ENDOGENOUS POPULATION IN A LAND-CONSTRAINED ECONOMY

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CORRECTIONS

p. 10, row 10: replace unfriend with unpriced

p. 12, row 14: replace steady with stationary

p. 18, last row: should read: ...., lead to reductions in fertility

p. 38, expression (23):

$$\hat{A} = [\frac{1}{B}]^{\frac{1}{\alpha}} \tag{23}$$

p. 46: delete footnote 34

p. 73, expression (47):

$$1 = Q - \frac{1}{1+\delta}\frac{\hat{A}F_A(\hat{A},\lambda)}{F_L(\hat{A},\lambda)-k} \tag{47}$$

p. 74, row 4: replace (53) with (54)

p. 92, row 3: replace (9) with (10)

p. 99, row 15: insert (increase) after the word "decrease"

p. 100: replace the text between "Define the expected value ... easily established that:" with "In Appendix 3 we show that a stochastic stationary state exists and that:"

p. 109, row 9: delete α

p. 114, expression (52'):

$$A_{t+1} = \frac{1}{\beta} \cdot \frac{1}{\theta_t} A_t^{1-\alpha} \tag{52'}$$

p. 114, RHS of expression (57'):

$$= - \sum_{j=-T}^{t-1} (\overset{\simeq}{B} + \overset{\simeq}{\theta}_j)(1-\alpha)^{-j-1}$$

p. 115, row 6: replace (55') with (54')

p. 115, expression (61'): replace $\overset{\simeq}{\Lambda}_t$ with $\overset{\simeq}{A}_t$

p. 116, expressions (70')–(73'):

replace expressions (70') – (73') with:

$$\lim_{t\to\infty} E\tilde{\bar{A}}_t = -\frac{\tilde{\bar{B}} + E\tilde{\bar{\theta}}}{\alpha} \tag{70'}$$

which proves that a stochastic stationary state exists.

From (66') follows:

$$A_t = G \exp\left[-\sum_{i=o}^{\infty} (1-\alpha)^i \tilde{\bar{\theta}}_{t-1-i}\right] \tag{71'}$$

$$= G \prod_{i=o}^{\infty} \theta_{t-1-i}^{-(1-\alpha)^i} \tag{72'}$$

where:

$$G = \exp\left[(1-\alpha)^t A_o - \frac{\tilde{\bar{B}}}{\alpha}\right] \tag{73'}$$

Since θ:s are independently distributed and:

$$E\,\theta_{t-1-i}^{-(1-\alpha)^i} = q \cdot 1^{-(1-\alpha)^i}\,(1-q)p^{-(1-\alpha)^i} \tag{74'}$$

the expected value of A_t is:

$$E\,A_t = G \cdot \prod_{i=o}^{\infty}\left[q + (1-q)\frac{1}{p(1-\alpha)^i}\right] \tag{75'}$$

Hence, an increase in p will lead to a reduction in EA_t and obviously also in $\lim_{t\to\infty} EA_t$.

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FOREWORD

The topic of this thesis emanates from my work with development issues in the Foreign Service and SIDA during the years 1977–1985. During the last years at SIDA, I was involved in the coordination of Swedish aid to Bangladesh. In the assistance to Bangladesh, emphasis was given to poverty reduction in rural areas. In practice, this was not easy to accomplish. The experience from Bangladesh made me interested in the determinants of changes in income distribution over time. In particular, I became interested in long-term changes, where demographic factors evidently played an important role. My attention was therefore directed towards economic models of fertility decisions, and to the literature that investigated the motives in fertility decisions in different parts of the world.

When thinking about the intellectual and moral support I have received over the years, my thoughts go foremost to my main supervisor, Karl Jungenfelt. His knowledge of, and great interest in, the fundamental issues of my field, combined with his enthusiastic interest in our own demographic and economic history, and his technical skill and intuition, have served as invaluable sources of learning and inspiration. Furthermore, his patience with interruptions in my work has made it possible to combine family life and research.

I also benefited greatly from discussions with and comments by several other people. At Princeton University, Ravi Kanbur provided insightful and constructive advice. Furthermore, I am indebted to Jonas Agell, who as the "opponent" to my licenciate thesis provided very useful ideas for the continuation of my research. The assistance provided by Harald Lang in guiding me into the technical aspects of different issues of the last chapter has been very helpful. Ulf Olsson, Anders Paalzow, Hans Wijkander, Anders Vredin as well as other present and former colleagues at the Department of Economics, provided many useful comments. Torbjörn Becker and Bo Andersson have also been kind enough to check the proofs. Advice from Karl–Göran Mäler, Tommy Bengtsson and Åke Blomqvist has been important in shaping the final product. For valuable comments and discussions I am also very grateful to Allen Drazen, Henrik Horn, George McNicoll, Marc Nerlove, Mårten Palme, Assaf Razin, Marc Rosenzweig, Efraim Sadka, David Starrett and Timothy Uinnane. Members of the Department of International Economics and Geography at the Stockholm School of Economics, and of the Institute for International Economic Studies, have at seminars, especially in the beginning of my work, provided me with a number of important comments. Kerstin Niklasson has improved my English, and assisted me in the editing and typing during the final weeks in an invaluable way.

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INTRODUCTION

1. The Purpose of the Thesis

The world's population continues to increase at a rapid rate and is estimated to reach 10 billion by 2050. More than 90 percent of the increase is expected to occur in the developing world, where at present more than one third of the population is younger than 15 years and thus just entering their reproductive years. It is obvious that an understanding of the mechanisms that are involved in fertility decisions in these countries is crucial to the design of economic policies that aim at improving the standard of living in these countries and that aim at reducing negative environmental damage of population growth. The purpose of this thesis is to contribute to this understanding, by shedding light on some mechanisms that might influence fertility decisions and population changes in developing countries.

The thesis consists of three analytical chapters. In this introduction we will first discuss some issues of relevance to all three chapters, then briefly describe what is done in each chapter and point to some of the limitations of our analysis.

2. Fertility from a Life–Cycle Perspective

From an economic point of view, a human life can be divided into three main broad phases[1]: During the first phase (childhood) an individual must be fed, dressed and reared,

[1] The length of each phase differs between different societies and changes as societies develop. Biology sets upper and lower limits for each phase but leaves a wide range for cultural and economic differences. As an example, see Cain (1977) and Cain (1982) for a discussion on at what age children in a village in Bangladesh become net producers and at what age they compensate for their own cumulative consumption. The length of the intervals is, however, not essential for the following analysis.

imposing costs that have to be born by someone else. During the second phase (adulthood) the individual earns a living in some form by which he can support himself. Some amount of the income is often saved for the last phase, provided there exists some mechanism by which resources can be transferred from adult to old age. Finally, in the third phase (old age), the income–generating ability of an individual is lower due to a number of different reasons such as retirement, age, sickness or inability. If the individual has transferred resources from the adult phase, such resources complement any income earned during this last phase. An individual can obviously not transfer resources from any other phase in life to the first phase, but must rely on someone else paying the cost for childhood. Why is someone willing to bear this cost?

There are clearly several reasons why people want to have children, and many of them are outside the realm of economic analysis. Economic models of fertility decisions point to a number of benefits from having children that explain why people are willing to accept the associated costs. These analyses assume that the fertility decision is made to trade off these costs and benefits, in order to maximize some objective function[2].

The costs associated with raising children are in most cultures born by parents, even though in some cultures, notably in sub–Saharan Africa, the costs of raising children often are diffused across the kinship, and sometimes also shared with non–relatives through the mechanisms of so–called fostering or child–development (Bledsoe, 1994).

The economic benefit of having children may differ between societies, as well as within societies over time. Four main motives for having children can be identified in the literature:

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[2] We refer to the dominating models of fertility behaviour inspired by the seminal contributions of Leibenstein (1957) and Becker (1960) who studied fertility behaviour within the framework of consumer theory, based on a theory of rational individuals maximizing utility under given constraints. In this thesis we stay within such neoclassical framework.

a. The Consumption Motive

The consumption motive provides the broadest type of motivation and "comprises a disparate set, ranging from the desire to have children because they are playful and enjoyable to the dictates of injunctions emanating from the cult of the ancestor, which sees religion as essentially the reproduction of the lineage" (Dasgupta, 1993, p. 357). This broad motive has been explored by several authors, for instance Becker (1960, 1981), Nerlove, Razin and Sadka (1987), Becker and Barro (1988). In those models parents are normally faced with a trade–off between own consumption and the quality of children and between the number of children and the quality of each child.

b. The Insurance Motive

During adult and old age, people's incomes might be reduced due to sickness, inability, unemployment or for some other reason. In developed societies, such insecurities have in most cases been taken care of by some type of social security scheme or through private insurances. In many developing countries no such schemes exist. Having a large family, with a large number of children contributing to the total income of the household, is one way of securing income (Cain, 1981). Obviously, this requires that risks to some extent are specific to individuals, and that household members share the income that is generated within the household. To spread risks, the household members may take up different types of jobs or work in different geographical areas where risk is not correlated.

c. The Investment Motive

Individuals aim at a smooth consumption over the life–cycle. As the income–generating ability normally is at its highest during the second phase of life, as defined above, individuals generally wish to transfer part of the income from that period to the last phase. A number of different institutional solutions exist by which resources accumulated in the second period of life can be transferred to the last period. Pension schemes and credit institutions constitute examples in industrialized societies. In many developing societies, and especially in poor rural areas, such institutions do not exist. Other types of storable assets by which transfers to old age could be executed, are often insecure and give a low expected rate of return.

A different way of transferring resources from active to adult age, would be to forego consumption in adult age by investing in the upbringing of children and arrange for resources to be transferred back from these children at old age. This is often called the old–age security motive for having children. Many observers have reported on its importance in fertility choices in developing countries and especially in rural areas (Nugent, 1985). Earlier theoretical models analyzing the old–age security motive in fertility decisions, for instance Neher (1971), Willis (1980), Hammer (1986), Nerlove, Razin and Sadka (1987), are typically one–factor models without any link between the fertility decisions of consecutive generations.

d. The Producer Goods Motive

In poor countries, children often start to contribute to the household's income early in age. But also as adults, offsprings are preferred by parents as labor in family enterprises, if such exist. Pollack (1985) gives a number of different reasons why farmers may prefer family labor on their farms. Investing in children with the purpose of using them as labor on the family farm when they have reached adult age, constitutes another possible motive in fertility decisions for landholding peasants. Such an arrangement requires some type of agreement that will make the children stay on the family farm and provide labor. A number of historical examples of such arrangements exist as well as examples of how such arrangements break down when they can no longer be enforced[3]. The producer goods motive, in terms of demand for child labor, has formally only been explored sparsely[4]. The

[3] For instance Sutch (1991) argues that the decline in fertility in North America was caused by a failure of the family–based income–transfer mechanism. That failure was induced by the opening of western lands to settlements at the beginning of the nineteenth century, making it possible for young adults to leave the family farm and start a new life far away.

[4] In Dasgupta and Mäler (1991) and in Nerlove and Meyer (1991) the motive is analysed in a context where there exists an explicit interaction between the quality of the environmental resource base and the demand for child labor.

producer goods motive, in terms of demand for the labor of grown–up offsprings, has been incorporated in models of old–age security (for instance Neher, 1971) or analyzed in combination with the consumption motive (for instance, Eckstein, Stern and Wolpin, 1989).

3. The Focus and Limitations of the Thesis

The discussion in the preceding section indicated that the costs and benefits of having children might differ between different types of societies, and also between different socio–economic groups within nations. As a consequence, the relative strength of different motives in fertility decisions will also differ. Hence, it is important to specify the type of society on which we are focusing our analysis.

a. Choice of Motives

The analysis of this thesis concentrates on rural societies in developing countries. In rural areas, old–age security is frequently reported to be the main motive in fertility decisions, particularly in agricultural subsistence economies (Nugent, 1985; Kagitcibasi, 1982). In such societies, grown–up children are considered valuable not only as providers of old–age care to their parents but also as labor on the family farm (Pearce and Warford, 1991, Dasgupta and Mäler, 1991).

In our model we have thus chosen to restrict ourselves to the analysis of fertility decisions of farmers who privately own their land, for whom the motivation for having children stems from the expectation that children will provide labor on the family farm once they grow up, and for whom there are no alternative means to transfer resources from adult to old age. We thus disregard all other motives for having children in order to highlight the implications of the investment (old–age security) and the producer goods motives (in terms of demand for labor from grown–up children).

b. Characteristics of Societies where Children are Important as Investment and Producer Goods

The problem of providing for consumption during old age and the characteristics of a society where children serve this purpose may be explored within the overlapping generations framework introduced by Samuelson (1958). In that model it can be demonstrated that the attractiveness of children as a device for smoothing consumption depends on the availability of alternatives to having children as well as on the possibilities of enforcing loyalty from children.

As already mentioned, social security, private pension policies, and markets for assets that help individuals to save today in order to spend tomorrow are virtually non-existent in rural areas in developing countries (Nugent and Gillaspy, 1983; Wildasin, 1983; Nugent, 1985). In the absence of such markets, land, cattle and children are the primary means for poor households to hold their savings (Dasgupta, 1993). Markets for land are, however, usually extremely thin (Nugent, 1985). Furthermore, land may require family labor to make it a secure and productive asset (Wildasin, 1983; World Bank, 1984). First, children might be needed to safeguard ownership as such (Adnan, 1988; Dasgupta, 1993). Second, land requires labor to make it a productive asset. There are several reasons why the cost, quality and availability of family labor differ from that of hired labor, even if an extensive labor market does exist (Stiglitz, 1988; Pollak, 1985; Rosenzweig, Binswanger and McIntire, 1987; Wildasin, 1983). Lower cost, higher quality or better availability thus often make farmers prefer family labor.

There is, however, no reason to invest in raising children for the purpose of getting family labor and old-age support, if retired parents cannot be sure of the loyalty of their children. One way to ensure loyalty is through some form of agreement or mutual understanding whereby labor in adult years is exchanged for a wage plus the right to inherit a certain amount of land upon the death of the parent. In a partible inheritance system, all children will inherit when a parent dies and thus have inheritance incentives to stay and provide labor on the family farm. In the case of an impartible system, only the favored child would have the inheritance incentive to stay, albeit a stronger incentive than he/she would have in a society with a partible system. Consequently, in comparing these two inheritance rules, a partible system could under certain circumstances be expected to create a relatively greater demand for children.

If the expected return to a job outside the family farm exceeds the expected value of the inheritance and the wage on the family farm, the value of the inheritance as a leverage on children would be smaller. The importance of the old–age security and producer goods motives in determining fertility patterns could therefore be expected to decline in fast developing segments of an economy with growing labor markets or where possibilities of migrating to outside labor markets open up (Sutch, 1991; Williamson, 1985).

In sum, the old–age security and producer goods motives in fertility decisions could be expected to be strong in societies characterized by the following conditions:

– no social or private security system, or low coverage and level, or low confidence in the duration of existing programs;

– underdeveloped markets for credit, land and other storable assets;

– partible inheritance systems with private ownership of the inheritable asset;

– slow expected changes in the growth of real income per capita;

– underdeveloped labor and service markets.

c. Assumptions, Limitations and Properties of the Models

The characteristics listed above have formed the basis for the formulation of the models of the thesis. The common theme is that we are studying a society in which the extended family constitutes the only life–cycle mechanism (social mechanism) available by which output can be transferred from the producing to the dependent generations. In the overlapping–generations models of this thesis, the extended family consists of three generations; children, adults and old. The dependents include the children and the old,[5] while the adults are producers.

The hypothetical peasant society in the models of Chapter 1 and Chapter 3 is made up of self–sufficient, extended families that produce one non–storable, subsistence good with inputs of labor and land. Each household lives in economic isolation, without any links to the rest of the economy. In Chapter 2, the society also contains a landholding class that does not work on the land but who hires labor and receives a share of its produce (landlords)[6]. The old maintain ownership of land and control over household food stock and there are no alternative markets for assets or social security. Children are thus the only means of saving for old age. The children stay and provide labor as adults in exchange for a wage and a bequest. When an old person dies, his land is shared equally among the (surviving) children, who in turn enter the period of old age. Parents provide their children with an exogenously determined amount of food[7] and the elderly consume an amount equal to total output minus costs for family labor[8].

Each model contains an institution that determines how the production on a farm is

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[5] The insurance motive, which is obviously also of great importance in poor rural societies, could be accommodated in the interpretation of our models, if we only discuss it in terms of having children to work for you in case of sickness, inability etc. In such a case the dependants would include the ill, the infirm, the involuntarily employed etc. If we were to discuss the dimension of risk–spreading, we would have to enrich the models with different probabilities attached to different types of incomes, which obviously would increase the difficulty to explicitly trace the dynamic paths of different variables in our models.

[6] The basic behaviourial assumptions below do not refer to landlords but only to extended peasant families.

[7] We thus assume that children are a net cost to their parents during childhood. Any income–generating activity during that phase is thus assumed to be valued less than the accumulated consumption cost of the child during childhood. In reviewing studies from different Asian countries, Cain (1982) concludes that it is most unlikely that the value of child labor alone justifies unlimited fertility in the countries concerned.

[8] In our models we have thus assumed that costs for children are born by the household and that benefits also stay within the same household. The analysis is thus not applicable to those societies where cost and benefits are spread over a larger group of people as is the case in a number of sub–Saharan societies.

divided between old and adults in a household[9]. In Chapter 1 and Chapter 3, a fix share of the production is allocated to the adults. In Chapter 2, the adults receive a lump sum payment. The analysis indicates that the basic analytical results do not depend on the choice of distribution rule. Nor does the level of the share or the size of the lump sum payment that is allocated to adults within the household have an impact on the qualitative results. Larger shares or higher lump sum payments allocated to adults do, however, increase the demand for children. This is explained by the fact that individuals aim at a smooth consumption over the life cycle. The only way to compensate for the fact that a larger share of the production is received in adult age, as compared to old age, is to increase the number of children, as such an action will increase consumption in old age while reducing it in adult age. Hence, even if there are no savings in the aggregate in our models, the adults in our economy do save, as their consumption falls short of their income. Consumption transfers to the children is foregone consumption, which represents savings and guarantees consumption in old age.

In our models we assume that individuals are selfish in the sense that they only have their own consumption in their utility function. In utility terms, the household is thus not extended as each individual excludes the utility of parents, siblings and children in his utility function. This assumption alludes to a vast literature on the existence or non–existence of altruism, its characteristics and direction. In most models where altruism is assumed, parents are concerned about their children. Such an assumption does not imply that children are loyal to their parents. This can only be assured by assuming altruism in

[9] An allocation of resources, at a certain moment in time, between family members based on age, hence between members belonging to different generations, may hide implicit intergenerational transfers. If we analyse the intra–household allocation of available food in our models from an angle where we look at transfers from the generation that controls assets and foodstock to the rest of the household, we can interpret the transfers as if adults are provided with a consumption insurance from their old parents in exchange for labor services. If we study the extended family in terms of transfers from producers to dependants, we observe transfers from the producers (adults) to dependants (old and children) at each point in time.

the direction from children to parents or by the presence of some type of social contract. In Dasgupta (1993), it is shown how a social norm similar to the one we incorporate in our models is self–enforcing without any need for social sanctions.

Furthermore, we restrict our analysis to changes in parental welfare, defined as the weighted sum of utility in adult and old age. By doing so we by–pass a discussion on how to treat "possible people" and the choice of a social welfare function[10].

As indicated by the structure of our model, the fertility decision of one generation has implications for the welfare of forthcoming generations. But the activity of having children is unfriend in terms of the effects on forthcoming generations, although it does affect production and consumption possibilities of future generations. A Pareto–efficient allocation normally requires that there exist competitive markets for every commodity. In our models there exists no market on which individuals belonging to different generations can act. Higher fertility will induce a decline in the returns to labor for future generations, a consequence which represents one example of an externality[11] of population growth in our models.

According to some observers, more people, for given levels of consumption, implies more stress on the environment, leading in some scenarios to degradation of natural resources, such as arable land ("a true externality"[12]). In the models of this thesis we do not incorporate any explicit interaction between population density and total availability of land. However, our analysis does show the impact of different policies on population density.

For the sake of analytical convenience we confine our analysis to one–sex models. It

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[10] See Nerlove, Razin and Sadka (1987) and Dasgupta (1993) for a discussion of these issues.

[11] See Willis (1994) and Birdsall (1994) on the ongoing discussion of whether such an externality shall be regarded as a "true" externality or as a " pecuniary" externality, which by definition only affects the distribution of income and which does not lead to inefficient outcomes.

[12] For a general discussion on population and externalities, see Birdsall (1994).

has been shown that women's reproductive goals do not differ notably from those of men in poor societies marked by gender–based asymmetry in employment opportunities and power (Mason and Taj, 1987). Cain (1982) argues that where women's economic dependence is extreme, women may even be more interested in maintaining high fertility than men, despite the situational advantage of the latter.

In the first two chapters of this thesis we disregard the fact that the investment in children is risky. However, in Chapter 3, we include uncertainty and study the effects of changes in child mortality. In a simulations study, May and Heer (1968) have calculated that, given the age–specific death rates in India, an average Indian couple in the 1960s needed to give birth to 6.3 children in order to be 95% sure of having one surviving son when the father reached the age of 65. In most regions of the less–developed world, crude death rates have lately dropped dramatically and much more rapidly than they did in the developed countries during their early industrialization. Furthermore, the reduction has to a much larger degree been concentrated to reductions in child mortality (World Bank, 1984; Schultz, 1981), highlighting the importance of an understanding of possible behavioral links between child mortality and fertility.

The fertility decision as such is highly simplified in our models. First, we assume that fertility is fully controllable[13]. Second, we assume that the number of children is continuous as do most other formal models of fertility decisions. Third, we assume that the number of children is a one–time decision. We thus ignore issues related to the spacing of children. As fertility choices are sequential by nature, for instance making it possible to replace a lost child, this simplification is an obvious shortcoming of the analysis, especially in the analysis of child mortality.

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[13] There is a common belief that observed differences in the populations of developing countries are largely due to deliberate choice on the part of the individual/couple (Lipton, 1983). It is also easily established that the fertility rates of developing countries are far below the biological maximum (Leibenstein, 1975). The fact that the reduction in fertility in the industrialized countries was achieved without presence of family planning programs serves as one evidence that traditional methods of family planning are effective.

4. Description of the Chapters

Above we have listed the some of assumptions and limitations of the models in the thesis. We will now give a brief summary of the purposes and results of each chapter.

Chapter 1: Endogenous Fertility and the Effects of Foreign Aid

The purpose of Chapter 1 is to develop a model which can be used to analyze the implications of the old–age security and producer goods motives for the relationships between size of landholdings, fertility and productivity and to investigate how fertility behavior affects living standards within families over time. By means of an overlapping generations framework and an assumption that the absence of a labor market makes children stay on the family farm, the model determines endogenously the development of fertility, population density, total farm output and per capita output over time. The model also enables us to distinguish between the immediate and stationary–state effects (short– and long–run effects) of government policy.

If per capita landholdings are initially greater than in steady state, the model predicts growth in both total output and population combined with declining per capita output. Within families, this process implies a gradual reduction in living standard over time. As individuals get poorer, in terms of income and inheritable assets, they can afford less children, which thus leads to a reduction in family size over time up to the point where the inherited amount of land stays the same generation after generation (stationary state).

At a given moment in time, families that are comparatively more wealthy in terms of the inheritable asset (hence, that belong to a dynasty which at the outset owned more land), are shown to have a larger number of children, higher consumption in adult and old age, as well as higher labor productivity and, hence, lower productivity per unit of land.

Several policy experiments are considered. It is shown that an increase in agricultural productivity, a decrease in the cost of raising children, and a transfer to the active population will increase the fertility rate in the short run, while a transfer to the old will decrease it. It is also found that permanent increases in agricultural productivity will leave stationary–state production and per capita consumption unchanged. The effects of such shifts are limited to an increase in population density. Permanent reductions in the cost of raising children and permanent increases in transfers to the active population will reduce per capita consumption in the new stationary state. Furthermore, such interventions reduce the share of consumption provided by own production and increase population density. Permanent increases in transfers to the old will, on the other hand, increase per capita consumption and reduce population density in the new stationary state.

The model thus indicates that certain types of development policies in rural areas in developing countries might cause an increase in fertility in the short run, an increase in population density in the long run as well as a decline in per capita consumption and in the share of consumption provided by own production in the new stationary state.

Chapter 2: Demographic Response to Agricultural Reform – Long–Run Effects on Consumption, Wages and Population Density

One simplification in the previous chapter was the assumption that no labor market exists. In this chapter we introduce a labor market and a different type of sharing rule within the family. To keep the model manageable, we have to restrict ourselves to a very simplified characterization of production relations in the agrarian sector. Following the discussion in Ghose (1983), we divide our agricultural sector in two clearly distinguishable agricultural classes – one comprising those who work on the land (the "poor"), and the other comprising those who do not work on the land but who receive a share in its produce (the "landlords"). The poor receive income from a combination of self–cultivation of own land and wage labor. The landlords, on the other hand, seek to maximize their share of the produce of the land, which they receive in the form of profit. We further assume that the choice of number of children by poor peasants is made rationally governed by the cost of raising children, the need of family labor on the family farm and what can be earned on the labor market, if family labor is partly allocated to that market. Within this model, the purpose is to study the effects of the introduction of land–augmenting "green revolution" technology and of a redistributing land reform.

It is shown how a redistributing land reform in favor of poor peasants raises life consumption in the new stationary state. Furthermore, such a reform increases the market wage while the population density is reduced. We also show how the effects of the land–augmenting green revolution may depend on its distributional characteristics. In short, a land–augmenting green revolution favoring rich landlords will induce growing population pressure, decreased welfare of the poor as well as a reduced market wage in the new stationary state. The direction of the causality in our model is opposite from what has usually been proposed, where "the green revolution can be considered an induced innovation in response to growing population pressure on land, without which both the wage rate and income level of the rural working population would have degenerated further" (Hayami, 1981, p.176).

If the aim is to increase the standards of living of the poor, the policy implications in our model are: In economies where large tracts of land are under the control of few individuals, make a redistribution of land titles a precondition for the introduction of land–augmenting technological assistance to the agricultural sector. If a redistribution of land titles is politically impossible to propose, target the assistance to the poor. If this is not possible, refrain from any assistance or the assistance will just cause further impoverishment.

Chapter 3: Consequences of Permanent Changes in Child Mortality on Parental Welfare and Population Density

In the first two chapters of this thesis there was no uncertainty in the rate of return of the investment in children. In this last chapter we introduce the possibility that children might die. The purpose of the chapter is to analyze the effects of a change in child mortality on parental welfare not only for the generation alive at the time when the change occurs, but also on the parental welfare of parents not yet born, in a rural economy of the type briefly described above. In the first part of the chapter we assume that a known and certain share of the children die before they reach adult age, and we choose to use a more general formulation of the utility function compared with earlier chapters.

It is shown that in the short run a decrease in child mortality will always increase parental welfare. The direction of the change in fertility choice will, however, depend on the relative risk aversion. If the relative risk aversion is higher than unity, a decrease in child mortality will cause a decrease in fertility. The opposite is true for a parent having a relative risk aversion lower than unity.

Next, we introduce the dynamics that arise from the fixed amount of land in combination with the inheritance rule. As an indirect effect, a decline in child mortality will make each individual inherit a smaller lot. We show that, independently of the degree of risk aversion, parental welfare will always decrease in the new stationary state as a consequence of a reduction in child mortality. This is explained by the fact that, even if a decrease in child mortality causes fertility to decline, it does not do so to fully compensate for the increase in the number of surviving children. In the long run, individuals who survive childhood will be worse off in the new stationary state as compared with individuals in the old stationary state, even though the "price" of a surviving child is lower. The fact that each individual will receive a lower wage and own a smaller amount of land in the new stationary state dominates over the welfare–increasing price effect.

The analysis further shows that fertility will eventually decline as a result from a reduction in child mortality, independently of the fertility response in the short run. This response is not only due to the fact that a lower number of births is needed to attain the desired family size. The choice of lower fertility is also caused by the fact that parents are poorer in an absolute sense.

One weakness in the analysis above is that a parent knows with full certainty how large fraction of his children that will survive to adult age. It is obvious that uncertainty aspects are not considered in such a set–up. In the second part of the chapter, we incorporate uncertainty in the model by assuming that the number of surviving children is stochastic: two outcomes are possible; that all children survive or that a certain share of the children die. A simplified interpretation would be that in each period of time the society faces the risk of being hit by an epidemic disease or a drought, and that if it is hit, a certain share of the children will die. It is shown that the effect on fertility choice in the short run and on population density in the long run (with a more restricted formulation of the utility function) stays qualitatively the same as in the model of Section 2.

In sum, we have formally shown one possible explanation of the covariation between improvements in child survival rates, lagged reductions in fertility, increased poverty and increased population density that is observed in some areas in developing countries.

5. Empirical Findings

All formal modeling necessitates a number of simplifications, and it is obvious that one has to be very careful before one can argue that a specific model explains the behavior of a certain society. This is especially important when the aim is to explain such complex decisions as fertility decisions. In this thesis we analyze a very special type of economy. In comparing with empirical findings it is obvious that such comparisons should only be made with findings from societies that resemble the model economy in a crude and acceptable way. At the end of Chapter 1, we argue that parts of rural Bangladesh[14] have some, but far from perfect, resemblance with the formal structure of the model. For instance, contrary to our assumptions in Chapter 1, it is evident that land, labor, and credits markets do exist in Bangladesh. However, we argue that various transaction costs make these markets far from perfect. With this qualification in mind we compare earlier empirical findings with the

[14] See footnote 33 in Chapter 1 for a detailed description of the arguments behind this conclusion.

results of our models and find quite a bit of consistency. This is obviously no proof of the accuracy of the model in describing the reality of rural Bangladesh, but rather an indication that our model might reveal mechanisms that are important in fertility decisions. Empirical testing of the results of the models remains to be done.

We will not list earlier empirical findings that are referenced in the different chapters here, but instead discuss somewhat speculatively, in the light of our analysis, an important observation that has had a great influence in the population debate and in policy decisions.

a. Income and Fertility

A statistical declining relationship between the fertility rate and national income per head has been observed both in time series and in statistical cross-country studies involving a large number of developing countries (Birdsall, 1988).

However, in studies of fertility patterns within the same socio-economic groups in developing countries, hence within groups that maybe can be believed to share the same costs and benefits of children, a positive relationship between income and fertility is generally found (Birdsall, 1988)[15]. Such a relationship has also been verified for Bangladesh by Muqtada and Alam (1985) and their explanation is: "The reasons for this are largely speculative and are often traced to socio-anthropological factors. For example, there is a well-known contention that fertility rates have a positive relationship with the general levels of living and 'prosperity of the household'" (Muqtada and Alam, 1985, p.25). In this thesis we have given a formal example of the mechanism proposed by Muqtada and Alam (1985). We have shown that in rural societies, where the investment and producer goods motive could be assumed to be strong, families that belong to rich family lines will have

[15] This statistical relationship is discussed surprisingly little in the literature. One reason might be that such a relationship could appear inconsistent with the statistical declining relationship that is found in cross-country studies and in time series for individual countries, as well as in some studies on differences in fertility between socio-economic groups within nations.

more children as compared to poorer ones, at a certain moment in time. Furthermore, we have shown that income–raising transfers could raise fertility in the short run, if it is directed towards children or adults.

The declining relationship between the fertility rate and national income per head in time series and in statistical cross–country studies could maybe be explained by the fact that rising per capita incomes changes a society in a number of ways, some of which may influence the costs and benefits of children, and hence the relative strength of different motives in fertility decisions. For example, development often brings new types of investment possibilities, through which individuals can transfer resources from adult to old age. Furthermore, labor mobility often increases. Hence, the availability of alternatives to children as a means to transfer resources from adult to old age, and as labor, increases while the possibility to enforce loyalty from children decreases. The attractiveness of children as producer goods and as investment for old age thus decreases, possibly leading to reductions in fertility. Under certain circumstances, such forces could in the aggregate show up as a correlation between rising per capita incomes and declining fertility[16].

The fact that there does exist a declining relationship in cross–country studies and in time series, has, however, inspired some observers to conclude that a rise in income will induce people to have fewer children. However, income raising transfers, without contemporary institutional changes that would alter the value of children as an investment, insurance and producer good, are likely to make people raise their fertility in the short run, if we are to believe the positive statistical relationship between income and fertility referred to above. One possible mechanism that could be driving such a response is demonstrated in our models. A reduction in income would, according to the same line of reasoning, lead to reduction fertility.

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[16] Differences in fertility within a nation but between different socio–economic groups, each group facing a different institutional setting with regard to the cost and benefits of children, might be explained with the same type of reasoning.

b. Recent Fertility Declines

Recent evidence suggests that birth rates in the developing world are falling. Some observers present these findings as unexpected and suggest that they dispute the notion that "development is the best contraceptive", a phrase that originated at the 1974 World Population Conference in Bucharest. As an argument they cite the fact that "fertility declined as many developing economies stagnated or lost ground in the 1980s" (Robey, Rutstein and Morris, 1993, p.34). Bangladesh is given as an example: "It is one of the world's poorest and most traditional agrarian countries. Infant mortality is high, women have low social status and most families depend on children for economic security. Nevertheless, fertility rates there declined 21 percent between 1970 and 1991: from seven to 5.5 children per woman." (Robey, Rutstein and Morris, 1993, p.34). According to the authors, the explanation of this decline is to be found in the increased use of contraceptives. During the period, the use of contraception among married women of reproductive age rose from 3 to 40 percent.

In our models we show how fertility will decrease over time, provided we start out from a situation where landholdings are greater than in stationary state. The driving force behind this pattern is the fact that, as time passes, people are getting poorer in an absolute sense ("a poverty–led demographic transition"). The fact that "fertility declined as many developing economies stagnated or lost ground in the 1980s" is thus not surprising in our framework. An increase in demand for contraceptives is the logical response to a decrease in the demand for children and must not necessarily be attributed to successful persuasion through effective marketing of family planning.

6. Policy

The overwhelming part of the literature in welfare economic theory and economics of poverty takes future numbers as given (World Bank, 1990; Dasgupta, 1993). But government policies can affect the size of future generations. Policies might also affect different generations in different directions. The thesis presents a number of examples of both these observations. The policy experiments share the property that the policies affect not only the welfare and fertility of the generation alive at the time of introduction of the policy but also the welfare and fertility of future generations. It is, however, much too early to draw any conclusions for real world policy from the policy experiments performed here. The models are too simplified in their structure or too restrictive in their parametric specifications[17], and they only highlight a few of all the possible links between fertility choices and the economic environment. But hopefully the study contributes to the understanding of mechanisms that are important in explaining fertility behavior and the development of population, income, wages and population density over time.

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Chapter 1

ENDOGENOUS FERTILITY AND THE EFFECTS OF FOREIGN AID

1. Introduction

Old–age security is frequently reported to be an important motive in fertility decisions in rural areas, particularly in agricultural subsistence economies (Nugent, 1985; Kagitcibasi, 1982). In such societies, grown–up children are considered valuable not only as providers of old–age care to their parents but also as labor on the family farm (Pearce and Warford, 1991; Dasgupta and Mäler, 1990; Dasgupta, 1993).

Given such a setting, the purpose of this paper is to develop a model which can be used to analyze the implications of the old–age security and producer goods motives in fertility decisions for the relationships between size of landholdings, fertility and productivity and to investigate how fertility behaviour affects consumption and parental welfare over time[1]. We also examine how common policy measures, intended to achieve goals other than fertility reduction, affect fertility, per capita consumption, parental welfare and population density in societies where the old–age security and producer goods motives are strong. An understanding of such relations is highly important, in cases where policy makers find that population growth in itself has negative externalities[2], or where policy intervention is aimed towards specific per capita goals.

These questions cannot be analyzed in earlier models of the old–age security motive in fertility decisions (for example, Neher, 1971; Willis, 1980; Hammer, 1986; Nerlove, Razin and Sadka, 1987). They are typically one–factor models without any link between the

[1] In this paper parental welfare is defined as the weighted sum of utility in adult and old age.

[2] Population growth is widely regarded as the dominant cause of environmental deterioration and overrapid use of resources in developing countries (World Bank, 1990; Pearce and Warford, 1991).

fertility decisions of consecutive generations. By means of an overlapping generations framework, and by assuming two factors of production – land and labor – in combination with a partible inheritance rule for land[3], our model determines endogenously the development of fertility, population density, total farm output and per capita output over time. This also enables us to distinguish between the short– and long–run effects of government policy[4].

The model developed here predicts growth in both total output and population, combined with declining per capita output, if per capita landholdings are initially greater than in steady state. Population growth will diminish over time, however, and eventually reach a level where each generation only reproduces itself (a stationary state). Within dynasties, this process implies a gradual reduction in per capita consumption over time. Households that at a certain moment in time are comparatively more wealthy in terms of land are shown to have a larger number of children, higher consumption in adult and old age, as well as higher labor productivity, and, hence, lower productivity per unit of land.

Several policy experiments are considered. For example, an increase in agricultural productivity, a decrease in the cost of raising children, and a transfer to the active population will increase the fertility rate in the short run, while a transfer to the old will decrease it. It is also found that permanent increases in productivity will leave stationary–state production and per capita consumption unchanged. The effects of such shifts are limited to an increase in population density. Permanent reductions in the cost of raising children and permanent increases in transfers to the active population will reduce per capita consumption in the new stationary state. Furthermore, such interventions

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[3] In a system of partible inheritance, land is divided equally among children upon their parents' death. The practice of partible inheritance is typical of South Asia and is becoming typical of Africa along with the emergence or strengthening of individual land ownership rights (Cain and McNicoll, 1988).

[4] Only the effects on current adults, at the point in time when a policy change is introduced, are included in the discussion of short–run effects. Long run is defined as the effects on stationary–state variables, and thus on the size and welfare of future generations.

reduce the share of consumption provided by own production and increases population density. Permanent increases in transfers to the old will, on the other hand, increase per capita consumption and reduce population density in the new stationary state.

The model is built on a number of assumptions concerning the structure of the economy as well as specified choices of functional forms. This implies that it is obviously too early to draw any policy recommendations from the analysis. The predictions of the model are, however, compared with empirical evidence mainly from Bangladesh, but also from other societies where the old–age security and producer goods motives are likely to prevail. Consistencies found between the formal results and empirical findings should, however, not be regarded as proofs of the accuracy of the model in describing the rural economies concerned, but rather as indications that the model might reveal some mechanism that could be present in fertility decisions in these societies.

The paper is organized as follows. The next section focuses on the conditions under which the old–age security motive as well as the producer goods motive may be important in fertility decisions in rural areas in developing countries. The model is outlined in Section 3. Section 4 is devoted to a partial equilibrium analysis of the household's decision problem and Section 5 to the dynamics of the model. The effects of different types of foreign aid are examined in Section 6. The theoretical predictions are compared with some empirical findings in Section 7. Section 8 contains concluding comments and a brief discussion of ways in which fertility behaviour could be changed.

2. Institutional Setting Favoring Children as an Old–Age Security Arrangement

The problem of providing for consumption during old age and the characteristics of a society where children serve this purpose may be explored within the overlapping generations framework introduced by Samuelson (1958). In this model it can be demonstrated how the equilibrium intertemporal allocation of resources can be Pareto inefficient when individuals belong to different generations. Samuelson shows that the existence of a financial asset such as money or bonds can serve as a store of value and make it possible to achieve Pareto–improving smoothing of consumption. A socialized tax and transfer system such as social security and a system of private inter–generational transfers within families may also improve efficiency. A system of private inter–generational transfers of goods within families, however, requires altruism on the part of children towards parents or the existence of some type of social contract that motivates children to transfer consumption goods to their old parents. Hence, the attractiveness of children as a device for smoothing consumption depends on the availability of alternatives to having children as well as on possibilities of enforcing loyalty from children.

Social security, private pension policies and markets for assets that help individuals to save today in order to spend tomorrow are virtually non–existent in rural areas in developing countries (Nugent and Gillaspy, 1983; Wildasin, 1983; Nugent, 1985)[5] [6]. In many such areas, the only available asset is land. Markets for land are, however, usually extremely thin (Nugent, 1985)[7]. Furthermore, land may require family labor to make it a secure and productive asset (Nugent, 1985; Wildasin, 1983; World Bank, 1984). First, children might be needed to safeguard ownership as such (Adnan, 1988). Second, land

[5] Several characteristic features of rural areas tend to undermine private and social old–age insurance. High information and transaction costs give rise to problems of adverse selection and moral hazard, thereby limiting supply (Rosenzweig, Binswanger and McIntire, 1987). On the demand side, reliability regarding the duration of old–age insurance is often undermined when governments break their promises to rural residents (Nugent, 1985).

[6] Currency and bank deposits, where available, are bad stores of value because of their vulnerability to inflation. Gold is subject to price uncertainty and theft. Commodities and livestock cannot be stored for long periods of time or are storable only at high cost. Dwellings and other structures can be destroyed by floods; they can also be subject to incompletely defined property rights or arbitrary changes in such rights over time (Nugent, 1985).

[7] Supply is limited due to the scarcity of profitable nonagricultural investment opportunities. Furthermore, the collateral value of land in societies where insurance is unavailable is such that the equilibrium price of land exceeds the present discounted flow produced from the land. Once land is lost, it cannot be rebought solely with borrowed funds, thereby limiting both supply and demand. These circumstances combined imply that sales are few and take place predominantly in bad crop years in the form of distress sales (Rosenzweig, Binswanger and McIntire, 1987).

requires labor to make it a productive asset. There are several reasons why the cost, quality and availability of family labor, e.g. children, might differ from that of hired labor, even if an extensive labor market does exist.

While the physical capacity of a worker may be assessed by inspection, his effort is not easily monitored. Since family members have a long–term relationship, deviations in terms of effort can be detected more easily and punished more effectively (Stiglitz, 1988). It is also evident that hiring and search costs for family labor are minimal (Pollak, 1985; Rosenzweig, Binswanger and McIntire, 1987). High positive covariance in the timing of agricultural operations across farms, and the importance of timeliness in these operations, also make it risky to rely solely on hired laborers. The productivity of small farms which use simple technology often depends critically on intimate familiarity with a particular plot (Wildasin, 1983). Knowledge of the specifics of the land owned by a family is more easily transferred to members of the same household. As family managers expect a continuing relationship with the farm, they are also less tempted to sacrifice long–run advantages for short–run gains (Pollak, 1985).

Apart from the need for agricultural labor of different qualities, a retired farmer requires old–age care. Satisfying the daily needs of the elderly in rural areas of developing countries is time consuming and laborious (Vatuk, 1980). This task can sometimes be performed by servants, but insecurity about their efficiency and loyalty often necessitates supervision that the elderly cannot provide. Preference that one's own children perform these tasks may also exist, thus reinforcing the demand for family labor.

However, there is no reason to invest in raising children for the purpose of old–age support if retired parents cannot be ensured of the loyalty of their children[8]. If we assume

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[8] This point alludes to a vast literature on the existence or non–existence of altruism, its characteristics and direction. In most models where altruism is assumed, parents are concerned about their children. But such an assumption does not imply that children are loyal to their parents. This can only be assured by assuming altruism in the direction from children to parents or by constructing some type of social contract that is self–enforcing or such whereby parents can enforce loyality from their children.

that individuals are selfish, both as parents[9] and as children, a landowning farmer needs some means of ensuring that his children will stay with him during his old age, till his land, hand over at least some of the proceeds and give him old–age care. One way this could be ensured is through some form of agreement or mutual understanding whereby labor in adult years is exchanged for a wage plus the right to inherit a certain amount of land upon the death of the parent[10]. If a child leaves the family farm when he reaches adult age, he will not inherit[11]. This agreement could be made between parents and one or several of their children. In a partible inheritance system, all children will inherit when a parent dies and thus have inheritance incentives to stay and provide labor on the family farm. In the case of an impartible system[12], only the favored child would have the inheritance incentive to stay, albeit a stronger incentive than he/she would have in a society with a partible system. Consequently, in comparing these two inheritance rules, a partible system could be expected to create a relatively greater demand for children[13].

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[9] Empirical studies from some developing countries claim that in situations of pressing poverty, parents have to discount their children's welfare extensively (Ahmad, 1984; Parson and Goldin, 1989). Ishikawa (1975) argues that altruism is income–dependent, and that what may seem like a lack of altruism in some societies, is rather a result of low incomes. People cannot afford to materialize their altruism. On the other hand, what may appear to be intergenerational altruistic transfers in other societies may instead be a result of strategic bequest motives. The predictive value of the altruism model has recently been questioned for the case of developed countries (Altonji, Hayashi and Kotlikoff, 1989).

[10] Such agreements could in principle be arranged with other relatives or persons outside the family lineage. In our model, we assume that such agreements cannot be made with anyone other than one's own children.

[11] The linkage between inheritance and fertility is longstanding among historical demographers (Lee, 1977). Bernheim, Schleifer and Summers (1985) give a theoretical rationale for the strategic bequest motive in general and present econometric evidence which strongly suggests that bequests are often used as compensation for services rendered by beneficiaries. If inheritance cannot be distributed freely by a parent, it obviously loses its enforcement capacity.

[12] In an impartible inheritance system, only one child inherits the land. In the case of primogeniture, the firstborn is the inheritor.

[13] In comparing regions with different inheritance rules in northern Europe, it was found that the existence of a partible inheritance rule limited rural emigration, increased rural overpopulation and increased the number, but decreased the size, of farms. Areas with impartible inheritance rules were, comparatively, incorporated much more easily into the

If the expected return to a job outside the family farm exceeds the expected value of the inheritance and the wage on the family farm, the value of the inheritance as a leverage on children would be smaller[14]. The importance of the old–age security motive in determining fertility patterns could therefore be expected to decline in fast developing segments of an economy with growing labor markets or where possibilities of migrating to outside labor markets open up.[15]

In sum, the old–age security and producer goods motives in fertility decisions could be expected to be strong in rural societies characterized by the following conditions:

(i) no social or private security system, or low coverage and level, or low confidence in the duration of existing programs;

(ii) underdeveloped markets for credit, land and other storable assets;

(iii) partible inheritance systems with private ownership of the inheritable asset;

(iv) slow expected changes in the growth of real income per capita;

(v) underdeveloped labor and service markets.

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industrializing sector (Habakkukk, 1955). Exactly the same patterns have been reported from historical Russia (Shanin, 1972). Hanley (1977) has studied the effects of a change in inheritance rules in seventeenth century Japan. In order to generate the maximum amount of tax income from agriculture and avoid the parcelling of land to the point where the tax rate would have to be lowered to permit mere subsistence, local administrators prohibited the subdividing of land below certain limits. This implied that, in the case of small farmers, only one son could inherit. This policy resulted in smaller families and slower growth in the number of households.

[14] Some value still exists if wages earned cannot be saved and where employment is unlikely in old age.

[15] In a study of rural fertility in nineteenth century England, Williamson (1985) found the growing labor market, which gave rise to an increase in rural out–migration, combined with a weak capital market, to be important explanations for the decline in fertility. This was also found by Sundstrom and David (1988) in antebellum America. Sutch (1991) argues that the decline in fertility in North America was caused by a failure of the family–based income–transfer mechanism. That failure was induced by the opening of western lands to settlements at the beginning of the nineteenth century. In micro level studies from contemporary Nepal, Goldstein and Bell (1981) have found that the growing industrial labor market in India makes children less safe as old–age insurance and that couples have begun to prefer smaller families.

3. The Model

The conditions discussed above are used as a starting point in formulating our model. Hence, we assume a rural economy where land is privately owned and where a partible inheritance rule is in force (condition (iii)). Children are assumed to be the only means of saving for old age; there are no alternative markets for assets or social security (conditions (i) and (ii)). Although we have discussed that there might be a need for a social contract in order to ensure loyalty from children, we do not explicitly model such a contract. Instead we study an economy in which the absence of a labor market makes children stay on the family farm (conditions (iv) and (v)). The only motivation for childbearing stems from the expectation that children will provide labor on the family farm. We thus suppress all other motives for having children in order to highlight the implications of old–age security and producer goods motives for fertility choice.

Each individual lives for three periods: as an improductive child; as a productive adult; and as retired and improductive in old age. At each point in time the extended family consists of three generations[16], and it owns and cultivates a given share of land. The property rights to the land are in the hands of the elderly[17]. When the retired person dies, the land is shared equally among the children, who in turn enter the period of old age. Our hypothetical peasant society is thus made up of self–sufficient, extended families that produce one non–storable, subsistence good (for instance food) with inputs of labor and land. Fertility decisions are taken at the beginning of the adult period[18]. The adults are paid

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[16] We regard all individuals as male–female pairs and we ignore mortality between the three periods.

[17] Caldwell (1978) outlines mechanisms by which patriarchs maintain control over children through monopoly on ownership of land. Cain (1982) finds the description accurate for South Asia.

[18] Fertility is thus assumed to be fully controllable. It is commonly believed that observed differences in the populations of the developing world are largely due to deliberate choice (Lipton 1983). It is also easily established that the fertility rates of developing countries are far below their biological maximum (Leibenstein 1975). There also seems to be overall agreement that most historical populations controlled their fertility either through age at marriage (more frequently) or by marital fertility control

a wage equal to their marginal product on the family farm. Parents provide their children with an exogenously determined amount of food that will ensure their survival to adult age. The elderly consume an amount equal to total output minus costs for family labor.

Let us assume that food (Y_t) is produced according to the following production function:

$$Y_t = F(A_t, N_t) \tag{1}$$

where Y_t is the aggregate production of food on the family farm at time t, A_t is the amount of land owned by the elderly at time t (born at t–2) and N_t is the number of adults at time t (born at t–1)[20]. According to the partible inheritance rule, the family–farm production function of the next generation will have the following form:

$$Y_{t+1} = F(\frac{A_t}{N_t}, N_{t+1}) \tag{2}$$

since

$$A_{t+1} = \frac{A_t}{N_t} \tag{3}$$

The individuals are selfish in the sense that only their own consumption is included

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(Lee 1977).

[19] If children were not a net cost to their parents during the period in which it is assumed that parents fully control the benefits of their children's productive activities, the maximum possible number of children, i.e., natural fertility, would be the optimal choice, as strongly advocated by Caldwell (1982). Any differential in fertility would then have to be explained by differentials in fecundity. Most studies on the subject seem to conclude that during their childhood, children are a net cost to their parents in peasant agriculture (Willis, 1980; Ahmad, 1984; Mueller, 1976; Cain, 1982). Our qualitative conclusions would not be affected by including child labor, provided the net cost of children remains positive.

[20] For simplicity, we assume that N is a continuous variable.

in their utility function[21]. The utility function for an adult born at t–1 is defined over consumption in period t and t+1:

$$U = U(c_t, c_{t+1}) \tag{4}$$

His consumption level in adulthood c_t is determined by the wage earned w_t, the number of children N_{t+1} he chooses to have and the exogenously determined level of consumption of each of his children c_0:[22]

$$c_t = w_t - N_{t+1} c_0 \tag{5}$$

The wage is set at the marginal product of labor on the farm:

$$w_t = F_N(A_t, N_t) \tag{6}$$

After having paid his children their wages, the individual in his old age, born at t–1, will consume the left–over production from his land. Assuming constant returns to scale, the budget constraint for the old–age period will have the following form:

$$c_{t+1} = A_{t+1} F_A(A_{t+1}, N_{t+1}) \tag{7}$$

where c_{t+1} is own consumption during old age, A_{t+1} the amount of land inherited and N_{t+1} the total number of children this individual decided to have in the preceding period.

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[21] In terms of utility the household is not extended. An adult individual does not include the utility of his parents, siblings or children in his utility function.

[22] We thus ignore the opportunity costs in terms of food production foregone by adult time devoted to child care. All adult persons work one unit of time regardless of how many children they have.

If we assume that the utility function is additively separable in consumption at t and t+1, the maximization problem for an individual born at time t–1 is:

$$\max U(c_t) + \delta U(c_{t+1}) \qquad (8)$$
$$\text{s.t. } (5), (6), \text{and } (7)$$

where δ is his subjective rate of time preference. The first–order condition to this problem is:

$$\frac{U'(c_t)}{U'(c_{t+1})} = \frac{\delta A_{t+1} F_{AN}(A_{t+1}, N_{t+1})}{c_0} \qquad (9)$$

According to (9), the optimal level of fertility occurs at the point where the marginal rate of substitution between consumption in the two periods is equal to the marginal rate of return on children. The right–hand side of (9) can be expressed as – (1+r), where r is the internal rate of return on children. It is not necessary for r to be positive in order to motivate positive fertility. As long as r > –1, there is a case for having children, provided there are no other available means of transferring resources from period t to period t+1[23].

In order to simplify the subsequent analysis, we will impose more structure on the utility and production functions. We assume that utility is represented by a loglinear function and that the production function is Cobb–Douglas:

$$F(A_t, N_t) = A_t^{\alpha} N_t^{1-\alpha} \qquad (10)$$

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[23] Wildasin (1983) indicates that a negative internal rate of return on children could be used as evidence against the old–age security hypothesis. According to expression (9), however, a negative rate of return on children is fully compatible with the old–age security hypothesis, where there are no alternatives with better returns.

$$(c_i) = \ln c_i \qquad i = t, t+1 \tag{11}$$

This enables us to rewrite the FOC (9) as:

$$\frac{c_{t+1}}{c_t} = \frac{\delta A_{t+1}(1-\alpha)\alpha A_{t+1}^{\alpha-1} N_{t+1}^{-\alpha}}{c_0} \tag{12}$$

After some manipulating, we get:

$$N_{t+1} = \frac{\delta(1-\alpha)^2 A_t^{\alpha} N_t^{-\alpha}}{c_0} - \delta(1-\alpha)N_{t+1} \tag{13}$$

From the inheritance rule it follows that:

$$A_t^{\alpha} N_t^{-\alpha} = A_{t+1}^{\alpha} \tag{14}$$

We then obtain the following expression, which relates the demand for children to the cost of children and the size of the landholding inheritance:

$$N_{t+1} = \frac{\delta(1-\alpha)^2}{[1+\delta(1-\alpha)]c_0} A_{t+1}^{\alpha} \tag{15}$$

In the subsequent analysis we use B as shorthand for the coefficient on the right–hand side of (15), i.e.:

$$B \equiv \frac{\delta(1-\alpha)^2}{[1+\delta(1-\alpha)]c_0} \tag{16}$$

4. Partial Results

In order to compare the fertility decisions of households that own different amounts of land[24], we differentiate N_{t+1} with respect to A_{t+1} in equation (15) and find:

$$\frac{dN_{t+1}}{dA_{t+1}} = \alpha B A_{t+1}^{\alpha-1} > 0 \tag{17}$$

$$\frac{d^2N_{t+1}}{dA_{t+1}{}^2} = \alpha(\alpha-1)B(A_{t+1})^{\alpha-2} < 0 \tag{18}$$

Labor and land productivity will also differ. Using the relationship $N_t=\phi(A_t)$, defined in (15) above, we find:

$$d(Y_t/N_t)/dA_t = \frac{d}{dA}\left[\frac{F[A,\phi(A)]}{\phi(A)}\right] = \frac{F_A}{N}(1-\alpha) > 0 \tag{19}$$

$$d(Y_t/A_t)/dA_t = \frac{d}{dA}\left[\frac{F[A,\phi(A)]}{A}\right] = (\alpha-1)BA^{\alpha}\frac{F_N}{A^2} < 0 \tag{20}$$

Hence, according to our model, households that are relatively richer in terms of land will have higher productivity per unit of labor. Higher average productivity per unit of labor implies a higher marginal product and hence higher wages and adult consumption. As seen in (7), rich households will also have higher old–age consumption. We have thus found:

Proposition 1 (P1). The more land a farmer owns, the more children he will decide to have, but at a decreasing rate. Households that are relatively richer in terms of land will have more land per working member of the household, higher productivity per unit of labor and higher consumption both in adult and old age. However, productivity per unit

[24] We are thus comparing households that belong to dynasties that at some point in time were endowed with different amounts of land.

of land will obviously be comparatively lower in richer households[25].

5. Dynamics

The dynamics of the system is determined by the fact that the total amount of land that the household owns is fixed in combination with the stipulated inheritance rule (3). Taking into account the FOC for the fertility decision, this gives the following relationship between the size of the landholding per household at time t+2 and the size of the landholding per household at time t+1:

$$A_{t+2} = \frac{1}{B} A_{t+1}^{1-\alpha} \tag{21}$$

The differential of A_{t+2} with respect to A_{t+1} is given by:

$$\frac{dA_{t+2}}{dA_{t+1}} = \frac{1}{B}(1-\alpha)A_{t+1}^{-\alpha} > 0 \tag{22}$$

The stationary–state level of landholding size per household $\hat{A}$ can be determined from (21) by setting $\hat{A}=A_{t+2}=A_{t+1}$:

$$\hat{A} = \left[\frac{1}{B}\right]^{\frac{1}{\alpha}} \tag{23}$$

From the fact that A_{t+2} is a monotone increasing function of A_{t+1}, and that

$$\frac{d^2A_{t+2}}{dA_{t+1}^2} < 0, \quad \lim_{A_{t+1}\to 0} \frac{dA_{t+2}}{dA_{t+1}} = +\infty, \text{ and } \lim_{A_{t+1}\to\infty} \frac{dA_{t+2}}{dA_{t+1}} = 0,$$

[25] This result stems from utility maximization behaviour. Richer households raise their adult consumption by having proportionally less children and hence less labor per acre in old age. Even though we have assumed CRS in production, our result thus shows decreasing returns to scale.

it follows that the stationary state is stable.

At the stationary–state value of landholding per household $\hat{A}$, the level of fertility is at the full reproduction rate[26], hence, the population reproduces itself. At landholdings above $\hat{A}$, population grows since the optimal choice of the number of children is above one. This implies that the farm size of each household becomes smaller over time. As seen from (23), output also grows above $\hat{A}$. But total output grows at a lower rate than the population. If households are identical, except for the initial endowment of land, these relationships imply that the distribution of land will be equalized over time. They also imply declining living standards within families over time. At landholdings below $\hat{A}$, the optimal fertility choice will be below one. In a two–sex setting, the choice of a single child constitutes one such example. This fertility choice would imply upward mobility in living standards over time. In sum, we have thus found:

Proposition 2 (P2). The stationary–state size of landholding per household $\hat{A}$ depends on the technological parameter α, the cost of bringing up children and the subjective rate of time preferences. Above $\hat{A}$, we will witness a growing population over time combined with declining per capita output. Below $\hat{A}$, the opposite will hold.

6. Consequences of Foreign Aid

Substantial amounts of foreign aid have been transferred to developing countries since the 1950s. In some recipient countries, subsistence societies comprise a segment of the national economy. In some of these societies, particularly those that share the characteristics identified in Section 2, the old–age security and producer goods motives can be expected to be strong in fertility decisions. We now turn to the effects on fertility, population density and parental welfare of different types of transfer policies in our stylized

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[26] To see this, insert (23) in (15). In the stationary state, the one–sex household of our model will consist of three persons: one child, one adult and one old person.

economy[27]. As we assume that the policies are financed from abroad, they only have a direct effect on the budget constraints of households reached by the policies.

Transfers to children, for example in the form of supplementary feeding programs or through primary health programs for children, are common in the Third World. In our model, a transfer program directed towards children could be captured as a reduction in the cost of raising children c_0. The effect of a temporary cut in costs of raising children can be found by differentiating equation (15) with respect to the cost of raising children:

$$\left.\frac{dN_{t+1}}{dc_0}\right|_{A_{t+1}=\text{const}} = -B\frac{1}{c_0}(A_{t+1})^{\alpha} < 0 \tag{24}$$

If the cut in costs is permanent, the effects are given by[28]:

$$\frac{d\hat{A}}{dc_0} = \frac{1}{\alpha}\left[\frac{1+\delta(1-\alpha)}{\delta(1-\alpha)^2}\right]^{\frac{1}{\alpha}} c_0^{\frac{1}{\alpha}-1} > 0 \tag{25}$$

$$\frac{d\hat{c}_1}{dc_0} = \frac{1}{\delta(1-\alpha)} > 0 \tag{26}$$

[27] We are here restricting ourselves to an analysis of the effects of relatively small changes in different parameters on parental welfare, a restricted form of the standard of living, the most relevant measure in this context. By looking at this measure we have obviously made a choice of one of many possible social objectives and by looking at small changes we avoid a discussion of optimal population. We are thus bypassing the recent literature on conflicts between different social objectives when population is not fixed and the value judgement involved in evaluating an additional hypothetical person. For a discussion of these and related issues see Dasgupta (1993) and Starett (1991).

[28] The stationary–state consumption in adult age, $\hat{c}_1$, the stationary–state consumption in old age, $\hat{c}_2$, and the production on each farm in the stationary state, $\hat{Y}$, are calculated by inserting (28) and $\hat{N}=1$ in (5), (8) and (11) respectively, utilizing the specific forms of the production and utility functions.

$$\frac{d\hat{c}_2}{dc_0} = \frac{\alpha[1+\delta(1-\alpha)]}{\delta(1-\alpha)^2} > 0 \tag{27}$$

$$\frac{d\hat{Y}}{dc_0} = \frac{1}{\delta(1-\alpha)^2} + 1 > 0 \tag{28}$$

In sum, we have thus shown:

Proposition 3 (P3). At any given A, an exogenously determined decrease in the cost of raising children will increase the number of children per adult. If such an intervention permanently reduces the cost of raising children, it will lower the stationary–state size of landholding per capita and thus increase population density. It will also reduce stationary–state consumption in both adult and old age, thus implying a lower parental welfare, once the new stationary state is reached. Production per capita will also be lower in the new stationary state.

Transfers to the adult generation may, for example, take the form of food rations or food–for–work programs[29]. In our model, we introduce such exogenous transfers by adding a parameter z_t to the budget constraint of the adult generation:

$$c_t = w_t - N_{t+1}c_0 + z_t \tag{29}$$

This gives us a modified demand equation:

$$N_{t+1} = BA^{\alpha}_{t+1} + \frac{z_t B}{1-\alpha} \tag{30}$$

Differentiating this equation with respect to z_t yields:

$$\left.\frac{dN_{t+1}}{dz_t}\right|_{A_{t+1}=\text{const}} = \frac{B}{1-\alpha} > 0 \tag{31}$$

[29] We assume that such programs do not influence the labor supply on the family farm.

Provided the transfer to the adults is permanent, we find[30]:

$$\frac{d\hat{A}}{dz} = -\frac{1}{\alpha}\left[\frac{1}{1-\alpha}\right]^{\frac{1}{\alpha}} z^{\left[\frac{1-\alpha}{\alpha}\right]} < 0 \tag{32}$$

$$\frac{d\hat{c}_1}{dz} = 0 \tag{33}$$

$$\frac{d\hat{c}_2}{dz} = -\frac{\alpha}{1-\alpha} < 0 \tag{34}$$

$$\frac{d\hat{Y}}{dz} = -\frac{1}{1-\alpha} < 0 \tag{35}$$

In sum:

Proposition 4 (P4). At any given A, transfers to the adult generation will increase fertility. Permanent transfers will result in lower per capita landholding and higher population density as well as in a reduction in per capita lifetime consumption in the new stationary state. The dependency on consumption provided from outside will increase. A "crowding–out" of consumption produced from own resources in favor of consumption provided from outside will occur.

Transfers to elderly are not common in foreign aid programs. However, in our model, the introduction of such programs would have a dampening effect on fertility in the short run and a positive effect on parental welfare in the long run, provided the transfers are permanent. In order to have an impact on fertility, the exogenous transfer must be trustfully promised to individuals in their childbearing age. Let us introduce such transfers by adding a parameter q_{t+1} in the budget constraint of the last period of life:

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30 The requirement for a stable stationary state is $z < \frac{1-a}{B}$. This implies that in the most restrictive case, z has to be less than or equal to c_0.

$$c_{t+1} = A_{t+1}F(A_{t+1},N_{t+1}) + q_{t+1} \tag{36}$$

Utilize (8), (10) and (11) and define the utility function:

$$\begin{aligned} Z(N,A,q) = & \ln[(1-\alpha)A^{\alpha}_{t+1} - N_{t+1}c_0] \\ & +\delta\ln[\alpha A^{\alpha}_{t+1}N^{1-\alpha}_{t+1} + q_{t+1}] \end{aligned} \tag{37}$$

Let us assume that an exogenous transfer to the old (q_{t+1}) is trustfully promised in time period t to individuals born in t–1, who thus in t are to decide on their optimal fertility choice, N_{t+1}. Since A_{t+1} is already given and thus unaffected by the promised transfer, $Z_{NN}dN + Z_{Nq}dq = 0$ should hold. According to the SOC, $Z_{NN} < 0$ and for positive values of q, $Z_{Nq} < 0$, hence:

$$\left.\frac{dN_{t+1}}{dq_{t+1}}\right|_{A_{t+1}=\text{const}} = - Z_{Nq}/Z_{NN} < 0 \tag{38}$$

The increase in consumption possibilities in old age is thus partly transferred to adult age through the only means available, a reduction in fertility. Provided the transfers to the elderly are permanent, and that we are investigating the stationary state, which implies that N is constant and that $Z_{NA}dA + Z_{Nq}dq = 0$ must hold, we can infer that[31]:

$$\frac{d\hat{A}}{dq} = - Z_{Nq}/Z_{NA} > 0, \tag{39}$$

implying that consumption is higher in both adult and old age in the new stationary state. Hence:

Proposition 5 (P5). At any given A, transfers to the elderly will decrease fertility.

[31] For positive values of q, $Z_{NA} > 0$.

Permanent transfers will result in lower population density and a higher individual standard of living in the new stationary state.

"Green revolution" technologies began to be implemented in many developing countries in the 1960s. The introduction of better seed and the distribution of pesticides or fertilizers are aimed at increasing productivity in agriculture. In our model, successful intervention of this kind will have an indirect impact on fertility as well as on the stationary–state amount of land per capita. In order to analyze this type of policy intervention, we introduce a Hicks–neutral productivity parameter β_t:

$$Y_t = \beta_t F(A_t, N_t) \tag{40}$$

This extension gives us the following demand equation for children:

$$N_{t+1} = \beta_t B A_{t+1}^{\alpha} \tag{41}$$

Assuming that β_t is increased as from t onwards, and that this increase was not foreseen by individuals that were adults in t–1 (A_{t+1} is not affected by the change), the first–period response to the change in productivity is determined by:

$$\left.\frac{dN_{t+1}}{d\beta_t}\right|_{A_{t+1}=\text{const}} = BA_{t+1} > 0 \tag{42}$$

If the productivity change is permanent, we find:

$$\frac{d\hat{A}}{d\beta} = -\frac{1}{\alpha}\left(\frac{1}{B}\right)^{\frac{1}{\alpha}}\frac{1}{\beta}^{\left(\frac{1-\alpha}{\alpha}\right)} < 0 \tag{43}$$

$$\frac{d\hat{c}_1}{d\beta} = 0 \tag{44}$$

$$\frac{d\hat{c}_2}{d\beta} = 0 \tag{45}$$

$$\frac{d\hat{Y}}{d\beta} = 0 \tag{46}$$

We have thus shown:

Proposition 6 (P6). An exogenous increase in the productivity of the factors of production in agriculture will result in a greater demand for children at any given A. A permanent increase in productivity will cause a decrease in the stationary–state amount of land per capita and thus cause an increase in population density, while production and consumption per capita will be unaffected[32].

7. Model Predictions and Empirical Evidence

In Section 2 we argued that it is only under certain specific institutional conditions that the old–age security motive and producer goods motive can be expected to be strong in fertility decisions. After having identified such conditions, we used them as a basis for the assumptions of our model. When evaluating the results of the model, it is obviously important to compare them with empirical findings from societies where the institutional set–up roughly matches the model assumptions. We have chosen to concentrate our comparisons with findings from rural Bangladesh, where some resemblance do exist[33].

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[32] In Chapter 2 we study the effects of the introduction of a land–augmenting technology in an economy that contains two different types of landholding classes.

[33] Land in Bangladesh is privately owned and divided among a large number of relatively small farms of varying size. The subsistence nature of production is manifested by a low marketed surplus (10%), a low share of hired labor (strictly casual), a small and diminishing share of rented land and primitive technology (Ahmad, 1984). In order to arrange for old–age security, farmers have to rely on private solutions. There is no social security for the elderly and local security arrangements, common a few decades ago, have practically disappeared (de Vylder and Asplund, 1979; Cain, 1978). Nor can relief from close kin or neighbours be counted on in times of general or personal crisis (Cain,

Hoewever, some of the assumptions concerning the structure in the model do not correspond to the characteristics of rural Bangladesh. For instance, it is evident that land, labor, and credits markets do exist in Bangladesh. However, various transactions costs make these markets far from perfect[34].

1977). Lateral bonds are very weak and a man can build a kin–based insurance network only through reproduction (Cain, 1982). Land is the most efficient store of value (Mahmud and McIntosh, 1980) and the single most important determinant of the standard of living (Westergaard, 1985). Land may be sold on the open market, but the majority of transactions take place through inheritance (Alamgir, 1978). Adult children are needed on the farm to safeguard ownership and to provide labor and old–age care. If an individual who reaches old age does not have mature sons to take over the farm, the risk for distress sale or even loss of land is very high. In the case of a patriarch's death, survivors risk appropriation of inherited land by other villagers (Adnan, 1988). There are no social institutions, such as joint production, that might compensate for inter–household labor imbalances (Cain, 1981). Labor for practically any service or activity can be purchased in Bangladesh, but such purchases often entail relatively high transaction costs (Cain, 1982). Labor from the existing spot–type labor market is used mainly as a complement to family labor in peak seasons (Boyce, 1989; Muqtada and Alam, 1985). Hiring–out of labor is highly complementary to work on the family farm. Nonagricultural employment is often fragmentary and the significance of such employment lies in the additional income it provides to a rural family (Muqtada and Alam, 1985). Out–migration to urban areas is in general no longer an attractive alternative, as expected urban wages are even lower than those on the rural spot market (Horn, 1987; Muqtada and Alam, 1985). A partible inheritance system is in force. According to Islamic law, land is inherited by all children in such a way that sons each receive an equal share and daughters half the share of the sons. However, daughters quite commonly forego their share of inherited land, primarily to remain on good terms with their brothers, on whom they may depend for help in the event of divorce or widowhood (Westergaard, 1985; Cain, 1977; Adnan, 1988). There is clear empirical evidence that inheritance is used as a leverage on children to provide expected labor and services (Torrey, 1988). Hence, most rural landowners regard children as the best available alternative for ensuring consumption in old age (Cain 1982). This conclusion is also confirmed by attitude surveys from Bangladesh (Nugent, 1985). The returns to child labor alone is not such that it would argue for unlimited high fertility in rural Bangladesh as children do not compensate for their consumption over the period that children typically remain members of their parents' household (for sons, the early twenties) (Cain, 1982). When the son of a landed family establishes his own household, he is usually given independent rights to cultivate a portion of his father's land – a form of ante mortem inheritance, but the title to the land will not usually be transferred until the father's death. Economically, the nuclear family lives in isolation and a son who has established his own household assumes independent responsibility for the welfare of his household (Cain, 1982).

[34] Land in Bangladesh is privately owned and divided among a large number of relatively small farms of varying size. The subsistence nature of production is manifested by a low marketed surplus (10%), a low share of hired labor (strictly casual), a small and diminishing share of rented land and primitive technology (Ahmad, 1984). In order to arrange for old–age security, farmers have to rely on private solutions. There is no social security for the elderly and local security arrangements, common a few decades ago,

The consistency found below, between earlier empirical findings and the results of the model, should obviously not be taken as a proof of the accuracy of the model in describing the structure of the rural economy in Bangladesh, but only as an indication that the model might reveal some mechanisms that are important in explaining fertility behaviour. Straightforward empirical testing of the results of our model, however, remains to be done.

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have practically disappeared (de Vylder and Asplund, 1979; Cain, 1978). Nor can relief from close kin or neighbours be counted on in times of general or personal crisis (Cain, 1977). Lateral bonds are very weak and a man can build a kin–based insurance network only through reproduction (Cain, 1982). Land is the most efficient store of value (Mahmud and McIntosh, 1980) and the single most important determinant of the standard of living (Westergaard, 1985). Land may be sold on the open market, but the majority of transactions take place through inheritance (Alamgir, 1978). Adult children are needed on the farm to safeguard ownership and to provide labor and old–age care. If an individual who reaches old age does not have mature sons to take over the farm, the risk for distress sale or even loss of land is very high. In the case of a patriarch's death, survivors risk appropriation of inherited land by other villagers (Adnan, 1988). There are no social institutions, such as joint production, that might compensate for inter–household labor imbalances (Cain, 1981). Labor for practically any service or activity can be purchased in Bangladesh, but such purchases often entail relatively high transaction costs (Cain, 1982). Labor from the existing spot–type labor market is used mainly as a complement to family labor in peak seasons (Boyce, 1989; Muqtada and Alam, 1985). Hiring–out of labor is highly complementary to work on the family farm. Nonagricultural employment is often fragmentary and the significance of such employment lies in the additional income it provides to a rural family (Muqtada and Alam, 1985). Out–migration to urban areas is in general no longer an attractive alternative, as expected urban wages are even lower than those on the rural spot market (Horn, 1987; Muqtada and Alam, 1985). A partible inheritance system is in force. According to Islamic law, land is inherited by all children in such a way that sons each receive an equal share and daughters half the share of the sons. However, daughters quite commonly forego their share of inherited land, primarily to remain on good terms with their brothers, on whom they may depend for help in the event of divorce or widowhood (Westergaard, 1985; Cain, 1977; Adnan, 1988). There is clear empirical evidence that inheritance is used as a leverage on children to provide expected labor and services (Torrey, 1988). Hence, most rural landowners regard children as the best available alternative for ensuring consumption in old age (Cain 1982). This conclusion is also confirmed by attitude surveys from Bangladesh (Nugent, 1985). The returns to child labor alone is not such that it would argue for unlimited high fertility in rural Bangladesh as children do not compensate for their consumption over the period that children typically remain members of their parents' household (for sons, the early twenties) (Cain, 1982). When the son of a landed family establishes his own household, he is usually given independent rights to cultivate a portion of his father's land – a form of ante mortem inheritance, but the title to the land will not usually be transferred until the father's death. Economically, the nuclear family lives in isolation and a son who has established his own household assumes independent responsibility for the welfare of his household (Cain, 1982).

We also cite evidence from other developing countries and historical populations when evidence from Bangladesh is meager or when the additional evidence is of general relevance. In these cases the resemblance between the structure of the model and the characteristics of the societies concerned might even be less than in the case of Bangladesh. Obviously any consistency found in these cases should be treated even more carefully. With these qualifications in mind, we now turn to the comparisons made.

There is strong empirical evidence of differential fertility (P1) in Bangladesh[35]. The number of children per family increases with the size of landholdings. Moreover, as predicted by our model, the number of children increases less than proportionally to the size of the landholding (Mahmud and McIntosh, 1980; Muqtada and Alam, 1985; Westergaard, 1985). In surveys of micro–level studies from other parts of the world, strong support is also found for a positive relationship between fertility and assets in the form of land (Anker and Knowles, 1982; Lipton, 1983). The same type of pattern has been observed in micro level studies from historical Europe. Families that own large amounts of inheritable assets (land, artisan equipment, etc.) have more children than families that own less inheritable assets (Andorka, 1978; Lipton, 1983; Shanin, 1972).

As for differences in land/worker ratios (P1), ' veral studies find a clear positive relationship between this ratio and the size of the landholdings in Bangladesh (Muqtada and Alam, 1985; Ahmad, 1984). On the Indian subcontinent in general (Boyce, 1989), as well as in developing countries in other parts of the world (Binzwanger and Rosenzweig, 1984), a substantial literature shows that smaller farm size is associated with greater intensity of labor use per acre. Another well–established fact is that the productivity per acre on smaller farms (P1) is higher than the productivity on larger farms (Binswanger and Rosenzweig, 1984). This finding is also confirmed for Bangladesh (Muqtada and Alam,

[35] The effect of malnutrition on the fecundity of poor women has not been found to be significant (Menken et al., 1981). "Supply constraints" thus seem unimportant in explaining fertility patterns.

1985; Boyce 1989).

Studies of intra–family distribution are scarce, making it difficult to find empirical evidence as to how consumption levels of adults and the elderly (P1) covary with the size of landholding. In Bangladesh, elderly males have been found to exhibit higher consumption in larger families (Mahmud and McIntosh, 1980). This fact supports a combination of two results in our model, i.e., the suggestions that richer families are larger, and that the elderly in these families consume more than the elderly in poorer families.

The model predicts equalization over time of the amount of land owned per person (P2). In studying the development of land ownership in Bangladesh, it is obvious that there has been a reduction in the number of large farms and greater concentration of smaller holdings, with a subsequent decline in the average size of holdings (Westergaard, 1985). The list of reasons for this development includes the proportionally higher population pressure on large farms and the subsequent subdivision of land through inheritance.

Anthropological and economic micro studies on the village level lend insight into the pattern of economic mobility within families over generations. As proposed by our model, the overall dominant trend during the twentieth century is downwards social and economic mobility (P2) (Ahmad, 1984; Westergaard, 1985; Mahmud and McIntosh, 1980). According to detailed micro level studies, the few who escape the downward trend have been "hit by chance" during the turmoil of wars or famines, or have managed to be in the right position to reap the benefits of growing foreign aid in recent decades (van Schendel, 1981).

The positive relationship between landholdings and fertility predicted by our model and confirmed above, in combination with the actual downward trend in inherited plot size may thus, as proposed in Mahmud and McIntosh (1980), result in a new poverty–led demographic transition, through which Bangladesh would eventually achieve zero population growth (stationary state in our model). But according to our model, zero population growth will not be maintained if there are <u>changes</u> in the cost of raising children, in transfers to adults or in agricultural productivity.

As for the effects of transfers to adults (P4), the positive relationship between fertility and income predicted by our model has been found in low income countries within the same socio–economic group, for instance among small farmers (Lipton, 1983; Birdsall, 1988; Wright, 1988).

Many studies have been performed to investigate the relationship between old age pensions and fertility patterns (P5) on an international cross–section basis. Such studies are questionable out of many reasons, making it more appropriate to base the empirical analysis of the old age pension/fertility relationship on experiences within single countries and even within fairly narrowly defined sub–national regions (Wildasin, 1983). In the studies available from rural areas in developing countries, suggestive evidence has been found that the introduction of pension schemes has a significant negative effect on fertility (Nugent and Gillaspy, 1983; Ridker, 1980).

According to our model, the introduction of productivity–increasing inputs would raise the demand for children in the short run and eventually reduce the steady–state level of arable land per household (P6). The introduction of green revolution technologies since the mid–1960s has resulted in overall increases in food production and yield per hectare in Bangladesh (Alauddin and Tisdell 1989). Areas with high productivity have also been found to have farms with smaller average size (Westergaard, 1985). The relationship between rural population density and the adoption of high–yielding varieties is also strikingly positive (Boyce, 1989).

8. Concluding Remarks

Even though the base case of our model predicts a stable population in the stationary state[36], the model can accommodate the observed general fact of persistent

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[36] This basic result accords with the homeostasis or "autoregulation" hypothesis. The homeostasis hypothesis is based on the observation that many pre–industrial societies

population growth. As we have shown, policies aimed at increasing agricultural productivity or providing consumption aid to certain age groups could initiate unintended increases in fertility. The effects of such interventions on parental welfare and its distribution among generations will depend on the permanency of intervention. As demonstrated, it might well be the case that certain types of intervention, shown to have positive effects on the parental welfare of one generation, could have a negative outcome on the parental welfare of subsequent generations. Ultimately, the choice of whether to introduce the policies or not in the framework of our model depends on the relative weight assigned to the welfare of generations not yet born.

In Bangladesh, agricultural production per capita remained roughly unchanged throughout the first half of this century (Ahmad, 1984). Only a modest decline in the land–person ratio occurred (Khan, 1988). The pattern changes in the late 1950s when population growth begins to accelerate sharply (Khan, 1988). Agricultural output increases too, but at a slower rate. As a result, agricultural production per capita falls. A growing share of total consumption is provided from abroad. If we were to analyze these trends within the framework of our model, we could speculate that the aggregated pattern may be explained by demographic responses to the introduction of the type of aid programs described above from the late 1950s onwards. According to our model, such programs would have a positive effect on parental welfare of generations first reached, thus giving rise to positive fertility responses. In the longer run, however, they would eventually cause reductions in per capita landholdings, consumption and production.

For reasons already mentioned it is too early to draw any policy recommendation from our analysis. But the analysis does highlight the importance of continued studies of demographic behavioral response to changes in institutions and economic conditions. If we find that higher population density in itself has negative externalities, we have to discuss

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incorporated some mechansim that regulated population size in relation to resources (Lee, 1977).

ways of altering fertility behaviour. In an institutional setting as described in the model, there appear to be two different ways of reducing fertility. The first way is to introduce reliable alternatives, such as social security[37] or markets for alternative, storable assets[38]. The costs of introducing such systems, as well as the quality demands on institutions to handle such alternatives, are too high in some developing countries to make such proposals realistic.

The second way would be to minimize the possibility of enforcing the social contracts required between parents and children. To the extent that inheritance is used as a device for securing binding obligations from children, the development of reliable and relatively well–paid job opportunities outside family farms would decrease the enforcing value of any inheritance and thereby maybe decrease fertility[39]. A third way of reducing the enforcement possibilities would be to convert the inheritance rule into an impartible rule or to tax inheritance heavily.

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[37] Proposition 5 and the calculations proving it, constitute one example to this effect.

[38] It is easily shown that the introduction of a capital market will make fertility a function of the difference between the rate of return on the alternative, storable asset and the internal rate of return to children. If the rate of return on the alternative asset is high enough, fertility will be reduced.

[39] In footnote 14 examples of such processes are given.

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Chapter 2

DEMOGRAPHIC RESPONSE TO AGRICULTURAL REFORM – LONG–RUN EFFECTS ON CONSUMPTION, WAGES AND POPULATION DENSITY

1. Introduction

In the late 1960s and early 1970s, great concern was expressed over the tendency for the growth of food supply to fall behind that of population growth in developing countries[1]. Until the middle of the twentieth century, the expansion of the land frontier had constituted an important source of increased agricultural output, but in many developing countries this possibility was no longer physically available or economically feasible (Kelley, 1988). The solution to the growing crisis was sought in what was to be called "the green revolution", i.e., the introduction of new high–yielding varieties, along with a package of farming practices on existing farm land, including high rates of fertilization and the controlled use of irrigation (Dorner, 1972). The assumption was that production did not grow fast enough because inputs necessary for such growth were not available to farmers and that this was the main cause behind signs of increasing rural poverty in developing countries at the time.

Evidence soon showed that rural poverty could not be alleviated only through supply side measures. In practice the green revolution in many cases only affected a minority of farmers because the majority lacked the resources, or were "institutionally" precluded from taking advantage of the new agricultural technology (Griffin and Ghose, 1979; Ladejinsky, 1970). In some cases the introduction of the green revolution even tended to worsen the problem of rural poverty (Ghose, 1983; Gourou, 1980).

Some observers began to regard rural poverty mainly as a result of an uneven

[1] For a number of references, see Ghose (1983). For a discussion on recent such claims, see Sen (1994).

distribution of agricultural land. Calls for land reform were put forward, and in the 1980s reforms in favor of the poorest continued to receive much attention in the development policy debate, especially in developing countries where land was the main productive asset and thus the basis for survival for the majority of the population. Equalizing land reforms were advocated for two main reasons. First, such a reform would improve the income opportunities of the poor, by transferring productive assets to them. Second, a reform would improve agricultural efficiency, as there existed empirical evidence of an inverse relationship between size of farm and output per acre.

In the discussions of the effects of the green revolution and of the effects of a redistributing land reform, surprisingly little space has been devoted to the relationship between the different measures proposed and to their possible demographic effects. The discussion of the green revolution has mainly seen it as a possible <u>response</u> to the problems caused by increasing population pressure. The following quotations illustrate the reasoning:

> "The green revolution is, of course, a necessary and soughtfor achievement. It offers promise to the more densely populated countries to sustain their growing numbers until they can achieve more effective control of population growth rates and expand their industrial sectors" (Dorner, 1972, p.25).

> "Societal institutions and government–designed policy can also play a role in mediating any effects of population change – for example through investment in a search for new technologies (the Green Revolution) to raise agricultural production" (Birdsall, 1988, p.484).

In the case of land reform, the limited discussion on the subject has mainly focused on the question whether <u>exogenously</u> determined population growth may turn viable post–reform holdings into unviable ones, through the process of the partitioning of land, as the generation alive at the reform is replaced by the next one:

> "...all is not lost for distributive reform just because population growth on the land causes average holding size later to fall below the level considered viable now. Indeed it could be argued that only the push to

yields given by land reform can provide the necessary safeguard against the impact of population growth" (Lipton, 1983, p. 305).

In this paper we will show that under certain circumstances green revolution measures and redistributing land reforms may initiate demographic responses with unintended effects on parental welfare[2], wages and population density, consequences that are not envisaged in the discussions referred to above. We base our model on the following reasoning: In rural areas in developing countries, land very often constitutes the main asset and the main determinant of income. In many of these areas it has empirically been found that family labor is the main labor input on small farms[3] and that there is a positive relationship between the size of landholdings and the number of children of landowners (Lipton, 1983; Birdsall, 1988)[4]. Taken together, this implies that the distribution of land, in combination with family size and the efficiency per unit of land, to a large extent determines the distribution of income among households in such areas at any particular moment in time.

Empirically it has also been found that a large part of land transactions in these societies are made through inheritance and that land markets for various reasons are underdeveloped (Nugent, 1985)[5]. This implies that the net fertility rate of individual farmers in combination with the inheritance rule of the society will be of crucial importance for the determination of changes in per capita income over time. For example, in societies where inheritance is equally divided among children[6], the number of children of a couple will determine both the size of the family labor force on the family farm, and the size of

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[2] Parental welfare is defined as the weighted sum of individual utility in adult and old age.

[3] For a discussion on why family labor is preferred and references on the subject, see Chapter 1.

[4] As stated in World Bank (1984, p.109): "Studies in Bangladesh, India, Iran, Nepal, the Philippines, and Thailand all show that fertility rises as farms get bigger".

[5] For a discussion on why markets for land are extremely thin, see e.g. Rosenzweig, Binswanger and McIntire (1987).

[6] a partible inheritance rule.

landholdings that each child will inherit. If we assume that the choice of number of children of a farming couple is partly determined by their need for labor on their own farm (the producer goods motive), it is obvious that any change in the distribution of land (or in the efficiency of land) will affect not only the distribution of land at the time of the reform but also fertility and hence the distribution of land, and thus income, in future generations.

Following this line of reasoning, a redistributing land reform and unevenly distributed green revolution measures could have an impact on the distribution of effective land holdings not only at the time of the reform but also in the long run in an economy where the total physical amount of land is fixed and, hence, where any increases in output must be accomplished through productivity increases.

We will study the effects of these reforms utilizing an overlapping–generations model in which it is possible to incorporate demographic responses. To keep the model manageable, we have to restrict ourselves to a very simplified characterization of production relations in the agrarian sector. Following the discussion in Ghose (1983), we divide our agricultural sector into two clearly distinguishable agricultural classes – one comprising those who work on the land (whom we will call "the poor"), and the other comprising those who do not work on the land but receive a share in its produce ("landlords")[7]. The poor receive income from a combination of self–cultivation of own land and wage labor. The landlords, on the other hand, seek to maximize their profit. We further assume that the choice of number of children by poor peasants is made rationally, taking into account the cost of raising children, the need of family labor on the family farm and what can be earned on the labor market if family labor is partly allocated to that market. Transactions in land are assumed to solely take place through inheritance, and we assume that a partible inheritance rule is in force. This makes it possible for us to study

[7] It can be empirically observed in many developing countries that those who work on the land often are identical with the group of rural poor, while those who do not work on the land but receive a share in its produce are identical to the group of rich landlords (Ghose, 1983).

the effects of the introduction of green revolution measures and of a redistributing land reform on future generations.

2. The Model

We consider a rural economy, where of the total amount of land A^T, A^R is owned by landlords and A^P by poor households. In each point in time the landholdings of all poor households are of equal size. Landlords only use hired labor on their farms. Moreover, the land of the landlords is not divided after the owner's death. All producers are price takers. Poor landowners only use family labor on their own farm but also get part of their income from hiring out labor on the labor market. We assume that both types of farms are operated with the same CRS technology.

Each individual of a poor household lives for three periods: as an improductive child, as a productive adult, and as retired and improductive in old age. In each period the extended family consists of three generations and owns and cultivates their share of land A_t. Production cannot be stored and income must be consumed during the same period as it is earned. The property rights to the land are in the hands of the elderly. When a retired person dies the land is shared equally among the children, who in turn enter the period of old age. We regard all individuals as male–female pairs and we ignore mortality. Fertility decisions are taken at the beginning of the adult period and fertility is assumed to be fully controllable. The old decide whether the adults are to work on the family farm or on the market, i.e., on the land of the landlords. If the adults work on the land of landlords, they are paid the equilibrium market wage. This income stays with the old[8]. Adults in return receive a fixed income from the old, independently of where they work. The adults provide

[8] There is clear empirical evidence that inheritance of land is used as a leverage on children to stay on the family farm and provide expected labor and services (Torrey, 1988). If we assume that the marginal utility of consumption in old age goes to infinity as consumption in old age approaches zero, adults will thus always obey their parents as the loss of inheritance in our model would imply zero consumption in old age (there is no labor market for old individuals on which they can earn a living).

their children with an exogenously determined amount of food that will ensure their survival to adult age. The old consume an amount equal to total output plus any income earned from family labor working on the market minus cost for family labor.

We now turn to the maximization problem of a representative poor household.

a. A Poor Household

Food (Y_t) is produced according to the following production function:[9]

$$Y_t = F(A_t, L_t) \tag{1}$$

where Y_t is the aggregate production of food on the family farm at time t, A_t is the amount of land owned by the elderly at time t (born at t–2) and L_t[10] is the number of adults working on the family farm at time t (born at t–1).

The total number of adults in a family at t is N_t, of which M_t work on the labor market and L_t work on the family farm:

$$N_t = M_t + L_t \tag{2}$$

According to the partible inheritance rule, land is divided equally among children. The output of each farm belonging to poor in period t+1 is then:

$$Y_{t+1} = F(\frac{A_t}{N_t}, L_{t+1}) \tag{3}$$

[9] We will eventually have to confine the analysis to a Cobb–Douglas technology. But, to keep the notation more readable we maintain a more general expression for the production function.

[10] For simplicity, we assume that L, and N and M, are continuous variables.

The individuals are selfish in the sense that only their own consumption is included in their utility function. The utility function for an adult born at t–1 is defined over consumption in period t and t+1:

$$U = U(c_t, c_{t+1}) \tag{4}$$

His consumption level in adulthood c_t depends on the exogenously determined wage he receives from his parent k, the number of children he chooses to have N_{t+1}, and the exogenously determined level of consumption of each of his children c_0:

$$c_t = k - N_{t+1}c_0 \tag{5}$$

The consumption in his old age c_{t+1}, is determined by the number of children that he decided to have in the preceding period N_{t+1}, the allocation of these children between the family farm and the market, the market wage w_{t+1}, the amount of land he inherited A_{t+1}, and finally the "intra–family wage" k:

$$c_{t+1} = F(A_{t+1}, L_{t+1}) - kN_{t+1} + w_{t+1}M_{t+1} \tag{6}$$

We assume that the utility function is additively separable in consumption at t and t+1, and that utility is represented by a loglinear function. As we are studying an individual born in t–1, he will at the beginning of adulthood at time t have to decide the optimal number of children N_{t+1}. In doing so he has to take into consideration how he, at old age, will optimally allocate these children between the family farm and the market, hence the choice of M_{t+1}. His maximization problem thus is:

$$\max_{N_{t+1},M_{t+1}} \ln c_t + \delta \ln c_{t+1} \tag{7}$$
s.t. (2), (5) and (6)

where δ is the subjective discount factor.

The expressions in (5) and (6) show that all period indices (subscripts) are referring to the same time period which means that we can ignore them at this stage and define the following intertemporal utility function:

$$\Omega(M,L) \equiv \ln[k-(M+L)c_0]+\delta\ln[F(A,L)-kL+(w-k)M] \tag{8}$$

The first–order conditions to (7) are thus found through the maximization of (8) with respect to M and L. The first–order conditions are:

$$\Omega_M(M,L) = \frac{1}{c_t}(-c_0) + \delta\frac{1}{c_{t+1}}(w-k) = 0 \tag{9}$$

$$\Omega_L(M,L) = \frac{1}{c_t}(-c_0) + \delta\frac{1}{c_{t+1}}(F_L-k) = 0 \tag{10}$$

It is easily shown that Ω_{MM} and $\Omega_{LL} < 0$ and that $\Omega_{MM}\Omega_{LL} - \Omega_{ML}^2 > 0$, and thus that any interior solution to (9) and (10) is a maximum[11]. Combining (9) and (10), we find:

$$F_L(A,L) = w \tag{11}$$

[11] We also see from (9) that w must always exceed k, for an interior solution to the problem. This simply reflects the fact that there is no idea to take the cost of bringing up a child c_0, if labor can be bought on the market at a wage rate equal to or lower than the intra–family wage.

All poor farmers thus have the same shadow price on labor and the same marginal product for land. Due to the CRS assumption, (11) gives us the following expression for the demand for labor on the family farm:

$$L = h(w)A \tag{12}$$

where $h \equiv F_L^{-1}$. Expression (9) can also be rewritten as the following expression relating the fertility choice to the size of the family farm and the market wage:

$$N = Q - \frac{1}{(1+\delta)}\left[\frac{AF_A(A,L)}{w-k}\right] \tag{13}$$

where:

$$Q \equiv \frac{\delta k}{(1+\delta)c_0} \tag{14}$$

b. Landlords

We now turn to the landlords in order to establish their demand for labor. Production in any time period is given by:

$$Y^R = F(A^R, M^D) \tag{15}$$

where Y^R is the total production on rich farms and M^D the amount of labor used. The profit maximization problem is:

$$\max_{M^D} \; Y^R - wM^D \tag{16}$$

The first–order condition is:

$$F_L(A^R, M^D) = w \tag{17}$$

The demand for labor is:

$$M^D = h(w)A^R \tag{18}$$

c. Labor Market Equilibrium

Total demand for labor in the economy at t+1 is determined by the demand on the labor market from landlords M^D_{t+1}, and the demand for family labor on each family farm L_{t+1}, multiplied with the number of such farms x_{t+1}. The number of family farms at t+1 is determined by dividing the total amount of land initially belonging to poor farmers by the actual size of each of these farms at t+1:

$$x_{t+1} = \frac{A^P}{A_{t+1}} \tag{19}$$

If we utilize (12), (18) and (19), total demand of labor in the economy at t+1 simplifies to:

$$M^D_{t+1} + x_{t+1}L_{t+1} = h(w_{t+1})A^T \tag{20}$$

The supply of labor at time t+1 is determined by the fertility choice of each poor family times the number of such families in t+1, hence $N_{t+1}x_{t+1}$.

Labor market equilibrium, in combination with (19), implies:

$$h(w_{t+1})A^T = \frac{A^P}{A_{t+1}} N_{t+1} \tag{21}$$

and thus:

$$w_{t+1} = h^{-1} \left(\frac{A^P}{A^T} \frac{N_{t+1}}{A_{t+1}}\right) \tag{22}$$

The market wage in each period thus depends on the relative share of land belonging to the poor, the size of each poor farm, and the number of children on each individual farm. Utilizing (12) and (18) we further find that the following relationships must always hold in equilibrium:

$$L_{t+1} = \frac{A^P}{A^T} N_{t+1} \tag{23}$$

$$M_{t+1} = \frac{A^R}{A^T} N_{t+1} \tag{24}$$

Let us denote the share of total land resources belonging to poor families as λ:

$$\lambda \equiv \frac{A^P}{A^T} \tag{25}$$

Inserting (14), (17), (21) and (25) in (13) gives the following equilibrium condition for any period:[12]

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[12] As all period indices refer to t+1, we can ignore them.

$$N = Q - \frac{1}{(1+\delta)}\left[\frac{AF_A(A, \lambda N)}{F_L(A, \lambda N)-k}\right] \tag{26}$$

which defines an implicit relationship between the number of children per poor household (per adult) at a given moment in time, and the size of the landholdings that each adult will inherit in the next period. As we have simplified our analysis to one–sex families, a constant population size requires N to equal one. This implies that as a condition for a stationary state, $Q > 1$ must hold, as the second term in (26) is positive. We assume this to be the case. The implicit relationship N(A) in (26) will now be used to derive the dynamics of the economy.

d. Dynamics

The dynamic element of our model is provided by the inheritance rule:

$$A_{t+1} = \frac{A_t}{N_t} \tag{27}$$

An increase in fertility in period t will thus decrease the size of each landholding in period t+1. It is easily shown that there exists a positive relationship between the optimal choice of number of children and the size of landholding to be inherited the forthcoming period, everything else equal[13]. A change in the size of the landholding will thus induce further changes in fertility, with further implication for the size of the landholding in the subsequent period, etc. But since fertility choices also depend on the market wage, and the wage in turn depends on the size of the work force, and thus on fertility decisions, a change in the size of landholdings will have further repercussions. In order to determine the

[13] If we investigate (26), using a Cobb Douglas production function, we find that the optimal number of children is a positive function of the size of landholdings. Any forseen change in the size of landholdings next period will thus initiate a higher number of children.

properties of this system, we start by determining the sign and size of:

$$\frac{dA_{t+1}}{dA_t} = \frac{1}{N_t^2}[N_t - A_t\frac{dN_t}{dA_t}] \tag{28}$$

Totally differentiating (26), we find that[14]:

$$\frac{dN_t}{dA_t} = -\frac{\frac{1}{1+\delta}\left[\frac{F_A}{F_L - k} + \frac{AF_{AA}}{F_L - k} - \frac{AF_A F_{LA}}{(F_L - k)^2}\right]}{1 + \frac{1}{1+\delta}\left[\frac{A\lambda F_{AL}}{F_L - k} - \frac{AF_A F_{LL}\lambda}{(F_L - k)^2}\right]} \tag{29}$$

Define the expression in the denominator of (29) as β_1 and the expression in the numerator as β_2. We can then rewrite (28) as:

$$\frac{dA_{t+1}}{dA_t} = \frac{1}{N^2\beta_1}[N\beta_1 - A\beta_2] \tag{30}$$

The expression in parenthesis in (30) simplifies to (see Appendix):

$$N\beta_1 - A\beta_2 = Q \tag{31}$$

which implies:

$$\frac{dA_{t+1}}{dA_t} = \frac{Q}{N_t^2\beta_1} \tag{32}$$

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[14] As all period indices refer to t+1, we can ignore them.

We now restrict our analysis of $\frac{dA_{t+1}}{dA_t}$ to a stationary state[15], where N is equal to one:

$$\left.\frac{dA_{t+1}}{dA_t}\right|_{\hat{A}} = \frac{Q}{\beta_1} \tag{33}$$

A necessary and sufficient condition for a stable stationary state is that:

$$0 < \left.\frac{dA_{t+1}}{dA_t}\right|_{\hat{A}} < 1 \tag{34}$$

After some manipulation, we find:

$$\left.\frac{dA_{t+1}}{dA_t}\right|_{\hat{A}} = \frac{Q}{1+\frac{(Q-1)}{F_A}\lambda F_{AL}\left[1+(Q-1)(1+\delta)\lambda^{-1}\right]} \tag{35}$$

The right hand side of expression (35) is always positive, as $Q > 1$ is assumed. To find conditions under which $\left.\frac{dA_{t+1}}{dA_t}\right|_{\hat{A}}$ is smaller than unity, we need to impose more structure on the problem. To this end assume a Cobb–Douglas–production function:

$$Y(A,L) = A^{1-\alpha}L^{\alpha} \tag{36}$$

After some manipulation, we find:

$$\left.\frac{dA_{t+1}}{dA_t}\right|_{\hat{A}} = \frac{Q}{1+(Q-1)\alpha\left[1+(Q-1)(1+\delta)\lambda^{-1}\right]} \tag{37}$$

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[15] Stationary state values of variables are marked ˆ.

The necessary and sufficient condition for local stability then simplifies to:

$$(Q-1)(1+\delta) > \frac{1-\alpha}{\alpha}\lambda \tag{38}$$

We assume this is fulfilled. If we insert (14) we can rewrite (38) as: $\delta(k-c_0) > \frac{1-\alpha}{\alpha}\lambda + c_0$. The expression implies that the amount paid to adults, **k**, must exceed the cost of bringing up a child, for a stable solution to exist. Furthermore, stability is more "likely" the larger the intra–family wage, the lower the cost of raising children, the lower the productivity of land, and the smaller the share of land belonging to poor farmers.

We now turn to the analysis of the existence of a stationary state. In a stationary state, (26) can be rewritten as:

$$\frac{AF_A(A,\lambda)}{F_L(A,\lambda)-k} = (Q-1)(1+\delta) \tag{39}$$

We thus have to investigate the properties of the left–hand side of (39) in order to determine the existence of a stationary state. First, let us define the left–hand side of (39) as:

$$H(A) \equiv \frac{AF_A(A,\lambda)}{F_L(A,\lambda)-k} \tag{40}$$

If we can prove that H(A) is monotone, and that H(A) can take on the value $(Q-1)(1+\delta)$ we have succeeded in proving that a unique stationary state exists. Note that $F_L(A,\lambda) > k$ is necessary for an interior solution to the poor household's maximization problem. Let A': $F_L(A',\lambda) = k$. Since $F_{LA} > 0$, relevant values of A are such that $A > A'$. We then have:

$$\lim_{A \to A'} H(A) = \infty \tag{41}$$

and,

$$\lim_{A \to \infty} H(A) = \frac{1-\alpha}{\alpha} \lambda \tag{42}$$

which according to (38) is smaller than $(Q-1)(1+\delta)$. Furthermore:

$$H_A'(A) = \frac{1}{F_L - k} F_A[1-\alpha-\alpha\lambda^{-1}(Q-1)(1+\delta)] \tag{43}$$

As $[(Q-1)(1+\delta)] > \frac{1-\alpha}{\alpha} \lambda$ by assumption, it follows that $H_A'(A) < 0$. We can thus conclude that (38) is a sufficient and necessary condition for the existence of a unique and stable stationary state.

The stationary equilibrium $(\hat{c}_1, \hat{c}_2, \hat{w}, \hat{A}, \hat{x}, \hat{L}, \hat{M})$ is given by the following system of equations, where $\hat{c}_1$ is the stationary–state consumption in adult age, $\hat{c}_2$ is the stationary–state consumption in old age, $\hat{w}$ is the stationary–state wage, $\hat{A}$ is the size of poor landholders' farms in the stationary state, $\hat{x}$ is the number of such farms in the stationary state, and, finally, $\hat{L}$ and $\hat{M}$, are the amount of labor on the family farm and labor market, respectively, in the stationary state:

$$\hat{c}_1 = k - c_0 \tag{44}$$

$$\hat{c}_2 = F(\hat{A},\lambda) - k + \hat{w} \frac{A^R}{A^T} \tag{45}$$

$$\hat{w} = h^{-1}\left(\frac{\lambda}{\hat{A}}\right) \tag{46}$$

$$1 = Q + \frac{1}{1+\delta}\frac{\hat{A}F_A(\hat{A},\lambda)}{F_L(\hat{A},\lambda)-k} \tag{47}$$

$$\hat{A} = \frac{A^P}{x} \tag{48}$$

$$\hat{L} = \lambda \tag{49}$$

$$\hat{M} = 1-\lambda \tag{50}$$

It is obvious from the system that the consumption during adult age is not sensitive to policy changes in the stationary state, while the consumption in old age is. We now turn to the analysis of policy changes.

3. Agricultural Reform

We now turn to our main issue; the consequences of a redistributing land reform and the green revolution. In the analysis we restrict ourselves to a comparison of stationary states. In the discussion of the mechanisms underlying our results, we assume that individuals can foresee changes in land distribution and productivity changes one period ahead[16].

a. A Redistributing Land Reform

In expression (25) above we define λ as the share of the total amount of land that belongs to poor households. A redistributing land reform in favor of poor households would obviously be translated into an increase in λ, with no change in the total amount of land in the economy. The effect of such a reform on the stationary–state parental welfare is determined by the effects on the stationary–state consumption in old age, $\hat{c}_2$. We can

[16] When discussing changes in population density, we only discuss changes in density on land belonging to poor households.

rewrite (45) as:

$$\hat{c}_2 = F(\hat{A},\lambda) - k + \hat{w}(1-\lambda) \tag{51}$$

The effects of a change in λ on the consumption in old age is then determined by:

$$\frac{d\hat{c}_2}{d\lambda} = F_A\frac{d\hat{A}}{d\lambda} + (1-\lambda)\frac{d\hat{w}}{d\lambda} + F_L d\lambda - \hat{w}d\lambda \tag{52}$$

As (11) must hold, the two last terms cancel. The effect of a redistributing land reform in favor of poor peasants on parental welfare of poor households in the stationary state thus depends on the effects on the market wage and on the size of landholdings in the new stationary state. In determining these effects we utilize (47) and define:

$$E(\hat{A},\lambda) \equiv 1 - Q + \frac{1}{1+\delta}\frac{\hat{A}F_A(\hat{A},\lambda)}{F_L(\hat{A},\lambda)-k} = 0 \tag{53}$$

By the implicit function rule:

$$\frac{d\hat{A}}{d\lambda} = -\frac{E_\lambda}{E_{\hat{A}}} > 0 \tag{54}$$

where the sign follows from:

$$E_\lambda = (Q-1)\ \alpha\lambda^{-1}\ [1+(Q-1)(1+\delta)\lambda^{-1}] > 0 \tag{55}$$

$$E_{\hat{A}} = \frac{\alpha(Q-1)}{\hat{A}}[\frac{1-\alpha}{\alpha} - (Q-1)(1+\delta)\lambda^{-1}] < 0 \tag{56}$$

In determining the sign of $d\hat{w}/d\lambda$, we utilize:

$$h(\hat{w}) = \frac{\lambda}{\hat{A}} \tag{57}$$

$$h' d\hat{w} = \frac{d\lambda}{\hat{A}} - \frac{\lambda}{\hat{A}^2} d\hat{A} \tag{58}$$

Rearranging and utilizing (53) we get:

$$\frac{d\hat{w}}{d\lambda} = \frac{1}{h'\hat{A}}\left[1+\frac{\lambda}{\hat{A}}\frac{E_\lambda}{E_A}\right] \tag{59}$$

and finally:

$$\frac{d\hat{w}}{d\lambda} = \frac{E_\lambda + E_A\frac{\hat{A}}{\lambda}}{E_A\frac{h'\hat{A}^2}{\lambda}} > 0 \tag{60}$$

where:

$$E_\lambda + E_A\frac{\hat{A}}{\lambda} = (Q-1)\lambda^{-1} > 0 \tag{61}$$

Hence, an increase in the share of total amount of land belonging to poor farmers will in the new stationary state result in lower population density (larger landholdings for each poor household), a higher wage on the labor market and higher consumption in old age. As noted above, consumption in adult age and childhood are not affected. We can thus conclude:

Proposition 1. The stationary–state parental welfare of the poor increases, population density decreases and the market wage increases from a redistributing land reform in favor of poor households.

The mechanism behind this result is that an increase in the amount of land that belongs to the poor can be characterized as a transfer of resources to the old. As we have noted above, children's and adults' consumption are not affected directly by the transfer. It follows from the assumptions in our model that an individual wishes to smooth consumption between adult and old age. An individual that foresees an increase in his landholding next period, and thus anticipates an increase in consumption in old age, wants to transfer part of those consumption possibilities to adult age. The only way to increase the consumption in adult age in our model is to cut costs, i.e., to have fewer children (see (5) above). A part of the increase in consumption possibilities in old age, achieved through the transfer of land, is thus transferred to adult age by not having as many children as a full compensation would predict. In the new stationary state each poor family thus ends up with a larger bit of land. Apart from the increase in consumption we can thus also witness a decreased population density on the land belonging to the poor in the new stationary state.

As to the effects on the labor market, the increase in the share of land belonging to poor makes each family use a larger share of the family labor on the farm (see (51) above), hence the supply of labor goes down. At the same time the loss of land by the rich makes the demand for labor shrink. We thus have two offsetting forces but, given our set–up, the reduction in the supply of labor dominates, and yields higher wages in the new stationary state.

A redistributing land reform in favor of poor households will obviously decrease the income of landlords, as we have modeled such a reform without a compensation mechanism.

b. Green Revolution

When we analyze the effects of the green revolution we assume that it takes the form of a land–augmenting[17] technology that is neutral with respect to scale[18]. The introduction of land–augmenting technology, that is unevenly distributed between the two landholding classes in our model, can be translated into changes in λ. If the land–augmenting technology is directed towards the poor, both A^P and A^T but also λ will increase. If the land–augmenting technology is directed towards the rich, both A^R and A^T will increase, while λ will fall. As can be seen from (52) above, the effects on the stationary–state consumption in old age depends only on changes in λ. The analysis of the previous section thus applies. The results in (52), (54) and (60) above thus imply that:

Proposition 2. A land–augmenting green revolution directed towards rich landowners will decrease the stationary–state consumption in old age in poor households, decrease the stationary–state market wage and increase population density in the new stationary state.

At the new stationary state, the increase in population density (in the number of poor households), in combination with larger market labor supply per household (see (50) above), increases the labor supply to such a degree that it dominates the increase in demand which results from the increased productivity on the land belonging to the rich. The net result is thus a downward pressure on the market wage as shown in (60), and a higher population density as shown in (54). A green revolution directed towards landlords, will obviously increase the income of landlords.

A land–augmenting "green revolution" directed towards the poor will have the opposite effects, and thus increase old age consumption and, hence, parental welfare as well

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[17] In Chapter 1 we study a technology change that is neutral with respect to factor use. As the assumptions of this paper include the assumption of two distinct classes of landholders, it is of interest to study the introduction of technologies which in their distribution could alter the distribution of land between the two classes.

[18] There seems to be a general agreement that the technology of the "green revolution" is neutral with respect to scale (Hayami 1981, Singh 1990).

as the market wage in the new stationary state. Furthermore, the size of each farm belonging to poor households will be larger in the new stationary state.

4. Concluding Comments

In this paper we have shown that a redistributing land reform in favor of poor peasants raises the parental welfare of the poor peasants in the new stationary state. Furthermore, the stationary–state market wage is increased and population density reduced. Each household thus ends up with a larger farm. However, the profit of landlords is reduced.

Very few empirical examples of extensive land reforms exist. Strong opposition from large landowners and urban populations, has in most cases forbidden reform to stray far from the status quo (World Bank, 1990). The experiences of South Korea, Taiwan and Japan, where extensive redistribution of assets such as land, in combination with other policies, was followed by a remarkable growth in the standards of living of the rural population, are, however, examples where one could argue that there is a consistency between the results of the model and empirical findings.

In this paper we have also shown how the effects of the green revolution may depend on its distributional characteristics. Observers seem to agree that the adoption of small farmers lagged behind that of large farmers in the initial stage of the introduction of the green revolution (Prahladachar, 1983). When we compare empirical findings with the results of our model we should therefore compare with the policy experiment where the introduction of land–augmenting green revolution techniques is directed towards landlords (we thus assume that those are identical with large farmers). In short, a green revolution favoring rich landlords will in our model induce growing population pressure, smaller landholding per poor household, increased poverty and inequality as well as increases in both the demand and supply of labor on the labor market but in a combination that reduces the equilibrium wage.

It is an established fact that the introduction of "green revolution" technologies in general resulted in a significant increase in labor demand (Hayami, 1981; Singh, 1990). Often, however, the labor absorption by agriculture using modern technology failed to keep up with the supply of labor with declining real wages as a result (Ahmed, 1976; Khan, 1984; Singh, 1990). In summary, the extensive literature on the structural response of the agrarian economy to the stimuli provided by the availability of modern inputs seems to conclude that the introduction of the "green revolution" often tended to be such as to further impoverish the poor (Ghose, 1983; Bardhan, 1985). For instance, in the case of Bangladesh, Aluaddin and Tisdell (1991, p.512) write: "All the potential welfare gains from productivity growth following the Green Revolution appear to have been swallowed up in supporting larger population in Bangladesh rather than in improving the standards of living, which appears to have declined for the masses"[19].

The direction of the causality between the introduction of "green revolution" measures and population pressure in the reasoning above as well as in our model is opposite from what was expressed in the quotations in the introduction and from what is expressed in the following:

> "the green revolution can be considered an induced innovation in response to growing population pressure on land, with which both the wage rate and income level of the rural working population would have degenerated further" (Hayami, 1981, p.176).

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[19] In Chapter 1 of this thesis, we argue that the basic characteristics of the model of that chapter are such that the model could be applicable to an analysis of a part of rural Bangladesh. One of the major questionable simplifications is, however, the assumption that no labor market exists. In this chapter we complement the basic model with one specific type of labor market, making it interesting to compare the results of the enriched model with empirical findings from Bangladesh. It is, however, important to highlight that the simplifications made in terms of the structure of the model and in the choices of functional forms are such that it is too early to draw any conclusions concerning policy recommendations.

Other empirical findings include the case of Turkey. Bayri and Furtan (1989, p.121) write: " These results show a typical example of rapid population growth offsetting the positive impact of HYV technology on labor demand. Although the demand for labor increased due to the technological change, it did not catch up with the increasing supply of labor."

To conclude, given that the aim is to increase the parental welfare of the poor households, the policy implications of our analysis could be summarized as follows: In economies where large tracts of land are under the control of few individuals, make a redistribution of land titles a precondition for productivity increasing assistance to the agricultural sector. If a redistribution of land titles is politically impossible to propose, target the productivity assistance to the poor. If this is not possible, refrain from any assistance or the assistance will just cause further impoverishment. In general, where institutional factors are favorable, in the sense that productivity increasing measures targeted towards poor actually reach the poor, the potential of new highly productive technologies is amplified by indirect demographic responses. Where they are not favorable, the costs of these indirect demographic responses could be high.

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APPENDIX

Utilizing (29) and the definitions of β_1 and β_2 we get:

$$N\beta_1 - A\beta_2 = N\left[1 + \frac{1}{1+\delta}\left[\frac{A\lambda\, F_{AL}}{F_L - k} - \frac{AF_A\, F_{LL}\, \lambda}{(F_L - k)^2}\right]\right. \tag{1'}$$

$$\left. + A\,\frac{1}{1+\delta}\left[\frac{F_A}{F_L - k} + \frac{A\, F_{AA}}{F_L - k} - \frac{AF_A\, F_{LA}}{F_L - k}\right]\right]$$

Utilizing the fact that

$$- NA\, F_A\, F_{LL}\, \lambda = F_A\, A^2\, F_{LA}, \tag{2'}$$

we can rewrite (1') as:

$$N\beta_1 - A\beta_2 = N + \frac{1}{1+\sigma}\cdot\frac{1}{F_L - k}\left[NA\lambda F_{AL} + \frac{F_A\, A^2\, F_{LA}}{F_L - k}\right]$$

$$+ \frac{1}{1+\sigma}\cdot\frac{1}{F_L - k}\left[AF_A + A^2 F_{AA} - \frac{A^2\, F_A\, F_{LA}}{F_L - k}\right] \tag{3'}$$

which simplifies to:

$$N\beta_1 - A\beta_2 = N + \frac{1}{1+\sigma}\cdot\frac{A\, F_A}{F_L - k} = Q \tag{4'}$$

Chapter 3

CONSEQUENCES OF PERMANENT CHANGES IN CHILD MORTALITY ON PARENTAL WELFARE AND POPULATION DENSITY.

1. Introduction

A debate in the medical literature on the effects of public health programs aimed at reducing child mortality in developing countries was initiated by King (1990). His concern was that in some areas rapidly growing populations would soon exceed the ability of their local environments to support them, with severe negative effects on the standards of living of future generations. Not only would population growth decrease the available per capita resources if the growth in such resources did not match the population growth. The increase in population density could also in itself have a negative effect on the size of the stock of natural resources, such as arable land, by environmental damage[1]. To ensure the well–being of future generations in these societies, certain sacrifices in the present generation were proposed, among them the withholding of public health programs such as vaccination against common childhood illnesses. The article was followed by an extensive debate in the medical literature and at aid policy headquarters (for a review of the debate see, for instance, Waldman, 1991).

In the classical view of the demographic transition, a decline in mortality in a given

[1] For given levels of consumption, more people translate into more stress on natural resources, including both sources (land, forests, water) and sinks (air,water). In the absence of prices which reflect the true scarcity value of these sources and sinks, there is likely to be excessive consumption of these "goods" from society's point of view (Birdsall, 1994).

country in the course of development is eventually matched by a later decline in fertility, until a rough balance is reached around the replacement level, the underlying assumption being some sort of causal influence between the mortality and fertility decline.[2]

The exact relationship between mortality and fertility is not well understood. Both behavioral and biological links between child mortality and fertility have been proposed. For example, in an environment where breast feeding is practiced, a link between infant death and mothers' susceptibility to conception has been established empirically (for references, see Ben Porath (1976) and Yamada (1984)). On the behavioral side, it is proposed that longer child survival may reduce the need for parents to "replace" children through additional births. Longer survival of children, as parents perceive it to spread within the community, may also reduce their motivation to "hoard" extra children as a guarantee against possible deaths. Both the replacement and hoarding reaction are possible reactions to mortality even if we assume that the preferred family size is not affected by changes in child mortality. But actual or expected death may also affect the desired number of surviving children, by influencing the actual or perceived cost per surviving child.

To predict the long–run effects of exogenous declines in child mortality on fertility, it is obvious that the behavioral effect must be isolated. Several authors have analyzed the effects of reduced child mortality on fertility in models where a couple makes a one–time

[2] In some parts of historical Europe the empirical support for a demographic transition along the lines of the classical view is weak, in other parts the evidence is less clear (Bengtsson and Olsson, 1994). In most regions of the less–developed world, a reduction in mortality, notably in child mortality, has recently taken place (Schultz, 1981). In comparing past and present transitions, crude death rates have dropped much more rapidly in low–income countries than they did in the developed countries during their early industrialization and have to a much larger degree been concentrated to reductions in child mortality (World Bank, 1984, Schultz, 1981). It is also true that on a global average level, total fertility (that is, average life time births per woman at then–prevailing rates of childbearing at each age) has decreased (from about 5.0 in 1960 to around 3.5 in 1990 (McNicoll, 1994)). But when aggregated figures are broken down a more diversed pattern emerges, and in some areas, notably in large parts of Africa and South Asia, fertility decline has been very modest.

decision concerning the number of births. What has typically been shown is that a decline in child mortality can lead both to an increase and a decrease in fertility, depending on factors such as the "price elasticity" of children (Schultz 1981, Ben–Porath and Welch, 1972). Sah (1992) was the first to model the fertility decision as a choice of a positive integer and to evaluate the effects on parental welfare from changes in child mortality. He showed that, under certain assumptions, a reduction in child mortality never causes fertility to rise and, furthermore, never causes parental welfare to decline. The analysis did not, however, include a discussion of the net effect of changes in child mortality on population growth. It is obvious that even if a decline in child mortality brings about a decline in fertility, the effect on population growth depends on the relative size of the two changes. If one believes or assumes, as did Malthus (and King (1990)) that some essential factor in the economy is fixed (or inelastic in supply) and that this same factor influences fertility choice, the effect on population growth from changes in child mortality also influences fertility choice and population growth in successive generations.

The purpose of this paper is to analyze the effects on parental welfare[3] not only for the generation alive at the time when a change in child mortality occurs, but also for future generations, in a rural economy where the total amount of land is fixed. The basic model is the one analyzed in Chapter 1, which builds on two assumptions: first, the main reason for having children in the rural economy is to provide labor to the family farm as well as to provide old age support, and secondly, the total amount of land is assumed to be fixed and land is divided equally among children upon death of parents.

In the basic model in the next section, each individual that survives childhood lives during three periods, as a costly, improductive child, as an adult worker on the family

[3] In this paper, parental welfare is defined as the weighted sum of utility in adult age and old age. We thus have a narrow definition of individual welfare, excluding the welfare of childhood, which of course would have implications for a case when we were to consider some sort of social welfare function, as would other fundamental questions such as the welfare of those not born. It is also obvious from the model that we completely ignore such aspects as the disutility of the loss of a child.

farm, and finally as a retired, improductive landowner of land inherited from parents. We start by assuming that a known and certain share of children die before they reach adult age, and the number of children is treated as a continuous variable. Within this framework we analyze the effects of exogenous changes in child mortality on fertility and parental welfare in the short run[4] and on fertility, population density and parental welfare in the long run[5].

It is shown that in the short run a decrease in child mortality will always increase parental welfare. However, the direction of the change in fertility choice will depend on the relative risk aversion[6]. If the relative risk aversion is higher than unity, a decrease in child mortality will cause a decrease in fertility. The opposite is true for a parent with relative risk aversion lower than unity.

Next, we introduce the dynamics that arises from the fixed amount of land in the economy and the inheritance rule. A decline in child mortality will as an indirect effect make each individual inherit a smaller lot. We show that, independently of the degree of risk aversion, parental welfare will always be lower in the new stationary state as a consequence of reduced child mortality. This is explained by the fact that, even if a decrease in child mortality causes fertility to decline, it does not do so to fully compensate for the increase in the number of surviving children. In the long run, individuals who survive childhood will be worse off in the new stationary state as compared with individuals at the "old" stationary state, even though the "price" of a surviving child is lower. The fact that each individual in the new stationary state will receive a lower wage

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[4] Only the effects on current adults, at the point in time when the change in child mortality occurs, are included in the discussion of short run effects.

[5] Long run is defined as the effects on stationary–state variables, and thus on the numbe. and welfare of future generations.

[6] In the first part of the paper, where mortality is treated as a deterministic variable, the "relative risk aversion" is better interpreted as the concavity of the instantaneous utility function or the price elasticity of the consumption in adult period of life expressed in the consumption in old age.

and own a smaller amount of land dominates over the welfare–increasing price effect. The analysis shows that fertility will eventually decline as a result of reduced child mortality, independently of the fertility response in the short run. This response is, however, not only due to the fact that a lower number of births is needed to attain the desired family size (corrected for the change in the relative price). The choice of lower fertility is also caused by the fact that parents are poorer in an absolute sense.

One weakness with the analysis in Section 2 is that a parent knows with full certainty how large fraction of his children will survive to adult age. It is obvious that uncertainty aspects are not considered in such a set–up. In Section 3, we incorporate uncertainty considerations into the model by assuming that the number of surviving children is stochastic: two outcomes are possible; that all children survive or that a certain share of the children die. A simplified interpretation would be that the society in each time period faces the risk of being hit by an epidemic disease or a drought, and that if it is hit, a certain share of the children die. We assume that the size of that share could be altered by some policy measure. It is shown that the effect on fertility choice in the short run and on population density in the long run (but with a more restricted formulation of the utility function) stays qualitatively the same as in the model of Section 2.

In sum, we have formally shown one possible explanation for the covariation between improvements in child survival rates, lagged reductions in fertility, increased poverty and increased population density, a pattern which is observed in some areas in developing countries (see Robey, Rutstein and Morris (1993) for a discussion on other explanations to recent such findings). Our results thus also constitute a formal illustration to some of the basic concerns in King (1990).

2. The Basic Model[7]

Individuals live for either one or three periods: all individuals go through childhood during which consumption is an exogenously determined amount of food provided by parents. A share, which is known with certainty, of the children die at the end of the childhood period. Those who survive enter productive adulthood and finally retire in old age. At each point in time the self–sufficient, extended household thus consists of three generations and it owns and cultivates a given share of land, producing a non–storable subsistence good – food – with inputs of labor and land. The property rights to the land are in the hands of the elderly. When the retired person dies, the land is shared equally among the adults who survived childhood, who in turn enter the period of old age. Adults work on the family farm. The wage paid to adults is equal to the value of their marginal product on the family farm. The elderly consume an amount equal to total output minus costs for family labor. Fertility decisions are taken at the beginning of the adult period and adults know the objective probabilities of survival. The decision is a one–time decision and replacement is thus ruled out[8].

Assume that food is produced according to the following production function:

$$Y_t = A_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where Y_t is the aggregate production of food on the family farm at time t, A_t is the amount of land owned by the elderly at time t (born at t–2), and L_t is available family labor:

[7] Several of the assumptions are discussed and motivated in the Introduction and Chapter 1 of this thesis.

[8] We thus bypass issues related to the spacing of children. As fertility choices are sequential by nature, implying that a lost child can be replaced, this simplification is an obvious shortcoming in the analysis.

$$L_t = p_t N_t \tag{2}$$

where p_t is the proportion of children that survives from t–1 to t and N_t is actual births in t–1. We assume an inheritance rule by which land is equally divided among adults who have survived childhood. The size of landholdings in t+1 will thus be determined by:

$$A_{t+1} = \frac{A_t}{p_t N_t} \tag{3}$$

The individuals are selfish in the sense that only their own consumption is included in their utility function[9]. The utility function for an adult born at t–1 is defined over consumption in period t and t+1 and we assume that the utility function is additively separable in consumption at t and t+1. Furthermore, the utility function is assumed to be of constant elasticity[10]:

$$U(c_i) \equiv \frac{1}{1-\rho} c_i^{1-\rho} \qquad i=t,t+1, \qquad 0< \rho \leq \infty \tag{4}$$

The consumption level in adulthood c_t, depends on the wage earned w_t, the choice of the number of children N_{t+1}, and the exogenously determined level of consumption of each of the children c_0:

$$c_t = w_t - N_{t+1} c_0 \tag{5}$$

where the wage is determined by the value of the marginal product of adult workers on the

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[9] Each adult of each household thus only maximizes his own utility, disregarding the utility of his parents, children and siblings.

[10] For the sake of presentation we will often write the utility and production function as U(·) and F(·), even though parametric forms are used.

family farm:

$$w_t = F_L(A_t, p_t N_t) \tag{6}$$

Consumption in the old–age period c_{t+1}, constitutes what is left of total production in t+1 when wages have been paid. It thus depends on the amount of land inherited A_{t+1}, the total number of children an individual decided to have in the preceding period N_{t+1}, and the share p_{t+1}, that survive from period t to t+1:

$$c_{t+1} = A_{t+1} F_A(A_{t+1}, p_{t+1} N_{t+1}) \tag{7}$$

Fertility choice is made at the beginning of adulthood at time t. The maximization problem is:

$$\max_{N_{t+1}} \; U(c_t) + \delta\, U(c_{t+1}) \tag{8}$$

subject to (5), (6) and (7),

where δ is the subjective rate of time preference. Utilizing (5) – (8) we can rewrite the problem as:

$$\max_{N_{t+1}} \; \Lambda(N_{t+1}, p_{t+1}, A_{t+1}, w_t) \equiv U(w_t - N_{t+1} c_0) + \delta U[A_{t+1} F_A(A_{t+1}, p_{t+1} N_{t+1})] \tag{9}$$

The first–order condition to this problem is:

$$\Lambda_N(N_{t+1},p_{t+1},A_{t+1},w_t) = -\,U'(w_t - N_{t+1}c_0)c_0 + \delta U'[A_{t+1}F_A(A_{t+1},p_{t+1}N_{t+1})]A_{t+1}p_{t+1}F_{AL}(A_{t+1},p_{t+1}N_{t+1}) = 0 \quad (10)$$

The second order condition requires that:

$$\Lambda_{NN} = U''(c_t)c_0^2 + \delta[U''(c_{t+1})p^2A^2F_{AL}^2 + U'(c_{t+1})Ap^2F_{ALL}] < 0 \quad (11)$$

which holds for the Cobb–Douglas production function.

a. Short–Term Consequences of Reduced Child Mortality

We start by considering the consequences of permanent reductions in child mortality on the welfare of parents alive at the time of the change (short–term). The reduction in child mortality could be the result of some policy action, such as the introduction of vaccines against infectious diseases common among children. Assume that child mortality falls from t+1 and onwards, thus $p_{t+1} > p_t$, and that in time period t parents can foresee this change in child mortality. As p_t remains unchanged, wages in time t, as well as the size of landholdings in t+1, are not affected by the change in child mortality in t+1 (see (3) and (6) above). Since $\Lambda_N = 0$ according to the FOC, the effect on parental welfare for individuals born in t–1 is[11]:

$$\frac{d\Lambda}{dp}\bigg|_{\overline{A},\overline{w}} = \Lambda_p \quad (12)$$

$$= \delta U'[AF_A(A,pN)]AF_{AL}N > 0 \quad (13)$$

Hence:

[11] Where all period indices are referring to the same period, we simplify the presentation by ignoring them.

Proposition 1. An improvement in child survival will unambiguously increase parental welfare of the generation of parents alive at the time of the change in child mortality.

Given the FOC in (9) above, and the assumption that A_{t+1} and w_t remain unchanged, $\Lambda_{Np}dp_{t+1}+\Lambda_{NN}dN_{t+1}= 0$ must hold. The effect on the optimal choice of fertility of parents born in t–1, N_{t+1}, from a change in child mortality from t+1 onwards is thus given by:

$$\frac{dN_{t+1}}{dp_{t+1}} = -\frac{\Lambda_{Np}}{\Lambda_{NN}} \tag{14}$$

The sign of Λ_{Np} is determined by:

$$\Lambda_{Np} = \delta AU'(c_{t+1})F_{AL}[\frac{U''(c_{t+1})c_{t+1}}{U'(c_{t+1})}\frac{F_{AL}L}{F_A} + 1 + \frac{LF_{ALL}}{F_{AL}}] \tag{15}$$

Let $r(c_{t+1})$ be the relative risk aversion :

$$r(c_{t+1}) \equiv -\frac{U''(c_{t+1})c_{t+1}}{U'(c_{t+1})} \tag{16}$$

Then:

$$\Lambda_{Np} = \delta AU'(c_{t+1})F_{AL}(1-\alpha)(1-r(c_{t+1})) \tag{17}$$

Hence, since the sign of $\frac{dN_{t+1}}{dp_{t+1}}$ depends on the sign of Λ_{Np}, we have:

Proposition 2. If the relative risk aversion is higher (lower) than unity, a reduction in child mortality will cause a decrease (increase) in fertility[12]. In the special case of a logarithmic utility function, fertility choice is independent of changes in child mortality[13].

b. Long–Run Effects of Reduced Child Mortality

We now turn to the effects of changes in child mortality in the long run restricting ourselves to a comparison of stationary states. The stationary–state value of landholdings $\hat{A}$, is given by the first order condition:

$$- U'[F_L(\hat{A},1) - \frac{c_0}{p}]\frac{c_0}{p} + \delta U'[\hat{A}F_A(\hat{A},1)]\hat{A}F_{AL}(\hat{A},1) = 0 \tag{18}$$

Appendix 1 shows that a stationary state exists and that, at the stationary state; $0 < \frac{dA_{t+2}}{dA_{t+1}} < 1$, independently of the degree of the relative risk aversion, i.e., that it is locally stable. Since $pN = 1$ in any stationary state, we immediately have:

Proposition 3. Reduced child mortality reduces stationary–state fertility.

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[12] This result is not restricted to the constant elasticity parametrization of the instantaneous utility function.

[13] At a first glance one could expect the effects from a change in child mortality to be similar to those of a change in the cost of bringing up children, as the former can be interpreted as an indirect way to change the price of a survived child. A comparison between the effects of a change in the cost of children in Chapter 1 and the logarithmic utility case in this paper shows that, in the short term, reduced costs of raising children increase the optimal number of children, while a reduction in child mortality, as formalized in this section, leaves the fertility choice unchanged. This is explained by the fact that a reduction in child mortality has two offsetting effects. It makes a surviving child cheaper but it also makes the number of births necessary to reach a desired number of surviving births smaller. In the logarithmic utility case the two effects fully offset each other. A reduction in the cost of raising children affects only the price of children, initiating an increase in the demand. In the long run, a change in child mortality does influence the amount of land that is transferred to the next generation. As more children survive, each child will inherit less land. In the case where only the cost of children is changed, the amount of inherited land is not influenced. But as fertility choice is increased in the short term as a response to the decrease in the cost of raising children, the actual amount of inherited land will decrease. We will thus also in this case witness a decrease in the stationary state amount of land. An important distinction, however, is the difference in the timing of the effects in the two cases.

The effect of a change - child mortality on the stationary–state value of landholdings is (see Appendix 2 fo. calculation):

$$\frac{d\hat{A}}{dp} = -\frac{c_0}{p^2 F_{LA}}\left[\frac{U'(\hat{c}_1) - U''(\hat{c}_1)\frac{c_0}{p}}{\delta U'(\hat{c}_2)\alpha(1-r(\hat{c}_2)) - U''(\hat{c}_1)\frac{c_0}{p}}\right] \tag{19}$$

where $\hat{c}_1$ is the stationary–state consumption in adult age and $\hat{c}_2$ is the stationary–state consumption in old age. The numerator within the parenthesis in (19) is always positive. The denominator is positive if:

$$r(\hat{c}_2) < 1 - \frac{c_0}{\delta\alpha p}\frac{U''(\hat{c}_1)}{U'(\hat{c}_2)} \tag{20}$$

With a constant elasticity utility function, (20) can be rewritten as:

$$\rho > \frac{1}{1 - \frac{F_L}{c_1}} \tag{21}$$

This condition is always fulfilled, since ρ can only take positive values and the expression of the right hand side of (21) is a negative number, as $F_L > c_1$. This implies that $\frac{d\hat{A}}{dp}$ is negative. Hence:

Proposition 4. Irrespective of the degree of the relative risk aversion, an increase in children's survival probabilities unambiguously reduces the stationary–state amount of landholding per household.

A reduction in child mortality will thus always result in higher population density in the new stationary state.

Let us now turn to the effects of a change in child mortality on parental welfare. Parental utility in stationary state is:

$$\hat{\Lambda}(p) \equiv U(\hat{c}_1(\hat{A}(p),p)) + \delta U(\hat{c}_2(\hat{A}(p),p)) \tag{22}$$

where the consumption in the adult and old age period in stationary state, $\hat{c}_1$ and $\hat{c}_2$ respectively, are given by:

$$\hat{c}_1(\hat{A}(p),p) \equiv F_L(\hat{A}(p),1) - \frac{1}{p}c_0 \tag{23}$$

$$\hat{c}_2(\hat{A}(p)) \equiv \hat{A}(p)F_A(\hat{A}(p),1) \tag{24}$$

The effect on parental welfare in stationary state is:

$$\frac{d\hat{\Lambda}}{dp} = U'(\hat{c}_1)\frac{d\hat{c}_1}{dp} + \delta U'(\hat{c}_2)\frac{d\hat{c}_2}{dp} \tag{25}$$

where:

$$\frac{d\hat{c}_1}{dp} = \frac{\partial\hat{c}_1}{\partial\hat{A}}\frac{d\hat{A}}{dp} + \frac{\partial\hat{c}_1}{\partial p} \tag{26}$$

$$\frac{d\hat{c}_2}{dp} = \frac{\partial\hat{c}_2}{\partial\hat{A}}\frac{d\hat{A}}{dp} \tag{27}$$

Consumption in adult age is thus affected by reduced child mortality in two ways. First, a reduction in child mortality has an impact on the stationary–state amount of land which determines the stationary–state wage and hence adult income. As the size of landholdings always will become smaller as an effect of reduced child mortality, the stationary–state income of adults will also be reduced. But, on the other hand, the total cost of children will go down as the number of children will be reduced as survival has increased. Some further calculation gives:

$$\frac{d\hat{c}_1}{dp} = \frac{c_0}{p^2}\left[1 - \frac{U'(\hat{c}_1) - U''(\hat{c}_1)\frac{c_0}{p}}{\delta U'(c_2)\alpha(1-r) - U''(c_1)\frac{c_0}{p}}\right] \tag{28}$$

It can be inferred from the calculations in Appendix 2 that:

$$\frac{U'(\hat{c}_1) - U''(\hat{c}_1)\frac{c_0}{p}}{\delta U'(c_2)\alpha(1-r) - U''(c_1)\frac{c_0}{p}} > 1 \tag{29}$$

We can therefore conclude that the consumption in adult age is reduced by reduced child mortality. Old age consumption is only affected through the effect on the stationary–state amount of land. Given the fact that the stationary–state size of landholdings always decreases when child mortality is reduced, it follows that the expression (27) is negative. Thus, in contrast to the short–run:

Proposition 5. The parental welfare of future generations, hence the welfare of parents not yet born, will always be reduced if child mortality is reduced, irrespectively of the degree of relative risk aversion.

3. Incorporation of Uncertainty

In the previous section it was assumed that a parent knows with full certainty how large fraction of his children will survive to adult age. It is obvious that uncertainty aspects are not considered in such a set–up. In order to get an indication as to whether the exclusion of uncertainty is decisive for the results above, let us analyze a case where uncertainty is incorporated in a simple, and therefore formally traceable, way. In order to make the analysis manageable, we restrict our attention to the case of constant relative risk aversion when analyzing the short–run effects and to a logarithmic utility function when analyzing the long–run effects on changes in the size of landholdings.

Let us assume that two different states can occur in each time period: in each time period the rural society can be hit by an exogenously determined shock, say a drought or an infectious disease. The probability that such a shock will occur is the same for all households[14] and known to be (1–q). In case the shock materializes, it hits all households and a known share of all children (1–p) die before entering adult age. If the shock does not materialize, all children survive. Hence, at the time of the fertility decision, the parent does not know whether some fraction of his children will die or not. Replacement of children who do not survive childhood is ruled out as the choice of family size is a one–time decision in our set–up. Let us define the stochastic variable θ, such that:

$$\theta \begin{array}{l} = 1 \text{ with probability } \quad q \\ = p \text{ with probability } \ (1-q) \end{array} \qquad\qquad p,q < 1 \qquad (30)$$

Define the following intertemporal expected utility function:

$$\Lambda^*(c_t, c_{t+1}, \bar{c}_{t+1}, q) \equiv \frac{1}{1-r}\{c_t^{1-r} + \delta[q\bar{c}_{t+1}^{1-r} + (1-q)c_{t+1}^{1-r}]\} \qquad (31)$$

where the consumption in adult age c_t, is given by (5) and (1), the consumption in old age if only a fraction p of the children survive from time period t to t+1, c_{t+1}, is determined by (7) and (1), and the consumption in old age if all children survive $\bar{c}_{t+1}$, is:

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[14] Analytically, it is a great advantage if all households are assumed to face the same shocks. We are thus only discussing aggregate risk, not risks which are specific to individuals.

$$\bar{c}_{t+1} = \alpha A_{t+1}^{\alpha} N_{t+1}^{1-\alpha} \tag{32}$$

We thus have:

$$c_{t+1} = p^{1-\alpha}\,\bar{c}_{t+1} \tag{33}$$

Hence, (31) can be rewritten as:

$$\Lambda^{*}(c_t,\bar{c}_{t+1},\delta^{*}) \equiv \frac{1}{1-r}\{c_t^{1-r} + \delta^{*}\,\bar{c}_{t+1}^{1-r}\} \tag{34}$$

where c_t and $\bar{c}_{t+1}$ are given in (1), (5) and (32), and:

$$\delta^{*}(p,q) = \delta[q + (1-q)p^{(1-\alpha)(1-r)}] \tag{35}$$

The first order condition to the maximization problem is:

$$\Lambda_N^{*} = c_t^{-r}\,\frac{dc_t}{dN_{t+1}} + \delta^{*}(p,q)\,\bar{c}_{t+1}^{-r}\,\frac{d\bar{c}_{t+1}}{dN_{t+1}} \tag{36}$$
$$= 0$$

The second order condition is given by:

$$\Lambda_{NN}^{*} = -rc_t^{-r-1}c_0{}^2 - \delta^{*}\alpha(1-\alpha)\bar{c}_{t+1}^{-r}A^{\alpha}N^{-\alpha}[r\bar{c}_{t+1}^{-1}N + \alpha] < 0 \tag{37}$$

which is fulfilled.

a. <u>Short–Term Consequences of Reduced Child Mortality</u>

When we study the short–term effects of a change in child mortality from t+1 onwards we assume that adults who are to take their fertility decision in time t can foresee the change in mortality in t+1. Their wages in time t as well as the size of their landholdings that they will inherit in t+1 remain unaffected by the change in mortality. Hence, given this, (1), (5), (32), and (34)–(36) together imply that $\Lambda^*_{NN}dN+\Lambda^*_{Np}dp = 0$ must hold. As Λ^*_{NN} is negative, the effect on the optimal choice of fertility from a change in child mortality from time period t+1 onwards, is determined by the sign of Λ^*_{Np}. It is easily shown:

$$\Lambda^*_{Np} = \frac{d\delta^*}{dp} K \tag{38}$$

where:

$$K = \bar{c}_{t+1}^{-r}(1-\alpha)\alpha A^{\alpha}N^{1-\alpha} > 0 \tag{39}$$

$$\frac{d\delta^*}{dp} = \delta(1-q)(1-\alpha)(1-r)p^{(1-\alpha)(1-r)-1} \tag{40}$$

The sign of the expressions in (40) is thus determined by the size of the relative risk aversion. Hence:

Proposition 6. If the risk aversion is higher (lower) than unity, an increase in the survival probability of children leads to a decrease in the optimal fertility choice.

The results so far in this section are thus equivalent to those in Section 2.

b. Effects on a Stochastic Stationary State of Reduced Child Mortality

In order to study the effects on the stochastic stationary–state value of landholdings we restrict ourselves to the case where the instantaneous utility function is logarithmic[15]. In Appendix 3 it is shown that the choice of fertility in such a case is independent of p in the short run. This is also evident from expressions (39) and (40) above, and in line with the results in Section 2 above. Define the expected value of landholdings in the stochastic steady state, $E\hat{A}$, as:

$$E\hat{A} \equiv \lim_{t \to \infty} EA_t \tag{41}$$

Appendix 3 shows that:

$$E\hat{A} = \left[\frac{1}{BE\theta}\right]^{\frac{1}{\alpha}} \tag{42}$$

where:

$$B = \frac{(1-\alpha)^2 \delta}{[1+\delta(1-\alpha)]c_0} \tag{43}$$

and

$$E\theta = q+(1-q)p \tag{44}$$

It is then easily established that:

[15] With a logarithmic utility function we get an explicit expression for the development of land holdings over time where the stochastic variable enters in a linear way. This is not possible, for instance, in the case with a constant elasticity utility function, making it difficult to evaluate the effects on the expected value of land holdings in the stationary state from changes in the expected value of the stochastic variable.

$$\frac{d}{dp} \lim_{t \to \infty} EA_t < 0 \tag{45}$$

We can thus conclude that:

Proposition 7. An improvement in children's survival probability reduces the expected size of land holding in the stochastic stationary state, independently of the short term effect on fertility.

Once again the results here conform with those of Section 2. However, it should be recalled that in the present section we have restricted the analysis to a case where the instantaneous utility function is logarithmic.

4. Conclusions and Discussion

Our results can be summarized as follows:

1. A reduction in child mortality will always increase parental welfare in the short run. This result is thus consistent with the results generated in the more general and realistic setting of Sah (1992).

2. The short run effect on fertility choice from a reduction in child mortality depends on the relative risk aversion. The results of empirical studies investigating parents' response to changes in subjective probabilities of survival levels in the community are equivocal, and the direction of the response is different in different societies (Bulatao and Elwan, 1983). Micro level studies that also include replacement behavior indicate that the fertility response to mortality is relatively weak. A deceased child is not fully replaced: the level of replacement is almost always below 0.5 and is typically around 0.25 (Bulatao and Elwan 1983).

3. In the long run, comparing stationary states, a reduction in child mortality always results in reduced fertility, independently of the size of relative risk aversion, and short–run fertility response. This long run response is thus consistent with the general

empirical observation that reduced child mortality in many societies eventually leads to reduced fertility. At the aggregated level, cross–national evidence shows that mortality levels are related to fertility levels declines but that fertility declines have tended to lag behind declines in mortality (Birdsall, 1988).

4. Irrespectively of the degree of relative risk aversion, an increase in children's survival probabilities unambiguously reduces the stationary–state amount of landholding per family and thus increases population density.

5. Reduced child mortality thus not only leads to reduced fertility in the new stationary state, but also to lower parental welfare in the long run, due to lower wages and smaller landholdings. Hence, even where the short–run fertility response is such that fertility is reduced by reductions in child mortality (as shown by Sah, 1992), the long–run effect on parental welfare in the new stationary state is negative.

6. The inclusion of elements of uncertainty did not alter the qualitative effects of reduced child mortality on fertility choice in the short run nor on the expected stationary–state amount of landholding.

In sum, we have formally shown one possible explanation of the covariation between improvements in child survival, reductions in fertility, increased poverty and population density witnessed in some areas in developing countries.

Our analysis thus shows that the welfare effect from reductions in child mortality will have different signs for different generations in our model. In line with the arguments in the medical literature cited above, continued efforts in reducing child mortality, with positive welfare effects on the first generation, will in our model in the long run make people worse off. This is the case even though we have not incorporated any negative externality, such as possible degradation of arable land due to stress from higher population density, in our model.

Our model is thus one highly simplified formal example where a policy maker would have to weigh the positive welfare effects on the present generation against negative welfare effects on future generations, where the policy choice variable is children's survival probabilities. Due to a number of simplifying assumptions it is obvious that the model is not suitable for any policy conclusions. It can only serve as an example of what type of mechanisms that could be in force in the demographic development of a society described by the model. In a real world context, it is also necessary to question whether future and uncertain benefits of a somewhat smaller population in the future would outweigh immediate welfare costs in terms of human suffering in case efforts to reduce child mortality were halted. Furthermore, lower child mortality might be a precondition for parents to change their behaviour in terms of investment in children regarding health and education, which in turn could change both cost and benefits of children with possible reductions in desired family size. An analysis of this last–mentioned type of relationship and behaviour would require a more sophisticated model, where child quality, for example child health status and education, and the number of children are jointly determined by parents.

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APPENDIX 1

A. Existence

Let x be defined by

$$x \equiv \hat{A}^{\alpha} \tag{1'}$$

The FOC in the stationary state (18) can then be written as

$$\frac{c_0}{p}(\alpha\ x)^{r-1} = \delta(1-\alpha)\left[(1-\alpha)x - \frac{c_0}{p}\right]^{r} \tag{2'}$$

or,

$$\frac{c_0\ \alpha^{r-1}}{p\delta(1-\alpha)} = x\left[1 - \alpha - \frac{c_0}{px}\right]^{r} \tag{3'}$$

Define the left–hand side of (3') by S, and the right–hand side by R(x):

$$R(x) = S \tag{4'}$$

Consumption in each period is necessarily positive. Hence, the minimum value x can take on is given by:

$$x_{min} = \frac{c_0}{p}\frac{1}{(1-\alpha)} \tag{5'}$$

Now, since S > 0 is finite, and R(x) is continuous, and

$$\lim_{x \to \frac{c_0}{p}\frac{1}{(1-\alpha)}} R(x) = 0 \tag{6'}$$

$$\lim_{x \to \infty} R(x) = \infty \tag{7'}$$

a stationary state exists.

B. Stability

The FOC

$$-U'(c_t)c_o + \delta U'(c_{t+1})pA_{t+1}F_{AL}(A_{t+1}, pN_{t+1}) = 0 \tag{8'}$$

where

$$A_{t+1}F_{AL} = A_{t+1}(1-\alpha)\alpha A_{t+1}^{\alpha-1}(pN_{t+1})^{-\alpha} = (1-\alpha)\alpha(A_{t+2})^{\alpha} \tag{9'}$$

implies

$$U'(c_t)c_o = \delta U'(c_{t+1})p(1-\alpha)\alpha\left[A_{t+2}\right]^{\alpha} \tag{10'}$$

Total differentiation of (10') yields:

$$-U''(c_t)c_o dc_t + \delta p\alpha(1-\alpha)A_{t+2}{}^{\alpha} U''(c_{t+1})dc_{t+1} + \\ + \delta U'(c_{t+1})p\alpha(1-\alpha)\alpha A_{t+2}{}^{\alpha-1} dA_{t+2} = 0 \qquad (11')$$

Divide (11') with LHS and RHS of (10'), respectively:

$$\frac{-U''(c_t)}{U'(c_t)} dc_t + \frac{U''(c_{t+1})}{U'(c_{t+1})} dc_{t+1} + \frac{\alpha}{A_{t+2}} dA_{t+2} = 0 \qquad (12')$$

Let r_i be the coefficient of relative risk aversion:

$$r_i = \frac{-U''(c_i)}{U'(c_i)} c_i \qquad (13')$$

Then, rewriting (12'):

$$r_t \frac{dc_t}{c_t} - r_{t+1} \frac{dc_{t+1}}{c_{t+1}} + \alpha \frac{dA_{t+2}}{A_{t+2}} = 0 \qquad (14')$$

To compute the differentials in (14'), note that:

$$c_t = w_t - N_{t+1}c_o = (1-\alpha)A_{t+1}{}^{\alpha} - \frac{A_{t+1}}{pA_{t+2}} c_o \qquad (15')$$

$$c_{t+1} = A_{t+1}F_A(A_{t+1}, pN_{t+1}) \qquad (16')$$

$$\frac{AF_{AA}}{F_A} = \alpha-1 \qquad (17')$$

$$\frac{F_{AL}}{F_A}\frac{A_{t+1}}{A_{t+2}} = 1-\alpha \tag{18'}$$

We get:

$$\frac{dc_t}{c_t} = \frac{1}{c_t}\left[\left[\alpha(1-\alpha)A_{t+1}^{\alpha-1} - \frac{c_o}{pA_{t+2}}\right] dA_{t+1} + \frac{A_{t+1}c_o}{pA_{t+2}^2} dA_{t+2}\right] \tag{19'}$$

$$= \frac{1}{c_t}\left[\left[\alpha w_t - N_{t+1}c_o\right]\frac{dA_{t+1}}{A_{t+1}} + \frac{N_{t+1}c_o\ dA_{t+2}}{A_{t+2}}\right] \tag{20'}$$

and:

$$\frac{dc_{t+1}}{c_{t+1}} = \frac{1}{A_{t+1}F_A}\left[\left[F_A + AF_{AA}\right]dA_{t+1} + AF_{AL}\ d\left[pN_{t+1}\right]\right] \tag{21'}$$

Taking into account:

$$d(pN_{t+1}) = \frac{A_{t+1}}{A_{t+2}}\frac{dA_{t+1}}{A_{t+1}} - \frac{A_{t+1}}{A_{t+2}}\frac{dA_{t+2}}{A_{t+2}} \tag{22'}$$

we finally get:

$$\frac{dc_{t+1}}{c_{t+1}} = \frac{dA_{t+1}}{A_{t+1}} - (1-\alpha)\frac{dA_{t+2}}{A_{t+2}} \tag{23'}$$

Insert (20) and (23) into (14), and assume that the instantaneous utility function U has constant elasticity function. Then,

$$r_t = r_{t+1} = r \tag{24'}$$

and

$$\left[\alpha w_t - N_{t+1}c_o - c_t\right] \frac{dA_{t+1}}{A_{t+1}} + \left[N_{t+1}c_o + c_t + c_t\alpha\left[\frac{1-r}{r}\right]\right]\frac{dA_{t+2}}{A_{t+2}} = 0 \quad (25')$$

In stationary state $A_{t+1} = A_{t+2}$, and $N_{t+1} = 1$. Hence, in a stationary state

$$\frac{dA_{t+2}}{dA_{t+1}} = (1-\alpha) \frac{1}{1 + \alpha\left[\frac{1}{r} - 1\right]\frac{c_t}{w_t}} \quad (26')$$

(26') implies that

$$0 < r < 1 \text{ is a sufficient condition for } 0 < \frac{dA_{t+2}}{dA_{t+1}} < 1$$

If $r \to \infty$,

$$\left.\frac{dA_{t+2}}{dA_{t+1}}\right|_{r \to \infty} = \frac{1 - \alpha}{1 - \frac{c_t \alpha}{w_t}} \quad (27')$$

As $w_t > \alpha c_t$ (see (8)), we find $0 < \frac{dA_{t+2}}{dA_{t+1}} < 1$

To conclude: the stationary state is stable, independently of the size of the relative risk aversion r.

APPENDIX 2

The FOC determines the stationary–state value of landholdings, $\hat{A}$:

$$-U'\left[F_L(\hat{A},1) - \frac{c_o}{p}\right]\frac{c_o}{p} + \delta\, U'\left[\hat{A}F_A(\hat{A},1)\right]\hat{A}F_{AL}(\hat{A},1) = 0 \qquad (28')$$

where the left–hand side LHS is a function LHS($\hat{A}$,p).

Differentiate totally:

$$\frac{d\hat{A}}{dp} = -\frac{\frac{\partial LHS}{\partial p}}{\frac{\partial LHS}{\partial \hat{A}}} \qquad (29')$$

where

$$\frac{\partial LHS}{\partial p} = -U''(\hat{c}_1)\,\frac{c_o}{p}\cdot\frac{c_o}{p^2} + U'(\hat{c}_1)\frac{c_o}{p^2} \quad > \quad 0 \qquad (30')$$

and where

$$\hat{c}_1 = F_L(\hat{A},1) - \frac{c_o}{p} \qquad (31')$$

$$\hat{c}_2 = \hat{A}F_A(\hat{A},1) = \alpha A^{\alpha} \qquad (32')$$

Furthermore:

$$\frac{\partial LHS}{\partial \hat{A}} = -U''(\hat{c}_1)\frac{c_o}{p} F_{LA} + \delta\Big[U''(\hat{c}_2)\alpha\ {}^2A^{\alpha-1}\alpha(1-\alpha)A^{\alpha} +$$

$$+ U'(\hat{c}_2)\alpha(1-\alpha)\alpha A^{\alpha-1}\Big]$$

$$= -U''(\hat{c}_1)\frac{c_o}{p} F_{LA} + \delta\ \alpha^2(1-\alpha)A^{\alpha-1}\Big[\alpha A^{\alpha}U''(\hat{c}_2) + U'(\hat{c}_2)\Big] \qquad (33')$$

Multiplying the first term in parenthesis with $\frac{U'(\hat{c}_2)}{U'(\hat{c}_2)}$, and utilizing $r = -\frac{U''(\hat{c}_2)}{U'(\hat{c}_2)}\hat{c}_2$:

$$\frac{\partial LHS}{\partial \hat{A}} = -U''(c_1)\ \frac{c_o}{p}\ F_{LA} + \delta\ \alpha\ F_{AL}\ U'(\hat{c}_2)[1-r] \qquad (34')$$

Insert (32') and (33') into (29'), divide all terms with $U'(\hat{c}_1)$ and multiply the last term in the numerator and the denominator with $\hat{c}_1$:

$$\frac{d\hat{A}}{dp} = -\frac{\frac{c_o}{p^2}}{F_{LA}}\left[\frac{1 - \frac{\hat{c}_1 U''(\hat{c}_1)}{U'(\hat{c}_1)}\frac{c_o}{p\hat{c}_1}}{\frac{\delta\alpha U'(\hat{c}_2)}{U'(\hat{c}_1)}\Big[1-r(\hat{c}_2)\Big] - \frac{\hat{c}_1 U'(\hat{c}_1)}{U'(\hat{c}_1)}\frac{c_o}{p\hat{c}_1}}\right] \qquad (35')$$

Noting:

$$\frac{U'(\hat{c}_2)}{U'(\hat{c}_1)} = \frac{c_o}{\delta p\alpha(1-\alpha)\hat{A}^{\alpha}} \qquad (36')$$

We get:

$$\frac{d\hat{A}}{dp} = -\frac{\frac{c_0}{p^2}}{F_{LA}} \left[\frac{1 + r\,\frac{c_0}{pc_1}}{(1-r)\underbrace{\frac{c_0}{p(1-\alpha)A^{\alpha}}}_{\frac{c_0}{pw}} + r\,\frac{c_0}{pc_1}} \right] \tag{37'}$$

which can be written as:

$$\frac{d\hat{A}}{dp} = -\frac{\frac{c_0}{p^2}}{F_{LA}} \left[\frac{1 + \frac{rc_0}{pc_1}}{\frac{c_0}{pw} - \frac{r\,c_0}{pw} + r\,\frac{c_0}{pc_1}} \right] \tag{38'}$$

Define:

$$\frac{c_0}{pc_1} \equiv x \tag{39'}$$

$$\frac{c_0}{pw} \equiv \beta \tag{40'}$$

and let the term in brackets in (38') be denoted

$$H(r) \equiv \frac{1 + rx}{\beta - r\beta + rx} \tag{41'}$$

$$H(o) = \frac{1}{\beta} = \frac{wp}{c_0} > 1 \tag{42'}$$

$$H'(r) = 0, \ \forall r \tag{43'}$$

Hence, since $H(r) > 1$ for all r, we have that $\frac{dA}{dp} < 0$.

APPENDIX 3

Assuming a logarithmic utility function, we rewrite (34) as:

$$\Lambda\left[c_t, \tilde{c}_{t+1}, c_{t+1}, q\right] \equiv \ln c_t + \delta\left[q \ln \tilde{c}_{t+1} + (1-q)\ln c_{t+1}\right] \tag{44'}$$

$$= \ln c_t + \delta\left[q \ln \tilde{c}_{t+1} + (1-q)[\ln p^{1-\alpha} + \ln \tilde{c}_{t+1}]\right] \tag{45'}$$

$$= \ln c_t + \delta(1-q)(1-\alpha) \ln p + \delta \ln \tilde{c}_{t+1} \tag{46'}$$

The first-order condition to (44') when maximizing over N_{t+1}, is:

$$\Lambda_N = - \frac{c_o}{(1-\alpha)A^{\alpha} - Nc_o} + \frac{\delta\alpha A^{\alpha}(1-\alpha)N^{-\alpha}}{\alpha A^{\alpha}N^{1-\alpha}} = 0 \tag{47'}$$

which, after some rearranging, can be expressed as:

$$N_{t+1} = \frac{(1-\alpha)^2\,\delta}{[1 + \delta(1-\alpha)]c_o} A^{\alpha}_{t+1} \tag{48'}$$

Define:

$$\frac{(1-\alpha)^2\delta}{[1 + \delta(1-\alpha)]c_o} \equiv B \tag{49'}$$

Combining (48') and (49') utilizing (31), and recalling that:

$$L_t = \theta_t N_t \tag{50'}$$

$$A_{t+1} = A_t / L_t \tag{51'}$$

we get:

$$A_{t+1} = \frac{1}{B} \cdot \frac{1}{\theta_t} A_t$$

where $\theta_t = \begin{cases} 1 & \text{with probability} \quad q \\ p & \text{with probability} \quad (1-q) \end{cases}$ (52')

Rewrite (52'):

$$\ln A_{t+1} = -\ln B - \ln \theta_t + (1-\alpha)\ln A_t \tag{53'}$$

Let superscript "$\simeq$" denote logs, and define y_t as:

$$\tilde{\tilde{A}}_t \equiv (1-\alpha)^t \; y_t \tag{54'}$$

Then, (53') can be rewritten as:

$$(1-\alpha)^{t+1} \; y_{t+1} = -\tilde{\tilde{B}} - \tilde{\tilde{\theta}}_t + (1-\alpha)^{t+1} \; y_t \tag{55'}$$

or,

$$y_{t+1} - y_t = -(\tilde{\tilde{B}} + \tilde{\tilde{\theta}}_t)(1-\alpha)^{-t-1} \tag{56'}$$

Replacing period index t by j in (56') and summing over both sides from j to t−1:

$$\sum_{j=-T}^{t-1} (y_{j+1} - y_j) = -\sum_{j=1}^{t-1} \left[\tilde{\tilde{B}} + \tilde{\tilde{\theta}}_j \, (1-\alpha)^{-j-1}\right] \tag{57'}$$

or,

$$y_t - y_{-T} = - \sum_{j=-T}^{t-1} (\tilde{\bar{B}} + \tilde{\bar{\theta}}_j)(1-\alpha)^{-j-1} \tag{58'}$$

Assume that the RHS of (58') satisfies:

$$\sum_{j=-\infty}^{t-1} |RHS| < \infty \tag{59'}$$

Hence $y_{-T} \to$ "A_0" when $T \to \infty$. Hence:

$$y_t = A_0 - \sum_{j=-\infty}^{t-1} (\tilde{\bar{B}} + \tilde{\bar{\theta}}_j)(1-\alpha)^{-j-1} \tag{60'}$$

By (55'):

$$(1-\alpha)^{-t} \tilde{\bar{\Lambda}}_t = A_0 - \sum_{j=-\infty}^{t-1} (\tilde{\bar{B}} + \tilde{\bar{\theta}}_j)(1-\alpha)^{-j-1} \tag{61'}$$

$$\tilde{\bar{A}}_t = (1-\alpha)^t A_0 - \sum_{j=-\infty}^{t-1} (\tilde{\bar{B}} + \tilde{\bar{\theta}}_j)(1-\alpha)^{t-j-1} \tag{62'}$$

To simplify, define $i \equiv t-1-j$. Then $i \to \infty$ when $j \to -\infty$, and

$i = 0$ when $j = t - 1$. Note $\sum_{\infty}^{0} = \sum_{0}^{\infty}$ (63')

This gives:

$$\overset{\simeq}{A}_t = (1-\alpha)^t \; A_o - \sum_{i=\infty}^{i=0} (\overset{\simeq}{B} - \overset{\simeq}{\theta}_{t-1-i})(1-\alpha)^i \tag{64'}$$

$$= (1-\alpha)^t \; A_o - \overset{\simeq}{B} \sum_{i=0}^{i=\infty} (1-\alpha)^i - \sum_{i=0}^{i=\infty} (1-\alpha)^i \; \overset{\simeq}{\theta}_{t-1-i} \tag{65'}$$

$$= (1-\alpha)^t \; A_o - \frac{\overset{\simeq}{B}}{\alpha} - \sum_{i=0}^{\infty} (1-\alpha)^i \; \overset{\simeq}{\theta}_{t-1-i} \tag{66'}$$

The expected value of $\overset{\simeq}{A}_t$ is then:

$$E\overset{\simeq}{A}_t = (1-\alpha)^t \; A_o - \frac{\overset{\simeq}{B}}{\alpha} - \sum_{i=0}^{\infty} (1-\alpha)^i \; E \; \overset{\simeq}{\theta}_{t-1-i} \tag{67'}$$

$$= (1-\alpha)^t \; A_o - \frac{\overset{\simeq}{B}}{\alpha} - \frac{E\overset{\simeq}{\theta}}{\alpha} \tag{68'}$$

where:

$$E\overset{\simeq}{\theta} = q \ln 1 + (1-q) \ln p = (1-q) \ln p \tag{69'}$$

We then have that:

$$\lim_{t \to \infty} E \; \overset{\simeq}{A}_t = - \frac{B + E\overset{\simeq}{\theta}}{\alpha} \tag{70'}$$

or

$$\lim_{t \to \infty} E \; A_t = \left(\frac{1}{BE\theta}\right)^{\frac{1}{\alpha}} = E \; \hat{A} \tag{71'}$$

where

$$E\theta = q + (1-q)p \tag{72'}$$

It is then easily seen that

$$\frac{d}{dp} \lim_{t \to \infty} EA_t = -\frac{1}{\alpha} \cdot \frac{1}{B} \cdot (1-q) \cdot \left[\frac{1}{BE\theta}\right]^{\frac{1}{\alpha} - 1} < 1 \tag{73'}$$