaneously, although

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1) The products are made jointly in the same process which, for technical reasons, cannot be divided into subprocesses for the respective products. The production model then encompasses the entire mine, sorting plant, dressing plant, etc. This is a case of multi-product processes or multi-ware production treated by Danø (1966, pp. 167 and 181-189) and Frisch (1965, pp. 269-289).

2) The model horizon is sufficient if external ore is prohibitively expensive. In other cases the increase in the costs of raw material (i.e. the crude ore) must be taken into account. In certain simple cases these cost increases may be treated as close-down payments, but ordinarily this will be too crude a method.

only two problems have been penetrated more thoroughly, i.e. the problems concerning rates of production and average grades. Other decision problems have been treated more superficially on the basis of the solutions of the first two problems.

The solutions have been derived on assumptions enumerated in section 42, which infringes the general validity of the solutions. Furthermore, the optimization methods are imperfect, which has been stressed in this chapter.

The optimization models have been constructed for the decision situation specified in the assumptions. It is now time to examine the influence of some changes in the decision situation.

CHAPTER 7

7 The optimization models in alternative decision situations

71 A review of decision situations

The alternative decision situations for which the optimization models should be adaptable have been stated in section 23. Some of the alternative situations have already been treated.

The first set of alternatives comprises decisions at a single point of time versus a series of interdependent decisions at various points of time. The problem has been discussed in section 34, and the two main optimization models cover both types of situations as the single decision is a special case of the more general problem with a sequence of interdependent decisions. The programs containing the optimization models can be used for both types of situations.

Where other decision problems than determining rates of production and average grades are concerned it has been assumed that the number of alternatives considered is finite and small, or that the results obtained concerning a small number of alternatives yield a sufficient precision from the point of view of the decision maker (section 58). This practically excludes the optimization of sequential interdependent decisions, if the interdependencies were to be taken into account in more detail. An approximate optimization is made instead, where that alternative (a set of present and future choices) is accepted as optimum, which yields the maximum capital value at the actual decision time, i.e. T_{11} (section 633). Hence, it is disregarded that a future choice yielding maximum capital value at time T_{11} may be different from a future choice yielding maximum capital value at the time the choice is actually made, i.e. it is disregarded that the problem is a dynamic optimization problem (section 342).

The second set of alternative decision situations comprises a single decision variable at each decision time versus simultaneous decisions concerning two or more decision variables. The two main optimization models have been combined in order to optimize simultaneous decisions concerning rates of production and average grades (section 57). They have also been used to find optimum combinations of e.g. technology or refinement-level variables and rates and grades (sections 58 and 633 to 65). However, it is disregarded that the problem is a dynamic optimization problem. A dynamic optimization model for a combined optimization of rates of production and average grades has been outlined in section 57, but considered too cumbersome for practical use.

The third set of alternative decision situations comprises factor and product prices which are constant over time versus such that are variable. The optimization models allow practically any time pattern in this respect (assumptions 28) and 39)).

The three types of variations in the decision situation treated above will not be discussed further in this study. Three other aspects of the decision situation have, however, so far been treated too narrowly. These will be expounded in the following sections. The alternative decision situations are:

4) Unlimited versus restricted rate of expansion of the rate of production.

5) A single ore deposit versus two or more deposits.

6) The mining company as a decision maker versus the community.

72 Restricted rate of expansion of the rate of production

721 The assumption of instantaneous and unlimited changes

It was assumed in assumptions 22) and 35) that all changes are instantaneous and unlimited in size. This may be a good approximation for small changes, but if the optimization yields as a result that e.g. the rate of production should be doubled or tripled instantaneously at the actual decision time, the change often cannot be realized. This may be due to the market where large additional sales may be obtained only through extreme price reductions. The market may also be able to absorb a certain additional quantity of ore, but no more. There may be restrictions on the ability of the mining company to increase the capacity of plants, railways, ports, etc.

Similar restrictions may exist concerning all decision variables, but the problem is most obvious concerning the rate of production. This particular problem will for that reason be examined more closely. It should, however, be viewed as an example of a more general problem, although the generalization is not carried out here.

The assumption of instantaneous changes introduces a deviation from the factual situation[1] but it is easily evaded, at least from a practical point of view. It may be assumed that in order to increase the rate of production an investment project is carried out and paid for during a given period of time. The decision time is set equal to the date when the investment project is completed. The rate of production is increased from this moment. The payment functions do not describe actual payments, but the relevant payments discounted to this fictitious decision time. The expansion investment includes losses and extra outlays which occur after the project is completed, in an initial period before the plant has been adjusted to optimum performance. Interpreted in this way the assumption of instantaneous changes is usually a fairly close approximation of reality.

722 Restricted rate of expansion and payment functions

The restrictions on the rate of expansion of the rate of production may in principle be inserted directly in the payment functions. An example is shown in Fig. 7:1. The maximum value of Q_n being γ_1 in a certain year, the payment function exists only in the interval $0 < Q_n < \gamma_1$ in this year. Correspondingly it is assumed that the function exists in the interval $0 < Q_n < \gamma_2$ in the next year, in the interval $0 < O_n < \gamma_3$ in a third year, etc. The function is for simplicity used in the interval $Q_n > 0$ in the optimization. The simplification is in accordance with the assumption of unrestricted rate of expansion.

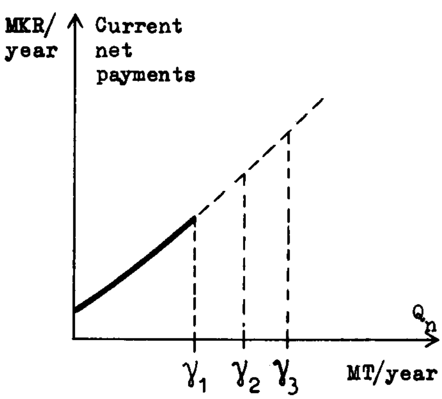

Fig. 7:1 Current net payments as a function of the rate of production.

The payment functions actually used in the programs can easily be restricted to a given interval for each year by a discontinuity with some prohibitive payment function inserted in the

1) Production usually takes time, and so do changes in the factors of production. Related problems have been discussed by e.g. Frisch (1965, pp. 29-38), who distinguishes between momentary and time-shaped production or, what amounts to almost the same distinction, between static and dynamic theories of production (ibid. p. 30).

interval of non-existence. However, this would not solve the problem of optimizing the rate of production because it is also assumed that the rate of production is constant during the production period of a zone. The interval of the payment function that happens to be relevant at time T_{1n} would be decisive. Assume that the interval $0 < Q_n < \gamma_1$ is relevant at time T_{1n}. The optimum would according to the model be to produce γ_1 (Fig. 7:1) throughout the production period of zone n, if the restriction were effective. On the other hand, the actual optimum would probably be to plan an increasing rate of production during this period. The optimization models constructed are not capable of dealing with payment functions as those of Fig. 7:1 in a proper way.

The payment functions must be assumed to exist for the entire range $Q_n > 0$ and the location of a discontinuity in the functions expressing the current payments must not change from one year to another as suggested in Fig. 7:1. Some other solution of the present problem must be found. In principle the problem could be solved by means of a model which permits more frequent, e.g. annual, changes in the rate of production, and rates of production which are smaller than the production capacity. Such a model would be rather complicated in comparison with the models contained in the programs EXRATE, CUTOFF, and CAPVAL, and has not been constructed. Instead, an approximate solution is obtained by means of the original models.

723 An approximate optimization

The optimization model in the program EXRATE yields the lower constancy limits and the expansion limits for all zones as a part of the results. The limits can be utilized in an approximate solution of the problem of determining optimum rates of production if the rate of expansion is restricted. It is assumed that the production can be increased continuously by a given rate, i.e. the maximum rate of expansion or the expansion potential, and that this maximum rate of expansion is fully utilized until the then relevant lower constancy limit is reached. The decisions are assumed to be made at the starting time of each zone as before. Some details of the method have to be examined.

The lower constancy limits and the expansion limits are in EXRATE determined on the condition of unrestricted expansion. The limits should now be determined on the condition that the rates of production in the first zones are determined by the maximum rate of expansion. This can be achieved by means of the initial guess, although only approximately. The optimization model for optimizing the rates of production is used, i.e. the program EXRATE (section 62). The rates of production of the initial guess are determined according to Fig. 7:2. The maximum rate of expansion may change over time, e.g. at a given point of time (α in Fig. 7:2). The assumptions of the original model are not changed in any other respect.

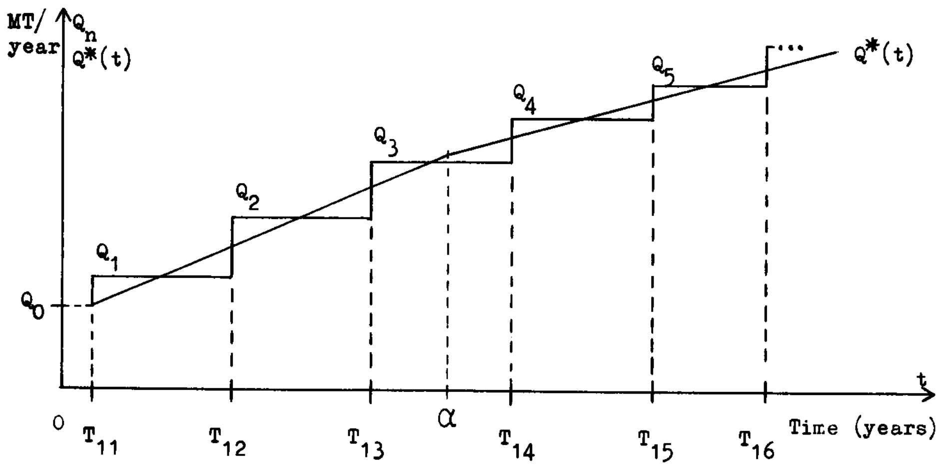

Fig. 7:2 Initial guess based on the maximum rate of expansion

$Q^*(t)$ is the maximum rate of production as a function of time t, which is reached if the maximum rate of expansion is fully utilized. $Q^*(t)$ is approximated by Q_n for n=1,2,...,N (the first five zones are shown in the figure). Q_n is determined by the equation

$$Q_n = \frac{1}{T_{1,n+1} - T_{1n}} \cdot \int_{T_{1n}}^{T_{1,n+1}} Q^*(t)dt \qquad (7.1)$$

Thus, Q_n is the mean rate of production during the production period of zone n. Other values than the mean value may be considered, e.g. values obtained if the utilization of the available expansion potential were optimized within each zone. Alternatives will not be discussed here as they do not influence the principles of the solution.

The first complete optimization is made with the initial guess as a starting point. The decision times are determined from the initial guess. The value of

Q_{n-1} of the initial guess determines the state at the decision time T_{1n} when the expansion limit and the lower constancy limit are being determined for zone n, provided that the value of Q_{n-1} does not exceed the lower constancy limit and that it is not too close to it (section 5431). Consequently, the expansion limit and the lower constancy limit in the zone during the production period of which the optimum course of action is to stop the expansion, are determined on the basis of the approximately correct rates of production in the preceding zones, which are given in the initial guess, and the corresponding decision times.

The two limits discussed are used to determine an optimal policy. The expansion continues until the rate of production equals or exceeds the lower constancy limit of the current zone[1]. It is assumed that the initial guess provides the optimal policy in all zones where the initial guess does not provide a rate of production which exceeds the lower constancy limit, and that these zones are the first ones, i.e. zones 1, 2, 3, ... Usually the limits in question have lower values in each successive zone (Fig. 6:1), whereas the initial guess provides increasing rates (Fig. 7:2). Hence, the expansion will not be resumed once the rate of production has reached a level where the lower constancy limit and the expansion limit have been effective[2].

The details of the application of the two limits in determining when and how to stop expanding, can be formed in several ways as the entire procedure is approximate. An application in line with the definitions of the limits is shown in Fig. 7:3. The expansion ceases in zone n if $Q^*(T_{1,n+1}) \geq (EL)$ where (EL) is the expansion limit in zone n. The expansion continues until $Q^*(t)=(EL)$ which occurs for $t=\gamma$. The mean rate of production in zone n is calculated by equation (7.2) which is similar to (7.1).

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1) An alternative method utilizing a static optimization model for determining the optimum rates of production at different future decision times after expansion from time T_{11} to the respective decision times, has previously been described by the author (Norén 1967, pp. 221-229). The present method yields a more reliable approximation of the optimal policy as dynamic aspects are considered in the optimization of the rates of production after the period of expansion, and is more easily applied as the results of one unrestricted optimization can be used in place of the results of many static optimizations.

2) Exceptions may occur where the payment functions vary with the time. The exceptions will not be treated here.

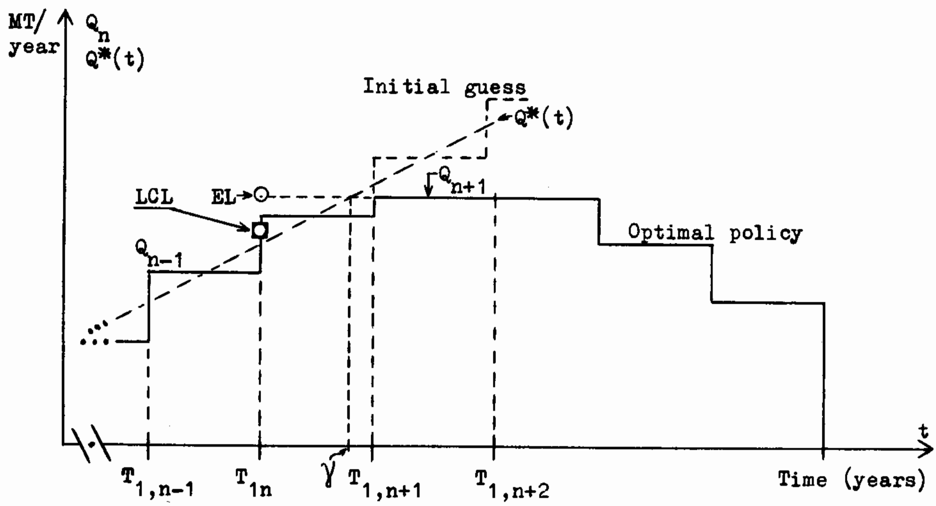

Fig. 7:3 The optimal policy approximated by the initial guess, the expansion limit (EL), and the lower constancy limit (LCL) in zone n.

$$Q_n = \frac{1}{T_{1,n+1}-T_{1n}} \cdot \left[\int_{T_{1n}}^{\gamma} Q^*(t)dt + Q^*(\gamma)\cdot(T_{1,n+1}-\gamma) \right]. \qquad (7.2)$$

It is assumed that at time $T_{1,n+1}$ the capacity of the mine equals $Q^*(\gamma)$, i.e. the rate of production when the expansion stops. Provided that $Q^*(\gamma)$ exceeds the lower constancy limit and is less than the upper constancy limit in zone n+1 the optimum decision at time $T_{1,n+1}$ is to make $Q_{n+1}=Q^*(\gamma)$.

There are a couple of further possibilities. If $Q^*(T_{1,n+1}) < (EL)$ the expansion is not stopped in zone n. It may occur that the expansion can continue during the entire production period of zone n-1 without reaching the expansion limit of this zone, but that the expansion causes $Q^*(T_{1n})$ to exceed the lower constancy limit (LCL) in zone n. In this case, thus, if $Q^*(T_{1n}) > (LCL)$ no expansion is made, and $\gamma = T_{1n}$ so that $Q_n = Q^*(T_{1n})$ and $Q_{n+1} = Q^*(T_{1n})$ if this rate of production lies between the two constancy limits in zone n+1.

The rates of production in subsequent zones are determined as usual according to the decision rules defined by the expansion limit, the contraction limit, and the constancy limits. The solid line indicates the optimal policy in Fig. 7:3. A new problem arises here.

The four limits have to be determined by repeated complete optimizations in certain cases (section 54). The repetitions are made automatically in EXRATE. The optimal policy obtained in the preceding complete optimization replaces the initial guess for all zones, and the optimal policy is determined on the assumption that the expansion is not restricted. Hence, the second and following complete optimizations are irrelevant for the present problem[1]. Repetitions are not necessary for zones 1, 2, ..., n-1, where n is the zone in which the expansion is stopped. However, repeated complete optimizations are necessary under the conditions discussed in section 54 for zones n, n+1, n+2, ..., N.

Relevant repetitions of the complete optimizations can be achieved in the following way: A new optimization is made with the optimal policy according to the first optimization as an initial guess. Thus, in this initial guess Q_i for $i=1,2,\ldots,n-1$ are determined according to the expression (7.1), Q_n is determined by (7.2), and Q_i for $i=n+1,n+2,\ldots,N$ are determined in accordance with the decision rules obtained in the first complete optimization. The new complete optimization is made with EXRATE. If new values are obtained for EL and LCL (Fig. 7:3) in zone n or for the limits determining the decision rules for subsequent zones, new values of Q_n, Q_{n+1}, etc. are determined accordingly. The procedure is repeated until worth-while improvements are no longer obtained.

The optimization is based on the same main principles as the optimization under unrestricted expansion, and the imperfections mentioned in section 62 exist here too. In addition, the maximum rate of expansion is assumed to be fully utilized during the period of expansion, although the production capacity may have to be increased in fairly large steps, and the continuous maximum increase is approximated by a discontinuous increase.

724 Some generalizations

The maximum rate of expansion may be a decision variable. It may be possible to increase it by changing the market policy, the product mix, or the investment policy. The payment functions are influenced by such changes, e.g. the

1) EXRATE automatically makes at least three complete optimizations (section 226 of Appendix B). Only the first one is relevant. A way of suppressing the irrelevant optimizations is to insert the statement "GO TO 960" between cards 0168 and 01685 in the main program of EXRATE (see Appendix E) or to allow so short a time on the computer that the job is terminated shortly after the first complete optimization.

prices of the final products may decrease, extra investment costs have to be paid, etc. It may also be possible to improve the payment functions by reducing the maximum rate of expansion.

Alternatives with different rates of expansion can be optimized as described in section 723, and the resulting capital values of the ore deposit be compared. The alternative giving the highest capital value is preferred. Another useful approach is in certain cases to calculate the capital value for a couple of optimal policies based on different maximum rates of expansion. The difference indicates what can be gained by finding a way to relax the restriction on the rate of expansion.

The optimization method can also be generalized in that the revised optimization model described in section 723 can replace the original model for optimizing the rate of production in the more general optimization methods described in sections 57 and 58, which are repeatedly referred to in section 633 and subsequent sections of Chapter 6.

725 An interpretation of the discontinuous expansion

The rate of production is in the models assumed to change only at the starting times of the zones. This is assumed in order to simplify the models, but the simplification is based on certain observations made by Billiet and Massé (section 3443). A factor of importance is also that the zone investment is often large and that the capacity of the mine is to a great extent determined during the production period of a zone by the installations made as zone investments (compare Fig. 1:1). For this reason expansion investments have been assumed to be coordinated with the zone investments.

It has been put forward in section 721 that changes are time-consuming and that an expansion may cause extra outlays in an initial period before the plant has been properly adjusted. These observations are readily extended to the assumption that the capacity is installed and taken into production successively during a period of time near the starting time of a new zone. It can also be assumed that ways of improving the utilization of the installations are successively found, so that the capacity can be increased also between the periods during which new installations are being taken into production.

Assuming this, the step-wise increasing rate of production is an approximation of a rather continuously increasing rate which fluctuates around a trend line, i.e. $Q^*(t)$. Fig. 7:4 shows an approximate optimal policy in such a case[1]. The optimal policy can only be roughly estimated by means of the present model, as a step-wise increase is assumed. Moreover, short-run considerations will influence the actual behaviour of the mining company. Fluctuations on the ore market, fluctuations in the supply and the prices of factors of production, etc. will alternately decelerate and accelerate the expansion. These considerations are

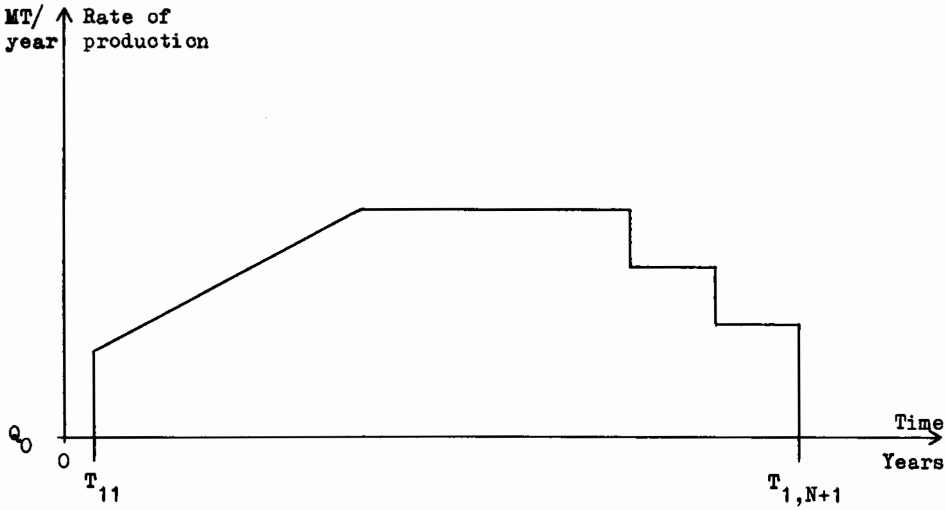

Fig. 7:4 An optimal policy roughly estimated by means of the optimization model.

here called short-run considerations because they are in effect only during short intervals in the production period of the deposit, although they may extend over several years. The fluctuations are not taken into account in the model when determining the expansion during the period of expansion. It is simply assumed that the maximum rate of expansion is a long-range average which is fully utilized.

1) The optimal policy of Fig. 7:4 has the general form predicted by Hotelling (1931, p. 164), and found empirically (ibid.).

73 Multiple ore deposits

731 Independent deposits

Until now it has been assumed that the mining company has only one ore deposit at its disposal (assumption 2)). A mining company having more than one deposit must co-ordinate the exploitation of the deposits so that the capital value of all deposits taken together is maximum, if the goal of the company is maximum capital value of future mining. The deposits may be independent of one another or interdependent. Both cases will be treated.

True independence may be non-existent as the financial effects of the exploitation of one deposit extend throughout the mining company. This influences the opportunity cost of capital and, hence, the rate of interest. However, the rate of interest is assumed to be a given constant (assumption 20)), and it is assumed to take the financial interdependencies into account. The rate of interest expresses the financial interdependence.

Each ore deposit[1)] can be optimized independently if no other interdependencies than the financial ones are assumed to exist. The single-deposit optimization models can be applied to each deposit. The capital values of the separate deposits are additive. Hence, the highest total capital value is obtained by simultaneous optimal mining of all deposits with positive capital values and other deposits worth mining (compane section 61). However, the conclusion is valid only on certain conditions which will be discussed below.

The payment functions may vary with time in such a way that the capital value at time zero of a deposit can be increased by postponing the opening of the deposit. This possibility is open only for deposits which are not being mined at time zero. For such deposits an optimum opening time can be determined. The optimization models are applied as before with the addition that the actual date of time zero is varied, e.g. successively set equal to the years 1970, 1975, 1980, 1990, and 2000. The payment functions are varied accordingly, and the optimum capital value is determined for each timing alternative. The capital values are then discounted to a common point of time, e.g. 1970, and the alternative yielding the highest discounted capital value is approximately the optimum.

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1) One ore deposit may consist of several ore bodies which are mined as one single mine. See section 633.

The optimization of the starting time of a deposit is relevant for companies having only one deposit as well as for companies having several deposits. In the latter case the payment functions relevant for different deposits may vary differently with time. Then it is no longer certain that the optimum action is to exploit all deposits simultaneously, but the starting time of each deposit can still be determined independently of the other deposits.

732 Interdependent deposits

The ores from different deposits may be processed in a common plant, transported on the same railway, shipped over the same port, sold on the same market, etc. The payment functions relevant for the different deposits influence each other. The details of the interdependencies will not be discussed.

The interdependencies can be taken into account by an application of the method described in section 58. Only the special features of the multiple-deposit problems will be treated here. Whatever the optimization problem, each deposit is optimized as if independent, with payment functions determined for given values of all decision variables concerning all other deposits. The given values should be preliminary estimates or guesses of the final optimum values. The optimum of the first deposit is determined for payment functions based on these assumptions. Then the payment functions of the second deposit are determined on the basis of the optimum of the first deposit and the originally estimated or guessed values of the decision variables concerning the other deposits. The optimum of the second deposit is determined for these payment functions. The other deposits are optimized similarly. The procedure is repeated until a stable state has been found or until the sum of the capital values of the individual deposits cannot be increased significantly from the point of view of the decision maker[1].

The method has the weak points of the optimization model described in section 633.

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1) Alternative models have previously been discussed by the author (Norén 1967, pp. 210–219). They are based on much more restrictive assumptions, which made it possible to find some comparatively simple solutions. They are not repeated here as the simplifications are no longer necessary when the computer programs presented here are available.

The method described presupposes that all deposits are mined simultaneously. As this may not be the optimum course of action, alternatives where the opening of certain deposits is postponed must be examined too. The method suggested in section 731 can be applied, although difficulties must be expected in defining a set of alternatives in which the optimal or approximately optimal alternative is a member.

The special case where the rate of expansion is restricted has been treated separately in the study of the single deposit. The solution presented can be extended to the present situation with two ore more interdependent deposits. It is assumed that the maximum rate of expansion is a common restriction for all deposits. Then the problem to be solved is how to allocate the available expansion potential between the deposits. A rough estimate is obtained by trying some alternative allocations and determining the total capital value of the deposits at the optimal policies of the deposits in each alternative.

The optimal policy and the capital value of each deposit is determined according to section 723. Payment functions as well as the maximum rate of expansion in each deposit have to be determined as described above, i.e. so that they are consistent with the values of the decision variables, which are assumed for the other deposits. Further details will not be discussed, except that it will be pointed out that the proportion of the total maximum expansion allocated to each deposit may change over time. This may even lead to postponing the opening of a new deposit.

Thus, a rapid expansion is motivated in a deposit where a certain increase in the rate of production yields a greater increment to the capital value of all deposits than the same increase in any other deposit. The incremental capital-value gain for the given increase in the rate of production decreases as the rate of production increases, and is zero or negative after the lower constancy limit has been reached. Hence, after a period of rapid expansion in the deposit, the incremental capital-value gain has decreased to the same level as the corresponding gains in other, more slowly expanding deposits. Then the rate of expansion should be decreased in the first deposit and correspondingly increased in the other deposits. Once the incremental capital-value gain is the same for all deposits for a given increase in the rate of production, the optimum allocation is the one maintaining this state of balance[1].

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1) This analysis is an application of the analysis usual in the theory of price discrimination, where the optimum distribution of a given output between the markets is the one yielding equal marginal revenues in all markets. See e.g. Henderson and Quandt (1958, p. 171). A formal analysis of some special (Continued)

The method suggested for determining an optimal policy concerning the rates of production in interdependent deposits with restricted expansion may be complicated and difficult to apply. For this reason a short-cut will be suggested. This is done quite parenthetically and the new sources of error introduced will not be discussed.

1) It is assumed that there are only two or three deposits. Start with the two deposits in which expansion appears most advantageous. EXRATE is used to determine the expansion limits and lower constancy limits.

2) If the total maximum expansion to be partitioned between the two deposits is 1 unit per annum, find first the optimum allocation if the deposits expand at a constant ratio, e.g. 0.6 and 0.4 units per annum in deposits 1 and 2, respectively, until either deposit ceases to expand at a time γ (Fig. 7:3), say γ_1 in deposit 1. After time γ_1 deposit 2 expands at the rate 1 unit per annum until a time γ_2. CAPVAL is used to evaluate the alternatives which have to be examined to find this allocation. The limits determined in 1) are used in constructing the alternatives. If necessary the limits are recalculated by means of EXRATE.

3) Examine if the total capital value of the two deposits can be increased by expanding at the rate 0 units per annum in one deposit and 1 unit per annum in the other during e.g. alternatively 3, 5, 10, and 15 years reckoned from time T_{11} of the expanding deposit. During subsequent years (i.e. 4 and following, 6 and following, etc.) the previously determined optimum allocation 0.6 and 0.4 units per annum, respectively, is assumed until either deposit ceases to expand. After that moment all expansion is concentrated on the still expanding deposit. The limits determined in 1) or 2) are used in determining when to stop the expansion. CAPVAL is used to evaluate the alternatives.

4) If 3) has yielded a new optimal solution, e.g. 10 years without expansion in deposit 1, it is established whether the allocation 0.6 and 0.4 units per annum is still optimal. Thus, new alternatives are evaluated in the same way as each alternative was evaluated in 3), and compared with the best alternative to date (10 years with the expansion 0 and 1, then 0.6 and 0.4 in the respective deposit).

5) Point 3) is repeated if 4) has yielded a new optimum. 4) is repeated then, if necessary. This is continued until the capital value cannot be increased significantly (from the point of view of the decision maker). New limits are determined by means of EXRATE if the limits are influenced by the alternatives.

6) It is examined whether a third deposit can compete with the first two deposits for expansion potential. The expansion in the first two deposits is reduced in proportion to their respective expansion during each year in order to permit expansion in the third deposit.

The short-cut has been used practically for two deposits, both being mined at time zero. A simple interdependence in addition to the common expansion restriction could be corrected for manually. The limits determined by EXRATE were not influenced by the alternatives considered so much that they had to be recalculated.

(Continued)
cases has previously been made by the author (Norén 1967, pp. 212-218). Where the expansion is unrestricted an optimum combination of rates of production in the deposits is one for which the incremental capital-value gain for any change is zero (or negative).

A special subprogram was written for CAPVAL, which determines the mean rates of production in each zone of each of two deposits for a given combination of maximum rates of expansion. The allocation of the rate of expansion must be of the following form: Expand at the rates x_1 and x_2 MT/year, respectively, in deposits 1 and 2 from time T_{11} to a given arbitrary time α, and at the rates y_1 and y_2 MT/year, respectively, from then on. From the moment one of the deposits ceases to expand, according to the rules stated in section 723 (Fig. 7:3), the whole expansion is transferred to the still expanding deposit, until this deposit ceases to expand too. After the expansion has stopped the rules stated in section 723 are followed (i.e. the usual decision rules utilizing the given expansion limits, contraction limits, and constancy limits for each deposit). The subprogram is not published in this report.

74 The models in social optimizations

It has been assumed that the mining company is the optimizing decision maker for whom the optimization models are constructed. The community has also an interest in the exploitation of ore deposits, and a few observations will be made concerning the utilization of the optimization models proposed here for social optimizations.

The capital value as a criterion in optimizations from the point of view of the community has been discussed in section 222. There it was also stated that the payments had to be adjusted. Thus, the payment functions have to be replaced by functions reflecting the economic effects on the community as a whole. This can be done within the framework of the models suggested. The rate of interest varies from one mining company to another depending on the financial structure of the company and available alternative investment opportunities. The rate of interest of the community must be found out, which may cause difficulties[1].

The community is interested in all ore deposits in the region or nation. The deposits are interdependent through factor and product markets. The single-deposit approach cannot be utilized without a study of these interdependencies in order to establish their significance. The deposits which have not yet been detected are also significant, as they too will be a part of the resources of the community when they are detected.

The model horizon may be too close from the point of view of the community. The mining industry and industries using the products of the mining industry may have

1) See e.g. Prest and Turvey (1965) and Werin (1968) for references and a discussion concerning social costs, rate of interest, etc.

to be replaced by other industries, and ore may have to be obtained from elsewhere when the deposits are exhausted. These problems have no place in the models presented, except as close-down payments.

The solutions of these problems will not be discussed. The problems have primarily been stated as a warning against rash generalizations regarding the use of the optimization models. The models are essentially constructed for optimizations on the company level.

CHAPTER 8

8 Conclusions

81 Two principal decision models

The main problem dealt with in this study is to construct decision models for an optimizing decision maker who has a limited and exhaustible supply of resources (an ore deposit) which constitutes the principal object of his activities, or else is independent of his other activities. If his mining activities and his other activities are interdependent, the study applies in the special case where the interrelations are restricted to the financing and a constant rate of interest can be determined. Two principal optimization models are presented, one for determining the optimum rate of exploitation (the rate of production) and one for determining the optimum level of exploitation, i.e. the degree to which the resources are utilized when the non-utilized amounts of the resources are assumed to be spoilt and useless for future use (the mining limit expressed as the average grade of the ore mined). Either of the models provides an answer to the question whether a certain resource should be exploited or not.

The two principal optimization models are based on the theory of dynamic programming. Efficient, although not optimally efficient, solutions have been found, which have simplified decision rules as their main components (section 6 of Appendix A, sections 5431 and 562). The decision rules concerning the rate of exploitation are based on the assumptions that changes in the rate are expensive, and hence to be avoided in a certain interval, and that the optimum change is to alter the rate to a certain value if a change is made. The decision rules concerning the level of exploitation (average grade) are based on the same assumptions concerning changes, except that it is assumed that small changes are not expensive.

The two principal optimization models are formed to computer programs, EXRATE and CUTOFF. The programs simulate the mining of an ore deposit from the time of the decision to the end of the production period of the deposit. The output of the programs describes the economic consequences of the mining thus simulated.

The two principal optimization models presuppose that only one source of resources (deposit) is involved. Thus, it is assumed that the decision maker controls only one source, two or more sources so highly interdependent that they can be treated as one source (ore bodies in an ore deposit)[1], or two or more independent sources. The single-deposit models can be used for these cases. The two principal models are also based on the assumption that a change of any size in rate or level of exploitation can be made over a very short time (unrestricted instantaneous changes).

82 Extensions of the two principal models

Optimization models are presented where the level of exploitation in other dimensions than the average grade of the ore mined, the technique of exploitation, and the refinement level of the final products are decision variables as well as the rate of exploitation and the level of exploitation expressed as the average grade. Further, the models discussed are extended to cases where the rate of expansion of the rate of exploitation is restricted, and two or more interdependent sources of the resources are controlled by the decision maker. The extended decision models are all based on the two principal optimization models or the capital-value model which is common to the two principal models. The capital-value model yields the capital value of future exploitation of the resources for a given plan of exploitation, and is contained in a separate computer program, CAPVAL.

The two principal optimization models are constructed and presented in full detail, whereas the extended optimization models are presented in more general terms. This reflects an important difference in the state of completion of the two principal models versus the extended models, i.e. the former optimize the decision variables automatically in computer programs, whereas the latter involve experimentation on the models of the decision problems and evaluation of the experiments by means of the former models.

83 Models for long-range decisions

The alternative courses of action considered by the decision maker are evaluated mainly with respect to their long-range consequences. Short-run optimizations of the decision variables are disregarded in the models. The optima obtained by means of the models should be interpreted as a framework for the short-run decisions.

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1) Compare the concept of limitational factors of production where the technology of the process prescribes fixed proportions of the factors in question. See e.g. Frisch (1965, pp. 225 ff.) or Danø (1966, pp. 16 ff.).

84 Sensitivity analysis

Sensitivity analysis is important for the decision maker in his evaluation and utilization of the results of the optimizations. Sensitivity analysis is easily performed where only the two principal models are involved by repeating the optimizations with varying assumptions concerning size and quality of the ore deposit, size of payments, relationships between decision variables and payments, and rate of interest. The programs are constructed so as to facilitate such variations. The sensitivity analysis is more complicated for the extended models as the effect of each change must often be evaluated by repeated applications of the two principal models.

85 Related problems remaining to be solved

Some problems are neglected in this study although they are relevant in certain decision situations where the exploitation of exhaustible resources is concerned. Exploring for new sources of the resources, e.g. exploring for new ore deposits, is not treated and the impact on present decisions of expectations that new sources will be found is disregarded. Short-run deviations from long-range optimal policies are not discussed[1]. The models do not allow for risk and uncertainty. These problems and many others offer a wide field where the models presented can be developed and improved, or replaced by others. Some other models already in existence have been referred to. In addition to this the extended models representing more complicated decision situations could probably be developed further. For example, multiple-deposit models in which expectations concerning the detection of new deposits could be taken into account should be of interest, especially for social optimizations.

It is felt, however, that before the models presented in this study are developed into more complex systems it would be of immense value to have the models put to the acid test - applications in a variety of actual decision situations. This will reveal weak points and supply material for more precise definitions of the problems to be solved.

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1) The case of restricted rate of expansion may be considered a study of such short-run deviations. However, it is also mainly concerned with the study of long-range effects. No short-run optimization is made.

Appendix A

Simplified graphical dynamic optimization

Contents | Page

1 Assumptions

A summary of the assumptions for the simplified dynamic optimization is given in section 1332. The graphical solution and the graphical technique of presentation necessitate some constraining assumptions. As this reduces the realism of the optimization model it should be observed that the constraints depend partly on the graphical method as such, and partly on a wish to make the description of the graphical method as simple as possible. Thus, the assumption 2) below is not necessary. Neither 1) nor 2) are necessary if the optimization is made by means of a suitable computer program. 3) only defines an example.

The following additional assumptions are made:

1) The capital calue of a zone is independent of the starting time of the zone, i.e. of the time passed since the mining started in the first zone. This excludes the possibility of payments, whose sizes are dependent on the time at which they are payable. Thus it is assumed that factor and product prices are constant over time. On the other hand the assumption does not prevent the payment from changing over time as a consequence of the progress of the mining, e.g. to new parts of the mine, or at a new rate of production.

2) The expansion and the contraction investments are proportional to the size of the change of the rate of production at a given moment.

3) The following data are given: An ore deposit is to be mined in three zones, each being of the size 50 MT. The expansion investment is 50 KR for each ton by which the annual rate of production is increased. The contraction investment is 20 KR for each ton by which the rate is decreased.

In section 1331 the general method of dynamic optimization in the case of three successive decisions has been summarized in three points. The points will be treated in sections 2 to 5 on the basis of the simple example of 3) above. The example represents a special case. A more general case will be treated in section 6. There the assumption 2) above will be relaxed to some extent.

2 Constant rate of production as a starting point

As a starting point it is assumed that the rate of production will be the same in all zones[1]. The capital values at the three decision times (the starting times of the zones, i.e. T_n for n=1,2,3) of the zones which are not yet mined at these points of time, are calculated, i.e. B_n for n=1,2,3[2]. The capital values are illustrated in Fig. A:1. As the rate of production is not changed during the production period neither expansion nor contraction investments are involved, except at time T_1.

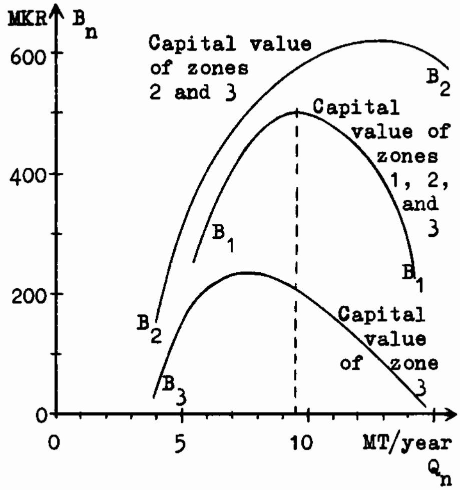

Fig. A:1 Capital values of future mining at times T_n, i.e. B_n, for n=1,2,3 as functions of Q_n if $Q_1=Q_2=Q_3$.

As $B_1 < B_2$ the capital value of zone 1, i.e. B'_1, is negative. This implies that expenditures exceed payments received. Naturally, the reason for this is the expansion investment at time T_1, which is common to all zones, but only reduces B_1, not B_2 or B_3.

The curve B_1 in Fig. A:1 directly corresponds to the capital-value curve of the static model (Fig. 1:5 in Chapter 1). They both show the capital value of the entire ore deposit at various rates of production, which are assumed to be constant during the whole production period. In a static optimization the curve yields the optimum rate of production 9.5 MT/year, i.e. at optimum $Q_1=Q_2=Q_3=9.5$ MT/year.

3 Dynamic optimization in the last (third) zone

On the present assumptions there is no reason to assume that the optimal policy is to produce at a constant rate of production. The conditions under which it is advantageous to change the rate, must be determined. Considering the simplifying assumptions introduced, the first point of the description

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1) In the computer program this starting point is replaced by an arbitrary set of rates of production in the zones, named the initial guess.

2) The symbols are listed and defined in Appendix F.

of the general method of dynamic optimization should be interpreted as follows (compare section 1):

1a) Optimize Q_3 for all possible values of Q_2.

This step is taken in Fig. A:2. B_3 is taken from Fig. A:1. First, suppose that Q_2=4 MT/year. Then the capacity available at the starting time of zone 3 is also 4 MT/year. If Q_3=4 MT/year, i.e. if the rate of production is not changed, B_3=28 MKR. An increase in the rate of production in zone 3 so that Q_3=4.5 MT/year, would yield B_3=93 MKR, if enough capacity had existed, or if the price of capacity had been zero. This is explained by the postulate of Fig. A:1 saying that B_3 is the capital value of zone 3 if Q_2=Q_3, i.e. if the capacity necessary for Q_3 has already been installed before time T_3.

If Q_2=4 MT/year an increase in the rate of production at time T_3 by 0.5 MT/year, gives Q_3=4.5 MT/year, and causes an expansion investment of 50 KR/T/year · 0.5 MT/year = 25 MKR. Then B_3=93–25=68 MKR. If Q_2=3.5 MT/year it is also easily

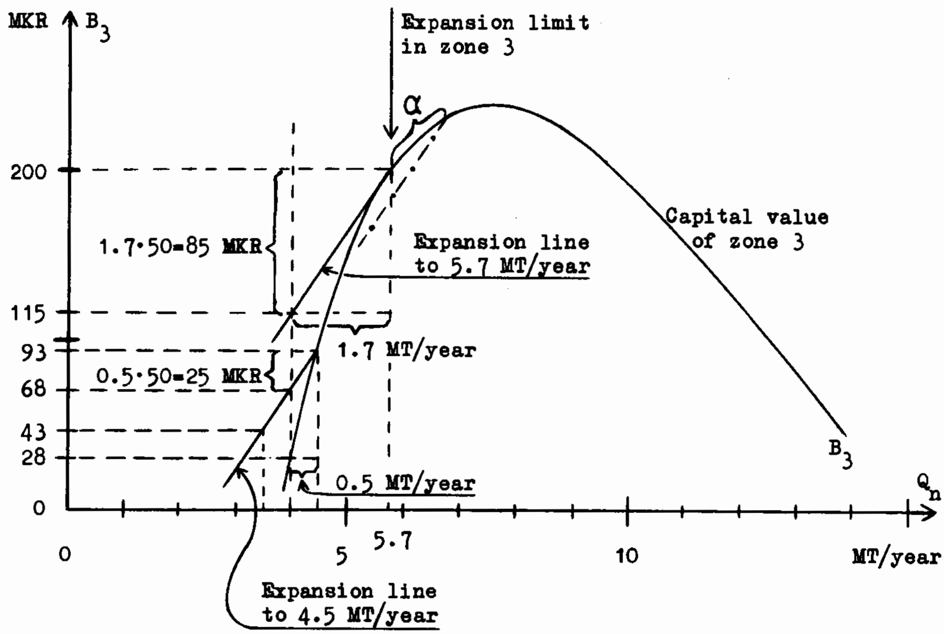


Fig. A:2 Expansion of rate of production at time T_3.

seen that another 25 MKR must be invested in order to increase the capacity so that Q_3=4.5 MT/year. Then B_3=93−50=43 MKR. A straight line can be drawn through the points (3.5,43), (4,68), and (4.5,93). This is the expansion line to 4.5 MT/year. It shows the capital value of zone 3, i.e. B_3, at Q_3=4.5 MT/year as a function of Q_2, if $0<Q_2<Q_3$. Q_2 is measured on the horizontal axis.

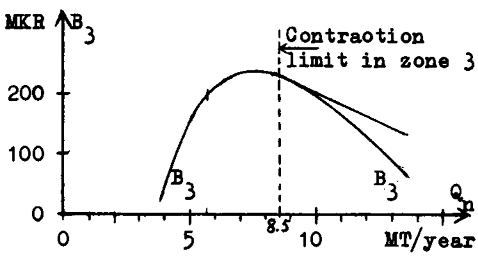
Again, let Q_2=4 MT/year. What happens if the rate of production at time T_3 is increased by a greater amount, e.g. by 1.7 MT/year so that Q_3=5.7 MT/year? If a sufficient capacity had already been installed at the time of the expansion, Fig. A:2 indicates that B_3=200 MKR. The expansion investment is, however, 50·1.7=85 MKR. Thus, after the expansion investment has been taken into account B_3=200−85=115 MKR. A straight line is drawn through the points (4,115) and (5.7,200). This is the expansion line to 5.7 MT/year. The line shows the capital value of zone 3, i.e. B_3, at Q_3=5.7 MT/year as a function of Q_2, if $0<Q_2<Q_3$. Q_2 is again measured on the horizontal axis. As the greater expansion yielded a capital value of 115 MKR it is apparently preferred to the smaller expansion, which yielded only 68 MKR.

In this way it is possible to construct an expansion line to an arbitrary value of Q_3. The line is drawn to the left from the point on the original curve B_3, where Q_3 has this arbitrary value, i.e. the rate of production after the expansion. The gradient is the same for all expansion lines, and is determined by the expansion investment. Thus, the gradient is 50 KR/T/year.

The greater the distance between the horizontal axis and an expansion line (at a given value of Q_2), the higher is the capital value of zone 3, and the more advantageous is the implied expansion. The expansion line to 5.7 MT/year is a tangent to the original curve. It thus offers the highest capital value at all relevant rates of production in zone 2, i.e. if $0<Q_2<5.7$ MT/year. Consequently, if $Q_2<5.7$ MT/year the optimum action at time T_3 is to increase the rate of production to 5.7 MT/year. To expand further would decrease the capital value. For example, the dotted and dashed expansion line indicates lower capital values than the expansion line to 5.7 MT/year. It also indicates capital values, which are lower than those indicated by the segment α of the original curve B_3. Similarly, every expansion line to a rate of production exceeding 5.7 MT/year will indicate lower capital values than their corresponding segments of the curve B_3. Consequently, it is advantageous not to increase the rate of production if it already exceeds or is equal to 5.7 MT/year in zone 2.

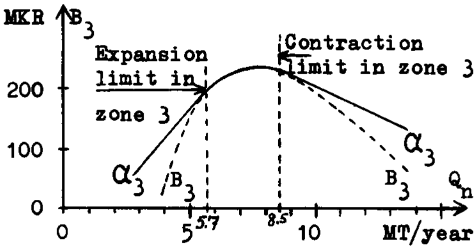
The rate of production at the point of tangency will be named the <u>expansion limit</u> in zone 3. It bears on the decision at time T_3, when the rate of production in zone 3 is determined.

In a similar manner it is determined whether it is profitable to decrease the rate of production at time T_3. <u>Contraction lines</u> may be drawn. Their gradient is determined by the contraction investment. Thus, in the present case it is -20 KR/T/year. The tangential contraction line shows, according to Fig. A:3, that the optimum rate of production in zone 3 is 8.5 MT/year, if $Q_2 > 8.5$ MT/year. This rate of production is the <u>contraction limit</u> in zone 3. The contraction line to 8.5 MT/year also shows the capital value of zone 3, i.e. B_3, at $Q_3 = 8.5$ MT/year as a function of Q_2, if $Q_2 > 8.5$ MT/year.



<u>Fig. A:3</u> Contraction of rate of production at time T_3.

The expansion line to the expansion limit, the contraction line to the contraction limit, and the segment of the curve B_3 situated between the two points of tangency, constitute a new curve, α_3, which indicates the capital value of zone 3, i.e. B_3, as a function of Q_2 at optimum values of Q_3 (Fig. A:4). Thus, the task of the first point - 1a) Optimize Q_3 for all possible values of Q_2 - has been completed.



<u>Fig. A:4</u> Optimum capital value at time T_3 of zone 3 as a function of Q_2.

The analysis implies the following optimum values of Q_3: $Q_3 = 5.7$ MT/year if $0 < Q_2 < 5.7$ MT/year, $Q_3 = Q_2$ if $5.7 \leq Q_2 \leq 8.5$ MT/year, and $Q_3 = 8.5$ MT/year if $Q_2 > 8.5$ MT/year.

4 Dynamic optimization in the penultimate (second) zone

The second point of the description of the general method of dynamic optimization was stated in section 1331. It is applied in the present case:

2) Apply the result (of point 1)) in optimizing Q_2, thus also Q_3, for all possible values of Q_1.

Thus, the problem is to find the optimum values of Q_2 at various values of Q_1. At all values of Q_2 the optimum values of Q_3 and the corresponding capital values of zone 3, i.e. B_3, are known from the preceding section, and applied in the solution of the present problem. The method is illustrated in Fig. A:5.

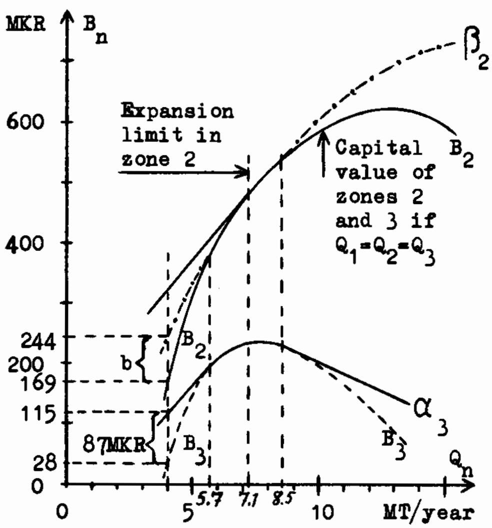


Fig. A:5 Adjusting the capital value of zones 2 and 3, i.e. B_2, at time T_2 for the optimization in zone 3. Expansion of rate of production at time T_2.
$b=87\cdot(1+r)^{-T_2}$.

The curve B_2 is taken from Fig. A:1, and must be adjusted for the new values of Q_3, which have been determined in the preceding optimization in zone 3. Thus, if Q_2=4 MT/year, B_3 could be increased by 87 MKR by optimizing Q_3. As the original curve B_2 contains the capital value at time T_2 of B_3, also the increase in B_3 is discounted to the same point of time. The discounted increase is added to the original value of B_2 at the given rate of production, i.e. to 169 MKR. Thus, owing to the optimization in zone 3

$B_2=169+87\cdot(1+r)^{-T_2}=244$ MKR, if Q_2=4 MT/year and Q_3=5.7 MT/year.

As all points on the curve B_2 are correspondingly adjusted a new curve indicating the capital value of zones 2 and 3 at optimum rates of production in zone 3, is obtained, i.e. β_2.

It is, however, still assumed that $Q_1=Q_2$. Expansion and contraction lines may be drawn from various points on the curve β_2 in the same way as they were drawn from various points on the curve B_3. The expansion line to 7.1 MT/year is the tangent, and consequently indicates optimum. The tangential and, thus, optimum contraction line is not shown, but Q_2=20 MT/year at the point of tangency. Hence, in zone 2 the expansion limit is 7.1 MT/year, and the contraction limit 20 MT/year.

The optimum rates of production in zone 2 are now determined: If $0<Q_1<7.1$ MT/year, Q_2=7.1, thus expansion. If $7.1\leq Q_1\leq 20$ MT/year, $Q_2=Q_1$. If $Q_1>20$ MT/year, Q_2=20 MT/year, thus contraction.

5 Dynamic optimization in the first zone

The third and last point of the description of the general method of dynamic optimization has been stated thus:

3) Apply the result (of point 2) in optimizing Q_1, thus also Q_2 and Q_3.

The curve B_1 in Fig. A:6 shows the total capital value at time T_1 of zones 1, 2, and 3, i.e. B_1, as a function of Q_n for n=1,2,3, provided that $Q_1=Q_2=Q_3$. However, it has been demonstrated that equal rates of production in all zones is an optimum policy only in a certain interval. Hence, the curve B_1 does not properly describe the capital value of future mining, i.e. the capital value which is the decision criterion in zone 1. The curve B_1 must be adjusted to indicate the relevant capital value B_1. At optimum rates in zones 2 and 3 the capital value at time T_2 of the two zones, i.e. B_2, is demonstrated by the curve α_2, which is derived from Fig. A:5 in the same manner as was the curve α_3 constructed in Fig. A:4. α_2 illustrates the optimum capital value B_2 as a function of Q_1. The difference in capital value between the two curves α_2 and B_2 for each possible value of Q_1, is obtained and discounted to time T_1, and added to the capital value indicated by the curve B_1 at the same value of Q_1. The operation gives the curve β_1.

β_1 indicates the capital value B_1 as a function of Q_1 at optimum values of Q_2 and Q_3. The maximum capital value is obtained if Q_1=10 MT/year. This is the optimum rate of production in zone 1[1).]

1) In zone 1 the rate of production is directly optimized without first determining the expansion and the contraction limit of zone 1. This is possible as it has been assumed that before time T_1 the rate of production, and thus also the production capacity, is a given constant. In the case discussed this constant is equal to zero. The principles of the optimization method are not affected if this capacity figure is given a positive value.

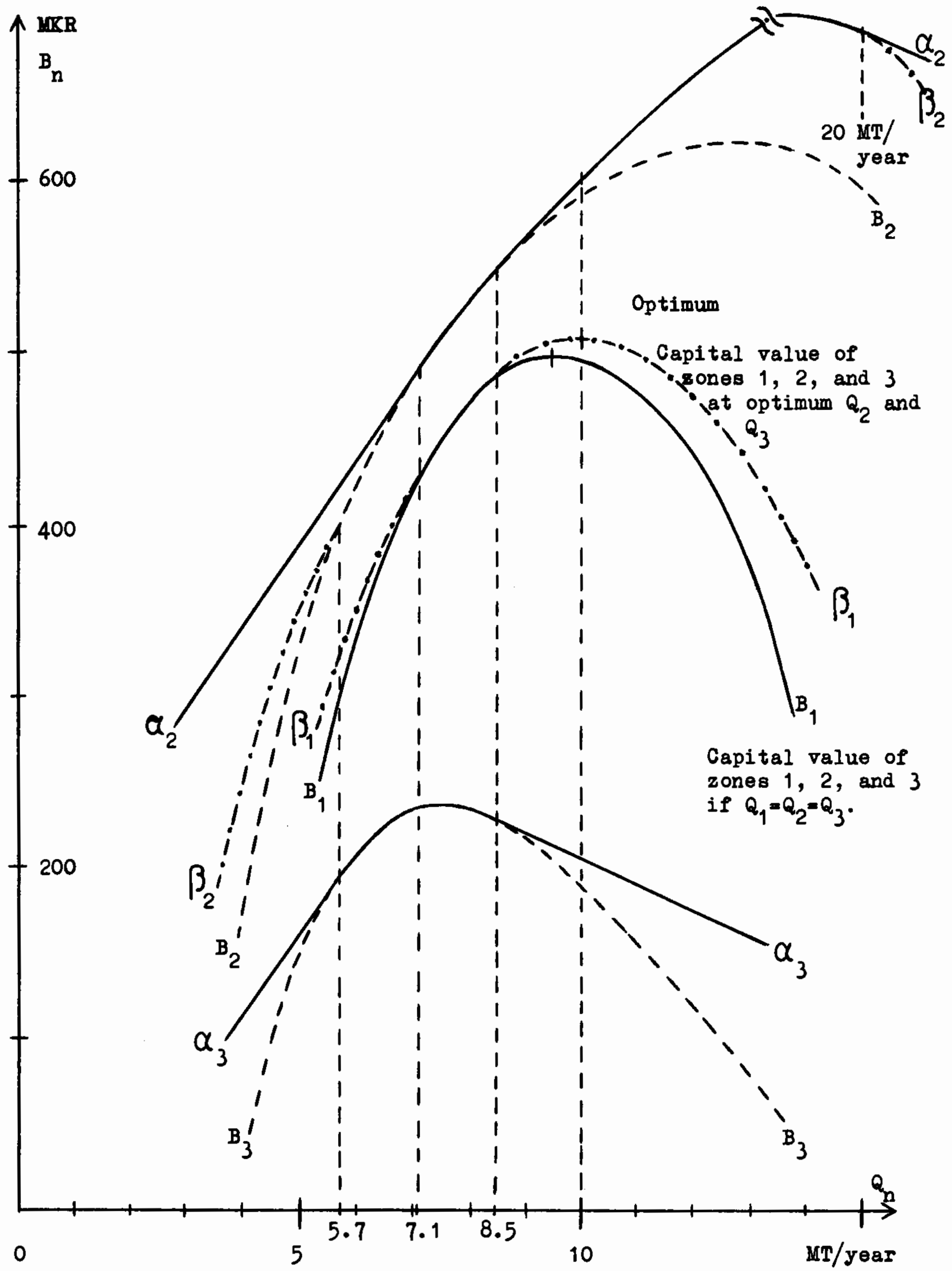


Fig. A:6 Adjusting the capital value of zones 1, 2, and 3, i.e. B_1, at time T_1 for the optimizations in zones 2 and 3. Optimum in zone 1.

As the optimum rate of production in zone 1 is determined as being 10 MT/year, it is assumed that the zones 2 and 3 are mined at optimum rates. According to the various expansion and contraction limits obtained, the optimum rate of production in zone 2 is also 10 MT/year, whereas in zone 3 it is 8.5 MT/year. However, the optima in the zones 2 and 3 are just reasonable estimates of future decisions, applied in optimizing the actual decision, i.e. in optimizing the rate of production in zone 1. The optimization described does not provide the information whereon the rates of production in the zones 2 and 3 are actually determined. Instead, when these decisions are to be made, new optimizations are carried out by applying the model to new data. These data should be the best available at the times of these decisions. In these new and future optimizations the zones will be renamed, so that zone 1 is the zone which is to be mined next. The subsequent zones (or zone, if there is any) are given the numbers 2, 3, etc.

The method described can be applied to any number of zones. The adjustments necessary to adapt the method to this more general case need no explanations. It will suffice to say that a fourth point in the description of the general dynamic optimization method of section 1331 is inserted between points 1 and 2. Considering the simplifications of section 1332 it can be stated as follows:

1b) Optimize Q_n for all possible Q_{n-1} on the assumption that optimum values of Q_{n+1}, Q_{n+2}, ..., Q_N are selected according to the results of the previous steps of the optimization procedure.

6 Constancy limits

In order to generalize the graphic dynamic optimization it is convenient to introduce the concepts of a lower constancy limit and a upper constancy limit. To explain the concepts the discussion concerning expansion and contraction limits has to be reopened. It has been demonstrated that the optimum decision is to increase the rate of production in zone n, i.e. Q_n $(n \neq 1)$, to the expansion limit in zone n, if Q_{n-1} is smaller than the expansion limit in zone n. Now, suppose that the expansion investment consists of a fixed amount, e.g. 20 MKR, in addition to the amount proportional to the increase in annual production, i.e. the expansion investment amounts to $20+50\cdot(Q_n-Q_{n-1})$ MKR[1], which should be compared with the expansion investment in the previous example, i.e.

1) Q_n in MT/year.

$50\cdot(Q_n-Q_{n-1})$ MKR. If the rate of production is not increased the expansion investment in both cases is assumed to be zero.

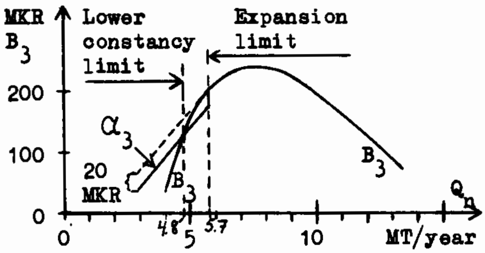


Fig. A:7 Lower constancy limit in zone 3.

Fig. A:7 illustrates the new situation as it will present itself in zone 3. The gradient of the expansion line has not changed. Consequently, the expansion limit remains unaffected. This is shown by the dashed line, which is the former expansion line. The dashed line also shows the capital value of zone 3 at time T_3 as a function of Q_2 for Q_3=5.7 MT/year under the assumptions of section 3 of this appendix (Fig. A:2), i.e. where no fixed amount is to be paid. However, if the rate of production is increased at all, even by a small quantity, in the present case 20 MKR have to be paid in addition to the 50 KR for each ton the annual rate of production is increased. Hence, the capital value of zone 3 is 20 MKR lower than the value indicated by the **dashed** line. The correct capital value is indicated by the line α_3, or the correct expansion line.

The curve B_3 shows the capital value at time T_3 of zone 3 if the rate of production is equally great in the zones 2 and 3. In the interval $4.8 < Q_2 < 5.7$ MT/year this capital value exceeds the capital value according to the line α_3. Consequently, no expansion should take place in this interval. The rate of production at the point where the expansion line α_3 intersects the curve B_3 is the lower constancy limit in zone 3. Only if the rate of production in zone 2 is lower than the lower constancy limit in zone 3, the optimum decision for zone 3 is to expand to the expansion limit.

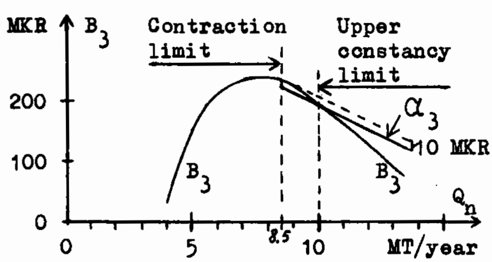


Fig. A:8 Upper constancy limit in zone 3.

By analogy, an upper constancy limit is obtained as demonstrated in Fig. A:8. It is assumed that any decrease in the rate of production incurs a contraction investment of $10+20\cdot(Q_2-Q_3)$ MKR. In this case the optimum decision is to decrease the rate of production in zone 3 to the contraction limit in zone 3 only if Q_2 exceeds the upper constancy limit.

Constancy limits are determined in the same way for all zones except the first, which is still optimized directly. Consequently, in the general case there are four limits to be determined for each zone n=2,3,...,N, N being the number of zones.

The case studied in sections 1 to 5 of this appendix is a special case, where the expansion limit coincides with the lower constancy limit, and the contraction limit with the upper constancy limit. However, the method applies also in the present, more general case. Only one point may need to be clarified: In the step from zone n to zone n-1 (section 4) the capital-value curve B_{n-1} (for which it is assumed that $Q_1=Q_2=\ldots=Q_N$) has to be adjusted, which gives the capital-value curve β_{n-1}. The method described in section 4 is applied in accordance with the example given in Fig. A:9. It should be noticed that the constancy limits, not the expansion and the contraction limit, determine where the curve β_{N-1} is separated from the curve B_{N-1} (or, in general, β_{n-1} from B_{n-1}). (Compare Fig. A:5, but note that the curves B_{N-1} and B_2 are not the same curves. The difference has no bearing on the present discussion.)

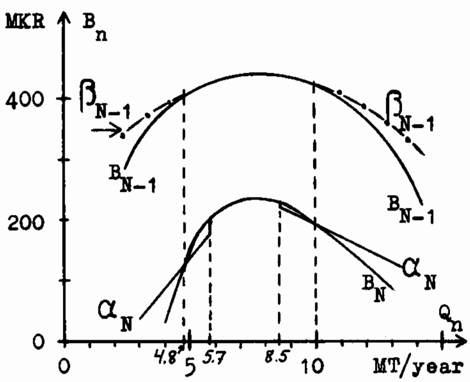


Fig. A:9 Adjusting the capital value of zones N-1 and N for the result of the optimization in zone N (compare Fig. A:5).

The four limits constitute a decision rule which ensures optimum decisions concerning the rates of production Q_n in the zones n=2,3,...,N: If Q_{n-1} is less than the lower constancy limit in zone n, expand the rate of production so that Q_n equals the expansion limit in zone n. If Q_{n-1} is greater than the upper constancy limit in zone n, reduce the rate of production so that Q_n equals the contraction limit in zone n. Finally, if Q_{n-1} is equal to any of the two constancy limits in zone n, or has a value between them, keep the rate of production unchanged, i.e. make $Q_n=Q_{n-1}$.

Appendix B

Description of the computer programs

Contents

1 Introduction

The optimization models concerning the single mineral deposit have been developed into three computer programs. They are described here together with two subsidiary programs. The names and principal purposes of the programs are:

CUTOFF	Long-run optimization of the average grade (relative contents of metal or other useful substances) of the ore extracted from the deposit.
EXRATE	Long-run optimization of the annual rate of production in the ore deposit.
CAPVAL	Computation of the capital value of the ore deposit if a complete set of values of all variables and parameters influencing the capital value is given. The program automatically treats a number of predetermined alternatives in this way. The program can be used to test the results obtained with CUTOFF or EXRATE, and to supply information needed for making an optimal choice between a number of fixed alternatives in other dimensions than the two optimized in CUTOFF and EXRATE, namely, the average grade of the ore and the rate of production.

The subsidiary programs are:

PAYMTS	Production of graphs of the functions which describe the payments flowing into and out of the mining company as a consequence of the development and exploitation of the ore deposit. These payments are current (annual) payments, investments, and payments connected with the final closing of the mine.
NEWPAR	Assignment of new values to the parameters of the payment functions and punching new data cards containing these new parameter values.

The programs are written in FORTRAN IV for IBM System/360[1]. In order to use CUTOFF, EXRATE, and PAYMTS no knowledge of the FORTRAN language is required. However, the subroutines CUT11, ANPAY1, ANPAY2, ANPAY3, and ANPAY4, which are parts of these programs, must often be rewritten to suit the particular problem of a user. In this case some acquaintance with FORTRAN is necessary[2]. This condition also holds for the use of CAPVAL and NEWPAR, which are not complete programs but merely frames of programs that are to be completed each time they are used.

The source programs are punched on cards in Extended Binary Coded Decimal Interchange Code (EBCDIC).

Some technical information might be useful for a user, for example program sizes, computing times, and output quantities.

The numbers of punched cards in the FORTRAN source programs are given below as a rough measure of the sizes of the programs. It should be added that the memory requirements of each program are less than 100K bytes[3].

The execution times and output quantities are largely dependent on the size of the mine, how many years the ore deposit is going to be mined, how many decisions are to be optimized, i.e. how the ore deposit is divided into zones and subzones, etc. The execution times, in minutes, and the output, in rows, are therefore given for two sample cases:

3·3 The ore deposit is divided into 3 zones, each of which is divided into 3 subzones. The production period of the deposit is 10–15 years. JTHOR=5.[4]

5·10 The ore deposit is divided into 10 zones, each of which is divided into 3 subzones. The production period is 20–45 years. JTHOR=5.

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1) The programs have been tested in an IBM System/360/75 only. Owing to the great number of possible paths through the programs it has not been practical to arrange a complete testing.

2) See e.g. IBM System/360 FORTRAN IV Language, Form C28-6515-5, IBM 1965, 1966.

3) This is a measure of memory requirements or storage space in an electronic computer. See e.g. reference in footnote 2.

4) The symbols are explained in Appendix C. Appendix B is mainly intended as a manual for the program user. Consequently, in order to make the description of the programs more closely related to the programs themselves the symbolic representation of the programs has been chosen here.

Program	Size, Cards	Compilation time, minutes	Execution time, minutes Case 3·3	Case 5·10	Output, rows[1] Case 3·3	Case 5·10
CUTOFF	950	0.3–0.4	0.1–0.2	2	2,000	7,500
EXRATE	1,180	0.4–0.5	0.1–0.2	2	1,700	5,500
CAPVAL	560	0.2–0.3	0.02[2]		300	500
PAYMTS	540	0.3–0.4	0.1–0.2		3,000	
NEWPAR	50	0.02[3]	0.03[3]		150 (+45 cards[3])	

Fig. B:1 Sizes, computing times, and output quantities of programs.

1) Exclusive of compiler output.

2) Inclusive of the compilation of the main program. Execution time and output refer to the one-alternative case.

3) Parameters for 5 years. Only 7 parameters are changed for each of the years.

2 Program CUTOFF

21 Applications

An ore deposit is assumed to contain a rich principal vein surrounded by, or extending into, poorer ore. As successively poorer ore is recovered through extending the mining limits, the average grade decreases and the ore reserve increases.[1] The ore deposit is partitioned into NMAX zones, and each zone is in turn partitioned into NSMAX subzones. They are mined in the order (NS,N)=(1,1),(2,1),...,(NSMAX,1),(1,2),...,(NSMAX,NMAX).
The program CUTOFF is intended for the optimization of the average grade of the ore mined in an ore deposit where this grade is a continuous variable. All other variables are assumed to be predetermined. The average grade is expressed as the fraction of the metal or the other useful material contained in the ore. It may vary with the time. Then it is assumed that the decision on a change of the average grade is made immediately before the change is actually made and that the average grade is subsequently constant until a new decision is made. The average grade is decided for each subzone at the starting time of the subzone, T(NS,N). The program is also valid for the special case of one single decision, i.e. if the grade is optimized only once, at the beginning of the production period of the mine, and then kept constant through the whole period.

The decision concerns the average grade of all the ore mined in the subzone. This should be carefully noted, as the decision variable in decisions of this type is often the grade of an incremental quantity of ore, a marginal grade. It is assumed that this marginal grade is deducible from the average grade. To emphasize this connection the optimum average grade is also called "the cut-off (mean) grade".

The program is essentially valid for all firms and ore deposits in optimizations of the type discussed. Exceptions are the subprogram CUT11 and the four ANPAY subprograms (see below), which might have to be altered to fit the particular case.

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1) Compare Fig. 1:3.

22 Structure and operation

221 Survey of the structure

The program CUTOFF consists of a main program with 13 specially designed subprograms. Standard library programs are also utilized.

The main program controls the input operations, directs the sequence of subzone optimizations, and usually decides when to stop the calculations. It also contains tests to establish the plausibility of the values of the decision variable - the average grade. In addition, the principal function of the main program is to control and coordinate the operations performed in the subprograms, whose names and main functions are listed below.

CUT1 with CUT11 calculates the starting time of each subzone and the time of the final closing of the mine. CUT11 transforms the tonnage figure of the equivalent ore reserve of the subzone into the corresponding actual ore reserve figure after the average grade of the subzone has been determined (see section 2 of Appendix D for definitions).

CUT2 selects successively new average grades in the subzone that is to be optimized, i.e. the current subzone. It uses the average grade of the initial guess or of the latest optimum obtained as a starting point. The successive average grades are alternatives, for which capital values are calculated. The alternative giving the highest capital value is to be chosen as an optimum. CUT2 selects the alternative grades using a comparatively large increment, i.e. DELTA1, first in a sequence of successively falling grades, then in a sequence of successively rising grades, in both cases beginning from the starting point mentioned above. CUT2 also decides when to stop selecting new alternative grades.

CUT3 chooses the average grade giving the highest capital value, from among the alternatives selected by CUT2. If the capital value has multiple maxima when expressed as a function of the average grade, CUT3 does not define only one maximum, but up to 5 different maxima. Together, CUT2 and CUT3 give the optimum or a set of potential optima in a first approximation, the exactness of which is determined by DELTA1.

CUT4 selects successively new average grades in the same manner as CUT2, but at smaller intervals, i.e. DELTA2. CUT4 uses successively the grades at each of the 1 to 5 potential optima obtained in CUT3 as a starting point. The capital value of each new alternative is calculated, and the alternative giving the highest capital value is taken as a more precise estimate of the potential optimum. Each of the 1 to 5 potential optima is thus refined in a second approximation. Finally, CUT4 chooses the potential optimum giving the highest capital value. The average grade of this alternative is defined as the optimum average grade of the current subzone (or the cut-off (mean) grade of the subzone, or the partial optimum of the subzone) at the current complete optimization (see DELTA3 in section 222 of this appendix and sections 225 and 226 of this appendix).

CUT5 calculates the capital value of the subzones not yet mined at the time of a given decision. Thus, the capital value calculated is that of the subzone being optimized and of all the subzones subsequently mined. The capital value is completely recalculated whenever CUT5 is called[1], e.g. every time CUT2 or CUT4 has selected a new alternative. The results of previous capital-value calculations are not utilized in any way, as they are normally rendered irrelevant by modifications of the payment flows or the times at which the investments occur. The capital value is always discounted to time 0. The interest is reckoned continuously.

ANPAY1, ANPAY2, ANPAY3, and ANPAY4 contain the payment models (see Appendix D), and thus give the different payments that are discounted in the capital value calculations on the assumptions specified in other parts of the program. These subprograms are called by CUT5 exclusively.

PRICOM prints the values of certain key variables if the execution of the program should be interrupted due to inconsistent results in the calculations.

1) In order to use a subprogram of this particular type a main program (or another subprogram) is said to call the subprogram.

INDATT performs a set of tests of the input data. If non-permissible data are encountered, an error indication stating the type of error is printed for each error, and the program execution is interrupted. The error indication code is explained in section 7 of this appendix.

PRICAP tabulates the pairs of associated average grades and capital values of the alternatives selected by CUT2 or CUT4.

222 Input data

The first step in the execution of the program is to read input data. A list of the data needed is given in section 6 of this appendix. Certain restrictions imposed on the values of some of the input variables will be explained here. The variables concerned will be treated in order of appearance in the program. The variables are defined in Appendix C. The initial guess concerns the average grades, H(NS,N) for NS=1,2,...,NSMAX and N=1,2,...,NMAX.

NMAX	Permissible numbers of zones: 1,2,...,14.
NSMAX	Each zone is assumed to be partitioned into the same number of subzones. The size of one or more subzones can, however, equal 0 MT. Exceptions are subzones with the ordinals 1, NSMAX-1, and NSMAX. In addition, two consecutive subzones cannot be of the size 0 MT. Permissible values: 1,2,...,20.
JTHOR	Permissible values of the data horizon: 2,3,...,50.
DELTA1 DELTA2	Permissible values: Positive fractions with DELTA1 > DELTA2. DELTA1 should always be an exact multiple of DELTA2.
DELTA3	A complete sequence of optimizations of the average grades of all the subzones, from the last subzone of the last zone (NSMAX,NMAX) to the first subzone of the first zone (1,1)[1] is named a complete optimization and results in a complete set of optima. If the capital value of the entire ore deposit at the latest complete set of optima is more than 1+DELTA3 times higher than the corresponding capital value at the complete

1) The subzone last mined is mentioned first as the subzones are optimized in the opposite order of mining. See section 224 of this appendix.

optimization preceding the latest one, the optimization is repeated once more.
Permissible values: No restrictions, but 0.01 to 0.10 might be appropriate in most cases.

HAEND1
HAEND2 — If the average grade of any of the alternatives selected for any of the subzones falls outside these two limits of plausible average grades, i.e. the lower and upper, respectively, the program execution is immediately interrupted.
Permissible values: Positive fractions with HAEND1+2·M·DELTA1 < HAEND2.

HBIN — If the ore deposit is not yet being mined at time T(1,1), HBIN=0.0 is compulsory. In other cases 0.0 < HBIN < 1.0.

JTOTMX — This is the maximum number of complete optimizations permitted in the particular case (compare DELTA3). The program execution is thus interrupted when JTOT has reached this value, regardless of the increase in the capital value at the optimum achieved through the latest complete optimization.
Permissible values: 3,4,...,20.

M — CUT2 selects new alternative average grades for a given subzone (the current subzone) using the average grade of the initial guess or of a previously calculated optimum as a starting point. The grades selected first decrease successively with the increment DELTA1. In this way new alternatives are selected until the capital values of M consecutive alternatives form a sequence of strictly falling values.

Then, using the same starting point CUT2 selects new alternatives whose average grades increase successively until, once more, the capital values of M consecutive alternatives form a sequence of strictly falling values. The incremental change is still DELTA1.

Permissible values: At least 3 and presumably no more than 20 or 30. The danger of arriving at non-permissible average grades (less than or equal to HAEND1, or greater than or equal to HAEND2) increases as M increases. It also increases as DELTA1 increases.

QBIN — If the ore deposit is not yet being mined at time T(1,1), put QBIN=0.0. In other cases QBIN ≥ 0.0.

R — Permissible values of the rate of interest: R>0.0.

RES(NS,N)[1] — This is the equivalent ore reserve mined prior to subzone NS of zone N (the cumulative equivalent ore reserve). The meaning of this can be clarified by means of an example: An ore deposit has been partitioned into 3 zones, each consisting of 4 subzones. The equivalent ore reserve in each subzone is 1.0 MT. Then RES(NS,N) can be tabulated according to Fig. B:2. Input values for NS=1,2,...,NSMAX and N=1,2,...,NMAX+1 must be given.

NS= \ N=	1	2	3	4
1	0.0	4.0	8.0	12.0
2	1.0	5.0	9.0	-
3	2.0	6.0	10.0	-
4	3.0	7.0	11.0	-

Fig. B:2 Cumulative equivalent ore reserve, RES(NS,N).

Permissible values: RES(1,1)=0.0. The cumulative equivalent ore reserve of one subzone must be larger than the cumulative equivalent ore reserve of the previously mined subzone. One exception is permitted: If a subzone containing 0 MT of ore (see NSMAX) is inserted, the cumulative equivalent ore reserves of two successive subzones will be equal. Fig. B:3 shows how the figures of the previous example will be changed if the size of subzone (2,2) is altered to 0 MT.

1) RES(NS,N) can be interpreted by means of the mathematical symbols in Appendix F in the following manner: Let RES(NS,N)=RES(α,β) where α=1,2,...,N', β=1,2,...,N+1, except that α=1 for β=N+1. Then

if $\alpha>1$:

$$RES(\alpha,\beta)=\sum_{n=1}^{\beta-1}\sum_{n'=1}^{N'}R_{n'n}+\sum_{n'=1}^{\alpha-1}R_{n'\beta}$$

where $n'=1,2,\ldots,N'$, $n'=1,2,\ldots,\alpha-1$,
and $n=1,2,\ldots,\beta-1$,
respectively.

if $\alpha=1$:

$$RES(\alpha,\beta)=\sum_{n=1}^{\beta-1}\sum_{n'=1}^{N'}R_{n'n}$$

where $n'=1,2,\ldots,N'$,
and $n=1,2,\ldots,\beta-1$.
Also, RES(α,β)=0 if $\alpha=\beta=1$.

NS= \ N=	1	2	3	4
1	0.0	4.0	7.0	11.0
2	1.0	5.0	8.0	-
3	2.0	5.0	9.0	-
4	3.0	6.0	10.0	-

Fig. B:3 RES(NS,N) if the size of subzone (2,2) is assumed 0 MT, i.e. if the number of subzones in zone 2 in reality is 3 instead of nominally 4.

HEQV(N) — The equivalent average grade must be given for N=1,2,...,NMAX. Permissible values: $0.0 < \mathrm{HEQV(N)} < 1.0$.

Q(N) — The rate of production must be given for N=1,2,...,NMAX. Permissible values: $\mathrm{Q(N)} > 0.0$.

H(NS,N) — The average grade (initial guess) must be given for NS=1,2,...,NSMAX and N=1,2,...,NMAX. Permissible values: $\mathrm{HAEND1} < \mathrm{H(NS,N)} < \mathrm{HAEND2}$. The absolute differences between H(NS,N) and the two extreme values should also exceed $|\mathrm{M \cdot DELTA1}|$ considerably[1].

Ci(N) i=1,2,3,4 — These parameters are used in CUT11 (see section 2 of Appendix D). They must be given for N=1,2,...,NMAX. Permissible values: No limitations except those of the computer system.

Ci i=5,6,7,8,9 — Parameters as Ci(N) but equal for all values of N.

PAR(JD,LT) — These parameters are used in the ANPAY subprograms (see section 4 of Appendix D). They must be given for JD=1,2,...,70 and LT=1,2,...,JTHOR.

In the example of possible payment functions, which has been used in the testing of the programs, and which is described in Appendix D, PAR(4,LT) denotes a standard or normal average grade of the ore mined. PAR(4,LT) should be given a value so that the payment functions can be deduced from empirical data as easily as possible. Consequently, it is usually the same every year, i.e. for all values of LT.

1) $|x|$ denotes the absolute value of x.

PAR(63,LT), PAR(64,LT),..., PAR(69,LT) are used as price-index numbers for different types of payments. They describe how the payments vary with the time, though in a most schematic way. Payments whose changes over time have been investigated more thoroughly can often be predicted in more detail. In such cases a more detailed description of the variation of the payments with the time can be made through the other parameters (JD$\leq$62).

PAR(70,LT) is reserved for a test of the input data and its value should be the subscript number of the year LT transformed into a real number (i.e. written with a decimal point, e.g. LT=5 gives PAR(70,5)=5.0).

In the models described in Appendix D parameters with JD=1,2,3, 5,6,...,21, 56,57,...,60, and 62, are used exclusively in models describing current payments and payments occurring at the final closing of the mine. These payments never occur during year 1. Thus, the parameters with the JD numbers listed above can be freed for year 1, i.e. the parameters PAR(1,1), PAR(2,1), etc. Then they can be utilized optionally[1]. For instance, they can be used to define discontinuities in the payment functions, or to extend the payment functions in other ways (see section 21 of this appendix).

Permissible values: No limitations except those of the computer system and the following: PAR(4,1)=PAR(4,JTHOR) (owing to a test in INDATT) and PAR(70,LT)=LT (see above).

After all input data have been read they are printed in tables, partly to enable the user of the program to assure himself that intended data have been used in the optimizations, partly to make it easier to distinguish between and recognize alternative optimizations that have been made on varying assumptions, e.g. in an analysis of the dependence of the optimum on the values of variables and parameters, that are not known with certainty (sensitivity analysis).

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1) To improve the reliability of the program it is then advisable to program a direct RETURN (before any operations are performed) in ANPAY1 and ANPAY4 if LT=1.

Input data are tested in INDATT at this stage of the execution of the program. If errors are detected they are indicated as described in section 7 of this appendix, and the program execution is stopped.

223 Capital value of the initial guess

The input data contain a complete production plan for the ore deposit, comprising its entire production period. The initial guess concerns the variables which are to be optimized, i.e. the average grade in each subzone, or H(NS,N) for NS=1,2,...,NSMAX and N=1,2,...,NMAX. On the assumptions thus implied the capital value of the ore deposit is calculated as a preliminary step, prior to the actual optimization. The capital value is discounted to time 0. Investments in machinery etc., which must be made before the mining can start, are presumed to be completed and paid at time 1.0. The mining then commences immediately after time 1.0. Hence, the capital value is discounted to a point of time that precedes the actual starting time by one year. This deviation from common practices in capital value computations is motivated by programming considerations only.

If the ore deposit is already being exploited at time 1.0 the optimization will embody only those parts of the ore deposit, which are to be mined after this time, and only the ore reserve remaining at this time.

The payments vary with the time according to the parameters of the payment functions, i.e. PAR(JD,LT). These parameters are fixed for each calendar year LT=1,2,...,JTHOR. Parameter values associated with LT=1 are used when payments at time 1.0 are calculated in the ANPAY subprograms, parameters with LT=2 are used for payments during the second year, i.e. from time 1.0 up to and including time 2.0, etc. For this reason and in order to discount the payments to time 0, precise information concerning the points of time at which the payments occur, must be obtained.

The current payments are assumed to constitute a continuous and constant flow during each of a series of time periods. The intensity of the flow can change at the points of time delimiting the periods. These points of time are 1) "New Year's Day", i.e. times 1.0, 2.0, ..., when the value of the subscript LT changes, thus causing the program to make use of a new set of parameters, PAR(JD,LT), and 2) the starting times of the subzones as the average grade is determined for each subzone, and any change of the average grade presumably influences the current payments, and 3) the starting times of the zones. This is because, in addition to the previous reason, the rate of production is given as a separate constant for each zone, and the rate of production is one of the factors influencing the current payments. The symbolic representation of the set of times indicated in 2) is T(NS,N) with NS=1,2,...,NSMAX and N=1,2,...,NMAX, and T(1,NMAX+1). This includes the set indicated in 3).

The zone investments and the expansion and contraction investments programmed in ANPAY2, are completed and paid at the starting times of the zones, i.e. T(1,N) for the zones N=1,2,...,NMAX. The grade-change investments programmed in ANPAY3 are completed and paid at the starting times of the zones or subzones, i.e. T(NS,N) for the subzones NS=1,2,...,NSMAX of the zones N=1,2,...,NMAX. The close-down payments programmed in ANPAY4 are paid at time T(1,NMAX+1), when the mine is finally closed.

It is now apparent that the times T(NS,N), including T(1,NMAX+1), have to be calculated before the capital value of the ore deposit can be determined. They are calculated in CUT1 and CUT11, and printed together with the values of some intermediate variables. Thus, a table containing the values of the following variables is produced:

HEQV(N) for N=1,2,...,NMAX
DRES for NS=1,2,...,NSMAX and N=1,2,...,NMAX[1)]
H(NS,N) for NS=1,2,...,NSMAX and N=1,2,...,NMAX
EXDRES for NS=1,2,...,NSMAX and N=1,2,...,NMAX[1)]
T(NS,N) for NS=1,2,...,NSMAX and N=1,2,...,NMAX, and T(1,NMAX+1).

The capital value is calculated in CUT5. The investments at time T(1,1)=1.0 are first determined through calling ANPAY2 and ANPAY3. These investments are then discounted to time 0. Then the current net payments of the first productive year, i.e. year 2, are obtained from ANPAY1. They are integrated over the calendar year and at the same time discounted to time 0. This process is repeated for each of the following calendar years up to but not including the year during which the first change between subzones occurs. The current net payments of the latter year are obtained by calling ANPAY1 as usual, but the current net payments are not integrated over the whole year. Instead, they are integrated only over that part of the year during which subzone (1,1) is being mined. Naturally, they are discounted to time 0.

---

1) Note that the variables DRES and EXDRES should logically be denoted DRES(NS,N) and EXDRES(NS,N), respectively. However, as they are non-subscripted variables in the program the original notation is retained.

Next, if NSMAX > 1 the grade-change investment at time T(2,1) is acquired from ANPAY3 and discounted to time 0. The current net payments in the current calendar year is once more obtained from ANPAY1 (the calendar year has not changed since ANPAY1 was last called). The payments are discounted and integrated over the remaining part of the year, i.e. from time T(2,1) to the end of the year. Then ANPAY1 is called for the next calendar year, etc.

The procedure is repeated until the end of the production period, T(1,NMAX+1), has been reached. The change between zones is treated in the same way as the change between subzones, but with the complementary operation of calling ANPAY2 and discounting the expansion or contraction investments thus determined to time 0.

Finally, the close-down payments are acquired from ANPAY4, and are discounted. The discounted values of the different payments are successively accumulated. Hence, the capital value of the ore deposit has now been calculated.

For each calendar year and after every change between zones or subzones the following current payments are printed as the capital value of the initial guess is calculated: SS, SK, SG, and SSKG[1].

The accumulated capital value at time 0 of payments which have occurred from time 1.0 to the end of the current year, is also printed annually.

Every time a change between zones takes place the investments calculated by ANPAY2 and ANPAY3 are printed, i.e. the values of SLM, SPAR, SE, SF, SH, and SEFHLM. For any change between subzones, which is not also a change between zones, the values of SLM and SPAR are printed. In this case SPAR is ignored when calculating the capital value.

The value of HEND is printed as it is obtained in ANPAY4.

Finally, the capital value of the initial guess is printed.

---

1) The printed amounts are identified with the variable names listed in Appendix C. Further, as a similar output will appear in other stages of the program execution, the names and values of the following variables are printed to identify the output: JTOT, NA, NSA, JA, N, NS, JT, HB, and QB.

224 Partial optimization in the subzone: The first approximation

The average grades are optimized for each subzone, starting with the ultimate subzone, i.e. the last subz one of the last zone to be mined. A first and a second approximation are calculated for each optimum. The method for the first approximation is described in this section. The method for the second approximation will be treated in the next section. The variables denoting the subscript numbers of the subzone that is currently being optimized are NSA and NA. Thus, in the beginning NSA=NSMAX and NA=NMAX. After the optimum of this subzone has been determined, the preceding subzone is optimized in the same manner. The subzone preceding the penultimate one, i.e. the subzone that will be mined before the current one, will then be optimized, and so on until the average grade of subzone (1,1) has been optimized. An example of the sequence of optimizations follows where the subscripts (NSA,NA) are given. NSMAX=3 and NMAX=3: (3,3), (2,3), (1,3), (3,2), (2,2), (1,2), (3,1), (2,1), and (1,1).

The calculations of the first approximation yield a rough optimum, or several (5 at most) equally rough potential optima. The main features of the optimization method have already been described in sections 221 (CUT2 and CUT3) and 222 (the variable M) of this appendix: A set of alternative average grades is selected for the current subzone. The difference between two adjacent alternatives is DELTA1. The capital value at time 0 of the subzones (NSA,NA), (NSA+1,NA), (NSA+2,NA), ..., (NSMAX,NA), (1,NA+1), (2,NA+1), ..., (NSMAX, NMAX) is calculated for each alternative. The alternative giving the highest capital value is the potential optimum.

A few details of some importance remain to be mentioned. CUT2 selects the alternative average grades. It is subsequently verified that the average grade of each alternative falls between the predefined extreme values HAEND1 and HAEND2. The number of alternatives must not exceed 93. The corresponding tests are successively performed as the alternatives are being defined. If the average grade of a new alternative falls outside the given limits, or if CUT2 attempts to define a 94th alternative, an error indication (PRICOM) is printed and the program execution is stopped. If the alternative is accepted, the starting times and the capital value of the alternative are subsequently calculated.

The starting time of the current subzone is determined by the initial guess or by the latest complete set of optima (section 222 of this appendix, the variable DELTA3). Hence, it is fixed and independent of the average grade of the current subzone. When a new alternative has been selected by CUT2, CUT1 is called and the starting times of the subzones mined after the current one are calculated on the new premises implied. The starting time of the current subzone is used as a starting point in these calculations.

The capital value is computed in CUT5 as described in the previous section. One essential difference exists: The calculations do not start with the subzone (1,1) but with the subzone (NSA,NA). The decision criterion is the capital value of the subzones which remain to be mined when the decision is made at time T(NSA,NA).

The capital value is for computational and programming reasons discounted to time 0. This does not influence the correctness of the optimum obtained, even if the optimization is regarded as a simulation of a future decision which will be made with the capital value discounted to the decision time as criterion. The reason for this is that the choice between the generated alternatives takes place at one single point of time, which is assumed to be independent of the choice. However, this point of time is not independent of the choice, as the latter will influence the optima of the previous subzones, which in turn will influence the decision time itself. This interrelation is not considered in the optimization of the current subzone. It will instead be taken into account in the next complete optimization. All the decision times are then revised according to the complete set of optima of the current complete optimization. Again, in the next complete optimization the decision times are fixed and assumed to be independent of the alternatives then being considered.

When CUT2 has selected a sufficient number of alternatives for the current subzone (section 223 of this appendix, the variable M) CUT3, among these, selects one or more whose capital values are maxima. The alternatives chosen are potential optima, which are subscripted JM=1,2,...,5. If the capital value, expressed as a function of the average grade, has more than 3 maxima at average grades inferior to a given limiting grade[1], the optimum subscripted 3, i.e. JM=3, will

1) The limiting grade is the average grade of alternative 1 (JA=1), which constitutes the starting point for the optimization of the current subzone (NSA,NA). The starting point is the average grade of the current subzone according to the initial guess if JTOT=0. It is the optimum average grade of the current subzone according to the latest complete optimization if JTOT > 0. Compare section 226 of this appendix.

contain the alternative with the highest capital value among the maxima encountered after[1] the first and the second maximum. In a similar manner multiple maxima at average grades superior to the same limiting grade are screened and collected in the optimum subscripted 5, i.e. JM=5, if the total number of maxima exceeds 5+k, k being the excess above 3 of the number of maxima encountered in the lower region of average grades (if less than 3 such maxima ore encountered, k=0).

In connection with the first approximation of the optimum of a current subzone the values of the following variables are printed for each alternative, i.e. for JA=1,2,...,≤93: MINE LIFE=T(1,NMAX+1), HA(JA), and B(JA). The latter two arrays are tabulated by means of the subprogram PRICAP.

The values of the following variables are printed for each potential optimum, i.e. for JM=1,2,...,5: HAMAX(JM) and BMAX(JM). If the total number of potential optima is less than 5, the value 0.0 is given to the superfluous elements of the two arrays.

No detailed information about the starting times of the subzones or about the various payments, which constitute the capital values, is given in this stage of the program execution.

If the potential optima with JM=3 or JM=5 contain multiple potential optima, this is indicated together with the value of the subscript JA of each alternative thus included as a potential optimum.

When the first approximation has been completed one or more potential optima of the current subzone (NSA,NA) are defined. These are now to be determined more exactly. If there exists more than one potential optimum, one of them must be selected.

225 Partial optimization in the subzone: The second approximation The complete optimization

Each of the potential optima selected in the first approximation is determined more exactly in a second approximation. The optimization method is essentially the same as in the first approximation. The most important differences are:

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1) Maxima at grades close to the average grade of alternative 1 (JA=1) are encountered first. Then maxima at grades which increasingly differ from this starting point are encountered.

1) The starting point (the first alternative with JA=1) is successively the average grade of the 1st, the 2nd, etc. of the potential optima of the first approximation.

2) New alternatives are in CUT4 generated with the increment DELTA2.

3) For each potential optimum of the first approximation only one more precise potential optimum is determined in the second approximation. This is done in CUT4.

4) New alternatives are selected until at least three successive alternative average grades with the increment DELTA2 give strictly falling capital values, and a fourth temporary alternative[1)] with an average grade, which differs from that of the previously selected alternative by 3·DELTA2, also has a lower capital value than the three other alternatives mentioned (e.g. $B(7) > B(8) > B(9) > B(12)$). The same rule applies to successively falling as well as successively rising average grades.

5) A maximum of 13 alternatives with successively decreasing average grades are selected and evaluated for each potential optimum JM. Including the alternatives with successively increasing average grades, the number of alternatives is increased to a maximum of 17. The additional alternatives are enumerated JA=17, 18, 19, 20 if the limit 13 has been reached. The temporary alternatives mentioned in 4) are not counted. If any of the two limiting numbers of alternatives is encountered before the decision rule of 4) applies, the generation of new alternatives in the direction in question is terminated. One of the alternatives already selected is accepted as a potential optimum in the second approximation[2)], namely the alternative constituting the capital-value <u>maximum</u> having the highest capital value. PRICOM is called, and gives an error indication, but the program execution continues. Capital values are printed only for JA=1,2,... ...,10, and 17, if both limits have been reached.

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1) This alternative is temporarily given the subscript number JA+3, JA being the subscript number of the third alternative with successively falling capital values. The temporary alternative JA+3 is not registered in the same manner as the other alternatives, and consequently it is not printed in the list containing the average grades and capital values of the alternatives (see 5)).

2) Note that a potential optimum which has been defined in this way, and chosen from among the alternatives 1 to 13, is replaced only by capital-value maxima encountered among the alternatives 17 to 20 giving a higher capital value. Alternatives with grades that are lower as well as higher than the average grade of alternative 1 are always selected and evaluated.

The reason for the limitation to a comparatively small number of alternatives in the seoond approximation[1] is that, it is feared, alternatives associated with two different potential optima from the first approximation might overlap. The overlapping alternatives will be selected and evaluated at least twice, which is quite unneccessary and perhaps expensive. The limitation diminishes this drawback, but it increases the possibility that the actual optimum will not be a member of the set of alternatives examined. Hence, the error indication should be interpreted as a warning that alternatives better than the optimum indicated might exist, i.e. that an erroneous suboptimization has taken place.

6) If there is only one potential optimum, it will constitute the optimum of the second approximation. If there exists more than one potential optimum, the one giving the highest capital value will form the optimum of the second approximation.

7) The time T(1,NMAX+1) and the average grade in the current subzone are printed as the alternatives are selected. The average grade and capital value of each alternative are printed by PRICAP when the calculations concerning the second approximation of a potential optimum have been completed. When the optimum of the seoond approximation has finally been chosen, the optimum average grade in the current subzone and the corresponding capital value are printed separately.

The optimization described is a partial optimization, and the optimum average grade of the second approximation is a partial optimum, as only one subzone, the current subzone (NSA,NA), is treated. The partial optimum of subzone (NSMAX,NMAX) is determined first, then successively those of subzones (NSMAX-1,NMAX), (NSMAX-2,NMAX),..., (1,NMAX), (NSMAX,NMAX-1), ..., (1,1). The complete set of partial optimizations is a <u>complete optimization</u>. The complete set of partial optima is called the <u>optimal policy</u>, or, in the computer program, the <u>total optimum</u>.

It is assumed that the average grades in the subzones mined before as well as after subzone (NSA,NA) are given constants, independent of the average grade in the current subzone. As the partial optima are determined, they are inserted successively as such constant average grades of the current subzones. However,

1) Compare the maximum number of alternatives in the first approximation, i.e. 93.

the assumptions made in the partial optimizations are rendered invalid as subsequently determined partial optima replace the initially assumed average grades. As this interdependence has not been taken into account, only a tentative total optimum has been found as yet. An iterative method has been applied to introduce the interdependences into the calculations.

226 Iterative complete optimizations

In the first complete optimization, i.e. in the first series of partial optimizations forming the first complete optimization, the initial guess defines the average grades in the subzones which are mined previous to the current subzone in every partial optimization. It also constitutes the starting points in the partial optimizations. As the partial optimizations are completed, the partial optima successively replace the initial guess. The value of H(NSA,NA) given in the initial guess is, accordingly, replaced by the partial optimum of the current subzone. When the first complete optimization has been carried out, all the average grades of the initial guess have been substituted in this way[1].

At this stage the capital value of the entire ore deposit is calculated in the same way as was the capital value of the initial guess, but with the complete set of optima inserted instead of the initial guess. The output described in section 223 of this appendix will be printed. This output will be identified with the sentence "TOTAL OPTIMUM NR 1", the number inserted being current value of JTOT.

The complete optimization is now repeated, the first complete set of optima substituting the initial guess. This results in a second complete set of optima, which replaces the first set in the same manner as the first set replaced the initial guess. Similar output data are printed. The capital value of the entire deposit is calculated anew. Again, the output described in section 223 occurs, now with JTOT=2.

In this way three complete optimizations are made. Then the program, according to the values of BTOT(JTOT), BTOT(JTOT-1), DELTA3, and JTOTMX (see section 222

1) When the first complete optimization has been carried out, JTOT is increased by one unit, from JTOT=0 to JTOT=1. As long as the calculations needed to find the first complete set of optima are still going on JTOT=0. Generally, JTOT is one unit less than the ordinal of the complete optimization, which is actually under way. This is important in identifying the intermediate output from the partial optimizations.

of this appendix), determines whether the optimizations should be stopped, or if a fourth, a fifth, etc. complete optimization should be made. When the optimizations have finally been ended, a list of the capital values of the ore deposit at the successively calculated total optima, or optimal policies, is printed. The program execution is then terminated.

3 Program EXRATE

31 Applications

An ore deposit is assumed to be partitioned into NMAX zones, each partitioned into NSMAX subzones. The program EXRATE is intended for the optimization of the rate of production in the exploitation of the deposit. All variables except the rate of production are assumed to be predetermined. The rate of production is expressed in millions of tons per year, or MT/year, of ore mined, i.e. crude ore leaving the deposit. The actual ore reserve in the deposit is measured correspondingly. Hence, the sum of the annual tonnages over the production period of the ore deposit coincides with this ore reserve. The accordance does not hold true for the equivalent ore reserve, if the actual average grades do not correspond to the equivalent average grades.

The rate of production may vary with the time in the sense that it can be changed when a new zone is being started. The zones are mined in the order 1, 2, ..., NMAX. It is assumed that the rate of production does not change during the production period of a zone, and that the decision for zone N is made at the starting time of the zone, i.e. T(1,N), where N=1,2,...,NMAX. The program is also valid for the special case of one single decision (NMAX=1).

The program is essentially valid for all firms and ore deposits in optimizations of the type discussed. Exceptions are the subprogram CUT11 and the four ANPAY subprograms (see below), which might have to be altered to suit the particular case.

32 Structure and operation

321 Survey of the structure

The program EXRATE consists of a main program with 15 specially designed subprograms. Standard library programs are also utilized. EXRATE has principally been constructed through reorganizing and extending CUTOFF. CUTOFF has been presented first as it is simpler. The discussion of EXRATE will mostly be confined to the alterations and extensions in comparison with CUTOFF. For this reason some knowledge of the contents of section 2 of this appendix is necessary, even for a reader mainly interested in EXRATE.

The main program has, in principle, the same functions as the main program of CUTOFF. However, the decision variable here is the rate of production. Many of the subprograms are also similar (see section 221). A general alteration has been made, which will be mentioned here before the individual subprograms are discussed: COMMON[1] has been extended in EXRATE, which implies that none of the subprograms of CUTOFF and EXRATE are identical.

EX1 with CUT11 is equivalent to CUT11 in CUTOFF. The output printed has been somewhat modified.

EX2 successively selects new alternative rates of production in the zone which is to be optimized, i.e. the current zone. When an expansion or a contraction limit is being determined, the starting point is a rate of production in the zone mined immediately before the current zone. This rate is given by the main program[2]. Successively higher rates of production in the current zone are selected in determining an expansion limit, and successively lower rates are selected in determining a contraction limit. These rates of production in the current zone are alternatives for which capital values are subsequently calculated. The alternative giving the highest capital value is the limit desired. As the alternatives are selected at comparatively large intervals, i.e. DELTA1, they are sufficient only for first approximations of the two limits (potential limits).

For zone 1 the optimum rate of production is determined directly. EX2 selects the alternatives which are to be examined. Starting point is Q(1) of the initial guess, or of the latest complete optimization (sections 322 and 326 below).

EX2 also decides when to stop the search for new alternative rates of production.

EX3 chooses the alternative giving the highest capital value from among those selected by EX2. If the capital value, expressed as a func-

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1) A memory space common to the main program and the subprograms.

2) See section 327 of this appendix.

tion of the rate of production, has multiple maxima, EX3 chooses up to 5 different alternatives, one for each maximum (potential limits). As EX2 in some respects differs from CUT2, EX3 also differs from CUT3. However, they are respective counterparts in EXRATE and CUTOFF.

EX4 determines the expansion or contraction limit (NA > 1), or the optimum rate of production in zone 1 (NA=1), more precisely (increment DELTA2) in a second approximation. In comparison with CUT4 in CUTOFF only the output printed is modified.

CUT5 is similar to CUT5 in CUTOFF.

EX6 calculates the optimum rates of production in those zones, which are to be mined after the current zone. EX6 is called when the rate of production in the current zone has been determined. The decision rules implied in the expansion and contraction limits and the two constancy limits are utilized in these calculations (Appendix A).

EX7 calculates the constancy limits through successive comparisons of alternatives in pairs:

Alternative 1: Equal rates of production in zones NA and NA-1, NA being the subscript of the current zone.

Alternative 2: a) In determining the lower constancy limit: Expansion from the rate of production of alternative 1 in zone NA-1 to the expansion limit in zone NA. The rate of production of Alternative 1 is put lower than the expansion limit in zone NA.

b) In determining the upper constancy limit: Contraction from the rate of production of Alternative 1 in zone NA-1 to the contraction limit in zone NA. The rate of production of Alternative 1 is put higher than the contraction limit in zone NA.

In determining the lower constancy limit new pairs of alternatives are selected with successively decreasing rates of production in both zones, i.e. NA and NA-1, in Alternative 1, and in zone NA-1 in Alternative 2. These three changeable rates are always equal to one another. In the first pair the changeable rates are DELTA1 smaller than the expansion limit in zone NA. In determining the upper constancy limit new pairs of alternatives are selected correspondingly, although with successively increasing changeable rates. Furthermore, the changeable rates in the first pair are DELTA1 higher than the contraction limit in zone NA. In both cases new pairs of alternatives are selected until the capital value of zones NA, NA+1, ..., NMAX in Alternative 1 is lower than or equal to the corresponding capital value in Alternative 2, i.e. until it is more advantageous to change the rate of production rather than to keep it constant.

The desired constancy limit is that changeable rate which makes the capital values of the two members of the pair of alternatives equal. Hence, the limit is a rate between the changeable rates selected in the last two pairs, or the changeable rate selected in the last pair.

For each successive pair the changeable rate changes by DELTA1 in a first approximation and by DELTA2 in a second approximation. In the second approximation the changeable rate in the penultimate pair of alternatives, is taken as a starting point. In addition, the changeable rate in one of the last two pairs selected, is accepted as the desired constancy limit in the second approximation. Which pair is accepted depends on the differences between the capital values of Alternatives 1 and 2, i.e. that pair is accepted in which the absolute difference is smallest. In other respects the method described previously applies in both approximations.

ANPAY1, ANPAY2, ANPAY3, and ANPAY4 are similar to the same subprograms in CUTOFF.

PRICOM has been extended to include the new variables in COMMON (see "The main program").

INDATT is similar to INDATT in CUTOFF.

PRICAP is similar to PRICAP in CUTOFF.

322 Input data

The same types of input data are needed in EXRATE as in CUTOFF (section 222 of this appendix). However, there are some differences. The initial guess concerns the rates of production, Q(N) for N=1,2,...,NMAX. In addition, some variables now express rates of production, MT/year, although they expressed average grades in CUTOFF:

DELTA1	This is the increment in the first approximations performed in EX2 and EX7. 2·DELTA1 is chosen as the increment for the rate of production in zone NA-1 in trying to find a sufficiently low or high rate of production in zone NA-1 when determining the expansion or contraction limit, respectively, in zone NA (compare section 324 of this appendix and M below). Permissible values: $0.0 < DELTA2 < DELTA1$.
DELTA2	This is the increment in the second approximations performed in EX4 and EX7. DELTA2 is also the minimum rate of production in zone NA-1 when the expansion limit in zone NA is being determined, and at the same time the minimum expansion limit. Permissible values: see DELTA1 and section 222.
HAEND1 HAEND2	Permissible values: $0.0 < HAEND1 < HAEND2$. HAEND2 should be considerably greater than HAEND1, e.g. 100 or 200 times higher. The rule given in section 222 of this appendix does not apply.

In other respects the meaning of input data and the limitations imposed on them are the same as in CUTOFF. One explanatory note might be added:

M	This is the minimum number of successive alternatives with strictly falling capital values required to make EX2 stop selecting new alternatives, as previously described. In connection with the first complete optimization, i.e. in partial optimizations for the first complete optimization, M is also utilized in calculating the expansion and contraction limits in the following manner:

Alternatives are tested successively. The difference between the value of Q(NA) and a constant value of Q(NA-1) increases steadily with the increment DELTA1 as new alternatives are selected. As the alternatives are tested successively, their capital values shall be increasing in at least the M consecutive alternatives 1, 2, ..., M in order to make the partial optimization in question accepted. This means that if among the M alternatives the capital value of the second of any two oonsecutive alternatives should be lower than that of the first, a new value of Q(NA-1) is selected, and the determination of the expansion or contraction limit again starts from the beginning. The new value of Q(NA-1) is 2·DELTA1 lower than the old value if the expansion limit is being determined, and 2·DELTA1 higher if the contraction limit is being determined.

The application of these constraints ensures that the capital value first increases over a comparatively large interval before it commences to decrease as the expansion or contraction of the rate of production is made successively greater. Similarly, the rule implied in the first paragraph concerning M (above), ensures that the capital value diminishes over a comparatively large interval as the change of the rate of production continues to increase. Between these two intervals there is either a maximum, which is the desired limit, i.e. the expansion or contraction limit (the first approximation), or an interval of either constant or partly increasing, partly decreasing capital values, which implies a set of potential limits (compare section 224 of this appendix). Consequently, this is a method of detecting and taking multiple maxima into consideration, and a way of avoiding erroneous suboptimizations.

After all input data have been read they are printed in tables and tested in INDATT in the same way as in CUTOFF.

323 Capital value of the initial guess

The initial guess, Q(N) for N=1,2,...,NMAX, is given as part of the input data. The capital value at time 0 of the entire ore deposit is calculated with these rates of production. The method used in CUTOFF is utilized, and the same output is printed (section 223 of this appendix). Note that the subzones are not eliminated, although the partial optimizations in EXRATE are concerned with the zones only. The subzones are of importance in calculating the capital value.

324 Partial optimizations in the zone: The expansion limit

The expansion limit in the current zone, NA, is determined in a first approximation, giving one or more potential optima, i.e. potential expansion limits, in EX2 and EX3 according to rules, which have been discussed in this appendix, i.e. in sections 321 (EX2 and EX3) and 322 (the variables DELTA1 and M). The limit is determined more exactly in a second approximation in EX4 as described in sections 225, 321 (EX4), and 322 (the variable DELTA2) of this appendix. If the first approximation should result in more than one potential expansion limit, each of them is treated in a second approximation, and the one giving the highest capital value is chosen.

For each new rate of production in zone NA, i.e. for each new alternative selected by EX2 or EX4, the optimum rates of production in zones NA+1, NA+2, ..., NMAX must be estimated, as these are dependent on the rate of production in zone NA. This is done by calling EX6. After that a complete production plan for the production period exists, and the capital value of zones NA, NA+1, ..., NMAX, which is the optimization criterion, can be calculated by first calling EX1, then CUT5.

In the second approximation the expansion limit in zone NA might be a rate of production which is smaller than the rate of production in zone NA-1[1]. As this is inconsistent with the concept of an expansion limit, an error indication is printed, a lower rate of production in zone NA-1 is selected, and a new attempt to find an expansion limit is made. The expansion limit might also coincide with the rate of the previous zone. As this is correct in some cases but incorrect in others, it cannot be automatically rejected. Here is a potential source of errors, which the program user should keep in mind. An error can be detected through the output printed: Coinciding rates are incorrect if

1) This cannot occur in the first approximation, where only rates of production in zone NA are selected, which are greater than the rate in zone NA-1 (section 321, EX2). There is no such limitation in the second approximation (section 225, paragraph 4)).

the capital value is an increasing function of the rate of production in zone NA over an interval just above the rate in zone NA-1.

The principal reason for errors of this type is that DELTA2 is large compared with the difference between the correct expansion limit and the rate of production in zone NA-1. If DELTA2 is greater than approximately one-fifth of this difference the alternative with Q(NA)=Q(NA-1) will appear among the selected alternatives. If in this case a payment appears which is, e.g., fixed and independent of the size of the expansion, but which also disappears if no expansion takes place, the alternative with Q(NA)=Q(NA-1) is favoured, and will probably be the optimum, i.e. the expansion limit. As no expansion takes place, this is incorrect.

The rules given for CUTOFF apply, where practicable, in other respects, e.g. regarding built-in limitations and tests (sections 224 and 225 of this appendix).

The values of the following variables are printed for each alternative selected in the first approximation, i.e. for JA=1,2,...,≤93[1]: MINE LIFE=T(1,NMAX+1), HA(JA), QX, and B(JA).

The values of the following variables are printed for each potential expansion limit, i.e. for JM=1,2,...,5: HAMAX(JM) and BMAX(JM). Note that HAMAX(JM) in EXRATE stands for a rate of production. In addition the comments on the output given in the last four paragraphs of section 224 also apply here.

The computer output in connection with the second approximation is the same as that of the first with the following exceptions: The values of HAMAX(JM) and BMAX(JM) are not printed. The same type of output is printed for each potential expansion limit. The expansion limit finally chosen is printed in a last message.

325 Partial optimizations in the zone: The contraction limit

The contraction limit is determined using methods similar to those utilized in determining the expansion limit. The difference is that the rates of production in zone NA of the alternatives selected in EX2 in the first approximation are

1) In the first approximation Q(NA)=Q(NA-1)+DELTA2 for JA=1.

successively smaller, and always less than the rate of production in zone NA-1. In the first approximation Q(NA)=Q(NA-1)-DELTA2 for JA=1.

The same type of output is printed as for the expansion limit.

326 Partial optimizations in the zone: The constancy limits. The complete optimization

The constancy limits are determined by EX7. The method is described in section 321 of this appendix. The values of the following variables are printed for each alternative of each pair of alternatives: MINE LIFE=T(1,NMAX+1), HA(JA), QX, and JA. The subscript JA here has a special significance:

JA=1	denotes Alternative 1 in the first approximation (see section 321 of this appendix, the subprogram EX7).
JA=2	denotes Alternative 2 in the first approximation.
JA=3	denotes Alternative 1 in the second approximation.
JA=4	denotes Alternative 2 in the second approximation.

Note that HA(2)=HA(4)=(the expansion limit in zone NA) when the lower constancy limit is being determined, and that HA(2)=HA(4)=(the contraction limit in zone NA) when the upper constancy limit is being determined. Note also that the values 1 and 2, or 3 and 4 reappear each time a new pair of alternatives is considered.

When the two constancy limits have been determined, the partial optimizations in the current zone are completed, and a summary of the results is printed. The summary contains the expansion and contraction limits and the two constancy limits in the current zone, i.e. the values of the variables QMIN(NA), QMAX(NA), QL(NA), and QH(NA), and the capital values of the zones NA, NA+1, ..., NMAX at the two constancy limits.[1)]

Partial optimizations resulting in the four limits are performed successively for all zones except the first in the order NMAX, NMAX-1, ..., 2. The rate of production is optimized directly for zone 1. The optimum rate of production in

1) If a constancy limit coincides with the expansion or the contraction limit, the corresponding capital value is not defined in the output.

zone 1 implies a set of optimum rates in zones 2, 3, ..., NMAX (see EX6 in section 321 of this appendix). The optimum in zone 1 and the corresponding optima in the succeeding zones form a complete set of (partial) optima, or an optimal policy. In the computer program the optimal policy is called a TOTAL OPTIMUM. It is further identified by an ordinal, i.e. the current value of JTOT, since the result of a single complete optimization is only a tentative optimum policy.

As in CUTOFF and for similar reasons the complete optimization is repeated a number of times. One difference in the reasons for the iterations will be mentioned: In EXRATE the effect of decisions concerning the current zone on the optimum rates in subsequently mined zones, is directly taken into account (in EX6). This is not the case in CUTOFF.

327 Iterative complete optimizations

The iterations are made by analogy to those described in section 226 of this appendix. Some differences are introduced, though:

Q(NA-1) is first set equal to Q(NA-1) according to the initial guess, when the expansion limit is being determined in the first complete optimization. Q(NA-1) is first set equal to QL(NA)+2·DELTA1 when the contraction limit is being determined in the first complete optimization (JTOT=0). These first values of Q(NA-1) are changed as described in section 322 of this appendix (the variable M) if they are not acceptable (see M). For the second and following complete optimizations, Q(NA-1) is primarily set equal to the optimum rate of production in zone NA-1 according to the latest complete optimization. If this rate exceeds or equals the lower constancy limit in zone NA, Q(NA-1) is set equal to this limit minus DELTA1 if the expansion limit is being determined. If the optimum rate in zone NA-1 is smaller than or equal to the upper constancy limit in zone NA, Q(NA-1) is set equal to the latter limit plus 4·DELTA1 if the contraction limit is being determined.

The output in connection with the termination of a complete optimization is in the first place of the same type as the output printed for complete optimizations in CUTOFF. In addition the four limits in each of the zones 2, 3, ..., NMAX are tabulated.

The program execution is also terminated as described in section 226 of this appendix.

4 Program CAPVAL

41 Applications

By means of the program CAPVAL the capital value at time 0 of one to ten predefined complete alternatives can be calculated. It is useful for several reasons. The two optimizing programs CUTOFF and EXRATE have some weak points. Therefore, it might be necessary to evaluate potentially optimal alternatives, which have been overlooked or eliminated in the automatic optimization procedures.

In CUTOFF and EXRATE continuously variable average grades and rates of production are optimized. A tool for calculating the capital value of an ore deposit in a number of given complete alternatives, is useful in other optimization problems. CAPVAL is intended as such a tool.

This program is, as the other programs, essentially valid for all firms and ore deposits in optimizations of the type discussed, except for CUT11 and the ANPAY subroutines.

42 Structure and operation

421 Survey of the structure

The program CAPVAL consists of a main program with 9 special subprograms. Standard library programs are also utilized.

The main program is only a frame of a program. It is not complete. In its simplest form, which is demonstrated in Appendix E, the program calculates the capital value in only one alternative, namely that defined by the formal input data.

New alternatives are defined by inserting new FORTRAN statements in the main program. In this way any of the original input data can be changed, thus creating new alternatives. As this involves changes in the main program, elementary knowledge of FORTRAN programming is necessary for the efficient use of the program.

VALGRD reads and prints input data, and also prints the values of the same variables and parameters for each new alternative. It calls CUT1 and CUT5 and prints the capital value of the ore deposit in each alternative.

CUT1, CUT11, CUT5, ANPAY1, ANPAY2, ANPAY3, ANPAY4, and PRICOM are all similar to the subprograms with the same names in CUTOFF.

422 Input data

Input data are treated in the same manner as in CUTOFF, except that they are not tested (INDATT is not included)[1]. Data cards intended for EXRATE can also be used. In that case the variables DELTA1, DELTA2, HAEND1, and HAEND2 should be interpreted as in EXRATE.

For each new alternative the values of the input variables are printed. To get this computer output correct, a special rule must be obeyed: If the number of zones is varied, the alternative comprizing the highest number of zones should be the first, which is defined in the formal input data. The array PAR(JD,LT), where JD=1,2,...,70 and LT=1,2,...,JTHOR, may be suppressed (see Appendix E, the comments in the source program of the main program of CAPVAL).

423 The capital-value calculation

The method described in section 223 (Capital value of the initial guess) is applied to all alternatives, and the same types of output are printed. However, the output of intermediate results can be suppressed (see comment in the main program in Appendix E). In that case only MINE LIFE=T(1,NMAX+1) and the capital value of the ore deposit are printed. Output with or without intermediate results can be selected separately for each alternative, or generally for all alternatives, or for any group of alternatives. In the version shown in Appendix E output without intermediate results has been selected.

When all the alternatives specified in the main program have been treated, the program execution is terminated.

1) If CAPVAL is used as a complement to CUTOFF or EXRATE, the input data are usually tested in these programs. The execution time of CAPVAL is also short, which reduces the cost of committing and correcting an error.

5 Subsidiary programs

51 PAYMTS - Graphic representation of payments

511 Applications

Thi sizes of all payments are determined in the subprograms ANPAY1, ANPAY2, ANPAY3, and ANPAY4. The payment functions programmed there are multi-dimensional and not easily grasped. In order to facilitate the understanding of the economic significance of the functions, and consequently to make it easier to understand, explain, and demonstrate the assumptions concerning the payments which are made in the optimizations, the program PAYMTS has been constructed. It shows graphically (via the printer) selected parts of the payment functions.[1)]

The parameters of the payment functions, PAR(JD,LT) with JD=1,2,...,70 and LT=1,2,...,JTHOR, determine the payment functions. Values must be found for the parameters, which make the functions as true representations of empirical facts as possible. The graphs produced by PAYMTS facilitate the comparison of the model with the facts. Discrepancies can be localized and corrected by adjusting the parameter values. Such a comparison might also give impulses to reprogram the ANPAY subprogram, e.g. to insert discontinuities into the payment funotions In some cases this necessitates ohanges in the main program of PAYMTS.

512 Structure and operation

The program PAYMTS consists of a main program with 6 special subprograms. Standard library subprograms are also utilized.

The main program reads and prints input data, determines all the variables in the payment functions, systematically varies the values of these variables, calls the ANPAY subprograms for calculating

1) CUT11 is a subprogram of a similar character. As it has been judged easy to visualize the corresponding function (the ore-reserve model; section 2 of Appendix D) a special program for "reading" it has not been constructed. If difficulties should arise, the output of CUT1 in CAPVAL could be utilized to tabulate the funotion.

the payments under the various conditions, and finally calls a curve-drawing subprogram. The main program also prints headings which label the graphs.

The ANPAY subprograms are the same as in the program CUTOFF, except that the last part of each subprogram now contains instructions for the curve drawing instead of printing instructions.

PRICOM has been simplified to an error indicator, which only tells that an error has been committed without giving any further information.

CURVE prints a curve or, more exactly, a series of asterisks approximating a curve, from a (2x50) matrix (50 points in a two-dimensional graph)[1].

A number of different types of payments are treated as dependent variables (section 514 of this appendix). All the different types of payments are functions of several independent variables, such as the rate of production, the average grade, and the expansion or the contraction of the rate of production (section 514 of this appendix). For each type of payment one graph is printed for each independent variable influencing the payment. The independent variables are those taken into account in the payment models of section 4 of Appendix D. In each graph the value of one independent variable is varied over a large interval. The values of the other independent variables are kept constant at preselected normal values.

The sizes of the payments often depend on the points of time at which the payments occur (section 222 of this appendix, the variable PAR(JD,LT)). For this reason the payment graphs are produced for two optional years from the years 2, 3, ..., JTHOR. In addition the graphs concerning ANPAY2 and ANPAY3 are always produced for year 1. If the two optional years are not explicitly defined, the years 2 and JTHOR are automatically selected.

513 Input data

PAYMTS reads input data from punched cards according to section 6 of this appendix. Only the first data card is specially designed for PAYMTS. The others

1) The underlying idea has been borrowed from Kallin (1967).

concern the parameters PAR(JD,LT), where JD=1,2,...,70 and LT=1,a,b. a and b stand for two optional years within the data horizon JTHOR. The parameters are read from the same cards as those used for the other programs.

The normal values of the independent variables are chosen in accordance with the instructions inserted into the source program (Appendix E). It should also be noted that normal values of especial interest in the acquisition of the input data, e.g. values representing the best known intervals, or estimated optimum values, are preferable. If the ore deposit is already being mined it might prove suitable to choose normal values close to the actual ones. All normal values must be greater than zero.

The following input data are needed:

HNORM — The normal value of the average grade should, if it is practicable, be an exact multiple of 0.0006, 0.006, or 0.06 depending on which of these three values is closest to, but below the desired normal value. The clearest output will be obtained in this way. If the limits of the ore body and the average grades in the subzones are fixed, set HNORM=PAR(4,LT).

QNORM — The normal rate of production should be an exact multiple of 10, 100, etc., according to the rule given for HNORM and for the same reason.

RNORM — RNORM represents the average depth of the mine, measured in cumulative equivalent ore reserve. It should be approximately half the total equivalent ore reserve of the deposit. Other values might be selected. See NMAX.

DRESET — The normal size of a zone is measured in equivalent ore reserve.

JTHOR — If graphs for year 1 and two optional years are desired, only parameters for these three years are read. The parameter cards for the three years must, in this case, be separated from the others. Put JTHOR=3. If parameters for all years are to be read, JTHOR denotes the actual data-horizon year (JTHOR$\geq$3). In this case graphs for years 1, 2, and JTHOR will be printed.

NMAX — The number of zones influences certain observation intervals. Therefore put NMAX=10, if possible. Then also put

	RNORM=5·DRESET. If the actual number of zones essentially differs from 10, say by ± 3 or 4 zones, NMAX should be the actual number and RNORM should be an exact multiple of DRESET, selected so that RNORM will be about half the total equivalent ore reserve of the deposit.
I	Depending on the value of this oontrol variable different sets of graphs will be produced: I=1: Graphs of payments acoording to ANPAY1 (current payments to and from the firm). I=2: Graphs of payments according to ANPAY2 (zone investments, expansion and contraction investments), and ANPAY3 (grade-change investments). I=3: Graphs of payments according to ANPAY4 (close-down payments). I≥4: Graphs of payments according to all ANPAY subprograms.
PAR(JD,LT)	Parameters of the payment functions for three years or for all years inside the data horizon are to be given (compare JTHOR).

The input data, except I, are printed in tables. They are not tested (compare section 422 of this appendix). None of the dependent variables (see section 514 of this appendix) may be put equal to zero for all examined values of the independent variables.

514 The graphs

Graphs are produced for a number of combinations of independent and dependent variables. The combinations are shown in Fig. B:4. For each graph the independent variable indicated is varied whereas the others are kept constant. The variables are listed below as they are called in the graphs:

Independent variables

1	RATE, MT/YR	Rate of production, MT/year. Q(N).
2	GRADE	Average grade, %/100. H(NS,N).
3	CUM. RESERVE, MT	The depth of the bottom of the current zone, measured in cumulative equivalent ore reserve. RES(1,N+1).
4	RATE CHGE, MT/YR	Change of rate of production, i.e. expansion or contraction. Q(N)-Q(N-1).
5	NIV. SIZE, MT	Zone size in millions of tons of equivalent ore reserve. RES(1,N+1)-RES(1,N).

6 GRADE CHANGE	Grade change in 1/100 percentage units. H(NS,N)-H(NS-1,N) if NS > 1 (-H(NSMAX,N-1) if NS=1).
7 ORE RESERVE, MT	Total equivalent ore reserve of the ore deposit. RES(1,NMAX+1).

<u>Dependent variables</u>

1 SALES, MKR/YR	Payments received for ore sold in one year in MKR/year. SS.
2 OP. PMTS, MKR/YR	Payments for annual current operating costs. SK.
3 REINVEST.,MKR/YR	Annual current reinvestments. SG.
4 CURR. NET,MKR/YR	Annual current net payments. SSKG=SS-SK-SG.
5 NIV. INVEST.,MKR	Zone investment. SE.
6 CAPCITY INV.,MKR	Expansion and contraction investments. SF and SH.
7 GRDECHGEINV.,MKR	Grade-change investment. SLM.
8 GRDCH SAVED, MKR	Payment to be subtracted from SLM if NS=1. SPAR.
9 CLOSE DOWN, MKR	Close-down payments. HEND.

Dependent variables	Independent variables 1	2	3	4	5	6	7	Sub-program
1	X	X	X					ANPAY1
2	X	X	X					
3	X	X	X					
4	X	X	X					
5	X	X	X	X	X			ANPAY2
6				X				
7						X		ANPAY3
8						X		
9	X	X					X	ANPAY4

<u>Fig. B:4</u> Combinations of independent and dependent variables for which graphs are produced.

The graphs do not illustrate all the interdependencies of the variables. It should be especially mentioned that the dependent variables 6, 7, and 8 are dependent also on the independent variable 1. Further, the relation between the independent variable 3 and the dependent variable 5 is influenced by the independent variable 1. In order to illustrate such relations the graphs can be produced in several editions, each with new normal values. In the above example a set of values of QNORM are selected. In addition, by putting I=2 the sets of graphs are confined to the interesting dependent variables. Thus, by utilizing the program repeatedly with varying values of HNORM, QNORM, etc., the implications of the payment functions can be studied more thoroughly.

52 NEWPAR - Revising the parameters of the payment functions

521 Applications

In order to find acceptable values of the parameters of the payment functions it is often a practical method to start from a set of roughly estimated parameters, which are then successively refined. It is also interesting to repeat optimizations under differing assumptions concerning the payment functions. In both cases a selected few of the parameters that can be very numerous, are altered each time. NEWPAR is a program for altering parameter values and punching new data cards for PAR(JD,LT). NEWPAR reads the values of the old parameters, alters the values of optional parameters, changes, if desired, the value of JTHOR, and determines parameter values for added years, if any. The parameter values for all years including the new horizon year are printed. At the same time a new set of data cards, including all the parameters PAR(JD,LT) for JD=1,2,...,70 and LT=1,2,...,JTHOR, is punched.

522 Structure and operation

The program NEWPAR consists of a main program. The program shown in Appendix E is only a frame of a program. To function it must be made complete through FORTRAN statements where the value of JTHOR for the old set of parameters, a new value of JTHOR (if desired), and the desired new values of parameters are stated.

The source program in Appendix E contains only the input and output statements, which are common to all usages of the program. The card deck produced is ready to be used in any of the other programs.

6 Data input

61 Common features

Input data are read from cards in fixed formats. The organization of the input-data cards will be described in these sections. The different programs use the same data, or the same types of data to a considerable extent. So most of the input-data cards can be used in any of the programs. They are described first. The special arrangements for each program will then be discussed.

The first common input variable is RES(NS,N). Values exist for NS=1,2,...,NSMAX, and N=1,2,...,NMAX+1. NSMAX$\leq$20. NMAX$\leq$14. Suppose first that NMAX=14. The input format of RES(NS,N) is illustrated in Fig. B:5a. The value of RES(1,1) is punched in the first 7 columns of the card, the value of RES(1,2) in the next 7 columns, etc. The positions of the decimal points are indicated[1]. The values of RES (1,N) for N=1,2,...,11 are placed on the first card, and the values for N=12,13,14 and 15 on the second. In the same manner the values of RES(2,N) for N=1,2,...,15 are placed on the 3rd and 4th cards, of RES(3,N) for N=1,2,...,15 on the 5th and 6th, and so on for NS=1,2,...,NSMAX.

If there are less than 14 zones, i.e. if NMAX$<$14, the highest value of N, NMAX+1, is less than 15. Then, in each successive pair of cards NMAX+1 values and not 15 values should be punched. If NMAX$\leq$9 no 2nd, 4th, etc. card is needed, and only one card for each value of N may be used. Note that if NMAX=10 the second card is necessary, though it will contain no data.

Columns 78–80 contain an identification code: e.g. the letter R followed by the value of the subscript NS. If two cards are needed for each NS the 2nd, the 4th, etc. card may be identified by an extra sign in a free column, e.g. by the figure 2 in column 77.

The values of HEQV(N) for N=1,2,...,NMAX are punched according to Fig. B:5b. Two cards are needed if NMAX$\geq$11. If NMAX=11 the second card contains no data. If NMAX$\leq$10 only one card may be used. The identification codes HEQ and 2HEQ are optional.

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1) The decimal point can be placed anywhere in the data field. However, the input data are reproduced on the printer with the indicated number of decimal places.

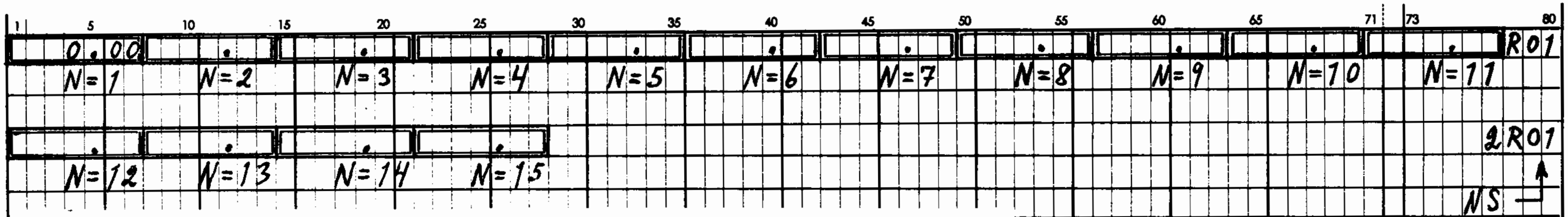

Fig. B:5a RES(NS,N)

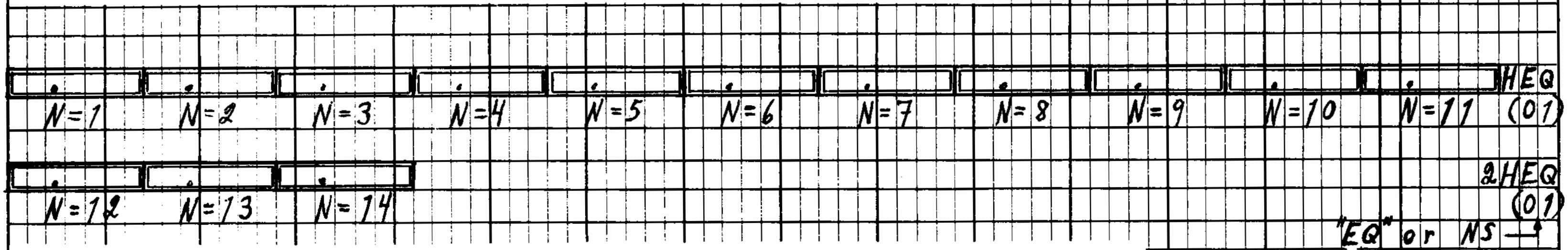

Fig. B:5b HEQV(N) and H(NS,N) (Note: H(NS,N) in after Q)

The same rules apply for H(NS,N), where NS=1,2,...,NSMAX and N=1,2,...,NMAX. Depending on NMAX, one or two cards are prepared for each NS. Columns 78–80 can contain an identification code, e.g. the letter H followed by the value of the subscript NS.

Q(N) for N=1,2,...,NMAX are to be transmitted between HEQV(N) and H(NS,N). The format is demonstrated in Fig. B:6a. Two cards are necessary if NMAX$\geq$11. If NMAX=11 the second card contains no data. If NMAX$\leq$10 only one card is used. The identifiers Q and 2Q are suggested.

The parameters C1(N), C2(N), C3(N), and C4(N) for N=1,2,...,NMAX are transmitted into the system according to Fig. B:6b. For C1(N) two cards are punched if NMAX$\geq$9, and one card is punched if NMAX$\leq$8. C1 and 2C1 are identifiers. C2(N), C3(N), and C4(N) are represented correspondingly, each on one or two cards according to the actual value of NMAX. Corresponding identifiers: C2, 2C2, C3, 2C3, C4, 2C4.

The parameters C5, C6, C7, C8, and C9 are contained in one card, identified as C9 (Fig. B:6b).

Fig. B:7 shows the format of the parameters PAR(JD,LT) for JD=1,2,...,70 and LT=1,2,...,JTHOR. The order of the parameter cards is:

1st card:	PAR(JD,LT)	for	JD=1,2,...,8	for LT=1
2nd card:	PAR(JD,LT)	for	JD=9,10,...,16	for LT=1
...	...	...	...	... ...
	PAR(JD,LT)	for	JD=57,58,...,64	for LT=1
	PAR(JD,LT)	for	JD=65,66,...,70	for LT=1
	PAR(JD,LT)	for	JD=1,2,...,8	for LT=2
	...	...	...	... ...
	PAR(JD,LT)	for	JD=65,66,...,70	for LT=2
	...	...	...	... ...
	PAR(JD,LT)	for	JD=65,66,...,70	for LT=JTHOR.

The free columns 73–80 can be used for identifying the cards, e.g. by punching the value of the index JD of the last parameter of each card in columns 77-78, and the value of LT (=year) in columns 79–80. The first card of the first year is thus identified as 0801, the last card of the first year 7001, etc.

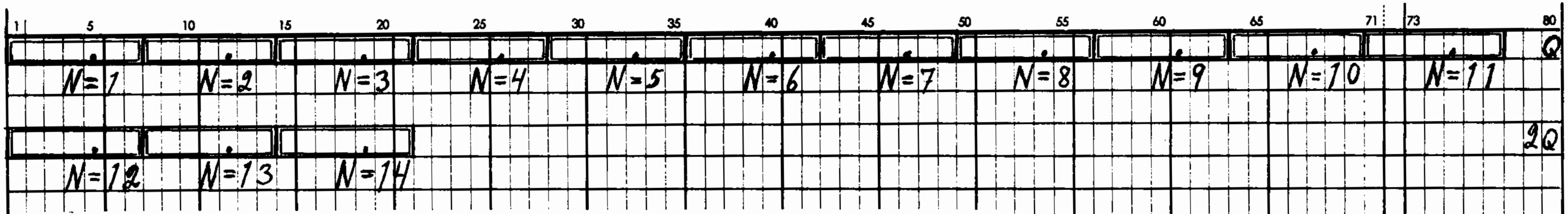

Fig. B:6a Q(N)

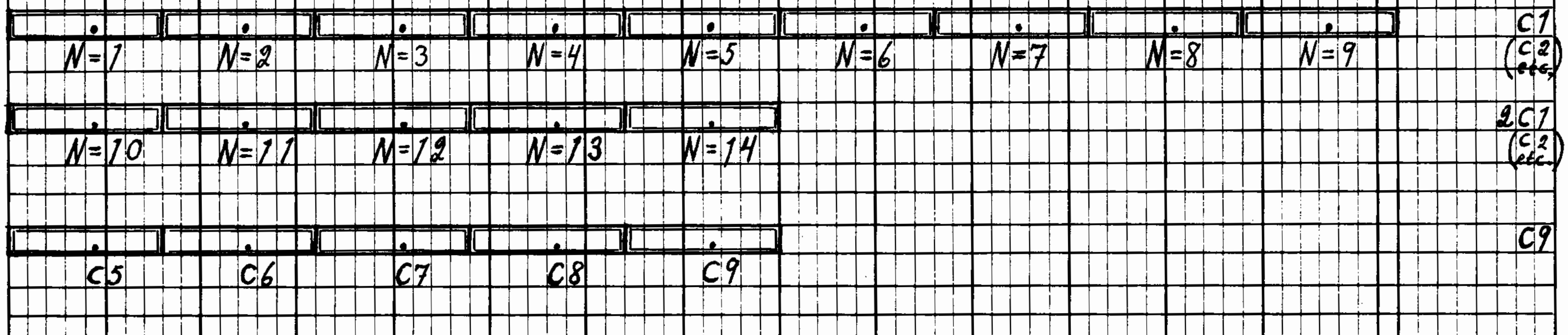

Fig. B:6b C1(N), C2(N), C3(N), and C4(N). C5, ..., C9.

JD= 1	JD= 2	JD= 3	JD= 4	JD= 5	JD= 6	JD= 7	JD= 8	0801
9	10	11	12	13	14	15	16	1601
17	18	19	20	21	22	23	24	2401
25	26	27	28	29	30	31	32	3201
33	34	35	36	37	38	39	40	4001
41	42	43	44	45	46	47	48	4801
49	50	51	52	53	54	55	56	5601
57	58	59	60	61	62	63	64	6401
JD=65	JD=66	JD=67	JD=68	JD=69	JD=70			7001

Last JD
Year LT

Fig. B:7 PAR(JD,LT)

These cards are common to the programs CUTOFF, EXRATE, and CAPVAL. The programs PAYMTS and NEWPAR use only the cards containing PAR(JD,LT). When all the common cards are used they should be sorted in the following order (only the identifiers are indicated):

RO1, 2RO1, RO2, 2RO2,..., R(NSMAX), 2R(NSMAX)
HEQ, 2HEQ
Q, 2Q
HO1, 2HO1, HO2, 2HO2,..., H(NSMAX), 2H(NSMAX)
C1, 2C1, C2, 2C2, C3, 2C3, C4, 2C4
C9
0801, 1601,...,70(JTHOR)

The cards with identification codes beginning with the figure 2 must be excluded for certain values of NMAX (see above).

62 CUTOFF

For CUTOFF one special card is needed. Its format and contents are defined in Fig. B:8a. The program name is used as identifier. This card is the first data card for CUTOFF, and it is placed immediately before the oommon input data cards. No further data cards are needed.

In fields lacking a decimal point the input consists of integers, which should be punched right-justified in the fields.

63 EXRATE

For EXRATE one special card is needed. Its format and contents are defined in Fig. B:8b. The program name is used as identifier. This card is the first data card for EXRATE, and it is placed immediately before the common input data cards. No further data cards are needed.

In fields lacking a decimal point the input consists of integers, which should be punched right-justified in the fields.

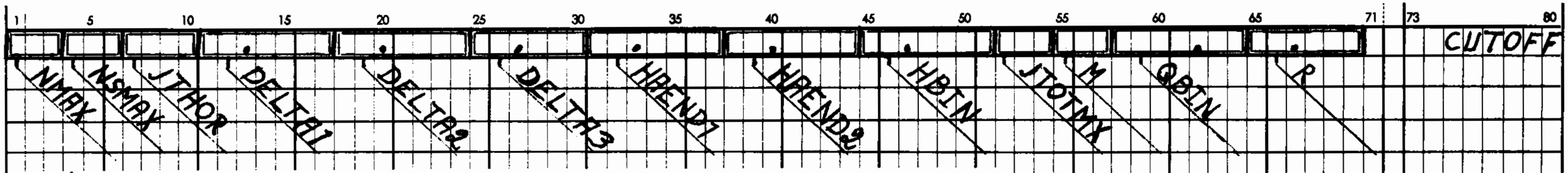

Fig. B:8a CUTOFF

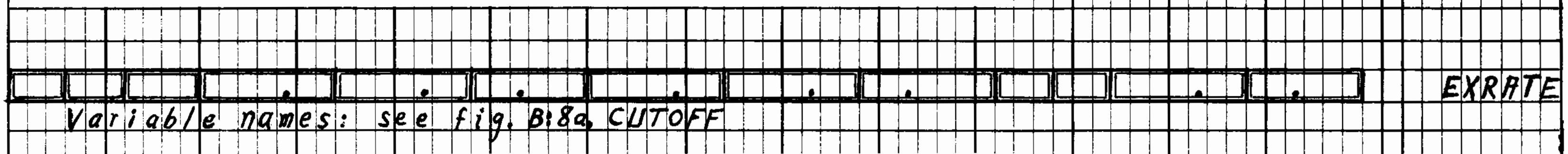

Fig. B:8b EXRATE

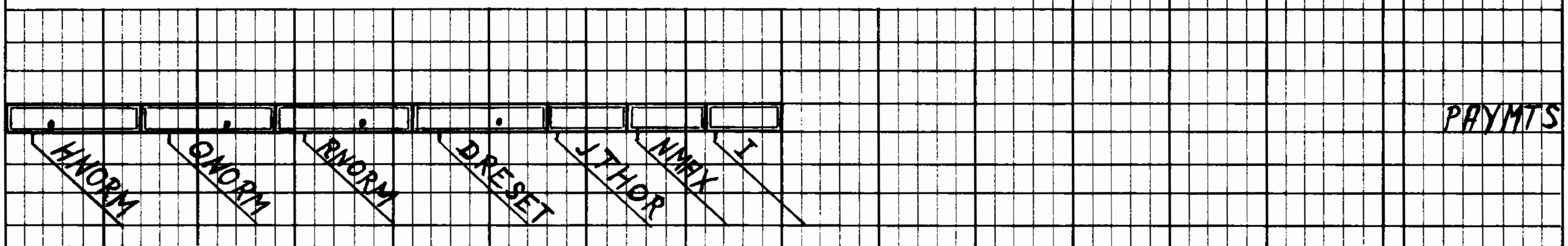

Fig. B:8c PAYMTS

64 CAPVAL

Any one of the complete sets of data cards for CUTOFF or EXRATE can be used for CAPVAL. However, all parameters listed in Fig. B:8a (i.e. on the first card of the data-card deck) are not necessary in this program. Only the values of NMAX, NSMAX, JTHOR, HBIN, QBIN, and R have to be defined.

65 PAYMTS

For PAYMTS one special card is needed. Its format and contents are defined in Fig. B:8c. The program name is used as identifier. Moreover, only the cards containing the values of the parameters PAR(JD,LT), i.e. either the cards 0801,1601,...,70(JTHOR) or some of them, namely the cards for LT=1 and for two further optional values of LT, are utilized. Note the constraint JTHOR$\geq$3. The special data card, identified as PAYMTS, should be placed immediately before the card 0801.

In fields that, according to Fig. B:8c have no decimal points, integer data should be punched right-justified.

66 NEWPAR

In NEWPAR only the input-data cards containing the values of the parameters PAR(JD,LT), i.e. the cards 0801,1601,...,70(JTHOR) are used. Note that the program must be completed with a number of FORTRAN statements.

7 Input-data errors. INDATT error codes.

The programs CUTOFF and EXRATE have in many cases comparatively long execution times. In order to avoid unnecessary erroneous program executions a test of oertain input data has been inserted before the real execution is started. The test is performed in the subroutine INDATT. A detected error causes an error indication and the disruption of the program execution. Furthermore, the computer system includes standard error detection facilities.

An error indication from INDATT refers to a code number. The code numbers are:

1 NMAX $>$ 14
2 NSMAX $>$ 20
3 JTHOR $>$ 50
4 DELTA2 $\geq$ DELTA1
5 DELTA3 $\geq$ 1.
6 JTOTMX $<$ 3 or JTOTMX $>$ 20
7 M $<$ 3
8 PAR(70,1) $>$ 1.
9 PAR(4,1) $>$ PAR(4,JTHOR)
10 R=0.

The tests for errors in INDATT are easily revized or extended.

8 Program or execution errors. PRICOM

At certain points in the programs the consistency and the plausibility of the operations or of the results of the operations of the programs, are tested. If these are not satisfactory PRICOM is called and the execution of the program is stopped. PRICOM prints "-STATE OF COMMON" followed by the values of some key variables in COMMON. The first of these, the value of the variable IDENT, is a reference to the point in the program where the error was detected. It is generally the sequence number of the program cards (the numbers to the extreme right in the source programs) of the FORTRAN statement preceding the CALL PRICOM statement. From that point the real cause of the trouble should usually be easily traced.

In addition to the output from PRICOM a few lines of the same general type beginning with "-STATE OF CUT5" are printed if the error is detected in the subprogram CUT5.

For formats of these messages, see PRICOM and CUT5.

This type of error can primarily be attributed to errors or inconsistencies in input data, source program errors not yet detected, or properties of the optimizing methods used.

Appendix C

Variables in the computer programs

The name of a variable consists of one or more signs (letters or figures). Subscripts are enclosed in brackets, e.g. x_{ij} is written as X(I,J). The variable names often have mathematical counterparts used in this report. These are written below the variable names, and are enclosed in brackets. A complete list of the mathematical symbols is found in Appendix F. Only variables encountered in the input or the output of the programs are listed. Other variables are internal to the programs, and implicitly defined in the programs.

B(JA) (B, B_{1n0}, $B_{n'n0}$, $B'_{n'0}$)	Capital value of the ore deposit or of part of it in the alternative JA. MKR.
BMAX(JM)	Capital value at the maximum JM. MKR.
BTOT(JTOT)	Capital value of the ore deposit at the complete set of optima (optimal policy or total optimum) JTOT. MKR.
C1(N) - C4(N) (C_{in}, i=1, 2,3,4)	Parameters for establishing the quantity of ore actually extracted in zone N as a function of the average grade and the equivalent ore reserve used in zone N, i.e. for transforming equivalent ore reserves into actual ore reserves.
C5 - C9 (C_i, i=5, 6,7,8,9)	Parameters as C1(N) - C4(N) but equal for all N.
DELTA1	Increment of the first approximation. In CUTOFF the average grade is incremented by DELTA1·100 percentage units. In EXRATE the rate of production is incremented by DELTA1 MT/year.
DELTA2	Increment of the second approximation. See DELTA1.
DELTA3	Minimum relative increase in the capital value of the ore deposit as a result of the latest complete optimization to induce the program to make another complete optimization. A fraction.

DRES ($\bar{R}_{n'n}$)	Equivalent ore reserve in subzone NS of zone N. MT.
DRESET	Normal equivalent ore reserve in one zone. MT.
EXDRES ($R_{n'n}$)	Actual ore reserve in subzone NS of zone N. MT.
H(NS,N) ($\bar{h}_{n'n}$)	Average grade in subzone NS of zone N. A fraction.
HA(JA)	In CUTOFF: Average grade in the current subzone NSA of the current zone NA in alternative JA. A fraction. In EXRATE: Rate of production in the current zone NA. MT/year.
HAEND1	In CUTOFF: The lowest plausible average grade of ore extracted from the deposit. A fraction. In EXRATE: The lowest plausible rate of production. MT/year.
HAEND2	In CUTOFF: The highest plausible average grade of ore extracted from the deposit. A fraction. In EXRATE: The highest plausible rate of production. MT/year.
HAMAX(JM)	In CUTOFF: Average grade in the current subzone NSA of the current zone NA at the capital-value maximum JM. A fraction. In EXRATE: Rate of production in the current zone NA at the capital-value maximum JM. MT/year.
HB ($\bar{h}_{n'-1,n}$ and $\bar{h}_{N',n-1}$)	Average grade in the previous subzone, i.e. in the subzone subscripted (NS-1,N) if NS≠1, and (NSMAX,N-1) if NS=1. A fraction.
HBIN ($\bar{h}_{00}$)	Average grade of ore mined immediately before starting in zone 1 when the program is applied to an ore deposit which is already being exploited. For fresh ore deposits it is arbitrarily assumed that HBIN=0.0. A fraction.
HEND ($H_{a,1,N+1}$)	Payments at time T(1,NMAX+1) in connection with the final closing of the mine (close-down payments). MKR.
HEQV(N) ($\bar{h}_n$)	Equivalent average grade in zone N. It is assumed to be equal in all the subzones of the zone. A fraction.

HNORM	Normal average grade. A fraction.
I	In PAYMTS: Execution control variable.
IDENT	Reference to the point in a program where the execution has been disrupted because of an error detected. This reference generally is the sequence number of a program card.
JA=1,2,...,99	Subscript denoting the ordinal of the alternative being processed or referred to. JA=95, or JA being negative, or represented by asterisks means that no specific alternative is being referred to.
JD=1,2,...,70 (i, see PAR(JD,LT))	Subscript of the parameter PAR(JD,LT) denoting the JDth parameter for the calendar year LT.
JM=1,2,...,5	Subscript denoting the ordinals of the capital-value maxima detected in CUT3 or EX3.
JT=1,2,... (t in certain cases)	Subscript denoting the calendar year of any point of time. Times +0.0 up to and including 1.0 occur during the year 1, i.e. JT=1, times after time 1.0 up to and including time 2.0 during the year 2, i.e. JT=2, etc.
JTHOR (A)	The data horizon of the payment functions expressing the number of years for which the parameters PAR(JD,LT) are specified annually and the last calendar year thus specified. For years beyond this horizon (JT>JTHOR) the parameters of the horizon year, JTHOR, are assumed to be valid.
JTOT=1,2,..., JTOTMX	Subscript denoting the ordinals of the complete optimizations. Before the complete optimization JTOT is terminated it is referred to as JTOT-1 in the data output related to it.
JTOTMX	The maximum number of complete optimizations desired.
LT=1,2,...,JTHOR (a)	Subscript of the parameter PAR(JD,LT) denoting the calendar year JT. If JT>JTHOR, LT=JTHOR is assumed.
M	Minimum number of strictly falling capital values in successive alternatives JA= 1,2,...,M,M+1,M+2,... to induce the program not to examine further alternatives in the current direction.

MINE LIFE ($T_{1,N+1}$)	This is not a variable name. It refers to the point of time T(1,NMAX+1) which is one year more than the production period of the mine, due to the fact that the ore extraction commences at time 1.0, not at time 0.
MKR	This is not a variable name. It is an abbreviation for "Millions of Swedish Kronor" but may, naturally, be interpreted as referring to any monetary unit.
MT	This is not a variable name. It is an abbreviation for "Millions of tons". It may, however, be interpreted as any ore-quantity measure, provided that all economic data are measured against the scale thus implied.
N=1,2,...,NMAX (n)	Subscript of zones. The zones are mined in the order 1,2, ...,NMAX.
NA	Subscript denoting the current zone, i.e. the current partial optimization is concerned with the zone NA or with a subzone of zone NA.
NIV	This is not a variable name. It stands for "Zone".
NMAX (N)	The number of zones into which the mine is partitioned and the subscript number of the one being mined last.
NS=1,2,...,NSMAX (n')	Subscript of subzones. In each zone the subzones are mined in the order 1,2,...,NSMAX.
NSA	Subscript denoting the current subzone, i.e. the current partial optimization is concerned with the subzone NSA of the current zone NA.
NSMAX (N')	The number of subzones into which each zone is partitioned and the subscript number of the subzone which is last mined in each zone.
PAR(JD,LT) (c_{ia})	Parameters of the payment functions.
Q(N) (Q_n)	Rate of production in zone N. It is assumed to be equal in all the subzones of the zone. MT/year.
QB (Q_{n-1})	Rate of production in the previous zone, i.e. in zone N-1. MT/year.
QBIN (Q_0)	Rate of production immediately before starting in zone 1. If the ore deposit is not previously mined, QBIN=0. MT/year.

QH(N)	Contraction limit in zone N. MT/year.
QL(N)	Expansion limit in zone N. MT/year.
QMAX(N)	Upper constancy limit in zone N. MT/year.
QMIN(N)	Lower constancy limit in zone N. MT/year.
QNORM	Normal rate of production. MT/year.
QX	Rate of production in the zone immediately preceding the current zone, i.e. in zone NA-1. If NA=1, QX=QBIN. MT/year.
R (j)	Rate of interest (continuous). R·100%/year.
RES(NS,N) (See reference)	Cumulative equivalent ore reserve, i.e. the equivalent ore reserve in all the zones mined before subzone NS of zone N. See section 222 of Appendix B. MT.
RNORM	Normal cumulative equivalent ore reserve, i.e. about half the total equivalent ore reserve of the deposit. See section 513 of Appendix B. MT.
SE (E_{a1n})	Zone investment in zone N at time T(1,N). MKR.
SEFHLM	Total investment at time T(1,N) (=SE+SF+SH+SLM). MKR.
SF (F_{a1n})	Expansion investment at time T(1,N). MKR.
SG ($G_{an'n}$)	Annual current reinvestments in the year JT during the production period of subzone NS of zone N. MKR/year.
SH (H_{a1n})	Contraction investment at time T(1,N). MKR.
SK ($K_{an'n}$)	Payments for annual current operating costs in the year JT during the production period of subzone NS of zone N. MKR/year.
SLM ($L_{an'n}+M_{an'n}$)	Grade-change investment at time T(NS,N). MKR.
SPAR	A payment reduction to be subtracted from SLM because the change in the average grade of the ore occurs at the same time as a zone investment. MKR.

SS ($S_{an'n}$)	Annual payments received for products sold in the year JT during the production period of subzone NS of zone N. MKR/year.
SSKG	Annual current net payments in the year JT during the production period of subzone NS of zone N (=SS-SK-SG). MKR/year.
SUBNIV	This is not a variable name. It stands for "Subzone".
T(NS,N) ($T_{n'n}$)	Starting time of subzone NS of zone N. Years from time 0.
T(1,NMAX+1) ($T_{1,N+1}$)	The time of the final closing of the mine. It is also referred to as MINE LIFE. It should be noted that the actual production period is one year shorter than MINE LIFE as no mining occurs during the first year. Years from time 0.
YR	This is not a variable name. It is an abbreviation for "Year".

Appendix D

Mathematical models in the computer programs

Contents Page

1 Introduction

In the computer programs CUTOFF, EXRATE, etc. some specific mathematical models are contained or implied. In order to facilitate the adaptation of the programs to individual cases the mathematical formulation of these models will be given. However, the over-all decision models will not be developed here. Instead the detailed models of the ore reserve, of the capital value of the ore deposit, and of the payments connected with the exploitation of the ore deposit will be expounded.

The capital-value model is a general model whereas the others describe special cases. The latter are mainly constructed for program testing and demonstration purposes. They are not intended to describe any existing mine or ore deposit, but they are such that there could exist a mine for which the models are fully relevant. It is expected that, if they do not suit a particular case, they will be useful starting points in the construction of more adequate models.

There are no restrictions regarding the reformulation of the ore-reserve model or the payment models, as long as such reformulations do not introduce variables and parameters not included in the original models described here. If a variable that has not previously been used in a certain model, is to be introduced into it, it must be established that the variable has been adequately defined at the points in the programs where the subprogram containing the model is called[1]. This necessitates some study of the source programs. If any of the subscripted parameters C or c is to be introduced into a model not previously containing it, it must be either freed from its former use[2], or applied as it stands. In the latter case the value of the parameter naturally is the same in all its uses (see e.g. c_{4a} in sections 42, 43, 44, 45, etc. of this appendix).

The symbols used are listed and defined in Appendix F.

---

1) The computer programs consist of main programs, each with a set of subprograms. In order to use a subprogram of this particular type a main program (or another subprogram) is said to call the subprogram.

2) Payments according to the subprograms ANPAY1 and ANPAY4 (section 41 of this appendix) do not occur in year 1, while the other types of payments (investments) usually do. The parameters for year 1 used in ANPAY1 and ANPAY4 can thus always be freed and introduced into the models programmed in the other ANPAY subprograms.

2 Ore-reserve model

An ore deposit is assumed to contain a rich principal vein surrounded by, or extending into, poorer ore. As successively poorer ore is recovered through extending the mining limits, the average grade decreases and the ore reserve increases.

The ore-reserve model defines how the actual ore reserve[1] in a subzone varies with the average grade of the ore in the subzone. The fixed quantity of ore that defines the size of the subzone is the equivalent ore reserve, i.e. the ore reserve of the subzone at a certain fixed average grade (the equivalent average grade). The ore-reserve model is placed in the subroutine CUT11.

For $\bar{h}_{n'n} \leq \hat{h}_n$ the following function is assumed[2]:

$$R_{n'n}=C_5\cdot\hat{R}_{n'n}\cdot e^{C_6+1-C_7\cdot\bar{h}_{n'n}}+(1-C_5)\cdot\hat{R}_{n'n}\cdot(1+C_{1n}\cdot e^{C_{2n}\cdot(\hat{h}_n-\bar{h}_{n'n})}-C_{1n})$$

For $\bar{h}_{n'n} > \hat{h}_n$ the same function applies, C_{1n} and C_{2n} having been replaced by C_{3n} and C_{4n}, respectively. Two parameters, C_8 and C_9, are available in the programs but have not been used here.

3 Capital-value model

The capital value of the ore extraction from the deposit after the point of time of a decision is the criterion of this decision. The capital values calculated in the programs are referred to time zero, i.e. the payments considered are discounted at the given rate of interest, j, to time 0. Interest is reckoned continuously and current payments are supposed to constitute a continuous flow. The subprogram CUT5 contains the capital-value model.

The decision times $T_{n'n}$, n' assuming any of the values 1, 2, ..., N', and n any of the values 1, 2, ..., N, are determined in CUT1 or EX1 as follows:

$R_{n'n}$ is defined in section 2 of this appendix.

---

1) The actual ore reserve in a subzone is the quantity of ore actually extracted or extractable from the subzone (i.e. exploitable ore. Compare section 21).

2) The first term of the expression is a direct application of a relationship for disseminated ore bodies given by Henning (1963).

$\Upsilon_{n'n}=\frac{R_{n'n}}{Q_n}$ is the production period of subzone n' of zone n.

$$T_{n'n}=1+\underbrace{\sum_{i=1}^{n-1}\sum_{k=1}^{N'}\Upsilon_{ki}}_{n>1}+\underbrace{\sum_{k=1}^{n'}\Upsilon_{kn}}_{n'>1} \qquad T_{1,N+1}=T_{N'N}+\Upsilon_{N'N}$$

where i=1,2,...
k=1,2,...

If a condition written directly under a term in an expression, e.g. n>1, is not fulfilled, the term is given the value zero.

Let $x_{n'na}$ be the flow of payments during one year, as measured in money units per year, provided that the subzone n' of zone n is being mined and that the payment functions for year a adequately define the payments during the actual year (year a if a< A. Compare below.). Let also $y_{n'na}$ be the payments (usually negative) at time $T_{n'n}$ according to payment functions for year a. These payments are discontinuous. They comprise investments and similar payments, such as payments in connection with the final closing of the mine. Finally, let b(t) stand for the point of time ending the calendar year during which t occurs,

thus: $b(t)=1$ for $0.0<t\leq 1.0$
$b(t)=2$ for $1.0<t\leq 2.0$
... ...

Then, for a decision at time $T_{\alpha\beta}$, α assuming any of the values 1, 2, ..., N', and β any of the values 1, 2, ..., N, the capital value forming the decision criterion is:

$$B_{n'n0}=y_{n'na}\cdot e^{-j\cdot T_{n'n}}+\int_{T_{n'n}}^{b(T_{n'n})} x_{n'na}\cdot e^{-j\cdot t}dt+\sum_{i=b(T_{n'n})}^{b(T_{n'+1,n})-2}\int_{i}^{i+1} x_{n'na}\cdot e^{-j\cdot t}dt$$

$$+\int_{b(T_{n'+1,n})-1}^{T_{n'+1,n}} x_{n'na}\cdot e^{-j\cdot t}dt+B_{n'+1,n,0}$$

where $i=1,2,\ldots,$

$n'=\alpha,\alpha+1,\alpha+2,\ldots,N'$ for $N=\beta$, $n'=1,2,\ldots,N'$ for $N>\beta$, and $n'=1$ for $n=N+1$, and

$n=\beta,\beta+1,\beta+2,\ldots,N+1,$

subject to the following rules:

The subscripts $1,n+1$ replace the subscripts $n'+1,n$ when $n'=N'$.

The upper limit of the first integral is altered to $T_{n'+1,n}$ if $b(T_{n'n})>T_{n'+1,n}$.

The value of an integral or of the sum $\sum_i f_i(t)$ is zero if the value of the lower limit exceeds or is equal to the value of the upper limit of the integral or the sum. $x_{n'na}=0$ if $n>N$.

Further, $a=1$ for $0.0<t\leq 1.0$

$a=2$ for $1.0<t\leq 2.0$

... ...

$a=A$ for $A-1.0<t\leq A$, and

$a=A$ for $A<t$.

One type of capital values appearing in the output of the programs should be especially mentioned: CUMULATIVE CAP VAL FROM TIME 0 B(JA)= ... . These capital values are intermediate values in calculations according to the above formula. They consist of the discounted values of payments from time 0 to the end of the year defined in the output printed, or, if the output appears in connection with a change between subzones or zones, from time 0 to the starting time of the new subzone or zone.

4 Payment models

41 Introduction

The payment models describe the payments connected with the exploitation of an ore deposit. Among these are the payments to the firm for ore sold, the payments going out from the firm for current production costs, and reinvestments. It is assumed that these payments constitute a continuous flow, and they are measured in money units per year (MKR/year). The surplus of the payments thus received over the two types of disbursements constitutes the annual current net payments. These are programmed in the subprogram ANPAY1.

Payments caused by and occurring at the time of moving the mining activities from one zone to another, i.e. payments caused by changing zones, are the zone investments of the new zone. At the same time the capacity of the mine might be altered, causing payments named expansion investments or contraction investments. It is assumed that these payments occur at times T_{1n}. The payments are programmed in ANPAY2.

Similarly, it is assumed that payments at the starting times of new subzones $T_{n'n}$, are caused by any simultaneously occurring change of the average grade of the ore mined (the average grade is fixed during the production period of each subzone). These grade-change investments are programmed in ANPAY3.

Payments occurring when the mine is finally closed at time $T_{1,N+1}$, i.e. the close-down payments, are programmed in ANPAY4.

A schematic mode of describing the functions will be used:

$\underbrace{\qquad\qquad}$ Fixed, Volume, etc.	The key words tell which dependencies the different parts of the payment functions are mainly intended to describe.

Certain interrelations are common to the different payment models:

$\bar{h}_{n'-1,n} = \bar{h}_{n'n}$ when $n'=1$ and $n=1$, if $\bar{h}_{00}=0.0$

$\bar{h}_{n'-1,n} = \bar{h}_{00}$ when $n'=1$ and $n=1$, if $\bar{h}_{00}>0.0$

$i=1,2,\ldots,n-1$ when the upper limit of a sum is $n-1$
$i=1,2,\ldots,n$ when the upper limit of a sum is n
$n'=1,2,\ldots,N'$
$n=1,2,\ldots,N$

$Q_{n-1} = Q_0$ when $n=1$

$\sum_{i=1}^{n-1} R_i = 0.0$ when $n=1$.

$n'-1,n$ becomes $N',n-1$ when $n'=1$.

42 Payments received for products sold

$$S_{an'n} = \Big[\underbrace{c_{1a}}_{\text{Basis price}} \underbrace{- \underbrace{c_{2a}\cdot Q_n^2}_{\text{Rate of production}} + \underbrace{c_{3a}\cdot(\bar{h}_{n'n}-c_{4a})}_{\text{Grade of the ore produced}} + \underbrace{c_{5a}\cdot\Big(\sum_{i=1}^{n-1}R_i + \sum_{i=1}^{n}R_i\Big)}_{\text{Part of the ore deposit currently mined}}}_{\text{influencing the price}}\Big]\cdot \underbrace{Q_n}_{\text{Rate of production}} \cdot \underbrace{c_{69,a}}_{\text{Price index}}$$

43 Payments for current operating costs

$$K_{an'n} = \Big\{\underbrace{c_{6a}}_{\text{Fixed}} + \underbrace{c_{7a}\cdot Q_n + c_{8a}\cdot e^{c_{9a}\cdot Q_n} - c_{8a}}_{\text{Rate of production}} + \Big[c_{10,a}\cdot(c_{4a}-\bar{h}_{n'n}) + \ldots$$

$$\ldots \underbrace{c_{11,a}\cdot\big(e^{c_{12,a}\cdot\bar{h}_{n'n}} - e^{c_{12,a}\cdot c_{4a}}\big)\Big]\cdot(Q_n+c_{13,a})}_{\text{Grade (and rate of production)}} +$$

$$\underbrace{\Big(c_{14,a}\cdot e^{c_{15,a}\cdot\sum_{i=1}^{n}R_i} - c_{14,a}\Big)\cdot(Q_n+c_{16,a})}_{\text{Part of the ore deposit currently mined (and rate of production)}}\Big\}\cdot\underbrace{c_{68,a}}_{\text{Price index}}$$

44 Current reinvestments

$$G_{an'n} = \Big[\underbrace{c_{17,a}}_{\text{Fixed}} + \underbrace{c_{18,a}\cdot Q_n}_{\text{Rate of production}} + \underbrace{c_{19,a}\cdot(\bar{h}_{n'n}-c_{4a})\cdot(Q_n+c_{20,a})}_{\text{Grade (and rate of production)}}\Big]\cdot\underbrace{c_{67,a}}_{\text{Price index}^{1)}}$$

If less than $c_{21,a}$ years remain of the production period of the mine $G_{an'n}$ is multiplied by the factor

---

1) In this index the first (the expanding) part of the reinvestment cycle during the first years of the production period of the ore deposit can be included (e.g. by analogy to the use of $c_{21,a}$: $c_{67,a}$=0.0, 0.2, 0.4, 0.6, 0.8, 1.0 for a=1,2,...,6, respectively).

$$\frac{T_{1,N+1}-b(t)}{c_{21,a}} \qquad \text{or, for the last calendar year, by 0,}$$

where $b(t)$ denotes the end of the calendar year of the payment date. The general definition of $b(t)$ in section 3 of this appendix also applies here.

45 Zone investments

$$E_{a1n}=\Big\{\underbrace{c_{22,a}}_{\text{Fixed}}+\underbrace{c_{23,a}\cdot Q_n+c_{24,a}\cdot e^{c_{25,a}\cdot Q_n}}_{\text{Rate of production}}+\underbrace{c_{26,a}\cdot(Q_n-Q_{n-1})\cdot Q_n}_{\text{Change of rate of production (and rate of production)}}+$$

$$\underbrace{\left[c_{27,a}\cdot(c_{4a}-\bar{h}_{n'-1,n})+c_{28,a}\cdot\left(e^{c_{29,a}\cdot\bar{h}_{n'-1,n}}-e^{c_{29,a}\cdot c_{4a}}\right)\right]\cdot\left[Q_n+(c_{30,a})\right]}_{\text{Grade (and rate of production)}}+$$

$$\underbrace{\left(c_{31,a}\cdot e^{c_{32,a}\cdot\sum_{i=1}^{n}R_i}-c_{31,a}\right)\cdot(Q_n+c_{61,a})}_{\text{Part of the ore deposit currently mined (and rate of production).}}+$$

$$\underbrace{\left(c_{33,a}\cdot e^{c_{34,a}\cdot R_n}-c_{33,a}\right)\cdot(Q_n+c_{35,a})\cdot e^{c_{36,a}\cdot(c_{4a}-\bar{h}_{n'-1,n})}}_{\text{Size of the zone (and rate of production and grade)}}\Big\}\cdot\underbrace{c_{66,a}}_{\text{Price index}}$$

46 Expansion investments

$$F_{a1n}=\underbrace{(c_{37,a}}_{\text{Fixed}}\underbrace{-c_{38,a}\cdot e^{c_{39,a}\cdot(Q_{n-1}-Q_n)}+c_{38,a})}_{\text{Change of rate of production}}\cdot\underbrace{e^{-c_{40,a}\cdot Q_{n-1}}}_{\text{Rate of production before the change}}\cdot$$

$$\underbrace{e^{c_{41,a}\cdot(c_{4a}-\bar{h}_{n'-1,n})}}_{\text{Grade}}\cdot\underbrace{c_{65,a}}_{\text{Price index}}$$

Further,

$F_{a1n}=0$ if $Q_n-Q_{n-1}\leq 0$, i.e. in case of a decreasing or constant rate of production.

Q_0=constant. Q_0 is the rate of production immediately before the starting time of zone 1. If the ore deposit is not previously mined $Q_0=0$.

47 Contraction investments

$$H_{a1n}=\underbrace{(c_{42,a}}_{\text{Fixed}}\underbrace{-c_{43,a}\cdot e^{c_{44,a}\cdot(Q_n-Q_{n-1})}+c_{43,a})}_{\text{Change of rate of production}}\cdot\underbrace{e^{-c_{45,a}\cdot Q_{n-1}}}_{\text{Rate of production before the change}}\cdot$$

$$\underbrace{e^{c_{46,a}\cdot(c_{4a}-\bar{h}_{n'-1,n})}}_{\text{Grade}}\cdot\underbrace{c_{65,a}}_{\text{Price index}}$$

Further,

$H_{a1n}=0$ if $Q_n-Q_{n-1}\geq 0$, i.e. in case of an increasing or constant rate of production.

Q_0 See section 46 of this appendix.

48 Grade-change investments

$\bar{I}_{an'n}$ is a variable, the value of which is calculated according to the formula:

$$\bar{I}_{an'n}=\underbrace{Q_n\cdot e^{\frac{c_{50,a}}{Q_n}}}_{\text{Rate of production}}\cdot\underbrace{e^{c_{51,a}\cdot(c_{4a}-\bar{h}_{n'-1,n})}}_{\text{Grade before the change}}\cdot\underbrace{e^{c_{52,a}\cdot\sum_{i=1}^{n}R_i}}_{\text{Part of the ore deposit currently mined}}$$

For an increase of the grade the following formula applies:

$$L_{an'n}=(\underbrace{c_{47,a}}_{\text{Fixed (relative to the size of the grade change)}}\underbrace{-c_{48,a}\cdot e^{c_{49,a}\cdot(\bar{h}_{n'-1,n}-\bar{h}_{n'n})}+c_{48,a}}_{\text{Grade change}})\cdot\bar{I}_{an'n}\cdot\underbrace{c_{64,a}}_{\text{Price index}}$$

For a decrease of the grade the following formula applies:

$$M_{an'n}=(\underbrace{c_{53,a}}_{\text{Fixed (relative to the size of the grade change)}}\underbrace{+c_{54,a}\cdot e^{c_{55,a}\cdot(\bar{h}_{n'-1,n}-\bar{h}_{n'n})}-c_{54,a}}_{\text{Grade change}})\cdot\bar{I}_{an'n}\cdot\underbrace{c_{64,a}}_{\text{Price index}}$$

In addition:

$L_{an'n}=0$ if $\bar{h}_{n'n}-\bar{h}_{n'-1,n}\leq 0$, i.e. in case of a decreasing or constant grade.

$M_{an'n}=0$ if $\bar{h}_{n'n}-\bar{h}_{n'-1,n}\geq 0$, i.e. in case of an increasing or constant grade.

$c_{47,a}=c_{53,a}=0.0$ when $n'=1$, i.e. if the grade change takes place at the starting time of a zone. Then the grade-change investment can be coordinated with the much more prominent zone investment and, because of this, a saving may be achieved.

49 <u>Close-down payments</u>

$$H_{a,1,N+1}=(\underbrace{c_{56,a}}_{\text{Fixed}}+\underbrace{c_{57,a}\cdot e^{c_{58,a}\cdot Q_N}-c_{57,a}}_{\text{Rate of production}})\cdot\underbrace{e^{c_{59,a}\cdot(c_{4a}-\bar{h}_{N'N})}}_{\text{Grade}}\cdot$$

$$(\underbrace{c_{62,a}}_{\substack{\text{Fixed}\\\text{(relative}\\\text{to the}\\\text{size of}\\\text{the ore}\\\text{deposit)}}}+\underbrace{c_{60,a}\cdot\sum_{i=1}^{N}R_i}_{\substack{\text{Size of the}\\\text{ore deposit}}})\cdot\underbrace{c_{63,a}}_{\substack{\text{Price}\\\text{index}}}$$

Appendix E

Source programs

Contents Page

Introduction

To most readers this appendix might appear unintelligible, and thus superfluous. In spite of this it has been included, mainly for two reasons:

1 There are occasions when it is necessary to look into the source programs, e.g. in case of errors (especially indicated through PRICOM) or if the programs are to be altered. And the program user usually has to adapt CUT11 and the ANPAY subprograms to the actual cases.

2 The source programs are the most complete descriptions of the various models oontained in the programs.

The source programs have been tested on some hypothetical examples as well as in practical applications. The number of zones and subzones has been varied from one of each to 10 and 5, respectively (oompare Fig. B:1 in Appendix B). The payment functions have been fixed over time, except in one case. Manual capital-value computations have been made in sore simple cases, and been compared with those obtained by means of the programs. The consistency of the optimizations has been controlled by inspection of the intermediate output. Certain capital values in this output have been recalculated by means of CAPVAL. The testing is not complete, and programming errors may still remain.

The source programs are available from the Economic Research Institute at the Stookholm Sohool of Economics.

```
C      PROGRAM CUTOFF
C      OPTIMUM CUT-OFF (MEAN) GRADES
C  VERSION 3, 15.8.68
       COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,
     2        T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,
     3        C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),
     4        HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),
     5        Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,
     6        NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,
     7        JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT
C  READ INPUT DATA
       READ (5,9020) NMAX, NSMAX, JTHOR, DELTA1, DELTA2, DELTA3, HAEND1,
     2              HAEND2, HBIN, JTOTMX, M, QBIN, R
       NMAXD=NMAX+1
       DO 21  NS=1,NSMAX
   21     READ (5,9021) (RES(NS,N), N=1,NMAXD)
       READ (5,9022) (HEQV(N), N=1,NMAX)
       READ (5,9023) (Q(N), N=1,NMAX)
       DO 22 NS=1,NSMAX
   22     READ (5,9022) (H(NS,N), N=1,NMAX)
       READ (5,9024)  (C1(N), N=1,NMAX)
       READ (5,9024)  (C2(N), N=1,NMAX)
       READ (5,9024)  (C3(N), N=1,NMAX)
       READ (5,9024)  (C4(N), N=1,NMAX)
       READ (5,9024)  C5, C6, C7, C8,C9
       DO 23 LT=1,JTHOR
   23     READ (5,9025) (PAR(JD,LT), JD=1,70)
C  WRITE INPUT DATA
       WRITE (6,9030) NMAX,NSMAX,JTHOR, DELTA1, DELTA2, DELTA3, HAEND1,
     2               HAEND2, HBIN, JTOTMX, M, QBIN, R
       WRITE (6,9031)
       WRITE (6,9090)
       DO 31 NS=1,NSMAX
   31     WRITE (6,9032) NS,(RES(NS,N), N=1,NMAXD)
       WRITE (6,9033)
       WRITE (6,9090)
       WRITE (6,9034) (HEQV(N), N=1,NMAX)
       WRITE (6,9035)
       WRITE (6,9090)
       WRITE (6,9036) (Q(N), N=1,NMAX)
       WRITE (6,9037)
       WRITE (6,9090)
       DO 32 NS=1,NSMAX
   32     WRITE (6,9038) NS,(H(NS,N), N=1,NMAX)
       WRITE (6,9039)
       WRITE (6,9041)  (C1(N), N=1,NMAX)
       WRITE (6,9042)  (C2(N), N=1,NMAX)
       WRITE (6,9043)  (C3(N), N=1,NMAX)
       WRITE (6,9044) (C4(N), N=1,NMAX)
       WRITE (6,9045)  C5, C6, C7, C8, C9
       KA=-4
       KB=0
   41  KA=KA+5
       KB=KB+5
       IF(JTHOR-KB) 42,43,43
   42  KB=JTHOR
   43  WRITE (6,9046)
       DO 45 JT=KA,KB
```

```
         WRITE (6,9047) JT, (PAR(JD,JT), JD=1,9)                          0060
         DO 44 KC=1,6                                                     0061
            KD=10*KC                                                      0062
            KE=KD+9                                                       0063
  44        WRITE (6,9048) KC,(PAR(JD,JT), JD=KD,KE)                      0064
  45     WRITE (6,9049) PAR(70,JT)                                        0065
      IF(JTHOR-KB) 46,46,41                                               0066
  46  WRITE (6,9200)                                                      0067
C  TEST INTERNAL CONSISTENCY OF INPUT DATA
      CALL INDATT                                                         00675
C  INITIAL VALUES                                                         0068
      JM=0                                                                0069
      JTOT=0                                                              0070
      JTOTED=0                                                            0071
      JA=1                                                                007150
      HA(JA)= H(1,1)                                                      007151
      T(1,1)=1.00                                                         0072
C  START FOR NEW TOTAL OPTIMUM                                            0073
   3  NA=1                                                                0074
      NSA=1                                                               0075
      JT=-1                                                               0076
      IPRO= 0                                                             00765
      GO TO 10                                                            0077
C  MAX 99 TRIALS. MAX 6 AFTER THIS POINT. SEE CUT4, STATEMENTS 320 AND    0079
C  326                                                                    0080
   9  IF(JA-93) 5,5,4                                                     0090
   4  IDENT=0091                                                          0091
      CALL PRICOM                                                         0092
      STOP                                                                0093
C  CHECK FOR PRACTICAL CONSTRAINTS                                        0095
   5  IF (HA(JA)-HAEND2) 7,4,4                                            0096
   7  IF (HA(JA)-HAEND1) 4,4,8                                            0097
C  NEW TRIAL. HA(JA) FROM CUT2 OR CUT4                                    0099
   8  H(NSA,NA)=HA(JA)                                                    0100
C  CALCULATE NEW STARTING TIMES                                           0101
  10  CALL CUT1                                                           0102
      IF (JT) 90,130,400                                                  0104
C  FIRST SUBNIV. TO OPTIMIZE                                              0106
  90  CONTINUE                                                            0107
C  COMPUTE AND PRINT CAP VAL OF INITIAL GUESS OR NEW OPT                  011610
      IF (JTOT) 94,95,95                                                  011611
  94  IDENT=11612                                                         011612
      CALL PRICOM                                                         011613
      STOP                                                                011614
  95  CALL CUT5                                                           011616
      WRITE(6,9218) B(JA)                                                 011619
C  TO WRITE MORE OF ANNUAL PAYMENTS,  REMOVE 01162
      IPRO= 1                                                             01162
      JA= 1                                                               01163
  96  NA=NMAX                                                             0117
      NSA=NSMAX                                                           0118
      JM=0                                                                0119
      GO TO 130                                                           0120
C  NEW SUBNIV. TO OPTIMIZE                                                0122
 100  IF (NSA-2) 110,120,120                                              0123
 110  NSA=NSMAX                                                           0124
      NA=NA-1                                                             0125
 112  JT=0                                                                0126
```

```
      JM=0
      GO TO 10
 120  NSA=NSA-1
      IF (NSA-2) 102,102,103
 103  IF (RES(NSA,NA)- RES(NSA-1,NA)- 0.001) 104,104,102
 104  H(NSA-1,NA)= H(NSA-2,NA)
 102  IF (RES(NSA+1,NA)- RES(NSA,NA)- 0.001) 101,101,112
 101  H(NSA,NA)= H(NSA+1,NA)
      GO TO 100
C  FIRST MEASUREMENT IN EVERY SUBNIV. IN FIRST APPROXIMATION
 130  HA(1)=H(NSA,NA)
      JA=1
      JAEND1=1
      GO TO 400
C  NEW TRIAL GRADES IN FIRST APPROXIMATION
 200  CALL CUT2
      GO TO (9,220),I
 220  WRITE (6,9201)
      CALL PRICAP
C  FIND 1 TO 5 MAXIMA IN FIRST APPROXIMATION
      CALL CUT3
C  FIND NEW TRIAL GRADES FOR REFINED OPTIMA FROM FIRST APPROXIMATION
C  (= SECOND APPROXIMATION)
 300  CALL CUT4
      GO TO (9,900),I
C  CALCULATE CAPITAL VALUE OF REMAINING ORE
 400  CALL CUT5
      IPRO=1
C  FIND NEW TRIAL GRADE FOR FIRST RESP. SECOND APPROX.
      IF (JM) 600, 200, 300
 600  STOP
C  IF NOT LAST SUBNIV., OPTIMIZE NEXT SUBNIV.
 900  IF (NSA+NA-2) 910,920,100
 910  IDENT=0163
      CALL PRICOM
      STOP
C  FIRST SUBNIV., NSA=NA=1, GIVES TOTAL OPTIMUM
 920  JTOT=JTOT+1
      BTOT(JTOT)=B(1)
      WRITE (6,9211) JTOT, JTOT, BTOT(JTOT)
      IF (JTOT-2) 3,3,930
C  OPTIMIZE ONCE MORE IF LAST INCREASE IN CAP. VAL. WAS GREAT,OR STOP
 930  IF (BTOT(JTOT)-BTOT(JTOT-1)-DELTA3*BTOT(JTOT-1)) 960,960,940
 940  IF (JTOT-JTOTMX) 3,960,960
 960  JTOTED=JTOT
      NA=1
      NSA=1
      IPRO=0
C  FIND STARTING TIMES FOR NIV. AND SUBNIV. IN LAST OPTIMUM
      CALL CUT1
      JM=0
      JA=1
C  COMPUTE AND SPECIFY CAPITAL VALUE OF LAST OPTIMUM
      CALL CUT5
      WRITE (6,9217) (JTOT,BTOT(JTOT), JTOT=1,JTOTED)
      STOP
 9020 FORMAT (2I3, I4, 2F7.4, F6.3, 3F7.4, 2I3, F7.2, F6.3)
 9021 FORMAT (11F7.2/4F7.2)
```

```
9022 FORMAT (11F7.4/3F7.4)                                                  0190
9023 FORMAT (11F7.2/3F7.2)                                                  0191
9024 FORMAT (9F8.3/5F8.3)                                                   0192
9025 FORMAT (8(8F9.3/), 6F9.3)                                              0193
9030 FORMAT (32H1  OPTIMUM CUT-OFF (MEAN) GRADES //13H  *INPUT DATA/        0195
    2          11H  *   NMAX=,I3, 9H   NSMAX=, I3, 9H   JTHOR=, I4/         0196
    3          13H  *   DELTA1=, F7.4,                                      0197
    4          10H   DELTA2=, F7.4,10H   DELTA3=, F6.3, 10H   HAEND1=,      0198
    5          F7.4, 10H   HAEND2=, F7.4, 8H   HBIN=, F7.4,                 0199
    6          10H   JTOTMX=,I3/ 8H  *   M=, I3, 8H   QBIN=, F7.2,          0200
    7          5H   R=,F6.3)                                                0201
9031 FORMAT (/68H0 *EQUIV CUMUL. ORE RESERVE MT TO BE EXTR. BEFORE (NS,     0202
    2N), RES(NS,N)  )                                                       0203
9032 FORMAT (1H ,11X, I2, 1X, 15F7.2)                                       0204
9033 FORMAT (/34H0 *GRADE OF EQUIV RESERVE, HEQV(N))                        0205
9034 FORMAT (15H    1... NSMAX ,14F7.4)                                     0206
9035 FORMAT (/29H0 *ANNUAL PRODUCTION MT, Q(N))                             0207
9036 FORMAT (15H    1... NSMAX , 14F7.2)                                    0208
9037 FORMAT (41H1 *INITIAL CUT-OFF (MEAN) GRADES H(NS,N)  )                 0209
9038 FORMAT (1H ,11X, I2, 1X, 14F7.4)                                       0210
9039 FORMAT (/36H0 *ORE RESERVE CONVERTING PARAMETERS/                      0211
    2         28H   1. DEPEND OF NIV, C-(N)  / 8H0     N=,4X,1H1,7X,1H2,    0212
    3         7X,1H3,7X,1H4,7X,1H5,7X,1H6,7X,1H7,7X,1H8,7X,1H9,6X,2H10,     0213
    4         6X,2H11, 6X,2H12,6X,2H13,6X,2H14)                             0214
9041 FORMAT (8H  C1(N)=,14F8.3)                                             0215
9042 FORMAT (8H  C2(N)=,14F8.3)                                             0216
9043 FORMAT (8H  C3(N)=,14F8.3)                                             0217
9044 FORMAT (8H  C4(N)=,14F8.3)                                             0218
9045 FORMAT (24H0   2. INDEP. OF NIV, C-/1H0,7X,3HC5=,F9.3,3X,3HC6=,        0219
    2         F9.3,3X,3HC7=,F9.3,3X,3HC8=,F9.3,3X,3HC9=,F9.3)               0220
9046 FORMAT (49H1 *ANNUALLY CHANGING ECONOMIC PARAMETERS, PAR(JD))          0221
9047 FORMAT (1H0,16X, 3HJD=,6X,2H*0,8X, 2H*1, 8X, 2H*2, 8X, 2H*3,           0222
    2         8X, 2H*4, 8X, 2H*5, 8X, 2H*6, 8X, 2H*7, 8X, 2H*8, 8X, 2H*9/   0223
    3         9H    YEAR ,I3, 3X, 5H*= 0 , 10X, 9F10.3)                     0224
9048 FORMAT (1H , 17X, I1, 1X, 10F10.3 )                                    0225
9049 FORMAT (1H , 17X, 2H7 ,F10.3)                                          0226
9090 FORMAT (1H0, 11X, 8HNIV N= 1, 6X, 1H2, 6X, 1H3, 6X, 1H4, 6X, 1H5,      0227
    2         6X, 1H6, 6X, 1H7, 6X, 1H8, 6X, 1H9, 5X, 2H10, 5X, 2H11,       0228
    3         5X, 2H12, 5X, 2H13, 5X, 2H14, 4X, 4H(15)/ 14H   SUBNIV NS=    0229
    4         )                                                             0230
9200 FORMAT (14H1**OUTPUT DATA, 10X, 3HAND/ 12H  -TEST DATA)                0231
9201 FORMAT (/54H0**CAPITAL VALUE B(JA) AT GRADE HA(JA)   FIRST APPROX.     0232
    2         )                                                             0233
9211 FORMAT (19H1**TOTAL OPTIMUM NR, I3, 3X, 5HJTOT=, I3, 3X,               0234
    2         11HBTOT(JTOT)=,F10.2)                                         0235
9217 FORMAT (/43H0**RECORD OF TOTAL OPTIMA, JTOT, BTOT(JTOT)/1H , 3X,       0240
    2         8(I3, 1H,,F10.2)/                                             0241
    3         1H ,3X, 8(I3,1H,,F10.2)/                                      0242
    4         1H ,3X, 4(I3,1H,,F10.2))                                      0243
9218 FORMAT (45H0**CAPITAL VALUE OF INITIAL GUESS OR NEW OPT ,F10.2)        02432
     END                                                                    0244
```

```
      SUBROUTINE CUT1                                                   1001
C     FIND STARTING TIMES FOR NIVS. AND SUBNIVS.                        1002
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                    CMN11
   10 N=NA                                                              1012
      IF (IPRO) 14,11,14                                                1013
   11 WRITE (6,9214) JTOT, JT                                           1014
   14 DO 29 NS=NSA,NSMAX                                                1015
         IF (NS-NSMAX) 25,21,20                                         1016
   20    IDENT=1017                                                     1017
         CALL PRICOM                                                    1018
         STOP                                                           1019
   21    DRES=RES(1,N+1)-RES(NS,N)                                      1020
         CALL CUT11                                                     1021
         T(1,N+1)=T(NS,N)+EXDRES/Q(N)                                   1022
         GO TO 26                                                       1023
   25    DRES=RES(NS+1,N)-RES(NS,N)                                     1024
         CALL CUT11                                                     1025
         T(NS+1,N)=T(NS,N)+EXDRES/Q(N)                                  1026
   26    IF(IPRO) 29,27,29                                              1027
   27    WRITE (6,9215) N, NS, HEQV(N), DRES, EXDRES, T(NS,N), H(NS,N)  1028
   29 CONTINUE                                                          1029
      IF(NA-NMAX) 35,50,33                                              1030
   33 IDENT=1031                                                        1031
      CALL PRICOM                                                       1032
      STOP                                                              1033
   35 NAD=NA+1                                                          1034
   36 DO 49 N=NAD, NMAX                                                 1035
         DO49 NS=1,NSMAX                                                1036
           IF (NS-NSMAX) 45,41,40                                       1037
   40      IDENT=1037                                                   1038
           CALL PRICOM                                                  1039
           STOP                                                         1040
   41      DRES=RES(1,N+1)-RES(NS,N)                                    1041
           CALL CUT11                                                   1042
           T(1,N+1)=T(NS,N)+EXDRES/Q(N)                                 1043
           GO TO 46                                                     1044
   45      DRES=RES(NS+1,N)-RES(NS,N)                                   1045
           CALL CUT11                                                   1046
           T(NS+1,N)=T(NS,N)+EXDRES/Q(N)                                1047
   46      IF (IPRO) 49,47,49                                           1048
   47      WRITE (6,9215) N,NS, HEQV(N), DRES, EXDRES, T(NS,N), H(NS,N) 1049
   49 CONTINUE                                                          1050
   50 WRITE (6,9216) T(1,NMAX+1), JA, HA(JA)                            1051
      RETURN                                                            1052
 9214 FORMAT (/100HO**EQUIVALENT (DRES) AND MINABLE (EXDRES) ORE IN SUBN 1053
     2IV, STARTING TIMES (T(NS,N) AND GRADES (H(NS,N)/1H ,3X, 5HJTOT=,  1054
     3       I3, 3X, 3HJT=,I4/1HO, 21X ,1HN, 4X, 2HNS, 3X, 7HHEQV(N),7X, 1055
     4       4HDRES, 5X, 6HEXDRES, 3X, 7HT(NS,N), 3X, 7HH(NS,N))        1056
 9215 FORMAT (1H ,19X,I3, I6, F10.4,2F11.2, F10.2, F10.4)               1057
 9216 FORMAT (13H **MINE LIFE=, F10.2, 11H   WHEN JA=, I3,              1058
     2       14H   AND HA(JA)=, F10.4)                                  10585
      END                                                               1059
```

```
      SUBROUTINE CUT11                                                  7101
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,    CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                     CMN11
      IF (H(NS,N)-HEQV(N)) 1,1,2                                        7112
    1 EXDRES=C5*DRES*EXP(C6+1.-C7*H(NS,N))+(1.-C5)*DRES*(1.+C1(N)*EXP   7113
     2       (C2(N)*(HEQV(N)-H(NS,N)))-C1(N))                           7114
      RETURN                                                            7115
    2 EXDRES=C5*DRES*EXP(C6+1.-C7*H(NS,N))+(1.-C5)*DRES*(1.-C3(N)*EXP   7116
     2       (C4(N)*(H(NS,N)-HEQV(N)))+C3(N))                           7117
      RETURN                                                            7118
      END                                                               7119
```

```
      SUBROUTINE CUT2                                                   2001
C     FIND NEW TRIAL GRADE IN ACTUAL SUBNIV. IN FIRST APPROX.           2002
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXORES, RES(20,15), HB,    CMN5
     2        T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6, CMN6
     3        C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),     CMN7
     4        HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),     CMN8
     5        Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,    CMN9
     6        NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7        JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                    CMN11
 200  IF (JAEND1-(M+1)) 201,202,202                                     2012
 201  IF (JA-1) 203,2011,2012                                           2013
 2011 WRITE (6,9210) JTOT, NA, NSA, T(1,NMAX+1)                         2014
 2012 JA=JA+1                                                           2015
      HA(JA)=HA(JA-1)-DELTA1                                            2016
      JAEND1=JAEND1+1                                                   2017
      I=1                                                               2018
      RETURN                                                            2019
 202  IF (JA-JAEND1) 203, 204,210                                       2020
 203  IDENT=2021                                                        2021
      CALL PRICOM                                                       2022
      STOP                                                              2023
 204  JCD=JA-(M-1)                                                      2024
      DO 205 JC=JCD,JA                                                  2025
         IF (B(JC)-B(JC-1)) 205,2012,2012                               2026
 205  CONTINUE                                                          2027
      JAEND2=JAEND1+1                                                   20285
      JA=JA+1                                                           2029
      HA(JA)=HA(1)+DELTA1                                               2030
      I=1                                                               2031
      RETURN                                                            2032
 210  IF (JAEND2-JAEND1-M) 211,212,213                                  2033
 211  JA=JA+1                                                           2034
      HA(JA)=HA(JA-1)+DELTA1                                            2035
      JAEND2=JAEND2+1                                                   2036
      I=1                                                               2037
      RETURN                                                            2038
 212  JCD=JA-(M-2)                                                      2039
      IF (B(JA-(M-1))-B(1)) 214,211,211                                 2040
 213  JCD=JA-(M-1)                                                      2041
 214  DO 215 JC=JCD,JA                                                  2042
         IF (B(JC)-B(JC-1)) 215,211,211                                 2043
 215  CONTINUE                                                          2044
      I=2                                                               2046
      RETURN                                                            2047
 9210 FORMAT (33H1**NEW SUBNIV. TO OPTIMIZE. JTOT=  ,I3,5H. NA= ,I3,    2048
     2        6H. NSA= ,I3/ 13H0  MINE LIFE= ,F10.2, 12H WHEN JA=  1  ) 2049
      END                                                               2050
```

```
      SUBROUTINE CUT3                                                   3001
C     FIND 1 TO 5 MAXIMA IN FIRST APPROX.                               3002
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,    CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                     CMN11
      JAMAX(1)=1                                                        30115
  220 DO 221 JM=2,5                                                     3012
  221    JAMAX(JM)=0                                                    3013
      JM=1                                                              3014
      DO 236 JA=2,JAEND1                                                3015
      IF (B(JA)-B(JA-1)) 236,230,230                                    3016
  230 IF (JAMAX(JM)-JA+1) 232,235,231                                   3017
  231 IDENT=3018                                                        3018
      CALL PRICOM                                                       3019
      STOP                                                              3020
  232 IF (JM-3) 234,233,231                                             3021
  233 WRITE (6,9012) JM, JTOT, NA, NSA, JA                              3022
      JAMAX3=JAMAX(3)                                                   3023
      IF (B(JA)-B(JAMAX3)) 236,235,235                                  3024
  234 JM=JM+1                                                           3025
  235 JAMAX(JM)=JA                                                      3026
  236 CONTINUE                                                          3027
      JMHMIN=JM                                                         3028
      JAD=JAEND1+2                                                      3029
      IF(B(JAEND1+1)-B(1)) 255,255,240                                  3030
  240 IF  (JAMAX(1)-1) 241,242,250                                      3031
  241 IDENT=3032                                                        3032
      CALL PRICOM                                                       3033
      STOP                                                              3034
  242 JAMAX(1)=JAEND1+1                                                 3035
      DO 246 JA=JAD,JAEND2                                              3036
         IF(B(JA)-B(JA-1))248,248,246                                   3037
  246 JAMAX(1)=JA                                                       3038
      GO TO 270                                                         3039
  248 JAD=JA+1                                                          3040
      GO TO 255                                                         3041
  250 JM=JM+1                                                           3042
      JAMAX(JM)=JAEND1+1                                                3043
  255 DO 266 JA=JAD,JAEND2                                              3044
         IF (B(JA)-B(JA-1)) 266,266, 260                                3045
  260    IF (JAMAX(JM)-JA+1) 262,265,266                                3046
  261    IDENT=3047                                                     3047
         CALL PRICOM                                                    3048
         STOP                                                           3049
  262    IF (JM-5) 264,263,261                                          3050
  263    WRITE (6,9012) JM, JTOT, NA, NSA, JA                           3051
         JAMAX5=JAMAX(5)                                                3052
         IF (B(JA)-B(JAMAX5)) 266,266,265                               3053
  264    JM=JM+1                                                        3054
  265    JAMAX(JM)=JA                                                   3055
  266 CONTINUE                                                          3056
  270 DO 274 JM=1,5                                                     3057
         IF (JAMAX(JM)) 271,272,273                                     3058
  271    IDENT=3059                                                     3059
```

```
         CALL PRICOM
      STOP
272      BMAX(JM)=0.
         HAMAX(JM)=0.
         GO TO 274
273      JA=JAMAX(JM)
         BMAX(JM)=B(JA)
         HAMAX(JM)=HA(JA)
274   CONTINUE
      WRITE (6,9207) JTOT, NA, NSA, (HAMAX(JM), JM=1,5), (BMAX(JM), JM=
     2      1,5)
      JM=0
      JA=0
      RETURN
9012 FORMAT (/18HO**MULT MAX IN JM=,I2, 10X, 5HJTOT=, I3, 3X, 3HNA=,I3,
     2       3X, 4HNSA=, I3, 3X, 3HJA=,I3)
9207 FORMAT (/23HO**OPTIMA FIRST APPROX./ 1H , 3X, 5HJTOT=, I3, 3X,
     2       3HNA=, I3, 3X, 4HNSA=, I3/ 1HO, 16X, 3HJM=, 16X, 1H1, 9X,
     3       1H2, 9X, 1H3, 9X, 1H4, 9X, 1H5/ 1H , 3X, 10HHAMAX(JM)=,
     4       16X, 5F10.4/ 1H , 3X, 9HBMAX(JM)=, 17X, 5F10.2/)
      END
```

```
      SUBROUTINE CUT4                                                        4001
C     FIND NEW TRIAL GRADE IN ACTUAL SUBNIV. AND MAXIMUM IN SECOND APPR.     4002
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,          CMN5
     2        T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,       CMN6
     3        C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),           CMN7
     4        HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),           CMN8
     5        Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,          CMN9
     6        NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,        CMN10
     7        JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                         CMN11
      IF(JA) 303,301,308                                                     4012
 301  JM=JM+1                                                                4013
      IF (JAMAX(JM)) 303,340,305                                             4014
 303  IDENT=4015                                                             4015
      CALL PRICOM                                                            4016
      STOP                                                                   41165
 305  HA(1)=HAMAX(JM)                                                        4017
      B(1)=BMAX(JM)                                                          4018
      JA=1                                                                   4019
      JAEND1=1                                                               4010
 308  IF (JAEND1-3) 309,310,310                                              4011
 3092 IDENT=4011                                                             401101
      CALL PRICOM                                                            401102
      GO TO 320                                                              401103
 309  IF (JA-13) 3091,3092,3092                                              401104
 3091 JA=JA+1                                                                401105
      HA(JA)=HA(JA-1)-DELTA2                                                 4012
      JAEND1=JAEND1+1                                                        4013
      I=1                                                                    4012
      RETURN                                                                 4015
 310  IF (JA-JAEND1)  312, 311,322                                           4016
 311  IF (HA(JA-1))  312, 318,313                                            4017
 312  IDENT=4018                                                             4018
      CALL PRICOM                                                            4019
      STOP                                                                   4020
 313  IF (B(JA)-B(JA-1)) 314,309,309                                         4021
 314  IF (B(JA-1)-B(JA-2)) 315,316,316                                       4022
 315  JA=JA+3                                                                4023
      HA(JA)=HA(JA-3)-3.*DELTA2                                              4024
      HA(JA-1)=0.                                                            4025
      JAEND1=JAEND1+3                                                        4026
      I=1                                                                    4027
      RETURN                                                                 4028
 316  IF (BMAX(JM)-B(JA-1)) 317,317,309                                      4029
 317  HAMAX(JM)=HA(JA-1)                                                     4030
      BMAX(JM)=B(JA-1)                                                       4031
      GO TO 309                                                              4032
 318  IF (B(JA)-B(JA-3)) 320,319,319                                         4034
 319  JA=JA-3                                                                4035
      JAEND1=JAEND1-3                                                        4036
      GO TO 309                                                              4037
 320  JAEND1=JAEND1-3                                                        4038
      DO 321 JC=1,3                                                          4039
         JA=JAEND1+JC                                                        4040
         INDEX=4-JC                                                          4041
         HA(JA)=HA(INDEX)                                                    4042
 321  B(JA)=B(INDEX)                                                         4043
 322  IF (HA(JA-1)) 323,330,324                                              4044
 323  IDENT=4045                                                             4045
```

```
      CALL PRICOM
      STOP
324   IF (B(JA)-B(JA-1)) 325,327,327
325   IF (B(JA-1)-B(JA-2)) 326,328,328
326   JA=JA+3
      HA(JA)=HA(JA-3)+3.*DELTA2
      HA(JA-1)=0.
      I=1
      RETURN
3272  IDENT=4055
      CALL PRICOM
      GO TO 330
327   IF (JA-20) 3271, 3272,3272
3271  JA=JA+1
      HA(JA)=HA(JA-1)+DELTA2
      I=1
      RETURN
328   IF (BMAX(JM)-B(JA-1)) 329,327,327
329   HAMAX(JM)=HA(JA-1)
      BMAX(JM)=B(JA-1)
      GO TO 327
330   IF (B(JA)-B(JA-3)) 338,331,331
331   JA=JA-3
      GO TO 327
338   JAEND2=JA-3
339   WRITE (6,9206)
      CALL PRICAP
340   IF (JM-5) 301,342,341
341   IDENT=4074
      CALL PRICOM
      STOP
342   H(NSA,NA)=HAMAX(JMHMIN)
      B(1)=BMAX(JMHMIN)
      DO 348 JM= 1,5
         IF (JAMAX(JM)) 345,348,346
345      IDENT=4081
         CALL PRICOM
         STOP
346      IF (B(1)-BMAX(JM)) 347,348,348
347      H(NSA,NA)=HAMAX(JM)
         B(1)=BMAX(JM)
348   CONTINUE
      WRITE (6,9208) NSA, NA, H(NSA,NA), B(1)
      JA= 95
      HA(JA)= H(NSA,NA)
      I=2
      RETURN
9206  FORMAT (/ 55H0**CAPITAL VALUE B(JA) AT GRADE HA(JA)   SECOND APPRO
     2X.)
9208  FORMAT (/39H0**OPTIMUM SECOND APPROX., H(NSA,NA)=H(,I2,1H,, I2,
     2      2H)=, F7.4, 22H.  CORRESP. CAP. VALUE, F10.2)
      END
```

```
      SUBROUTINE CUT5                                                   5001
C     FIND CAPITAL VALUE OF REMAINING ORE                               5002
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                    CMN11
      IPAR=T(1,NMAX+1)+0.9999                                           5012
      TE=IPAR                                                           5013
      B(JA)=0.                                                          5014
C  PAR FOR RIGHT YEAR, LT                                               5015
      LT=T(NSA,NA)+0.9999                                               5016
      IF (JTHOR-LT) 403,403,406                                         5017
 403  LT=JTHOR                                                          5018
 406  IF(NSA-1) 411,412,416                                             5019
 411  IDENT=5020                                                        5020
      GO TO 593                                                         5021
 412  IF (NA-1)  411,414,415                                            5022
 414  HB=HBIN                                                           5023
      GO TO 417                                                         5024
 415  HB=H(NSMAX,NA-1)                                                  5025
      GO TO 417                                                         5026
 416  HB=H(NSA-1,NA)                                                    5027
 417  IF (NA-NMAX) 421,421,418                                          5028
 418  IDENT=5029                                                        5029
      GO TO 593                                                         5030
 421  DO 599 N=NA,NMAX                                                  5031
         IF(N-NA) 422,423,424                                           5032
 422     IDENT=5033                                                     5033
      GO TO 593                                                         5034
 423     NSB=NSA                                                        5035
         GO TO 425                                                      5036
 424     NSB=1                                                          5037
 425     IF (NSB-NSMAX) 427,427,426                                     5038
 426     IDENT=5039                                                     5039
      GO TO 593                                                         5040
 427     DO 598 NS=NSB,NSMAX                                            5041
            JTA=T(NS,N)+0.9999                                          5042
            TA=JTA                                                      5043
            IF(NS-NSMAX) 450,451,452                                    5044
 450        JTC=T(NS+1,N)+0.9999                                        5045
            TD=T(NS+1,N)                                                5046
      GO TO 453                                                         5047
 451        JTC=T(1,N+1)+0.9999                                         5048
            TD=T(1,N+1)                                                 5049
            GO TO 453                                                   5050
 452        IDENT=5051                                                  5051
      GO TO 593                                                         5052
 453        TC=JTC                                                      5053
            JT=JTA                                                      5054
C  TO WRITE -STATE SF CUT5, REMOVE 50555, INSERT SUBROUTINE IFPRI  AND
C  CALL IFPRI.  CHANGE 5104 AND 5123 ACCORDING TO COMMENTS THERE.
C  CALL IFPRI DELETED AND REPLACED BY 505550-53                         5055
      IF (IPRO) 456,455,456                                           505550
 455        I= 0                                                      505551
            GO TO 458                                                 505552
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```
 456        I= -1                                                     505553
C  CHECK IF PAR FOR RIGHT YEAR                                        5056
 458        IPAR=PAR(70,LT)                                           5057
            IF(JT-IPAR) 459,460,457                                   5058
 457        IF(JTHOR-IPAR) 459,460,459                                5059
 459        IDENT=5060                                                5060
      GO TO 593                                                       506001
 460        TB=JT                                                     5061
            CALL ANPAY1(SSKG)                                         5062
            IF (JT-JTA) 462,470,490                                   5063
 462        IDENT=5064                                                5064
      GO TO 593                                                       5065
 470        IF (NS-1) 471,472,480                                     5066
 471        IDENT=5067                                                5067
      GO TO 593                                                       5068
 472        CALL ANPAY2 (SEFHLM)                                      5069
            IF (JTA-JTC) 475,474,473                                  5070
 473        IDENT=5071                                                5071
      GO TO 593                                                       5072
C  NEW NIV. + NEW SUBNIV. + PROD. TILL NEW SUBNIV., ALL IN SAME YEAR  5073
 474        B(JA)=B(JA)+(SSKG/R)*(1./EXP(R*T(NS,N))-1./EXP(R*TD))-    5075
     2      SEFHLM/EXP(R*T(NS,N))                                     5076
            GO TO 590                                                 5077
C  NEW NIV. + NEW SUBNIV. + PROD. TILL END OF YEAR                    50775
 475        B(JA)=B(JA)+(SSKG/R)*(1./EXP(R*T(NS,N))-1./EXP(R*TA))-    5078
     2      SEFHLM/EXP(R*T(NS,N))                                     5079
            GO TO 495                                                 5080
 480        CALL ANPAY3 (SLM,SPAR)                                    5081
            IF (JTA-JTC) 485,484,483                                  5082
 483        IDENT=5083                                                5083
      GO TO 593                                                       5084
C  NEW SUBNIV. + PROD. TILL NEW SUBNIV., ALL IN SAME YEAR             5085
 484        B(JA)=B(JA)+(SSKG/R)*(1./EXP(R*T(NS,N))-1./EXP(R*TD))-    5086
     2      SLM/EXP(R*T(NS,N))                                        5087
            GO TO 590                                                 5089
C  NEW SUBNIV. + PROD. TILL END OF YEAR                               5090
 485        B(JA)=B(JA)+(SSKG/R)*(1./EXP(R*T(NS,N))-1./EXP(R*TA))-    5091
     2      SLM/EXP(R*T(NS,N))                                        5092
            GO TO 495                                                 5093
 490        IF (JT-JTC)  493,492,491                                  5094
 491        IDENT=5095                                                5095
      GO TO 593                                                       5096
C  PROD. FROM BEG. OF YEAR TILL NEW SUBNIV. IN SAME YEAR              5098
 492        B(JA)=B(JA)+(SSKG/R)*(1./EXP(R*(TC-1.))-1./EXP(R*TD))     5099
            GO TO 590                                                 5100
C  PROD. DURING ONE WHOLE YEAR IN SAME SUBNIV.                        5101
 493        B(JA)=B(JA)+(SSKG/R)*(1./EXP(R*(TB-1.))-1./EXP(R*TB))     5102
 495        IF (I) 497,494,498                                        5103
C  5104-07 REMOVED. REINSERT FOR  -STATE OF CUTS
 494  WRITE (6,9104) B(JA)                                            5104
 497        JT=JT+1                                                   5108
            IF(JT-JTHOR) 496,496,458                                  5109
 496        LT=LT+1                                                   5110
            GO TO 458                                                 5112
 498        IDENT=5113                                                5113
      GO TO 593                                                       5114
 590        IF(I) 598,592,591                                         5115
 591        IDENT=5116                                                5116
```

```
 593        WRITE (6,9103) JTOT, JA, NA, NSA, N, NS, NSB, NMAX, NSMAX,     5117
    2              JM, JT, JTA, JTC, JTMAXD, PAR(1,LT), PAR(2,LT),          5118
    3              PAR(70,LT),HB, Q(N), B(JA), SEFHLM, SLM, SSKG,           5119
    4              T(NS,N), TA, TB, TC, TD, T(1,NMAX+1), TE                 5120
            CALL PRICOM                                                     5121
            STOP                                                            5122
C  5123-26 REMOVED. REINSERT FOR -STATE OF CUT5
 592        WRITE (6,9104) B(JA)                                            5123
 598     HB=H(NS,N)                                                         5127
 599  CONTINUE                                                              5128
      CALL ANPAY4 (HEND)                                                    5129
      B(JA)=B(JA)-HEND/EXP(R*T(1,NMAX+1))                                   5130
      RETURN                                                                5131
 9103 FORMAT (/16H0 -STATE OF CUT5, 9I4,I3, I6, 3I5, 2F10.3, F7.1,          5132
    2        F8.4, F8.2/ 1H ,7X, 4F11.2, 7F8.2)                             5133
 9104 FORMAT (40H0**CUMULATIVE CAP VAL FROM TIME 0 B(JA)= , F11.2)          51335
      END                                                                   5134
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      SUBROUTINE ANPAY1(SSKG)                                             8101
C     FIND PAYMENTS IN YEAR JT FOR ORE SALES, RUNNING PRODUCTION AND RE-  8102
C     INVESTMENTS                                                         8103
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,      CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,    CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),        CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),        CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,       CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,     CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                      CMN11
      HD=H(NS,N)                                                          8112
      QD=Q(N)                                                             8113
      SS=(PAR(1,LT)-PAR(2,LT)*QD*QD                                       8114
     2 +PAR(3,LT)*(HD-PAR(4,LT))                                          8115
     3 +PAR(5,LT)*(RES(1,N)+RES(1,N+1)))*QD                               8116
      SS=SS*PAR(69,LT)                                                    8117
      SK=PAR (6,LT)+PAR(7,LT)*QD+PAR(8,LT)*EXP(PAR(9,LT)*QD) -PAR(8,LT)   8118
     2 +(PAR(10,LT)*(PAR(4,LT)-HD)+PAR(11,LT)*(-EXP(PAR(12,LT)            8119
     3 *PAR(4,LT))+EXP(PAR(12,LT)*HD)))*(QD+PAR(13,LT))                   8120
     4 +(PAR(14,LT)*EXP(PAR(15,LT)*RES(1,N+1))-PAR(14,LT))*(QD            8121
     5 +PAR(16,LT))                                                       8122
      SK=SK*PAR(68,LT)                                                    8123
      IF (TE-TB) 7,8,9                                                  812350
    7 IDENT=812351                                                      812351
      CALL PRICOM                                                       812352
      STOP                                                              812352
    8 SG=0.                                                             812354
      GO TO 2                                                           812355
    9 SG=PAR(17,LT)+PAR(18,LT)*QD                                         8124
     2 +PAR(19,LT)*(HD-PAR(4,LT))*(QD+PAR(20,LT))                         8125
      SG=SG*PAR(67,LT)                                                    8126
      IF (TE-TB-PAR(21,LT)) 1,1,2                                         8127
    1 SG=SG*(T(1,NMAX+1)-TB)/PAR(21,LT)                                   8128
    2 SSKG=SS-SK-SG                                                       8129
      IF (IPRO) 5,3,5                                                     8130
    3 WRITE (6,9231) JTOT, NA, NSA, JA, N, NS, JT, HB, SS, SK, SG, SSKG   8131
    5 RETURN                                                              8132
 9231 FORMAT (/16H0**ANPAY1  JTOT=,I3, 4H NA=, I3, 5H NSA=, I3, 4H JA=,   8134
     2       I3, 3H N=, I3, 4H NS=,I3, 4H JT=,I4/ 1H ,3X, 3HHB=,F7.4,     8135
     3       4H SS=,F10.2, 4H SK=,F10.2,4H SG=,F10.2, 6H SSKG=,F10.2)     8136
      END                                                                 8137
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```
      SUBROUTINE  ANPAY2 (SEFHLM)                                       8201
C     FIND PAYMENTS IN YEAR JT FOR INVESTMENTS EXCEPTING GRADE CHANGE   8202
C     INVESTMENTS                                                       8203
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2      T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3      C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4      HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5      Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6      NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,     CMN10
     7      JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                      CMN11
      QD=Q(N)                                                           8212
      IF (N-1) 1,2,3                                                    8213
    1 IDENT=8214                                                        8214
      CALL PRICOM                                                       8215
      STOP                                                              8216
    2 QB=QBIN                                                           8217
      IF (HBIN) 1,5,9                                                   8218
    5 HB=H(NS,N)                                                        8219
      GO TO 9                                                           8220
    3 QB=Q(N-1)                                                         8221
    9 SE=PAR(22,LT)+PAR(23,LT)*QD+PAR(24,LT)*EXP(PAR(25,LT)*QD)         8222
     2  +PAR(26,LT)*(QD-QB)*QD                                          8223
     3   +(PAR(27,LT)*(PAR(4,LT)-HB)+PAR(28,LT)*(EXP(PAR(29,LT)*HB)     8224
     4  -EXP(PAR(29,LT)*PAR(4,LT))))*(QD+PAR(30,LT))                    8225
     5  +(PAR(31,LT)*EXP(PAR(32,LT)*RES(1,N+1))-PAR(31,LT))             8226
     6  *(QD+PAR(61,LT))                                                8227
     7  +(PAR(33,LT)*EXP(PAR(34,LT)*(RES(1,N+1)-RES(1,N)))-PAR(33,LT))  8228
     8  *(QD+PAR(35,LT))*EXP(PAR(36,LT)*(PAR(4,LT)-HB))                 8229
      SE=SE*PAR(66,LT)                                                  8230
      IF (Q(N)-QB+0.005) 11,10,10                                       82305
   10 IF (Q(N)-QB-0.005) 12,12,13                                       8231
   11 SF=0                                                              8232
      SH=(PAR(42,LT)-PAR(43,LT)*EXP(PAR(44,LT)*(QD-QB))+PAR(43,LT))     8233
     2  *EXP(-PAR(45,LT)*QB)                                            8234
     3  *EXP(PAR(46,LT)*(PAR(4,LT)-HB))                                 8235
      SH=SH*PAR(65,LT)                                                  8236
      GO TO 20                                                          8237
   12 SF=0                                                              8238
      SH=0                                                              8239
      GO TO 20                                                          8240
   13 SF=(PAR(37,LT)-PAR(38,LT)*EXP(PAR(39,LT)*(QB-QD))+PAR(38,LT))     8241
     2  *EXP(-PAR(40,LT)*QB)                                            8242
     3  *EXP(PAR(41,LT)*(PAR(4,LT)-HB))                                 8243
      SF=SF*PAR(65,LT)                                                  8244
      SH=0                                                              8245
   20 CALL ANPAY3 (SLM,SPAR)                                            8246
      SEFHLM=SE+SF+SH+SLM-SPAR                                          8247
      IF (IPRO) 25,23,25                                                8248
   23 WRITE (6,9232) JTOT, NA, NSA, JA, N, NS, JT, QB, SE, SF, SH,      8249
     2                SLM, SEFHLM                                       8250
   25 RETURN                                                            8251
 9232 FORMAT (/16H0**ANPAY2  JTOT=,I3,4H NA=,I3, 5H NSA=,I3, 4H JA=,I3, 8252
     2       3H N=,I3, 4H NS=,I3, 4H JT=,I4/ 1H ,3X,3HQB=,F7.2, 4H SE=, 8253
     3       F10.2, 4H SF=,F10.2, 4H SH=,F10.2, 5H SLM=,F10.2,          8254
     4       8H SEFHLM=,F10.2)                                          82545
      END                                                               8255
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```
      SUBROUTINE ANPAY3 (SLM,SPAR)                                      8301
C     FIND PAYMENTS IN YEAR JT FOR INVESTMENTS FOR GRADE CHANGE         8302
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                    CMN11
      HD=H(NS,N)                                                        8312
      QD=Q(N)                                                           8313
      IF (NS+N-2) 1,2,4                                                 8314
    1 IDENT=8315                                                        8315
      CALL PRICOM                                                       8316
      STOP                                                              8317
    2 IF (HBIN) 1,3,4                                                   8318
    3 HB=H(NS,N)                                                        8319
    4 SA=QD*EXP(PAR(50,LT) /QD)                                         8321
     2  *EXP(PAR(51,LT)*(PAR(4,LT)-HB))                                 8324
     3  *EXP(PAR(52,LT)*RES(1,N+1))                                     8325
      IF(H(NS,N)-HB+0.00005) 13,6,6                                    83255
    6 IF(H(NS,N)-HB-0.00005) 12,11,11                                   8326
   11 SLM=PAR(47,LT)-PAR(48,LT)*EXP(PAR(49,LT)*(HB-HD))+PAR(48,LT)      8327
      SLM=SLM*SA*PAR(64,LT)                                             8328
      SPAR=PAR(47,LT)*SA*PAR(64,LT)                                     8329
      GO TO 14                                                          8330
   12 SLM=0                                                             8331
      SPAR=0.                                                          83315
      GO TO 14                                                          8332
   13 SLM=PAR(53,LT)+PAR(54,LT)*EXP(PAR(55,LT)*(HB-HD))-PAR(54,LT)      8333
      SLM=SLM*SA*PAR(64,LT)                                             8334
      SPAR=PAR(53,LT)*SA*PAR(64,LT)                                     8335
   14 IF (IPRO) 25,23,25                                                8336
   23 WRITE (6,9233) JTOT, NA, NSA, JA, N, NS, JT, HB, H(NS,N), SLM,    8337
     2       SPAR                                                      83375
   25 RETURN                                                            8338
 9233 FORMAT (/16H0**ANPAY3  JTOT=,I3, 4H NA=,I3, 5H NSA=,I3, 4H JA=,I3, 8339
     2       3H N=,I3, 4H NS=,I3, 4H JT=,I4, 4X, 3HHB=,F7.4,            8340
     3       9H H(NS,N)=,F7.4, 5H SLM=,F10.2, 6H SPAR=, F7.2)          83405
      END                                                               8341
```

```
      SUBROUTINE ANPAY4 (HEND)
C     FIND PAYMENTS IN YEAR JT FOR FINAL CLOSING DOWN OF MINE
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT
      QD=Q(NMAX)
      HEND=(PAR(56,LT)+PAR(57,LT)*EXP(PAR(58,LT)*QD)-PAR(57,LT))
     2    *EXP(PAR(59,LT)*(PAR(4,LT)-HB))
     3 *(PAR(62,LT)+PAR(60,LT) *RES(1,NMAX+1))
      HEND=HEND*PAR(63,LT)
      IF (IPRO) 5,3,5
    3 WRITE (6,9234)  JTOT, NA, NSA, JA, N, NS, JT, HEND
    5 RETURN
 9234 FORMAT (/16HO**ANPAY4   JTOT=,I3, 4H NA=, I3, 5H NSA=,I3, 4H JA=,
     2       I3, 3H N=,I3,4H NS=,I3,4H JT=,I4, 4X, 5HHEND=,F10.2)
      END
```

```
      SUBROUTINE PRICOM
C  PRINT KEY VARIABLES IN COMMON OF CUTOFF AND SUBROUTINES
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRC, LT
C
      WRITE(6,9101) IDENT, JTOT, JA, NA, NSA, N, NS, NMAX, NSMAX, M,
     2              JAEND1, JAEND2, JAMAX(1), JAMAX(3), JAMAX(5),
     3              JTOTMX, JTOTED, JC, JT, JTHOR, JM, JMHMIN, I, IPRO,
     4              C1(1), C1(NMAX), C1(14), C2(1), C2(NMAX), C3(1),
     5              C3(NMAX), C4(1), C4(NMAX), C4(14), C5, C6, C7,
     6              C8, C9, B(1), B(2), B(99), BMAX(1), BMAX(3), BMAX(5),
     7              BTOT(1), BTOT(3), BTOT(20), DELTA3
C
      WRITE(6,9102) DELTA1, DELTA2, H(1,1), H(NSMAX,1), H(2,2),
     2              H(NSMAX,NMAX), H(20,14), HA(1), HA(2), HA(99),
     3              HAEND1, HAEND2, HAMAX(1), HAMAX(3), HAMAX(5), HB,
     4              HBIN, HEQV(1), HEQV(NMAX), HEQV(14), LT, PAR(1,1),
     5              PAR(70,JTHOR), PAR(70,50), Q(1),  Q(NMAX), Q(14),
     6              QBIN, R, DRES, EXDRES, RES(1,1), RES(NSMAX,1),
     7              RES(2,2),  RES(NSMAX,NMAX), RES(20,15), T(1,1),
     8              T(NSMAX,1), T(2,2), T(NSMAX,NMAX), T(20,15),
     9              T(1,NMAX+1), TB, TE
C
      RETURN
 9101 FORMAT (/18H0 -STATE OF COMMON, I6, 17I4, I6, I5, 4I3/ 1H , 7X,
     2       10F10.3/ 1H , 7X, 5F10.3/ 1H , 7X, 9F11.2, F7.3)
C
 9102 FORMAT (1H , 7X, 12F8.4/ 1H , 7X, 8F8.4, I4, F10.3, 2F6.1/ 1H ,
     2       7X, 4F8.2, F7.3, 7F9.2/ 1H , 7X, 8F8.2)
C
      END
```

```
      SUBROUTINE INDATT                                                     9141
C     CHECK INTERNAL CONSISTENCY OF INPUT DATA                              9142
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,         CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,       CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),           CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),           CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,          CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,        CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                         CMN11
      JA= 0                                                                 9153
      IF(NMAX-14) 2,2,1                                                     9154
    1 JA= 1                                                                 9155
      WRITE (6,9029) JA                                                     9156
    2 IF(NSMAX-20) 4,4,3                                                    9157
    3 JA= 2                                                                 9158
      WRITE (6,9029) JA                                                     9159
    4 IF(JTHOR-50) 6,6,5                                                    9160
    5 JA= 3                                                                 9161
      WRITE (6,9029) JA                                                     9162
    6 IF(DELTA2- DELTA1) 8,7,7                                              9163
    7 JA= 4                                                                 9164
      WRITE (6,9029) JA                                                     9165
    8 IF(DELTA3- 1.) 10,9,9                                                 9166
    9 JA= 5                                                                 9167
      WRITE (6,9029) JA                                                     9168
   10 IF(JTOTMX- 20) 12,12,13                                               9169
   12 IF(JTOTMX- 3)  13,14,14                                               9170
   13 JA= 6                                                                 9171
      WRITE (6,9029) JA                                                     9172
   14 IF(3- M) 16,16,15                                                     9173
   15 JA= 7                                                                 9174
      WRITE (6,9029) JA                                                     9175
   16 IF(PAR(70,1)- 1.00005) 18,17,17                                       9176
   17 JA= 8                                                                 9177
      WRITE (6,9029) JA                                                     9178
   18 IF(PAR(4,1)- PAR(4,JTHOR)-0.00005) 20,19,19                           9179
   19 JA= 9                                                                 9180
      WRITE (6,9029) JA                                                     9181
   20 IF(R) 21,21,22                                                      918110
   21 JA= 10                                                              918111
      WRITE (6,9029) JA                                                   918112
   22 GO TO 100                                                             9182
  100 IF(JA) 105,101,105                                                    9183
  101 WRITE (6,9028)                                                        9184
      RETURN                                                                9185
  105 STOP                                                                  9186
 9028 FORMAT (/59H0 *NO ERRORS IN INPUT DATA DETECTED, BUT LOOK FOR YOUR    9187
     2SELF. )                                                               9188
 9029 FORMAT (/16H .*ERROR OF TYPE ,I3,24H DETECTED IN INPUT DATA. )        9189
      END                                                                   9190
```

```
      SUBRCUTINE PRICAP                                                  9201
C     PRINT TABLE OF CAPITAL VALUES                                      9202
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,      CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,    CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),        CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),        CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,       CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,     CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                      CMN11
    1 KB=9                                                               9212
      IF (JAEND2-KB)2,3,3                                                9213
    2 KB=JAEND2                                                          9213
    3 WRITE (6,9202) JTOT, JM, NA, NSA, (HA(JA), JA=1,KB)                9214
      WRITE (6,9203) (B(JA), JA=1,KB)                                    9215
      IF(JAEND2-9) 11,11,8                                               92155
    8 DO 10 KC=1,9                                                       9216
        KA=10*KC                                                         921651
        KB=KA+9                                                          921652
        IF (JAEND2-KB) 4,5,5                                             9217
    4   KB=JAEND2                                                        9218
    5   WRITE (6,9204) KC, (HA(JA), JA=KA,KB)                            9219
        WRITE (6,9205) KC, (B(JA),JA=KA,KB   )                           9220
        IF (JAEND2-KB) 11,11,10                                          9221
   10 CONTINUE                                                           9222
   11 RETURN                                                             9223
 9202 FORMAT (1H ,3X,5HJTOT=,I3, 3X, 3HJM=,I2, 3X, 3HNA=,I3, 3X,         9224
     2       4HNSA=,I3/ 1H0, 16X, 3HJA=, 6X, 2H*0, 8X, 2H*1, 8X, 2H*2,   9225
     3       8X, 2H*3, 8X, 2H*4, 8X, 2H*5, 8X, 2H*6, 8X, 2H*7, 8X, 2H*8, 9226
     4       8X, 2H*9/ 11H0    HA(JA)=, 4X, 5H*= 0 ,10X,9F10.4)          9227
 9203 FORMAT (10H     B(JA)=, 5X, 5H*= 0 , 10X, 9F10.2)                  9228
 9204 FORMAT (11H0    HA(JA)=, 4X, 2H*=,I2, 1X, 10F10.4)                 9229
 9205 FORMAT (10H     B(JA)=, 5X, 2H*=,I2, 1X, 10F10.2)                  9230
      END                                                                9231
```

```
C      PROGRAM EXRATE
C      OPTIMUM RATES OF PRODUCTION, Q(N)
C  VERSION 2, 8.8.68
       COMMON QMIN(14), QL(14), QH(14), QMAX(14),QX
       COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,
     2        T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,
     3        C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),
     4        HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),
     5        Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,
     6        NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,
     7        JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX
       DIMENSION QOPT(14)
C  READ INPUT DATA
       READ (5,9020) NMAX, NSMAX, JTHOR, DELTA1, DELTA2, DELTA3, HAEND1,
     2              HAEND2, HBIN, JTOTMX, M, QBIN, R
       NMAXD=NMAX+1
       DO 21  NS=1,NSMAX
   21     READ (5,9021) (RES(NS,N), N=1,NMAXD)
       READ (5,9022) (HEQV(N), N=1,NMAX)
       READ (5,9023) (Q(N), N=1,NMAX)
       DO 22 NS=1,NSMAX
   22     READ (5,9022) (H(NS,N), N=1,NMAX)
       READ (5,9024)  (C1(N), N=1,NMAX)
       READ (5,9024)  (C2(N), N=1,NMAX)
       READ (5,9024)  (C3(N), N=1,NMAX)
       READ (5,9024)  (C4(N), N=1,NMAX)
       READ (5,9024)  C5, C6, C7, C8,C9
       DO 23 LT=1,JTHOR
   23     READ (5,9025) (PAR(JD,LT), JD=1,70)
C  WRITE INPUT DATA
       WRITE (6,9030) NMAX,NSMAX,JTHOR, DELTA1, DELTA2, DELTA3, HAEND1,
     2              HAEND2, HBIN, JTOTMX, M, QBIN, R
       WRITE (6,9031)
       WRITE (6,9090)
       DO 31  NS=1,NSMAX
   31     WRITE (6,9032) NS,(RES(NS,N), N=1,NMAXD)
       WRITE (6,9033)
       WRITE (6,9090)
       WRITE (6,9034) (HEQV(N), N=1,NMAX)
       WRITE (6,9035)
       WRITE (6,9090)
       WRITE (6,9036) (Q(N), N=1,NMAX)
       WRITE (6,9037)
       WRITE (6,9090)
       DO 32 NS=1,NSMAX
   32     WRITE (6,9038) NS,(H(NS,N), N=1,NMAX)
       WRITE (6,9039)
       WRITE (6,9041)  (C1(N), N=1,NMAX)
       WRITE (6,9042)  (C2(N), N=1,NMAX)
       WRITE (6,9043)  (C3(N), N=1,NMAX)
       WRITE (6,9044) (C4(N), N=1,NMAX)
       WRITE (6,9045)  C5, C6, C7, C8, C9
       KA=-4
       KB=0
   41  KA=KA+5
       KB=KB+5
       IF(JTHOR-KB) 42,43,43
   42  KB=JTHOR
```

```
   43  WRITE (6,9046)
       DO 45 JT=KA,KB
          WRITE (6,9047) JT, (PAR(JD,JT), JD=1,9)
          DO 44 KC=1,6
             KD=10*KC
             KE=KD+9
   44        WRITE (6,9048) KC,(PAR(JD,JT), JD=KD,KE)
   45     WRITE (6,9049) PAR(70,JT)
       IF(JTHOR-KB) 46,46,41
   46  WRITE (6,9200)
C   TEST INTERNAL CONSISTENCY OF INPUT DATA
       CALL INDATT
C   INITIAL VALUES
       JM=0
       JTOT=0
       JTOTED=0
       JA=1
       HA(JA)= Q(1)
       T(1,1)=1.00
       QOPT(1)= Q(1)
       QX= QBIN
C   START FOR NEW TOTAL OPTIMUM
    3  NA=1
       NSA=1
       IPRO = 0
C   FIND STARTING TIMES FOR NIV. AND SUBNIV.
       CALL EX1
       JM=0
       IF(JTOT) 94,95,95
   94  IDENT=89
       GO TO 135
C   FIND CAPITAL VALUE OF INITIAL GUESS
   95  CALL CUT5
       WRITE (6,9218) B(JA)
       IPRO=1
C   OPTIMIZE LAST NIV., NMAX
       JA= 1
   96  NA= NMAX
       GO TO 130
C   OPTIMIZE PREVIOUS NIV., NA-1
  100  NA= NA - 1
       JM= 0
C   LOWER LIMIT = EXPANSION LIMIT
C   FIND LOWER LIMITS FOR RATE
  130  JA= 1
       IX= 1
       IF(JTOT) 131,140,132
  140  IF(NA-1) 131,133,136
  131  IDENT=105
  135  CALL PRICOM
       STOP
  132  IF(NA-1) 131,133,134
C   FIND OPTIMUM RATE IF NA=1
  133  QX= QBIN
       HA(1)= QOPT(1)
       JAEND1= 1
       WRITE (6,9225) JTOT
       GO TO 9
```

```
C  FIND LOWER LIMITS FOR RATE, CONT.
 134   IF(QOPT(NA-1)- QMIN(NA)) 150,151,151                              113
 150   QX= QOPT(NA-1)                                                    1131
       GO TO 152                                                         1132
 151   QX= QMIN(NA)-DELTA1                                               1133
 152   WRITE (6,9223) JTOT,NA                                            1134
 145   Q(NA-1)= QX                                                       114
       HA(1)= QX + DELTA2                                                115
       JAEND1= 0                                                         116
       JAEND2= 1                                                         117
       GO TO 9                                                           118
 136   QX= Q(NA-1)+2*DELTA1                                              119
       WRITE (6,9223) JTOT,NA                                            1195
 137   QX= QX - 2. * DELTA1                                              120
 138   IF(QX-DELTA2) 142,141,141                                         121
 142   QX= DELTA2                                                        122
 141   Q(NA-1)= QX                                                       123
       HA(1)= QX + DELTA2                                                124
       JAEND1= 0                                                         125
       JAEND2= 1                                                         126
       GO TO 9                                                           127
C  FIND OPTIMUM RATE OR OPTIMUM LOWER OR UPPER LIMIT. FIRST APPROX.
 200   CALL EX2                                                          128
       GO TO (9,220), I                                                  1301
 220   WRITE (6,9201)                                                    1302
       CALL PRICAP                                                       1303
       CALL EX3                                                          1304
C                                                   SECOND APPROX.
 300   CALL EX4                                                          1307
       GO TO (9,304),I                                                   13095
C  GO TO CONSISTENCY TEST OF LOWER (305) AND UPPER (310) LIMITS
 304   GO TO (305,310), IX                                               1310
C  GO TO 920 VIA 510 IF NA=1 FOR DECISION ON NEW OPTIMIZATION
 305   IF(NA-1) 306,510,307                                              1311
 306   IDENT= 1312                                                       1312
       GO TO 135                                                         1313
 307   IF(Q(NA)- Q(NA-1)) 308,309,309                                    1314
C  LOWER LIMIT NOT ACCEPTED - TRY AGAIN
 308   WRITE (6,9222) JTOT,NA,IX,QX,Q(NA)                                1315
       IF(QX- 1.5 * DELTA2) 309,309,303                                  1316
 303   JM= 0                                                             1317
       JA= 1                                                             1318
       QX= QX- DELTA1                                                    1319
       GO TO 138                                                         1320
C  LOWER LIMIT ACCEPTED - FIND UPPER LIMIT (510)
 309   QL(NA)= Q(NA)                                                     1321
       GO TO 510                                                         1322
 310   IF(NA-1) 311,319,312                                              1323
 311   IDENT= 1324                                                       1324
       GO TO 135                                                         1325
 312   IF(Q(NA)- Q(NA-1)) 319,319,313                                    1326
C  UPPER LIMIT NOT ACCEPTED - TRY AGAIN
 313   WRITE (6,9222) JTOT,NA,IX,QX,Q(NA)                                1327
       JM= 0                                                             1328
       JA= 1                                                             1329
       QX= QX+ DELTA1                                                    133
       GO TO 518                                                         134
C  UPPER LIMIT ACCEPTED
```

```
 319  QH(NA)= Q(NA)                                                        135
C  FIND OPTIMUM LIMITS FOR KEEPING RATE UNCHANGED
      CALL EX7                                                             136
C  DECIDE ON NEW NIV. TO OPTIMIZE
      GO TO 100                                                            139
C  CHECK FOR PRACTICAL CONSTRAINTS
   9  IF(JA-93) 5,5,4                                                      140
   4  IDENT= 141                                                           141
      GO TO 135                                                            142
   5  IF(HA(JA) - HAEND2) 7,4,4                                            143
   7  IF(HA(JA) - HAEND1) 4,4,8                                            144
   8  Q(NA)= HA(JA)                                                        145
C  DECIDE FUTURE OPTIMUM RATES
      CALL EX6                                                             146
C  FIND STARTING TIMES FOR NIV. AND SUBNIV.
      CALL EX1                                                             147
C  COMPUTE CAPITAL VALUE OF FUTURE MINING (=OF REMAINING NIVS.)
 400  CALL CUT5                                                            150
      IPRO= 1                                                              153
C  DECIDE ON NEXT STEP ACCORDING TO VALUES OF JTOT, JM, JA, NA, IX
      IF(JTOT) 401,402,403                                                 154
 402  IF(JM) 401,410,300                                                   155
 403  IF(JM) 401,200,300                                                   156
 401  IDENT= 157                                                           157
      GO TO 135                                                            158
 410  IF(JA-M) 411,415,200                                                 159
 411  IF(JA-1) 401,200,415                                                 1600
 415  IF(NA-1) 416,200,420                                                 1601
 416  IDENT= 1602                                                          1602
      GO TO 135                                                            1603
 420  IF(B(JA) - B(JA-1)) 430,430,200                                      1604
 430  IF(QX- 1.5 * DELTA2) 200,200,431                                     1605
 431  JA= 1                                                                1606
      GO TO (137,517), IX                                                  1607
C  UPPER LIMIT = CONTRACTION LIMIT
C  FIND UPPER LIMITS FOR RATE
 510  IX= 2                                                                1608
      JM= 0                                                                16085
      JA= 1                                                                1609
C  OPTIMIZATION COMPLETED IF NA=1. DECIDE ON NEW OPTIMIZATION IN 920
C  VIA 520 OR 512
      IF(JTOT) 511,520,512                                                 1610
 511  IDENT= 1611                                                          1611
      GO TO 135                                                            1612
 520  IF(NA-1) 511,920,516                                                 1613
 512  IF(NA-1) 511,920,514                                                 1614
 514  IF(QOPT(NA-1)- QMAX(NA)) 521,521,522                                 1615
 521  QX= QMAX(NA)+DELTA1*4                                                16151
      GO TO 523                                                            16152
 522  QX= QOPT(NA-1)                                                       16153
 523  Q(NA-1)= QX                                                          1616
      HA(1)= QX- DELTA2                                                    1617
      JAEND1= 1                                                            1618
      WRITE (6,9224)                                                       16185
      GO TO 9                                                              1619
 516  QX= QL(NA)                                                           1620
      WRITE (6,9224)                                                       16205
 517  QX= QX+ 2. * DELTA1                                                  1622
```

```
 518  Q(NA-1)= QX
      HA(1)= QX- DELTA2
      JAEND1= 1
      GO TO 9
C  DEFINE OPTIMUM AND DECIDE ON NEW OPTIMIZATION
 920  JTOT= JTOT + 1
      QOPT(1)= Q(1)
      CALL EX6
      JA= 95
      BTOT(JTOT)= B(1)
      WRITE (6,9211) JTOT,JTOT,BTOT(JTOT)
      WRITE (6,9220) JTOT
      DO 925 N=2,NMAX
      COPT(N)= C(N)
 925  WRITE (6,9221) N,QMIN(N),CMAX(N),QL(N),QH(N)
C  OPTIMIZE AT LEAST 3 TIMES
      IF (JTOT-2) 3,3,930
C  OPTIMIZE ONCE MORE IF LAST INCREASE IN CAP. VAL. WAS GREAT,OR STOP
 930  IF (BTOT(JTOT)-BTOT(JTOT-1)-DELTA3*BTOT(JTOT-1)) 960,960,940
 940  IF (JTOT-JTOTMX) 3,960,960
 960  JTOTED=JTOT
      NA=1
      IPRO=0
C  FIND STARTING TIMES FOR NIV. AND SUBNIV. IN LAST OPTIMUM
      CALL EX1
      JM=0
C  COMPUTE AND SPECIFY CAPITAL VALUE OF LAST OPTIMUM
      CALL CUT5
      WRITE (6,9218) B(JA)
      WRITE (6,9217) (JTOT,BTOT(JTOT), JTOT=1,JTOTED)
      STOP
 9020 FORMAT (2I3,I4,2F7.2,F6.3,2F7.2,F7.4,2I3,F7.2,F6.3)
 9021 FORMAT (11F7.2/4F7.2)
 9022 FORMAT (11F7.4/3F7.4)
 9023 FORMAT (11F7.2/3F7.2)
 9024 FORMAT (9F8.3/5F8.3)
 9025 FORMAT (8(8F9.3/), 6F9.3)
 9030 FORMAT (32H1  OPTIMUM RATE OF PRODUCTION    //13H  *INPUT DATA/
     2        11H  *   NMAX=,I3, 9H   NSMAX=, I3, 9H   JTHOR=, I4/
     3        13H  *   DELTA1=, F7.2,
     4        10H   DELTA2=, F7.2,10H   DELTA3=, F6.3, 10H   HAEND1=,
     5        F7.2, 10H   HAEND2=, F7.2, 8H   HBIN=, F7.4,
     6        10H   JTOTMX=,I3/ 8H  *   M=, I3, 8H   QBIN=, F7.2,
     7        5H   R=,F6.3)
 9031 FORMAT (/68H0 *EQUIV CUMUL. ORE RESERVE MT TO BE EXTR. BEFORE (NS,
     2N), RES(NS,N)  )
 9032 FORMAT (1H ,11X, I2, 1X, 15F7.2)
 9033 FORMAT (/34H0 *GRADE OF EQUIV RESERVE, HEQV(N))
 9034 FORMAT (15H    1... NSMAX ,14F7.4)
 9035 FORMAT (/29H0 *ANNUAL PRODUCTION MT, Q(N))
 9036 FORMAT (15H    1... NSMAX , 14F7.2)
 9037 FORMAT (41H1 *INITIAL CUT-OFF (MEAN) GRADES H(NS,N)  )
 9038 FORMAT (1H ,11X, I2, 1X, 14F7.4)
 9039 FORMAT (/36H0 *ORE RESERVE CONVERTING PARAMETERS/
     2       28H   1. DEPEND OF NIV, C-(N)  / 8H0     N=,4X,1H1,7X,1H2,
     3       7X,1H3,7X,1H4,7X,1H5,7X,1H6,7X,1H7,7X,1H8,7X,1H9,6X,2H10,
     4       6X,2H11, 6X,2H12,6X,2H13,6X,2H14)
 9041 FORMAT (8H  C1(N)=,14F8.3)
```

```
9042 FORMAT (8H  C2(N)=,14F8.3)
9043 FORMAT (8H  C3(N)=,14F8.3)
9044 FORMAT (8H  C4(N)=,14F8.3)
9045 FORMAT (24H0   2. INDEP. OF NIV, C-/1H0,7X,3HC5=,F9.3,3X,3HC6=,
    2       F9.3,3X,3HC7=,F9.3,3X,3HC8=,F9.3,3X,3HC9=,F9.3)
9046 FORMAT (49H1 *ANNUALLY CHANGING ECONOMIC PARAMETERS, PAR(JD))
9047 FORMAT (1H0,16X, 3HJD=,6X,2H*0,8X, 2H*1, 8X, 2H*2, 8X, 2H*3,
    2       8X, 2H*4, 8X, 2H*5, 8X, 2H*6, 8X, 2H*7, 8X, 2H*8, 8X, 2H*9/
    3       9H    YEAR ,I3, 3X, 5H*= 0 , 10X, 9F10.3)
9048 FORMAT (1H , 17X, I1, 1X, 10F10.3 )
9049 FORMAT (1H , 17X, 2H7 ,F10.3)
9090 FORMAT (1H0, 11X, 8HET. N= 1, 6X, 1H2, 6X, 1H3, 6X, 1H4, 6X, 1H5,
    2       6X, 1H6, 6X, 1H7, 6X, 1H8, 6X, 1H9, 5X, 2H10, 5X, 2H11,
    3       5X, 2H12, 5X, 2H13, 5X, 2H14, 4X, 4H(15)/ 14H    SUBNIV NS=
    4       )
9200 FORMAT (14H1**OUTPUT DATA, 10X, 3HAND/ 12H  -TEST DATA)
9201 FORMAT (/53H0**CAPITAL VALUE B(JA) AT RATE HA(JA).  FIRST APPROX.)
9211 FORMAT(/19H0**TOTAL OPTIMUM NR, I3, 3X, 5HJTOT=, I3, 3X,
    2       11HBTOT(JTOT)=,F10.2)
9217 FORMAT (/43H0**RECORD OF TOTAL OPTIMA, JTOT, BTOT(JTOT)/1H , 3X,
    2       8(I3, 1H,,F10.2)/
    3       1H ,3X, 8(I3,1H,,F10.2)/
    4       1H ,3X, 4(I3,1H,,F10.2))
9218 FORMAT (45H0**CAPITAL VALUE OF INITIAL GUESS OR NEW OPT  ,F10.2)
9220 FORMAT (/83H0**OPTIMUM RATES. CHANGE FROM OUTSIDE QMIN(N) AND QMAX
    2(N) TO QL(N) AND QH(N). JTOT=  ,I3/)
9221 FORMAT (6H    N= ,I3,29X,F7.2,5X,F7.2,3X,F7.2,3X,F7.2)
9222 FORMAT (68H0**QL(NA) OR QH(NA) WRONG WHEN JTOT, NA, IX, QX, AND Q(
    2NA) ARE RESP  ,3I3,2F7.2,23H  NEW TRIAL IF QX LARGE  )
9223 FORMAT ( 30H1**FIND EXPANSION LIMIT. JTOT= ,I3,5H. NA=,I3/)
9224 FORMAT ( 25H **FIND CONTRACTION LIMIT / )
9225 FORMAT (27H1**FIND OPTIMUM RATE. JTOT= ,I3,7H. NA= 1/)
     END
```

SUBROUTINE CUT11
CUT5
ANPAY1
ANPAY2
ANPAY3
ANPAY4
INDATT

These subroutines are, excepting COMMON, identically the same in EXRATE and CUTOFF. Therefore, see CUTOFF.

```
      SUBROUTINE EX1                                                    1001
C     FIND STARTING TIMES FOR NIVS. AND SUBNIVS.                        1002
      COMMON QMIN(14), QL(14), QH(14), QMAX(14),QX                      CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,    CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX                 CMN11
   10 N=NA                                                              1012
      IF (IPRO) 14,11,14                                                1013
   11 WRITE (6,9214) JTOT, JT                                           1014
   14 DO 29 NS=NSA,NSMAX                                                1015
         IF (NS-NSMAX) 25,21,20                                         1016
   20    IDENT=1017                                                     1017
         CALL PRICOM                                                    1018
         STOP                                                           1019
   21    DRES=RES(1,N+1)-RES(NS,N)                                      1020
         CALL CUT11                                                     1021
         T(1,N+1)=T(NS,N)+EXDRES/Q(N)                                   1022
         GO TO 26                                                       1023
   25    DRES=RES(NS+1,N)-RES(NS,N)                                     1024
         CALL CUT11                                                     1025
         T(NS+1,N)=T(NS,N)+EXDRES/Q(N)                                  1026
   26    IF(IPRO) 29,27,29                                              1027
   27 WRITE (6,9215) N,NS,HEQV(N),DRES,EXDRES,T(NS,N),Q(N)              1028
   29 CONTINUE                                                          1029
      IF(NA-NMAX) 35,50,33                                              1030
   33 IDENT=1031                                                        1031
      CALL PRICOM                                                       1032
      STOP                                                              1033
   35 NAD=NA+1                                                          1034
   36 DO 49 N=NAD, NMAX                                                 1035
         DO49 NS=1,NSMAX                                                1036
           IF (NS-NSMAX) 45,41,40                                       1037
   40      IDENT=1037                                                   1038
           CALL PRICOM                                                  1039
           STOP                                                         1040
   41      DRES=RES(1,N+1)-RES(NS,N)                                    1041
           CALL CUT11                                                   1042
           T(1,N+1)=T(NS,N)+EXDRES/Q(N)                                 1043
           GO TO 46                                                     1044
   45      DRES=RES(NS+1,N)-RES(NS,N)                                   1045
           CALL CUT11                                                   1046
           T(NS+1,N)=T(NS,N)+EXDRES/Q(N)                                1047
   46      IF (IPRO) 49,47,49                                           1048
   47 WRITE (6,9215) N,NS,HEQV(N),DRES,EXDRES,T(NS,N),Q(N)              1049
   49 CONTINUE                                                          1050
   50 WRITE (6,9216) T(1,NMAX+1), JA, HA(JA), QX                        1051
      RETURN                                                            1052
 9214 FORMAT (/100H0**EQUIVALENT (DRES) AND MINABLE (EXDRES) ORE IN SUBN 1053
     2IV, STARTING TIMES (T(NS,N) AND RATES  (Q(N))  /1H ,3X, 5HJTOT=,  1054
     3       I3, 3X, 3HJT=,I4/1H0, 21X ,1HN, 4X, 2HNS, 3X, 7HHEQV(N),7X, 1055
     4       4HDRES, 5X, 6HEXDRES, 3X, 7HT(NS,N), 3X, 7HQ(N)    )       1056
 9215 FORMAT (1H ,19X,I3, I6, F10.4,2F11.2, F10.2, F10.2)               1057
 9216 FORMAT (13H **MINE LIFE=, F10.2, 11H   WHEN JA=, I3,              1058
     2      10H   HA(JA)=, F10.2, 10H   AND QX= , F10.2)                10585
      END                                                               1059
```

```
      SUBROUTINE EX2                                                  2001
C     FIND NEW TRIAL RATE IN ACTUAL NIV. IN FIRST APPROX.             2002
      COMMON QMIN(14), QL(14), QH(14), QMAX(14),QX                    CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX               CMN11
      IF(NA-1) 20,200,40                                              20112
   20 IDENT= 20113                                                    20113
   21 CALL PRICOM                                                     20114
      STOP                                                            20115
   40 GO TO (210,200), IX                                             20116
  200 IF (JAEND1-(M+1)) 201,202,202                                   2012
  201 IF (JA-1) 203,2011,2012                                         2013
 2011 CONTINUE                                                        2014
 2012 JA=JA+1                                                         2015
      HA(JA)=HA(JA-1)-DELTA1                                          2016
      JAEND1=JAEND1+1                                                 2017
      I=1                                                             2018
      RETURN                                                          2019
  202 IF (JA-JAEND1) 203, 204,210                                     2020
  203 IDENT=2021                                                      2021
      CALL PRICOM                                                     2022
      STOP                                                            2023
  204 JCD=JA-(M-1)                                                    2024
      DO 205 JC=JCD,JA                                                2025
         IF (B(JC)-B(JC-1)) 205,2012,2012                             2026
  205 CONTINUE                                                        2027
      IF(NA-1) 230,231,232                                            202820
  230 IDENT= 202821                                                   202821
      CALL PRICOM                                                     202822
      STOP                                                            202823
  231 JAEND2=JAEND1+1                                                 20285
      JA=JA+1                                                         2029
      HA(JA)=HA(1)+DELTA1                                             2030
      I=1                                                             2031
      RETURN                                                          2032
  210 IF (JAEND2-JAEND1-M) 211,212,213                                2033
  211 JA=JA+1                                                         2034
      HA(JA)=HA(JA-1)+DELTA1                                          2035
      JAEND2=JAEND2+1                                                 2036
      I=1                                                             2037
      RETURN                                                          2038
  212 JCD=JA-(M-2)                                                    2039
      IF (B(JA-(M-1))-B(1)) 214,211,211                               2040
  213 JCD=JA-(M-1)                                                    2041
  214 DO 215 JC=JCD,JA                                                2042
         IF (B(JC)-B(JC-1)) 215,211,211                               2043
  215 CONTINUE                                                        2044
      GO TO 234                                                       20444
  232 JAEND2= JAEND1                                                  20445
  234 I=2                                                             2046
      RETURN                                                          2047
      END                                                             2048
```

```
      SUBROUTINE EX3                                                    3001
C     FIND 1 TO 5 MAXIMA IN FIRST APPROX                                3002
      COMMON QMIN(14), QL(14), QH(14), QMAX(14),QX                      CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,    CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX                 CMN11
      JAMAX(1)=1                                                       30115
  220 DO 221 JM=2,5                                                     3012
  221    JAMAX(JM)=0                                                    3013
      JM=1                                                              3014
      DO 236 JA=2,JAEND1                                                3015
      IF (B(JA)-B(JA-1)) 236,230,230                                    3016
  230 IF (JAMAX(JM)-JA+1) 232,235,231                                   3017
  231 IDENT=3018                                                        3018
      CALL PRICOM                                                       3019
      STOP                                                              3020
  232 IF(NA-1) 231,202,201                                             30204
  201 IF(JM-5) 234,233,231                                             30205
  202 IF(JM-3) 234,233,231                                              3021
  233 WRITE (6,9012) JM, JTOT, NA, NSA, JA                              3022
      JAMAX3=JAMAX(3)                                                   3023
      IF (B(JA)-B(JAMAX3)) 236,235,235                                  3024
  234 JM=JM+1                                                           3025
  235 JAMAX(JM)=JA                                                      3026
  236 CONTINUE                                                          3027
      JMHMIN=JM                                                         3028
      IF(NA-1) 231,210,205                                             30284
  205 GO TO (210,206), IX                                              30285
  206 JMHMIN= 1                                                        30286
      GO TO 270                                                        30287
  210 JAD= JAEND1+ 2                                                    3029
      IF(B(JAEND1+1)-B(1)) 255,255,240                                  3030
  240 IF   (JAMAX(1)-1) 241,242,250                                     3031
  241 IDENT=3032                                                        3032
      CALL PRICOM                                                       3033
      STOP                                                              3034
  242 JAMAX(1)=JAEND1+1                                                 3035
      DO 246 JA=JAD,JAEND2                                              3036
         IF(B(JA)-B(JA-1))248,248,246                                   3037
  246 JAMAX(1)=JA                                                       3038
      GO TO 270                                                         3039
  248 JAD=JA+1                                                          3040
      GO TO 255                                                         3041
  250 JM=JM+1                                                           3042
      JAMAX(JM)=JAEND1+1                                                3043
  255 DO 266 JA=JAD,JAEND2                                              3044
         IF (B(JA)-B(JA-1)) 266,266, 260                                3045
  260 IF(JAMAX(JM)- JA +1) 262,265,266                                  3046
  261    IDENT=3047                                                     3047
         CALL PRICOM                                                    3048
         STOP                                                           3049
  262    IF (JM-5) 264,263,261                                          3050
  263    WRITE (6,9012) JM, JTOT, NA, NSA, JA                           3051
         JAMAX5=JAMAX(5)                                                3052
```

```
          IF (B(JA)-B(JAMAX5)) 266,266,265
264       JM=JM+1
265       JAMAX(JM)=JA
266    CONTINUE
270    DO 274 JM=1,5
          IF (JAMAX(JM)) 271,272,273
271       IDENT=3059
          CALL PRICOM
       STOP
272       BMAX(JM)=0.
          HAMAX(JM)=0.
          GO TO 274
273       JA=JAMAX(JM)
          BMAX(JM)=B(JA)
          HAMAX(JM)=HA(JA)
274    CONTINUE
       WRITE (6,9207) JTOT, NA, NSA, (HAMAX(JM), JM=1,5), (BMAX(JM), JM=
     2      1,5)
       JM=0
       JA=0
       RETURN
9012 FORMAT (/18H0**MULT MAX IN JM=,I2, 10X, 5HJTOT=, I3, 3X, 3HNA=,I3,
     2         3X, 4HNSA=, I3, 3X, 3HJA=,I3)
9207 FORMAT (/23H0**OPTIMA FIRST APPROX./ 1H , 3X, 5HJTOT=, I3, 3X,
     2         3HNA=, I3, 3X, 4HNSA=, I3/ 1H0, 16X, 3HJM=, 16X, 1H1, 9X,
     3         1H2, 9X, 1H3, 9X, 1H4, 9X, 1H5/ 1H , 3X, 10HHAMAX(JM)=,
     4         16X, 5F10.2/ 1H , 3X, 9HBMAX(JM)=, 17X, 5F10.2/)
       END
```

```
      SUBROUTINE EX4                                                    4001
C     FIND NEW TRIAL RATE IN ACTUAL NIV. AND MAXIMUM IN SECOND APPROX.  4002
      COMMCN QMIN(14), QL(14), QH(14), QMAX(14),QX                      CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,    CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX                 CMN11
      IF(JA) 303,301,308                                                4012
  301 JM=JM+1                                                           4013
      IF (JAMAX(JM)) 303,340,305                                        4014
  303 IDENT=4015                                                        4015
      CALL PRICOM                                                       4016
      STOP                                                             41165
  305 HA(1)=HAMAX(JM)                                                   4017
      B(1)=BMAX(JM)                                                     4018
      JA=1                                                              4019
      JAEND1=1                                                          4010
  308 IF (JAEND1-3) 309,310,310                                         4011
 3092 IDENT=4011                                                      401101
      CALL PRICOM                                                     401102
      GO TO 320                                                       401103
  309 IF (JA-13) 3091,3092,3092                                       401104
 3091 JA=JA+1                                                         401105
      HA(JA)=HA(JA-1)-DELTA2                                            4012
      JAEND1=JAEND1+1                                                   4013
      I=1                                                               4012
      RETURN                                                            4015
  310 IF (JA-JAEND1)  312, 311,322                                      4016
  311 IF (HA(JA-1))  312, 318,313                                       4017
  312 IDENT=4018                                                        4018
      CALL PRICOM                                                       4019
      STOP                                                              4020
  313 IF (B(JA)-B(JA-1)) 314,309,309                                    4021
  314 IF (B(JA-1)-B(JA-2)) 315,316,316                                  4022
  315 JA=JA+3                                                           4023
      HA(JA)=HA(JA-3)-3.*DELTA2                                         4024
      HA(JA-1)=0.                                                       4025
      JAEND1=JAENC1+3                                                   4026
      I=1                                                               4027
      RETURN                                                            4028
  316 IF (BMAX(JM)-B(JA-1)) 317,317,309                                 4029
  317 HAMAX(JM)=HA(JA-1)                                                4030
      BMAX(JM)=B(JA-1)                                                  4031
      GO TO 309                                                         4032
  318 IF (B(JA)-B(JA-3)) 320,319,319                                    4034
  319 JA=JA-3                                                           4035
      JAEND1=JAEND1-3                                                   4036
      GO TO 309                                                         4037
  320 JAEND1=JAEND1-3                                                   4038
      DO 321 JC=1,3                                                     4039
         JA=JAEND1+JC                                                   4040
         INDEX=4-JC                                                     4041
         HA(JA)=HA(INDEX)                                               4042
  321 B(JA)=B(INDEX)                                                    4043
  322 IF (HA(JA-1)) 323,330,324                                         4044
```

```
323   IDENT=4045
      CALL PRICOM
      STOP
324   IF (B(JA)-B(JA-1)) 325,327,327
325   IF (B(JA-1)-B(JA-2)) 326,328,328
326   JA=JA+3
      HA(JA)=HA(JA-3)+3.*DELTA2
      HA(JA-1)=0.
      I=1
      RETURN
3272  IDENT=4055
      CALL PRICOM
      GO TO 330
327   IF (JA-20) 3271, 3272,3272
3271  JA=JA+1
      HA(JA)=HA(JA-1)+DELTA2
      I=1
      RETURN
328   IF (BMAX(JM)-B(JA-1)) 329,327,327
329   HAMAX(JM)=HA(JA-1)
      BMAX(JM)=B(JA-1)
      GO TO 327
330   IF (B(JA)-B( JA-3)) 338,331,331
331   JA=JA-3
      GO TO 327
338   JAEND2=JA-3
339   WRITE (6,9206)
      CALL PRICAP
340   IF (JM-5) 301,342,341
341   IDENT=4074
      CALL PRICOM
      STOP
342   Q(NA)= HAMAX(JMHMIN)
      B(1)=BMAX(JMHMIN)
      DO 348 JM= 1,5
         IF (JAMAX(JM)) 345,348,346
345      IDENT=4081
         CALL PRICOM
         STOP
346      IF (B(1)-BMAX(JM)) 347,348,348
347   Q(NA)= HAMAX(JM)
         B(1)=BMAX(JM)
348   CONTINUE
      WRITE (6,9208) NA,Q(NA),B(1)
      JA= 95
      HA(JA)= Q(NA)
      I=2
      RETURN
9206  FORMAT (/53H0**CAPITAL VALUE B(JA) AT RATE HA(JA). SECOND APPROX.)
9208  FORMAT (/36H0**OPTIMUM SECOND APPROX.  Q(NA)= Q(,I2,2H)=,F7.2,
    2        22H.  CORRESP. CAP. VALUE, F10.2/1H1)
      END
```

```
      SUBROUTINE EX6                                                    6001
C     FIND OPTIMUM RATES IN REMAINING NIVS., GIVEN RATE IN ACTUAL NIV.  6002
      COMMON QMIN(14), QL(14), QH(14), QMAX(14),QX                      CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,    CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),        CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),        CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,       CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,     CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX                 CMN11
      IF(NA-NMAX) 2,60,1                                                6012
    1 IDENT= 6012                                                       6013
      CALL PRICOM                                                       6014
      STOP                                                              6015
    2 NAD= NA+ 1                                                        6016
      DO 40 N= NAD,NMAX                                                 6017
      IF(Q(N-1)- QMIN(N)) 10,10,15                                      6018
   10 Q(N)= QL(N)                                                       6019
      GO TO 40                                                          6020
   15 IF(Q(N-1)- QMAX(N)) 20,30,30                                      6021
   20 Q(N)= Q(N-1)                                                      6022
      GO TO 40                                                          6023
   30 Q(N)= QH(N)                                                       6024
   40 CONTINUE                                                          6025
   60 RETURN                                                            6026
      END                                                               6027
```

```
      SUBROUTINE EX7                                                    7001
C     FIND OPTIMUM LIMITS FOR KEEPING RATE UNCHANGED                    7002
      COMMON QMIN(14), QL(14), QH(14), QMAX(14),QX                      CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,    CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX                 CMN11
      WRITE (6,9220)                                                    7012
      YSIGN= 1.                                                         7013
      HYL= QL(NA)                                                       7014
    5 Q(NA-1)= HYL                                                      7015
      B(99)= 0.                                                         7016
   10 Q(NA-1)= Q(NA-1)- DELTA1* YSIGN                                   7017
      IF(Q(NA-1)) 40,40,11                                              7018
   11 Q(NA)= Q(NA-1)                                                    7019
      JA= 1                                                             7020
   20 CALL EX6                                                          7021
C  QX AND HA(JA) FOR OUTPUT IN EX1
      QX= Q(NA-1)                                                       702151
      HA(JA)= Q(NA)                                                     702152
      CALL EX1                                                          7022
      CALL CUT5                                                         7025
      GO TO (30,35,60,70), JA                                           7028
   30 Q(NA)= HYL                                                        7029
      JA= 2                                                             7030
      GO TO 20                                                          7031
   35 IF(B(1)- B(2)) 40,40,10                                           7032
   40 Q(NA-1)= Q(NA-1)+ DELTA1* YSIGN                                   7033
   50 Q(NA-1)= Q(NA-1)- DELTA2* YSIGN                                   7034
      IF(Q(NA-1)) 95,95,51                                              7035
   51 Q(NA)= Q(NA-1)                                                    7036
      JA= 3                                                             7037
      GO TO 20                                                          7038
   60 Q(NA)= HYL                                                        7039
      JA= 4                                                             7040
      GO TO 20                                                          7041
   70 B(98)= B(3)- B(4)                                                 7042
      IF(B(98)) 80,80,75                                                7043
   75 B(99)= B(98)                                                      7044
      B(93)= B(3)                                                       7045
      GO TO 50                                                          7046
   80 IF(YSIGN) 210,82,81                                               7047
   81 IF(B(98)+ B(99)) 100,90,90                                        7048
   82 IDENT= 7049                                                       7049
      CALL PRICOM                                                       7050
      STOP                                                              7051
   90 QMIN(NA)= Q(NA-1)                                                 7052
      B(11)= B(3)                                                       7053
      GO TO 200                                                         7054
   95 QMIN(NA)= DELTA2                                                  705431
      B(11)= -0.                                                        705432
      GO TO 200                                                         705433
  100 QMIN(NA)= Q(NA-1)+ DELTA2                                         7055
      B(11)= B(93)                                                      7056
  200 YSIGN= -1.                                                        7057
```

```
      HYL= QH(NA)
      GO TO 5
 210  IF(B(98)+ B(99)) 230,220,220
 220  QMAX(NA)= Q(NA-1)
      B(21)= B(3)
      GO TO 250
 230  QMAX(NA)= Q(NA-1) - DELTA2
      B(21)= B(93)
 250  WRITE (6,9219) JTOT,NA,QMIN(NA),QMAX(NA),QL(NA),QH(NA),B(11),B(21)
C  CAPITAL VALUES AT LAST POINT WHERE NO CHANGE OF RATE OCCURS
      RETURN
 9219 FORMAT ( 80HO**OPTIMUM RATES. CHANGE FROM OUTSIDE QMIN(NA) AND QMA
     2X(NA) TO QL(NA) AND QH(NA)  /1HO,3X,5HJTOT=,I3,3X,3HNA=,I3,17X,
     3F7.2,6X,F7.2,4X,F7.2,4X,F7.2/1HO,3X,28HCORRESPONDING CAPITAL VALUE
     4S  ,4X,F10.2,3X,F10.2,3X, 9HAT LIMITS )
 9220 FORMAT ( 82HO**FIND OPTIMUM LIMITS FOR KEEPING RATE UNCHANGED. QX=
     2QMIN(NA) AND QMAX(NA) RESP.    /)
      END
```

```
      SUBROUTINE PRICOM                                                 9101
C     PRINT KEY VARIABLES IN COMMON OF EXRATE AND SUBROUTINES           9102
      COMMCN QMIN(14), QL(14), QH(14), QMAX(14),QX                     CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX                CMN11
C                                                                       9112
      WRITE(6,9101) IDENT, JTOT, JA, NA, NSA, N, NS, NMAX, NSMAX, M,    9113
     2              JAEND1, JAEND2, JAMAX(1), JAMAX(3), JAMAX(5),       9114
     3              JTOTMX, JTOTED, JC, JT, JTHOR, JM, JMHMIN, I, IPRO, 9115
     4              C1(1), C1(NMAX), C1(14), C2(1), C2(NMAX), C3(1),    9116
     5              C3(NMAX), C4(1), C4(NMAX), C4(14), C5, C6, C7,      9117
     6              C8, C9, B(1), B(2), B(99), BMAX(1), BMAX(3), BMAX(5), 9118
     7              BTOT(1), BTOT(3), BTOT(20), DELTA3                  9119
C                                                                       9120
      WRITE(6,9102) DELTA1, DELTA2, H(1,1), H(NSMAX,1), H(2,2),         9121
     2              H(NSMAX,NMAX), H(20,14), HA(1), HA(2), HA(99),      9122
     3              HAEND1, HAEND2, HAMAX(1), HAMAX(3), HAMAX(5), HB,   9123
     4              HBIN, HEQV(1), HEQV(NMAX), HEQV(14), LT, PAR(1,1),  9124
     5              PAR(70,JTHOR), PAR(70,50), Q(1),  Q(NMAX), Q(14),   9125
     6              QBIN, R, DRES, EXDRES, RES(1,1), RES(NSMAX,1),      9126
     7              RES(2,2),  RES(NSMAX,NMAX), RES(20,15), T(1,1),     9127
     8              T(NSMAX,1), T(2,2), T(NSMAX,NMAX), T(20,15),        9128
     9              T(1,NMAX+1), TB, TE                                 9129
C                                                                       9130
      WRITE (6,9104) QMIN(2), QMIN(NMAX),QMIN(14),QMAX(2),QMAX(NMAX),  91301
     2               QMAX(14),QL(2),QL(NMAX),QL(14),QH(2),QH(NMAX),QH(14) 91302
     3               ,QX,IX                                            91303
      RETURN                                                            9131
 9101 FORMAT (/18H0 -STATE OF COMMON, I6, 17I4, I6, I5, 4I3/ 1H , 7X,   9132
     2       10F10.3/ 1H , 7X, 5F10.3/ 1H , 7X, 9F11.2, F7.3)           9133
C                                                                       9134
 9102 FORMAT (1H , 7X, 12F8.4/ 1H , 7X, 8F8.4, I4, F10.3, 2F6.1/ 1H ,   9135
     2       7X, 4F8.2, F7.3, 7F9.2/ 1H , 7X, 8F8.2)                    9136
C                                                                       9137
 9104 FORMAT (8X,13F8.2,I3)                                            91371
      END                                                               9138
```

```
      SUBROUTINE PRICAP                                                 9201
C     PRINT TABLE OF CAPITAL VALUES                                     9202
      COMMON QMIN(14), QL(14), QH(14), QMAX(14),QX                      CMN4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2      T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3      C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4      HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5      Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6      NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,     CMN10
     7      JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT, IX                  CMN11
    1 KB=9                                                              9212
      IF (JAEND2-KB)2,3,3                                               9213
    2 KB=JAEND2                                                         9213
    3 WRITE (6,9202) JTOT, JM, NA, NSA, (HA(JA), JA=1,KB)               9214
      WRITE (6,9203) (B(JA), JA=1,KB)                                   9215
      IF(JAEND2-9) 11,11,8                                              92155
    8 DO 10 KC=1,9                                                      9216
         KA=10*KC                                                       921651
         KB=KA+9                                                        921652
         IF (JAEND2-KB) 4,5,5                                           9217
    4    KB=JAEND2                                                      9218
    5    WRITE (6,9204) KC, (HA(JA), JA=KA,KB)                          9219
         WRITE (6,9205) KC, (B(JA),JA=KA,KB   )                         9220
         IF (JAEND2-KB) 11,11,10                                        9221
   10 CONTINUE                                                          9222
   11 RETURN                                                            9223
 9202 FORMAT (1H ,3X,5HJTOT=,I3, 3X, 3HJM=,I2, 3X, 3HNA=,I3, 3X,        9224
     2      4HNSA=,I3/ 1H0, 16X, 3HJA=, 6X, 2H*0, 8X, 2H*1, 8X, 2H*2,   9225
     3      8X, 2H*3, 8X, 2H*4, 8X, 2H*5, 8X, 2H*6, 8X, 2H*7, 8X, 2H*8, 9226
     4      8X, 2H*9/ 11H0   HA(JA)=, 4X, 5H*= 0 ,10X,9F10.2)           9227
 9203 FORMAT (10H    B(JA)=, 5X, 5H*= 0 , 10X, 9F10.2)                  9228
 9204 FORMAT (11H0   HA(JA)=, 4X, 2H*=,I2, 1X, 10F10.2)                 9229
 9205 FORMAT (10H    B(JA)=, 5X, 2H*=,I2, 1X, 10F10.2)                  9230
      END                                                               9231
```

```
C       PROGRAM CAPVAL
C       COMPUTE CAPITAL VALUE OF MINE
C   VERSION 2, 15.8.68
C   DATA CARDS OF CUTOFF AND EXRATE EQUALLY USABLE
        COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,        CMN5
       2        T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,     CMN6
       3        C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),         CMN7
       4        HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),         CMN8
       5        Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,        CMN9
       6        NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,       CMN10
       7        JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                        CMN11
C   INSERT IPRO= 0 FOR ANNUAL OUTPUT OR IPRO= 1 FOR MINIMUM OUTPUT
        IPRO= 1
        JTOTED= 1
        JTOT= 0
  101   CALL VALGRD
        JTOT= JTOT+ 1
        GO TO (1,2,3,4,5,6,7,8,9,10), JTOT
C   INSERT DATA + GO TO 101 CARDS OR CONTINUE CARDS BELOW
C   INSERT JTOTED= 2  BEFORE  GO TO 101  CARD TO SUPPRESS TABLE OF
C   PAR(JD,LT). TO REOPEN, INSERT JTOTED= 1  BEFORE ACTUAL GO TO 101 CARD.
    1   CONTINUE
    2   CONTINUE
    3   CONTINUE
    4   CONTINUE
    5   CONTINUE
    6   CONTINUE
    7   CONTINUE
    8   CONTINUE
    9   CONTINUE
   10   STOP
        END
```

SUBROUTINE CUT1
CUT11
CUT5
ANPAY1
ANPAY2
ANPAY3
ANPAY4

These subroutines are identically the same in CAPVAL and CUTOFF. Therefore, see CUTOFF.

```
      SUBROUTINE VALGRD                                                 1.0
C     COMPUTE CAPITAL VALUE OF MINE
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,   CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                   CMN11
      IF(JTOT) 1,2,4                                                   1150
    1 IDENT= 1151                                                      1151
      CALL PRICOM                                                      1152
      STOP                                                             1153
C  READ INPUT DATA
    2 READ (5,9020) NMAX, NSMAX, JTHOR, DELTA1, DELTA2, DELTA3, HAEND1,  12.0
     2              HAEND2, HBIN, JTOTMX, M, QBIN, R                   13.0
      NMAXD=NMAX+1                                                     14.0
      DO 21  NS=1,NSMAX                                                15.0
   21    READ (5,9021) (RES(NS,N), N=1,NMAXD)                          16.0
      READ (5,9022) (HEQV(N), N=1,NMAX)                                17.0
      READ (5,9023) (Q(N), N=1,NMAX)                                   18.0
      DO 22 NS=1,NSMAX                                                 19.0
   22    READ (5,9022) (H(NS,N), N=1,NMAX)                             20.0
      READ (5,9024)  (C1(N), N=1,NMAX)                                 21.0
      READ (5,9024)  (C2(N), N=1,NMAX)                                 22.0
      READ (5,9024)  (C3(N), N=1,NMAX)                                 23.0
      READ (5,9024)  (C4(N), N=1,NMAX)                                 24.0
      READ (5,9024)  C5, C6, C7, C8,C9                                 25.0
      DO 23 LT=1,JTHOR                                                 26.0
   23    READ (5,9025) (PAR(JD,LT), JD=1,70)                           27.0
C  WRITE INPUT DATA                                                    28.0
    4 WRITE (6,9030) NMAX,NSMAX,JTHOR, DELTA1, DELTA2, DELTA3, HAEND1,  29.0
     2               HAEND2, HBIN, JTOTMX, M, QBIN, R                  30.0
      WRITE (6,9031)                                                   31.0
      WRITE (6,9090)                                                   32.0
      DO 31 NS=1,NSMAX                                                 33.0
   31    WRITE (6,9032) NS,(RES(NS,N), N=1,NMAXD)                      34.0
      WRITE (6,9033)                                                   35.0
      WRITE (6,9090)                                                   36.0
      WRITE (6,9034) (HEQV(N), N=1,NMAX)                               37.0
      WRITE (6,9035)                                                   38.0
      WRITE (6,9090)                                                   39.0
      WRITE (6,9036) (Q(N), N=1,NMAX)                                  40.0
      WRITE (6,9037)                                                   41.0
      WRITE (6,9090)                                                   42.0
      DO 32 NS=1,NSMAX                                                 43.0
   32    WRITE (6,9038) NS,(H(NS,N), N=1,NMAX)                         44.0
      WRITE (6,9039)                                                   45.0
      WRITE (6,9041)  (C1(N), N=1,NMAX)                                46.0
      WRITE (6,9042)  (C2(N), N=1,NMAX)                                47.0
      WRITE (6,9043)  (C3(N), N=1,NMAX)                                48.0
      WRITE (6,9044) (C4(N), N=1,NMAX)                                 49.0
      WRITE (6,9045)  C5, C6, C7, C8, C9                               50.0
      GO TO (5,46), JTOTED                                             51.0
    5 KA=-4                                                            52.0
      KB=0                                                             53.0
   41 KA=KA+5                                                          54.0
      KB=KB+5                                                          55.0
```

```
      IF(JTHOR-KB) 42,43,43
   42 KB=JTHOR
   43 WRITE (6,9046)
      DO 45 JT=KA,KB
         WRITE (6,9047) JT, (PAR(JD,JT), JD=1,9)
         DC 44 KC=1,6
            KD=10*KC
            KE=KD+9
   44       WRITE (6,9048) KC,(PAR(JD,JT), JD=KD,KE)
   45    WRITE (6,9049) PAR(70,JT)
      IF(JTHOR-KB) 46,46,41
   46 WRITE (6,9200)
      JM=0
      JA=1
      HA(JA)= H(1,1)
      T(1,1)=1.00
    3 NA=1
      NSA=1
      JT=-1
   10 CALL CUT1
C     COMPUTE AND PRINT CAP VAL OF INITIAL GUESS
   95 JA=1
      CALL CUT5
      WRITE(6,9218) B(JA)
 9020 FORMAT (2I3, I4, 2F7.4, F6.3, 3F7.4, 2I3, F7.2, F6.3)
 9021 FORMAT (11F7.2/4F7.2)
 9022 FORMAT (11F7.4/3F7.4)
 9023 FORMAT (11F7.2/3F7.2)
 9024 FORMAT (9F8.3/5F8.3)
 9025 FORMAT (8(8F9.3/), 6F9.3)
 9030 FORMAT (32H1  CAPITAL VALUE OF MINE          //13H  *INPUT DATA/
    2          11H  *   NMAX=, I3, 9H   NSMAX=, I3, 9H   JTHOR=, I4/
    3          13H  *   DELTA1=, F7.4,
    4          10H   DELTA2=, F7.4,10H   DELTA3=, F6.3, 10H   HAEND1=,
    5          F7.4, 10H   HAEND2=, F7.4, 8H   HBIN=, F7.4,
    6          10H   JTOTMX=,I3/ 8H  *   M=, I3, 8H   QBIN=, F7.2,
    7          5H   R=,F6.3)
 9031 FORMAT (/68H0 *EQUIV CUMUL. ORE RESERVE MT TO BE EXTR. BEFORE (NS,
    2N), RES(NS,N)  )
 9032 FORMAT (1H ,11X, I2, 1X, 15F7.2)
 9033 FORMAT (/34H0 *GRADE OF EQUIV RESERVE, HEQV(N))
 9034 FORMAT (15H    1... NSMAX ,14F7.4)
 9035 FORMAT (/29H0 *ANNUAL PRODUCTION MT, Q(N))
 9036 FORMAT (15H    1... NSMAX , 14F7.2)
 9037 FORMAT (41H1 *INITIAL CUT-OFF (MEAN) GRADES H(NS,N)   )
 9038 FORMAT (1H ,11X, I2, 1X, 14F7.4)
 9039 FORMAT (/36H0 *ORE RESERVE CONVERTING PARAMETERS/
    2        28H   1. DEPEND OF NIV, C-(N)  / 8H0      N=,4X,1H1,7X,1H2,
    3        7X,1H3,7X,1H4,7X,1H5,7X,1H6,7X,1H7,7X,1H8,7X,1H9,6X,2H10,
    4        6X,2H11, 6X,2H12,6X,2H13,6X,2H14)
 9041 FORMAT (8H  C1(N)=,14F8.3)
 9042 FORMAT (8H  C2(N)=,14F8.3)
 9043 FORMAT (8H  C3(N)=,14F8.3)
 9044 FORMAT (8H  C4(N)=,14F8.3)
 9045 FORMAT (24H0   2. INDEP. OF NIV, C-/1H0,7X,3HC5=,F9.3,3X,3HC6=,
    2        F9.3,3X,3HC7=,F9.3,3X,3HC8=,F9.3,3X,3HC9=,F9.3)
 9046 FORMAT (49H1 *ANNUALLY CHANGING ECONOMIC PARAMETERS, PAR(JD))
 9047 FORMAT (1H0,16X, 3HJD=,6X,2H*C,8X, 2H*1, 8X, 2H*2, 8X, 2H*3,
```

```
    2       8X, 2H*4, 8X, 2H*5, 8X, 2H*6, 8X, 2H*7, 8X, 2H*8, 8X, 2H*9/
    3       9H    YEAR ,I3, 3X, 5H*= 0 , 10X, 9F10.3)
 9048 FORMAT (1H , 17X, I1, 1X, 10F10.3 )
 9049 FORMAT (1H , 17X, 2H7 ,F10.3)
 9090 FORMAT (1H0, 11X, 8HNIV N= 1, 6X, 1H2, 6X, 1H3, 6X, 1H4, 6X, 1H5,
    2       6X, 1H6, 6X, 1H7, 6X, 1H8, 6X, 1H9, 5X, 2H10, 5X, 2H11,
    3       5X, 2H12, 5X, 2H13, 5X, 2H14, 4X, 4H(15)/ 14H    SUBNIV NS=
    4       )
 9200 FORMAT (14H1**OUTPUT DATA, 10X, 3HAND/ 12H  -TEST DATA)
 9218 FORMAT (34H0**CAPITAL VALUE OF MINE           ,F10.2)
      END
```

```
C      PROGRAM PAYMTS
C      COMPUTING AND PRINTING PAYMENTS FROM ANPAY1-4
C   VERSION 3, 26.8.68
C   HNORM. MAKE RELATION (MINIMUM GRADE)/HNORM/(MAXIMUM GRADE) 3/6/8
C   QNORM. MAKE RELATION (MINIMUM RATE)/QNORM/(MAXIMUM RATE) 1/10/50
C   RNORM. CHOOSE GENERALLY 5*DRESET
C   DRESET.MEAN NIV. SIZE (EQUIV.) OR, ALTERNATIVELY, 1/10 OF MINE SIZE
C   JTHOR. CHOOSE GENERALLY 3 AND INSERT PAR DATA FOR 3 OPTIONAL YEARS
C   NMAX. (MINE SIZE, EQUIV.)/DRESET. ORDINARILY= (NR OF NIVS.)= 10
C   I= 1,2,3 OR 4, MEANING ANALYSING SUBROUTINES ANPAY1, 2+3, 4 OR 1-4
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT
      COMPLEX*16 TEXT(20)
      DATA TEXT/
     1'RATE, MT/YR      ',
     2'GRADE            ',
     3'CUM. RESERVE, MT',
     4'RATE CHGE, MT/YR',
     5'NIV. SIZE, MT    ',
     6'GRADE CHANGE     ',
     7'ORE RESERVE, MT  ',
     8'                 ',
     9'                 ',
     1'SALES, MKR/YR    ',
     1'OP. PMTS, MKR/YR',
     2'REINVEST.,MKR/YR',
     3'CURR. NET,MKR/YR',
     4'NIV. INVEST.,MKR',
     5'CAPCITY INV.,MKR',
     6'GRDECHGEINV.,MKR',
     7'GRDCH SAVED, MKR',
     8'CLOSE DOWN, MKR  ',
     9'                 ',
     1'                 '/
      READ(5,9310)
     2                HNORM,QNORM,RNORM,DRESET,JTHOR,NMAX,I
      DO 23 LT=1,JTHOR
   23    READ (5,9025) (PAR(JD,LT), JD=1,70)
      WRITE(6,9315)
      WRITE(6,9311)
     2                HNORM, QNORM, RNORM, DRESET, JTHOR, NMAX
      KA=-4
      KB=0
   41 KA=KA+5
      KB=KB+5
      IF(JTHOR-KB) 42,43,43
   42 KB=JTHOR
   43 WRITE (6,9046)
      DO 45 JT=KA,KB
      KT= PAR(70,JT)
      WRITE (6,9047) KT, (PAR(JD,JT), JD=1,9)
         DO 44 KC=1,6
            KD=10*KC
```

```
            KE=KD+9
 44         WRITE (6,9048) KC,(PAR(JD,JT), JD=KD,KE)
 45   WRITE (6,9049) PAR(70,JT), JT
      IF(JTHOR-KB) 46,46,41
 46   CONTINUE
      N= 2
      NS= 1
      H(NS,N)= HNORM
      HB= HNORM
      Q(N)= QNORM
      Q(N-1)= QNORM
      RES(1,N+1)= RNORM
      RES(1,N)= RNORM- DRESET
      TE=21.0
      TB=1.0
      GO TO (101,201,401,101),I
101   LT=2
 10   JT=LT
      Q(N)= Q(N)/10.
      IPRO=20
      DO 20 JA= 1,50
      CALL ANPAY1(SSKG)
      PAR(JA,10)= Q(N)
 20   Q(N)= Q(N)+ QNORM/10.
      Q(N)= QNORM
      H(NS,N)=H(NS,N)/2.0
      IPRO= 24
      DO 30 JA= 1,50
      CALL ANPAY1(SSKG)
      PAR(JA,11)= H(NS,N)
 30   H(NS,N)= H(NS,N)+ HNORM/60.
      H(NS,N)= HNORM
      RES(1,N+1)= DRESET
      RES(1,N)=0.0
      IPRO=28
      DO 40 JA=1,50
      CALL ANPAY1(SSKG)
      PAR(JA,12)= RES(1,N+1)
      RES(1,N+1)= RES(1,N+1)+ DRESET/4.
 40   RES(1,N)= RES(1,N)+ DRESET/4.
      RES(1,N+1)= RNORM
      RES(1,N)= RNORM- DRESET
      KT= PAR(70,JT)
      KB= 16
      DO 142 JEKB=10,13
      DO 141 JEKA=1,3
      KB= KB+4
      KA= JEKA+9
      GO TO (144,145,146),JEKA
144   KC= 20
      GO TO 143
145   KC= 31
      GO TO 143
146   KC=17
143   WRITE (6,9321) TEXT(JEKB),TEXT(JEKA),KT,TEXT(JEKB),TEXT(JEKB),
     2                  TEXT(JEKB),TEXT(JEKA)
141   CALL CURVE (KA,KB,KC)
142   KB= KB-11
```

```
      GO TO (50,51), LT
      GO TO 52
 50   LT= 2
      GO TO 10
 51   LT=JTHOR
      GO TO 10
 52   GO TO (452,452,452,201),I
201   LT= 1
210   JT=LT
      Q(N)= Q(N)/10.
      Q(N-1)=Q(N)
      IPRO= 32
      DO 220 JA=1,50
      CALL ANPAY2(SEFHLM)
      PAR(JA,10)= Q(N)
      Q(N)= Q(N)+ QNORM/10.
220   Q(N-1)= Q(N)
      Q(N)= QNORM
      Q(N-1)= QNORM
      HB= HB/2.0
      H(NS,N)= HB
      IPRO= 33
      DO 230 JA=1,50
      CALL ANPAY2(SEFHLM)
      PAR(JA,11)= HB
      HB= HB+ HNORM/60.
230   H(NS,N)=HB
      H(NS,N)= HNORM
      HB= HNORM
      RES(1,N+1)= DRESET
      RES(1,N)=0.0
      IPRO= 34
      DO 240 JA=1,50
      CALL ANPAY2(SEFHLM)
      PAR(JA,12)= RES(1,N+1)
      RES(1,N+1)= RES(1,N+1)+ DRESET/4.
240   RES(1,N)= RES(1,N)+ DRESET/4.
      RES(1,N+1)= RNORM
      RES(1,N)= RNORM- DRESET
      Q(N)= Q(N)/2.
      IPRO= 35
      DO 260 JA=1,41
      CALL ANPAY2(SEFHLM)
      DQ= Q(N) - Q(N-1)
      PAR(JA,13)= DQ
260   Q(N)= Q(N)+ 0.1* QNORM
      Q(N)= Q(N)- 0.1* QNORM
      DO 265 JA= 42,50
      Q(N)= Q(N)+ 0.2* QNORM
      CALL ANPAY2(SEFHLM)
      DQ= Q(N) - Q(N-1)
265   PAR(JA,13)= DQ
      Q(N)= QNORM
      RES(1,N)= RNORM- 0.08* DRESET
      IPRO= 36
      DO 270 JA=1,50
      CALL ANPAY2(SEFHLM)
      DR= RNORM - RES(1,N)
```

```
      PAR(JA,14)= DR
270   RES(1,N)= RES(1,N)- 0.08* DRESET
      RES(1,N)= RNORM- DRESET
      H(NS,N)= HNORM/2.
      IPRO= 40
      DO 280 JA=1,50
      CALL ANPAY3(SLM,SPAR)
      DH= H(NS,N) -HB
      PAR(JA,15)= DH
280   H(NS,N)= H(NS,N)+ HNORM/60.
      H(NS,N)= HNORM
      KT= PAR(70,JT)
      JEKB= 14
      DO 241 JEKA=1,5
      KA= JEKA+ 9
      KB= JEKA+ 31
      GO TO (242,243,244,245,246),JEKA
242   KC= 20
      GO TO 247
243   KC= 31
      GO TO 247
244   KC= 17
      GO TO 247
245   KC= 6
      GO TO 247
246   KC= 13
247   WRITE (6,9321) TEXT(JEKB),TEXT(JEKA),KT,TEXT(JEKB),TEXT(JEKB),
     2                TEXT(JEKB),TEXT(JEKA)
241   CALL CURVE(KA,KB,KC)
      WRITE (6,9321) TEXT(15),TEXT(4),KT,TEXT(15),TEXT(15),TEXT(15),
     2                TEXT(4)
      CALL CURVE(13,37,26)
      JEKA= 6
      KA= 15
      KC= 21
      DO 248 JEKB=16,17
      KB= JEKB+ 24
      WRITE (6,9321) TEXT(JEKB),TEXT(JEKA),KT,TEXT(JEKB),TEXT(JEKB),
     2                TEXT(JEKB),TEXT(JEKA)
248   CALL CURVE(KA,KB,KC)
      GO TO (250,251),LT
      GO TO 252
250   LT= 2
      GO TO 210
251   LT= JTHOR
      GO TO 210
252   GO TO (452,452,452,401),I
401   N=NMAX
      Q(N)= QNORM
      LT=2
      RES(1,NMAX+1)=NMAX * DRESET
410   JT=LT
      Q(N)= Q(N)/10.
      IPRO= 42
      DO 420 JA=1,50
      CALL ANPAY4(HEND)
      PAR(JA,10)= Q(N)
420   Q(N)= Q(N)+ QNORM/10.
```

```
      Q(N)= QNORM
      HB= HB/2.0
      IPRO= 43
      DO 430 JA=1,50
      CALL ANPAY4(HEND)
      PAR(JA,11)= HB
430   HB= HB+ HNORM/60.
      Z= RES(1,NMAX+1)*0.2
      HB= HNORM
      RES(1,NMAX+1)=Z
      IPRO= 44
      DO 440 JA=1,50
      CALL ANPAY4(HEND)
      PAR(JA,16)= RES(1,NMAX+1)
440   RES(1,NMAX+1)= RES(1,NMAX+1)+ Z*0.25
      RES(1,NMAX+1)=NMAX * DRESET
      KT= PAR(70,JT)
      JEKB= 18
      DO 441 KD=1,3
      KB= KD+ 41
      GO TO (442,443,444),KD
442   KA= 10
      KC= 20
      GO TO 445
443   KA= 11
      KC= 31
      GO TO 445
444   KA= 16
      KC= 17
445   JEKA= KA- 9
      WRITE (6,9321) TEXT(JEKB),TEXT(JEKA),KT,TEXT(JEKB),TEXT(JEKB),
     2               TEXT(JEKB),TEXT(JEKA)
441   CALL CURVE(KA,KB,KC)
      GO TO (450,451),LT
      GO TO 452
450   LT= 2
      GO TO 410
451   LT= JTHOR
      GO TO 410
452   STOP
 9025 FORMAT (8(8F9.3/), 6F9.3)
 9046 FORMAT (49H1 *ANNUALLY CHANGING ECONOMIC PARAMETERS, PAR(JD))
 9047 FORMAT (1H0,16X, 3HJD=,6X,2H*0,8X, 2H*1, 8X, 2H*2, 8X, 2H*3,
     2       8X, 2H*4, 8X, 2H*5, 8X, 2H*6, 8X, 2H*7, 8X, 2H*8, 8X, 2H*9/
     3       9H    YEAR ,I3, 3X, 5H*= 0 , 10X, 9F10.3)
 9048 FORMAT (1H , 17X, I1, 1X, 10F10.3 )
9049  FORMAT (1H ,17X,2H7 ,F10.3,33H  THIS YEAR IS REFERRED TO BY JT=,
     2       I3)
9310  FORMAT (F7.4,3F7.2,3I4)
 9311 FORMAT (40H0 *INPUT DATA FOR TEST OF PAYMENT MODELS/
     2       11H0 *  HNORM=,F7.4,3X,6HQNORM=,F7.2,3X,6HRNORM=,F7.2,
     3       3X,7HDRESET=,F7.2,3X,6HJTHOR=,I4,3X,5HNMAX=,I4)
9315  FORMAT (63H1  PAYMENTS ACCORDING TO PAYMENT MODELS IN SUBROUTINES
     2ANPAY1-4 )
9321  FORMAT (17H1**PAYMENT TYPE: ,2A8,5X,13HFUNCTION OF: ,2A8,5X,
     2        15HDATA FOR YEAR: ,I3//
     3        1H0,2A8,5X,2A8,3H...,64X,3H...,2A8/1H ,12X,2A8)
      END
```

```
      SUBROUTINE ANPAY1(SSKG)                                           8101
C     FIND PAYMENTS IN YEAR JT FOR ORE SALES, RUNNING PRODUCTION AND RE-8102
C     INVESTMENTS                                                       8103
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,    CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                     CMN11
      HD=H(NS,N)                                                        8112
      QD=Q(N)                                                           8113
      SS=(PAR(1,LT)-PAR(2,LT)*QD*QD                                     8114
     2 +PAR(3,LT)*(HD-PAR(4,LT))                                        8115
     3 +PAR(5,LT)*(RES(1,N)+RES(1,N+1)))*QD                             8116
      SS=SS*PAR(69,LT)                                                  8117
      SK=PAR (6,LT)+PAR(7,LT)*QD+PAR(8,LT)*EXP(PAR(9,LT)*QD) -PAR(8,LT) 8118
     2 +(PAR(10,LT)*(PAR(4,LT)-HD)+PAR(11,LT)*(-EXP(PAR(12,LT)          8119
     3 *PAR(4,LT))+EXP(PAR(12,LT)*HD)))*(QD+PAR(13,LT))                 8120
     4 +(PAR(14,LT)*EXP(PAR(15,LT)*RES(1,N+1))-PAR(14,LT))*(QD          8121
     5 +PAR(16,LT))                                                     8122
      SK=SK*PAR(68,LT)                                                  8123
      IF (TE-TB) 7,8,9                                                812350
    7 IDENT=812351                                                    812351
      CALL PRICOM                                                     812352
      STOP                                                            812352
    8 SG=0.                                                           812354
      GO TO 2                                                         812355
    9 SG=PAR(17,LT)+PAR(18,LT)*QD                                       8124
     2 +PAR(19,LT)*(HD-PAR(4,LT))*(QD+PAR(20,LT))                       8125
      SG=SG*PAR(67,LT)                                                  8126
      IF (TE-TB-PAR(21,LT)) 1,1,2                                       8127
    1 SG=SG*(T(1,NMAX+1)-TB)/PAR(21,LT)                                 8128
    2 SSKG=SS-SK-SG                                                     8129
C SPECIAL FOR PROGRAM PAYMTS:                                           8130
      PAR(JA,IPRO)= SS                                                  8131
      PAR(JA,IPRO+1)= SK                                                8132
      PAR(JA,IPRO+2)= SG                                                8133
      PAR(JA,IPRO+3)= SSKG                                              8134
      RETURN                                                            8135
      END                                                               8136
```

```
      SUBROUTINE  ANPAY2 (SEFHLM)                                        8201
C     FIND PAYMENTS IN YEAR JT FOR INVESTMENTS EXCEPTING GRADE CHANGE    8202
C     INVESTMENTS                                                        8203
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,     CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,   CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),       CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),       CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,      CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,     CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                      CMN11
      QD=C(N)                                                            8212
      IF (N-1) 1,2,3                                                     8213
    1 IDENT=8214                                                         8214
      CALL PRICOM                                                        8215
      STOP                                                               8216
    2 QB=QBIN                                                            8217
      IF (HBIN) 1,5,9                                                    8218
    5 HB=H(NS,N)                                                         8219
      GO TO 9                                                            8220
    3 QB=Q(N-1)                                                          8221
C   SPECIAL FOR PROGRAM PAYMTS: 822141 - 8222 AND 823041 - 42            822140
    9 AQD= QD                                                            822141
      AQB= QB                                                            822142
      QD= QB                                                             822143
      QB= QD+ AQB- AQD                                                   822144
      SF=PAR(22,LT)+PAR(23,LT)*QD+PAR(24,LT)*EXP(PAR(25,LT)*QD)          8222
     2  +PAR(26,LT)*(QD-QB)*QD                                           8223
     3   +(PAR(27,LT)*(PAR(4,LT)-HB)+PAR(28,LT)*(EXP(PAR(29,LT)*HB)      8224
     4  -EXP(PAR(29,LT)*PAR(4,LT))))*(QD+PAR(30,LT))                     8225
     5  +(PAR(31,LT)*EXP(PAR(32,LT)*RES(1,N+1))-PAR(31,LT))              8226
     6  *(QD+PAR(61,LT))                                                 8227
     7  +(PAR(33,LT)*EXP(PAR(34,LT)*(RES(1,N+1)-RES(1,N)))-PAR(33,LT))   8228
     8  *(QD+PAR(35,LT))*EXP(PAR(36,LT)*(PAR(4,LT)-HB))                  8229
      SE=SE*PAR(66,LT)                                                   8230
      QD= AQD                                                            823041
      QB= AQB                                                            823042
      IF (Q(N)-QB+0.005) 11,10,10                                        82305
   10 IF (Q(N)-QB-0.005) 12,12,13                                        8231
   11 SF=0                                                               8232
      SH=(PAR(42,LT)-PAR(43,LT)*EXP(PAR(44,LT)*(QD-QB))+PAR(43,LT))      8233
     2  *EXP(-PAR(45,LT)*QB)                                             8234
     3  *EXP(PAR(46,LT)*(PAR(4,LT)-HB))                                  8235
      SH=SH*PAR(65,LT)                                                   8236
      GO TO 20                                                           8237
   12 SF=0                                                               8238
      SH=0                                                               8239
      GO TO 20                                                           8240
   13 SF=(PAR(37,LT)-PAR(38,LT)*EXP(PAR(39,LT)*(QB-QD))+PAR(38,LT))      8241
     2  *EXP(-PAR(40,LT)*QB)                                             8242
     3  *EXP(PAR(41,LT)*(PAR(4,LT)-HB))                                  8243
      SF=SF*PAR(65,LT)                                                   8244
      SH=0                                                               8245
   20 CALL ANPAY3 (SLM,SPAR)                                             8246
      SEFHLM=SE+SF+SH+SLM-SPAR                                           8247
C   SPECIAL FOR PROGRAM PAYMTS:                                          8248
      IF (IPRO-35) 24,23,24                                              8249
   23 PAR(JA,35)= SE                                                     8250
      PAR(JA,37)= SF+SH                                                  8251
```

```
      RETURN
24    PAR(JA,IPRO)= SE
      RETURN
      END
```

```
      SUBROUTINE ANPAY3 (SLM,SPAR)                                      8301
C     FIND PAYMENTS IN YEAR JT FOR INVESTMENTS FOR GRADE CHANGE         8302
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,      CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,    CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),        CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),        CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,       CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,     CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                      CMN11
      HD=H(NS,N)                                                        8312
      QD=Q(N)                                                           8313
      IF (NS+N-2) 1,2,4                                                 8314
    1 IDENT=8315                                                        8315
      CALL PRICOM                                                       8316
      STOP                                                              8317
    2 IF (HBIN) 1,3,4                                                   8318
    3 HB=H(NS,N)                                                        8319
    4 SA=QD*EXP(PAR(50,LT) /QD)                                         8321
     2 *EXP(PAR(51,LT)*(PAR(4,LT)-HB))                                  8324
     3 *EXP(PAR(52,LT)*RES(1,N+1))                                      8325
      IF(H(NS,N)-HB+0.00005) 13,6,6                                    83255
    6 IF(H(NS,N)-HB-0.00005) 12,11,11                                   8326
   11 SLM=PAR(47,LT)-PAR(48,LT)*EXP(PAR(49,LT)*(HB-HD))+PAR(48,LT)      8327
      SLM=SLM*SA*PAR(64,LT)                                             8328
      SPAR=PAR(47,LT)*SA*PAR(64,LT)                                     8329
      GO TO 14                                                          8330
   12 SLM=0                                                             8331
      SPAR=0.                                                          83315
      GO TO 14                                                          8332
   13 SLM=PAR(53,LT)+PAR(54,LT)*EXP(PAR(55,LT)*(HB-HD))-PAR(54,LT)      8333
      SLM=SLM*SA*PAR(64,LT)                                             8334
      SPAR=PAR(53,LT)*SA*PAR(64,LT)                                     8335
C  SPECIAL FOR PROGRAM PAYMTS;                                          8336
   14 IF (IPRO-40) 25,23,25                                             8337
   23 PAR(JA,40)= SLM                                                   8338
      PAR(JA,41)= SPAR                                                  8339
   25 RETURN                                                            8340
      END                                                               8341
```

```
      SUBROUTINE ANPAY4 (HEND)                                          8401
C     FIND PAYMENTS IN YEAR JT FOR FINAL CLOSING DOWN OF MINE           8402
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,    CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,  CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),      CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),      CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,     CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,   CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                    CMN11
      QD=Q(NMAX)                                                        8412
      HEND=(PAR(56,LT)+PAR(57,LT)*EXP(PAR(58,LT)*QD)-PAR(57,LT))        8413
     2    *EXP(PAR(59,LT)*(PAR(4,LT)-HB))                               8414
     3 *(PAR(62,LT)+PAR(60,LT) *RES(1,NMAX+1))                          8415
      HEND=HEND*PAR(63,LT)                                              8416
C  SPECIAL FOR PROGRAM PAYMTS:                                          8417
      PAR(JA,IPRO)= HEND                                                8418
      RETURN                                                            8419
      END                                                               8420
```

```
      SUBROUTINE PRICOM
C     DUMMYROUTINE
      WRITE(6,9104)
      RETURN
 9104 FORMAT (21HO**COMMON PRINT DUMMY )
      END
```

```
      SUBROUTINE CURVE (KA,KB,KC)                                        9301
C     DRAWING CURVES FROM TABLES                                         9302
C  KA=INDEX OF INDEPENDENT VARIABLE                                      9303
C  KB=INDEX OF DEPENDENT VARIABLE                                        9304
C  KC=INDEX OF NORMAL VALUE                                              93041
      COMMON B(99), BMAX(5), BTOT(20), DRES, EXDRES, RES(20,15), HB,      CMN5
     2       T(20,15), TB, TE, C1(14), C2(14), C3(14), C4(14), C5, C6,    CMN6
     3       C7, C8, C9, DELTA3, DELTA1, DELTA2, H(20,14), HA(99),        CMN7
     4       HAEND1, HAEND2, HAMAX(5), HBIN, HEQV(14), PAR(70,50),        CMN8
     5       Q(14), QBIN, R, IDENT, JTOT, JA, NA, NSA, N, NS, NMAX,       CMN9
     6       NSMAX, M, JAEND1, JAEND2, JAMAX(5), JTOTMX, JTOTED, JC,      CMN10
     7       JD, JT, JTHOR, JM, JMHMIN, I, IPRO, LT                       CMN11
      DIMENSION A(70,50), AR(102)                                        9312
      EQUIVALENCE (PAR(1,1),A(1,1))                                      9313
      DATA AST/1H*/,ABL/1H /,APL/1H+/                                    9314
      AL=ALOG10(ABS(2.*A(KC,KB)))                                        9315
      KAL=AL                                                             9316
      IF (AL) 30,31,31                                                   93164
   30 KAL=KAL-1                                                          93165
   31 ADL=AL-KAL                                                         9317
      IF (ADL-0.301) 4,2,1                                               9318
    1 IF (ADL-0.699) 2,3,3                                               9319
    2 AM=5.*10.**KAL                                                     9320
      GO TO 5                                                            9321
    3 AM=1C.*10.**KAL                                                    9322
      GO TO 5                                                            9323
    4 AM=2.*10.**KAL                                                     9324
    5 IF(AM-5000000.)6,9,8                                               9325
    6 IF(AM-5.)7,10,9                                                    9326
    7 IF(AM-0.005)8,8,10                                                 9327
    8 DO11KD=1,10                                                        9328
   11 AR(KD)=KD*AM/10.                                                   9329
      WRITE(6,91)(AR(KD),KD=1,10)                                        9330
      GO TO 13                                                           9331
    9 KE=AM/10.+0.1                                                      9332
      KAM=AM+0.1                                                         9333
      WRITE(6,92)(KD,KD=KE,KAM,KE)                                       9334
      GO TO 13                                                           9335
   1C DO12KD=1,10                                                        9336
   12 AR(KD)=KD*AM/10.                                                   9337
      WRITE(6,93)(AR(KD),KD=1,10)                                        9338
   13 WRITE(6,94)                                                        9339
      AR(1)=APL                                                          9340
      AR(101)=APL                                                        9341
      AR(102)=ABL                                                        9342
      DO14KD=2,100                                                       9343
   14 AR(KD)=ABL                                                         9344
      KJ=0                                                               9345
      DO22KF=1,10                                                        9346
      DO17KG=1,4                                                         9347
      KJ=KJ+1                                                            9348
      KD=100.*ABS(A(KJ,KB))/AM+1.5                                       9349
      IF(KD-101)16,16,15                                                 9350
   15 KD=102                                                             9351
   16 ARA=AR(KD)                                                         9352
      AR(KD)=AST                                                         9353
      WRITE(6,95) A(KJ,KB),A(KJ,KA),AR                                   9354
   17 AR(KD)=ARA                                                         9355
```

```
      KJ=KJ+1
      KD=100.*ABS(A(KJ,KB))/AM+1.5
      IF(KD-101)19,19,18
   18 KD=102
   19 ARA=AR(KD)
      DO20KG=11,91,10
   20 AR(KG)=APL
      AR(KD)=AST
      WRITE(6,95) A(KJ,KB),A(KJ,KA),AR
      AR(KD)=ARA
      DO21KG=11,91,10
   21 AR(KG)=ABL
   22 CONTINUE
      WRITE(6,94)
      RETURN
   91 FORMAT(24H0  DEPEND.   INDEP.    00,10E10.3)
   92 FORMAT(24H0  DEPEND.   INDEP.    00,10I10)
   93 FORMAT(24H0  DEPEND.   INDEP.    00,10F10.3)
   94 FORMAT(1H ,21X,1H+,10(10H....*....+))
   95 FORMAT(1H ,E10.3,F10.4,1X,102A1)
      END
```

```
C      PROGRAM NEWPAR
C      ALLOCATING NEW VALUES TO PAR(JD,LT) AND PRODUCING NEW PAR DECK
       DIMENSION PAR(70,50)
C   INSERT OLD HORIZON, JTHOR=   .
       IPRO= 1
       DO 23 LT=1,JTHOR
  23      READ (5,9025) (PAR(JD,LT), JD=1,70)
       KA=-4
       KB=0
  41   KA=KA+5
       KB=KB+5
       IF(JTHOR-KB) 42,43,43
  42   KB=JTHOR
  43   WRITE (6,9046)
       DO 45 JT=KA,KB
          WRITE (6,9047) JT, (PAR(JD,JT), JD=1,9)
          DO 44 KC=1,6
             KD=10*KC
             KE=KD+9
  44         WRITE (6,9048) KC,(PAR(JD,JT), JD=KD,KE)
  45      WRITE (6,9049) PAR(70,JT)
       IF(JTHOR-KB) 46,46,41
  46   GO TO (47,48), IPRO
  47   CONTINUE
C   INSERT NEW HORIZON (MIN=2, MAX=50) AND REQUIRED NEW PAR VALUES
       IPRO= 2
       KA= -4
       KB= 0
       GO TO 41
  48   WRITE (6,9028)
       DO 50 LT= 1,JTHOR
       DO 49 KB= 8, 64, 8
       KA= KB- 7
       WRITE (6,9026) ((PAR(JD,LT), JD=KA,KB),KB,LT)
  49   WRITE (7,9026) ((PAR(JD,LT), JD=KA,KB),KB,LT)
       WRITE (6,9027) ((PAR(JD,LT), JD=65,70),LT)
  50   WRITE (7,9027) ((PAR(JD,LT), JD=65,70),LT)
       STOP
 9025  FORMAT (8(8F9.3/), 6F9.3)
 9026  FORMAT (8F9.3,4X,2I2)
 9027  FORMAT (6F9.3,22X,2H70,I2)
 9028 FORMAT (17H1**NEW DATA CARDS)
 9046 FORMAT (49H1 *ANNUALLY CHANGING ECONOMIC PARAMETERS, PAR(JD))
 9047 FORMAT (1H0,16X, 3HJD=,6X,2H*0,8X, 2H*1, 8X, 2H*2, 8X, 2H*3,
     2        8X, 2H*4, 8X, 2H*5, 8X, 2H*6, 8X, 2H*7, 8X, 2H*8, 8X, 2H*9/
     3        9H    YEAR ,I3, 3X, 5H*= 0 , 10X, 9F10.3)
 9048 FORMAT (1H , 17X, I1, 1X, 10F10.3 )
 9049 FORMAT (1H , 17X, 2H7 ,F10.3)
      END
```

Appendix F

Selected symbols

Symbols used in the computer programs, Appendix B, and Appendix E are listed in Appendix C.

A	Data horizon.
$\bar{A}$	Intermediate variable defined in section 48 of Appendix D.
a	Calendar year within the data horizon. a=1,2,...,A
B_n	Capital value (present value) at time T_n of future mining.
$B_{n'n}$	Capital value at time $T_{n'n}$ of future mining.
$B_{n'n0}$	Capital value of future mining at time $T_{n'n}$ discounted to time 0.
B'_n	Capital value at time T_n of zone n.
$B'_{n'n}$	Capital value at time $T_{n'n}$ of subzone n'n.
$B'_{n'n0}$	Capital value of subzone n'n discounted to time 0.
$b(t)$	End of the calendar year during which time t occurs.
C_i	Technical coefficients relevant for all zones. i=5,6,7,8,9.
C_{in}	Technical coefficients relevant for zone n. i=1,2,3,4.
c_{ia}	Coefficients of the payment functions relevant for year a. i=1,2,...,70.
E_{a1n}	Zone investment in zone n at time T_{1n}.
e	Base of natural logarithm.
F_{a1n}	Expansion investment at time T_{1n}.
$f(x)$	Function of x.
$G_{an'n}$	Current reinvestments (annual amount) according to payment function for year a during the production period of subzone n'n.

H_{a1n}	Contraction investment at time T_{1n}.
$H_{a,1,N+1}$	Close-down payments at time $T_{1,N+1}$.
$\bar{h}_{n'n}$	Average grade of the ore mined in subzone n'n.
$\bar{h}_{00}$	Average grade of ore mined immediately before starting in zone 1.
$\bar{\bar{h}}_n$	Equivalent average grade in zone n.
i	General subscript.
j	Continuous rate of interest.
$K_{an'n}$	Payments for current operating costs (annual amount) according to payment function for year a during the production period of subzone n'n.
$L_{an'n}$	Grade-change investment (increased average grade) at time $T_{n'n}$.
$M_{an'n}$	Grade-change investment (decreased average grade) at time $T_{n'n}$.
N	Number of zones in an ore deposit.
n	Subscript of zones. n=1,2,...,N. (Also N+1 in certain cases.)
N'	Number of subzones in each zone.
n'	Subscript of subzones. n'=1,2,...,N'. n'n denotes subzone n'n, i.e. subzone n' of zone n.
Q_n	Rate of production (annual amount) in zone n.
Q_0	Rate of production immediately before starting in zone 1.
$Q^*(t)$	Maximum rate of production as a function of time t, which is reached if the maximum rate of expansion is fully utilized.
$R_{n'n}$	Ore reserve (actual ore reserve) in subzone n'n.
$\mathbf{R}_n$	Equivalent ore reserve in zone n.
$\mathbf{R}_{n'n}$	Equivalent ore reserve in subzone n'n.
r	Discontinuous rate of interest.
$S_{an'n}$	Payments received for products sold (annual amount) according to payment function for year a during the production period of subzone n'n.

$T_n = T_{1n}$	Starting time of zone n.
$T_{n'n}$	Starting time of subzone n'n.
$T_{1,N+1}$	End of the production period of an ore deposit, i.e. the time of the final closing of a mine.
t	Time.
$\alpha, \beta, \gamma, \varrho$	Defined in the context.
τ_n	Production period of zone n.
$\tau_{n'n}$	Production period of subzone n'n.

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1) Published in English

2) With a summary in English

Nils-Erik Norén
Long-range Decision Models in Mining

<u>E r r a t a</u>

<u>p. 83</u>: Expression (3.1) is more easily understood if written

$$\underset{Q_{n\ldots N}}{\text{Maximum}}\ B_n = \underset{Q_{n\ldots N}}{\text{Maximum}}(B'_n + \underset{Q_{n+1\ldots N}}{\text{Maximum}}\ B_{n+1} \cdot e^{-j \cdot T_n}) \qquad (3.1)$$

The expressions (3.4) (p. 87), (4.3) (p. 116), and (4.4) (p. 117) may be rewritten correspondingly.

<u>p. 84, lines 11 to 16</u> are replaced by:

...

first period. The restriction in the principle of optimality has been introduced in order to permit a certain simplification in the payment functions without simultaneously complicating the optimization procedure. The restriction is said to be effective if it exerts influence on the optimum obtained. It exerts influence only where the optimum decision for a period n, as determined at the beginning of period n, differs from the optimum decision for period n in conjunction with decisions for preceding periods, the combined optimum being determined at the beginning of a period preceding n. The optima will differ only if the assumption is abandoned, that B'_n is independent of Q_i for $i=n+1,n+2,\ldots,N$. An example of the simplification in question will be given.[1] It is obtained from the payment model of section 44 in Appendix D, which represents the only case in this study where the restriction is effective (through $T_{1,N+1}$, i.e. T_{N+1}). However, if the remaining production period (through $T_{1,N+1}$) is inserted into other payment functions as a variable, this will cause further cases. An optimization regarding

...

<u>p. 86, lines 9 to 18</u> are replaced by:

...

model in order to avoid complications in the calculations. A sensitivity analysis will show whether the error introduced influences the optimum significantly. In that case some alternative approximation will have to be found. For example, the reinvestments may be formally assumed independent of the remaining production period, and the decrease in the reinvestments during the last years of the production period may be treated as a part of the close-down payments. This part should equal the difference between the formally assumed reinvestments and the actually assumed decreasing reinvestments, the difference being discounted to the end of the production period, and accumulated over time.

...

<u>p. 142, line 11</u>: "The contraction limit" is replaced by "The expansion limit"

<u>p. 188, line 15</u> is replaced by:

...

is optimized as if independent, but with the capital value of all deposits together as the optimization criterion. A way of achieving this is to let the payment functions of the deposit which is currently being optimized, include the effect of the value of the current decision variable upon payments pertaining to the other deposits. The payment functions should be determined for given

...

<u>p. 245, line 32, and p. 256, line 11</u>: "JTHOR $\geq$ 3" is replaced by "3 $\leq$ JTHOR $<$ 10".

6.5.69