

Hedging of Contracts, Anticipated Positions, and Tender Offers



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Hedging of Contracts, Anticipated Positions, and Tender Offers

A Study of Corporate Foreign Exchange
Rate Risk and/or Price Risk

Catharina Lagerstam



A Dissertation for the
Doctor's Degree in Business Administration
Stockholm School of Economics 1990

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ISBN 91-7258-306-1

Keywords:

Risk Management
Hedging
Options
Currency Options
Tender Offers
Foreign Exchange

Gotab, Stockholm 1990

Acknowledgements

I am greatly indebted to Professor Bertil Näslund of the Stockholm School of Economics for all the support, comprehensive ideas and knowledgeable comments provided when writing this thesis. Despite a heavy workload, Professor Näslund always managed to find the time to discuss any problems arising in this research.

I am also indebted to Professor Gunnar Hedlund and to Professor Anders Westlund, members of the PhD committee at the Stockholm School of Economics, for providing valuable comments in their respective fields.

Furthermore, I am very grateful towards Mr Peter Carlsson, Managing Director, and Mr Per Holmertz, Manager of Trading, at ABB Treasury Center (Sweden) AB, subsidiary of ABB Asea Brown Boveri Ltd. They have both shown a great insight into the problem of hedging, and have always unselfishly made themselves available for discussions. I am especially grateful to Mr Carlsson, who ensured that all information at ABB required during the research was made available. I am also thankful to the innumerable people at ABB who have taken their time to discuss the practical aspects of hedging in business. The experience acquired at ABB proved invaluable for this thesis.

The three-dimensional figures and the figures based on simulation are produced by Mr Per Johansson, a PhD-student at the Department of Statistics at the Stockholm School of Economics. I am very grateful for and impressed by the skill and the speed of his assistance.

I am also very grateful towards all colleagues at the Department of Finance at the Stockholm School of Economics for their unflagging support, shared knowledge, and willingness to discuss any part of the research. A special thanks is owed to Mr Anders Warne. Thanks also to Mrs Kerstin Lindskog for providing everything from administrative assistance to good company during the late evenings, weekends and other holidays used for writing this paper.

Thanks also to the Members of the Professional Communication Skills Unit at the Stockholm School of Economics for their excellent and impressively speedy assistance which went far beyond duty.

The generous financial support from the Humanistisk-Samhälls-
vetenskapliga Forskningsrådet (HSFR) is gratefully acknowledged.

Finally, I am very grateful to my parents for all the support given - both financially, domestically and otherwise - during the years of studies and research leading to this thesis.

I remain solely responsible for any faults/omissions that may occur.

Stockholm, July 5, 1990

Catharina Lagerstam

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Glossary of Symbols

$P_{1,FW}$	The price at time FW of one unit of the domestic risky asset number one. (One or both of the indices may be suppressed.) $p_{2,x}$ denotes the exercise price of an option on domestic risky asset number two.
e	The direct exchange rate (domestic currency per one unit of foreign currency, DC/FC).
g	The price of a gathered asset.
a	The generic variable for p , e , or g .
$B_D[t_0, t_B]$	The value at time t_0 of a risk free bond with the face value of one unit of the domestic currency (D) to be paid at time t_B .
$F_a[t_0, t_F]$	The futures price of a futures contract on one unit of asset 'a', initiated at time t_0 with time of exercise t_F . ($F_x[t_F]$ denotes the futures price of the futures contract received upon exercising an option on a futures contract.)
$FW_a[t_0, t_{FW}]$	The forward price of a forward contract, initiated at time t_0 and exercisable at t_{FW} . ($FW_x[t_{FW}]$ denotes the forward price of the forward contract received upon exercising an option on a forward contract.)
$C_a[t_C]$	The initial value of a call option for one unit of the underlying asset 'a'. t_C denotes the date of expiration of the option. t_C is often substituted for the more general t_x . a_x denotes the exercise price.
$P_a[t_p]$	The initial value of a put option on one unit of asset 'a'. (See C_a for the indices.)
$VA[·;t]$	The value at time t of a prespecified amount of assets or of a strategy.
$CF[·;t]$	The cash flow at time t from a strategy or position.

$dV_{a,B}[t]$	The value of a portfolio at time t containing the risky asset 'a' and bonds. d denotes a desired position after a potential reallocation. (No subscript 'd' implies the de facto value prior to a reallocation.)
$COST[t]$	The hypothetical, total cost at time t of assuring the ownership of the total amount of risky assets in the position.
$REMCOST[t]$	The cost of a hypothetical purchase to eliminate the remaining short position.
$ACCCOST[t]$	The accumulated cost from previous purchases deducted by proceeds from sales.
$FL[t]$	The floor at time t .
$CL[t]$	The ceiling at time t .
T_a	The time left until expiration for asset a, $T_a = t_a - t_0$.
t	The prevailing time, general.
ϵ	A short period of time.
Φ	The probability, estimated at time t_0 , that the anticipatory position will realize.
Θ	The probability, estimated at time t_0 , that the tender offer will be accepted.
Pn_i	The number of domestic assets (p) number 'i' in the position. The sign of the position is measured prior to the final transaction. $n < 0$ implies a short position in the risky asset, and $n > 0$ a long position in the risky asset.
q_a	The number of units of a specific, predefined asset 'a'.
$Ph_i[FW]$	The fraction of domestic assets (p) number 'i' to be hedged by using a forward contract. The superscript is suppressed when the hedging fraction concerns q_a .
i	The inflation rate.
r_D	The continuously compounded domestic (D) risk-free interest rate.
R_F	The discretely compounded foreign (F) risk-free interest rate.
m	The multiplier in a leveraged position.

b The parameter spanning the domain of integration.
X The stochastic variable in the pricing equation.
 $X \sim N[0,1]$.
std[·] or σ_a The standard deviation. σ_a is used as a short notation for the standard deviation of the continuously compounded rate of return of asset 'a' per one unit of time.
var[·] or σ_a^2 The variance.
cov[·,·] The covariance.
corr[·,·] or τ The correlation coefficient. τ is used as a short notation when no doubt about the stochastic variables implied may arise.
V The covariance matrix.
 μ The drift.
 μ' The matrix containing the drift parameters.
N[] The standard cumulative normal distribution function.
 **x_F
g** The parameter of the cumulative normal distribution function. The subscript denotes the object.
E[·;t] The expectations operator.
prob[] The probability of an event occurring.
|· Conditional on ·.
max[·,·] The maximum of the two expressions.
min[·,·] The minimum of the two expressions.
|·| The determinant of ·.
J The Jacobian.
Z' The matrix containing natural logarithms.
 δ A change in a variable. δ is also used to denote a derivation, e.g. $\delta C/\delta p$.
e The base for natural logarithms.
ln[] The natural logarithm.
exp[] The natural antilogarithm.

CP	Contractual position
RAP	Realized anticipatory position.
NRAP	Non-realized anticipatory position
EAP	Expected outcome of anticipatory position
ATO	Accepted tender offer
RTO	Refused tender offer
ETO	Expected outcome of tender offer
B&H	The buy-and-hold strategy.
CPPI	The constant proportion portfolio insurance.
TIPP	The time-invariant portfolio protection.
SL	The stop-loss hedging strategy.
[]	Specifies parameters of a function.
()	'Normal' brackets.
{ }	'Normal' brackets or inserted explanations.
√	Square root. Only the first factor or the first expression within brackets is operated upon.

Subscripts

D	domestic
F	foreign
C	call option
P	put option
FW	forward
F	futures
p	domestic risky asset
e	currency (possibly invested in foreign bonds)
g	gathered asset
B	bonds
0,s,t,l	specific points in time (zero, short, target, long), $t_0 < t_s < t_t < t_l$, t_0 = time of initiation.
0,1,2	a type index
x	exercise price of option, or end of relevant time period
d	desired position, denoting the position after reallocation at one point in time
diff	difference between two values
CRP	end of contractual risk period
TORP	end of tender offer risk period

Superscript

'	a related, but not identical object or a changed interest rate
p,e,g	the sort of asset implied

1 Introduction and Background

1.1 Introduction

An organization involved in transactions is susceptible to the risk incurred by stochastic prices, measured in the domestic currency unit. The random characteristic stems from a stochastic domestic price or from a stochastic foreign exchange rate possibly combined with a locally stochastic price. The risk may be reduced through initiation of hedging activities.

Today, there is no enveloping method on the market to guide the actors in the choice among the abundance of hedging methods and in the decision of hedging level. This dissertation aims at trying to provide such a decision support method.

The method developed must be easily applicable, time efficient and general in order to be implementable on the market. Hence, the method will not be based on any arcane utility function. The omission of utility functions implies that the actor himself must choose the optimal hedge. The fundamental idea of the method developed is to provide the actor with the probability distribution of the outcome of the total position, conditional on any combination of hedging vehicles chosen. Thus, by analyzing and by possibly altering the hedging mix, the actor may mould a risk profile of the total position that satisfies his disposition.

1.2 A Short History

1.2.1 The Foreign Exchange Market

World Wide¹

Gold coins were used in Europe as a means of payment in Roman times, but were not used again until the thirteenth century. (The first, the genoin of Genoa, was minted in 1252.) As long as the currencies were made out of an unambiguous asset - gold, silver,

¹ Notes appear at the end of each chapter.

or copper for instance - the exchange rate simply consisted of the difference in weight of the coins. Bank notes were introduced in 1661 in Sweden. Bills of exchange implied a fixed conversion rate to the underlying asset in the beginning, i.e. to gold. The pound sterling was fixed in 1717 at a gold standard² and was not finally abandoned until 1931. (Silver was also used, so called bimetallism.) The French relied mainly on the silver standard.

In 1944, the Bretton Woods agreement³ stabilized the exchange rates. The US dollar was pegged to the price of gold, USD 35/ounce⁴. The dollar and the British pound would serve as reserve currencies, although in practice it turned out that only the former did so. The other currencies were pegged to the dollar within a limit of $\pm 1\%$ of the par value, partly by the help of loans from the International Monetary Fund (IMF). Furthermore, any changes in excess of the stated limits had to be approved of by the IMF. From the date of the agreement until 1958, the currencies were not freely convertible, although in 1958, current account convertibility was implemented.⁵ Due to a prohibition beginning in 1956 of the lending of pound sterling abroad, a market for dollars was opened in London, so called Eurodollars.⁶ Due to restrictions imposed in the USA in 1963, the interest equalization tax, and in 1965, the foreign restraint program, the Eurocurrency market was given a strong impetus. This development was followed by other Eurocurrencies.

The goal of the gold pool, created in 1961, was to obtain joint interventions on the market to keep the surplus of dollars outside the USA, as the surplus was used to demand conversion to gold, e.g. France in June 1967. The USA adopted a two-tier system in March 1968 under which the price of gold for private persons was allowed to fluctuate freely. In August 1971, the dollar was made inconvertible into gold, and in December the Smithsonian Agreement was entered. Under this agreement, the price of gold was set to USD 38/ounce, the value of the currencies was corrected, and the exchange band was expanded to $\pm 2.25\%$. In February 1973, the agreement collapsed and the foreign exchange markets were closed. Following this, the currencies were floated in a managed way, 'dirty floating'.⁷ In April 1972, the Snake (or the Snake in the Tunnel) came into being, under which the fluctuations were narrowed to $\pm 1\ 1/8\%$. The European Monetary System, EMS, was created in March 1979. It imposes two restrictions on changes in currency rates. Firstly, the currencies are not allowed to deviate by more than $\pm 2.25\%$ (6% for the Spanish peseta) from their par value measured in ECU compared to each other's currency. Both parties are compelled to intervene. Secondly, each currency rate may not deviate by more than 1.69% from its central rate (4.5% for the peseta). If so, the habitat country must intervene alone.⁸ Apart from the EMS, there are several exchange rate arrangements or pegging relationships in operation. (See appendix D.)

The Swedish Market

In the seventeenth century, there were three sorts of coins in Sweden: a gold coin, 'dukat', a silver coin, 'riksdaler', and a copper coin.⁹ In 1661, the first bank note in the world was issued. During the end of the nineteenth century, the gold standard became dominant over the silver standard and over the bimetallism. In 1931, it was abandoned.

After having participated in the Bretton Woods Agreement and in the Smithsonian Agreement, Sweden joined the Snake. In October 1976, the Swedish krona was devalued by 3% and in April 1977 by 6%.¹⁰ After a devaluation in August 1977 by 10%, Sweden had to leave the Snake. The krona was then pegged to a currency basket. In September 1981, the krona was devalued again by 10%¹¹ and in October 1982 by 16%.

1.2.2 Introduction of Hedging Tools on the Market

World Wide

Options as hedging tools have existed for hundreds of years.¹² In the beginning, the primary asset consisted of a commodity. Nowadays, it is mainly a money market instrument or a stock. The development of and transition to the latter financial instruments has taken place recently.

Trading of options increased during the 1970s after a decline in the USA during the 1930s. Standardized trade on call options on stock was introduced in 1973 following the opening of the Chicago Board of Exchange.¹³ In 1976, three more markets were opened for stock option trading: the Pacific Stock Exchange, the American Stock Exchange, and the Philadelphia Stock Exchange (PHLX). In 1977, put options on stock were introduced on all option exchanges. Trade on standardized currency options was introduced in Amsterdam on the European Options Exchange in 1978,¹⁴ and in 1982 the Montreal Exchange in Canada and the PHLX followed. The liquidity on the markets however did not become satisfactory until 1984/-85, though.¹⁵ The contracts traded were DEM/USD and GBP/USD. Nowadays, the main market places for currency options are the PHLX and the Chicago Mercantile Exchange (Merc). The options are both of the European and of the American type. The main currencies are: pound sterling, Deutschmarks, Swiss franc, Japanese yen, French franc, Canadian dollars, ECUs and Australian dollars.¹⁶

The first financial futures contract on currency was introduced in 1972 on the Chicago International Monetary Market (IMM).¹⁷ In 1981, IMM introduced futures on Eurodollars. In 1982, LIFFE (London International Financial Futures Exchange) introduced futures on pound sterling, Deutschmark, Swiss franc and Japanese yen. Options on futures were introduced in 1982.¹⁸ The

first options on currency futures were introduced in 1984 (on Eurodollar futures, denominated in Deutschmark). Options on forward contracts are not as common. There are, however, some options on forward contracts on Treasury notes.¹⁹

The Swedish Market

Trading of standardized call options on stock was introduced in 1985;²⁰ and of put options on stock in 1987. Standardized currency options were not introduced until 1988.²¹ These were not denominated in the domestic currency, but on the exchange rate USD/DEM. All options were of the European type.²² In 1989, trading of standardized dollar currency options denominated in the domestic currency began.

The forward contract is the major instrument for hedging in Sweden, as the futures contract does not exist. A standardized market for forward contracts on currencies has existed since the beginning of 1989.²³ Prior to the opening of the market, Swedish banks had provided forward contracts for a long period of time, subject to some legal restrictions. Options on forward contracts do not, to the author's knowledge, exist on the Swedish market yet.

1.2.3 Theoretical Background of Hedging Methods

The development of the theoretical background, e.g. pricing formulas, is of a recent date, although the foundation of the formulas was laid earlier. The underlying stochastic processes were developed by Louis Bachelier in 1900 in his dissertation 'Theorie de la Speculation'. The Brownian motion, also called the Wiener process, was developed by Brown²⁴ in the nineteenth century. Its mathematical properties were studied by Wiener in the 1920s and applied to the Brownian motion by Einstein.²⁵

The seminal work of Black & Scholes [1973] provided a formula for pricing options on stocks when interest rates were deterministic.²⁶ In the same year, Merton [1973] relaxed the assumption and introduced stochastic interest rates.²⁷ The pricing formula for options on foreign currency was developed by Garman & Kohlhagen [1983], and extended to include stochastic interest rates by Grabbe [1983]. The foundation for pricing of options on commodity futures contracts under a constant interest rate assumption was developed by Black [1976]. However, the assumption of coinciding maturity dates of the option and of the underlying contract which Black made has not yet been relaxed. It will be done in this thesis. The pricing of options on forward contracts is a subject not yet thoroughly treated in the literature²⁸. The theory of dynamic hedging methods is also of recent origin: the idea of option replication was implicitly stated in the Black & Scholes article [1973], and it was further elaborated by Leland & Rubinstein in the same year.²⁹ The constant proportion portfolio insurance is based on an idea

presented by Merton [1971],³⁰ elaborated by Perold [1986], and by Black & Jones [1987].³¹ The time-invariant portfolio protection was developed by Estep & Kritzman [1988], as a slight modification of the constant proportion portfolio insurance.

1.3 Purposes

The purpose of the dissertation is twofold. The overriding purpose is to develop a decision support method, easily implementable in business, for hedging of price and/or foreign exchange rate risk and to analyze and illustrate its applicability.

In order to pursue the first purpose, the existing theory is extended and a new concept of hedging tool is introduced. This constitutes the second purpose.

1.4 Limits³²

Type of Option

The study is limited to the European type of option. Thus, no premature exercise is allowed.

Primary Assets

Three primary assets are discussed: domestic risky asset, foreign currency, and foreign (locally) risky asset. The decision support method designed is illustrated by a foreign currency position, but it is also applicable for the other assets as well as mixes of the three sorts. The hedging vehicles are discussed for the three assets in order to facilitate the obvious generalization of the illustration of the hedging method.

Interest Rate

The interest rates are assumed to be deterministic³³ and, to simplify the formulas, also constant.

Tax

Tax is not considered.

Settlement of Purchases and Sales

It is assumed that any cash transfer following a purchase or a sale takes place immediately, if not otherwise stated.

Market Structure

The markets are assumed to be liquid and each actor is assumed to be small. Everyone is a price taker. Hence, no separate action is assumed to influence the market prices. Furthermore, it is assumed that no consensus exists on the market as to when to trade and/or how to trade. Consequently, there will be no actor initiated fluctuations in the market prices. Finally, it is assumed that a potential introduction of new instruments based on the concept 'gathered asset' does not influence the prevailing market prices. The pricing formulas are derived as if the instruments already exist on the market.

Purpose of Transactions

The purpose of a transaction on the market is of one of three kinds: arbitrage, speculation or commercial hedging.³⁴ The focus of this dissertation will be on commercial hedging, implying a non-speculative restraint. However, the method developed is applicable for a speculative position as well.

Currency of Reference

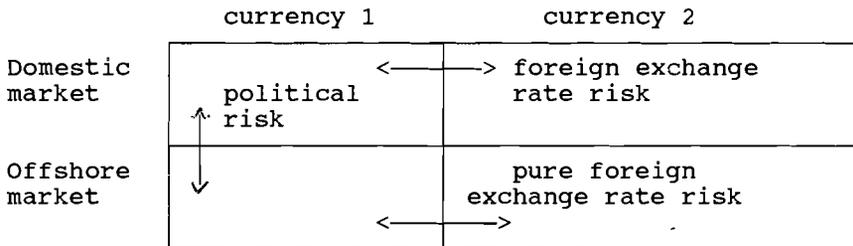
The performance of an organizational unit is measured in a specific currency, often the local one. If the total group performance should be evaluated in another currency, for example the consolidated income statement presented in another currency unit, it may prove better to evaluate the different group units in that same currency. Consequently, the reference currency may be foreign. This complex management problem does not have any direct bearing on the design of the hedging decision method as the risk is estimated conditional on a reference currency. Naturally, the choice of reference currency will have an enormous impact on the level of hedge and on the hedging vehicles chosen. Thus, the decision support method developed will not be influenced. The domestic currency is henceforth assumed, out of convenience, to be the reference currency.

Risk to be Analyzed

The company risk may be split into three non-exclusive groups: business risk, foreign exchange risk and political risk.³⁵ The first one being associated with the competitive situation and the domestic money market. The second risk originates indirectly or directly from the effect of deviations from the expected currency exchange rates. The political risk involves the risk of specific regulations or actions imposed by a country such as

expropriation, confiscation and changes in the taxation system or in specific regulations. The non-exclusive characteristic may be illustrated by the political risk influence on the foreign exchange rates. This phenomenon is a noteworthy feature on the currency markets where a foreign exchange rate differs between the domestic market place and in an offshore market place.³⁶ See figure 1.1. Henceforth, the political risk will not be considered although the author is well aware of its large impact on hedging decisions in companies. The only business risk considered is the stochastic price of assets to be purchased or sold.

Figure 1.1 Pure foreign exchange risk versus political risk.



The foreign exchange exposure is often divided into three groups: translation exposure, transaction exposure and economic exposure.³⁷ Translation exposure arises from the necessity to restate a balance sheet given in one currency into another currency whereby the entity's assets and liabilities are influenced. The exposure has no real impact on the value of the firm, should the tax system not be distortive. The transaction exposure is the impact of a change in exchange rates on existing obligations. Economic exposure stems from anticipated future cash flows for which no obligations exist. It is a result of economic analysis.³⁸ The focus in the forthcoming discussions is on the transaction exposure plus the change in value of anticipated transactions. However, changes in the volume of assets inherent in the transactions (for instance due to a change in competitiveness) are not included.

Real/Nominal Value

The risk inherent in a position denominated in a foreign currency is influenced by inflation. A distinction may therefore be made between foreign exchange rate risk and currency risk. The former measure is concerned with the dependency on the future exchange rate of the net present/future value of the position. The latter measure focuses on the value of the money, i.e. the real net present/future value.³⁹ A similar definition was given by Adler & Simon [1986]: "Currency risk is to be identified with statistical quantities that summarize the probability that the actual domestic purchasing power of home or foreign currency on a given future date will differ from its originally anticipated value."⁴⁰

The currency risk may be subdivided into three risks: inflation risk, foreign exchange risk, and a relative price risk.⁴¹

Thus, when measuring the value of a position, the focus may either be the real value exempt from the inflationary impact or the nominal value. In choosing the nominal domestic value as a benchmark for a foreign position, the domestic inflationary risk is included both in the benchmark and in the exchange rate. The domestic inflationary risk may be treated separately by comparing the domestic nominal value and the domestic real value.⁴² In the forthcoming discussions, only the nominal value is considered. The concept foreign exchange risk is used interchangeably with currency risk.

Level of Hedging

The capital asset pricing model states that the owners of a company may themselves diversify, i.e. eliminate non-systematic risks, implying that there is no reason for the company to hedge against currency risk. A costly hedge implies irrational behaviour, whereas a non-cost hedge is neither advantageous nor disadvantageous. However, there are two major situations in which hedging is considered rational.⁴³

First, in tightly held companies, the risk bearing capacity may be limited, due to lack of diversification possibilities.⁴⁴ Hence, should the owners be risk averse, it would be optimal to hedge within the company.

Second, the risk of bankruptcy renders intertemporal risk aversion,⁴⁵ as the long run perspective may not be applied. The rationale of hedging when facing a risk of bankruptcy emanates from three sources.⁴⁶ Firstly, financial distress may result in dysfunctional behaviour by the management. (At least the behaviour would lead to adverse effects, should the company survive.) The impetus to invest in R&D, in promotional activities, and in promising product development/production is reduced at the same time as there is a substantial risk that the quality of the production is allowed to deteriorate and that investments are made in high risk activities. Secondly, bankruptcy involves costs by itself. Thirdly, a company in financial distress is likely to face higher operating costs due to demands for compensation for the extra risk faced by the personnel, by the customers, by the suppliers, and by the bond holders. Besides, should the information be asymmetric, even the investors (owners) would demand higher compensation, and the cost of capital would increase.

In this dissertation no attempt is made to generate a normative model, i.e. to provide a decision as to how much of a position should be hedged. As will be seen, the method will simply quantify the risk, subject to a chosen level of hedge. Consequently, any of the above mentioned aspects must be included heuristically by the user of the decision support method.

Levels and Methods of Position Adjustment

There are three levels of which the position or potential position may be adjusted.⁴⁷ The first one is associated with the prerequisites of entering a position. If for instance a currency is highly uncertain, the underlying commercial position may not be allowed at all should the currency risk not be possible to handle. The second level is to include the currency problem when negotiating the commercial contract, whereby the possibility of including advantageous currency clauses⁴⁸ is balanced by loss of competitiveness. The third level of intervention constitutes the corrective method, in which the foreign currency position is given exogenously. The adjustment potential is reduced to different methods of taking offsetting positions or buying insurance. In this dissertation, only the corrective method is considered. It is assumed that the possibilities of negotiating advantageous currency clauses, have been exhausted.

The risk-reducing methods to be discussed belong to two groups: tools and dynamic hedging strategies. The main methods in each group have been chosen. In the first group are: forward contracts,⁴⁹ futures contracts, options, options on forwards and options on futures⁵⁰. In the second group are: substitutes, CPPI, TIPP and stop-loss. In illustrating how the risk-reducing methods may be applied in the decision support method developed, futures and options on futures have been excluded as they do not exist on the Swedish market.⁵¹ Substitutes copy other instruments more or less perfectly, and are therefore excluded. TIPP is shown to be very similar to CPPI under the numerical assumptions made, and is therefore excluded.

The following limitations are made when analyzing three different decision situations: In a contractual position, only forwards are relevant due to the non-speculative requirement. In an anticipatory position, forward contracts, options,⁵² CPPI, and stop-loss are analyzed separately. Two principally different approaches are illustrated. The first approach accommodates the case when using methods having a fixed final value conditional on the stochastic price of the underlying asset, e.g. forwards and options. The second approach accommodates the situation when the final value given the stochastic price of the asset may be stochastic, e.g. CPPI and stop-loss. In the former method, an semi-analytical approach is applicable whereas simulation must be applied in the latter method. The tender-offer position is illustrated by analyzing the risk when applying options on forwards or options followed by forwards. Due to the similarity between a tender offer position and an anticipatory position, the discussion will often refer to the analysis of the latter position in order to reduce redundant formulas/figures. The possibility of simultaneously applying more than one risk reducing method for an underlying position is illustrated by options and forwards.

Organizational Structure

Management of risk may either be centralized or decentralized. A mixture of the two principles may also be applied. Some of the advantages of a centralized system are:⁵³

- * matching of different units' positions
- * total group net position is easily obtained
- * larger resources for the centralized department, enabling employment of specialists
- * decisions optimal on group level
- * exploitation of taxation differences between countries
- * evasion of potential national controls on currency flows
- * overall perspective, implying avoidance of overreactions

Some of the disadvantages of a centralized system are:

- * motivational consequences
- * decrease of risk awareness within the organization
- * increase in communication need
- * potential neglect of knowledge of local market conditions
- * difficulty of evaluating performance at operating level

The two systems may be mixed, for example, by first letting the operating level apply their own risk principles and then making a risk management decision on group level to obtain an acceptable total risk structure.⁵⁴ An elaboration of this mixture would be to create an internal market for hedging instruments,⁵⁵ whereby the operational units obtain the desired risk level by buying/selling the instruments from/to the central unit. The central unit subsequently purchases/sells, possibly a different amount of, instruments on the market. The motivational incentive is retained at the cost of redundant work. The dilemma of how the risk management is to be organized will not be discussed any further, as the decision support method presented is not dependent on any specific form of organizational structure.

1.5 Definitions

Asset, etc.

A **contingent claim** is a "security whose value derives solely from the values of other securities"⁵⁶. Contingent claim is used interchangeably with derivative assets.

An **underlying asset** is the contract or asset which is 'closest in line' for a contingent claim, whereas the **primary asset** is the 'final' asset involved in the contract. In an option on a forward contract on currency for instance, the forward contract

constitutes the underlying asset whereas the currency constitutes the primary asset. For an option on currency, the underlying asset and the primary asset are the same - namely currency.

In 'gathered risk' the word 'gathered' is intended to show that two sequential risks⁵⁷ have been gathered into one single entity. Thus, a **gathered asset** is an asset involving two sources of risk. Consider, for example, a foreign commodity, valued in the domestic currency unit of the owner, which is submitted to foreign exchange rate risk and commodity price risk. A **gathered instrument**, e.g. a gathered option, is defined by its primary asset being gathered.⁵⁸

Risk Management, etc.

Risk management involves the question of what should be insured, as opposed to the concept of hedging which is the technology of how the hedging is to be carried out.⁵⁹ This very strict definition is not adhered to. The method for risk evaluation developed in Part Three may be used for choosing which of the existing positions to insure as well as to decide the level of insurance. Hence, the concept hedge will include both items.

Asset Allocation is defined as the generic name of the two methods for allocating your assets among different investment opportunities and possibly rebalancing them over time. Fixed asset allocation is a method whereby an initial position is not rebalanced. Dynamic asset allocation implies, in contrast to the fixed allocation strategy, that the initial position may be rebalanced an arbitrary number of times. A fixed asset allocation, may be regarded as a special case of dynamic asset allocation, i.e. when the number of rebalances decreases to zero. Dynamic asset allocation is defined in the wider sense in this dissertation.

Portfolio Insurance/Protection is an application of the concept of asset allocation, should the portfolio consist of currently owned assets.⁶⁰ However, the portfolio may consist of an arbitrary number of different positions. Buying portfolio insurance, or simply insuring/protecting a portfolio, is defined as "managing it in a way that alters the probabilities of particular outcomes, compared with the risk profile of the unprotected or reference portfolio"^{61,62} The management is to be in such a way that the value of the portfolio is protected to some extent. Selling portfolio insurance is the opposite position.⁶³

Hedging is used interchangeably with insurance and protection.

Hedging Method, etc.

A **hedging method** is defined as a possibly combined hedge, consisting of hedging tools and/or dynamic hedging activities. The concept of hedging method is used interchangeably with

hedging mix and hedging combination.

A **hedging tool** consists of a contract entered in order to hedge. Examples are forward contracts, futures contracts and options.

A **substitute** is an action aiming at mimicking the outcome of another hedging action. The substitute is said to be perfect if the outcome is a blueprint of the target outcome. A **replica** is a perfect substitute for a tool, and it is a static strategy. A **synthetic** is a perfect substitute that may require continuous trading, and it is a dynamic strategy.

Risk/Exposure

Risk is defined differently from uncertainty. Risk includes a notion of the probability distribution of the event whereas uncertainty does not. In a hedging context only the risk is relevant as there are always some ideas about the probability distribution.

A distinction must be made between risk and exposure. Risk is associated with the probability of outcome, whereas **exposure** "should be defined in terms of what one has at risk"⁶⁴, e.g. the future quantity of local currency.⁶⁵ Hence, the exposure consists of the amount of local currency which is susceptible to the relevant risk. The notion of Adler & Simon [1986] that exposure must have a definite target date is contended with. They base the statement on the necessity of having an implementable (measurable and hedgeable) measure. Whether or not the exposed position is hedgeable, it is an exposure, and so the definition in this dissertation will deviate slightly.

Position, etc.

A position is defined either as a commitment to submit to a future cash flow or as an anticipated future cash flow. A **total position** may consist of an arbitrary number of separate, possibly contingent, positions, each of which is called **subposition**. Positions are used generically for a subposition or a total position in situations where no confusion may arise. The subpositions may either stem from the underlying position, i.e. the position to be insured, or from the hedging activity. The former subposition is assumed to be exogenous whereas the latter one is endogenous. (See Section 7.2.)

The judgement of whether a position is **long** or **short** is made just prior to the final transaction, i.e. before the asset is sold or bought in order to obtain the target allocation. A **present long position** implies that the asset is already owned, whereas a **future long position** implies that the asset will be obtained at a future point in time, but prior to the final transaction date.

Local Currency

Local currency is defined as domestic currency if the point of reference is the domestic country, otherwise it is defined as the foreign currency.

Cash Flow Versus Value

The **cash flow** is strictly defined as the amount of cash that is transacted at a specific point in time.

The **value** of a hedging tool equals the amount of cash that it could be bought or sold for assuming perfect markets. The value of a hedging activity using tools includes both the prevailing value of the tools and the capitalized value of the potential initial cash flow required.⁶⁶ The value of a hedged position consists of the capitalized value of any cash flow generated.

Forward Premium and Basis

The **forward premium** (discount) can be defined in two ways: as an amount of currency units per the relevant maturity, forward price - spot price,⁶⁷ or as a percentage of the prevailing spot price per year.⁶⁸ The **basis** is defined as the spot price subtracted by the futures price.⁶⁹

Method Versus Model

By **method** is meant either a hedging method (see definition) or the entire decision support design. A **model** is defined as an algorithm which renders the results used for making decisions according to the hedging method. **Model** is also used for the PC-programme calculating the relevant algorithm.

1.6 Outline and Summary of Contents

The body of this dissertation is divided into three parts, apart from this introductory chapter and a concluding chapter. The first part consists of a presentation of the underlying assets and an analysis of the hedging tools. In the second part, non-tool hedging methods are discussed. In the third part, the design of the hedging decision support method is presented and applied in different hedging situations.

In Chapter Two, the three underlying assets analyzed in the dissertation are described as to the pricing formula and the risk. The assets are: domestic risky asset, foreign currency and foreign risky asset. It is shown that the two risks involved in the last asset (foreign exchange risk and local price risk) may

be gathered into one single risk entity. If this is done, the asset is called a 'gathered asset', and the risk a 'gathered risk'. Subsequent to this is a short discussion of the correlation coefficients.

The hedging tools discussed in the dissertation are the derivative assets: forward contracts, futures contracts, options, options on forward contracts and options on futures contracts. They are elaborated on in Chapter Three by each of the three assets introduced in Chapter Two, respectively.

Chapter Four begins with an examination of multiple-risk situations. Two conceptually different types are defined: the parallel risk structure, and the sequential risk structure. This is followed by a discussion of the hedging allocation possibilities in a two-component sequential risk setting. Finally, the effects of using a gathered option instead of simple options are investigated. The main conclusion is that a gathered option has a significantly lower speculative characteristic than a simple options alternative.

In Chapter Five, an introduction to the second part of the dissertation is given through a classification of the dynamic hedging methods, a presentation of properties for evaluation and a discussion of the alternatives on how to set a floor or a ceiling. Subsequent to this is a short description of the non-substitute dynamic hedging methods: the constant proportion portfolio insurance, the time-invariant portfolio protection and the stop-loss strategy. They are adapted to include foreign exchange and to accommodate short positions.

In Chapter Six, the focus is on how to substitute a desired tool. The methods discussed are: application of a tool with non-coinciding maturity, cross-hedging, and synthetics/replicas.

Chapter Seven sets the framework for the discussion in Part Three. The first subject addressed is the risky position to be hedged. It is positioned within a system of classification derived, and a subset to be analyzed is chosen. The second subject discussed is the rationale for choice of decision support method; criteria for evaluating the method are given, the implication of cash flow distributions from hedges are commented on, and the applicability of existing methods critically analyzed. Thirdly, the design of the decision support method is presented in general terms. Finally, the problem of calculating the probabilities in the method is examined thoroughly. The main finding of Chapter Seven is that existing approaches for making hedging decisions are not suitable. The method suggested implies the provision of a probability distribution of the outcome of the position, conditional on any hedging mix the decision maker wishes to analyze.

In Chapter Eight, the unhedged contractual position, i.e. when the time of payment and the volume - not necessarily the price - are certain, and the unhedged anticipatory position are analyzed regarding cash flow and probability distribution. Hedging

activities are then introduced, and the combined position of the underlying position plus hedge is analyzed using the method introduced in Chapter Seven. A simulation approach is applied to obtain the probabilities when the final cash flow is stochastic conditional on the prices of the risky assets, and a semi-analytical calculation approach is used when the final cash flow is deterministic.

In Chapter Nine, the tender offer position is analyzed. The problem of allocating the hedge between the tender offer risk period and the contractual risk period is discussed as well as the choice of the point in time for evaluation of the risk/-outcome. The generated imperfections of hedging by options followed by forwards instead of by options on forwards are investigated.

A numerical example of a decision process using the hedging method is given in Chapter Ten. It is followed by a short examination and description of an implementation of the method which the author made in a Swedish company. Finally, the design of the hedging method is evaluated based on the criteria given in Chapter Seven.

A short summary of the contributions to research made in this dissertation is given in Chapter Eleven. Suggestions of topics for further research are also given.

In order to enable actors on the market to directly use the results presented in the dissertation, formulas and derivations of the equations are provided in the appendices. Shorter comments are given in the notes, which are placed at the end of each chapter.

Notes

1. If not otherwise stated, the section is based on Kindleberger [1984].
2. There were two sorts of gold standards, the 'Gold Specie Standard' which implied the use of gold as means of payment, and the 'Gold Bullion Standard' under which the notes used could be exchanged for gold at the issuing bank. (Elmér, Jakobsson & Lundin [1987], p. 118.)
3. The discussion about Bretton Woods is based on Grabbe [1986], pp. 3-26.
4. For gold: one ounce equals one troy ounce, oz, equals 1/12 troy pound equals 31.1 grammes. (Otherwise, one ounce equals 1/16 pound equals 28.35 grammes.)
5. Current account convertibility concerns transactions associated with international trade. In capital account convertibility, investments in foreign bonds and stock are also allowed. (Grabbe [1986], p. 12.)
6. There are some alternative explanations for the emergence of the Eurodollar. (See Kindleberger [1984], p. 450.)
7. Prior to 1970, only Canada (1950-1962) and Lebanon (1950-) had used floating rates. (Aliber [1980], p. 18.)
8. Eiteman & Stonehill [1983], p. 67.
9. Myhrman [1982], p. 34.
10. Elmér, Jakobsson & Lundin [1987], p. 122.
11. SOU 1985:52, p. 220.
12. See Rawls III & Smithson [1989] for an excellent presentation of the evolution of risk management products.
13. Cox & Rubinstein [1985], p. VII.
14. Recent Innovations in International Banking [1986], p. 220.
15. Sutton [1988], p. 3.
16. Sutton [1988], pp. 7-10.
17. The facts in this chapter about the international markets stem from Walmsley [1988], pp. 5-6 and 100, if nothing else is explicitly stated.
18. Ball & Torus [1986], p. 857.
19. Jarrow & Oldfield [1988], p. 16.

20. Kinnunen [1988], p. 103.
21. Prior to the trading of standardized options on currency, the instruments could be bought from the larger banks on request if no legal restrictions existed.
22. Brochures from Stockholms Optionsmarknad OM Fondkommission AB.
23. Ibid.
24. Named after Robert Brown, a British biologist in the nineteenth century. The phenomenon studied was the irregular movements of small particles in a liquid or in a gas. (Rosén [1982], p. 125.)
25. Justusson [1979], pp. 19-21.
26. See Black [1989a].
27. See Smith's [1976] review article for the development and the extension of the Black & Scholes formula.
28. Jarrow & Oldfield [1988] treated the problem with American call options and stochastic interest rate, but no closed form formula was presented.
29. Leland & Rubinstein [1988], p. 4. Their most famous paper in the area was published in 1981, Rubinstein & Leland [1981].
30. Perold & Sharpe [1988], p. 26.
31. The papers were written contemporaneously and independently. (Perold [1986], p. 2.)
32. Only the overriding limitations are presented in Section 1.4. The more specific limitations are presented on an ad hoc basis in the text.
33. The effects of one single unexpected change in the interest rates are discussed for a specific situation in Chapter Nine. Some extensions as to stochastic interest rates are made in Lagerstam [1989b].
34. Uggla [1971], p. 21.
35. Oxelheim [1987], pp. 19-22, made a slightly different division. His four groups were country risk, financial risk (interest rates and suspensions of payments), currency risk (including inflationary risk) and commercial risk (prices and sales volume).
36. Aliber [1980], pp. 46-52. Aliber also noted that "the interest differentials between domestic and offshore deposits denominated in the same currency provide an estimate of the marginal investor's assessment of the

political risk" (p. 149).

37. Eiteman & Stonehill [1983], pp. 146-151, Levi [1983], pp. 1-5, or Prindl [1976], pp. 21. Eiteman & Stonehill added a fourth exposure, the tax exposure, due to the fact that any change in the foreign exchange rate has an impact on the tax situation.
38. The division may be made in another way. For instance Ankrom [1979], p. 383, stated that "translation exposure ... pertains to actions already past" and that transaction exposure was "arising from future actions that are contained in sales and profit plan assumptions". Ankrom then stated that "combination of translation and transaction exposure is referred to as economic exposure".
39. Elvestedt [1980], p. 10.
40. Adler & Simon [1986], p. 44.
41. Oxelheim [1985], p. 69.
42. See Lessard & Sharp [1988], pp. 123-124, for real versus nominal currency movements.
43. The agency theory is another explanation to why hedging is undertaken. It might be personally more advantageous for a manager to hedge due to work security or bonuses, for instance. However, in a perfect information world this reason does not exist.
44. Mayers & Smith [1986], p. 235.
45. Elvestedt [1980], p. 86.
46. See for instance Shapiro & Titman [1986], pp. 217-222, Mayers & Smith [1986], or Rawls III & Smithson [1990], p. 9.
47. Ugglå [1971], pp. 3 and 248.
48. For example: choice of contractual currency, or maximum allowed change of foreign exchange rate without compensating payments.
49. Second-generation forwards are not included. Examples of these are cylinders (also called collars), and range forwards (cylinders having zero initial cash flow). See Srinivasulu [1987] or Warren [1987] for a discussion of these tools.
50. The special instruments targeting the tender offer situation "the tender-to-contract (TTC) problem" (Warren [1987]) are excluded. An example of such an instrument is where the payoff "is dependent upon whether or not the commercial contract is awarded" (Warren [1987]). Another example is the Scout, share currency option under tender, in which the

currency option contract is awarded to the party winning the contract in the bidding process.

51. Should they be included, the simulation approach must be used due to their stochastic final value. See Chapter Three.
52. Option on forwards are not relevant here.
53. Elvestedt [1980], pp. 122-124, Uggla [1971], pp. 146-147, Lessard & Lorange [1979], p. 556, and Oxelheim [1984], pp.227-229.
54. Elvestedt [1980], p. 124.
55. Åhlander, interview about results to be presented in forthcoming PhD dissertation. See also Lindholm [1987], pp. 167-169, for a discussion of this method applied in Swedish Match.
56. Granito [1988], p. 202.
57. Sequential risks imply that one risk influences the other, but not the other way around. See Chapter Four.
58. See Lagerstam [1989a] or Chapter Four for a further discussion of this instrument.
59. Bookstaber & Langsam [1988], p. 15.
60. Grannis [1988], p. 49.
61. Granito [1988], p. 197.
62. Bierman [1988] pointed out that 'insurance' means that the probability of loss is reduced to zero whereas 'portfolio insurance' is limited to mean that the return distribution is altered.
63. Buying portfolio insurance is part of the strategies which are called 'momentum strategies' because of their propensity to maintain/enhance market movements. (Hill & Jones [1988], p. 29.) Selling portfolio insurance is part of the value based strategies. (Ibid., p. 29.)
64. Adler & Dumas [1984], p. 42.
65. Adler & Simon [1986], p. 45.
66. Compare this to the concept 'net profit' that Giddy [1983], p. 146, used for options.
67. Oldfield & Messina [1977], p. 476.
68. Eiteman & Stonehill [1983], p. 99, and Levi [1983], p. 39.

69. Grabbe [1986], p. 103, or Walmsley [1988], p. 114.

Part One
The Hedging Tools

2 The Underlying Risky Assets: Presentation and Formulas

2.1 Introduction

Future transactions, anticipated or compulsory, may not be fixed with regards to the amount of domestic cash involved. Reasons for this may be: that the price of the asset is stochastic although denominated in the domestic currency, that the price is fixed in a foreign currency, or finally that the price is stochastic and in addition denominated in a foreign currency. The three assets are called domestic risky asset, foreign currency, and foreign risky asset.

In this chapter, the formulas of the risky assets will be derived. They will serve as a foundation for deriving the formulas of the tools in Chapter Three, for simulating the values of the different methods in Chapter Five, and for calculating the probabilities in Part Three. The formulas for the price of the foreign risky asset formalize the idea of gathering the risks versus separating them. This has not been addressed in previous literature, and the formulas will serve as a basis for a new group of tools¹ to be introduced.

2.2 Distributional Assumptions: the General Case

Changes in prices are often assumed to follow a random walk, also known as a martingale.² The random walk implies two assumptions, namely the non-correlation of successive price changes and the distribution of the price changes.³

Two different levels of acceptable autocorrelation are given by Fama [1965].⁴ The first level is governed by whether traders are able to increase profits if they have knowledge of the correlation. The second level is whether the correlation is "sufficient" to account for some particular property of the distribution" (p.35). The rationale for using a submartingale as an approximation for stocks was given by Fama [1970].⁵ The possibility of a supermartingale for currency follows from the

fact that the interest differential may be both positive and negative.

The distribution of the prices is assumed to be lognormal,⁶ although this has been questioned empirically. (See the following sections.)

2.3 Domestic Risky Asset

The risk associated with a domestic asset to be bought or sold stems from the stochastic price, as alluded to earlier. In order to formalize the risk and to develop a hedging method, the behaviour of the price must be approximated by a stochastic process. For stocks and commodities, the lognormal distribution is often assumed⁷ although it is refutable empirically.⁸ Should the position consist of any other goods, it is assumed that the price either follows a lognormal process or that it is approximately deterministic and therefore not risky.⁹

Denote the price of the domestic asset at time t by p_t , the momentaneous standard deviation of the continuously compounded rate of return σ_p , the drift per unit of time μ_p , and the standard wiener process by X_p . The distributional properties assumed may be formalized as equation (2.1) through (2.3).¹⁰

$$(2.1) \quad \frac{dp_t}{p_t} = \mu_p dt + \sigma_p \sqrt{dt} X_p; \quad X_p \sim N[0,1].$$

Hence:

$$(2.2) \quad p_t = p_0 e^{\mu_p T + \sigma_p \sqrt{T} X_p}; \quad T = t - t_0$$

The expected value is:¹¹

$$(2.3) \quad E[p_t] = p_0 e^{\mu_p T + \frac{1}{2} \sigma_p^2 T}$$

2.4 Foreign Currency

The stochastic distribution of the foreign exchange rate is often assumed to follow a lognormal distribution.¹² To denote the exchange rate at time t by e_t , the drift by μ_e , the standard deviation by σ_e , and the standard wiener process by X_e , the formulas used later in this analysis may be stated as equation (2.4).^{13 14} In order to align the expected future spot rate with the interest rate differential, μ_e must equal $(r_D - r_F - \frac{1}{2} \sigma_e^2)$.

$$(2.4) \quad e_t = e_0 e^{\mu_e T + \sigma_e \sqrt{TX} e} = e_0 e^{(r_D - r_F - \frac{1}{2}\sigma_e^2)T + \sigma_e \sqrt{TX} e}$$

$$(2.5) \quad E[e_t] = e_0 e^{\mu_e T + \frac{1}{2}\sigma_e^2 T} = e_0 e^{(r_D - r_F)T}; \quad T = t - t_0$$

The normal distributional assumption of price changes has been subject to criticism based on empirical findings. Several studies¹⁵ have shown that the distribution is leptocurtic, i.e. that it is more peaked and has fatter tails.¹⁶ Westerfield [1977] showed that the distributions are also symmetric. Giddy & Dufey [1975] showed that the changes in the exchange rates may be approximated by a random walk of the submartingale¹⁷ sort. The probability distribution chosen to approximate the changes is most often¹⁸ of the non-normal stable Paretian family.¹⁹

Empirical studies also show that the currency market is approximately efficient in the weak form sense,²⁰ i.e. that no serial correlation (autocorrelation) exists.²¹ However, "successive exchange rate changes do have some "memory" - but a memory that is short-lived and weak"²². Cornell [1977] found the empirical material inconclusive. Burt, Kaen & Booth [1977]²³ found serial correlation for one out of three exchange rates. Swanson & Caples [1987] and Kritzman [1989b] on the other hand found autocorrelation in their studies. Hence, as an approximation, a zero autocorrelation is assumed.

2.5 Foreign Risky Asset

The price of a foreign risky asset, measured in local currency units, follows the same stochastic distribution as a domestic risky asset. The currency of denomination is not the same as the currency of measure, and the price in domestic currency will be influenced by the exchange rate. If the price measured in local currency is denoted by p_t , the price measured in domestic currency by g_t , and the exchange rate by e_t (DC/FC), the relationship becomes:

$$(2.6) \quad g_t = p_t \cdot e_t$$

Insertion of (2.2) and (2.4) renders:

$$(2.7) \quad g_t = p_0 e_0 e^{(\mu_p + \mu_e)T + \sigma_p \sqrt{TX} p + \sigma_e \sqrt{TX} e}$$

Note that $\sigma_p \sqrt{TX} p \sim N[0, \sigma_p^2 T]$ and that $\sigma_e \sqrt{TX} e \sim N[0, \sigma_e^2 T]$. The sum of the two normally distributed expressions is $N[0, (\sigma_p^2 + 2\tau\sigma_p\sigma_e + \sigma_e^2)T]$. τ is the correlation coefficient between X_p and X_e .²⁴ Denote:

$$(2.8) \quad \mu_g = \mu_p + \mu_e$$

$$(2.9) \quad \sigma_g^2 = \sigma_p^2 + 2\tau\sigma_p\sigma_e + \sigma_e^2$$

Inserting (2.8) and (2.9) into (2.7) and substituting for a new Wiener process, $X_g \sim N[0,1]$, equation (2.10) is obtained.²⁵

$$(2.10) \quad g_t = g_0 e^{\mu_g T + \sigma_g \sqrt{TX} g}$$

Thus, through substitution, the foreign risky asset can mathematically be treated like a domestic risky asset. The foreign risky asset will be called a gathered asset when the two stochastic components have been gathered into one single unit.²⁶ This new concept will prove useful when constructing hedges and hedging tools.

2.6 A Short Note on the Correlation Coefficients

There are three correlation coefficients involved when applying the hedging decision method developed: between the price of an asset denominated in a foreign currency and the foreign exchange rate, between two foreign exchange rates, and between the price of two different assets.²⁷ An analysis of the size and sign of these correlation coefficients constitutes a vast and very complex subject. There are several theories trying to explain the relationships,²⁸ but the empirical validation may prove difficult.²⁹

Some empirical studies have been made to quantify the correlation coefficients. The correlation coefficient between foreign stock and exchange rate was found to be positive, small, and not statistically significant in a study by Thomas [1988]. Wasserfallen [1988] also found small coefficients of correlation although the sign was inconclusive. Adler & Simon [1986] found that the "local stock price indexes and exchange rates were positively correlated after October 1979, whereas they had been largely independent before" (p. 49). See also Eun & Resnick [1988], p. 200, who found significant positive correlation coefficients. Oxelheim [1985], pp. 166-170, estimated the correlation coefficients for several pairs of foreign exchange rates, inclusive and exclusive of changes in the Swedish krona. A study of stock market returns has been made by Eun & Resnick [1988].

Notes

1. According to Thomas [1988], forward contracts on currency where the volume expands and contracts with foreign equity prices are already offered on the market. This tool eliminates the risk of having only part of the foreign risky position hedged as to the currency risk. However, it is not a 'gathered' instrument according to the definition. During the very last stage of writing this dissertation, it has come to the author's knowledge that options on the West German stock index (FAZ) denominated in the Swedish krona have been written by Bankers Trust International in London. (Ekwall [1990].)
2. In denoting the price by 'a', an arbitrary point in time by 't', and one period later by t+1, the martingale distributional assumption may be formalized as $(E[a_{t+1}|a_t] = a_t)$. If there is a trend in the change of the price, the martingale turns into a submartingale for upward trends $(E[a_{t+1}|a_t] \geq a_t)$, and into a supermartingale for downward trends $(E[a_{t+1}|a_t] \leq a_t)$. (See Malliaris & Brock [1982], pp. 16-19.) The term submartingale is occasionally used although the existence of a downward trend is noticed. See for instance Giddy & Dufey [1975], p. 11.
3. Stevenson & Bear [1970], p. 68.
4. Fama [1965], p. 35.
5. Fama [1970], p. 386.
6. This was suggested by Osborne [1959] for the stock market, "common-stock prices... can be regarded as an ensemble of decisions in a statical equilibrium, with properties quite analogous to an ensemble of particles in statical mechanics... the distribution function ... is precisely the probability distribution for a particle in Brownian motion" (p.100). However, Osborn noted that the fit was not perfect: "the distributions are nearly normal" (p. 106).
7. Black & Scholes [1973], Black [1976], Osborne [1959], or Oldfield & Messina [1977].
8. For instance Praetz [1972], who showed that the scaled t-distribution was better than the stable Paretian distribution, which in turn was shown to fit the empirical material better than a normal distribution. Fama [1965, 1970] on the other hand, settled for the non-normal stable Paretian distribution developed by Mandelbrot [1963]. (See Fama [1965] or Mandelbrot [1963] for a discussion of the Paretian family of distributions.) Mandelbrot showed the better fit of a stable Paretian for commodity.

The existence of autocorrelations is found in a study by Lo & MacKinlay [1988]. It was stated in the excellent review

article by Fama [1970] that autocorrelations are found in some studies but not in others, but as an overall conclusion, they are small and often have positive signs. In Fama's article [1965], it was also concluded that "there is little evidence... of any large degree of dependence..." (p. 80).

9. When applying the method, the lognormal distribution assumption must be submitted to an empirical investigation for each good.
10. The mean is $\mu_p T$ whereas the stationary mean is $\mu_p T + \frac{1}{2}\sigma^2_p T$. (Black & Perold [1987].) The variance and the probability density function are: (Lindgren [1976], pp. 190-191. See Appendix B.2 for a derivation.)

$$\text{Var}[p_t] = p_0^2 e^{2\mu_p T + \sigma_p^2 T} (e^{\sigma_p^2 T} - 1)$$

$$\text{pdf}[p_t] = \frac{1}{\sigma_p p_t \sqrt{2\pi T}} e^{-(\ln[p_t/p_0] - \mu_p T)^2 / (2\sigma_p^2 T)}$$

11. Lindgren [1976], pp. 190-191. See also Appendix B.2 for a derivation.
12. Garman & Kohlhagen [1983], Grabbe [1983].
13. The possible existence of risk premiums is discussed in Chapter Three.
- 14.

$$\text{Var}[e_t] = e_0^2 e^{2\mu_e T + \sigma_e^2 T} (e^{\sigma_e^2 T} - 1)$$

$$\text{pdf}[e_t] = \frac{1}{\sigma_e e_t \sqrt{2\pi T}} e^{-(\ln[e_t/e_0] - \mu_e T)^2 / (2\sigma_e^2 T)}$$

If a cash position in foreign currency is invested, e.g. in risk-free foreign bonds, the measure stated above must be corrected for the continuously compounded foreign interest rate, r_F . As a result, μ_e is substituted for $\mu_e + r_F$.

15. For instance: Westerfield [1977]; Rogalski & Vinso [1978]; Giddy & Dufey [1975]; Logue & Sweeney [1977]; or Logue, Sweeney & Willett [1978]. See also the excellent summary by Kohlhagen [1978b]. Wasserfallen [1988] found this to be the case for intraday changes.
16. See Chou [1975], pp. 142-143, and Mandelbrot [1963] for definitions.
17. The simple random walk (martingale) is complemented by information about the interest rate differential in the submartingale. Nieuwland, Verschoor & Wolff [1989] showed that the random walk assumption can not be rejected, even if

the currencies belong to the European Monetary System. No mean-reversion was detected in their study. This result has an impact through analogy, on the assumption of the distribution of the Swedish krona.

18. The student-t was for instance chosen by Rogalski & Vinso [1978]. See Chou [1975], pp. 318-321, for the distribution.
19. See Cornell & Dietrich [1978].
20. The efficient market property may be divided into three forms. The weak form implies that future price changes are independent of past price changes. The semi-strong form implies that all publicly known information is already included in the price. The strong form implies that all information is included in the prices.
21. For instance: Logue, Sweeney & Willett [1978]; Cornell & Dietrich [1978]; and Rogalski & Vinso [1978]. Wasserfallen [1988] found this to be the case for monthly data and even for intraday changes. Sweeney [1986] showed that "the autocorrelation function ... is approximately white save for spikes at lags 8 and 10" (p. 169). The same is valid if the series is adjusted for the interest rate differential.
22. Giddy & Dufey [1975], p. 27. As the possibility of detecting correlations is said to depend on the method used, the results may be misleading. (Logue & Sweeney [1977].)
23. See Miles & Wilford [1979] and Burt, Kaen & Booth [1977] for a plausible reason for serial correlation.
24.
$$\tau = \text{corr}[X_p, X_e] = \frac{\text{cov}[X_p, X_e]}{\text{std}[X_p]\text{std}[X_e]} = \text{cov}[X_p, X_e]$$
25. See Thomas [1988] for a study of investment in a foreign stock market.
26. See Chapter Four for a more general description.
27. In addition, there is a correlation between specific asset prices measured in a foreign currency and any other foreign exchange rate through changes in the strength of the own currency for instance. An example would be if a country is exceptionally dependent on import/export of a good priced in foreign currency. See Maloney [1990], pp. 31-32.
28. See Krueger [1983], Chapters Three and Four, for an excellent discussion. See also Appendix C for a set of theories.
29. Shapiro [1983], p. 295, for instance.

3 Pricing and Valuation of the Hedging Tools

3.1 Introduction

Hedging of a risky position involves a choice of a hedging tool and/or a dynamic hedging method.¹ The main tools are: forward contracts, futures contracts, options, options on forward contracts and options on futures contracts. If these do not exist on the market, they may be substituted through a dynamic strategy. (See Chapter Six.)

In this chapter, the price and value of the five tools are derived for the three assets under consideration. These formulas are essential when the tools in the decision support method in Part Three of this dissertation are included. In deriving the formulas, the existing theory will be extended by six issues. Firstly, the price of an option on a forward contract is derived. Secondly, the price of an option on a futures contract is derived, should the maturity of the futures contract exceed that of the options contract. Thirdly, the price of an option on a currency futures contract is derived. Fourthly, the put-call parity for options on forward contracts is derived. Fifthly, the put-call parity for options on futures contracts is derived, should the dates of maturity not coincide. The sixth extension consists of the fact that the price and value of all five tools for a gathered asset are analogous to the formula for a domestic risky asset, should the variance and drift be substituted.

3.2 Separation of the Underlying Position and the Hedge

The total position consists of the underlying position and the positions generated through hedging activities. A partially hedged position may mathematically be divided into two different parts, in order to facilitate the geometrical illustrations and/or the formulas. There are two principally different ways of

performing this division.

One alternative is to separate the cash flow of the underlying position from the value of the hedging activities. This method will be used when analyzing the tools and dynamic methods as well as the hedging approach for contractual positions and anticipatory positions.² A noteworthy feature of this first method is that in a three-dimensional geometric setting with two stochastic variables (see Chapter Seven), the separation implies that the overall position corresponds to 'the sum' of two surfaces: one depicting the value of the underlying position and the other depicting the value of the hedging activity.

The second alternative is to separate the hedged fraction of the total position from the unhedged fraction of the underlying position. This notion will have a bearing on the possibility of risk shifting in an anticipatory position and in a tender offer position.

3.3 Forward Contract³

In a forward contract, two parties have agreed to exchange an underlying asset at the day of expiration, t_{FW} , at a specific price.⁴ The forward price is specified at the time of initiation, t_0 , and will not be subject to any further alterations. By convention, the forward price is set in order to render the contract a zero initial value.⁵ At expiration, the party who is long the forward contract (the buyer) will receive the underlying asset and will pay the party who is short the forward contract (the seller) the prespecified price, irrespective of the prevailing spot price of the asset.⁶ The contractual price is henceforth assumed to be the prevailing forward price and is denoted by $FW_a[t_0, t_{FW}]$ where 'a' is a generic subscript for p, e, and g. The forward price is given by the covered interest rate arbitrage restriction.⁷

Insertion of the distribution of the stochastic price of the primary asset into the forward price⁸ and comparison of the results to the formulas of the price of the primary assets render (3.1).⁹ The variance of the relative change in forward price is therefore equal to the variance of the relative change in the price of the underlying asset.¹⁰ This finding will be used in deriving the price of an option on a forward contract.

$$(3.1) \quad \sigma_{FW_a}^2 = \sigma_a^2$$

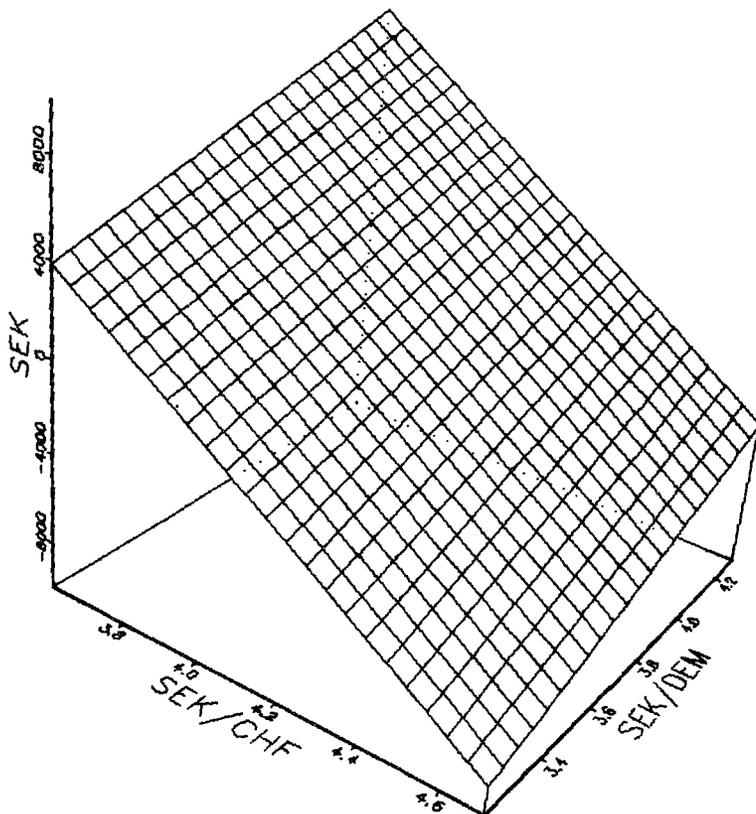
At any point in time, $t_0 \leq t \leq t_{FW}$, the value of a forward contract equals the net present value of the prevailing, accrued payoff to be paid at maturity. Due to the possibility of entering an offsetting contract, the payoff is valued as being risk-free.¹¹ The value of a forward hedge consists of the value of the

total number of long forward contracts plus the number of short forward contracts. The former equals the number of assets in a short position times the hedging fraction, $q_2 h_2[FW]$, and the latter equals the number of long assets times the hedging fraction, $q_1 h_1[FW]$. In setting $h_1[FW]=h_2[FW]=1$, the total forward hedge value at maturity may be written as (3.2).

$$(3.2) \quad VA[FW[t_{FW}]; t_{FW}] = q_1(FW_1[t_{FW}]-a_1) + q_2(a_2-FW_2[t_{FW}])$$

Note that the value-surface in figure 3.1 is not broken up in any smaller parts, and that it is flat over the whole region. The value at $\{(e_1, e_2): (4.6, 3.4)\}$ is for instance $-6,658,416$ SEK $\{11,510,937(4.1440-4.6) + 4,924,630(3.4-3.6862)\}$. (See Section 7.4.1 for the background of the numbers.)

Figure 3.1 Value of a forward hedge.



3.4 Futures Contract

A futures contract is an agreement between the contractual parties to trade an underlying asset at expiration at a prespecified price, the futures price. At the end of each trading day the contract will be rewritten, whereby the new contract will be defined by the prevailing futures price. The difference between the daily consecutive futures prices will be paid to the winning party by the losing party. Thus, at expiration the cash flow will equal the prevailing spot price plus the last resettlement cash flow.¹² The futures price is defined to be the price that renders a zero initial value of the contract. After each settlement, the contract will be based on the concurrent futures price and the value will be restored to zero. To achieve a zero value contract, the present value of the future expected cash flow must equal zero. The futures price at time t of a contract that expires at t_F will be denoted by $F[t, t_F]$. If the interest rates are deterministic and no arbitrage profits exist, the futures price will equal both the forward price¹³ and the expected future spot price.¹⁴ (See Appendix A.2.) Thus, the futures price follows the same distribution as the forward price.¹⁵ Consequently, the variance is equal to the variance of the price of the primary asset.

$$(3.3) \quad \sigma_{F_a}^2 = \sigma_a^2$$

Upon resettlement of the futures contract at the end of each trading day, the value will be restored to zero. Consequently, the value of a contract entered an arbitrary number of days earlier is equal to a contract entered the previous day.¹⁶ The total value at any arbitrary point in time, $t_0 \leq t - \epsilon \leq t_F$, including the value of the previous resettlements, equals (3.4). This constitutes the value of a futures contract plus the capitalized value of all previous cash flow. The value is path dependent, and hence stochastic even conditional on the future spot price of the underlying asset.

$$(3.4) \quad \begin{aligned} \text{Total value} & & t-2 & & r_D(t-\epsilon-t_i-1) \\ \text{of initial} & = & \sum_{t_i=t_0}^{t-2} & (F_a[t_i+1, t_F] - F_a[t_i, t_F])e & + \\ \text{contract} & & t_i=t_0 & & \\ & & & & + e^{-r_D \epsilon} (F_a[t-\epsilon, t_F] - F_a[t-1, t_F]) \end{aligned}$$

3.5 Option

If interest rates are deterministic,¹⁷ standard deviation constant, and transaction costs¹⁸ zero, the option price of a domestic risky asset and of a gathered asset will follow the original Black & Scholes [1973] formula. The option price of

foreign currency will follow Garman & Kohlhagen [1983].¹⁹ The results for constant interest rates are stated in Appendix A.3 for reference.²⁰

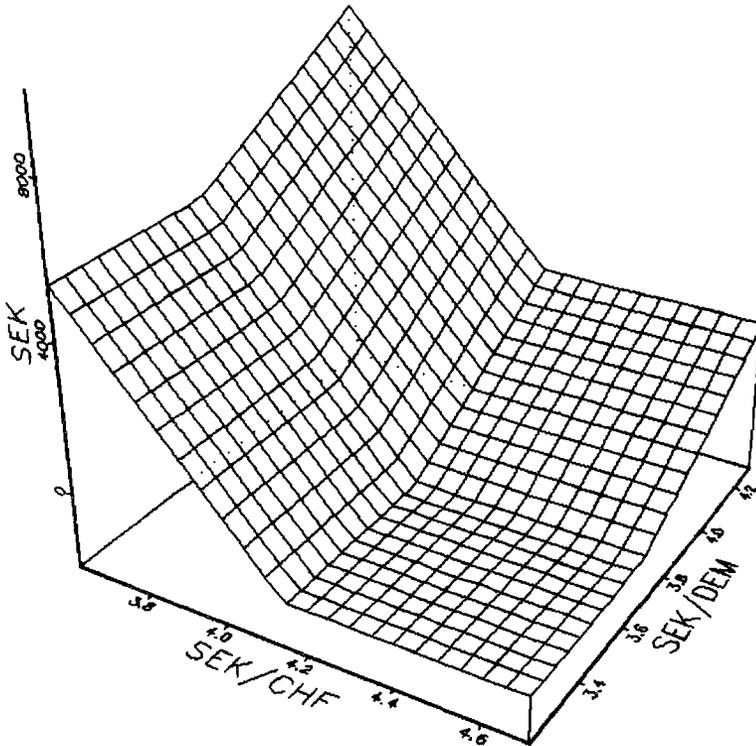
The formula for the price of an option on a gathered asset is analogous to that of a domestic asset, should the price be substituted for the gathered price and the variance for $\sigma_g^2 = \sigma_p^2 + 2r\sigma_p\sigma_e + \sigma_e^2$. The put-call parity for gathered options is analogous to that for domestic asset options.

The value of a long position in an option, 'the net value'²¹, must include the capitalized value of the initial payment. Let $a_{1,x}$ denote the exercise price and t_x the time of exercise of the two types of options. Illustrating with a two subposition example, q_1 put options on asset number one and q_2 call options on asset number two, the value is defined according to equation (3.5). In a three-dimensional figure, equation (3.5) looks like figure 3.2 for currency.

$$(3.5) \quad VA[C\&P;t_x] = q_1(\max[0, a_{1,x} - a_1] - P_e[t_x]e^{r_D T_x}) + q_2(\max[0, a_2 - a_{2,x}] - C_e[t_x]e^{r_D T_x}); \quad T_x = t_x - t_0$$

Figure 3.2 Value of an option hedge.

(See Section 7.4.1 for the value of the parameters.)



The value-surface consists of four flat surfaces, connected at the vertical projection of the exercise prices. The horizontal surface is for instance situated at the level $-787,814.72$ SEK, $\{- (11,510,937 \cdot 4.1 + 4,924,630 \cdot 3.64) 1.034 \cdot 1.17\}$. The value at $(3.8, 4.2)$ is $5,702,222.50$ SEK, $\{11,510,937(4.1440 - 3.8) + 4,924,630(4.2 - 3.6862) - 787,814.72\}$.

3.6 Option on a Forward Contract

As options on forward contracts have not been discussed thoroughly in the literature a more comprehensive elaboration is necessary in order to derive the formulas, to be used in Chapter Nine.

An option on a forward contract is a right to acquire or sell a forward contract of predetermined maturity which has a specified forward price, namely the exercise price of the option, $FW_x[t_{FW}]$.²² As the profit or loss of a forward contract is settled at maturity, no money changes hands at the date of expiration of the option unless it coincides with the date of maturity of the forward contract.

Call Option

To derive formulas for the price of an option on a forward contract, the payoffs of the option are compared with those of an option on the primary asset. The law of one price may be applied following some minor alterations. (See Appendix A.4.) The premium becomes: (For p and g : set $r_F=0$ and substitute e for p or g respectively.)

$$(3.6) \quad C_{FW}^e [t_C] = e_0 e^{-r_F T_{FW}} N[x_{FWe}] - FW_x[t_{FW}] e^{-r_D T_{FW}} N[x_{FWe} - \sigma_{FWe} \sqrt{T_C}]$$

$$x_{FW}^e = \frac{\ln[e_0 / FW_x[t_{FW}]] + (r_D - r_F) T_{FW} + \frac{1}{2} \sigma_{FWe}^2 T_C}{\sigma_{FWe} \sqrt{T_C}}$$

Put Option

The formulas for put options can be derived in two different ways. The first way is to compare the payoff from an option on the primary asset (analogously to the derivation for a call option). The second way is to apply the put-call parity theory and insert the call option pricing formula. By comparing the payoff of a put option on a forward contract to the payoff on a put option on the primary asset, the same substitutions must

obviously be made as for call options. Thus, the pricing formula will be obtained by making these substitutions in the put option formula on a primary asset, followed by insertion of the relationship between spot price and forward price. (For p and g: set $r_F=0$ and substitute e for p or g respectively. x_{FW} is according to the formula given for a call option if T_C is substituted for T_P .)

$$(3.7) \quad P_{FW} [t_P] = e_0 e^{-r_F T_{FW}} (N[x_{FW} e] - 1) - FW_X [t_{FW}] e^{-r_D T_{FW}} (N[x_{FW} e^{-\sigma_{FW} \sqrt{T_P}}] - 1)$$

Put-Call Parity

The put-call parity theory for options on forward contracts can be derived in either of two ways: by comparing the outcome of two different portfolios, or by substituting into the put-call parity theory for options on the primary asset.²³ (See Appendix A.4. Set $r_F=0$ and substitute e for p and g to obtain the parity for options on a forward on a domestic risky asset or a foreign risky asset.)

$$(3.8) \quad P_{FW} [t_{FW}] + e_0 e^{-r_F T_{FW}} = C_{FW} [t_{FW}] + FW_X [t_{FW}] e^{-r_D T_{FW}}$$

Value of an Option on Forward Hedge

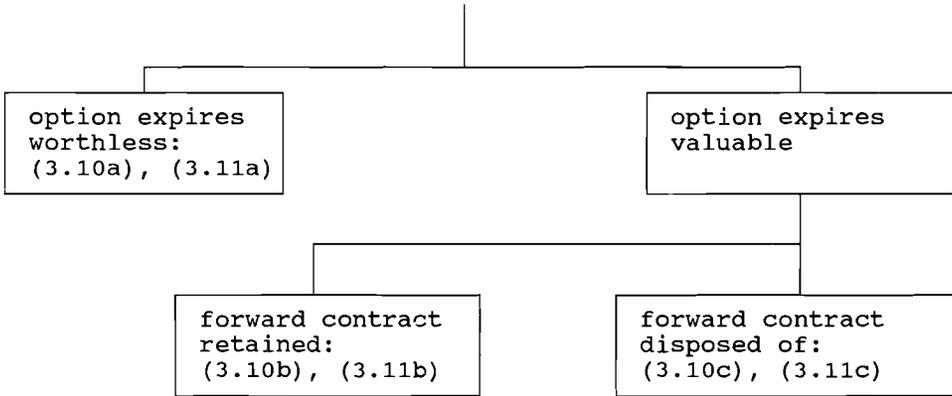
The value of a hedge using options on forward contracts may be measured at two different points in time: the first upon expiration of the option, t_X , and the second one at the time when the potential forward contract matures, t_{FW} . If the valuation takes place at time t_X , the value follows equation (3.9) in a two-subposition example. The value of the hedge at time t_X equals the value of the option minus the capitalized value of the initial price of the option. Geometrically, the value has the same structure as the value of an option hedge.

$$(3.9) \quad VA[C_{FW} & P_{FW}; t_X] = q_1 \{ \max[0, e^{-r_D(t_{FW}-t_X)} (FW_X [t_{FW}] - FW_a [t_X, t_{FW}])] - P_{FW} [t_X] e^{r_D(t_X-t_0)} \} + q_2 \{ \max[0, e^{-r_D(t_{FW}-t_X)} (FW_a [t_X, t_{FW}] - FW_X [t_{FW}])] - C_{FW} [t_X] e^{r_D(t_X-t_0)} \}$$

However, should the valuation of the hedge take place at the time of maturity of the potential forward contract, the value will depend on whether the forward contract has been retained or not. At time t_X , the options will only be exercised if profitable, irrespective of whether the forward contracts obtained are needed

or not. Should they not be needed, they may be disposed of at a profit. Hence, the final hedging value is dependent on the market situation and actions at time t_x . The three final outcomes to consider are depicted in figure 3.3.

Figure 3.3 Hedging value alternatives for an option on a forward contract. (Measured at time t_{FW} .)



$$(3.10) \quad VA[C_{FW}[t_C]; t_{FW}] =$$

$$(3.10a) \quad = - C_{FW}[t_C]e^{r_D(t_{FW}-t_0)}$$

$$(3.10b) \quad = a_{FW} - FW_X[t_{FW}] - C_{FW}[t_C]e^{r_D(t_{FW}-t_0)}$$

$$(3.10c) \quad = FW_a[t_C, t_{FW}] - FW_X[t_{FW}] - C_{FW}[t_C]e^{r_D(t_{FW}-t_0)}$$

$$(3.11) \quad VA[P_{FW}[t_P]; t_{FW}] =$$

$$(3.11a) \quad = - P_{FW}[t_P]e^{r_D(t_{FW}-t_0)}$$

$$(3.11b) \quad = FW_X[t_{FW}] - a_{FW} - P_{FW}[t_P]e^{r_D(t_{FW}-t_0)}$$

$$(3.11c) \quad = FW_X[t_{FW}] - FW_a[t_P, t_{FW}] - P_{FW}[t_P]e^{r_D(t_{FW}-t_0)}$$

Naturally, the differences between (3.10b) and (3.10c) and between (3.11b) and (3.11c) consist of the value of a forward contract entered at time t_C or t_P , maturing at time t_{FW} .

3.7 Option on a Futures Contract

The underlying asset of an option on a futures contract is a futures contract having a specified time before maturity. The exercise price of the option will serve as the futures price of the contract, and it is therefore not a sum to be paid on exercising. If the futures price of the contract received does not equal the prevailing futures price on the market, the correction is accomplished by immediately marking to market.²⁴

The pricing formula of an option on a commodity futures contract was first derived by Black [1976]. In that formula, it was assumed that the date of maturity of the option coincided with the date of maturity of the futures contract. Ramaswamy & Sundaresan [1985]²⁵ pointed out that the Black formula is applicable even though the dates of expiration/maturity differ. No revised formula was given though. This is achieved by substituting the futures price for domestic risky asset/gathered asset $F[t_0, t_x]$ for $F[t_0, t_F] \exp[-r_D(T_F - T_x)]$ and for currency $F_e[t_0, t_F] \exp[-(r_D - r_F)(T_F - T_x)]$.

An option on a futures contract can be priced either by using the same approach as when pricing options on primary assets,²⁶ or by comparing its payoffs to the payoffs of options on the primary asset. The latter method is used to obtain the formulas. (See Appendix A.5.)

Call Option

The premium is: (For p and g: set $r_F = 0$ and substitute e for p or g respectively.)

(3.12)

$$C_{F_e}[t_C] = e^{r_D(T_F - T_C) - r_F T_F} e_0 N[x_{F_e}] - F_x[t_F] e^{-r_D T_C} N[x_{F_e} - \sigma_{F_e} \sqrt{T_C}]$$

$$x_{F_e} = \frac{\ln[e_0 / F_x[t_F]] + (r_D - r_F) T_F + \frac{1}{2} \sigma_{F_e}^2 T_C}{\sigma_{F_e} \sqrt{T_C}}$$

By comparing (3.12) to the price of an option on the primary asset, it is obvious that there is little difference in buying an option on a futures from buying an option on the primary asset. This is due to the fact that on exercise, the difference between the prevailing futures price and the exercise price (the futures price of the contract received) is earned immediately and that entering a new futures contract is costless. Thus, the only difference (with deterministic interest rates) compared to an option on an underlying asset is that the futures price differs from the spot price by a factor $\exp[(r_D - r_F)(T_F - T_C)]$.

When the time of expiration of the option is equal to the maturity of the futures contract, $t_C = t_F$, the price of the option on the futures contract is exactly equal to the price of the option on the primary asset.²⁷ This is an obvious result due to the fact that the payoffs will be identical.²⁸

Put Option

The price of a put option can be obtained in two ways, by using the put-call parity theory, or by comparing the payoff to the payoff of an option on the primary asset. As the methods are very similar to the ones used for call options, they are omitted. The price of the put option is given in equation (3.13). (For p and g: set $r_F=0$ and substitute e for p or g respectively. x_F according to the formula given for a call option, should T_C be exchanged for T_P .)

$$(3.13) \quad P_F [t_P] = e^{r_D(T_F-T_P)-r_F T_F} e_0(N[x_F] - 1) - F_X[t_F] e^{-r_D T_P} (N[x_F - \sigma_F \sqrt{T_P}] - 1)$$

Put-Call Parity

By comparing two strategies rendering the same final cash flow (see Appendix A.5), the put-call parity is easily obtained.²⁹

$$(3.14) \quad P_F [t_X] + F_a[t_0, t_F] e^{-r_D T_X} = C_F [t_X] + F_X[t_F] e^{-r_D T_X}$$

Value of an Option on Futures Hedge

The value of an option on futures hedge is derived analogously to the value of an option on forwards hedge. The only modification needed is to let the value be dependent on the path of the futures price, should the option be exercised and the futures contract retained.

Notes

1. By a tool a physical contract is implied. (See Section 1.5.)
2. This was used in Folks [1973] for a forward position.
3. A swap is an agreement to exchange one asset (long or short) for another asset at one point in time, and at a future point in time it is changed back again at a prespecified price. The swap is therefore a combination of a purchase/sale of an asset, of a short/long forward position in the same asset, and of a position in bonds. In this way, the swap agreement is better analyzed through its components.
4. For simplicity, it is assumed that each contract refers to one unit of the underlying asset.
5. The forward price of a contract may naturally differ from the forward price defined as that which renders a zero value contract, as the parties may decide upon any price they like. Henceforth, it is assumed that each contract is written on the zero value contract forward price, unless otherwise stated.
6. Some forward contracts have cash settlements instead of physical delivery of the underlying asset. As the mode of delivery has no bearing on the price determination and valuation, it will be disregarded.
7. See Appendix C and Appendix A.1.
8. Set $r_F=0$ for p and g.

$$FW_a[t, t_{FW}] = a_0 e^{\mu_a(t-t_0) + \sigma_a \sqrt{t-t_0} X_a + (r_D - r_F)(t_{FW} - t)}$$

9. The lognormal assumption for the forward price on currency and for the spot exchange rate has been criticized. (See Rogalski & Vinso [1978].) However, Cornell [1977] found that the market sets the forward price "as if the exchange could be characterized by a diffusion process with trend" (p. 55).
10. Empirically, this has been criticized. In a study by Rogalski & Vinso [1978], it was implied that the variance for a currency forward was larger than for the underlying spot exchange rate. On the other hand, Oxelheim [1987] (p. 40) argued that they were roughly equal.
11. Hence, the value follows the equations below for a long forward position and for a short forward position respectively, under a constant interest rate assumption. ($t_0 \leq t \leq t_{FW}$.) Insertion of $t=t_0$ renders the initial zero-value condition.

$$\text{VA}[\text{FW}_a[t_0, t_{\text{FW}}]; t] = e^{-r_D(t_{\text{FW}}-t)} (\text{FW}_a[t, t_{\text{FW}}] - \text{FW}_a[t_0, t_{\text{FW}}])$$

(long)

$$\text{VA}[\text{FW}_a[t_0, t_{\text{FW}}]; t] = - \text{VA}[\text{FW}_a[t_0, t_{\text{FW}}]; t]$$

(short) (long)

12. A thorough description of how the futures contract is defined is given in Richard & Sundaresan [1981], p.348.
13. Jarrow & Oldfield [1981], pp. 378-380, and Richard & Sundaresan [1981], p. 364 or Cox, Ingersoll & Ross [1981], p. 325. Levy [1989] showed that if the price of a bond maturing at the same time as the forward/futures contract is known one period in advance, then the forward price must equal the futures price. The conclusion is valid even if interest rates are stochastic. Cornell & Reinganum [1981] showed in an empirical study that the difference between the forward price and the futures price was small on the foreign exchange market but large in the bond market. French [1983] showed that "significant differences between these prices" (p. 312) existed for commodities.
14. The question whether the futures price is an unbiased estimate of the expected future spot price was discussed by Keynes in 1923 and in 1930, and by Hicks in 1946 in terms of normal backwardation (The New Palgrave [1987], pp. 169-170) and by Telser in 1958 (Richards & Sundaresan [1981], pp. 349-350) in terms of contango. Normal backwardation implies, in short, that the demand for short futures contracts is larger than the demand for long futures contracts, due to the greater need for producing corporations to hedge their production than for the more flexible consumers to hedge purchases. For this reason, the futures price should be lower than the expected future spot price. However, the theory is not applicable for foreign exchange and financial instruments. Contango is the reverse effect, i.e. that the futures price is an upward bias estimate of the expected future spot price.
15. Empirically, changes in the futures price are sometimes assumed to follow the stable Paretian model instead of the normal distribution due to 'fatter tails'. (Dusak [1973], for futures contracts on commodities.) The serial correlation was shown to be approximately zero in the same study. Stevenson & Bear [1970] found that the changes in the commodity futures markets had "a tendency for negative dependence in short periods of time and positive dependence over longer periods" (p. 79). They got a leptocurtic distribution.
16. In denoting any point in time during day t by $t-\epsilon$, the value can be formalized as:

$$\text{VA}[\text{F}_a[t_0, t_F]; t-\epsilon] = \text{VA}[\text{F}_a[t-1, t_F]; t-\epsilon]; \quad 0 \leq \epsilon \leq 1$$

If an offsetting position is entered, the payoff becomes certain.

$$VA[F_a[t_0, t_F]; t-\epsilon] = e^{-r_D \epsilon} (F_a[t-\epsilon, t_F] - F_a[t-1, t_F])$$

(long)

17. The case of stochastic interest rates for a domestic risky asset was elaborated by Merton [1973] and for foreign currency by Grabbe [1983]. See also Hilliard, Madura & Tucker [1989].
18. See Stoll [1988] for a discussion of the components of the transaction cost.
19. Derivations and explanations of those formulas can be found in Lagerstam [1989a].
20. Should the interest rates be deterministic but non-constant, substitute r_T for the integral of r over time.
21. Giddy [1983] called this value 'net profit'.
22. Ramaswamy & Sundaresan [1985] argued that there are two ways in which an option on a forward contract might be defined. The first definition is similar to an option on a future, i.e. that on exercise the owner receives the difference between the prevalent forward price and the exercise price whereby he receives a newly created forward contract. The second definition is consistent with all other options, and therefore the definition chosen in this paper. It says that on exercise, the owner of the option will receive a forward contract in which the exercise price will be the contract's forward price. No money will then change hands on expiration.
23. As the method is rather simple, the substitutions will only be outlined. Start with the put-call parity theory for options on the primary asset. Insert the relationship between the spot price and the forward price. Make the substitutions as described in the section for call options. Thereby, the price of the option on the asset will be equal to the price of the option on the forward contract. Hence the option prices may be substituted for the option prices on the forward contracts. Rearranging and simplifying renders the put-call parity theory for options on forward contracts.
24. Fabozzi [1985], p. 76.
25. Note 7, p. 1326. However, Ramaswamy & Sundaresan [1985] did not pursue the matter any further than stating the applicability.
26. See Brenner, Courtadon & Subrahmanyam [1985], p. 1306, or Black [1976], p. 177.

27. As was derived, an option position on a futures contract is similar to an option on the underlying asset. Then, why do options on futures contracts exist? Fabozzi [1985] (pp. 13-14) has given three reasons. Firstly, options on futures have no dividends to correct for. Considered by Walmsley [1988], p. 181, to be an important factor as it makes the trading easier. Secondly, delivery difficulties are reduced due to the fact that the futures contract is not on a specific asset, but rather on an asset with specific characteristics. Thirdly, futures and options on futures are often traded on the same market place. Thus, pricing information is more readily available.
28. This is also noted in Hilliard, Madura & Tucker [1989], p. 1, and in Brenner, Courtadon & Subrahmanyam [1985], p. 1305.
29. Brenner, Courtadon & Subrahmanyam [1985], p. 1305, presented the parity for $t_C = t_F$.

4 Gathered Risk Versus Separated Risks

4.1 Introduction

A foreign risky asset involves two risk components, as was noted in Section 2.5, the price risk of the asset measured in local currency, and the foreign exchange rate risk. The two risks may be gathered into one concept, 'gathered risk', or kept separate, 'separated risks'.

An instrument hedging against the gathered risk, i.e. against both risks at the same time, will be called a gathered instrument. An instrument dealing with only one of the two risks will be called a simple instrument.¹ A gathered stock/currency option for instance renders the owner the right to buy/sell a foreign stock at an exercise price denominated in domestic currency. In this way, the downside risk is eliminated.

The purpose of Chapter Four is to analyze and to quantify the difference between hedging with a gathered instrument as opposed to hedging with simple instruments. This shows that using a gathered instrument has a lower speculative characteristic than using simple instruments. Regarding the risk-reducing purpose in this dissertation, only the gathered instrument is considered in the following chapters.

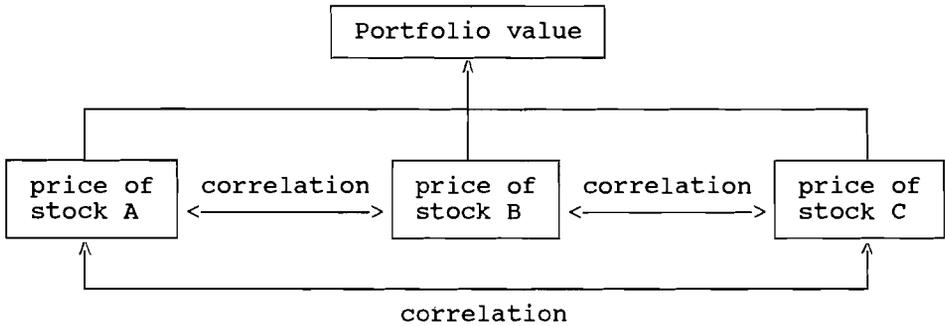
The discussion in Chapter Four is divided into three sections. The first one relates the gathered risk to other multi-risk concepts. The second section focuses on the variety of hedging possibilities when having two risks. In the third section, the gathered option is analyzed and compared to a strategy of simple options.

4.2 The Gathered Risk: Related

In a situation involving two risks, the risks may be parallel or sequential.

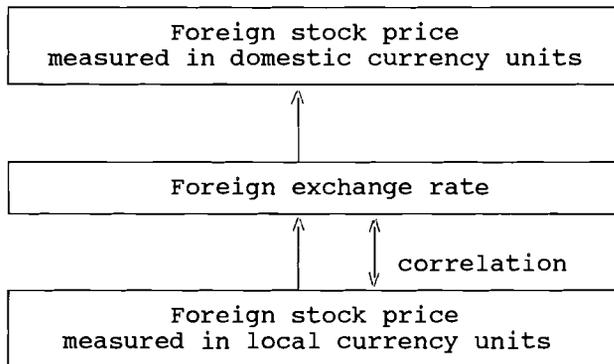
In a parallel risk situation, the two risks influence the value of the asset separately, but may be correlated (see figure 4.1). An example of a price submitted to parallel risks is a portfolio of stock. The price of each stock influences the value directly. In a parallel risk structure, the risks are additive.

Figure 4.1 Parallel risk structure



The sequential risk situation is characterized by one risk influencing the final price through the other risk. (See figure 4.2.) An example of a sequential risk situation is the price of a foreign good denominated in local currency units but measured in another currency². The local price risk of the good influences the price of the good measured in another currency unit through

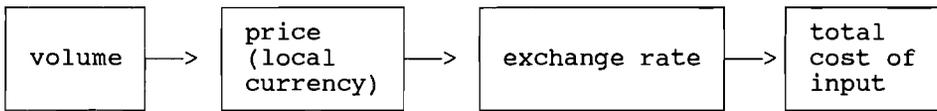
Figure 4.2 Sequential risk structure



the foreign exchange rate. Hence, the risks are situated in a sequence. Naturally, the risks may be correlated as well. The first risk in the sequence will henceforth be referred to as risk number one, and the second risk as number two. In a sequential risk structure, the risks are multiplicative. The concept gathered risk consists of a set of sequential risks.

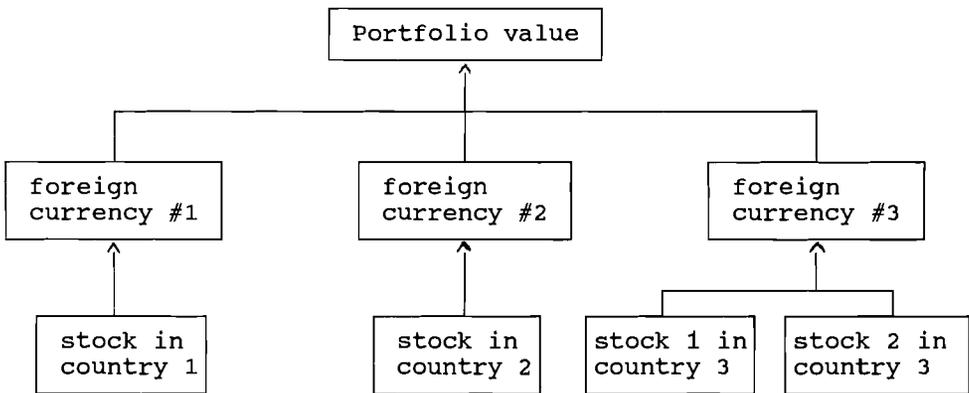
The total sequential risk may involve more than two elements. Take for instance a tender offer situation where the price of the input necessary to honour a potential contract is uncertain. The cost side of the tender offer is then submitted to the following risk structure.

Figure 4.3 Three-component sequential risk structure



The parallel risk structure and the sequential risk structure may naturally be combined in a number of ways. Consider for instance a portfolio of foreign stock.

Figure 4.4 Combined sequential and parallel risk structure



4.3 Hedging Possibilities in a Two-Component Sequential Risk Setting

When making a hedging decision in a two-component sequential risk setting, two issues arise. Firstly, due to the fact that there are two risks involved, the hedge may be targeted against both risks equally, against merely one of the risks, or against both risks but to an unequal extent. Consequently, by choosing the hedging method, any remaining risk may be allocated almost arbitrarily between the two risk components.³ (See Section 4.3.1.) Secondly, if the position is to be hedged equally, either simple instruments or gathered instruments may be applied due to the sequential risk structure. Which of these two alternatives is to be applied? The fundamentals of a gathered instrument will be outlined in Section 4.3.2. The gathered option will be addressed in Section 4.4.

4.3.1 Hedging Allocation Possibilities

If the alternatives of hedging allocation are limited to the dichotomy: hedge both risks equally, and hedge only one of the risks, the alternatives in table 4.1 will emerge if using forward contracts and options as hedging methods. The difference between the hedging principles is best illustrated by a contractual position to sell a foreign stock at a specific point in time, and which is hedged by forward contracts. (Risk #1: stock price risk, risk #2: foreign exchange risk.) Hedging of both risks

Table 4.1 Allocation alternatives of hedge in a two-component sequential risk setting.

Method	Risk #1	Risk #2	Comments
1	-	-	hedge none of the risks
2a 2b	forwards options	- -	hedge the first risk only
3a 3b	- -	forwards ⁴ options	hedge the second risk only
4a 4b	forwards options	forwards options	hedge both risks by the same instrument
4c 4d	forwards options	options forwards	hedge both risks by different instruments

eliminates the risk totally as the final cash flow will equal the product of the two forward prices. The first risk may be hedged separately, where the certain amount of foreign currency remains susceptible to the foreign exchange risk. The second risk may not be hedged separately, however, as the amount of currency forward contracts to enter will be unknown.

4.3.2 Fundamentals of a Gathered Instrument

A gathered instrument is an instrument hedging against two sequential risks at the same time. A gathered goods/currency forward contract implies a forward to buy/sell foreign goods at a fixed price measured in domestic currency. A gathered goods/currency option implies an exercise price which is denominated in domestic currency.

A hedge using simple forwards for the first risk and second risk respectively will fix the outcome completely. Using a gathered forward with the forward price equal to the product of the two simple forward prices, renders an identical final cash flow. Hence, the two hedging methods are identical.

A hedge using a simple option for the first risk and the second risk respectively, will set a floor/ceiling for the future cash flow. By setting the exercise price of the gathered option equal to the product of the exercise prices of the simple options ($g_x = p_x \cdot e_x$), the same floor/ceiling is obtained. However, the final cash flow of the gathered option is a function of the exercise price and the product of the prevailing prices. Thus, an increase in one of the prevailing prices may be offset by a decrease in the other. The value of the simple options is a function of the exercise prices and the prevailing prices, compared separately. Hence, the final cash flow of a gathered option may differ from that of simple options.

4.4 Gathered Option Versus Simple Options

The purpose of this section is to analyze the new instrument introduced. In this way it will be shown that in a two component sequential risk setting, a gathered option strategy will dominate a simple option strategy, if risk reduction is desirable. Therefore, only gathered options need to be included in the following chapters.

The discussion will first focus on the final cash flow of the two strategies. Subsequent to this, the difference in premiums will be derived. Finally, the implication of the premium difference and the cash flow difference is discussed.

In order to compare a gathered option strategy to a simple option strategy, it must be assumed that the same floor/ceiling prevails. Denote the exercise price of the gathered option by g_x , of the simple options by p_x and e_x respectively.

$$(4.1) \quad g_x = p_x e_x$$

Furthermore, one unit of the gathered option (e.g. for one foreign stock) must be compared to one simple option on the first risk, for example one unit of stock, plus p_x units of options on the foreign currency.

4.4.1 Final Cash Flow of Option Strategies

Throughout this section the final cash flows of option strategies are derived exclusive of the option premiums inherent in an option strategy. The premiums will be discussed separately in Section 4.4.2.

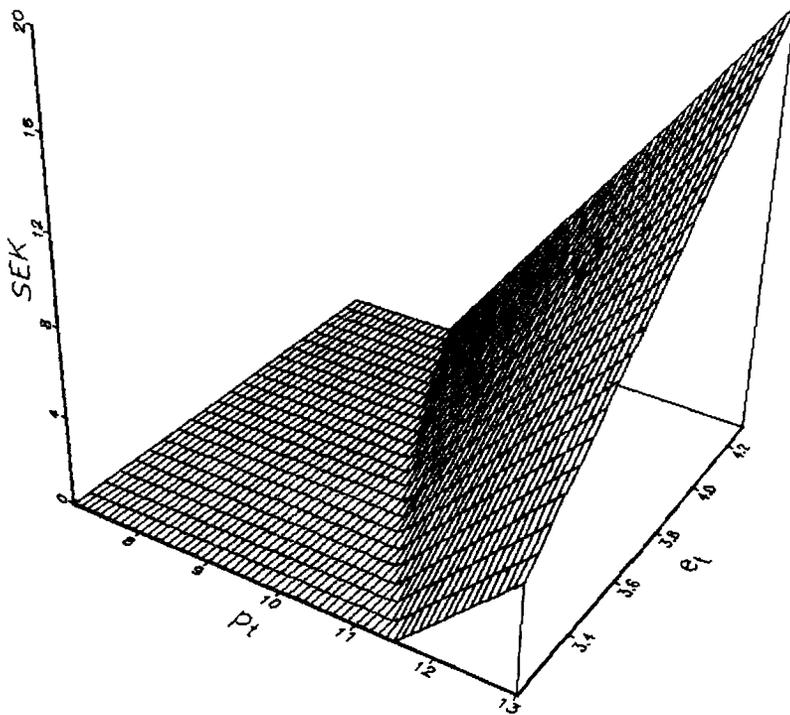
Call Option Strategy

The final cash flow of a gathered call option has the same form as that of a simple option, should the underlying stock price $g_t = p_t e_t$ be considered as one risk variable, equation (4.2). However, to be able to make a comparison, it is necessary to measure the cash flow as a function of each of the two variables, equation (4.3) or figure 4.5. Hence, the call will be exercised if $p_t e_t > g_x$. The limit corresponds to the projection of intersection between surface $CF = p_t e_t$ and horizontal surface $CF = g_x$.

$$(4.2) \quad CF = \max[0, g_t - g_x]$$

$$(4.3) \quad CF = \max[0, p_t e_t - p_x e_x]$$

Figure 4.5 Final cash flow of a gathered call option⁵
 ($p_X = 10, e_X = 3.6862$)



The final cash flow from a contractual position consisting of a future purchase of one unit of a foreign risky asset and a call option hedge is represented in (4.4). A comparable simple call option strategy consists of one option on the locally risky asset and p_X options on the foreign currency. The final cash flow will be according to equation (4.5) and figure 4.6. Hence, the domain of exercise of the option strategy is set separately for the two options, i.e. when $p_t > p_X$ and $e_t > e_X$ respectively.

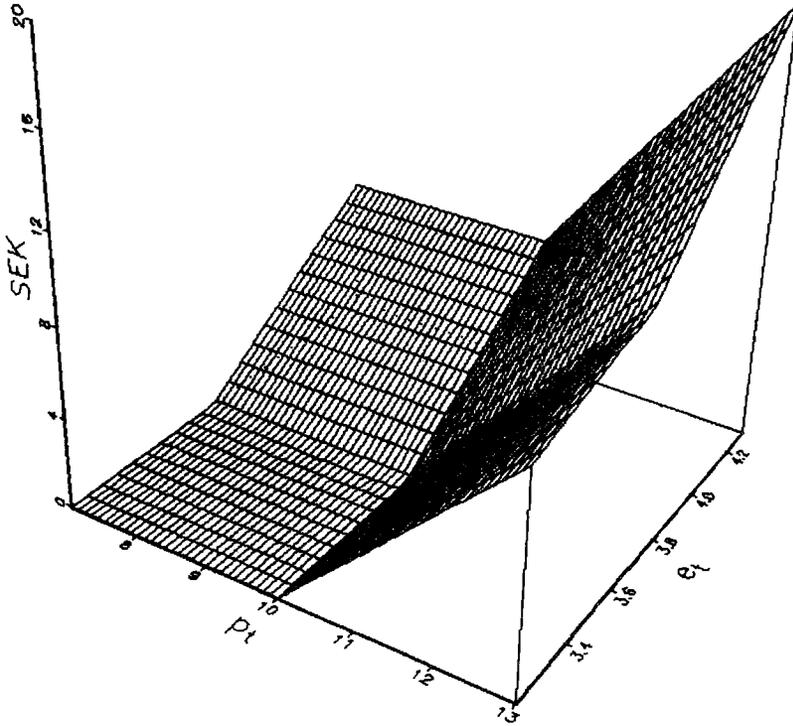
$$(4.4) \quad CF = -p_t e_t + \max[0, p_t e_t - p_X e_X] = \max[-p_X e_X, -p_t e_t]$$

$$(4.5) \quad CF = \max[0, p_t - p_X] e_t + \max[0, e_t - e_X] p_X$$

In a hedged contractual situation to purchase one unit of the foreign asset, the cash flow will follow equation (4.6).

$$(4.6) \quad CF = -p_t e_t + \max[0, p_t - p_X] e_t + \max[0, e_t - e_X] p_X$$

Figure 4.6 Cash flow of a simple call option strategy
 ($p_x = 10$, $e_x = 3.6862$)



The final cash flow from a gathered call option will differ from that of a simple call option strategy due to the fact that there are some occasions when it is optimal to exercise only one of the simple options. (See Appendix A.3.) The differences will occur for the combinations marked in figure 4.7. In combinations two and three, the cash flow from the simple option strategy (pure option position or combined with contractual position) will be higher than for a gathered option strategy. In combinations one and four, the final cash flows will be identical. It is important to note, however, that the gathered option strategy always satisfies the condition of generating a maximum cost for the future purchase of the foreign asset. The surplus from a simple call option strategy is given in table 4.2.

Figure 4.7 Combinations of p and e when cash flow will differ.
 (Numbers as in table 4.2.)

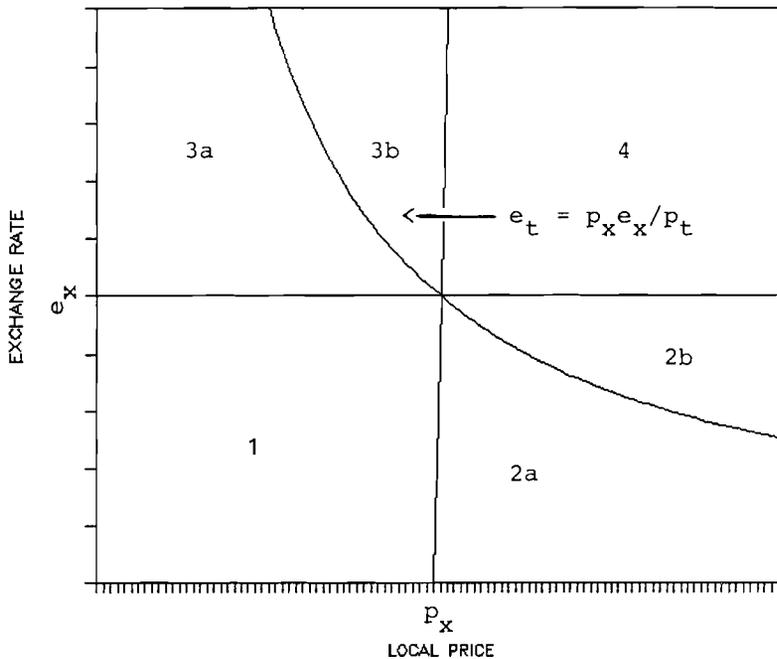


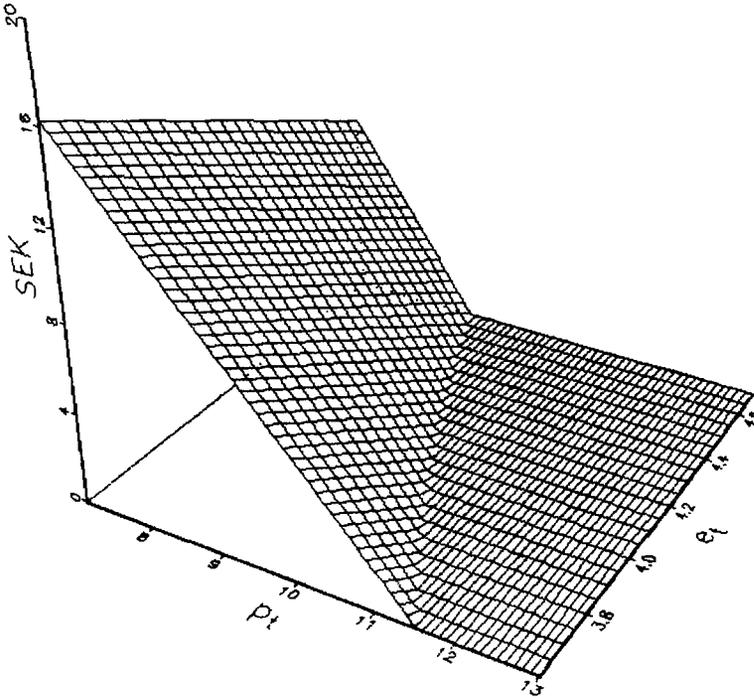
Table 4.2 Surplus cash flow from a simple call options strategy compared to a gathered call options strategy

outcome	surplus
1	-
2a	$(p_t - p_x) e_t$
2b	$(e_x - e_t) p_x$
3a	$(e_t - e_x) p_x$
3b	$(p_x - p_t) e_t$
4	-

The final cash flow of a gathered put option is given in equation (4.7) and in figure 4.8. The domain of exercise consists of the area where $g_x > g_t$. The bordering line, $g_x = g_t$, is given by the projection of the intersection between surface $CF = p_t e_t$ and the horizontal surface $CF = g_x$.

$$(4.7) \quad CF = \max[0, g_x - g_t] = \max[0, p_x e_x - p_t e_t]$$

Figure 4.8 Final cash flow of a gathered put option
 ($p_x = 10, e_x = 4.1440$)



A contractual situation to sell one unit of the foreign asset renders the following cash flow if hedged by put options.

$$(4.8) \quad CF = p_t e_t + \max[0, p_x e_x - p_t e_t] = \max[p_x e_x, p_t e_t]$$

The cash flow of a corresponding simple put options position will be according to the equation (4.9) and figure 4.9. The domain of exercise is set separately for the foreign asset option ($p_t < p_x$) and the currency option ($e_t < e_x$). In a hedged contractual situation to sell one unit of the foreign asset, the cash flow will be according to (4.10).

$$(4.9) \quad CF = \max[0, p_x - p_t] e_t + \max[0, e_x - e_t] p_x$$

$$(4.10) \quad CF = p_t e_t + \max[0, p_x - p_t] e_t + \max[0, e_x - e_t] p_x$$

The put option strategies will differ in final cash flow, exempt of premium, for the same combinations of p and e as the call option strategies, figure 4.7. The cash flow surplus is given in table 4.3. (See Appendix A.3, table A.2 for the cash flow of each outcome.)

Figure 4.9 Final cash flow of a simple put option strategy
 ($p_x = 10, e_x = 4.1440$)

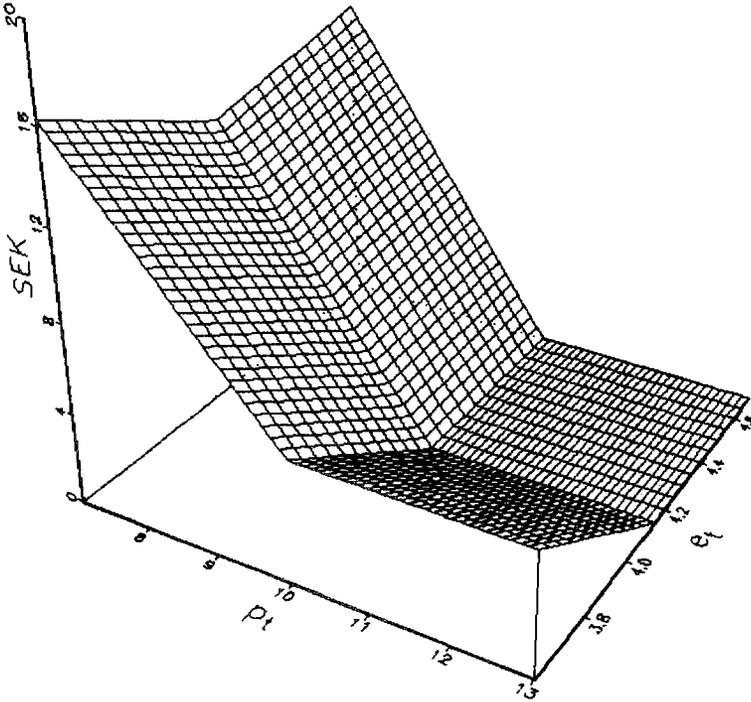


Table 4.3 Surplus cash flow from a simple put options strategy compared to a gathered put options strategy

outcome	surplus
1	-
2a	$(p_t - p_x)e_t$
2b	$(e_x - e_t)p_x$
3a	$(e_t - e_x)p_x$
3b	$(p_x - p_t)e_t$
4	-

In combinations one and four, the cash flow will be identical for the two strategies. In combinations two and three, however, the cash flow of the strategy using simple options will be higher than that of the gathered option strategy. As the gathered options strategy satisfies the initial condition of a floor for the cash flow, the difference in cash flow constitutes a surplus. In a risk reducing perspective, it carries no significance.

In comparing tables 4.2 and 4.3 it is seen that irrespective of

the outcome, the surplus will be equal for a put option strategy and a call option strategy.

4.4.2 The Premium

The premiums of each of the options were derived in Chapter Three. The total initial cash flow of a gathered option strategy equals C_g or P_g respectively. The total amount of premiums of a simple option strategy amounts to $p_x C_e + C_p e_0$, or $p_x P_e + P_p e_0$ respectively. Hence, the difference amounts to:⁶

$$(4.11a) \quad C_{diff} = p_x C_e + C_p e_0 - C_g$$

$$(4.11b) \quad P_{diff} = p_x P_e + P_p e_0 - P_g$$

4.4.3 Concluding Remarks

The simple options strategy renders a final cash flow which is higher than, or equal to, the final cash flow of a gathered option strategy. Consequently, the premium must be higher.

The difference in premium may be interpreted as a premium for an option giving the right to exercise only one of the simple options, compared to the gathered option alternative. This is equivalent to saying that the premium difference constitutes the premium of an option giving a final cash flow (with appropriate probabilities) according to table 4.2 and table 4.3 respectively. Hence, the difference in premium constitutes the price of increasing the upside potential of a position, without any impact on the floor/ceiling. In a risk-reducing context, the premium difference is purely speculative.

The speculative proportion of a simple options hedge compared to a gathered option hedge may be quantified through the premium difference:

$$(4.12a) \quad \frac{C_{diff}}{p_x C_e + C_p e_0}$$

$$(4.12b) \quad \frac{P_{diff}}{p_x P_e + P_p e_0}$$

The simple options method will not be considered in future chapters.

Notes

1. The effect of using two simple instruments will differ significantly from the effect of using a gathered instrument.
2. For instance, the price of gold or oil denominated in USD for an organization, which measures its performance in a non-dollar currency. See Maloney [1990].
3. The possibility of dividing the risk between two items will also occur in a tender offer situation. There, the basis for division is time. (To be discussed in Chapter Nine.)
4. Note that risk number two may never be eliminated as long as risk number one remains unhedged. This is due to the fact that the amount of forward contracts to enter is unknown.
5. If the premium is included, the surface is lowered vertically by the capitalized value of the premium.
6. See Lagerstam [1989a] for an analysis of the sensitivity of the difference.

Part Two
Dynamic Hedging Methods
and Substitutes

5 General Discussion and Non-Substitute Dynamic Hedging Methods

5.1 Introduction

Dynamic asset allocation methods abound on the market. The models developed are targeted for hedging a portfolio of assets, i.e. a long present position. Should the long position not arise until later, i.e. a long future position, the dynamic hedging methods may be applied anyway. The modifications necessary will be discussed. For foreign currency for instance, interest is explicitly received in a long present position (bond) but not in a long future position. Furthermore, the literature does not treat any dynamic hedging methods for short positions, except for the option based portfolio insurance replicating call options. The methods will be extended to incorporate this position. The methods developed in the literature concerns domestic stock.¹ No alterations are needed to transform the methods of dealing with other domestic risky assets.² Neither dynamic hedging methods for foreign currency nor for foreign risky assets are treated in the literature. This will be done in this chapter, and it constitutes the last modification necessary in order to be able to initiate the dynamic hedging as an alternative in the decision support method developed in Part Three.

The discussion in Chapter Five will focus on three of the most common non-substitute dynamic hedging techniques in the literature, and which have a protective impact on a position; the constant proportion portfolio insurance, the time-invariant portfolio protection, and the stop-loss method.³ The general aspects of dynamic hedging such as classification of the methods, general properties, how a floor or ceiling is set, and potential implementation problems are also discussed. In Chapter Six, dynamic hedging methods, whose value profiles mimic those of the hedging tools, are discussed.

5.2 Classification and Properties of Dynamic Hedging Methods

Classification

There is no single system of classification which encompasses all the different dynamic hedging methods. The most illustrative way to classify, is according to whether the payoff curve for a long position is convex, concave, or if it is a straight line.⁴ A strategy resulting in a convex value curve represents a purchase of portfolio insurance.⁵ Examples of dynamic portfolio insurance methods are: constant proportion portfolio insurance, time-invariant portfolio protection, and the replicating strategy option based portfolio insurance. If the initial portfolio is not rebalanced, a fixed asset allocation strategy, its value at the end will be a linear function of the price of the risky asset and the amount initially invested in it. An example of a strategy having a linear value is the buy-and-hold. With a wider definition the stop-loss strategy may be included, albeit two straight value lines.⁶ The value curve will be concave, should the initial long underlying position be rebalanced in such a way that the risky asset is bought during a decline and sold during a rise in market price. No downside protection exists and the upside potential is low. This is equivalent to selling portfolio insurance.⁷ Examples of value-based strategies are the constant-mix strategy and selling option based portfolio insurance.⁸ As the focus is upon buying insurance, strategies having concave value curves will not be discussed further.

Properties

There are three properties according to which different hedging methods may be measured, namely: path-independence, time-invariance, and process-freedom.

Path-independence implies that the value of a portfolio is independent of the particular path taken by the portfolio over the course of the hedging period.⁹ The strong form states that the value is completely independent of the behaviour of the price of the asset. The weak form of path-independence says that the value is dependent on the volatility of the price during the hedging period. The strong form of path-independence is a prerequisite for complete portfolio insurance as the value must be a function solely on the price of the risky asset at the end of the period. The distinction between the strong form of path-independence on the one hand and the weak form of path-independence on the other hand has a bearing on calculation method applied in Part Three. If the final value is stochastic, conditional on the stochastic final price of the asset, a simulation approach must be applied. Otherwise, a semi-analytic approach may be applied. Examples of the strong form of path-independent strategies are the buy-and-hold strategy and the option based portfolio insurance. The

constant proportion portfolio insurance is of the weak form. Finally, the stop-loss and the time-invariant portfolio protection are path dependent.

A strategy is said to be time-invariant if it does not "depend on a fixed time horizon, or on the time remaining in the programme"¹⁰. The buy-and-hold, the CPPI, the TIPP and the stop-loss strategies are time-invariant. The option based portfolio insurance is time dependent.

A strategy which does not explicitly depend on any stochastic processes assumed to govern the value of the portfolio, is defined as process-free. Bookstaber & Langsam [1988]¹¹ argue that a process-free strategy is inferior to a strategy incorporating the extra information contained in a correctly stated stochastic process. The buy-and-hold, the CPPI, the TIPP and the stop-loss are process-free.

5.3 Setting of the Floor and of the Ceiling

The purpose of setting a floor is to ascertain that the value of a position does not fall below an acceptable level, either continuously during a period of arbitrary length (potentially infinite), or at a fixed point in time. The same idea governs the ceiling. It serves as a restraint for how high costs are allowed to rise. Due to the obvious analogy between setting a floor and setting a ceiling, only the former will be discussed in the remainder of this section.

The continuous floor serves as a momentaneous triggering point for rebalancing of the position, and it will be called the momentaneous triggering level. In order to meet the floor at a future point in time, the portfolio may have to be rebalanced in advance. The triggering level is consequently set in order to meet the floor at the future point in time, and it is called the anticipatory triggering level. The two triggering levels may coincide, but on the other hand they may not.¹² If the floor is fixed only for a specific point in time, it has an anticipatory triggering level,¹³ but it does not have a momentaneous triggering level for any time separate from the point of time of restriction.

A floor set at a specific future point in time may have two origins. Firstly, it may result from a need to meet an obligation. The level of the floor must then at least equal the obligation, corrected for potential interest rate. The lowest possible level of the floor is consequently set exogenously. Secondly, the floor may result from a wish to sell a good, whereby the floor will be set entirely by the seller. There is no theoretical basis for setting the floor as long as it is between

zero and one hundred percent of the initial position and as long as it is, if possible, above a potential, exogenously given limit.

In the following discussions, the floor is assumed to rise at the rate of the domestic risk-free interest rate.¹⁴ Hence, the question whether it is of the anticipatory or momentaneous sort is irrelevant.

Denote the floor at time t by $FL[t]$, and at the end of period by $FL[t_x]$. In denoting the domestic risk-free interest rate by r_D , the relationship may be formalized as equation (5.1).

$$(5.1) \quad FL[t] = FL[t_x] \cdot e^{-r_D(t_x-t)} \quad ; \quad t \leq t_x$$

5.4 Potential Implementation Problems

When applying dynamic hedging methods, there are some aggravating circumstances in addition to the caveats implied in Chapter Three. The first problem is inherent to the prerequisite of dynamic hedging, i.e. the opportunity of continuous adaption of the portfolio. Should a sudden decline in the price of the risky asset occur, the trading technique might fail due to lack of time for trading. A similar problem arises, should the markets be discontinuous.¹⁵ A discrepancy between the capitalization factor of the floor and the payoff of a risk-free asset may cause a violation of the floor. Furthermore, if non-identical instruments are used as substitutes in an offsetting transaction¹⁶, an imperfect correlation between the values of the two assets may impair the usefulness of the trading process.¹⁷ Finally, option replication requires that the volatility is certain,¹⁸ as the price of an option depends upon the expected volatility, i.e. the ex ante volatility. Should a change in volatility occur, the payoff will, ceteris paribus, remain the same for an option. Hence, the owner is insensitive to changes in the volatility.¹⁹ However, in a dynamic strategy, the option 'copied' will depend on the realized volatility.²⁰

5.5 Constant Proportion Portfolio Insurance

In the literature, the constant proportion portfolio insurance is only analyzed for long present positions, i.e. it is mainly treated as an asset allocation strategy. In order to be able to apply the strategy in the more general hedging context in part three, the formulas/trading rules must be extended in three ways:

to include foreign currency and foreign risky assets, to accommodate long future positions, and finally to accommodate short positions. The CPPI may be constructed to act on the sum of a short and a long subposition. But in order to have the method comparable to an option or forward hedge, the CPPI is assumed to treat each subposition separately.

In a long present position, the principle idea of the CPPI is to increase the proportion of the total portfolio value held in risky assets as the price of the risky asset rises, and vice versa.

In a long future position, the CPPI is not directly applicable due to non-existence of portfolio to trade. By assuming an initial borrowing, the actor extricates himself. Successive to this, the actor may construct an initial portfolio, which in turn may be submitted to the CPPI strategy. At the end of the hedging period, the loan is repaid with interest, using the cash flow received inherent in the long future position. By borrowing the discounted value of the future cash flow, this position will turn out equivalent to the long present position.

In a short position, the risky position must be considered as the amount of risky assets short, not the amount of risky assets owned in the portfolio. Hence, a purchase of the risky asset reduces the risk as the short position is reduced. The adapted CPPI strategy implies that the risky asset is to be bought as the price rises and to be sold when the price falls.

All three positions are to be discussed as to the trading, to be referenced in Part Three.

Long Position

Consider a portfolio consisting of risky assets and risk-free assets. (For a long future position, the value of the debt will be subtracted in the 'value of the strategy'.) Denote the total value at time t of the portfolio by $V_{a,B}[t]$, the amount held in risk free assets by $V_B[t]$, and in risky assets $V_a[t]$.²¹ Define the cushion at time t , $c[t]$, as the difference between the total value of the portfolio and the floor, $FL[t]$. Furthermore, denote the multiplier by m . The CPPI states that the desired (subscript 'd') amount held in risky assets, i.e. after a reallocation of the assets, is a function of the multiplier²² and the cushion.²³

$$(5.2) \quad dV_a[t] = m \cdot c[t]$$

$$(5.3) \quad c[t] = V_{a,B}[t] - FL[t]$$

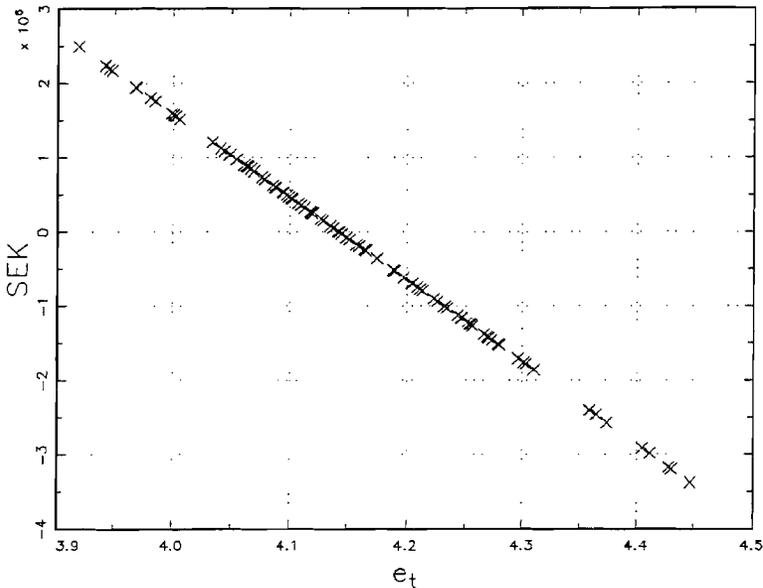
$$(5.4) \quad V_{a,B}[t] = V_a[t] + V_B[t] = dV_a[t] + dV_B[t]$$

A decrease in total value of the portfolio results in a proportionate decrease in the risky assets. If the total value falls to the level of the floor, the portfolio will consist only of risk-free assets. If the value of the portfolio increases very much, two different sorts of trading rules may be applicable. The

first one was introduced by Black & Jones [1987]. They apply a limit for the maximum percentage of the portfolio value to be invested in the risky assets.²⁴ Once the limit has been reached, no trading will take place until the desired percentage exposure falls below the limit again. The effect is that the CPPI will, temporarily, become a buy-and-hold strategy²⁵, i.e. to be "in a buy-and-hold state"²⁶. A second trading rule is not to impose a restriction, i.e. to allow unlimited leverage. Hence, if the desired exposure exceeded the total value of the portfolio, a short position in the risk-free asset would supply the means. A limit above 100% implies a speculative behaviour. Due to the non-speculative approach in this dissertation, a limit at 100% is assumed henceforth. Initiating a limit on the exposure prevents a derivation of the final value of the portfolio.²⁷ A simulation approach must be applied. As a conclusion, the CPPI is time-invariant, weak path-independent (path-dependent if limiting the leverage) and process-independent.

The value of the final position consists of the portfolio value for both the long present position and the long future position. The value of the CPPI strategy consists of the value of the final position subtracted by the value of an unhedged reference portfolio, $V_p[t_0]P_t/P_{t_0}$, $V_e[t_0]\exp[r_F(t-t_0)]e_t/e_{t_0}$, and $V_g[t_0]g_t/g_{t_0}$ respectively, as it is stochastic. Consider figure 5.1, the scatter of dots illustrating the final value of a CPPI-hedge on a long position, reallocated once a day. (Due to the

Figure 5.1 Value of a 100% limited CPPI hedge of long foreign currency position. (The floor is 1.17% below the initial value, $m=3$, 100 simulations. See Section 7.4.1 for the value of the remaining parameters.)



stochastic characteristic, a long position is best illustrated separately from a short position, i.e. not in a three-dimensional graph.) Due to the very restrictive floor level, 98.83%²⁸, and the conservative multiplier, 3, the strategy will resemble a buy-and-hold strategy as the difference between the CPPI and an unhedged strategy then roughly consists of 96.49% of the initial foreign position.²⁹ Hence, the seemingly linear curve. Note though, that at $E[e_t]=4.1440$, the value of the CPPI is negative due to the volatility cost.

Short Position

The CPPI may be adapted to accommodate a short position in risky assets. By purchasing the risky asset as the price rises, and selling when it falls, the risk will be reduced.

Denote the maximum cost allowed at time t to close the short position by $CL[t]$, the ceiling. Denote the accumulated costs incurred from buying/selling the risky assets by $ACCCOST[t]$,³⁰ and the costs necessary to purchase the remaining short position $REMCOST[t]$, the hypothetical purchasing cost. The total hypothetical cost equals the sum of the two aforementioned costs. Furthermore, let $c[t]$ denote the cushion and m the multiplier. The amount to leave unhedged ${}_dREMCOST[t]$ equals the discounted final short position to cover minus $m \cdot c[t]$.

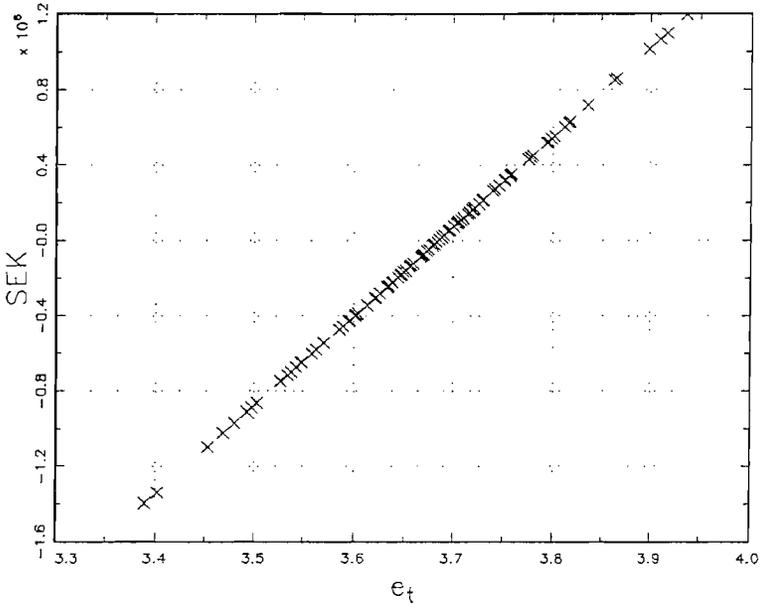
$$(5.5) \text{ COST}[t] = \text{ACCCOST}[t] + \text{REMCOST}[t]$$

$$(5.6) c[t] = CL[t] - \text{COST}[t]$$

$$(5.7) {}_dREMCOST[t] = q_2 e^{-r_F(t_X-t)} e - m \cdot c[t]$$

The value of the short position CPPI hedge consists of the difference between the cost incurred if using the CPPI and the cost incurred if purchasing the short assets for delivery at the end of the period, an unhedged reference position. Due to the restrictive ceiling, 1.17% above expected cost (to align with an option strategy), and the low multiplier, 3, the strategy will resemble a buy-and-hold strategy.³¹ Hence, the seemingly linear curve. The volatility cost may be seen in the figure at $E[e]=3.6862$. The position is corrected daily.

Figure 5.2 Value of a 100% limited CPPI hedge of short foreign currency position. (The ceiling is 1.17% above the initial value of the position, $m=3$, 100 simulations. See Section 7.4.1 for the values of the remaining parameters.)



5.6 Time-Invariant Portfolio Protection

The time-invariant portfolio protection (TIPP) resembles CPPI to a large extent. The formulas for trading are identical. The difference however, is how the floor and the ceiling are set. The level of the floor is changed so that it will be a prespecified percentage of the highest total value the portfolio has reached.³² Analogously, the ceiling is changed so that it will be a prespecified percentage above the lowest total cost the short position has ever reached.³³ Apart from this adaption, the floor and ceiling will be increased by the domestic interest rate. The idea is that the floor/ceiling should be based on an 'alternative cost mentality' or 'what could have been achieved' mentality, rather than on a fundamental analysis. Once the floor/ceiling has been set, the portfolio is balanced according to the same principles as the CPPI, using the cushion and the multiplier.

Estep & Kritzman [1988] consider the fact that CPPI may have to be levered as a problem. Furthermore, they state that this is evaded by using TIPP. It has not been pointed out, however, that the TIPP may require leverage if the percentage of the maximum value of the portfolio set as a floor is small and/or the multiplier large. Furthermore, some of the upside potential will be lost, which is not pointed out in the literature. A higher level of the floor results in a lower cushion and hence in a lower amount of risky assets.

TIPP is time-independent, process-free, but it is path-dependent.³⁴

The value of a TIPP-hedge, daily reallocated, is plotted in figure 5.3 for a long position and in figure 5.4 for a short position. Naturally, the 100% limit as for CPPI is maintained. The TIPP-value will be very similar to the CPPI value as the new, additional limits for the floor and ceiling are wide, considering the moderate volatility and time period.

Figure 5.3 Value of a 100% limited TIPP hedge of long foreign currency position. (The floor is 1.17% below initial value, or 10% below the highest value, $m=3$, 100 simulations. See Section 7.4.1 for the values of the remaining parameters.)

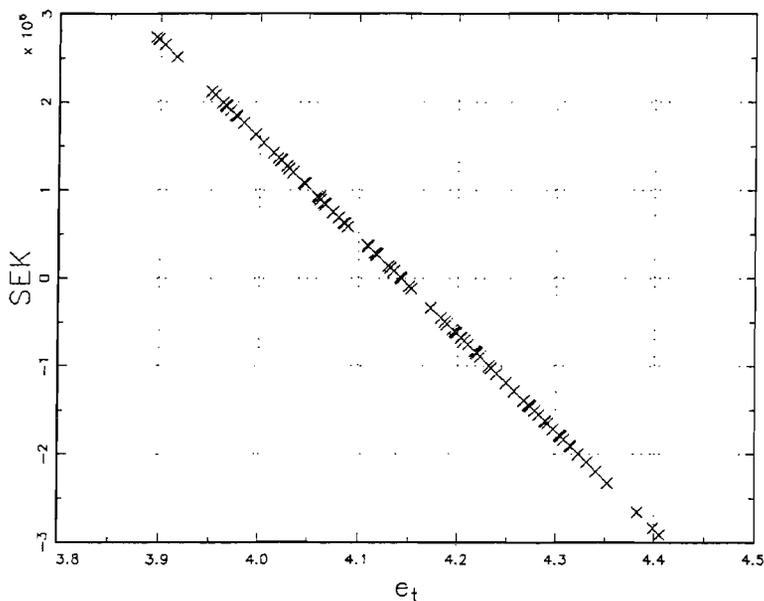
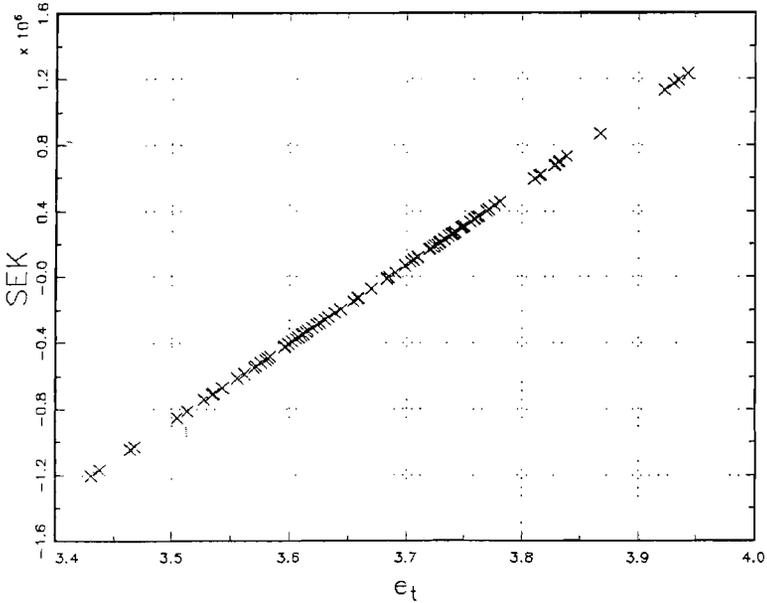


Figure 5.4 Value of a 100% limited TIPP hedge of a short foreign currency position. (The ceiling is 1.17% above initial cost or 10% above lowest cost obtained, $m=3$, 100 simulations. See Section 7.4.1 for the value of the remaining parameters.)



5.7 Stop-Loss

The principle of a stop-loss strategy is to be fully exposed initially. If, at any future point in time prior to the horizon, the value (cost) of the position falls to (raises to) the floor (ceiling),³⁵ the risky position is immediately, and permanently eliminated.³⁶ Hence, a stop-loss strategy involves, at the most, one single irreversible transaction during the hedging period.³⁷ Naturally, any interest accrued on domestic risk-free positions or foreign currency, assumed invested in locally risk-free assets, must be included.

The value of the stop-loss constitutes the difference between the hedged position and a unhedged reference position, see figure 5.5 and 5.6. The conditional probability, given the stochastic price, of having performed a stop-out is not known, however.³⁸ The evident exception is when the floor/ceiling has been exceeded, when the probability is 100%.

If the position consists of a long and a short subposition, there are two ways to define the stop-loss trading rule. The first way is to set a stop-loss rule for each subposition separately,

Figure 5.5 Value of a stop-loss hedge on a long foreign currency position.

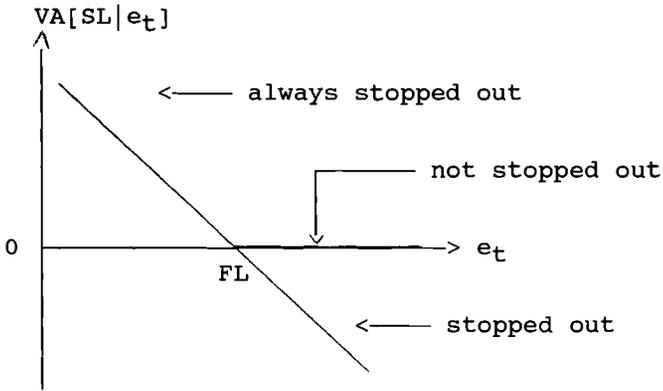
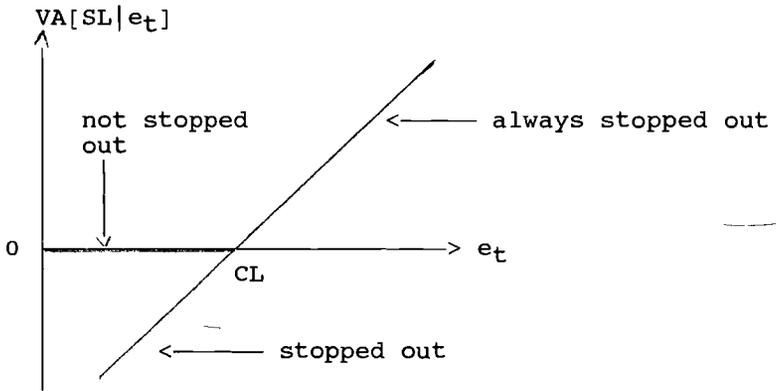


Figure 5.6 Value of a stop-loss hedge on a short foreign currency position.



'separate stop-loss rules'. The second way is to consider both subpositions together, and set a stop-loss for the sum of the values, 'total stop-loss rule'. The final value will differ if one or two of the separate stop-losses have fallen out and the total stop-loss has not. Henceforth, the separated stop-loss rule is applied, in order to align the method with a forward or option hedge. Furthermore, if a position consists of a foreign risky asset, the two risks involved may be hedged separately or together. This is analogous to the gathered option versus simple option hedging choice discussed in Chapter Four. The gathered approach is chosen.

To summarize, the stop-loss is time-invariant as the trading rule will not vary over time, it is path-dependent³⁹ and it is process-free.

Notes

1. Perold & Sharpe [1988], Bookstaber & Langsam [1988] and Estep & Kritzman [1988] for example.
2. As was stated in Chapter Two, the stochastic distribution of the value of a domestic risky asset is assumed to be lognormal.
3. The constant proportion portfolio insurance (CPPI) on the other hand, involves a continuous rebalancing according to prespecified trading rules. The proportion of risky assets in the portfolio is increased in a rising market and decreased in a falling market. The time-invariant portfolio protection is a modification of the CPPI strategy, as to how the lowest potential value of the position (the floor) is changed over time. A stop-loss strategy sets a lower limit for the value of the position. If this limit is exceeded, the risky assets will be disposed of in a long position and bought in a short position. The position will merely consist of one type of asset at a time. The buy-and-hold strategy involves an initial purchase or sale of a combination of assets. No rebalancing transactions take place during the hedging period. For this reason, it is a fixed asset allocation strategy. Due to its similarity with a forward contract, it will be discussed in Chapter Six.
4. Another way to group the methods would be according to the trading rules. If the amount of risky assets in a long position is a constant multiplier, m , of the difference between the value of the portfolio and the floor, the strategy would be called a 'constant proportion strategy'. Examples are the CPPI ($m > 1$), the B&H ($m = 1$), and the constant mix ($0 < m < 1$, floor = 0). (Perold & Sharpe, p. 17.) These strategies are also defined as linear. (See for instance Bookstaber & Langsam [1988], p. 17.)
5. If the portfolio is rebalanced to increase the content of risky assets when the value of the risky asset is increasing and vice versa, the portfolio will capture part of the upside potential and reduce the downside risk. Due to the trading rules inherent in a portfolio insurance strategy, the market movements are enhanced or maintained. Therefore, the methods are also called 'momentum' strategies. (Hill & Jones [1988], p. 29.)
6. A final rebalance may be triggered on an arbitrary point in time, whereby the payoff will consist of two alternative curves.
7. To offset this seemingly unprofitable strategy, the value during trendless, volatile periods is high. Strategies of this kind are called 'value-based' as the trade is based on the value of the risky asset rather than on the direction of the price movements.

8. In the former strategy, a constant relationship between the value of the risky asset and the value of the risk-free asset within the portfolio is held. The latter strategy is the opposite of the option based portfolio insurance (put), which will be discussed in Chapter Six.
9. Bookstaber & Langsam [1988] proved that "all path-independent strategies are option-replication strategies" (proposition 1, pp. 18-19, common letters are substituted by the author). This follows from the fact that "all payoffs may be created with the appropriate mix of option", that "option strategies are path-independent" and that any option value may be mimicked by dynamic trading.

Furthermore, Bookstaber & Langsam state that "a dynamic hedging strategy that has the potential for requiring the investor to be fully invested, or moving the investor to be fully cashed out, is a path-dependent strategy" (proposition 2, p. 19, capital letters replaced by the author). It is assumed that 'fully invested' implies that no further investment in the risky asset may be carried out. This restriction may be due to a limitation of the leverage necessary to carry out the required rebalance. Thus, the trading rule may no longer be adhered to, and the value will turn path dependent.

10. Bookstaber & Langsam [1988], p. 17.
11. Proposition 3, pp. 20-21.
12. A momentaneous floor is defined as a situation where a binding condition exists at each point in time during a period of, possibly, indefinite length. The momentaneous triggering level may increase at a speed different from the risk-free interest rate (i.e. the anticipatory triggering level) depending on the purpose of the floor. Consequently, the two triggering levels may not coincide. Should the momentaneous triggering level rise quicker than the risk-free interest rate, future obligations may only be honoured if the hedging period is fixed. The floor is met, despite an unprofitably large margin being held prior to the horizon of the hedging period. At any subsequent point in time, the value will have fallen below the floor. If the momentaneous floor rises slower than the anticipatory, the former one will always be the binding restraint.
13. In order to meet a floor at a future fixed point in time, the portfolio must be rebalanced in such a way that it will never fall below the discounted value (at the risk-free interest rate) of the final floor.
14. The identical assumption is made in Perold [1986] and Black & Perold [1987].
15. Granito [1988], p. 205.

16. For example futures to offset a position in the primary asset. Rubinstein [1985], p. 50.
17. Black & Jones [1987], p. 49.
18. See for instance Brenner & Galai [1989] for a suggestion of financial instruments to hedge against changes in volatility.
19. Hill, Jail & Wood [1988], p. 24.
20. Asay & Edelsburg [1986], p. 68. "The market extracts a higher price for the synthetic option if the ex post volatility is greater than the ex ante volatility irrespective of the accuracy of the hedge ratio." (Asay & Edelsburg [1986], p. 24.) This is also noted in Goodman, Ross & Schmidt [1985], p. 359.

"Proper implementation of the replicating dynamic asset allocation strategy retains full loss protection, but the upside capture now depends on the realized volatility over the year. The greater the volatility, the less the upside capture. This introduces a form of path-dependence into the outcome." (Rubinstein [1985], p. 49.)

21. The interest earned on the domestic risk-free assets will increase the value of V_B . The interest earned on the foreign currency (as the money is assumed to be invested) will augment V_A .

The portfolio may be considered as composed of two subportfolios. The division may be done in two ways. The first division partitioning the portfolio into a risk-free subportfolio and a risky subportfolio. The second division will let one subportfolio consist of the floor value in risk-free asset, and the other of leveraged risky asset. (Perold [1986], p. 1. The risky leveraged subportfolio is called "the surplus portfolio", p. 5.)

22. To ascertain a purchase when the price rises and vice versa, the proportionality factor (= leverage factor, m) must exceed unity. There is no theoretical foundation concerning the choice of multiplier, except for the condition of insurance stating that the multiplier must be larger than one. Three factors influencing the choice are indicated by Black & Rouhani [1987]. The value m should be higher the more risk-seeking the portfolio owner, the higher the expected payoff on the risky asset, and the lower the expected volatility. See also Black & Jones [1988]. A disadvantage following a high m is that the risk of violating the floor increases for sharp price changes. No violation occurs as long as the decline, without trading possibilities, is less than or equal to the inverse of m .

$$(\text{decline} \cdot V_A) \leq \text{cushion} = V_A/m \quad \Rightarrow \quad \text{decline} \leq 1/m$$

As m becomes larger and there is a limited leverage, the amount of time spent in B&H-state will approach 100%. If m is large, the portfolio will be fully invested initially. As the cushion goes to zero, the portfolio is transformed completely into risk-free assets. Hence, we have a stop-loss strategy. (Black & Perold [1989], p. 16.)

23. Black & Jones [1987], p. 48, and Perold & Sharpe [1988], p. 22.
24. See also Black & Rouhani [1987] and Black & Perold [1987, 1989].
25. See Appendix A.5.
26. Black & Perold [1989], p. 11.
27. Perold & Sharpe [1988], p. 27. Should no leverage limit be imposed, the conditional value of a domestic risky asset hedged by CPPI is derived as below stated. The loss due to volatility is called volatility cost or slippage. It is proportional to $\exp[(1-m)\frac{1}{2}m\sigma^2(t-t_0)]$. As is easily seen from the formula, a flat ($p_t \approx p_0$), oscillating (σ^2 is large) market is disadvantageous for the CPPI. If the market rise or fall in a straight forward manner, the CPPI will do well.

$$V_{p,B}[t] = FL[t] + (V_{p,B}[t_0] - FL[t_0]) \cdot (p_t/p_0)^m \cdot \exp[(1-m)(r_D + \frac{1}{2}m\sigma^2)(t-t_0)]$$

If a leverage limit is imposed, the trading will be reduced during the buy-and-hold state implying reduced/unchanged volatility cost. If the maximum leverage is 100%, no trading will occur at the buy-and-hold state. Hence, the volatility cost will be zero. For a leverage limit above 100%, the volatility cost will be larger than zero as trading is needed to keep the proper leverage. (Black & Perold [1989], p. 13.) The same applies if the leverage limit is below 100%, a fact which was not noted by Black & Perold. A further effect of limited leverage is the impaired upside potential.

28. To align the CPPI method with the option strategies, the floor at time t was chosen as the forward price minus the premium paid for the option, i.e. 98.83% of the forward price.
29. With a floor 1.17% below the initial value and a multiplier of 3, 3.51% is kept in the foreign currency. The reallocating characteristic of the CPPI is negligible in comparison to the difference to an unhedged strategy. Should the exchange rate rise to 4.3 SEK/CHF for instance, the value of the strategy should become roughly 11,510,937.96.49%(4.1440-4.3) \approx -1.7 MSEK.
30. Any proceeds from the holding will reduce ACCCOST. ACCCOST constitutes a loan in domestic currency.

31. The difference consists of approximately of $100 - 3 \cdot 1.17\% = 96.49\%$ of the initial value of the foreign exchange required. The value of the strategy would therefore equal approximately $4,924,630 \cdot 96.49\% (3.8 - 3.6862) \approx 541$ kSEK, should the foreign exchange rate rise to 3.8, for example.
32. Estep & Kritzman [1988].
33. As for the CPPI, should the underlying position consist of both a long and a short position, the TIPP is assumed to act on each subposition separately.
34. This path-dependence is the reason for the conclusion made by Choie & Seff [1989] that "TIPP's results are time-sensitive with respect to the starting date".
35. These are increased by the domestic risk-free interest rate.
36. The long future position is treated analogously to the one described in the CPPI strategy. Consider a portfolio consisting of a risky asset. The value of the portfolio constitutes the stochastic value of the asset. Suppose that the value of the portfolio is not permitted to fall below a specific, increasing level, the floor. One way to ascertain that this condition is met is to sell off the risky asset if its value falls to the floor and subsequently to invest the proceeds in risk-free assets. In a short position, the cost of acquiring the asset to be delivered later on, must not rise to the ceiling. If it does, the risky asset is purchased immediately. The maximum cost of the position has been limited.
37. A modified version of the stop-loss strategy involving a gradual transferral of funds from the risky asset to the risk-free asset was outlined by Bird, Dennis & Tippett [1988]. Hereby, the path-dependence is lowered.
38. The ex ante probability that no stop-out occurs before time t was derived by Black & Perold. (The esoteric derivation of the formula for stock is given in Black & Perold [1987], pp. 24-25. See also [1989].)
39. The level of path dependence has been measured by Rubinstein [1985] for a numerical example using a binomial distribution. The measure used was "the expected absolute deviation of the rate of return conditional of the level of the" (p. 45) price of the risky asset.

6 Substitutes

6.1 Introduction

The hedging tool selected by the decision support method presented here may not exist on the market. This may be the effect of for example an inappropriate maturity date, an inappropriate exercise price, lack of a correct underlying asset, or that market liquidity is so poor that trading is immobilized.¹ Thus, the problem stems from an imperfect or incomplete market. An opportunity for substitution will expand the prevailing options open to the actor, and enhance the realism and usefulness of the decision support method to be presented. Substitutes to the option on forward contracts and options on futures contracts are not discussed in the literature. Hence, the purpose of this chapter is partly to supply those derivations and partly to comment on the substitutability of the other instruments.

The purpose of a substitute is to "duplicate the gains and losses that would have been achieved by holding a hypothetical ... with the desired terms"². A perfect substitute must give the same return, involve the same initial investment and be self-financing.³ Henceforth, a synthetic implies a perfect substitution where continuous trading may be needed, whereas a replica will be defined as a perfect substitute where no trading is required subsequent to the initial transactions. The substitute may either be created through a combination of existing instruments or through dynamic hedging. Unless the payoff of the selected tool is replicable by a dynamic or a static method, the extrication may come from applying either a tool with non-coinciding maturity, or from applying a tool on a different primary asset, 'cross-hedging'.⁴

6.2 Application of a Tool with Non-Coinciding Maturity

If no instrument with the proper maturity exists, an instrument having a longer or a shorter maturity may be chosen instead. The disadvantage is that the ideally certain value at the target maturity may become uncertain. The issue will be exemplified by forward contracts, futures contracts, and options.⁵ Denote the

shorter maturity t_s , the target maturity t_t , and the longer maturity t_l , $t_0 \leq t_s \leq t_t \leq t_l$.

Forward Contract

Let a long forward position illustrate the effect of a deviation in maturity. The target value at time t_t is according to equation (6.1).

$$(6.1) \text{VA}[\text{FW}_a[t_t]; t_t] = a_t - \text{FW}_a[t_0, t_t] \\ \text{(long)}$$

If a longer contract is entered into, there will be no difference in value for the domestic risky asset and the foreign risky asset provided that interest rates do not change unexpectedly. By altering the number of forward contracts to enter, the target value will also be reached for foreign currency.⁶

On the date of maturity of a shorter contract, either a new forward contract is entered into (it is assumed that a contract exists which maturity corresponds to the time remaining to the target maturity), or the position is left unhedged. If a new forward contract is not entered into, any subsequent changes in the spot price will change the final value compared to the target value.⁷ If a new forward contract is entered into, any unexpected change in the price will be compensated for by the new forward contract. Adaption of the number of forward contracts traded is needed for currency.⁸ Naturally, a change in the interest rate will alter the final value.

Futures Contract

The target value of a long futures contract is with daily settlements, i.e. including the interest accrued on the cash flow from the resettlements:

$$(6.2) \text{VA}[\text{F}_a[t_0, t_t]; t_t] = \\ = \sum_{t_i = t_0}^{t_t-1} (\text{F}_a[t_{i+1}, t_t] - \text{F}_a[t_i, t_t]) e^{r_D(t_t - (t_i+1))}$$

If a longer maturity futures contract is entered into, the target final value may be obtained by correcting the number of contracts traded.⁹ The interest rate must remain unchanged. If a shorter maturity is chosen, a new futures contract may be entered into when the first contracts mature. Should new contracts be entered into, the target value may be obtained, but not otherwise.¹⁰

Option

The target value inclusive of the premium of a call option and of a put option is:

$$(6.3a) \quad \max[0, a_t - a_x] - C_a[t_t] e^{r_D(t_t - t_0)}$$

$$(6.3b) \quad \max[0, a_x - a_t] - P_a[t_t] e^{r_D(t_t - t_0)}$$

An option having a longer maturity does not provide full protection. The reason is that the option price at the target maturity date will not equal the difference in spot price compared to the expected spot price.¹¹ Should a shorter maturity be chosen, the remaining period may be left unhedged, which incurs extra risk,¹² or a new option may be purchased. In the latter case, the target value is not obtained due to the non-linearity of the option premium. The value is the sum of the value of two successive options, capitalized to time t_t .

6.3 Cross-Hedging

Cross-hedging is defined as hedging "with instruments that are not locally perfectly correlated with"¹³ the desired instrument. The cross-hedge may either consist of instruments on one single different sort of asset, a single cross-hedge, or it may consist of instruments on multiple different assets, a multiple cross-hedge. The cross-hedge is inferior due to the non-perfect correlation, and it should only be considered if "there are no futures or forward markets in a currency"¹⁴. The absolute dominance is only valid for markets without transaction costs. Should these differ between a perfect hedge and an imperfect cross-hedge, the risk of the cross-hedge might be balanced by a lower transaction cost.¹⁵ Another advantage of the cross-hedging technique is that it does not depend on the target market. Effects of deficiencies in the money market are consequently eliminated.

The effectiveness of cross-hedging is given by the correlation between the prices of the assets. The correlation will influence the probability of obtaining a lower value at the horizon date than if a perfect hedge had been applied. Consider the situation of selling one unit of the risky asset at a future point in time, t . A perfect same asset hedge would imply shorting a forward contract at $FW_a[t_0, t]$. Consequently, the final value will be the forward price. Should the hedge be obtained through a different asset forward, indicated by $'$, the final value will follow equation (6.4).¹⁶ The risk profile is given by the stochastic part. q_{FWa}' is the number of substituting forward contracts to enter into and constitutes a scaling factor the purpose of which is to equate the values under a perfect correlation assumption. The probability that the value will fall below an arbitrary level, $FL[t]$, may be obtained through (6.5).¹⁷

$$(6.4) \quad a_t + q_{FW_{a'}} (FW_{a'}[t_0, t] - a_t');$$

$$q_{FW_{a'}} = \frac{FW_{a'}[t_0, t]}{FW_{a'}[t_0, t]}$$

$$(6.5) \quad \text{prob}[a_t + q_{FW_{a'}} (FW_{a'}[t_0, t] - a_t') < FL[t]] =$$

$$\int_{[a_t, a_t': a_t - q_{FW_{a'}} a_t' < FL[t] - q_{FW_{a'}} FW_{a'}[t_0, t]]} \int \text{pdf}[a_t, a_t'] da_t da_t'$$

The cumulative distribution function may be obtained by calculating the integral for different floor levels. Note that by calculating the probabilities for different levels of $q_{FW_{a'}}$, the actor may choose a suitable hedging level for a correlation different from unity. (The principle is identical to the calculation method to be introduced in Chapter Seven.)

6.4 Synthetics and Replicas

Formulas for synthetics and replicas have been derived in the literature for forward contracts, futures contracts and options, with an emphasis on domestic risky assets. To the author's knowledge, options on forwards and options on futures have not yet been discussed, hence the more thorough derivations.

6.4.1 Forward Contract

The condition of non-existing arbitrage opportunity renders the strategy of entering a long forward contract tantamount to the strategy of buying the underlying asset plus borrowing the discounted value of the forward price (a partial buy-and-hold strategy).¹⁸ Thus, the latter strategy constitutes a perfect substitute to a forward contract.

6.4.2 Futures Contract

Due to the resettlements adherent to a futures position, the value profile will be dependent on the path followed by the futures price during the hedging period. (See equation (6.2).)

The formula of the futures price under constant interest rates was derived in Chapter Three. It is easily seen that the value from a futures contract may be duplicated by a borrowing and investment program. (See Appendix A.9.)

6.4.3 Option

An option may be substituted in one of two ways, depending on whether a corresponding put/call option (identical exercise price and time to maturity) exists or not. If it does, a static perfect synthetic (replica) may be created by using the put-call parity. Thus, buying a call option is tantamount to buying a put option, buying the underlying asset and borrowing the net present value of the exercise price. Analogously, a put option value may be mimicked by a long call, lending the net present value of the exercise price and a short position in the underlying asset. If no corresponding option exists, a dynamic approach may be applied. (See Rubinstein & Leland [1981]. See Appendix A.9 for derivations.)

6.4.4 Option on a Forward Contract

In deriving the put-call parity theory for an option on a forward contract, it was established that buying a put option is tantamount to buying a call, lending/borrowing money, and shorting a forward contract. The replicating call is derived analogously.

In absence of a corresponding put/call option, a dynamic approach must be applied. There are two possibilities, either to trade in the forward contract, or to trade in the primary asset.

Call Option Synthetic: Trading in the Forward Contract

Create a portfolio consisting of q_{FW} forward contracts (the time of initiation is irrelevant) and one call option on a forward contract having the forward price $FW_x[t_{FW}]$. The value is as below stated. ($t \leq t_x \leq t_{FW}$)

$$(6.6) \quad V_{FW_a, C_{FW_a}} [t] = q_{FW} VA[FW_a[t_{FW}]; t] + VA[C_{FW_a}[t_x]; t]$$

The portfolio is insulated to a change in the prevalent forward price if:

$$(6.7) \quad q_{FW} \frac{\delta VA[FW_a[t_{FW}]; t]}{\delta FW_a[t, t_{FW}]} + \frac{\delta VA[C_{FW_a}[t_x]; t]}{\delta FW_a[t, t_{FW}]} = 0$$

$$(6.8) \quad \frac{\delta VA[FW_a[t_{FW}]; t]}{\delta FW_a[t, t_{FW}]} = e^{-r_D(t_{FW}-t)}$$

Insertion of (6.8) into (6.7) renders the amount of long forward contracts used in a replicating strategy.

$$(6.9) \quad \frac{\delta VA[C_{FW_a}[t_x]; t]}{\delta FW_a[t, t_{FW}]} e^{r_D(t_{FW}-t)}$$

In substituting the fraction for the derivative, the number of long forward contracts to hold at time t is $N[x_{FW}^a]$.

Call Option Synthetic: Trading in the Primary Asset

The replicating portfolio of the primary asset may be derived in two ways, either by applying the forward replicating technique presented in Section 6.4.1, or by applying a portfolio protection argument.

A long forward contract may be replicated by borrowing $FW_x[t, t_{FW}] \cdot \exp[-r_D(t_{FW}-t)]$, and by buying one unit of the domestic risky asset/the foreign risky asset or $\exp[-r_F(t_{FW}-t)]$ units of the foreign currency. The number of units of the primary asset to hold become (6.10). (Set $r_F=0$ for p and g .)

$$(6.10) \quad N[x_{FW}^a] e^{-r_F(t_{FW}-t)}$$

To derive the relationship directly, consider a portfolio according to (6.11). The number of long assets to hold is according to (6.12), and is naturally equal to (6.11). (Set $r_F=0$ for p and g .)

$$(6.11) \quad V_{a, C_{FWa}}[t] = q_a a_t + VA[C_{FWa}[t_x]; t]$$

$$(6.12)$$

$$\frac{\delta VA[C_{FWa}[t_x]; t]}{\delta FW_a[t, t_{FW}]} \cdot \frac{\delta FW_a[t, t_{FW}]}{\delta a_t} = \frac{\delta VA[C_{FW}[t_x]; t]}{\delta FW_a[t, t_{FW}]} e^{(r_D-r_F)(t_{FW}-t)}$$

Put Option Synthetic

A portfolio replicating a put option using forward contracts may be constructed analogously. The number of forward contracts to hold is $(N[x_{FWa}^a]-1) < 0$. If the replicating portfolio consists of the primary asset, the number of units to acquire is according to (6.13). (Set $r_F=0$ for p and g .)

$$(6.13) \quad (N[x_{FWa}^a]-1) e^{-r_F(t_{FW}-t)}$$

6.4.5 Option on a Futures Contract

In replicating an option on a futures contract, the same two methods may be used as for options on forward contracts. Firstly, following the derivation of the put-call parity, buying a

put option is equivalent to buying a call, to lending/borrowing money, and to entering into a short futures contract. The strategy for replicating a call is derived analogously. Secondly, a dynamic approach may be applied by trading in the futures contracts or in the primary asset.

Call Option Synthetic: Trading in the Futures Contracts

The derivation of the substituting portfolio of futures is analogous to the derivation of the corresponding portfolio for options on forward contracts. Only the differences will therefore be pointed out.

The change in value of a futures contract with respect to a change in the futures price is unity, $\delta VA[F_a[t, t_{FW}]; t] / \delta F_a[t] = 1$, should there be continuous settlement or if a sale of the contract renders an immediate cash flow. Hence, the number of futures contracts to hold is $\delta VA[C_{Fa}[t_x]; t] / \delta F_a[t]$,¹⁹ which equals:

$$(6.14) \quad N[x_F]_a e^{-r_D(t_x - t)}$$

Call Option Synthetic: Trading in the Primary Asset

Application of the futures replicating technique in Section 6.4.2 renders: (Set $r_F = 0$ for p and g.)

$$(6.15) \quad N[x_F]_a e^{r_D(t_F - t_x) - r_F(t_F - t)}$$

The same result is obtained by creating a portfolio of q_a primary asset and one call option, equation (6.16). The number of long primary assets is according to (6.17). Inserting the derivative of $\delta F_a[t_x] / \delta a_t$ and the equation (6.14) renders (6.15).

$$(6.16) \quad V_{a,C} [t] = q_a a_t + VA[C_F [t_x]; t]$$

$$(6.17) \quad \frac{\delta VA[C_{Fa}[t_x]; t]}{\delta F_a[t, t_F]} \cdot \frac{\delta F_a[t, t_F]}{\delta a_t}$$

Put Option Synthetic

The equations for put options are identical to the call option replication, should $N[x_{Fa}]$ be substituted for $(N[x_{Fa}] - 1)$.

Notes

1. Grannis [1988], p. 50.
2. Hill & Jones [1988], p. 34. This differs from the statement by Greenleaf [1989], p. 71, "the modern synthetic ... is not aimed at hedging, however, but at creating a new financially positive investment or borrowing situation".
3. Rubinstein & Leland [1981], p. 64.
4. As was noted by Shalen [1989], p. 216, using tools with non-coinciding maturities may be defined as a sort of cross-hedging.
5. The implication of a non-coinciding date of maturity for options on forward contracts and options on futures contracts is analyzed analogously to an option on the primary asset. The value of the forward/futures contract received will be a variable of interest.

6. The value of a long contract is: (Set $r_F=0$ for p and g.)

$$\begin{aligned}
 VA[\text{FW}_a[t_0, t_1]; t_t] &= e^{-r_D(t_1-t_t)} (\text{FW}_a[t_t, t_1] - \text{FW}_a[t_0, t_1]) = \\
 (\text{long}) & \\
 &= (a_t - \text{FW}_a[t_0, t_t]) e^{-r_F(t_1-t_t)}
 \end{aligned}$$

A comparison with the target value shows that there is no difference in value for the domestic risky asset and the foreign risky asset. However, there is a stochastic loss (profit) from having too long a contract in foreign currency, amounting to:

$$(e_t - \text{FW}_e[t_0, t_t]) (1 - e^{-r_F(t_1-t_t)})$$

By trading $\exp[r_F(t_1-t_t)]$ units of the contract instead of one unit, the payoff will be identical, as the loss pertains to the deficiency of contracts, $\exp[r_F(t_1-t_t)] - 1$. Hence, full protection is obtained.

7. When not entering into a new contract, the value of the initial long contract at the target date, t_t , is: (Set $r_F=0$ for p and g.)

$$\begin{aligned}
 VA[\text{FW}_a[t_0, t_s]; t_t] &= e^{r_D(t_t-t_s)} (a_s - \text{FW}_a[t_0, t_s]) = \\
 (\text{long}) & \\
 &= a_s e^{r_D(t_t-t_s)} - \text{FW}_a[t_0, t_t] e^{r_F(t_t-t_s)}
 \end{aligned}$$

The stochastic difference in value compared to the target value stems from the difference between a_t and a_s . (The other difference may be eliminated by changing the number of contracts entered into.) Remember that the position is

unhedged during $[t_s, t_t]$. The stochastic loss of having too short a contract becomes: (Set $r_F=0$ for p and g.)

$$a_t - a_s e^{r_D(t_t-t_s)} + FW_a[t_0, t_t] (e^{r_F(t_t-t_s)} - 1)$$

8. If a new contract is entered into, the value at time t_t will be: (Set $r_F=0$ for p and g.)

$$VA[FW_a[t_0, t_s]; t_t] + VA[FW_a[t_s, t_t]; t_t] =$$

$$= e^{r_D(t_t-t_s)} (a_s - FW_a[t_0, t_s]) + a_t - FW_a[t_s, t_t] =$$

$$= a_t - FW_a[t_0, t_t] e^{r_F(t_t-t_s)} + a_s e^{r_D(t_t-t_s)} (1 - e^{-r_F(t_t-t_s)})$$

Any unexpected change in the price level will be compensated for by the new forward contract in a domestic or foreign risky asset position. However, the loss (profit) of entering a shorter contract in currency is stochastic and equal to:

$$(FW_e[t_0, t_s] - e_s) e^{r_D(t_t-t_s)} (e^{(r_D-r_F)(t_t-t_s)} - 1)$$

By trading in $\exp[-r_F(t_t-t_s)]$ units of the initial forward contract and one unit of the contract issued at t_s , the payoff will be identical to that of a target contract. It is therefore possible to obtain full protection. The loss (profit) according to the last equation pertains to the excess quantity of the first contract, $1 - \exp[-r_F(t_t-t_s)]$.

9. $VA[F_a[t_0, t_1]; t_t] = \{\text{set } r_F=0 \text{ for p and g}\} =$

$$= \sum_{t_i = t_0}^{t_t-1} (F_a[t_{i+1}, t_t] - F_a[t_i, t_t]) e^{r_D(t_t-(t_i+1))} \cdot e^{(r_D-r_F)(t_1-t_t)}$$

The factor $\exp[r_D(t_1-t_t)]$ or $\exp[(r_D-r_F)(t_1-t_t)]$ makes up the whole difference. If the factor is larger than one the absolute value of the payoff is increased, otherwise it is decreased. Hence, if not one contract but $\exp[-r_D(t_1-t_t)]$ or $\exp[-(r_D-r_F)(t_1-t_t)]$ contracts were bought, the value would become exact.

10. When no new futures contract is entered into, the value will be: (Set $r_F=0$ for p and g.)

$$VA[F_a[t_0, t_s]; t_t] =$$

$$= \sum_{t_i = t_0}^{t_s-1} (F_a[t_{i+1}, t_t] - F_a[t_i, t_t]) e^{r_D\{t_t-(t_i+1)\} - (r_D-r_F)(t_t-t_s)}$$

Thus, the difference partly constitutes the factor $\exp[-r_D(t_t-t_s)]$ or $\exp[-(r_D-r_F)(t_t-t_s)]$ respectively which can be eliminated by trading in an appropriate amount of contracts, and partly by the fact that the summation is done over a fewer amount of time periods. The latter fact is due to the non-hedged position over the time period $[t_s, t_t]$.

If entering into a new futures contract at t_s , and if $\exp[r_D(t_t-t_s)]$ or $\exp[-(r_D-r_F)(t_t-t_s)]$ units of futures contracts are entered into at time t_0 , and one unit of futures contracts entered into at time t_s , the value will be identical to the payoff from entering into one target contract initially.

11. The stochastic difference in value for call options are: (Put options are treated analogously.)

$$VA[C_a[t_t];t_t] - VA[C_a[t_1];t_t]$$

12. If no new contracts are entered into, the value will be at time t_t :

$$VA[C_a[t_s];t_t] = \max[0, a_s - a_x] e^{r_D(t_t-t_s)} - C_a[t_s] e^{r_D(t_t-t_s)}$$

$$VA[P_a[t_s];t_t] = \max[0, a_x - a_s] e^{r_D(t_t-t_s)} - P_a[t_s] e^{r_D(t_t-t_s)}$$

If $a_x \exp[-r_F(t_t-t_s)]$ (set $r_F=0$ for p and g) options are entered into, the risk pertains to the difference $a_s \exp[(r_D-r_F)(t_t-t_s)] - a_t$ and the difference in initial option price. Note that a_x above is the forward price for time t_s , whereas a_x in equation (6.3) is the price for time t_t .

13. Adler & Simon [1986], p. 51.
14. Eaker & Grant [1987], p. 85. The same idea is presented in Anderson & Danthine [1981].
15. Braga, Martin & Meilke [1989].
16. Naturally, a one-to-one hedge is not ideal unless the forward prices are equal.
17. See Appendix B.1.2 for the bivariate probability density function.
18. Compare to Appendix A.8. See Appendix A.9 for a derivation of the forward contract replica.
19. See Appendix A.6 for the derivations.

Part Three
Risks and Hedging

7 Risks and Choice of Decision Support Method

7.1 Introduction

The overriding purpose of Part Three is to present a design of a decision support method for hedging against foreign exchange rate risk and/or price risk. The method is described in general terms as well as illustrated for three different hedging positions.

Chapter Seven focuses on two subjects. The first one is to present the taxonomy of the risky position and to choose three positions to analyze by the hedging method. The second subject is to present the design of the hedging method and its rationale where the probability calculations are especially emphasized.

In Chapter Eight, the hedging method is applied to a contractual position and to an anticipatory position. The relevant hedging tools and dynamic non-replicating methods are applied separately. Two different approaches are used. The first one involves hedging by methods where the final values are deterministic conditional on the outcome of the stochastic variable, leading to an semi-analytical derivation of the probability distribution. The second approach involves hedging by methods where the final values are path dependent. Hence, the probabilities must be obtained by simulation.

In Chapter Nine, a further dimension in the problem of hedging is introduced. In a contractual position, the amount of assets to be traded is known initially. In an anticipatory position, the amount to be traded is assumed to be known at a future point in time coinciding with the final cash flow. However, in a tender offer position, the point in time when the amount of assets to be traded becomes certain lies in between the time of initiation and the time of the potential cash flow hence the probability change during the hedging period. This phenomenon has a bearing on the risk situation as well as the applicability of hedging tools. In order to analyze only the new aspects, the hedging methods applied are limited to the path-independent tools.

In Chapter Ten, a numerical example illustrates how a decision may be made using the decision support method (tender offer, two

foreign currencies, options and forward contracts, 100% forward hedge of potential contractual period). This example is followed by a description and an evaluation of an implementation of the method at ABB Asea Brown Boveri AB.

7.2 Classification and Choice for Analysis

7.2.1 Classification of the Risky Position

The concept of position encompasses all the commitments, anticipatory positions and hedging positions which are to be considered as one risk entity. Each position may consist of many subpositions signifying one of the above mentioned positions. When no confusion may arise, 'position' is occasionally used as a generic. An organization may therefore have an arbitrary number of risk positions, depending on how broadly each position is defined. Thus, the total corporate position consists of a sum of positions, each position consisting of a sum of subpositions. The subpositions are of two kinds depending on their origin, an underlying subposition (assumed exogenously given) or a hedging subposition (endogenous).

The classification may be made more specific¹ by taking into account seven factors affecting a risky subposition, four of them having a certain value and the remaining three an unknown value.

- | | |
|----------------|---------------------------------------|
| Unknown value: | 1. Size of each subposition |
| | 2. Time of transaction |
| | 3. Sort of asset |
| Certain value: | 4. Number of subpositions included |
| | 5. Direction of cash flow |
| | 6. Committing/non-committing position |
| | 7. Ownership of asset |

Size of Subposition

The size of the subposition may be measured in the local currency or in the domestic currency unit. The same division will emerge although a subposition may be differently positioned. The latter basis of division is chosen. The size is either known as to the probability distribution or not. In the former situation, four special cases of practical use may be derived. (This risk is also called the quantity risk.²)

A certain amount: e.g. a contract to buy or sell at a specific price.

Two alternative amounts: e.g. a tender offer which may be accepted or refused.

A known probability distribution: e.g. the intent to buy or sell a commodity at a future, stochastic price.

A known probability distribution combined to an alternative amount: e.g. a conditional purchase at a stochastic price on acceptance of a tender offer.

Time of Transaction

Relevant alternatives as to the time of transaction are:

A certain date: e.g. a purchase.

A known probability: e.g. a tender offer to a potential customer. (The time limits of tender offers are often disregarded by customers.) In this group the case of a potential selling of an asset is also subsumed.

Sort of Asset

A subposition may involve more than one asset, where the choice of asset is made by one of the contracting parties at a future point in time. Subpositions of this kind are for instance convertible bonds, the choice being stock and cash, or multicurrency loans where the final payback may be made in any of a prespecified set of currencies. In a business situation, a tender offer submitted in local currency units to several subsidiaries of a multinational corporation is another example if a purchase by one of the subsidiaries excludes any other purchase.³

Number of Subpositions Included

The number of subpositions to include in the position may span from one single transaction, e.g. a contracted sale or purchase, to an almost infinite number, e.g. budgeted sales and purchases in a globally active corporation. The factors influencing the decision are the following.

The time span of the position. This factor governs the period of time over which the aggregation will be made. A longer period will render a more comprehensive position, but on the other hand the autocorrelations and intratime dependencies between asset prices will impair the usefulness due to model complexity and computational difficulties.

The variety of assets to include, and the relationship between subpositions. The position may be limited to related subpositions, such as conditional purchases in a

tender offer. The position may also be constructed based on the similarity between subpositions, e.g. denominated in the same foreign currency.

The probability of realization. A subposition having a low probability of realization may better be excluded to avoid a too comprehensive position. The marginal cost in complexity and computational time required may be too high. The factor governs where to draw the borderline between an anticipatory position and a potential non-included position.

Direction of Cash Flow

Evidently, it would be foolish to consider this anything but certain.

Committing/Non-Committing Position

As alluded to earlier, this factor describes whether a commitment exists or not. If no commitment exists, the position calls for a different treatment. A tender offer is a committing position if it is legally binding, or if it is a competitive necessity.

Ownership of Asset

A position to sell an asset in the future, i.e. a long future position, may not necessarily correspond to a long present position. The asset may be acquired during the period remaining until the sale. An example from business is a mining company. The ore is quarried and the metal produced at a known speed. The good is therefore not owned in the sense that it is tradeable at the point in time when it is known that the good will be available for sale.

In assuming that only one sort of asset is included in each subposition and that only practically relevant alternatives are included, each subposition can be classified according to table 7.1.

During the period of analysis, the subposition will not always retain the same set of factor values. Hence, the classification is only valid at a specific point in time.

Table 7.1 Relevant factor combinations for a subposition

			SIZE OF SUBPOSITION (DC)			
			certain		fixed size / 0	
			DATE OF TRANSACTION		DATE OF TRANSACTION	
			known	probability distribution	known	probability distribution
COMMITTING SUB- POSITION	CF IN	n. owned	1	2	3	4
		owned	9	10	11	12
	CF OUT		17	18	19	20
NON- COMMITTING SUB- POSITION	CF IN	n. owned	25	26	27	28
		owned	33	34	35	36
	CF OUT		41	42	43	44

n.owned = not owned

(cont'd.)

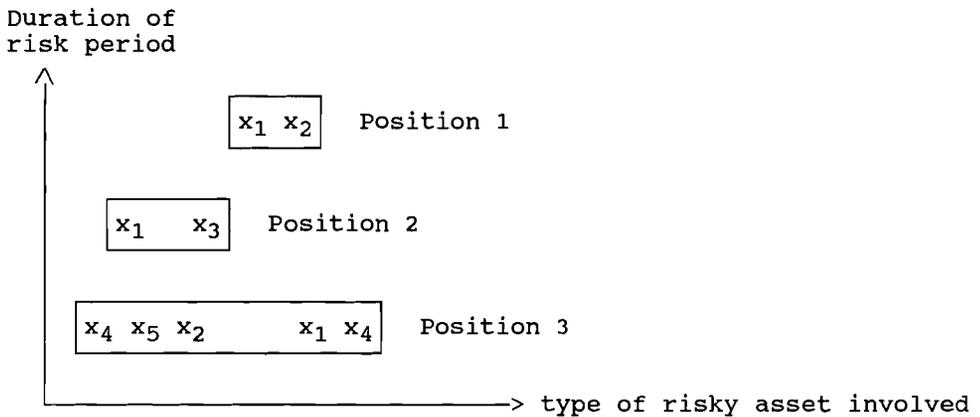
			SIZE OF SUBPOSITION (DC)			
			probability distribution		probability distribution / 0	
			DATE OF TRANSACTION		DATE OF TRANSACTION	
			known	probability distribution	known	probability distribution
COMMITTING SUB- POSITION	CF IN	n. owned	5	6	7	8
		owned	13	14	15	16
	CF OUT		21	22	23	24
NON- COMMITTING SUB- POSITION	CF IN	n. owned	29	30	31	32
		owned	37	38	39	40
	CF OUT		45	46	47	48

7.2.2 Subsets to be Analyzed

The three subsets to be analyzed are contracts, anticipatory positions, and tender offers, all three possibly consisting of many subpositions.

Many independent contracts may be gathered into one single contractual position in the forthcoming analysis as long as the cash flows will take place at approximately one single point in time. Should the cash flows not coincide, the contract must be split into different positions. See figure 7.1. A position consisting of contracts will consist of an arbitrary number of exogenous subpositions of squares number 1, 5, 9, 13, 17, 21 from table 7.1. Number one contains the sale of a long future asset at a known domestic price. Number nine corresponds to a sale of a long present asset at a fixed price in domestic currency units. Furthermore, a sale denominated in a foreign currency or at a stochastic price corresponds to square five and thirteen. The contract may also be a commitment to buy, square number seventeen or twenty-one.

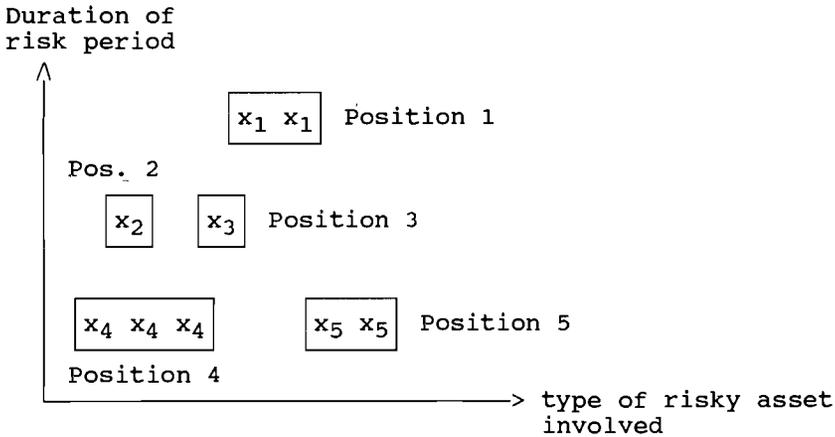
Figure 7.1 Position grouping for contracts (x_i denotes the exogenous subposition from contract 'i')



The time of the transactions is assumed to be known and to coincide for tender offers and anticipatory positions. This is a rather far-fetched assumption, but necessary in order to isolate the main problem.⁴ Furthermore, each underlying position will be analyzed separately as total contingency is required.⁵ The subposition grouping will be according to figure 7.2.

An anticipatory position implies that the position as such is not fixed. This may occur for instance if the sale/purchase is only anticipated. The combinations of interest are square number 27, 31, 35, 39, 43 or 47 if only one point in time is considered, as assumed.

Figure 7.2 Position grouping for anticipatory positions and tender offers (x_i denotes the exogenous subposition from anticipatory position number 'i' or from tender offer number 'i')



The third and last position to analyze is when submitting a binding tender offer which may or may not cause contingent purchases. Hence, the positive cash flow will consist of alternative 3, 7, 11 or 15, and the purchases of square number 19 or 23, if only one point in time is considered.

The method developed may accommodate any combination of the three types of assets mentioned: domestic risky asset, foreign currency, and foreign risky asset. The positions chosen for graphical illustration will only consist of two foreign currencies. Due to the time required for calculations, the method is best suited to handling of few different assets. Hence, the focus will be on larger, separate projects.

7.2.3 Situations to Consider

The transaction is certain to take place in a contractual position [CP]. Consequently, only one situation of interest will arise, i.e. the fulfilment of the contract. In an anticipatory position, the occurrence of the sale/purchase is not certain. Consequently, three situations to be taken into consideration emerge. The expected outcome is a weighted average of the RAP and the NRAP. See Section 7.4.2.1. It is assumed that all subpositions within the enveloping position are conditional on each other, i.e. that the totality is realized or not.⁶

- * Realized anticipatory position [RAP]
- * Non-realized anticipatory position [NRAP]
- * Expected outcome of anticipatory position [EAP]

A similar set of situations emerges in a tender offer position depending on whether it is accepted or refused. The situation 'expected outcome' is conditional on that the tender offer is issued, and it consists of a weighted average of an acceptance and a refusal.

- * Accepted tender offer [ATO]
- * Refused tender offer [RTO]
- * Expected outcome of tender offer [ETO]

7.3 Rationale for Choice of Decision Support Method for Hedges

7.3.1 Necessary Criteria for the Method

Three criteria are of the utmost importance when constructing a model for hedging in companies: realism of assumptions, operationability, and implementability.⁷ The second assumption is associated with generation of the decision parameters whereas the last assumption is associated with the acceptance of the model within the corporation. The two latter criteria may be subdivided into:

- Operationability:
 - availability of input data
 - acceptable processing time and ease of use
 - availability and use of hedging methods
- Implementability:
 - comprehensibility for the everyday user
 - cost of operation

7.3.2 Implications of Distribution of Hedge Value

An all encompassing definition of a purchase of a hedge is a conscious shift of the probability distribution of the outcome to decrease the downside potential.⁸ The downside risk is reduced/eliminated while the upside potential is retained to some extent. Depending on the hedging method used, the symmetry of the probability distribution may be lost. Hedging with options for instance, generates a very skewed distribution. "The traditional method of comparing risk and return by using just mean and standard deviation ... is not well suited to return distributions altered by the use of options"⁹. Furthermore, it has been pointed out that "mean-variance analysis is not valid when the portfolio return is non-linearly related to market

returns"¹⁰. The conclusion is that the mean-variance ought not to be used in distinguishing between hedging methods. Thus, the method chosen must be void of a final variance criterion.

7.3.3 A Short Comment on Probability Distribution Preferences

Hedging implies a reduction of the upside potential in order to limit the downside risk. The ultimate hedge implies a completely certain final cash flow, and constitutes the 100% non-speculative behaviour.

In a contractual situation, only a forward contract will render this certain outcome. A long position in an option is speculative, as a premium is paid initially in order to conserve the upside potential. In this way, the 'hedge' is like a lottery ticket. However, the hedge is not speculative in the sense that an unknown downside risk remains as the initial negative cash flow is certain. The upside potential causes a variance in the portfolio, which is speculative but does not have a negative characteristic. Hence, the variance is not a proper measure for the quality of the hedge.

In an uncertain position, completely non-speculative behaviour may not be achieved. It would require a zero-cash flow in case of non-realization/refusal and a certain cash flow in case of realization/acceptance. This is not possible due to the properties of the hedges available. Thus, there will always be a trade-off either between the protection obtained in a RAP/ATO and the risk generated in a NRAP/RTO (e.g. through a forward/-futures), or between the initial cost and protection (ATO/RAP) or profit (NRAP/RTO) obtained (e.g. options).

The trade-off preference is unique for each actor due to diverging utility functions. For this reason, the illustrating approach to be presented is preferable to a conclusive choice.

7.3.4 A Short Comment on Existing Methods

Most models for hedging decisions of currency risk are very similar. They usually assume a fixed exogenous position, that forwards and/or futures and bonds are used and finally that the mean-variance approach may be applied, possibly together with a utility function.¹¹ The hedging effectiveness is often measured in terms of reduction of variance.¹²

Soenen [1979] allowed forwards, the Eurocurrency market and the local money market. He derived an efficient frontier for hedging costs/foreign exchange portfolio¹³ (= risk). A similar approach was used by Lietaer [1971] for hedging against devaluation risk. The choice was then to be made by applying a utility function. Kritzman [1989a] used forwards/futures in order to hedge a risky position, after which a utility approach was used in order to

obtain the optimal mix. Howard & Antonio [1984] used a mixture of spot position, futures contracts, and risk-free assets to derive an efficient frontier, i.e. a mean-variance approach. Black [1989b] implicitly included only instruments with linear payoffs.¹⁴ Babbel [1983] allowed forwards and borrowing and applied a utility function. Futures contracts have been analyzed by Marmer [1986], by Ederington [1979] and by Dale [1981]. Kwok [1987] applied the framework for hedges using both futures and forwards. Garcia & Gould [1987] applied the mean-variance approach on hedges carried out by synthetic puts.

The inclusion of higher moments, for instance the skewness and the kurtosis, still requires a trade-off for the actor, and entering this into a utility function may prove difficult.¹⁵

Those models are not possible for a more general framework for several reasons. Firstly, the variance as a measure of risk is not acceptable for several hedging methods. Secondly, the individual utility function is not operationable as it is hard to derive as well as that the method will most certainly be non-implementable due to incomprehensibility for the user. Thirdly, the models do not indicate any rules of action, should the position not consist of an existing asset or liability.¹⁶

A method of more discussant character was advanced by Wheelwright [1975]. The underlying criteria for choice between different amounts of hedging were whether the long run perspective was tolerated by the corporation, whether risks not associated with the main line of business of the corporation were accepted, and the implications of the agency theory.¹⁷

A method more adapted to market needs was introduced by Kohlhagen [1978a], who explicitly denounced the use of utility functions and exchange rate projections. Kohlhagen's major idea was to present the user of the model with the payoff of different strategies for different values of the variables. The hedging tools used were forward contracts and the money market. One of the drawbacks of the model is the fact that the probability of the outcome was not estimated, although Kohlhagen mentioned that it would be a possible extension of the model.

Lewent & Kearney [1990] discussed a method developed¹⁸ for making hedging decisions in an export company. Including different levels of currency option hedging, exchange rate dynamics, and forecasted but stochastic future foreign cash flow, the model is claimed to generate frequency plots, means, standard deviations and confidence levels.

The author is of the opinion that the obvious solution is to let the model generate the probability distribution of the outcome for each hedging combination the actor wishes to analyze, after which the actor may apply his risk preferences implicitly and make his choice.¹⁹ Hence, any arcane utility function is eliminated. The portfolio effect is implicitly included for a multiple subposition situation, i.e. that correlation between the stochastic components are included.²⁰ This solution is more

easily implementable than valuing the hedge portfolio according to several different criteria and consolidating them into one measure. (For example: mean and variance of implementation cost, ability to avoid negative returns, and degree of path-dependence.²¹) The author is well aware that the results obtained by the user may be coincidental, due to the absence of any formal rule of how to choose the hedging mixes to analyze. Hence, the method does not purport to render the optimal mix (exempt of other factors than the probability distribution), but rather a satisfactory mix to the user.

7.4 Presentation of Decision Support Method for Hedging: the General Case

The overall idea of the method is to provide the user of the model with the probability distributions of the position outcome for each of the different hedging combinations under consideration. By successively altering the hedging combination, the actor can design the probability distribution that closest matches his preferences, and it will constitute the optimal hedge. The target variable to analyze is the final cash flow of the underlying position plus the value of the hedging activity. This variable is henceforth referred to as the value of the total position. Note that the position does not include the goods, but only the intertemporally adjusted cash flow generated.

The probability of the outcome may be presented in two different ways. Firstly, the final value may be given for each combination of the stochastic variables in a multidimensional position as an equation, and for a two-subposition presentation also as a surface in a three-dimensional setting. The probability of the combination occurring may also be given as an equation. For a bivariate problem it may be drawn in a three-dimensional figure. However, such a presentation is not ideal for decision making. By combining the multidimensional probability density and the final value, a two-dimensional figure may be obtained, having the value on the horizontal axis and a probability measure on the vertical axis. This constitutes the best way of presentation. Two probability measures may be used in this model, the probability density function (pdf) and the cumulative distribution function (cdf).²² The information content is naturally identical, but due to the different presentation, an actor may prefer one to the other. Due to the fact that the position may be uncertain as to whether or not a transaction will take place, the following situations are to be monitored.

Contract:	pdf[VA[CP]]	cdf[VA[CP]]
Anticipatory position:	pdf[VA[RAP]]	cdf[VA[RAP]]
	pdf[VA[NRAP]]	cdf[VA[NRAP]]
	pdf[VA[EAP]]	cdf[VA[EAP]]
Tender offer:	pdf[VA[ATO]]	cdf[VA[ATO]]
	pdf[VA[RTO]]	cdf[VA[RTO]]
	pdf[VA[ETO]]	cdf[VA[ETO]]

7.4.1 Two-Subposition Example for Illustration

In order to visualize the dependence on the stochastic variable of the final cash flow and the value of different hedges, a two-subposition situation is used. Here, a three-dimensional figure may be used which has the value of the risky variables on each of two axis, and a performance variable (value/cash flow) on the third axis.

Three different positions are to be analyzed: the contractual, the anticipatory and the tender offer. Let each position consist of a long subposition in a foreign currency, a short subposition in a different foreign currency and of a subposition in domestic currency. Denote the amount of long currency by q_1 , the amount of short currency (different) by q_2 , and the amount of domestic currency by q_0 ($q_0 < 0$ implies a short position). For an unhedged contract, the final cash flow will follow (7.1).

$$(7.1) \quad q_1 e_1 - q_2 e_2 + q_0$$

For an anticipatory position the position will be identical, with the exception that the probability of realization of the anticipatory position has to be introduced, ϕ . The expected cash flow will follow the equation (7.2). In case of realization, the position turns out identical to the contract position. In case of non-realization, the cash flow will be zero.

$$(7.2) \quad \phi(q_1 e_1 - q_2 e_2 + q_0)$$

A position consisting of a tender offer is also submitted to a probability, the probability of acceptance of the tender offer, θ . The expected cash flow follows equation (7.3).

$$(7.3) \quad \theta(q_1 e_1 - q_2 e_2 + q_0)$$

The values chosen, unless otherwise stated, are as follows for the contractual position and the anticipatory position: $q_1=11,510,937$ CHF; $q_2=4,924,630$ DEM; $q_0=-26,343,064$ SEK; $\phi=55\%$; $T = 91$ days; strike price = forward price. (For the tender offer, see the numerical example in Chapter Ten.) The market data chosen are the spot prices 410 SEK/CHF and 364 SEK/DEM, the correlation coefficient +0.6; the volatilities 6% per year; the nominal interest rates (three months) 13.6% for SEK, 9.2% for CHF and 8.4% for DEM. (See Chapter Ten for a discussion of the

values chosen. For the contractual position and the anticipatory position, the numbers are chosen so that they correspond to the discounted values of the cash flow in the numerical example for the tender offer.)

7.4.2 Comments on the Calculation of Probabilities Within the Model

The probability density function and the cumulative density function for the total outcome of a position may not be derived analytically except for a few extreme cases. The reason is twofold. Firstly, the probability density function of the joint distribution of the prices of the risky assets is not analytically integrable. Secondly, the domain of integration is difficult to specify into a formula.²³ The second problem also rules out any numerical approximations of/formulas for the integration.

The problem of domain-fixing will first be discussed in depth through a two-subposition problem. Subsequent to this is a suggestion of how to handle the calculations. (For a more comprehensive discussion of the outcomes and values, see Chapter Eight.)

7.4.2.1 Analysis of Domain-Fixing for a Two-Subposition Foreign Currency Problem

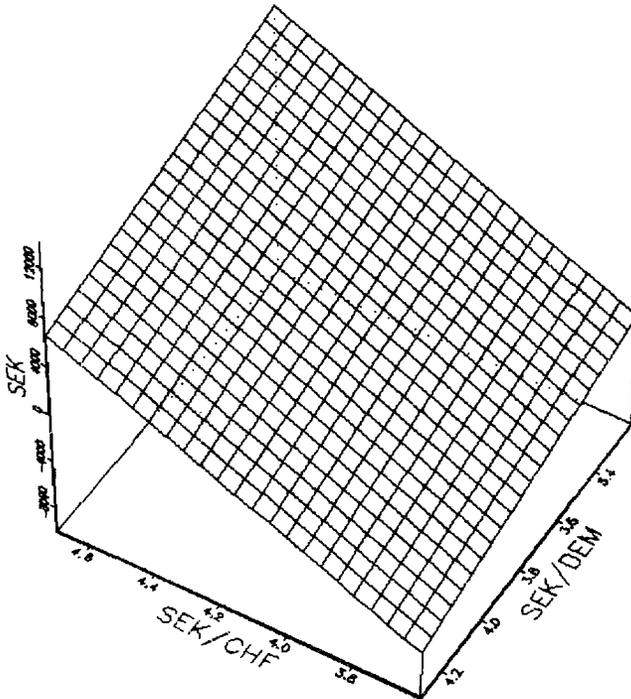
Consider a position consisting partly of an anticipatory commitment to sell a product at a fixed price denominated in the foreign currency number one at time t , q_1 . The cash inflow will be $q_1 e_1$ or zero. In order to honour the agreement, inputs must be purchased. The second subposition consists of a conditioned commitment to purchase foreign assets at a fixed local price in currency number two, q_2 , and domestic inputs to the amount $-q_0$. The total outflow will be $(q_2 e_2 - q_0)$ or zero. The total cash flow from the underlying position follows equation (7.4). Equation (7.4a) and figure 7.3 illustrate the intuitively obvious fact that the cash flow increases with e_1 and decreases with e_2 . The surface in the figure will be shifted upwards or downwards in proportion to the size and sign of q_0 .

$$(7.4a) \quad CF[RAP] = q_1 e_1 - q_2 e_2 + q_0$$

$$(7.4b) \quad CF[NRAP] = 0$$

Let the position be partly hedged. In order to simplify the understanding of the domain problem, only two hedging methods are included, namely forwards and options. (Stop-loss brings the problem of a probability distribution of the outcome for specific values of e_1 and e_2 . CPPI and TIPP do not have any formulas for the final conditional value.) Denote the fraction of the cash flow in to be hedged through forwards by $e_{h_1}[FW]$ and

Figure 7.3 Final cash flow from a realized, unhedged anticipatory position. (See Section 7.4.1 for the parameters.)



through put options $e_{h_1}[P]$. In the same way, denote the fraction of the cash flow out to be hedged through forwards by $e_{h_2}[FW]$ and through call options by $e_{h_2}[C]$. $e_{i,x}$ denoted the strike price of the options. Assume that a non-speculative restriction is put on the hedges, i.e. that the total hedge and separate hedges must be between 0 and 100%, equation (7.5).²⁴ The value of the hedge then becomes according to (7.6):

$$(7.5a) \quad 0 \leq e_{h_1}[FW], e_{h_2}[FW], e_{h_1}[P], e_{h_2}[C] \leq 1$$

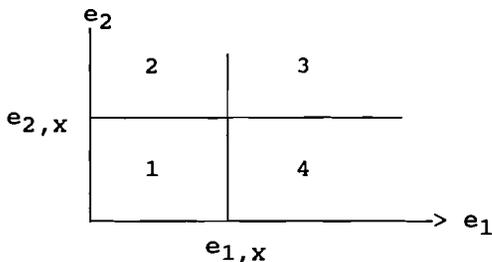
$$(7.5b) \quad 0 \leq e_{h_1}[FW] + e_{h_1}[P] \leq 1$$

$$(7.5c) \quad 0 \leq e_{h_2}[FW] + e_{h_2}[C] \leq 1$$

$$(7.6) \quad VA[hedge; t_x] = q_1 \{ e_{h_1}[FW] (FW_1[t_0, t_x] - e_1) + \\ + e_{h_1}[P] (\max[0, e_{1,x} - e_1] - P_e[t_x] e^{r_D T_x}) \} + \\ + q_2 \{ e_{h_2}[FW] (e_2 - FW_2[t_0, t_x]) + \\ + e_{h_2}[C] (\max[0, e_2 - e_{2,x}] - C_e[t_x] e^{r_D T_x}) \}$$

Equation (7.6) clearly points to the first problem of integration. The function of the final value differs depending on which side of the exercise price of the option you are, due to the maximum functions. With a n-subposition problem, 2^n different domains will emerge. Consider figure 7.4.

Figure 7.4 Domains having different functions for the value of an option hedge



In a three-dimensional figure (see figure 7.5), the value corresponds to four surfaces having different slopes. Naturally, the surfaces meet at the vertical projections of $e_1=e_{1,x}$ and of $e_2=e_{2,x}$ respectively. The implication for an integration is the necessity to consider, at the most, four different domains.

The form of the four-part surface must be like a non-regular pyramid turned upside down. The reason is as follows: (The subscript of the value corresponds to the number of the square in figure 7.4.)

$$(7.7a) \quad \frac{\delta VA[\text{hedge}]_1}{\delta e_1} = \frac{\delta VA[\text{hedge}]_2}{\delta e_1} = -q_1 e_{h_1}[\text{FW}] - q_1 e_{h_1}[\text{P}]$$

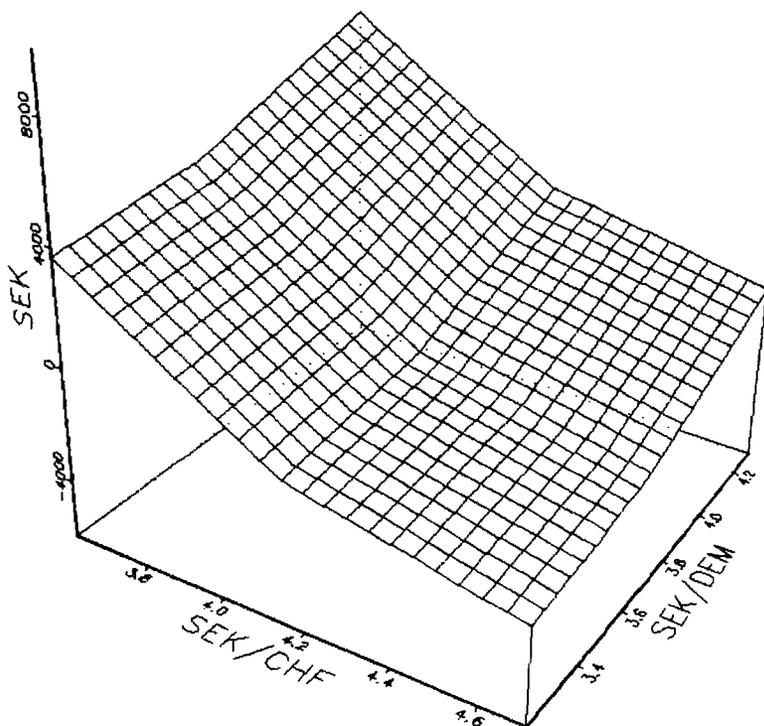
$$(7.7b) \quad \frac{\delta VA[\text{hedge}]_4}{\delta e_1} = \frac{\delta VA[\text{hedge}]_3}{\delta e_1} = -q_1 e_{h_1}[\text{FW}]$$

$$(7.7c) \quad \frac{\delta VA[\text{hedge}]_1}{\delta e_2} = \frac{\delta VA[\text{hedge}]_4}{\delta e_2} = q_2 e_{h_2}[\text{FW}]$$

$$(7.7d) \quad \frac{\delta VA[\text{hedge}]_2}{\delta e_2} = \frac{\delta VA[\text{hedge}]_3}{\delta e_2} = q_2 e_{h_2}[\text{FW}] + q_2 e_{h_2}[\text{C}]$$

The hedging value decreases less in square number four than in square number one with an increase in e_1 . The value increases more in square number two than in square number one with an increase in e_2 . The value decreases less in square number three than in square number two with an increase in e_1 . The value increases more in square number three than in square number four with an increase in e_2 . Hence, the overturned pyramid.

Figure 7.5 Value of a non-realized anticipatory position hedged by a mix of options and forwards. ($h_1[\text{FW}] = h_2[\text{FW}] = 25\%$, $h_1[\text{P}] = h_2[\text{C}] = 50\%$. See Section 7.4.1 for the value of the other parameters.)

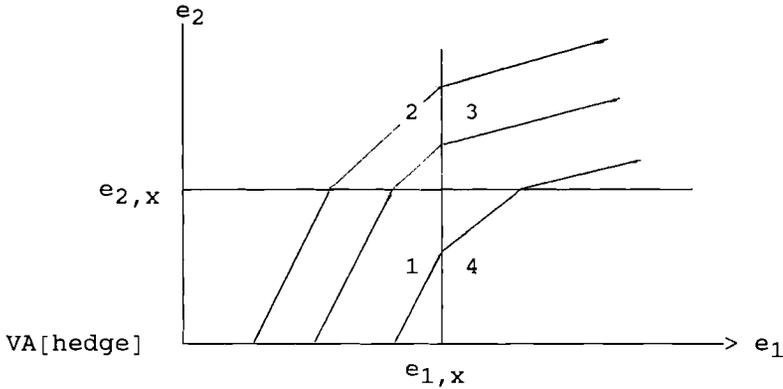


Non-Realized Anticipatory Position

The value of the hedge in figure 7.5 not only depicts the hedging value, but also the total value should the anticipatory position not realize. The domain of underscoring a specific floor conditional on a non-realization is found by inserting a horizontal floor level. The intersecting curve renders the iso-value curves in figure 7.6.

The area under the iso-value curve, corresponding to the floor, is the domain of integration. The area has the equation according to (7.8), where [1&2] signifies that the factor is only to be applied for square number one and two. Whether the combination of e_1 and e_2 obtained are in accordance with the formula used, i.e. the [1&2] or [2&3], must be analyzed.

Figure 7.6 Alternative iso-value curves and domains of underscoring: NRAP



$$(7.8) \quad e_2 \leq \{FL[t_x] -$$

$$- q_1\{e_{h_1}[FW]FW_1[t_0, t_x] + e_{h_1}[P](e_{1,x}[1\&2] - P_e[t_x]e^{r_D T_x})\} +$$

$$+ q_2\{e_{h_2}[FW]FW_2[t_0, t_x] + e_{h_2}[C](e_{2,x}[2\&3] + C_e[t_x]e^{r_D T_x})\} /$$

$$/ \{q_2(e_{h_2}[FW] + e_{h_2}[C][2\&3])\} + \frac{q_1(e_{h_1}[FW] + e_{h_1}[P][1\&2])}{q_2(e_{h_2}[FW] + e_{h_2}[C][2\&3])} e_1$$

From the equation, it is obvious that:

- * Two different bordering lines in the same square (from two different floor values) are parallel.
- * The slope is always equal to or larger than zero.
- * The slope is lower or equal in square two than in square one and lower/equal in square three than in square two. The slope is lower/equal in square four than in square one. Finally, the slope is lower in square three than in square four. ($0 \leq \#3 \leq \#4 \leq \#2 \leq \#1$)

Realized Anticipatory Position

If the position is realized, the total final value is:

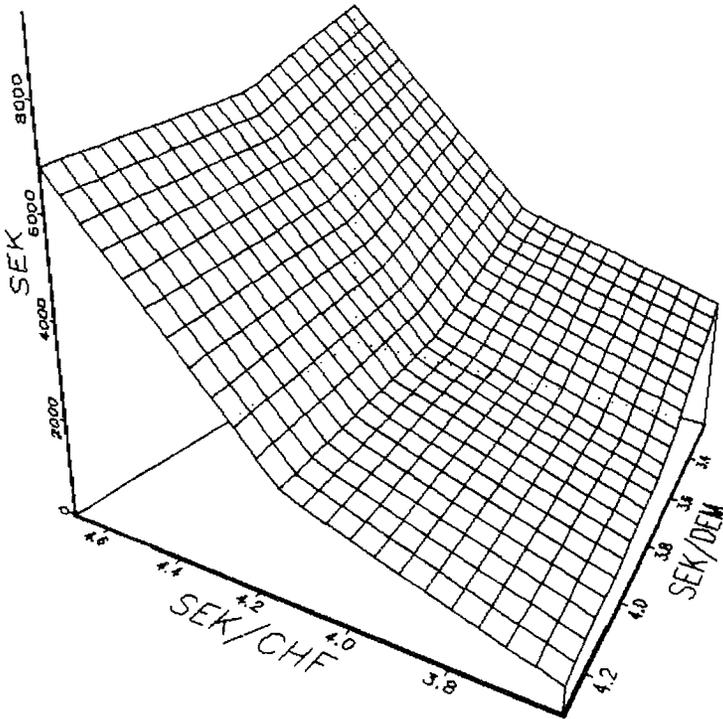
$$(7.9) \quad VA[RAP] = VA[unhedged] + VA[hedge] =$$

$$= q_1 e_1 - q_2 e_2 + q_0 + VA[hedge]$$

Hence, the value at each combination of e_1 and e_2 is the sum of

the unhedged position and the hedging value. Geometrically, this is equivalent to 'adding' a sloping surface corresponding to an unhedged position, figure 7.3., and the surface in figure 7.5. The resulting figure will look like figure 7.7.

Figure 7.7 Value of realized anticipatory position hedged by a mix of options and forwards. ($h_1[\text{FW}] = h_2[\text{FW}] = 25\%$, $h_1[\text{P}] = h_2[\text{C}] = 50\%$. See Section 7.4.1 for the other parameters.)



The slope of the total value surface is:

$$(7.10a) \quad \frac{\delta VA[\text{RAP}]_1}{\delta e_1} = \frac{\delta VA[\text{RAP}]_2}{\delta e_1} = q_1(1 - e^{-h_1[\text{FW}]} - e^{-h_1[\text{P}]})$$

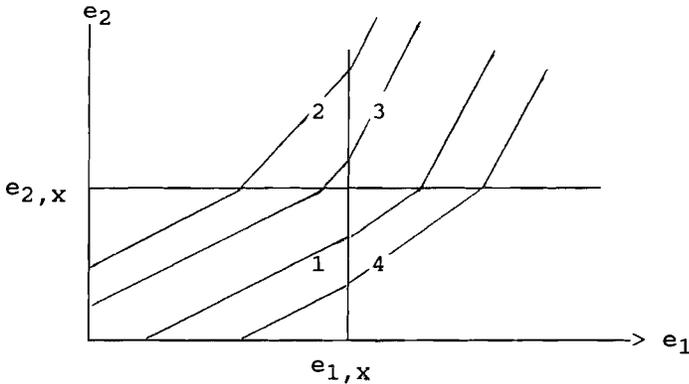
$$(7.10b) \quad \frac{\delta VA[\text{RAP}]_4}{\delta e_1} = \frac{\delta VA[\text{RAP}]_3}{\delta e_1} = q_1(1 - e^{-h_1[\text{FW}]})$$

$$(7.10c) \quad \frac{\delta VA[\text{RAP}]_1}{\delta e_2} = \frac{\delta VA[\text{RAP}]_4}{\delta e_2} = -q_2(1 - e^{-h_2[\text{FW}]})$$

$$(7.10d) \quad \frac{\delta VA[\text{RAP}]_2}{\delta e_2} = \frac{\delta VA[\text{RAP}]_3}{\delta e_2} = -q_2(1 - e^{-h_2[\text{FW}]} - e^{-h_2[\text{C}]})$$

Under the non-speculative restrictions in equation (7.5) it is seen that the total value increases with e_1 and at a higher rate in square four than in square one. Furthermore, the total value decreases with e_2 and at a lower rate in square two than in square one. Thus, the 'pyramid' has tipped over on the other side. The area of integration is above the relevant iso-value curve in figure 7.8. The equation of the integration area is according to (7.11).

Figure 7.8 Alternative iso-value curves and domains of underscoring: RAP



$$\begin{aligned}
 (7.11) \quad e_2 \geq & \{- FL[t_x] + q_0 + \\
 & + q_1\{e^{h_1}[FW]FW_1[t_0, t_x] + e^{h_1}[P](e_{1,x}[1\&2] - P_e[t_x]e^{r_D T_x})\} - \\
 & - q_2\{e^{h_2}[FW]FW_2[t_0, t_x] + e^{h_2}[C](e_{2,x}[2\&3] + C_e[t_x]e^{r_D T_x})\} \} / \\
 & / \{q_2(1 - e^{h_2}[FW] - e^{h_2}[C][2\&3])\} + \frac{q_1(1 - e^{h_1}[FW] - e^{h_1}[P][1\&2])}{q_2(1 - e^{h_2}[FW] - e^{h_2}[C][2\&3])} e_1
 \end{aligned}$$

From the equation, it becomes evident that under the non-speculative restriction, the iso-value curve in square three has a slope larger than the one in square two. The slope of the curve in square two is larger than the one in square four, which in turn is larger than the one in square one. All slopes are larger than zero. ($\#3 \geq \#2 \geq \#4 \geq \#1 \geq 0$.) Furthermore, the iso-value curves are parallel within each square. Naturally, the relations above will only hold, should the intercept be such that the curve is actually positioned in the square analyzed.

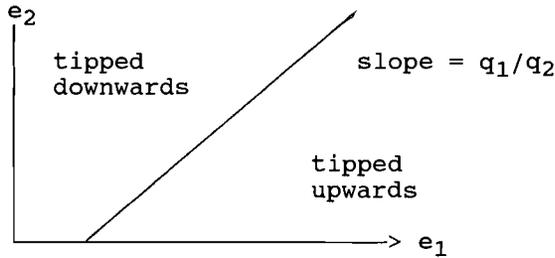
Expected Anticipatory Position

The expected outcome of the anticipatory position implies that the 'overturned pyramid' will not be in one of the two extreme positions discussed above. It will stop at a proportion ϕ , signifying the probability of realization.

The first question is how and around which axis the tipping will take place, and the second question is what the domain of integration will look like.

As the value of the hedge will not change with ϕ , it is only the underlying position that alters value. Hence, no change will take place where $VA[\text{underlying position}] = 0$, i.e. around $e_2 = (e_1 q_1 + q_0) / q_2$. Hence, each of the four total-value surfaces will change under the restriction that the intersection of the projection of the $e_2 = (e_1 q_1 + q_0) / q_2$ -line does not change.

Figure 7.9 Tipping due to an increase in ϕ
(Compare figures 7.5 ($\phi=0$) and 7.7 ($\phi=1$)).



The domain of integration varies depending on how the probability of the expected anticipatory position is defined. Two alternatives emerge.

The first alternative consists of the answer to the question "what is the probability of obtaining a value which falls below x ". The probability obtained is the weighted average of the probability of underscoring should the position realize and the probability of underscoring should the position not realize, hence (7.12).²⁵ The two probabilities on the right hand side of the equation are obtained from the two previous sections.

$$(7.12) \text{ prob}[VA[EAP] \leq x] = \phi \text{ prob}[VA[RAP] \leq x] + (1-\phi) \text{ prob}[VA[NRAP] \leq x]$$

The second alternative for defining the probability is to ask the question "what is the probability that the expected outcome will fall below x ". The rationale for choosing this alternative would be a focus on the expected outcome instead of the expected value of different outcomes. The probability calculated is (7.13).²⁶ A reason for applying this would be if the performance was

evaluated prior to the outcome occurring on the basis of expected value.

$$(7.13) \text{ prob}[VA[EAP] \leq x] = \text{prob}[\phi VA[RAP] + (1-\phi)VA[NRAP] \leq x]$$

Henceforth, the former alternative is chosen as definition. (The calculations necessary for the second alternative are occasionally given in the notes.)

7.4.2.2 Suggestion of Probability Calculation

In order to overcome the problem of finding the integration domain and integrating the function, the continuous integration technique is exchanged for a summation of probabilities. Furthermore, all combinations are tested as to whether they are to be included in the summation or not, i.e. if the value is less than or equal to the floor.²⁷ Here, no analytically defined domain is needed. The probability of underscoring a specific floor level may be calculated as:

$$(7.14) \text{ cdf}[FL] = \sum_{e_1} \sum_{e_2} \dots \sum_{p_1} \dots \sum_{p_1} \text{ IF}[VA \leq FL] : \text{pdf}[e_1, \dots, p_1] \dots de_1 \dots dp_1 \dots$$

The method generates two questions: How large is each summation area to be made, i.e. $de_1 \dots dp_1 \dots$? Over which values of each variable is the summation to be made? The first question is given by the computational speed of the computer used and by the accuracy needed. The second question is given by the percentage of the possible outcomes to include. A quick approximation would be to include the variable values at a certain number of standard deviations around the expected value. (b=1 includes for a non-correlated variable 68.27% of all outcomes, b=2 95.45% and b=3 99.73%.²⁸)

$$(7.15) \text{ a: } a_t = e^{\frac{\mu_a T}{\sigma} + \frac{1}{2} \sigma^2 T \pm b \sigma \sqrt{T}}$$

A cautious application is required, especially if the correlation between the stochastic variables is high. The probability density is then much concentrated, and the approximations made may cause unacceptably large errors. Under a ceteris paribus assumption, the approximation error will increase with τ .

For a two-subposition, the computational transformation is:

$$(7.16) \int_{\substack{e_1 \\ [D]}} \int_{e_2} \text{pdf}[e_1, e_2] de_1 de_2 \rightarrow \sum_{e_1} \sum_{e_2} \text{IF}[VA \leq FL] : \text{pdf}[e_1, e_2] de_1 de_2$$

$$e_1: E[e_1] e^{+b\sigma\sqrt{T}}$$

$$e_2: E[e_2] e^{+b\sigma\sqrt{T}}$$

$$e_1: E[e_1] e^{-b\sigma\sqrt{T}}$$

$$e_2: E[e_2] e^{-b\sigma\sqrt{T}}$$

If the squares, de_1 and de_2 , are made small enough, the problem of 'misfit' at the delimiting lines is negligible, as well as the approximation of using one value of $\text{pdf}[]$ for the whole square.

Notes

1. Toevs & Jacob [1986] presented a similar idea for assets and liabilities, including the two factors: currently held/anticipated position and known/uncertain period of time. They also noted that the amount to be received might be uncertain.
2. Giddy [1988], p. 84.
3. This is noted by Giddy [1983], p. 148.
4. Kwok [1987] analyzed the problem of hedging multiple cash flows in one foreign currency which were distributed over time. He found that hedging each cash flow separately using forwards and futures, would only be slightly inferior to using an integrative approach. Hence, the independent approach implicitly used, or rather the assumption made, in this dissertation may be considered acceptable.
5. In a generalization of the method to be presented, the inclusion of multiple interdependent tender offers and/or anticipatory positions is achieved by including the correlation coefficient between the probabilities of realization/acceptance. The same approach is to be used, should the occurrence of cash flows in an anticipatory position not be totally interdependent, as assumed. The present method may only be used if the correlation is unity.
6. Naturally, the volume might be treated as a stochastic variable analogous to the local price or exchange rate. A domestic position must then be considered as 'gathered' as the value in currency units consists of two sequential stochastic variables. A foreign risky position may be treated analogously, where three stochastic variables are included.
7. See Little [1970], p. 470, for further characteristics that may be required.
8. Clarke & Arnott [1987], p. 39, stresses the function of the floor by saying ... "the entire purpose of portfolio insurance, to eliminate the risk of losses below the designated floor". A definition relying on a minimization of variance (Toevs & Jacob [1986], p. 60) is not acceptable due to loss of symmetry of the distribution.
9. Clarke & Arnott [1987], p. 43. See also Bookstaber & Clarke [1984, 1985] for a thorough discussion.
10. Dybvig [1988], p. 67. Stop-loss was among the methods mentioned.

11. A general framework for utility-based hedging is given in Sharpe [1987].
12. See Ederington [1979], Dale [1981], or Overdahl & Starleaf [1986]. The two former measured the variance based on changes from the spot rate whereas it was measured as a deviation from expected changes in the last paper.
13. I.e. the portfolio model of hedging. (See Ederington [1979], p. 161.)
14. Black [1989b]. A most interesting finding presented in Black's article was that the hedge may be considered as a "free lunch" (p. 17), due to Siegel's paradox.
15. See Kraus & Litzenberger [1976], who included skewness.
16. Folks [1973] mentioned the problem, should the size of the position be uncertain. However, no solution was given. Lypny [1988] discussed the problem of an "n-currency spot portfolio", p. 703, in a mean-variance framework using futures as a hedging tool.
17. The five methods for setting the level of the hedge and their rationale are (Wheelwright [1975], pp. 43-44): Never hedge (perfect market, willingness "to play the long-run averages"); always hedge (short run perspective, no willingness to take on risks not "associated with its industry"); hedge amounts exceeding a certain level (larger losses not tolerated); hedge if the probability of devaluation exceeds a certain level ("manager viewed unfavorably any time there was a devaluation and he had not hedged"); lowest expected cost (assumes that the corporation can beat the market).
18. The computer model was developed with the guidance of Professor Duffie (p. 27).
19. Bookstaber & Clarke [1983] used a similar approach for a portfolio hedged by call options and put options.
20. The opposite approach is called 'the isolation strategy' (see Lypney [1988]) and implies that all subpositions are to be treated separately.
21. Bird, Dennis & Tippett [1988].
22. The proper analytical function is not obtainable, but the curve may be drawn. This curve is henceforth called the 'function'.
23. The integration must take place for both the pdf and the cdf for the total outcome, as the joint binomial pdf is not directly linked to the outcome.
24. This is a common rule in non-financial companies. The same assumption was also made in Kritzman [1989a].

25. (7.12) is an unorthodox way of writing, as $VA[ETO]$ may not be substituted for $\Phi VA[ATO] + (1-\Phi)VA[RTO]$.

26. For: $(\Phi - e^{h_2}[FW] - e^{h_2}[C][2\&3]) > 0$

$$e_2 \geq \{- FL[t_x] + \Phi q_0 + q_1 \{ e^{h_1}[FW] FW_1[t_0, t_x] + e^{h_1}[P] (e_{1,x}[1\&2] - P_e[t_x] e^{r_D T_x}) \} - q_2 \{ e^{h_2}[FW] FW_2[t_0, t_x] + e^{h_2}[C] (e_{2,x}[2\&3] + C_e[t_x] e^{r_D T_x}) \} \} / \{ q_2 (\Phi - e^{h_2}[FW] - e^{h_2}[C][2\&3]) \} + \frac{q_1 (\Phi - e^{h_1}[FW] - e^{h_1}[P][1\&2])}{q_2 (\Phi - e^{h_2}[FW] - e^{h_2}[C][2\&3])} e_1$$

For $(\Phi - e^{h_2}[FW] - e^{h_2}[C][2\&3]) < 0$, the direction of the inequality is changed, hence we have $e_2 \leq \dots$

Consequently, four times four combinations to be analyzed emerge, depending upon whether the contents within the respective brackets in the last fraction are larger than zero, less than zero, equal to zero, or change signs when shifting square.

27. Naturally, it is possible to use the domains defined in Section 7.4.2.1. However, should the second definition of the probabilities for the expected tender offer be applied instead, this trial-and-error method is definitely preferable due to the multiple types of domains that may occur.

28. Chou [1975], p. 64.

8 Hedging of Contractual and Anticipatory Positions

8.1 Introduction

The positions under consideration in Chapter Eight are contracts and anticipatory positions. The former constitute commitments, i.e. the volume of asset is certain. The latter constitute planned transactions, i.e. the volume is uncertain. A position may consist of an arbitrary number of subpositions.¹ The discussion will be based on all-encompassing multi-subposition formulas, and illustrated by a simplified position in a two-dimensional probability distribution graph, or in three-dimensional figures, where two axes depict one risky variable each and the third axis a performance variable, the cash flow or value.

The method introduced in the previous chapter is applied both on separate hedging tools and on separate dynamic hedging methods. In the former case, the final contingent cash flow may be derived analytically. In the latter case no such solution is available and a simulation approach is, therefore, applied.

8.2 Unhedged Contractual Position

Cash Flow: Multiple Risky Subpositions

A certain position may consist of a commitment to buy ($n < 0$) or sell ($n > 0$) $|P_{n_i}|$ units of the domestic risky asset 'i' at a specific future point in time at the stochastic price p_i . Furthermore, there may be an obligation to pay (the currency must be bought, $n < 0$) or right to receive (hence, they must be sold, $n > 0$) $|e_{n_j}|$ units of currency 'j' for certain. The cash flow is $e_{n_j} e_j$ per subposition. The last kind of risky position is a commitment to buy or sell $|g_{n_{ij}}|$ units of the foreign risky asset 'i' denoted in currency 'j'. The cash flow is $g_{n_{ij}} p_i e_j$. Finally, a position may contain a certain cash flow. This is included by letting $j=0$ denote the domestic currency unit ($e_0=1$), and e_{n_0} denote the amount of domestic currency. Note that the time index

is suppressed in the entire chapter.

The cash flow of a position at time t is the sum of the cash flows of the subpositions spread over the three different assets.

$$(8.1) \quad CF[CP] = \sum_{i=1} p_{n_i} p_i + \sum_{j=0} e_{n_j} e_j + \sum_{i=1} \sum_{j=1} g_{n_{ij}} p_i e_j$$

Cash Flow: Two Risky Subpositions

Consider a position according to the example given in Section 7.4.1, consisting of two risky components, a cash flow of e_{n_1} units of currency one and e_{n_2} units of currency two. Assume an extra certain cash flow of e_{n_0} in the domestic currency unit. Let $e_{n_1}=q_1>0$, $e_{n_2}=-q_2<0$ and $e_{n_0}=q_0<0$, e.g. an order denoted in currency one, requiring purchases of inputs in currency two and in the domestic currency. The position is simplified into:

$$(8.2) \quad CF[CP] = e_{n_1} e_1 + e_{n_2} e_2 + e_{n_0} e_0 = q_1 e_1 - q_2 e_2 + q_0$$

In a three-dimensional orthogonal coordinate system, the cash flow is depicted by a sloping, non-kinked surface. (See figure 7.3, Section 7.4.2.1.) The impact of q_0 is simply a vertical shift of the surface, orthogonally to the surface $CF[CP]=0$.

Probability Distribution: Multiple Risky Subpositions

The probability of falling below a specific cash flow level is calculated in two steps. Firstly, by finding the combinations of the risky variables which render this result. Secondly, by calculating the probability through integration over the variable combinations from step one. The domain of combinations of p_i and e_j rendering a cash flow equal to or below a floor level, is denoted by D ,² and the probability is given by the integral:

$$(8.3) \quad \int_{p_1} \int_{\dots} \int_{e_1} \int_{\dots} pdf[p_1, \dots, e_1, \dots] dp_1 \dots de_1 \dots$$

[$(p, e): D$]

pdf[...] is the multivariate lognormal probability distribution. (See appendix B.1.1.) The integral may be used to render a cumulative distribution function by letting the floor, FL, successively change value. The probability density for a specific floor level may either be derived through division of the difference between two consecutive cumulative distributions by the difference in cash flow, or by integration over [($p_1 \dots e_1 \dots$): CF = analyzed floor].

Probability Distribution: Two Risky Subpositions

The combinations of e_1 and e_2 rendering a value at or below FL , are obtained by inserting a horizontal surface at the level FL , i.e. parallel to the surface $CF=0$, finding the intersection between this surface and the cash flow-surface, projecting the intersecting curve onto the horizontal surface $CF=0$. The first quadrant is split into two parts (or one part for an extreme situation.) The part fulfilling the condition $CF \leq FL$ is chosen. The intersecting line and its projections are defined by equation (8.4). All lines parallel to the projection are iso-cash flow lines. They are obtained by changing the floor level.

$$(8.4) \quad e_2 = - \frac{FL - q_0}{q_2} + \frac{q_1}{q_2} e_1$$

The combinations of e_1 and e_2 rendering a cash flow below, or equal to, the floor are:

$$(8.5) \quad (e_1, e_2) = [e_2 \geq - \frac{FL - q_0}{q_2} + \frac{q_1}{q_2} e_1; \quad e_1 \geq 0; \quad e_2 \geq 0]$$

The probability that the value will be at, or below the floor level is obtained by integrating the joint probability density over the area indicated in equation (8.5). Assuming a lognormal distribution,³ the probability becomes:

$$(8.6) \quad \text{prob}[CF \leq FL] = \int_{e_1} \int_{e_2} \text{pdf}[e_1, e_2] de_1 de_2$$

$[D]$

$$\text{pdf}[e_1, e_2] = \{e_1 e_2 2\pi \sigma_1 \sigma_2 T \sqrt{1 - \tau^2}\}^{-1} \cdot \exp[-(X_1^2 - 2\tau X_1 X_2 + X_2^2) / 2(1 - \tau^2)]$$

$$X_1 = (\ln[e_1/e_{1,0}] - \mu_1 T) / \sigma_1 \sqrt{T}$$

$$X_2 = (\ln[e_2/e_{2,0}] - \mu_2 T) / \sigma_2 \sqrt{T}$$

Assuming a correlation separated from ± 1 or 0 , equation (8.6) is not analytically solvable. As stated above, the integral may be used to render the cumulative distribution function and the probability density function. In the latter case, the domain of integration is the dividing line obtained earlier. The pdf may naturally also be obtained through changes in the cdf, a discrete approximation approach. The cumulative distribution function will be as shown in figure 8.1. The probability distribution depends to a large extent on the correlation between changes in currency one and in currency two. (For a perfect positive correlation, only the net cash flow need be considered for hedging.) The curves in figure 8.2 illustrate the obvious fact that the higher the correlation, the less risky the position. This is only valid as long as the total position consists of a long and short position.

Figure 8.1 Cumulative distribution function for unhedged contractual position.⁴ (See Appendix E for exact numbers.)

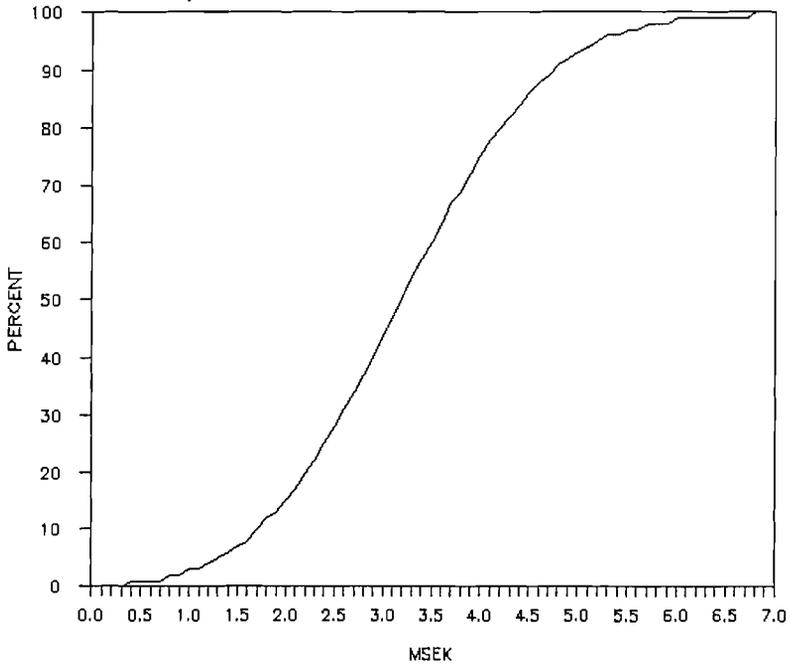
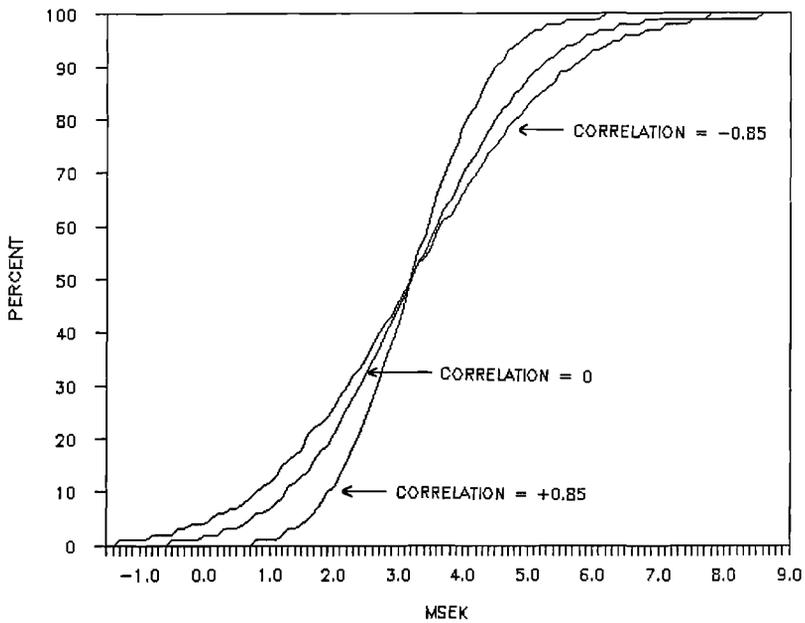


Figure 8.2 Cumulative distribution function for unhedged contractual positions, having different correlations. (See Appendix E for exact numbers.)



8.3 Unhedged Anticipatory Position

Cash Flow: Multiple Risky Subpositions

The expected cash flow at time t is according to equation (8.7), where ϕ denotes the probability of realization. The cash flow conditional on realization is obtained by setting $\phi=1$, and on non-realization by setting $\phi=0$.

$$(8.7) \quad E[CF] = \phi \left(\sum_i P n_i p_i + \sum_j e_j \right) + \sum_i \sum_j q_{ij} p_i e_j$$

Cash Flow: Two Risky Subpositions

The initiation of two alternative outcomes correspond to two alternative surfaces in a three-dimensional coordinate system. The first one being identical to the surface introduced in figure 7.3, and the second surface being horizontal at the $CF=0$ level. Taking the expected value of the cash flow corresponds to reducing the slope of the initial surface proportionally to the size of the probability ϕ . (This is easily realized by studying the expected value at each combination of e_1 and e_2 .) The axis around which the surface is to be tipped, is the intersection between the initial surface and the $CF=0$ surface, as the cash flow is identical to the expected cash flow there.

$$(8.8) \quad E[CF[AP]] = CF[EAP] = \phi CF[RAP] + (1-\phi) CF[NRAP] = \\ = \phi (q_1 e_1 - q_2 e_2 + q_0)$$

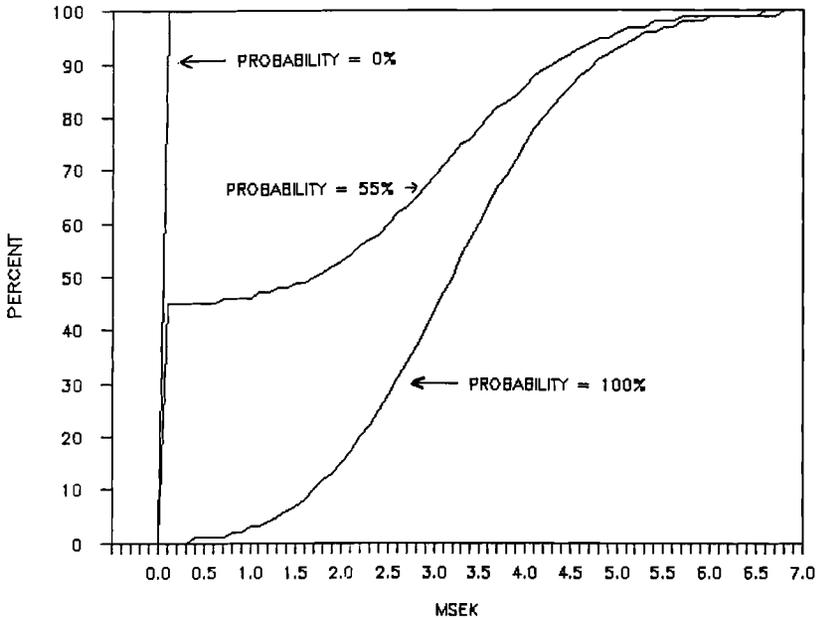
Probability Distribution

The probability of getting a cash flow at or below the floor conditional on realization, is obtained exactly as in Section 8.2, as it is equivalent to setting $\phi=1$. The probability of getting a cash flow at or below FL conditional on non-realization is either 0 % ($FL < 0$) or 100 % ($FL \geq 0$). The probability of getting a cash flow below the floor, is the weighted average, the weights being ϕ and $(1-\phi)$ respectively, of the probabilities conditional on realization and conditional on non-realization.⁵

The cumulative distribution for different probabilities of realization will be as shown in figure 8.3. In the extreme situation of zero probability, the cash flow will evidently be zero, a vertical straight line. In the other extreme situation, the position is identical to an unhedged contractual position. Should the probability of realization be 55%, there is 45% probability of getting an outcome at exactly zero MSEK. Should the anticipatory position realize, probability 55%, the probability of falling below zero is approximately zero. Hence, the curve will make a jump from zero to 45% at zero MSEK. The remaining probability will be allocated according to the open position, scaled by 55%. For any other probability of

realization, the curve will be positioned in proportion to the probability in the continuous spectra delimited by the two extreme curves depicted in the figure.

Figure 8.3 Cumulative distribution function for unhedged expected anticipatory position. (See Appendix E for exact numbers.)



8.4 Choice of Hedge

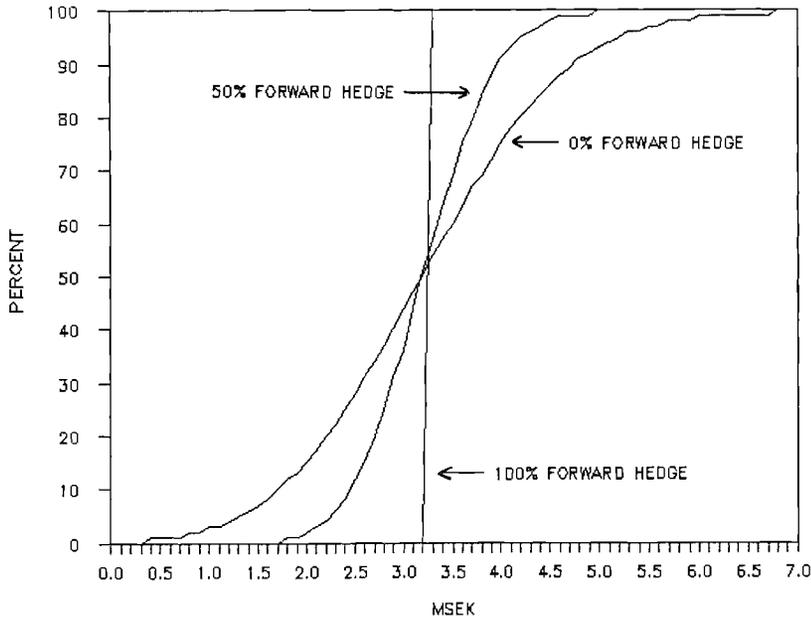
To hedge a position of the contractual or anticipatory kind, three tools may be used: forward contracts, futures contracts and/or options. Due to the similar structure of the forward and the futures and due to the path-dependence of the latter, only the forward contract will be included. (Options on forward and futures contracts are not of interest, as the underlying contract will be due after the payments are to be made/received. Hence, they contain an undesired speculative component for these specific positions.) The CPPI and the stop-loss are included. TIPP is excluded due to the similarity to the CPPI strategy. The cumulative distribution function is chosen to illustrate the probability distribution. The underlying position is assumed to consist of two risky subpositions, q_1 long units of currency one and q_2 short units of currency two. In addition, there is a position of q_0 domestic currency units.

8.4.1 Contractual Position

In a contractual position, the ultimate hedge may be obtained.⁶ The final cash flow may be fixed, irrespective of the value of the stochastic variables, by entering forward contracts. Any other hedging method would introduce a speculative component, which ought to be considered separately from the hedging activity by using the forward hedged position as a benchmark. A lower percentage hedge will render a stochastic cash flow, and is dependent on the correlation between the stochastic prices.⁷ The value of a partially hedged position is according to equation (8.9).⁸ Figure 8.4 shows the two polar positions of a totally hedged position and of an open position. The 100% forward position renders a certain final cash flow of $11,510,937 \cdot 4.1440 - 4,924,630 \cdot 3.6862 - 26,343,064 = 3,205,088$ SEK ≈ 3.2 MSEK. The 50% hedge curve is naturally situated between the other two.

$$(8.9) \quad VA[CP] = q_1 \{ e^{h_1[FW]} FW_1[t_0, t] + (1 - e^{h_1[FW]}) e_1 \} - q_2 \{ e^{h_2[FW]} FW_2[t_0, t] + (1 - e^{h_2[FW]}) e_2 \} + q_0$$

Figure 8.4 Cumulative distribution function for contractual positions, having different levels of forward hedge.
(See Appendix E for exact numbers.)



8.4.2 Anticipatory Position

A perfect hedge in an anticipatory position would render a zero value if non-realized, and the forward prices multiplied by the quantities if realized. An open position would fulfil the first condition but not the second. A forward hedge would fulfil the second condition but not the first one. No other hedging method would fulfil any of the two criteria, but as they have positive features they may nevertheless be used. In this section, the methods are analyzed separately in order to discuss their effects.

8.4.2.1 Forward Hedge

Consider an anticipatory position where the fractions $e_{h_1}[\text{FW}]$ and $e_{h_2}[\text{FW}]$ are hedged. Set $\phi=1$ for the conditional realized situation and $\phi=0$ for the conditional non-realized situation. It is easily seen that a forward hedge simply shifts the risk between the RAP-situation and the NRAP-situation. See figure 8.5.

$$(8.10) \quad \text{VA}[\text{EAP}] = \phi(q_1 e_1 - q_2 e_2 + q_0) + q_1 e_{h_1}[\text{FW}] (\text{FW}_1 - e_1) + \\ + q_2 e_{h_2}[\text{FW}] (e_2 - \text{FW}_2)$$

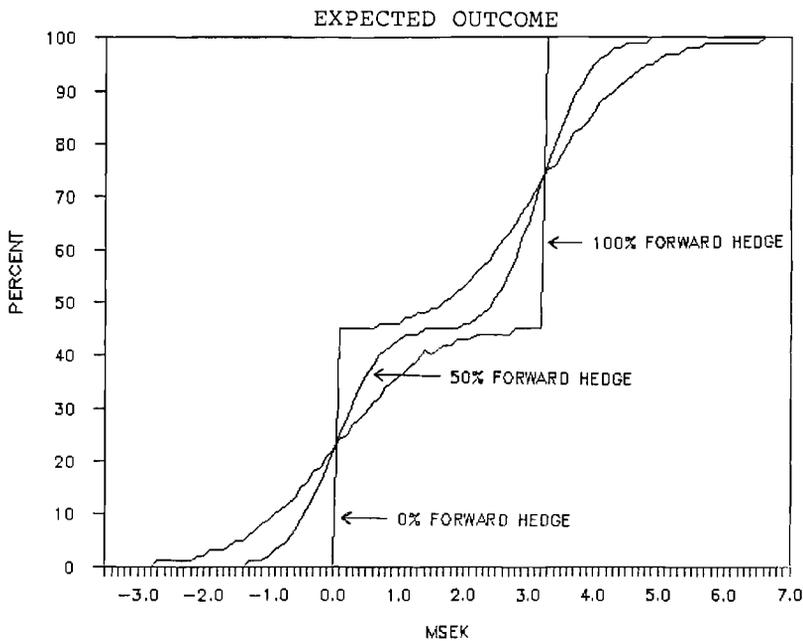
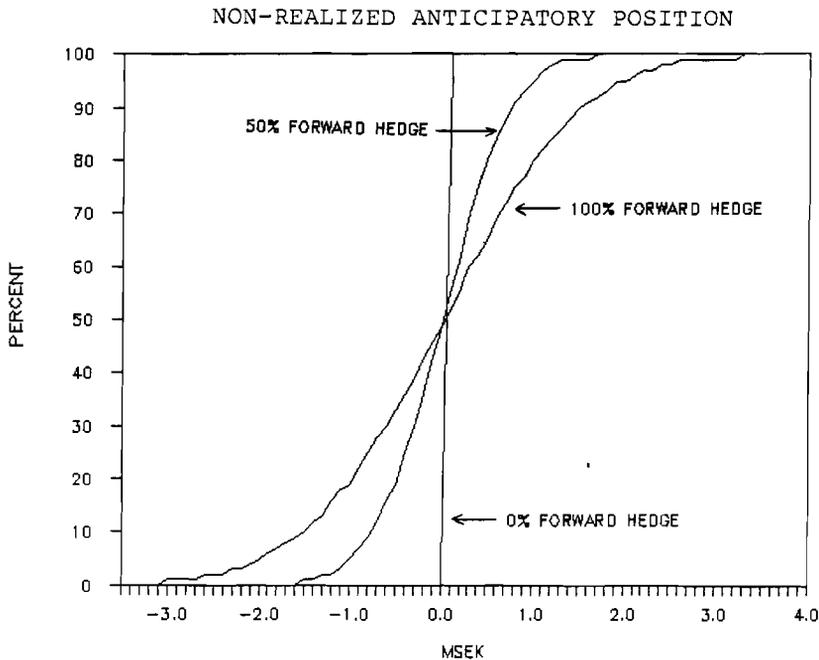
If the anticipatory position does not realize, the forward contracts will constitute an open position, i.e. be risky. A large forward hedge will therefore have a large upside potential and a large downside potential. A zero hedge naturally renders a zero value, i.e. a vertical line. The 50% hedge renders a final cash flow exactly in between the two extreme hedging levels. The expected outcome for the 100% forward hedge consists of a risky part around the value zero SEK, emanating from the non-realization situation, and a certain outcome at 3.2 MSEK (55% of the cumulative probability) adhering to the situation of realization. The open position has 45% at zero SEK and 55% distributed around 3.2 MSEK.

8.4.2.2 Option Hedge

Consider an anticipatory position, where the proportion $e_{h_1}[\text{P}]$ is hedged by put options and the proportion $e_{h_2}[\text{C}]$ hedged by Call options. The final value is according to (8.11). (Set $\phi=1$ for $\text{VA}[\text{RAP}]$ and $\phi=0$ for $\text{VA}[\text{NRAP}]$.)

$$(8.11) \quad \text{VA}[\text{EAP}] = \phi(q_1 e_1 - q_2 e_2 + q_0) + q_1 e_{h_1}[\text{P}] (\max[0, e_{1,x} - e_1] - \\ - P_e[t_x] e^{r_D T_x}) + q_2 e_{h_2}[\text{C}] (\max[0, e_2 - e_{2,x}] - C_e[t_x] e^{r_D T_x})$$

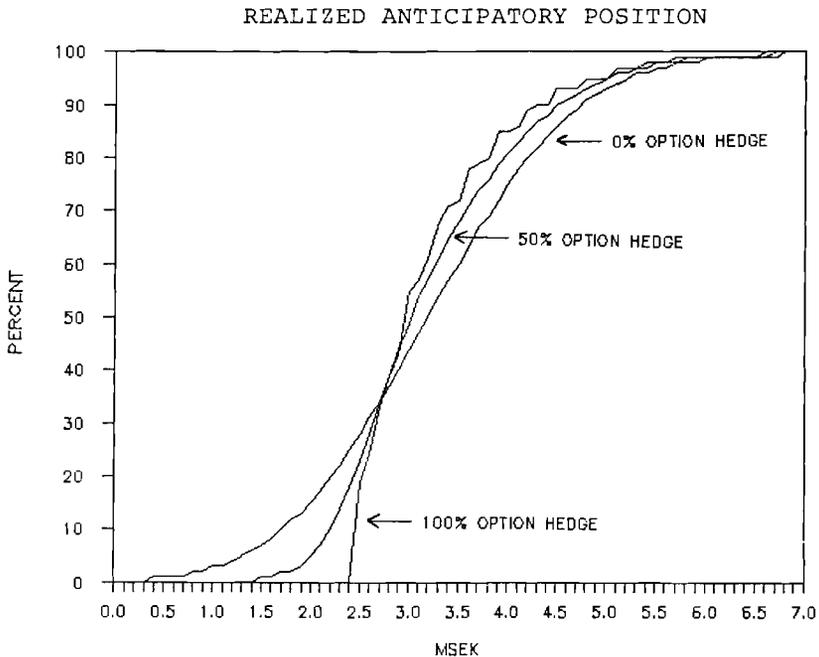
Figure 8.5 Cumulative distribution function for anticipatory positions, having different levels of forward hedge.
 (See figure 8.4 for the RAP position and Appendix E for exact numbers.)



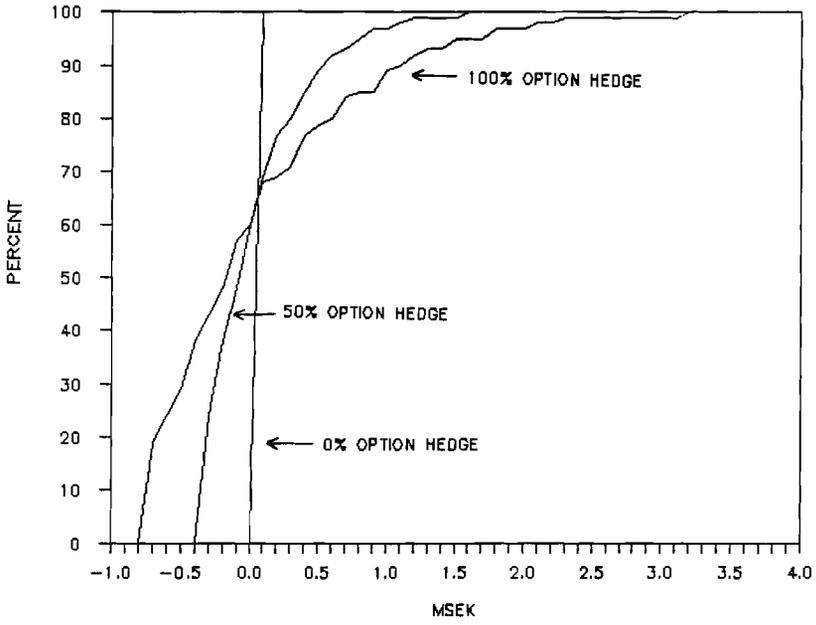
The cumulative distribution function is depicted in figure 8.6. In a realized anticipatory position, the 100% option hedged final cash flow may never fall below 2.4 MSEK (3.2 MSEK - 0.8 MSEK, the premium being 787,814.72 SEK). The upside potential is naturally reduced by the same amount. Hence, the 100% curve is found above the others on the extreme right of the figure. The curve will rise sharply (it should actually be vertical) at 2.4 MSEK, as the change in cumulative probability corresponds to the probability that both the call options and the put options will expire valuable, i.e. that the exchange rate SEK/CHF will fall below 4.1440 and the SEK/DEM will surpass 3.6862. In that case, the hedged total value will be constant.

Should the anticipatory position not realize, the 100% option hedge is equivalent to a purchased lottery ticket. An initial investment has been made, 0.8 MSEK, with a potential to win. Break even is found at approximately 65%.

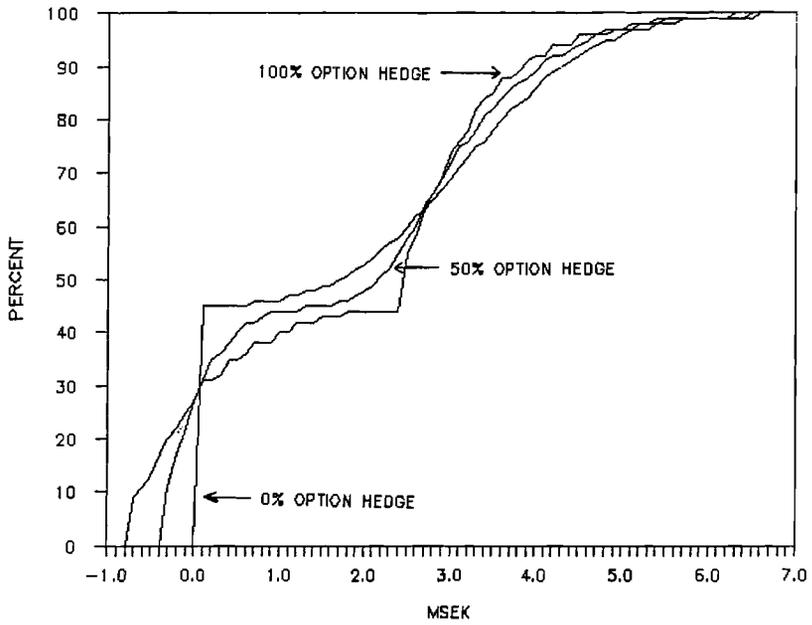
Figure 8.6 Cumulative distribution function for anticipatory positions, having different levels of option hedge. (See Appendix E for exact numbers.)



NON-REALIZED ANTICIPATORY POSITION



EXPECTED OUTCOME



8.4.2.3 Dynamic Hedge

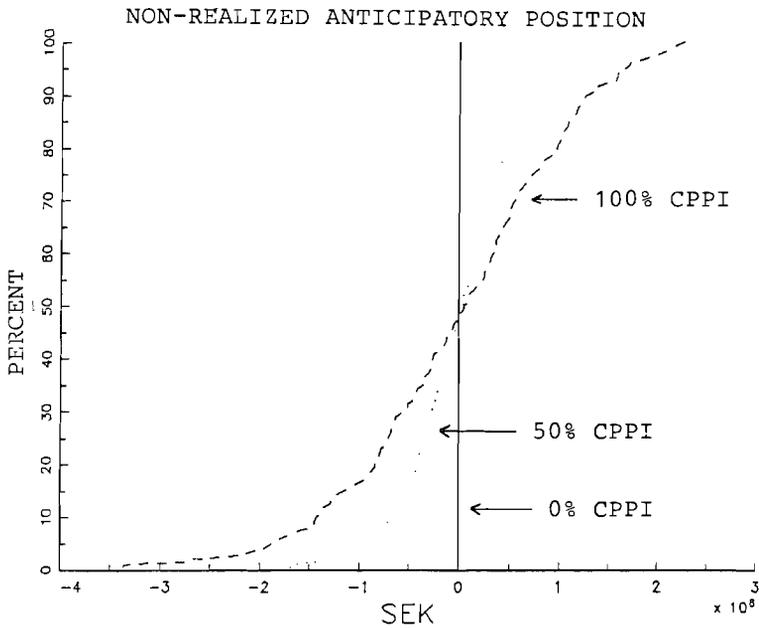
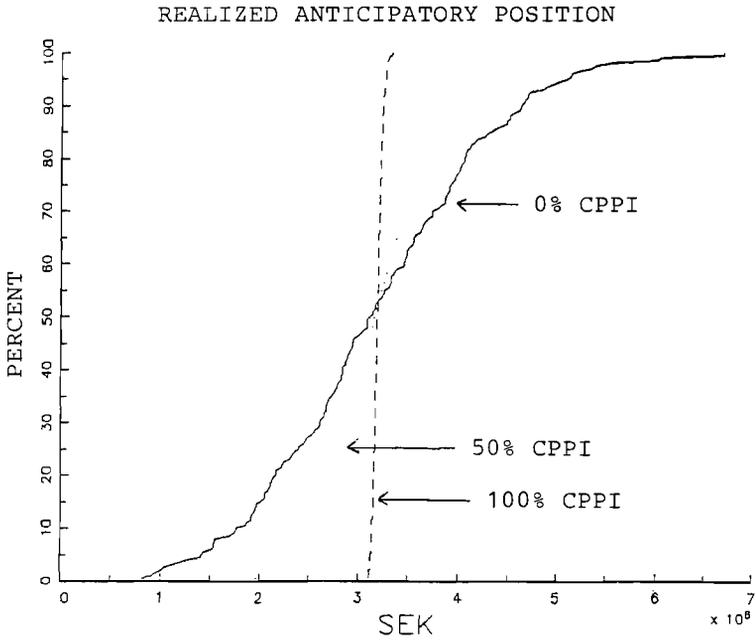
Consider the same underlying subpositions. The CPPI and the stop-loss are chosen to solely represent the dynamic hedging methods, as the TIPP turned out identical to the CPPI under the assumptions made (see Chapter Five). The final value is derived through simulation. Two hundred simulations are used. The potential trading is done once a day. The parameters are as given in Section 7.4.1. The results are depicted in figure 8.7 for CPPI and in 8.8 for stop-loss.

The 100% limited CPPI strategy implies a daily trading in order to keep the total value above the lowest acceptable. Having set the floor and the ceiling in harmony with the protection from an option hedge, the result ought not to fall below 2.4 MSEK for the numerical example. The floor level is set 1.17% below the initial value of the Swiss franc position, and the ceiling is set at 1.17% above the initial estimated cost to acquire the Deutschmarks required. The initial position of Swiss francs is therefore $11,510,937 \cdot 1.17\% \cdot 3 / 1.023 \approx 394,950$ CHF. The remaining short position of Deutschmarks after the initial allocation is $4,924,630 \cdot 1.17\% \cdot 3 / 1.021 \approx 169,299$ DEM.

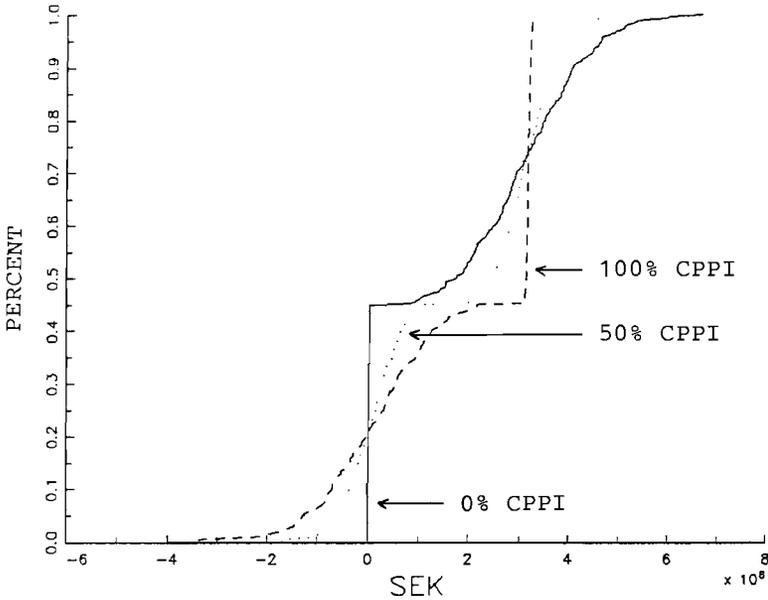
If the underlying position is realized, only the Swiss francs and Deutschmarks mentioned above, are susceptible to the stochastic exchange rates. Consider a large, momentaneous and disadvantageous change in the rates to gain a notion of the sensitivity of the 100% hedged position. Let the SEK/CHF fall to 3.9, and the SEK/DEM rise to 4.0. According to figures 5.1 and 5.2, these outcomes are very rare if considered separately from each other, as was done in the figures. In this position, the exchange rates are positively correlated by +0.6. Hence, the joint disadvantageous outcome is even less probable. The value of the position will fall by $394,950(4.1440 - 3.9) + 160,299(4.0 - 3.6862) \approx 149,494$ SEK. This is seen in the figure 8.7 as the final value will have a very limited downside risk. Should the multiplier m be increased, the distribution would naturally widen. A small upside potential may also be discerned from the figure. The position is naturally very similar to a buy-and-hold strategy or a forward hedge strategy. The floor may only be abused if the Swiss exchange rate on one day falls below 1/3 of the previous rate. The ceiling is abused if the German rate rises by more than 1/3 of the previous value. Both changes are unlikely to occur.

If the position is not realized, the CPPI strategy may incur large losses and gains, as the strategy is similar to an open forward position as a consequence of the assumptions made.

Figure 8.7 Cumulative distribution function for anticipatory positions, having different levels of CPPI hedge.

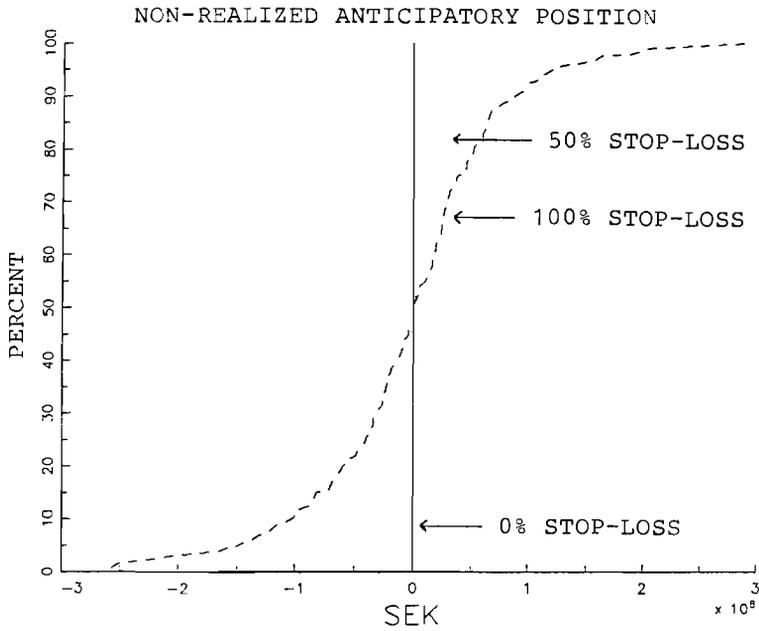
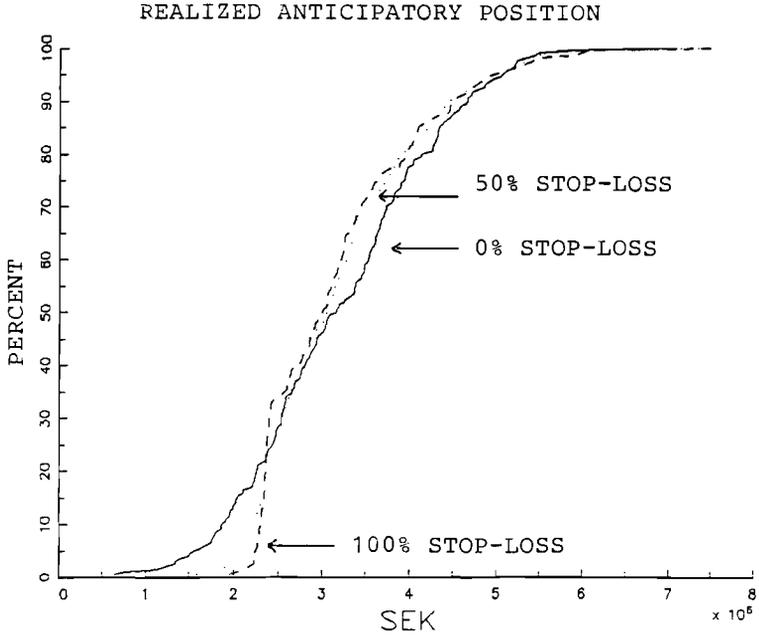


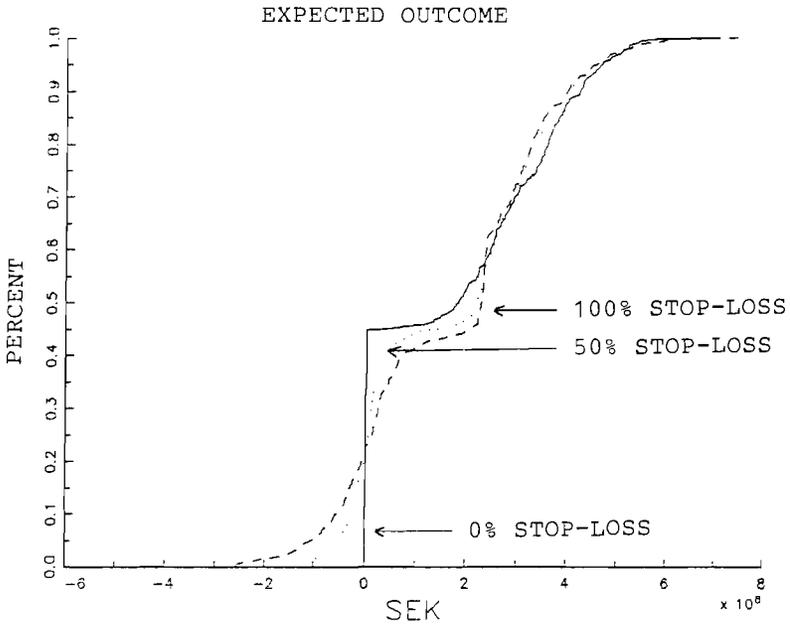
EXPECTED OUTCOME



The stop-loss strategy implies a daily control of whether the expected cash flow has fallen below/to the floor or risen above/to the ceiling. If that is so, the risky position is eliminated. Hence, the lowest acceptable result is 2.4 MSEK for the numerical example. However, due to the non-continuous trading the limit may be abused. For example, consider a daily movement of the Swiss franc of 0.032 SEK and of the Deutschmark of 0.025 SEK. The lowest acceptable result may then be underscored by approximately 0.49 MSEK. If the underlying position is not realized, the value of the final position merely constitutes the value of the stop-loss strategy. From the figures 5.5 and 5.6, it follows that the stop-loss value will always be positive if the Swiss franc falls below 4.1440 SEK and the Deutschmark exceeds 3.6862 SEK. If, for instance, both rates become 3.9, the profit will be 3.8 MSEK. However, this is not very likely. The relatively small probability of getting a hedge value at exactly zero is approximately 5%. This will occur if four conditions are fulfilled, namely: if the Swiss franc ends above 4.1440 SEK, if it has never fallen to/below the continuously changing floor, if the Deutschmark ends below 3.6862 SEK, and if it has never risen to/above the ceiling. Due to the very non-speculative limits chosen, the stop-loss will often fall out.

Figure 8.8 Cumulative distribution function for anticipatory positions, having different levels of stop-loss hedge.





8.4.2.4 Concluding Remarks

The probability distribution of the final cash flow may be altered by choosing different fractions for the hedging methods. By regarding the cumulative distributions of the separate methods as building blocks, Sections 8.4.2.1 through 8.4.2.3, the actor may mould a probability distribution according to his own preferences. However, special attention must be paid to the shifting of the risk between the situation of realization and non-realization.

Notes

1. No initial transactions are assumed except for hedging activities, in order to clarify the discussion. However, they are easily included by subtracting/adding them to the floor value, i.e. to the benchmark.
2. Should the position be such that the cash flow out dominates, FL is negative, and signifies the maximum allowable negative cash flow that the position may take. Hence, the discussion is valid irrespective of the sign of the position and the floor level.
3. The bivariate lognormal distribution is derived in Appendix B.1.2.
4. A noteworthy feature is that the maximum value of the corresponding pdf does not occur at the expected future cash flow. The reason is that the pdf[e_1, e_2] is not at (e_1, e_2 : $E[e_1], E[e_2]$). See Appendix B.3. The maximum point of the pdf[e_1, e_2] will occur at:

$$(e_1, e_2): (E[e_1]e^{-\frac{3\sigma^2 T}{2} - \tau\sigma_1\sigma_2 T}, E[e_2]e^{-\frac{3\sigma^2 T}{2} - \tau\sigma_1\sigma_2 T})$$

The probability of falling below the expected value will be slightly above 50%, due to the skewed (lognormal) distribution. For a one-subposition, the 50% level is at $E[e]\exp[-\frac{1}{2}\sigma^2 T] < E[e]$, i.e. where the stochastic variable X equals zero.

5. The probability of getting an expected cash flow below the floor is obtained analogously to the case of a certain position ($\phi=1$) although the domain of integration is obtained through comparison to CF[EAP]. The projection of the intersecting line between a floor level, FL, and the expected cash flow is given by:

$$e_2 = -\frac{FL - \phi q_0}{\phi q_2} + \frac{q_1}{q_2} e_1$$

It is immediately obvious that the 'tipping' will take place around this line, setting FL=0. Hence, the domain of integration, $CF[EAP] \leq FL$, is given by:

$$(e_1, e_2): [e_2 \geq -\frac{FL - \phi q_0}{\phi q_2} + \frac{q_1}{q_2} e_1; e_1 \geq 0; e_2 \geq 0]$$

6. The traditional insurance theory would demand a 100% hedge, a so called naive hedge. (See Folks [1973].)

$$\begin{aligned}
7. \quad VA = & \sum_i P_{n_i} \{ P_{h_i}[FW] FW_p[t_0, t_{FW}] + (1 - P_{h_i}[FW]) P_{t_{FW}} \} + \\
& + \sum_j e_{n_j} \{ e_{h_j}[FW] FW_e[t_0, t_{FW}] + (1 - e_{h_j}[FW]) e_{t_{FW}} \} + \\
& + \sum_{ij} g_{n_{ij}} \{ g_{h_{ij}}[FW] FW_g[t_0, t_{FW}] + (1 - g_{h_{ij}}[FW]) g_{t_{FW}} \}
\end{aligned}$$

8. The multiple, general formula of a 100% hedge is:

$$CF = \sum_i P_{n_i} FW_p[t_0, t] + \sum_j e_{n_j} FW_e[t_0, t] + \sum_i \sum_j g_{n_{ij}} FW_g[t_0, t]$$

9 Hedging of Tender Offers

9.1 Introduction

In Chapter Nine, the design introduced in Section 7.4 will be applied to the problem of hedging a tender offer. The emphasis will be on the implications of the principally different structure of the position.

A tender offer differs from both a contract and an anticipatory position in one major way. In a contract, the volume of the asset to be traded is known initially. In an anticipatory position, the probability of realization of the volume (which is assumed fixed) is known initially and no subsequent information is assumed to be released until the final date. Hence, the probability of acceptance/refusal is fixed over time. However, in a tender offer the reply (acceptance/refusal) is received after the initial day and prior to the point in time when the cash flow is received from the potential contract. The perceived probability of entering into a contract for a fixed volume is therefore changed during the duration of the total risk period. This characteristic generates some new aspects, not covered in the contractual position and/or the anticipatory position. Two issues will be addressed in this chapter. Firstly, the cash flow will prove dependent on the market at two different points in time. Consequently, the method derived in Chapter Seven must be modified. Secondly, the imperfections generated from hedging by forwards and options instead of from hedging by options on forward contracts are discussed.

The underlying position to be analyzed is assumed to consist of one single submitted tender offer. If the tender offer is accepted, a contract is entered which implies future cash inflow (payments for the goods) and cash outflow (contingent purchases to be able to honour the contract). In order to focus on the two issues mentioned above as well as on the application of the hedging decision technique, some simplifying assumptions are made. Firstly, the total amount of cash inflow from a potential contract is assumed to occur at one point in time, namely at the end when all the contractual obligations have been fulfilled. The common practice in business of an initial payment is disregarded. See the numerical example in Chapter Ten, for a relaxation of this assumption. Furthermore, the positive cash flow is assumed to consist of only one foreign currency. Secondly, the negative cash flows generated by contingent purchases are also assumed to occur at one single point in time,

namely at the end of the contractual period.¹ (Relaxed in the numerical example, Chapter Ten.) Hence, all parts of the total cash flow are assumed to coincide in time. Furthermore, it is assumed that at the time of submission of the tender offer, the foreign currency involved in the contingent purchase is known.² Thirdly, it is assumed that the reply from the potential customer is received exactly at the end of the duration of the tender offer period, which is known at the time of submission. The length of the tender offer risk period is fixed. Consequently, the contractual risk period is fixed in time. Fourthly, only a few of the hedging methods will be used. The tool used for the main analysis is the option on forward contract plus potential extra forward contracts unless otherwise stated. Options on the underlying asset and forward contracts are analyzed separately. Although the former method is the most ideal, the latter are the most commonly used.

In Chapter Nine, the position serving as illustration will consist of q_1 long units of foreign currency number one, q_2 short units of currency number two, and possibly q_0 long units of the domestic currency ($q_0 < 0$ implies a short position).³

9.2 Structure of the Total Risk Period

The period of risk when submitting a binding tender offer, defined at the initial point in time, spans from the initial point in time to the time of the final cash flow of the potential contract. (As mentioned earlier, the cash flow emanating from a contract is assumed to occur at one single point in time.)⁴ The total risk period may be split into two parts. The first one constitutes the time period between submission and the point in time when the tender offer is accepted or refused.⁵ Henceforth, this concept will be called the tender offer risk period, or, more briefly, as the tender offer period.⁶ The end of the period is denoted by t_{TOPP} . The second part of the total risk period constitutes the time between the reply and the final cash flow of the potential contract. It is defined as the contractual risk period, or, more briefly, as the contractual period. The end of the period is denoted by t_{CRP} . Denote the time of submission, the initial point in time, by t_0 . The course of events and the concepts defined may be as depicted in figure 9.1.

The splitting of the total risk period mentioned before may appear subtle and a mere question of semantics. However, it has a bearing on the evaluation process. A split total risk period implies that the risk may be assessed at two different points in time. (See Section 9.3.)

A '100% forward hedge' is often used in the industry. It involves an arbitrary hedge by options during the tender offer risk period and a 100% forward hedge during the contractual risk period, should the tender offer be accepted. Hence, any insufficiency in forward contracts must be met through the market at the beginning of the contractual period. The rationale for this special strategy is the opinion that industrial companies ought not to indulge in speculation.

Point in Time for Evaluation

The choice of point in time for evaluating the outcome is more of an illustrative problem than an emerging computational problem for a one hundred-percent hedge during the contractual period (ATO/RTO/ETO) or for a lower hedge ratio (only RTO). The estimations of the probabilities must be carried out at the point in time when the options on the forward contracts expires. However, the risk profile may be stated at different points in time by capitalizing/discounting the values, holding the probabilities constant.

The best point in time for illustrating the 100% hedged tender offer position varies depending on whether it is accepted, refused, or whether it is the expected tender offer to be analyzed. If the tender offer is accepted, the cash flow from the contract is assumed to occur at time t_{CRP} . As the major position is the underlying position, it is intuitively most appealing to choose the end of the contractual period as the point in time for evaluation of the risk and cash flow. Any potential intermediate cash flow must then be capitalized. Assuming that the purpose of the activities is to reduce the risk, a negative reply implies closing all the hedging subpositions as any remaining subpositions constitute a pure speculative activity. Hence in a refused tender offer position, no single subposition remains during the contractual period. The most illustrative point in time for evaluation is then the end of the tender offer risk period. Analyzing the expected tender offer implies that the accepted tender offer and the refused tender offer situations are to be integrated. Analogously to the discussion for the ATO case, the most convenient point in time for assessing the ETO is at the end of the contractual risk period. However, the point in time may be shifted rather easily by simply multiplying the cash flow by the discounting/capitalizing factor.

For a hedge ratio separate from 100% during the contractual period, the same ideal points in time for evaluating the outcome emerge, although computational problems arise unless the initial hedge ratio is zero. The approach necessary will be outlined in Section 9.5.

9.4 Unhedged Tender Offer

The cash flow involved in a tender offer may be mathematically represented by the cash flow of an unhedged anticipatory position, should the probability of realization be substituted for the probability of acceptance and the number of long subpositions be limited to one. Hence, previous results are entirely applicable. The same analogy is applicable for the probability distribution.

9.5 Hedging With Options on Forwards: the Consistent Approach

The consistent approach involves splitting the risk evaluation into two parts, one pertaining to the tender offer risk period and the other pertaining to the contractual risk period conditional on the outcome of the tender offer risk period. In the ATO and ETO, the value will depend, as will be shown below, on the market prices at two different points in time if the hedge ratio during the contractual risk period is different from 100% and 0%. For this reason, the method developed in Chapter Seven requires a modification.

Calculation Method for Intratemporally Dependent Cash Flow

In calculating the probability that the final cash flow will fall below a specific level, a two-step method must be applied. Consider a position consisting of q units of long foreign currency.⁷

$$(9.1) \text{VA}[\text{ATO}; t_{\text{CRP}}] = (1-h[\text{PFW}])q e_{\text{CRP}} + qh[\text{PFW}](\text{FW}_e[t_{\text{TORP}}, t_{\text{CRP}}] + \max[0, \text{FW}_x[t_{\text{CRP}}] - \text{FW}_e[t_{\text{TORP}}, t_{\text{CRP}}]] - \text{PFW}[t_{\text{TORP}}]e^{r_D(t_{\text{CRP}} - t_0)})$$

The calculating principle remains the same as in Chapter Seven and Eight. For each combination of e_{CRP} and $\text{FW}_e[t_{\text{TORP}}, t_{\text{CRP}}]$ calculate the pdf $[\text{FW}_e[t_{\text{TORP}}, t_{\text{CRP}}], e_{\text{CRP}}]$ times the integrating area and add to the cdf, if the value rendered is below the floor level.

Accepted Tender Offer

The value of a partially hedged position may be stated as (9.2). If the tender offer is accepted, the final value is dependent on the market both at time t_{TORP} and at time t_{CRP} . Equation (9.2) shows that the unhedged part of the underlying position is

affected by the market price at time t_{CRP} through $e_{1,CRP}$ and $e_{2,CRP}$ respectively. Furthermore, the hedged part of the underlying position including the hedge value depends on the market at time t_{TORP} through the variable $FW_e[t_{TORP}, t_{CRP}]$. Initially, the aforementioned variables are stochastic. In addition, they are dependent as the outcome of e_{TORP} will have an impact on the outcome of e_{CRP} .

$$\begin{aligned}
 (9.2) \quad VA[ATO; t_{CRP}] = & q_1(1-h_1[P_{FW}])e_{1,CRP} - q_2(1-h_2[P_{FW}])e_{2,CRP} + \\
 & + q_0 + q_1h_1[P_{FW}](\max[FW_1[t_{TORP}, t_{CRP}], FW_{1,x}[t_{CRP}]] - \\
 & \quad - P_{FW}[t_{TORP}]e^{r_D(t_{CRP}-t_0)}) + \\
 & + q_2h_2[C_{FW}](\max[-FW_2[t_{TORP}, t_{CRP}], -FW_{2,x}[t_{CRP}]] - \\
 & \quad - C_{FW}[t_{TORP}]e^{r_D(t_{CRP}-t_0)})
 \end{aligned}$$

Refused Tender Offer

The value at time t_{TORP} of a hedged position using options on forward contracts is:

$$\begin{aligned}
 (9.3) \quad VA[RTO; t_{TORP}] = & q_1h_1[P_{FW}](\max[0, FW_{1,x}[t_{CRP}] - \\
 & - FW_1[t_{TORP}, t_{CRP}]] \cdot e^{-r_D(t_{CRP}-t_{TORP})} - P_{FW}[t_{TORP}]e^{r_D(t_{TORP}-t_0)}) + \\
 & + q_2h_2[C_{FW}](\max[0, FW_2[t_{TORP}, t_{CRP}] - FW_{2,x}[t_{CRP}]] \cdot \\
 & \cdot e^{-r_D(t_{CRP}-t_{TORP})} - C_{FW}[t_{TORP}]e^{r_D(t_{TORP}-t_0)})
 \end{aligned}$$

Thus, the value is not dependent on the market at any time but t_{TORP} . Consequently, the probability distribution may be derived analogously to the anticipatory position in Chapter Eight. In calculating the probability density function and the cumulative distribution function for the refused tender offer, the method presented in Chapter Seven is applicable. A slight modification is, however, necessary in the implementation. The probability calculation is made at t_{TORP} , using the discounted value of the final reference value for comparison. Hence, the values of the risk profile pertain to time t_{CRP} , whereas the probabilities are calculated at time t_{TORP} . Although the modification is of principal importance, the effect on the calculations is merely that the value will be altered according to the time value of money adherent to the time period $[t_{TORP}, t_{CRP}]$.

Expected Tender Offer

The expected value of the tender offer is derived by weighting the value from the RTO and from the ATO, measured at one point in time, by the probability of acceptance, θ . The probability distribution is, as stated before, not calculated on this expected value. (See Section 7.4.2.1.)

$$(9.4) \quad VA[ETO;t_{CRP}] = (1-\theta)VA[RTO;t_{TORP}]e^{r_D(t_{CRP}-t_{TORP})} + \theta \cdot VA[ATO;t_{CRP}]$$

Concluding Remarks

An analysis of the ATO and ETO requires an excessive number of calculations due to the intratemporal dependence. Consequently, running a computer model will be very time consuming for any position consisting of more than one hedged subposition.

One way to handle the problem would be to assess the risk pertaining to the tender offer risk period separately from the contractual risk period. The former risk is derived by assuming that the contractual period is 100% hedged by forward contracts, and is preferably estimated at time t_{TORP} . The latter risk stems from the de facto unhedged part of the potential contract and is estimated at time t_{CRP} . The overall analysis must then be carried out in a heuristic manner. There is, therefore, no formal integration resulting in a single risk profile. (The risk profile of a 100% forward hedge is analyzed in the following section. The risk profile from a partially forward hedged contract has been derived in Chapter Eight. Hence, no further elaboration is necessary.) As each of the two risks are only dependent on the market at one single point in time, the number of calculations to be carried out is reduced significantly.

9.6 Hedging With Options on Forwards: the 100% Forward Approach

The 100% forward strategy implies that the tender offer risk period is hedged by an arbitrary fraction of options on forward contracts. If the tender offer is refused, the potentially acquired forward contracts are disposed of at t_{TORP} . If the tender offer is accepted, the discrepancy between the potentially acquired forward contracts through the option and the required 100% is acquired on the market. Hence, at time t_{TORP} no more risk remains and the value at the end of the tender offer risk period follows equation (9.5). Let $h_1[P_{FW}]$ and $h_2[C_{FW}]$ denote the hedging fraction during the tender offer risk period. Set $\theta=0$

for VA[RTO] and $\theta=1$ for VA[ATO]. Note that the only two stochastic variables, $e_{1,TORP}$ and $e_{2,TORP}$, adhere to the same point in time, namely t_{TORP} . The probability distribution is easily obtained by the same method as in Chapter Seven and Eight.

$$\begin{aligned}
 (9.5) \quad VA[ETO;t_{TORP}] &= \\
 &= \theta(q_1 e_{1,TORP}^{-r_{F1}(t_{CRP}-t_{TORP})} - q_2 e_{2,TORP}^{-r_{F2}(t_{CRP}-t_{TORP})}) + \\
 &\quad + q_0 e^{-r_D(t_{CRP}-t_{TORP})} + \\
 &\quad + q_1 h_1 [P_{FW}] (\max[0, e_{1,x}^{-e_{1,TORP}}] e^{-r_{F1}(t_{CRP}-t_{TORP})} - \\
 &\quad - P_{FW}[t_{TORP}] e^{r_D(t_{TORP}-t_0)}) + \\
 &\quad + q_2 h_2 [C_{FW}] (\max[0, e_{2,TORP}^{-e_{2,x}}] e^{-r_{F2}(t_{CRP}-t_{TORP})} - \\
 &\quad - C_{FW}[t_{TORP}] e^{r_D(t_{TORP}-t_0)})
 \end{aligned}$$

$$(9.6) \quad FW_x[t_{CRP}] = e_x e^{(r_D - r_F)(t_{CRP} - t_{TORP})}$$

$$(9.7) \quad e_x = e_0 e^{(r_D - r_F)(t_{TORP} - t_0)}$$

9.7 Hedging With Options and Forwards: Generated Imperfections

Should options on forward contracts not exist, they may be replaced by options on the underlying assets and by forward contracts. The strategy now involves buying options expiring at t_{TORP} , and then to enter forward contracts maturing at time t_{CRP} should the tender offer be accepted.

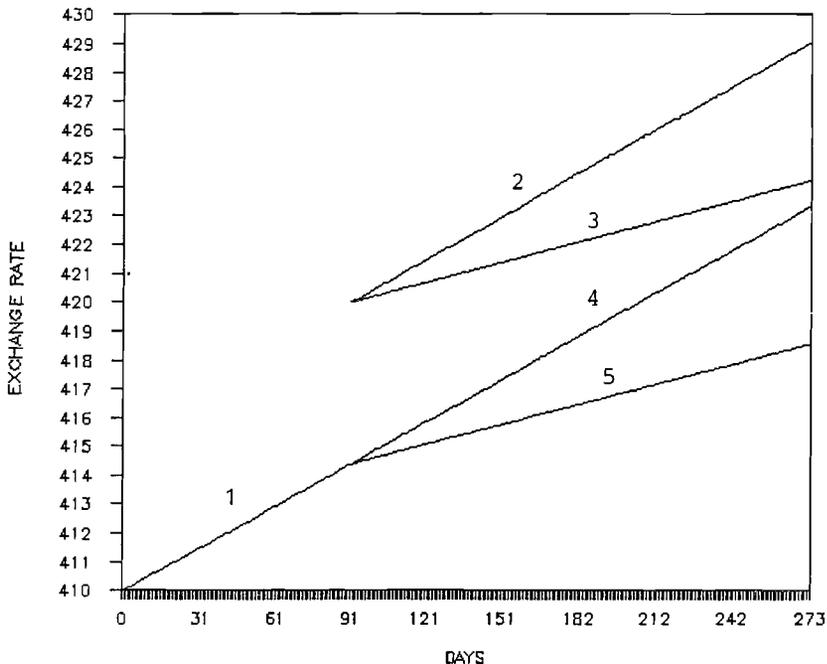
The purpose of this section is to derive the value and the probability distribution of the outcome and, more particularly, to analyze the difference in outcome compared to hedging by options on forward contracts. Both the consistent hedge and the 100% forward hedge are analyzed.

The underlying principle is as follows: The purpose of the hedge during the tender offer period is to compensate for the difference between the expected (measured at initiation) forward price to be obtained at t_{TORP} and the forward price de facto obtained at t_{TORP} , should the contract be awarded. See figure 9.2. (As was stated in Section 1.4, the nominal value is used as

benchmark.)

A change in the spot rate for time t_{TORP} compared to the estimated value, will shift the starting point vertically for the new curve depicting the expected future spot rate. If the interest rate differential remains the same, the initially estimated curve (number four) will be submitted to a parallel, vertical shift, to number two for example. A change in the interest rate differential alters the slope of the curve. Having a zero spot rate change, the curve is merely tipped up or down, from number four to number five for example. The hedge will not compensate for this. Should both a spot rate change and a change in the interest rate differential occur, both the starting point and the slope of the curve are shifted. The curve number four is substituted for number three. The hedge is limited to the vertical shift.

Figure 9.2 Hedging potential of option plus forward on a tender offer. Curve 1: estimated change of spot rate, measured at t_0 . Curve 2-5: estimated change in spot rate, estimated at t_{TORP} , where there has been a change during $[t_0, t_{TORP}]$. The change is for curve 2: spot rate; 3: spot rate plus interest rate differential; 4: no changes; 5: interest rate differential.



9.7.1 The Consistent Hedge

The value at time t_{CRP} of the consistent hedge using options and forwards is according to equation (9.8). Set $\theta=0$ for RTO and $\theta=1$ for ATO. $h_1[P_e] = h_1[FW_e] = h_1$, $h_2[C_e] = h_2[FW_e] = h_2$, where the hedging fractions adhere to different points in time.

$$(9.8) \quad VA[ETO;t_{CRP}] = \theta\{(1-h_1)q_1e_{1,CRP} - (1-h_2)q_2e_{2,CRP} + q_0\} + \\ + q_1h_1(\max[0, e_{1,x}-e_{1,TORP}]e^{r_D(t_{CRP}-t_{TORP})} - \\ - P_e[t_{TORP}]e^{r_D(t_{CRP}-t_0)} + \theta e_{1,TORP}e^{(r_D-r_{F1})(t_{CRP}-t_{TORP})}) + \\ + q_2h_2(\max[0, e_{2,TORP}-e_{2,x}]e^{r_D(t_{CRP}-t_{TORP})} - \\ - C_e[t_{TORP}]e^{r_D(t_{CRP}-t_0)} - \theta e_{2,TORP}e^{(r_D-r_{F2})(t_{CRP}-t_{TORP})})$$

By comparing equation (9.8) to (9.2), (9.3) and (9.4) respectively, it is easily seen that the difference in value stems from the values of the options. The value of the option on the forward contract is lower than the value of an option, by the factor $\exp[-r_F(t_{CRP}-t_{TORP})]$. Furthermore, the capitalized value of the initial option price differs as the C_e and P_e may differ from C_{FW} and P_{FW} respectively.

A Synthesis

The difference between hedging by options on forward contracts and hedging by options on the underlying asset has its habitat in the foreign risk free interest rate and in the initial price of the options. The former source of divergence is only relevant for foreign currency and may be compensated for by altering the amount of options bought. It is immediately obvious that $\exp[-r_F(t_{CRP}-t_{TORP})]qh[C_e \text{ or } P_e]$ options are to be bought, i.e. the discounted value (using the local interest rate) is to be hedged.⁸ The identical hedging fraction during the contractual risk period must, however, be retained, i.e. the number of forward contracts. The remaining discrepancy stems from the initial premium of the options. If the corrections for the number of options bought are included, the following relationships emerge:

$$(9.9a) \quad P_{FW}[t_{TORP}]e^{r_D(t_{CRP}-t_0)} \quad \leftrightarrow \quad P_e[t_{TORP}]e^{-r_F(t_{CRP}-t_{TORP}) + r_D(t_{CRP}-t_0)}$$

$$(9.9b) \quad C_{FW}[t_{TORP}]e^{r_D(t_{CRP}-t_0)} \quad \leftrightarrow \quad C_e[t_{TORP}]e^{-r_F(t_{CRP}-t_{TORP}) + r_D(t_{CRP}-t_0)}$$

The expressions in equation (9.9) must be identical if no risk-

free arbitrage exists. (The equality is shown in Appendix A.7.) From the equation it also follows that the initial price of an option on a forward contract is lower than the initial price of an option on the underlying asset.

Concluding Remarks

As has been shown above, an option on forward contracts was perfectly replaceable by an option on the underlying asset under the assumptions made. A point of interest, however, is the robustness of the substitutability concerning interest rates.⁹

In an option on forward hedge, the forward price is based on the initial estimation of the interest rates. Hence, the final lowest value is completely certain for the hedged part. In an option hedge, the forward price is stochastic and depends on the new, changed interest rates. Hence, the final lowest value is stochastic. The difference is due to the error made in predicting the future forward price. Furthermore, an adverse effect of this is that the corrective factor (the number of options to buy in order to equate the two methods) will be unknown.

9.7.2 The 100% Forward Hedge

The 100% forward hedge may be divided into a consistent hedge combined with an extra forward hedge. The extra forward hedge is always obtained at time t_{TORP} and the forward price is that prevailing on the market.

The extra forward hedge is identical under a hedge by options on forward contracts and a hedge by options on the underlying asset. Hence, the value difference between the two methods will be identical to that in a consistent hedge. Naturally, the value in each of the hedging methods will change except for a 100% option on a forward hedge.

Notes

1. See Section 9.2 for definitions of the risk period concepts.
2. This is not always in accordance with reality, but usually, at least the major foreign currency is known.
3. The formulas derived are easily generalized to the other two assets, by setting $r_F=0$.
4. If the tender offer is refused, the period of risk will naturally be simultaneously discontinued.
5. The author is aware that this assumption is a considerable simplification of the situation in business where deviations (usually extensions of the period) by 50 to 100% are not uncommon. The effect of allowing the length of the tender offer risk period become stochastic may be an area for future research. However, the decision support method encompasses this extension.
6. The concept tender-to-contract exposure, denoting "the contingent risk arising from the time gap between bid and possible award of contract" was used by Warren [1987].
7. Naturally, the method may be generalized to incorporate two or more subpositions. This simplification is analogous to the reduction from multiple subpositions to two-subposition made in Chapter Eight. The essence is to allow only two risky variables to remain. In this case e_{CRP} and $FW_e[t_{TORP}, t_{CRP}]$. For a two-subposition problem, the number of stochastic variables will be four. Hence, the integration domain may not be depicted in a figure. Should the calculus be necessary, however, the method from the one-subposition is to be combined to the two-subposition method described in previous chapters. Hence, for each conceivable combination of $e_{1,CRP}$, $e_{2,CRP}$, $FW_1[t_{TORP}, t_{CRP}]$ and $FW_2[t_{TORP}, t_{CRP}]$ which renders a final cash flow below the reference level, calculate the pdf [$e_{1,CRP}$, $e_{2,CRP}$, $FW_1[t_{TORP}, t_{CRP}]$, $FW_2[t_{TORP}, t_{CRP}]$].
8. This result becomes obvious if thought of in terms of a money market hedge to be initiated at time t_{TORP} . The amount to borrow/lend is the discounted value of the final cash flow.
9. Assume that the interest rates change unexpectedly (the initial pricing of the options does not change) just prior to time t_{TORP} . Denote the new interest rates by r_D and r_F . The values become, ceteris paribus:

Options on forwards:

$$\begin{aligned}
 VA[ETO;t_{CRP}] &= \theta\{(1-h_1[P_{FW}])q_1e_{1,CRP} - (1-h_2[C_{FW}])q_2e_{2,CRP} + \\
 &+ q_0\} + q_1h_1[P_{FW}](\max[0, e_{1,x}e^{(r_D-r_{F1})(t_{CRP}-t_{TORP})} - \\
 &- e_{1,TORP}e^{(r_D-r_{F1})(t_{CRP}-t_{TORP})}] - \\
 &- P_{FW}[t_{TORP}]e^{r_D(t_{TORP}-t_0)+r_D(t_{CRP}-t_{TORP})} + \\
 &+ \theta e_{1,TORP}e^{(r_D-r_{F1})(t_{CRP}-t_{TORP})}) + \\
 &+ q_2h_2[C_{FW}](\max[0, e_{2,TORP}e^{(r_D-r_{F2})(t_{CRP}-t_{TORP})} - \\
 &- e_{2,x}e^{(r_D-r_{F2})(t_{CRP}-t_{TORP})}] - \\
 &- C_{FW}[t_{TORP}]e^{r_D(t_{TORP}-t_0)+r_D(t_{CRP}-t_{TORP})} - \\
 &- \theta e_{2,TORP}e^{(r_D-r_{F2})(t_{CRP}-t_{TORP})})
 \end{aligned}$$

Options and forwards: (Consistent hedge: $h_1[P_e] = h_1[FW_e] = h_1$, $h_2[C_e] = h_2[FW_e] = h_2$.)

$$\begin{aligned}
 VA[ETO;t_{CRP}] &= \theta\{(1-h_1)q_1e_{1,CRP} - (1-h_2)q_2e_{2,CRP} + q_0\} + \\
 &+ q_1h_1(\max[0, e_{1,x}e_{1,TORP}e^{r_D(t_{CRP}-t_{TORP})} - \\
 &- P_e[t_{TORP}]e^{r_D(t_{TORP}-t_0)+r_D(t_{CRP}-t_{TORP})} + \\
 &+ \theta e_{1,TORP}e^{(r_D-r_{F1})(t_{CRP}-t_{TORP})}) + \\
 &+ q_2h_2(\max[0, e_{2,TORP}e_{2,x}e^{r_D(t_{CRP}-t_{TORP})} - \\
 &- C_e[t_{TORP}]e^{r_D(t_{TORP}-t_0)+r_D(t_{CRP}-t_{TORP})} - \\
 &- \theta e_{2,TORP}e^{(r_D-r_{F2})(t_{CRP}-t_{TORP})})
 \end{aligned}$$



10 Implementation of the Decision Support Method

10.1 Introduction

The purpose of this chapter is twofold: firstly to illustrate how a currency hedging decision would be made using the decision support method developed and secondly critically evaluate the method. As a background to the evaluation, a short description of an actual implementation of the method is given.

10.2 A Numerical Example

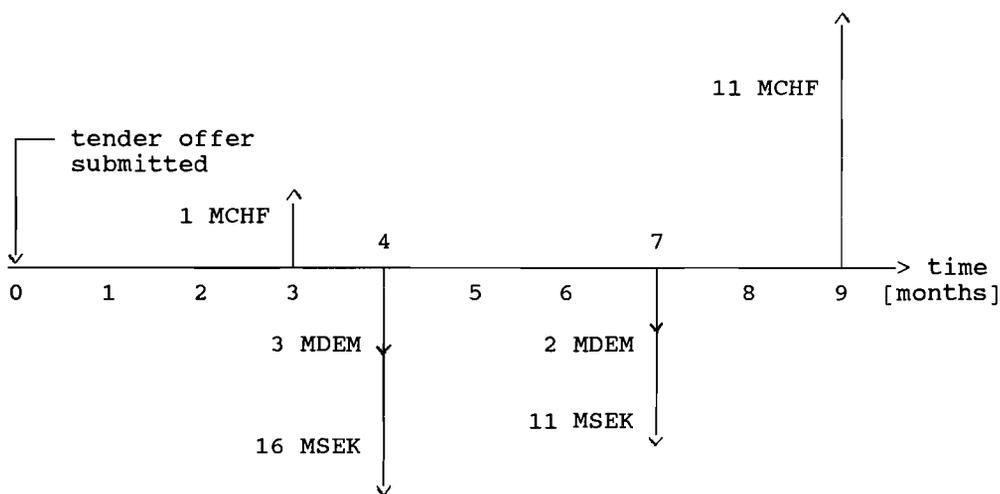
The purpose of this section is to illustrate how a decision may be made when the decision support method suggested in Chapter Seven is used. The position chosen for this purpose consists of a tender offer,¹ and the currency for evaluation is the Swedish krona. The selling company is henceforth called X AB. The illustration will be made in the following order. Firstly, the tender offer will be described. Secondly, the market situation will be given. Thirdly, some initial calculations will be made. Finally, the position is analyzed using two sets of three hedging combinations in succession. Exact values for the curves in the figures are given in Appendix E.

10.2.1 The Tender Offer

Assume that the tender offer period is three months. If the contract is awarded, the customer will make a down payment of one million Swiss francs immediately. When the goods have been delivered/constructed six months after signing the contract, the customer will make a final payment of eleven million Swiss francs. Thus, the cash inflow to the selling company consists of foreign currency. In order to be able to produce the goods to be delivered, X AB must make some purchases. These will partly be made in the domestic currency and partly in Deutschmarks. It is assumed that the prices of the inputs are fixed, if measured in

local currency units. The first purchases will be made immediately, and assuming the condition of '30-days net', the first payments will be made one month after signing the contract. Assume that this cash outflow consists of three million Deutschmarks and 16 million Swedish kronas. Further purchases are considered necessary during the production. ('Purchase' may be interpreted as any cost incurred causing cash outflow, e.g. wages.) To simplify the example, assume that the remaining payments to be made amount to two million Deutschmarks and eleven million Swedish kronas, to be paid four months after signing the contract. Thus, the allocation of the cash flow over time will be as depicted in figure 10.1. If the tender offer is refused, the cash flow is assumed to be zero, for the sake of simplicity. The probability of being awarded the contract is estimated as 55%. If the contract is awarded, it is assumed that the cash flows must be one hundred percent forward hedged.

Figure 10.1 Cash flow of accepted tender offer



10.2.2 The Market

Market information consists of foreign exchange rates (spot), interest rates, volatilities of exchange rates and the correlation coefficient. In this example, it is assumed that no spread exists,² and that:³

Spot exchange rate: ⁴	410.00 SEK/CHF
	364.00 SEK/DEM

Nominal interest rate: ⁵	SEK: 13.6%
(3 months)	CHF: 9.2%
	DEM: 8.4%

Forward exchange rate: (3 months) ⁶	414.4086 SEK/CHF
	368.6347 SEK/DEM
Volatility: ⁷	SEK/CHF: 6%
	SEK/DEM: 6%
Correlation coefficient: ⁸	SEK/CHF - SEK/DEM: +0.6

10.2.3 Initial Calculations

The discounted values at time 3-months, using the local interest rate for discounting, of the cash flows are as stated below.⁹ (See Section 9.7.1 for reasons why it is the discounted values that are to be hedged.) The expected total value at time 3-months is 3,205,354 SEK.¹⁰ The margin of the project is 6.7%.¹¹ The proportion of foreign purchases is 40.8%.¹² The option premiums, exercise price being set to the forward rate, are 1.17% on CHF (≈ 0.0480 SEK per one CHF) and 1.17% on DEM (≈ 0.0426 SEK per one DEM).¹³

SEK:	26,343,064 SEK
CHF:	11,510,937 CHF ($\approx 47,702,313$ SEK)
DEM:	4,924,630 DEM ($\approx 18,153,895$ SEK)

10.2.4 Application of the Method¹⁴

The method will be illustrated by analyzing two sets, each set comprising three different hedging combinations. The first set consists of three extreme hedging mixes: no hedge, a pure forward hedge and a pure options hedge respectively. The second set is chosen based on the findings from the first set. The cumulative distribution function is chosen to depict the risk.¹⁵

Set One

In the first set, the three hedging methods below are applied. Calculating the cumulative distribution function¹⁶ renders table 10.1. If the probabilities are illustrated graphically, figure 10.2 is obtained for the outcomes: accepted tender offer, refused tender offer and expected outcome respectively.

- #1: no hedge at all (100% open)
- #2: an 100% overall forward hedge¹⁷
- #3: an 100% overall options hedge
(strike price = forward price)

The table shows that a non-hedged tender offer, hedging mix number one, is submitted to a three per cent risk of falling below one million Swedish kronas in value, measured at time 91 days, should the contract be awarded. There is a 72% probability (100-28) that the value will be 2.5 MSEK or higher on a contract. Should the tender offer be refused, there is no risk of falling below zero. The value will naturally be exactly zero. By weighting the outcome of an accepted tender offer (55% probability of acceptance) by the outcome of a refused tender offer, the expected outcome is obtained. Thus, if the tender offer is submitted, X AB enjoys a 22% probability (100-78, $78\% = 55\% \cdot 60 + 45\% \cdot 100$) of getting a value at or over 3.5 MSEK.

Figure 10.2 clearly shows the obvious fact that if the tender offer is left unhedged, the remaining risk is entirely sourced in the acceptance outcome. If the underlying position is 100% hedged by forward contracts, the outcome in case of acceptance will be certain and approximately equal to 3.2 MSEK. (See Section 10.2.3.) However, in case of refusal, X AB will have two open forward positions, a short one in Swiss francs and a long one in Deutschmarks. Those contracts render the risk shown in table 10.1, and in figure 10.2. For example, X AB will face a 19% risk of losing more than one million Swedish kronas and a 20% chance of gaining more than or equal to 1 MSEK if the tender offer is refused. Should the tender offer be submitted, the risk of losing more than 1 MSEK is 9% ($55\% \cdot 0 + 0.45\% \cdot 19$). The risk stems entirely from the refused tender offer situation. Hence, by comparing an open position to a forward hedge in figure 10.2 it is obvious that the risk is shifted from the situation of acceptance to the situation of refusal.

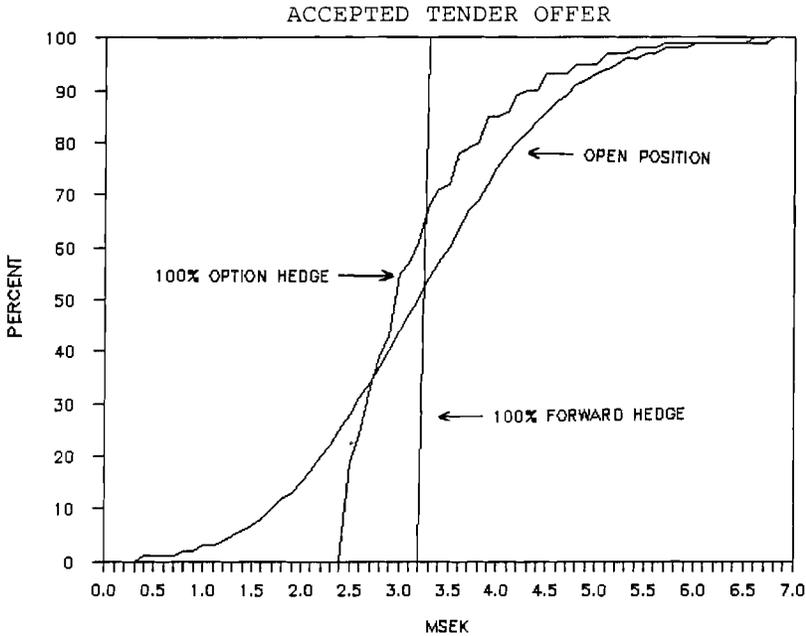
The third hedging mix consists of a pure option hedge. The capitalized value of the initial premium amounts to 787,815 SEK.¹⁸ Hence, the worst possible value if the contract is awarded is approximately 2.4 MSEK and in case of refusal approximately -0.8 MSEK.

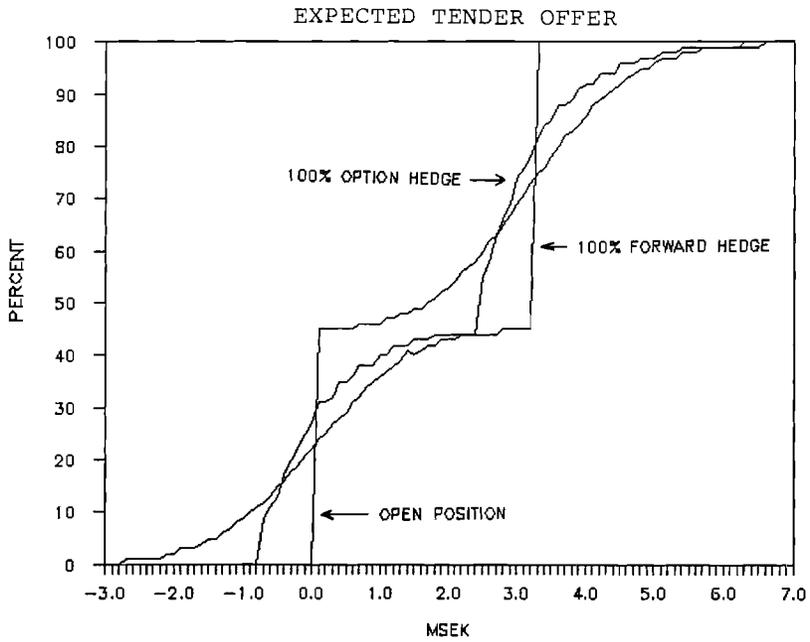
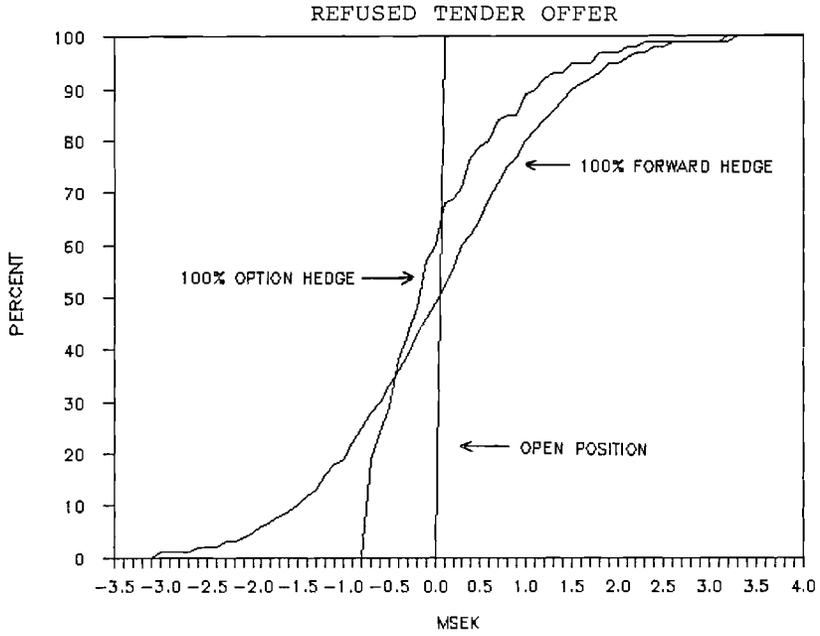
When the three hedging alternatives are compared, the open position may be considered too risky if the contract is awarded, e.g. 15% risk of getting a value under +2 MSEK, whereas a forward hedge may be considered too risky if the tender offer is refused, e.g. 49% risk of loss. The more conservative approach of a complete option hedge has the disadvantage of an initial premium. Thus, alternative mixes must be analyzed.

Table 10.1 Cumulative distribution for hedging set number one.
 (Selection of values. See Appendix E.)

	#1: 100% open			#2: 100 forw.			#3: 100% opt.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
< -3,500,000	0	0	0	0	0	0	0	0	0
< -3,000,000	0	0	0	0	1	0	0	0	0
< -2,000,000	0	0	0	0	5	2	0	0	0
< -1,000,000	0	0	0	0	19	9	0	0	0
< - 800,000	0	0	0	0	25	11	0	0	0
< - 700,000	0	0	0	0	28	12	0	19	9
< ± 0	0	0	0	0	49	22	0	60	27
< 500,000	1	100	45	0	65	29	0	79	35
< 1,000,000	3	100	46	0	80	36	0	89	40
< 2,000,000	15	100	53	0	95	43	0	97	44
< 2,500,000	28	100	60	0	98	44	19	99	55
< 3,000,000	44	100	69	0	99	45	55	99	74
< 3,500,000	60	100	78	100	100	100	72	100	85
< 5,000,000	93	100	96	100	100	100	95	100	97
< 7,000,000	100	100	100	100	100	100	100	100	100

Figure 10.2 Cumulative distribution for hedging set number one.



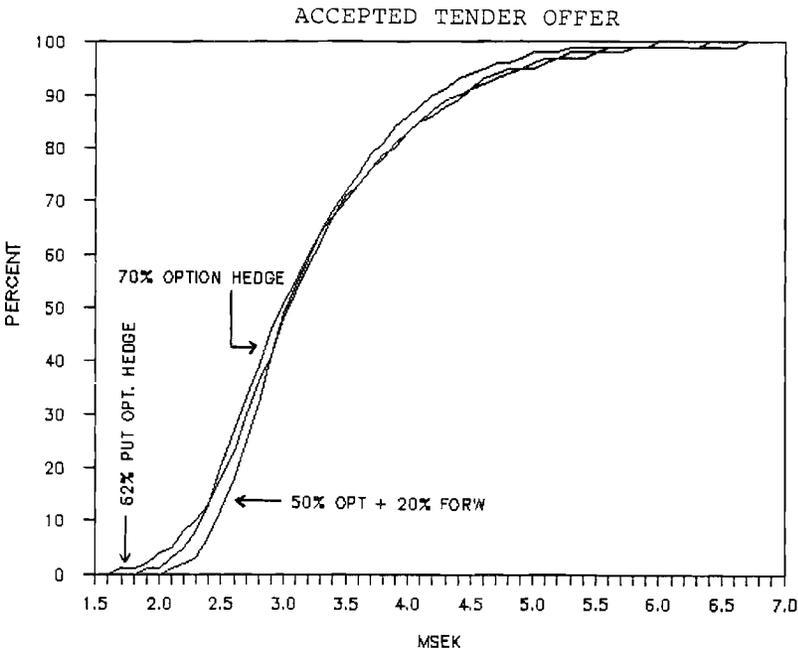


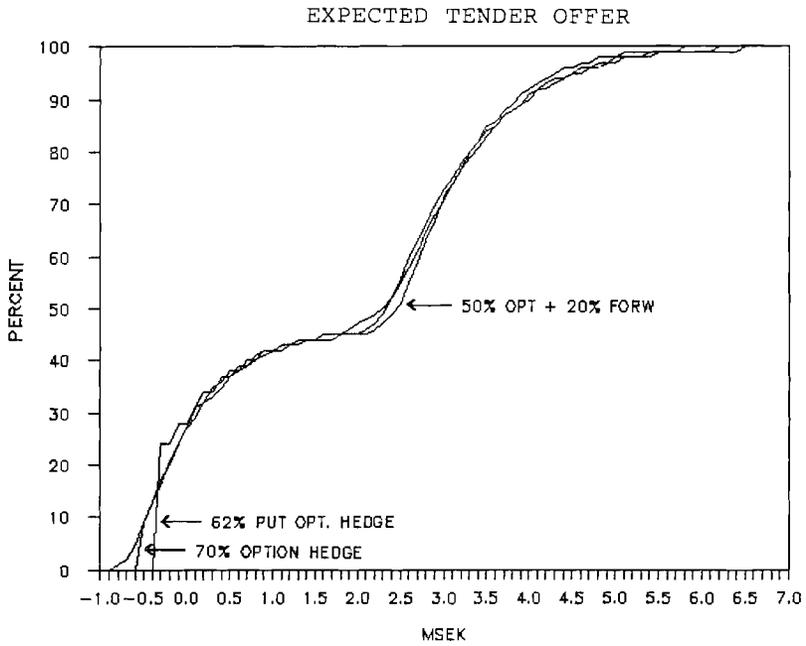
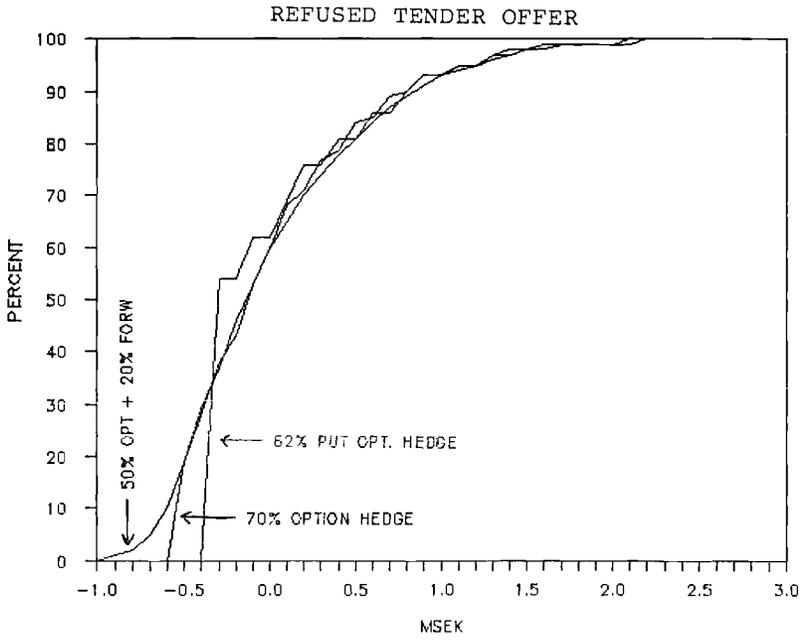
Set Two

Assume that the option risk reducing effect shown in the previous section is desirable, but that the initial premium is considered too high. To reduce the total premium either hedge a lower overall percentage of the underlying position or lower the hedging proportion unevenly.¹⁹ To illustrate the former alternative, set the overall hedge ratio to 70% for example. The capitalized value of the premium will be reduced to 551,470 SEK from 787,815 SEK. One way to lower the percentages unevenly would be to hedge only the net amount of foreign cash flow, after matching the positively correlated currencies. The hedge ratio would amount to 62% of the Swiss francs.²⁰ The capitalized value of the premium is 353,991 SEK.²¹ A third way of choosing a hedging mix is to combine the different hedging alternatives. An arbitrary choice may be to hedge using 50% overall options and 20% overall forward contracts. In this way, 30% of the underlying position remains open. The capitalized value of the premium is 393,908 SEK. Thus, the hedging combinations of set two are:

- #1: 70% overall options hedge
- #2: 62% put options on Swiss francs
- #3: 50% overall options plus
20% overall forward contracts

Figure 10.3 Cumulative distribution for hedging set number two.





By comparing mix number one and number two in table 10.2 or figure 10.3, it is seen that the risk in case of acceptance is marginally higher for number two, a maximum of five percentage units in deviation at +2.9 MSEK. In case of refusal, the 70% option hedge incurs a loss exceeding 0.4 MSEK by 19% due to the premium, whereas combination number two has zero percent risk. Values less than -300,000 renders a risk of 38% for number one compared to 54% for number two. Due to the non-perfect correlation between the currencies, the 62% put option hedged position may underscore 2.8 MSEK, (3.2-0.35). By profiting from the positive correlation, the premium may be reduced from 551 kSEK to 354 kSEK with a reasonable reduction in safety.

If the option hedge amounts to 50% and the forward hedge to 20%, the downside risk in case of acceptance is reduced compared to mix number one. This is so because 20% of the option hedge, and the incurred premium, has been exchanged for the ideal forward contract. Naturally, the upside potential has been reduced. Should the tender offer be refused, the third mix renders a larger risk of losing more than 600 kSEK than mix one and two.

Table 10.2 Cumulative distribution for hedging set number two.
(Selection of values. See Appendix E.)

	#1: 70% option			#2: 62% put			#3: 50% opt. 20% forw.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
< -1,000,000	0	0	0	0	0	0	0	0	0
< - 600,000	0	0	0	0	0	0	0	10	5
< - 500,000	0	19	9	0	0	0	0	19	9
< - 400,000	0	28	13	0	0	0	0	29	13
< - 300,000	0	38	17	0	54	24	0	37	16
< - 200,000	0	43	20	0	54	24	0	46	21
< - 100,000	0	53	24	0	62	28	0	53	24
< ± 0	0	60	27	0	62	28	0	60	27
< 500,000	0	84	38	0	81	37	0	81	37
< 1,000,000	0	93	42	0	93	42	0	93	42
< 2,000,000	1	99	45	4	99	47	0	99	45
< 2,500,000	20	100	56	18	100	55	12	100	51
< 2,900,000	46	100	70	41	100	68	41	100	67
< 3,000,000	51	100	73	48	100	71	49	100	72
< 5,000,000	95	100	97	96	100	98	98	100	98
< 7,000,000	100	100	100	100	100	100	100	100	100

Concluding Remarks

From the discussions it follows that the probability of being awarded the contract has a significant bearing on the choice of hedging vehicle, if the expected outcome is considered i.e. the total risk. If the probability of acceptance is large, the weight of the acceptance-outcome will increase, which subsequently implies that the incentive to shift the risk to the refusal-outcome will increase. Consequently, the use of forward contracts will increase. The use of forward contracts will decrease with a decreased probability of acceptance, as the risk is best allocated to the situation of acceptance as that outcome will occur less often. The proportion of the position left open ought to increase. In the intermediate ranges, between the polar probabilities of almost 100% probability of acceptance and almost 100% probability of refusal, the option alternative must be considered.

The discussions in the two previous sections illustrate the trade-off between upside potential and downside risk. Furthermore, the possibility of allocating the risk between the situation of acceptance and refusal is pointed out. The hedging decision must always be made by the party who will bear the consequence of the risk. The company utility function will thereby be included implicitly.

10.3 Empirical Indication of Implementability – ABB Asea Brown Boveri AB

The decision support method developed in this thesis has been implemented by ABB Asea Brown Boveri AB (the Swedish part of the ABB group), henceforth called ABB. The implementation will be described as a background for the evaluation of the method in Section 10.4. A special focus is put on the discrepancies between the assumptions of the model and the realities in business life.

The discussion will be initiated by a short description of the structure of the ABB group. This is followed by a short discussion of the choice of currency to measure the performance with. The third part of the description will focus on the structure of decision making with regards to the currency risk in ABB. Finally, the implementation will be analyzed.

10.3.1 Structure of the ABB-Group²²

Due to the recent merger of ASEA and BBC (January 5, 1988), the structure of the newly created company has not yet stabilized. Hence, this description must be interpreted with some care.

However, it will serve the purpose of illustrating the complex situation when making a hedging decision.

Figure 10.4 Ownership of the ABB-Group

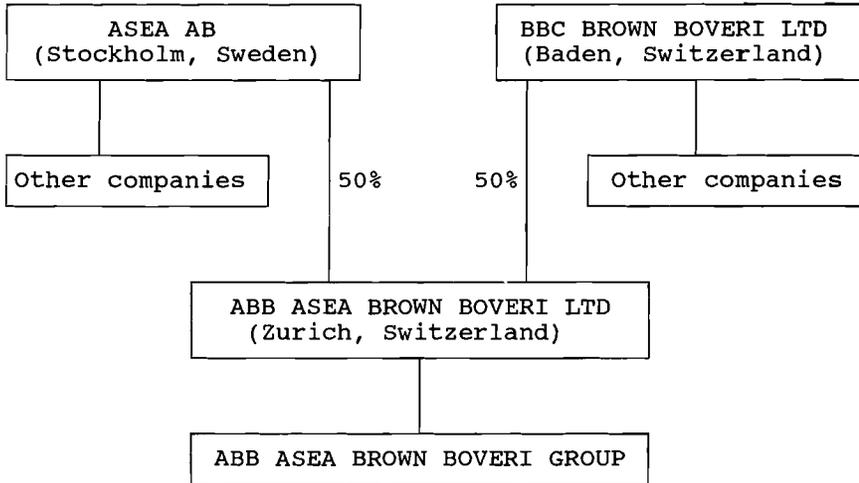


ABB Asea Brown Boveri Ltd is a holding company for the ABB group. The group comprising approximately 800 companies is organized in a matrix structure with eight business segments subdivided into approximately 50 business areas and divided into five geographical regions.²³ The Swedish companies of the group (approximately 130) are owned by ABB Asea Brown Boveri AB.

The regions are:

- * Western Europe - European Community
- * Western Europe - EFTA
- * North America
- * Asia and Australasia
- * Others

The business segments are:

- * Power Plants
- * Power Transmission
- * Power Distribution
- * Industry
- * Transportation
- * Environmental Control
- * Financial Services
- * Various Activities

The business areas within the Financial Services are:

- * Treasury Centers
- * Leasing and Financing
- * Insurance
- * Trading and Trade Finance
- * Stock Brokerage and Investment Management
- * Other Financial Services

The Treasury Centers (Sweden, Norway, Finland, Germany, Switzerland, Italy, USA) are coordinated by ABB World Treasury Center in Zurich. The objectives of the centres comprise acting as internal banks, managing the group's liquid assets, borrowing and foreign exchange transactions.

10.3.2 Choice of Currency for Evaluating the Performance

The different entities of the ASEA/BBC/ABB present their income statement and balance sheets in different currency units. ASEA AB uses Swedish kronas, BBC Ltd Swiss francs, ABB Ltd Swiss francs, the ABB group (consolidated) US dollars whereas most separate ABB companies use the local currency.²⁴ The performance of the ABB companies are evaluated in these local currencies. The ultimate owners of the group are the stock holders of ASEA AB (listed in Stockholm, Copenhagen, Helsinki, London, Freiverkehr (OTC, München) and NASDAQ (NY)) and of BBC Ltd (listed in Zurich, Basel, Geneva, Frankfurt and Vienna).

The dilemma of finding the best currency unit to evaluate each entity is left as an issue for future research. In the forthcoming discussions, the currency chosen for measuring risk/performance in the Swedish companies is the one in which the companies state their income/balance, i.e. the Swedish krona.

10.3.3 Structure of Decision Making of Currency Hedging

The ABB headquarters in Zurich issue a financial policy. This policy sets the limits for the currency hedging decisions of all the companies world wide. It might be developed into a more detailed, regional policy within each country or an arbitrary wider region, should it be required. Within the limits set by the policy, each ABB company is fully autonomous regarding hedging decisions, and will fully bear the consequences of its hedging decisions. The risk is totally decentralized. No hedge is made on behalf of a company at regional headquarters level or at group level.²⁵ The companies are encouraged to confer with the local Treasury Centres to profit from their specialized knowledge, before making a major hedging decision. The ultimate decision and responsibility remain at company level.

The decision making within each company varies. In some cases, the total foreign exchange risk and decision making are pooled

together in the financial department. In other companies, the hedging decisions are made by the people who incur the risks (purchasers, contract negotiators, tender offer personnel etc). Furthermore, a mixed method is also used. This implies that the department which incurs the risks makes a fictitious hedging decision, where the fictitious tools are obtained from the financial department. The former department will then be evaluated on the project inclusive of the hedge. The latter department subsequently purchases the hedge on the market or leaves the position exposed if it is considered better at company level.

The complex problem of deciding where to position the decision power is left as a question for further research as was alluded to in the Section 1.4. In future discussions, the existing decision level is accepted, ad hoc.

10.3.4 Implementation in ABB Asea Brown Boveri AB

In implementing the decision support design developed, it must be adapted to the specific requirements prevailing. It must also be made easy to use. For this reason, the implementation was split into three stages: the preparation stage, the education stage, and the follow up stage.²⁶

The preparation stage consisted of adapting the design to comprize the special hedging requirements of the policy, of constructing a PC-programme²⁷ in order to facilitate the use of the design and finally of writing an extensive documentation/-teaching material.

The teaching stage constituted an effort to enhance the knowledge of the foreign exchange rate, the hedging tools, and the risks inherent in a foreign exchange position. Furthermore, the design and the PC-programme was introduced and a case of currency hedging of tender offer was solved by the participants using the PC-programme.²⁸ The target group of this activity was the personnel in the ABB companies which were exposed to foreign exchange rate risk.

The follow up stage, not yet conducted, will consist of a discussion with each company on how the decision support design may be implemented in terms of each company's specific foreign exchange rate exposure. Adaption of the initial PC-programme will also be made.

10.3.4.1 Specific Requirements of ABB

The decision situation targeted by the decision support method was primarily the tender offer position. Furthermore, any contractual situation has to be hedged according to the policy,

i.e. a potential contractual period following the tender offer period. Although an option hedge is not forbidden for this purpose in the policy, it is excluded from the model due to its higher speculative content than an equivalent forward hedge. Finally, no hedging position may be entered where an underlying position does not exist.

10.3.4.2 The PC-Programme

The PC-programme developed for ABB consists of three parts: the probability distributions assumed for future foreign exchange rates, an analysis of the hedged position, and a part for option analysis. The first part illustrates the probability distribution of the future foreign exchange rates which are implicitly assumed by the programme. The purpose of this first part is to give an inexperienced user a notion of the risks involved. In the second part, the underlying position is analyzed with regards to the future probability distribution given any hedging combinations the user wishes to test. (In accordance with the design presented in Chapter Nine, but without any three dimensional figures. Both pdf and cdf are given.) In the third and last part, the user is able to analyze the relevant options as to the premium, the time value and the intrinsic value by choosing different maturities and exercise prices. The purpose is to facilitate the location of an option on the market whose characteristics correspond to the one chosen in the former part.²⁹

10.3.4.3 Evaluation of the Implementation

Due to the fact that the decision support method is being implemented at the time this thesis is written, an extensive analysis of the results is not possible to make. The third stage 'follow up' has not yet begun. A few indications may, however, be given as well as some problems pointed out, based on the results from the teaching period. The reader is referred to Little [1970], pp. 467-468, for a discussion of implementation problems.

The teaching stage of the implementation, or more precisely, the solving of the case on the PC, gave the opportunity to estimate the receptivity of the participants with regards to the decision support method. The receptivity was considered good although no pretensions were made for analyzing the results strictly. However, the author is of the opinion that a follow up stage is necessary in order to help the participants 'over the threshold' in using the method in their hedging decisions.

The major problem when introducing the method was to obtain the critical mass of knowledge on behalf of the participants during the very short time period available (eight hours). A basic knowledge in the statistical field is necessary in order to

interpret the figures generated by the programme. A knowledge of the tools is an absolute necessity in order to know how to change the hedging mix to alter the risk structure in the desired direction. An understanding of the three outcomes: acceptance, refusal and expected outcome respectively, is essential. The two first concepts caused no problem. The usefulness and the principle of the last one however were difficult to convey.³⁰

A minor problem was that some participants did not have the necessary hardware at their disposal.³¹

Referring to the criteria for evaluating the method, Section 7.3.1, the following conclusions may be drawn for ABB:

Realism of Assumptions

It was necessary to make three major simplifications,³² resulting in a discrepancy between the model and the reality, in order to obtain a PC-programme whose processing time is reasonable.

Firstly, it is assumed that the receiver of the tender offer replies at the point in time which the hedging decision maker assumes. However, this is rarely true in business. Delays in replying of up to fifty percent of the total tender offer period is not uncommon. By substituting the time stated in the tender offer for the time estimated by ABB, the problem may be reduced. Theoretically, it would not be a problem to include the time of reply as a stochastic variable in the model if the probability distribution could be estimated. However, the processing time would become unacceptable.

Secondly, the amount of currencies involved is limited to three: two foreign currencies and a domestic currency. This seemingly unrealistic assumption is in fact in harmony with business life. Only one more foreign currency apart from the tender offer currency is involved at the time of submitting the tender offer. This is due to the fact that often only one contingent purchase abroad is decided upon. The other purchases are not fixed as to where they will be made and therefore budgeted in Swedish kronas. Hence, only two foreign currencies are fixed at the time of submission, i.e. one generating cash flow in, and the other generating cash flow out. Extending the model to include more currencies, i.e. extra stochastic variables, would not cause any problem theoretically, but again the processing time would become unacceptable.

Finally, as only one tender offer is considered at a time the portfolio approach may not be applied. As for the simplifications mentioned before, the principle of the decision support design comprises even this extension, but the processing time would be excessive.

Operationability

The operationability of the method is considered good. The input data is easily available,³³ and the processing time is considered acceptable.³⁴ The hedging tools indicated by the method are however not always available due to market illiquidity. The problem is overcome by choosing a volume of hedging tools for analysis that may be obtained on the market. The criteria that the model ought to include all tools/hedging methods available on the market may be contended with. The model introduced at ABB includes options and forward contracts. The stop-loss approach would be a realistic alternative to include, but not the dynamic hedging methods due to the cost involved (spread plus time for personnel).

Implementability

The implementability with regards to the cost of operation appears to be good. The cost of purchasing the hardware is within limits. The most difficult part of the decision support method, as already mentioned, is to make the everyday user fully understand the model. The process of achieving this may well prove costly.

10.3.4.4 Concluding Remarks

Several interesting issues appeared when the model was implemented at different ABB companies. The first issue constituted that there is a definite belief among the industrial companies that the prevailing forward rate does not correspond to the expected future spot rate.³⁵ Whether their deviation was in harmony with the 'general market estimations' was hard to discern, but it is evident that some companies believed that they could 'beat the market'. Hence, the incentive to use forward contracts as hedging tool was affected. Several companies were instead inclined to speculate and remain unhedged. However, should the expectations theory not hold, and should the company wish to speculate on the superiority of their guess, this speculation would easily be separated from the hedging component of the position. The second issue that was often discussed during the implementation was the premium. It constituted the source of three major questions. The first problem was where to allocate the premium; should it be allocated to the project level or to higher levels? The second question was whether the premium ought to be included as a cost when constructing the tender offer.³⁶ The third question originating in the premium discussion was whether each company could afford to consider 'a long time perspective', i.e. whether it was reasonable to make the calculations (evaluation of the refused tender offer situation) on the basis that 'in the long run the premium will be recovered exclusive of the spread'. The third issue stemmed from the partial approach of the method, i.e. merely to focus on the foreign exchange problems. The participants in the implementation

stage requested some ideas on how to integrate the results with the other risks, such as the political risk and the risk that the potential customer did not reply at the time agreed upon.³⁷

10.4 Evaluation of the Design

The evaluation of the design follows the criteria given in Section 7.3.1. The discussion will focus on the design, and not on any specific application of it. (See for example the preceding section).³⁸

Realism of Assumptions

The realism of the design is almost entirely a function of the adaptations made for a special application. In principle, the design encompasses any statistical distribution, any position, any hedging tool/method and any other stochastic variable the user wishes to include (such as the date of reply as to whether the tender offer is accepted or refused). However, in order to make the model usable, some limiting assumptions must be made. A statistical distribution must for instance be chosen for each stochastic variable that enables the derivation of a joint probability distribution. A specific number of positions must be separated from the rest of the positions to be treated separately in order to limit the amount of stochastic variables. Thus, there is a trade-off between the amount of assumptions (lower realism) and the processing time of the PC-model necessary to carry out the computations. The preceding section showed that an acceptably realistic model might be obtained.

Operationability

The operationability of the design is a function of the assumptions made: the stricter the assumptions, the higher the operationability.

The availability of input data varies according to which stochastic variable is included, i.e. which position is being analyzed. Assuming that the lognormal probability distribution is being used, the volatility, the trend and the correlation coefficient must be quantified apart from the size and intertemporal allocation of the potential future cash flow. Furthermore, should the position consist of an anticipatory transaction or of a tender offer, the probability of realization must be estimated. This estimation is a source of great uncertainty. Should the position consist of foreign exchange rate risk, the availability of data would be considered good.³⁹

Due to the fact that the probabilities may not be calculated analytically (see Section 7.4), the processing time will be very

long if a large number of stochastic variables are included.⁴⁰ Thus, this criterion poses a very strict limit on the applicability of the method. The author's tentative conclusion is that this criterion is the most limiting for the decision support method.

The first part of the criteria 'availability and use of hedging methods' may be fulfilled by including only the hedging methods available on the market.⁴¹ The second part is nevertheless problematic. By including a hedging method which has a stochastic final value (stop-loss,⁴² CPPI or TIPP), the simulation approach needed will cause the processing time to rise.

Implementability

The acceptance of the design by a potential user is a function of whether the user can understand the approach, or at least interpret the results, and the cost of using the method. Due to the fact that there is no need on behalf of the user to understand the computations and the equations, but merely interpret the results and be able to enter the correct data into the PC-programme, the decision support method fulfils the basic requirements for being comprehensible.⁴³ However, as was pointed out in Section 10.3, the user must usually submit to some teaching. The cost of operation may be split into two parts: one originating in the initiation of the method, and the other originating in the everyday use of the PC-programme. Initiating the method requires a purchase of a computer including a mathematical processor and colour monitor, a purchase/-construction of the PC-programme necessary to carry out the computations, and teaching the user how to run/understand the programme. Running the programme entails cost of personnel due to the time requirements.

Notes

1. The figures given in the tender offer in the example are not taken from any of the tenders seen during the author's study of ABB. Any similarity of numbers is purely coincidental.
2. The spread is easily included by choosing the appropriate bid price/offer price on the market.
3. The numbers given correspond approximately to the market as of April 1990. However, they are not intended to depict the situation on any specific point in time.
4. Per 100 units of the foreign currency.
5. The equivalent interest rates per three months are 3.4% (13.6/4), 2.3% and 2.1% respectively. It is assumed that all months consist of the same amount of interest rate days in this simplified initial calculation.
6. $410.00 \cdot 1.034 / 1.023 \approx 414.4086$
 $364.00 \cdot 1.034 / 1.021 \approx 368.6347$
(Rounded to four decimal digits.)
7. Standard deviation in percent per year.
8. The correlation coefficient is not obtained from the market. In Oxelheim [1985], pp. 166-170, the coefficient was estimated in various ways, giving numbers such as 0.51, 0.55, 0.38 and 0.64. The three former ones were measured by comparing them to the basket valuing the Swedish krona, while the last number included changes in the basket (including devaluations). No information was given as to how the trend components of the currencies were eliminated, i.e. the expected change in foreign exchange rate. The correlation coefficient in the model must include changes in the basket, hence a more positive number than the three first values is chosen. As the period studied, the whole or part of 1974-84, contains several large devaluations, a lower number than 0.64 is chosen. The figure +0.6 is chosen arbitrarily given the restrictions above. The author does not claim that this number is the correct one. Reasons for deviations are the possible unification of Germany, how accurately the coefficients were calculated by Oxelheim, the effect of the EMS etc.
9. SEK: $16 / (1.034^{1/3}) + 11 / (1.034^{4/3}) \approx 26,343,064$ SEK
CHF: $1 + 11 / (1.023^{6/3}) \approx 11,510,937$ CHF
 $11,510,937 \cdot 414.4086 / 100 \approx 47,702,313$ SEK
DEM: $3 / (1.021^{1/3}) + 2 / (1.021^{4/3}) \approx 4,924,630$ DEM
 $4,924,630 \cdot 368.6347 / 100 \approx 18,153,895$ SEK
10. $47,702,313 - 18,153,895 - 26,343,064 = 3,205,354$

11. $3,205,354/47,702,313 \approx 6.7\%$
12. $18,153,895/(18,153,895 + 26,343,064) \approx 40.8\%$
13. Following the Garman & Kohlhagen [1983] formula. Following the put-call parity, the premium of the put option and call option are identical if the exercise price equals the forward price. However, only the call option is used for DEM and put option for CHF.
14. The program used to calculate the probabilities uses 365 interest days per year. The tender offer period is chosen as 91 days.
15. The probability densities are easily obtained from the cumulative distribution.
16. The probability calculations have been carried out by the trial-and-error method described in Section 7.7.2. The span of exchange rate outcomes included in the calculation is $e_{t0} \cdot \exp[(r_D - r_F - \frac{1}{2}\sigma^2)T \pm 3\sigma\sqrt{T}]$. The areas of summation are defined by generating equal squares, measured in the stochastic variable X. Each square measures 0.2 times 0.2 in the stochastic normally distributed variable. Hence, de_1de_2 in the summation becomes for instance around $X_1=1.5$ and $X_2=$
 $= 1.0$:

$$e_1 \exp[(r_D - r_{F1} - \frac{1}{2}\sigma^2_1)T + 1.5\sigma_1\sqrt{T}] \cdot \{\exp[0.1\sigma_1\sqrt{T}] - \exp[-0.1\sigma_1\sqrt{T}]\} \cdot e_2 \exp[(r_D - r_{F2} - \frac{1}{2}\sigma^2_2)T + \sigma_2\sqrt{T}] \cdot \{\exp[0.1\sigma_2\sqrt{T}] - \exp[-0.1\sigma_2\sqrt{T}]\}$$
17. I.e. the discounted values of the CHF and DEM cash flows are hedged separately by forward contracts.
18. $1.17\% \cdot 1.034(4.10 \cdot 11,510,937 + 3.64 \cdot 4,924,630) \approx 787,815$
19. A third way would be to change the strike price of the options.
20. $(47,702,313 - 18,153,895)/47,702,313 \approx 62\%$
21. $62\% \cdot 1.17\% \cdot 4.10 \cdot 11,510,937 \cdot 1.034 \approx 353,991$
22. The sources are: annual statements, information brochures and interviews with Ms Piscator (Controller) and with Mr Carlsson (Managing Director) at ABB Treasury Center (Sweden) AB.
23. The group is, principally, divided into regional holding companies which in turn own the ABB companies within the region.

24. An exception is for instance the high-inflation countries in South America, where the local currency is substituted for the US dollar.
25. The remaining exposure is however reported to the regional headquarters in order to obtain the total risk level.
26. See Gershefski [1970], p. 310, for another set of steps in model development. The reader is referred to Schultz & Slevin [1975], pp. 12-13, for models of implementation processes; to Huysmans [1975], pp. 278-281, Manley [1975], pp. 183-187, Harvey [1970], pp. 314-316, and Churchman & Scheinblatt [1965], pp. 69-70, for factors affecting implementation success; to Huysmans [1975], p. 273, Reisman & Kluyver [1975], p. 291, and Hildebrandt [1980], pp. 10-11, for suggestions regarding implementation.
27. During the development of the model, there was a close interaction between the author and the future users of the model. (See Schultz & Slevin [1975], pp. 33-35, for different model building approaches.)
28. The course given (eight hours) by the author consisted of the following. A description of the two statistical measures necessary to analyze the outcome of a position (probability density function and cumulative distribution function) and two statistical measures defining the currency situation (volatility and correlation). A discussion of the exchange rates as to pricing and correlation. The forward contract was analyzed regarding price, hedging potential and value. The option contract was discussed more thoroughly, due to the participants' relative unfamiliarity with the tool. Especially focused upon were factors affecting the premium, and risk reduction potential. Finally, the PC-programme was described and the participants had the opportunity to work with a realistic case, i.e. what hedging decision to make. The positions analyzed were the contractual position, the anticipatory position and the tender offer position. Special attention was given to the change in risk when altering the hedging strategy.
29. The programme is under reconstruction to include options of longer maturity in the position analysis.
30. Compare to the discussion in Little [1970], p. 469.
31. The hardware demand is a personal computer having a colour monitor and a mathematical processor.
32. Further simplification includes non-inclusion of risk of devaluation, non-inclusion of transaction costs (spread), problems in execution of the hedging mix analyzed by the PC-programme due to market imperfections (e.g. relative illiquidity of the market leading to limits on the available size of hedging contracts and limits on maturity of options).

33. The correlation coefficient may not be directly obtainable on the market.
34. The processing time for each set of three hedging combinations to analyze was approximately one minute.
35. The expectations theory did not hold.
36. The problem is whether to consider the initial cash outflow as a cost when calculating the tender offer conditions in the bidding process. If the premium is included in the price quote in the tender offer, the potential customer will bear the cost for it. The advantage of the option acquired will benefit the bidder, as in the long run, the premium will be recuperated. Thus, the bidder will profit in the long run but the competitive edge may be impaired unnecessarily.
37. However, as was pointed out earlier, the model implemented is not intended to be a panacea. For this reason, the integration at the moment must be made heuristically, using the good judgement of the decision maker.
38. No attempt has been made to assess the performance of the model by comparing the value of a hedging decision when using the model to a decision when not using the model. See Taylor & Iwanek [1980], pp. 184-187, regarding how this may be done. However, the "hard dollar benefits of a model are difficult to measure" (Gershefski [1970], p. 307). A utility approach may also be applied, for instance Watson & Brown [1978], but the relative uncomprehensibility will reduce the applicability.
39. Three kinds of input are needed, facts about the exogenous position, information about the market prices, and specification of the hedging combinations. The first kind of input consists of size and time of cash flow, currency units of denominations, type of risky assets, and probability of realization/acceptance. The second input consists of risk-free interest rate for the habitat of the currencies involved, spot prices of assets, spot exchange rates, volatilities and correlation coefficients. The third kind of input that the model requires is the amount of hedging activity that the actor wishes to analyze. None of these factors will cause any problem. However, the variances and correlations are not directly available on the market. The former one is usually obtainable from the banks. The correlation, nevertheless, must either be computed by the actor, or obtained from published studies.
40. The processing time depends on the number of subpositions to include, the accuracy needed and the power of the computer. (A model built by the author for a two-subposition problem in Quick Basic for option and forward hedges, took about one minute to run for a set of three hedging combinations per cdf or pdf.)

41. As the actor may specify the hedging methods to use, a choice of existing methods eliminated this problem.
42. As of today, the final value of the stop-loss must be simulated. However, it is not impossible that future research will provide the formula for the stochastic distribution of the future value given the price of the asset.
43. The only thing that a user must master is the interpretation of a probability density or a cumulative distribution. To simplify the choice of successive hedging combinations to test, a basic knowledge of the hedging methods is advisable.

11 Summary of Contributions to Research and of Topics for Further Research

11.1 Contribution to Research

Contribution to research has been made in two areas: in the theoretical area and in the area of applied business research.

The Theoretical Area

In the theoretical area, several minor contributions have been made. These are of three different kinds: adapting an existing theory to accommodate a different asset, e.g. foreign exchange instead of stock; changing a parameter, e.g. letting the forward contract mature after the option on it; or introducing a new concept, gathered asset/tool. Some of these are of very basic nature, although, to the author's knowledge, they have not been published. The author does not claim to be the first to derive the results, but the derivations of the results were made independently.

The contributions consist of the derivation of: the pricing formula for options on forward contracts; the pricing formula for options on futures contracts for non-coinciding maturity of the option and the underlying futures contract; the pricing formula for options on currency futures; the put-call parity for options on forward contracts; the put-call parity for options on futures contracts for non-coinciding dates of maturity; of the elaboration of the concept of gathered asset and analysis of the effects of hedging by a gathered tool versus hedging by separate tools; and of adapting the existing dynamic hedging methods to accommodate currency and short positions.

The Area of Applied Business Research

In the area of applied business research, the major contribution has been the design of an integrated approach of a decision support method when hedging by non-linear methods. The contributions in the design will be pointed out below, as well as smaller contributions which emerged during the illustration/-construction of the method. Connected to the afore-mentioned is the empirical indication of implementability, i.e. that the method actually has been/is being implemented in business life. The implementation that the author carried out conveyed several important aspects which confirmed the advantages/problems of the method which had been anticipated.

A new classification of risky positions based on the stochastic factors was necessary in order to be able to position the hedging target in the total risk environment of the organization. Furthermore, a multiple-risk situation proved possible to divide into the new concepts of parallel risk and sequential risk, which facilitated the correct choice of hedging instrument.

The design contributes to the research in two ways. Firstly it has the potential of accommodating practically all risky positions and hedging methods within an organization, at least theoretically. Secondly, based on the shown possibility of condensing the risky situation into a two-dimensional context, the results may be presented to the user by means of an easily interpreted graph. Hence, the design will not be esoteric. Furthermore, it has been shown how three-dimensional graphs may be used for two stochastic variables. A three-dimensional graph is a useful tool when illustrating profit/loss, break-even or sensitivity of the result to changes in exchange rates.

Some smaller results have been achieved during the derivation of the decision support method. Firstly, a hedged position is demonstrated to be separable in terms of the underlying position and the hedging activity in two different ways, which facilitate the calculations. Secondly, the hedging activity (potentially interpreted as the absence of a hedge) is shown to be dividable into a hedge component and a speculative component. This is especially evident when comparing a hedge by gathered options to a hedge by simple options, when comparing the alternatives option hedge/forward hedge on contractual situations, and when making a hedging decision when the expectations theory does not hold for the decision maker. This result is useful for instance when evaluating a business project as the speculative component may be separated. Furthermore, the result is useful in an organization when limits for speculation for different entities are set. Thirdly, the frequently used method of hedging a tender offer position by options plus forward contracts instead of the option on forward tool, has been analyzed for generated imperfections. Fourthly, the idea of using the expected outcome as a measure of the total risk of an anticipatory position and tender offer situation has been launched. Two different ways of defining the expected outcome and their rationale were given, as well as the totally different calculation approaches needed. Fifthly, the

problem of achieving a time-efficient calculation programme for the probabilities has been described, with special focus on the domain-fixing dilemma.

11.2 Topics for Further Research

Further research may take place in both the theoretical field and in the applied business field.

In the theoretical area, the stop-loss, the CPPI and the TIPP may be further elaborated on to render a formula for the expected value conditional on the outcome of the stochastic variables. Furthermore, the stable paretian distribution may be included to align the hedging method assumptions with the empirical results.

In the field of applied business research, the analysis may be extended in several ways: to include stochastic interest rates; to include other hedging instruments, such as futures and American options; to include market imperfections such as transaction cost (especially important for dynamic hedging methods), discontinuous trading and information imperfections (e.g. autocorrelation); to explicitly include the possibility of allocating risk intertemporally¹; to let the length of the tender offer risk period become stochastic; to redefine/extend the underlying positions to include different points in time for the cash flow and to include more than one tender offer/anticipatory position and their contingent sales/purchases.

Notes

1. I.e. to condense risks over several different periods into one measure. An example of this is the possibility to measure the effect of shifting the risk between the tender offer period and the potential subsequent contractual period. Another example is the possibility of analyzing the total position of two partially hedged future cash flows which do not coincide in time.

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Appendices

A Basic Formulas

A.1 Forward Contracts

By convention, the forward price is set in order to render the contract a zero initial value. In denoting the value of the contract at time t by $VA[FW_a[t_0, t_{FW}]; t]$ the condition can be formalized as:

$$VA[FW_a[t_0, t_{FW}]; t_0] = 0, \quad t_0 \leq t_{FW}$$

Consequently, the forward price must be equal to the expected future spot price at time t_{FW} of the underlying asset.¹

$$FW_a[t_0, t_{FW}] = E[a_{FW}]$$

The equation implies that the forward price of a contract initiated at time t with zero maturity, a 'zero-term forward contract', is equal to the expected spot price at time t ,

¹ Giddy & Dufey [1975] argued that the speculation in a free market would drive the rates so that the forward rate equals the expected future spot rate. In the modern theory of forward exchange, they argue, the covered interest arbitrage possibilities and the net supply/demand of currency from commercial hedgers must be included as well. They state that empirically "forward exchange rates have remained at interest rate parity with respect to Eurocurrency interest rates throughout the recent periods of international monetary turmoil" (p. 17).

From the review article by Kohlhagen [1978b], it becomes clear that there is no empirical evidence contradicting the forward rate as an unbiased estimate of the future spot rate. Eun & Resnick [1988] stated that "forward exchange premium is known to be a nearly unbiased predictor of the future change of the exchange rate" (pp. 198-199). However, the forward rate does not always perform better than the current spot rate as predictor according to Agmon & Amihud [1981].

As the forward price refers to the expected spot price at expiration, any known dividends prior to the expiration day are implicitly subtracted in the formula.

estimated at time t .² The price is the current spot price, $FW_a[t, t] = a_t$.

Acquiring a long position in a forward contract is equivalent to buying a call option and selling a put option if the exercise prices of the options equal the forward price and if the dates of maturity coincide, $a_x = FW_a[t_0, t_{FW}]$. As the forward contract has no initial value, the price of the call option, C_a , must equal the price of the put option, P_a . Substituting the equation into the put-call parity, and remembering that the spot price equals the forward price on a zero-maturity contract, renders: (Set $r_F=0$ for p and g .)

$$FW_a[t_0, t_{FW}] = FW_a[t_0, t_0] \cdot e^{(r_D - r_F)(t_{FW} - t_0)}$$

The equation states that the forward price equals the current spot price capitalized to expiration day at the risk-free interest rate³ or at the differential of the two risk-free interest rates.⁴

² Jarrow & Oldfield [1988], p. 16. See Levich [1989] for a discussion of forward rates as the optimal future spot rate forecast.

³ Cox & Rubinstein [1985], p. 61.

⁴ If the interest rate is deterministic but not constant, i.e. varies in a predetermined way, $(r_D - r_F)(t_{FW} - t_0)$ is substituted for the integral of $(r_D - r_F)$ over time. Let r_{D_i} denote the domestic continuously compounded interest rate at time t_i and r_{F_i} the foreign equivalence. This renders:

$$\int_{t_i = t_0}^{t_{FW}} r_{D_i} dt_i \quad \text{and} \quad \int_{t_i = t_0}^{t_{FW}} (r_{D_i} - r_{F_i}) dt_i$$

If the interest rate is stochastic, the capitalization factor is substituted for $B_F[t_0, t_B]/B_D[t_0, t_B]$, as the value of the bond incorporates the market's expectation of the stochastic risk-free future interest rate during the time to maturity. $B_D[t_0, t_B]$ denotes the value of a domestic bond at time t_0 that will certainly give one unit of domestic currency at time t_B . B_F denotes the foreign corresponding bond. ($t_B = t_{FW}$) (Put-call parity theory will hold even though the interest rates are stochastic.)

A.2 Futures Contracts

Set $r_F=0$ for p and g.

$$F_a[t_0, t_F] = a_0 e^{(r_D - r_F)(t_F - t_0)}$$

A.3 Options

Denote the price of a call option on the underlying asset 'a' by C_a , the current time t_0 , the time to maturity $T_X (= t_X - t_0)$, and the exercise prices by p_X , e_X , and g_X . The formulas can now be written as:⁵

$$C_p[t_X] = p_0 N[x_p] - p_X e^{-r_D T_X} N[x_p - \sigma_p \sqrt{T_X}]$$

$$x_p = \frac{\ln[p_0/p_X] + r_D T_X + \frac{1}{2} \sigma_p^2 T_X}{\sigma_p \sqrt{T_X}}$$

$$C_e[t_X] = e^{-r_F T_X} e_0 N[x_e] - e_X e^{-r_D T_X} N[x_e - \sigma_e \sqrt{T_X}]$$

$$x_e = \frac{\ln[e_0/e_X] + (r_D - r_F) T_X + \frac{1}{2} \sigma_e^2 T_X}{\sigma_e \sqrt{T_X}}$$

$$C_g[t_X] = g_0 N[x_g] - g_X e^{-r_D T_X} N[x_g - \sigma_g \sqrt{T_X}]$$

⁵ The derivation of the premium for an option on gathered assets follows from the fact that the asset price has the same structure as the price of a domestic asset. (See Section 2.5.) For a more thorough discussion, see Lagerstam [1989a].

The price of the option is derived as if the option already existed on the market. The assumption implies that the parameters of the price equation are allocated the values/prices prevailing on the market. Hereby, it is implicitly assumed that a future introduction of the option on the market does not change the values/prices.

Mason & Merton [1985] state that if issuing the option would "significantly change the macro investment opportunity set available to capital market investors" (p. 39), an estimation error will be introduced.

$$x_g = \frac{\ln[g_0/g_x] + r_D T_x + \frac{1}{2}\sigma_g^2 T_x}{\sigma_g \sqrt{T_x}}$$

$$\sigma_g^2 = \sigma_p^2 + 2\tau\sigma_p\sigma_e + \sigma_e^2$$

The formulas for put options are as follows, where x is given in the previous equations.

$$P_p[t_x] = p_0(N[x_p]-1) - p_x e^{-r_D T_x} (N[x_p - \sigma_p \sqrt{T_x}] - 1)$$

$$P_e[t_x] = e^{-r_F T_x} e_0 (N[x_e]-1) - e_x e^{-r_D T_x} (N[x_e - \sigma_e \sqrt{T_x}] - 1)$$

$$P_g[t_x] = g_0(N[x_g]-1) - g_x e^{-r_D T_x} (N[x_g - \sigma_g \sqrt{T_x}] - 1)$$

The put-call parity may be stated as:

$$P_p[t_x] + p_0 = p_x e^{-r_D T_x} + C_p[t_x]$$

$$P_e[t_x] + e_0 e^{-r_F T_x} = e_x e^{-r_D T_x} + C_e[t_x]$$

$$P_g[t_x] + g_0 = g_x e^{-r_D T_x} + C_g[t_x]$$

The tables A.1 and A.2 on the following pages pertain to the discussion in Chapter Four and consist of a comparison of a gathered options strategy and a simple options strategy.

Table A.1 Final cash flow of call option strategies and of hedged contractual positions

outcome		strategy	call option cash flow	contractual position cash flow
1	$p_t \leq p_x, e_t \leq e_x$ $p_t e_t \leq p_x e_x$	gathered: no exercise simple: no exercise	0 0	$-p_t e_t$ $-p_t e_t$
2a	$p_t > p_x, e_t \leq e_x$ $p_t e_t \leq p_x e_x$	gathered: no exercise simple: exercise p	0 $(p_t - p_x) e_t$	$-p_t e_t$ $-p_x e_t$
2b	$p_t > p_x, e_t \leq e_x$ $p_t e_t \geq p_x e_x$	gathered: exercise simple: exercise p	$p_t e_t - p_x e_x$ $(p_t - p_x) e_t$	$-p_x e_x$ $-p_x e_t$
3a	$p_t \leq p_x, e_t > e_x$ $p_t e_t \leq p_x e_x$	gathered: no exercise simple: exercise e	0 $(e_t - e_x) p_x$	$-p_t e_t$ $-p_t e_t + (e_t - e_x) p_x$
3b	$p_t \leq p_x, e_t > e_x$ $p_t e_t \geq p_x e_x$	gathered: exercise simple: exercise e	$p_t e_t - p_x e_x$ $(e_t - e_x) p_x$	$-p_x e_x$ $-p_t e_t + (e_t - e_x) p_x$
4	$p_t > p_x, e_t > e_x$ $p_t e_t \geq p_x e_x$	gathered: exercise simple: exercise p and e	$p_t e_t - p_x e_x$ $p_t e_t - p_x e_x$	$-p_x e_x$ $-p_x e_x$

Table A.2 Final cash flow of put option strategies and of hedged contractual positions

outcome	strategy	put option cash flow	contractual position cash flow
1 $p_t \leq p_x, e_t \leq e_x$ $p_t e_t \leq p_x e_x$	gathered: exercise simple: exercise p and e	$p_x e_x - p_t e_t$ $p_x e_x - p_t e_t$	$+p_x e_x$ $+p_x e_x$
2a $p_t > p_x, e_t \leq e_x$ $p_t e_t \leq p_x e_x$	gathered: exercise simple: exercise e	$p_x e_x - p_t e_t$ $(e_x - e_t) p_x$	$+p_x e_x$ $+p_x e_x + (p_t - p_x) e_t$
2b $p_t > p_x, e_t \leq e_x$ $p_t e_t \geq p_x e_x$	gathered: no exercise simple: exercise e	0 $(e_x - e_t) p_x$	$+p_t e_t$ $+p_t e_t + (e_x - e_t) p_x$
3a $p_t \leq p_x, e_t > e_x$ $p_t e_t \leq p_x e_x$	gathered: exercise simple: exercise p	$p_x e_x - p_t e_t$ $(p_x - p_t) e_t$	$+p_x e_x$ $+p_x e_x + (e_t - e_x) p_x$
3b $p_t \leq p_x, e_t > e_x$ $p_t e_t \geq p_x e_x$	gathered: no exercise simple: exercise p	0 $(p_x - p_t) e_t$	$+p_t e_t$ $+p_t e_t + (p_x - p_t) e_t$
4 $p_t > p_x, e_t > e_x$ $p_t e_t \geq p_x e_x$	gathered: no exercise simple: no exercise	0 0	$+p_t e_t$ $+p_t e_t$

A.4 Options on Forward Contracts

The value of a call option on a forward contract is at expiration of the option t_C :⁶ (Set $r_F=0$ for p and g .)

$$\begin{aligned} VA[C_{FW}[t_C];t_C] &= \max[0, VA[FW_X[t_{FW}];t_C]] = \\ &= \max[0, a_{ce}^{-r_F(t_{FW}-t_C)} - FW_X[t_{FW}]e^{-r_D(t_{FW}-t_C)}]; \quad t_{FW} \geq t_C \end{aligned}$$

Domestic Risky Asset

The payoff of an option on a forward is similar in structure to the payoff of an option on the primary asset, for a call option: $\max\{0, p_C - p_X\}$. If the exercise price p_X is substituted for $FW_X[t_{FW}] \exp[-r_D(t_{FW}-t_C)]$, exactly the same payoff will emerge if interest rates are deterministic. Thereby, the price of the option on the forward contract must be equal to the price of an option on the underlying domestic asset according to the law of one price.

Inserting a new exercise price into the pricing formula of an option on the primary asset and substituting the standard deviation according to equation (3.1), renders the price of the option. The equation implies that the price of the option depends on the interest rate during the length of time until payment is received, T_{FW} . Intuitively, this is explained by the alternative to keep the forward contract until maturity or to sell the contract at a discount prior to maturity. The variance of the forward price affects the price of the option only during the maturity of the option, as by the time of exercise the cash flow can always be ascertained by selling the contract.

$$\begin{aligned} C_{FW}[t_C] &= p_0 N[x_{FWp}] - FW_X[t_{FW}] e^{-r_D T_{FW}} N[x_{FWp} - \sigma_{FWp} \sqrt{T_C}] \\ x_{FWp} &= \frac{\ln[p_0 / FW_X[t_{FW}]] + r_D T_{FW} + \frac{1}{2} \sigma_{FWp}^2 T_C}{\sigma_{FWp} \sqrt{T_C}} \end{aligned}$$

Foreign Currency

The payoff of a call option on a currency maturing at t_C and with the exercise price e_X is:

$$VA[C_e[t_C];t_C] = \max[0, e_C - e_X]$$

⁶ It is easy to note that if $t_{FW}=t_C$, buying an option on the underlying asset is identical to buying an option on a forward contract on the underlying asset. See Feiger & Jacquillat [1979], p. 1130, or Ramaswamy & Sundaresan [1985], p. 1322.

When compared to the payoff of an option on a forward contract, it is seen that not only the exercise price will differ, which is easily corrected, but also the first term. If scaling the number of options on currency by $\exp[-r_F(T_{FW}-T_C)]$ and adjusting the exercise price to be $FW_X[t_{FW}] \cdot \exp[-(r_D-r_F)(T_{FW}-T_C)]$, the total price will equal the price of an option on a forward contract on currency as they render identical values at exercise.

$$C_{FW_e} [t_C] = e_0 e^{-r_F T_{FW}} N[x_{FW_e}] - FW_X[t_{FW}] e^{-r_D T_{FW}} N[x_{FW_e} - \sigma_{FW_e} \sqrt{T_C}]$$

$$x_{FW_e} = \frac{\ln[e_0/FW_X[t_{FW}]] + (r_D-r_F)T_{FW} + \frac{1}{2}\sigma_{FW_e}^2 T_C}{\sigma_{FW_e} \sqrt{T_C}}$$

Foreign Risky Asset

The formula is derived analogously to the formula for domestic risky assets.

$$C_{FW_g} [t_C] = g_0 N[x_{FW_g}] - FW_X[t_{FW}] e^{-r_D T_{FW}} N[x_{FW_g} - \sigma_{FW_g} \sqrt{T_C}]$$

$$x_{FW_g} = \frac{\ln[g_0/FW_X[t_{FW}]] + r_D T_{FW} + \frac{1}{2}\sigma_{FW_g}^2 T_C}{\sigma_{FW_g} \sqrt{T_C}}$$

Put-Call Parity

Consider the two strategies in table A.3 on the following page. According to the law of one price:

$$P_{FW_a} [t_x] + FW_a[t_0, t_{FW}] e^{-r_D T_{FW}} = C_{FW_a} [t_C] + FW_X[t_{FW}] e^{-r_D T_{FW}}$$

Transforming the equation renders:

$$P_{FW_p} [t_x] + FW_p[t_0, t_0] = C_{FW_p} [t_x] + FW_X[t_{FW}] e^{-r_D T_{FW}}$$

$$P_{FW_e} [t_x] + FW_e[t_0, t_0] e^{-r_F T_{FW}} = C_{FW_e} [t_x] + FW_X[t_{FW}] e^{-r_D T_{FW}}$$

$$P_{FW_g} [t_x] + FW_g[t_0, t_0] = C_{FW_g} [t_x] + FW_X[t_{FW}] e^{-r_D T_{FW}}$$

Table A.3 Put-call parity for options on forward contracts.

STRATEGY	INITIAL CASH FLOW	VALUE/CASH FLOW AT t_x IF:	
		$FW_x[t_{FW}] \geq FW_a[t_x, t_{FW}]$	$FW_x[t_{FW}] < FW_a[t_x, t_{FW}]$
NO 1: Buy put:	$-P_{FW_a}[t_x]$	$(FW_x[t_{FW}] - FW_a[t_x, t_{FW}]) \cdot e^{-r_D(T_{FW} - T_x)}$	0
Buy bonds:	$-FW_a[t_0, t_{FW}]e^{-r_D T_{FW}}$	$FW_a[t_0, t_{FW}] \cdot e^{-r_D(T_{FW} - T_x)}$	$FW_a[t_0, t_{FW}] \cdot e^{-r_D(T_{FW} - T_x)}$
Enter forward contract:	0	$(FW_a[t_x, t_{FW}] - FW_a[t_0, t_{FW}]) \cdot e^{-r_D(T_{FW} - T_x)}$	$(FW_a[t_x, t_{FW}] - FW_a[t_0, t_{FW}]) \cdot e^{-r_D(T_{FW} - T_x)}$
Total: strategy no 1	$-P_{FW_a}[t_x] - FW_a[t_0, t_{FW}]e^{-r_D T_{FW}}$	$FW_x[t_{FW}]e^{-r_D(T_{FW} - T_x)}$	$FW_a[t_x, t_{FW}] \cdot e^{-r_D(T_{FW} - T_x)}$
NO 2: Buy call:	$-C_{FW_a}[t_x]$	0	$(FW_a[t_x, t_{FW}] - FW_x[t_{FW}]) \cdot e^{-r_D(T_{FW} - T_x)}$
Buy bonds:	$-FW_x[t_{FW}]e^{-r_D T_{FW}}$	$FW_x[t_{FW}] \cdot e^{-r_D(T_{FW} - T_x)}$	$FW_x[t_{FW}] \cdot e^{-r_D(T_{FW} - T_x)}$
Total: strategy no 2	$-C_{FW_a}[t_x] - FW_x[t_{FW}]e^{-r_D T_{FW}}$	$FW_x[t_{FW}]e^{-r_D(T_{FW} - T_x)}$	$FW_a[t_x, t_{FW}]e^{-r_D(T_{FW} - T_x)}$

A.5 Options on Futures Contracts

The payoff of a call option on a futures contract follows the equation below as the difference in futures price will be settled immediately on the day of expiration of the option. (Set $r_F=0$ for p and g .)

$$\begin{aligned} VA[C_F [t_C]; t_C] &= \max[0, F_a[t_C, t_F] - F_X[t_F]] = \\ &= \max[0, a_{ce} \frac{(r_D - r_F)(T_F - T_C)}{p} - F_X[t_F]] \end{aligned}$$

Comparing the equation to the payoffs from an option on the underlying asset, applying the law of one price, and finally inserting the volatility equivalence, equation (3.3), renders:⁷

$$\begin{aligned} C_{F_p} [t_C] &= e^{\frac{r_D(T_F - T_C)}{p}} \frac{p_0 N[x_{F_p}] - F_X[t_F] e^{-r_D T_C}}{p} N[x_{F_p} - \sigma_F \sqrt{T_C}] \\ x_{F_p} &= \frac{\ln[p_0 / F_X[t_F]] + r_D T_F + \frac{1}{2} \sigma_{Fp}^2 T_C}{\sigma_F \sqrt{T_C}} \end{aligned}$$

$$\begin{aligned} C_{F_e} [t_C] &= e^{\frac{r_D(T_F - T_C) - r_F T_F}{e}} \frac{e_0 N[x_{F_e}] - F_X[t_F] e^{-r_D T_C}}{e} N[x_{F_e} - \sigma_F \sqrt{T_C}] \\ x_{F_e} &= \frac{\ln[e_0 / F_X[t_F]] + (r_D - r_F) T_F + \frac{1}{2} \sigma_{Fe}^2 T_C}{\sigma_F \sqrt{T_C}} \end{aligned}$$

$$\begin{aligned} C_{F_g} [t_C] &= e^{\frac{r_D(T_F - T_C)}{g}} \frac{g_0 N[x_{F_g}] - F_X[t_F] e^{-r_D T_C}}{g} N[x_{F_g} - \sigma_F \sqrt{T_C}] \\ x_{F_g} &= \frac{\ln[g_0 / F_X[t_F]] + r_D T_F + \frac{1}{2} \sigma_{Fg}^2 T_C}{\sigma_F \sqrt{T_C}} \end{aligned}$$

Put-Call Parity

Consider the strategies in table A.4 on the following page.

$$P_{F_a} [t_X] + F[t_0, t_F] e^{-r_D T_X} = C_{F_a} [t_X] + F_X[t_F] e^{-r_D T_X}$$

⁷ The equations are consistent with Black [1976], p. 177, if it is assumed that the futures contract has the same date of maturity as the option on the futures contract.

Table A.4 Put-call parity for options on futures contracts.

STRATEGY	INITIAL CASH FLOW	VALUE/CASH FLOW AT t_x IF:	
		$F_X[t_F] \geq F_a[t_x, t_F]$	$F_X[t_F] < F[t_x, t_F]$
NO 1: Buy put:	$-P_F [t_x]_a$	$F_X[t_F] - F_a[t_x, t_F]$	0
Buy bonds:	$-F_a[t_0, t_F] e^{-r_D T_x}$	$F_a[t_0, t_F]$	
Enter futures contract:	0	$F_a[t_x, t_F] - F_a[t_0, t_F]$	
Total: strategy no 1	$-P_F [t_x]_a - F_a[t_0, t_F] e^{-r_D T_x}$	$F_X[t_F]$	$F_a[t_x, t_F]$
NO 2: Buy call:	$-C_F [t_x]_a$	0	$F_a[t_x, t_F] - F_X[t_F]$
Buy bonds:	$-F_X[t_F] e^{-r_D T_x}$	$F_X[t_F]$	
Total: strategy no 2	$-C_F [t_x]_a - F_X[t_F] e^{-r_D T_x}$	$F_X[t_F]$	$F_a[t_x, t_F]$

A.6 The Partial Derivatives of Option Prices

Options

$$\frac{\delta C_p[t_C]}{\delta p_0} = N[x_p] ; \quad \frac{\delta C_e[t_C]}{\delta e_0} = e^{-r_F T_C} N[x_e] ; \quad \frac{\delta C_g[t_C]}{\delta g_0} = N[x_g]$$

Options on Forward Contracts

$$\frac{\delta C_{FWa}[t_C]}{\delta FW_a[t_0, t_{FW}]} = e^{-r_D T_{FW}} N[x_{FW_a}]$$

Options on Futures Contracts

$$\frac{\delta C_{Fa}[t_C]}{\delta F_a[t_0, t_F]} = e^{-r_D T_C} N[x_{F_a}]$$

A.7 Relationship Between the Price of an Option on a Forward Contract and the Price of an Option on the Underlying Asset⁸

Due to the similarity between call and put options, only one of them needs to be proven. Furthermore, if the proof is valid for foreign currency, it is also valid for domestic risky asset and foreign risky assets. It becomes evident by setting $r_F=0$. Hence, to be proven:

$$C_{FW}[t_C] = e^{-r_F(t_{FW}-t_C)} e_c[t_C]$$

Where:

$$FW_x[t_{FW}] = e_x e^{(r_D-r_F)(t_{FW}-t_C)}$$

The price of a call option on a forward contract is restated below, modified according to the above stated.

$$C_{FWe}[t_C] = e_0 e^{-r_F T_{FW}} N[x_{FWe}] - e_x e^{(r_D-r_F)(T_{FW}-T_C) - r_D T_{FW}} N[x_{FWe} - \sigma_{FWe} \sqrt{T_C}]$$

The price of a call option was given in appendix A.3. Hence:

$$C_e[t_C] e^{-r_F(T_{FW}-T_C)} = e^{-r_F T_{FW}} e_0 N[x_e] - e_x e^{(r_D-r_F)(T_{FW}-T_C) - r_D T_C} N[x_e - \sigma_e \sqrt{T_C}]$$

By comparing the two last equations, it is easily seen that it only remains to show that $x_e = x_{FWe}$ as $\sigma_e = \sigma_{FWe}$ according to equation (3.2). By inserting $FW_x[t_{FW}]$ into the definition of x_{FWe} it is easily seen that it equals x_e .

$$\begin{aligned} x_{FW_e} &= \frac{\ln[e_0/FW_x[t_{FW}]] + (r_D-r_F)T_{FW} + \frac{1}{2}\sigma^2 FWeT_C}{\sigma_{FWe}\sqrt{T_C}} = \\ &= \frac{\ln[e_0/e_x] + (r_D-r_F)T_C + \frac{1}{2}\sigma^2 FWeT_C}{\sigma_{FWe}\sqrt{T_C}} \\ x_e &= \frac{\ln[e_0/e_x] + (r_D-r_F)T_C + \frac{1}{2}\sigma_e^2 T_C}{\sigma_e\sqrt{T_C}} \end{aligned}$$

8

The rationale for the statement to be proven in this appendix was given in Section 9.7.1.

A.8 The Buy-and-Hold Strategy

A buy-and-hold strategy (B&H) involves creating an initial long or short position consisting of a risk-free asset and/or a risky asset. The asset may be any of the three sorts discussed. No subsequent modification is made.⁹

The B&H strategy for a long position in stock was developed thoroughly by Perold & Sharpe [1988]. Their analysis is applicable for a domestic asset and a foreign risky asset, but it needs a slight modification of the floor to accommodate foreign currency. If the long present position consists of currency, it will be invested at the foreign risk-free interest rate. Hence, the asset actually consists of foreign bonds. The interest accrued on the foreign currency (bonds) must naturally be included. The amount invested in the risk-free asset plus the certain interest rate accrued on the investment will constitute the lowest potential value of the subposition. The relative value, $V_{a,B}[t]/V[t_0]$, is a linear function of the price of the risky asset at time t .¹⁰

The B&H strategy may also be applied to a long future position which is not linked to a long present position. To achieve this, a loan is obtained, the good is purchased immediately, the buy-and-hold strategy applied, and at the end the good received in the emerging future position is sold on the market. The value of the B&H will not be influenced. The value of the strategy is therefore identical to that of a long present position. The value of the strategy for a long position is the difference between the position value and the value of a hypothetical reference portfolio not submitted to the strategy.

If an asset must be purchased at or before time t , e.g. as an input in production, a ceiling for the future cost may never be obtained unless the entire amount of the asset is bought initially. However, the risk may be reduced by immediately buying part of the asset needed. The value of a buy-and-hold hedge for a short position equals the difference between the cost of an unhedged reference portfolio and the cost of a hedged portfolio.

The strategy is completely independent of the path that the price of the risky asset follows during the time $[t_0, t]$. The strategy is also independent of the process governing the price of the risky asset. It is process-independent. Finally, the strategy is not dependent upon the time horizon. The B&H strategy is identical to a partial forward hedge.

⁹ If an asset must be disposed of at time t , the risk may be reduced by selling off part of the asset immediately.

¹⁰ The upside capture is defined as "how much of the market's appreciation the strategy lets you capture" (Black & Rouhani [1987], p. 13).

A.9 Synthetics and Replicas

Forwards

Table A.5 Forward replicating strategy

STRATEGY	C A S H F L O W (at t_0 or t_{FW})		
	Domestic asset	Foreign currency	Foreign risky asset
at t_0 : =====			
No 1: Buy asset:	$-p_0$	$-e_0 e^{-r_F T_{FW}}$	$-g_0$
Borrow:	$+FW_x e^{-r_D T_{FW}}$	$+FW_x e^{-r_D T_{FW}}$	$+FW_x e^{-r_D T_{FW}}$
No 2: enter forward	0	0	0
at t_{FW} : =====			
No 1 = no 2	$p_{FW} - FW_x$	$e_{FW} - FW_x$	$g_t - FW_x$

Futures

Table A.6 on the following page illustrates the strategies for the first period.¹¹

Should the resettlement be continuous, the payoff from holding a futures contract is δF_a . Holding a portfolio of $-\delta F_a / \delta a_t$ units of the primary asset per long futures contract will therefore render the portfolio insured. Hence a substituting portfolio will consist of $\delta F_a / \delta a_t$ units of the primary asset. (Set $r_F=0$ for p and g.)

$$\frac{\delta F_a[t, t_F]}{\delta a_t} = e^{(r_D - r_F)(t_F - t)}$$

¹¹ As the strategy for a gathered asset is identical to the one for a domestic risky asset it is omitted.

B.1.2 The Bivariate Lognormal Distribution

The distribution will be derived for two currencies: number one and number two. (In setting the foreign interest rate to zero, the equation will be applicable for domestic and foreign risky asset.) See Chapter Two for the formulas for the exchange rate.

Step 1: The Bivariate Normal Distribution

X_1 and X_2 are the stochastic components in the currency exchange rate. $X_1, X_2 \sim N[0,1]$

$$\text{corr}[X_1, X_2] = \frac{\text{cov}[X_1, X_2]}{\text{std}[X_1] \cdot \text{std}[X_2]} = \text{cov}[X_1, X_2]$$

$$-1 \leq \text{corr}[X_1, X_2] = \tau \leq 1$$

Suppressing the subscript 'e', the logarithm of the future stochastic exchange rate is:

$$\ln[e_t] = \ln[e_{t0}] + \mu T + \sigma \sqrt{T} X = Z$$

The distribution is:

$$\begin{aligned} \ln[e_t] = Z &= N[\ln[e_{t0}] + \mu T, \sigma^2 T] = \{ \ln[e_{t0}] + \mu = \mu' \} = \\ &= N[\mu', \sigma^2 T] \end{aligned}$$

The bivariate distribution is:

$$(Z_1, Z_2) = N[(\mu'_1, \mu'_2), (\sigma_1^2 T, \sigma_2^2 T, \tau)]$$

τ denotes the correlation between X_1 and X_2 , and is equal to the correlation between Z_1 and Z_2 .

The probability distribution of a bivariate normal distribution is:³

$$\begin{aligned} \text{pdf}[Z_1, Z_2] &= \frac{1}{2\pi\sigma_1\sigma_2 T \sqrt{1-\tau^2}} \exp\left[-\frac{1}{2(1-\tau^2)} \left[\frac{(Z_1 - \mu'_1)^2}{\sigma_1^2 T} - \right. \right. \\ &\quad \left. \left. - \frac{2\tau(Z_1 - \mu'_1)(Z_2 - \mu'_2)}{\sigma_1\sigma_2 T} + \frac{(Z_2 - \mu'_2)^2}{\sigma_2^2 T} \right] \right] \end{aligned}$$

Step 2: The Bivariate Lognormal Distribution

To obtain the bivariate lognormal distribution, e_t is transformed. (y_1 and y_2 are lognormal)

³ Hogg & Craig [1978], pp. 405.

$$Y_1 = e^{Z_1}$$

$$Y_2 = e^{Z_2}$$

The Jacobian becomes:⁴

$$J = \begin{bmatrix} \frac{\delta Z_1}{\delta Y_1} = \frac{1}{Y_1} & \frac{\delta Z_1}{\delta Y_2} = 0 \\ \frac{\delta Z_2}{\delta Y_1} = 0 & \frac{\delta Z_2}{\delta Y_2} = \frac{1}{Y_2} \end{bmatrix}$$

The determinant is:

$$|J| = \begin{vmatrix} 1/Y_1 & 0 \\ 0 & 1/Y_2 \end{vmatrix} = (1/Y_1)(1/Y_2) = 1/(Y_1 Y_2)$$

The density function is:

$$\begin{aligned} \text{pdf}_{Y_1, Y_2}[Y_1, Y_2] &= \text{pdf}_{Z_1, Z_2}[\ln[Y_1], \ln[Y_2]] \cdot |J| = \\ &= \frac{1}{Y_1 Y_2 2\pi \sigma_1 \sigma_2 T \sqrt{1-\tau^2}} \exp\left[-\frac{1}{2(1-\tau^2)} \left(\frac{(\ln[Y_1] - \mu_1')^2}{\sigma_1^2 T} - \right. \right. \\ &\quad \left. \left. - \frac{2\tau(\ln[Y_1] - \mu_1')(\ln[Y_2] - \mu_2')}{\sigma_1 \sigma_2 T} + \frac{(\ln[Y_2] - \mu_2')^2}{\sigma_2^2 T} \right) \right] \end{aligned}$$

Rewriting the equation renders:

$$\begin{aligned} \text{pdf}[e_1, e_2] &= \{e_1 e_2 2\pi \sigma_1 \sigma_2 T \sqrt{1-\tau^2}\}^{-1} \cdot \\ &\quad \cdot \exp[-(X_1^2 - 2\tau X_1 X_2 + X_2^2)/2(1-\tau^2)] \end{aligned}$$

$$X_1 = (\ln[e_1/e_1, t_0] - \mu_1 T)/\sigma_1 \sqrt{T}$$

$$X_2 = (\ln[e_2/e_2, t_0] - \mu_2 T)/\sigma_2 \sqrt{T}$$

⁴ Lindgren [1976], pp. 454-455.

B.1.3 The Monovariate Lognormal Distribution

The monovariate lognormal distribution is easily obtained from the bivariate distribution by setting the correlation coefficient equal to zero and noting that then, $\text{pdf}[e_1, e_2] = \text{pdf}[e_1] \cdot \text{pdf}[e_2]$.

$$\text{pdf}[e] = \{e\sigma\sqrt{2\pi T}\}^{-1} \cdot \exp[-\frac{1}{2}X^2]$$

$$X = (\ln[e/e_{t0}] - \mu T) / \sigma\sqrt{T}$$

$$\mu = r_D - r_F - \frac{1}{2}\sigma^2$$

B.2 Expected Value and Variance

Let a_t be lognormally distributed with the instantaneous standard deviation σ and the drift μ . Denote the standard wiener process by X . The expected value of ' a_t ' is:⁵

$$E[a_t] = E[a_{t_0} e^{\mu T + \sigma\sqrt{T}X}] = a_{t_0} e^{\mu T} E[e^{\sigma\sqrt{T}X}]$$

$$E[e^{\sigma X}] = \frac{1}{\sqrt{2\pi T}} \int_{-\infty}^{\infty} e^{\sigma X} e^{-X^2/2T} dX = e^{\frac{1}{2}\sigma^2 T}$$

$$E[a_t] = a_{t_0} e^{\mu T + \frac{1}{2}\sigma^2 T}$$

The variance is given by:

$$\begin{aligned} \text{Var}[a_t] &= E[a_t^2] - E^2[a_t] = \\ &= a_{t_0}^2 e^{2\mu T} (E[e^{2\sigma\sqrt{T}X}] - E^2[e^{\sigma\sqrt{T}X}]) = a_{t_0}^2 e^{2\mu T} (e^{2\sigma^2 T} - e^{\sigma^2 T}) = \\ &= a_{t_0}^2 e^{2\mu T + \sigma^2 T} (e^{\sigma^2 T} - 1) \end{aligned}$$

5

See Ross [1985], pp. 61-63. The last equality may be proven by the Poisson Integral and polar coordinates. (See Lagerstam [1989a] or Ullemar [1972].)

B.3 The Maximum Point of the Lognormal Density Function

The maximum of the bivariate lognormal function is found where $\delta \text{pdf}[e_1, e_2] / \delta e_1$ and $\delta \text{pdf}[e_1, e_2] / \delta e_2$ are both zero. Hence:

$$-1 + \frac{\tau X_2 - X_1}{(1 - \tau^2) \sigma_1 \sqrt{T}} = 0 \quad \text{and} \quad -1 + \frac{\tau X_1 - X_2}{(1 - \tau^2) \sigma_2 \sqrt{T}} = 0$$

Some algebra renders that, at pdf-max:

$$X_1 = -(\sigma_1 + \tau \sigma_2) \sqrt{T}$$

$$X_2 = -(\sigma_2 + \tau \sigma_1) \sqrt{T}$$

Hence:

$$(e_1, e_2): (E[e_1] e_1^{-3\sigma_1^2 T/2 - \tau \sigma_1 \sigma_2 T}, E[e_2] e_2^{-3\sigma_2^2 T/2 - \tau \sigma_1 \sigma_2 T})$$

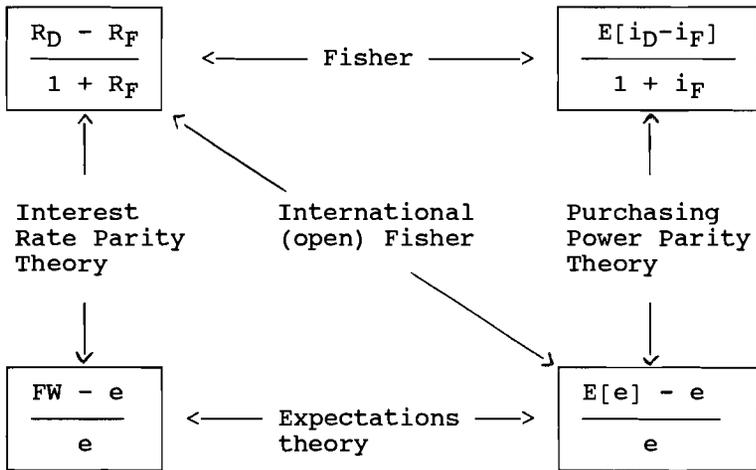
The maximum of the monovariate lognormal density function is found where:

$$e_1: E[e_1] e_1^{-3\sigma_1^2 T/2} < E[e_1]; \quad \text{if } \sigma_1 > 0$$

C Basic Relationships Between Inflation, Interest Rate and Exchange Rate

The basic relationships are given in figure C.1 below for reference. Derivations of the relationships may be found in almost any basic text book in finance.¹ Note that the exchange rate must be given in the European way, i.e. domestic currency units per unit of foreign currency.² (R = discrete risk-free interest rate, i = inflation.)

Figure C.1 Basic relationships between inflation, interest rate and exchange rate



1 See for instance Brealey & Myers [1982], pp. 686-695, or Eiteman & Stonehill [1983], pp. 123-135.

2 Should that not be the case, care must be taken in order to avoid Siegel's paradox: $E[1/X] \neq 1/E[X]$. (See Siegel [1972].)

D Exchange Rate Arrangements¹

Table D.1 Exchange Rate Arrangements (March 31, 1989)²

US Dollar	CURRENCY PEGGED TO:			
	French Franc	Other currency	SDR	Other composite ³
Afghanistan	Benin	Bhutan	Burma	Aleria
Antigua & Barbuda	Burkina Faso	(Indian Rupee)	Burundi	Austria
Bahamas, The	Cameroon	Kiribati	Iran, I.R. of	Bangladesh
Barbados	C.African Rep.	(Australian dollar)	Jordan	Botswana
Belize	Chad		Libya	Cape Verde
Djibouti	Comoros	Lesotho	Rwanda	Cyprus
Dominica	Congo	(South African Rand)	Seychelles	Fiji
El Salvador	Côte d'Ivoire		Zambia	Finland
Ethiopia	Equatorial Guinea	Swaziland		Hungary
Grenada		(South African Rand)		Iceland
Guatemala	Gabon			Israel
Guyana	Mali			Kenya
Haiti	Niger	Tonga		Kuwait
Honduras	Senegal	(Australian dollar)		Malawi
Iraq	Togo			Malaysia
Liberia				Malta
Nicaragua				Mauritius
Oman				Nepal
Panama				Norway
Peru				Papua New Guinea

(cont'd)

- 1 This is a reprint from International Financial Statistics, Volume XLII, No. 7, July 1989, p. 22. Spain has joined the EMS since March 1989.
- 2 Excluding the currency of Democratic Kampuchea, for which no current information is available. For members with dual or multiple exchange markets, the arrangement shown is that in the major market.
- 3 Comprises currencies which are pegged to various "baskets" of currencies of the members' own choice, as distinct from the SDR basket.

CURRENCY PEGGED TO:				
US Dollar	French Franc	Other currency	SDR	Other composite
St.Kitts & Nevis				Poland
St.Lucia				Romania
St.Vincent				Sao Tome & Principe
Sierra Leone				Solomon Islands
Sudan				Somalia
Suriname				Sweden
Syrian Arab.Rep.				Tanzania
Trinidad & Tobago				Thailand
Uganda				Vanuatu
Viet Nam				Western Samoa
Yemen Arb.Rep.				Zimbabwe
Yemen, P.D.Rep.				

FLEXIBILITY LIMITED IN
TERMS OF A SINGLE
CURRENCY OR GROUP OF
CURRENCIES

MORE FLEXIBLE

Single currency ⁴	Cooperative arrangements ⁵	Adjusted according to a set of indicators ⁶	Other managed floating	Independently floating
Bahrain	Belgium	Brazil	Argentina	Australia
Qatar	Denmark	Chile	China, P.R.	Bolivia
Saudi Arabia	France	Colombia	Costa Rica	Canada
United Arab Emirates	Germany	Madagascar	Dominican Rep.	Gambia, The Ghana
	Ireland	Portugal	Ecuador	Japan
	Italy		Egypt	Lebanon
	Luxembourg		Greece	Maldives
	Netherlands		Guinea	New Zealand
			Guinea- Bissau	Nigeria
			India	Paraguay
			Indonesia	Philippines
			Jamaica	South Africa
			Korea	Spain
			Lao P.D.Rep.	United Kingdom
			Mauretania	United States
			Mexico	Uruguay
			Morocco	Venezuela
			Mozambique	Zaire
			Pakistan	
			Singapore	
			Sri Lanka	
			Tunisia	
			Turkey	
			Yugoslavia	

4 Exchange rates of all currencies have shown limited flexibility in terms of the U.S.dollar.

5 Refers to the cooperative arrangement maintained under the European Monetary System.

6 Includes exchange arrangements under which the exchange rate is adjusted at relatively frequent intervals, on the basis of indicators determined by the respective member countries.

E Exact Numbers Underlying Figures¹

Figure 8.1 See table for figure 10.2, ATO unhedged.

Table E.1 Numbers for figure 8.2.

Less than:	Correlation:		
	-0.85	0	+0.85
-1,500,000	0	0	0
-1,400,000	0	0	0
-1,300,000	1	0	0
-1,200,000	1	0	0
-1,100,000	1	0	0
-1,000,000	1	0	0
- 900,000	1	0	0
- 800,000	2	0	0
- 700,000	2	0	0
- 600,000	2	0	0
- 500,000	2	1	0
- 400,000	3	1	0
- 300,000	3	1	0
- 200,000	4	1	0
- 100,000	4	1	0
± 0	4	2	0
100,000	5	2	0
200,000	6	2	0
300,000	6	3	0
400,000	7	3	0

(cont'd.)

Less than:	Correlation:		
	-0.85	0	+0.85
500,000	7	3	0
600,000	8	4	0
700,000	9	5	0
800,000	10	6	1
900,000	11	6	1
1,000,000	12	7	1
1,100,000	13	8	1
1,200,000	15	9	2
1,300,000	16	11	3
1,400,000	17	12	3
1,500,000	18	13	4
1,600,000	20	14	5
1,700,000	22	16	6
1,800,000	23	18	8
1,900,000	24	19	10
2,000,000	26	21	11
2,100,000	28	23	13
2,200,000	30	26	16
2,300,000	32	28	19
2,400,000	33	30	21

(cont'd.)

¹ The same underlying numbers are sometimes used for more than one figure. In order to minimize redundant information, the reader is occasionally referred to different tables. The accuracy of the numbers in the tables is approximately ± 1 percentage unit.

Less than:	Correlation:		
	-0.85	0	+0.85
2,500,000	35	32	24
2,600,000	38	35	28
2,700,000	40	37	31
2,800,000	42	40	35
2,900,000	43	42	38
3,000,000	46	45	42
3,100,000	48	47	46
3,200,000	50	50	51
3,300,000	53	53	55
3,400,000	54	55	58
3,500,000	56	58	62
3,600,000	59	60	66
3,700,000	61	63	69
3,800,000	62	65	73
3,900,000	64	67	75
4,000,000	66	70	79
4,100,000	68	72	81
4,200,000	70	74	83
4,300,000	72	76	86
4,400,000	74	78	88
4,500,000	75	80	90
4,600,000	77	82	91
4,700,000	79	83	93
4,800,000	80	85	94
4,900,000	81	86	95
5,000,000	83	88	96
5,100,000	84	89	97
5,200,000	85	90	97
5,300,000	86	91	98
5,400,000	87	92	98
5,500,000	89	93	98
5,600,000	89	94	99
5,700,000	90	94	99

(cont'd.)

Less than:	Correlation:		
	-0.85	0	+0.85
5,800,000	91	95	99
5,900,000	92	96	99
6,000,000	93	96	99
6,100,000	93	97	99
6,200,000	94	97	100
6,300,000	95	97	100
6,400,000	95	98	100
6,500,000	96	98	100
6,600,000	96	98	100
6,700,000	96	98	100
6,800,000	97	99	100
6,900,000	97	99	100
7,000,000	97	99	100
7,100,000	98	99	100
7,200,000	98	99	100
7,300,000	98	99	100
7,400,000	98	99	100
7,500,000	99	99	100
7,600,000	99	99	100
7,700,000	99	99	100
7,800,000	99	100	100
7,900,000	99	100	100
8,000,000	99	100	100
8,100,000	99	100	100
8,200,000	99	100	100
8,300,000	99	100	100
8,400,000	99	100	100
8,500,000	99	100	100
8,600,000	100	100	100
8,700,000	100	100	100
8,800,000	100	100	100
8,900,000	100	100	100
9,000,000	100	100	100

(cont'd.)

Figure 8.3 See table for figure 10.2, 100% open, RTO ($\phi=0$), ETO ($\phi=0.55$) and ATO ($\phi = 1$).

Table E.2 Numbers for figure 8.4.

Less than:	Forward hedge:		
	0%	50%	100%
± 0	0	0	0
100,000	0	0	0
200,000	0	0	0
300,000	0	0	0
400,000	1	0	0
500,000	1	0	0
600,000	1	0	0
700,000	1	0	0
800,000	2	0	0
900,000	2	0	0
1,000,000	3	0	0
1,100,000	3	0	0
1,200,000	4	0	0
1,300,000	5	0	0
1,400,000	6	0	0
1,500,000	7	0	0
1,600,000	8	0	0
1,700,000	10	0	0
1,800,000	12	1	0
1,900,000	13	1	0
2,000,000	15	2	0
2,100,000	17	3	0
2,200,000	20	4	0
2,300,000	22	6	0
2,400,000	25	8	0
2,500,000	28	12	0
2,600,000	31	15	0
2,700,000	34	20	0
2,800,000	37	25	0
2,900,000	40	31	0
3,000,000	44	36	0
3,100,000	47	44	0
3,200,000	50	50	0
3,300,000	54	57	100
3,400,000	57	63	100
3,500,000	60	69	100

Less than:	Forward hedge:		
	0%	50%	100%
3,600,000	63	75	100
3,700,000	67	79	100
3,800,000	69	84	100
3,900,000	72	88	100
4,000,000	75	91	100
4,100,000	78	93	100
4,200,000	80	95	100
4,300,000	82	96	100
4,400,000	84	97	100
4,500,000	86	98	100
4,600,000	88	99	100
4,700,000	89	99	100
4,800,000	91	99	100
4,900,000	92	99	100
5,000,000	93	100	100
5,100,000	94	100	100
5,200,000	95	100	100
5,300,000	96	100	100
5,400,000	96	100	100
5,500,000	97	100	100
5,600,000	97	100	100
5,700,000	98	100	100
5,800,000	98	100	100
5,900,000	98	100	100
6,000,000	99	100	100
6,100,000	99	100	100
6,200,000	99	100	100
6,300,000	99	100	100
6,400,000	99	100	100
6,500,000	99	100	100
6,600,000	99	100	100
6,700,000	99	100	100
6,800,000	100	100	100
6,900,000	100	100	100
7,000,000	100	100	100

(cont'd.)

Table E.3 Numbers for figure 8.5.

Less than:	NRAP, % forw.			EAP, % forw.		
	0%	50%	100%	0%	50%	100%
-5,000,000	0	0	0	0	0	0
-4,000,000	0	0	0	0	0	0
-3,500,000	0	0	0	0	0	0
-3,400,000	0	0	0	0	0	0
-3,300,000	0	0	0	0	0	0
-3,200,000	0	0	0	0	0	0
-3,100,000	0	0	0	0	0	0
-3,000,000	0	0	1	0	0	0
-2,900,000	0	0	1	0	0	0
-2,800,000	0	0	1	0	0	0
-2,700,000	0	0	1	0	0	1
-2,600,000	0	0	2	0	0	1
-2,500,000	0	0	2	0	0	1
-2,400,000	0	0	2	0	0	1
-2,300,000	0	0	3	0	0	1
-2,200,000	0	0	3	0	0	1
-2,100,000	0	0	4	0	0	2
-2,000,000	0	0	5	0	0	2
-1,900,000	0	0	6	0	0	3
-1,800,000	0	0	7	0	0	3
-1,700,000	0	0	8	0	0	3
-1,600,000	0	0	9	0	0	4
-1,500,000	0	1	10	0	0	5
-1,400,000	0	1	12	0	0	5
-1,300,000	0	2	13	0	1	6
-1,200,000	0	2	16	0	1	7
-1,100,000	0	3	18	0	1	8
-1,000,000	0	5	19	0	2	9
- 900,000	0	7	22	0	3	10
- 800,000	0	9	25	0	4	11
- 700,000	0	12	28	0	5	12
- 600,000	0	16	30	0	7	13
- 500,000	0	19	33	0	9	15
- 400,000	0	25	36	0	11	16
- 300,000	0	30	39	0	13	18
- 200,000	0	36	43	0	16	19
- 100,000	0	43	46	0	19	21
± 0	0	49	49	0	22	22
100,000	100	56	52	45	25	24
200,000	100	62	56	45	28	25
300,000	100	69	60	45	31	27
400,000	100	75	62	45	34	28
500,000	100	80	65	45	36	29
600,000	100	84	69	45	38	31
700,000	100	88	72	46	40	32
800,000	100	91	75	46	41	34
900,000	100	93	77	46	42	35

(cont'd.)

Less than:	NRAP, % forw.			EAP, % forw.		
	0%	50%	100%	0%	50%	100%
1,000,000	100	95	80	46	43	36
1,100,000	100	97	82	47	44	37
1,200,000	100	98	84	47	44	38
1,300,000	100	99	86	48	44	39
1,400,000	100	99	88	48	45	40
1,500,000	100	99	90	49	45	40
1,600,000	100	99	91	49	45	41
1,700,000	100	100	92	50	45	42
1,800,000	100	100	93	51	45	42
1,900,000	100	100	95	52	45	43
2,000,000	100	100	95	53	46	43
2,100,000	100	100	96	54	46	43
2,200,000	100	100	97	56	47	44
2,300,000	100	100	97	57	48	44
2,400,000	100	100	98	58	49	44
2,500,000	100	100	98	60	51	44
2,600,000	100	100	99	62	53	44
2,700,000	100	100	99	63	56	44
2,800,000	100	100	99	65	58	45
2,900,000	100	100	99	67	62	45
3,000,000	100	100	99	69	65	45
3,100,000	100	100	99	71	69	45
3,200,000	100	100	99	73	73	45
3,300,000	100	100	100	75	76	100
3,400,000	100	100	100	76	80	100
3,500,000	100	100	100	78	83	100
3,600,000	100	100	100	80	86	100
3,700,000	100	100	100	82	89	100
3,800,000	100	100	100	83	91	100
3,900,000	100	100	100	84	93	100
4,000,000	100	100	100	86	95	100
4,100,000	100	100	100	88	96	100
4,200,000	100	100	100	89	97	100
4,300,000	100	100	100	90	98	100
4,400,000	100	100	100	91	98	100
4,500,000	100	100	100	92	99	100
4,600,000	100	100	100	93	99	100
4,700,000	100	100	100	94	99	100
4,800,000	100	100	100	95	99	100
4,900,000	100	100	100	95	99	100
5,000,000	100	100	100	96	100	100
5,100,000	100	100	100	97	100	100
5,200,000	100	100	100	97	100	100
5,300,000	100	100	100	97	100	100
5,400,000	100	100	100	98	100	100
5,500,000	100	100	100	98	100	100
5,600,000	100	100	100	98	100	100
5,700,000	100	100	100	99	100	100

(cont'd.)

Less than:	NRAP, % forw.			EAP, % forw.		
	0%	50%	100%	0%	50%	100%
5,800,000	100	100	100	99	100	100
5,900,000	100	100	100	99	100	100
6,000,000	100	100	100	99	100	100
6,100,000	100	100	100	99	100	100
6,200,000	100	100	100	99	100	100
6,300,000	100	100	100	99	100	100
6,400,000	100	100	100	99	100	100
6,500,000	100	100	100	99	100	100
6,600,000	100	100	100	100	100	100
6,700,000	100	100	100	100	100	100
6,800,000	100	100	100	100	100	100
6,900,000	100	100	100	100	100	100
7,000,000	100	100	100	100	100	100

Table E.4 Numbers for figure 8.6.

Less than:	RAP, % opt.=			NRAP, % opt.=			EAP, % opt.=		
	0%	50%	100%	0%	50%	100%	0%	50%	100%
-5,000,000	0	0	0	0	0	0	0	0	0
-4,000,000	0	0	0	0	0	0	0	0	0
-3,500,000	0	0	0	0	0	0	0	0	0
-3,400,000	0	0	0	0	0	0	0	0	0
-3,300,000	0	0	0	0	0	0	0	0	0
-3,200,000	0	0	0	0	0	0	0	0	0
-3,100,000	0	0	0	0	0	0	0	0	0
-3,000,000	0	0	0	0	0	0	0	0	0
-2,900,000	0	0	0	0	0	0	0	0	0
-2,800,000	0	0	0	0	0	0	0	0	0
-2,700,000	0	0	0	0	0	0	0	0	0
-2,600,000	0	0	0	0	0	0	0	0	0
-2,500,000	0	0	0	0	0	0	0	0	0
-2,400,000	0	0	0	0	0	0	0	0	0
-2,300,000	0	0	0	0	0	0	0	0	0
-2,200,000	0	0	0	0	0	0	0	0	0
-2,100,000	0	0	0	0	0	0	0	0	0
-2,000,000	0	0	0	0	0	0	0	0	0
-1,900,000	0	0	0	0	0	0	0	0	0
-1,800,000	0	0	0	0	0	0	0	0	0
-1,700,000	0	0	0	0	0	0	0	0	0
-1,600,000	0	0	0	0	0	0	0	0	0
-1,500,000	0	0	0	0	0	0	0	0	0
-1,400,000	0	0	0	0	0	0	0	0	0
-1,300,000	0	0	0	0	0	0	0	0	0

(cont'd.)

Less than:	RAP, % opt.=			NRAP, % opt.=			EAP, % opt.=		
	0%	50%	100%	0%	50%	100%	0%	50%	100%
-1,200,000	0	0	0	0	0	0	0	0	0
-1,100,000	0	0	0	0	0	0	0	0	0
-1,000,000	0	0	0	0	0	0	0	0	0
- 900,000	0	0	0	0	0	0	0	0	0
- 800,000	0	0	0	0	0	0	0	0	0
- 700,000	0	0	0	0	0	19	0	0	9
- 600,000	0	0	0	0	0	24	0	0	11
- 500,000	0	0	0	0	0	29	0	0	13
- 400,000	0	0	0	0	0	38	0	0	17
- 300,000	0	0	0	0	24	43	0	11	20
- 200,000	0	0	0	0	38	48	0	17	22
- 100,000	0	0	0	0	48	57	0	22	25
± 0	0	0	0	0	60	60	0	27	27
100,000	0	0	0	100	69	68	45	31	31
200,000	0	0	0	100	77	69	45	35	31
300,000	0	0	0	100	80	71	45	36	32
400,000	1	0	0	100	85	77	45	38	35
500,000	1	0	0	100	89	79	45	40	35
600,000	1	0	0	100	92	80	45	42	36
700,000	1	0	0	100	93	84	46	42	38
800,000	2	0	0	100	95	85	46	43	38
900,000	2	0	0	100	97	85	46	44	38
1,000,000	3	0	0	100	97	89	46	44	40
1,100,000	3	0	0	100	98	90	47	44	40
1,200,000	4	0	0	100	99	92	47	44	42
1,300,000	5	0	0	100	99	93	48	45	42
1,400,000	6	0	0	100	99	93	48	45	42
1,500,000	7	1	0	100	99	95	49	45	43
1,600,000	8	1	0	100	100	95	49	45	43
1,700,000	10	2	0	100	100	95	50	46	43
1,800,000	12	2	0	100	100	97	51	46	44
1,900,000	13	3	0	100	100	97	52	47	44
2,000,000	15	5	0	100	100	97	53	48	44
2,100,000	17	7	0	100	100	98	54	49	44
2,200,000	20	10	0	100	100	98	56	51	44
2,300,000	22	14	0	100	100	99	57	52	44
2,400,000	25	18	0	100	100	99	58	55	44
2,500,000	28	23	19	100	100	99	60	58	55
2,600,000	31	28	24	100	100	99	62	60	58
2,700,000	34	34	33	100	100	99	63	64	63
2,800,000	37	39	39	100	100	99	65	66	66
2,900,000	40	44	43	100	100	99	67	69	69
3,000,000	44	49	55	100	100	99	69	72	74
3,100,000	47	54	57	100	100	99	71	75	76
3,200,000	50	57	61	100	100	100	73	76	78
3,300,000	54	61	68	100	100	100	75	78	82
3,400,000	57	65	71	100	100	100	76	81	84
3,500,000	60	68	72	100	100	100	78	82	85

(cont'd.)

Less than:	RAP, % opt.=			NRAP, % opt.=			EAP, % opt.=		
	0%	50%	100%	0%	50%	100%	0%	50%	100%
3,600,000	63	71	78	100	100	100	80	84	88
3,700,000	67	74	79	100	100	100	82	86	88
3,800,000	69	76	80	100	100	100	83	87	89
3,900,000	72	79	85	100	100	100	84	88	91
4,000,000	75	81	85	100	100	100	86	89	92
4,100,000	78	83	86	100	100	100	88	91	92
4,200,000	80	85	89	100	100	100	89	92	94
4,300,000	82	87	90	100	100	100	90	92	94
4,400,000	84	88	90	100	100	100	91	93	94
4,500,000	86	90	93	100	100	100	92	94	96
4,600,000	88	91	93	100	100	100	93	95	96
4,700,000	89	92	93	100	100	100	94	96	96
4,800,000	91	93	95	100	100	100	95	96	97
4,900,000	92	94	95	100	100	100	95	97	97
5,000,000	93	95	95	100	100	100	96	97	97
5,100,000	94	96	97	100	100	100	97	97	98
5,200,000	95	96	97	100	100	100	97	98	98
5,300,000	96	97	97	100	100	100	97	98	98
5,400,000	96	97	98	100	100	100	98	98	99
5,500,000	97	98	98	100	100	100	98	99	99
5,600,000	97	98	98	100	100	100	98	99	99
5,700,000	98	98	99	100	100	100	99	99	99
5,800,000	98	99	99	100	100	100	99	99	99
5,900,000	98	99	99	100	100	100	99	99	99
6,000,000	99	99	99	100	100	100	99	99	99
6,100,000	99	99	99	100	100	100	99	99	99
6,200,000	99	99	99	100	100	100	99	99	99
6,300,000	99	99	99	100	100	100	99	99	100
6,400,000	99	99	99	100	100	100	99	99	100
6,500,000	99	99	99	100	100	100	99	100	100
6,600,000	99	99	100	100	100	100	100	100	100
6,700,000	99	100	100	100	100	100	100	100	100
6,800,000	100	100	100	100	100	100	100	100	100
6,900,000	100	100	100	100	100	100	100	100	100
7,000,000	100	100	100	100	100	100	100	100	100

Table E.5 Numbers for figure 10.2.

Less than:	#1: 100% open			#2: 100% forw.			#3: 100% opt.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
-5,000,000	0	0	0	0	0	0	0	0	0
-4,000,000	0	0	0	0	0	0	0	0	0
-3,500,000	0	0	0	0	0	0	0	0	0
-3,400,000	0	0	0	0	0	0	0	0	0
-3,300,000	0	0	0	0	0	0	0	0	0
-3,200,000	0	0	0	0	0	0	0	0	0
-3,100,000	0	0	0	0	0	0	0	0	0
-3,000,000	0	0	0	0	1	0	0	0	0
-2,900,000	0	0	0	0	1	0	0	0	0
-2,800,000	0	0	0	0	1	0	0	0	0
-2,700,000	0	0	0	0	1	1	0	0	0
-2,600,000	0	0	0	0	2	1	0	0	0
-2,500,000	0	0	0	0	2	1	0	0	0
-2,400,000	0	0	0	0	2	1	0	0	0
-2,300,000	0	0	0	0	3	1	0	0	0
-2,200,000	0	0	0	0	3	1	0	0	0
-2,100,000	0	0	0	0	4	2	0	0	0
-2,000,000	0	0	0	0	5	2	0	0	0
-1,900,000	0	0	0	0	6	3	0	0	0
-1,800,000	0	0	0	0	7	3	0	0	0
-1,700,000	0	0	0	0	8	3	0	0	0
-1,600,000	0	0	0	0	9	4	0	0	0
-1,500,000	0	0	0	0	10	5	0	0	0
-1,400,000	0	0	0	0	12	5	0	0	0
-1,300,000	0	0	0	0	13	6	0	0	0
-1,200,000	0	0	0	0	16	7	0	0	0
-1,100,000	0	0	0	0	18	8	0	0	0
-1,000,000	0	0	0	0	19	9	0	0	0
- 900,000	0	0	0	0	22	10	0	0	0
- 800,000	0	0	0	0	25	11	0	0	0
- 700,000	0	0	0	0	28	12	0	19	9
- 600,000	0	0	0	0	30	13	0	24	11
- 500,000	0	0	0	0	33	15	0	29	13
- 400,000	0	0	0	0	36	16	0	38	17
- 300,000	0	0	0	0	39	18	0	43	20
- 200,000	0	0	0	0	43	19	0	48	22
- 100,000	0	0	0	0	46	21	0	57	25
± 0	0	0	0	0	49	22	0	60	27
100,000	0	100	45	0	52	24	0	68	31
200,000	0	100	45	0	56	25	0	69	31
300,000	0	100	45	0	60	27	0	71	32
400,000	1	100	45	0	62	28	0	77	35
500,000	1	100	45	0	65	29	0	79	35
600,000	1	100	45	0	69	31	0	80	36
700,000	1	100	46	0	72	32	0	84	38
800,000	2	100	46	0	75	34	0	85	38
900,000	2	100	46	0	77	35	0	85	38

(cont'd.)

Less than:	#1: 100% open			#2: 100% forw.			#3: 100% opt.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
1,000,000	3	100	46	0	80	36	0	89	40
1,100,000	3	100	47	0	82	37	0	90	40
1,200,000	4	100	47	0	84	38	0	92	42
1,300,000	5	100	48	0	86	39	0	93	42
1,400,000	6	100	48	0	88	40	0	93	42
1,500,000	7	100	49	0	90	40	0	95	43
1,600,000	8	100	49	0	91	41	0	95	43
1,700,000	10	100	50	0	92	42	0	95	43
1,800,000	12	100	51	0	93	42	0	97	44
1,900,000	13	100	52	0	95	43	0	97	44
2,000,000	15	100	53	0	95	43	0	97	44
2,100,000	17	100	54	0	96	43	0	98	44
2,200,000	20	100	56	0	97	44	0	98	44
2,300,000	22	100	57	0	97	44	0	99	44
2,400,000	25	100	58	0	98	44	0	99	44
2,500,000	28	100	60	0	98	44	19	99	55
2,600,000	31	100	62	0	99	44	24	99	58
2,700,000	34	100	63	0	99	44	33	99	63
2,800,000	37	100	65	0	99	45	39	99	66
2,900,000	40	100	67	0	99	45	43	99	69
3,000,000	44	100	69	0	99	45	55	99	74
3,100,000	47	100	71	0	99	45	57	99	76
3,200,000	50	100	73	0	99	45	61	100	78
3,300,000	54	100	75	100	100	100	68	100	82
3,400,000	57	100	76	100	100	100	71	100	84
3,500,000	60	100	78	100	100	100	72	100	85
3,600,000	63	100	80	100	100	100	78	100	88
3,700,000	67	100	82	100	100	100	79	100	88
3,800,000	69	100	83	100	100	100	80	100	89
3,900,000	72	100	84	100	100	100	85	100	91
4,000,000	75	100	86	100	100	100	85	100	92
4,100,000	78	100	88	100	100	100	86	100	92
4,200,000	80	100	89	100	100	100	89	100	94
4,300,000	82	100	90	100	100	100	90	100	94
4,400,000	84	100	91	100	100	100	90	100	94
4,500,000	86	100	92	100	100	100	93	100	96
4,600,000	88	100	93	100	100	100	93	100	96
4,700,000	89	100	94	100	100	100	93	100	96
4,800,000	91	100	95	100	100	100	95	100	97
4,900,000	92	100	95	100	100	100	95	100	97
5,000,000	93	100	96	100	100	100	95	100	97
5,100,000	94	100	97	100	100	100	97	100	98
5,200,000	95	100	97	100	100	100	97	100	98
5,300,000	96	100	97	100	100	100	97	100	98
5,400,000	96	100	98	100	100	100	98	100	99
5,500,000	97	100	98	100	100	100	98	100	99
5,600,000	97	100	98	100	100	100	98	100	99
5,700,000	98	100	99	100	100	100	99	100	99

(cont'd.)

Less than:	#1: 100% open			#2: 100% forw.			#3: 100% opt.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
5,800,000	98	100	99	100	100	100	99	100	99
5,900,000	98	100	99	100	100	100	99	100	99
6,000,000	99	100	99	100	100	100	99	100	99
6,100,000	99	100	99	100	100	100	99	100	99
6,200,000	99	100	99	100	100	100	99	100	99
6,300,000	99	100	99	100	100	100	99	100	100
6,400,000	99	100	99	100	100	100	99	100	100
6,500,000	99	100	99	100	100	100	99	100	100
6,600,000	99	100	100	100	100	100	100	100	100
6,700,000	99	100	100	100	100	100	100	100	100
6,800,000	100	100	100	100	100	100	100	100	100
6,900,000	100	100	100	100	100	100	100	100	100
7,000,000	100	100	100	100	100	100	100	100	100

Table E.6 Numbers for figures 10.3.

Less than:	#1: 70% option			#2: 62% put			#3: 50% opt. 20% forw.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
-5,000,000	0	0	0	0	0	0	0	0	0
-4,000,000	0	0	0	0	0	0	0	0	0
-3,500,000	0	0	0	0	0	0	0	0	0
-3,000,000	0	0	0	0	0	0	0	0	0
-2,500,000	0	0	0	0	0	0	0	0	0
-2,000,000	0	0	0	0	0	0	0	0	0
-1,500,000	0	0	0	0	0	0	0	0	0
-1,000,000	0	0	0	0	0	0	0	0	0
- 900,000	0	0	0	0	0	0	0	1	0
- 800,000	0	0	0	0	0	0	0	2	1
- 700,000	0	0	0	0	0	0	0	5	2
- 600,000	0	0	0	0	0	0	0	10	5
- 500,000	0	19	9	0	0	0	0	19	9
- 400,000	0	28	13	0	0	0	0	29	13
- 300,000	0	38	17	0	54	24	0	37	16
- 200,000	0	43	20	0	54	24	0	46	21
- 100,000	0	53	24	0	62	28	0	53	24
± 0	0	60	27	0	62	28	0	60	27
100,000	0	68	31	0	69	31	0	65	29
200,000	0	71	32	0	76	34	0	70	32
300,000	0	77	35	0	76	34	0	74	33
400,000	0	79	36	0	81	37	0	78	35
500,000	0	84	38	0	81	37	0	81	37
600,000	0	85	38	0	86	39	0	84	38

(cont'd.)

Less than:	#1: 70% option			#2: 62% put			#3: 50% opt. 20% forw.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
700,000	0	89	40	0	86	39	0	87	39
800,000	0	90	40	0	90	41	0	89	40
900,000	0	93	42	0	93	42	0	91	41
1,000,000	0	93	42	0	93	42	0	93	42
1,100,000	0	95	43	0	95	43	0	94	42
1,200,000	0	95	43	0	95	43	0	95	43
1,300,000	0	97	44	0	97	44	0	96	43
1,400,000	0	97	44	0	98	44	0	97	44
1,500,000	0	98	44	0	98	44	0	98	44
1,600,000	0	99	44	0	99	45	0	98	44
1,700,000	0	99	44	1	99	45	0	99	44
1,800,000	0	99	45	1	99	45	0	99	45
1,900,000	1	99	45	2	99	46	0	99	45
2,000,000	1	99	45	4	99	47	0	99	45
2,100,000	3	99	46	5	100	48	1	99	45
2,200,000	5	100	47	8	100	49	2	100	46
2,300,000	8	100	49	10	100	50	3	100	47
2,400,000	13	100	52	13	100	52	7	100	49
2,500,000	20	100	56	18	100	55	12	100	51
2,600,000	27	100	60	23	100	58	18	100	55
2,700,000	33	100	63	30	100	61	25	100	59
2,800,000	39	100	66	36	100	65	32	100	63
2,900,000	46	100	70	41	100	68	41	100	67
3,000,000	51	100	73	48	100	71	49	100	72
3,100,000	55	100	75	53	100	74	54	100	74
3,200,000	60	100	78	58	100	77	59	100	77
3,300,000	64	100	80	62	100	79	64	100	80
3,400,000	67	100	82	67	100	81	68	100	82
3,500,000	71	100	84	70	100	83	72	100	85
3,600,000	73	100	85	73	100	85	75	100	86
3,700,000	76	100	87	76	100	87	79	100	88
3,800,000	79	100	88	78	100	88	81	100	89
3,900,000	80	100	89	81	100	89	84	100	91
4,000,000	83	100	90	83	100	91	86	100	92
4,100,000	85	100	92	85	100	92	88	100	93
4,200,000	86	100	92	87	100	93	90	100	94
4,300,000	88	100	93	89	100	94	91	100	95
4,400,000	89	100	94	90	100	94	93	100	96
4,500,000	91	100	95	91	100	95	94	100	96
4,600,000	92	100	95	93	100	96	95	100	97
4,700,000	93	100	96	94	100	96	96	100	97
4,800,000	94	100	96	95	100	97	96	100	98
4,900,000	95	100	97	95	100	97	97	100	98
5,000,000	95	100	97	96	100	98	98	100	98
5,100,000	96	100	98	97	100	98	98	100	99
5,200,000	97	100	98	97	100	98	98	100	99
5,300,000	97	100	98	98	100	98	99	100	99

(cont'd.)

Less than:	#1: 70% option			#2: 62% put			#3: 50% opt. 20% forw.		
	ATO	RTO	ETO	ATO	RTO	ETO	ATO	RTO	ETO
5,400,000	97	100	98	98	100	99	99	100	99
5,500,000	98	100	99	98	100	99	99	100	99
5,600,000	98	100	99	99	100	99	99	100	99
5,700,000	98	100	99	99	100	99	99	100	99
5,800,000	99	100	99	99	100	99	99	100	100
5,900,000	99	100	99	99	100	99	99	100	100
6,000,000	99	100	99	99	100	99	100	100	100
6,100,000	99	100	99	99	100	99	100	100	100
6,200,000	99	100	99	99	100	100	100	100	100
6,300,000	99	100	99	99	100	100	100	100	100
6,400,000	99	100	99	100	100	100	100	100	100
6,500,000	99	100	100	100	100	100	100	100	100
6,600,000	99	100	100	100	100	100	100	100	100
6,700,000	100	100	100	100	100	100	100	100	100
6,800,000	100	100	100	100	100	100	100	100	100
6,900,000	100	100	100	100	100	100	100	100	100
7,000,000	100	100	100	100	100	100	100	100	100

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